Description: Beginning to generalize mathematical expressions to generate cases for towers 4-cubes tall selecting from green and blue cubes **Parent Tape: Early Algebra Ideas About Binomial Expansion, Stephanie's Interview Six of Seven (student view)** **Date: 1996-03-27** **Location: Union Catholic** **Researcher: Professor Carolyn Maher**	**Transcriber(s): Aboelnaga, Eman** **Verifier(s): DeLeon, Christina** **Date Transcribed: Spring 2009** **Page: 1 of 13**

1	R2	Okay. Very fine. Okay. Should we try the big step?
2	R1	Why not? Are you very tired?
3	Stephanie	Umm.
4	R2	How are you doing?
5	Stephanie	I'm fine.
6	R2	Okay. How are you doing? Okay? So, now, look what we've gone from.
7	R1	So we just woke – um – Professor Spieser up. Notice how animated he is. [*All laugh.*]
8	R2	Yeah. I'm starting to get lively. I'm very excited by the way you're moving through this. Um, so we started, let's remember what we did. We started with the towers that had one green.
9	Stephanie	Yeah.
10	R2	And then by putting in a second green, we built the towers that had two greens. So, they came up in pairs.
11	Stephanie	Yes.
12	R2	That's the way it happened. That's what we found.
13	Stephanie	Um hm.
14	R2	Okay. So now we've got, but now I think we've got a hold on, the six different towers with the two greens. Or the twelve when they came up in pairs. Okay. So now let's take those six. Okay, so, let's forget that, we can

		use these duplicates as extra blocks if we find we need them.
15	Stephanie	Okay.
16	R2	Okay. Now, these are the old ones [*the six towers with two green and two blue*], and we're gonna build a generation of new ones that have three green.
17	Stephanie	Okay.
18	R2	Okay. And we're expecting, maybe, that something interesting will happen. Okay, so first of all, how many do we produce?
19	Stephanie	Um, with three greens?
20	R2	Um hm. Well, how many would you produce from each of the towers first and
21	Stephanie	Well, you'd produce two from each.
22	R2	then how much from…two from each! Why?
23	Stephanie	Well, because there's only two places for you to move. [*Stephanie points to the blue blocks in the* $\begin{bmatrix}G\\G\\B\\B\end{bmatrix}$ *tower.*]
24	R2	Excellent.
25	Stephanie	You can't
26	R2	And you're looking, you're pointing at the blues.

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27	Stephanie	Yeah. You can only put a green here or here for that one.
28	R2	So, how many new towers would that produce?
29	Stephanie	Well, you'd get two from each.
30	R2	Two from each. So…
31	Stephanie	So. And we have six, so you'd get twelve.
32	R2	So, we'd get twelve. Okay. I'm correcting you. [*R2 moves two towers in line with the others.*] Right. But, yeah. Looks like twelve to me.
33	Stephanie	Um hm.
34	R2	Good. Uh. Now the question is, um, are they all, are these twelve new towers all different?
35	Stephanie	I don't think so. But, um
36	R2	Okay. What do you think?
37	Stephanie	Um. There'll probably be pairs again.
38	R2	You think there will be pairs again. Okay.
39	Stephanie	Uh maybe. 'Cause I mean – [*Stephanie sighs heavily.*]
40	R2	Okay. So here's a tower with three greens. [*the* $\begin{bmatrix} G \\ G \\ B \\ G \end{bmatrix}$ *tower*] So, it's one of the new ones. Right?

41	Stephanie	Um hm.
42	R2	Which would be the old ones that gave it?
43	Stephanie	Um, this one [*the* $\begin{bmatrix} G \\ G \\ B \\ B \end{bmatrix}$ *tower*]
44	R2	Um hm.
45	Stephanie	or, this one [*the* $\begin{bmatrix} B \\ G \\ B \\ G \end{bmatrix}$ *tower*]
46	R2	Um hm.
47	Stephanie	or this one. [*the* $\begin{bmatrix} G \\ B \\ B \\ G \end{bmatrix}$ *tower*]
48	R2	You found three.
49	Stephanie	Oh. Okay. So, maybe they'll be, um, groups of three?
50	R2	So, you think they might be in groups of three.
51	R1	Okay. Now explain to me how that happened.

