ASSIMILATION AS ATTRACTION: COMPUTING DISTANCE, SIMILARITY, AND LOCALITY IN PHONOLOGY

by

Adam Wayment

A dissertation submitted to Johns Hopkins University in conformity with the requirements for the degree of Doctor of Philosophy

Baltimore, Maryland
September, 2009

©2009 Adam Wayment
All Rights Reserved
Abstract

This dissertation explores similarity effects in assimilation, proposing an Attraction Framework to analyze cases of parasitic harmony where a trigger-target pair only results in harmony if the trigger and target agree on other features. Attraction provides a natural model of these effects by relating the pressure for assimilation to the representational distance between segments: the more similar a trigger-target pair, the stronger the attraction force between them. Attraction grammars in Optimality Theory (OT; Prince & Smolensky, 2004) are rigorously compared to those of Harmonic Grammar (HG; Legendre, Miyata, and Smolensky, 1990). A condition for equality of attraction in OT and HG converges with empirical considerations by prohibiting unattested patterns of disjunctive parasitic harmony.

Another goal of this work is to investigate how similarity preconditions interact with the locality effects common to harmony. Long-distance consonant harmony, blocking and transparency in vowel harmony, and strictly local assimilation receive a unified explanation in the Attraction Framework by hypothesizing that like features, locality can contribute to a general notion of similarity. A positional similarity hypothesis maintains that string-proximate segments are under greater pressure to assimilate than distal segments. General similarity subsumes aspects of autosegmental phonology, since mapping to a region of a general similarity space parallels projecting to a feature-tier. However, similarity spaces are more powerful, having the flexibility to analyze both consonant intervention and non-intervention in vowel harmony.
Moreover, since the Attraction Framework derives from Burzio’s (2002a,b; 2005) system of representational entailments, it benefits from a strong connectionist underpinning which derives grammatical attraction from network principles, like Hebbian learning (Hebb, 1949) and Harmony maximization (Smolensky, 1986). This dissertation presents neural network simulations of assimilation which illustrate (i) how positional and feature similarity may be related, respectively, to roles and fillers in a system of tensor product representations (Smolensky & Legendre, 2006), (ii) how local and non-local harmony derive from weighting positional and feature similarity, and (iii) how the Attraction Framework is typologically consistent, since networks are unable to learn unattested patterns of anti-parasitic harmony.

Advisors: Luigi Burzio, Robert Frank, Colin Wilson
Acknowledgements

It simply would not have been possible to navigate the difficulties of graduate school and the transition to independent researcher without the support of many kind souls along the way. Furthermore, a project of this size does not occur in isolation; there are many individuals without whom this undertaking would have been quite fruitless. If this work makes any advances in the cognitive science of phonology, then it is only because there were many who contributed bits of sunshine here and there to help the work grow. Of course, any deficiencies that remain stem from my own failure to catch enough light.

Firstly, much credit goes to my primary advisor Luigi Burzio whose consummate patience and truly keen insight kept me motivated and moving forward. I most appreciate how Luigi was so eager to explore new domains with me and how he was always thinking about what to do next.

My secondary advisors Robert Frank and Colin Wilson also provided invaluable advice. I will be ever grateful to Bob for helping me develop a passion for formal methods and for polishing my capacity to apply them, and I was constantly amazed at Colin’s breadth of knowledge—he not only helped me explore connections between my work and other researchers, but also had a capacity to quickly and precisely identify crucial theoretical issues. I am grateful, as well, to the other members of my committee, Lisa Zsiga and Sanjeev Khudanpur, whose corrections and challenging questions improved this work.

Others deserving of special mention include Don Mathis who played a prominent role in assisting the development of the simulation paradigm and Paul Smolensky whose
influence as my master’s thesis advisor extended to this work in numerous ways both practical and theoretical.

I am also grateful for the care and concern of the other members of the Department of Cognitive Science at Johns Hopkins University. Staff members, especially Barb Fisher and Sue Potterfield, were always there when I needed help managing the university bureaucracy. Other faculty members, notably Geraldine Legendre, Barbra Landau, and Brenda Rapp, always provided timely encouragement. Likewise, my fellow JHU students deserve much thanks for their friendship and for hearing me out, especially Sara Finley, Manny Vindiola, Ariel Goldberg, Banchi Dessalegn, Becca Morely, Adam Buckwald, Joan Chen-Main, Gaja Jarosz-Snover, Tamara Nicol-Medina, Jared Medina, Simon Fischer-Baum, Deepti Ramadoss, and Ehren Reilly. I will always cherish the sense of camaraderie, community, and support that I found during my time in Baltimore. Outside of JHU, I am grateful for influential conversations with Gunnar Hansson, Rachel Walker, Jason Riggle, Erik Baković, Matt Goldrick, Andrew Nevins, Jennifer Cole, Joe Pater, Paul Boersma, Bruce Morén, and Elliot Moreton.

Lastly and most importantly, I wish to thank my family without whose constant faith, love, and support this work would not have been possible. My parents, Don and Vivian Wayment, and my in-laws, Louis and Lois Larson provided much needed aide to my family in both time and love. I am in debt to all of them for their unending kindness. To Corwyn and Emmery, I am grateful for all the reminders of the good things in life, for the appreciation of small joys like bouncing balls and chocolate cookies that provided a welcome distraction to the rigors of research. To my wife, Becky—who may be the only
person to read this work twice–I am grateful for all the help in editing and providing
time, space, and encouragement for me to work. Becky is the real gardener in our home;
despite managing two young kids and a too-often ornery researcher, she somehow
managed to provide enough love for all of us to keep growing.
# Table of Contents

1. Introduction ..................................................................................................................... - 2 -
   1.1. Preview ....................................................................................................................... - 2 -
       1.1.1. The problem: Preconditions on Assimilation .................................................. - 2 -
       1.1.2. The solution: Attraction .................................................................................. - 4 -
       1.1.3. Unification under attraction ............................................................................. - 5 -
   1.2. Overview .................................................................................................................... - 7 -

2. The empirical case for prerequisite feature similarity ........................................... - 10 -
   2.1. Introduction .............................................................................................................. - 10 -
   2.2. Similarity preconditions on long-distance consonant agreement ......................... - 12 -
       2.2.1. Ineseño Chumash coronal sibilant harmony .................................................... - 13 -
       2.2.2. Nasal harmony in Ngbaka and Kikongo (Rose & Walker, 2004) .................... - 14 -
       2.2.3. Laryngeal agreement in Yucatec Mayan and Chaha ........................................ - 17 -
       2.2.4. Conclusions of LDCA ...................................................................................... - 21 -
   2.3. Parasitic vowel harmony parallels LDCA ............................................................. - 22 -
       2.3.1. Rounding harmony (RH) .................................................................................. - 26 -
       2.3.2. Height dependencies in ATR harmony ............................................................... - 38 -
       2.3.3. Palatal (Front/Back) harmony in Finnish .......................................................... - 47 -
       2.3.4. Vowel height harmony ..................................................................................... - 55 -
       2.3.5. Conclusion ......................................................................................................... - 59 -
   2.4. Similarity effects in strictly local assimilations ..................................................... - 60 -
       2.4.1. Spirantization in Sudanese Arabic ................................................................. - 62 -
       2.4.2. Word-medial spirantization in Castilian .......................................................... - 62 -
       2.4.3. Sanskrit: minor coronal place assimilation in external sandhi ......................... - 64 -
       2.4.4. /il/ prefixation in Maltese .................................................................................. - 65 -
       2.4.5. Turkish condition on coalescence for vowel hiatus resolution ....................... - 65 -
       2.4.6. Catalan nasal place agreement ....................................................................... - 67 -
       2.4.7. /in/ prefixation in English ................................................................................. - 68 -
       2.4.8. /in/ lateral agreement in Italian and English ..................................................... - 72 -
   2.5. Conclusion ................................................................................................................. - 75 -
3. Formalizing ATTRACTION ................................................................. - 76 -
   3.1. Introduction ............................................................................. - 76 -
   3.2. Representational distance in an attraction system ..................... - 79 -
   3.3. Attraction and the Entailment Framework .................................. - 83 -
   3.4. Dependency, Parasitism, and Phonetic Similarity ....................... - 91 -
   3.5. ATTRACTION correspondence constraints in OT ....................... - 100 -
      3.5.1. Entailment persistence ...................................................... - 100 -
      3.5.2. Direction of persistence .................................................... - 107 -
      3.5.3. Defining ATTRACTION in Optimality Theory ..................... - 108 -
      3.5.4. Equality with entailments under conjunctive additivity .......... - 113 -
      3.5.5. Universal rankings for subset similarity relations ................. - 118 -
      3.5.6. Deriving prerequisite similarity effects with faithfulness thresholding .............................................................. - 123 -
   3.6. Weighting ATTRACTION constraints ........................................ - 132 -
      3.6.1. Defining Harmonic Grammar for CON BA ............................ - 133 -
      3.6.2. The (non)-issue of superadditivity ..................................... - 138 -
      3.6.3. Thresholding in Harmonic Grammar ................................. - 141 -
      3.6.4. Harmonic Grammar vs. OT in the arena of ATTRACTION ........ - 143 -
   3.7. ATTRACTION in regular harmony spaces ................................... - 150 -
      3.7.1. Language specific similarity .............................................. - 151 -
      3.7.2. Regular harmony spaces ................................................... - 153 -
      3.7.3. Converting a regular feature space into an ATTRACTION hierarchy .... .............................................................. - 158 -
      3.7.4. Faithfulness in regular harmony spaces ............................... - 163 -
   3.8. Avoiding disjunction ................................................................ - 167 -
      3.8.1. Typology and pathology of a ‘Principle of Similarity’ ............ - 167 -
      3.8.2. Disjunctive agreement ....................................................... - 172 -
      3.8.3. Intersection completion for the elimination of disjunctive agreement. ............................................................................. - 176 -
      3.8.4. Typological numbers and motivations .................................. - 179 -
   3.9. Conclusion ............................................................................... - 186 -
4. **Attraction at a distance** .............................................................. - 188 -
   4.1. Introduction .............................................................................. - 188 -
   4.2. Extending **Attraction** for sensitivity to locality .................... - 192 -
      4.2.1. Locality ‘features’ via segment-to-segment correspondence ..... - 194 -
      4.2.2. Violable locality for blocking and transparency .................. - 199 -
      4.2.3. Features, locality, and general similarity ............................ - 212 -
   4.3. Integrating preconditions on parasitic vowel harmony .......... - 220 -
      4.3.1. Contrast in parasitism: rounding harmony in Yawelmani and Kirgiz
      ...................................................................................................... - 221 -
      4.3.2. Blocking by interrupting the parasitic domain .................... - 228 -
      4.3.3. Strictly local parasitic assimilations in Turkish ................... - 240 -
      4.3.4. Transparency under Attraction: Is Finnish palatal harmony a case of
        true transparency? ........................................................................ - 243 -
      4.3.5. Discussion & Conclusion ...................................................... - 256 -
   4.4. Locality contrasts in nasal harmony ........................................ - 258 -
      4.4.1. Patterns of blocking and transparency in nasal harmony ...... - 261 -
      4.4.2. Similarity in nasal harmony .................................................. - 270 -
      4.4.3. Parasitic long-distance vowel agreement in Moba Yoruba ...... - 288 -
      4.4.4. Nasality a mixed bag of C’s and V’s ................................. - 304 -
      4.4.5. Conclusions of nasal harmony ............................................. - 314 -
   4.5. Recasting autosegmental feature tiers as prerequisite similarity
      relations .................................................. - 315 -
      4.5.1. Delineating undergoers and non-undergoers with similarity ..... - 315 -
      4.5.2. Prerequisite similarity solutions to C-V interactions ............ - 329 -
      4.5.3. Avoiding the representational challenges of feature geometry.. - 339 -
      4.5.4. Dependency as similarity clustering ..................................... - 344 -
   4.6. Summary and Conclusion ..................................................... - 347 -

5. **Similarity bias in Entailment Networks** ...................................... - 350 -
   5.1. Introduction .............................................................................. - 350 -
   5.2. Methods for attraction in Entailment Networks ....................... - 354 -
   5.3. Representing positions with a role-filler system ....................... - 361 -
   5.4. Simulation #1: Motivations for Assimilation ......................... - 366 -
      5.4.1. Methods .............................................................................. - 366 -
      5.4.2. Results .............................................................................. - 369 -
“Isaac Newton’s breakthrough was not that apples fall to the earth because of gravity; it was that the planets are constantly falling toward the sun for exactly the same reason.”

– Nathaniel Page Stites, 2004

“It would be quite interesting to consider whether VH [vowel harmony] and CH [consonant harmony] are not more alike than we have heretofore believed.”

– Larry Hyman, 2002
1. Introduction

1.1. Preview

1.1.1. The problem: Preconditions on Assimilation

It is common across languages for the realization of some phonemes to depend on other phonemes in the same domain. This work focuses on patterns known as *assimilation or harmony* where the presence of a *triggering* element causes a *target* element to become more similar to the trigger. For example, in the English prefix *in-*, meaning ‘not’, the nasal assimilates in place to subsequent stops. Thus, as shown below in (1)(a), *in-* before a [p]-trigger surfaces as [ɪm], but before a [t]-trigger *in-* surfaces as [ɪn] and before a [k]-trigger *in-* surfaces as [ɪŋ].

(1) *English nasal place assimilation:*

<table>
<thead>
<tr>
<th>word</th>
<th>phoneme</th>
<th>context</th>
</tr>
</thead>
<tbody>
<tr>
<td>impossible</td>
<td>[ɪm]</td>
<td>labial stop context</td>
</tr>
<tr>
<td>intolerable</td>
<td>[ɪn]</td>
<td>coronal stop context</td>
</tr>
<tr>
<td>inconceivable</td>
<td>[ɪŋ]</td>
<td>velar stop context</td>
</tr>
</tbody>
</table>

One major argument this dissertation advances is that languages tend to set feature similarity preconditions on harmony between a trigger and a target. ¹ In particular, languages may require an amount of *prerequisite feature similarity*, where harmony only

---

¹ Others have observed patterns about similarity in assimilation, although not quite in the same terms (see Steriade, 1981; Cole & Trigo, 1988; Archangeli & Pulleyblank, 1994, 2007; Rose & Walker, 2004; Hansson, 2001; Bakovic, 2007; Kaun, 1995; Hong, 1994; etc.)
obtains if triggers and targets agree on certain features. As Baković (2007) points out, such a feature precondition must exist for English nasal place assimilation because [+continuant] consonants do not trigger harmony on [−continuant] nasals:

(2) Lack of nasal place assimilation in English when trigger and target disagree in continuancy:

\[
\begin{align*}
\text{insurmountable} & [\text{in}] \quad \text{coronal fricative context} \\
\text{infallible} & [\text{in}] \quad \text{labiodental fricative context} \\
\text{inhospitable} & [\text{in}] \quad \text{glottal fricative context}
\end{align*}
\]

Taken together, (1) and (2) show that agreement on [−continuant] is prerequisite to agreement on [nasal] in English.

The data surveyed in this work suggest that such feature preconditions are more common than anticipated (Hansson, 2001; Rose & Walker, 2004), having a wide application in (i) a number of strictly local processes, like English where the harmonic feature is parasitic on agreement along some other feature, (ii) parasitic vowel harmony processes where syllable-adjacent vowels undergo harmony if they agree on other features, and (iii) non-local consonant harmony processes where the trigger and target need not be proximate, but they must be similar in order for harmony to obtain.

In light of this data, feature similarity preconditions must be able to apply to harmony processes with various locality preconditions. This calls for a more thorough investigation of how locality and feature similarity interact as preconditions on harmony. A goal of this dissertation is to argue that locality also ought to be understood as a kind of similarity on par with feature similarity. From the perspective of representations, agreement in proximity and/or features can yield similar representations of segments.
Another main contribution of this work is an understanding of why some kinds of parasitic dependencies readily occur, but others are unattested. I present data suggesting that parasitic dependency is always grounded in an amount of phonetic similarity between parasitic and harmonic features. For example, in English and many other languages, nasality is known to be antagonistic to continuancy (Ohala & Ohala, 1993) because the high pressure drop needed for the generation of frication noise is undermined by a lowered velum, which opens an air pathway that circumvents the stricture point. Thus, for these aerodynamic reasons [−continuant] is more similar to [nasal] than [+continuant], and so phonetic similarity rightly predicts that [−continuant]-parasitic [nasal] harmony exists, but [+continuant]-parasitic [nasal] harmony does not.

In sum, similarity plays multiple roles in the preconditions of harmony processes. Assimilation is sensitive to the general similarity preconditions that can take the form of features, proximity, or both. Furthermore, the typology of parasitic dependencies is constrained by the existence of an amount of phonetic similarity between parasitic and harmonic features. Because these aspects of the preconditions on harmony are related to similarity, all of the above (feature similarity preconditions, their grounding in phonetic similarity, and locality conditions on assimilation) ought to be reducible to a general cognitive process which is sensitive to similarity.

1.1.2. The solution: Attraction

This dissertation explains why similarity has an essential role in assimilation by hypothesizing that aspects of subsymbolic, connectionist computation are directly active in the patterns of phonology. This general computational process is based on the principles of Hebbian learning (Hebb, 1949) in a Hopfield network (1982), where the
units that “fire together, wire together.” This dissertation shows how generalizations about assimilation can be reduced to the fact that when representations consist of similar components, then the connections between the components are reinforced. These reinforced connections create a pressure for partially similar representations to be even more alike. Thus, these connectionist forces can be described as an attraction relationship where the distance between representations determines the strength of the force of attraction.

Because these forces of attraction are blind to whether units are encoding features or positions, attraction explains both why harmony is more likely to occur under proximity and also why harmony is more likely to occur under feature and/or phonetic similarity. Burzio (2002a, b, 2004; 2005; Burzio & Tantalou, 2007) has shown that principles of attraction are made available to a constraint framework, like Optimality Theory (OT; Prince & Smolensky, 2004) or Harmonic Grammar (HG; Legendre, Miyata, & Smolensky, 1990) by considering a set of entailments between the components of a representation. This dissertation shows how to extend these concepts of attraction to give the segment-to-segment dependencies seen in patterns of assimilation, fully integrating both feature and proximity preconditions on harmony.

1.1.3. Unification under attraction

A major ramification of this attraction-based perspective on assimilation is that attraction affords unification across (i) empirical domains and (ii) multiple levels of explanation. These aspects of unification are considered in turn. Concerning unification across domains, a diverse set of harmony drivers have been posited to explain the differences between consonant and vowel harmony. The present work supersedes many
of these drivers by providing a formal vocabulary which is rich enough to describe consonant and vowel harmony as well as spreading and non-local agreement. A large, diverse set of cases of assimilation are analyzed with a proposed family of ATTRACTION constraints. The remaining differences between the harmony phenomena are reduced to a sensitivity to different aspects of similarity. For example, patterns of blocking are understood as exploiting positional similarity, whereas patterns of transparency ignore positional similarity in favor of feature similarity. Attraction enables a degree of unification across empirical domains which has been, heretofore, unobtainable.²

Turning to unification across explanatory levels, the attraction-based solution which I advocate allows for multiple levels of explanation (Smolensky & Legendre, 2006). There is an explanation available at a higher, phonological, grammatical level and another at a lower, connectionist level. In the present work, a number of formal results show how a higher, OT-level description can be derived from a Harmonic Grammar, which, in turn, follows from a low-level similarity space. However, a number of connectionist simulations confirm the importance of a subsymbolic grounding for the sensitivity to distance expressed at the grammar-level.

For instance, I show that OT with local constraint conjunction can perform the additivity of a subsymbolic space, but the specified rankings are unmotivated and the predicted typology is not restrictive without further grounding in computational principles, like sensitivity to low-level and phonetic similarity and biases on learning. Another example of how connectionism informs the grammar can be found in a study of

² In fact, there are even a number of arguments in the literature on long-distance consonant harmony that unification should not occur (Gafos, 1996; Hansson, 2001; Rose & Walker, 2004). However, the present work and others (Hansson, 2007; Hyman, 2002; and Walker, 2009) suggest that the differences between consonant and vowel harmony are weaker than originally supposed.
tensor product representations in role-filler systems (Smolensky, Miyata, & Legendre, 1992). This study demonstrates that features and proximity combine as equal partners in maximizing similarity because there are shared resources across positions and shared resources across features with tensor product representations.

On the other hand, the higher level can also add insight into the lower level. In particular, I show how the mapping between phonological strings and the low-level similarity space where harmony obtains can be understood as kind of tier projection, where similar elements map to the same region of the harmony space. Under attraction, regions of this harmony space act like feature tiers, which allow triggers to induce harmony on targets; segments which map to different regions behave exactly as if they project to different feature tiers. Thus, because of a common notion of attraction, levels of explanation can be mutually informing (see Smolensky & Legendre, 2006 for other benefits of having multiple levels of explanation).

1.2. Overview

This dissertation illustrates how the cognitive science of phonology reaps increasing benefits as studies become more interdisciplinary. This dissertation applies various methodological aspects of theoretical phonology, formal mathematical reasoning, computational linguistics, and, to a lesser extent, experimental phonetics, to the problems of preconditions on assimilation. The employed methods and chapter summaries are provided below.

Chapter 2 is a cross-linguistic survey of a wide number of phenomena that exhibit feature preconditions on assimilation. These theoretical/typological methods confirm that there is a role for phonetics in driving the relationship between parasitic and
harmonic features. Furthermore, Chapter 2 confirms that there are parasitic assimilations in a wide number of phenomena across a diverse set of localities.

Chapter 3 presents the basics of the Attraction Framework and, through a series of formal theorems, develops a thresholding theory, whereby the ranking of IO-FAITH relative to ATTRACTION constraints provides a cut-off for the similarity preconditions on harmony. Furthermore, Chapter 3 explores the following: (i) how entailment satisfaction can derive the strength of parasitic dependency, (ii) the conditions under which attraction at the OT-level (which uses a limited form of local constraint conjunction) is identical to attraction at the HG-level (which uses numerical additivity of constraints), and (iii) the low-level nature of a pathology of disjunctive preconditions on harmony.

Chapter 4 contains the central theoretical arguments for how the Attraction Framework integrates both locality and feature preconditions on harmony. For a number of cases of parasitic vowel harmony and nasal harmony, phonetic similarity is shown to play an essential role in identifying otherwise unrelated sets of harmony participants. Chapter 4 also contains a discussion of how an attraction grammar relates to machine learning kernel methods with the conclusion that, because of a high-dimensional similarity space, the Attraction Framework ably subsumes both autosegmental feature tiers and the co-harmony which is elsewhere due to a common parent in a feature geometry (e.g. Clements & Hume, 1995).

Finally, Chapter 5 presents a number of simulations in connectionist networks which confirm that (i) forces of attraction are due to principles of Harmony maximization (Smolensky, 1986) in a neural network, (ii) tensor product representations in a role-filler
system explain why features and proximity interact as equal partners in determining similarity preconditions, and (iii) the similarity perspective engendered by these representations is restrictive, since an Entailment Network is unable to learn unattested patterns of anti-parasitic or anti-local harmony.
2. The empirical case for prerequisite feature similarity

Chapter Overview: This chapter reviews the empirical facts about similarity effects in assimilation from the perspective of prerequisite similarity. In a wide number of cases, the trigger-target relationship, where a target assimilates to a trigger, is shown to be predicated on the (prerequisite) similarity between the trigger and target. Only targets which are similar “enough” assimilate to triggers. Thus, a prerequisite similarity cut-off delineates a set of undergoers from a set of non-undergoers. The empirical landscape suggests that different languages utilize distinct ways of computing the similarity between triggers and targets, and that these similarity effects are found in all of consonant harmony, vowel harmony, and strictly local assimilations for both vowels and consonants.

2.1. Introduction

This chapter reviews evidence that suggests a prominent role of similarity in assimilatory phenomena. It aims to show that similarity effects occur systematically in assimilatory phenomena across a wide variety of languages and language families. This evidence will require the development of a more complete formalism, capable of describing the more general function of similarity in assimilation. While Chapters 3&4 present the formalism, this chapter presents the abundance. A number of studies focus on the role of similarity in assimilation (notably Cole & Trigo, 1988; Hansson, 2001; Rose & Walker, 2004; Baković, 2007), but this is the first effort to perform a side-by-side comparison of the similarity properties of consonant harmony, vowel harmony, and strictly local assimilations. The main goal of this chapter is to argue that there are parallels among these diverse phenomena in terms of how trigger-target similarity affects whether a target undergoes harmony.
Now, within phonological forms, segments may differ from one another in various respects, including syllable position, sequential proximity, phonological features, exact articulatory realization, and acoustic properties. This creates a plurality of ways to measure the similarity of two segments. Ultimately, I will argue for a general notion of representational similarity that can combine multiple similarity measures, but first I focus on understanding the role of feature similarity. By standard definition, the result of an assimilatory process is increased feature similarity: disagreeing segments come to agree on the harmonic feature after assimilation applies. However, feature similarity is not only an end product of assimilation. This chapter confirms that feature similarity has an additional role as a precondition to assimilation. In the cases reviewed here, input segments which are sufficiently similar become even more alike at the surface, while segments which are not sufficiently similar do not show any change. To contrast these input and output roles of similarity, I refer to similarity preconditions on assimilation as the prerequisite feature similarity.

This chapter reviews data from long-distance consonant agreement, parasitic harmony, and exclusively local assimilations supporting a two-part view. First, prerequisite feature similarity plays a role in a broad set of phenomena, which spans both consonant and vowel harmony and both local and non-local assimilations. Second, languages differ in the amount of feature similarity they require for assimilation to obtain. Some languages exhibit a relatively high prerequisite similarity, only allowing assimilation if the trigger and target differ exactly on the harmonic feature, but other languages are much less stringent, allowing targets to assimilate even though they differ from a trigger in several respects. In the rest of this chapter, I review examples of
prerequisite feature similarity in long-distance consonant agreement (§2.2), parasitic vowel harmony (§2.3), and strictly local assimilations (§2.4).

2.2. Similarity preconditions on long-distance consonant agreement

In comprehensive surveys, Hansson (2001) and Rose and Walker (2004) argue that sensitivity to similarity is among the primary characteristics of long-distance consonant agreement (LDCA). As the name implies, the non-local nature of LDCA is one of its distinguishing characteristics. I address such locality issues in Chapter 4. However, LDCA tends to also be subject to similarity preconditions, so here I review a few examples of LDCA to illustrate prerequisite feature similarity.

Following Hansson (2001) and Rose and Walker (2004), I assume that both phonotactics within a morpheme and alternation in heteromorphemic contexts derive from the same family of harmony constraints. Thus, some of the examples below exhibit morpheme structure constraints (MSCs) that prohibit combinations of segments which agree on some features but not others. Other examples are productive assimilation processes in which the segments of one morpheme alternate to agree with the features of segments in another morpheme.3

§2.2.1 illustrates prerequisite feature similarity in Inseño Chumash long-distance coronal harmony. §2.2.2 shows how Ngbaka and Kikongo differ in the degree of similarity required for long-distance nasal agreement. Finally, §2.2.3 demonstrates that

---

3 I attribute the existence of both MSCs and productive harmony to some factor, perhaps, specific faithfulness, that is independent of the harmony driver. See McCarthy (2007) for further discussion of the relevance of morphemic context in consonant harmony.
prerequisite feature similarity must be determined on a language-specific basis to account for subtle differences in Yucatec Mayan and Chaha laryngeal agreement.

2.2.1. Ineseño Chumash coronal sibilant harmony

Coronal harmony in Ineseño Chumash (Hansson, 2001; McCarthy, 2007; references therein) is a prototypical case of LDCA, exhibiting a right-to-left dependency for the feature [anterior].4 Below, (1)(a-c) show that consonants which share the features [+continuant, +coronal] with the rightmost [s] or [ʃ], also agree with the place feature [±anterior] of that [s] or [ʃ], but, as seen in (1)(d), non-coronal and non-continuant obstruents are not required to agree. Here and elsewhere, triggers and targets are underlined, while alternating segments are in bold font.

(1) | surface form | underlying form | gloss |
--- | --- | --- | ---
(a) | kʃuʃojin | /k-suʃoʃin/ | ‘I darken it’ |
(b) | fapitʃolit | /ʃ-apiʃo-it/ | ‘I have a stroke of good luck’ |
(c) | sistisijepus | /s-iʃiʃi-jep-us/ | ‘they (2) show him’ |
(d) | haʃxintilawafaʃ | /haʃ-xintila-wafaʃ/ | ‘his former Indian name’ |


In (1)(d), word-medial /x/ does not assimilate even though it is a fricative; likewise, /t/ does not undergo place assimilation even though it is a coronal obstruent. Thus, both

---

4 Because theories of phonological features vary, the particular scalar value used to indicate the feature similarity certainly depends on the choice of a particular theory of phonological features. But feature similarity can still be defined in broad, theory-neutral terms: the feature similarity of two segments is related to the number of phonological feature specifications those segments have in common—the more features in common, the greater the similarity. Therefore, some aspects of feature similarity are independent of the choice of a particular feature theory (see, in particular, the discussion of subset similarity in §3.5.5). By and large, this chapter aims to discuss feature similarity in these neutral terms, so unless otherwise noted, any indications of particular features in this chapter are solely for expository purposes and faithfulness to the existing literature. Thus, here the choice of [anterior] instead of other possible features, like [distributed] or [alveolar], is not critical to the discussion of feature similarity preconditions.
[coronal] and [continuant] are part of the prerequisite similarity for minor place agreement in Ineseño Chumash. For Ineseño Chumash, participating segments agree on all features except [anterior]. Differing on any other feature is enough to prevent a segment from assimilating. I will say that a target segment participates in an assimilation process, if there exists a context in which that segment undergoes harmony. In Chumash, a high prerequisite feature similarity separates the participating segments from the non-participating, transparent, segments. This relatively high prerequisite feature similarity, where the only difference between triggers and targets is the harmonic feature, is somewhat typical of coronal harmony.

2.2.2. Nasal harmony in Ngbaka and Kikongo (Rose & Walker, 2004)

However, for other harmonic features, it is not uncommon for languages to differ in the similarity preconditions placed on assimilation. For example, Ngbaka is another language that exhibits high prerequisite similarity, whereas Kikongo is a language with less feature preconditions on assimilation. As I review, Ngbaka has a homorganicity requirement on nasal assimilation, but Kikongo does not.

Unlike Chumash, in the Niger-Congo language, Ngbaka (Broe, 1995; Rose & Walker, 2004), the harmonizing feature is [nasal] and harmony is expressed as a morpheme structure constraint. Here, I follow (Rose & Walker, 2004) in observing that this MSC operates along a voicing-nasality continuum. Ngbaka has a four-way contrast between oral voiceless stops, oral voiced stops, prenasalized voiced stops, and full nasals, e.g. /p/-/b/-/mb/-/m/. In non-compound words, homorganic stops cannot co-occur with

---

5 I may use “participating segments” as others use “possible target”. I prefer the former term as it avoids confusion between the typological possibilities and the individual facts of a language. Furthermore, the goal of prerequisite similarity is to explain why only some segments undergo harmony, even though, in principle, all segments are potential targets.
neighbors on this continuum. Thus, while [boma] ‘how’ is attested, *[bo^mba] and *[m^boma] are prohibited. As (2) shows, there seems to be an exception for identical, homorganic stops: sequences which agree on both place and nasality are allowed.

(2) a. nan^e ‘today’
    m^b^e m^b^e ‘snail’

b. [m^a^ng^a] ‘net’
    [mini] ‘tongue’

(2)(a) illustrates that both homorganic nasal stops and homorganic prenasalized stops are permitted. Furthermore, (2)(b) confirms that unlike homorganic stops, non-homorganic stops are free to agree or disagree on the voicing-nasality continuum. Rose and Walker (2004), thus, conclude that “the generalization is that nasal and prenasal stops which match in place must also agree in nasality, i.e. both must be full nasal or both (partially) oral” (p. 502). Their description of Ngbaka makes crucial use of prerequisite feature similarity: where homorganicity fails to occur, there is no requirement of nasal agreement, so homorganicity is prerequisite to nasal agreement. To ignore the prerequisite feature similarity would be to miss something essential about Ngbaka nasal agreement. Ngbaka receives further treatment in the analysis of nasal harmony in §4.4.

Contrastingly, the feature similarity preconditions on Ngbaka nasal agreement are stronger than the preconditions of the Bantu Language, Kikongo (Rose & Walker, 2004). As Rose & Walker (2004) show, Kikongo lacks the near-identity precondition of Ngbaka, requiring even non-homorganic stops to participate in agreement. The data in (3) confirm

---

6 As noted by Rose & Walker (2004), the status of the conditions on the co-occurrence of non-continuum neighbors remains unclear, although /b/-/m/ pairs are attested, /p/-/m/ combinations are rare. The relation between /p/-/m/ is not reported. (Frisch, 2004) suggests that while identical sequences are always permitted, gradient interaction may be needed to explain the co-occurrence of other pairs on the voicing-nasality continuum.
that the perfective active suffix /idi/ emerges as [ini] when there is another nasal in the stem.

(3) a. m-[bud-idi]_{stem} ‘I hit’
    n-[suk-idi]_{stem} ‘I washed’

b. tu-[nik-ini]_{stem} ‘we ground’
    tu-[kun-ini]_{stem} ‘we planted’

c. [sim-ini]_{stem} ‘prohibited’
    [futumuk-ini]_{stem} ‘resuscitated (intr.)’

(Rose & Walker, 2004, p. 503)

(3)(a) shows that assimilation is limited to within stems. (3)(b) provides examples of homorganic agreement. (3)(c) indicates that heteroganic agreement also occurs. Thus, among the differences between Ngbaka and Kikongo nasal agreement is a difference in prerequisite similarity: homorganicity is prerequisite in Ngbaka, but not Kikongo.

Also note that while prerequisite feature similarity is lower in Kikongo, it is not wholly absent. Feature similarity still plays a role in separating participants from nonparticipants: prerequisite feature similarity can explain why the suffix consonant alternates, but not the vowels, if it is assumed that participating segments must agree on [consonantal] before assimilation can occur. Thus, as discussed at more length in Chapter 4, prerequisite similarity has the potential to subsume the work done by more traditional autosegmental feature tiers by deriving the notion of ‘tiers’ from similarity relationships. Not only does similarity account for the general non-participation of consonants in vowel harmony (cf. Ni Chiosáin & Padgett, 2001), it also predicts C-V interactions in harmony on the basis of similarity (see §4.5 for examples and analysis). However, the main point in these cases above is that prerequisite feature similarity is not
only useful in describing the set of participants within a particular process, like Ngbaka or Kikongo nasal agreement, but it is also useful in describing how sets of participants vary across languages: Ngbaka has a higher prerequisite feature similarity than Kikongo.

2.2.3. Laryngeal agreement in Yucatec Mayan and Chaha

The contrast between Yucatec Mayan and Chaha provides another clear example of contrast based on prerequisite feature similarity, but further argues that prerequisite feature similarity is determined on a language-specific basis. The evidence for laryngeal agreement in Yucatec Maya (Yip, 1989; Straight, 1976; Rose & Walker, 2004) has been stated as a generalization over non-sonorants in CVC roots. The non-sonorant inventory of Yucatec Maya is indicated in (4):

(4) Oral obstruents:

<table>
<thead>
<tr>
<th>Labial</th>
<th>Coronal</th>
<th>Dorsal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dental</td>
<td>Alveolar</td>
</tr>
<tr>
<td>p</td>
<td>t</td>
<td>c</td>
</tr>
<tr>
<td>p’</td>
<td>t’</td>
<td>c’</td>
</tr>
<tr>
<td></td>
<td>s</td>
<td>š</td>
</tr>
</tbody>
</table>

Here, ’ denotes that segments carry a glottalized laryngeal articulation, ejective, which is indicated with the feature [+constricted glottis]; [c], [č] are affricates.

Yip notes, concerning co-occurrence restrictions over CVC roots, that “If both consonants are stops, and only one is glottalised, they must differ either in Place or in Manner, or both…If both Cs are glottalised, they must be totally identical” (1989, pp. 363-364; original emphasis). Additionally, if both Cs are [+continuant] (affricates or fricatives), then they must be totally identical. These facts are presented in the co-occurrence table below, in (5), which indicates that among the 25 possible C’VC’ roots, only five, the identity roots, are permitted: p’Vp’, t’Vt’, c’Vc’, č’Vč’, and k’Vk’.
Among 36 possible $C_{[+\text{cont.}]} V C_{[+\text{cont.}]}$ sequences, only the six identity roots are allowed:
cVc, c’Vc’, sVs, čVč, č’Vč, and šVš.

(5) *Yucatec Mayan morpheme structure constraint (MSC).* Shaded cells indicate impossible $C_1VC_2$ roots. Dark lines separate places of articulation.

(5) also illustrates that these tendencies toward $C_1C_2$-identity are more pronounced among glottalized than non-glottalized stops. For example, [p] may occur with any segment, except for its glottalized alternative [p’]. On the other hand, [p’] may only occur with [p’] and non-glottal, non-sonorants ([t], [c], [s], [č], [š], and [k]). In a binary feature theory, this asymmetry requires prerequisite feature similarity to make explicit reference to a single feature value, e.g. homorganicity is prerequisite to [+constricted glottis] agreement (but not [−constricted glottis] agreement). However, because there is no evidence of [−constricted glottis] agreement in Yucatec Mayan, it is likely that [constricted glottis] is a privative, monovalent, feature.

Because Yucatec Mayan agreement is manifest as an MSC, exact input-output relations are not available, making it more difficult to determine which features are prerequisite and which are harmonic. In a sense, the MSC’s tendency toward identity
simply requires that homorganicity and glottalization co-occur; it does not indicate the
direction of dependency. If it is assumed that prerequisite place agreement triggers [c. g.]
agreement, then the prohibition on *[pVp’] is derived, but it would remain unclear why
*[p’Vt’] sequences are disallowed. On the other hand, if it is assumed that prerequisite
laryngeal agreement triggers place agreement, then the *[p’Vt’] prohibition is derived,
yet *[pVp’] would remain unexplained. Thus, both directions of prerequisite similarity
are necessary, so Yucatec Mayan exhibits mutual prerequisite similarity: place
agreement is prerequisite to laryngeal agreement and laryngeal agreement is prerequisite
to place agreement. Likewise, bidirectional implication is necessary to describe the
identity restrictions on affricates and fricatives: if C₁ and C₂ must agree in [+cont], then
agreement on place is enforced, ruling out *[cVč] roots, and if C₁ and C₂ must agree in
place, then total agreement on [+cont.] is enforced, eliminating *[cVs] roots. In sum,
Yucatec Mayan has two forms of identity agreement where homorganicity is mutually
prerequisite with, respectively, [constricted glottis] and [+continuant]. Note that without
a notion of similarity (or identity), this mutual dependence must be implemented with
unrelated rules or unrelated constraints.

Now, other languages are free to form different kinds of dependencies between
features. For example, Rose and Walker’s (2004) analysis of laryngeal agreement in
Chaha (Ethiopian Semitic) notes that there is a strong tendency for non-labial, oral stops
in a root to agree on laryngeal features (voiceless, voiced, and ejective). In both Yucatec
Mayan and Chaha, the active MSCs are limited to non-sonorants, but elsewhere, Chaha
contrasts with Yucatec Mayan in terms of the prerequisite features, the harmonic features,

---

7 If laryngeal agreement were prerequisite on place agreement, then prerequisite feature similarity
would not be met in *[p’Vc’], so laryngeal agreement would not apply, meaning in this case, that *[p’Vc’]
should remain unchanged, and therefore emerge faithfully.
and the preference for identity. First, as (6) below shows, unlike Yucatec Mayan, homorganicity is not prerequisite to laryngeal agreement in Chaha. Any sequence of coronals and velars in the root which disagree on laryngeal features are dispreferred.

(6)  Restrictions on oral, non-labial stops in Chaha roots:

<table>
<thead>
<tr>
<th>C₁…Cⱼ</th>
<th>t</th>
<th>k</th>
<th>t’</th>
<th>k’</th>
<th>d</th>
<th>g</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>k</td>
<td></td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>t’</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td></td>
<td>*</td>
</tr>
<tr>
<td>k’</td>
<td>*</td>
<td>*</td>
<td></td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>d</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>g</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td></td>
<td>*</td>
</tr>
</tbody>
</table>

Second, Rose and Walker attribute the neutrality of labials to an idiosyncratic inventory in which all labials are underlyingly sonorant (/m/, /β/), and surface oral labial stops [p] and [b] are allophones of /β/, but the neutrality of Chaha fricatives must be derived from similarity because they can minimally contrast for voicing, /f β s z x/. Thus, while continuancy and laryngeal agreement revealed similar patterns, they did not directly interact in Yucatec Mayan, but for the case of Chaha, laryngeal agreement must be prerequisite on [−continuant] agreement. Third, unlike Yucatec Mayan, which exhibited agreement for privative [constricted glottis], in Chaha, all laryngeal features ([±voice], [constricted glottis]) are active in agreement, not just [constricted glottis]. Fourth, identity between root consonants is treated rather differently than in Yucatec Mayan: identical segments are completely avoided in Chaha. This derives from the preservation of Semitic MSCs which disallow homorganic stops in a root. These differences are surmised in the table below in (7):
Comparison of Yucatec Mayan and Chaha:

<table>
<thead>
<tr>
<th></th>
<th>Yucatec Mayan laryngeal agreement</th>
<th>Chaha laryngeal agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Place</td>
<td>Participating segments <strong>must</strong> agree in place.</td>
<td>Participating segments <strong>may</strong> agree in place.</td>
</tr>
<tr>
<td>Stricture</td>
<td>[cont.] agreement <strong>parallels</strong> laryngeal agreement.</td>
<td>[cont.] agreement <strong>prerequisite to</strong> laryngeal agreement.</td>
</tr>
<tr>
<td>Harmonic features</td>
<td>Only [constricted glottis]</td>
<td>All laryngeal features, [c. g.] and [±voice]</td>
</tr>
<tr>
<td>Total segment identity</td>
<td>Identity is <strong>preferred</strong>.</td>
<td>Identity is <strong>avoided</strong>.</td>
</tr>
</tbody>
</table>

Taken together, even though Yucatec Mayan and Chaha are both instances of laryngeal agreement, the contrasts between them argue for a *language-specific* definition of prerequisite similarity: mutually prerequisite place in Yucatec Mayan vs. prerequisite [−continuant] in Chaha. Additionally, Yucatec Mayan and Chaha have a language-specific interplay between prerequisite feature similarity and harmonic features: privative [c. g.] agreement in Yucatec Mayan vs. agreement on all laryngeal features in Chaha. Thus, languages are somewhat free to define the dependency between a harmonic feature and its prerequisite feature similarity. This freedom, however, is not absolute; as I show below in §2.3, some dependencies are more prone to occur than others. Chapters 3 & 4 formally illustrate how cross-linguistic differences in similarity dependency are naturally explained from distinct weightings (or rankings) of the factors of similarity.

### 2.2.4. Conclusions of LDCA

The role of prerequisite similarity in defining the trigger-target relationship in LDCA should now be apparent: only those targets that agree with a trigger on other features are under pressure to become even more similar through assimilation. In the remainder of this chapter, I argue (contra Rose & Walker, 2004 and Hansson, 2001) that such similarity-based dependencies are not unique to consonant harmony, and not even...
restricted to non-local harmony phenomena more generally. Therefore, prerequisite similarity ought to be seen as a broad driving force in assimilation, and not a part of a more narrowly active constraint that only applies to long-distance consonant harmony.

2.3. Parasitic vowel harmony parallels LDCA

This section aims to show that prerequisite feature similarity also plays a prominent role in vowel harmony. So-called ‘parasitic’ vowel harmony (Steriade, 1981; Cole, 1987; Cole & Trigo, 1988; Mester, 1988; van der Hulst, 1988; Hong, 1994; Kaun, 1995; van der Hulst & van de Weijer, 2001) exhibits strong parallels to LDCA in terms of sensitivity to similarity. Furthermore, this sections illustrates that even harmony systems which are not traditionally analyzed as parasitic benefit from a parasitic analysis because parasitic harmony allows for directly considering the similarity of trigger-target pairs. In particular, non-participation of interveners is more naturally explained by a failure to meet similarity preconditions on harmony.

Parasitic vowel harmony (PVH) is defined by a parasitic feature and a harmony feature: when vowel harmony for a feature $f$ only occurs if vowels already agree on some other feature $g$, then $f$-harmony is parasitic on $g$. This mirrors the terminology of prerequisite feature similarity: a parasitic feature is a feature similarity precondition consisting of a single feature. Prerequisite feature similarity is slightly more general than parasitic harmony, since prerequisite feature similarity can be used to describe systems where agreement on more than one feature is required for harmony to take place (see Ineseño Chumash in §2.2.1 for a clear example).

Such multiply parasitic systems rarely occur in vowel harmony (although see Oroch in Tolskaya, 2007 for an exception), so in terms of norms, prerequisite similarity is
generally higher for consonant harmony than vowel harmony. This similarity difference likely stems from the fact that consonant inventories are generally larger than vowel inventories, and therefore, there are more contrastive features among consonants than among vowels. For example, it is not unusual (Maddieson, 1989) for consonants at a major place of articulation to contrast in all of stricture, nasal, and laryngeal features; richer systems add contrast in any of secondary articulation, minor places of articulation, or sonority. On the other hand, vowels at a given height and backness may contrast in rounding or ATR, but not usually both. Rich vowel inventories that add contrast in nasality and voicing are less common than rich consonant inventories.

Since there are fewer contrasting features among vowels, there are usually fewer differences between vowels, so it would take fewer similarity preconditions to restrict harmony to environments where trigger and target are already identical vowels. I refer to such situations as trivial or vacuous harmony because no violations of faithfulness are needed to satisfy harmony constraints. It is not surprising that in productive vowel harmony, prerequisite feature similarity is usually limited to a single parasitic vowel feature because further restrictions would tend to result in vacuous harmony system where assimilatory pressures are only applied to already identical segments.

Furthermore, when exceptional multiple preconditions exist on vowel harmony, multiple harmonic processes are frequently at work. For example, as I will show below, in several Turkic languages, vowels only agree in rounding if they agree in both height and backness, but these languages exhibit independent palatal (front/back) harmony, so forms which activate the backness precondition rarely occur. §4.4 and §4.5 provide a formal account of these differences.

8 Tungusic-Manchu languages are a notable exception, having both ATR and rounding harmony.
These overall differences between vowel and consonant harmony notwithstanding, parasitic vowel harmony has two parallels with LDCA that are of interest here. First, in both LDCA and PVH, some languages have higher prerequisite similarity conditions than others for a given harmony feature. For instance, the prerequisite similarity contrast between nasal agreement in Ngbaka and Kikongo (§2.2.2) which was shown to differ in a homorganic precondition, is mirrored by contrasts between parasitic and non-parasitic examples of vowel harmony. Second, in both LDCA and PVH, languages differ on their choice of the parasitic feature for a given type of harmony. As Yucatec Mayan and Chaha (§2.2.3) differ in their prerequisite feature similarity for laryngeal agreement, so do the various languages with PVH differ in the agreement preconditions placed on harmony.

The prospects of a similarity based analysis of parasitic vowel harmony are hinted at in the literature on similarity (Rose & Walker, 2004; Burzio, 2005; Frisch, Pierrehumbert, & Broe, 2004; Hansson, 2007), but none provide a formal account. Because a more formal analysis of PVH is presented in §4.3, this section merely introduces cases of PVH with an eye towards cross-linguistic typology. While a complete survey of all attested cases of parasitic vowel harmony is beyond the scope of this dissertation, which additionally considers LDCA, nasal harmony, and local assimilations, the sample discussed in this section confirms that parasitic vowel harmony (PVH) is somewhat pervasive. The kinds of PVH discussed are previewed in (8). Here, shaded cells indicate impossible dependencies (vacuous harmony systems), blank cells indicate unattested cases, and example languages indicate attested cases.
Typology of parasitic vowel harmony:

<table>
<thead>
<tr>
<th>Parasitic Feature</th>
<th>[round]</th>
<th>[ATR]</th>
<th>[back]</th>
<th>[hi]</th>
<th>[lo]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[round]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ATR]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[back]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[hi]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[lo]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Thus, parasitic harmony exists for a variety of vowel contrasts: prevalently for rounding and advanced tongue root (ATR), rarely for backness, and somewhat marginally for height. The prevalence of PVH indicates that sensitivity to similarity is certainly not an exclusive property of consonant harmony and calls for a general account of the role of similarity in assimilation.

Among the themes that emerge from this review of PVH is that the relationship between a harmonic feature and its parasitic feature is not arbitrary: despite the above variation, the harmonic feature and the parasitic feature usually share some commonality in terms of their phonetic correlates. Furthermore, vowel harmony systems with neutral vowels, which have not always been traditionally viewed as PVH, benefit from considering the similarity of the trigger-target pairs under a parasitic analysis, since directly controlling for trigger-target similarity gives more descriptive power.

I consider the types of parasitic vowel harmony in turn, in order of harmonic feature. A discussion of parasitic rounding harmony (§2.3.1) introduces the importance of phonetic grounding in parasitic harmony. Height-parasitic ATR harmony (§2.3.2)
illustrates how prerequisite similarity interacts with inventory markedness to determine trigger-target pairs. A case study of parasitic palatal harmony in Finnish (§2.3.3) suggests that harmony can be directly parasitic on the phonetic space, and further illustrates how prerequisite similarity determines the set of neutral segments. Finally, §2.3.4 argues that the more limited existence of parasitic height harmony supports a role for the phonetic grounding of parasitic dependencies.

2.3.1. Rounding harmony (RH)

Detailed surveys of rounding harmony (van der Hulst, 1988; Hong, 1994; Kaun, 1995) overwhelmingly conclude that instances of parasitic rounding harmony are vastly more frequent than instances of non-parasitic harmony. As Kaun notes, “rounding harmony rules nearly always impose conditions on the participating vowels which make reference to the dimension of height and/or backness” (1995, p. 1). However, distilling the role of similarity is complicated by a rich set of typological facts concerning the trigger-target relationship. These complexities are best exemplified by a review of height-dependent rounding vowel harmony.

2.3.1.1. Height interactions in parasitic rounding harmony

The typology of interaction between rounding harmony and vowel height is consistently described by both Hong and Kaun and restated below in (9). Here, I combine information from their respective tables (Hong, 1994, p. 14; Kaun, 1995, p. 69). ‘yes’ and ‘no’ indicate whether harmony is permitted in the height context listed in the header row. Therefore, each column reflects a possible height combination of trigger and target.
(9)  **Typology of height-dependent rounding harmony:**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>no harmony</td>
<td>English</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>I</td>
<td>[+hi]-parasitic</td>
<td>Hixkaryana</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>II</td>
<td>[-hi]-parasitic</td>
<td>Eastern Mongolian/</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Tungusic languages</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>[α-hi]-parasitic</td>
<td>Yawelmani</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>IV</td>
<td>[+hi]-target</td>
<td>Turkish</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td></td>
<td>favoring</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>[+hi]-target</td>
<td>Yakut</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td></td>
<td>favoring and</td>
<td>[α-hi]-parasitic</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VI</td>
<td>no conditions</td>
<td>Kirghiz (Conrie, 1981)</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

This section reinterprets this typology from the perspective of prerequisite feature similarity. As I argue below, at least three forces are needed to describe the typological interaction of rounding harmony and vowel height shown in (9):

1) **Prerequisite Similarity:** when trigger and target agree on height, harmony is more likely to occur (Types I, II, III, and V).

2) **Articulatory Markedness:** [+hi] vowels are preferred targets of rounding harmony (Types I, IV, and V).

3) **Licensing of weak-contrast:** rounding harmony facilitates the perception of rounding on [−hi] vowels. (Types II and probably VI).

I now discuss how similarity interacts with the articulatory markedness and contrast-enhancing forces to give the typology of height-interactions in rounding harmony.

As (9) shows, most instances of rounding harmony have prerequisite conditions related to height. In fact, the only known RH system to be completely independent of height (Hong, 1994, p. 14; Kaun, 1995, pp. 5-7) is a dialect of Kirgiz (Conrie, 1981), which constitutes the entirety of Type VI. The Kirgiz vowel inventory is typical of the
Turkic family, being fully contrastive on the features [back], [hi], and [round], as shown in (10) below.

(10) *Kirgiz Vowel Inventory:*

```
<table>
<thead>
<tr>
<th></th>
<th>Front</th>
<th>Back</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unround</td>
<td>Round</td>
</tr>
<tr>
<td>High</td>
<td>i</td>
<td>ü[y]</td>
</tr>
<tr>
<td>Non-high</td>
<td>e</td>
<td>ö[ø]</td>
</tr>
</tbody>
</table>
```

Examples of Kirgiz vowel harmony are found in (11) and (12) below. (11) shows how each possible root-vowel interacts with a high target; (12) shows how root-vowels interact with low targets.

(11) **Hi targets**

a. bir 'one' bir-intʃi 'first'

b. beʃ 'five' beʃ-intʃi 'fifth'

c. alti 'six' alti-ntʃi 'sixth'

d. ʒijirma 'twenty' ʒijirma-ntʃi 'twentieth'

e. ütʃ 'three' ütʃ-untʃi 'third'

f. tört 'four' tört-untʃi 'fourth'

g. toguz 'nine' toguz-untʃu 'ninth'

h. on 'ten' on-untʃu 'tenth'

(12) **Low targets**

a. ifj 'work' ifj-ten 'work-ABL'

b. et 'meat' et-ten 'meat-ABL'

c. ʒil 'year' ʒil-dan 'year-ABL'

d. alma 'apple' alma-dan 'apple-ABL'

e. üj 'house' üj-dön 'house-ABL'

f. köl 'lake' köl-dön 'lake-ABL'

g. tuz 'salt' tuz-don 'salt-ABL'

h. tokoj 'forest' tokoj-don 'forest-ABL'
Clearly, Kirgiz is not parasitic on height: there is a full-spectrum of interaction – any vowel can trigger rounding harmony in any other vowel-height. However, as noted, this sort of symmetry which ignores height features is exceptional in rounding harmony. More common are Types I-III, which involve prerequisite similarity, and Types IV and V, which although dependent on height, are not solely described by similarity.

Among Type III rounding harmony languages, the Yawelmani dialect of Yokuts (Archangeli, 1985; Kenstowicz, 1994; van der Hulst & van de Weijer, 2001) is a prototypical instance of rounding harmony parasitic on height. Hence, its prerequisite feature similarity includes agreement on height. This is schematically illustrated in (13).  

9 Because backness and rounding correlate, it might also be possible to characterize Yawelmani RH as [back] or [color] harmony. However, because of the height dependent interaction, Yawelmani is typologically more like other rounding harmony systems.
Such an observed dependency between rounding and vowel height is relatively well-understood based on articulatory considerations. Phonetic evidence shows (for a review see Hong, 1994; Kaun, 1995) that vowel height and degree of rounding correlate: round, high vowels are reliably more rounded than round, low vowels on a variety of measures, including vertical aperture, horizontal aperture, lip protrusion, lip compression, and pressing of the sides of the lips. Hong and Kaun each argue that the jaw is the bridge between the otherwise independent articulators of the tongue and lips. Jaw opening and tongue height correlate to minimize the tongue distortion needed for high vowel articulation, and a narrow jaw opening brings the lower lip closer to the upper lip, facilitating both lip compression and protrusion. Furthermore, both rounding and high vowel articulation narrow the cross-sectional area of the aperture at the lips. Thus, for reasons of articulatory facilitation, round, high vowels are strongly preferred to round, non-high vowels, in both phoneme inventories and as possible targets of harmony.

These phonetic facts provide a partial explanation for the typological distribution of height dependent rounding shown in (9): the preference for [+hi]-targets shown in Types IV and V derives the lack of low vowel interaction from the articulatory incompatibility of [+round] and [−hi]. However, this hypothesis cannot alone explain why there are Type-II languages and, in fact, whole language groups (Mongolian-Tungusic) where only non-hi vowels undergo rounding harmony. These cases are problematic under the articulatory account because if rounding is less marked on high

---

10 Note that some Type IV languages and all Type V languages which allow round, non-high vowels in prominent contrastive positions, like initial or stressed syllables (Kiparsky & Pajusalu, 2003). Such patterns require positional faithfulness or positional licensing (e.g. Walker, 2001) to explain why the markedness considerations against round, non-high vowels are only active in harmony and not the phonemic inventory (see also Smolensky, 2006 for a discussion of the interaction between inventory and harmony).
vowels than non-high vowels, then it follows that if non-high vowels are good enough targets with respect to markedness, then high vowels ought to be even better targets.

With these problems in mind, explanations for the exclusion of [+hi] vowels from participating in rounding harmony have eschewed articulatory grounding in favor of an account which seeks to optimize the perceptibility of weak contrast (Flemming, 1995; Kaun, 1995, 1997; Walker, 2001, 2005; Finley, 2008). The central idea is that vowel harmony increases the perceptibility of weak contrasts by expanding the duration of relevant auditory cues across multiple segments. Thus, high vowels can fail to participate in harmony because the more extreme rounding on high vowels makes for a sufficiently salient minimal contrast, i.e., the duration of a single segment provides enough time for the perceptual system to determine the value of rounding when it is expressed on a high vowel. On the other hand, because of a less extreme articulation, the rounding contrast between non-high vowels is insufficiently salient, so harmony occurs, facilitating the perception of rounding by providing the perceptual system with more data about the expression of rounding across the duration of multiple syllables.

Unfortunately, however, weak-contrast triggering is only a partial solution to the existence of Type II languages. If weak-contrast licensing were uniquely paramount, then that would predict the existence of languages where non-high vowels are the only triggers, and non-high vowels induce rounding harmony on both high and non-high vowels, i.e. languages where the rounding of the non-high vowel is licensed by any adjacent round syllable.\(^{11}\) While such cross-height licensing would satisfy the need to

\(^{11}\) Walker (2001) reviews cases of bisyllabic triggering in Altaic, but the description and analysis make crucial use of a [-hi]-parasitic spreading constraint. Thus, the typological issue is not whether or not syllabic-based licensing is allowed, but whether cross-height licensing is permitted. If the motivation for
prolong the duration of the rounding gesture, I found no such language in the available surveys. Because Type II rounding harmony does not allow low vowels to trigger harmony on high vowels, it is not such a language where licensing is undominated, and it seems similarity is still playing a role as a precondition: a weak-rounding contrast among non-high vowels is dispreferred to a weak-rounding contrast in a cross-height context.

In review, the forces needed to describe the typological interaction of rounding harmony and vowel height (see (9)) include the following:

1) **Prerequisite Similarity:** when trigger and target agree on height, harmony is more likely to occur (Types I, II, III, and V).

2) **Articulatory Markedness:** [+hi] vowels are preferred targets of rounding harmony (Types I, IV, and V).

3) **Licensing of weak-contrast:** rounding harmony facilitates the perception of rounding on [−hi] vowels. (Types II and probably VI).

While none of these forces dominates to the exclusion of the others, prerequisite similarity is the one force that is never completely ignored. As discussed above, Type II RH violates the markedness preference for [+hi] targets, so markedness can be violated. Furthermore, Types I and IV ignore the preference for non-high, round vowels to agree with other round vowels for perceptual reasons, so the licensing of weak-contrast can be violated. But as (9) illustrates, the presence of cross-height harmony (right two columns) implies the presence of the corresponding within-height harmony (left two columns). This confirms that prerequisite similarity is never violated in rounding harmony and licensing were solely to extend the duration of a rounding gesture, then cross-height licensing ought to exist, but I found no reliable cases in the available surveys.

---

12 van der Hulst (1988) hints in passing that such a language exists, but does not give any examples. Neither Hong (1994) nor Kaun (1995) reports such a language.
previews the typological generalizations to come (see §3.8.1), suggesting the following universal implicational relation, concerning the feature similarity of triggers and targets: all else being equal, any triggering context in which less similar targets assimilate, necessarily implies the assimilation of more similar targets in that same context.

2.3.1.2. Back interaction in parasitic rounding harmony

In addition to height, rounding harmony can also be parasitic on backness. Again, there is a clear phonetic grounding for the aspects of round-back interaction in vowel harmony (Odden, 1991). Backness and rounding are independent articulators, but they have a similar phonetic correlate: both backing and rounding affect the peak of the second spectral formant, F2 (Stevens, Keyser, & Kawasaki, 1986; Stevens, 2000). This is due to the fact that F2 is for the most part determined by the length of the front cavity. Backing the tongue increases the length of the front cavity, as does the lip protrusion associated with lip rounding. Thus, a contrast in backing is enhanced by additional contrast in rounding, so it is common for languages to have phonological inventories that only allow rounding on back vowels (Lindblom, 1986), e.g. canonical \{i e a o u\}. In this respect, Turkic languages are unusual in that they have both a front series and a back series of minimally contrastive round and unround vowels, here, denoted \{i ü e ö i u a o\}. As for cases of height-parasitic vowel rounding harmony (esp. Type II languages in (9)), I assume rounding harmony parasitic on backness is motivated by enhancing the duration of these otherwise weak contrasts.

Now, there are instances of color harmony, where back and round are co-harmonic (Odden, 1991), i.e. vowels in a form always agree for both backness and
rounding. These systems often lack a minimal contrast in rounding at a given backness, making it impossible to separate rounding harmony from backness harmony. Therefore, in order to differentiate [back]-parasitic rounding harmony from [back], [round] co-harmony, it is crucial to consider inventories with a minimal rounding contrast at both front and back. Among such languages, there are many examples of back harmony without rounding harmony (Kaun, 1995), exemplified by the alternations /e/ + /i/ → [eɪ] and /o/ + /i/ → [oɪ]. However, I found no instances of rounding harmony which ignore backness when there is a minimal contrast in rounding, e.g. a case where */o/+ /i/ → [oʊ], when [i], [ʊ], [i], and [u] are in the surface inventory.

Thus, the attested alternations, assuming a non-high, round vowel trigger, /o/, and a high target, /i/, are as follows in (14):

(14) Possible round-back interactions:

a. /o/+ /i/ → [oʊ], both BH and RH are allowed.
b. /o/+ /i/ → [oɪ], RH is blocked, but BH is allowed.
c. /o/+ /i/ → [o.i], both BH and RH are blocked.
d. */o/+ /i/ → [oʊ], BH is blocked, but RH is allowed.

Note this pattern implies the following generalization: rounding harmony can only occur if back harmony obtains. Thus, when minimal contrasts exist in rounding, RH is parasitic on backness.

The [back]-parasitic nature of rounding harmony is particularly well-illustrated in the case of Turkish disharmonic roots (Clements & Sezer, 1982; Polgardi, 1999). Data from (Polgardi, 1999) provides evidence for rounding harmony to be parasitic on backness. Turkish has palatal (front/back) harmony, which is lexically controlled: some

\[13\] In fact, Kirgiz, in (11) and (12) and, Yawelmani, in (13), present such cases.
roots are harmonic, others are not; some affixes participate in harmony, others do not.

(15) shows regular Turkish palatal harmony.

(15) **Turkish vowel harmony:**

<table>
<thead>
<tr>
<th>NOM. SG.</th>
<th>GEN. SG. [+hi]</th>
<th>NOM. PL. [−hi]</th>
<th>gloss</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>ip</td>
<td>ip-ı̞n</td>
<td>ip-ler</td>
</tr>
<tr>
<td>b.</td>
<td>kiz</td>
<td>kiz-ı̞n</td>
<td>kiz-ı̞r</td>
</tr>
<tr>
<td>c.</td>
<td>yüz</td>
<td>yüz-ı̞n</td>
<td>yüz-ı̞r</td>
</tr>
<tr>
<td>d.</td>
<td>pul</td>
<td>pul-ı̞n</td>
<td>pul-ı̞r</td>
</tr>
<tr>
<td>e.</td>
<td>el</td>
<td>el-ı̞n</td>
<td>el-ı̞r</td>
</tr>
<tr>
<td>f.</td>
<td>sap</td>
<td>sap-ı̞n</td>
<td>sap-ı̞r</td>
</tr>
<tr>
<td>g.</td>
<td>köy</td>
<td>köy-ı̞n</td>
<td>köy-ı̞r</td>
</tr>
<tr>
<td>h.</td>
<td>son</td>
<td>son-ı̞n</td>
<td>son-ı̞r</td>
</tr>
</tbody>
</table>

Note in (15) how back harmony obtains in both the GEN. SG. and NOM. PL, but rounding harmony only obtains in GEN. SG. Furthermore, Turkish rounding harmony is a height-conditioned harmony (Type IV from (9)), where high targets agree with preceding vowels in rounding, so because the GEN. SG. is [+hi] it has rounding harmony, while [−hi] NOM. PL does not.

Now, consider the disharmonic roots in (16), indicating a general lack of harmony among roots in both native words and loans (Clements & Sezer, 1982; Polgardi, 1999):

(16) **Disharmonic roots:**

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>a/i</td>
<td>takvim</td>
<td>‘calendar’</td>
<td>fiat</td>
<td>‘prince’</td>
</tr>
<tr>
<td>b.</td>
<td>a/e</td>
<td>haber</td>
<td>‘news’</td>
<td>hesap</td>
<td>‘estimate’</td>
</tr>
<tr>
<td>c.</td>
<td>o/i</td>
<td>polis</td>
<td>‘police’</td>
<td>pilot</td>
<td>‘pilot’</td>
</tr>
<tr>
<td>d.</td>
<td>o/e</td>
<td>otel</td>
<td>‘hotel’</td>
<td>metot</td>
<td>‘method’</td>
</tr>
<tr>
<td>e.</td>
<td>i/u</td>
<td>billur</td>
<td>‘crystal’</td>
<td>muhit</td>
<td>‘area’</td>
</tr>
<tr>
<td>f.</td>
<td>u/e</td>
<td>kudret</td>
<td>‘power’</td>
<td>mebus</td>
<td>‘deputy’</td>
</tr>
</tbody>
</table>

All of (16)(a-f) are disharmonic with respect to backness, and (16)(c-f) are disharmonic with respect to rounding. Of particular interest are the forms *muhit* and *polis* because these are exactly the contexts in which rounding harmony is expected to apply on the basis of the height condition (cf. GEN. SG. in (15)(d) and (h), respectively). These data
contradict a view of rounding harmony as being independent of backness, else *muhüüt and *polüis would be expected, since ü is permitted in non-initial syllables.

It is possible to view the failure of rounding harmony as another instance of lexical control (Polgardi, 1999) by positing that the vowels which mismatch in rounding are lexically specified as such. However, a compact explanation is available if rounding harmony is parasitic on backness: lexical specification for disagreeing backness is sufficient for the blocking of both backing and rounding harmony in Turkish roots. Vowels, which mismatch in backness, prevent the prerequisite similarity conditions from being met, so rounding harmony cannot take place, predicting that root vowels will never disagree in rounding unless they also disagree in backness.

This parasitic hypothesis is supported by the fact that rounding harmony occurs in a proper subset of forms that exhibit back harmony, e.g. both the GEN. SG. and NOM. PL. have palatal harmony, but only the GEN. SG. has rounding harmony. Furthermore, if lexical specification were the cause of the blocking of rounding harmony, then lexical specification would be free to create disharmonic forms which disagree in backness, but agree in rounding (the typologically dispreferred surface form of (14)(d)). The active resistance to such lexical specifications is reflected in the fact that they are only found in words with variable pronunciation, such as in (17)(a-c), which as Polgardi (1999) notes, tend ‘regularize’ so as to agree in backness:

(17) a. püro ~ puro ‘cigar’
    b. nüzul ~ nüzül ‘paralysis’
    c. kupür ~ küpür ‘denomination, clipping’
Contrastingly, forms which disagree in rounding, but not backness are abundant and unvarying throughout the GEN. PL., (15)(c-d,g-h), and elsewhere. Thus, while there is ample evidence that lexical specification blocks palatal harmony, there is no such evidence for rounding harmony. Therefore, this language-internal evidence supports the view that the failure of rounding harmony in disharmonic Turkish stems is caused by rounding harmony being parasitic on backness and not lexical specification.

2.3.1.3. Conclusions of rounding harmony

Rounding harmony among vowels is always parasitic on either height or backness. Non-similarity dependencies between height and rounding are also available, so the degree and manner of parasitism varies from language to language. The round-height interactions are grounded in articulatory similarity (both rounding and height favor a narrow jaw opening), while the round-back interactions are grounded in acoustic similarity (both round and back lower F2). I know of no languages where rounding harmony is parasitic on ATR or nasality. The lack of interaction between rounding and ATR or rounding and nasality presumably derives from a lack of similarity between the features in terms of articulation and acoustics.

Therefore, I conclude that parasitic vowel harmony, where $f$-harmony is parasitic on $g$, derives from an amount of similarity between $f$ and $g$ in a multi-dimensional phonetic space expressing the acoustic and articulatory correlates of $f$ and $g$. The more $f$ and $g$ are phonetically similar, the more likely languages are to develop prerequisite similarity relations between $f$ and $g$. The typology of parasitic rounding harmony

---

14 The similarity of features is measured by distance in an articulatory/acoustic space, and is therefore not to be confused with the similarity of segments measured in terms of feature specifications. I have and will continue to call the similarity between segments feature similarity, and I will refer to the similarity between features as phonetic similarity.
supports this hypothesis that prerequisite feature similarity is grounded in phonetic similarity: the attested parasitic features for rounding harmony (height and backing) are exactly those features that are phonetically similar to rounding.

2.3.2. Height dependencies in ATR harmony

The typology of parasitic advanced tongue root (ATR) vowel harmony (Archangeli & Pulleyblank, 1994; Baković & Wilson, 2000; Pulleyblank, 2002; Baković, 2003; Poliquin, 2006; Mahanta, 2008; O’keefe, 2006; Tolskaya, 2008) reinforces phonetic similarity as a driver for dependency in parasitic harmony. Additionally, this discussion illustrates how prerequisite feature similarity can play a role in determining whether segments are triggers, even when inventory considerations such as structure preservation (Kiparsky, 1985; Archangeli & Pulleyblank, 2007) determine whether segments are targets. Thus, similarity and the shape of the inventory combine to identify the trigger-target pairs.

ATR harmony systems occur within languages that contrast vowels for the position of the tongue root. The literature has used any of the contrastive features [±ATR], [±RTR(retracted tongue root)], ‘tense’ vs. ‘lax’, and [±dorsal] to indicate a change in the quality of vowels as determined by the position of the dorsal/pharyngeal region of the tongue, i.e. the ‘root.’ I will consistently use the feature [±ATR] in this section.

2.3.2.1. Similarity determines trigger-target pairs

Although the participants of vowel harmony always depend on a language’s phonological inventory, this is particularly true of ATR harmony, where the set of participants in ATR harmony is usually the set of minimally ATR-contrastive segments.
in the phonological inventory (Archangeli & Pulleyblank, 2007). Therefore, it is essential to allow the forces which shape the inventory (markedness, dispersion, feature economy, etc.; see Flemming, 1995; Clement, 2003 for a review) to also play an active role in harmony. Otherwise, it would be merely accidental that the non-participating vowels in harmony are exactly the set of vowels without an opposing minimal pair, e.g. the fact that [a][-ATR] does not participate in ATR harmony, as in Akan below, ought not to be coincidental with the absence of an opposing [+ATR], low vowel, [æ][+ATR]. Rather, [a] does not undergo harmony because its harmonic alternate [æ] is prohibited by inventory considerations, in this case the markedness of [+ATR, +low] vowels.

Smolensky’s (2006) Harmony/Inventory Theorem confirms that if a language has an [aF] segment, s[aF], and a [−aF]-harmony process, but s[aF] never undergoes [−aF]-harmony, then s[−aF] (the segment identical to s[aF], but differing on the harmonic feature) is not a member of the inventory of surface contrasts. In other words, the Harmony/Inventory Theorem confirms that vowel harmony cannot result in an allophone that is not elsewhere permitted by the surface inventory. Thus, the set of undergoing segments is usually a subset of the set of contrasts allowed in triggering positions (cf. Kiparsky & Pajusalu, 2003).

Therefore, the main role of prerequisite feature similarity is not to designate a set of ATR harmony targets, which can arise from the same forces which shape the inventory (i.e. when harmony is ‘structure preserving’ Kiparsky, 1985; Archangeli & Pulleyblank, 2007), but rather is to ensure that the segments which exhibit allophony, are also allowed in other positions.

---

15 A brief proof by contradiction: if s[−aF] were independently surface viable in an environment E, then faithfulness to [F] must dominate the markedness (and other) forces which disprefer s[−aF]. In turn, harmony constraints must dominate FAITH([F]) or else there would be no harmony at all, and so if s[−aF] is allowed in environment E, then it must be available as a possible result of harmony, and so s[aF] must undergo harmony, contradicting the assumption that s[aF] does not undergo [−aF]-harmony.

16 There are vowels which are only licensed in harmonic contexts (for cases, see Walker, 2001; Kiparsky & Pajusalu, 2003), but these vowels are seen as the triggers not the targets of harmony, and so the results of harmony, those segments which exhibit allophony, are also allowed in other positions.
2007), but rather to designate the trigger-target relationships where harmony obtains. That is, given a set of triggers and a set of targets based on distributional and inventory considerations, prerequisite feature similarity determines which triggers induce harmony on which targets.

In this regard, the cases of Degema and Akan provide a useful contrast. In Degema (Archangeli & Pulleyblank, 2007), all vowels come in \([\pm \text{ATR}]\) pairs: \([\text{+ATR}]\) \{i, e, ə, o, u\} contrast, respectively, with \([-\text{ATR}]\) \{ɪ, ɛ, a, ɔ, ʊ\}. In Degema, ATR harmony is unrestricted. As (18) shows, all possible vowels trigger ATR harmony.

(18) *Degema ATR harmony:*

a. \([\text{+ATR}]\)

- \text{u-bi-ə} ‘state of being black’
- \text{u-der-əm} ‘cooking’
- \text{o-gədəγə} ‘mighty’
- \text{i-sor-ə} ‘passing liquid feces’
- \text{u-pu-əm} ‘closing’

b. \([-\text{ATR}]\)

- \text{a-k₁} ‘pot’
- \text{ɔ-dɛdɛ} ‘chief’
- \text{ɔ-kpəkəraka} ‘tough’
- \text{u-bɔm-əm} ‘beating’
- \text{u-fu-ə} ‘state of being white’

As with rounding harmony, the kind of symmetry in (18) is more an aberration than norm simply because most languages lack an ATR contrast at some height. For example, Akan (Archangeli & Pulleyblank, 1994; Archangeli & Pulleyblank, 2007; Polgardí, 2007; Kenstowicz, 1994) lacks an ATR contrast on low vowels. Akan has \([\text{+ATR}]\) \{i, e, ə, o, u\}
and [−ATR] {ɪ, ɛ, a, ɔ}. What is surprising is that /a/, which is specified as [−ATR], freely combines with both [+ATR] and [−ATR] vowels in roots, as shown in (19):

(19)  

Akan low vowel /a/ (Kenstowicz, 1994, p. 354):

a. kari ‘to weigh’
   bisa ‘to ask’

b. yarɪ ‘to be sick’
   pira ‘to sweep’

c. prako ‘pig’
   fuńanì ‘to search’

(19)(a) confirms that /a/ occurs with [+ATR] vowels, (19)(b) confirms that /a/ occurs with [−ATR] vowels, and (19)(c) shows that /a/ can occur with roots with both a [+ATR] and a [−ATR] vowel. Therefore, /a/ is inert to root ATR harmony: it is neither a target nor a trigger.

Now, the fact that /a/ does not undergo [+ATR] harmony can be understood by a common markedness prohibition against [+ATR, +low] vowels (Archangeli & Pulleyblank, 1994; O’Keefe, 2006; Polgarditi, 2007), but that does not explain why /a/ is not a trigger. Intra-segmental feature co-occurrence restrictions cannot determine the well-formedness of sequences of segments. For example, the markedness constraint *+[ATR, +low] may prefer disharmonic [ɪ[+ATR]: æ[−ATR]] to harmonic [ɪ[+ATR]: æ[+ATR]], preventing [a] from undergoing [+ATR] harmony, but *+[ATR, +low] does not distinguish [æ[−ATR]: ɪ[+ATR]] from [æ[+ATR]: ɪ[−ATR]], since they both lack [æ] and so satisfy *+[ATR, +low]. Nevertheless, only [æ[−ATR]: ɪ[−ATR]] is harmonic. Therefore, whether or not [a] is a trigger of harmony is independent of the fact that [a] does not undergo harmony due to high ranking inventory markedness *+[ATR, +low].
In sum, inventory markedness alone provides an incomplete picture of the trigger-target relationships involved with the inertness of /a/ in Degema. Here, the addition of a prerequisite feature similarity perspective holds promise: as originally indicated in (Cole & Trigo, 1988), the failure of /a/ to trigger root harmony is predicted by understanding Akan ATR harmony as being parasitic on the vowel height feature [low]. The failure of interaction between /a/ and /i/ in (19)(a) derives from their failure to agree on [low].

From a prerequisite similarity perspective, Degema (18) and Akan (19) contrast as follows: Akan requires agreement on [low] before ATR harmony can take place, Degema does not. This contrast is independent of the inventory distinctions between Akan and Degema because it is possible that even though there is no low ATR contrast, Akan- /a/ could still be a possible ATR-trigger. And in fact, while not true of roots, Akan- /a/ triggers harmony in suffixes (Polgardi, 1999) as shown in (20).

(20) /a/ blocks suffix from carrying [+ATR]
   a. o-bisa⁻¹ ‘he/she asked (it)’
   b. œ-kari⁻¹ ‘he/she weighed (it)’

Therefore, while it is not accidental that the non-contrastive vowel [a] is the only inert vowel in roots (since [æ] is not permitted by *[+ATR, +low]), inventory prohibitions against certain segments cannot fully determine whether two segments are in a trigger-target relation. On the other hand, prerequisite feature similarity provides a principled means for understanding the nature of trigger-target relationships: only segments which agree in [low] are similar enough to participate in Akan root harmony, but in suffix allomorphy, this precondition is relaxed, allowing for trigger-target pairs which disagree in [low], as in (20).
As I will show in §3.5.6, faithfulness ‘thresholding’ provides a natural means for controlling how much prerequisite feature similarity is required for interaction. Thus, the difference between Akan roots and suffixes can be analyzed as a difference in the ranking of faithfulness constraints: a higher ranking root-faithfulness compared to general faith allows for the higher feature similarity requirements to apply in roots. For a formal analysis along these lines, see §4.3.2.2.

Now, low vowels are the most common inert vowels in ATR harmony, but height interactions in ATR harmony are not limited to [low] and prerequisite feature similarity plays a role in other interactions as well. A non-exhaustive typology of parasitic ATR harmony follows in (21), highlighting the possible ATR-height interactions between triggers and targets. ‘yes’ and ‘no’ indicate whether vowels at the specified heights are in a trigger-target relationship for [ATR]; e.g., a ‘no’ under ‘non-low/low’ indicates that the language does not permit ATR harmony interactions between low and non-low vowels, and a ‘yes’ indicates that there is some trigger-target relationship between low and non/low vowels. No interactions were found in other possible height pairs (hi/low, low/mid, etc.).
(21) Height parasitic ATR harmony

<table>
<thead>
<tr>
<th>Example Language</th>
<th>Description</th>
<th>hi/hi</th>
<th>hi/mid</th>
<th>mid/mid</th>
<th>non-low/low</th>
<th>low/low</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degema (Archangeli &amp; Pulleyblank, 2007)</td>
<td>Full-interaction.</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Degema (1994)</td>
<td>Low vowels are inert.</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Wolof (Kenstowicz, 1994)</td>
<td>Both trigger and target must be mid.</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Pulaar (Archangeli &amp; Pulleyblank, 2007)</td>
<td>[+ATR] hi vowels trigger harmony on mid vowels.</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Canadian French (Poliquin, 2006), Menominee (Cole &amp; Trigo, 1988) (Archangeli &amp; Pulleyblank, 1994)</td>
<td>Both trigger and target must be high.</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Eastern Javanese (Hong, 1994)</td>
<td>Both trigger and target must be [ahi, −lo].</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
</tbody>
</table>

For the languages in (21), the set of undergoers is largely predicted by the vowel inventory, but the set of trigger-target pairs is better explained by prerequisite similarity. For example, in Eastern Javanese, ATR harmony is fully parasitic on height (Hong, 1994); vowels only agree in [−ATR] if they also agree in height. Eastern Javanese ATR harmony is fed by the laxing of [+ATR] vowels in closed final syllables, so each [+ATR] vowel {i, e, o, u} has a [−ATR] allophone {ɪ, ɛ, ɔ, ʊ}, which emerges (i) if in a closed, final syllable, or (ii) if followed by a [−ATR] vowel that agrees in height. Note laxing must take place in the final syllable before harmony can obtain in preceding syllables, so ATR harmony in Eastern Javanese is a derived environment effect. As with Akan, the low vowel /a/ is inert to harmony. Examples of Eastern Javanese ATR harmony from Hong (1994, p. 69) are indicated in (22). (22)(a) shows high vowel interaction, (22)(b)
shows mid vowel interaction, and (22)(c) confirms the absence of cross-height ATR harmony.

<table>
<thead>
<tr>
<th>(22)</th>
<th><strong>Open final syllable</strong></th>
<th><strong>Closed final syllable</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>buri ‘back’</td>
<td>mʊɾɪt ‘student’</td>
</tr>
<tr>
<td></td>
<td>turu ‘sleep’</td>
<td>plʊpɪɾ ‘edge’</td>
</tr>
<tr>
<td>b.</td>
<td>bodo ‘stupid’</td>
<td>lɛɾɛɾn ‘stop’</td>
</tr>
<tr>
<td></td>
<td>kere ‘beggar’</td>
<td>ɡoɫɛʔ ‘get’</td>
</tr>
<tr>
<td>c.</td>
<td>iʃɛɾn ‘alone’</td>
<td>*iʃɛɾn</td>
</tr>
<tr>
<td></td>
<td>e^nɪɾʔ ‘obtain’</td>
<td>*e^nɪɾʔ</td>
</tr>
</tbody>
</table>

As in Yawelmani rounding harmony in (13), the trigger-target relationships of such purely parasitic systems receive a straightforward account with prerequisite feature similarity. Eastern Javanese ATR harmony requires vowels to agree in the height features [ahi] and [−low]. A [ahi] precondition prevents cross-height interaction in (22)(c) and a [−low] precondition preserves the inertness of low /a/. Prerequisite similarity, thus, provides a principled way of separating the triggering segments from the non-triggering segments in Eastern Javanese, Akan, and elsewhere.

2.3.2.2. **Phonetic basis for parasitic ATR harmony**

In terms of the phonetic grounding for height-ATR interactions, since the tongue root is connected to the tongue body, it is not surprising that there should be dependencies between [ATR] and vowel height. There are two strong cross linguistic markedness tendencies (Archangeli & Pulleyblank, 1994; Baković, 2003; O’keefe, 2006): (i) [+ATR] high vowels are preferred to [−ATR] high vowels, and (ii) [−ATR] low vowels are preferred to [+ATR] low vowels. Because the tongue root cannot be retracted or advanced independently of the tongue body, these markedness relations derive from
tongue anatomy: retracting the tongue root has a tendency to lower the tongue body and advancing the tongue root has a tendency to raise the tongue body.

Besides these articulatory correlations, there is also an acoustic confound. Along with vowel height, ATR contrasts are primarily, though not exclusively, indicated with a distinction along F1. The acoustic confound has been confirmed by Gick et al.’s (2006) phonetic investigation of ATR harmony in Kinande, which shows that there is heavy F1 overlap between [−ATR] high vowels and [+ATR] mid vowels, but there is much less overlap between high and mid vowels which agree in ATR. Therefore, ATR harmony can be seen as enhancing an F1 contrast by eliminating the co-occurrence of overlapping acoustic categories in sequential vowels. Also, these acoustic factors may be in conflict with articulation because articulatory markedness can override ATR harmony that might otherwise enhance a weak F1 contrast. Therefore, the motivations for a given type of height-parasitic ATR harmony must include both articulatory and acoustic considerations.

I know of only two languages with ATR harmony dependent on a feature that is not related to height. First, Mahanta (2008) reports that in Karajá, [+ATR] harmony is blocked by nasal vowels that lack a contrast in ATR. A prerequisite feature similarity analysis could therefore assume that ATR harmony is parasitic on [−nasal], and the failure of ATR harmony derives from the increased distance between nasal and non-nasal vowels. The basis of such an [ATR]-[nasal] link is might be due to the nasal murmur masking low frequency formant cues (in this case F1) that would otherwise indicate a [+ATR] source (see §4.4 and §4.5.2.4 for further discussion of the interactions between F1 and nasality).
Second, Oroch (Tolskaya, 2008) has an unusual vowel inventory that only has ATR contrast in [+back] vowels ( [+ATR]: {u, ə} → [−ATR]: {ʊ, a, ɔ}). A [ə] in the input maps to [a] in unround contexts and [ɔ] in round contexts, so Oroch also has independent rounding harmony. Here, the point is that while the back vowels undergo ATR harmony, the front vowels {i, æ} are neutral to ATR harmony. This is the only language I know of where ATR harmony is preconditioned on backness. However, because of this remarkable inventory, I consider backness-dependent ATR harmony a typological anomaly.

In sum, there is a strong phonetic dependency between height and ATR which predicts the attested abundance of height-dependent ATR harmony. There are weaker articulatory relationships between ATR and nasality and between ATR and backness (especially among low vowels), and perhaps a few cases that might indicate a carry over into parasitic ATR harmony. The lack of a known phonetic relationship between ATR and rounding predicts the attested lack of instances of round-parasitic ATR harmony. Thus, this review of parasitic ATR harmony also supports a role of phonetic similarity in grounding the relationship between parasitic and harmonic features.

2.3.3. Palatal (Front/Back) harmony in Finnish

In this section, I review cases of back harmony which exhibit a degree of parasitism, arguing even more strongly for the role of phonetic similarity in driving harmony. A study of Finnish palatal harmony previews a more formal account presented in §4.3.4 and reveals that phonetic similarity can trump feature-based similarity.

Cross-linguistically, palatal (front/back) harmony is somewhat rare (Harrison, 2001), but it is readily attested in Turkic and Finno-Urgic languages. In Turkic
languages, agreement on backness is almost never constrained by prerequisite similarity, as exemplified by the normal Turkish suffix harmony shown in (15) and repeated here in (23).

(23)  

Turkish vowel harmony:

<table>
<thead>
<tr>
<th>NOM. SG.</th>
<th>GEN. SG. [+hi]</th>
<th>NOM. PL. [−hi]</th>
<th>gloss</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>ip</td>
<td>ip-ın</td>
<td>ip-ler</td>
</tr>
<tr>
<td>b.</td>
<td>kiz</td>
<td>kiz-ın</td>
<td>kiz-ler</td>
</tr>
<tr>
<td>c.</td>
<td>yüz</td>
<td>yüz-ı̈n</td>
<td>yüz-ler</td>
</tr>
<tr>
<td>d.</td>
<td>pul</td>
<td>pul-ın</td>
<td>pul-ler</td>
</tr>
<tr>
<td>e.</td>
<td>el</td>
<td>el-ın</td>
<td>el-ler</td>
</tr>
<tr>
<td>f.</td>
<td>sap</td>
<td>sap-ın</td>
<td>sap-ler</td>
</tr>
<tr>
<td>g.</td>
<td>köy</td>
<td>köy-ı̈n</td>
<td>köy-ler</td>
</tr>
<tr>
<td>h.</td>
<td>son</td>
<td>son-ın</td>
<td>son-ler</td>
</tr>
</tbody>
</table>

In (23), suffix vowels always agree with the root vowel in backness in both within height and cross-height contexts. There are instances where palatal harmony fails to apply in Turkish roots, but these exceptions are best explained via lexicalization of disharmonic backness values (Polgardi, 1999), rather than as a failure to satisfy prerequisite similarity. It is somewhat typical for prerequisite similarity to play a limited role in palatal harmony, so, unlike rounding harmony and ATR harmony, fully symmetric harmony systems, where all vowels come in front-back alternating pairs, like Turkish, are not at all exceptional in palatal harmony.

In contrast, some Finno-Ugric languages exhibit palatal harmony over asymmetric phoneme inventories. These systems vary somewhat in the exact shape of their inventories and how the unpaired vowels behave with respect to harmony in term of opacity vs. transparency (see Kiparsky & Pajusalu, 2003 for a more complete review).

17 I follow Clements & Sezer (1982) and Polgardi (1999) in concluding that in modern Turkish productive backness harmony is no longer active in roots. Thus, both roots which express disharmony and roots which are fully harmonic are specified as such in the lexicon. (23) shows productive backness harmony is still available in suffix allomorphy.
Therefore, for reasons of clarity and because parasitic harmony is exceptional among palatal vowel harmony, I forgo a full-typology and focus on the case of Finnish palatal harmony, which is typical of the pattern of interest. The basic generalization, which I will explain with prerequisite similarity, is that unpaired vowels, \( i \) and \( e \) are neither trigger, nor targets in Finnish palatal harmony. To my knowledge, Finnish has not previously been analyzed as parasitic vowel harmony. The Finnish phonemic vowels are indicated below in (24):

(24) Finnish vowel inventory. Unpaired (neutral) vowels are in shaded cells.

<table>
<thead>
<tr>
<th></th>
<th>Front</th>
<th></th>
<th>Back</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unround</td>
<td>Round</td>
<td>Unround</td>
</tr>
<tr>
<td>High</td>
<td>( i )</td>
<td>( ü ) [y]</td>
<td></td>
</tr>
<tr>
<td>Mid</td>
<td>( e )</td>
<td>( ö ) [( \text{o} )]</td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>( ä ) [( \text{æ} )]</td>
<td></td>
<td>( a ) [( \text{a} )]</td>
</tr>
</tbody>
</table>

Finnish palatal harmony is relevant because the neutral vowels cannot be distinguished from harmonizing vowels by means of a single, articulatory-based distinctive feature, so the set of participating vowels (\( u \sim ü \), \( o \sim ö \), and \( a \sim ä \)) is an apparent unnatural class, being a disjoint set across features [low] and [round]: unround low vowels and round non-low vowels are harmonizing, so Finnish palatal harmony is not parasitic on any single articulatory feature or any combination thereof. If palatal harmony were parasitic on rounding, then it would preclude the attested cross-rounding interaction. Likewise, if palatal harmony were parasitic on [low], then the attested cross-height interaction would be eliminated. To resolve this dilemma, I will argue based on phonetic studies by Wiik (1965) and Kim (2005) that Finnish back harmony is parasitic on an acoustic correlate common to the harmonic vowels (lower F2), which may be characterized with the acoustic feature [lower F2]. Thus, Finnish palatal harmony shows
that PVH can be directly dependent on distances in a phonetic space, supporting emerging research which argues for a role of phonetic similarity in determining natural classes (Mielke, 2004, 2005), for the availability of phonetic detail in harmony processes (Benus & Gafos, 2006; 2007), and for a more prominent place for acoustic features (Flemming, 1995; Boersma, 1997; Steriade, 2001).

The basic facts of Finnish vowel harmony (Ringen & Orvokki, 1999; Valimaa-Blum, 1999; Kim, 2005; Kiparsky & Pajusalu, 2003) are indicated in (25). These data show that neutral vowels freely combine with other vowels in roots. Otherwise roots tend observe palatal harmony as a MSC. Moreover, the neutral vowels have no effect on the left-to-right harmonization of suffixes.

(25)  

**Fully harmonic roots:**

a. pöüdä-llä ‘table’, adessive  
b. pouda-lla ‘clear weather’, adessive

**Roots with neutral vowels and front vowels:**

c. isä-llä ‘father’, adessive  
d. säde-tta ‘ray’, partitive  
e. tädi-llä ‘aunt’, adessive

**Roots with neutral vowels and back vowels:**

f. kesto-a ‘of duration’  
g. sade-tta ‘rain’, partitive  
h. Kätı-lla (woman’s name), adessive

**Roots with only neutral vowels:**

i. peili-ssä ‘in the mirror’  
j. sii-nä ‘it’, essive
(25)(a-b) confirm that harmony is both cross-height and cross-rounding, as suffix low, unround vowels (a~ä) agree in backness with non-low, round vowels in roots. (25)(c-e) show that i and e are transparent to front harmony; (25)(f-h) show transparency to back harmony. (25)(i-j) indicates that all-neutral roots prefer front suffixes.

