Description: Developing the correspondence among Towers, selecting from two colors, Pascal's Triangle, and the symbolic algebraic expansions of (a+b) squared and (a+b) cubed **Parent Tape: Early Algebra Ideas About Binomial Expansion, Stephanie's Interview Six of Seven** **Date: 1996-03-27** **Location: Union Catholic** **Researcher: Professor Carolyn Maher**	**Transcriber(s): Aboelnaga, Eman** **Verifier(s): DeLeon, Christina** **Date Transcribed: Spring 2009** **Page: 1 of 7**

1	Stephanie	And that's how you can get (inaudible) Should I keep going with that?
2	R2	Did you do that last night?
3	Stephanie	Last
4	R2	Last time
5	Stephanie	Um
6	R2	Did you carry it further?
7	Stephanie	Yeah, I think we went a little bit. I think 'cause what happened was we were doing this problem like before that, like way before we started this, on 'a' plus 'b' quantity squared [*writes* $(a + b)^2$]
8	R2	Um hm.
9	Stephanie	And at first, I did um [*writes* $a^2 + b^2$] but that was proved wrong, and it was *a* squared plus two *ab*.
10	R2	Two *ab*!
11	Stephanie	Plus *b* squared [*writes* $a^2 + 2ab + b^2$] and um we kept going like I think I got up to like six like *a* plus *b* quantity squared, quantity like to the sixth power.
12	R2	Ah.
13	Stephanie	And I think see this is where I forgot and um, I think with the numbers let's see (inaudible) [*draws Pascal's triangle until the sixth row*] There. I think that's one…zero, one, two, three, four, five, six. [*Stephanie points to each row as she counts.*] All right. That's six, and um, I think, using that see this is where I forget, I think she figured out the exponents or something to some of the numbers or like you know that there's going to be an *a* but I think she figured

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		out like what the numbers were going to be up here [*indicates the position of the exponents*]. The exponents, is that what you did? I don't
14	R1	I don't know. I don't remember it myself and I didn't look at the tape, but I have a question now. You just wrote down what *a* plus *b* quantity squared was. Why don't you write it on the top of this paper? [*gives Stephanie a new piece of paper and she writes* $(a+b)^2 = a^2 + 2ab + b^2$]
15	Stephanie	Okay.
16	R1	And I guess my question now is can that at all be related to the triangle or what you built with your
17	Stephanie	I'm sorry. [*Stephanie moves a tower that was in the way.*]
18	R1	With, with your tow- with your cubes, can you take each of those terms in that expansion *a* squared, 2*ab*, *b* squared and see any relationship to the towers or any of those lines of the triangle or any part of the triangle – column, line, diagonal, anything.
19	Stephanie	I guess like here [*takes the towers two high*] there's, I don't, I don't, I mean, not with the exponents. Like I don't see how *a* squared
20	R1	Tell us what you do see.
21	Stephanie	Well, I guess cause like there's two with an *a* and a *b*. [*indicates* $\begin{bmatrix} G \\ B \end{bmatrix}$ *and* $\begin{bmatrix} B \\ G \end{bmatrix}$] Like
22	R1	What's an *a* and a *b*?
23	Stephanie	If green was *a*. And
24	R1	Okay. Lets call green *a* and lets call blue *b*.

25	Stephanie	[*lifts* $\begin{bmatrix} G \\ B \end{bmatrix}\begin{bmatrix} B \\ G \end{bmatrix}$] you have two with green is *a*, and blue is *b*.
26	R1	Okay.
27	Stephanie	You know like one of each.
28	R1	Okay, so you have an *ab* and a *ba* or 2*ab*. [*points to the towers* $\begin{bmatrix} G \\ B \end{bmatrix}$ *and* $\begin{bmatrix} B \\ G \end{bmatrix}$ *that Stephanie put aside*]
29	Stephanie	You have one that's all *a* [*indicates* $\begin{bmatrix} G \\ G \end{bmatrix}$] and one that's all *b*. [*indicates* $\begin{bmatrix} B \\ B \end{bmatrix}$]
30	R1	Ok but this says what do you mean by all *a*? This is an *a* and a *b* [*indicates* $\begin{bmatrix} G \\ B \end{bmatrix}$] and an *a* and a *b* [*indicate* $\begin{bmatrix} B \\ G \end{bmatrix}$] .
31	Stephanie	Yeah, well
32	R1	*aa* [*points to* $\begin{bmatrix} G \\ G \end{bmatrix}$] *bb* [*points to* $\begin{bmatrix} B \\ B \end{bmatrix}$]
33	Stephanie	Yes.
34	R1	So what do you mean *aa*? What could these *aa* and *ab* mean? Is that *a*
35	Stephanie	Oh. I get to, oh, well if you're saying that this is *a* [*takes one green cube*] and two of them would like *aa* would be like *a* squared. [*lifts* $\begin{bmatrix} G \\ G \end{bmatrix}$]
36	R1	Could be (inaudible) how many of those do you have?

