Towards Uncertainty in Micro-grids: Planning, Control and Investment

by

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ABSTRACT OF THE DISSERTATION

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The advent of micro-grids and their potential participation in the wholesale market makes the development of new business models necessary. It is anticipated that the future wholesale market for electricity and its ancillary services will include the existing major players as well as micro-grids, which will act both as buyers and sellers. The wholesale market will shift more towards a distributed system with the centers of gravity dynamically changing depending on how the smaller micro-grids play out their supply and demand and also how the market aggregation takes place. Having the obligation to fully satisfy its demand at each point of time, any shortage in available power supply within the micro-grid will lead to the purchase of electricity from the macro-grid at spot market price. The sources of variability rising from the forecast of renewable energy resources and electricity demand, would introduce uncertainty in making decisions at planning layer. Also, a micro-grid may fail to honor its contractual obligations to supply to the

market due to these volatilities.To optimize the planning and operation of micro-grids, it is important for the planner and controller to take into account uncertainties inherent to the micro-grid and the overall supply and demand network, including the energy market place. With high capital costs involved in building a micro-grid, sequential investment strategies, which promote gradual increase in capacity of generation, are needed.

This work aims to develop a set of models and tools that address the optimal decision-making processes involved in both operation and investment in micro-grids. These models account for short-term operational and long-term investment uncertainties in decision making and adopt the following analytics:

- Two-stage stochastic programming and certainty equivalent models to obtain optimal decisions for day-ahead planning in micro-grid's operation;
- Contingent claim analysis and Monte Carlo simulation to examine the value in delaying the investment due to uncertainty around the investment;

Capital budget planning model along with Monte Carlo stochastic scenario generation to derive the optimal investment decisions for micro-grid's portfolio considering its optimal operation under uncertainty.

Acknowledgement

It is a treat to be able to express my gratitude to people and organizations that provided me with generous support throughout my graduate study.I would like to thank DNV KEMA for their partial funding of my thesis and CAIT for their continuous support during my PhD years. Among those who guided and/ or accompanied me in this journey, with so much regret, Professor TyfurAltiok is no longer with us. It was an honor for me to have him as a teacher and a member of my PhD committee. I will never forget his positive outlook and will dearly miss his warm smile.

When I look back to the past five years, I cannot stop thinking about individuals who had such significant impact on my life, education and career. I would like to take my opportunity here to let them know that how much I appreciate their presence in my life and that I am committed to keep up the hard work and to pass on the values and dreams they have given to me. I simply cannot afford letting them down!

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I am so grateful to have Dr Ralph Masiello as my co-advisor and as my mentor at work. He is one of the smartest people I ever met and his passion for what he does is a huge motivation for me.Working for him is a constant learning experience. I am amasedby his knowledge and expertise in energy industry. I appreciate his amazing personality and sense of humor; he generously refuses to take any credit for himself and rather to give it to you once you truly know that it was him who came up with such a great idea to solve the problem in the first place. Ralph thanks so much for giving me the opportunity to be part of DNV KEMA and for being able to work with so many brilliant people in such a vibrant environment. I truly hope that I could be as enthusiastic as him and to someday be able to be an awesome leader as he is.

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Finally, no words can convey my love to my dearest parents and brother. They are the most precious reason of my life and my best friends with their unconditional love and support. My parents taught us all the core values that we hold today. I would not have made it thus far without them.I have the smartest and wisest mother ever. I am amazed of her understanding of different situations which makes me rely on her opinions drastically. I have an easy-going, yet very reliable father. My dad cheers me up with his incredible wittiness even at rough times when he assures me that he always has my back no matter what. I have a wonderful brother and I have been lucky to have him with me at Rutgers. Past few years have not been an easy ride for me and he has been a great source of my peace and comfort during my good and bad times. To my family, thank you, for having faith in me and for helping me to instill confidence. I love you so much!

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Chapter 1 - Introduction and Research Background

1.1 Objectives

This thesis intends to deliver solutions to the following problems:

1. Optimal planning and operational control of micro-grids under uncertainties due to electricity demand, spot pricing, and renewable generation. The following models will be considered:
    a. Certainty equivalent model where expected values are used;
    b. Risk neutral model where the above uncertainties are included, but the model excludes risks attributed to the decision maker;
    c. Risk-averse model, which repeats case (b) and includes the decision making risks.
2. Characterization of micro-grids according to a set of resource configuration and volatility parameters. The characterization is intended for micro-grid benchmarking and selection of decision models that best match exogenous and endogenous operating environment of micro-grids.
3. Extension of the current state of art in real option investment models (both analytical and Monte Carlo simulations):
    a. To simultaneously consider stochasticity in technology cost (specifically for PV) and price of natural gas;
    b. To include correlation between gas price and PV investment cost;
    c. To include speculations with respect to shut down and reactivation of existing micro-grid resources.

4. Development of a comprehensive micro-grid portfolio optimization model with gas-fired generation, PV, wind turbine, battery storage and sell-back to the grid.

1.2 Brief Overview of Thesis Accomplishments

Chapter two covers discussion on the development of a decision support system for micro-grid operators in order to optimize their one-day-ahead plans and daily micro-grid operations. Different sources of uncertainties are included in the formulation. This includes stochastic electricity demand and volatile renewable resources. Three different regimes for decision making are examined and compared:

1. Certainty equivalent regime, which utilizes expected values for planning cost minimization. Then the planning decisions are fed into an operation cost minimization model which computes the overall planning/operation average cost and standard deviation.
2. Risk-neutral stochastic programming regime: two-stage stochastic programming is used where the first stage is for planning and the second stage for the next day operation with the objective of optimizing planning decisions under all possible operational scenarios.
3. Risk-averse stochastic programming regime: this model is similar to risk-neutral stochastic programming regime; however, the variation of cost is also controlled via minimizing conditional value at risk.

This chapter also includes introduction of two sets of parameters, namely configuration and volatility parameters. These parameters can be used to compute *difference statistics* between micro-grids, hence for benchmarking and comparison.

Chapter 3 extends the current state of art in real option approach to investment in micro-grids with two sources of random variations with and without correlations. This work builds on prior work on real option models of distributed generation. Both analytical solutions with certain functional forms and Monte Carlo simulations are presented and compared.

In Chapter 4 we present a novel approach to micro-grid portfolio optimization. The model uniquely integrates short-term risks and uncertainties due to day-to-day planning and operations and long terms risks due to investment and technology costs and returns. A two-stage solution approach is presented: (i) A conditional functional form is built and fed into an investment model which utilizes stochastic scenario generation techniques to accommodate the uncertainty in investment; (ii) A capital budgeting model is developed to obtain the optimal investment decisions across all possible stochastic scenarios created.

1.3 Synopsis of Contributions

1.3.1 Optimal Planning and Operation Control of Micro-grids under Uncertainty (Chapter 2)

In this chapter we develop optimal planning and control models for micro-grids under uncertainty. A micro-grid is defined by a collection of local renewable and non-renewable generators, and is subject to correlated sources of random variations, namely, solar, wind, spot price of electricity, and internal demand. In cases of expected shortfalls in its generation, the micro-grid commits itself to purchase electricity from the macro-grid that it connects to. We formulate a two-stage stochastic programming optimization model, where the first stage decisions (i.e., prior purchase agreement) can be altered by a set of optimal recourse actions in the second stage. Three variations of this model, namely certainty equivalent regime, risk neutral regime, and risk-averse regime, are presented and compared. Finally, we introduce a parametric characterization of micro-grids according to their mean capacity configuration and operational volatility. A dissimilarity measure defined on this probability space determines the relative volatility between two micro-grids. The contribution of this chapter is to examine the sensitivity of optimal plan and control to these parameters. Also, the comparison of the three regimes is done based on these characteristic parameters to help with choosing the appropriate model that provides a fair balance between simplicity and performance.

1.3.2 A Real Option Model of Micro-grid Investment under Uncertainty (Chapter 3)

In this chapter we present a real option approach to investment in micro-grids where the micro-grid owner is given the option of delaying investment decisions depending on market driven exogenous factors, such as price of fuel and cost of technology, and indigenous factors, such as electricity load. This work extends the current state of investment modeling by considering: (i) Simultaneous investment in more than one asset with uncertain behavior; (ii) Multiple sources of uncertainties along with more realistic probability distributions. Within the context of the above extensions, the following practical contributions are also made: (a) Operation flexibility including switching between investment, suspension and reactivations, (b) Investment strategy changes due to the level of interdependency between fuel prices and the price of electricity, (c) Realistic natural gas prices; and (d) Optimal operation of micro-grid is considered once the interdependency exists between fuel and electricity.

1.3.3 Micro-grid Portfolio Optimization under Uncertainty (Chapter 4)

In this chapter we propose an integrated two-step approach to micro-grid power generation portfolio optimization under uncertainty. This work extends the current literature in two major ways: (i) It takes a holistic approach to investment by including different types of distributed generations with portfolio, (ii) It directly includes short-term planning and operational risks and long-term investment and pricing risks and integrates them into a single two-step optimization model. Finally, the solution approach uniquely combines a general binomial lattice with mixed integer quadratic model for budgeting and a regression model that estimates cost of operation and planning micro-grid with its current resources and load. The practical benefits of this work are enormous; the methodology can be easily implemented in commercial software and used by investors and planners in distributed generation industry.

1.4 Motivation

The reliability and aging of the US power grid, capacity constraints on transmission lines and the need for greener and more sustainable electricity have been driving the technological advancements in this field. The penetration of renewable energy into the grid is increasing at a rapid pace and some states in the union (e.g., California) is already requiring up to 33% renewable generation capacity across their distribution network. Carbon tax credits and emission control regulations, and the desire for higher degree of geographical proximity of generation to load are slowly changing the face of the grid. Smart metering, dynamic pricing at retail level, residential solar unit of generation are shaping the supply and demand of electricity, and the changes are expected to happen at much higher pace between now and 2050.

There are many advocates to using the Moor's law to make electricity personal. Many believe that electricity will take the same journey that computing took in the last 30 years, transforming from a dispersed system ("users far away, not easily accessible, and to be done by professionals") to today's connected computing ("everywhere we go, part of our lives, everyone does it"). Many envision new communities (residential and commercial; university or industrial campuses, military installations, etc) with self-sufficient energy production, islanding capabilities if necessary, and sufficient capacity to sell to any macro-grid in order to mitigate and reduce their risks.

With this background in mind, we are motivated to build the necessary tools to design and operate micro-grids. The existing decision models lack the necessary elements and ingredients to correctly capture the dynamics of micro-grids and interactions with larger macro-grids. To optimize the planning and operation of micro-grids, it is important for the planner and controller to take into account uncertainties inherent to the micro-grid and the energy market place. With high capital costs involved in building a micro-grid, non-traditional investment strategies, which promote gradual increase in capacity of generation, are needed.

1.5 Brief Introduction on Micro-grids

According to Wikipedia, a micro-grid (figure 1) is a localized grouping of electricity generation, energy storage, and loads that normally operate connected to a traditional centralized grid or

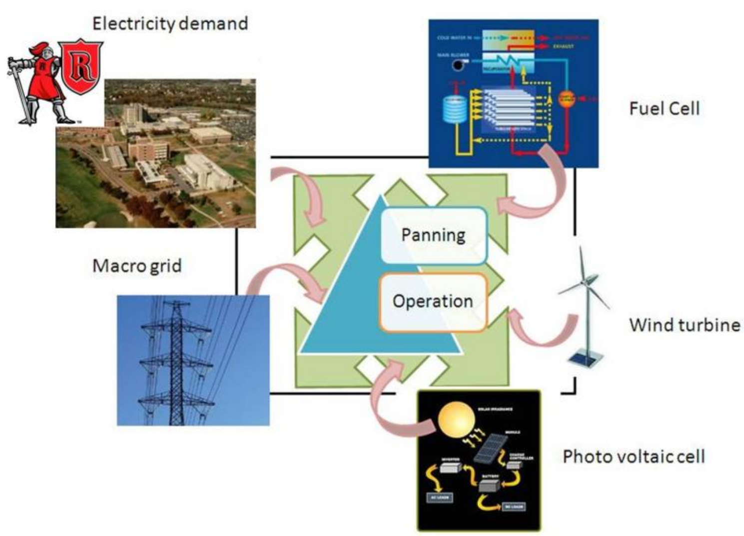

Figure 1 Micro-grid sample

macro-grid. Generation and loads in a micro-grid are usually interconnected at low voltage. Micro-grid generation resources can include fuel cells, wind, solar, natural gas turbines, or other energy sources. The multiple dispersed generation sources and ability to isolate the micro-grid from a larger network provides highly reliable electric power. Byproduct heat from generation sources such as micro-turbines could be used for local process heating or space heating, allowing flexible tradeoff between the needs for heat and electric power. For a macro-grid operator, the micro-grid works as single point node in the network, with islanding capability and capable of running on its own resources if necessary.

The introduction of micro-grids is expected to drastically change the dynamics of the future electricity market. Micro-grids define dual purpose nodes in electricity supply chain networks. On one hand, a micro-grid can act as a generator and sell power to a larger macro-grid in the wholesale market. On the other hand, it can buy from the macro-grid when its internal demand exceeds its residual supply capacity or the electricity market price is right. While distributed generation in the overall market may reduce volatility to high peak demands, micro-grids add their own risks to the market in supply and demand terms. Due to unique feature of micro-grids (*i.e.*, being dual purpose nodes and using renewable resources), new modeling approaches are needed to capture the action/reaction behaviors of micro-grids with respect to the market, including dynamically changing economics, finance, and regulatory requirements [6].

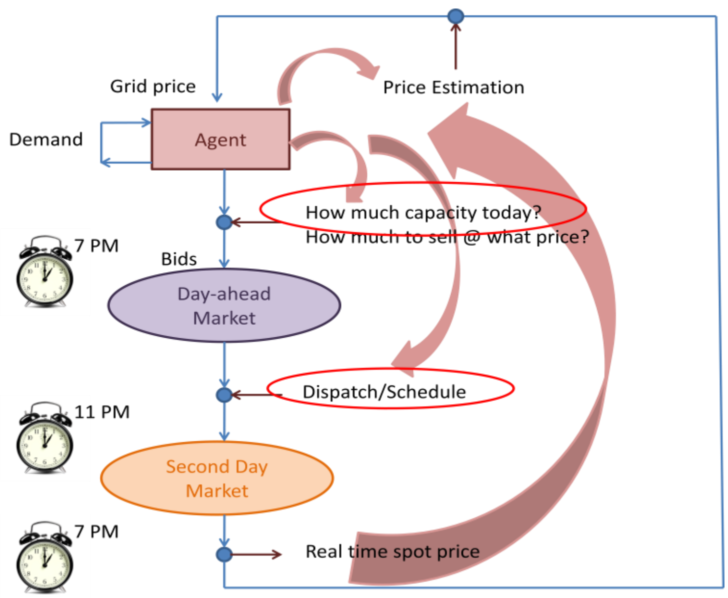

Figure 2 Interaction with wholesale market

Having the obligation to fully satisfy its demand at each point of time, any shortage in available power supply within the micro-grid will lead to the purchase of electricity from the macro-grid at spot market price. The sources of variability rising from the forecast of renewable energy resources and electricity demand, would introduce uncertainty in making decisions at planning layer. Also, a micro-grid may fail to honor its contractual obligations to supply to the market due to these volatilities. Current practice in wholesale market is one-day-ahead bidding and commitment. With the micro-grids, however, one could think of longer time windows for contracting (both buy and sell) to increase the negotiation power in a pre-set price and to avoid spot market price as much as possible. The drawback of a longer contractual window could be the increase in the uncertainty level of forecasts and the likelihood of disruptive events. To be able to satisfy the internal and external demand at minimum cost, one should then take into account the risks due to variation in demand, weather forecasts, spot price, unexpected disruptive events and failures.

The following chapters are organized as follows: Chapter 2 focuses on planning and operation of micro-grid in day-ahead horizon under uncertainty. In Chapter 3, real options models are used to consider delay in investment due to uncertainty in investment. A limited portfolio of micro-grid is assumed and both analytical and simulation based solution approaches are used in this chapter. Finally, Chapter 4 provides a model for portfolio optimization of micro-grid under uncertainty. A more enhanced portfolio is considered compared to the one in Chapter 3. Operational models presented in Chapter 2 are used in tandem with capital budgeting model to close the loop between optimal investment and operation of micro-grid. Monte Carlo simulation is used to solve the investment under all possible stochastic scenarios that can be realized for certain random investment parameters. Finally, Chapter 5 touches upon some application examples and explains possible venues that can be explored as the extension of this work.

2 CHAPETR2 - OPTIMAL PLANNING AND OPERATION CONTROL OF MICRO-GRIDS UNDER UNCERTAINTY

Abstract

In this article we present optimal planning and control models for micro-grids under uncertainty. A micro-grid is defined by a collection of local renewable and non-renewable generators, and is subject to correlated sources of random variations, namely, solar, wind, spot price of electricity, and internal demand. In cases of expected shortfalls in its generation, the micro-grid commits itself to purchase electricity from the macro-grid that it connects to. The traditional deterministic mixed integer programming (MIP) used in wholesale market fails to produce adequate results, especially when variations are high. Hence, we formulate a two-stage stochastic programming optimization model, where the first stage decisions (i.e., prior purchase agreement) can be altered by a set of optimal recourse actions in the second stage. Three variations of this model, namely certainty equivalent regime, risk neutral regime, and risk-averse regime, are presented and compared. The motivation for this comparison is to balance out the computational complexity of the decision-making process and risks due to the underlying sources of uncertainty. Finally, we introduce a parametric characterization of micro-grids according to their mean capacity configuration and operational volatility. The mean capacity configuration is a design parameter set taking into account the micro-grid's internal generation capacity of renewables and non-renewables, and the ratio of this capacity to its average internal demand. The volatility measure is a parameter set defined by a multivariate distribution on the probability space determined by the four sources of randomness. A dissimilarity measure defined on this probability space determines the relative volatility between two micro-grids. We numerically examine the sensitivity of optimal plan and control solutions and associated costs to these parameters.

2.1 Introduction

The reliability and aging of the US power grid, capacity constraints on transmission lines and the need for greener and more sustainable electricity have been driving the technological advancements in this field. The penetration of renewable energy into the grid is increasing at a rapid pace and some states in the union (e.g., California) is already requiring up to 33% renewable generation capacity across their distribution network. Carbon tax credits and emission control regulations, and the desire for higher degree of geographical proximity of generation to load are slowly changing the face of the grid. Smart metering, dynamic pricing at retail level, residential solar unit of generation are shaping the supply and demand of electricity, and the changes are expected to happen at much higher pace between now and 2050.

There are many advocates to using the Moor's law to make electricity personal. Many believe that electricity will take the same journey that computing took in the last 30 years, transforming from a dispersed system ("users far away, not easily accessible, and to be done by professionals") to today's connected computing ("everywhere we go, part of our lives, everyone does it"). Many envision new communities (residential and commercial; university or industrial campuses, military installations) with self-sufficient energy production, islanding capabilities if necessary, and sufficient capacity to sell to any macro-grid in order to mitigate and reduce their risks. According to Fiona Sim (Director – Intel Open Energy Initiative), micro-grids are here to empower the communities of the future and to provide greener and more sustainable electricity for many years to come [1].

We recognize that the existing decision models (*e.g.*, MIP used in electricity wholesale market) lack the necessary elements to correctly capture the uncertainties and stochasticity that surround micro-girds. With this in mind and with the growing market anticipation for micro-grids, we are motivated to build the necessary tools to optimally plan and operate micro-grids. In this work, we will fix the generation portfolio of micro-grid to include {purchase from the grid, wind turbines, PVs, and gas-fired generation}. We will formulate a two-stage stochastic programming model, and present and compare three variations of it, namely certainty equivalent regime, risk neutral regime, and risk-averse regime. The motivation for this comparison is to balance out the computational complexity of the decision-making process and risks due to the underlying sources of uncertainty. We will also introduce mean capacity configuration and volatility measures to characterize micro-grids, and examine how change pattern in these measures impact the overall operation cost of the micro-grid. Mean capacity configuration is defined on design parameters of micro-grid, and volatility measures relate to the operational environment and constraints of the micro-grid. This latter set of results will be especially important if a decision maker is seeking simple and fast feedback on approximate quantification of changes in operational cost due to capacity enhancement or degradation and/or higher market or weather related risks.

2.2 Literature Review

The problems of micro-grid planning and operations have received some attention in the literature. Immonen [2] classifies the optimization needs into three categories: Long Term Optimization (LTO), Short Term Optimization (STO) and Real Time Optimization (RTO). LTO focuses on long term planning horizon, *e.g.*, a year or more, and includes energy contracts, long term operation and maintenance schedules and energy budgets. STO finds the best combination of energy resources to satisfy the predicted energy demand, hour by hour, over the entire planning period. Because of price volatility, any evaluation of the purchases and sales need to be repeated frequently, taking into account the previous decisions and all the new offers available.

As for RTO, the optimization evaluation needs to be performed repeatedly, usually at 5-15 minutes intervals. RTO needs the real time measurement of loads for its input, to account for the rapid variations in energy demand and energy price. An example of RTO is RT-OPTICOM which uses mixed integer linear programming approach [3]. The model generates a finite number of random scenarios on energy loads, electricity prices, DG availability, and solar insulation. The optimization problem is discretized into time-steps in the range of minutes to an hour. The objective of RT-OPTICOM is to minimize the expected energy cost for a period (such as a month or a year) calculated over all scenarios.

Distributed Energy Resources Customer Adoption Model (DER-CAM) developed by Lawrence Berkeley National Lab [4] is an optimization tool intended for minimizing the cost of supplying electric and heat loads of a specific customer site by optimizing the installation and operation of distributed generation, combined heat and power, and thermally activated cooling equipment. The model chooses which DG and/or CHP technologies a customer should adopt and how that technology should be operated hourly based on specific site load and price information and performance data for available equipment options. DER-CAM assumes that no deterioration in output or efficiency during the lifetime of the equipment takes place. The model does not include start-up and ramping constraints, nor does it include reliability measures.

A two-stage stochastic programming model was proposed in Beraldi's work [5, 6], where the uncertain market price is represented by a random variable defined on a given probability space. Here the objective function is weighted mean-risk sum, considering the trade-off between revenue and risk. Their risk measure is the semi-variance from a given target weighted by a user-defined trade-off parameter accounting for the risk aversion attitude. This risk measure was originally introduced by Markovitz [7] as variant of the well-known mean-variance criterion. It is known to be consistent with stochastic dominance. Moreover, it enjoys the convexity property. To solve this model, Beraldi proposes a method, which adopts the traditional Branch and Bound framework and uses Interior Point algorithm as solver for each sub-problem generated along the exploration of the search space. To make the computation reusable from one node to another in the Branch and Bound tree, a warm start procedure has been implemented. Also an early branching strategy is implemented for the estimation of the integrality of the optimal solution and the early termination of the solution process. This model is needed if energy prices, customer demands and power input from renewable resources are uncertain. Typically, this information is determined up to sometime interval t, and followed by stochastic information in the remaining time intervals. For this model, a decomposition method is usually implemented [8].

To deal with incomplete information, Schultz et al [9-11] propose an extension of traditional stochastic programming methodology. They address the inclusion of integer decision variables and the transition from risk neutral models to those incorporating risk aversion. A decomposition algorithm is also proposed in their work to solve the mixed integer linear programming problem. They assume a system with dispersed generation of power and heat and present computational results showing the superiority of their decomposition algorithm over a standard mixed-integer linear programming solver.

2.3 Problem Statement and Preliminaries

We assume that our micro-grid includes gas-fired generation (GF), Wind Turbines (WT) and Photovoltaic solar cells (PV) for its internal renewable resources, and access to external power grid. No storage is considered. It is assumed that micro-grid is subject to several sources of variations: (i) variations in weather forecast, which leads to variation in the availability of renewable resources, (ii) variations in demand, and (iii) variation in spot prices. The following assumptions are made:

- Next-day spot prices, electricity demand, solar radiation and wind speed are assumed to have distributions with mean and variance estimated from historical data.
- The above random variables are correlated in their mean values but not in their variances.
- To set up the model, some key inputs are needed:
    - End-user demand profiles
    - Wind speed profile
    - Solar intensity profile
    - Economic data, such as the energy prices and fuel costs for gas-fired generator

The model outputs include:

- Commitment for electricity purchase from the grid,
- The electricity generation of different internal resources and the amount of spot electricity purchase from the grid,
- The total cost of supplying electricity both from internal resources and the grid.

Nomenclature:

Tables 1 and 2 illustrate the variables and their descriptions in the models.

Table 1 Nomenclature

Demand	
$demand_t^s$	Electricity demand at time t & scenario s (Continuous)
$\overline{demand}_t$	Expected electricity demand at time t (Continuous)
Price	
$C_{preset,t}$	Preset electricity price from wholesale market at time t (Continuous)
$C_{spot}{}_t^s$	Spot market electricity price at time t (Constant)
$C_{penalty,t}$	Penalty paid for the unused electricity purchased from the grid (Constant).
C_{fuel}	Fuel unit price (Constant)
C_{up}	Start up cost (Constant)
C_{down}	Shut down cost (Constant)

Table 2 Nomenclature

	Grid		Gas-fired Generation		Wind Turbine		Photo Voltaic	
Generation	$gridcom_t$	Prior commitment to the grid at time t (Continuous)	gfc_t^s	Electricity generation at time t/scenario s(Continuous)	gwt_t^s	Wind turbine generation at time t/scenario s(Continuous)	gpv_t^s	PV generation at time t & scenario s(Continuous)
	$gridreturn_t^s$	Returned electricity to the grid at time t & scenario s(Continuous)	uup_t^s	Start up indicator at time t/scenario s (Binary)	$\overline{gwt}_t$	Mean wind turbine electricity generation at time t(Continuous)	$\overline{gpv}_t$	Expected PV electricity generation at time t(Continuous)
	$gridspot_t^s$	Spot purchase from the grid at time t & scenario s(Continuous)	$udown_t^s$	Shut down indicator at time t/scenario s (Binary)	WS_t^s	Wind speed at time t/scenario s(Continuous)	SI_t^s	Solar intensity at time t/scenario s (Continuous)
	$gridmin$	Min. grid purchase (Constant)	S_t^s	Status indicator at time t/scenario s (Binary)	$\overline{\mathrm{WS}}_t$	Mean wind speed at time t (Continuous)	$\overline{\mathrm{SI}}_t$	Mean solar intensity at time t (Continuous)
	$gridmax$	Max. grid purchase(Constant)	$gfcmin$	Min. operation capacity(Constant)	WTA_t^s	WT availability at time t/scenario s (Binary)	PVA_t^s	PV availability at time t (Binary)
			$gfcmax$	Max. operation capacity(Constant)	$gwtmin$	Min. WT operation capacity(Constant)	$gpvmin$	Min. PV operation capacity(Constant)
			UT	Min. up time(Constant)	$gwtmax$	Max. WT operation capacity(Constant)	$gpvmax$	Max. PV operation capacity(Constant)
			DT	Min. down time(Constant)				
			R	Ramp up				

			time(Constant)	
		$S_{initial}$	Initial status of (Constant)	

2.3.1 Distributed Generation Components

Gas-fired generation units provide reliable flow of electricity and are characterized by their somewhat extensive ramp up/down periods (D) during which power is generated at a level below its minimum output level (see [12]). Wind power converts wind energy into electricity with wind turbines. Electricity generated from wind power can be highly variable at several different timescales. To assess the frequency of wind speeds at a particular location, a probability distribution function (*i.e.*, Reyleigh distribution) is often used to fit to observed data. The Reyleigh model closely mirrors the actual distribution of hourly wind speeds at many locations. The power function $\mathrm{gwt}(\mathrm{v})$ is then given by:

$$gwt(v) = \frac{1}{2\alpha\rho\pi r^2} v^3$$

Where $\mathrm{gwt}(\mathrm{v})$ is power in watts, α is an efficiency factor determined by the design of turbine, ρ is mass density of air, r is radius of the wind turbine and v is the velocity of air (see [12]). However, no wind turbine in practice would generate electricity based on above equation. A more realistic approximation suggests using v^2 instead of v^3 in wind turbine power generation formula. For photo voltaic (PV), we will use

$$\mathrm{gpv(I) = kI}$$

to compute the amount of power generated by PV with solar intensity, I, where k is a constant related to PV type. We assume no cost associated to generation from the wind or solar energy.

2.4 Scenario Generation

Regimes II and III are modeled by two-stage stochastic programming. To avoid non-linearity arising from the underlying continuous distribution functions, we will discretize and generate samples of random scenarios from these distributions. For large number N of scenarios, the sampled discrete probability distribution will approach to the initial continuous distribution. A correlation matrix is considered in the sampling process, which enables us to capture the correlation that exists between the three random sources. Clearly, solar intensity and wind speed are correlated; there is often a negative correlation between wind speed and solar intensity. Similarly, electricity demand is correlated with the weather data, for example, during summer times, there is often a positive correlation between solar intensity and electricity demand and a negative correlation between wind speed and electricity demand. The correlation matrix is defined to mimic the dependency among electricity demand, solar intensity and wind speed. Table 3 gives an example of correlation matrix.

Table 3 Sample correlation matrix

	Solar Intensity	**Wind Speed**	**Electricity Demand**
Solar Intensity	1	-0.5	0.7
Wind Speed	-0.5	1	-0.2
Electricity Demand	0.7	-0.2	1

In next subsections, the sampling methods for uncertainty analysis are explained:

2.4.1 Sampling methods for uncertainty analysis

2.4.1.1 Simple random sampling

Simple random sampling involves generating random vectors of parameters from given probability distributions repeatedly. For example, a normally distributed random variable X with mean μ and standard deviation σ can be generated by:

$$x^* = \sigma r_n + \mu$$

where r_n is a normally distributed random number with mean 0 and variance 1.

A multivariate normal distribution with variance-covariance matrix V can be sampled using the lower and upper triangular matrix (LU) decomposition method [14]. The variance-covariance matrix V is first decomposed by Cholesky factorization:

$$V = LL^T$$

where L is the lower triangular matrix. To generate the random variables vector $\vec{X}$, matrix L is multiplied by vector, $\overrightarrow{r_n}$, of independent normal random numbers with mean 0 and variance 1:

$$\vec{X} = L\overrightarrow{r_n} - \vec{\mu}$$

To generate random vectors for a sample of size, ns, the above procedure is repeated resulting in a set of random vectors with mean vector, $\vec{\mu}$, and expected variance-covariance matrix: $Lcov(\overrightarrow{r_n})L^T$. Since the random numbers are independent, the covariance matrix $cov(\overrightarrow{r_n})$ is equal I (the identity matrix):

$$Lcov(\overrightarrow{r_n})L^T = LIL^T = LL^T = V$$

2.4.1.2 Latin hypercube sampling

Latin hypercube sampling (LHS) is a stratified random procedure, which provides an efficient way of sampling variables from their distributions ([15, 16]). In LHS ns values are sampled according to the distribution of each k random variables, $x_1, x_2, \dots, x_k$. The cumulative distribution for each variable is divided into N equi-probable intervals. A value is then selected randomly from each interval. The next step is to randomly pair the N values obtained for each variable with the other variables. With this method, one would ensure that the range of each variable is fully covered by maximally stratifying each marginal distribution (Figure 3).

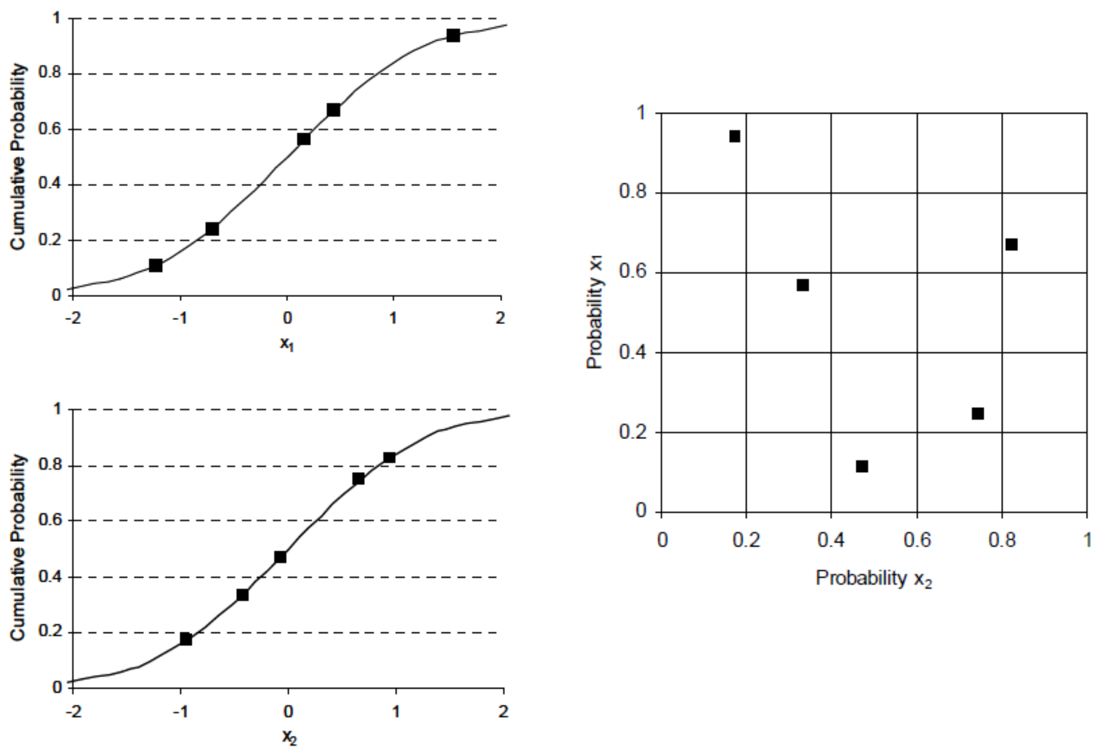

Figure 3 Example of LHS: Random stratified sampling of variables x1 and x2 at 5 intervals (Left) and random pairing of sampled x1 and x2 forming a Latin hypercube (Right)

The following explains the algorithm for LHS:

- The cumulative distribution of each variable is divided into N equi-probable intervals;

- Select a value randomly from each interval. For the i^{th} interval, the sampled cumulative probability is given by [17]:

$$Pr_i = \frac{1}{N} r_u + \frac{(i-1)}{N}$$

where r_u is uniformly distributed random number ranging from 0 to 1;

- Map the probability values into random variables using the inverse of cumulative distribution function, F^{-1}:

$$x_i = F^{-1}(Pr_i)$$

- The N values obtained for each variable x are paired randomly (equally likely combinations) with the N values of the other variables.

The algorithm explained above has the underlying assumption of independent variables. For the case of correlated random variables, this algorithm could result in infeasible combinations and also tends to bias the uncertainty.

