DEEPENING TEACHERS' AWARENESS OF STUDENTS' MATHEMATICAL REASONING THROUGH VIDEO STUDY IN AN ONLINE COURSE

By MARJORY FAN PALIUS

A dissertation submitted to

The Graduate School of Education

Rutgers, The State University of New Jersey

in partial fulfillment of the requirements

for the degree

Doctor of Education

Graduate Program in Mathematics Education

Approved by

____

Carolyn A. Maher, Chair

____

Alice S. Alston

____

Linda J. Anthony

New Brunswick, New Jersey

January 2013

ABSTRACT OF THE DISSERTATION

DEEPENING TEACHERS' AWARENESS OF STUDENTS' MATHEMATICAL REASONING THROUGH VIDEO STUDY IN AN ONLINE COURSE

By MARJORY FAN PALIUS

Dissertation Director: Dr. Carolyn A. Maher

This research explored the hypothesis that teachers may be able to learn how to attend to children's productions of proof-like justifications if provided with illustrative examples of how students' mathematical reasoning develops through engagement in problem solving with appropriate pedagogical facilitation. An experimental online course was designed and implemented to measure how studying videos from a research collection and related literature might serve to deepen teachers' awareness of students' mathematical reasoning.

Three studies investigated teacher learning across two implementations of the course using a mixed methods approach that included analysis of a video-based pre/post assessment and discourse analysis. The assessment was administered to participants at the beginning and end of their instructional intervention. Data were collected from experimental and comparison conditions as part of a larger research project. Assessment data for various conditions were analyzed to measure change from pre to post in describing the features of mathematical arguments expressed by children in the video. Assessment data for course participants also were analyzed by individual within group for examining change in relation to what small groups discussed online in a course unit featuring different children sharing their reasoning to the same mathematical task as in the assessment video. Discourse analysis was performed on two other units with identical assignments for both implementations.

Data analysis revealed evidence of learning with 91% of teachers in the course attending sufficiently well to the details of children articulating their mathematical reasoning in the assessment video for two different argument forms. This finding was supported by a higher post-assessment growth rate for these argument forms by participants in the experimental course than those in three different comparison conditions. Discourse analysis that coded for details of attending to children's mathematical activities showed how teachers learned collaboratively by engaging in discussions online about the development of students' mathematical reasoning in through studying and reflecting on multimedia artifacts. Findings reveal the depth and breadth of their learning about children's reasoning about fraction ideas, how it develops, and how they view what they have learned as being highly relevant to teaching practices at elementary through secondary grades and beyond.

ACKNOWLEGEMENTS

I am very appreciative of the people who have supported my dissertation research. First, I would like to thank and acknowledge the members of my committee, chaired by Professor Carolyn A. Maher, and including Dr. Alice S. Alston and Dr. Linda J. Anthony. All three patiently encouraged me as I went through a lengthy process of defining the scope of my research; and all were willing to learn along with me about both the complexities of engaging in discourse analysis and how theories from the learning sciences can shape an analysis of online discussion data in the field of mathematics teacher education.

I also suspect that I would not have persevered to complete my dissertation research were it not for the encouragement of my peers in the doctoral program, most of who finished before me. I am especially grateful for my special friendship with Dr. Kate O'Hara, to whom I would be indebted had I not already supported her through the process a couple of years ago. Yet my circle of friends who have cheered me on and inspired me to join their ranks extends to so many others. In that respect, I am truly blessed since I could probably fill a page if I were to name them all.

I also owe many thanks to members of the research team for Video Mosaic Collaborative (VMC), including Robert Sigley for data management and countless other aspects of research support, Dr. James A. Maher for preliminary analyses of assessment data, and especially Dr. Elena P. Steencken for working with me to code video assessment data in the fractions strand and attain good inter-rater reliability. Yet, fundamentally, it is the vast experience and stellar reputations of my fellow investigators, Dr. Carolyn A. Maher, Associate University Librarian Grace Agnew, and Dr. Cindy E. Hmelo-Silver that enabled us to be awarded the VMC research and development grant, and therefore I am particularly appreciative for the opportunity to be their collaborator.

I would like to acknowledge the support of the National Science Foundation[1] for the current grant award, as well as for the four previous awards that have supported the long-term research agenda of Professor Carolyn Maher and several colleagues at the Robert B. Davis Institute for Learning, dating back to when this group organized their work under the moniker Mathematics Projects with Schools. My dissertation research would not have been possible without the many, many studies that were conducted by numerous scholars over decades of prior research.

Finally, I owe much gratitude to the graduate students at Rutgers University who participated in the first two implementations of the experimental online course and provided me with excellent data for research. I also appreciate the participation of other students at Rutgers who, in different courses, agreed to become part of the research by giving their responses to the video assessment as a pre- and post-test.

[1] Note that the National Science Foundation sponsors a research project for its intellectual merit and broader impact, as well as the perceived capacity of its personnel to conduct the project. While the VMC research and development project has been supported with funds from the NSF, my dissertation research is not sponsored by NSF funding. All views expressed in this dissertation are my own and do not necessarily coincide with the views of any funding agency that has sponsored the work of the Video Mosaic Collaborative.

TABLE OF CONTENTS

LIST OF TABLES AND FIGURES

CHAPTER ONE: INTRODUCTION

The research presented here addresses issues in the fields of mathematics teacher education and learning in asynchronous, online environments. The studies described in the following chapters lie at the intersection of those fields by investigating the nature of teachers' learning online from studying multimedia artifacts that illustrate how fourth graders constructed meaning of fraction ideas when researchers had created classroom conditions to support those students' critical thinking and reasoning. That is, the studies are intended to shed light on both (1) what teachers attend to when studying videos and related literature about students' mathematical learning and (2) how the design of an online learning environment may provide affordances to support teacher education. This research provides an in-depth analysis of a particular context for mathematics teacher education and is connected to a larger, long-term research agenda.

Investigating how teachers can learn about students' mathematical reasoning by studying video episodes from a seminal collection generated from more than two decades of longitudinal studies on children learning mathematics in various settings has been the subject of a research and development grant funded by the National Science Foundation[1]. The author has been a Co-Principal Investigator on the grant, and her dissertation research has been carved out of the much larger scope of work. The research team engaged in the large-scale project has two main objectives: (1) building the Video Mosaic Collaborative (VMC) repository (see www.videomosaic.org) to make publically accessible a unique video collection featuring students, followed in cohorts over time, developing mathematical ideas

[1] Research supported by the National Science Foundation grant DRL-0822204, directed by C. A. Maher with G. Agnew, C. E. Hmelo-Silver, and M. F. Palius. The views expressed in this paper are those of the author and do not necessarily reflect those of the funding agency.

and forms of reasoning; and (2) conducting design research studies on how VMC videos are used in instructional interventions with pre-service and in-service teachers. The following is a brief history of the long-term research agenda that yielded the video collection in order to give context for the overarching research hypothesis that these videos are particularly useful for teachers to learn about the development of students' mathematical reasoning.

The VMC videos come from a larger collection (about 4,500 hours of video) that is a product of multiple prior grants from the National Science Foundation,[2] which funded investigations including (1) a yearlong study in a fourth-grade classroom where students make sense of fraction as number and build conceptual understanding for operations with fractions; (2) a three-year study of informal mathematics learning with urban, middle-school students in an after-school program; and (3) a seminal longitudinal study that followed a cohort of students from elementary through high school and beyond to adulthood. The video-based research features students engaged in mathematical problem solving across multiple content strands in classroom and informal settings where the researchers established conditions to be particularly supportive of the development of reasoning. Those conditions pertain to the design of problem-solving tasks, time for exploration and revisiting, and establishing norms for both social and mathematical behaviors (Maher, 2005). Thus, researchers provided students with special opportunities to extend their mathematical learning, typically by engaging them in problem solving before formal introduction of the underlying mathematical content and algorithmic procedures in the regular curricula at their schools (Maher, Powell & Uptegrove, 2010;

---

[2] Prior research on children's mathematical learning was supported by three grants from the National Science Foundation: MDR-9053597 directed by R. B. Davis, C. A. Maher; REC-9814846 directed by C. A. Maher; and REC-0309062 directed by C. A. Maher, A. B. Powell, and K. Weber.

Maher & Steencken, 2003). Capturing students' activities on video with multiple camera views and collecting their written work has enabled detailed studies of how certain mathematical ideas were built and how forms of reasoning naturally emerged in the various learning environments. Well-documented examples of students' mathematical reasoning, which initially were identified in case studies conducted by many scholars, now populate the VMC repository.

Concurrent with construction of the VMC, researchers began using the video clips, as they were being prepared for the repository, in instructional interventions with pre-service and in-service teachers at three university sites in New Jersey and some nearby schools. The overarching research framework for these investigations has been design research as it is defined in the learning sciences (e.g., Barab, 2004, Bielaczyc, 2006; Kelly, 2003;). Briefly, design research involves developing a theoretical rationale for how people learn, which guides the design of a learning environment in order to implement that design in authentic contexts, collect and analyze data to measure the predicted learning, and co-evolve theory and design while progressing through subsequent implementations to refine the design. A goal of design research is not only to systematically engineer the learning environment, but also to generate and test hypotheses about the effects that result from systematic changes in learning interventions.

The theoretical rationale that has guided the interventions draws from particular views on the learning and teaching of mathematics (Davis, 1990; Davis & Maher, 1990; Maher, 1988; Maher, 1998; Noddings, 1990) and broader views about how people learn (Bransford, Derry, Berliner, & Hammerness, 2005). Interventions using VMC video clips have focused on the mathematical strands of counting and fractions, with pre and post assessment data to measure impact on teachers' beliefs about learning and teaching mathematics (Maher, Palius & Mueller, 2010; Maher, Landis & Palius, 2010) and on teachers' abilities to recognize students' reasoning (Maher, 2011; Maher, Palius, Maher & Sigley, 2012). Teacher learning interventions have been implemented in the context of a university course or a school-based workshop series, and generally followed one of two models (Palius & Maher, 2011). Across intervention designs, an essential component has been focus on both the provision of convincing arguments for solutions to problem-solving tasks and the details of representations that learners construct to support their articulation of arguments. Earlier intervention designs took place in face-to-face settings. Subsequently, however, several of the course-based interventions have included an online component that uses threaded discussions for reflecting on learners' own problem-solving activities, assigned videos, and related literature. Online discussions also have served as a forum to talk about implications for bringing problem solving tasks into teachers' own classrooms or even sharing results from having done so.

Researchers also designed an intervention in the fractions strand to be almost entirely online, with just a single in-person meeting near the start of the term. While a benefit of this meeting may have included giving participants an opportunity to personally connect with one another, its primary design purposes were to provide time for being introduced to building models for problem solutions with the Cuisenaire rods, experiencing how they can be used to explore fraction ideas by learning a particular way of working with them (i.e., that the rods have permanent color names but their number names are changeable), and receiving a set for continued use at home (e.g., for following videos of the children's model building, one's own further exploration, etc.).

The experimental online course, *Topics in Mathematics Education: Critical Thinking and Reasoning*, was co-developed and co-taught by PI Maher and Co-PI Palius during the Spring 2010 semester. The course was subsequently modified for instruction during a four-week summer session in 2011. This dissertation research examines teacher learning about student reasoning across the two implementations of the online course through analysis of discourse and performance on pre/post assessment measures. Chapter 2 provides an overall methodology. Chapters 3, 4 and 5, respectively, describe self-contained yet interrelated studies that build upon one another by starting with the clearly quantifiable results of analyzing assessment data on identifying specific features of children's mathematical arguments and then delving into the complexities of examining discursive data for insights into how participants are learning through online course activities. Each of the three chapters has its own set of research questions, which are framed by particular theoretical perspectives that guide the study. Although each of the study chapters ends with a discussion section, this dissertation includes a concluding chapter with an overall discussion and implications of the research.

CHAPTER TWO: METHODOLOGY

2.1 Course Design

As indicated in the previous chapter, the experimental online course, *Topics in Mathematics Education: Critical Thinking and Reasoning*, was designed as a particular learning intervention for teachers as part of a larger program of design research. The course was offered as an elective for graduate students, open to any masters or doctoral program area as well as to anyone admitted for non-degree study at the university. It was anticipated that the majority of graduate students enrolling in the course would likely be certified for secondary mathematics education and have some experience in teaching math at the middle or high school level. Hence, this course was designed for teachers with relatively strong math backgrounds so that the focus was not about learning math content or math methods but rather for learning how a particular class of children reasoned about fraction ideas through studying videos and related literature as products of research.

The videos used in the course were episodes from a yearlong study in a 4th grade classroom, in which the children engaged in problem solving prior to formal instruction of operations with fractions in the school's curriculum. These video data had been studied extensively by former doctoral students (Steencken, 2001; Bulgar, 2002; Reynolds, 2005; Yankelewitz, 2009; Schmeelk, 2010), and their collective work identified critical events in the children's building of fraction ideas and their development of mathematical reasoning, along with transcripts of video data. This work laid the foundation for the research team to prepare video clips for the VMC repository. Since the design research was underway concurrently, researchers made the video clips available for use in the experimental online course prior to their being cataloged and made publically accessible on the VMC site.

The course syllabus stated its objectives as introducing participants to literature on the obstacles to fraction learning and asking them to examine, in detail, how some students build understanding of fundamental fraction concepts and operations by exploring those ideas as a sense-making activity. An explicit intent was provoking their consideration of the way that fractions are introduced in the elementary and secondary grades by examining the critical thinking and reasoning in younger students through studying video clips and related readings from research on children's problem solving with fractions. Participants also were informed that they will be asked to consider the relationship, if any, between learning fractions and learning other topics in mathematics and the implications of meaningful mathematical learning for teacher education and professional development.

2.2 Course Implementations and Design Modification

Initial implementation of the course occurred during a 15-week semester in Spring 2010. It contained eight course units with 15 videos and ten articles assigned for study and discussion, plus a ninth unit for a group project that made available five more videos of clips still being prepared while the course was running. Units 1 through 8 each lasted one or two weeks. Each unit included one week of small group discussion; the two-week units shifted to a class-wide discussion in the second week, which happened three times. The in-person meeting to work with the Cuisenaire rods occurred at the beginning of week five, a week later than planned due to a snowstorm, which is what led to the first online class-wide discussion. The ninth unit for group project work spanned three weeks. Assessments were administered to study participants during the first week as a pre-test measure and during the final week as a post-test measure. The video assessment is described in section 2.4.1 of this chapter.

Between the first and second implementations of the course, some changes were made to evolve its design based on preliminary analysis of the online discourse and other factors. Two video clips, which had been used in Unit 8, were eliminated from the initial 15 because they were deemed unsatisfactory for their intended purpose by the researchers. One theoretical article (Pirie & Kieren, 1992) that had been in Unit 2 also was eliminated. The five video clips that had been used for group project work were paired with a recently published research paper (Schmeelk & Alston, 2010) to comprise another course unit. However, the biggest change was the modification from a semester-long duration to a much shorter summer-session period.

The second implementation of the experimental online course occurred over a four-week period during Summer 2011. It contained seven course units with 17 videos and ten articles but no group project and no class-wide discussions. With the compressed schedule, each unit lasted three to four days. The pre-assessment data collection occurred in the first unit, which also gave an assigned reading to discuss online. The face-to-face meeting for exploration with the Cuisenaire rods took place on day three, overlapping with the end of the first online unit. The post-assessment data collection occurred after the seventh unit had ended.

2.3 Participants

A total of 25 graduate students participated in the two implementations of the experimental online course. They are considered to be a population of teachers because the vast majority of them had teaching experience, ranging from less than a year of student teaching to more than 20 years teaching mathematics, and spanning across the middle, secondary and post-secondary levels, although a few were still preparing to teach. In each implementation, participants were assigned to a discussion group for the entire term.

2.3.1 Participants in the First Implementation (Spring 2010)

The first implementation of the online course enrolled 14 graduate students, split as 79% doctoral and 21% masters, and as 35.7% female and 64.3% male. One of them was in the learning sciences concentration and the other 13 were in the mathematics education concentration. Nearly all of them had some teaching experience, and two worked in staff development positions in their schools. Study participants were divided into four small groups of three or four people for online discussions. One of the doctoral students enrolled in the course was a member of the research team. He was an active contributor to online discussions; however he did not take the assessments since he assisted with their design, managed the assessment database, and analyzed data collected from interventions in the counting strand. Also, post-assessment data from one of the other study participants was discovered to be invalid. The Spring 2010 intervention thus yielded a set of 12 pre and post video-assessments for analysis in this research.

2.3.2 Participants in the Second Implementation (Summer 2011)

The second implementation enrolled 11 graduate students, who were 27% doctoral, 55% masters, and 18% non-degree study. Two of the doctoral students were taking their last elective for the educational administration and supervision program; the third was advanced in course work for mathematics education doctorate. The masters students were all in mathematics education concentration, some taking this course early in their program and others toward the end. One non-degree student was enrolled in a Master of Arts in Teaching program at a nearby university as a career change from business. The other non-degree student also was taking courses at another institution. One participant was looking ahead to student teaching in the fall. Three had 1-2 years teaching experience; four had 4-6 years; and three had 9 or more years experience as an educator. Two participants were special education teachers and eight were or had been teachers of mathematics. All of them taught at the middle or high school levels. The 11 participants were divided into two small groups for online discussion. Due to missing data from one participant, the Summer 2011 intervention yielded a set of 10 pre and post video-assessments for analysis.

2.4 Data Sources

2.4.1 Video-based Assessment for the Fractions Strand

A video-based assessment was administered to all participants in the fractions strand experimental and comparison groups in conducting the larger program of design research for the VMC project. Study participants viewed the same video as a pre and post-test measure at the beginning and end of the experimental intervention or comparison learning condition. It was administered online by providing a link to the assessment video and its transcript, sufficient time to respond to the prompts, and specific criteria that would be used to evaluate their responses. The video featured children working on and then supporting their solutions to a particular fraction task with Cuisenaire rods, which asked them to find a rod that could be given the number name one-half when the blue rod has the number name one. This particular video was chosen for the assessment because research that investigated the reasoning of urban sixth graders, featured in the assessment video, and compared it with reasoning of the fourth graders from the yearlong classroom study found that the task tends to elicit certain forms of reasoning (Maher, Mueller, & Yankelewitz, 2009). The prompt for administering the video assessment and the transcript of that video are provided in Appendix A. One also could refer to the studies presented in Chapter 3, Section 3.4.2, and Chapter 4, Section 4.2.3, both of which provide a detailed explanation of the video assessment and the particular context in which it was used.

2.4.2 Coding and Reliability for Video-based Assessment Data

The research team developed a detailed rubric to code the open-ended responses generated by study participants on the video assessment. The rubric defined features comprising four mathematical arguments made by children in the video, one of which took the form of reasoning by upper and lower bounds and three of which were reasoning by cases. Data were coded for the presence or absence of each argument mentioned in the participant's response as well as for which specific features of the argument were mentioned. Two researchers independently coded the video assessment data with 90.4% inter-rater reliability. Appendix B contains the rubric for scoring the fractions strand video assessment. Also, the study presented in Chapter 4, Section 4.2.4, provides the details from the rubric for how specific features combine to form a complete argument for each of the four different arguments.

2.4.3 Data for Discourse Analysis

Given the modifications made to the online course between its first and second implementation, there are three online course units with identical content that enable discourse analysis to make comparisons across all six small discussion groups. Table 2.1 shows the specific video and reading assignments in the comparison course units along with where in the sequence of units they occurred in the two instructional interventions.

Table 2.1: Comparable Course Units for Discourse Analysis

Course Unit Content	**Spring 2010**	**Summer 2011**
Video: "David's Upper and Lower Bound" Video: "New Rod Set" Reading: *Children's Explorations Leading to Proof* (Maher & Davis, 1995)	Unit 5	Unit 3
Video: "Meredith 1/5=2/10 part 1" Video: "Meredith 1/5=2/10 part 2" Video: "Which is larger, 1/2 or 1/3?" Video: "Brian challenges the girls' argument" Video: "Meredith 1/6 = 2/12 interview" Video: "Meredith 1/6 = 2/12 class discussion" Reading: *Tracing Fourth Graders' Learning of Fractions"* (Steencken & Maher, 2003)	Unit 6	Unit 4
Video: "Placing Fractions on a Number Line" Video: "Placing 1/3 on the Number Line" Reading: *Children's Use of Alternative Structures* (Alston, Davis, Maher & Martino, 1994) Reading: *Children's Different Ways of Thinking About Fractions* (Maher, Martino & Davis, 1994)	Unit 7	Unit 5

Even though adjustments were made in terms of the duration of the course units, and hence the amount of time that participants had to study materials and engage with one another online, the content and discussion prompts were identical for both of the course implementations. While building the Video Mosaic repository has been an ongoing process of construction, the videos used in the experimental online course had been cataloged and made available on the VMC website (www.videomosaic.org) by the time of the second implementation. As part of the cataloging process, our research team adopted a naming convention to standardize the structure of titles as part of the overall design for metadata in the VMC. Thus, what began as our names for videos based mainly on contents of clips we made (e.g., titles in Table 2.1) became standardized and enabled us to indicate when clips were part of the same session that was video recorded during prior research in school locations. Once cataloged, these videos became permanent resources with open source access, and each video has a persistent URL that serves as a direct link to its location. The Video Mosaic (VM) title and persistent URL for the video clips used in the course were provided in the course syllabus for the second implementation, although links also were provided as unit content items in the eCollege course site, which is how the videos were made available during the first implementation. Table 2.2 provides complete references for the videos, including their persistent URLs, and the related research literature that were assigned in the three course units on which discourse analysis was conducted. Also included in Table 2.2 are the prompts that were used to stimulate online discussion among the participants in their small groups.

Table 2.2: Assigned Materials and Discussion Prompts for Three Course Units

First Course Unit Analyzed (Unit 5 in Spring 2010 and Unit 3 in Summer 2011)	
Assigned Materials	*Discussion Prompts*
Assigned readings: (**1**) Maher, C. A. & Davis, R. B. (1995). Children's explorations leading to proof. In C. Hoyles and L. Healy (eds.), *Justifying and proving in school mathematics*, pp. 87-105. London: Mathematical Sciences Group, Institute of Education, University of London. **Study video clips**: (**1**) URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000054465 VM Title: *Fractions, Grade 4, Clip 1 of 4: David's upper and lower bound argument* (**2**) URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000054751 VM Title: *Fractions, Grade 4, Clip 4 of 4: Designing a new rod set* **Contribute to online discussion** per course guidelines and topical questions	For this week watch Video 1 and 2 and read *Children's Exploration Leading to Proof*. As you study the videos, pay attention to the children's sense making and arguments. Discuss the form of the arguments they make and the evidence they provide - either verbal, supported by models, or otherwise. Discuss the children's problem solving in terms of your readings thus far, citing connections where possible. For example, is there evidence of "understanding"? If so, what is that evidence? Is their evidence of obstacles? If so, what might they be?

Second Course Unit Analyzed (Unit 6 in Spring 2010 and Unit 4 in Summer 2011)	
Assigned Materials	*Discussion Prompts*
Assigned readings: (**1**) Steencken, E. P. & Maher, C. A. (2003). Tracing fourth graders' learning of fractions: Episodes from a yearlong teaching experiment. *The Journal of Mathematical Behavior, 22* (2), 113-132. **Study video clips**: (**1**) URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000059681 VM title: *Introducing Fraction Equivalence and an Exploration of Fraction Comparison, Clip 1 of 4: Equivalent fractions, a debate* (**2**) URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000059685 VM title: *Introducing Fraction Equivalence and an Exploration of Fraction Comparison, Clip 2 of 4: An introduction to proportional reasoning*. (**3**) URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000059691 VM title: *Introducing fraction equivalence and an exploration of fraction comparison, Clip 3 of 4: Proportional Reasoning Continued* (**4**) URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000059695 VM title: *Introducing fraction equivalence and an exploration of fraction comparison, Clip 4 of 4: Finding the number name for the difference between one half and one third* **Contribute to online discussion** per course guidelines and topical questions	(1) Provide a brief description of the main idea(s) in each clip. (2) Discuss your views on how teachers might (or not) find observing the videos of the children's problem solving useful. (3) Discuss the main ideas in the Steencken and Maher paper as they relate to children's building an understanding of fraction ideas.
Third Course Unit Analyzed (Unit 7 in Spring 2010 and Unit 5 in Summer 2011)	
Assigned Materials	*Discussion Prompts*
Assigned readings: (**1**) Alston, A. S., Davis, R. B., Maher, C. A., & Martino, A. M. (1994). Children's use of alternative structures. In J. P. da Ponte and J. F. Matos (Eds.), *Proceedings of the 18th Annual Conference of the International Group for the Psychology of Mathematics Education, (2)*, 248-255. Lisboa, Portugal: University of Lisboa. (**2**) Maher, C. A., Martino, A. & Davis, R. B., (1994). Children's different ways of thinking about fractions. In J. P. da Ponte and J. F. Matos (Eds.), *Proceedings of the 18th Annual Conference of the International Group for the Psychology of Mathematics Education, (3)*, 208-215. Lisboa, Portugal: University of Lisboa. **Study video clips**: (1) URL:http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000055290 VM Title: *Fraction problems, Sharing and Number Lines, Clip 3 of 5: Comparing unit fractions* (2) URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000055292 VM Title: *Fraction problems, Sharing and Number Lines, Clip 5 of 5: Placing fractions on the number line* **Contribute to online discussion** per course guidelines and topical questions	This unit includes two readings and two videos. Consider, in the video episodes, the fraction ideas that the children are exploring. (1) Provide a brief description of the main idea(s) in each clip. (2) Discuss how teachers might (or not) find observing the videos of the children's problem solving useful. (3) Discuss the main ideas in the two PME conference papers as they relate to children's building an understanding of fraction ideas.

2.5 Outline of Data Analysis in Subsequent Chapters

Each of the following three chapters is comprised of a self-contained study that investigates the nature of teacher learning about students' mathematical reasoning. The overall methodology is one of progressively deeper analysis. Chapter 3 is a study that considers the combined set of participants from both implementations of the experimental online course as a single population of teacher learners who experience an instructional intervention designed to deepen their awareness of how students develop mathematical reasoning. The measure of their learning is based on how, if at all, they change in terms of noticing specific features of children's arguments from pre to post on the video assessment. Chapter 4 is a study that probes more deeply into the process of learning by considering how interacting with other teachers in a small group to discuss online what they saw in the videos and read in the literature may shed light on measures of their learning from analysis of video assessment data. Specifically, this study analyzes the discourse of six small groups across the two implementations in the first course unit shown in Table 2.2 in conjunction with an analysis of how the members in each of those six small groups performed on their video-based assessments. Notably, this course unit features video of the fourth graders working on the same task as the sixth-grade children featured in the assessment video. Thus, the study in Chapter 4 enables direct comparison of describing children's arguments for particular forms of reasoning in two different contexts. The study in Chapter 5 delves even more deeply into the process of teacher learning through communication. It is based on a detailed discourse analysis of the six small groups' online discussions for the second and third course units shown in Table 2.2. Note that each of the studies presented in the following three chapters includes a section on methodology tailored to the specific nature of the study it reports.

