```
Description: Generating Unifix-cube towers 4
cubes tall from exactly 2 to exactly }3\mathrm{ red cubes
and from exactly }3\mathrm{ yellow cubes to the tower
with all 4 yellow cubes
Parent Tape: Early Algebra Ideas About Binomial
Expansion, Stephanie's Interview Seven of Seven
Date: 1996-04-17
Location: Union Catholic
Researcher: Professor Carolyn Maher
```

| 1 | R1 | four things taken one at a time to four things taken two at a time. |
| :---: | :---: | :---: |
| 2 | Stephanie | Um hm. |
| 3 | R1 | Okay. |
| 4 | R2 | Okay. |
| 5 | R1 | So you were going to ask another question - but you were going to do something? |
| 6 | Stephanie | No. I was just going to keep building. |
| 7 | R1 | So what would you do - be building next? |
| 8 | Stephanie | Um. Towers with three reds? |
| 9 | R1 | Four taken three - can you tell us what you think is going to happen and why before you do it? |
| 10 | Stephanie | Well, it would be six [Stephanie points to the row of six pairs.] times two. Because |
| 11 | R1 | Hold on a minute. |
| 12 | Stephanie | Oh. Well, here it was |
| 13 | R1 | Tell me - explain what you're doing. |
| 14 | Stephanie | 'Cause here it was four times three to get the twelve, because you could have red here [She picks up $\left[\begin{array}{c}Y \\ Y \\ Y \\ R\end{array}\right]$ and points to the three yellow positions on the tower one at a time as she speaks.], a red here or a red here. So that's |

```
Description: Generating Unifix-cube towers 4
cubes tall from exactly 2 to exactly }3\mathrm{ red cubes
and from exactly }3\mathrm{ yellow cubes to the tower
with all 4 yellow cubes
Parent Tape: Early Algebra Ideas About Binomial
Expansion, Stephanie's Interview Seven of Seven
Date: 1996-04-17
Location: Union Catholic
Researcher: Professor Carolyn Maher
```

Transcriber(s): AboeInaga, Eman
Verifier(s): DeLeon, Christina
Date Transcribed: Spring 2009
Page: 2 of 9

|  |  | three - it'll produce three. And here [This time she picks up $\left[\begin{array}{l}Y \\ R \\ Y \\ R\end{array}\right]$ ] you can only put a red here [the top position] or here [the third position]. So it'll produce two. And so |
| :---: | :---: | :---: |
| 15 | R1 | And so from the six you could produce |
| 16 | Stephanie | You could produce two. |
| 17 | R1 | two. Any duplicates? |
| 18 | Stephanie | Yeah. There'll be duplicates. There'll be um two duplicates for each. So you divide by two - No. - Will there be two for each? I forget how many there were. |
| 19 | R1 | You see. That that's the question. |
| 20 | Stephanie | I forgot how many there were. So you have to like build it. |
| 21 | R1 | Okay. So could you think about it for a minute without doing it and predict? |
| 22 | Stephanie | Oh. Four. |
| 23 | R1 | You predict there would be |
| 24 | Stephanie | Four. Oh. Three duplicates and then there'll be four. |
| 25 | R1 | You think there would be three duplicates and then four of them. |
| 26 | Stephanie | Yeah. Because it's just the opposite of that. |
| 27 | R1 | What do you mean? |

```
Description: Generating Unifix-cube towers 4
cubes tall from exactly 2 to exactly }3\mathrm{ red cubes
and from exactly }3\mathrm{ yellow cubes to the tower
with all 4 yellow cubes
Parent Tape: Early Algebra Ideas About Binomial
Expansion, Stephanie's Interview Seven of Seven
Date: 1996-04-17
Location: Union Catholic
Researcher: Professor Carolyn Maher
```


