

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 = a^4 + 4a^3b + 4ab^3 + \underline{6a^2b^2}$$

Green - A

Blue - B

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from 3 green to 4 green we found  
4 & divided by 4.

from 2 green to 3 green we found  
12 & divided by 3.

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$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$C_2^4 = 6$$

$$C_1^4 \binom{4}{1} = 4$$

$$C_2^4 = 6 \quad C_3^4 = 4$$

$$C_4^4 = 1 \quad C_0^4 = 1$$

$$C_r^4 =$$

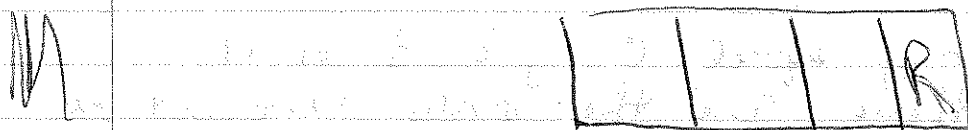
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## Combinations

In this project we started like the towers project only we were building trains. The trains were 4 cubes long. I had two different colors to choose from, red & yellow. I was shown that if I was selecting 1 red from 4 cubes I could write it in 2 ways,  $C_1^4$  or  $(4)$ . We found that there was four ways to do this

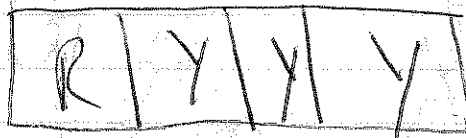


$$C_1^4 = 4$$

If I chose 2 red ( $C_2^4$ ) I had 6 combinations

$$C_2^4 = 6$$

there was  $C_3^4 = 4$ , this is the opposite of 1 red. Because if there is 1 red there are 3 yellows.



So both  $C_1^4$  &  $C_3^4$  have the same number of combinations.

