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1	R1	I think so too. I think she can too. I guess um what we're to explore from here. There's lots of ways. But I'm sort of interested in maybe exploring this way [*indicates with her finger moving horizontally across a row in the triangle*]. At least on one of the rows or so you want to look at that exploration?
2	R2	Uh, I'd be delighted, uh, let me think about, let me think for a moment about how I'd start – what I'm curious about and maybe you can help me is um let's look at the at the towers that are three high where you have one, three, three, and one here in the different cases. [*indicated the fourth row of the triangle*] Um, now, uh, let's see, um, this is the case where there are no blues. [*points to the left one in the one, three, three, one row*]
3	Stephanie	Um hm.
4	R2	This is the case where there's one blue, okay? [*points to the left three in the one, three, three, one row*]
5	Stephanie	Yes.
6	R2	Now, what I'm interested in is reading this this row of numbers from the left to the right. How do we get from one number to the next?
7	Stephanie	Like, like
8	R2	I'm looking for a new idea.
9	Stephanie	Okay.
10	R2	Okay. In other words, suppose we know that – okay. Suppose we start with what we do know - that if there are towers three high with one green, there are exactly three of them. If I remember your explanation, it was because there were only three places where we can put that one green.

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11	Stephanie	Yes.
12	R2	So we're very sure of that. Now suppose we wanted to start with that knowledge. In other words, not just that there were three towers but also remembering what the towers were.
13	Stephanie	Okay.
14	R2	Okay, and then um the next question is, okay, imagine one of those towers, okay?
15	Stephanie	Um hm.
16	R2	Um. The next number in this row, if we didn't know it, we know what it's supposed to count. It's supposed to count the towers with two greens. So now we've got a tower with one green.
17	Stephanie	Oh, okay.
18	R2	Okay. Now let's imagine trading
19	Stephanie	Switching this
20	R2	one of the blues for a new green.
21	Stephanie	Okay.
22	R2	Okay, how many different ways could we do that? How many new towers could we…
23	Stephanie	Well, I know there's three but like all you if you're saying these with one green.

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		[*takes* $\begin{bmatrix} B \\ G \\ B \end{bmatrix}$]
24	R2	Yeah, take one of the towers with one green.
25	Stephanie	If I could picture this one okay, all I'd have to really do is picture it back like if instead of green have this be blue [*indicates the green cube*] and have these two be green [*indicates the blue cubes*]. Oh you want with two green.
26	R2	Yeah.
27	Stephanie	Oh.
28	R2	But starting with the one green and then taking one of the blues and trading it for a green.
29	Stephanie	All right let me start.
30	R2	How many different towers could we make that way?
31	R1	From the one you have in your hand…just worry about the one you have in your hand.
32	Stephanie	All right, um.
33	R2	Just that one.
34	Stephanie	Well, I just couldn't I just imagine it the opposite like if I imagined it as um two green and one blue 'cause there's they all have an opposite one.
35	R1	Okay, let's stop for a minute. I think I understand what the problem is. That one in your hand

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36	Stephanie	Okay.
37	R1	has one green. That one green can't move.
38	Stephanie	Okay.
39	R1	'Cause you've picked it and you've picked this one with the one green. That one green can't move. Right?
40	Stephanie	Okay.
41	R1	But now you know what to change this tower to have two greens.
42	Stephanie	Okay.
43	R1	So obviously that one.
44	Stephanie	Well, you can put a green on the top or a green on the bottom.
45	R1	Okay.
46	R2	Good so there are two ways.
47	Stephanie	Yes.
48	R1	So there are two ways that you can change that one to have exactly two greens from a one green.
49	Stephanie	Yes.
50	R1	Okay, now is that the only one green tower that you can make two greens?
51	Stephanie	No.
52	R1	What are the others?