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52	Stephanie	Mmm, because, here I could either, I could have one here [*points to* $\begin{bmatrix} G \\ G \\ B \\ B \end{bmatrix}$]. Which would make this one [*the* $\begin{bmatrix} G \\ G \\ B \\ G \end{bmatrix}$ *tower*]. I could have here, I could have one here, another green here [*points to* $\begin{bmatrix} G \\ B \\ B \\ G \end{bmatrix}$], which would make this one. And here, I could put another green here [points to $\begin{bmatrix} B \\ G \\ B \\ G \end{bmatrix}$], which would make this one. So, there's three of them that can make that one [*the* $\begin{bmatrix} G \\ G \\ B \\ G \end{bmatrix}$ *tower*]. [*pause*] So, um, I guess there'll be groups of three, maybe?
53	R2	Ah. You're guessing.
54	Stephanie	So, I guess
55	R2	Would you like to

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56	Stephanie	it would come up, the answer would be like four. There would be four without duplicates, if there's groups of three. 'Cause
57	R2	Four, ah.
58	Stephanie	you'll come up with twelve, and then, if there's groups of three, you'll get four. 'Cause four divided by three is twelve.
59	R1	In triples?
60	R2	So we're multiplying by two and dividing by three.
61	Stephanie	Yes.
62	R2	And, you're saying there, there are four. Well, we started with four and then we found six, and then we found four. Okay.
63	Stephanie	There's only, if you wanted, (inaudible)
64	R2	Okay. So now we've got- Okay, let's. Do we have the four?
65	Stephanie	Um, the four would be
66	R2	Can you build them?
67	Stephanie	(inaudible) over here
68	R2	Have we got them already? Um.
69	Stephanie	With three green. Here's one. Um. [*She picks up the* $\begin{bmatrix} G \\ G \\ B \\ G \end{bmatrix}$ *tower.*]

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70	R2	You're looking for three greens?
71	Stephanie	Yeah.
72	R2	Here's one. [*The* $\begin{bmatrix} B \\ G \\ G \\ G \end{bmatrix}$ *tower*]
73	Stephanie	Thank you.
74	R2	Here's another. [*The* $\begin{bmatrix} G \\ B \\ G \\ G \end{bmatrix}$ *tower*]
75	Stephanie	That one we already have.
76	R2	Are they all different?
77	Stephanie	No, I'll just make one.
78	R2	Oh. Okay.
79	Stephanie	There. [*Stephanie has made the* $\begin{bmatrix} G \\ G \\ G \\ B \end{bmatrix}$ *tower.*]
80	R2	Okay.

81	Stephanie	There's your four. Now
82	R2	Okay. So suppose we play another round.
83	Stephanie	There's only one way to do that one.
84	R1	But, suppose we're getting it from there [*The row of towers four tall containing three greens and one blue*] though. We know the answer
85	R2	But, we're gonna start with these old ones.
86	R1	We're making it from there.
87	Stephanie	Well, there's only
88	R2	How do we produce it?
89	Stephanie	You're only gonna get one from each, because there's only one place you can put the green.
90	R2	Green.
91	Stephanie	And you're gonna get four all green, and so you're going to come up with one 'cause
92	R2	Oh.
93	Stephanie	they're all the same.
94	R1	But, you got really four.
95	Stephanie	Well, yeah.
96	R1	How many duplicates did you get?