2.4.1.3 Latin hypercube sampling for correlated random variables

Several methods are proposed to include correlation in LHS. Stein [18] proposed a method for sampling dependent variables based on the rank of a target multivariate distribution. The algorithm runs as follows:

- Matrix R is generated using simple random sampling of k variables with a sample size N;
- Matrix U is constructed with k columns and N rows containing the order or rank corresponding to the target correlation matrix;
- The Latin hypercube sample $x_{ij}(i = 1, \dots, N; j = 1, \dots, k)$ is generated by

$$x_{ij} = F^{-1}(\frac{u_{ij-r_u}}{N})$$

With this transformation, the sample could approximate the joint distribution. More detain description is available in works by Heuvelink [19] and Zhang and Pinder [20].

For the illustrative examples, random variables are generated on an hourly basis for electricity demand, solar intensity and wind speed. The underlying distributions are generated from normal distributions with mean and variance estimated from historical data. For simplicity, random values are generated independently for each hour; however, the mean values across hours represent a rationale trend for the variables.

2.5 Technical Approach

Three different solution techniques or regimes are examined and compared:

1. Certainty equivalent regime (CE): random variables are presented by their expected values and cost minimization problem is solved. Furthermore, the planning decisions under this regime are fed into operation cost minimization model for each possible random scenarios to obtain planning/operation average cost and its standard deviation.
2. Risk-neutral stochastic programming regime (RNSP): two-stage stochastic programming technique is used for this regime. The decisions are decomposed into first stage (planning) and second stage (operation) decisions. The objective is to find optimal planning decisions under all possible scenarios.
3. Risk-averse stochastic programming regime (RASP): this model is similar to risk-neutral stochastic programming regime; however, the variation of cost is also controlled via minimizing conditional value at risk.

Regimes II and III are modeled by two-stage stochastic programming. To avoid non-linearity arising from the underlying continuous distribution functions, we discretize and generate samples of random scenarios from these distributions. For large number N of scenarios, the sampled discrete probability distribution approaches to the initial continuous distribution. A correlation matrix is considered in the sampling process, which enables us to capture the correlation that exists between the random sources. Clearly, solar intensity and wind speed are correlated; there is often a negative correlation between wind speed and solar intensity. Similarly, electricity demand is correlated with the weather data, for example, during summer times, there is often a positive correlation between solar intensity and electricity demand and a negative correlation between wind speed and electricity demand. Finally, there is a positive correlation between electricity demand and spot price.

2.5.1 Certainty Equivalent Regime

In this model, the expected values of electricity demand, wind speed and solar intensity are estimated from historical data.

Objective Function

The cost of electricity to satisfy the demand has the following components:

- Purchase cost of electricity from the grid at one-day-ahead planning stage, $C_{preset,t}$;
- Spot price of electricity to be purchased the next day as needed, $C_{spot,t}$;
- Penalty cost to be paid to the grid if micro-grid fails to honor its commitment to purchase electricity, $C_{penalty,t}$;
- Cost of electricity generation from the gas-fired unit, C_{fuel}.

The objective function is to minimize the summation of the above cost components over a period of T hours (*i.e.*, 24 hours). It is given by:

$$\min \sum_{t=1}^{T} [C_{preset,t} \times gridcom_t + E[C_{spot}{}_t^s] \times gridspot_t + (C_{penalty,t} - C_{preset,t}) \times gridreturn_t + C_{fuel} \times gfc_t]$$

Constraints

The prior commitment should be more than a certain value and cannot exceed a certain limit:

$$ggridmin \leq gridcom_t \leq ggridmax \quad t = 1, \dots, T$$

The same constraint holds for spot purchase from the grid:

$$ggridmin \leq gridspot_t \leq ggridmax \quad t = 1, \dots, T$$

The amount of electricity not purchased from the grid cannot exceed the prior commitment:

$$gridreturn_t \leq gridcom_t \quad t = 1, \dots, T$$

The electricity generation from the gas-fired unit is constrained by the generator's technology. $S_t \in \{0,1\}$ is status indicator that GF generator is in operation or not in time interval $t = 0,1, \dots, T-1$:

$$S_t gfcmin \leq gfc_t \leq S_t gfcmax \quad t = 1, \dots, T$$

Status constraint is imposed by start-up and shut-down indicators:

$$S_t - S_{t-1} = u_{t-R}^{up} - u_t^{down}$$

$$u_t^{up} + u_t^{down} \leq 1$$

$$S_0 = S_{initial} - u_0^{down}$$

$$u_t^{up}, u_t^{down} \in \{0,1\}$$

$$S_{initial} \in \{0,1\}$$

Moreover, there are constraints for minimum up time and minimum down time, *i.e.*, if the GF is turned on, it should remain on for a minimum time and also if it is shut down, it should be off for at least a certain amount of time:

$$\sum_{n=0}^{UT-1} (S_{t+n} - u_{t-R}^{up}) \geq 0, t = R, R+1, \dots, T-UT$$

$$\sum_{i=1}^{UT-k} (S_{T-i} - u_{T-UT+k-R}^{up}) \geq 0, k = 1, \dots, UT-1$$

$$\sum_{n=0}^{UT-1} (1 - S_{t+n} - u_t^{down}) \geq 0, t = 0,1, \dots, T-DT$$

$$\sum_{i=1}^{DT-k} (1 - S_{T-i} - u_{T-UT+k}^{down}) \geq 0, k = 1, \dots, DT-1$$

Generation from wind turbine and photo voltaic is also restricted by the operational range of the respective equipment. The wind speed and solar intensity should be within the acceptable range in order to have wind turbine or photovoltaic available for generation. Two binary variables are defined, WTA_t and PVA_t. WTA_t is wind turbine availability at time t (binary indicator); $WTA_t = 1$ if $WS_{min} \leq WS_t \leq WS_{max}$, otherwise $WTA_t = 0$. PVA_t is PV availability at time t (binary indicator); $PVA_t = 1$ if $SI_{min} \leq SI_t \leq SI_{max}$, otherwise $PVA_t = 0$. Therefore, generation from wind turbine is:

$$WTA_t(\frac{1}{2\alpha\rho\pi r^2}\overline{WS_t}^2) \leq \overline{gwt}_t \leq WTA_t(\frac{1}{2\alpha\rho\pi r^2}\overline{WS_t}^2)$$

And, generation from photo voltaic cell is:

$$PVA_t(k\bar{S}I_t) \leq \overline{gpv}_t \leq PVA_t(k\bar{S}I_t)$$

The Energy balance must be maintained - Different resources of electricity consisting of prior commitment, spot purchase, GF generation, wind turbine generation and photo voltaic generation should collectively satisfy the expected demand electricity. Therefore, we have,

$$gridcom_t - gridreturn_t + gridspot_t + gfc_t + \overline{gwt}_t + \overline{gpv}_t \geq \overline{Demand}_t \quad t = 1, \dots, T$$

One should note that this is a planning model assuming optimal operation. The only binding decision in this stage is prior commitment to the grid ($gridcom_t$). On the next day, once any of random scenarios is realized, a set of optimal operational decisions will be made given the planning decisions. To be able to evaluate the consequence of planning decisions under this certainty equivalent regime and to set up a fair comparison with the other two regimes, we take the planning decision from this model and set up a similar optimization model for the operation. The only difference is that now $gridcom_t$ is not a decision anymore and is known from the planning stage. This operation model is run under all possible random scenarios and expected cost and its variation are calculated.

2.5.2 Stochastic Regimes

Two-stage stochastic programming technique [9] is used to formulate and solve Regime (II) and Regime (III). We will start with general notions that apply to both regimes. A synopsis of the decisions made in the two stages follows:

- In the first stage, day-ahead plans are made to commit to the grid for a certain amount of purchase. The decision is made taking into account all sources of uncertainty.
- The second stage includes observation of scenarios and taking recourse decisions for each scenario. The recourse decisions are made in terms of:

a. How much of the prior commitment should really be purchased ($gridreturn_t^s$);

b. How much spot electricity should be purchased ($gridspot_t^s$) and

c. How much electricity from gas-fired unit should be generated (gfc_t^s).

These recourse decisions are corrective actions to the first stage decisions for each hour depending on which random scenario is realized.

The above two-stage stochastic programming problem is formulated such that the first stage decision is optimal under all possible scenarios realized in the second stage. Mathematically speaking, we have:

$$\min\{c^T x + q(\omega)^T y : T(\omega)x + W(\omega)y = z(\omega), x \in X, y \in Y\}$$

where x is the set of first stage decisions and ω is the random component. The decision on x is followed by observation $(q, T, W, z)(\omega)$ and the second stage decision on y. Clearly, y depends on x and ω.

<u>Non-Anticipatively Constraint:</u> This constraint allows for decisions on x without anticipation of the random data$(q, T, W, z)(\omega)$. It means that decisions that share the same history must have the same first stage plans. In discrete sampling cases, all the scenario-dependent decisions made at the second stage share the same history from the first stage, *i.e.*, $x^1 = \cdots = x^s = x^N = x$.The above dynamics becomes more explicit by the following reformulation:

$$\min_x\left\{c^T x + \min_y\{q(\omega)^T y : W(\omega)y = z(\omega) - T(\omega)x, y \in Y\}:, x \in X\right\}$$
$$= \min_x\{c^T x + \Phi(x, \omega) : x \in X\}$$

<u>Risk-Averse Formulation:</u> For the risk-averse case we use mean-risk approach, which is based on a weighted sum of expected value and a parameter $\mathfrak{R}$ (to be specified later) reflecting the risk. Let $f(x,\omega) = c^T x + \Phi(x,\omega)$ then we have the following mean-risk SP:

$$\min\{Q_{MR}(x) \colon x \in X\}$$

where

$$Q_{MR}(x) \coloneqq (E + \rho\mathfrak{R})[f(x,\omega)] = E[f(x,\omega)] + \rho\mathfrak{R}[f(x,\omega)] = Q_E(x) + \rho Q_{\mathfrak{R}}$$

with some fixed weight $\rho>0$. The term $\rho Q_{\mathfrak{R}}$ includes risk aversion into the objective. ”$min\{Q_{MR}(x)\}$” is a non-linear optimization problem, where the objective function is a multivariate integral whose integrand is the optimal value of another (namely the second-stage) optimization problem. This leads to numerical difficulties when computing using complicated probability distributions function values or gradients of Q_{MR}. The latter does not necessarily exist. Far worse, Q_{MR} is in general discontinuous, and the objective may have a finite greatest lower bound that might not be achieved. This may happen for example, when $\mathfrak{R}$ is specified as variance. Thus, the risk measure $\mathfrak{R}$, besides being meaningful for the practitioner, should also fulfill requirements regarding the mathematical tractability with the structure it induces in Q_{MR}. Furthermore; non-linearity must be avoided to the extent possible so that the problem remains viable and attractive for practical reasons. Even within the constraints of linearity, it is important that the model retains a block structure amenable to decomposition or approximation.

Several measures of risk are proposed in the literature fulfilling these requirements: excess probability, conditional value at risk (CVAR), expected excess and semi-deviation. We will choose CVAR as our risk measure $\mathfrak{R}$ [10].We recall that our objective was to plan and operate the micro-grid with minimum cost subject to a set of constraints. Taking a risk-neutral approach, one's concern would be to minimize the expected value of cost which is the cost of prior commitment to the grid (initial decisions) and the expected cost of spot purchase, return to the grid and fuel cell generation over all possible scenarios (recourse decisions). However, a mean risk-averse approach with CVAR will also include a measure of variation for cost and therefore would control the cost variation by minimizing this measure.

2.5.2.1 Risk-neutral Two-stage Stochastic Programming Regime

Objective Function

The objective function of this regime is the sum of the cost of first stage decision and expected cost of the second stage. The first stage cost is the cost of electricity that the micro-grid commits to purchase from the grid, and is given by

$$\sum_{t=1}^{T} C_{preset,t} \times gridcom_t$$

The second stage objective function is:

$$\sum_{s=1}^{N} P_s \left(\sum_{t=1}^{T} [C_{spot}{}_t^s \times gridspot_t^s + (C_{penalty,t} - C_{preset,t}) \times gridreturn_t^s + C_{fuel} \times gfc_t^s]\right)$$

where P_s is the probability of each scenario which is assumed to be 1/N over a discrete sampling plan. The overall objective function is then:

$$\sum_{t=1}^{T} C_{preset,t} \times gridcom_t + \sum_{s=1}^{N} P_s \times \left(\sum_{t=1}^{T} [C_{spot}{}_t^s \times gridspot_t^s + (C_{penalty,t} - C_{preset,t}) \times gridreturn_t^s + C_{fuel} \times gfc_t^s]\right)$$

Constraints

The prior commitment at each time should be more than a certain value and cannot exceed a certain limit

$$ggridmin \leq gridcom_t \leq ggridmax \quad t = 1, \dots, T$$

Note that in this framework, the constraints should be repeated for every scenario once a recourse decision is involved. Scenario-based constraints for a specific scenario are relevant only for the recourse decisions in that scenario and other scenario-based constraints become irrelevant for these decisions. For example, the equivalent set of constraints for spot purchase (which is a recourse decision and scenario-dependent) would be:

$$ggridmin \leq gridspot_t^s \leq ggridmax \quad t = 1, \dots, T \; and \; s = 1, \dots, N$$

Note that s indicates scenario s. Other constraints will be the same as those explained in certainty equivalent regime but repeated for each scenario:

The amount of electricity not purchased from the grid cannot exceed the prior commitment:

$$gridreturn_t^s \leq gridcom_t \quad t = 1, \dots, T \; and \; s = 1, \dots, N$$

The electricity generation from the GF at each time and under each scenario s is constrained by the fuel cell technology:

$S_t^s \in \{0,1\}$ is status indicator that GF is in operation or not in time interval $t = 0,1, \dots, T - 1$ and $s = 1, \dots, N$:

$$S_t^s gfcmin \leq gfc_t^s \leq S_t^s gfcmax \quad t = 1, \dots, T and \; s = 1, \dots, N$$

Status constraint is imposed by start-up and shut-down indicators:

$$S_t^s - S_{t-1}^s = uup_{t-R}^s - udown_t^s$$

$$uup_t^s + udown_t^s \leq 1$$

$$S_0 = S_{initial} - u_0^{down}$$

$$uup_t^s, udown_t^s \in \{0,1\}$$

$$S_{initial} \in \{0,1\}$$

The constraints for minimum up time and minimum down time:

$$\sum_{n=0}^{UT-1} (S_{t+n}^s - uup_{t-R}^s) \geq 0, t = R, R+1, \ldots, T - UT \text{and } s = 1, \ldots, N$$

$$\sum_{i=1}^{UT-k} (S_{T-i}^s - uup_{T-UT+k-R}^s) \geq 0, k = 1, \ldots, UT - 1 \text{ and } s = 1, \ldots, N$$

$$\sum_{n=0}^{UT-1} (1 - S_{t+n}^s - udown_t^s) \geq 0, t = 0,1, \ldots, T - DT \text{ and } s = 1, \ldots, N$$

$$\sum_{i=1}^{DT-k} (1 - S_{T-i}^s - udown_{T-UT+k}^s) \geq 0, k = 1, \ldots, DT - 1 \text{ and } s = 1, \ldots, N$$

Generation from wind turbine and photo-voltaic is also restricted by the operational range of equipment: The wind speed and solar intensity should be within the acceptable range of the respective equipment in order to have wind turbine or photovoltaic available for generation. Two binary variables are defined here, namely, WTA_t^s and PVA_t^s. WTA_t^s is wind turbine working availability at time t (binary indicator) and $WTA_t^s = 1$ if $WS_{min} \leq WS_t^s \leq WS_{max}$, otherwise $WTA_t^s = 0$. PVA_t^s is PV working availability at time t (binary indicator). $PVA_t^s = 1$ if $\mathrm{SI}_{\mathrm{min}} \leq SI_t^s \leq \mathrm{SI}_{\mathrm{max}}$, otherwise $PVA_t^s = 0$. Therefore, generation from wind turbine is:

$$WTA_t^s(\frac{1}{2\alpha\rho\pi r^2}(WS_t^s)^2) \leq gwt_t^s \leq WTA_t^s(\frac{1}{2\alpha\rho\pi r^2}(WS_t^s)^2)$$

And, generation from photo voltaic cell is:

$$PVA_t^s(k(SI_t^s)) \leq gpv_t^s \leq PVA_t^s(k(SI_t^s))$$

For energy balance at each time period and under each scenario s we must ensure that all sources of electricity, consisting of prior commitment, spot purchase, GF generation, wind turbine generation and photo voltaic generation, collectively satisfy the expected demand electricity. That is,

$$gridcom_t - gridreturn_{t_t}^s + gridspot_t^s + gfc_t^s + gwt_t^s + gpv_t^s \geq Demand_t^s$$

$$t = 1, \dots, T \; and \; s = 1, \dots, N$$

2.5.2.2 Risk-averse Two-stage Stochastic Programming Regime

Objective Function

This regime is very similar to the risk-neutral regime; the objective is not only to minimize the expected cost but also to minimize the variations of cost with a weight factor, which is the risk-averseness measure of the decision maker. From a decision maker point of view, any variation from the expected cost is unpleasant and its degree increases with the risk-averseness measure.

We use *conditional values at risk* (CVAR) as a measure of cost variation, which for a given probability level $\alpha \in (0,1)$, reflects the $(1-\alpha) \times 100\%$worst outcomes. CVAR is defined as:

$$E[cost \; in \; (1-\alpha) \times 100\% \; of \; worst \; cases]$$

Consider the following distribution:

$$Pr\{random \; cost \leq \eta\}$$

We are looking for

$$\eta_\alpha = min\{\eta : Pr\big(random \; cost \leq (1-\alpha)\big)\}$$

CVAR would then be:

$$E[random\ cost | random\ cost \geq \eta_\alpha]$$

Using the expected value of the truncated distribution for approximation, we will have

$$\eta_\alpha + \frac{1}{1-\alpha} E[max\{random\ cost - \eta_\alpha\}]$$

Hereafter, we drop the subscript α from η_α for simplicity and introduce η_t as a first-stage variable at each hour. Furthermore, we define v_s as the additional continuous second-stage variables stemming from a resolution of the above max-expression. The above expression can be re-formulated as a linear programming by adding the following term to the objective function and a constraint, which will be explained later [see 11]:

$$\rho\{\sum_{t=1}^{T} \eta_t + \frac{1}{1-\alpha} \sum_{s=1}^{N} P_s\, v_s\}$$

Hence, the objective function for this regime is given by:

$$\sum_{t=1}^{T} C_{preset,t} \times gridcom_t + \sum_{s=1}^{N} P_s (\sum_{t=1}^{T} [C_{spot}{}_t^s \times gridspot_t^s$$

$$+ \left(C_{penalty,t} - C_{preset,t}\right) \times gridreturn_t^s + C_{fuel} \times gfc_t^s\big] + \rho\{\sum_{t=1}^{T} \eta_t$$

$$+ \frac{1}{1-\alpha} \sum_{s=1}^{N} P_s\, v_s\}$$

The additional constraint mentioned earlier to define conditional value at risk will be:

$$\sum_{t=1}^{T} C_{preset,t} \times gridcom_t + C_{spot}{}_t^s \times gridspot_t^s + (C_{penalty,t} - C_{preset,t}) \times gridreturn_t^s$$

$$+ C_{fuel} \times gfc_t^s - \eta_t \leq v_s \qquad s = 1, \dots, N$$

2.6 Micro-grid Parametric Characterization

Planning and control of a micro-grid and its operation cost over a time horizon are influenced by a host of factors, including type and capacity of its generation resources, and the underlying multivariate distribution of wind, solar, demand and spot prices. We assume that the power generation portfolio is fixed at {purchase from grid, wind turbines, PVs, and gas-fired generation}. Hence, we are able to only focus on the mean capacity configuration and volatility measures, and numerically examine their statistical significance on planning and control of micro-grids. As we will see shortly, the mean capacity and volatility measures do not fully point to the same type of elements. Thus, it will make more sense if treat them independently. Therefore, the comparison between two micro-grids, per se, MG_1 and MG_2 will be characterized by$(\theta_1 - \theta_2; D(\mathbb{X}, \mathbb{Y}))$ where θ_i is the mean capacity configuration measure and $D(\mathbb{X}, \mathbb{Y})$is defined on the basis of statistical probability distributions associated with two variance vectors of $\mathbb{X}$ and $\mathbb{Y}$.

2.6.1 Micro-grid Mean Capacity Configuration Measure

A three-tuple $\theta = \left(I_{on-site-generation}\,;\ I_{fuel-renewable}; I_{renewable}\right)$ is defined to represent the configuration of a micro-grid. The first index is a measure of the total on-site generation capacity of micro-grid. It is defined as the ratio of total on-site generation capacity over the average demand:

$$I_{on-site\ gen.} = \frac{On - stie\ generation\ capacity}{Average\ electricity\ demand} = \frac{gfcmax + C_{WT} \times E[\overline{WS^2}] + C_{PV} \times E[\overline{SI}]}{Average\ electricity\ demand}$$

where C_{WT} is wind turbine constant (*i.e.*, $C_{WT} = \frac{1}{2\alpha\rho\pi r^2}$) and C_{PV} is photo voltaic constant (*i.e.*, $C_{PV} = k$).

The second index is fuel-renewable index, which is defined as the ratio of fuel-fired generation capacity over total renewable capacity:

$$I_{fuel-renewable} = \frac{Fuel - fired\ generation\ capacity}{Renewable\ capacity} = \frac{gfcmax}{C_{WT} \times E[\overline{WS^2}] + C_{PV} \times E[\overline{SI}]}$$

Finally, the renewable index is the ratio of wind turbine capacity to PV capacity:

$$I_{renewable} = \frac{Wnd\ turbine\ capacity}{Photo\ voltaic\ capacity} = \frac{C_{WT} \times E[\overline{WS^2}]}{C_{PV} \times E[\overline{SI}]}$$

2.6.2 Micro-grid Volatility Measure

Since we are dealing with multivariate distributions with correlation among individual variables, we use *distance measure* as a statistic to identify statistical significance between any two sets of populations. This measure calculates the dissimilarity between the two populations using principal component analysis (PCA) (see [21, 22]).The measure consists of three components that separately take into account differences in the correlations and variances of the data sets to be compared. It is also possible to weight the components differently, if needed.

The first component of the dissimilarity index measures the degree by which data set $\mathbb{X}$ must be rotated such that its principle components point in the same direction as those of $\mathbb{Y}$.We consider $\mathbb{X}$ and $\mathbb{Y}$ to be most similar to each other when their principal components, paired according to their ranks, are aligned and most dissimilar when all of the components of $\mathbb{X}$ are orthogonal to those of $\mathbb{Y}$. More formally, given a data set $\mathbb{X}$, consider the singular value decomposition (SVD) of its covariance matrix:

$$cov(\mathbb{X}) = U\Lambda_X X^T$$

where the columns of X are the principal componentsof the data set $\mathbb{X}$, arranged from left to right in the order of decreasing variance in their respective directions, and Λ_X is the diagonal matrix of singular values (eigenvalues). Note that one can also find the singular value decomposition of the correlation matrix of $\mathbb{X}$ as an alternative to the covariance matrix. To determine the rotation dissimilarity between the two data sets $\mathbb{X}$ and $\mathbb{Y}$, we measure the angles between their principal components (by using SVD on the correlation matrix). Since the columns of X and Y are unit vectors, the diagonal of matrix X^TY isthe cosine of the angles between the corresponding principal components. Thus, the proposed dissimilarity measure is

$$D_r(\mathbb{X}, \mathbb{Y}) = trace(cos^{-1}(abs(X^TY)))$$

$\mathrm{D_r}$ accounts for some aspects of difference in the covariances of $\mathbb{X}$ and $\mathbb{Y}$. However, we still must account for the amount of variance of each random variable or the shape of data sets. It may be the case that the principal components of $\mathbb{X}$ and $\mathbb{Y}$ are completely aligned, but they still have very different shapes. To account for these differences in the shapes of the data sets, we examine the difference in the distributions of the variance over the principal components of $\mathbb{X}$ and $\mathbb{Y}$. More formally, consider the random variable$\mathrm{V_{\mathbb{X}}}$ having the probability mass function:

$$Pr(V_{\mathbb{X}} = i) = \frac{\lambda_i^X}{trace(\Lambda_X)}$$

where Λ_X is the diagonal matrix of singular values, and λ_i^X is the ith singular value (using covariance matrix). $Pr(V_{\mathbb{X}} = i)$ is then the proportion of the variance in the direction of the ith principal component. We can then compare the distributions of $V_{\mathbb{X}}$ and $V_{\mathbb{Y}}$ by finding Kullback-Leibler divergence. Since this is a non-symmetric measure, we use the following to obtain a symmetric metric:

$$SRKL(V_{\mathbb{X}}, V_{\mathbb{Y}}) = 1/2(KL(V_{\mathbb{X}}||V_{\mathbb{Y}}) + KL(V_{\mathbb{Y}}||V_{\mathbb{X}}))$$

where:

$$KL(V_{\mathbb{X}}||V_{\mathbb{Y}}) = \sum_i \Pr(V_{\mathbb{X}} = i) \ln \frac{\Pr(V_{\mathbb{X}} = i)}{\Pr(V_{\mathbb{Y}} = i)}$$

We can then define the variance dissimilarity as:

$$D_v(\mathbb{X}, \mathbb{Y}) = SRKL(V_{\mathbb{X}}, V_{\mathbb{Y}})$$

Total dissimilarity index then can be simply obtained by

$$D_{Tot} = D_r(\mathbb{X}, \mathbb{Y}) + D_v(\mathbb{X}, \mathbb{Y})$$

2.7 Model Verification and Experiments

We attempt to pursue the following two specific goals:

- To determine if and when the planning and control complexity can be reduced without significantly influencing the micro-grid operational cost distribution.
- To determine the change pattern in micro-grid's cost distribution as a function of the mean capacity configuration and volatility measures.

The three regimes are formulated as linear programming and solved using commercially available linear programming solvers. We used MATLAB optimization toolbox to solve them.

2.7.1 Model Verification

The optimization models are exact, except for the distributional and data assumptions. While the models can incorporate any probability distribution type, the generation of scenarios poses limitations on how much of these distributions can be actually experienced. The correlation matrix requires that data for wind, solar and demand satisfy geographical proximity and similar climate constraints. The electricity pricing data on day-ahead basis and spot pricing can be forecasted using historical data, but may not necessarily be from the same location or time assumed for solar and wind. Taking these issues into account, the model verification will attempt to verify that:

1. The models are sensitive to changes in their significant parameters and that these change follow correct patterns;
2. The models produce results that are intuitively justifiable.

2.7.2 Input data

To demonstrate the results with some real data, the following input data is used:

- Solar data is taken from National Solar Radiation database provided by National Renewable Energy Laboratory [23].
- Wind speed data is taken from Easter Wind Dataset provided by National Renewable Energy Laboratory.
- Input data for electricity demand, day-ahead electricity price and real time prices are taken from NYISO website [24].
- GF generator cost is assumed to be 70 $/MWh.

The data corresponds to July 15, 2005. These inputs are taken to be the mean values regarded as forecasts at each hour for any of our four random variables. Table 4 gives the simulated correlation matrix. It is assumed that solar radiation and wind speed are negatively correlated for a day in July and solar radiation and demand are positively correlated. Figure 4 demonstrates the historical data; the existing correlation is obvious from the plot. Moreover, the difference between day ahead and spot price reflects the high variability in electricity spot price. It is

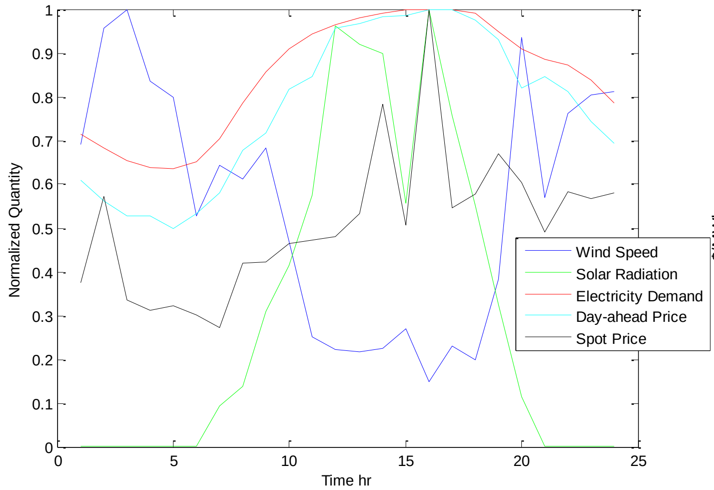

Figure 4 Normalized wind speed, solar radiation, electricity demand, day-ahead and spot price

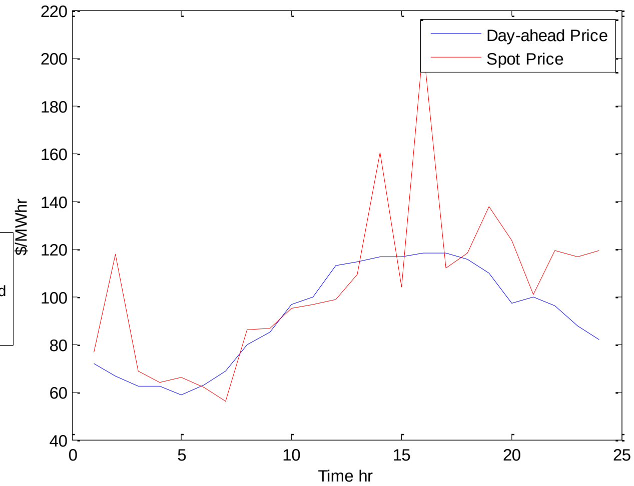

Figure 5 Hourly day-ahead and spot price

assumed that electricity demand, wind speed and solar radiation can be forecasted with more accuracy than electricity spot price. Hence, a higher variance is associated to electricity price and lower variances associated to the rest.

Hourly day-ahead and spot prices are plotted in Figure 5, and Table 5 gives the micro-grid capacity configuration.

Table 4 Correlation matrix

	SI	WS	ED	EP
SI	1	-0.5	0.7	0.4
WS	-0.5	1	-0.2	-0.1
ED	0.7	-0.2	1	0.7
EP	0.4	-0.1	0.7	1

Solar Intensity	SI
Wind Speed	WS
Electricity Demand	ED
Electricity Spot Price	SP

Table 5 On-site generation units

Generation Unit	Characteristics
Fuel Cell	Max. Capacity = 3.99 MW
Wind Turbine	Average generation = 1.33MW
Photo Voltaic	Average generation = 2.66 MW

Using input data described above along with specific stochastic parameters (covariance matrix), Latin Hypercube sampling is employed to generate random scenarios and discretize the continuous stochastic distributions [14-16, 18-20, 25].

2.7.3 Results

Figures 6, 7 and 8 show the planning schedule under three regimes. Figure 9 plots the average optimal planning and operation decisions under these planning decisions but considering all possible scenarios. For RASP, the decision maker intends to avoid 1% of worst costs.

Figure 10 illustrates the total operation cost obtained from the three regimes. Clearly, CE yields higher cost and higher variance. Under this input data set we do not see any significant difference between RNSP and RASP data, however; by increasing the demand variance, the difference between the two will be more profound as shown in Figures 11 and 12.

Figures 9 shows the average daily decisions made under each regime. It can be seen how decisions on a daily average could be different under each regime. It is interesting that as we move from CE regime to RNSP and RASP, the planning decision move towards more prior commitments and less spot purchase and leading to lower expected cost and variance (shown in Figure 10). This would not be the case for other settings; for example, as the existing uncertainty increases, one would expect significant difference between the two models. It turns out that the difference and the way the three models make decisions depend on several factors such as micro-grid’s configuration and variability resources. Therefore, a careful examination of existing settings would help the decision maker to choose the appropriate model for planning and operation to make sure that ignoring uncertainty (*i.e.*, choosing CE model) does not have an adverse impact in terms of increasing the variation of planning decisions, or considering uncertainty would not lead to a more complicated and costly model without having a noticeable benefit for the decision maker.