The apparent obstacle for a similarity analysis is as follows: less similar vowels (e.g. u~ü and a~ä, which disagree on both height and rounding) are not permitted to disagree on backness, but more similar vowels (e.g. i and a~ä, which only disagree on height) are permitted to disagree on backness (compare (25)(a) to (25)(h)). Put another way, ä is more featurally similar to i than to ü, but ä patterns like less similar ü, not more similar i. It would seem, then, that prerequisite feature similarity based solely in articulation cannot simultaneously account for the neutrality of non-low, front vowels and the participation of ä.

However, a prerequisite similarity solution remains viable if acoustic properties are available for similarity computations. I have previously argued (§2.3.1, §2.3.2) that the motivations for vowel harmony often reside in facilitating the perception of weak contrasts (Flemming, 1995; Walker, 2001, 2005; Finley, 2008), and have argued at some length that the interactions between parasitic and harmonic features in rounding and ATR harmony are predicted by the phonetic similarity of vowel features. Therefore, I consider it a small step to suggest that parasitic interactions may directly compute similarity in terms of the acoustic properties of segments. In the remainder of this section, I argue that Finnish palatal harmony is such a case, specifically that Finnish palatal harmony, while not parasitic on any articulatory feature, is parasitic on location in an acoustic space.
Because the acoustic correlate of backness is the lowering of the spectral peak of the second formant (F2) (due to the lengthening of the front cavity (Stevens, Keyser, & Kawasaki. 1986), it is appropriate to examine the F2 properties of Finnish vowels, which follow in (26).

(26) Average F2 values of long-vowels produced by male Finnish speakers; data from Wiik (1965). Vowels are ordered according to descending F2:

<table>
<thead>
<tr>
<th>Vowel</th>
<th>F2 (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>2495</td>
</tr>
<tr>
<td>e</td>
<td>2240</td>
</tr>
<tr>
<td>ü</td>
<td>1995</td>
</tr>
<tr>
<td>ä</td>
<td>1840</td>
</tr>
<tr>
<td>ö</td>
<td>1805</td>
</tr>
<tr>
<td>a</td>
<td>1240</td>
</tr>
<tr>
<td>o</td>
<td>905</td>
</tr>
<tr>
<td>u</td>
<td>605</td>
</tr>
</tbody>
</table>

Height-F2 interactions among front vowels are well-known (see for instance Stevens, 2000) and are depicted in the standard vowel quadrangle, so it is not surprising to find in (26) that lower, Finnish vowels have a lower average-F2 than otherwise identical higher vowels, e.g. F2(i)=2495 Hz. > F2(e) = 2240 Hz. > F2(ä) = 1840 Hz. However, it is notable that the F2-difference between high and low vowels is greater than the F2-difference due to rounding among front, high vowels: \( \Delta F2(i-ä) = 655 \text{ Hz.} > \Delta F2(i-ü) = 500 \text{ Hz.} \) Therefore, front, unround, low vowels (ä) have a lower average F2 (1840 Hz.) than front, round, high vowels (average F2(ü) = 1995). I hypothesize that because front, low vowels (ä), inhabit a region of F2 space containing other harmonizing vowels,

---

18 The statistical significance of this difference could not be determined with the available data, but suggesting that low, unround vowels are acoustically within the harmonizing region of F2 space does not depend on statistical difference. If the F2 values for ä and ü are not significantly different, then they would be more similar, not less. If the F2 values are significantly different, then the direction is such that ä lies within the harmonizing region.
(i) ä is acoustically similar to round, non-low vowels (ü and ö), and (ii) this acoustic similarity causes ä to behave like ü and ö with respect to palatal harmony.

As the shading in (26) indicates, the distinction between transparent and harmonizing vowels can be easily gleaned from the acoustic properties of the vowels: transparent and harmonizing vowels are linearly separable along a single dimension, F2, with a boundary region at about F2 = 2100 Hz. Thus, even though in terms of articulation there is no available prerequisite feature similarity capable of explaining palatal harmony in Finnish, acoustic similarity can be exploited for the purpose of delineating participants from non-participants. Evidently, an acoustic feature, which may be termed [lower F2] (where [lower F2] corresponds to having a lower spectral peak of the second formant), more aptly characterizes the Finnish vowel inventory for the purposes of harmony as indicated in (27), than the articulatory description previously given in (24).

(27) Revised vowel space for Finnish palatal harmony. Neutral categories are shaded.

<table>
<thead>
<tr>
<th></th>
<th>[−lower F2]</th>
<th>[+lower F2]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Front</td>
<td>Front</td>
</tr>
<tr>
<td>Hi</td>
<td>i</td>
<td>ü [y]</td>
</tr>
<tr>
<td>Mid</td>
<td>e</td>
<td>ö [ø]</td>
</tr>
<tr>
<td>Low</td>
<td>ä [æ]</td>
<td>a</td>
</tr>
</tbody>
</table>

Now, (27) does not say that the articulatory features are unavailable, but rather, some articulatory features (in this case [round]) are not relevant to palatal harmony, while other acoustic features (in this case [lower F2]) are more relevant. In this hybrid acoustic/articulatory space, Finnish palatal harmony receives a straightforward explanation: prerequisite similarity demands that vowels which agree on [lower F2] must
also agree in backness, i.e. Finnish palatal harmony is parasitic on the [lower F2] of vowels.

This parasitic analysis of Finnish palatal harmony predicts (i) the failure of i and e to be targets of palatal harmony (neutrality), and (ii) the failure of i and e to block the harmony of subsequent targets to previous targets, i.e. the transparency of i and e.

Empirically, neutrality is confirmed by (25)(c-h) in which i and e freely combine with both front and back vowels. Transparency is evidenced by (25)(d-e,g-h), which show that neutral vowels allow non-local interaction between root initial vowels and subsequent suffixes. §4.3.4 presents arguments that this Finnish VH data represents an instance of true transparency, as opposed to accounts which allow transparent segments to partially (sub-phonemically) undergo harmony (Gafos, 1996; Ní Chiosáin & Padgett, 2001; Walker, 2003; Gafos & Benus, 2006).

Note that standard inventory markedness constraints which rule out i and ô (back i and back e respectively), only predict the neutrality of i and e, but, like in ATR harmony §2.3.2, inventory markedness has no way of evaluating what should happen once vowels do not participate, i.e. whether or not they are triggers (blocking) or non-triggers (transparency). Prerequisite similarity predicts both neutrality and transparency. Neutrality follows from a failure of i and e to satisfy the prerequisite similarity precondition of [lower F2] agreement when in the context of root-initial u, o, or a. Transparency follows from a failure of subsequent u, o, or a to agree with preceding i and e on [lower F2]. As discussed at more length in Chapter 4, the agreement of suffix vowels to root-initial vowels across fully transparent vowels requires non-local application of harmony constraints. However, for present purposes, the important point
is that the neutral vowels are not triggers of harmony by virtue of the same failure to meet prerequisite similarity that prevents them from being targets of harmony. In this regard, Finnish palatal harmony is on par with LDCA phenomena such as Inseño Chumash(§2.2.1), in which any number of intervening consonants may be neutral and transparent to harmony, because neutrality and transparency are two sides of the same prerequisite similarity coin.

Transparency in both Finnish palatal harmony and LDCA receives a straightforward description via prerequisite feature similarity (parasitic harmony), exactly because prerequisite feature similarity is well-equipped to handle transparency. This derives from the fact that the failure of two segments to meet prerequisite feature similarity does not depend on their status as triggers or targets. Thus, if as in Finnish palatal harmony, *i* and *a* are not prerequisitely similar, then *i* cannot undergo harmony to *a*, and *a* cannot undergo harmony to *i*, giving both the neutrality and transparency of *i* in back domains.

In conclusion, this examination of Finnish palatal harmony suggests that acoustic features must be available to maintain a prerequisite similarity solution. By viewing Finnish palatal harmony as parasitic on [lower F2], it is possible to unify aspects of transparency in both vowel and consonant harmony under a single prerequisite feature similarity solution. A more formal analysis and further discussion of Finnish vowel harmony can be found in §4.3.4.

2.3.4. Vowel height harmony

Unlike other harmonic features, prerequisite similarity plays a limited role in vowel height harmony (VHH), as parasitic VHH is not readily attested. This section
explores why vowel height is exceptional in this regard. In VHH, adjacent vowel sequences which disagree on the features [lo] and [hi] are dispreferred to vowel sequences which match on both [lo] and [hi]. Both raising and lowering are attested; in raising, non-high vowels come to agree with high triggers. In lowering, high vowels come to agree with non-high triggers. VHH is widely found in Bantu languages (Hyman, 1999) and in romance metaphony (Campos-Astorkiza, 2007). VHH is also attested in Buchan Scots English (Paster, 2004), Basque and Woleanian (Parkinson, 1996). With some exceptions, VHH languages tend to lack interior vowels (besides schwa), so there is rarely minimal contrast in rounding in VHH harmony languages. Therefore, the only available features for parasitism are (i) a [back] or [color] feature contrasting front and back non-low vowels, (ii) an [ATR] feature which may be used in inventories with more than three contrasting vowel heights, or (iii) other height features, either [hi] or [lo]. I consider each potential parasitic feature in turn.

Of these features, the literature has given the most attention to understanding the interaction between height and backness (Beckman, 1997; Hyman, 1999; Riggle, 1999; Campos-Astorkiza, 2007; Finley, 2008). The basic cross-linguistic pattern which has been explored is the tendency for /i/ to lower before both /e/ and /o/, but /u/ to only lower before /o/. Current explanations for this front-back asymmetry reside in a markedness preference, which disfavors [Ce.Co] sequences for reasons due to perceptual optimization ([Co.Ce] are not avoided to the same extent). No other strong cross-

---

19 In non-Bantu languages, there are lowering harmonies in which mid vowels lower to agree with low vowels, e.g. Basque (Parkinson, 1996), and raising systems in which low vowels raise to agree with mid vowels, e.g. Aller (Romance) (Campos-Astorkiza, 2007). However, alternating on the feature [low] is typologically much less frequent than harmony alternating on [hi].

20 In standard 5-vowel inventories /i e a o u/ only a 3-way height contrast exists, so two features [hi] and [lo] are sufficient, but for 7-vowel /i e a ɔ o u/ and 9-vowel inventories /i e ɛ a ɔ o ʊ u/ another feature is needed for contrasting the, respective, 4-way and 5-way distinctions in height.
linguistic interactions between height and back emerge, and certainly none approaching parasitism.

Because there is a weak interaction between height and backness in front vowels (as shown in the previous section on Finnish palatal harmony), one might expect there to be instances where back and height develop a strong dependency resulting in parasitic harmony. However, the absence of such a dependency presumably derives from the relative independence of backing and height, when, unlike Finnish, minimal F2 contrasts due to rounding are not available. In other words, in peripheral inventories, the height-F2 interaction is small relative to the over-all F2-contrast between front, unrounded and back, rounded vowels, so the smaller F2-interaction due to height does not result in an acoustic overlap of vowel categories, and, therefore, height is independent from backness for the purposes of VHH.

The magnitude of contrast is also important for understanding the relationship between ATR and VHH. While there are several instances of height-parasitic ATR harmony, there are no attested cases of ATR-parasitic VHH. This asymmetry follows from the ATR contrast being small (both perceptually and articulatorally) relative to major vowel height. Where both height and ATR are contrastive, it is the ATR contrast that results in more acoustic category overlap, so through height-parasitic ATR harmony, some overlapping categories are avoided. However, ATR-parasitic VHH holds no such advantage, since F1-overlapping categories would not be eliminated and harmony would only occur over vowels which already contrast readily along F1. For example, in a hypothetical ATR-parasitic VHH, the F1-overlapping sequence [Cɛ.Ce] would not satisfy the ATR precondition, so no harmony would obtain, but already F1-contrasting vowels
[Čɛ.Cɛ] would neutralize to [Čɛ.Cɛ]. The absence of ATR-parasitic VHH supports a typological generalization in which smaller contrasts tend to be parasitic on larger contrasts, and not vice versa. This generalization is supported by the existence of LDCA systems in which agreement on minor place of articulation is parasitic on major place agreement, but no systems in which major place agreement is prerequisite on minor place agreement (Hansson, 2001).

Unlike [back] and [ATR], there are marginal cases in which [hi] harmony is parasitic on [-lo], where low vowels are neither triggers nor targets of harmony. For example, consider the Shona (Bantu) (Beckman, 1997; Riggle, 1999; Hyman, 1999) data below in (28). Relative to (28)(a), (28)(b) shows productive lowering of /i/ to /e/ when preceded by a mid vowel, but (28)(c) shows that low /a/ does not trigger harmony.

(28) Shona VHH harmony (Beckman, 1997)

<table>
<thead>
<tr>
<th>Root + applicative</th>
<th>Root</th>
</tr>
</thead>
<tbody>
<tr>
<td>‘be evil’</td>
<td>ip-ira ‘be evil for’</td>
</tr>
<tr>
<td>‘sew’</td>
<td>son-era ‘sew for’</td>
</tr>
<tr>
<td>‘itch’</td>
<td>vav-era ‘itch at’</td>
</tr>
</tbody>
</table>

There are separate cases (e.g. Basque; Parkinson, 1996) where [lo] harmony is parasitic on [−hi], but the situation is the same: it is common in VHH harmony for one extreme of the height continuum to be non-participatory and the other extreme to be the only possible targets. The lack of participation by one extreme could follow from harmony along one height feature being parasitic on another height feature. For Shona, non-participating /a/ could follow from [−lo]-parasitic height harmony.
However, such cases are marginal instance of PVH because there are no cases of fully-parasitic VHH–where [+F] segments only trigger harmony in other [+F] segments and [−F] segments only trigger harmony in other [−F] segments. The lack of fully parasitic VHH derives from the descriptive asymmetry in the feature system, whereby vowels can be [+hi] or [+lo] but not both, i.e. it is impossible to have a non-vacuous system where vowels which already agree on [αlo] must agree on [βhi] because [+lo] vowels cannot disagree on [hi].

In sum, while prerequisite similarity is never violated in height harmony, it plays a more limited role than in other vowel harmonies where both fully-parasitic systems and larger contrasts are available. However, understanding why VHH lacks robust parasitism confirms the role of phonetic similarity as the underlying motivation for parasitic vowel harmony. Specifically, VHH highlights the importance of relative contrast as (i) height-F2 interactions available with the minimal-F2 contrasts due to rounding are not available with the larger-F2 contrasts due to backing and (ii) while both ATR and height affect F1, only the smaller F1 contrast (ATR) may ever be parasitic on the larger contrast (height).

2.3.5. Conclusion

This section has presented empirical data from parasitic harmony, which confirms that prerequisite similarity is typologically useful in areas besides LDCA. At least in terms of sensitivity to feature-similarity, parasitic harmony seems to parallel LDCA, accepting a range of feature-similarity preconditions to harmony, which determine the trigger-target relationship. However, these preconditions are not arbitrary, being heavily dependent on the phonetic similarity of harmonic and parasitic features. This section has

- 59 -
also previewed how prerequisite similarity is crucial for understanding the transparency of interveners.

One important goal of the present work is to understand why phonetic similarity yields parasitic dependency. I will show that parasitic dependency is directly derivable from a computational framework that is sensitive to low-level phonetic similarity. This framework operates on principles of attraction (Burzio, 2002a, b, 2004, 2005; Burzio & Tantalou, 2007; Wayment, Burzio, Mathis, & Frank, 2008), and this low-level similarity can drive the strengths of attraction relations between features. Features which have phonetically similar correlates are, therefore, under pressure to cluster together. More specific discussion on the formal implementation of phonetic dependency can be found in §3.4, §4.3, and §4.4.

### 2.4. Similarity effects in strictly local assimilations.

This dissertation hypothesizes that prerequisite similarity drives assimilation, and, therefore, similarity effects ought to be found in even strictly local assimilations. However, both Rose & Walker (2004) and Hansson (2001) suggest that similarity preconditions are relatively unique to long-distance consonant agreement (LDCA, see §2.2). For instance: “The similarity requirement on agreeing segments in LDCA is not systematically found in other kinds of assimilatory systems involving consonants, and this sets LDCA apart” (Rose & Walker, p. 492).

But as first hinted at by (Burzio, 2005), parasitic assimilations also exist for exclusively local assimilations. This section presents data which illustrates that similarity effects are indeed found in contexts where trigger and target are required to be adjacent in the surface form, confirming that similarity preconditions have broader application than
previously considered.\textsuperscript{21} If similarity requirements are found at both extremes of locality (surface adjacency and long-distance interaction), then the phonological system must be fundamentally sensitive to the similarity of segments.

As the table in (29) previews, these similarity-sensitive, local assimilations have a wide range of harmonic and parasitic features, involving a number of place, stricture, and laryngeal features. Note that many of these examples constitute assimilatory gemination processes (e.g. Hayes, 1986), by which a segment undergoes complete neutralization to another (usually subsequent) segment. Because these identical segments are adjacent, they must undergo structural coalescence in order to avoid violations of the obligatory contour principle (OCP; McCarthy, 1986).\textsuperscript{22} For the moment, the concern is not to explain why some contexts result in coalescence and others only partial assimilation, but rather to explore the range of similarity preconditions that exist on assimilatory repairs, no matter the ultimate nature of the repair.

(29) \textit{Strictly local parasitic assimilations:}

\begin{tabular}{|c|c|c|c|}
\hline
\textbf{Section} & \textbf{Language} & \textbf{Harmonic Features} & \textbf{Prerequisite Features} \\
\hline
2.4.1 & Sudanese Arabic & Continuant & Major consonantal place \\
2.4.2 & Castilian Spanish & Approximate & Voicing \\
2.4.3 & Sanskrit & Minor coronal place & Coronal, Continuant \\
2.4.4 & Maltese & Continuant, Nasal, Voice & Coronal \\
2.4.5 & Turkish & Vowel Height & Round, Back \\
2.4.6 & Catalan & Minor labial place & Labial, Consonantal \\
2.4.7 & English & Nasal place & Non-continuant \\
2.4.8 & Italian & Lateral, Nasal & Sonorant \\
\hline
\end{tabular}

\begin{footnotesize}
\textsuperscript{21} Such strictly local assimilations are not formally excluded by frameworks which incorporate locality into correspondence constraints. Both Rose & Walker (2004) and Hansson (2001) allow for such interaction, so this section’s cases of assimilation constitute empirical, not formal, counter-examples.

\textsuperscript{22} While the repairs are certainly assimilatory, it is possible that the OCP—rather than agreement per se—is, in fact, the driver for harmony. Similarity effects are robustly attested in OCP phenomena (Frisch, Pierrehumbert, & Broe, 2004), so it would not be surprising to find them in OCP-driven gemination. The relationship between the OCP and assimilation is not taken up in earnest in the present work. Although see \textbf{Chapter 6} for suggestions that the OCP and assimilation reduce to the same principles of repairing pressures of similar elements to be even more alike.
\end{footnotesize}
Below, I briefly consider each case of strictly local assimilation as outlined in (29). The last two examples, English and Italian, will provide an opportunity to preview part of the formal implementation of prerequisite similarity discussed in more detail in Chapter 3.

2.4.1. Spirantization in Sudanese Arabic

In Sudanese Arabic (Kenstowicz, 1994; refs therein), word final stops assimilate to subsequent fricatives. This assimilation is subject to a homorganicity precondition, and is illustrated in the possessive context shown below in (30) (Fáthi, Samúr, Šarí, Xáalid, and Ḥásan are the possessing nouns). Instances of spirantization are indicated by shaded cells.

<table>
<thead>
<tr>
<th>Context</th>
<th>Target</th>
<th>Labial final</th>
<th>Coronal final</th>
<th>Velar final</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>kitáab ‘book’</td>
<td>bit ‘daughter’</td>
<td>sámak ‘fish’</td>
<td></td>
</tr>
<tr>
<td>Labial</td>
<td>kitáaf Fáthi</td>
<td>bit[t] Fáthi</td>
<td>sáma[k] Fáthi</td>
<td></td>
</tr>
<tr>
<td>Alveolar</td>
<td>kitáap Samúr</td>
<td>bit[s] Samúr</td>
<td>sáma[k] Samúr</td>
<td></td>
</tr>
<tr>
<td>Post-alveolar</td>
<td>kitáap Šarí</td>
<td>bit[ʃ] Šarí</td>
<td>sáma[k] Šarí</td>
<td></td>
</tr>
<tr>
<td>Velar</td>
<td>kitáap Xáalid</td>
<td>bit[t] Xáalid</td>
<td>sáma[ʃ] Xáalid</td>
<td></td>
</tr>
<tr>
<td>Glottal</td>
<td>kitáap[ʃ] Ḥásan</td>
<td>bit[t] Ḥásan</td>
<td>sáma[k] Ḥásan</td>
<td></td>
</tr>
</tbody>
</table>

Clearly only word-final segments that already agree in major place with subsequent segments assimilate in continuancy. This would seem to parallel examples of LDCA with homorganic preconditions (see Ngbaka in §2.2.2), except that the assimilation only occurs in strictly local contexts. Longer distance neutralization, where coalescence is not available, is prohibited by the common Arabic OCP.

2.4.2. Word-medial spirantization in Castilian

Inter-vocalic spirantization in Castilian Spanish (Burzio, 2005) provides another clear example of local, parasitic assimilation. In some languages, e.g. Florentine Italian (Dalcher, 2006), both voiced and voiceless stops spirantize, but in Castilian, voicing is
prerequisite to spirantization: voiced stops spirantize, (31)(a), while unvoiced stops do not spirantize, (31)(b):

(31)

  a. Cu[β]a ‘Cuba’
  b. co[p]a, *co[ʃ]a ‘cup’

Spirantization is understood as assimilation to a preceding approximant or vowel (Shosted and Willgohs, 2006), but because voiced stops and their approximant counterparts are in complementary distribution, the following argument is needed to confirm that this effect is exclusively local: Martínez-Celdrán (2004) reports that voiced stops do not spirantize following nasals. Shosted and Willgoh’s (2006) air-flow study shows that nasals do not spirantize in intervocalic position, but instead pattern like voiceless stops. Since nasals do not participate in spirantization, the failure of a stop to spirantize following a nasal can be understood as a blocking of the spreading of some spirantizing feature from a preceding approximant or vowel. Exactly which feature spreads depends on the available Feature Geometry (Clements, 1985), but it is likely an oral stricture feature such as [+approximant] carried by both vowels and liquids, but not nasals. Strict adjacency is enforced thusly: when an approximant source is immediately followed by a voiced stop, prerequisite similarity is satisfied and the stop spirantizes, but as in VNC_{[+voice]} sequences, if even one non-approximant intervenes between the source and the voiced stop, there is no alternation. Spanish spirantization is, thus, an exclusively local assimilation with a precondition based on similarity: approximate harmony is parasitic on voicing.
The phonetic reasons for this dependency may be related to the relative short duration of voiced stops, as compared to unvoiced stops: the compressed time scale for voiced stops leads to greater articulatory undershoot, and therefore incomplete closure, resulting in frication (see Shosted and Willgohs, 2006). That the dependency between voicing and approximant has a basis in a durational phonetic space is consistent with how similarity preconditions in parasitic vowel harmony may refer to phonetics. On the other hand, phonetics will be insufficient for all the cases of strictly local assimilation, but prerequisite feature similarity is sufficient.

2.4.3. Sanskrit: minor coronal place assimilation in external sandhi

Sanskrit has a notorious external sandhi process, whereby word final /s/ undergoes assimilation to subsequent coronals, and ‘elsewhere’ emerges as /h/ (Kiparsky, 1973). This is presented as the rule in (32) below (based on Kenstowicz, 1994).

(32)  a. /s/→[s]/__#t
     /s/→[ʂ]/__#ʂ
     /s/→[ɕ]/__#ɕ

   b. /s/→[h], otherwise

The ‘elsewhere’ condition and its formal implications have received most of the attention related to this process, but, here it is worth noting the prerequisite similarity that unifies the alternating patterns (32)(a) and separates them from the elsewhere condition (32)(b). Word-final /s/ is coronal; when followed by other coronals, /s/ undergoes minor place of articulation assimilation, coming to agree in alveolar, retroflex, or palatal with subsequent coronals, otherwise the rule for the elsewhere condition applies. Thus, word-final /s/ assimilation in Sanskrit is parasitic on coronal, and feature similarity naturally delineates
the harmonizing contexts (coronal) from those where the elsewhere condition applies (non-coronal).

2.4.4. /il/ prefixation in Maltese

In Maltese, the definite article for consonant initial noun phrases is generally indicated with /il/ (Cipollone, 2001, p. 101) as seen in (33)(a) below. (33)(b), however, shows that when affixed to a coronal initial root, the /l/ in the prefix neutralizes to the subsequent coronal consonant, coming to agree in voicing, nasality, and continuancy.

(33) a. non-coronal context b. coronal context

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>il-fellus</td>
<td>‘the chicken’</td>
<td>it-tiin</td>
</tr>
<tr>
<td>il-mara</td>
<td>‘the woman’</td>
<td>id-dawl</td>
</tr>
<tr>
<td>il-kelb</td>
<td>‘the dog’</td>
<td>is-shab</td>
</tr>
<tr>
<td>il-?attus</td>
<td>‘the cat’</td>
<td>in-natura</td>
</tr>
<tr>
<td>il-?itan</td>
<td>‘the walls’</td>
<td></td>
</tr>
</tbody>
</table>

Here, like Sanskrit, prerequisite similarity naturally distinguishes assimilating from non-assimilating contexts by placing a precondition of agreement on coronal on any further assimilation.

2.4.5. Turkish condition on coalescence for vowel hiatus resolution

Probably because of the marked nature of onsetless syllables, instances of strictly adjacent vowel interaction are somewhat rarer than local consonant interaction, and those adjacent vowel interactions, which are sensitive to similarity conditions, are even rarer. However, a case of vowel hiatus resolution in Turkish (Kabak, 2007) provides evidence that similarity can also act as a precondition to strictly local vowel assimilations. There are a number of conditions on this interaction that are not all based on similarity, but the
basic generalization is that in a derived environment, an optional coalescence repair to vowel hiatus is preconditioned on agreement on both rounding and backing (compare (34)(a) vs. (34)(b)). The facts are summarized in (34) (data from Kabak, 2007):

(34)

a. **V2 assimilates to V1 following medial consonant deletion:**

<table>
<thead>
<tr>
<th>Word</th>
<th>V2</th>
<th>V1</th>
<th>Type</th>
<th>Vowel</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>ağır →</td>
<td>[air]</td>
<td>~ [aar]</td>
<td>* [iir]</td>
<td>‘heavy’</td>
<td></td>
</tr>
<tr>
<td>yoğurt →</td>
<td>[yourt]</td>
<td>~ [yoort]</td>
<td>*[yuurt]</td>
<td>‘yogurt’</td>
<td></td>
</tr>
<tr>
<td>öğür →</td>
<td>[öür]</td>
<td>~ [öör]</td>
<td>*[üürt]</td>
<td>‘to retch’</td>
<td></td>
</tr>
</tbody>
</table>

b. **No alternation unless V1 and V2 agree on rounding and backness:**

<table>
<thead>
<tr>
<th>Word</th>
<th>V2</th>
<th>V1</th>
<th>Type</th>
<th>Vowel</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>tavuk →</td>
<td>[tauk]</td>
<td>~ *[taak]</td>
<td>*[taok]</td>
<td>‘chicken’</td>
<td></td>
</tr>
<tr>
<td>döviz →</td>
<td>[döiz]</td>
<td>~ *[dööz]</td>
<td>*[döez]</td>
<td>‘foreign currency’</td>
<td></td>
</tr>
<tr>
<td>sair →</td>
<td>[şair]</td>
<td>~ *[şaar]</td>
<td>*[şaer]</td>
<td>‘poet’</td>
<td></td>
</tr>
<tr>
<td>soğan →</td>
<td>[soan]</td>
<td>~ *[soon]</td>
<td>*[saan]</td>
<td>‘onion’</td>
<td></td>
</tr>
<tr>
<td>süüt →</td>
<td>[süüt]</td>
<td>~ *[süüt]</td>
<td>*[siit]</td>
<td>‘suite’</td>
<td></td>
</tr>
</tbody>
</table>

c. **V2 must be high:**

<table>
<thead>
<tr>
<th>Word</th>
<th>V2</th>
<th>V1</th>
<th>Type</th>
<th>Vowel</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>siğar →</td>
<td>[siar]</td>
<td>~ *[siir]</td>
<td>*[saar]</td>
<td>‘fits’</td>
<td></td>
</tr>
<tr>
<td>.cgiğer →</td>
<td>[cier]</td>
<td>~ *[ciir]</td>
<td>*[ceer]</td>
<td>‘liver’</td>
<td></td>
</tr>
</tbody>
</table>

d. **Sequence [e.i] does not alternate:**

<table>
<thead>
<tr>
<th>Word</th>
<th>V2</th>
<th>V1</th>
<th>Type</th>
<th>Vowel</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>beyit →</td>
<td>[beit]</td>
<td>~ *[beet]</td>
<td>*[biit]</td>
<td>‘couplet’</td>
<td></td>
</tr>
<tr>
<td>nehir →</td>
<td>[near]</td>
<td>~ *[neer]</td>
<td>*[niir]</td>
<td>‘river’</td>
<td></td>
</tr>
</tbody>
</table>

While the exceptional nature of [e.i] sequences, (34)(d), is apparently idiosyncratic, the height condition in (34)(c) follows the basic rules of Turkish vowel harmony discussed in §2.3.1, but unlike normal, productive rounding VH, hiatus resolution results in lowering.
The repairs in (34)(a) may, thus, be termed vowel height assimilation. I have already shown how Turkish rounding VH is parasitic on backness, and apparently height harmony (for hiatus resolution) adds a further precondition of rounding agreement, otherwise the forms in (34)(b) would alternate. Though height interacts with other aspects of harmony, there is no evidence of non-adjacent vowel height harmony in Turkish, so (34) exemplifies a case of strictly local vowel assimilation sensitive to prerequisite feature similarity. A formal analysis of this data is presented in §4.3.3.

2.4.6. Catalan nasal place agreement

In continental Catalan (Wheeler, 2005), underlying /m/ emerges completely faithfully in NC clusters, except when followed by labiodentals /l/ or /v/. This contrasts with the Majorica dialect of Catalan (Wheeler, 2005) where NC clusters never disagree in place. Consider the contrasting data in (35); all of the bolded segments are orthographically represented as ‘m’:

(35)

<table>
<thead>
<tr>
<th></th>
<th>Continental Catalan</th>
<th>Majorica Catalan</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.  so[ɲ] feliçcos</td>
<td>‘we are happy’</td>
<td>so[ɲ] vint</td>
</tr>
<tr>
<td></td>
<td>‘there are twenty of us’</td>
<td></td>
</tr>
<tr>
<td>b.  pre[m]sa</td>
<td>‘press’</td>
<td>pre[n]sa</td>
</tr>
<tr>
<td></td>
<td></td>
<td>‘press’</td>
</tr>
</tbody>
</table>

(35)(a) shows that both continental and Majorica varieties allow minor place of assimilation to other labials: /m/→[ɲ]/__C[labial, labiodental], but as (35)(b) shows, only Majorica allows major place of articulation assimilation /m/→[n]/__C[coronal].

This contrast is best explained as a difference in similarity preconditions. Like Sanskrit final-/s/ assimilation, continental Catalan requires major place agreement before minor place agreement takes place. Majorica has no such homorganicity requirement, so
trigger and target only need be adjacent NC sequences, no further preconditions are placed on assimilation. This contrast in homorganic preconditions mirrors the contrast between Ngbaka and Kinande LDCA (discussed in §2.2.2), where a homorganicity precondition was found on nasal agreement in Ngbaka, but not Kinande. Therefore, the Catalan data show that the kinds of contrasts in similarity requirements found in LDCA are also attested in strictly local assimilations.

2.4.7. /\textit{in/} prefixation in English

As the data in (36) shows, English nasal place assimilation (Baković, 2007; refs. therein) differs from Catalan in that nasals in standard, careful speech do not assimilate to fricatives (labiodental or otherwise). This is illustrated with the prefix \textit{in-}:

(36) \textit{English nasal place assimilation:}

\begin{itemize}
\item a. inapplicable [\textit{\textipa{1n}}] \textit{vowel context}
\item b. impossible [\textit{\textipa{1m}}] \textit{labial stop context}
\item intolerable [\textit{\textipa{1n}}] \textit{coronal stop context}
\item inconceivable [\textit{\textipa{1n}}] \textit{velar stop context}
\item c. insurmountable [\textit{\textipa{1n}}] \textit{coronal fricative context}
\item infallible [\textit{\textipa{1n}}] \textit{labiodental fricative context}
\item inhospitable [\textit{\textipa{1n}}] \textit{glottal fricative context}
\end{itemize}

(36)(a) shows that the default realization is [\textit{\textipa{1n}}]; (36)(b-c) shows that underlying /\textit{n/} assimilates to subsequent stops, but not to subsequent fricatives. Here, prerequisite

---

23 In casual speech, English allows minor place of assimilation to subsequent continuants. Thus, \textit{\textipa{te[\textipa{\textipa{n}}]th}}, \textit{\textipa{[\textipa{n}] your dreams}}, and \textit{\textipa{ar[\textipa{m}]ful}} are not uncommon. However, in careful speech, where gestural blending (Browman & Goldstein, 1992) is less likely to occur, place assimilation to subsequent fricatives is unlikely, especially across major places of articulation. For example, in my dialect of Western American English, \textit{\textipa{i[m]fallible}} or \textit{\textipa{i[m]fallible}} are marked as a citation form for \textit{infallible}.
similarity is active as non-continuant nasals only assimilate to other non-continuant obstruents.

Although independently developed, Baković (2007) parallels some of the formalism this dissertation will build on for analyzing prerequisite similarity. The present work derives from Burzio’s (2002a, b, 2005) earlier proposal of representational entailment (see Chapter 3 for more details), but Baković seems to have arrived at the same conclusion, namely: some conditions on assimilation are best characterized by directly incorporating dependencies between features into harmony constraints. This dissertation argues that the dependency between features is based in phonetic similarity, and additionally, that such dependencies are robustly available as a general recipe for determining the feature preconditions on assimilation.

In order to account for the non-interaction between nasals and fricatives in nasal place assimilation in American English (and other languages), Baković proposes the following in (37):

\[
\text{(37) \ \text{STR/PL (PL→STR):} \quad \text{(Baković, 2007)}}
\]

Adjacent output segments that have the same place feature value must also have the same value of the stricture feature [±cont].

This constraint penalizes [ɱf] sequences because the segments are both labial, but only /f/ is continuant. To make clear the direction of dependency, I will rename the constraint with an arrow. Thus, Baković’s STR/PL may be renamed PL→STR because according to the formalization in (37), stricture agreement depends on place agreement.

---

24 Dependency in phonology has traditionally been handled in representations, as characterized by of Ni Chiosáin & Padgett’s (2001) Feature Class theory, earlier Feature Geometry (Clements & Hume, 1995), and others. Bakovic moves the dependencies from representations to explicit constraints, and I will follow suit.
PL→STR can be used to block general place agreement, which would otherwise prefer NC_{±cont} sequences to agree in place, and can be used in tandem with a NONASALFRICATIVE constraint to produce the correct output, for the input /nf/, as shown in the tableau in (38):

<table>
<thead>
<tr>
<th>/nf/ →[nf]</th>
<th>NONASALFRIC</th>
<th>PL→STR</th>
<th>AGREE(place)</th>
<th>IDENT(place)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. [ñf]</td>
<td></td>
<td>*!</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>b. [nf]</td>
<td></td>
<td></td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>c. [ nf]</td>
<td>*!</td>
<td></td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

The crucial point is that PL→STR enforces a feature-similarity precondition: segments which agree in place, like (38)(a), [ñf], must also agree in [±cont.]; segments which do not agree in place, like (38)(b), [nf], are under no such pressure to agree in [±cont.]. The facts of English can, thus, be analyzed as making use of place-parasitic continuancy harmony to block general place assimilation.

Now, for a number of reasons, the proposed constraints which implement prerequisite feature similarity in Chapter 3 are different form PL→STR, but they both enforce a context sensitive pressure for agreement, where the context is determined by the similarity of trigger and target. For English, [ñf] sequences are similar enough (because they share place) to activate PL→STR, whereas [nf] sequences are not.

As a further illustrative example, this notion of “similar enough”, or sufficient similarity, allows for a direct reformulation of English place agreement in terms of prerequisite similarity without the need for any blocking constraints. If the constraints PL→STR and AGREE(place) are replaced with STR→PL, as formulated in (39), then nasal place assimilation may be directly derived without blocking by other constraints.
(39) $\text{STR} \rightarrow \text{PL}$:
Adjacent output segments that have the same stricture specification ($[{\pm \text{cont.}}]$) must also have the same specification for place features.

$\text{STR} \rightarrow \text{PL}$ differs from $\text{PL} \rightarrow \text{STR}$ in terms of the direction of dependency, i.e. the parasitic and harmonic features are switched. While both constraints are likely at work, and here both can be used to achieve the same ends, $\text{STR} \rightarrow \text{PL}$ is truer to the descriptive way in which I have viewed prerequisite similarity in English place assimilation: the dependent feature is the harmony feature and the antecedent feature is the parasitic, or prerequisite feature. English place agreement is more straightforwardly described as dependent on continuancy agreement, so I prefer $\text{STR} \rightarrow \text{PL}$, which can be used to give the attested non-interaction between nasals and fricatives, as shown in (40), without any need for a blocking constraint to general agreement:

(40)

<table>
<thead>
<tr>
<th></th>
<th>/nfl/$\rightarrow$[nf]</th>
<th>NONASALFRIC</th>
<th>$\text{STR} \rightarrow \text{PL}$</th>
<th>IDENT(place)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>[nfl]</td>
<td></td>
<td>$\text{STR} \rightarrow \text{PL}$</td>
<td>!*</td>
</tr>
<tr>
<td>b.</td>
<td>[nf]</td>
<td></td>
<td>$\text{STR} \rightarrow \text{PL}$</td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td>![f]</td>
<td>*!</td>
<td>$\text{STR} \rightarrow \text{PL}$</td>
<td>*</td>
</tr>
</tbody>
</table>

Here, nasal fricative sequences do not activate the $\text{STR} \rightarrow \text{PL}$ agreement constraint, enforcing the same preference as blocking general agreement with $\text{PL} \rightarrow \text{STR}$ in (38). Additionally, $\text{STR} \rightarrow \text{PL}$ gives standard nasal place assimilation to non-continuants:

---

25 There are, of course, reasons to believe that general agreement constraints are active in other languages and other parts of English. However, as this survey and the other examples in Chapter 4 confirm, harmony with feature similarity preconditions seems to be more the norm than harmony without feature similarity preconditions.
Therefore, [np], (41)(a), is similar enough with respect to continuancy to activate 
STR→PL, whereas [nf], (40)(b) is not. In sum, this section previews how by directly 
encoding feature dependency into the constraints it is possible to use ranking in 
Optimality Theory to set the threshold of similarity which activates agreement constrains, 
thereby, capturing prerequisite similarity effects.

2.4.8. /in/ lateral agreement in Italian and English

The Italian prefix in- assimilates before non-continuants, as in English (shown by 
the cognates in (42)(a)). Additionally, in Italian and to some extent in English, in- is 
realized as [il] before /l/-initial stems and [ir] before r-initial stems, as shown in (42)(b).

(42) a. [im]possibile ‘impossible’
[in]fallibile ‘infallible’
[in]tollerabile ‘intolerable’

b. [ir]razionale, *[in]razionale ‘irrational’
[il]legale, *[in]legale ‘illegal’

The most basic description is that besides a constraint STR→PL, there is also a 
requirement that adjacent consonants which share sonority agree in nasality, liquid, and 
lateral, i.e. nasal, liquid, and lateral agreement are each dependent on sonorant agreement. 
This can be formalized with the constraints in (43):
(43) a. \( \text{SON} \rightarrow \text{NAS} \):

Adjacent output segments that have the same sonority specification
([±son.]) must also have the same specification for nasality [±nasal].

b. \( \text{SON} \rightarrow \text{LIQ} \):

Adjacent output segments that have the same sonority specification
([±son.]) must also have the same specification for [±liquid].

c. \( \text{SON} \rightarrow \text{LAT} \):

Adjacent output segments that have the same sonority specification
([±son.]) must also have the same specification for [±lateral].

These constraints capture the dependencies necessary to give rise to full neutralization of
nasals when followed by liquids (see §4.5 for how cases of total neutralization
traditionally accounted for with node spreading in a feature geometry can be recast as
similarity relations). As the tableau below in (44) and (45) show, each of these
constraints is necessary because they rule out a unique candidate that would otherwise
allow for partial neutralization to the liquid.

(44)

<table>
<thead>
<tr>
<th></th>
<th>[nl]→[ll]</th>
<th>( \text{SON} \rightarrow \text{LIQ} )</th>
<th>( \text{SON} \rightarrow \text{LAT} )</th>
<th>( \text{SON} \rightarrow \text{NAS} )</th>
<th>( \text{IDENT}([\text{liq.}], [\text{lat.}], [\text{nas}.]) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>[nl]</td>
<td>**</td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>[ll]</td>
<td></td>
<td>*</td>
<td>**</td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td>[rl]</td>
<td></td>
<td>*</td>
<td>**</td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td>[d_{[+\text{son}l]}]</td>
<td>**</td>
<td></td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>e.</td>
<td>[ll]</td>
<td></td>
<td></td>
<td>**</td>
<td></td>
</tr>
</tbody>
</table>

(45)

<table>
<thead>
<tr>
<th></th>
<th>[nr]→[rr]</th>
<th>( \text{SON} \rightarrow \text{LIQ} )</th>
<th>( \text{SON} \rightarrow \text{LAT} )</th>
<th>( \text{SON} \rightarrow \text{NAS} )</th>
<th>( \text{IDENT}([\text{liq.}], [\text{lat.}], [\text{nas}.]) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>[nr]</td>
<td>**</td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>[rr]</td>
<td></td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td>[lr]</td>
<td></td>
<td>*</td>
<td>***</td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td>[d_{[+\text{son}r]}]</td>
<td>**</td>
<td></td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>e.</td>
<td>[rr]</td>
<td></td>
<td></td>
<td>**</td>
<td></td>
</tr>
</tbody>
</table>
(44) and (45) only differ in the lateral value of the input. \([d_{f+son}]\) in (44)(d) and (45)(d) is intended to indicate a non-nasal, non-liquid sonorant, the phonetic realization of which is, of course, ruled out by independent markedness constraints against oral-sonorant obstruents (not indicated in the tableau) but also eliminated by the dependency \(\text{SON} \rightarrow \text{LIQ}\) because adjacent \([d_{f+son}]\) agree on [sonorant], but not [liquid].

Nasal-liquid sequences are dispreferred throughout the English and Italian lexica, but especially within “Level 1” morphology to which the prefix \(\text{in-}\) belongs. However, a number of nasal-liquid sequences arise through “Level 2” affixation. For example, in English: ‘timeless’ \([\text{taim.les}]\), ‘thinly’ \([\text{θin.li}]\), etc. Furthermore, \(\text{in-}\) does not readily attach to glide initial words in English, the L2 prefix \(\text{un-}\) applying instead, e.g.: \(\text{unworkable}, \ast \text{inworkable} \text{ and unyielding}, \ast \text{inyieding}\). Thus, a full account of \(\text{in-}\) prefixation would require the constraints in (43) to have a lower rank (relative to faithfulness; see Burzio, 2002a, b, 2004, 2005) for Level 2 morphology, and a separate account of the ineffability of \(\text{in-+}\) glide combinations.

These details aside, this simplified case shows how multiple prerequisite similarity conditions may be instantiated in separate dependency constraints, working in tandem to result in total neutralization in a case of strictly local assimilation sensitive to similarity. Agreeing on the feature sonorant is enough to activate all of nasal, liquid, and lateral agreement. The sufficient similarity for the activation on \(\text{in-}\) prefixation is \([\text{continuant}]\) agreement for nasal-stop assimilation and [sonorant] agreement for nasal-liquid assimilation. Thus, even within the alternation of a single morpheme, prerequisite similarity is playing a diverse and essential role.
2.5. Conclusion

This chapter explored a wide range of assimilatory phenomena from consonant harmony, vowel harmony, and local assimilations, and, without exception, has shown that similarity may act as a precondition to assimilation. Strong parallels were found between LDCA, parasitic vowel harmony, and a number of strictly local assimilations, demonstrating that languages vary greatly in the way they form dependencies between harmonic and parasitic features, but these dependencies generally exploit the articulatory and acoustic similarity of the participating features. Because of this abundance, similarity is of central, not peripheral, importance to a formal account of assimilation, and therefore, this notion is presumably central to phonology.

I have argued at length – and sketched analyses of English and Italian in-prefixation (§2.4.7-2.4.8) – that it is essential for constraints to be able to express the notion of sufficient similarity. All of this chapter’s examples, perhaps most notably the cases of parasitic vowel harmony discussed in §2.3, are preconditioned on triggers and targets being similar enough to activate harmony, while in other contexts, triggers are not similar enough to potential undergoers to activate agreement. A flexible threshold for similarity can, thus, serve to define the trigger-target pairs which allow harmony to obtain, thereby, predicting the class of neutral segments. However, similarity cut-offs can only be employed as an effective tool for analyzing assimilation if language-specific similarity is allowed. Nonetheless, the set of available dependencies is grounded in the phonetic properties that define the universal articulatory/acoustic space in which segments are realized. The remainder of this dissertation formalizes these principles and expands on these similarity themes.
3. Formalizing ATTRACTION

**Chapter overview:** This chapter formally presents the basic properties of ATTRACTION constraints and illustrates that ATTRACTION naturally predicts and accounts for prerequisite feature similarity effects in assimilation by implementing the notion of representational distance. Attraction is evaluated at multiple levels including low-level harmony spaces, Harmonic Grammar, and Optimality Theory. Finally, a number of restrictions are proposed to ensure a restrictive typology and formal equality across these levels.

3.1. **Introduction**

The focus of the previous chapter was prerequisite similarity; the focus of this chapter is attraction. The mediating concept that spans both similarity and attraction is a notion of representational distance. Representational distance argues for a more subsymbolic notion of computation than is traditionally available in theoretical linguistics (Burzio, 2002a, b, 2004, 2005; Burzio & Tantalu, 2007). This chapter explores how representational distance can be expressed at multiple levels of explanation. Using multiple levels of explanation is a general method of scientific exploration in cognitive science (cf. Marr’s ‘Levels of Analysis’ (1982) for visual processing). Smolensky & Legendre (2006) argue that among the benefits of multiple levels of explanation—in linguistics and elsewhere—is that the levels can be mutually informing.
A higher, more abstract *phonological level* can describe important generalizations about phonological inventories, the conditions on allomorphy, and the phonotactic distribution of segments. These generalizations are very difficult to find when working directly with more detailed connectionist descriptions that include units, activation spreading, and distributed representations. However, the lower *connectionist level* can have profound impact on the kinds of computations available which drive the generalizations of the abstract level. For example, there are optimization processes which are known to exist in connectionist networks. These harmony maximizing networks (Smolensky, 1986) inspired a numerical kind of grammar, *Harmonic Grammar* (HG; Legendre *et al.*, 1990; Smolensky & Legendre, 2006), where constraints are weighted and the grammar produces the ‘optimal’ output with respect to these weights. HG evolved into *Optimality Theory* (OT; Prince & Smolensky, 2004), which provides a general non-numerical way of performing similar optimization computations over constraints.

This chapter introduces the Attraction Framework, which provides the basis for the analysis of parasitic harmony and connectionist simulations which follow in subsequent chapters. This chapter pursues the strategy of multiple-levels of explanation in order to understand attraction. In particular, I focus on the relationship between the OT-level, the HG-level, and a numerical similarity space, postponing most issues related to connectionist realization until Chapter 5. For a family of proposed *Attraction* constraints, I formally demonstrate a tight coupling between HG and OT, and describe conditions under which the two grammatical systems are identical. This ensures a way to transfer the generalizations of the phonological level (e.g. typological distributions) to
connectionist implementation and a way for connectionist level principles of computation (e.g. additivity, distance, and similarity) to be imported into Optimality Theory.

Another main conclusion along these lines is that while the OT-level can express and grammatically implement generalizations related to parasitism and distance, these generalizations are more aptly described by a more subsymbolic kind of representational description. That is, without the lower-level grounding of a subfeatural level engendered by connectionism, the solutions at the OT-level are little more than descriptive generalizations. In particular, I address the following issues: (i) why parasitic and harmonic features tend to be phonetically similar (as introduced in Chapter 2), (ii) why harmony obeys a ‘Principle of Similarity’ whereby participation by less similar segments implies participation by more similar segments, and (iii) a pathology of the higher level, where conditions to harmony can be disjunctive. While somewhat stipulative, universal rankings could solve these problems at the OT-level, but at the lower-level the problematic rankings are naturally avoided by plausible low-level assumptions, respectively, Hebbian learning, constraint additivity, and a bias in learning towards generalization.

This chapter is outlined as follows: §3.2 and §3.3 introduce the essential properties of attraction necessary for prerequisite similarity effects and show how these properties derive from Burzio’s Entailment Framework (2002a, b, 2005; Burzio & Tantalou, 2007). §3.4 illustrates how the low-level entailments available in Hebbian learning predict that parasitic dependencies are based on phonetic similarity. §3.5 provides a formalization of attraction in Optimality Theory, presenting a family of ATTRACTION constraints that implement distance effects, and introduces a theory of
faithfulness thresholding to describe the behavior of a grammar. §3.6 shows how these same ATTRACTION constraints can be derived from Harmonic Grammar. §3.7 explores the interaction of faithfulness and attraction from the multiple levels of OT constraints, HG constraints, and lower-level feature spaces. §3.8 proposes a restriction to ensure that ATTRACTION constraints do not overgenerate and produce disjunctive parasitic harmony.

3.2. Representational distance in an attraction system

In a conventional attraction system, like household magnets, an attractor exerts a force which tends to pull other objects toward the attractor. Attraction forces become weaker as the distance between attractors and potential undergoers increases. In the face of a force opposing attraction, like friction, an attractor will succeed in attracting some objects and not others depending on their relative distances from the attractor. A small magnet may pick up a nearby scrap of metal, but not attract the same scrap from across the room. Thus, only objects which are sufficiently nearby an attractor will undergo attraction.

In a representational attraction space, some representations are attractors. A representational attractor exerts a force which tends to pull other representations toward the attractor. A variety of factors will determine whether or not these other representations will ultimately be attracted, and depending on the language domain, i.e. the representational space, “being attracted” could take on various meanings. Morphological syncretism (Burzio, 2002), non-derived environment blocking (Burzio, 2005), phonetic enhancement (Wayment et al., 2007), and irregular stress patterns (Burzio & Tantalou, 2007) have all been described as representational attraction. Like conventional attraction systems, the strength of the force of attraction falls off with the
distance between representations, but unlike conventional systems, “distance” no longer has an intuitive reference to usual three-dimensional, Euclidean space, rather some suitable notion of representational distance is required. There are many ways in which to measure the distance between two linguistic forms because there are many different semantic, pragmatic, phonological, and other factors that affect what might constitute the neighborhood of a form. However, since this dissertation only concerns itself with segment-to-segment interactions, unless otherwise stated, all distances will refer to a standard distance (either Euclidean or Manhattan) in a multidimensional space encoding the acoustic and articulatory correlates of the features which bundle together to form phonological segments. To ensure tractable analysis, I will often only consider a subset of this space and assume that other features do not affect the relevant distances between representations.

The central hypothesis of this dissertation is that assimilation can be understood as an attraction system, where triggering segments are attractors that exert attraction forces on targets. Triggers of assimilation are viewed as sources of attraction, but targets also undergo other forces that may oppose attraction to a particular source. These opposing forces may take the form of faithfulness, markedness, or even attraction to another possible trigger.26 Whether or not a particular target ultimately assimilates to a trigger depends on the interaction of all the forces acting on the target, but if a target is sufficiently close, it will be attracted.

26 Note that faithfulness has also been argued to reduce to attraction (Burzio, 2002a, b, 2004, 2005; Burzio & Tantalou, 2007), and it may well be that markedness can be described as attraction since unmarked feature values attract marked feature values, so it is likely the case that the conventional forces which oppose assimilation can also be described in terms of an attraction landscapes. The summation of these different landscapes would yield a final landscape that determines whether assimilation obtains.
The cumulative effect of the forces acting on potential targets can be visualized in a ‘gravity-well’ diagram, or attraction landscape (see (1)). The center of each well is an attracting trigger and the strength of each attractor is related to both the radius and depth of the well. Target representations will fall into the gravity-well of a particular attractor if they are sufficiently close to a trigger.

(1) **Attraction Landscapes.** In $L_{\text{harmony}}$, on the left (a), $\text{Trig}$ attracts $\text{Targ}$. This contrasts with $L_{\text{disharmony}}$ on the right (b), where $\text{Trig}$ does not attract $\text{Targ}$. In this illustration, $\text{Trig}$ is a stronger attractor in $L_{\text{harmony}}$ than in $L_{\text{disharmony}}$.

I postulate, as in OT, that both the forces and the representational space are universal with the typology of possible languages deriving from the possible interactions of these forces. Some languages exhibit harmony to a particular feature and others do not, so different languages have different attraction landscapes over the same articulatory and acoustic space. In the attraction framework, differences in attraction arise from a language specific weighting of the many competing forces on targets (see §3.7). Thus, as (1) illustrates, in a language, $L_{\text{harmony}}$, (1)(a), which harmonizes on a feature $f$, the $f$-attraction force is stronger than the force for faithfulness to $f$. On the other hand, in a disharmonic language, $L_{\text{disharmony}}$, (1)(b), the $f$-attraction force is weaker than the force for faithfulness to $f$. This contrast in attraction strength is indicated by the deeper, larger
gravity-well in (1)(a) than in (1)(b). Thus, a source in $L_{\text{harmony}}$, $\text{Trig}$, attracts a target, $T$, but the same source $\text{Trig}$ does not attract $\text{Targ}$ in $L_{\text{disharmony}}$. While the absolute distance between $\text{Trig}$ and $\text{Targ}$ has not changed, $L_{\text{harmony}}$ and $L_{\text{disharmony}}$ have differing attraction landscapes and therefore differing notions of sufficiently close for attraction to $\text{Trig}$. Because $\text{Targ}$ is sufficiently close to $\text{Trig}$ in $L_{\text{harmony}}$, but not sufficiently close to $\text{Trig}$ in $L_{\text{disharmony}}$, $L_{\text{harmony}}$ and $L_{\text{disharmony}}$ differ on whether or not $\text{Targ}$ harmonizes to $\text{Trig}$ on the feature-$f$.

Thus, distance provides the bridge between the concepts of attraction and similarity because similarity is inversely related to distance. More similar representations in the multi-dimensional acoustic/articulatory space are closer to one another; less similar representations are farther apart. Because the size of the basin of attraction for a trigger differs in distinct languages, languages differ on what makes up the prerequisite similarity for attraction to that source. Large basins of attraction correspond to low prerequisite similarity. Small basins correspond to high prerequisite similarity. If the basins are small enough, then no other segments are attracted giving a non-harmonic language. However, targets which do undergo attraction alternate to feature-values like those of the attracting trigger, and so surface similarity increases after attraction; only targets which meet the prerequisite similarity (are close enough to sources) are attracted. Thus, an attraction framework can derive in a most natural fashion the fact that languages utilize prerequisite similarity to condition assimilation: targets which are close enough to triggers will be attracted, and those attracted targets assimilate as they move closer to

\footnote{“Absolute” since the locations of $\text{Trig}$ and $\text{Targ}$ in the representational space are static across (1)(a-b).}
triggers. The table in (2), below, provides an overview of the analogy between assimilation and conventional attraction systems, like magnetism.

(2) \textit{Attraction in physics and in phonology:}

<table>
<thead>
<tr>
<th>Elements of Attraction</th>
<th>Physics, e.g. Magnetism</th>
<th>Phonology, e.g. Assimilation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Result of attraction</td>
<td>Physical motion</td>
<td>Segmental alternation</td>
</tr>
<tr>
<td>Sources of attraction</td>
<td>Household magnets</td>
<td>Triggers of assimilation</td>
</tr>
<tr>
<td>Potential undergoers</td>
<td>Ferrous metal shavings</td>
<td>Targets of assimilation</td>
</tr>
<tr>
<td>Space where distance is measured</td>
<td>Conventional 3D-Euclidean space</td>
<td>Acoustic/articulatory feature space</td>
</tr>
<tr>
<td>‘Sufficiently close’ to undergo attraction is based on</td>
<td>Physical proximity</td>
<td>Representational proximity, i.e. Prerequisite similarity</td>
</tr>
<tr>
<td>Other forces</td>
<td>Friction, etc.</td>
<td>Faithfulness, markedness, etc.</td>
</tr>
<tr>
<td>Formal model</td>
<td>Mathematical equations</td>
<td>Grammatical constraints</td>
</tr>
</tbody>
</table>

3.3. \textit{Attraction and the Entailment Framework}

This section sketches how a formal model of attraction can be derived from a set of constraints based on Burzio’s Entailment Framework (Burzio, 2002a, b, 2004, 2005; Burzio & Tantalo, 2007). The Entailment Framework is closely related to Hebbian learning (Hebb, 1949) in a connectionist network because it posits connections, i.e. entailments, between elements which co-occur. Thus, the main hypothesis of the Entailment Framework is that representations are associated with entailments, as indicated in (3) below:

(3) \textit{Representational Entailments Hypothesis (REH):}

Mental representations of linguistic expressions contain sets of entailments. E.g. a representation consisting of A, B corresponds to the entailments:

$A \rightarrow B$, $B \rightarrow A$. (if A then B; if B then A).

A space for representations is specified by identifying a set of components – such as features – that make up representations. (3), then, states that the Entailment Framework
ments that associated with each representation is a set of (logical) entailments among the components. For example, suppose the vowel phoneme /i/ in the inventory, in (4) below, is indicated by the feature bundle, [−cons., +hi, +ATR, −back].

(4) Hypothetical inventory of non-low vowels:

<table>
<thead>
<tr>
<th></th>
<th>+ATR</th>
<th>−ATR</th>
<th>+ATR</th>
<th>−ATR</th>
</tr>
</thead>
<tbody>
<tr>
<td>[+hi]</td>
<td>i</td>
<td>i</td>
<td>u</td>
<td>u</td>
</tr>
<tr>
<td>[−hi]</td>
<td>e</td>
<td>e</td>
<td>o</td>
<td>o</td>
</tr>
</tbody>
</table>

(4) exhausts the possibilities for vowels in this representational space. By the Representational Entailments Hypothesis, each feature entails every other feature, so associated with /i/ is the following corresponding set of entailments, as follows in (5):

(5) [−cons]→[−cons] [−cons]→[+hi] [−cons]→[+ATR] [−cons]→[−back] [+hi]→[−cons] [+hi]→[+hi] [+hi]→[+ATR] [+hi]→[−back] [+ATR]→[−cons] [+ATR]→[+hi] [+ATR]→[+ATR] [+ATR]→[−back] [−back]→[−cons] [−back]→[+hi] [−back]→[+ATR] [−back]→[−back]

This set of entailments, in (5), is sufficient for demonstrating the basic attraction properties of the Entailment Framework. The key for attraction is to allow representational entailments to act as constraints, which may be either violated or satisfied. Thus, when confronted with alternative feature configurations, each entailment has a logical value that is either true or false. The entailment $p \rightarrow q$ is true of a configuration $X$, when, if $p$ is a component of $X$, $q$ is also a component of $X$.

Furthermore, these entailments can be understood as constraints in an OT-type framework, so configurations which are logically true satisfy the entailment, and false
configurations are *entailment violations*. (6) lists the logical truth values and the entailment violation profiles of possible candidate configurations in context of the entailment [−cons]→_{/i/} [+ATR]. The subscript _/i/_ indicates that this entailment is associated with the representation of _/i/_.

(6) **Logic of entailment constraints:**

<table>
<thead>
<tr>
<th>Candidate configurations</th>
<th>Truth value of [−cons]→_{/i/} [+ATR]</th>
<th>Violation profile of [−cons]→_{/i/} [+ATR]</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. [−cons, +ATR]</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>b. [−cons, −ATR]</td>
<td>F</td>
<td>*</td>
</tr>
<tr>
<td>c. [+cons, +ATR]</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>d. [+cons, −ATR]</td>
<td>T</td>
<td></td>
</tr>
</tbody>
</table>

Candidate (6)(b), [−cons, −ATR], violates the entailment [−cons]→_{/i/} [+ATR] because the antecedent is true of (6)(b), while the consequent is false. As in this case, an entailment is violated if and only if a candidate has the antecedent feature value, but lacks the consequent feature value.  

Crucially the Entailment Framework also maintains that entailment violation is additive. Consider, in (7), the cumulative entailment violations of possible representations under three _/i/_ entailments [−cons]→_{/i/} [+hi], [−cons]→_{/i/} [+ATR], and [−cons]→_{/i/} [−back]. All [+cons.] representations are left off the table since they trivially satisfy these entailments because the antecedent is false.  

(7) shows that because of

---

28 This is true of formal logic generally. Novice logic students often mistake entailments, or implications, as bi-conditionals, assuming the statement _p_→_q_ is false whenever _p_ and _q_ disagree. Entailment constraints, however, are faithful to the formal definition of implication in logic. Thus, the implication _p_→_q_ is not violated if _p_ is false of a configuration and _q_ is true of that configuration (cf. (6)(c)), but only violated if _p_ is true and _q_ is false (cf. (6)(b)).

29 This is not to say that [+cons] segments can or cannot carry the features [hi], [back], [ATR], but rather that, whatever the specifications of consonants, they cannot violate the entailments in (7), because they disagree on the antecedent [−cons.] and so trivially satisfy the entailments.
entailment additivity, entailment violation can have the gradient quality of differentiating many candidates.

(7) **Entailment additivity distinguishes candidates’ similarity to /i/:**

<table>
<thead>
<tr>
<th>Candidate configurations</th>
<th>[−cons]→/i/[+hi]</th>
<th>[−cons]→/i/[+ATR]</th>
<th>[−cons]→/i/[−back]</th>
<th>Total # of violations</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. /i/=[−cons, +hi, +ATR, −back]</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>b. /e/=[−cons, −hi, +ATR, −back]</td>
<td>*</td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>c. /u/=[−cons, +hi, −ATR, −back]</td>
<td></td>
<td>*</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>d. /u/=[−cons, +hi, +ATR, +back]</td>
<td></td>
<td></td>
<td>*</td>
<td>1</td>
</tr>
<tr>
<td>e. /o/=[−cons, −hi, −ATR, −back]</td>
<td></td>
<td>*</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>f. /o/=[−cons, −hi, +ATR, +back]</td>
<td>*</td>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>g. /o/=[−cons, +hi, −ATR, +back]</td>
<td>*</td>
<td></td>
<td>*</td>
<td>2</td>
</tr>
<tr>
<td>h. /ɔ/=[−cons, −hi, −ATR, +back]</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>3</td>
</tr>
</tbody>
</table>

The Entailment Framework maintains that the system of mental representations optimizes so as to seek to minimize entailment violation. This axiom is akin to harmony maximization in OT, which holds that the mental system aims to minimize constraint violations. Also like OT, Entailment Theory allows for the possibility that some entailment violations remain even in viable outputs because of competition among the competing constraints.

However, unlike traditional OT, in the entailment framework the content of some of the constraints is tied to the representation of linguistic forms. For example, the constraint [−cons.]→/i/[+ATR], as in (6), derives its content from the representation of the vowel /i/. While all vowels in an inventory like (4) have an entailment between [cons.] and [ATR], only tense vowels, like /i/, have a representational specification which requires that [−cons.] entails [+ATR], i.e. a vocalic configuration entails an advanced tongue root configuration as well. In contrast, lax vowels have a different entailment,
e.g. \([-\text{cons.}] \rightarrow /i/[-\text{ATR}]\), a vocalic configuration entails a retracted tongue root configuration.

By allowing the entailments from triggering segments to apply to targets, entailments may play the role of agreement constraints, as follows: if \([-\text{cons.}] \rightarrow /i/ [+\text{ATR}]\) is applied to other vowels, then the /i/-entailment creates pressure for those vowels to agree with /i/ on [ATR], since they already agree with /i/ on [cons.]. In some other context – for instance /ɪ/, which has the opposite dependency, \([-\text{cons.}] \rightarrow /i/[-\text{ATR}]\), the entailment content requiring agreement to /i/ is absent. Applying trigger entailments to targets is, thus, compatible with feature co-occurrence constraints whose content depends on a particular trigger. From an attraction perspective, since the constraints depend on the representations, different inputs may have different attraction landscapes with different entailment violation minima, which therefore yield distinct outputs.

(8), below, illustrates that minimizing entailment violation is tantamount to hypothesizing that the linguistic system is a system of attractors. Under the indicated system of three entailments, only candidate (8)(a), /i/, violates zero entailments, so complete entailment satisfaction is only non-trivially possible with absolute neutralization. But beyond attraction as absolute neutralization, it is important to

---

30 It may be possible to view entailments context sensitivity as an instance of lexically controlled variation, through reranking, i.e. in the context of /ɪ/, \([-\text{cons.}] \rightarrow [+\text{ATR}] \gg [−\text{cons.}] \rightarrow [−\text{ATR}]\), but in the context of /i/, the ranking is reversed. While this may be necessary for cases of variable pronunciation, I will assume that in the context of only /i/, the entailment \([-\text{cons.}] \rightarrow [−\text{ATR}]\) is not just low ranked, but wholly absent. Of course, this move is not material to the analysis of parasitic assimilation, as there is minimal difference between constraints ranked below relevant faithfulness and absent constraints.

31 As discussed below in §3.4-3.5, aspects of attraction may be given different weight relative to IO-faithfulness, so although /i/ is unmarked with respect to attraction force to /i/, total neutralization may not always obtain because of the stronger faithfulness forces which block attraction along particular features.
ensure the entailment framework has another important property of attraction, namely, that attraction forces weaken with distance.

In a full set of entailments, like (5) above, total violation does not correlate absolutely with the force of attraction because representations which differ in similarity may have the same total number of violations. For example, similar /e=/[−cons, −hi, +ATR, −back] violates three /i/-entailments: [−cons]→[+hi], [+ATR]→[+hi], and [−back]→[+hi]. On the other hand, dissimilar /ɔ=/[−cons, −hi, −ATR, +back] also violates three /i/-entailments: [−cons]→[+hi], [−cons]→[−ATR], [−cons]→[−back]. There is a difference, however, all violations of /e/ are localized to a single consequent [hi], whereas the violations of /ɔ/ are spread across multiple components [hi], [ATR], and [back].

I now show that the change in the number of violations per component is relatable to the force of attraction. By way of illustration, consider each possible vowel candidate in the space of representations containing /i/. (8) lists (i) the number of components different from /i/, which relates to the total similarity between /i/ and other configurations, (ii) the total number of /i/ entailment violations of a candidate configuration, and (iii) the violations per relevant consequent – indicated by leaving the antecedent blank, so the column →_i/+hi] contains the number of entailment violations where [+hi] is the consequent of an /i/-entailment.
The pressure for attraction is the gradient of entailment violations with respect to an antecedent feature:

<table>
<thead>
<tr>
<th>Candidate configurations</th>
<th># of components differing from /i/</th>
<th>Total # of /i/ violations</th>
<th>$\rightarrow_{\partial} [+hi]$</th>
<th>$\rightarrow_{\partial} [+ATR]$</th>
<th>$\rightarrow_{\partial} [-back]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. /i/=[-cons, +hi, +ATR, -back]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>b. /ɛ/=[-cons, -hi, +ATR, -back]</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>c. /ɪ/=[-cons, +hi, -ATR, -back]</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>d. /u/=[-cons, +hi, +ATR, +back]</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>e. /ɛ/=[-cons, -hi, -ATR, -back]</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>f. /ɔ/=[-cons, -hi, +ATR, +back]</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>g. /ʊ/=[-cons, +hi, -ATR, +back]</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>h. /ɔ/=[-cons, -hi, -ATR, +back]</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

(8) demonstrates that candidates similar to /i/ have most entailment violations on a single component (light shaded cells), whereas dissimilar candidates have their violations spread out evenly among multiple components (dark shaded cells). The violation per consequent is the number of violated entailments that would be satisfied, if that component of the candidate were to assimilate. The violations per consequent are, thus, relatable to the pressure on that component to be attracted to – agree with – /i/.

Candidates which only differ from /i/ on exactly one feature have more pressure for that differing feature to assimilate than candidates which differ by more than one feature. A candidate which partially assimilates on one component would then face increased pressure to assimilate on the other features; e.g. if in the context of triggering /i/, the target /ɛ/ assimilated on [ATR] to become /ɛ/, there would then be increased pressure for target’s [-hi] specification to become [+hi]. Likewise, a candidate which partially dissimilates on one component would face decreased pressure to assimilate on the other features; e.g. if /ɛ/ dissimilated on [back] to become /ɔ/, there would be decrease pressure...
for [−hi] to become [+hi]. Thus, the Entailment Framework preserves the weakening of attraction forces at a distance.

If total entailment violation is the height of the target in an attraction landscape, the change in entailment violations better reflects the slope of the gravity-well: targets are more strongly pulled to sources of attraction, where the slope is steeper. (9), below, is an illustration of the attraction landscape over the vowel space indicate in (4) of the set of entailments associated with /i/, where each entailment receives equal weight.

(9) *The attraction landscape of the set of entailments for /i/ as given in (5).*

The vertical axis is the total number of entailments violated. The slope of the surface is the gradient of changes in entailment violation, which corresponds to the pressure for attraction. The trigger, the source of attraction, /i/, is at the bottom, other possible targets are above.

In (9), consider /u/ and /ɔ/, which have the same height in the figure because they both violate the same number of entailments, in this case, three (see (8)(d) and (8)(h), respectively). However, /u/ has a gradient toward /i/, /ɔ/ has a zero gradient (there is no force of attraction from /i/ to /ɔ/), and /ɔ/ has a gradient away from /i/. This landscape is in some respects only schematic because the four-dimensional space (three features plus one entailment violation score) is difficult to embed in this piece of paper. Distances
relative to /i/ are preserved as much as possible, but the exhibited distances between targets are not true, since /ɔ/ is equidistant from all of /ʊ/, /o/, and /ɛ/. Nevertheless, the more similar vowels are to /i/ the steeper the slope of the gravity-well, and the stronger the force of attraction.

Among the central contributions of this dissertation on the understanding of assimilation is that the way in which a trigger attracts a target is that the entailments of a trigger are imposed upon potential targets. From a landscape perspective, a target is ‘dropped’ on an attraction landscape formed by the entailment violations of the trigger, like (9), and the other forces of the system, markedness and faithfulness. The markedness and faithfulness forces are constant across inputs, but a trigger’s attraction properties deform the markedness and faithfulness landscape, subtly making the behavior of a possible target depend on the trigger.

### 3.4. Dependency, Parasitism, and Phonetic Similarity

Thinking about attraction in a visual fashion has merit, but because deciding whether or not a target will undergo attraction can be very difficult to visually determine in a higher dimensional space with more constraints, the remainder of this chapter presents methods and principles for a formal implementation of segment-to-segment attraction in Optimality Theory and Harmonic Grammar. An essential component of this Attraction Framework is a family of ATTRACTION constraints which enforce a dependency between parasitic and harmonic features through entailment. An introductory definition of ATTRACTION constraints is given below in (10):
(10) \( \text{ATTRACT}(p \rightarrow q) \) (Definition to be revised)

\[
\text{IF} \quad \text{segments } x, y \text{ in a surface form have the same specification for } p, \\
\text{THEN} \quad \text{incur one mark unless } x, y \text{ also have the same specification for } q.
\]

\( \text{ATTRACT}(p \rightarrow q) \) applies the entailment \( p \rightarrow q \) such that if two segments agree on the prerequisite feature, \( p \), then they must also agree on the harmony feature \( q \). Thus, as I show in more detail in §3.5, \textit{Attraction} constraints provide a way for the entailments of triggers to apply to targets. However, as formulated in (10), it is not clear whether \( x \) should agree with \( y \) or the other way around, so this definition must ultimately be revised to account for directionality and locality effects (see §3.5, \textit{Chapter 4}).

Nevertheless, (10) is sufficient to encode a parasitic dependency between \( p \) and \( q \). This can be illustrated by considering how the constraint \( \text{ATTRACT}([\text{hi}] \rightarrow [\text{round}]) \) evaluates harmonic versus non-harmonic candidates as in (11), which shows how left-to-right height-parasitic rounding harmony, not unlike Yawelmani (discussed in more detail in §2.3.1.1 and §4.3), can arise from attraction.

(11) \textit{Attraction constraints prefer harmony when similarity preconditions are met:} 

<table>
<thead>
<tr>
<th></th>
<th>( \text{ATTRACT}([\text{hi}] \rightarrow [\text{round}]) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. u u</td>
<td></td>
</tr>
<tr>
<td>b. u i</td>
<td>*!</td>
</tr>
<tr>
<td>c. u o</td>
<td></td>
</tr>
<tr>
<td>d. u e</td>
<td></td>
</tr>
</tbody>
</table>

The main point of (11) is to show that \( \text{ATTRACT}([\text{hi}] \rightarrow [\text{round}]) \) is only violated when trigger and target agree on the parasitic feature, in this case \([\text{hi}]\), but disagree on the harmonic feature, \([\text{round}]\). Note that (11)(d) is as disharmonic as (11)(b) with respect to rounding, but penalties for non-harmony are only incurred when the trigger and target
agree [hi]. In this way, \textit{Attraction} constraints are sufficient to encode the dependencies needed for parasitic assimilation.

This section formally explores why some kinds of parasitic dependencies are cross-linguistically robust, while others are unattested. Here, I take as a case study the fact that [\textit{back}]-parasitic rounding harmony and [\textit{height}]-parasitic rounding harmony are somewhat common, while [\textit{ATR}]-parasitic rounding harmony is unattested (§2.3.1). This is tantamount to suggesting that some kinds of \textit{Attraction} constraints are strong, such as \textit{Attract}([\textit{back}] \rightarrow [\textit{round}]) and \textit{Attract}([\textit{hi}] \rightarrow [\textit{round}]), while others, like \textit{Attract}([\textit{ATR}] \rightarrow [\textit{round}]), are universally weak to the point of never applying in harmony. Thus, the dilemma is to explain why some kinds of dependencies are stronger than others, even though the constraints which implement dependency are of equal formal complexity: a single prerequisite feature $p$ and a single parasitic feature $q$.

The solution to this dilemma is a grounding in phonetic similarity that derives the strength of parasitic dependency. As rehearsed in \textbf{Chapter 2}, there is an overwhelming tendency for parasitic features to be similar to harmonic features in terms of articulation, acoustics, or both. For instance, rounding harmony on vowels was found to be dependent on vowel height or vowel backness, but not on ATR or nasality. The phonetic explanation is that rounding primarily affects the peak of the second spectral formant (F2). Vowel backness also strongly correlates with F2; both rounding and backing lower F2 because they both lengthen the front cavity. This similarity along F2 gives rise to a higher-ranked \textit{Attract}([\textit{back}] \rightarrow [\textit{round}]). At the same time, height also has a moderate effect on F2, as lower front vowels have a lower F2 than higher front vowels. There is also an articulatory link between vowel height and rounding for both front and back
vowels, as a raised jaw position facilitates a reduction in the amount of tongue distortion required for high vowel positions and narrows the closure at the lips to facilitate the lip protrusion associated with rounding. This phonetic similarity explains the existence of grammars where ATTRACT([hi]→[round]) is active. However, there are no such phonetic links between rounding and ATR because they are implemented with independent articulators and have unrelated effects on the acoustic signal (ATR primarily affects F1) and so ATTRACT([ATR]→[round]) is universally low-ranked.

Because phonetic similarity predicts the strength of attraction constraints, one could stipulate that only those phonetically similar features result in relatively strong ATTRACTION constraints. This could be succinctly stated as below in (12):

(12) **Phonetic grounding of attraction:**
    
    If p and q are features with an amount of phonetic similarity, and p′ and q are orthogonal in terms of phonetic similarity, then ATTRACT(p→q) is universally much stronger than ATTRACT(p′→q).

    While such a ranking principle would certainly be well-motivated, it would offer little more than a formal stipulation of the observed pattern of parasitic dependencies. Thus, while formally adequate, a principle like (12) altogether avoids what is perhaps the more important question: why do only phonetically similar features depend on one another in parasitic systems?

    Fortunately, because of the Entailment Framework, the present attraction system can provide more insight than descriptive ranking principles. In Wayment et al. (2007), it was shown that the notion of entailment violation depends on similarity. In particular, we showed how the entailment framework could give another kind of dependency
between features: the phonological inventory effect known as phonetic enhancement. Phonetic enhancement (Stevens, Keyser, & Kawasaki, 1986) explains why inventories contain some combination of features, but not others. For instance, it is common for inventories to have [a back, a round] vowels, while lacking [a back, −a round] vowels. At the descriptive level, enhancement follows from the aforementioned phonetic similarity of backing and rounding (both having a lower [F2]), since [a back, a round] vowels enhance the contrast along F2, while intermediate [a back, −a round] vowels provide conflicting cues related to F2. Wayment et al. showed that in a model that allows subfeatural components of representation to indicate the phonetic similarity of [back] and [round], the set of entailments prefers [a back, a round] to [a back, −a round], even when provided equal evidence of all possible combinations of backing and rounding. This confirmed that the Entailment Framework is inherently sensitive to the kind of low-level similarity that gives phonetic enhancement.

A similar analysis is available to explain the phonetic grounding of the strength of ATTRACTION constraints. It suffices to derive (12) to show that by considering entailments among subcomponents, the pressure for attraction is greater given similar p and q than for dissimilar p′ and q. Thus, the phonetic grounding of parasitic interaction is an instance of Burzio’s ‘binding corollary’ (2005; see also Wayment et al., 2007) which states that the strength of the entailment p→q is related to the similarity of p and q. In the case of phonetic enhancement, binding occurred within a segment, but in the case of parasitic assimilation, binding occurs across segments in a string as the entailments of triggers apply to targets.
For a concrete illustration of how the binding corollary gives the dependency needed for parasitic harmony, consider the subfeatural distributed representations of the features [back], [round], and [ATR] in (12) below. In what follows, I will show that the entailment [back]→[round] is stronger than [ATR]→[round] because of representational similarity along these subfeatures. In (12), a “✓” denotes that the phonological feature is associated with the phonetic subfeature and a blank cell denotes that the phonological feature is not associated with the phonetic subfeature. The exact value the subfeature takes depends on the specifications of the features with which it is associated. For example, [F2] depends on the values of [back] and [round], but not [ATR], so [+back, +round, +ATR] and [+back, +round, −ATR] each denote [lower F2].

(13) \textit{Distributed representations of features:}

<table>
<thead>
<tr>
<th>Phonetic Subfeature</th>
<th>Gesture</th>
<th>[back]</th>
<th>[round]</th>
<th>[ATR]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[tongue dorsum retraction]</td>
<td>✓</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>[tongue root retraction]</td>
<td></td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>[lip protrusion]</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Cue</td>
<td>[F1]</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[F2]</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

With these plausible low-level representations of features, it is clear that [back] and [round] are similar, since they share cue features (indicated by shading), while [round] and [ATR] are not similar since they do not share either cue features or gesture features.

In order to compare the entailment between similar elements, [back]→[round], with the entailment between dissimilar elements, [ATR]→[round], there must be a way to compute entailments when there are subfeatures. Wayment \textit{et al.} showed that (i) an entailment \( p \rightarrow q \) can be defined as the set of entailments among the subcomponents, i.e.
for \( p = \{ p_1, \ldots, p_m \} \) and \( q = \{ q_1, \ldots, q_n \} \), the entailment \( p \to q \) is a set of sub-entailments, \( \{ p_i \to q_j \mid \forall i,j \text{ such that } 1 \leq i \leq m, 1 \leq j \leq n \} \) and (ii) the strength of the attraction force imposed by \( p \to q \) is related to the number of its violated sub-entailments. In Wayment et al. and elsewhere in this dissertation, the number of violated sub-entailments can be correlated with the Harmony (Smolensky, 1986; Smolensky & Legendre, 2006) of patterns in a connectionist network. However, for the sake of encapsulating the discussion of network properties, I postpone a discussion of network Harmony to Chapter 5. Instead, here, I directly compute and compare the sub-entailments of [back]→[round] and [ATR]→[round], as shown below in (14).

(14) Sub-entailments of [back]→[round] and [ATR]→[round]:

<table>
<thead>
<tr>
<th>[back]→[round]</th>
<th>[round]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gesture</td>
<td>Cue</td>
</tr>
<tr>
<td>[lip protrusion]</td>
<td>[F2]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>[back]</th>
<th>[tongue dorsum retraction]</th>
<th>[t.d.r.]→[l.p.]</th>
<th>[t.d.r.]→[F2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gesture</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[F2]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cue</td>
<td>[F2]</td>
<td>[F2]→[l.p.]</td>
<td>[F2]→[F2]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>[ATR]→[round]</th>
<th>[round]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gesture</td>
<td>Cue</td>
</tr>
<tr>
<td>[lip protrusion]</td>
<td>[F2]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>[ATR]</th>
<th>[tongue root retraction]</th>
<th>[t.r.r.]→[l.p.]</th>
<th>[t.r.r.]→[F2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gesture</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[F1]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cue</td>
<td>[F1]</td>
<td>[F1]→[l.p.]</td>
<td>[F1]→[F2]</td>
</tr>
</tbody>
</table>

Note that I only indicate entailments between specified subfeatures, as unspecified feature values cannot have a causal effect as either antecedent or consequent; for instance, [ATR] denotes nothing about [F2], so the entailment [ATR]→[F2] cannot be violated. This is sensible since having an [ATR] specification provides no information about [F2] and vice versa.
Now, [back]→[round] is stronger than [ATR]→[round] because in the case of subfeatural similarity, there are aspects of the sub-entailments internal to a feature that are duplicated in the sub-entailments across features. The internal entailments are described by an auto-associative entailment. For instance, since [back] has the configuration [tongue dorsum retraction] and [F2], the self-entailment [back]→[back] gives the sub-entailments internal to [back]: [t.d.r.]→[t.d.r.], [t.d.r.]→[F2], [F2]→[t.d.r.], and [F2]→[F2]. By comparing this set to the sub-entailments of [back]→[round] in (14), it is clear that both [back]→[back] and [back]→[round] have the sub-entailment [t.d.r.]→[F2]. This and other duplicated entailments are indicated with arrows in (14).

In a representational system that performs Hebbian learning of features (represented with their detailed acoustic and articulatory correlates), the within-feature sub-entailments are learned alongside the across-feature sub-entailments. This is because Hebbian learning does not discriminate between internal and across-feature subfeatures, associating any elements which co-occur independent of which features give rise to them (see Wayment et al., 2007; Chapter 5). Therefore, in the case of phonetic similarity between phonological features, the within-feature sub-entailments reinforce the across-feature sub-entailments making the overall entailment [back]→[round] stronger than the entailment [ATR]→[round] which lacks any such overlap of sub-entailments. Unlike [back]→[round], [ATR]→[round] does not reinforce any internal entailments, [ATR]→[ATR] or [round]→[round], because the distributed representations of [ATR] and [round] are orthogonal.

Put another way, since segments x and y must agree on backness to activate ATTRACT([back]→[round]), if x, y violate ATTRACT([back]→[round]) then x, y also
violate aspects of the representation of [back] that must be independently available at each of \(x\) and \(y\). Thus, satisfying \(\text{ATTRACT}([\text{back}] \rightarrow [\text{round}])\) enhances the contrast between phonological strings in the same way that satisfying [back]→[round] enhances a phonological contrast in the inventory.\(^{32}\) Of course, harmony may work to satisfy an enhancement effect, like [back]→[round], even if the inventory violates it. On the other hand, if \(x, y\) violate \(\text{ATTRACT}([\text{ATR}] \rightarrow [\text{round}])\), then \(x, y\) do not violate any aspects of the representation that must be independently available at each of \(x\) and \(y\). There is no pressure for enhancement between [ATR] and [rounding] in the inventory, nor is there additional pressure for agreement on rounding given that \(x\) and \(y\) agree on [ATR].

In sum, the dependency between parasitic features and the clustering of phonological inventories due to phonetic enhancement have the same motivation: clustering across phonetic similarity. This computational explanation due to entailment satisfaction among subfeatural representations makes clear why phonetic similarity drives the strength of \(\text{ATTRACTION}\) constraints. Hereafter, it is assumed that only those dependencies which derive from non-orthogonal features are strong enough to motivate harmony. Furthermore, this computational motivation of the phonetic grounding of parasitic dependency provides a clear example of the role of additivity and violation in entailments in predicting clustering. Other related aspects of feature clustering are discussed in §4.4 where phonetic similarity explains the attested patterns of parasitism in both local and non-local nasal harmony and §4.5 where phonetic similarity can predict whether features spread together as a unit in a feature geometry.

---

\(^{32}\) As discussed in Chapter 2, one of the motivations for harmony is perceptual optimization which eliminates overlapping categories within a morpheme. Harmony, thus, has the effect of enhancing the contrast between morphemes as [+F][−F] sequences are avoided in favor of [+F][+F] and [−F][−F] sequences. This notion of harmony as enhancing lexical contrast is not further developed in this work.
3.5. **ATTRACTION correspondence constraints in OT**

With the motivations for parasitic dependency in place, this section presents the basic principles and hypotheses which allow for an analysis of parasitic assimilation within Optimality Theory in terms of an attraction model. I introduce a family of formal ATTRACTION constraints and show that these constraints are consistent with Burzio’s Entailment Framework, using conjunctive additivity. I also propose a ranking restriction based on subsets of preconditions in order to ensure that ATTRACTION is sensitive to prerequisite similarity in a fashion consistent with the distance properties of an attraction model. Finally, I formally demonstrate that the rank of faithfulness determines the relevant threshold for prerequisite similarity in an ATTRACTION system.

3.5.1. **Entailment persistence**

As previewed above, the mechanism that successfully derives prerequisite similarity effects relies upon the entailments of a trigger applying to other segments in a surface form. This section, further, develops the idea of entailments extending from one segment to another, and sketches the motivation for this hypothesis. Other issues related to entailment persistence are discussed in Chapter 4 and 5.

In previous entailment models in morphology and prosody, a surface form is subject to the entailments of other, independent surface forms (Burzio, 2002a, b); e.g., for cases of morphological syncretism, additional semantic specification creates entailment pressure for all of the distinct outputs which are members of that paradigm to have similar expressions. The important point is that, in previous work, entailment violations only occurred when an *entire surface form* (in a morphological/semantic category) was compared to another form (usually, in that same category). However, in order to analyze
assimilation with entailments, it is essential that entailment violations occur as one segment (in a surface form) is compared to other segments (in the same surface form).

This segment-to-segment interaction is tantamount to positing entailment persistence as indicated by the axiom in (15), below:

(15) **Entailment Persistence Hypothesis**

The entailments among the components of triggers persist to other segments in a surface, and therefore a trigger’s entailments may be violated by other segments in a surface form.

Under entailment persistence, entailments continue to persist beyond a trigger, playing an active role in determining the surface realization of other segments.\(^{33}\)

Ultimately, entailment persistence will be derived from representational entailments (see Chapter 5) in a neural network using a role-filler system with tensor product representations (Smolensky & Legendre, 2006), where the basic idea is that in a role-filler system different positions can share network resources, which allow the entailments from one segment to persist until another by way of the connections along these shared resources. However, this system is not easily employed in the kinds of formal OT analysis typical of theoretical phonology because of the many assumptions about units and activation patterns needed to implement a role-filler system, so here and in Chapter 4 I advocate a higher-level perspective based on segment-to-segment correspondence (Hanson, 2001; Rose & Walker, 2004) that can be used to the same effect in evaluating candidates. As discussed in more detail in Chapter 4, correspondence provides a way for formally controlling the locality conditions which apply to a harmony

\(^{33}\) Entailments persist in both progressive and regressive harmony. Thus, entailment persistence is not durational (starting at a certain time and persisting for a certain number of cycles), but rather representational (starting at a certain trigger and persisting for a certain number of potential targets).
process. While the Attraction Framework uses correspondence at the higher level of
description, like the strength of ATTRACTION constraints (see §3.4), these segment-to-
segment correspondences derive from aspects of subsymbolic representation and
entailment satisfaction in a connectionist network.

One possible way to view entailment persistence at the more abstract level would
be to think of it in terms of traditional spreading machinery, where the entailments of
triggers would spread to other segments. Under this perspective, targets would be
evaluated against the attraction properties of the trigger because the trigger’s attraction
landscape has spread to targets. This is schematically indicated in (16) below, which
could typify the process of iterative ATR harmony, such as in (Yoruba, Akan, and others)
whereby input /Ci.Cɛ...Cʊ/→/Ci.Cɛ...Cʊ/.

(16)  *Spreading of entailments in ATR harmony:*

\[
\sigma_1 \quad \sigma_2 \quad \ldots \quad \sigma_k
\]

\[
\text{[−cons]} \rightarrow_{n/}^{i/} [\text{+ATR}]
\]

Entailment spread of [−cons]→_{n/}^{i/} [+ATR] would motivate ATR harmony as [−ATR] [ɛ]
and [ʊ] violate the entailment, but [+ATR] [e] and [u] do not.

However, because this dissertation aims to unify the disparate phenomena of
long-distance consonant agreement (LDCA; Hanson, 2001; Rose & Walker, 2004; §2.2),
parasitic vowel harmony (PVH; Cole & Trigo, 1988; §2.3, §4.3), and local parasitic
assimilation (§2.4), I favor an alternative to spreading more closely related to Agreement
by Correspondence (ABC; Rose & Walker, 2004; Hansson, 2001; 2007; Walker, 2009a).

The basic property of ABC distinguishing it from spreading is that ABC requires
segments in a surface form that are in a formal correspondence relation to agree on
harmonic features. The present proposal differs from ABC because correspondence is
used exclusively to express locality conditions on assimilation, whereas in traditional
ABC correspondence is also affected by similarity.

Thus, a better scheme of how entailments apply from segment to segment is given
in (17) below.

(17)  *Correspondence-based agreement with entailments:*

\[
\begin{align*}
\sigma_1 & \quad \sigma_2 \quad \ldots \quad \sigma_k \\
\mathfrak{C}_i & \quad \mathfrak{C}_\varepsilon \quad \ldots \quad \mathfrak{C}_\upsilon
\end{align*}
\]

Note in this scheme each segment is independently in correspondence with – and,
therefore, independently affected by – the triggering source of attraction. From this
perspective, when a target is in correspondence with a trigger, the target is subject to the
attraction properties of triggers because the correspondence provides the means for
triggers to deform the landscape that would otherwise affect the target. Correspondence
links targets to the attraction properties of triggers, so a candidate with \(x \mathcal{R} y\), posits that
then entailments of \(x\) persist to \(y\).\(^{34}\) Correspondence provides a formal means for
manipulating the domain of entailment persistence.

---

\(^{34}\) Generally, the notation \(x \mathcal{R} y\) denotes “\(x\) is related to \(y\)”, but, unless otherwise stated, the relation \(\mathcal{R}\) refers to a segment-to-segment correspondence, so \(x \mathcal{R} y\) denotes that “for segments \(x,y\) in a surface form, \(\alpha, x\) corresponds with \(y\).”
As indicated in (17) and elsewhere in this chapter, entailments persist indefinitely. This is largely for the sake of exposition, as it allows issues of prerequisite similarity to be separated from the complex and abundant locality concerns (e.g. transparency, blocking, domain of interaction, etc.) which so often appear in assimilatory phenomena. Technically, these constraints are not “non-local”, rather, they are alocal because their formal definition makes no reference to the locality of correspondence or persistence of entailments.

Ultimately, however, alocal entailment persistence proves insufficient. As discussed at length in Chapter 4, there are a number of important typological differences between spreading and ABC, perhaps the most important of which is the contrast in blocking and transparency: ABC naturally predicts transparency of non-undergoing interveners, while spreading naturally predicts blocking by non-undergoing interveners. I will show by properly controlling the correspondence relations over which entailments operate, it is possible to restate this and other differences, as a difference in the locality of entailment persistence: blocking requires that entailments only persist locally, while long-distance interaction allows non-local persistence of entailments (see Chapter 4 for more details). However, for the basic feature-parasitic similarity effects discussed in this chapter alocal persistence is sufficient.

In order to address the motivations for entailment persistence, it is essential to consider the dichotomous motivations for vowel and consonant harmony. As argued in Chapter 2, the motivations for vowel harmony are often grounded in perceptual optimization. Vowel harmony avoids sequences which minimally contrast in some acoustic property, thereby, extending the perception of less salient features across
multiple vowels. Parasitic vowel harmony limits such perceptual optimization to the cases where vowels already agree on some features, ruling out only the particularly problematic sequences where the less salient parasitic feature is the only contrast between vowels, but allowing other sequences where there are multiple contrasts between vowels.