CHAPTER THREE: TEACHERS LEARNING ABOUT STUDENT REASONING THROUGH VIDEO STUDY

3.1 Introduction

As indicated in Chapter 1, the study of teacher learning presented in this chapter is part of a larger project of current research[1] that builds on what has been a decades-long research agenda. The magnitude and importance of the body of prior research merit its further discussion in the upcoming section on background. For now suffice it to say that the prior research yielded a major video collection on students' mathematical learning that holds promise for current research on teachers learning about student reasoning through video study. Researchers engaged in a large-scale project have been simultaneously building a repository that makes the video collection publically accessible and conducting design research in various teacher education contexts. One such context is an experimental online course designed to deepen teachers' awareness of how students develop their reasoning of fraction ideas when videos and related literature are studied and discussed critically. Using a video-based assessment instrument, the study presented in this chapter examines what change, if any, occurred in teachers' recognition of students' reasoning from pre to post with the online course as an intervention.

3.2 Background

3.2.1 Origin of a Unique Video Collection

An extensive program of longitudinal and cross-sectional research has been ongoing for a quarter century at Rutgers University to investigate how students build mathematical

[1] Research supported by the National Science Foundation grant DRL-0822204, directed by C. A. Maher with G. Agnew, C. E. Hmelo-Silver, and M. F. Palius. The views expressed in this paper are those of the authors and not necessarily those of the funding agency.

ideas and forms of reasoning when invited to work on cognitively challenging tasks under conditions that support and encourage student engagement (Palius & Maher, 2011). The research has produced a unique video collection with over 4,500 hours of raw video that both fuels current work and already has sourced data for analyses resulting in numerous publications and over 30 doctoral dissertations that report on students' mathematical learning. Research detailing the development of student mathematical thinking generated an evolving model for video data analysis involving seven, non-linear interlacing steps (Davis, Maher & Martino, 1992; Powell, Francisco & Maher, 2003). Resulting from this analytical method, numerous studies yielded transcripts of video data and identified critical events in the development of mathematical thinking and reasoning. The current project makes videos, transcripts, and student work (when available) that emanate from seminal, long-term research accessible over the World Wide Web through efforts of a multidisciplinary team on the Video Mosaic Collaborative (www.videomosaic.org).

3.2.2 Video Mosaic Collaborative Repository and Design Research

Video Mosaic Collaborative (VMC) is a multifaceted research and development project. The teams at the Robert B. Davis Institute for Learning and at Rutgers University Libraries have been building a digital repository of videos and related resources, which mathematics education researchers have been using to conduct design research studies in teacher education. The VMC video collection features students engaged in mathematical problem solving across multiple content strands in both classroom and informal settings. The VMC collection was generated by providing students with opportunities to extend their mathematical thinking, typically by engaging them in problem solving before formal introduction of the underlying mathematical content and algorithmic procedures in the regular curricula at their schools, while capturing their activities on video and collecting their written work. Use of these data and the analytical methods described above have enabled detailed studies of how certain mathematical ideas were built and how forms of reasoning naturally emerged in the various learning environments (e.g., Maher, Powell & Uptegrove, 2010; Francisco, 2005; Maher, 2005; Steencken & Maher, 2003; Maher & Martino, 1996). Supported through multiple grants[2] from the U.S. National Science Foundation, this unique collection of video-based research includes (1) a yearlong study in a fourth-grade classroom where students make sense of fraction as number and build conceptual understanding for operations with fractions; (2) a three-year study of informal mathematics learning with urban, middle-school students in an after-school program; and (3) a seminal longitudinal study that followed a cohort of students from elementary through high school and beyond to adulthood. Well-documented examples of students' mathematical reasoning, which were initially identified through research conducted by many scholars, have now been prepared by the VMC team and populate a searchable database called the Video Mosaic.

The current design research on growth in teacher knowledge has deep roots that extend back more than 30 years. An intervention aimed at helping teachers move from a traditional, direct teaching approach of transferring mathematical information to one in which the students are actively engaged in building understanding of the mathematics was guided by a multi-part model of teacher development (Maher, 1988). Grounded in the experiences of that initial project and subsequent teacher professional development work

---

[2] Prior research on children's mathematical learning was supported by three grants from the National Science Foundation: MDR-9053597 directed by R. B. Davis, C. A. Maher; REC-9814846 directed by C. A. Maher; and REC-0309062 directed by C. A. Maher, A. B. Powell, and K. Weber.

that made use of videos (Maher, 1988; Maher, Davis & Alston, 1992; Maher, Alston, Dann & Steencken, 2000), Maher and colleagues have adapted models that utilize VMC resources in order to help teachers to attend to students' mathematical reasoning. As reported at an international conference (Palius & Maher, 2011), one model has focused on the education of pre-service teachers, and the second has been geared towards professional development of in-service teachers. These models have guided the implementation of courses and workshops that engage teachers in face-to-face settings. Researchers also have implemented a different kind of intervention model that was created specifically for use in the context of online learning with digital resources (Maher, Palius, Maher & Sigley, 2012).

In order to situate a particular study to be presented of teacher learning about student reasoning in an online course environment, key theoretical perspectives are outlined that both underlie the long-term research program and guide the current research in teacher education. The online course setting and use of videos as a context for research are then described before presenting methods used in this study.

3.3 Theoretical Perspectives

3.3.1 Constructivist Views on the Learning and Teaching of Mathematics

A multi-decade program of research led by Professors Carolyn A. Maher and Robert B. Davis has been guided by their constructivist views on the learning and teaching of mathematics (Davis & Maher, 1990; Maher & Davis, 1990; Davis, 1990). When mathematics is viewed as a sense-making activity, learners cycle through a process of creating representations, retrieving or constructing relevant knowledge, mapping representations to knowledge, and using them as a basis for action toward problem solving. Teachers, or researchers functioning in their stead, assume the role of facilitator in the learning environment, with the goal of having students view mathematics as a subject where one thinks creatively and builds understanding of important ideas. This perspective comes about from active engagement in building meaning, sometimes by making mistakes along the way, in order to learn what does and does not work to solve problems (Maher, 1988). Teachers, in their role as facilitators of student learning, can encourage students to discover various ways to solve problems and learn the big mathematical ideas and ways of reasoning, as they orchestrate opportunities for students to work together and learn from one another (Maher, 1998). It also is the teacher's role to encourage learners to create personally meaningful representations, as a sense-making basis for subsequent introduction of mathematical notation, and to foster students' abilities to communicate explanations in justification of strategies used and solutions found (Maher & Weber, 2010; Maher, Powell & Uptegrove, 2010). Supporting the growth of these functions is how mathematics teachers help their students develop along the continuum from reasoning, explaining, and justifying towards articulation of formal proof (Yackel & Hanna, 2003).

Within this constructivist perspective, pedagogy is based on creating an appropriate assimilation paradigm (Davis, 1990; Davis, 1984) as an experience through which students build representations and map to existing knowledge for constructing new ideas. It thus provides a learning opportunity in which students actively discover key ideas in their experiences. As Davis (1990, p. 102) asserts, "Your mental representations must give you the power to see new possibilities and new constraints in new situations." Teachers responsible for supporting students' ability to verify, explain, and communicate in the context of doing mathematics are faced with the challenge of developing their own adaptive expertise as educator (Bransford, Derry, Berliner, & Hammerness, 2005). That is, good teaching demands the ability to spontaneously and flexibly identify, critically evaluate, and respond in appropriate ways to instances of children's learning. Of particular concern to the field of math teacher education is helping teachers to attend to emerging forms of reasoning as children express justifications using their own language. It is from this pedagogical perspective that our studies in teacher education have been designed.

3.3.2 Conditions of the Learning Environment

Over many years and in multiple settings, Maher working with Davis and other collaborating researchers created conditions in the learning environments where their studies were conducted that supported students' building of mathematical ideas and fostered the development of their reasoning. Conditions include the design of problem-solving tasks, time for exploration and revisiting, and the establishment of norms for both social and mathematical behaviors (Francisco & Maher, 2005; Maher, 2005). Creating those conditions has been a consistent element across all of these studies (see footnote 2, p.18), whether the learners be children or adults, situated in formal or informal settings, from urban or suburban communities (Palius & Maher, 2011).

In the long-term studies with children, facilitation and questioning by researchers prompted students to offer convincing arguments, which evolved into the expectation that it should be a mathematically sound and convincing argument (Maher, 2005). Importantly, the researcher was never the sole arbitrator of what was convincing, as students also had to convince one another that solutions were valid. Thus, the learning environments supported the development of students' reasoning and justification (Maher, Powell & Uptegrove, 2010). Videotaping with multiple cameras captured the mathematical activity of the students and enabled the developmental process of their reasoning to be studied carefully. Findings from such studies inform the research community (e.g., Maher & Martino, 1996; Francisco & Maher, 2011); and videos of students reasoning and justifying in their problem solving serve as an important tool in mathematics teacher education (Maher, 2008).

3.3.3 Videos of Student Reasoning for Teacher Education

Research now seeks to identify evidence documenting whether teachers, making use of the VMC videos with transcripts of episodes that illustrate children's reasoning in mathematical problem solving, are able to build the mathematical and pedagogical knowledge necessary for recognizing components of children's reasoning. Instructional interventions with teachers are premised on the notion that they must not only know how to solve math problems, but they must also come to recognize and understand the reasoning that justifies valid solutions to those problems (Palius & Maher, 2011). The perspective of this research is that teachers must move beyond their own way of approaching and solving a problem to recognize that more than one solution strategy and form of reasoning may be valid when working on a particular task, as all students do not think alike and one of our goals is development of adaptive expertise (Bransford et al., 2005). A hypothesis of the overarching agenda of design research is that the VMC collection may provide affordances to support teachers as they develop such skills.

Studies that have been conducted thus far focused on the mathematical strands of counting/combinatorics and fractions, with pre and post assessment data to measure impact on teachers' beliefs about learning and teaching mathematics (Maher, Palius & Mueller, 2010; Maher, Landis & Palius, 2010) and on teachers' abilities to recognize students' reasoning (Maher, 2011; Maher, et al., 2012). The interventions generally followed one of two models (Palius & Maher, 2011), and some were implemented in a fully face-to-face setting while others included an online component. The study presented here uses an online course as a context for design research of an intervention that targets the fractions strand. The guiding question for this study is: What change, if any, occurs in the participating teachers' recognition of students' reasoning from pre to post, as measured by a video-based assessment instrument, with the online course as an intervention?

3.4 Methods

3.4.1 Online Course Design

Researchers designed and implemented an online course in mathematics education to be taken as an elective by graduate students. The course, *Critical Thinking and Reasoning*, is almost entirely online, with just a single in-person meeting near the start of the term. This intervention focuses on the fractions strand with tasks that involve modeling numerical situations and relationships with Cuisenaire rods (see Figure 3.1). The face-to-face meeting serves to give participants an opportunity to personally connect with one another and have time to explore the rods and how they might be used to explore fraction ideas. The problem solving with rods begins by establishing the condition that the rods have permanent color names but that their number names can vary. A few examples of tasks are given to demonstrate how the rods could be used for problem solving about fraction ideas, for instance: If we call the Orange rod one, what number name would be given to the Yellow rod? And what number name would we give to the Red rod? Now if we call the Blue rod one, what number name would be given to the Light Green rod? The intent of these activities is for participants to become comfortable with the idea of using rods and combinations of rods to define a unit, so that fractional relationships can be examined and fractions can be compared and examined using rod models (Palius & Maher, 2011).

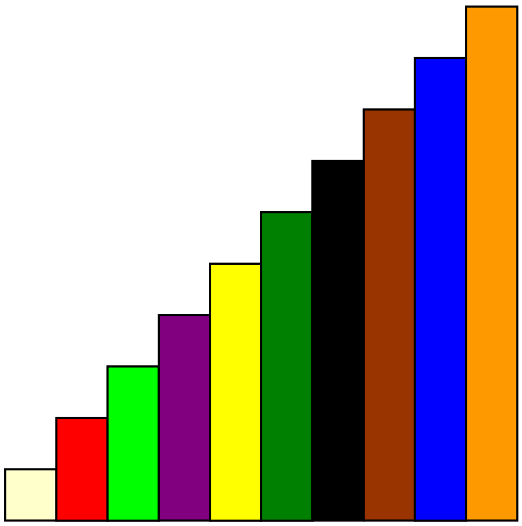


Figure 3.1: Staircase model of the set of Cuisenaire rods

An objective of this activity for the teachers is to set the stage for their subsequent viewing of children's mathematical activities captured on video and studied in the online course. The videos selected from the VMC collection feature earlier research on how the rods and corresponding tasks were used with children before they were formally introduced to fraction operations. The videos reveal that conceptual understanding of operations with fractions can naturally emerge using the rods as tools to make models, through collaboration and sharing, and through teacher facilitation of problem solving when appropriate classroom conditions are in place (Steencken & Maher, 2003).

Thus, the online course was designed to make available video clips as a tool for mathematics teacher education. One purpose was to stimulate participants to consider how children in a fourth-grade classroom study engaged in deep and critical mathematical thinking through problem solving on tasks in the fractions strand. A second purpose was to illustrate how the researcher serving as classroom teacher engaged the children and facilitated the learning environment. Participants in the course also had an opportunity to learn that certain tasks tend to elicit particular forms of reasoning (Yankelewitz, Mueller & Maher, 2010). Research literature connected to the video content was assigned as readings to comprise course units around which online discussions were focused. Consistent with the larger program of design research, the study examined teachers' attention to children's reasoning before and after the intervention. Explicitly investigated was the nature of teacher growth in identifying specific features that comprise a complete argument for different forms of reasoning expressed by children in a video-based assessment.

3.4.2 Assessment

As a pre and post–assessment, the participants were shown a video of children supporting their solutions to fraction tasks. The assessment prompted study participants to describe as completely as they can the reasoning that the children put forth, whether each argument offered by children is convincing, and why or why not are they convinced. The video includes footage from research conducted in an after-school enrichment program for sixth graders in an urban community, where children engaged in many of the same tasks that were explored by children in the fourth-grade classroom study (Maher, Mueller, & Yankelewitz, 2009). It contains short clips of children working in groups on a task to find a Cuisenaire rod in the set (see Figure 1) that could be given the number name one-half when the Blue rod has been given the number name one. It also contains short clips of children explaining their solution ideas with rod models as justification to the whole class (Maher, Mueller, & Palius, 2010). Participants were provided with a transcript for the video and were given sufficient time to respond. The assessment prompt informed participants that their responses would be evaluated by the following criteria: recognition of children's arguments, their assessment of the validity or not of children's reasoning, evidence to support their claims, and whether the warrants they give are partial or complete.

It should be noted that the children in the assessment video offered various explanations for why they found that there is no rod in the set that can be called one half when the Blue rod is called one. Some of the explanations took the form of reasoning by cases, which included identifying rods as being even (i.e., another rod in the set could be its half) or odd (i.e., there was no rod in the set that could be its half) and making an argument why the Blue rod, in particular, does not have a half because it is odd. However, one of the arguments took the form of reasoning by upper and lower bounds, which was an argument that had emerged in a prior research setting as well as the one with urban middle-school students (Maher & Davis, 1995; Yankelewitz, Mueller & Maher, 2010). An interesting aspect of the assessment video was that more than one child's discourse contributed to the articulation of certain argument forms. It was of particular interest to investigate the extent to which teachers would recognize that children were expressing in their own language that the solution for half of Blue is bounded by the Yellow and Purple rods, with Yellow being the least upper bound and Purple being the greatest lower bound (i.e., that there is no rod in between them).

3.4.3 Online Course Interventions

There were two interventions done in an online, graduate course: one in the spring semester and the other during the summer session. There were 14 graduate students enrolled in the first intervention and 11 enrolled in the second intervention, however for 3 of them there was not a set of pre and post assessment data. For the 22 study participants, 50% were male and 50% were female; 50% were master's students and 50% were doctoral; 19 specialized in mathematics education, 2 specialized in educational administration, and 1 specialized in learning sciences. All but one had teaching experience at middle school, high school, or post-secondary level, and the amount of experience ranged from less than one year to more than twenty.

The two implementations of the online course were very similar in content despite varying durations. The first intervention was semester-long and contained eight course units with 15 videos and readings plus a ninth unit for a group project that made available five more videos. The second, month-long, contained seven course units with 17 videos and readings but no group project. Notably, both interventions contained a unit that focused specifically on children's mathematical reasoning about the fractions task in the video assessment. Students were assigned to study two videos:
Fractions, Grade 4, Clip 1 of 4: David's upper and lower bound argument
(http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000054465)
and *Fractions, Grade 4, Clip 4 of 4: Designing a new rod set*
(http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000054751). The reading assignment for this unit was a book chapter that discussed children's mathematical exploration that leads toward proof-like reasoning, which included the example of David's

upper and lower bounds argument (Maher & Davis, 1995). The prompt for group online discussions was open-ended and suggested that attention be paid to forms of children's arguments and the evidence they provide, as well as consideration of what may be evidence of obstacles encountered by the children or evidence of their understanding of the mathematical ideas. Students were assigned to small groups for engaging in online discussions about the videos they had viewed and the related literature they had read.

3.4.4 Coding and Reliability

A detailed rubric was developed by the research team in order to code the data by the components of the arguments that were articulated by the children in the assessment video. Two researchers independently scored assessment data with 90.4% inter-rater reliability. For the upper and lower bounds argument, there were four components of the children's reasoning that could combine in three different ways to be a complete argument (a, b, and c; a, b, and d; or a, b, c, and d):

a. The Yellow rod is (1/2 of one White rod) longer than half of Blue; (AND)
b. Purple is (1/2 of one White rod) shorter than half of Blue; (AND)
c. There is no rod with a length that is between Yellow and Purple; (OR)
d. The White rod is the shortest rod and the difference between the Yellow rod and the Purple rod is one White rod.

Participant responses that did not mention any of the above components or that mentioned only one or two of them were deemed to be incomplete. Although it was considered a possibility that a participant might identify the upper and lower bounds argument simply by naming as such (i.e., not describing any of its features: a, b, c or d), that did not occur with this set of participant data.

For the case argument, there were three components of the children's reasoning that could combine in three different ways to make a complete argument (f and g; f and h; or f, g, and h).

f. The Blue rod is the same length as 9 White rods; (AND)

g. Nine is an odd number and cannot be divided evenly. "Halving it" would result in a decimal or remainder; (OR)

h. A rod half as long as the Blue rod would have to be the length of 4.5 White rods and there are no rods that are of "decimal" length.

Mentioning only one of these components, or g and h without f, was deemed to be incomplete, as was not mentioning any of the features of this argument. The coded data were analyzed quantitatively.

3.5 Results

The data in this study indicate that from pre to post-assessment the majority of teachers as learners in the online course implementations were able to recognize and describe in complete detail two different arguments expressed by children in the assessment video. The analysis reports the significant growth rate achieved by the 22 study participants from pre to post-video assessments. Findings are first presented for the argument by upper and lower bounds, with details of pre-assessment results in Table 3.1, detailed post-assessment results in Table 3.2, and a transition matrix in Table 3.3 to show how and where the growth in recognizing children's mathematical reasoning occurred. Note that transition was only tabulated for those participants who did not express a complete argument on the pre-assessment, as that is where there was a potential for improving in recognition and description of the reasoning.

Table 3.1: Pre-Assessment Classification Scores for Argument by Upper and Lower Bounds

Count by Number and Percentage	Complete Detailed Description			Partial Description				No Description	Total
	a,b,c	a,b,c	a,b,c,d	a	a,b	c	d	None	
Study Participants	4 18.18	2 9.09	3 13.64	1 4.55	1 4.55	1 4.55	2 9.09	8 36.36	22

Table 3.2: Post-Assessment Classification Scores for Argument by Upper and Lower Bounds

Count by Number and Percentage	Complete Detailed Description			Partial Description					No Description	Total
	a,b,c	a,b,d	a,b,c,d	b	d	a,b	a,d	b,d	None	
Study Participants	4 18.18	3 13.64	5 22.73	2 9.09	1 4.55	4 18.18	1 4.55	1 4.55	1 4.55	22

As shown in Table 3.1 above, 9 of 22 (40.9%) of study participants in the online course implementations were successful in providing a detailed complete description of the argument by upper and lower bounds on the video-based pre-assessment, and 13 of the 22 failed to do so. While Table 3.2 reports post-assessment results for all participants, Table 3.3 focuses on post-assessment outcomes for only those who failed to describe a complete argument on their pre-assessment. Of these 13 graduate students, 3 provided a detailed complete description on the post assessment; 8 others provided additional component details on their post-assessment written description of children's reasoning in the video. In total, 11 of the 13 participants exhibited growth on their post-assessment. Thus, the growth rate on description of argument by upper and lower bounds is 84.6%. A 95% confidence interval for this growth rate is 57.8% to 95.7%. Only 1 student of the 13 (7.69%) failed to observe any features of this argument.

Table 3.3: Argument by Upper and Lower Bounds Transition Matrix: Pre vs. Post Assessment Classification for Participants Who Did Not Provide A Detailed Complete Description on the Pre-Assessment*

Pre-Assessment Classification	Post-Assessment Classification									Row Totals
	Complete Detailed Description			Partial Description					No Description	
Count by Number and Percentage	a,b,c	a,b,d	a,b,c,d	b	d	a,b	a,d	b,d	None	
a							**1 7.69**			1
c	**1 7.69**									1
d					**1 7.69**			**1 7.69**		2
a,b		**1 7.69**								1
None			**1 7.69**	**2 15.38**		**4 30.77**			**1 7.69**	8
Column Totals	1	1	1	2	1	4	1	1	1	13

* Red signifies growth to a complete detailed description; green signifies growth to a more complete description; and yellow signifies no growth.

Turning to the case argument, a parallel analysis of video assessment data is reported in the following tables. The details of pre-assessment results for the case argument are shown in Table 3.4, detailed post-assessment results in Table 3.5, and a transition matrix in Table 3.6 to show how and where the growth in recognizing children's mathematical reasoning occurred. Again, note that transition was only tabulated for those participants who did not offer a complete argument on the pre-assessment because that is where there was a potential for improving in recognition and description of the reasoning.

Table 3.4: Pre-Assessment Classification Scores the Case Argument

Count by Number and Percentage	Pre-Assessment Argument Description Classification			Total
	Complete Detailed Description	Partial Description	No Description	
	f,g	h	None	
Study Participants	16 72.73	2 9.09	4 18.18	22

Table 3.5: Post-Assessment Classification Scores for the Case Argument

Count by Number and Percentage	Post-Assessment Aggregate Description Classification					Total
	Complete Detailed Description			Partial Description	No Description	
	f,g	f,h	f,g,h	f	None	
Study Participants	17 77.27	2 9.09	1 4.55	1 4.55	1 4.55	22

Table 3.6: Case Argument Transition Matrix: Pre vs. Post Assessment Classification for Participants Who Did Not Provide A Detailed Complete Description on the Pre-Assessment*

Pre-Assessment Classification	Post-Assessment Description Classification					Row Totals
	Complete Detailed Description			Partial Description	No Description	
Count by Number and Percentage	f,g	f,h	f,g,h	f	None	
H		**1** **16.67**	**1** **16.67**			2
None	**1** **16.67**	**1** **16.67**		**1** **16.67**	**1** **16.67**	4
Column Totals	1	2	1	1	1	6

* Red signifies growth to a complete detailed description; green signifies growth to a more complete description; and yellow signifies no growth.

As shown in Table 3.4, 16 of 22 (72.7%) of the study participants were successful on the pre-assessment in providing a detailed complete description of the case argument and 6 of 22 failed to do so. While Table 3.5 reports post-assessment results for all participants, Table 3.6 focuses on post-assessment outcomes for only those who failed to describe a complete argument on their pre-assessment. Of these 6 participants, 4 provided a detailed complete description on the post assessment and 1 other provided additional component details on the written description for their post-assessment. In total, 5 of these 6 graduate students exhibited growth on the post-assessment in describing the case argument; thus, the growth rate for this argument is 83.3%. A 95% confidence interval for participant growth rate on the case argument is 35.9% to 99.6%. Only 1 of 22 (4.5%) participants failed to mention the children's case argument on the post-assessment.

3.6 Discussion

The data provide evidence that by the completion of the post-assessment the majority of teachers as graduate students in the combined implementations of the online course, specifically 12 out of 22 or 54.5%, recognized and described in complete detail the children's argument by upper and lower bounds from the video assessment. By the completion of the post-assessment, all but one student provided at least a partial description of one or more major components of this argument. The data also provide evidence that by the completion of the post-assessment over 90%, specifically 20 out of 22, were able to recognize and describe in complete detail the children's case argument as presented on the video assessment.

Situating these findings in the context of the elective, online graduate course in mathematics education, the following considerations merit explicit discussion. First, it should be noted that direct instruction in recognizing and describing forms of reasoning that constitute valid mathematical arguments did not occur because it was not an objective of the course. Consistent with the theoretical perspectives outlined in this paper, the researchers who also served as instructors were interested in seeing how teachers as learners would build knowledge that they did not already have about how children as students develop and express mathematical reasoning through problem solving by having opportunities to study the process in detail through videos. That is, similarly to how the children in the videos were situated in a learning environment that supported their mathematical development, the graduate students were invited to participate in an online learning environment with tools, resources, and conditions designed to support their knowledge construction through purposeful activity.

A second consideration involves the strengths that participants may have brought to the online course in terms of their own mathematical content knowledge and prior teaching experience. For both interventions, nearly everyone had at least some teaching experience, although, for most participants, it occurred at the secondary or post-secondary level, and some at the upper-elementary or middle grades. However, the intervention that took place in the summer session included two participants taking the course as an elective for their doctoral program in educational administration and supervision. Of those two, one indicated in her personal survey (collected from all students at the start of the course) that in her nine years of being a special education teacher for grades 6-8 that she had taught mathematics although was not doing so at the time enrolled in the course. The other indicated that he has been “an educator for almost 20 years” but did not specifically indicate whether teaching math was included in his experiences. We can associate math content knowledge with teaching experience by noting that certification to teach secondary math requires minimally 30 credits of undergraduate mathematics, and teaching at the post-secondary level requires a master's degree in mathematics. Thus, the 72.7% of participants who described complete case argument and 40.9% who described complete argument by upper and lower bounds on their pre-assessment were not surprising results, given that their task was to make sense of children expressing the mathematical reasoning in their own words, as it was emerging as knowledge before instruction that introduced more formal mathematical language. Nor was it a surprising result that the one study participant who failed to describe any features of either argument was the school administrator who may never have taught mathematics at any grade level.

Further exploration of relationships among mathematical content, pedagogical knowledge and performance on video-based assessments of children's reasoning is an area for future study. It also will be interesting to examine what course participants discuss online in their study of video episodes with related readings from research literature and how, if at all, these outcomes might influence their performance on the video assessments to identify mathematical reasoning. Detailed analyses can provide further insight into how the use of VMC resources in teacher education can focus more teacher attention on the development students' mathematical reasoning, essential knowledge for teaching practice that builds on children's reasoning.