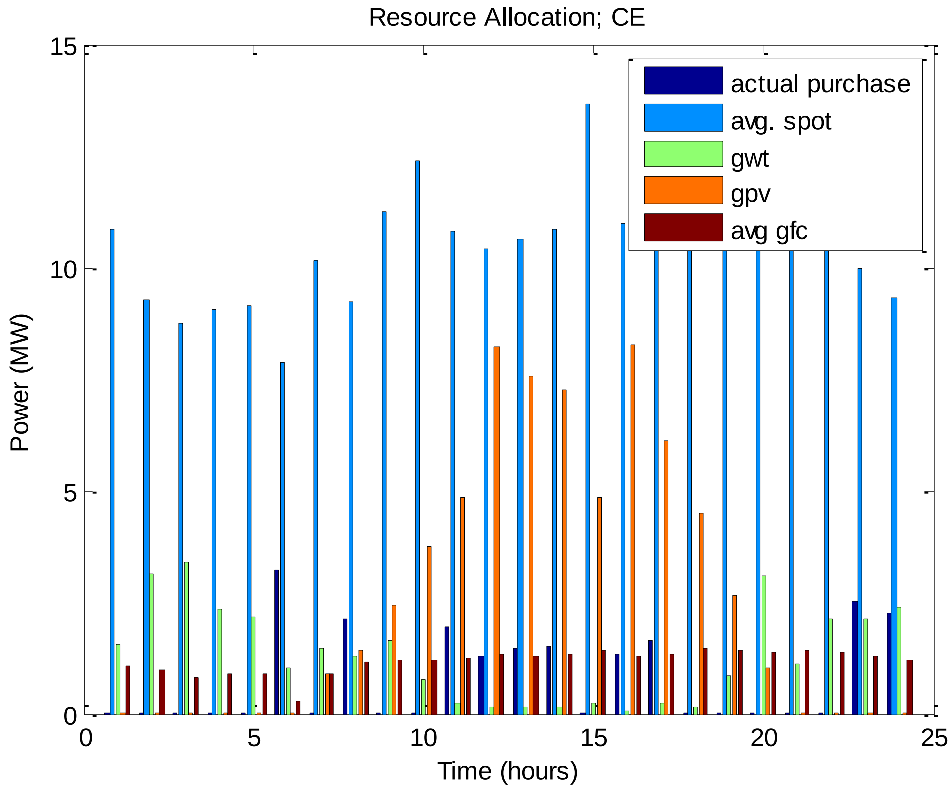

Figure 6 Average hourly planning coupled with anticipated operation under CE regime

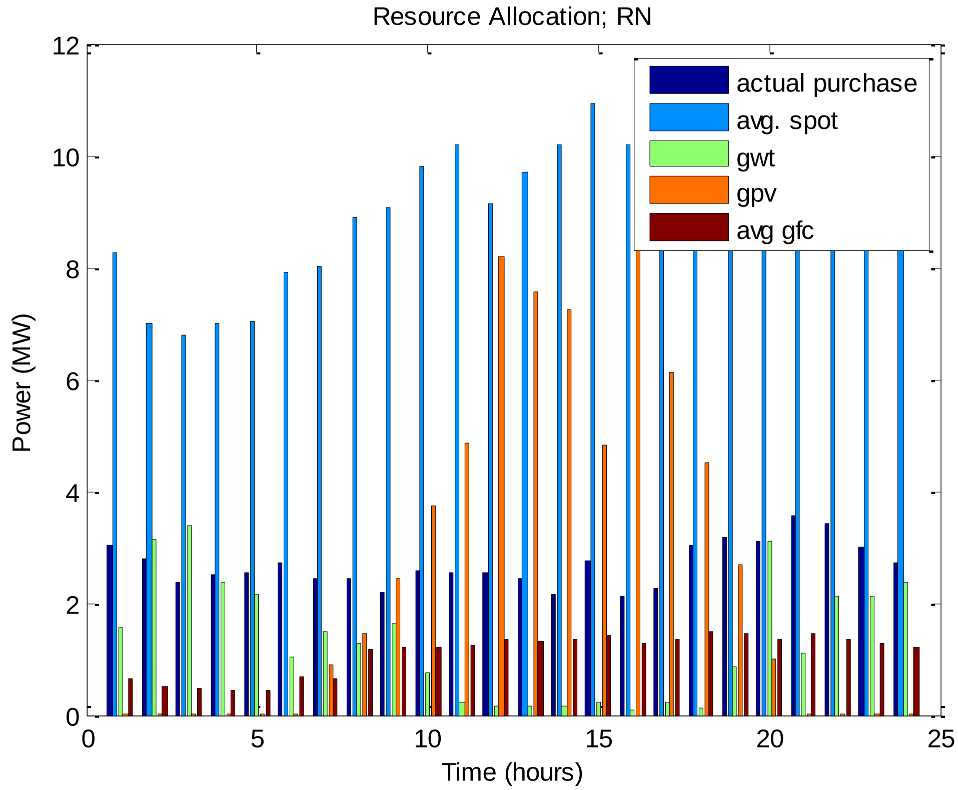

Figure 7 Average hourly planning/operation schedule under RNSP

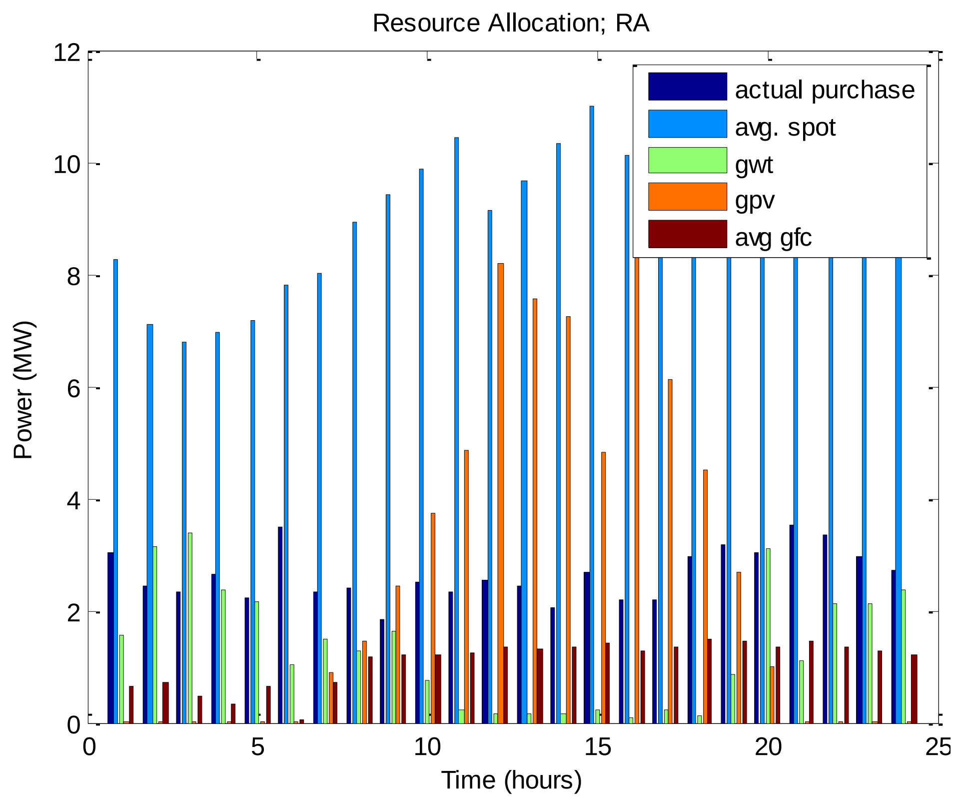

Figure 8 Average hourly planning/operation schedule under RASP, alpha = 0.99

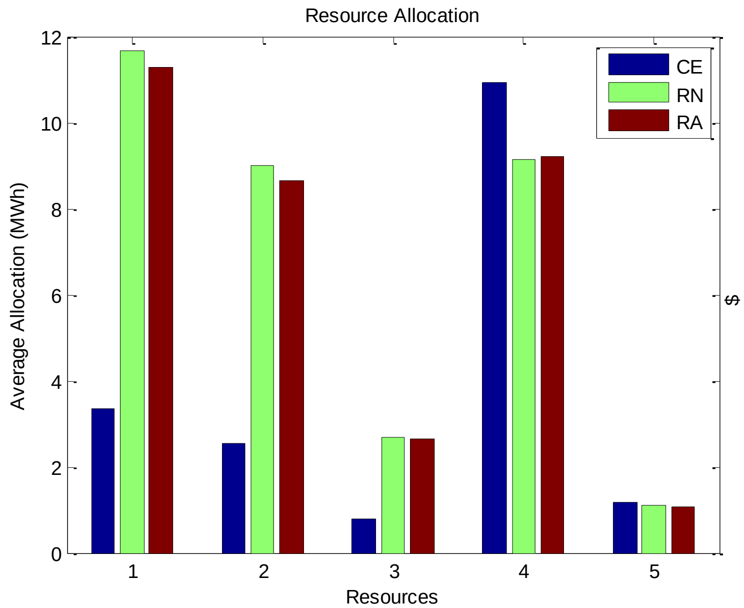

Figure 9 Average resource allocation; 1) prior commitment; 2) return; 3) actual purchase from commitment; 4) spot purchase and 5) GF generation

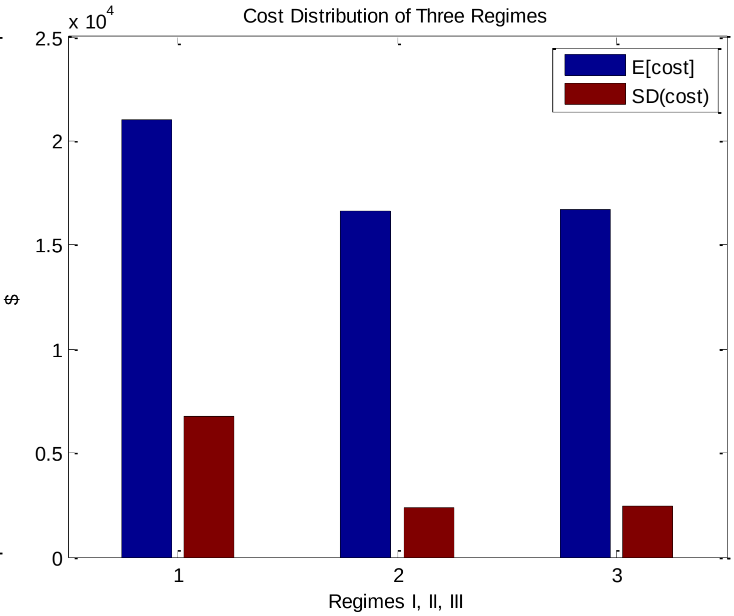

Figure 10 Cost distributions under three regimes

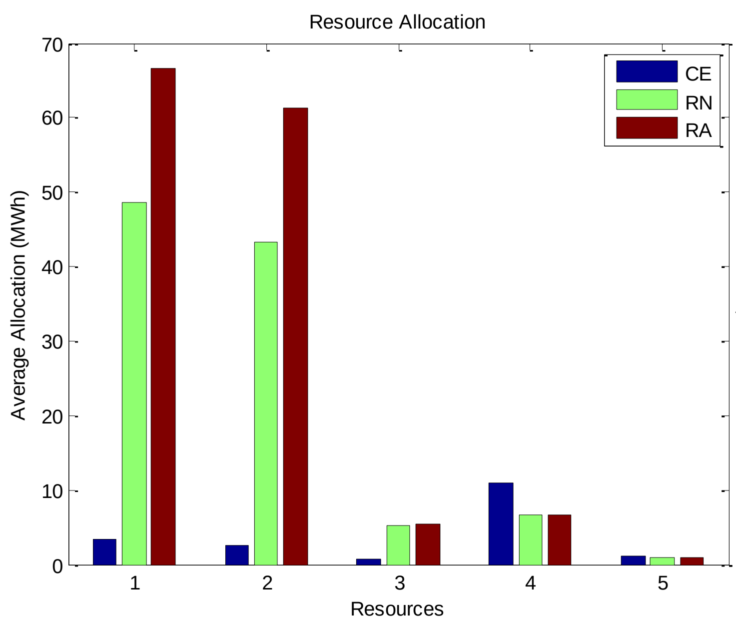

Figure 11 Average resource allocation; 1) prior commitment; 2) return; 3) actual purchase from commitment; 4) spot purchase and 5) GF generation

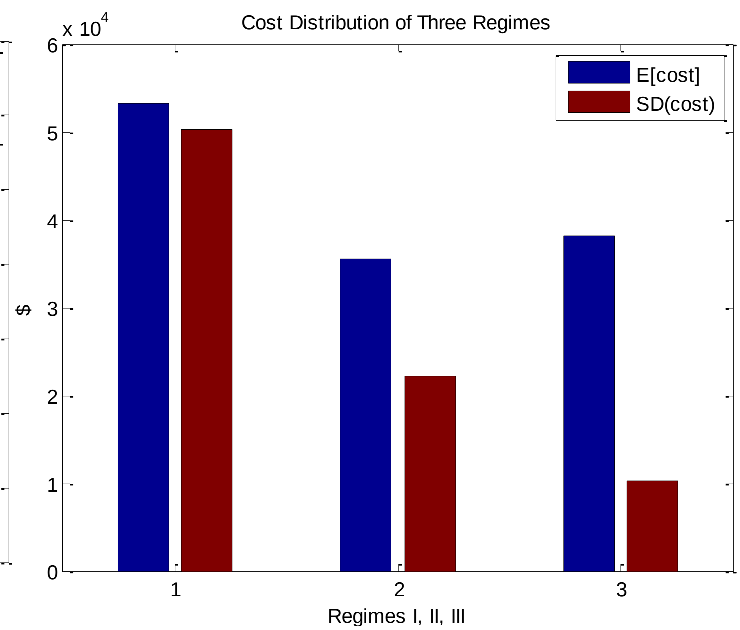

Figure 12 Cost distributions under three regimes

2.7.3.1 Impact of Mean Capacity Configuration Parameters

In this section, the impact of mean capacity configuration of micro-grid is evaluated in terms of planning/operation cost distribution across the three regimes of decision making. For this purpose, some parameters such as mean and co-variance of stochastic variables (wind speed, solar intensity, electricity demand and price) are kept to be constant. An illustrative set of model parameters (kept constant) are shown in Tables 6 and 7.

Table 6 Fixed mean values and variances

	Wind Speed	Solar Intensity	Electricity Demand	DA Electricity Price	Fuel Price
Daily Avg.	5.5 m/s	222 W/m2	16 MW	91.65 $/MWh	70 $/MWh
Variance	Medium	Medium	High	High	NA

Table 7 Fixed correlation matrix

	SI	**WS**	**ED**	**SP**
SI	1	-0.5	0.7	0.4
WS	-0.5	1	-0.2	-0.1
ED	0.7	-0.2	1	0.7
SP	0.4	-0.1	0.7	1

Solar Intensity	SI
Wind Speed	WS
Electricity Demand	ED
Electricity Spot Price	SP

Multiple cases with different configuration parameters are run considering all combinations of $I_{on-site\ gen.} = (0.5,1,1.5,2), I_{fuel-renewable} = (0,0.5,1,1.5,2)$ and $I_{renewable} = (0.5,1,1.5,2)$. The major findings are:

- The micro-grid's cost distribution is significantly influenced by its dependency on the macrogrid (as determined by $I_{on-site\ gen}$). The statistical significance of this dependency depends on the electricity pricing and its variation. For the illustrative example, as shown in Figures 13 and 14, lowering $I_{on-site\ gen.}$ increases the expected cost of planning/operation and its variance. Moreover, in such cases, CE regime leads to higher variability in the cost (Figures 15 and 16).
- As the micro-grid's onsite capacity (as determined by $I_{on-site\ gen}$) increases, the micro-grid's cost distribution starts being less sensitive to the type of planning and control model that is used to calculate it. In other words, risks and uncertainty starts becoming less important in the micro-grid planning and control. This effect is adversely influenced by the penetration of more renewables (as determined by $I_{fuel-renewable}$ and $I_{renewable}$) and positively influenced by the penetration of more fuel fired generation within the micro-grid.
- Statistically speaking, ignoring higher levels of risks, *i.e.*, using CE regime, gives higher cost of planning/operation compared to the two stochastic regimes.

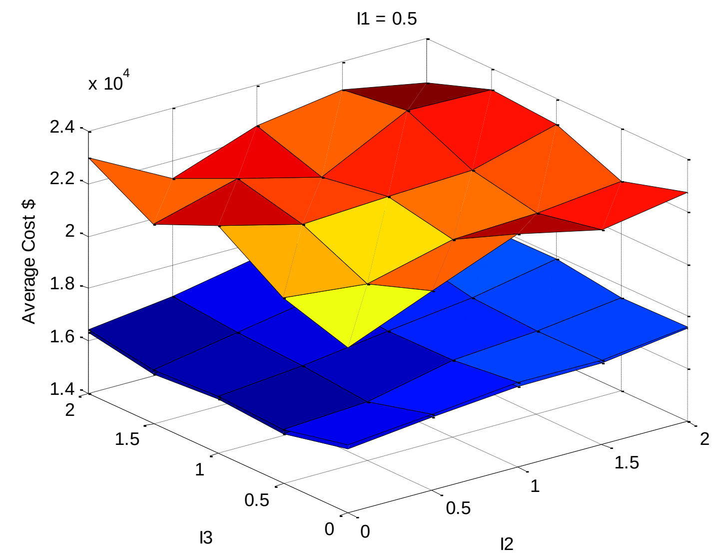

Figure 13 Average cost; I1 = 0.5

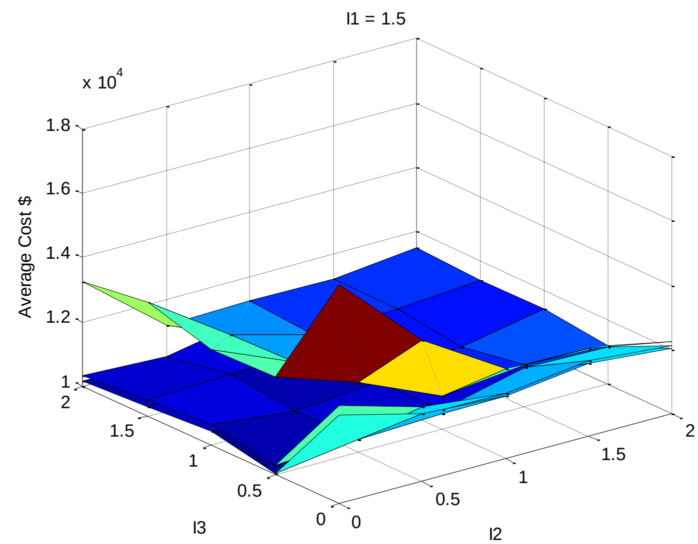

Figure 14 Average cost; I1 = 1.5

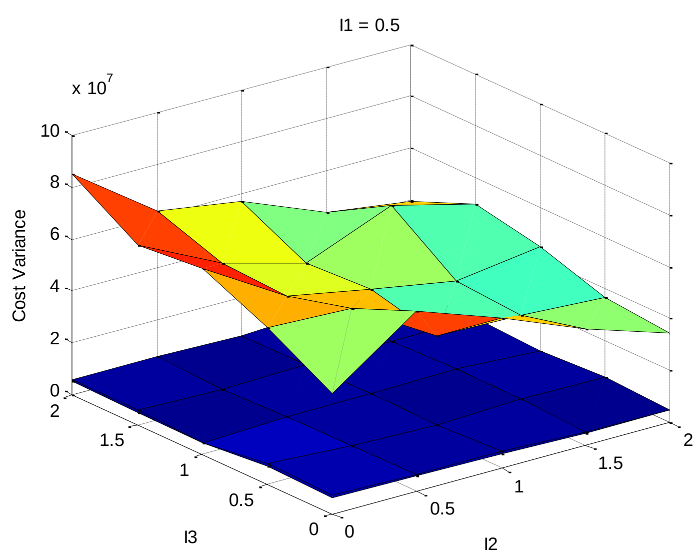

Figure 15 Cost variance; I1 = 0.5

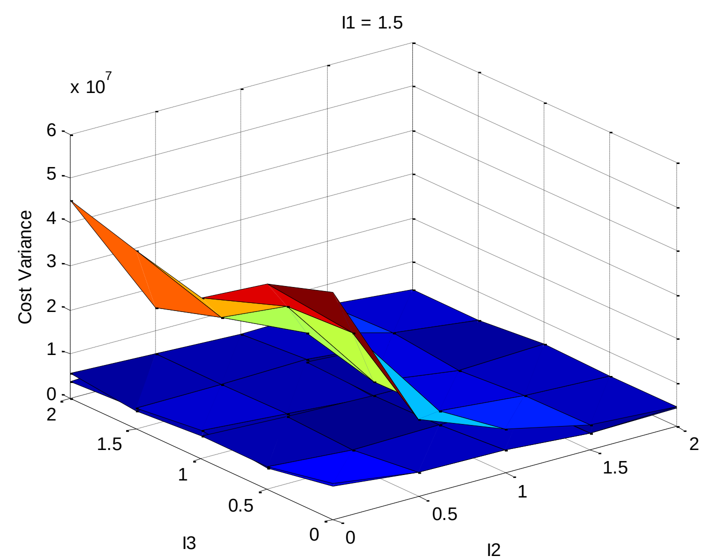

Figure 16 Average cost; I1 = 1.5

2.7.3.2 Impact of Covariance Matrix

The idea here is to compare two micro-grids, which are similar on their mean capacity configuration, but their operational volatility measures differ. For illustrative purposes, consider a reference micro-grid, say MG_1, which has low variance on spot electricity prices and low correlation matrix between the random sources. Table 8 provides different levels of variances for spot price, demand, wind, solar and the associated costs. Table 9 provides physical characteristics of this micro-grid.

Table 8 Mean values and variances

	Wind Speed	Solar Intensity	Electricity Demand	DA Electricity Price	Fuel Price
Daily Avg	5.5 m/s	222 W/m2	16 MW	91.65 $/MWh	70 $/MWh
Variance	Medium	Medium	High	Low, Med,High	NA

Table 9 Fixed on-site generation and average demand

Generation Unit	**Characteristics**
Fuel Cell	Max. Capacity = 4.8 MW
Wind Turbine	Avg. generation = 1.06MW
Photo Voltaic	Avg. generation = 2.12 MW
Avg. Demand	16 MW

In Figure 17 MG_1 relates to point designated as (1,1). By moving along the correlation and price variance axes in Figure 17, we are able to examine micro-grids which have distance from MG_1 measured by the dissimilarity index, $D_{Tot.}$ Figures 19, 20 and 21 demonstrate the rotation and distribution difference components of dissimilarity index as well as the total dissimilarity index. Multiple cases with different DA price variance (low, medium and high) and correlation matrices (low, medium, high and mixed) are evaluated:

Figures 17 and 18 show how the cost distribution changes for these different micro-grids (as price variance and correlation change). The cost distribution changes along both axis or as the total dissimilarity measures change. These changes are more profound in the cases of low or mixed correlation and high variance. In the above example, spot price variability happens to be the most significant volatility factor. However, this can change depending on the other random variables.

Based on a number of scenarios that we have run, the following general conclusions can be drawn:

- Operational volatility is an important measure influencing the true cost distribution of micro-grids (see Figures 17 and 18).Using dissimilarity measure, it is possible to calculate perturbations in cost distribution as operational conditions (as measured by volatility) changes. Our initial results show that the relationship between dissimilarity measure between two micro-grids and the difference between their cost distributions is non-linear. Further studies will be required to examine this further.
- If operational volatility changes due to decreasing correlation factors, the random sources of the micro-grid will start behaving more independently (see Figure 18). Consequently, the variability of the micro-grid cost distribution will become more sensitive to the individual four variances. The ratio of individual significant variances to the overall dissimilarity measure will then be a reasonable weighing factor for planning and control

optimization. Finally, the micro-grid planning and control will be more sensitive to whether or not the underlying risks are accounted for.

- If operational volatility changes so that correlation factors increase and only one source of variation (say spot price) becomes dominant, the planning and control of micro-grids become more dependent on one single source of uncertainty (see Figure 18). This source, if changed, will cover most of the dissimilarity measure. The optimization in such cases will be simpler due to a single variable optimization problem.
- If operation environment changes such that correlation factors move from uniform to more mixed structure, the impact of individual variances becomes more magnified. Consequently, more sources of risks and uncertainties must be accounted for in the planning and control of micro-grids.

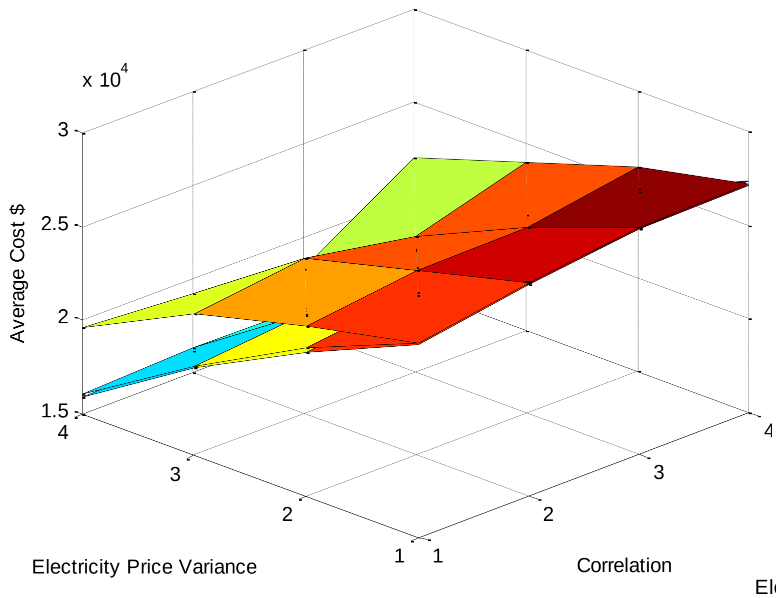

Figure 17 Average cost; I1 = 0.5, I2 = 1; I3 = 0.5

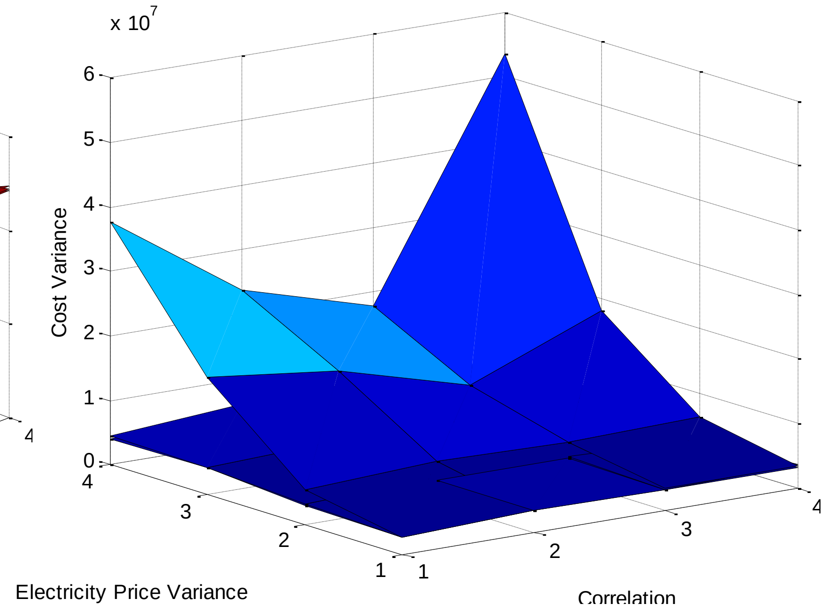

Figure18 Cost variance; I1 = 0.5, I2 = 1; I3 = 0.5

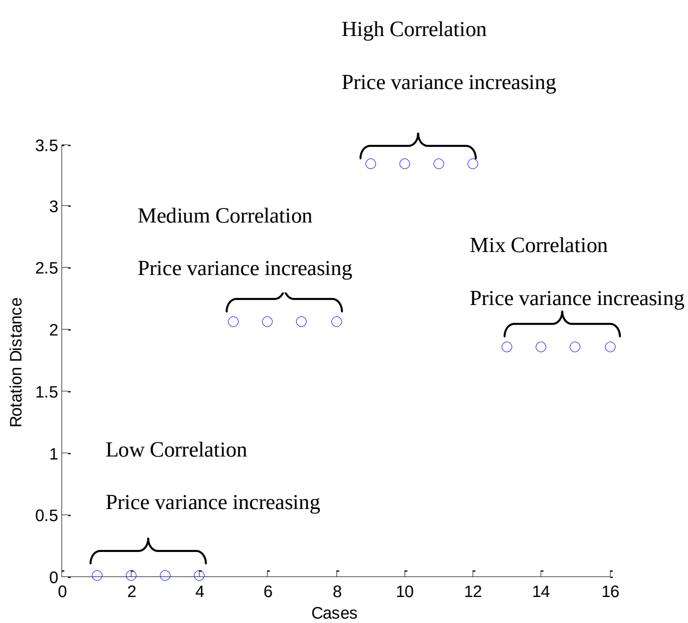

Figure 19 Rotation dissimilarity; I1 = 0.5, I2 = 1; I3 = 0.5

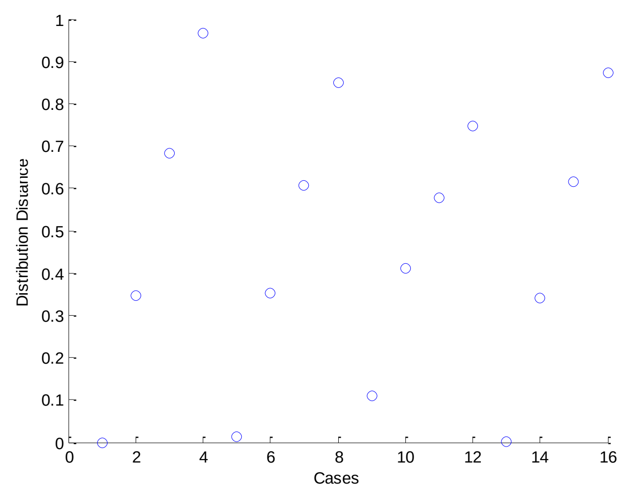

Figure 20 Variance dissimilarity; I1 = 0.5, I2 = 1; I3 = 0.5

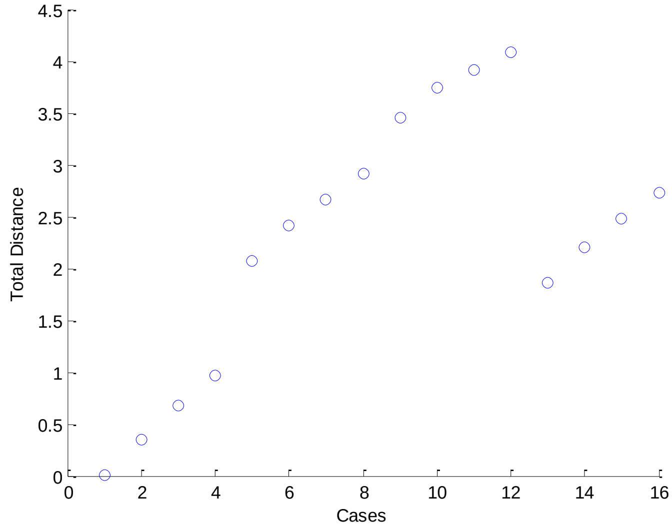

Figure 21 Total dissimilarity; I1 = 0.5, I2 = 1; I3 = 0.5

2.8 Conclusion

In this chapter we presented optimal planning and control models for micro-grids under uncertainty. The micro-grid portfolio includes purchase from the grid, wind turbines, solar PVs, and fuel-fired generation capacity. We formulated a two-stage stochastic programming optimization model, where the first stage decisions (*i.e.*, purchase agreement) can be altered by a set of optimal recourse actions in the second stage. Three variations of this model, namely certainty equivalent regime, risk-neutral regime, and risk-averse regime, are presented and compared. We compared these models under a variety of experimental conditions and concluded that the risk-neutral and risk-averse two-stage stochastic programming models are more appropriate when variations are too high. We also introduced a parametric characterization of micro-grids according to their mean capacity configuration and operational volatility. The mean capacity configuration was defined on the basis of a design parameter set taking into account the micro-grid's internal generation capacity of renewables and non-renewables, and the ratio of this capacity to its average internal demand. The volatility measure was defined on the basis of a parameter set defined by a multivariate distribution on the probability space determined by the four sources of randomness. A total dissimilarity measure defined on this probability space determines the relative volatility between two micro-grids. We numerically examine the sensitivity of optimal plan and control solutions and associated costs to these parameters. We also present preliminary ideas on how the total dissimilarity measure can be used for optimization.

3 Chapter 3 - A Real Option Model of Micro-grid Investment under Uncertainties

Abstract

In this paper we present a real option approach to investment in micro-grids where the micro-grid owner is given the option of delaying investment decisions depending on market driven exogenous factors, such as price of fuel and cost of technology, and indigenous factors, such as electricity load. This work is motivated by the recent growth in micro-grids, and the fact that the traditional net present value (NPV) approach to investment in micro-grid assets does not take into account the inherent uncertainties in fuel prices, cost of technology, and micro-grid load profile. The proposed micro-grid portfolio includes solar PVs and gas-fired generation assets. It is assumed that the capacity configuration of these assets is parametrically fixed, so that the investor must only deal with the timing of the investment. The model also accounts for speculations and includes options on suspension or reactivation of invested assets. An analytical solution using contingent analysis is formulated under certain assumptions and conditions, and compared to traditional NPV solutions. A Monte Carlo simulation model is also presented under more general assumptions. Under common assumptions the two models agree in their decisions. This work extends the current state of investment modeling by considering: (i) Simultaneous investment in more than one asset with uncertain behavior; (ii) Multiple sources of uncertainties along with more realistic probability distributions. Within the context of the above extensions, the following practical contributions are also made: (a) Operation flexibility including switching between investment, suspension and reactivations, (b) Investment strategy changes due to the level of interdependency between fuel prices and the price of electricity, (c) Realistic natural gas

prices; and (d) Optimal operation of micro-grid is considered once the interdependency exists between fuel and electricity.

3.1 Introduction

In this work, we utilize real option theory to model micro-grid investment taking into account a number of uncertainty sources. Two solution approaches are presented, an analytical approach using specific functional forms on the value of investment, and a Monte Carlo simulation. The analytical approach utilizes contingent claim analysis and solves stochastic differential equations that govern the dynamics of project and option values. The second approach utilizes Monte Carlo simulation and least squares approach to solve for optimal timing and investment thresholds. For the comparison purposes and to demonstrate the impact of considering uncertainty in the form of opportunity cost, we compare the analytical results to the results from a stochastic NPV model, where the rate of return on investment is the risk-adjusted rate of return. To the best of our knowledge, this work expands the current state of art as follows:

- Some commonly made assumptions are relaxed. In particular, we relax the assumption of having geometric Brownian motion for random variables once we present the simulation-based approach.
- Both fuel price and investment cost are considered to be stochastic. This complicates investment threshold calculations.
- The operational flexibility in terms of optimal switching between investment, suspension and re-activation is examined.
- The impact of interdependency between fuel price and electricity price is examined. It is shown that it could lead to a different investment strategy.

While we consider our analytical model as the main contribution of this work, its pros and cons are well understood. The analytical approach is easy to use whenever its assumptions apply, or whenever it provides reasonably accurate approximations. On the other hand, the Monte Carlo simulation is more general but computationally expensive, requiring long runs to ensure validity of results and long setups prior to experimentation.

Investment in micro-grids is subject to exogenous and indigenous sources of uncertainties, and unless these sources are accounted for in the decision-making process, significant risk exposures can be expected. While the micro-grid investment trend and financing sources are not fully determined at this time, both public and private fund share expected to be used for micro-grid investments. A private investor would expect to have the necessary tools to hedge against as many foreseeable sources of uncertainties as possible.

A typical micro-grid includes renewable (solar PVs and wind turbines) and non-renewable generation assets (e.g., natural gas-fired generation assets), each with its own contributing uncertainties. For gas-fired generation, price of natural gas is driven by market dynamics and can experience short and long term drifts and volatilities. The cost of technology for PVs and wind turbines drop with technology innovations and market penetration, and government regulations and tax incentives. We will refer to these sources by "*supply side uncertainty*". This category further includes price of electricity. There is also "*demand side uncertainty*" which includes the micro-grid demand profile over a day or longer period of time. Weather and other, often, non-controllable factors influence the demand profile too.

Traditionally, net present value analysis (NPV) is used to evaluate engineering projects. This methodology ignores the opportunity cost due to delay in investment under uncertainty. The option to postpone the investment gives the decision maker the opportunity to wait for more information about the uncertain future. If the owner of the property has the exclusive right to invest, and if for example, the price is expected to rise and/or there is uncertainty about future prices, there can be an added value on postponing the investment in a micro-grid to sometime in the future. The value of the option to postpone is not included in a static NPV analysis; this can adversely impact the investment decision. Using real options paradigm would make it possible to examine if there is positive value in postponing the investment (i.e., if the discounted value of the future net present value is higher than the one today). In other words, if there is uncertainty about the future, the postponement of the decision may reveal new information. In that case the investor or owner always has the option to invest if the stochastic parameters move in a favorable direction and the ability to not invest if they are not favorable. With real option approach further operational flexibility can be formulated into the decision-making process. For example, an irreversible investment on a gas-fired generator can benefit from an operational option of switching between suspension and re-activation. This can give the decision maker more insight into the investment and its possible future values.

