DAMAGE MECHANISMS OF MATRIX CRACKING AND INTERFACIAL DEBONDING IN RANDOM FIBER COMPOSITES UNDER DYNAMIC LOADINGS

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ABSTRACT OF THE DISSERTATION

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By considering the wide applications of composite materials, it is necessary to have a proper knowledge of dynamic behavior as well as static behavior reflecting the damage in composite materials. Strain rates have significant effects on dynamic behavior in composite materials when they are under dynamic loadings.

In this thesis, a multiscale numerical approach with finite element code ABAQUS is developed to characterize failure criteria to express static and dynamic damage mechanisms of matrix cracking and interfacial debonding under uniaxial tensile loadings for composite materials. The random epoxy/glass composite material is investigated under three strain rates: quasi-static, intermediate and high, corresponding to $10^{-4}$, 1 and 200 s$^{-1}$, respectively. A representative volume element (RVE) of a random glass fiber composite is employed to analyze microscale damage mechanisms of matrix cracking and interfacial debonding, while the associated damage variables are defined and applied in a mesoscale stiffness reduction law. The macroscopic response of the homogenized damage model is investigated using finite element analysis and validated through
experiments. The random epoxy/glass composite specimens fail at a smaller strain; there is less matrix cracking but more interfacial debonding accumulated as the strain rate increases. The dynamic simulation results of stress strain response are compared with experimental tests carried out on composite specimens, and a respectable agreement between them under the low strain rate is observed. Finally, a case study of a random glass fiber composite plate containing a central hole subjected to tensile loading is performed to illustrate the applicability of the multiscale damage model.
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Dedications

To my wife, Mrs. Lu Su.

To my parents, Mr. Congkai Yang and Mrs. Fenzhi Yu.
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Chapter 1
Introduction

1.1 Random Glass Fiber Composite

Many efforts have been made toward the lightweight composite materials to increase the energy and economic efficiencies for engineering applications. Unidirectional Composite Materials have been widely used as the substituting materials for traditional ones for they have higher specific stiffness and strength [1], but they are very expensive to fabricate and there are only few applications. Then random fiber composite materials have come into use for their comparable or even better services, such as energy absorption and cost reduction due to the programmable powder preform process (P4) and structure reaction injection molding (SRIM) technique [2, 3, 4, 5, 6]. Composites manufactured with the above techniques are widely used in automotive and aerospace industries [7, 8].

Random Fiber Composites (RFCs) are structures composed of a dominant matrix material that embeds randomly orientated fibers with significantly superior mechanical properties. RFCs are attractive structural materials for many industrial applications that require low weight, good crashworthiness, and low fabrication cost. For example, RFCs are being used to construct bus body panels, urban train seats, shells for watercraft and vehicle parts due to their light weight and the ease with which complex shapes can be achieved.
The increasing applications of composite materials expose them to extreme loading and severe environmental conditions and bring new challenges to the composite designs. To advantageously use these materials and optimize the design process, both the elastic properties and progressive damage up to failure of the material have to be characterized. Damage in RFCs is a very complicated phenomenon and extremely difficult to predict due to the complexity of the microstructure. At the microscale, damage can start with the occurrence of matrix microcracks that propagate and weaken the fiber-matrix interfaces and lead to debonding. Excessive matrix cracking and fiber-matrix decohesion at higher loadings will cause fiber pull out and breakage. These mechanisms can occur at the same time or successively and greatly reduce the overall properties of the composite. The composite structure will fail as damage accumulates.

1.2 Research Goals

There are three major damage mechanisms for RFCs which are matrix cracking, interfacial debonding and fiber breakage when they are under loadings, but only matrix cracking and interfacial debonding will be taken into account in this thesis since fiber will not break before the final failure of RFCs.

The main objective of this doctoral thesis is to study the damage mechanisms including matrix cracking and interfacial debonding in RFCs under dynamic loadings of different strain rates using finite element method validated by experimental tests. A random epoxy/glass composite material is investigated under three strain rates, quasi-static, intermediate and high strain rate, corresponding to $10^{-4}$, 1 and 200 s$^{-1}$, respectively. A multiscale modeling approach to damage will be undertaken shown in Figure 1.1 for
each strain rate. At the microscale, damage can start with matrix microcracks, and then interfacial debonding subsequently as microcracks propagate and weaken the fiber-matrix interfaces. At the mesoscale, a representative volume element (RVE) [9] of the random glass fiber composite will be used for the analysis of the damage mechanisms such as matrix cracking and fiber-matrix interfacial debonding, at the same time, the associated damage variables will be defined and the stiffness reduction law versus these variables will be established. Then the macroscopic response of the homogenized damage model will be implemented in finite element analysis and validated through the experiments. Finally, an analysis on damage of the random glass fiber composite plate containing a central hole subjected to a tensile loading will be conducted through the damage model.

![Figure 1.1 Schematic of multiscale modeling approach to random glass/epoxy composites](image)

1.2.1 Effective Stiffness
In this research, the undamaged elastic properties of the glass-RFCs are computed by applying numerical analysis with proper boundary conditions to the RVE, which was generated by using modified random sequential absorption (RSA) method [10, 11, 12], and is composed of E-glass fibers and epoxy matrix. The fiber bundle is treated as a fiber having the bundle’s equivalent homogenized properties, with fiber aspect ratio the fiber bundle’s ratio of 20 and 38% fiber volume fraction in the homogenized glass-RFC. The RVE of a glass-RFC material should be large enough to represent the material properties. Since a large RVE is computationally costly, so in order to consider both macroscopic properties representation and computational efficiency, there are 114929 nodes, 513670 tetrahedral elements, and 62075 6-node cohesive elements in the numerical model of RVE.

1.2.2 Damage Mechanisms

For three decades, many papers have been published on the mechanical behavior and damage modeling of RFCs using micromechanical and experimental approaches. While there are some micromechanical models [13-15] offering very good predication of the composite elastic properties, based on the Eshelby equivalent inclusion method [16] and the Mori-Tanaka model [17], there is little literature available on the nonlinear behavior due to damage for composites. In a study by Taya and Mura the effects of fiber end cracking on properties of an aligned short fiber composite were evaluated by the Eshelby-Mori-Tanaka theory [18]. A modified Mori-Tanaka model coupled with micromechanical analysis was also employed by Meraghni et al. to investigate the effects of interfacial debonding and matrix degradation on the overall behavior of glass-RFC specimens subjected to tensile loading [19-20]. Continuation of this work appears in Reference [21]
where a micromechanical Eshelby-Mori-Tanaka based model uses a statistical approach to predict damage evolution in randomly oriented short fiber composites. The above-mentioned studies involved glass/epoxy composites contrasting Dano’s et al. [22] investigation in which the tensile behavior of a randomly oriented short glass/polyester composite was examined to provide some information about the different damage mechanisms. Among many distinguished experimental studies for the characterization of the mechanical behavior of random short fiber composites, Hour and Sehitoglou’s [23] and Dano and Gendron’s [24] are most relevant to our approach since they focused on the description and representation of damage in the glass-RFCs.

In Nguyen’s et al. [25-27] studies the stiffness of the reference aligned fiber composite was used to derive the stiffness of the random fiber composite containing matrix microcracks and fiber-matrix interfacial decohesion using the modified Mori-Tanaka micromechanical model. The stiffness of the undamaged and damaged random short fiber composite was obtained from the stiffness of the reference composite which was averaged over all possible orientations and weighted by an orientation distribution [24-25]. Acoustic emission experiments were employed to determine matrix cracking and fiber–matrix debonding during damage, and identify the failed regions in the RFC plate, so as to validate the model predictions.

In this study, a numerical method is developed in ABAQUS [28], based on micromechanical and continuum damage mechanics, which characterizes the damage mechanisms of matrix cracking and fiber-matrix debonding in glass-RFCs and predicts the composite’s macroscopic response. The damage evolution generally consists of two stages. The first stage is controlled by a progressive degradation of the composite due to
matrix cracking leading to interfacial debonding and the second stage refers to the final failure for which the composite is unable to carry load. The elastic properties of undamaged and damaged random glass-RFCs are calculated from RVE in which the fiber orientation is statistically determined, thus improving the statistical representation of fiber random distribution increasing the simulation accuracy. The homogenized damage model is further validated by experiments.

1.3 Outline

Chapter 2 delivers a review of the current research approaches undertaken on modeling damage mechanisms of RFC materials. This chapter includes two sections. The first section summarizes the methods that have been applied to characterize the damage mechanisms of RFCs, including Micromechanics analytical approach, micro-macro mechanistic approach, and experimental method. The second section introduces the generation of RVE. The last section describes residual thermal stress effect on performance of composite materials.

In chapter 3, a hybrid composite structure is designed to consider cost, stiffness, and energy absorption for automotive application using analytical and finite element methods, then an analytical solution for the energy absorption of the hybrid structure is derived.

In chapter 4, the numerical model is developed to characterize the damage mechanisms of matrix cracking and interfacial debonding in RFCs subjected to quasi-static tensile loading, in which two damage variables are defined, and the RVE is used to develop the homogenized damage model including stiffness reduction law, damage evolution relation.
In chapter 5, the strain rates effect is incorporated for the elastic modulus and strengths of matrix to characterize the dynamic damage mechanisms as well as static damage mechanisms of matrix cracking and interfacial debonding for RFCs under dynamic loadings.

Chapter 6 concludes the application of a hybrid composite structure in auto industry, the damage behavior of RFCs, and the strain rates effect on damage in RFCs. It also includes suggestions for future research on damage in RFCs.
Chapter 2

Literature Review

Random fiber reinforced composite materials have been widely used in engineering applications for they can provide comparable or even better services compared to traditional ones, such as energy absorption and cost reduction. In order to expand their range of applications, both the elastic properties and behavior of the materials under dynamic loadings inducing damage up to failure have to be studied. The difficulties in fully exploring the capabilities of the RFCs lie in the obstacle to effectively model the damage from micro to macro scales when they are under dynamic loadings. In this chapter, research work related to characterization of damage in RFCs is reviewed.

2.1 Damage Model Methods

There were three major methods developed to study the damage in RFCs which are micromechanics analytical approach based on Eshelby inclusion theory and Mori-Tanaka method, micro-macro mechanistic approach and macroscopic approach. The following three subsections summarize the aforementioned methods.

2.1.1 Micromechanics Analytical Approach

In the work by Meraghni [19-20], the interfacial dobonding effects on the overall behavior of randomly oriented discontinuous fiber composites containing matrix microcracks was modeled. In the work by Weng [29-30], the effective properties was predicted for two- or three-phase composites and the thermomechanical properties was evaluated for a composite having a variable fiber aspect ratio with various orientation
distributions. A micromechanical analysis based on Eshelby inclusion and the Mori-Tanaka method has been performed. It was intended to evaluate the interfacial stress tensor by considering the local perturbation due to the onset and the growth of matrix cracking. However, as developed previously, the interfacial degradation effects were modeled by using the damage participation rate. The latter was determined on the basis of an experimental damage methodology of acoustic emission. This methodology was based on the amplitude treatments and microscope observations, which led to an identification and a schematic classification of damage mechanisms.

The model developed was then used to predict stiffness reduction and to simulate the behavior of a composite with degraded interfaces and containing matrix microcracks. The model simulations agreed well with the experimental results, notably for the materials referred to as 600 tex and 1200 tex. Indeed, the simultaneous integration of matrix degradation and interfacial debonding had improved the numerical results because for these materials the damage development was mainly governed by both failure processes as confirmed by the experimental findings.

The elastic stiffness tensors of the matrix and the bundle are denoted \( C_m \) and \( C_B \), respectively. From the mean stress field theory [31], the average stress in the matrix differs from the applied macrostress, \( \Sigma \), by a perturbed stress denoted \( \langle \sigma \rangle_{D-\Omega-\Omega_m} \). This difference was due to the disturbance caused by the presence of all inclusions. Therefore, at a finite bundle concentration, the average stress tensor in the matrix was given by:

\[
\Sigma + \langle \sigma \rangle_{D-\Omega-\Omega_m} = \Sigma + \langle \sigma \rangle_{\Omega_m} = C_m : (E_0 + \tilde{\varepsilon})
\]

(2.1)
Where $E_0$ was the uniform macrostrain field, $\langle \sigma \rangle_{\Omega_m}$ denotes the volume average of the total stress in the matrix, and $\bar{\varepsilon}$ referred to the average disturbance in strain of the matrix due to the presence of all inclusions.

When a bundle was introduced into a RVE, the stress tensor inside the bundle $\sigma_B$ differs from the mean stress in the matrix by a perturbed value $\sigma^{pt}$, which generated a strain designated $\varepsilon^{pt}$ representing the strain disturbance due to the presence of the introduced bundle.

$$\sigma_B = \sum + \langle \sigma \rangle_{\Omega_m} = C_m : (E_0 + \bar{\varepsilon} + \varepsilon^{pt}) \quad (2.2)$$

By using Eshelby’s equivalence principle [16], the reinforcement is transformed to an equivalent inclusion having the matrix property $C_m$, so that:

$$\sigma_B = C_B : (E_0 + \bar{\varepsilon} + \varepsilon^{pt}) = C_m : (E_0 + \bar{\varepsilon} + \varepsilon^{pt} - \varepsilon^*) \quad (2.3)$$

Where $\varepsilon^*$ was the eigenstrain of the bundle expressed in the global coordinate system and $S_B$ was the transformed fourth order Eshelby tensor of the bundle. So the equation (2.3) could be written as follows:

$$\sigma_B = C_B : (E_0 + \bar{\varepsilon} + \varepsilon^{pt}) = C_m : (E_0 + \bar{\varepsilon} + (S_B - I) : \varepsilon^*) \quad (2.4)$$

Considering the assumption that the volume average of the mean stress field $\sigma_D$ over the entire RVE domain $D$ must be zero, the homogenization relations could be written as:

$$\frac{1}{D} \int_D \sigma dv = \frac{1}{D} (\int_\Omega \sigma dv + \int_{\Omega_m} \sigma dv + \int_{\Omega_\mu} \sigma dv) = 0 \quad (2.5)$$
In the case of an equiprobable distribution of reinforcement and microcracks, the stress tensor inside the inclusion became:

\[
\sigma^{in} = C_m : [E_0 + (S_B - I) : \epsilon^* - \sum_{B=1}^{n} f_B (S_B - I) : \epsilon_B^* - \sum_{\mu=1}^{n} f_{\mu} (S_{\mu} - I) : \epsilon_{\mu}^* ]
\]  

(2.6)

Where \( f_B \) and \( f_{\mu} \) referred to the distribution of bundles and microcracks, respectively. They were assumed to uniform functions:

\[
f_B = \frac{(V_B)_C}{N} \]

\[
f_{\mu} = \frac{V_{\mu}}{N}
\]

(2.7)

Where \((V_B)_C\) was the global volume fraction for the bundles in the composite materials, and \(V_{\mu}\) was the damage parameter relative to the matrix microcracking and was evaluated by means of AE technique. \(N\) was the total representative orientation number in the plane of the material. The random distribution of the reinforcement could be represented by 18 orientations \((N=18)\).

