Description: Comparing Towers, selecting from two colors, built inductively and corresponding to the addition rule of Pascal's Triangle **Parent Tape: Early Algebra Ideas About Binomial Expansion, Stephanie's Interview Six of Seven** **Date: 1996-03-27** **Location: Union Catholic** **Researcher: Professor Carolyn Maher**	**Transcriber(s): Aboelnaga, Eman** **Verifier(s): DeLeon, Christina** **Date Transcribed: Spring 2009** **Page: 1 of 17**

1	R2	Um hm.
2	Stephanie	And then – um – then I think we started to build – um – we figured out all of them like from like from this.
3	R2	All of?
4	Stephanie	Like if you started out with you see from one from zero
5	R2	Oh.
6	Stephanie	And we said that would equal one, ‘cause
7	R2	Um hm.
8	Stephanie	And then – um –one – and you figured that out all the way up to um
9	R2	Um hm
10	Stephanie	four. And then, she showed me how to build the triangle – one -
11	R2	Okay. Um –Tell me a little more about the triangle. Um. What is this number?
12	Stephanie	That’s
13	R2	What does that count?
14	Stephanie	That’s how many you can get if you take zero from zero.
15	R2	So that’s the zero-zero.
16	Stephanie	Yes.
17	R2	And then these two ones?
18	Stephanie	That’s –um –zero out of one or one out of one.

19	R2	Um hm.
20	Stephanie	That's zero out of two.
21	R2	Uh huh.
22	Stephanie	One out of two, two out of two.
23	R2	Two out of two?
24	Stephanie	Um hm.
25	R2	So this counts the ways – so which towers – okay – does this have to do with towers?
26	Stephanie	Yeah. I – it wou-
27	R2	Show me.
28	Stephanie	It would be – *[Stephanie grabs the towers two tall.]*
29	R2	Okay.
30	Stephanie	And this one - those
31	R2	So these are the towers that are two high.
32	Stephanie	Yeah.
33	R2	-two blocks high and then um how do find the one, the two, and the one?
34	Stephanie	It would be –um – if you're selecting green, it would be – one well if you're selecting blue, it would be one with no selections of blue.
35	R2	Right.
36	Stephanie	Two with one selection of blue and one with um one's with

37	R2	Okay.
38	Stephanie	all selections of blue.
39	R2	Oh. Okay. Okay. So this has more to do
40	Stephanie	It’s like the towers
41	R2	It’s like the way you’d organized
42	Stephanie	Um hm.
43	R2	the towers before.
44	Stephanie	Yeah.
45	R2	Uh – I was interested in how – Do you remember if you had – if these were the original order in which you arranged them – you know when you had them here – or whether you had rearranged them?
46	Stephanie	When we were there, I think it was here, here, here, and there. [*the towers are arranged from her left to her right:* $\begin{bmatrix} B \\ B \end{bmatrix}\begin{bmatrix} G \\ B \end{bmatrix}\begin{bmatrix} B \\ G \end{bmatrix}\begin{bmatrix} G \\ G \end{bmatrix}$]
47	R2	and there.
48	Stephanie	Yeah. Because this has all the blue. And
49	R2	Um hm.
50	Stephanie	on the bottom. *[Pause]*
51	R2	Oh. Very fine. Yeah.
52	Stephanie	Actually [*Stephanie reverses the pattern:* $\begin{bmatrix} G \\ B \end{bmatrix}\begin{bmatrix} B \\ B \end{bmatrix}\begin{bmatrix} B \\ G \end{bmatrix}\begin{bmatrix} G \\ G \end{bmatrix}$] There. I think I

		messed this row up. *[She is talking about the towers three cubes high.]* When I moved I think I moved these around.
53	R2	Okay. I'm still understanding this one. Um. These two came from this green one by putting
54	Stephanie	Yes.
55	R2	different tops on it. And similarly *[R2 indicates the two with blue bottom cubes.]*
56	Stephanie	Um hm.
57	R2	those two. *[pause]* Ah ha. It's interesting because the way you arranged them to show the one, two, and the one, switched these two.
58	Stephanie	I think I just messed them up when I was making these. I couldn't see and I had to move those.
59	R2	Oh. But they're a different
60	Stephanie	Yeah.
61	R2	But they're different choices anyway, but it is interesting, it was interesting to me
62	Stephanie	Um, but
63	R2	how you see the -
64	Stephanie	It's just
65	R2	Yeah. How would you organize the next row, so that it makes more sense?
66	R1	(inaudible)

