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| 1 | R2 | I think it's hard to go backwards. Let's, maybe we can, you want to try going forwards |
| 2 | Stephanie | Okay. |
| 3 | R2 | and then see if we can meet in the middle and then put all our information together. Okay. We started with four towers that had one green |
| 4 | R1 | Let's get another piece of paper. |
| 5 | Stephanie | Okay. |
| 6 | R2 | And, then, um – one green and three blues. Ready? |
| 7 | Stephanie | All right. |
| 8 | R2 | We started with four towers |
| 9 | Stephanie | Yes. |
| 10 | R2 | that had one green and three blues. [*R2 points to the '4' in Pascal's Triangle.*] |
| 11 | Stephanie | All right. |
| 12 | R2 | Right? Now those were the old ones in that round. |
| 13 | Stephanie | Um hm. |
| 14 | R2 | To produce new ones, what did we do? |
| 15 | Stephanie | We, um, added another green. To any other part of the tower. |

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| 16 | R2 | So how many choices? |
| 17 | Stephanie | Three. |
| 18 | R2 | Three. So we multiplied by three, the four towers we had by three, we multiplied by three choices. |
| 19 | Stephanie | Um hm. |
| 20 | R2 | And then we found |
| 21 | Stephanie | That we had duplicates and we divided it by, um. You divided it by three, right? Or did we, we divided it by four? |
| 22 | R2 | I think you found that |
| 23 | R1 | Well, why don't we |
| 24 | R2 | the number of duplicates was the number of greens. |
| 25 | R1 | Let's, let's um, maybe it would help Bob |
| 26 | R2 | If I remember it. |
| 27 | R1 | if you did the writing and Stephanie did the thinking. |
| 28 | R2 | Okay. So. Well, let me swing around so that we're actually sort of sitting straight up. [*R2 moves his chair next to Stephanie's chair.*] |
| 29 | R1 | So, you could write down what Stephanie's saying. Right. |
| 30 | R2 | Okay. So, we started – can you read my writing? |
| 31 | Stephanie | Yes. |

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| 32 | R2 | Good. Okay. We started with four towers with one green [*R2 writes this on the paper.*] and three blues. And then |
| 33 | R1 | Here they are. |
| 34 | R2 | Okay. So that was the, that was the first one. And then, from each. |
| 35 | R1 | From each. Here's one. [*the* $\begin{bmatrix} B \\ B \\ B \\ G \end{bmatrix}$ *tower*] |
| 36 | R2 | We built |
| 37 | R1 | How many Stephanie? |
| 38 | Stephanie | From each, we built three. |
| 39 | R1 | Okay, this one you built three [*R1 points to the* $\begin{bmatrix} B \\ B \\ B \\ G \end{bmatrix}$ *tower.*] |
| 40 | R2 | We built three. |
| 41 | R1 | This one you built three [*the* $\begin{bmatrix} B \\ B \\ G \\ B \end{bmatrix}$ *tower*], this one you built three [*the* $\begin{bmatrix} B \\ G \\ B \\ B \end{bmatrix}$ |

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| | | *tower*], this one you built three [*the* $\begin{bmatrix} G \\ B \\ B \\ B \end{bmatrix}$ *tower*] |
| 42 | Stephanie | Well- |
| 43 | R1 | Right? |
| 44 | Stephanie | How many green, we're adding how many greens on though? |
| 45 | R1 | Exactly one green. |
| 46 | Stephanie | Like? Yeah. |
| 47 | R1 | Okay. So- right? So from |
| 48 | Stephanie | 'Cause I have three spaces to put it. |
| 49 | R1 | 'Cause you have three spaces to put it. |
| 50 | Stephanie | Yeah. |
| 51 | R1 | So, from this you got |
| 52 | Stephanie | Um, three. |
| 53 | R1 | three. |
| 54 | Stephanie | Yeah. I got three from all of them. So, I got twelve. |
| 55 | R1 | Three from the blue spaces, three from the blue spaces… So from the four |

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| 56 | Stephanie | Um hm. |
| 57 | R1 | you tripled it. |
| 58 | Stephanie | (inaudible) |
| 59 | R1 | You started with the four, you tripled it. Right? |
| 60 | Stephanie | Yeah. |
| 61 | R1 | And you got twelve? |
| 62 | Stephanie | Yes. |
| 63 | R1 | But, you know that, there aren't |
| 64 | Stephanie | Um hm. There's three of each kind. |
| 65 | R1 | exactly two green. You know there aren't twelve. |
| 66 | Stephanie | Yes. |
| 67 | R1 | Well, how many are there? |
| 68 | Stephanie | There's four, so you divided it by |
| 69 | R1 | No, think a minute, think a minute. When you have exactly two green |
| 70 | Stephanie | Um hm. |
| 71 | R1 | how many are there? |
| 72 | Stephanie | When I have exactly two green? |