A micromechanical analysis had been conducted to model the effects of interfacial degradation on the overall behavior of a random discontinuous fiber composite containing matrix microcracks. The amplitude analysis of AE signals had been used to identify and classify the dominate damage mechanisms. And a good agreement between the theoretical results and the experimental data for in-plane mechanical properties had been achieved. However, in order to determine all of the components of the stiffness tensor characterizing the damaged material experimentally, it was necessary to perform others types of mechanical tests.
2.1.2 Micro-Macro Mechanistic Approach

In the work by Nguyen [25-27], a micro-macro mechanistic approach to damage in random fiber composites was developed. At microscale, a reference aligned fiber composite was considered for the analysis of the damage mechanisms such as matrix cracking and fiber-matrix interfacial debonding using the modified Mori-Tanaka model. The associated damage variables are defined and the stiffness reduction law dependent on these variables was established. The stiffness of a random fiber composite containing matrix cracking and imperfect interface was then obtained from that of the reference composite, which was averaged over all possible orientations and weighted by an orientation distribution function. The macroscopic response was determined using a continuum damage mechanics approach and finite element analysis. The model was validated using the experimental results found in literature as well as the results obtained for a random glass-vinyl ester composite. Acoustic emission techniques were used to characterize the amount and the type of damage during quasi-static testing.

Consider a short fiber composite in which the fibers and fiber shape matrix cracking are unidirectional. If fiber bundles exist, they were regarded as equivalent fibers with associated mechanical properties. The defined composite served as the reference composite to compute the crack density. During the damage development prior to the final failure, if the linking-up process of matrix cracking didn’t happen, the crack density in a random fiber composite could be computed from the solution for the aligned fiber composite system which included the same number of matrix cracking parallel to each other. The Eshelby-Mori-Tanaka method was used to matrix cracking problem of the reference composite which was regarded as a hybrid inclusion composite system in which
the fibers were the first type inclusions which the matrix cracking were modeled as the second type of inclusions with zero stiffness. So the stiffness matrix of such as composite could be expressed as:

\[
C = \left( f_m C_m + f_1 C_1 : A_1 + f_2 C_2 : A_2 \right) : \left( f_m I + f_1 A_1 + f_2 A_2 \right)^{-1}
\]  

(2.8)

Where \( C_m, C_1 \) and \( C_2 \) were the stiffness tensors of the matrix material and of the fibers and matrix cracking inclusions whose volume fractions were \( f_1 \) and \( f_2 \), respectively. \( A_i \) \((i = 1, 2)\) were the inclusion concentration matrices defined as:

\[
A_i = \left[ I + S_i : C_m^{-1} : (C_i - C_m) \right]^{-1}
\]

(2.9)

And \( S_i \) were the Eshelby tensors. Since the first type of inclusion was the actual fibers while the second type of inclusion represents matrix cracking with zero stiffness, Equation (2.8) becomes:

\[
C = \lim_{C_2 \to 0} \left( f_m C_m + f_1 C_1 : A_1 + f_2 C_2 : A_2 \right) : \left( f_m I + f_1 A_1 + f_2 A_2 \right)^{-1}
\]  

(2.10)

In order to keep numerical stability, the components of \( C_2 \) were taken to be very small, so that the matrix cracking was with zero stiffness.

When degradation of matrix occurs at the interface and was important to cause the fiber-matrix interface to debond. Accordingly, the stiffness of a composite containing aligned ellipsoidal inclusions with weakened interfaces was given by:

\[
C = \lim_{C_2 \to 0} \left( f_m C_m + \sum_{i=1}^{n} f_i C_i : A_i \right) : \left( f_m I + \sum_{i=1}^{n} f_i A_i + \sum_{i=1}^{n} f_i H_i : C_i : A_i \right)^{-1}
\]

(2.11)
Where the tensor $H$ had the dimension of a compliance matrix, and depended on the interface property and the geometry of the inclusion [32, 33].

$$A_i = \left[ I + S_i: C_m^{-1} : (C_i - C_m) \right]^{-1}$$

(2.12)

$$S^* = S + (I - S) : H : C_m : (I - S)$$

By limiting equation (2.11) to a three phase composite in which the third phase was microcracks that are parallel to fibers and to each other, the stiffness of such a composite was obtained by taking the limit of the microcrack stiffness to zero.

$$C = \lim_{C_2 \to 0} \left( f_mC_m + f_1C_1 : A_1 + f_2C_2 : A_2 \right)$$

$$: \left( f_mC_m + f_1A_1 + f_2A_2 + f_1H_1 : C_1 : A_1 + f_2H_2 : C_2 : A_2 \right)^{-1}$$

(2.13)

Finally, the stiffness of the random fiber composite having matrix cracking and fiber-matrix interfacial debonding was obtained from the stiffness of the reference composite given by equation (2.13), which was averaged over all possible orientations and weighted by an orientation distribution function:

$$\bar{C}(\alpha) = \frac{\int_{-\pi/2}^{0} R(\alpha, \theta) \lambda e^{-\lambda(\theta)} d\theta + \int_{0}^{\pi/2} R(\alpha, \theta) \lambda e^{-\lambda(\theta)} d\theta}{2\int_{0}^{\pi/2} \lambda e^{-\lambda(\theta)} d\theta}$$

(2.14)

$\alpha$ was the volume fraction for matrix cracking, $R(\alpha, \theta)$ was the global stiffness matrix of the reference composite obtained by transforming $C(\alpha)$ into the global coordinate system which was defined such that the fibers were assumed in the layer plane.

The damage evolution law in terms of the local strains $\varepsilon$ could be obtained using thermodynamics of continuous media and a continuum damage mechanics formulation.
The elastic deformation energy was used as the thermodynamic potential energy expressed by:

$$\phi(\varepsilon, \alpha) = \frac{1}{2} C_{ij}(\alpha) \varepsilon_i \varepsilon_j$$  \hspace{1cm} (2.15)$$

From equation (2.15), the constitutive relations and the thermodynamics force associated with the damage variable were obtained as:

$$\sigma_i = \frac{\partial \phi(\varepsilon, \alpha)}{\partial \alpha} = C_{ij} \varepsilon_j$$ \hspace{1cm} (2.16)$$

$$F(\varepsilon, \alpha) = \frac{\partial \phi(\varepsilon, \alpha)}{\partial \alpha} = \frac{1}{2} \frac{\partial C_{ij}}{\partial \alpha} \varepsilon_i \varepsilon_j$$ \hspace{1cm} (2.17)$$

As damage was an irreversible process, Clausius-Duhem’s inequality expressing the total dissipation must be satisfied. In this case, it could be written as:

If \(-F(\varepsilon, \alpha) \dot{\alpha} \geq 0 \Rightarrow F < 0 : \dot{\alpha} > 0\) Damage was in Progression

If \(-F(\varepsilon, \alpha) \dot{\alpha} \geq 0 \Rightarrow F \geq 0 : \dot{\alpha} = 0\) Damage was stable  \hspace{1cm} (2.18)$$

Where dot symbol referred the rate of the damage propagation. Relations in equation (2.18) imposed the conditions for damage evolution based on the second law of thermodynamics. It was assumed that damage was governed by a criterion dependent on a damage threshold function, \(F_c\), which was in function of the damage variable \(\alpha\):

$$f(F, \alpha) = F_c(\alpha) - F \leq 0$$ \hspace{1cm} (2.19)$$

Using the criterion in equation (2.19), the damage evolution equation was derived from the consistency conditions. Mathematically, these condition were defined by \(f = 0\)
and $df = 0$. So the increase of the damage variable was directly expressed below in terms of the current strain and strain increment.

$$d\alpha = -\frac{dC_{ij}}{d\alpha} \varepsilon_i d\varepsilon_j$$

$$= \frac{1}{2} \frac{d^2C_{ij}}{d^2\alpha} \varepsilon_i \varepsilon_j - \frac{dF_c}{d\alpha}$$

(2.20)

The application of equation (2.20) required the computation of the damage threshold function $F_c$ which could be based on experimental crack density versus applied stress data.

The participation rates of matrix cracking, $p_i^m$ and of both matrix cracking and fiber-matrix interfacial debonding, $p_i$ at a given applied stress level, $i$ were defined, respectively, as:

$$P_i^m = \frac{a_i^m}{A_{tot}}; P_i = \frac{A_i}{A_{tot}}$$

(2.21)

Where $a_i^m$ refer to the area of the matrix cracking distribution, and $a_i$ was total area for matrix cracking and fiber-matrix interfacial debonding distribution, $A_{tot}$ was the total area of all the damage mechanisms distribution before the final failure of the composite specimens. So the matrix cracking volume fraction (crack density) was given by:

$$a = p_i^m f_m = \frac{p_i^m}{1 + p_i^m} (1 - f_b)$$

(2.22)

Where $f_b$ was the fiber bundle volume fraction in the composite.
A micro-macro mechanistic approach to damage in short fiber polymer composites had been developed starting from the microscale related to the constituents, microcracks, and interfacial defects and extending to the scale of a macroscopic composite structure. Before final failure, matrix cracking and interfacial debonding had been accounted for the reduction of the composite stiffness. The elastic and reduced properties of the composite had been obtained from micro mechanical modeling based on the modified Mori-Tanaka model. Two damage variables were defined for matrix cracking and interfacial debonding, respectively. The continuum composite modeled through this homogenization had been used to establish the constitutive relation and the damage evolution. The damage model had been implemented into the ABAQUS finite element code to analyze the macroscale response of composite specimens. The model validity had been illustrated through the tensile stress-strain response simulations for the random glass/epoxy and glass/vinyl ester composites. Numerical results had shown that model provided good predictions of the tensile stress-strain responses and crack density versus applied stress for these composites.

2.1.3 Macroscopic Approach

In the work by Dano [34], a theoretical damage mechanics model was developed to analyze the damage in random short glass fiber reinforced composites. This model was based on a macroscopic approach using internal variables together with a thermodynamic potential expressed in the stress space. Induced anisotropic damage, nonsymmetric tensile/compressive behavior and residual effects were taken into account. The anisotropic damage was represented with second order tensor internal variable D. The unilateral effect due to microcrack closure in compression was introduced by generalizing the hypothesis of the complementary elastic energy equivalence.
In the case of an isotropic material, the complementary elastic energy is defined as:

$$U_0^c = \frac{1}{2E} \sigma : \sigma - \frac{\nu}{2E} \left( (\sigma : I_2) - \sigma : \sigma \right)$$  \hspace{1cm} (2.23)

Where $I_2$ was a second-order unit tensor, $\sigma$ was the stress tensor, $E$ was Young’s modulus, and $\nu$ was the Poisson’s ratio. It was assumed that the complementary elastic energy of the damaged materials had the same form as an equivalent undamaged material by replacing the usual stress variable by an effective one, except for the energy linked to the compression that was responsible for the crack closure. Then the complementary elastic energy could be written as:

$$U_0^c (\sigma, D) = \frac{1}{2E} \tilde{\sigma}_n^+ : \sigma_n^+ + \frac{1}{2E} \tilde{\sigma}_n^- : \sigma_n^- + \frac{1}{2E} \tilde{\sigma}_{ij} : \sigma_{ij} |_{\nu_i} - \frac{\nu}{2E} \left( \tilde{\sigma}_{ij} \tilde{\sigma}_{ij} - \tilde{\sigma}_{n} \tilde{\sigma}_{n} \right)$$  \hspace{1cm} (2.24)

Where $\tilde{\sigma}_n^+$ referred to the tensile stress (positive), while $\tilde{\sigma}_n^-$ was the compressive stress (negative) normal to the plane of the microcracks system. In other words, the compressive stress $\sigma^-$ was defined as:

$$\sigma^- = H \left( -n_i \sigma n_i \right) \left( n_i \sigma n_i \right) n_i \otimes n_i = P^{(\sigma,D)^-} : \sigma$$

$$P^{(\sigma,D)^-} = H \left( -n_i \sigma n_i \right) \left( n_i \otimes n_i \otimes n_i \otimes n_i \right)$$  \hspace{1cm} (2.25)

Where $n_i$ were the principal directions of the damage and $H$ was the Heaviside function.

The quantity $P^{(\sigma,D)^-}$ was a fourth–order tensor corresponding to a negative projection operator. In the same way, the positive projection operator could be expressed as:

$$P^{(\sigma,D)^+} = H \left( n_i \sigma n_i \right) \left( n_i \otimes n_i \otimes n_i \otimes n_i \right)$$  \hspace{1cm} (2.26)
The fourth-order damage tensor $M(D)$ had a canonical form [35] which was defined as:

$$M(D) = \frac{1}{\sqrt{(1-D_i)(1-D_j)}} n_i \otimes n_j \otimes n_i \otimes n_j$$  \hspace{1cm} (2.27)$$

Where $D_i$ was the damage eigenvalue in the principle direction $i$. The canonical form was used because the fourth-order tensor $M(D)$ was a real symmetric tensor and its second-order eigentensors associated with the real positive eigenvalues $1/\sqrt{(1-D_i)(1-D_j)}$ were $n_i \otimes n_j$.

Substituting equations (2.25), (2.26) and (2.27) into equation (2.24), the complementary elastic energy could be updated as:

$$U^e(\sigma,D) = \frac{1}{2E} \sigma : (M : M) : \sigma - \frac{V}{2E} \left[ \left( tr(M : \sigma) \right)^2 - \sigma : (M : M) : \sigma \right]$$

$$- \frac{1}{2E} \sigma : \left[ P^{(\sigma,D)} : (M : M - I_4) : P^{(\sigma,D)^{-1}} \right] : \sigma$$  \hspace{1cm} (2.28)$$

Where $I_4$ was a fourth-order unit tensor, and the last term represented the restoration of the system rigidity due to crack closure. Then the definition corresponding to the last term in the above equation was written as follows:

$$\frac{1}{2E} \sigma : \left[ P^{(\sigma,D)^{-1}} : (M : M - I_4) : P^{(\sigma,D)^{-1}} \right] : \sigma = \frac{1}{2E} \sigma : \tilde{M}(D) : \sigma$$  \hspace{1cm} (2.29)$$

The fourth order operator was defined as:

$$\tilde{M}(D) = H (-n_i \sigma n_i) \frac{D_i (2 - D_i)}{(1 - D_i)^2} n_i \otimes n_i \otimes n_i \otimes n_i$$  \hspace{1cm} (2.30)$$
Using equations (2.26), (2.27) and (2.30), the complementary elastic energy could be expressed as:

$$U^c(\sigma, D) = \frac{1}{2} \sigma : \tilde{C}^{-1} - \frac{1}{2E} \sigma : \tilde{M}(D) : \sigma$$

(2.31)

Where \( \tilde{C}^{-1} = M : C^{-1} : M \) was the fourth-order elastic compliance tensor of the damaged material and could be written in the principal coordinate system of damage as:

\[
\tilde{C}^{-1} = \begin{bmatrix}
\frac{1}{E(1-D_1)} & \frac{-\nu}{E(1-D_1)(1-D_2)} & \frac{-\nu}{E(1-D_1)(1-D_3)} & 0 & 0 & 0 \\
\frac{-\nu}{E(1-D_2)(1-D_1)} & \frac{1}{E(1-D_2)} & \frac{-\nu}{E(1-D_2)(1-D_3)} & 0 & 0 & 0 \\
\frac{-\nu}{E(1-D_3)(1-D_1)} & \frac{-\nu}{E(1-D_3)(1-D_2)} & \frac{1}{E(1-D_3)} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G(1-D_1)} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G(1-D_2)(1-D_1)} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G(1-D_3)(1-D_1)}
\end{bmatrix}
\]

(2.32)

When loads were applied on a composite structure, cracks and other damage were induced in the material. The level of degradation was quantified through the second-order damage tensor \( D \). On the contrary, during the unloading phase, the microcracks progressively close up to a certain extent. The residual effect was attributed to the state of damage, and the existence of a potential, noted \( U^p(\sigma, D) \), which was a function of the damage tensor \( D \) and stress tensor was assumed as:

$$U^p(\sigma, D) = \sigma : A : D$$

(2.33)
Where $A$ was a symmetric fourth-order tensor whose coefficients were material constants. The energy could be viewed as frozen by the microcracks. The thermodynamic potential could be rewritten as:

$$U^e(\sigma, D) = \frac{1}{2} \sigma : \tilde{C}^{-1} - \frac{1}{2E} \sigma : \tilde{M}(D) : \sigma + \sigma : A : D$$  \hspace{1cm} (2.34)$$

The elastic constitutive law of the damaged material was obtained by differentiating the dual potential with respect to the stress tensor. The strain was defined in two parts due to the Heaviside function in the potential expressions:

$$\varepsilon = \tilde{C}^{-1} : \sigma - \frac{1}{E} \tilde{M} : \sigma + A : D$$  \hspace{1cm} (2.35)$$

The total strain included an elastic part ($\varepsilon^e$) and a permanent part ($\varepsilon^p$) which were defined as:

$$\varepsilon^e = \left( \tilde{C}^{-1} - \frac{1}{E} \tilde{M} \right) : \sigma$$  \hspace{1cm} (2.36)$$

$$\varepsilon^p = A : D$$

The elastic compliance tensor of the damaged material had incorporated the unilateral effect. The thermodynamic model was physically consistent and figure out through the experiments.