67	R2	So it makes the most sense for you?
68	Stephanie	Oh.
69	R1	It works for the chart.
70	R2	Could it work for the chart? Yeah. You want to try that?
71	Stephanie	For the chart?
72	R1	You can come around here.
73	R2	Yeah.
74	Stephanie	Well for the chart it would be um *[Stephanie writes]* wait – *[writes some more]* So
75	R2	How did you know to write those numbers?
76	Stephanie	'Cause - - one goes to one and one and then one goes here. One plus one is two.
77	R2	Oh.
78	Stephanie	One goes here. One.
79	R2	So you do it by adding.
80	Stephanie	Yeah. One plus two is three. One plus two is three. And one goes there. It…it that's how you figure it out.
81	R2	Ahh so that so that's how you got this row.
82	Stephanie	Yes.
83	R2	Okay.

84	Stephanie	That's how I got it.
85	R2	Did you explore why the adding works?
86	Stephanie	Um, I don't know, I, I mean we, um, we worked it out like this on paper, but -
87	R1	What, what's – that's a good question. You just took these four *[points to the row of towers two high:* $\begin{bmatrix} B \\ B \end{bmatrix}\begin{bmatrix} G \\ B \end{bmatrix}\begin{bmatrix} B \\ G \end{bmatrix}\begin{bmatrix} G \\ G \end{bmatrix}$]
88	Stephanie	Um hm.
89	R1	and you explained how the adding works for this, for the row one-two-one.
90	Stephanie	Um hm.
91	R1	Isn't that what you just did?
92	Stephanie	Yeah.
93	R1	Any you had them like this [*rearranges the row so that the towers are in* $\begin{bmatrix} G \\ B \end{bmatrix}\begin{bmatrix} B \\ B \end{bmatrix}\begin{bmatrix} B \\ G \end{bmatrix}\begin{bmatrix} G \\ G \end{bmatrix}$ *order*] Right?
94	Stephanie	Um hm.
95	R1	But you said it could also go like this [*rearranges the row so that the towers are back in* $\begin{bmatrix} B \\ B \end{bmatrix}\begin{bmatrix} G \\ B \end{bmatrix}\begin{bmatrix} B \\ G \end{bmatrix}\begin{bmatrix} G \\ G \end{bmatrix}$ *order*]. It didn't matter
96	Stephanie	Not really.
97	R1	with this design.

98	Stephanie	No.
99	R1	It still works.
100	Stephanie	Yeah, it still works.
101	R1	Why?
102	Stephanie	Because you're taking there's two choices you can do from each. There are two blue [*picks up the* $\begin{bmatrix} B \\ B \end{bmatrix}$ *tower.*] If you're building up, you have a blue on the bottom and a blue on the top or a blue on the bottom and a green on the top. [*Stephanie indicates towers* $\begin{bmatrix} B \\ B \end{bmatrix}\begin{bmatrix} G \\ B \end{bmatrix}$.]
103	R1	Okay. So it works for here. [*indicates* $\begin{bmatrix} B \\ B \end{bmatrix}\begin{bmatrix} G \\ B \end{bmatrix}$] It also works [*indicates* $\begin{bmatrix} B \\ G \end{bmatrix}\begin{bmatrix} G \\ G \end{bmatrix}$]
104	Stephanie	Yes.
105	R1	I thought I heard you say that here you have, this would be the one [*lifts* $\begin{bmatrix} G \\ G \end{bmatrix}$]
106	Stephanie	Right.
107	R1	because that says from the two you have no blue and here you have one with one blue [*indicates* $\begin{bmatrix} B \\ G \end{bmatrix}$] and one with one blue [*indicates* $\begin{bmatrix} G \\ B \end{bmatrix}$] So together you could, that gives you the two. [*pushes the* $\begin{bmatrix} G \\ B \end{bmatrix}$ *and* $\begin{bmatrix} B \\ G \end{bmatrix}$ *closer together.*]

