Description: Investigating numerical and algebraic representations for each case for the number of Unifix-cube towers 4 cubes tall selecting from red and yellow cubes **Parent Tape: Early Algebra Ideas About Binomial Expansion, Stephanie's Interview Seven of Seven** **Date: 1996-04-17** **Location: Union Catholic** **Researcher: Professor Carolyn Maher**	**Transcriber(s): Aboelnaga, Eman** **Verifier(s): DeLeon, Christina** **Date Transcribed: Spring 2009** **Page: 1 of 10**

1	R1	So when we divided by two – all this stuff – this is the row we ended up with.
2	Stephanie	Um hm.
3	R1	Isn't that right? We had a row of six. Alright. Now before doing it – see if you can use the same kind of reasoning that we just used with the four times three divided by in that case two to think about what's happening here?
4	Stephanie	Um.
5	R1	Okay.
6	Stephanie	Well. It's six, because there's six towers.
7	R1	Um hm.
8	Stephanie	And it's going to be six times um, three because – wait – no it was six times two because there's two places to put it. Because it was four times three. Yeah.
9	R1	It's getting a little trickier. Huh?
10	Stephanie	It's it's six times two because there's two places to put it.
11	R1	Okay. So you have two places to put it. You have your six towers four high. Right?
12	Stephanie	Yes.
13	R1	And so you're saying there are two possible ways – places of putting it as you go from two to three.
14	Stephanie	Um hm.

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15	R1	So that's – two available positions – but see now – the the height of the tower seems to have nothing to do with this anymore. Does it?
16	Stephanie	Um hm.
17	R1	It's weird. You know what I'm saying.
18	Stephanie	Yeah.
19	R1	It had something to do with it before and it doesn't have anything to do with it now. Um. And I sorta you know before you know you had gee that was very nice. You had the height of the tower. Right?
20	Stephanie	Um hm.
21	R1	Times the available positions. Right?
22	Stephanie	Um hm.
23	R1	Divided by the duplicates. Right? I sort of liked that. The height of the tower, right? That was kind of nice.
24	Stephanie	Um hm.
25	R1	The tower is still high – right – but that's, now if we talked about available positions, right? It wouldn't it, would be two – isn't that right?
26	Stephanie	Yeah.
27	R1	It sorta doesn't quite – it would give us eight.
28	Stephanie	um
29	R1	Right. And you're saying that what we have here is – um – height of the tower doesn't seem to enter into it. The available positions does. It's two. Right? But there's six towers. So maybe you have to think about this as the number of towers? Right. You started with four towers with exactly one of

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		a color. Maybe that's what this has to be thought of. What do you think?
30	Stephanie	Yeah, you could
31	R1	'Cause if you want to be consistent
32	Stephanie	You can still get from four to eight – though. You'd have to divide by two. But – I don't know where the two comes from.
33	R1	Well, remember you want to get twelve up here.
34	Stephanie	Um hm
35	R1	You said from each of the six
36	Stephanie	Um.
37	R1	that have exactly two of a color, right?
38	Stephanie	Um hm.
39	R1	You have two available positions.
40	Stephanie	Yes.
41	R1	You multiply by two. –or you can say in exactly four with exactly one of a color
42	Stephanie	Um hm.
43	R1	Right? You have three available positions – you multiply by three. That's consistent. If you thought about it that way.
44	Stephanie	Um.
45	R1	It's a little bit different though.

46	Stephanie	Yeah.
47	R1	See it's a way to think about it. I don't know. But the problem then is you still have duplicates. Right?
48	Stephanie	Um hm. You have three of each.
49	R1	So here you divided by
50	Stephanie	three
51	R1	three. Right? And you ended up with four.
52	Stephanie	Um hm.
53	R1	Well. Let's use that again. You ended up with four. Right?
54	Stephanie	Yes.
55	R1	And you have how many available positions?
56	Stephanie	Two. – For which one?
57	R1	When you ended up with four. One – [*Dr. Maher points to the groups of four – each with three red and one yellow.*]
58	Stephanie	Oh! You have one available position there.
59	R1	So you have one available position and then you produce
60	Stephanie	four
61	R1	And how many
62	Stephanie	[*speaking at the same time as R1*] There are duplicates.
63	R1	duplicates? How many?

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64	Stephanie	So you divide – four -
65	R1	You div – you have four then.
66	Stephanie	Yes.
67	R1	And that works. Right?
68	Stephanie	Um hm.
69	R1	Four times three is twelve, divided by two – you got six.
70	Stephanie	Um hm.
71	R1	Is that right? Did you get six the first time?
72	Stephanie	Yeah.
73	R1	And six times two is twelve divided by three, you got four. And four times one is four divided by four, you got one.
74	Stephanie	Um hm.
75	R1	So that was an interesting way to think about it.
76	Stephanie	Yes.
77	R1	But this number meant how many of that kind. Now if we used that same thinking here – I wonder if that works. Right?
78	Stephanie	Um hm.
79	R1	With with no reds, you'd have how many?
80	Stephanie	You'd have all yellow.
81	R1	But how many? - We said here this is how many with exactly one red.