CHAPTER FOUR: EXAMINING TECHNOLOGY AFFORDANCES FOR TRIGGERING TEACHERS' LEARNING ABOUT STUDENTS' MATHEMATICAL REASONING

4.1 Introduction

In the inaugural issue of the International Journal of Computer Supported Collaborative Learning, Suthers (2006) offered a proposal for integrating a research agenda for the field that centered on technology affordances for intersubjective meaning making. Through consideration of similarities and differences among theoretical perspectives and methodologies being put forth by a range of investigative scholars, it was suggested that multiple vantage points might be drawn upon for taking "an eclectic approach" to tackle the complex problem of "what counts as interpretive acts and what those acts mean for the learning of individuals and groups" (Suthers, 2006, p. 332). This might be viewed as a call for creative framing of research by drawing relevant elements from multiple theories of learning and combining methodologies to harness the strengths of different approaches. It is suggested that doing so may shed light on the complexities of examining how learning transpires in online environments and other situations where people make use of technology tools and multimedia resources.

The study presented here is part of a larger research project[1] to investigate how teachers can learn about students' mathematical thinking and reasoning through studying videos of students engaged in mathematical activities. The videos have been purposefully selected from a unique collection emanating from a quarter century of research on how mathematical thinking and reasoning develop in students through engagement in problem

[1] Research supported by the National Science Foundation grant DRL-0822204, directed by C. A. Maher with G. Agnew, C. E. Hmelo-Silver, and M. F. Palius. The views expressed in this paper are those of the authors and not necessarily those of the funding agency.

solving under conditions that support collaborative work, sense making, communication, and critique of ideas (Maher, Powell & Uptegrove, 2010; Francisco & Maher, 2005; Maher, 2005; Maher & Martino, 1996; Davis, Maher & Martino, 1992). The videos have been made publically accessible for use by researchers, teacher educators, and teachers through Video Mosaic Collaborative (VMC, see http://videomosaic.org/). Investigators have been conducting design research on teacher learning through use of videos and related resources in a variety of instructional interventions, typically based on one of two models implemented in a course or workshop setting (Palius & Maher, 2011). Studies that our team conducted in earlier iterations were situated in face-to-face learning environments (Maher, Palius & Mueller, 2010; Maher, Landis & Palius, 2011). Subsequent designs have made increasing use of online platforms for implementing interventions for teachers to study and learn from video resources.

The scope of the larger project not only involves design research but also employs empirical methods that include pre and post-assessments as well as comparison groups for experimental interventions. Thus, the larger project has provided opportunity for a study within it that synthesizes iterative design, empirical methods, and descriptive methods to take a deep and multifaceted look at how participants accomplish learning.

Situated in an online, graduate-level mathematics education course, the study reported here analyzes participants' discourse and assessment data to examine the extent to which they attend to and describe students' mathematical reasoning on a problem-solving task in the fractions strand. This task, described in section 4.2.3, was selected as focal context for study because prior research comparing the problem solving of two different populations of students has shown that the task tends to elicit particular forms of reasoning (Yankelewitz, Mueller & Maher, 2010). Yet this study is not constrained by sole focus on argument features that comprise such forms of reasoning, as it also examines the nature of what the participants chose to discuss online in their small groups. In sum, this study examines participant learning as intersubjective meaning making both in terms of particular, discipline-specific knowledge and in terms of "discovering affinities with others, orienting attention, expressing viewpoints, exposing conflict and consensus, and supporting debate and negotiation" (Suthers, 2006, p. 332).

4.1.1 Framing the Study

Intersubjective meaning making, or "the joint composition of interpretations" (Suthers, 2006, p. 321), is a useful perspective to take for examining how small groups of participants engage jointly in the activity of sharing their thoughts about a common set of stimuli, to which they can also bring to bear their distinct, individual experiences. The joint activity of participants entails viewing videos and reading articles related to the videos, then sharing their interpretations via threaded discussions in an online learning environment. Hence, this study is guided by an online interaction learning model (Benbunan-Fich, Hiltz & Harasim, 2005) that considers characteristics of the technology, instructor, individual students, and course context as four inputs that influence how the online course is conducted. These four input variables shape the learning processes and characterize the mode in which the technology is adapted and used. The learning processes mediate the input variables and lead to outcomes. While the model specifies five outcomes, including faculty and student satisfaction, access, and cost effectiveness, student learning is the only one relevant to the study reported here.

In the online interaction learning model, it is suggested that examination of the learning processes might include questions about the amount and type of interactions as well as the number, type and length of contributions made by the students and the instructor (Benbunan-Fich, Hiltz & Harasim, 2005). Characterizing the learning process in this manner points toward descriptive methods of data analysis. While such findings could be summarized with categorical counts, it is argued that presenting results that way does little to reveal the active process of collaborative construction of knowledge (Suthers, 2006). Thus, there is merit in augmenting quantified contributions with descriptive details that serve as examples illustrating the nature of particular interactions, which may vary across the small groups.

While some elements of course context and instructor characteristics are indicative of methodological choices made by researchers, certain aspects reflect underlying theories about learning processes. Permeating this study is an enduring adherence to constructivist views on the learning and teaching of mathematics (Davis, Maher & Noddings, 1990), which have deeply influenced not only the design of the online course but also two decades of prior research on student mathematical learning that yielded the video collection, and scholarly analyses based on that video data (e.g., Maher, 2005; Steencken & Maher, 2003; Maher & Martino, 1996) from which course materials were selected. Briefly, this view of mathematical learning is described as the process of assimilating new experiences into the knowledge built up from previous experiences (Davis & Maher, 1990). Learners are actively involved in the knowledge construction process, which takes place through actions such as problem solving and interactions such as communication with the instructor or with peers when participating in a group endeavor. Within this view of learning, the role of the teacher is to listen carefully to students' ideas, to facilitate discussions, and to "probe for better understanding of student thinking through appropriate questioning that is related to students' constructions" (Maher, 1998, p. 39). Group learning affects cognitive processes through seeking resolution for disagreements, internalizing knowledge from interactions with more knowledgeable peers, and deepening one's own understanding through providing explanations to others (Webb, 1982). Constructivist perspectives on learning mathematics, and the effects of group learning in particular, are taken from face-to-face environments and applied to online learning environments. Moreover, processes deemed effective for learning mathematics are similarly valued for mathematics teacher education. In addition to teachers engaging as learners in mathematical problem solving, teachers studying videos of students engaged in solving the same tasks where they, too, are encouraged to provide justifications for solutions is an essential component of the teacher education model (Palius & Maher, 2011). The hypothesis that a particular video collection serves as a useful resource for teachers to become more attentive to students' reasoning (Maher, 2008) underlies the larger research project as well as the study presented here.

4.1.2 Teachers Learning from Videos

Several researchers have been using videos in mathematics teacher education in a variety of contexts that include math methods courses for pre-service teachers (McCrory, Putnam & Jansen, 2008; Santagata, Zannoni & Stigler, 2007; Star & Strickland, 2008) as well as video clubs (Sherin & Han, 2004), lesson study (Alston, Basu, Morris & Pedrick, 2011), and other school-based professional development (Borko, Jacobs, Eiteljorg & Pittman, 2008; Nemirovsky & Galvis, 2004). Although some contexts for teacher learning from video are conducted entirely face-to-face, others incorporate a blending of classroom sessions for targeted learning with online video analysis activities (Santagata, Zannoni & Stigler, 2007) or are conducted fully online (Koc, Peker & Osmanoglu, 2009; McCrory, Putnam & Jansen, 2008; Nemirovsky & Galvis, 2004). Given the richness of video as an educational medium, it is not surprising that a common concern across contexts is focusing teachers' attention on particular aspects of classroom events observable from the video, even though it varies what is targeted for focus and what methods are used for directing focus of attention.

For example, Star and Strickland (2008) argue that when educating pre-service teachers there is a fundamental need to develop first their abilities to identify what is noteworthy about classroom situation (van Es & Sherin, 2002). Thus, researchers assessed a wide range of what participants did and did not notice, from relatively trivial classroom features to substantive mathematical activities, in two lessons from the TIMSS videos. Although the research was limited to what changed in pre-service teachers' noticing about classroom video in five observation categories from beginning to end of the methods course, researchers speculated that an assignment involving a second viewing of pre-assessment video and use of an observational framework may have contributed to improved noticing (Star & Strickland, 2008).

In contrast, professional development with classroom teachers tends to specifically target attending to students' mathematical thinking (Alston, Basu, Morris & Pedrick, 2011; van Es & Sherin, 2008). Earlier research involving in-service teachers as participants in video clubs defines "noticing" as attending to what is important in a particular situation, making connections between specific events and broader ideas, and using what one knows about the context to reason about situations (van Es & Sherin, 2002). A more recent study with seven elementary grade teachers participating in a series of video club meetings with facilitating researchers examined video filmed in their classrooms (van Es & Sherin, 2008). The researchers selected and transcribed excerpts that focused on students' mathematical thinking so that the five-to-seven minute videos served as artifacts for group discussion. By prompting teachers with questions ranging from what do you notice, to what evidence in the video supports your idea, to how do you interpret the child's thinking, researchers found an increase in percentage of comments focused on mathematical thinking over the ten sessions as well as three different developmental pathways for learning to notice (van Es & Sherin, 2008). An adaptation of lesson study that focuses teacher attention onto task design and uses students' written work as well as video artifacts in debriefing discussions also has successfully elicited teacher commentary on students' mathematical thinking (Alston, Basu, Morris & Pedrick, 2011). In both of these models of teacher professional development, the role of the researcher in facilitating group discussion in a face-to-face setting is a significant factor contributing to growth of teachers' observational skills.

Engaging teachers in use of video analysis tools is another mechanism for focusing their attention. Such tools have been tested with teachers pursuing alternate route to certification to probe the extent to which the software scaffolds their noticing skills (van Es & Sherin, 2002) and implemented over a two-year period as part of a teacher education program at an Italian university (Santagata, Zannoni & Stigler, 2007). While the former study focused on how teachers selected video and corresponding transcript as evidence to support interpretations of observed events (van Es & Sherin, 2002), the latter study delved into analysis of lessons, identification of learning goals, and analysis of teaching strategies with suggestions of how they might be improved (Santagata, Zannoni & Stigler, 2007).

Another contrast between the two studies just mentioned is the analysis of video from teachers' own classrooms versus analyzing lessons taught in someone else's classroom. In science teacher education, researchers have examined the affordances and limitations of teachers studying videos from their own classroom versus videos with others teaching (Seidel, Stürmer, Blomberg, Kobarg & Schwindt, 2011; Zhang, Lundeberg, Koehler & Eberhardt, 2011). An interesting compromise is the use of a video case where the recorded teacher participates in the learning environment neither as student nor instructor, but as a knowledgeable person who responded to but never initiated topics of discussion (Koc, Peker & Osmanoglu, 2009). Yet it seems that the most salient concern is whether the video episodes selected for use in teacher education contain material well suited for achieving the desired instructional goals. For instance, when searching for appropriate segments of video from teachers' own classrooms, it was found that "some clips seemed to provide greater access to student thinking than other clips" (van Es & Sherin, 2008, p. 263).

Concerning use of video in online learning environments for teachers, another scaffold deployed by facilitating researchers is to "seed" discussion threads (Nemirovsky & Galvis, 2004). That is, to foster online discussions grounded in the video cases being studied by participating teachers, the researchers contributed to the discussion area by posing a question that elicited responses from participants. This generated data that enabled analysis of which seeds produced what type of responses, ranging from general views on a topic, to remarks about overall quality of video case or general characteristics of participants' own classroom, to commentary grounded in specific happenings of the video case or specific events in participants' own classroom. Although the researchers adjusted their moderation of the first group so that the second group had more discussion seeds, the number of grounded postings increased but still constituted less than 20% of total postings (Nemirovsky & Galvis, 2004). This finding corroborates what McCrory et al. (2008, p. 177) found in analyzing online discussion data from their teacher education courses, namely that "Students control how they participate and they can exercise that control regardless of what structures are in place or what assignments are given." The implication from their research suggests that online courses should seek to establish norms that invite student participation and guidelines to support their engagement.

4.1.3 Research Questions

Returning to the particular research context for offering the online course in which the study presented here is situated, it is important to emphasize that instructional design is constructivist and research-oriented to see what teacher learners will do and say in their participation with minimal intervention by the instructor. Based on views that a particular video collection serves as a useful resource for teachers to become more attentive to students' reasoning (Maher, 2008), the following questions were investigated: (1) What evidence, if any, supports the hypothesis that participation in online group discussions can deepen teachers' awareness of students' mathematical reasoning when stimulated by study of video and related research literature? (2) How, if at all, do results from pre- and post-assessments differ between those participating in the experimental online course and participants in comparison learning environments? (3) What is the nature of teachers' intersubjective meaning making as participants in online threaded discussions?

4.2. Methodology

4.2.1 Online Course Design

An online course was designed to examine the nature of teacher learning about students' mathematical thinking and reasoning through study of videos, related readings, and group discussion of ideas. The course, entitled Critical Thinking and Reasoning, focused on student learning of fraction ideas through study of videos from a longitudinal study in a fourth grade classroom, in which students engaged in problem solving prior to formal instruction of operations with fractions in the school's curriculum. With the exception of a single on-campus meeting, all course work was recorded within the eCollege online course platform.

Initial implementation of the course occurred during a 15-week semester in Spring 2010. The course was subsequently revised and implemented in a four-week Summer Session during 2011 with the same instructor. The two implementations were very similar in content despite varying durations. The first contained eight course units with 15 videos and ten readings plus a ninth unit for a group project that made available five more videos. The second contained seven course units with 17 videos and ten readings but no group project. Course objectives focused on learning about how children reason about fraction ideas through introducing participants to literature on the obstacles to fraction learning and asking them to examine, in detail, how students build understanding of fundamental fraction concepts and operations. Participants were given access to videos and related data from studies of children's problem solving with fractions that are stored on the Video Mosaic Collaborative (VMC) site. It was explicitly stated that the course also is intended to provoke consideration of the way that fractions are introduced in the elementary and secondary grades by examining the critical thinking and reasoning in younger students by studying video clips and related readings, working on strands of fraction problems, and examining the learning of fraction ideas as a sense-making activity. Course objectives also included asking participants to consider the relationship, if any, between learning fractions and learning other topics in mathematics and the implications of meaningful mathematical learning for teacher education and professional development.

For both sections, course work consisted of engaging in online discussions about assigned readings and videos, attending one face-to-face meeting early in the term to learn a particular way of working with the Cuisenaire rods (i.e., that the rods have permanent color names but their number names are changeable) and get a set for use at home, and writing a final reflection paper. Course work included taking the pre and post-tests using assessment instruments created for the VMC design research in the fractions strand. The video-based assessment, in which participants provide written description of children's mathematical reasoning on a task from the fractions strand, is one focal point of this study and outlined in the upcoming section on data sources.

The study's second focal point is a course unit that focused specifically on children's mathematical reasoning about the same problem-solving task that was being explored by a different group of children in the assessment video. Participants were assigned to study two videos, *Fractions, Grade 4, Clip 1 of 4: David's upper and lower bound argument* (http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000054465) and *Fractions, Grade 4, Clip 4 of 4: Designing a new rod se*t (http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000054751). The videos were paired with a reading assignment about children's mathematical exploration that leads toward proof-like reasoning, which features the example of David's upper and lower bounds argument (Maher & Davis, 1995). The prompt for each group's online discussion suggested that attention be paid to forms of children's arguments and the evidence they provide, as well as consideration of what may be evidence of obstacles encountered by the children or of their understanding of the mathematical ideas.

4.2.2 Participants

4.2.2.1 Experimental Groups

These study participants are graduate students enrolled in one of two sections of the online course, which were offered in different terms, as described above. The initial section enrolled 14 students, all but one from mathematics education concentration, with 79% doctoral and 21% masters, and with 35.7% female and 64.3% male. Nearly all of them had some teaching experience, ranging from less than a year of student teaching to more than 20 years teaching mathematics, at the middle, secondary and post-secondary levels. Two worked in staff development positions in their schools. These students were divided into four small groups of three or four people for online discussions (groups 1, 2, 3 and 4).

The subsequent section of the course enrolled a slightly smaller number of students with more varied backgrounds. The 11 graduate students were 27% doctoral, 55% masters, and 18% non-degree study. Two of the three doctoral students were taking their last class, an elective, for the educational administration and supervision program; and the third was advanced in course work for mathematics education doctorate. The six masters students were all in mathematics education concentration, some taking this course early in their program and others toward the end. One of the non-degree students was enrolled in a Master of Arts in Teaching program at a nearby university and was making career change from business to education. The other non-degree student also was taking courses at another institution. The participants also varied in their teaching experience. One had no prior experience and was looking ahead to student teaching in the fall. Three had 1-2 years teaching experience; four had 4-6 years; and three had 9 or more years experience as an educator. Two participants were special education teachers and eight were or had been teachers of mathematics. All of them taught at the middle or high school levels (grades 6-12). The students, 63.6% female and 36.4% male, were divided into two small groups for online discussion (groups 5 and 6).

4.2.2.2 Comparison Groups

Other mathematics education courses, in which participants did not specifically study children's mathematical reasoning from VMC videos and related literature, served as comparison learning contexts for experimental conditions in the larger project of design research. Given the focus on the fractions strand in the online course, students enrolled in elementary math methods courses formed the comparison groups. One instructional context was a graduate-level course in which the instructor did not use any videos, and assessment data were collected from 16 post-baccalaureate, pre-service teachers as study participants. The other instructional contexts were two sections of an undergraduate-level course, with two different instructors and varying conditions. In one section, the instructor used different videos than those available on the VMC web site as a resource for teacher learning, and assessment data were collected from 13 pre-service teachers as participants in the study. In the other section, the instructor gave an assignment where students had the option of viewing some VMC videos as part of lesson planning. Assessment data were collected from 27 pre-service teachers as study participants in this instructional context.

4.2.3 Data Sources

Study participants in both the experimental and comparison groups took a pre- and post–assessment in which they viewed a video of children supporting their solutions to a particular fraction task: Find a Cuisenaire rod in the set (see Figure 4.1) that could be given the number name one-half when the Blue rod has the number name one. Participants took the assessment as an online activity where they were given a link to the video and its transcript, sufficient time to respond, and the criteria for evaluating their responses. The assessment prompted study participants to describe as completely as they can the reasoning that the children put forth, whether each argument offered by children is convincing, and why or why not are they convinced. The video includes footage from research conducted in an after-school enrichment program for 6th graders in an urban community, where children engaged in many of the same tasks that were explored by children in the 4th grade classroom study (Maher, Mueller, & Yankelewitz, 2009). The assessment video includes short clips of children working in small groups on the task prior to clips of children explaining their solution ideas with rod models as justification to the whole class (Maher, Mueller, & Palius, 2010).

Instructional design choices for the course are considered to be an input variable on the learning process, per the online interaction model (Benbunan-Fich, Hiltz & Harasim, 2005), and they are reflected in the use of eCollege platform, the specific video and text resources given as assignments, and the wording of prompts for group discussion. The threaded discussions that emerged in each of six groups are a primary source of data for examining the nature of the learning process in this online course.

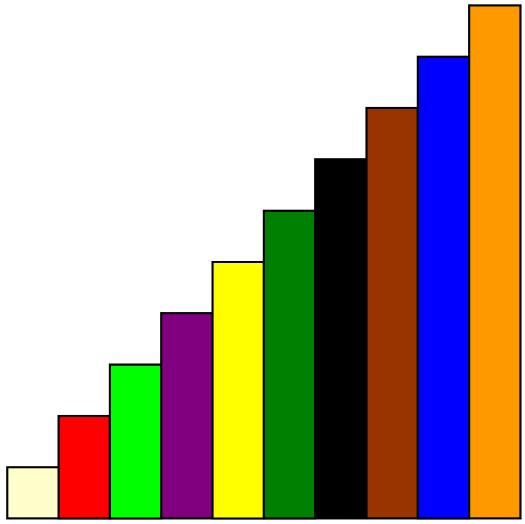


Figure 4.1: Staircase model of the set of Cuisenaire rods

4.2.4 Analysis

As part of the larger research project, we developed a detailed rubric to code the open-ended responses generated by study participants on the video assessment. We defined features comprising four mathematical arguments made by children in the video, one of which took the form of reasoning by upper and lower bounds and three of which were reasoning by cases. Data were coded for presence or absence of each argument as well as which specific features were mentioned, thus enabling fine-grained analysis. Two researchers independently coded the video assessment data with 90% inter-rater reliability.

For the upper and lower bounds argument, there were four components of the children's reasoning that could combine in three different ways to be a complete argument (a, b, and c; a, b, and d; or a, b, c, and d):

a. The Yellow rod is (1/2 of one White rod) longer than half of Blue; (AND)

b. Purple is (1/2 of one White rod) shorter than half of Blue; (AND)

c. There is no rod with a length that is between Yellow and Purple; (OR)

d. The White rod is the shortest rod and the difference between the Yellow rod and the Purple rod is one White rod.

Participant responses that did not mention any of the above components or that mentioned only one or two of them were deemed to be incomplete. Although it was considered a possibility that a participant might identify the upper and lower bounds argument simply by naming as such (i.e., not describing any of its features: a, b, c or d), that did not occur with this set of participant data.

For the first case argument, that the Blue rod is of odd length, there were three components of the children's reasoning that could combine in three different ways to make a complete argument (f and g; f and h; or f, g, and h).

f. The Blue rod is the same length as 9 White rods; (AND)

g. Nine is an odd number and cannot be divided evenly. "Halving it" would result in a decimal or remainder; (OR)

h. A rod half as long as the Blue rod would have to be the length of 4.5 White rods and there are no rods that are of "decimal" length.

Mentioning only one of these components, or g and h without f, or making reference to the argument without noting its features (coded as feature i) constitutes partial recognition.

For the second argument by cases, based on an exhaustive collection of the four two-rod trains (i.e., two rods placed end-to-end) with each train as long as the blue rod, the only way to make a complete argument is by combining features j and k.

j. For the ten rods, there are exactly four two-rod trains that are the same length as the Blue rod (White and Brown, Red and Black, Light Green and Dark Green, and Yellow and Purple); (AND)

k. No one of these trains is made up of two rods that are the same length.

l. Any of the following trains of two rods: White and Brown, Red and Black, Light Green and Dark Green, and Yellow and Purple is the same length as the Blue Rod.

Mentioning either feature j or k by itself, mentioning feature l by itself or in combination with feature k, or making reference to the argument without noting its features (coded as feature m) constitutes partial recognition.

For the third argument by cases, based on categorizing all of the rods into two sets: those rods that have other rods that are half as long and those that do not, the only way to make a complete argument is by combining features n and o.

n. Five of the rods: Red, Purple, Dark Green, Brown and Orange, can be built from trains of two smaller rods of the same length. (Two Yellows are the same length as Orange. Two Purple rods are the same length as Brown. Two Light Green rods are the same length as Dark Green. Two Red rods are the same length as Purple. Two White rods are the same length as Red). (AND)

o. No one of the remaining five rods: Blue, Black, Yellow, Light Green and White, can be built by trains of two identical rods. (There are no rods with lengths between White, Red, Light Green, Purple and Yellow.)

Mentioning either n or o by itself or making reference to the argument without noting its features (coded as feature p) constitutes partial recognition.

The coded video assessment data were analyzed quantitatively to measure change from pre- to post-test. For each of the four mathematical arguments, aggregated data were analyzed for empirical evidence that the experimental condition of the online course had an effect that differed from the three comparison conditions. For the study participants in the online course, the data also were disaggregated to examine variation in changes across the six discussion groups.

Detailed discourse analysis was performed to code the online discussion data from the focal course unit for all six groups. The coding scheme began as *a priori* to capture the forms of reasoning and specific argument features discussed by participants and the extent to which discourse adhered to the instructional prompt of addressing the children's sense making by providing evidence of mathematical understanding or obstacles to their understanding. However, the coding scheme also evolved through grounded analysis to capture connections made to assigned readings, agreement or disagreement with what someone else posted, reflections on teaching practice, and a catch-all category for other topics appearing in the online discussions. The intent was to ensure that all aspects of the participants' intersubjective meaning making were included in the discourse analysis. Thus, every online post received at least one code and some of the posts received multiple codes. Discussion group was used as unit of analysis to enable comparison across groups.

4.3 Results

The following three sections present results of data analysis. First are the findings from fine-grained analysis of the extent to which teachers noticed the reasoning of the children in the video assessment data and how their recognition of argument features changed from pre to post-test. Data are presented with discussion group as the unit of analysis for each of the four mathematical arguments. Next are aggregated results of growth from pre to post for the participants in the online course as an experimental group in relation to three different comparison groups. Then focus shifts to the results of discourse analysis for considering how much of the course participants' online discussion addressed children's mathematical reasoning, both for the four arguments and for the process of sense making through being engaged in problem solving, as well as the extent to which other topics were discussed.

4.3.1 Findings from Analysis of Video Assessment Data

Presented in a series of four tables are results of analyzing coded video assessment data to show which argument features were described by the teachers participating in this study. The total of 22 participants reflects the fact that two individuals enrolled in the course did not provide either a pre or a post test and one other was purposefully excluded from assessments given his role as member of the research team. Results are reported by group for pre and post-test to show change. Each argument is in a separate table, with cells for no features and features comprising partial recognition of the argument above the horizontal line and feature combinations constituting complete argument below the line.

As Table 4.1 shows, one or more teachers in Groups 1, 2, 3, 5 and 6 demonstrated positive change or growth in their recognition of features of students' reasoning by Upper and Lower Bounds. As Table 4.2 shows, because 100% of teachers in Groups 2 and 3 and 80% of teachers in Group 6 recognized a complete argument for the students' reasoning that "the blue rod is of odd length", there was relatively little room for growth across study participants. Yet teachers in Groups 4 and 5 demonstrated growth in recognition of this argument form, and 86% of teachers in the study described the complete argument on their post-test.

Table 4.1: Recognition of Argument by Upper and Lower Bounds

	Group 1		Group 2		Group 3		Group 4		Group 5		Group 6	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Features												
none	1		2		2				2	1	1	
a						1			1			
b				1				1				
c					1							
d							1		1	2		
ab		1		1	1	1				1		3
abc			1			1	1	1	1		1	1
abd						1	1	1			1	1
abcd	1	1		1						1	2	
Total	2	2	3	3	4	4	3	3	5	5	5	5

Table 4.2: Recognition of Argument by Cases that "The blue rod is of odd length"

	Group 1		Group 2		Group 3		Group 4		Group 5		Group 6	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Features												
none									3	1	1	
f		1										1
g												
h	1						1					
fg	1	1	3	3	4	4	2	3	2	3	4	4
fh										1		
fgh												
Total	2	2	3	3	4	4	3	3	5	5	5	5

As Table 4.3 shows, the cases argument based on the exhaustive set of two-rod trains with same length as the Blue rod went largely unnoticed by the study participants. Even though no one described a complete argument, there was some growth in recognizing this form of reasoning as evinced by participants describing at least one of the two-rod trains in the set (i.e., feature l). Such growth occurred in 14% of participants across the six groups.

