VENDOR FINANCING AND ITS IMPACT ON VENDOR'S OPTIMAL POLICIES

by

JOSE BENEDICTO B. DUHAYLONGSOD

A thesis submitted to the

Graduate School-New Brunswick

Rutgers, The State University of New Jersey

In partial fulfillment of the requirements

For the degree of

Master of Science

Graduate Program in Operations Research

Written under the direction of

DR. BENJAMIN MELAMED

DR. BEN SOPRANZETTI

and approved by

New Brunswick, New Jersey

May 2013

ABSTRACT OF THE THESIS

Vendor Financing and Its Impact on Vendor's Optimal Policies

By JOSE BENEDICTO B. DUHAYLONGSOD

Thesis Directors:

DR. BENJAMIN MELAMED

DR. BEN SOPRANZETTI

This research aims to elucidate how vendor financing impacts the business strategy of the vendor and to shed light on the resulting optimal inventory and dividend policies. We consider a vendor employing a Make-to-Stock inventory policy and selling to a particular set of buyers facing product demand. The vendor is constrained by a fixed amount of capital available for purchasing inventory and incurs a variety of costs. Since the buyers are also financially constrained, the vendor offers financing to the buyers in the form of trade credits, and receives the corresponding incremental orders, which would not be placed with the vendor in the absence of vendor financing. This thesis makes two primary contributions: (1) the suboptimal supply chain policies that arise from implementing vendor financing are explored; and (2) a stochastic optimization model and the attendant objective function from the perspective of the vendor are formulated and solved for optimal financial and inventory policies, simultaneously. The objective function maximizes the expected discounted dividends generated by the vendor, given its initial inventory and capital, subject to capital constraints. This is compared and contrasted with the case wherein the vendor utilizes an inventory policy, but no vendor financing and cases wherein the vendor uses vendor financing but has less available access to external funds. Analyses and insights are provided thereafter.

ACKNOWLEDGEMENTS

I would like to express my deep gratitude to Dr. Benjamin Melamed and Dr. Ben Sopranzetti, my thesis advisers, for their contribution, guidance, and invaluable critiques of this research.

I would also like to extend my special thanks Pedro Pontes, Paolo Trinidad and Joonhee Lee for their help in offering resources and support during my semesters here in the US.

Finally, I wish to thank my family, especially my mother, for the unwavering support, encouragement and belief in me throughout the course of my study.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

1. Introduction

Vendor financing is the practice of vendors serving as monetary intermediaries that fund customer purchases in lieu of a bank or financial institution. A case in point took place in early 2000, when Motorola, one of the major telecommunication manufacturers in the US, extended vendor financing of close to $2 billion to TelSim, a privately owned Turkish telecommunications provider. This was part of Motorola's initial growth strategy to enter emerging markets as the telecommunications industry was booming in the late 1990's. However, the political and economic instability in Turkey took a toll on TelSim and led the company to default on the $728 million it owned Motorola on April 30, 2001.

This research aims to elucidate how vendor financing impacts the business strategy of the vendor and to shed light on the resulting optimal inventory and dividend policies. We consider a vendor employing a Make-to-Stock inventory policy and selling to a particular set of buyers (or end-customers) facing product demand. The vendor is constrained by a fixed amount of capital available for purchasing inventory and incurs a variety of costs. The vendor firm's corporate treasury manages all its financial transactions. Since the buyers are also financially constrained, the vendor offers financing to the buyers in the form of trade credits, and receives the corresponding incremental orders, which would not be placed with the vendor in the absence of vendor financing.

Vendor financing is a ubiquitous and important source of short-term working capital in the United States (Rajan & Petersen 1997), in European markets (Wlison & Summers 2002) and in less developed countries (Fisman & Love 2003). Basically, in vendor financing (or trade credits), the vendor assumes the role of a typical financial institution and provides funding for a buyer that is unable to gain access to external funds due to its credit worthiness. The vendor typically enjoys a cost advantage over banks due to the following reasons: (a) ability to get more information on the buyer; (b) ability to exert control over the buyers in terms of operations and production; (c) ability to carry out better salvaging and reselling of unsold products in cases of buyer defaults; and (d) ability to price discriminate among customers and reduce transaction costs. For buyers, the choice of vendor financing essentially stems from the bank's unwillingness to provide credit to a risky firm. Further, empirical evidence shows that optimal trade credit contracts are generally cheaper as compared to bank financing (Kouvelis & Zhao 2012), and that small firms in financial distress gain a degree of security and safety when associating with a vendor that provides financing (Evans & Koch 2007).

Vendor decisions to extend financing are primarily motivated by increased vendor sales, based on price discrimination among their buyers (Brennan, Maksimovic & Zechner 1988), or by their subsequent ability to exert a measure of control over buyers (Rajan & Petersen 1997). However, there is a dearth of literature addressing the effects and implications of vendor financing to the policies and financial performance of the vendor. Offering vendor financing would tie up the vendor's limited capital, and consequently, increase its chance of bankruptcy. But a more exigent issue arises when credit limits are ignored – an apparent resultant sub-optimality of the vendor's supply chain policies.

This paper is motivated by the observation that to attain global optimization, one should integrate and optimize logistics performance and financial performance simultaneously. Under such an integrated framework, the focal party is a vendor firm which must balance and optimize the conflicting goals of revenue enhancement and lending risk, subject to capital availability. Vendor financing enhances revenue as incremental demand is added in the form of financed sales that might otherwise be delayed or lost. The downside of vendor financing is that the vendor's capital is tied up and encumbered by lending risk. For the vendor to generate the same incremental sales, it might have to consider external financing to supplement limited internal capital, but external financing is typically a more expensive source of capital, thereby increasing bankruptcy risk.

As the vendor assumes the role of a lender in this financing scheme, the following questions arise:

1. How does the vendor's decision to offer vendor financing impact operational decisions, such as optimal inventory and dividend policies?
2. Does vendor financing augment the baseline demand it initially faces?
3. Are there any unintended consequences stemming from the vendor's decision to extend financing?

This thesis makes two primary contributions to the literature. Firstly, we explore the suboptimal supply chain policies that arise from implementing vendor financing. This results from the tension between the increased revenue generated by the vendor due to its decision to offer vendor financing and the concomitant reduction in capital available to implement inventory policies. Secondly, we formulate a stochastic optimization model and the attendant objective function from the perspective of the vendor, and solve for optimal financial and inventory policies, simultaneously. More specifically, our objective function aims to optimize the vendor's stock price by maximizing the total expected discounted dividends generated by the vendor, given its initial inventory and available capital, subject to capital constraints. This is later compared and contrasted with the following scenarios: (1) case wherein the vendor utilizes an inventory policy, but no vendor financing; and (2) cases wherein the vendor uses vendor financing but has less available access to external funds.

We shall use the following notation. For any real x, $x^+ = \max\{x, 0\}$ and $x^- = \min\{x, 0\}$.

This thesis is organized as follows. Section 2 contains a literature review. Sections 3 and 4 discuss, respectively, the conceptual description of the system considered and mathematical model formulation. Section 5 solves the combined inventory and financial optimization problem and presents the results and analysis. Finally, Section 6 provides the conclusion of the thesis and offers insights.

2. Literature Review

2.1 Vendor Financing

The literature of vendor financing can be divided into two categories: (1) reasons for vendors and buyers to utilize vendor financing; and (2) mathematical models thereof.

The apparent advantages of vendor financing to both vendor and buyer have been treated in numerous publications. The most referenced paper appears to be Rajan & Petersen (1997), which includes an extensive review and empirical study of the reasons that underlie vendor financing. The paper posits a number of conjectures, based on empirical evidence, such as the vendor's ability to capture and hold onto future business prospects if it provides credit to buyers and acquires industry information at lower costs.

Vicente Cuñat (2006) argues that the rise of vendor financing in the competitive banking sector is a natural result of interactions among buyers and vendors. More specifically, vendors are in a better position to demand debt repayment from buyers as compared to a bank due to the control the vendors exert over the supply of specific intermediate goods, and this specificity makes substitution expensive for the buyer. Consequently, vendors provide liquidity to their buyers when these buyers encounter liquidity problems or when losing buyers is overly costly. The paper further points out that the level of trade credit builds up as time goes by and as the relationship between buyer and vendor moves forward, typically accompanied by increasing sales volumes and trust. Finally, the paper provides empirical results that exhibit the usage of trade credit as a form of financing of last resort.

Fabbri and Klapper (2011) posits two hypotheses regarding what motivates a vendor to extend trade credits to its buyers. First, the paper opines that vendors offer financing to their buyers as a competitive gesture due to weak vendors' market power or the presence of a high degree of competition in the market. This motivates the provision of better credit terms to buyers and subsequent increase of sales on credit. Second, the paper states that vendors offer trade credits similar to the amount and terms they receive from their suppliers. The paper points out that a vendor utilizes the payables as a means to manage risk as it matches its payables and receivables terms.

Several publications model vendor financing from the perspective of the vendor under different assumptions, and offer explanations for opting for vendor financing. Brennan, Maksimovic and Zechner (1988) proposes an optimization framework for analyzing the optimality of vendor financing from the perspective of a monopolistic vendor. Optimality is attained through product price discrimination when there is evident discrepancy in the reservation prices of cash and credit buyers or when there is asymmetric information among supply chain members that precludes customized contracts for buyers with different credit risks, even in a perfectly competitive banking environment. The model is extended to oligopolistic markets where optimality is retained due to the fact that vendor financing can reduce competition among vendors. Although this paper aims to maximize profit, it does not constrain the vendor's initial capital and ignores tying up of capital in lending transactions. It does not address the optimal inventory policy as it assumes that the vendor is always able to fulfill the demand of its buyers.

Kouvelis and Zhao (2012) constructs a model that assumes a newsvendor setting of a buyer and a vendor, both of which are capital constrained and subject to bankruptcy risk. The paper identifies the optimal supply contract (optimal wholesale price and interest rate) using a game theoretic-approach (Stackelberg Game). Here, a financially constrained buyer (e.g., one that has limited access to financing due to low credit worthiness) would prefer vendor financing over bank financing. The reason is that vendor financing ultimately improves the efficiency of the supply chain by giving rise to more orders from buyers, which in turn increases overall profitability. The model closely resembles ours in that it takes into consideration vendor capital and inventory decisions. However, the paper focuses more on modeling the strategic interactions between buyer and vendor to derive the optimal supply contract.

Wang (2011) presents another examination of vendor financing and other financing schemes, all from the vendor's viewpoint. It uses a Stackelberg Game approach to model and characterize the performance of supply chain members under three financing schemes: independent financing, vendor financing and inventory subsidy. It then compares the effects of these practices on various performance metrics, such as profits, wholesale price, expected sales volume, etc., and clarifies the selection and implementation of financial arrangements. The paper also shows that buyer and vendor preferences for vendor financing depend on whether the vendor's cost of capital is below that of the buyer's. These results are shown to be robust with respect to certain assumptions on the demand distribution.

2.2 Stochastic Programming – Solution Quality Assessment

Optimality conditions of solutions are essential ingredients of optimization. In particular, tests for assessing the nearness to optimality of a given solution generally have a higher computational complexity in the context of stochastic programming because of the experimental error induced by random variables, and the need to use approximation methods to solve some of these problems [8, 9].

Bayraksan and Morton (2006) presents four Monte-Carlo sampling-based procedures to assess a solution obtained from approximation methods for solving stochastic programming problems. The stochastic programming problem considered there is

$$z^* = \min_{x \in X} \mathbb{E}[f(x, \Xi)] \tag{2.1}$$

where f is a real-valued objective function, x is the decision vector, Ξ is a random vector whose distribution is known, $X \in \mathbb{R}^d$ is the set of constraints and z^* is the optimal value of (2.1). A sampling-based approximation for the model is given by

$$Z_n^* = \min_{x \in X} \frac{1}{n} \sum_{i=1}^{n} f(x, \xi_i) \tag{2.2}$$

where the ξ_i have the same distribution as Ξ and the Z_n^* is the optimal value of (2.2). The optimal solution of (2.2) is denoted as x_n^*. This sampling and approximation approach is used when the dimension of the random vector Ξ is very large and the exact solution of (2.1) becomes too difficult to obtain. The asymptotic correctness of the solution estimate obtained from (2.2) has been discussed extensively in various other literatures (referenced in [1, 2]).

The following assumptions are made regarding the stochastic programming problem:

(1) $f(.,\Xi)$ is continuous on X, with probability one

(2) $\mathbb{E}[\sup_{x \in X} f^2(x, \Xi)] < 0$

(3) $X \neq \emptyset$ and is a compact set

The test procedures are based on the idea of using the optimality gap of a given estimated solution $\hat{x}$, namely,

$$\mu_{\hat{x},} = \mathbb{E}[f(\hat{x}, \Xi)] - z^* \tag{2.3}$$

as a measure of quality of the solution. A sufficiently small gap implies that $\hat{x}$ is near optimal. Since exact computation of $\mathbb{E}[f(\hat{x}, \Xi)]$ and z^* is difficult, we merely estimate an upper bound of the optimality gap, given by $\mathbb{E}[f(\hat{x}, \Xi)] - \mathbb{E}[Z_n^*]$. This is obtained from the derived statistical lower bound of z^*, $\mathbb{E}[Z_n^*]$, and the upper bound of z^*, $\mathbb{E}[f(\hat{x}, \Xi)]$, due to the general suboptimality of $\hat{x}$. The upper bound for the optimality gap is estimated by

$$g_n(\hat{x}) = \frac{1}{n} \sum_{i=1}^{n} f(\hat{x}, \xi_i) - \min_{x \in X} \frac{1}{n} \sum_{i=1}^{n} f(x, \xi_i)$$

The first term of $g_n(\hat{x})$ converges to $\mathbb{E}[f(\hat{x},\Xi)]$ by the Strong Law Of Large Numbers with probability 1, and the second term is a lower bound estimate of z^*. Using these estimates, a one-sided $100(1-\alpha)\%$ confidence interval on the estimated optimality gap is constructed via four procedures described in the paper: Multiple Replications Procedure (MRP), Single Replication Procedure (SRP), Independent 2-Replication Procedure (I2RP) and Averaged Two-Replication Procedure (A2RP). The first has been presented in an earlier publication [12], while the remaining three are discussed in this paper. The last two procedures are variants of SRP.

The methodology underlying the four procedures above is similar. One starts with an estimated solution, $\hat{x}$, to be assessed for optimality. Next, one samples a vector ξ_i from the known random vector Ξ, which are used in turn to solve (2.2). With the solution obtained from (2.2) and the estimated solution $\hat{x}$, one constructs the estimated gap, sample variance (s_n^2) and a one-sided confidence interval, using Student's t-distribution with n degrees of freedom and prescribed confidence level, $100(1-\alpha)\%$. The four procedures differ in their sampling methods, number of simulation replications, and formulae for the estimated gap and sample variance. Finally, the paper provides numerical examples (Newsvendor Problem and Two-Stage Stochastic Programming Problem). It also describes potential problems and guidelines for improving the performance of the procedures. Refer to [1] for the detailed steps of each procedure.

3. Conceptual Model

We consider a vendor employing a Make-to-Stock (MTS) inventory policy and selling to a particular set of buyers. The vendor is constrained by a prescribed initial amount of capital available for purchasing inventory and paying other costs, and faces demands it wishes to satisfy. The vendor's corporate treasury manages all financial transactions. The interaction between the supply chain members extends over an infinite time horizon, divided into periods of equal lengths. Inventory and/or financial transactions take place at the boundary points of periods.

