ESSAYS IN INDUSTRIAL ORGANIZATION

by

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ABSTRACT OF THE DISSERTATION

Essays in Industrial Organization

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This dissertation contains three chapters on topics in industrial organization. Chapter 2 examines the accuracy of the Hypothetical Monopolist Test, a test for market definition which is often used in the legal analysis of mergers. Chapter 3 analyzes the use of market share discounts by firms and the effect of these discounts on profits and consumer welfare. Chapter 4 investigates pricing practices where a service is offered for a fixed fee plus a multipart usage fee.

In Chapter 2, the accuracy of the Hypothetical Monopolist Test (HMT) is examined. In applying the test, two separate models are considered. In each model, the true extent of the product market is known. First, the HMT is applied in a linear differentiated demand model with *n* firms each producing one symmetrically differentiated product. The test consistently underestimates the number of products in the relevant market in this setting. The second model utilizes a quasi-linear utility specification, and I derive conditions under which the number of products in the market is overestimated.

Chapter 3 analyzes the practice of a manufacturer offering a discount or rebate to a retailer if the sales of that manufacturer's product achieve certain benchmarks. Using a vertical model with linear differentiated demand, I find that the welfare effects of market share discount plans can often be positive. The optimal market share discount plan is closely approximates the outcome of vertical integration between the manufacturer and the retailer. This constructive vertical integration can eliminate double marginalization and encourage competing manufacturers to decrease their own wholesale prices. As such, when one manufacturer offers a market share discount plan, the prices of all products may fall.

In Chapter 4, I examine pricing plans that include buckets of free units, as are common in the wireless telephone industry. A simple demand structure is established which allows for direct comparisons of the profits and consumer welfare under different pricing plans. The free bucket pricing plans are shown to generate the same profits for the monopolist and the same consumer surplus as two-part tariffs, even when multiple plans are offered to consumers.

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Chapter 1

Introduction

My dissertation consists of three chapters in the field of industrial organization. Chapter 2, A Critical Analysis of the Hypothetical Monopolist Test, looks at the fundamental shortcomings of the most commonly used method of market definition for analyzing the competitive effects of mergers under antitrust law. Chapter 3, Market Share Discounts, also looks at a current issue in antitrust law, the use of pricing plans dependent on the seller's percentage of a reseller's total sales. Chapter 4, Service Pricing with Free Buckets and Demand Uncertainty, looks at the use of increasing block tariffs. The use of these plans is common in telecommunications, but pricing theory suggests that these plans will not be optimal for the seller.

In Chapter 2, I examine the accuracy of the Hypothetical Monopolist Test (HMT). Models are presented where the true extent of the product market is known. The test is then applied to these markets and the conditions under which the test performs poorly are derived. The analysis here is in the spirit of Monte Carlo techniques in econometrics. The goal is to determine the accuracy of the test.

In applying the test, two separate models are considered. First, the HMT is applied in a linear differentiated demand model with n firms each producing one symmetrically differentiated product. In this setting, the equilibrium outcome from a cartel of m firms (where $m \leq n$) is calculated. Applying the HMT, conditions are derived whereby the m collusive firms would be considered to comprise a product market. I then examine the settings where the HMT understates market size and by how much. In the second model, I posit a quasi-linear utility specification, in which the products enter the utility function additively. The only interaction between the two products occurs through the income effects of the budget constraint. Yet under certain conditions, if the HMT is applied to all of the producers of a particular product, the smallest possible product market must include at least one producer of the other product. Therefore, depending on the underlying parameters of the utility function, the HMT will either over or underestimate the size of the relevant product market. This shortcoming calls into question the usefulness of the HMT in the analysis of mergers and other antitrust cases.

Chapter 3 looks at market share discounts in a vertical setting. The model used involves a manufacturer offering a discount or rebate to a retailer if the sales of that manufacturer's product achieve certain benchmarks. Using a vertical model with linear differentiated demand, I find that the welfare effects of market share discount plans can often be positive. The optimal market share discount plan is closely approximates the outcome of vertical integration between the manufacturer and the retailer. This constructive vertical integration can eliminate double marginalization and encourage competing manufacturers to decrease their own wholesale prices. As such, when one manufacturer offers a market share discount plan, the prices of all products may fall. When these price changes occur, the welfare effect of a market share discount plan will be positive.

Chapter 4 looks at three part pricing plans. Currently most cellular phone plans include a monthly access fee, a low usage price (typically zero) for the first block of minutes, and a higher usage price (overage price) for minutes beyond the initial block. This is an increasing block tariff. Pricing theory suggests that optimal pricing should take the form of a decreasing block tariff. The reason is that an increasing block tariff can cause two types of inefficiencies. First, consumers typically pay less than marginal cost for minutes within their initial block. For consumers with lower demands for usage who tend to consume the entire initial block, this will lead to inefficient overconsumption of minutes within the initial block. Second, overage prices are typically much higher than a firm's marginal cost for minutes beyond the initial block. This will lead to the traditional deadweight loss as consumers with high demand will not purchase some minutes beyond the initial block for which their willingness to pay exceeds marginal cost.

Prior analysis of these plans by Perry (2005) suggests that these plans do not necessarily improve on two part tariffs. His model assumed that consumers know their level of demand prior to any purchasing decision. This paper extends that model to one of demand uncertainty prior to the purchasing decision. With demand uncertainty at the time of the purchase decision, consumers will be consuming less than the initial block during some periods and more during other periods. Thus, an increasing block tariff will reduce the variance of consumption. However with constant marginal costs, the inefficiencies remain, and the monopolist cannot improve on a standard two-part tariff with the increasing block tariff. When firms are restricted to offering three part pricing plans with an overage price higher that the price for the first block of units, the optimal three part pricing plans exactly mirror the optimal two part pricing plans. In addition, it is clear that in some cases an overage price below the price for the first block will increase profits for the firm.

Chapter 2

A Critical Inquiry into the Hypothetical Monopolist Test

2.1 Introduction

Market definition is one of the most critical and controversial steps for analysis of the competitive effects of mergers. The competitive effects of a merger will depend on the number of firms and the range of products which comprise the relevant product and geographic markets. In addition, market definition also plays an important role in other antitrust cases. In cases under Section 2 of the Sherman Act, the defendant must first hold monopoly power in a relevant product market before being found guilty of an anticompetitive practice. This requires determining the relevant product market in which the defendant competes. In collusion cases under Section 1 of the Sherman Act, the relevant product market will determine whether the conduct of the colluding firms could have any anticompetitive effects.

In this paper, we examine product market definition in the context of models where the exact nature of product substitution is known. In particular, we examine the methodology of the *Horizontal Merger Guidelines* (*Guidelines*) used jointly by the Antitrust Division of the Department of Justice and by the Federal Trade Commission. The question addressed is how well this methodology from the *Guidelines* captures the product market defined by the theoretical specification of a demand model.

Under the *Guidelines*, merger analysis begins by defining the relevant product and geographical markets of the merging firms and then calculating the market shares of firms in those markets.[1] Once a market is defined, the Herfindahl-Hirschman Index (HHI) is calculated by summing the squared market shares of each of the firms producing products in the market.[2] If the merger increases the HHI by more than 100 points, the merger may raise concerns within the agencies about a potential increase in market power and anticompetitive effects. Mergers which raise such competitive concerns are much more likely to be challenged by the government and more likely to be found anticompetitive by the federal courts. Other factors are also considered by the agencies and courts. These factors include the ease of entry into the market, the financial condition of the merging firms, and the possible unilateral price effects of the merger. Despite these other factors, the effect of a merger on industry concentration in the relevant market is often the most significant factor in the determination of whether or not the agencies will examine the merger more closely for any anticompetitive effects.[3] Since the HHI is derived from the market shares within the relevant product market, market definition is a critical part the analysis of mergers.

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[1] The newest revision of the *Guidelines* separates the analysis of mergers into market concentration, unilateral effects, and coordinated effects. Market definition plays a role in the analysis under the market concentration and coordinated effects frameworks, but it is sometimes bypassed in unilateral effects analysis.

[2] See §5.3 of the *Guidelines*. Mergers which increase the HHI by less than 100 points are considered unlikely to have an anticompetitive effect and usually no further analysis is undertaken. In addition, if the HHI is below 1500 after the merger, mergers are again considered to be unlikely to have anticompetitive effects and usually no further analysis is undertaken. If the post-merger HHI is between 1500 and 2500, mergers that increase the HHI by more than 100 points are said to potentially raise significant competitive concerns and to often warrant scrutiny. If the post-merger HHI is above 2500, mergers that raise the HHI by 100 to 200 points also potentially raise significant competitive concerns and often warrant scrutiny. Mergers which increase the HHI by more than 200 points create a rebuttable presumption of enhancing market power. The increase in the HHI that results from a merger is equal to double the product of the market share of the merging firms.

[3] The market share cutoffs serve as a safe harbor for mergers. If the market share cutoffs are such that the merger does not warrant further scrutiny, typically no further action is taken. When more scrutiny is warranted, the agencies will examine the unilateral effects of the merger, that is, the direct impact of the merger on the prices charged in the market. This analysis normally does not require market definition to proceed. The agencies will also examine the coordinated effects of the merger, which is the impact the merger may have on future coordination between market participants. This analysis usually does involve market definition.

According to the *Guidelines* a product market will include the smallest set of products for which a hypothetical monopolist over all of these products would choose to impose a "small but significant and nontransitory" increase in price (SSNIP) on at least one of these products. This process is often referred to as the Hypothetical Monopolist Test (HMT). Fundamentally, the test asks whether a single firm pricing all of the products optimally would significantly increase the price of at least one of the products above the current market price. The smallest set of products for which the hypothetical monopolist would profitably impose a SSNIP on one of the products is determined to be the relevant product market for the analysis of the merger. The algorithm for determining this product market is straightforward. The agencies start with each product produced by a merging firm and ask whether the loss in sales that accompanies a price increase is sufficient to make the price increase unprofitable. If the price increase is unprofitable, the next closest substitute product is added to the potential product market and the same question is asked regarding an increase in price for either or both of the products. Again, if the increase is unprofitable, the next closest substitute product is added to the potential product market and the question is repeated. This process continues until the hypothetical monopolist would profitably impose a SSNIP and the resulting set of products is deemed to be the relevant product market. Since this test is performed for each product sold by the merging firms, it is often quite likely that it will result in several distinct product markets in which the firms compete. An analysis is performed for each product market that results from the test. Some mergers may raise concerns for only a subset of the products which the merging firms sell.

Historically, the outcome of numerous mergers has been determined by the process of defining the product market. A classic example is the case of the proposed merger between Coke and Dr Pepper. Experts for the firms argued for a large market, which included carbonated soft drinks as well as fruit juices and bottled water. With this product market, the HHI would have been well below the threshold for an unconcentrated market and even Coke would have had a small market share. On the other side, experts for the FTC argued that the firms competed in the smaller market for carbonated soft drinks. This product market would have an HHI above the threshold for a highly concentrated market and, combined with Coke's large market share, this would create the presumption that the merger would have anti-competitive effects. Ultimately, the court agreed with the FTC and a preliminary injunction was issued preventing the firms from merging pending a full trial. Given this ruling, the firms abandoned their plans to merge.

This pattern of events is typical of challenged mergers. If the agencies expect anticompetitive effects from the merger, the agencies will seek a preliminary injunction to block the merger from being consummated. At the preliminary injunction hearing, experts from both sides will present their reports and testimony, each with differing views regarding the relevant product market. These differences mostly arise from different methods for determining just how much demand would decline in response to a price increase, more generally known as defining the level of critical loss.[4] Much of the disagreement in these cases revolves around the proper application of the HMT using the critical loss methodology. Many papers have considered which method of calculating

[4] Usually, it is the ruling at this level which will determine the fate of the merger. With few exceptions, if an injunction is not issued, the government will not pursue the case any further and if one is issued, the companies involved will abandon their plans to merge.

this critical loss is most appealing analytically and which method best captures the spirit of the hypothetical monopolist analysis.

In this paper, we examine the accuracy of the HMT itself. Basically, the HMT has severe shortcomings which, without regard to the application of the critical loss methodology, can cause it to significantly underestimate or overestimate the relevant product market in even simple demand models. This analysis is similar to a Monte Carlo study, in which an econometrician knows the true distribution generating the data and asks whether a new econometric method can recognize that distribution. In this paper, we know the true market for the products from the demand model and we test whether or not the HMT can accurately identify this market.

2.2 Literature Review

Early papers on market definition proposed a variety of tests to estimate relevant geographic and product markets (Elzinga and Hogarty (1973); Stigler and Sherwin (1985)). These papers were generally written before the critical loss methodology was formalized by Harris and Simons in 1989. They discussed the shortcomings of the HMT, but their criticisms focused on the ability of the agencies to implement the test. Once critical loss analysis became a standardized methodology for implementing the HMT, the test became generally accepted by the agencies and in the literature. (See Werden (2003))

Most of the more recent papers concerning product market definition have focused on either the correct application of the critical loss methodology (e.g. Coate and Fischer (2008)) or the appropriateness of using critical loss at all (e.g. Murphy and Topel (2008)). In either case, the overall effectiveness of the HMT is not really considered. Even the papers which have focused on alternative market definition methods typically do not directly address the HMT, but instead examine situations where the HMT may be hard to implement (Shapiro and Farrell (2010)).

The primary problem with the HMT is that it may identify a product market which is too small or too large relative to the true product market. In the analysis to follow, the test will fail to properly identify simple product markets which have no close substitutes outside the market. Using a setup with *n* symmetric firms producing symmetrically differentiated products, the HMT will find that the product market may include only a small subset of the products. In addition, a representative consumer model will show that the income effects of the budget constraint are sufficient to for the HMT to overestimate the relevant product market and include products which are not direct substitutes.

2.3 Underestimation of Product Market

Consider an industry with *n* symmetrically differentiated products and one firm producing each product. Each firm has constant average and marginal cost *c* of producing a differentiated product. Demand is characterized by the linear differentiated model where the inverse demand for product *j* given by

$$P_j(x_1, x_2, ..., x_n) = a - b_j x_j - \sum_{i \neq j} d_{ji} x_i \ . \tag{1}$$

Here, x_i is the output of firm *i* and d_{ji} is a measure of the substitutability between products *j* and *i*. In order to construct the clearest illustration, we assume that the products are symmetric, that is $b_i=b$ and $d_{ji}=d$ for all products. Thus all products are equal substitutes for all other products. When $b=d$, the products are homogeneous because the price will depend only on the total output of all firms. Here, we will assume that $b>d$, assuring that the products are differentiated. The products are substitutes for one another because the last term of the inverse demand function ensures that the willingness to pay for a particular product decreases as the consumption of other products increases.