The thermodynamics force, which was known as the damage strain energy release rate, must be associated with the damage tensor. In the case wherein the principal directions of the damage should not change during loading, for each eigenvalue of the damage $D_i$, there was an associated thermodynamic force $Y_i$, defined as:
In fact, taking into account the possibility the interaction between two principal damage directions (two dimensional case), a correction must be done to the expression of the thermodynamic forces. A weighted sum of the two thermodynamic forces was thus used as:

\[ 
\bar{Y}_i = Y_i + bY_j, \quad i, j = 1, 2, i \neq j 
\]  

(2.38)

Where the parameter \( b \) is a material constant ranging from 0 to 1 which reflects the interaction between the two damage variables.

The thermodynamic forces drive the evolution of the internal variable characterizing the damage up to failure. Those forces must satisfy the Clausius-Duhem inequality due to damage:

\[ 
Y : \dot{D} \geq 0 
\]  

(2.39)

The evolution (quasi-static) law satisfying this inequality was chosen to be of the following form:

\[ 
D_i = f \left( Y_s^i \right), \quad i = 1, 2 
\]

\[ 
Y_s^i (t) = \max \left\{ Y_o, \sup \left( \bar{Y}_i (\tau) \right) \right\}, \quad \tau \leq t 
\]  

(2.40)

The relationship between the principal damage function and the thermodynamic force was expressed as:

\[ 
f \left( Y^s \right) = a \left( Y^s - Y_0 \right) 
\]  

(2.41)
Where $Y^5$ was the thermodynamic force defined in Equation (2.40), $Y_0$ was the initial value at which damage initiated, and $\alpha$ was a material parameter. These two parameters could be identified.

In the principal coordinate system of damage, the matrix $A$ was reduced to the following expression using Voigt’s notation due to symmetry of matrix.

$$A = \begin{bmatrix} \alpha & -\beta \\ -\beta & \alpha \end{bmatrix} \quad (2.42)$$

The permanent strains due to damage were obtained by equation (2.36) as follows:

$$\varepsilon_{11}^p = \alpha D_1 - \beta D_2$$
$$\varepsilon_{22}^p = -\beta D_1 + \alpha D_2 \quad (2.43)$$

Where $\alpha$ and $\beta$ are unknown material parameters to be identified using loading tests results.

Equations (2.37) and (2.38) could also be simplified in the case of uniaxial loading:

$$\tilde{Y}_1 = \frac{\sigma_1^2}{E(1-D_1)^3} + (\alpha - b\beta)\sigma_1$$
$$\tilde{Y}_2 = (b\alpha - \beta)\sigma_1 + b\frac{\sigma_1^2}{E(1-D_1)^3} \quad (2.44)$$

A loading/unloading test allowed the estimation of the secant damage Young’s modulus. Then the damage value could be figured out using Equation (2.32):

$$D_1 = 1 - \sqrt[3]{\frac{E}{E_1}} \quad (2.45)$$
At the same time, the damage variable $D_2$ was evaluated by using Equations (2.40), (2.41) and (2.44):

$$D_2 = a \left[ (b\alpha - \beta)\sigma_1 + b \frac{\sigma_1^2}{E(1-D_1)^3} - Y_0 \right]$$

(2.46)

Finally, Equation (2.43) could be used to express the permanent strain as a function of the five unknown parameters $Y_0, a, b, \alpha$, and $\beta$ which were determined using a constrained optimization technique.

$$\varepsilon_{11}^p = \alpha D_1 - \beta a (b\alpha - \beta)\sigma_1 - b\beta \frac{\sigma_1^2}{E(1-D_1)^3} + \beta a Y_0$$

$$\varepsilon_{22}^p = -\beta D_1 + \alpha a (b\alpha - \beta)\sigma_1 + b\alpha \frac{\sigma_1^2}{E(1-D_1)^3} - \alpha a Y_0$$

(2.47)

A damage model based on the fundamental principal of the thermodynamics of irreversible process was presented for random short glass fiber reinforced composites. The model had incorporated the unilateral effect of crack closure and the permanent strains after residual effect. Using experimental results and an appropriate identification procedure, all unknown parameters had been solved. There was a good agreement between the simulation and the experimental results after implementing this damage model in finite element method. It concluded that this damage model was appropriate to simulate damage evolution in random short glass fiber reinforced composite structures.

2.2 RVE Generation
In the investigation of the RFCs using finite element method, one usual procedure is the numerical generation of a representative volume element (RVE). And the size of the RVE must be large enough so that effective properties characterized by the RVE can effectively represent the material properties of the heterogeneous composite; at the same time the RVE should be as small as possible for computational efficiency.

The Random Sequential Absorption (RSA) technique is employed to generate the geometry of an RVE of a random fiber composite. Two algorithms are developed, the first of which is RSA algorithm that is designed for straight fibers and can only achieve the RVE with small fiber volume fractions which are less than 20%, the second of which is modified RSA algorithm which is developed for depositing the straight and curved fibers to achieve higher volume fractions which are between 35% to 40%. The RSA algorithm will be applied to generate RVES with 38% in volume fraction for damage analysis in RFCs.

### 2.2.1 Mathematical Implementation

In the following, the 2D fiber intersection will be discussed, and then the 3D intersection analysis algorithm will be introduced. For the two dimensional analysis, please check the figure 2.1, in which the two relative fibers’ projections A and B are illustrated in the x-y plane. The length of each fiber has been normalized to one. Given one end point \( P_E \) and the unit direction \( \bar{n} \) of a fiber’s generic line segment, points on the lime segment can be expressed as:

\[
P(\lambda) = P_E + \lambda L\bar{n}, 0 \leq \lambda \leq 1
\]  

(2.48)
Where $L$ is the length of the fiber and the scalar coefficient $\lambda$ is the normalized distance measured along the segment from one end point. So the distance between two points on two different generic line segments can be given by:

$$d(\lambda_1, \lambda_2) = \|P_1(\lambda_1) - P_2(\lambda_2)\|, 0 \leq \lambda_1 \leq 1 \text{ and } 0 \leq \lambda_2 \leq 1$$

(2.49)

In order to determine the minimum the distance between two fibers, the constrained nonlinear two variable function $d(\lambda_1, \lambda_2)$ described in above equation needs to be minimized by solving

$$D_{\text{min}} = \text{Min} \left[ d(\lambda_1, \lambda_2) \right]$$

$$0 \leq \lambda_1 \leq 1$$

$$0 \leq \lambda_2 \leq 1$$

(2.50)

Figure 2.1 Schematic illustration of side (a) and top (b) views of over-crossing fibers
For the 3D problem, it is noted that a straight fiber is represented as a convex prism with two dodecagonal faces, while a curved one has several convex irregular polyhedral. A convex polyhedral with \( n = 14 \) surfaces can be defined algebraically as the set of solution to a system of linear inequalities:

\[
H\bar{x} \leq b
\]  

Where \( H \) is a real \( 14 \times 3 \) matrix, \( b \) is a real \( 14 \times 1 \) vector and \( \bar{x} \) is a vector corresponding to the point coordinates in three dimensional system. The linear matrix inequality (LMI) in Equation (2.51) can be computed from the vertices of the polyhedron. Then the distance between any two points from two different polyhedral is expressed as:

\[
d(\bar{x}_1, \bar{x}_2) = \|
\bar{x}_1 - \bar{x}_2 \|, \quad H_1\bar{x}_1 \leq b_1 \text{ and } H_2\bar{x}_2 \leq b_2
\]  

The minimum distance between two convex polyhedral can be determined by solving the convex optimization problem shown in the Equation (2.53):

\[
D_{\text{min}} = \text{Min}\left[d(\bar{x}_1, \bar{x}_2)\right]
\]

\[
H_1\bar{x}_1 \leq b_1
\]

\[
H_2\bar{x}_2 \leq b_2
\]  

The numerical solution of the two convex optimization problems described above can be obtained by applying the existing free-ware CVX, which is available in Matlab and Python codes.

The RVE length, \( L \), fiber length, \( l \), semi axes, \( a \) and \( b \), and fiber volume fraction, \( V_f \), are user input parameters. An RVE with fiber volume fraction of 35.1% is generated and shown in the Figure 2.2a. In this work, the \( L/l=2 \) as the RVE size, and the RVE has dimensions of \( 80b \times 80b \times 12.7b \), where \( b \) is the semi-minor axis of the elliptical cross.
section of fiber, and there are 174 straight and curve fibers with aspect ratio 20 which is expressed as $l/2b=20$. The cross section of a fiber is a dodecagon approximation of an ellipse with $a/b=2$ shown in Figure 2.2. The in-plane fiber orientation distribution is shown in Figure 2.3.

Figure 2.2 Dodecagonal approximation of elliptical cross section with major axis $2a$ and minor axis $2b$ of fiber
Figure 2.3 RVE of the RFCs with curved fiber in ABAQUS, its fiber volume fraction is about 35.1%: (a) RVE with curved fibers and (b) Fiber orientation distribution.

In order to save computation time, an RVE containing only one layer fiber is also generated with fiber volume fraction of 38% since one loading case takes about two weeks to be completed if REV with two layers fiber.
2.3 Residual Thermal Stress

All materials have the tendency to increase or decrease their volume in the presence of temperature difference due to their characteristic thermal properties. When a composite comprised of materials with disparate thermal expansion coefficients is subjected to a uniform change in temperature during the manufacture process, it will develop the residual thermal stresses. These residual thermal stresses are contingent upon the mismatch of thermal expansion coefficients for the fiber and the matrix as well as the constraint of the bonded interface. These residual thermal stresses may cause warpage and shrinkage of the manufactured products and they may induce reduction of the tensile and fatigue strengths which are very important properties for fiber reinforced composites.

The residual thermal stresses will change to accommodate the fracture surfaces when the cracks are developed in a composite. One effect of the redistribution of residual thermal stress is a change in the effective thermal expansion coefficient of the composite, and the redistribution of residual and mechanical stressed releases strain energy that can drive crack propagation. The residual thermal shear stress developed during the manufacture process may cause damage to the fiber-matrix interfaces [36, 37]. In the work by Eason [36], due to processing temperature, matrix experiences stressed of 37 MPa and up to 97.7 MPa as calculated computationally. The research on stress distribution and failure mechanisms on the surface and in the interior of short fiber composite conducted by Choi [38] also supports this point. They observed that shear cracks in the fiber length direction occurred preferentially for longer fibers. Residual stresses may cause a composite to fail sooner than expected, to have greater susceptibility
to solvents, or to have reduced durability due to accelerated aging or fatigue damage mechanisms.

As a result, residual thermal stresses have detrimental effect on the matrix fiber load transfer mechanisms and on the mechanical properties of the composite, and there are considered in the following research.
Chapter 3

A Hybrid Composite structure in Automotive Application

A finite element method is employed to numerically evaluate the stiffness and energy absorption properties of an architecturally hybrid composite material consisting of unidirectional and random glass fiber layers. An LS-DYNA finite element model of a composite hollow square tube is developed in which the position of the random fiber layers varies through the thickness. The assessment of the stiffness and energy absorption is performed via three-point impact and longitudinal crash tests at two speeds, 15.6 m/s (35 Mph) and 29.0 m/s (65 Mph), and five strain rates, $\dot{\varepsilon} = 0.1 \text{ s}^{-1}$, $1 \text{ s}^{-1}$, $10 \text{ s}^{-1}$, $20 \text{ s}^{-1}$ and $40 \text{ s}^{-1}$. It is suggested that strategic positioning of the random fiber microstructural architecture into the hybrid composite increases its specific absorption energy and therefore enhances its crashworthiness. The simulation data indicate that the composite structure with outer layers of unidirectional lamina followed by random fiber layers is the stiffest. Moreover, it was illustrated that the specific energy absorption increases with the increased ratio of impact contact area over cross section area. Of all the parameters tested the thickness of the unidirectional laminate on the specific energy absorption did not appear to have a significant effect at the studied thickness ratios.

3.1 Introduction

The application of structural composite materials in the auto industry began in the 1950s. Since those early days, fiber reinforced polymers have been used for many automotive applications. The Automotive Composite Consortium (ACC) is interested in investigating the use of random fiber reinforced composites as crash energy absorbers primarily
because of the low costs involved in their manufacturing thus making them cost effective for automotive applications. The energy absorption capability of a composite material is important in developing improved human safety in an automotive crash.

In auto industry, the ability to absorb impact energy and reduce fatality of the occupants during survivable crashes is called “crash worthiness” of the structure. Current auto legislation requires that vehicles be designed such that occupants of the passenger compartment should not experience a resulting force that produces a deceleration greater than 20 g [39] when an impact at the speed of the 15.6 m/s (35 Mph) with a solid and immovable object occurs. In order to reduce the overall weight and improve fuel economy composite materials have been used to replace metallic structures.

The crashworthiness of a composite structure is described in terms of its specific energy absorption (SEA), which is characteristic to that particular structure. It is defined as the energy absorbed, per unit mass, of a crushed composite structure, namely [40],

\[ {SEA} = \frac{W}{m} \quad \text{or} \quad {SEA} = \frac{W}{\rho \cdot A \cdot l} \] (3.1)

where \( m \) is the total mass of the crushed composite structure calculated as the product of cross section area (A) of the composite structure, \( l \) is the displacement of steel panel, \( \rho \) is the material density, and \( W \) is the total energy absorbed calculated by integrating the impact force-deflection curve:

\[ w = \int F \, dl \] (3.2)

The energy absorption characteristics of various continuous [41,42,43] and random [44, 45, 46] fiber reinforced composites using experimental approaches have been
substantially investigated. Studies on the behavior and crashworthiness characteristics of continuous fiber square composite tubes subjected to static and dynamic axial compression exerted by a hydraulic press and a drop-hammer were performed in Ref. [41]. Ramakrishna [42] examined the energy absorption behavior of knitted glass fiber fabric/epoxy and knitted carbon fiber fabric/epoxy composite tubes, and the effects of fiber content, fiber orientation and testing speed on the crush zone morphology and discussed their specific energy absorption capabilities. The dynamic crush data of edgewise-loaded composite plates were studied with a modified crush test fixture adapted by Lavoie et al. in Ref. [43]. The influence of stacking sequence, trigger mechanism, and thickness on energy absorption was evaluated for graphite/thermoplastic, graphite/epoxy, and a hybrid Kevlar/graphite epoxy. Significant differentiations were observed between the energy absorption during dynamic and quasi static testing of the three material systems.