On the other hand, consonant harmony has been grounded in the processing related to speech planning (Rose & Walker, 2004; Hansson, 2001; Walker, 2007). These proponents of correspondence review studies of speech errors arguing that there are trends which are constant across consonant speech errors and consonant harmony, including (i) a tendency toward anticipatory directionality for both consonant harmony and speech errors and (ii) a bias for similar segments to interact as both trigger and target and as intended utterance and actual, errant utterance. Rose & Walker suggest that both harmony and speech planning exhibit “the tendency … to improve ease of production-related processing by overriding the differences between the consonants and making some or all of their properties match…LDCA is a phonologized means of accomplishing such matching for individual features” (p. 486).

Thus, the tendencies are for consonant harmony to arise in order to facilitate speech production and vowel harmony to arise in order facilitate speech perception.36

35 Optimizing speech planning is not the same as minimizing articulatory effort. Clearly, in terms of planning, there are increased costs for resetting the articulatory targets for similar segments, e.g. every time /s/ follows /ʃ/ the anterior/palatal feature specification must be changed but this change is avoided if /s/ never follows /ʃ/, as in coronal harmony (see §2.2.1). However, unchanging articulations may not reduce total (physical) effort when some harmonic articulation is effortful (marked). For example, to maintain a vocal tract configuration for [+]round, the cross sectional area of the aperture at the lips must be decreased by raising the jaw and protruding the lips, this is more effortful than the more neutral [−round] configuration that requires less movement of the jaw and lips, so rounding harmony does not minimize articulatory effort, though it does optimize speech planning (see §2.3.1).

36 These tendencies are clearly not mutually exclusive. There certainly are production benefits to vowel harmony (illustrated by speech errors with the tongue twister consisting of quickly repeating toy boat), and also perception benefits to some types of consonant harmony (laryngeal harmony in a system of three way contrast would cause less overlap in voice onset times). However, overlap in motivation only
Fortunately, as a general method of expressing clustering the Attraction Framework has the wherewithal to provide a formal vocabulary rich enough to subsume both effects. That, perhaps, attractors are more based in articulation for consonant harmony and more based in perception for vowel harmony is incidental to the more general computational process, for which I argue. By postulating that the space of attraction includes both articulatory and acoustic components, then the phonologization of a correspondence can arise from production, perception, or both.

To preview the explanation given in Chapter 5, the computational motivation for entailment persistence resides in how both production and perception are sequential processes. Spreading activation models (for instance Dell, 1987) of sequential processes have long provided insight into priming effects based on similarity: because of shared processing nodes, the activation for one segment may spread to other similar segments. In a spreading activation model, entailment persistence arises from the persistence of activation among these shared processing resources. For example, the entailment, \([-\text{cons.}] \rightarrow [+\text{ATR}]\), persists because activation in the processing nodes for \([-\text{cons.}]\) and \([+\text{ATR}]\) does not immediately decay, so other vowels are biased towards also being \([+\text{ATR}]\). Entailment persistence through activation persistence constitutes a kind of representational inertia, where resetting articulatory or perceptual targets comes at a cost (cf. Hansson, 2007). Phonologization of entailment persistence as correspondence is, then, the entrenchment of an unwillingness to pay these reset costs. By hypothesis, these costs exist for all sequential processes, and by extension both production and perception are subject to the diachronic/synchronic forces that yield entrenchment of persistence.

---

further argues for unification, as assimilation generally has consequences on both production and perception.
3.5.2. Direction of persistence

That there is direction to harmonic processes seems uncontroversial. However, it remains somewhat unclear why systems differ in their patterns of directionality. This is related to the complexity of predicting whether a segment will be a trigger or a target of harmony. Morphology provides one sort of answer, as special (morphologically indexed) faithfulness constraints allow some classes of morphemes to be designated as triggers and others as targets. Default directionality provides another, for example, in default regressive (Right to Left) harmony, the rightmost $x$, is always the trigger. However, it is unclear what principles would predict whether a system will use morphologically-based or default directionality.

Determining trigger status is further complicated by the prosodic and other positional factors, which have been shown to have a diverse effect on triggering. For instance, Walker (2005) has shown that unstressed positions may trigger harmony which contrasts with the more conventional harmonic spreading from stressed to unstressed syllables (van der Hulst & van de Weijer, 2001). Default directionality, morphology, and prosody all seem to interact to determine the direction of harmony.

Furthermore, studies of directionality in harmony (Baković, 2000; Hansson, 2001) have shown that there are a number of differences in the tendencies for directionality in vowel and consonant harmony: consonant harmony is more likely to be default regressive directionality than vowel harmony, and vowel harmony is more likely to exhibit morphological dominance (e.g., stem to affix). However, these tendencies are not absolute (Hyman, 2002; Rose & Walker, 2004), suggesting that both default directionality and morphological dominance must remain available for both consonant and vowel harmony, and, therefore, directionality is not an empirical obstacle to a unified
theory of assimilation under attraction. Since progressive and regressive harmony are independently needed, I will not derive directionality from any more simple principles, considering it an orthogonal issue to understanding the role of similarity in assimilation. Thus, describing a typology of the diverse factors that affect directionality goes beyond the scope of this dissertation, which focuses on similarity and locality in assimilation.

Formally, as I discuss more in §4.2, directionality may be tied to correspondence by allowing correspondence to be an asymmetric relation ($xRy$, but $y$ is not related to $x$). Thus, like in ABC, in the attraction framework, there are multiple directional versions of the constraints which enforce agreement. The highest ranking of which determines the direction of agreement. These constraints are indicated with a subscript on the correspondence relation: $R_{R\rightarrow L}$ for regressive harmony and $R_{L\rightarrow R}$ for progressive harmony.

3.5.3. Defining ATTRACTION in Optimality Theory

As discussed in §2.4.7 and §2.4.8, Baković (2005) has proposed a STR/PLACE constraint, restated below in (18),

(18)  \text{STR/PLACE:}

"Adjacent output segments that have the same place feature value must also have the same value of the stricture feature."

(Baković, 2007)

Furthermore, Reiss (2001) has proposed VOI/NAS in:

(19)  “Voice link implies nasal link VOI/NAS:

Adjacent [output] segments which agree in voicing must agree in nasality.”

(Reiss, 2001)
With the backdrop of attraction, STR/PLACE and VOI/NAS are better seen as examples of the use of entailment persistence. STR/PLACE creates an implicational dependency between place and stricture features: if segments agree in place, then they must also agree in stricture. The trigger’s entailment \( \text{place} \rightarrow \text{stricture} \) reapply to the target, requiring stricture agreement exactly when the trigger and target already agree on place features. Likewise, for VOI/NAS, the triggers entailment \([\text{voice}] \rightarrow [\text{nasal}]\) reapply to the target, requiring nasal agreement exactly when the trigger and target already agree on voicing.

This section presents a more general family of ATTRACTION constraints that, like STR/PLACE and VOI/NAS, directly enforce implicational pressures between segments in a surface form. These Optimality Theory (Prince & Smolensky, 2004) constraints are defined in (20) below:

\[(20) \quad \text{Define: } \text{ATTRACTION}(R, P=\{p_1, \ldots, p_k\} \rightarrow q):\]

where \( R \) is a segment-to-segment correspondence relation, \( P \) is a set of prerequisite features, and \( q \) is a harmonic feature:

Let \( x, y \) be segments in a surface form, \( \alpha \).

If

(i) \( xRy, \)

(ii) \( \forall p_i \in P, x \) agrees with \( y \) on \( p_i, \)

Then \( \alpha \) incurs one mark if \( x \) and \( y \) do not agree on feature \( q \).

(20) defines ATTRACTION as a segment-to-segment correspondence constraint, which is violated when corresponding segments in a surface form meet the specified prerequisite similarity conditions, but fail to agree on the specified harmonic feature.
For example, Baković’s STR/PLACE can be restated as the following attraction constraint: \( \text{ATTRACT}(R_{\text{Adj}}, \{ \text{place} \} \rightarrow \text{stricture}) \), where \( R_{\text{Adj}} \) is an adjacency correspondence relation, and \( \text{place} \) and \( \text{stricture} \) are generic features (or feature nodes in a feature geometry, e.g. Clements & Hume, 1995), which encode respectively place of articulation and stricture of consonants.\(^{37,38}\) This constraint is specified in detail in (21) below, with some annotating comments in parentheses:

(21) **Define**: \( \text{ATTRACT}(R_{\text{Adj}}, \{ \text{place} \} \rightarrow \text{stricture}) \):

Let \( x, y \) be segments in a surface form, \( \alpha \).

If 

- \( (i) \quad xR_{\text{Adj}}y \), \( (x \text{ is adjacent to } y) \)
- \( (ii) \quad \forall p_i \in \{ \text{place} \}, \; x \text{ agrees with } y \text{ on } p_i \), \( (x, y \text{ agree in } \text{place}) \)

Then \( \alpha \) incurs one mark if \( x \) and \( y \) do not agree on \( \text{stricture} \).

Although the language and notation is different, it should be clear that \( \text{ATTRACT}(R_{\text{Adj}}, \{ \text{place} \} \rightarrow \text{stricture}) \) is a restatement of STR/PLACE: they are both violated exactly when an output form contains adjacent segments which agree on place features, but fail to agree on stricture features. The ATTRACTION family, however, additionally posits the existence of many other instances of implicational feature dependency with a strength

\(^{37}\) Such generic features have the benefit of being *universal*, applying to both consonants and vowels. Generic \( \text{place} \) may be considered multi-valued taking on the places \([\text{labial}]\), \([\text{coronal}]\), or \([\text{dorsal}]\), and in general, a universal \( \text{stricture} \) feature is also multi-valued, capable of distinguishing multiple vowel heights, and is, therefore, sometimes subsumed or dominated by other nodes like \( \text{aperture} \) or \( \text{manner} \) but, here, \( \text{stricture} \) serves as a placeholder for \([\pm \text{continuant}]\) (cf. Clements & Hume, 1995).

\(^{38}\) Unlike a feature bundle, feature geometry naturally and restrictively allows multiple features to spread at once, and it is this linked spreading that argues for the existence of a feature node, but in terms of preconditions the differences are less pronounced. Using feature bundles, a similarity precondition activates if segments have the same specification of a feature, say the place feature, \([\text{dorsal}]\). Using feature geometry, a similarity precondition activates if segments have the same child specified for a node, e.g. \( \text{place} \) dominates \([\text{dorsal}]\). If feature organization (possible nodes and relationship between them) is universal, then under either theory segments either agree in \([\text{dorsal}]\) or they do not, and preconditions activate in the same circumstances. See §4.5 for more details on the interplay between Feature Geometry and ATTRACTION.
according to their phonetic similarity (see §3.4). This formal richness is required to account for the diverse set of phenomena presented in Chapter 2.

Now, the arguments of ATTRACTION constraints indicated in (20) – R, P, and q – correspond to aspects of attraction already outlined in this chapter.\(^{39}\) R instantiates both entailment persistence (§3.5.1) and directionality (§3.5.2). The condition in (20).\((i)\) implicitly states that ATTRACTION is only active if segments x and y correspond in the specified manner. For example, if \(R = R_{L \rightarrow R}\), then only segments where \(x\) precedes \(y\) will be in the kind of correspondence necessary to activate the constraint. On the other hand, if \(R = R_{R \rightarrow L}\), then only segments where \(y\) precedes \(x\) will be in the correct correspondence, so \(\text{ATTRACT}(R_{L \rightarrow R}, P \rightarrow q)\) enforces a different directionality of harmony than \(\text{ATTRACT}(R_{R \rightarrow L}, P \rightarrow q)\). Manipulating and controlling \(R\) for locality effects is the focus of Chapter 4.

The argument, \(P\), instantiates prerequisite similarity, since ATTRACTION will only be violated if two segments agree on all of the feature in \(P\) (see (20).\((ii)\)). Failure of segments \(x\) and \(y\) in a candidate \(\alpha\) to agree on even one of these \(p_i\) is a failure to meet the prerequisite similarity conditions specified by this constraint, causing the ATTRACTION constraint to be inactive. Now, there may be some other ATTRACTION constraint, with a different prerequisite feature set, violated by the same \(x, y \in \alpha\), so each ATTRACTION constraint has its own specified prerequisite similarity. For example, for phenomena where there is zero prerequisite similarity required for agreement, \(\text{ATTRACT}(R, \emptyset \rightarrow q)\) is equivalent to general agreement (cf. \textsc{Agree}(\textsc{[F]}): Baković, 2000 and \textsc{Ident-CC}(\textsc{[F]}): Hansson, 2001; Rose & Walker, 2004), because when \(P = \emptyset\), condition

\(^{39}\) The mnemonics are imperfect: \(R\) is for correspondence Relation, \(P\) is for Prerequisite similarity, but \(q\) emphasizes the role of the harmonic feature as a dependent consequent in the standard implicational notation, \(p \rightarrow q\). Of course, here, that implication is really \(P \rightarrow q\) or \(p_1 \land \ldots \land p_k \rightarrow q\).
(20).\textit{(ii)} is vacuously satisfied. Thus, $P = \{\}$ specifies that no agreement on prerequisite features is necessary for $q$ harmony to obtain. Controlling the members of the set $P$ is a crucial part of the analysis of parasitic assimilation presented in this and subsequent chapters.

The argument, $q$, instantiates the existence of a harmonic feature, and its inclusion as an argument of $\text{ATTRACTION}$ forces $q$-harmony to depend on $R$ and $P$. To avoid tautological constraints it is necessary to require that $q \notin P$. Otherwise, useless tautological self-parasitic constraints such as $\text{ATTRACT}(R, \{q\} \rightarrow q)$ would be posited to exist. Thus, $\text{ATTRACTION}$ implements dependencies between harmonic features and other parasitic features. \textbf{Chapter 2} argued that these dependencies are almost always based on phonetic similarity, and §3.4 showed how this role of phonetic similarity can be derived from entailment satisfaction among subfeatural representations.

Now, if the conditions in (20).\textit{(i)} and (20).\textit{(ii)} are met and inalterable, then the only way to repair violations to $\text{ATTRACTION}$ is to have agreement on $q$.\footnote{40} Therefore, the ranking of $\text{ATTRACT}(R, P \rightarrow q)$ and $\text{IDENT-IO}(q)$, a standard input-output correspondence constraint (McCarthy & Prince, 1995, 1999), is decisive in determining whether or not candidates are attracted. If $\text{ATTRACT}(R, P \rightarrow q) \gg \text{IDENT-IO}(q)$, then, when the correspondence and prerequisite similarity conditions are met, $\text{ATTRACT}(R, P \rightarrow q)$ motivates $q$-agreement. However, if $\text{IDENT-IO}(q) \gg \text{ATTRACT}(R, P \rightarrow q)$, then, even if the correspondence and prerequisite similarity conditions are met, $q$-agreement is blocked by the higher ranking faithfulness constraint.

\footnote{40} Inalterability of the preconditions and correspondence follows from higher-ranked constraints, like perhaps special IO-\textsc{Faith}, that keep the trigger from changing to agree with the target. As discussed in more detail in \textbf{Chapter 4}, determining trigger status is tied to directionality effects which by and large are beyond the scope of this dissertations. For further discussion of issues related to directionality see Baković (2000), Hansson (2001), Hyman (2002), Finley (2008), and the references therein.
3.5.4. Equality with entailments under conjunctive additivity

The formalization of ATTRACTION in (20) is of a slightly different form than that of the Representational Entailment Hypothesis, (3), so this section presents rigorous arguments for viewing the ATTRACTION family as an OT-instantiation of the REH with entailment additivity and entailment persistence. However, although formally equivalent, ATTRACTION constraints have at two largely aesthetic advantages over more basic entailment constraints: (i) correspondence provides a means for directly controlling which entailments apply where and (ii) ATTRACTION constraints enforce entailment without making reference to a particular entailment of a particular trigger.

Now, one entailment principle is trivially equal under both ATTRACTION and traditional entailment models: clearly, ATTRACT(R, P→q) implements entailment persistence if R is a segment-to-segment correspondence relation. As previously discussed (see §3.5.1), xRy explicitly posits that the entailments of x persist to y. However, despite the name, it is not as formally transparent that ATTRACTION faithfully instantiates other aspects of the Entailment Framework, especially the REH and entailment additivity.

The basic complexity hinges on the cardinality of the set P. ATTRACTION constraints where |P| = 1 are termed *simple* or *pairwise*. As I show, simple ATTRACT(R, {p}→q) has the same violation profile as when the x-entailment, p→x, q, is evaluated against the configuration of y, but when ATTRACTION is not simple, there is no one-to-one correspondence between the constraint and pairwise, REH entailments. Therefore, complex ATTRACTION constraints with |P|>1 must be seen as invoking entailment additivity.
Crucially, the existence of $p \rightarrow x q$ follows directly from the REH, as $p$ and $q$ are components of segments, and with entailment persistence via $xRy$, the $x$-entailment persists to $y$, so $\text{ATTRACT}(R, \{p\} \rightarrow q)$ may be directly compared to $p \rightarrow x q$. This can be concretely demonstrated by considering $\text{ATTRACT}(R_{\text{Adj}}, \{\text{place}\} \rightarrow \text{stricture})$, from (21), and the entailment, $\text{place} \rightarrow x \text{stricture}$. For clarity, consider a generic inventory $\{P, F, T, S\}$, and suppose $T$ and $P$ are obstruents $[-\text{cont}]$ and $F$ and $S$ are fricatives, $[+\text{cont}]$.

Furthermore, let $T$ and $S$ share coronal place, and let $P$ and $F$ share labial place. As (14) shows, in brute force fashion, $\text{ATTRACT}(R_{\text{Adj}}, \{\text{place}\} \rightarrow \text{stricture})$’s violation profile is identical to $y$’s violation profile of the $x$-entailment, $\text{place} \rightarrow x \text{stricture}$, which persists from $x$ to adjacent $y$.

(22) **Equivalence between ATTRACT and entailments when $|P|=1$:**

<table>
<thead>
<tr>
<th>$xR_{\text{Adj}}y$</th>
<th>$\text{ATTRACT}(R_{\text{Adj}}, {\text{place}}, \text{stricture})$</th>
<th>$x$-entailment $\text{place} \rightarrow \text{stricture}$</th>
<th>$y$’s violation of $\text{place} \rightarrow x \text{stricture}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. P P</td>
<td></td>
<td>$[\text{lab}] \rightarrow [\text{lab}]$</td>
<td></td>
</tr>
<tr>
<td>b. P F</td>
<td>*</td>
<td>$[\text{lab}] \rightarrow [\text{lab}]$</td>
<td>*</td>
</tr>
<tr>
<td>c. P T</td>
<td>$[\text{lab}] \rightarrow [\text{lab}]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. P S</td>
<td>$[\text{lab}] \rightarrow [\text{lab}]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e. F P</td>
<td>*</td>
<td>$[\text{lab}] \rightarrow [\text{lab}]$</td>
<td>*</td>
</tr>
<tr>
<td>f. F F</td>
<td></td>
<td>$[\text{lab}] \rightarrow [\text{lab}]$</td>
<td></td>
</tr>
<tr>
<td>g. F T</td>
<td></td>
<td>$[\text{lab}] \rightarrow [\text{lab}]$</td>
<td></td>
</tr>
<tr>
<td>h. F S</td>
<td></td>
<td>$[\text{lab}] \rightarrow [\text{lab}]$</td>
<td></td>
</tr>
<tr>
<td>i. T P</td>
<td>$[\text{cor}] \rightarrow [\text{cor}]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>j. T F</td>
<td>$[\text{cor}] \rightarrow [\text{cor}]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>k. T T</td>
<td>$[\text{cor}] \rightarrow [\text{cor}]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>l. T S</td>
<td>*</td>
<td>$[\text{cor}] \rightarrow [\text{cor}]$</td>
<td>*</td>
</tr>
<tr>
<td>m. S P</td>
<td>$[\text{cor}] \rightarrow [\text{cor}]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n. S F</td>
<td>$[\text{cor}] \rightarrow [\text{cor}]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>o. S T</td>
<td>*</td>
<td>$[\text{cor}] \rightarrow [\text{cor}]$</td>
<td>*</td>
</tr>
<tr>
<td>p. S S</td>
<td>$[\text{cor}] \rightarrow [\text{cor}]$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

However, when $|P|>1$, it is not immediately clear how to compare $\text{ATTRACT}(R, P \rightarrow q)$ to an entailment established by the REH. The form of implication posited directly
by the REH is pairwise \((p \rightarrow q)\), but the form of implication posited by Attraction, via (20), is of the form of Horn clauses, where for \(P = \{p_1, \ldots, p_k\}\), Attract\((R, P \rightarrow q)\) posits an entailment \(p_1 \land \ldots \land p_k \rightarrow q\).\(^{41}\) The difference is that the antecedents of simple entailments are single literals, but the antecedents of the implications enforced by complex Attraction are conjunctions of single literals.

Here, as a lemma in (23), I show that Attract\((R, P = \{p_1, \ldots, p_k\} \rightarrow q)\) can be expressed as a local conjunction (denoted ‘\&’; Smolensky, 1993, 1995; Smolensky & Legendre, 2006) of Attract\((R, \{p_1\} \rightarrow q)\) \& \ldots \& Attract\((R, \{p_k\} \rightarrow q)\). I call this style of additivity in OT conjunctive additivity because it makes use of constraint conjunction.

\[ (23) \textbf{Conjunctive Additivity Lemma:} \text{ Attract}(R, P = \{p_1, \ldots, p_k\} \rightarrow q) \text{ is equivalent to } \text{Attract}(R, \{p_1\} \rightarrow q) \& \ldots \& \text{Attract}(R, \{p_k\} \rightarrow q). \]

\textbf{Proof Sketch:} Among the benefits of Horn clauses is that the conjunction of antecedents has the same truth-value as a disjunction of pairwise implications: \((p_1 \land \ldots \land p_k) \rightarrow q \iff (p_1 \rightarrow q) \lor \ldots \lor (p_k \rightarrow q)\).\(^{42}\) In \((p_1 \rightarrow q) \lor \ldots \lor (p_k \rightarrow q)\), the terms are logical entailments, which individually correspond, respectively, to Attract\((R, \{p_1\} \rightarrow q)\), \ldots , and Attract\((R, \{p_k\} \rightarrow q)\), so in terms of constraints, a false implication, \(p_i \rightarrow q\), is a violation of the corresponding simple Attraction constraint. A logical disjunction of implications is only false when every term is false, so a logical disjunction is the same as a conjunction of constraint violations (e.g. Hansson, 2001; refs therein).\(^{43}\) Thus, \((p_1 \rightarrow q) \lor \ldots \lor (p_k \rightarrow q)\) is only false

\(^{41}\) The definition of Attraction, in (20), states that IF \((x, y \text{ agree on } p_i)\) AND \((x, y \text{ agree on } p_2)\) \ldots AND \((x, y \text{ agree on } p_n)\), THEN \(x\) and \(y\) must agree on \(q\), so Attraction has the same structure as a Horn clause.

\(^{42}\) \((p_1 \land \ldots \land p_k) \rightarrow q \iff \neg(p_1 \land \ldots \land p_k) \lor q \iff \neg(p_1 \lor \ldots \lor p_k) \lor q \iff \neg(p_1 \lor q) \lor \ldots \lor (p_k \lor q) \iff (p_1 \rightarrow q) \lor \ldots \lor (p_k \rightarrow q)\)

\(^{43}\) This move from logic to constraints is an application of De Morgan’s Law: \((p_1 \rightarrow q) \lor \ldots \lor (p_k \rightarrow q)\) can be expressed as \(\neg(\neg(p_1 \rightarrow q) \land \ldots \land \neg(p_k \rightarrow q))\). In logic, the truth of the entailment is of interest, but in a constraint system, it is the false, or violating, states that are of interest (see (6)). Applying De Morgan’s Law makes this conversion, and explains why logical disjunction becomes constraint conjunction.
when every simple $\text{ATTRACT}(\mathbf{R}, \{p_i\} \rightarrow q)$ constraint is violated, and so $\text{ATTRACT}(\mathbf{R}, \mathbf{P} \rightarrow q)$ has the same violation profile as $\text{ATTRACT}(\mathbf{R}, \{p_1\} \rightarrow q) \& \ldots \& \text{ATTRACT}(\mathbf{R}, \{p_k\} \rightarrow q)$. ■

The Conjunctive Additivity Lemma is important because it provides a way for complex ATTRACTION constraints to be expressed as a number of violations of simple entailments directly posited by the REH. These complex ATTRACTION constraints are only violated when all of the conjoined simple entailments are violated. Thus, the family of ATTRACTION constraints implements entailment additivity in OT by positing the existence of constraints which are the local conjunction of individual entailment constraints.

For example, a back-parasitic rounding harmony among vowels, as in Turkish (§2.3.1.2), may be implemented with $\text{ATTRACT}(\mathbf{R}, \{\text{cons}, \text{back}\} \rightarrow \text{[round]}) \gg \text{IDENT-IO([round])} \gg \{\text{ATTRACT}(\mathbf{R}, \{\text{cons}\} \rightarrow \text{[round]}), \text{ATTRACT}(\mathbf{R}, \{\text{back}\} \rightarrow \text{[round]})\}$. The higher ranking ATTRACTION constraint is only violated when both $\text{[cons]} \rightarrow x \text{[round]}$ and $\text{place} \rightarrow x \text{[round]}$ are violated, neither entailment violation alone is enough to override faithfulness, so in complex ATTRACTION, the effects of violating two or more separate entailments are summed through constraint conjunction. Therefore, although stated somewhat differently, ATTRACTION is tightly bound to the violations of entailments: simple ATTRACTION corresponds to violations of single pairwise entailments and complex ATTRACTION corresponds to the additive effect of simultaneously violating multiple pairwise entailments.

Here, a warning is in order: this section has shown that entailment additivity can be successfully implemented in Optimality Theory with local constraint conjunction.
(OTLCC). However, this should not necessarily be seen as evidence in favor of local conjunction, since the more basic numerical summation available in Harmonic Grammar is truer to the subsymbolic position for which I argue. Nevertheless, the Conjunctive Additivity Lemma confirms that there is a way of using OTLCC that is consistent with this lower-level approach.

The results below in §3.6 show that under certain conditions the grammatical attraction properties of OTLCC are identical to the attraction properties of HG. However, there are a number of empirical and formal studies (see Pater et. al, 2007; Coetzee & Pater, 2008; Pater 2009) which raise doubts about the general applicability of constraint conjunction. One particular dilemma is worth mentioning here: in the Conjunctive Additivity Lemma above, only simple ATTRACTION constraints were conjoined. At the level of OTLCC there is no independent reason why only ATTRACTION constraints can add up, but violations of, say, ATTRACTION and faithfulness constraints cannot. Nevertheless, the assumption of only conjoining ATTRACTION constraints with the same consequent is crucial to the results which follow. This is symptomatic of a general problem of determining what kinds of constraints are allowed to conjoin. To date, there does not seem to be any general solution, since the literature is mixed as to even the basic question of whether markedness and faithfulness constraints can conjoin much less the more detailed question of which kinds of markedness and faithfulness constraints participate in conjunctions.

In contrast, in Harmonic Grammar there is no such problem of determining which constraints add up, since all violations are cumulative. Therefore, at the HG-level, the dilemma is reformulated as the task of determining which violations are cumulatively
strong versus those which are cumulatively weak. As discussed in §3.4, phonetic similarity exactly predicts which parasitic dependencies are strong enough to motivate alternation, so the Entailment Framework makes significant inroads on this HG-level problem by being sensitive to the low-level similarity of representations. Thus, the OTLCC level must arbitrarily stipulate that only ATTRACTION constraints may conjoin, and so OTLCC vastly benefits from a lower-level explanation, where sensitivity to subsymbolic similarity can predict which dependencies are likely to occur in active ATTRACTION constraints.

3.5.5. Universal rankings for subset similarity relations

By viewing complex ATTRACTION as conjunctive entailment additivity, we may exploit a universal ranking principle of local conjunction theory. Namely, conjoined constraints must universally dominate each of the conjuncts; e.g., \( A&B \gg_{UG} \{A, B\} \) (Smolensky, 1993, 1995, 2006). This section shows that this ranking principle simultaneously greatly constrains the typology and successfully incorporates a notion of distance, which is essential to the analysis of parasitic assimilation.

Because of the way complex ATTRACTION constraints are composed of simple ATTRACTION constraints, the consequences of invoking \( A&B \gg_{UG} \{A, B\} \) are that constraints with higher prerequisite similarity must universally dominates constraints with lower prerequisite similarity. The specified rankings are explicitly given by the Subset Similarity Ranking Principle in (24):

---

44 This principle automatically falls out from Harmonic Grammar, so again the use of \( A&B \gg_{UG} \{A, B\} \) does not provide evidence for or against local conjunction versus numerical ranking. If anything the robustness of \( A&B \gg_{UG} \{A, B\} \) suggests something general about the nature of additivity that might be more naturally expressed in HG, where all constraint violations add up.
Subset Similarity Ranking Principle (SSRP):

If \( P' \subseteq P \), then \( \text{ATTRACT}(R, P \rightarrow q) \) universally dominates \( \text{ATTRACT}(R, P' \rightarrow q) \).

(24) suggests that for each member of the power set of a feature set, \( \mathcal{P}(P) \), there is an ATTRACTION constraint, and also provides a partial order among this set of constraints. This partial order is determined by principles of local conjunction theory, which must apply to ATTRACTION by the Conjunctive Additivity Lemma. The SSRP, as stated in (24), is more general than the universal domination required by pairwise local conjunction because the SSRP posits a ranking relationship for every proper subset relationship. However, by recognizing that local conjunction is associative and commutative, e.g., \( A \& B \& C = A \&(B \& C) = (A \& B)\&C = (A \& C)\&B \), and, therefore, \( A \& B \& C \) must universally dominate each of \( A \& B \), \( A \& C \), and \( B \& C \), the SSRP follows directly from an application of the Conjunctive Additivity Lemma.

To illustrate the SSRP, I now show how properties of the SSRP can be related to lattice theory, providing another tool for understanding and investigating the interaction of ATTRACTION constraints. (25), below, shows the power set of the prerequisite feature set, \( P = \{[\text{cons}], \text{manner}, \text{place}\} \). Generic manner and place are introduced now for the sake of continuity. These features were explicitly chosen as typical of unified feature theories (Clements & Hume, 1995; Ní Chiosáin & Padgett, 2001; Morén, 2006), extensively explored in Feature Geometry (Clements, 1985; Sagey, 1986; Halle, 1992), which make allowances for processes involving both consonants and vowels. This departure from more SPE (Chomsky & Halle, 1968) style feature bundles is introduced to account for a number of CV effects in harmony discussed in detail in Chapter 4, which

\[\text{Note that script, italic } \mathcal{P} \text{ denotes the power set, and non-script, bold } P, \text{ denotes the set of prerequisite features. } \mathcal{P}(P) \text{ is the set of all subsets of } P = \{p_1 \ldots p_k\}, \text{ including } P \text{ and the empty set, } \{\}.\]
are uniquely predicted under a unified account of consonant and vowel harmony in ATTRACTION. Unified features also highlight the important of ATTRACTION theory being restrictive in how it uses distance in thresholding (see §3.8)

This chapter uses examples from the domain of parasitic rounding harmony on vowels, so manner nearly corresponds to vowel height, place corresponds to vowel backness, and a feature [labial] would correspond to the harmonic feature in vowel rounding harmony, with the following caveat: these features are not only vowel features, but may also be carried by consonants, so harmony constraints must make reference to [cons] in order to limit harmony to vowels. Thus, manner indicates the shape of the oral constriction, i.e. the height of vowels or the continuancy of consonants, and may have dependents like [open], [hi], or [±continuant]. Place indicates the place of articulation and has dependents like, [labial], [coronal], and [dorsal]. Ultimately, I am more committed to the notion of unification of consonant and vowel harmony, than to the advancement of a particular geometry of phonological features.

The partial order of ATTRACTION constraints given by the SSRP is strictly defined based on a subset relationship, as the example figures, in (25)-(26) below, illustrate:

(25) Subset lattice for $P = \{[\text{cons}], \text{manner}, \text{place}\}$, lines indicate subset relationship:
Given the subset lattice in (25), it is possible to apply the SSRP to the ATTRACTION constraints for a prerequisite feature set, \( P = \{ [\text{cons}], \text{manner}, \text{place} \} \) and a harmonic feature \( q = [\text{labial}] \), as indicated in (26):

\[(26) \quad \text{Lines among } \text{ATTRACT}(R, P \rightarrow q) \text{ constraints indicate universal dominance:} \]

\[
\begin{align*}
\text{ATTRACT}(R, \{ [\text{cons}], \text{manner}, \text{place} \} \rightarrow [\text{labial}]) \\
\text{A}(R, [\text{cons}], \text{manner} \rightarrow [\text{lab}]) & \quad \text{A}(R, [\text{cons}], \text{place} \rightarrow [\text{lab}]) & \quad \text{A}(R, \text{manner}, \text{place} \rightarrow [\text{lab}]) \\
\text{A}(R, \{ [\text{cons}] \} \rightarrow [\text{labial}]) & \quad \text{A}(R, \{ \text{manner} \} \rightarrow [\text{labial}]) & \quad \text{A}(R, \{ \text{place} \} \rightarrow [\text{labial}]) \\
\text{ATTRACT}(R, \{ \} \rightarrow [\text{labial}])
\end{align*}
\]

Now, assuming the harmonic feature [labial] depends on these features in \( P \), then, the lattice-hierarchy in (26) expresses the set of universal rankings that exist by the SSRP.

In both (25) and (26), the supremum, top, of the lattice hierarchy depends on the choice of the set \( P \), while proper subsets of \( P \) fall on lower levels. The supremum ATTRACTION constraint requires agreement between segments that only differ on the harmonic feature. The infimum ATTRACTION constraint requires agreement between all segments, irrespective of differences.\(^{46}\) In (26), a node in the lattice with an arc to a node on a lower level, universally dominates the connected, lower-level node. For example, whatever the ranking of \( \text{ATTRACT}(R, \{ [\text{cons}] \} \rightarrow [\text{labial}]) \), \( \text{ATTRACT}(R, \{ [\text{cons}], \text{place} \} \rightarrow [\text{labial}]) \) is necessarily higher ranked because \( \{ [\text{cons}] \} \subset \{ [\text{cons}], \text{place} \} \).

\(^{46}\) The infimum ATTRACTION constraint exists by the power set operation, but it is universally low ranked, since there is no phonetic similarity between the empty set \( \{ \} \) and [labial].
Nodes in the lattice-hierarchy that share an arc differ on their prerequisite similarity requirements by exactly one feature, but because universal ranking is transitive, the SSRP, also specifies dominance between prerequisite-feature sets that differ on several features. For instance, \text{ATTRACT}(\mathbf{R}, \{\text{cons}, \text{manner, place}\} \rightarrow \text{[labial]}) \gg \text{ATTRACT}(\mathbf{R}, \{\text{cons}, \text{place}\} \rightarrow \text{[labial]}) \gg \text{ATTRACT}(\mathbf{R}, \{\text{cons}\} \rightarrow \text{[labial]}) \gg \text{ATTRACT}(\mathbf{R}, \{\} \rightarrow \text{[labial]}). These constraints, as are all monotonic paths from the supremum to infimum, are universally ordered in a specific to general fashion, and as such, \text{ATTRACTION} constraints have a subset of the violations of the more general constraints which they universally dominate.\footnote{Constraint conjunction can be implemented with violation profile intersection, so more complex \text{ATTRACTION} constraints can never expand the set of candidates which violate less complex constraints, rather they can only decrease the number of candidates which violate \text{ATTRACTION}.}

In this way, the universal rankings stipulated by the SSRP stratify \text{ATTRACTION} such that agreement for similar targets dominates agreement for dissimilar targets. Distance between trigger and target is, therefore, expressed in a pattern of constraint violations, as indicated in the example (27), which considers progressive rounding harmony in the context of a high, back vowel-trigger [u].

\begin{align*}
\text{(27)} \quad &\text{Distance under \text{ATTRACTION} for the SSRP ranking,} \\
&\text{\text{ATTRACT}(\mathbf{R}, \{\text{cons}, \text{manner, place}\} \rightarrow \text{[labial]})} \\
&\quad \gg \text{ATTRACT}(\mathbf{R}, \{\text{cons}, \text{place}\} \rightarrow \text{[labial]}) \\
&\quad \gg \text{ATTRACT}(\mathbf{R}, \{\text{cons}\} \rightarrow \text{[labial]}) \\
&\quad \gg \text{ATTRACT}(\mathbf{R}, \{\} \rightarrow \text{[labial]})
\end{align*}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
 & \text{ATTRACTION}(\mathbf{R}_{\mathbf{L} \rightarrow \mathbf{R}}, \ldots \rightarrow \text{[labial]}) & \{\text{cons}, \text{manner, place}\} & \{\text{cons}, \text{place}\} & \{\text{cons}\} & \{\} \\
\hline
a. & u \ddagger & * & * & * & * \\
b. & u \ddagger a & * & * & * \\
c. & u \ddagger e & * & * \\
d. & u \ddagger t & * & \\
\hline
\end{tabular}
\caption{Constraint conjunction can be implemented with violation profile intersection, so more complex \text{ATTRACTION} constraints can never expand the set of candidates which violate less complex constraints, rather they can only decrease the number of candidates which violate \text{ATTRACTION}.}
\end{table}
As (27) shows, the SSRP forces the most similar targets to violate the highest ranking constraints with dissimilar targets only violating lower ranked constraints. This shows that the SSRP implements the desired property of an attraction model that the force of attraction falls off with distance. In (27), as the distance between trigger and target grows (27)(a) to (27)(d), the lower the ranking of the violated agreement constraints.

Because each prerequisite feature is a conjunctive precondition to agreement, more specific ATTRACTION constraints focus the violations of ATTRACTION in a smaller region around the trigger. For example, ATTRACT(R, {[cons]} → [labial]) allows a vowel-trigger to attract both back and non-back vowels, but ATTRACT(R, {[cons], place} → [labial]) limits attraction to those vowels that agree in place, eliminating the variation of participating targets along place allowed by ATTRACT(R, {[cons]} → [labial]). Thus, preconditions place barriers to attraction across the variation on the specified feature dimension. As the number of barriers increases, less variation is allowed between triggers and targets and, therefore, the closer a target must be to a trigger in order to be attracted.

3.5.6. Deriving prerequisite similarity effects with faithfulness thresholding

In the present approach, whether or not a harmony obtains depends on the ranking of ATTRACTION relative to IO-FAITHFULNESS. Of course, ATTRACTION is itself a form of segment-to-segment faithfulness, identity being required between corresponding segments that meet prerequisite similarity, and these ATTRACTION constraints may certainly compete with one another when there are multiple harmonic features. But the more usual situation is that ATTRACTION competes with the faithfulness between input and output segments. Here, the most relevant IO-FAITH constraint is IDENT-IO(q) from
the Correspondence Theory of Faithfulness (McCarthy & Prince, 1995, 1999), which is defined in (28) below:

(28) **IDENT-IO(q)**, where q is a feature.48

Let I be an input and O an output, such that I corresponds with O in a candidate \( \alpha = <I, O> \), according to the correspondence relation \( R_{IO} \).

If \( aR_{IO}b \), where \( a \in I \) and \( b \in O \)

Then, \( \alpha \) incurs one mark, unless \( a, b \) agree on feature q.

**IDENT-IO(q)** penalizes outputs segments which do not agree with corresponding input segments on the feature q. In this chapter, I assume that other correspondence constraints are undominated such that output segments are in a one-to-one, contiguous, linear correspondence with segments in the input. Thus, deletion, coalescence, insertion, metathesis, and other restructuring repairs are not available, meaning segmental alternation (violation of **IDENT-IO**) is the only possible repair to segment-to-segment attraction.

This section shows how within this limited domain, the interaction of **ATTRACTION** and **IO-Faithfulness** in OT results in similarity thresholding, faithfulness

---

48 It is important to note that **IO-Faith** itself can be viewed as an attraction relation (Burzio & Tantalo, 2007; Burzio, 2002a, b, 2004, 2005), having the property that small deviations from the input are under greater pressure to be fully faithful than larger deviations. This explains, for example, the well-known effect of ’Non-derived Environment Blocking’ (see Burzio & Tantalo, 2007; Burzio, 2005) which describes cases where a process \( \phi \) is blocked unless another process has already taken place. **IO-Faith** as **ATTRACTION** explains this as follows: \( \phi \) is only stronger than **IO-Faith** in the derived environment because in the non-derived environment the output is more similar to the input, and so the force of **IO-ATTRACTION** is stronger and blocks \( \phi \).

However, for the harmony phenomena discussed in this dissertation, such gradience in faithfulness is not needed because the crucial comparisons occur along a single feature. Both **IO-ATTRACTION([F])** and conventional **IO-Faith([F])** prefer a candidate that agrees with the input on a feature [F] to one that disagrees on [F]. Therefore, although as part of the general framework all faithfulness (input-output, output-output, and segment-segment) is attraction, the more conventional correspondence-based **IO-Faith([F])** (McCarthy & Prince, 1995) is used throughout this dissertation for the sake of reader familiarity.
acting acts as a cut-off, or threshold, which determines the active preconditions to harmony. The methods which develop this threshold theory are more rigorous than previous parts of the dissertation as elements of set theory, including intersection, union, and closure, are applied to various aspects of attraction. The discussion proceeds as follows: I first introduce a basic attraction system of constraints and show how the behavior of a ranking of these constraints can be summarized by considering the set of trigger-target similarities which activate high-ranking ATTRACTION constraints. I then show that instead of considering all possible trigger-target similarities which result in harmony, it is possible to exactly describe the behavior of a basic attraction grammar with a threshold set, indicating the minimum similarity needed for harmony to obtain.

While the ranking of some ATTRACTION constraints is universally determined according to the SSRP, IDENT-IO(q) may be freely ranked with respect to ATTRACTION. This interaction is analyzed in a simplified domain, where assimilation of the target is the only possible repair to violations of ATTRACTION. Thus, the set of relevant constraints for basic attraction, CONBA, are as follows:

(29) **Def:** Let \( \mathcal{P}(\text{ATTRACT}(R, P \rightarrow q)) \) denote the set of ATTRACT(R, P \rightarrow q) constraints posited by the SSRP for a prerequisite feature set, \( P = \{p_1, \ldots, p_k\} \), i.e. \( \mathcal{P}(\text{ATTRACT}(R, P \rightarrow q)) = \{ \text{ATTRACT}(R, P' \rightarrow q) \mid \forall P' \in \mathcal{P}(P) \} \).

(30) **Def:** Let \( \text{CONBA} = \{ \mathcal{P}(\text{ATTRACT}(R, P \rightarrow q)) \} \cup \{ \text{IDENT-IO}(q) \} \)

As previewed above, under a set of basic attraction constraints, on non-harmonic inputs, IDENT-IO(q) is in direct conflict with some ATTRACTION constraints. When IDENT-IO(q) dominates all of the constraints in \( \mathcal{P}(\text{ATTRACT}(R, P \rightarrow q)) \), then agreement never takes place and all inputs emerge faithfully. Likewise, when all members of
\( \mathcal{P}(\text{ATTRACT}(R, P \rightarrow q)) \) dominate IDENT-IO\((q)\), then all outputs are fully harmonic for a feature \( q \), regardless of input. Though based on the discussion in §3.5 where the strength of attraction is related to similarity, one might expect that such a system where all attraction constraints dominate faithfulness is very unlikely to occur.

IDENT-IO\((q)\) discourages targets from altering, so if IDENT-IO\((q)\) dominates some but not all of \( \mathcal{P}(\text{ATTRACT}(R, P \rightarrow q)) \), then only those higher ranked ATTRACTION constraints are satisfied at the expense of faithfulness. The preconditions imposed by these higher ranked constraints can be formally identified as a set of active preconditions, \( U \), defined below in (31):

\[
\text{(31) Def: A set of active preconditions, } U, \text{ for a ranking of } \text{CON}_{BA}, \text{ as follows:}
\]

\[
U = \{ u_i | u_i \in \mathcal{P}(P) \text{ and } \text{ATTRACT}(R, u_i \rightarrow q) \text{ dominates } \text{IDENT-IO}(q) \}.
\]

Separate rankings of \( \text{CON}_{BA} \) have different sets of active preconditions, so \( U \) may be indicated to depend on a particular ranking, \( R \), as \( U_R \). However, the results of this section are universally quantified over all rankings of \( \text{CON}_{BA} \) subject to the SSRP, so the subscript is generally omitted. Note \( U \) is a set of sets of prerequisite features.

For example, given \( \mathcal{P}(P=\{p_1,p_2\}) = \{ \{p_1,p_2\}, \{p_1\}, \{p_2\}, \{\} \} \) and the ranking \( \text{ATTRACT}(R, \{p_1,p_2\} \rightarrow q) \gg \text{ATTRACT}(R, P=\{p_1\} \rightarrow q) \gg \text{IDENT-IO}(q) \gg \text{ATTRACT}(R, \{p_2\} \rightarrow q) \gg \text{ATTRACT}(R, P=\{\} \rightarrow q) \), then the set of active preconditions is \( U = \{ \{p_1, p_2\}, \{p_1\} \} \) because \( \text{ATTRACT}(R, \{p_1,p_2\} \rightarrow q) \) and \( \text{ATTRACT}(R, P=\{p_1\} \rightarrow q) \) each dominate IDENT-IO\((q)\).

The following theorem, below in (33), confirms that, for a given ranking of \( \text{CON}_{BA} \), \( U \) exactly determines the set of trigger-target pairs that result in assimilation.
Def: $\mathcal{A}(x, y)$ is a function that computes the set of agreeing features for segments $x$ and $y$.

Theorem: Given $xRy$, $\mathcal{A}(x, y) \in U$ if and only if $y$ assimilates to $x$ on feature $q$.

Proof: (If) By the definition of $U$ and $\mathcal{A}$, if $\mathcal{A}(x, y) \in U$, then $x$ and $y$ are similar enough to activate some ATTRACTION constraint which dominates IDENT-IO$(q)$, so given $xRy$, $y$ must assimilate to $x$ on $q$.

(Only if) If $y$ (non-trivially) assimilates to $x$, then the input must map to some unfaithful output, and so the optimal candidate, $a_{opt}$, incurs a violation of IDENT-IO$(q)$. The only way $a_{opt}$ could win is if the faithful candidate, $a_{f}$, contains a violation of some ATTRACTION constraint, say $\text{ATTRACT}(R, p', q)$, which dominates IDENT-IO$(q)$ motivating $y$’s assimilation to $x$, so by definition $p' \in U$. $\text{ATTRACT}(R, p', q)$ must be activated by trigger $x$ and target $y$, so $\mathcal{A}(x, y) = p'$, meaning $\mathcal{A}(x, y) \in U$. ■

Thus, by (33), $U$ contains the full set of feature-similarity preconditions which result in assimilation, and since $U$ is defined based on the relative ranking of ATTRACTION and IDENT-IO, it is clear that for a fixed ranking of ATTRACTION, the ranking of IDENT-IO exactly determines which trigger-target pairs result in assimilation. From a landscape perspective, $U$ may be termed the participation basin, as all segments which are ultimately attracted to a trigger have input positions located in a basin, which corresponds to $U$. The higher the rank of faithfulness, the smaller the size of the participation basin, so IDENT-IO prevents segments not in the basin from assimilating.

The set of active preconditions, $U$, can also be associated with a similarity threshold set, $T$, as defined in (34) below, which more concisely describes the behavior of an attraction grammar by listing the minimum preconditions that must be satisfied for harmony to obtain:
A threshold set, $T = \{ t_i \mid t_i \in U, \forall t_i' \subseteq t_i, t_i' \not\in U \}$, where $U$ is a set of active preconditions.

Thus, a threshold set is the set of preconditions, $T = \{ t_i \}$, for which there are no more general ATTRACTION constraints which also dominate faith. $T$ is, thus, the minimal description of the behavior of a ranking of $\text{CON}_{BA}$. For example, $U = \{ \{\text{cons}, \text{manner}, \text{place} \}, \{\text{cons}, \text{manner} \}, \{\text{cons}, \text{place} \}, \{\text{cons} \} \}$ has a threshold $T = \{\{\text{cons} \}\}$ meaning that all segments which agree on $\text{[cons]}$ must agree on the harmonic feature. In this case, violating $\text{ATTRACT}(R, \text{[cons]}, q)$ is sufficient to motivate harmony. Moreover, the other non-threshold constraints always incur a subset of the violations of $\text{ATTRACT}(R, \text{[cons]}, q)$, and so the non-threshold constraints do not need to be evaluated to determine whether a target undergoes harmony.

In terms of a subset lattice, like in (25), the elements of $T$ are nodes, which are in $U$, but do not have any children that are in $U$, so if the supremum is the root, then $T$ is the set of leaf nodes of $U$. Hence, $T$ consists of the minimum preconditions necessary for assimilation to obtain, which means that $\text{ATTRACT}(R, t_i \rightarrow q) \gg \text{IDENT-IO}(q)$ exactly determines the similarity boundary between the segments which are attracted to a trigger and those which are not. If $x$ and $y$ agree on the features in $t_i$, then $x$ and $y$ cross the similarity threshold and are subject to assimilation. For a particular ranking of $\mathcal{P}(\text{ATTRACT}(R, P \rightarrow q))$, the higher the relative ranking of $\text{IDENT-IO}(q)$, the more specific the threshold set, and therefore, the greater the similarity needed for assimilation to obtain. Thus, for a fixed ranking of $\mathcal{P}(\text{ATTRACT}(R, P \rightarrow q))$ the ranking of $\text{IDENT-IO}(q)$ exactly determines the prerequisite similarity threshold for assimilation.

A major theme of the remainder of this chapter is to explore how the threshold sets of attraction grammars in OT and HG stack up to empirical generalization about
thresholding. As I show in more detail in §3.8, the unfettered ranking of faithfulness and ATTRACTION constraints seems to predict threshold sets with more than one element.

Threshold sets with $|T| > 1$ are termed disjunctive because if any of the $t_i \in T$ are satisfied, then harmony obtains. For instance, consider $T = \{ \{ \text{cons}, \text{manner} \}, \{ \text{cons}, \text{place} \} \}$. With this threshold set harmony obtains if trigger and target agree on [cons], manner OR if trigger and target agree on [cons], place. This threshold set would exemplify a height-OR-back parasitic vowel harmony. However, as discussed below in §3.8, no such fully disjunctive parasitic systems are known to exist. Instead, empirically speaking, similarity preconditions tend to be conjunctive so that the threshold set is a single element, consisting of a set of conjunctive preconditions. Languages with the following thresholds are attested $T = \{ \{ \text{cons}, \text{manner} \} \}$, $T = \{ \{ \text{cons}, \text{place} \} \}$, and even $T = \{ \{ \text{cons}, \text{manner}, \text{place} \} \}$, but there are no systems that allow harmony to be parasitic on the disjunctive combination thereof.

What is perhaps most surprising is that eliminating disjunctive harmony by ensuring $|T| = 1$ imposes a strong condition on the ranking of ATTRACTION constraints with respect to IDENT-IO$(q)$, namely that the set of active preconditions is closed.

(35) **Def:** $U$ is closed under p-intersection, $\cap^p$, if and only if $\forall u_i, u_j \in U, u_i \cap^p u_j \in U$.

By the SSRP, $U$ is always closed under set union, i.e. if $u_i, u_j \in U$, then $u_i \cup u_j \in U$, but not necessarily closed under precondition-intersection (p-intersection). However, if and only if $U$ happens to be closed under p-intersection, then, as the following theorem in

---

49 Proof: $u_i \subseteq u_i \cup u_j \subseteq P$. By definition, $\text{ATTRACT}(R, u_i, q) \gg \text{IDENT-IO}(q)$, so by the SSRP and transitivity of dominance, $\text{ATTRACT}(R, u_i \cup u_j, q) \gg \text{ATTRACT}(R, u_i, q) \gg \text{IDENT-IO}(q)$. Thus, $u_i \cup u_j \in U$.

50 Let $P = \{ p_1, p_2, p_3 \}$. Suppose $\{ p_1, p_2 \}, \{ p_2, p_3 \} \in U$. $\{ p_1, p_2 \} \cap \{ p_2, p_3 \} = \{ p_2 \}$. $\{ p_2 \}$ is not necessarily an element of $U$ because IDENT-IO might intervene exactly between $\{ \text{ATTRACT}(R, \{ p_1, p_2 \}, q), \text{ATTRACT}(R, \{ p_2, p_3 \}, q) \}$ and $\text{ATTRACT}(R, \{ p_2 \}, q)$. 

- 129 -
(37) shows, the threshold set must consist of a single element which is the minimum of the set $U$:

\[
\text{(36) \hspace{1cm} Def: } u_{\text{min}} = \bigcap_{i}^{p} u_{i}.
\]

\[
\text{(37) \hspace{1cm} Theorem: } U \text{ is closed under p-intersection if and only if } T = \{ u_{\text{min}} \}.
\]

\[
\text{Proof: } (\text{If}) \hspace{1cm} \text{The existence of } u_{\text{min}} \text{ follows from } U \text{ being closed under p-intersection. That this } u_{\text{min}} \text{ is the only member of the threshold set, } T, \text{ follows from the subset properties required for membership in } T.
\]

\[
\text{Case 1: If there exist } u_{i}, u_{j} \text{ such that } u_{i} \cap u_{j} = \emptyset, \text{ then } \{ \} \text{ must be an element of } U, \text{ and by the definition of p-intersection } \forall u_{k} \in U,
\]

\[
u_{k} \cap \{ \} = \{ \}, \text{ so } u_{\text{min}} = \bigcap_{i}^{p} u_{i} = \{ \}. \text{ Here, } u_{\text{min}} \text{ must be in the threshold set, since there are no } t' \text{ such that } t' \subset \{ \}, \text{ and, thus, the definition of an element of a threshold set in (34) is vacuously satisfied. Because } \{ \} \text{ is a proper subset of all other members of } U, \text{ no other elements of } U \text{ may be in the threshold set, and so } u_{\text{min}} = \{ \} \text{ is the only element of } T.
\]

\[
\text{Case 2: If no such } u_{i}, u_{j} \text{ exist such that } u_{i} \cap u_{j} = \emptyset, \text{ then }
\]

\[
\text{By closure of } U, \forall u_{i}, u_{j} \in U, u_{i} \cap u_{j} \in U, \text{ so because of the associative properties of set-intersection, every successive intersection in } \bigcap_{i}^{p} u_{i} \text{ is non-empty and yields an element of } U, \text{ so } u_{\text{min}} = \bigcap_{i}^{p} u_{i} \text{ exists.}
\]

Suppose there exists some $t' \subset u_{\text{min}}$ such that $t' \in U$. Then, $u_{\text{min}} \cap t' = t'$ requires that $u_{\text{min}} = \bigcap_{i}^{p} u_{i} \subset t'$, but this contradicts that $u_{\text{min}}$ is the minimum of $U$ and $t' \subset u_{\text{min}}$, so $u_{\text{min}}$ must be in $T$. Suppose there were some other $u' \in T$, $u' \neq u_{\text{min}}$, then by intersection closure, $u' \cap u_{\text{min}}$ is in $U$, but $u' \cap u_{\text{min}} \subset u'$ since $u' \neq u_{\text{min}}$ which contradicts that $u' \in T$. Hence, $u_{\text{min}}$ is the only element of $T$.

(Only if) Assume $T$ is a singleton set $\{ u_{\text{min}} \}$. Let $U'$ be the set of supersets of $u_{\text{min}}$. $U' = \{ u' \mid u_{\text{min}} \subset u' \in P \}$. I will show that $U = U'$.

\[
U' \subset U: \text{ All supersets of } u_{\text{min}} \text{ are elements of } U, \text{ since by the SSRP } U \text{ is closed under union.}
\]
$U \subseteq U'$: Now, suppose there were some $v \in U$ such that $v \notin U'$. $v \notin U'$ implies $v \neq u_{\text{min}}$, and so $v \notin T$ since $T = \{u_{\text{min}}\}$. Therefore, by the definition of $T$, there must exist some subset $v' \subseteq v$ such that $v' \notin U'$. $v' \notin U'$, otherwise $v$ would be in $U'$, and so $v' \neq u_{\text{min}}$. Therefore, $v' \notin T$, so there must exist some subset $v'' \subseteq v'$, where $v'' \in U$. Similarly, $v'' \notin U'$ and $v'' \neq u_{\text{min}}$, so $v'' \notin T$. Eventually, by finiteness of $P$, there will be no more available subsets of $v$, so there exists $v^* \subseteq \ldots \subseteq v$ such that $v^* = \{\}$, $v^* \in U$, $v^* \neq u_{\text{min}}$ (else $v \in U'$), and, therefore, $v^* \notin T$, which of course contradicts that if $\{\} \in U$, $\{} \in T$, because $\{}$ vacuously satisfies the conditions for membership in $T$. Thus, there are no $v \in U$ such that $v \notin U'$.

Therefore, $U = U'$.

It remains to show that $U' = \{u' \mid u_{\text{min}} \subseteq u' \subseteq P\}$ is closed under p-intersection. Since $U'$ only contains supersets of $u_{\text{min}}$, $\forall u', u'_j \in U'$, $u'_i \cap \cap u'_j$ is a superset of $u_{\text{min}}$ and so is by definition an element of $U'$.

From the perspective of the SSRP, if $U$ is closed under intersection, then $u_{\text{min}}$ is the node which is universally dominated by every other element of $U$, and, in a subset-lattice, all possible ancestors of $u_{\text{min}}$ are elements of $U$. Moreover, if the minimal preconditions for assimilation can be succinctly described as a single conjunctive set of preconditions—which appears to be the case cross-linguistically—then $T$ must be a singleton set, and therefore $U$ is closed under p-intersection.

However, if $U$ is not closed under intersection, then, in general, the only way to determine $T$ is to check each subset of each member of $U$ in brute force fashion. Thus, the closure of the set of active preconditions ensures a means for concisely describing and therefore evaluating an attraction grammar. This efficiency argument might explain why cross-linguistically there is a preference for avoiding disjunction. There are, however, other benefits to a closed $U$: the next section (§3.6) shows that closure under p-intersection additionally guarantees equality for ATTRACTION in HG and OT. Beyond
these formal niceties, as I show in §3.8, there are also a number of empirical reasons related to learning that make it desirable to eliminate the possibility of disjunctive parasitic harmony.

### 3.6. Weighting ATTRACTION constraints

The local conjunctive additivity that underscored many of the above results contrasts with a more numerical kind of additivity allowed by Harmonic Grammar (HG; Legendre, Miyata, & Smolensky, 1991; Smolensky & Legendre, 2006), which might be termed *weighted additivity* because each constraint is assigned a numerical strength. Entailment Theory has generally been more closely related to weighted additivity than conjunctive additivity (Wayment *et al*., 2007; Burzio, 2002a, b), so it remains of interest to determine how local conjunction relates to weighted additivity for the case of basic attraction.

This section defines a Harmonic Grammar for basic attraction and identifies the effective numerical strength of complex ATTRACTION constraints in HG (§3.6.1). I, then, explore, in §3.6.2, the in principle arguments of (Smolensky, 2006), which suggest that local conjunction requires superadditivity of a sort not generally available in HG, but, as I show, superadditivity is not required for a conjunction of ATTRACTION, so a HG implementation of ATTRACTION remains available. §3.6.3 illustrates that the above properties of faithfulness thresholding and active preconditions are preserved in HG implementations of ATTRACTION. However, in §3.6.4, I show that while in general conjunctive additivity is strictly more powerful than weighted additivity, there is equality under the p-intersection closure of the set of active preconditions.
3.6.1. Defining Harmonic Grammar for CONB_{A}

Harmonic Grammar (HG; Legendre, Miyata, Smolensky, 1990; Smolensky & Legendre, 2006) is a constraint optimization system, which is somewhat similar to its more familiar descendent, Optimality Theory (OT; Prince & Smolensky, 2004). Both HG and OT require relative strengths to be assigned to the constraints over which they optimize, but in OT, these assignments are made with strict domination, while in HG, these assignments are made with numerical weighting.

Under strict domination, no number of violations of lower ranked constraints can motivate a single violation of a higher ranked constraint, prohibiting so-called “ganging up” effects by which the lower-ranked constraints “out vote” a higher ranked constraint. Thus, in OT, the penalty for violating a higher ranked constraint is infinitely higher than the penalty for violating the lower ranked constraint, so counting the total amount of violation of a candidate requires ordinal arithmetic. In general, there is no single natural or real number that may fill in the blank in the predicate: “The OT-Harmony of candidate α is ____.” Rather, an ordinal vector, or violation profile must be used (Prince, 2002).

For example, the violations of constraints A>>B>>C may be indicated as an ordinal vector, \( \mathbf{v}(\alpha) = (v_A(\alpha), v_B(\alpha), v_C(\alpha)) \), where \( v_x(\alpha) \) is a violation function, which gives the number of violations of constraint \( X \) by candidate \( \alpha \). In OT, the harmony of a candidate \( \alpha \) is its violation profile, \( \mathbf{v}(\alpha) \), because violation profiles order the candidates from most to least harmonic. Suppose a candidate \( \beta \) has \( \mathbf{v}(\beta) = (1, 0, 0) \) and \( \gamma \) has \( \mathbf{v}(\gamma) = (0, 10, 1) \), then \( \gamma \) is more harmonic than \( \beta \), denoted \( \gamma \succ_{\text{OT}} \beta \) or \( H_{\text{OT}}(\gamma) > H_{\text{OT}}(\beta) \). The arithmetic of OT is ordinal because under strict domination even if \( H_{\text{OT}}(\gamma) = (0, \infty, 1) \), \( H_{\text{OT}}(\gamma) > H_{\text{OT}}(\beta) = (1,0,0) \). In OT, a standard means of determining the optimum is to use conventional
OT tableaus, which provide an effective means of performing the necessary ordinal comparisons.

In a HG, however, the assignment of the relative strengths of constraints are made through *numerical weighting*: each constraint, $C_i$, is assigned a numerical strength $s_i$. Thus, a Harmonic Grammar may be defined as in (38), below:

\[(38) \quad \text{Def: } \text{HG is a three-tuple: } (\text{GEN, CON} = \{C_i\}, S=\{s_i\}), \text{ where} \]

\[
\begin{align*}
\text{GEN}(I) & \quad \text{is a generating function, which generates possible candidates for an input } I. \\
\text{CON} = \{C_i\} & \quad \text{is a set of violable constraints.} \\
S=\{s_i\} & \quad \text{is a set of numerical weights, } s_i \geq 0, \text{ corresponding to the constraints in } \text{CON}.
\end{align*}
\]

Unlike in OT, in HG, a single number exists that indicates the total amount of constraint violation and can be directly determined as a linear sum of $s_i \cdot v_{C_i}(\alpha)$, so no ordinal arithmetic is needed. Usually, in HG, instead of ordering candidates from smallest to largest penalties ($\beta \succ_{HG} \alpha$ means $\beta$ has less total penalty than $\alpha$) candidates are ordered from largest to smallest harmonies, denoted $H_{HG}$, ($\beta \succ_{HG} \alpha$ means $\beta$ is more harmonic than $\alpha$), so harmony is the negative of the total penalty. Thus, $\text{EVAL}_{HG}$ may be given in terms of harmony as below in (39):

\[(39) \quad \text{Def: } \text{EVAL}_{HG}(I) = \text{arg max}_{\alpha \in \text{GEN} (I)} H_{HG} (\alpha) \\
= \text{arg max}_{\alpha \in \text{GEN} (I)} \sum_{C_i \in \text{CON}} -s_i \cdot v_{C_i} (\alpha)
\]
Among the primary benefits of EVAL\textsubscript{HG} is that it ensures a neurally plausible implementation (Smolensky & Legendre, 2006). For instance, if a HG is faithfully instantiated in a harmony maximizing neural network, EVAL\textsubscript{HG} produces the same results as evaluating a network. In Chapter 5, I present connectionist network simulations of assimilation, so it is important at the outset to ensure that there is some transfer between symbolic OT analyses and more connectionist simulations. HG can provide that bridge between these various levels of description of ATTRACTION (see Smolensky & Legendre, 2006 for other benefits of multiple levels of description).

However, because HG departs from ordinal arithmetic, it does not always implement the basic OT tenet of strict dominance (although see Prince, 2002 for sufficient conditions for a weighting in HG to give strict dominance). For example, if CON = \{A, B, C\} and \(s_A = 10, s_B = 2, s_C = 1\), then \(\gamma\) with \(v(\gamma) = (0, 10, 1)\) has \(H_{HG}(\gamma) = -s_A \cdot v_A(\gamma) + -s_B \cdot v_B(\gamma) + -s_C \cdot v_C(\gamma) = -10 \cdot 0 + -2 \cdot 10 + -1 \cdot 1 = -21\); on the other hand, \(\beta\) with \(v(\beta) = (1, 0, 0)\) has \(H_{HG}(\beta) = -10 \cdot 1 + -2 \cdot 0 + -1 \cdot 0 = -10\). Since \(H_{HG}(\beta) = -10 > H_{HG}(\gamma) = -21\), \(\beta \succ_{HG} \gamma\), even though \(\beta\) has a violation of the higher ranked constraint. Therefore, it is crucial to understand how ATTRACTION in OT relates to ATTRACTION in HG because it is not a foregone conclusion that there will necessarily be any transfer at all. As a step towards comparing ATTRACTION in HG and OT, the remainder of this section shows that the ganging up properties of HG may be exploited for the weighted additivity of ATTRACTION constraints.

In §3.5, I showed how complex ATTRACTION constraints can be expressed as a local conjunction of simple ATTRACTION constraints. These local conjunction operations create new constraints, which are part of the basic set of attraction constraints, CON\textsubscript{BA} =
\{ \mathcal{P}(\textsc{Attract}(R, P=\{p_1, \ldots, p_k\} \rightarrow q)) \} \cup \{ \textsc{Ident}-\textsc{IO}(q) \}. \) However, in standard HG, local conjunction operations are not possible, so local conjunction constraints do not exist, and so cannot be ranked independently. Hence, the constraint set for an OT-grammar with local conjunctions must be distinct from the constraint set for its HG implementation. \( \textsc{CON}_{\textsc{HG}} \) is simpler than \( \textsc{CON}_{\textsc{OT}} \), as it only admits the constraints that are not local conjunctions. The constraints in \( \mathcal{P}(\textsc{Attract}(R, P=\{p_1, \ldots, p_k\} \rightarrow q)) \) that are not local conjunctions are \( \textsc{Attract}(R, \{p_1\} \rightarrow q) \), \( \ldots \), \( \textsc{Attract}(R, \{p_k\} \rightarrow q) \), and \( \textsc{Attract}(R, \{\} \rightarrow q) \). Thus, define \( \textsc{CON}_{\textsc{BA-simple}} \) in (40), as follows:

(40) \[ \textbf{Def:} \ \textsc{CON}_{\textsc{BA-simple}} = \{ \textsc{Attract}(R, \{p_1\} \rightarrow q), \ldots, \textsc{Attract}(R, \{p_k\} \rightarrow q), \\textsc{Attract}(R, \{\} \rightarrow q), \textsc{Ident}-\textsc{IO}(q) \} \]

This smaller constraint set does not mean that conjunctive constraints have no effects on the optimization. However, unlike OT, in HG, the penalties for violating conjunctive constraints are not independently rankable.

HG asserts that candidates which violate multiple constraints incur the linear sum of the penalties for violating each constraint, so penalties for violating conjunctive constraints are still available, but are not independent parameters. Thus, in a HG, the effective strength of a conjoined constraint \( A \& B \) is \( e_{A \& B} = s_A + s_B \). Because of the Conjunctive Additivity Lemma, (23), in the \textsc{Attraction} framework is it possible to identify the effective strength of every constraint in \( \textsc{CON}_{\textsc{BA}} \) given the numerical weighting of \( \textsc{CON}_{\textsc{BA-simple}} \), as shown in (41). For succinctness, the associated strengths of \( \textsc{CON}_{\textsc{BA-simple}} = \{ \textsc{Attract}(R, \{p_1\} \rightarrow q), \ldots, \textsc{Attract}(R, \{p_k\} \rightarrow q), \textsc{Attract}(R, \{\} \rightarrow q), \textsc{Ident}-\textsc{IO}(q) \} \) are denoted \( s_{\textsc{BA-simple}} = \{ s_{p_1}, \ldots, s_{p_k}, s_{\{\}}, s_{\textsc{Ident}(q)} \} \).
Effective Strength Lemma: Let $P' = \{p'_1 \ldots p'_m\} \subseteq P = \{p_1 \ldots p_k\}$. The effective strength, $e_{P'}$, of $\text{ATTRACT}(R, P' \rightarrow q) \in \mathcal{P}(\text{ATTRACT}(R, P \rightarrow q)$ is

$$
e_{P'} = \begin{cases} 
s_{p'_1} + \ldots + s_{p'_m} + s() & \text{if } |P'| > 0 \\
        s() & \text{if } |P'| = 0 \end{cases}
$$

Proof: If $|P'| = 0$, $P' = \{\}$, so clearly $e_{P'} = s()$, since the general agreement constraint is available in $\text{CON}_{\text{BA-simple}}$. When $|P'| = 1$, the given effective strength is true, $e_{P'} = s_{p'_1} + s()$, since a candidate must violate both the specific $\text{ATTRACT}(R, \{p'_1\} \rightarrow q)$ and the more general $\text{ATTRACT}(R, \{\} \rightarrow q)$. If $|P'| > 1$, because of the Conjunctive Additivity Lemma, $\text{ATTRACT}(R, P' \rightarrow q) = \text{ATTRACT}(R, \{p'_1\} \rightarrow q) \& \ldots \& \text{ATTRACT}(R, \{p'_m\} \rightarrow q)$, so the complex attraction constraint is violated exactly when each of its simple constraints are violated. In HG, these simple constraints have strengths $s_{p'_1}, \ldots, s_{p'_m}$. Additionally, any candidate which disagrees on $q$ incurs a violation of general attraction, $\text{ATTRACT}(R, \{\} \rightarrow q)$, so $e_{P'} = s_{p'_1} + \ldots + s_{p'_m} + s()$. ■

(41) shows how the violations of conjoined constraints in $\text{CON}_{\text{BA}}$ are scored in HG, confirming that the violations of simple ATTRACTION combine through weighted additivity to give the effective strength of complex ATTRACTION.

For example, if $P = \{[\text{cons}], \text{manner}, \text{place}\}$ and $q = \text{labial}$, then $\text{CON}_{\text{BA-simple}} = \{\text{ATTRACT}(R, \{[\text{cons}]\} \rightarrow \text{labial}), \text{ATTRACT}(R, \{\text{manner}\} \rightarrow \text{labial}), \text{ATTRACT}(R, \{\text{back}\} \rightarrow \text{labial}), \text{ATTRACT}(R, \{\} \rightarrow \text{labial})\}$ with associated strengths $S = \{s_{[\text{con}]}, s_{\text{manner}}, s_{\text{place}}, s(), s_{\text{IDENT-IO(labial)}}\}$. The effective strength of $\text{ATTRACT}(R, \{[\text{con}], \text{manner}\} \rightarrow \text{labial})$ is $e_{[\text{con}], \text{manner}} = s_{[\text{con}]} + s_{\text{manner}} + s()$ and $e_{\text{manner}, \text{place}} = s_{\text{manner}} + s_{\text{place}} + s()$. 

- 137 -
3.6.2. The (non)-issue of superadditivity

This section focuses on the following question: Does weighted additivity in HG yield the same results as conjunctive additivity in OT? As Smolensky (2006) shows, in general, conjunctive additivity differs from weighted additivity in at least one important respect: local constraint conjunction implements superadditivity, whereby the strength of a conjunctive violation is strictly greater than the sum of the penalties for violating the conjuncts. However, I argue that for the present case where local conjunction is limited to ATTRACTION constraints with the same harmonic feature, $q$, this difference between HG and local conjunction is not relevant because superadditivity is simply not required for complex ATTRACTION constraints to motivate alternation.

For the sake of contrast, I review Smolensky’s (2006) argument for superadditivity of local conjunction. It may be summarized as follows: suppose there are two markedness constraints *[+F] and *[+G], which may be respectively be repaired by violations to FAITH(F) and FAITH(G). Further, suppose that FAITH(F) $>$ *[+F] and FAITH(G) $>$ *[+G]. Therefore, all candidates emerge faithfully to F and G. A HG, which preserves these assumptions must posit that $s_{FAITH(F)} > s_{*[+F]}$ and $s_{FAITH(G)} > s_{*[+G]}$, where $s_i$ is the strength of the corresponding constraint.

Further suppose that it is desirable to eliminate inputs from being faithful, exactly when they violate both *[+F] and *[+G]. In OT$_{LCC}$, this is directly possible with *[+F]&*[+G], but HG must rely on effective strengths. Note that the assumptions, $s_{FAITH(F)} > s_{*[+F]}$ and $s_{FAITH(G)} > s_{*[+G]}$, must still apply in order to keep other inputs ( [−F, +G], [+F, −G], [−F, −G]) from altering. In HG, conjunctive penalties arise from the weighted sum of the penalties of the conjuncts (as in the Effective Strength Lemma), so a faithful candidate, [+F, +G], which violates both *[+F] and*[+G], incurs a penalty with
effective strength, $e_{*[F]\&*[+G]} = s_{*[+F]} + s_{*[+G]}$. Crucially, by $s_{\text{FAITH}(F)} > s_{*[+F]}$ and $s_{\text{FAITH}(G)} > s_{*[+G]}$, the penalty $s_{*[+F]} + s_{*[+G]}$ is less than each of $s_{\text{FAITH}(F)} + s_{*[+G]}$, $s_{*[+F]} + s_{\text{FAITH}(G)}$, and $s_{\text{FAITH}(F)} + s_{\text{FAITH}(G)}$, so the faithful candidate, $[+F, +G]$, is HG-optimal over all other candidates that are the least bit unfaithful, even though it violates both $*[+F]$ and $*[+G]$. This is true for all possible weights of the constraints, so there is no HG that implements the local conjunction of $*[+F]$ and $*[+G]$ for the case, where $s_{\text{FAITH}(F)} > s_{*[+F]}$ and $s_{\text{FAITH}(G)} > s_{*[+G]}$.

This contrasts with a local conjunction in OT: suppose $*[+F]\&*[+G]$ dominates the other constraints, then a candidate which violates the conjoined constraint is less optimal than every other candidate. As Smolensky notes, in order for HG to perform such local conjunction, the penalty for violating the conjunctive candidate must be $s_{*[+F]\&*[+G]} = s_{*[+F]} + s_{*[+G]} + \varepsilon$, where $\varepsilon$ is a number such that, $s_{*[+F]} + s_{*[+G]} + \varepsilon$ is greater than each of $s_{\text{FAITH}(F)} + s_{*[+G]}$, $s_{*[+F]} + s_{\text{FAITH}(G)}$, and $s_{\text{FAITH}(F)} + s_{\text{FAITH}(G)}$. Thus, local conjunction requires superadditivity, a penalty above and beyond the combined penalties of the parts.

One way of understanding superadditivity is through the notion of transitivity of conjunctive violations. HG posits all conjunctive violations with transitivity, so the effective strength of $*[+F]\&*[+G]$, $e_{*[+F]\&*[+G]} = s_{*[+F]} + s_{*[+G]}$ must necessarily be less than $e_{\text{FAITH}(F)\&\text{FAITH}(G)} = s_{\text{FAITH}(F)} + s_{\text{FAITH}(G)}$, if $s_{\text{FAITH}(F)} > s_{*[+F]}$ and $s_{\text{FAITH}(G)} > s_{*[+G]}$. However, local conjunction in OT does not posit all conjunctive violations with transitivity. For instance, depending on the theory of local conjunction, $\text{FAITH}(F)\&\text{FAITH}(G)$ may not even be required to exist, much less dominate, $*[+F]\&*[+G]$, simply because $\text{FAITH}(F) >> *[+F]$ and $\text{FAITH}(G) >> *[+G]$. Thus, in OT, local conjunctions are freely rerankable.
with respect to one another, but in HG there is no possible way to rerank such “second order” constraints; the order among the second order is wholly dependent on the order among the first.

The crucial difference between this example and the ATTRACTION framework is that, in the above example requiring superadditivity, the conjoined markedness constraints are repaired by independent violations of faithfulness, but ATTRACTION constraints which are conjoined are, fortunately, repaired by the same violation of faithfulness. As discussed, only constraints with the same consequent may be conjoined and since the conjuncts all have the same consequents, they have the same repair, IDENT-IO(q). Thus, as shown in the tableau in (42), a faithful candidate, α, (42)(a), which violates a complex ATTRACTION constraint (and its relevant simple ATTRACTION constraints), may be repaired to a candidate β, (42)(b), which does agree on q, by incurring a single violation of IDENT-IO(q), but no other violations of other constraints.

(42) Total repair with a single violation of Faith

<table>
<thead>
<tr>
<th></th>
<th>ATTRACT (R, {p₁, p₂} → q)</th>
<th>IDENT-IO(q)</th>
<th>ATTRACT (R, {p₁} → q)</th>
<th>ATTRACT (R, {p₂} → q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. α:</td>
<td>xRy, x, y agree on p₁ and p₂, but disagree on q</td>
<td>*!</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>b. β:</td>
<td>xRy, x, y agree on p₁, p₂, and q</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

In Smolensky’s superadditive example, repairing the violation of each markedness conjunct incurred its own penalty, but (42) shows that the two simple ATTRACTION violations and one more complex ATTRACTION are repaired at the price of a single faithfulness violation. Therefore, while in general local conjunction, the penalty for violating conjuncts may need to be larger than the penalty of two faithfulness violations (e.g. s_{FAITH(F)} + s_{FAITH(G)}), under ATTRACTION, violating conjuncts must only ever be greater
than a single violation of faith (e.g. \( s_{\text{IDENT}-\text{IO}(q)} \)), so a HG does exist that satisfies the assumptions, \( s_{\text{IDENT}-\text{IO}(q)} > s_{p_1 \rightarrow q} \), \( s_{\text{IDENT}-\text{IO}(q)} > s_{p_2 \rightarrow q} \), and \( s_{p_1 \rightarrow q} + s_{p_2 \rightarrow q} > s_{\text{IDENT}-\text{IO}(q)} \). As (43) demonstrates, these conditions are met with \( s_{\text{IDENT}-\text{IO}(q)} = 0.5 \), \( s_{p_1 \rightarrow q} = 0.4 \), and \( s_{p_2 \rightarrow q} = 0.3 \), and, therefore, the conjunctive candidate (43)(a) is also correctly eliminated in HG.

(43) HG that gives the same result as conjunctive additivity

<table>
<thead>
<tr>
<th>S: ( \text{IDENT}-\text{IO}(q) )</th>
<th>( \text{ATTRACT (R, } {p_1} \rightarrow q) )</th>
<th>( \text{ATTRACT (R, } {p_2} \rightarrow q) )</th>
<th>Total Harmony</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( \alpha ): ( xRy ), ( x, y ) agree on ( p_1 ) and ( p_2 ), but disagree on ( q )</td>
<td>-0.5</td>
<td>-0.4</td>
<td>-0.3</td>
</tr>
<tr>
<td>( \Rightarrow ) b. ( \beta ): ( xRy ), ( x, y ) agree on ( p_1 ), ( p_2 ), and ( q )</td>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.5 + 0.0 + 0.0 = -0.5</td>
</tr>
</tbody>
</table>

This line of reasoning defuses the in principal arguments which might suggest that some sort of equality between HG and OT\textsubscript{LCC} implementations of ATTRACTION is impossible, laying the foundation for the main result of §3.6.4, which investigates whether all OT-rankings of CON\textsubscript{BA} may be expressed as a HG over CON\textsubscript{BA-simple}, and vice versa.

3.6.3. Thresholding in Harmonic Grammar

Before proceeding to these results, I restate the above OT\textsubscript{LCC}-level theorems from §3.5.6 about the SSRP, active preconditions, and threshold sets. These theorem can be directly applied to a HG based on the observation that \( e_{p'} > s_{\text{IDENT}-\text{IO}(q)} \) has the same force in a HG of CON\textsubscript{BA-simple} as \( \text{ATTRACTION}(R, P' \rightarrow q) >> \text{IDENT}-\text{IO}(q) \) under CON\textsubscript{BA} in OT\textsubscript{LCC}. Namely, if \( xRy \) are in correspondence and activate an ATTRACTION constraint with effective strength, \( e_{p'} > s_{\text{IDENT}-\text{IO}(q)} \), there is an incentive for assimilation in HG because the faithful candidate, \( \alpha_F \), has a penalty greater than or equal to \( e_{p'} \), while an unfaithful candidate, \( \alpha_{\text{opt}} \) that has agreement between \( x \) and \( y \) incurs a lesser penalty since \( e_{p'} > s_{\text{IDENT}-\text{IO}(q)} \), so HG gives the same preference as OT\textsubscript{LCC}: \( \alpha_{\text{opt}} \succ \alpha_F \). Likewise, \( s_{\text{IDENT}-\text{IO}(q)} \)
IO(q) > \epsilon_P yields the same blocking of assimilation under CONBA-simple as IDENT-IO(q)

>>> \text{OT ATTRACTION}(R, P', q) does under CONBA. Therefore, membership in the set of active preconditions, \( U_{HG} \), as defined in (44), has the same results as membership in \( U_{OT} \) as defined in (31), so the definitions and theorems of \$3.5.6$ are included without proof.

(44) **Def:** A set of active preconditions, \( U_{HG} \), for a ranking of CONBA-simple, as follows:
\[
U_{HG} = \{ u_i \mid u_i \in \mathcal{P} \text{ and } e_{u_i} > s_{\text{IDENT-IO}(q)} \},
\]
where \( e_{u_i} \) is given by the Effective Strength Lemma, (43).

(45) **Theorem:** Given \( x \text{R} y, \mathcal{A}(x, y) \in U_{HG} \) if and only if \( y \) assimilates to \( x \) on \( q \).

(46) **Theorem:** \( U_{HG} \) is closed under p-intersection if and only if \( T = \{ u_{\text{min}} \} \).

These proofs are identical to those previously given, except the terminology of “constraint dominance” must be replaced with “greater effective strength”. However, these proofs do rely on the SSRP (both implicitly and explicitly), so it is worth restating in terms of HG as in (47):

(47) **Subset Similarity Ranking Principle (SSRP)\text{HG}:**
\[
\text{If } \mathcal{P}' \subset \mathcal{P}, \text{ then } \epsilon_{\mathcal{P}} \succeq_{\text{UG}} \epsilon_{\mathcal{P}'}.
\]

Note in OT\text{LCC}, the SSRP follows from the theory of local conjunction and the Conjunctive Additivity Lemma. However, in HG, the SSRP may be derived from the workings of conjunctive violation and the Effective Strength Lemma. Briefly, in HG, a superset violates all the same simple constraints as a proper subset, and additionally
violates some other simple constraint, with strength $s_{pi}$, so $e_p = e_{p'} + s_{pi}$, and since $s_{pi}$ is required to be non-negative, $e_p \geq e_{p'}$, with equality only when $s_{pi} = 0$.

3.6.4. Harmonic Grammar vs. OT in the arena of ATTRACTION

Normally, showing equality of HG and OT requires showing that for all inputs $I$, the optimal HG output is equivalent to the optimal OT output, i.e. $\forall$ inputs $I$, $\text{EVAL}_{HG}(\text{GEN}_{HG}(I)) = \text{EVAL}_{OT}(\text{GEN}_{OT}(I))$. However, having theorems, as above, which relate the qualitative behavior of HG or OT$_{LCC}$ to a set of active preconditions allows HG and OT$_{LCC}$ to be directly compared without having to consider the evaluation of individual inputs. The main results of this section are that in a basic attraction system that (i) every HG of ATTRACTION with weighted additivity has a corresponding OT$_{LCC}$-grammar of ATTRACTION with conjunctive additivity, (ii) but, the reverse is not true, so, in general, conjunctive additivity of ATTRACTION is more powerful than weighted additivity of ATTRACTION, (iii) however, if the set of active preconditions is closed under p-intersection, then there does exist a HG for every OT$_{LCC}$-grammar of ATTRACTION.

A few more assumptions must be explicitly stated before stating the theorems of this section: let $\text{GEN}_{BA}$ be a candidate generator function common to both OT$_{LCC}$ and HG, such that violations of ATTRACTION may only be repaired through the assimilation of the target to the trigger on feature $q$. Thus, $\text{GEN}_{BA}$ preserves trigger inalterability and requires that that if targets are in a position where harmony is licensed, then targets are in correspondence with triggers. Therefore, in this basic attraction system, an OT$_{LCC}$ or HG grammar can only determine which targets, if any, in an input form will participate in

---

51 Only $s_{(1)}$ is ever set to zero, so the SSRP yields strictly greater effective strengths. While not essential, allowing $s_{(1)}$ to be equal to zero simplifies some of the proofs which follow.
harmony. Other possible repairs are eliminated by GEN\textsubscript{BA}, including, deletion, insertion, metathesis, breaking correspondence, or changing the identity of non-\(q\) features.\textsuperscript{52}

Let \(\text{CON}\textsubscript{BA}\), as in (30), be a set of basic attraction constraints, and let \(\text{CON}\textsubscript{BA-simple}\) be the corresponding simple attraction constraints in \(\text{CON}\textsubscript{BA}\), as in (40). Thus, \(\text{CON}\textsubscript{BA}\) and \(\text{CON}\textsubscript{BA-primitive}\) operate over the same set of prerequisite and harmonic features. Furthermore, let \(S = \{s_l\}\) be a positive, numerical weighting of \(\text{CON}\textsubscript{BA-simple}\), and let \(R\) be a strict ranking of \(\text{CON}\textsubscript{BA}\), where both \(S\) and \(R\) conform to the SSRP, as in (47) and (24), respectively. Finally, let \(U\textsubscript{OT}\) be the set of active preconditions for \(R\), as defined in (31), and let \(U\textsubscript{HG}\) be the set of active preconditions for \(S\), as defined in (44). With these definitions in place, it is possible to state a lemma confirming that equality in terms of sets of active preconditions yields grammatical equality in terms of evaluation, as in (48).

\[
\text{(48) Equivalence Lemma: } HG(\text{GEN}\textsubscript{BA}, \text{CON}\textsubscript{BA-simple}, S) = OT\textsubscript{LCC}(\text{GEN}\textsubscript{BA}, \text{CON}\textsubscript{BA}, R) \text{ if and only if } U\textsubscript{HG} = U\textsubscript{OT}.
\]

\textbf{Proof sketch:} (If) if \(HG=OT\textsubscript{LCC}\), then \(\forall \text{ inputs } I, \text{ EVAL}\textsubscript{HG}(\text{GEN}(I)) = \text{ EVAL}\textsubscript{OT}(\text{GEN}(I))\), and so for all inputs, \(HG\) and \(OT\textsubscript{LCC}\) have the same result. If \(xRy \in \text{GEN}\textsubscript{BA}(I)\) and upon \(\text{EVAL}\textsubscript{HG}(\text{GEN}(I)) = \text{ EVAL}\textsubscript{OT}(\text{GEN}(I))\), \(y\) assimilates to \(x\) on feature \(q\), then by the theorem in (33), \(A(x, y) \in U\textsubscript{OT}\) and by (45) \(A(x, y) \in U\textsubscript{HG}\). Likewise, if \(xRy \in \text{GEN}\textsubscript{BA}(I)\), and upon \(\text{EVAL}\textsubscript{HG}(\text{GEN}(I))\) = \(\text{EVAL}\textsubscript{OT}(\text{GEN}(I))\), \(y\) does not assimilate to \(x\) on feature \(q\), then by (33), \(A(x, y) \notin U\textsubscript{OT}\) and by (45) \(A(x, y) \notin U\textsubscript{HG}\). Therefore, \(U\textsubscript{OT} = U\textsubscript{HG}\).

(Only if) if \(U\textsubscript{HG} = U\textsubscript{OT}\), then for all \(xRy\), \(A(x, y) \in U\textsubscript{OT} \Leftrightarrow A(x, y) \in U\textsubscript{HG}\), so \(\forall \text{ inputs } I, \text{ if } xRy \in \text{GEN}\textsubscript{BA}(I)\) and \(A(x, y) \in U\textsubscript{OT}\), then \(A(x, y) \in U\textsubscript{HG}\), so by (33) and (45), on input \(I\), both \(HG\) and \(OT\) yields assimilation of \(y\) to \(x\) on feature \(q\). If,

\textsuperscript{52} It is not known whether these strong conditions are needed in general, but they make the analysis much more tractable. Even with these conditions, the proofs are not trivial, and, of course, determining trigger and target status seems far beyond the modest aims of the basic attraction system, which is the focus of this section. Furthermore, systems that allow these other repairs are not just complex, they are also not assimilation, so they fall outside the focus of this chapter.
However, \( xRy \in \text{GEN}_{BA}(I) \) and \( \mathcal{A}(x, y) \notin U_{OT} \), then \( \mathcal{A}(x, y) \notin U_{HG} \), so by (33) and (45), on input \( I \), both HG and OT \( LCC \) do not yield assimilation of \( y \) to \( x \) on feature \( q \). Therefore, \( \text{EVAL}_{HG}(\text{GEN}_{BA}(I)) = \text{EVAL}_{OT}(\text{GEN}_{BA}(I)) \).

This Equivalence Lemma hinges on limiting the basic attraction system to assimilatory repairs. Without that limitation, a candidate could have a trigger and target which are similar enough to be in the set of active preconditions, but the target still fails to assimilate because some other repair violates a lower ranked constraint. In that case, \( U_{OT} = U_{HG} \), but OT and HG might differ on the lower ranked repairs, and so have different outcomes. The Equivalence Lemma feeds directly into the first result, in (49) below, that shows that every HG of \( \text{CON}_{BA\text{-simple}} \) has a corresponding OT \( LCC \) grammar of \( \text{CON}_{BA} \), and therefore, for the purposes of \( \text{ATTRACTION} \), HG \( \subseteq \text{OT}_{LCC} \).

(49) **Theorem:** HG(\( \text{GEN}_{BA}, \text{CON}_{BA\text{-simple}}, S \)) \( \subseteq \text{OT}_{LCC}(\text{GEN}_{BA}, \text{CON}_{BA}, R) \).

**Proof:** I will show that \( \forall \) numerical weightings, \( S \), of \( \text{CON}_{BA\text{-simple}} \) with corresponding \( U_{HG} \), \( \exists \) a strict ranking, \( R \), of \( \text{CON}_{BA} \) with corresponding \( U_{OT} \), such that \( U_{OT} = U_{HG} \). If this is true, then by application of the Equivalence Lemma (48), HG(\( \text{GEN}_{BA}, \text{CON}_{BA\text{-simple}}, S \)) \( \subseteq \text{OT}_{LCC}(\text{GEN}_{BA}, \text{CON}_{BA}, R) \).

The proof of the existence of an OT-ranking for every HG-weighting is based on constructing \( R \) from \( U_{HG} \). For each \( u_i \in U_{HG} \), strictly rank \( \text{ATTRACT}(\text{R}, u_i \rightarrow q) \in \text{CON}_{BA} \) above \( \text{IDENT\text{-}IO}(q) \) in \( R \). For each \( v_j \in \mathcal{P}(\mathcal{P}) - U_{HG} \), strictly rank \( \text{ATTRACT}(\text{R}, v_j \rightarrow q) \in \text{CON}_{BA} \) below \( \text{IDENT\text{-}IO}(q) \) in \( R \). Thus, \( R = \{ \text{ATTTRACT}(\text{R}, u_i \rightarrow q) \} > \{ \text{ATTTRACT}(\text{R}, v_j \rightarrow q) \} \). And so, clearly \( R \), has a corresponding \( U_{OT} = U_{HG} \).