With symmetry, the demand function for product j will be

$$x_j(p_1, p_2, ..., p_n) = \frac{(b-d)a - [b+(n-2)d]p_j + d\sum_{j\neq i} p_i}{(b-d)[b+(n-1)d]}. \qquad (2)$$

To apply the HMT, we will compare the equilibrium that results from each firm independently maximizing its own profit to the case where a subset of firms jointly maximize profits as a partial cartel. This cartel represents the hypothetical monopolist for applying the HMT. The cartel will take account of the cross effects in setting a common price for all products produced by its members. The smallest cartel which increases its price by the necessary amount to satisfy the HMT will comprise the product market as described in the *Guidelines*.[5]

Let m firms in the industry form a cartel and jointly maximize profits with a common price for each of their products. If m=1, this equilibrium is simply the pre-merger industry from which the HMT will be applied. We first examine the profit maximizing price of the non-cartel members.

Let p_c be the price charged by firms within the cartel for each of their products and let p_n be the price charged for all products outside the cartel. For a non-cartel product, profits can be expressed as

$$\pi_j(p_j, p_c, p_n, m) = (p_j - c)\cdot\frac{(b-d)a - [b+(n-2)d]p_j + d[m\cdot p_c + \sum_{i\neq j, i\notin cartel} p_i]}{(b-d)[b+(n-1)d]}. \qquad (3)$$

The symmetric first-order condition ($p_j = p_n$) for profit maximization is

[5] Here, we apply the test with the cartel raising the prices of all of the products produced by cartel firms. If we only examine the increase in price to an individual product, the underestimation of the product market will be more extreme.

$$a \cdot (b-d) - (p_n - c) \cdot [b+(n-2)d] - [b+(n-2)d] \cdot p_n + d \cdot [m \cdot p_c + (n-m-1) \cdot p_n] = 0 \tag{4}$$

This condition defines the equilibrium price for the non-cartel members as a function of the number of firms in the cartel and the cartel's price.

For the cartel, profits are given by

$$\sum_{i=1}^{m} \pi_i(p_c, p_n, m) = m(p_c - c) \cdot \frac{(b-d)a - [b+(n-2)d]p_c + d[(m-1)p_c + (n-m)p_n]}{(b-d)[b+(n-1)d]}. \tag{5}$$

The first order condition for the cartel price is:

$$a \cdot (b-d) - (p_c - c) \cdot \{[b+(n-2)d] - d(m-1)\} - [b+(n-2)d] \cdot p_c + d \cdot [(m-1)p_c + (n-m)p_n] = 0. \tag{6}$$

The equilibrium prices for the cartel and non-cartel products can be obtained by solving (4) and (6) simultaneously. The resulting equilibrium prices are:

$$p_n(m;n) = \frac{a \cdot (b-d) \cdot \{2[b+(n-2)d] - d(m-2)\} + c \cdot [b+d(n-m-1)] \cdot \{2[b+(n-2)d] + dm\}}{2\{2[b+(n-2)d] - d(n-m-1)\} \cdot [b+d(n-m-1)] - d^2 m(n-m)} \tag{7}$$

$$p_c(m;n) = \frac{a \cdot (b-d) \cdot (2[b+(n-2)d] + d) + c\{2[b+(n-2)d]^2 - d(2m-3) \cdot [b+(n-2)d] + d^2 \cdot (n-m-1) \cdot (m-1)\}}{2\{2[b+(n-2)d] - d(n-m-1)\} \cdot [b+d(n-m-1)] - d^2 m(n-m)}. \tag{8}$$

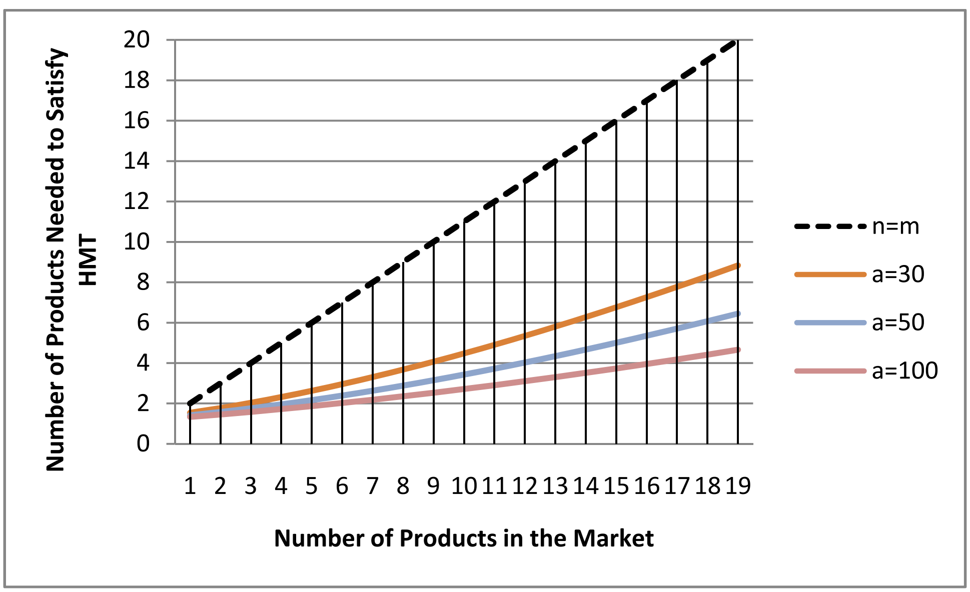
Let $p_c(m;n)$ be the price that a cartel composed of m products out of n total products will charge in equilibrium. In the case where there is no cartel (m=1) then

$$p_n = p_c(1;n) = \frac{a(b-d) \cdot \{2[b+(n-2)d] + d\} + c\{2[b+(n-2)d]^2 + d[b+(n-2)d]\}}{2[2b+(n-2)d] \cdot [b+(n-2)d] - d^2(n-1)}. \tag{9}$$

This price provides the reference point from which to apply the HMT. Using a 5% price increase as the standard for a SSNIP, the test will define a market as the smallest value for m that satisfies $p_c(m;n) \geq (1.05)\, p_c(1;n)$. Solving this equality yields the minimum number of products for which a hypothetical monopolist will profitably impose a price increase of 5%.[6] For a given total number of firms, *n*, the above condition will determine the number of firms that satisfy the HMT and define a product market under the *Guidelines*. We will denote this number as $m^*(n)$.

We can now illustrate market definition for different values of the demand and cost parameters. Consider the case where *a*=50, *b*=3, *c*=10, and *d*=1. By varying *n* and solving for the value of *m* that satisfies the above condition, the shortcomings of the HMT become evident. Figure 2.1 below shows the values of $m^*(n)$ (vertical axis) that would constitute a market using the HMT as a function of *n* (horizontal axis) for different values of *a*. Because the model consists of *n* identical firms, the dashed line representing *n*=*m* indicates the true size of the market.

Figure 2.1 Accuracy of HMT as Intercept Varies



[6] Note that this is the threshold number of products for which a profit maximizing cartel will raise the price 5%. It is possible to have a smaller number of products which could profitably raise price by 5%. It could easily be the case that a smaller number of products maximizes profit by increasing price slightly less than 5%, but would earn a higher profit at a 5% price increase than by charging the non-cartel price. In this sense, the model here understates the degree to which the HMT can underestimate the relevant product market.

For each example value of a, the HMT consistently understates the size of the market. As a percentage of the true market size, the HMT performs worse as a increases. For a given value of a, the test performs worse as n increases, up to some point (n=18 for a=100, n=13 for a=50, and n=10 for a=30), but beyond that point the test performs better with larger values of n. However, even with n=100, the test still captures at most 77.1% of the market.

The test will perform worse for higher values of a because as a increases relative to c, the potential margin between price and marginal cost will increase. As this potential margin increases, the cartel price will increase from c more quickly when the cartel expands than with a smaller gap between the demand function and marginal cost. The test does improve when the demand intercept is lower, but it still underestimates the size of the market by over 20% even in the best case presented and over the range of examples captures only between 23.3% and 77.1% of the market.

While the HMT performs better as a decreases, at some point it will begin to perform poorly by overestimating the product market. At the limit as a approaches c, the markup by even the grand cartel comprised of all the firms in the industry will fail to satisfy the HMT. As such, the market including all of the products would not be sufficient to define a market within the framework of the *Guidelines*.

Repeating the same exercise, only varying c or d yields similar results, which are given in Figures 2.2 and 2.3.

Figure 2.2 Accuracy of HMT as Degree of Product Differentiation Varies

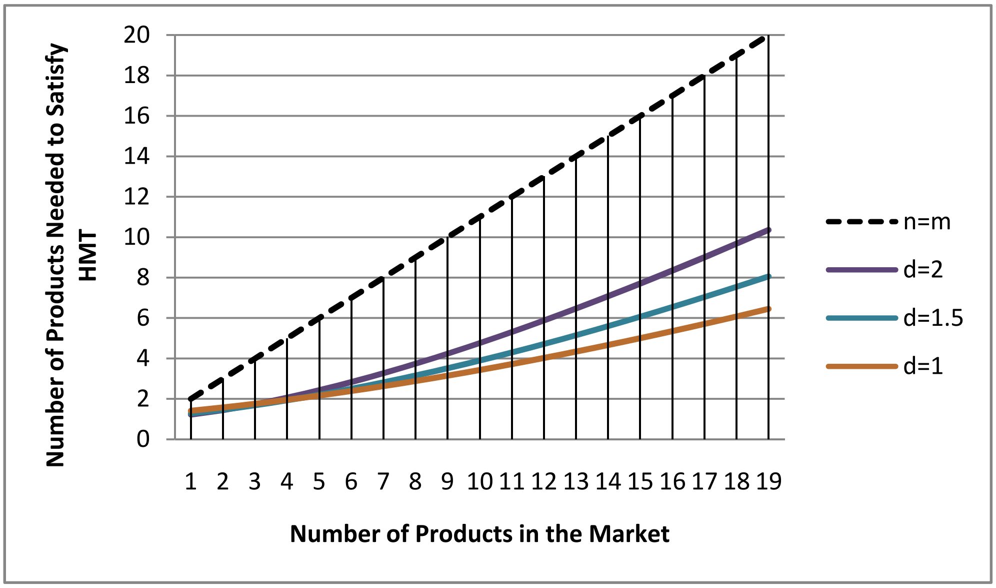
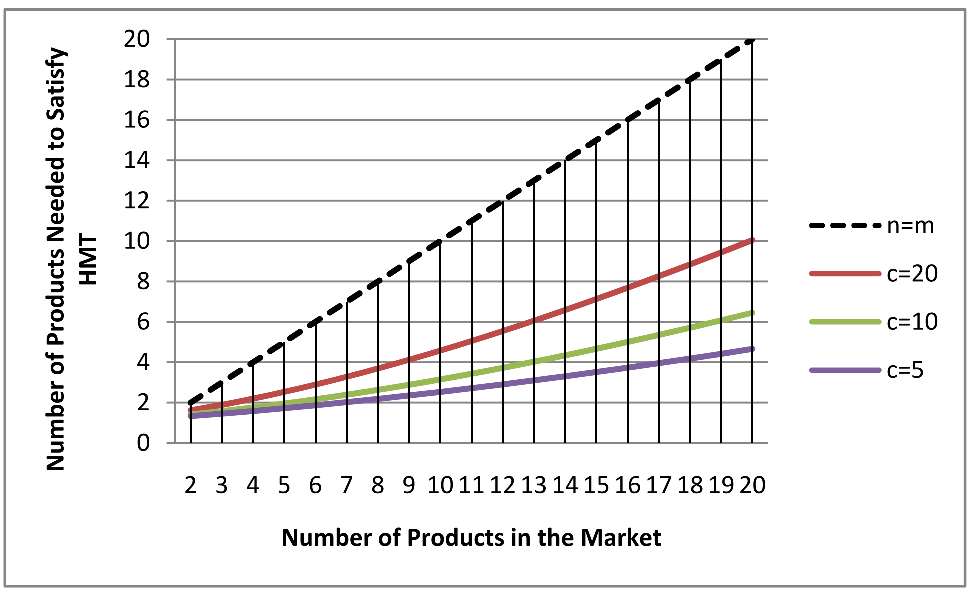


Figure 2.3 Accuracy of HMT as Cost Varies



In both cases the HMT significantly understates the market size, capturing between 23.2% and 81.1% of the market. The test performs more poorly as the products become more differentiated (as *d* decreases with *b* constant). In addition, the greater the net value (*a*-*c*) of the products, the less accurate the market definition becomes.

When a=c, the test also captures all of the relevant products in the estimated market.[7] In this case, there is no ability to price any product above marginal cost without losing all of the sales of that product. Likewise, when b=d, the test exactly identifies the relevant product market, but as the products become more differentiated, the HMT is much less accurate. This results from price setting with homogeneous products. The price on any individual product cannot be increased without losing all of the sales of that product, so the only cartel that could profitably increase price must include every product in the market. Thus, the HMT is only accurate when the product market is obviously homogeneous or nonexistent.

When the products have a value above marginal cost for some consumers and when the products become more differentiated, the HMT is much less accurate and will include a small subset of products in the market. Moreover, the products included could be many combinations of the other products not produced by one of the two merging firms.

To further illustrate the inaccuracy of the HMT consider an alternative for comparison. Let a vary such that for each value of n the competitive price is equal to \$100. The HMT will define the market as the smallest number of products for which a cartel would charge a price of at least \$105. Table 2.1 presents the results. Table 2.2 presents a similar comparison where the competitive price is equal to \$20 and the smallest number of products in the market is such that the price is at least \$21. In these situations, the test performs poorly across market sizes and the performance is worse as n

[7] In this case, the HMT will actually overestimate the product market and require products not included in this demand specification within the relevant product market.

grows.[8] Consistently, the HMT includes less than one quarter of the actual products for $n \geq 8$.

Table 2.1 Accuracy of HMT as *n* Varies

Varying *a* so that equilibrium price (m=1) = 100		
n	m*	P(m=m*)
3	2	127.826087
4	2	121.4705882
5	2	117.4468085
6	2	114.6774194
7	2	112.6582278
8	2	111.122449
9	2	109.9159664
10	2	108.943662
20	3	107.2251868
30	4	106.5955285
40	5	106.2688777
50	6	106.0689388

Table 2.2 Accuracy of HMT as *n* Varies (Alternative Normalization)

Varying A to such that equilibrium price (m=1) = 20		
n	m*	P(m=m*)
3	2	23.47826087
4	2	22.64705882
5	2	22.12765957
6	2	21.77419355
7	2	21.51898734
8	2	21.32653061
9	2	21.17647059
10	2	21.05633803
20	4	21.16740088
30	6	21.22080679
40	7	21.05536861
50	9	21.1102156

[8] Note that in the prior discussion, *a* was fixed as market size changed. Here *a* is allowed to increase in order to hit the targeted competitive price for the various market sizes.

In order to capture the entire market, the price increase threshold for the HMT would have to be greater than the price increase imposed by a cartel made up of *n*-1 firms. Table 2.3 compares the number of firms in the market to the percentage price increase that would be imposed by *n*-1 firms. For the previous base case with *a*=50, *b*=3, *c*=10, and *d*=1, the minimum price increase that captures the entire market ranges from 19% for *n*= 3 firms to 115% for *n*= 50 firms.