Table 4.3: Recognition of Argument by Cases for “two-rod trains same length as Blue rod”

	Group 1		Group 2		Group 3		Group 4		Group 5		Group 6	
Features	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
none	2	1	2	2	3	2	2	1	2	2	4	5
j										1		
k												
l		1	1	1	1	2	1	2	1		1	
m									2	2		
jl												
Total	2	2	3	3	4	4	3	3	5	5	5	5

As Table 4.4 shows, only one teacher in each of Groups 3, 5 and 6 demonstrated any growth in their recognition of the student’s argument by cases for rods that either do or do not have another rod in the set as its half. None of the teachers described a complete argument, which would have entailed listing the colors of rods in each of the two cases.

Table 4.4: Recognition of Argument by Cases for “rods that have halves and rods that don’t”

	Group 1		Group 2		Group 3		Group 4		Group 5		Group 6	
Features	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
none	2	2	3	3	3	2	3	3	4	3	3	2
n										1	1	1
o									1			
p					1	2				1	1	2
no												
Total	2	2	3	3	4	4	3	3	5	5	5	5

A useful way to summarize the findings presented above is to focus on those participants who failed to recognize a complete argument on their pre-test and highlight the percentage that exhibited growth by describing additional argument features on their post-test. For the argument by upper and lower bounds, out of the 13 with incomplete on the pre-test, three provided a detailed complete description on the post assessment and eight provided additional components of it. Thus, the growth rate is 84.6% and it has a 95% confidence interval from 57.8% to 95.7% (See Chapter 3, Section 3.5). For the cases argument that the Blue rod is of odd length, out of the six with incomplete on the pre-test, four provided a detailed complete description on the post assessment and one provided additional components of it. Thus, the growth rate is 83.3% and it has a 95% confidence interval from 35.9% to 99.6% (See Chapter 3, Section 3.5). Interestingly, only one of the teachers failed to notice any of the features for both of these arguments on the post-test. For the other two arguments by cases, the growth rate was substantially lower.

4.3.2 Empirical Findings: Experimental versus Comparison Groups

To consider the impact of participating in the experimental online course as a means for triggering teachers' awareness of children's mathematical reasoning, it is illuminating to compare assessments results from teacher participants in various instructional conditions. The tables that follow present results for the argument by upper and lower bounds and the three arguments by cases for four different study populations. Instructional context one (I-1) is the experimental intervention that had two implementations in the online course with the same instructor. Instructional context two (I-2) is the first of two sections of the same undergraduate elementary math methods course, in which the instructor made use of different videos than those available on the VMC web site.

Instructional context four (I-4) is the second of two sections of that undergraduate elementary math methods course, in which a different instructor gave teachers an assignment where they viewed some VMC videos but for lesson planning rather than focusing on the children's mathematical reasoning. Instructional context three (I-3) is a section of a graduate elementary math methods course, in which yet another instructor did not use any videos.

Similar to what has been presented above, the focus is on growth in recognizing children's mathematical reasoning as they express an argument by upper and lower bounds and three different arguments by cases, as measured by change in participants' descriptions of the details of four arguments from their pre to post-assessment responses. Again, by focusing on those participants that did not describe a complete argument on their pre-assessment, analysis measured no change, growth to partial description, and growth to complete description. Results are presented as aggregate growth rates for the four different instructional conditions for each of the four mathematical arguments, with each argument in its own table.

Table 4.5: Post Aggregate Growth Rates by Online and Comparison Groups for the Argument by Upper and Lower Bounds

Intervention Context	Instructor	Aggregate No Change	Aggregate Growth: Partial	Aggregate Growth: Complete	Aggregate Growth: Partial or Complete	Odds Growth	Relative Odds Growth
Online	I-1	2/13 (15.4%)	8/13 (61.5%)	3/13 (23.1%)	11/13 (84.6%)	5.5 to 1	8.43
Comparison	I-2	5/9 (55.6%)	2/9 (22.2%)	2/9 (22.2%)	4/9 (44.4%)	0.65 to 1	
	I-3	6/9 (66.3%)	2/9 (22.2%)	1/9 (11.1%)	3/9 (33.3%)		
	I-4	12/20 (60%)	4/20 (20%)	4/20 (20%)	8/20 (40%)		

Table 4.6: Post Aggregate Growth Rates by Online and Comparison Groups for the Cases Argument (that the Blue rod is odd)

Intervention Context	Instructor	Aggregate No Change	Aggregate Growth: Partial	Aggregate Growth: Complete	Aggregate Growth: Partial or Complete	Odds Growth	Relative Odds Growth
Online	I-1	1/6 (16.7%)	1/6 (16.7%)	4/6 (66.7%)	5/6 (83.3%)	5 to 1	3.33
Comparison	I-2	2/5 (40%)	0/5 (0.0%)	3/5 (60%)	3/5 (60%)	1.5 to 1	
	I-3	0/3 (0.0%)	1/3 (33.3%)	2/3 (66.6%)	3/3 (100%)		
	I-4	4/7 (57.1%)	1/7 (14.3%)	2/7 (28.6%)	3/7 (42.9%)		

Table 4.7: Post Aggregate Growth Rates by Online and Comparison Groups for the Cases Argument (based on an exhaustive collection of two rods with each train the same length as the Blue rod)

Intervention Context	Instructor	Aggregate No Change	Aggregate Growth: Partial	Aggregate Growth: Complete	Aggregate Growth: Partial or Complete	Odds Growth	Relative Odds Growth
Online	I-1	17/22 (77.3%)	5/22 (22.7%)	0/22 (0.0%)	5/22 (22.7%)	0.29 to 1	0.75
Comparison	I-2	7/11 (63.6%)	4/11 (36.4%)	0/11 (0.0%)	4/11 (36.4%)	0.39 to 1	
	I-3	12/16 (75%)	4/16 (25%)	0/16 (0.0%)	4/16 (25%)		
	I-4	19/26 (73.1%)	6/26 (23.1%)	1/26 (3.8%)	7/26 (26.9%)		

Table 4.8: Post Aggregate Growth Rates by Online and Comparison Groups for Cases Argument (based on categorizing all of the rods into two sets: those rods that have another rod that is half as long and those that don't)

Intervention Context	Instructor	Aggregate No Change	Aggregate Growth: Partial	Aggregate Growth: Complete	Aggregate Growth: Partial or Complete	Odds Growth	Relative Odds Growth
Online	I-1	18/22 (81.8%)	3/22 (13.6%)	1/22 (4.5%)	4/22 (18.2%)	0.22 to 1	1.04
Comparison	I-2	11/13 (84.6%)	2/13 (15.4%)	0/13 (0.0%)	2/13 (15.4%)	0.21 to 1	
	I-3	12/14 (85.7%)	0/14 (0.0%)	2/14 (14.3%)	2/14 (14.3%)		
	I-4	19/24 (79.2%)	5/24 (20.8%)	0/24 (0.0%)	5/24 (20.8%)		

To summarize these results, the data presented in Tables 4.5 and 4.6 support the claim that the post-assessment growth rate for the Experimental Online (I-1) condition is higher than that of the Comparison Groups (I-2, I-3 and I-4) for both the argument by upper and lower bounds and the cases argument that the Blue rod is of odd length. However, the data do not provide evidence that the post-assessment growth rate of the Experimental Online condition is higher than that of the Comparison Groups for the other two cases arguments, as shown in Tables 4.7 and 4.8.

4.3.3 Findings from Discourse Analysis

Results from discourse analysis show that all teachers were active participants in online discussion among their group in the focal unit analyzed. There was considerable variation among the groups in terms of the range of topics discussed as well as how deeply the topics raised were explored. Before presenting highlights of what each group discussed, an overview of their online conversations is shown in Table 4.9 below. Note that Argument 1 is the argument by upper and lower bounds and Arguments 2, 3, and 4 are, respectively, the three arguments by cases for the Blue rod is of odd length, the exhaustive set of two-rod trains with length same as Blue rod, and the sets of rods that either have or do not have another rod that is half its length.

As Table 4.9 shows, a relatively small proportion of the posts made to the online discussions specifically addressed the four mathematical arguments. However, in Groups 2, 3 and 5, which experienced the greatest growth in recognizing the argument by upper and lower bounds, participants explicitly discussed features of that argument. In some instances, the same individual attending to details of the child's argument was the one to experience growth; in other instances it was another member of the group who may have benefited from a peer making such observations in their discussion. For Group 5, more than one person mentioned specific features of this argument, but only one person did in Groups 2, 3, 4, and 6. The argument by cases, that there are rods that have another rod as half and rods that don't, was discussed by Groups 2, 3, and 6. Two of those three groups happened to experience growth in recognizing this form of reasoning.

Table 4.9: Summary of Discourse Analysis

	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Total
No. members	3	4	4	3	5	6	25
Total No. posts	24	36	16	10	29	34	149
Posts by Instr.	1	3	1	2	1	2	10
No. threads	7	10	4	2	6	6	35
Longest thread	6	12	6	6	14	9	n/a
Posts coded for:							
Argument 1	0	1	1	1	4	1	8
Argument 2	0	0	0	0	1	0	1
Argument 3	0	1	0	0	0	0	1
Argument 4	1	1	2	0	0	1	5
Sense Making	4	14	7	7	15	13	60
Readings	7	8	5	0	2	1	23
Reflect Practice	2	12	6	5	7	8	40
Other topics	6	9	0	2	5	9	31
Agree/Disagree	7	7	7	1	5	13	40

A greater proportion of participants' posts attended to the children's sense making as they engaged in problem solving. In particular, the teachers discussed the extent to which specific children understood the full mathematical definition of what it means to be one-half, namely that the two parts comprising the whole have to be equal parts. They made reference to the rod models built by the children as well as to what the children said in the video for giving evidence in support of claims about mathematical understanding or what appeared to be obstacles to understanding.

The teachers also made connections to the readings, both for attending to features of mathematical arguments (as detailed in Davis & Maher, 1995) and in their commentary about children's sense making and the development of mathematical understanding. There were instances in Groups 1 and 2 of participants making reference to readings from a previous course unit and in Group 6 of referring to a topical article in the daily newspaper.

Particularly interesting was the extent to which participants offered reflections on teaching practices. These covered a wide range of themes, from student reactions to encountering a math problem with no solution (i.e., that the solution is the null set) to ways of teaching fractions that consider auditory learning strategies, such as musical beats per measure. In one group, a participant who was having his first experience as instructor of basic math at a community college, after previously having taught at the high school level, excitedly posted about how well it went using the Cuisenaire rods to teach fractions to his students. His posting resulted in a conversation where other teaching strategies were exchanged. Teachers from other groups also mentioned classroom experiences, extending discussion into topics such as student learning of formal proofs and the art of knowing when to intervene and when to let a student struggle through a cognitive challenge.

Reflections on teaching practices typically happened spontaneously, as it was not included in the course unit's prompt for discussion, although in three instances they were elicited in response to a post made by the instructor. It is notable that the instructor made relatively few posts in any of the group discussions. Out of ten total posts, three were solely to clarify or correct the name of the child in the video being discussed and two were a brief affirmation of what was being discussed (i.e., "Right on!" and "I so very much enjoyed reading your conversation."). Two other posts made by the instructor were to ask questions about ideas raised by a participant and invite everyone in the group to offer further commentary.

4.4. Discussion

By considering the study participants' discussion group as a unit of analysis, there is evidence that engaging in online discussion about videos and related readings that feature the mathematical reasoning of a classroom of fourth graders resulted in greater awareness of mathematical arguments expressed by some sixth graders who worked on the same task in the assessment video. The evidence is strongest for the argument by upper and lower bounds, where Groups 2, 3, 4, 5 and 6 discussed features of this argument online (see Table 4.9, posts coded for Argument 1) and demonstrated growth from pre- to post-assessment (see Table 4.1). In Group 2, three teachers recognized additional features of this argument, which the fourth group member, for whom there is no assessment data, described completely in the group's discussion. All four members of Group 3 demonstrated growth, two from none to partial and the other two from partial to complete, after one of the teachers in this group posted in great detail about this form of reasoning. Two of three teachers in Group 4 described a complete argument on both their pre-test and post-test, and the third noticed a different argument feature on her post-test than on her pre-test. Group 5 also experienced growth by recognizing additional features of this argument, and one of the teachers dramatically changed from none on pre-test to describing all four features on the post-test following discussion of this argument by one of her peers in their online conversation. Although most of the teachers in Group 6 already had recognized this complete argument on their pre-test, the one who had not demonstrated growth on the post-test after the reasoning was discussed online. Thus, for this argument form, we clearly see solid evidence that the nature of group learning reported by Webb (1982) in face-to-face environments also can occur in an online environment. While the evidence is not as compelling for the argument by cases that the Blue rod is of odd length, the only group that discussed online this form of reasoning was one of two groups to demonstrate growth from pre- to post-assessment.

Further evidence that the opportunity to discuss children's mathematical reasoning with other educators in an online environment catalyzes teachers' learning comes from the empirical data that contrasts outcomes between experimental and comparison groups of study participants. Again, this evidence is strongest for the argument by upper and lower bounds but also is apparent for the cases argument that the Blue rod is odd, as shown in Tables 4.5 and 4.6, respectively. It is worth noting that obtaining growth in the other two cases arguments would have entailed making an exhaustive listing of specific colors of rods that belong to each of the cases, and merely viewing the assessment video a second time could have enabled participants to notice more of such details. Attending to particular features of the argument by upper and lower bounds, however, is not very likely to manifest without some kind of targeted intervention between two viewings of the assessment video. Indeed, even a single-unit intervention may not be sufficient to promote awareness of the least upper bound and the greatest lower bound as key components of this argument form (Maher, Palius, Maher & Sigley, 2012). However, given that approximately two-thirds of the participants in the experimental online course have the certification to teach secondary mathematics, this characteristic of the experimental study population may contribute to their greater recognition of salient argument features than the comparison population of pre-service elementary teachers. Yet mathematical content knowledge may not be the only explanatory variable because the video assessment also serves as a measure of how well someone is listening to the children as they express their reasoning and the extent to which the children's language gets mapped to relevant mathematical knowledge. In short, explaining the differing performances on the video assessment is a complicated issue that is beyond the scope of this study but an interesting area for future research.

The design of the experimental online course, which paired selected videos with a related article from the research literature (Maher & Davis, 1995), provided the study participants with a rich context of multimedia from which to learn. The discourse analysis revealed that some participants noticed and discussed certain aspects of children's mathematical reasoning from the videos while others specifically cited what they had read in the article. The pairing of video episodes with related research literature also afforded participants with the opportunity to learn more deeply from the readings, as shown in the following excerpt from one teacher's post to the online discussion. He mentions how the research article (Maher & Davis, 1995) includes a quote from David Tall (1995) that is well connected with both a prior reading assignment and the currently assigned videos, posting:

> The other aspect of this quote seems to fit better with the videos we watched for this week. It is the idea of "the visuo-spatial becoming verbal and leading to proof." After watching the videos, I could not help but appreciate this quote, but I couldn't help but think that without watching the videos I would not have regarded it as much. I guess I still needed some visuals before I could accept Tall's verbal claim.

Thus, this participant provides compelling evidence of how the design of this online course stimulated deep consideration of how mathematical reasoning develops, which was triggered not only by the videos themselves but also by reflecting on the readings in light of that which could be observed in videos as children engaged in problem solving with the rods.

In terms of intersubjective meaning making, the experimental online course served as a powerful stimulus for participants to make connections among videos and text resources, their own educational experiences, and the experiences of others as shared and discussed within small groups. While there was variation across groups, discussion of the children's sense making through mathematical activity as seen on the videos, connections to readings, and reflections on teaching practices emerged as dominant themes in their posts. The summary of discourse analysis as presented in Table 4.9 does not do justice to the richness of the online conversations. However, the findings from categorical coding of the participants' posts from the focal unit in the experimental online course are similar to what we found in analyzing discourse in a single unit from a hybrid course using VMC videos and text resources in the counting/combinatorics strand (Hmelo-Silver, Maher, Palius & Sigley, 2012). An interesting finding from both studies is that relatively little scaffolding from the instructor was given to specifically focus teachers' noticing as they viewed and discussed videos. This is reflected in the small number of instructor posts for each group, which tended to occur towards the end of the discussion period in the course unit.

The research presented above attempts to shed light on the complex phenomenon we call learning as it transpires in an online learning environment that offers an array of technology affordances. Although the VMC video collection itself has been hypothesized to be useful in promoting teachers' awareness of how mathematical reasoning develops in students, the pairing of videos with related research literature and providing general discussion prompts for participants to support their interpretations with evidence have shown to be an effective catalyst for learning. Yet the process of intersubjective meaning making merits greater attention and suggests directions for future research. In particular, the discourse analysis needs to be extended for examining trends over the duration of the experimental online course. As Suthers (2006) suggested, it is worth looking closely at which ideas are picked up on by others; and it would be interesting to see whether some ideas are perhaps not addressed immediately but rather re-emerge after some time has elapsed. Also interesting would be deeper exploration of the kinds of evidence that participants provide for their interpretations, perhaps drawing from the work of Nemirovsky and Galvis (2004) to see whether claims are grounded in specifics or generalities of certain videos, texts or teachers' own classroom experiences. Examining how teachers interact with one another and the resources for reasoning about children's mathematical activities has much to inform mathematics teacher education in online environments replete with technology affordances.

CHAPTER FIVE: LEARNING ABOUT STUDENTS' MATHEMATICAL REASONING THROUGH COMMUNICATION

5.1 Introduction

Communication has become widely recognized as an important part of mathematics education, as has been the intent of influential academics, chief school officers, and policy makers leading the standards movement. The National Council of Teachers of Mathematics specifies communication as one of the process standards, both in terms of articulating one's own mathematical ideas and evaluating those expressed by others, and a second process standard identifies the use of representations to communicate ideas (NCTM, 2000). The NCTM Standards informed development of the more recent Common Core State Standards, which describe clearly and with specificity what a mathematically proficient student does at particular grade levels and defines overall Standards for Mathematical Practice. However, the Common Core State Standards do not describe how a mathematics educator should act to develop the mathematical practices and proficiencies in students. Instead, it is suggested that when the word "understand" begins one of the expectations that it presents a good opportunity to connect the practices to the content. Indicating explicitly how mathematics instruction could make such connections is a responsibility assigned to "designers of curricula, assessments, and professional development" (NGACBP & CCSO, 2010, p. 8).

Concurrent with development of standards has been production of standards-based curricular materials and an evolution of professional development programs. During the 1990's, mandatory participation in partial-day or day-long workshops sponsored by the school district was a common mode of teacher professional development, yet teacher lore and the limited research available both indicate that such programs were often irrelevant and not much valued by teachers (Wilson & Berne, 1999). Compared to earlier programs, a study initiated in 2005 to analyze the quality of professional development programs for mathematics and science found that the large majority of programs in the sample of 25 leading efforts from across 14 states were heavily focused on content knowledge and included active methods of learning for teachers (Blank, de las Alas & Smith, 2007). Yet only one of the programs in the study addressed the matter of communication as a change in instructional practices with an instrument for observing student discourse.

Thus, while the importance of communication in the mathematics classroom has come to be recognized, it may not be receiving adequate attention in the context of teacher professional development for informing instructional practices. In particular, teachers are not getting the opportunity to learn enough about how to facilitate communication in their classroom in ways that foster the development of reasoning through meaning-making activities. Asking students to give explanations that justify solutions is an important part of the communication process, but it only represents have of it since "genuine communication (is not) involved unless someone attempted to interpret the explanation or justification" (Yackel & Hanna, 2003, p. 229). Moreover, teachers cannot assume sole responsibility for interpreting student explanations if the Standards seek student proficiency in responding to the mathematical arguments of others; but how will teachers learn how to foster that? A challenge facing teacher educators, therefore, is the provision of professional learning opportunities that expose teachers to classroom environments that illustrate the vital role that communication plays in the Common Core's Standards of Mathematical Practice being enacted by students along with the pedagogical facilitation that elicits such enactment.

5.1.1 Innovations for Teachers' Professional Learning

Constraints of time, distance, money and access make it impractical for widespread use of live observations of exemplary instructional practices in teacher education contexts; but fortunately other resources are available for innovative teacher learning opportunities. Video is a powerful medium for making learning environments visible, thereby making instructional practices available for study, interpretation and discussion. Video has been used in several models of teacher professional development, including video clubs (Sherin & Han, 2004), lesson study (Alston, Basu, Morris & Pedrick, 2011), and other school-based programs that engage teachers in reflective discussions about classroom practices (Borko, Jacobs, Eiteljorg & Pittman, 2008; Nemirovsky & Galvis, 2004). Depending on the model, the videos selected for study either come from the participating teachers' own classrooms or get provided as video cases by the professional developers. Since the source of the video episodes can be both an enabling and a constraining factor (Seidel, Stürmer, Blomberg, Kobarg & Schwindt, 2011; Zhang, Lundeberg, Koehler & Eberhardt, 2011), it is optimal to align the professional development goals with what can be learned from studying the video resources chosen.

A particular video collection, gathered from more than twenty years of longitudinal and cross-sectional research on the development of mathematical ideas in students, may be especially valuable for studying how children learn to reason mathematically (Maher, 2008). The theory is now being tested in a research and development project[1] that makes publically available videos from the collection (see www.videomosaic.org) and investigates

---

[1] Research supported by the National Science Foundation grant DRL-0822204, directed by C. A. Maher with G. Agnew, C. E. Hmelo-Silver, and M. F. Palius. The views expressed in this paper are those of the author and do not necessarily reflect those of the funding agency.

their use in courses and workshops aimed at deepening teachers' awareness of students' mathematical reasoning. The initial settings for implementing instructional interventions occurred in face-to-face learning environments (Maher, Palius & Mueller, 2010; Maher, Landis & Palius, 2011; Palius & Maher, 2011). As researchers proceeded with iterations of design research, some instructors chose to make increasing use of online platforms for teachers to study and learn from video resources (Maher, Palius, Maher & Sigley, 2012).

5.1.2 Situating the Study

The study presented here examines the nature of learning through analysis of participants' discourse in an experimental online course offered as an elective, Topics in Math Education: Critical Thinking and Reasoning, for graduate students in mathematics education and other education program areas. The intent of the study is to investigate the process of teacher knowledge building by analyzing their communication, which includes their observations and reflections on the communication of others as viewed in the videos, and how they form interpretations through discussion with one another. A question that guides this study is: *What does discourse analysis reveal about the process of constructing knowledge about the developmental process of learning to reason mathematically and the conditions of the learning environment that have been shown to support such development?*

5.2 Theoretical Perspectives

A fundamental and unifying theme for the environment of this study is the notion of building or constructing. It pertains not only to the long-term research agenda that yielded the video collection, but also to the design of the online learning environment where the videos were studied as well as to the theory of discourse analysis being used to examine the online interactions of the study participants. The relevance of this theme to each aspect of the work is described in the following subsections.

5.2.1 Building Mathematical Ideas Over Time

Drawing from literature in the fields of psychology and philosophy, Noddings (1990) characterizes constructivism as assuming that human beings are knowing subjects who behave in a purposive manner and have the ability to organize their knowledge through a combination of innate capacity and developmental processes. That presumption is basis for “the claim that there is an essential connection between purposive activity and the development of cognitive structure (Noddings, 1990, p. 9). More simply said, through doing things we build what we come to know. This view of learning guided the researchers’ actions through many years of engaging students in problem solving within and/or across strands cognitively challenging mathematical tasks that enabled them to create personally meaningful representations that they could use to convince others about the validity of their ideas, conjectures, and solutions (Maher, Powell & Uptegrove, 2010; Maher & Weber, 2010; Maher, 2005; Steencken & Maher, 2003; Davis & Maher, 1997; Maher & Martino, 1996).

Motivated by a combination of curiosity and optimism about the extent of children’s innate capacity for mathematical thinking, the researchers introduced tasks for students to explore before they had formal instruction on those topics in their regular math lessons at school (Maher, Powell & Uptegrove, 2010; Steencken & Maher, 2003). During a series of problem-solving sessions that took place in either classroom or informal settings, researchers created conditions in the learning environment that included plenty of time for exploration and revisiting, establishing norms for both social and mathematical behaviors, and refraining from sole arbitration of what was convincing by encouraging students to listen carefully to and critique one another's ideas (Maher, Powell & Uptegrove, 2010; Francisco & Maher, 2005; Maher, 2005). Recording the sessions with multiple cameras captured the mathematical work and talk of the children as well as researcher actions as designer and facilitator of learning in a constructivist setting (Maher 1998; Maher 1988).

5.2.2 Building an Experimental Online Course

The view of constructivism that guided the research studies yielding the videos and scholarly literature analyzing them also underlies the design and implementation of the experimental online course; yet the research context is different in two significant ways. First, the purposive activities for constructing organized knowledge shift from students' building of mathematical ideas to teachers' building of awareness of how mathematical reasoning develops in students over time through their learning experiences. Second, the learning environment is transformed from a face-to-face setting mediated by a facilitator who draws upon both verbal and non-verbal cues for coordinating communication to one that relies on asynchronous, computer-mediated communication that is largely text-based. Note that it is largely, rather than entirely, text-based because the online course instructor happens to be the main researcher featured in the videos being studied by the graduate students. Thus, although quite different from an instructor using video podcasts to create a personal presence with students enrolled in an online course, viewing their instructor in the role of classroom teacher to fourth-graders working on problems to build fraction ideas serves as unique mode of non-text communication with particular salience to this study.

It is these aspects of course context and technology, along with characteristics of the instructor and individual students who participate, that form the basis for interactions that shape the process for learning in an online environment (Benbunan-Fich, Hiltz & Harasim, 2005). Ideally, the design of the learning environment should be constructed in a way that supports collaborative interaction (Sammons, 2007). Garrison, Anderson and Archer (2001, p. 7) argue that creating a community of inquiry that “involves (re)constructing experience and knowledge through critical analysis of subject matter, questioning, and the challenging of assumptions” is a very valuable and perhaps essential context for higher-order learning to occur through computer-mediated communication. Moreover, the online learning environment should be built so that it elicits a substantial amount of student-to-student communications that occur independent of the instructor in order for interactions to be truly collaborative (Sammons, 2007).

5.2.3 Building Tasks of Language

Examining the nature of online interactions that are predominantly text based necessarily involves discourse analysis. Therefore, this study is grounded in a theory of language-in-use, wherein language serves “to support the performance of social activities and social identities and to support human affiliation within cultures, social groups, and institutions” (Gee, 2005, p.1). Language viewed thusly is about conveying and finding meaning, and interactions build *situated meanings* that vary with use in specific contexts and by specific people. Interpreting situated meanings through language-context analysis gives rise to the reflexive property of language because, simultaneously, “an utterance influences what we take the context to be and context influences what we take the utterance to mean” (Gee, 2005, p.57).

Gee (2005, pp. 11-13) defines seven building tasks of language that occur with every spoken or written utterance to construct situated meanings. A person’s use of language imparts meaning or value to something in ways that builds its *significance*. Language use gets one recognized as engaging in something that builds an *activity* that is taking place at the time. It also gets one recognized as assuming a certain role that serves to build an *identity* in that moment. How we use language builds the kind of social *relationships* we have, or perhaps are trying to have, with the audience with whom we are communicating. One's use of language also builds a perspective on *politics (the distribution of social goods)* by communicating one's discernment about value, status, correctness, normalcy, power and so forth. Use of language can build *connections* or relevance among things. Lastly, the way one uses language in a given situation builds privilege for a *sign system or knowledge* claim by making it more relevant or prestigious than another type of language form (Gee, 2005). Taken together, these building tasks of language reveal who is doing what while the interactions are taking place. As participants interact with one another in a specific context, they produce, reproduce and transform situated meanings through their utterances.