108	Stephanie	Um hm.
109	R1	And then, that's the one [*indicates* $\begin{bmatrix} B \\ B \end{bmatrix}$]. And I think you were asking the question, Bob, can – how does that work for the next row? [*indicates the row of towers three high*]
110	Stephanie	Like
111	R1	How how do you get the one-three-three-one out of the next row?
112	Stephanie	Oh, all right. [*Stephanie sweeps away the rows of towers four high.*]
113	R1	I think that was (inaudible) this one here. [*points to the third row of towers still remaining*]
114	R2	Yes.
115	Stephanie	'Cause this, oops
116	R2	Oops *[Some of the towers three high fall over.]*
117	Stephanie	There's millions of these
118	R2	This was part of the row
119	Stephanie	All right.
120	R2	that we wanted, right? [*lifts the* $\begin{bmatrix} B \\ B \\ B \end{bmatrix}$ *that was knocked over*]
121	R1	Why why don't we move them away? [*moves away the four high fallen towers*]

122	Stephanie	There's the one.
123	R2	Okay.
124	Stephanie	Here's one [*indicates* $\begin{bmatrix} B \\ B \\ B \end{bmatrix}$] if you've selected none, uh, no greens out of towers of three you have all blue.
125	R2	That's right.
126	Stephanie	Then if you're selecting one green out of the towers it can be um [*pause*]
127	R2	Um hm.
128	Stephanie	It could be these three. [*takes* $\begin{bmatrix} G \\ B \\ G \end{bmatrix} \begin{bmatrix} B \\ G \\ B \end{bmatrix} \begin{bmatrix} G \\ B \\ B \end{bmatrix}$]
129	R2	Um hm.
130	Stephanie	No these three. [*exchanges* $\begin{bmatrix} G \\ B \\ G \end{bmatrix}$ *for* $\begin{bmatrix} B \\ B \\ G \end{bmatrix}$]
131	R2	One green.
132	Stephanie	With one green.
133	R2	Good.

Description: Comparing Towers, selecting from two colors, built inductively and corresponding to the addition rule of Pascal's Triangle Parent Tape: Early Algebra Ideas About Binomial Expansion, Stephanie's Interview Six of Seven Date: 1996-03-27 Location: Union Catholic Researcher: Professor Carolyn Maher	Transcriber(s): Aboelnaga, Eman Verifier(s): DeLeon, Christina Date Transcribed: Spring 2009 Page: 10 of 17

134	Stephanie	And then if you're selecting two green it would be these [*takes* $\begin{bmatrix} G \\ G \\ B \end{bmatrix} \begin{bmatrix} B \\ G \\ G \end{bmatrix} \begin{bmatrix} G \\ B \\ G \end{bmatrix}$]
135	R2	Those three. Okay.
136	Stephanie	And then if you're selecting all green, there'd be one way to do it. [*takes* $\begin{bmatrix} G \\ G \\ G \end{bmatrix}$]
137	R2	Um hm.
138	Stephanie	So I guess.
139	R1	So, so I guess – Let's go back, I think you were asking this question but I'm not sure you were – um – can these work also this pattern- can you both patterns work at the same time?
140	Stephanie	Yeah. Uh, I mean
141	R1	Where would you place these so that they fit the pattern you're building here as well as looking like that. Is it possible, is that, was that your question. I don't know if that's it.
142	R2	This was- it was really curious to me. I wanted to understand the addition. For example, we're adding these two [*indicates* $\begin{bmatrix} G \\ B \end{bmatrix}$ *and* $\begin{bmatrix} B \\ G \end{bmatrix}$] and this one [*indicates* $\begin{bmatrix} G \\ G \end{bmatrix}$] to get- isn't it these three? [*indicates* $\begin{bmatrix} G \\ G \\ B \end{bmatrix} \begin{bmatrix} B \\ G \\ G \end{bmatrix} \begin{bmatrix} G \\ B \\ G \end{bmatrix}$]

Description: Comparing Towers, selecting from two colors, built inductively and corresponding to the addition rule of Pascal's Triangle Parent Tape: Early Algebra Ideas About Binomial Expansion, Stephanie's Interview Six of Seven Date: 1996-03-27 Location: Union Catholic Researcher: Professor Carolyn Maher	Transcriber(s): Aboelnaga, Eman Verifier(s): DeLeon, Christina Date Transcribed: Spring 2009 Page: 11 of 17