Table 2.3 Minimum Price Increases Needed TO Satisfy HMT

Number of firms and minimum price increse necessary to capture entire market	
a=50 b=3	c=10 d=1
Number of firms	% Price increase of a cartel with *n* -1 firms
3	19.25466
4	26.90058
5	33.82353
6	40.05957
7	45.67901
8	50.7556
9	55.35714
10	59.54317
20	86.91679
30	101.1022
40	109.7606
50	115.5927

Finally, the failure of the test to include all of the products in the market can have significant impacts on the outcome of a merger application. Consider an industry consisting of 20 identical single product firms with *a*=50, *b*=3, *c*=10, and *d*=1. The equilibrium price in this market prior to any merger is $13.48. If two of the firms merge, these firms will charge a price of $13.67 after the merger. This is a price increase of only 1.4%, which is not generally viewed as anticompetitive. The premerger HHI in this industry would be 500 and the merger would increase it to 550 at most.[9] Applying the structural approach of the *Guidelines*, this merger would not be likely to raise anticompetitive concerns.

Applying the HMT to this market shows that a cartel of six firms would charge a price of $14.19. This is an increase of 5.31%. Because the six firms can profitably increase price by more than 5%, the test would define these six products as the market. The two merging firms would then represent a third of the defined market, and a merger such as this would possibly be challenged by the government. In fact, using the same example parameters, a merger of two firms in a six firm industry would lead to a 8.44% increase in price over the six firm equilibrium. Likewise, the premerger HHI would be 1667 and a merger of two of the six firms would increase the HHI to 2222. Under the *Guidelines*, this large increase in the HHI would potentially raise significant competitive concerns and quite possibly be challenged.

Repeating the same exercise with a=100, the HMT would identify the product market as including only four products and the corresponding HHI increases would be sufficient to create a presumption of being likely to enhance market power. So, by incorrectly identifying the breadth of the product market, proper application of the HMT will lead to an appearance of an anticompetitive merger in a case where the merger is of relatively little impact on the market.

The linear differentiated demand system is just one of several common demand models, but within this system we have demonstrated that the HMT underestimates the market and thus can fail to properly address anticompetitive concerns for which it was

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[9] The post merger HHI calculation assumes that the merged firm would retain all of the sales for the products produced by the merging firms. If some sales are lost due to the post merger price increase, the actual post merger HHI would be lower.

created. The HMT will not only define the product market to include fewer than the true number of products, but will also define the product market to be some subset including *m* of the *n* products. As such, an application of the test could result in finding that the two merging firms are located in separate product markets. In addition, because the test performs more poorly as product differentiation becomes more pronounced, then the test cannot accurately define product markets when that definition is most needed by the agencies and the courts.

2.4 Overestimation of Product Market

In this section, we show that the HMT can overestimate the product market. We present an example with two independent products which are not direct substitutes but might be included in the same product market using the HMT. In particular, under some conditions, the producers of one product will not by themselves comprise the product market for that product. As such, the producers of the other product will be included in the product market for the first product, even though the other product is not a direct substitute.

Consider the case where a representative consumer has a quasi-linear utility function. That is, utility is given by $U(x, y) = f(x) + g(y)$, where $f(x)$ is a non-linear function of the consumption of product x and $g(y)$ is a linear function of the consumption of product y. This model is helpful for analyzing the shortcomings of the HMT because in this setting products x and y enter the utility function independently. In other words, the consumption of one product has no impact on the utility derived from the other product. Thus, the only substitution between the products will result from income effects through the budget constraint of the consumers. Now we consider a more specific example of the general model.

Let utility be given by $U(x, y) = \ln(x) + \alpha \cdot y$. Utility is concave in *x* with diminishing marginal utility and utility is linear in *y* with constant marginal utility. The parameter *α* determines the relative importance of product *y* on overall utility.

For income *I*, the demand functions for products *x* and *y* are given by

$$x(I, p_x, p_y) = \frac{p_y}{\alpha \cdot p_x} \text{ , and} \tag{10}$$

$$y(I, p_x, p_y) = \frac{I}{p_y} - \frac{1}{\alpha}. \tag{11}$$

Note that the demand for *y* is not a function of the price of product *x*. Thus, the cross-price elasticity of demand for *y* is 0. The cross-price elasticity of demand for *x* is 1, but only because of the income effect.

The inverse demand for product *y* can be expressed as

$$p_y(I, x, y) = \frac{\alpha \cdot I}{\alpha \cdot y + 1}. \tag{12}$$

Suppose there are *n* firms which produce product *y*. Since *y* is a homogeneous product, these firms should comprise the entire product market for this product. However, the HMT may find that together they do not satisfy the condition for a product market. Thus, other products such as *x* which are not direct substitutes would be included in the product market with *y*. Using the HMT, we first identify the conditions for which all of the firms producing product *y* are included in the product market. This occurs where the cartel of all firms producing product *y* would set a price less than 105% of the equilibrium price in which all these firms compete separately. The conditions under which this will occur depend on I, c, and α.

2.4.1 Cournot Equilibrium for the Firms Producing Product *y*

Suppose there are n firms producing product y and they compete in a Cournot game. Assume each firm has no fixed costs and constant marginal costs are given by c. Each firm will choose output level y_i to maximize profits

$$\pi_i(I,c,y_i,y_j) = (p_y - c)\cdot y_i = \left[\frac{\alpha \cdot I}{1+\alpha\cdot\sum_{j=1}^{n} y_j} - c\right]\cdot y_i\,. \tag{13}$$

The resulting first-order condition for profit maximization is then

$$\frac{\alpha \cdot I}{1+\alpha\cdot\sum_{j=1}^{n} y_j} + \left[\frac{-\alpha^2\cdot I}{\left[1+\alpha\cdot\sum_{j=1}^{n} y_j\right]^2}\right]\cdot y_i - c = 0\,. \tag{14}$$

In the symmetric equilibrium, each firm will choose an output defined by

$$y_i = \frac{\alpha\cdot I(n-1) - 2cn + \sqrt{\alpha^2 I^2 (n-1)^2 + 4\alpha I c n}}{2\alpha c n^2}\,. \tag{15}$$

The resulting total output of the firms producing y and price of y are given by

$$\sum_{i=1}^{n} y_i = \frac{\alpha\cdot I(n-1) - 2cn + \sqrt{\alpha^2 I^2 (n-1)^2 + 4\alpha I c n}}{2\alpha c n} \tag{16}$$

and $$p_y = \frac{2\alpha I c n}{\alpha I(n-1) + \sqrt{\alpha^2 I^2 (n-1)^2 + 4\alpha I c n}}\,. \tag{17}$$

Since there is no cross-elasticity with product x, neither the price of product x nor its quantity appears in the equilibrium price for y.[10]

2.4.2 Monopolist Pricing

In order to identify the conditions under which the product market must be expanded to include more than all of the producers of product y, the profit maximizing price for a monopolist producer of product y must also be calculated. The monopolist will choose price p_y to maximize profits

$$\pi(I,c,y) = (p_y - c)y = (p_y - c)\left[\frac{I}{p_y} - \frac{1}{\alpha}\right]. \tag{18}$$

The resulting first-order condition for the profit maximization is then

$$(p_y - c)\left(\frac{-I}{p_y^2}\right) + \frac{I}{p_y} - \frac{1}{\alpha} = 0.\text{[11]} \tag{19}$$

This yields the profit maximizing price of

$$p_y = \sqrt{\alpha I c}\,. \tag{20}$$

2.4.3 Application of the HMT

With both the n firm Cournot and monopoly prices defined, it is straightforward to apply the HMT. Let γ be the percentage increase in price necessary to satisfy the SSNIP standard of the test (plus one). That is, if we consider a 5% price increase, γ is equal to 1.05. Under this standard, the n firms will constitute the market for product y if and only if

[10] This assumes that there is an interior solution. A corner solution will occur when α is sufficiently small and the consumer purchases only good x.
[11] The second order condition is satisfied for all non-negative values of I and c.

$$\sqrt{\alpha Ic} \geq \gamma \cdot \frac{2\alpha Icn}{\alpha I(n-1) + \sqrt{\alpha^2 I^2 (n-1)^2 + 4\alpha Icn}}. \tag{21}$$

Solving for α, representing product *y*'s share of utility, the expression simplifies to

$$\alpha \geq \frac{c \cdot \left(n\gamma^2 - 1\right)^2}{I \cdot (n-1)^2 \cdot \gamma^2}. \tag{22}$$

If this condition is satisfied then the *n* firms will satisfy the HMT and will comprise a product market on their own. However, if this condition is not satisfied, then the *n* firms producing product *y* will not satisfy the HMT by themselves, and the product market for product *y* must include at least one firm which produces product *x*. To see this, assume that all of the producers of products *x* and *y* were included in the test. Here, the test will be trivially satisfied. As the firms increase their prices, total combined revenue will remain constant at *I*, but the total cost will decrease as consumers purchase fewer units. Thus, profits must increase for any percentage price increase, including a 5% increase.

Therefore, if the producers of product *y* do not constitute a product market under the HMT, at least one firm producing product *x* must be included in the product market under the HMT. Therefore, the HMT will overestimate the size of the relevant product market if $\alpha < \frac{c}{I} \cdot \frac{\left(n\gamma^2 - 1\right)^2}{(n-1)^2 \gamma^2}$. This result will hold for any structure for product *x*. In particular, the market for product *x* could be competitive or imperfectly competitive and the result would hold in either case.

The HMT will overestimate the product market for product *y* as long as α is small enough relative to the ratio of production costs to consumer income. The term $\frac{\left(n\gamma^2 - 1\right)^2}{(n-1)^2 \gamma^2}$

will decrease as n increases for values of $\gamma > 1$. For $\gamma = 1.05$, this term will take a value of 1.31 for n=2, and decrease for larger values. Therefore, with fewer firms producing product y, the inequality is more likely to be satisfied. Thus when product y makes a sufficiently small contribution to consumer utility, the HMT will overstate the size of the relevant product market. This failure will occur more often for lower values of n, which is precisely the situation where the anticompetitive effect of a merger is more likely to be significant.

The fundamental failing of the HMT in this case is that it can force the estimated product market for product y to include some products of good x. The only relationship between these two products is that the representative consumer derives utility from both of them. Just as the underestimated product market in the previous section included an arbitrary subset of related products, the overestimated product market here could include an arbitrary subset of unrelated products. Suppose firms which produce product x but not product y are included in the HMT product market for product y. This will make the market appear to be less concentrated than it actually is. This mistake may allow a merger which greatly increases market power in market y to appear to have a much smaller effect.

2.5 Concluding Remarks

The above analysis demonstrates the fundamental failings of the HMT. In the case of the linear differentiated demand model, the HMT consistently overestimates the size of the relevant product market. This result did not require any asymmetry or unusual assumptions on the parameters of the model. Given a simple demand setup, the HMT would include only a relatively small subset of the products in the true product market.

In addition, using the case of the quasi-linear utility function, we derived the conditions under which the HMT would overestimate the size of the relevant product market. In these situations, at least one producer of another independent product would be included in the product market which results from the HMT.

The newest merger guidelines indicate that the agencies are moving away from structural presumptions towards analyzing the unilateral effects of a merger. That is, instead of investigating how a merger makes the market more concentrated, they hope to focus on how the merger affects the pricing incentives of the merging firms. This approach would allow the agencies to sidestep the difficulties surrounding market definition. However, the *Guidelines* do not carry the force of law, and it remains to be seen whether or not the courts will accept merger challenges which do not include product market definition. Litigators in the field expect that the courts will still require the agencies to apply the standard market definition and concentration analysis.[12]

In addition, market definition still plays a critical role in other antitrust areas, especially the prosecution of claims under §2 of the Sherman Act. In these cases, the government (or a private party seeking damages) must show that the defendant has monopoly power in a relevant product market. In theory, the market in which the defendant competes must then be defined.

Despite attempts to reduce the role of market definition in merger analysis, it remains a critical element of antitrust litigation. The weaknesses of the Hypothetical Monopolist Test call into question its usefulness in defining product markets.

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[12] See Fry, McGuire, and Schmierer (2011) pp. 823-827 for a discussion of the impact of the 2010 *Guidelines* on litigation of merger cases.

Chapter 3

Market Share Discounts

3.1 Introduction

Market share discounts are programs in which a downstream firm receives a discount on the purchase price of a product, conditional on the resale of the product comprising some minimum quantity or percentage of the total sales of similar products by the downstream firm.[13] In industries with predictable demand, the nature of the target will not make a difference. Whether the upstream firm demands a quantity or a market share, it is essentially choosing a point on the demand curve. In either case, retail prices and quantities will remain the same. Market share discounts are a specialized form of a non-linear tariff. The upstream firm allows the downstream purchaser to choose between competing price-quantity pairs in making its purchasing decisions.

Market share discounts are controversial because they may have opposing welfare effects. On one hand, market share discounts generally reduce the marginal price paid by distributors. On the other hand, market share discounts could be used to exclude competitors. As a result the limited literature on market share discounts tends to be contradictory and inconclusive. Likewise, different jurisdictions have taken different approaches to evaluating the discount plans. In this paper, the analysis will focus on examining the effects of market share discounts on equilibrium outcomes and the legal treatment of market share discount plans.

In practice, market share discounts often take the form of rebates. Once the downstream firm reaches its threshold level of sales or dollar value for a discount, the

[13] Market share discounts are similar in effect to loyalty discounts. However, market share discounts typically reward purchases in the current period where loyalty discounts reward past purchases.

discount is applied to all of the units purchased and then paid in the form of a rebate. This particular method has drawn significant attention, mainly focused on the incentives near the threshold level for the discount. As a firm reaches the threshold, the effective price can be negative for units of the product beyond the threshold. For example, suppose a firm faces a pre-discount price of $100. The price is discounted to $80 if the firm purchases at least 100 units. A firm purchasing 90 units would face a total cost of $9,000. However, if the firm purchases 10 additional units and receives the discount, total expenditures would fall to $8,000. Essentially, the firm pays -$1,000 for the last 10 units. These plans raise concerns about predatory pricing by the firm offering the market share discount.

The discount need not take the form of a rebate for this effect to occur. Suppose an upstream firm offers a pricing menu where the price is p_1 for firms that purchase less than X units and p_2 for firms that purchase at least X units. As long as $p_1 > p_2$, then there will be a similar point where marginal revenue is negative for the upstream firm. Even if the discount is strictly offered as a choice between competing bundles, it will have the same effect as a rebate.[14] As such, these pricing plans would be similar to other pricing plans that have drawn significant antitrust scrutiny.