Since, \( \text{CON}_{BA\text{-simple}} \) is subject to the SSRP, \( R \) can be made to be SSRP conforming, by ranking \( \{ \text{ATTTRACT}(\text{R}, u_i \rightarrow q) \} \) according to the SSRP, putting \( \text{IDENT\text{-}IO}(q) \) in a lower strata, and then independently ranking \( \{ \text{ATTTRACT}(\text{R}, v_j \rightarrow q) \} \) according to the SSRP, the highest ranking \( \text{ATTTRACT}(\text{R}, v_j \rightarrow q) \), being
place in the strata below \textsc{Ident-Io}(q). Therefore, \( \forall S, \exists \) a \textsc{Ssrp}-conforming \( R \), and so \( \text{HG}(\text{GEN}_{BA}, \text{CON}_{BA\text{-simple}}, S) \subseteq \text{OT}_{LCC}(\text{GEN}_{BA}, \text{CON}_{BA}, R) \).\[\]

The proof of (49) relies on the ability of \( \text{OT}_{LCC} \) to independently rank distinct conjoined constraints. Therefore, whatever the effective strengths of complex \textsc{Attraction} are in \( \text{HG} \), an \( \text{OT}_{LCC} \) model of attraction can literally be fit to those effective strengths by appropriately ranking the complex \textsc{Attraction} constraints. However, because \( \text{HG} \) has fewer parameters (only the members of \( \text{CON}_{BA\text{-simple}} \) may be explicitly ranked), it is not generally possible to fit a \( \text{HG} \) to \( \text{OT}_{LCC} \), as the following counter-example in (50) below shows:

\begin{equation}
\text{Corollary: } \text{OT}_{LCC}(\text{GEN}_{BA}, \text{CON}_{BA}, R) \not\subseteq \text{HG}(\text{GEN}_{BA}, \text{CON}_{BA\text{-simple}}, S).
\end{equation}

\textbf{Proof:} Consider the \textsc{Ssrp} conforming lattice-hierarchy of \( \text{CON}_{BA} \) for \( \text{P} = \{ p_1, p_2, p_3, p_4 \} \) below:

\begin{center}
\begin{tikzpicture}
    \node (1) at (0,0) {\textsc{Attract}(\textbf{R}, \{p_1, p_2, p_3, p_4\}, q)};
    \node (2) at (-3,-1) {\{p_1, p_2, p_3\}};
    \node (3) at (-1,-1) {\{p_1, p_2, p_4\}};
    \node (4) at (1,-1) {\{p_1, p_3, p_4\}};
    \node (5) at (3,-1) {\{p_2, p_3, p_4\}};
    \node (6) at (0,-3) {\textsc{Ident-Io}(q)};
    \node (7) at (-3,-4) {\{p_1\}};
    \node (8) at (-1,-4) {\{p_1, p_4\}};
    \node (9) at (1,-4) {\{p_2, p_3\}};
    \node (10) at (3,-4) {\{p_2, p_4\}};
    \node (11) at (0,-6) {\textsc{Attract}(\textbf{R}, \{\}, q)};
    \path (1) edge (2)
          (1) edge (3)
          (1) edge (4)
          (1) edge (5)
          (6) edge (7)
          (6) edge (8)
          (6) edge (9)
          (6) edge (10)
          (11) edge (1)
          (11) edge (2)
          (11) edge (3)
          (11) edge (4)
          (11) edge (5);
\end{tikzpicture}
\end{center}
Of interest is that disjoint \( \{p_1, p_2\} \) and \( \{p_3, p_4\} \) are in \( U_{OT} \), but \( \{p_1, p_3\}, \{p_1, p_4\}, \{p_2, p_3\}, \) and \( \{p_2, p_4\} \) are not. There is no HG that gives this effect. The reason is that because \( \{p_1, p_2\} \in U_{HG} \) requires \( e_{\{p_1, p_2\}} = s_{p_1} + s_{p_2} + s_I \geq s_{\text{IDENT-IO}} \), either \( s_{p_1} \) or \( s_{p_2} \) must be larger than \( \frac{s_{\text{IDENT-IO}}(q) - s_I}{2} \) (if both \( s_{p_1}, s_{p_2} < \frac{s_{\text{IDENT-IO}}(q) - s_I}{2} \), then \( s_{p_1} + s_{p_2} < s_I \)) and therefore, \( s_{p_1} + s_{p_2} + s_I < s_{\text{IDENT-IO}}(q) \), which contradicts that \( \{p_1, p_2\} \in U_{HG} \). Similarly, because \( \{p_3, p_4\} \in U_{HG} \), \( s_{p_3} \) or \( s_{p_4} \) must be larger than \( \frac{s_{\text{IDENT-IO}}(q) - s_I}{2} \). Without loss of generality, assume \( s_{p_1} \) and \( s_{p_3} \) are greater than \( \frac{s_{\text{IDENT-IO}}(q) - s_I}{2} \). Then, \( s_{p_1} + s_{p_3} > (s_{\text{IDENT-IO}}(q) - s_I) \Leftrightarrow s_{p_1} + s_{p_2} + s_I \geq s_{\text{IDENT-IO}}(q) \), which means that \( e_{\{p_1, p_3\}} > s_{\text{IDENT-IO}}(q) \) and therefore \( \{p_1, p_3\} \in U_{HG} \).

Thus, there is no way for HG to include disjoint \( \{p_1, p_2\}, \{p_3, p_4\} \) in \( U_{OT} \), and simultaneously exclude all of \( \{p_1, p_3\}, \{p_1, p_4\}, \{p_2, p_3\}, \) and \( \{p_2, p_4\} \). Thus, \( \exists R \) such that \( \forall S, U_{OT} \neq U_{HG} \). Therefore, \( OT_{LCC}(\text{GEN}_{BA}, \text{CON}_{BA}, R) \not\subseteq \text{HG}(\text{GEN}_{BA}, \text{CON}_{BA\text{-simple, } S}) \).

Together, these results confirm that \( \text{HG} \subset \text{OT}_{LCC} \). Because complex \text{ATTRACTION} can be explicitly ranked in \( \text{OT}_{LCC} \), but not HG, it is, perhaps, not altogether surprising that \( \text{OT}_{LCC} \) is strictly more powerful than HG. However, the above results ensure that working directly in terms of similarity numbers as in §3.7 or in terms of network simulations as in Chapter 5 does not produce typologically different results than \( \text{OT}_{LCC} \) models of \text{ATTRACTION}, since the higher-level of description is strictly more powerful.

Furthermore, there are a number of empirical reasons related to learning, which might explain why disjoint threshold sets, such as in the above counter-example, (50), are cross-linguistically dispreferred (see §3.8), suggesting that the completely unfettered reranking of \text{IDENT-IO} and complex \text{ATTRACTION} in \( \text{OT}_{LCC} \) is too powerful. Some of these problems persist in HG, but being less powerful than \( \text{OT}_{LCC} \) with local conjunction is likely more of a virtue than a debit (see also Pater, 2009).
Closure of \( U \) under \( p \)-intersection is sufficient to rule out disjoint threshold sets, because it requires the threshold set is a singleton set, \( T = \{ u_{\text{min}} \} \). Therefore, it remains of interest to determine if under \( p \)-intersection closure whether or not \( OT_{\text{LCC}} \) is still more powerful than HG. Theorem (51) below – the last of this section’s formal results – shows, that if the set of active preconditions is closed under \( p \)-intersection, then there is equality between \( OT_{\text{LCC}} \) and HG.

(51) **Theorem:** If \( U_{\text{OT}} \) is closed under \( p \)-intersection, then \( OT_{\text{LCC}}(\text{GEN}_{\text{BA}}, \text{CON}_{\text{BA}}, R) = HG(\text{GEN}_{\text{BA}}, \text{CON}_{\text{BA-simp}}) \).

**Proof:** I will show that \( \forall \) strict ranking, \( R \), of \( \text{CON}_{\text{BA}} \) with corresponding \( U_{\text{OT}} \), such that \( U_{\text{OT}} \) is closed under \( p \)-intersection, \( \exists \) a numerical weighting, \( S \), of \( \text{CON}_{\text{BA-simp}} \) with corresponding \( U_{\text{HG}} \) such that \( U_{\text{HG}} = U_{\text{OT}} \). If this is true, then by application of the Equivalence Lemma (48), then \( OT_{\text{LCC}}(\text{GEN}_{\text{BA}}, \text{CON}_{\text{BA}}, R) \subseteq HG(\text{GEN}_{\text{BA}}, \text{CON}_{\text{BA-simp}}, S) \). Together with Thm. (49)\((HG \subseteq OT_{\text{LCC}})\), this would show \( OT_{\text{LCC}}(\text{GEN}_{\text{BA}}, \text{CON}_{\text{BA}}, R) = HG(\text{GEN}_{\text{BA}}, \text{CON}_{\text{BA-simp}}, S) \).

The proof of the existence of an HG-ranking for closed \( U_{\text{OT}} \) is based on constructing \( S \) from \( U_{\text{OT}} \). As corollary (50) confirms, the challenge is setting the weighting \( S \) so that each \( u_i \in U_{\text{OT}} \) is in \( U_{\text{HG}} \) and simultaneously each \( v_j \in V = \wp(P) \) – \( U_{\text{OT}} \) is not in \( U_{\text{HG}} \). By the \( p \)-intersection closure of \( U_{\text{OT}} \), \( T_{\text{OT}} = \{ u_{\text{min}} \} \), \( u_{\text{min}} = \bigcap_{U_{\text{OT}}} u_i \) (Thm. (37)).

The strategy, here, is to sort out the cases of assigning \( S \) based on the cardinality of \( u_{\text{min}} \). Let \( u_{\text{min}} = \{ a_1, \ldots, a_m \} \subseteq \wp(P) = \{ p_1, \ldots, p_k \} \). Note \( |P| = k \). Let These separate cases are needed because the proofs make reference to \( \frac{1}{m-1} \), which is not well defined if \( m = 1 \).

**Case 1:** \( m = 0 \). Then \( u_{\text{min}} = \{ \} \) and \( U_{\text{OT}} = \wp(P) \), so to get \( U_{\text{HG}} = \wp(P) \), set \( s_0 > s_{\text{IDENT-IO}(q)} \) \( U_{\text{HG}} = U_{\text{OT}} \).
\textbf{Case 2: } m=1. Then $u_{\min} = \{a_1\}$. Therefore, because $u_{\min}$ is a singleton set, $U_{OT}$ is the set of $u_i$ which contain $a_1$, and all $v_i \in V=\mathcal{P} - U_{OT}$ do not contain $a_1$.

The following numerical weighting gives $U_{HG} = U_{OT}$: set $s_{\text{IDENT-IO(q)}} > 0$, $s_{a_1} > s_{\text{IDENT-IO(q)}}, s_{p_j = a_1} < \frac{1}{k} \cdot s_{\text{IDENT-IO(q)}}$, and $s_{\emptyset} = 0$. Now, because $\forall u_i \in U_{OT}$ contain $a_1$, $e_{u_i} \geq s_{a_1} > s_{\text{IDENT-IO(q)}}$, which means $u_i \notin U_{HG}$. Consider $v = \{b_1, \ldots, b_n\} \in V$, $v$ must not equal $P$ because $v$ does not contain $a_1$. Therefore, $|n| \leq k - 1$, which means that $e_v = s_{b_1} + \ldots + s_{b_n} + s_{\emptyset} = s_{b_1} + \ldots + s_{b_n} + (0)$. And since $b_j \neq a_1$, each term $s_{b_1} + \ldots + s_{b_n}$ must be less than $\frac{1}{k} \cdot s_{\text{IDENT-IO(q)}}$, so collectively $e_v < \frac{k-1}{k} \cdot s_{\text{IDENT-IO(q)}} < s_{\text{IDENT-IO(q)}}$. Hence, $v \notin U_{HG}$, and so $U_{OT}=U_{HG}$.

\textbf{Case 3: } m>1.

Let $a = u_{\min} = \{a_1 \ldots a_m\}$ and let $b = P - u_{\min} = \{b_1, \ldots, b_k \ldots m\}$. $\forall u_i \in U_{OT} u_{\min} \subseteq u_i$ because $u_{\min} = \bigcap_{U_{OT}} u_i$. Therefore, $U_{OT}$ is the set of $u_i$ which contain all the elements of $a$, and all $v_i \in V=\mathcal{P} - U_{OT}$ do not contain some element of $a$, and so members of $U_{OT}$ contain at least $m$ elements of $a$, and members of $V$ contain at most $(m - 1)$ elements of $a$.

Thus, the hardest case for inclusion in $U_{HG}$ is $u_{\min}$ (because any other member of $U_{OT}$ would incur some additional penalty $s_{b_j}$ for failing to agree), and the hardest case for exclusion from $U_{HG}$ is $v_{\max}$ which contains $(m - 1)$ elements of $a$ and also contains all the elements of $b$ (because any other member of $V$ would incur less penalty by having fewer elements of $b$ and/or $a$). Hence, the relevant bounds are $e_{u_{\min}} > s_{\text{IDENT-IO(q)}}$ and $e_{v_{\max}} < s_{\text{IDENT-IO(q)}}$.

If there is a weighting $S$, that satisfies these bounds, $U_{HG}=U_{OT}$, because any other $u_i$ must have greater effective strength, and any other $v$ must have less than or equal effective strength. Define such an $S$: set $s_{\text{IDENT-IO(q)}} = 1$ and $s_{\emptyset} = 0$.

Set $s_{a_1}$ as follows: $\frac{1}{m} < s_{a_1} < \frac{1}{m-1}$. Set $s_{b_j} < \frac{1 - (m-1) \cdot \max \{s_{a_1}\}}{(k-m)}$.

$s_{a_1} > \frac{1}{m}$ ensures that if there are at least $m$ elements of $a$ in $u_i \in U_{OT}$, $e_{u_i} \geq m \cdot \min(s_{a_1}) > m \cdot \frac{1}{m} = 1 = s_{\text{IDENT-IO(q)}},$ so $u_i \in U_{HG}$.
\[
s_{bj} < \frac{1 - (m - 1) \cdot \max \{s_{ai}\}}{(k - m)}
\]
ensures that if \(v \in V\) there are at most \((m-1)\) elements of \(a\) and at most all \((k - m)\) elements of \(b\),
\[
e_v \leq (m - 1) \cdot \max(s_{ai}) + (k - m) \cdot \max(s_{bj})
\]
\[
< (m - 1) \cdot \max(s_{ai}) + (k - m) \cdot \frac{1 - (m - 1) \cdot \max \{s_{ai}\}}{(k - m)}
\]
\[
= (m - 1) \cdot \max(s_{ai}) + 1 - (m - 1) \cdot \max(s_{ai})
\]
\[
= 1 = s_{\text{IDENT-IO}(q)}.
\]

Note, \(s_{ai} < \frac{1}{(m-1)}\) ensures that \(s_{bj} < \frac{1 - (m - 1) \cdot \max \{s_{ai}\}}{(k - m)}\) is positive, which is necessary for SSRP conformity. Thus, \(\forall v \in V, e_v < s_{\text{IDENT-IO}(q)} \Rightarrow v \not\in U_{HG}\) and \(U_{OT}=U_{HG}\).

In every case, there exists a weighting \(S\) of \(\text{CON}_{BA\text{-}\text{simple}}\), such that \(U_{HG} = U_{OT}\), so \(\text{OT}_{LCC}(\text{GEN}_{BA}, \text{CON}_{BA}, R) \subseteq \text{HG}(\text{GEN}_{BA}, \text{CON}_{BA\text{-}\text{simple}}, S)\). Together with Thm. (49) \((\text{HG} \subseteq \text{OT}_{LCC})\), this demonstrates that when \(U_{OT}\) is closed under p-intersection \(\text{OT}_{LCC}(\text{GEN}_{BA}, \text{CON}_{BA}, R) = \text{HG}(\text{GEN}_{BA}, \text{CON}_{BA\text{-}\text{simple}}, S)\)\n
Theorem (51) confirms that closure under p-intersection is a sufficient condition for the equality of \(\text{OT}_{LCC}\) and \(\text{HG}\). P-intersection closure is not, however, a necessary condition,\(^{53}\) but because of converging evidence suggesting that p-intersection closure is advantageous, it is the only condition for equality of \(\text{OT}_{LCC}\) and \(\text{HG}\) that I have considered.

3.7. **Attraction in regular harmony spaces**

The above results closely link the behavior of **Attraction** in \(\text{HG}\) and \(\text{OT}_{LCC}\), providing a way for numerical analyses to be imported into \(\text{OT}\) and a way for some \(\text{OT}\) analyses to be interpreted as numerical. Thus, symbolic grammatical analysis can be directly linked to the subsymbolic, or subphonemic, forces at work in parasitic harmony.

\(^{53}\) The errant \(\text{HG}\) example from (50) which had active preconditions \(\{p_1, p_3\}, \{p_1, p_2\}, \{p_3, p_4\}\) is not closed under p-intersection, since none of \(\{p_1\}, \{p_3\}, \{p_4\}\) are in \(U_{HG}\), but by (49) there remains an equivalent \(\text{OT}\) grammar.
By modeling the space of attraction in terms of the distances and similarities of segments and/or features, the subsymbolic can also have a direct result on the assimilatory properties of a grammar.

As discussed in §3.4, phonetic dependencies are the basis for the phonological dependencies of parasitic assimilation, whereby agreement on one feature feeds agreement on another. Given, then, that the phonetic affects the phonological, it is important for more numerical properties like distance and similarity to be expressible in grammar. The usefulness of distance and similarity in phonetics, phonology, and morphology has been argued at length in (Stevens, Keyser, & Kawasaki, 1986; Lindblom, 1986; Steriade, 1997, 2001; Flemming, 1995; Boersma, 1997; Mielke 2004, 2005; Burzio, 2002a, b, 2004, 2005; Burzio & Tantalou, 2007; Frisch, Pierrehumbert, & Broe, 2004; etc.). From the perspective of attraction phonology, the most natural way for such numerical properties to be imported into OT are by way of HG models of attraction.

In particular, this section shows how a HG ATTRACTION grammar allows the different factors that determine similarity to be directly modeled and how to use lattice diagrams to convert a numerically weighting of the factors of similarity into a ranking of OT ATTRACTION constraints. This section proceeds as follows: I introduce the notion of language specific similarity, then show how specific numerical values for the factors of similarity describe a regular similar space, and conclude by showing how these spaces may be encoded as a grammar in HG or OT.

3.7.1. Language specific similarity

Allowing feature dependencies to be assigned different strengths is useful because languages differ in how individual features contribute to overall prerequisite similarity.
Essentially, different languages have different metrics of similarity.\textsuperscript{54} In some language, a pair of segments may be similar enough to result in assimilation, while in another language those ‘same’ segments fail to interact because the metrics of similarity are different in the two languages. Some cross-linguistic differences in similarity can be grounded in the distinct acoustic/phonetic properties of the segments. Other differences may result from language-specific properties of phonologically abstract representations, which presumably arise from facts about phonological inventories (dense vs. sparse inventories) and syllable phonotactics (e.g. onset vs. coda contrast).

In the spirit of OT, these cross-linguistic differences arise from a distinct weighting of the factors of similarity, which are seen as both universal and competitive.\textsuperscript{55} In the spirit of HG, the weighting of these factors is numerically continuous and gradient. While all languages have the same set of potential similarity links (the same dimensions in a harmony space), languages ascribe different weights to the individual connections, thereby, exploiting different aspects of similarity. For example, every language has a link between rounding and height, since both affect the size of the lip opening and to a degree \( F_2 \) (even in English lower [o] is less rounded than higher [u]), but not every instance of rounding harmony is parasitic on height. In such cases, the round-height phonetic link is not absent, but merely weighted such that height contributes little to the overall similarity of participating segments for the purposes of rounding harmony. The reason for the low-weighting of the height-round link may certainly be grounded in the phonetic details of a particular language, such as the absence of front round vowels. However, the low-

\textsuperscript{54} See §4.5 for a discussion of how these different metrics relate to different grammar-kernels.

\textsuperscript{55} ‘Universal’ is not the same as ‘nativist.’ That some phonetic dependencies are universally available to speakers of diverse languages, I take as uncontroversial (see e.g. Wilson, 2007). Whether or not these dependencies derive from specific genetic and modular factors due to language is an open question.
weighting may solely exist at a more abstract level of representation. For instance, Turkish has a full-series of contrastive round/unround front vowels, but no height-parasitic rounding harmony.

While allowing weighting of a link in purely abstract terms, the availability of a possible link must still be grounded in phonetic universals. Thus, as shown in §3.4, the set of potential dependencies is limited because some links are universally weaker than others due to their correlation along phonetic dimensions. For example, as noted, ATR and rounding are not dependent on one another in any phonetic way, nor in any assimilation in the present survey, so evidently ATR agreement is universally unimportant to determining the similarity of segments for the purposes of rounding harmony.

3.7.2. Regular harmony spaces

As a starting point, in this section, I consider cases where the space for harmony is regular. A harmony space is the space in which the attraction landscape is embedded. Despite the limitations listed below, regular harmony spaces have the important property that feature dependencies may be assigned different strengths. A regular harmony space, denoted $S_{RH}$, is a multi-dimensional, rectilinear box,\footnote{Here, a multidimensional, rectilinear box denotes having parallel faces and right angles at all the vertices; it differs from a hypercube in that the sides do not have to be of equal length.} where phonemes are located at the vertices. Semantically, each dimension in the space is a potential prerequisite feature, $\{p_i\}$, for harmony on a feature $q$.

In a regular harmony space, prerequisite features are designated by orthogonal basis vectors, so the prerequisite features are independent for the purpose of $q$-harmony;
relaxing this feature orthogonality assumption yields *irregular feature spaces*.\(^57\) The length of this basis vector is the distance between two segments which agree on all features except \(p_i\) with respect to a harmonic feature \(q\) denoted \(d_{q|p_i}\) or alternatively \(d_{p_i}\) where the harmonic feature is known. The independence assumption means that the distance between two segments \(x, y\) which agree on all features except \(p_i\) does not depend on which feature values \(x\) and \(y\) share, so for example if *place* is independent of *manner*, then the distance between segments disagreeing on backness does not depend on whether those segments agree on [+hi] or [−hi]. This regularity assumption is clearly an approximation that is ultimately rejected in favor of irregular feature spaces (see §4.5), but regular feature space are extremely illustrative in explaining how the weighting of the factors similarity works in a low-level numeric space.

Consider the following regular harmony space, (52), which can be used to give back parasitic rounding harmony in vowels based on the prerequisite feature set \{[con], *manner*, *place*\}.

---

\(^{57}\) The assumption is not that \(p_i\) and \(p_j\) have to be independent everywhere, but just with respect to the harmonic feature \(q\). For example [hi] and [back] may not be fully independent in the phonetic space because of a small interaction between F2 and height, since front low vowels have a lower F2 than front high vowels. However, if the distances between [i] and [ii] and between [i] and [u] are identical, and so are the distances between [e] and [œ] and between [æ] and [o], then the difference with respect to rounding given height agreement is independent of the value of [back], and so for rounding harmony, [hi] is conditionally orthogonal from [back].
Regular harmony space $\mathbf{P} \rightarrow [\text{labial}]$ for $\mathbf{P} = \{[\text{con}], \text{manner}, \text{place}\}$, with $d_{[\text{labial}]\rightarrow [\text{cons}]} = 9$, $d_{[\text{labial}]\rightarrow \text{manner}} = 1$, and $d_{[\text{labial}]\rightarrow \text{place}} = 5$.

In (52), the space was weighted with $d_{[\text{labial}]\rightarrow [\text{cons}]} = 9$, $d_{[\text{labial}]\rightarrow \text{manner}} = 1$, and $d_{[\text{labial}]\rightarrow \text{place}} = 5$. These distances denote the spacing when disagreeing on only $p_i$, so because similarity is the inverse of distance these same numbers also denote the amount of similarity when agreeing on only $p_i$; segments which agree on $p_i$ are exactly $d_{q\rightarrow p}$ more similar than segments which disagree on $p_i$. Thus, $d_{q\rightarrow p} = \text{sim}_{q,p}$. Similarity is generally unitless, so these numbers are arbitrary, i.e. the specific numbers are not as important as the relationship between them. Here, $\text{place}$ is five-times as important as $\text{manner}$ for the purposes of labial harmony, and [cons] is almost twice as strong as $\text{place}$ in the same context. These relative distances are expressed in the lengths of the sides of the rectilinear box, above.

[labial] is not a dimension of the figure above because [labial] is the assumed harmonic feature context. Therefore, vertices are labeled with both round and unround alternates, e.g. [i]/[ü] indicates the spatial position of high, front vowels. The similarities are measured in context of [labial], so the strengths may be denoted with subscripts, e.g. $s_{[\text{labial}]}|\text{manner} = 1$, $s_{[\text{labial}]}|\text{place} = 5$, and $s_{[\text{labial}]}|[\text{cons}] = 9$, to indicate that these similarities are
measured with respect to the harmonic feature being [labial]. Because of the strong weighting of [cons], segments which agree in [cons] are closer to one another than those that disagree on [cons]. In this way, conditional similarity simultaneously affects the distance between both agreeing and non-agreeing segments in a regular harmony space.

(53), below, gives the Manhattan, or Taxicab, distance corresponding to this regular harmony space. The Manhattan distance, $d_M$, is the distance travelled between vertices along the edges of the rectilinear box. To compute $d_M$, denote segments with their coordinates in the harmony space: a segment $x = (x_{[con]}, x_{manner}, x_{place})$, where $x_i \in \{0, s_i\}$. For $d_M$ in a regular harmony space, it is irrelevant which feature specification is associated with 0 and which with $s_i$, as long as the assignment is consistently applied. For example, let $x = [i]/[\ddot{u}] = (s_{cons}, s_{manner}, 0)$ then $y = [a]/[o]$ has the coordinates $(s_{cons}, 0, s_{place})$. $d_M(x, y)$ is the sum of the difference between dimensions: $d_M(x, y) = |x_{[con]} - y_{[con]}| + |x_{manner} - y_{manner}| + |x_{place} - y_{place}|$. Thus, $d_M(([i]/[\ddot{u}], [a]/[o])) = d_M(x = (s_{cons}, s_{manner}, 0), y = (s_{cons}, 0, s_{place})) = |s_{cons} - s_{cons}| + |s_{manner} - 0| + |0 - s_{place}| = 0 + s_{manner} + s_{place} = (1) + (5) = 6.$

Manhattan distance correlates very well with conventional Euclidean distance, is simpler to compute, and, most importantly, allows regular harmony spaces to be directly mapped onto a weighting of ATTRACTION constraints.

(53) Manhattan distances in the regular harmony space for rounding in (52)

<table>
<thead>
<tr>
<th></th>
<th>[i]/[ɪ]</th>
<th>[i]/[u]</th>
<th>[ɛ]/[ɨ]</th>
<th>[a]/[o]</th>
<th>[t]/[ɾ]</th>
<th>[k]/[ɾ]</th>
<th>[s]/[ɾ]</th>
<th>[x]/[ɾ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ɪ]/[ɪ]</td>
<td>0</td>
<td>5</td>
<td>1</td>
<td>6</td>
<td>9</td>
<td>14</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>[i]/[u]</td>
<td>5</td>
<td>0</td>
<td>6</td>
<td>1</td>
<td>14</td>
<td>9</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>[ɛ]/[ɨ]</td>
<td>1</td>
<td>6</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>9</td>
<td>14</td>
</tr>
<tr>
<td>[a]/[o]</td>
<td>6</td>
<td>1</td>
<td>5</td>
<td>0</td>
<td>15</td>
<td>10</td>
<td>14</td>
<td>9</td>
</tr>
<tr>
<td>[t]/[ɾ]</td>
<td>9</td>
<td>14</td>
<td>10</td>
<td>15</td>
<td>0</td>
<td>5</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>[k]/[ɾ]</td>
<td>14</td>
<td>9</td>
<td>15</td>
<td>10</td>
<td>5</td>
<td>0</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>[s]/[ɾ]</td>
<td>10</td>
<td>15</td>
<td>9</td>
<td>14</td>
<td>1</td>
<td>6</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>[x]/[ɾ]</td>
<td>15</td>
<td>10</td>
<td>14</td>
<td>9</td>
<td>6</td>
<td>1</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>
These distances can be converted directly into total conditional similarities, as shown in 0, using the transformation \( \text{sim}(x, y) = \max d_M - d_M(x, y) \). It is straightforward to show that \( \max d_M - d_M(x, y) \) gives the total conditional similarity between \( x \) and \( y \). Non-normalized similarity \( \text{sim}(x, y) \) of points in the regular harmony space for rounding in (52).

![Table]

(53) and 0 are numerical representations of the regular harmony space in (53). These tables crucially show that because of the weighting of conditional similarities, \( s_{\text{cons}} > s_{\text{place}} > s_{\text{manner}} \), the closest (most similar) non-identical segments are those which agree on both \([\text{cons}]\) and \( \text{place} \). Thus, the above regular harmony space favors \( \text{place} \)-parasitic vowel harmony over \( \text{manner} \)-parasitic vowel harmony.

---

58 Note that \( \max d_M = \max_{a,b \in S_{\text{hil}}} d_M(a, b) = d_M\left(\begin{array}{l} a = (0, \ldots, 0), b = (d_{p_1}, \ldots, d_{p_k}) \end{array}\right) = \sum_{p_i} d_{p_i} \). Also, note that \( d_M(x, y) \) is the sum of the distance along the differing coordinates, so \( d_{p_i} \) is added to \( dM \) exactly when \( x, y \) differ on \( p_i \). Thus, \( d_M(x, y) = \sum_{p_i} (1 - \mathcal{A}_{p_i}(x, y)) \cdot d_{p_i} \), where

\[
\mathcal{A}_{p_i}(x, y) = \begin{cases} 1 & \text{if } x, y \text{ agree on } p_i \\ 0 & \text{otherwise} \end{cases}
\]

With these facts:

\[
\text{sim}(x, y) = \max d_M - d_M(x, y) = \sum_{p_i} d_{p_i} + \sum_{p_i} (1 - \mathcal{A}_{p_i}(x, y)) \cdot d_{p_i} = \sum_{p_i} (d_{p_i} + (1 - \mathcal{A}_{p_i}(x, y)) \cdot d_{p_i}) = \pi \mathcal{A}_{p_i}(x, y) \cdot d_{p_i} = \pi \mathcal{A}_{p_i}(x, y) \cdot \text{spit}.
\]
3.7.3. Converting a regular feature space into an ATTRACTION hierarchy

This section shows that if the \( \{ s_i \} \), which correspond to the distances in a regular harmony space, are taken as the strengths of simple ATTRACTION constraints in a HG, then that HG also favors place-parasitic vowel harmony over manner-parasitic vowel harmony. This can be demonstrated concretely by recognizing that a regular harmony space has the same topology as a subset lattice, where the vertices of the harmony space are the nodes of the lattice. Thus, the rectilinear box in (52) may be converted into a similarity lattice, by choosing a segment for the supremum, say [i]/[u], and then drawing arcs between segments which differ by exactly one feature as in (54), below. If each feature is given equal weight then, rank in the lattice corresponds to similarity with the trigger, as the rank is the number of feature differences from the supremum.

(54) Regular harmony lattice, with [i]/[u] chosen as supremum. Nodes also indicate which features are shared with [i]/[u].

Now, in a regular space, the harmonic feature, \( q \), is non-privative and non-dominant, i.e. the penalties for disagreeing on \([+q]\) are equivalent to disagreeing on \([-q]\). Also, two
[+\(p_i\)] segments activate the same ATTRACTION constraint as two \([-\(p_i\)]\) segments, so it is impossible to model asymmetric feature dependencies, such as semi-parasitic harmony (Hong, 1994), where rounding harmony is parasitic on vowel height but harmony is only required among high vowels. Furthermore, second-order feature dependencies are impossible to express. For example, there is no dependency between height and rounding that is limited to only front vowels, because in a regular feature space, height is independent of backness.

Based on these assumptions, [+F] has the same effect as [-F] for both prerequisite and harmonic features, so the common features across arcs in a regular harmony lattice are the same for all choices of the supremum. Thus, it is possible to construct an agreement lattice, which instead of picking a particular trigger as supremum is true for all triggers, so the nodes of an agreement lattice are labeled with the set of features shared with the supremum, as below in (55):

(55) Agreement lattice for the regular harmony space with \(P = \{\text{con}, \text{manner}, \text{place}\}\) and \(q = \text{labial}\).

Level 0:

\[
\begin{array}{ccc}
\text{place, manner, [cons]} \\
\end{array}
\]

Level 1:

\[
\begin{array}{ccc}
\text{manner, [cons]} & \text{place, [cons]} & \text{place, manner} \\
\end{array}
\]

Level 2:

\[
\begin{array}{ccc}
\text{[cons]} & \text{manner} & \text{place} \\
\end{array}
\]

Level 3:

\[
\begin{array}{ccc}
\text{[\{} & \text{\}} \\
\end{array}
\]

Of course, the features in a regular harmony space are not necessarily of equal weight, so it is possible to assign nodes a numerical distance from the supremum based on the
distances in the harmony space, (53). Note because a regular harmony space is rectilinear all paths between nodes in the weighted agreement lattice in (56) have the same distance.

(56) Agreement lattice for the regular harmony space with \( P = \{\text{[con]}, \text{manner}, \text{place}\} \), \( s_{\text{[cons]}} = 9 \), \( s_{\text{manner}} = 1 \), \( \text{place} = 5 \), and \( q = \text{[labial]} \).

Level 0:

\[
\begin{align*}
\text{[back], manner, [cons]} & = 0 \\
\text{place} & = 5 \\
\text{manner} & = 1 \\
\text{[con]} & = 9
\end{align*}
\]

Level 1:

\[
\begin{align*}
\{\text{manner, [cons]}\} & = 5 \\
\{\text{back}, [cons]\} & = 1 \\
\{\text{back}, \text{manner}\} & = 9
\end{align*}
\]

Level 2:

\[
\begin{align*}
\{\text{[cons]}\} & = 6 \\
\{\text{manner}\} & = 14 \\
\{\text{back}\} & = 10
\end{align*}
\]

Level 3:

\[
\begin{align*}
\{\} & = 15
\end{align*}
\]

However, in order for an agreement lattice to be made to correspond with a lattice-hierarchy of ATTRACTION constraints, lattice nodes should be labeled with the total conditional similarity as given by 0, as indicated in (57) below. Note there is no difference in the arc labels between (56) and (57), the only difference is whether similarity or distances are counted.

(57) Similarity lattice for the regular harmony space with \( P = \{\text{[con]}, \text{manner}, \text{place}\} \), \( s_{\text{[cons]}} = 9 \), \( s_{\text{manner}} = 1 \), \( \text{place} = 5 \), and \( q = \text{[labial]} \).

Level 0:

\[
\begin{align*}
\{\text{place, manner, [cons]}\} & = 15 \\
\text{place} & = 5 \\
\text{manner} & = 1 \\
\text{[con]} & = 9
\end{align*}
\]

Level 1:

\[
\begin{align*}
\{\text{manner, [cons]}\} & = 10 \\
\{\text{place, [cons]}\} & = 14 \\
\{\text{place, manner}\} & = 6
\end{align*}
\]

Level 2:

\[
\begin{align*}
\{\text{[cons]}\} & = 9 \\
\{\text{manner}\} & = 1 \\
\{\text{place}\} & = 5
\end{align*}
\]

Level 3:

\[
\begin{align*}
\{\} & = 0
\end{align*}
\]
A similarity lattice of a regular harmony space, as above in (57), can then be directly related to an ATTRACTION lattice-hierarchy by way of a correspondence between nodes in the similarity lattice and constraints in the lattice-hierarchy, as in (26).

This move requires the observation that the similarity of a lattice-node corresponds exactly to the effective strength of a complex HG-ATTRACTION constraint, where the numerical strengths of the HG’s simple ATTRACTION constraints in CONBA-simple are identical to the weights assigned in the regular harmony space and additionally \( s_{\{\}} = 0 \). This is shown in the numerical ranking of HG-ATTRACTION constraints in

\[ P(\text{ATTRACT}(R, \{\{\text{cons} \}, \text{manner}, \text{place}\} \rightarrow q)) \]

according to \( S = \{ s_{\{\text{cons}\}} = 9, s_{\text{manner}} = 1, s_{\text{place}} = 5, s_{\{\}} = 0 \} \), as given below in (58):

\[
\begin{align*}
  e_{\text{ATTRACT}}(R, \{\{\text{cons} \}, \text{manner}, \text{place}\} \rightarrow q) &= s_{\{\text{cons}\}} + s_{\text{manner}} + s_{\text{place}} + s_{\{\}} = 9 + 1 + 5 + 0 = 15 \\
  e_{\text{ATTRACT}}(R, \{\{\text{cons} \}, \text{place}\} \rightarrow q) &= s_{\{\text{cons}\}} + s_{\text{place}} + s_{\{\}} = 9 + 5 + 0 = 14 \\
  e_{\text{ATTRACT}}(R, \{\{\text{cons} \}, \text{manner}\} \rightarrow q) &= s_{\{\text{cons}\}} + s_{\text{manner}} + s_{\{\}} = 9 + 1 + 0 = 10 \\
  e_{\text{ATTRACT}}(R, \{\{\text{cons}\}\} \rightarrow q) &= s_{\{\text{cons}\}} + s_{\{\}} = 9 + 0 = 9 \\
  e_{\text{ATTRACT}}(R, \{\{\text{manner}, \text{place}\}\} \rightarrow q) &= s_{\text{manner}} + s_{\text{place}} + s_{\{\}} = 1 + 5 + 0 = 6 \\
  e_{\text{ATTRACT}}(R, \{\{\text{place}\}\} \rightarrow q) &= s_{\text{place}} + s_{\{\}} = 5 + 0 = 5 \\
  e_{\text{ATTRACT}}(R, \{\{\text{manner}\}\} \rightarrow q) &= s_{\text{manner}} + s_{\{\}} = 1 + 0 = 1 \\
  e_{\text{ATTRACT}}(R, \{\{\}\} \rightarrow q) &= s_{\{\}} = 0 = 0 
\end{align*}
\]

Therefore, the HG with \( S = \{ s_{\{\text{cons}\}} = 9, s_{\text{manner}} = 1, s_{\text{place}} = 5, s_{\{\}} = 0 \} \) operates in the regular harmony space given by (52) and shown in the similarity lattice in (57). As the spacing in (58) illustrates, the effective strength is the total conditional similarity, as listed in 0, across the set of shared features between triggers and targets.
In a very real sense, Harmonic Grammars compute the total weighted conditional similarity between triggers and targets, as shown below in (59):

(59) **Link between Harmonic Grammar and regular harmony spaces:**

If the numerical weights of a HG of simple \textsc{attraction} are the same as the conditional similarities of a regular harmony space, then the HG computes the sum of conditional similarities, i.e. $H_{HG}$ is related to the total conditional similarity between a trigger and a target.

**Proof sketch:**

Let $P = \{p_1, \ldots, p_k\}$ and let $\text{CON}_{A} = \{\textsc{Attract}(R, \{p_i\} \rightarrow q)\}$. Given $xRy$ in a candidate $\alpha$ and $P' = \mathcal{A}(x,y)$, a set of features on which $x$ and $y$ agree, then the harmony of $\alpha$, $H_{HG}(\alpha) = \sum_{c_i \in \text{CON}_{A}} s_{p_i} \cdot \nu_{c_i}(\alpha) = \sum_{p_i \in P'} s_{p_i} \cdot \mathcal{A}_{p_i}(x,y) = e_{P'}$.

This is true because $C_i = \textsc{Attract}(R, \{p_i\} \rightarrow q)$ is violated exactly when $\mathcal{A}_{p_i}(x,y) = 1$. ■

Among the ramifications of this linking between HG and similarity is that different numerical weightings of simple \textsc{attraction} constraints correspond to different representational spaces and, therefore, cross-linguistic differences in the assimilatory properties arise from differences in representation – or more precisely, a difference in the weighting of the factors of representations.

For example, the HG in (58) prefers back-parasitic vowel harmony to height-parasitic vowel harmony because of the difference in effective strengths, $e_{[\text{cons}, \text{place}]} > e_{[\text{cons}, \text{manner}]}$, so any grammar that allows harmony among segments which agree in \textit{manner} also allows harmony among segments which agree in \textit{place}, but not vice versa.

The linking of the HG to the regular feature space, by (59), adds the insight that this difference in numerical weighting arises from how phonemes are represented. Since $s_{[\text{labial}]\text{place}} > s_{[\text{labial}]\text{manner}}$, agreement on \textit{place} is more closely linked to agreement on
than agreement on manner, as if the features [labial] and place are encoded in a more similar fashion than the features [labial] and manner.

3.7.4. Faithfulness in regular harmony spaces

This section shows how to incorporate IDENT-IO into the regular harmony space, and how to convert the given numerical ranking into a stratified OT ranking. As discussed in §3.5.6 and §3.6.3, in HG, IDENT-IO can be directly assigned a weight, which acts as a threshold for assimilation. For example, if $s_{\text{IDENT-IO}([\text{labial}])} = 13$, then the numerical ranking given in (58) is divided as follows in (60):

\begin{align*}
\mathcal{P}(\text{ATTRACT}(R, P = \{\text{[cons], manner, place}\} \rightarrow \text{[labial]}) \cup \text{IDENT-IO} ([\text{labial}])) & \text{ with } S = \{s_{\text{[cons]}} = 9, s_{\text{manner}} = 1, s_{\text{place}} = 5, s_{\{\}} = 0, s_{\text{IDENT-IO}([\text{labial}])} = 13\} \\
\epsilon_{\text{ATTRACT}}(R, \{\text{[cons], manner, place}\} \rightarrow \text{[labial]}) & = 15 \\
\epsilon_{\text{ATTRACT}}(R, \{\text{[cons], place}\} \rightarrow \text{[labial]}) & = 14 \\
\epsilon_{\text{ATTRACT}}(R, \{\text{[cons]}\} \rightarrow \text{[labial]}) & = 10 \\
\epsilon_{\text{ATTRACT}}(R, \{\text{manner, place}\} \rightarrow \text{[labial]}) & = 9 \\
\epsilon_{\text{ATTRACT}}(R, \{\text{manner}\} \rightarrow \text{[labial]}) & = 6 \\
\epsilon_{\text{ATTRACT}}(R, \{\text{place}\} \rightarrow \text{[labial]}) & = 5 \\
\epsilon_{\text{ATTRACT}}(R, \{\} \rightarrow \text{[labial]}) & = 1 \\
\epsilon_{\text{ATTRACT}}(R, \{\} \rightarrow \text{[labial]}) & = 0 \\
\end{align*}

Of course, under different values of $s_{\text{IDENT-IO}([\text{labial}])}$ different grammatical results obtain, e.g. $s_{\text{IDENT-IO}([\text{labial}])} = 8$ gives general rounding harmony on vowels, but with $U_{\text{HG}} = \{\{\text{[cons], manner, place}\}, \{\text{[cons], place}\}\}$, as indicated in (60), the HG implements place, [cons]-parasitic rounding harmony.
This HG can be instantiated in \( \text{OT}_{LCC} \) by using the general method presented in Thm. (49) \((\text{HG} \subseteq \text{OT}_{LCC})\). If \( e_{\text{ATTRACT}(R, P' \rightarrow \text{[labial]})} > s_{\text{IDENT-IO}([\text{labial}])} \), then \( \text{ATTRACT}(R, P' \rightarrow \text{[labial]}) \) should be placed in a strata above \( \text{IDENT-IO}([\text{labial}]) \). If \( e_{\text{ATTRACT}(R, P' \rightarrow \text{[labial]})} < s_{\text{IDENT-IO}([\text{labial}])} \), then \( \text{ATTRACT}(R, P' \rightarrow \text{[labial]}) \) should be placed in a strata below \( \text{IDENT-IO}([\text{labial}]) \). These strata can be separately made SSRP conforming, by total ordering according to effective strength. Therefore the \( \text{OT}_{LCC} \) ranking equivalent to the HG grammar in (60) is trivially given by the effective strengths, as shown below in (61):

(61) \( \text{OT ranking equivalent to the HG in (60):} \)

\[
\text{ATTRACT}(R, \{\text{[cons], manner, place}\} \rightarrow \text{[labial]}) \\
\quad \text{ ATTRACT}(R, \{\text{[cons], place}\} \rightarrow \text{[labial]}) \\
\quad \text{ IDENT-IO([labial])} \\
\quad \text{ATTRACT}(R, \{\text{[cons]}, \text{manner}\} \rightarrow \text{[labial]}) \\
\quad \text{ IDENT-IO([labial])} \\
\quad \text{ATTRACT}(R, \{\text{[cons]}\} \rightarrow \text{[labial]}) \\
\quad \text{ATTRACT}(R, \{\text{manner, place}\} \rightarrow \text{[labial]}) \\
\quad \text{ IDENT-IO([labial])} \\
\quad \text{ATTRACT}(R, \{\text{place}\} \rightarrow \text{[labial]}) \\
\quad \text{ATTRACT}(R, \{\text{manner}\} \rightarrow \text{[labial]}) \\
\quad \text{ ATTRACT}(R, \{} \rightarrow \text{[labial]})
\]

Therefore, because \( \text{HG} \subseteq \text{OT}_{LCC} \), the linking between regular harmony spaces and HG is also a link between regular harmony spaces and \( \text{OT}_{LCC} \). The gradient quality of more numerical HG is expressed in \( \text{OT}_{LCC} \) through the specific to general ranking \( \text{ATTTRACTION} \) constraints. Thus, without explicitly referring to distance, a ranking of OT \( \text{ATTTRACTION} \)
constraints can adequately express the relevant similarity effects for parasitic assimilation.

Just as the strengths of constraints can be linked to regular harmony spaces, so can the grammatical faithfulness threshold be instantiated in regular harmony spaces. All that is required is to perform the inverse of the transformation from distance to total conditional similarity that expressed the effective strengths of HG constraints. Recall, 
\[ \text{sim}(x, y) = \max d_M - d_M(x, y) \] and for assimilation to take place 
\[ P' = A(x, y) e_P > s_{\text{IDENT-IO(labial)}}. \] Because the effective strength of \textit{ATTRACTION} is the sum of the conditional similarities, assimilation takes place if \[ \text{sim}(x, y) > s_{\text{IDENT-IO(labial)}}. \] Therefore, the attraction basin for a trigger \( x \) is the set of \( y \) such that \( (\max d_M - d_M(x, y)) > s_{\text{IDENT-IO(labial)}} \)

\[ \Leftrightarrow (\max d_M - d_M(x, y) - \max d_M) \Leftrightarrow d_M(x, y) < (\max d_M - s_{\text{IDENT-IO(labial)}}) = \theta_F. \]

This \textit{faithfulness threshold} can be interpreted in a regular harmony space as a radius around a trigger. Only segments that are within this \( \theta_F \) ball are under pressure to assimilate.

For example, for the HG with \( S = \{ s_{\text{cons}} = 9, s_{\text{manner}} = 1, s_{\text{place}} = 5, s_{\text{labial}} = 0, s_{\text{IDENT-IO(labial)}} = 13 \} \), the \( \theta_F = \max d_M - s_{\text{IDENT-IO(labial)}} = (s_{\text{cons}} + s_{\text{manner}} + s_{\text{place}}) - s_{\text{IDENT-IO(labial)}} = 9 + 1 + 5 - 13 = 2 \). Therefore, all segments that are less than 2 similarity units away are going to be under pressure to assimilate. (62) graphically presents the corresponding attraction landscapes of two high, vowel triggers overlaid on top of the regular feature space from (52).
(62) Attraction basins around [i]/[ü], on the left, and [i]/[u], on the right, with $\theta_F = 2$

(62) confirms visually that the HG weighting in (60) and therefore the equivalent OT ranking in (61) implement place-parasitic rounding harmony among vowels. The basin for each high, vowel trigger covers non-high targets which match in place and [cons], but no other segments. Lower ranking faithfulness would have larger basins of ATTRACTION that would allow all the vowels to be covered by a single basin. For example, suppose $s_{\text{IDENT-IO}}(\text{labial}) = 8$, then $\theta_F = 7$ for this regular harmony space. The maximum distance between vowels is 6, so with that threshold all vowels would be mutually attracting.

This section has shown the relationship between multiple levels of description (see Wayment, et.al, 2007; Smolensky & Legendre, 2006) of ATTRACTION phenomena. These different levels provide different tools to inform the analyst. Harmony spaces provide a visual medium to express attraction landscapes and the numerical similarity of the dependencies between prerequisite and harmonic features. Lattices give another medium for understanding similarity relationships. Harmonic Grammar provides a numerical, yet grammatical, way to express concepts like similarity and faithfulness thresholding in a way that ensures a connectionist implementation. Optimality Theory
provides a purely symbolic means to express the distance and similarity effects of the lower levels. OT-level analysis hides some of the intricacy required by lower-level implementations, allowing for the interaction of the constraints to be distilled in term of phonological alternation. However, the OT-level also hides some of the motivations for dependencies, in particular, why some dependencies, but not others are available for parasitic interactions.

3.8. Avoiding disjunction

3.8.1. Typology and pathology of a ‘Principle of Similarity’

Normal parasitic assimilation places a distance requirement between trigger and target, such that if \( d(\text{trigger}, \text{target}) \) is less than a threshold, \( \theta_F \), the target assimilates to the trigger. The formulation of ATTRACTION eliminates systems that would allow other kinds of languages. For instance, imagine an anti-threshold language which systematically allows a dissimilar segment to alternate, but does not allow a more similar segment to alternate. Such languages place a lower bound, such that the distance between trigger and target must be greater than a threshold, \( d \geq \theta_F \). No such lower bounding languages were found in the present survey. Furthermore, these unattested systems are incompatible with the motivations for harmony: for instance, \( d \geq \theta_F \)-languages only allow harmony where sounds are already perceptually distinct or articulatorily unrelated, but prohibit it where sounds are quite similar. The generalization from Chapter 2 is quite the opposite: parasitic features were overwhelmingly found to be phonetically similar to harmonic features.

The subjection of ATTRACTION constraints to the Subset Similarity Ranking Principle (SSRP, (24)and (47)) eliminates these anti-threshold languages. The SSRP
requires that whatever the ranking of an ATTRACTION constraint violated by less similar segments, an ATTRACTION constraint violated by more similar segments is necessarily higher ranked. Hence, under the SSRP if the lower ranked constraint dominates faith, so does the higher ranked constraint, so any system that allows interaction between less similar segments necessarily allows interaction between more similar segments.

Thus, among the distance effects predicted by ATTRACTION is an implicational universal, which is related to what has elsewhere been called, the “Principle of Similarity” (as established by Hutcheson, 1973; Hong, 1994; refs therein; see also Hansson, 2007; Walker, 2009b for applications of the Principle of Similarity to ABC):

(63)  Principle of Similarity
Let \( x, y, \) and \( z \) be segments such that \( \text{sim}(x, y) > \text{sim}(x, z) \).
If \( z \) assimilates to \( x \), then \( y \) must also assimilate to \( x \).

Although intuitively appealing, Hong (1994) shows that there are metrics of similarity for which (63) is demonstrably incompatible with empirical data. Especially problematic are metrics that give all features equal weight: for example, if the large differences between consonants and vowels count equally with differences in place of articulation, then a general vowel rounding harmony process which allows participating targets to vary in features related to manner and place (up to 2 feature differences) ought to also predict spreading from vowels to consonants of similar place and manner (only 1 feature difference). No such language is known to exist.

However, using subset similarity as the relevant metric for (63), as dictated by the SSRP, preserves the Principle of Similarity by making much more narrow predictions about its application (cf. ‘Target-Output Closeness’ in Reiss, 2001).\(^59\) Subset similarity can formally be defined as follows in (64):

---

\(^{59}\) Reiss (2001) proposes “Target-Output Closeness (TOC): Suppose that in a language \( L \) there is a phonological process (a rule or set of rules) \( P \), by which a segment \( x \) becomes \( z \). If a segment \( y \) is closer to \( z \) than \( x \) is, \( y \) will also be a target of \( P \) in \( L \) and also becomes \( Z \).” Reiss indentifies the subset principle as
(64) **Subset Similarity**

For all segments \(x, y, w, \) and \(z\), \(sim_{\prec}(x, y) < sim_{\prec}(w, z)\) if and only if \(\mathcal{A}(x, y) \subset \mathcal{A}(z, w)\), where \(\mathcal{A}(\alpha, \beta)\) gives the set of agreeing features for segments \(\alpha\) and \(\beta\).

Subset similarity, as defined in (64), does not indicate the similarity of a pair of segments, but rather describes how the similarity of one pair may be compared to another pair, and dictates that the such comparison are only predictive if there exists a subset relationship among the agreeing features. Furthermore, under subset similarity, a pair of segments \((x, y)\) are more similar than some other pair, \((w, z)\) only if \((x, y)\) share all the features of \((w, z)\), and additionally agree on at least one other feature.

Under subset similarity, the Principle of Similarity can be restated, as in (65), indicating a subset threshold for attraction

(65) **Subset Threshold for Attraction**

Let \(x, y,\) and \(z\) be segments such that \(sim_{\prec}(x, y) > sim_{\prec}(x, z)\).

All else being equal, if \(z\) assimilates to \(x\), then \(y\) must also assimilate to \(x\).

In words, (65) requires that if \(z\) assimilates to \(x\), and there exists some \(y\) such that \(y\) agrees with \(x\) on all the features \(z\) does, then \(y\) must also assimilate to \(x\).\(^60\) From the perspective of distance, subset similarity requires that if some \(z\) is attracted to \(x\), and some \(y\) is located between \(x\) and \(z\), then \(y\) must also assimilate. The existence of some alternating \(z\)

---

\(^60\) Of course, the alternate of \(y\), which agrees with \(x\), must be surface viable in the specified position or else harmony would be blocked despite the meeting the similarity threshold. Hence, the qualifier “all else being equal.”
indicates a similarity threshold for attraction that if met or exceeded by \( y \) results in assimilation.

This contrasts acutely with naïve metrics, which posit a threshold for attraction based on the number of feature differences, as schematically shown in the agreement lattice figures below. Recall that in an agreement lattice, the trigger is located at the supremum, nodes are labeled based on which prerequisite features they share with the trigger, and that the number of feature differences as compared to the supremum corresponds to the level on the lattice. Using these lattices, the difference between subset similarity and naïve feature counting may be clearly shown:

(66) Differences between feature counting and subset similarity.

a. Feature counting

\[ \mathcal{A}(x, x) = \{[\text{cons}], [\text{manner}], [\text{place}]\} \]

Level 2: \[ \mathcal{A}(x, y) = \{[\text{cons}], [\text{manner}]\} \quad \{[\text{cons}], [\text{place}]\} \quad \{\text{manner}, [\text{place}]\} \]

Level 1: \[ \mathcal{A}(x, z) = \{[\text{cons}]\} \quad \{\text{manner}\} \quad \{[\text{back}]\} \]

Level 0: \{\}

b. Subset similarity

\[ \mathcal{A}(x, x) = \{[\text{cons}], [\text{manner}], [\text{place}]\} \]

Level 2: \[ \mathcal{A}(x, y) = \{[\text{cons}], [\text{manner}]\} \quad \{[\text{cons}], [\text{place}]\} \quad \{\text{manner}, [\text{place}]\} \]

Level 1: \[ \mathcal{A}(x, z) = \{[\text{cons}]\} \quad \{\text{manner}\} \quad \{[\text{back}]\} \]

Level 0: \{\}
Feature counting, (66)(a), assumes that attraction landscapes operate in terms of lattice levels, so if some $z$ assimilates to $x$, then under feature counting all segments at a higher level must also assimilate. But as seen in (66)(b), a subset similarity threshold, only posits agreement for segments that are at a higher lattice level and also agree on all the features $x$ and $z$ share. In (66)(b), all the $y$’s are between $x$ and $z$ in the sense that if $z$ were to incrementally, but fully agree with $x$, the alternation of $z$ would pass through one of the $y$’s. However, in (66)(a), the step from $z = \{[\text{cons}]\}$ to $\{\text{manner, place}\}$ requires removing the agreement on $[\text{cons}]$, only to add it back in the final step to $x$, so a path from $z$ to $x$ through $\{\text{manner, place}\}$, does not monotonically increase the similarity of $z$, and, therefore, $\{\text{manner, place}\}$ is not between $x$ and $z$. Paths from a lower level node, $z$, to a higher level node, where $z$ is not a subset of $y$ are always non-monotonic.

In this case, the difference between feature counting and subset similarity may seem small, only a single different arrow ($\{[\text{cons}]\}$ to $\{\text{manner, place}\}$), but this one arrow is exactly the empirical problem outlined above. As under subset similarity, there is nothing unusual about general vowel harmony requiring that vowels assimilate, when they also agree on height or backness, but there is something very unsettling about general vowel harmony requiring place and manner matching consonants to also agree, as the extra arrow under feature counting requires. Such patently false predictions have led some researchers (notably Hong, 1994) to eschew the Principle of Similarity, raising doubts about the prospects of similarity-based analyses of assimilation. However, the Subset Threshold for ATTRACTION does not make such false predictions, and so the Attraction Framework restores a restrictive role for similarity in assimilation.
3.8.2. Disjunctive agreement

Unlike the ‘Principle of Similarity,’ these typological anomalies are not required to exist under subset similarity. However, the unattested cases are not ruled out by the SSRP or other principles stated so far. The reason the SSRP does not, alone, eliminate feature counting grammars is that, although a lattice-hierarchy contains all the possible subset similarity relations that can be exploited by an ATTRACTION system, the rankings are only a partial order. No universal assumptions about the rankings of ATTRACTION constraints are made if there is no subset relationship between the sets of prerequisite features.\(^6^1\) For example, while the constraints ATTRACT(R,\{[cons], manner\}→[labial]) and ATTRACT(R,\{[cons], place\}→[labial]) overlap in their preconditions – both depend on [cons], these constraints are not in a subset relationship, so they are predicted to be independently rankable. And in fact, both height-parasitic and back-parasitic rounding vowel harmony are independently attested (see §2.3.1.1 and §2.3.1.2, respectively).

Since not all languages exploit potential similarity dependencies in the same way, there are good reasons not to universally specify a total order, allowing some ATTRACTION constraints to be freely ranked relative to others.

However, this independence still allows some potentially spurious rankings: e.g., ATTRACT(R,\{[cons]\}→[labial]) and ATTRACT(R,\{manner, place\}→[labial]) are not in a subset relationship, so independence of ranking allows them both to be highly ranked, resulting in behavior analogous to the unattested feature-counting. Thus, while Subset Similarity Ranking is imperative to derive the correct thresholding effects in

---

\(^6^1\) As §3.4 discusses, there may be other specific rankings based on certain features that because of phonetic grounding ought to be universally higher or lower ranked, but in general without specific language data in mind, the SSRP only provides a set of universal rankings among subsets.
preconditions to assimilation, other restrictions on ATTRACTION are still needed to adequately restrict the typology of interactions between ATTRACTION and FAITHFULNESS.

How do these unattested systems differ from the attested systems? It is a matter of disjunction versus conjunction in the preconditions. The attested thresholding in parasitic harmony is conjunctive: if segments agree on \( p_1 \) AND \( p_2 \), then they must agree on \( q \). Whereas, the unattested thresholding in parasitic harmony is disjunctive: if segments agree on \( p_1 \) OR \( p_2 \), then they must agree on \( q \).

Conjunctive preconditions are true of all parasitic vowel harmonies, which require agreement on [cons] AND some other vowel feature. This conjunctive nature of prerequisite similarity is also required in cases of consonant harmony. For instance, in Inseño Chumash (§2.2.1), the rightmost /s/ or /ʃ/ triggers regressive [anterior] harmony. Examples, like /ha-ʃ-xintila-wa/ → [haʃxintilawa], confirm that only other continuant, coronal consonants participate in harmony, as none of /x/, /t/, or /i/ alternate. Thus, Chumash sibilant harmony is prerequisite on place AND manner AND [cons].

A harmony space investigation further explains why the problematic rankings, which derive from distance thresholds, allow disjunctive preconditions to agreement. First, consider the numerical ranking of the same regular harmony space as before, in (62), with the change that the strength of \( s_{\text{IDENT-IO}}([\text{labial}]) = 5.5 \):
(67) Numerical ranking of \( \mathcal{N}(\text{ATTRACT}(R, \mathcal{P} = \{\text{cons}, \text{manner}, \text{place}\} \rightarrow \text{labial})) \cup \text{IDENT-IO} ([\text{labial}]) \) with \( S = \{s_{\text{cons}} = 9, s_{\text{manner}} = 1, s_{\text{place}} = 5, s_{\text{I}} = 0, s_{\text{IDENT-IO}([\text{labial}])} = 5.5\} \).

\[
\begin{align*}
\text{e}_{\text{ATTRACT}}(R, \{\text{cons}, \text{manner}, \text{place}\} \rightarrow \text{labial}) &= 15 \\
\text{e}_{\text{ATTRACT}}(R, \{\text{cons}, \text{place}\} \rightarrow \text{labial}) &= 14 \\
\text{e}_{\text{ATTRACT}}(R, \{\text{cons}, \text{manner}\} \rightarrow \text{labial}) &= 10 \\
\text{e}_{\text{ATTRACT}}(R, \{\text{cons}\} \rightarrow \text{labial}) &= 9 \\
\text{e}_{\text{ATTRACT}}(R, \{\text{manner}, \text{place}\} \rightarrow \text{labial}) &= 6 \\
\text{s}_{\text{IDENT-IO}([\text{labial}])} &= 5.5 \\
\text{e}_{\text{ATTRACT}}(R, \{\text{place}\} \rightarrow \text{labial}) &= 5 \\
\text{e}_{\text{ATTRACT}}(R, \{\text{manner}\} \rightarrow \text{labial}) &= 1 \\
\text{e}_{\text{ATTRACT}}(R, \{\} \rightarrow \text{labial}) &= 0
\end{align*}
\]

(67) shows that there are weighting of the factors of similarity relative to faithfulness, which yield disjunctive harmony. This particular weighting (and the corresponding OT grammar) gives the odd disjunctive harmony where if segments agree in [\text{cons}] OR on both \text{manner} and \text{place}, then segments must agree in rounding.

A harmony space description illustrates why this is so. In (68) below, where the critical subspace of the regular harmony space (the \( s_{\text{con}} \times s_{\text{place}} \) plane) is depicted, \( \theta_F = \max d_M - s_{\text{IDENT-IO}([\text{labial}])} = 15 - 5.5 = 9.5 \). As indicated, this threshold is exactly large enough for [i]/[ü] to attract all vowels and [t]/[t''], but no other consonants.
A number of CV effects in rounding harmony are attested in the literature (see §4.5), including vowels triggering rounding on consonants, so the problem in (68) is not that the system gives CV-interactions, but rather that it gives a disjunctive interaction, which selectively includes only the nearest consonants or all vowels.

Ultimately, the pathology of disjunctive agreement stems from the shape of the threshold boundary being (spherical), which allows for the assimilation of some, but not all members of a natural class.\textsuperscript{62} As (68) confirms, spherical attraction basins allow for the creation of assimilation systems which exclude only the most distant possible targets, where ‘most distant’ is dependent on the trigger. Thus, in height-OR-back parasitic rounding harmony on vowels, trigger-[ü] attracts [e]/[ö] and [i]/[u], but not [a]/[o], and trigger-[ö] attracts [i]/[ü] and [a]/[o], but not [i]/[u]. Now, there certainly are languages where some possible targets never participate in harmony. For example, it would not be

\textsuperscript{62} If the space were restructured so that \( s_{\text{cons}} \) were smaller, it would be possible for CV interactions to be parasitic on place or manner forming a precondition that does not include all the vowels. The necessity of such CV-classes is strongly attested in Serbian (see Morèn, 2006) and includes cases such as affrication of /t/ before /l/, place→[continuant]. Thus, CV interactions are not the dilemma, or even classes that include both consonants and vowels, rather it is the combination of a narrow CV class with wider vowel harmony, all assimilating to the same features, that is unattested.
unusual for [a]/[o] to never be a target of rounding harmony, but there are no attested languages in the present survey where context sensitive exclusion is systematically limited to only the most distant.

Furthermore, the predicted disjunctive assimilatory grammars are not few in number. For instance, with the regular harmony space above, $s_{\text{IDENT-IO}([\text{labial}])} = 9.5$ gives harmony conditions ([cons] and place) OR ([cons] and manner), $s_{\text{IDENT-IO}([\text{labial}])} = 4$ gives (place OR [cons]), and $s_{\text{IDENT-IO}([\text{round}])} = 0.5$ give ([cons] OR manner OR place).

Therefore, the free ranking of IDENT-IO relative to ATTRACTION readily allows disjunctive agreement, and so is not typologically restrained.

It is worth noting that any framework which maintains simple distance thresholding ($d \leq \theta_F$) will, likewise, allow disjunctive agreement. This includes systems like Agreement by Correspondence (Rose & Walker, 2004; Hansson, 2001). Therefore, the over-generation follows from the basic concept of distance/similarity, rather than from something particular about the way distance is implemented in ATTRACTION constraints.

3.8.3. Intersection completion for the elimination of disjunctive agreement

To correct for this liability of spherical thresholds, this section proposes a restriction on how ATTRACTION is ranked relative to faithfulness based on the set of active preconditions, $U$, being closed under p-intersection, $\cap^p$. This section makes use of the definitions and theorems presented in §3.5-6, and essentially provides a proof sketch that disjunctive agreement is prohibited if and only if $U$ is closed under $\cap^p$.

Closure of the set of active preconditions, or attraction basin, $U$, under p-intersection guarantees that the threshold set, $T$, only has a single member, $u_{\text{min}}$, (Thm.
(46)) and, therefore, the preconditions for harmony are conjunctive. Under $\cap^P$-closed $U$, harmony only takes place if trigger and target agree on all of the conditions of $u_{\text{min}}$. If trigger and target disagree on any prerequisite feature in $u_{\text{min}}$, then harmony fails to obtain. Thus, closed $U$ is exactly the set of all supersets of $u_{\text{min}}$: $U = \{ u | u_{\text{min}} \subseteq u \subseteq \wp(P) \}$.

With such a $U$, the most distant segment from a trigger $x$ is the segment, $z$, which only agrees on $u_{\text{min}}$, other segments, $y$, are necessarily between $x$ and $z$, i.e., $u_{\text{min}} = \cap^P x \subseteq y \cap^P x \subseteq x \cap^P x = u_{\text{max}} = P$. Hence, if $U$ is a set of all supersets, then it corresponds to a rectilinear hyperbox containing the trigger and all targets agreeing on $u_{\text{min}}$. The rectilinearity of closed sets is confirmed by the examples below in (69), which based on $P = \{ p_1, p_2, p_3, p_4 \}$, presents successively larger $U$ as lattices. Only supremum and infimum are given, because of space concerns, but the labels for the other nodes follow from the SSRP. E.g., $u_{\text{min}} = \{ p_1, p_2 \}$ is necessarily immediately dominated by $\{ p_1, p_2, p_3 \}$ and $\{ p_1, p_2, p_3, p_4 \}$, since those nodes are between $u_{\text{min}}$ and $P$.

(69) Rectilinearity of sample closed $U$ in subset-lattice form.

Because the threshold set has a single element, $u_{\text{min}}$, all and only those $u_i$ such that $u_{\text{min}} \subseteq u_i$ are elements of $U$, and so $U$ is a sublattice with infimum $u_{\text{min}}$ and supremum $u_{\text{max}}$. 

- 177 -
= \mathbf{P}, and all nodes between \( u_{\text{min}} \) and \( u_{\text{max}} \). (69) makes it clear that the set of all nodes between two nodes in a lattice necessarily has the same topology as a rectilinear.

Therefore, \( U \) is closed under p-intersection if and only if \( U \) is a rectilinear basin. Rectilinear basins can be succinctly described with a conjunction of preconditions, so rectilinear basins are exactly like those attested in the present survey.

That rectilinear \( U \) is a necessary condition to avoid disjunctive agreement derives from disjunctive agreement having threshold sets that have more than one element. As schematically, illustrated in the examples below in (70):

(70) Examples showing that non-rectilinear \( U \) gives disjunctive agreement:

\[
\begin{align*}
(70)(a) \text{ is disjunctive because it gives } & (p_1, p_2, p_3 \text{ OR } p_2, p_3, p_4) \rightarrow q, \text{ which excludes a most distant segment that agrees on } p_2 \text{ and } p_3 \text{ but not } p_1 \text{ or } p_4. & \text{ Note if } \{p_2, p_3\} \text{ were an element of } U, \text{ then } (70)(a) \text{ would be closed and correspond to the threshold with solely conjunctive } \{p_2, p_3\} \rightarrow q. \text{ Disjunctive agreement exactly excludes such potential } u_{\text{min}}, \text{ which would lead to rectilinearity and } \cap^{p} \text{-closure. For similar reasons, } (70)(b) \text{ and } (70)(c) \text{ are also disjunctive.}

\text{In sum, if } U \text{ is restricted to be closed under p-intersection then, } U \text{ has a single element threshold set, so } U \text{ is a rectilinear subspace, and, therefore, disjunctive agreement}
\end{align*}
\]
is prohibited. Furthermore, if disjunctive agreement is prohibited, then \( U \) is rectilinear, and so \( U \) is \( \cap^p \)-closed. Therefore, \( \cap^p \)-closure of \( U \) is a necessary and sufficient condition to ensure that disjunctive agreement is not permitted, and so a restrictive theory of \textsc{Attraction} must implement a restriction equivalent to \( \cap^p \)-closure of \( U \).

3.8.4. Typological numbers and motivations

As §3.8.3 shows, if the \textsc{Attraction} framework can incorporate \( \cap^p \)-closure of \( U \), then it is guaranteed to be restrictive in its predictions: only allowing disjunctive harmony. At the outset, there seem to be several alternatives for how to ensure \( \cap^p \)-closure of \( U \): (i) modify the constraints \textsc{Iden}t-\textsc{IO} or \textsc{Attraction}, (ii) stipulate a ranking principle, or (iii) hypothesize some other bias against disjunctive harmony. For the reasons presented in this section I hypothesize that the \( \cap^p \)-closure of \( U \) derives from principles which govern the acquisition of phonological processes, rather than the application of universal rankings, or from the formulation of the constraints themselves.

Firstly, it is not clear how to modify the behavior of faithfulness or \textsc{Attraction} constraints in a distance framework so that threshold boundaries are rectilinear not spherical. However, it is clear that if only the faithfulness constraint were modified and not the \textsc{Attraction} constraints, and if faithfulness may be freely ranked against \textsc{Attraction}, then no matter how faithfulness is implemented, there always remains the prediction of disjunctive harmony. Furthermore, modifying the \textsc{Attraction} constraints alone is not likely to be beneficial, because each individual constraint is conjunctive, and independently rankable conjunctive constraints are robustly needed and attested.

Thus, it is only in the interaction with faith, that the \textsc{Attraction} system gives disjunctive harmony. Therefore, the \( \cap^p \)-closure of \( U \) is tantamount to a restriction on
how IDENT-IO may be ranked relative to ATTRACTION constraints, i.e. \( \cap^p \)-closure of \( U \) requires a restriction on the interaction of ATTRACTION and IDENT-IO and not on the formulation of any of the constraints.

A grammar restriction of this sort is impossible to state in terms of universal rankings, because the ranking of some ATTRACTION constraints relative to faith depends on the rank of other ATTRACTION constraints, which need to be freely rerankable to cover the cross-linguistic variation. Also, a ranking requirement for the \( \cap^p \)-closure of \( U \) can only be stated in terms of intersection closure, \( \forall u_i, u_j \in U, u_i \cap^p u_j \in U \), or in terms of constraints, \( \forall u_i, u_j \in \mathcal{P}(P) \), such that ATTRACT(\( R, u_i, q \)) and ATTRACT(\( R, u_j, q \)) dominate IDENT-IO(\( q \)), ATTRACT(\( R, u_i \cap^p u_j, q \)) also dominates IDENT-IO(\( q \)). By the sheer complexity, such a ranking principle might inevitably appear stipulative in nature if postulated to be part of Universal Grammar.

However, as the remainder of this section shows, there are motivations for such a restriction based on learning. I hypothesize that the closure of \( U \) derives from principles associated with a conjunctive bias in learning. The learning benefits of eliminating disjunctive parasitic harmony can be highlighted with the following hypothetical data, in (71) below, for an unattested disjunctive harmony system, which requires progressive harmony of [labial] if target vowels agree on height OR backness. Assume the usual Turkic inventory \{i, ü, e, ö, i, u, a, o\}, and that [labial] vowels are the only triggers.
(71) Example data for unattested height OR back parasitic rounding harmony on vowels. Bold inputs do not assimilate.

<table>
<thead>
<tr>
<th>Trigger:</th>
<th>Hi, front</th>
<th>Non-hi, front</th>
<th>Hi, back</th>
<th>Non-hi, back</th>
</tr>
</thead>
<tbody>
<tr>
<td>ü.i → ü.ü</td>
<td>ö.i → ö.ü</td>
<td>u.i → u.ü</td>
<td>o.i → o.ü</td>
<td></td>
</tr>
<tr>
<td>ü.e → ü.ö</td>
<td>ö.e → ö.ö</td>
<td>u.e → u.e</td>
<td>o.e → o.ö</td>
<td></td>
</tr>
<tr>
<td>ü.i → ü.u</td>
<td>ö.i → ö.i</td>
<td>u.i → u.u</td>
<td>o.i → o.u</td>
<td></td>
</tr>
<tr>
<td>ü.a → ü.a</td>
<td>ö.a → ö.o</td>
<td>u.a → u.o</td>
<td>o.a → o.o</td>
<td></td>
</tr>
</tbody>
</table>

As (71) shows, the majority of evidence (75%) indicates that a target vowel always agrees with the trigger. Worse yet, the context where a target fails to assimilate is different for each possible target, so disjunctive parasitic harmony requires learning a separate exception to alternation for each possible trigger vowel (or alternatively each target).

Thus, learning disjunctive harmony is more difficult because it eliminates most generalization from one trigger-target pair to another. In contrast, conjunctive harmony affords ample generalization, since all triggers and targets that agree on $u_{\text{min}}$ assimilate, as shown in back-parasitic rounding harmony in (72), below:

(72) Training data for back parasitic rounding harmony on vowels (cf. Turkish §2.3.1.2). Bold inputs do not assimilate.

<table>
<thead>
<tr>
<th>Trigger:</th>
<th>Hi, front</th>
<th>Non-hi, front</th>
<th>Hi, back</th>
<th>Non-hi, back</th>
</tr>
</thead>
<tbody>
<tr>
<td>ü.i → ü.ü</td>
<td>ö.i → ö.ü</td>
<td>u.i → u.ü</td>
<td>o.i → o.ü</td>
<td></td>
</tr>
<tr>
<td>ü.e → ü.ö</td>
<td>ö.e → ö.ö</td>
<td>u.e → u.e</td>
<td>o.e → o.e</td>
<td></td>
</tr>
<tr>
<td>ü.i → ü.i</td>
<td>ö.i → ö.i</td>
<td>u.i → u.u</td>
<td>o.i → o.u</td>
<td></td>
</tr>
<tr>
<td>ü.a → ü.a</td>
<td>ö.a → ö.a</td>
<td>u.a → u.o</td>
<td>o.a → o.o</td>
<td></td>
</tr>
</tbody>
</table>
Here, the pattern of the trigger [ü] can be generalized without exception to the trigger [ö], and the pattern of [u] can be generalized to [o], and vice versa.

The relative difficulty of disjunctive versus conjunctive harmony is also supported by considering the number of possible typologically distinct OT rankings eliminated by requiring $\cap^p$-closure of $U$. As shown in the following table in (73):

(73) Ranking combinatorics:

| $|P|$ | $|\beta(P)|$ | a. Number of possible rankings of ATTRACTION. | d. Number of SSRP conforming rankings of ATTRACTION. | e. Number of typological distinct rankings when IDENT-IO is added freely. | f. Number of typologically distinct rankings with $\cap^p$-closure of $U$. |
|---|---|---|---|---|---|
| Eq: | $2^{|P|}$ | $(2^{|P|})!$ | $??$ | $??$ | $2^{|P|} + 1$ |
| 1 | 2 | 2 | 1 | 3 | 3 |
| 2 | 4 | 24 | 2 | 6 | 5 |
| 3 | 8 | 40320 | 48 | 20 | 9 |
| 4 | 16 | $2.0923 \times 10^{13}$ | $1.68 \times 10^9$ | 168 | 17 |
| 5 | 32 | $2.6313 \times 10^{15}$ | $??$ | $??$ | 33 |

Note when no restrictions are placed on the relative ranking of FAITH and ATTRACTION, IDENT-IO is permitted to freely rank in a SSRP-conforming ATTRACTION hierarchy, so the count in (73)(e) includes both conjunctive and disjunctive harmony, whereas (73)(f) only counts conjunctive harmony. Of most note in (73) is that the number of typologically distinct rankings, when both disjunctive and conjunctive parasitic harmony are allowed, (73)(e), grows exponentially in $2^{|P|}$, but the number of conjunctive parasitic harmonies, (73)(f), grows linearly. Therefore, allowing disjunctive harmony makes the

---

A recursive program was written to count the number of SSRP conforming rankings and the number of those rankings which are typologically distinct when IDENT-IO was freely added. This program was not smart! It generated all possible rankings, making it $O((2^{|P|})!)$, and upon creation checked each for SSRP-conformity. There are very real space challenges ($2.6313 \times 10^{15}$ rankings) that made non-recursive solutions unavailable. However, the algorithm was as optimized as possible and can be shown to not miss any rankings. The estimated time for simultaneously counting the number of SSRP- rankings and number of typologically distinct rankings on a new MacBook Pro purchased in 2008, when $|P|=5$, is on the order of weeks. Therefore, the numbers for $|P|=5$ are unknown and indicated with ‘???’.
set of possible languages exponentially larger than the set of languages, which have $\cap^p$-closure of $U$, so a learner which assumes only conjunctive parasitic harmony has a much smaller hypothesis space to search.

In the domain of formal learning theory, conjunctive hypothesis spaces have been known for decades (see, e.g., Mitchell, 1997 for a broad review) to provide learning advantages. This work in formal learning theory has shown that learners that are biased towards assuming the hypothesis space is conjunctive, implement a version of Occam’s Razor, which presumes that simpler hypotheses are better than more complex hypotheses. This can be directly implemented in terms of ATTRACTION by assuming that the size of the threshold set correlates with hypothesis complexity: conjunctive parasitic harmony can be described by a single element threshold set, $T = \{ u_{\min} \}$, but disjunctive parasitic harmony may require as many as $|P|$ elements, e.g. $T = \{ \{ p_1 \}, \ldots, \{ p_k \} \}$. From this perspective, the learning advantage of a restriction on $\cap^p$-closure of $U$ is that such simpler hypotheses are guaranteed to be true of the languages being learned, facilitating language transmission by simplifying the task for the learner.

Another difference between conjunctive and disjunctive hypothesis spaces is that learners which are biased towards conjunctivity are biased towards ranking the most general ATTRACTION constraint above faith. For example, presented with examples that, respectively, require $(p_1 \land p_2) \rightarrow q$ and $(p_1 \land p_3) \rightarrow q$, a conjunctively-biased learner concludes that the correct description of the harmony is $p_1 \rightarrow q$ (the only hypothesis that satisfies $(p_1 \land p_2) \rightarrow q$ AND $(p_1 \land p_3) \rightarrow q$), and therefore, more general ATTRACT($R$, $\{ p_1 \}$, $q$) ought to be ranked above IDENT-IO($q$). In contrast, when given the same evidence, a disjunctively-biased learner concludes that the correct hypothesis is exactly the examples
seen: \((p_1 \land p_2) \lor (p_1 \land p_3)\)→q, and so there is no incentive for the elevation of \(\text{ATTRACT}(R, \{p_1\}, q)\).

In this way, a bias towards conjunctivity is a bias towards generalization, the benefits of which are only now being debated in the work on phonological learning. For instance, Albright & Hayes (2006) show that correctly learning Navajo sibilant harmony requires a bias which elevates the rank of more general constraints, otherwise their learner tends to postulate rules which overfit the training data. In contrast, (Hale and Reiss, ms) argue child learners are initially very restrictive in their assignment of categories, and so are biased away from generalization. Thus, the role of a generalization bias in phonological acquisition is not completely resolved.

There are, however, a growing number of examples which suggest that some biases must be available to artificial language learners (both human and computer) (Wilson, 2006; Hayes & Wilson, 2008; Albright & Hayes, 2006; Albright in prep), suggesting that biases to learning are universally available, being active in both synchronic and diachronic grammar. In sum, the appeal of \(\cap^P\)-closure of \(U\) being derived from a bias in learning is that it avoids overly stipulative ranking principles, allows for the normal operation of \(\text{ATTRACTION}\) and \(\text{IDENT-IO}\), and adds to the growing work on the role of bias in phonological learning.

What possible languages remain after the application of the SSRP and the requirement for the \(\cap^P\)-closure of \(U\) to \(\text{CON}_{BA}\)? The answer is that there is a distinct language corresponding to a set of preconditions for each element of \(\mathcal{P}(P)(a u_{\text{min}}\text{ for each } P' \subseteq P)\), and additionally a language that does not allow harmony at all. Thus, for a prerequisite feature set, \(P = \{[\text{con}], \text{manner}, \text{place}\}\) and a harmonic feature \(q = [\text{labial}],\)
the following languages are predicted for different weighting of the conditional similarities. Here, in (74), these languages are ordered from most to least stringent preconditions:

(74)

<table>
<thead>
<tr>
<th>Predicted languages</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>No [labial] harmony.</td>
<td>English</td>
</tr>
<tr>
<td>( [\text{con}] \land \text{manner} \land \text{place} ) parasitic [labial] harmony.</td>
<td>Height parasitic vowel rounding harmony in Yawelmani (§2.3.1.1).</td>
</tr>
<tr>
<td>( [\text{con}] \land \text{manner} ) parasitic [labial] harmony.</td>
<td>Back parasitic vowel rounding harmony in Turkish (§2.3.1.2).</td>
</tr>
<tr>
<td>( \text{manner} \land \text{place} ) parasitic [labial] harmony.</td>
<td>General vowel rounding harmony in Khirgiz</td>
</tr>
<tr>
<td>( [\text{con}] ) parasitic [labial] harmony.</td>
<td>Labial attraction in Igbo/Turkish (/api/→/ap/)</td>
</tr>
<tr>
<td>( \text{place} ) parasitic [labial] harmony.</td>
<td>General [round] harmony</td>
</tr>
<tr>
<td></td>
<td>Round spreading in Nawuri (Casali, 1995).</td>
</tr>
</tbody>
</table>

Clearly, not every possibility is attested in (74), but the vacant cells can be shown to be necessary for other types of harmony. For example, \( [\text{con}] \land \text{manner} \land \text{place} \) parasitic [anterior] harmony exists in Chumash sibilant harmony (§2.2.1) and a number of \( \text{manner} \land \text{place} \) parasitic assimilations exist in Serbian (Morén, 2006). Therefore, the absence of certain types of parasitic rounding harmony is not because the formal expressiveness of \textit{Attraction} is still somehow too powerful, but rather because other facts about how rounding interacts with these other features.

Therefore, (74) shows that \( \cap^p \)-closure of \( U \) allows for a typologically restrictive Attraction Framework, affording distance related effects, like those seen in \textbf{Chapter 2}, but also previewing the kinds of CV interactions to be discussed at length in \textbf{Chapter 4}.  

- 185 -
3.9. Conclusion

I have argued that together two principles, the Subset Similarity Ranking Principle and the $\cap^p$-closure of $U$, guarantee all of the following, in (75):

\begin{equation}
\text{(75) Benefits of the SSRP and $\cap^p$-closure of $U$}
\end{equation}

\begin{enumerate}
\item \textbf{Distance:} A preservation of the notion of representational distance consistent with the Entailment framework and other models of attraction (§3.5.4-3.5.5).
\item \textbf{Simplicity:} A single element threshold set $T = \{u_{\text{min}}\}$ (§3.5.6), providing a relationship to a learning bias like Occam’s Razor (§3.8.4).
\item \textbf{Implementation:} Equality between HG and OT (§3.6), affording multiple levels of analysis, including symbolic grammars and subsymbolic harmony spaces (§3.7).
\item \textbf{Restrictiveness:} The avoidance of unattested disjunctive parasitic assimilation (§3.8.3), which greatly constrains the typology of predicted languages.
\end{enumerate}

Taken together, this collection of formal and semi-formal results cements the essential status of these principles in the Attraction Framework. This formalization of attraction prepares a fertile ground for the phonological analysis of assimilation in subsequent chapters.

Furthermore, this chapter showed how the strategy of multiple levels of explanation (Prince & Smolensky, 2006) can be applied to attraction. Forces of attraction were shown to be expressed in $\text{OT}_{\text{LCC}}$ (§3.5), in HG(§3.6), and in low-level harmony.
spaces (§3.7). This chapter highlighted a number of ways in which the HG and harmony spaces derived aspects of the Attraction Framework that can only be stated as descriptive stipulations at the level of OT\textsubscript{LCC}. In particular, the phonetic grounding of parasitic dependency derives from clustering across similarity among subsymbolic representations (§3.4), and the interplay between spherical attractions and rectilinear harmony spaces explained why disjunctive harmony is available in HG and OT\textsubscript{LCC}(§3.8). These underpinnings form a basis for the OT-level discussion of the next chapter, which focuses on generalizations at the phonological level.
4. ATTRACTION at a distance

4.1. Introduction

Phonological segments are the symbols that combine to form strings of phonological material. There is a long history in phonology of using minimal contrast in the segmental makeup of morphemes as evidence for contrastive phonological features. For example, in English, [b]it and [p]it contrast in both meaning and in the nature of the first symbol: [b] vs. [p]. Phoneticians have shown that this minimal pair contrast is due to a difference in the onset timing of the vibration of the vocal cords, and phonologists formally described such a contrast with a feature, like [±voice].

However, the contrast between morphemes is not limited to feature contrasts: sequential position also plays a critical role. In English, [p]it and til[p] are sequential position minimal pairs, containing the same phonemes, but in a different order. Such positional pairs show that string order is a contrastive ‘feature’ in English. This observation is hardly revolutionary, but it underscores how it is possible to take for granted the existence of sequential contrasts when working with phonological strings, where positional information is implicitly encoded.

Research in connectionist networks has shown that explicitly representing order information is non-trivial. In particular, models of the English past tense (Rumelhart &

---

Portions of this chapter were presented at the 45th Annual Meeting of the Chicago Linguistic Society, held in April 2009, under the title “Integrating preconditions on parasitic vowel harmony.”
McClellan, 1986) and the ensuing debate (Pinker & Prince, 1988; etc.) about ‘wickelphones’ has confirmed the non-trivial nature of the so-called binding problem (see Bechtel & Ambrahmsen, 2002; Smolensky & Legendre, 2006 for reviews), which is the problem of determining how to encode strings in a network such that positional and segmental information are bound together. The technical, connectionist solution to the binding problem I adopt is presented in Chapter 5, but for present purposes, the main lesson is that representing strings requires associating segments with positions.

Furthermore, this association is not unlike the association between features and segments: a phonological feature, like [±voice], must be associated with the representation of segments in order to give rise to a [b]it~[p]it distinction, likewise, a positional contrast between [p]it and ti[p] requires that positional information, say word-initial or word-final, must be encoded in the representation of the segments which make up phonological strings. In English, the availability of positional information is confirmed by the fact that word-initial and word-final stops differ in phonetic detail: Word final stops are not released, yielding aspiration allophony ([pʰ]it, *[p]it and ti[p], *[ti[pʰ]]). More broadly, the cross-linguistic existence of directionality effects and metathesis confirms that, in the human phonological system, segments must be associated with information about sequential position. Therefore, in some form or another, phonological representations must have access to the positional information of the segments in morphemes.

While there are a number of aspects of the representation of phonological position worthy of exploration (prosody, syllable structure, edge-effects, morphological
boundaries, etc.), this dissertation develops but one hypothesis about how this sequential information is encoded. This hypothesis is indicated in (1), below:

(1) **Positional Similarity Hypothesis:**

There exists a similarity relation among the representation of positions such that proximate segments are more similar than distal segments.

Segments with more feature values in common are more similar than segments with fewer feature values in common. Likewise, (1) indicates that segments which are sequentially close (proximate) are “more similar” than the same segments at a greater distance. For example, by this hypothesis, [p] and [t] in *pit* are representationally more similar than [p] and [t] in *posit* because the segments are more proximate in the shorter string.

Now, there is a formal explanation presented in detail in **Chapter 5**, for how proximity must be encoded in a connectionist network such that the hypothesis of positional similarity is true. The basic notion is that the motivations for positional similarity might derive from similarity along a temporal dimension: all else being equal the pattern of activation at time $T$ is more similar to a pattern at time $T+1$ than at $T+1+k$. And so, positional similarity can be grounded in connectionist computation by positing that, the activation pattern encoding a segment at root position $X_i$ is more similar to the activation pattern encoding the same segment at $X_{i+1}$ than at $X_{i+1+k}$. Thus, positional similarity can be implemented by controlling the representation of the role vectors in a role-filler system (Smolensky & Legendre, 2006) that encode phonological strings.

This chapter, however, develops arguments from the perspective of theoretical phonology for using proximity-based similarity as the driver for locality effects in
phonology. In particular, this chapter focuses on how the availability of a similarity-based encoding of position provides a way for the Attraction Framework to be naturally extended to the abundant locality affects in assimilation. When attraction is applied to positional similarity, proximate segments are more strongly attracted than distal segments. This chapter presents evidence suggesting that allowing attraction forces to act on positional similarity yields all of the following:

(2) **Benefits of Attraction and Positional Similarity:**

1. An explanation for both transparency and blocking by neutral segments.
2. A unified analysis of consonant and vowel harmony.
3. A typological description of the range of expected interactions between locality and parasitic harmony.
4. The flexibility to analyze languages where phonetic similarity derives otherwise unnatural classes of participants.
5. The availability of similarity solutions to both local and non-local interactions.
6. A general similarity perspective, which can recast autosegmental feature-tiers as similarity attraction regions in a harmony space.

Taken together, this coverage of different aspects of assimilation constitutes a body of evidence for the merits of the Attraction Framework of which positional similarity is an integral part. To my knowledge, this is the first work to take a similarity-based perspective on locality, and the only available constraint framework that fully integrates both locality and feature-based preconditions on assimilation (although see also Suzuki, 1998; Gafos, 1996; Ní Chiosáin & Padgett, 2001; Hansson, 2001; Pulleyblank, 2004; Frisch, Pierrehumbert, & Broe, 2004; Rose & Walker, 2004; Ajibóyê & Pulleyblank, ms. 2008 for influential work regarding similarity and/or locality in phonology).
This chapter is outlined as follows: §4.2 presents methods and issues related to extending attraction for sensitivity to locality, §4.3 applies these principles to a number of parasitic vowel harmony patterns with differing locality conditions, §4.4 directly compares and contrasts local and non-local nasal harmony, suggesting that feature similarity plays a role in both, §4.5 demonstrates how attraction in a multidimensional similarity space can subsume both autosegmental feature tiers and feature geometric representations, §4.6 concludes.

4.2. Extending ATTRACTION for sensitivity to locality

Recall from Chapter 3, that in an attraction model (Burzio, 2002a, b, 2004, 2005; Burzio & Tantalou, 2007; Wayment et. al, 2007) of assimilation, triggers are seen as sources of attraction and targets are subject to these attractive forces. These attractive forces are related to distance in a representational space such that the closer a target is to a trigger, the greater the pressure to assimilate. Thus, attraction derives how similarity acts as a precondition on assimilation: only targets which are close enough to triggers are attracted, and attracted targets assimilate as they move closer to triggers.

Formally, attraction can be implemented in Optimality Theory (Prince & Smolensky, 2004) with a proposed family of ATTRACTION constraints, restated, here, in (3):
Let $R$ be a correspondence relation between segments in a surface form, $\alpha$. $P = \{p_1, p_k\}$ is a set of parasitic features, and $q$ is a harmonic feature.

Let $x, y$ be segments in $\alpha$.

**IF**

(i) $xRy$,

(ii) $\forall p_i \in P$, $x$ agrees with $y$ on $p_i$,

**THEN**

(iii) $x$ and $y$ must agree on feature $q$.

As (3) indicates, ATTRACTION constraints are segment-to-segment faithfulness constraints, and as detailed in §3.5, these constraints must be ranked according to the Subset Similarity Ranking Principle (SSRP), which requires that if $P \supset P'$, then $\text{ATTRACT}(R, P \rightarrow q) \gg_{\text{UG}} \text{ATTRACT}(R, P' \rightarrow q)$. The SSRP stratifies ATTRACTION constraints in a specific to general fashion, giving rise to distance effects and preserving the weakening of attractive forces as trigger and target become less featurally similar. As shown below, by properly interleaving ATTRACTION and IDENT-IO, it is possible to analyze parasitic assimilation as an application of feature-based similarity preconditions.

