OPTIMAL PROCUREMENT POLICY FOR

COST CONSCIOUS RETAILER

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ABSTRACT OF THE THESIS

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Manufacturers whose products primarily relay on expensive raw materials need to make well informed procurement decisions in order to reduce their production costs. Decision making is especially difficult when there is uncertainty about the raw material cost as well as the product demand. This research considers an optimum procurement and production planning problem under uncertainty. Basic raw material is a rare metal with a dynamically and stochastically changing price. The manufacturer has the dual sourcing option: buy from the spot market; or to sign long term contracts. Long term contracts can be signed only at the beginning of every year with a yearly duration. In addition, the contract price depends on the average spot price during some interval. We first consider a simplified version of the problem without long-term contract, and assume a base-stock procurement and inventory control policy. We model the problem as a Markov chain and present some numerical examples demonstrating the effect of various parameters on the optimum base-stock value. Then, we investigate a case with dual procurement option, and present our results.

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DEDICATION

This thesis is dedicated to my parents who have given me the opportunity of an education from the best institutions and support throughout my life. They have always stood by me and gave me the strength to carry out my studies.

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Chapter 1

Introduction

Procurement problems, particularly direct procurement problems concerning mainly raw material and production goods, are becoming more and more prominent in supply chain management these days because they greatly affect the total expenditure of many companies. This is especially true when the raw material is an expensive rare metal. This research considers an optimum procurement and production planning problem under raw material price and demand uncertainty. The raw material used to produce an intermediate product is a rare metal with dynamically and stochastically changing prices. The demand for the intermediate product is also random but reasonably stationary.

The manufacturer has dual sourcing options; they can either buy from the spot market, or sign a long term forward contract. That is, dual sourcing is comprised of forward contract buying and spot market buying. Forward contract buying provides a long term stable supply at a fixed price for a fixed delivery time. While spot buying, although is more flexible with the delivery times and the order quantity, is more risky and uncertain. For instance, you might not able to purchase a large volume when the price is good because other companies already took the advantage first. According to Zhang et al. (2011), in recent years spot markets have emerged for a wide variety of commodities, and companies are starting to use it to incorporate with the traditional forward contracts especially in dynamic random access memory procurements. The market has been a mix of private, bilateral contracts and spot market trading for years, where 80% of the intermediate products are sold through a fixed price long term contract.

For large manufacturers, especially in chemical and pharmaceutical industries whose products primarily relay on expensive raw materials, their production costs can be largely reduced if good strategies be adopted when making procurement decisions. However, these complex strategies of decision making are very difficult considering the demand uncertainty and randomness of the raw material costs. Most of the mathematical models fail to predict a jump or slump on the prices because sudden events that cause such a jump or slump to happen are impossible to predict sometimes. Relatively, the demand of products is also uncertain but may be easier to predict since we have a more stable requirement for the materials globally. Thus the strategy that is applied in order to compensate the possible failure of forecasting raw material price becomes very important for the dual sourcing scenario.

In Chapter 2, we present the literature survey. Chapter 3 considers a simplified model of the procedure for a problem without long term contract. The effect of various parameters on the optimum order quality is investigated. A case in which both long term contract and spot buying are available is studied in Chapter 4 with real world data and situations. Finally, in Chapter 5, conclusions and future research are discussed.

Chapter 2

Literature Review

Many optimization approaches based on periodic updates have been used in commodity procurement. Secomandi and Kekre (2009) consider the problem of reselling the commodity back to the market in the forward process. They suggest not only to update the forward price, but also to update the demand forecast. Thus the whole dynamic optimization can provide a better solution overall. Optimal cost value function is provided through a Markov decision process formulation.

For a typical periodic-review inventory control model with stochastic demand, Goel and Gutierrez (2012) characterized the optimal procurement policy and developed a computational algorithm to obtain the optimal policy thresholds. The model they developed has a total cost consists of procurement cost and a market-determined economic cost of holding inventory, and this economic cost can be significantly reduced according to the numerical results from this paper. Also they considered a dynamic program with a risk-neutral valuation approach for minimizing the total procurement cost.

In comparing the forward contract with the total order quantity commitment, the latter is more flexible according to the paper of Zhang, Chen, Hua and Xue (2011). Hence, it is easier to use it and take advantage of the dual sourcing. In this paper they suppose a stochastic demand and the spot price can be stationary or non-stationary, but it is independent of the demand process.

Two of the papers focused on a dual sourcing scenario of periodic-review inventory model. First one by Inderfurth, Kelle and Kleber (2013) assume random stationary demand and a capacity reservation contract for the sourcing. In this modeling part, they used dynamic programming recursive equations. However, when formulating the models, this paper focuses more on the structure analysis, because of the special reservation capacity decision needed to be considered. The model can be extended to be optimal for infinite horizon, too. A heuristic approach is used for determining iterative parameters step by step. The second one by Seifert, Thonemann and Hausman (2004) mentioned that in these problems the demand and the spot price are usually positively correlated, and thus assume a bivariate distribution rather than a normal distribution. Their paper then deals with dynamic pricing and determines the contract-to-spot ratio for purchases, where in spot market we need to consider a hedging situation since it allows salvage. Based on the assumed distribution, we can adjust the multiplier of the deviation to set different policies for different level of risks you wish to take. Also, a spot price premium case was studied, as well as a pure contract sourcing case.

Other two papers focused on a multi-period model with a single product. Nagarajan and Rajagopalan (2009) first consider a comprehensive environment and the equilibrium order quantities have the special property such that each player ignores the strategy of his opponent under reasonable conditions on the cost parameters. Secondly, they show that under certain conditions, the equilibrium quantities in the finite horizon game can be reduced to a multi-period single-product inventory model for each player. Then they approach to the equilibrium by proving the deterministic demand identifies a unique Nash equilibrium while stochastic demand derived from a decoupling result. Thus, the equilibrium strategies of two players is simply to solve a single-product in n-period stochastic inventory model with partial backlogging instead of lost sales, where the demand of non-substituting customers from the current period is back logged to the next period.

Hwang and Hahn (2000) studied a periodic review inventory model for a single perishable product and demand rate of the item is dependent on the current inventory level. The proof first shows some properties about the ordering quantity. Since the item have a fixed life time, two different kind of order up to values are introduced depending on the outdated items. They approach to solve the two different situations by using linear programming and then apply enumeration method which has a similar approach to dynamic programming.

The last optimal procurement policy paper we reviewed here by Polatoglu and Sahin (2000), studied a (s,S) type inventory control model under price-dependent demand relationship. The main difference in the model of this paper is that the price is a decision variable here and brings two opposing cost-related effects. The solution approach is also dynamic programming. However, an n-period pseudo-profit function is introduced instead. And finally, the problem is solved by investigating the critical inventory level of (s,S) under certain sufficient condition. Some special situations such as No-fixed-ordering-cost, Non-stationary extensions and deterministic demand are also considered in the end of this paper.

Chapter 3

A simplified Markov Chain Model of the Inventory/Production Control of a Cost Conscious Retailer

In this section our goal is to build a Markov chain model for base-stock type procurement policies, and then analyze the effects of various parameters on the optimum order quantity. Dual sourcing will not be considered in this chapter but will be discussed in detail in Chapter 4.

3.1 Assumptions and Notations

We begin modeling by making some necessary assumptions for the inventory/production control policy. The procurement policy is adopted depending on the price level and the inventory on hand. We assume the order of each month's operations are as follows: Production facility consumes items from the inventory to meet demand, notice demand is always met here since we have the minimum inventory level that is equal to the maximum daily demand. Then we observe the current price level and the number of items in stock, in order to decide on how many items to purchase. Lastly, after we finish the purchasing stage, we check our new inventory level and set it as the next inventory level.

To simplify the problem, we assume only two price levels: H, denoting the state of high price level, and L, denoting the state of low price level. The unit cost for low price level is denoted as Y_L and for high price level is denoted as Y_H.

Let $C(n)$ denote the raw material price at time n. Assume that the price transition probability matrix P is given as follows:

P=

	H	L
H	1-a	a
L	b	1-b

Table 3.1: Price Inventory transition probability matrix

Here, a denote the conditional probability that the price will be low at the next time period given that it is currently high, i.e., $a = P\{C(n+1) = L\ /\ C(n) = H\}$. b is similarly defined as $b = P\{C(n+1) = H\ /\ C(n) = L\}$.

Also, we assume that there are historical data such that we could use to obtain information about the price distribution. Assume that the historical data indicates that in the long-run 20% of the time the price is at the high state and 80% of the time it is at the low state. Next, we set the expected cycle length to be 25 days which means in every 25 days we will have the above probability distribution for the high and low prices.

We assume the demand for raw material is distributed in a very simplified fashion; demand on day n is equal to 1 item with probability p and 2 items with probability (1-p). That is, let D_n denote the demand quantity at time n, we have:

$$P\{D_n = 1\} = p, \qquad P\{D_n = 2\} = 1 - p$$

When the price is low, we purchase enough items to stock up to the maximum inventory of K items. When the price is high, we do not purchase unless the number of items we have on hand are less than the maximum daily demand. More specifically, if the price level is high we only purchase when the inventory level is below 2 and if so, we purchase to bring the inventory level back to the minimum of 2 items.

Lastly, we assume that the total cost of inventory/production management consists of the inventory holding cost and the procurement cost, where the holding cost is $h/day per item and the procurement cost is the total cost of all the purchases made.

3.2 Model Description

Now we are ready for the modeling of our inventory management problems. The objective of our model is to optimize the inventory/production policy we utilized, and therefore to minimize the expected total cost under such a policy. Later on we will see that the expected total cost is decided by the base-stock value, which is K. However, we would like to obtain the actual values of a and b before we calculate the expected total cost and start the optimization.

Let π_H and π_L denote the stationary probability of price states H and L respectively. In Table 3.1 matrix we can obtain all the transition probabilities between the states by having the following relations:

$$\begin{cases} \pi_H = 0.2 \text{ and } \pi_L = 0.8 \\ a \geq 0, b \geq 0 \\ [\pi_H, \pi_L] = [\pi_H, \pi_L] * \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix} \end{cases}$$

$$\Longrightarrow a = 4b \cdots\cdots\cdots\cdots\cdots (1)$$

$$\text{and } a \geq 0, b \geq 0$$

Next, we use the fact that the cycle length is on the average of 25 days, where 20% of a cycle is at the high state, and 80% of a cycle is at the low state, in order to calculate the exact a and b values. Let the first day be day 0 and since a cycle is 25 days, next cycle starts at day 25. Consider the probability that the first time a high state goes to low state; it is exactly the probability for the number of consecutive days that a high state stay at high, before it goes to low. The corresponding expected values for the number of consecutive days at the high state before it goes to low is exactly equal to the total number of days at high state.

$$P\,\{X{=}n\} = P\,\{\text{the first time H goes to L at day n}\} = (1-a)^n * a$$

Let X denotes the number of consecutive days at H before it goes to L in one cycle. Since we know that for a geometrically distributed random variable X, the expected value is:

$$E(X) = \frac{1-a}{a}$$

It means that the expected number of days at H is equal to $\frac{1-a}{a}$ as well. Similarly, we can obtain the expected number of days at L is equal to $\frac{1-b}{b}$. Now, we also know that the expected cycle length is 25days, so:

$$\frac{1-a}{a} + \frac{1-b}{b} = 25 \cdots\cdots\cdots\cdots\cdots (2)$$

We can then obtain a, b values by combining equations (1) and (2) above, getting results for the price transition probability matrix:

$$a = \frac{20}{108}, b = \frac{5}{108}$$

Now we can start to build the detailed model of the inventory control problem. We want to have this matrix because we can then use it to compute the expected total cost which is ultimately what we intend to optimize. Let $\{C(n)I(n)\}\ n \geq 0$ denote the two-dimensional state, where C(n) is the price state and I(n) is the inventory on hand at the beginning of nth time period. For example, Hm denotes the state of high price level with inventory of m items, and similarly Lm denotes the state of low price level with inventory of m items.

The matrix size of the probability transition matrix of $\{C(n)I(n)\}\ n \geq 0$ depends on the base-stock value K. Thus the matrix will be defined as P_k. This also implies that our main optimization of the procurement policy is based on value K as well.

First, let K =2 which is the minimum case, we have the following matrix that is similar to Table 3.1:

$P_2 =$

	H2	L2
H2	1-a	a
L2	b	1-b

Table 3.2: Price Inventory transition probability matrix for K=2

Next, we let K=3 and obtain the Table 3.3 below. However, we notice that the states H3 and L2 are not accessible from any states, not even themselves. This means their values are zeros so we do not have to consider them at all. Thus we should simplify the matrix in a more compact fashion on the following:

$P_3 =$

	H2	H3	L2	L3
H2	1-a	0	0	a
H3	1-a	0	0	a
L2	0	0	0	0
L3	b	0	0	1-b

$\Longrightarrow$

$P_3 =$

	H2	L3
H2	1-a	a
L3	b	1-b

Table 3.3: Price Inventory transition probability matrix for K=3

Notice the above matrix is very similar to the Table 3.2 matrix the inventory levels. Then we look at Table 3.4 for K=4 below, in this case we have the probability to jump to either H2 or H3 from L4 , because the demand consumptions are $P\{D_n = 1\} = p$ and $P\{D_n = 2\} = 1 - p$. We show only the simplified matrix after we deleted all the columns and rows of inaccessible states:

$P_4 =$

	H2	H3	L4
H2	1-a	0	a
H3	1-a	0	a
L4	b*(1-p)	b*p	1-b

Table 3.4: Price Inventory transition probability matrix for K=4

Next, let us look at K=5 and K=6 on Table 3.5, where a pattern on the price inventory transition probability matrix can be found. Because when we reach a low state, we always purchase up to stock level K, therefore there will always be only one low state that is accessible, the LK. In contrast, we will never be able to reach HK because no states could ever access HK.