Market share discount plans can force buyers to make discrete choices in the quantity of a product that they purchase. With a single discount offered, a downstream firm or a consumer will have the choice of two competing quantities. They could purchase the optimal quantity given the higher non-discounted price or choose the

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[14]In fact for any multi-unit bundle with a lower average price than is available for a single unit, there will typically be some point where at least one unit of the good has a negative marginal price. This assumes that there are only a limited number of bundles available. As long as there are gaps in the size of the available bundles and average price reduces as the bundle size increases, then there will be points around the size of the bundle where marginal revenue is negative.

optimal quantity at the lower post-discount price. The larger the discount is, the greater the gap between these two quantities will be. It is possible that large enough discounts will force the buyer to choose between two price-quantity pairs that are not socially optimal. By restricting the choice space for potential buyers, it could be the case that market share discounts have a negative impact on consumer welfare.

3.2 Literature Review

The literature directly related to market share discounts is relatively thin, and many of the relevant papers are recent. Mills (2004) explores a setting where market share discounts serve to improve merchandising services, leading to a welfare improvement. Greenlee and Reitman (2004) show that it can be profitable for firms to target certain consumers with loyalty discounts and that, in equilibrium, only one firm's loyalty discount program will be accepted. Kolay, Shaffer, and Ordover (2004) look at a monopolist offering an all-units discount on a single product. They find that these discounts eliminate double marginalization in full information settings and can extract surplus more efficiently than a menu of two-part tariffs with imperfect information.

Caminal and Claici (2007) show possible consumer benefits for intertemporal loyalty discounts. Finally, Carlton and Waldman (2006) and Ordover and Shaffer (2007) look at the use of loyalty discounts to exclude potential entrants through denying their opportunity to achieve economies of scale. Because the Intel case has attracted so much recent attention, there is a renewed interest in the effects of market share discounts.

3.3 Model

Let manufacturer 1 (M1) and manufacturer 2 (M2) be the producers of products 1 and 2 respectively. These manufacturers sell their products to a common retailer who is a monopolist in the retail market. The products are imperfect substitutes for one another.

The inverse demand functions are given by $p_1 = a_1 - b_1x_1 - dx_2$ and $p_2 = a_2 - b_2x_2 - dx_1$, where p_i is the price of product i, a_i is the intercept for the inverse demand for product i, b_i is the slope of the inverse demand function for product i, and d is a common parameter capturing the substitution between the two goods. The goods are more differentiated when d is smaller relative to b_i.

M1 and M2 produce products 1 and 2 with constant marginal and average costs c_1 and c_2, respectively. Assume that M1 is the more dominant firm, captured by a strict inequality of one or more of the following parameter restrictions: $a_1 \geq a_2$, $b_1 \leq b_2$ and $c_1 \leq c_2$. If $a_1 > a_2$ or if $b_1 < b_2$, then consumers will have a higher willingness to pay for product 1 than product 2 for any given quantity. If $c_1 < c_2$, then M1 will have lower marginal costs of production than M2 for any output level. M1 and M2 sell to the common retailer at prices r_1 and r_2. The retailer then chooses p_1 and p_2 and sells to consumers.

The demand functions are given by

$$x_1(p_1, p_2) = \frac{b_2(a_1 - p_1) - d(a_2 - p_2)}{b_1b_2 - d^2} \text{ and}$$

$$x_2(p_1, p_2) = \frac{b_1(a_2 - p_2) - d(a_1 - p_1)}{b_1b_2 - d^2}. \tag{1}$$

Suppressing the denominator $b_1b_2-d^2$ for now, we can express the profits for the common retailer and M1 and M2 as:

$$\pi_{CR} = [(a_1 - r_1) - (a_1 - p_1)][b_2(a_1 - p_1) - d(a_2 - p_2)]$$
$$+ [(a_2 - r_2) - (a_2 - p_2)][b_1(a_2 - p_2) - d(a_1 - p_1)],$$
$$\pi_1 = [(a_1 - c_1) - (a_1 - r_1)][b_2(a_1 - p_1) - d(a_2 - p_2)], \text{ and}$$
$$\pi_2 = [(a_2 - c_2) - (a_2 - r_2)][b_1(a_2 - p_2) - d(a_1 - p_1)].^{15} \tag{2}$$

Since the denominator only depends on the parameters, it will not affect the profit maximizing prices. This denominator will be reintroduced when equilibrium quantities are discussed later.

3.3.1 Equilibrium without Market Share Discounts

Solving for the profit maximizing prices for the retailer, we obtain the usual monopoly margins of

$$(a_1 - p_1) = \frac{(a_1 - r_1)}{2} \text{ and}$$
$$(a_2 - p_2) = \frac{(a_2 - r_2)}{2}.^{16} \tag{3}$$

Substituting the retail prices into the demand functions yields the derived demand functions for products 1 and 2:

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[15] Profits are written in terms of $(a_1 - r_1) - (a_1 - p_1)$ rather than $(p_1 - r_1)$ because we will solve for equilibrium prices in terms of $(a_1 - p_1)$. Presenting the prices as margins will simplify the algebra and present more intuitive solutions.

[16] These margins assume an interior solution. The full solution to the retail pricing problems is:

$$(a_1 - p_1), (a_2 - p_2) = \begin{cases} 0,0 & if\ r_1 > a_1\ and\ r_2 > a_2 \\ \frac{(a_1 - r_1)}{2}, \frac{(a_2 - r_2)}{2} & if\ 0 \le d(a_1 - r_1) < b_1(a_2 - r_2)\ and\ 0 \le d(a_2 - r_2) < b_2(a_1 - r_1) \\ \frac{(a_1 - r_1)}{2}, \frac{d(a_1 - r_1)}{2b_1} & if\ d(a_1 - r_1) \ge b_1(a_2 - r_2)\ and\ 0 \le d(a_2 - r_2) < b_2(a_1 - r_1) \\ \frac{d(a_2 - r_2)}{2b_2}, \frac{(a_2 - r_2)}{2} & if\ 0 \le d(a_1 - r_1) < b_1(a_2 - r_2)\ and\ d(a_2 - r_2) > b_2(a_1 - r_1) \end{cases}$$

The third and fourth pricing solutions identify corner solutions where the demand for products 2 and 1, respectively, will be zero.

$$x_1(r_1, r_2) = \frac{1}{2}[b_2(a_1 - r_1) - d(a_2 - r_2)], \text{ and}$$

$$x_2(r_1, r_2) = \frac{1}{2}[b_1(a_2 - r_2) - d(a_1 - r_1)]. \tag{4}$$

Given the derived demand functions, M1 and M2 will choose r_1 and r_2 to maximize their profits. The resulting Nash equilibrium prices will be:

$$(a_1 - r_1) = \frac{2b_1b_2(a_1 - c_1) + b_1d(a_2 - c_2)}{4b_1b_2 - d^2},$$

$$(a_2 - r_2) = \frac{2b_1b_2(a_2 - c_2) + b_2d(a_1 - c_1)}{4b_1b_2 - d^2},$$

$$(a_1 - p_1) = \frac{2b_1b_2(a_1 - c_1) + b_1d(a_2 - c_2)}{2(4b_1b_2 - d^2)}, \text{and}$$

$$(a_2 - p_2) = \frac{2b_1b_2(a_2 - c_2) + b_2d(a_1 - c_1)}{2(4b_1b_2 - d^2)}.^{17} \tag{5}$$

Substituting these prices into the demand functions yields the resulting equilibrium outputs, with the constant term in the denominator now reintroduced:

$$x_1 = \frac{b_2(2b_1b_2 - d^2)(a_1 - c_1) - b_1b_2d(a_2 - c_2)}{2(4b_1b_2 - d^2)(b_1b_2 - d^2)}, \text{and}$$

$$x_2 = \frac{b_1(2b_1b_2 - d^2)(a_2 - c_2) - b_1b_2d(a_1 - c_1)}{2(4b_1b_2 - d^2)(b_1b_2 - d^2)}. \tag{6}$$

3.3.2 Market Share Discounts

In this setting, M1 will choose a market share, s^*, such that if $\frac{x_1}{x_1+x_2} \geq s^*$, then M1 will sell to the retailer at a reduced wholesale price. By ensuring that M1 will obtain

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[17] The Nash equilibrium prices do not assume an interior solution. Consider the full solution to the retailer's pricing problem, *supra* note 16. In the third listed solution, M2 will receive a payoff of zero. The retail will choose these prices whenever $d(a_1 - r_1) \geq b_1(a_2 - r_2)$. Because M2 will receive a positive payoff if $d(a_1 - r_1) < b_1(a_2 - r_2)$, it will lower its price and move into the interior solution. The same logic applies to M1 in the fourth region. As such, wholesale pricing that leads to a corner solution for the retailer will not occur in equilibrium.

a specific market share of the retailer's business, the retailer will receive a discount on the units of product 1 that it sells. Market share discounts are often implemented with a rebate that is returned to the retailer if the market share condition is satisfied over some time period. The market share discount is viewed as an offer by M1 to the retailer. The retailer can choose to either accept the offer and participate in the market share discount plan of M1 or reject the offer and earn the Nash equilibrium profits given in the previous section. Instead of defining a discount off of a list price, this model focuses directly on the wholesale price of M1 assuming that the retailer reaches the market share threshold.[18]

Since the retailer may reject the offer, the manufacturer will choose the price and market share that maximize its profits while ensuring that the retailer makes at least as much profit as in the Nash equilibrium. M1 must then choose the price and share to induce the retailer to participate.

The game played by the firms is:

1) M1 sets $s*$,
2) M1 and M2 choose r_1 and r_2, and
3) Retailer chooses p_1 and p_2.

Wholesale pricing is modeled as a simultaneous game given the market share threshold, but we will see that the equilibrium outcome will not be sensitive to the timing of the wholesale pricing decision. As long as the M2 chooses $r_2$after M1 has chosen $s*$, M2's reaction function, choosing the optimal r_2 as a function of r_1, will be the same for any alternative timing of M1's and M2's pricing decisions. Solving for equilibrium through backward induction, the retailer will choose p_1 and p_2 in the final stage of the game.

[18] Note that the list price would be very high (a_1 or above) with only one retailer. With multiple retailers, the list price would be defined by the prices charged to other smaller retailers.

3.3.2.1 Retail Pricing

There are two possible interior solutions to the retailer's profit maximization problem. First, wholesale prices and the market share threshold might be such that the retailer's optimal prices satisfy the market share threshold without any adjustment in the retail prices. In this case, the retailer may set prices such that the market share constraint is not binding ($s>s*$, where s is the actual market share).[19] For computational simplicity, define k such that $(a_2 - p_2) = k(a_1 - p_1)$. Then,

$$k = \frac{(1-s)b_2 + sd}{sb_1 + (1-s)d}. \tag{7}$$

The value k ensures that the retail prices are such that the market shares satisfy the requirements of the market share discount plan offered by M1.

The optimal prices for the retailer are then:

$$(a_i - p_i) = \frac{(a_i - r_i)}{2}\ . \tag{8}$$

The retailer will set these prices whenever $(a_2 - r_2) \leq k \cdot (a_1 - r_1)$.

Conversely, if $(a_2 - r_2) \geq k \cdot (a_1 - r_1)$, then the above prices will not satisfy the market share constraint. In this case, the retailer will have to adjust the retail prices in order to ensure that the market share constraint is satisfied. The optimal prices for the retailer will satisfy:

$$(a_1 - p_1) = \frac{(a_1 - r_1)\cdot(b_2 - dk) + (a_2 - r_2)\cdot(b_1 k - d)}{2[(b_2 - dk) + k\cdot(b_1 k - d)]}, \tag{9}$$

$$\text{and } (a_2 - p_2) = k \cdot (a_1 - p_1). \tag{10}$$

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[19] The term "market share constraint" refers to the inequality that the retailer must satisfy in order to receive the market share discount. The term "market share threshold" refers to the value of $s*$which must be reached in order to receive a discount.

In addition, there is one possible corner solution that the retail may choose.[20] If $d(a_1 - r_1) \geq b_1(a_2 - r_2)\ and\ 0 \leq d(a_2 - r_2) < b_2(a_1 - r_1)$, then the retailer will choose prices such that:

$$(a_1 - p_1) = \frac{(a_1 - r_1)}{2}, \tag{11}$$

$$\text{and } (a_2 - p_2) = \frac{d(a_1 - r_1)}{2b_1}. \tag{12}$$

With these prices, x_2=0, and the market share constraint will always be satisfied. Together equations (8) - (12) will characterize the optimal retail prices given the wholesale prices.

Note that when $(a_2 - r_2) = k \cdot (a_1 - r_1)$ the two retail prices given by (8) and the retail prices given by (9) and (10) are the same. So when this condition is satisfied, the market share discount constraint will bind with the monopolist markups.

3.3.2.2 Wholesale Pricing

First, consider the profit maximization problem for M2 assuming that s* < 1. There are two possible cases. In the first case, the wholesale prices are such that the retailer simply charges the monopoly markup on each good and the market share constraint is satisfied. In this case, the market share constraint is not necessarily binding. In the second case, the usual monopoly markup on each good does not satisfy the market share constraint, so the retailer adjusts prices in order to satisfy the constraint. In this case, the constraint will bind.

<u>Case 1: Market Share discount constraint does not necessarily bind.</u> $[(a_2 - r_2) \leq k \cdot (a_1 - r_1)]$

[20] It is assumed that M1 will never choose a market share threshold where s*=0, so we will ignore the corner solution that results in x_1=0.

