Description: Developing numerical representations for each case for the number of Unifix-cube towers 5 and 6 cubes tall selecting from red and yellow cubes **Parent Tape: Early Algebra Ideas About Binomial Expansion, Stephanie's Interview Seven of Seven** **Date: 1996-04-17** **Location: Union Catholic** **Researcher: Professor Carolyn Maher**	**Transcriber(s): Aboelnaga, Eman** **Verifier(s): DeLeon, Christina** **Date Transcribed: Spring 2009** **Page: 1 of 9**

1	R1	Yeah. So I guess the next thing is to try this with five. I mean you know what the answers are. So you know what you're working for.
2	Stephanie	Okay.
3	R1	You may not even have to build them. 'Cause if you think of the number of positions. I mean you just have to do it for one.
4	Stephanie	Alright. For the first one – if you're
5	R1	Oh! You want to do it now?
6	Stephanie	Well. Oh!
7	R1	Okay. We can if you want to. Wow! Um. Alright. Let's do it. For the first one. Okay. We need some more. We'll build the basic ones. This destroys all the things I worked on. I tried this before I came. But we can rebuild these. So if we're going five – this is what we're starting with, right?
8	Stephanie	Um hm.
9	R1	One of these. There. [*R1 builds* $\begin{bmatrix} Y \\ Y \\ Y \\ Y \\ Y \end{bmatrix}$.] Okay.
10	Stephanie	Okay.
11	R1	So I'll let you be recorder. This is my attempt at recording. So we're going to start with the case with
12	Stephanie	The first one's one

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13	R1	There's one of those
14	Stephanie	times five
15	R1	Why five?
16	Stephanie	'Cause there's five positions.
17	R1	Okay.
18	Stephanie	Divided by one, 'cause they come in groups of one.
19	R1	Um hm.
20	Stephanie	Five.
21	R1	Okay. So that's five things taken one at a time.
22	Stephanie	Yes. The second one
23	R1	Why don't we write that down? Five things taken – equals five things taken one at a time. [*Stephanie writes.*]
24	Stephanie	Okay. For the second one – um – there's four spaces. But there's – out of five – so it's five times four and they'll come in groups of – I don't know – um – that's what we don't know though.
25	R1	Alright. So. Let's – Can we make these five? - Just - here
26	Stephanie	Well – maybe they'll come in groups of two?
27	R1	One – Let's think about at least one of these.
28	Stephanie	They might come in groups of two, I guess.
29	R1	Hm. Interesting. It's not easy to imagine what they come in. That - - one

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		more do we need? [*They have built* $\begin{bmatrix} R \\ Y \\ Y \\ Y \\ Y \end{bmatrix} \begin{bmatrix} Y \\ R \\ Y \\ Y \\ Y \end{bmatrix} \begin{bmatrix} Y \\ Y \\ R \\ Y \\ Y \end{bmatrix} \begin{bmatrix} Y \\ Y \\ Y \\ R \\ Y \end{bmatrix}$.]
30	Stephanie	Yeah.
31	R1	(inaudible)
32	Stephanie	One red on the bottom. [*They add* $\begin{bmatrix} Y \\ Y \\ Y \\ Y \\ R \end{bmatrix}$ *to the row of towers.*]
33	R1	Alright. So. That's these five, right?
34	Stephanie	Yes.
35	R1	Okay. so what you're saying here - - move some of this aside - - um – okay. Let's think of that one. [*R1 indicates* $\begin{bmatrix} R \\ Y \\ Y \\ Y \\ Y \end{bmatrix}$.]
36	Stephanie	Okay.
37	R1	There are five.

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38	Stephanie	Yes. [*Stephanie begins to build.*] You have one like that. [*builds* $\begin{bmatrix} R \\ R \\ Y \\ Y \\ Y \end{bmatrix}$], one like that [*builds* $\begin{bmatrix} R \\ Y \\ R \\ Y \\ Y \end{bmatrix}$]
39	R1	Well, can you predict before you do it?
40	Stephanie	Yeah. There's going to be four from each. There's gonna be
41	R1	four from each.
42	Stephanie	Yeah.
43	R1	Okay. So – and what's the each? How many make up each?
44	Stephanie	How – wh – what do you mean?
45	R1	You're saying – it's four from this.
46	Stephanie	Yeah. Four from
47	R1	What does
48	Stephanie	one
49	R1	each mean in this case?