This theory of language-in-use also sheds light on how a community of learners gets built in an online environment. Per the conceptual framework for a community of inquiry, the educational experience lies at the intersection of social presence, cognitive presence, and teaching presence (Garrison, Anderson & Archer, 2000). Social presence is defined as the ability of learners to project their personal characteristics into the community. With every contribution to the conversation, each learner is (re)constructing personal identity and views on social relationships, as well as building significance, relevance, and value for certain ideas, feelings, and ways of knowing. The definition for cognitive presence is grounded in literature on critical thinking and made operational by a four-phase model of practical inquiry (Garrison, Anderson & Archer, 2001). The process begins with a triggering event emerging from experience, which is then explored through private critical reflection and social discourse. As exploration transitions to integration, meaning is constructed as learners build connections among ideas through repeated movement between reflection and discourse. To the extent that consensus occurs, some kind of resolution may be reached. It is thus clear how developing one's cognitive presence relies on the activity of thinking critically and talking with others to build significance of ideas, connections among them, and knowledge. Teaching presence is defined as "the design, facilitation and direction of cognitive and social processes for the purpose of realizing personally meaningful and educationally worthwhile learning outcomes" (Anderson, Rourke, Garrison & Archer, 2001). Again, this is achieved through the reflexive function of language-in-use to build significance, activities, identity, social relationships, politics, connections, and knowledge.

5.2.4 Building as a Unified Metaphor That Guides Research

To summarize, researchers can examine how situated meanings get built through conducting language-context analysis. The specific intent of this study is to shed light on the process of constructing knowledge about the developmental process of learning to reason mathematically and the conditions of the learning environment that have been shown to support such development. It examines participant learning as intersubjective meaning making both in terms of particular, discipline-specific knowledge and in terms of "discovering affinities with others, orienting attention, expressing viewpoints, exposing conflict and consensus, and supporting debate and negotiation" (Suthers, 2006, p. 332). The analysis occurs in a specific context of who is doing what as teachers-as-learners in an experimental online course engage in intersubjective meaning making about children's critical thinking and reasoning about fraction ideas. The guiding research questions are:

(1) What discursive patterns emerge among small groups of teachers-as-learners as they study videos and related literature to attend to children's building of fraction ideas?
(2) What specific ideas are discussed pertaining to the children's mathematical activities?
(3) How, if at all, do participants attend to theoretical ideas from the literature and the actions of the researcher in the role of classroom teacher in the video episodes?
(4) How, if at all, do the videos and literature stimulate participants to reflect on teaching practices?

5.3 Methodology

The following subsections describe notable characteristics of the course context, the technology, the instructor, and the graduate students participating in the experimental online course. In so doing, attention is drawn to how those characteristics potentially contribute to the social presence, cognitive presence, and teaching presence as factors influencing the participants' educational experiences. The subsection following those descriptions explains the methods used to analyze the data from online discussions.

5.3.1 Course Characteristics

The experimental online course was designed as an intervention context for design research investigating how teachers learn about students' mathematical thinking and reasoning by studying video episodes from the collection and related research literature, and then communicating about their observations and reflections via asynchronous online discussions. Course content focused on provoking consideration of how fractions are introduced in school and included literature on the obstacles to fraction learning. The featured videos came from yearlong study conducted in a 4th grade classroom where students were introduced to a particular way of working with Cuisenaire rods (they had permanent color names but their number names could be changed), thereby making them a flexible manipulative for building models to support conjectures and claims about fraction ideas (Steencken & Maher, 2003).

The course was first implemented during a 15-week semester in Spring 2010. It was subsequently revised and implemented in a four-week Summer Session during 2011. Even though length of term varied, the two implementations were very similar in content. The first contained eight course units with 15 videos and ten articles plus a ninth unit for a group project that made available five more videos. The second contained seven course units with 17 videos and ten articles (nine in common) but no group project. Another difference was that the semester-long course included class-wide discussions following small-group discussions during the second week of a two-week course unit, which happened with three units. That did not occur during the much shorter summer course.

The selection of content represents the intersection of teaching presence and cognitive presence (Garrison, Anderson & Archer, 2000). While overall the content was similar between the two implementations, the adaptations made for design research limits how many course units are identical for purposes of directly comparing the participants' educational experience through discourse analysis. There were three such units, but since one has been reported on extensively elsewhere (See Chapter 4, Sections 4.2.1, 4.2.3 and 4.3.3 of this dissertation), two focal units are analyzed in the study presented here.

The first focal unit contained six videos of children engaged in fraction comparison tasks for which they built rod models, discussed differences among models, debated claims about number names, and discovered the idea of equivalent fractions. The assigned article traced the children's building of fraction ideas over the first seven sessions of the 4th grade classroom study, directly connecting to the assigned videos and filling in with some details of what happened in videotaped sessions not viewed by course participants (Steencken & Maher, 2003). Titles of the assigned materials, including the persistent URLs for video resources stored on the VMC Repository, appear in Table 5.1.

Table 5.1: Assigned Materials and Discussion Prompts for First Focal Course Unit

First Focal Course Unit (Unit 6 in Spring 2010 and Unit 4 in Summer 2011)	
Assigned Materials	*Discussion Prompts*
Assigned readings: (**1**) Steencken, E. P. & Maher, C. A. (2003). Tracing fourth graders' learning of fractions: Episodes from a yearlong teaching experiment. *The Journal of Mathematical Behavior, 22* (2), 113-132. **Study video clips**: (**1**) URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000059681 VM title: *Introducing Fraction Equivalence and an Exploration of Fraction Comparison, Clip 1 of 4: Equivalent fractions, a debate* (**2**) URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000059685 VM title: *Introducing Fraction Equivalence and an Exploration of Fraction Comparison, Clip 2 of 4: An introduction to proportional reasoning*. (**3**) URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000059691 VM title: *Introducing fraction equivalence and an exploration of fraction comparison, Clip 3 of 4: Proportional Reasoning Continued* (**4**) URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000059695 VM title: *Introducing fraction equivalence and an exploration of fraction comparison, Clip 4 of 4: Finding the number name for the difference between one half and one third* **Contribute to online discussion** per course guidelines and topical questions	(1) Provide a brief description of the main idea(s) in each clip. (2) Discuss your views on how teachers might (or not) find observing the videos of the children's problem solving useful. (3) Discuss the main ideas in the Steencken and Maher paper as they relate to children's building an understanding of fraction ideas.

The second focal unit contained two videos of children engaged in comparison of unit fractions and beginning to consider placement of fractions on a number line, which also elicited lively discussions facilitated by the researcher in role of classroom teacher. There were two assigned articles that addressed content overlapping with the videos, examined number line representations created by a child in a different study, and discussed theoretical ideas about children developing conceptual understanding of fractions (Alston, Davis, Maher & Martino, 1994; Maher, Martino & Davis, 1994). Titles of the assigned materials, including the persistent URLs for video resources stored on the VMC Repository, appear in Table 5.2.

Table 5.2: Assigned Materials and Discussion Prompts for Second Focal Course Unit

Second Focal Course Unit (Unit 7 in Spring 2010 and Unit 5 in Summer 2011)	
Assigned Materials	*Discussion Prompts*
Assigned readings: (**1**) Alston, A. S., Davis, R. B., Maher, C. A., & Martino, A. M. (1994). Children's use of alternative structures. In J. P. da Ponte and J. F. Matos (Eds.), *Proceedings of the 18th Annual Conference of the International Group for the Psychology of Mathematics Education, (2)*, 248-255. Lisboa, Portugal: University of Lisboa. (**2**) Maher, C. A., Martino, A. & Davis, R. B., (1994). Children's different ways of thinking about fractions. In J. P. da Ponte and J. F. Matos (Eds.), *Proceedings of the 18th Annual Conference of the International Group for the Psychology of Mathematics Education, (3)*, 208-215. Lisboa, Portugal: University of Lisboa. **Study video clips**: (1) URL:http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000055290 VM Title: *Fraction problems, Sharing and Number Lines, Clip 3 of 5: Comparing unit fractions* (2) URL: http://hdl.rutgers.edu/1782.1/rucore00000001201.Video.000055292 VM Title: *Fraction problems, Sharing and Number Lines, Clip 5 of 5: Placing fractions on the number line* **Contribute to online discussion** per course guidelines and topical questions	This unit includes two readings and two videos. Consider, in the video episodes, the fraction ideas that the children are exploring. (1) Provide a brief description of the main idea(s) in each clip. (2) Discuss how teachers might (or not) find observing the videos of the children's problem solving useful. (3) Discuss the main ideas in the two PME conference papers as they relate to children's building an understanding of fraction ideas.

5.3.2 Technology Characteristics

The technology used was the eCollege online course platform. Instructors use the tools within it to create course units with unit content items, compose introductions to each unit, create groups and assign particular students to them, collect student work in dropboxes, decide which items become accessible to students by what dates, and enact various other activities. Clicking on the unit title displays the introductory message, which explains the assignment and timeframe for doing it, and reveals the unit content items, which can include downloadable or embedded resources, links to web-based multimedia, and threaded discussions. In short, the technology supports the educational experience.

As mentioned above, two focal units are pertinent to the study being reported here. The video and reading assignments were made available as unit content items accessible to students via hyperlinks to the resources that they could click on from expanded view of the unit. Students were assigned to groups for discussion, and although they could only access the threaded discussions for their own group, each group had identical initiating prompts, which appear in the right columns of Tables 5.1 and 5.2, respectively, for the two units.

Note that a new thread gets initiated when a student clicks the respond button to the instructor's prompt for online discussion, which is composed in advance and then made available to students as part of a course unit. The student gives a subject name to the new thread, which is an opportunity to be creative and give focus to the discussion but could be as broad and unimaginative as "response," and then posts his or her commentary. A thread could contain a single post if no one else responds; otherwise the thread becomes extended when someone posts a response to a classmate's post. Technology characteristics of online threaded discussions thus provide opportunity for students to show their social presence as well as build cognitive presence through their interactions with one another and the instructor. Threaded discussions enable analysis of discursive patterns among participants and discourse analysis of the contents of each post that contributes to a thread and overall communication within the course unit.

5.3.3 Instructor Characteristics

The instructor is a prominent scholar in the field of mathematics education with an international reputation for her longitudinal research on the development of mathematical thinking and reasoning in students and longstanding views on constructivist learning. As indicated earlier, the views guiding her actions as a facilitator of mathematics learning in a research context similarly shaped the facilitation of this experimental online course. She was supported in making this transition by the author as a protégée and co-investigator on the design research to examine teacher learning about student reasoning from studying videos. Per a shared set of views, careful thought went into using language in the syllabus, introductory text to units, and discussion threads that invited and encouraged participant actions. For instance, course objectives let participants know that they will be asked to consider the relationship, if any, between learning fractions and learning other topics in mathematics and the implications of meaningful mathematical learning for teacher education and professional development. Prompts for discussion indicated that questions were posed as a guide and that raising other ideas stimulated by the materials was welcomed. Also notable is that the instructor purposefully held back from engaging in the online conversations until the graduate students had ample time to express their views and exchange ideas with one another. It was both a research goal and a pedagogical style to see what would happen with minimal instructor intervention.

5.3.4 Individual Student Characteristics

A total of 25 graduate students participated in the two implementations of the experimental online course. In the aggregate, they are considered teachers-as-learners because the vast majority of them had teaching experience, ranging from less than a year of student teaching to more than 20 years teaching mathematics, and spanning across the middle, secondary and post-secondary levels; although a few were still preparing to teach.

In each implementation, participants were assigned to a group for the entire term. Those in the semester-long course were divided into four small groups of three or four people for online discussions (Groups 1, 2, 3 and 4).

Group 1 consisted of two males and one female, all doctoral students in mathematics education. One male had not taught but had experience in tutoring mathematics; the others were experienced high school teachers, and the female also worked as a math coach to develop teachers in her district. Group 2 consisted of four male doctoral students, three concentrating in mathematics education and the fourth in learning sciences. One had never taught but the other three were experienced teachers at the secondary and post-secondary levels, the latter spanning a wide range from developmental math to advanced math and statistics courses. Group 3 consisted of three males who were teaching at a middle school or high school and one female with a modest amount of teaching experience. One male was in a master's program, the other three were in a doctoral program, all concentrating in math education. Group 4 consisted of three females who were all in a master's program for mathematics education; one only had experience in student teaching, one was teaching mathematics in a middle school, and one was teaching high school mathematics. These four groups had opportunities to interact with one another in class-wide discussions, twice occurring as the latter portion of assigned activity in the focal course units.

The four-week summer implementation included slightly fewer yet somewhat more diverse participants who were divided into two small groups for online discussion (Groups 5 and 6). Group 5 consisted of three females and two males. One male was taking his last course toward a doctorate as an elective outside the specialization area of educational administration and supervision; the other was about mid-way through a master's in mathematics education. Two of the females also were in a master's program for math education, and the third was not in a degree program. Four of the five each had between four and six years of teaching experience at the middle and high school levels, including in special education. It was not clear whether the administrator had classroom experience, but if he did then it was not likely as a teacher of mathematics. Group 6 consisted of four females and two males. One male was in a doctoral program and other was in a master's, both for mathematics education, with ten years and two years, respectively, of teaching experience at two different independent high schools. One female also was taking her last course toward a doctorate as an elective outside the specialization area of educational administration and supervision, and she had nine years of teaching experience in special education, some of which included mathematics. The other three females were in a master's program, and each had two years or less of teaching experience at the middle or secondary level. These two groups did not interact during any of the online discussions.

5.3.5 Data Analysis

The data consist of small group discussion threads in two focal course units across two implementations of the experimental online course. The methodology for discourse analysis is grounded in the language-in-use theory discussed earlier. Yet the methods also include quantitatively summarizing both the amount of discursive interaction and the richness of the online communication by tallying the number of posts and how many coding categories were evidenced in each post.

The coding categories emerged from a combination of the instructor's prompts for group discussion, the specific content of the assigned videos and research literature, and the discourse of participants as they engaged with the content and one another. Across the two course units, 28 coding categories were applied to the discourse; 18 of those categories were applicable to both units, three were specific to the first unit, and seven were specific to the second unit. Ten of the coding categories pertain to participants' descriptions of what they viewed in the videos and read in the literature, which included describing the children's representations, reasoning and sense-making activities as they worked on tasks involving comparison of fractions and placing fractional numbers on the number line. Sixteen of the codes pertain to participants' reflections on what they viewed in the videos and read in the literature, which included reflecting on the children's sense-making activities, the developmental process of reasoning, the actions of the researcher in the role of classroom teacher, their own classroom practices, and more general views on teaching and learning mathematics. The other two codes pertain to functional aspects of the course, which were planning a group post to whole-class discussion and the raising and resolving of technical issues. A more detailed explanation of the coding categories with illustrating examples from the online discussion posts is presented in the upcoming section with results. It also will become apparent in the following section how the seven building tasks of language (Gee, 2005) both manifest and are reflected in the results of the discourse analysis.

5.4 Results

5.4.1 First Focal Course Unit

5.4.1.1 Overall Discursive Patterns

As can be seen in Table 5.3 below, all groups show evidence of engaging in the activity of posting their ideas in the discussion threads and responding to the ideas posted by other group members during the first focal course unit. These were largely student-to-student interactions, with the percentage of instructor posts ranging from 9% to 19% of total contributions to the threaded discussions in this unit. Instructor posts tended to be very brief, however, and served one or more of the following discursive functions that reflect teaching presence: give positive feedback, ask a question or prompt for clarification, provide more information about the 4th grade classroom study or the research literature, and invite others to respond to something specific that one group member had to say. Student posts varied greatly in their length (i.e., from a short sentence to approximately one page) and in how their language-in-use contributed to social presence and cognitive presence, a matter to be discussed shortly.

Table 5.3: Summary of Participation in the First Focal Course Unit

	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6
Number of Members	3	4	4	3	5	6
Posts per Thread						
First Thread	8	5	1	6	2	2
Second Thread	3	2	4	2	1	9
Third Thread	5	3	3	3	4	10
Fourth Thread	4	1	2	2	11	4
Fifth Thread	3	4	4	3	3	3
Sixth Thread	3	6	9		1	3
Seventh Thread	5	1	3		1	1
Eighth Thread	11	1	2			10
Ninth Thread	4	3	5			4
Tenth Thread						1
Total Number of Posts	46	26	33	16	23	47
Posts by Instructor	7	5	3	3	2	7
Threads per Group	9	9	9	5	7	10
Longest Thread	11	6	9	6	11	10

But first another aspect of the summarized participation merits attention, namely the number of discussion threads that contain only a single post. Looking at Table 5.3, one might infer that singleton posts denote instances where someone's ideas were not being addressed by anyone else. While there is a factual basis to such an assumption, it is a gross oversimplification that does not take into account mediating factors, such as the same ideas being discussed in a different thread or a having a different purpose to the communication.

5.4.1.2 Teachers' Descriptive Discourse

The tally of posts primarily serves as a context for reporting on the content of the communication among participants in their small groups. All the teachers discussed the mathematical behaviors of the children in the videos. They attended to the details of children's representations, which included describing specific Cuisenaire rod models constructed by particular children and descriptions of specific mathematical arguments that children articulated. Comparing the columns in the first two rows of Table 5.4 shows variation in how frequently members of the six groups made such representations significant by discussing them in detail. In a related category, the teachers also remarked on how using the rods to build physical models enabled children to explain their fraction ideas and supported development of mathematical reasoning. Such comments tended to occur with observations about the children's activities in comparing fractions and finding equivalent fractions. For example, the math coach in Group 1 replied to a post detailing rod models and arguments by saying, "A teacher would absolutely find Brian's use of a physical model to justify that 2/10 and 1/5 are 'both correct' number names interesting. Their equivalence is not easily understood without this particular physical representation."

Table 5.4: Summary of Coded Discourse in the First Focal Course Unit

	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	TOTAL
Posts coded for:							
Describe Rod Models	13	3	2	1	3	6	28
Describe Arguments	8	4	2	2	4	8	28
Rod-based Reasoning	1	2	5	3	9	5	25
Comparing Fractions	11	2	4	3	2	10	32
Equivalent Fractions	13	6	8	4	5	6	42
Switching the Unit	3	1	4	0	1	4	13
Operator vs. Number	1	0	3	0	0	2	6
Language Issues	0	1	2	2	1	7	13
Symbolic Notation	2	1	1	0	4	5	13
Assimilation Paradigm	3	2	6	1	0	1	13
Candy Bar metaphor	2	0	3	0	0	2	7
Relate Article/Video	1	3	4	1	2	0	11
Researcher Actions	4	7	4	2	8	6	31
Q's re: Before or After	3	2	0	1	0	7	13
Affective Reaction	2	0	3	2	6	3	16
Using Rods (oneself)	0	1	1	0	3	0	5
Idea from Prior Unit	1	4	2	0	2	0	9
Idea Other Literature	4	0	3	0	0	0	7
Reflect on Practice	3	1	8	7	9	15	43
Benefit to Teachers	5	4	5	4	5	4	27
Plan Group Post	12	7	14	0	0	1	34

An interesting point of variation among groups concerns making significant (or not) an instance of faulty reasoning in the video when three girls shared their rod model with the class as a solution to the task, which is larger, one-half or one-third, and by how much? Groups 1, 3 and 6 had substantial communication about how the girls built a valid model for the first part of the problem but then represented the difference as a part of a part rather than in relation to the rod length that was originally given the number name one. This topic is reflected in the codes *Switching the Unit* and *Candy Bar metaphor* in Table 5.4, with the latter referring to a video episode not viewed but described in the related research article from this unit. Notably, the instances of the former being coded in Groups 2 and 5 both occurred in a singleton post.

5.4.1.3 Teachers' Reflective Discourse

A conversation that arose across all groups, yet manifested in different ways, deals with which sign systems and forms of knowledge should be given prestige. Such interaction is reflected in the codes *Language Issues* and *Symbolic Notation*, yet often also was coded for *Researcher Actions* and/or *Reflect on Practice*, depending on what types of connections were being made in individual participants' commentary. Certain activities in the videos served as triggering events. One such trigger was the researcher's decision not to introduce the word "equivalent" when the children in her classroom were trying to articulate that concept in their own words. The special education teacher in Group 6 initiated a thread:

> It could be my old school ways, but I feel like these clips may be frustrating to me because the students don't have the vocabulary they need to fully and clearly explain what they are trying to say when they prove equivalence. … For the first set, I feel like Meredith didn't know it was ok to have equivalent fractions because she didn't know the terminology. She could clearly see that 1/5 = 2/10, but since she couldn't say it, I don't think she could express herself past her first finding.

One of her peers responded with description of an instance of children sharing a rod model featuring two halves, three thirds, and the unit rod all being the same length, and noted that they got stuck at that point. She then said, "I think the concept of equivalence would have worked perfectly – but they didn't know the term. Yet, I think the goal was to see what the students can think of organically and therefore, they shouldn't know equivalence but arrive to the term on their own." The debate about when and whether to introduce standard vocabulary into developing ideas continued yet did not get resolved among the group. In addition to one member pointing out that "the teacher would ultimately have to introduce the vocabulary to them," another, more experienced educator indicated that he would have remained quiet. He opined that when students are allowed to express themselves without having to remember the correct vocabulary that it can empower them to gain confidence in their arguments. He then connected it to his own practices by saying, "I don't know how often I am telling students when I see that they are struggling to find the 'mathy' way to say things, 'Just say it in your own words.' Then the idea is able to come out."

Not all the groups' conversations about language and mathematical notation were so controversial. For instance, an inexperienced teacher in Group 2 noticed that not having the math language made it hard for students to communicate to each other but recognized that having the rods supported students who were struggling to explain themselves. His interpretation was that, "This doesn't show lack of understanding, just maybe they don't know the terms yet to relate the rods to proper fraction ideas." Other events from the videos triggered positive commentary, particularly when the researcher wrote down some of the children's fraction ideas towards the end of the sixth clip. One teacher in Group 5 carefully described the rod model that Meredith had built to represent the difference between two thirds and one half, which generated a debate among the fourth graders because the difference was represented both as a red rod given the number name one sixth and two white rods given the number name two twelfths. This teacher pointed to the crux of the learning challenge by saying that "The class as a whole seems to have difficulty comprehending that there are two answers that mean the same thing." Then she actually included in her post the number sentences that were written on the overhead projector

1/12 + 1/12 = 2/12   1/2 of 1/6 = 1/12   1/6 = 2/12

and observed that "now is the time that the students can begin to make the connection between words and symbols" as the children had agreed that those fraction ideas are true. Two group members concurred that they liked the way that rods, words, and symbols were being connected; and one further observed that the students were developing the idea of reducing fractions. Teachers in Groups 1, 2, 3 and 6 also noticed how the idea of reducing fractions emerged from the children's activity, some with delighted appreciation for Erik's statement that "they're the same… but it's easier to call it one sixth." However, one of the post-secondary teachers in Group 2 seemed to think that the children must have had prior knowledge of this concept. This claim received a somewhat challenging inquiry from the course instructor, who posted "What prior knowledge? In the process of problem solving, can new knowledge be built?"

Yet another theme among the coding categories in Table 5.4 concerns teachers' reflections on theoretical ideas discussed in the research literature. The assigned reading in this focal course unit reintroduced the idea of *Assimilation Paradigm*, which participants had read about in an article from a previous course unit (Davis, 1992). Another important idea is the distinction of fraction as *Operator vs. Number*, also introduced in a paper from a prior course unit (Davis, Hunting & Pearn, 1993). Note that posts discussing these two ideas are reflected in their respective specific coding category because of their direct connection to the articles in first and second focal course units and what can be seen in the videos for the two focal units. Teachers in most of the groups posted about the significance of these ideas and connected them to what they observed about children's mathematical activities, with Group 3 doing so most frequently. No one in Group 5 made any reference to these theoretical ideas, nor did any one discuss ideas from other research literature. One teacher in Group 4 mentioned assimilation paradigm, but only in an unelaborated list that mirrored the headings for the findings in the assigned article by Steencken and Maher (2003). Teachers in Groups 1 and 3 made connections and built significance for ideas in research literature from an earlier assigned reading through considering how children in the videos might be progressing through layers of understanding and folding back as part of the process (Pirie & Kieren, 1992).

Another way in which similar connections were made is represented in the coding category *Relate Article/Video*, which captured language-in-use about specific associations between the research artifacts. One teacher in Group 5 posted about how “In the reading, the discussion about children using their prior knowledge and personal experiences was important. The constructivist approach led me to reflect on the videos and consider how powerful the learning of these students had become.” Although most teachers who posted on this topic saw the article and videos as equilibrating, one teacher in Group 2 claimed

> Watching the video and reading the paper gave me somewhat different impressions of what was happening in the class… reading the paper gave me more of an impression of continuity of development of thought for the students generating and accepting two different answers for two white blocks as 2/10 and 1/5, and after discussion deciding they were equivalent, while watching the video segments gave me less of an impression of spontaneous development of the equality – it seemed to be facilitated by the teacher’s question making two answers acceptable.

He then asked whether anyone else shared his interpretation and experienced the two sources as different. One group member indicated that the idea was more likely built up over time than spontaneous, to which the instructor responded, “The idea had to have an origin; then the idea spreads. Often it’s the teacher’s idea that is the original – and the hope is that it will spread.”