3.2 Literature Review

Investment in micro-grid has been the topic of research in recent years. Deregulation of the electric power industry provides incentives for the adoption of distributed generation (DG) and combined heat and power (CHP) by micro-grids. Although the electric-only efficiency of DG is lower than that of central-station generation, the former becomes economically attractive when CHP applications are utilized to meet heat loads via heat exchangers (HXs). Furthermore, the persistence of relatively high tariff rates for electricity consumption makes DG attractive to commercial and industrial entities that may be able to organize themselves into micro-grids. Investment in micro-grids opens up many options for the investor and different decisions to make. The economics of the investment including its rate of return, the benefits or value associated with the micro-grid and its cost would determine whether or not the investment makes sense. Moreover, as mentioned before, a micro-grid is a portfolio of distributed energy resources. One of the aspects of investment decisions should address the optimal combination of this portfolio. Both stochastic discounted cash flow and real options are utilized in the literature to address the investment decisions in micro-grids.

Hybrid Optimization Model for Electric Renewable (HOMER) [26] is a product of the National Renewable Energy Laboratory, which evaluates design options for both off-grid and grid-connected power systems for remote, stand-alone and distributed generation applications. HOMER's optimization and sensitivity analysis algorithms allow users to evaluate the economic and technical feasibility of a large number of technology options and to account for variation in technology costs and energy resource availability. HOMER performs an hourly time series simulation of every possible combination of components entered and ranks the feasible systems according to user-specified criteria, such as net present cost or cost of energy. It does not consider changes over time, such as load growth or the deterioration of battery performance with aging (see [27]).

It turns out that sometimes it makes more sense to retain the option to invest even for a project that is “in the money" from the deterministic discounted cash flow (DCF) perspective. To formulate this type of investment model real options has been used in literature. This approach is appropriate because it trades off in time between the benefits from immediate investment (present value of money) and consequent costs stemming from uncertainty. Specifically, the real options approach includes not only direct investment costs such as the capital cost, but also the opportunity cost of exercising the option to invest, which is the loss of the discretion to wait for more information about the future and delaying the investment (see [28]).Real options theory adopts its concept from the pricing of financial call options (see [29]). In the real options approach we construct a risk-free portfolio by acquiring a short position on the underlying asset and a long position in the option to invest in micro-grid. Option pricing is used to obtain the value of option and to find the investment threshold at which the investment is triggered. Appendix 1 provides a brief description on option theory on physical asset investments.

Siddiqui*et al* ([30] and [31]) performed detailed economic and thermodynamic analysis of DG investment and operation in purely deterministic settings based on a cost-minimizing mixed-integer linear program. In almost all of the case studies they conducted for California, the adoption of gas-fired DG turned out to be attractive, with on-site generators typically covering a large fraction of the electric load (as well as a large fraction of overall energy needs).

Another approach the investor could take to hedge its investment against uncertainty is to modularize its investment decisions by proceeding in a sequential approach. Various investment and upgrade strategies can be evaluated under different stochastic conditions. Siddiqui et al [32]. formulate real option on investing in DG with operational flexibility. The real options approach constructs a risk-free portfolio by taking short position on the underlying asset and equates its expected appreciation (net of any dividend payments) to the instantaneous risk-free rate that could have been earned by investing in the portfolio. For a perpetual option, the resulting partial differential equation (PDE) from this “no-arbitrage” condition becomes an ordinary differential equation (ODE), which is solved analytically using boundary conditions. As part of the solution, an investment threshold price for the underlying asset is obtained, at which investment is triggered.

Asano *et al* [33] discuss investment strategies in a micro-grid consisting of cogeneration and renewable power generation under uncertainty in the natural gas price. They take the real options approach to analyze investment decision. By varying price volatility they show that the optimal investment strategy depends on the level of uncertainty. As volatility increases, strategies with installation option of renewable power generation, here, photovoltaic generation, become attractive in terms of risk reduction.

Fleton et al [34], in their work, present a model to obtain optimal investment thresholds (timing and capacity) for DG. They consider renewable resources in the portfolio.

3.3 Analytical Approach to Real Option

We consider investment in a micro-grid portfolio that consists of PV renewable generation, fuel-fired generation and connection to the grid. The micro-grid generation capacity configuration is assumed to beparametric and is fixed prior to making investment decision. The uncertainty in future natural gas prices and PV investment cost are included in making the investment decisions. The way we treat the relationship between prices of electricity and natural gas is expected to lead to fundamentally different behaviors of micro-grid value as a function of natural gas price. If there is no interdependence between the two prices, the value of micro-grid decreases as natural gas price increases. This is an abstraction for the current wholesale electricity market, but it is appropriate for cases where cost of on-site electricity production is independent of the grid electricity market. Examples are bio-fuel or fuel cell electric generators. In the case of interdependence, the behavior of micro-grid value would depend on how the two prices compare.

Nomenclature:

C	Natural gas price(*$/mmBtu*)
I	Investment cost (*$/kW*)
α	Annual percentage growth rate
σ	Annual percentage volatility
ρ	Correlation factor between two random variables
r	Risk-free rate of return
μ	Risk-adjusted rate of return
δ	Convenience yield
Z	Standard Geometric Brownian Motion
$I_i; i = GF, PV$	Resource index (%)
Cap_{GF}	Gas-fired generation capacity (*kW*)
Cap_{PV}	PV capacity (*kW*)
ϵ	Heat rate (*mmBtu/kW*)
η_{PV}	PV efficiency (%)
NPV	Net present value ($)
C_e	Electricity price (*$/kW)*
ϕ	Risk-free portfolio
F	Option value ($)
V_{MG}	Value of active micro-grid ($)
V_M	Value of active micro-grid with mothballed GF($)
M	Maintenance cost during suspension ($)
E_M	Fixed cost paid for suspension ($)
R	Fixed cost paid for re-activation ($)
C_M	Gas price threshold for suspension (*$/mmBtu*)
C_R	Gas price threshold for re-activation (*$/mmBtu*)

3.3.1 Dynamics of Uncertainty

For fuel-fired generation, we assume natural gaswith market price dynamics described commonly by a Brownian motion with a deterministic mean growth rate and random volatility. In this model the tendency of price to revert around a long-term average cost of production is exploited. The price can be modeled using one-factor or two-factor schemes. Two-factor models include short-term variations and provide a better fit for historical data. However, it has been commonly argued that for long-term investment decisions the one-factor model (i.e., Geometric Brownian Motion or GBM) are sufficiently accurate [35]. Moreover, Schwartz and Smith [36] show that the long-term factor is more decisive element in long-term investment decisions. The change in natural gas price over a short time interval using GBM is then given by:

$$dC = \alpha_{N.G.} C dt + \sigma_{N.G.} C dZ_{N.G.} \qquad (1)$$

where$C$ is natural gas price ($/mmBtu), $\alpha_{N.G.}$is the natural gas annual percentage growth rate and $\sigma_{N.G.}$is the natural gas annual percentage volatility. Furthermore, Z_t is standard GBM,$dZ_t^{N.G.} = e\sqrt{dt}$and$e \sim N(0,1)$. We assume that $\delta_{N.G.}$is the convenience yield on natural gas.Since, gas-fired generation is a quite mature technology; we assume that its investment cost is constant over the planning horizon.

There is no stochastic process identified for PV investment cost. We assume a decreasing trend according to a GBM:

$$dI = \alpha_{PV} I dt + \sigma_{PV} I dZ_{PV} \qquad (2)$$

where$I$ is Photo Voltaic investment cost ($), α_{PV} is the investment cost annual percentage growth rate and σ_{PV} is the electricity demand annual percentage volatility.

Z_{PV} is standard Brownian motion. In addition, we assume that δ_{PV} is the convenience yield on PV.

3.3.2 Micro-grid Configuration

A three-tuple $\theta = \left(I_{on-site-generation};\ I_{fuel-renewable}; I_{renewable}\right)$ is defined to represent the configuration of a micro-grid. The first index is a measure of the total on-site generation capacity of micro-grid. It is defined as the ratio of total on-site generation capacity over the average demand:

$$I_{on-sitegen.} = \frac{On-site\_generation\_capacity}{Average\_electricity\_demand} \quad (3)$$

The second index is fuel-renewable index, which is defined as the ratio of fuel-fired generation capacity over total renewable capacity:

$$I_{fuel-renewable} = \frac{Fuel-fired\_generation\_capacity}{Renewable\_capacity} \quad (4)$$

Finally, the renewable index is the ratio of wind turbine capacity to PV capacity:

$$I_{renewable} = \frac{Wind\_turbine\_capacity}{PhotoVoltaic\_capacity} \quad (5)$$

3.3.3 Stochastic Discounted Cash Flow–Independent Electricity and Natural Gas Prices

The present value from an operating micro-grid is the cost savings from its own generation compared to the case where all electricity demand is purchased from the grid discounted to the present. Since natural gas is assumed to be stochastic, the cost associated to gas-fired generation unit is discounted using risk-adjusted rate $\mu_{N.G.}$ of return,. Cost of supplying electricity from grid and savings from PV are discounted with risk-free rate of return, r, since we assume electricity price to be constant and also there is no stochasticity associated to the operation of PV. We note that PV technology pricing is stochastic, but once it is realized, there is no further stochasticity with PV pricing. We have:

$$Saving_{MG} = Saving_{GT} + Saving_{PV}$$
$$= \{Cap_{GF} \times C_e - Cap_{GF} \times \epsilon_{GF} \times E[C_t|C_{N.G.}]\} + \{\eta_{PV} \times Cap_{PV} \times C_e\} \quad (6)$$

$$NPV_{stoch} = 365 \times (\int_0^{\infty} Cap_{GF} \times C_e \times e^{-rt} dt$$
$$- \int_0^{\infty} Cap_{GF} \times \epsilon_{GF} \times C_{N.G.} \times e^{\alpha_{N.G.} t} \times e^{-\mu_{N.G.} t} dt$$
$$+ \int_0^{\infty} \eta_{PV} \times Cap_{PV} \times C_e \times e^{-rt} dt - I_{GF} - I_{PV})$$

$$NPV_{stoch} = 365 \times \left(\frac{Cap_{GF} \times C_e}{r} - \frac{Cap_{GF} \times \epsilon_{GF} \times C_{N.G.}}{\delta_{N.G.}} + \frac{\eta_{PV} \times Cap_{PV} \times C_e}{r} \right) - I_{GF} - I_{PV} \quad (7)$$

where I_{GF}, I_{PV} are investment costs of GF and PV.

The investment is profitable as long as net present value is positive. Therefore, the investment thresholds, $C^*_{N.G.}$andI^*_{PV}, must satisfy:

$$I^*_{PV} = -365 \times \left(\frac{Cap_{GF} \times \epsilon_{GF} \times C^*_{N.G.}}{\delta_{N.G.}} + \frac{Cap_{GF} \times C_e}{r} + \frac{\eta_{PV} \times Cap_{PV} \times C_e}{r} \right) - I_{GF} \quad (8)$$

3.3.4 Postponing Investment

While making investment decisions under uncertainty, investment decisions can significantly benefit if real option strategies were adopted. In the following sections, we present two formulations using contingent claim approach [32], assuming that: (i) the grid electricity and gas prices are independent and (ii) these two prices are interdependent.

3.3.4.1 Independent Electricity and Natural Gas Prices

Here we assume that grid electricity price is exogenously determined and is independent of natural gas price movements. We formulate the investment problem with real options, and examine optimal switching to suspension and re-activation of gas-fired generator. The equations for different value functions are solved with appropriate boundary conditions to obtain the desired thresholds for investment in PV and GF, suspension of GF and re-activation of GF. The value functions are:

- Value of *idle project* in which the option to invest in PV and GF are kept alive;
- Value of *active micro-grid* (with and without provisions for suspension and reactivation) where savings are compared to a *no-micro-grid* state;
- Value of *micro-grid* with *mothballed* gas-fired generator.

3.3.4.1.1 Value of Idle Project

We construct a risk-free portfolio, $\emptyset$, which includes long position in one unit F of option to invest in both PV and GF, and short position of appropriate units of some *assets* that span the stochasticity in natural gas price and PV investment cost. These *spanning assets* have stochastic behavior similar to PV or natural gas. Let $m \; and \; n$ be the number of spanning assets for natural gas and PV, respectively. Therefore, the value $\emptyset$ is given by:

$$\emptyset = F(C_{N.G.}, I_{PV}) - m \times C_{N.G.} - n \times I_{PV} \quad (9)$$

where $C_{N.G.}$ and I_{PV} are correlated with a correlation factor of ρ, and each follow a GBM process according to (1) and (2) above. Under no-arbitrage condition, the instantaneous rate of return on a risk-free portfolio equals its expected appreciation less any dividend. Over a short time interval dt, the return on the above portfolio will be (we drop $N.G.$ and PV for simplicity):

Capital gain: $d\emptyset = dF - m \times dC - n \times dI$ (10)

Short sell cost: $m\delta_{C.}C + n\delta_I I$ (11)

Since F is a function of $C_{N.G.}$ and I_{PV}, Ito's Lemma [37] can be used to find dF. We will adopt these notations: $F_C = \frac{\partial F}{\partial C}, F_I = \frac{\partial F}{\partial I}, F_{CI} = \frac{\partial^2 F}{\partial C \partial I}$, $F_{CC} = \frac{\partial^2 F}{\partial C^2}$ and $F_{II} = \frac{\partial^2 F}{\partial I^2}$. We then have:

$$dF = F_C dC + F_I dI + \frac{1}{2}(\sigma_C^2 C^2 F_{CC} + 2\rho\sigma_C\sigma_I F_{CI} + \sigma_I^2 I^2 F_{II})dt \quad (12)$$

Using Eqs. (1), (2), (10) and (11) we obtain

$$d\emptyset = F_C(\alpha_C C dt + \sigma_C C dZ_C) + F_I(\alpha_I I dt + \sigma_I I dZ_I) + \frac{1}{2}(\sigma_C^2 C^2 F_{CC} + 2\rho\sigma_C\sigma_I F_{CI} + \sigma_I^2 I^2 F_{II})dt -$$

$$m(\alpha_C C dt + \sigma_C C dZ_C) - n(\alpha_I I dt + \sigma_I I dZ_I) - m\delta_C C dt - n\delta_I I dt \quad (13)$$

The expression above can be rearranged as follows:

$$d\emptyset = (F_C\alpha_C C - m\alpha_C C + F_I\alpha_I I - n\alpha_I I - m\delta_C C - n\delta_I I + \frac{1}{2}(\sigma_C^2 C^2 F_{CC} + 2\rho\sigma_C\sigma_I F_{CI} +$$

$$\sigma_I^2 I^2 F_{II}))dt + (F_C\sigma_C C - m\sigma_C C)dZ_C + (F_I\sigma_I I - n\sigma_I I)dZ_I \quad (14)$$

Since the portfolio should be risk-free we set m and n such that the stochastic components diminish. That is,

$$m = F_C$$

$$n = F_I$$

Substituting for m and n yields:

$$d\emptyset = \left(-F_C\delta_C C - F_I\delta_I I + \frac{1}{2}(\sigma_C^2 C^2 F_{CC} + 2\rho\sigma_C\sigma_I F_{CI} + \sigma_I^2 I^2 F_{II})\right)dt \quad (15)$$

Since the portfolio is risk-free, this return should equal the risk-free rate of return:

$$d\emptyset = r\emptyset dt$$

Therefore:

$$\left(-F_C\delta_C C - F_I\delta_I I + \frac{1}{2}(\sigma_C^2 C^2 F_{CC} + 2\rho\sigma_C\sigma_I F_{CI} + \sigma_I^2 I^2 F_{II})\right)dt = r(F - F_C C - F_I I)dt \quad (16)$$

dt is cancelled out from both sides of Eq. (16). We re-arrange and obtain the following partial differential equation, which governs the dynamics of idle project value (i.e., investment option):

$$\frac{1}{2}(\sigma_C^2 C^2 F_{CC} + 2\rho\sigma_C\sigma_I F_{CI} + \sigma_I^2 I^2 F_{II}) + (r - \delta_C)CF_C + (r - \delta_I)IF_I - rF = 0 \quad (17)$$

The boundary conditions for Eq. (17) are obtained using value-matching and smooth pasting conditions [28]. Value-matching condition implies that at investment threshold defined by C^* and I^*, the value of option should be equal to the value $V_{MG}$of an active micro-grid minus the investment costs. Smooth pasting conditions guarantee that these two values match continuously at those thresholds. These are free boundary conditions (C^* and $I^*$are to be determined as part of the solution). In addition, as C and I go to infinity, the value of option goes to zero. We have:

$$F(C^*, I^*) = V_{ActiveMG}(C^*) - I^* \quad (18)$$

$$\frac{\partial F}{\partial C}|_{C^*} = \frac{\partial(V_{ActiveMG} - I)}{\partial C}|_{C^*}$$

$$\frac{\partial F}{\partial I}|_{I^*} = \frac{\partial(V_{ActiveMG} - I)}{\partial I}|_{I^*}$$

PDE with free boundary conditions appear in many disciplines such as fluid mechanics. In some cases analytical solutions exist to such problems and one way to obtain it is to assume a specific form for the function which satisfies the PDE and solve for free boundary conditions. In some areas (e.g., fluid mechanics), the assumption of functional form comes from the knowledge of physical behavior. We are looking for a solution that both satisfies the PDE and also behaves as expected with the changes in C and I.

We choose a product form value function, namely $F = A(CI)^{\beta}$. With this function, the PDE holds and the change in option value is consistent with the changes in C and I. As C and I increase, the option value decreases and vice versa. By substituting $F = A(CI)^{\beta}$ into Eq. 17, its characteristic function is derived; $\beta_1 > 0$ and $\beta_2 < 0$ are the roots of following characteristic function.

$$(\tfrac{1}{2}\sigma_C^2 + \rho\sigma_C\sigma_I + \tfrac{1}{2}\sigma_I^2)\beta(\beta - 1) + (2r - \delta_C - \delta_I)\beta - r = 0 \qquad (19)$$

As C and I grow significantly high, the option value is expected to decline. Therefore the coefficient associated to positive β should be zero:

$$F = A_2(CI)^{\beta_2} \qquad (20)$$

It is left to calculate the value of active micro-grid, which is described next.

3.3.4.1.2 Value of active micro-grid with no possibility of later suspension/re-activation, V_{MG}

Here, we derive the value of an active micro-grid with both PV and GF. The micro-grid value is a contingent or derivative asset whose payoffs depend on the natural gas price. Thus, the value of the active micro-grid is a function $V_{MG}(C)$ of the natural gas price. We use contingent claims valuation to compute value of the microgrid. We construct a riskless portfolio by taking suitable combinations of the active micro-grid and the underlying asset C. The riskless portfolio is expected to earn the riskless rate of return. Portfolio $\emptyset$ is then constructed at time t by taking long position in one unit of active micro-grid, $V_{MG}(C)$, and a short position of n units of an underlying asset that spans the stochasticity in natural gas price. Therefore, the value $\emptyset$ of the portfolio is:

$$\emptyset = V_{MG}(C) - n \times C \qquad (21)$$

We consider holding this portfolio over a small interval of time $(t, t + dt)$. The holder of the active micro-grid will earn $Savings \times dt$ in payoffs from cost savings (use of micro-grid compared to purchase from the grid). Also, the holder of the short position must pay the holder of the respective long position the dividend or convenience yield in the amount of $\delta_C C dt$. Therefore, the net dividend of holding the above portfolio would be $(Savings - \delta_C C)dt$. Moreover, the portfolio will yield a capital gain equal to (details are similar to Eqns. 9-11):

$$d\emptyset = dV_{MG}(C) - n \times dC = (\frac{dV_{MG}}{dC} - n)dC + 1/2\sigma_C^2 C^2 \frac{d^2 V_{MG}}{dC^2} dt$$

$$= (\frac{dV_{MG}}{dC} - n)(\alpha_C C dt + \sigma_C C dZ_C) + 1/2\sigma_C^2 C^2 \frac{d^2 V_{MG}}{dC^2} dt$$

$$d\emptyset = \{\alpha_C C(\frac{dV_{MG}}{dC} - n) + 1/2\sigma_C^2 C^2 \frac{d^2 V_{MG}}{dC^2}\}dt + (\frac{dV_{MG}}{dC} - n)\sigma_C C dZ_C \quad (22)$$

Now we choose $n = \frac{dV_{MG}}{dC}$ so that the stochastic term (i.e., the coefficient of dZ_C) disappears and portfolio becomes riskless. Assuming a riskless portfolio we then have:

$$\left[1/2\sigma_C^2 C^2 \frac{d^2 V_{MG}}{dC^2} + Savings(C) - \delta_C C\right] dt = r\,\emptyset dt \quad (23)$$

Furthermore, the following PDE governs the dynamics of the active micro-grid:

$$1/2\sigma_C^2 C^2 \frac{d^2 V_{MG}}{dC^2} + (r - \delta_C)C \frac{dV_{MG}}{dC} - rV_{MG} + Savings(C) = 0 \quad (24)$$

It is now left to calculate $Savings(C)$. Assuming no interdependence between electricity and gas price and no possibility of further suspension, the gas-fired generation is only operational in the region which $C < C_e/\epsilon$, i.e., cost of on-site generation is less than grid electricity price. In order to relate the value of idle project and active micro-grid (through boundary conditions), the value of micro-grid in operational region is used (i.e., region $C < C_e/\epsilon$). In this region, the savings from the operation of both GF and PV become:

$$Savings(C) = Saving_{GF} + Saving_{PV}$$
$$= 365 \times (Cap_{GF} \times C_e - Cap_{GF} \times \epsilon_{GF} \times C) + 365 \times \eta_{PV} \times Cap_{PV} \times C_e \quad (25)$$

Substituting Eq. (25) into Eq. (24), a two-part (homogeneous and non-homogeneous) solution can be obtained:

$$V_{MG} = V_{MG,h} + V_{MG,nh} \quad (26)$$

The homogeneous part of solution is:

$$V_{MG,h} = G_1 C^{\lambda_1} + G_2 C^{\lambda_2} \quad (27)$$

where $\lambda_1 > 0$ and $\lambda_2 < 0$ are the roots of the following characteristic function:

$$\tfrac{1}{2}\sigma_C^2 \lambda(\lambda - 1) + (r - \delta_C)\lambda - r = 0 \quad (28)$$

To the homogenous solution, we add any particular solution of the full equation as follows:

$$V_{MG} = G_1 C^{\lambda_1} + G_2 C^{\lambda_2} + 365 \times \frac{(Cap_{GF} + \eta_{PV} \times Cap_{PV}) \times C_e}{r} - 365 \times \frac{Cap_{GF} \times \epsilon_{GF} \times C}{\delta_C} \quad (29)$$

where $G_1$and $G_1$are to be determined. Since there is a positive probability that natural gas price goes beyond electricity price, there is a speculative term in the form of $G_1 C^{\lambda_1} + G_2 C^{\lambda_2}$. As C tends to zero, the possibility of increasing natural gas price over electricity price diminishes and therefore, $G_2 = 0$.

For region $C > C_e/\epsilon$, microgrid's savings will be from the operation of PV only:

$$Savings(C) = Saving_{PV} = 365 \times \frac{\eta_{PV} \times Cap_{PV} \times C_e}{r} \quad (30)$$

Following the same line of proof as above, we can obtain the value of active micro-grid in region $C > C_e/\epsilon$ as shown below:

$$V_{MG} = F_1 C^{\lambda_1} + F_2 C^{\lambda_2} + 365 \times \frac{\eta_{PV} \times Cap_{PV} \times C_e}{r} \quad (31)$$

The first two terms refer to possible decrease in natural gas price below the electricity price. With C going to infinity, this possibility is remote and $F_1 = 0$. Two unknown coefficients, G_1 and F_2, are obtained using the fact that at $C = C_e/\epsilon$ these two values should match tangentially (value-matching and smooth pasting conditions). That is,

$$G_1 = 365 \times \frac{\lambda_2}{\lambda_1 - \lambda_2} Cap_{GF} \times \epsilon^{\lambda_1} C_e^{(1-\lambda_1)} \left(\frac{1}{r} + \frac{1}{\delta_C}\left(\frac{1-\lambda_2}{\lambda_2}\right)\right) \quad (32)$$

3.3.4.1.3 Value of active micro-grid with possibility of later suspension/re-activation, $\boldsymbol{V_{MG}}$

In this section we assume that there is a possibility of later suspension of GF if the gas price movement is such that it makes no economic sense to generate electricity form GF generator. By suspension of GF, the micro-grid will incur a one-time sunk cost, E_M, and a maintenance cost M during the time the GF is suspended. Since, the operation of the micro-grid is independent of PV investment cost; the value of micro-grid is only a function of natural gas price. The same approach as previous section is used to obtain the governing equation for V_{MG}, except that there is no need to consider two separate regions, due to future suspensions and re-activations. We have:

$$\frac{1}{2}\sigma_C^2 C^2 \frac{d^2 V_{MG}}{dC^2} + (r - \delta_C) C \frac{dV_{MG}}{dC} - rV_{MG} + Savings = 0 \quad (33)$$

where the micro-grid saving from the operation of both PV and GF is:

$$Savings = Saving_{GF} + Saving_{PV}$$
$$= 365 \times \{Cap_{GF} \times C_e - Cap_{GF} \times \epsilon_{GF} \times C\} + 365 \times \{\eta_{PV} \times Cap_{PV} \times C_e\} \quad (34)$$

The solution to the above PDE consists of two parts: homogenous part, $V_{MG,h}$, which includes speculative bubble reflecting the value of micro-grid once GF is suspended, and non-homogenous part, $V_{MG,nh}$, which includes the fundamental value equal to the savings from the operation of PV and GF generations.

The homogeneous part of the solution is:

$$V_{MG,h} = B_1 C^{\lambda_1} + B_2 C^{\lambda_2} \quad (35)$$

$where\ \lambda_1 > 0$ and $\lambda_2 < 0$ are the roots of the following characteristic function:

$$\frac{1}{2}\sigma_C^2 \lambda(\lambda - 1) + (r - \delta_C)\lambda - r = 0 \quad (36)$$

As C goes to infinity the possibility of suspension becomes remote, thus, $B_2 = 0$. $V_{MG,nh}$ is the fundamental value of micro-grid given by savings from PV and GF:

$$V_{MG,nh} = 365 \times \frac{Cap_{GF} \times C_e}{r} - 365 \times \frac{Cap_{GF} \times \epsilon_{GF} \times C}{\delta_C} + 365 \times \frac{\eta_{PV} \times Cap_{PV} \times C_e}{r} \quad (37)$$

$$V_{MG} = B_1 C^{\lambda_1} + 365 \times \frac{(Cap_{GF} + \eta_{PV} \times Cap_{PV}) \times C_e}{r} - 365 \times \frac{Cap_{GF} \times \epsilon_{GF} \times C}{\delta_C} \quad (38)$$

3.3.4.1.4 Value of active micro-grid with mothballed gas-fired generator, V_M

Once the investment is triggered and micro-grid is active, its value is stochastic because of stochastic natural gas price. Under certain conditions, it makes economic sense to suspend the operation of gas-fired generation. Using the same analysis as before, the value of a micro-grid with mothballed GF is calculated to obtain the optimal switching to suspension and further re-activation. It should be noted that during mothballed stage, the only on-site resource of micro-grid is PV and therefore the savings are due to the operation of this component:

$$Savings = Saving_{PV} = 365 \times \eta_{PV} \times Cap_{PV} \times C_e \qquad (39)$$

Micro-grid's value with suspended GF is then:

$$V_M = K_1 C^{\lambda_1} + K_2 C^{\lambda_2} + 365 \times \frac{(\eta_{PV} \times Cap_{PV}) \times C_e}{r} - \frac{M}{r} \qquad (40)$$

$K_1 C^{\lambda_1} + K_2 C^{\lambda_2}$ is the re-activation value and $\frac{M}{r}$ is the capitalized maintenance cost, assuming that GF remains in the mothballed state forever. As C grows infinitely high, the possibility of re-activation is remote and $K_1 = 0$.

Suspension and re-activation thresholds, C_M and C_R, are obtained using value-matching and smooth pasting boundary conditions. Value-matching condition indicates that upon suspension, the value of active micro-grid is equal to V_M minus a one-time sunk cost, E_M, for suspension. This sunk cost could be seen as any charges that micro-grid incurs for changes in its contractual agreement with the gird or any cost associated to infrastructure upgrades/modifications. We have

$$V_{MG}(C_M) = V_M(C_M) - E_M \qquad (41)$$

$$\frac{dV_{MG}}{dC}|_{C_M} = \frac{dV_M}{dC}|_{C_M}$$

For re-activation threshold, value-matching condition suggests that V_M is equal to the value of active micro-grid minus a sunk cost incurred upon re-activation, R. We have

$$V_M(C_R) = V_{MG}(C_R) - R \qquad (42)$$

$$\frac{dV_M}{dC}|_{C_R} = \frac{dV_{MG}}{dC}|_{C_R}$$

3.3.4.1.5 Solution Technique

There are total of 7 unknowns, namely, A_2, B_1, K_2, C^*, I^*, C_M and C_R along with 7 value-matching and smooth pasting conditions (Equations 18, 41 and 42). They construct a system of non-linear equations that can be solved numerically to obtain the thresholds. Four equations associated to suspension and re-activation can be solved separately to obtainB_1, K_2, C_M and C_R:

$$\begin{cases} B_1 C_M^{\lambda_1} - K_2 C_M^{\lambda_2} - 365 \times \dfrac{Cap_{GF} \times \epsilon_{GF}}{\delta_C} C_M + 365 \times \dfrac{Cap_{GF} \times C_e}{r} + \dfrac{M}{r} - E_M = 0 \\ B_1 \lambda_1 C_M^{\lambda_1 - 1} - K_2 \lambda_2 C_M^{\lambda_2 - 1} - 365 \times \dfrac{Cap_{GF} \times \epsilon_{GF}}{\delta_C} = 0 \\ B_1 C_R^{\lambda_1} - K_2 C_R^{\lambda_2} - 365 \times \dfrac{Cap_{GF} \times \epsilon_{GF}}{\delta_C} C_R + 365 \times \dfrac{Cap_{GF} \times C_e}{r} + \dfrac{M}{r} - R = 0 \\ B_1 \lambda_1 C_R^{\lambda_1 - 1} - K_2 \lambda_2 C_R^{\lambda_2 - 1} - 365 \times \dfrac{Cap_{GF} \times \epsilon_{GF}}{\delta_C} = 0 \end{cases}$$

These equations are nonlinear in terms of thresholds C_M and C_R; however, it can be proved that a unique solution exists (see [28]). We solve this system numerically using available commercial solvers (e.g., MATLAB).

There are three other boundary conditions of new investment explained earlier in the value of idle project. We recapture them here:

Value matching condition:

$$F(C^*, I^*) = V_{ActiveMG}(C^*) - I_{GF} - Cap_{PV} I^*$$

and smooth pasting conditions:

$$\frac{\partial F}{\partial C}|_{C^*} = \frac{\partial (V_{ActiveMG} - I_{GF} - Cap_{PV} I)}{\partial C}|_{C^*}$$

$$\frac{\partial F}{\partial I}|_{I^*} = \frac{\partial (V_{ActiveMG} - I_{GF} - Cap_{PV} I)}{\partial I}|_{I^*}$$

Three other unknowns, namely, A_2, C^* and I^* are determined using these three equations. We use numerical schemes to solve these equations.

3.3.4.2 Interdependence between Grid Electricity and Natural Gas Prices

In the following subsections we assume that electricity price at peak hour is driven by natural gas price. The cost saving from micro-grid is a function of natural gas price and depends on the optimal operation of micro-grid against grid electricity price. Hourly electricity price, $ElecPr(h)$, is therefore a function of natural gas price and assumed to be:

$$ElecPr(h) = C_{N.G.} \times \frac{Peak\ Elec\ Price}{GF\ Prod\ Cost} \times Profile_{ElecPr}(h) \qquad (43)$$

where, $Profile_{ElecPr}(h)$ is the grid electricity price as a percentage of daily electricity peak price and is shown in Figure 22.

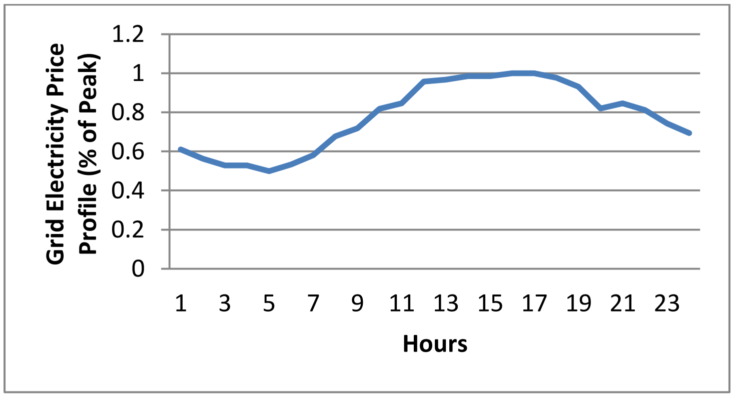

Figure 22: Daily grid electricity price profile

We also assume that daily electricity demand has the following profile (Figure 23):

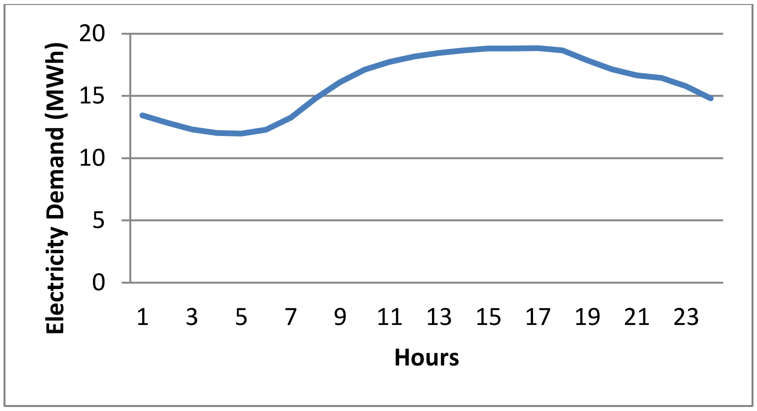

Figure 33: Daily electricity profile

We assume that a single year can be represented by a few representative average profiles. This approach is adopted to simplify presentation and can be easily expanded to more elaborate electricity demand and price over the course of a year. Since there is no cost associated to PV production, electricity demand is primarily supplied by available PV production. Any remaining demand is optimized between gas-fired production and purchase from the grid. The optimization takes into account grid electricity price and gas-fired production cost and accounts for GF generator's capacity. This is a simplified version of optimization that was presented in Chapter 2. The PV production is assumed to follow the profile given in Figure 24.