$P_5 =$

	H2	H3	H4	L5
H2	1-a	0	0	a
H3	1-a	0	0	a
H4	(1-a)*(1-p)	(1-a)*p	0	a
L5	0	b*(1-p)	b*p	1-b

$P_6 =$

	H2	H3	H4	H5	L6
H2	1-a	0	0	0	a
H3	1-a	0	0	0	a
H4	(1-a)*(1-p)	(1-a)*p	0	0	a
H5	0	(1-a)*(1-p)	(1-a)*p	0	a
L6	0	0	b*(1-p)	b*p	1-b

Table 3.5: Price Inventory transition probability matrix for K=5 and K=6

Lastly we conclude the detailed price inventory transition probability matrix for $K \geq 6$ in the following Table 3.6:

	H2	H3	H4	…	$H(K-1)$	LK
H2	1-a	0	0	…	0	a
H3	1-a	0	0	…	0	a
H4	(1-a)*(1-p)	(1-a)*p	0	…	0	a
…	…	…	…	…	…	…
$H(K-1)$	0	0	(1-a)*(1-p)	(1-a)*p	0	a
LK	0	0	0	b*(1-p)	b*p	1-b

Table 3.6: Price Inventory transition probability matrix for $K \geq 6$

Finally, we will define the expected total cost in one cycle with respect to K. Also, we let the probability of states with high price level and m items in inventory denote as π_{Hm}, while the probability of states with low price level and m items denote as π_{Lm}. We have:

$$\text{E (total cost)} = \text{E (inventory cost)} + \text{E (procurement cost)} \quad \cdots\cdots\cdots\cdots\cdots (3)$$

And:

$$\text{E (inventory cost)} = h * [(k * \pi_{LK}) + \sum_{i=2}^{K-1} (i * \pi_{Hi})] \quad \cdots\cdots\cdots\cdots\cdots (4)$$

Also:

$$\text{E (procurement cost)} = \text{E (procurement cost on } Y_L) + \text{E (procurement cost on } Y_H)$$

With:

E (procurement cost on Y_L)

$$= Y_L * [\pi_{LK} * (1 - b) * [1 * p + 2 * (1 - p)] + Y_L * \sum_{i=2}^{K-1} \{\pi_{Hi} * a * [(k - i + 1) * p + (k - i + 2) * (1 - p)]\} \quad \cdots\cdots\cdots\cdots\cdots (5)$$

E (procurement cost on Y_H)

$$= Y_H * \{\pi_{H2} * (1 - a) * [1 * p + 2 * (1 - p)] + \pi_{H3} * (1 - a) * [0 * p + 1 * (1 - p)]\} \quad \cdots\cdots\cdots\cdots\cdots (6)$$

Therefore, we plug in the equations (4), (5), (6) into equation (3) to get the expected total cost function below:

$$\text{E (total cost)} = h * [(k * \pi_{LK}) + \sum_{i=2}^{K-1} (i * \pi_{Hi})] + Y_L * [\pi_{LK} * (1 - b) * [1 * p + 2 * (1 - p)] + Y_L * \sum_{i=2}^{K-1} \{\pi_{Hi} * a * [(k - i + 1) * p + (k - i + 2) * (1 - p)]\} + Y_H * \{\pi_{H2} * (1 - a) * [1 * p + 2 * (1 - p)] + \pi_{H3} * (1 - a) * [0 * p + 1 * (1 - p)]\} \quad \cdots\cdots\cdots\cdots\cdots (7)$$

In equation (7), parameters a, b, p, h, Y_L and Y_H are all actual values. Also, π_{Hi} and π_{LK} are variables that depend on the value of K. Thus the expected total cost depends solely on the base-stock value K, which means to optimize the expected total cost we simply need to optimize the value of K.

3.3 Optimization

After obtaining the expected total cost function, we will begin this section with the minimization of the expected total cost and solve the optimal inventory/production management problem with respect to K. That is, we will be able to optimize our procurement policy if we can find the optimal value of the base-stock value K*.

In order to minimize equation (7) with respect to K, we need to first obtain all the stationary probability of state π_{Hi} and π_{LK}, $i = 2 \ldots k-1$. We can do so by analyzing the price inventory transition probability matrices. For K=2, the stationary probability of the matrix is obtained by solving the balance equations and the normalization equations.

$$\begin{cases} (1-a) * \pi_{H2} + b * (\pi_{L2}) = \pi_{H2} \\ a * \pi_{H2} + (1-b) * \pi_{L2} = \pi_{L2} \\ \pi_{H2} + \pi_{L2} = 1 \end{cases}$$

Thus, we apply a linear transformation to the above system of linear equations, so that they are represented in the matrix form equations:

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ a & -b \end{bmatrix} * \begin{bmatrix} \pi_{H2} \\ \pi_{L2} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Notice one of the equations above is redundant when we are performing the transformation.

Similarly, we approach the other equations respect of K in the same way and present their matrix equations in the following:

When K= 3,

$$\begin{bmatrix} 1 & 1 \\ -a & b \end{bmatrix} * \begin{bmatrix} \pi_{H2} \\ \pi_{L3} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Notice in this case the solutions for the states are essentially the same as K=2 case.

When K=4,

$$\begin{bmatrix} 1 & 1 & 1 \\ -a & 1-a & b*(1-p) \\ 0 & -1 & b*p \\ a & a & -b \end{bmatrix} * \begin{bmatrix} \pi_{H2} \\ \pi_{H3} \\ \pi_{L4} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

However, we notice that in the above matrix,

$$[-a \quad 1-a \quad b*(1-p)] + [0 \quad -1 \quad b*p] = [a \quad a \quad -b]$$

This means there exist linear dependency between the rows, and thus we can reformulate the equations for K=4 again after omitting the linearly dependent row:

$$\begin{bmatrix} 1 & 1 & 1 \\ -a & 1-a & b*(1-p) \\ 0 & -1 & b*p \end{bmatrix} * \begin{bmatrix} \pi_{H2} \\ \pi_{H3} \\ \pi_{L4} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

We will do this linear dependency check for all the other cases and reformulate them in a cleaner fashion when necessary. So when $K \geq 5$, in general we will have:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ -a & 1-a & (1-a)*(1-p) & 0 & \cdots & 0 \\ 0 & -1 & (1-a)*p & (1-a)*(1-p) & & 0 \\ 0 & 0 & -1 & (1-a)*p & & 0 \\ & & \ddots & \ddots & \ddots \quad \ddots & \vdots \\ \vdots & \vdots & & & & b*(1-p) \\ 0 & 0 & \cdots & \cdots & \cdots & b*p \end{bmatrix} * \begin{bmatrix} \pi_{H2} \\ \pi_{H3} \\ \pi_{H4} \\ \vdots \\ \vdots \\ \pi_{L(K-1)} \\ \pi_{LK} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

Since the solutions of π_{Hi} and π_{LK}, $i = 2 \ldots k - 1$ can be solved from above nonhomogeneous matrix equations with $p = 0.5$ and the values of $a = \frac{20}{108}$, $b = \frac{5}{108}$ we obtained before. We can finally compute equation (7) with respect to K by substituting the values of all the πs. Also, recall we assumed the following parameters for calculations before: $Y_L = 15$, $Y_H = 25$, $h = 0.5$.

In order to find the optimum K*, we use complete enumeration. We utilize Matlab to numerically evaluate the expected total cost for K=1, 2, 3, 4… until the minimum cost value is detected, since the cost is monotonically changing with respect to K. In our case, the optimum base-stock value K*=9 with E (total cost) = 23.89.

The details of the coding from Matlab program for a general case that $K \geq 5$ can be found at Appendix A.

3.4 Numerical Results

Next, we present numerical results showing the effect of various parameters on optimum base-stock value K*. We choose parameters π_H, Y_H, p and h, then we present their influences in pairs on the sensitivity of K*.

Firstly, we begin by changing the stationary probability of high states, π_H and high price value, Y_H. Let $\pi_H = 10\%, 20\%, 40\%, 80\%$, and Y_H=20, 25, 50, 100. Since increasing the probability of high states, and increasing high price value would force the procurement policy to be more efficient at the low price state, both suggests an increase in the optimal base-stock value, K*. Table 3.7 presents K* values in response to the changes in π_H and Y_H while keeping p and h constant. Figure 3.1 presents the visualization of such response for K*. We can see that K* is more sensitive to π_H than Y_H.

Y_H / π_H	20	25	50	100
10%	4	5	7	10
20%	7	9	14	19
40%	16	20	31	41
80%	63	70	92	114

Table 3.7: Effect of π_H and Y_H on K*

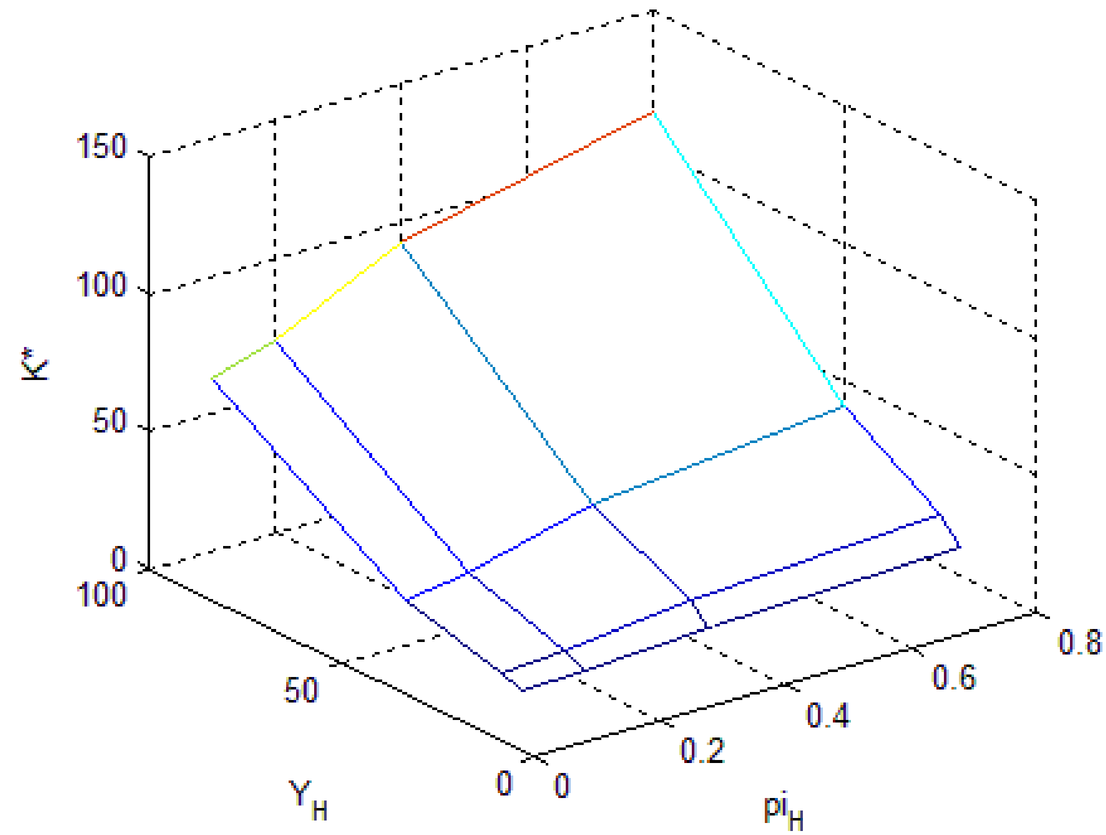


Figure 3.1: Visualization of the dependence of K* on π_H and Y_H

Next, we change the stationary probability of high states π_H and the probability (1-p) that demand is equal to 2 items and then evaluate K*. We let $\pi_H = 10\%, 20\%, 40\%, 80\%$ and 1-p=0.2, 0.5, 0.7, 0.9. Table 3.8 provides K* values to the changes in π_H and (1-p) while keeping Y_H and h constant. Figure 3.2 presents the visualization of such response for K*. We see that (1-p) only have a small impact on K* compared to π_H. Thus K* is more sensitive to π_H than (1-p).