143	Stephanie	Um hm.
144	R2	How did that, how does that happen?
145	Stephanie	Oh. All right. There's the one with the two on the bottom. [*indicates* $\begin{bmatrix} B \\ G \\ G \end{bmatrix}$] There's the one with the blue in the middle. [*indicates* $\begin{bmatrix} G \\ B \\ G \end{bmatrix}$] And there's the one with the blue on the bottom, see how you're adding like on. [*indicates* $\begin{bmatrix} G \\ G \\ B \end{bmatrix}$]
146	R2	Okay. Let me see if I see it. So these two both had a green placed on top which keeps the one blue, right?
147	Stephanie	Um hm.
148	R2	And then these two greens had a had to have a blue on top in order to get
149	Stephanie	Yes.
150	R2	one with one blue.
151	Stephanie	And those would be the three
152	R2	And the (inaudible)
153	Stephanie	Yeah.
154	R1	Is there any other way you can do it though how do I know there's not another

		way you can do it.
155	Stephanie	Because.
156	R1	Do you understand my question? I, I, I believe you can do these to get
157	R2	Yeah.
158	R1	to keep the one blue. But how do I know there's one we haven't missed in our counting? Do you know my question?
159	R2	Ahhhh!
160	Stephanie	Yeah. Um, oh, well I think you can – can't I just do this again? Like, cause there's one blue - I can put the one blue on the top.
161	R2	Oops.
162	Stephanie	I can put the one blue on the top. I can move it down one to the middle. I can move it down one to the bottom. I can't move it up or down anymore. [*rearranges the towers to:* $\begin{bmatrix} B \\ G \\ G \end{bmatrix} \begin{bmatrix} G \\ B \\ G \end{bmatrix} \begin{bmatrix} G \\ G \\ B \end{bmatrix}$]
163	R2	Right.
164	Stephanie	There's no more blocks.
165	R1	So you're using the position argument.
166	Stephanie	Yeah, I can't, there's nothing else.
167	R1	Is that ok with you?
168	R2	It's ok with me. This, yeah, the position is fine. I'm convinced that these are the only ones that have one blue, no doubt about it. I was convinced before.

		I'm convinced again. Um. What I was interested in was that uh was where these came from
169	Stephanie	Oh. Well
170	R2	You know it's like a family tree.
171	Stephanie	Well, they keep building up. That's the whole
172	R2	Yeah.
173	Stephanie	thing.
174	R2	Yeah.
175	Stephanie	Like if I placed them there, there, and there it's just building up [*replaces* $\begin{bmatrix} G \\ G \\ B \end{bmatrix}$ $\begin{bmatrix} G \\ B \\ G \end{bmatrix}\begin{bmatrix} B \\ G \\ G \end{bmatrix}$ *in the triangle of towers*].
176	R2	Um hm.
177	Stephanie	And I also place um this here
178	R2	Uh huh
179	Stephanie	and this here and this here. So now the row is [*replaces towers:* $\begin{bmatrix} B \\ B \\ B \end{bmatrix}\begin{bmatrix} B \\ G \\ B \end{bmatrix}$

		$\begin{bmatrix} G \\ G \\ B \end{bmatrix}\begin{bmatrix} B \\ B \\ G \end{bmatrix}\begin{bmatrix} G \\ B \\ G \end{bmatrix}\begin{bmatrix} B \\ G \\ G \end{bmatrix}$]
180	R2	Because of the way they built up.
181	R1	That bothers me because it's messed up the three you wanted to keep together. Is there any way you keep my three together and not mess up your pattern? Because this bothers me a lot.
182	Stephanie	Um.
183	R1	See what I'm saying?
184	Stephanie	You mean like
185	R1	I like, I like patterns.
186	Stephanie	separate the ones with two and the ones with one. [*pulls* $\begin{bmatrix} G \\ G \\ B \end{bmatrix}$ *and* $\begin{bmatrix} B \\ B \\ G \end{bmatrix}$ *in front*]
187	R1	Yeah. Is there a way of doing it and still keep the pattern and keeping, in other words, keeping both at the same time? I don't know. Is it possible?
188	R2	Is it possible?
189	Stephanie	They mix in the middle though. I mean they're gonna
190	R1	Do they have to mix in the middle? There's no way of avoiding it?
191	Stephanie	Oh. I have to put one here and one like, [*pause*] like if you're talking; about how they build up.