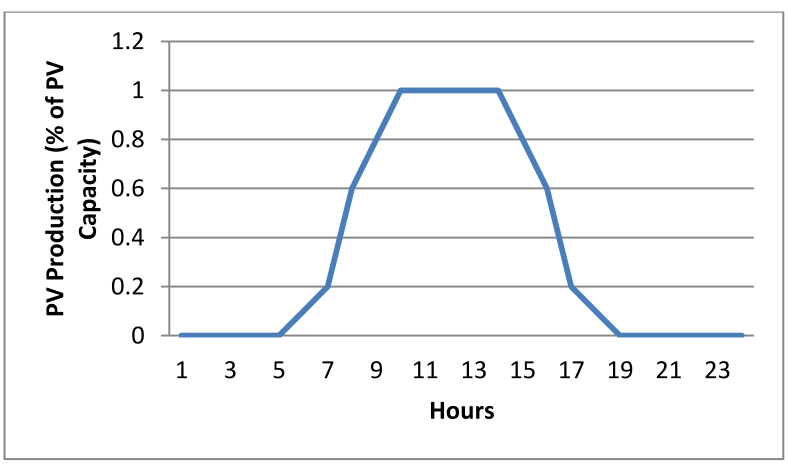

Figure 24:Daily PV production profile

3.3.4.2.1 Value of idle project

Following the same line of proof as before, we have the following PDE governing the value of option to invest:

$$\frac{1}{2}\sigma_I^2 I^2 \frac{\partial^2 F}{\partial I^2} + \rho\sigma_C\sigma_I CI \frac{\partial^2 F}{\partial C \partial I} + \frac{1}{2}\sigma_C^2 C^2 \frac{\partial^2 F}{\partial C^2} + (r - \delta_I) I \frac{\partial F}{\partial I} + (r - \delta_C) C \frac{\partial F}{\partial C} - rF = 0 \qquad (44)$$

The boundary conditions are given by:

$$F(C^*, I^*) = V_{ActiveMG}(C^*) - I_{GF} - Cap_{PV} I^* \qquad (45)$$

$$F(C^*, I^*) = V_{ActiveMG}(C^*) - \ I^*$$

$$\frac{\partial F}{\partial C}|_{C^*} = \frac{\partial (V_{ActiveMG} - I)}{\partial C}|_{C^*}$$

$$\frac{\partial F}{\partial I}|_{I^*} = \frac{\partial (V_{ActiveMG} - I)}{\partial I}|_{I^*}$$

We assume functional form of $F = I.f\left(\frac{C}{I}\right) = If(p)$, where $f()$ is to be determined. The rationale behind this selection is that as C and I increase the value of active micro-grid increases and so does the investment cost. By substituting $F = If(p)$ into Eq. (45) we obtain the following ODE for which a closed form solution exists:

$$\frac{1}{2}(\sigma_C^2 + 2\rho\sigma_C\sigma_I + \sigma_I^2 I^2)p^2 f_{pp} + (\delta_I - \delta_C)pf_p - \delta_I f = 0 \qquad (46)$$

The solution has the form of:

$$f = A_1 p^{\gamma_1} + A_2 p^{\gamma_2} \qquad (47)$$

$$F = I(A_1 \left(\frac{C}{I}\right)^{\gamma_1} + A_2 \left(\frac{C}{I}\right)^{\gamma_2})$$

where $\gamma_1 > 0$ and $\gamma_2 < 0$ are the roots of characteristic function:

$$\left(\frac{1}{2}\sigma_C^2 + \rho\sigma_C\sigma_I + \frac{1}{2}\sigma_I^2\right)\gamma(\gamma - 1) + (\delta_I - \delta_C)\gamma - \delta_I = 0 \qquad (48)$$

When p is very small (i.e., near zero), the prospect of it rising to threshold is quite remote. In other words, the option is worthless at this extreme and $A_2 = 0$. A_1 and the optimal threshold p^* are to be determined using value matching and smooth pasting conditions imposed on active micro-grid value.

3.3.4.2.2 Value of active micro-grid

Using Eqns. (21-23), the SDE which governs the behavior of micro-grid value function, V_{MG}, is:

$$1/2\sigma_C^2 C^2 \frac{d^2 V_{MG}}{dC^2} + (r - \delta_C)C\frac{dV_{MG}}{dC} - rV_{MG} + Savings = 0 \qquad (49)$$

To obtain a functional form to substitute for savings in the above equation, we calculate the micro-grid cost savings under different natural gas prices and fit a linear regression of the form:

$$Saving_{MG} = \beta C_{N.G.}$$

By substituting this functional form into above ODE, micro-grid value function becomes:

$$V_{MG} = B_1 C^{\lambda_1} + \frac{\beta C_{N.G.}}{\delta_C} \qquad (50)$$

We ignore the possibility of further suspension or re-activation and omit the speculation bubble term:

$$V_{MG} = \frac{\beta C_{N.G.}}{\delta_C} \qquad (51)$$

There is one value matching and two smooth pasting conditions explained earlier. Any of two can be used to solve for A_1 and the optimal threshold p^*. Substituting $F = IA_1\left(\frac{C}{I}\right)^{\gamma_1}$ into value-matching condition and the first smooth pasting condition gives the following relationship for investment thresholds:

$$\frac{C^*}{I^*} = \frac{\gamma_1 \delta_C}{(\gamma_1 - 1)\beta} \qquad (52)$$

It should be noted that I^* is the total investment cost which includes both PV and GF investment cost:

$$I^* = I_{GF} + Cap_{PV} \times I_{PV}^*$$

Since we considered GF cost to be deterministic and constant, we need to only find optimal thresholds for gas price and PV investment cost. Substituting for I^* in Eq. (52), we have:

$$\frac{C^*}{I_{GF} + Cap_{PV} \times I_{PV}^*} = \frac{\gamma_1 \delta_C}{(\gamma_1 - 1)\beta}$$

Rearranging the expression above yields:

$$I_{PV}^* = \frac{(\gamma_1 - 1)\beta}{\gamma_1 \delta_C Cap_{PV}} C^* - \frac{I_{GF}}{Cap_{PV}} \qquad (53)$$

3.3.4.3 NPV vs. Real Options – Independent Natural gas and electricity prices

We examine how the two approaches respond to the volatility of natural gas and PV investment cost on investment timing. This is examined under different volatility conditions and using a fundamental parameter that absorbs changes in volatility. No future suspension or re-activation is assumed to ensure fair comparison between the two models. (Note that NPV is incapable of capturing such flexibility in investment.)

Suppose that the riskless rate of return r is exogenously specified, such as the return on government bonds. Then the fundamental condition of equilibrium from the capital asset pricing model (CAPM), see [38] and [39], says that:

$$\mu_C = r + \varphi \sigma_C \rho_{C,m} \qquad (54)$$

$$\mu_I = r + \varphi \sigma_I \rho_{I,m}$$

where φ is an aggregate market parameter (the market price of risk) that is exogenous and $\rho_{x,m}(x = C \; or \; I)$ is the coefficient of correlation between return on the particular asset x and the whole market portfolio m. We assume that spanning holds, i.e., the uncertainty over future values of gas price and PV investment cost can be replicated by existing assets. Thus μ_x is risk-adjusted expected return on these underlying assets. We also assume that μ_x is greater than α_x because if this was not the case, the option holder is always better off waiting. δ_x denotes the difference between μ_x and α_x. By using an analogy to financial call option, we can elaborate on δ_x and its role. Suppose that x is the price of a share of a stock and δ_x is the dividend paid on that stock. The total expected return on the stock is $\mu_x = \alpha_x + \delta_x$. If the dividend payment were zero, there would have been no cost to keep the option alive due to the fact that the entire return on the stock is captured by its price movement. The dividend can be therefore seen as an opportunity cost that one gives up by holding the option rather than the stock itself. In our investment problem, δ_x is an opportunity cost of delaying investment and instead keeping the option alive. As other parameters in the model such as σ_x change, δ_x changes too. We always assume that r is fixed by the whole capital market regardless of what happens to individual assets. By the same token, aggregate market price of risk φ is held fixed. Now with the increase in σ_x, μ_x will increase, but to preserve the equality in $\delta_x = \mu_x - \alpha_x$, either δ_x or α_x must change. This will be used to conduct experiments on changing volatility – see Figure 5 below.

For the illustration, we consider a micro-grid with PV as the only renewable resource, i.e., $I_{renewable}$=0. We fix $I_{on-site\ gen}$ and $I_{fuel-renewable}$ which means that both investments would happen at the same time. Configurations shown in Table 10 are considered:

Table 40 Configuration parameters

Case	$I_{on-sitegen.}$	$I_{fuel-renewable}$	Average Demand (kW)	GF Capacity (kW)	GF Heat Rate (mmBtu/kWh)	PV Capacity (kW)	PV Efficiency
1	0.5	1	2000	500	0.03	1250	0.4
2	0.5	2	2000	666	0.03	833	0.4

We assume that once a unit is installed, its lifetime will be infinite if maintained properly. This assumption is further justified by the fact that the discrepancy between present value of perpetuity and the present value of an annuity due would decrease as the length of investment time horizon increases. Natural gas price and PV investment cost are each assumed to follow a GBM (in real options approach) with the stochastic parameters listed in Table 11:

Table 11 Stochastic parameters

	α	σ	δ
GP	0.1	0.06	0.06
PV	-0.6	0.06	0.06

Risk-free discount factor is set to $r = 6\%$ and electricity price to $C_e = 1\ \$/kW$. Figure 25 shows the threshold line for investment in PV investment cost and natural gas price. Two sets of lines are shown corresponding to $I_{fuel-renewable} = 1$ or 2. As expected, real options suggests lower thresholds (i.e., delay in investment) for triggering the investment. It should be noted that the negative region for PV investment cost refers to infeasible region for investment, *i.e.*, if gas price runs high, investment only makes sense when investment cost is negative. Figure 26 shows the impact of increasing volatilities on the investment thresholds. As uncertainty about the future increases, the postponement in investment gains more value. This effect can be captured using contingent claims analysis and not in traditional NPV.

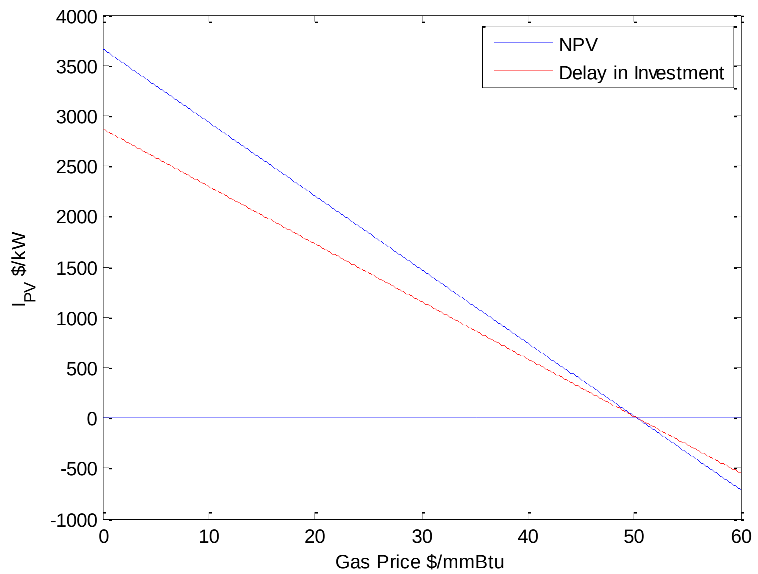

Figure 25: Threshold line using NPV and Contingent Claim Analysis

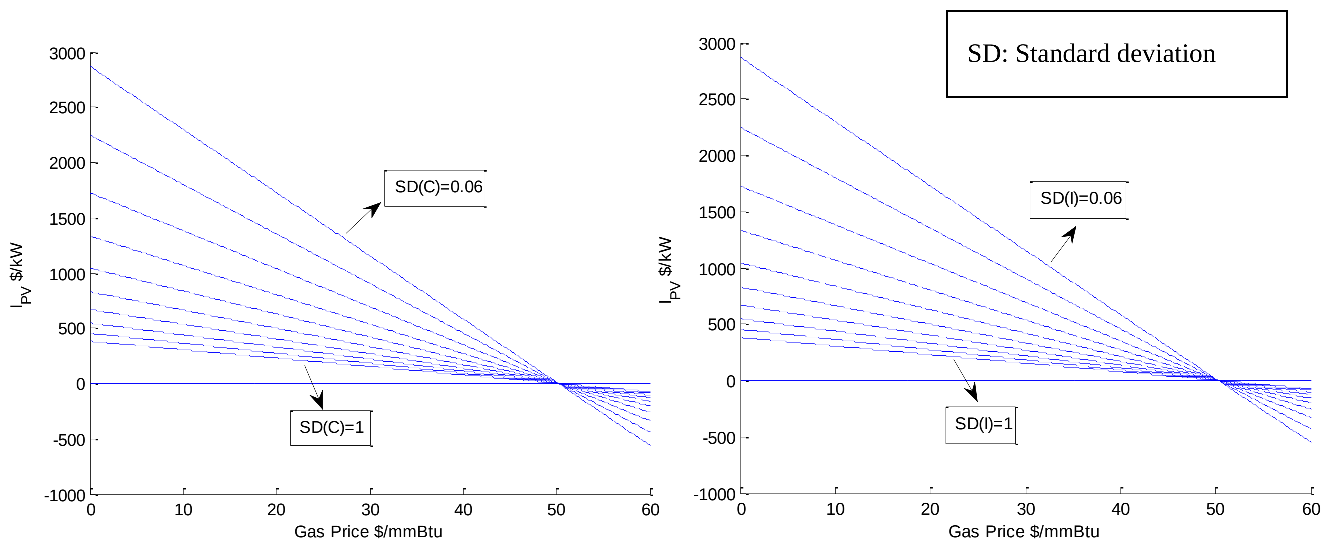

Figure 26: Optimal thresholds under different volatilities

Figure 27 shows the impact of different configuration of micro-grid on the thresholds (Case 2 in Table 10). In this case we increase $I_{fuel-renewable}$ to 2. This moves the thresholds to an earlier investment, which means that the investment is still exercised in higher PV investment cost and gas price.

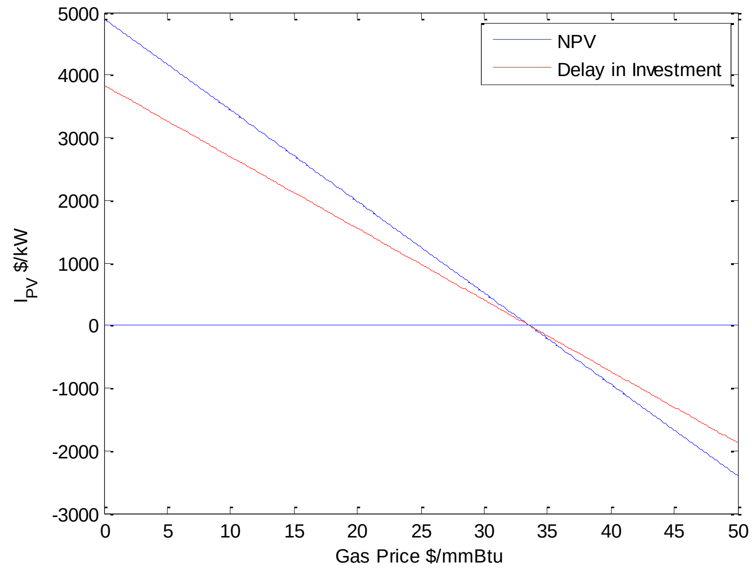

Figure 27: Threshold line using NPV and Contingent Claim Analysis.

3.4 Monte Carlo Simulation of Real Option

In this section, we present Monte Carlo simulation to solve real option investment in micro-grids. The idea is adopted from Longstaff and Schwartz [40] who used Monte Carlo and least-squares (Monte Carlo LSM) approach to evaluate American style financial options.

The problem of optimal timing in investment with option to postpone the investment is similar to financial American style option. At each exercise date, the option holder optimally decides whether to immediately exercise his/her option (i.e., trigger the investment) or to keep the option alive until some other dates in future. Therefore, the optimal exercise strategy is determined by comparing the immediate payoff from investment in a micro-grid (with fixed configuration) and the conditional expectation of payoffs from continuing the option. This conditional expectation is estimated from cross-sectional information in the simulation using least squares regression. We regress the future realized payoffs from continuation on the values of state variables (gas price and PV investment cost). This function is then used to calculate the conditional expectation of continuation at each exercise date. Starting from the last time period in the investment horizon, we work backward to determine the optimal investment timing along each path. The methodology is applied over all generated paths and conditional distributions of investment thresholds (i.e., natural gas price and PV investment cost) are obtained. A numerical example is provided in the next section to illustrate the methodology.

3.4.1 Simulation vs. Analytical Results - Interdependent gas and electricity prices and GBM processes for gas price and PV investment cost

It is assumed that the micro-grid does not exercise suspension/re-activation options. Furthermore, electricity and natural gas prices are interdependent, and GBM governs both natural gas price and PV investment cost. The parameters of the two processes are listed in Table 12:

Table 12 Stochastic parameters of GBM for natural gas price and PV investment cost

	$\propto$	σ
GP	0.1	0.06
PV	-0.6	0.6

At each exercise date, $t-1$, regression on state variables of that time is used to estimate the conditional expectation of continuation until time t:

$$E\left[Continutation_t \middle| X_{1,t-1}, X_{2,t-1}\right] = \beta_{0,t-1} + \beta_{1,t-1}X_{1,t-1} + \beta_{2,t-1}X_{2,t-1} + \beta_{3,t-1}X_{1,t-1}^2 + \beta_{4,t-1}X_{2,t-1}^2 + \beta_{5,t-1}X_{1,t-1}X_{2,t-1} \qquad (55)$$

Table 13 illustrates the results for 10 sample paths. Moreover, at each exercise date, the conditional distribution of thresholds can be obtained as shown in figures 28 and 29. For example, based on all simulation paths, Figure 28 shows the distribution of natural gas price threshold that triggers the investment if investment time is in years 2, 3 and 4. Therefore, the expectation of optimal thresholds can be calculated from these conditional distributions, $f(X^*|\tau = t)$, using :

$$E[X^*] = \int_0^\infty X^* f(X^*) dX^* = \int_0^\infty X^* f(X^*|\tau = t) \times \Pr(\tau = t)\, dX^* \quad (56)$$

Let us take one example path in the simulation and walk through the solution. For example, in path one, natural gas prices and PV investment costs are shown as they realize over 4 years. In years 1 and 2, the savings from micro-grid (due to respective gas price and therefore electricity price) are 1.53 and 1.85 million dollars respectively. Immediate investment on those years would lead to net present value of around 14 million dollars. However the conditional expectation of continuation (i.e., wait instead of immediate investment) is higher for both years (27 and 21 million dollars respectively). Therefore, the investor continues to wait and does not undertake the investment. In year 3, the value of immediate investment turns out to be higher than its continuation (14 versus 9 million dollars). This means that the investor is better off if he/she immediately exercises the option to invest and not to wait until year 4 or beyond. The decision indicator would become 1 for year 3 and 0 for any other years.

Based on the decision matrix, the probability of investment exercise at each year over 200 scenarios is:

$$\Pr(\tau = 1) = 0, \Pr(\tau = 2) = 30/200, \Pr(\tau = 3) = 90/200, \Pr(\tau = 4) = 80/200$$

Consequently the expected thresholds are calculated by:

$$E[C^*] = 10.24\ \$/mmBtu$$

$$E[I^*] = 4905\ \$/kW$$

Table 13 Sample paths simulated in Monte Carlo

	Natural Gas Price ($/mmBtu)					PV Investment cost ($/kW)			
Path	year 1	year 2	year 3	year 4	Path	year 1	year 2	year 3	year 4
1	5.158	6.2194	7.886	8.8116	1	5827.3	6301.1	6491.5	6403.6
2	6.4235	9.3383	18.053	38.423	2	6077.5	5581.1	4823.6	4692.6
3	4.6986	5.7278	7.616	9.1477	3	6110.7	5479.1	4598.6	3791.7
4	3.9935	5.0561	6.1723	8.1445	4	5804.8	5456.7	5340.4	5294.5
5	5.4921	7.3165	11.118	12.044	5	5972.7	5927.1	5438.3	5560.7
6	4.8807	5.1176	5.9211	7.5439	6	5513.2	5134.6	4905.7	4895.2
7	4.6526	6.5299	9.5136	13.015	7	6076.1	6055.6	5353.7	4299.2
8	4.5752	5.2606	6.3869	9.6902	8	5688.1	5056.7	4655.1	3919.5
9	4.016	4.9888	7.018	11.609	9	5925	5365.5	4845.3	4650
10	4.859	6.6581	9.8576	13.995	10	6375.6	5971.5	5286	5851.1

	Annual Micro-grid Cost Savings ($)					Net Present Value ($)			
Path	year 1	year 2	year 3	year 4	Path	year 1	year 2	year 3	year 4
1	1.53E+06	1.85E+06	2.35E+06	2.62E+06	1	1.43E+07	1.43E+07	1.47E+07	3.59E+05
2	1.91E+06	2.78E+06	5.37E+06	1.14E+07	2	1.39E+07	1.75E+07	2.31E+07	1.61E+08
3	1.40E+06	1.70E+06	2.27E+06	2.72E+06	3	1.33E+07	1.67E+07	2.07E+07	1.05E+07
4	1.19E+06	1.50E+06	1.84E+06	2.42E+06	4	1.40E+07	1.66E+07	1.79E+07	3.91E+05
5	1.63E+06	2.18E+06	3.31E+06	3.58E+06	5	1.40E+07	1.58E+07	1.91E+07	2.00E+07
6	1.45E+06	1.52E+06	1.76E+06	2.24E+06	6	1.52E+07	1.76E+07	1.92E+07	0
7	1.38E+06	1.94E+06	2.83E+06	3.87E+06	7	1.34E+07	1.51E+07	1.89E+07	2.92E+07
8	1.36E+06	1.57E+06	1.90E+06	2.88E+06	8	1.46E+07	1.79E+07	2.02E+07	1.29E+07
9	1.19E+06	1.48E+06	2.09E+06	3.45E+06	9	1.37E+07	1.69E+07	1.97E+07	2.07E+07
10	1.45E+06	1.98E+06	2.93E+06	4.16E+06	10	1.25E+07	1.54E+07	1.92E+07	2.94E+07

	Conditional Expected Continuation					Decision			
Path	year 1	year 2	year 3	year 4	Path	year 1	year 2	year 3	year 4
1	2.70E+07	2.08E+07	8.87E+06	0	1	0	0	1	0
2	6.59E+07	5.76E+07	1.25E+08	0	2	0	0	0	1
3	1.94E+07	2.03E+07	1.62E+07	0	3	0	0	1	0
4	1.99E+07	1.62E+07	4.26E+06	0	4	0	1	0	0
5	3.36E+07	3.12E+07	3.96E+07	0	5	0	0	0	1
6	2.52E+07	1.73E+07	3.15E+06	0	6	0	1	0	0
7	1.91E+07	2.39E+07	2.73E+07	0	7	0	0	0	1
8	2.10E+07	1.85E+07	6.58E+06	0	8	0	0	1	0
9	1.89E+07	1.60E+07	1.09E+07	0	9	0	1	0	0
10	2.07E+07	2.53E+07	3.05E+07	0	10	0	0	0	1

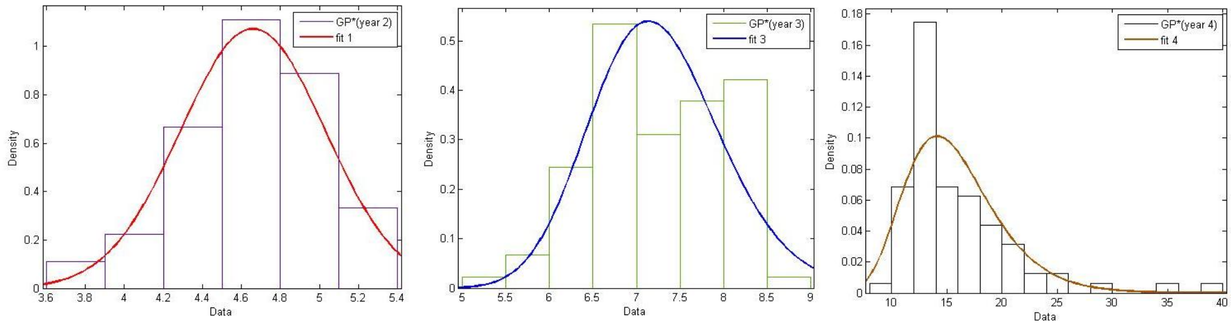

Figure 28 Conditional distribution of optimal natural gas price threshold given exercise date on years 2, 3 and 4

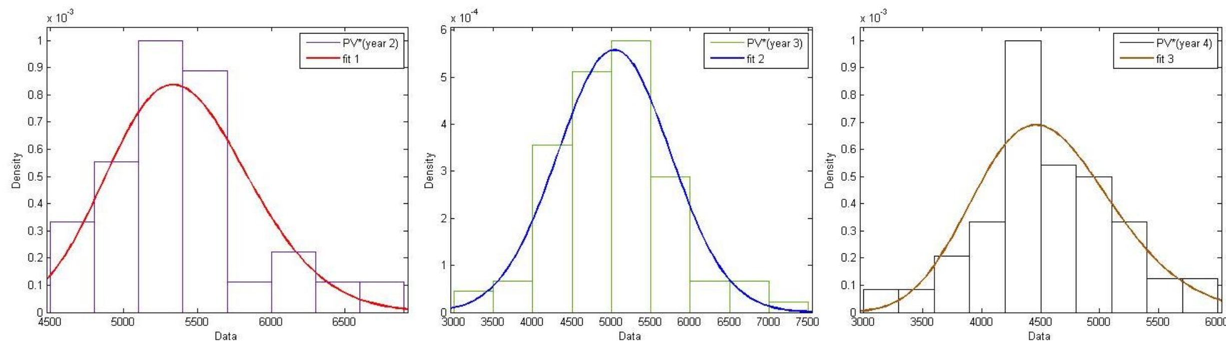

Figure 29 Conditional distribution of optimal PV investment cost threshold given exercise date on years 2, 3 and 4

Analytical solution suggests that the optimal thresholds lie on the following line:

$$\frac{C^*}{I^*} = \frac{\beta_1 \delta_C}{(\beta_1 - 1)\beta}$$

By substituting, for example, $E[C^*] = 10.24$ obtained from simulation solution for C^* into the above equation, optimal threshold for $E[I]^* = 4629\ \$/kW$. The result from simulation is 4905$/kW, so the error is less than 6%.

3.4.1.1 Impact of volatility

Both simulation (see Table 14) and analytical results (see Figure 26) show that as volatility increases, waiting for more information about uncertain future would add value to the investment and leads to lower triggering thresholds.

Table 14 Impact of volatility on delaying the investment (simulation results)

σ_C	Frequency of stopping time*	$E[C^*]$	$E[I^*]$
0.06	(0,30,90,80)	10.24	4905
0.07	(0,17,88,95)	10.267	4835.75
0.08	(0,30,85,85)	11.031	4870.51
0.09	(0,21,100,79)	11.06	4913
0.1	(5,14,113,68)	11.27	4953

*Indicates the frequency of investment exercise in each year

3.4.1.2 Impact of correlation

So far, we have assumed the natural gas prices and PV investment cost are independent from each other. We now examine cases where this assumption is relaxed. Figure 30 shows the threshold lines for different levels of correlation factor between gas price and PV investemtn cost. Recall that this is the case where electricity and gas price are dependent and as gas price increases, electricity price increases as well resulting in a higher value of micro-grid (or on-site generation). With positive correlation between gas price and PV capital cost, investment makes sense in even more expensive PV capital cost given the same gas price. The same argument holds and in the case of negative correlation, investment should be delayed until lower capital cost for PV is reached. Simulation cannot capture this impact.

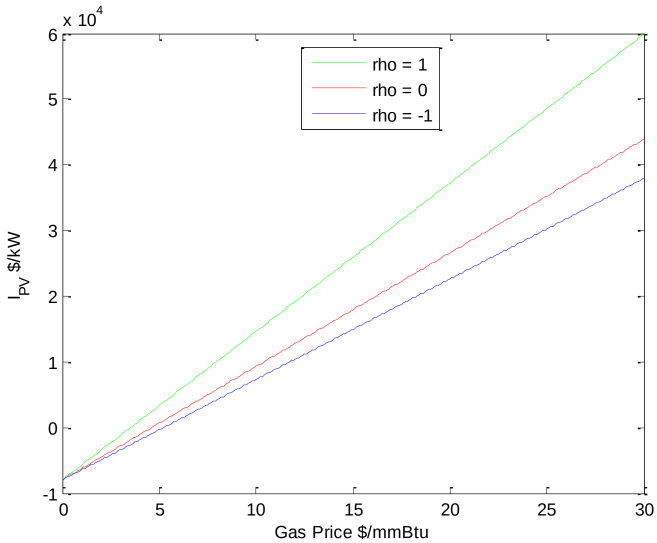

Figure 30 Impact of correlation in delaying investment

3.4.2 Mean Reverting Process for Natural Gas Price

We use Ornstein-Uhlenbeck Brownian motion with mean reverting drift to model natural gas prices. Ornstein-Uhlenbeck model is a special case of a Hull-White-Vasicek model [41] with constant volatility. The discrete-time equation of this model can be written as:

$$\Delta C_t = \propto_{N.G.} (\mu_{N.G.} - C_t)\Delta t + \sigma_{N.G.} dz_t \ , \qquad dz_t \sim N\left(0, \sqrt{\Delta t}\right)$$

where $\propto_{N.G.}$ = mean reversion rate, $\mu_{N.G.}$ = mean level and $\sigma_{N.G.}$ = volatility.

The equation can be rewritten as

$$\frac{\Delta C_t}{\Delta t} = -\propto_{N.G.} C_t + \propto_{N.G.} \mu_{N.G.} + \frac{\sigma_{N.G.}}{\Delta t} dz_t \qquad (57)$$

This model is fitted to the historical data set, which contains spot prices of natural gas at Henry Hub from 2000 to 2008 [42]. A linear regression is fitted between the natural logarithm of price and its first difference scaled by the time interval parameter. The mean reversion parameters and instantaneous volatility are then calculated and used to simulate stochastic paths. The HWV

(Hull-White-Vasicek) constructor in MATLAB enables us to setup a stochastic differential equation (SDE) by using the above estimated parameters ($\propto_{N.G.}, \mu_{N.G.}, \sigma_{N.G.}$). Figure 31 shows one sample path containing both historical and simulated prices.

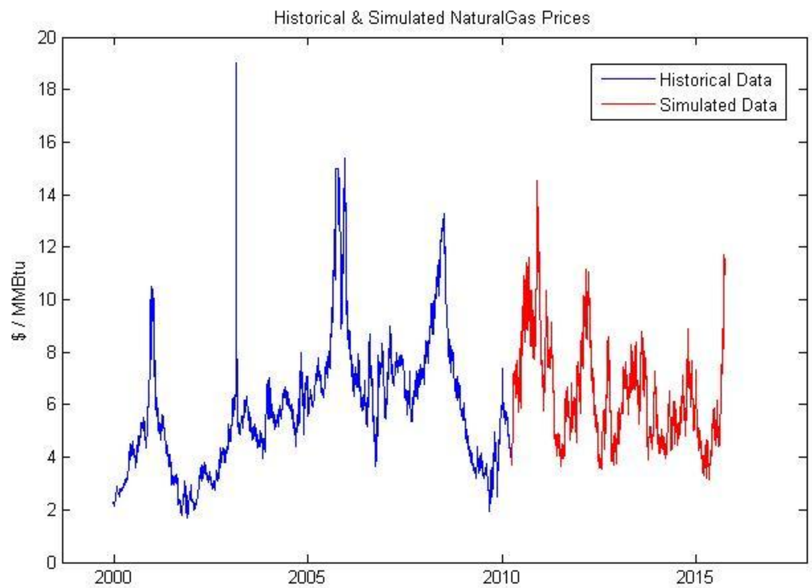

Figure 31: Historical natural gas price and fitted Orenstein-Uhlenbeck process

Since there is no sufficient historical data to estimate a stochastic process for PV investment cost over time, we consider a decreasing trend and assign a binomial probability mass function to the annual rate of decrease:

$$I_{PV}(t) = \propto_{PV} I_{PV}(t-1)$$

$$\propto_{PV} = \begin{cases} D_1 & with\ \ Probablity\ P \\ D_2 & with\ \ Probablity\ 1-P \end{cases} \quad (58)$$

The parameters are simulated numbers and no historical data is used to calibrate them.

Table 15 lists the on-site generation configuration, heat rates and the ratio of peak electricity price and gas-fired production cost for an illustrative example:

Table 15 Input parameters

Capacity (MW)	
Gas Turbine	7.975
Photovoltaic	3.2

Peak electricity price / Gas-fired production cost	1.25
Average grid heat rate (mmBtu/kWh)	0.03
Gas-fired generator heat rate (mmBtu/kWh)	0.03

Hourly electricity price, $ElecPr(h)$, is a function of natural gas price and assumed to be:

$$ElecPr(h) = C_{N.G.} \times \frac{Peak\ Elec\ Price}{GF\ Prod\ Cost} \times Profile_{ElecPr}(h) \quad (59)$$

where, $Profile_{ElecPr}(h)$ is the grid electricity price as a percentage of daily electricity peak price. We consider similar days over a year which is an abstraction of reality and it only simplifies our computations. The model is easily expandable to more elaborate electricity demand and price over the course of a year.

To calculate the value of active micro-grid, we consider no production cost for PV, thus, PV is the primary generation source for our micro-grid. Any remaining electricity demand is then optimally planned according to grid electricity price, gas-fired production cost, and GF generation capacity. For hourly electricity price, electricity demand and PV production profiles we use the same data as before shown in Figures 22-24. Table 16 lists the stochastic parameters of Orenstein-Uhlenbeck process for natural gas price and binomial process for PV cost.

Table 16 Stochastic parameters for OU and binomial processes

$\propto_{N.G.}$	1.7696
$\mu_{N.G.}$	1.7043
$\sigma_{N.G.}$	0.74451

D_1	0.8
D_2	0.6
$P(D_1)$	2/3

At each exercise date, $t-1$, regression on state variables of that time is used to estimate the conditional expectation of continuation until time t (Eqn. 55).

Table 17 illustrates the results for 10 sample paths. Realized natural gas price and PV investment costs are obtained by Monte Carlo simulation. Based on these realized values and optimal operation of micro-grid, annual cost savings from the active micro-grid is calculated. For each time period the net present value of micro-grid, given that the investment is to be triggered at that time, is calculated by discounting the perpetual savings from the micro-grid and netting out the investment cost. As explained above, regression is used to find the conditional expectation of continuation as a function of state variables at each time period. By comparing the immediate payoff from the investment and its conditional expected continuation, an optimal decision is reached as shown in decision tables below. Finally, the distribution of optimal threshold, $f(X^*|\tau = t)$, is obtained and its expectation is calculated using Eqn. 56.