The coding category *Benefit to Teachers* ties back to the unit’s discussion prompt for participants to comment on how teachers might (or not) find observing the videos of the children's problem solving useful. As Table 5.4 shows, this theme was addressed by all six groups and nearly always it was discussed in conjunction with one or more other coding categories, most frequently with *Reflect on Practice*. For example, the female teacher in Group 3 said, "From watching these videos teachers can gain perspective on how students can build strong understandings of concepts many students struggle with." In Group 5, one of the female teachers commented, "I think that one of the most important points that the videos offer is the power of explanation. 'Convince me' is a wonderful tactic that they use." A female teacher in Group 4 claimed, "The communication allowed extension of knowledge. I think that is another reason these videos are useful for teachers/preservice teachers to notice how and why communication is so important." However, some of the other teachers expressed more ambivalence, perhaps associated with their having relatively more years of teaching experience:

> I believe that a teacher will find observing the videos very useful, but would feel somewhat impatient about the flow of ideas as presented. In the interest of time, a teacher would probably not remain in the more passive role that the researchers remain in, rather, take a more active role in facilitating understandings. (Male teacher in Group 2)
>
> The videos are certainly open to interpretation. A teacher looking to better understand the kinds of questions students would have about fractions could learn a lot. On the other hand, some teachers may see the videos as examples of students getting confused because they were taught in a non-traditional way. (Male teacher in Group 3)
>
> I think that watching student videos of problem solving is especially useful for teachers who may not be comfortable with math. Since many of the teachers who would be introducing and teaching these topics are often elementary school teachers, the general perception is that most of them are not well versed in understanding math conceptually let alone guiding students through conceptual problems. I would think these videos would give teachers a guide for what they should do and also to gain a clearer understanding of how children with little prior knowledge would approach problems. I think a lot of teachers want to teach for understanding, but if they were not taught in this structure and have a low level of comfortability (sic)

> in conveying math topics, it is easier to resort to drill and kill. By watching these videos, it provides a model for how to run problem solving sessions and also for the types of reasoning, both productive and invalid, that students create. (Female teacher in Group 4)
>
> Since the students did develop and defend two ideas and then find a way to relate them, there would be utility on several levels. First the surface level of exposure to the content interpreted through Cuisenaire rods would be useful for some teachers to ground their own conception of fractions in the manipulatives. Second, the fine job the students did in presenting and defending their claims would be useful to some teachers to raise their expectations of students. Third, the instructional methodology demonstrated by the teacher in the video of questioning and facilitating discussion to stimulate student thinking could have utility for some teachers. Finally, observing the development of the students' understandings of the fraction names and what behaviors indicated coming to those understandings would have utility as well. (Male teacher in Group 2)
>
> For many teachers in k-12 schools the world is very small. There is never enough time, learning how to use new resources can be a bit scary and very difficult the first time you are using them in class. I have heard too many times, “They won’t work in my class,” or “Like I have the time to use those.” Watching these videos where a person is using the rods with success makes them seem much more realistic to use in class than if they just were given a set and told to try them out. Teachers are more likely to take a risk if they have seen someone else have results with them. (Male teacher in Group 6)

As can be seen from the quotations from teachers above, several of them recognized that the videos portray *Researcher Actions* in the role of classroom teacher, which entitled her to a degree of freedom that may be viewed as not afforded to the typical teacher. Some of these quotes express both accolades and apprehensions, reflecting that practical inquiry does not always get resolved. Another aspect of researcher actions that was explored but not to the point of resolution concerned the sequencing of tasks. Thus, the videos not only prompted participants’ reflection on certain choices the researcher made in terms of what to do when and how she facilitated discussion among the 4th graders but also stimulated them to raise *Questions* about what may have happened *Before or After* the video episodes they viewed. In addition to being posed as questions, sometimes these comments were phrased as wonderings or speculative statements about trajectories of occurrences.

> I found Alan's response of '3 wholes' interesting. Given that there are three red rods with the number name 1, as a teacher I would've expected him to count 1-2-3 ... and simply call the brown rod '3'. That he says '3 wholes' leads me to believe that he's had prior instruction in part to whole relationships, and is transferring that knowledge to this situation. (Female teacher in Group 1)

> I believe these videos all took place before the students had any formal introduction to fractions and their operations aside from fraction as operator. I could be wrong though. Either way, they did not know LCD (lowest common denominator), as fraction operations are usually taught in 5th and 6th grade. It's actually crucial/for the best that these exercises are done before LCD is "taught" or introduced. Otherwise, it is tough to break the mold of arguments like "well, 1/5 is equal to 2/10 because 1 • 2 is 2 and 5 • 2 is 10". (Female teacher in Group 4)

> I'm interested to see how students would react to the thought of improper fractions. Such a situation arose when students were asked, "If the train of yellow and light green is 1, what is the number name for red?" During Erik's explanation, he states, "if you have one, there'd be two halves, but if you have two it's two halves plus two halves which would be four halves." (Male teacher in Group 6)

> I am curious to see what followed. Did the students realize they were adding, multiplying, and coming up with equivalent fractions? How long did it take to get them to the point were they realized the procedures for operations with fractions (without manipulatives). (Female teacher in Group 6)

Note that some of the above comments also hint at one's personal values. A couple of the coding categories Table 5.4 encompass more direct reflections of one's identity into the social presence of online discussion. One is by interjecting an *Affective Reaction* to the videos. Largely these were positive affect and included being amazed, impressed or excited about how the children were developing arguments, expressing reasoning, building fraction ideas, and engaging with "remarkable maturity" in critical thinking as a classroom community. A male teacher in Group 5 said "I really enjoyed the way these various clips built upon one another." Some of the teachers exclaimed "I love it!" or "These kids are too funny!" as segue into their posts about the children's mathematical arguments and lively discussions debating them. However, two teachers expressed negative affect. The special education teacher in Group 6 was worried about children who might be getting left behind because they were not as verbal or not finding it helpful to work with the rods as a manipulative. Also, in contrast to more positive comments about the pedagogical tactic of saying "I'm confused" to elicit better explanations, a male teacher in Group 1 said "I don't know how these students survived all the confusion from teachers: I feel like I would be constantly under negative affect in this situation." Despite a potential inference that this comment was meant to be sarcastic, there was no emoticon or other evidence of humor being injected into his post; hence it is interpreted as sincere.

The second strong indicator of personal identity appeared in posts where teachers made statements about *Using Rods* themselves. A few teachers posted about using the rods to create models in conjunction with the tasks that children in the video were doing, some of which included reflective commentary about their building a conceptual understanding for what they knew in a procedural way about fractions. Other teachers posted about wanting to use the rods in their own classrooms, notably with Algebra II classes and in developmental math with adult learners, as well as for middle school mathematics lessons.

5.4.1.4 Teachers' Functional Discourse

The last coding category in Table 5.4 is *Plan Group Post*. Groups 1 through 4 had an extended period for their focal unit in which the second week was for bringing ideas from the small groups into a class-wide discussion. These four groups varied greatly in how they approached the assignment for sharing, which is evidenced in their respective use of discussion threads for that purpose. The eighth and longest thread for Group 1 focused exclusively on planning, yet its posts dealt predominantly with the logistics and format for sharing and included little discussion about the contents of what would be shared. They interpreted the assignment literally, as worded by the instructor, to be a group description of the videos and main ideas of the paper. The sixth and longest thread for Group 2 also focused exclusively on planning, but in addition to coordinating the effort they talked about the substance of what to include, reaching a decision that a summary of their group's ideas was more suitable than summarized description of the videos and reading. Group 3 was most collaborative in how they handled the assignment. Their sixth and longest thread was about planning for group discussion, and the teacher who initiated it also included what he saw as important ideas across group members' earlier posts in the unit. The next teacher to post elaborated by including more specifics about what would be good to share. The remainder of this thread was about coordinating their activities. Then, their seventh and eighth threads each began with a draft of what two members had agreed to write about, for which they solicited and got feedback from peers to refine. Their ninth thread was about their completed group entry, on which they worked together to finalize. At the other end of the spectrum, Group 4 did not do any planning together online. Lastly, note that the single post in this category by Group 6 was a suggested, and quickly dismissed, notion of dividing up the workload in their unit assignment.

5.4.1.5 Richness of Teachers' Discourse

As mentioned earlier in the section on overall discursive patterns, there was much variation in the length of posts that teachers made to the online discussion, which ranged from a short sentence to approximately a full page of text. Particularly with regard to the posts that initiated a new discussion thread, they serve as an indicator of that individual's social and cognitive presence in terms of what one chooses to raise for discussion and how one organizes his or her ideas for presenting to the group. Ideas that get raised are either picked up on, or not, by others who respond to that post, if anyone responds at all. As Table 5.5 shows, teachers' discussion posts tended to address multiple topics, with the lengthier ones weaving seven or more into an integrated narrative. With the exception of one teacher in Group 1 whose pattern was to describe main ideas from a video and points of relevance to teachers in a choppy style with bullet points, the teachers presented their ideas in coherent prose that made connections as various themes captured by the coding categories emerged in their discourse. Thus, the number of different coding categories that were applied to each post serves as an indicator of the richness of teachers' discourse. While the results in Table 5.5 are informative on their own, they are more meaningful when considered in conjunction with the results of descriptive and reflective discourse reported in the previous sections and the numerical summaries of discursive patterns and thematic codes presented earlier in Tables 5.3 and 5.4.

Table 5.5: Comparative Richness of Communications in the First Focal Unit

	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6
Posts with 0 codes	3	4	2	0	0	2
Posts with 1 code	13	7	13	2	5	14
Posts with 2 - 3 codes	11	3	9	8	8	13
Posts with 4 - 6 codes	12	5	2	3	6	11
Posts with 7 - 9 codes	0	2	2	0	2	0
Posts with 10 + codes	0	0	2	0	0	0

5.4.2 Second Focal Course Unit

5.4.2.1 Overall Discursive Patterns

As can be seen in Table 5.6 below, all groups show evidence of engagement in posting their ideas and discussing them online during the second focal course unit. Again, these were predominantly student-to-student interactions, with the percentage of instructor posts ranging from 0% to 14% of total contributions to the threaded discussions in this unit. Note that the instructor engaged far less with the four groups in the semester-long implementation, ranging from 0% to 6%, than with the two groups in the shorter summer implementation, which ranged from 13% to 14%. These posts again tended to be brief and fell into one or more of the four categories reported in Section 5.4.1.1 of results.

Table 5.6: Summary of Participation in the Second Focal Course Unit

	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6
Number of Members	3	4	4	3	5	6
Posts per Thread						
First Thread	1	4	5	4	5	7
Second Thread	6	2	1	2	2	6
Third Thread	3	1	10	1	4	2
Fourth Thread	5	2	2	2	3	6
Fifth Thread	1	2	8	6	2	9
Sixth Thread	3	2	4	1	4	2
Seventh Thread	4	5			1	
Total Number of Posts	23	18	30	16	21	32
Posts by Instructor	1	1	1	0	3	4
Threads per Group	7	7	6	6	7	6
Longest Thread	6	5	10	6	5	9

5.4.2.2 Teachers' Descriptive Discourse

Many of the discussion themes that appeared in the first focal unit re-emerged in the second focal unit, as the mathematical content of the assigned videos and articles shifted from fraction comparison tasks that evoked considering the idea of equivalency to ones that involved the placement of fractions on the number line. Table 5.7 provides a summary of the coding categories applied to each group's online discussion from the second focal unit. Since the meaning of many categories already was discussed extensively in previous sections, the following reports on the new codes that appeared in the second focal unit and how the teachers connected these ideas to the categories that carried forth from earlier conversations.

As occurred in the prior course unit, teachers engaged in substantial communication about children's mathematical representations. Particularly interesting were their discussions of how the tasks in which rod models were built to justify claims about fraction ideas seemed to be influencing the children's activities as viewed in this unit's assigned videos. Receiving the codes *Describe Drawings*, *Rod-based Reasoning*, and *Comparing Fractions* were teachers' posted observations about how, in the first video, David went about sharing his ideas about the problem, Can you tell which is largest: one half, one third, one fourth or one fifth? Five of the six groups discussed this topic but there was variation in how teacher talk emerged and the extent of the connections made regarding children's developing mathematical reasoning.

Table 5.7: Summary of Coded Discourse in Second Focal Course Unit

	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	TOTAL
Posts coded for:							
Describe Rod Models	0	0	0	0	1	3	4
Describe Arguments	3	2	4	2	1	5	17
Describe Drawings	3	3	5	1	3	5	20
Rod-based Reasoning	0	2	3	1	3	8	17
Comparing Fractions	3	0	1	5	2	3	14
Equivalent Fractions	0	0	0	0	0	1	1
Number Line	6	5	5	11	8	10	45
Ruler Analogy	0	2	5	2	3	3	15
Candy Bar Metaphor	0	0	2	0	1	0	3
Operator vs. Number	2	5	9	4	8	5	33
Counting vs. Quantity	2	4	1	4	2	3	16
Language Issues	1	7	1	0	0	0	9
Stages of Understanding	2	2	2	4	0	1	11
Concept Stability	0	0	4	0	0	0	4
Relate Article/Video	1	3	4	1	2	2	13
Researcher Actions	4	5	7	11	6	3	36
Q's re: Before or After	1	0	2	5	3	5	16
Affective Reaction	0	0	1	1	7	2	11
Using Rods (oneself)	0	0	0	0	2	0	2
Idea from Prior Unit	1	1	2	0	0	3	7
Idea Other Literature	2	2	4	0	0	1	9
Reflect on Practice	7	4	10	7	4	12	44
Benefit to Teachers	0	1	7	6	1	3	18
Plan Group Post	3	6	10	1	0	0	20
Online Tech. Problems	3	1	0	0	2	0	6

Two teachers in Group 2 independently posted observations that David essentially draws the rods to create his representation of one half, one third, one fourth and one fifth in the first video (See Table 5.2). Although two male teachers in Group 1 refer to David's actions in drawing a model, neither of them remark on the nature of his representation with the insights that the two teachers in Group 2 did. In contrast, the female teacher in Group 3 observed that:

> This video allowed us to see David's use of mental image to describe and conclude the comparison of unit fractions in magnitude, and then to draw his mental image which he clearly built from the his work with the Cuisenaire rods.

The above quote was part of a lengthy post; however, this aspect of her contribution to the discussion happened not to be what stimulated further conversation by group members. One of the teachers in Group 4 indicated that she liked the way the child began "by drawing his own fraction bars and relating each fraction to one whole. He showed how each fraction relates to a whole and to each other." Her post received no responses. In Group 5, a pair of female teachers talked about David's drawing. The first remarked that she found it very interesting how he drew a unit length and then partitioned it into two halves, then drew another unit beneath portioned into three equal pieces, continuing the process through fifths. She then observed, "By David beginning the lesson like this, it seemed easier for the students to create a number line below with all the unit fractions from 1/2 to 1/5 accurately placed." Her peer responded to concur that what David did was an interesting way to start rather than by drawing the number line and added "I thought that when he was drawing the parts that he was drawing 'rods'." Similarly, an interactive discussion took place in Group 6. One of the female teachers initiated a thread highlighting the children's conversation about making small adjustments to where the fractional numbers should be placed on the number line and claiming that it illustrates that they are still thinking "in concrete modeling terms. It appears that in order to place the numbers on the line, they must be actual representations of the 'splitting' of the whole." She saw this as evidence that the children were still viewing fractions as operators. One of the male teachers responded that he agreed that the children "were still thinking of the numbers in terms of the blocks" but thought that their wanting to adjust placement of numbers reflected "squishing numbers into a small space." The other male teacher agreed with that interpretation and wondered about whether distinct numbers being very close together on a number line would be a difficult idea for the children. A second female teacher agreed that the children were in the fraction-as-operator stage yet some had a good idea of what to do with unit fractions since "David was able to draw rod representations of each of the fractions, and make his number line look similar."

Several of the teacher posts cited above that described David's representations and mathematical reasoning also were coded for *Number Line*, which was the most frequently applied code in this unit, as shown in Table 5.7. Given that the two videos and two research articles assigned in this unit focused on where certain fractions would be placed on the number line (See Table 5.2), one naturally would expect that this topic was made significant in the conversations among the teachers. Yet what makes this finding interesting is the way in which the teachers' descriptive discourse was intertwined with their reflective discourse.

5.4.2.3 Teachers' Reflective Discourse

Without exception, teacher posts coded for *Number Line* also were coded for at least one of the other categories *Operator vs. Number*, *Researcher Actions*, and *Reflect on Practice*. Looking across the six groups, these four coding categories appeared 45, 33, 36 and 44 times, respectively. This evidence suggests that the teachers, in the aggregate, were making strong connections among how the activities being enacted by the researcher in role of classroom teacher to engage the fourth graders in the tasks of placing fractions on the number line related to forms of knowledge being built about fractions and how that relates to the activity of teaching mathematics. Moreover, there is evidence that the teachers also were reflecting on their identity as teachers of mathematics in terms of actions they have taken or might wish to take in their own classrooms. Some examples from discussions are:

> I found the (classroom) discussion that took place (in the second video) to be incredibly interesting because it highlights the multiple ways that fractions are used. While these students are thinking about how to place 1/3 on a number line, they are still ultimately discussing a matter of how to label length and in essence discussing fractions as operators rather than as a numerical value. … As a teacher I can honestly say that this is an eye open experience because while I have a thorough understanding of fractions because of this I have been able to move fluidly back and forth through the ideas of fraction as an operator and fraction as a number, but because of that never fully made the distinction or saw the need to. Granted I do not teach fractions in my grade level so I have never been faced with these challenges, but I could only imagine that prior to being exposed to these materials I would have never anticipated the obstacle created by the multiple representations of fractions. (Female High School Teacher, Group 4)

> As an aside, I just wanted to say how impressed I am with these students and the discussions they had. It is amazing how carefully and thoughtfully planned out problems and classroom situations can yield such amazing results. When (our instructor) was discussing that the research focused on the difference between fraction as operator and fraction as number, I did not fully understand the difference. By watching the videos and reading the articles, I now understand the difference. It amazes me to watch these students who have not had any formal training in adding fractions, that they were able to discuss the nature of 0/3, 1/3, 2/3, 3/3. The difference between the length of a segment 1/3 versus the actual number 1/3. It is wonderful to see this. I think that activities like this could be used as a means to help high school students even. I know that many of my students struggled with fractions and really didn't understand them. It's powerful. (Teacher, Group 5)

It is also noteworthy that the above quotes were taken from posts that received additional codes. Both were coded for *Affective Reaction*, the former for *Benefit to Teachers*, and the latter for *Relate Articles / Videos*.

The second video in this unit (See Table 5.2) also triggered discussion about how the children's prior activities in building rod models may have been influencing the representations they were creating and the mathematical arguments they were making about placing one-third on the number line. The groups varied, however, in the extent to which teachers verbalized particular connections, made certain ideas significant, and gave privilege to certain forms of knowledge and ways of knowing. For instance, in Group 4, one of the teachers remarked on the *Researcher Actions* in facilitating classroom discussion among the fourth graders when their work was revealing different placements of one third on the number line. By relating comments about *Comparing Fractions* and *Number Line* she made significant what the researcher did in offering the *Ruler Analogy* to help the children see why it would not be helpful to put one-third in multiple places, as one boy argued could be done. She said, "I think comparing fractions to a ruler was a very good connection. It allowed the students to see that even though each inch-mark is one inch away from the last, we must label them in order or a ruler would be useless." A group member responded to this insight by indicating how she saw those actions as fostering knowledge building about *Counting vs. Quantity*:

> (The researcher as teacher) kept emphasizing to Alan "okay, you are telling me that has a length 1/3 and I can clearly see you've proven that with the rods, but can we call that mark the number name 1/3". To me, this was a subtle way of drawing everyone's attention to the two different issues at stake here: length of a segment versus accumulating quantity on a number line.

Similar observations were made among four teachers interacting in Group 6 in their fourth thread, which was initiated by a male teacher discussing Alan's struggle in the video.

> Alan was still thinking about the fractions in terms of the blocks, and not translating them to the number line. He is having trouble because he is taking each distance as 1/3, so he was willing to put 1/3 in any of 3 spots. He is seeing each block as an individual length not as an accumulation of length. Putting 3 reds down were 3 1/3's not 1/3, 2/3, 3/3. Others in class were saying that he needs to start at 0, but he held true to his assertions that each distance is 1/3 from the last mark on the paper. He needs to be convinced about the importance of 0. The ruler analogy was a good one. He began to get the importance of beginning at zero and counting up.

The other male teacher responded by bringing up the related topic of fraction as *Operator vs. Number* that had been discussed in prior units and then wondering if Alan "does not understand what is being asked of him, or if he knows that their is some validity to his argument and will persist until this validity is acknowledged?" Although the third teacher participating in this conversation mainly re-voiced things that her peers had already said, doing so gave them further significance. However, a different female teacher contributed with new information by introducing ideas from the related reading assignment and responding to the question raised by her peer by observing, "it might be both," and offering evidence to substantiate her claims.

Three of the teachers in Group 5 discussed the ruler analogy in conjunction with placing the fraction one third on the number line as part of their third thread. However, two of them also mentioned the homework assignment given by the researcher at the end of this video, raising the *Question* about what happens *After* by wondering what individual children's number lines would look like as they placed numbers between zero and two. There were similarities among teacher posts in Groups 2 and 3 in the sense that five of eight teachers mentioned the *Ruler Analogy* in lengthy, rich posts that received between four and twelve other codes. While this particular topic did not generate interactive conversation among teachers in these two groups, other topics did recur in responding posts. Although the three teachers in Group 1 discussed the second video, one pointed out that it was "fully covered" in one of the assigned readings and another focused only on student actions by judging them as mathematically correct or faulty. While the third commented on *Researcher Actions* and connected a flawed idea from the fourth grade classroom to a similar mistake made by high school students, she did not mention the *Ruler Analogy*.

The two reading assignments (See Table 5.2) provided participants with a few interrelated themes pertaining to cognitive development of fraction ideas, which are encompassed in the coding categories of *Stages of Understanding, Counting vs. Quantity, Number vs. Operator*, and *Concept Stability*. As shown in Table 5.7, teachers incorporated these topics into their group discussions with noticeable variation across groups. In Group 1, the two male teachers conversed about how an idea they had talked about the week before was being extended with regard to development from counting to quantity in that “children are building from their previous level (here number name) to a greater understanding (accumulation of length), and using knowledge of both simultaneously.” Also, in a singleton post, one of them interpreted the drawings that David made in the first video in light of a certain theoretical framework for growth of mathematical understanding (Pirie & Kieren, 1992); this is a type of connection back to an *Idea from Prior Unit* that the teacher made repeatedly throughout the course. However, a markedly different pattern of discourse emerged in Group 2. Each of the four members produced a lengthy post addressing five, seven or ten different coding categories (See Table 5.8) that connected theoretical constructs to classroom activities seen on the videos and described in one of the readings. Yet there was little interaction among them in discussing these ideas. Instead, one of the teachers honed in on *Language Issues* as being a unifying theme, and he made identical responding posts in three different threads to direct attention to a new thread for discussing his idea. He then copied and pasted quotes from each person’s postings, including one of his own, and offered the following:

> To summarize, what ties these together for students of math, or for students of math teaching (us!), is that student thinking is at least somewhat bounded by the thinking space defined visually, and the thinking space defined by vocabulary. I suggest that is why using the words "number name" has power to shape student responses; it opens a thinking space.

Other groups focused in on different ideas. Group 3 was the only one to discuss the notion of *Concept Stability and Instability*, which was introduced in one of the readings (Maher, Martino & Davis, 1994) to contrast the Piagetian view of *Stages* of development that was more heavily emphasized in the other paper (Alston, Davis, Maher & Martino, 1994). In a lengthy post, the female teacher drew attention to the idea of concept stability and instability by relating it to her own practice and invited commentary from her peers:

> With some students I have seen them be on the brink or even seem to grasp a concept and then the next session or even a few minutes later when they are faced with a new question seem to have forgotten it. I rather think they forgot that they do know it or perhaps its that they have not made the connection yet and see the concept solely linked to the particular problem that lead them to it in the first place… what do you think?

Two of the other teachers commended her for raising the issue, indicating its relevance to their own classroom experiences. One noted, "In fact, when I first read the article I skimmed over that because it was foreign to me, but now that I am thinking about it, I am very interested in the issue." He then theorized that perhaps students are not getting – or taking – the opportunity to reflect on their learning experiences to internalize them.

Group 4 chose to make other ideas significant in their discussion of the articles. One of the teachers responded to her peer's post on facilitating communication about students' different representations and valuing explanations in students' own words by adding that:

> I also felt that the articles highlighted the mini stages that students go through in coming to an understanding of fractions. I thought it important that the alternative structures article noticed that these mini-stages are nonlinear: often occurring in conjunction with each other or even recurring.

> Alan's dilemma would be an example of this as well. However, in traditional procedural ways of teaching, these mental mappings often go unaddressed. They are not explicitly verbalized by students and discussion about them is not facilitated by teachers. I think this is because it can be tough to help students grapple with those ministages or multiple representations without telling them the "conventional" or "correct" ways of interpreting.

Thus, she connected the theoretical constructs with a specific example from the videos and then offered reflections on teaching practices. The same teacher gave another example of mini-stages in a different thread by referencing the children's debate about placing one third both to the left and to the right of one half because the length of the segment between them is one third. This time, however, her comment was offered in the context of how the videos could be a *Benefit to Teachers* because the cognitive development issues could be viewed and discussed in relation to how they might be handled in the classroom.

Groups 5 and 6 chose to use the language of *Operator vs. Number* most frequently to discuss children's developing fraction ideas, doing so a total of 13 times across the threads, as shown in Table 5.7. The terminology *Counting vs. Number* appeared less frequently, twice and three times, respectively, in Groups 5 and 6. The only reference to the concept *Stages of Understanding* occurred in the last post of Group 6's fourth thread about Alan's Struggle, in which one of the female teachers cited one of the readings and claimed:

> The first article explains, "Fractions...are typically thought of as operators for an extended period of time, often years, before they come to be thought of as numbers." Alan seems to be stuck in this first phase, thinking of 1/3 as an operators, 1/3 of the whole and not 1/3 the number.

As this quote illustrates, even the use of the word "phase" as analogous to what the article refers to as a stage of understanding is meshed with what appears to be these participants' preferred terminology of fraction as operator and fraction as number.

5.4.2.4 Teachers' Functional Discourse

The only other coding category that emerged anew in the second focal unit dealt with *Online Tech. Problems*. During the semester-long implementation, there were some network upgrades made at the university during spring break that interfered with the participants' ability to get access to the online course. Group 1, in particular, made a point to check in with one another to be sure that the problem was not being individually experienced. During the summer implementation, one of the teachers was having difficulty getting videos to play and then expressed thanks when his problem got resolved.

For the semester-long implementation only, there was the assignment to *Plan Group Post* based on what they discussed in small groups for sharing with the whole class online. As occurred in the first focal unit, Groups 1, 2, 3 and 4 varied in how they enacted this activity; yet a couple groups showed some consistency in their approach. Again, Group 3 was the most collaborative in doing their planning online. The same teacher took initiative in prompting his peers to prepare their group post and began with his own contributions. Interestingly, he made explicit his choice to re-phrase his ideas as questions, rather than restating what he had written, along with his hope that doing so would be a better lead into discussion. Building on this thread, each member pulled together ideas to share and suggested ways to integrate with one another where it made sense to do so. One of the teachers also decided to "vote for" his preference to make their posts by topic so that it would make it easier for classmates to respond to what specifically interested them. In Group 2, the thread about "Vocabulary and Thinking Space" mentioned above became the basis for planning their group post. One teacher nominated the originator of that thread to take the lead in planning, which more or less had occurred already, and another teacher indicated that they all could add on ideas once conversation moved into class-wide discussion. There also was a delegation of responsibility in Group 1, where one of the male teachers volunteered the female to prepare the group post because she had not done one yet, but he went on to tell her specifically how to do it in terms of which ideas to take from the posts they had made. She agreed to do it, and he thanked her. In Group 4, which did no planning online in the previous unit, one teacher organized and posted a summary of their group's ideas but no one else responded to what she did.

5.4.2.5 Richness of Teachers' Discourse

Some of the findings reported above made reference to the comparative richness of the online posts by mentioning how some of the lengthier ones included content spanning seven or more coding categories. Table 5.8 shows how the groups varied in terms of how many of their posts received a comparative smaller or larger number of coding categories based on thematic content discussed in the second focal unit of the online course. Again, these data should be interpreted in conjunction with the data reported in Tables 5.6 and 5.7 yet also grounded in the analysis of language-in-use within the online discourse itself. Such interpretation follows as part of the upcoming discussion section.

