Description: Making sense of the Pascal Triangle addition rule with Unifix cube towers **Parent Tape: Early Algebra Ideas About Binomial Expansion, Stephanie's Interview Seven of Seven** **Date: 1996-04-17** **Location: Union Catholic** **Researcher: Professor Carolyn Maher**	**Transcriber(s): Aboelnaga, Eman** **Verifier(s): DeLeon, Christina** **Date Transcribed: Spring 2009** **Page: 1 of 15**

1	Stephanie	Um hm.
2	R1	'Cause I was interested in something like this piece of the triangle. Let's take – um – this is a four. [R1 draws in the diagonal lines from one to four and three to four.] Right?
3	Stephanie	Um hm.
4	R1	Or this piece. Right? Or this piece. – um – what did I do wrong here? Um. [lines drawn in the opposite direction]
5	Stephanie	(inaudible)
6	R1	What did I put there?
7	R3	It's two from the top, one from the bottom.
8	R1	I know it. Which is – I wanted to do this one – Thank you, Steve. I want to do the ten. The four to the ten. And um, do you understand what I'm saying?
9	Stephanie	Um hm.
10	R1	We went – we went this way. And what we still have to think about is – we have a pattern. We have a rule. We have a way of generating it and we know what to divide by because of a pattern, but I – and you said groupings.
11	Stephanie	Um hm.
12	R1	And I'm not so sure I follow the groupings stuff yet. Like I see the um – let's say here: six times two divided by three. Right?
13	Stephanie	Um hm.
14	R1	But I didn't see those groupings of three until you were all done and grouped them as three.

15	Stephanie	Yeah.
16	R1	And I would've liked to have known why there'd be groupings of three before you did it and just counted. – I'd like to be able to have some way of thinking about that, to know that.
17	Stephanie	Um.
18	R1	Do you understand what I'm saying?
19	Stephanie	Yeah. But it's like because there's three places that have the same – one in the same positions. Like -
20	R1	So you're telling me that when you move from here to here [R1 indicates from the towers that are two reds and two yellows to the towers that are three reds and one yellow.]
21	Stephanie	Like this $\begin{bmatrix} R \\ R \\ R \\ Y \end{bmatrix}$ one
22	R1	from two reds to three reds.
23	Stephanie	[as she moves the appropriate towers] This can be from this $\begin{bmatrix} R \\ Y \\ R \\ Y \end{bmatrix}$, it can be from this $\begin{bmatrix} R \\ Y \\ Y \\ R \end{bmatrix}$. Oh no, it can't be from that. It can be from this $\begin{bmatrix} Y \\ R \\ R \\ Y \end{bmatrix}$ or it

		can be from this $\begin{bmatrix} R \\ R \\ Y \\ Y \end{bmatrix}$.
24	R1	So how do you know that in advance?
25	Stephanie	I guess you could look at 'em and say these – have I guess this space in common, but
26	R1	Okay. So it came from those three.
27	Stephanie	Yes.
28	R1	Um hm. I could see that. And so I have to imagine that this $\begin{bmatrix} R \\ R \\ Y \\ Y \end{bmatrix}$ could come, could come from the shorter one like that or the other one.
29	Stephanie	Um hm.
30	R1	And only two.
31	Stephanie	Yeah. (inaudible)
32	R1	Uh huh. – Okay. So – so – you would, you would know, for instance, something like this. – That there are five positions that could've generated this? [R1 points out $\frac{15 \cdot 2}{5}$.] How would you know that?
33	Stephanie	Um.
34	R1	It could've come from you know what I'm saying that [Stephanie sneezes.] God bless you.

35	Stephanie	Thank you.
36	R1	Hmmm.
37	Stephanie	I guess you'd – I mean – you'd really I guess have to be looking at it. I really – I probably
38	R1	A visual picture helps?
39	Stephanie	I probably couldn't like say well that's gonna come from that, that, that, that, and that.
40	R1	Yeah. Let's go back to this, so I can make a neater picture. [redraws Pascal's triangle on a new sheet of paper] I'm doing it again. I keep making this a six and I want it to be a four. Okay. Um. Let's explore um – which one should we explore? Let's do this one. [R1 selects 1 3] \ / 4 Okay?
41	Stephanie	Um hm.
42	R1	Do you know what this one means? If you had to build this one, what would that tower look like?
43	Stephanie	That one?
44	R1	What would that tower look like? What would these two look like?
45	Stephanie	[There is a pause as Stephanie begins building towers.] I think that one would be like this and – that one [Stephanie indicates the one in R1's selection from Pascal's triangle. Stephanie has built this tower. $\begin{bmatrix} Y \\ Y \\ Y \end{bmatrix}$] And that one

