AN INVESTIGATION OF TWO ENGAGEMENT STRUCTURES IN MIDDLE SCHOOL MATHEMATICS CLASSROOMS

by

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ABSTRACT OF THE DISSERTATION

An Investigation of Two Engagement Structures in

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Engagement and motivation have become increasingly important to mathematics education, particularly as concern about the educational success of students in the US grows. As a way to describe the patterns cognitive, behavioral, and affective engagement of students, Goldin, Epstein, Warner, and Schorr (2011) developed the theoretical concept of engagement structures. *Engagement structures* are idealized, recurring highly dynamic affective patterns inferred from behavior. This study focuses on characterizing two of these patterns of behavior: Let Me Teach You (LMTY) and Look How Smart I Am (LHSIA) in the context of urban middle-school students working on a conceptually challenging problem. These structures were selected because they both are likely to focus on the mathematical content as well as require interaction between students.

This dissertation reports on a qualitative study with video and audio data as well as survey responses of 55 seventh- and eighth-grade students in four classes in a large urban school district in NJ. These students worked in small groups (3-4 students) on the same task. The students' survey responses were compared to the findings based on the audio and video analysis. The results indicate that students appear to help one another and are often, though not always, willing to accept help from classmates. Students who attempt to impress others in some way may be trying to prove they are smarter than others or may wish to be perceived at least as smart as their classmates. Behavioral features for each engagement structure emerged from the data and describe the different behaviors that seem to be in service of the same goal. The features associated with the LMTY structure are: 1) teaching to clarify, 2) teaching procedures, 3) teaching or explaining strategies, and 4) checking for understanding. The features associated with the LHSIA structure are: 1) expressing an idea, 2) stating an answer or answers, 3) correcting others, 4) stating, "I'm smart," or "We're smart," 5) keeping up with others in the group. This study, an initial investigation of two patterns of engagement, contributes to a goal of better understanding the complexity of "in the moment" engagement in the mathematics classroom.

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Dedication

For my beautiful mother, my first teacher

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Chapter 1: Introduction

1.1 The importance of student engagement in mathematics education

In the report *Helping Children Learn Mathematics* (Kilpatrick & Swafford, 2002), the National Research Council (NRC) identifies five interdependent and interwoven strands of mathematical proficiency: “1) understanding mathematics; 2) computing fluently; 3) applying concepts to solve problems; 4) reasoning logically; and 5) engaging with mathematics” (p. 1). The report goes on to say that student engagement is “the key to success” (p. 9), suggesting that engagement plays a crucial role in the development of students’ mathematical knowledge and skills. Engagement is commonly defined as a commitment to or involvement in some activity. When students engage with the mathematics, they become involved in finding solutions, thereby improving their reasoning and comprehension of concepts and ideas (Kilpatrick & Swafford, 2002).

The importance of student engagement was described by the NRC in its earlier report *Adding It Up* (Kilpatrick, Swafford, & Findell, 2001), where they state that engagement[1], “is a major factor in determining [students’] educational success” (p. 131). According to this report, when students are engaged, they develop confidence. When students successfully perform operations and apply mathematical concepts to problems, they become more confident in their ability to understand mathematics. Through these interwoven processes – understanding, computing, applying, reasoning, and engaging – mathematics comes to make sense to students and is viewed as useful, meaningful, and

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[1] *Helping Children Learn Mathematics* (Kilpatrick & Swafford, 2002), based on the book *Adding It Up* (Kilpatrick, Swafford, & Findell, 2001), uses the term “engaging” to simplify the phrase “productive disposition” in the earlier report. These labels are intended to be interchangeable. I continue to use the words “engaging “or “engagement.”

worthwhile. The NRC emphasizes that continually engaging with the mathematics can help all students become mathematically proficient (Kilpatrick, Swafford, & Findell, 2001; Kilpatrick & Swafford, 2002).

However, many students may not be engaged in school or, specifically, in their mathematics classes. As the National Council of Teachers of Mathematics (NCTM) states, “Too many students disengage from school mathematics, which creates a serious problem not only for their teachers but also for a society that increasingly depends on a quantitatively literate citizenry” (NCTM, 2000, p. 371). Students may develop negative attitudes and beliefs toward mathematics as they progress through their schooling (Kilpatrick et al., 2002). Some individuals may find mathematics challenging and as a result, come to believe they are not smart enough to do mathematics. Many students consider mathematics to be a boring set of facts and algorithms to memorize. Students who might be otherwise interested in mathematics may disengage due to pressure from their peers who discourage positive attitudes toward mathematics and other academic activities (Kilpatrick & Swafford, 2001). These students may avoid challenging mathematics courses “at the expense of precluding careers in science, technology, medicine, and other fields that require a high level of mathematical proficiency” (Kilpatrick & Swafford, 2001, p. 132).

The National Science Board (NSB, 2007) published a set of recommendations in their report, *A National Action Plan*, aimed at improving science, technology, engineering, and mathematics (STEM) education for all U.S. students. The *Plan* recommends providing opportunities to “enhance the Nation’s ability to produce a numerate and scientifically, and technologically literate society” (p. 2) and preparing individuals to join the STEM workforce. For example, the NSB asserts that educational programs ought to engage students of all ages in STEM curricula and similar opportunities. This recommendation is echoed in NSB's 2010 report *Preparing the Next Generation of STEM Educators*. In this document, NSB points out that some programs that already exist are typically offered only to high-performing students who have already demonstrated interest in science, technology, engineering, and mathematics.

The NSB also recommends that schools and communities work together to foster supportive environments for students (NSB, 2007; 2010). For example, they suggest that teachers and parents take steps to ensure they do not pass on their own fears or aversions to mathematics and other STEM-related subjects. They suggest offering support by "display[ing] a positive attitude toward learning and discovery to their children" (NSB, 2010, p. 23). NCTM echoes this sentiment, suggesting that, "Disengagement is too often reinforced in both overt and subtle ways by the attitudes and actions of adults who have influence with students" (NCTM, 2000, p. 371). By recognizing the influence of their own attitudes on students, teachers and parents can nurture students' enthusiasm about learning mathematics. In this way, educators and others can promote student engagement and student interest in mathematics and science-related fields.

The NSB reports convey concern over student performance and proficiency in mathematics and science. For example, they cite statistics stating that over 30% of first-year college students are placed into remedial mathematics and science courses because they are not academically ready to enroll in college-level courses. This is disappointing but not necessarily surprising, given the low number of mathematics courses required for some students and the low percentage of students who have taken higher-level mathematics courses (e.g., pre-calculus) in high school. The National Center for Education Statistics (NCES) presents details about the mathematics courses that high school students have taken, among other things, in an annual report, the *Condition of Education.* The 2007 report stated that, as of 2005, students in only 27 states were required to take 3 or more years of mathematics coursework, though some districts and schools may have additional requirements (Planty, Provasnik, & Daniel, 2007). The most recent report found that in 2009, 76% of graduating high school students reported taking an Algebra II course, up from 54% in 1990, and 35% had taken a pre-calculus course, up from only 13% in 1990 (Aud et al., 2012, p. 7). These increases in the percentage of students taking higher mathematics courses are encouraging, but do not necessarily demonstrate that students are preparing themselves for college or for work in the STEM fields.

Results of student performance on national assessments such as National Assessment of Educational Progress (NAEP) indicate that many students are not proficient in mathematics. Success rates on NAEP decline for students in upper grades, compared to their counterparts in earlier grades. In 2009, 39% of fourth-graders were at or above proficiency, while 34% of eighth-grade students were at or above proficiency. Only 26% of twelfth-grade students were at or above proficiency. The number of students who report enjoying mathematics declines as well. On that same assessment, about 62% of fourth-grade students and 64% of eighth-grade students reported liking math, compared to 55% of twelfth-grade students. While test scores provide only a partial measure of mathematics learning, these numbers suggest a need to improve proficiency and increase student interest in mathematics across all grades within the United States.