1-p / π_H	0.2	0.5	0.7	0.9
10%	4	5	6	6
20%	7	9	10	10/11
40%	16	20	22	25
80%	56	70	81	91

Table 3.8: Effect of π_H and 1-p on K*

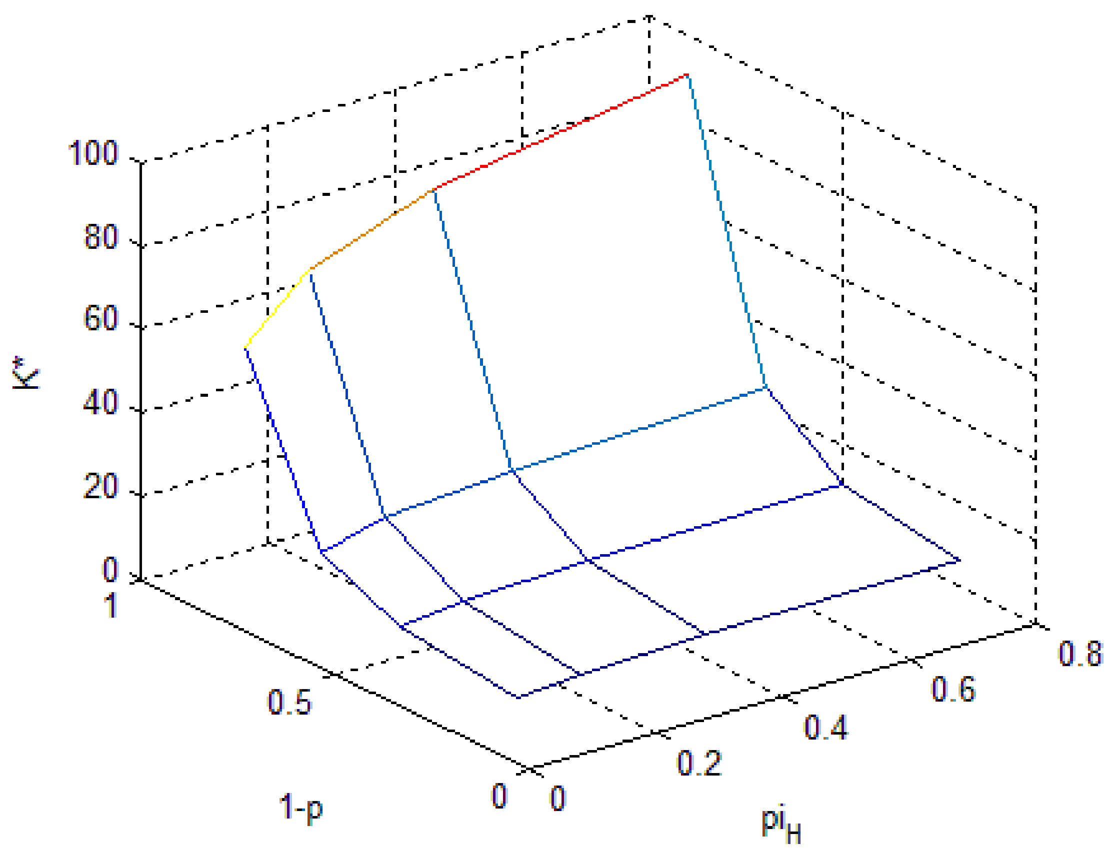


Figure 3.2: Visualization of the dependence of K* on π_H and 1-p

Next, we change the stationary probability of high states π_H and the holding cost of h and then evaluate K*. We let $\pi_H = 10\%, 20\%, 40\%, 80\%$ and h=0.1, 0.5, 2, 5. Table 3.9 provides K* values to the changes in π_H and h while keeping Y_H and (1-p) constant. Figure 3.3 presents the visualization of such response for K*. We see that both π_H and h have considerable influence on K*. Thus K* is sensitive to both π_H than h.

h / π_H	0.1	0.5	2	5
10%	10	5	2	2
20%	21	9	2	2
40%	45	20	4	4
80%	121/122	70	26	4

Table 3.9: Effect of π_H and h on K*

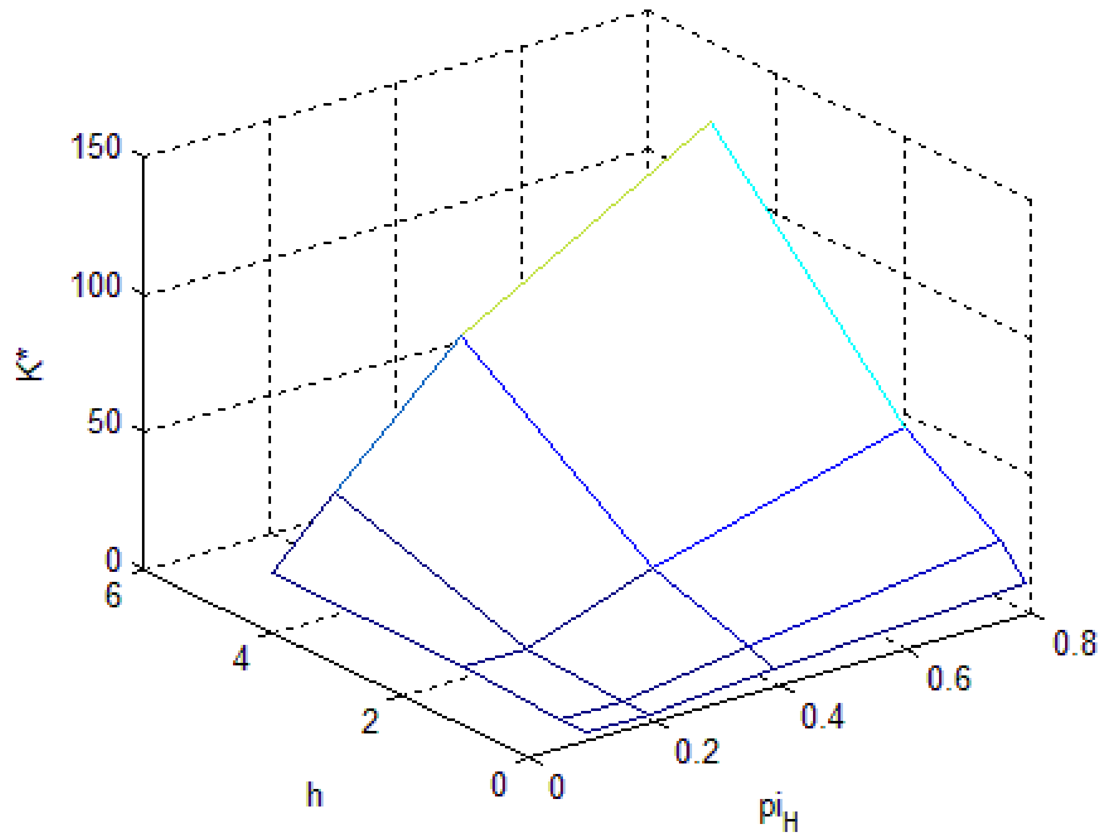


Figure 3.3: Visualization of the dependence of K* on π_H and h

Next, we change high price value Y_H and the probability (1-p) that the demand is equal to 2 items and then evaluate K*. We let Y_H=20, 25, 50, 100 and 1-p=0.2, 0.5, 0.7, 0.9. Table 3.10 provides K* values to the changes in Y_H and (1-p) while keeping π_H and h constant. Figure 3.4 presents the visualization of such response for K*. We see that (1-p) only have a small impact on K* compared to Y_H. Thus K* is more sensitive to Y_H than (1-p).

1-p / Y_H	0.2	0.5	0.7	0.9
20	6	7	8	8
25	7	9	10	10/11
50	11	14	16	17
100	16	19	21	24

Table 3.10: Effect of 1-p and Y_H on K*

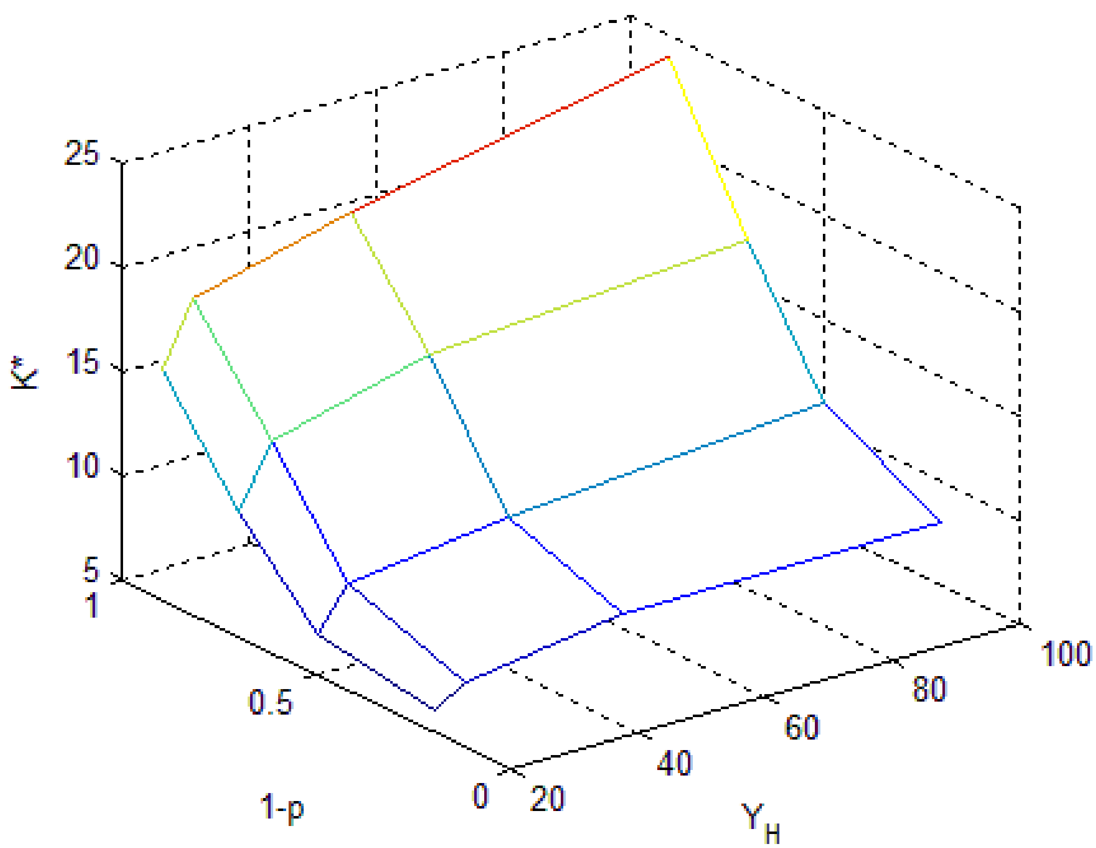


Figure 3.4: Visualization of the dependence of K* on 1-p and Y_H

Next, we change high price value Y_H and the holding cost of h and then evaluate K*. We let Y_H=20, 25, 50, 100 and h=0.1, 0.5, 2, 5. Table 3.11 provides K* values to the changes in Y_H and h while keeping π_H and (1-p) constant. Figure 3.5 presents the visualization of such response for K*. We see that Y_H have some impact on K*, and h have even greater influences on K*. Thus K* is more sensitive to h than Y_H.

h / Y_H	0.1	0.5	2	5
20	19	7	2	2
25	21	9	2	2
50	26	14	4	2
100	31	23	9	4

Table 3.11: Effect of h and Y_H on K*

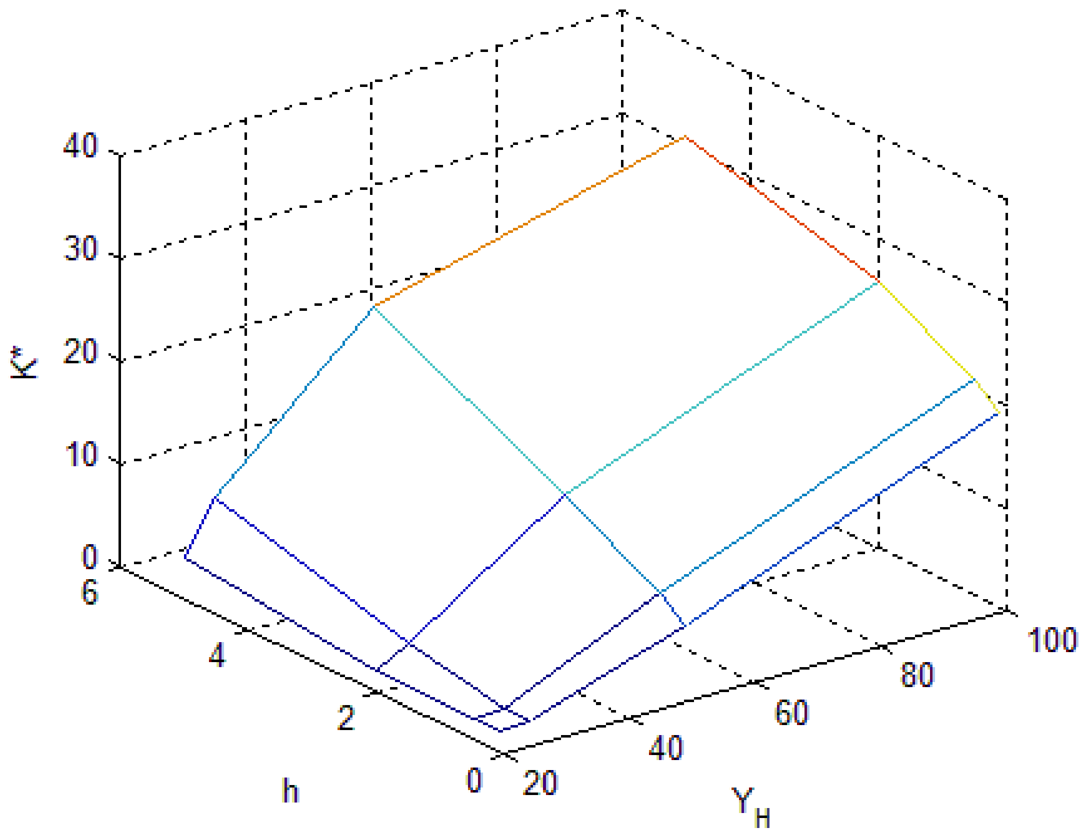


Figure 3.5: Visualization of the dependence of K* on h and Y_H

Lastly, we change the holding cost of h and the probability (1-p) that the demand is equal to 2 items and then evaluate K*. We let h=0.1, 0.5, 2, 5 and 1-p=0.2, 0.5, 0.7, 0.9. Table 3.12 provides K* values to the changes in h and (1-p) while keeping π_H and Y_H constant. Figure 3.6 presents the visualization of such response for K*. We see that (1-p) only have a small impact on K* compared to h. Thus K* is more sensitive to h than (1-p).