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| 73 | R1 | Towers four high. |
| 74 | Stephanie | Towers four high, and exactly two green, I'm building on or that's how many I have? |
| 75 | R1 | Just, just tell me what, you know the result of that. When you have exactly two green. |
| 76 | Stephanie | I have four. No, wait, no. I have six. |
| 77 | R1 | It should be six, right? |
| 78 | R2 | That's right. |
| 79 | Stephanie | Yeah. |
| 80 | R1 | But when you, you started with the four, you ended up with four times three, or twelve. You're supposed to have six… |
| 81 | Stephanie | Um hm. |
| 82 | R1 | So, how many duplicates, did you have? |
| 83 | Stephanie | Two. |
| 84 | R2 | Two. So they came in |
| 85 | R1 | That first time you did it, there were only, there was only one duplicate for each one. I couldn't remember all of this. But, doesn't that make sense? Here's the six. Right? [*R1 points to the '6' on Pascal's Triangle.*] |
| 86 | Stephanie | Yeah. |

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| 87 | R1 | But you didn't get six, you got twelve. So, and then if you pulled them out and you did… We weren't recording as we went along, and that's what's hard. |
| 88 | R2 | Let me check out what I'm writing and see if it makes sense to you. |
| 89 | Stephanie | Okay. |
| 90 | R2 | And, then, um, what I'd like to do is, is correct it if I need to so that it begins to look like what you're really thinking. |
| 91 | Stephanie | All right. |
| 92 | R2 | Okay, because what I'm thinking may be different from what you're thinking. And I really want to understand <u>your</u> thinking. |
| 93 | Stephanie | Um hm. Okay. |
| 94 | R2 | Okay? Okay. Ah. We built three towers with two greens – from each, okay, we started with four towers with one green and from each of those four, we built three towers with two greens. This gave four times three, which was twelve towers, but they came in pairs of two. [*R2 reads this from the paper on which he has been writing.*] |
| 95 | Stephanie | Okay. |
| 96 | R2 | These are the duplicates. So, there are really, it seems like, four times three is twice as many as we should have had. So, that's four times three, is twice as many. |
| 97 | Stephanie | Yeah. |
| 98 | R2 | So, we had to divide it by two, |

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| 99 | Stephanie | Um, hm. |
| 100 | R2 | and that gives |
| 101 | Stephanie | Six. |
| 102 | R2 | Six. |
| 103 | R1 | Okay. I'd like you to look at this [*R1 points to the towers.*] again Stephanie. Because, it helps me when I see four. Right? |
| 104 | Stephanie | Um hm. |
| 105 | R1 | Three. Right? Three blue? |
| 106 | Stephanie | Um hm. |
| 107 | R1 | Imagining twelve. And then when you looked at them and pulled them together, you saw the duplicates. |
| 108 | Stephanie | Um hm. Okay. |
| 109 | R1 | But, it may be hard to remember, because each of these were chunked separately. |
| 110 | Stephanie | All right. |
| 111 | R2 | Where do you think |
| 112 | R1 | Do you think you'll remember that? You're not sure, you're not really sure where you got how many duplicates each time, I think. |
| 113 | Stephanie | It, ehh. I understand that like, from these you're going to get three. |

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| 114 | R2 | Right. |
| 115 | R1 | Right. |
| 116 | Stephanie | Like, 'cause, oops. [*Stephanie knocks over some towers and sighs.*] |
| 117 | R1 | Um hm. |
| 118 | Stephanie | 'Cause, there's only three places for you to move them. |
| 119 | R1 | Um hm. |
| 120 | Stephanie | And then, I think what messed me up was how many duplicates you were going to get from each of them. |
| 121 | R2 | Okay. |
| 122 | R1 | Um hm. You know there have to be a total of six when you're done. |
| 123 | Stephanie | Yeah. But, I just |
| 124 | R1 | Right. Sure. |
| 125 | Stephanie | I |
| 126 | R2 | Yeah. There was a step that we talked about at that point. And, um, that was if we took any one of these with the two greens, how many of the old ones did it come from? |
| 127 | Stephanie | Two. |
| 128 | R2 | And it was because there were… |
| 129 | Stephanie | Oh. 'Cause there were two, two of, that would have the possibility, like two |