In short, it is widely recognized that students' level of proficiency in mathematics needs to be improved in the United States. However, many politicians and administrators use test scores, such as those cited above, as the only indicator of proficiency and learning. Kilpatrick & Swafford (2002) note that on such standardized assessments, procedural skills have often been tested exclusively. These tests may be neglecting to assess other important skills taught in mathematics classes such as conceptual understanding and problem-solving skills needed outside of school. Even though several state-wide, national, and international assessments currently include items which require written responses, many items are still multiple-choice. Very often other indicators of student learning, such as student portfolios which could include student work throughout a school year, are not considered or recognized by school districts or states when reporting the success or failure of student performance in mathematics. In addition, in a brief review of recent literature, Ashcraft and Moore (2009) offer evidence that math and test anxiety can cause students to perform worse on tests, particularly in a timed, high-stakes environment, including the setting in which such standardized tests are given.

Even if test scores improve, there is no guarantee that interest in mathematics will improve. Many educators and policy makers have made recommendations to increase both the level of proficiency and student interest in mathematics, which include implementing different programs or curricula and better preparing K-12 teachers to teach mathematics. Among these recommendations, one common goal emerges: to increase student engagement in the classroom. The question then arises how to foster and support student engagement, particularly for those students who believe they lack natural mathematical ability.

In 2008, the National Mathematics Advisory Panel (NMAP) published a report aimed at addressing concerns about mathematics literacy. The Panel's recommendations to improve the teaching and learning of mathematics for students include suggestions regarding mathematical content and instructional practices. Moreover, the report advises teachers to emphasize "the vital importance of effort in learning mathematics" (p. 31) to students and their parents. NMAP notes that many people believe that mathematical ability is innate. The report counters this idea, stating that students who recognize that putting forth effort can lead to mathematical success are more likely to engage with mathematics.

Several classroom factors are believed to lead to productive engagement, including teacher attitudes and behaviors and the tasks given to students (NCTM, 1991; 2000). NCTM suggests that mathematics teachers should not only have a deep knowledge and understanding of mathematics, but also ought to provide a challenging and supportive environment for the students (NCTM, 2000). According to NCTM, teachers should support and challenge their students by communicating the belief that all students can learn mathematics. Additionally, they recommend that mathematical tasks should "promote the development of all students' dispositions to do mathematics" (NCTM, 1991, p. 24). Many educators believe that students may be more likely to engage with the mathematics when the task relates to their interests and experiences (Middleton & Jansen, 2011; NMAP, 2008), and challenges them intellectually (NSB, 2010). When teachers encourage students while working on stimulating and demanding tasks, they may be able to provide an environment in which students can feel confident about themselves as doers of mathematics.

NCTM (2000) and the NRC (2001; 2002) endorse student collaboration which can be fostered through small-group work and whole-class discussions. The NMAP (2008) report found evidence that students may improve their computation skills in cooperative learning, particularly when students assist one another in heterogeneous groups. Students who work with their classmates or participate in discussions with others may be continually engaged during their interactions (Bransford, Brown, & Cocking, 1999; NCTM, 2000). NCTM promotes the idea that students should learn to communicate about mathematics by sharing strategies, asking classmates to explain or defend their strategies, building on others' suggestions, and working together to arrive at a solution. According to the NRC, small group work can "promote positive social interactions among students" (Kilpatrick & Swafford, 2002, p. 27).

Students who engage with mathematics in the classroom are, at least sometimes, motivated to do so for some identifiable reasons. Many of the reports shared here discuss motivation as either *intrinsic* or *extrinsic*. Intrinsically motivated students are driven to learn something because the topic is genuinely interesting or because the student enjoys learning. Students who are extrinsically motivated are working for a reward or some external goal, such as praise, a grade, or a prize for doing well. But when students are labeled as being either intrinsically or extrinsically motivated to learn, educators may be overlooking the complexity of the students' interactions, engagement, and motivation in the classroom. For example, a student who typically engages with mathematics strictly to earn a passing grade may become excited about a task that relates to his favorite sport. On the other hand, a student who usually enjoys math class may be assigned to work with students she does not like. This student may feel annoyed with her classmates and may disengage from the group's work. Many similar examples are familiar to experienced teachers.

Motivation and engagement continue to be areas of interest for educators and researchers. For example, NCTM published their seventy-third yearbook on this issue, titled *Motivation and Disposition: Pathways to Learning Mathematics* (Brahier, 2011). This book covers topics ranging from different perspectives on motivation to ways to motivate and engage different populations of students in the classroom to suggestions on professional development for teachers. In this book and in other reports, motivation and engagement are discussed mainly with respect to long term traits. Certainly, students have enduring traits, leading educators to label some students as "self-motivated" and others as "disengaged." However, a student may find himself in a situation that influences his state and behavior, which may change moment to moment. The classroom contexts and environments, as well as a variety of circumstances, all impact a student's motivation and engagement at any given time.

The study presented here examines student engagement "in the moment" as it occurs within the classroom as students work on a mathematical task. By viewing engagement as involvement in some activity, I seek to go beyond the general descriptors "engaged" and "disengaged." I aim to describe specific patterns of verbal and nonverbal behaviors as students involve themselves in mathematical activity.

1.2 Background of the study

This study is part of a larger project that was conducted by MetroMath: The Center for Mathematics in America's Cities[2], an NSF-funded partnership between Rutgers University, the City University of New York, and the University of Pennsylvania, directed by Prof. Gerald Goldin. Research conducted by the MetroMath Center focused on the teaching and learning of mathematics in urban school districts. Faculty, graduate students, and other researchers involved with MetroMath represented various disciplines, including mathematics education, urban education, mathematics, and cognitive science. As a MetroMath graduate student fellow starting in 2006, I became a member of the research team exploring student and teacher affect (e.g., attitudes, mood, emotions) in urban middle school mathematics classrooms. Investigating students in middle school classrooms is particularly important because, "During this time, many students will solidify conceptions about themselves as learners of mathematics— about their competence, their attitude, and their interest and motivation" (NCTM, 2000, p. 211).

In a first, exploratory study, three urban classrooms in predominantly African-American and Hispanic, low-income communities were videotaped several times over the course of the 2006-2007 school year. Following each observed class session, video based stimulated recall interviews were conducted and recorded with selected students asking them about different events that occurred during the class. Upon investigation and analysis of the data, we inferred that students chose to engage in a mathematics class in a multitude of ways, particularly when assigned to work in small groups. Several patterns

[2] This research was supported by the U.S. National Science Foundation (NSF), grant no. ESI-0333753 (MetroMath: The Center for Mathematics in America's Cities). Any opinions, findings, and conclusions or recommendations are those of the author and do not necessarily reflect the views of the NSF.

of behavior and engagement were identified. For example, some students pretended to be working when they were not. Other students worked independently, sharing only final solutions with others. On the other hand, some students were excited to share ideas and came to solutions through collaborative effort.

These various forms of engagement may be based on a student's in-the-moment as well as longer-term goals and desires. A student's engagement in a mathematics class varies from moment to moment. For example, a student may be trying to impress classmates with his answer in one minute, and in the next, may be defending his solution because someone challenged his explanation. The underlying motivation of a student's engagement may be influenced by circumstances which are specific to the day and the environment. For example, a student who is typically not interested during math class may on occasion find a particular activity challenging and be motivated to work toward a solution.

In the present study, I examine two particular patterns of engagement, called *engagement structures*, which may occur when small groups are assigned to work on a conceptually challenging task. Goldin, Epstein, Schorr, and Warner (2011) have identified and described nine such structures and suggest others likely to exist. The first pattern I investigate is called "Let Me Teach You" (LMTY). It entails a student trying to help a classmate understand some aspect of the problem or the mathematics involved. The second pattern is called "Look How Smart I Am" (LHSIA). In this scenario, a student wants others (e.g., classmates or a teacher) to recognize that he or she is smart, intelligent, has mathematical ability, or is knowledgeable about the problem or the mathematics. These two engagement structures and several others are described in more detail in Chapter 2. The purpose of this study was to further characterize and compare "Let Me Teach You" and "Look How Smart I Am" as these patterns of engagement were inferred to occur during the mathematical problem solving activity of 55 middle school students in one of four classes in an urban district.

1.3 Research Questions

1) How can we characterize interactions in which urban middle-school students attempt to teach or explain something to one another during small group work?
2) How can we characterize interactions in which urban middle-school students attempt to impress another student or the teacher with his or her ideas about the mathematics problems, or with his or her intelligence or ability?
3) What distinctions, if any, can be made observationally between the interactions in which a student attempts to teach others, and the interactions in which a student attempts to impress others, as described above?