Table 5.8: Comparative Richness of Communications in the Second Focal Unit

	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6
Posts with 0 codes	3	0	2	0	0	0
Posts with 1 code	10	10	8	0	4	9
Posts with 2 - 3 codes	5	2	12	5	6	9
Posts with 4 - 6 codes	3	1	4	10	8	9
Posts with 7 - 9 codes	1	2	1	1	0	1
Posts with 10 + codes	0	2	2	0	0	0

5.6 Discussion

The open-ended nature of the discussion forum gave the teachers participating in the experimental online course free reign to structure their conversations as they wished. Although the instructor's teaching presence was apparent primarily through course design, in no way did her relatively small number of contributions to the online discourse result in low participation among the graduate students enrolled in either implementation of the course. To the contrary, there is evidence that a constructivist view of the teacher's role as a designer and facilitator of learning (Maher, 1988; Maher 1998) in a face-to-face setting made a successful transition to an asynchronous online environment. That is, provision of stimulating multimedia resources in an open and inviting forum with minimal instructor intervention yielded substantial cognitive presence as learners interacted collaboratively.

A couple different discursive patterns emerged among the small groups of teachers-as-learners as they studied videos and related literature to attend to children's building of fraction ideas. Within Groups 2 and 3, we saw a tendency for participants to compose longer posts initiating a thread that addressed multiple themes, either in a sequential review of the main ideas from a few videos and readings or in an integrated narrative that wove together many ideas to tell its own interpretive story. By contrast, Groups 1 and 4 tended to initiate their threads with a more focused discussion of a single resource. While those often were fairly rich conversations, they typically did not address more than six coding categories. Similar patterns emerged in Groups 5 and 6. It is important to emphasize that these are only prevailing trends as observed across the two focal course units.

The videos and related literature also prompted the participating teachers to attend to specific details of the children's mathematical activities, notably their representations. In their threaded discussions, teachers described particular models that children constructed with the Cuisenaire rods and specific arguments they articulated as they reasoned from the rod models. Similarly detailed attention was given to describing drawings and number line representations that appeared in the videos and research papers. The teachers also focused carefully on the language that the children used as they reasoned and debated ideas. The details of these various verbal and visual representations were used by the teachers to make justifiable inferences about the children's cognitive development as they were engaged in the mathematical actions making sense of fraction ideas through exploration, communication and discovery.

The discourse analysis also shows how the teachers drew upon theoretical ideas from the literature, both to interpret the children's mathematical activities and to express aspects of how their own knowledge of fractions was being deepened through participation in this course. In particular, several teachers explicitly stated that they developed a better understanding of fraction as number, including some who indicated that they had not really thought about it much before or even realized that it had been a gap in their knowledge. This is a remarkable admission from teachers with strong mathematical backgrounds, in two significant ways. First, that they were willing to mention it suggests a high degree of comfort within a very positive, supportive social presence among the participants in the online course. Second, that certified teachers of middle and secondary grades mathematics could discover that they had such a gap in their knowledge points to an urgent need to change the way that fractions are taught so that learners develop number sense as well as procedural fluency in operations. Yet, by the teachers' own comments, the knowledge gap was not obvious from just reading the literature; studying the videos of children grappling with the idea of fraction as number in a supportive, communicative classroom environment is what made the point crystal clear for them. To that end, it was participants' attention to the actions of the researcher in the role of classroom teacher in the video episodes that helped them see the educational merits of an exploratory approach to building fraction ideas with an appropriate manipulative for constructing physical models and reasoning.

The videos and related literature also were a powerful stimulant for participants to reflect on teaching practices, both in terms of what they saw the researcher do in the role of classroom teacher and in terms of what they have done themselves or would consider doing differently in the future. As indicated above, the teachers commented on the use of Cuisenaire rods as a manipulative to build models for exploring the meaning of fraction ideas. They discussed this pedagogical approach in general as well as use of particular tasks to foster learning of fraction as number and fraction as operator, which in relation to the number line activities also was talked about as counting versus quantity or accumulated length. All the teachers expressed positive views on this pedagogical approach, however some tempered their views with concerns about classroom teachers having the patience or finding the time to enact lessons like those seen in the videos. Also, the special education teacher was particularly concerned about some students not being "visual learners" or not wanting to handle the rods. Yet a few teachers said that they intend to use the rods in their own classrooms, and one indicated that she made such a recommendation to her colleague.

Another recurring theme about the videos and how viewing them might benefit teachers is in participants' commentary about communication in the classroom. They spoke about how teachers should see and hear how children are expressing themselves as they build fraction ideas, observed the maturity with which children debated arguments, and pointed to several instances where the researcher actions effectively fostered deeper exploration of conjectures. Although some participants were eager to see the researcher introduce terms like equivalence or mathematical symbols when operations with fractions were naturally falling out of the children's explorations, other participants praised the choice to hold back, let children's own language emerge, and validate their talk by writing down what they said. Perhaps the most profound insight about communication arose in the Group 2 discussion, when the doctoral student in the learning sciences introduced a term from his field of study. He conjectured that introducing the vocabulary of "number name" (e.g., the task: If we give the blue rod the number name one, what number name would we give the light green rod?) opened up a "thinking space" for the students. That is, he saw how the language-in-use of the researcher-as-teacher created a way for the classroom to communicate by moving back and forth between reflection and discourse so that students could build an understanding of fraction as number.

In closing, the research presented here is a case study of teachers learning about students' mathematical reasoning in an online course as measured through discourse analysis. While the results are not conclusive or generalizable, they offer an intriguing example for how teacher educators might go about exposing teachers to the kind of "genuine communication" (Yackel & Hanna, 2003, p. 229) that illustrates how to enact the Standards of Mathematical Practice while fostering understanding of key ideas in the Common Core State Standards in the mathematical content area of fractions.

CHAPTER SIX: CONCLUDING DISCUSSION

6.1 Summary of the Research

The research that has been presented in the preceding chapters examined the nature of teachers' learning online from studying multimedia artifacts, which included (1) video episodes from a yearlong classroom research project with fourth graders exploring fraction ideas prior to formal instruction in those topics, (2) text materials drawn from related research articles and literature addressing theoretical perspectives on children's learning of fractions, and (3) written communications from both their peers as they engage with and reflect on those materials and the feedback provided by instructor of the experimental online course. Teacher learning has been investigated through multiple methods using quantitative and qualitative analyses. The first two studies included detailed analysis of video-based assessment measures, in which open-ended participant responses were coded for presence or absence of certain mathematical arguments and the extent of their completeness, as defined by the constituent features for each of four arguments. The coded data became quantified through analyzing the extent to which there was change in individual participants' responses from pre-test to post-test. The second two studies used methods of discourse analysis to define, categorize and count the utterances made by participants in online group discussions. In so doing, this research used a mixed methods approach (1) to qualitatively quantify the nature of what teachers learned online with both pre and post measurements to determine what changed in their knowledge as a outcome of participating in the course, and (2) to examine the teachers' intersubjective meaning making as a learning process while the course was being enacted by the participants.

6.2 Key Findings from the First Study

The study in Chapter 3 examined the video assessment data from the set of 22 participating teachers across both implementations of the experimental online course. Those findings focused on aggregated results for recognizing details of the children's argument by upper and lower bounds and their cases argument that the Blue rod is odd. For the latter argument, since 73% of the teachers already had given a complete argument description on their pre-assessment, there was limited potential in the aggregate for growth. Yet four out of six described the argument completely on their post-test and one other provided an argument feature not described on the pre-test, yielding a growth rate of 83.3% for this argument. For the argument by upper and lower bounds, 41% described a complete argument on their pre-test. Out of the 13 teachers who did not, three provided a complete description on their post-test and eight others provided an additional feature of the argument, resulting in a growth rate of 84.6% for the argument by upper and lower bounds. Taken together, these findings show clear evidence of learning in that all but two teachers were attending sufficiently well to the details of children articulating their mathematical reasoning in the video episode to describe it partially or completely for two different argument forms on an open-ended, post-assessment measure.

Aggregated results, however, provide a limited view on the nature of teacher learning in the online course. Some may be particularly wary of claims of learning when, at the onset of the course, two out of every five teachers already showed that they could describe one argument completely and four out of every five could describe the other argument completely. Yet it is important to keep in mind that the majority of participating teachers were undergraduate mathematics majors and had varying amounts of experience in teaching, so in an ideal world one might expect all of them to be able to understand sixth graders explaining their reasoning in support of solution to a task in the fractions strand. However, this is not necessarily the case, and there are serious issues to address regarding communication in the math classroom and how teachers might be educated to improve classroom practices. Doing so necessitates dealing with complex issues in math education and learning sciences.

6.3 Key Findings from the Second Study

The study in Chapter 4, therefore, delved into the complexities of what might be revealed about teachers' learning about students' mathematical reasoning in an online course. Thus, the activity of participants viewing videos and reading related articles, then sharing their ideas online to jointly build interpretations through interactive discussions, became the perspective for examining how small groups of teachers were building knowledge about children's mathematical learning. Small groups were the unit of analysis for comparing results of discursive interactions aimed at sense making with results of their performance on video-based assessments. One of the online course units, which featured videos and an article focused on the same mathematical task that appeared in the assessment video, was used for discourse analysis. This enabled direct comparison of (a) how six small groups of teachers talked with one another about the representations and arguments made by the fourth graders in support of their solutions, with (b) the extent to which those same groups of teachers individually described the representations and arguments made by the sixth graders in their problem solving on the same task.

Analysis of the data showed that a relatively small proportion of the posts made to the online discussions specifically addressed the four mathematical arguments appearing in the assessment video, even though the both argument by upper and lower bounds and the cases argument that there are rods that have another rod as half and rods that don't were featured prominently in one of the videos and the reading in the focal course unit. However, there was some intersection between the groups that did discuss online the features of those two arguments, as well as for the case argument that the Blue rod is odd, and the groups whose members showed some improvement in describing those arguments from pre-assessment to post-assessment. The correspondence was strongest for the argument by upper and lower bounds, particularly as exhibited by Groups 2, 3 and 5.

Two other aspects of the study in Chapter 4 also are of interest. First is the analysis of aggregated assessment results for the combined sections of the experimental online course with three different comparison conditions. Those results support the claim that the post-assessment growth rate for the Experimental Online (I-1) condition is higher than that of the Comparison Groups (I-2, I-3 and I-4) for both the argument by upper and lower bounds and the cases argument that the Blue rod is of odd length, with relative odds growth of 8.43 for the former argument and 3.33 for the latter. Thus, there is promising evidence of the theorized effectiveness for the design of the online course intervention, which used VMC videos in the fraction strand for group discussion and reflection, in that it resulted in the participating teachers demonstrating a greater awareness of children's mathematical reasoning than three different alternate educational experiences: using VMC videos for lesson planning, using other video resources, and not using any video resources.

Secondly, it is interesting what the discourse analysis revealed about the variety of topics that teachers discussed online in their small groups. Even though specific argument forms that aligned with the video assessment were not discussed much by the teachers, less than 11% of their total posts, they did devote much of their conversations, 43.2%, to children's sense making through problem solving. Topics within that category included (1) the extent to which specific children understood the full mathematical definition of what it means to be one-half, namely that the two parts comprising the whole have to be equal parts; (2) reference to the rod models built by the children; (3) describing what the children said in the video for giving evidence in support of claims about mathematical understanding; and (4) discussing what appeared to be obstacles to children's mathematical understanding. The teachers also made connections to the readings in 16.5% of their posts, which sometimes overlapped with the coding categories for features of arguments (as detailed in Davis & Maher, 1995) and children's sense making. Another large chunk of online conversations, 28.8%, were coded for reflections on teaching practices, which typically happened spontaneously as teachers were interactively making meaning of the multimedia artifacts in terms of "discovering affinities with others, orienting attention, expressing viewpoints, exposing conflict and consensus, and supporting debate and negotiation" (Suthers, 2006, p. 332).

6.4 Key Findings from the Third Study

The richness of teachers' online conversations in the focal course unit that was analyzed in Chapter 4 provided both motivation and methodological basis for extending the discourse analysis in the study reported in Chapter 5. A grounded analysis of discursive interactions considered the prompts for group discussions and the contents of assigned videos and readings and then investigated in detail what the teachers chose to make significant and how they did so through their language in use as participants in the online course. The two other course units that had identical assignments across the two implementations of the experimental online course became focal units in this third study that compared the conversations that had occurred in each of six small discussion groups. With a more fine-grained lens for examining how teachers talked about children's sense making and mathematical reasoning in the two focal course units analyzed in Chapter 5, it was possible to reveal the variety of topics discussed and the richness of the teachers' online conversations.

6.4.1 Key Findings from First Focal Course Unit

In the first focal course unit, there were 21 coding categories applied to the teachers' discursive interactions, and 12 of those 21 pertained to defining the nature of teachers' discussed observations and reflections on various aspects of the children's mathematical activities. One of the categories dealt with planning the group post to a class-wide discussion, impacting only Groups 1 – 4. Of the remaining eight coding categories, one denoted their affective reaction to the videos, and the other seven captured their reflections on the 4th grade classroom study, connections made to research literature and ideas expressed in prior course units, reflections on their own teaching practices (including using the Cuisenaire rods themselves), and considering how teachers might benefit from seeing the videos.

Across the six groups of teacher participants, they generated a total of 164 online posts in the first focal course unit. In reporting what they discussed, the findings were presented in terms of teachers' descriptive discourse, their reflective discourse, and their functional discourse. The latter corresponded to conversations about planning their group's post to the upcoming class-wide online discussion, which took place in 34 of the 164 posts. Teachers' descriptive discourse was encompassed by seven of the coding categories applied to this course unit's online discussions. Those categories were codes for describing details of children's representations, including specific rod models or the words used to express arguments, and their mathematical activities, such as comparing fractional numbers and reasoning from their representations. As one might imagine, descriptions of representations frequently coincided with descriptions of mathematical activities, which is reflected in the seven descriptive discourse coding categories being applied a total of 175 times across 164 posts. Teachers' reflective discourse was encompassed by 13 of the coding categories and spanned a variety of ways in which participants reflected on the videos and readings, as well as their own teaching practices and mathematics teaching and learning more generally. Many of these codes also were applied simultaneously to a single post in the online conversations, which happened frequently as evidenced by the 13 codes being applied 207 times in the aggregate across 164 posts.

The frequency of concurrent application of multiple codes, as an indicator of how teachers tended to make connections among ideas, becomes clearer when examining the full set of data. Eleven of the 164 total posts, or 7%, did not receive any codes and 54 posts, or 33%, received just a single code. However, with 60% of the total teacher posts to the online discussions falling under two or more of the coding categories, there is solid evidence of the richness of their discourse about children's mathematical learning and the implications for teaching practices. It is especially compelling for the 29% of online postings that addressed four or more interrelated topics in an integrated narrative of participant reflections on learning and teaching.

6.4.2 Key Findings from Second Focal Course Unit

In the second focal unit, there were 25 coding categories applied to the teachers' discursive interactions, and 15 of those 25 pertained to defining the nature of teachers' discussed observations and reflections on various aspects of the children's mathematical activities, six of which were identical to categories in the first focal unit. Also, there were eight identical coding categories that captured affective reactions, connections to ideas from the literature, reflections on teaching practices, and considerations of how teachers might find the videos useful. There also was a code for planning group post, impacting only four of the groups, and a category for denoting posts about technical problems that occurred. These latter two categories were reported as teachers' functional discourse. Nine of the coding categories were reported as teachers' descriptive discourse, and the remaining 14 coding categories were reported as teachers' reflective discourse.

Across the six groups of teacher participants, they generated a total of 130 online posts in the second focal course unit. Teachers' descriptive discourse, which attended to the details of children's representations, including specific rods models, drawings, and verbally expressed arguments, and their related mathematical activities, such as comparing fractions and placing fractional numbers on a number line, was coded for 136 times in the aggregate. This compares with teachers' reflective discourse, spanning topics they explored in the videos and readings as well as their reflections on teaching and learning, which occurred a total of 213 times across conversations in six small discussion groups. Teachers' functional discourse occurred least frequently, in only 26 of the online discussion posts.

Only five of 130 posts, or 4%, did not receive any codes and 41 posts, or 31%, received just a single code. However, 30% of their posts addressed topics in two or three different coding categories, and 35% of posts discussed four or more interrelated topics in a reflective narrative about children's learning and implications for teaching practices. Again, results from analyzing the participants' discourse in the second focal course unit provides compelling evidence of teachers learning about students' mathematical reasoning through the process of intersubjective meaning making.

6.5 Conclusions and Limitations of the Research

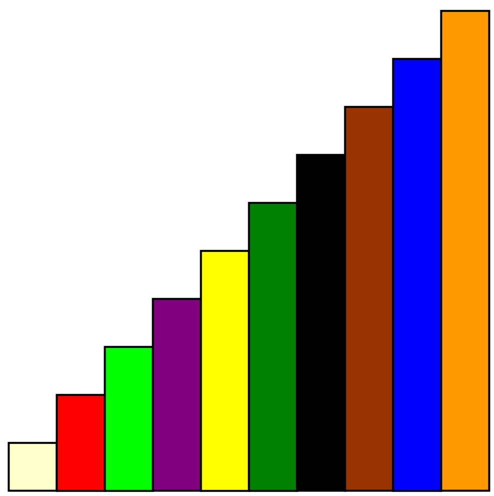
As a whole, the three studies reveal a detailed picture regarding the nature of how teachers learn collaboratively by engaging in rich, thoughtful discussions online about the development of students' mathematical reasoning in the fractions strand through jointly studying and reflecting on multimedia artifacts. The research demonstrates the depth and breadth of their learning about children's reasoning about fraction ideas, how it develops, and how they view what they have learned as being highly relevant to teaching practices at the elementary through secondary grades and beyond. The claim of teachers' learning through collaborative interaction in the experimental online course is further corroborated by analysis of empirical data from a video-based assessment instrument. Results from the data analysis show improvement in teachers' recognition of students' mathematical reasoning for an argument by upper and lower bounds and an argument by cases. Not only did the participants in the experimental online course improve by providing further details of the children's arguments on their post-assessment, but they also did notably better from pre to post than those participating in three different comparison learning environments.

Moreover, this research has implications for the larger program of design research about teachers' learning through studying videos of children's reasoning. Despite the limitations of small sample size and specificity of the population of teachers participating in the three studies reported in this dissertation, the research gives strong evidence that the online forum for interaction with (1) video resources, (2) relevant research literature, and (3) stimulating and reflective feedback from both peers and the instructor, combine to catalyze participant learning in an effective manner. While face-to-face interactions may have their own particular benefits, the asynchronous online environment affords time for individual reflection, which may serve to enhance the discussion among participants.

APPENDIX A: Video Assessment Prompt and Transcript

Title: If I name the Blue Rod “1” ……..

Context: This episode is composed of two segments from an after-school enrichment class for 6th grade students in a middle school in a small urban district in New Jersey. The students are investigating relationships among numbers using Cuisenaire Rods (pictured below). As the episode begins, the students have been presented with the following task: *If I give the Blue Rod the number name: “1”, find a rod whose number name is “1/2”.* The first segment shows the students working in small groups to solve the task and then to develop and record justifications for their solutions to share with the class. In the second segment, which occurs at the end of the 90-minute session, the students are called together for whole-group discussion during which several of the students present their arguments to the class.



After viewing the video of the children explaining and justifying their approaches to the problems, please describe as **completely** as you can: (1) each example of reasoning that a child puts forth; (2) whether or not the argument is convincing; and (3) why or why not you are convinced. Give evidence from the video episode to support any claims that you make. You may refer to the attached transcript as needed.

Each response will be evaluated according to the following criteria:

- Recognition of children’s arguments
- Your assessment of the validity or not of children’s reasoning
- Evidence to support your claims
- Whether the warrants you give are partial or complete

Investigating Fractions with Cuisenaire Rods
Transcript for Episode

This episode is composed of two segments from an after-school enrichment class for 6th grade students in a middle school in a small urban district. The students are investigating relationships among numbers using Cuisenaire Rods. As the episode begins, the students have been presented the following task: *If I give the Blue Rod the number name: "1", find a rod whose number name is "1/2".*

The first segment shows the students working in small groups to solve the task and then to develop and record justifications for their solutions to share with the class. In the second segment, which occurs at the end of the 90-minute session, the students are called together for whole-group discussion during which several of the students present their arguments to the class.

Segment One: Students are working independently in small groups with occasional interventions by the facilitating researchers (R1 and R2).

Chris's Group: (Danielle, Chris, Brittany, Jeffrey)

Brittany	This is what we did – we came from the mint –
	It's something about the blue –
Danielle	But you carry the width –
Brittany	What do you call it? In combo -
Chris	See. I made one like this.
Jeffrey	Yeah. I already made four –
Chris	I made one like that –
Jeffrey	I mean I already made 3.
Danielle	I'm going to put a brown one and I want to put –
Chris	This is the brown and -

Dante's Group: (Michael, Chanel, Dante and Shirelle)

Shirelle	Blue?
R1	Blue is called "one", I want a rod that is called one-half. Blue.
Michael	Blue is "one". Right?
R1	Blue is one, find one rod that you would call one-half.
Chanel	Yellow.
Michael	Nope. Purple and yellow
R1	Purple and yellow? Which one is half, purple or yellow?

Dante	No, yellow is not good …
Michael	I said purple and yellow
R1	Purple or yellow, so which would you call one-half?
Michael	Purple.
R1	You would call Purple one-half?
Chanel	No [she has lined up two yellow next to the blue]
R1	Chanel doesn't agree with you, why not Chanel?
Chanel	Because if you put two purple together, it's still smaller than the other, than the blue [she built a model of two purples lined up next to blue and points to the space that is remaining]
Michael	Purple and yellow [he has a model of a purple and yellow lined up next to the blue]
Chanel	I know, I know, there is none! [Holds up her hand and tries to get R1's attention.]

Chris's Group: (Danielle, Chris, Brittany, Jeffrey)

Chris	[Chris has built a model with nine white rods lined up next to the blue rod] If you take out four that's an even number but if you put the four back … that's not a half because it's only nine and nine is an odd number
Danielle	It's the same as this one this one is even too [she has built the following combinations of rods each lined up next to a blue rod: black and red, purple and yellow, light green and dark green, brown and white, three light greens]
Chris	Which one?
Danielle	This one is even too. What did she say the blue one was?
Chris	One, so you can't find a half of the blue one because if you put all white you only have nine so for nine you can't really do it [he is rearranging his model of nine white rods lined up next to the blue rod]
Brittany	But I can explain it because I can say you can't do it because if you put a black and then a red you can't do it because they are not a half and they are not the same size [she has built the following models of rods lined up next to the blue rod: yellow and purple, red and black, light green and dark green, brown and white, two purple and white,
Chris	And also if you put the white ones you its an odd number which is nine and you can't do it
Danielle	And you take away one
Danielle	Hold on, let me see one of your blue
Jeffrey	You already took one of my blue ones
Danielle	And I gave it back [she is lining up two purple rods next to the blue rod]
Jeffrey	No you didn't. You need a white
Chris	See that's not a half so you can't use blue, it's an odd number
Danielle	So you can't use this one, one, two, three, so for the whole thing you can't use it because of the white ones, you can't really divide an odd number…..If you had like, say if you had two yellows that would equal

	the blue right? That would equal the blue that's one-half, no I know it wouldn't equal the blue I'm just saying if the yellow did go into half the blue [she is pointing to Brittany's model of two yellow rods lined up next to the blue rod]
Chris	Overall you can't do it because if you use a white one it is an odd number so you can't divide by two
Jeffrey	Unless you get a decimal or a remainder
Chris	And you wouldn't be able to do it anyway because none of these are even
Danielle	If only we had the two yellows but the yellows were shorter
Jeffrey	If purple was bigger
Danielle	Yeah if the purple was bigger, then the green was kind of shorter so that the same color green could fit on it
Brittany	Like three of them

Segment Two: Researcher 1 (R1) asks for the class to share what they have found. Students take turns presenting at the overhead projector.

Justina	I was just making half of the colors – I just made this picture with like – umm – half of the Orange would be Yellow. Half of the Brown would be Purple. Half of Dark Green would be Light Green – and the same for those two up there
R2	Justina. Can you say why you decided to look at those rods?
Justina	Um – I didn't really feel like doing the one that we had. I just wanted to do the ones that had halves.
R2	The ones that had halves? And the other ones – you say do not have halves?
R1	Chris!
Chris	Um like y'all was saying the white little rods won't be able to do it but since there's nine white little rods you can't really divide that into a half so you can't really divide by two because you get a decimal or remainder so there is really no half, no half of blue because of the white rods.
R1	That's an interesting explanation, what do the rest of you think about that? Yes over here. Hold on, Dante, nice and loud
Dante	And plus there's one more that she didn't put up there, because she could do
R1	Come up and show us. [He goes up to overhead
Dante	Instead of doing blue and white you could do yellow and purple [He builds this at the overhead.]
R1	Okay, so what do we have here, we have a half, we have two halves right? [Students say no] no? Why not?

Dante	It’s not half its just equal – its not half because yellow is bigger than the purple. [He picks up the rods and shows that the yellow is bigger]
R1	By how much?
Dante	By one white, by one white piece
R1	Okay, do you want to put those down, so your saying it’s not a half because he said yellow is bigger than purple by one white, right? How does that persuade you that it is not a half, maybe there is something else, you just didn’t find it.
Dante	That’s it
R1	Why is that it?
Dante	Because we tried all we can because if usually for the blue piece, it would usually be purple or yellow but yellow would be one um one white piece over it and the pink would be, I mean purple would be one white piece under it.
R1	Okay, so there’s nothing in between in these rods. That’s very interesting.
R1	Okay – nice and loud.
Lorren	We gave a title, its called singles because these, well the ones that have an arrow towards it those are the ones that don’t have any um rods that make up a half of it
R2	And which ones are those?
Lorren	Um, Yellow, Light Green, White, Black, and um Blue. So what I did was I showed the ones would be most practical to being a half and I showed that there is no half even how practical it may seem, its not
R1	So why?
Lorren	Because when I put these shapes together well when I did Black there was a Purple rod here, I mean a Purple Rod, so another Green Rod, another Green Rod would make a Black instead of two Purples and that’s the same with every one of these. [She shows this at the OH as she is explaining.]
R2	When you say instead of two Purples so why were you looking for two Purple Rods?
Lorren	Well I was looking for more than just Purple Rods because here I have Purple and Green, Light Green, and Yellow …
R1	Is there some question.
Chanel	Did you say that Purple is single by itself?
Lorren	No. I said Black. Here’s another one (picture) of Rods that have halves.
R1	Well. Our time has run out -

APPENDIX B: Rubric for Scoring the Fractions Strand Video Assessment

Instructions for Using the Rubric to Code Participant Responses to the FRACTIONS VIDEO ASSESSMENT prompt

When using the *online form* in the scoring database to code participant responses, a participant ID number is assigned to each unique response (i.e., a pre or post test) in the fractions video assessment database. Your coding is automatically associated with the particular participant ID number when you enter codes via the online form. If you are temporarily unable to work online, you can record codes on a scoring worksheet. In those instances, write the participant's ID at the top of the column on each worksheet page. There are four code entry columns per page, which permit up to four assessments to be scored on the multipage worksheet. Circle codes and provide written text as needed in the designated places. All coding done on worksheets must be transferred by the scorer to the database using the online form. You will find radio buttons for Yes / No rubric items, check boxes for features, and text boxes for recording salient information not captured by the coding options described in detail below.