From section I, in this case the retailer will set prices such that:

$$(a_i - p_i) = \frac{(a_i - r_i)}{2}.$$

These are the retail prices that maximize profit for the retailer absent a market share discount plan. Since the market share constraint is satisfied with these prices, the profit maximization problems for each firm will be exactly the same as when no market share discount plan is offered. M2's best response to a given r_1 is:

$$(a_2 - r_2) = \frac{(a_2 - c_2)}{2} + \frac{d}{2b_1} \cdot (a_1 - r_1). \tag{13}$$

Given the second order conditions of M2's profit maximization problem, for any r_2 greater than the value defined by the above best response function, profit is strictly decreasing in r_2. So, when the value of r_2 that makes the market share constraint bind is greater than the value characterized above, it will always be the case that, given a market share constraint, M2 will choose its price such that:

$$(a_2 - r_2) = k \cdot (a_1 - r_1). \tag{14}$$

Essentially, M2's profit maximization problem has a unique maximum for a given r_1. In this case, it is assumed that $(a_2 - r_2) \leq k \cdot (a_1 - r_1)$. If the lowest value of r_2 that satisfies this inequality is greater than the best response value given in equation (13), then within Case 1 M2's profit is strictly decreasing in r_2. As such, M2 prefers setting price such that $(a_2 - r_2) = k \cdot (a_1 - r_1)$ to setting any higher price. This will be true when:

$$(a_1 - r_1) \cdot \left[k - \frac{d}{2b_2}\right] < \frac{(a_2 - c_2)}{2} \tag{15}$$

<u>Case 2: Market Share discount constraint necessarily binds.</u> $[(a_2 - r_2) \geq k \cdot (a_1 - r_1)]$

With $(a_2 - r_2) \geq k \cdot (a_1 - r_1)$, the monopolist markup will not necessarily satisfy the market share constraint. As the constraint forces the retailer farther away from the monopolist markup, retailer profit will fall. As such, the retailer will set prices such that the constraint is satisfied with equality. Thus, it will always be the case that $(a_2 - p_2) = k \cdot (a_1 - p_1)$. Then the retailer will set prices such that:

$$(a_1 - p_1) = \frac{(a_1 - r_1) \cdot (b_2 - dk) + (a_2 - r_2) \cdot (b_1 k - d)}{2[(b_2 - dk) + k \cdot (b_1 k - d)]} \tag{16}$$

Now, given r_1, M2's best response function is:

$$(a_2 - r_2) = \frac{(a_2 - c_2)}{2} - \frac{(b_2 - dk)}{2(b_1 k - d)} \cdot (a_1 - r_1) \tag{17}$$

Given the second order conditions of M2's profit maximization problem, for any r_2 less than the best response value, profit is strictly increasing in r_2. So when the value of r_2 that would make the market share constraint exactly bind is lower than the best response value, M2 will prefer the highest price possible such that the constraint exactly binds. That is, M2 will choose price such that:

$$(a_2 - r_2) = k \cdot (a_1 - r_1). \tag{18}$$

As in Case 1, M2's profit maximization problem has a unique maximum for a given r_1. In this case, it is assumed that $(a_2 - r_2) \geq k \cdot (a_1 - r_1)$. If the highest value of r_2 that satisfies this inequality is lower than the best response value given in equation (17), then within Case 2 M2's profit is strictly increasing in r_2. As such, M2 prefers setting price such that $(a_2 - r_2) = k \cdot (a_1 - r_1)$ to setting any lower price. This will be true when:

$$(a - r_1) \cdot \left[k + \frac{(b_2 - dk)}{2(b_1 k - d)}\right] > \frac{(a_2 - c_2)}{2} \tag{19}$$

If conditions (15) and (19) are both met, then the best response function for M2 in the wholesale pricing game is to choose r_2 such that $(a_2 - r_2) = k \cdot (a_1 - r_1)$, whenever

s*<1. It will be established below that these conditions are satisfied in equilibrium. Note that this best response function for M2 is built solely around two factors. First, M1 has already set $s*$. Second, M2 knows the retailer's profit maximizing markups. M2 will always know the retailer's profit maximizing markups. So, whenever $s*$ is set prior to the wholesale pricing game, M2's best response will be $(a_2 - r_2) = k \cdot (a_1 - r_1)$.

To ensure that the retailer will accept the market share discount plan, M1 must choose $r_1(r_2, k)$ such that the retailer's profit under the MSD plan is at least as large as the retailer's profit in Nash equilibrium. The profits to the retailer with the market shares discount plan are:

$$\pi_{CR} = \frac{(a_1 - r_1)^2(b_2 - dk) + (a_1 - r_1)(a_2 - r_2)(b_1 k - d)}{4(b_1 b_2 - d^2)} \tag{20}$$

M1 will choose $r_1$such that the above profits are equal to the retailer's profits in the Nash equilibrium without market share discounts. So, M1's best response function is characterized by:

$$\frac{(a_1 - r_1)^2(b_2 - dk) + (a_1 - r_1)(a_2 - r_2)(b_1 k - d)}{4(b_1 b_2 - d^2)} = \pi_{CR}^{Nash\ Equilibrium} \tag{21}$$

Given a value for k, the above equation will determine M1's best response function in the wholesale pricing game. Taken together, equations 18 and 21 characterize the interior solution to the wholesale pricing portion of the game. The solutions are:

$$(a_1 - r_1) = \sqrt{\frac{4(b_1 b_2 - d^2)\pi_{CR}^{Nash\ Equilibrium}}{(b_2 - dk) + k(b_1 k - d)}},$$

$$(a_2 - r_2) = k \cdot \sqrt{\frac{4(b_1 b_2 - d^2)\pi_{CR}^{Nash\ Equilibrium}}{(b_2 - dk) + k(b_1 k - d)}} \tag{22}$$

Then the corresponding retail prices are:

$$(a_1 - p_1) = \sqrt{\frac{(b_1 b_2 - d^2)\pi_{CR}^{Nash\ Equilibrium}}{(b_2 - dk) + k(b_1 k - d)}},$$

$$(a_2 - p_2) = k \cdot \sqrt{\frac{(b_1 b_2 - d^2)\pi_{CR}^{Nash\ Equilibrium}}{(b_2 - dk) + k(b_1 k - d)}} \tag{23}$$

The above expressions give the wholesale pricing equilibrium if $s^*<1$. If $s^*=1$, then M1's profit maximization problem remains the same. It must ensure that the retailer earns at least as much profit as in the Nash equilibrium outcome. For M2, profits will be zero regardless of its choice of prices. Therefore any response, including the response function in equation 18, is a best response for M2 when s*=1. So, let the prices in equation 22 characterize equilibrium in the wholesale pricing game.

3.3.2.3 Market Share Threshold

The retail prices will determine the demand for product 1 and product 2. Together with the wholesale prices, these demand functions will define M1's profit as a function of *k*. Given the wholesale prices, M1's profit function is strictly decreasing in *k*. As *k* decreases, *s* increases, and M1's profit is maximized when $s = 1$ and $k = \frac{d}{b_1}$.[21]

In order for this to be an equilibrium, conditions (15) and (19) must both be satisfied. At $k = \frac{d}{b_1}$, condition (19) is always satisfied. Plugging this value of *k* into (15), it will also be satisfied when

$$\frac{(a_2 - c_2)}{(a_1 - c_1)} > \frac{d(2b_2 - b_1)}{2b_1 b_2 - d^2}.$$

[21] This result holds even with different timing in the wholesale pricing stage. Whether the firms move simultaneously or either firm moves first, so long as the market share threshold is already set, the wholesale pricing equilibrium will lead to full exclusion in the market share threshold stage of the pricing game.

When this condition is satisfied, then the equilibrium is for M1 to completely exclude M2 from the market. When this condition is not satisfied, it implies that M2's optimal market share is less than 0. Therefore, the condition will always be satisfied and the above prices, given by (22) and (23), represent the equilibrium of this game. This indicates that the equilibrium of this game is for M1 to completely exclude M2 from the market by setting a market share threshold of $s^*=1$.

Full exclusion can occur for a few reasons. First, for a given r_1, as *k* decreases (and *s* increases), M2 responds by increasing its price r_2. In most pricing games, as one firm decreases price, competitors will decrease their own prices. Within the market share threshold, the best response of M2 in the wholesale pricing game is reversed. Now, when M1 increases s^*, *k* decreases and M2 increases r_2. In order to satisfy the profit constraint, M1 will have to reduce its wholesale price as s^* increases. So, M2 essentially responds to a decrease in the wholesale price of product 1 by increasing the wholesale price of product 2. Because of this price response, there is less of a constraint on M1 with respect to wholesale pricing. Normally M1 would have to lower its wholesale price in order to increase its market share. Now as M1 increases the market share threshold, M2 increases its own wholesale price. This reduces the amount that M1 must reduce its wholesale price and will increase the incentive for a higher market share threshold.

Normally we would think of exclusion as not profitable or inefficient. In order to exclude a competitor from the market, a firm may need to sacrifice profits in the short run. However, if exclusion arises from an inefficient scenario, it may be profitable. In this case, the inefficiency generated by the double marginalization reduces the joint profits of the retailer and M1 enough that by reducing the double marginalization, they can increase their joint profits. As long as the joint profits of the retailer and M1 are greater under exclusion than under the Nash equilibrium without market share discounts, then the exclusion result is possible.

Next, exclusion is possible because of the timing of the game. M1 sets s^* before M2 sets r_2. However, if M2 could set or commit to r_2 in advance of the setting of s^*, then exclusion may not be optimal. One option that is not available to M2 in the above timing is setting a price such that M1 prefers an interior solution. With a low enough r_2, M1 will prefer that the retailer continue to sell some positive quantity of product 2. As the retailer earns some profit on the resale of product 2, M1 does not need to reduce its price as much to ensure that the retailer accepts the market share discount plan.

In the above game, because M2 acts after s^* is set, when s^* is set to 1, M2's actions are irrelevant. Consider the version of the game where M2 can commit to a price (possibly through a long term contract) as a defense to the imposition of a MSD regime. In that setting, M2 may be able to choose a wholesale price such that exclusion is not optimal for M1. We will next consider one such scenario.

3.4 Market Share Discounts as Vertical Integration

Now, let M2 choose a wholesale price prior to the offer of a market share discount plan. If M2 can anticipate how M1 will react to a price, it may be the case that exclusion no longer results in equilibrium. The market share threshold will determine the relative prices of products 1 and 2 in the retail market. Without exclusion, M2's decision in the wholesale pricing game can determine how many total units are produced and sold to consumers. As a result, M2 will indirectly determine the overall size of the market when it chooses its wholesale price. Given the wholesale price of M2, M1 will maximize profits by choosing the prices and threshold that maximize the joint profits of itself and the retailer. Essentially, the market share discount model is identical to a vertically integrated model where the firms that are offering and receiving the discounts have effectively merged.

In the vertically integrated model, the discounted wholesale price will serve merely as an internal transfer for the firm. M1 will choose this discounted price so that it guarantees that the retailer earns the same profits with the market share discount plan as it earns in the Nash equilibrium without the discounts. M1 will also choose retail prices to maximize its joint profit with the retailer. M1 will then ensure that the retailer accepts the offer and that its profits are maximized under the market share discount plan.

By itself, a market share discount plan does not allow M1 to fully implement the vertically integrated solution. It will only allow M1 to define the ratio of p_1 to p_2. Another control, such as a lump sum transfer, is needed in order to fully implement the vertically integrated solution. This additional control is implicitly defined by the difference between the high pre-discount wholesale price for product 1 and the lower post-discount price. In practice, this discount is typically credited to retailers as a rebate or as a credit once the market share threshold is reached.[22] This additional control will allow M1 to ensure that the vertically integrated outcome is achieved. M1 cannot earn a higher profit in the original model than it can with the model in which M1 effectively integrates with the retailer. If M1 has the controls at its disposal to implement the higher

[22] In practice, manufacturers also use marketing expenditures and product support as additional controls to ensure that market share thresholds are reached. Some of these controls are described in the complaint against Intel.

profit solution, then it will prefer this solution to the exclusion outcome arrived at in the prior section.[23]

This scenario may seem to be an overly stylized example, but it may also be the most likely timing of the game to occur. Market share discount plans are not offered in a vacuum or in an abstract game starting at period 0. They are offered in dynamic markets where large companies often contract with each other over long periods of time. As such, it may often be the case that prior to the offer of a market share discount by M1, M2 will have already chosen its wholesale prices. In addition, the constructive vertical integration solution will always offer the greatest possible profit to M1. It is reasonable to expect this solution to also solve problem of choosing the optimal market share discount plan. So, while this is a stylized model, it may be that this model better characterizes firm behavior than the above model

M1 will choose a market share discount plan that solves the joint profit maximization problem:

$$\max_{p_1,p_2} \ \pi_1 + \pi_{CR} =$$

$$[(a_1 - c_1) - (a_1 - p_1)] \cdot [b_2(a_1 - p_1) - d(a_2 - p_2)]$$
$$+[(a_2 - r_2) - (a_2 - p_2)] \cdot [b_1(a_2 - p_2) - d(a_1 - p_1)]. \tag{24}$$

Substituting in for p_2, the problem becomes:

$$\max_{p_1,k} \ \pi_1 + \pi_{CR} = [(a_1 - c_1) - (a_1 - p_1)] \cdot (a_1 - p_1) \cdot (b_2 - dk)$$
$$+[(a_2 - r_2) - k(a_1 - p_1)] \cdot (a_1 - p_1) \cdot (b_1 k - d). \tag{25}$$

[23] It is worth noting that while market share discounts can allow M1 to arrive at the vertical integration solution, there could be other mechanisms that will arrive at this same solution.

Profits are then maximized where $k = \frac{(a_2 - r_2)}{(a_1 - c_1)}$ and $(a_1 - p_1) = \frac{(a_1 - c_1)}{2}$. The retail price for product 2 is then $(a_2 - p_2) = \frac{(a_2 - r_2)}{2}$, with the usual monopoly margin.[24]

Derived demand for product 2 is:

$$x_2(r_2) = \frac{1}{2}[b_1(a_2 - r_2) - d(a_1 - c_1)]. \tag{26}$$

M2 will then choose r_2 to maximize:

$$\pi_2 = \frac{1}{2}[(a_2 - c_2) - (a_2 - r_2)][b_1(a_2 - r_2) - d(a_1 - c_1)]. \tag{27}$$

The resulting profit maximizing price for M2 is defined by:

$$(a_2 - r_2) = \frac{(a_2 - c_2)}{2} + \frac{d(a_1 - c_1)}{2b_1}. \tag{28}$$

This will lead to the equilibrium solution to the market share discount problem with retail prices and quantities:

$$(a_1 - p_1) = \frac{(a_1 - c_1)}{2},$$

$$(a_2 - p_2) = \frac{(a_2 - c_2)}{4} + \frac{d(a_1 - c_1)}{4b_1},$$

$$x_1 = \frac{(a_1 - c_1)(2b_1b_2 - d^2)}{4b_1(b_1b_2 - d^2)} - \frac{d(a_2 - c_2)}{4(b_1b_2 - d^2)},$$

$$x_2 = \frac{b_1(a_2 - c_2) - d(a_1 - c_1)}{4(b_1b_2 - d^2)},$$

$$k = \frac{b_1(a_2 - c_2) + d(a_1 - c_1)}{2b_1(a_1 - c_1)},$$

[24] Because the retailer and M1 maximize profits jointly, the market share threshold is implicitly determined in the retail pricing stage. As such, the order of the profit maximization decisions is different from the order in the simple model given earlier where M2 can adjust r_2 after the market share threshold has been set. That alternative model results in full exclusion of product 2 from the market and zero profits for M2. Therefore, even if the above model allows M2 to adjust r_2 after s^* has been set, M2 will not want to change this price and M1 and the retailer will know that M2 will not choose to change its price.

and

$$s = \frac{(a_1 - c_1)(2b_1b_2 - d^2) - b_1d(a_2 - c_2)}{(a_1 - c_1)(2b_1b_2 - d^2) - b_1d(a_1 - c_1) + b_1(a_2 - c_2)(b_1 - d)}.$$

(29)

Note that for x_2 to be positive and for s to be less than 1, we need the following condition on the parameters to be satisfied:

$$\frac{b_1}{d} > \frac{(a_1 - c_1)}{(a_2 - c_2)}.$$

(30)

This condition requires that the degree of product differentiation is large enough relative to the cost difference between the firms that consumers will still demand units of product 2 at the optimal prices for M1. When this condition fails, M1 will use the market share discount plan to force M2 to exit from the market. If the condition is satisfied the result allows a direct comparison of the welfare results before and after the imposition of the market share discount plan. The comparison that follows will focus on the more interesting case when the condition is satisfied.