97	Stephanie	You got four dup - well
98	R1	So you have four
99	Stephanie	I guess you
100	R1	but you have four duplicates
101	Stephanie	Yeah.
102	R1	So you have to, you got four times and you have to undo the duplicates, what do you do to undo the duplicates?
103	Stephanie	You get rid of ‘em?
104	R1	By dividing by what?
105	Stephanie	Oh, by four.
106	R1	By four.
107	R2	Terrific.
108	R1	You divided, you got rid of the duplicates the last time by dividing by three. You got rid of the duplicates the last time by dividing by two. Isn’t that right?
109	Stephanie	Yeah. Okay. So that would give you the, um, all green ones.
110	R2	Okay. So let’s just review a little bit.
111	R1	Maybe we should write down what we say.
112	R2	In each round we multiplied by one number

113	Stephanie	Okay.
114	R2	and we divided by another number.
115	Stephanie	Yes.
116	R2	Okay. So why don't we write, just to remember what happened, why don't we write down at each stage the number that we multiplied by and the number we divided by.
117	Stephanie	All right. For – should I start with like from here and work backwards?
118	R2	If that's easiest for you, that's fine.
119	Stephanie	All right.
120	R2	Okay. As long as we remember exactly what, you know, which round of the, the process we were in.
121	Stephanie	From, I guess from, oops [*Stephanie knocks over a tower*] – three green to four green. [*Stephanie writes this on the paper.*] We multiplied by, um, well we multiplied, I guess we divided by four, and I guess we multiplied it by, we only got one of each. So, we came up with four.
122	R1	So, you had four.
123	Stephanie	Yeah, so it was…um. Found, I guess four and divided by four? Um, should, I guess, go on to the next one.
124	R1	Does it help to think about that as having the one tower, go to that one tower with blue. There are four of them. So from one you have four.
125	Stephanie	Um hm. All right.

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126	R1	One times four, and then you have the duplicates and you divide by four.
127	Stephanie	Okay. Um.
128	R1	Does that help Bob? To think of it that way, or to represent it that way?
129	R2	It helps me.
130	R1	So, you might think of that, from that one, I don't know, you might, if you're looking for a way of generalizing a rule or a pattern. You might think of that one tower with the blue, any one you want. You can think of any one you want.
131	Stephanie	Um hm.
132	R1	But, from that one, right?
133	Stephanie	There were
134	R1	You can generate
135	Stephanie	Like, well
136	R1	The green, all green.
137	Stephanie	Yeah.
138	R1	But you can do that from the one, four times.
139	Stephanie	Four, okay.
140	R1	But, then you get the duplicates. I don't know –if that's helpful or not. See this is, see it's just as easy. I mean what Stephanie was doing right now, that's intriguing to me is that she's

141	R2	It looks like
142	R1	She's
143	R2	Yeah.
144	R1	Yeah. It looks like she's counting.
145	R2	It looks like she's multiplying by one, 'cause each
146	Stephanie	Yeah, see that's
147	R1	Exactly.
148	R2	(inaudible) old tower produced one new tower. And then we needed to divide by four in order to take care of the duplication.
149	Stephanie	Um hm.
150	R2	Okay. So. What about, okay, so why don't we just go back to one step before that?
151	Stephanie	One step before that was from, um
152	R2	Let me move this a little bit so you can actually see what you wrote on the paper.
153	Stephanie	From two green…two green…[*Stephanie writes.*] Um, we found four and we divided – well, no we found more than four. We found twelve, and we divided by three.
154	R2	Excellent.
155	Stephanie	…found twelve… [*Stephanie writes more on the paper.*]

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156	R2	So, how did we find the twelve? Did we find, didn't we find the twelve by multiplying?
157	Stephanie	Well, what happened was, they were duplicates. They were groups of three.
158	R2	Yes. That's right.
159	Stephanie	So…
160	R1	Where did the twelve come from?
161	Stephanie	They came from, um, from these. [*Stephanie points to a group of towers with two green and two blue.*] Oh, well, they didn't come from all of these, they came from like (inaudible)
162	R1	I think it's hard to go backwards.
163	R2	I think it's hard to go backwards. Let's, maybe we can, you want to try going forwards