Based on the decision matrix, the probability of investment exercise at each year over 200 scenarios is obtained and consequently the expected thresholds are calculated as:

$$E[C^*] = 7.1\ \$/mmBtu$$

$$E[I^*] = 1600\ \$/kW$$

Table 17 Sample paths simulated in Monte Carlo

	Natural Gas Price ($/mmBtu)					PV Investment cost ($/kW)			
Path	**year 1**	**year 2**	**year 3**	**year 4**	**Path**	**year 1**	**year 2**	**year 3**	**year 4**
1	6.4986	8.793	6.377	3.323	**1**	3768	2260.8	1356.5	1085.2
2	3.3474	4.1667	5.8587	7.371	**2**	3768	2260.8	1808.6	1446.9
3	5.769	8.2798	6.6111	7.1061	**3**	3768	3014.4	2411.5	1446.9
4	4.4296	5.5476	6.2118	4.0478	**4**	3768	2260.8	1356.5	813.89
5	4.769	6.9286	7.2688	4.7887	**5**	3768	2260.8	1356.5	1085.2
6	6.342	6.4704	4.6081	8.6327	**6**	5024	4019.2	3215.4	1929.2
7	4.6465	6.7041	8.0542	12.835	**7**	5024	4019.2	3215.4	1929.2
8	6.4694	8.0372	6.1388	7.7298	**8**	5024	4019.2	3215.4	1929.2
9	8.4038	6.3968	8.787	5.1456	**9**	3768	2260.8	1356.5	1085.2
10	6.1303	5.4997	9.8925	6.3315	**10**	5024	3014.4	1808.6	1446.9

	Annual Micro-grid Cost Savings ($)					Net Present Value ($)			
Path	**year 1**	**year 2**	**year 3**	**year 4**	**Path**	**year 1**	**year 2**	**year 3**	**year 4**
1	1.93E+06	2.62E+06	1.90E+06	9.89E+05	**1**	0	0	1.29E+06	0
2	9.96E+05	1.24E+06	1.74E+06	2.19E+06	**2**	0	0	0	8.60E+06
3	1.72E+06	2.46E+06	1.97E+06	2.11E+06	**3**	0	0	0	7.21E+06
4	1.32E+06	1.65E+06	1.85E+06	1.20E+06	**4**	0	0	1.25E+06	0
5	1.42E+06	2.06E+06	2.16E+06	1.42E+06	**5**	0	0	1.56E+06	0
6	1.89E+06	1.92E+06	1.37E+06	2.57E+06	**6**	0	0	0	1.37E+07
7	1.38E+06	1.99E+06	2.40E+06	3.82E+06	**7**	0	0	0	3.58E+07
8	1.92E+06	2.39E+06	1.83E+06	2.30E+06	**8**	0	0	0	8.95E+06
9	2.50E+06	1.90E+06	2.61E+06	1.53E+06	**9**	0	0	2.01E+06	0
10	1.82E+06	1.64E+06	2.94E+06	1.88E+06	**10**	0	0	8.98E+05	3.14E+06

	Conditional Expected Continuation					Decision			
Path	**year 1**	**year 2**	**year 3**	**year 4**	**Path**	**year 1**	**year 2**	**year 3**	**year 4**
1	0	0	4.58E+05	0	**1**	0	0	1	0
2	0	0	0	0	**2**	0	0	0	1
3	0	0	0	0	**3**	0	0	0	1
4	0	0	7.07E+05	0	**4**	0	0	1	0
5	0	0	-3.74E+05	0	**5**	0	0	1	0
6	0	0	0	0	**6**	0	0	0	1
7	0	0	0	0	**7**	0	0	0	1
8	0	0	0	0	**8**	0	0	0	1
9	0	0	1.82E+05	0	**9**	0	0	1	0
10	0	0	8.97E+05	0	**10**	0	0	1	0

3.5 Conclusion

The work presented in this chapter aims to shed light at one aspect of uncertainty in investment in micro-grids in a controlled set-up. The concept is adopted from option value for financial assets and it is applied to the investment in real assets. Investment in a micro-grid can be seen as an American-style call option and contingent claim analysis is used to obtain the value of option to invest rather than immediately undertaking the investment in the face of uncertainty. Comparing the results with stochastic net present value shows that considering the option to delay the investment leads to postponing the investment to a lower thresholds of gas price and PV investment cost. The delay becomes more significant as the volatility in the investment increases.

While the mathematics behind the contingent claim analysis is pretty solid, the analysis becomes intractable as the number of decision variables (i.e., investment timing of several resource) and the number of stochastic variables increase. Furthermore, stochastic processes other than GBM make the resulting stochastic differential equations computationally intractable. Therefore, a simulation based approach along with regression is used to address more general stochastic processes such as mean reverting and discrete binomial processes. It is shown that the results from simulation agrees with analytical approach under common assumptions.

4 Chapter 4 - Micro-grid Portfolio Optimization Under Uncertainty

Abstract

In this chapter we propose an integrated two-step approach to micro-grid power generation portfolio optimization under uncertainty. The portfolio includes PVs, Wind turbine, gas-fired generation, storage and purchase from the grid. The model uniquely integrates short-term uncertainties rising from micro-grid operation, and the long-term uncertainties due to future natural gas prices, investment in renewable assets, and financing costs. This work extends the current literature in two major ways: (i) It takes a holistic approach to investment by including different types of distributed generations within portfolio, (ii) It directly includes short-term planning and operational risks and long-term investment and pricing risks and integrates them into a single two-step optimization model. Finally, the solution approach uniquely combines a general binomial lattice with mixed integer quadratic model for budgeting and a regression model that estimates cost of operation and planning micro-grid with its current resources and load. The practical benefits of this work are enormous; the methodology can be easily implemented in commercial software and used by investors and planners in distributed generation industry.

4.1 Introduction

We are seeking solutions to optimal investment on a micro-grid power generation portfolio under uncertainty. A micro-grid portfolio can include PVs, wind turbine, gas-fired generation, storage and purchase from the grid. We intend to uniquely integrate into a single model the short-term uncertainties rising from micro-grid operation, and the long-term uncertainties due to future natural gas prices, investment in renewable assets, and financing costs. This work extends results from chapters two and three in a number of ways: (i) Larger portfolio of power generation assets with options to purchase from grid or sell to the grid; (ii) Optimal selection of portfolio over the course of planning horizon; and (iii) Optimal incremental investment in each resource over the course of planning horizon. To our knowledge, this work extends the current literature as it solves for optimal investment decisions while considering a portfolio of electricity generation and storage assets and also capturing short-term operational and long-term investment uncertainties.

We are motivated by the fact that a proper mix of power generation resources and timely investment on these resources is important design and operational planning decisions for micro-grids. These decisions can significantly impact micro-grid long-term and short-term objectives, namely saving in energy costs, reliable and secure energy supply, reducing risks for grid on blackouts and brownouts, and the use of renewables in generation portfolio. Furthermore, higher levels of exposure to the grid and market volatility can be avoided if the portfolio is optimized in response to its short-term load and market conditions.

The value of the micro-grid portfolio depends on the return on the investment and its growth on operational savings. For financial assets portfolio, the investment payoff depends on assets' prices which embed in them often sufficient aggregate information on operation and financial health of their respective company and/or industry. Historical data on prices and insight into company and/or industry futures usually drive investment in financial assets. For the micro-grid, the investment pay off is directly linked to the operation of the physical assets and return on investment is directly linked to how these operations are optimized in short-term. As indicated in Chapter 2, the savings from a micro-grid could be significantly under- or over-estimated if the underlying risks were not taken into account. The long-term value of the micro-grid will also depend on when (in terms of market conditions) investments were made and also on the amount and investment financing costs. Different parameters such as finance charge rate, finance term, and relative relationship between finance rate and discount factor would result in different optimal investment decisions. Hence, the model proposed in this chapter integrates short-term and long-term risks into a single decision-making loop. The loop works as follows: (i) An optimization model of a daily micro-grid operation runs and calculates a functional form of to-be-designed micro-grid, and (ii) The functional form is fed into a stochastic long-term investment model which decides when to invest on micro-grid components and expansions. The operation optimization model is an extension of what was offered in Chapter 2, but focuses only on risk-neutral approach. The investment model is a stochastic mixed integer programming (SMIP). A Monte Carlo simulation approach is taken where several sample path realizations over the course of planning horizon are generated, and a deterministic model for investment optimization for each sample path is solved. At the end, probabilistic characteristics of investment decisions along with optimal cash flow are obtained considering all sample paths.

4.2 Literature Review

The problem of optimal investment in real assets is the focus of engineering economics. Many concepts from investment in financial assets are adopted and modified to address the investment decisions in real assets. The methodologies used are around maximizing net present value or an alternative criterion representing the financial cost and benefits of the investment. A micro-grid is a diverse portfolio of different energy generation resources, energy storage, and demand response and energy efficiency technologies. Cost and benefit streams and therefore investment in such a system is tightly coupled with the operation of its resources. Literature lacks sufficient work addressing optimal investment uncertainty in an enhanced micro-grid portfolio considering the optimal operation.

Hybrid Optimization Model for Electric Renewable (HOMER) [43] is a product of the National Renewable Energy Laboratory, which evaluates design options for both off-grid and grid-connected power systems for remote, stand-alone and distributed generation applications. HOMER's optimization and sensitivity analysis algorithms allow users to evaluate the economic and technical feasibility of a large number of technology options and to account for variation in technology costs and energy resource availability. This tool does not include an operational optimization and different design configurations can be evaluated by comparing their operating cost/benefits and their investment costs. El Khattam et al [44], study the capacity investment in distributed generation (DG) from a distribution company's perspective. The decision is therefore to optimize the sizing and siting for DG capacity. The objective function is the company's investment and operating costs as well as payment toward loss compensation. Their proposed heuristic method helps to avoid the use of binary variables and makes the computation more convenient.

There are some elements of investment that are stochastic and ignoring this uncertainty can lead to poor results in investment decisions. Bruno et al [45] consider the problem of optimal investment portfolio for a company that purchases, sells and distributes gas and owns a network of gas pipelines. They propose a two-stage stochastic programming to solve the problem and consider the demand to be stochastic. They also use a coherent measure of risk, i.e., conditional value at risk to control the variability of the decisions.

Real option is another approach, which is popular to address uncertainty in investment and including value of delay in investment into the problem (this was elaborated more in Chapter 3). Asano et al [46], discuss investment strategies in a micro-grid consisting of cogeneration system and renewable resources under uncertainty in natural gas price. They examine the sensitivity of optimal investment decision to the level of uncertainty in gas price. They show that as volatility of gas price increases, investment in renewable resources (i.e., PV in their case) becomes more attractive, and reduces the investment risk. Although real option is very powerful in handling uncertainties, its applications are limited for small-scale problems due to complexities in the solution methodology, unless numerical results are sought.

4.3 Problem Statement and Preliminaries

Consider a micro-grid generation resource portfolio including gas-fired generation, photo voltaic, wind turbine, electricity storage, and purchase from the grid. Our investment problem particularly aims at what capacity of each resource, if any, at each time period should be purchased within planning horizon of $\boldsymbol{\tau}$. The objective is to maximize the cash flow due to investment in the micro-grid at the end of the horizon, which includes cash flows due to investments and operational savings prior to the end of horizon, end-of-horizon and beyond-horizon projected investment cash flows. This is a stochastic asset portfolio optimization problem under short-term and long-term uncertainties. The investment decisions are subject to a set of constraints:

- A functional form that describes the short-term benefit growth of the micro-grid under short-term (operational) uncertainties, e.g., stochasticity of electricity demand, electricity spot price, solar intensity and wind speed
- Long-term uncertainty due to investment stochasticity including investment cost (e.g., PV and storage) and natural gas price. Available funds for investment dynamically changes over time. We assume that electricity price at peak is driven by natural gas price.
- Constraints on micro-grid resources (i.e., on-site generation and energy storage).

One approach to account for operation is to formulate the two problems, i.e., investment and operation as one single optimization problem. Alternatively, one can decompose the two problems by first determining a functional form of micro-grid operation cost and then feeding this function into the investment optimization problem. We take the second approach with a cost function defined at time *t* by

$$Cost_{MG,t} = f(I_{GF,t}, I_{PV,t}, I_{WT,t}, I_{WT,t}) \qquad (60)$$

where *f()* accounts for micro-grid uncertainty in both day-ahead planning and same day operation, and its argument vector defines the microgrid characteristics. The two models work in a closed loop to compute the optimal portfolio over a planning horizon. Results from Chapter 2 can be used to compute *f()* in Eqn. 60, but we will use a simpler but extended variation of it here. The investment problem is formulated as a capital budgeting model using a formal optimization framework. The value of micro-grid is defined as its electricity cost savings and earned revenue compared to no-micro-grid case. We assume yearly investment decisions in terms of asset purchase. We also allow for borrowing funds for purchase and also alternative investment of available cash. The objective is to maximize the accumulated cash at the end of the investment horizon accounting for beyond-the-horizon cash flows as well. Based on the dynamics of investment stochasticity, we run the investment model for possible random scenarios and examine the probability distribution of investment decisions.

<u>Nomenclature</u>

$Capinc_{i,t};\ i = GF, PV, WT, ST; t = 1, \dots, \tau$	Incremental capacity purchased(MW)
$OwnCapinc_t;\ \ t = 1, \dots, \tau$	Incremental PV capacity purchased and installed on own land(MW)
$ExPVCapinc_t;\ \ t = 1, \dots, \tau$	Incremental PV capacity purchased and installed on extra land (MW)
$Capex_{i,t}; i = GF, PV, WT, ST; t = 1, \dots, \tau$	Unit Capacity Cost (*$/MW*)
$CBS_t;\ t = 1, \dots, \tau$	Cash spent to procure resource (*$*)
$B_t; t = 1, \dots, \tau$	Borrowed fund (*$*)
$CI_t;\ t = 1, \dots, \tau$	Cash invested in alternative (*$*)
$I_i; i = GF, PV, WT, ST$	Resource index (%)
$MaxI_i; i = GF, WT, ST$	Maximum resource index (%)
$PVReqLand$	Land required for PV (*acres/MW*)
$LandPr$	Land price (*$/acres*)
$LandDens$	*Portion of land covered by PV (%)*
$AvLand$	Own available land for PV (*acres*)
PV	Present value ($)
NPV	Net present value ($)

$CF_t;\ t = 1, \dots, \tau$	Cash flow ($)
$CBS_t;\ t = 1, \dots, \tau$	Cash spent to procure resource ($)
$B_t;\ t = 1, \dots, \tau$	Borrowed fund ($)
$CI_t;\ t = 1, \dots, \tau$	Cash invested in alternative ($)
$CB_t;\ t = 1, \dots, \tau$	Cash balance ($)
$MGSaving_t;\ t = 1, \dots, \tau$	Micro-grid savings ($)
ROI	Investment rate of return in an alternative investment (%)
FC	Finance charge (%)
FT	Finance term (years)
$BLimit$	Maximum Borrowing Limit ($)

4.4 Solution Approach

Two models will be formulated and solved here: (i) Micro-grid cost model (e.g., Eqn. 60); (ii) Capital budgeting, we first need to characterize a micro-grid according to its assets.

4.4.1 Micro-grid Characterization

Each asset is characterized by a one or more parameters: Gas-fired generator is specified with its capacity of electricity generation $GFCap$(MW) and its heat rate ϵ_{GF} (mmBtu/kW). We assume a fixed heat rate and an index I_{GF}, which represents its unit capacity:

$$I_{GF} = \frac{GFCap}{E[D]}$$

PV electricity production at each hour gpv_h is assumed to be:

$$gpv_h = C_{PV} \times SI_h$$

where C_{PV} is PV constant and SI_h is solar intensity at each hour. We define a PV index which corresponds to PV capacity:

$$I_{PV} = \frac{Average\ Daily\ PV\ Electricity\ Production}{Average\ Daily\ Demand} = \frac{PVCap}{E[D]} = \frac{C_{PV} \times E[SI]}{E[D]}$$

where $E[D]$ and $E[SI]$ are daily expected values for electricity demand and solar intensity.

For wind turbine electricity generation, we use expression:

$$gwt_h = C_{WT} \times \eta_{WT} \times WS_h^2$$

where C_{WT} and η_{WT} are wind turbine constant and efficiency respectively. A representative index for WT capacity is defined by:

$$I_{WT} = \frac{Average\ Daily\ WT\ Electricity\ Production}{Average\ Daily\ Demand} = \frac{WTCap}{E[D]} = \frac{C_{WT} \times \eta_{WT} \times E[WS^2]}{E[D]}$$

To avoid higher orders in investment optimization, we assume fixed efficiency for wind turbine. Finally, electricity storage is characterized by two parameters, namely, its charging/discharging rate $STCap$ (MW) (assumed to be the same for charge and discharge) and its charging duration $STDur$ (hr). We assume that storage assets all have the same charging duration. The storage representative index is therefore:

$$I_{ST} = \frac{STCap}{E[D]}$$

Note that investment decision must be made per each period between $t = 1, \ldots, t$. Therefore, capacity variables, namely, $GFCap_t$, $PVCap_t$, $WTCap_t$, and $STCap_t$ are all function of *t*.

4.4.2 Calculation of $Cost_{MG,t}$

Here we will derive Eq. (1) assuming that the micro-grid investment portfolio includes gas-fired generation (GF), Wind Turbines (WT) and Photovoltaic solar cells (PV) for its internal renewable resources, and access to external power grid. We also allow for electricity (battery) storage. Moreover, the micro-grid is able to sell back to the grid if it makes economic sense. It is assumed that micro-grid is subject to several sources of variations: (i) variation in weather forecast, which leads to variation in the availability of renewable resources, (ii) variation in demand, and (iii) variation in spot prices.

Peak electricity price on each day is assumed to be driven by natural gas price, i.e.,

$$Peak\ Elec\ Price = Ratio_{EG} \times C_{N.G.} \times \epsilon_{Grid}$$

where $Ratio_{EG} > 1$ accounts for transmission and distribution cost at grid level and ϵ_{Grid} is grid average heat rate for generation of electricity from natural gas. Assuming a daily profile for day-ahead electricity price ($Profile_{ElecPr,h}$) as a percentage of peak price, hourly electricity price over the course of a day is obtained by:

$$C_{preset,h} = C_{N.G.} \times \frac{Peak\ Elec\ Price}{GF\ Prod\ Cost} \times Profile_{ElecPr,h}$$

Next-day spot prices, electricity demand, solar radiation and wind speed are assumed to have distributions with mean and variance estimated from historical data. These random variables are correlated in their mean values but not in their variances. End-user daily electricity demand, wind speed and solar intensity profiles are inputs to the model. The annual net cost of micro-grid operation deducts any revenue from the electricity sell-back to the macro grid, computed at spot prices. Moreover, there are no operation cost of PV, WT and storage. Any planned purchase made by the microgrid is calculated at a day-ahead price, whereas spot purchases are charged at spot prices. We allow for later modification of purchase commitment by paying a penalty as a percentage of pre-set prices.

Similar to Chapter 2, we formulate the microgrid planning and operation optimization problem as a Two-stage stochastic programming problem [47]. A synopsis of the decisions made in the two stages follows:

- In the first stage, day-ahead plans are made to commit to the grid for a certain amount of purchase. The decision is made taking into account all sources of uncertainty.
- The second stage includes observation of realized operational scenarios and taking recourse decisions for each scenario. The recourse decisions are made in terms of:
    - d. How much of the prior commitment should really be purchased ($gridreturn_h^s$);
    - e. How much spot electricity should be purchased ($gridspot_t^s$) and
    - f. How much electricity from gas-fired unit should be generated (gfc_h^s).
    - g. How much electricity should be charged to storage (cst_t^s).
    - h. How much electricity should be dis-charged from storage (dst_h^s).
    - i. How much electricity should be sold to the grid (sb_h^s).

These recourse decisions are corrective actions to the first stage decisions for each hour depending on which random scenario is realized. In chapter two we developed three regimes to solve for the operation of micro-grid. In this chapter we use risk-neutral two-stage stochastic programming framework to solve the micro-grid planning/operation. It should be noted that while the operation model utilizes the same methodology as of chapter two, it also enhances that operational optimization model to include electricity storage and sell back to the grid.

Objective Function

The objective function of this regime is the sum of the cost of first stage decision and expected net cost (i.e., cost minus revenue) of the second stage. The first stage cost is the cost of electricity that the micro-grid commits to purchase from the grid, and is given by

$$\sum_{\mathrm{h}=1}^{24} C_{preset,h} \times gridcom_h$$

The second stage objective function is:

$$\sum_{\mathrm{s}=1}^{\mathrm{N}} \mathrm{P_s}\left(\sum_{\mathrm{h}=1}^{24} {C_{spot}}_h^s \times gridspot_h^s + (C_{penalty,h} - C_{preset,h}) \times gridreturn_h^s + C_{N.G.} \times \epsilon_{GF} \times gfc_h^s - {C_{spot}}_h^s \times sb_h^s\right)$$

where $\mathrm{P_s}$ is the probability of each scenario which is assumed to be 1/N over a discrete sampling path. The overall objective function is then given by:

$$\sum_{h=1}^{24} C_{preset,h} \times gridcom_h + \sum_{s=1}^{N} P_s \times \left(\sum_{h=1}^{24}\left[{C_{spot}}_h^s \times gridspot_h^s + (C_{penalty,h} - C_{preset,h}) \times gridreturn_h^s + C_{N.G.} \times \epsilon_{GF} \times gfc_h^s - {C_{spot}}_h^s \times sb_h^s\right]\right)$$

Constraints

The prior commitment at each time should be more than a certain value and cannot exceed a certain limit $gridpurmax$:

$$gridmin \leq gridcom_h \leq gridpurmax \quad h = 1, \ldots, 24$$

Note that in this framework, the constraints should be repeated for every scenario once a recourse decision is involved. Scenario-based constraints for a specific scenario are relevant only for the recourse decisions in that scenario and other scenario-based constraints become irrelevant for these decisions. For example, the equivalent set of constraints for spot purchase (which is a recourse decision and scenario-dependent) would be:

$$gridmin \leq gridspot_h^s \leq ggridmax \quad h = 1, \dots, 24 \; and \; s = 1, \dots, N$$

Amount sold back should also be within the allowable limit $gridsbmax$:

$$sb_h^s \leq gridsbmax$$

Note that s indicates scenario s. Other constraints will be the same as those explained in certainty equivalent regime but repeated for each scenario:

The amount of electricity not purchased from the grid cannot exceed the prior commitment:

$$gridreturn_h^s \leq gridcom_h \quad h = 1, \dots, 24 \; and \; s = 1, \dots, N$$

Total purchase from the grid at each hour should not exceed the grid maximum purchase limit:

$$gridcom_h - gridreturn_h^s + gridspot_h^s \leq gridpurmax \quad h = 1, \dots, 24 \; and \; s = 1, \dots, N$$

The electricity generation from the GF is constrained to the minimum operation level and maximum capacity of GF:

$$gfcmin \leq gfc_h^s \leq gfcmax \quad h = 1, \dots, 24 \; and \; s = 1, \dots, N$$

Since there is no cost associated to the operation of PV and WT, electricity production from these resources is dictated by the availability of renewable resources (i.e., solar intensity and wind speed).

$$gwt_h^s = C_{WT} \times \eta_{WT} \times {WS_h^s}^2$$

And, generation from photo voltaic cell is:

$$gpv_h^s = C_{WT} \times SI_h^s$$

For energy balance at each time period and under each scenario "s" we must ensure that all sources of electricity, consisting of prior commitment, spot purchase, GF generation, wind turbine generation and photo voltaic generation, and battery discharge collectively satisfy the expected demand electricity, sell-back commitments and battery charging. That is,

$$gridcom_h - gridreturn_h^s + gridspot_h^s + gfc_h^s + gwt_h^s + gpv_h^s + dst_h^s$$
$$\geq Demand_h^s + sb_h^s + cst_h^s$$
$$h = 1, \ldots, 24 \;\; and \;\; s = 1, \ldots, N$$

We also need a set of constraints for the electricity storage device. At the end of each time period (i.e., hour), the available energy kept in storage is conserved by:

$$sl_h^s = sl_{h-1}^s + cst_h^s - dst_h^s \qquad h = 1, \ldots, 24 \; and \; s = 1, \ldots, N$$

where cst_h^s and dst_h^s are charging and discharging from storage during each hour. Charging and discharging to/from storage is constrained by maximum charge/discharge rate of the device$STCap$:

$$cst_h^s \leq STCap \qquad h = 1, \ldots, 24 \; and \; s = 1, \ldots, N$$

$$dst_h^s \leq STCap \qquad h = 1, \ldots, 24 \; and \; s = 1, , N$$

Moreover, charging is constrained by the remaining space left in the storage and discharging is constrained by the available energy in the storage from the previous hour:

$$cst_h^s \leq (STCap \times STDur - sl_{h-1}^s)/STDur \qquad h = 1, \ldots, 24 \; and \; s = 1, \ldots, N$$

$$dst_h^s \leq sl_{h-1}^s/STDur \qquad h = 1, \ldots, 24 \; and \; s = 1, \ldots, N$$

Finally, energy stored in the device cannot exceed the maximum energy limit:

$$sl_h^s \leq STCap \times STDur \qquad h = 1, \ldots, 24 \; and \; s = 1, \ldots, N$$

The last set of constraint indicates that selling back should be supplied from either on-site generation or discharge from storage at each hour:

$$sb_h^s \leq gfc_h^s + gwt_h^s + gpv_h^s + dst_h^s \qquad h = 1, \ldots, 24 \; and \; s = 1, \ldots, N$$

Above formulation can be extended to solve for more than one day. However, to avoid lengthy computations and to demonstrate the concept, we solve the model for three representative days over a year and extrapolate the annual cost, $Cost_{MG,t}$, based on the cost of each representative day and their weight factors:

$$Cost_{MG,t} = \gamma_{,Spring/Fall} \times Cost_{MG,Spring/Fall} + \gamma_{,Summer} \times Cost_{MG,Summer} + \gamma_{,Winter} \times Cost_{MG,Winter}$$

Solving the above optimization problem according to a design of experiment yields a functional form as shown below:

$$Cost_{MG,t} = \beta_{0,t} + \beta_{1,t} I_{GF,t} + \beta_{2,t} I_{PV,t} + \beta_{3,t} I_{WT,t} + \beta_{4,t} I_{ST,t} + \beta_{5,t} I_{GF,t} \times I_{PV,t} + \beta_{6,t} I_{GF,t} \times I_{WT,t} + \beta_{7,t} I_{GF,t} \times I_{ST,t} + \beta_{8,t} I_{PV,t} \times I_{WT,t} + \beta_{9,t} I_{PV,t} \times I_{ST,t} + \beta_{10,t} I_{WT,t} \times I_{ST,t} \qquad (61)$$

In the above equation $\beta_{0,t}$ can be interpreted as the cost of electricity supplied by the grid. The rest of the expression in Eqn. 61 refers to micro-grid's cost saving or revenue resulted from on-site resources.

4.4.3 Capital Budgeting Model

Economic analysis can be applied to different types of capital budgeting problems such as single investment project, mutually exclusive projects, incremental investment and machine replacement analysis [48]. The analysis is on the basis of cash flow reflecting the actual outflows and inflows of monetary values. It requires proper identification of costs and benefits resulting from the investment, including any marginal values introduced by the investment to the system. We include sunk cost incurred by a new investment and opportunity cost, which is the benefit forgone if the investment is undertaken. An opportunity cost is incurred if the asset or resource can be used in some alternative way with some positive return. We do not include taxation and depreciation of assets.

Net present value (NPV) analysis is one approach to evaluate whether a project is economically justified. To perform an NPV analysis the cash flow streams of the investment over a certain horizon should be calculated. In practice, actual cash flows take place throughout a given year. However, in cash flow analysis, it is assumed that each flow occurs at the end of each year. Figure 32 illustrates the cash flow diagram.

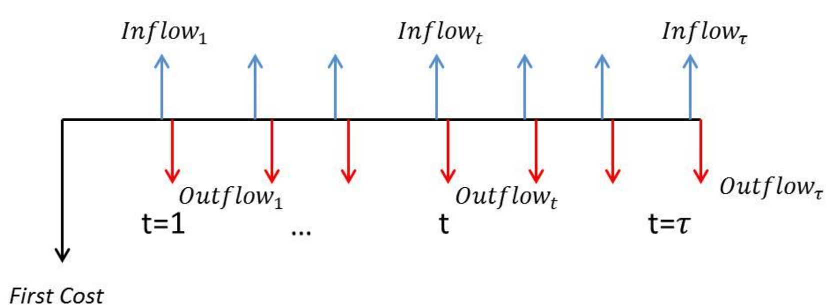

Figure 32 Cash flow diagram

Net cash flow in each period t is given by:

$$CF_t = Inflow_t - Outflow_t \qquad t = 1, \dots, \tau$$

Present value of the cash flow streams over the investment horizon is therefore:

$$PV = \sum_{t=1}^{\tau} \frac{CF_t}{(1 + DF)^t}$$

where DF is the discount factor by which the present value of the future cash flow is computed. The discount factor is calculated on the basis of project financing cost. The discount factor generally reflects the time value of money, inflation risk and business risk. In private sector, funding resources include both borrowed funds and equity capital. While the rate charged due to the borrowed fund is determined externally, the financing cost of equity funds can be considered as the return that average investor in the firm expects to receive. Exact computation of expected return for the equity capital and its rationale can be found in finance textbooks (see for example, Brealey et al [49]). Here we will compute DF as a weighted average of debt and equity financing.

The present value is netted out by subtracting first cost:

$$NPV = PV - First\ Cost = \sum_{t=1}^{\tau} \frac{CF_t}{(1+DF)^t} - First\ Cost$$

Once we deal with multiple investment options that can be undertaken over an investment horizon, DF calculation is not straightforward. Baumol and Quandt [50] suggest the use of discount rates between periods determined by the model itself. These rates are referred to as consistent discount rates. Other considerations such as outside investment would introduce criticism into NPV approach. An alternative approach is the horizon model in which present value is ignored and the focus is on the accumulated cash at the end of the investment horizon. Many of conceptual issues such as the choice of appropriate discount factor could be avoided using this approach.

In this work, we make use of horizon model based on Weingartner's work [51]. Therefore, cash flow at the end of horizon plus the value of beyond-horizon cash flows at the end of horizon is the investment criterion to be maximized within an optimization model. We look at incremental investment decisions to form a micro-grid over a specific horizon. The decision would be what capacity of each resource (i.e., GF, PV, WT and electricity storage), if any, should be purchased at each time period (i.e., one year). Within each period, the active micro-grid yields a payoff in the form of cost savings and possible revenue. There would be initial cash available in the first period and it is assumed that in the beginning of each period any available cash can be either used to purchase assets or to spend in other investment opportunities. We also assume that cash inflow resulting from micro-grid's revenue will be added to the available cash in each period.

The following assumptions are made:

- For on-site generation resources we consider only one decision variable for each. This is to avoid high orders in the optimization model. For example, WT efficiency and electricity storage duration are considered fixed. An asset purchased in a period will only be active at the beginning of the next period. The life of assets are assumed to be infinite and no asset depreciation is considered.
- Borrowing is available at a constant finance charge for a constant finance period. Funds borrowed in each period can only be invested in the same period. Outside investment is available at a fixed rate of return, and any cash invested outside in each period can be re-invested in the next period. We assume a fixed maximum borrowing limit in each period.
- There is a maximum limit on installed capacity of GF, WT and electricity storage. Certain land space is available for PV installation; additional capacity could be installed by paying for extra space needed.

The formal presentation of the model follows:

4.4.3.1 Investment Optimization Model

We now formulate a mixed integer quadratic programming model of investment model. Input variables are categorized in three groups: "On-site Resources", "Financial Parameters" and "Resource Investment Constraints". Tables 18-20 list all input variables needed for the model:

Table 58 On-site resource parameters

On-Site Resources	
WT efficiency	η_{WT}
Electricity Storage Duration	$STDur$

Table 19 Financial and cost input parameters

Financial& Cost Parameters	
Initial Cash Available	CB_0 ($)
Investment Rate of Return (in an alternative investment)	ROI (%)
Finance Charge	FC (%)
Finance Term	FT $(years)$
Maximum Borrowing Limit	$BLimit$($)
Unit Capacity Cost of GF	$Capex_{GF,t}$ ($\$/MW$)
Unit Capacity Cost of PV	$Capex_{PV,t}$ ($\$/MW$)
Unit Capacity Cost of WT	$Capex_{WT,t}$ ($\$/MW$)
Unit Capacity Cost of ST	$Capex_{ST,t}$ ($\$/MW$)
Price of Extra Land for PV	$LandPr$ ($\$/acres$)
Portion of Land Covered by PV	$LandDens$(%)

Table 20 Constraints input parameters

Resource Investment Limits	
Maximum GF Index	$MaxI_{GF}$
Maximum WT Index	$MaxI_{WT}$
Maximum Storage Index	$MaxI_{ST}$
Own Available Land for PV	$AvLand\ (acres)$
Land Required for PV	$PVReqLand\ (acres/MW)$

Decision variables

$Capinc_{i,t};\ t = 1, \dots, \tau,$ is the capacity purchased from each resource in each period (continuous variable); where i refers to resources GF, PV, WT and storage.

$OwnCapinc_t; t = 1, \dots, \tau,$ is the PV capacity purchased and installed in each period on own land at no cost.

$ExPVCapinc_t;\ t = 1, \dots, \tau,$ is the PV capacity purchased and installed in each period on additional land incurred an extra cost.

$CBS_t;\ t = 1, \dots, \tau,$ is the cash spent to purchase resource in each period; continuous variable.

$B_t; t = 1, \dots, \tau,$fund borrowed in each period; continuous variable.

$CI_t;\ t = 1, \dots, \tau,$ alternative investment from cash available in the beginning of each period; continuous variable:

Objective function

The objective is to maximize the end of horizon cash flow plus the horizon time value of any cash flows beyond the horizon. End-of-horizon cash flow is the net of cash inflow and outflow at τ:

$$CF_\tau = CIF_\tau - COF_\tau = MGSaving_\tau + ROC_\tau + B_\tau - FP_\tau - CBS_\tau$$

where $MGSaving_\tau$ is savings/revenue from micro-grid compared to no-micro-grid state (explained in earlier)

$$\begin{aligned} MGSaving_\tau = {} & \beta_{1,t} I_{GF,t} + \beta_{2,t} I_{PV,t} + \beta_{3,t} I_{WT,t} + \beta_{4,t} I_{ST,t} + \beta_{5,t} I_{GF,t} \times I_{PV,t} + \beta_{6,t} I_{GF,t} \times I_{WT,t} \\ & + \beta_{7,t} I_{GF,t} \times I_{ST,t} + \beta_{8,t} I_{PV,t} \times I_{WT,t} + \beta_{9,t} I_{PV,t} \times I_{ST,t} + \beta_{10,t} I_{WT,t} \times I_{ST,t} \end{aligned}$$

We ignore the higher orders of interaction between the factors to have a computationally tractable model in the context of optimization. At the same time, our numerical studies show that the second order terms sufficiently explain the interactions of resources.

