

Description: Stephanie revisits combinatorics notation for building Unifix-cube Towers selecting from two colors Parent Video: Early Algebra Ideas About Binomial Expansion, Stephanie's Interview Six of Seven (student view) Date: 1996-03-27 Location: Union Catholic Researcher: Professor Carolyn Maher	Transcriber(s): Aboelnaga, Eman Verifier(s): DeLeon, Christina Date Transcribed: Spring 2009 Page: 1 of 14
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0:00	1	R1	So as much as you can remember what last time was about um – and I have paper here and pens and things if you need them. Any way you can be helpful.
	2	Stephanie	All right.
	3	R1	And if you need to come closer, I'll just move back. How we even started this discussion – which I can't even remember. I'll help – if I can be helpful.
	4	R2	What was it about?
	5	Stephanie	I think. Did you – you started with um explaining that if you had like – four – like a towers of four –
	6	R1	I'm going to let you move up.
	7	Stephanie	Or
	8	R1	So you can
	9	Stephanie	trains of four
	10	R1	(inaudible)
	11	R2	Okay.
	12	R1	switch positions, Bob.
	13	Stephanie	that um

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	14	R2	Thank you.
	15	Stephanie	and you have two different choices.
	16	R2	Two different colors?
	17	Stephanie	Yeah. Like
	18	R2	Um hm.
	19	Stephanie	if it was [<i>Stephanie gets some Unifix cubes</i>] like that
	20	R2	Okay.
	21	Stephanie	Um. I think it started with her explaining that um if you took one of the four colors [<i>Stephanie pauses, she rolls her eyes, and appears to be thinking – recalling the last interview.</i>] Yeah. One of the fours. Oh. One color.
	22	R2	Um hm.
	23	Stephanie	How many different combinations you could make – out of four high. Like you could have
	24	R2	You mean they'd be four high with one green?
	25	Stephanie	Yeah.
	26	R2	Somewhere.
	27	Stephanie	Yes.
	28	R2	Okay.

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	29	Stephanie	So. Well, it would – we did trains so it would be four like this.
	30	R2	Okay.
	31	Stephanie	Or um. Oh. Four like this [<i>Stephanie builds a train.</i>] or four like um – [<i>she builds another train</i>] this or four like [<i>continues building</i>] this or four [<i>builds a fourth train</i>] like that.
	32	R2	(inaudible)
	33	Stephanie	And that taking one out of four, like one out of four choices was the same as um – [<i>Stephanie writes C_1^4 and $\binom{4}{1}$ on the paper before her</i>]- or – I think – that's how we started. [<i>Four trains are now visible on the table. They are arranged in a row on the table in front of Stephanie from Stephanie's right to left.</i>]
	34	R2	Okay. So this is the way you would write –
	35	Stephanie	Yeah.
	36	R2	What?
	37	Stephanie	Um that –
	38	R2	What do those symbols stand for?
	39	Stephanie	That – well – that means that you're selecting one out of four

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	40	R2	four
	41	Stephanie	choices.
	42	R2	Out of four choices.
	43	Stephanie	Uh hm. Yeah. I think that's how we started. And then what happened was
	44	R2	Um hm.
	45	Stephanie	she asked like two out of - if I had two green?
	46	R2	Um hm.
	47	Stephanie	What would it be? And it – how many choices would there be? And for one, there was four. [<i>Stephanie writes on the paper in front of her.</i>]
	48	R2	And they're the four that you've shown?
	49	Stephanie	Yeah. There's no more.
	50	R2	And there are no more.
	51	Stephanie	Yeah. You can't make any more.
	52	R2	Okay. I'm ready to believe that.
	53	Stephanie	Okay.
	54	R2	Uh. When you start – when you work with two, though, the question might be interesting.