Figure 1 depicts schematically the system under consideration. It consists of a focal vendor firm that sells inventory to buyers, is replenished by a tier-1 supplier, is provided with external capital by a funding entity and disgorges dividends to its owners.

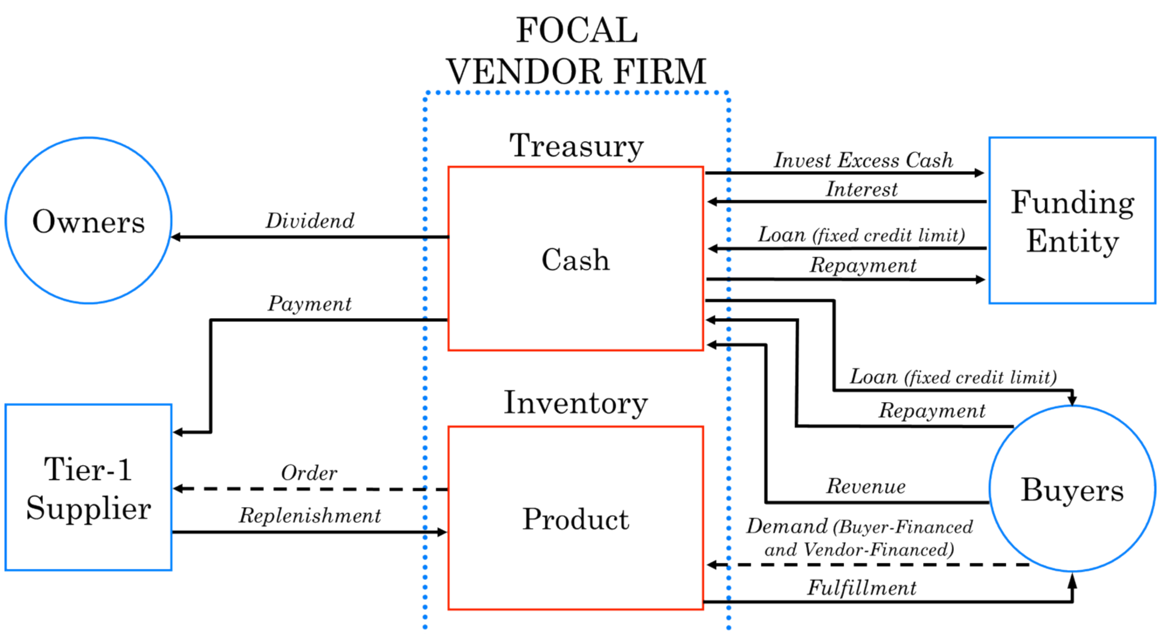

Figure 1. Schematic of a generic system utilizing vendor financing

Here, the focal vendor firm boundary is denoted by the dotted line, and the boxes inside it stand for the vendor's inventory and treasury components. These two components are joined through interactions with external entities such as buyers, a tier-1 supplier (or supplier for short), a funding entity and the owners of the vendor. The solid arrows represent tangible flows, such as product and money, while the dashed arrows represent flows of information. The elements and rules of operation of the system are described as follows:

Inventory operations

- **Demand.** The vendor faces random demand from buyers. Demand is filled at the end of the each period, subject to limited backordering, that is, the number of unfilled orders that could be carried over to the next period is constrained and the rest is lost. Furthermore, the vendor backorders only that portion of the demand shortfall that can be funded at the time of backordering. This backorder quantity is fulfilled (here, defined as delivered to the buyer and paid for by the buyer) at the end of the next period and is funded first before any newly-arrived demand in that period. Any unfilled backorders are carried over to the next period.
- **Replenishment.** At the beginning of each period, the vendor orders inventory for replenishment through a supplier with unlimited capacity. However, there is a 1-period lead time, that is, the actual delivery occurs at the end of the period. The vendor's replenishment order is computed to bring the inventory level as close as possible (funding permitting) to the prescribed MTS base stock level. Orders are financed by first utilizing the vendor's cash on hand, and if insufficient, the balance is financed by debt, which is interest-bearing. The unit cost of ordered product is less than the unit selling price of the product.

Financial operations

- **Net Cash.** The vendor's net cash is the difference between its cash on hand and its debt. Consistent with the Myers and Majluf (1984) Pecking Order theory, the firm will first use cash on hand to fund inventory and then debt. Consequently, if there is any debt outstanding, the firm will immediately use any cash on hand to retire as much of it as possible. As a result, cash on hand and debt are mutually exclusive of one another. The net cash, if positive, earns interest for the previous period, and if negative, pays interest for the previous period.

- **Financing of Buyers Purchases.** Buyers are able to self-fund demand up to a level, which represents the buyers' bank–extended credit limit. Self-funded demand is paid for by the buyers immediately. Any demand above this trade credit limit but less than a vendor-determined threshold (called the vendor–extended credit limit) is financed by the vendor. In all purchases, the buyers' bank–extended credit limit is used up first before dipping into the buyers' vendor–extended credit limit. This vendor financing is in the form of 1-period loans (secured by the purchased goods), each of which is repaid in a lump sum at the end of the respective period plus interest. The interest rate charged by the vendor is higher than the vendor's cost of financing (described below).
- **Financing of Vendor Operations.** The vendor cannot sell additional equity, but has access to external debt capital, subject to a credit limit, called the vendor–borrowing credit limit. If the vendor does not have enough cash on hand to pay for an order, it will borrow at most the amount needed to cover the cost of the MTS-based order, subject to the vendor–borrowing credit limit. This type of borrowing will be referred to as vendor minimal borrowing.
- **Dividend Payout.** All cash exceeding a certain threshold, called the dividend threshold, is distributed to the vendor's shareholders as dividends at the end of each period. Dividends are paid only from the vendor's cash on hand only after all other liabilities are satisfied. It is assumed that the vendor pays out dividends first to the shareholders before checking its inventory level and issuing the replenishment order for the next period.

- **Vendor Bankruptcy.** Bankruptcy occurs at period boundary points whenever vendor resources (the vendor's current cash position plus credit limit plus liquidation value of the vendor's inventory) are insufficient to cover outstanding costs. In the event of bankruptcy, the vendor is liquidated, and otherwise, operations continue (see Section 4 for details).

In our model, the decision variables of the optimal policy are the MTS base stock level and dividend threshold. The goal of the thesis is to identify the optimal policy in terms of these decision variables that maximizes the total expected discounted dividends of the vendor, given its initial inventory and capital and subject to its capital constraints. The optimal MTS policy with vendor financing will be compared and contrasted with: (1) the optimal policy of a vendor employing MTS but offering no vendor financing; and (2) the optimal policy with vendor financing but the vendor has a smaller credit limit.

4. Mathematical Model

In this section, we formulate the mathematical model and establish our notation. The infinite time horizon $\bigcup_{i=1}^{\infty}(\tau_{i-1}, \tau_i]$ is divided into equally-spaced periods, where $(\tau_{i-1}, \tau_i]$ denotes period i, the τ_i are the period boundary points, and $\tau_0 = 0$.

The model uses the following parameters:

- $e_0 > 0$ is the initial cash on hand provided by the owners of the vendor.
- $u_0 > 0$ is the initial size of the vendor's inventory.
- $r_e > 0$ is the simple earned interest rate on the vendor's cash on hand over each period.
- $r_b > 0$ is the interest rate paid by the buyers to the vendor.
- $r_v > 0$ is the interest rate paid by the vendor to the funding entity. We assume $r_e < r_v < r_b$.
- K is the fixed cost incurred by the vendor for each period.
- $c > 0$ is the unit cost of ordered product by the vendor.
- $p_s > 0$ is the unit price of product sold by the vendor. We assume that $c < p_s$.
- $p_f > 0$ is the unit price of forced-sale product. We assume that $p_f < p_s$.
- $h > 0$ is the inventory holding cost per unit inventory per period.
- $g > 0$ is the backordering penalty cost per unit of backordered inventory per period.
- $G_v > 0$ is the vendor–borrowing credit limit of the vendor when borrowing from the funding entity.
- $G_b > 0$ is the vendor–extended credit limit of the buyers when borrowing from the vendor
- $H_b > 0$ is the buyers' bank–extended credit limit.
- $B_{max} > 0$ is the maximal number of backorders. Any orders beyond this value are lost.
- $0 < \beta < 1$ is the discount factor per period.

The exogenous source of randomness is the i.i.d. demand process $\{D_i\colon\ i \geq 0\}$, where D_i is the demand size in period i with the convention $D_0 = 0$. The value of the D_i becomes known at τ_i.

The random processes derived from the exogenous one are defined as follows:

- $\{X_i : i \geq 0\}$ denotes the inventory-size process, where X_i is the vendor's ending inventory at τ_i.
- $\{Z_i : i \geq 0\}$ denotes the inventory order process, where Z_i is the number of units ordered by the vendor at τ_i.
- $\{R_i : i \geq 1\}$ denotes the period revenue process, where R_i is the number of units sold by the vendor multiplied by the unit price of product sold at τ_i.
- $\{C_i : i \geq 0\}$ denotes the ending cash process, where C_i is the vendor's ending cash balance at τ_i. Accordingly, $C_i = C_i^+ + C_i^-$, where C_i^+ and C_i^- are the corresponding vendor's cash on hand and outstanding debt balance, respectively.
- $\{V_i : i \geq 1\}$ denotes the dividends process, where V_i is the amount of dividends paid out (if any) by the vendor at τ_i.

The decision variables of the vendor are as follows.

- $S \geq 0$ denotes the MTS base stock level, which characterizes the inventory policy.
- $T \geq 0$ denotes the dividend threshold which characterizes the dividend policy.

The evolution of the MTS system is described in terms of a state process $\{S_i : i \geq 0\}$, where the state at τ_i is given by

$$S_i = (X_i, C_i, D_i), \tag{4.1}$$

Since the third component above is exogenous, it suffices to describe the system's evolution in terms of the first two components only.

4.1 Mathematical Model with Vendor Financing

Following are the state transitions for the case of vendor financing:

1. **Initialization at time $\boldsymbol{\tau_0 = 0}$.** The system starts with initial cash on hand e_0 and initial inventory u_0. The following transactions are carried out:

   1. Let

   $$X_0 = u_0$$

   $$C_0 = e_0$$

   2. Let

   $$L_0^{(v)} = \min\{[c(S - X_0) - C_0]^+, G_v\}$$

   where $L_0^{(v)}$ is the initial loan (if any) borrowed by the vendor from the funding entity. The equation above follows from the inequalities

   $$L_0^{(v)} \leq [c(S - X_0) - C_0]^+ \text{ and } L_0^{(v)} \leq G_v$$

   3. Finally, let

   $$Z_0 = \min\left\{[S - X_0]^+, \frac{L_0^{(v)} + C_0}{c}\right\}$$

   The equation above follows from the inequalities

   $$Z_0 \leq [S - X_0]^+ \text{ and } Z_0 \leq \frac{L_0^{(v)} + C_0}{c}$$

   since the right hand side in the first inequality is the nominal MTS-based order size, while its counterpart in the second inequality is the order size the vendor can afford.

B. State Transition to $\boldsymbol{\tau_1 = 1}$.

1. Let

$$X_1^{(b)} = X_0 + Z_0$$

$$C_1^{(b)} = C_0 - cZ_0$$

where $X_i^{(b)}$, $i \geq 1$, is the vendor's intermediate inventory level after replenishment arrives at τ_i and $C_i^{(b)}$, $i \geq 1$, is the vendor's intermediate cash balance after subtracting the cost of that replenishment.

2. Let

$$D_1^{(f)} = \min\left\{D_1, \frac{H_b + G_b}{p_s}\right\} \tag{4.2}$$

where $D_i^{(f)}$, $i \geq 1$, is the total fundable demand that could be financed by the buyers through their bank-extended line of credit as well as any vendor-extended financing at τ_i.

3. Let

$$D_1^{(atf)} = \min\left\{X_1^{(b)}, D_1\right\}$$

where $D_i^{(atf)}$, $i \geq 1$, is the portion of demand that is available for immediate fulfillment from the vendor's inventory on hand at τ_i.

4. Let

$$D_1^{(bf)} = \min\left\{D_1^{(atf)}, \frac{H_b}{p_s}\right\}$$

$$D_1^{(vf)} = \min\left\{D_1^{(atf)} - D_1^{(bf)}, \frac{G_b}{p_s}\right\} \tag{4.3}$$

where $D_i^{(bf)}$, $i \geq 1$, is the portion of D_i that is actually financed by the buyers at τ_i, and $D_i^{(vf)}$, $i \geq 1$, is the portion of D_i that is actually financed by the vendor at τ_i. Note that the sum, $D_i^{(bf)} + D_i^{(vf)}$, is the amount of demand actually fulfilled at τ_i.

5. Let

$$D_1^{(bo)} = \min\left\{D_1^{(f)} - \left(D_1^{(bf)} + D_1^{(vf)}\right), B_{max}\right\} \tag{4.4}$$

where $D_i^{(bo)}$, $i \geq 1$, is the portion of D_i that is actually backordered by the vendor at τ_i.

6. Let

$$X_1^{(d)} = X_1^{(b)} - \left(D_1^{(bf)} + D_1^{(vf)}\right) - D_1^{(bo)} \tag{4.5}$$

where $X_i^{(d)}$, $i \geq 1$, is vendor's intermediate inventory level after subtracting fulfillment (if any) and backorders (if any) at τ_i.

7. Let

$$L_1^{(b)} = p_s D_1^{(vf)} \tag{4.6}$$

where $L_i^{(b)}$, $i \geq 1$, is the loan (if any) extended by the vendor to the buyers at τ_i.

8. Let

$$R_1 = p_s D_1^{(bf)}$$

9. Let

$$C_1^{(d)} = C_1^{(b)} - \left[hW_1 + K + gD_1^{(bo)} + r_v L_0^{(v)}\right] + \left[R_1 + r_e C_0^+\right]$$

where $C_i^{(d)}$, $i \geq 1$, is the intermediate cash after subtracting period costs and penalties (holding and fixed costs, backordering penalties and interest owed) and adding period earnings (revenue from fulfilled demand and interest(s) earned) and $W_i = \frac{X_{i-1}^+ + \left[X_i^{(d)}\right]^+}{2}$, $i \geq 1$, is an approximate inventory time average over period i.

10. Let the vendor's bankruptcy condition be given by

$$C_1^{(d)} + G_v + p_f \left[X_1^{(d)}\right]^+ < 0$$

Namely, the sum of the vendor's resources (intermediate cash, vendor's credit limit, and the liquidation value of vendor's inventory on hand) is negative. If this condition holds, then the vendor declares bankruptcy, and the system transitions to an absorbing state. Otherwise, operations continue.

11. Let

$$X_1^{(s)} = \frac{-\left[C_1^{(d)} + G_v\right]^-}{p_f}$$

where $X_i^{(s)}$, $i \geq 1$, is the minimal forced-sale portion of $X_1^{(d)}$ that raises just enough funds to avoid bankruptcy (that is, to cover debt in excess of the vendor's credit limit) at τ_i.