The first research question asks about those classroom moments in which one student tries to help another student understand something, in a way that may be similar to peer tutoring. There may be benefits to peer tutoring, as the National Mathematics Advisory Panel (NMAP, 2008) notes. For example, the student who is tutoring or explaining may develop a greater understanding of a concept when successfully explaining it to a classmate. Also, it may be the case that a student can better understand an explanation from a peer rather than a teacher because the student may identify with the classmate' confusion. If we suppose that this is true, at least in some cases, then it behooves us to explore student engagement in more detail during these interactions. To answer this first research question, I looked at and described in detail the behaviors of students who were helping others. I sought to discover whether other activities or interactions may be influencing the behaviors of the tutor and the tutee. I also searched for cues that could indicate how the student determined that a classmate needed assistance.

The second research question asks about those classroom moments in which a student tries to impress his classmates and/or teacher with his mathematical ability, intelligence, or knowledge. For example, one student may be constantly looking for acknowledgement of his ideas from others. Another student who is often reluctant to share ideas may feel confident about her strategy and wants others to recognize her contribution toward a solution. Given that there may be a range of behaviors for a student who wants to impress others or is looking for recognition of his mathematical ability, we ought to learn more about student engagement in these situations. To answer this second research question, I looked at and described the behaviors of students who were trying to impress others. I sought to discover whether other activities or interactions may be influencing the behaviors of the student of interest and his classmates. I also searched for cues that could indicate why a student attempted to impress others or seek recognition from others.

The third research question asks about those instances in which it may not be immediately clear to an observer whether a student is trying to help a classmate or impress his classmates. For example, a student may be explaining an idea or strategy to his classmates, but doing so with the intention of earning recognition for his intelligence or knowledge, rather than genuinely trying to help fellow students. Alternatively, a student who is explaining a concept may want her classmates to understand but may also be thinking to herself, “I want my classmates to think that I am smart so that they listen to my ideas.” If the distinctions between these and many other situations are made explicit, both engagement structures Let Me Teach You (LM TY) and Look How Smart I Am (LHSIA) can be better understood. To answer this research question, I aimed to describe those behaviors which appeared to be similar, and determine if the student’s intentions could be inferred from the behavior. I strove to distinguish those situations in which both engagement structures were active at the same time from those in which only one of the two was active for a student. I consider the context and other interactions which may be influencing the students’ behaviors in that moment.

1.4 Overview of results, limitations, and significance

In addressing the first question, I was able to characterize interactions when students help one another by identifying moments when one student acted on his or her belief that a classmate needed some kind of explanation. Using video and audio recordings, I observed verbal and non-verbal behaviors where students provided explanations about the mathematics or the problem at hand. Some of these behaviors include telling a classmate how to do something directly, offering a specific strategy, and sharing representations that helps to clarify the problem. Across the four classes, 55% of students were observed as willing at some time to help their classmates by explaining something about the mathematics or the problem at hand.

After analyzing the video and audio data, I analyzed students’ responses from a questionnaire administered to all students at the end of the problem-solving sessions. I was able to confirm my interpretations of whether a student had an active LMTY structure by comparing the analyses of the video and the questionnaire responses. For 30 out of the 51 students who indicated they had an active LMTY structure according to their questionnaire, I recorded at least one instance this structure being activated. I did not find behavioral evidence of this activation for the other 21 students. There were 3 students who did not indicate having an active LMTY structure or motivating desire on their questionnaire. More details of the questionnaire and specific items are given in subsequent chapters.

With respect to the second question, the interactions in which a student attempted to impress others with some mathematical knowledge or ability were sometimes more subtle than those in which one student tried to help others. When observing verbal and non-verbal behaviors, a desire to impress others was not always obvious. After reviewing my video data multiple times, I observed 75% of the students in the four classes exhibiting behaviors at some time that indicated they may have desired recognition from others. These behaviors included making statements such as, “This problem is easy,” and looking for confirmation of one’s ideas from a classmate.

I was able to confirm my interpretations of whether a student had an active LHSIA structure by analyzing his or her questionnaire responses. For 37 of the 48 students who indicated they had an active LHSIA structure according to their questionnaire, I recorded at least one instance this structure being activated. For the 11 other students who indicated an active LHSIA structure, I did not find behavioral evidence that this engagement structure was active.

Finally, my findings relating to the third question demonstrate that there are instances for which both the LMTY and LHSIA structure appear to be active for the same student at the same time. Over the four classes, I inferred that a student may have had both an active LMTY and LHSIA structure at the same time on 13 occasions. In addition, I inferred that a student with an active LMTY structure branched into an active LHSIA structure only once, and that a student with an active LHSIA structure branched into an active LMTY structure on three occasions. The questionnaire responses are not able to support or refute this, as these instances of active structures occur dynamically over the problem-solving session whereas the questionnaire is given at the end of the class and does not ask students to pinpoint when they experienced certain thoughts or feelings.

As a result of this study, I was able to construct a set of behavioral characteristics, or features, for these engagement structures based on evidence from middle school mathematics classrooms. I used this evidence to elaborate on the current definitions of these two engagement structures (see Chapters 2 and 5). I also provide examples of classroom episodes to offer a more complete characterization of each engagement structure. In doing so, I provide rich descriptive detail to support the identification of active Let Me Teach You and Look How Smart I Am structures. Additionally, I was able to use these distinct characteristics to help differentiate between classroom episodes in which Let Me Teach You was active and those in which Look How Smart I Am was active for a student. I use these distinctions to identify episodes in which both structures appeared to be simultaneously active for a student.

There are several limitations of this study. One such limitation is that the study is not comprehensive enough for one to generalize these findings broadly. The engagement behaviors of the students in these urban middle school classrooms may or may not be comparable to the behaviors of students in other communities. A second limitation lies in the reliability of my own inferences of student behavior and engagement, which are based on video and audio recordings of the classroom. The students and I may even have different interpretations of the same classroom events. A third potential limitation sits with the students' questionnaire responses. Students may have misunderstood some questions or may have recalled only certain events from the class and not others. Despite these limitations, I believe this study offers a deeper understanding of in-the-moment student engagement and insight into certain observable student behaviors.

Future research can address some of these limitations by investigating the engagement structures of students from other backgrounds (e.g., suburban and rural as well as urban schools) and at different grade levels. Conducting interviews with students may provide researchers with more detail about the students' motivating desires and behaviors during problem-solving sessions, allowing a better insight into the activation of certain engagement structures. In addition, the questionnaire instrument can be refined in a way to concentrate on certain aspects of the students' experiences, behaviors, and motivating desires. Finally, other engagement structures need to be investigated, and I anticipate that the methods and coding schemes used here will provide a basis for this research.

The significance of the findings of this research can have implications both for teaching and on a theoretical level. After additional research has been conducted, I anticipate that teachers can be trained to identify different engagement structures, including LMTY and LHSIA. By learning the operational definitions and viewing classroom examples of activated structures, teachers can identify those students who are likely to either help classmates or students who are likely to seek recognition for their contributions and mathematical ideas. Teachers can then guide students to engage in ways that are productive. For example, a teacher may encourage a student to explain the problem or solution strategy to a struggling classmate.

Teachers can also look for students who appear to be continually talking over others or disagreeing with classmates' ideas or suggestions. The teacher could then take action that discourages negative behavior, such as providing this student with a specific role, or the teacher might acknowledge the student's correct ideas, providing the student with the recognition he may be seeking. By identifying the characteristics of LMTY, LHSIA, and other engagement structures, teachers may have new opportunities to engage their students and improve the learning environment for everyone. Professional development might be offered while researchers continue to study how to best support current teachers and teachers-in-training as they learn and make use of the engagement structures in their classrooms.

My findings include specific features that describe behavioral characteristics of each of these two engagement structures. These features have theoretical implications in that they may complement some the characteristics Goldin and colleagues (2011) have described as belonging the construct of engagement structure. The features of the LMTY structure are: 1) teaching to clarify, 2) teaching procedures, 3) teaching or explaining strategies, and 4) checking for understanding. These behaviors recurred throughout the data and provide the major behavioral distinctions of the students. The following behaviors, or features, were identified for LHSIA: 1) expressing an idea, 2) stating an answer or answers, 3) correcting others, 4) stating, "I'm smart," or "We're smart," 5) keeping up with others in the group.