## Transcriber(s): AboeInaga, Eman <br> Verifier(s): DeLeon, Christina <br> Date Transcribed: Spring 2009 <br> Page: 3 of 9

| 28 | Stephanie | Well, like this is one red and three yellow [Stephanie picks up a tower $\left[\begin{array}{l}Y \\ Y \\ R \\ Y\end{array}\right]$ ]. It'll be three red and one yellow. So it'll be just the opposite. |
| :---: | :---: | :---: |
| 29 | R1 | Should we try it? [R2 says something inaudible.] [Stephanie immediately begins to build towers.] What do you think? [Pause] So tell us what you're doing while you're doing it. |
| 30 | Stephanie | Alright. Well, for this one |
| 31 | R1 | Um hm |
| 32 | Stephanie | It has to have two red on top. So I put a red down there [third position] and now it's going to go $\left[\begin{array}{l}R \\ R \\ Y \\ R\end{array}\right]$ ] so there are two for that one. And for the second one - that one there $\left[\begin{array}{l}R \\ R \\ R \\ Y\end{array}\right]$ ] and $\left[\left[\begin{array}{l}R \\ Y \\ R \\ R\end{array}\right]\right.$ ] that one there. The third one |
| 33 | R1 | Now let me see. The second one - help me understand this. You have to have a red - |
| 34 | Stephanie | [The mistakenly placed tower is corrected.] Oooh. - This is - |
| 35 | R1 | Does that belong there? |


| Description: Generating Unifix-cube towers 4 |
| :--- |
| cubes tall from exactly 2 to exactly 3 red cubes |
| and from exactly 3 yellow cubes to the tower |
| with all 4 yellow cubes |
| Parent Tape: Early Algebra Ideas About Binomial |
| Expansion, Stephanie's Interview Seven of Seven |
| Date: 1996-04-17 |
| Location: Union Catholic |
| Researcher: Professor Carolyn Maher |

Transcriber(s): AboeInaga, Eman
Verifier(s): DeLeon, Christina
Date Transcribed: Spring 2009
Page: 4 of 9

| 36 | Stephanie | No. [Stephanie switches $\left[\begin{array}{l}Y \\ R \\ Y \\ R\end{array}\right]$ with $\left[\begin{array}{l}R \\ Y \\ R \\ Y\end{array}\right]$.] |
| :---: | :---: | :---: |
| 37 | R1 | Okay. - Okay. So, let's see. Let's go through this again. So you have the red there and there [R1 indicates the top and third positions.] |
| 38 | Stephanie | Um hm. [She builds $\left[\begin{array}{l}R \\ R \\ Y \\ R\end{array}\right]$ while R1 is speaking.] |
| 39 | R1 | That's still the same. You put it in the middle and here and here. Then you put it on the bottom. |
| 40 | Stephanie | Um hm. [Then she builds $\left[\begin{array}{l}R \\ Y \\ R \\ R\end{array}\right]$. She continues building towers very confidently and quite quickly. She produces $\left[\begin{array}{l}R \\ R \\ R \\ Y\end{array}\right]\left[\begin{array}{l}Y \\ R \\ R \\ R\end{array}\right]\left[\begin{array}{l}R \\ R \\ Y \\ R\end{array}\right]\left[\begin{array}{l}Y \\ R \\ R \\ R\end{array}\right]$.] And then, um, this one [begins to collect the duplicates from among the towers $\left[\begin{array}{l}R \\ R \\ R \\ Y\end{array}\right]$ |

Description: Generating Unifix-cube towers 4 cubes tall from exactly 2 to exactly 3 red cubes and from exactly 3 yellow cubes to the tower with all 4 yellow cubes
Parent Tape: Early Algebra Ideas About Binomial Expansion, Stephanie's Interview Seven of Seven Date: 1996-04-17
Location: Union Catholic
Researcher: Professor Carolyn Maher

Transcriber(s): Aboelnaga, Eman
Verifier(s): DeLeon, Christina
Date Transcribed: Spring 2009
Page: 5 of 9