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53	Stephanie	Um-
54	R1	You don't have to show me if you can tell me without showing me and then you can go to the towers.
55	Stephanie	Um, the one let me see. [*takes* $\begin{bmatrix} G \\ B \\ B \end{bmatrix}$]
56	R1	Okay, that's one.
57	Stephanie	Yeah you cause you can make put a green here. [*indicates blue on the bottom*]
58	R1	Right, that'll make it two greens.
59	Stephanie	You can put it here too [*indicates blue in the middle*] but you already did that.
60	R1	Well, forget about that one for a minute.
61	R2	Don't worry about that.
62	Stephanie	Well, here you have two ways too. They all have two ways.
63	R2	Good.
64	R1	I agree with you that we've already done that. That that's good that you remember that's wonderful, but from that one you could do it two ways, right?
65	Stephanie	Yeah.
66	R2	So let's see where we are now.
67	Stephanie	Okay.

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68	R2	From this one you made two new ones. [*indicates* $\begin{bmatrix} B \\ G \\ B \end{bmatrix}$]
69	Stephanie	Yes.
70	R2	And from this one you made also two new ones. [*indicates* $\begin{bmatrix} G \\ B \\ B \end{bmatrix}$] And you also noticed
71	R1	You have a
72	R2	that there is a duplicate.
73	R1	Very good.
74	R2	Okay. Good. Okay. This is all strong. Okay there's one tower left.
75	Stephanie	Okay.
76	R2	It's this one. [*takes* $\begin{bmatrix} B \\ B \\ G \end{bmatrix}$]
77	Stephanie	Yes.
78	R2	Um, how many ways can you?
79	Stephanie	Two.

80	R2	Two.
81	Stephanie	Um, hum but they're both duplicates.
82	R2	But they're both duplicates
83	Stephanie	Yes.
84	R2	Okay. Very good. Okay.
85	R1	How's that?
86	Stephanie	Well if you put one on the top, you have this one with the one on the top, and one on the bottom. [*points to* $\begin{bmatrix} G \\ B \\ B \end{bmatrix}$] If you put one there, you have this one with the one there, and the one there. [*points to* $\begin{bmatrix} B \\ G \\ B \end{bmatrix}$] So that's just that doesn't really do anything.
87	R2	Okay. Good. Okay. So if so we built six. We imagined building six towers but we noticed that they came in pairs.
88	Stephanie	Yeah.
89	R2	Is that right?
90	Stephanie	Um hum.
91	R2	Okay, so um so what's the real number of towers?

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92	Stephanie	Three.
93	R2	There's three because…
94	Stephanie	Because you can have one with two green.
95	R2	Um hm.
96	Stephanie	Here.
97	R2	Yes.
98	Stephanie	One with two green, one here, and one here [*lifts* $\begin{bmatrix} G \\ B \\ B \end{bmatrix}$]. One with one green here and one green here. [*lifts* $\begin{bmatrix} B \\ G \\ B \end{bmatrix}$]
99	R2	Okay.
100	Stephanie	That's it without having any like duplication.
101	R2	Okay, so the duplicates seem to come up two at a time.
102	Stephanie	Um hm.
103	R2	Right?
104	Stephanie	They come like
105	R2	Is that right?

106	Stephanie	Oh. I wouldn't like two at a time you mean like two from the same one or cause this one they're they're just
107	R1	You know what? Let's do it.
108	R2	Okay.
109	R1	Let's do it.
110	Stephanie	Oh you mean like let's build a tower.
111	R1	Let's start all over again cause this is too confusing for me. [*brushes aside all towers*]
112	R2	Yeah. This is getting lovely. Let's build it, okay?
113	R1	Let's do it.
114	R2	Okay. Good.
115	Stephanie	Start with this one. [*indicates* $\begin{bmatrix} B \\ G \\ B \end{bmatrix}$]
116	R2	Okay.
117	Stephanie	You can have one like that. [*takes* $\begin{bmatrix} B \\ G \\ G \end{bmatrix}$]