These findings and discussion of the engagement structures “Let Me Teach You” and “Look How Smart I Am” are intended to contribute to a better understanding of student engagement and motivation of urban middle school students in their mathematics class. The National Research Council (NRC; Kilpatrick & Swafford, 2002) report and National Council of Teachers of Mathematics (NCTM, 1991; 2000) documents suggest increasing students’ engagement may lead to increasing their mathematical success. As mathematics educators learn more about student engagement in the classroom, I anticipate more opportunities will arise to develop student interest in mathematics.

Chapter 2: Literature Review and Theoretical Framework

This study examined the engagement of middle school students working in small groups on a non-routine algebraic problem. In this chapter I review the literature on several topics: engagement, motivation, affect, small group work in mathematics classrooms, and algebraic thinking in middle schools. Engagement, motivation, and affect are each such vast topics that they require a discussion in both the educational psychology field and within mathematics education specifically. The final section of this chapter presents the theoretical framework, describing engagement structures in detail, particularly those which are the focus of my study: “Let Me Teach You” and “Look How Smart I Am.”

Prior to the discussion of the literature I share some definitions, since terms like *engagement* and *motivation*, are used differently within the literature. Further clarification will be given as needed throughout the chapter.

By *engagement*, I refer to the in-the-moment involvement with an academic activity. Other definitions supported by other researchers are described in the context of those specific studies.

Motivation generally refers to the reason(s) a person decides to do something, such as a student who decides to engage in a mathematical task. Brophy (1983) views motivation as both a longer-term trait as well as a situation-specific state. That is, motivation may refer to a stable disposition in which a student values learning. Alternatively, a student may be motivated in the classroom to learn a skill at that moment. Middleton and Jansen (2011) assert that motivation is “in-the-moment,” as a student may be motivated by an interesting task or by an opportunity to learn and be appropriately challenged. In their discussions of theories or studies, many researchers do not clarify whether they are looking at longer-term or in-the-moment motivation.

Affect, which is discussed in much of the work on engagement and motivation, is also associated with a range of definitions in the literature. Many researchers describe affective states as emotions and moods (Linnenbrink & Pintrich, 2002) and affective traits as predispositions or personality-like characteristics. My work follows the definitions of McLeod (1992) and DeBellis and Goldin (2006) who define affect, particularly in mathematics education, as including emotions, attitudes, beliefs, and values in such a way that each component interacts with the other three.

Though each of the following three sections focuses on its primary topic (i.e., engagement, motivation, or affect), there is much overlap as researchers investigate the relationships between and among engagement, motivation, and affect. Many studies reviewed here include constructs from engagement, motivation, and affect, or some combination of these concepts. This suggests that these constructs are all related and may be difficult to separate. Such studies also shed light on the influence that engagement, motivation, and affect each have on student learning. To that end, the studies in this chapter focus primarily on the student, student learning or achievement, and several factors that may impact student engagement, motivation, and affect. Studies which focus on teachers and their behaviors are beyond the scope of this chapter, as the current study does not emphasize the role of the teacher.

2.1 Engagement

Researchers have given a variety of definitions of student engagement, with most including aspects of behavioral, cognitive, and emotional or affective engagement. For example, Skinner and Belmont (1993) include behavior and emotion within their definition, offered in their study on the effect of teacher behavior on student engagement: "behavioral involvement in learning activities accompanied by a positive emotional tone" (p. 572). In their review of research on school engagement, Fredricks, Blumenfeld, and Paris (2004) provide separate definitions of behavioral, cognitive, and emotional components of engagement, reflecting the distinctions made by other researchers (e.g., Jimerson, Campos, & Grief, 2003; Stipek, 2002). *Behaviorally engaged* students participate in academic activities. *Emotional engagement*, on the other hand, "encompasses positive and negative reactions to teachers, classmates, academics, and school and is presumed to create ties to an institution and influence willingness to do the work" (Fredricks, Blumenfled, & Paris, 2004, p. 60). *Cognitive engagement* describes "the level of thinking skills" (Skinner & Belmont, 1993, p. 572) or the "willingness to exert the necessary effort to comprehend complex ideas and master difficult skills" (Fredericks, Blumenfeld, & Paris, 2004, p. 60). These are among several definitions used in the studies reviewed in this chapter.

One view of engagement looks at *school engagement,* which Finn (1993) describes with his participation-identification model. By *participation*, Finn refers to the behavioral aspect of engagement, including actions such as attending school and responding to the teacher's questions in class. *Identification* is defined by Finn as the psychological or affective aspect of engagement, which consists of a sense of belonging in school. Two related studies analyzed the relationship between student engagement and achievement, using the participation-identification model.

Finn's first study draws on data from the National Educational Longitudinal Study in 1988 and uses survey responses from a sample of 15,737 eighth-grade public school students. Both the students and their teachers reported on student behaviors, which measured student participation. The responses on items about class attendance/absenteeism, school activities such as homework and extracurricular activities, and preparation for class were used to categorize students into one of four participation groups: high (n = 6,506 or 41.3%), mid-high (n = 5606 or 35.6%), mid-low (n = 2802 or 17.8%), and low (n = 823 or 5.2%). Students were also administered achievement tests in reading comprehension, mathematics, science, and history/geography/ civics. Finn reported a 0.75 effect size, based on a multivariate analysis of variance using mean differences, for a positive linear association between participation level and average achievement scores, which is evidenced by the mean scores given by subgroup, though it is not clear how these scores were calculated. For example, in mathematics, the high participation group had a mean score of 2.293, the mean score of the mid-high participation group was –0.292, the mid-low participation group's mean score was –2.689, and the low participation group had a mean score of –4.215. The author suggests that a student who appears to be failing and does not receive support from the school will eventually disengage from school.

Finn's second study drew upon 5,945 eighth-grade students included in the first study. Students were deemed at-risk if they: spoke a language other than English at home, were in the lower third distribution on the socioeconomic status (SES) index and whose family had 5 or more members, or indicated they were Asian, Hispanic, or Black and attended school in an urban area.. Finn analyzed these students' responses to the surveys to determine what, if any, behavioral differences could be found between those students who were considered academically successful and the other students who were considered to be just passing or unsuccessful. The reading and mathematics test scores were used to determine the at-risk students to be successful (19.6% scoring above the national mean), passing (14.9% scoring higher than one-half standard deviation below the national mean on both tests but less than the national mean), or unsuccessful (65.4% scoring lower than one-half standard deviation below the mean on one or both tests). The results showed that the race, language, gender, and SES factors were not predictors of success. Rather, the successful students were found to participate more in and out of the classroom, exhibiting behaviors such as watching less television than their unsuccessful peers, reading for pleasure, participating in extracurricular activities and being prepared for class. For example, 86.6% of successful eighth-grade students in this sample report reading outside of school for at least one hour per week, whereas only 72.7% of the successful students report the same behavior. In addition, the means of the "behavior" variable are –0.093 for the successful students, –0.078 for the passing students and 0.051 for the unsuccessful students (lower scores correlate to "preferred" behavior). Identifying that classroom engagement behaviors, rather than unchangeable variables such as race or gender, affect a student's academic achievement is important, as Finn says, "Engagement behaviors are more amenable to influence than traditional status indicators and should be the focus of educators and researchers" (p. vi).

Steinberg, Brown, and Dornbusch (1996) also looked at school engagement via a large-scale, longitudinal study in which they collected data for over 20,000[1] students over 3 years. The students were enrolled in one of 9 high schools in 2 states – California and Wisconsin – that were selected "to capture the diversity of communities that American students come from" (p. 20). These students came from middle-class suburban schools, suburban schools with primarily Black and White students, and city schools with large minority students (e.g., African-American, Latino, Asian), as well as rural schools. The 20,000–plus students were surveyed, and 600 of these students participated in focus groups or personal interviews to learn why many students exhibit a lack of engagement in the classroom and with school in general. The study found that about 40%[2] of the surveyed students reported that when they attend class, they are "neither trying very hard nor paying attention" (p. 67). Approximately one-third of students appeared to be disengaged, as they exhibited wandering minds, a lack of interest; half the students claimed their classes were "boring." Still, the most engaged students claimed that they enjoy going to school, and felt proud and accomplished when they were successful in their classes.