h / 1-p	0.1	0.5	2	5
0.2	17	7	2	2
0.5	21	9	2	2
0.7	23	10	4	2
0.9	26	10/11	4	2

Table 3.12: Effect of h and 1-p on K*

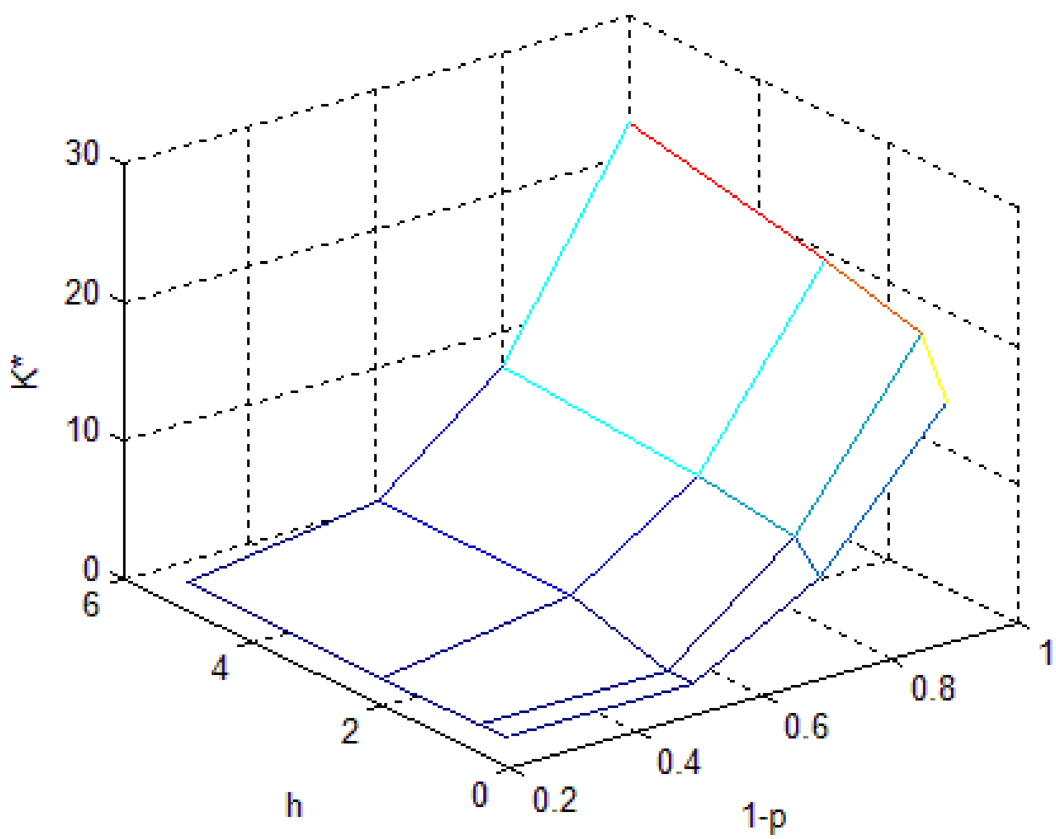


Figure 3.6: Visualization of the dependence of K* on h and 1-p

In summary, we can conclude that the sensitivity of K* is contributed mostly by π_H and h, and moderately by Y_H, while the impact of p on K* is rather small compared to the others. Table 3.13 shows the changes in K* by keeping one parameter constant and raising the value of the others. In conclusion, for solving the optimal procurement policy regarding optimal K* value, we find the percentages of different price states and the holding cost are the most important factors to be considered. The price values of states are next to consider, while the probability of different demand consumptions has relatively less impact on the system.

K*	π_H ↑	Y_H ↑	p ↑	h ↑
π_H		↑	↓	↓
Y_H	↑		↓	↓
p	↑	↑		↓
h	↑	↑	↓	

Table 3.13: Effect of π_H, Y_H, h and p on K*

Chapter 4

A Case Study

4.1 Introduction

We consider a company, producing an intermediate material that is used in manufacturing military as well as consumer products. An expensive rare metal is required in the production as the raw material. Since the intermediate material is used in manufacturing military products, we do not allow any back ordering or lost sales. The procurement problem of rare metal, Y, is to minimize the total cost of the company including inventory holding, spot buying and contract buying costs during a finite time horizon.

We have made our case study based on the following assumptions: The procurement frequency is monthly. Spot buying is a purchase made in the current month and contract buying is a purchase made at the beginning of every year that lasts 12 months. That means if we make a long term contract at the beginning of the year, we pay the total cost at the beginning and receive a delivery every month throughout the year. Let C(m) denote the spot price at month m, C’(m) denote the contract price at month m, Q(m) denote the total contract amount for the year of month m and S(m) denote the amount of spot buying at month m. Assume that the contract price has a discount of d dollars per kg on the spot price. All purchases are made at the beginning of the month while demands, D(m), come after the purchases are in the beginning of the month as well. At the end of the month we count the new inventory level I, also notice all costs at the present have an interest rate of r% as discount factor and the holding cost are h dollars per kg. It is required that we keep the inventory level at least twice as large as the last month's demand. Also, we assume the inventory level starts with initially 1000kg of rare metal Y, and there is a capacity limit for spot buying so that the maximum amount that could be bought every month is 3000kg.

We use a monthly updated linear/dynamic programming model to optimize this procurement problem. However, some of the data we need are the predicated values that are coming from a time series autoregressive integrated moving average (ARIMA) model. The ARIMA model can be used to predict a series of future values when previous values over a certain time period is available. It fits the distribution of past data and builds a model to predict future values. More details of the times series model and ARIMA model can be found in Brockwell & Davis (2002). We use ARIMA to predict the demand and spot price for a 24month time period in 2010-2011. We utilize the actual data for those 2 years to validate our forecasting model. Lastly, we present the optimum procurement policy and compare it to the actual procurement policy of company X.

4.2 Data Preparation and Forecasting

The raw data sets we obtained from company X are weekly reports of the spot prices offered from Asian Metal, monthly consumption reports of the final product, frequency reports of company X's purchasing history and their monthly inventory report.

As the first step of the data preparation, we focus on building a forecasting model to predict spot prices. So, it is necessary to clean the weekly spot price data and sort them by dates from 1/1/2001-7/30/2012. We could pivot the report to monthly now but since we want to use a time series model, more historical data is always better. Thus we keep it as weekly until we finish the prediction part and then we will turn the data into a monthly report. Figure 4.1 presents this weekly historical data for spot prices with a trend line. It shows rare metal Y spot price is very volatile since its usage is relatively narrow and is affected heavily by global politics. Its price is also affected by disasters, for example 2011 Japan's earthquake and tsunami at Tōhoku made the price jump greatly from March to May as shown on the figure because Japan is a large consumer of rare metal Y.

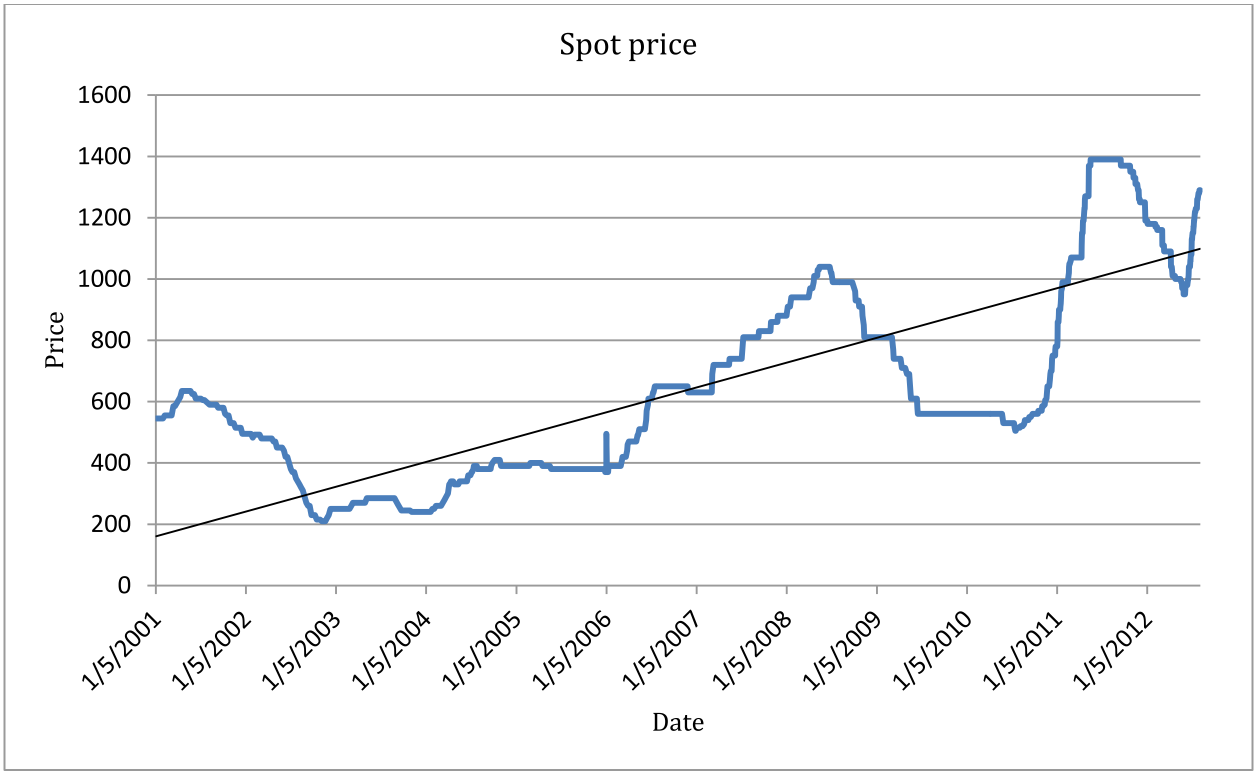
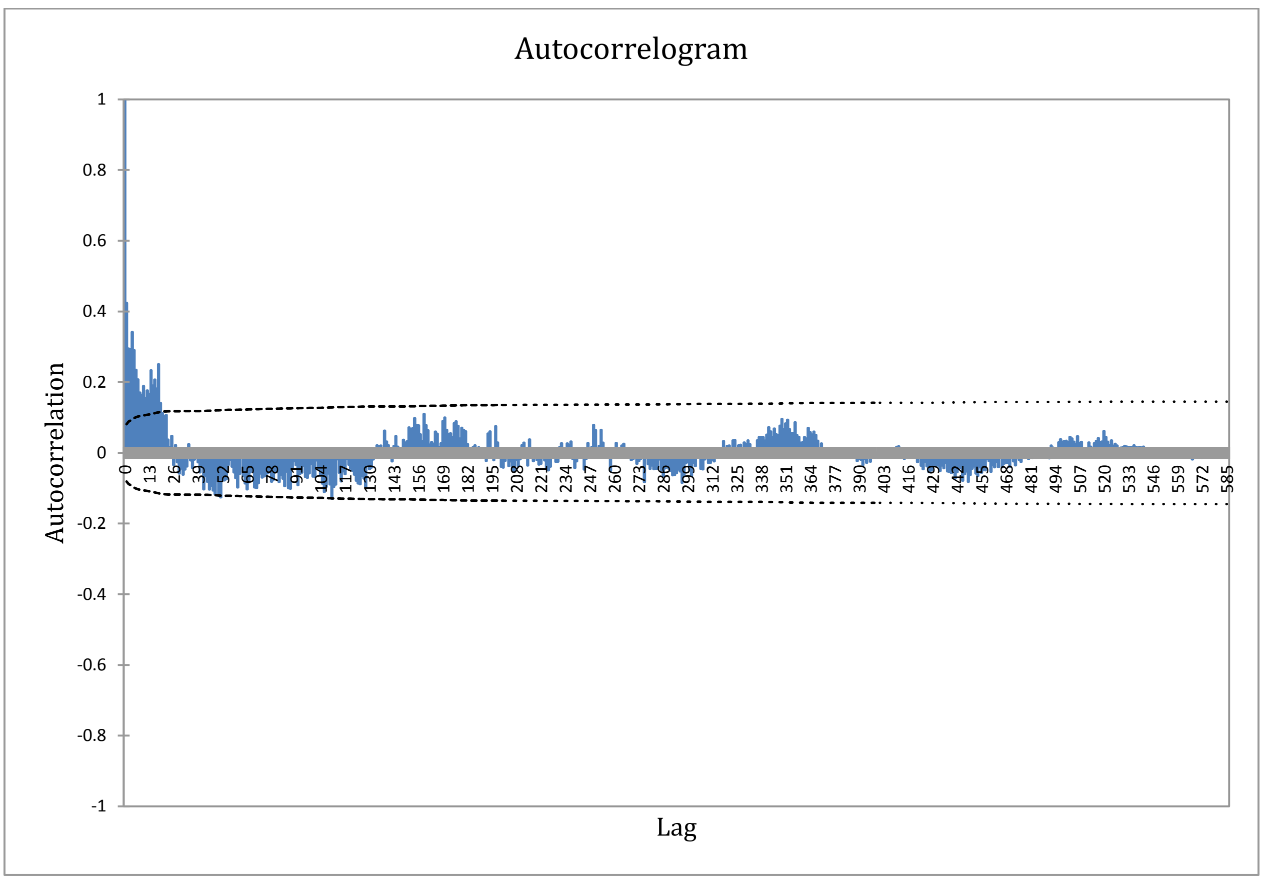


Figure 4.1: Weekly historical data of spot prices

Also, we want to make sure that the time series model is indeed a good choice here for our regression analysis and forecasting. So we attempt to find if there exists a liner/nonlinear relation between the data variables we have so far, which are the spot prices, consumptions, purchases and inventory. However, a correlations matrix and regression results would suggest there are no significant relations exist between the variables and maybe we should look into the relations between past week spot prices and current week spot prices. Then we see a strong linear relation between last week and current week spot prices with an adjusted R square > 0.95 here. So this confirms that we should consider a times series model for forecasting because of the strong relations between past and current week spot prices.