192	R1	Yeah.
193	Stephanie	They go together. *[replaces the towers she had put in front so they are grouped:* $\begin{bmatrix} G \\ B \\ B \end{bmatrix}$ - $\begin{bmatrix} G \\ G \\ B \end{bmatrix}\begin{bmatrix} B \\ G \\ B \end{bmatrix}$ - $\begin{bmatrix} B \\ B \\ G \end{bmatrix}\begin{bmatrix} G \\ B \\ G \end{bmatrix}$ - $\begin{bmatrix} B \\ G \\ G \end{bmatrix}$]
194	R1	So they're always gonna have to
195	Stephanie	Even if I – there's
196	R2	(inaudible)
197	Stephanie	There's no matter what you do they're gonna be
198	R1	No matter what you do there's gonna be
199	Stephanie	They're gonna touch.
200	R2	Okay. Yeah. I think I see it. This one has one blue and one green [*indicates* $\begin{bmatrix} B \\ G \end{bmatrix}$].
201	Stephanie	Um hm.
202	R2	So what can happen to the number of blues and greens when we build on top of it?
203	Stephanie	They (inaudible) two blues or two greens
204	R2	and so the two cases get shuffled
205	Stephanie	Yes.
206	R2	That way they - it seems that they have to.

Description: Comparing Towers, selecting from two colors, built inductively and corresponding to the addition rule of Pascal's Triangle **Parent Tape: Early Algebra Ideas About Binomial Expansion, Stephanie's Interview Six of Seven** **Date: 1996-03-27** **Location: Union Catholic** **Researcher: Professor Carolyn Maher**	**Transcriber(s): Aboelnaga, Eman** **Verifier(s): DeLeon, Christina** **Date Transcribed: Spring 2009**

207	Stephanie	Um hm.
208	R2	Yeah.
209	Stephanie	Oh, so
210	R2	Yeah.
211	Stephanie	you want to keep these
212	R2	We wanted to keep those
213	Stephanie	Over here and these over here? *[arranges towers:* $\begin{bmatrix} G \\ B \\ B \end{bmatrix}\begin{bmatrix} G \\ B \\ G \end{bmatrix}\begin{bmatrix} B \\ G \\ B \end{bmatrix}\begin{bmatrix} B \\ B \\ G \end{bmatrix}\begin{bmatrix} G \\ G \\ B \end{bmatrix}\begin{bmatrix} B \\ G \\ G \end{bmatrix}$*; exchanges towers to make this arrangement]* Or um. Yeah, those would have to go the other way.
214	R1	This one bothers me now [*indicates* $\begin{bmatrix} G \\ G \\ B \end{bmatrix}$] 'cause there's a blue on the bottom and it's next to the green. That really bothers me.
215	Stephanie	Yeah, but then I'd have to move them again and
216	R1	Exactly.
217	Stephanie	it would still- [*Stephanie rearranges the towers back to the first groupings.*]
218	R2	So it looks like there's different…
219	R1	organizations

Description: Comparing Towers, selecting from two colors, built inductively and corresponding to the addition rule of Pascal's Triangle **Parent Tape: Early Algebra Ideas About Binomial Expansion, Stephanie's Interview Six of Seven** **Date: 1996-03-27** **Location: Union Catholic** **Researcher: Professor Carolyn Maher**	**Transcriber(s): Aboelnaga, Eman** **Verifier(s): DeLeon, Christina** **Date Transcribed: Spring 2009** **Page: 17 of 17**

220	R2	organizations
221	Stephanie	They, they all work but
222	R2	They all work.
223	R1	But, but they're different, aren't they?
224	Stephanie	Yeah.
225	R2	But they seem to do something different, okay, but that looks like a kind of a victory in its own way (inaudible)
226	Stephanie	And then um [*places the* $\begin{bmatrix} B \\ B \\ B \end{bmatrix}$ *and* $\begin{bmatrix} G \\ G \\ G \end{bmatrix}$ *at the ends of the row of towers.*]
227	R2	(inaudible) they had to be different (inaudible) good
228	Stephanie	And that's how you can get (inaudible) Should I keep going with that?