The main focus of this section is to develop the hypothesis that sequential proximity, or locality, may also be expressed as a similarity relationship and that locality is on par with feature-based similarity as a precondition to harmony. This chapter shows that, like the cases in **Chapter 2**, where an amount of feature-based similarity is prerequisite to assimilation, in this chapter, an amount of positional similarity, i.e. locality, may be prerequisite to assimilation. Positional similarity predicts that when locality is a significant component of overall similarity, the Attraction Framework correctly yields blocking, but when the locality preconditions are relaxed, the Attraction Framework predicts transparency. Formally, these locality preconditions are expressed
as a set of segment-to-segment correspondence constraints (Rose & Walker, 2004; Hansson, 2001).

4.2.1. Locality ‘features’ via segment-to-segment correspondence

Note that ATTRACT as specified in (3) does not stipulate that \( x \) and \( y \) be in correspondence, but merely uses correspondence to drive prerequisite feature similarity effects.\(^{65}\) Such correspondences are explicitly posited by a separate family of correspondence constraints, which require correspondence between segments which satisfy locality condition \( l \), as defined in (4) below:

\[
\text{(4)} \quad \text{CORR}(l)
\]

Let \( x, y \) be segments in surface form \( \alpha \).

\[
\begin{align*}
\text{IF} & \quad x, y \text{ satisfy locality precondition } l \\
\text{THEN} & \quad xRy.
\end{align*}
\]

Different CORR constraints require the existence of different correspondences, allowing ATTRACTION constraints to operate at various grains of locality. Therefore, in addition to the relationships previously discussed, the Attraction Framework is a variation on the framework posited by Rose & Walker (2004) and Hansson (2001, 2007) for Agreement by Correspondence (ABC), having both a segment-to-segment faithfulness component, ATTRACTION, and a correspondence component, CORR. In addition to significant conceptual differences mentioned in this chapter, there are a number of subtle formal difference between the Attraction Framework and ABC. The main formal difference is that Rose & Walker (2004) and Hansson (2001) use feature similarity to derive a ranking

---

\(^{65}\) Of course, it would be possible to create more complicated attraction constraints that integrate both locality and similarity preconditions into a single attractor, but there would be limited typological gains. Furthermore, separating locality and similarity preconditions between, respectively, CORR and ATTRACT constraints creates a division of labor that makes it easier to interpret tableaus.
of correspondence constraint and have a unitary IDENT-CC constraint for segment-to-segment faithfulness. Here, correspondence is only determined by locality and it is the segment-to-segment faithfulness (ATTRACTION) which is ranked by similarity.  

Determining and controlling for locality preconditions is an important component of the analysis in this chapter. The positional similarity hypothesis in (1) suggests that harmony can depend on agreement on locality ‘features’, like say string adjacency. However, unlike phonological features, proximity is not a property of individual segments, but rather a property of pairs of segments. Thus, these locality ‘features’ are enforced in the Attraction Framework by way of the CORR constraints.

As discussed in §3.5, correspondence is related to the persistence of entailments, and so if segments $x, y$ are in correspondence, then $xRy$ posits that the entailments of $x$ persist until $y$. Different locality preconditions constitute different amounts of entailment persistence. For example, a correspondence constraint that requires adjacent segments to be in correspondence posits that entailments only persist in a strictly local fashion. Thus, while correspondence is not directly derived from entailments, correspondence directly implements a hypothesis about how the entailments of one segment may be applied to another. In particular, when $x$ is not in correspondence with $y$, then the entailments of $x$ do not persist to $y$, so $y$ is under no pressure to agree with $x$, even if it otherwise meets the feature similarity preconditions specified in an ATTRACTION constraint. Chapter 5 discusses in more explicitly computational terms how the persistence of entailments derives from connectionist representations.

---

66 Walker (2009) and Rhodes (2008) use the original formulation of ABC where correspondence constraints do the work of feature-based similarity, and so for them transparency occurs by a lack of correspondence. In this dissertation, correspondence constraints perform a different labor (enforcing locality) and so there is a different consequence (blocking) when there is a lack of correspondence.
What are the relevant locality preconditions on harmony? The evidence suggests that in addition to two directionalities, \( R \rightarrow L \) (regressive harmony) and \( L \rightarrow R \) (progressive harmony), there are at least three proximity preconditions on which harmony may depend: segmental adjacency, syllabic adjacency, and no adjacency. These different proximities constitute different constraints as indicated in (5), below.\(^{67}\)

(5) Possible locality preconditions on harmony:

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Locality precondition</th>
<th>Example Languages</th>
<th>Correspondence Template</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{CORR}(X_i X_{(i+1)})_{L \rightarrow R} )</td>
<td>Segmental adjacency</td>
<td>Turkish (\S4.3.3)</td>
<td>( \text{cV, cV, cV} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ikwere (\S4.4.2)</td>
<td></td>
</tr>
<tr>
<td>( \text{CORR}(\sigma_i \sigma_{(i+1)})_{L \rightarrow R} )</td>
<td>Syllable adjacency</td>
<td>Yawelmani (\S4.3.1-2)</td>
<td>( \text{cV, cV, cV} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Moba (\S4.4.4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Yoruba (\S4.4.4)</td>
<td></td>
</tr>
<tr>
<td>( \text{CORR}_{L \rightarrow R} )</td>
<td>No adjacency required</td>
<td>Finnish (\S4.3.4)</td>
<td>( \text{cV, cV, cV} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ngbaka (\S4.4.5)</td>
<td></td>
</tr>
</tbody>
</table>

These constraints are defined more precisely in (6) below. Note that the indicated precedence relations in the definitions are necessary because all of the constraints in (5) and (6) have Left to Right directionality (\( L \rightarrow R \)).

\(^{67}\) The relevance of syllabic adjacency in LDCA is noted by Rose & Walker (2004) and enforced with their PROXIMITY constraint. Pulleyblank (2002) and Ajibọyẹ & Pulleyblank (ms, 2008) argue for distinct agreement constraints for vowel harmony that apply under adjacency and non-adjacency. Citing Suzuki (1998) and Walker (2000), Hansson (2001) provides a proximity hierarchy like (7), which additionally allows for \( \mu-\mu \) correspondence.
Defining Right to Left CORR constraints:

a. \( \text{CORR}(X_i X_{i+1})_{L \rightarrow R} \)
   if \( x \) immediately precedes \( y \),
   then \( xRy \).

b. \( \text{CORR}(\sigma_i \sigma_{i+1})_{L \rightarrow R} \)
   Let segments \( x \) and \( y \) be respectively dominated by syllables, \( \sigma_i \) and \( \sigma_j \).
   if \( x \) precedes \( y \),
   \( \sigma_i = \sigma_j \) \( \text{OR} \) \( \sigma_i \) immediately precedes \( \sigma_j \),
   then \( xRy \).

c. \( \text{CORR}_{L \rightarrow R} \)
   if \( x \) precedes \( y \),
   then \( xRy \).

Now, directionality is not an issue directly addressed in the present work, so like the original correspondence framework (Rose & Walker, 2004; Hansson, 2001), directionality is implemented as argument of correspondence constraints rather than derived from other principles. Also, note that Hansson (2001) shows that asymmetrical correspondence does not alone give directionality effects because disagreeing \( xRy \) can satisfy \text{ATTRACTION} by altering either \( x \) or \( y \). As Hansson shows, directionality effects require a split of segment-to-segment faithfulness constraints. In the Attraction Framework, that split implemented by making \text{ATTRACT}(R, P \rightarrow [+q]) distinct from

---

68 In a representation, an element \( \alpha \) precedes an element \( \beta \), if the position of \( \alpha \) is to the left of the position of \( \beta \) in a phonological string; e.g., in \textit{pit}, [p] precedes both [i] and [t]. Furthermore, \( \alpha \) immediately precedes an element \( \beta \), if the position of \( \alpha \) is to the left of the position of \( \beta \) and \( \alpha \) is adjacent to \( \beta \). In \textit{pit}, [p] immediately precedes [i]. Right to Left versions are available by replacing “precedes” with “follows.”

69 Under \text{CORR}(\sigma_i \sigma_{i+1}) (cf. \text{PROXIMITY} in Rose & Walker, 2004), if segments are co-syllabic (\( \sigma_i = \sigma_j \)) or if segments are in adjacent syllables (\( \sigma_i \) immediately precedes \( \sigma_j \)) then a correspondence is required. Thus, \text{CORR}(\sigma_i \sigma_{i+1}) requires a correspondence if segments are \textit{at least} as positionally similar as being in neighboring syllables.

70 In ABC, it was the segment-to-segment faithfulness constraints that had the directionality argument, not the correspondence constraints, as here.
ATTRACT(\(R, P \rightarrow [-q]\)) or in the case of a monovalent feature, [F], \(\text{ATTRACT}(R, P \rightarrow [F])\) is only violated if \(xRx\) and \(x\) is specified as [F]. Since directionality is not central to the claims of this dissertation, and to avoid the details of this solved problem, it is generally assumed that asymmetric correspondence does indeed give rise to directionality of harmony.

As with sets of prerequisite similarity features, there is a subset relation among the correspondence constraints in (6). For example, segmental adjacency \((V_1, V_2)\) implies at least syllabic adjacency, but not vice versa \((V_1, cV_2)\). Likewise, a correspondence between segments in adjacent syllables is required by both \(\text{CORR}(\sigma_i \sigma_{(i+1)})_{L \rightarrow R}\) and \(\text{CORR}_{L \rightarrow R}\), but \(cV_1, cV_2, cV_3\) with \(V_1RV_3\) is only required by \(\text{CORR}_{L \rightarrow R}\). Because of this subset relationship, by the Subset Similarity Ranking Principle (3.5.5), these constraints must be universally ranked in specific to general fashion as in (7):

\[
(7) \quad \text{Similarity ranking of \text{CORR} constraints:} \\
\text{CORR} (X_i X_{(i+1)})_{L \rightarrow R} \gg \text{UG CORR}(\sigma_i \sigma_{(i+1)})_{L \rightarrow R} \gg \text{UG CORR}_{L \rightarrow R}
\]

Under this ranking of \text{CORR} constraints, proximate segments are under more pressure to be in correspondence than distal segments. Thus, the hierarchy of \text{CORR} constraints is an application of attraction to the positional similarity hypothesis. (7) enforces the positional similarity hypothesis through ranking, since adjacent segments are more similar than those same non-adjacent segments in adjacent syllables, which, in turn, are more similar than those same segments elsewhere in a surface form.\(^{71}\)

\(^{71}\) There may be other cases (see Rose & Walker, 2004; Hansson, 2001) where it is important to distinguish between adjacent segments in the same syllable from adjacent segments in separate syllables. Such a co-syllabic correspondence constraint allows harmony from coda to onsets, but not to other syllables. For most of the cases discussed in this chapter, no such correspondence is needed (although see §4.4.3). Even more generally, it may be important to allow for a role of other constituents like Foot, \(\mu\),
4.2.2. Viable locality for blocking and transparency

The hierarchy in (7) above instantiates the generalization that non-local interaction implies the availability of local interaction. The reverse, however, is not true: languages may allow local assimilation without any amount of non-local assimilation. This section explores how blocking and transparency relate to locality. Following Suzuki (1998) and Pulleyblank (2002), I sketch how the availability of non-local interaction allows transparency, and the avoidance thereof derives blocking.

4.2.2.1. A formal duplication for transparency and blocking

In order to provide some context for the above proposal, I now briefly review recent research on transparency and blocking in assimilation. Both transparent segments and blocking segments do not assimilate to triggers, so by definition they are neutral to harmony.\(^{72}\) Transparent and blocking segments diverge, however, in how they affect whether other targets assimilate to triggers. Neutral segments are transparent if subsequent targets undergo harmony. Neutrals segments are blockers if subsequent targets do not undergo harmony.\(^{73,74}\)

The importance of locality to blocking has been understood, since at least the advent of Autosegmental Phonology (Goldsmith, 1979), where a constraint against lines

---

\(^{72}\) Neutral segments can carry the harmonic feature underlying, and so may vacuously agree with a trigger. Non-vacuous assimilation requires alternation. Therefore, a segment is only neutral if there exists a triggering context in which it fails to express harmonic features. If harmony is manifest as a Morpheme Structure Constraint (see §2.2.2), then neutrality is attested by the co-occurrence of segments which disagree on harmonic features (see Kiparsky & Pajusalu, 2003).

\(^{73}\) Often blockers act as triggers of harmony, causing eligible targets to undergo harmony to the blocker instead of the ‘original’ trigger. These triggering blockers are defined as neutral because they occur in a position where other eligible targets do undergo harmony, but fail to undergo harmony.

\(^{74}\) In an unfortunate over-application of terminology, with analogy to transparency blockers have been called ‘opaque,’ but ‘opacity’ also has meaning in a derivational context that is more naturally associated with transparent segments, so I strictly use ‘blocker.’
crossing prohibited non-local interaction, and also predicted blocking. In various forms, strict versions of locality have been imported into OT (see Gafos, 1996; Baković, 2000; Walker, 2003; Benus & Gafos, 2005; Ní Chiosáin & Padgett, 2001; Smolensky, 2006; McCarthy, 2004). However, assuming strict locality creates a challenge because transparency seems to indicate that triggers are able to reach across neutral segments in order to affect subsequent targets, suggesting there is something inherently non-local about true transparency.

In order to preserve locality and its explanation for blocking, phonologists have employed additional machinery to analyze transparency. For instance, one strategy is to posit covert harmony (Gafos, 1996; Ní Chiosáin & Padgett, 2001; Benus & Gafos, 2005; Walker & Mpiranya, 2005, to appear), where transparent segment do not phonemically participate in harmony, but at a phonetic level there are fine grained articulatory differences in how transparent segments are realized depending on the harmonic context. These small differences allow harmony to propagate locally and reflect a non-linearity in the harmony system, whereby small phonetic differences at one point may yield large phonemic differences at another (Gafos & Benus, 2006). Under these explanations, transparent segments do undergo harmony, but they do so covertly, sub-phonemically.

Another, slightly different explanation for transparency is found in Span Theory (McCarth, 2004; O’Keefe, 2006) and Headed Feature Domains (Smolensky, 2006), where transparent segments reside in so-called embedded domains. For instance, to analyze an ATR harmony like Wolof (O’Keefe, 2006), where high vowels are transparent to [–ATR] harmony, the [+ATR] high vowel is embedded in a [–ATR] domain, \( [u]_{+\text{ATR}}, [\varepsilon]_{-\text{ATR}} \). Note transparent [u] is in a [+ATR] domain, but this domain is dominated

- 200 -
by a [−ATR] domain, where [−ATR] harmony is strictly local. These hierarchical structures are a representational version of the derivational accounts explicitly advocated by Walker (2003) and implied by Targeted Constraint analysis (Baković & Wilson, 2000): at an earlier phonological level, transparent segments are full-participants in harmony, but at a later phonetic level the contrast on the harmonic feature is neutralized on transparent vowels. Under these hierarchical/derivational accounts, transparency is an instance of derivational opacity (Kiparsky & Pajusalu, 2003; Baković, 2006).

These differing accounts of transparency under locality are not mutually exclusive. Covert harmony offers an explanation for how harmony can obtain locally without the phonemic alteration of neutral segments: even subphonemic distortion of a neutral segment is sufficient to trigger harmony in subsequent targets. Complementarily, a hierarchical/derivational account explains how phonological harmony can fail to result in phonemic alteration in neutral segments: harmony takes place at a more abstract level, where transparent segments express harmonic features, but at a lower, phonetic level the contrast on the harmonic feature is neutralized, and so only the contrast expressed in the phonological inventory is available to the harmony process.

In sharp contrast to these proposals based on local feature spreading, strict locality is patently disregarded in the theory of Agreement by Correspondence (ABC; Rose & Walker, 2004; Hansson, 2001, 2007). Therefore, ABC naturally accounts for massive transparency, where even a large number of interveners may be transparent to the long-distance interactions between triggers and targets, e.g. in Inseño Chumash (Hansson, 2001) /ha-xintila-waʃ/ → [haʃxintilawaʃ] (see §2.2 for other cases).

Turbidity theory (Finley, 2008) also provides a multi-leveled account of transparency through the availability of different ‗projections‘, confirming that the availability of an intermediate-level where harmony takes place reconciles transparency and strict locality.
Advocates of ABC initially struggled with whether ABC could even derive blocking (Rose & Walker, 2004; Hansson, 2001), and proclaimed its apparent lack of blocking a virtue in the context of the long-distance consonant harmonies that seemed to eschew blocking. However, Hansson (2007) showed that under certain ranking conditions a number of blocking effects are in fact predicted by ABC, as the correspondence which might allow assimilation is avoided in order to satisfy other higher-ranked constraints.

Thus, within the current theoretical landscape, assuming a formal system with a bias towards locality, like those derived from autosegmental spreading, naturally accounts for blocking effects, but additional machinery/explanation is required for transparency. On the other hand, assuming a formal system with a bias towards non-locality, like ABC, naturally accounts for even massive transparency, but also, necessarily allows a number of blocking effects. Therefore, the literature presents an analytical duplication in terms of the available machinery for transparency and blocking.

Now, Optimality Theory supports “homogeneity of target/heterogeneity of process” (McCarthy, 2002), so it is not obvious that such duplication is ipso facto evidence for one proposal or another. Perhaps, languages are free to employ any of a number of grammatical processes that can give rise to transparency and/or blocking. In fact, McCarthy (2007) argues that both local spreading and non-local agreement may be independently needed for different morphological aspects of the same language.

Usually in OT, the “homogeneity of target” derives from a single markedness constraint and “the heterogeneity of process” follows from distinct rankings of faithfulness. However, the consequence of the duplicative machinery is that for assimilation the situation is somewhat reversed: homogeneous faithfulness and
heterogeneous markedness. The homogeneous target is an assimilatory repair, i.e. violations of a single faithfulness constraint, IO-FAITH (McCarthy & Prince, 1994). Any plurality of process follows from the diversity of available harmony drivers, including ALIGNMENT constraints (McCarthy & Prince, 1993; Prince & Smolensky, 2004), AGREE constraints (Baković, 2000), frameworks like ABC (Rose & Walker, 2004; Hansson, 2001) and contextual markedness constraints (Suzuki, 1998; Pulleyblank, 2002; Ajíbóyè & Pulleyblank, ms, 2008), feature domain systems (Cole & Kisseberth, 1994; Smolensky, 2006; McCarthy, 2004), and others. These harmony constraints are in many ways unrelated markedness constraints, which would, therefore, seem no more likely to pair together in terms of repair than, say, *NC̆ and ONSET. It is true that voiceless consonants can delete in order to satisfy *NC̆ (Pater, 2001) and that vowels may also delete in order to avoid hiatus and satisfy ONSET (McCarthy, 2002), but surely there is no deep relationship between *NC̆ and ONSET in a theory of deletion. Since the drivers are independent, it would seem, likewise, accidental that the multiple markedness constraints for harmony tend to prefer the same kind of repair: violations of IO-FAITH (see Finley, 2008 for a review of evidence suggesting other faithfulness constraints like DEP or MAX have limited interaction with harmony). Thus, without a unified solution one needs many different markedness constraints to explain assimilation, precisely like before the advent of OT, many different rules were available, and so any commonality was an unexpected ‘conspiracy’.76 Thus, there is great risk that a formal dichotomy between spreading and long-distance agreement is that it might also be a conspiracy.

76 To this point, Walker (2009b) argues that none of the available solutions is sufficient to cover the available assimilatory data.
4.2.2.2. Unification

With the above background, a basic claim of this dissertation is that the similarity of representations can predict which processes (spreading or long-distance agreement, blocking or transparency) will apply. However, a stronger claim of the present work is that by allowing locality to be a violable notion, there is a parsimonious, unified explanation of transparency and blocking aspects of assimilation in the Attraction Framework. Thus, the empirical dichotomy, whereby some languages exploit locality and others ignore it, does not necessarily require a formal dichotomy of unrelated families of harmony constraints. This section previews how the Attraction Framework can unify blocking and transparency, by controlling for positional similarity through the ranking of correspondence constraints.

Before discussing this unifying framework, it is important to directly address the empirical claims that have supported such a dichotomy based on the differences between (long-distance) consonant and vowel harmony (Rose & Walker, 2004; Hansson, 2001).77 Rose & Walker and Hansson each argue that there are differences in terms of sensitivity to similarity, default directionality, and availability of blocking. In particular, they argue that (i) similarity is more exclusively germane to long-distance consonant harmony than, say the spreading harmony, exemplified by spreading nasal harmony, (ii) blocking is unattested in long-distant consonant harmony, but readily available in vowel harmony, and (iii) long-distance consonant harmony, unlike vowel harmony, exhibits a default bias in directionality (Right-to-Left), while on the other hand vowel harmony tends to exhibit dominance (only Hansson (2001) embraces this point of view). Additionally, there is an implied claim which follows from the formalism where only consonants may be in

77 For other arguments against the dichotomy, see Ajibóyè & Pulleyblank (ms, 2008).
correspondence, which is that (iv) vowel harmony is governed by different harmony
drivers than long-distance harmony. This point is a crucial one because it has sometimes
been assumed since Clements & Keyser (1983) that consonants and vowels reside on
separate skeletal tiers and are governed by different constraints. 78

However, I will argue that the above claims are undermined by the following
considerations. The possibility of different drivers for consonant and vowel harmony is
untenable because

1) The literature describes a growing number of harmony processes that
show interactions between consonants and vowels in terms of blocking
(§4.4, §4.5), so harmony constraints do not privilege one major class over
another by segregation of consonants and vowels into separate
tiers/constraints (see esp. Clement & Hume, 1995; Clements, 2003; Ní
Chiosáin & Padgett, 2001 for similar arguments).

2) There are phenomena suggesting long-distance vowel agreement is
attested (§4.3.3, 4.4.3) eliminating the possibility that differences between
consonant and vowel harmony are universal.

The exclusivity of similarity to non-local consonant harmony is undermined because

3) There are cases of parasitic vowel harmony that like consonant harmony
employ feature-similarity (§2.3, §4.3), so feature similarity is also an
important factor in vowel harmony.

78 It is not clear that this view is necessarily held by Hansson (2001) and Rose & Walker (2004),
since they each hint at the prospects of ABC for an analysis of parasitic vowel harmony. It is clear,
however, that they view vowel harmony which has been analyzed as spreading to be unrelated to long-
distance consonant harmony.
4) There exist strictly local assimilations for both consonants and vowels where feature similarity seems to play a role (§2.4), so (contra Rose & Walker, 2004; Hansson, 2001) feature similarity preconditions are not exclusive to long-distance assimilation.

Likewise, research shows that the differences in terms of directionality (Hansson, 2001) are not absolute, but only tendencies

5) Both consonant and vowel harmony exhibit default directionality, and both exhibit dominance (Rose & Walker, 2004; Hyman, 2002).

Thus, the strongest remaining difference, which might motivate a formal dichotomy, is a tendency to avoid blocking in consonant harmony and the ready allowance of blocking in vowel harmony (Hansson, 2001; Rose & Walker, 2004). However, the tendency for non-locality in consonant harmony was never claimed to be universal: a number of the cases in (Rose & Walker, 2004; Hansson, 2001) were known to require syllabic adjacency of trigger and target, so Agreement by Correspondence already makes allowances for an amount of locality. Furthermore, there are languages where consonant harmony can be blocked, notably Kinyarwanda (Walker & Mpiranya, 2005; Walker, Byrd, & Mpiranya, 2009) and Sanskrit (Gafos, 1996; Hansson, 2001; Rose & Walker, 2004), which have been shown through both theoretical and phonetic studies to implement a kind of local spreading.

Thus, a careful review of the evidence suggests that the presence of blocking is not a difference between consonant and vowel harmony, but rather a typological diagnostic, which determines the importance of locality. In other words, it is not the case
that blocking is one parameter of description and locality is another (free) parameter of description. If that were true, then the full cross product of blocking and locality ought to be attested, as indicated below in (8) with roughly uniform distribution in each cell. Here, ‘Adjacent’ denotes the (iterative) locality conditions that operate in spreading harmony and ‘Long-distance’ denotes the absence of such conditions in non-local agreement processes.

(8) Prediction if blocking is independent of locality (✓ = attested, * = not attested)

<table>
<thead>
<tr>
<th></th>
<th>Blocking</th>
<th>No Blocking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjacent</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Long-distance</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

In contrast, if there were full dependency between blocking and locality, then a different strongly diagonal typology is predicted, as shown below (9):

(9) Prediction if blocking is dependent of locality (✓ = attested, * = not attested)

<table>
<thead>
<tr>
<th></th>
<th>Blocking</th>
<th>No Blocking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjacent</td>
<td>✓</td>
<td>*</td>
</tr>
<tr>
<td>Long-distance</td>
<td>*</td>
<td>✓</td>
</tr>
</tbody>
</table>

Among the generalizations that emerges from this dissertation is that no sample of consonant harmony or vowel harmony is like (8), rather the distribution of harmony processes for both consonants and vowels more closely resembles (9) with a strong correlation between the proximity of interaction and the occurrence of blocking.

Of course, there are reasons to expect an asymmetric sensitivity to locality in consonants and vowels: as Gafos (1996) argues, vowel gestures may persist through the closures associated with consonants, but consonant closures cannot persist through nucleic vowels, which require an open vocal tract configuration. Thus, V-to-V coarticulation has a tendency towards strict locality in ways that C-to-C coarticulation
does not (see also Ni Chiosáin & Padgett, 2001 and additional discussion in §4.4.4). However, there are cases of vowel harmony where locality conditions are more relaxed allowing for substantial transparency as in Moba Yoruba (§4.4.3) and Finnish (§4.3.4). There are also abundant cases of CC-interactions where strict adjacency is required and, as such, any neutral intervener is a blocker (see §2.4), so Gafos’ observation reflects a strong – but not universal – tendency.

In spite of these difference, it would be a mistake to classify any harmony system with blocking (whether consonant or vowel harmony) as requiring non-local interaction, since long-distance interaction is inherently at odds with blocking. Blocking requires sensitivity to locality, but non-local agreement, by definition, ignores locality. Hence, the empirical contrast of avoiding blocking in non-local interactions borders on tautology: if a system has blocking, then it is not a long-distance agreement process, otherwise the trigger would reach beyond the blocker to affect subsequent targets, and if a system has non-local interaction, then it does not have blocking because the trigger can reach beyond interveneres. In sum, the observation that long-distance consonant agreement does not have blocking is really the observation that evidently, some languages, like Kinyarwanda and Sanskrit, prohibit non-local interaction in ways that other systems, like Inseño Chumash, do not. The contrast between Kinyarwanda, Sanskrit, and Chumash is particularly relevant, since all are coronal harmony systems that involve /s/-/ʃ/ alternations.

In the Attraction Framework, locality differences do not require unrelated families of harmony constraints, but rather a distinction in sensitivity to positional similarity: distinctions in sensitivity to locality are unified within the Attraction Framework through
contrasts in the ranking of CORR constraints. In an extreme form, the contrast between feature spreading and long-distance agreement may be stated as follows: the former postulates that locality is inviolate, and the later postulates that locality is ever violated. Allowing for the ranking of violable locality constraints provides a middle way: for the analysis of those blocking phenomena or other processes where strict locality is important, CORR(X_iX_{i+1}) can be highly ranked and those constraints which would allow non-local interaction (CORR(σ_iσ_{i+1}), CORR) can be ranked below relevant faithfulness constraints. With such a ranking, only local correspondence can motivate alternation, rendering strict locality inviolate. On the other hand, for the analysis of massive transparency or other process where locality is completely ignored, less local correspondence constraints, CORR(σ_iσ_{i+1}) and/or CORR, may be ranked above relevant faithfulness constraints, allowing ATTRACTION to motivate non-local interaction.

Therefore, the hypothesized grains of locality in the CORR constraints allow for a range of blocking and transparency phenomena, as indicated below in (10).

(10) Predicted transparency and blocking if CORR(l) (from (4)) dominates IDENT-IO

<table>
<thead>
<tr>
<th>Corr constraint</th>
<th>Transparent Element(s)</th>
<th>Blocking Element(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CORR(X_iX_{i+1})</td>
<td>None</td>
<td>Single Neutral Segment</td>
</tr>
<tr>
<td>CORR(σ_iσ_{i+1})</td>
<td>Multiple Neutral Segments</td>
<td>Single Neutral Syllable</td>
</tr>
<tr>
<td>CORR</td>
<td>Multiple Neutral Syllables</td>
<td>None</td>
</tr>
</tbody>
</table>

In (10), above, there is a tradeoff between transparency and blocking: systems with massive transparency eschew blocking and systems with rampant blocking eschew transparency. This tradeoff follows directly from locality because both blocking and transparency are determined by how much distance between a trigger and target is permitted for a correspondence and, therefore, for harmony to obtain. When faith or
other constraints dominate CORR constraints, then the demands for strictly local, σ-local, or non-local correspondence may be violated, even in output forms. Thus, if under some ranking, a CORR(l) constraint is never satisfied, then for that ranking no l-interaction is permitted. For instance, if IDENT-IO([q]) dominates CORR(σiσi+1) then even if some ATTRACTION constraint dominates IDENT-IO([q]), assimilation that is not strictly local is avoided in order to satisfy IDENT-IO([q]) by failing to have correspondence between non-adjacent triggers and targets. Without correspondence, ATTRACTION is not active, and so, in such situations, cannot motivate violations of IDENT-IO([q]) non-locally.

Thus, it is possible for the Attraction Framework to implement strictly local spreading that disallows σ-σ-interaction, where even a single neutral intervener blocks harmony. Strictly local spreading occurs when CORR(XiXi+1) is the only correspondence constraint ranked above IO-Faith. This happens the same way it does in other strictly local frameworks: because of strict locality, the trigger cannot directly affect subsequent targets, so these targets are under no pressure to assimilate, and so faithfulness demands disharmony. This is a typical pattern of the spreading nasal harmony phenomena discussed in §4.4.

With additionally CORR(σiσi+1) ranked above IO-faith, then a single neutral intervener is transparent, and possibly multiple interveners, but not if the neutral intervener(s) constitute an entire syllable. There is no way for a trigger to reach across a neutral syllable, since the constraint that would posit such a correspondence (CORR) is ranked below faith. Thus, in vowel harmony without transparency (see §4.3), Vx.c.Vx is a possible template for correspondence, but Vx.c.Vx is not. Now, if CORR also happens
to be ranked above IO-faith, then no blocking is predicted and even massive transparency is allowed.\textsuperscript{79}

As a concluding note on unification, violability of locality is consistent with the original spirit of Optimality Theory. As Prince & Smolensky note:

“Because they are ranked, constraints are regularly violated in the grammatical forms of a language. Violability has significant consequences not only for the mechanics of description, but also for the process of theory construction: a new class of predicates becomes usable in the formal theory, with a concomitant shift in what we can think the actual generalizations are. \textit{We cannot expect the world to stay the same when we change our way of describing it.”} -- (Prince & Smolensky, 2004, pp. 8, italics added).

Perhaps, a commitment to inviolate strict locality would represent a resistance to a changing of world views. This, however, is not to say that locality is unimportant! Strict locality requires the participation (either covert or overt) of all segments in a surface form, including both consonants and vowels. This is exactly the right generalization for a number of blocking and spreading phenomena (Gafos, 1996; Ní Chiosáin & Padgett, 2001; Walker, 2003), but locality is not the only generalization. For example, trigger-target feature similarity also plays a prominent role, and, therefore, it is not surprising to find cases of true transparency, where the demands of locality surrender to the demands

\textsuperscript{79}Hansson (2007) shows that in ABC such “global” correspondence as allowed by CORR can result in a limited amount of blocking, but not due to locality, rather it is the result of feature-similarity: under CORR $\gg$ Ident-IO($q$) blocking only occurs if the neutral intervener is more similar to the target than the trigger is, and both $[+q]$ and $[-q]$ induce harmony. The same is true of the present work, but overgeneration is much less likely. Because similarity is a more fluid notion than the feature counting of Frisch similarity used by Hansson, different aspects of phonetic similarity may be exploited for the same segments, so blocking by similarity is hardly fatal.
of other constraints, as exemplified by long-distance agreement. Thus, employing ranked violable locality through correspondence constraints does not discount the importance of strict locality, rather it discounts a universal requirement that all assimilations persist through strictly local means, allowing strict locality to apply to some cases, perhaps even most cases, but not necessarily all cases of assimilation.

4.2.3. Features, locality, and general Similarity

4.2.3.1. General prerequisite similarity in the Attraction Framework

The more similar two representations are the stronger the attraction tension between them (Burzio, 2002a, b). The sufficient similarity for attraction pressures to result in assimilation is called the prerequisite similarity. Chapter 2 argued at length that features are preconditions that determine prerequisite similarity, and that prerequisite similarity is determined on a language specific basis. Throughout this chapter, I consider language data which shows how locality is another precondition which determines (a more general) prerequisite similarity and, likewise, how locality preconditions are determined on a language specific basis. Thus, feature similarity and positional similarity interact as equal partners in determining the strengths of the attraction tension between triggers and targets. In this section, I develop a notion of general prerequisite similarity (GPS), which is influenced by both feature similarity and positional similarity. A theory of general prerequisite similarity allows the computational principles of attraction to derive the effects of both locality and feature similarity.

From the GPS perspective, proximity can be viewed as a kind of similarity on par with feature-based similarity. However, proximity is implemented in a different fashion (CORR constraints) than prerequisite feature similarity (ATTRACT constraints), so it is
useful to explain how the positional similarity hypothesis, (1), is related to correspondence. As previously discussed in §3.5.1, correspondence is grounded in elements of speech planning, where there are penalties for resetting articulatory targets (Hansson, 2001; Rose & Walker, 2004; Walker, 2005). Computationally speaking, a correspondence is related to the persistence of entailments (Burzio, 2002a, b): a correspondence $x \text{R} y$ posits that the entailments of $x$ persist until $y$. Correspondence encapsulates the hypothesis that segments which are proximate are in the same “temporal activation window” of network (see Chapter 5), and so are representationally similar. Hence, segments which are in correspondence have one aspect of their representation that is similar (position), so there is attraction pressure to also be similar in other aspects (features) in order to avoid resetting the elements of speech planning. In a language where entailments only persist locally, the window of overlapping activation is small, and so segments in distal positions are more prone to disagree on features. Nevertheless, languages may differ in the locality conditions which incur the penalties for resetting articulatory targets, so the ranking of CORR constraints instantiates the hypothesis that differing amounts of positional similarity are allowed.

As with ATTRACTION and feature-based similarity, the ranking of CORR constraints relative to IDENT-IO($q$) instantiates a preferences for how important locality is to determining the GPS of segments in a surface form. The lower the ranking of IDENT-IO($q$) relative to the hierarchy of CORR constraints in (7), the less important proximity-based similarity is to determining the general prerequisite similarity. In this way,

---

80 In addition to avoiding the resetting or articulatory targets, harmony can also be motivated by avoiding the resetting of acoustic targets on the assumption that cues of longer duration are more easily perceived. In this way, harmony can enhance acoustic contrasts that are otherwise weak (Walker, 2001; Finley, 2008; see also Chapter 2).
although formally somewhat different, positional similarity receives a ready implementation at the descriptive level of OT. However, I note that at the lower entailment level, both positional similarity and feature similarity derive from the forces of harmony maximization which avoid entailment violations (see Chapter 5).

4.2.3.2. A GPS tradeoff between features and proximity

§4.2.2.2 discussed how sensitivity to positional similarity predicts a tradeoff between transparency and blocking. This section discusses how sensitivity to a more general notion of similarity predicts the attested tradeoff between positional similarity and feature-based similarity, indicated below in (11):

(11) **Tradeoff between features and proximity:**

Local elements interact under feature similarity or feature dissimilarity, but non-local elements only interact under feature similarity; featurally similar elements interact either locally or non-locally, but featurally dissimilar elements only interact locally.

This tradeoff between feature-similarity and proximity is observed in the literature on assimilation (Pulleyblank, 2002; Archangeli & Pulleyblank, 2007; Ajíbóyè & Pulleyblank, ms, 2008), although there were earlier indications that such a tradeoff can be found based on dissimilatory phenomena (Suzuki, 1998; Frisch et al., 2004).

In terms of a cross-linguistic typology, features and proximity are independent components of general similarity. Languages may have stringent feature preconditions without stringent locality preconditions (see cases of long-distance consonant agreement §2.2) and vice versa (see cases of spreading nasal harmony §4.4). Therefore, to a large
extent, the sufficient similarity for assimilation (GPS) may be achieved either by having high positional or high feature similarity. Assigning a precondition of high positional similarity allows featurally dissimilar segments to interact, and assigning a precondition of high feature similarity allows positionally dissimilar segments to interact (see also Archangeli & Pulleyblank, 2007; Ajibóyè & Pulleyblank, ms, 2008), but if segments meet neither positional nor feature similarity preconditions, then the segments are representationally very dissimilar, and so attraction pressures are too weak to overcome the demands of faithfulness, thereby, prohibiting harmony.

This tradeoff is found in the typology below in (12), which shows that there are no attested cases without either feature or proximity preconditions. The absence of such similarity blind languages are discussed, following the presentation of a typology of GPS.

4.2.3.3. A typology of general prerequisite similarity

The positional similarity hypothesis allows for the development of a typology of assimilation based on the range of possibilities for overall, or general, prerequisite similarity. By allowing languages to attribute different weights (rankings) to feature-based similarity and positional similarity for computing the strength of attraction tension between triggers and targets in a space of general prerequisite similarity, the Attraction Framework derives a cross-linguistic typology seen, below, in (12). This free ranking follows from either independent ranking in OT or independent weighting in HG.

In (12), cells are filled with example assimilatory processes and the section where the examples may be found. The axes indicate the relative importance of features and proximity for determining the general prerequisite similarity of trigger-target pairs. The horizontal axis expresses the importance of proximity, ranging from “+” strictly local
assimilations where only $\text{CORR}(X_i X_{i+1})$ dominates faith to “0” $\sigma$-local assimilations where $\text{CORR}(\sigma_i \sigma_{i+1})$ also dominates faith to “−” non-local assimilations where general $\text{CORR}$ dominates faith. The vertical axes expresses the importance of features, ranging from “+” very stringent feature similarity preconditions to “0” somewhat stringent feature preconditions to “−” almost no feature-based similarity preconditions. The cells in the table may be indexed by $(L, F)$ pairs, e.g. $(L+, F0)$ denotes the case where locality is strongly weighted, but features are only moderately weighted.

(12) A typology of general prerequisite similarity

<table>
<thead>
<tr>
<th>A general prerequisite similarity typology of assimilation</th>
<th>How important is locality $(L)$?</th>
<th>More Important</th>
<th>Less Important</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{CORR}(X_i X_{i+1})$</td>
<td></td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>$\text{CORR}(\sigma_i \sigma_{i+1})$</td>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$\text{CORR}$</td>
<td></td>
<td>−</td>
<td></td>
</tr>
<tr>
<td>$\text{ATTRACT} ({p_1, \ldots, p_n}, q)$</td>
<td>$\text{ATTRACT} ({}, q)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strictly local assimilations with feature preconditions. ($\S$2.4, $\S$4.3.3)</td>
<td>Strictly local assimilations with feature preconditions. ($\S$2.4, $\S$4.3.3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parasitic vowel harmony with transparent C’s and blocking by neutral V’s. ($\S$4.3.2)</td>
<td>Parasitic vowel harmony with transparent C’s and blocking by neutral V’s. ($\S$4.3.2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long-distance consonant harmony. ($\S$2.2)</td>
<td>Long-distance consonant harmony. ($\S$2.2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vowel harmony blocked by consonants. ($\S$4.5.2)</td>
<td>Vowel harmony blocked by consonants. ($\S$4.5.2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vowel harmony with transparent C’s and blocking by neutral V’s. ($\S$4.4.3)</td>
<td>Vowel harmony with transparent C’s and blocking by neutral V’s. ($\S$4.4.3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vowel harmony with transparency. ($\S$4.3.4)</td>
<td>Vowel harmony with transparency. ($\S$4.3.4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spreading nasal harmony with blocking. ($\S$4.4.1-4.4.2)</td>
<td>Spreading nasal harmony with blocking. ($\S$4.4.1-4.4.2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spreading nasal harmony with transparency. ($\S$4.4.1-4.4.2)</td>
<td>Spreading nasal harmony with transparency. ($\S$4.4.1-4.4.2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No attested languages.</td>
<td>No attested languages.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As similarity is the inverse of distance in the representational space, in the Attraction Framework, high similarity corresponds to low distance, and, therefore, the strongest attraction forces. From this general similarity perspective, where there is blocking, proximity must be playing a stronger role in computing the general similarity of segments, and where there is transparency proximity must be playing a lesser role in
determining general similarity. Thus, blocking by a single neutral segment is indication that a harmony process belongs under the (L+) column in (12), while transparency of a single neutral segment with blocking by larger neutral substrings indicates (L0), and massive transparency indicates (L−) (see also (10)).

As (12) indicates in (L+, F+), languages may have assimilatory processes where both adjacency and feature similarity are prerequisite for triggers to attract targets. Such is usually the case in non-harmony languages. In non-harmonic languages, the assimilation triggers fail to attract possible targets which are featurally similar, but not adjacent. Likewise, triggers fail to attract targets which are adjacent, but not featurally similar. For example, in American English nasal place assimilation (see §2.4.8), there is no non-local nasal place assimilation (map contrasts with nap, so there are no [NVp]→[mVp] alternations), nor do nasals assimilate to continuants (*[Nf]→[mf]), so both feature similarity (agreeing on continuancy) and locality (strict adjacency) are needed to give the attested [Np]→[mp] patterns, like impossible.

On the other hand, languages with spreading harmony (L+, F−) reflect an asymmetric sensitivity to proximity over features in computing general similarity, allowing sources to attract almost any adjacent target (see cases of spreading nasal harmony below in §4.4.3). In contrast, long-distance harmonies like long-distance consonant harmony, (L−, F+), place feature-based similarity at a premium, allowing triggers to attract targets at any distance as long as they have a sufficient number of features in common. In terms of GPS, there are a range of phenomena in between spreading harmony and long-distance consonants harmony, so while the table has discrete

---

81 However, while feature similarity preconditions are certainly very low in spreading nasal harmony, §4.4 argues that these feature similarity preconditions are not at absolute zero. Feature similarity plays a dominant role in determining the trigger-target pairs in nasal harmony.
categories, a more detailed sample would reveal a more continuous distribution across the dimensions. Chapter 5 demonstrates how this typological tradeoff between features and locality may be derived from attraction properties of a neural network.

4.2.3.4. Impossible languages

As the shaded cell in (12) indicates, there are no attested processes that seem blind to both the feature content and the sequential proximity of triggers and targets. Such a similarity blind language would allow for the arbitrary interaction of segments at any distance, e.g. the height of a vowel determines the nasality of a stop at any distance: */e/+lato/→[e.la.to] and */i/+lato/→[i.la.no]. No similarity blind languages were found in the present survey, suggesting that sensitivity to similarity is a robust universal principle of the phonological processes at work in human cognition. Such interactions could never be motivated by Attraction because there is such little similarity between trigger and target.

With the aim of defusing any worries that attraction is a too useful notion, so it must also be too powerful, I now briefly consider a few other systems that are impossible from the perspective of general prerequisite similarity.

In the present survey, I found no cases of anti-parasitic harmony, where harmony only occurs if trigger and target disagree on other features. Anti-parasitism is not the same as non-parasitism. Non-parasitic systems allow triggers and targets to interact when the prerequisite similarity between them is very low. Most of these, however, at least require [cons] agreement, so even non-parasitic systems require something more than zero feature similarity. Also, asymmetric harmony systems, where certain feature values privilege triggers or targets, are not anti-parasitic. E.g., it is not uncommon for
only high or only non-high vowels to trigger rounding harmony (Kaun, 1995; Walker, 2001). However, for true anti-parasitism both high and non-high vowels would have to trigger and undergo harmony, but the trigger-target pairings would have to be anti-parasitic. This can be illustrated with “anti-Yawelmani” (cf. real Yawelmani §4.3.1), in (13) below.

(13) *Anti-Yawelmani rounding vowel harmony (unattested). Shaded cells mark alternating affixes.

<table>
<thead>
<tr>
<th>Triggers(↓) \ Targets(→)</th>
<th>High</th>
<th>Non-high</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[hin]/[hun] ‘non-future’</td>
<td>[al]/[ol] ‘might’</td>
</tr>
<tr>
<td>High</td>
<td></td>
<td></td>
</tr>
<tr>
<td>/xil/ ‘tangles’</td>
<td>xil-hin</td>
<td>xil-ol</td>
</tr>
<tr>
<td>/dub/ ‘lead by the hand’</td>
<td>dub-hin</td>
<td>dub-ol</td>
</tr>
<tr>
<td>Non-high</td>
<td></td>
<td></td>
</tr>
<tr>
<td>/xat/ ‘eat’</td>
<td>xat-hin</td>
<td>xat-ol</td>
</tr>
<tr>
<td>/bok/ ‘find’</td>
<td>bok-hun</td>
<td>bok-ol</td>
</tr>
</tbody>
</table>

What makes (13) so strange from the perspective of attraction is that, /dub/ can induce harmony across height on [al]/[ol], but /dub/ cannot induce harmony within height on [hin]/[hun]. Segments must be dissimilar in height, before they can assimilate in rounding. To my knowledge, there are no such attested cases of anti-parasitism.

Likewise, there are no *anti-local* languages, allowing harmony at greater distances, but not when the same trigger and target are more proximate. There are systems where certain specific positions are targets of harmony from a non-adjacent trigger, where intervening segments cannot undergo harmony (Pulleyblank, 2002; Walker, 2001, 2009b), but these systems derive from interactions with prosody rather than locality per se. True anti-local harmony would allow harmony at any distance greater than some threshold, say, productive ATR harmony if and only if trigger and target are more than two syllables apart. No such systems are known to exist.
The absence of anti-local harmony and anti-parasitic harmony follows directly from general prerequisite similarity and the “Principle of Similarity” (see §3.8.1). If a target undergoes harmony, then a more similar segment is also under attraction pressure because it exceeds what ever similarity preconditions were met by the dissimilar segment in order to alternate. A proximate segment is necessarily more GPS-similar to the trigger than a distal segment, so anti-local harmony is impossible. A featurally similar segment is also necessarily more GPS-similar to the trigger than a featurally dissimilar segment, so anti-parasitic harmony is impossible.

Thus, there are certain unattested patterns whose absence follows directly from the perspective of general prerequisite similarity. These unattested cases are explored more in Chapter 5, where a connectionist simulation confirms that the Attraction Framework cannot generate anti-similarity languages.

However, there is still a fair amount of attested typological variability predicted by GPS as shown in (12). To illustrate how the Attraction Framework can account for this variability, I now consider two case studies in detail. Firstly, an analysis of different instance of parasitic vowel harmony (§4.3), which confirms the existence of feature-based similarity effects at all localities. Secondly, an analysis of local and non-local nasal harmony (§4.4) which explores assimilation across a range of localities and feature similarities.

4.3. Integrating preconditions on parasitic vowel harmony

*Parasitic vowel harmony* (PVH; Steriade, 1981; Cole, 1987; Cole & Trigo, 1989; Mester, 1988; van der Hulst, 1988; Hong, 1994; Archangeli & Pulleyblank, 1994; Kaun, 1995; van der Hulst & van de Weijer, 2001; etc.) occurs when agreement on a *harmonic*
feature has a feature similarity precondition of agreement on some other parasitic feature. §2.3 reviewed a number of the parallels between PVH and long-distance consonant agreement (LDCA). In particular, this survey of PVH found vowel assimilations for which prerequisite feature similarity plays a dominant role in governing the relationship between trigger and targets. Furthermore, the survey suggests that, cross-linguistically, this sensitivity to phonological similarity is robust: there are a number of parasitic systems for a range of harmonic features; rounding and ATR exhibiting the strongest tendencies toward parasitism. Additionally, there is overwhelming evidence for the existence of a phonetic motivation grounding the relationship between parasitic and harmonic features. §3.4 and §4.4.2 show how this phonetic grounding predicts the dependency between parasitic and harmonic features.

This section expands on these feature similarity themes, focusing on how to integrate the similarity preconditions of PVH with the locality concerns common to vowel harmony. Now, the prospects for a similarity-based analysis of parasitic vowel harmony are hinted at by Rose & Walker (2004), Frisch et al. (2004), Burzio (2005), and Hansson (2007), but none provide a formal account. This section shows how a formal attraction model with violable locality can be successfully applied to parasitic vowel harmony.

4.3.1. Contrast in parasitism: rounding harmony in Yawelmani and Kirgiz

The formal properties of faithfulness thresholding were developed in Chapter 3. Before turning to locality contrasts, this section provides a concrete example of thresholding to illustrate the difference between parasitic and non-parasitic vowel harmony. The Yawelmani dialect of Yokuts is a prototypical example of PVH. The
Yawelmani data below in (14), show that rounding agreement is only required if vowels already match in height (shaded cells).

(14) Height parasitic rounding vowel harmony in Yawelmani (data from Cole & Kisseberth, 1997).

<table>
<thead>
<tr>
<th>Triggers(↓) \ Targets(→)</th>
<th>High</th>
<th>Non-high</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[hin]/[hun] ‘non-future’</td>
<td>[al]/[ol] ‘might’</td>
</tr>
<tr>
<td>High</td>
<td>xil-hīn</td>
<td>xil-āl</td>
</tr>
<tr>
<td>/xil/ ‘tangles’</td>
<td></td>
<td></td>
</tr>
<tr>
<td>/dub/ ‘lead by the hand’</td>
<td>dub-hūn</td>
<td>dub-āl</td>
</tr>
<tr>
<td>Non-high</td>
<td>xat-hīn</td>
<td>xat-āl</td>
</tr>
<tr>
<td>/xat/ ‘eat’</td>
<td></td>
<td></td>
</tr>
<tr>
<td>/bok/ ‘find’</td>
<td>bok-hīn</td>
<td>bok-ōl</td>
</tr>
<tr>
<td>/bok/ ‘find’</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In this way, parasitic features act as a precondition to harmony. Moreover, the existence of PVH suggests that the phonological system is affected by the similarity of representations, since, here in (14), the tendency for harmony is related to the similarity of triggers and targets: height similar [u] and [i] interact, but height dissimilar [u] and [a] do not. Yawelmani also expresses the tendency for parasitic features to be phonetically similar to harmonic features: height/rounding interaction has a basis in both acoustics and articulation (Hong, 1994; Kaun, 1995, 1997, 2004). In terms of acoustics, height (especially among front vowels) and rounding each affect the spectral peak of the second formant (F2), and from the perspective of articulation, the tongue raising associated with high vowels and the lip protrusion associated with round vowels are each facilitated by a closed jaw position.

Now, Yawelmani contrasts with the Turkic language, Kirgiz, which is exceptional among languages with rounding harmony because it shows no interaction between vowel heights and rounding. In Kirgiz, high vowels trigger harmony in low targets, and low
vowels trigger harmony in high targets. As the data in (15) confirm, there is a full spectrum of cross-height interaction in Kirgiz rounding harmony, so all cells are shaded.

As typical of other Turkic languages, Kirgiz differs from Yawelmani in other respects: Kirgiz has an inventory which permits interior vowels (front, round [ü], [ö] and back, unround [i], [a]), and Kirgiz has productive palatal harmony. These differences are orthogonal to the point here, which is to show how the ranking of IDENT-IO relative to ATTRACTION constraints determines whether or not harmony is parasitic on height.

A few analytical details are now in order. For the moment, I postpone a discussion of blocking effects in order to focus on the contrast in feature similarity preconditions, so in the preliminary tableau in (17), for Yawelmani, and (18), for Kirgiz, there is a high-ranking CORR\textsubscript{L→R} constraint, which requires that if \( x \) linearly precedes \( y \) in a surface form, then \( x \) is related to \( y \).\textsuperscript{82} Although CORR\textsubscript{L→R} is a general non-local correspondence constraint, for the sake of simplicity only relevant correspondences

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\textbf{Triggers(↓) \ Target(→)} & \textbf{High} & \textbf{Non-high} \\
\hline
\textbf{High} & bir-int[ĭ] ‘first’ & if-ten ‘work-ABL’ \\
& toguz-int[û] ‘ninth’ & tuz-don ‘salt-ABL’ \\
\textbf{Non-high} & bef-int[ĭ] ‘fifth’ & et-ten ‘meat-ABL’ \\
& on-int[û] ‘tenth’ & tokoj-don ‘forest-ABL’ \\
\hline
\end{tabular}
\end{table}

\textsuperscript{82} It is likely the case that Kirgiz like other Turkish languages implements strict spreading where consonants also undergo harmony (see Ni Chiosáin & Padgett, 2001), so having only CORR\((X, X_{(i+1)}h_{→L})\) dominate faith would more apt for a complete analysis of Kirgiz. Also, as I show, CORR\((\sigma, \sigma_{(i+1)})_{→L}\) must be the most general correspondence constraint to dominate faith in Yawelmani. Yawelmani blocking is considered in the next section, §4.3.2.
between vowels are considered. Thus, for the purposes of this discussion of parasitic vowel harmony, $\text{CORR}_{L\rightarrow R}$ is denoted as $\text{CORR-} VV_{L\rightarrow R}$, which only requires a correspondence between vowels. §4.4-4.5 take up the issue of what happens when $\text{CORR-} VV_{L\rightarrow R}$ is replaced with the general correspondence constraints like those introduced in §4.2, which require correspondences between consonants and vowels.

Here, the shift from $\text{CORR}$ to $\text{CORR-} VV$ is exclusively for expository purposes.

Also, the evidence from studies of rounding harmony (Odden, 1991; Steriade, 1995; Hong, 1994; Kaun, 1995, 1997, 2004) suggests that only [+round] spreads, and so I assume that a privative feature [labial] is the harmonic feature, and, therefore, the faithfulness constraint $\text{IDENT-IO}([\text{labial}])$ in (17) and (18) could also be characterized as a combination of $\text{DEP}([\text{labial}])$ and $\text{MAX}([\text{labial}])$. Furthermore, although the default labial specification of suffix vowels is not available from the data because suffix vowels always agree with preceding vowels, it is consistent with the absence of [−round] harmony to assume that the default realization of suffix vowels in both Yawelmani and Kirgiz lacks a [labial] specification. Let /l/ denote a non-labial, high vowel and /A/ denote a non-labial, non-high vowel.\footnote{If the default specification of /hlh/ were [+round], /hun/, then /dub+/hun/ is harmonic in the input, so the interesting case is /xil+/hun/, which surfaces as [xil-hin]. Such [−round] harmony could be accounted for in an attraction analysis, by assuming the harmonic feature is [±round]; the ranking $\text{ATTRACT}([\text{cons}, \text{manner} \rightarrow [\text{round}]) \gg \text{IDENT-IO}([\text{round}])$ gives the right results. Thus, the assumption of unround suffix vowels in the input is critical because privative [labial] is the right feature to characterize rounding harmony, not because a successful attraction analysis requires certain input specifications.}

Finally, by the Subset Similarity Ranking Principle (SSRP; §3.5.5) there exists a universal ranking of $\text{ATTRACTION}$ constraints describing the height-rounding interaction of vowel harmony as follows, in (16). Note there are a number of different proposals for the feature corresponding to vowel height (e.g., Clements & Hume, 1995; Harrison,
Expressing vowel height with the feature [±hi], as here, instead of an aperture node or [open] feature is not critical to an ATTRACTION analysis of PVH.

(16) Hierarchy of ATTRACTION constraints for height interactions in labial harmony, given by the SSRP for prerequisite feature sets {cons, [hi]} ⊇ {cons} ⊇ {}: 

\[
\text{ATT}([\text{cons}, [\text{hi}]] \rightarrow \text{labial}) \gg \text{UG ATT}([\text{cons}] \rightarrow \text{labial}) \gg \text{UG ATT}([\emptyset] \rightarrow \text{labial})
\]

Under the ranking in (16), segments which agree in both [cons] and [hi] are under more pressure to agree on [labial], than segments which only agree on [cons]. Placing IDENT-IO([labial]) in a stratum between ATTRACT([cons, [hi]] \rightarrow [labial]) and ATTRACT([cons] \rightarrow [labial]), allows only the higher ranking similarity condition to be active, resulting in height-parasitic labial harmony, as shown with the analyzed Yawelmani forms in (17), below. (17) illustrates that when the higher prerequisite feature similarity (height agreement in addition to [cons] agreement) is met harmony obtains, (17)(a-b) and (17)(g-h), but where vowels mismatch in height no harmony is allowed, (17)(c-d) and (17)(e-f), because higher ranking IDENT-IO([labial]) prevents the weaker ATTRACTION constraints from motivating harmony.
(17) Attraction analysis of Yawelmani PVH (to be modified).

<table>
<thead>
<tr>
<th></th>
<th>CORR-</th>
<th>ATTRACT ({{cons}[hi]}→[labial])</th>
<th>IDENT-</th>
<th>ATTRACT ({{cons}→[labial]}</th>
<th>ATTRACT ({}→[labial])</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>VV→R</td>
<td></td>
<td>IO</td>
<td></td>
<td></td>
</tr>
<tr>
<td>/dub/+hIn/→</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[dub-hun]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. dub-hin</td>
<td></td>
<td>*!</td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>b. dub-hun</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>/dub/+/Al/→</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[dub-al]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. dub-al</td>
<td></td>
<td></td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>d. dub-ol</td>
<td></td>
<td></td>
<td>*!</td>
<td></td>
<td></td>
</tr>
<tr>
<td>/bok/+hIn/→</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[bok-hin]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e. bok-hin</td>
<td></td>
<td></td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>f. bok-hun</td>
<td></td>
<td></td>
<td>*!</td>
<td></td>
<td></td>
</tr>
<tr>
<td>/bok/+/Al/→</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[bok-ol]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>g. bok-ol</td>
<td></td>
<td></td>
<td>*!</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>h. bok-ol</td>
<td></td>
<td></td>
<td>*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In contrast, if IDENT-IO([labial]) is demoted one stratum, the similarity threshold for attraction is lowered, so ATTRACT(/{cons}→[labial]) becomes active, motivating the cross-height harmony exhibited by Kirgiz, shown in the tableau in (18). Here, the crucial contrast is that unlike Yawelmani, (17)(c-d) and (17)(e-f), Kirgiz allows cross-height harmony, (18)(c-d) and (18)(e-f), because of the lower ranking of faith.
The above examples show that the Attraction Framework makes allowances for different levels of feature similarity preconditions (some parasitic and some not) through the relative ranking of ATTRACTION and faithfulness constraints. In Yawelmani, (17), higher-ranked faithfulness requires greater trigger-target similarity for ATTRACTION constraints to become active and motive harmony. In Kirgiz, (18), lower-ranked faithfulness allows less similar trigger-target pairs to result in harmony. The next section shows how faithfulness thresholding also applies to locality preconditions.
4.3.2. Blocking by interrupting the parasitic domain

The Attraction Framework also makes allowances for different levels of positional similarity preconditions through the relative ranking of correspondence and faithfulness constraints. In particular, if CORR-VV\textsubscript{L→R} is ranked below IDENT-IO then correspondence can be avoided in order to satisfy faith, eliminating the possibility of non-local interaction. I now show how this lower-ranking of non-local correspondence yields an analysis of blocking effects in Yawelmani and Akan.

4.3.2.1. Breaking feature similarity: Yawelmani rounding harmony

Yawelmani again provides a telling example. As the data in (19) demonstrates, Yawelmani exhibits a locality precondition of at least strict syllable locality:

(19) Blocking by non-height matching intervening vowels:
   a. bok’-k’o ‘find (it)!’
   b. bok’-sit-k’a ‘find (it) for him!’

(19)(a) confirms that harmony is available in adjacent syllables, but not, as in (19)(b), when an intervening syllable fails to undergo harmony. Of course, because rounding harmony is parasitic on height, the reason the intervening syllable fails to undergo harmony is due to a failure to meet prerequisite feature similarity. Thus, the intervening high vowel [i] in (19)(b) interrupts the contiguous non-high parasitic domain in (19)(a).

As previewed above, such blocking receives a straightforward analysis by the lower ranking of non-local correspondence as shown in the tableau in (20). As before, correspondences are indicated with arrows, and these correspondences are what allow attraction to apply between segments.
(20) **ATTRACTION** analysis of blocking in Yawelmani PVH.

<table>
<thead>
<tr>
<th></th>
<th>CORR-VV</th>
<th>ATTRACTION</th>
<th>IDENT-IO</th>
<th>CORR-VV_L→R</th>
<th>ATTRACTION</th>
<th>ATTRACTION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(σ[σ(i+1)]_L→R)</td>
<td></td>
<td>([cons],[hi])</td>
<td>([labial])</td>
<td></td>
<td>([labial])</td>
</tr>
<tr>
<td>/bok'/+/k'/'A/→</td>
<td>[bok'-k'o]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. bok'-k'a</td>
<td></td>
<td>*!</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. bok'-k'o</td>
<td></td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>/bok'/+/sit+/k'/'A/→</td>
<td>[bok'-sit-k'a]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. bok'-sit-k'a</td>
<td></td>
<td>*</td>
<td></td>
<td>* (o→i)</td>
<td>* (o→i)</td>
<td></td>
</tr>
<tr>
<td>d. bok'-sit-k'a</td>
<td></td>
<td>*! (o→a)</td>
<td></td>
<td>* (o→i), *</td>
<td>* (o→i), *</td>
<td></td>
</tr>
<tr>
<td>e. bok'-sit-k'o</td>
<td></td>
<td>*!</td>
<td></td>
<td>* (o→i), *</td>
<td>* (o→i), *</td>
<td></td>
</tr>
<tr>
<td>/bok'/+/hIn/→</td>
<td>[bok-hin]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f. bok-hin</td>
<td></td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>g. bok-hun</td>
<td></td>
<td>*!</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(20) shows how violations of IDENT-IO are incurred exactly when both locality preconditions and similarity preconditions are met (20)(a-b). If the trigger and target are not proximate enough (20)(c-e) or if the trigger and target are not featurally similar enough (20)(f-g) no harmony takes place.

As the difference between (17) and (18) is the ranking of faith compared to weaker ATTRACTION constraints, the difference between (17) and (20) is the ranking of faith compared to weaker correspondence constraints. Since IDENT-IO dominates CORR-VV_L→R, (20)(e) is less harmonic than (20)(c), and non-local harmony is not allowed. In this way, the ranking of CORR-VV_L→R relative IDENT-IO to is critical to determining
whether non-undergoing vowels are transparent or blockers. Hence, the ranking of IDENT-IO determines the attraction threshold for both feature-based similarity and proximity-based similarity.

Furthermore, among the more powerful aspects of such blocking by interrupting the parasitic domain (as in (20)(c-e)) is that it allows for context-sensitive neutrality. In high contexts, [i] is a perfect target of harmony (cf. /dub-hin/ → [dub-hun]), but in non-high contexts, [i] does not undergo harmony (cf. /bok’-sit-k’a/ → [bok’-sit-k’a]), and is a blocker. In the Attraction Framework, this context-sensitivity derives from the implementation of ATTRACTION as segment-to-segment faithfulness.

Yawelmani’s context-sensitive neutrality is in sharp contrast to languages like Akan (discussed in the next section), which can be analyzed as a context-free neutrality. Akan is context-free because the shape of the phonological inventory is mirrored in the ‘structure preserving’ harmony system (Kiparsky, 1985; Archangeli & Pulleyblank, 1994, 2007), so high ranking markedness constraints predict both the shape of the inventory and the neutrality to harmony, as the harmonic alternates of neutral segments are never permitted. No such broad appeals to inventory markedness are available in Yawelmani because the non-participating vowel, [i], has a harmonically opposed vowel that is surface viable, namely, [u]. Thus, the blocking by [i] cannot follow from inventory markedness, but instead must be due to contextual markedness, in which the blocking only occurs in environments with a mismatch in height. Such fully-parasitic systems with contextual blocking provide the best evidence for a formalism like attraction that can reflect both feature and proximity roles of similarity.
4.3.2.2. Low vowels in Akan-ATR harmony: breaking the parasitic domain or satisfying locality?

Akan (Archangel & Pulleyblank, 1994; Kenstowicz, 1994; Polgardi, 2006; O’Keefe, 2003; refs therein) lacks a [+ATR], low vowel, denoted [æ], and it is, therefore, no surprise that [−ATR] [a] is not a target of ATR harmony. The goal of the discussion of Akan is to illustrate two different types of blocking available in the Attraction Framework. One kind is blocking by segments which because of similarity preconditions are not possible triggers of harmony, so like in Yawelmani above, blocking occurs because of the interruption of the parasitic domain. Another kind of blocking follows from structure preservation effects, in which high-ranking markedness constraints, e.g. *[æ], prevent a segment from undergoing harmony, but allow that neutral segment to trigger harmony on subsequent targets. Akan is an interesting case because the same segment, the low vowel [a], performs both kinds of blocking depending on the morphological context.

The neutrality of [a] derives from the shape of the phonological inventory, since [+ATR, +low] vowels are avoided, so Akan conforms to other cases where non-participating vowels overwhelmingly tend to be those vowels which occur without another vowel which minimally contrasts for the harmonic feature (e.g. Kiparsky, 1985; Archangeli & Pulleyblank, 1994, 2007; Baković, 2000; Walker, 2003; Smolensky, 2006; Polgardi, 2006; etc.). Thus, neutral vowels tend to be unpaired. This section shows how these inventory effects are consistent with the Attraction Framework by discussing the case of unpaired [a] in Akan, showing that, even with unpaired neutral segments, similarity is still playing a vital role in determining the trigger-target pairs. As previously discussed in §2.3.2, Akan has differing similarity conditions in distinct morphological
environments, this morphological sensitivity to similarity determines whether or not [a] is a trigger of harmony.

The present analysis differs from previous work in trying to account for both root and affix harmony in Akan, which is complicated because the same vowel [a] behaves differently in these morphological contexts. As I show, at the root level, [a] is transparent, but at the affix level [a] is a blocker. The basic facts of Akan from §2.3.2 are repeated in (21) below.

(21) Akan [ATR] harmony

a. **Phonological Inventory:** [+ATR] { i, e, o, u} and [−ATR] { i, e, a, ɔ, ʊ}. [a] does not contrast with a [+ATR] vowel, [æ].

b. **Harmony:** There are a few disharmonic roots, but, in general, adjacent non-low vowels agree in [ATR], so ATR-disharmonic sequences are dispreferred in both roots and affixes. Both suffixes and prefixes undergo harmony, so harmony is bidirectional.

   ɔ-ﬁti-ı ‘he/she pierced (it)’ ɔ-ciɨ-ı ‘he/she showed (it)’

c. **Root-Neutrality:** In roots, [a][−ATR] freely occurs with both [+ATR] and [−ATR] non-low vowels, so [a] is neither a trigger nor a target of root [ATR] harmony.

   kari ‘to weigh’ bisa ‘to ask’
   yari ‘to be sick’ pira ‘to sweep’

d. **Affix-blocking:** Affix vowels adjacent to [a][−ATR] are [−ATR], so [a] is a trigger and a blocker of otherwise productive [ATR] harmony in affixes.

   ɔ-kari-ı ‘he/she weighed (it)’ ɔ-bisa-ı ‘he/she asked (it)’

---

Polgardi (2006) reports that [a] can raise to [æ] preceding [+ATR] vowels, but this raising is independent, non-contrastive, gradient, and, therefore, post-lexical, so for the purposes of [ATR] harmony [a] is unpaired.
e. **Limit on the number of domains:** In both roots and morphologically complex words, at most two [ATR] domains, are permitted. Thus, [+ATR][−ATR] and [−ATR][+ATR] sequences are allowed, but [+ATR][−ATR][+ATR] are not.

\[
\begin{align*}
\text{[fun]}_{[+]}\text{[an]}_{[−]} & \quad \text{‘to search’} \\
*\text{[fu]}_{[+]}\text{[ña]}_{[−]}\text{[n]}_{[+]} & \\
\text{[pra]}_{[−]}\text{[ko]}_{[+]}. & \quad \text{‘pig’} \\
*\text{[pi]}_{[+]}\text{[ra]}_{[−]}\text{[ko]}_{[+]}. & 
\end{align*}
\]

Only (21)(e) contains information not already discussed in §2.3.2, so a word on the domain limits is in order. Kenstowicz (1994) argues for a non-linear analysis of Akan root ATR harmony, suggesting that Akan disallows more than two autosegments.\(^{86}\) O’keefe (2006) shows how this Akan limit on autosegments can be derived from the interaction of heads in Span Theory (McCarthy, 2004), and although I borrow the notation from the similar theory of Headed Feature Domains (Smolensky, 2006), for the sake of exposition, the domains here are not headed, and instead I use a brute-force, descriptive *STRUCTURE constraint, *>2DOMAINS, to reflect the fact that Akan limits the number of domains.\(^{87}\) Because of this limit on the number of domains, wherever [a] occurs medially segments to the left or right will agree in [−ATR], explaining the fact that [i]_{[+]} [a]_{[−]} [i]_{[+]} sequences are not attested. However, this limit on domains cannot explain why [u]_{[+]} [i]_{[−]} sequences are dispreferred, so there is still an explanatory role for [ATR] harmony and therefore similarity in describing the trigger-target pairs. In what follows, I show that [a] is neutral to this root-harmony process, and furthermore that

---

85 Here, the convention (Smolensky, 2006) is to bracket domains and label them with a subscript. Because the feature of domains in this section is always [±ATR], I label domains [+] for [+ATR] domains and [−] for [−ATR] domains.

86 As Kenstowicz (1994) notes the non-linear analysis is also supported by suppletive forms, like [s’an] ‘to come down’, which take [+ATR] prefixes, [o-s’an-i] ‘he came down’. Historically, [s’an] derives from [sian], under a non-linear analysis, the restructuring resulted in a floating [+ATR] autosegment, predicting the prefix.

87 Representations that include all of heads, domains, and correspondence are certainly possible, but not easily accessible, as there becomes a combinatorial explosion of candidates that consist of the same segments, but differ in these other representational elements, so for the sake of analytical tractability headedness is not pursued in tandem with correspondence.
inventory restrictions, *[æ], and *>2DOMAINS only provide a partial explanation to the distributional patterns.

Now, Akan harmony is structure preserving: the fact that [a] does not undergo root [ATR] harmony, (21)(e), is tied to the high ranking of some *[æ] constraint which prohibits [+ATR, +low] vowels. Furthermore, some attraction constraint must dominate faithfulness in order for there to be any harmony at all. As (22) below shows, if that harmony constraint is a more general vowel-to-vowel agreement constraint, ATTRACT([cons], [ATR], since harmony is bidirectional, if *[æ] prevents [a] from changing, (22)(c), then general harmony would prefer for the root to fully agree with [a], errantly predicting *[kari], (22)(b), instead of the attested [kari], (22)(a).

(22) Insufficiency of inventory markedness

<table>
<thead>
<tr>
<th></th>
<th>kari</th>
<th>*[æ]</th>
<th>CORR-VV σiσi+1</th>
<th>ATTRACT ([{cons}]→[ATR])</th>
<th>IDENT-IO ([ATR])</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>kaₐᵣᵢₐ</td>
<td></td>
<td></td>
<td>*!</td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>kaₐᵣᵢᵢ</td>
<td></td>
<td></td>
<td></td>
<td>*</td>
</tr>
<tr>
<td>c.</td>
<td>kₐæᵣᵢᵢ</td>
<td>*!</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td>kaᵣᵢ</td>
<td></td>
<td>*!</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Thus, while the shape of the phonological inventory expressed in constraints is an important part of understanding non-participation in vowel harmony, it is not the whole story. Context-free markedness constraints, like *[æ], are blind to the harmony process at work, so while *[æ] can prevent [a] from undergoing harmony it has no effect on whether or not [a] is a trigger of harmony. (22) shows that a more general attraction constraint errantly predicts that [a] should trigger harmony in [i]. As Cole & Trigo (1989) show for Akan, a height-parasitic harmony analysis avoids this dilemma. Akan

---

88 Following Rose & Walker (2004), the letter subscripts indicate that the segments are in bidirectional correspondence, so [kaₐᵣᵢₐ] indicates that [a]R[i] and [i]R[a]. This bidirectional correspondence is needed to account for prefix and suffix harmony.
ATR harmony is a cross-height harmony system, (21)(b), where high vowels and mid vowels induce harmony on one another, so only the low vowels are excluded from participation. Hence, I make reference to the more fine grained distinctions engendered by [±lo] and [±hi].\(^89\) (23) below shows how a [low]-parasitic analysis of root harmony predicts the disharmonic [kari]. Like the parasitic rounding harmony, this analysis of Akan uses similarity to discriminate the pairs of segments, where harmony obtains (all pairs of non-low vowels) from those where harmony does not obtain (mixed pairs of low and non-low vowels).

\[(23) \text{ [low]-parasitic root harmony} \]

<table>
<thead>
<tr>
<th>kari</th>
<th>*[æ]</th>
<th>CORR-VV (σ_iσ_{i+1})</th>
<th>ATTRACT ({\text{[cons]}, \text{[low]}} \rightarrow \text{[ATR]})</th>
<th>IDENT-IO ({\text{[ATR]}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ka(_r) r(_i)</td>
<td>![Cross]</td>
<td>![Cross]</td>
<td>![Cross]</td>
<td>![Cross]</td>
</tr>
<tr>
<td>b. ka(_r) r(_i)</td>
<td>![Cross]</td>
<td>![Cross]</td>
<td>![Cross]</td>
<td>![Cross]</td>
</tr>
<tr>
<td>c. ka(_r) r(_i)</td>
<td>![Cross]</td>
<td>![Cross]</td>
<td>![Cross]</td>
<td>![Cross]</td>
</tr>
<tr>
<td>d. ka(_r) r(_i)</td>
<td>![Cross]</td>
<td>![Cross]</td>
<td>![Cross]</td>
<td>![Cross]</td>
</tr>
</tbody>
</table>

As typical of a parasitic analysis under attraction, if vowels agree on [±low], then there is pressure to harmonize on ATR. (23)(a) and (23)(b) show that with a parasitic analysis [i] is not under pressure to agree with [a] and [a] is not under pressure to agree with [i], so [kari] faithfully emerges. This contrasts with the analysis, in (24) below, of trisyllabic roots containing [a], which do have pressure for [i] to agree with [a], but crucially this pressure does not come from the harmony process, but instead from the higher-ranked *\text{>2DOMAINS}.

\(^89\) An analysis using, Clements & Hume’s (1995), privative [open] and [closed] could work as well. Though note that in their system there are no unspecified vowel heights, mid vowels are both [open] and [closed]. There is both raising and lower harmony, and for both rounding harmony and ATR harmony, there are systems where only mid vowels are triggers and systems where only mid vowels are targets. Thus, it seems right to me that, in general, mid vowels should not be unspecified, even if privative height features are used.
(24) Trisyllabic roots with [a]: why tri-domainal /u a i/ emerges unfaithfully but not fully harmonically (cf. [fu]_[+]ñan]_[−] * [fu]_[+]na]_[−]ni]_[+], see (20)(a), and [piral]_[−]ko]_[+], *[pi]_[+]ra]_[−]ko]_[+, see (20)(b)).

<table>
<thead>
<tr>
<th></th>
<th>/u a i/</th>
<th>*&gt;2DOMS</th>
<th>*[\sigma]</th>
<th>CORR-VV \sigma,\sigma_{i+1}</th>
<th>ATTRACT ([icons], [low] \rightarrow [ATR])</th>
<th>IDENT-IO ([ATR])</th>
<th>CORR-VV</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>[u]<em>[+]a]</em>[−]i]_[−]</td>
<td>✓</td>
<td></td>
<td></td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>[u]<em>a]</em>[+][i]_[+]</td>
<td>✓</td>
<td></td>
<td></td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td>[u]<em>a]</em>[−][i]_[−]</td>
<td></td>
<td></td>
<td></td>
<td>**!</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td>[u]<em>[+]a]</em>[−][i]_[+]</td>
<td></td>
<td></td>
<td></td>
<td>*!</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e.</td>
<td>[u]<em>[+]a]</em>[−][i]_[+]</td>
<td></td>
<td></td>
<td></td>
<td>*!</td>
<td>**</td>
<td></td>
</tr>
<tr>
<td>f.</td>
<td>[u]<em>æ]</em>[+][i]_[+]</td>
<td></td>
<td></td>
<td></td>
<td>*!</td>
<td></td>
<td></td>
</tr>
<tr>
<td>g.</td>
<td>[u]<em>[+]a]</em>[−][i]_[−]</td>
<td></td>
<td></td>
<td></td>
<td>*!</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(24) shows why despite the neutrality of [a], there are no forms which show [a] as being transparent: the faithful candidate, (24)(d), with transparent [a] is eliminated by the prohibition against more than two feature domains. Other possible forms (24)(e-g), having a non-local correspondence that might motivate transparency, also violate one of the higher-ranked constraints. Furthermore, (24) illustrates how the parasitic nature of the harmony means that in roots containing [a], it is left to faithfulness to determine, which of (24)(a), (b), or (c) is optimal, and in fact each of these forms are attested, and with a different input could emerge as the exclusive optimum. Note optimal (24)(a,b) have segments in correspondence that do not, however, agree in ATR. This underscores how correspondence explicitly represents locality, but the similarity of corresponding segment still determines whether there is pressure for assimilation. In forms where there
are no [+low] interveners it is the combination of this correspondence and attraction, which results in harmony.

Now, if a more general harmony process were available to drive [a] and [i]/[u] agreement, then, as in (22), the fully harmonic candidate (24)(c) would be required to resolve the demands of the high-ranking markedness constraints. Clearly, this cannot be the case, as disharmonic tri-syllabic roots are readily attested. In sum, [a] must be neutral and transparent to root harmony, and so, faithfulness largely determines how roots which contain [a] satisfy *>2DOMAINS and *[æ].

However, this determination of the optimum by the faithfulness constraints is problematic for an analysis of Akan affix harmony. Unlike in roots, where non-low vowels freely combine with [a], affixes which are adjacent to [a] are always [−ATR], (21)(d). That fully [−ATR]-harmonic roots which contain [a] never take [+ATR] affixes cannot follow from *>2DOMAINS because, e.g., *[ɔ-][yari-ı][-] does not violate *>2DOMAINS, nor does *[ɔ-yari][-][-][-], so some other factor must explain why only [ɔ-yari-ı][-] is attested. If input-output faithfulness is allowed to cast the deciding vote, as in root harmony, then by Richness of the Base (Prince & Smolensky, 2004), both forms like [ɔ-yari-ı][-] and [o-][yari-ı][-] ought to be attested (presumably for different affixes). Nevertheless, stems always determine the [ATR] value of non-low affixes, so some aspect of harmony must be higher ranked than the faithfulness that would allow affix-stem disharmony. Therefore, [a] must be the trigger of [−ATR] harmony when it occurs in both harmonic stems like [yari] and disharmonic stems like [kari] and [bisa], which respectively give [ɔ-ka][-][ri-ı][+]+ and [a-bi][+][sa-ı][-]. In this way, while [a] is transparent to Akan root harmony, [a] is also a trigger and a blocker of affix harmony.
In order to express these morphologically controlled processes, I take the standard approach of assuming root faithfulness dominates general faithfulness (McCarthy & Prince, 1995; see Kager, 1999 for a review, and O’Keefe, 2006 for application to Akan), even though morphological control is not always so easily characterized (see Burzio & Tantalou, 2007; refs therein). Then, the interleaving of specific and general faithfulness with specific and general attraction gives the correct results. In particular, as before, only the parasitic attraction constraint dominates root faithfulness, but non-parasitic attraction must dominate general faithfulness in order to give [a]’s triggering of [−ATR] harmony in affixes. This triggering by initiating a new harmonic domain blocks any amount of assimilation to [+ATR] vowels. Thus, in addition to parasitism, locality is also playing an important role, and non-local correspondence is strictly avoided.

In (25), below, I consider an input which is more likely to result in transparency of [a], [o−] [+ka]−[ri -i]+, and show that even without the high ranking * >2DOMAINS, because locality contributes to general similarity, only adjacent correspondences drive the harmony, and so [a] can trigger harmony in the prefix because of the low rank of general IDENT-IO([ATR]), relative to vowel-to-vowel attraction. Here, some marks are labeled with the offending segment, or pair of segments. Also, note that the underlying, or default, [ATR] values of affixes are not available because they always agree, so the strategy, here and elsewhere, is to assume that the input contains the opposite feature value of the surface realization and show how the constraints can overcome even preferential treatment due to faithfulness (see Finley, 2008 for similar strategies). In this way, if the faithfulness advantages for disharmonic inputs were removed, by in fact being already harmonic, then the system would still produce the right output. This “assuming
the worst” strategy avoids any unnecessary complication or theoretical baggage from otherwise assuming affix underspecification.\(^90\)


<table>
<thead>
<tr>
<th></th>
<th>o- kari -</th>
<th>ATTRACT( {[cons]} →[ATR])</th>
<th>IDENT-IO(_{ ROOT}) ([ATR])</th>
<th>ATTRACT( {[cons]} →[ATR])</th>
<th>IDENT-IO ([ATR])</th>
<th>CORR-VV</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>[ɔ-ka] [-] [ri-i] [+i]</td>
<td>*[x]</td>
<td>*(a, i)</td>
<td>*(ɔ, i)</td>
<td>***</td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>[ɔ- a] [-] [i] [+i]</td>
<td>![i] (i, i)</td>
<td>*(a, i)</td>
<td>*(i, i)</td>
<td>*(ɔ)</td>
<td>***</td>
</tr>
<tr>
<td>c.</td>
<td>[ɔ-] [a] [-] [i] [+i]</td>
<td>![i] (i, i)</td>
<td>*(0, a)</td>
<td>*(i, i)</td>
<td>***</td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td>[ɔ-] [a] [-] [i] [+i]</td>
<td>![i] (i, i)</td>
<td>*(0, a)</td>
<td>*(i, i)</td>
<td>***</td>
<td></td>
</tr>
<tr>
<td>e.</td>
<td>[ɔ- æ] [+a] [-i] [+i]</td>
<td>![i] (i)</td>
<td>*(æ)</td>
<td>*(æ)</td>
<td>***</td>
<td></td>
</tr>
<tr>
<td>f.</td>
<td>[ɔ- a] [-] [i]</td>
<td>![i] (i)</td>
<td>*(a)</td>
<td>*(i)</td>
<td>***</td>
<td></td>
</tr>
</tbody>
</table>

Above, fully harmonic strings are eliminated by high ranking root faithfulness and *[x]*, (25)(e-f), so non-local correspondence, (25)(d), only increase the violations of \(\text{ATTRACT}([[\text{cons}]} \rightarrow\text{[ATR]}]\), which dominates the general faith that applies to affixes, resulting in [a] triggering harmony in the [ɔ]/[æ]-affix, (25)(a).

Understanding this harmony without \(*>2\text{DOMAINS}*, as above, is important because of the cases like in (26), where it has no say in whether or not affixes agree with [a], leaving to \(\text{ATTRACT}([[\text{cons}]} \rightarrow\text{[ATR]}]\) to dictate harmony. In (26), non-local correspondences are omitted because they cannot motivate harmony.

---

\(^90\) As similarly argued by Kiparsky & Pajusalu (2003), this assumption is not to necessarily say that underspecification is wrong (although there are reasons to doubt underspecification of ATR, Steriade, 1995), but only to say that harmony obtains independent of the assumptions about the representations of affixes.
(26) Triggering by [a] in bi-domainal structures.

<table>
<thead>
<tr>
<th>o- yari -i→ [ɔ-yari-i]_{[-]}</th>
<th>*&gt;2 DOMS</th>
<th>*[a]</th>
<th>CORR-VV</th>
<th>ATTRACT( ([cons],[low]) → [ATR])</th>
<th>IDENT-IO_{Root} ([ATR])</th>
<th>ATTRACT( ([cons]) →[ATR])</th>
<th>IDENT-IO ([ATR])</th>
<th>CORR-VV</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. [ɔ a i]<em>{[i]}</em>{[i]}_{[-]}</td>
<td>![image]</td>
<td></td>
<td></td>
<td>![image]</td>
<td>![image]</td>
<td>![image]</td>
<td>![image]</td>
<td>![image]</td>
</tr>
<tr>
<td>b. ![image] [ɔ a i]<em>{[i]}</em>{[i]}_{[-]}</td>
<td>![image]</td>
<td></td>
<td></td>
<td>![image]</td>
<td>![image]</td>
<td>![image]</td>
<td>![image]</td>
<td>![image]</td>
</tr>
<tr>
<td>c. [ɔ]<em>{[i]}+[a i]</em>{[i]}<em>{[i]}</em>{[-]}</td>
<td>![image]</td>
<td></td>
<td></td>
<td>![image]</td>
<td>![image]</td>
<td>![image]</td>
<td>![image]</td>
<td>![image]</td>
</tr>
<tr>
<td>d. ![image] [ɔ æ i]<em>{[i]}</em>{[i]}_{[i]}</td>
<td>![image]</td>
<td></td>
<td></td>
<td>![image]</td>
<td>![image]</td>
<td>![image]</td>
<td>![image]</td>
<td>![image]</td>
</tr>
</tbody>
</table>

Summarizing this analysis of Akan, for root level harmony, [a] interrupts the parasitic domain which might allow fully agreeing roots, and so because of high ranking *>2DOMAINS, despite the neutrality of [a], this interruption prevents [a] from occurring in forms where there is disharmony on both sides, i.e. where it is trivially demonstrably transparent. However, in affix harmony, [a] operates like any other root vowel as a trigger of harmony, so no parasitic domain is interrupted by [a] and blocking occurs in order to satisfy the demands of locality.

4.3.3. Strictly local parasitic assimilations in Turkish

§4.3.1 explored the role of feature similarity, and §4.3.2 explored how positional similarity gives blocking. The independence of the components of the formalism for these different kinds of similarity (respectively, correspondence and attraction constraints) predicts the existence of exclusively local parasitic assimilations, where segments must be proximate and featurally similar in order for harmony to obtain. Such a case is found in optional repairs to vowel hiatus in Turkish (Kabak, 2007; §2.4.5). Not all the conditions on this interaction are based on similarity, but the basic generalization
is that in a derived environment, an optional coalescence repair to vowel hiatus has a
precondition of agreement on both rounding and backing (compare (27)(a) vs. (27)(b)).

(27)  

a.  V2 assimilates to V1 following medial consonant deletion:

\[
\begin{align*}
\text{ağır} & \rightarrow \quad [\text{air}] \sim [\text{aar}], *[\text{iir}] \quad \text{‘heavy’} \\
yo\text{ğurt} & \rightarrow \quad [\text{yourt}] \sim [\text{yoort}], *[\text{yuurt}] \quad \text{‘yogurt’} \\
ö\text{ğür} & \rightarrow \quad [\text{öür}] \sim [\text{öör}], *[\text{üürt}] \quad \text{‘to retch’}
\end{align*}
\]

b.  No alternation unless V1 and V2 agree on rounding and backness:

\[
\begin{align*}
dö\text{viz} & \rightarrow \quad [\text{döiz}] \sim *[\text{dööz}], *[\text{döez}] \quad \text{‘foreign currency’} \\
ş\text{air} & \rightarrow \quad [\text{şair}] \sim *[\text{şaar}], *[\text{şaer}] \quad \text{‘poet’} \\
\text{saur} & \rightarrow \quad [\text{saur}] \sim *[\text{saar}], *[\text{saor}] \quad \text{‘meal bf. fasting’} \\
\text{soğan} & \rightarrow \quad [\text{soan}] \sim *[\text{soon}], *[\text{saan}] \quad \text{‘onion’} \\
\text{suit} & \rightarrow \quad [\text{süit}] \sim *[\text{süüt}], *[\text{siit}] \quad \text{‘suite’}
\end{align*}
\]

Unlike normal, productive Turkish Vowel Harmony, hiatus resolution results in lowering.
The repairs in (27)(a) may, thus, be termed vowel height assimilation. Turkish rounding
vowel harmony is parasitic on backness (see §2.3.1.2), and apparently height harmony for
hiatus resolution adds a further precondition of rounding agreement, otherwise the forms
in (27)(b) would alternate. Though height interacts with other aspects of harmony (only
high vowels are targets of rounding harmony), there is no evidence of non-adjacent vowel
height harmony in Turkish, so (27) exemplifies a case of strictly local vowel height
assimilation also sensitive to feature similarity preconditions. The analysis of optional
repairs for vowel hiatus resolution can be explored directly in three tableaus. (28) shows
that without the velar deletion that puts vowels in hiatus context, harmony does not
obtain even though segments in syllabic correspondence meet prerequisite feature
similarity. (29) shows that in a hiatus context, harmony fails to obtain even in segments
in root-adjacent correspondence if they fail to meet prerequisite feature similarity. Thus,
as (30) confirms, only forms that meet both feature (rounding and back agreement) and positional (root adjacency) similarity preconditions result in harmony.

(28) No height harmony when only prerequisite feature similarity is met:

<table>
<thead>
<tr>
<th>[yoğurt]→[yoğurt]</th>
<th>CORR-VV (xᵢxᵢ₊₁)L→R</th>
<th>ATTRACT ([[labial], [back]]→[hi])</th>
<th>IDENT-IO ([hi])</th>
<th>CORR-VV (σᵢσᵢ₊₁)L→R</th>
<th>ATTRACT ([[back]]→[hi])</th>
</tr>
</thead>
<tbody>
<tr>
<td>d. yoğurt</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>*</td>
</tr>
<tr>
<td>e. yoₐğurt</td>
<td></td>
<td>*!</td>
<td></td>
<td></td>
<td>*</td>
</tr>
<tr>
<td>f. yoₐğurt</td>
<td></td>
<td></td>
<td>*!</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(29) No height harmony when only prerequisite positional similarity is met:

<table>
<thead>
<tr>
<th>[saur]→[saur]</th>
<th>CORR-VV (xᵢxᵢ₊₁)L→R</th>
<th>ATTRACT ([[labial], [back]]→[hi])</th>
<th>IDENT-IO ([hi])</th>
<th>CORR-VV (σᵢσᵢ₊₁)L→R</th>
<th>ATTRACT ([[back]]→[hi])</th>
</tr>
</thead>
<tbody>
<tr>
<td>g. saur</td>
<td></td>
<td>*!</td>
<td></td>
<td></td>
<td>*</td>
</tr>
<tr>
<td>h. saₐuₐrt</td>
<td></td>
<td>*!</td>
<td></td>
<td></td>
<td>*</td>
</tr>
<tr>
<td>i. saₐₐuₐrt</td>
<td></td>
<td></td>
<td>*!</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(30) Height harmony if both positional and feature similarity preconditions are met:

<table>
<thead>
<tr>
<th>[yourt]→[yoort]</th>
<th>CORR-VV (xᵢxᵢ₊₁)L→R</th>
<th>ATTRACT ([[labial], [back]]→[hi])</th>
<th>IDENT-IO ([hi])</th>
<th>CORR-VV (σᵢσᵢ₊₁)L→R</th>
<th>ATTRACT ([[back]]→[hi])</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. yourt</td>
<td></td>
<td>*!</td>
<td></td>
<td></td>
<td>*</td>
</tr>
<tr>
<td>b. yoₐuₐrt</td>
<td></td>
<td>*!</td>
<td></td>
<td></td>
<td>*</td>
</tr>
<tr>
<td>c. yoₐₐuₐrt</td>
<td></td>
<td></td>
<td>*!</td>
<td></td>
<td>*</td>
</tr>
</tbody>
</table>

The existence of strictly local parasitic assimilations, like in Turkish vowel hiatus and in the other examples presented in §2.4, strongly argues for a formal system capable of expressing both positional and feature similarity. Other examples of strictly local assimilation would receive a similar analysis: only strictly local correspondence and parasitic attraction dominate faith. Furthermore, as discussed, strictly local assimilation where feature similarity plays a role undermines the empirical claims (Rose & Walker, 2004; Hansson, 2001) that motivate a formal dichotomy between (long-distance)
consonant harmony and vowel harmony. Feature similarity can play an important role in both consonant and vowel harmony, in both local and non-local processes.

4.3.4. Transparency under Attraction: Is Finnish Palatal harmony a case of true transparency?

All the cases so far in this section have shown how positional similarity can be a precondition to assimilation. However, because correspondence and faith are freely rankable, this system predicts that there ought to be vowel harmony systems where positional similarity is not an important precondition. I argue that Finnish palatal vowel harmony (Ringen & Heinamäki, 1999; Valimaa-Blum, 1999; Kim, 2005; Kiparsky & Pajusalu, 2003) is such a case where locality preconditions are relaxed, and that Finnish is best analyzed as an instance of long-distance vowel agreement. In particular, I argue that Finnish palatal vowel harmony is (i) an instance of parasitic vowel harmony, and (ii) a case of true transparency, requiring non local-application of attraction pressures. For the sake of encapsulation, some element of arguments in §2.3.3 are repeated here.