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118	R1	No. Let's leave these alone. [*pushes away all towers except* $\begin{bmatrix} B \\ G \\ B \end{bmatrix}$] Let's start with the three with one
119	R2	Let's save all the originals.
120	R1	Let's leave all the originals by themselves and let's just start with the three with one.
121	Stephanie	Okay.
122	R1	Let's pull those aside and let's start all over again. [*Stephanie places* $\begin{bmatrix} G \\ B \\ B \end{bmatrix}$ *and* $\begin{bmatrix} B \\ B \\ G \end{bmatrix}$ *on the sides of* $\begin{bmatrix} B \\ G \\ B \end{bmatrix}$.] We could take apart (excuse us) okay. Now.
123	Stephanie	From this one [*indicates* $\begin{bmatrix} G \\ B \\ B \end{bmatrix}$], you could have one like that [*indicates* $\begin{bmatrix} G \\ G \\ B \end{bmatrix}$]. Or one like that [*indicates* $\begin{bmatrix} G \\ B \\ G \end{bmatrix}$]. That's without moving the top one.
124	R1	Replacing…
125	R2	Replacing…

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126	R1	each blue
127	R2	one of the blues
128	R1	one or the other ones?
129	R2	Good.
130	R1	I believe that…
131	R2	Now let's go on to the next one. [$\begin{bmatrix} B \\ G \\ B \end{bmatrix}$ *is the next one.*]
132	Stephanie	You can have one like that. [*indicates* $\begin{bmatrix} G \\ G \\ B \end{bmatrix}$]
133	R2	Um hm.
134	Stephanie	Or (inaudible) um (inaudible)…one like that. [*indicates* $\begin{bmatrix} B \\ G \\ G \end{bmatrix}$]
135	R2	Okay
136	Stephanie	And then the next one – you can either – one like that [*indicates* $\begin{bmatrix} G \\ B \\ G \end{bmatrix}$] or one

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		like that [*indicates* $\begin{bmatrix} B \\ G \\ G \end{bmatrix}$]
137	R2	Okay.
138	Stephanie	and that's it.
139	R2	Okay, now which are duplicates in this row? You know in this new row that you constructed?
140	Stephanie	These two, [$\begin{bmatrix} G \\ B \\ G \end{bmatrix}\begin{bmatrix} G \\ B \\ G \end{bmatrix}$] these two, [$\begin{bmatrix} G \\ G \\ B \end{bmatrix}\begin{bmatrix} G \\ G \\ B \end{bmatrix}$] and these two [$\begin{bmatrix} B \\ G \\ G \end{bmatrix}\begin{bmatrix} B \\ G \\ G \end{bmatrix}$].
141	R2	Aha, so they do come in pairs.
142	Stephanie	Yes, oh okay like that.
143	R2	Yeah, that's what we meant.
144	Stephanie	Okay.
145	R2	I think that's what Carolyn meant.
146	R1	Right.
147	R2	Right. Okay, now let's put them back with the parents. It's okay to call these the parents
148	Stephanie	Yeah.
149	R2	and the new ones the children?

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150	R1	Different kind of parents. I'm getting very mixed up.
151	R2	Oh, I'm sorry. Okay, would you call them step one and step two or something like that. But um…
152	Stephanie	Um. [*Stephanie replaces duplicates. The towers are now arranged:* $\begin{bmatrix}G\\B\\B\end{bmatrix}\begin{bmatrix}B\\G\\B\end{bmatrix}$ $\begin{bmatrix}B\\B\\G\end{bmatrix}$ $\begin{bmatrix}G\\G\\B\end{bmatrix}\begin{bmatrix}G\\B\\G\end{bmatrix}\begin{bmatrix}G\\G\\B\end{bmatrix}\begin{bmatrix}B\\G\\G\end{bmatrix}\begin{bmatrix}G\\B\\G\end{bmatrix}\begin{bmatrix}B\\G\\G\end{bmatrix}$
153	R2	Let me see. Good, okay so…