The aforementioned studies were performed on continuous fibers (unidirectional and/or woven fabrics) in which the specific absorption energy is significant. Nevertheless, when investigators started taping the random fiber composites they noticed considerable superior SEA performance of these materials majorly due to their random fiber micro-architecture. Starbuck et al. [44] embarked on a long-term study of random fiber composites and their energy absorption characteristics. His team performed quasi-static progressive crush tests on composite plates manufactured from carbon fiber with an epoxy resin system using compression molding techniques [44]. The material parameters considered during this program were fiber length, fiber volume fraction, and areal density, while specimen width, profile radius, and profile constraint were also carefully examined.
The same group continued their studies in Ref. [45] in which quasi-static progressive crush strip tests were conducted on randomly oriented carbon fiber composite materials to evaluate the effect of various material parameters (fiber volume fraction, fiber length and fiber tow size) on their energy absorption capability. A noteworthy experimental set up was developed Jacob et al. [46] for discerning the deformation behavior and damage mechanisms that occur during the progressive crushing of composites. Furthermore, Jacob’s team characterized the strain rate effects on the energy absorption of a random carbon fiber P4 composite. It was found, that for the particular random fiber structures studied, the SEA was dependent on the loading rate.

Although the SEA capability of random fiber composites is noteworthy the load carrying capacity of these materials is very limited. To this end, and reinforced by the fact that the literature available on the energy absorption and stiffness characteristics of hybrid unidirectional/random glass fiber composite structures is limited, in this paper, we will study the energy absorption and stiffness characteristics of the aforementioned composites via architecturally modifying the layup (thus the local -micro- content of random versus continuous fiber), and we will quantified them using a finite element (FE) model. At the same time, the effect of contact surface and geometry on the energy absorption and stiffness will also be investigated.

3.2 Finite Element Model

The FE simulation is performed in three main steps, the first is the model building using Altair’s pre-process software Hypermesh v8.0. Then the nonlinear analysis is carried out using LS-DYNA v971R5.0. Finally, the results are assessed and presented using Altair’s post-process Hyperview v8.0 software.
The geometry of the hybrid composite laminate is of a hollow tube with a square cross-section (20mm × 20mm) and 200mm length. The thickness of the structure is 5mm and consists of five layers fabricated by two unidirectional and three random glass fiber plies. Positioning of the random fiber layers through the thickness of the laminate is numerically investigated using three-point impact simulations, so as to determine the effect of the architectural layup on the structural integrity (crashworthiness) of the composite.

Three point impact testing was selected to evaluate the stiffness of the structure in locations normal to the lateral surface of a car simulating side impacts. In the 1980-2000 period, research on car crashworthiness and development of frontal car crash protection systems led to decreasing driver death rates by 52% while for side impacts the same parameter was reduced by merely 27% [47] indicating the need for additional studies [48]. Furthermore, the front crash modality was selected to evaluate the specific energy absorption ability of the proposed hybrid structure since front impacts in different locations (varying contact area) are the most common accident cases.

3.2.1 Three Point Impact Testing

The mesh scheme of the three-point impact testing is shown in Figure 3.1a. The model consists of the composite structure, two bases, and one impact cylinder. The element types used in this model are four-node shell element, six-node solid element and eight-node solid element. There are 42,440 elements and 51,330 nodes in the model.

3.2.2 Energy Absorption Testing

Figure 3.1b illustrates the mesh scheme of the energy absorption test. The model accounts
for the composite structure and one impact steel panel. There are 41,600 four-node shell and eight-node solid elements, and 50,281 nodes in the model.

Figure 3.1 Finite element model of (a) three point impact, and (b) energy absorption testing for a hybrid (2-ply) unidirectional/(3-ply) random glass fiber composite tube. Dimensions are 20mm × 20mm × 200mm (width × depth × length) and 5mm thickness.

3.2.3 Material Models and Properties

Two LS-DYNA material models are used for the FE analysis of the hybrid composite, namely Material Model 2 and Material Model 161 [49]. The moving impact cylinder, steel panel, and stationary bases are modeled via Material Model 20.

The unidirectional and random glass fiber plies can be treated as transversely isotropic materials whose mechanical properties are determined as in Refs. [50] and [51], respectively. Briefly, the fiber material chosen for this study is E-glass unidirectional lamina with Young’s modulus $E_{11}=39.0$ GPa for the longitudinal, and $E_{22}=8.60$ GPa, and $E_{33}=8.6$ GPa for the transverse directions. The Young’s modulus for the E-glass fibers used in the random plies are $E_{11}=11.95$ GPa, $E_{22}=11.70$ GPa, and $E_{33}=7.95$ GPa.

In LS-DYNA, the Material Model 2 (Material Orthotropic Elastic Solid Model) is chosen for the random glass fiber laminate in the three-point impact testing, while the Material Model 161 (Material Composite MSC) is selected for the unidirectional glass
fiber laminate in the energy absorption testing. The later model is a Hashin-based failure criterion [52], which not only includes fiber tensile/shear mode, fiber compression mode, fiber crush mode, perpendicular matrix mode, parallel matrix model (delamination), but also considers the strain rate effect. In order to account for the strain rate effect in the random fiber mats we assume that the Hashin’s failure criterion is activated only for the perpendicular matrix mode. The latter assumption holds true for thermoset-type random fiber composites used here since matrix cracking is the major mode of damage for structures subjected to impact loads.

The criteria used for fiber failure consider the following loading modes: tension/shear, compression, and crush under pressure. The relevant failure criteria for the three cases are:

Fiber tensile/shear mode:

\[ f_1 = \left( \frac{\sigma_a}{S_{at}} \right)^2 + \left( \frac{\tau_{ab} + \tau_{ca}}{S_{FS}} \right) - 1 = 0 \] (3.3)

Fiber compression mode:

\[ f_2 = \left( \frac{\sigma_a}{S_{ac}} - 1 \right) = 0, \quad \sigma_a' = -\frac{\sigma_a + \sigma_c}{2} \] (3.4)

Fiber crush mode:

\[ f_3 = \left( \frac{P}{S_{FC}} \right)^2 - 1 = 0, \quad P = -\frac{\sigma_a + \sigma_b + \sigma_c}{3} \] (3.5)

where \( \{ \) \) are Macaulay Brackets, \( S_{at} \) and \( S_{ac} \) are the tensile and compressive strengths in the fiber direction, and \( S_{FS} \) and \( S_{FC} \) are the layers strengths associated with the fiber shear
and crush failure, respectively.

Matrix mode failures should occur without fiber failure, and they will occur on planes parallel to the fibers. For simplicity, only two failure planes are considered: one perpendicular to the plies and one parallel to them. Thus, the matrix failure criteria are depicted by:

Perpendicular matrix mode:

\[
f_4 = \left( \frac{\sigma_b}{S_{bT}} \right)^2 + \left( \frac{\tau_{bc}}{S_{bc}} \right)^2 + \left( \frac{\tau_{ab}}{S_{ab}} \right)^2 - 1 = 0
\]  \(3.6\)

Parallel matrix mode (delamination):

\[
f_5 = S^2 \left( \left( \frac{\sigma_c}{S_{bT}} \right) + \left( \frac{\tau_{bc}}{S_{bc}} \right) + \left( \frac{\tau_{ca}}{S_{ca}} \right) \right)^2 - 1 = 0
\]  \(3.7\)

where \(S_{bT}\) is the transverse tensile strength, and \(S\) is a factor to control the delamination.

Based on the Coulomb-Mohr theory, the shear strengths for the transverse shear failure and the two axial shear failure modes are depicted by:

\[
S_{ab} = S_{ab}^{(0)} + \tan(\varphi)\{-\sigma_b\}
\]  \(3.8\)

\[
S_{bc} = S_{bc}^{(0)} + \tan(\varphi)\{-\sigma_b\}
\]  \(3.9\)

\[
S_{ca} = S_{ca}^{(0)} + \tan(\varphi)\{-\sigma_c\}
\]  \(3.10\)

\[
S_{bc}^{''} = S_{bc}^{(0)} + \tan(\varphi)\{-\sigma_c\}
\]  \(3.11\)

where \(\varphi\) is a material constant as \(\tan(\varphi)\) is similar to the coefficient of friction, and \(S_{ab}^{(0)}\), \(S_{bc}^{(0)}\) and \(S_{ca}^{(0)}\) are the shear strength values of the corresponding tensile modes. Note that
for the unidirectional model, $a$, $b$ and $c$ denote the fiber, in-plane transverse and out of plane transverse directions, respectively. Moreover, the parameters used for the elastic modulus reduction in the Material Model 161, such as, Coulomb friction angle, element eroding axial strain, limit damage parameter and so on, are based on Refs. [53, 54].

The moving impact cylinder, steel panel, and two stationary bases are modeled using Material Model 20 (Material Rigid.) The reason for modeling these parts as rigid objects lies in that these parts have much higher stiffness and density compared to the composite structure when interacted. Since, there is no storage assumed in these rigid parts, the rigid material type is very computationally efficient in our simulations.

The contact interface type “automatic surface to surface” is used to define the contact between the rigid parts and the composite structure. This contact type is selected in order to prevent the occurrence of negative volume during the simulation since the stiffness penalty scale factor can be applied to offset the stiffness difference between the composite structure and the rigid parts.

3.2.4 Boundary Conditions

The boundary conditions refer to the moving impact cylinder and steel panel. During the three-point impact test, the moving cylinder is modeled as a rigid body with five constraints. There are no displacements along the model global $y$ and $z$-axes, and no rotations about the three global basic axes. Therefore, only the displacement along $x$-axis is allowed. Our approach is similar to [55] and [56], in which the impact cylinder has two dynamic loading conditions: one of low velocity of 15.6 m/s and one of high velocity of 29.0 m/s which are obtained from speed limits of 35 Mph and 65 Mph, respectively.
During the energy absorption test, the moving steel panel is also modeled as a rigid body with five constraints. There are no displacements along the model global x and y-axes, and no rotation about the three global basic axes. Only displacement along the global z-axis is permitted. In the simulation, our approach is similar to [57, 58], that is the impact steel panel has two different constant velocities resulting in different strain rate cases.

3.3 Simulation Results

In order to consider both material cost and adequate structural stiffness, only two layers of unidirectional glass fibers are employed to fabricate the composite structure. The notation of the layup abides to the following guidelines: the number of unidirectional plies followed by the number of random fibers sequenced by position of unidirectional fibers, where the plies are separated by “/”; for example, 2U3R_1/3 indicates that the laminate has two (2) unidirectional (U) plies positioned between the 1\textsuperscript{st} and 3\textsuperscript{rd} layers (counting from the outer perimeter of the cross sectional area of the hybrid tube) through the thickness, and three (3) random (R) fiber plies.

The considered hybrid composite structure is composed of five layers from which any two are fabricated by unidirectional glass fiber material and three by random glass fiber plies, resulting in ten different combinations. Once all random (5R) and all unidirectional (5U) composite structures are included there are twelve different architectural layups, see cross sections in Figure 5.2, which need to be simulated.

Two sets of simulations were performed, namely, three-point impact tests on all 12 layup configurations and energy absorption tests on the optimal layup configuration in terms of composite structure’s stiffness.
Figure 3.2 Cross sections of twelve different arrangements of the hybrid (2-ply unidirectional (2U), 3-ply (3R) random) glass fiber composite tube. Red and blue colors refer to unidirectional lamina and random fiber lamina, respectively. Position of the unidirectional glass plies is marked as x/x notation, counting from the outer perimeter of the cross section.

3.3.1 Three Point Impact Testing

Given that the first layer is a unidirectional lamina there are four different arrangements to be tested, i.e., 1/2 (first and second layers), 1/3, 1/4, and 1/5. Figures 3.3 and 3.4 illustrate the simulation results of deflection and impact force versus time for low velocity 15.6 m/s and high velocity 29.0 m/s impact, respectively. In terms of completeness, the three-point impact simulation in all unidirectional (5U) and all-random (5R) glass fiber plies are performed.

The resulting simulation data illustrate that the random plies (5R) exhibit the highest deflection values for both the low, Figure 3.3(a), and high, Figure 3.4(a), velocity impacts while the unidirectional plies (5U) exhibit the lowest deflections, further indicating the superior (to all random plies) stiffness of the latter layup architecture. All hybrid architectures 2U3R_1/x (x=2,3,4,5) closely follow the (5U) trends demonstrating the significance of the two unidirectional plies in the overall stiffness of the hybrid structure.
Figures 3.3(b) and 3.4(b) depict the effect of the 2U3R_1/x hybrids and pure (5R) and (5U) architectures on impact force under low (15.6 m/s) and high (29.0 m/s) velocity impacts. The trends are reversed when compared to those of the deflections in Figures 3.3(a) and 3.4(a). Specifically, the unidirectional (5U) layups display significantly higher values of impact force than the random (5R) ones, closely followed by the hybrid layups. Again, the deviation of the impact force values of the hybrids and (5U) architectures from the (5R) layups increases at higher velocities. Evidently, the results denote that the use of random fiber plies (in a ratio of three to five of total plies) does not significantly alter their performance, in terms of impact force tolerance or deflection, of the hybrid structure from that of the unidirectional one, while it can result in notably cheaper composites.

Figure 3.3 (a) The deflection curves of the four 2U3R_1/x hybrids, in addition to the (5R) and 5(U) layups under low (15.6 m/s) velocity impact; (b) the impact forces of the same layups under low velocity impact. All random (5R) and all unidirectional (5U) layups form an envelope of values for the hybrid structure, with (5R) presenting the highest deflections and (5U) the lowest deflections. Inverse trends are observed for the impact load.
Figure 3.4 (a) The deflection curves of the four 2U3R_1/x hybrids, in addition to the (5R) and 5(U) layups under high (29.0 m/s) velocity impact; (b) the impact forces of the same layups under high velocity impact. All random (5R) and all unidirectional (5U) layups form an envelope of values for the hybrid structure. Deflection and impact force values are higher than the observed ones of 15.6 m/s (Figure 3.3 a-b) but trends are similar. The above-mentioned observations are further supported in the remaining six cases of 2U3R_2/x (i.e., 2U3R_2/3, 2U3R_2/4, and 2U3R_2/5), 2U3R_3/x (i.e., 2U3R_3/4, 2U3R_3/5) and 2U3R_4/5 hybrid layup architectures. Due to space limitation the graphs are not presented here, however the results for low velocity (15.6 m/s) and high velocity (29.0 m/s) impacts are reported in Tables 3.1 and 3.2, respectively. The tabulated data indicate that the hybrid configurations depart from the (5U) layup deflection and impact force values as the unidirectional plies are placed closer to the inner perimeter of the cross section and further away from the loading surface. For example, one may compare the impact force values (in parenthesis) for 5U (40635.6 N) and 2U3R_1/2 (39426.8 N) versus 2U3R_4/5 (34641.4 N) at high velocity impact. In summary, the optimal hybrid architecture in terms of high stiffness and tolerance to higher impact forces, at the discussed velocity ranges, appears to be the composite structure with first and second
layers of unidirectional lamina, namely 2U3R_{1/2}.