|  |  | $\left[\begin{array}{l}R \\ R \\ R \\ Y\end{array}\right]\left[\begin{array}{l}R \\ R \\ R \\ Y\end{array}\right]$, for example] this one and this one are the same. This one [she picks up $\left[\begin{array}{l}R \\ R \\ Y \\ R\end{array}\right]$ ] |
| :---: | :---: | :---: |
| 41 | R1 | So you tripled them. |
| 42 | Stephanie | This one |
| 43 | R1 | Triplicates. |
| 44 | Stephanie | Yeah. And that one. That one, that one, that one, and these three. So you have four. [She continues to sort the towers into triples. The result is: $\left[\begin{array}{l} R \\ R \\ R \\ Y \end{array}\right]\left[\begin{array}{l} R \\ R \\ R \\ Y \end{array}\right]\left[\begin{array}{l} R \\ R \\ R \\ Y \end{array}\right]\left[\begin{array}{l} R \\ R \\ Y \\ R \end{array}\right]\left[\begin{array}{l} R \\ R \\ Y \\ R \end{array}\right]\left[\begin{array}{l} R \\ R \\ Y \\ R \end{array}\right]\left[\begin{array}{c} R \\ Y \\ R \\ R \end{array}\right]\left[\begin{array}{c} R \\ Y \\ R \\ R \end{array}\right]\left[\begin{array}{c} R \\ Y \\ R \\ R \end{array}\right]\left[\begin{array}{c} Y \\ R \\ R \\ R \end{array}\right]\left[\begin{array}{c} Y \\ R \\ R \\ R \end{array}\right]\left[\begin{array}{c} Y \\ R \\ R \\ R \end{array}\right]$ |
| 45 | R1 | So. - Is that what you predicted? |
| 46 | Stephanie | Yes. |
| 47 | R1 | You predicted you would get triplicates. |
| 48 | Stephanie | I said there would be four, so it would be groups of three. |
| 49 | R1 | So what would you be dividing by? |

```
Description: Generating Unifix-cube towers 4
cubes tall from exactly 2 to exactly }3\mathrm{ red cubes
and from exactly }3\mathrm{ yellow cubes to the tower
with all 4 yellow cubes
Parent Tape: Early Algebra Ideas About Binomial
Expansion, Stephanie's Interview Seven of Seven
Date: 1996-04-17
Location: Union Catholic
Researcher: Professor Carolyn Maher
```

Transcriber(s): AboeInaga, Eman
Verifier(s): DeLeon, Christina
Date Transcribed: Spring 2009
Page: 6 of 9

| 50 | Stephanie | Three |
| :---: | :--- | :--- |
| 51 | R1 | By three. And before that you were dividing by? |
| 52 | Stephanie | Two. |
| 53 | R1 | And before that you were dividing by? We didn't do before that. |
| 54 | Stephanie | We didn't do that. |
| 55 | R1 | We could've. We could've started with all yellow. |
| 56 | Stephanie | Um. Well. Yeah, but, we're gonna |
| 57 | R1 | No red. - Let's see how that works with all yellow. Sorta I like (inaudible) <br> explain (inaudible) from the beginning. Do you know what I'm saying? |
| 59 | R1 | Um hm. <br> [Stephanie sneezes.] God bless you. |
| 60 | Stephanie | Thank you. <br> 61 |
| R1 | You need some tissues? [Stephanie goes on building towers. This time <br> every tower (four of them) are entirely yellow.] Before you do it, why don't <br> you predict what will happen? |  |
| 62 | Stephanie | Oh. There'll be one. |
| 63 | R1 | Stephanie |
| Why? |  |  |
| Because you're going to get yellow - all yellow from all four. 'Cause there's <br> only one space you can put a yellow. So you're going to get one all yellow <br> from here, one all yellow from here, one all yellow from here and one all <br> yellow from here. So there's going to be one. [pause; Stephanie lines up |  |  |

```
Description: Generating Unifix-cube towers 4
cubes tall from exactly 2 to exactly }3\mathrm{ red cubes
and from exactly }3\mathrm{ yellow cubes to the tower
with all 4 yellow cubes
Parent Tape: Early Algebra Ideas About Binomial
Expansion, Stephanie's Interview Seven of Seven
Date: 1996-04-17
Location: Union Catholic
Researcher: Professor Carolyn Maher
```