12. Let

$$V_1 = \left[C_1^{(d)} - T\right]^+$$

13. Let

$$X_1 = X_1^{(d)} - X_1^{(s)}$$

$$C_1 = \left(C_1^{(d)} + p_f X_1^{(s)}\right) - V_1$$

14. Let

$$L_1^{(v)} = \begin{cases} 0, & \text{if } c(S - X_1) \le C_1 \\ \min[c(S - X_1) - C_1, G_v], & \text{if } 0 < C_1 < c(S - X_1) \\ \min[c(S - X_1), G_v + C_1], & \text{if } -G_v < C_1 \le 0 \\ 0, & \text{if } C_1 \le -G_v \end{cases}$$

where $L_i^{(v)}$, $i \ge 1$, is the loan (if any) borrowed by the vendor from the funding entity at τ_i.

15. Let

$$Z_1 = \min\left\{[S - X_1]^+,\ \frac{L_1^{(v)} + C_1^+}{c}\right\}$$

This equation follows from the inequalities $Z_i \le [S - X_1]^+$ and $Z_i \le \frac{L_i^{(v)} + C_1^+}{c}$, since the right hand side in the first inequality is the nominal MTS–based order size, while its counterpart in the second inequality is the order size the vendor can fund.

16. Finally, let

$$B_1^{(d)} = \min\{-X_1^-, Z_1\}$$

where $B_i^{(d)}$, $i \ge 1$, is portion of the backorder quantity that could be fulfilled by the order Z_i which arrives as replenishment at τ_{i+1}.

C. State Transition to $\tau_i, i \ge 2$.

1. Let

$$X_i^{(b)} = X_{i-1} + Z_{i-1}$$

$$C_i^{(b)} = C_{i-1} - cZ_{i-1}$$

2. Let

$$D_i^{(f)} = \min\left\{D_i, \left[\frac{H_b + \left(G_b - L_{i-1}^{(b)}\right)}{p_s} - B_{i-1}^{(d)}\right]^+\right\} \tag{4.7}$$

Recall that funding backorders from the previous period takes precedence over any newly-arrived demand in the current period.

3. Let

$$D_i^{(atf)} = \min\left\{\left[X_i^{(b)}\right]^+, D_i\right\}$$

Note that if there are still unfilled backorders after replenishment, that is, $X_i^{(b)} < 0$, then the vendor has no available inventory for immediate fulfillment of new demand.

4. Let

$$D_i^{(bf)} = \min\left\{D_i^{(atf)}, \left[\frac{H_b}{p_s} - B_{i-1}^{(d)}\right]^+\right\}$$

$$D_i^{(vf)} = \min\left\{D_i^{(atf)} - D_i^{(bf)}, \frac{\left(G_b - L_{i-1}^{(b)}\right)}{p_s} + \left[\frac{H_b}{p_s} - B_{i-1}^{(d)}\right]^-\right\} \tag{4.8}$$

where we use the funds pecking order of Section 3 in the equations above, that is, the buyer uses its credit line with the bank before any vendor-extended credit, which is more expensive.

5. Let

$$D_i^{(bo)} = \min\left\{D_i^{(f)} - \left(D_i^{(bf)} + D_i^{(vf)}\right), B_{max} + \left[X_i^{(b)}\right]^-\right\} \tag{4.9}$$

This equation follows from the fact that any unfilled backorders are carried over to the next period (recall from Section 3), so the maximum amount that the vendor can backorder for the current period is $B_{max} + \left[X_i^{(b)}\right]^-$, where $\left[X_i^{(b)}\right]^-$ is the unfilled backorders from period $i - 1$.

6. Let

$$X_i^{(d)} = X_i^{(b)} - \left(D_i^{(bf)} + D_i^{(vf)}\right) - D_i^{(bo)} \tag{4.10}$$

7. Let

$$L_i^{(b)} = p_s D_i^{(vf)} \tag{4.11}$$

8. Let

$$R_i = p_s\left[D_i^{(bf)} + D_{i-1}^{(vf)} + B_{i-1}^{(d)}\right] \tag{4.12}$$

Note that the revenue above includes payment for buyer-financed demand for the current period, the principal of the vendor financing from the previous period, and payment for the filled backorders that were rolled over from the previous period.

9. Let

$$C_i^{(d)} = C_i^{(b)} - \left[hW_i + K + gD_i^{(bo)} + r_v\left(-C_{i-1}^{-} + L_{i-1}^{(v)}\right)\right] + \left[R_i + r_e C_{i-1}^{+} + r_b L_{i-1}^{(b)}\right] \tag{4.13}$$

Note that period costs and penalties include holding and fixed costs, backordering penalties and interest owed from the vendor's total outstanding debt, and period earnings include the previously computed revenue and the interest earned from vendor's cash on hand and vendor financing.

10. The vendor's bankruptcy condition,

$$C_i^{(d)} + G_v + p_f\left[X_i^{(d)}\right]^{+} < 0 \tag{4.14}$$

is checked. If this condition holds, then the vendor declares bankruptcy, and the system transitions to an absorbing state. Otherwise, operations continue.

11. Let

$$X_i^{(s)} = \frac{-\left[C_i^{(d)} + G_v\right]^{-}}{p_f}$$

12. Let

$$V_i = \left[C_i^{(d)} - T\right]^{+}$$

13. Let

$$X_i = X_i^{(d)} - X_i^{(s)}$$

$$C_i = \left(C_i^{(d)} + p_f X_i^{(s)}\right) - V_i$$

14. Let

$$L_i^{(v)} = \begin{cases} 0, & \text{if } c(S - X_i) \le C_i \\ \min[c(S - X_i) - C_i, G_v], & \text{if } 0 < C_i < c(S - X_i) \\ \min[c(S - X_i), G_v + C_i], & \text{if } -G_v < C_i \le 0 \\ 0, & \text{if } C_i \le -G_v \end{cases}$$

15. Let

$$Z_i = \min\left\{[S - X_i]^+,\ \frac{L_i^{(v)} + C_i^+}{c}\right\}$$

16. Finally, let

$$B_i^{(d)} = \min\{-X_i^-, Z_i\}$$

Figure 2 summarizes the sequence of inventory and monetary transactions at each period boundary point. The columns of Figure 2 outline a series of transactions that take place at time at τ_i in the order of the vertical arrows. The horizontal arrow points to the contiguous period to which the transactions are associated with.

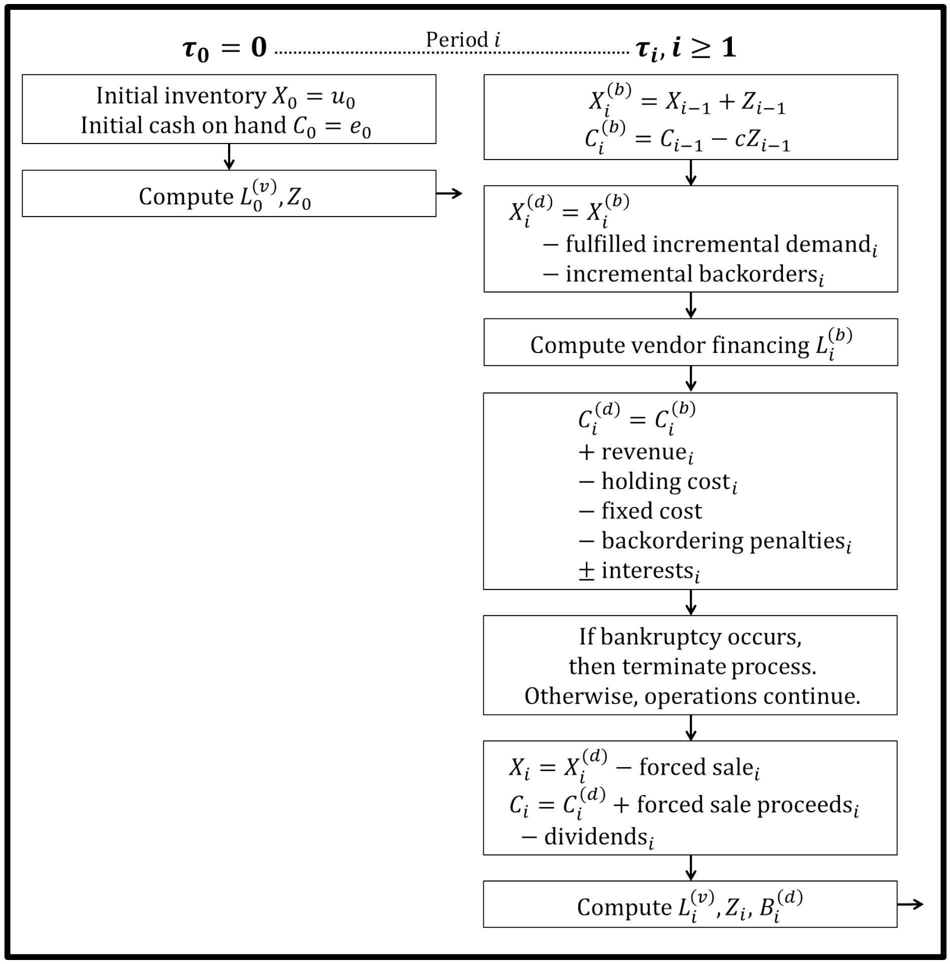

Figure 2. Outline of inventory and monetary transactions at period boundaries (with vendor financing)[1]

The objective function is the sum of the conditional expected discounted dividends paid out by the vendor over an infinite time horizon, given the initial inventory size and initial capital, that is,

$$J(S,T) = \mathbb{E}\left[\textstyle\sum_{i=1}^{\infty} \beta^i V_i \middle| S_0 = s_0\right]$$

The goal is to find the optimal pair (S^*, T^*) that optimizes the objective function above, yielding the optimal objective function value

$$J^* = J(S^*, T^*).$$

[1] Note that the computation for revenue at τ_1 differs from its computation at $\tau_i, i > 1$ (see Equation 4.13 for details).

4.2 Mathematical Model without Vendor Financing

The state transitions for the case of no vendor financing are the same as for the case of vendor financing but with the following exceptions:

Deletions. The following equations are not included:

1. Equation (4.3)
2. Equation (4.6)
3. Equation (4.8)
4. Equation (4.11)

Modifications. The following equations (and inequalities) are modified as follows:

1. Equation (4.2) becomes

$$D_1^{(f)} = \min\left\{D_1, \frac{H_b}{p_s}\right\} \tag{4.15}$$

2. Equation (4.4) becomes

$$D_1^{(bo)} = \min\left\{D_1^{(f)} - D_1^{(bf)}, B_{max}\right\} \tag{4.16}$$

3. Equation (4.5) becomes

$$X_1^{(d)} = X_1^{(b)} - D_1^{(bf)} - D_1^{(bo)} \tag{4.17}$$

4. Equation (4.7) becomes

$$D_i^{(f)} = \min\left\{D_i, \left[\frac{H_b}{p_s} - B_{i-1}^{(d)}\right]^+\right\} \tag{4.18}$$

5. Equation (4.9) becomes

$$D_i^{(bo)} = \min\left\{D_i^{(f)} - D_i^{(bf)}, B_{max} + \left[X_i^{(b)}\right]^-\right\} \tag{4.19}$$

6. Equation (4.10) becomes

$$X_i^{(d)} = X_i^{(b)} - D_i^{(bf)} - D_i^{(bo)} \tag{4.20}$$

7. Equation (4.12) becomes

$$R_i = p_s\left[D_i^{(bf)} + B_{i-1}^{(d)}\right] \tag{4.21}$$

8. Finally, equation (4.13) becomes

$$C_i^{(d)} = C_i^{(b)} - \left[hW_i + K + gD_i^{(bo)} + r_v\left(-C_{i-1}^{-} + L_{i-1}^{(v)}\right)\right] + \left[R_i + r_e C_{i-1}^{+}\right] \tag{4.22}$$

Figure 3 summarizes the sequence of inventory and monetary transactions at each period boundary point. The columns of Figure 3 outline a series of transactions that take place at time at τ_i in the order of the vertical arrows. The horizontal arrow points to the contiguous period to which the transactions are associated with.

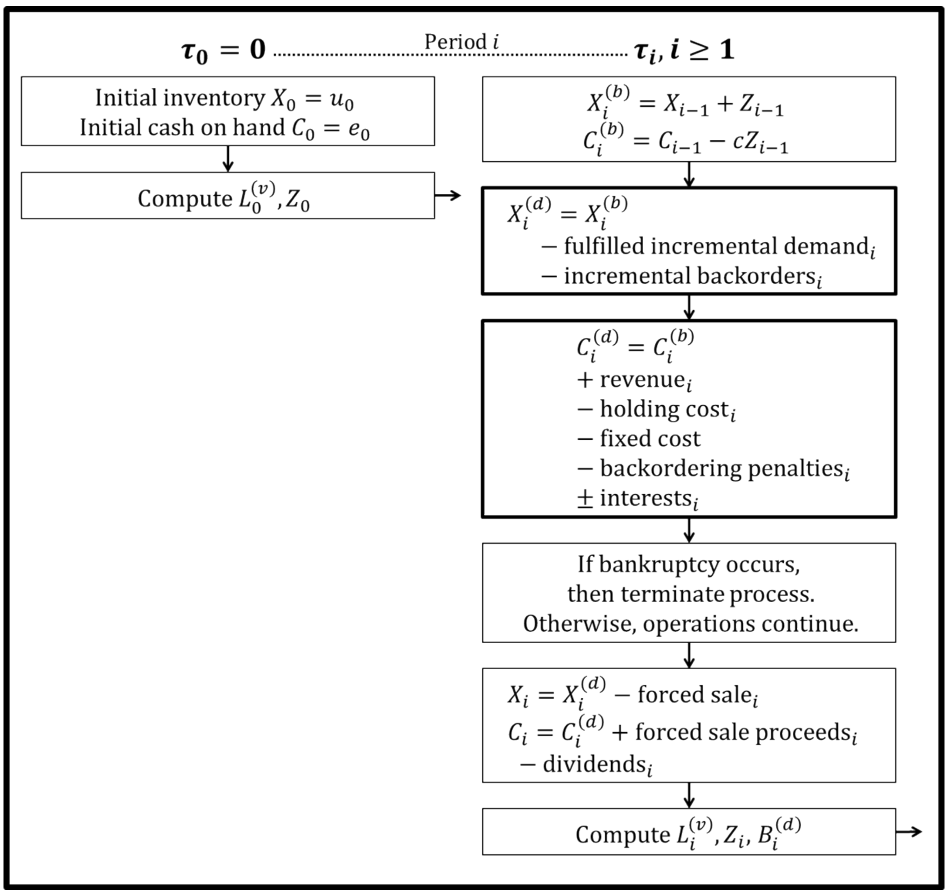

Figure 3. Outline of inventory and monetary transactions at period boundaries (no vendor financing)[2]

[2] The transactions with thicker borders represent the modified steps for this case (see Section 4.2 for details)

5. Results and Analysis

This section describes the solution methodology, presents the results and provides an analysis thereof.

5.1 Methodology

This subsection describes the simulation-based optimization methodology used to obtain the optimal base stock level and dividend threshold, solution quality-assessment procedures, and approach to application of a naïve policy – a policy wherein there is no restriction or reduction in the baseline vendor-borrowing credit limit

First, eleven cases of the stochastic optimization model are considered and solved: 10 cases with vendor financing (called Case 1 through Case 10) and one case with no vendor financing (Case 11). The set of parameters used for all 11 cases are identical except for Case 2 – 10, where vendor-borrowing credit limits differ, ranging from 90% to 10% of Case 1. The Palisade Corporation's DecisionTools Suite - RiskOptimizer 5.5 [16] was used as an optimization tool. RiskOptimizer 5.5 uses Monte-Carlo simulation and Genetic Algorithm to generate random samples and optimize the decision variables. For each case, the optimization engine was run for over 2000 simulations (more than 2.5 hours per case). The summary logs of each optimization were included in the Appendix.