The area moment of each layer’s cross section can be calculated using mechanics of materials method, which are I_1=16278.7 \text{mm}^4, I_2=13140.0 \text{mm}^4, I_3=10433.3 \text{mm}^4, I_4=8126.7 \text{mm}^4, I_5=6188.0 \text{mm}^4, and the total area moment of the cross section I=54166.7 \text{mm}^4. In material of mechanics, the bending stiffness (K=EI) is used to account for the structure property subject to loads. In the three point bending test, E_U=8.6 \text{GPa}, E_R=7.95 \text{Gpa}, there are ten different configurations since the structure is composed by two plies unidirectional fiber lamina and three plies random fiber lamina. So each configuration has a different bending stiffness. Then a formula is formulated to describe each configuration’s bending stiffness as follow:

\[ K = E_{eff} I \]  
(3.12)

\[ K = E_1 I_1 + E_2 I_2 + E_3 I_3 + E_4 I_4 + E_5 I_5 \]  
(3.13)

From above equations, the effective elastic modulus for each configuration can be expressed:

\[ E_{eff} = E_1 \frac{I_1}{I} + E_2 \frac{I_2}{I} + E_3 \frac{I_3}{I} + E_4 \frac{I_4}{I} + E_5 \frac{I_5}{I} \]  
(3.14)

Where \( E_i \) (i=1-5) is either the elastic modulus of unidirectional fiber lamina (E_U) or random fiber lamina (E_R) depending on its configuration. So the corresponding effective elastic modulus are E_{1/2eff}=8.30 \text{GPa}, E_{1/3eff}=8.27 \text{GPa}, E_{1/4eff}=8.24 \text{GPa}, E_{1/5eff}=8.22 \text{GPa}, E_{2/3eff}=8.23 \text{GPa}, E_{2/4eff}=8.20 \text{GPa}, E_{2/5eff}=8.18 \text{GPa}, E_{3/4eff}=8.17 \text{GPa}, E_{3/5eff}=8.15 \text{GPa}, E_{4/5eff}=8.12 \text{GPa}.

In summary, the optimal hybrid architecture in terms of high stiffness and tolerance to higher impact forces, at the discussed velocity ranges, appears to be the composite structure with first and second layers of unidirectional lamina, namely 2U3R_{1/2} from
the above analysis. Consequently, the energy absorption testing (crash simulation) is performed to this hybrid composite.
<table>
<thead>
<tr>
<th>Description</th>
<th>Deflection (mm)</th>
<th>Difference Percentage</th>
<th>Impact Force (N)</th>
<th>Difference Percentage</th>
</tr>
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<td>34.6%</td>
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</table>

Table 3.1 Deflection and impact force data for the 10 glass fiber hybrid and (5R), (5U) composite structures subjected to low velocity (15.6 m/s) impact
<table>
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<tr>
<th></th>
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<th>Impact Force (N)</th>
<th>Difference Percentage</th>
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</table>

Table 3.2 Deflection and impact force data for the 10 glass fiber hybrid and (5R), (5U) composite structures subjected to high velocity (29.0m/s) impact
3.3.2 Energy Absorption Testing

The objective of this study is to identify and quantitatively evaluate a hybrid composite structure, from what have been considered above, in terms of energy absorption during a crash test. The motivation for using random fiber hybrid composites emerges from the need to have a high stiffness, high strength, and lightweight composite structure while reducing the manufacturing cost. Once the three point impact simulations were performed, evaluation of the resulting data indicated that the optimal layup configuration is the 2U3R_1/2 in relation to both low cost and high stiffness, designating it as the prime candidate for the specific energy absorption simulations. Three different effects are investigated during the energy absorption test, i.e., strain rate, contact area, and lamina thickness, in order to better understand the hybrid’s crashworthiness.

The energy absorption simulations are performed by impacting a steel panel on the hybrid composite tube. The boundary conditions are as described on section 2.4. The hybrid tube is crushed 8 mm in the fiber direction at five different strain rates, namely \( \dot{\varepsilon} = 0.1 \text{ s}^{-1} \), \( \dot{\varepsilon} = 1 \text{ s}^{-1} \), \( \dot{\varepsilon} = 10 \text{ s}^{-1} \), \( \dot{\varepsilon} = 20 \text{ s}^{-1} \), and \( \dot{\varepsilon} = 40 \text{ s}^{-1} \). The impact force-displacement curves at the above strain rates are shown in Figure 3.5. As illustrated, increase of the strain rate results in higher impact force values achieved at larger displacements. The total energy absorption at each strain rate is calculated by integrating its impact force-displacement curve ensuing in 291.5J (\( \dot{\varepsilon} = 0.1 \text{ s}^{-1} \)), 407.0J (\( \dot{\varepsilon} = 1 \text{ s}^{-1} \)), 461.1J (\( \dot{\varepsilon} = 10 \text{ s}^{-1} \)), 563.1J (\( \dot{\varepsilon} = 20 \text{ s}^{-1} \)), and 681.7J (\( \dot{\varepsilon} = 40 \text{ s}^{-1} \)). An appropriate deduction from the above data is that at higher strain rates the hybrid composite material’s total absorption capacity increases since the total energy absorbed is increased (due to the higher impact forces that are generated at faster compression rates.)
The crashworthiness of a structure is defined in terms of its specific energy absorption [10]. The specific energy absorption (SEA) is delegated as the energy absorption normalized by the crushed mass, which in this study is 0.007516kg, leading to corresponding specific energy absorption (SEA) values of 38.8kJ/kg (\( \dot{\varepsilon} = 0.1 \text{ s}^{-1} \)), 54.2kJ/kg (\( \dot{\varepsilon} = 1 \text{ s}^{-1} \)), 61.3kJ/kg (\( \dot{\varepsilon} = 10 \text{ s}^{-1} \)), 74.9kJ/kg (\( \dot{\varepsilon} = 20 \text{ s}^{-1} \)), and 90.7kJ/kg (\( \dot{\varepsilon} = 40 \text{ s}^{-1} \)) for the 2U3R_1/2 glass fiber hybrid, with the respective strain rates noted in parenthesis. It is seen that at higher strain rate values the specific energy absorption increases. In particular there is a 133% increase in the SEA values for \( \dot{\varepsilon} = 0.1 \text{ s}^{-1} \) to 40 s^{-1} strain rate increase, expressing high crashworthiness values at high strain rates.

Figure 3.5 Impact force versus displacement curves at different strain rates during 8 mm crashing in the fiber direction of the 2U3R_1/2 hybrid. Increase of the strain rate results in higher impact force values achieved at larger displacements.

The energy absorption testing is performed under the same strain rate loading of \( \dot{\varepsilon} = 20 \text{ s}^{-1} \), chosen as a computationally efficient -still representative of results- rate, with four different contact areas between the impact steel panel and the composite structure by
crushing 8mm in the unidirectional fiber direction. The four different contact areas are quarter contact, half contact, three quarters contact, and full area contact. The full area contact is the same as the cross sectional area of the tubular composite structure.

The impact force versus displacement curves for the four different contact areas between the steel plate and the 2U3R_1/2 hybrid are illustrated in Figure 3.6. The total energy absorption of each case is calculated by integrating its impact force-displacement curve resulting in the following values 169.6J, 337.2J, 457.5J and 563.1J for quarter, half, three quarter, and full contact areas, respectively. It is noted that increase of the contact area results in higher impact force values achieved at larger displacements denoting that the structure can withstand larger crushing forces apportioned to larger damage areas.

Since the crushed mass is 0.007516kg in each case, the corresponding SEA values are 22.6kJ/kg, 44.9kJ/kg, 60.9kJ/kg and 74.9kJ/kg for the four aforementioned contact areas. The SEA is inverse analogous to the size of the impactor’s contact area values as seen in Eq. (1) and analogous to the energy absorbed by the hybrid material, given that the crushing length and the density of the material remain constant. By further studying the simulation data, at full-scale contact the energy absorbed by the hybrid structure almost quadruples in value of the one of the quarter contact area (see Figure 3.6) indicating the efficiency by of the hybrid structure in crush energy absorption.
Figure 3.6 Impact force versus displacement curves at different steel impactor contact area sizes during 8 mm crashing in the fiber direction of the 2U3R_1/2 hybrid. Strain rate of $\dot{\varepsilon} = 20 \, \text{s}^{-1}$ during loading. Increase of the contact area results in higher impact force values achieved at larger displacements.

Figure 3.7 Impact force-deflection curves for four different unidirectional lamina thicknesses of the 2U3R_1/2 hybrid (overall thickness is 5mm.) The increased percentage of unidirectional fibers in the structure results in slightly higher impact forces in the begging of the 8mm crash simulations at $\dot{\varepsilon} = 20 \, \text{s}^{-1}$.
The total thickness of 5mm of the hybrid composite tube depicts the thickness of the two unidirectional and the three random glass fiber plies. In order to test the crushed weight effect a parametric study on the ratio of the unidirectional over the hybrid thickness is undertaken. Note that increasing unidirectional ply thickness increases the volume fraction (tighter packed fibers), and ultimately the crushed weight, of the hybrid. The energy absorption simulations are performed at $\dot{\varepsilon} = 20 \text{ s}^{-1}$ strain rate with at four different unidirectional lamina thicknesses, to be exact, 0.8mm, 1.0mm, 1.2mm, and 1.4mm, by crushing 8mm in fiber direction. Note, that there are two unidirectional laminas in the 5-ply hybrid structure. For the studied cases the unidirectional/total thickness ratio varied from 32% (1.6 mm/5.0mm), 40% (2.0mm/5.0mm), 48% (2.4mm/5.0mm) to 56% (2.8mm/5.0mm). As the total thickness of the structure remains constant (at 5mm) and the thickness of the unidirectional lamina increases, the crushed mass increases since the volume of the carbon fibers (due to the thicker unidirectional plies) within the hybrid composite increases. The resulting impact force-displacement curves are shown in Figure 3.7. The total energy absorption of each case is calculated by integrating its impact force-deflection curve, and they are 534.6J, 563.1J, 655.3J, and 663.0J, with crushed masses 0.00738kg, 0.007516kg, 0.007648kg and 0.007776kg, respectively. The corresponding specific energy absorptions are 72.4kJ/kg, 74.9kJ/kg, 86.6kJ/kg, and 85.2kJ/kg. As seen, the SAE is not considerably affected by the thickness of the unidirectional lamina during a crust test along the fiber direction. The SEA values tentatively followed the increasing trend of the unidirectional plies up to the 48% increase while after that percentage no significant results are recorded. For example, by comparing the crash mass values of 0.007648kg and 0.007776kg and their respective
SEA values of 86.6kJ/kg and 85.2kJ/kg, one may argue that as the ratio of unidirectional fibers increases (higher volume fraction) in the hybrid structure, the crushed mass also increases up to a threshold value subsequent to which the large fiber content in the hybrid delays the crack propagation in the matrix of the hybrid. This is in agreement with the crush studies for random fiber composites conducted in Ref. [44] in which the high volume fractions of random fibers caused an increase in the SAE of the random composite.

From the above finite element simulation results, the analytical solution for energy absorption can be described in terms of \( f(A) \), and \( f(\varepsilon, E_{\text{eff}}, F_{\text{ICU}}, F_{\text{ICR}}) \).

\[
W = f(\varepsilon, E_{\text{eff}}, F_{\text{ICU}}, F_{\text{ICR}}) \times f(A)
\]  
(3.15)

The effective longitudinal elastic modulus can be described using the following equation.

\[
E_{\text{eff}} = V_U E_{1U} + (1 - V_U) E_{1R}
\]  
(3.16)

Where \( V_U \) is the volume fraction of the unidirectional fiber lamina, \( E_{1U} \) and \( E_{1R} \) are the corresponding longitudinal elastic modulus of unidirectional fiber lamina and random fiber lamina, respectively. At the same time, the effective compression strength can be figured out using the following equation.

\[
F_{\text{ICeff}} = V_U F_{\text{ICU}} + (1 - V_U) F_{\text{ICR}}
\]  
(3.17)

Where \( V_U \) is the volume fraction of the unidirectional fiber lamina, \( F_{\text{ICU}} \) and \( F_{\text{ICR}} \) are the corresponding longitudinal compression strength of unidirectional fiber lamina and random fiber lamina, respectively.

From the discussion on the effect of contact sizes, the energy absorptions with different contact sizes under the specific strain rate are almost linear to their contact size
ratios. The formula for different contact sizes can be fit using Matlab.

\[ f(A) = 0.92 \frac{A}{A_{CS}} + 0.1 \]  \hspace{1cm} (3.18)

Where \( A \) is contact size, \( A_{CS} \) is the area of the cross section of the composite tube.

From the above discussions on the effect of strain rate and effective longitudinal elastic modulus and strength, the effective longitudinal elastic modulus keeps constant while the strength becomes larger as strain rate increases. So the function \( f(\varepsilon, E_{eff}, F_{ICU}, F_{ICR}) \) can be fit using Matlab.

\[ f(\varepsilon, E_{eff}, F_{ICU}, F_{ICR}) = \frac{F_{ICR}}{F_{IC_{eff}}} (0.0065\dot{\varepsilon} + 0.26) \]  \hspace{1cm} (3.19)

\[ \varepsilon = \frac{V}{t}, \dot{\varepsilon} = \frac{V}{L} \]  \hspace{1cm} (3.20)

Where \( V \) is the impact speed, \( t \) is the impact time, and \( L \) is the length of the composite tube. Then introducing the equations (3.15) to (3.19) into (3.1), the specific energy absorption can be expressed.

\[ SEA = \frac{E_{eff} \varepsilon F_{ICR}(0.0065\dot{\varepsilon} + 0.26)(0.92 \frac{A}{A_{CS}} + 0.1)}{V \rho_U + (1-V)\rho_R} \]  \hspace{1cm} (3.21)

Where \( \rho_U \) and \( \rho_R \) are densities of unidirectional fiber lamina and random fiber lamina, respectively.
Chapter 4

Damage Behavior of RFCs

The development of our finite element damage model requires definition of the damage variables which can physically describe the damage mechanisms, expression of the stiffness reduction as a function of the damage variables, determination of the damage evolution relation, and validation of the homogenized damage model.

The epoxy matrix is modeled as a linear elastic material and the damage is governed by the Modified Von Mises criterion. Its stiffness is reduced gradually after damage initiation. The fiber-matrix interface is treated by a cohesive zone technique.

The undamaged elastic properties, stiffness reduction law and damage evolution of the glass-RFCs are computed by applying numerical analysis to the RVE with proper boundary conditions shown in Figure 4.1.

![RVE for random glass/epoxy composites](image)

Figure 4.1 RVE for random glass/epoxy composites
4.1 Damage Variables

Prior to the final failure of the composites due to matrix cracking and interfacial debonding, it is assumed that the damage happens sequentially and interfacial deterioration has not yet led to fiber pull-out. The matrix microcracks initiate damage, which then propagates among the fiber bundles in the RVE; when they reach the fiber-matrix interfaces they cause interfacial decohesion. In order to characterize the damage mechanisms in composites, two damage variables $\alpha$ and $\beta$ are employed for matrix cracking and interfacial debonding, respectively.

4.2 Matrix Failure Criterion

The degradation rule for the matrix cracking is developed by using the ABAQUS User Defined Field (USDFLD) and the User Defined Material (UMAT) subroutines, which are provided in ABAQUS/Standard [28]. In the USDFLD and UMAT, a solution dependent state variable can be defined to characterize the degradation of each matrix element.
Polymeric resins generally have isotropic mechanical properties and show higher compressive than tensile strength [59], therefore hydrostatic and deviatoric stresses are incorporated to model their yield behavior [60-61]. The Raghava’s modified Von Mises Criterion [62] is applied as follows:

\[
\left( \frac{\sigma_{eq}^{cr}}{\sigma_{eq}^{cr}} \right)^2 + \frac{I_1^{cr}}{I_1^{I}} = 1
\]  

(4.1)

Where \( \sigma_{eq} = \sqrt{\frac{3}{2} S : S } \) is the Von Mises equivalent stress, \( I_1 = \text{trace}(\sigma) \) is the first invariant of the stress tensor, and \( S = \sigma - \frac{I_1}{3} I \) is the deviatoric stress tensor.