46	R1	(inaudible) three high, no red.
47	Stephanie	[Stephanie continues to build and move towers.] like this. [She makes $\begin{bmatrix} R \\ Y \\ Y \end{bmatrix}$ $\begin{bmatrix} R \\ R \\ Y \end{bmatrix}\begin{bmatrix} R \\ Y \\ R \end{bmatrix}$.]
48	R1	Okay. Three high. – Exactly one red.
49	Stephanie	Yes.
50	R1	Okay.
51	Stephanie	Oh! Wait! [Stephanie corrects the two towers with two red cubes and builds the missing tower. $\begin{bmatrix} R \\ R \\ Y \end{bmatrix}\begin{bmatrix} R \\ Y \\ R \end{bmatrix}$ to $\begin{bmatrix} Y \\ R \\ Y \end{bmatrix}\begin{bmatrix} Y \\ Y \\ R \end{bmatrix}$.]
52	R1	Okay, makes you dizzy after a while, doesn't it? 'Cause I think I see exactly one also. Even when you make it, I just believe you're gonna do it. Okay. Now. When we – doing this [1 3] \ / 4
53	Stephanie	Um hm.
54	R1	What's different about these and this tower here [tapping the number four from Pascal's triangle] that I call four? There

55	Stephanie	Well – it's four high.
56	R1	Okay. So there's one of these. [indicates $\begin{bmatrix} Y \\ Y \\ Y \end{bmatrix}$] That's one. Right?
57	Stephanie	Yes.
58	R1	There are three of these. [indicates the towers with two yellows and one red]
59	Stephanie	Yes.
60	R1	And that's exactly one red.
61	Stephanie	Um hm.
62	R1	And and that's four, but what else about it?
63	Stephanie	Like?
64	R1	They're four high.
65	Stephanie	What else now?
66	R1	What else can you tell me about this? They're four tall.
67	Stephanie	Um hm.
68	R1	What about the coloring of this?
69	Stephanie	Well, there's going to be three of one color and one of the other instead of two and one like for three.
70	R1	Okay. So these are going to be four tall.
71	Stephanie	Um hm.

72	R1	And next, and there's going to be three of one color.
73	Stephanie	And one of another.
74	R1	And what's what's the – what's the one – what's the one color?
75	Stephanie	One can be red.
76	R1	Well
77	Stephanie	And three could be
78	R1	Well, we have to be consistent.
79	Stephanie	Alright. One is red. Three is yellow.
80	R1	One is red and three is yellow.
81	Stephanie	Yes.
82	R1	Okay. Now study that.
83	Stephanie	You want me to tell you why those give you four.
84	R1	I want to know from here
85	Stephanie	Uh huh
86	R1	What would you do to these
87	Stephanie	Well
88	R1	to get me, to get me
89	Stephanie	Well, I'd build them higher.
90	R1	Well, don't don't do it yet. Just think about it for a minute. Remember what they're going to look like.
91	Stephanie	Yeah.

92	R1	There's going to be exactly one red.
93	Stephanie	This would go here. [moves the $\begin{bmatrix} Y \\ Y \\ Y \end{bmatrix}$ over] and there would be red
94	R1	No. No. We start with these [R1 indicates the four again and moves the $\begin{bmatrix} Y \\ Y \\ Y \end{bmatrix}$ back.] I don't need to touch these. I want you to tell me what you're gonna do to these so that when you're all done
95	Stephanie	Um hm.
96	R1	you end up with exactly one red. But you got to make them all four tall.
97	Stephanie	I'm going to put a yellow here [points to $\begin{bmatrix} R \\ Y \\ Y \end{bmatrix}$]
98	R1	Okay.
99	Stephanie	I'm gonna put a yellow there. [points to $\begin{bmatrix} Y \\ R \\ Y \end{bmatrix}$]
100	R1	Right.
101	Stephanie	I'm going to put a yellow there. [points to $\begin{bmatrix} Y \\ Y \\ R \end{bmatrix}$] and I'm gonna put a red

		there. [points to $\begin{bmatrix} Y \\ Y \\ Y \end{bmatrix}$]
102	R1	Okay. So how many ways – how many do you end up with?
103	Stephanie	Four
104	R1	Four. – So from the one three tall with no reds
105	Stephanie	Um hm.
106	R1	And the three three tall with one red, right?
107	Stephanie	Yes.
108	R1	You end up with four four tall with one red.
109	Stephanie	Um hm.
110	R1	Isn't that neat?
111	Stephanie	Yeah.
112	R1	Okay. Let's do another one. Which one should we do? Um. [6 4] \ / 10
113	Stephanie	Okay.
114	R1	That's a little hard.
115	Stephanie	Um. That's – well we had that. That would be -