Second, three procedures were implemented to assess solution quality (as discussed in Section 2.2), namely, SRP, I2RP and A2RP. For each procedure, $N=200$ random samples were generated per replication to construct one-sided confidence intervals for the optimality gap (2.3) at confidence level $\alpha=0.05$. For variance reduction purposes [1], the same random samples were used in each procedure and each case. The Palisade Corporation's DecisionTools Suite - Evolver 5.5 [15] was used to solve the approximate stochastic problem (2.2) in each procedure and construct the aforementioned confidence intervals. The computations involved in the construction of the confidence intervals (optimal gap estimate, sample variance etc.) in each case (as discussed in [1]) were included in the Appendix.

Finally, the optimal policy of Case 1 was applied to Case 6 – 10 in other to gauge the sub-optimality of the objective function values when a naïve policy was applied. A Monte-Carlo simulation (5000 iterations) was run for each case in order to compute random samples of the objective function. For each case, the resultant histogram of random objective function values and their respective mean, standard deviation and coefficient of variation were collected, as well as the percentage deviation from the optimal values. The Palisade Corporation's DecisionTools Suite - @Risk 5.5 [17] was used to run the Monte-Carlo simulations. The histograms were shown in the Appendix.

The following model parameters were used in all cases: 100 periods, $e_0 = 10$, $u_0 = 10$, $r_e = 0.02$, $r_b = 0.08$, $r_v = 0.06$, $K = 3$, $c = 1$, $p_s = 1.4$, $p_f = 0.84$, $h = 0.2$, $g = 0.1$, $G_b = 12$, $H_b = 10$, $B_{max} = 7$, $\beta = 0.10$. Each demand process $\{D_i\colon\ i \geq 0\}$ was assumed to follow Poisson distribution with rate $\lambda = 15$. Finally, $G_v = 20$ was the baseline vendor-borrowing credit limit used in Case 1, with Case 2 – 10 using a decreasing percentage of this value.

5.2 Results

This subsection presents the results of the aforementioned optimization, solution assessment and simulation procedures.

Table 1 displays the results of the 11 cases: the optimal decision variables, S^* and T^*, and the optimal objective function value, J^*, as well as the one-sided 95% confidence for the optimality gap for each case. Additionally, Figure 4 depicts the optimal decision variables and corresponding optimal objective function values as function of the vendor-borrowing credit limit.

	S*	T*	J*	SRP CI	I2RP CI	A2RP CI
Case 1 – with vendor financing	11.418	0	22.633	[0.0, 0.005]	[0.0, 0.005]	[0.0, 0.004]
Case 2 – with vendor financing (90% of Borrowing Credit Limit)	11.428	0	22.660	[0.0, 0.00007]	[0.0, 0.004]	[0.0, 0.002]
Case 3 – with vendor financing (80% of Borrowing Credit Limit)	11.421	0	22.616	[0.0, 0.002]	[0.0, 0.005]	[0.0, 0.003]
Case 4 – with vendor financing (70% of Borrowing Credit Limit)	11.428	0	22.937	[0.0, 0.0002]	[0.0, 0.0007]	[0.0, 0.003]
Case 5 – with vendor financing (60% of Borrowing Credit Limit)	11.427	0.003	23.354	[0.0, 0.002]	[0.0, 0.002]	[0.0, 0.002]
Case 6 – with vendor financing (50% of Borrowing Credit Limit)	11.429	1.429	23.262	[0.0, 0.00008]	[0.0, 0.00008]	[0.0, 0.00007]
Case 7 – with vendor financing (40% of Borrowing Credit Limit)	11.437	3.427	22.993	[0.0, 0.004]	[0.0, 0.003]	[0.0, 0.003]
Case 8 – with vendor financing (30% of Borrowing Credit Limit)	11.429	5.429	22.607	[0.0, 0.00005]	[0.0, 0.00005]	[0.0, 0.00004]
Case 9 – with vendor financing (20% of Borrowing Credit Limit)	11.435	7.427	22.240	[0.0, 0.003]	[0.0, 0.003]	[0.0, 0.002]
Case 10 – with vendor financing (10% of Borrowing Credit Limit)	11.435	9.425	21.846	[0.0, 0.003]	[0.0, 0.003]	[0.0, 0.003]
Case 11 – with no vendor financing	7.143	0	16.244	[0.0, 0.00004]	[0.0, 0.00004]	[0.0, 0.00004]

Table 1. Optimization results and 95% confidence intervals for the optimality gap

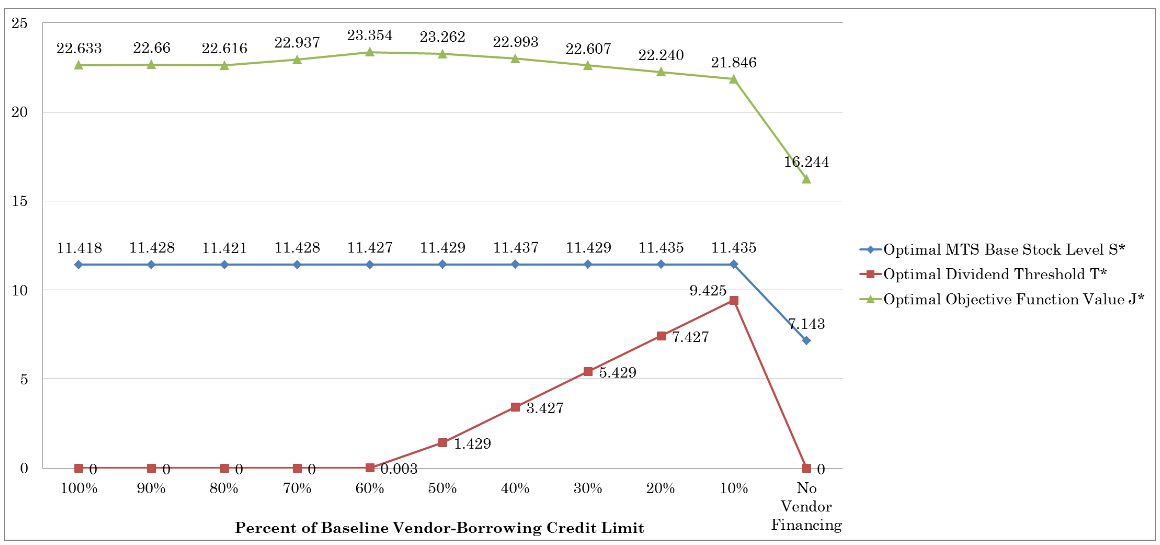

Figure 4. Graph of the optimal decision variables and optimal objective values

The results of Table 1 and Figure 4 indicate that S^* is generally constant in the vendor-borrowing credit limit for cases with vendor financing. For T^*, Case 1 – 4 have optimal values of 0, but for Case 5 – 10, the optimal values are positive and exhibit an increasing trend. For

J^*, the optimal values exhibit a gentle increasing trend for Case 1 – 5 and a gentle decreasing trend for Case 6 – 10. However, for Case 11, S^* and J^*, are significantly lower.

Table 2 displays the results when the optimal policy in Case 1 is applied to cases with 50% to 90% reductions in the vendor-borrowing credit limits (Case 6 – 10). The values of J^* are the mean of the resultant distributions from the aforementioned simulation runs. Table 2 also includes the standard deviation and coefficient of variation of these distributions and the percentage deviation from the initial values of J^*.

	Mean J*	Standard Deviation	Coefficient of Variation	Percent Decrease
Case 6 – with vendor financing (50% of Borrowing Credit Limit)	18.729	0.785	0.0419	19.49%
Case 7 – with vendor financing (40% of Borrowing Credit Limit)	16.855	0.360	0.0214	26.70%
Case 8 – with vendor financing (30% of Borrowing Credit Limit)	17.110	0.508	0.0297	24.32%
Case 9 – with vendor financing (20% of Borrowing Credit Limit)	16.503	0.384	0.0233	25.96%
Case 10 – with vendor financing (10% of Borrowing Credit Limit)	16.051	0.260	0.0162	26.60%

Table 2. Suboptimal values of J^* (Case 1 policy applied to Case 6 – 10)

The results of Table 2 indicate that for Case 6 – 10, there is at least a 19% decrease in J^*. Case 7 displays the largest percent decrease, while Case 8 – 10 exhibit a gentle increasing trend.

5.3 Analysis

This subsection presents the analysis and discussion of the results in four parts: (1) assessment of the computed solutions; (2) comparison and contrast between vendor financing and no vendor financing; (3) comparison and contrast of cases with vendor financing but with varying vendor-borrowing credit limit; and (4) implications of the applied naïve policy.

5.3.1 Assessment of Computed Solutions

The narrow-width confidence intervals for each case indicate that the computed solution (S^*, T^*) are of high quality or near optimal, given the assigned confidence level of 95%. Recall that the constructed confidence intervals are for the estimated upper bound of the optimality gap, defined in (2.3). Therefore, tight confidence intervals are good solution-quality assessors.

5.3.2 Vendor Financing vs. No Vendor Financing

We first consider the two extremal cases, Case 1 and 11, which correspond to vendor financing and no vendor financing, respectively. Contrasting the results in Table 1, we see that the two aforementioned cases differ dramatically in their values of S^* and J^*. More specifically, Case 1 produced much larger values for S^* and J^* than Case 11. These results highlight the primary effect of vendor financing – enhanced profits. With vendor financing, the vendor is able to finance the satisfaction of additional demand that would otherwise be lost. The vendor orders more inventory to capitalize on the additional demand and enjoys a higher gain in the process. However, when there is no vendor financing, the vendor only orders inventory to the extent that its buyers can afford.

We also observe an optimal dividend threshold of 0 for Case 1 and 11. This result is in accordance with our objective of maximizing the total expected discounted dividends. With a discount factor of $\beta = 0.10$, relatively cheap cost of capital (6%) and no restriction on the vendor-borrowing credit limit, the vendor would be motivated to pay out more dividends so as to maintain a cash balance of 0 at the end of each period, and just borrow money to finance any orders for the next period.

5.3.3 Vendor Financing With Varying Vendor-Borrowing Credit Limit

Next, we consider the cases with vendor financing but with varying vendor-borrowing credit limits (Case 1 – 10). We observe that the 10 cases produced values of S^* that varied little, despite the diminishing vendor-borrowing credit limits.

Analyzing the results for T^*, we observe that Case 1 – 4 yield an optimal value of 0. This suggests that the reduced vendor-borrowing credit limit in these cases did not constitute binding constraints on vendor borrowing (that is, optimal vendor borrowing does not exceed its credit limit). Thus, the vendor is able to continue operations optimally while maintaining a zero cash balance at the end of each period. However, once these reductions in the vendor-borrowing credit limit lower it sufficiently (as in Case 5 – 10), the relative constancy of S^* is accompanied by a higher value of T^* to maintain optimality. Table 1 shows that the effect of reduced vendor-borrowing credit limit is more conspicuous in T^* than in S^*.

We next consider the optimal policies for Case 5 – 10. As mentioned above, a higher reduction in the vendor-borrowing credit limit leads to a higher T^*, which drives the vendor to hoard some cash. Table 1 shows that this higher T^* was accompanied by a small increase in S^*.

As mentioned before, to maintain optimality, reduction in the vendor-borrowing credit limit is accompanied by an increase in T^*. However, this increase in T^* reduces the value of J^*, because less cash is allocated to dividends in each period. Table 1 shows how T^* impacts J^*. On the other hand, the aforementioned gentle decreasing trend in J^* implies that the vendor is still able to pay out relatively large dividends despite having very low credit limit.

The results of Table 1 highlight the tension between increased profits and extended vendor financing. The potential gain from vendor's enhanced profits is offset not only by delayed payments from buyers but also by a diminishing vendor-borrowing credit limit. The vendor is forced to hold on to more cash due to delayed receivables from vendor financing and reduced borrowing power, which leads to lower dividend payouts.

5.3.4 Vendor Financing – Application of the Naïve Policy

Finally, we applied the naïve policy of Case 1 to Case 6 – 10 where the reduction in the vendor-borrowing credit limit diminishes J^*. The results of Table 2 show that applying this naïve policy give rise to suboptimal values of J^* because the vendor disgorges more cash as dividends than optimality calls for. In the same table, we observe a higher percent decrease in Case 7 – 10 than in Case 6. We observed that in Case 7 – 10, higher reductions in the credit limit and deviations of the naïve policy from the optimal policy lead to a more substantial decrease of J^*.

The results of Table 2 demonstrate the consequences of deviating from the optimal policy by substituting for it the naïve one, thereby ignoring the effect of the vendor-borrowing credit limit. In such cases, the vendor's attempt to increase profitability by extending vendor financing to buyers is severely impacted by substantial reductions in the vendor-borrowing credit limit.

6. Conclusion

The published literature does not address vendor financing and its interaction with vendor-borrowing credit limits. To fill this lacuna, we developed a detailed and realistic model of a vendor's inventory and treasury, subject to a joint supply/financial policy concerning optimal replenishment and optimal dividend distribution.

We found that vendor financing is beneficial as evidenced by the results of Section 5.2, but careful consideration of the financial capabilities of both the vendor and the buyers is called for. The results of Table 1 and 2 demonstrate the potential risks inherent in vendor financing in the presence of limited vendor-borrowing credit limits, as well as uncritical application of a naïve supply/finance policy. Our results underscore the importance of coordinating logistical and financial decisions to attain improved optimization of a supply chain performance.

To sum up, the following insights have been gleaned from our analysis of vendor financing:

(1) Vendor financing enhances profitability provided there is no substantial reduction in the vendor-borrowing credit limit.

(2) Higher reductions in the vendor-borrowing credit limit have a major impact on the optimal dividend threshold. In our model, the vendor maintains a particular optimal MTS base stock level, but simultaneously increases the optimal dividend threshold.

(3) Naïve inventory and dividend policies can yield substantially suboptimal dividend payouts, especially in the presence of substantial reductions in the vendor-borrowing credit limit.

7. References

[1] Bayraksan, Güzin, and David P. Morton. "Assessing Solution Quality in Stochastic Programs." *Mathematical Programming* A and B 108.2 (2006): 495-514.

[2] Bayraksan, Güzin, David P. Morton, and Amit Partani. "Simulation-Based Optimality Tests for Stochastic Programs." Ed. Gerd Infanger. *Stochastic Programming: The State of the Art, in Honor of George B. Dantzig*. 2011 ed. Vol. 150. New York: Springer, 2011. 37-55. International Ser. in Operations Research & Management Science. Web.

[3] Brennan, Michael J., Vojislav Maksimovic, and Josef Zechner. "Vendor Financing." *The Journal of Finance* 43.5 (1988): 1127-141.

[4] Cuñat, Vicente. "Trade Credit: Suppliers as Debt Collectors and Insurance Providers." *The Review of Financial Studies* 20.2 (2007): 491-527.