To start the time series analysis, initially we would like to examine the autocorrelation (AC) plot and partial autocorrelation (PAC) plot with a default 95% confidence level on the weekly historical data (Figure 4.2). Also, notice that ARMA is used to build a stationary linear time series model while ARIMA is used to build a non-stationary linear model with trend in the mean. Combing Figure 4.1 and Figure 4.2, we can see that our data is probably non-stationary. Also from Figure 4.2 we can see a large set of correlated lags that suggests an ARIMA with 1-difference. Thus we might prefer an ARIMA model than an ARMA model for our data but that requires further investigation and comparison of both models, which we will perform on later stages.



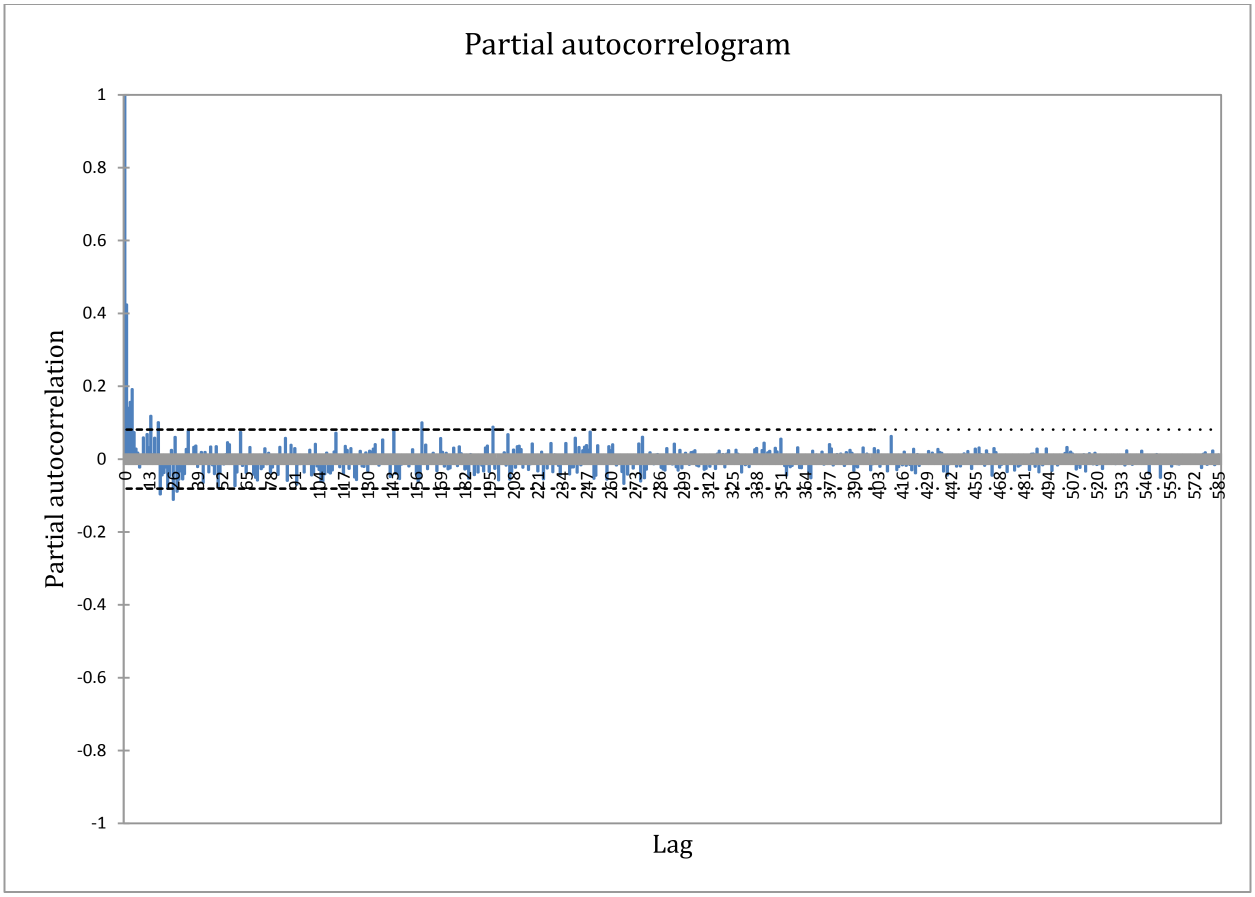


Figure 4.2: AC and PAC plots of original data

So next we perform 1 differencing and mean subtraction on the original spot prices data and obtain the residuals as new data on Figure 4.3. Let $\widetilde{C(m)}$ denote the mean subtracted price at month m, $\overline{C}$ denote mean value of the spot prices and W(m) denote the new residual price at month m.

$$\widetilde{C(m)} = C(m) - \overline{C}$$

$$W(m) = \widetilde{C(m)} - C(\widetilde{m-1})$$

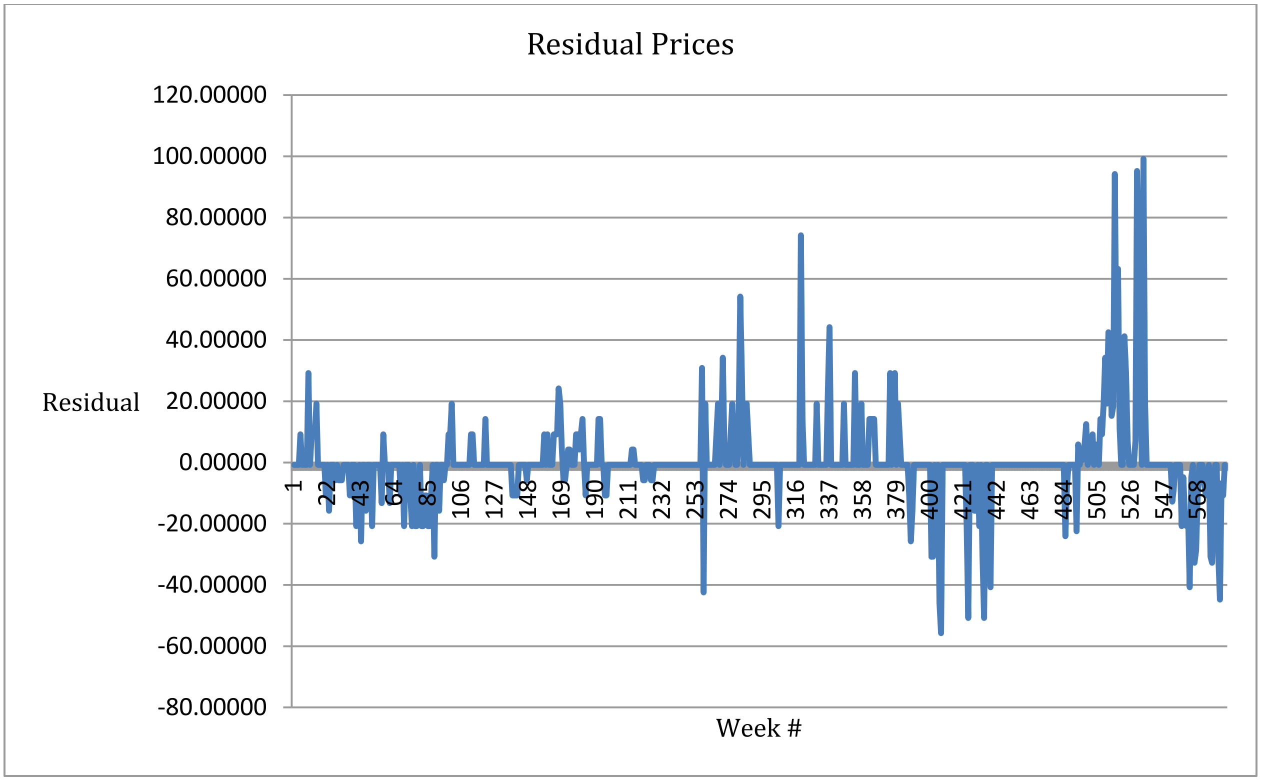
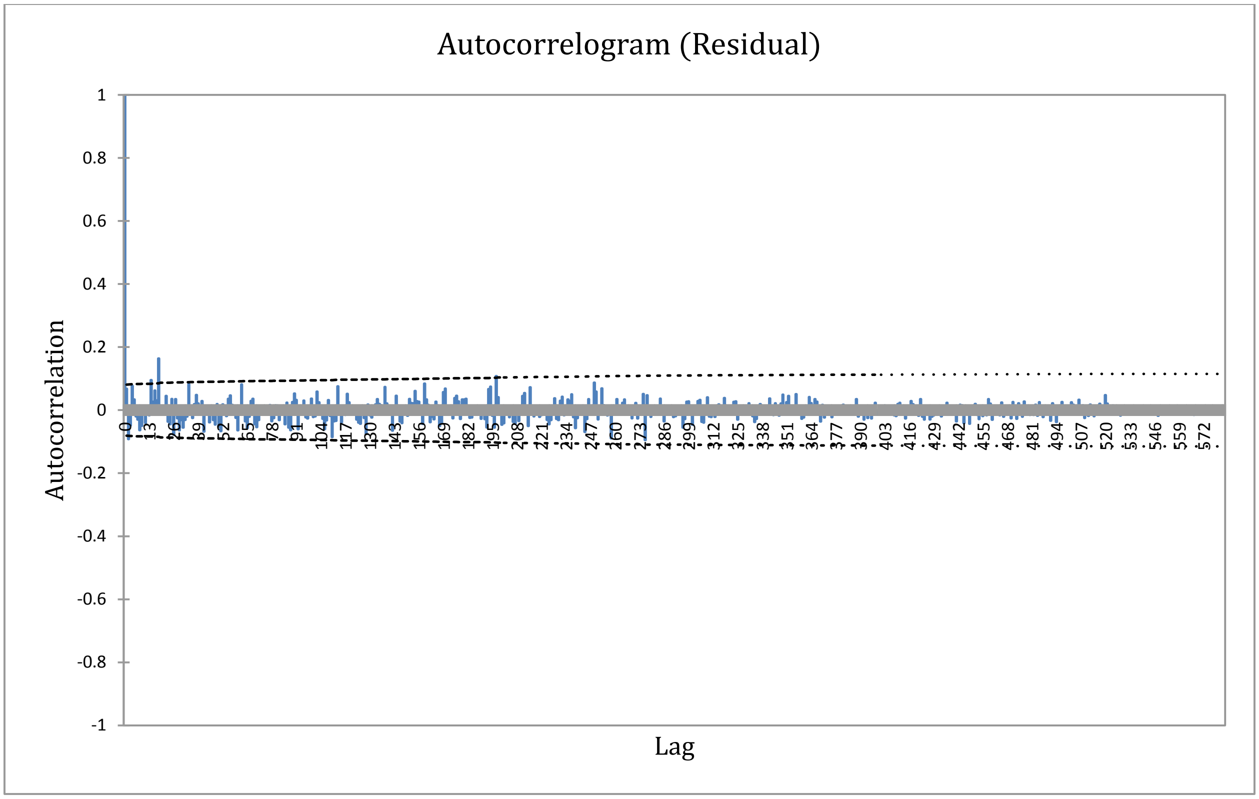


Figure 4.3: 1-differenced and mean subtracted data of spot prices

Again we check its AC and PAC plots (Figure 4.4) which shows a good result with value of 1 as the correlation ratio for the first lag. Similarly if we do 2-difference and mean subtract on the data we get a worse result on the lags because of over-differencing. This indicates us to use the ARIMA model with exactly one time differenced. Next we can proceed to build our ARMA/ARIMA model with all the above information.



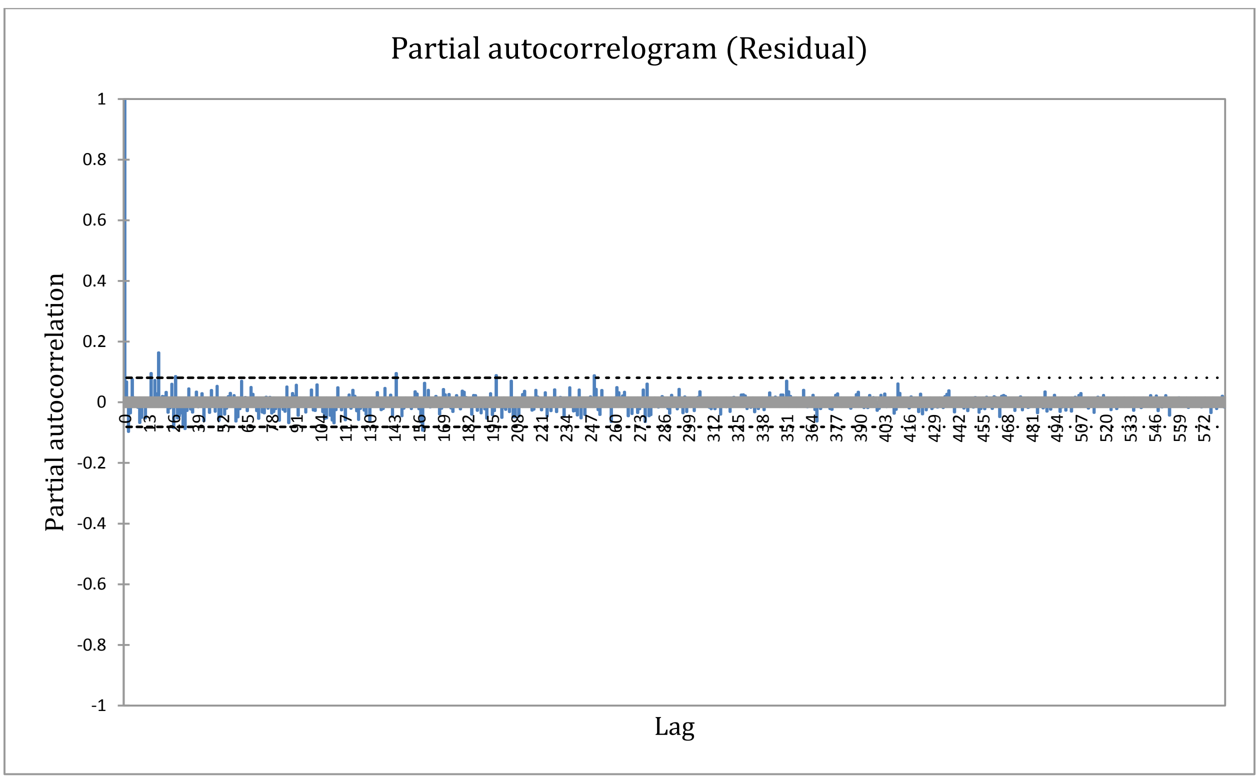


Figure 4.4: AC and PAC plots of 1-differenced and mean subtracted data

Let $\widehat{C(m)}$ denote the predicted spot price on month m, $\widehat{C'(m)}$ denote the predicted contract price on month m, W(m) denote the 1-differenced data on month m, and d is a discount of contract price. Following are the formulas that we are going to use for our times series ARMA/ARIMA modeling:

$$W(m) = \widehat{C(m)} - C(\widehat{m-1})$$

$$\widehat{C'(m)} = \widehat{C(m)} - d$$

For a ARMA(p,q) model with autoregressive of order p and moving average of order q on our data we will have:

$$\widehat{C(m)} = \sum_{i=1}^{p}(\Phi_i \cdot C(\widehat{m-\imath})) + \sum_{i=1}^{q}(\Theta_i \cdot \ \varepsilon(m-i)) \ + \ \varepsilon(m)$$

Where, $\varepsilon(m)$ is white noise, $\Phi_i + \Theta_i \neq 0$ for all i, where Φ_i and Θ_i are parameters to be determined by the ARMA model we build to fit for data.