116	R1	So, what's this one? Tell me what this one is. [R1 points to the six.]
117	Stephanie	Those are four high with two red.
118	R1	Okay. And there are how many of those?
119	Stephanie	Six of them.
120	R1	Okay. I can help you a little bit. [They pull already built towers from the pile on the table.] Um. Here's another one.
121	Stephanie	We already have this one.
122	R1	A bunch of duplicates here.
123	Stephanie	Two, three, four, five. We need one more. What one do we need?
124	R1	Which one is missing?
125	Stephanie	Um. The one with two on the bottom. I'll just make it.
126	R1	Here.
127	Stephanie	Oh! Okay. Oh! Wait! [Stephanie sees that the tower is upside down.]
128	R1	Oh wait! [She reverses the order of the cubes.] There you go.
129	Stephanie	Alright.
130	R1	Okay. We better move these a little bit. Are you sure we have them all?
131	Stephanie	Yes. There's the six. And this – [Stephanie points to the four.] is um one with three red and one um one yellow.
132	R1	Okay. Three red and one yellow. These are the same?
133	Stephanie	Yes. [R1 moves over towers as Stephanie builds.] Um. (inaudible) one and – that one

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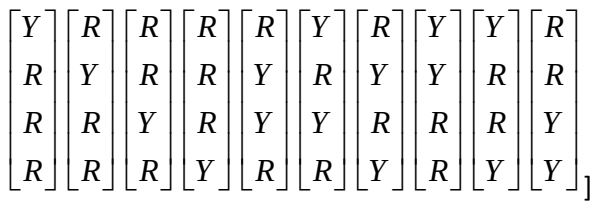
134	R1	So there are four of them.
135	Stephanie	Um hm.
136	R1	And in order to help me [R1 rearranges the order of the towers.] Do you mind?
137	Stephanie	Yeah. Go ahead. [The towers are arranged: ]
138	R1	Okay.
139	Stephanie	Okay.
140	R1	So I believe that. In combina – combinatorics how would you write this? This six?
141	Stephanie	How would I write that one?
142	R1	Yeah.
143	Stephanie	Um.
144	R1	Just write it with an arrow and tell me what this is – what these numbers are.
145	Stephanie	The six the six is the two – so that would be be um – you want me to write it here?
146	R1	Sure.

147	Stephanie	[writes C_2^4] Four. Two.
148	R1	And this one is? [Stephanie writes C_3^4 .] Four three and we're adding them together. Plus.
149	Stephanie	Oh.
150	R1	to get
151	Stephanie	to get [Stephanie writes C^5 .] um. [She then adds the three: C_3^5 .]
152	R1	Is that three?
153	Stephanie	Um. That's
154	R1	Another one, two, three.
155	Stephanie	Yeah.
156	R1	It is three. Okay. So. Um. What does that mean? What is -
157	Stephanie	That means you have four and you're selecting two. You're taking – well you're taking two red
158	R1	Okay. Exactly two red.
159	Stephanie	and
160	R1	And then you have exactly three red.
161	Stephanie	Yes.
162	R1	And now you're making them – how tall?
163	Stephanie	Five tall.
164	R1	Five tall. And how many reds are there going to be?

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165	Stephanie	Three.
166	R1	So how can you make them five tall with three reds?
167	Stephanie	Red there. Red there. Red there.
168	R1	So here you get three ways, right?
169	Stephanie	A red there. A red there. A red there. A yellow there. A yellow there, a yellow there and a yellow there.
170	R1	There's your ten.
171	Stephanie	Yes.
172	R1	Isn't that neat!
173	Stephanie	Um hm.
174	R1	That's what it is. I think that's so neat.
175	R2	It is.
176	R1	Do you like that?
177	R2	Yes.
178	R1	So the question is – think about these in general ways – you know. Are there general ways to be it? - And you see, we, we could do it in arithmetic with these combinatorics. We're saying four things two plus four things three is five things three.
179	Stephanie	Um hm.
180	R1	It's kind of arithmetic, isn't it? [Stephanie laughs.] I mean if you just start writing these as combina – or do we say here – we said – this is which row?
181	Stephanie	Um. That's the three. So that [writes C^3]

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182	R1	Three things – each one is
183	Stephanie	That's none? [writes C_0^3]
184	R1	None. Right.
185	Stephanie	And that means one.
186	R1	plus
187	Stephanie	Three. One. [writes C_1^3]
188	R1	Right? And we said that's gonna give you
189	Stephanie	Four. One. [writes C_1^4]
190	R1	Isn't that an interesting kind of arithmetic?
191	Stephanie	Um hm.
192	R1	Now what I'm going to ask you to do – to think about
193	Stephanie	Okay.
194	R1	is to, is to write as many of these and convince yourself and see if you can come up with a general rule with your n's and n minus one's or whatever.
195	Stephanie	Okay.
196	R1	You can call – what you can do is call this n and this r. [writes C_r^n] Right? Or you can call this r and then you can call this r minus one.
197	Stephanie	Okay.
198	R1	Right? If this is r, this is one mi – whatever – you understand? If this is r, this is r minus one. If this is r, this is r plus one.

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199	Stephanie	Yes.
200	R1	You can do it any way you want to.
201	Stephanie	Yes.
202	R1	Do you see what I'm saying?
203	Stephanie	Yeah.
204	R1	You could go one either way or the other. And see if you can develop a prediction of a rule.
205	Stephanie	Okay.
206	R1	Won't that be – see if you can play around with the algebra.
207	Stephanie	Okay.