Steinberg et al. (1996) were particularly critical of how students spent their time, particularly since not much time was spent in school during the year. The authors state that up to 60% of the time that students are in school is spent in study hall and break times as well as ancillary classes, though it was not clear which classes were considered "ancillary". They found that many students had time to do homework while at school,

[1] An exact *n* is not given by the authors, who repeat "more than 20,000 different students participated" (p.21) throughout the report.
[2] The authors provide percentages and fractional values to represent the number of students, rather than the exact number of students who reported such statements.

which meant less time was spent on schoolwork outside the standard school hours. Overall, Steinberg and colleagues expressed concern for students' behavior and engagement, or lack thereof, in school.

Furrer and Skinner (2003) examined relationships between student engagement and a sense of relatedness or belonging, which is similar to Finn's idea of identification. The participants were 641 students (95% Caucasian) in grades 3 through 6 who attended elementary school in a suburban-rural district. Students completed self-report questionnaires which asked about a sense of relatedness to their parents, teacher, classmates, and friends and about the level of control they perceived to have over their academic success or failure. Both teachers and students responded to items regarding the level of engagement (e.g., active, constructive, persistent) and disaffection (e.g., alienated, apathetic, rebellious). Among the results, the authors found significant correlations ($p < 0.01$) between a reported sense of belonging and emotional engagement (teacher report $r^2 = 0.29$; student self-report $r^2 = 0.61$) as well as between a sense of belonging and behavioral engagement (teacher report $r^2 = 0.29$; student self-report $r^2 = 0.57$) in school. Furrer and Skinner state that students who reported they felt unimportant or rejected were more likely to feel frustrated, bored, and alienated from learning activities, which in turn hindered their academic progress. The authors found that relatedness to peers (e.g., classmates and friends), as determined by student reports, was a predictor of students' behavioral ($\beta = 0.16$[3]) and emotional engagement ($\beta = 0.26$), whereas earlier research (e.g., Goodnow, 1993; Ryan, Stiller, & Lynch, 1994) did not find significant effects for peer support. Relatedness to peers did not predict students'

[3] Betas (β) are standardized regression coefficients.

emotional engagement as reported by the teachers. Overall, this report suggests that students are more likely to be engaged in academic activity if they feel a sense of belonging to and in school and relate well to their parents, teacher, and peers.

Newmann (1992) has another take on student engagement, stating, "Engagement involves psychological investment in learning, comprehending, or mastering knowledge, skills, and crafts, not simply a commitment to complete assigned tasks or to acquire symbols of high performance such as grades or social approval" (p. 12). Several researchers (e.g., Klem & Connell, 2004; Marks, 2000; Shernoff et al., 2003) refer to Newmann's theoretical perspective which identifies 3 primary factors influencing student engagement in academic work: (1) a central need to develop and express competence, which does not just include academic competence, but can also refer to interpersonal skills, or excellence in arts, crafts, hobbies, etc.; (2) school membership, which is shown through fair treatment, personal support, experience of success; (3) authentic work, in which tasks are "meaningful, valuable, significant, and worthy of one's effort" (Newmann, Wehlage, & Lamborn, 1992, p. 23). Authentic work has also been highly suggested in mathematics education, for example, when Davis (1992) critiqued school mathematics curricula, stating that it does not always foster interest when it contains, "meaningless bits and pieces" (p. 730), rather than provides meaningful problems that relate to the real world or students' lives.

Using Newmann's theory of student engagement, Marks (2000) focused on classroom factors to investigate student engagement across different grade levels. She obtained survey data on students' attitudes, behaviors, and experiences for 3,696 students in mathematics and social studies classrooms in the fifth-grade (*n* = 1,348), eighth-grade

(n = 1,151), and tenth-grade (n = 1,170). She looked for the effect of various factors such as subject matter and authentic tasks as described by Newmann (1992), on student engagement across these grade levels. Though specific examples of what might be considered authentic work were not given, Marks stated, “Authentic academic work involves students intellectually in a process of disciplined inquiry to solve meaningful problems, problems with relevance in the world beyond the classroom and of interest to them personally” (p. 158). To Marks, student engagement specifically refers to the amount of attention, interest, investment, and effort applied by students to their academic work.

The participating students attended purposefully-selected schools so as to include 8 elementary, 8 middle, and 8 high schools which were involved in restructuring to enhance student engagement. The author found that students in the higher grades reported lower levels of engagement, based on four items which students rated on a scale of 1 to 5 (e.g., “In social studies/mathematics class, how often do you try as hard as you can?” [Marks, 2000, p. 176]). The mean of the fifth-grade students’ responses was 3.95, the eighth-grade students reported a mean of 3.80, and the mean of the tenth-grade students’ responses was 3.69. The reported levels of engagement also differed depending on the academic subject. The students in the mathematics classes (n = 1,765) reported a mean of 3.92 on these items whereas those in the social studies classes (n = 1,904) reported a mean of 3.72. Marks notes that a significant difference occurred only for the elementary and high school students, but not for the eighth-grade students. The author suggests that students in mathematics classes may have been given more authentic and challenging tasks than those in the social studies class. Marks states, “Authentic instructional work contributes strongly to the engagement of all students" (p. 173), asserting that higher order thinking, relevance to the world outside the classroom, and substantive discussions about the content contribute to greater engagement with academic work.

Shernoff, Csikszentmihalyi, Schneider & Shernoff (2003) draw on Csikszentmihalyi's theory of flow in their study of student engagement. Csikszentmihalyi (1990) describes "flow" as a state of deep concentration, enjoyment, and interest in an activity that is worth doing for its own sake. For example, athletes and artists report the experience of flow when working on their performance or piece of art. Students may also experience flow, particularly when the level of challenge is appropriately matched to the level of skills needed for the activity or task.

Shernoff et al. sampled 526 tenth- and twelfth-grade students from across the U.S. who participated in a longitudinal study conducted from 1992 to 1997. The researchers investigated how high school students spent their time in school, and which factors (e.g., challenge, skill, instructional method) were associated with student engagement. They collected data using the Experience Sampling Method (ESM). Participating students were given pre-programmed watches which signaled participants at random intervals throughout the day. When signaled, students were expected to complete a form asking for their location (e.g., which class), thoughts, and primary and secondary activities in which they were engaged. This form also asked students to report their feelings and moods at the time they were signaled. Students completed these forms over the course of one week, up to eight times in one day.

Shernoff et al. found that students spent approximately one-third of their in-class time in a passive, non-interactive activity such as listening to a lecture or watching a video. Approximately 21% of class time was reported to be spent on individual work, and only 14% of class time was spent on interactive activities such as class discussion or group work. The students reported that they felt most engaged during individual or group work in class[4]. Student engagement was also increased when an appropriately perceived challenge was matched with an appropriate skill level, and when the student felt that the instruction was relevant (e.g., important to student, important to student's future goals). Shernoff et al. note that at least some high school students in this study did not have many opportunities to be interactive or to participate in their classrooms. Therefore, they recommend the use of classroom activities which are academically intense and foster positive emotions such as enjoyment in order to encourage greater student motivation and engagement.

For the remainder of this section, the discussion is limited to student engagement in mathematics classrooms exclusively. Several studies (e.g., Connell, Spencer, Aber, 1994; Finn, 1993; Wentzel, 1997) have demonstrated a relationship between student engagement and achievement. Since engagement and achievement in mathematics is a concern to teachers, mathematics educators, and many others, it is worthwhile to consider the behavior of students during mathematics class as well as some of the reasons why students choose to engage with the mathematics. Studies investigating engagement and motivation of mathematics students are not necessarily conducted by mathematics educators. Researchers in the fields of psychology and educational psychology may have a primary interest in student engagement and/or motivation. Their rationale for conducting studies in mathematics classrooms or with mathematics students is

[4] These findings are reported to be significant, $p < .001$, using various statistical tests, namely, ANOVAs, *T*-tests, and chi-square analyses.

summarized by Pajaras and Graham (1999), "Mathematics holds a valued place in the academic curriculum; it is prominent on high-stakes measures of achievement generally used for level placement, for entrance into special programs, and for college admission" (p.124).