82	Stephanie	Um hm.
83	R1	This was how many with exactly two reds. This was how many with exactly three reds. [*points to the rows of towers on the table*]
84	Stephanie	With no reds, you'd have four. With no reds, you'd have all yellow. You'd have all - - I gu – okay here zer-
85	R1	You'd have one. That's the only one.
86	Stephanie	Oh! Well -
87	R1	Alright. Isn't that right?
88	Stephanie	Yeah.
89	R1	Does that work? You'd have one.
90	Stephanie	Um hm.
91	R1	And from this one – right?
92	Stephanie	Um hm.
93	R1	How many available positions do you have for exactly one red?
94	Stephanie	Four.
95	R1	So from that one you have four.
96	Stephanie	Yeah.
97	R1	But you don't divide by – right? – right?
98	R2	That's a tough one (inaudible)
99	R1	[*to R2*] What would you do if you had a situation like that?

100	Stephanie	Wouldn't you divide, though? 'Cause -
101	R1	By what?
102	Stephanie	Well. –Wait. But if you're doing it with each one -
103	R1	We're supposed to end up with four, remember?
104	Stephanie	Oh. Yeah. That's right. I was thinking 'cause we had all four. Forget it.
105	R1	We're supposed to end up with one, two, three, four. And you're starting with no reds.
106	Stephanie	Um hm.
107	R1	[*to R2*] What do you do when you teach this? What do you tell your students? Do you hand wave it? Most textbooks hand wave it.
108	R2	Well. I guess you look at it as how many places you have to move things around.
109	R1	How many places you have to move things
110	Stephanie	You have four.
111	R1	around
112	Stephanie	We have four places. But – we can't divide by that.
113	R1	Well. Here you had three places to move things around. That was the three. There's four places to move things around.
114	Stephanie	Um hm.
115	R1	Right? So that works.
116	R2	Maybe if we think about how you grouped things when you were finished.

		If they're related.
117	R1	Here you divided by two. To make this work – what would you have to divide by here?
118	Stephanie	Oh! These are groups of one!
119	R1	Okay. So here you divided by two. Here you divided by three. Here you divided by four.
120	Stephanie	Oh!
121	R1	To make this work – what would you have to divide by
122	Stephanie	Yeah. But – oh – 'cause here we divided by the groups. 'Cause here there were groups of two. Here there were groups of three. Here there's groups of one.
123	R1	I don't understand. Help me.
124	Stephanie	All right. For this one. For like the second one, where there were four times three. There were groups of two. Like they came in pairs. There were two of these. Right?
125	R1	Um hm.
126	Stephanie	So they came in groups of two. So we divided by two.
127	R1	Um hm.
128	Stephanie	And here for the six, they came in groups of three. So you divided by three.
129	R1	Um hm.
130	Stephanie	But here
131	R1	How would you know what they come in groups of? – Unless you were all

		done?
132	Stephanie	Because there were three duplicates. Here's the two duplicates.
133	R1	How would you know before you start how many duplicates there would be?
134	Stephanie	You mean -
135	R1	I mean here you divided by one; here you divided by two; here you divided by
136	Stephanie	The number of reds in it?
137	R1	But isn't that nice? It goes one, two, three, four.
138	Stephanie	Um hm.
139	R1	Say I wonder if we were doing it the next way, would it be one, two, three, four, five? You know if we were going five high?
140	Stephanie	Um hm. Uh.
141	R1	Do you know what I'm saying?
142	Stephanie	Yeah.
143	R1	That's an interesting question, isn't it?
144	R2	It is.
145	R1	And if we were doing six high, would it be divided by one, two
146	Stephanie	And
147	R1	Or does it – is it a symmetry thing? Does it come back? I'm curious about that. Wouldn't that be something to explore?

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148	Stephanie	Um hm.
149	R1	Are you getting sick of towers or is it interesting?
150	Stephanie	No. It's interesting. And 'cause they come in groups of one. Like this.
151	R1	Yeah. [*to R2*] Do your students think about groups like that?
152	R2	Some of them do.
153	R1	Some of them do.
154	R2	Yeah.
155	R1	Some kids will think about it. Okay.
156	R2	This is a very interesting way, though.
157	R1	To think about it?
158	R2	Yeah.
159	R1	Yeah. So I guess the next thing is to try this with five. I mean you know what the answers are. So you know what you're working for.