\( \sigma_{eq}^{cr} = \sqrt{C_m T_m} \) and \( I_1^{cr} = \frac{C_m T_m}{C_m - T_m} \), with \( C_m \) and \( T_m \) being the compressive and tensile strengths of the matrix, respectively. The elastic modulus of the material is reduced to approximate zero once the failure criterion is met, while the shear modulus is reduced to 20% of the original value [63-64]. The degradation of matrix, is accounted by two damage indices \( D_E \) and \( D_G \) that correspondingly reflect the reduction of elastic and shear moduli, which are governed by the maximum principle strain criterion at the material point by:

\[
D_i = A_i \phi, \quad \phi = \left( \frac{\varepsilon_{p}^{\text{max}} - \varepsilon_y}{\varepsilon_f - \varepsilon_y} \right)^{\eta}, \quad i = E, G
\]  

(4.2)

Where \( \varepsilon_{p}^{\text{max}} \) is the current maximum principal strain, \( \varepsilon_y \) is the maximum principal strain at initial yield (which is calculated directly from ABAQUS subroutine USDFLD and treated as a flag for degradation initiation of matrix), and \( \varepsilon_f \) is the maximum principal strain at
failure. Parameter $A$ represents the reduction parameter of the modulus, which maintains the stability of the numerical computation while parameter $\eta$ controls the rate of degradation. The state variable $\phi$ is defined and used in the subroutines USDFLD and UMAT, which represent the degradation process of each matrix element. Note that the degradation of a solid matrix element corresponds to the state $0 \leq \phi \leq 1$, the virgin state of a solid matrix element corresponds to the state $\phi = 0$, while failure of a solid matrix element corresponds to the state $\phi = 1$. This model describes the matrix degradation process in Wood et al [65] as depicted in Figure 4.3. Then the damage variable $\alpha$ describing the matrix crack density can be expressed as:

$$\alpha = \frac{\sum_{k=1}^{n} \phi_k V_k}{V}$$  (4.3)

Where $V_k$ is the volume of the matrix element $K$, $V$ is the whole volume of the RVE, and $\phi_k (0 \leq \phi_k \leq 1)$ is the state variable characterizing the degradation of the matrix element $K$. 
4.3 Interfacial Failure Criterion

Concurrently a cohesive zone model (CZM) is applied to characterize the fiber-matrix interfacial properties, which are governed by a traction-separation law. The cohesive element is employed to simulate the interfacial behavior. The tetrahedron element, C3D4, is used for the fiber and the matrix due to complex geometry, at the same time, the six-node cohesive element, COH3D6, is used for the fiber-matrix interface. The cohesive model utilized in this paper initially exhibits linear elastic behavior followed by initiation and exponential evolution of damage. The initial linear traction-separation law is written in terms of an elastic constitutive matrix that relates the nominal stresses to the nominal strains across the interface by interfacial penalty stiffnesses $K_n$, $K_s$, and $K_t$, where subscripts $n,s,t$ represent the normal, first shear and second shear directions of the interface. Damage initiation refers to the beginning of degradation of a material point. In this work, initiation of damage is governed by the quadratic nominal strain criterion defined by:

$$f(\varepsilon_n, \varepsilon_s, \varepsilon_t) = \left(\frac{\varepsilon_n}{\varepsilon_n^0}\right)^2 + \left(\frac{\varepsilon_s}{\varepsilon_s^0}\right)^2 + \left(\frac{\varepsilon_t}{\varepsilon_t^0}\right)^2 = 1$$  \hspace{1cm} (4.4)$$

with nominal strains $\varepsilon_n = \delta_n / T, \varepsilon_s = \delta_s / T, \varepsilon_t = \delta_t / T$ and $T$ is the constitutive thickness of the cohesive element, superscript “0” represents the peak values of nominal strain for pure loading in each direction, and the Macaulay bracket $\langle \rangle$ is used to signify that a pure compressive deformation or stress state does not initiate damage.
Once the damage initiation criterion is satisfied, the exponential softening model (after the damage has been initiated) can be defined with a scalar stiffness degradation variable $d$ as:

$$
t_n = \begin{cases} 
(1-d)\bar{t}_n, & \bar{t}_n > 0 \\
\bar{t}_n, & \bar{t}_n < 0 
\end{cases}
$$

$$
t_s = (1-d)\bar{t}_s$$

$$
t_i = (1-d)\bar{t}_i$$

(4.5)

and

$$
d = \int_{\delta_{0}^{f}}^{\delta_{0}^{T}} \frac{T_{\text{eff}}}{G_{c} - G_{0}}$$

(4.6)

Where $\bar{t}_n$, $\bar{t}_s$, and $\bar{t}_i$ are the stress components predicted by the elastic traction-separation behavior for the current strains without damage. $t$ and $\delta$ are the traction and the separation of the interface, respectively. Correspondingly, $\delta_{eff}^{0}$ and $\delta_{eff}^{f}$ are the effective separations at damage initiation and final failure of the interface. $G_c$ is the mixed-mode fracture energy and $G_0$ is the elastic energy at damage initiation. In Equation (4.6), the scalar stiffness degradation variable $d$ evolves exponentially. Note that degradation of a cohesive element refers to the state $0 \leq d \leq 1$, the virgin state of a cohesive element refers to the state $d = 0$, while failure of a cohesive element refers to the state $d = 1$. Then the damage variable $\beta$ describing the fiber-matrix interfacial debonding ratio can be expressed below, and fiber-matrix interfacial debonding and matrix cracking are dependent mechanisms because fiber will carry more loads as the matrix start to degrade, so how much of interfacial debonding between fiber and matrix interface depends on how fast the matrix cracking propagates, a relationship between them can be established such
as \( \beta = \beta(\alpha) \) through the numerical simulations on the RVE. Note that residual thermal stress developed during the manufacturing process may cause damage to the fiber-matrix interface, so the effect of residual thermal stress is also considered in the damage modeling of RVE.

\[
\beta = \frac{\sum_{k=1}^{n} d_k S_k}{S}
\]

(4.7)

Where \( S_k \) is the area of the cohesive element \( K \), \( S \) is the whole area of all the cohesive elements, and \( d_k(0 \leq d_k \leq 1) \) is the scalar stiffness degradation variable for cohesive element \( K \).

The material and interfacial properties used in our model were based on Ref. [66]. The glass fiber and epoxy Young’s moduli \( E \) are 80 GPa and 3.25 GPa, respectively, and the Poisson’s ratio \( v \) for the glass fiber is 0.2 and for the epoxy is 0.35. The tensile and compressive strengths are \( T_m = 2.150 \) MPa and \( C_m = 1.450 \) MPa for the glass fiber, and \( T_m = 80 \) MPa and \( C_m = 120 \) MPa for the epoxy, while their thermal expansion coefficients are \( 5 \times 10^{-6} \) and \( 81 \times 10^{-6} \) /°C for the glass fiber and the epoxy. The penalty stiffness components are \( K_n = 3.5 \times 10^{15} \), \( K_s = 1.3 \times 10^{15} \), and \( K_t = 1.3 \times 10^{15} \) N/m\(^3\), the nominal strains at damage initiation are \( \varepsilon_n = 2.5 \times 10^{-5} \), \( \varepsilon_s = 6.65 \times 10^{-5} \), and \( \varepsilon_t = 6.65 \times 10^{-5} \), and the critical fracture energies are \( G_n = 350 \), \( G_s = 700 \), and \( G_t = 700 \) J/m\(^2\) for the normal and two shear directions, respectively.

### 4.4 Stiffness Reduction Law

Random glass fiber composites can be treated as transversely isotropic materials having five independent stiffness components; however, there is a 10 deg. bias in the out-of-
plane direction when generating the RVE driven by the manufacturing process. In general, as the loading increases and damage accumulates in the material, the stiffness reduces due to the matrix cracking coupled with fiber-matrix debonding, represented by the damage variables $\alpha$ and $\beta$, respectively. Since the fiber-matrix interface depends on the matrix cracking, designating $\beta = \beta(\alpha)$, then the stiffness reduction law for each component can be a function of the damage variable $\alpha$. In order to suitably represent the composite stiffness while accounting for both matrix cracking and fiber-matrix debonding, a cubic polynomial of the form in equation (4.8) is chosen [27]:

$$\bar{C}_{ij}[\alpha, \beta(\alpha)] = \bar{C}_{ij}(\alpha) = \bar{C}_{ij}^0 (1 + a_\alpha \alpha + b_\alpha \alpha^2 + c_\alpha \alpha^3)$$  (4.8)

The superscript “0” indicates the undamaged state, and $a_{ij}, b_{ij}$ and $c_{ij}$ are the stiffness reduction parameters which are obtained from the numerical simulations on the RVE and shown in the Appendix A. As seen in equation (4.8), the reduction law is governed by the parameter $a_{ij}$ for the small values of the crack density, $\alpha$, so a linear variation of the composite stiffness represents a first approximation by ignoring the quadratic and cubic items. However, in advanced stages in which the crack density is important and large enough to induce the fiber-matrix decohesion, the stiffness reduction law is governed by the other two parameters $b_{ij}$ and $c_{ij}$, which reflect the importance of the interaction of the two mechanisms when both matrix cracking and fiber-matrix debonding are present. Note that the summation rule for repeating indices is not applied in (4.8), so that each stiffness reduction coefficient is related to its corresponding stiffness component.

### 4.5 Damage Evolution Relation
The effective stiffness of a glass-RFC containing matrix cracking and interfacial debonding, for a given value of the crack density or damage variable, can be determined through the numerical characterization of damage. In order to predict damage accumulation and propagation, it is necessary to formulate the constitutive relations accounting for the damage evolution. To achieve this goal, a functional form of the damage variable in terms of the local strains $\varepsilon_i$, is depicted in equation (4.9). The ensuing damage evolution law can be derived by taking the derivative on both sides of the equation (4.9) as described in equation (4.10):

$$\alpha = f(\varepsilon_i), \ i = 1 \sim 6 \quad (4.9)$$

$$d\alpha = \frac{df}{d\varepsilon_i} d\varepsilon_i, \ i = 1 \sim 6 \quad (4.10)$$

In this process, every derivative can be obtained through numerical simulations of the RVE. For example, the derivative $d\varepsilon_i/d\varepsilon_i$ can be determined through tensile loading simulation on the RVE.

The stiffness reduction law, damage evolution relation and homogenized damage model established at the micro and meso scales are then implemented in a finite element analysis for the glass-RFC at the macroscale.

**4.6 Results and Validation**

The computational procedure consists of two parts. First, the undamaged elastic properties, stiffness reduction parameters, and the damage evolution law are determined for each different strain rate. Second, a finite element analysis is carried out in ABAQUS utilizing the user subroutine UMAT to implement the damage evolution law and the constitutive relations.
The static analysis corresponding to the loading cases which are tensile loadings in three normal and two shear directions and homogenization procedure were performed on the RVE using a finite element model in ABAQUS. In the model, tensile strain is applied in five directions independently, which are three normal and two shear directions, to calculate the stiffness components. In the very early stage there is no damage in the RVE, so the undamaged stiffness matrix of the glass-RFC is expressed in equation (4.11). At the same time, the stiffness reduction law for each component starting from the undamaged state to the failure state of the composite corresponding to a given value of the crack density is also determined. The reduction of the stiffness matrix components is depicted in Figure 4.4.

\[
\bar{C} = \begin{bmatrix}
16.44 & 6.91 & 5.69 & 0 & 0 & 0 \\
6.91 & 16.24 & 5.50 & 0 & 0 & 0 \\
5.69 & 5.50 & 13.88 & 0 & 0 & 0 \\
0 & 0 & 0 & 6.76 & 0 & 0 \\
0 & 0 & 0 & 0 & 3.61 & 0 \\
0 & 0 & 0 & 0 & 0 & 3.61 \\
\end{bmatrix}
\]  

Figures 4.5-4.6 show the damage variables of matrix cracking and interfacial debonding in function of applied strain under the tensile loading in the two in-plane normal directions. Then, the stiffness reduction parameters in equation (4.8), the damage (crack density) evolution law as a function of the applied strain in equation (4.10), and the fiber-matrix interfacial debonding ratio versus crack density are obtained. For example, Figure 4.7 illustrates the relationship between the interfacial damage variable \( \beta \) and the crack density \( \alpha \) when the composite is subjected to in-plane tensile loading. Initially the increase of the interfacial damage variable is relatively small; as the matrix
cracking increases and damage is initiated at the fiber-matrix interfaces the slope of $\beta/\alpha$ increases. Numerical simulations indicate that crack density larger than 6% in the $x$-axis will catastrophically damage the composite structure.

Figure 4.4 Stiffness reductions due to matrix cracking and fiber-matrix interfacial debonding computed for the random glass/epoxy composite material.
Figure 4.5 Matrix cracking versus strain for the glass-RFC material under the tensile loading in the two in-plane normal directions

Figure 4.6 Interfacial Debonding versus strain for the glass-RFC material under the tensile loading in the two in-plane normal directions

Figure 4.7 Evolution of the interfacial debonding $\beta$ in function of the crack density $\alpha$ under the tensile loading in the two in-plane normal directions
The damage model consequently is incorporated into an ABAQUS finite element code to simulate tensile stress-strain responses, or can be employed to analyze the damage evolution of the homogenized glass-RFC subjected to quasi-static loading. In the latter case, the user subroutine UMAT is applied to implement the damage evolution law and constitutive relations established in the section 4.5.

The simulation of the tensile stress-strain responses for the homogenized random glass/epoxy composites (glass-RFC) is presented next. For this tensile testing simulation, the specimen dimensions used were 40mm × 40mm × 2mm (length × width × thickness), in which there are 32,805 nodes and 25,600 8-node brick elements.

Concurrently, the damage model formulated with ABAQUS is experimentally validated. To this end, An Instron 8821S machine was employed to conduct quasi-static tensile tests for glass-RFC specimens at a constant head speed of 2 mm/min. The specimens were cut from plate with a band saw at the following dimensions: 25.4mm × 2.5mm × 155mm (width × thickness × length), and then were milled. Strain was measured using an extensometer, and loading was recorded from a load cell mounted in-line with the tensile specimen.

The predicted tensile stress-strain response for the glass-RFC specimens using the homogenized damage model and the experimental data from three individual experiments are illustrated in Figure 4.8. A good agreement between the homogenized damage model and the experiments is observed. In the homogenized damage model, the specimens are catastrophically impaired as the tensile strain reaches 2%, which is higher than the 1.5% strain reported in Ref. [27]. (In that study, the Acoustic Emission (AE) events above 1.5%
strain were ignored in the computation, while in the experiment, the specimens broke as the tensile strain reaches about 2.2%.

Figure 4.8 Predicted (continuous line) and experimental (dotted lines) tensile stress-strain responses of the gass-RFC specimens

After a fairly linear behavior, the crack density accumulates in the nonlinear stress-strain response, and the homogenized damage model captures this effect very well. The ultimate value of the crack density refers to the highest value of the damage variables for which the composite material can’t carry any further load. Physically, this phenomenon is illustrated by the initiation and propagation of a macroscopic crack that subsequently generates interfacial debonding, and is reflected in the response by a drastic stress reduction.
Chapter 5

Strain Rates Effect

Strain rates have significant effects on dynamic behavior in composite materials when they are under dynamic loadings. In this chapter, a numerical approach with finite element code ABAQUS is developed to characterize dynamic failure criteria to express dynamic damage mechanisms of matrix cracking and interfacial debonding under uniaxial tensile loading for composite materials. A random epoxy/glass composite material is investigated under three strain rates; quasi-static, intermediate and high strain rate, corresponding to $10^{-4}$, 1, and 200 s$^{-1}$.