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50	Stephanie	Oh! Like there's going to be four from this one. Four from that one. Four from that one. Four from that one. Four from that one. Four from that one.
51	R1	Okay. So how many eaches?
52	Stephanie	There's five.
53	R1	Five eaches. Okay.
54	Stephanie	Yeah.
55	R1	Alright. So – so that – you say – five times four
56	Stephanie	Yes. I have that. I just don't know what the -
57	R1	right
58	Stephanie	bottom part. It
59	R1	So - - and by groups you mean. The groupings you mean.
60	Stephanie	Groups like – one after we've put all of them out. Like how many groups there are going to – come in -
61	R1	I don't know. I'm
62	Stephanie	duplicates?
63	R1	I'm wondering – when you say you divide by
64	Stephanie	Oh! 'Cause that's the number of duplicates. That there are.
65	R1	But how do you know that before hand? Do you think there's a way?
66	Stephanie	Oops. [*She is building towers.*]
67	R1	So if this um, is going to be a pattern to this – the five times four – what do

		you think you would divide by?
68	Stephanie	Five times four – what do I think I'd – um – maybe two.
69	R1	You would guess two? Let's see, what does Pascal's triangle say? Do it quick. Right? [*R1 draws Pascal's Triangle as she speaks.*] Have to make that a four. Okay. – Is that right? Did I do that right?
70	R2	Um hm.
71	Stephanie	You've – well – there's the ten, so …if – I guess this is the
72	R1	So this is
73	Stephanie	Five, 'cause that's zero. So that's one. That's like this one. [*It is not apparent to which she is referring from this vantage point.*] Right? Or that's
74	R1	This is
75	Stephanie	Like -
76	R1	Um hm.
77	Stephanie	That's this one. [*She points to the row of towers of four yellow and one red.*]
78	R1	Alright. So if we were writing a formula -
79	Stephanie	This is this. [*points to the one and then to the all yellow tower.*]
80	R1	Um hm.
81	Stephanie	This is these. [*points to the five and then to the row of the five towers, each with four yellows and one red*] So that – it – you would divide by two.
82	R1	So it works with the result. Okay. We'll explore that later.

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83	Stephanie	Um.
84	R1	Okay.
85	Stephanie	And the next one, there's three spaces to put it. And there's
86	R1	But you have how many of them?
87	Stephanie	Ten. So it would be ten times three and you divide by three. [*Stephanie writes as she speaks.*]
88	R1	And it worked?
89	Stephanie	Yeah. And the next one, there is two spaces to put it and you have ten. So that's ten times two and you divide by two? [*Stephanie writes on the paper in front of her.*] And the last one – there's one space to put it – it's five times one divided by five equals one.
90	R1	Okay. Why did you switch to dividing by two here? Why didn't you divide by four? Why didn't you go one, two, three, four, five? Here you went two and here you went five.
91	Stephanie	Because – um – well, I was kind of thinking that if there there was one red and there were two reds and – I don't know – I guess I should've divided by four. Oh! – Duh – Yeah. [*Stephanie changes the two she wrote to a four.*] Yeah. That's right. It should be a four. I just wrote – I wasn't dividing right.
92	R1	So is that right then?
93	Stephanie	Yeah.
94	R1	And then you started with the one here, because there was one way of doing that?
95	Stephanie	Um hm.

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96	R1	Should we try the next row? You think -
97	Stephanie	Okay.
98	R1	You think that makes sense? One – six – fifteen – twenty – fifteen – six – one. So we know the one and six. That's easy, right?
99	Stephanie	Times six divided by one – six – [*Stephanie writes.*] The next one is six times five divided by two. That's fifteen. The next one is fifteen times four divided by three. Gosh. Fifteen times four – sixty divided by three – twenty. The next one is twenty times three divided by four. Oops. Sixty. Fifteen. Next is fifteen times oh and there's two spaces. That's thirty um divided by five. That's um six – six – [*Stephanie is writing very quickly as she is speaking.*] is one. Yeah. That works.
100	R1	Okay. What do you think?
101	Stephanie	Um hm.
102	R1	Um. There's a nice pattern here. Um. Why are those – that many duplicates coming up?
103	Stephanie	Um.
104	R1	You know what I'm saying? We – we know – you know it after you do it and then you can count them.
105	Stephanie	Um hm.
106	R1	You understand what I'm saying?
107	Stephanie	Um hm.
108	R1	You know it after you do it and you count the next one. Right?
109	Stephanie	Um hm.

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110	R1	But the question is – um – which is not a trivial question, is – how can you think about those duplicates if you'd – and I bet you can justify it by going through all the tediousness of finding it, but I don't think any of us want to do that, right? Um. But it would be nice to know in advance, how many duplicates come up. And that's – we have a good conjecture. It matches this and this, so it's a good tool, but we still haven't thought about why that works. Let's put that aside for a minute. Okay? Can I switch gears?