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	55	Stephanie	Yeah.
	56	R2	So let's see what happens.
	57	Stephanie	Well, for two there's um...this one. [<i>Stephanie builds</i> [G B B]]
	58	R2	Um hm.
	59	Stephanie	And there's –this one. [<i>She builds</i> [B B G G]] And there's –
	60	R2	Um hm.
	61	Stephanie	This one. [<i>She builds</i> [G B G B]] and there's –[<i>builds</i> [B G B G]]
	62	R2	Um hm.
	63	Stephanie	[<i>Stephanie builds</i> [B G G B] and [G B B G]] That's it.
	64	R2	Six?
	65	Stephanie	Um hm. There's no more.
	66	R2	How do you know that?
	67	Stephanie	Um. 'Cause I tried all the combinations-
	68	R2	Um.
	69	Stephanie	-possible – like – um alright. If you start out with – um – two blue on top, [<i>Stephanie picks up that tower</i>] there,

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			you can have – if you start out with two blue together, you can
	70	R2	Yes.
	71	Stephanie	put them on top. You can put them in the middle – you can move them one down-
	72	R2	Um hm.
	73	Stephanie	-or you can put them on the bottom. [<i>Stephanie rearranges the towers, lining them up in the following order:</i> $\left[\begin{array}{c} B \\ B \\ G \\ G \end{array} \right] \left[\begin{array}{c} G \\ B \\ B \\ G \end{array} \right] \left[\begin{array}{c} G \\ G \\ B \\ B \end{array} \right]]$
	74	R2	Yes.
	75	Stephanie	If you start with them separated by a green
	76	R2	Um hm.
	77	Stephanie	There'd be one – on top- or like this. [<i>Stephanie shows the two towers:</i> $\left[\begin{array}{c} B \\ G \\ B \\ G \end{array} \right] \left[\begin{array}{c} G \\ B \\ G \\ B \end{array} \right]]$. You can't move it anymore, because you only have four spaces to move it.
	78	R2	Um hm.

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05:00-09:59	79	Stephanie	And there's only one like that. <i>{Stephanie indicates the</i> $\begin{bmatrix} B \\ G \\ G \\ B \end{bmatrix}$ <i>tower.}</i>
	80	R2	How would you describe this one?
	81	Stephanie	It's separated by two green.
	82	R2	So here it's like they're separated by no greens?
	83	Stephanie	Yeah.
	84	R2	And here separated
	85	Stephanie	By one.
	86	R2	The two blues are separated by
	87	Stephanie	one green.
	88	R2	one green. And here, they're separated by two.
	89	Stephanie	Um hm.
	90	R2	Um. – Is it possible that there could be another tower that you haven't built yet?
	91	Stephanie	Oh. Yeah. No. No. Un uh.
	92	R2	How would you explain that?
	93	Stephanie	All right. Wait. Let me think. <i>[Stephanie writes something on the paper in front of her.]</i> Yeah, because

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			you can't move them any more. –There's only four spaces for you to move them.
	94	R2	Um hm.
	95	Stephanie	And – like with this one – if they're separated by none, you can have them here
	96	R2	Yes.
	97	Stephanie	up top. You could move them down one and have them here.
	98	R2	Right.
	99	Stephanie	And you can down them move them down another – You can't move them down any more. There's no more
	100	R2	Because
	101	Stephanie	spaces for you to move them.
	102	R2	That's true.
	103	Stephanie	Here, you have
	104	R2	separated by one
	105	Stephanie	separated by one green, you can have them here. You can move them down one and have them here. You can't move them down any more.
	106	R2	That's true.
	107	Stephanie	Because there's only four. If they're separated by two. You can't move them – you have one on the top and one

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			of the bottom and that's it. You can't do anything else to it.
	108	R2	Okay.
	109	Stephanie	So there's six.
	110	R2	I think – I think I'm convinced.
	111	Stephanie	Okay.
	112	R2	Okay. Good.
	113	Stephanie	So then it was -
	114	R2	Did you do this last week?
	115	Stephanie	Yeah.
	116	R2	Yeah. Okay. So you found six.
	117	Stephanie	<p>Um hm. And then with three – <i>[Stephanie begins building towers with three green and one blue. She first</i></p> <p><i>builds</i> $\begin{bmatrix} B \\ G \\ G \\ G \end{bmatrix}$] three, there's only – you can have one at</p> <p>the top – Oh. No. – You can have one at the bottom.</p> <p><i>[builds</i> $\begin{bmatrix} G \\ B \\ G \\ G \end{bmatrix}$] You can have – there. <i>[builds</i> $\begin{bmatrix} G \\ G \\ B \\ G \end{bmatrix}$] And</p>