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| | | parents, or two, like these two [*the* $\begin{bmatrix} G \\ B \\ B \\ B \end{bmatrix}$ *and* $\begin{bmatrix} G \\ G \\ B \\ B \end{bmatrix}$ *towers*]. |
| 130 | R2 | Yeah. And how do you count, okay, what is it about this tower [*the* $\begin{bmatrix} G \\ G \\ B \\ B \end{bmatrix}$ *tower*] that counts the number of parents? |
| 131 | Stephanie | It has a green in two places where |
| 132 | R2 | Excellent. So, it's that two [*the two green blocks*] which counts the parents. |
| 133 | Stephanie | Okay. |
| 134 | R2 | And then it's this two [*the two blue blocks*] that count the next one. |
| 135 | Stephanie | Yeah. So, like here I divide by three, because there's three green? |
| 136 | R2 | Excellent. Excellent. |
| 137 | Stephanie | Okay. |
| 138 | R2 | Okay. In the next step – uh- we took each of the six towers with two greens. [*R2 writes this on the paper.*] Right? |
| 139 | Stephanie | Um hm. |
| 140 | R2 | And produced how many new ones? |

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| 141 | Stephanie | Um. We produced, from the six with two greens? |
| 142 | R2 | Yeah. How many new ones would you get from this one [*the* $\begin{bmatrix} B \\ G \\ B \\ G \end{bmatrix}$ *tower*], for example. |
| 143 | Stephanie | Two. |
| 144 | R2 | Two. Any different from the others? |
| 145 | Stephanie | No. So you produce twelve again. |
| 146 | R2 | Okay. What were you counting when you got the two? |
| 147 | Stephanie | The spaces left over. There were two blue spaces. |
| 148 | R2 | The blues that were left over. |
| 149 | Stephanie | Yeah. |
| 150 | R2 | Okay. Now, you just |
| 151 | R1 | Maybe you ought to put in parentheses blue, parentheses green, when it's appropriate. No? |
| 152 | R2 | Come and write. [*R2 laughs.*] Okay. Um. Okay, so we. Uh. So, each of the six produce two which gives, six times two equals twelve. But, you just told me that, uh, that these twelve, uh, came up in |
| 153 | Stephanie | Are duplicates? |

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| 154 | R2 | And how many came up at a time? |
| 155 | Stephanie | Two. Um. Two at a time. Well, like. You'll get two from each, but you have to divide it by three 'cause there's three green? |
| 156 | R2 | Aha. There's three green in the next generation. |
| 157 | Stephanie | Yes. |
| 158 | R2 | Okay. [*R2 writes.*] Okay. So this gives, uh, six times two divided by three actual towers, with, now it's three greens, right? |
| 159 | Stephanie | Yes. |
| 160 | R2 | Okay. So, the first time we multiplied by three, the second time we multiplied by two. |
| 161 | Stephanie | Um hm. |
| 162 | R2 | The first time we divided by two, and then, the second time we multiplied by |
| 163 | Stephanie | By three. |
| 164 | R2 | three. Can you guess what will happen? |
| 165 | Stephanie | You'll multiply by um four and divide by one. Oh, wait, no! The opposite. You multiply by one and you divide by four. |
| 166 | R2 | Okay. So in the next step… [*R2 writes.*] Prediction. This is by you. [*Stephanie laughs.*] Okay. We, we'd multiply by |
| 167 | Stephanie | By one |
| 168 | R2 | one and |

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| 169 | Stephanie | and divide by four. |
| 170 | R2 | and divide by four. Okay. How did you guess one and how did you guess four? |
| 171 | Stephanie | 'Cause it decreased on |
| 172 | R2 | Or how did you predict one? |
| 173 | Stephanie | Um, like I guess, the numerator, decreased. And the denominator, increased. |
| 174 | R2 | Increased. Terrific. Suppose we do that |
| 175 | Stephanie | Okay. |
| 176 | R2 | and let's just see what turns up. So, the actual number of towers here is six times two over three, which is? |
| 177 | Stephanie | Um, six, oh, three, four. [*Stephanie laughs and covers her face.*] |
| 178 | R2 | Four. So that's, so what happens when we multiply by one and divide by four? |
| 179 | Stephanie | (inaudible) |
| 180 | R2 | Which is? |
| 181 | Stephanie | One. |
| 182 | R2 | Is that what you found? |
| 183 | Stephanie | Yes. |

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| 184 | R2 | So, the prediction is that there are this many towers. |
| 185 | Stephanie | Um hm. |
| 186 | R2 | with four greens. |
| 187 | Stephanie | Yes. |
| 188 | R2 | Which is an old story. Okay. But, um. Now the next, the final question is this. Okay. Um, here are the actual four towers with the three greens. Right? |
| 189 | Stephanie | Um hm. |
| 190 | R2 | How do you see them multiplying by one and dividing by four when we make the next generation? |
| 191 | Stephanie | Well. Each one gives off one new one, one with four green, 'cause there's only one place for you to put the green. |
| 192 | R2 | Excellent. |
| 193 | Stephanie | And because there's four greens, you divided by four. Like the new generation has four greens. You divided by four. |
| 194 | R2 | You know what? |
| 195 | Stephanie | What? |
| 196 | R2 | I'm convinced. |
| 197 | Stephanie | Oh, good. |