Also for a generalized ARIMA(p,1,q) model with autoregressive of order p, 1-differenced and moving average of order q on our data we will have:

$$W(m) = \sum_{i=1}^{p}(\Phi_i \cdot W(m-i)) + \sum_{i=1}^{q}(\Theta_i \cdot \ \varepsilon(m-i)) \ + \ \varepsilon(m)$$

$$\Rightarrow \widehat{C(m)} - C(\widehat{m-1})$$

$$= \ \Phi_1 \cdot \left(C(\widehat{m-1}) - C(\widehat{m-2})\right) + \Phi_2 \cdot \left(C(\widehat{m-2}) - C(\widehat{m-3})\right) + \cdots$$

$$+ \Phi_p \cdot \left(C(\widehat{m-p}) - C(m\widehat{-p}-1)\right) + \sum_{i=1}^{q}(\Theta_i \cdot \ \varepsilon(m-i)) \ + \ \varepsilon(m)$$

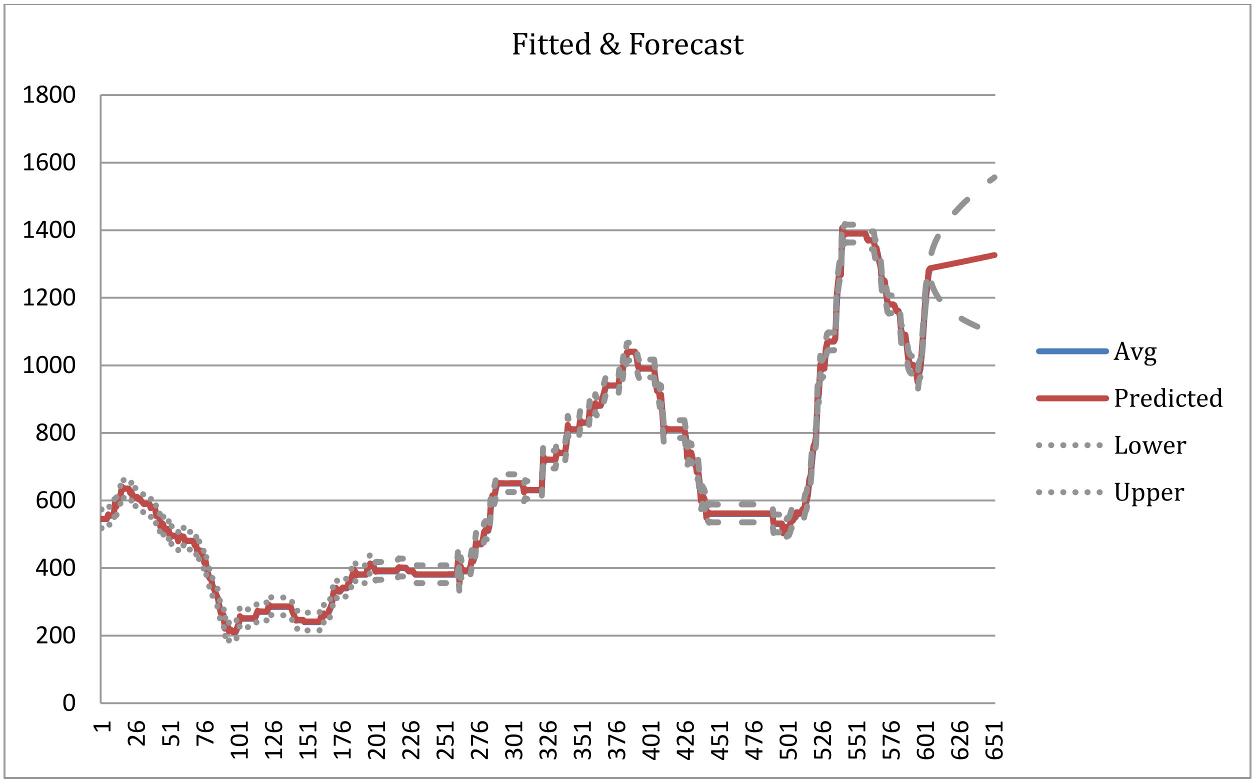
$$\Rightarrow \widehat{C(m)} = C(\widehat{m-1}) + \Phi_1 \cdot C(\widehat{m-1}) - \Phi_1 \cdot C(\widehat{m-2}) + \Phi_2 \cdot C(\widehat{m-2}) - \Phi_2 \cdot C(\widehat{m-3}) + \cdots + \Phi_p \cdot C(\widehat{m-p}) - \Phi_p \cdot C(m\widehat{-p}-1) + \sum_{i=1}^{q}(\Theta_i \cdot \varepsilon(m - i)) + \varepsilon(m)$$

$$\Rightarrow \widehat{C(m)} = C(\widehat{m-1}) \cdot (1 + \Phi_1) + \sum_{i=2}^{p} C(\widehat{m-\imath}) \cdot (\Phi_i - \Phi_{i-1}) - \Phi_p \cdot C(m\widehat{-p}-\imath) + \sum_{i=1}^{q}(\Theta_i \cdot \varepsilon(m-i)) + \varepsilon(m)$$

Here, $\varepsilon(m)$ is white noise, $\Phi_i + \Theta_i \neq 0$ for all i, and Φ_i and Θ_i are parameters to be determined by the ARIMA model we build to fit for data.

Now we will use some statistical software to test our ARMA/ARIMA models and see which one will fit best for our data. Here we are using SPSS, Eviews, R and a time series plugin for Excel to compare their results, and therefore choose the relatively better fit among all.

Firstly we show the “autoarima” function from SPSS’s ARIMA fit and predictions on Figure 4.5, the X axis stands for the week number as time period and the Y axis stands for the rare metal spot price. Also in the later ARMA/ARIMA model figures from different software fits and predicts, unless specified, the X axis is always the time line for week number and the Y is always the spot price.



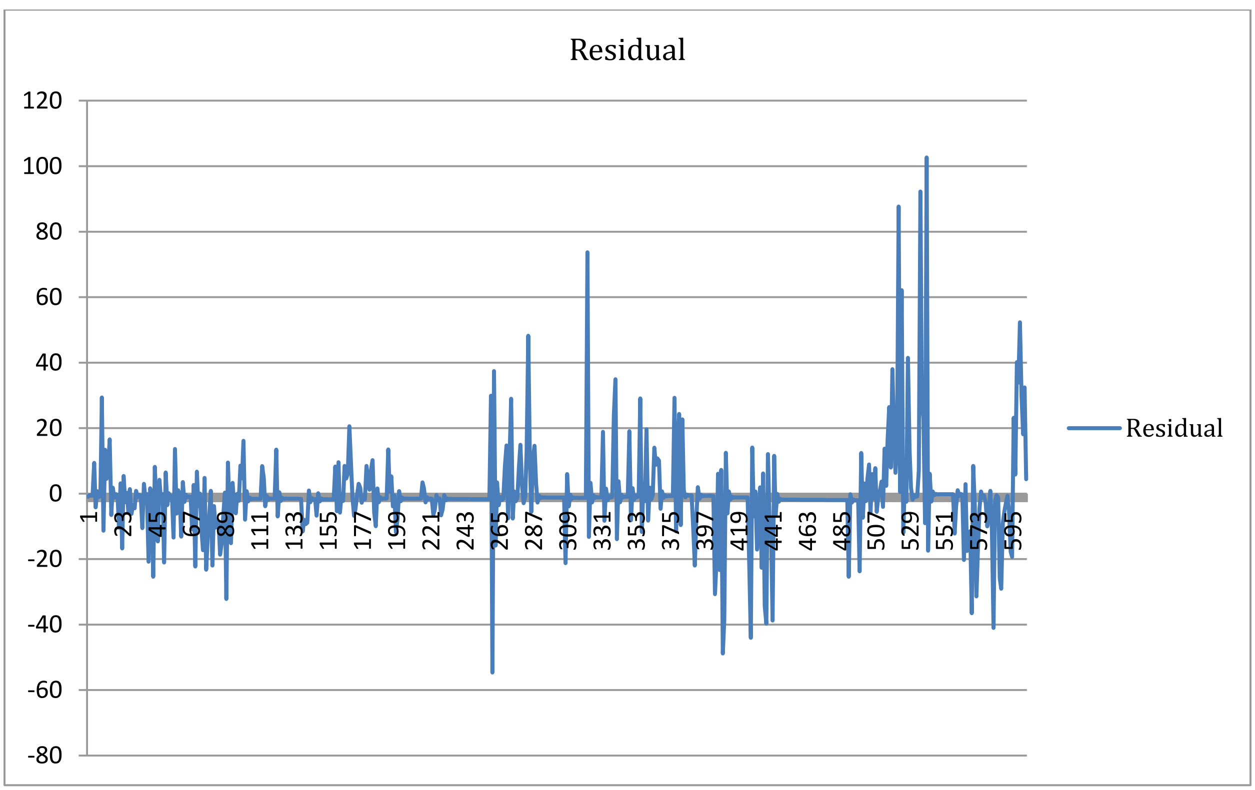
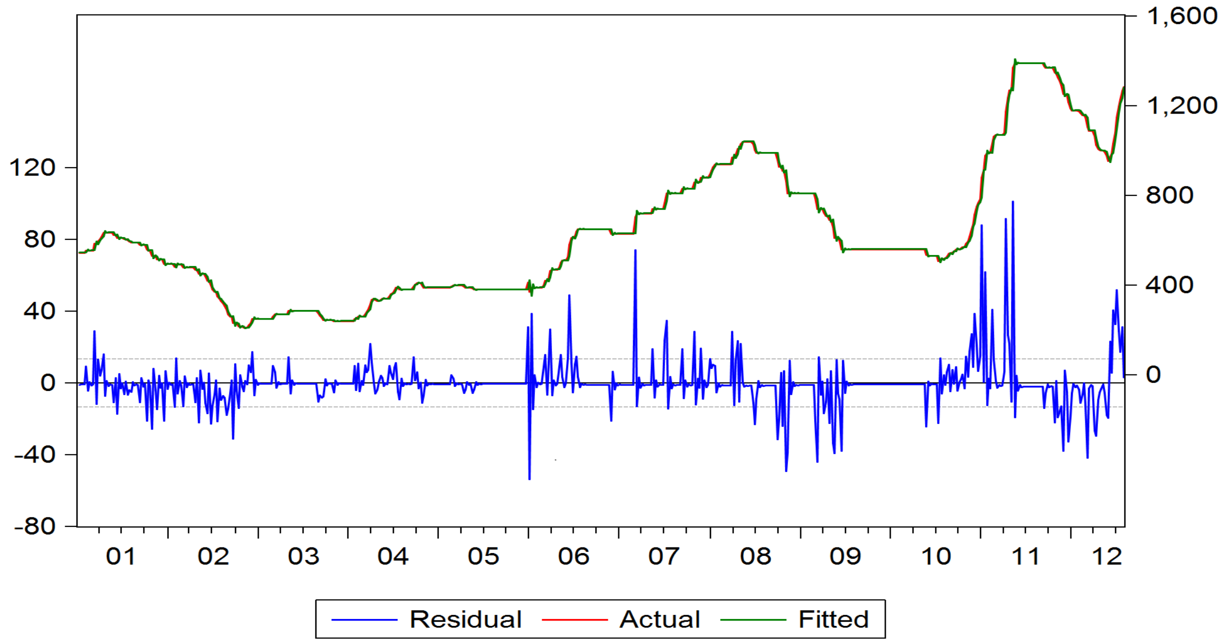


Figure 4.5: SPSS "autoarima" fit and forecast

Next let us look at Eviews' ARMA(1,1) model on Figure 4.6, where for the first graph, the left Y axis is the residual thresholds and the right Y axis is the spot price. And the second graph is the forecasting plot for the spot price of 12 periods with a two standard deviation line.