Relative to the Nash equilibrium, a few issues are of interest. First, we examine what happens to prices. In theory, the retailer can increase the market share of the dominant firm by lowering the price of the dominant firm's output, or by increasing the price of the other firm's output. Second, it is interesting to identify when the optimal contract tends towards exclusion versus allowing the second firm to remain in the market. Finally, the effect of the plan on profits for the dominant firm should be addressed. If the plan is, in and of itself, profit increasing, then it should not be the case that it involves purely predatory pricing.[25]

A few key results are clear. First, the retail price of each product falls. As a result, consumer welfare will unambiguously increase. Fundamentally, this addresses the question of how the desired market share is obtained. The margin over the wholesale price for product 2 remains the same before and after the market share discount plan. As a result, the desired market share is obtained with a decrease in the retail price of product 1. Alternatively, the price of product 1 falls because the implicit vertical integration of the market share discount plan allows M1 to eliminate the double marginalization on the price of product 1 that occurs in the non-integrated setting. In addition, M2's strategic response to the decrease in price of product 1 is to decrease its own price. Together these two factors ensure that prices fall and consumer welfare increases once the market share discount regime begins.

Second, the optimal market share discount plan may or may not exclude the non-dominant firm from the market. Exclusion occurs when the degree of product differentiation is small relative to the differences in costs for the firms. When the products are more differentiated exclusion will not occur. An increase in the price of product 2 will decrease the quantity demanded for product 2 and increase the demand for product 1. When the products are less related, the negative effect of increasing the retail price of product 2 becomes relatively more significant. For each lost sale of product 2,

[25] Predatory pricing typically assumes that the firm offering a low price is doing so with the intention of driving a competitor from the market. However, if the low price is profit increasing, then the firm will pursue the lower price without regard for its effects on competing firms. Any price decrease will harm a firm's competitors. Only price decreases which also do not help the price cutter in the short term should be subject to antitrust scrutiny.

the dominant manufacturer will have to compensate the retailer with a lower wholesale price. This will be necessary to ensure that the retailer accepts the discount plan.

When the two firms have similar costs, exclusion is also less likely. With similar costs, it is not profitable to exclude the competing product from the market in terms of lost resale profit for the retailer. Because the dominant manufacturer wants to maximize the joint profits of itself and the retailer, it will often want to earn some profits through the resale of product 2. As such, in many situations exclusion will not be optimal for the dominant manufacturer.

In this model M2 is allowed to adjust its price response to the market share discount plan of M1. However, we do not allow M2 to offer a competing market share discount plan. If either firm is able to offer a market share discount plan, each would choose a market share threshold at least as large as its market share in the Nash equilibrium. As a result, the retailer can only accept one of the offers and can only receive a discount on one of the two products. Because of this limited choice, the retailer will choose the plan that offers it the highest profits. Since M1 is assumed to have lower costs of production and stronger demand for its product, M1 will be able to offer the plan that is preferred by the retailer in every case. Since the optimal plan will incorporate the results of the vertically integrated problem, retail prices and the consumer surplus will remain unchanged whether or not firm 2 can offer a plan. The only outcome that will change is the internal transfer in the vertical model.

This internal transfer will only have an impact on the result in the retail market if it causes M1's profits to fall enough that M1 would prefer not to offer any market share discount plan at all. However, this would imply that M2 has offered a plan such that M1 would rather operate under M2's plan's regime than exist under its optimal competing plan. Since M1's set of responses to M2's plan can include offering a plan that yields the exact same market share and prices as M2's plan, it must be that the optimal plan offers M1 at least as much profit as its best response given that the retailer has accepted M2's plan. Therefore, M1 will always choose to offer a market share discount plan.

In addition, while the ability of M2 to offer a competing plan may change the allocation of total profits between M1 and the retailer, the total profits of the two will always be maximized and the market result will remain unchanged. Effectively M2 can offer a plan that will reduce M1's profits, but will never have an effect on its own profits or the market outcome. This is because M2's plan will never be accepted in equilibrium. Every plan offered by M1 will solve the joint maximization problem of M1 and the retailer. Only the internal transfer between M1 and the retailer will change when M2 offers a plan. Both manufacturers will produce the same output level and retail prices will not change. Therefore M2's profits and consumer welfare will remain the same in equilibrium regardless of whether M2 offers a plan or not.

Since the introduction of a competing plan does not impact consumer welfare, we do not need to separately consider the case where both manufacturers offer competing plans. Instead, the Bertrand-Nash equilibrium should be used as the basis for comparison in the analysis of the market share discount plan. In the event that the dominant manufacturer cannot institute a market share discount plan, the Bertrand-Nash equilibrium will give the outcome of the game between the two manufacturers.

Much of the legal analysis of market share discounts focuses on the pre-discount prices that the dominant firm charges instead of the equilibrium outcome. Focusing on the pre-discount price is a fundamental error in this analysis. The dominant firm will have a strong incentive to increase the pre-discount price as much as possible in order to entice the retailer to accept the discount. However, this high pre-discounted price is a non-credible threat. Given that a retailer refuses a market share discount agreement, it will not be optimal for the dominant firm to maintain the high pre-discount price. In fact, the outcome after the offer has been refused should be the Bertrand-Nash equilibrium, and in any subgame perfect equilibrium, the Bertrand-Nash equilibrium should be played in the event the market share discount offer is not accepted by the retailer.

3.5 Analysis

The models presented here are very simple. In the games, both firms have complete information and linear differentiated demand curves. The advantage of such models is that they are computationally straightforward and deliver clear results. Their primary purpose is to offer some insight into the dynamics that occur in the implementation of such plans. In the simple application of market share discounts, complete exclusion occurs. When market share discounts are modeled as a tool for creating a constructive vertical integration, exclusion is still possible, but will not always occur. In practice, market share discount plans rarely fully exclude smaller manufacturers. This result could be an attempt by dominant firms to avoid antitrust scrutiny, but it could also be the case that exclusion is not optimal. In the vertically integrated model, M2 can set a price in anticipation of a market share discount plan by entering into a long term contract with the retailer. The ability to effective act first in the market share discount game allows M2 to protect itself against full exclusion.

In the vertical integration result, market share discounts do not necessarily drive the equilibrium outcome. Any mechanism that allows for vertical integration will yield the same result. However, because of the exclusionary effect that occurs with simultaneous wholesale pricing after the market share discount plan, M2 now has the incentive to force M1 into this game. In an indirect way, market share discounts can act to endogenize the timing of the game played by M1 and M2. Because firms have tools at their disposal (long term contracts) which can change the timing of the pricing decisions, M2 can defend against exclusionary market share discounts. It is only natural that they will do so. While the outcome may not be unique to market share discounts, the vertically integrated outcome, with M2 acting first, may be the most likely outcome. In other words, this outcome is not the equilibrium of the game primarily because of M1's optimization. Instead, it results from M2's defensive actions.[26]

The most significant of the results in the vertical integration game, that retail prices fall and that the profit for the dominant firm rises, are likely to arise in a variety of demand structures. In any demand structure, the dominant firm will want to maximize the joint profits of itself and the retailer. Because the dominant firm does not need to - increase the expected profits of the retailer beyond some trivial amount to ensure compliance with the plan, then the plan that maximizes joint profit will also maximize profit for the dominant manufacturer. As such, market share discount plans should always increase profits for the dominant manufacturer, regardless of the demand specification.

[26] It is reasonable to anticipate that M1 can implement full exclusion once M2's contracts expire. This will give M2 an incentive to enter into longer contracts than would otherwise be optimal. The defense contracts could generate inefficiencies themselves, and could be an area of interest to antitrust regulators.

Because market share discounts allow for the implementation of a constructive vertical integration by M1, the retail price of product 1 should always fall as M1 eliminates the double marginalization that occurs in the Bertrand-Nash equilibrium. M2 will then have a strategic incentive to reduce the wholesale price for product 2 in most demand settings. In situations such as the model presented here, where consumers are identical, M2 will generally choose to cut its price in anticipation of the market share discount offered by M1. Because M1 is implementing a plan that maximizes joint profits, the retail margin on product 2 remains unchanged, so the retail price of product 2 falls.

One can envision situations where M2 may desire to raise its wholesale price in response to a market share discount plan offered to the retailer by M1. For example, if there are different types of consumers and only a subset with highly inelastic demand will purchase product 2 after the price of product 1 falls, then it could be optimal for firm 2 to increase its price in response to the market share discount plan. Consider the case of two brands of a similar differentiated product, brand A and brand B, where consumers fall into three categories. Each product has partisans who only consume their preferred brand with highly inelastic demand. In addition, there is a third set of consumers who are relatively indifferent between the two brands and choose which brand to purchase based on prices. If brand A can offer a market share discount that will reduce price enough to capture all or most of the relatively indifferent consumers, then brand B may be left facing a substantially inelastic residual demand curve and may choose to increase its price in response to the market share discount plan. Since the model here does not allow for such results, a logical extension of this paper is to consider the use of market share discounts with heterogeneous consumers.

The welfare impact of market share discount plans should be decomposable into two competing effects. First, there will be a positive price effect for the product produced by the dominant firm. Even with the retailer placed in a more competitive environment, the dominant firm will always want to offer a market share discount plan that maximizes the joint profit of the retailer and the manufacturer. At the limit, with a perfectly competitive retailer, this effect may be zero but it will never be negative. The manufacturer will never choose a plan that increases the price of its product beyond the price that occurs in the Nash equilibrium absent the discount plan.

The second effect can be positive or negative. As the market share threshold of the dominant firm increases, the other manufacturer will be excluded from an increasing portion of the market. In the vertical integration model, the second firm reduces its price in response to the plan. As such, even the consumers who prefer product 2 in Nash equilibrium and switch to product 1 after the market share discount plan will not suffer. They are still given the option of purchasing product 2, and the price of product 2 is lower. Therefore, it must be that they receive more surplus from purchasing product 1 than from purchasing product 2. Because the price of product 2 has fallen, they will necessarily receive more surplus from purchasing product 1 at the discounted price than they would from purchasing product 2 at the Nash equilibrium price.

If competing manufacturers are driven from the market entirely, or if they increase their prices in response to market share discount plans, then the exclusion effect on consumer welfare may be negative. Some consumers will purchase product 1 at the discounted price even though they will prefer product 2 at the Nash equilibrium price. In addition, some consumers who may prefer product 2 will not make any purchases at all. Either of these cases will cause a decrease in surplus to a subset of consumers.

Likewise, if constant marginal costs are relaxed, then the exclusion effect could lead to an increase in price for product 2. The introduction of a market share discount plan for product 1 will decrease the output of product 2 in equilibrium. If marginal costs are decreasing over a large enough range of output, then for M2 the marginal cost of expanding output under the market share discount plan will be greater than the marginal cost in the Bertrand-Nash equilibrium. All else equal, this effect will cause M2 to increase the wholesale price of product 2. If the increase in marginal costs is significant enough, this effect will dominate any strategic incentive to cut the wholesale price of product 2 in response to a decrease in the wholesale price of product 1. As such, when marginal costs are not constant, it is possible that the use of a market share discount plan by M1 will lead to an increase in the wholesale and retail price of product 2.

3.6 Concluding Remarks

The models presented here suggest that market share discount plans may offer some benefit to manufacturers and consumers. Manufacturers can use market share discounts to eliminate double marginalization and increase their profits. Consumers may benefit from the elimination of double marginalization and the strategic response by other manufacturers, both of which will decrease retail prices.

The next logical step in this line of research should be to further examine the impact of market share discounts on consumer welfare. The welfare improvement that is observed here is dependent upon the structure of the model. If the retailer has less market power, the benefits of eliminating the double marginalization will be mitigated. In addition, it could be the case that for certain demand specifications competing manufacturers will increase their prices in response to a market share discount plan.

Market share discount plans are the subject of litigation around the world and much of the analysis of the plans has grown out of these cases. In the model presented here, the plans can have a positive impact on consumer welfare and on the profits for the dominant manufacturer. This would support the idea that there plans can be pro-competitive and should, at the very least, be subject to a rule of reason analysis as to their legality. Since the plans increase the profits for the firms offering them, they should not be considered as predatory pricing, even though some marginal prices may be negative. The ultimate evaluation of any plan should depend on applying the facts of a given case to determining whether the exclusion effect on consumer welfare is positive or negative, and, if negative, whether or not it dominates the price effect of the plans.

Chapter 4

Service Pricing with Free Buckets and Demand Uncertainty

4.1 Introduction

In the wireless telephone industry it is common for consumers to be offered a pricing plan which takes the form of an increasing-block tariff with access fees. Typically, the consumer will pay an access fee and receive some fixed number of units of usage at a very low price, often zero. Once the consumer uses units beyond this fixed amount, the per unit usage price increases sharply. Perry (2005) calls into question the performance of such plans both in terms of profit maximization and overall welfare. In his paper, Perry investigates pricing plans with buckets of free units, using wireless calling plans as the motivating example. The analysis compares the results of pricing under various types of plans, finding that pricing with free buckets can result in identical profits for the monopolist, compared to unit pricing or to access pricing. However, he finds that the net surplus for consumers can vary across the plans. Perry then shows that pricing with the free bucket is identical to a two-part tariff. Here, his model is extended to include demand uncertainty for the consumers.

In Perry, there is a distribution of consumer types, but each type has a fixed demand function for usage. In this paper, each consumer has a distribution of demand functions and the type parameterizes the distribution over the range of possible demand functions. Each consumer is uncertain about his exact demand function realization and only knows the distribution function at the time he subscribes to the service. The ultimate goal of this research is to understand the role of multiple free bucket plans. With uncertain consumer demand, multiple free bucket plans may have the advantage of signaling to the consumer when to switch to a plan with a larger bucket of free units. Since the price of units beyond the initial bucket is typically large, consumers with different distributions on their demands will eventually sort themselves into the free bucket plans with different size buckets. As such, the three-part pricing plans with free buckets may better serve the firm and the consumers.

4.2 Overview

We consider the case of a monopolist choosing among alternative non-linear pricing plans. Any pricing plan will consist of an access fee, which allows the consumer the right to use the service, a free bucket of some specified number of units, and a per unit charge for units beyond the free bucket. By setting any two of these three variables equal to zero, we can identify the single dimensional pricing strategy that a monopolist would employ. These pricing plans are well known and serve as the reference strategies for a free bucket plan.

The concern with free bucket pricing is that it may create two types of inefficiencies. If the marginal cost of usage is positive, when consumers have low demand, they will use the service to a point where their valuations for the last units in the bucket are below the marginal cost of the monopolist. This is a deadweight loss from excessive usage. Second, when consumers have high demands, they will purchase units beyond the free bucket at an overage price greater than marginal cost. These consumers will under utilize the service, resulting in the standard deadweight loss.