37	Stephanie	Well one of *a*
38	R1	So where's the one I don't see the one in this.
39	Stephanie	Well, the one's just there. [*points in front of* a^2 *on her paper*]
40	R1	So imagine there's a one
41	Stephanie	Yeah.
42	R1	in front of that *a* squared.
43	Stephanie	I mean I could put it
44	R1	Yeah. Put it somewhere okay? [*Stephanie writes ones on the paper in front of* a^2 *and* b^2.] So now, now help me see what that might mean.
45	Stephanie	Okay, there's one with two *a*'s with like *aa* or *a* squared. [*lifts* $\begin{bmatrix} G \\ G \end{bmatrix}$]
46	R1	Two factors of *a*.
47	Stephanie	Yeah, and there's two with *ab*, with *a* and *b*. [*indicates* $\begin{bmatrix} G \\ B \end{bmatrix}$ *and* $\begin{bmatrix} B \\ G \end{bmatrix}$]
48	R1	One factor of *a* and one factor of *b.*
49	Stephanie	One factor of *b*. And there's one with two factors of *b*.
50	R1	So, so that relates to the *a* plus *b* quantity squared. What about the triangle?
51	Stephanie	One, two, one. [*points to the third row of the triangle*]
52	R1	Okay, tell me what you think *a* plus *b* quantity cubed will be. Without having to work out all the details of it now. Using your cubes and using what you just

		told me.
53	Stephanie	I guess it would be
54	R1	'Cause you didn't like multiplying those out all the time. That was a lot of hard work.
55	Stephanie	I know there'll be an *a* cubed and a *b* cubed. [*writes* a^3 *and* b^3 *on the paper leaving a large space between them.*]
56	R1	How do you know that?
57	Stephanie	Because there's a one and a one [*points to the fourth row of the triangle*] and besides I mean
58	R1	What's the *a* cubed? Which cube, which tower is this? Don't make new ones. You have them made, I think.
59	Stephanie	That would be that. [*indicates* $\begin{bmatrix} G \\ G \\ G \end{bmatrix}$]
60	R1	Oh, okay.
61	Stephanie	And the *b* would be that. [*indicates* $\begin{bmatrix} B \\ B \\ B \end{bmatrix}$]
62	R1	That was easy.
63	Stephanie	And there's gonna be, I guess, three *a* squared *b* cubed and three *ab* squared.
64	R1	Ok. Why don't you write that down and then see if we can find them. [*Stephanie writes:* $3ab^2$] Tell me why you think that.

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65	Stephanie	All right. Here. The *a* is the green. So here's the [*5 second pause; then picks up* $\begin{bmatrix} G \\ G \\ B \end{bmatrix}$ *and* $\begin{bmatrix} B \\ G \\ G \end{bmatrix}$] Am I missing one?
66	R1	How many do you want? How many towers three high should you have and let's, let's find them. How many should you have altogether?
67	Stephanie	I should have eight.
68	R1	Okay. I see eight. There's four here and then you have four up there. [*indicates towers three high*] Let's get these out of the way. [*pushes away the towers two high*] Right, here's eight of them. Right? [*R2 uprights four towers that have fallen*.]
69	Stephanie	Zero, one, two, three, yeah, that's three high. Oh here [*takes* $\begin{bmatrix} G \\ B \\ G \end{bmatrix}$] um okay so.
70	R1	Tell me what's *a* and what's *b* again. I keep forgetting.
71	Stephanie	Green is *a*.
72	R1	Why don't you write that down what *a* is. I get [*Stephanie writes: Green – A, Blue – B*] Okay, green is *a*, blue is *b*.
73	Stephanie	I have three with two factors of *a* and one factor of *b*. [*Stephanie indicates* $\begin{bmatrix} G \\ G \\ B \end{bmatrix}\begin{bmatrix} G \\ B \\ G \end{bmatrix}\begin{bmatrix} B \\ G \\ G \end{bmatrix}$]

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74	R1	Okay.
75	Stephanie	And I have three with two factors of *b* and one factor of *a* [*indicates* $\begin{bmatrix} G \\ B \\ B \end{bmatrix}\begin{bmatrix} B \\ G \\ B \end{bmatrix}\begin{bmatrix} B \\ B \\ G \end{bmatrix}$] so I guess it would be *a* cubed plus three *a* squared *b* plus three *ab* squared plus *b* cubed. [*inserts plus signs so that her paper now reads:* $a^3 + 3a^2b + 3ab^2 + b^3$]
76	R1	So how do you know there can't be a *c* in here?
77	Stephanie	Because I only have two colors.
78	R1	Oh.
79	Stephanie	If I had a third color there could be a *c*, but
80	R1	That's interesting. That's something to explore later. [*Stephanie writes* $(a + b)^3$ *before the expansion she has written previously.*] We could look into that. Okay so now could you tell me about another one of those binomials raised to a power?