20/5/20  
If you take 4 out of 4

$$C_4^4 = 1$$

None out of 4

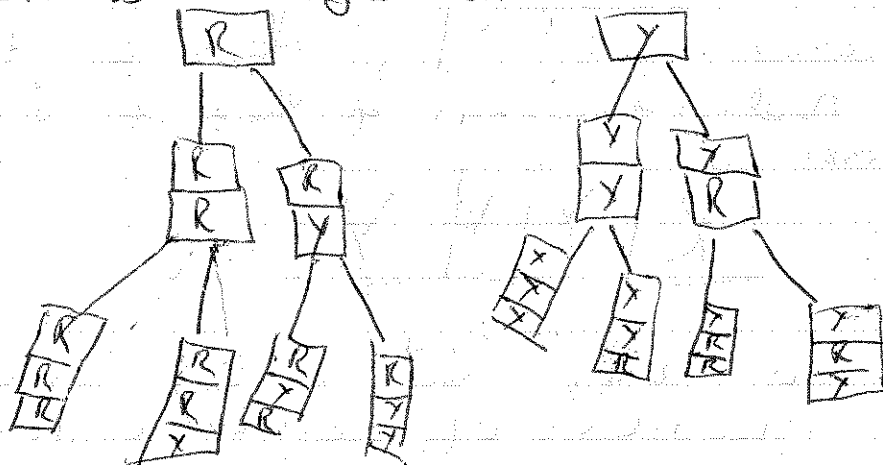
$$C_0^4 = 1$$

Next we added a variable  $r$ .  
And the problem became  $r$  out  
of 4

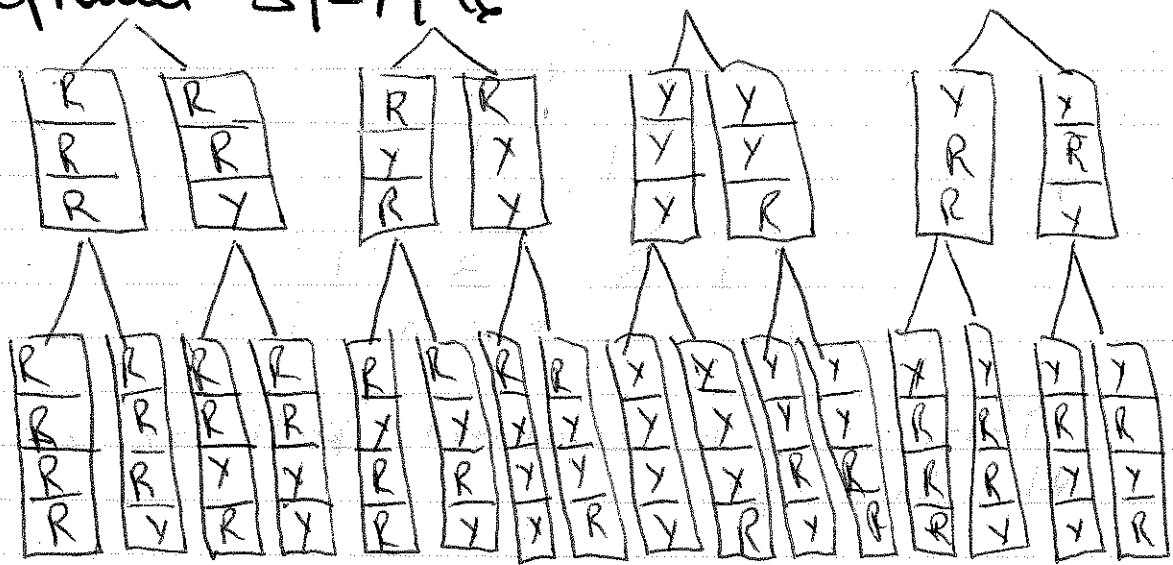
$$C_r^4$$

$r$  could only equal 0, 1, 2, 3, or 4  
because these are the only amount of  
things you can select from 4.

Now we referred back to the train problem  
but we turned it into the tower problem.  
I was asked if I remembered building  
a family of towers. I did. It started  
with towers 1 high with 2 cubes.  
then it progressed.



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I knew that this would turn out to be 16.

This -  $C_0^4 = 1$  also equals 16.

$$C_1^4 = 4$$

$$C_2^4 = 6$$

$$C_3^4 = 4$$

$$C_4^4 = 1$$

Next we incorporated this into Blaise Pascal's triangle.

$$C_0^0 = 1 \quad C_2^3 = 3$$

$$C_0^1 = 1 \quad C_3^3 = 1$$

$$C_1^1 = 1 \quad C_0^4 = 1$$

$$C_0^2 = 1 \quad C_1^4 = 4$$

$$C_1^2 = 2 \quad C_2^4 = 6$$

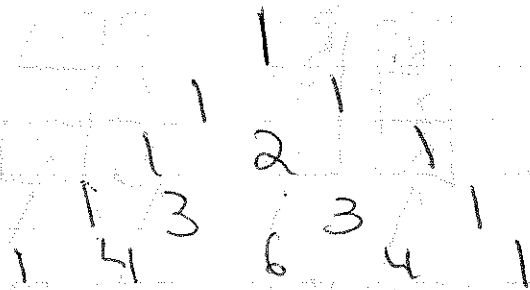
$$C_2^2 = 1 \quad C_3^4 = 4$$

$$C_0^3 = 1 \quad C_4^4 = 1$$

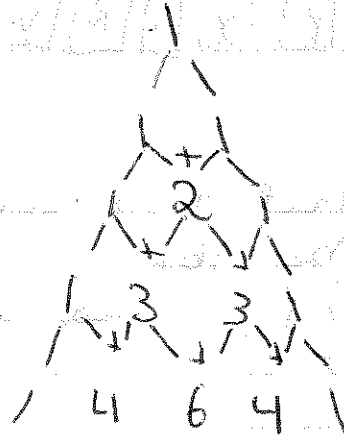
$$C_3^3 = 3$$

now we put the answers in the triangle.

20/15/2 mix 40/2



There is a distinct pattern in the triangle

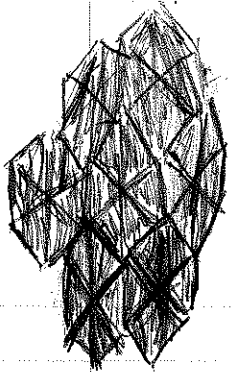


after that we changed numbers into combinations

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 2 \\ 2 \end{pmatrix} \\ \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad \begin{pmatrix} 3 \\ 3 \end{pmatrix} \\ \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 4 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 4 \\ 2 \end{pmatrix} \quad \begin{pmatrix} 4 \\ 3 \end{pmatrix} \quad \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

here we also discovered a pattern.

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$



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The question of what would

$$\binom{m}{0} + \binom{m}{1} =$$

after some time we found the answer

$$\binom{m}{0} + \binom{m}{1} = \binom{m+1}{1}$$

Now I was asked to take out my paper on  $(a+b)^2$ . I had worked with this all the way up to  $(a+b)^6$ .

Without even looking at my paper Dr. Maher told me all of the exponents in one of the problems, using only the numbers in the triangle.



$$(A+B)^0 = 1$$

$$(A+B)^1 = A+B$$

$$(A+B)^2 = A^2 + 2ab + b^2$$

$$(a+b)^3 = A^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 = A^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$(a+b)^5 = A^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

$$(a+b)^6 = A^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + B^6$$

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