In addition to the inefficiencies, we examine whether or not free bucket pricing is optimal for the monopolist. We observe free bucket pricing in certain industries, such as telecommunications. However, free bucket pricing is effectively an increasing-block tariff, and the literature on non-linear pricing suggests that this should not be an optimal strategy.[27] In particular, the marginal price in an optimal pricing plan should decline to marginal cost for the largest user. In the next section of this paper, the basic model is presented, providing the structure of consumer demand and a framework analyzing any pricing plan. Then several basic pricing strategies are examined and the resulting prices and profits are compared.

4.3 The Model

Consumer demand, during the period covered by the access fee, is a function of the parameter $\theta \in [0,1]$. The parameter θ measures the extent of demand, with higher θ meaning a higher quantity demanded at every usage price. The consumer does not know the value of θ at the time he or she must purchase the service and pay the monthly charge. The consumer's specific level of demand is realized after purchasing the service. We assume that the demand function is given by the linear specification:

$$x(p;\theta) = \frac{a-p}{b} \cdot \theta \qquad\qquad \theta \in [0,1] \tag{1}$$

p is the usage price of using the service once the consumer has subscribed. The inverse demand function of every consumer is linear with a constant intercept a, but the slope, $\frac{b}{\theta}$, is uncertain. For realizations of θ closer to one, the demand curve will rotate outward with a lower slope.

A consumer of type t will observe a realization of θ from the distribution function $G(\theta,t)= t\cdot\theta + (1-t)\cdot\theta^2$, with the corresponding density function $g(\theta,t)=t + 2(1-t)\cdot\theta$ for $t \in [0,2]$. Consumer types t are distributed uniformly over [0,2]. This particular

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[27] See Wilson, Nonlinear Pricing, pp. 91-96. The optimal n-part tariff usually takes the form of a fixed access fee and n-1different "block declining" marginal usage prices.

distribution function for θ is convenient because it allows for integration of consumer demand and an analytical solution to the monopolist's profit maximization problem. A consumer with higher t will have a lower probability of higher demand at any price where demand is positive. For a consumer with t=1, the distribution of θ is uniform between zero and one. If t=0, the consumer has a high probability of realizing a high value of θ and conversely if t=2.

This demand structure allows for a comparison between the resulting price, profit, and consumer surplus that arise from different pricing strategies. The monopolist has a constant marginal cost c, which may be zero. If the access fee and the size of the free bucket were both set to zero, the monopolist would set the simple monopoly usage price for using the service.

With this demand structure we can calculate the consumer surplus for a consumer of type t. We first specify the model without a free bucket in which the monopolist sets an access fee and a usage price. Let r be the access or subscription fee that a consumer pays for the service, and p be the usage price for the service. The net surplus for a consumer with a realized demand given by θ is

$$ncs(p,r;\theta) = \frac{(a-p)^2}{2b} \cdot \theta \ - r \,. \tag{2}$$

The resulting expected net surplus for a consumer of type t is obtained by integrating over the demand functions that might be realized after the consumer has purchased access:

$$E[ncs(p,r;\theta,t)] = \frac{(a-p)^2}{2b} \cdot \int_0^1 \theta \cdot [t + 2(1-t)\theta] d\theta \ - r \,. \tag{3}$$

Integrating with respect to θ, the expected net surplus is

$$E[ncs(p,r;t)] = \frac{(a-p)^2}{2b} \cdot \left(\frac{2}{3} - \frac{t}{6}\right) - r \,. \tag{4}$$

This expected net surplus is a decreasing function of t because a higher type t corresponds to a higher probability of lower demand.

We can extend this format to three-part tariff pricing plans, which would include one bucket of units with a low usage price, possible zero. Consumers pay an access fee of r and then pay a usage price of p_0 for an initial bucket of s units, called the first bucket. They pay an overage price $p > p_0$ for units beyond s. The variable s represents the fixed size of the first bucket.

For different realizations of θ, we need different expressions for consumer surplus. If θ is less than $\frac{bs}{a-p_0}$, then $x(p_0;\theta) < s$ and at θ the consumer surplus from usage is:

$$cs_1(p_0;t) = \frac{(a-p_0)^2}{2b} \cdot \theta. \tag{5}$$

Integrating with respect to θ, the expected consumer surplus is:

$$E[cs_1(p_0;t)] = \int_0^{\frac{bs}{a-p_0}} \left(\frac{(a-p_0)^2}{2b} \cdot \theta\right) \cdot [t + 2(1-t)\cdot\theta] \cdot d\theta. \tag{6}$$

If θ is greater than $\frac{bs}{a-p_0}$ but less than $\frac{bs}{a-p}$, then $x(p_0;\theta) \geq s > x(p;\theta)$ and at θ the consumer surplus from usage is:

$$cs_2\left(p_0, s; t\right) = (a-p_0)\cdot s - \frac{bs^2}{2\theta}\,. \tag{7}$$

Integrating with respect to θ, the expected consumer surplus is:

$$E[cs_2(p_0,s;t)] = \int_{\frac{bs}{a-p_0}}^{\frac{bs}{a-p}} \left((a-p_0)\cdot s - \frac{bs^2}{2\theta} \right) \cdot [t + 2(1-t)\cdot\theta]\cdot d\theta. \tag{8}$$

Finally, if θ is greater than $\frac{bs}{a-p}$, then $x(p;\theta) > s$ and at θ the consumer surplus from usage is:

$$cs_3(p_0, s, p; t) = s(p-p_0) + \frac{(a-p)^2}{2b}\cdot\theta. \tag{9}$$

Integrating with respect to θ, the expected consumer surplus is:

$$E[cs_3(p_0,s,p;t)] = \int_{\frac{bs}{a-p}}^{1} \left(s(p-p_0) + \frac{(a-p)^2}{2b}\cdot\theta \right)\cdot [t + 2(1-t)\theta]\cdot d\theta. \tag{10}$$

Over all values of θ, the expected net surplus for a consumer of type t is equal to:

$E[cs_1] + E[cs_2] + E[cs_3] - r =$

$$\left(\frac{b^2s^3(1-t)}{3}\right)\cdot\left(\frac{1}{a-p_0} - \frac{1}{a-p}\right) + s\cdot(p-p_0) + \frac{(4-t)\cdot(a-p)^2}{12b} + \left(\frac{bs^2}{2}\right)\cdot \ln\left(\frac{a-p}{a-p_0}\right) - r. \tag{11}$$

A consumer will only purchase the service if the expected net surplus is positive. As such, we can define the marginal consumer as the consumer with an expected net surplus equal to zero. Let $\bar{t}$ denote this consumer. All consumers of type less than $\bar{t}$ will purchase the service, while all consumers of type greater than $\bar{t}$ will not purchase the service. If either p_0=p or s=0, then the three-part tariff collapses into the two-part tariff. Now that the structure of individual demand has been established, we can define the overall demand for access and usage of the service.

4.4 Consumer Demand

The demand for access to the service will simply be the number of subscribers. Because higher type consumers will have a higher expected surplus, all consumers with a type less than the marginal consumer will demand access to the service. As such, we can express the number of subscribers as:

$$N(p_0, s, p, r) = \left[\frac{\bar{t}(p_0, s, p, r)}{2}\right]. \tag{12}$$

To obtain demand for usage by consumers who have purchased access, we integrate the individual expected demands, $x(p;t)$ over the distribution of types. For the two-part tariff, with access fee r and usage price p, the demand for usage is given by:

$$X(p;t) = \frac{a-p}{b} \cdot \left(\frac{\bar{t}}{3} - \frac{\bar{t}^2}{24}\right), \tag{13}$$

where the marginal consumer type $\bar{t}$ is given by $\bar{t} = 4 - \frac{12br}{(a-p)^2}$.

With the three-part tariff, the calculation is more complicated. As before, there are three ranges of demand for every type of consumer who purchases access. With realizations of θ ranging from 0 to $\frac{bs}{a-p_0}$, the consumer will purchase less than s units at price p_0:

$$x(p_0;\theta < \frac{bs}{a-p_0}) = \frac{(a-p_0)}{b} \cdot \theta. \tag{14}$$

Next, over some intermediate range of θ, $[\frac{bs}{a-p_0}, \frac{bs}{a-p}]$, the consumer will purchase exactly s units given prices p_0 and p:

$$x(p_0;\ \frac{bs}{a-p_0}) = x(p;\ \frac{bs}{a-p}) = s. \tag{15}$$

Finally, for higher θ, from $\frac{bs}{a-p}$ to 1, the consumer will purchase more than s units at price p:

$$x(p;\theta > \frac{bs}{a-p}) = \frac{(a-p)}{b} \cdot \theta. \tag{16}$$

The usage demand can then be divided into the demand for two buckets, first the bucket of size s, at usage price p_0, and second the bucket containing all units beyond s at price p. The usage demand for the first bucket is:

$$X_1(p_0, s, r) = s \cdot \left(\frac{\bar{t}}{2} - \frac{\bar{t}^2}{8}\left(\frac{bs}{a-p_0}\right) - \frac{\bar{t}}{6}\left(\frac{bs}{a-p_0}\right)^2 + \frac{\bar{t}^2}{12}\left(\frac{bs}{a-p_0}\right)^2 \right). \tag{17}$$

The usage demand for the second bucket is then

$$X_2(p_0, s, p, r) = \left(\frac{a-p}{b}\right)\left(\frac{\bar{t}}{3} - \frac{\bar{t}^2}{24}\right) - s \cdot \left(\frac{\bar{t}}{2} - \frac{\bar{t}^2}{8}\left(\frac{bs}{a-p_0}\right) - \frac{\bar{t}}{6}\left(\frac{bs}{a-p_0}\right)^2 + \frac{\bar{t}^2}{12}\left(\frac{bs}{a-p_0}\right)^2 \right). \tag{18}$$

For the monopolist, each bucket will have a different revenue stream associated with it. The margin on the first bucket is $p_0 - c$, which may be positive or negative. With a free bucket, $p_0 = 0$, so the monopolist will incur a unit loss of c over the s units in the first bucket. For the second bucket, the monopolist will receive a per unit return of $p - c$, which is positive. Now we will address the optimal pricing of the monopolist, given consumer demand for various pricing plans.

4.5 Usage Pricing

In the simplest reference case, the monopolist charges only a single price *p* for usage. The access fee *r* is set to zero and there are no buckets. With no access fee, all consumers will purchase some units, so $\bar{t} = 2$. The profits to the monopolist are:

$$\Pi_M(p) = (p-c)\cdot X(p) = (p-c)\cdot\left(\frac{a-p}{2b}\right). \tag{19}$$

The profit-maximizing price is

$$p_M = \frac{a+c}{2}. \tag{20}$$

Given the demand structure, the monopoly price is a function of the parameters *a* and *c* but independent of the distribution of consumers. This is a consequence of the linearity of demand and the common intercept *a*. With this price, profits and the net consumer surplus are:

$$\Pi_M = \left(\frac{a-c}{2}\right)^2\left(\frac{1}{2b}\right) = \frac{(a-c)^2}{8b}, \text{ and}$$

$$CS_M = \frac{1}{2}\cdot\Pi_M. \tag{21}$$

This outcome will serve as a benchmark with which to compare the results of other pricing plans.

4.6 Access Pricing

In this reference case, the monopolist will not earn any profits on the usage of the service, but instead derive all of his profits from the access fee *r* for the service. We will address two cases. In the first case, the monopolist will set the price of usage *p* equal to its marginal cost *c*. As such, there are no profits or losses on the usage, and all profits must be earned from the access fee. We assume that there is no marginal cost to the monopolist for providing access and that fixed costs are zero. This is the simplest two-part tariff, and will serve as a reference point for other pricing strategies.

With $p=c$, the profits of the monopolist are:

$$\Pi_{A(p=c)}(r) = r \cdot \left[\frac{\bar{t}(p,r)}{2} \right] \tag{22}$$

where the marginal consumer type $\bar{t}$ is given by $\bar{t} = 4 - \frac{12br}{(a-c)^2}$.

Solving for the optimal access fee, the resulting profit-maximizing access fee and marginal consumer are:

$$r_{A(p=c)} = \frac{(a-c)^2}{6b}, \text{ and}$$

$$\bar{t}_{A(p=c)} = 2. \tag{23}$$

The profits and consumer surplus are:

$$\Pi_{A(p=c)} = \frac{(a-c)^2}{6b}, \text{ and}$$

$$cs_{A(p=c)} = \frac{(a-c)^2}{12b}. \tag{24}$$

Access pricing with $p=c$ generates higher profits for the monopolist than usage pricing and a higher net surplus for consumers. With both usage pricing and access pricing with $p=c$, all consumers purchase access to the service.

In the second case, $p=0$. Now the monopolist must earn sufficient profit from the access fee to cover the cost of the usage by the consumers. This is the case of access pricing with a single unlimited free bucket of usage. With this pricing plan, the profits are:

$$\Pi_{A(p=0)}(r) = r \cdot \left[\frac{\bar{t}(p,r)}{2}\right] - c \cdot \int_0^{\bar{t}} X(p=0;t) \cdot h(t)dt . \tag{25}$$

where the marginal consumer type $\bar{t}$ is given by $\bar{t} = 4 - \frac{12br}{a^2}$. The profit-maximizing access fee and marginal consumer are:

$$r_{A(p=0)} = \frac{a^3}{6b(a-c)}, \text{ and}$$

$$\bar{t}_{A(p=0)} = 4 - \frac{2a}{(a-c)}. \tag{26}$$

The profits and consumer surplus are:

$$\Pi_{A(p=0)} = \frac{a^3 \cdot [a - c(c+2)]}{6b(a-c)^2}, \text{ and}$$

$$cs_{A(p=0)} = \frac{a^2 \cdot (a^2 - 6ac + 4c^2)}{12b(a-c)^2}. \tag{27}$$

For all values of the parameters a, b, and c, access pricing with $p=c$ generates a higher profit and a higher consumer surplus than usage pricing. If $2c+c^2 > a$, then the profits for access pricing with $p=0$ are negative. The profit increases monotonically as c decreases, but for all positive values of c, it is less than the profits from access pricing with $p=c$. With $c=0$, the two pricing plans are identical.

In addition, with $c>0$, the marginal consumer for access pricing with $p=0$ is less than 2. The access fee is higher than in the case where $p=c$ and the consumers with the lowest probability of realizing high demand will choose not to purchase access to the service.

With usage pricing and with access pricing, the monopolist generates higher profit with $p > 0$ than with p=0. Neither the linear pricing nor the two-part tariff allow the monopolist to limit the usage of consumers, and the higher access fee for access pricing with p=0 does not compensate for the negative margin on usage when p=0.