[5] Evans, Jocelyn, and Timothy Koch. "Surviving Chapter 11: Why Small Firms Prefer Supplier Financing." *Journal of Economics and Finance* 32.2 (2007): 186-206.

[6] Fabbri, Daniela, and Leora F. Klapper. “Trade Credit and the Supply Chain.” Working Paper (2011). Cass Business School.

[7] Fisman, Raymond, and Inessa Love. "Trade Credit, Financial Intermediary Development, and Industry Growth." *The Journal of Finance* 58.1 (2003): 353-74.

[8] Gassmann, Horand I. "MSLIP: A Computer Code for the Multistage Stochastic Linear Programming Problem." *Mathematical Programming* 47.1-3 (1990): 407-23.

[9] Kleywegt, Anton J., and Alexander Shapiro. "The Sample Average Approximation Method for Stochastic Discrete Optimization." *SIAM Journal on Optimization* 12.2 (2002): 479-502.

[10] Kouvelis, Panos, and Wenhui Zhao. "Financing the Newsvendor: Supplier vs. Bank, and the Structure of Optimal Trade Credit Contracts." *Operations Research* 60.3 (2012): 566-80.

[11] Li, Lode, Shubik, Martin and Sobel, Matthew J., “Control of Dividends, Capital Subscriptions, and Physical Inventories” (February 2005). Available at SSRN: http://ssrn.com/abstract=690864

[12] Mak, Wai-Kei, David P. Morton, and R. Kevin Wood. "Monte Carlo Bounding Techniques for Determining Solution Quality in Stochastic Programs." *Operations Research Letters* 24 (1999): 47-56.

[13] Myers, Stewart C., and Nicholas S. Maljuf. "Corporate Financing and Investment Decisions When Firms Have Information Investors Do Not Have." *Journal of Financial Economics* 13 (1984): 187-221.

[14] Moffett, Michael H., Arthur I. Stonehill, and David K. Eiteman. "Mini-Case: Motorola & TelSim of Turkey." *Fundamentals of Multinational Finance*. Boston, MA: Prentice Hall, 2011. 398-99. Print

[15] Palisade Corporation. *Guide to Using Evolver*. N.p.: Palisade Corporation, 2010. *Palisade*. Sept. 2010. Web. <http://www.palisade.com/downloads/manuals/EN/Evolver5_EN.pdf>.

[16] Palisade Corporation. *Guide to Using Risk Optimizer*. N.p.: Palisade Corporation, 2010. *Palisade*. Sept. 2010. Web. <http://www.palisade.com/downloads/manuals/EN/ RISKOptimizer5_EN.pdf>.

[17] Palisade Corporation. *Guide to Using @Risk*. N.p.: Palisade Corporation, 2010. *Palisade*. Sept. 2010. Web. <http://www.palisade.com/downloads/manuals/EN/RISK5_EN.pdf>.

[18] Petersen, Michell A., and Raghuram G. Rajan. "Trade Credit: Theories and Evidence." *The Review of Financial Studies* 10.3 (1997): 661-91.

[19] Wang, Min. "Supply Chain Management: Supplier Financing Schemes and Inventory Strategies." Diss. Columbia University, 2011. *Columbia University Academic Commons*. Web. May 2012. <http://academiccommons.columbia.edu/catalog/ac:142635>.

[20] Williams, Tish. "Vendor Financing Turns Lose-Lose for Motorola." *TheStreet.com*. N.p., 15 May 2001. Web. 11 Nov. 2012. <http://www.thestreet.com/story/1429838/1/vendor-financing-turns-lose-lose-for-motorola.html>.

[21] Wilson, N., B. Summers. “Trade credit firms offered by small firms: Survey evidence and empirical analysis.” *Journal of Business Finance & Accounting* 29.3 (2002): 317-351.

8. Appendix

Simulation	Elapsed Time	Iterations	Result	Goal Cell Statistics				Adjustable Cells	
				Mean	Std. Dev.	Min.	Max.	B33	B34
1	0:00:03	200	14.5273	14.5273	1.0988	10.0136	16.3032	15	1
14	0:00:14	200	22.0105	22.0105	1.4801	17.1727	24.8004	11.63681975	1
144	0:01:54	200	22.2237	22.2237	1.4621	17.4581	24.9824	11.63681975	0
155	0:02:03	200	22.3234	22.3234	1.4430	17.6808	25.0740	11.25	1
172	0:02:18	200	22.5386	22.5386	1.4259	17.9579	25.2560	11.25	0
476	0:09:01	1200	22.5899	22.5899	1.2917	16.0843	25.0818	11.09091391	0
545	0:12:09	1200	22.5930	22.5930	1.2967	16.0690	25.0987	11.10636146	0
573	0:13:23	1200	22.6080	22.6080	1.3203	15.9977	25.1774	11.17818073	0
590	0:14:14	1300	22.6122	22.6122	1.3200	15.9903	25.1854	11.18554943	0
623	0:15:58	1300	22.6152	22.6152	1.3244	15.9769	25.2003	11.19914059	0
640	0:16:40	1300	22.6242	22.6242	1.3383	15.9352	25.2462	11.24106227	0
958	0:38:19	1500	22.6305	22.6305	1.3846	15.7559	25.4604	11.41806172	0.01013788
970	0:39:28	1500	22.6327	22.6327	1.3844	15.7597	25.4623	11.41806172	0

Table 3. Summary of optimization progress steps (Case 1)

Simulation	Elapsed Time	Iterations	Result	Goal Cell Statistics				Adjustable Cells	
				Mean	Std. Dev.	Min.	Max.	B33	B34
1	0:00:03	200	19.3433	19.3433	1.1989	13.7048	21.5174	13	1
3	0:00:05	200	21.1694	21.1694	0.8041	17.3980	22.3243	9.4806078	1
20	0:00:18	200	21.6526	21.6526	0.9644	17.7236	23.1972	10.0092868	1
116	0:01:31	200	21.9069	21.9069	0.9549	17.9952	23.4487	10.0092868	0
207	0:02:49	200	22.4982	22.4982	1.2500	17.7639	24.6855	10.92413988	0
429	0:06:45	400	22.5028	22.5028	1.3640	15.9671	25.0504	11.2403039	0
502	0:08:11	400	22.5118	22.5118	1.3808	15.9276	25.1017	11.29361545	0
546	0:09:06	800	22.6305	22.6305	1.3689	15.8538	25.3487	11.3931188	0
591	0:10:24	800	22.6352	22.6352	1.3801	15.8271	25.3842	11.42879506	0
661	0:12:52	1100	22.6474	22.6474	1.2946	15.9345	25.3886	11.28429832	0
714	0:15:43	1800	22.6521	22.6521	1.3241	15.8707	25.5081	11.37037163	0
723	0:16:12	1800	22.6600	22.6600	1.3423	15.8271	25.5884	11.42879506	0
792	0:20:13	1800	22.6604	22.6604	1.3422	15.8278	25.5884	11.42819829	0

Table 4. Summary of optimization progress steps (Case 2)

Simulation	Elapsed Time	Iterations	Result	Goal Cell Statistics				Adjustable Cells	
				Mean	Std. Dev.	Min.	Max.	B33	B34
1	0:00:03	200	21.3558	21.3558	1.3876	15.9844	24.0670	12	1
19	0:00:25	200	21.9322	21.9322	1.4209	16.5866	24.7460	11.67216015	1
101	0:01:50	200	22.5192	22.5192	1.4064	17.4847	25.3739	11.45545145	0
151	0:02:31	200	22.5326	22.5326	1.3280	17.7237	25.1450	11.20390219	0
187	0:02:59	200	22.5470	22.5470	1.3570	17.6554	25.2446	11.28260264	0
215	0:03:25	200	22.5561	22.5561	1.3851	17.5885	25.3441	11.36130308	0
229	0:03:37	200	22.5605	22.5605	1.3994	17.5552	25.3939	11.4006533	0
243	0:03:49	200	22.5627	22.5627	1.4066	17.5387	25.4188	11.42032842	0
287	0:04:34	200	22.5631	22.5631	1.4080	17.5355	25.4237	11.4241904	0
408	0:08:31	1200	22.5830	22.5830	1.3749	16.9035	25.5769	11.42449882	0
476	0:11:34	1300	22.5841	22.5841	1.3847	16.8997	25.5818	11.42786456	0
609	0:19:32	2100	22.5846	22.5846	1.3594	16.9017	25.5663	11.41998569	0.021187587
618	0:20:26	2100	22.5862	22.5862	1.3606	16.8980	25.5737	11.42449882	0.01695007
623	0:20:41	2100	22.5865	22.5865	1.3617	16.8943	25.5784	11.42772015	0.01736891
629	0:21:16	3500	22.5945	22.5945	1.3143	16.4711	25.3668	11.28318466	0.01736891
634	0:21:46	3500	22.5987	22.5987	1.3136	16.4754	25.3700	11.28318466	0
654	0:23:33	3500	22.5998	22.5998	1.3160	16.4720	25.3816	11.29108111	0
657	0:24:03	3500	22.6024	22.6024	1.3271	16.4540	25.4335	11.32758258	0.008027782
693	0:27:14	3500	22.6062	22.6062	1.3390	16.4364	25.4895	11.36636687	0.012105445
704	0:28:27	3500	22.6091	22.6091	1.3385	16.4394	25.4918	11.36636687	0
749	0:34:18	3500	22.6096	22.6096	1.3510	16.4192	25.5444	11.40449079	0.017312518
757	0:35:29	3500	22.6097	22.6097	1.3564	16.4105	25.5668	11.42077877	0.025092618
765	0:36:13	3500	22.6104	22.6104	1.3563	16.4112	25.5673	11.42077877	0.02232741
782	0:38:10	3500	22.6139	22.6139	1.3557	16.4147	25.5699	11.42077877	0.008027782
810	0:42:05	3500	22.6142	22.6142	1.3557	16.4151	25.5702	11.42077877	0.006599292
844	0:46:04	3500	22.6159	22.6159	1.3554	16.4167	25.5714	11.42077877	0

Table 5. Summary of optimization progress steps (Case 3)

Simulation	Elapsed Time	Iterations	Result	Goal Cell Statistics				Adjustable Cells	
				Mean	Std. Dev.	Min.	Max.	B33	B34
1	0:00:03	200	21.5389	21.5389	1.2304	18.5268	24.0391	12	1
9	0:00:12	200	22.2433	22.2433	1.0499	19.1010	24.2307	10.7149756	1
116	0:02:18	200	22.6235	22.6235	0.9882	19.6319	24.4407	10.7149756	0
156	0:03:20	200	22.6864	22.6864	1.2331	19.6019	25.2261	11.3574878	0.5
158	0:03:22	200	22.8783	22.8783	1.2008	19.8489	25.3192	11.3574878	0
451	0:08:48	700	22.9092	22.9092	1.2546	18.4296	25.3599	11.38881615	0
458	0:09:00	700	22.9126	22.9126	1.2611	18.4193	25.3851	11.40814557	0
652	0:14:49	900	22.9286	22.9286	1.2363	18.4481	25.3150	11.35421787	0
671	0:15:24	900	22.9339	22.9339	1.2463	18.4317	25.3548	11.38488703	0
879	0:25:09	1600	22.9344	22.9344	1.2662	17.3100	25.5248	11.41602728	0
938	0:28:31	1600	22.9349	22.9349	1.2670	17.3096	25.5279	11.41844862	0
1519	1:35:42	1600	22.9352	22.9352	1.2676	17.3093	25.5299	11.42007317	0
1522	1:36:13	1600	22.9362	22.9362	1.2692	17.3084	25.5361	11.42490816	0
1527	1:37:07	1600	22.9366	22.9366	1.2699	17.3080	25.5386	11.42690804	0
1579	1:47:07	1600	22.9367	22.9367	1.2700	17.3079	25.5393	11.42739718	0
1663	2:02:05	1600	22.9368	22.9368	1.2702	17.3079	25.5397	11.42777029	0

Table 6. Summary of optimization progress steps (Case 4)

Simulation	Elapsed Time	Iterations	Result	Goal Cell Statistics				Adjustable Cells	
				Mean	Std. Dev.	Min.	Max.	B33	B34
1	0:00:01	200	21.9333	21.9333	1.0995	17.9025	23.8248	12	1
102	0:01:40	200	22.4651	22.4651	1.0895	17.1258	24.0831	12	0
128	0:02:05	200	22.4697	22.4697	1.1454	18.2193	24.4702	11.68493983	1
138	0:02:16	200	22.8016	22.8016	1.1770	18.4849	24.8890	11.47935276	1
148	0:02:27	200	23.2681	23.2681	1.1955	17.4754	25.2175	11.47935276	0
644	0:16:45	1600	23.2783	23.2783	1.2830	15.4879	25.4247	11.41618269	0.046875
647	0:16:58	1600	23.2823	23.2823	1.2888	15.4867	25.4357	11.44063073	0
648	0:17:03	1600	23.2878	23.2878	1.2769	15.5167	25.4010	11.38306914	0
654	0:17:23	1600	23.2945	23.2945	1.2858	15.5027	25.4361	11.40981504	0
727	0:21:15	1600	23.2961	23.2961	1.2880	15.4994	25.4444	11.41618269	0
752	0:23:18	2600	23.3108	23.3108	1.2591	15.4796	25.5830	11.43224686	0.046875
758	0:24:01	2600	23.3120	23.3120	1.2462	15.5090	25.5301	11.38433148	0.028919313
764	0:25:00	2300	23.3235	23.3235	1.2596	15.5160	25.4564	11.38433148	0
771	0:25:36	2300	23.3266	23.3266	1.2734	15.4900	25.4955	11.43417864	0
787	0:27:00	2600	23.3267	23.3267	1.2609	15.4936	25.5863	11.42103095	0.013430891
790	0:27:24	2300	23.3279	23.3279	1.2657	15.5069	25.4762	11.40192282	0
866	0:34:27	2600	23.3306	23.3306	1.2619	15.4986	25.5841	11.41775047	0
934	0:41:18	5500	23.3388	23.3388	1.2369	15.4852	25.5874	11.43224686	0.023647395
951	0:43:42	5500	23.3443	23.3443	1.2299	15.5104	25.5512	11.39513341	0
964	0:45:51	5500	23.3445	23.3445	1.2302	15.5099	25.5525	11.39602282	0
979	0:49:00	5500	23.3472	23.3472	1.2404	15.4910	25.5918	11.43224686	0
986	0:50:20	5500	23.3523	23.3523	1.2411	15.4932	25.5992	11.4281538	0
1508	1:52:28	9400	23.3524	23.3524	1.2411	14.4182	25.5939	11.42518321	0.005269175
1510	1:52:57	9400	23.3531	23.3531	1.2426	14.4080	25.5966	11.43003419	0
1526	1:57:45	9400	23.3537	23.3537	1.2420	14.4142	25.5969	11.42691059	0.002918223
1574	2:15:29	9400	23.3538	23.3538	1.2420	14.4144	25.5966	11.42666851	0.002634588

Table 7. Summary of optimization progress steps (Case 5)