4.3.4.1. Arguments for lower F2-parasitic palatal harmony

The phonological inventory of Finnish vowels is given in (31), below.

(31) Finnish vowel inventory. Unpaired (neutral) vowels are in shaded cells.

<table>
<thead>
<tr>
<th></th>
<th>Front</th>
<th>Back</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unround</td>
<td>Round</td>
</tr>
<tr>
<td>High</td>
<td>i [i]</td>
<td>ü [y]</td>
</tr>
<tr>
<td>Mid</td>
<td>e [e]</td>
<td>ö [ø]</td>
</tr>
<tr>
<td>Low</td>
<td>ä [æ]</td>
<td></td>
</tr>
</tbody>
</table>

The relevant data for palatal harmony are presented in (32), below:
The relevant generalizations from the data in (32) are (i) \( i \) and \( e \) are neutral, neither triggers nor targets of root harmony, (32)(c-e, g-i), (ii) stems only containing \( i \) and \( e \) trigger front agreement in suffixes, (32)(e-f), and (iii) \( i \) and \( e \) are transparent to affix harmony, suffixes agree with whatever non-\( i/e \) vowel happens to be in the root, (32)(c-e, g-i). Thus, in certain morphological respect Finnish is like Akan, §4.3.2.2, neutral vowels having different properties for roots and affix harmony.

However, a feature-similarity account is more complicated in Finnish because the set of participating vowels (\( u-\ddot{u}, o-\ddot{o}, \) and \( a-\ddot{a} \)) does not form a natural class, being a disjoint set across features [low] and [round]. Therefore, at first blush, it might seem that Finnish palatal harmony cannot be parasitic on any single feature or any combination, thereof. The proposed solution previewed in §2.3.3 is to argue, based on phonetic studies by (Wiik, 1965; Kim, 2005), that Finnish back harmony is parasitic on an acoustic correlate common to the harmonic vowels (lower F2), which may be characterized with an acoustic feature [lower F2]. The ramification is that Finnish palatal harmony is directly parasitic on the phonetic correlates of distinctive features.

Because of the associated lengthening of the front cavity, the acoustic correlate of backness is a lowering of the spectral peak of the second formant (F2) (Stevens, Keyser,
& Kawasaki, 1986). Vowel height also has an effect on F2, therefore, it is appropriate to examine the F2 values of Finnish vowels as in (33) below:

(33) Average F2 values of long-vowels produced by male Finnish speakers; data from (Wiik, 1965). Vowels are ordered according to descending F2:

<table>
<thead>
<tr>
<th>Vowel</th>
<th>F2 (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>2495</td>
</tr>
<tr>
<td>e</td>
<td>2240</td>
</tr>
<tr>
<td>ü</td>
<td>1995</td>
</tr>
<tr>
<td>ä</td>
<td>1840</td>
</tr>
<tr>
<td>ö</td>
<td>1805</td>
</tr>
<tr>
<td>a</td>
<td>1240</td>
</tr>
<tr>
<td>o</td>
<td>905</td>
</tr>
<tr>
<td>u</td>
<td>605</td>
</tr>
</tbody>
</table>

Note the F2-difference between high and low vowels is greater than the F2-difference due to rounding among front, high vowels (ΔF2(i − ä) = 655 Hz. vs. ΔF2(i − ü) = 500 Hz.). Because front, low vowels (ä), inhabit a F2 region containing other harmonizing vowels, I hypothesize that (i) ä is acoustically similar to round, non-low vowels (ü and ö), and (ii) this acoustic similarity causes ä to behave like ü and ö with respect to palatal harmony. Therefore, an acoustic feature, which may be termed [color] (see Odden, 1991), but is here called [lower F2] more aptly characterizes the Finnish vowel inventory for the purposes of harmony as indicated in (34).

(34) Revised vowel space for Finnish palatal harmony. Neutral categories are shaded.
In this space, Finnish palatal harmony receives a straightforward explanation as parasitic vowel harmony. Attraction demands that vowels which agree on [lower F2] must also agree in backness, i.e. $\text{ATTRACT}([[\text{cons}],[\text{lower F2}]] \rightarrow \text{[back]}) \gg \text{IDENT-IO([back])}$. Note that the differences between assuming a bivalent [±back], instead of two privative [coronal] and [dorsal] features for vowel place are subtle (see discussion in Clements & Hume, 1995), but not critical to an attraction analysis, since both front and back spread to affixes. Hence, it is possible to rank a single $\text{ATTRACT}([[\text{cons}],[\text{lower F2}]] \rightarrow \text{[back]})$ above $\text{IDENT-IO([back])}$ or to rank $\text{ATTRACT}([[\text{cons}],[\text{lower F2}]] \rightarrow \text{[coronal]})$ and $\text{ATTRACT}([[\text{cons}],[\text{lower F2}]] \rightarrow \text{[dorsal]})$, respectively, above $\text{IDENT-IO([coronal])}$ and $\text{IDENT-IO([dorsal])}$. For the sake of tableau conciseness, I assume [±back] is the relevant harmonic feature.

A [lower F2]-parasitic analysis of Finnish palatal harmony predicts the neutrality and transparency of $i$ and $e$ in root harmony, as indicated, respectively in (35) and (36), below.

(35) Root neutrality: $i$ and $e$ are not targets of palatal harmony, c.f. (32)(h) [sade].

<table>
<thead>
<tr>
<th></th>
<th>/sade/</th>
<th>CORR-VV$_L$→R</th>
<th>ATTRACT</th>
<th>IDENT-IO([back])</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>sa de</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>sa dö</td>
<td></td>
<td>*!(ä→ö)</td>
<td>*(ö)</td>
</tr>
<tr>
<td>c</td>
<td>sa do</td>
<td></td>
<td></td>
<td>*(ö)</td>
</tr>
<tr>
<td>d</td>
<td>sä de</td>
<td>!</td>
<td></td>
<td>*(ä)</td>
</tr>
</tbody>
</table>
(36)  *Root transparency:* $i$ and $e$ are not triggers of palatal harmony, c.f. (32)(g) [kesto].

<table>
<thead>
<tr>
<th>/kesto/</th>
<th>CORR-VVL→R</th>
<th>ATTRACT ({[[\text{cons}],\text{[lower F2]]}\to[\text{back}]} )</th>
<th>IDENT-IO_Root ({[back]})</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ke\SATO]</td>
<td>ke sto</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. ke stö</td>
<td></td>
<td></td>
<td>*!(6)</td>
</tr>
</tbody>
</table>

Hence, root neutrality follows from a failure of $i$ and $e$ to satisfy the precondition of \([\text{lower F2}]\) agreement when in the context of root-initial $u$, $o$, or $a$. Transparency follows from a failure of subsequent $u$, $o$, or $a$ to agree with preceding $i$ and $e$ on \([\text{lower F2}]\).

Before turning to the transparency of $i$ and $e$ in suffix harmony, I consider the fact that all-neutral roots trigger harmony in suffixes. Like in Akan, this can be expressed by a morphological split of faithfulness and a subterranean non-parasitic V-to-V attraction constraint. Note, in (37), while the parasitic and harmonic features are different, the ranking of attraction and faithfulness is identical to that of Akan: \(\text{ATTRACT}_{\text{parasitic, V-to-V}} \gg \text{IDENT-IO}_{\text{Root}} \gg \text{ATTRACT}_{\text{V-to-V}} \gg \text{IDENT-IO}_{\text{General}}\). Again, I assume the worst possible input for the resulting harmony, and show how the constraints, nevertheless, overcome faithfulness to give harmony, avoiding any stipulation of the nature of the default realization of affixes.
(37) All-neutral roots trigger front harmony

<table>
<thead>
<tr>
<th></th>
<th>sii-na→ [sii-nä]</th>
<th>CORR-VV L→R</th>
<th>ATTRACT ([cons],[lower F2]) →[back])</th>
<th>IDENT-IO Root ([back])</th>
<th>ATTRACT ([cons]) →[back])</th>
<th>IDENT-IO ([back])</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>sii na</td>
<td></td>
<td></td>
<td>*![ii→a]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>sii nä</td>
<td></td>
<td></td>
<td></td>
<td>*![ä]</td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td>su na</td>
<td></td>
<td></td>
<td>*![u]</td>
<td>*(u)</td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td>su na</td>
<td>*!</td>
<td></td>
<td>*(u)</td>
<td>*(u)</td>
<td></td>
</tr>
</tbody>
</table>

In (37), the parasitic attraction constraint is never activated because the root vowel and the suffix vowel do not agree on [lower F2], so it is only the more general attraction constraint that can motivate harmony (37)(a) vs. (37)(b).

In Akan, when the general attraction constraint was combined with the low ranking of non-local correspondence, the result was blocking, but for Finnish, high-ranking CORR-VV L→R allows root neutrality to yield the transparency of i and e to suffix harmony, as in (38), below. To elucidate, the marks have been labeled by the violating segments, and the correspondence arrows have been color coded. Black, solid arrows are correspondences between segments which agree on [back], gray, dashed are correspondences between segments which do not agree on [back]. This makes it clear that because of feature similarity (and the related elevated ranking of the parasitic attraction constraint), the non-local correspondence between the initial syllable and the suffix wins out over the more local correspondence between the neutral vowel and the suffix, see (38)(a) and (38)(b).
(38) *Transparency*: *i* and *e* do not block the harmony of subsequent targets to previous triggers, e.g. (32)(h) [sade-tta].

<table>
<thead>
<tr>
<th>sade-ttä→ [sade-tta]</th>
<th>CORR-VV_{L→R}</th>
<th>ATTRACT ({[cons],[lower F2]}→[back])</th>
<th>IDENT-IO_{Root} ({[cons]}→[back])</th>
<th>ATTRACT ({[cons]}→[back])</th>
<th>IDENT-IO ({[back]})</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. sa de -ttä</td>
<td></td>
<td>*!(a→ä)</td>
<td>*!(a→e), *(e→a)</td>
<td>*(a)</td>
<td></td>
</tr>
<tr>
<td>b. sa de -ttä</td>
<td></td>
<td></td>
<td>*!(a→ä)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. sa de -ttä</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. sa do -ttä</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Therefore, as previously discussed, the difference between blocking and transparency is sensitivity to positional similarity. In Finnish, positional similarity does not contribute to general similarity, but in Akan it does (non-local correspondence is prohibited). To be concrete, the difference between the ranking of Akan and Finnish constraints is presented below.

(39)

**Akan**
parasitic feature: [low]
harmonic feature: [ATR]

<table>
<thead>
<tr>
<th>CORR-VV(σ_{i}, σ_{i+1})_{L→R}</th>
<th>ATTRACT_{parasitic, V-to-V}</th>
<th>IDENT-IO_{Root}</th>
<th>ATTRACT_{V-to-V}</th>
<th>IDENT-IO_{General}</th>
<th>CORR-VV_{L→R}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&gt;&gt;</td>
<td>&gt;&gt;</td>
<td>&gt;&gt;</td>
<td>&gt;&gt;</td>
<td>&gt;&gt;</td>
</tr>
</tbody>
</table>

**Finnish**
parasitic feature: [lower F2]
harmonic feature: [back]

<table>
<thead>
<tr>
<th>CORR-VV(σ_{i}, σ_{i+1})_{L→R}</th>
<th>ATTRACT_{parasitic, V-to-V}</th>
<th>IDENT-IO_{Root}</th>
<th>ATTRACT_{V-to-V}</th>
<th>IDENT-IO_{General}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&gt;&gt;</td>
<td>&gt;&gt;</td>
<td>&gt;&gt;</td>
<td>&gt;&gt;</td>
</tr>
</tbody>
</table>

Also, note how degraded feature similarity is required for transparency, if the suffix vowel were more similar to *e* than initial *a* in the root *sade*, then transparency would not
occur, and agreement along the more local correspondence would win out. Crucially, however, for Finnish there are no such vowels where the important dimension of feature similarity is [lower F2] agreement. A vowel which is more similar to [e] than [a] must lack a lower F2, and so would never be predicted to agree with [a], as vowels without a lower F2 are always transparent. The point is that non-local correspondence always employs an element of prerequisite feature similarity, which obviates interveners from participating.

Again, standard inventory markedness constraints, which rule out [i] and [õ] (back /i/ and back /e/ respectively), only predict the neutrality of i and e, but inventory markedness has no way of evaluating what should happen once vowels do not participate, i.e. whether or not they are triggers(blocking) or non-triggers(transparency). Transparency is available because CORR-VV_{L→R} >> IDENT-IO([back]). If the ranking were reversed, as in Akan, syllabic locality would be enforced resulting in blocking.

In sum, in parasitic vowel harmony neutral vowels are not triggers of harmony by virtue of the same failure to meet similarity preconditions that prevents them from being targets of harmony. In regards to locality, this analysis of Finnish palatal harmony is on par with LDCA phenomena in which any number of intervening segments may be neutral and transparent to harmony. The next section argues that non-local correspondence is the right way to view transparency in Finnish.

4.3.4.2. Arguments for true transparency

In order to satisfy the strict locality conditions necessary for blocking, other proposals have suggested that transparent segments are covert undergoers (Gafos, 1996; Ní Chiosáin and Padgett, 2001; Walker and Mpiranya, 2005). Targeted Constraints
(Baković & Wilson, 2000), sympathy theory (Walker, 2003), embedded feature domains (Smolensky, 2006), and non-linear dynamics (Benus & Gafos, 2007) have all been used to wrestle with the issues of locality vs. transparency. These frameworks all submit to some notion of derivational opacity roughly defined as follows (see discussion in Baković, 2006): At some higher ‘phonological’ level, transparent segments carry harmonic features, but, at a lower ‘phonetic’ level, these distinctions disappear (presumably due to markedness).

Although derivational opacity is an independent problem, which might seem to lend some support to these approaches, in general, I am agnostic on the existence of an abstract level where transparent vowels participate in harmony, serving as bridges between triggers and surface distinct targets. However, there are two reasons to doubt that Finnish palatal harmony is such a case of covert harmony, one phonological, the other phonetic. Together, these support a view of Finnish palatal harmony as a case of true transparency, which supports the analysis of non-local agreement shown above and confirms the prediction of strong parallels between consonant and vowel harmony.

Firstly, if there were some late-stage markedness force causing the neutrality of i and e, then, at an earlier stage, /i/ and /õ/ would be available as possible triggers of harmony. In fact, this is exactly what would be predicted to occur through local spreading. For example, as (40) shows, a surface form like sadetta ‘ray (partitive)’ would have an abstract level of representation /sadõ-tta/, in which because of strict locality, the partitive suffix /ttA/ gets its backness value from root final /õ/.

(40) \[
\begin{array}{ccc}
\text{Input} & \text{Output} \\
/sade-ttA/ & \rightarrow /sadõ-tta/ & \rightarrow sade-tta \\
\end{array}
\]
Therefore, while /õ/ is not surface viable (at the phonetic level [õ] neutralizes to [e]), /õ/ would be a trigger of harmony at the more abstract, phonological level.

However, by Richness of the Base (Prince & Smolensky, 2004), allowing such triggering by [õ] predicts the existence of forms where surface neutral vowels are [+back] at the abstract level, and so some all-neutral vowel roots ought to require back vowels suffixes, e.g. */dõ-tta/→*de-tta. Yet, as in (32)(e-f), all-neutral stems always require front suffixes (Valimaa-Blum, 1999; Kiparsky & Pajusalu, 2003), falsifying this prediction. In this way, Finnish contrasts with Hungarian (Benus & Gafos, 2007), which also has transparent i and e, but in which there are some all neutral stems which reliably take back suffixes and other all neutral stems which reliably take front suffixes.

The literature on Finnish harmony notes the basically exceptionless regularity of all neutral roots requiring front suffixes, but also points out a few potential counterexamples that are worth mentioning. Välimaa-Blum (1999) reports that, in native vocabulary, there are two exceptions to suffix harmony, “both in the partitive case only. In all other grammatical cases … suffix harmony works normally” (p. 249) These exceptions are the roots meri ‘sea’ and veri ‘blood’, which take the back partitive suffix: mertal/*mertä ‘of the sea’ and vertal/*vertä ‘of the blood’. However, because all the other cases take predictably front forms, e.g. the inessive, meressäl/*meressa ‘in the sea’ and veressäl/*veressa ‘in the blood’. These roots are not exceptional in the way that is predicted by derivation. If at a more abstract level meri and veri were [+back], then that would predict that all suffixes (not just the partitive) should emerge with back suffixes.
Therefore, the prediction of all-neutral roots which only take [+back] suffixes is not expressed by *meri*, *veri*, or for that matter any other word in the native vocabulary.\footnote{Loan word phonology is much less predictable, see (Valimaa-Blum, 1999) and Ringen & Heinämaa, 1999).}

Other potential counter-examples are presented by Kiparsky & Pajusalu (2003), suggesting that a nominalizing suffix marker, -o or -u, is always [+back] in the context of monosyllabic, neutral roots. Thus, *el-o ‘liv-ing’, tiet-o ‘know-ledge’, and pes-u ‘wash-ing’. However, my own search of Finnish reference materials, show that, like *meri* and *veri*, everywhere else these monosyllabic roots take front suffixes. Furthermore, most of these stems show a full spectrum of contrast at the position of the suffix, e.g. *ele ‘sign’, elö ‘organism’, elo ‘living’, so I speculate that it is more likely a more complex morphological process that is preventing this nominalizing suffix from harmonizing. Thus, I conclude that all of these potential counter-examples are driven by the idiosyncratic behavior of the suffix, not because of the properties of the root. It would be consistent, root-driven [+back] harmony from neutral vowels roots that would support the derivational account. This simply does not occur in Finnish. Thus, there are phonological reasons to eschew analyzing Finnish transparency as derivational.

Secondly, researchers (Gafos, 1996; Benus & Gafos, 2007) have generally used phonetic evidence of V-to-V co-articulation to argue for covert harmony. This evidence shows that the variation in the realization of neutral vowels reliably depends on harmonic context, minimally demonstrating that transparent vowels are affected by the harmonic context. However, there is a known confound due to the independent existence of V-to-V coarticulation common to both harmonic and disharmonic languages: phonetic variation based on harmonic context can neither confirm nor refute the hypothesis that transparent
vowels covertly undergo harmony, unless that phonetic variation can be shown to be different from the variation predicted by the standard V-to-V coarticulation found in even disharmonic languages. With this in mind, Benus & Gafos (2007) find some supporting evidence for covert harmony in Hungarian by showing a reliable articulatory difference in the citation form (without affixation) of neutral, monosyllabic stems between those which take [+back] suffixes from those which take [−back] suffixes. They argue that because these forms are monosyllabic there is no way that standard V-to-V coarticulation could predict the difference, and instead abstract speaker knowledge must be determining the articulatory realization of the neutral vowels.

However, because in Finnish neutral roots always require front suffixes, this strategy is not available. Kim (2005) performed a phonetic investigation aimed exactly at determining how the variation in neutral, Finnish vowels in harmonic contexts relates to standard V-to-V coarticulation. The strategy is to explore the variance of co-articulation, as expressed by the acoustic correlate F2. The hypothesis is that in conventional V-to-V coarticulation, say, in a disharmonic language, like English, there is gradiency and overlap (between and within distinct ‘harmonic’ contexts), but in V-to-V harmony differences are prone to be more categorical and distinct. For Finnish, Kim reports that for harmonic pairs, like ä~a, F2 values vary categorically and distinctly in response to harmonic context, but there is much more overlap and gradiency in the F2 values of neutral e in differing harmonic contexts.

Kim also found that the amount of coarticulation (as expressed by F2 values), decreased with the distance between trigger and neutral vowels, for example, in forms, like tuotteeensa ‘product’, the first long /e/ showed more V-to-V coarticulation than the
second long /e/. Thus, the path that the first long /e/ takes to phonetic realization is distinct from the path taken by the second long /e/, disconfirming a derivational account in which both long /e/’s are [+back] at an early level and independently neutralize at the phonetic level. On the other hand [a]/[ä] varied categorically independent of the distance from the target. These facts suggest that the phonetic behavior of neutral vowels is qualitatively different than harmonizing vowels, so the available phonetic evidence does not support covert harmony of transparent vowels in Finnish.

Of course, the existence of true transparency in Finnish vowel harmony does not preclude other languages from having sub-phonemic harmonic alternations, but the true transparency argument is important because of the influential work (notably Gafos, 1996 and Ní Chiosáin and Padgett, 2001) suggesting otherwise, and also because the existence of true transparency in vowel harmony affirms a parallel to consonant harmony systems where consonants are truly transparent. Thus, a non-local interaction of the sort permitted by CORR_L→R, and shown above, is preferred to a covert transparency account with strict locality.

In fairness, however, this section has only argued against the multi-leveled accounts of transparency (e.g. Walker, 2003) and has not sought to dismiss dynamical theories, like Benus & Gafos (2007), which are, in a sense, quite compatible with the correspondence approach. Positional similarity instantiated as correspondence can provide a way to model in OT the durational extent of V-to-V coarticulation due to harmony. For strictly local spreading, if a segment does not harmonize, then no

---

92 Gafos & Benus argue that such ‘count effects’ (Hayes & Londe, 2006), as in Hungarian, where a single neutral vowel is transparent, but multiple neutral vowels block harmony, support permeability (not transparency) of the neutral vowels because they show that the intervening vowels can affect harmony. However, under positional similarity it is the distance from the trigger not the permeability of the neutral vowels which drives ‘count effects’.
subsequent segments will, but for non-local interaction, the pressure for co-articulation persists even though neutral interveners do not (fully) undergo harmony. True transparency in Finnish and Moba Yoruba (to come in §4.4.3) raise the possibility that vowels are not obliged to covertly undergo harmony, and like the intervening consonants in LDCA it is possible for vowels to be completely neutral and transparent to harmony. That neutral vowels do, evidently, covertly undergo harmony in Hungarian is orthogonal to the broader common theme between this and Benus & Gafos of harmony directly appealing to the nature of the phonetic space: for Hungarian it is fine grained articulatory detail, whereas in Finnish it is the availability of phonetic categories like [lower F2], which allow for a parasitic analysis of palatal harmony.

4.3.5. Discussion & Conclusion

This attraction analysis of parasitic vowel harmony designates a similarity-based feature ‘tier’, a harmonic region embedded in a rich articulatory/acoustic space that extends across vowel sequences. The tunneling illustrated in the schematic attractors in (41) is possible because attraction integrates both locality and feature preconditions. Positional similarity dictates attraction across positions (planes), so, in Finnish (41)(a), the attractor persists across multiple vowels and consonants, but in Turkish, strict adjacency is required, so the attractor persists for exactly one segment. Feature similarity determines the shape of the attractor within a plane, so in Finnish (41)(a) only vowels with [lower F2] are mutually attracting, but in Turkish (41)(b) the similarity requirements are higher, and only vowels that agree on both backness and rounding attract one another.
One contribution of this dissertation is to illustrate how such language specific attractors can derive the feature tiers assumed in autosegmental and derivative frameworks. §4.5, in particular addresses how these similarity-based attractors can be used to explain a number of C-V interactions and non-interactions.
Taken together, the data in this section argues that each language makes its own weighting of the factors of general similarity (proximity and features), which determines the shape of the harmony attractor. In this chapter, this weighting is expressed through a ranking of OT constraints, but as shown in Chapter 3, this weighting can be also be expressed numerically in Harmonic Grammar with limited differences in typology. Violable, or rankable, locality is an essential element of that language specific weighting. When positional similarity is given more weight, the result is blocking, but when proximity precondition are relaxed, transparency is allowed. Pulleyblank (2002) and Suzuki (1998) also note the importance of locality for blocking, and the lack thereof for transparency, but the present work is the first to recognize how these locality effects parallel those of feature similarity, and therefore the first to provide a general mechanism, attraction, for integrating similarity and locality preconditions on assimilation.

4.4. Locality contrasts in nasal harmony

This section presents attraction analyses of patterns of assimilation involving the harmonic feature [nasal], which characterizes the lowering of the velum that allows airflow into the nasal passages.\footnote{As Hall (2007) reviews, the literature presents arguments for and against the binarity of nasality. Cohn (1990) argues that a specified articulatory target for a raised velum must be available for oral segments in English, French, and Sundanese. However, following Steriade (1995) it seems that [−nasal] never spreads productively in harmony, and so for the purposes of harmony I assume a privative [nasal] feature.} Previous studies of nasal harmony have been compartmentalized by locality, focusing on either cases of spreading nasal harmony (Piggott, 1992, 2003; Cole & Kisseberth, 1994; Walker, 2000, 2003; Clements, 2003; refs therein) or cases of long-distance agreement (Rose & Walker, 2004; Hansson, 2001; Archangeli & Pulleyblank, 2007; refs therein). To my knowledge, this is the first effort
to consider both spreading and long-distance nasal harmony in tandem (though Suzuki, 1998; Ní Chiosáin & Padgett, 2001; Hansson, 2001; Pulleyblank, 2002; Rose & Walker, 2004; Walker, 2006; Archangeli & Pulleyblank, 2007; Ajíbóyè & Pulleyblank ms, 2008 and others investigate issues related to locality and harmony more generally).

This section describes a typology of nasal harmony with the finding that nasal harmony occurs at various grains of locality, ranging from spreading harmony where segmental adjacency is required to long-distance harmony where no locality conditions apply. As predicted by the Attraction Framework, this variation in locality permits a range of behaviors in regards to transparency and blocking. As in the vowel harmony examples of the previous section (§4.3), strictly local propagation prohibits transparency and allows a single neutral segment to block harmony, while non-local interaction prohibits blocking and allows massive transparency.

Looking beyond locality, because nasality is readily carried by both consonants and vowels, a study of nasal harmony directly confronts some of the primary issues for unifying consonant and vowel harmony. Here, I present patterns of nasal assimilation where trigger and target must be vowels (long-distance vowel agreement in Moba Yoruba §4.4.3), patterns where trigger and target must both be consonants (LDCA in Ngbaka and Kikongo §2.2.2), and also patterns where triggers and targets can be either consonants or vowels (spreading harmony in Jahore Malay and Applecross Scottish Gaelic (§4.4.2.4)). Therefore, a unified analysis must allow only C-C or only V-V interaction, but not to the universal exclusion of C-V interaction. Because C-V interaction is sometimes, but not always permitted, I argue that nasal harmony is incompatible with a traditional autosegmental account (see similar arguments in Cole &
Kisseberth, 1994; Clements & Hume, 1995; Clements, 2006). Therefore, a unified analysis of nasal harmony requires a more flexible formalism. Attraction provides this flexibility, enabling a number of subsymbolic solutions to nasal harmony where the representational similarity between segments can be determined on a language specific sensitivity to the phonetic correlates of nasality.

Furthermore, spreading nasal harmony provides a critical case to test the merits of unification under attraction because feature-based similarity has been thought not to play a prominent role (Rose & Walker, 2004; Hansson, 2001). Hence, analyzing nasal harmony among both consonants and vowels requires showing how similarity may be co-opted for a number of asymmetries between local and non-local nasal harmony. For instance, similarity must derive both the “nasal compatibility” (Walker, 2003) of spreading harmony and the parasitic requirements of non-local agreement in LDCA and LDVA. Like in parasitic vowel harmony, the present similarity perspectives on nasal harmony argue for a prominent role of phonetics in determining the available dependencies that may act as preconditions to harmony.

Related to the issue of unification, this section shows how regular feature spaces (§3.7) predict that harmony among vowels errantly predicts harmony among consonants, the so-called “mirror pathology,” which is inconsistent with the fact that nasal systems generally allow vowel harmony without consonant harmony, and vice versa. Therefore, I argue for the importance of irregular feature spaces, where the conditional similarity of segments agreeing on [+cons] may not be the same as the conditional similarity of segments agreeing on [−cons]. This separation further argues for a subsymbolic component (below that of phonological features) of the representations which undergo
attraction, as dependencies are not only formed between features, but are also formed between the phonetic correlates of feature values.

This section is outlined as follows: §4.4.1 presents the basic typology of blocking and transparency in nasal harmony, §4.4.2 argues for a phonetic basis of similarity solutions in both spreading harmony and non-local agreement, §4.4.3 analyzes the transparency of nasal consonants in V-to-V nasal harmony in Moba Yoruba, §4.4.4 discusses issues related to the interplay between consonants and vowels in autosegmental feature tiers, and §4.4.5 concludes.

4.4.1. Patterns of blocking and transparency in nasal harmony

A full-scale survey of nasal harmony languages is beyond the scope of this dissertation. Furthermore, there already exist a number of high-quality surveys (e.g. Piggott, 1992, 2003; Walker, 2000, 2003; Hansson, 2001; Rose & Walker, 2004). Therefore, this section focuses more on the patterns of nasal harmony rather than the data of individual languages. To this end, the contrasts between example languages are presented in table form, in (43)-(45), below. This section explains these tables, and argues that the generalizations from this sample support the locality predictions of the present Attraction Framework.

What are those predictions? As already discussed in the preceding sections, the correspondence constraints which implement locality preconditions, CORR(Xi,X(i+1)), CORR(σi,σ(i+1)), and CORR, make different allowances for sensitivity to locality. If only CORR(Xi,X(i+1)) dominates faith then strictly locality is enforced, if CORR(σi,σ(i+1)) also dominates faith then syllable-local interaction is allowed, and if even non-local correspondence, CORR, dominates faith then no locality preconditions apply to harmony.
These grains of locality make the predictions, in (42) below, in regard to transparency and blocking.

(42) Predicted transparency and blocking if $\text{CORR}(l) \gg \text{IDENT-IO}$ (repeated from (10)).

<table>
<thead>
<tr>
<th>Correspondence constraint</th>
<th>Transparent Element(s)</th>
<th>Blocking Element(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{CORR}(X_iX_{i+1})$</td>
<td>None</td>
<td>Single Neutral Segment</td>
</tr>
<tr>
<td>$\text{CORR}(\sigma_i\sigma_{i+1})$</td>
<td>Multiple Neutral Segments</td>
<td>Single Neutral Syllable</td>
</tr>
<tr>
<td>$\text{CORR}$</td>
<td>Multiple Neutral Syllables</td>
<td>None</td>
</tr>
</tbody>
</table>

A brief survey of nasal harmony confirms that the typological predictions of these grains of locality are correct, as each pattern is attested. The table in (43), below, summarizes cases of nasal harmony where a single neutral segment blocks. (44), below, shows cases of nasal harmony where transparency of single segments is allowed, but multiple neutral interveners block.

(45), below, reviews cases of nasal harmony where locality conditions are ignored, resulting in massive transparency and a lack of blocking. A detailed explanation follows the presentation of the tables.

---

$^94$ $\text{CORR}(\sigma_i\sigma_{i+1})$ interacts with syllable structure to determine the number of interveners permitted to be transparent and the number of interveners required for blocking. If only CV syllables are allowed, as in many of these cases of nasal harmony, then two neutral interveners constitute a neutral syllable and so multiple neutral interveners are blockers. However, if CVC syllables are allowed, then $C_xV.CVC_x$ is a required correspondence, and so as many as four segments (VC.CV) may be neutral. Further investigation into the syllable structures permitted by the languages in (44) will likely reveal a need for further division in locality preconditions, as $C_xV.CVC_x$ interaction is sometimes allowed, while $C_xV.CVC_x$ is not. However, $\text{CORR}(\sigma_i\sigma_{i+1})$ does not discriminate between the two cases. These structure effects related to onsets and codas are not further discussed here, though see Smolensky (2006) and Kaplan (2008) for a discussion of Lango where vowel harmony is allowed for (C)V.CV structures, but not if a coda intervenes (C)VC.CV.
(43) \( \text{CORR}(X_i, X_{i+1}) \gg \text{IO-FAITH (nasal)} \): cases of nasal harmony with no transparency, and a single neutral segment blocks.

<table>
<thead>
<tr>
<th>Language</th>
<th>Selected Refs</th>
<th>Triggers</th>
<th>Targets</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>N ( \checkmark )</td>
<td>( \hat{V} )</td>
<td>V</td>
</tr>
<tr>
<td>Sundanese</td>
<td>(Walker, 2000, 2003)</td>
<td>( \checkmark )</td>
<td>R</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>Orejon</td>
<td>(Cole &amp; Kisseberth, 1994)</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>Johore Malay</td>
<td>(Walker, 2000, 2003)</td>
<td>( \checkmark )</td>
<td>R</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>Kolokuma Ijo</td>
<td>(Walker, 2000, 2003)</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>Ikwere</td>
<td>(Clements, 2003)</td>
<td>R</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>Applecross Gaelic</td>
<td>(Walker, 2000, 2003)</td>
<td>R</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
</tr>
</tbody>
</table>

Orejon has two nasal harmony processes, glides only undergo harmony to nasal vowels.

(44) \( \text{CORR}(X_i, X_{i+1}) \gg \text{CORR}(\sigma_i, \sigma_{i+1}) \gg \text{IO-FAITH (nasal)} \): cases of nasal harmony with some transparency, but multiple neutral segments block.

<table>
<thead>
<tr>
<th>Language</th>
<th>Selected Refs</th>
<th>Triggers</th>
<th>Targets</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>N ( \checkmark )</td>
<td>( \hat{V} )</td>
<td>V</td>
</tr>
<tr>
<td>Tuyuca</td>
<td>(Walker, 2000, 2003)</td>
<td>R</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>Barasana</td>
<td>(Piggott, 2003)</td>
<td>R</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>Moba Yoruba</td>
<td>(Piggott, 2003)</td>
<td>T</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>(Archangeli &amp; Pulleyblank, 2007)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Hansson, 2001)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note nasal Cs are transparent, confirming that this is a non-adjacent interaction. Harmony is parasitic on [+hi], mid and low vowels block.

Strictly trans-vocalic harmony. A single non-participating C blocks harmony.
(45) \( \text{CORR}(X_i X_{i+1}) \gg \text{CORR}(\sigma_i \sigma_{i+1}) \gg \text{CORR} \gg \text{IO-FAITH([nasal])}: \) cases of nasal harmony with massive transparency, where no blocking occurs.

<table>
<thead>
<tr>
<th>Language</th>
<th>Selected Refs</th>
<th>Triggers</th>
<th>Targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ngbaka</td>
<td>(Rose &amp; Walker, 2004) (Hansson, 2001)</td>
<td>✓ T</td>
<td>T T T ✓ T T T</td>
</tr>
</tbody>
</table>

In addition to nasal harmony at different localities, these tables contain information about the trigger-target pairs. This is encoded as follows: triggering segments are nasal stops (N) and/or nasalized vowels (Ṽ). A check ‘✓’ under the ‘Trigger’ heading denotes that the segment is a trigger, ‘R’ denotes that the segment is not a trigger, but occurs allophonically as a result of harmony, and ‘T’ denotes that the segment is not a trigger but freely co-occurs with the harmonic context, i.e. it is transparent. For example, as discussed in §4.4.3, nasal stops are transparent to V-to-V nasal harmony in Moba Yoruba, and so are marked ‘T’ in (44), above.
The possible targets are arranged by sonority because sonority is known to play an important role in nasal harmony (Walker, 2000, 2003; Clements, 2003; Piggott, 1992, 2003). The key for these segments is ‘V’ – vowels, ‘W’ – glides, ‘L’ – liquids, ‘D[+son]’ – a rare class of prenasalized and non-explosive stops that are phonologically and phonetically sonorant, and so participate in harmony in distinct ways from conventionally released, voiced stops – ‘D[−son]’, ‘S’ – fricatives, and ‘T’ – voiceless stops. The dashed line between ‘S’ and ‘D[−son]’ indicates that unlike the other segments the sonority relationship between them is not universally determined. A ‘✓’ under the ‘Target’ heading means that the segment undergoes harmony to the indicated triggers emerging as nasal in the output. ‘B’ indicates that the segment is neutral and blocks harmony to subsequent targets. ‘T’ indicates that the segment is neutral and does not block harmony to subsequent targets. ‘T?’ is used to indicate the probable behavior of certain segments although not explicitly listed as such in the available refs. When even the status in the inventory is not available, a plain ‘?’ denotes the limits of the sources. Blank cells indicate that the sonority class is not attested in the language.

Now, (43)-(45) confirm that there are attested phenomena exhibiting each predicted locality precondition, solidifying a role for locality in determining general prerequisite similarity not only for the purposes of nasal harmony, but also for assimilation more generally, since a similar typology was demonstrated for parasitic vowel harmony. Like the analysis of those vowel harmony phenomena, the variation in sensitivity to locality in nasal harmony is explained in the attraction framework by ranking the appropriate correspondence constraints above faith. Thus, the locality differences between nasal harmony processes derive from the same forces of attraction.

<table>
<thead>
<tr>
<th>Target</th>
<th>Vowel</th>
<th>Glide</th>
<th>Liquid</th>
<th>Pre-nasalized</th>
<th>Voiced Stop</th>
<th>Fricative</th>
<th>Voiceless Stop</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W</td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L</td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D[+son]</td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D[−son]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S</td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>T’</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
</tbody>
</table>
that operate in assimilation generally, and so differences between nasal harmony systems in terms of locality only receive incidental formal treatment.

However, there are important generalizations from the above tables, which are parameterized by locality. In certain respects, local harmony behaves in an opposite fashion to non-local harmony, and so these generalizations are not obviously compatible with a similarity analysis. For instance, attraction excels at explaining parasitism as in non-local harmony, but the languages that enforce strict locality do not seem to require parasitism, raising the question of why locality affects the availability of parasitism. In the next section, I challenge the conventional thinking (Rose & Walker, 2004; Hansson, 2001) that similarity and parasitism has little role in spreading nasal harmony, but, first, based on (43)-(45), I present the main generalizations which must be explained by any account. In terms of feature preconditions, there are several strong asymmetries between spreading – local nasal harmony (LNH) – and long-distance interaction – non-local nasal harmony (NLNH).

First, obstruents are systematically dispreferred as targets of local nasal harmony (Walker, 2000, 2003; Piggott, 2003), but obstruents are the preferred targets of non-local nasal harmony. This generalization is confirmed by (43), which shows that obstruents are strongly avoided as targets of LNH. However, observe from (45) that

---

95 Throughout the remainder of this section, unless otherwise noted “local” refers to the local, iterative harmony processes indicative of spreading and “strictly local” refers to non-iterative, local nasal spreading. Non-iterative nasal harmony may not disprefer obstruents to the same degree as iterative nasal harmony. However, that is not to say similarity conditions do not apply in non-iterative harmony: in Sestwana (Cole, 1985) and Catalan (Mascaró, 1976) only allow spreading between homorganic consonants ([mb]→[mm]).
96 Phonetic studies (Piggott, 2003; Clements, 2003) of stops which undergo local nasal harmony show that the participating stops are not conventional, explosive stops, like those found in English. Rather, stops which undergo nasal harmony tend to exhibit a spontaneous voicing akin to sonorancy due to (partial) prenasalization or non-explosive release. Therefore, the relevant generalization is that phonetic obstruents
obstruents are systematically privileged as targets of NLNH. Note syllable local systems, (44), are mixed in this regard, but I know of no fully non-local system that disallows harmony between nasal stops and homorganic, voiced stops. Thus, non-local harmony opposes the local systems with blocking, where there are no systems that allow obstruents (including voiced, homorganic stops) to undergo harmony.

Second, sonority is very important to describing local nasal harmony, lending to the following implicational universals: (i) if a less sonorous segment undergoes harmony, then so do more sonorous segments, and (ii) if a more sonorous segment blocks harmony, then so do less sonorous segments (Walker, 2000, 2003; Clements, 2003; Piggott, 2003). However, sonority is of limited importance to NLNH. It is certainly not true, that if a less sonorous segment undergoes NLNH, then a more sonorous segment undergoes NLNH, since vowels and glides are never targets of non-local nasal consonant harmony. It is also not true that if a more sonorous segment undergoes NLNH, then a less sonorous segments undergoes NLNH (e.g. in Kikongo, /n…l/ sequences are avoided, but /n…s/ sequences are allowed, even though /l/ is generally assumed to be more sonorous than /s/). Hence, unlike LNH, no implicational relations involving sonority are available to NLNH.

Walker (2000, 2003) argues that “nasal compatibility” is not the same as sonority, since nasal stops top the hierarchy of nasal compatibility, but they are less sonorous than stops, glides, and liquids. For the phonetic reasons explored below, I argue that a correlate of sonority, resonance, is the primary component of similarity which determines “compatibility” for spreading nasal harmony, which is to say that vowels enhance certain which undergo nasal harmony are actually phonological sonorants, and so the generalization remains that phonological obstruents do not participate in local nasal harmony.
characteristics of nasality beyond that of even nasal stops. Furthermore, from the present perspective, the differences between compatibility and sonority are conceptually significant, as sonority allows for a similarity analysis to obtain, which grounds and predicts harmonic interactions in a much less stipulative manner (see §4.4.3).

Finally, as previously indicated, local nasal harmony is never parasitic in the same ways as non-local nasal harmony. NLNH can be parasitic on voicing (Kikongo (45)), place (Ngbaka (45)), or height (Moba Yoruba (44)), but LNH is never parasitic on any of these features. It is worth noting that Mascaro (1976) reports that in Catalan /mp/ sequences emerge as geminate nasals /mm/, but /np/ sequences do not change. This, however, if borne out, would be another case of exclusively local parasitic assimilation (the parasitic feature is place), and is non-iterative in character because it is confined to intervocalic contexts (see §2.4 for other examples). Crucially no spreading nasal harmony language exhibits place-parasitism of the sort where [m] only induces harmony on round vowels or [n] only induces harmony on front vowels/glides while [ŋ] only triggers harmony on back vowels/glides.

The above asymmetries due to differences in feature-based preconditions to harmony are summarized below in (46):

(46) Feature-based asymmetries between local and non-local nasal harmony

<table>
<thead>
<tr>
<th></th>
<th><strong>Local (Spreading)</strong></th>
<th><strong>Non-local (long-distance)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nasal Harmony</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obstruents:</td>
<td>Obstruents are avoided as targets of harmony.</td>
<td>Obstruents are privileged as targets of harmony.</td>
</tr>
<tr>
<td>Sonority:</td>
<td>There are implicational relations involving sonority.</td>
<td>There are no implicational relations involving sonority.</td>
</tr>
<tr>
<td>Parasitism:</td>
<td>Harmony is not parasitic on voicing, place, or vowel height.</td>
<td>Harmony can be parasitic on voicing, place, or vowel height.</td>
</tr>
</tbody>
</table>
From the perspective of similarity, the contrast in the above generalizations is striking because pairs of segments which tend to interact in LNH do not interact in NLNH, and vice versa. Examples of these contrasts are shown in the following sample of data from various nasal harmony languages in (47), below.

(47) Contrasts in interaction between local and non-local nasal harmony

a. **Harmony between N and D[−son] in non-local nasal harmony**:
   - **Kikongo**: tu-\(\text{n}\)-\(\text{ik}\)\(-\text{di} \rightarrow [tu-\text{n}-\text{ik}-\text{i}]\) ‘we ground’
   - **Ngbaka**: \([^m]\text{be}[^m]\text{be}]^{*},[^m]\text{be}[^m]\text{be}[^m]\text{be}]^{*},[^m]\text{be}[^m]\text{me}]^{*} ‘snail’

b. **Lack of harmony between N and D[−son] in local nasal harmony gives blocking**:
   - **Johore Malay**: \(\text{pom}^\text{a}\text{n}^\text{a}\text{dan}^\text{an} \rightarrow [\text{pom}^\text{a}\text{n}^\text{a}\text{dan}^\text{an}]\) ‘scenery’
   - **Kolokuma Ijo**: \(\text{um}^\text{b}^\text{a} \rightarrow [\text{um}^\text{b}^\text{a}]\) ‘breath’
   - **Applecross Gaelic**: \(\text{s}^\text{n}^\text{a}^\text{d}^\text{a}^\text{n} \rightarrow [\text{s}^\text{n}^\text{a}^\text{d}^\text{a}^\text{n}]\) ‘thread’

(47) Contrasts in interaction between local and non-local nasal harmony


c. **Harmony between N and V, N and W in local nasal harmony**:
   - **Johore Malay**: \(\text{p}^\text{a}\text{n}^\text{a}\text{wa}^\text{s}^\text{a}^\text{n} \rightarrow [\text{p}^\text{a}\text{n}^\text{a}\text{wa}^\text{a}^\text{san}]\) ‘supervision’
   - **Sundanese**: \(\text{n}^\text{a}\text{tur} \rightarrow [\text{n}^\text{a}\text{tur}]\) ‘to arrange’

d. **Lack of harmony between N and V, N and W in non-local nasal harmony**:
   - **Kikongo**: \([tu-\text{n}-\text{ik}\text{di}] \rightarrow [\text{tu}-\text{n}-\text{ik}-\text{i}],[^*tu-\text{n}-\text{ik}-\text{i}]\) ‘we ground’
   - **Ngbaka**: \([^m]\text{be}[^m]\text{be}]^{*},[^m]\text{be}[^m]\text{be}[^m]\text{be}]^{*},[^m]\text{be}[^m]\text{be}[^m]\text{be}]^{*} ‘snail’
   - **Yaka**: \([\text{j}^\text{a}^\text{n}^\text{i}-\text{i}] \rightarrow [\text{j}^\text{a}^\text{n}^\text{i}-\text{i}],[^*\text{j}^\text{a}^\text{n}^\text{i}-\text{i}]\) ‘to cry out’

The challenge for an attraction account is to explain how pairs of segments like [n] and [d] or [m] and [b], which on the one hand, (47)(a), must be similar in an attraction analysis of non-local nasal harmony, can on the other hand, (47)(b), also be dissimilar in an analysis of local nasal harmony. Likewise, an attraction account must simultaneously explain how pairs of segments like [n] and [a] or [n] and [w] can be similar for local nasal harmony, (47)(c), while also being dissimilar for non-local harmony, (47)(d).
next section addresses how the similarity differences between local and non-local nasal harmony, reflect a language-specific difference in the sensitivity to aspects of nasality.

4.4.2. Similarity in nasal harmony

This section reviews aspects of nasality that have been independently argued to exist to account for the representation of nasal contours (Steriade, 1993; see also Clements & Hume, 1995; Cole & Kisseberth, 1995) which fully account for all of the asymmetries between non-local nasal harmony and local nasal harmony. Steriade’s (1993) observation is that the contrast between prenasalized (\( mb \)) and postnasalized (\( bm \)) stops is due to the timing of the lowering of the velum. Recall that stops consist of a closure period during which the oral cavity is completely closed, allowing for the buildup of intraoral pressure. This pressure quickly dissipates after the closure is released. In prenasalized stops, the velum is lowered during the closure period, but raised before the release, and in postnasalized stops, the velum is raised during the closure period, but lowered during the release. The contrast between prenasalized and postnasalized stops, therefore, requires positing two kinds of root nodes: “closure” nodes and “release” nodes to which nasality may link. Prenasalized stops only have a [nasal] link during closure, and postnasalized stops only have a [nasal] link during release; full-nasal stops are represented by linking [nasal] to both closure and release.

In what follows, I argue that local nasal harmony is based on the sonority of [nasal] during the closure period, whereas non-local nasal harmony (which is limited to harmony among consonants) is based on similarity to the correlates of [nasal] during and immediately after release, related to nasal stops being [–continuant].

- 270 -
4.4.2.1. Subfeatural methods

The solution to the above asymmetries in (46) is to allow the representation of segments to have aspects which are *subfeatures*. The Attraction Framework allows phonological features to be decomposed into subfeatures corresponding to the acoustic and articulatory correlates of features. Forces of attraction which drive harmony may then operate directly on these subfeatures. In the case of Finnish, §4.3.4, the availability of reference to a lower F2, allowed a sub-feature, \[\text{lower F2}\], to cut across distinctive feature boundaries, more naturally explaining why the disjoint classes \([-\text{low}, +\text{round}]\) vowels and \([+\text{low}, -\text{round}]\) vowels pattern together as triggers and targets of palatal harmony (see §3.4 for other examples).

In a way, the relationship between features and sub-features is like the relationship between segments and autosegments. Recall that among the primary contributions of autosegmental phonology (Goldsmith, 1979; see Kenstowicz, 1994 for a review) is that it allows for the representation of structure that is smaller or larger than a single segment. "Larger" autosegments are useful for tonal and other systems where the relevant representational feature cannot be localized to a single root position. "Smaller" autosegments explain contour segments like affricates where more than one feature must be linked to a single root position.

In like manner, among the contributions of subsymbolic phonology is that it allows for the representation of structure internal to a segment that is smaller or larger than a single phonological feature.\(^{97}\) For instance, the \([\text{lower F2}]\) feature that characterizes Finnish palatal harmony defines a natural class which is "larger" than a

\(^{97}\) §4.5 discusses how, although perhaps similar in spirit, subsymbolic representations differ in critical ways from Feature Geometry and derivatives.
single feature because [lower F2] is an auditory-cue correlate of multiple features, including height, backness, and rounding. This section further shows how the subsymbolic approach also allows phonological processes to operate on subfeatures which are "smaller" than a single distinctive feature.

Note if [n] and [d] always differ in the same way, [nasal], then they should behave the same way in both local and non-local harmony. However, the fact that [n] and [d] tend to interact in non-local nasal harmony and tend to not interact in local nasal harmony means that [n] and [d] must be represented in such a way that [n] and [d] are similar in some respects, but not others. If [nasal] can be associated with features N₁ and N₂, [n] and [d] only sharing N₂, then there is the possibility that similarity along N₁ could be exploited for local nasal harmony while similarity along N₂ could be exploited for non-local nasal harmony.

Fortunately, [n] and [d] differ on N₁ = [+sonorant] and share N₂ = [−continuant]. Even so, in order to fully derive the asymmetries between LNH and NLNH, it will prove crucial to refer to subfeatural aspects of sonority and continuant, in particular the resonant quality of sonorants and the release of non-continuant closure with its accompanying burst. To that end, I now discuss nasal sonority and nasal non-continuancy in turn.

4.4.2.2. Nasal resonance during closure

The usual single-tube model of resonance that is suitable for oral stops and vowels must be abandoned in favor of a two tube-model for nasals because opening the velum creates a separate channel of airflow which begins at the velum and ends at the nostrils (Stevens, 2000; Johnson, 2004). The relevance of the different tubes depends on the place of articulation. A uvular closure completely blocks the oral side branch, and so
the vocal tract configuration can be approximated by a single tube from the glottis to the nostrils. However, a supra-uvular closure creates a closed oral side branch that results in a number of anti-formants, while a pre-uvular constriction (pharyngeal or glottal) allows both oral and nasal branches to be simultaneously available.

Despite these differences, certain characteristics are common to all nasal stops during the period of constriction. Unlike other stops, no cessation of voicing occurs during the closure period of a nasal stop. This is true because transglottal pressure remains relatively unchanged: while the oral cavity is completely blocked, airflow passes freely through the open velum into the nasal passageways. Like the open vocal tract configurations of vowels, this open configuration through the nasal passageways prevents the build-up of intraoral pressure, leading to spontaneous voicing and the classification of nasal stops as [+sonorant] (Jakobson, Fant, & Halle 1952; Maddiesson & Ladefoged, 1993; Stevens, 2000; Walker, 2000, 2003; Piggott, 1992, 2003; Clements, 2003; etc.).

Here, I follow Clements (2006; refs therein) in presuming that resonance, i.e. clear formant structure, is a main indicator of sonority. While resonance is facilitated by loudness and a voiced source, as Clements notes, it is different from each, e.g. a click may be loud but not have resonance. I hypothesize that it is [+sonorant] and related aspects of resonance that are most relevant for local nasal harmony.

Importantly, nasal stops have resonance during the entire stop-closure period. In particular, nasals have a strong formant around 250-300 Hz, and other resonances at higher frequencies. This high-intensity, low-frequency formant is the so-called “nasal formant” or “nasal murmur.” Because of heavy damping due to impedance in the nasal passages, the harmonics of nasals tend to be of lower intensity, but broader spectrum,
than those of vowels. Regardless, unlike other stops, nasals as sonorants yield a spectrum with both strong peaks and valleys during the closure period.

It should also be said that, like nasal stops, nasalized vowels also have a number of formants and anti-formants, although because the oral branch is more open than the nasal branch, it is the nasal branch that becomes the closed side branch. The perception of the F1 values of vowels is heavily influenced by nasality (Johnson, 2004) because of the strong low frequency formant. These influences are shown to have a phonological impact in Moba Yoruba (§4.4.3), where the nasal murmur interacts with vowel height (also characterized by F1).

4.4.2.3. [−continuant] and nasal release

Before turning to how sonority is recruited by local nasal harmony, I discuss the contrasting qualities that occur upon release of the nasal closure. For all stops (voiceless, voiced, aspirated, and nasal), the period immediately following release is associated with rapid formant transitions. The nature of these formant transitions is largely determined by the place of articulation, but for nasals, these transitions are more complicated because the opening of the oral pathway yields a quick shifting in the anti-resonances (due to the previously closed oral side-branch). In addition to the emergence of a number of high frequency formants, of particular mention is F2, which, following the release of nasal stops, emerges at a place where there was a previous anti-resonance (Stevens, 2000). Furthermore, the F2 transitions for nasal stops mirror those of other stops.

Now, the place of nasal consonants is known to be a weak cue, so it is common for nasal consonants to assimilate in place to nearby stops (see Baković, 2007; refs therein). The weakness of this cue derives from (i) the lack of pressure (due to the open
nasal pathway), which might otherwise give a stronger, more sustained burst and (ii) the shifting resonances and anti-resonances that can mask the usual formant transitions indicative of place.

The dilemma is that for nasal stops, the duration and completeness of the closure period and the timing of the release are roughly comparable to oral stops, but, nevertheless, because of a lack of intraoral pressure, nasal stops at release lack the burst which usually denotes place features in non-continuants. Thus, articulatorily, nasal stops are \([-\text{continuant}]\), but acoustically they lack the associated burst. I hypothesize that it is the burst-like release of \([-\text{continuant}]\) associated with the articulation of nasal stops, which is recruited in non-local nasal harmony.

**4.4.2.4. Resolving locality asymmetries with subfeatures**

Articulatorily, the goal of nasalization is to have a specification \([\text{nasal}]\) which requires a lowered velum (see Walker & Pullum, 1996 for arguments that it is velum lowering and not nasal airflow that is the target of \([\text{nasal}]\)), but there are other ways to be similar to nasal stops. Oral stops agree on \([-\text{continuant}]\) but not \([+\text{sonorant}]\). Likewise, vowels might agree on \([+\text{sonorant}]\) but not \([-\text{continuant}]\).

I contend that targets of local nasal harmony are \([+\text{sonorant}]\), tending to have clear formant structure, and targets of non-local nasal harmony are \([-\text{continuant}]\), tending to have a burst which denotes a preceding fully closed vocal tract configuration. In this way, local nasal harmony is parasitic on sonority and non-local harmony is parasitic on \([-\text{continuant}]\). Formally, for local harmony, \(\text{ATTRACT}([\text{sonorant}] \rightarrow [\text{nasal}])\) is active, but for non-local harmony, \(\text{ATTRACT}([\text{continuant}] \rightarrow [\text{nasal}])\) is the harmony driver. I now show how this hypothesis derives all of the asymmetries summarized in (46) by
considering the subfeatures and phonotactics of sonority and continuancy. Each asymmetry, obstruent, sonority, and parasitism, is considered in turn.

4.4.2.4.1. Obstruent asymmetries

For obvious reasons, [sonorant]-parasitic nasal harmony explains why obstruents do not undergo local nasal harmony (see (43)), and of course, locality predicts that the neutral segments are blockers. This is briefly illustrated with the tableau below, in (48), which considers the Johore Malay form pəməndanjan→[pəməndanjan] ‘scenery’.

Correspondences between relevant segments are indicated with letter subscripts; for succinctness, not all correspondences, nor their L→R directionality, are indicated.

(48) Blocking in Johore Malay by non-resonant obstruents.

<table>
<thead>
<tr>
<th>pəməndanjan→[pəməndanjan]</th>
<th>CORRL→R X,X_{i+1}</th>
<th>ATTRACT( { [sonorant] } → [nasal] )</th>
<th>IO-FAITH(^{98}) ([nasal])</th>
<th>CORRL→R (\sigma_i \sigma_{i+1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. pəm,ə,n, d,ə n,ə,n</td>
<td>*!(m→a₁), *(n→a₃)</td>
<td>*(n₁→a₂)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. pəm,ə,n, d,ə n,ə,n</td>
<td>*(n→a₃)</td>
<td>*(n₁→a₂)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. pəm,ə,n, d,ə n,ə,n</td>
<td>*(n→a₃)</td>
<td>*(n₁→a₂)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. pəm,ə,n, n,ə,n</td>
<td>*(n→a₃)</td>
<td>*(n₁→a₂)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e. pəm,ə,n d a n,ə,n</td>
<td>*(n,d)</td>
<td>*(n₁→a₂)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f. pəm,ə,n,d,ə,n,ə,n</td>
<td>*(n,d)</td>
<td>*(n₁→a₂)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The critical candidate pair above is (48)(c) and (48)(d). Because harmony is parasitic on [sonorant], [d] is under no pressure to agree in nasality,\(^{99}\) and so it is neutral

\(^{98}\) [nasal] is privative, so again, IO-FAITH([nasal]) enforces MAX([nasal]) and DEP([nasal]).

\(^{99}\) Because of the privative nature of the harmonic feature, non-nasal segments cannot induce harmony on other segments, so, for instance, [m] is under no pressure to agree with the preceding, non-
to harmony. This analysis differs from those of Walker (2003), Piggott (2003), and Clements (2003) because instead of a compatibility hierarchy preventing targets from undergoing harmony, it is a lack of trigger-target similarity with regard to sonority that predicts neutrality. In both systems, locality demands that neutral segments are blockers (cf. (48)(c),(e),(f)).

Of course, (48) assumes that, in Johore Malay, nasal stops and vowels are [+sonorant], while voiced stops are not. This could be independently confirmed phonetically for Malay, but even without a specific phonetic study of Malay, there is good reason to think these representations hold. A division between sonorant and non-sonorant stops is critical to spreading nasal harmony in Ikwere (Clements, 2003) and Barasana (Piggott, 2003), where phonetically confirmed [+sonorant] stops do undergo harmony, while phonetically confirmed [−sonorant] stops do not. Evidently, the voiced stops of Johore Malay are [−sonorant], since unlike certain stops in Ikwere and Barasana, they are not targets of local nasal harmony.

By definition, released stops are [−continuant], so [continuant]-parasitism explains why obstruents are the preferred targets for non-local harmony. The constraint which demands that stops agree in nasality is \textsc{ATTRACT}([[continuant]] \rightarrow [nasal]). For forms like Kikongo, /tu-\text{nìk}/stem+/i\text{di}/ \rightarrow [tu-\text{nìk}-\text{imì}] `we ground’, the lower ranked affix faithfulness allows long-distance harmony between [n] and [d]. Furthermore, the transparency of suffix vowels derives from their antagonism to [−continuant]: vowels are perhaps universally [+continuant], and so \textsc{ATTRACT}([[continuant]] \rightarrow [nasal]) does not mark candidates where vowels disagree with stops in nasality.

\textit{nasal} [ə]. Formally, privative features work in \textsc{ATTRACT} constraints by only requiring [f] agreement if x\textit{Ry} and x carries a specification for [f].
Furthermore, consider the case of Ngbaka, which has non-local nasal harmony among consonants, but where underlying nasalized vowels freely combine with nasal, prenasal, voiced and unvoiced stops (Rose & Walker, 2004). Given that even [nasal] vowels are not [+continuant], it is not surprising to find that in Ngbaka, [nasal] vowels are transparent to [nasal] harmony among consonants. Since nasal consonant harmony is parasitic on [continuant], vowels are never triggers or targets of harmony independent of whether they are specified as [nasal].

4.4.2.4.2. Sonorancy asymmetries

Turning to the asymmetry along sonorancy, there are strong implicational relations among the undergoers of local nasal harmony in terms of sonorancy, while there are no such implicational relations in non-local harmony. This also follows from [sonorant]-parasitic spreading nasal harmony and [continuant]-parasitic long-distance harmony.

Sonority can be a gradient quality, enabling implicational relations along the resonance continuum. From an attraction based perspective, these implications follow the ‘Principle of Similarity’ where if less similar segments (in terms of resonance) undergo harmony then so do more similar segments (see §3.8.1), and if more similar segments fail to undergo harmony then so do less similar segments.

However, there is some subtlety in that similarity for sonority is not computed as ‘differences in sonority’, i.e. where the compatibility hierarchy follows because the sonority difference between nasals and vowels is $X$, and the sonority difference between nasals and glides is $X + \Delta$. This is not possible because nasals and glides are more alike in their sonority qualities (duration, intensity, type of resonance, etc.). Rather, nasals are
seen as phonologically [+sonorant], and harmony seeks to enhance that sonority as much as possible, so sonority differences are computed from an ideal sonorant, \( \alpha \), where \( \text{sim}(\alpha, V) \) is necessarily greater than \( \text{sim}(\alpha, W) \).

Thus, the difference between nasal compatibility hierarchies (Walker, 2000, 2003) and sonority hierarchies is that vowels, not nasal stops, top the sonority hierarchy. This is crucial because there can be systems like Moba Yoruba (§4.4.3) and Terena (Cole & Kisseberth 1995; Piggott, 1997), where nasal stops can be transparent to harmony without violating any implicational relations that might otherwise suggest nasals are obligatorily non-neutral. Put another way, there are systems (like Moba Yoruba and Terena) where nasal stops are neutral to sonority-parasitic harmony, but there are no systems where nasal vowels are neutral to sonority-parasitic harmony.

On the other hand, [continuant] does not seem to lend itself to the same degree of gradiency, since it piggybacks on the release nature of stops. Either a consonant has a complete articulatory closure or it does not, so there are no implicational relations at all based on [−continuant] in non-local nasal harmony. Nor is there any reason to expect sonority-based generalizations when sonority is antagonistic to [−continuant], differing in whether there is a buildup of intraoral air pressure.

4.4.2.4.3. Parasitic asymmetries

The last asymmetry between local and non-local nasal harmony is based on parasitism. Under the present perspective both local and non-local harmony are parasitic, LNH is [sonorant]-parasitic and NLNH is [continuant]-parasitic, so the difference between local and non-local harmony is not whether they allow parasitism, but rather it is what features may combine to form multiple parasitic systems. Ngbaka is parasitic on
place and Kikongo is parasitic on voicing (see §2.2.2). None of these features are available for dependency in strictly local nasal harmony. This lack of feature preconditions is perhaps why similarity has been thought to play a limited role in spreading nasal harmony (Rose & Walker, 2004; Hansson, 2001).

In regard to parasitism, there are two explanatory dilemmas for the Attraction Framework: (i) why the reranking of correspondence constraints does not give [sonorant]-parasitic NLNH and [continuant]-parasitic LNH, which are unattested, and (ii) why there is a difference in the availability of multiply-parasitic systems; for instance, local nasal harmony is never parasitic on place, even though ATTRACT({place, [sonorant]}→[nasal]) is as formally complex as ATTRACT({place, [continuant]}→[nasal]). One solution to these dilemmas is to deny the antecedent, showing that some cases of nasal harmony are actually parasitic in predicted ways. Another solution is to argue that independent phonotactic or markedness considerations restrict the available kinds of parasitism. Both strategies are employed below.

Strictly local spreading in [continuant]-parasitic harmony is avoided because of phonotactic considerations based on the Obligatory Contour Principle (OCP; McCarthy, 1986; Frisch et. al, 2004). The OCP prohibits multiple release points from being adjacent. For this reason, cross-linguistically there is an avoidance of doubly released aspirates *[tʰh], in favor of long-closures with a single release [tʰ], and likewise, cross-linguistically there is a strong preference for NC consonant clusters to have as few releases as possible, giving the well attested nasal-place agreement in English, Italian,
and Catalan, e.g. $i[mp]ossible$, $^*i[np]ossible$ (see §2.4 for details). Thus, in order for adjacent segments to share [−continuant], they must share the same closure, but the OCP prohibits multiple adjacent release points, so unlike [sonorant], [continuant] is not available for parasitic spreading nasal harmony. However, at longer distances, there are no such prohibitions against multiple release points, so [continuant]-parasitic systems are exclusively available to NLNH.

On the other hand, the absence of [sonorant]-parasitic NLNH is somewhat illusory, as I believe there are cases that can be viewed as [sonorant]-parasitic NLNH. [sonorant]-parasitic NLNH would be confirmed by a nasal stop determining the nasality of a non-local vowel, e.g. $\text{na.k.a.ti} \rightarrow \text{na.k.a.ti}$. However, such interaction is unlikely to be attested because intervening vowels are also [sonorant]. If basic syllable structure constraints require there to be some [sonorant] segment in any sequence of significant length, then it is phonotactically impossible for the non-locality of [sonorant] parasitic harmony to be attested, since intervening resonants should also undergo harmony, e.g. $\text{na.k.a.ti} \rightarrow \text{nã.kã.î}$. Is more harmonic than $\text{na.k.a.ti} \rightarrow \text{na.k.a.î}$.101

The prediction of [sonorant]-parasitic NLNH, then, is that there should be systems where only non-sonorant segments are transparent. This is exactly the independent conclusion reached by Piggott (2003) based on a study of Barasana, where obstruents, voiceless stops and fricatives are transparent to nasal harmony. Similar phenomena are found in Tuyuca (Walker, 2000, 2003) and Moba Yoruba (§4.4.3), although for Yoruba

100 Furthermore, it is quite likely that even heterorganic NC clusters still only have one release. For me, the [n] in [l[mp]ut] is unreleased. The lack of release would explain why heterorganic NC clusters are so unstable: without a release, only limited information is conveyed about the place of the nasal.

101 True non-locality could only be found if [obstruents] could serve as the syllable nucleus as in Berber (Prince & Smolensky, 2004), e.g. $\text{n.t.x.ti} \rightarrow \text{n.t.x.ti}$, but with such radical sequences of non-sonorants, there is no reason to expect Berber representations to encode [sonority] in a way that might yield parasitic harmony.
the class of less-sonorants for Wd-nasal harmony is expanded to include all consonants (see (44) above). Thus, there probably are cases of [sonorant]-parasitic NLNH, but they do not readily reveal their non-locality because syllable phonotactics prohibit long strings of non-resonant material. Furthermore, since syllables tend to have at least one [sonorant] nucleus, NLNH and syllable-local nasal harmony are indistinguishable.

Now, I turn to the issue of which features may combine to form multiply-parasitic systems. I consider four features on which nasality may depend: place, [voice], [continuant], and [sonorant], and discuss why some feature combinations are preferred to others. The choice of features is not accidental as [nasal] is known to readily depend on place and [voice] in both consonant harmony systems (Hansson, 2001; Rose & Walker, 2004) and patterns of speech errors (Walker, 2007). The following table, in (49), summarizes the phonetic dependencies between features which might be available to form phonological dependencies for parasitic attraction. The strength of the dependency is denoted with color (lighter-color = stronger dependency) and symbols ((++) is strongest, (−−) weakest).102

\[\text{Table 49}\]

\begin{tabular}{|c|c|c|c|}
\hline
Feature & Place & Voice & Sonorant \\
\hline
Place & ++ & + & −
\hline
Voice & + & ++ & −
\hline
Sonorant & − & − & +
\hline
\end{tabular}

\[\text{Legend:}\ (++) \text{strongest, (−−) weakest}\]
(49) Pair-wise phonetic dependencies between the feature *place*, [voice], [sonorant], and [continuant].

<table>
<thead>
<tr>
<th></th>
<th>place</th>
<th>[voice]</th>
<th>[continuant]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[sonorant]</td>
<td>(+): Steady formant structure denoting resonance also indicates the place of constriction in vowels.</td>
<td>(++): Sonorous segments tend to be voiced.</td>
<td>(→) [+sonorant] = lack of intraoral pressure, [−continuant] = complete closure and therefore quick buildup of intraoral pressure</td>
</tr>
<tr>
<td>[continuant]</td>
<td>(++): Rapidly changing formants during the burst associated with the release of stops are prime indicators of consonant place.</td>
<td>(+): Cues for voicing (lie VOT) are enhanced by the burst associated with a release.</td>
<td></td>
</tr>
<tr>
<td>[voice]</td>
<td>(0): Cues may temporally co-occur, during the release, but the cues are non-overlapping, duration for voice and formant transitions for <em>place</em>, so this is only a weak dependency.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The main point of this table is to demonstrate that not all of the possible dependencies are of equal strength, so even though doubly-parasitic attraction constraints, at the level of OT, of the form $\text{ATTRACT}(\{p_1, p_2\}, q)$ are all of equal formal complexity, there is no reason in the Attraction Framework to think that they are all of equal typological likelihood. Subsymbolic properties predict which constraints are likely to join forces in a dependency. In particular, those features that are similar, i.e. that have a common phonetic correlate, are expected to form the strongest dependencies. §3.4 discusses how this similarity sensitive dependency derives from the operation of entailments among subfeatures. This work predicts that only the stronger dependencies are expected to yield attested cases of parasitic harmony, the weaker cases may not occur at all.

Therefore, if the Attraction Framework with its sensitivity to such subsymbolic information is on the right track, then based on (49), cross-linguistically, the following multiply-parasitic systems are expected to exist: $\{\text{[sonorant], place}\}$, $\{\text{[sonorant], voice}\}$,
([continuant], *place*), and {([continuant], [voice]). As the table in (50) below indicates, there are attested cases that are consistent with each parasitic analysis.

(50) Multiply-parasitic nasal harmony; \textsc{Attract}({\{p_1, p_2\}→[nasal]}) driving harmony.

<table>
<thead>
<tr>
<th>$p_1$</th>
<th>$p_2$</th>
<th>Attested Cases</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>[sonorant]</td>
<td><em>place</em></td>
<td>Moba, Yoruba</td>
<td>When <em>place</em> is associated with the resonance associated with sonority, it has much in common with vowel-place. High vowels, glides, and liquids participate in nasal harmony. Other segments including nasal stops are transparent. This arises from the lower F1 associated with high vowels, glides, and the nasal murmur.</td>
</tr>
<tr>
<td>[sonorant]</td>
<td>[voice]</td>
<td>Tuyuca, Barasana</td>
<td>Voiceless segments, which are only fricatives and stops, are transparent. There are no voiced fricatives in either language.</td>
</tr>
<tr>
<td>[continuant]</td>
<td><em>place</em></td>
<td>Ngbaka, Ganda</td>
<td>Homorganic stops must agree with preceding nasal stops. For Ngbaka, this prevents sequences like *[m] … [&quot;b] and *[&quot;b] … [b].</td>
</tr>
<tr>
<td>[continuant]</td>
<td>[voice]</td>
<td>Kikongo, Yaka</td>
<td>Only voiced stops and liquids undergo harmony. [voice], [continuant] better characterizes the class of participants than either [voice] (which would include voiced fricatives) or [continuant] alone (which would include voiceless stops).</td>
</tr>
</tbody>
</table>

The dependencies predicted to be avoided are \textsc{Attract}({[sonorant], [continuant]}) →[nasal]) and \textsc{Attract}({*place*, [voice]}→[nasal])). The former consists of an overtly antagonistic dependency, so it is universally low-ranked. The later is presumably too weak to drive harmony, because it ignores the elsewhere essential qualities of [sonorant] or [continuant], which is to say that [sonorant] and [continuant] count more for nasal conditional similarity than *place* and [voice] combined. Consistent with the prediction of weak dependencies, I know of no languages where \textsc{Attract}({[sonorant], [continuant]}) →[nasal]) or \textsc{Attract}({*place*, [voice]}→[nasal])) could be said to be active in harmony.

This example illustrates how typological restrictiveness in the present parasitic perspective on nasal harmony follows from understanding the phonetic relationships between features. Thus, attraction theory critically relies on the availability of low-level,
subfeatural phonetic information, which if it were not somehow available to language learners (either synchronically or diachronically) would allow for the free reranking of Attraction constraints, giving unattested systems. The problem of unattested constraints of similar formal complexity is not unique to attraction. *ONSET is presumably as complex as NOCODA, yet only the latter seems to be readily active (although see Boersma, 2009 for indications of the potential value of CODA and *ONSET). Unlike traditional OT, using low-level phonetic detail, the Attraction Framework has the unique ability to predict which constraints are likely to be active.

Returning to issues of asymmetry, all of the phenomena described in (50) are σ-local or non-local, so there is still room for phonotactics in describing the absence of multiply-parasitic harmony in spreading nasal harmony. First, like singly parasitic \{[continuant]\} harmony, doubly parasitic\{[continuant], place\} and \{[continuant], [voice]\} are avoided because of the OCP restrictions on adjacent releases previously mentioned. Second, the difference between \{[sonorant], [voice]\} and \{[sonorant]\} is that the later allows participation by voiceless resonants, while the former prohibits it. However, voicelessness is cross-linguistically marked on sonorants, including nasals, as the lack of intraoral pressure lends to a transglottal pressure drop that facilitates voicing. Thus, it would take a very unique inventory, which has voiceless vowels, glides, or liquids, and independent nasal harmony to really test whether \{[sonorant], [voice]\} does not apply to LNH.\footnote{The same might be said of non-local \{[sonorant], [voice]\} nasal harmony. Tuyuca (Walker, 2000, 2003) provides the only example of a contrast within a sonority class where voicing determines a contrast in nasal harmony participation (voiced obstruents undergo harmony, voiceless obstruents are transparent).} Aside from cases where non-resonant voiceless stops and fricatives do not undergo harmony, but voiced stops do undergo harmony, I know of no system
where a voicing contrast yields a contrast in undergoing nasal harmony. It is not clear whether this is because agreement on [sonorant] is driving most of the attraction or because [voice] contrasts on resonants are too marked to allow for the low-ranking of faithfulness which gives rise to nasal harmony.

Lastly, as indicated in the notes in (50), [[sonorant], place] really expresses a dependency akin to vowel-place, as the formant structure indicating the place of vocalic constriction is facilitated by the resonance associated with sonority. In fact, there are no [sonorant]-parasitic nasal harmony systems among consonants, and [sonorant]-parasitic harmony always requires the participation of vowels. Therefore, the reason there is no strictly local nasal harmony parasitic on [[sonorant], V-place] is because long sequences of vowels are marked. Formally, this can also be expressed using the OCP or syllable theory constraints like ONSET.

In sum, the availability of subfeatural aspects of features together with independently needed phonotactic restrictions, like the OCP and syllable structure, explains all of the locality-based asymmetries regarding parasitism.

4.4.2.5. Summary & Conclusions

This section argued that similarity and parasitism play a stronger role in spreading nasal harmony than previously anticipated (Rose & Walker, 2004; Hansson, 2001). This section has shown how the availability of the acoustic correlates of distinctive features allows for a parasitic description of nasal harmony. While there are a few individual exceptions, local nasal harmony was found to be parasitic on [sonorant], while non-local nasal harmony was found to be parasitic on [continuant].
This parasitic division of labor explained a number of typological asymmetries between LNH and NLNH involving obstruents, sonority, and multiple-parasitism. Note that under the previous non-unified account such asymmetries are seen as accidental, different formalisms with different properties deriving each. However, under attraction, these differences derive from the formalisms’ sensitivity to distinct aspects of subsymbolic representations of nasality, [sonorant] and [continuant]. For instance, a typology of doubly-parasitic systems was described with the finding that only those features which have similar correlates combine to form multiply-parasitic systems. The Attraction Framework is sensitive to these similarities, deriving why some attraction constraints are typologically more active than others. Furthermore, [sonorant]-parasitic harmony explains why LNH always targets vowels and may also target [sonorant]-consonants, while [continuant]-parasitic harmony explains why NLNH preferentially targets consonants.

This study of similarity in nasal harmony also shows that languages are somewhat free to determine the strength of individual dependencies. For example, a language may exploit [sonorant] but not [continuant] for nasal harmony. However, nasal harmony also shows that the formation of dependency is always phonetically grounded. Languages never take two abstract, unrelated features and form a dependency for parasitic harmony. For instance, there are no [+continuant]-parasitic nasal harmony systems, and not coincidentally, nasality and [+continuant] are known to be aerodynamically opposed (Ohala & Ohala, 1993; Baković, 2007) because the pressure required for turbulent noise is lost through the nasal pathway. Crucially, cases where fricatives do undergo harmony, as in Applecross Gaelic and Tuyuca (Walker, 2000, 2003), do so based on an appeal to
the sonority properties of fricatives not because of their continuancy. In sum, some aspects of similarity may be given more importance than others (e.g. [sonorant] over [continuant]), but similarity cannot be created where there is no common subsymbolic correlate (e.g. fricatives and nasals). Acoustic subfeatures provide a vernacular for expressing and computing such similarity relationships.

4.4.3. Parasitic long-distance vowel agreement in Moba Yoruba

Ajíbóyè & Pulleyblank (ms, 2008) also provide arguments in favor of a more prominent role for similarity in nasal harmony based on the case of Moba Yoruba, which is considered in this section. Earlier descriptions of this pattern can be found in Ajíbóyè (2002), Pulleyblank (2002), and Archangeli & Pulleyblank (2007). The case of Moba Yoruba is critical for at least three reasons:

1) Because nasal consonants are transparent, Moba Yoruba is incompatible with even a covert spreading account, so generic autosegmental representations are insufficient.

2) Moba Yoruba provides another solid case of long-distance vowel agreement (LDVA), which like Finnish (§4.3.4), parallels phenomena that have been previously thought to be by in large exclusive to consonant harmony (Hansson, 2001; Rose & Walker, 2004).

---

104 Based on this earlier work, which did not mention the importance of similarity for understanding the phenomenon, the phonetic basis of the analysis was developed independently from Ajíbóyè & Pulleyblank(ms, 2008). In the course of developing this dissertation, I became aware of the unpublished paper by Ajíbóyè & Pulleyblank (ms, 2008), which greatly improved the present analysis with the additional data and insights related to domains of harmony.  

105 Although see Hansson (2007) and Walker (2009a) for indications that the long-distance nature of LDCA may be necessary for some vowel harmony processes.
3) Moba Yoruba is a case of parasitic harmony, where the availability of an acoustic subfeature related to nasal resonance, a spectral peak in the low frequency range (<350 Hz), exactly predicts the set of trigger-target pairs and the neutrality of non-sonorant consonants and non-high, oral vowels.

The first and second points are adroitly argued for in Archangeli & Pulleyblank (2007) and Ajíbóyè & Pulleyblank (ms, 2008), so there is consensus between Ajíbóyè & Pulleyblank and the present work on many fronts, including, the trade-off between similarity and proximity, the inadequacy of a formal dichotomy between local and non-local harmony, and the usefulness of exploiting sonority-based similarity in Moba Yoruba. However, the third point represents a significant divergence from Ajíbóyè & Pulleyblank (ms, 2008), who argue that similarity in regards to sonorancy and moraic properties is important for Moba Yoruba. This section argues in favor of this parasitic perspective, postponing issues related to the deficiencies of autosegmental representations until §4.5.

Now, the present analysis could be seen as an attempt to ground Ajíbóyè & Pulleyblank’s similarity claim in phonetic relationships, but in the context of the subsymbolic framework that I am arguing for, the standard nasal compatibility scales, e.g. *NASOBSSTOP >> *NASFRICATIVE >> *NASLIQUID>>*NASGLIDE >>*NASVOWEL (Walker, 2000, 2003; Piggott, 2003), and *MID/NAS >> *LOW/NAS >> *HI/Nas (Ajíbóyè

---

106 Ajíbóyè & Pulleyblank (ms, 2008) also explain a number of locality differences between Standard and Moba Yoruba, suggesting that Standard Yoruba has strictly σ-local, sonorant-parasitic nasal harmony, but those locality conditions are relaxed in Moba Yoruba; these differences are not explored here.

107 Cole & Kisseberth (1995) raise similar points based on Terena where nasals are also transparent to harmony, though unlike Moba Yoruba, there are quite possibly significant morphological forces at work in Terena (Piggott, 1997), so while it is not a case of LDVA, it is still problematic for traditional autosegmental spreading.
& Pulleyblank, ms, 2008) should not only be grounded, but wherever possible compatibility should be replaced with subfeature similarity because it has more explanatory power. Thus, the main difference between Ajíbóyè & Pulleyblank and the analysis presented, here, is that the parasitic nature of the pattern is predicted from the acoustic properties of representations, rather than stipulated as part of the constraints.

For Moba Yoruba, a single phonetic fact explains which segments are triggers, which segments are targets, and which segments are neutral. A markedness scale only makes such predictions indirectly through the ranking of a scale and independent constraints. The insufficiency of segmental markedness was confirmed in the cases of parasitic vowel harmony (see §4.3), where even structure preserving harmony (Kiparsky, 1985) requires additional machinery to explain the trigger-target pairs. Hence, grounding a scale can explain why the markedness constraints are ranked in a particular way, but that grounding still only has indirect application to independent, freely-ranked harmony constraints. With scales, predictions about triggers, targets, and neutral segments are decoupled, risking that any relationships between predictions about triggers, targets, and neutral segments are accidental. On the other hand, as I show below, under Attraction, feature preconditions and harmonic features are bound together based on their subsymbolic similarity, so similarity gives a scale for harmony.

Moba Yoruba nasal harmony provides an extremely illustrative case in this regard. The analysis proceeds as follows §4.4.3.1 presents the facts to be explained, §4.4.3.2 argues for a subsymbolic solution, and §4.4.3.3 concludes.
4.4.3.1. Facts of Moba Yoruba nasal harmony

Like Standard Yoruba, the Moba dialect of Yoruba has twelve vowels: seven oral \{i u e o e a\} and three nasals \{i̥ u̥ ḁ\}. No nasal mid vowels are permitted in roots or in derived environments. Unlike Standard Yoruba, where nasal harmony is confined to a syllable, in Moba Yoruba, all nasal vowels trigger harmony on vowels in a preceding syllable. This is shown with the examples in (51), below:

(51) *All nasal vowels, [i̥], [u̥], and [ḁ], trigger harmony:*

a.  u̥r̥i  *ur̥i*  ‘iron’
b.  i̥w̥i  *uw̥i*  ‘spirit’
c.  ūj̥a  *ūj̥a*  ‘famine’
d.  ũj̥a  *ij̥a*  ‘pounded yam’
e.  ũf̥u  *if̥u, *i̥f̥u*  ‘intestine’
f.  ũg̥u  *ug̥u, *i̥g̥u*  ‘corner (of a house)’

Although here the pattern holds of morphemes, it is also expressed in morphologically complex environments. (51)(a-d) also illustrate that concerning consonants, liquids and glides undergo harmony, while fricatives and consonants do not (51)(e-f). It is likely the case that [n] is a nasalized allophone of [l] as in Standard Yoruba (Clements & Sonaiya, 1990), so in Moba Yoruba there is only one phonemic nasal consonant, [m], and all other nasal consonants are derived.

Below, (52) confirms that the high-targets in (51) are not accidental, as only high vowels undergo harmony. While the neutrality of mid-vowels, (52)(a-h), is not surprising given the inventory lacks nasal mid vowels, the neutrality of the low vowel, (52)(i-k), is unexpected, considering that the nasal low vowel, [ã], participates in the process as a trigger.
(52) Non-high vowels are neutral to harmony:

a. erî ‘elephant’

b. orî ‘song’

c. ogû ‘war’

d. èsî ‘reproach’

e. èrâ ‘meat’

f. erî ‘mouth’

g. źřa ‘matter’

h. ôdû ‘festival’

i. âři KIND OF GAME

j. ârû ‘disease’

k. ârâ ‘velvet’

Not only are the non-high vowels neutral, but (53), below, shows that neutral vowels block high-vowels in preceding syllables from undergoing harmony. Note tri-syllabic nasals are phonotactically legal, e.g. [îṣugbî], ‘traditional singers’, so the non-participation of the initial syllables is tied to the intervening oral, non-high vowel.

(53) Non-high vowels block nasal harmony:

a. ìrègû ‘reproaching’

b. ùrôỳî ‘news’

c. ilègû NAME OF A COMPOUND

d. ùròrû ‘peace of mind’

e. isasû KIND OF POT

f. úsàmî ‘baptism’

Finally, nasal consonants do not trigger harmony on preceding vowels, and so are transparent to harmony across syllables. This includes nasal stops as shown in (54) and (53).

(54) Nasal stops occur in nasal vowel contexts:

a. îmû ‘nose’

b. îmà ‘palm leaf’

c. ùmâlê ‘light’

- 292 -
(55) *Nasal stops occur in oral vowel contexts:*
   a. ùmoji  NAME OF A VILLAGE
   b. imélé  laziness
   c. ùmórù  PERSONAL NAME

That nasal stops do not block nasal harmony, (54), is unremarkable, what is remarkable is that nasal stops do not trigger harmony to a preceding high vowel, (55), which is otherwise a suitable target of harmony. Ajibóyè & Pulleyblank report a morpheme-level MSC, which states that within a syllable nasal consonants never occur with an oral vowel, so nasal consonant-oral vowel sequences, ČV, as in (55), are only possible, in derived environments that follow from vowel deletion and other processes.

The above facts can be summarized as follows in (56)

(56) **Summary of Moba Yoruba nasal harmony:**
   a. **Triggers:** All nasal vowels, {i̯, ù, ā}, trigger nasal harmony on preceding vowels.
   b. **Targets:** All high vowels, {i, u}, glides, {j, w} and liquids, {r, l}, undergo harmony to a following nasal trigger.
   c. **Non-triggers:** Nasal consonants, {j̯, ̯w, ŋ̣, n, m}, do not trigger harmony on preceding vowels.

(60 continued)
   d. **Neutrals:** All non-high vowels, {e, o, ɛ, ɔ, ə, a} and non-sonorant consonants, {b, f, t, d, s, ʃ, k, ɡ, kp, gb, h}, do not undergo harmony to a preceding or following nasal trigger.
   e. **Locality:** An adjacent-syllable precondition, allows neutral C’s to be transparent, but requires neutral V’s to be blockers.
4.4.3.2. Analysis

This section argues for a [low frequency formant]-parasitic harmony of nasal harmony in Moba Yoruba. As previously discussed in §4.4.2.2, nasal resonance is associated with a nasal murmur, a broad, low frequency formant in the 250-300 Hz range, so it is plausible to hypothesize that the feature [nasal] has an acoustic subfeature [low frequency formant]. Because 250-300 Hz is also the range of the peak of the first formant (F1) on oral high vowels, it is, therefore, not accidental that only high vowels undergo harmony. The asymmetry between high and non-high vowels may be described as a difference in representation: only high vowels are specified for the subfeature [low frequency formant]. In Moba Yoruba, segments which agree on having this low frequency formant are under attraction pressure to also agree in nasality. Formally, this can be instantiated in the attraction constraint \( \text{ATTRACT}([\text{low frequency formant}] \rightarrow [\text{nasal}]) \), and so, like Finnish, §4.3.4, Moba Yoruba exemplifies a case where harmony shows a direct low-level interaction between acoustics, [low frequency formant], and articulation, [nasal].

Now, there is only a limited amount of phonetics research on vowel harmony in Moba Yoruba. However a study by Przezdziecki (2005), while not specifically looking at nasality, did show that F1 dependencies are important for [ATR] harmony, so there is an independent reason to believe that F1 and therefore the associated subfeature [low frequency formant] is playing an active role in the harmony processes of Moba Yoruba.

For four Moba speakers, Przezdziecki reports the following average F1 values for the first oral vowel in \( V_1CV_2 \) sequences. \( V_1 \) is a position in which nasal harmony obtains:
(57) Mean F1 of vowels in the target position for speakers of Moba Yoruba (from Przezdziecki, 2005):

<table>
<thead>
<tr>
<th>Speaker</th>
<th>High</th>
<th>Upper-mid</th>
<th>Lower-mid</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[i]</td>
<td>[e]</td>
<td>[ɛ]</td>
<td>[a]</td>
</tr>
<tr>
<td></td>
<td>[u]</td>
<td>[o]</td>
<td>[ɔ]</td>
<td></td>
</tr>
<tr>
<td>Mb5</td>
<td>276</td>
<td>387</td>
<td>509</td>
<td>702</td>
</tr>
<tr>
<td>Mb8</td>
<td>273</td>
<td>378</td>
<td>528</td>
<td>713</td>
</tr>
<tr>
<td>Mb9</td>
<td>331</td>
<td>380</td>
<td>485</td>
<td>615</td>
</tr>
<tr>
<td>Mb10</td>
<td>315</td>
<td>369</td>
<td>508</td>
<td>666</td>
</tr>
</tbody>
</table>

(57) confirms that across all speakers only the high vowels have a mean F1 in the range of the nasal murmur. Note the nasal murmur is of broad enough bandwidth to somewhat overlap with the F1 of mid-vowels, especially upper-mids, but not so broad as to overlap with the F1 of low vowels. Therefore, the reason nasal low vowels, [â], trigger harmony is being nasal they have a distinct pole denoting the low frequency nasal murmur. In contrast, oral low vowels, [a], and mid vowels, [e], [o]. [ɛ], and [ɔ], do not undergo harmony because they lack a formant peak in the lowest-frequency range. Exactly determining the category boundary would take additional phonetic work that is beyond the scope of this present work, but a formant peak boundary of around 350Hz would separate the mean F1 of high vowels for all speakers from the mean F1 of all non-high vowels for all speakers (see (57)).

This murmur-parasitic analysis makes a different prediction than Ajíbóyè & Pulleyblank’s scale for nasal vowels, *Mid/Nas >> *Low/Nas >> *Hi/Nas: mid-vowels should be more likely than low-vowels to undergo nasal harmony, i.e. for the purposes of harmony, *Low/Nas >> *Mid/Nas. This follows from the F1 of mid vowels being more like the nasal murmur than the F1 for low vowels. Unfortunately, this hypothesis cannot

---

108 In the case of [â], the “first” lowest frequency formant is the nasal murmur, 250-300 Hz, so F1, here, denotes the first formant due to the constriction in the oral cavity. For [a], that is in the 650 -750 Hz range. Thus, although not of equal amplitude, [â] has two spectral peaks less than 1000Hz. Also, it is expected that the zeros from nasality can have a diverse effect on F1: the F1 on [i] and [u] is somewhat higher than [i] and [u], while the F1 for [â] is somewhat lower than [a] (Johnson, 2004).
be tested in Yoruba since it lack nasal vowels. However, if true, this would bring nasal vowel harmony more in line with ATR vowel harmony and other systems, where cross high-low harmony without mid-participation is strongly avoided, unless – like in Moba Yoruba, which prohibits mid-nasal vowels – there are no mid-vowels in the inventory which carry the harmonic feature. The reason mid-nasal vowels tend to be avoided in phonological inventories is that the nasal murmur interferes with the perceptions of height contrasts (Johnson, 2004), effectively shrinking the height space, so smaller contrasts in height in nasal vowels are not as stable, as the same contrasts in oral vowels.

Lacking specific phonetic studies of [j]~[j], [w]~[w̃], [r]~[r̃], and [l]~[n], it is impossible to know the precise F1 for these Moba Yoruba segments. However, Stevens (2000) reports that both glides and liquids have a low frequency formant, although for liquids, on average, it is somewhat higher, 300-400 Hz., than for the glides, [j] and [w], which for English have a pole in the 250-350 Hz. range. Thus, [low frequency formant]-parasitic harmony also explains why these sonorant consonants are targets of harmony.

Other neutral consonants are not sonorous enough to have much resonance at all, much less a specific kind of resonant structure in the low frequencies, so the set of neutral segments includes non-high vowels and obstruents (see (56) for a complete list). Without an appeal to low-level detail, this class of neutral segments would be an ad hoc, unnatural class, but with an understanding of the phonetics of nasality, this class may be succinctly described as the set of segments which lack a low frequency formant. Note, however, that non-high vowels and obstruents lack the low frequency formant for different reasons: non-high vowels lack formant structure in the right frequency range, while obstruents do not have much formant structure at all. Thus, [low frequency formant]-parasitism exactly
explains the set of triggers (segments which have a low frequency formant caused by nasalization), the set of targets (segments which have a low frequency formant not caused by nasalization are under attraction pressure to be nasalized), and the set of neutral segments (segments which do not have a low frequency formant: non-low vowels and obstruents). All of the above are predicted by a harmony driver of the form, \( \text{ATTRACT}([\text{low frequency formant}] \rightarrow \text{nasal}) \).

There is, however, one important outlier to the above analysis: nasal stops have a nasal murmur during the closure period, so if [low frequency formant] were the only feature precondition, then, that predicts that nasal stops should trigger nasal harmony. Nevertheless, the evidence suggests that nasal stops never trigger harmony on a preceding vowel within morphemes or in derived environments.\(^{109}\) To resolve this conflict, I posit that harmony must be parasitic on a measure of sonority, where only those segments more sonorous than nasal consonants participate in right-to-left harmony.