Rubric item 1 consists of three Yes / No questions – 1(a), 1(b) and 1(c) – that always will be answered. Coding Yes for 1(b) or 1(c) requires that you also answer 1(d) and 1(e) and provide student name(s), if mentioned, and the participant's reason for student solution being convincing or not convincing. Coding No for both 1(b) and 1(c) means that you skip to rubric item 2.

Rubric items 2 through 5 are for coding the four different argument types presented by children in the video: **Upper/Lower Bounds, Odd/Even, and two different arguments by Cases. First Cases** refers to an argument based on an exhaustive collection of the four two-rod trains with each train as long as the Blue rod. **Second Cases** refers to an argument based on categorizing all of the rods into two sets: the (5) rods that have other rods that are half as long and the (5) rods that do not. This rubric contains a scoring key (see below) that lists the salient features and their corresponding codes for each argument. Note that a participant's remarks about an argument type should only be scored as "convincing" if they are specifically stated by the participant as convincing somewhere in the response. Similarly, the participant must remark specifically that an argument is "not convincing" for you to code positively to that item.

Rubric item 6, **General Features**, is intended to code for participant response information that is relevant to the video but is not part of one of the four arguments. For example, feature r "*Yellow and Purple are the same length as Blue*" is only to be selected as a General Feature when the participant's response indicates that remark is not part of the Upper/Lower Bounds argument or the First Cases argument.

Rubric items 1 through 6 include text boxes so that, when necessary, additional information about what was written in a participant response can be recorded to aid in explaining outlier data points or capturing important aspects of a response not already included in the coding scheme for scoring assessment data.

Scoring Holistically

Although prompted to describe each argument put forth by the children in the video, participant responses are totally open ended. Therefore, all contributions to formulating the arguments should be considered when scoring. For the argument types Upper/Lower Bounds, Odd/Even, First Cases, and Second Cases: If the participant provides **an appropriate combination of salient features** of that argument type – as specified by the (AND) and (OR) denotations on the scoring key – in his/her written response, then that argument should be scored as a **Complete** description regardless of where in the response those criteria appear. If **at least one feature** of the argument is noted (but not an appropriate combination of them), then the description should be scored as **Partial**. The completeness of a participant's argument should be scored in light of considering his/her response in its entirety.

Scoring Key for Features

A. Essential features for complete arguments

1. **Upper-Lower Bounds Argument**

    a. **The Yellow rod is (1/2 of one White rod) longer than half of Blue; (AND)**

    b. **Purple is (1/2 of one White rod) shorter than half of Blue; (AND)**

    c. **There is no rod with a length that is between Yellow and Purple; (OR)**

    d. **The White rod is the shortest rod and the difference between the Yellow rod and the Purple rod is one White rod.**

    e. **Argument cited without noting any features.**

2. **Odd/Even Argument**

    f. **The Blue rod is the same length as 9 White rods; (AND)**

    g. **Nine is an odd number and cannot be divided evenly. "Halving it" would result in a decimal or remainder; (OR)**

    h. **A rod half as long as the Blue rod would have to be the length of 4.5 White rods and there are no rods that are of "decimal" length.**

    i. **Argument cited without noting any features.**

3. **First Cases Argument based on a collection of all possible trains of two rods with each train the same length as the Blue rod.**

    j. **For the ten rods, there are exactly four two-rod trains that are the same length as the Blue rod (White and Brown, Red and Black, Light Green and Dark Green, and Yellow and Purple); (AND)**

    k. **No one of these trains is made up of two rods that are the same length.**

    l. **Any of the following trains of two rods: White and Brown, Red and Black, Light Green and Dark Green, and Yellow and Purple is the same length as the Blue Rod.**[1]

---

[1] **Feature l.** for the **Cases One argument** is intended to describe a response that notes that students have **included one or more, but not all of the four trains** that are the same length as Blue. This feature, even with **Feature k.**, is <u>not</u> sufficient for a complete description. The response should be coded as **partial.**

m. **Commentary made re: two-rod trains equaling blue without any example (i.e., no specific rod colors) of student findings – hence, none of the above features mentioned. (CODE AS A PARTIAL DESCRIPTION)**

4. **Second Cases Argument based on categorizing all of the rods into two sets: those rods that have other rods that are half as long and those that do not.**

n. **Five of the rods: Red, Purple, Dark Green, Brown and Orange, can be built from trains of two smaller rods of the same length. (Two Yellows are the same length as Orange. Two Purple rods are the same length as Brown. Two Light Green rods are the same length as Dark Green. Two Red rods are the same length as Purple. Two White rods are the same length as Red). (AND)**

o. **No one of the remaining five rods: Blue, Black, Yellow, Light Green and White, can be built by trains of two identical rods. (There are no rods with lengths between White, Red, Light Green, Purple and Yellow.)**

p. **Commentary made re: rods that have halves and those that do not, without any example (i.e., no specific rod colors) of student findings – hence, none of above features mentioned. (CODE AS A PARTIAL DESCRIPTION)**

B. **General features noted in participant responses (not essential for mathematical arguments).**

q. **If the Blue rod is called "1", for a rod to be called "1/2", two of that rod taken together must be the same length as the Blue rod.**

r. **Yellow and Purple are the same length as Blue.**

s. **Each of the rods can be measured evenly by the White rod.**

t. **A solution was modeled with manipulatives.**

u. **A solution was found using trial and error.**

Participant ID:	
Item - Video Assessment	**Assessment of Participant Response**
1(a) Did the participant address the prompt given in this assignment?	Y or N
1(b) Did the participant give only a student(s)' name(s) but no mathematical argument(s) in describing the student(s') solution(s)?	Y or N Student's name(s):
1(c) Did the participant indicate that student(s) solution(s) were confusing and give no mathematical argument(s) in describing the student(s') solution(s)?	Y or N Student's name(s):
If 1(b) or 1(c) is Yes, then answer 1(d) and 1(e). If both are No, then skip to 2.	
1(d) Did the participant claim that a solution or solutions was convincing but with no described argument? Circle **Y** if Yes and **N** if No. Specify any student's name(s) mentioned and note why the participant considered the solution convincing. (*Text limit: 250 characters*)	Y or N **Student's Name(s):** Reason given for convincing:
1(e) Did the participant claim that a solution or solutions was not convincing but without noting a mathematical argument as evidence? Circle **Y** if the solution was noted to be unconvincing and **N** if no such statement was made. Specify any student's name(s) mentioned and note why the participant considered the solution not convincing. (*Text limit: 250 characters*)	Y or N **Student's Name(s):** Reason given for unconvincing:

2(a). Did the participant indicate that an Upper-Lower Bounds argument was used? Circle **Y** if the argument was provided and **N** if no such argument was made. Circle code(s) for salient argument features noted by the participant.	Y or N **Argument Features noted:** a b c d e
If Yes, then answer A through 2(d). If No, then skip to Item 3.	
A. Partial argument description provided	Y or N
B. Complete argument description provided	Y or N
2(b). Did the participant claim that an Upper-Lower Bounds argument was convincing? Circle **Y** if the participant claims this argument was convincing and **N** if not. Circle code(s) for salient features noted by the participant as part of a convincing argument.	Y or N **Features noted as convincing:** a b c d e
2(c) **Did the participant claim that the Upper-Lower Bounds argument was not convincing?** Circle **Y** if the argument was noted to be unconvincing and **N** if no such statement was made. Circle any salient features that the participant indicated were missing from the argument.	Y or N **Features noted as missing from the argument:** a b c d e
2(d) **Did the participant describe any additional features or make other comments** salient to this argument? (*Text limit: 250 characters*)	Y or N If Y, describe:

3(a). Indicates that an Odd-Even argument was used. Circle **Y** if the argument was provided and **N** if no such argument was made. Circle code(s) for salient argument features noted by the participant.	Y or N **Argument Features noted:** f g h i
If Yes, then answer A through 3(d). If No, then skip to Item 4.	
A. Partial argument description provided	Y or N
B. Complete argument description provided	Y or N
3(b). Did the participant claim that an Odd - Even argument was convincing? Circle **Y** if the participant claims this argument was convincing and **N** if not. Circle code(s) for salient features noted by the participant as part of a convincing argument.	Y or N **Features noted as convincing:** f g h i
3(c) Did the participant claim that the Odd-Even argument was not convincing? Circle **Y** if the argument was noted to be unconvincing and **N** if no such statement was made. Circle any salient features that the participant indicated were missing from the argument.	Y or N **Features noted as missing from the argument:** f g h i
3(d) Did the participant describe any additional features or make other comments salient to this argument? (*Text limit: 250 characters*)	Y or N If Y, describe:

4(a). Did the participant indicate that a First Cases argument (based upon a collection of all possible trains as long as the Blue rod) was used? Circle **Y** if the argument was provided and **N** if no such argument was made. Circle code(s) for salient argument features noted by the participant.	Y or N **Argument Features noted:** j k l m
If Yes, then answer A through 4(d). If No, then skip to Item 5.	
A. Partial argument description provided	Y or N
B. Complete argument description provided	Y or N
4(b). Did the participant claim that a First Cases argument (based upon a collection of all possible trains as long as the Blue rod) was convincing? Circle **Y** if the participant claims this argument was convincing and **N** if not. Circle code(s) for salient features noted by the participant as part of a convincing argument.	Y or N **Features noted as convincing:** j k l m
4(c) Did the participant claim that a First Cases argument (based upon a collection of all possible trains as long as the Blue rod) was not convincing? Circle **Y** if the argument was noted to be unconvincing and **N** if no such statement was made. Circle any salient features that the participant indicated were missing from the argument.	Y or N **Features noted as missing from the argument:** j k l m
4(d) Did the participant describe any additional features or make other comments salient to this argument? (*Text limit: 250 characters*)	Y or N If Y, describe:

5(a). Did the participant indicate that a Second Cases argument (based on categorizing all of the rods into two sets: those rods that have another rod that is half as long and those that do not) was used? Circle **Y** if the argument was provided and **N** if no such argument was made. Circle code(s) for salient argument features noted by the participant.	Y or N **Argument Features noted:** n o p
If Yes, then answer A through 5(d). If No, then skip to Item 6.	
A. Partial argument description provided	Y or N
B. Complete argument description provided	Y or N
5(b). Did the participant claim that a Second Cases argument (based on categorizing all of the rods into two sets: those rods that have another rod that is half as long and those that do not) was convincing? Circle **Y** if the participant claims this argument was convincing and/or provides it as an answer to question 3 and **N** if not. Specify any student's name(s) mentioned and circle code(s) for salient features noted by the participant as part of a convincing argument.	Y or N **Features noted as convincing:** n o p
5(c) Did the participant claim that a Second Cases argument (based on categorizing all of the rods into two sets: those rods that have another rod that is half as long and those that do not) was not convincing? Circle **Y** if the argument was noted to be unconvincing and **N** if no such statement was made. Circle any salient features that the participant indicated were missing from the argument.	Y or N **Student's Name(s):** **Features noted as missing from the argument:** n o p
Did the participant describe any additional features or make other comments salient to this argument? (*Text limit: 250 characters*)	Y or N If Y, describe:

6(a) Did the participant include general features, not essential for establishing a complete mathematical argument, in describing student(s) solutions? Circle **Y** if any general features were noted and **N** if not. Circle code(s) for features noted by the participant.	Y or N **General Features noted:** q r s t u
6(b) Did the participant describe any non-coded features or general comments that are *not* part of the four arguments and are not included in 6(a)? (*Text limit: 250 characters*)	Y or N If Y, describe:

Rubric last edited on 08/16/2011 by M. F. Palius

REFERENCES

Alston, A., Basu, R., Morris, K. and Pedrick, L (2011). Lesson Study as a tool for developing teachers' close attention to students' mathematical thinking. In L. Hart, A. Alston and A. Murata (Eds.) *Lesson Study Research and Practice in Mathematics Education: Learning Together* (pp. 135-151). New York: Springer.

Alston, A. S., Davis, R. B., Maher, C. A., & Martino, A. M. (1994). Children's use of alternative structures. In J. P. da Ponte and J. F. Matos (Eds.), *Proceedings of the 18th Annual Conference of the International Group for the Psychology of Mathematics Education, (2)*, 248-255. Lisboa, Portugal: University of Lisboa.

Anderson, T., Rourke, L., Garrison, D. R. & Archer. W. (2001). Assessing teaching presence in a computer conferencing context. *Journal of Asynchronous Learning Networks 5,* (2), 1–17.

Barab, S. A. (2004). Design-based research: Clarifying the terms. [Special Issue] *Journal of the Learning Sciences, 13*, 1-38.

Benbunan-Fich, R., Hiltz, R. S. & Harasim, L. (2005). The online interaction learning model: An integrated theoretical framework for learning networks. In S. R. Hiltz and R. Goldman (Eds.), *Learning together online: Research on asynchronous learning networks* (pp. 19-37). Mahwah, NJ: Lawrence Erlbaum Associates.

Bielaczyc, K. (2006). Designing social infrastructure: Critical issues in creating learning environments with technology. *Journal of The Learning Sciences, 15*, 301-329.

Blank, R. K., de las Alas, N., & Smith, C. (2007). Analysis of the quality of professional development programs for mathematics and science teachers: Findings from a cross-state study. *Washington, DC: Council of Chief State School Officers. Retrieved February 29, 2008.*

Borko, H., Jacobs, J., Eiteljorg, E., & Pittman, M. (2008). Video as a tool for fostering productive discussions in mathematics professional development. *Teaching and Teacher Education, 24*(2), 417-436.

Bransford, J. D., Derry, S. J., Berliner, D. C. & Hammerness, K. (2005). Theories of learning and their roles in teaching. In L. Darling-Hammond & J. Bransford (Eds.), *Preparing teachers for a changing world: What teachers should learn and be able to do.* San Francisco: Jossey-Bass.

Bulgar, Sylvia (2002). *Through a teacher's lens: Children's constructions of division of fractions.* Unpublished doctoral dissertation, Rutgers University.

Davis, G., Hunting, R. P. & Pearn, C. (1993). What might fractions mean to a child and how would a teacher know?" *The Journal of Mathematical Behavior, 12*(1), 63-76.

Davis, R. B. (1984). *Learning mathematics: The cognitive science approach to mathematics education.* Norwood, NJ: Ablex Publishing Company.

Davis, R. B. (1990). Discovery learning and constructivism. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), Constructivist Views on the Teaching and Learning of Mathematics, *Journal for Research in Mathematics Education Monograph No. 4*, 93-106. Reston, VA: National Council of Teachers of Mathematics.

Davis, R. B. (1992). Understanding 'understanding'. *The Journal of Mathematical Behavior, 11*, 225-241.

Davis, R. B. & Maher, C. A. (1990). What do we do when we do mathematics? In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), Constructivist Views on the Teaching and Learning of Mathematics, *Journal for Research in Mathematics Education Monograph No. 4*, 65-78. Reston, VA: National Council of Teachers of Mathematics.

Davis, R. B., Maher, C. A. & Martino A. M. (1992). Using videotapes to study the construction of mathematical knowledge by individual children working in groups. *Journal of Science, Education and Technology*, *1*(3), 177-189.

Davis, R. B., Maher, C. A. & Noddings, N. (1990). Constructivist views on the teaching and learning of mathematics. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics: Journal for Research in Mathematics Education, Monograph No. 4*, 1-3. Reston, VA: National Council of Teachers of Mathematics.

Francisco, J. M. & Maher, C. A. (2005). Conditions for promoting reasoning in problem solving: Insights from a longitudinal study. Special Issue: Mathematical problem solving: What we know and where we are going (Guest Editors: Cai, J, Mamona-Downs, J. & Weber, K.) *The Journal of Mathematical Behavior, 24*(3-4), 361-372.

Francisco, J. M. & Maher, C. A. (2011). Teachers attending to students' mathematical reasoning: Lessons from an after-school research program. *Journal of Mathematics Teacher Education 14* (1), 49-66.

Garrison, D. R., Anderson, T. & Archer, W. (2000). Critical inquiry in a text-based environment: Computer conferencing in higher education. *The Internet and Higher Education 2* (2-3), 1-19.

Garrison, D. R., Anderson, T. & Archer, W. (2001). Critical thinking, cognitive presence, and computer conferencing in distance education. *The American Journal of Distance Education 15* (1), 7-23.

Gee, J. P. (2005). *An introduction to discourse analysis, second edition*. Routledge: New York.

Hmelo-Silver, C. E., Maher, C. A., Palius, M. F. & Sigley, R. (2012). Resources for reasoning: Use of video and text in a CSCL environment. *The future of learning: Proceedings of the 10th International Conference of the Learning Sciences (ICLS 2012) – Volume 2*, (pp. 537-538). Sydney, Australia: ISLS

Kelly, A. E. (2003). Theme issue: The role of design in educational research. *Educational Researcher 32* (1), 3-34.

Koc, Y., Peker, D. & Osmanoglu, A. (2009). Supporting teacher professional development through online video case study discussions: An assemblage of preservice and inservice teachers and the case teacher. *Teaching and Teacher Education 25*, 1158-1168.

Maher, C. A. (1988). The teacher as designer, implementer, and evaluator of children's mathematical learning environments. *The Journal of Mathematical Behavior, 6*, 295-303.

Maher, C. A. (1998). Constructivism and constructivist teaching - can they co-exist? In Ole Bjorkqvist (Ed.), *Mathematics teaching from a constructivist point of view* (pp.29-42). Finland: Abo Akademi, Pedagogiska fakulteten.

Maher, C. A. (2005). How students structure their investigations and learn mathematics: Insights from a long-term study. *The Journal of Mathematical Behavior, 24*(1) 1-14.

Maher, C. A. (2008). Video recordings as pedagogical tools in mathematics teacher education. In D. Tirosh and T. Wood (Eds.), *International Handbook of Mathematics Teacher Education: Vol. 2: Tools and Processes in Mathematics Teacher Education* (pp. 65-83). Rotterdam, The Netherlands: Sense Publishers.

Maher, C. A. (2011). Supporting the development of mathematical thinking through problem solving and reasoning. In B. Ubuz (Ed.), *Proceedings of 35th Conference of the International Group for the Psychology of Mathematics Education 1*, 85-90. Ankara, Turkey: PME.

Maher, C. A., Alston, A., Dann, E., & Steencken, E. (2000). *Private Universe Project in Mathematics Series Content Guide: A Professional Development Workshop Series for K-12 Teachers of Mathematics.* Produced by Harvard-Smithsonian Center for Astrophysics and the Robert B. Davis Institute for Learning, Smithsonian Institution Astrophysical Observatory, Cambridge, MA.

Maher, C. A. & Davis, R. B. (1990). Building representations of children's meanings. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), Constructivist Views on the Teaching and Learning of Mathematics: *Journal for Research in Mathematics Education Monograph No. 4*, 79-90. Reston, VA: National Council of Teachers of Mathematics.

Maher, C. A. & Davis, R. B. (1995). Children's explorations leading to proof. In C. Hoyles and L. Healy (eds.), *Justifying and proving in school mathematics* (pp. 87-105). London: Mathematical Sciences Group, Institute of Education, University of London.

Maher, C. A., Davis, R. B. & Alston, A. (1992). Teachers paying attention to students' thinking. *Arithmetic Teacher, 39*(9), 34-37.

Maher, C. A., Landis, J. H. & Palius, M. F. (2010). Teachers attending to students' reasoning: Using videos as tools. *Journal of Mathematics Education 3* (2), 1-24.

Maher, C. A. & Martino, A. M. (1996). The development of the idea of mathematical proof: A 5-year case study. In F. Lester (Ed.), *Journal for Research in Mathematics Education, 27* (2), 194-214.

Maher, C. A., Martino, A. & Davis, R. B., (1994). Children's different ways of thinking about fractions. In J. P. da Ponte and J. F. Matos (Eds.), *Proceedings of the 18th Annual Conference of the International Group for the Psychology of Mathematics Education, (3)*, 208-215. Lisboa, Portugal: University of Lisboa.

Maher, C. A., Mueller, M. & Palius, M. F. (2010). Using an informal learning environment to promote mathematical reasoning among middle-school students. *Mathematics Bulletin, 49*, 64-72.

Maher, C. A., Mueller, M. & Yankelewitz, D. (2009). A comparison of fourth and sixth grade students' reasoning in solving strands of open-ended tasks. In M. Tzekaki, M. Kaldrimidou & H. Sakonidis (Eds.), *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education, (4)*, 73-80, Thessaloniki, Greece.

Maher, C. A., Palius, M. F., Maher, J. A. & Sigley, R. (2012). Teachers' identification of children's upper and lower bound reasoning. In L. R. Van Zoest, J. J. Lo, & J. L. Kratky (Eds.), *Proceedings of the 34th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, 460-467. Kalamazoo, MI: Western Michigan University.

Maher, C. A., Palius, M. F. & Mueller, M. (2010). Challenging beliefs about children's mathematics learning through video study. In P. Brosnan, D. Erchick, and L. Flevares (Eds.), *Proceedings of the 32nd annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (4), 885-992, Columbus, OH: The Ohio State University.

Maher, C. A., Powell, A. B. & Uptegrove, E. (Eds.), (2010). *Combinatorics and reasoning: Representing, justifying and building isomorphisms.* New York: Springer Publishers.

Maher, C. A. & Weber, K. (2010). Representation Systems and Constructing Conceptual Understanding. *Special Issue of the Mediterranean Journal for Research in Mathematics Education 9*(1), 91-106.

McCrory, R., Putnam, R. & Jansen, A. (2008). Interaction in online courses for teacher education: Subject matter and pedagogy. *Journal of Technology and Teacher Education 16*(2), 155-180.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). Common Core State Standards Initiative: About the standards. Retrieved from http://www.corestandards.org/about-the-standards

Nemirovsky, R. & Galvis, A. (2004). Facilitating grounded online interactions in video-case-based teacher professional development. *Journal of Science Education and Technology 13*(1), 67-79.

Noddings, N. (1990). Constructivism in mathematics education. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), Constructivist Views on the Teaching and Learning of Mathematics: *Journal for Research in Mathematics Education Monograph No. 4*, 7-18. Reston, VA: National Council of Teachers of Mathematics.

Palius, M. F. & Maher, C. A. (2011). Teacher education models for promoting mathematical thinking. In Ubuz, B. (Ed.), *Proceedings of 35th Conference of the International Group for the Psychology of Mathematics Education 1*, 321-328. Ankara, Turkey: PME.

Pirie, S. E. B. & Kieren, T. E. (1992). Watching Sandy's understanding grow. *The Journal of Mathematical Behavior*, 11, 243-257.

Powell, A. B., Francisco, J. M., & Maher, C. A. (2003). An analytical model for studying the development of learners' mathematical ideas and reasoning using videotape data. *The Journal of Mathematical Behavior, 22*(4), 405-435.

Reynolds, Suzanne L. (2005). *Study of fourth grade students' explorations into comparing fractions.* Unpublished doctoral dissertation, Rutgers University.

Sammons, M. (2007). Collaborative interaction. In M. G. Moore (Ed.), *Handbook of Distance Education, Second edition* (pp. 311-321). Mahwah, NJ: Lawrence Erlbaum Associates.

Santagata, R., Zannoni, C., & Stigler, J. W. (2007). The role of lesson analysis in pre-service teacher education: An empirical investigation of teacher learning from a virtual video-based field experience. *Journal of Mathematics Teacher Education 10*, 123-140.

Schmeelk, Suzanna (2010). *An Investigation of Fourth-Grade Student's Growing Understanding of Rational Numbers.* Unpublished doctoral dissertation, Rutgers University.

Schmeelk, S. & Alston, A. (2010). From equivalence to rational numbers: The case of Meredith. In P. Brosnan, D. B. Erchick & L. Flevares (Eds.), *Proceedings of the 32nd annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (6),* 720-727. Columbus, OH: The Ohio State University.

Sherin, M. G., & Han, S. Y. (2004). Teacher learning in the context of a video club. *Teaching and Teacher Education, 20*(2), 163-183.

Seidel, T., Stürmer, K., Blomberg, G., Kobarg M. & Schwindt, K. (2011). Teacher learning from analysis of videotaped classroom situations: Does it make a difference whether teachers observe their own teaching or that of others? *Teaching and Teacher Education 27*, 259–267.

Star, J. R. & Strickland, S. D. (2008). Learning to observe: using video to improve preservice mathematics teachers' ability to notice. *Journal of Mathematics Teacher Education 11*, 107-125.

Steencken, Elena P. (2001). *Tracing the growth in understanding of fraction ideas: A fourth grade case study*. Unpublished doctoral dissertation, Rutgers University.

Steencken, E. P. & Maher, C. A. (2003). Tracing fourth graders' learning of fractions: Episodes from a yearlong teaching experiment. *The Journal of Mathematical Behavior, 22* (2), 113-132.

Suthers, D. D. (2006). Technology affordances for intersubjective meaning making: A research agenda for CSCL. *Computer-Supported Collaborative Learning 1*, 315-337.

Swan, K. & Shea, P. (2005). The development of virtual learning communities. In S. R. Hiltz and R. Goldman (Eds.), *Learning together online: Research on asynchronous learning networks* (pp. 239-260). Mahwah, NJ: Lawrence Erlbaum Associates.

Tall, D. O. (1995). Cognitive growth in elementary and advanced mathematical thinking. In L. Meira and D. Carraher (Eds.), *Proceedings of the 19th Conference of the International Group for the Psychology of Mathematics Education (3)*, 61-75. Recife, Brazil: Universidade Federal Pernambuco.

van Es, E. A. & Sherin, M. G. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education 10*(4), 571-596.

van Es, E. A. & Sherin, M. G. (2008). Mathematics teachers' "learning to notice" in the context of a video club. *Teaching and Teacher Education 24*, 244–276.

Webb, N. (1982). Student interaction and learning in small groups. *Review of Educational Research, 52*(3), 421-445.

Wilson, S. M. & Berne, J. (1999). Teacher learning and the acquisition of professional development knowledge: An examination of research on contemporary professional development. *Review of Research in Education 24*(1), 173-209.

Yackel, E. & Hanna, G. (2003). Reasoning and proof. In J. Kilpatrick, G. W. Martin, and D. Schifter, (Eds.), *A Research Companion to Principles and Standards for School Mathematics* (pp. 227-236). Reston, VA: National Council of Teachers of Mathematics.

Yankelewitz, Dina (2009). *The development of mathematical reasoning in elementary school students' exploration of fraction ideas*. Unpublished doctoral dissertation, Rutgers University.

Yankelewitz, D., Mueller, M. & Maher, C. (2010). A task that elicits reasoning: A dual analysis. *Journal of Mathematical Behavior 29*(2), 76-85.

Zhang, M., Lundeberg, M., Koehler, M. J., & Eberhardt, J. (2011). Understanding affordances and challenges of three types of video for teacher professional development. *Teaching and Teacher Education, 27*(2), 454-462.