Simulation	Elapsed Time	Iterations	Result	Goal Cell Statistics				Adjustable Cells	
				Mean	Std. Dev.	Min.	Max.	B33	B34
1	0:00:01	200	22.2914	22.2914	1.4335	17.1684	23.8732	12	1
77	0:00:59	200	22.5415	22.5415	1.1308	18.6081	24.2857	12	1.6843813
263	0:05:26	200	22.8789	22.8789	1.1386	19.0409	24.7819	11.23828643	2.245390531
286	0:05:51	200	22.9787	22.9787	1.1676	19.0931	24.8387	11.61914321	1.622695266
334	0:06:43	400	22.9981	22.9981	1.4404	13.9952	25.1370	11.87785643	1.341041046
378	0:07:37	400	23.1475	23.1475	1.2537	14.3021	25.1929	11.35941637	1.543790591
447	0:09:11	400	23.1599	23.1599	1.2937	14.2114	25.2518	11.4552492	1.475972914
531	0:11:13	400	23.2006	23.2006	1.3083	14.0908	25.3111	11.44404797	1.430913072
742	0:17:56	800	23.2114	23.2114	1.2895	14.1375	25.3116	11.43357062	1.446700562
767	0:19:31	800	23.2118	23.2118	1.2945	14.0521	25.3013	11.44440991	1.416866221
790	0:20:50	1300	23.2124	23.2124	1.3294	14.0819	25.4359	11.41040196	1.423889646
831	0:22:58	1400	23.2147	23.2147	1.3339	14.0522	25.4274	11.44404797	1.416866221
901	0:27:59	1400	23.2175	23.2175	1.3307	14.0683	25.4393	11.42953976	1.421090223
921	0:29:15	1900	23.2239	23.2239	1.2817	14.2118	25.3962	11.3886882	1.468656527
938	0:30:32	1900	23.2348	23.2348	1.2955	14.1712	25.4381	11.41392016	1.45675234
952	0:31:40	1900	23.2350	23.2350	1.2941	14.1566	25.4284	11.40575775	1.45052695
980	0:33:48	1900	23.2385	23.2385	1.3018	14.1517	25.4495	11.41987581	1.450328965
998	0:35:03	1900	23.2418	23.2418	1.3100	14.0920	25.4593	11.43700127	1.430539568
1027	0:37:09	1900	23.2449	23.2449	1.3101	14.0944	25.4636	11.43249058	1.430913072
1094	0:42:58	1900	23.2455	23.2455	1.3104	14.0925	25.4646	11.43249058	1.43022335
1395	0:56:17	1900	23.2456	23.2456	1.3115	14.0760	25.4577	11.43700127	1.424738774
1408	0:57:54	1900	23.2462	23.2462	1.3115	14.0776	25.4596	11.43624994	1.425247771
1430	0:59:29	1900	23.2473	23.2473	1.3109	14.0884	25.4668	11.43249058	1.42874362
1511	1:05:18	1900	23.2473	23.2473	1.3106	14.0850	25.4642	11.42813703	1.426995696
1523	1:06:05	1900	23.2484	23.2484	1.3106	14.0942	25.4690	11.42924905	1.430460514
1542	1:07:20	1900	23.2488	23.2488	1.3106	14.0966	25.4701	11.42851343	1.431263571
1574	1:10:14	1900	23.2490	23.2490	1.3105	14.0944	25.4692	11.42764173	1.430381459
1581	1:10:53	1900	23.2495	23.2495	1.3114	14.0875	25.4698	11.43008065	1.42814352
1586	1:11:24	1900	23.2495	23.2495	1.3112	14.0890	25.4704	11.42935313	1.428609523
1616	1:14:31	5900	23.2501	23.2501	1.3281	13.2582	25.4634	11.4337394	1.430104555
1645	1:17:15	5900	23.2508	23.2508	1.3280	13.2589	25.4644	11.4323439	1.430421193
1648	1:17:46	5900	23.2517	23.2517	1.3283	13.2594	25.4657	11.43190868	1.429869362
1658	1:18:56	5900	23.2525	23.2525	1.3286	13.2597	25.4669	11.43188974	1.429125516
1676	1:20:50	5900	23.2537	23.2537	1.3294	13.2597	25.4673	11.43301683	1.427393107
1698	1:24:50	5900	23.2539	23.2539	1.3283	13.2606	25.4693	11.4290786	1.430683906
1708	1:27:09	5900	23.2541	23.2541	1.3285	13.2609	25.4692	11.42931262	1.429662593
1730	1:31:52	5900	23.2544	23.2544	1.3289	13.2608	25.4697	11.43029839	1.428547245
1740	1:34:21	9300	23.2577	23.2577	1.3404	13.2574	25.4586	11.43878682	1.424843861
1772	1:41:38	9300	23.2585	23.2585	1.3389	13.2618	25.4599	11.43051567	1.425715088
1812	1:51:37	9300	23.2593	23.2593	1.3399	13.2592	25.4626	11.43484069	1.426059732
1824	1:54:19	9300	23.2595	23.2595	1.3395	13.2602	25.4628	11.43283715	1.426236304
1843	1:57:23	9300	23.2614	23.2614	1.3394	13.2608	25.4683	11.43077789	1.42770215
1873	2:01:01	9300	23.2620	23.2620	1.3392	13.2618	25.4701	11.42864063	1.428299367
2035	2:14:49	9300	23.2622	23.2622	1.3391	13.2616	25.4710	11.42851383	1.4289592
2118	2:22:57	9300	23.2623	23.2623	1.3393	13.2616	25.4710	11.42884646	1.428505661

Table 8. Summary of optimization progress steps (Case 6)

Simulation	Elapsed Time	Iterations	Result	Goal Cell Statistics				Adjustable Cells	
				Mean	Std. Dev.	Min.	Max.	B33	B34
1	0:00:01	200	15.2096	15.2096	1.7027	8.9273	17.4900	20	1
10	0:00:15	200	15.5647	15.5647	0.4932	12.9669	15.9794	3.4335658	1
12	0:00:17	200	16.1395	16.1395	1.5153	9.5481	17.8226	20	0.1596907
113	0:02:17	200	16.5744	16.5744	0.8856	12.3778	17.5377	14.4890046	1
138	0:02:37	200	17.8025	17.8025	0.9061	13.1551	18.5260	14.4890046	0
175	0:03:10	300	18.0380	18.0380	0.8138	13.8591	18.9375	11.4966858	1.909970511
257	0:04:59	200	20.1301	20.1301	0.6602	16.6248	20.9293	10.3215042	2.546423838
366	0:07:54	200	20.4479	20.4479	0.9958	15.8521	21.3491	11.34765697	2.535504876
372	0:08:01	200	21.7613	21.7613	1.1018	17.5872	23.3879	11.96781114	4.06783415
528	0:11:45	300	22.4145	22.4145	1.0488	18.4318	24.1147	10.9658704	3.266051106
612	0:13:39	200	22.5105	22.5105	1.1804	17.1355	24.1944	11.95953888	3.449925712
619	0:13:46	300	22.5324	22.5324	1.0296	18.7416	24.1918	10.9658704	3.796565598
715	0:15:38	200	22.7667	22.7667	1.1789	18.3224	24.5348	11.42570355	3.308738858
785	0:16:56	300	22.8564	22.8564	1.2153	17.6476	24.8532	11.69262122	3.379332285
1001	0:22:05	400	22.8940	22.8940	1.1824	18.5641	24.8548	11.39629944	3.500142529
1058	0:24:05	400	22.8953	22.8953	1.1879	18.6304	24.8406	11.49150297	3.38175167
1101	0:25:27	600	22.9511	22.9511	1.1874	15.7550	24.8932	11.49150297	3.40431541
1107	0:26:00	600	22.9746	22.9746	1.1877	15.7954	24.9257	11.42570355	3.42822572
1354	0:36:48	900	22.9817	22.9817	1.1628	15.7684	24.9816	11.46995274	3.414472365
1388	0:40:09	900	22.9934	22.9934	1.1634	15.7895	25.0223	11.43684881	3.427395638

Table 9. Summary of optimization progress steps (Case 7)

Simulation	Elapsed Time	Iterations	Result	Goal Cell Statistics				Adjustable Cells	
				Mean	**Std. Dev.**	**Min.**	**Max.**	**B33**	**B34**
1	0:00:04	200	17.7815	17.7815	1.0048	13.8294	18.5738	15	1
25	0:00:29	300	18.0279	18.0279	2.2978	13.0310	22.2444	15	4.9870997
74	0:01:12	300	18.2078	18.2078	2.4496	13.2257	22.8150	15	5.2797178
116	0:01:49	200	18.2643	18.2643	1.1013	14.7385	19.0826	15	0
123	0:01:55	200	22.1603	22.1603	1.0000	19.1769	23.7460	10.95248846	6.208682764
474	0:09:13	300	22.2423	22.2423	1.0148	18.8945	23.8172	10.95248846	5.723423777
526	0:10:23	400	22.5085	22.5085	1.1295	18.0871	24.3096	11.33298576	5.656205006
730	0:16:23	600	22.5548	22.5548	1.2109	16.7405	24.4405	11.43367298	5.542908923
870	0:21:19	600	22.5733	22.5733	1.2104	16.5024	24.3422	11.54249588	5.371290529
959	0:24:59	800	22.5817	22.5817	1.2326	15.2227	24.4345	11.5462626	5.40346789
960	0:25:02	800	22.5856	22.5856	1.2291	15.3140	24.4728	11.43367298	5.471993223
1319	0:49:40	1700	22.5898	22.5898	1.2439	14.6793	24.5635	11.47388099	5.420862512
1344	0:51:44	1700	22.5976	22.5976	1.2419	14.7313	24.5778	11.43769018	5.429559823
1350	0:52:24	1700	22.6005	22.6005	1.2425	14.7026	24.5730	11.44447603	5.423188685
1480	1:09:51	1700	22.6024	22.6024	1.2419	14.7417	24.5857	11.4293004	5.432863145
1489	1:11:10	1700	22.6042	22.6042	1.2424	14.7254	24.5836	11.43240775	5.426793606
1576	1:30:42	1700	22.6050	22.6050	1.2425	14.7292	24.5861	11.43055487	5.427608264
1587	1:34:21	1700	22.6058	22.6058	1.2426	14.7332	24.5890	11.42887257	5.428561721
1858	2:45:19	14000	22.6062	22.6062	1.2328	9.5661	24.7365	11.42930232	5.428122187
1920	3:05:58	14000	22.6064	22.6064	1.2328	9.5686	24.7378	11.42848865	5.428925364
1950	3:17:14	14000	22.6065	22.6065	1.2328	9.5682	24.7377	11.42834567	5.428787218
2089	3:48:12	14000	22.6066	22.6066	1.2329	9.5678	24.7380	11.42856108	5.428652631

Table 10. Summary of optimization progress steps (Case 8)

Simulation	Elapsed Time	Iterations	Result	Goal Cell Statistics				Adjustable Cells	
				Mean	Std. Dev.	Min.	Max.	B33	B34
2	0:00:04	200	15.9399	15.9399	0.5314	13.2774	16.3909	11.5	1
3	0:00:05	200	18.8807	18.8807	1.1225	12.1212	19.7818	11.5	6.308631536
5	0:00:07	200	21.6539	21.6539	1.3131	16.6695	23.7984	11.5	7.99994148
14	0:00:14	200	21.9141	21.9141	1.3262	14.7779	23.7477	11.5	7.289740728
44	0:00:36	200	22.0203	22.0203	1.3330	16.4556	24.1146	11.5	7.447518144
100	0:01:19	200	22.0653	22.0653	1.3410	16.1730	24.1178	11.5	7.391850424
225	0:03:34	200	22.0754	22.0754	1.3449	16.2184	24.1517	11.5	7.40253071
295	0:05:09	1500	22.0946	22.0946	1.2207	16.6015	24.0735	11.5	7.474733032
304	0:05:43	1600	22.1613	22.1613	1.2090	16.0907	24.0513	11.5	7.370902047
316	0:06:31	1600	22.1657	22.1657	1.2316	16.7157	24.1779	11.42681826	7.501968074
320	0:06:50	1600	22.1734	22.1734	1.2160	16.1409	24.0928	11.5	7.383975588
326	0:07:18	1600	22.1905	22.1905	1.2297	16.5725	24.1881	11.44609643	7.435389906
344	0:08:32	1600	22.1969	22.1969	1.2252	16.5397	24.1748	11.43241663	7.41619447
403	0:13:15	1700	22.1992	22.1992	1.2401	15.8755	24.2051	11.42116562	7.422853763
427	0:15:07	1700	22.2059	22.2059	1.2423	15.8934	24.2206	11.42184422	7.433834273
466	0:18:31	1700	22.2071	22.2071	1.2439	15.8881	24.2264	11.43330704	7.429643873
498	0:21:40	1700	22.2074	22.2074	1.2423	15.8769	24.2141	11.43708663	7.423156079
499	0:21:45	1700	22.2108	22.2108	1.2443	15.8909	24.2316	11.42898648	7.42985229
512	0:22:57	1900	22.2133	22.2133	1.2308	15.8564	24.1854	11.45409876	7.414719598
521	0:24:02	1900	22.2171	22.2171	1.2348	15.8640	24.2105	11.45744433	7.421407671
531	0:25:11	1900	22.2181	22.2181	1.2345	15.8672	24.2100	11.45409876	7.421256513
532	0:25:17	1900	22.2184	22.2184	1.2348	15.8911	24.2242	11.43199901	7.432157712
540	0:26:25	1900	22.2201	22.2201	1.2356	15.8748	24.2230	11.44576696	7.425450193
578	0:32:07	1900	22.2244	22.2244	1.2356	15.8914	24.2319	11.42817718	7.430019871
601	0:35:32	2100	22.2257	22.2257	1.2308	15.8807	24.2220	11.43875142	7.425085133
619	0:38:45	4600	22.2273	22.2273	1.2212	15.8762	24.3354	11.43487772	7.42259864
694	0:52:15	4600	22.2295	22.2295	1.2234	15.8875	24.3532	11.43273329	7.428621422
813	1:16:16	11400	22.2324	22.2324	1.2078	14.9275	24.3388	11.45909367	7.422833889
853	1:24:27	11400	22.2334	22.2334	1.2045	14.9344	24.3311	11.4304363	7.422130717
857	1:25:36	11400	22.2360	22.2360	1.2074	14.9335	24.3418	11.44841121	7.423661401
871	1:29:24	11400	22.2360	22.2360	1.2065	14.9317	24.3352	11.44720949	7.42179998
880	1:32:55	11400	22.2367	22.2367	1.2073	14.9346	24.3420	11.44627152	7.423748275
895	1:37:59	11400	22.2369	22.2369	1.2071	14.9346	24.3410	11.44555878	7.423433648
918	1:45:08	11400	22.2377	22.2377	1.2079	14.9416	24.3526	11.43900303	7.42732392
951	1:56:33	11400	22.2378	22.2378	1.2071	14.9364	24.3422	11.44261396	7.423839044
966	2:01:33	11400	22.2378	22.2378	1.2071	14.9366	24.3428	11.44266329	7.424006262
992	2:11:49	11400	22.2383	22.2383	1.2073	14.9381	24.3449	11.44076829	7.424646674
994	2:12:29	11400	22.2384	22.2384	1.2073	14.9385	24.3454	11.44035845	7.424813976
1020	2:21:04	11400	22.2399	22.2399	1.2079	14.9436	24.3536	11.43492395	7.427333832

Table 11. Summary of optimization progress steps (Case 9)