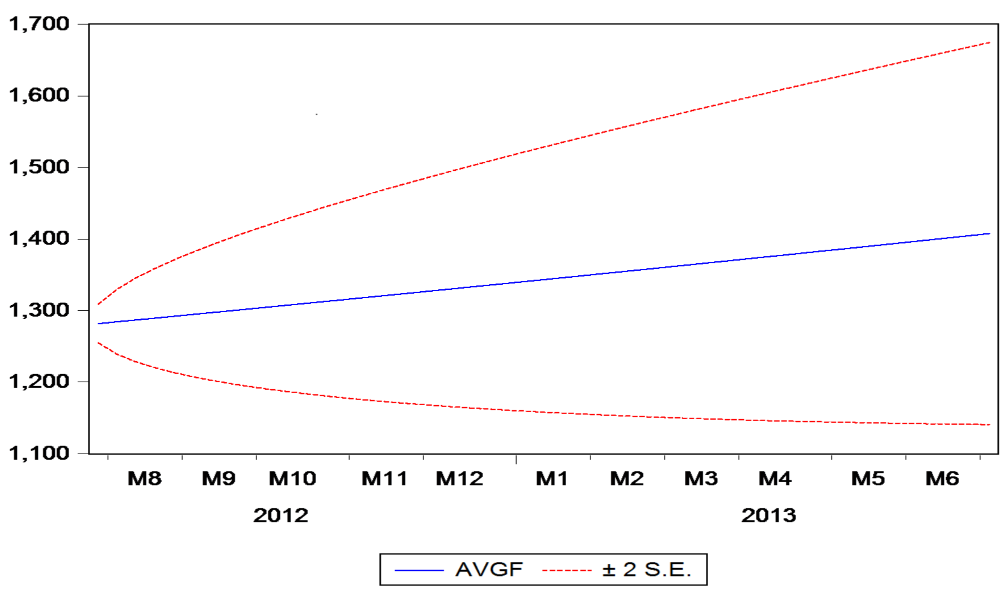
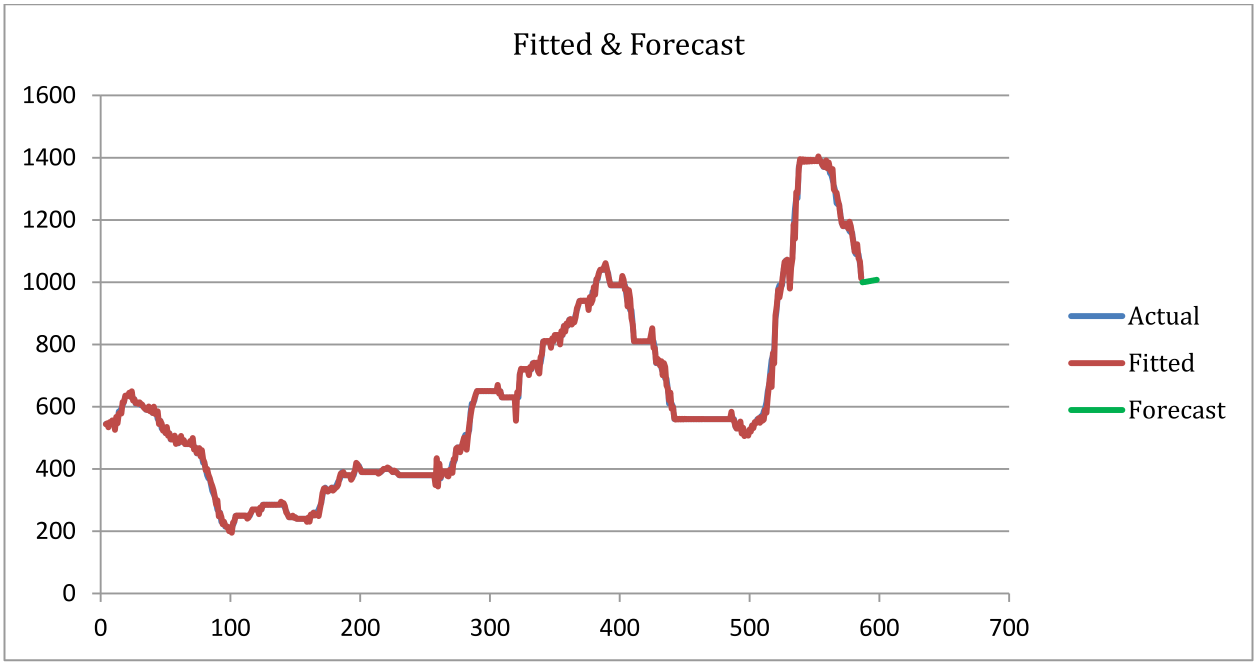


Figure 4.6: Eviews ARMA(1,1) fit and forecast

Next we will exam the ARMA model from the time series add-in of Excel, that is ARMA(3,1) on Figure 4.7. Also we adjust the time period range a little here with 2 months less comparing to the previous model to see how it reflects on the new one.



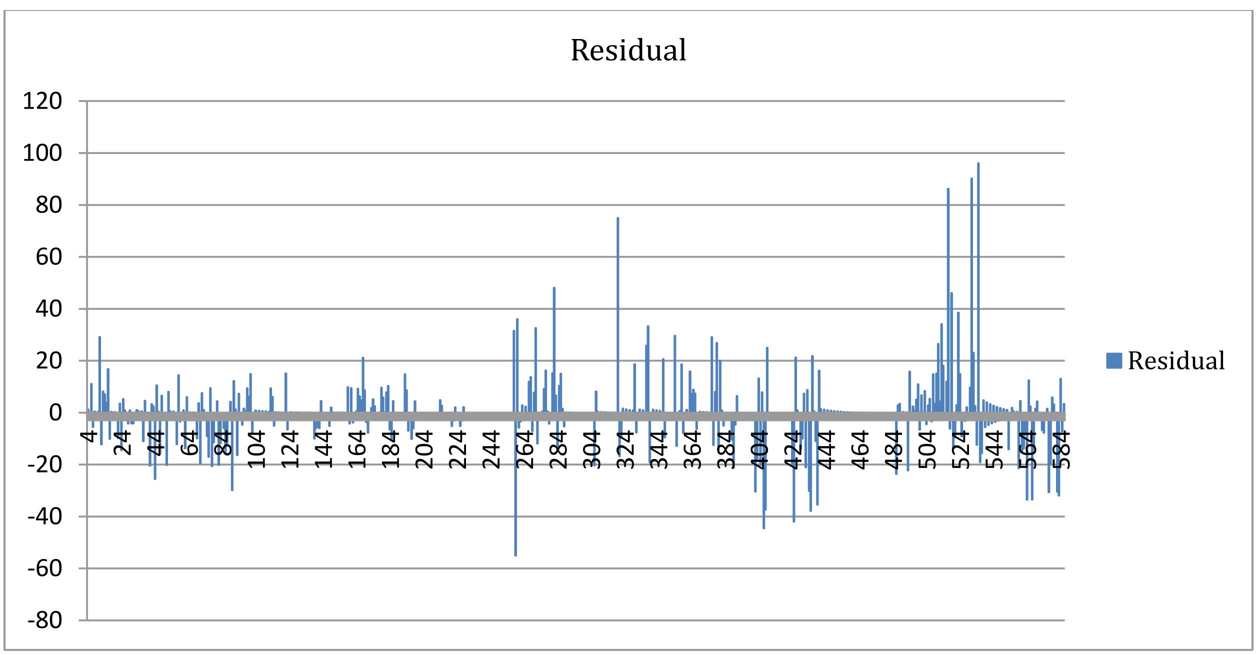
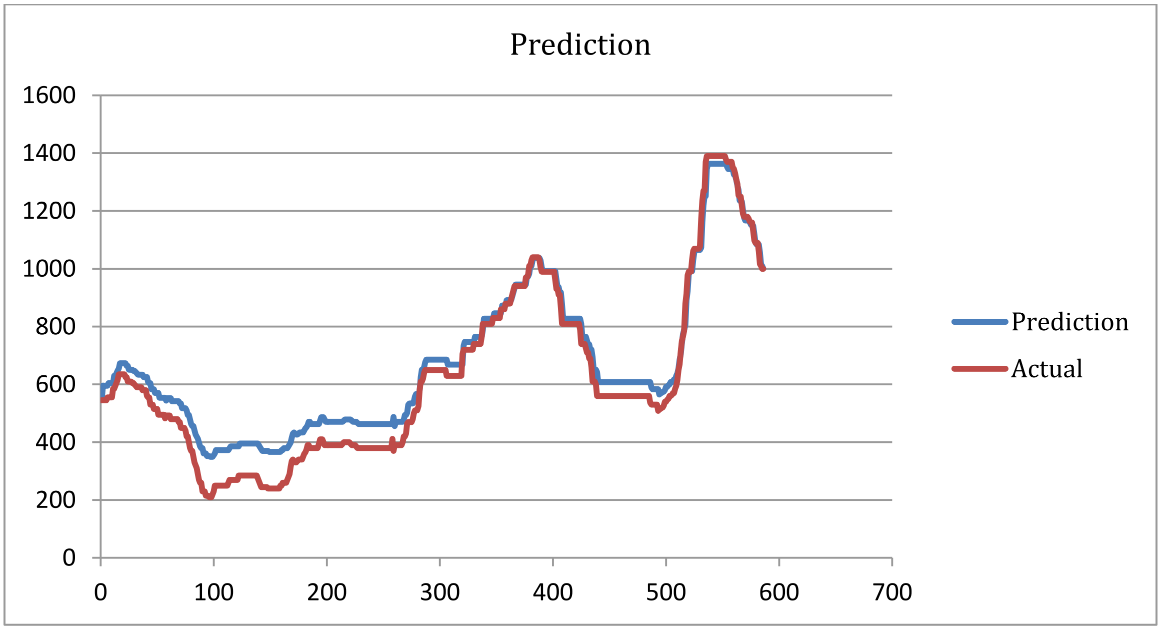


Figure 4.7: Excel ARMA(3,1) fit and forecast

Lastly, a GARCH model (Figure 4.8) using R to build is also considered here but the fits are significantly worse than all the ARMA/ARIMA models and thus should not be our choice. Again we adjust the time period range with 2 months less to see how it reflects on the new one.



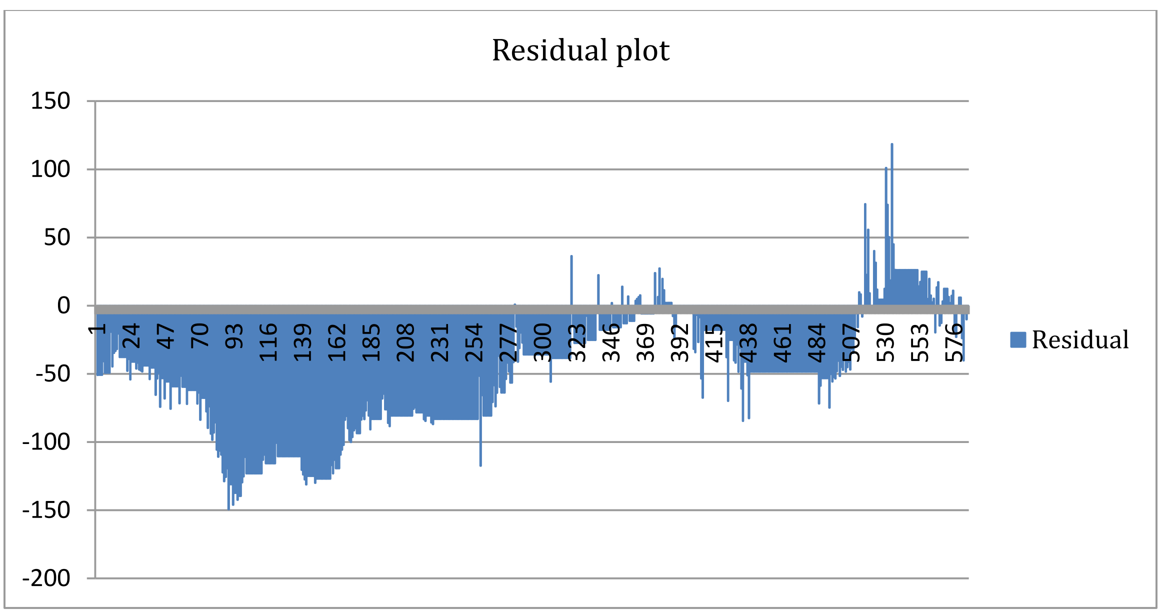


Figure 4.8: R GARCH fit and forecast

In summary, the time series model we choose to use for our forecasting should not only be a best fit of past data, but also a most accurate and correct model for forecasting data. Notice that from all of the above models we see a huge misfit around week 500-540. That is around year 2011 when Tōhoku earthquake and tsunami happened in Japan and its aftermath affected the market price of rare metal Y. We understand that such an unpredictable event happened all the time but it should not affect our model in a general sense. Now considering the fitting part, we conclude that ARMA(3,1) from Excel and “autoarima” from SPSS have a better performance. In order to see which forecast data performs better, since year 2010-2011 are the most unpredictable two years, we take the historical data from 2001-2009 and let all models to forecast 2010-2011 data to compare them with the actual data we have. Therefore although both ARMA(3,1) and “autoarima” were providing models with good fit for past data, “autoarima” from SPSS showed us a better forecasting ability. Thus we choose “autoarima” function from SPSS to do our forecasting with the results below on Table 4.1. As you can see, in a long turn, the price forecasting could make a large error comparing to the actual values. However, in our dynamic model for optimization we would only require for the next time period predicted value which usually is accurate enough and acceptable.

We can also do similar analysis on the demand forecasting part which contains a much smaller historical data set. Since we assume the company is always going to fulfill the demand, demand values are not as sensitive as the price. Thus we omit the procedures of the demand forecasting and provide the result directly on Table 4.2 below.