ROC_τ is the return on cash invested in an alternative investment other than micro-grid in the previous period:

$$ROC_\tau = CI_{\tau-1} \times (1 + ROI)$$

and B_τ is the fund borrowed in the last period. $FP_\tau$is the total finance charges made on any earlier borrowed funds. Fund borrowed in period j would entitle the borrower to payment flows in coming periods. This payment is calculated according to normal annuity:

$$P_{j,t} = \frac{FC \times B_j}{1 - (1 + FC)^{FT}} \qquad t = j + 1, \dots, j + FT$$

Therefore, the total payment in each period would be:

$$FP_t = \sum_{j=1}^{t-1} P_{j,t} \qquad\qquad t = j+1, \dots, j+FT$$

Beyond-horizon cash flows are discounted to obtain their horizon time value, $\widehat{CF}_\tau$, and include perpetual savings from the micro-grid:

$$\widehat{MGSaving}_\tau = \frac{1}{1+DF} \sum_{s=0}^{\infty} \frac{MGSaving_{\tau+1}}{(1+DF)^s} = \frac{MGSaving_{\tau+1}}{DF}$$

The return on cash invested in the last period:

$$\widehat{ROC}_{\tau+1} = \frac{1}{(1+DF)} ROC_{\tau+1}$$

And finally the remainder of finance charges:

$$\widehat{FP}_\tau = \sum_{t=\tau+1}^{\tau+FT} \frac{FP_t}{(1+DF)^t}$$

The objective is therefore:

$$\max(CF_\tau + \widehat{CF}_\tau) = \max\{MGSaving_\tau + ROC_\tau + B_\tau - FP_\tau - CBS_\tau$$

$$+\widehat{MGSaving}_\tau + \widehat{ROC}_{\tau+1} - \widehat{FP}_\tau\}$$

Constraints - Cash flow in each period is calculated as:

$$CF_t = CIF_t - COF_t = MGSaving_t + ROC_t + B_t - FP_t - CBS_t \quad t = 1, \dots, \tau$$

ROC_t is the return in period t on cash invested outside in the previous period:

$$ROC_t = CI_{t-1} \times (1 + ROI)\ t = 1, \dots, \tau$$

There is a fixed maximum limit on the amount to be borrowed in each period:

$$B_t \leq Blimit$$

There is a constraint for cash invested outside based on the availability of cash:

$$CI_1 \leq CB_0 - CBS_1$$

$$CI_t \leq ROC_t + MGRevenue_t - CBS_t \; t = 2, \dots, \tau$$

$MGRevenue_t$ is the micro-grid revenue from selling back to the grid. In other words if micro-grid's cost is negative, then it is actually making money. We define a binary variable $BR_t$to determine whether micro-grid's cost is negative in each period:

$$BR_t = \begin{cases} 1 & Cost_{MG,t} < 0 \\ 0 & Cost_{MG,t} \geq 0 \end{cases} \qquad t = 1, \dots, \tau$$

Funds borrowed are restricted to be used to purchase on-site resources and cannot be invested on an alternative option. Therefore, total investment in micro-grid in each period is constrained by funds available through borrowing and amount spent from available cash:

$$PVTotCapex_t + \sum_{i=1}^{3} Capex_{i,t} \times Capinc_{i,t} = B_t + CBS_t \; t = 1, \dots, \tau \; and \; i = GF, WT, ST$$

where $PVTotCapex_t$ is total PV investment cost at each period. The reason it is treated separately from other resources is due to the fact that available land for PV installation at no cost imposes another constraint on PV. If land requirement for the PV capacity to be installed exceeds initial available no-cost land, extra space should be acquired at a specific price to install PV excessive capacity. To be able to formulate this constraint linearly, we decompose incremental investment in PV capacity into two separate decisions, namely, incremental capacity on own land ($OwnCapinc_t$) and incremental capacity in extra land ($ExPVCapinc_t$):

$$Capinc_{2,t} = OwnCapinc_t + ExPVCapinc_t \; t = 1, \dots, \tau$$

As mentioned earlier, it is assumed that resources purchased in each period will not be active until the next period, therefore, in each period, the operating capacity of each resource is:

$$Cap_{i,1} = Cap_{i,0} i = GF, PV, WT, ST$$

$$Cap_{i,t} = Cap_{i,t-1} + Capinc_{i,t-1} t = 2, \dots, \tau + 1 \; and i = GF, PV, WT, ST$$

There is also limit on the installed capacity in each period for GF, WT and storage:

$$Cap_{i,t} \leq MaxI_i t = 1, \dots, \tau \; and i = GF, WT, ST$$

To keep track of how much excessive PV capacity we are allowed to invest in before hitting the own land availability, we need the following constraint:

$$Capinc_{2,t} + OwnCapinc_t \leq AvLand \times LandDens \times PVReqLand \qquad t = 1, \dots, \tau$$

The investor should pay an additional cost on top of investment cost for PV. Therefore the total capital cost for PV should include the price of extra land:

$$PVTotCapex_t = Capex_{i,t} \times Capinc_{2,t} + ExPVCapinc_t \times \frac{PVReqLand}{LandDens} \times LandPr \quad t = 1, \dots, \tau$$

There are of course additional non-negativity constraints for variables that cannot take negative values:

$$Capinc_{i,t} \geq 0 \qquad t = 1, \dots, \tau$$

$$OwnCapinc_t \geq 0 \qquad t = 1, \dots, \tau$$

$$CBS_t \geq 0 \qquad t = 1, \dots, \tau$$

$$B_t \geq 0 \qquad t = 1, \dots, \tau$$

$$CI_t \geq 0 \qquad t = 1, \dots, \tau$$

And constraints for binary variables:

$$BR_t \in \{0,1\} \qquad\qquad t = 1, \dots, \tau$$

There are of course additional non-negativity constraints for variables that cannot take negative values. Objective function along with constraints explained above will form a mixed integer quadratic programming. Lingo optimization platform is used to solve the problem and obtain global optimal solution.

4.4.3.2 Solution approach using Stochastic Scenario Generation

To account for uncertainty, the above investment model is solved under different stochastic scenarios. Natural gas price, PV and storage costs are considered to be random variables with specific stochastic processes. Random paths rising from underlying processes are then constructed over the investment horizon with yearly time intervals that correspond to time periods in which investment decisions are made. Random paths represent the possible realization of uncertain variables. In this section, uncertainty dynamics of investment variables along with the scenario generation are explained.

4.4.3.2.1 Dynamics of Natural Gas Price; A Symmetric Lattice Binomial Approach

Volatile gas prices impacts the investment timing and GF capacity. Different models are studied in the literature to explain the long-term behavior of natural gas [see 52]. Extensive historical data is available to calibrate the parameters of each model. In this work, we assume a simple geometric Brownian motion for gas price:

$$dC = \alpha_{N.G.} C dt + \sigma_{N.G.} C dZ_{N.G.}$$

where C is natural gas price (\$/mmBtu), $\alpha_{N.G.}$ is the natural gas annual percentage growth rate and $\sigma_{N.G.}$ is the natural gas annual percentage volatility. Z_t is standard Brownian motion (GBM) and $dZ_t^{N.G.} = e\sqrt{dt}$ and $e \sim N(0,1)$. Once random scenarios are constructed, the investment model explained earlier applies.

A popular approach used to model one-factor Markov processes is Lattice of Cox, Ross & Rubinsterin [53]. Based on this methodology, first the deterministic part of the equation is used and then the variance is added in a symmetrical lattice. Their approach therefore converges weakly to a GBM motion. The expected value expression can be written as ("$\mathrm{N.G.}$" is dropped for simplicity):

$$E[\mathrm{C}_t] = \mathrm{C}_0 e^{\alpha_C (t-t_0)}$$

If we take $x_t = Ln(\mathrm{C}_t)$ and $\Delta t = t - t_0$, the above expression can be re-written as:

$$E[x_t] = x_{t-1} + (\alpha_C - \frac{\sigma_C^2}{2})\Delta t$$

$$Var(x_t) = \sigma_C^2 \Delta t$$

A binomial step of Δt is considered in which u and d are multipliers associated to up and down movements of price in each step with probabilities of p and $1-p$ shown in Figure 33:

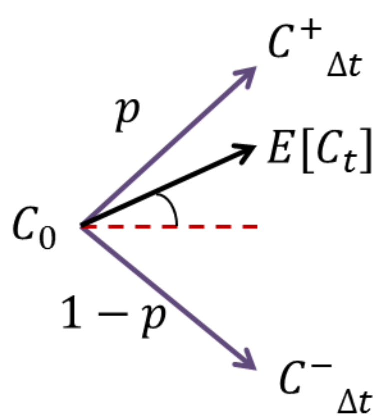

Figure 33 GBM binomial step

The following values are proposed by Cox et al to match the first and second moments of GBM model:

$$u = e^{\sigma_C\sqrt{\Delta t}}, d = e^{-\sigma_C\sqrt{\Delta t}} = \frac{1}{u}$$

$$p = \frac{1}{2}(1 + \frac{(\alpha_C - \sigma_C^2/2)}{\sigma_C}\Delta t = \frac{1 + \alpha_C - d}{u - d}$$

The expected value becomes:

$$E[\mathrm{C}_t] = C_0 \times u \times p + C_0 \times d \times (1 - p)$$

The drift parameter of GBM is presented in up and down probabilities and the lattice values model the volatility of the process. For a risk neutral approach, these probabilities should be adjusted.

In this work we use an alternative approach proposed by Bastian-Pinto et al [54] which is equivalent to Cox et al., but applies to more general stochastic processes. Furthermore, this model is more intuitive compared to a similar model proposed by Nelson and Ramswamy [55]. The same principle is used which is to match the first and second moments of the process by using the deterministic expression of the expected value (first moment) in the lattice mean value and modeling the volatility (second moment) through lattice up and down movement. We assume that $x_t = x_t^* + x_t'$, where x_t' is the deterministic component representing the expected value: $x_t' = x_{t-1}' + (\alpha_C - \frac{\sigma_C^2}{2})\Delta t$ and x_t^* is the value of additive lattice which models an arithmetic Brownian motion with zero drift and with u and d as its up and down increments. The diagram shown in Figure 34 represents the whole process.

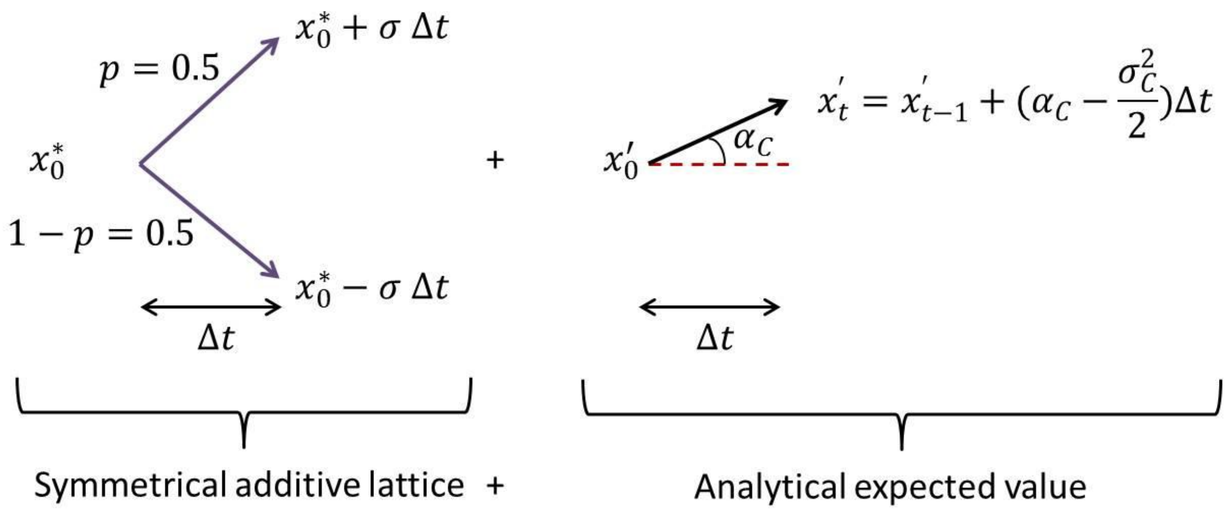

Figure 44 Symmetrical binomial lattice

Figure 35 demonstrates an example where a GBM is modeled using symmetrical lattice approach. As it is shown the branches are symmetrical around the expected value.

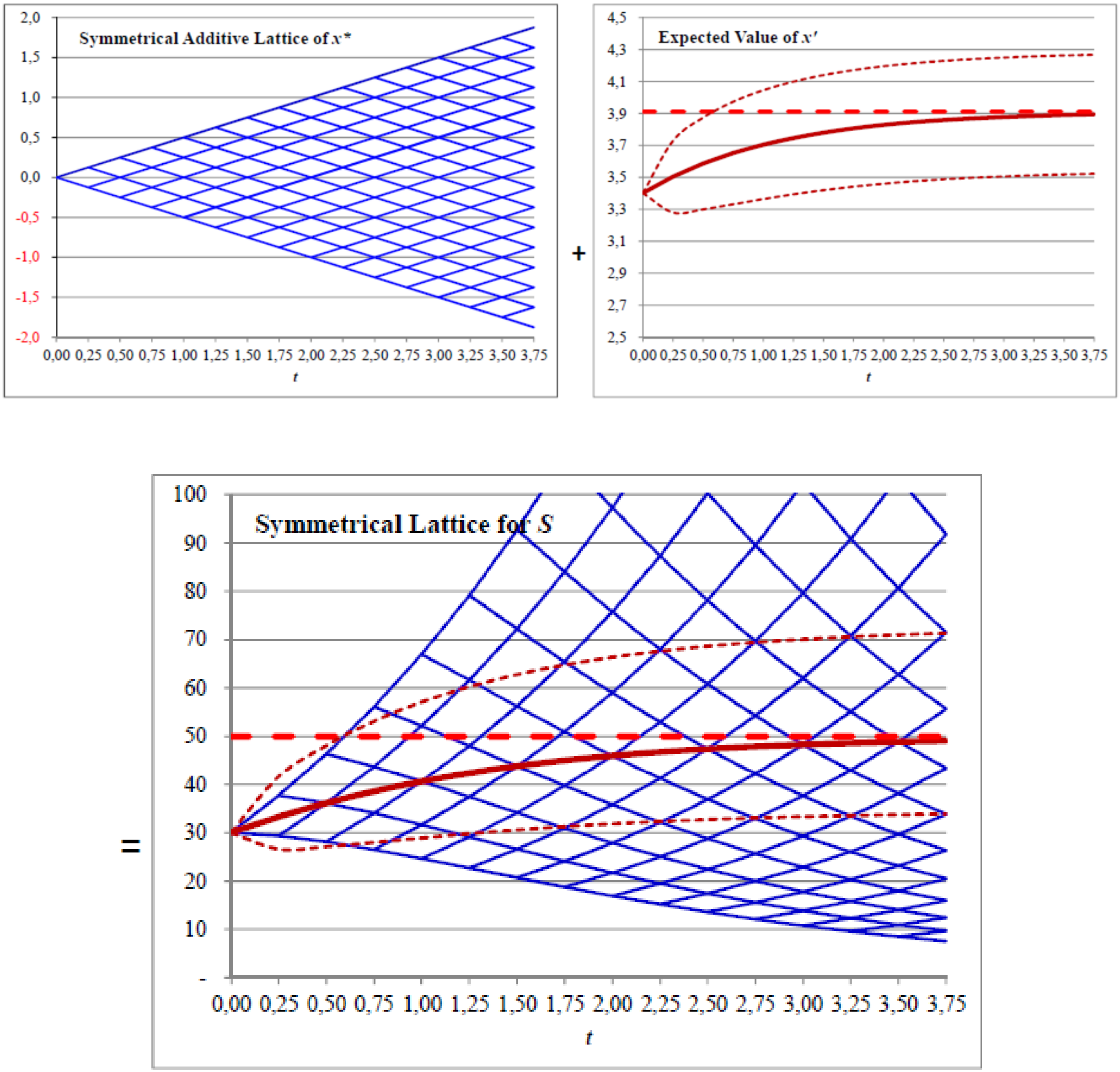

Figure 35 Combination of symmetrical lattice and expected value to represent a GBM

Note that the above approach applies to mean reversion processes and more generic processes.

4.4.3.2.2 Dynamics of PV and Storage Investment Cost; Discrete Probability Distributions

There is no stochastic process identified for PV and electricity storage investment cost. Since there is no sufficient historical data to estimate a stochastic process for PV and electricity storage investment cost over time, we assume a decreasing trend and assign a binomial probability mass function to the rate by which the investment cost decreases by each year:

$$I_{PV}(t) = \propto_{PV} I_{PV}(t-1)$$

$$\propto_{PV} = \begin{cases} D_{PV,1} & with\ \ Probablity\ P_{PV} \\ D_{PV,2} & with\ \ Probablity\ 1 - P_{PV} \end{cases}$$

$$I_{ST}(t) = \propto_{ST} I_{ST}(t-1)$$

$$\propto_{ST} = \begin{cases} D_{ST,1} & with\ \ Probablity\ P_{ST} \\ D_{ST,2} & with\ \ Probablity\ 1 - P_{ST} \end{cases}$$

The parameters are simulated numbers and no historical data is used to calibrate them.

4.5 Model Validation

The investment model is built upon a mathematical optimization and the methodologies to calculate periodical cash flows, finance charges and time value of money are adopted from well-established concepts in finance and engineering economics.

The only simulated term in this model is micro-grid's savings which is obtained from a two-stage stochastic optimization model explained in Chapter 2. This model is convex and unique optimal solution is guaranteed for it. The functional form that defines micro-grid's savings is a linear regression model and its accuracy and goodness of fit can be examined with appropriate statistics used in linear regression.

Therefore, the objective function along with the constraints for investment model would form a convex mixed integer linear or quadratic programming and unique optimal solution is guaranteed for such problem.

4.6 Illustrative Results

We expect that the investment decisions, namely, investment amount and timing, are dependent on economic benefits from micro-gird and capital expenditure spent on investment. Moreover, uncertainty in capital cost and operational stochasticity would impact the investment decisions.

In this section, we aim at demonstrating such impacts through illustrative examples. In particular, among all factors, the significance of the following is of great interest:

- Impact of uncertainty in investment decisions
- The functional form of micro-grid's savings and the contribution of different on-site resources to the savings

As mentioned earlier, the investment model is run for all possible stochastic scenarios for random capital costs and natural gas prices. Therefore, investment results will be given in probability distribution terms and certain statistics such as mean and variance value will be interpreted out of that.

This section is organized as follows: first necessary inputs to set up the investment model will be explained and presented. Second, numerical results for an illustrative case will be provided to demonstrate the concepts of investment model under uncertainty. In this section the same investment case study will be solved using two different approaches: (i) Using stochastic scenarios and (ii) Using expected value of random variables and creating a deterministic version of the problem. The objective is to show the significance of considering uncertainty in investment decision making process. Finally, a sensitivity analysis is performed to examine the significance of uncertainty in capital cost and the functional form of savings on the investment decisions.

4.6.1 Input Data

The investment decisions are sought to form a micro-grid over a 4-year horizon. On-site resources for the micro-grid are to be selected from GF, PV, WT and electricity storage. GF and WT investment costs are considered to be deterministic and known over the course of four years. Gas price, PV and storage investment costs are considered to be stochastic and each uncertainty is captured via stochastic scenario generation as explained in section 4.4.3.2. No correlation is assumed among these random variables. Electricity price is assumed to be driven by gas price during peak hours. The electricity price for off-peak hours is to be obtained via a profile as a percentage peak price (details can be found in section 3.3.4.2). Table 21 lists the parameters that define the dynamics of random variables.

Table 21 Parameters of stochastic processes for PV investment cost, BS investment cost and gas price

PV Investment Cost	
$D_{PV,1}$	0.9
$D_{PV,2}$	0.6
P_{PV}	2/3
$I_{PV,0}$	6750000 ($/MW)

Storage Investment Cost	
$D_{ST,1}$	0.9
$D_{ST,2}$	0.85
P_{ST}	2/3
$I_{ST,0}$	5200000($/MW)

Gas Price	
α_C	0.045
σ_C	0.2
C_0	7 ($/mmBtu)

Given the above input, possible scenarios realized over the course of four years for each random

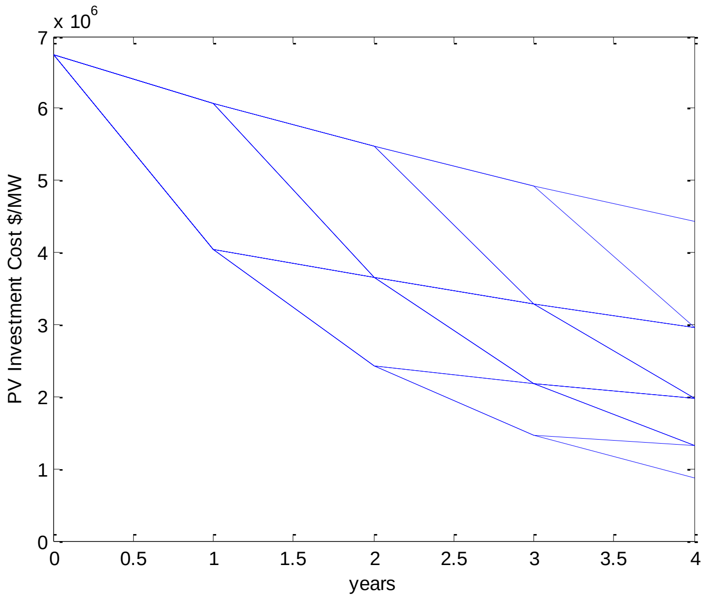

Figure 36 Stochastic PV investment cost

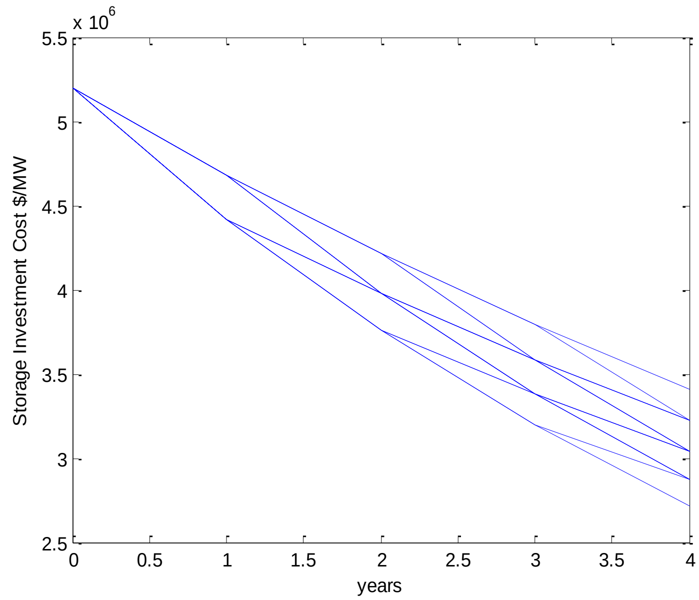

Figure 37 Stochastic storage investment cost

variable are shown in figures 36, 37 and 38:

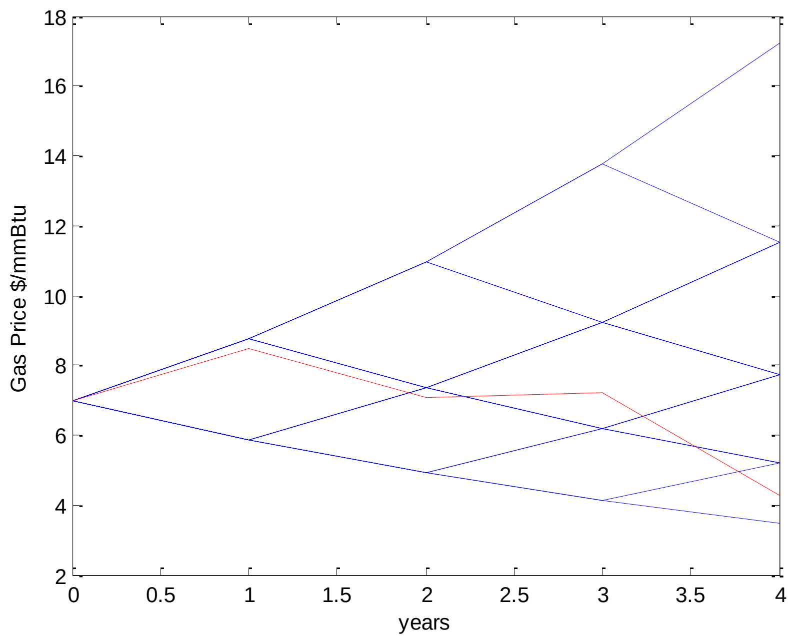

Figure 38 Stochastic gas price

Another set of inputs to the investment model is the capital cost of gas-fired generation unit and wind turbine, which are considered to be deterministically known for the next four years (Figure 39).

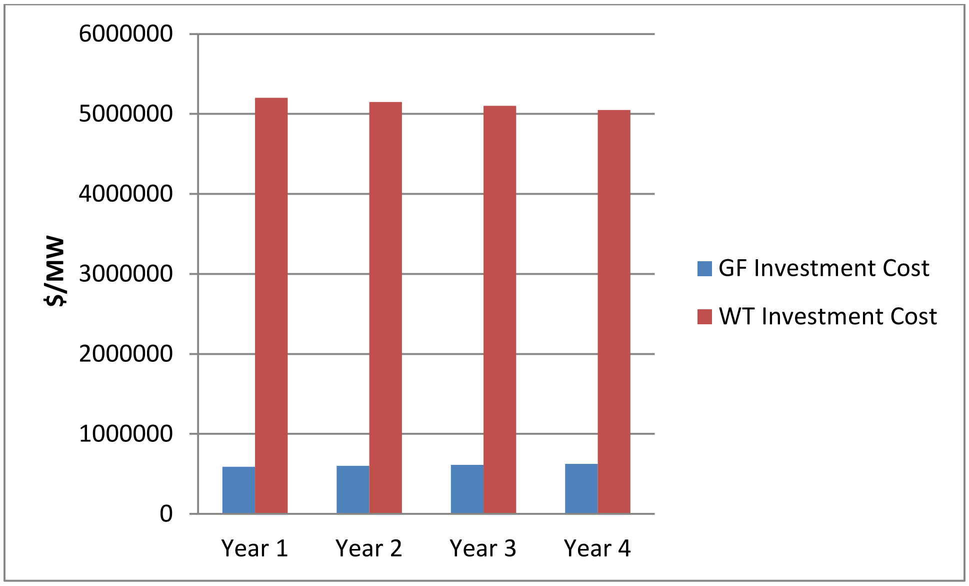

Figure 39 GF and WT investment cost

Some operational characteristics such as wind turbine efficiency and battery storage duration are fixed to keep the optimization problem computationally tractable. Throughout the illustrative cases we assume $\eta_{WT} = 39\%$ and battery storage duration to be 1 hour.

There are also a set of financial parameters that are inputs to the investment model (see Table 22):

Table 22 Input for financial parameters and resource limits

Financial Parameters	
CB_0 (\$)	1,000,000
ROI (%)	2
FC (%)	4
FT (years)	5
$BLimit$(\$)	5,000,000
$LandPr$ (\$/acres)	20,000
$LandDens$(%)	100

Resource Investment Limits	
$MaxI_{GF}$	0.75
$MaxI_{WT}$	0.75
$MaxI_{ST}$	0.075
$AvLand$ (acres)	2
$PVReqLand$ (acres/MW)	4

The last set of inputs refers to the restrictions imposed to invest in on-site resources. These will form the constraints that directly determine the maximum capacity of each resource that can be installed in the micro-grid.

4.6.2 Illustrative Example: Impact of uncertainty is significant

In this section, an illustrative example is presented for investment in micro-grid over a time horizon of $\tau = 4$ years. A linear regression is built to explain the annual cost of micro-grid conditioned on natural gas price. The following functional form turns out to be the appropriate fit to the conditional annual cost of micro-grid:

$$Cost_{MG,t} = \beta_{0,t} + \beta_{1,t} I_{GF,t} + \beta_{2,t} I_{PV,t} + \beta_{3,t} I_{WT,t} + \beta_{4,t} I_{ST,t}$$

For example, when gas price equals 3.48 \$/mmBtu, the coefficients are as shown in Table 23:

Table 23 Coefficients of micro-grid's cost function

GP=3.48	Coefficients
Intercept	10024872.8
I_{GF}	-597642.28
I_{PV}	-6850356.5
I_{WT}	-9196434.9
I_{ST}	-920318.47

Expected optimal incremental investments over the four years are plotted in Figure 40. More investment in wind turbine is due to the fact that its contribution to micro-grid's savings is more than that of the other resources. Moreover, due to the interdependency between gas and electricity prices, the savings from micro-grid increases when gas prices increase. This would lead to more investment in on-site resources once the gas price grows higher. Figure 41 depicts the financial activities over the investment horizon. The decision on whether to use own cash or borrowed fund for resource procurement depends on rate of return on invested cash and finance charge. The expected cash flow at the end of horizon (including beyond-horizon projected cash flow) along with its standard deviation is shown in Figure 42. High volatility of cash flow is due to high variance of PV investment cost and gas price.

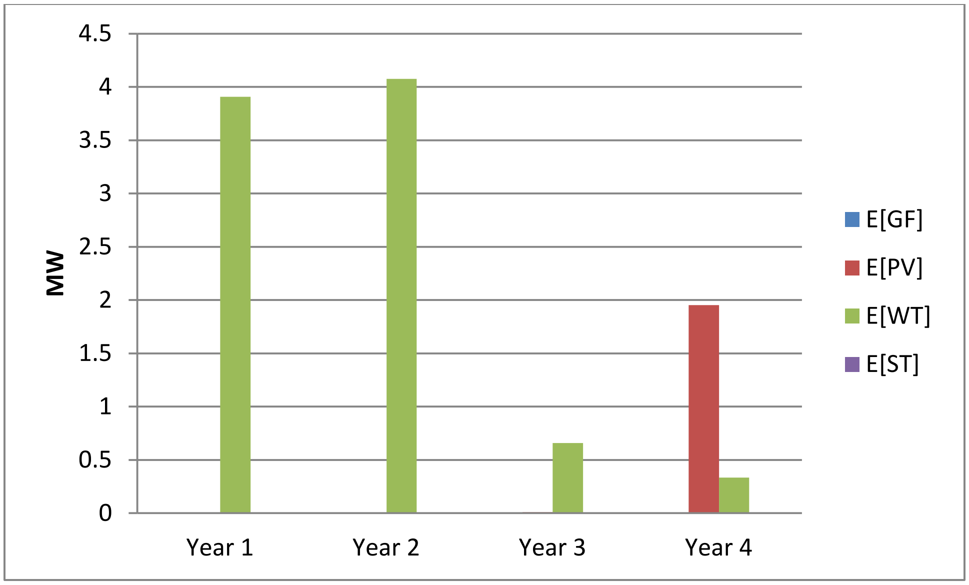

Figure 40 Optimal incremental capacity investments over 4 years; average over all scenarios

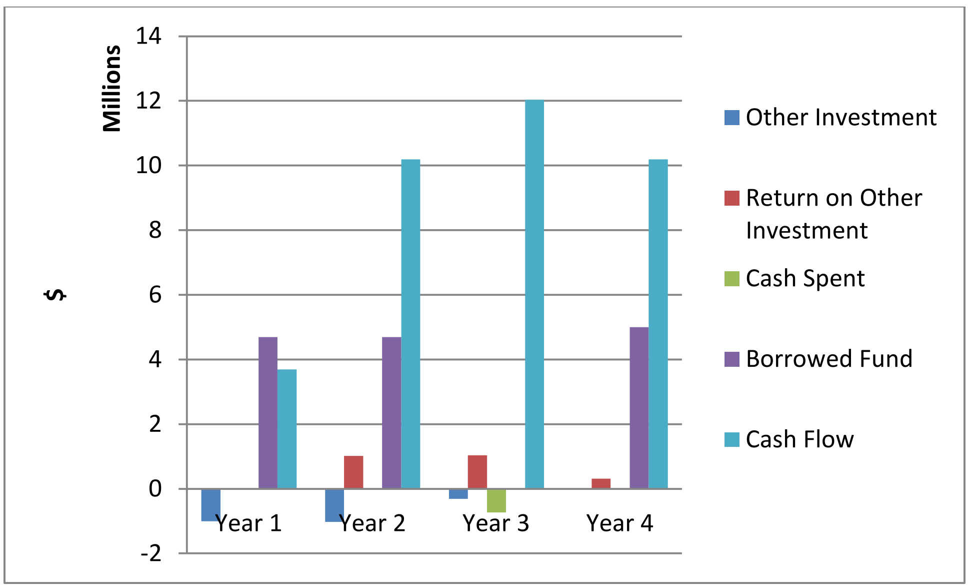

Figure 41 Average financial activities over 4 years average over all scenarios

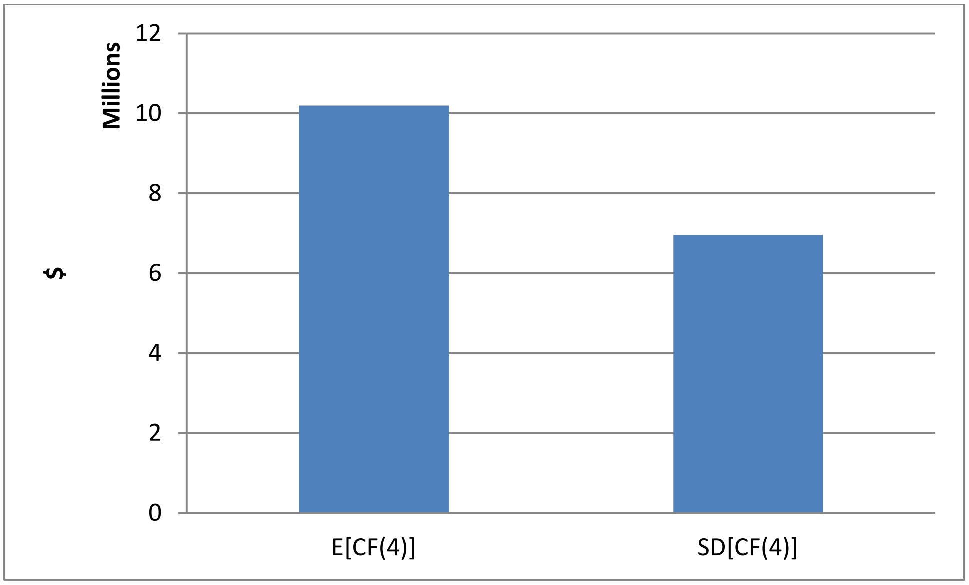

Figure 42 Distribution of cash flow at the end of horizon average over all scenarios

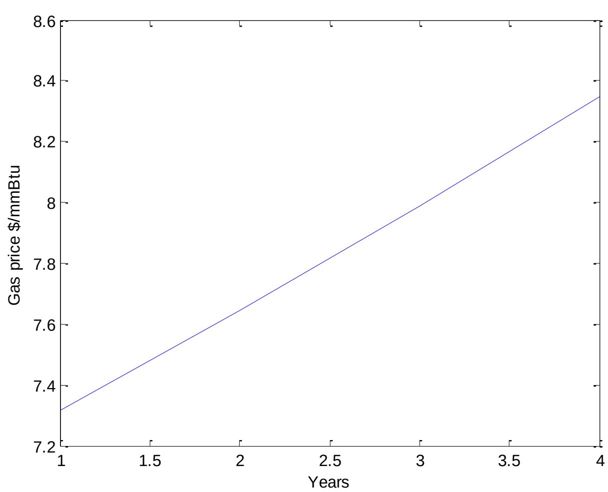

Figure 43 Expected gas price over investment horizon

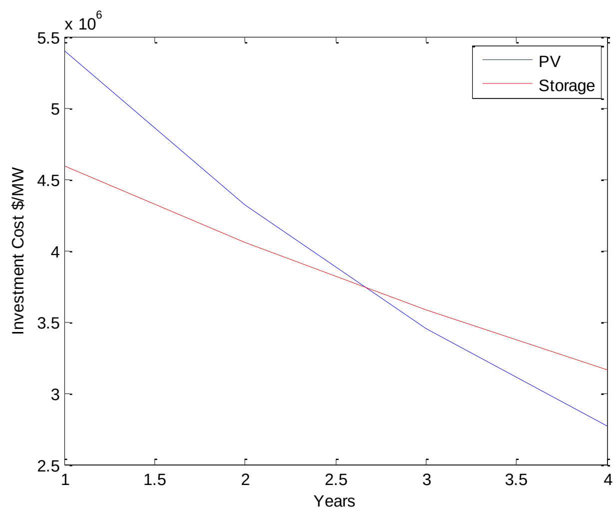

Figure 44 Expected PV and Storage investment costs over investment horizon

Next, we examine the results if we did not consider the uncertainty and represent the random values with their respective expected values. Figures 43 and 44 graph the expected value of gas price, and storage investment costs over four years of investment horizon.