Simulation	Elapsed Time	Iterations	Result	Goal Cell Statistics				Adjustable Cells	
				Mean	Std. Dev.	Min.	Max.	B33	B34
1	0:00:04	200	15.4484	15.4484	0.3654	12.2874	15.9942	11.5	1
2	0:00:05	200	15.4784	15.4784	0.3778	12.2931	16.0430	11.59480787	1
7	0:00:10	200	17.2632	17.2632	0.6815	12.4835	17.9736	11.5	7.973954016
48	0:00:39	200	21.6409	21.6409	1.2663	17.3558	23.5591	11.5	9.538697242
67	0:00:54	200	21.6538	21.6538	1.2656	17.3627	23.5686	11.5	9.519707626
88	0:01:08	200	21.6641	21.6641	1.1850	17.5280	23.2853	11.5	9.287111962
130	0:01:44	200	21.7047	21.7047	1.1943	17.5486	23.3300	11.6	9.287111962
170	0:02:19	200	21.7701	21.7701	1.2735	17.4761	23.6975	11.56162198	9.412904602
177	0:02:27	200	21.7983	21.7983	1.2680	17.4747	23.6991	11.42982586	9.412904602
244	0:03:34	200	21.8027	21.8027	1.2752	17.4564	23.7295	11.41294248	9.443555981
278	0:03:59	200	21.8061	21.8061	1.2795	17.4564	23.7480	11.42982586	9.443555981
317	0:04:54	900	21.8306	21.8306	1.2894	14.2842	23.8438	11.46947402	9.412904602
390	0:09:40	900	21.8417	21.8417	1.2945	14.3407	23.8886	11.42982586	9.428230291
570	0:36:40	4000	21.8420	21.8420	1.2647	13.3706	23.9383	11.43921817	9.419354143
605	0:46:25	4000	21.8439	21.8439	1.2663	13.3686	23.9462	11.44037746	9.421761617
679	1:07:25	4000	21.8463	21.8463	1.2677	13.3701	23.9548	11.43510166	9.424995954

Table 12. Summary of optimization progress steps (Case 10)

Simulation	Elapsed Time	Iterations	Result	Goal Cell Statistics				Adjustable Cells	
				Mean	Std. Dev.	Min.	Max.	B31	B32
1	0:00:03	200	13.523	13.523	0.238	11.458	13.558	10	1
19	0:00:17	200	14.223	14.223	0.102	13.368	14.414	3.89254045	1
66	0:00:51	200	14.288	14.288	0.108	13.368	14.480	4.07344425	1
68	0:00:53	200	14.958	14.958	0.138	13.755	14.992	5.93529105	1
101	0:01:17	200	15.714	15.714	0.147	14.468	15.738	7.690009561	0
220	0:02:42	200	15.840	15.840	0.157	14.368	15.865	7.198463126	0.40726418
228	0:02:48	200	16.184	16.184	0.158	14.694	16.209	7.198463126	0
401	0:05:09	300	16.234	16.234	0.147	14.751	16.256	7.125635805	0
670	0:09:41	300	16.240	16.240	0.147	14.753	16.262	7.142710709	0
722	0:10:40	400	16.240	16.240	0.139	14.753	16.262	7.141595338	0
855	0:13:11	400	16.240	16.240	0.139	14.753	16.262	7.142710709	0
940	0:16:04	1500	16.242	16.242	0.130	14.468	16.260	7.142431866	0.001953125
945	0:16:17	1500	16.244	16.244	0.130	14.470	16.262	7.142753353	0

Table 13. Summary of optimization progress steps (Case 11)

SRP	
$x_{n,1}^*$	$(11.429, 0)$
$g_n(\hat{x})$	0.00138
$s_n^2(x_{n,1}^*)$	0.00119
one-sided CI, $\left[0, g_n(\hat{x}) + \frac{t_{n-1,\alpha} \cdot s_n(x_{n,1}^*)}{\sqrt{n}}\right]$	$[0, 0.00541]$
I2RP	
$x_{n,2}^*$	$(11.429, 0)$
$g_n(\hat{x}) = g_{n,1}(\hat{x})$	0.00138
$s_n^2(x_{n,2}^*)$	0.00118
one-sided CI, $\left[0, g_{n,1}(\hat{x}) + \frac{t_{n-1,\alpha} \cdot s_n(x_{n,2}^*)}{\sqrt{n}}\right]$	$[0, 0.00540]$
A2RP	
$g_{n,2}(\hat{x})$	0.00125
$g_n'(\hat{x}) = \frac{1}{2}\left(g_{n,1}(\hat{x}) + g_{n,2}(\hat{x})\right)$	0.00132
$s_n' = \frac{1}{2}\left(s_n^2(x_{n,1}^*) + s_n^2(x_{n,2}^*)\right)$	0.00118
one-sided CI, $\left[0, g_n'(\hat{x}) + \frac{t_{2n-1,\alpha} \cdot \sqrt{s_n'}}{\sqrt{2n}}\right]$	$[0, 0.00415]$

Table 14. Confidence Interval Computations (Case 1)

SRP	
$x_{n,1}^*$	$(11.429, 0)$
$g_n(\hat{x})$	0.0000476
$s_n^2(x_{n,1}^*)$	4.747×10^{-8}
one-sided CI, $\left[0, g_n(\hat{x}) + \frac{t_{n-1,\alpha} \cdot s_n(x_{n,1}^*)}{\sqrt{n}}\right]$	$[0, 0.0000730]$
I2RP	
$x_{n,2}^*$	$(11.428, 0)$
$g_n(\hat{x}) = g_{n,1}(\hat{x})$	0.0000476
$s_n^2(x_{n,2}^*)$	0.00114
one-sided CI, $\left[0, g_{n,1}(\hat{x}) + \frac{t_{n-1,\alpha} \cdot s_n(x_{n,2}^*)}{\sqrt{n}}\right]$	$[0, 0.00400]$
A2RP	
$g_{n,2}(\hat{x})$	0.0000218
$g_n'(\hat{x}) = \frac{1}{2}\left(g_{n,1}(\hat{x}) + g_{n,2}(\hat{x})\right)$	0.0000347
$s_n' = \frac{1}{2}\left(s_n^2(x_{n,1}^*) + s_n^2(x_{n,2}^*)\right)$	0.000571
one-sided CI, $\left[0, g_n'(\hat{x}) + \frac{t_{2n-1,\alpha} \cdot \sqrt{s_n'}}{\sqrt{2n}}\right]$	$[0, 0.00200]$

Table 15. Confidence Interval Computations (Case 2)

SRP	
$x^*_{n,1}$	$(11.429, 0)$
$g_n(\hat{x})$	0.00109
$s_n^2(x^*_{n,1})$	0.0000235
one-sided CI, $\left[0, g_n(\hat{x}) + \frac{t_{n-1,\alpha} \cdot s_n(x^*_{n,1})}{\sqrt{n}}\right]$	$[0, 0.00166]$
I2RP	
$x^*_{n,2}$	$(11.429, 0)$
$g_n(\hat{x}) = g_{n,1}(\hat{x})$	0.00109
$s_n^2(x^*_{n,2})$	0.00116
one-sided CI, $\left[0, g_{n,1}(\hat{x}) + \frac{t_{n-1,\alpha} \cdot s_n(x^*_{n,2})}{\sqrt{n}}\right]$	$[0, 0.00509]$
A2RP	
$g_{n,2}(\hat{x})$	0.000979
$g_n'(\hat{x}) = \frac{1}{2}\left(g_{n,1}(\hat{x}) + g_{n,2}(\hat{x})\right)$	0.00104
$s_n' = \frac{1}{2}\left(s_n^2(x^*_{n,1}) + s_n^2(x^*_{n,2})\right)$	0.000594
one-sided CI, $\left[0, g_n'(\hat{x}) + \frac{t_{2n-1,\alpha} \cdot \sqrt{s_n'}}{\sqrt{2n}}\right]$	$[0, 0.00304]$

Table 16. Confidence Interval Computations (Case 3)

SRP	
$x^*_{n,1}$	$(11.429, 0)$
$g_n(\hat{x})$	0.000137
$s_n^2(x^*_{n,1})$	2.617×10^{-7}
one-sided CI, $\left[0, g_n(\hat{x}) + \frac{t_{n-1,\alpha} \cdot s_n(x^*_{n,1})}{\sqrt{n}}\right]$	$[0, 0.000197]$
I2RP	
$x^*_{n,2}$	$(11.429, 0)$
$g_n(\hat{x}) = g_{n,1}(\hat{x})$	0.000137
$s_n^2(x^*_{n,2})$	0.00308
one-sided CI, $\left[0, g_{n,1}(\hat{x}) + \frac{t_{n-1,\alpha} \cdot s_n(x^*_{n,2})}{\sqrt{n}}\right]$	$[0, 0.00662]$
A2RP	
$g_{n,2}(\hat{x})$	0.000122
$g_n'(\hat{x}) = \frac{1}{2}\left(g_{n,1}(\hat{x}) + g_{n,2}(\hat{x})\right)$	0.000130
$s_n' = \frac{1}{2}\left(s_n^2(x^*_{n,1}) + s_n^2(x^*_{n,2})\right)$	0.00154
one-sided CI, $\left[0, g_n'(\hat{x}) + \frac{t_{2n-1,\alpha} \cdot \sqrt{s_n'}}{\sqrt{2n}}\right]$	$[0, 0.00336]$

Table 17. Confidence Interval Computations (Case 4)

SRP	
$x^*_{n,1}$	$(11.428, 0)$
$g_n(\hat{x})$	0.00133
$s^2_n(x^*_{n,1})$	0.0000166
one-sided CI, $\left[0, g_n(\hat{x}) + \frac{t_{n-1,\alpha} \cdot s_n(x^*_{n,1})}{\sqrt{n}}\right]$	$[0, 0.00181]$
I2RP	
$x^*_{n,2}$	$(11.428, 0)$
$g_n(\hat{x}) = g_{n,1}(\hat{x})$	0.00133
$s^2_n(x^*_{n,2})$	0.0000172
one-sided CI, $\left[0, g_{n,1}(\hat{x}) + \frac{t_{n-1,\alpha} \cdot s_n(x^*_{n,2})}{\sqrt{n}}\right]$	$[0, 0.00181]$
A2RP	
$g_{n,2}(\hat{x})$	0.00140
$g'_n(\hat{x}) = \frac{1}{2}\left(g_{n,1}(\hat{x}) + g_{n,2}(\hat{x})\right)$	0.00136
$s'_n = \frac{1}{2}\left(s^2_n(x^*_{n,1}) + s^2_n(x^*_{n,2})\right)$	0.0000169
one-sided CI, $\left[0, g'_n(\hat{x}) + \frac{t_{2n-1,\alpha} \cdot \sqrt{s'_n}}{\sqrt{2n}}\right]$	$[0, 0.00170]$

Table 18. Confidence Interval Computations (Case 5)

SRP	
$x^*_{n,1}$	$(11.428, 1.429)$
$g_n(\hat{x})$	0.0000240
$s^2_n(x^*_{n,1})$	2.649×10^{-7}
one-sided CI, $\left[0, g_n(\hat{x}) + \frac{t_{n-1,\alpha} \cdot s_n(x^*_{n,1})}{\sqrt{n}}\right]$	$[0, 0.0000841]$
I2RP	
$x^*_{n,2}$	$(11.428, 1.429)$
$g_n(\hat{x}) = g_{n,1}(\hat{x})$	0.0000240
$s^2_n(x^*_{n,2})$	2.135×10^{-7}
one-sided CI, $\left[0, g_{n,1}(\hat{x}) + \frac{t_{n-1,\alpha} \cdot s_n(x^*_{n,2})}{\sqrt{n}}\right]$	$[0, 0.0000780]$
A2RP	
$g_{n,2}(\hat{x})$	0.0000419
$g'_n(\hat{x}) = \frac{1}{2}\left(g_{n,1}(\hat{x}) + g_{n,2}(\hat{x})\right)$	0.0000329
$s'_n = \frac{1}{2}\left(s^2_n(x^*_{n,1}) + s^2_n(x^*_{n,2})\right)$	2.392×10^{-7}
one-sided CI, $\left[0, g'_n(\hat{x}) + \frac{t_{2n-1,\alpha} \cdot \sqrt{s'_n}}{\sqrt{2n}}\right]$	$[0, 0.0000733]$

Table 19. Confidence Interval Computations (Case 6)

SRP	
$x_{n,1}^*$	$(11.428, 3.428)$
$g_n(\hat{x})$	0.00316
$s_n^2(x_{n,1}^*)$	0.0000499
one-sided CI, $\left[0, g_n(\hat{x}) + \frac{t_{n-1,\alpha} \cdot s_n(x_{n,1}^*)}{\sqrt{n}}\right]$	$[0, 0.00398]$
I2RP	
$x_{n,2}^*$	$(11.430, 3.430)$
$g_n(\hat{x}) = g_{n,1}(\hat{x})$	0.00316
$s_n^2(x_{n,2}^*)$	0.0000165
one-sided CI, $\left[0, g_{n,1}(\hat{x}) + \frac{t_{n-1,\alpha} \cdot s_n(x_{n,2}^*)}{\sqrt{n}}\right]$	$[0, 0.00363]$
A2RP	
$g_{n,2}(\hat{x})$	0.00131
$g_n'(\hat{x}) = \frac{1}{2}\left(g_{n,1}(\hat{x}) + g_{n,2}(\hat{x})\right)$	0.00223
$s_n' = \frac{1}{2}\left(s_n^2(x_{n,1}^*) + s_n^2(x_{n,2}^*)\right)$	0.0000332
one-sided CI, $\left[0, g_n'(\hat{x}) + \frac{t_{2n-1,\alpha} \cdot \sqrt{s_n'}}{\sqrt{2n}}\right]$	$[0, 0.00271]$

Table 20. Confidence Interval Computations (Case 7)

SRP	
$x_{n,1}^*$	$(11.429, 5.429)$
$g_n(\hat{x})$	0.0000441
$s_n^2(x_{n,1}^*)$	8.296×10^{-9}
one-sided CI, $\left[0, g_n(\hat{x}) + \frac{t_{n-1,\alpha} \cdot s_n(x_{n,1}^*)}{\sqrt{n}}\right]$	$[0, 0.0000548]$
I2RP	
$x_{n,2}^*$	$(11.429, 5.429)$
$g_n(\hat{x}) = g_{n,1}(\hat{x})$	0.0000441
$s_n^2(x_{n,2}^*)$	5.336×10^{-9}
one-sided CI, $\left[0, g_{n,1}(\hat{x}) + \frac{t_{n-1,\alpha} \cdot s_n(x_{n,2}^*)}{\sqrt{n}}\right]$	$[0, 0.0000527]$
A2RP	
$g_{n,2}(\hat{x})$	0.0000284
$g_n'(\hat{x}) = \frac{1}{2}\left(g_{n,1}(\hat{x}) + g_{n,2}(\hat{x})\right)$	0.0000363
$s_n' = \frac{1}{2}\left(s_n^2(x_{n,1}^*) + s_n^2(x_{n,2}^*)\right)$	6.816×10^{-9}
one-sided CI, $\left[0, g_n'(\hat{x}) + \frac{t_{2n-1,\alpha} \cdot \sqrt{s_n'}}{\sqrt{2n}}\right]$	$[0, 0.0000431]$

Table 21. Confidence Interval Computations (Case 8)