Transcriber(s): Aboelnaga, Eman
Verifier(s): DeLeon, Christina
Date Transcribed: Spring 2009
Page: 7 of 9

|  |  | four towers each built using four yellow Unifix cubes.] |
| :---: | :---: | :---: |
| 65 | R1 | Um. But there are four of them there. So... |
| 66 | Stephanie | You divide by four. |
| 67 | R1 | Okay. So - If you were trying to help me know what to divide by, is there anything that helps you? |
| 68 | Stephanie | You - um - (inaudible) [Stephanie repeats the question.] What - is there anything that helps you |
| 69 | R1 | Um hm. |
| 70 | Stephanie | like to know what to divide by? |
| 71 | R1 | Um hm. |
| 72 | Stephanie | Um - the number of red, I guess? ‘Cause here you divided by one. [Stephanie points to the towers with one red.] Here you divided by two. [Now she indicates the towers with two reds.] And here you divided by three. [She indicates the towers built with three reds.] |
| 73 | R1 | Here the number of red is zero. [R1 points to the four all yellow towers.] |
| 74 | Stephanie | No. But I'm saying here [pointing to the area between the all yellow towers and the towers with one red.] you divided by one. Like to get that - |
| 75 | R1 | Um hm |
| 76 | Stephanie | I don't know. |
| 77 | R1 | Does that work? For all of them? |
| 78 | Stephanie | I guess not for this one, but like - the number of --[long pause] |
| 79 | R1 | Well, how does it work from here to here? [R1 points from the towers with |

```
Description: Generating Unifix-cube towers 4
cubes tall from exactly 2 to exactly }3\mathrm{ red cubes
and from exactly }3\mathrm{ yellow cubes to the tower
with all 4 yellow cubes
Parent Tape: Early Algebra Ideas About Binomial
Expansion, Stephanie's Interview Seven of Seven
Date: 1996-04-17
Location: Union Catholic
Researcher: Professor Carolyn Maher
```

Transcriber(s): AboeInaga, Eman
Verifier(s): DeLeon, Christina
Date Transcribed: Spring 2009
Page: 8 of 9

|  |  | three reds to the non-existent towers of four red.] |
| :---: | :---: | :---: |
| 80 | Stephanie | Well. The same way it works there. It's just all red. |
| 81 | R1 | All red. So-- |
| 82 | Stephanie | And it would be four times one divided by one, 'cause there's only one spot. |
| 83 | R1 | So it works here. So -- |
| 84 | Stephanie | Yeah. |
| 85 | R1 | It's one spot here. One spot - - [R1 points to the one yellow cube in each of the towers with three reds and one yellow.] |
| 86 | Stephanie | Um hm. |
| 87 | R1 | Um hm. So it works there. |
| 88 | Stephanie | Yeah. |
| 89 | R1 | It's just that this situation here is - a little bit different. If you wanted it to be nice and consistent, you would -sometimes that forces |
| 90 | Stephanie | Um hm. |
| 91 | R1 | people to make definitions particular ways. |
| 92 | Stephanie | Um |
| 93 | R1 | 'Cause you know you can't divide by zero. -Um - - so - |
| 94 | Stephanie | D - um - the number of spots you're pu - filling in - like - to get these [She indicates going from three yellow and one red to all yellow.] You put a yellow in one spot. Like - um - you know - I don't - |
| 95 | R1 | Yeah. |


| Description: Generating Unifix-cube towers 4 |
| :--- |
| cubes tall from exactly 2 to exactly 3 red cubes |
| and from exactly $\mathbf{3}$ yellow cubes to the tower |
| with all 4 yellow cubes |
| Parent Tape: Early Algebra Ideas About Binomial |
| Expansion, Stephanie's Interview Seven of Seven |
| Date: 1996-04-17 |
| Location: Union Catholic |
| Researcher: Professor Carolyn Maher |

Transcriber(s): Aboelnaga, Eman
Verifier(s): DeLeon, Christina
Date Transcribed: Spring 2009
Page: 9 of 9

| 96 | Stephanie | Like to get these [pointing to the towers with two reds and two yellows] you <br> put - there's - like - [Stephanie sounds frustrated.] I don't - |
| :---: | :--- | :--- |
| 97 | R1 | No. You can't force it. It's very interesting. What do you think about it? <br> [to R2] |
| 98 | R2 | It's interesting. |
| 99 | R1 | It is. Do you have a question? |
| 100 | R2 | I feel like it's something missing though (inaudible) - with the four yellows. <br> Is this the end here? |
| 101 | Stephanie | No. You can do four reds. <br> 102 |
| R1 | Now. Maybe what we should do is write out the cases that work. We <br> wrote out one of them really very clearly here. [R1 takes out some papers.] <br> Right? You did the one here um where you started with um exactly one <br> when it was green and blue in this case, right? |  |