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			<p>one there. [builds $\begin{bmatrix} G \\ G \\ G \\ B \end{bmatrix}$] [Stephanie looks at R2.] And that's it. {Stephanie has built her 'traditional' "staircase". $\begin{bmatrix} B \\ G \\ G \\ G \end{bmatrix} \begin{bmatrix} G \\ B \\ G \\ G \end{bmatrix} \begin{bmatrix} G \\ G \\ B \\ G \end{bmatrix} \begin{bmatrix} G \\ G \\ G \\ B \end{bmatrix}$}</p>
	118	R2	That's it?
	119	Stephanie	Um hm. 'Cause if there's – well, what it is, is it's the opposite of this one. [Stephanie indicates the towers with three blues and one green.]
	120	R2	Ah! – Are you – are you saying that three greens is the same as one blue?
	121	Stephanie	Yeah.
	122	R2	Ah?
	123	Stephanie	They're the opposite. Because here it's blue separated by one green and here's it's green and one blue.
	124	R2	So by opposite, you mean – wherever there's a green on this side, you put a blue on this side.
	125	Stephanie	Yeah.
	126	R2	And wherever there's blue on this side, you put a green on that side.

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	127	Stephanie	Yeah.
	128	R2	And then - - Yeahh - -
	129	Stephanie	And so you have four.
	130	R2	And so four.
	131	Stephanie	<i>[writing]</i> And there's only two ways to do this one. Oops -
	132	R1	Let's leave those.
	133	Stephanie	No. There's only one way to do this one.
	134	R1	Why don't we leave these?
	135	Stephanie	If it's blue, you can only do it like this. <i>[builds</i> $\left[\begin{array}{c} B \\ B \\ B \\ B \end{array} \right]$ <i>]</i>
	136	R2	I see.
	137	Stephanie	You're selecting four blue out of four.
	138	R2	Four.
	139	Stephanie	So you can't do any thing else.
	140	R2	Um hm.
	141	Stephanie	And zero is the opposite. You're selecting no blue, so

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			you're selecting all green. [<i>builds</i> $\begin{bmatrix} G \\ G \\ G \\ G \end{bmatrix}$]
	142	R2	Yeah.
	143	Stephanie	That's it.
	144	R2	Those seem so much simpler.
	145	Stephanie	Yeah.
	146	R2	But again, I've seen you're using the opposite in order to
	147	Stephanie	Um hm.
	148	R2	connect them together. Uh. Let me just ask you one more question
	149	Stephanie	Okay.
	150	R2	about when you um I think it was when you built these six towers. Uh. It looked to me like you were making pairs of opposites
	151	Stephanie	Yeah.
	152	R2	at the beginning, when you were constructing them,
	153	Stephanie	Um hm.
	154	R2	but then when you were explaining to me how many there were you organized them differently.

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	155	Stephanie	Um hm.
	156	R2	Um. Could you say a little more about that?
	157	Stephanie	Oh. Well. – Because it's easier for me to look at them as opposites when I'm building them.
	158	R2	Um hm.
	159	Stephanie	Then – 'cause I know – 'cause it's like pairing them up – like – if there's one separated on top, there's one – you know -
	160	R2	Yeah -
	161	Stephanie	But, it's easier for you to look at them when they're done if they're like this. So you can see the pattern that they make. That you can't build down any more.
	162	R2	Um hm.
	163	Stephanie	Or you can't build up any more, 'cause there's no more to – do it.
	164	R2	So it was more for your explanation that
	165	Stephanie	Um hm.
	166	R2	you rearranged them.
	167	Stephanie	Like you could see it better like this, than if I said – I mean – 'cause when we first did the towers problems, we went through – I mean there were tons of Unifix cubes and all it was was those two are opposites. "Well, how do you know?"

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	168	R2	Um hm.
	169	Stephanie	And I don't know. I didn't know how to explain it. So it's easier for you to see that – there's the – you know – because it goes down – you can't build anymore. That's why.
	170	R2	Thank you.