However, as noted by Ajibóyè & Pulleyblank, [m\(V\)\(+hi\)] sequences are strongly avoided, suggesting nasal consonants can trigger left-to-right harmony within the same syllable.\(^{110}\) This complicates a correspondence account because there is a directionality

\(^{109}\) Ajibóyè & Pulleyblank present data which in derived, morphologically complex, environments shows that Moba allows [\(V[+hi],N\)] sequences, see (55), so nasal stops never trigger nasal harmony on vowels in a preceding syllable.

\(^{110}\) In general, the *\([\tilde{C}V\text{oral}]\ast\) MSC prevents such harmony from being visible productively in morphemes, but in vowel deletion environments there is evidence both for and against triggering by nasal consonants. In casual speech, given, [m\(V\)\(_1,V\)\(_2\)], either \(V\)\(_1\) or \(V\)\(_2\) may be deleted. Of relevant interest are cases where \(V\)\(_2\) is high. If \(\tilde{V}\)\(_1\) = [\(\tilde{a}\)], then only \(V\)\(_2\) deletion is allowed, even though [m\(V\)\(_2\)] is phonotactically unmarked. One explanation for this blocking of deletion follows from the hypothesis that [m] cannot trigger harmony, so the marked \(\tilde{C}V\)\(_2\) would be the only possible result of \(V\)\(_1\) deletion. As in (55), if \(V\)\(_2\) is not high, then \(\tilde{C}V\)\(_2\) sequences are allowed.

On the other hand, when \(V\)\(_1\) and \(V\)\(_2\) are both high, most attested forms tend to have the same place of articulation for \(V\)\(_1\) and \(V\)\(_2\), so the end result is derivable by either (i) \(\tilde{V}\)\(_1\) deletion followed by nasal spreading \(V\)\(_2\) ([m.i.i] \(\rightarrow [m] \rightarrow [m\tilde{i}]\)) or (ii) only \(V\)\(_2\) deletes ([m.i.i] \(\rightarrow [m\tilde{i}]\)). The only form presented by Ajibóyè & Pulleyblank, where \(\tilde{V}\)\(_2\) and \(V\)\(_2\) are not identical in place does express a contrast based on deletion, resulting in either [m\(V\)\(_1\)] or [m\(V\)\(_2\)], [m\(\tilde{u}\) i\(g\)i] \(\rightarrow [m\tilde{u}\tilde{g}\tilde{i}]\) OR [m\(\tilde{u}\) i\(g\)i] \(\rightarrow [m\tilde{u}\tilde{g}\tilde{i}]\) ‘take stick’, the latter suggesting that [nasal] can spread from [m] when \(V\)\(_1\) deletes.
asymmetry in the data that cannot be expressed with unitary correspondence. The
problem is explained as follows: for vowel harmony with consonantal transparency,
there must be a $L \rightarrow R$, non-local correspondence $V_x.CV_x$. The constraint that permits that
correspondence, $\text{CORR}_{L \rightarrow R}((i)_{i+1})$, also requires the existence of the strictly local
correspondence $V_y.C_y.V$. For there to be harmony within a syllable, the $R \rightarrow L$
correspondence $V.C_y.V_x$ must exist. Hence, the dilemma is that $[mV[i+hi]]$ sequences
harmonize, while $[V_{hi}.m]$ sequences do not, even though there are identical similarity
conditions and correspondence in both cases. Thus, if, as in this case, both a $L \rightarrow R$
(within a syllable) and a $R \rightarrow L$ (between syllables) correspondence are required, then
$R \rightarrow L$ triggering implies $L \rightarrow R$ triggering. This is because ATTRACTION constraints are
blind to the directionality of correspondence; they are merely sensitive to the existence of
some correspondence.

Thus, the only remedy seems be to make a direct appeal to syllable structure.
Here, I follow Ajibóyè & Pulleyblank’s strategy, which is to propose two separate
processes for nasal harmony, one describing harmony within a syllable for which all
sonorant segments are triggers and targets, and another describing harmony between
syllables for which only vowels are triggers and targets. This domain-centric solution
could be imported into segment-to-segment correspondence theory by suggesting there
are different kinds of correspondence constraints for different domains, and each domain
correspondence has its own attraction constraints that operate independently of other
domains. In this way, within-syllable harmony could have different properties than
between-syllable harmony. For Moba Yoruba, what is needed is some correspondence

\footnote{In any case, $[mV[i+hi]]$ sequences never occur. It is not clear to what extent this pattern generalizes to other nasal consonants.}
between segments within a syllable and another more general correspondence between segments within a word.

(58), below, shows the desired structure that should emerge from such correspondences. The word-level correspondence is in gray, while the correspondence within syllables is in black. Note there is no σ-correspondence between [u] and [m] even though they are adjacent, nor is there any left-to-right Wd-correspondence between [u] and [i].

(58) Desired structure to predict that in $V_1.mV_2$ sequences $V_2$ directly triggers harmony on both $V_1$ and [m], while [m] only triggers harmony on $V_2$. 

The relevant correspondence constraints are, therefore, (i) $Wd$-$CORR_{R\rightarrow L}(\sigma_i\sigma_{i+1})$, which requires a correspondence between segments in word at no greater distance than adjacent syllables in a right-to-left fashion and (ii) $\sigma$-$CORR(X_iX_{i+1})$ which requires a correspondence between segments in the same syllable in both directions.

In such a system, σ-correspondence could have different attraction properties than Wd-correspondence, and this is clearly what is needed since [m] triggers σ-harmony, but not Wd-harmony. However, for Moba Yoruba, both of the relevant ATTRACTION constraints depend on [low frequency formant]: $\sigma$-$ATTRACT([\text{low frequency} \ldots \text{low frequency} \ldots \text{low frequency}])$.
formant]) → [nasal]) and WD-ATTRACT([low frequency formant], [cons]) → [nasal]). An additional [cons] precondition on WD-ATTRACT is needed because, otherwise, Wd-correspondences would require harmony between nasal consonants and preceding high vowels. From the similarity perspective, only segments which agree on [cons] and [low frequency formant] are similar enough to trigger non-local harmony, while the conditions within a syllable are relaxed to a precondition of only [low frequency formant]. This includes nasal stops, which while sonorant enough to trigger harmony within a syllable, are fully-transparent to the word level harmony process.

To trigger harmony, both the correspondence constraint and the attraction constraint must dominate faithfulness, so the ranking { WD-CORR_R→L(σ_iσ_{i+1}), WD-ATTRACT([low frequency formant], [cons]) → [nasal]), σ-CORR(X_iX_{i+1}), σ-ATTRACT([low frequency formant]) → [nasal]) } >> IO-FAITH([nasal]) suffices. These five constraints fully describe the nasal harmony patterns in Moba Yoruba. Like Ajíbóyè & Pulleyblank’s solution similarity and domains play a prominent role in this analysis, but unlike Ajíbóyè & Pulleyblank’s analysis there is no role for compatibility scales, which are subsumed by [low frequency formant]-parasitism in both within syllable and word level harmony.

The tableau, below in (59), shows how the above ranking gives triggering by [â] on vowels through Wd-harmony, and on [low frequency formant] consonants through σ-harmony. Here and elsewhere, correspondence is color coded: Wd-correspondences are in grey and σ-correspondences are in black.

---

111 The differences between [cons] and syllable structure features, like [syllabic] or the [moraic] used by Ajíbóyè & Pulleyblank, are not critical to the analysis, since any of the above features exclude nasal stops from being triggers of Word Harmony. Here, glides behave as consonants so they are classified as such.
Nasal harmony triggered by [â]: \[\text{i̇j̄â} \rightarrow \text{̃i̇j̄â} \text{`pounded yam'}\]

\[\begin{array}{|c|c|c|c|c|}
\hline
\text{ij̄â} & \text{̃i̇j̄â} & \text{WD-CORR}_{\sigma, \sigma_{i,1}} & \text{WD-ATRACT} (\{\text{l.f.f.}, \text{cons}\} \rightarrow \text{nasal}) & \text{IO-FAITH} (\text{nasal}) \\
\hline
\text{a.} & \text{i̇j̄} & \text{!i\rightarrowi,} & \text{*(a→i)} & \text{*(j, i)} \\
\hline
\text{b.} & \text{i̇j̄} & \text{!*i\rightarrowa,} & \text{*(a→i)} & \text{*(i̇j̄)} \\
\hline
\text{c.} & \text{i̇j̄} & \text{!*a→j,} & \text{*(a→i)} & \text{*(i̇j̄)} \\
\hline
\text{d.} & \text{i̇j̄} & \text{!*a→i} & \text{*(j)} & \text{*(i̇j̄)} \\
\hline
\text{e.} & \text{i̇j̄} & \text{*(j)} & \text{*(i̇j̄)} & \text{*(i̇j̄)} \\
\hline
\end{array}\]

Above, (59)(a-c) demonstrate that the desired correspondence structure from (58) is required by the high-ranking correspondence constraints. (59)(c-e) confirm that this correspondence structure allows attraction constraints to demand harmony between nasal vowels and adjacent sonorant consonants and between nasal vowels and non-adjacent vowels. Also note in (59)(d) that even though [i] is in correspondence with [j] it is not under attraction pressure to agree in nasality because [i] and [j] disagree in sonority.

The neutrality of non-high vowels and obstruents is illustrated in (60), below. For clarity, not all correspondences are drawn, but they are assumed, below, to exist in the form of (58), except for (60)(d), which has a long-distance correspondence between final [̃u] and initial [i]. As shown, this correspondence can only be demanded by a lower-ranked correspondence constraint, so it never wins out over IO-FAITH, and so when neutrality occurs there must be blocking (cf. (60)(a) vs. (60)(d)). As usual in the Attraction Framework, neutrality follows from a failure to meet similarity preconditions, and any nasalization of obstruents or non-high vowels is not motivated, since these segments lack a [low frequency formant].
(60) Neutrality, blocking, and transparency in Moba Yoruba: *isasu* ‘kind of pot’

<table>
<thead>
<tr>
<th>isasu → isasu</th>
<th>WD-Corrρ_{-L} σ_{iσ_i+1}</th>
<th>σ-Corr X_{X_{i+1}} →[nasal]</th>
<th>WD-ATTRACT([l.f.f.], [cons]) →[nasal])</th>
<th>σ-ATTRACT([l.f.f.]) →[nasal])</th>
<th>IO-FAITH([nasal])</th>
<th>WD-Corrρ_{-L}</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>i s a s u</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>*(u→i)</td>
</tr>
<tr>
<td>b.</td>
<td>i s a s u</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>*(u→i)</td>
</tr>
<tr>
<td>c.</td>
<td>i s a s u</td>
<td></td>
<td></td>
<td>*(ā), *(i)</td>
<td></td>
<td>*(u→i)</td>
</tr>
<tr>
<td>d.</td>
<td>i s a s u</td>
<td></td>
<td></td>
<td>*(i)</td>
<td></td>
<td>*(i)</td>
</tr>
</tbody>
</table>

Finally, I consider the hypothetical inputs */ime/ and */mu/*, and show that domain sensitivity only allows the lower similarity conditions for harmony to apply to [mi]. In (61), */ime/* does not have the output *[īmē]* because [m] does not agree with [i] on [cons] and [m] does not agree with [ɛ] on [low frequency formant], giving the non-triggering of nasal harmony on high vowels in preceding syllables and on non-high vowels in the same syllable (cf. imélé ‘laziness, (55)(b)).

(61) Neutrality of [m] to word harmony: */ime/* →*ime*

<table>
<thead>
<tr>
<th>ime → ime</th>
<th>WD-Corrρ_{-L} σ_{iσ_i+1}</th>
<th>σ-Corr X_{X_{i+1}} →[nasal]</th>
<th>WD-ATTRACT([l.f.f.], [cons]) →[nasal])</th>
<th>σ-ATTRACT([l.f.f.]) →[nasal])</th>
<th>IO-FAITH([nasal])</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>i m e</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>i m ē</td>
<td></td>
<td></td>
<td></td>
<td>*(ē)</td>
</tr>
<tr>
<td>c.</td>
<td>i m ē</td>
<td></td>
<td></td>
<td></td>
<td>*(i)</td>
</tr>
<tr>
<td>d.</td>
<td>i m ū</td>
<td></td>
<td></td>
<td></td>
<td>*(i), *(ē)</td>
</tr>
</tbody>
</table>

In contrast, (62) confirms that if a high vowel and [m] are in the same syllable then harmony must obtain.
4.4.3.3. Conclusions

This analysis has shown that the complexities involving domains and directionality effects are a non-trivial formal detail. Unfortunately, further discussion of these issues is beyond the scope of the present work. Nevertheless, the main point of the analysis is to show that parasitic harmony and the availability of phonetic correlates can supersede nasal compatibility scales providing a better similarity-based explanation.

There are unusual aspects of the Moba pattern: (i) low vowels trigger but do not undergo nasal harmony, (ii) non-high vowels pair with obstruents as neutral segments, and (iii) nasal consonants do not trigger harmony on preceding syllables, but do trigger harmony within a syllable. The first two of these asymmetries directly derive from a similarity explanation involving [low frequency formant], which is associated with the nasal murmur, while the last asymmetry also requires a role for domains and sonority: segments having a [low frequency formant] are triggers and target of σ-harmony, but, for Wd-harmony, both [low frequency formant] and highest sonority are required for participation. Nasal consonants are transparent to Wd-harmony even though they are

(62) Triggering by [m] in syllable harmony: /mu/ → [mũ] (cf. [mũ] ‘drink’).

<table>
<thead>
<tr>
<th>mu→mũ</th>
<th>WD-CORR_{R→L}</th>
<th>σ-CORR_{X_iX_{i+1}}</th>
<th>WD-ATTRACT({[l.f.f.], [cons]} → [nasal])</th>
<th>σ-ATTRACT({[l.f.f.]} → [nasal])</th>
<th>IO-FAITH ([nasal])</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td><img src="image1" alt="Image" /></td>
<td><img src="image2" alt="Image" /></td>
<td>*!(m→u)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td><img src="image3" alt="Image" /></td>
<td><img src="image4" alt="Image" /></td>
<td><img src="image5" alt="Image" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td><img src="image6" alt="Image" /></td>
<td><img src="image7" alt="Image" /></td>
<td><img src="image8" alt="Image" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td><img src="image9" alt="Image" /></td>
<td><img src="image10" alt="Image" /></td>
<td><img src="image11" alt="Image" /></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
specified as [nasal], so Moba Yoruba again confirms the role of prerequisite similarity in determining the trigger-target pairs. The differences between Wd-harmony and \( \sigma \)-harmony confirm that the lower similarity conditions only apply in the more local domain (the \( \sigma \)), providing evidence (as noted by Ajidoye \& Pulleyblank) for the typological tradeoff between proximity and feature similarity discussed in §4.2.3.

Moba Yoruba Wd-harmony also constitutes another reliable case of long-distance vowel harmony on par with consonant harmony phenomena (Rose \& Walker, 2004; Hansson, 2001), providing further the evidence that patterns of consonant and vowel harmony should be unified (see §4.2.2). The strength of the attraction analysis comes from it being sensitive to different kinds of similarity, in this case within different domains. Thus, in the Attraction Framework, being specified for the harmonic feature is not enough to trigger harmony, a segment must also agree with targets on locality, feature, and domain preconditions in order for harmony to obtain.

4.4.4. Nasality a mixed bag of C's and V's

The case of Moba Yoruba shows a language internal pattern which is also attested cross-linguistically in nasal harmony: non-local harmony is always within major consonantal class, but local harmony can be across consonantal class. This section argues that this within-class and across-class duality renders nasal harmony incompatible with both traditional autosegmental accounts (Clements \& Keyser, 1983; see Kenstowicz, 1994 for a review) that assume separate tiers for consonants and vowels and more recent feature geometric models (Sagey, 1986; Halle, 1992; Clements \& Hume, 1995; Halle, Vaux, \& Wolfe, 2000) and other systems (Archangeli \& Pulleyblank, 1994; Padgett, 2001) that assume nasality links directly to the root (§4.4.4.1). While the Attraction
Framework’s richer representations avoid these problems, 4.4.4.2, considers and proposes solutions for another dilemma, the so-called “mirror pathology” in which CC-harmony predicts VV-harmony.

4.4.4.1. Inflexibility of autosegmental representations

The preceding discussion, in §4.4.3, shows how Moba Yoruba simultaneously requires C-V interaction in σ-harmony and no C-V interaction in Wd-harmony, so in Moba Yoruba consonants are sometimes, but not always, triggers and targets of nasal harmony. Linking nasality becomes a paradox in these situations because root-linkage is needed for the C-V interaction, but root-linkage must be avoided for strictly V-V or C-C interaction. The paradox is that Moba Yoruba needs both.

Some early autosegmental representations assumed inviolably separate C-V tiers (Clements & Keyser, 1983; see Kenstowicz, 1994 for a review). For Moba Yoruba separate tiers naturally explain the transparency of C’s in Wd-harmony, but fails to explain C-V interaction in σ-harmony because there is no way for nasality to spread from a consonant to a vowel. Thus, segment-to-segment dependency in σ-harmony does not support having separate nasal tiers for consonants and vowels.

When this skeletal structure is abandoned, in favor of root linkage of nasality, as common in feature geometric approaches (Sagey, 1986; Halle, 1992; Clements & Hume, 1995; Halle et al., 2000; Padgett, 2001), then C-V interaction is permitted, but universally so. For example, the spreading of nasality in a form like [īmū] ‘nose’, which marks oral-initial high vowels across a nasal stop, *[imū] can only follow from spreading from the nasal stop, since spreading directly from [ū] violates the constraint against lines crossing, as shown in (63).
Crucially the [m] in [imű] must be specified as [nasal], since nasality is contrastive among labial stops and because [m] is a source of σ-harmony, so no underspecification or covert harmony solutions are available. Given then, that an intervening nasal consonant must be the source of nasality on preceding high vowels, then [m] should always trigger harmony. However, as discussed above, a preceding high vowel can be either oral or nasal in the context of a subsequent [m], see (55), so [m] is not a trigger of Wd-harmony. Thus, the segment-to-segment dependency in Moba Yoruba Wd-nasal harmony does not support root-linkage of nasality.

The dilemma is really that assuming separate C-V tiers for [nasal] fails to predict certain attested interactions between consonants and vowels, while assuming root-linkage for [nasal] fails to predict certain non-interactions between consonants and vowels. Feature tiers are not flexible enough to have both interactions and non-interactions between consonants and vowels. Evidently in nasal harmony, there are instances where consonants and vowels are on the same nasal feature tier, allowing harmony to spread freely from nasal consonants to oral vowels and vice versa. There are other instances where consonants and vowels are not on the same nasal feature tier, even when both are specified as [nasal]!}

---

112 This particular representation also violates the OCP. Note [m] must be linked with a [nasal] feature, since it is contrastive, so even if harmony spreads by delinking the nasal specification for [m] and then relinking [m] with [ĩ], then harmony can still spread from [m] to [i]. On an autosegmental spreading account, there is no way for [imũ] and [imɛ] to give different results because of strict locality of spreading.
Moba Yoruba was challenging for the correspondence solution because these needs were simultaneously present in different domains, but the Attraction Framework has the flexibility to allow for both interactions and non-interactions as needed. Non-interactions derive from increasing similarity preconditions (adding [cons] to the set of prerequisite features), interactions come from lowering similarity preconditions (removing [cons] from the set of prerequisite features). It should be clear that assuming a single universal tier structure is doomed to failure when confronted with paradoxical linking systems, like Moba Yoruba.113

This same linking paradox is also present across nasal harmony languages. Cross-linguistically, only C-C interaction, only V-V interaction, and C-V interaction are each attested in nasal harmony in individual languages: cases of non-local nasal harmony (NLNH) usually involve just consonants, as shown in (44)-(45). As mentioned, in Ngbaka, nasal vowels are transparent to C-C nasal harmony, confirming that this interaction is truly non-local (Rose & Walker, 2004). Likewise, nasal consonants are transparent to the V-V nasal harmony at the level of words in Moba Yoruba, so Moba Yoruba also represents a case of truly non-local interaction (Ajíbóyè & Pulleyblank, ms, 2008). On the other hand, cases of local nasal harmony (LNH) usually involve the participation of both consonants and vowels, as in (43)-(44). Examples of C-to-V nasal harmony include Jahore Malay, Sundanese, Applecross Gaelic, Barasana, Tuyuca, Ikwere, and others (Walker, 2000, 2003; Piggott, 1992, 2003; Clements, 2003). Hence, an inflexible tier structure is likewise found wanting in terms of the typology of nasal harmony. Now, linking paradoxes are found for other feature besides nasality, some of

113 Cf. Clements (2001), who argues that feature geometry delimits the set of dependencies, but languages differ in which of those dependencies are active.
which are discussed in §4.5, but because [nasal] is readily carried by both consonants and vowels, it highlights the problem without worrying about how vowel features might be expressed on consonants and vice versa.

Of course, there are more sophisticated solutions to these linking paradoxes, which become available if one assumes a more complicated geometry in which there are separate nodes for C-Nasal and V-Nasal with a dominance relationship between them, like the more conventional C-Place and V-Place (c.f. Ní Chiosáin & Padgett, 1993; Clements & Hume, 1995; Clements, 2001, 2006; Morén, 2006). These node-subdivision strategies are discussed more fully in §4.5. However, I will argue that there are conceptual reasons why similarity ought to subsume these node subdivisions.

For now, it suffices to note that if C’s and V’s each have their own [nasal] feature tier, then the only C-C and only V-V interaction is predicted, but C-V interaction is impossible. However, if there is a [nasal] tier which links to both consonants and vowels then there is an explanation for C-V interaction, but errantly predicts that there should always be C-V interaction. Neither case is true for all cases of nasal harmony, providing additional evidence that representations are constructed on a language specific basis.

4.4.4.2. The Mirror Pathology

Turning momentarily from the issues of how the Attraction Framework relates to Autosegmental Phonology and Feature Geometry, this section is an important technical aside that resolves a pathology involving CC-harmony predicting VV-harmony, suggesting that in the case of binary features, it is feature values not the features themselves that participate in attraction relationships.
The problem can be elucidated by considering a hypothetical language, which might be called Mirror-Moba. In Mirror-Moba, Wd-harmony results in [low frequency formant] consonants agreeing in nasality, so an input, like, /remi/ with non-adjacent sonorant consonants harmonizes to [ɾemĩ]. The mid-vowel is under no pressure to agree in nasality since it is not [low frequency formant]. Furthermore, suppose like in regular Moba Yoruba, in Mirror-Moba, consonants do not trigger harmony on vowels, so /u-remi/ → [uɾemĩ]. What attraction constraint would give this interaction in Mirror-Moba? It is the exact same constraint that was used in the above analysis of regular Moba Yoruba, ATTRACT({[low frequency formant], [cons]} → [nasal]), which states that segments that agree in having a [low frequency formant] and on the feature [cons] are under pressure to also agree in [nasality]. It would seem then that the analysis of Moba Yoruba is identical to Mirror-Moba, predicting that they should co-occur. Of course, in real Moba Yoruba, there is no direct CC-nasal harmony. Hence, the mirror pathology.

The mirror pathology is closely tied to the binarity of features. When a binary feature, [±F], is a precondition to harmony on [H], then the constraint ATTRACT({[F]} → [H]) implements complete parasitism. Under ATTRACT({[F]} → [H]), when two corresponding segments agree on [+F], then they must also agree on [H], and when two corresponding segments agree on [−F], then they must also agree on [H]. Note an analogous situation can arise with monovalent features if they all link to the same node, e.g. if agreement on Place is a precondition to harmony, then fully-parasitic harmony is predicted, since [labial], [coronal], and [dorsal] are generally mutually exclusive daughters of Place, so in Place parasitic nasal harmony only coronals assimilate to nasal coronals, labial to nasal labials, etc.
Fully parasitic harmony does readily occur (see example is Chapter 2) for a number of different features, and at a range of localities, but I know of no process that is fully-parasitic on [±consonantal]. It certainly is not true of nasal harmony that nasal consonants only trigger harmony on other consonants, while at the same time nasal vowels only trigger harmony on vowels. Furthermore, while there certainly are consonantal interactions in vowel harmony, they are exceptional, and I know of no instance of long-distance consonant harmony, where vowel alternations cannot be reduced to independent harmony processes, i.e. different harmony drivers with their own set of preconditions. The absence of [cons]-mirror harmony is tied to [cons] never being a fully-parasitic precondition, so there is a need for a slight reformulation of attraction constraints.

To this end, I posit that attraction constraints may be parasitic on a feature value instead of a binary feature. In this way, attraction could be limited to only consonant or only vowels as is normally the case, by being parasitic on [+cons] or [−cons], but not generally both. This constitutes separating the dependency between [+F] and [H] from a dependency between [−F] and [H], allowing each to have their own strength. Formally, this separation is achieved through splitting \( \text{ATTRACT}([F] \rightarrow [H]) \) into \( \text{ATTRACT}([+F] \rightarrow [H]) \) and \( \text{ATTRACT}([-F] \rightarrow [H]) \). This splitting could also be used to analyze cases of ‘semi-parasitic’ harmony (Hong, 1994), where only one feature value is parasitic, e.g. in Turkish vowels which agree in [+hi] also agree in rounding, but vowels that are [−hi] do not undergo rounding harmony. Higher ranked \( \text{ATTRACT}([+hi] \rightarrow [+round]) \) addresses this partial parasitism. As with all attraction
constraints, such feature value to feature value dependencies ought to be phonetically grounded.

For prerequisite [±cons], a separation may have a basis in the system of phonological contrasts, where the larger inventory of consonants discounts the similarity contribution of a single feature, predicting that on average the force of attraction between vowels, \( \text{ATTRACT}([\neg \text{cons}] \rightarrow \text{H}) \) is stronger than the force of attraction between consonants, \( \text{ATTRACT}([+\text{cons}] \rightarrow \text{H}) \). However, for Ngbaka and Kikongo (§2.2.2) only \( \text{ATTRACT}([+\text{cons}] \rightarrow \text{H}) \) is active, so this relative strength is tendential rather than absolute.

There is also an asymmetry in the kinds of harmony features for consonants and vowels. Relatively small contrasts in the advancement of the tongue root (ATR) are cross-linguistically common feature for harmony on vowels, while somewhat related [dorsal] harmony among consonants is not nearly as robust (Hansson, 2001). Likewise, coronal harmony systems which distinguish dental/alveolar obstruents from alveolopalatal obstruents are among the most common kinds of long-distance consonant agreement (Gafos, 1996; Hansson, 2001), while I know of no cases of central/peripheral harmony among front vowels without corresponding changes in height or ATR.

As with the other asymmetries discussed in this chapter, this one also benefits from phonetic grounding. It is likely the case that some features are more similar, or enhanced by, the phonetic correlates of vowels than those of consonants (and vice versa). For instance, F1 is affected by [±ATR] (Gick et al., 2006; Przedziecki, 2005), so it denotes a kind of resonance that is enhanced by vowels, but not consonants. Small deviations in the articulation of [coronal] are not as contrastive on vowels because of the
quantal nature of vocalic resonance (Stevens, 2000; Benus & Gafos, 2007), so it is not surprising that coronal harmony should only happen with [+cons].

In sum, there are good reasons both typologically and phonetically to think that in general the contrast along a harmonic feature [±H] given [+cons] is not the same as the contrast along [±H] given [−cons]. In Chapter 3, these sorts of relationships were expressed as conditional similarities. The conditional similarity of a feature H given F is how H-similar two segments are if they agree on F. The hypothesis being that the strength of attraction may be related to the conditional similarities which can in turn be grounded in phonetics. The separation strategy above is really the assumption that

\[ \text{sim}_{[H][+\text{cons}]}(x, y) \neq \text{sim}_{[H][+\text{cons}]}(x, y). \]

Allowing dependencies directly between feature values constitutes an abandonment of regular harmony spaces with their corresponding rectilinear shape, which assume \[ \text{sim}_{[H][+\text{cons}]}(x, y) = \text{sim}_{[H][+\text{cons}]}(x, y). \] The contrast between regular and irregular spaces is illustrated in the figure below in (64):

(64)  Regular vs. Irregular Harmony Spaces (2 dimensions)

a. Regular

\[
\begin{array}{c|c|c}
[F, +H] & [F, +H] \\
\hline
[F, -H] & [-F, +H] \\
\hline
[F, -H] & [-F, -H] \\
\end{array}
\]

b. Irregular

\[
\begin{array}{c|c|c}
[F, +H] & [-F, +H] \\
\hline
[F, -H] & [F, -H] \\
\end{array}
\]
For the sake of clarity, imagine that triggers are [+H] and potential targets are [−H] (attraction forces pull upwards). In (64)(a), the distance between the [+H] and [−H] harmonic feature values does not depend on the distances between agreeing on [+F] or [−F] (the vertical sides of the rectangle are the same length), so an attractor of sufficient strength to allow H-assimilation given agreement on [+F] necessarily allows H-assimilation given agreement on [−F]. In contrast, in (64)(b), the distance between [+H] and [−H] does depend on whether or not segments agree on [+F] or [−F] (the vertical sides of the trapezoid are not the same length), so H-assimilation given [+F] agreement does not imply H-assimilation given [−F] agreement. Regular harmony spaces assume that the distance along a harmonic feature is conditionally independent of which feature values triggers and targets share, so parasitic harmony in regular feature spaces is always fully parasitic. In contrast, irregular harmony spaces assume that the distance along a harmonic feature is conditionally dependent on the shared feature values, and so no mirror pathology is predicted. It should also be noted that irregular harmony spaces are strictly more powerful than regular feature spaces, since every regular feature space can be expressed as an irregular feature space, where the conditional similarities happen to be identical. Of course, like with regular feature spaces, the kinds of dependencies available in irregular feature spaces are restrained by the phonetics: only phonetically similar feature values develop strong dependencies (cf. §3.4).

This discussion shows that in order to unify consonant and vowel harmony, it becomes necessary to relax the regularity assumptions, in particular regarding [±cons] and postulate instead that similarity is computed directly between feature values.
4.4.5. Conclusions of Nasal Harmony

One major result of this section is showing the unification of consonant and vowel harmony is possible in the Attraction Framework. Another conclusion is that it is also possible to unify local and non-local harmony processes. Any remaining differences between C-harmony, V-harmony, CV-harmony, local harmony, and non-local harmony in terms of sonorancy, obstruents, blocking, transparency, or parasitism can all be explained by an appeal to independent subsymbolic factors of similarity regarding the phonetic aspects of nasality.

In this way, the Attraction Framework was shown to be highly grounded with the strength of the dependency between parasitic and harmonic features being mediated by similarity along articulatory and acoustic correlates. For instance, differences in local and non-local nasal harmony were reduced to differences in appealing to the acoustic correlates of [sonorant] and [continuant]. A phonetic aspect of nasal resonance [low frequency formant] was shown to play a dominant role in describing the sets of triggers, targets, and neutral segments in Moba Yoruba.

Moba Yoruba was also used to shown the insufficiencies of traditional autosegmental representations. Internal to Moba Yoruba, and cross-linguistically nasal harmony requires a more flexible formalism that can allow for both the participation and non-participation of consonants and vowels. Such flexibility provides strong evidence that the similarity relations between segments matter on a language specific basis.
4.5.  Recasting autosegmental feature tiers as prerequisite similarity relations

4.5.1.  Delineating undergoers and non-undergoers with similarity

Turing away from the analysis of specific cases of assimilation, this section explores more broadly how the Attraction Framework enables a perspective on assimilation related to similarity-based classification. Here, I further explore how general prerequisite similarity separates undergoers from non-undergoers with strong analogy to kernel methods in machine learning. This similarity perspective is then compared to how that classification is made in autosegmental frameworks. I conclude that autosegmental feature tiers have much less power and are ungrounded in significant ways.

4.5.1.1. GPS and kernel methods

General prerequisite similarity (GPS) is the sufficient representational similarity needed for harmony to obtain. The individual factors which make up feature similarity derive from the phonetic correlates of those features in terms of acoustic and articulatory subfeatures (§3.4). As with other constraints in OT, these factors receive different rank/weighting in individual languages. Proximity is, likewise, given language-specific importance. As shown in the cases of parasitic vowel harmony and nasal harmony, general prerequisite similarity is determined by the ranking of positional similarity constraints (CORR) and feature similarity constraints (ATTRACT) relative to faithfulness (IDENT-IO). If the preconditions determined by those rankings are met, then the similarity constraints become active and motivate harmony. In this way, using a language-specific weighting of the factors of similarity, general prerequisite similarity predicts the set of undergoing segments (based on feature similarity requirements) and the positions of interaction (based on positional similarity requirements) for a harmony.
process. Furthermore, given a trigger, general prerequisite similarity classifies the members of a phonological inventory into sets of eligible target segments and ineligible targets and into sets of eligible positions and ineligible positions. The intersection of eligible targets and eligible positions is the set of contexts where harmony obtains.

Along these lines, a grammar for harmony can be viewed as a classification or pattern completion task. This task is to classify the set of all trigger-target pairs, i.e. for a particular trigger $x$, what segments and positions are under pressure to agree with $x$? GPS plus attraction is a solution to this problem: if a target is similar enough to a trigger, then the target is under pressure to be more like the trigger because of attraction forces. Thus, a grammar which implements GPS is a classifier that divides inputs into two categories: harmonizing and non-harmonizing. The attractor around a trigger is the category boundary between harmonizing contexts and non-harmonizing contexts. GPS defines the basin of attraction (the set of harmonizing contexts) around a trigger by ranking feature and proximity preconditions relative to faithfulness. The harmonizing inputs meet positional and feature preconditions resulting in harmony, and the non-harmonizing inputs do not meet positional and feature preconditions, and so do not result in harmony. In the Attraction Framework, the criterion for classification is trigger-target similarity.

Now, similarity-based classification has a rich history in the literature on machine learning, so an analogue to these machine learning methods is instructive. Some of these machine learning methods which use similarity include $K$-Nearest Neighbor methods (Cover & Hart, 1967; see Mitchell, 1997 for a review), Radial Basis Networks (Moody & Daren, 1989; see Haykin, 1999; Howlett & Jain, 2001 for reviews) and other kernel methods, like Support Vector Machines (SVM; Boser et. al, 1992; see Shawe-Taylor &
Cristianini, 2000 for review). The kernel methods solution is of particular relevance to the present discussion. Briefly, the motivations for kernel methods are that there are a number of learning algorithms that are very effective at learning to classify categories as long as the categories are *linearly separable* (the categories can be divided by a hyperplane). Unfortunately, however, example data are rarely encoded in a way that lends to a linear separation in the *input space*. Kernel methods solve this problem by mapping the data onto a higher dimensional *feature space* where linear separability is easier to achieve. The *kernel* is the function which maps examples from the input space to the higher dimensional space. The SVM is a known method for finding a boundary in the higher dimensional space that generalizes well to novel data. Despite these sophisticated learning algorithms, much of the success of a kernel method comes from choosing the right kernel for a particular domain. However, reviewing the possibilities for kernels and the inner workings of an SVM or other learning methods are not important for the present discussion. What is important is (*i*) distance and similarity are primary notions in the development of kernel methods and (*ii*) the basic concept of mapping from an input space to a higher dimensional space in which the categorization problem is more easily solved.

Kernel methods relate to the present work because I contend that GPS provides a principled way for mapping the set of input forms into a higher dimensional space where harmony is more naturally explained (for reasons of avoiding terminological confusion involving “feature”, I call this higher-dimensional, image space the *harmony space*). From this perspective, a grammar of ATTRACTION constraints can be seen as similarity-based kernel. The successful analysis of a particular assimilatory phenomena involves
the construction of a GPS kernel which maps triggers and targets to points which are close together in harmony space, while also mapping triggers and neutrals to points which are far apart in harmony space. This analytical work includes weighting the different aspects of similarity (positions, individual feature dependencies, etc.) such that only harmonizing elements map close to one another, while the non-harmonizing elements map further apart.

Because the free parameters in attraction kernel construction are a set of weights/rankings of ATTRACT and CORR constraints, the set of possible kernel functions is limited. If the set of available kernels were unconstrained, then quite literally any kind of harmonic alternation would be possible. However, these attraction kernels are not just abstract mathematical tools, but rather grammatical constructs which are constrained by principles such as the aforementioned Positional Similarity Hypothesis, the Subset Ranking Principle, and the phonetic grounding of dependency.

In the case of Finnish, I argued that an acoustic subfeature, [lower F2], explains why a disjoint set of vowels (low, unround and non-low round) pattern together as neutral. In the case of Moba Yoruba, I argue that another subfeature, [low frequency formant], explains why non-high vowels and obstruents pattern together as neutral, while high vowels and sonorant consonants are targets. In both cases, those segments which agree on the acoustic subfeature are similar enough in harmony space to undergo attraction, while those which do not agree on the acoustic subfeature are not similar enough for attraction pressures to apply.

The additional dimension of the acoustic subfeature makes it easier to separate participants from non-participants. This is clearly analogous to the mapping from input...
to higher dimensional space in kernel method solutions to classification. Crucially, however, subfeatures were not chosen at random, but rather were grounded in the phonetic details of the particular language. Thus, the kinds of kernels available to a GPS solution are powerful and flexible, but certainly not arbitrary. A GPS solution should not only succeed in separating undergoers from non-undergoers, but should also add insight into why harmonizing category boundaries take their ultimate shape.

4.5.1.2. Feature tiers as regions of harmony space

A simple implication of the above perspective is that forces of assimilation apply in a rich harmony space, not the input space. Therefore, the cognitive reality of the representation of phonological strings is much more like a high-dimensional similarity space than a linear, feature-tiered representation. This section supports this claim by exploring the differences between locality in string space and locality in harmony space.

The input space is an understudied aspect of OT phonology. This may be because the principle of “Richness of the Base” (Prince & Smolensky, 2004) has led to the collective opinion that all possible inputs must be considered and dealt with. However, since faithfulness is one of the central tenants of OT, the status of the input can have profound consequences on an analysis. Burzio (2002a, b) and Burzio & Tantalou (2007) have shown that there are significant theoretical advantages for explaining phenomena such as morphological syncretism, by assuming that the input is an independently occurring, fully specified output form. Other important input space issues include the availability of syllable structure, prosodic information, geometry, detailed phonetic information, etc. Clearly, these structural elements can be derived from constraints, so the input space issue is whether that structural detail can be available for faithfulness
relations. Based on the work of Burzio and others, it is evident that more detail, rather than less, needs to be available for faithfulness relations.

Here, however, because the problem domain is limited to harmony, I take a much more simplistic view and assume that the inputs and outputs of phonology are strings of phonological segments, but I recognize that this is probably only an illustrative approximation of the input space. Even so, how are strings encoded in such a space? One straightforward approach is to represent a string as a sequence of points in a two dimensional space, one dimension giving the linear position and the other giving the segments. This encoding is not too different from the ASCII/Unicode encoding of the text in this document. Different symbols are assigned a number based on something like lexicographic order, and the sequence of numbers across different positions gives a phonological string. This is the string space. This space is intuitive and compact, and nothing like the actual representations used to compute phonological alternations.\textsuperscript{114}

In what follows, I suggest that major advances in phonological theory have come from remapping the set of phonological strings onto a space that posited more highly structured representations of phonological forms. This discussion aims to show that the similarity-methods of this dissertation are a natural next step in the evolution of phonological theory and also highlights a number of differences between this work and other methods. Jakobson, Fant, & Halle’s (1952) influential work was the first comprehensive remapping, transforming the set of phonological strings into a much higher dimensional space where segments are bundles of features (see also Chomsky &

\textsuperscript{114} In fact, it may be somewhat suspect whether the linear sequence of phonemes is even the goal of speech planning as there are fairly strong interactions in terms of gestural overlap and coarticulation in even non-harmonic languages. However, I will not consider these phonetic details and assume that a specified phonological string is the goal of the interface between phonology and phonetics.
Halle, 1968). These feature bundles can be expressed as a feature score, which is a set of independent features, each of which is actually a separate dimension of representation. In this higher dimensional space, generalizations about harmony can be more naturally stated as rules which change the specifications of a segment along one or more features dimensions, instead of a set of unrelated rewrite rules where one element of a phonological string is replaced with another.

Later, with the advent of autosegmental phonology (Goldsmith, 1979; McCarthy, 1980; Clements & Keyser, 1983; Kenstowicz, 1994 for a review), the set of phonological strings was again mapped to a feature bundles space, but instead of a sequence of feature bundles, feature tiers allowed feature specifications to persist across multiple root nodes by linking root nodes to tiers. Feature tiers and linking provide a way for segments which are not adjacent in string space to be adjacent on a feature tier. This primarily occurs through assumptions of underspecification and/or skeletal tiers. Segments which do not project to a feature tier do not link to that feature tier, so the unspecified forms cannot interfere with tier-adjacency. For example, skeletal C-V structures stipulated that the features which link to consonants are of a different sort than those that link to vowels, so there is no cross talk between consonants and vowels that could otherwise interfere with tier adjacency (see also §4.4.4 on this point). Thus, feature tiers provide a way for similar elements (those which project to a tier) to interact as though they were adjacent even though they are not adjacent in the space of phonological strings.¹¹⁵ In an autosegmental harmony space, segments are close together if they are adjacent on a feature tier, independent of the string distance between them. Thus, from the kernel

¹¹⁵ My thanks to Paul Smolensky who first suggested a perspective along these lines.
perspective, feature tiers provide a mapping that allows non-adjacent segments to be proximate in the image space, giving a sort of clustering across positions.

On the other hand, feature geometry (Clements, 1985; Sagey, 1986; Halle, 1988; Odden, 1991; Clement & Hume, 1995; Halle et. al, 2000; Clemens, 2003; etc.) provides a restrictive mapping for clustering across features. In a feature score, even with autosegment feature tiers, it is difficult to express instances of neutralization where multiple features spread together. Cross-linguistically, some features tend to spread together, while others do not. For example, it is quite common for the vowel features back and round to cluster together in inventories and in assimilation processes (Stevens, Keyser, & Kawasaki, 1986; Odden, 1991). In Yawelmani, unround, front [i] alternates to round, back [u]. In contrast, there are no languages where backing and nasality spread together. There is no way for rules to predict such asymmetries when all features are independent, since a rule which spreads backing and rounding seems just as likely as a rule which spreads backing and nasality.

Feature geometry explains the attested dependency patterns in neutralization by positing a tree-structure hierarchy where some features can link to the same non-terminal node, and so spread together, while other features cannot. For example, [back] and [round] can have a common parent, color (Odden, 1991) or V-Place (Clements & Hume, 1995) but [back] and [nasal] do not. The tree structure expresses how some features can depend on one another. Thus, unlike an SPE-style feature score, feature geometry is a kernel which maps phonological strings to a space where features are not necessarily independent. Below in §4.5.4, I show how entailment theory predicts that after harmony applies non-orthogonal features should cluster together, deriving the constituency of
feature geometric models, i.e. ‘feature classes’ (Padgett 2002). For now, the important point is that features which are children of the same parent map to similar regions in harmony space, so a feature geometry is a kernel where a neighborhood in harmony space is based on the clustering of features in a hierarchical structure.

These kernel perspectives on aspects of phonological theory are relevant because the Attraction Framework provides a general computational means of understanding clustering in phonology (Wayment, et. al, 2007) based on avoiding the violation of representational entailments (Burzio, 2002 a, b). Therefore, the kernels engendered by the Attraction Framework can supersede the clustering principles of autosegmental phonology (different positions being tier adjacent) and feature geometry (different features spreading together). The remainder of this section focuses on deriving feature tiers; §4.5.2.6 focuses on deriving dependency in feature geometry.

However, an attraction kernel is not merely the sum of feature geometry and feature tiers; rather, it is a much more general method of determining whether triggers and targets interact. It is for this reason that the Attraction Framework is able to analyze cases of non-local harmony processes like long-distance consonant harmony (Rose & Walker, 2004; Hansson, 2001) and long-distance vowel agreement (§4.3.4, §4.4.3, Ajíbóyè & Pulleyblank ms. 2008), alongside the more usual spreading based phenomena in nasal harmony (Walker, 2000) and vowel harmony (Ni Chiosáin & Padgett, 2001).

A GPS kernel defines a region around a trigger in a harmony space where attraction forces apply, so there is a very relevant notion of neighborhood in harmony space. The existence of non-local harmony confirms that it is proximity in a GPS-harmony space that governs harmony processes, and not proximity in the input string
space that is the driving force of assimilation. With a notion of distance in harmony space, it is possible to derive the effects of autosegmental feature tiers: under a GPS kernel, differences between segments in distinct positions may be blurred as feature similarity yields increasing similarity in harmony space. Hence, with attraction kernels, segments which are tier-adjacent in an autosegmental analysis are neighbors in the harmony space because of the same similarity that allows both segments to project to the same feature tier.

For some attraction kernels, when a string of C’s and V’s is pumped through the kernel function, regardless of string position, the consonants map to points close to one another and the vowels map to points close to one another. Thus, in this harmony space it is easy to define a C-region and V-region, see (65) below, deriving the notion of C-V skeletal tiers.

The example here is based on the Finnish input form sadettä, which must map to sadetta ‘rain (partitive)’ to give the harmonic alternation of the affix across the transparent vowels and consonants. Constructing the appropriate kernel for Finnish illustrates how additional dimensions ease the classification task, and how regions of harmony space are a more powerful notion than feature tiers.
A kernel that gives the map in (65) must take into consideration the value of the feature \([\pm \text{cons}].\) It must also discount differences in terms of positions in the string relative to differences in \([\pm \text{cons}].\) A ranking like \(\{\text{ATTRACT}([[-\text{cons}]] \rightarrow [\text{H}]), \text{CORR}_{L\rightarrow R}\}\) >> \(\text{IDENT-IO}([\text{H}])\) has the necessary preconditions. In fact, this is true for all possible harmonic features, \([\text{H}].\)

The picture in (65) refers to the relevant subspace for [back] harmony in Finnish and the division above implies that, for the purposes of [back] harmony, vowels are more similar to one another than consonants. Therefore, the attraction framework derives the neutrality of consonants that is stipulated on an assumption of C-V skeletal tiers, or similar accounts where only vowels project to the [back] tier. A conditional similarity subspace is the analogue of a feature tier in autosegmental phonology.

This subspace encodes how far apart segments are for the purposes of [back] harmony, successfully deriving the neutrality of consonants, but (65) above is clearly not sufficient since neutral [e] is closer to the affix [ä] than the trigger [a]. This proximity
would predict that [ä] is more strongly attracted to [e] than [a], but in Finnish a non-string local attraction relation allows [a] to induce harmony on [ä], sadettä → sadetta. Note [e] must have a specification for [back] because it triggers harmony in the absence of [lower F2] vowels (see §4.3.4), so [e] must map to somewhere in the subspace, but as drawn above, where only [±cons] and string position were taken into consideration, [e] is not linearly separable from [a] and [ä].

Like kernel methods in machine learning which ease learning by adding dimensions, an analysis of Finnish benefits from an additional dimension. As argued in §4.3.4, the correct additional dimension seems to be the acoustic subfeature [lower F2], which captures the otherwise unnatural class of low, unrounded and non-low, rounded vowels. By adding a [lower F2] dimension to the attraction kernel, i.e.

\{\text{ATTRACT}([\text{−} \text{cons}], [\text{lower F2}]) \rightarrow [\text{back}], \text{CORR}_{L \rightarrow R} \} \gg \text{IDENT-IO}([\text{back}])\}

neutral [e] becomes trivially separable from triggers and targets, as shown below in (66):

(66)  \textit{C-V and [lower F2] regions in the harmony subspace for [back]}:

![Diagram showing C-V and [lower F2] regions in the harmony subspace for [back].]
The kernel that gives the mapping above not only discounts positional similarity, but is utterly blind to the sequential order in the string space. Distances in harmony space are being determined exclusively by feature similarity, which is here represented hierarchically with a [lower F2] region inside the [−cons] region. Because [a] and [ä] map to points which are proximate in the [back] harmony subspace, there is strong pressure to agree in backness. Put another way, no matter what their position, the representations of [a] and [ä] are not particularly distinct in regards to backness, so harmony easily obtains in Finnish. In contrast, [e] is more distinguishable from [a] and [ä] and so does not trigger or undergo harmony in a root. Other non-local harmony processes receive a similar explanation with attraction kernels that mostly ignore proximity in the input string in favor of feature similarity.

If the subspace is the attraction analogue of a feature-tier, then what are the embedded regions? The embedded regions in a harmony space are related to projection on a feature tiers: a segment which is in the neighborhood of a trigger is under pressure to assimilate, likewise a segment which links to the same feature tier as a trigger is under pressure to assimilate. Attraction kernels, thus, provide a similarity-based account of tier projection, but with a few important improvements.

One difference between tier projection and kernel mapping is that while tier projection is explanatory, it is not predictive because there is not always an independent reason for why segments should project to some tiers and not others. For example, if the feature tier [back] is stipulated to only associate to vowels, it grammatically derives the fact that consonants do not participate in [back] harmony, but why consonants do not project to the [back] tier remains somewhat mysterious. In contrast, within a harmony
space, a lack of interaction between consonants and vowels (or any other pairs of segments) is derived from an attraction kernel’s sensitivity to the differences between consonant and vowels which are mapped to different parts of the harmony space. Note in (66) that the consonants are not part of the neighborhood of the triggering [a].

Another critical difference is that a feature tier is indivisible, making it more difficult to explain parasitic interactions. For example, in Yawelmani, high vowels only trigger [labial] harmony on other high vowels and non-high vowels only trigger [labial] harmony on other non-high vowels. If both high and non-high vowels project to the same feature tier, then it is more reasonable to expect cross-height harmony. However, with an attraction kernel, small attraction basins predict that the set of undergoing segments might be non-overlapping, and so if the attractor covering the high vowels is distinct from the attractor covering the non-high vowels, then there is no reason to expect cross-height harmony. This essentially amounts to tier sub-categorization, since there is a [labial] attractor for high vowels and a separate [labial] attractor for non-high vowels. Distinct tiers are not required to have the same projections, so subcategorization resolves the expectation of cross-height harmony. Crucially, however, the Attraction Framework derives subcategorization from trigger-target similarity. Lacking a direct notion of similarity, it is not clear how to augment feature tiers with subcategorization in a non-stipulative manner. Thus, unlike traditional feature tiers, harmony spaces can be divided in many different ways, so it is possible for the targets of one trigger to be unrelated to the targets of another.

This discussion shows that GPS provides a more principled, yet flexible, means for drawing association lines between segments and the tiers, or regions, in which the
pressures for harmony apply. The flexibility is essential because there are many kinds of parasitic interactions. Furthermore, there are attested cases where consonants do indeed interact with harmony, so wholly separate regions for C’s and V’s (as in (66)) cannot be universal. These CV interactions are the focus of the next section, §4.5.2, where the principle of phonetic grounding remains essential because not all possible interactions are attested: the set of interactions, including those where consonants and vowels interact, is exactly the subset where there is some other phonetic similarity that underlies the harmony process.

As discussed in §4.4.4, it is a challenge for autosegmental clustering to simultaneously explain both participation and non-participation. However, from the present perspective, participation and non-participation are merely different attraction kernels with distinct weightings of the factors of similarity. While feature tiers are inflexible, harmonic regions are much more pliant in the face of complex interactions and non-interactions.

Therefore, I conclude that feature tiered accounts, in particular with stipulative CV-skeletons, are only an approximate harmony kernel. For many cases, CV interactions are not relevant, so modeling harmony with a CV skeleton provides a measure of explanation. However, as I review below, this approximate solution cannot be the actual state of affairs, as there are a growing number of examples in the literature of CV interaction in harmony.

4.5.2. Prerequisite similarity solutions to C-V interactions

This section reviews a number of attested cases of complex CV interactions and sketches similarity solutions. The main point of this small survey is to show that
although consonants and vowels do not always participate in harmony, there are enough cases of CV interaction to warrant a replacement of conventional tier projection with the more suitable alternative offered by attraction kernels described above.

The cases previously presented in this chapter in regard to CV interactions largely constituted instances of non-participation, where participation should be readily predicted by an autosegmental account. For example, contrastively nasal consonants are transparent to V-to-V nasal harmony in Moba Yoruba and contrastively nasal vowels are transparent to C-to-C nasal harmony in Ngbaka. In this section, I focus on cases of participation where consonants block the spread of V-to-V harmony. An appeal to the phonetic space suggests that this blocking is not arbitrary or random, rather consonants block vowel harmony exactly when, for reasons of phonetic similarity, they map to the region of harmony space of vowel harmony.

There are attested cases of consonant blocking of vowel harmony for each of the major vowel harmony features: backing, rounding, ATR, and height. These features are considered in turn. Taken together, these cases reject the notion that consonants do not participate in vowel harmony because of some ad hoc tier separation between consonants and vowels, suggesting that vowel harmony can obtain through strictly local means (cf. Gafos, 1996; Ní Chiosáin & Padgett, 2001; Mahanta, 2008). Recall, however, the previously discussed cases of massive transparency in long-distance consonant and vowel harmony, confirming that strict locality is not the only means of harmonic interaction. These cases again support a role for phonetic similarity in determining the strength of attraction relations, as there is a phonetic basis for each interaction.
4.5.2.1. Velar consonants block in Turkish palatal harmony

Turkish is among the best known cases of consonant intervention in vowel harmony (Clements & Sezer, 1982; Clements & Hume, 1995; Ni Chiosáín & Padgett, 2001). Regular Turkish palatal harmony, which requires adjacent syllables to agree in backness (see more details in §2.3.1.2), is blocked by a class of consonants that contrast in palatalization. The non-palatalized consonants are \{k, g, r, l\} and their respective palatalized counterparts are \{k\textsuperscript{j}, g\textsuperscript{j}, r\textsuperscript{j}, l\textsuperscript{j}\}. As indicated in the suffix alternations below in (67), palatalized consonants block harmony in back harmony domains, (67)(a), and non-palatalized consonants block harmony in front domains, (67)(b):

\[(67)\quad\text{Consonant blocking in Turkish back harmony}\]

\begin{itemize}
  \item[a.] \quad \text{idrak\textsuperscript{j}-i} ‘perception’
  \quad \text{har\textsuperscript{j}f-i} ‘letter’
  \quad \text{sual\textsuperscript{j}-i} ‘question’
  \item[b.] \quad \text{tasdik-i} ‘confirmation’
\end{itemize}

This Turkish data has provided strong evidenced that consonants are available as full-participants in harmony, and became one of the driving forces for frameworks which support CV interaction (e.g. Clements & Hume, 1995; Ni Chiosáín & Padgett, 2001), rejecting earlier proposals (Clements & Keyser, 1982; Clements, 1991; Odde\n, 1991; Clements and Hume, 1992) which postulated separate skeletal tiers for consonants and vowels.

The phonetic basis of this CV interaction is clear: palatalized and non-palatalized consonants differ in the same articulatory gesture as front and back vowels, namely, the position of the tongue dorsum. This specific phonetic similarity explains why the other potentially interacting consonants (labial and coronals) cannot block harmony. Thus, \{k, g, r, l\} are represented in a region of harmony space also occupied by back vowels, while
{k̂, ĝ, r̂, l̂} are represented in a region of harmony space associated with front vowels. This allows the consonants to trigger harmony on subsequent vowels, blocking harmony to preceding vowels.

4.5.2.2. Labial attraction and color harmony in Warlpiri

Warlpiri provides another case of consonant interaction in vowel harmony. Warlpiri (McCarthy, 2003; refs. therein) has a limited vowel space consisting of {i, a, u}. As shown in (68)(a), suffix vowels which are underlyingly /u/ emerge as [i] in the context of a root final [i].

(68)  Warlpiri vowel harmony
a.  maliki-kirli-ri-li-ji-li  ‘dog-COMIT-ERG-then-me-they’
    minija-kurlu-ru-lku-ju-ju  ‘cat-id.’
    kurdu-kurlu-ru-lku-ju-ju  ‘child-id.’

b.  ḋamirni-puraji  ‘uncle-your’
    ḋali-wurruru  ‘we two (incl.)-EMPH’

However, as shown in (68)(b) this color harmony is blocked by an intervening labial consonant. As McCarthy (2003) notes, this blocking is related to the well-known effect of ‘Labial Attraction’ (Lee, 1966; Campbell, 1974; Clements & Sezer, 1982; Ito & Mester, 1995), where the labial specification of consonants spreads to subsequent vowels. Although labial attraction is not fully productive in Warlpiri (e.g. wapirri-mi ‘ABS conceal, cover up DAT’), forms tend to satisfy the demands of labial attraction at the expense of harmony to [i].

---

116 As McCarthy (2003) shows, this must be a derived environment effect, since violations of labial attraction are permitted in the input, but are not licit as the result of harmony. Burzio (2002a,b) demonstrates how Entailment Theory can explain such instances of ‘non-derived environment blocking’, though this case is somewhat more complicated. A preliminary analysis might proceed as follows: labial attraction is blocked by faithfulness to an independent output. However, this constraint is inactive if the
Labial attraction locks in the input environment where harmony might otherwise obtain, and strict locality demands that subsequent vowels agree in color with the frozen back, round vowel. The motivations for labial attraction are clearly related to articulation (Clements & Sezer, 1982) because unlike [p-u] sequences, [p-i] sequences require resetting the [labial] target from consonant to vowel. Furthermore, I suspect that explosive release favors lip protrusion (associated with rounding on vowels) over lip constriction because the inertial force of release tends to push the lips outwards.

4.5.2.3. Labial consonants block rounding harmony in Nawuri

Unlike Warlpiri, where labial consonants block harmony to unround, front vowels, in Nawuri (Casali, 1995), labial consonants block harmony to round, back vowels. Nawuri labial harmony can be illustrated by considering the single noun class prefix /gI/.

Like other Nawuri affixes, /gI/ takes its [ATR] specifications from the stem initial vowel, but unlike other affixes, /gI/ is [labial] if the stem initial vowel is round or if the stem begins with the labial glide w. This [labial] and [ATR] harmony is shown in (69)(a) below. Interconsonantally, front, short vowels centralize (I denote this centralization with a double underline ‘_’ to distinguish it from the triggers which are denoted with a single underline ‘_’). In this derived environment, centralized vowels are under pressure to agree in rounding with subsequent labial vowels. Here, bold font marks when /gI/ emerges as [labial].

environment is derived and so, in derived environments, labial attraction is available to block vowel harmony.
(69) Nawuri labial spreading

a. gi-ɲi  ‘tooth’
   gi-ba:  ‘hand’
   gʊ-wa:  ‘doing’
   gʊ-su  ‘ear’
   gu-jo  ‘yam’

b. casual speech         careful speech
   gu-mu      gi-mu      ‘heat’
   gu-pulala gi-pulala ‘burial’
   gu-fufuli gi-fufuli ‘white’

(69)(b) presents forms, which during careful speech, are likely to exhibit blocking by the less sonorant labial consonants. This evidence from Nawuri is more complicated than previous examples because the labial stops express the harmonic feature, but are nevertheless blockers of harmony to that feature. This means that the failure of [i] to undergo harmony to [u], unlike the other cases, cannot be seen as a tendency to undergo harmony to the intervening consonant instead. Therefore, the labial stops are not triggers, but nevertheless interfere with that ability of [u] to trigger harmony. A possible explanation for the interference by labial consonants is that (locally) the transmission of [labial] cues must come from the consonant, but they are not similar enough to vowels to result in harmony. On the other hand, non-labial consonants do not map to a [labial] region of harmony space and so do not interfere.

4.5.2.4. Nasal consonants block ATR harmony in Assamese

A stronger case of interference can be found in Assamese [ATR] harmony. Mahanta (2008) reports that Assamese has productive ATR harmony within and across morphemes which is blocked by nasal consonants [n], [m], and [ŋ]. Of particular interest
is the fact that ATR harmony is only blocked when the trigger is a high vowel. This
difference can be seen in (70), where (70)(a) shows the blocking by an intervening nasal
stop when it immediately precedes a high vowel trigger, and (70)(b) shows a lack of
blocking by a nasal stop when it immediately precedes a non-high vowel trigger.

(70) Assamese ATR harmony

a. sɛkɔni ‘strainer’
   xɔmɔnia ‘colleague’
   pɔtɔni ‘dumping ground’
   kɔmɔr ‘leavening agent’

b. pɔnoru ‘onion’
   somokit ‘frightened suddenly’

Mahanta (2008) accounts for these facts with an admittedly unmotivated contextual
markedness constraint, *[V₁[hi,+ATR]NV₁[hi,+ATR]], denoted *[oNi]. Note *[oNi] is not a
constraint by which the nasal stop triggers a [−ATR] value on preceding non-high
vowels, otherwise, the forms in (70)(b) with non-high vowels would be [−ATR] before
the nasal stops. Thus, like Nawuri blocking, Assamese consonants are not triggers, and
so it must be the case that *[oNi] marks the relationship between triggers and targets: the
nasal stop interferes with the ability of [i] to trigger [+ATR] harmony on non-high
vowels, which obtains in all other contexts. Such interferences raise two related
questions, (i) why do nasal stops interfere with [ATR] harmony? and (ii) why is only
triggering by high vowels affected?

Attraction over phonetic similarity answers these queries, giving the sought after
grounding of the *[oNi] constraint. As discussed in detail in §4.4, nasality and vowel
heights are known to affect formant structure, especially in the low frequency range

- 335 -
associated with high vowels (<300 Hz). Like vowel height, differences in ATR are also primarily indicated by the frequency of the first formant (Gick et al., 2006; Przezdziecki, 2005). Thus, under an account of strictly local propagation, the nasal resonance during the closure of the stop can mask some of the formant structure that indicates the [+ATR] trigger. Johnson (2004) argues that nasalization on high vowels causes them to be perceived with a higher F1. With this background, the best explanation for Assamese blocking is that a preceding nasal consonant is having a similar effect on high vowels, filtering the local propagation of [ATR] cues and causing a slightly higher F1 that would be associated with a [−ATR] high trigger. Therefore, a [+ATR] high vowel preceded by a nasal stop, actually yields a [−ATR] attractor in the acoustic space.

Under this explanation, the reason nasal stops only interfere with high vowels derives from the relative overlap of the nasal murmur with the (oral) first formant. For high vowels, this overlap is almost complete, but for non-high vowels the nasal formant only partially overlaps with the first formant, so, unlike high vowels, the [+ATR] value of the non-high vowels may persist through the closure period. A [+ATR] non-high vowel preceded by a nasal stop yields a [+ATR] target because the filtering aspects of nasal resonance during closure only affect lower frequencies. In this way, phonetic similarity provides a much better account of Assamese blocking, predicting the context where blocking by nasal consonants occur. Also, Assamese is an interesting contrast to Moba Yoruba where similar phonetic facts combine with a relaxation of locality requirements to yield transparency of nasal stops (see §4.4.3).
4.5.2.5. Consonant blocking of height harmony in Buchan Scots

Paster (2004) presents data from Buchan Scots further suggesting that CV interactions are based in phonetic similarity. Buchan Scots has a vowel height harmony process by which word-final /i/ lowers to [e] when preceded by a stressed non-high vowel. This is illustrated in (71) where stressed [e], [ɛ], and [a] trigger lowering of /i/ to [e], but in (71) there is no alternation because the triggers are high, [i], [u].

(71) Buchan Scots height harmony

a. \textit{vere} \quad ‘very’
   \textit{merse} \quad ‘mercy’
   \textit{megne} \quad ‘many’
   \textit{kafe} \quad ‘coffee’

b. \textit{pi\textbar{i}} \quad ‘pity’
   \textit{rili} \quad ‘really’
   \textit{bjui\textbar{i}} \quad ‘beauty’

This normal harmony process is blocked by a diverse set of consonants and consonant clusters as shown in (72):

(72) Blocking by voiced consonants

\textit{mebi} \quad ‘maybe’
\textit{den\textbar{i}} \quad ‘dainty’
\textit{em\textbar{i}} \quad ‘empty’
\textit{redi} \quad ‘ready’
\textit{agli} \quad ‘ugly’
\textit{pozi} \quad ‘posey’

Following Paster, the best class to define this diverse set of blocker is [+voice, –sonorant], although blocking does occur by a number of C[+son]C[–son, –voiced] clusters.
In line with the present similarity proposal, Paster argues that it is a phonetic link between voicing in obstruents and vowel height that motivates the blocking of harmony. There is a known effect of laryngeal lowering (Denning, 1989; Stevens, 2000; Paster, 2004; Bauer & Parker, 2008), whereby in order to compensate for the build-up of intraoral air pressure, speakers lower the larynx to increase the size of the supraglottal cavity. This facilitates voicing during a stop closure which is otherwise antagonistic to the transglottal pressure drop needed for voicing. Phonetic studies robustly show that laryngeal lowering affects the peak of the first formant which is the primary indicator of vowel height (e.g. Bauer & Parker, 2008). This laryngeal lowering account is advantageous because it unifies an otherwise disparate set of blockers and exactly predicts the unusual interaction between consonants and vowels. This unification is only available in the phonetic space where similarity along F1 predicts the interactions.\footnote{Paster (2004) explicitly argues against acoustic features based on blocking by NT and LT clusters, but this perspective fails to consider other F1 effects besides those due to lowering. Both nasals and liquids have an effect on the low frequency formant structure (Stevens, 2000) that could, likewise, block harmony.}

4.5.2.6. Conclusions of CV interactions

The above cases plus the others elsewhere discussed in this dissertation (in particular Chapter 2, §4.3, and §4.4.4) call for a system that is able to operate on and compute the phonetic similarity between phonological features. As I have shown, the Attraction Framework excels at describing such similarity effects. The rest of this section explores the relationship between representation and similarity with the aim of understanding what the consequences of similarity-based representations are for feature geometric and derivative representations.
In particular, in §4.5.3 I take issue with the concept of node subdivision, which is the usual feature-geometric approach to solving the participation/non-participation dilemma previously discussed in §4.4.4. In light of the above data on consonant intervention in vowel harmony, this dilemma may be surmised as follows: consonants sometimes, but not always, interact with vowel harmony. Even more importantly, the specific kind of similarity which describes a CV-interaction is sometimes, but not always exploited. I argue that similarity networks are a superior alternative to geometries because a geometry’s rigid hierarchy is ill-equipped to express the multiple facets of similarity-based dependency that are necessary to explain the present survey.

Another dilemma addressed in this section is understanding why some features cluster together as daughters of a common parent in a feature geometry, but other features do not cluster. §3.4 confirmed that because of the availability of entailments among subsymbolic representations, the Attraction Framework predicts another kind of clustering: only phonetically similar features interact in parasitic assimilation. A similar analysis is presented in §4.5.4, which shows that similar features are under pressure to spread together, giving the kind of clustering represented by the hierarchical nature of feature geometry.

4.5.3. Avoiding the representational challenges of feature geometry

The available feature geometric solution to place-interactions between consonants and vowels is to posit a node to which both consonants and vowels may link place features and a separate node to which only vowel features link. It is somewhat standard (Clements & Hume, 1995; Clements, 2003; Morén, 2006) to call the node for spreading between vowels and consonants C-place and the node for spreading from vowel-to-vowel
**V-place.** Here, I follow Clements & Hume (and other geometric proposals) and assume that when a consonant is associated with a specified \(V\)-place constituent, it has a secondary place of articulation. As (73) and (74) show, these nodes are available for both [+cons] and [−cons] segments, allowing for the formulation of a unified place theory, where place features \{[labial], [coronal], [dorsal]\} are constant across consonants and vowels, although the locations of linkage differ between consonants and vowels.

(73) Partial geometries for [labial] contrasts among consonants:

- \([p]\) \hspace{1cm} \([k]\) \hspace{1cm} \([k^w]\)
- \([+cons]\) \hspace{1cm} \([+cons]\) \hspace{1cm} \([+cons]\)
- \(C\)-place \hspace{1cm} \(C\)-place \hspace{1cm} \(C\)-place
- [labial] \hspace{1cm} [dorsal] \hspace{1cm} [dorsal]
- \(V\)-place \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \[labial]\)

(74) Partial geometries for [labial] contrasts among vowels:

- \([i]\) \hspace{1cm} \([u]\)
- \([+cons]\) \hspace{1cm} \([+cons]\)
- \(C\)-place \hspace{1cm} \(C\)-place
- \(V\)-place \hspace{1cm} \(V\)-place
- [dorsal] \hspace{1cm} \[dorsal] \hspace{1cm} \[labial]\)

These subcategorized nodes allow for CV-interaction if a specification for \(C\)-place can spread to a \(V\)-place. Labial attraction where \([pi]\)→[pu] (as needed in Nawuri §4.5.2.2) requires such a spread from \(C\)-place to \(V\)-place. Palatalization of consonants in the context of vowel harmony (as in Turkish§4.5.2.1) requires \(V\)-place to spread to \(C\)-place. Thus, as noted in (Clements, 2003), node subdivision allows the range of CV-interactions
and non-interacts as follows in (75). Though, here, I generalize from place and consider a feature [F] with non-terminal node subdivisions $C-F$ and $V-F$.

(75) Participation and non-participation in Feature Geometry:

<table>
<thead>
<tr>
<th>Type of interaction</th>
<th>Feature geometry description</th>
<th>Spreading rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Only CC-interaction</td>
<td>[F] can only link to $C-F$ nodes</td>
<td>$X_i \rightarrow X_{i+1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$C-F \rightarrow C-F$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V-F \rightarrow V-F$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[F]</td>
</tr>
<tr>
<td>Only VV-interaction</td>
<td>[F] can only link to $V-F$ nodes</td>
<td>$X_i \rightarrow X_{i+1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$C-F \rightarrow C-F$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V-F \rightarrow V-F$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[F]</td>
</tr>
<tr>
<td>CV-interaction</td>
<td>[F] can link to both $C-F$ and $V-F$.</td>
<td>$X_i \rightarrow X_{i+1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$C-F \rightarrow C-F$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V-F \rightarrow V-F$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[F]</td>
</tr>
</tbody>
</table>

As already argued in this section, the Attraction Framework provides a more general way of understating feature projection, i.e. linkage, predicting that processes requiring subdivision explanations should be readily attested beyond just place features. As discussed in §4.4.4, nasal harmony certainly requires such a subdivision in order to predict the non-interaction of nasal consonants in $V-V$ nasal harmony in Yoruba, as well as the non-interaction of nasalized vowels in $C-C$ nasal harmony in Ngbaka, in addition
to the more common C-V nasal spreading harmony. The other cases of CV-interaction in §4.5.2 would, likewise, require a subdivision account to explain why consonants do not always block vowel harmony.

The case of Buchan Scotts (Paster, 2004) provides a solid example of how the work of node subdivision is a crude technique for representing the aspects of similarity that are really driving the process. The relevant forms here are [mebi], *[mebe] ‘maybe’, [vere], *[veri] ‘very’, and [kafe], *[kafi] ‘coffee’. Lowering harmony is only blocked in the first case where the intervening consonant is a voiced obstruent. In order to give the transparency of voiceless obstruents and voiced sonorants in lowering harmony, it would be necessary to posit a subdivision between voiced and voiceless stops such that the aperture features which encode height only link to Obs-Voiced, but not Obs-Voiceless or Son-Voiced. An analysis along these lines is shown below:

(76) Possible analysis of voiced consonant blocking of vowel height harmony in feature geometry with an active no-lines crossing prohibition.

Note an analysis which fails to split sonorant voicing from non-sonorant voicing would miss the generalization that only obstruents block harmony. Likewise, an analysis which fails to split voiced obstruents from voiceless obstruents would miss the generalization
that only voiced obstruents, but not voiceless obstruents, are blockers. Thus, the indicated subdivisions in (76) are minimally necessary.

The primary criticisms of this sort of feature geometric node-subdivision follow. First, subdivision has been too narrowly applied, being primarily limited (e.g. Clements & Hume, 1995) to place; although some have argued in favor of a manner subdivision as well (e.g. Morén, 2006; refs therein). If the descriptions given in (75) hold for many different kinds of harmonic processes, as indicated in this dissertation, then that suggests there is a need for another more general explanation for linkage than the ad hoc stipulation of subdivision for some features and not others. An adequate geometry must subdivide place, manner, [nasal], voicing, obstruency, and likely other features.

As the case of Buchan Scots shows, sometimes these dependencies are multi-faceted, meaning binary subdivision cannot be guaranteed to suffice. While there is ready use of subdivision, there is a lack of a theory of subdivision that might predict what kinds of subdivisions are likely to occur. The theory of feature geometry needs to be augmented with principles that can independently determine when dependency is likely to occur, and a computational framework that is sensitive to these principles. As shown in the next section, entailments can predict when dependency will occur based on a sensitivity to phonetic detail.

A second criticism follows from the fact that a feature must be allowed to link to more than one kind of parent. This is minimally necessary to give C-V interaction, but also occurs whenever there is participation between subdivided nodes. When this occurs, a geometry is no longer truly hierarchical, and the tree structure is replaced with something much more closely resembling a spidergram or sparsely connected graph.
Such structures are better thought of as similarity networks where features link to some nodes and not others. In (76), V-place dominates aperture, but so does Obs-Voice. For a geometric approach, such linkages seem cumbersome and arbitrary as there is little relationship between the dominating nodes. However, in a similarity network, every node has the potential to link to any other node. The only issue in a similarity space is to determine why there is a link between voicing and vowel height, but such a connection is known to exist based on phonetics: laryngeal lowering in order to compensate for the reduction in transglottal pressure needed for voicing during closure is known to affect F1 (Paster, 2004; Bauer & Parker, 2008). Perhaps subdivision can be used to express such similarity dependencies in feature geometry, but if, as I have argued, similarity is of central importance then it is more natural to start at the outset with a dependency framework such as a similarity network or the present attraction proposal where similarity is not merely epiphenomenal but rather a basic element of computation.

4.5.4. Dependency as similarity clustering

The above arguments make the case for a more general use of representational similarity, and that the work of arbitrary node subdivision in feature geometry should be replaced with computations over phonetic similarity. However, as mentioned in §4.5.1, feature geometry was not introduced for the sake of similarity. It was introduced to express harmony patterns where multiple features neutralize at the same time or in the same ways. Parent-child dependencies between a place node and place specifications explain why the conditions on harmony for each of the specifications is the same even though the features are distinct. Furthermore, children which share the same parent can be co-harmonic if the parent node spreads, e.g. color harmony where backing and
rounding spread together (Odden, 1991). This section shows how the entailment framework can predict this sort of co-harmony from the (sub-)entailments along the phonetic correlates of features.

For a concrete example, consider the phonological processes over feature F, G, and H in the rules below in (77):

(77) Patterns of spreading in a hypothetical language

(a) **Harmonic features F, G**

<table>
<thead>
<tr>
<th>Feature</th>
<th>Rule</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>[−F]→[+F]/V[+F]C___</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>[−G]→[+G]/V[+G]C___</td>
<td></td>
</tr>
</tbody>
</table>

(b) **Non-harmonic feature H**

<table>
<thead>
<tr>
<th>Feature</th>
<th>Rule</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>[−H]→[+H]/V[−H]C___</td>
<td></td>
</tr>
</tbody>
</table>

Feature geometry explains why features F and G spread together but F and H do not. Suppose the geometry includes a node which dominates F and G but not F and H. Such an **FG-node**, where \([±F] [±G]_{FG-node}\), explains why the patterns of harmony for F and G are identical.

Researchers in feature geometry have strived to ground the creation of non-terminal nodes in phonetic similarity. For example, Halle *et al.*, (2000) argue that all non-terminal nodes are related to common articulation, and Odden (1991) posits the existence of a color node based on the common F2 correlate of backing and rounding. If this research is on the right track, then the motivations for features being co-harmonic, like F and G above in (77), is usually grounded in phonetic similarity. However, I recognize that perhaps not all structure in the feature geometry literature may be reduced to phonetic similarity. It goes beyond the scope of this work to even know whether all geometry structure should be reduced to phonetic similarity, so it suffices to say that
some kinds of nodes which are posited for co-harmony are naturally and appropriately derived from phonetic similarity.

As shown particularly in §3.4, the entailment framework has the capacity to capture such clustering along phonetic correlates based on the satisfaction of the subentailments among these subsymbolic correlates. A similar account is available for the explanation of clustering in co-harmony. In brief, suppose that F, G, and H have the distributed, subfeature specifications below:

(78) Distributed representations of F, G, and H. Similarity is indicated by shading and common within-feature and between-feature entailments are highlighted with arrows.

<table>
<thead>
<tr>
<th>Phonetic Subfeature</th>
<th>Gesture</th>
<th>Phonological Feature</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[tongue dorsum retraction]</td>
<td>F=[back]</td>
</tr>
<tr>
<td></td>
<td>[tongue root retraction]</td>
<td>G=[labial]</td>
</tr>
<tr>
<td></td>
<td>[lip protrusion]</td>
<td>H=[ATR]</td>
</tr>
<tr>
<td></td>
<td>[F1]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[F2]</td>
<td></td>
</tr>
</tbody>
</table>

With these representations, there are common subentailments between the cue features (here, [F2]) and the gesture features ([tongue dorsum retraction] and [lip protrusion]), so when F and G have similar encodings, the between-feature entailments enhance within-feature entailments, creating increased pressure for output segments which agree on one of F or G to agree on both F and G. Agreeing on F but not G would violate the entailments between cues and gestures. However, because F and H differ on both articulation and acoustics there is no additional pressure for H agreement given G agreement.
Thus, given similar F and G and dissimilar F and H, for any context $c$, where F-harmony obtains, there is increased pressure (in terms of segment-to-segment entailments) for G harmony in $c$, but no increased pressure for H harmony in $c$, explaining why F and G pattern together as if children of a common $FG$-node. In this way, entailments explain why the co-harmony allowed by feature geometry derives from phonetic similarity, and why there is a propensity for co-harmony among phonetically similar features.

In sum, this result suggests that the Attraction Framework can subsume a portion of the clustering achieved through geometry without any loss, as long as the posited feature geometric nodes can be reinterpreted as activating an amount of phonetic similarity. Therefore, the solution available with GPS-kernels is not only more general than feature geometry (able to account for non-local interaction), but also a conceptual improvement because, unlike feature geometry, it can explain why some features cluster, but not others (via representational similarity).

4.6. **Summary and Conclusion**

This chapter has presented a novel method for analyzing locality and similarity effects in assimilation from the perspective of representational attraction (Burzio, 2002a, b, 2004, 2005; Burzio & Tantalo, 2007). The Attraction Framework introduces a family of segment-to-segment faithfulness constraints, $\text{ATTRACT}(P \rightarrow q)$, which encode a dependency between parasitic and harmonic features. These constraints preserve the operation of entailments between segments in a surface form by way of a family of correspondence constraints, $\text{CORR}$, which, like $\text{ATTRACTION}$, are ranked according to similarity.
The investigation of parasitic vowel harmony and local and non-local nasal harmony illustrates how the Attraction Framework can unify aspects of consonant and vowel harmony by providing a single family of harmony drivers. These studies also show how this unification does not come at a cost of empirical adequacy in the face of complex transparency and blocking phenomena. Transparency occurs when positional similarity conditions are relaxed but feature similarity conditions are increased. In contrast, blocking occurs when positional similarity is a more important component of representational similarity. Lastly, these studies show the importance of the availability of low-level phonetic detail to explain patterns of participation and non-participation. For instance, phonetic similarity explains why low, unround and non-low round vowels pattern together in Finnish vowel harmony (both groups of vowels have a [lower F2]), and phonetic similarity also explains the participation patterns of V-to-V harmony in Moba Yoruba, which while triggered by both high and non-high vowels, only allows high targets (nasal and high vowels, but not low vowels, yield a [low frequency formant]).

This chapter also discussed how the Attraction Framework’s general similarity perspective has ties to kernel methods in machine learning, where a grammar is a kernel which manages to map triggers and targets to proximate points in harmony space, but triggers and neutrals to distal points in the same space. Locality and features play equal parts in the GPS kernels. Therefore, the Attraction Framework can subsume both the clustering of non-local elements described in autosegmental theory by tier projection and the clustering of similar features described in feature geometry due to node subdivision. Both clustering effects are merely the result of similarity in a representational space.
The ubiquity of prerequisite similarity effects in assimilation (either in terms of locality preconditions, feature preconditions, or both) confirms that the generality of the approach is on the right track. Furthermore, parasitic and other dependencies only develop between features which are phonetically similar. Likewise, non-local interaction is only possible in the face of increased feature similarity because, otherwise, more proximate neutral interveners would be available as triggers and targets. Therefore, the OT-rankings of this chapter should be seen as an instantiation of the weighting of aspects of representational similarity, so any concerns about restrictiveness are repudiated by a grounding in similarity. Dependency occurs where, and only where, phonetic similarity exists.

Taken together, these results argue that subsymbolic representations and processes can play a dominant role in the higher level phonological patterns known as assimilation. Many aspects of an assimilation pattern are due to language specific factors, including the choice of parasitic features and harmonic features, the exploited phonetic dependency between parasitic and harmonic features, and the positional similarity requirements. Nevertheless, the computational underpinnings that predict the weakening of attraction forces as general representational similarity decreases are common across all patterns of assimilation.
5. Similarity bias in Entailment Networks

5.1. Introduction

This chapter presents methods and simulations which explore the similarity biases of a kind of connectionist network that implements principles of attraction. I call these networks *Entailment Networks* because the method of learning instantiates entailments in the connections between units. The theme of this chapter is to derive central assumptions of the theoretical analysis in Chapters 2-4 from lower-level connectionist principles. I aim to show that, in many ways, these grammar-level concepts are more naturally framed within a connectionist implementation (see Smolensky & Legendre, 2006 for other benefits of having multiple levels of explanation).

For example, the theoretical analysis postulates the existence of a family of *ATTRACTION* constraints whose strength depends on the similarity of triggers and targets. At the higher level, these attraction pressures can be analyzed using representational entailments (Burzio, 2002a, b, 2004, 2005; Burzio & Tantalo, 2007). The connectionist level can inform this grammar-level concept by offering insight into why these attraction constraints have the form that they do. The learning rule of Entailment Networks is based on Hebbian learning (Hebb, 1949) which conceptually can be stated as ‘units that fire together, wire together.’ Besides simplicity, Hebbian learning has the virtue that it gives a fully-connected network which guarantees that the network is Harmony maximizing (Smolensky, 1986; Smolensky & Legendre, 2006). The focus of §5.2 is to
confirm that Harmony maximization in an Entailment Network exhibits properties of attraction, where the pressure for change falls off with the distance between representations.

Thus, by reducing a grammar-level concept (OT-style entailment attraction constraints) to a connectionist implementation (Harmony maximization in Entailment Networks), it is not only possible to definitively test whether the grammar-level concept has desired the consequences (e.g. sensitivity to similarity), but it is also possible to understand why the grammar-level concept is active in the first place. The grammar works how it does because the computational system has specific properties that give rise to the grammatical behavior. Furthermore, unlike the grammar-level concepts, these computational properties could be tested against additional computational criteria including biological plausibility, efficiency, and complexity. Success on these criteria validates the grammar-level assumptions against processing needs, providing further evidence for grammar-level concepts.

In this manner, the computational work in this chapter is used to support the theoretic analysis in the rest of the dissertation. In particular, I propose to support four elements of the theoretical analysis through computational modeling, namely:

1) The existence of entailment-based attraction constraints.
2) The existence of segment-to-segment correspondence.
3) The relationship between parasitism and sensitivity to similarity.
4) The typology of the interaction between proximity and feature-based similarity.

The table in (1) below provides a preview of how these grammar-level concepts are reduced to a connectionist-level concept. Each reduction is discussed in the indicated section.
(1) Multiple levels in a theory of attraction

<table>
<thead>
<tr>
<th>Grammar-level Concept</th>
<th>Network-level Concept</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entailment-based attraction constraints</td>
<td>Hebbian learning representations with Harmony maximization</td>
<td>§5.2</td>
</tr>
<tr>
<td>Segment-to-segment correspondence</td>
<td>Tensor product representations with similar role vectors.</td>
<td>§5.3-5.4</td>
</tr>
<tr>
<td>Parasitic dependency and similarity</td>
<td>Harmony maximization over tensor product representations.</td>
<td>§5.5</td>
</tr>
<tr>
<td>Typology of proximity and feature-based similarity</td>
<td>Biases of Entailment Networks</td>
<td>§5.6</td>
</tr>
</tbody>
</table>

In order to encode phonological strings, I adopt the tensor product representations (Legendre, Miyata, & Smolensky, 1992) of a role-filler system in the Integrated Connectionist-Symbolic (ICS; Smolensky & Legendre, 2006) cognitive architecture. The main result of §5.3 is to show that the similarity of two role-filler pairs is the product of the similarity of fillers and the similarity of the roles. A direct consequence of this result is the hypothesis that Entailment Networks with role-filler systems should be sensitive to similarity in the ways Attraction was used in Chapters 3 and 4.

Following the presentation of these methods, this chapter presents a series of simulations which explore the similarity biases of Entailment Networks. Unlike problem sets in machine learning, where the training sets are often large and varied, here, the training is tightly controlled in order to better understand the behavior of the networks. Thus, the aim of this chapter is not to show that the network can learn particular data sets, but rather to reveal the inherent biases in the network through simulations. Simulation #1 (§5.4) confirms that the motivations for harmony are relatable to a kind of entailment persistence that only exists in a network where there is an amount of similarity between positions (role vectors). Simulation #2 (§5.5) shows how parasitic dependencies develop in the context of increased similarity between segments (fillers), confirming the previous
arguments in this dissertation that (i) parasitic dependency derives from an amount of phonetic similarity and (ii) increased feature similarity is needed for the neutrality of intereners in non-local interaction (on this point see also Archangeli & Pulleyblank, 2007; Ajibóyè & Pulleyblank, ms, 2008). Finally, Simulation #3 (§5.6) tests the hypothesis that Entailment Networks are inherently sensitive to similarity, and so certain classes of unattested languages ought to remain unlearnable. The results of Simulation #3 confirm that Entailment Networks with tensor product representations are biased against learning anti-similarity languages, such as anti-parasitic harmony or anti-local harmony.

Simulations #1 and #2 use an experimental paradigm based on the property of ‘Richness of the Base’ (Prince & Smolensky, 2004), where the network is exhaustively trained equally on all possible inputs (Wayment, Burzio, Mathis, & Frank, 2007). In such a case, any biases that emerge must derive from inherent properties of the network and do not follow from any factors of the training data. In contrast, Simulation #3 uses a paradigm related to Wilson’s (2006) “Poverty of the Stimulus” (POTS) method of training subjects in an artificial grammar learning task. In POTS-training, a network is trained on only a subset of the possible inputs. However, the network is tested on all possible forms, so any patterns of behavior that emerge on the novel test forms reveal an inherent generalization bias of the network. Therefore, the results of Simulations #1, #2, and #3 ought to be seen as broadly applicable in deriving the behavior of ATTRACTION constraints used elsewhere in this dissertation. These subsymbolic foundations provide insight into the sources of harmony and the existence of feature similarity and positional similarity preconditions on harmony.

Finally, §5.7 discusses some of the implications of these connectionist underpinnings on the analysis of the preceding chapters and in relation to others’ work, in particular, I consider the role of phonetic grounding, the viability of formal correspondence, and directions for future research involving simple recurrent networks.
Methods for attraction in Entailment Networks

Previous work (Wayment, Burzio, Mathis, and Frank, 2007) has shown that an advantage of using attraction in the entailment framework is a natural computational implementation in a neural network. The kinds of optimization computations in Chapter 4 that were performed in Optimality Theory can also be tested in network simulations, which instantiate entailments in the connections between units. This section highlights the relationship between the Entailment Framework (Burzio, 2002a,b, 2005) and a special class of networks, which I call Entailment Networks, which exhibit attraction forces by instantiating entailments in the connections between units. The reported results confirm that there exists a tight coupling between Entailment Theory and connectionism.

Formally, the Representational Entailment Hypothesis (REH) (Burzio, 2002a, b) may be defined as in (2) below (see Wayment et al, 2007).

(2) **Def:** A representation $R=(C, E)$ consists of

i. A set of components $C=\{A_1, A_2, \ldots A_n\};$

ii. A set of (logical) entailments, $E$, such that for all $1 \leq i, j \leq n$, the entailment $A_i \rightarrow A_j$ is an element of $E$.

For example, a representation of the back, round, high vowel [u] has feature-value components $\{[+back], [+round], [+hi]\}$ as well as the following entailments:

(3) **Entailments of** $[u] = \{[+back], [+round], [+hi]\}$

<table>
<thead>
<tr>
<th>+back $\rightarrow$ [+back]</th>
<th>+round $\rightarrow$ [+back]</th>
<th>+hi $\rightarrow$ [+back]</th>
</tr>
</thead>
<tbody>
<tr>
<td>+back $\rightarrow$ [+round]</td>
<td>+round $\rightarrow$ [+round]</td>
<td>+hi $\rightarrow$ [+round]</td>
</tr>
<tr>
<td>+back $\rightarrow$ [+hi]</td>
<td>+round $\rightarrow$ [+hi]</td>
<td>+hi $\rightarrow$ [+hi]</td>
</tr>
</tbody>
</table>
An entailment can be interpreted as a co-occurrence dependency: for an entailment $A_i \rightarrow A_j$ the presence of $A_i$ as a component of a representation entails that $A_j$ is also a component of that representation. Thus, an entailment of $R$ is violated by another representation $R'$ if $A_i$ is a component of $R'$ but $A_j$ is not. By postulating that the system of mental representations is organized so as to minimize entailment violations, the REH has been successfully used to account for phenomena in phonology and morphology (Burzio, 2002a, b, 2004, 2005; Burzio and Tantalou, 2007). Minimizing entailment violation derives attraction effects because the strength of the attraction force between two representations can be measured by the change in entailment violation when a single differing component is altered (see §3.2-3.4 for more discussion of entailments and how they relate to ATTRACTION constraints).

The remainder of this section shows how entailments can be instantiated in a special kind of connectionist network, called an Entailment Network. An Entailment Network is a fully-connected, Hopfield network (Hopfield, 1982) with symmetric connections. Entailment Networks are an extension of Hopfield networks which support superposition representations (Smolensky & Legendre, 2006) and use Hebbian learning (Hebb, 1949) to learn the weights on the connections between units.

Hopfield networks are known to maximize Harmony (Smolensky, 1986; Smolensky & Legendre, 2006). Harmony is a way of measuring the degree to which a pattern of activation agrees with the weights of a network. Harmony maximizing networks utilize activation functions which tend to move the network from lower Harmony to higher Harmony states. Because Hopfield networks have symmetric
connections, the network dynamics are such that Hopfield networks maximize the standard quadratic Harmony function given in (4) as activation flows around the network:

\[
\text{Def: } H_{W}(\textbf{A}) = \textbf{A} \cdot \textbf{W} \cdot \textbf{A}^T = \sum_{i,j} a_i w_{ij} a_j .
\]

Intuitively, Harmony is the sum of how much the activation of each pair of units \((a_i \text{ and } a_j)\) agrees with the weight between them \((w_{ij})\), so high Harmony means that \(\textbf{A}\) generally agrees with the weights between units, while low Harmony means that \(\textbf{A}\) generally disagrees with the weights between units.

For example, if \(w_{ij} = -1\), then there is an inhibitory connection between \(a_i\) and \(a_j\). Thus, if \(\textbf{A}\) has \(a_i = 1\) and \(a_j = -1\), then the pattern expresses that inhibition of \(w_{ij}\) and so \(\textbf{A}\) agrees with \(w_{ij}\) and has a Harmony increased by \(a_i w_{ij} a_j = (1) \cdot (-1) \cdot (-1) = 1\). On the other hand if \(\textbf{A}'\) has \(a'_i = 1\) and \(a'_j = 1\), then \(\textbf{A}'\) ignores the inhibition denoted by \(w_{ij} = -1\), and so \(\textbf{A}'\) does not agree with \(w_{ij}\) and has a Harmony decreased by \(a'_i w_{ij} a'_j = (1) \cdot (-1) \cdot (1)\). If there are no other connections or units then, Harmony expresses a difference in the agreement to the connections, since \(H(\textbf{A}) = 1 > H(\textbf{A}') = -1\).

A Harmony landscape gives a pictorial representation of the Harmony of different network states, an example of which is shown in (4). Because the network maximizes Harmony, higher Harmony states can be viewed as attractors, which pull nearby less harmonic patterns towards the Harmony maxima. Learning can therefore be understood as instantiating a set of attractors. Harmony then measures which network states are the strongest attractors.

\[118\text{ N.b. case marking on harmony and Harmony. The former is a phonological process of assimilation. The later is fitness to the connections in a network.}\]
Harmony landscape. The peaks in this landscape are the attractors of this network.