SRP	
$x^*_{n,1}$	$(11.429, 7.429)$
$g_n(\hat{x})$	0.00200
$s^2_n(x^*_{n,1})$	0.0000193
one-sided CI, $\left[0, g_n(\hat{x}) + \frac{t_{n-1,\alpha} \cdot s_n(x^*_{n,1})}{\sqrt{n}}\right]$	$[0, 0.00251]$
I2RP	
$x^*_{n,2}$	$(11.429, 7.429)$
$g_n(\hat{x}) = g_{n,1}(\hat{x})$	0.00200
$s^2_n(x^*_{n,2})$	0.0000191
one-sided CI, $\left[0, g_{n,1}(\hat{x}) + \frac{t_{n-1,\alpha} \cdot s_n(x^*_{n,2})}{\sqrt{n}}\right]$	$[0, 0.00251]$
A2RP	
$g_{n,2}(\hat{x})$	0.00198
$g'_n(\hat{x}) = \frac{1}{2}\left(g_{n,1}(\hat{x}) + g_{n,2}(\hat{x})\right)$	0.00199
$s'_n = \frac{1}{2}\left(s^2_n(x^*_{n,1}) + s^2_n(x^*_{n,2})\right)$	0.0000192
one-sided CI, $\left[0, g'_n(\hat{x}) + \frac{t_{2n-1,\alpha} \cdot \sqrt{s'_n}}{\sqrt{2n}}\right]$	$[0, 0.00235]$

Table 22. Confidence Interval Computations (Case 9)

SRP	
$x^*_{n,1}$	$(11.429, 9.428)$
$g_n(\hat{x})$	0.00248
$s^2_n(x^*_{n,1})$	0.0000384
one-sided CI, $\left[0, g_n(\hat{x}) + \frac{t_{n-1,\alpha} \cdot s_n(x^*_{n,1})}{\sqrt{n}}\right]$	$[0, 0.00320]$
I2RP	
$x^*_{n,2}$	$(11.429, 9.429)$
$g_n(\hat{x}) = g_{n,1}(\hat{x})$	0.00248
$s^2_n(x^*_{n,2})$	0.0000434
one-sided CI, $\left[0, g_{n,1}(\hat{x}) + \frac{t_{n-1,\alpha} \cdot s_n(x^*_{n,2})}{\sqrt{n}}\right]$	$[0, 0.00325]$
A2RP	
$g_{n,2}(\hat{x})$	0.00258
$g'_n(\hat{x}) = \frac{1}{2}\left(g_{n,1}(\hat{x}) + g_{n,2}(\hat{x})\right)$	0.00253
$s'_n = \frac{1}{2}\left(s^2_n(x^*_{n,1}) + s^2_n(x^*_{n,2})\right)$	0.0000409
one-sided CI, $\left[0, g'_n(\hat{x}) + \frac{t_{2n-1,\alpha} \cdot \sqrt{s'_n}}{\sqrt{2n}}\right]$	$[0, 0.00306]$

Table 23. Confidence Interval Computations (Case 10)

SRP	
$x^*_{n,1}$	$(7.143, 0)$
$g_n(\hat{x})$	0.0000309
$s^2_n(x^*_{n,1})$	4.299×10^{-9}
one-sided CI, $\left[0, g_n(\hat{x}) + \frac{t_{n-1,\alpha} \cdot s_n(x^*_{n,1})}{\sqrt{n}}\right]$	$[0, 0.0000386]$
I2RP	
$x^*_{n,2}$	$(7.143, 0)$
$g_n(\hat{x}) = g_{n,1}(\hat{x})$	0.0000309
$s^2_n(x^*_{n,2})$	5.333×10^{-9}
one-sided CI, $\left[0, g_{n,1}(\hat{x}) + \frac{t_{n-1,\alpha} \cdot s_n(x^*_{n,2})}{\sqrt{n}}\right]$	$[0, 0.0000394]$
A2RP	
$g_{n,2}(\hat{x})$	0.0000355
$g'_n(\hat{x}) = \frac{1}{2}\left(g_{n,1}(\hat{x}) + g_{n,2}(\hat{x})\right)$	0.0000332
$s'_n = \frac{1}{2}\left(s^2_n(x^*_{n,1}) + s^2_n(x^*_{n,2})\right)$	4.816×10^{-9}
one-sided CI, $\left[0, g'_n(\hat{x}) + \frac{t_{2n-1,\alpha} \cdot \sqrt{s'_n}}{\sqrt{2n}}\right]$	$[0, 0.0000389]$

Table 24. Confidence Interval Computations (Case 11)

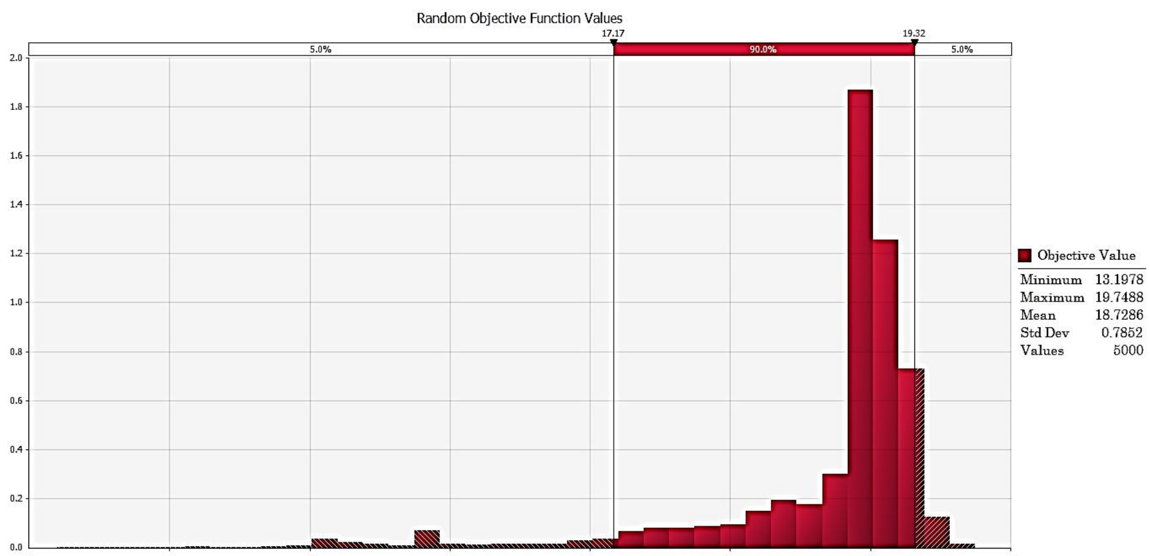

Figure 5. Histogram (Case 1 policy applied to Case 6)

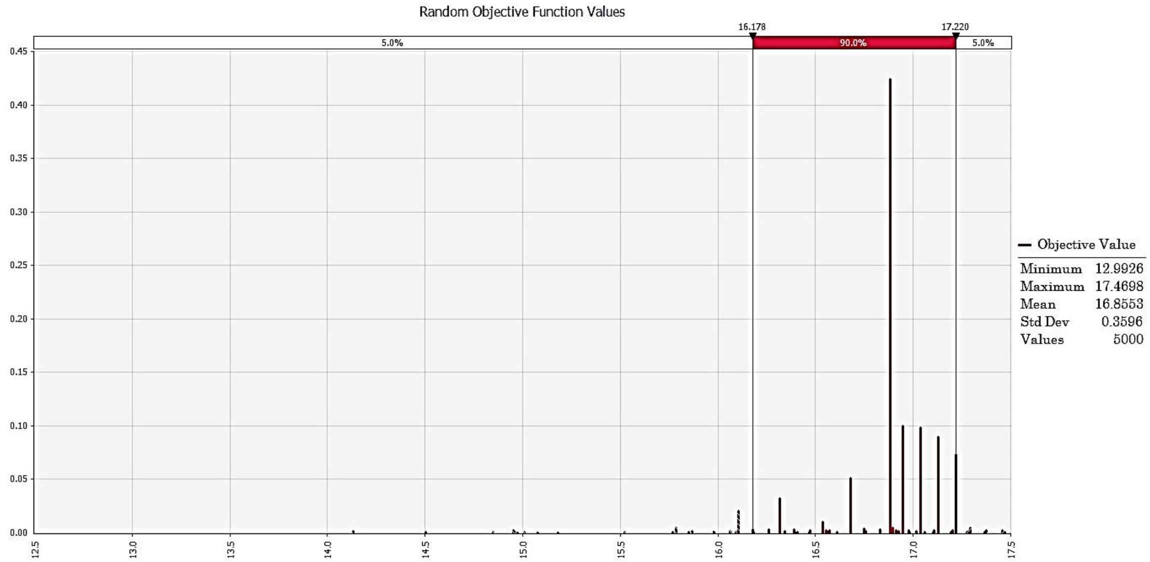

Figure 6. Histogram (Case 1 policy applied to Case 7)

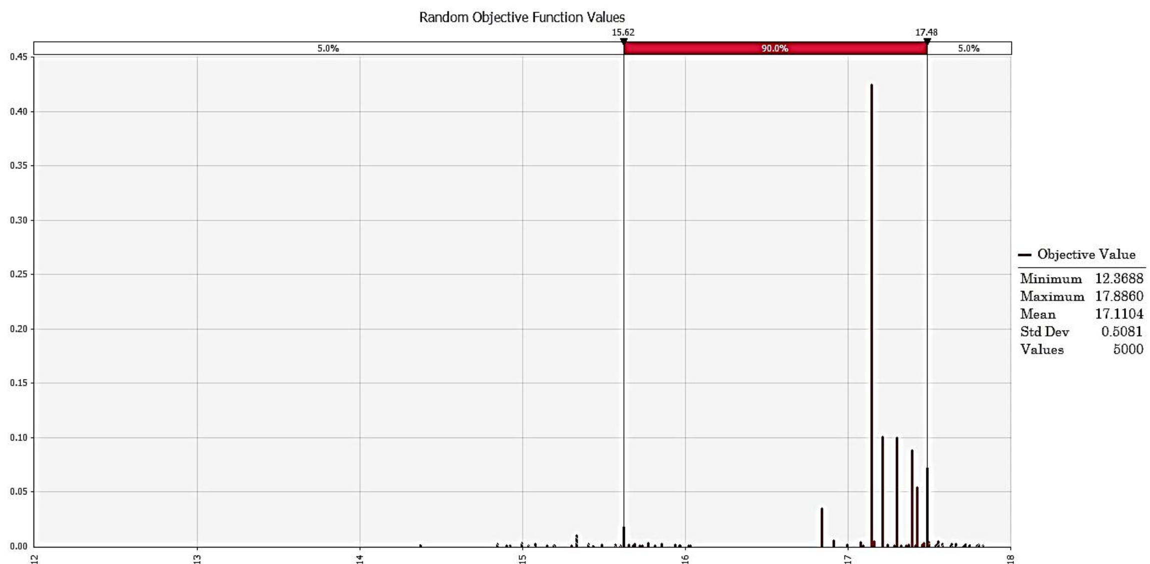

Figure 7. Histogram (Case 1 policy applied to Case 8)

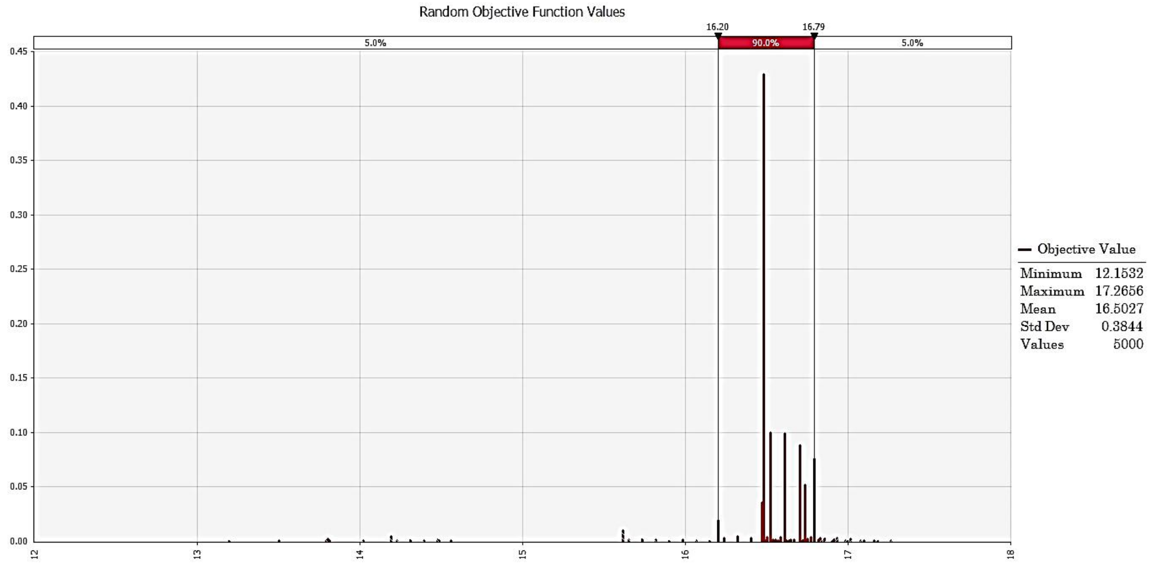

Figure 8. Histogram (Case 1 policy applied to Case 9)

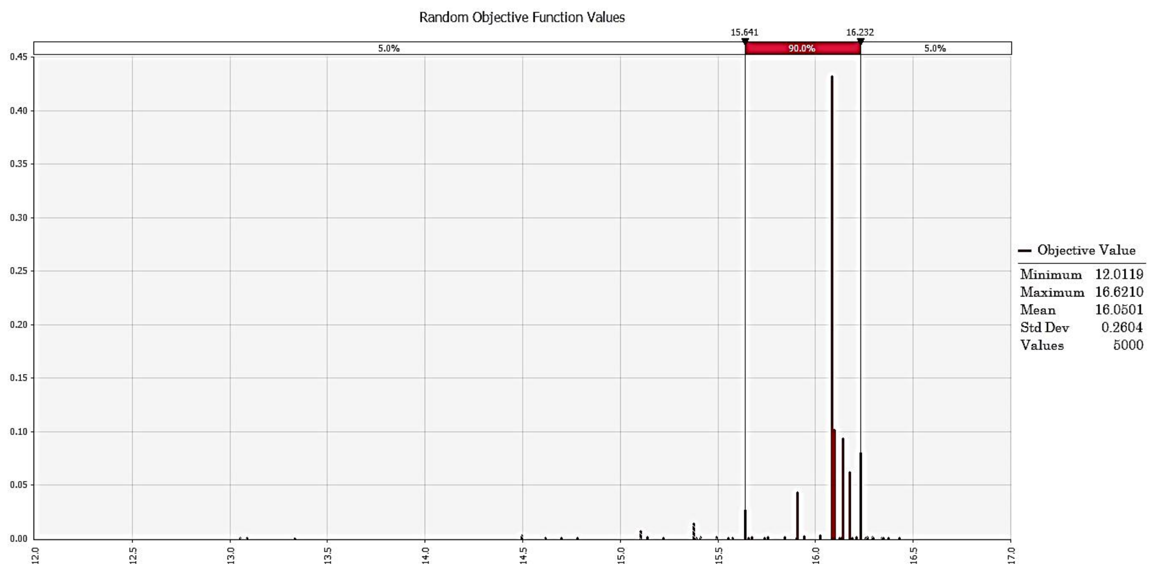

Figure 9. Histogram (Case 1 policy applied to Case 10)