Studying engagement in mathematics classes gained popularity in the 1980s, as this study by McIntyre, Copenhaver, Byrd, Norris (1983) illustrates. The authors note that previous research has demonstrated that time-on-task, or engaged time, led to higher achievement which was typically measured by test scores. McIntyre and colleagues, looking for more details about the engagement behaviors, investigated 173 third- (*n* = 60), fifth- (*n* = 72), and seventh-grade (*n* = 41) students in mathematics classes within one school district. Data was collected via classroom observations over six weeks in the early spring. Observers looked for behavioral evidence that students were engaged by recording students' activities such as paying attention or listening to instruction, writing, reading, answering or asking questions and talking to classmates about the subject matter. Behaviors which indicated that a student was not engaged were also recorded and included talking to a teacher or classmate about something other than the subject matter, playing, and not paying attention or listening to the teacher. The authors found that when the students were engaged, their actions mostly consisted of listening to or paying attention to the teacher, writing, or reading. For students at all grade levels, there were few or no observations of students asking or answering questions or talking to classmates about mathematics. This may correspond to the finding that most in-class activity involved either teacher-led instruction or seatwork. For students in grades 3 and 5, approximately twice as much time was spent on seatwork than on teacher instruction.

However, students in grade 7 spent an average of 10 more minutes on direct instruction than on seatwork.

McIntyre et al. (1983) observed a slight decrease in the amount of time students appeared to be engaged during class as students moved from third grade (77% of time) to seventh grade (73% of time). This decrease correlates with the reported shift from seatwork to teacher-led activities during class, which the authors suggest may partly account for the decrease in the engagement rate. However, there is no evidence of this, and numerous factors – both related and unrelated – may contribute to amount of time students appear to be engaged. Though the engagement rate is over 70%, it is important to note that 'attending' (i.e., paying attention and listening) was the primary behavior during teacher–led activities for all three grade levels. As this is based on observations, these determinations may be subjective, and do not necessarily indicate the level of affective or cognitive engagement of the students. McIntyre and colleagues suggest that future work should look at "relationships between time-on-task, instructional activities and/or achievement," (p. 59) particularly for secondary students.

In an investigation of the relationship between achievement in mathematics and student engagement for first grade students, Park (2005) used national survey data collected from 1991 to 1994. This study used the data collected from the teachers (*n* = 446) and parents of first-grade students (*n* = 6,208). The teachers rated items intended to determine student engagement; the items ranged from the number of days the student did not attend or was late to school, to asking questions and participating during class discussions. Additional classroom variables were considered as well, including the teacher's highest degree, content covered, and the student's gender and race/ethnicity.

Using statistical hierarchical data analysis, the author found that student engagement ($\beta = 1.4638$, $p > 0.01$) was the most significant factor in achievement gains ($\beta = 12.0668$, $p > 0.01$), which in this case measured student performance month to month. Park found that the average growth rate per month was 2.7839 standard deviations ($p < 0.001$). The author notes the importance of the change or growth, as it emphasizes the importance of the learning process and environment, supporting the important role that engagement plays in student learning. Park observes that minority students made greater improvement than the non-minority students, noting that these findings are of interest since the students are in their first year of elementary school. Based on these findings, the author suggests, "Student engagement should be emphasized in a school and in educational policy" (p. 94).

Helme and Clarke (2001) investigated the cognitive engagement of Year 8 Australian mathematics students using qualitative data from classroom observations and follow-up interviews with students. This study sought to identify several behavioral and verbal cues to determine student engagement. Based on a larger project, this study focuses on four Year 8 lessons taught by the same teacher and the associated student and teacher interviews. The authors provide evidence of cognitive behaviors by including classroom excerpts and interview samples from four classroom situations: 1) individuals working in parallel; 2) collaborative small group activity; 3) small group interactions with the teacher; and 4) whole class interactions with the teacher. Within these excerpts, several behaviors indicating cognitive engagement were identified, including: verbalizing thinking, seeking feedback, exchanging ideas, gesturing, asking and answering questions, and contributing ideas. The authors suggest that the behaviors vary depending on the classroom situation, the characteristics of the task, and the personal experiences of the individual students. By providing evidence for these specific behaviors, the authors intended to demonstrate that cognitive engagement can be inferred and observed in classroom settings. They state that the next step is to empirically demonstrate a relationship between cognitive engagement and learning.

Brown (2009) also conducted a qualitative study to identify and detail several factors affecting student engagement. Employing multiple case studies, she drew on attribution theory and the construct of self-efficacy[5] to follow four high school mathematics students over the course of one school year. Brown uses attribution theory to investigate and understand "the behavioral and environmental factors that students attribute [to] their engagement" (p. 11). Self-efficacy is described by Brown as, "an individual's belief that he or she can perform a particular task or behavior" (p. 15). Data was collected via classroom observations, students' journals, and several interviews with participants. Brown found that for all four case study participants, the factors which led to increased student engagement were an increased level of competence and perceived ability, as well as tasks that were meaningful, authentic, and/or non-routine. On several occasions, these students would collaborate with other mathematically engaged students. However, there were other times when working with peers led to off-task conversations rather than increased student engagement. Of the three dimensions of engagement – behavioral, cognitive, and affective – Brown found that behavioral engagement alone was not sufficient for learning and retaining the material. She identified four major themes which contributed to student engagement for her four participants: a) moods, feelings,

[5] Attribution theory and self-efficacy are discussed in more detail in section 2.2.

and/or physical conditions; b) effort; c) behavioral engagement; d) teacher's approach to instruction. Using students' own voices as evidence, Brown's investigation into these factors demonstrates that student engagement is multifaceted and complex.

The research cited here, as well as other studies, indicate that there are many ways of thinking about student engagement as well as many factors influencing student engagement. Student engagement, to some (e.g., Finn, 1993; Steinberg et al. 1996), means the level of participation and the sense of belonging in school. It is disconcerting to realize that many students are engaged in non-academic activities or are engaged in passive activities throughout much of the school day (e.g., Shernoff et al.; Steinberg et al., 1996). Others (e.g., Furrer & Skinner, 2003) consider engagement encompassing three dimensions: behavioral, affective, and cognitive. Newmann (1992) and Marks (2000) describe three basic factors of student engagement: competence, school membership, and authentic work. Brown (2009), who also found these factors to influence student engagement with four high school mathematics students, determined more specific factors (i.e., affect, effort, behavioral engagement, and teacher approach to instruction) which may influence "in-the-moment" engagement, perhaps more so than Newmann's three factors. These researchers and others agree that increased engagement leads to increased achievement (defined in various ways), particularly in mathematics. Several investigations indicate that student engagement appears to decrease as students move into the upper grades.

2.2 Motivation

The research on motivation has covered a range of concepts and theories (Wentzel & Wigfield, 2009). Here, some of these concepts and theories are briefly introduced, followed by a discussion of several studies which have used one or more of these constructs and theories. In particular, the constructs of intrinsic and extrinsic motivation and self-efficacy are presented prior to a discussion on three major theories of motivation: 1) Achievement Goal theory, 2) Expectancy-value theory, and 3) Attribution theory.

Two of the more basic distinctions of motivation are *intrinsic motivation* and *extrinsic motivation* (Ryan & Deci, 2000). Students who are intrinsically motivated do something because it is interesting or enjoyable. Students participate in an activity for its own sake and may get a sense of satisfaction from the experience (Deci, Vallerand, Pelletier, & Ryan, 1991; Turner et al., 1998). Extrinsically motivated students do something for an external reason, such as a reward or praise. The activity or task is done because it will lead to a separable outcome. These outcomes, rewards, or praises can take a variety of forms, such as avoiding punishment, receiving approval from others, or leading to other goals (e.g., preparing for a test leads to earning a good grade in class, which in turn contributes to being accepted into the college of one's choice).

While educators may wish for all their students to be intrinsically motivated, this is not a realistic expectation. Not all students are inherently interested in or enjoy the same activities. Many students who participate in an assigned academic activity will be extrinsically motivated to do so, but may have different reasons for getting involved. Ryan and Deci (2000) note that students may be looking for a reward or are looking to avoid punishment; they may be seeking praise or may act to maintain a sense of self-esteem. A student who recognizes the value of an activity is more likely to internalize the activity than the student looking for some reward. The first student may engage more with the activity and may feel more confident in his ability to complete the task successfully. This student is more autonomous in his behavior, similar to the student who is intrinsically motivated to participate in the activity.