Harmony measurements demonstrate that Hebbian learning suffices for instantiating a set of entailments in the connections between units. Consider representations with two units (components), A and B. Suppose a representation \( R_{AB} = [+A, -B] \) is encoded as \( A = 1, B = -1 \), or equivalently as the vector \( R_{AB} = [1 -1] \). The set of entailment corresponding to this representation by applying (2) is given below:

\[
\begin{align*}
A = 1 &\implies A = 1 \\
B = -1 &\implies A = 1 \\
A = 1 &\implies B = -1 \\
B = -1 &\implies B = -1
\end{align*}
\]

Hebbian learning increases the strength of the connections between units which have the same activation values, and decreases the strength of connections between units with opposing activation values. As shown in Smolensky & Legendre (2006), Hebbian learning of a pattern \( \mathbf{R} \) in a Hopfield network corresponds to the weight update equation in (7).
(7) **Def:** The Hebbian learning rule is \( \Delta W = \eta \mathbf{R} \otimes \mathbf{R} \), where \( \eta \) is the learning rate, and \( \otimes \) is the outer, or ‘tensor’ product.\(^{119}\)

(8) gives the weight matrix when an instance of Hebbian learning is performed on the representation \( \mathbf{R}_{AB} \) with \( \eta = 1 \).

\[
W_{\mathbf{R}_{\overline{A}B}} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}
\]

Under Harmony measures, the weight matrix in (8) preserves the preferences given by the set of entailments in (6). Below, (9) shows how the Harmony due to the connection from A to B \( w_{AB} \) in \( \mathbf{W}_R \) matches the violation profile of the R-entailment \( A=1 \rightarrow B=-1 \) for different values of A and B.

(9) *The entailment constraint \( A \rightarrow B \) derived from Harmony:*

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>A=1→B=-1</th>
<th>( \mathbf{H}<em>{AB} = \mathbf{A} \cdot \mathbf{W}</em>{AB} \cdot \mathbf{B} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>( \mathbf{R}_{AB} = 1 )</td>
<td>1</td>
<td>*</td>
<td>-1</td>
</tr>
<tr>
<td>b.</td>
<td>( \mathbf{R}_{AB} = 1 )</td>
<td>-1</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>c.</td>
<td>( \mathbf{R}_{AB} = -1 )</td>
<td>1</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>d.</td>
<td>( \mathbf{R}_{AB} = -1 )</td>
<td>-1</td>
<td></td>
<td>-1</td>
</tr>
</tbody>
</table>

(9)(a) violates the \( \mathbf{R}_{AB} \)-entailment \( A=1 \rightarrow B=-1 \) because \( A=1 \), but \( B = 1 \), not the -1 demanded by the entailment. (9)(a) also has lower Harmony: \( \mathbf{H}_{\overline{A}B}([1 \ 1]) = 1 \cdot -1 \cdot 1 = -1 \), so lower Harmony can be related to violating entailments. Both (9)(b) and (9)(c) do not violate the entailment and have higher Harmony, so the entailment and Harmony score have the same preferences for (9)(a-c). Note, however, that (9)(d) does not violate the

\(^{119}\) An outer product differs from the perhaps more familiar inner or dot product. In a dot product \( \mathbf{A} \cdot \mathbf{B} \) the components of A and B are pairwise multiplied and added together: \( \mathbf{A} \cdot \mathbf{B} = a_1b_1 \ldots a_nb_n \). In a tensor product, the components are again multiplied, but this time all possible pairs are multiplied and each multiplication becomes a component in a new matrix. \( \mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_1b_1 & \ldots & a_1b_n \\ \vdots & \ddots & \vdots \\ a_nb_1 & \ldots & a_nb_n \end{bmatrix} \).
entailment because the antecedent is false, while (9)(d) still has lower Harmony,

\[ H_{AB}([-1 -1]) = -1 -1 -1 -1 = -1. \]

This case shows that the Harmony along a single connection is close to but not exactly like the corresponding entailment, because connections in a network enforce a biconditional via Harmony, e.g. \( A = 1 \leftrightarrow B = -1 \). Fortunately, the REH hypothesizes mutual entailments. Thus, by the definition in (2), for each entailment \( A_i \rightarrow A_j \) there exists an entailment \( A_j \rightarrow A_i \). Although an individual entailment does not correspond to the Harmony along a single connection, the information encapsulated in the set of entailments is manifest in the total Harmony of an Entailment Network. In fact, Harmony \( H = \# \) of satisfied entailments \( - \# \) of violated entailments.

Because of this close coupling between Harmony and entailment violation, it is not surprising that Harmony in an Entailment Network is sensitive to similarity in the same way that entailment constraints in the OT analysis in Chapter 3-4 were also sensitive to similarity. The following theorem which is proven in the Appendix (§5.9) expresses the relationship between Harmony and similarity.

\[(10) \text{ Theorem:} \quad \text{If } T_1, \ldots, T_n \text{ are a set of patterns that are presented to an Entailment Network, } H, \text{ which learns these patterns through Hebbian learning, then for all patterns of activation } A, \quad H_H(A) = \sum_i (A \cdot T_i)^2.\]

(10) demonstrates that Harmony is a measure of the average squared similarity (measured by the inner product, \( \cdot \)) between the test pattern and the training patterns. The averaging comes from the cumulative properties of Hebbian learning in Hopfield networks. This theorem confirms that Entailment Networks are indeed sensitive to similarity between the test pattern and the training patterns because the more typical a pattern is of the training data, the higher its Harmony will be. It remains of interest to determine whether
Entailment Networks are sensitive to similarity in ways consistent with linguistic generalizations.

In Wayment et al. (2007), we argued that phonetic enhancement effects (Stevens, Keyser, & Kawasaki, 1986) could be derived from sensitivity to similarity. For example, in vowel inventories, [+round, +back] vowels tend to contrast with [−round, −back] vowels without further contrast because of the mutually enhancing nature of the features round and back (Stevens, Keyser, & Kawasaki, 1986). [+round] and [+back] contribute to a lower second formant (F2). Therefore, a preference for phonetically enhanced features ought to disperse the categories defined by backing and rounding at maximal F2 distances. We aimed to show that by encoding rounding and backing on an F2 scale, [+round] and [+back] would be similar (likewise [−round] and [−back] would be similar), and vowels which matched on backing and rounding would, therefore, be preferred by the Entailment Network in terms of higher Harmony.

To test this hypothesis, the set of training patterns was controlled so as to investigate whether the inherent biases of Entailment Networks were consistent with phonetic enhancement. In particular, to test whether Entailment Networks exhibited a preference for the enhanced combinations [+round, +back] and [−round, −back], an Entailment Network was presented with vowels corresponding to all possible combinations of backing and rounding. In this case, if there was any difference in the Harmony of the different vowels, then it must result from a bias in internalizing the training patterns, not because of any asymmetry in frequency of presentation. This explored the biases of Entailment Networks by examining the network’s behavior in a “Richness of the Base” paradigm, in which all patterns have equal evidence. The results
of training equally on combinations of backing and rounding were that the enhanced combinations have higher Harmony than the unenhanced combinations. We therefore concluded that the Entailment Network’s sensitivity to similarity was sufficient to give dispersion-like preferences without any special system which computes the distance between categories, because while Entailment Networks do act so as to maximize Harmony, they do not explicitly compute any sort of distance between learned patterns.

§3.4 and §4.5.4 provide similar explanations (although at a less technical level) for how the clustering of phonetic enhancement relates to the clustering of, respectively, parasitic dependency in cases of phonetic similarity and co-harmony à la feature geometry, where a non-terminal node allows features to spread together.

These results and the theorem in (10) give reason for optimism that the kinds of similarity based generalizations in Chapters 2-4 can be modeled in Entailment Networks, but as illustrated in §5.3, there is still one significant technical hurdle that must be overcome before the work of Wayment et al. (2007) can be extended to assimilatory phenomena.

### 5.3. Representing positions with a role-filler system.

The previous work modeling phonetic enhancement differs from assimilation in an important way. The enhancement task of Wayment et al. (2007) required the network to learn a distribution over the universe of vowels that can be part of a phonological inventory. This is somewhat different from learning a distribution over strings of phonological material because the network not only has to represent the combination of features as phonemes, but additionally the network must bind feature bundles to structural positions. It is a network engineering problem to find an encoding for positions
that is consistent with phonological hypotheses. As discussed below, I propose to instantiate a similarity sensitive role-filler system by using tensor product representations (Legendre, Miyata, & Smolensky, 1992; Smolensky & Legendre, 2006).

The analysis of parasitic vowel harmony in §4.3 and nasal harmony in §4.4 showed that by allowing segments to participate in attraction relationships, it is possible to derive the kinds of long-distance dependencies that give rise to assimilatory processes. This attraction relationship is posited to exist because of entailment persistence (§3.5.1) which is instantiated at the OT-grammar level with correspondence (§4.2). This section takes up the issue of how segments in different positions might come to be in segment-to-segment attraction relationships. The connectionist solution is also grounded in tensor product representations.

Tensor product representations (Legendre, Miyata, & Smolensky, 1992; Smolensky & Legendre, 2006) were proposed to solve the binding problem, i.e. how the content of different structural positions may be distributed across a set of units without losing the important information concerning which content was bound to which position. For example, the string \(ba\) differs from the string \(ab\) despite the fact that both strings contain one \(a\) and one \(b\) because the content (\(a\) and \(b\)) is associated with different structural positions (first position and second position) in the two strings. A network that is unable to bind content to positions loses this information about the position of content, and is therefore unable to distinguish \(ba\) and \(ab\).

Binding is not merely a technical worry. A binding failure is catastrophic in terms of matching the phonological generalizations for harmony languages. For example, in a directional harmony system, a failure to bind makes it impossible for the network to
distinguish sources from possible targets, making it impossible for the network to
determine which vowels should undergo assimilation. In ATR harmony, if [αATR]
spreads right, then /ui/ → [i] and /ui/ → [ii], but if binding fails, then /ui/ and /ui/ have
identical representations, so because it is impossible for a deterministic network to give
different outputs for an input, /ui/ and /ii/ will have identical outputs. Therefore, even this
simplest of harmony languages is impossible to model in a network without binding
*fillers* (content) to *roles* (structural positions).

A tensor product representation is constructed by taking the tensor product of a
role-filler pair. Throughout this discussion fillers are phonological segments and roles
are positions in strings. For example, if the filler /i/ is represented with distributed vector
representation \( i = [-1 1 -1] \), where the components correspond to respectively [±cons.],
[±hi], and [±ATR], and if the first position in a string is encoded with the vector
\( X_1 = [1 0 1] \), then binding \( i \) to \( X_1 \) is performed by taking their tensor product as given in
(11).

(11) **Tensor product representation of \( i \) in the first string position, \( X_1 \):**

\[ i \otimes X_1 = [-1 1 -1] \otimes [1 0 1] = \]

\[
\begin{array}{c|c|c}
\otimes & 1 & 0 & 1 \\
\hline
-1 & (-1 \cdot 1) = -1 & (-1 \cdot 0) = 0 & (-1 \cdot 1) = -1 \\
1 & (1 \cdot 1) = 1 & (1 \cdot 0) = 0 & (1 \cdot 1) = 1 \\
-1 & (-1 \cdot 1) = -1 & (-1 \cdot 0) = 0 & (-1 \cdot 1) = -1 \\
\end{array}
\]

Thus, the tensor product of two vectors is a matrix where each entry in the matrix is a
pair-wise product of the components of the vectors. The matrix in (11) corresponds to a
distributed pattern of activation across an entire network, encoding the binding of \( i \) to
position \( X_1 \). This binding of a single filler to a single role is called a role-filler pair.

Strings are formed from role-filler pairs by taking the superposition (direct sum) of role-filler pairs. Thus, the string /ii/ is encoded as \((i \otimes X_1) \oplus (i \otimes X_2)\), where \( i \) and \( i \) are distributed phoneme-filler vectors and \( X_1 \) and \( X_2 \) are positional-role vectors.

An alternative to tensor product representations for binding fillers to roles is to localize specific positions in a group of units that refer to the possible fillers in that position. In such a network, there is a copy of the filler network for each possible role. If this were the mental organization used for human phonology, it seems unlikely that there should arise much dependency between different positions because each structural position is a unique sub-network. Demonstrating this fact is the focus of Simulation #1 in §5.4. Fortunately, tensor product representations reject this kind of localization by distributing a role-filler pair across the entire network. Therefore, it is possible for different positions to directly interact because different positions share network resources, predicting that segment-to-segment dependencies might arise in order to ensure compatibility among these shared resources. This hypothesis can be made more concrete by using the language of similarity and Harmony from §5.2 as in (12).

(12) **Role-filler similarity hypothesis:** All else being equal, a network will have higher Harmony if similar roles are bound to fillers which are similar to one another, than if those similar roles are bound to fillers which are dissimilar.

If (12) holds, then the existence of the segment-to-segment attraction relationships used in *Chapter 4* can be derived from the use of a role-filler system in a network which is sensitive to similarity. Verifying (12) confirms whether or not fillers are in attraction relationships among the shared resources by testing whether similar fillers have higher Harmony than dissimilar fillers. If the network is sensitive to the Harmony, \( H \),
everywhere, not just among the shared resources of similar roles, then attraction in the
shared part of the network might lead to attraction in all parts of the network.
Definitively validating (12) requires (i) a mathematical understanding of similarity in
role-filler systems and (ii) testing those similarity relations in a system known to be
sensitive to similarity like Entailment Networks.

On this point, some mathematical work investigating similarity in role-filler
systems is worth reporting here:

(13) **Theorem (Similarity of role-filler pairs):** Let $f_1 \ldots f_n$ be a set of fillers and $r_1 \ldots r_n$ a set of roles, where filler $f_i$ is bound to role $r_x$ by the tensor product $f_i \otimes r_x$. If similarity in this role-filler system is computed using the inner product, $\bullet$, i.e.

$$sim(f_i, f_j) = f_i \bullet f_j$$
$$sim(r_i, r_j) = r_i \bullet r_j$$
$$sim(f_i \otimes r_x, f_j \otimes r_y) = (f_i \otimes r_x) \bullet (f_j \otimes r_y)$$

Then $sim(f_i \otimes r_x, f_j \otimes r_y) = sim(f_i, f_j) \cdot sim(r_x, r_y)$.

The proof of (13), which follows a straightforward derivation using linear algebra, is
found in the **Appendix**(§5.9). This theorem guarantees that the similarity of a filler and
role vector independently contribute to the similarity of filler-role pairs. (13) predicts
that (12) should hold in an Entailment Network because it is known that the Harmony of
these networks is maximized by increasing average similarity (see (10)), so if the roles
are somewhat similar ($sim(r_x, r_y) \geq 1$) then greatest similarity is obtained when fillers are
also similar ($sim(f, f) \geq 1$). Therefore, in a “Richness of the Base” paradigm, assimilated
strings which place similar fillers in similar roles are predicted to have higher Harmony
than strings which place dissimilar fillers in the similar roles. A study testing this
hypothesis is presented in the next section.
5.4. Simulation #1: Motivations for Assimilation

5.4.1. Methods

This dissertation posits a novel set of drivers for assimilation based on principles of attraction. These ATTRACTION constraints derive from the entailments of a trigger placing pressure on targets which meet feature and proximity preconditions to agree on harmonic features. This segment-to-segment interaction was hypothesized to exist through the persistence of entailment instantiated in OT by way of segment-to-segment correspondence constraints. Simulation #1 shows how segment-to-segment interaction can be derived from the similarity of roles in a role filler system.

This simulation also confirms the hypothesis in (12), which suggests that similar fillers should line up in similar roles. Role-filler similarity directly relates to segment-to-segment interaction because if there is no similarity between roles, then there is no motivation for harmony as positions are represented with distinct separate resources. However, if there is an amount of similarity between positional roles, then there is pressure for patterns to satisfy the connections to these roles by agreeing in other aspects as well.

The problem domain for this simulation is a simple spreading harmony system among strings that conform to the template CVCVC. In this domain, example surface forms contain the specifications for a harmonic feature, [F], along these CVCVC sequences. Thus, \( C_{[\+]} V_{[\+]} C_{[\+]} V_{[\+]} C_{[\+]} \) is fully harmonic, but \( C_{[\+]} V_{[\+]} C_{[\-]} V_{[\-]} C_{[\-]} \) is not. Hence, the set of possible fillers is \( \{ C_{[\+]}, C_{[\-]}, V_{[\+]}, V_{[\-]} \} \). A straightforward distributed encoding of these fillers is given below:
(14) **Filler vectors for Simulation #1:**

<table>
<thead>
<tr>
<th>±cons</th>
<th>C_+</th>
<th>C_-</th>
<th>V_+</th>
<th>V_-</th>
</tr>
</thead>
<tbody>
<tr>
<td>±F</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

Furthermore, I assume for the sake of clarity that the pattern to be learned is harmony to the first vowel, i.e. that the first vowel is a trigger, so in what follows, I focus on agreement to V_1. Note that with these vectors the similarities may be computed with the dot product, and as shown in the table in (15), both C_+ and V_+ are more similar than, respectively, C_- and V_-.

(15) **Similarity of Simulation #1 fillers:**

<table>
<thead>
<tr>
<th>sim(A, B) = A•B</th>
<th>C_+</th>
<th>C_-</th>
<th>V_+</th>
<th>V_-</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_+</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>C_-</td>
<td>0</td>
<td>2</td>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>V_+</td>
<td>0</td>
<td>-2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>V_-</td>
<td>-2</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

Therefore, if the network can learn a segment-to-segment dependency and the network is sensitive to similarity, then if V_1=V_+ there should be pressure for all positions to become [+] in order to agree with the trigger. Likewise, if V_1=V_- there should be pressure for all positions to become [−]. In particular, if the network learns an ATTRACTION constraint, then C_+V_+C_+V_+C_+ should be more harmonic than every other combination of C_±V_±C_±V_±C_± where V_1 is [+F]. Similarly, C_-V_-C_-V_-C_- should be more harmonic than every other combinations of C’s and V’s with V_1 = V_+.

In order to test that hypothesis, however, the network must have equal opportunity to learn the other patterns, so I employ the “Richness of the Base” paradigm (Wayment, et. al, 2007), where the network sees each possible example in the space with equal probability. Thus, the training set for Simulation #1 is a uniform distribution over the
examples below in (16). If the learning rate is held constant across each epoch of training, then the result of many iterations of training will have the same pattern (in the limit) as if the network were presented with each example exactly once. As a short cut to this end, here training consists of one-shot Hebbian learning of each string in (16), with the learning rate, \( \eta = 1/(\# \text{ of training examples}) \). Note because addition is commutative the effects are not dependent on order of presentation.

(16) \textit{Training set for Simulation \#1:}

Examples where \( V_1 = [+F] \)  

<table>
<thead>
<tr>
<th>( X_1 )</th>
<th>( X_2 )</th>
<th>( X_3 )</th>
<th>( X_4 )</th>
<th>( X_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C[+ )</td>
<td>( V[+] )</td>
<td>( C[+ )</td>
<td>( V[+] )</td>
<td>( C[+ )</td>
</tr>
<tr>
<td>( C[+ )</td>
<td>( V[+] )</td>
<td>( C[+ )</td>
<td>( V[-] )</td>
<td>( C[-] )</td>
</tr>
<tr>
<td>( C[+ )</td>
<td>( V[+] )</td>
<td>( C[-] )</td>
<td>( V[+] )</td>
<td>( C[+ )</td>
</tr>
<tr>
<td>( C[+ )</td>
<td>( V[-] )</td>
<td>( C[-] )</td>
<td>( V[-] )</td>
<td>( C[-] )</td>
</tr>
</tbody>
</table>

Examples where \( V_1 = [-F] \)  

<table>
<thead>
<tr>
<th>( X_1 )</th>
<th>( X_2 )</th>
<th>( X_3 )</th>
<th>( X_4 )</th>
<th>( X_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C[+ )</td>
<td>( V[-] )</td>
<td>( C[+ )</td>
<td>( V[+] )</td>
<td>( C[+ )</td>
</tr>
<tr>
<td>( C[+ )</td>
<td>( V[-] )</td>
<td>( C[-] )</td>
<td>( V[+] )</td>
<td>( C[-] )</td>
</tr>
<tr>
<td>( C[-] )</td>
<td>( V[+] )</td>
<td>( C[+] )</td>
<td>( V[-] )</td>
<td>( C[-] )</td>
</tr>
<tr>
<td>( C[-] )</td>
<td>( V[-] )</td>
<td>( C[+] )</td>
<td>( V[-] )</td>
<td>( C[-] )</td>
</tr>
</tbody>
</table>

One important simulation detail remains: determining what should be the role vectors \( X_1, \ldots, X_5 \) to encode the fillers from (14) in the required positions for each example in (16). For \textit{Simulation \#1}, I take the strategy of allowing the roles to control for positional similarity. Thus, consider a set \( X \) of role vectors which are not similar and a set \( Y \) of role vectors which have some similarity for adjacent positions, as defined below in (17):
Role vectors for Simulation #1:

<table>
<thead>
<tr>
<th>X role vectors are orthogonal:</th>
</tr>
</thead>
<tbody>
<tr>
<td>X₁</td>
</tr>
<tr>
<td>X₂</td>
</tr>
<tr>
<td>X₃</td>
</tr>
<tr>
<td>X₄</td>
</tr>
<tr>
<td>X₅</td>
</tr>
</tbody>
</table>

Y role vectors are not orthogonal Yᵢ•Yᵢ+1 = 1

| Y₁ | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| Y₂ | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| Y₃ | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| Y₄ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| Y₅ | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |

The above encoding is such that the X-vectors are orthogonal but the Y-vectors are not (the shading of cells above indicates cases where Y role vectors overlap).

The role-filler positional similarity hypothesis, (12), can be applied to the data set in (16) with the fillers in (15) and the roles in (17). The prediction is that similar fillers should only be under pressure to line up for the set of Y-roles which have similarity, but under the orthogonal X-vectors there should be no pressure for similar fillers to be adjacent. Thus, the simulation should show that when similar Y role vectors are used, the network learns an ATTRACTION constraint that prefers patterns which agree on [F] to those that do not, but when vectors without such positional similarity, like X, are used as role vectors, then the network should have no preference for assimilation given that the data contains equal examples of harmony and disharmony.

5.4.2. Results

The column below X in (18) gives the network harmonies for each pattern with respect to a set of weights for an Entailment Network that is presented with each pattern in (16) encoded with the fillers in (15) and the orthogonal X-roles from (17). Under the Y column, the network harmonies are shown for another Entailment Network that is
presented with the same patterns with the difference being that the similar Y-roles from (17) are used to encode the training patterns. Absolute harmonies between the X-network and the Y-network are not directly comparable because the X and Y vectors are of different length. To ameliorate, this dilemma I have also presented the “H%”, the Harmony percentile, where H% = 0.00 is the lowest Harmony pattern(s) and H% = 1.00 is the highest pattern(s).

(18) **Simulation #1.** Harmonies of surface patterns as measured by (4) in an Entailment Network which was instantiated with “Richness of the Base” training on the forms in (16) encoded with the roles from (15) and the roles from either X or Y in (17). Shaded fillers do not agree with the trigger ([+F] in (18)(A) and [−F] in (18)(B)):

A. \( V_1 = V_{[+]} \)

<table>
<thead>
<tr>
<th>Candidate surface forms:</th>
<th>X: ( H )</th>
<th>X: ( H% )</th>
<th>Y: ( H )</th>
<th>Y: ( H% )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. C[+] V[+] C[+] V[+] C[+]</td>
<td>30 1.00</td>
<td>118 1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. C[+] V[+] C[+] V[+] C[−]</td>
<td>30 1.00</td>
<td>94 0.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. C[+] V[+] C[+] V[−] C[+]</td>
<td>30 1.00</td>
<td>70 0.59</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. C[+] V[+] C[+] V[−] C[−]</td>
<td>30 1.00</td>
<td>86 0.73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e. C[+] V[+] C[−] V[+] C[+]</td>
<td>30 1.00</td>
<td>62 0.53</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f. C[+] V[+] C[−] V[+] C[−]</td>
<td>30 1.00</td>
<td>46 0.39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>g. C[+] V[+] C[−] V[−] C[+]</td>
<td>30 1.00</td>
<td>62 0.53</td>
<td></td>
<td></td>
</tr>
<tr>
<td>h. C[+] V[+] C[−] V[−] C[−]</td>
<td>30 1.00</td>
<td>86 0.73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>i. C[−] V[+] C[+] V[+] C[+]</td>
<td>30 1.00</td>
<td>94 0.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>j. C[−] V[+] C[+] V[+] C[−]</td>
<td>30 1.00</td>
<td>70 0.59</td>
<td></td>
<td></td>
</tr>
<tr>
<td>k. C[−] V[+] C[+] V[−] C[+]</td>
<td>30 1.00</td>
<td>46 0.39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>l. C[−] V[+] C[+] V[−] C[−]</td>
<td>30 1.00</td>
<td>62 0.53</td>
<td></td>
<td></td>
</tr>
<tr>
<td>m. C[−] V[+] C[−] V[+] C[+]</td>
<td>30 1.00</td>
<td>46 0.39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n. C[−] V[+] C[−] V[+] C[−]</td>
<td>30 1.00</td>
<td>30 0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>o. C[−] V[+] C[−] V[−] C[+]</td>
<td>30 1.00</td>
<td>46 0.39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p. C[−] V[+] C[−] V[−] C[−]</td>
<td>30 1.00</td>
<td>70 0.59</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(18 continued)

B. \( V_1 = V_{[-]} \)

<table>
<thead>
<tr>
<th>Candidate surface forms:</th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H</td>
<td>H%</td>
</tr>
<tr>
<td>a. C[-] V[-] C[-] V[-] C[-]</td>
<td>30</td>
<td>1.00</td>
</tr>
<tr>
<td>b. C[-] V[-] C[-] V[-] C[+]</td>
<td>30</td>
<td>1.00</td>
</tr>
<tr>
<td>c. C[-] V[-] C[-] V[+] C[-]</td>
<td>30</td>
<td>1.00</td>
</tr>
<tr>
<td>d. C[-] V[-] C[-] V[+] C[+]</td>
<td>30</td>
<td>1.00</td>
</tr>
<tr>
<td>e. C[-] V[-] C[+] V[-] C[-]</td>
<td>30</td>
<td>1.00</td>
</tr>
<tr>
<td>f. C[-] V[-] C[+] V[-] C[+]</td>
<td>30</td>
<td>1.00</td>
</tr>
<tr>
<td>g. C[-] V[-] C[+] V[+] C[-]</td>
<td>30</td>
<td>1.00</td>
</tr>
<tr>
<td>h. C[-] V[-] C[+] V[+] C[+]</td>
<td>30</td>
<td>1.00</td>
</tr>
<tr>
<td>i. C[+] V[-] C[-] V[-] C[-]</td>
<td>30</td>
<td>1.00</td>
</tr>
<tr>
<td>j. C[+] V[-] C[-] V[-] C[+]</td>
<td>30</td>
<td>1.00</td>
</tr>
<tr>
<td>k. C[+] V[-] C[-] V[+] C[-]</td>
<td>30</td>
<td>1.00</td>
</tr>
<tr>
<td>l. C[+] V[-] C[-] V[+] C[+]</td>
<td>30</td>
<td>1.00</td>
</tr>
<tr>
<td>m. C[+] V[-] C[+] V[-] C[-]</td>
<td>30</td>
<td>1.00</td>
</tr>
<tr>
<td>n. C[+] V[-] C[+] V[-] C[+]</td>
<td>30</td>
<td>1.00</td>
</tr>
<tr>
<td>o. C[+] V[-] C[+] V[+] C[-]</td>
<td>30</td>
<td>1.00</td>
</tr>
<tr>
<td>p. C[+] V[-] C[+] V[+] C[+]</td>
<td>30</td>
<td>1.00</td>
</tr>
</tbody>
</table>

These results show that when there is no similarity among role vectors, as in the Entailment Network for X role vectors, there is no pressure for assimilation because all possibilities are equally Harmonic. However, if even a small amount of positional similarity is available, then the Entailment Network prefers patterns which are fully harmonic to the triggering initial vowel. Thus, entailment training with Y-roles and the given fillers yields a general, strictly local, agreement constraint, like \textsc{attract}({}_{} \rightarrow [F]), where even though C’s and V’s differ in their specification of \([\pm \text{con}]\) there is still pressure for agreement on [F] between adjacent positions. Note the only difference between the simulations is the difference in the encoding of the role vectors, all other aspects including the training set, the filler vectors, and the learning rate remained constant.
It should also be said the X-Entailment Network did learn something. For example, the sequence $V_{[+]V_{[+]V_{[+]V_{[+]}}}}$ has a much lower Harmony ($H=4$) than any of the training patterns because it violates the phonotactics of the CVCVC template. The X-Entailment Network only failed to learn harmony. An examination of the X and Y patterns of activation for the optimal form $C_{[+]V_{[+]C_{[+]V_{[+]C_{[+]}}}}}$ illustrates why the X-network does not learn harmony. These patterns are shown in (19) below. Because the X-role vectors are orthogonal, a tensor product representation of a string in the X-network has localized representations of the fillers. These localized representations are listed below as columns with position labels, Pos$i$.


| [±con] | 1 | -1 | 1 | -1 | 1 | 0 | 0 | 0 | 0 |
| [±F] | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| a     | Pos$_1$ | Pos$_2$ | Pos$_3$ | Pos$_4$ | Pos$_5$ | Pos$_{1,2}$ | Pos$_{2,3}$ | Pos$_{3,4}$ | Pos$_{4,5}$ |

| [±con] | 1 | -1 | 1 | -1 | 1 | 0 | 0 | 0 | 0 |
| [±F] | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 |
|       | Pos$_1$ | Pos$_2$ | Pos$_3$ | Pos$_4$ | Pos$_5$ | Pos$_{1,2}$ | Pos$_{2,3}$ | Pos$_{3,4}$ | Pos$_{4,5}$ |

The representation of $C_{[+]V_{[+]C_{[+]V_{[+]C_{[+]}}}}}$ for Y has these same localized representations, but there also is activation on groups of units that are only non-zero if adjacent positions agree on the indicated feature on the vertical axis. Thus, after learning all the X-patterns, there are no connections to the Pos$_{i,i+1}$ units (since Hebbian learning on zero values results in no weight change), but after learning all the Y-patterns, there are connections between the Pos$_{i,i+1}$ and Pos$_i$ units that are only satisfied if the filler in Pos$_i$ and the filler in Pos$_{i+1}$ agree on [F].
5.4.3. Discussion

Simulation #1 confirms that Entailment Networks which learn patterns in a role-filler system are sensitive to the similarity of role vectors and the similarity of filler vectors. If role vectors are non-orthogonal, then there is increased pressure for filler vectors to agree. Thus, a set of reasonable general assumptions – (i) a simple Hebbian learning algorithm, (ii) equal presentation of possible patterns, (iii) a role-filler system to give binding, and (iv) an amount of similarity among role vectors for adjacent positions – derives the existence of spreading harmony from an inalterable trigger.

Other more technical assumptions also help this simulation achieve its results. First, similarity effects are enhanced by superposition, where, as in tensor product representations, the co-occurrence of two role-filler pairs is represented with their direct sum, denoted $\oplus$. This is because, as (19) shows, where role-filler pairs have similarity, activations add up, but where role-filler pairs have dissimilarity activations cancel. An increased activation makes a unit more impactful on the weight changes of Hebbian learning and therefore Harmony.

Second, these superposition representations are not normalized which is otherwise a somewhat standard network technique (see Bechtel & Abrahamsen, 2002). However, the lack of normalization is crucial because scaling patterns to be of unit length would obliterate some similarity differences. For instance, consider the superposition reps $A = [0 1]$, $B = [0 2]$, and $C = [0 3]$. $B$ and $C$ are more similar than $A$ and $B$ because $B$ and $C$ have more activation in the same direction (along the second unit). However, after normalizing $A = B = C = [0 1]$, so the similarity differences between $A,C$ and $B,C$ are lost by normalizing the representations.
Thirdly, of course, that there is a difference between \( \text{sim}(A,B) \) and \( \text{sim}(B,C) \) in the first place hinges on the assumption of dot-product similarity. Other similarity metrics, such as cosine similarity, would predict that there is no difference between A, B, and C. It remains an open question what kinds of similarity metrics allow networks to learn harmony over tensor product representations. Dot-product similarity is sufficient, but it is unknown whether it is necessary.

Fourth, dot-product similarity has a profound effect on Harmony if the Harmony function is the standard quadratic Harmony function (see (4)). This is because quadratic Harmony is roughly quadratic in the length of the vector and so is dot-product similarity. Thus, because A, B have the same relationship between units but are of different length (\(|A|= 1 \) and \(|B|= 2\)), the magnitude of the Harmony of B, \( |H(B)|_{\text{abs}} \), is necessarily greater than or equal to the magnitude of the Harmony of A independent of the weight matrix. This is, perhaps, a natural postulate because the dynamics of a network should respond to the strongest connections and Hebbian learning forms the strongest connections among the most active units, but it is also a crucial assumption that allows the use of difference in Harmony as an indicator of network fitness. If the same network representations were instantiated in a network with different Harmony dynamics, then there is no guarantee that the fully-harmonic pattern would be the most Harmonic in the network.

Fifth, the network in Simulation #1 successfully learned a dependency between adjacent segments for both [+F] and [−F]. This is possible because the fillers encoding [F] were chosen such that [+F] = - [−F]. [+F] = - [−F] affects Hebbian learning because Hebbian learning multiplies the values of connected units in order to determine the weight change, so when a pair of [+F] fillers in one example and a pair of [−F] fillers
another example occur in the same roles, then they will have an identical effect on the changes to the weight matrix. For instance, Hebbian learning a pattern $[1 \ 1]$ increases the connection between the units $(1 \cdot 1 = 1)$, likewise Hebbian learning a pattern $[-1 \ -1]$ also increases the connection between the units $(-1 \cdot -1 = 1)$.

Thus, it is a natural, but crucial assumption to posit that for a bivalent feature the different feature values take on exact opposite representations in a network. Note, however, that the results of learning $[+F]$ and $[-F]$ also obtain if $[+F]$ is orthogonal to $[-F]$, but then the network would actual develop into two subnets one that learns $[+F]$ harmony and another that learns $[-F]$ harmony. $[+F] = - [-F]$ forces the network to learn both $[+F]$ and $[-F]$ harmony using the same connections and under Hebbian learning it succeeds because of the sign properties of multiplication. Without $[+F] = - [-F]$, there would be interference instead of reinforcement between $[+F]$ and $[-F]$ harmony.

Now, returning to how the results of Simulation #1 relate to harmony more generally, note that the above Y-network only learned a segment-to-segment attraction constraint. Other constraints, like markedness or IO-faithfulness, would need to be independently instantiated. It is for this reason that full harmony is preferred to any amount of disagreement along the harmonic feature in the table in (18). Note that the fewer number of harmonic domains, the higher the Harmony.

This simulation also vindicates the hypothesis of positional similarity, since the correspondence constraints (used in Chapter 4 to give locality effects) can be expressed as role vectors with varying degrees of similarity. A language without harmony encodes strings with role vectors more like orthogonal X-vectors, while a language with spreading harmony encodes strings with role vectors more like similar Y-vectors. Furthermore,
entailment persistence follows directly from the sharing of resources (the Pos_{ij} units) in the network.

Tensor product representation can also express how harmony can be motivated by a penalty for resetting articulatory targets (Hansson, 2001, 2007; Rose & Walker, 2004) which explains a number of speech errors which pattern with harmony (Walker, 2007). These penalties are directly expressible in terms of Harmony: fillers that share overlapping network resources (like the Pos_{ij} units), but disagree in harmonic features, incur a Harmony penalty because of conflicting activation of both [+F] and [−F] along these shared resources. Of course, if these Harmony penalties are to persist non-locally as is needed for non-local consonant and vowel harmony, the encoding of non-adjacent positions must also share network resources by having similar role vectors. A study of non-adjacent harmony follows in the next section.

5.5. **Simulation 2: Parasitic assimilation**

5.5.1. **Methods**

In the infinite space of activation vectors, it would be possible to construct role vectors such that non-adjacent Pos_{i} and Pos_{i+2} are more similar than adjacent Pos_{i} and Pos_{i+1}. However, cross-linguistically the availability of non-local harmony implies the possibility of local harmony; there are no known anti-local harmony systems that allow harmony at greater distances without also permitting identical targets to undergo harmony in proximate positions. Such an unattested anti-local vowel harmony would allow harmony across a transparent vowel, but not when the trigger and target are
For this reason, it seems that the way positional similarity is expressed in role vectors is through the use of overlapping windows of persistence, where resources which are shared by Pos_i and Pos_{i+2} are necessarily also shared by Pos_{i+1}. Thus, the role vectors that allow non-adjacent harmony must be of the sort seen below in (20).

Again, shading indicates the overlap that determines the shared resources between the positions indicated in the Pos_i labels.

(20)  Role vectors for Simulation #2:

<table>
<thead>
<tr>
<th></th>
<th>Pos_1</th>
<th>Pos_2</th>
<th>Pos_3</th>
<th>Pos_4</th>
<th>Pos_5</th>
<th>Pos_{1,2}</th>
<th>Pos_{2,3}</th>
<th>Pos_{3,4}</th>
<th>Pos_{4,5}</th>
<th>Pos_{1,2,3}</th>
<th>Pos_{2,3,4}</th>
<th>Pos_{3,4,5}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z_1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Z_2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Z_3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Z_4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Z_5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

With this fixed set of Z role vectors that allow non-adjacent harmony, Simulation #2 illustrates that harmony can only obtain if there is increased similarity between the filler vectors. Put another way, because languages encode positions such that the resources which are shared between Pos_1 and Pos_3 are also shared by Pos_2, the only way for non-local harmony from Pos_1 to Pos_3 to obtain (without also assimilation by Pos_2) is if the similarity between the fillers in Pos_1 and Pos_3 is much stronger than the forces that might otherwise demand strictly local harmony.

---

120 Anti-local harmony differs from domain sensitive systems, where certain domains are privileged as targets of harmony, e.g. stressed (Walker, 2001) or unstressed position (Pulleyblank, 2002). In anti-local vowel harmony, it is distance and no other factors that mitigate the conditions on harmony.

121 This may express a temporal component of representation (as in a recurrent network) that is not explored further in the present work.

122 The necessity of these sort of role vectors with increasing overlap provides a way for first, second, and even third order statistics to be stored through Hebbian learning. Thus, the simple learning algorithm benefits from the representational richness. Furthermore, unlike learning with hidden units, in an Entailment Network, the learning process does not have to discover which statistics are important. The representations directly inform which kinds of statistics (the locality based statistics) are important. Also, it is not surprising that increasing the dimensionality of role vectors allows for more computational power, which emerges here as an ability to express non-local harmony (see the discussion in §4.5 on the relationship between kernel methods and the Attraction Framework).
**Simulation #2** fixes the role vectors to the Z-role vectors in (20) and shows a contrast in non-local harmony for the less similar fillers in (21) and the more similar fillers in (22). Only the increased similarity provided by the addition of feature similarity here an abstract prerequisite feature \([p_3]\), allows for non-local harmony to take place. This confirms why parasitism is more dominant in non-local harmony (an issue raised by Rose & Walker, 2004; Hansson, 2001), and also explains an attested tradeoff, whereby as the distance between triggers and targets increases, the feature similarity must also increase in order for harmony to obtain (Archangeli & Pulleyblank, 2007; Ajibóyè & Pulleyblank, ms. 2008).

(21) *Less similar filler vectors, LS, for Simulation #2:*

<table>
<thead>
<tr>
<th></th>
<th>(C_{[+]})</th>
<th>(C_{[-]})</th>
<th>(V_{[+]})</th>
<th>(V_{[-]})</th>
</tr>
</thead>
<tbody>
<tr>
<td>([\pm\text{cons}])</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>([\pm F])</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>([p_1])</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>([p_2])</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

\[\text{sim}(A, B) = A \bullet B\]

<table>
<thead>
<tr>
<th></th>
<th>(C_{[+]})</th>
<th>(C_{[-]})</th>
<th>(V_{[+]})</th>
<th>(V_{[-]})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_{[+]})</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>(C_{[-]})</td>
<td>0</td>
<td>2</td>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>(V_{[+]})</td>
<td>0</td>
<td>-2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>(V_{[-]})</td>
<td>-2</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

(22) *More similar filler vectors, MS, for Simulation #2:*

<table>
<thead>
<tr>
<th></th>
<th>(C_{[+]})</th>
<th>(C_{[-]})</th>
<th>(V_{[+]})</th>
<th>(V_{[-]})</th>
</tr>
</thead>
<tbody>
<tr>
<td>([\pm\text{cons}])</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>([\pm F])</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>([p_1])</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>([p_2])</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>([p_3])</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

\[\text{sim}(A, B) = A \bullet B\]

<table>
<thead>
<tr>
<th></th>
<th>(C_{[+]})</th>
<th>(C_{[-]})</th>
<th>(V_{[+]})</th>
<th>(V_{[-]})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_{[+]})</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>(C_{[-]})</td>
<td>0</td>
<td>2</td>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>(V_{[+]})</td>
<td>0</td>
<td>-2</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>(V_{[-]})</td>
<td>-2</td>
<td>0</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>
Note that the relative similarities between consonants and vowels has not changed between Simulation #1 and Simulation #2, nor does it change between the LS and MS sets of vectors. The only differences are the relative similarity between vowels which disagree on the harmonic feature. As the results below show, only the addition of similarity along a third prerequisite subfeature, here denoted $[p_3]$, allows non-local harmony to take place.

5.5.2. Results

Because the focus of Simulation #2 is whether or not non-local harmony could take place and because Simulation #1 shows that networks can learn both $[+F]$ and $[-F]$ harmony, this simulation only sets $V_1 = V_{[+]}$ as an inalterable trigger and permits other segments to vary freely on $[\pm F]$. If the network learns non-local harmony then $H(C_{[\alpha]}V_{[+]}C_{[\beta]}V_{[+]}C_{[\gamma]})$ will be greater than the identical form but with mismatching V’s $H(C_{[\alpha]}V_{[+]}C_{[\beta]}V_{[-]}C_{[\gamma]})$, so only candidates that differ in $V_2$ are compared. The difference between whether $C_{[\alpha]}V_{[+]}C_{[\beta]}V_{[+]}C_{[\gamma]}$ is more harmonic than some other CVCVC string with different consonants is due to some other factor than non-local, V-to-V harmony. The crucial test cases will be the examples where harmony must obtain non-locally because an intervening $C_{[\_\_]}$ could block local harmony. From this perspective, non-adjacent V-to-V harmony can only obtain if the vowels share more resources (due to similarity) than the adjacent consonants and vowels, despite their higher proximity.

The results of Simulation #2 are presented below. Again, the Harmonies cannot be directly compared, since filler vectors are of different lengths, so the percentiles are given as well in (23) below.
Results of Simulation #2. Harmonies of surface patterns as measured by (4) in an Entailment Network which was instantiated with “Richness of the Base” training on the forms in (16) encoded with the Z-roles from (20) and the fillers from either less similar (LS)(21) or more similar (MS)(22). Fillers that do not agree with the [+F] trigger are shaded:

<table>
<thead>
<tr>
<th>Candidate surface forms:</th>
<th>LS</th>
<th>MS</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. C[+] V[+] C[+] V[+] C[+]</td>
<td>2238 1.00</td>
<td>3414 1.00</td>
</tr>
<tr>
<td>b. C[+] V[+] C[+] V[-] C[+]</td>
<td>1786 0.63</td>
<td>2914 0.67</td>
</tr>
<tr>
<td>c. C[-] V[+] C[+] V[+] C[+]</td>
<td>1822 0.66</td>
<td>2902 0.66</td>
</tr>
<tr>
<td>d. C[-] V[+] C[+] V[-] C[+]</td>
<td>1410 0.32</td>
<td>2442 0.35</td>
</tr>
<tr>
<td>e. C[+] V[+] C[-] V[+] C[+]</td>
<td>1430 0.33</td>
<td>2462 0.37</td>
</tr>
<tr>
<td>f. C[+] V[+] C[-] V[-] C[+]</td>
<td>1282 0.21</td>
<td>2266 0.23</td>
</tr>
<tr>
<td>g. C[-] V[+] C[-] V[+] C[+]</td>
<td>1134 0.09</td>
<td>2070 0.10</td>
</tr>
<tr>
<td>h. C[-] V[+] C[-] V[-] C[+]</td>
<td>1026 0.00</td>
<td>1914 0.00</td>
</tr>
<tr>
<td>i. C[+] V[+] C[+] V[+] C[-]</td>
<td>2078 0.87</td>
<td>3254 0.89</td>
</tr>
<tr>
<td>j. C[+] V[+] C[+] V[-] C[-]</td>
<td>1778 0.62</td>
<td>2906 0.66</td>
</tr>
<tr>
<td>k. C[-] V[+] C[+] V[+] C[-]</td>
<td>1670 0.53</td>
<td>2750 0.56</td>
</tr>
<tr>
<td>l. C[-] V[+] C[+] V[-] C[-]</td>
<td>1410 0.32</td>
<td>2442 0.35</td>
</tr>
<tr>
<td>m. C[+] V[+] C[-] V[+] C[-]</td>
<td>1390 0.30</td>
<td>2422 0.34</td>
</tr>
<tr>
<td>n. C[+] V[+] C[-] V[-] C[-]</td>
<td>1394 0.30</td>
<td>2378 0.31</td>
</tr>
<tr>
<td>o. C[-] V[+] C[-] V[+] C[-]</td>
<td>1102 0.06</td>
<td>2038 0.08</td>
</tr>
<tr>
<td>p. C[-] V[+] C[-] V[-] C[-]</td>
<td>1146 0.10</td>
<td>2034 0.08</td>
</tr>
</tbody>
</table>

Again there is an overwhelming preference for total harmony, but there are also other preferences for non-local harmony. Of particular note are (23)(e-h) and (23)(m-p) where harmony must obtain across an intervening neutral C. According to the filler specifications given in (21) and (22), consonants can express [+F]. Furthermore, because of “Richness of the Base” training, the network sees ample evidence (exactly half of the training data) with a C in the intervening position. In spite of this evidence, the Entailment Network is clearly biased towards similarity, because if the filler vectors become similar enough, as in (22), then V-to-V harmony obtains across a segment that could express the harmonic features, but does not.

The differences between (23)(e-h) and (23)(m-p), where non-local harmony obtains even for the LS-set of fillers in (23)(e-h) are likely due to the independent
availability of CV assimilation, since there are still Pos_{i,i+1} resources in the tensor product representations. Thus, in the context of C_{[-]}_C_{[-]}, an assimilating V_{[+]} incurs more Harmony penalty, (23)(m-p), because of disagreement on both sides, than an assimilating V_{[+]} in the context of C_{[-]}_C_{[+]}.

5.5.3. Discussion

Evidently, the similarity between differences in the LS and MS filler sets is near the threshold because the Harmony differences between (23)(o-p) for MS are so small. For this reason, the specific details of the exact similarity needed for MS fillers to yield non-local harmony is an artifact of many factors including the number of units dedicated to encode Pos_i, Pos_{i,i+1}, Pos_{i,i+1,i+2}, the length of the C vectors, and perhaps even the length of the string. Therefore, the crucial point is that there exists a threshold.\(^{123}\) If the trigger and target are similar enough, then there is pressure for non-local harmony, even in the face of a neutral intervener that could carry the harmonic feature. Of course, in a more natural learning environment where non-local V-to-V harmony might be present in the input but local C-V harmony is not, there would be even

\(^{123}\) Let A = C_1V_1C_2V_{2,A}C_3 denote a candidate where V_1 and V_2 agree on [F]. Let D = C_1V_1C_2V_{2,D}C_3 denote a candidate where V_1 and V_2 disagree on [F]. V_1-to-V_2 harmony obtains when H(A) > H(D). Consider harmony in the worst case, where C_1, C_2, C_3 all disagree with V_1 (cf. (23)(o-p)). We can express A = C_1V_1C_2V_{2,A}C_3 as C_1\oplus V_1 \oplus C_2 \oplus V_{2,A} \oplus C_3, where Filler denotes the tensor product of Filler in role Z_c : H(A) = (C_1\oplus V_1 \oplus C_2 \oplus V_{2,A} \oplus C_3) \cdot W \cdot (C_1\oplus V_1 \oplus C_2 \oplus V_{2,A} \oplus C_3). By expanding terms H(A) = C_{1\cdot W \cdot C_1} + C_{1\cdot W \cdot V_1} + \ldots and so by cancelling common terms in H(A) and H(D) and assuming V_1 and V_2 are of equal length, then H(A) > H(D) iff C_{1\cdot W \cdot V_{2,A}} + C_{2\cdot W \cdot V_{2,A}} + C_{3\cdot W \cdot V_{2,A}} + V_{1\cdot W \cdot V_{2,A}} > C_{1\cdot W \cdot V_{2,D}} + C_{2\cdot W \cdot V_{2,D}} + C_{3\cdot W \cdot V_{2,D}} + V_{1\cdot W \cdot V_{2,D}}. For A, C_1 and V_{2,A} disagree and V_1 and V_{2,A} agree, but for D, C_1 and V_{2,D} agree but V_1 and V_{2,D} disagree on F. Thus, by rearranging terms in H(A) > H(D) if the component of harmony that denotes the reward for agreement to V_1, call it H_{V_1} = V_{1\cdot W \cdot V_{2,A}} - V_{1\cdot W \cdot V_{2,B}} is greater than the sum of the rewards for having V_2 agree with the C_i, call it H_{C_i} = C_{i\cdot W \cdot V_{2,D}} + C_{2\cdot W \cdot V_{2,D}} + C_{3\cdot W \cdot V_{2,D}} - (C_{1\cdot W \cdot V_{2,D}} + C_{2\cdot W \cdot V_{2,D}} + C_{3\cdot W \cdot V_{2,D}}), then V-to-V harmony obtains iff H_{V_1} > H_{C_i}. Therefore, given any similarity C_i\bullet V_i it is possible to find a similarity V_i\bullet V_2 such that V-to-V harmony obtains. Hence, a similarity threshold for V-to-V harmony always exists if the role vectors posit some similarity between non-adjacent positions (i.e. H_{V_1} > 0), but the exact number depends on the values of the other parameters, including the encoding of Z, the length of Cs and Vs, and the similarity of Cs and Vs.
more pressure for assimilation. But even in the difficult “Richness of the Base” task, there is still evidence that Entailment Networks and, more broadly, the Attraction Framework are inherently sensitive to similarity.

This similarity threshold result of Simulation #2 explains why non-local harmony systems exploit additional factors of similarity. In the case of Simulation #2, the more similar two segments become, the more network resources they share in tensor product representations, and the greater attraction pressures that exist for the segments to harmonize. Thus, Simulation #2 with MS features derives an ATTRACTION constraint of the form $\text{ATTRACT}([p_1, p_2, p_3] \to [F])$ – or equivalently a single strongly weighted $\text{ATTRACT}([p] \to [F])$, where $[p]$ has subfeatures $[p_1]$, $[p_2]$, and $[p_3]$. Of course, in a more natural system, $[p]$ may simply indicate an amount of additional similarity due to agreeing on $[-\text{cons}]$ (see §4.4.4) or a much stronger contrast than $[F]$, such as height versus ATR (see §2.3.2). Where only agreement on $[p_1]$ and $[p_2]$ obtains there is a chance harmony will not obtain, but even in the LS case there is a clear trend toward non-local harmony in the face of partial similarity as $V_2$ harmonizes to $V_1$ in (23)(e-h).

Simulation #2, thus, expresses a clustering property, Burzio’s “Binding Corollary” (see §3.4 and §4.5.3), that has been used in this dissertation to explain why parasitic dependencies form between similar elements and why feature co-harmonies arise in the context of low-level similarity. Although the relevant representations are somewhat different, the crucial point is that when representations share network resources through either one or both of subfeatures and tensor product representations, so there is a pressure for similar elements to occur to the degree allowed by markedness and
faithfulness. Hence, the pressure for parasitic assimilations given in \textsc{attraction} constraints directly derive from Harmony maximization in an Entailment Network.

5.6. Simulation #3: Typological restrictiveness

5.6.1. Anti-similarity\textsuperscript{124}

This section turns from the reduction of individual \textsc{attraction} constraints to deriving an aspect of cross-linguistic typology. In all the various phenomena across consonant and vowel harmony, I found no cases which indicate an antagonism to similarity, i.e. cases where less similar segments interact, but more similar segments cannot. Any exceptions to this generalization were independently due to idiosyncratic inventories that excluded the more similar segments. This would give the following generalization:

\begin{quote}
\textbf{(24) Anti-similarity languages are impossible:}

There are no cases of anti-similarity harmony, where harmony only occurs if trigger and target disagree in other respects.
\end{quote}

There are two kinds of anti-similarity languages worth considering: (i) \textit{anti-parasitic harmony} would violate (24) because harmony could only obtain if trigger and target differed on other features and (ii) \textit{anti-local harmony} would violate (24) because assimilation could only obtain if trigger and target differed in proximity.

Anti-parasitism can be illustrated with “anti-Yawelmani” (cf. real Yawelmani §4.3.1), in (25) below.

\textsuperscript{124} For the sake of encapsulation some of the discussion of anti-similarity in §4.2.3.4 is repeated here.

<table>
<thead>
<tr>
<th>Triggers(↓) \ Targets(→)</th>
<th>High</th>
<th>Non-high</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[hin]/[hun] ‘non-future’</td>
<td>[al]/[ol] ‘might’</td>
</tr>
<tr>
<td>High</td>
<td></td>
<td></td>
</tr>
<tr>
<td>/xil/ ‘tangles’</td>
<td>xil-hin</td>
<td>xil-al</td>
</tr>
<tr>
<td>/dub/ ‘lead by the hand’</td>
<td>dub-hin</td>
<td>dub-al</td>
</tr>
<tr>
<td>Non-high</td>
<td></td>
<td></td>
</tr>
<tr>
<td>/xat/ ‘eat’</td>
<td>xat-hin</td>
<td>xat-al</td>
</tr>
<tr>
<td>/bok/ ‘find’</td>
<td>bok-hun</td>
<td>bok-al</td>
</tr>
</tbody>
</table>

What makes (25) so strange from the perspective of attraction is that, /dub/ can induce harmony across height on [al]/[ol], but /dub/ cannot induce harmony within height on [hin]/[hun]. Segments must be dissimilar in height, before they can assimilate in rounding. To my knowledge, there are no such attested cases of anti-parasitism.

Likewise, there are no anti-local languages allowing harmony at greater distances, but not when the same trigger and target are more proximate. True anti-local harmony would allow harmony at any distance greater than some threshold, for example productive ATR harmony, if and only if trigger and target are more than two syllables apart. No such systems are known to exist.

The absence of anti-local harmony and anti-parasitic harmony follows directly from general prerequisite similarity and the “Principle of Similarity” (see §3.8.1). If a target undergoes harmony, then a more similar segment is also under attraction pressure because it exceeds whatever similarity preconditions were met by the dissimilar segment in order to alternate. A proximate segment is necessarily more GPS-similar to the trigger than a distal segment, so it maps to an even more proximate position in harmony space, so anti-local harmony is impossible. A featurally similar segment is also necessarily more GPS-similar to the trigger than a featurally dissimilar segment, so anti-parasitic harmony is impossible.
Thus, there are certain unattested patterns whose absence follows directly from the perspective of general prerequisite similarity. These unattested cases are explored more in this section, where they prove crucial for determining whether the proposed system for Entailment Networks is restrictive.

5.6.2. Methods

Simulation #3 uses a different training paradigm to explore the biases of Entailment Networks than Simulation #1 orSimulation #2. Here, instead of presenting the network with all (possibly conflicting) examples and seeing which patterns it best internalized, the network is given a very small training set that leaves out crucial test patterns. Because of the absence of essential data, the paradigm is called “Poverty of the Stimulus” training (POTS training), following Wilson’s (2006) use of a POTS training paradigm in human artificial grammar learners to explore the accessibility of universal phonological biases.

For the anti-similarity issue at hand, the crucial examples to be withheld during training are those for which triggers and targets are more similar than other trigger target pairs in training set. Thus, after training on the less similar pairs, the generalization bias of the network is tested on both more similar and less similar trigger-target pairs. If an Entailment Network cannot learn anti-similarity languages, then even when presented with training data that only provides evidence of anti-similarity harmony, the network should generalize to harmony on the more similar forms. For example, if Entailment Networks have a bias against anti-similarity, then when given data suggesting anti-Yawelmani across height harmony, at the test phase, the network should prefer within-
height harmony as well. Likewise, when given data suggesting anti-local harmony, at test, the network should prefer local harmony as well.

**Simulation #3** consists of presenting an Entailment Network with a few examples suggesting anti-local, anti-parasitic Yawelmani. However, the results show that even with this impoverished data set, the Entailment Network generalizes to local, parasitic Yawelmani.

The full range of test pattern on which the network is tested are found in (26):

(26) **Test set for Simulation #3:**

<table>
<thead>
<tr>
<th>CuCiC</th>
<th>CuCuC</th>
</tr>
</thead>
<tbody>
<tr>
<td>CuCeC</td>
<td>CuCoC</td>
</tr>
<tr>
<td>CuCCiC</td>
<td>CuCCuC</td>
</tr>
<tr>
<td>CuCCeC</td>
<td>CuCCoC</td>
</tr>
<tr>
<td>CuCCCiC</td>
<td>CuCCCuC</td>
</tr>
<tr>
<td>CuCCCeC</td>
<td>CuCCCCoC</td>
</tr>
</tbody>
</table>

The test phase consists of measuring the Harmony of forms with V-to-V vowel rounding harmony as compared to forms that lack V-to-V assimilation. \( V_1 = u \) is the trigger, so it is fixed and \( V_2 \) is the eligible target. If the network has a bias toward similarity, then surface forms which satisfy assimilation to \( u \) should have a higher Harmony than non-assimilatory forms.

However, the training set is a much more limited set consisting only of cases of non-local, cross-height interaction, as shown in (27). Because Entailment Networks are known to be sensitive to similarity (formally demonstrated in (10), and confirmed by **Simulations #1 and #2**) I expect that even though the network is presented with both a
harmonic and a non-harmonic pattern, the network will prefer the pattern with V-to-V harmony, CuCCCoC.

(27) Training set for Simulation #3

CuCCCeC

CuCCCoC

The question is how will the network generalize from these patterns? Will harmony in other positions, e.g. \(H(\text{CuCoC}) > H(\text{CuCeC})\), be licensed? Will other possible targets be allowed to undergo harmony, e.g. \(H(\text{CuCCCuC}) > H(\text{CuCCCiC})\)? If Entailment Networks have a bias against anti-similarity harmony, then generalization should occur.

In order to answer these questions, the simulation must be given specific role and filler vectors to encode the patterns in (26) and (27). The filler vectors are the same \(Z\)-vectors from Simulation #2, only with two more positions, since the strings can be longer:

(28) Role vectors for Simulation #3:

<table>
<thead>
<tr>
<th>(Z_i)</th>
<th>(P_1)</th>
<th>(P_2)</th>
<th>(P_3)</th>
<th>(P_4)</th>
<th>(P_5)</th>
<th>(P_6)</th>
<th>(P_7)</th>
<th>(P_{1,2})</th>
<th>(P_{2,3})</th>
<th>(P_{3,4})</th>
<th>(P_{4,5})</th>
<th>(P_{5,6})</th>
<th>(P_{6,7})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Z_1)</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(Z_2)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(Z_3)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(Z_4)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(Z_5)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>(Z_6)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(Z_7)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

(29) Filler vectors for Simulation #3:

<table>
<thead>
<tr>
<th>C</th>
<th>(u)</th>
<th>i</th>
<th>o</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pm\text{cons})</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(\pm\text{hi})</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>(\pm\text{round})</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>1</td>
</tr>
</tbody>
</table>
However, the role vectors are quite different than those previously given, since consonants cannot carry the harmonic feature. Having shown in Simulation #2 how true transparency of consonants can be achieved by having increased \(V\to V\) similarity, this lack of specification is a simplifying assumption, but an essential one. If assimilation can obtain locally (as if by spreading), then the generalization from anti-local to local harmony might follow from learning that all positions should express the harmonic feature, [+round], but if the consonants cannot express harmonic features, then it is much more difficult for a network to generalize from \(H(CuCCCoC) > H(CuCCCCeC)\) to \(H(CuCoC) > H(CuCeC)\), since the intervening consonants are under no pressure for vowel rounding harmony. For the case where interveners do not carry harmonic features, if the Entailment Network is still antagonistic towards anti-local harmony, then that would confirm the general bias against anti-similarity independent of the other biases in favor of local harmony if it is available (see Simulation #1 and #2).

5.6.3. Results & Discussion

An Entailment Network learned each of the patterns in the training set (27), which were encoded using the role vectors in (28) and the fillers in (29) through Hebbian learning. With this impoverished stimulus, the viability of assimilation was then tested at other positions and for other vowels.

(30) Simulation #3. Harmony measures of \(V\to V\) harmonic and \(V\to V\) non-harmonic forms.

<table>
<thead>
<tr>
<th>Non-harmonic</th>
<th>(H)</th>
<th>(H%)</th>
<th>Harmonic</th>
<th>(H)</th>
<th>(H%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(CuCiC)</td>
<td>1402</td>
<td>0.00</td>
<td>(CuCuC)</td>
<td>1666</td>
<td>0.08</td>
</tr>
<tr>
<td>(CuCeC)</td>
<td>2642</td>
<td>0.40</td>
<td>(CuCoC)</td>
<td>2986</td>
<td>0.51</td>
</tr>
<tr>
<td>(CuCCiC)</td>
<td>1618</td>
<td>0.07</td>
<td>(CuCCuC)</td>
<td>1898</td>
<td>0.16</td>
</tr>
<tr>
<td>(CuCCCeC)</td>
<td>2938</td>
<td>0.49</td>
<td>(CuCCCoC)</td>
<td>3298</td>
<td>0.61</td>
</tr>
<tr>
<td>(CuCCCCiC)</td>
<td>2500</td>
<td>0.35</td>
<td>(CuCCCCuC)</td>
<td>2836</td>
<td>0.46</td>
</tr>
<tr>
<td>(CuCCCCeC)</td>
<td>4100</td>
<td>0.86</td>
<td>(CuCCCCoC)</td>
<td>4516</td>
<td>1.00</td>
</tr>
</tbody>
</table>
As (30) shows, in every case, the network learned the pattern of V-to-V rounding harmony preferring the V₂-harmonic surface form to the V₂-disharmonic surface form. The network clearly has other preferences as well, such as, V₂ should be non-high and there should be three intervening consonants. However, if there were independent evidence that those positions or syllabic templates were licensed, then Simulation #3 shows that an Entailment Network would have an automatic bias towards having harmony in those positions given that harmony is allowed under less proximity and less feature similarity.

Because of the POTS training, the only the way the network could generalize from the anti-local, anti-parasitic harmony data to preferring harmony in more local, more similar contexts is if the network has a bias against anti-similarity languages. Entailment Networks, indeed, have such a bias, so the Attraction Framework simultaneously derives why anti-similarity languages are unattested and why, on the other hand, there are an abundance of locality and parasitic conditions on assimilation.

5.7. **Relation to other frameworks and Future research**

Simulations #1, #2, and #3 confirm that Entailment Networks are biased towards a sensitivity to similarity in ways that are consistent with human language. The biases include (i) a tendency toward harmony in the face of positional similarity (§5.4), (ii) the possibility of non-local interaction given increased feature similarity (§5.5), and (iii) the impossibility of anti-similarity harmony (§5.6).

These results solidify the computational grounding of the Attraction constraints used in the other parts of this dissertation. For example, the Subset-Similarity Ranking Principle (SSRP; 3.8) is a grammar level stipulation needed to disallow anti-similarity
harmony. In a network, the SSRP is explained by the interaction of Harmony maximization, Hebbian learning, and tensor product representations. These principles ground the SSRP in aspect of cognition that are independently needed to simultaneously account for the symbolic aspects of grammar and a connectionist underpinning of computation in the human brain (see Smolensky & Legendre, 2006). Furthermore, these results generalize well to other features and syllabic templates because the simulations confirmed a bias in learning a language, not merely the capacity to learn a pattern.

Thus, the general computational principles of attraction provide a unified explanation for similarity preconditions in terms of locality and features, across both consonant and vowel harmony. The remainder of this section discusses the implications of these connectionist foundations outline in the chapter on relating the Attraction Framework to other frameworks and discusses possible directions for future research.

5.7.1. Phonetic grounding vs. computational grounding

The influential work in Hayes, Kirchner, & Steriade (2004; refs therein) and Boersma (1998) exemplifies the trend in phonological research towards grounding the rules and constraints governing phonological processes in the phonetic properties of acoustics and/or articulation. This dissertation argues that phonetic information is not only available to determine the strength of Attraction constraints, but phonetic correlates may also be directly available for the similarity computations which drive assimilation. This section reviews how this dissertation has used phonetic grounding and discusses the subtle differences between phonetic grounding and computational grounding, concluding that the later, not the former is a better characterization of the Attraction Framework.
In Chapter 2, I argued that parasitic features overwhelmingly tend to be phonetically similar to their harmonic features, and in §3.4, I showed how the entailments between subfeatures explain why phonetic similarity determines the available feature dependencies that give parasitic harmony. Furthermore, in §4.5.4, I argued that the dependencies in parasitic harmony mirror some of the dependencies given for the analysis of co-harmony in Feature Geometry (e.g. Clement & Hume, 1995). Thus, co-harmony was also shown to be derivable from the Entailment Framework’s ability to explain clustering in the face of phonetic similarity. Phonetic similarity, thus, seems to play a primary role in determining the strength/rank of ATTRACTION constraints.

In §4.3.4 and §4.4.3, I argued that long-distance vowel agreement in Finnish and Moba Yoruba benefits from the harmony constraints having direct access to the phonetic space. Phonetic subfeatures—[lower F2] for Finnish and [low frequency formant] in Moba Yoruba—explained the set of participant and non-participants in those harmony processes. As noted in §4.5, such similarity (kernel) methods that use the strategy of phonetic subfeatures are extremely powerful. Therefore, it is necessary to posit that the set of available subfeatures must be closed, but at this juncture, it remains unclear exactly what bounds the set of subfeatures.

Subfeatures are certainly grounded in phonetics, but not every phonetic aspect is necessarily available for parasitic dependency. Consider the possible phonetic subfeatures [audibly loud], [long duration], [oral], [pulmonic], etc., which are probably too large a phonetic category to benefit from the usual outcomes of harmony: perceptual optimization and facilitating speech planning. For instance, [long duration] might denote the class of vowels, fricatives, and geminate consonants, while excluding non-geminate
consonants. It is not clear what perceptual or articulatory process could benefit from ensuring that this unusual group of segments agree in some way, so it would be very surprising for this class to act as a group in a harmony system. Thus, while phonetics delimits the space of possible subfeatures, merely having an amount of phonetic similarity is not sufficient to motivate the existence of a subfeature. Incorporating theories for how auditory cues, speech planning, and speech production determine the availability of phonetic features would further elucidate how subfeatures behave in harmony systems.

Nevertheless, it is clear that the forces which give rise to harmony systems have access to phonetic information. Rejecting the availability of phonetics would leave the classes of participants and the motivations for harmony in Finnish and Moba Yoruba unexplained. Furthermore, the hypothesized features were corroborated by phonetic studies that showed that the features relate to cues that are part of the speakers’ knowledge: for Finnish, Kim (2005) explored speakers’ knowledge of harmony through F2, and for Moba Yoruba, Przedziecki (2005) showed that F1 was an indicator of ATR-harmony.

Thus, in one broad sense, this work endorses the role of detailed phonetic information in phonological processes. This commitment to phonetic information supports a view of representations as distributed representations in a multi-dimensional space. Such distributed representations are at the heart of the connectionist movement (Rumelhart & McClelland, 1986).

---

However, in another sense, the Attraction Framework is altogether a different animal than phonetic grounding. The difference between the Attraction Framework and phonetics is like the difference between operators and their operands. It would be inappropriate to consider an investigation of multiplication a theory of integers because multiplication can also operate on real or imaginary numbers, matrices, and even transfinite numbers. Likewise, it is inappropriate to consider the Attraction Framework (only) a theory of phonetic grounding because attraction has been argued to play a diverse role in both phonology and morpho-phonology. Burzio (2002a,b) and Burzio & Tantalou (2007) demonstrate how attraction applies to morphological syncretism. Burzio (2005, 2009) show how attraction also applies in derived environment effects, like non-derived environment blocking. Wayment et al. (2007) apply attraction to phonetic enhancement while this dissertation applies attraction to parasitic harmony. The operands are distinct in these different domains (only a subset of which require phonetic information) but the operator, attraction via representational entailments, is common to all domains.

Therefore, the conclusion of the simulations in this chapter is that parasitic harmony can be grounded in attraction computations in a connectionist network. These simulations have not sought to model the particular phonetic properties of a particular language, but the hypothesis is that if one were to do so correctly, the same attraction properties which drive harmony in Simulations #1-3 would give rise to the harmony phenomena analyzed in Chapters 2-4. Of course, it is no small matter to find the right model of phonetic detail in connectionist networks, so this is another instance where the Attraction Framework benefits from multiple levels of explanation (Smolensky &
Legendre, 2006). The formulation of ATTRACTION constraints in OT allows for the analysis of language data without constructing detailed patterns of activation.

This dissertation makes a very strong commitment to the kinds of computations involved in phonological alternation by the hypothesis of attraction. In Chapter 4, I argued in detail that the kinds of similarity effects which apply to features can also be understood as applying to positions and localities. Chapter 4 also showed that as a consequence of the general clustering properties of attraction, certain aspects of autosegmental phonology and feature geometry can be understood as attraction operating in a general similarity space. The commitments to particular feature sets, geometries, or kinds of phonetic grounding should be seen as secondary to the primary hypothesis of assimilation as attraction.

This move has the advantage that any deficiencies in the present work likely stem from inadequacies in the secondary hypotheses. With more accurate information about how a particular language computes the similarity of segments, attraction constraints can still describe the harmony process. For instance, suppose that further phonetic data suggested that there is a need to adjust the reported numerical values of formants which define [lower F2] and [low frequency formant]. Clearly, the attraction strategy does not hinge on particular values, so the operands, in this case the subfeatural categories, can be modified while the general operation of attraction remains unchanged.

Worries that this move towards operator neutrality necessarily render the Attraction Framework too unrestrained are averted by some central, specific principles about how attraction operates in phonology. These principles include sensitivity to phonetic similarity (§3.4, §4.5), the Subset Similarity Ranking Principle (introduced in
§3.5.5 and confirmed in Simulation #3 to derive directly from the biases of Entailment Networks) and the hypothesis that phonological learners are biased towards generalization and so tend to form conjunctive parasitic—not disjunctive parasitic—harmony systems (§3.5.6, §3.8).

5.7.2. Extending Entailment Networks to SRNs

Turning to another area where the simulations of this chapter allow for a novel take on the rest of this dissertation, this section considers the ramifications of Entailment Networks giving rise to assimilation by way of resource sharing (Simulation #1-3). While these shared resources allow for a dependency between positions, this connectionist dependency is somewhat different than the formal correspondence between segments (Rose & Walker, 2004; Hansson, 2001) instantiated with the correspondence constraints used for locality effects in Chapter 4. Fully resolving these differences must remain part of future work, but some of the prominent issues are listed in this and the subsequent section.

First, the OT-level correspondence constraints require correspondence between segments within a locality window \( l \), but in the Entailment Networks, the notion of a window is artificial. At the network level, the locality window is an artifact of how the role vectors were chosen. As noted, the role vectors could have been chosen so that first and last position share some resources, but intermediate positions do not share those same resources. Such representations must be impossible in order to be consistent with principles of entailment persistence and the positional similarity hypothesis (see §3.5.1, §3.5.2, §4.2), which yield the lack of anti-local harmony. Likewise, representations of roles were chosen such that the similarity of Pos\(_1\) and Pos\(_2\) is equal to that of Pos\(_3\) and
While sensible, there is no network level reason why this should be so, since the role vectors for Pos\textsubscript{1} and Pos\textsubscript{2} were chosen independently of Pos\textsubscript{3} and Pos\textsubscript{4}. Nonetheless, **Simulation #2** confirms that when segments are in positions that are represented in a similar fashion, there will be forces which prefer assimilation. Furthermore, when segments are in more proximate positions, there is greater incentive for harmony (only disharmonic C\textsubscript{1}, (23)(c) has less Harmony than only disharmonic C\textsubscript{5}, (23)(i)).

Hence, the network naturally predicts that attraction forces should fall off with distance, but can only instantiate the universal that non-local harmony implies local-harmony (**Simulation #3**) if the representations are of the right sort. In contrast, at the OT-level, correspondence preserves the implication from non-local to local harmony because of the way the constraints are formulated, but only instantiates positional similarity through the (SSRP) ranking of locality constraints. A better model would be one where computational principles derive both (\textit{i}) the implication between non-local and local harmony and (\textit{ii}) the fall off of attraction forces with lesser proximity.

As part of future work, it will be worth exploring whether truly recurrent networks, like Elman’s (1991) Simple Recurrent Networks (SRN), are better able to capture these properties. In SRNs, positions are represented in time, so there is reason to think that attraction forces fall off with sequential distance, since the activation on the context units will be most similar at adjacent time steps. Furthermore, windowing might be naturally instantiated in SRNs with a momentum term that keeps the representations of positions similar across a sequence. In such a framework, entailment persistence becomes representational inertia as adjacent segments have incentives to minimally change the activation on the context units. Since momentum is constant across positions,
the equal similarity of all adjacent positions in naturally derived. However, despite these positive indications, there is reason to worry that SRNs may prove too powerful a learning system, so unattested patterns like anti-similarity harmony would be crucial test cases.

5.7.3. Phonological consequences of positional features

This section argues that (i) the Entailment Network simulations suggest that instead of analyzing locality with correspondence, positional information may be directly available to phonological processing via positional features and (ii) outlines future work on how the availability of positional information allows neutralization, dissimilation, and metathesis to be encapsulated as attraction.

Chapter 4 suggested that positions in strings could be like features and showed how aspects of entailment persistence, including positional similarity, may be instantiated with correspondence. In the network, however, the relationship between features and positions is much stronger than analogy. Features are patterns of activation across the network; positions are also distributed patterns of activation. Moreover, Harmony does not discriminate between whether positional or feature information gives rise to the connections which promote assimilation. Therefore, since the network is blind to whether roles or fillers are associated with a unit, perhaps the analysis should aim for greater symmetry in the constraints that govern locality and feature preconditions. If true, then the locality preconditions described by formal correspondence in OT ought to be revisited as positional information features. That is to say, correspondence ought to be superseded by ATTRACTION constraints which use positional features as a formal feature in entailment relations.
For example, there are specific components of role vectors (see (28)) which encode each position orthogonally from all other positions. A feature \([\text{pos}_i]\) may refer to activation on these units in the same way that \([\text{cons}]\) refers to activation on another set of units. Now, for the role vectors of the simulations, such a \([\text{pos}_i]\) must be a multi-valued feature that can take on a value of each position in a string. Thus, \([\text{pos}_i=X_3]\) is a segment associated with the third position in the string exactly like \([\text{cons}=+]\) is a segment associated with being consonantal. Here, I use the shorthand \(X_i\) to denote the value of the positional feature. Other multi-valued, positional features, such as \([\text{pos}_{i,i+1}]\), might allow the formal system to distinguish segments which are adjacent from those in the same position.

Even as a multi-valued feature, segments can agree on \([\text{pos}_i]\) by having the same specification for \([\text{pos}_i]\). Therefore, positional features can operate in the Attraction system with the modification that the correspondence condition is removed. Now, if \([\text{pos}_i]\) is an antecedent in an \textsc{Attraction} constraint, \textsc{Attract}(\{[\text{pos}_i]\}→[F]) it has limited effect because a segment in a position always agrees with itself on [F]. However, \textsc{Attract}(\{[\text{pos}_{i,i+1}]\}→[F]) denotes a general agreement constraint between adjacent segments because it requires segments which agree on \([\text{pos}_{i,i+1}]\) to agree on [F]. Of course, as a matter of representation (at both the connectionist and symbolic levels), segments only agree on \([\text{pos}_{i,i+1}]\) if the segments are adjacent. Thus, the locality effects of \textbf{Chapter 4} could in principle be re-evaluated with positional features.

However, instead of performing this formal remapping from correspondence to positional features, this section shows how positional features provide a fertile area for
future research. Here, I sketch analyses for neutralization, dissimilation, and metathesis based on positional features.

5.7.3.1. Positional Neutralization

Consider the familiar case of final-stop voicing neutralization in Dutch and German (see Steriade, 1995, 1997; Flemming 1995, 2004; and refs therein) by which, when in final position, voiced [d] emerges as unvoiced [t]. The constraint
\[ \text{ATTRACT}([\{\text{pos}_{i},[-\text{cont.}]\}]\rightarrow[\text{voice}]) \]
can be seen as motivating such neutralization if the attraction constraint applies across morphemes (cf. Burzio 2002a,b). Furthermore, perhaps not all positions are of equal similarity, since the cues of some positions, like coda, are weaker than others (Steriade, 1995, 1997; Flemming, 1995, 2004); weakened cues denote less distinctiveness and, hence, greater similarity. In this way, \text{ATTRACT} constraints may be sensitive to the phonetic similarity of segments using positional features, and so \[ \text{ATTRACT}([\{\text{pos}_{\text{Coda}},[-\text{cont.}]\}]\rightarrow[\text{voice}]) \] can motivate positional neutralization if the relevant faithfulness constraints are lower-ranked.

5.7.3.2. Dissimilation (OCP)

This dissertation focuses on assimilatory phenomena. However, much ado has been made of trying to link assimilation and dissimilation. Dissimilation and OCP (Obligatory Contour Principle) phenomena parallel parasitic assimilation in having a sensitivity to similarity (Frisch, Pierrehumbert, & Broe, 2004). Furthermore, Suzuki (1998), Nevins (2004), and Pulleyblank (2002) draw important analogies between assimilation and dissimilation, but only succeed in reducing the choice of assimilation or dissimilation to a choice of whether or not a rule/constraint chooses assimilation or dissimilation as a repair. This section briefly explores how the present attraction proposal
has the potential to derive dissimilation effects from the same family of constraints that drive assimilation.

With only the modification that positional features are directly available, the Attraction Framework has the capacity to describe OCP phenomena as a result of the ranking of faithfulness. This is true because given a state of attraction tension, segments can resolve that tension by decreasing the distance between them (assimilation) or by increasing the distance between them (dissimilation).

Consider the illustrative case of rightward /l/-/r/ dissimilation in Latin, where no two liquids with the same value of [±lateral] are permitted. The following alternations are tolerated: /l…l r/ → /l…l r/ and /r…r l/ → /r…r l/. Suppose a dependency between position and [±lateral] is instantiated as the constraint ATTRACT([lateral]→[pos]) where, as with all ATTRACTION constraints, the values of [lateral] and [pos] come from the trigger. Below, (31) illustrates how different candidates violate this ATTRACTION constraint.

(31)  Positional features in ATTRACTION:

<table>
<thead>
<tr>
<th></th>
<th>Trigger</th>
<th>Target</th>
<th>ATTRACT([lateral]→[pos])</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[pos]</td>
<td>[lateral]</td>
<td></td>
</tr>
<tr>
<td>a.</td>
<td>X₁⁺</td>
<td>X₁⁺</td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>X₁⁺</td>
<td>X₁⁻</td>
<td>*</td>
</tr>
<tr>
<td>c.</td>
<td>X₁⁺</td>
<td>X⁻₁⁺</td>
<td>*</td>
</tr>
<tr>
<td>d.</td>
<td>X₁⁺</td>
<td>X⁻₁⁻</td>
<td></td>
</tr>
</tbody>
</table>

(31)(b-c) violate ATTRACT([lateral]→[pos]) because the constraint prohibits segments which disagree in [±lateral] to disagree in position. Importantly, there are two candidates, (31)(a,d), which fully satisfy ATTRACT([lateral]→[pos]). (31)(a) satisfies the constraint because the candidate matches the trigger on both positional and lateral
features. Candidate (31)(d) satisfies the constraint because the antecedent of the ATTRACTION constraint does not hold, rendering the entailment inactive.

Thus, (31) shows that there are two possible repairs to an attraction relationship specified by entailments with positional features: full-agreement or full-disagreement, and as usual in OT, which repair is ultimately realized depends on the lowest ranked faithfulness constraint. LINEARITY (McCarthy & Prince 1995) is a faithfulness constraint that governs position. In the case of Latin, evidently IO-FAITH([lateral]) is lower-ranked than LINEARITY and ATTRACT({[lateral]} →[pos_i]) as shown by the tableau in (32) below which derives the correct output. Note the subscripts X_1, X_2, X_3 refer respectively to the first, second, and third position in the string, and input positions are denoted by order in the candidate.

(32) Dissimilation with ATTRACTION:

<table>
<thead>
<tr>
<th>/IVl/</th>
<th>LINEARITY</th>
<th>ATTRACT({[lateral]} →[pos_i])</th>
<th>IO-FAITH([lateral])</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>I_{X_1}V_{X_2} I_{X_1}</td>
<td>*!</td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>I_{X_1}V_{X_2} I_{X_1}</td>
<td>*!</td>
<td>*</td>
</tr>
<tr>
<td>c.</td>
<td>I_{X_1}V_{X_2} I_{X_3}</td>
<td>*!</td>
<td></td>
</tr>
<tr>
<td>✓= d.</td>
<td>I_{X_1}V_{X_2} I_{X_3}</td>
<td>*</td>
<td></td>
</tr>
</tbody>
</table>

High-ranking faithfulness to position blocks full-agreement, (32)(a), which is a neutralization along position, meaning that full-dissimilation, (32)(d), is the only possible way to satisfy the demands of attraction. Traditionally, dissimilation and assimilation are driven by separate families of markedness constraints. Uniquely, in the attraction proposal, they can derive from the same family of attraction constraints. Dissimilation is, thus, a natural extension of the Attraction Framework worthy of future investigation.
5.7.3.3. Metathesis

If proximity is on par with feature similarity in determining the prerequisite similarity conditions that create attraction pressures, then it is natural to posit that proximity is on par with feature similarity in resolving those attraction pressures. Unlike (32) above, where LINEARITY was high-ranked, the Attraction Framework predicts that LINEARITY can also be lower-ranked, allowing segments to exchange their linear order in response to attraction pressures. It is beyond the scope of this study to fully investigate whether or not there are attested cases of metathesis that would support this predicted repair. However, data from the Austronesian language Hiligaynon (Wolfenden, 1971) illustrate that featurally similar segments can be placed in closer position, a kind of harmony metathesis. In Hiligaynon, segments which agree in the feature [+nasal] undergo metathesis to bring the nasals into closer proximity. Furthermore, the alternative repair for alleviating attraction tension, increasing string distance, is also attested in dissimilatory metathesis. Languages like Lithuanian and Faroese exhibit a dissimilatory phenomenon in which adjacent stops are repulsed when preceded by a fricative, e.g. STT→TST. These data support the hypothesis that proximity is a kind of similarity, but a full analysis must remain part of future work.

If the indications of these analyses of neutralization, dissimilation, and metathesis are on the right track, then there is greater reason to believe that positional features, not correspondence, are the real drivers of interaction in locality, which is to say, in the context of Simulation #1-3, that the sharing of network resources across roles is the same as the sharing of resources across features: an instance of representational similarity.
5.8. Conclusion

Herein, similarity effects are explored in many different ways, including cross-linguistic surveys (Chapter 2), mathematical linguistics (Chapter 3), theoretical phonology (Chapter 4), and connectionist simulations (Chapter 5). These methods provide converging evidence that (i) similarity is a useful vocabulary for understanding the interactions between triggers and targets in phonological alternations and (ii) attraction is a useful paradigm for capturing those similarity relationships in a linguistic analysis. Furthermore, this work confirms the merit of Burzio’s (2002a, b; 2004; 2005) Entailment Framework for understanding the nature of attraction in linguistics at multiple levels of explanation. The connectionist and grammatical levels interplay in identifying the important generalizations and providing solutions.

For the problem of assimilation, this dissertation confirms the relevance of representing strings of phonological material in a multi-dimensional, general similarity space that is sensitive to both phonetic distance and positional information. This space makes possible the unification of long-distance consonant harmony, vowel harmony with blocking, vowel harmony with true transparency, spreading nasal harmony, and a number of strictly local assimilations. This novel effort at unification represents a repudiation of the formal dichotomy inherent in the current literature between non-local correspondence-based drivers (Rose & Walker, 2004; Hansson, 2001, 2007; Walker, 2009a) and local spreading-based constraints (Walker, 2000; Baković, 2000; Ní Chiosáin & Padgett, 2001). Moreover, this work reinterprets autosegmental feature tiers (Goldsmith, 1979) and some nodes in Feature Geometry (Clement & Hume, 1995) as an
effect of clustering in a general similarity space that is weighted on a language-specific basis.

The associations engendered by the Entailment Framework and preserved by Hebbian learning in an Entailment Network ensure that Harmony in a network is sensitive to representational similarity independent of whether the source of that similarity is proximity or features. Along with the arguments of §4.5, the computational grounding of this chapter provides strong evidence that the space of phonological representations is a general similarity space that encodes both features and positional information by reducing features to fillers, positions to roles, and similarity to activation in a connectionist network.

5.9. Appendix

This appendix proves the two theorems presented in (10) and (13) above. The proof of (10) applies the definition of Hebbian learning (7) and Harmony (4) with linear algebra to rearrange terms.

(from 10)

Theorem: If \( T_1, \ldots, T_n \) are a set of patterns that are presented to an Entailment Network, \( \mathcal{N} \), which learns these patterns through Hebbian learning, then for all patterns of activation \( A \), \( H_\mathcal{N}(A) = \sum_i (A \cdot T_i)^2 \).

Proof:

Ignoring factors related to learning rate and initial zero values on all connections in \( \mathcal{N} \), the weight matrix \( W_{\text{final}} \) following training on patterns \( T_1 \) through \( T_n \) is \( \otimes \)

\[
W_{\text{final}} = \sum_i T_i \otimes T_i
\]
Thus, the harmony $H_N$ of a pattern $A$ is $H_N(A) = A \bullet W_{\text{final}} \bullet A = A \bullet \sum_i T_i \otimes T_i \bullet A$

$$= \sum_i (A \bullet (T_i \otimes T_i) \bullet A) = \sum_i \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} \bullet \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} \begin{bmatrix} t_{i,1,1} \cdot t_{i,1,1} & \cdots & t_{i,1,1} \cdot t_{i,1,n} \\ \vdots & \ddots & \vdots \\ t_{i,n,1} \cdot t_{i,1,1} & \cdots & t_{i,n,1} \cdot t_{i,1,n} \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$$

$$= \sum_i \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} \bullet \begin{bmatrix} a_1 \cdot t_{i,1,1} + \cdots + a_n \cdot t_{i,1,n} \\ \vdots \\ a_1 \cdot t_{i,n,1} + \cdots + a_n \cdot t_{i,n,n} \end{bmatrix}$$

$$= \sum_i \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} \bullet \begin{bmatrix} t_{i,1,1} \cdot (a_1 \cdot t_{i,1,1} + \cdots + a_n \cdot t_{i,1,n}) \\ \vdots \\ t_{i,n,1} \cdot (a_1 \cdot t_{i,1,1} + \cdots + a_n \cdot t_{i,1,n}) \end{bmatrix} = \sum_i \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} \bullet \begin{bmatrix} t_{i,1,1} \cdot A \bullet T_i \\ \vdots \\ t_{i,n,1} \cdot A \bullet T_i \end{bmatrix}$$

$$= \sum_i (a_1 \cdot t_{i,1} \cdot A \bullet T_i + \cdots + a_n \cdot t_{i,n} \cdot A \bullet T_i) = \sum_i (a_1 \cdot t_{i,1} + \cdots + a_n \cdot t_{i,n}) \cdot A \bullet T_i$$

$$= \sum_i A \bullet T_i \cdot A \bullet T_i = \sum_i (A \bullet T_i)^2$$

The following theorem concerning the similarity of role-filler pairs was presented in (13). The proof of which follows from further application of linear algebra. The theorem/proof here is given in slightly different variables to resolve matters of indexing, but the results generalize to the statement in (13). The results also generalize to other similarity metrics related to the inner product, such as cosine distance.

(from 13)

**Theorem (Similarity of role-filler pairs)**: Let $\{A, B\}$ be a set of fillers and $\{X, Y\}$ be a set of roles, where a filler is bound to a role by the tensor product $\otimes$. If similarity in this role-filler system is computed using the inner product, $\bullet$, i.e.

$$\text{sim}(A, B) = A \bullet B$$

$$\text{sim}(X, Y) = A \bullet Y$$
\[ \text{sim}(A \otimes X, B \otimes Y) = (A \otimes X) \bullet (B \otimes Y) \]

Then \( \text{sim}(A \otimes X, B \otimes Y) = \text{sim}(A, B) \cdot \text{sim}(X, Y) \).

**Proof:**

Let \(|A| = |B| = m\) and let \(|X| = |Y| = n\).

\[
\text{sim}(A \otimes X, B \otimes Y) = (A \otimes X) \bullet (B \otimes Y)
\]

\[
= \begin{bmatrix}
    a_1 \cdot x_1 & \cdots & a_1 \cdot x_n \\
    \vdots & \ddots & \vdots \\
    a_m \cdot x_1 & \cdots & a_m \cdot x_n
\end{bmatrix}
\bullet
\begin{bmatrix}
    b_1 \cdot y_1 & \cdots & b_1 \cdot y_n \\
    \vdots & \ddots & \vdots \\
    b_m \cdot y_1 & \cdots & b_m \cdot y_n
\end{bmatrix}
\]

The matrixes can be converted to vectors to take their dot product:

\[
= \begin{bmatrix}
    a_1 \cdot x_1 \\
    a_1 \cdot x_n \\
    \vdots \\
    a_m \cdot x_1 \\
    a_m \cdot x_n
\end{bmatrix}
\bullet
\begin{bmatrix}
    b_1 \cdot y_1 \\
    \vdots \\
    b_m \cdot y_1 \\
    \vdots \\
    b_m \cdot y_n
\end{bmatrix}
\]

\[
= (a_1 \cdot x_1) \cdot (b_1 \cdot y_1) + \cdots + (a_1 \cdot x_n) \cdot (b_1 \cdot y_n) + \cdots + (a_m \cdot x_1) \cdot (b_m \cdot y_1) + \cdots + (a_m \cdot x_n) \cdot (b_m \cdot y_n)
\]

\[
= (a_1 \cdot b_1) \cdot (x_1 \cdot y_1) + \cdots + (a_1 \cdot b_1) \cdot (x_n \cdot y_n) + \cdots + (a_m \cdot b_m) \cdot (x_1 \cdot y_1) + \cdots + (a_m \cdot b_m) \cdot (x_n \cdot y_n)
\]

\[
= (a_1 \cdot b_1) \cdot [(x_1 \cdot y_1) + \cdots + (x_n \cdot y_n)] + \cdots + (a_m \cdot b_m) \cdot [(x_1 \cdot y_1) + \cdots + (x_n \cdot y_n)]
\]

\[
= \{ (a_1 \cdot b_1) + \cdots + (a_m \cdot b_m) \} \cdot [(x_1 \cdot y_1) + \cdots + (x_n \cdot y_n)]
\]

\[
= A \bullet B \cdot X \bullet Y
\]

\[
= \text{sim}(A, B) \cdot \text{sim}(X, Y). \quad \blacksquare
\]
References


- 418 -
Curriculum Vitae

Adam Wayment was born in Layton, Utah, U.S.A. on July 4, 1979. In his youth, Adam lived in Utah, Missouri, Ohio, New York, and Idaho. He graduated from Burley High School in 1997 and then immediately began his undergraduate studies at Brigham Young University in Provo, Utah. Adam temporarily postponed his studies to serve a mission for the Church of Jesus Christ of Latter-day Saints in Milan, Italy, completing his B.S. in Computer Science in 2002. He began his graduate studies in the Department of Cognitive Science at Johns Hopkins University in 2003, completing his Master’s Thesis in 2005 under the supervision of Paul Smolensky. During his time in Baltimore, Adam met his wife Rebekah Larson, and their first son Corwyn was born in 2006; their second son Emmery was born in 2008. Adam’s dissertation was primarily supervised by Luigi Burzio with Robert Frank and Colin Wilson playing additional advisory roles. Upon the completion of his Ph.D., Adam will begin a post-doctorate research position at Carnegie Mellon University in Pittsburgh, Pennsylvania.