Spot Buy Price																										
Date	Period	Actual	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17	P18	P19	P20	P21	P22	P23	P24
(2010)1	P1	560	560																							
2	P2	560	560	560																						
3	P3	560	561	560	560																					
4	P4	560	561	561	560	560																				
5	P5	549	562	561	561	560	549																			
6	P6	530	562	562	561	561	523	530																		
7	P7	517	563	562	561	561	511	527	517																	
8	P8	529	563	563	562	561	502	522	509	529																
9	P9	554	564	563	563	562	495	518	502	544	554															
10	P10	566	565	564	563	562	490	516	498	550	566	566														
11	P11	623	565	565	564	563	485	514	494	555	575	575	623													
12	P12	754	566	565	564	564	482	512	491	559	582	582	697	754												
(2011)1	P13	950	567	566	565	564	479	510	488	562	586	587	738	834	950											
2	P14	1039	567	567	566	565	476	509	485	564	590	591	769	893	1044	1039										
3	P15	1070	568	567	566	566	473	508	483	565	593	594	791	932	1116	1112	1070									
4	P16	1190	569	568	567	566	471	507	481	566	596	596	808	963	1175	1171	1082	1190								
5	P17	1362	569	569	568	567	469	505	479	567	598	599	825	992	1230	1227	1102	1347	1362							
6	P18	1390	570	569	568	567	466	504	477	569	600	601	839	1017	1277	1274	1122	1453	1439	1390						
7	P19	1390	571	570	569	568	464	503	475	569	602	603	850	1036	1313	1312	1139	1540	1514	1409	1390					
8	P20	1390	572	571	570	569	462	502	473	570	604	605	862	1055	1350	1349	1158	1630	1590	1442	1399	1390				
9	P21	1377	572	571	570	569	459	501	471	571	606	607	873	1074	1384	1384	1176	1714	1661	1474	1416	1395	1377			
10	P22	1361	573	572	571	570	457	499	469	572	608	609	884	1092	1416	1416	1195	1792	1726	1505	1436	1409	1370	1361		
11	P23	1297	574	573	572	571	455	498	467	573	610	611	895	1109	1446	1447	1214	1866	1786	1535	1458	1427	1376	1343	1297	
12	P24	1227	575	574	572	571	453	497	465	574	611	612	904	1124	1472	1474	1231	1928	1837	1562	1479	1445	1387	1341	1221	1227

Table 4.1: Predicted price from SPSS "autoarima"

Demand																										
Date	Period	Actual	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16	D17	D18	D19	D20	D21	D22	D23	D24
(2010)1	D1	654	654																							
2	D2	388	841	388																						
3	D3	523	858	608	523																					
4	D4	621	868	798	754	621																				
5	D5	794	878	806	793	720	794																			
6	D6	863	888	813	801	781	822	863																		
7	D7	1084	899	821	808	788	798	821	1084																	
8	D8	1085	909	829	815	794	805	811	955	1085																
9	D9	689	919	836	822	801	812	818	854	925	689															
10	D10	1150	929	844	829	807	819	825	862	879	749	1150														
11	D11	917	939	852	836	814	825	832	870	888	857	1062	917													
12	D12	778	950	859	844	820	832	839	878	897	865	915	848	778												
(2011)1	D13	902	960	867	851	827	839	846	887	906	873	924	907	873	902											
2	D14	786	970	874	858	833	846	853	895	915	881	934	916	908	922	786										
3	D15	764	980	882	865	840	853	860	903	923	889	943	925	916	920	852	764									
4	D16	924	990	890	872	846	860	867	911	932	896	953	934	925	928	913	869	924								
5	D17	850	1001	897	880	853	867	874	920	941	904	962	943	934	937	921	911	939	850							
6	D18	856	1011	905	887	859	873	881	928	950	912	971	951	942	946	929	919	925	881	856						
7	D19	929	1021	913	894	866	880	888	936	958	920	981	960	951	955	938	927	933	924	911	929					
8	D20	891	1031	920	901	872	887	895	944	967	928	990	969	959	963	946	935	942	932	929	938	891				
9	D21	877	1041	928	908	878	894	902	953	976	935	999	978	968	972	954	943	950	940	937	939	916	877			
10	D22	1037	1052	935	915	885	901	910	961	985	943	1009	987	977	981	962	951	958	948	945	947	942	923	1037		
11	D23	1115	1062	943	923	891	908	917	969	994	951	1018	996	985	989	971	959	966	955	953	954	950	946	1002	1115	
12	D24	1023	1072	951	930	898	915	924	977	1002	959	1027	1004	994	998	979	967	974	963	960	962	957	953	965	1020	1023

Table 4.2: Predicted demand from SPSS "autoarima"

4.3 Modeling

After we did our data preparation including the data forecasting, we had all the data we need for the model. Now we have all the data including actual demand, predicted demand, actual spot price, predicted spot price, actual contract price, predicted contract price, actual procurement purchases and actual inventory levels. Notice for the spot price and demand in our dynamic programming, they will not all be the actual values but as a hybrid of both actual values and predicted values. And with the updates move on, our dynamic system will substitute the old predicted values with the recently obtained actual values and require new predicted values from ARMA/ARIMA forecasting model.

Next, we shall start to build our linear programming model to minimize the expected total cost of each month.

Total cost of month m:

$$\mathrm{T(m)} = \begin{cases} \mathrm{C(m) \cdot S(m) + C'(m) \cdot Q(m) + h \cdot I(m),} \\ \quad \text{for m is a beginning month of the year} \\ \mathrm{C(m) \cdot S(m) + h \cdot I(m),} \text{ for m otherwise} \end{cases}$$

$\mathrm{C'(m)}$ and $\mathrm{Q(m)}$ are determined by the beginning of the year since it is a long term contract price and will be decide at the beginning of month 1, 13, 25…etc.

The following constraints will ensure the validity of the assumptions.

Inventory level of month m:

$$I(m) = I(m-1) + S(m) + Q(m) - D(m)$$

$$I(m) \geq \frac{1}{2} D(m-1)$$

Procurement policy of month m:

$$0 \leq S(m) \leq 2$$

$$Q(m) > 0$$

Contract buy and spot buy relations at month m:

$$C'(m) \;= C(m) - d$$

With this linear program of every month m, we can then extend it to a multi period mathematical programming model with the objective of minimizing the total cost of all time periods in a finite horizon. Let M be the total number of months. Thus, we have the mathematical programming model as the following.

Minimize the total cost of all M months considering interest rate r:

$$T(M) = \sum_{m=1}^{M} \left(\frac{1}{1+r}\right)^{m} [C(m) \cdot S(m) \cdot +h \cdot I(m)] + \sum_{f=1,13,25\ldots} \left(\frac{1}{1+r}\right)^{f} C'(f) \cdot Q(f)$$

Such that:

$$I(m) = I(m-1) + S(m) + Q(m) - D(m)$$

$$I(m) \geq \frac{1}{2} D(m-1)$$

$$C'(m) = C(m) - d$$

$$0 \leq S(m) \leq 2, Q(m) > 0$$

Notice that all the values can be imported from our data preparation stage, and then plugged into the above system starting for m=1…M.

This means we have to run the linear programming repeatedly while updating the predicted data and plug in new actual data to solve the problem M times until the end. We approach to do so by using Xpress Mosel while incorporating it with updated ARIMA models imported from SPSS. Now let us try our algorithm based on all the assumptions we have made before and test its efficiency for the year of 2010-2011, since they are the most unpredictable years, and would sort of serve us as the worst case scenario. In this case M=24 and January of 2010 will be m=1.

The codes we use to build the above single time period model in Xpress Mosel can be found in Appendix B, where the spot price C and demand D are in a hybrid fashion containing both actual values from historical data and predicted values from ARIMA model.

4.4 Optimization Result

Eventually, we want to compare our optimum procurement policy with the actual procurement company X made during 2010-2011 and by inserting different interest rate or holding cost we can also simulate the policy under different scenarios. Here we would like to show one of the most basic scenarios with h = 10, d = 50 and r = 0.0006. Also, since it is a simulation for year 2010-2011, M = 1…24 and F = 1…2. Since we are doing this linear programming optimization with updates, only every month's spot buying strategy will be adopted and the inventory will rely on the actually demand in the future rather than the predicted values from ARIMA model. That is, using the new spot price we observe, demand predictions from the ARIMA model and the policy we have adopted last month via the linear programming model, we run the linear program to compute a new inventory level, thus decide on the order quantity. The linear programming model is applied repeatedly in order to provide the next month's procurement policy as well as calculate the new inventory level. Eventually, we will reach our last time period and then we can consolidate the entire spot buy, contract buy and inventory level control record for all months.

In summary, by running the above Xpress Mosel model over 24 months with each time importing a new updated data from ARMA/ARIMA, we have the result for every month's spot buying and contract buying amount, as well as the inventory levels. Finally, the optimum expected total cost is: $15,922,901, with a total purchase of 21937 kg of rare metal Y and average of $726/kg, the detailed optimal policy is at the following.

Spot buying quantity of month 1-24: all zeros kg.

Contract buying quantity of year 2010 and 2011: 11167kg and 10770kg.

Inventory level of the month 1-24: 1276kg, 1819kg, 2227kg, 2536kg, 2673kg, 2740kg, 2587kg, 2432kg, 2674kg, 2455kg, 2468kg, 2621kg, 2616kg, 2728kg, 2861kg, 2835kg, 2882kg, 2924kg, 2892kg, 2899kg, 2919kg, 2780kg, 2562kg, 2437kg.

When we compare the optimum procurement policy to company X's actual purchases, we see that they have made a total purchase of 19983kg of rare metal Y and total cost of $17,165,381 with average purchase of $ 859/kg, we have spent 7.8% less on the total cost and 18.3% less per kg on rare metal of Y while doing more purchasing. Thus, it strongly suggests that our approach would provide considerable savings for the total cost.

Chapter 5

Conclusions

In this thesis, we study various methods for optimal procurement policy under different circumstances of our own. Firstly, in chapter 3, we use a policy that its form is given as a base-stock policy and try to find the best base-stock value K*. Also, we investigate the impact of various parameters on the optimal base-stock value. Secondly, in chapter 4, we approach a real life scenario where we have to provide an optimal procurement policy from historical data. We applied time series modeling to provide raw material price and demand forecasts that will be used in a mathematical program. This mathematical program is solved recursively by updating the price and demand forecasts at each time step. In this case study, we show that inventory/procurement costs can be reduced by implementing our algorithm. In future, we plan to investigate properties and structure of the optimum policy for procurement and production planning. We are especially interested in threshold type policies that are easy to implement.

Appendix A

The coding for the expected total cost respect to different values of K in Matlab when K ≥ 5 is below.

```
%K >= 5
function total = totalcost(K)


% Parameters
a = 20/108;
b = 5/108;
p = 0.5;
k = K-1;
h = 0.5;
YL = 15;
YH = 25;


% Build the matrix
A = [ones(1,k); zeros(1,k); zeros(k-2,1) (diag(-ones(1,k-2))+diag((1-
a)*p*ones(1,k-3),1)+diag((1-a)*(1-p)*ones(1,k-4),2)) zeros(k-2,1)];


A(2,1) = -a;
A(2,2) = 1-a;
```

```
A(2,3) = (1-a)*(1-p);

A(k-1,k) = b*(1-p);
A(k,k) = b*p;

B = zeros(k,1);
B(1) = 1;

% Solve all the pi
mypiT = A\B;

mypi = (mypiT)';

display(mypi);

% Calculate total cost with respect K
total1 = 0;
total2 = 0;
for i = 2:k
   total1 = h*(mypi(k)*K+sum(i*mypi(i-1)));
   total2 = YL*(mypi(k)*(1-b)*(p+2*(1-p)))+YL*sum(mypi(i-1)*a*((K-i+1)*p+(K-
i+2)*(1-p)));
end
```

```
total3 = YH*(mypi(1)*(1-a)*(p+2*(1-p))+mypi(2)*(1-a)*(1-p));

total = total1 + total2 + total3;

display(K);
```

Appendix B

The coding in Xpress Mosel of a single time period mathematical model with data imported from ARIMA is below.

```
model ModelName

uses "mmxprs";



!parameters

declarations

M = 1..24 !insert all time periods

F = 1..2 !insert the number of years

h = 10 !insert holding cost here

r = 0.0006 !insert interest rate here

d = 50 !insert contract buy discount here

C: array(M) of real

dC: array(F) of real

D: array(M) of real

end-declarations
```

```
!import data from ARIMA model

initializations from 'OptimalPolicy.dat'

C D

end-initializations



!setup values

declarations

S: array(M) of mpvar

Q: array(F) of mpvar

I: array(M) of mpvar

end-declarations



!constraints

dC(1):= C(1) - d

dC(2):= C(13) - d

!insert more if there are more time period
```

```
I(1)= 1000 + S(1) + Q(1)/12 - D(1)

forall(m in 2..12) I(m)= I(m-1) + S(m) + Q(1)/12 - D(m)

forall(m in 13..24) I(m)= I(m-1) + S(m) + Q(2)/12 - D(m)

!insert more if there are more time period



forall(m in 2..24) I(m) >= 2 * D(m-1)

!insert more if there are more time period



forall(m in M) S(m) <= 3000

forall(f in F) Q(f) >= 0



!objective

cost:= dC(1) * (1/1+r)^1 * Q(1) + dC(2) * (1/1+r)^13 * Q(2) +

!insert more if there are more time period

        sum(m in M) C(m) * (1/1+r)^m * S(m) +

        sum(m in M) h * (1/1+r)^m * I(m)
```

```
!solve
minimize(cost)


writeln("Begin running model")
writeln("The minimum of the total cost is: ", getobjval)
forall(m in M) writeln("Spot buy quantity of month(", m, "): ", getsol(S(m)))
forall(f in F) writeln("Contract buy quantity of year(", f, "): ", getsol(Q(f)))
forall(m in M) writeln("Inventory level of the month(", m, "): ", getsol(I(m)))
writeln("End running model")


end-model
```

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