Self-efficacy is a motivation construct which has been investigated alongside other constructs and with respect to various theories and perspectives. Bandura (1994) defines self-efficacy as, "people's beliefs about their capabilities to produce designated levels of performance" (p. 71). Self-efficacy has been demonstrated to influence how people think and what behaviors they undertake with regards to certain tasks. Schunk (1991) points to research demonstrating that self-efficacy predicts motivation and perseverance for students of varying levels of ability, particularly when faced with challenges and potential setbacks. Individuals with high self-efficacy tend to work harder and persist longer, whereas those with a lower sense of self-efficacy may attempt to avoid the work and have low aspirations of their performance. Researchers investigating various motivational constructs tend to include a measure of self-efficacy, as it is thought to be predictive of levels of motivation as well as achievement (Pajares, 1996; Schunk 1991).

Middleton and Spanias (1999), in their review of literature on motivation in mathematics education, remind the reader that early theories of motivation were centered around behavior. For example, researchers have learned that students appear to be motivated by success and incentives (e.g., reward for individuals or groups), but more recent research suggests that intrinsic motivation may be reduced when students work for a reward or praise, or to avoid punishment or guilt (Deci, Vallerand, Pelletier, & Ryan, 1991). Another limitation of behaviorist theories is the reliance of time-on-task as an observable measure, particularly when a difficult or realistic problem (that is, items that are not multiple-choice or well-defined problems) may not lend itself to an immediate solution or may require additional time. More current research on academic motivation considers beliefs, affect, values, and perceptions of self as well as individual's behavior (Eccles & Wigfield, 2002; Middleton & Spanias, 1999).

Achievement goal theory, or goal theory, investigates the primary reasons why a student participates and engages in an academic task. Ames (1992) states that an *achievement goal* "defines an integrated pattern of beliefs, attributions, and affect that produces the intentions of behavior (Weiner, 1986) and that is represented by different ways of approaching, engaging in, and responding to achievement-type activities (Ames, 1992b; Dweck & Leggett, 1988)" (p. 261). Two major goal orientations are called *mastery*, *or learning, goal orientation* and *performance goal orientation* (Ames, 1992; Anderman & Wolters, 2006; Dweck & Leggett, 1988; Schunk, Pintrich & Meece, 2008). Mastery goal orientation refers to a "focus on learning, mastering the task according to self-set standards or self-improvement" (Schunk, Pintrich, & Meece, 2008, p. 184). Pintrich (2000) mentions that mastery goals "have been related to a number of adaptive outcomes, including higher levels of efficacy, task value, interest, positive affect, effort and persistence, the use of more cognitive and metacognitive strategies, as well as better performance" (p. 544).

On the other hand, performance goals place an emphasis on recognition and praise. Success is defined by grades and out-performing others (Ames, 1992). An individual with a performance goal may engage in superficial learning strategies, such as memorizing without understanding or processing the information (Ames, 1992).

Additional distinctions can be made within each goal orientation, particularly *performance-approach goals* and *performance-avoid goals* (Anderman, Austin, & Johnson, 2002; Middleton & Midgley, 1997). A performance-approach goal orientation suggests that the student desires to demonstrate his or her competence, whereas a student who has a performance-avoid goal orientation prefers to avoid appearing incompetent to others (Anderman & Wolters, 2006). A student with a performance goal orientation, particularly performance-avoid orientation, may exhibit maladaptive patterns, such as attributing failure to a lack of ability or intelligence, exhibiting negative affect, engaging in discussion or activity unrelated to a task, and demonstrating a decrease in performance over time (Dweck & Leggett, 1988; Middleton & Midgley, 1997).

Expectancy-value theory as described by Eccles, Wigfield, and colleagues (Anderman & Wolters, 2006; Wigfield & Eccles, 2002) attempts to explain how a student's expectancies and values influence one's engagement with an academic task. Wigfield and Eccles (2000) describe the construct *expectancy* as belief of how one will do on an upcoming task. They contrast this with *ability beliefs* which emphasize present ability, rather than one's ability to perform in the future. To frame this difference, Wigfield and Eccles (2000) share sample questionnaire items. A statement intended to ascertain a student's expectancies asks, "How well do you expect to do in math this year?" (p. 70), whereas an item that measures a student's ability beliefs is phrased, "How good in math are you?" (p. 70). The term *value* refers to a set of four subjective values: attainment value (the importance of doing well), intrinsic value (enjoyment in the task), utility value (usefulness for the future), and cost (emotional cost, and the effort needed to engage in the task; Wigfield & Eccles, 2002).

Attribution theory of motivation and emotion, commonly used in education research, has been formulated and advanced by Weiner (1985). Within this perspective, students attribute their success or failure on some achievement outcome to perceived causes or reasons. Individuals retrospectively ask, "Why did I succeed or fail?" (Pekrun, 2009). Three causal dimensions that have been identified are: (a) locus of causality (cause is internal or external), (b) stability (cause can vary or is constant), and (c) controllability (amount of control individual has). For example, a student who has succeeded on a quiz may claim she was "lucky," attributing her success to external, unstable, and uncontrollable forces. On the other hand, another student may claim that the quiz was too difficult, attributing failure to the task, which is external, stable, and uncontrollable. Many students may attribute their success or failure to ability or competence, which they may perceive as internal, stable, and uncontrollable (Graham & Williams, 2009). The second and third dimensions, stability and controllability, both depend on one's perspective of whether one believes one can learn to improve one's competence (Dweck & Leggett, 1988; Graham & Williams, 2009). A fourth common attribution, effort, is typically viewed as internal, unstable, and controllable, suggesting that one can improve one's performance when additional effort is exerted.

Attribution theory is distinct from the previously discussed theories in that it relies on an affective component as well. According to Weiner (1985), specific emotions or feelings may be linked to or even dependent on particular attributions. For example, a person's pride or self-esteem is related to the locus of causality; a student who attributes his success to effort (an internal cause) may take pride in his work (Graham & Williams, 2009; Pekrun, 2009). Other feelings, such as shame, anger, or guilt are related to the stability and controllability dimensions. Shame may be felt by a student who feels responsible for the failure and attributes the outcome to an internal, controllable cause such as effort. On the other hand, a student who believes the failure is due to an external and uncontrollable cause such as task difficulty may become angry at the teacher who assigned the task. Future motivation and behavior are believed to be guided by the related expectancy of future success and the associated emotions (Weiner, 1985).

Each of the studies discussed in detail below use the mathematics classroom as its setting or approach motivation from a mathematics education perspective. As Turner and Meyer (2009) point out, studying motivation in mathematics classrooms is important because students may be uniquely motivated to participate in or not engage in mathematical activities. Their motivation to engage with other subject areas may be different and may be related to other factors as already described in this section.

Pintrich (2000) studied the adaptive outcomes of 150 mathematics students in the eighth and ninth grades, with the point of view that a student may have *both* mastery and performance goal orientations. This differs from several researchers who previously suggested that the mastery and performance goal orientations were in contrast to one another (e.g. Ames, 1992; Elliot & Dweck, 1988). Students completed a questionnaire, which was an adapted version of the Motivated Strategies for Learning Questionnaire (MSLQ; Pintrich & DeGroot, 1990). This version, administered three times over two academic years, measured eleven different scales, including mastery goals, approach performance goals, self-efficacy, negative and positive affect. Based on students' responses, their goal orientations were categorized as: high mastery/high performance (n = 45), high mastery/low performance (n = 30), low mastery/high performance (n = 35), or low mastery/low performance (*n* = 40). Previous researchers (Dweck & Legget, 1988; Harackiewicz, Barron, & Elliot, 1998) suggested that students who had a mastery goal orientation were more likely to succeed and adapt (e.g., put forth more effort) when faced with possible failure. At the same time, students who were oriented with performance goals were likely to report lower levels of motivation, affect, and strategy use (Ames, 1992). Pintrich found that students with high performance goals coupled with high mastery goals had increased levels of self-efficacy (means 5.90, 5.63, 5.25 out of 7 at times 1, 2 and 3, respectively), more positive affect (means 5.09, 4.22, 4.79) and less negative affect, and higher levels of task value than those in the low mastery groups. The low mastery/high performance group had self-efficacy means of 5.47, 5.02, and 4.59 at times 1, 2, and 3 respectively; the low mastery/low performance ratings of self-efficacy were 4.88, 5.15, and 4.78. This demonstrated that a high performance goal can have positive effects when linked with a high mastery goal. Unfortunately, students with low mastery/high performance and those with low mastery/low performance orientations demonstrated less confidence, less positive affect ($p < 0.05$), and reported waning effort and engagement as tasks became more difficult.