5.1 Background

The increasing applications of composite materials expose them to extreme loading and severe environmental conditions and bring new challenges to the composite designs. In many engineering applications the strain rate at which composite materials are loaded ranges from low quasi-static loading to high dynamic loading. Damage is a very complicated phenomenon involving many interacting failure mechanisms including matrix cracking, interfacial debonding and fiber breakage in composite materials. Several approaches have been introduced to characterize the damage for composite materials, including fracture mechanics, damage mechanics, and macroscopic failure criteria. Current developed failure criteria are still not fully reliable, especially under dynamic loading conditions. The material response occurs in very short time scales of milliseconds or even microseconds under such dynamic loadings, so the response of a structure
designed with static properties might be too conservative. The main reason is that mechanical properties of composites vary significantly as strain rate changes. Therefore, it is important to characterize the dynamic behavior as well as static behavior to develop appropriate failure criteria for composite materials containing matrix cracking and fiber-matrix interfacial debonding.

It is very necessary to characterize the strain rates effect on mechanical properties of composite materials under dynamic loadings from above argument. Many experimental methods have been introduced and developed for dynamic characterization of composite materials [67-76]. A hydraulic testing machine was used for quasi-static and low strain rates up to approximately 10 s\(^{-1}\), a drop tower apparatus was applied for strain rates between 10 s\(^{-1}\) and approximately 200 s\(^{-1}\), and a split Hopkinson pressure bar [77,78] was utilized for higher strain rates up to and exceeding 1000 s\(^{-1}\). In the study [79], the behavior of unidirectional glass/epoxy composites under uniaxial loading was determined at quasi-static and intermediate strain rates of 0.01-100 s\(^{-1}\) using experimental method; the visual inspection of the failed specimens showed significant changes in the fracture surface with increased strain rate. The change in the mechanical response as strain rate increased was associated with a change in the failure modes. In the reference [80], the quasi-static and dynamic behavior of composite materials was characterized and the failure theories to describe static and dynamic failure were developed under multiaxial states of stress. Experiments on unidirectional carbon/epoxy material were conducted at three strain rates; quasi-static, intermediate and high strain rate, referring to \(10^4\), 1 and 180 – 400 s\(^{-1}\), respectively. Both the moduli and strengths varied linearly with the logarithm of strain rate. In reference [81], the strain rates effect was conducted to study
the energy absorption for the hybrid composite structures using a numerical method, in which an analytical solution was also derived.

5.2 Model Formulation

In order to consider the strain rates effect on damage mechanisms of matrix cracking and interfacial debonding in RFC materials, two parameters $C_1$ and $C_2$ are introduced to incorporate the strain rates effect on elastic modulus and strengths of matrix [82], which are described in the equations (5.1) and (5.2), respectively. While the strain rates effect is not applied for fiber and fiber-matrix interface since their stiffnesses are very large.

\[ E = E_0 (1 + C_1 \ln \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}) \]  
\[ S = S_0 (1 + C_2 \ln \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}) \]

Where $E_0$ is the elastic modulus, and $S_0$ refers to the compressive and tensile strengths of the matrix at the reference strain rate $\dot{\varepsilon}_0$ of 1×10^{-4} s^{-1}. Then the strain rate effect on critical Von Mises equivalent stress and first invariant of the stress tensor of matrix can be derived by substituting equation (5.2) into their expressions in terms of compressive and tensile strengths described in chapter 4, and are expressed:

\[ \sigma_{eq}' = \sigma_{eq/0}' (1 + C_2 \ln \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}) \]  
\[ I_{eq}' = I_{eq/0}' (1 + C_2 \ln \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}) \]

Where $\sigma_{eq/0}'$ and $I_{eq/0}'$ are the critical Von Mises equivalent stress and first invariant of the stress tensor of matrix at the reference strain rate $\dot{\varepsilon}_0$ of 1×10^{-4} s^{-1}, respectively.
5.3 Results

By applying numerical method with corresponding boundary conditions to the RVE, the undamaged stiffness matrices of the glass-RFC at three different strain rates are expressed in equations (5.5-5.7), in which there is a little difference between the stiffness components C11 and C22 due to the bias in the fiber distribution causing the RFC materials not completely random. At the same time, the stiffness reduction laws for each component at three different stain rates starting from the undamaged state to the failure state of the composite corresponding to a given value of the crack density are also determined. The reductions of the normalized stiffness matrix components versus crack density are depicted in Figures 5.1-5.3 for three different strain rates, which are fully explained later. A numerical method can be applied to solve any stiffness component at any strain rate by applying interpolating of polynomial method.

\[
\overline{C} = \begin{bmatrix}
16.44 & 6.91 & 5.69 & 0 & 0 & 0 \\
6.91 & 16.24 & 5.50 & 0 & 0 & 0 \\
5.69 & 5.50 & 13.88 & 0 & 0 & 0 \\
0 & 0 & 0 & 6.76 & 0 & 0 \\
0 & 0 & 0 & 0 & 3.61 & 0 \\
0 & 0 & 0 & 0 & 0 & 3.61 \\
\end{bmatrix} \quad (5.5)
\]

\[
\overline{C} = \begin{bmatrix}
19.32 & 8.32 & 7.07 & 0 & 0 & 0 \\
8.32 & 19.10 & 6.86 & 0 & 0 & 0 \\
7.07 & 6.86 & 15.96 & 0 & 0 & 0 \\
0 & 0 & 0 & 7.41 & 0 & 0 \\
0 & 0 & 0 & 0 & 3.91 & 0 \\
0 & 0 & 0 & 0 & 0 & 3.91 \\
\end{bmatrix} \quad (5.6)
\]
Figure 5.1 Stiffness reductions due to matrix cracking and fiber-matrix interfacial debonding computed for the glass-RFC material at low strain rate of $1 \times 10^{-4}$ s$^{-1}$. 

\[
\bar{C} = \begin{bmatrix}
20.88 & 9.09 & 7.84 & 0 & 0 & 0 \\
9.09 & 20.62 & 7.62 & 0 & 0 & 0 \\
7.84 & 7.62 & 17.56 & 0 & 0 & 0 \\
0 & 0 & 0 & 7.75 & 0 & 0 \\
0 & 0 & 0 & 0 & 4.05 & 0 \\
0 & 0 & 0 & 0 & 0 & 4.05 \\
\end{bmatrix}
\]
Figure 5.2 Stiffness reductions due to matrix cracking and fiber-matrix interfacial debonding computed for the glass-RFC material at intermediate strain rate of 1 s\(^{-1}\)

Figure 5.3 Stiffness reductions due to matrix cracking and fiber-matrix interfacial debonding computed for the glass-RFC material at high strain rate of 200 s\(^{-1}\)

The stiffness reduction law, damage evolution relation, and homogenized damage model established at the micro and meso scales are then implemented in a finite element analysis of glass-RFC structure at macroscale for each different strain rate.

There are five loading cases which are three normal directions (two in-plane directions and one out of plane normal direction) and two shear directions (in-plane shear direction and out of plane shear direction). The epoxy/glass composite specimens break at a smaller strain; there is less matrix cracking but more interfacial debonding as the strain rate increases which can be observed from the following figures.

In the Figure 5.4, the predicted stress-strain responses from homogenized damage models for the glass-RFC material at low, intermediate, and high strain rates of 10\(^{-4}\), 1, and 200 s\(^{-1}\) under unidirectional tensile loading are shown, which conclude that the glass-
RFC specimens break at a smaller strain as strain rate increases. In physical meaning, there is much more interfacial debonding but there is less matrix cracking as the strain rate of tensile loading increases, because fiber will carry much more loading at higher strain rate, which causes more damage in the interface dominating the stiffness of RFC materials. This phenomenon is illustrated by the initiation and propagation of a macroscopic crack followed by the interfacial debonding generated, and is reflected in the response by a drastic stress reduction. Figures 5.5-5.6 show the relations between damage variables and applied strain under the tensile loading in the in-plane normal directions at three different strain rates. In physical meaning, there are corresponding values of matrix cracking and interfacial debonding at a certain applied strain since these two damage mechanisms are in function of applied loading. Figure 5.7 shows the relationship between the interfacial debonding and matrix cracking since the two damage mechanisms are dependent at three different strain rates, which are determined through the numerical simulations on the RVE. In physical meaning, there is much more interfacial debonding generated at a certain value of matrix cracking as the strain rate of tensile loading increases, because fiber will carry much more loading at higher strain rate, which causes more damage in the fiber-matrix interface and less damage in the matrix.
Figure 5.4 Homogenized damage models for the glass-RFC material at low, intermediate, and high strain rates of $10^{-4}$, 1, and 200 s$^{-1}$ under unidirectional tensile loading in the in-plane normal directions.

Figure 5.5 Matrix cracking versus strain for the glass-RFC material at the low, intermediate, and high strain rates of $10^{-4}$, 1, and 200 s$^{-1}$ under the tensile loading in the in-plane normal directions.
Figure 5.6 Interfacial debonding versus strain for the glass-RFC material at low, intermediate, and high strain rates of $10^{-4}$, 1, and 200 s$^{-1}$ under the tensile loading in the in-plane normal directions.

Figure 5.7 Evolution of the interfacial damage variable $\beta$ in function of the crack density $\alpha$ at low, intermediate, and high strain rates of $1\times10^{-4}$, 1, 200 s$^{-1}$ under the tensile loading in the in-plane normal directions.
Figures 5.8-5.9 show the damage variables of matrix cracking and interfacial debonding in function of applied strain at the low, intermediate, and high strain rates of $10^{-4}$, 1, and 200 s$^{-1}$ under the tensile loading in the out of plane normal direction. In physical meaning, there are corresponding values of matrix cracking and interfacial debonding at a certain applied strain since these two damage mechanisms are in function of applied loading. While, Figure 5.10 indicates the relationship between the interfacial debonding and matrix cracking at three different strain rates through the numerical simulations on the RVE. In physical meaning, there is much more interfacial debonding generated at a certain value of matrix cracking as the strain rate of tensile loading increases, because fiber will carry much more loading at higher strain rate, which causes more damage in the fiber-matrix interface and less damage in the matrix.

![Figure 5.8 Matrix cracking versus strain for the glass-RFC material at the low, intermediate, and high strain rates of $10^{-4}$, 1, and 200 s$^{-1}$ under the tensile loading in the out of plane normal direction](image)
Figure 5.9 Interfacial debonding versus strain for the glass-RFC material at low, intermediate, and high strain rates of $10^{-4}$, 1, and 200 s$^{-1}$ under the tensile loading in the out of plane normal direction.

Figure 5.10 Evolution of the interfacial damage variable $\beta$ in function of the crack density $\alpha$ at low, intermediate, and high strain rates of $1\times10^{-4}$, 1, 200 s$^{-1}$ under the tensile loading in the out of plane normal direction.
Figure 5.11 Matrix cracking versus strain for the glass-RFC material at the low, intermediate, and high strain rates of $10^{-4}$, 1, and 200 s$^{-1}$ under the tensile loading in the in-plane shear direction

Figure 5.12 Interfacial debonding versus strain for the glass-RFC material at low, intermediate, and high strain rates of $10^{-4}$, 1, and 200 s$^{-1}$ under the tensile loading in the in-plane shear direction
Figure 5.13 Evolution of the interfacial damage variable $\beta$ in function of the crack density $\alpha$ at low, intermediate, and high strain rates of $1 \times 10^{-4}$, 1, and 200 s$^{-1}$ under the tensile loading in the in-plane shear direction.

Figures 5.11-5.12 show the damage variables of matrix cracking and interfacial debonding in function of applied strain at the low, intermediate, and high strain rates of $10^{-4}$, 1, and 200 s$^{-1}$ under the tensile loading in the in-plane shear direction. In physical meaning, there are corresponding values of matrix cracking and interfacial debonding at a certain applied strain since these two damage mechanisms are in function of applied loading. While, Figure 5.13 indicates the relationship between the interfacial debonding and matrix cracking at three different strain rates, which can be determined through the numerical simulations on the RVE. In physical meaning, there is much more interfacial debonding generated at a certain value of matrix cracking as the strain rate of tensile loading increases, because fiber will carry much more loading at higher strain rate, which causes more damage in the fiber-matrix interface and less damage in the matrix.
Figure 5.14 Matrix cracking versus strain for the glass-RFC material at the low, intermediate, and high strain rates of $10^{-4}$, 1, and 200 s$^{-1}$ under the tensile loading in the out of plane shear directions.

Figure 5.15 Interfacial debonding versus strain for the glass-RFC material at low, intermediate, and high strain rates of $10^{-4}$, 1, and 200 s$^{-1}$ under the tensile loading in the out of plane shear directions.
Figure 5.16 Evolution of the interfacial damage variable $\beta$ in function of the crack density $\alpha$ at low, intermediate, and high strain rates of $1 \times 10^{-4}$, 1, 200 s$^{-1}$ under the tensile loading in the out of plane shear directions.

Figures 5.14-5.15 show the damage variables of matrix cracking and interfacial debonding in function of applied strain at the low, intermediate, and high strain rates of $10^{-4}$, 1, and 200 s$^{-1}$ under the tensile loading in the out of plane shear directions. In physical meaning, there are corresponding values of matrix cracking and interfacial debonding at a certain applied strain since these two damage mechanisms are in function of applied loading. While, Figure 5.16 indicates the relationship between the interfacial debonding and matrix cracking at three different strain rates, which are determined through the numerical simulations on the RVE. In physical meaning, there is much more interfacial debonding generated at a certain value of matrix cracking as the strain rate of tensile loading increases, because fiber will carry much more loading at higher strain rate, which causes more damage in the fiber-matrix interface and less damage in the matrix.
From Figures 5.17-5.22, the normalized stiffness components reduction rates for C11, C22, C12, C23, C13 increase as the strain rate increases since more interfacial debonding is generated at higher strain rate in which fiber dominates these five stiffness components. There is much more interfacial deboning generated at the same matrix crack density as strain rate increases, the reason of which lies in the stress components in the effective traction $T_{\text{eff}}$ in the equation (4.6) are much larger due to higher elastic modulus of matrix at higher strain rates, which causes damage parameter $d$ much larger as strain rates increase. So it is concluded that the damage variable $\beta$ for interfacial debonding is larger based on its definition in equation (4.7). The normalized stiffness components reduction rates for C33, C55, C66, out of plane normal and shear stiffness components decrease as strain rate increase since less matrix cracking is generated at higher strain rate in which matrix dominates these three stiffness components. For stiffness components C55 and C66, it is showed that there is a drop for the stiffness reduction rate when crack density reaches about 5.4% at high strain rate of 200 s$^{-1}$, the reason of which lies in the effect of fiber overrides the effect of matrix on the stiffness components C55 and C66 although these two stiffness components are dominated by the matrix when the interfacial deboning is large enough to reverse the trend of stiffness reduction rate. However, the normalized stiffness component reduction rate for C44, the in-plane shear stiffness component decreases as the strain rate increases because there is not as much interfacial debonding generated for C44 as for C11 and C22 because of different loading conditions at three different strain rates, in which the effect of matrix overrides the effect of fiber on the stiffness component C44 although it is dominated by the fiber, which are shown in
Figures 5.6 and 5.12 at three different strain rates, in which the matrix still actually dominates this stiffness component.

Figure 5.17 Normalized stiffness component reductions computed for C11 and C22 under low, intermediate, and high strain rates of $1 \times 10^{-4}$, 1, 200 s$^{-1}$.