4.7 Access Pricing with Free Bucket

In the simplest three-part tariff setting, the consumer pays price $p_0 = 0$ for the first bucket of size s. For units greater than s the consumer pays a very high price, $p \geq a$, such that no consumer would demand more than s units of usage. This simple three-part tariff will also serve as a reference outcome. Unlike the two-part tariff pricing with $p = 0$, consumers are now limited in the units that they will consume. Thus, with marginal cost $c > 0$, this pricing plan should prove more profitable than access pricing with p=0. The profits from this pricing plan are:

$$\Pi_B = r \cdot \left(\frac{\bar{t}}{2} \right) - c \cdot X_1(r, s, p). \tag{28}$$

The monopolist must now choose the access fee r and the bucket size s. The profit-maximizing bucket size, access fee, and marginal consumer are:

$$s_B = \frac{a - p_0}{b} = \frac{a}{b},$$

$$r_B = \frac{a^3}{6b(a-c)}, \text{ and}$$

$$\bar{t}_B = 4 - \frac{2a}{(a-c)}. \tag{29}$$

The profits and the expected consumer surplus are:

$$\Pi_B = \frac{a^3 \cdot [a - c(c+2)]}{6b(a-c)^2}, \text{ and}$$

$$cs_B = \frac{a^2 \cdot (a^2 - 6ac + 4c^2)}{12b(a-c)^2}. \quad (30)$$

This result is exactly the same as the profit-maximizing two-part tariff with $p=0$. The size of the optimal free bucket is equal to demand at price p_0 for the highest possible realization of θ. As a result, this three-part tariff does not generate any inefficiencies beyond those which occur with the two-part tariff. The consumer would never consume more units than are contained in the free bucket and the profits and the surplus expected from this plan are the same as with the two-part tariff with $p=0$.

4.8 More general two-part tariff

The earlier two-part tariff examples use two particular values for p. If p is allowed to take on any value, we can find the optimal two-part tariff. The monopolist is now free to make an incremental profit or take a loss on the usage of a consumer. The access fee is the expected surplus of the marginal consumer. Again, we assume that there is no marginal cost to the monopolist for providing access and that fixed costs are zero. With this pricing plan, the profits are:

$$\Pi_{2PT}(p,r) = r \cdot \left[\frac{\bar{t}(p,r)}{2}\right] + (p-c) \cdot \int_0^{\bar{t}} X(p;t) \cdot h(t)dt \,. \quad (31)$$

where the marginal consumer type $\bar{t}$ is given by $\bar{t} = 4 - \frac{12br}{(a-p)^2}$. The profit-maximizing access fee, marginal consumer, and usage price are:

$$r_{2PT} = \frac{3(a-c)^2}{32b},$$

$$\bar{t}_{2PT} = 2, \text{and}$$

$$p = \frac{a+3c}{4}. \tag{32}$$

The profits and consumer surplus are:

$$\Pi_{2PT} = \frac{3(a-c)^2}{16b}, \text{and}$$

$$cs_{2PT} = \frac{3(a-c)^2}{64b}. \tag{33}$$

This profit is always greater than the profit for either of the other two candidate two-part tariffs. Even though the expected consumer surplus for the marginal consumer and the access fee are lower than the two-part tariff with $p=c$, the monopolist earns sufficient profit on the usage price to make up this difference. Unlike the two-part tariff with $p=c$, this pricing plan will result in a deadweight loss due to the usage price being greater than marginal cost.

4.9 More general three-part tariff

For a general three-part tariff, we relax the requirement that $p_0=0$ and that $p \geq a$. Instead, we only assume that $p \geq p_0$. With $p \geq p_0$, the monopolist chooses the increasing-block tariff that maximizes profit. The profits are:

$$\Pi_{3PT} = r \cdot \left(\frac{\bar{t}}{2}\right) + (p_0 - c) \cdot X_1(r, s, p_0) + (p - c) \cdot X_2(r, s, p). \tag{34}$$

Now, the monopolist must choose the access fee r, the bucket size s, the usage price for units within the bucket p_0, and the usage price for units beyond the bucket p. The profit-maximizing bucket size, access fee, marginal consumer, bucket price, and overage price are:

$$s_{3PT} = \frac{a-p_0}{b} = \frac{3(a-c)}{4b},$$

$$r_{3pt} = \frac{3(a-c)^2}{32b},$$

$$\bar{t}_{3PT} = 2,$$

$$p_0 = \frac{a+3c}{4}, \text{ and}$$

$$p = a. \tag{35}$$

The profits and the expected consumer surplus are:

$$\Pi_{3PT} = \frac{3(a-c)^2}{16b}, \text{ and}$$

$$cs_{3PT} = \frac{3(a-c)^2}{64b}. \tag{36}$$

The optimal three-part tariff results in the exact same prices, usage, and profits as the optimal two-part tariff.[28] Because $s_B = \frac{a - p_0}{b}$, the overage price does not have to equal a. Consumer demand at price p_0 is less than or equal to s_B for all realizations of θ, so with $p \geq p_0$, no overage units are purchased. As with the optimal two-part tariff, the usage price is greater than marginal cost, so this plan will create a deadweight loss.

The monopolist could eliminate some of this deadweight loss and increase profits by choosing an overage price between the bucket price and the marginal cost.[29] With this lower overage price, the monopolist is able to make some incremental profit on units

[28] The optimal two-part tariff and the optimal three-part tariff are also the same when we allow p to vary and calculate the optimal plan with a free bucket (p_0=0). In this case, the monopolist chooses s=0 and sets the overage price and access fee equal to the usage price and access fee in the optimal two-part tariff.
[29] This is not the case with the free bucket three-part tariff discussed in footnote 2. In that case, the two-part tariff result is duplicated through the overage price, which will not allow for the adjustment to achieve a higher profit.

consumed beyond the bucket and consumers will receive a higher surplus from usage, allowing the monopolist to charge a higher access fee. Therefore, a three-part increasing-block tariff cannot be optimal. In addition, any plan which includes a usage price of zero for the first bucket cannot be optimal.[30]

Logically, the purpose of a pricing plan such as a three-part tariff is to allow the monopolist to price discriminate, charging a higher price to those consumers who value the service more. However, this can be achieved through the two-part tariff and the results demonstrate that the three-part tariff and the two-part tariff yield the same profit for the monopolist. Moreover, the immediately preceding results show that a decreasing block tariff will yield a higher profit than an increasing-block tariff.

4.10 Multiple plans

Now we compare two and three-part tariffs where the monopolist offers a pair of plans that consumers can purchase. With a pair of two-part tariff offered to consumers, profit is:

$$\Pi_{2PT(2)}(p_1,p_2,r_1,r_2) = r_1 \cdot \left[\frac{\bar{t}_1(p_1,p_2,r_1,r_2)}{2}\right] + r_2 \cdot \left[\frac{\bar{t}_2(p_1,p_2,r_1,r_2) - \bar{t}_1(p_1,p_2,r_1,r_2)}{2}\right] + (p_1 - c)\cdot\int_0^{\bar{t}_1} X(p_1;t)\cdot h(t)dt + (p_2 - c)\cdot\int_{\bar{t}_1}^{\bar{t}_2} X(p_2;t)\cdot h(t)dt\,. \tag{37}$$

The monopolist offers plans 1 and 2 which have access fees and usage prices r_1 and p_1 and r_2 and p_2, respectively. The marginal consumer for the second plan, $\bar{t}_2$, is indifferent between purchasing plan 2 and not purchasing access at all. Similarly, the marginal

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[30] A pricing plan with a usage price of zero for the first bucket must fall into one of two categories. It could be access pricing with p=0 or it could be an increasing-block tariff. Neither of these pricing plans is optimal for the monopolist.

consumer for plan 1, $\bar{t}_1$, is indifferent between purchasing plan 1 and plan 2. Each consumer with type less than $\bar{t}_1$ purchases plan 1 and each consumer with type between $\bar{t}_1$ and $\bar{t}_2$ purchases plan 2. Because expected demand is higher for lower values of *t*, plan 1 is purchased by higher expected demand users than plan 2.

It is expected that plan 2 will have a higher usage price and a lower access fee than plan 1. The prices and marginal consumers which maximize profit are:

$$r_1 = \frac{61(a-c)^2}{384b}, \qquad r_2 = \frac{25(a-c)^2}{384b}$$

$$\bar{t}_1 = 1, \qquad \bar{t}_2 = 2,$$

$$p_1 = \frac{a+7c}{8}, \qquad p_2 = \frac{3a+5c}{8}. \tag{38}$$

The profits and consumer surplus are:

$$\Pi_{2PT(2)} = \frac{37(a-c)^2}{192b}$$

$$cs_{2PT(2)} = \frac{31(a-c)^2}{384b} \quad . \tag{39}$$

As expected, the profits with a pair of two-part tariff plans are higher than the profits under a single plan. In addition, the access fee for plan 1 is greater than the access fee under a single plan, which is greater than the access fee for plan 2. The usage prices also satisfy the expected relationships. The usage price for plan 1 is less than the usage price under a single plan, which is less than the usage price under plan 2.

With multiple three-part tariff plans, we assume only that the overage price for each plan is greater than the price charged within the bucket of minutes. If the three-part pricing plan allows for more effective price discrimination, we expect that the marginal consumers will be different than with a pair of two-part tariffs, and that the profits from each plan will improve. Profits for the monopolist who offers two three-part pricing plans are:

$$\begin{aligned}\Pi_{3PT(2)}(p_{0,i}, p_i, r_i, s_i) &= r_1 \cdot \frac{\bar{t}_1}{2} + r_2 \cdot \frac{\bar{t}_2 - \bar{t}_1}{2} \\ &+ (p_{0,1} - c) \cdot \int_0^{\bar{t}_1} X_1(p_{0,1}; t) \cdot h(t)dt + (p_{0,2} - c) \cdot \int_{\bar{t}_1}^{\bar{t}_2} X_1(p_{0,2}; t) \cdot h(t)dt \\ &+ (p_1 - c) \cdot \int_0^{\bar{t}_1} X_2(p_{0,1}, p_1, s_1; t) \cdot h(t)dt + (p_2 - c) \cdot \int_{\bar{t}_1}^{\bar{t}_2} X_2(p_{0,2}, p_2, s_2; t) \cdot h(t)dt \end{aligned} \tag{40}$$

$\bar{t}_1$ is the consumer who is indifferent between plan 1 and plan 2, and $\bar{t}_2$ is the consumer who indifferent between purchasing plan 2 or not purchasing either plan. $p_{0,1}$ and $p_{0,2}$ denote the bucket prices for plan 1 and plan 2 respectively. s_1 and s_2 denote the sizes of the buckets for plan 1 and plan 2 respectively. $\bar{t}_1$ and $\bar{t}_2$ are functions of the bucket prices, overage prices, access fees, and bucket sizes for the two three-part tariffs. The profit-maximizing bucket sizes, access fees, marginal consumers, bucket prices, and overage prices are:

$$\begin{aligned} s_1 &= \frac{a - p_{0,1}}{b} = \frac{7(a-c)}{8b}, & s_2 &= \frac{a - p_{0,2}}{b} = \frac{5(a-c)}{8b}, \\ r_1 &= \frac{61(a-c)^2}{384b}, & r_2 &= \frac{25(a-c)^2}{384b}, \\ \bar{t}_1 &= 1, & \bar{t}_2 &= 2, \\ p_{0,1} &= \frac{a+7c}{8}, & p_{0,2} &= \frac{3a+5c}{8}, \\ p_1 &= a, \text{and} & p_2 &= a. \end{aligned} \tag{41}$$

The profits and consumer surplus are:

$$\Pi_{3PT(2)} = \frac{37(a-c)^2}{192b}$$

$$cs_{3PT(2)} = \frac{31(a-c)^2}{384b}. \tag{42}$$

Again, a pair of three-part tariffs exactly duplicates a pair of two-part tariffs. In addition, the size of each bucket is again the maximum amount that a consumer might demand given the bucket price, so the precise value of the overage price is irrelevant. With each plan, so long as the overage price is greater than the bucket price, consumers will never purchase overage units. As such, a pair of three-part tariffs is not superior to a pair of two-part tariffs. In addition, the optimal three-part tariff has usage prices within the first bucket which are greater than marginal cost. As such, a three-part tariff with a free bucket offers less than maximum profits to the monopolist even when the marginal cost of usage is zero.

4.11 Concluding Remarks

In the wireless telecommunications industry, pricing plans typically include an access fee, a bucket of units available with a usage price of zero, and a higher overage price for units beyond the first bucket. However, Perry (2005) finds that profits from pricing with two-part tariffs are identical to profits from pricing with three-part, increasing-block tariffs. Here, we extend the Perry model and introduce demand uncertainty for consumers. The demand uncertainty does not change the outcome. The optimal three-part, increasing-block tariff generates the same profits and consumer surplus as the optimal two-part tariff. Moreover, the optimal size of the first bucket of units is sufficiently large to ensure that consumers never purchase units at the overage price. In addition, the optimal pair of three-part, increasing-block tariffs exactly duplicates the prices, profits, and consumer surpluses of the optimal pair of two-part tariffs.

If we relax the assumption that the overage price must be greater than the usage price for the first bucket, the three-part tariffs can increase profits for the monopolist. Yet, most cellular telephone service providers continue to offer three-part, increasing-block tariffs for their services. In addition, as wireless data plans have become common in the industry many plans continue to exhibit an increasing-block structure.[31] Since the results indicate that it is never optimal for the monopolist to offer the first bucket of units with a usage price of zero, the continued use of these plans is surprising.

The fundamental question remains: Why do companies offer services to consumers using three-part, increasing-block tariffs? It cannot be that the costs of monitoring and billing usage within the free bucket are high enough that firms would rather not charge for these units. Because the firms charge for overage units, they must monitor usage within the first bucket and bill their customers regardless. In other industries where increasing-block tariffs are used, namely water and electricity, the tariffs are designed to reduce consumption or provide cheap utilities to poorer customers. [(Borenstein 2008) and (Boland and Whittington 1998)] In the cellular telephone

[31] See, for example, AT&T's DataConnect 3G plan which charges $60 per month for the first 5GB of usage, and an overage rate of $0.05 per MB for additional usage. This is a three-part, increasing-block tariff with a usage price of $0 for the first bucket and a higher overage price for additional units. In this case, for a consumer who uses more than 5GB of data in a given month, the average price of usage is $12.00 per GB for the first bucket and $51.20 per GB for units beyond the first bucket. (www.wireless.att.com)

Verizon offers plans which take a similar form, an access fee with a bucket of free units and an overage price. In the Verizon plans, the overage price is constant across different bucket sizes. For cheaper, smaller free buckets (2 GB for $30), the $10 per additional GB overage price is cheaper than the average price within the bucket. For larger, more expensive free buckets (10 GB for $80), the $10 per GB overage price is more expensive than the average price within the bucket. (www.verizonwireless.com) industry, firms do not have the same incentives as publicly regulated utilities. Wireless firms have an incentive to increase consumption of their services.

Extensions to this paper should check for robustness with respect to the distribution function of consumer types. This could be addressed by taking a more general mechanism design approach to the pricing problem. An additional extension is to examine competition between firms offering non-linear pricing plans for services.

It remains to be shown how three-part, increasing-block tariffs are superior to alternative pricing plans. Yet these plans are the dominant pricing strategy in the cellular telephone industry. In the model presented here, the use of three-part, increasing-block tariffs does not increase profits compared to a simple two-part tariff. In addition, the offering of a first bucket with a usage price of zero does not maximize profits.

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