Optimal incremental investment decisions are shown in Figure 45. While the decisions on average are not significantly different, considering all stochastic scenarios leads to more distributed investment over 4 years. This would be a better strategy in the presence of uncertainty.

Accumulated cash flow at the end of the horizon (including projected future cash flows) is also slightly different compared to the case when uncertainties are considered (Figure 46).

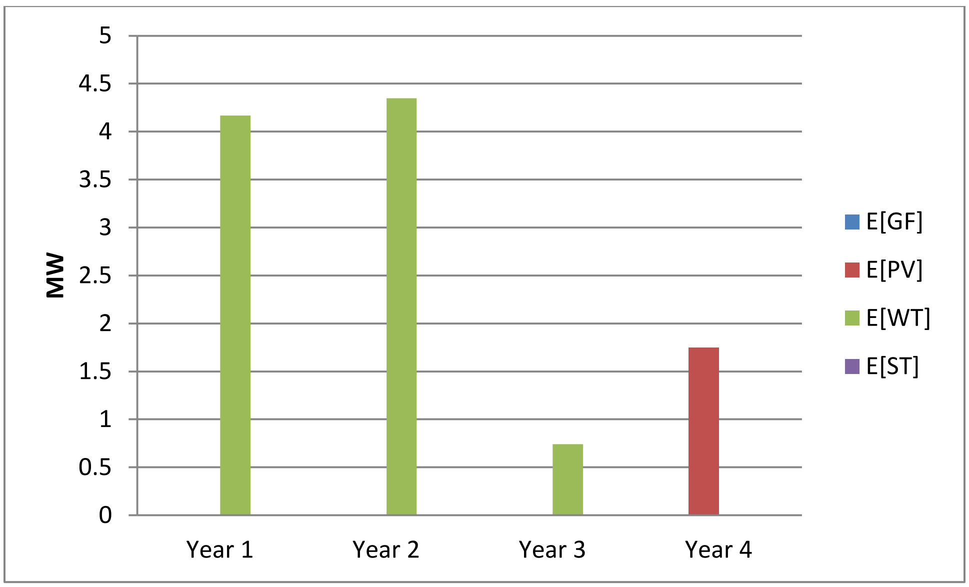

Figure 45 Optimal incremental investment over 4 years; deterministic

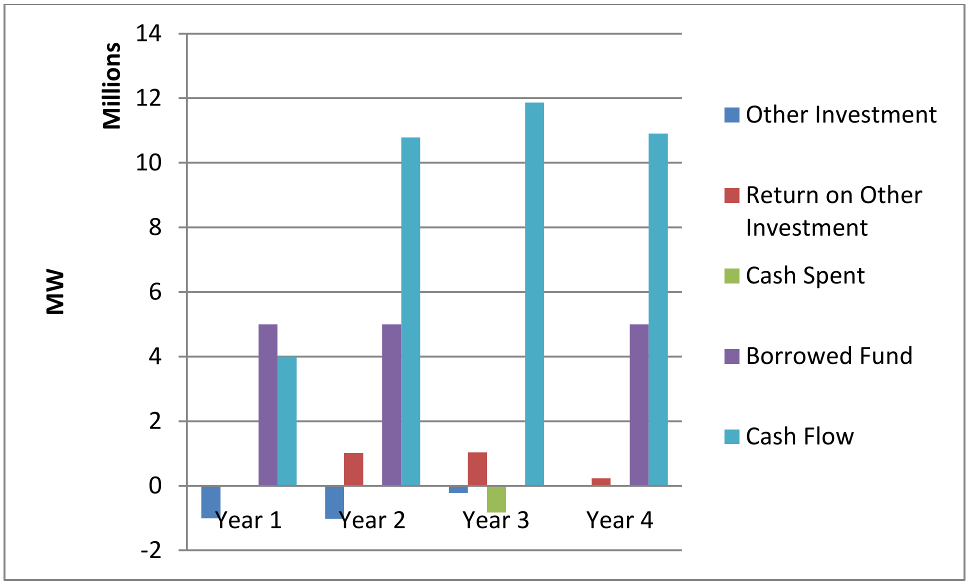

Figure 46 Financial activities over 4 years; deterministic

As shown in Figure 40, the standard deviation of cash flow at the end of the horizon is significant. The uncertainty increases as the variation of random variables increases. Figures 47-49 depict the results for a case where the variance of PV capital cost is higher than that of previous case (i.e., the case presented in Figures 40-42).

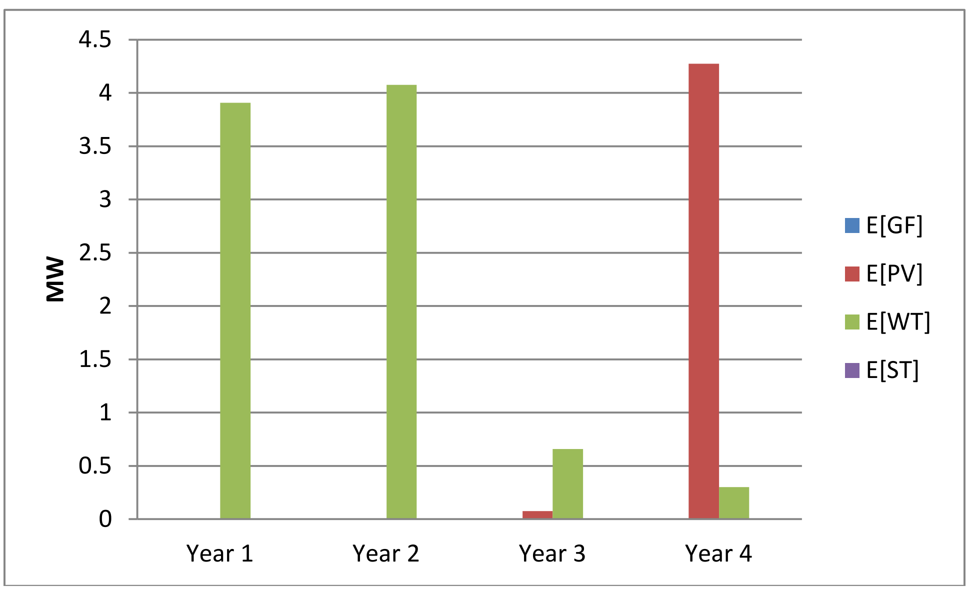

Figure 47 Optimal incremental capacity investments over 4 years; average over all scenarios

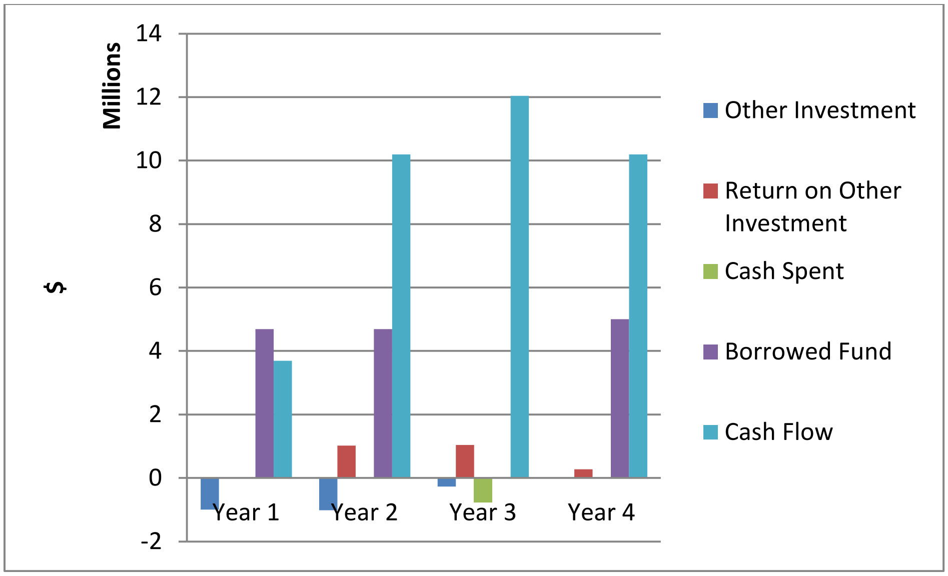

Figure 48 Average financial activities over 4 years; average over all scenarios

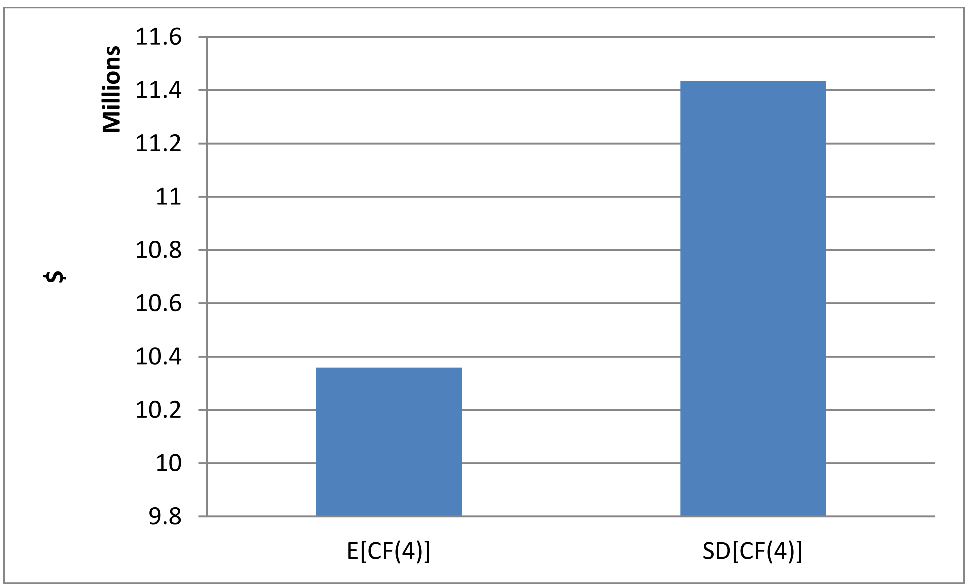

Figure 49 Distribution of cash flow at the end of horizon average over all scenarios

4.6.3 Different Operational Forms

In this section different functional forms of micro-grid's cost are considered in order to show how investment decisions change as the contribution of each resource to micro-grid's savings varies. Two different functional form are examined: 1) savings with both linear and interaction terms, and 2) savings with only interaction terms. The results are shown for one sample path for which gas prices shown in Table 24 are realized:

Table 24 Sample natural gas prices over 4 years

Year	1	2	3	4
GP ($/mmBtu)	8.77	10.98	13.75	17.22

Investment costs for various resources are shown in Figure 50.

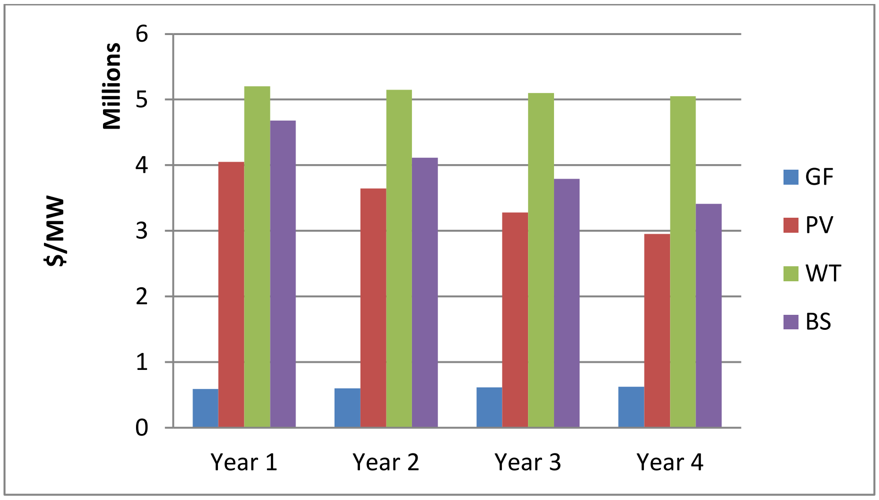

Figure 50 Capital cost for resources over 4 years

The first functional form we study is an example of micro-grid where the contribution of GF unit to the savings is linear and the savings from the other resources are significant only if we consider the following interactions (coefficients are shown in Figure 51):

$$Cost_{MG,t} = \beta_{0,t} + \beta_{1,t} I_{GF,t} + \beta_{2,t} I_{PV,t} \times I_{WT,t} + \beta_{3,t} I_{WT,t} \times I_{ST,t}$$

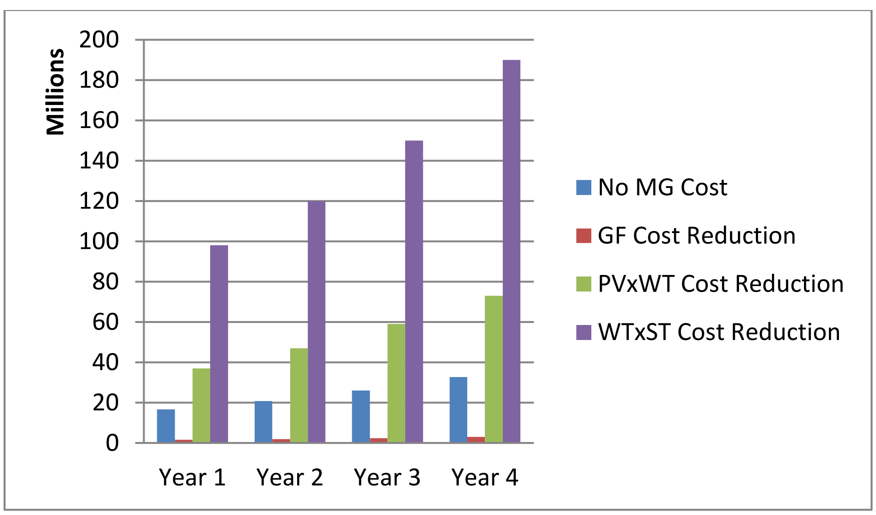

Figure 51 Coefficients of micro-grid's cost function over 4 years

The incremental investment decisions in various resources are shown in Figure 52. The interaction between WT and BS forces the investment to be taken over for these resources simultaneously in year 3. WT dominates the investment because of the higher contribution of this resource to the savings. Once the value of storage in micro-grid is increased by expanding its application, it can becomes more attractive for investment. Having an interaction term between WT and BS can represent a case where BS is not only used for arbitrage but also coupled with WT's production.

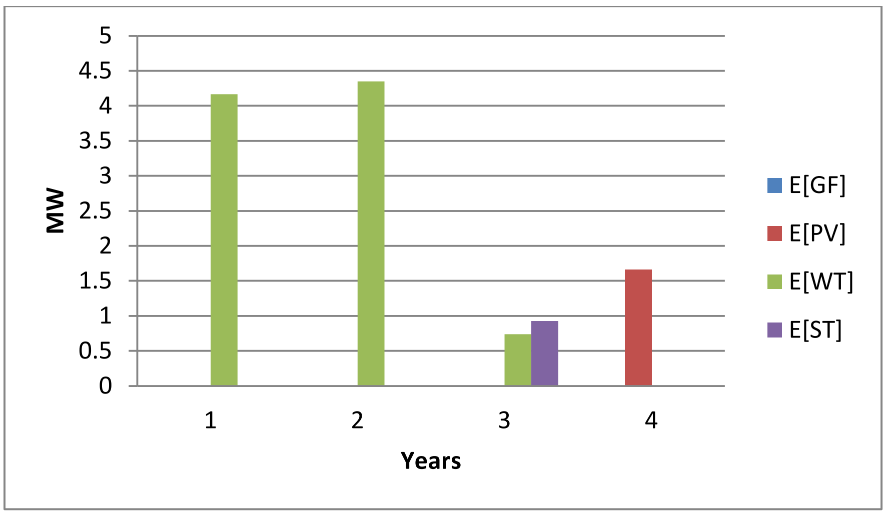

Figure 52 Incremental investment decisions over 4 years

Second example is the case where only the interactions of resources are considered in micro-grid's investment:

$$Cost_{MG,t} = \beta_{0,t} + \beta_{1,t} I_{GF,t} \times I_{ST,t} + \beta_{2,t} I_{PV,t} \times I_{WT,t} + \beta_{3,t} I_{PV,t} \times I_{ST,t} + \beta_{4,t} I_{WT,t} \times I_{ST,t}$$

Incremental investment in resources is shown in Figure 53. Investment in PV and BS are more significant from the previous example as the contribution of these resources to the savings increases.

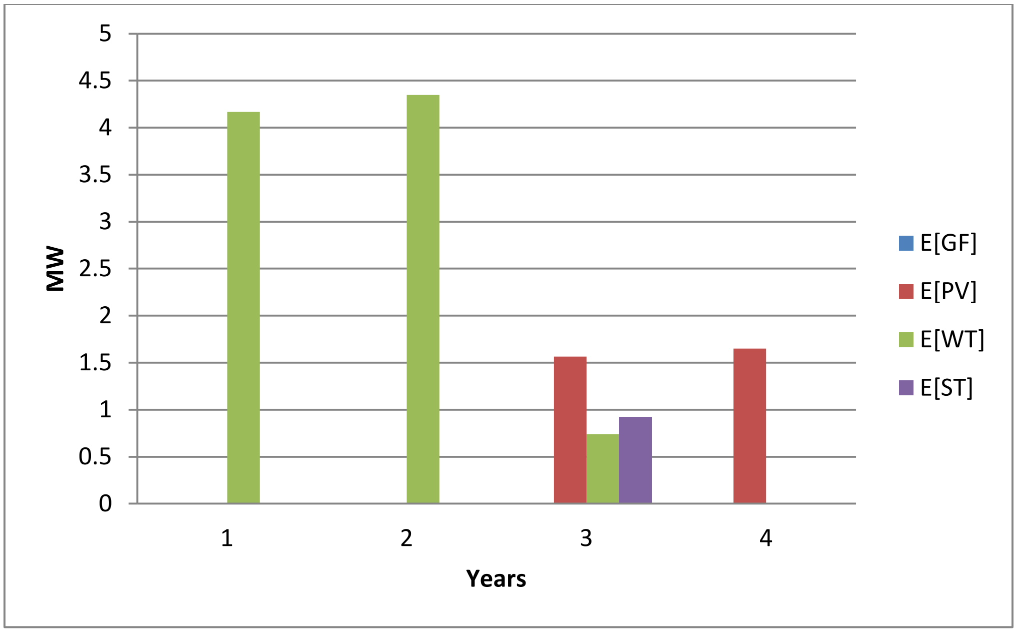

Figure 53 Incremental investment decisions over 4 years

4.7 Conclusion

In this chapter an investment model was developed for micro-grid's portfolio optimization. The return on investment in a micro-grid is highly dependent on the operation of various on-site resources coupled with the interchange with the utility grid. In this work, the loop between the investment and the operation model was closed using operation models developed in chapter 2. We also considered two types of uncertainty in investment decisions: 1) short-term or operational variations were built into optimal operation models, and 2) long-term uncertainty associated with investment was addressed by solving the investment model for all possible stochastic scenarios that could be realized over the investment horizon. It was shown that the impact of uncertainty is significant and investment strategy which is more distributed over the horizon could be a better alternative in hedging against the uncertainty in the future.

Moreover, different functional forms for micro-grid's cost (or savings) were examined. The results show that enhancing the application of more expensive resources such as battery storage can lead to more savings associated with those resources. Therefore, investment would become more attractive in such resources.

5 CHAPTER 5 - APPLICATIONS AND FUTURE WORK

In this chapter we will present two practical examples of the models presented in chapters two to four. We will also give our vision on the future extensions of this work.

5.1 Practical Applications

5.1.1 Optimal Switching to Suspension and Re-activation

The operation of a micro-grid with GF unit is valuable in a certain range of gas price. With uncertain natural gas price, there is a possibility that the operation of GF does not economically make sense. In such conditions, the operator might decide to mothball the unit and wait until prices move in a favorable direction. A one-time cost might be incurred once the micro-grid's operator decides to suspend its own GF generation and purchase from the grid. Moreover, continuous maintenance cost is needed for the mothballed unit and upon the re-activation some fixed cost might be involved. The possibility of mothballing and re-activation would lead to different thresholds for initial investment and therefore switching could be taken into account once solving for investment thresholds.

In this section, we present numerical results for optimal switching equations developed earlier in section 3.3.4.3. Let us consider the micro-grid configuration listed in Table 25.

Table 25 Micro-grid's configuration

$I_{on-sitegen.}$	$I_{fuel-renewable}$	**Average Demand (kW)**	**GF Capacity (kW)**	**GF Heat Rate (mmBtu/kWh)**	**PV Capacity (kW)**	**PV Efficiency**
0.5	2	2000	500	0.03	1250	0.4

Electricity price (C_e) and other costs such as GF investment cost (I_{GF}), maintenance cost (M), suspension and re-activation sunk costs $(E_M$ and $R)$ are given in Table 26.

Table 26 Cost and price inputs

$C_e(\$/kWh)$	$I_{GF}(\$/kW)$	$M(\$)$	$E_M(\$)$	$R(\$)$
1	1200	1000	0	500

Solving Equations (18), (41) and (42) of Chapter 3, the following thresholds are obtained for new investment (C^*, I^*), suspension of GF (C_M) and its re-activation (C_R) (see Table 27). It is shown that the possibility of suspension and re-activation along with associated cost to them would lead to lower thresholds of initial investment ,i.e., more delay in investment.

Table 27 Investment and optimal switching thresholds

$C^*(\frac{\$}{mmBtu})$	$I^*(\frac{\$}{kW})$	$C_M(\frac{\$}{mmBtu})$	$C_R(\frac{\$}{mmBtu})$
29	551	35	32

Mothballing requires sunk cost denoted by E_M. In addition, the asset needs maintenance once it is suspended which costs an annual amount of M. The operation of GF can later be re-activated at a sunk cost of R. Suspension only makes sense not only when the maintenance cost is less than the operation cost (which depends on electricity and gas prices) but also when the re-activation cost is less than the fresh investment. As it is shown above, in the case of increase in gas price beyond C_M, GF is suspended to prevent expensive on-site electricity generation from GF. However, if the gas price movement is favorable in the future, GF operation can be re-activated. This threshold is still higher than new investment threshold. These thresholds are affected by various costs C_e, I_{GF}, M, E_M and R, as well as the volatilities of gas price and PV investment cost.

5.1.2 Different cost functions

The models of chapters 2 and 4 were applied to illustrative cases to demonstrate the concept. For example, we understand that the functional form derived for micro-grid's cost (or savings) is highly sensitive to a set of input parameters. This includes the price interdependency between natural gas and electricity, operational specifications of resources such as PV efficiency, heat rates and battery storage duration, limits on the operational capacity of resources (e.g., gas-fired unit and battery) and operational variability in both demand and supply side. Models in Chapter 2 can be used with different parameters and yield different functional forms. Using a different term for micro-grid's cost (or savings) might lead to different investment decisions (Chapter 4). Exploring different functional forms are practical applications of models developed in Chapters 2 and 4.

5.2 Future Work

In designing analytical tools to support decision making in planning of a micro-grid, many elements should be modeled in tandem. In summary, the contribution of this work is to develop and implement analytics to make optimal decisions for micro-grid's investment under optimal operation, and in the presence of both short- and long-term uncertainties. However, we understand that this work can be expanded in several dimensions:

5.2.1 Enhancement of Micro-grid's Portfolio

Micro-grids are moving towards the path to take advantage of every possible technology or programs that enable them to better manage their energy consumption and production. Micro-grid's activities are not only constrained to their local boundaries but also are expandable across their boundaries to interact with utility grid and/or other micro-grids. That being said, the portfolio of micro-grid can be expanded to include enrollment in demand response programs, energy efficiency and load automation technologies. On the energy production side, the portfolio can be expanded to consider technologies such as CHP and thermal storage.

5.2.2 Demand-side Model

In our work an aggregate profile is used for to represent the demand with no further break-down of end-uses. A demand side model can be developed to replace the current aggregate forecast of load profile. This requires a model that can capture the dynamics of demand and its sensitivity to weather and price variations. Such model would enable us to capture the evolution of demand as some input drives change over time.

Furthermore, in the context of a campus-like micro-grid, it is not only important to consider energy demand and resource management but also other elements such as water resource and land-use management.

5.2.3 Long-term Causal Relationships: Investment and Demand

While we tried to close the loop between operation and investment, another causal relationship was ignored which is the obvious dependency between the investment and demand growth. An interesting dynamics might evolve once we study the longer term behavior of demand growth as new investment are made that offer less expensive electricity or offer ways to save energy.

5.2.4 More Comprehensive Real Options Models

Numerically solving the partial differential equations derived in Chapter 3 can provide benchmarks to analytical solutions obtained using contingent claim analysis approach. The simulation model in chapter 3 can also be modified and expanded to accommodate for a larger portfolio of micro-grid and more uncertain investment parameters. Moreover, a dividend payment analogy can be introduced to account for other opportunity cost incurred due to delay in investment.

Appendix

Contingent claims analysis builds on ideas from financial economics. Begin by observing that an investment project is defined by a stream of costs and benefits that vary through time and depend on the unfolding of uncertain events. The firm or individual that owns the right to an investment opportunity, or to the stream of operating profits from a completed project, owns an asset that has a value. A modern economy has markets for quite a rich menu of assets of all kinds. If our investment project or opportunity happens to be one of these traded assets, it will have a known market price. However, even if it is not directly traded, one can compute an implicit value for it by relating it to other assets that are traded.

All one needs is some combination or portfolio of traded assets that will exactly replicated the pattern of returns from our investment project, at every future date and in every future uncertain eventuality. (The composition of this portfolio need not be fixed; it could change as the prices of the component assets change.) Then the value of the investment project must equal to the total value of that portfolio, because any discrepancy would present an arbitrage opportunity: a sure profit by buying the cheaper of the two assets or combinations, and selling the more valuable one. Implicit in this calculation is the requirement that the firm should use its investment opportunity in the most efficient way, again because if it did not, an arbitrager could buy the investment opportunity and make a positive profit. Once we know the value of the investment opportunity, we can find the best form, size, and timing of investment that achieves this value, and this determine the optimal investment policy.

Consider F to be the market value of an asset that entitles the owner to the firm's future profit flows π.

Financial economics has developed sophisticated theories describing the decisions of investors, the market equilibria resulting from the aggregation of such decisions, and the equilibrium prices of assets. The basic setting is an economy with rich menu of traded assets with different return and risk characteristics. Ro value a new asset, we try to replicate its return and risk characteristics through a portfolio of existing traded assets. The price of the new asset must then equal the market value of its portfolio. Therefore, price discrepancies for equivalent assets or portfolios could not persist in equilibrium. The asset held in continuation region of our analysis can be analyzed this way. Much of this theory has assumed that the underlying uncertainty can be described by an Ito process.

Replicating Portfolio

We begin with the simplest setting. Suppose the profit flow depends on a variable x; think of it as the firm's output price. Since we will be dealing with proportional rates of returns, it is convenient to assume that x follows a geometric Brownian motion:

$$dx = \alpha x dt + \sigma x dz$$

whereα is the growth rate, σ is the proportional variance parameter, and dz is the increment of standard Wiener process.

Now we assume that firm's output can itself be traded as an asset in financial markets. This would literally be the case if the output is a commodity like oil or copper. It can be shown that it is sufficient that the risk in the dynamics of x, namely the dz term above, can be replicated by some portfolio of traded assets.

Like any asset, the output is held by investors only if it provides sufficiently high return. Part of the return comes in the form of the expected price appreciation, α. Another part may also come in the form of a dividend, directly (the product might be a tree that grows more wood) or indirectly (the holder of oil or copper might be a firm that plans to use these inputs and finds it convenienct to hold its own inventory rather than rely on the spot market; then the dividend is implicit "convenience yield"). We simply stipulate that there is a dividend and denote its rate by δ. Let $\mu = \alpha + \delta$ denote the total expected rate of return.

This expected return must be enough to compensate the holders for risk. Of course it is not risk as such that matters, buy only nondiversifiable risk. The whole market portfolio provides the maximum available diversification, so it is the covariance of the rate of return on the asset with that on the whole market portfolio that determines the risk premium.

Throughout our analysis we assume that the riskless rate of return r is exogenously specified, for example as the return on government bonds. Then the fundamental condition of equilibrium from the capital asset pricing model (CAPM) says that

$$\mu = r + \varphi \sigma \rho_{x,m}$$

where φ is an aggregate market parameter (the market price of risk) that is exogenous and $\rho_{x,m}$ is the coefficient of correlation between return on the particular asset x and the whole market portfolio m.

We find the value of $F(x,t)$ of a firm with profit flow $\pi(x,t)$ by replicating its return and risk characteristics using traded assets of known value. Specifically, consider investing a dollar in the riskless asset and also buying n units of the firm's output; we will choose n shortly to achieve the desired replication. This portfolio costs $(1+nx)$ dollar. Hold for a short interval of time dt. I this time, the riskless asset pays the sure return rdt, while the other asset pays a dividend $n\delta xdt$ and has a random capital gain of $ndx = n\alpha xdt + n\sigma xdz$. Thus the total return per dollar invested is:

$$\frac{r + n(\alpha + \delta)x}{1 + nx} dt + \frac{\sigma nx}{1 + nx} dz$$

Compare this with holding ownership of the firm for the same short interval of time dt. This costs $F(x,t)$ to buy. The dividend is the profit $\pi(x,t)dt$; this involves no uncertainty since x is known when the initial decision is being made. The asset also yields a random capital gain, which we calculate using Ito's Lemma as

$$dF = \left[F_t(x,t) + \alpha x F_x(x,t) + \frac{1}{2}\sigma^2 x^2 F_{xx}(x,t)\right] dt + \sigma x F_x(x,t) dz$$

The total return per dollar invested is

$$\frac{\pi(x,t) + F_t(x,t) + \alpha x F_x(x,t) + \frac{1}{2}\sigma^2 x^2 F_{xx}(x,t)}{F(x,t)} dt + \frac{\sigma x F_x(x,t)}{F(x,t)} dz$$

If our portfolio is to replicate the risk of owning the firm, we must therefore choose

$$\frac{nx}{(1 + nx)} = \frac{x F_x(x,t)}{F(x,t)}$$

However, in the market, two assets with identical risk must earn equal return. Therefore this choice must also ensure

$$\frac{\pi(x,t) + \alpha x F_x(x,t) + \frac{1}{2}\sigma^2 x^2 F_{xx}(x,t)}{F(x,t)} = \frac{r + n(\alpha + \delta)x}{(1 + nx)}$$

Substituting for $nx/(1+nx)$, the right-hand side becomes:

$$r\left[1-\frac{xF_x(x,t)}{F(x,t)}\right]+(\alpha+\delta)\frac{xF_x(x,t)}{F(x,t)}$$

On simplification, the return equation becomes a partial differential equation for the value:

$$\frac{1}{2}\sigma^2x^2F_{xx}(x,t)+(r-\delta)xF_x(x,t)+F_t(x,t)-rF(x,t)+\pi(x,t)=0$$

An alternative and equivalent way to derive the same result is to construct a portfolio that consists of the firm and n units of short position in the asset x. The n is chosen to make this portfolio riskless. This is algebraically much simpler, so we will generally use this method. However, the one given above demonstrates the concept of constructing a replicating portfolio more directly and clearly.

Value Matching and Smooth Pasting

The above analysis assumes only that the various assets are held during a very short interval of time dt. What happens after time $t+dt$ is of no concern, and does not affect the validity of the partial differential equation (PDE) obtained above. However, solutions to this equation require boundary conditions, and therefore some attention to longer time spans.

If the firm that is being valued above has a fixed time horizon T when it is forced to take a termination payoff $\Omega(x_T,T)$, then we can solve the PDE subject to the boundary conditions $F(x,T)=\Omega(x,T)$ for all x. Likewise, the firm may be forced to take the termination payoff at an earlier time t if the state variable hits a threshold $x^*(t)$. Here the boundary condition for all t is clearly

$$F(x^*(t),t)=\ \Omega(x^*(t),t)$$

This is called value-matching condition.

Sometimes the firm can choose its termination optimally, knowing is termination payoff function. This decision will be made so as to maximize the firm's value. Such choice determines a threshold or free boundary$x^*(t)$, and that the appropriate additional condition is the smooth-pasting property for all t:

$$F_x(x^*(t),t)=\Omega_x(x^*(t),t)$$

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