Figure 5.18 Normalized stiffness component reductions computed for C12 under low, intermediate, and high strain rates of $1 \times 10^{-4}$, 1, 200 s$^{-1}$.
Figure 5.19 Normalized stiffness component reductions computed for C13 and C23 under low, intermediate, and high strain rates of $1 \times 10^{-4}$, 1, 200 s$^{-1}$

Figure 5.20 Normalized stiffness component reductions computed for C33 under low, intermediate, and high strain rates of $1 \times 10^{-4}$, 1, 200 s$^{-1}$
Figure 5.21 Normalized stiffness component reductions computed for C44 under low, intermediate, and high strain rates of $1 \times 10^{-4}$, 1, 200 s$^{-1}$.

Figure 5.22 Normalized stiffness component reductions computed for C55 and C66 under low, intermediate, and high strain rates of $1 \times 10^{-4}$, 1, 200 s$^{-1}$.

5.4 Application in Damage Analysis

A practical application of this model is illustrated through the damage analysis of a glass-RFC plate containing a central hole and subjected to tensile loading. With the
homogenized damage model, it is possible to simulate more complex shapes, such as the
damage analysis of a glass-RFC plate containing a central hole subjected to a tensile
loading under three different rates. The plate is 200 mm in length, 100 mm in width, 4
mm in thickness, and the radius of the hole is 20 mm. Figure 5.23 illustrate the 3-D finite
element mesh of the plate for the finite element analysis using ABAQUS with the UMAT
subroutine, in which the area near to the hole has a finer mesh since it is vulnerable to
stress concentration. There are 67,410 nodes and 52,916 solid elements in the finite
element model. This damage analysis will help optimize engineering design to prevent
catastrophic failure because it predicts the fatigue crack growth and interfacial debonding
characteristics in engineering composite structures.

Figure 5.24 illustrates the accumulation of damage in the glass-FRC plate around the
hole in regions where the longitudinal stresses $\sigma_{11}$ are very high at low strain rate which
refers to quasi static, $10^{-4}$ s$^{-1}$. Cracks initiate and propagate perpendicular to the loading
direction and continue their progression by branching. The damage distribution at failure
due to matrix cracking (damage variable $\alpha$) and fiber-matrix interfacial debonding
(damage variable $\beta$) at low strain rate are shown in Figures 5.25 and 5.26, respectively.
The longitudinal stresses in this region have reached the critical value 186 MPa at failure
while the remote tensile stress has attained only about 60 MPa. As illustrated in Figures
5.25-5.26, more damage including matrix cracking and interfacial debonding accumulates
in the area near to the hole because of stress concentration, in which the longitudinal
stress has attained the critical value (about 186 MPa at failure).
Figure 5.23 Finite element model used for the damage analysis of the glass-RFC plate containing a central hole subjected to tensile loading.

Figure 5.24 Contour of the longitudinal stress (MPa) $\sigma_{11}$ in the composite plate at the failure under the applied stress of about 60 MPa at low strain rate.
Figure 5.25 Distribution of the matrix microcrack density (damage variable $\alpha$) in the composite plate under the applied stress of about 60 MPa at low strain rate.

Figure 5.26 Damage accumulation of the interfacial debonding (damage variable $\beta$) in the composite plate under the applied stress of about 60 MPa at low strain rate.
Figure 5.27 illustrates the accumulation of damage in the glass-FRC plate around the hole in regions where the longitudinal stresses $\sigma_{11}$ are very high at intermediate strain rate which refers to $1 \text{s}^{-1}$. Cracks initiate and propagate perpendicular to the loading direction and continue their progression by branching. The damage distribution at failure due to matrix cracking (damage variable $\alpha$) and fiber-matrix interfacial debonding (damage variable $\beta$) at intermediate strain rate are shown in Figures 5.28 and 5.29, respectively. The longitudinal stresses in this region have reached the critical value 209 MPa at failure while the remote tensile stress has attained only about 65 MPa. As illustrated in Figures 5.28-5.29, more damage including matrix cracking and interfacial debonding accumulates in the area near to the hole because of stress concentration, in which the longitudinal stress has attained the critical value (about 209 MPa at failure).

Figure 5.27 Contour of the longitudinal stress (MPa) $\sigma_{11}$ in the composite plate at the failure under the applied stress of about 65 MPa at intermediate strain rate
Figure 5.28 Distribution of the matrix microcrack density (damage variable $\alpha$) in the composite plate under the applied stress of about 65 MPa at intermediate strain rate

Figure 5.29 Damage accumulation of the interfacial debonding (damage variable $\beta$) in the composite plate under the applied stress of about 65 MPa at intermediate strain rate
Figure 5.30 illustrates the accumulation of damage in the glass-FRC plate around the hole in regions where the longitudinal stresses $\sigma_{11}$ are very high at high strain rate which refers to 200 s$^{-1}$. Cracks initiate and propagate perpendicular to the loading direction and continue their progression by branching. The damage distribution at failure due to matrix cracking (damage variable $\alpha$) and fiber-matrix interfacial debonding (damage variable $\beta$) at high strain rate are shown in Figures 5.31 and 5.32, respectively. The longitudinal stresses in this region have reached the critical value 217 MPa at failure while the remote tensile stress has attained only about 68 MPa. As illustrated in Figures 5.31-5.32, more damage including matrix cracking and interfacial debonding accumulates in the area near to the hole because of stress concentration, in which the longitudinal stress has attained the critical value (about 217 MPa at failure).

Figure 5.30 Contour of the longitudinal stress (MPa) $\sigma_{11}$ in the composite plate at the failure under the applied stress of about 68 MPa at high strain rate
Figure 5.31 Distribution of the matrix microcrack density (damage variable $\mathcal{\alpha}$) in the composite plate under the applied stress of about 68 MPa at high strain rate.

Figure 5.32 Damage accumulation of the interfacial debonding (damage variable $\mathcal{\beta}$) in the composite plate under the applied stress of about 68 MPa at high strain rate.
Chapter 6

Conclusions and Future Work

6.1 Conclusions

In this study, a multiscale numerical approach based on finite element code ABAQUS is established to model damage in random glass fiber composites. At microscale, composite damage can occur in the form of matrix cracking and fiber-matrix interfacial debonding. A representative volume element of a random glass fiber composite is employed at mesoscale to analyze microscale damage mechanisms such as matrix cracking and interfacial debonding while the associated damage variables are defined and stiffness reduction and damage evolution laws are derived. At macroscale, the homogenized damage model is applied to predict the stress-strain response of the composite structure using finite element method. The predicted stress-strain result is compared to experimental tests, and a good agreement has been achieved between the homogenized damage model and experimental results. Finally, a case study of a random glass fiber composite plate containing a central hole subjected to tensile loading is performed to illustrate the applicability of the homogenized damage model.

Conclusions on the application of a hybrid composite structure in auto industry, damage behavior of RFCs, and strain rates effect on damage in RFCs can be drawn as follows.

6.1.1 Application of a Hybrid Composite Structure
Many hollow composite tubes have been used in industry, especially within automotive and aviation fields. Energy absorption is dependent on many parameters like fiber type, matrix type, fiber architecture, specimen geometry, processing conditions, fiber volume fraction, and so on. Random fiber lamina is composed of a dominant matrix material embedding a smaller volume fraction of fibers with better mechanical properties which can absorb more energy whereas unidirectional fiber has better stiffness properties. So the hybrid composite tube has been chosen for the study considering both stiffness and energy absorption ability.

The crashworthiness of a hybrid unidirectional/random glass fiber composite is numerically investigated using a finite element model based on LS-DYNA. Simulations of three-point impact testing and energy absorption testing are performed in order to determine the most advantageous in terms of cost, adequate structural stiffness, and layup architecture for a two unidirectional and three random ply (2U3R) hybrid glass structure.

Specifically, three-point impact simulations were performed in order to identify the stiffest hybrid structure. The data reveals that the hybrid composite with first and second layers of unidirectional lamina, 2U3R_1/2, is the stiffest from all hybrid configurations. Consequently, energy absorption simulations were carried out in order to determine the crashworthiness of the identified hybrid. The resulting data illustrates increase of the specific energy absorption (SEA) values with increased strain rate. Furthermore, the SEA is almost a linear function of the strain rate and possibly relates to the intrinsic micromechanical damage present in the complex architecture of the hybrid structure.

The ratio of the steel impactor’s contact area with respect to the hybrid tube’s cross sectional area was also investigated in terms of the SEA values. It was concluded that the
SEA increases with the increased ratio of contact area over the hybrid’s cross sectional area. Moreover, although high forces accompany the full contact area impact, the damage appears to be localized in the first half (0-4mm) of the crushed zone (8mm) of the hybrid tube.

The last parameter studied in terms of the SEA values was the thickness ratio of the unidirectional plies into the hybrid. The SEA is not considerably affected by the thickness of the unidirectional lamina during a crust test along the fiber direction. The SEA slightly increased with the increased unidirectional ply thickness since the volume fraction of the fibers is up to a threshold value after which no significant changes were observed. Finally, the analytical solution can provide some important insights on the determinations of crashworthiness for other composite structures.

6.1.2 Damage Behavior of RFCs

A multiscale numerical approach to model damage in random glass fiber composites (glass-RFCs) has been developed that accounts for matrix microcracks and matrix/fiber interfacial debonding, and their contribution to the macroscale damage response of the composite structure. Matrix cracking and interfacial debonding (microscale) responsible for the reduction of the composite stiffness components have been attributed to strain levels (3% strain) leading to the operational failure of the composite. Two damage variables have been defined characterizing the matrix crack density and the degradation of the fiber-matrix interface. The constitutive relation, damage evolution, and stiffness components reduction law have been established through a numerical homogenization (mesoscale) scheme. Consequently, the homogenized damage model is implemented into ABAQUS coupled with the UMAT subroutine to conduct the composite structural
analysis (macroscale). The soundness of this damage model was demonstrated through tensile stress-strain response simulations for glass-RFC specimens. The numerical simulations indicated that the damage model could accurately predict the tensile stress-strain response when compared to experimental data from composite specimens. Finally, the damage analysis was performed for a glass-RFC plate containing a central hole that was subjected to tensile loading. Future plans involve inclusion of matrix strain rate effects into the multiscale damage model.

6.1.3 Strain Rates Effect on Damage in RFCs

The random epoxy/glass composite material is investigated under three strain rates: quasi-static, intermediate, and high strain rate, corresponding to $10^{-4}$, 1 and 200 s$^{-1}$, respectively. A representative volume element of a random glass fiber composite is employed to analyze microscale damage mechanisms of matrix cracking and interfacial debonding, while the associated damage variables are defined and applied in a mesoscale stiffness reduction law. The macroscopic response of the homogenized mesoscale damage model is investigated using finite element analysis and validated through experiments at low strain rate.

6.2 Future Work

The present study has demonstrated the robustness of this approach. In this section, directions for future efforts are suggested to the further understanding of the mechanical behavior of RFCs.

6.2.1 Application of Hybrid Composite Structures
Currently an extensive experimental program is underway to evaluate the crashworthiness of the hybrid structures and to further validate the simulation results. Furthermore, the effect of the micro-architecture in the energy absorption of the hybrid composites at different strain rates will be further investigated.

6.2.2 Damage Behavior of RFCs

In future works, acoustic emission method for the quasi static test will be performed to characterize the matrix cracking and interfacial debonding, which will be used to validate them predicted by the numerical method applied on RVE, other fiber and matrix materials will be studied by this multiscale numerical method to develop the general constitutive damage model for RFCs.

6.2.3 Strain Rates Effect on Damage in RFCs

In future works, the uniaxial tensile tests will be performed on the glass-RFCs at intermediate and high strain rate of 1 and 200 s\(^{-1}\) to validate the homogenized damage models under these two strain rates, and the tensile tests for the damage analysis will be performed on the composite plate with a central hole to characterize the damage and failure progressions in the form of matrix cracking and interfacial debonding, which will be used to validate the results predicted by the homogenized damage models at three different strain rates of \(10^{-4}\), 1, and 200 s\(^{-1}\). And the dynamic homogenized models will be generalized to others RFC materials.
Appendix A

Stiffness Reduction Law at Low Strain Rate

IF (STATEV(1) .LE. ZERO) THEN

C(1,1) = C11
C(2,2) = C22
C(3,3) = C33
C(1,2) = C12
C(2,1) = C21
C(1,3) = C13
C(3,1) = C13
C(2,3) = C23
C(3,2) = C32
C(4,4) = C44
C(5,5) = C55
C(6,6) = C66

ELSE IF (STATEV(1) .LE. A1) THEN

C(1,1) = C11 * (ONE - 1000.D0*STATEV(1)**3 + 52.D0*STATEV(1)**2 - 3.0D0*STATEV(1))

C(2,2) = C22 * (ONE - 1000.D0*STATEV(1)**3 + 52.D0*STATEV(1)**2 - 3.0D0*STATEV(1))

$ -3.0D0*STATEV(1))$
\begin{align*}
C(3,3) &= C_{33} \times (\text{ONE} - 3.5\times\text{STATEV}(1)) \\
C(1,2) &= C_{12} \times (\text{ONE} - 2.1\times\text{STATEV}(1)) \\
C(1,3) &= C_{13} \times (\text{ONE} - 0.87\times\text{STATEV}(1)) \\
C(2,3) &= C_{23} \times (\text{ONE} - 0.87\times\text{STATEV}(1)) \\
C(4,4) &= C_{44} \times (\text{ONE} - 1.2\times\text{STATEV}(1)) \\
C(5,5) &= C_{55} \times (\text{ONE} - 5.5\times\text{STATEV}(1)) \\
C(2,2) &= C(1,1) \\
C(2,1) &= C(1,2) \\
C(3,1) &= C(1,3) \\
C(3,2) &= C(2,3) \\
C(6,6) &= C(5,5) \\
\text{ELSE} \\
C(1,1) &= C_{11} \times (1.627 - 784\times\text{STATEV}(1)) \\
C(2,2) &= C_{22} \times (1.627 - 784\times\text{STATEV}(1)) \\
\end{align*}
$ \text{*STATEV(1)**2-22.D0*STATEV(1)}$

\[ C(3,3) = C33 \times (\text{ONE} - 4.D0 \times \text{STATEV(1)**3-11.D0*STATEV(1)**2} - 3.5D0 \times \text{STATEV(1)}) \]

$ -3.5D0 \times \text{STATEV(1)}$

\[ C(1,2) = C12 \times (\text{1.83D0-1750.D0*STATEV(1)**3+362.D0}) \]

$ \text{*STATEV(1)**2-32.4D0*STATEV(1)}$

\[ C(1,3) = C13 \times (\text{ONE+150.D0*STATEV(1)**3-44.D0*STATEV(1)**2}) - 0.87D0 \times \text{STATEV(1)}) \]

$ -0.87D0 \times \text{STATEV(1)}$

\[ C(2,3) = C23 \times (\text{ONE+150.D0*STATEV(1)**3-44.D0*STATEV(1)**2}) - 0.87D0 \times \text{STATEV(1)}) \]

$ -0.87D0 \times \text{STATEV(1)}$

\[ C(4,4) = C44 \times (\text{ONE-31.D0*STATEV(1)**3+6.7D0*STATEV(1)**2}) - 1.2D0 \times \text{STATEV(1)}) \]

$ -1.2D0 \times \text{STATEV(1)}$

\[ C(5,5) = C55 \times (\text{ONE+1900.D0*STATEV(1)**3-150.D0}) \]

$ \text{*STATEV(1)**2-5.5D0*STATEV(1)}$

\[ C(2,2) = C(1,1) \]

\[ C(2,1) = C(1,2) \]

\[ C(3,1) = C(1,3) \]

\[ C(3,2) = C(2,3) \]

\[ C(6,6) = C(5,5) \]

ENDIF
References


[54] Center for Composite Materials, University of Delaware, Delaware, USA.