Miller et al. (1996) conducted two studies to explore the motivation students might have for engaging in academic work. The five goals measured were mastery (or learning) goals, performance goals, obtaining future consequences, pleasing the teaching, and pleasing the family. The authors sought to investigate the relationships among these goals and perceived ability. Both studies were conducted with high school mathematics students from a large middle-class suburban school in the mid-South. The authors state that they selected mathematics students because they “anticipated great variation in future consequences and perceptions of ability” (p. 391).

The first study sampled 297 tenth through twelfth grade students enrolled in Geometry, Algebra II, Trigonometry, Pre-Calculus or Advanced Placement Calculus. During the last week of school, students completed an 83-item instrument, the Attitude Toward Mathematics Survey, which was developed by Miller et al. (1996) and colleagues. Items which had been used in previous work as well as those developed for these studies measured goals for doing the academic work assigned in class, self-perceptions of ability, self-regulation and cognitive strategy use in studying, and effort and persistence. During the following school year, data for the second study was collected from 296 high school students enrolled at the same school and in the same classes. (Some students may have participated in both studies.) The Attitude Toward Mathematics Survey was revised and split into two instruments. A portion of the items were administered in early March and the remaining items were given in May prior to final exams.

From both studies, Miller et al. (1996) found that goals beyond mastery and performance goals also contribute to a student’s engagement in mathematical academic tasks, most significantly, pleasing the teacher (Study 1: $\beta = 0.15$, $p < 0.005$, Study 2: $\beta = 0.18$, $p < 0.01$) and future consequences (Study 1: $\beta = 0.15$, $p < 0.005$, Study 2: $\beta = 0.21$, $p < 0.01$). Those who reported pleasing the teacher as a goal were likely to use self-regulatory processes (e.g., setting goals, monitoring progress; Study 1: Pearson correlation coefficient $r = 0.35$, $p < 0.001$, Study 2: $r = 0.26$, $p < 0.001$). The apparent relationship between pleasing the teacher and other goals such as wanting to outperform classmates and avoiding the appearance of being incompetent suggests that pleasing the teacher may have been a means to service those other goals. An additional finding suggests that students who reported higher levels of perceived ability and high mastery goals also reported higher levels of persistence (mean of 3.66 on 5-point Likert scale) than those who did not convey high levels of perceived ability (mean of 3.41 for average perceived ability, mean of 3.17 for low perceived ability). The authors note that each of these findings might not be generalizable to other populations, such as middle school students, or to other subject areas.

Eccles et al. (1993) shared results from two-year study in which they suggest that several factors of the traditional middle grade school (or junior high school) contributes to the decline in students' motivation during these years. Recognizing that there are a number of motivational constructs and theories, the authors focused on the following constructs: self-concept of ability, expectancies for success, efficacy beliefs, and achievement values. The participants were students (n = 2,500) from 12 middle/lower-middle-income school districts in southeastern Michigan who transitioned from elementary school (grade six) to the junior high school for grade seven. The student questionnaire, asked about students' self-concepts of ability and how they valued academic activities and sports, among other constructs. One portion of this questionnaire was the Student Classroom Environment Measure (SCEM; developed by the authors), which measured certain dimensions such as teacher control, methods of evaluation, and opportunities for student-to-student interaction. Similar questionnaires were completed by the teacher and classroom observers. The questionnaires were administered to students in their mathematics class during the fall and spring of both the sixth and seventh grades.

Eccles et al. (1993) reported generalized results based on ANOVAs and *t*-tests, noting that the findings they presented were significant at the 0.01 level; detailed statistical information was not included. From the combination of teacher, student, and observer questionnaires, the authors found that there was an increase in teacher control in the classroom as well as an increase in the number of classes which were grouped by ability, suggesting that more seventh-grade classes were homogenous. Those students who felt they were able to make fewer class-related decisions in the seventh grade than in the sixth grade demonstrated a decline in their interest in mathematics as they progressed to the seventh grade. The authors concluded that overall student motivation declined as they moved from elementary school to the middle grades. Schools which include any of the middle grades may be changing, such as having grades K-8, or other combinations (Combs et al., 2011).

Mathematics anxiety has often been studied and has led way to more investigations of affect in mathematics education (see next section). Meece, Wigfield, and Eccles (1990) investigated math anxiety of students in grades 7 through 9 ($n = 250$) from the perspectives of self-efficacy and expectancy-value theory. The goals of this investigation were to identify specific predictors of math anxiety for students in these grade levels, and to determine if there was a predictive influence of math anxiety on students' plans to enroll in future mathematics courses. The students participated in the study over two academic years, during which they completed a questionnaire during each spring. Eccles's Student Attitude Questionnaire asks students to evaluate their expectancies for success, perceived values, perceived ability, as well as report on items relating to the affective and cognitive dimensions of mathematics anxiety.

Students' achievement was measured by the students' end-of-year grades for each year (e.g., letter grades with A+ as the highest grade and F as the lowest). The authors found performance expectancies (e.g, how well one expects to do in math that year) to directly affect students' math anxiety (e.g., nervous about math; correlation –0. 32, $p < 0.01$), as well as predict their end-of-year grades (correlation 0.25 for Year 1, 0.47 for Year 2, $p < 0.01$). Math anxiety was also directly related to students' mathematics ability perceptions (correlation 0.20, $p < 0.01$) and their value perceptions (correlation 0.20, $p < 0.01$). Finally the relationship between expectancy of success and value judgments was determined to be positive (correlation 0.46, $p < 0.01$), which the authors note is important as it had been previously been thought to be a negative relationship. The positive relationship means that a student would expect to perform well on a highly valued task.

Pajares and Graham (1999) measured several motivational constructs, including self-efficacy, mathematics self-concept, value of mathematics, and engagement of sixth-grade mathematics students. The participants were 273 students attending their first year of a Southern suburban public middle school. Pajares and Graham measured these constructs using and adapting items from several existing questionnaires: Betz's (1978) Mathematics Anxiety Scale, the Academic Self-Description Questionnaire II (Marsh 1990), Bandura's Children's Multidimensional Self-Efficacy Scales (Zimmerman, et al., 1992), and the Student Attitude Questionnaire (Eccles, 1983). In addition, students' previous academic achievement was determined by two measures which were provided by the school: the mathematics portion of the Iowa Test of Basic Skills, a national standardized assessment, and the students' class grades from the fifth and sixth grades.

The mathematics self-efficacy instrument not only asked students to rate their general confidence in mathematics, but also specifically to rate their confidence to solve 20 given mathematics problems. These problems were prepared by the students' mathematics teachers and were similar to those given on an end-of- unit test. Thus, students were guided to think about certain kinds of problems as they rated their confidence levels. This seems to differ from other studies measuring self-efficacy or self-concept in that the students were focused on the same mathematical problems, rather than thinking about math in general. Participants in other studies may have a range of mathematical tasks they call to mind when responding to such questionnaires; some may think of the most difficult problems, others may recall problems they enjoyed working on, or students may consider the most recent mathematics problem they were solving.

Because these surveys were administered to students in October and the following April, the authors were able to compare the results over the course of a single school year. The statistical analyses revealed that mathematics self-efficacy independently predicted student performance at both the beginning and end of the school year ($\beta = 0.267$ fall; $\beta = 0.272$ spring, $p < 0.001$). To respond to the authors' objective, "to discover the extent to which mathematics self-beliefs begin to change during the first year of middle school," (Pajares & Graham, 1999, p. 126) students reported a decrease in engagement (mean difference –0.37, $p < 0.002$), as determined by effort and persistence, as well as a decrease in the value of mathematics (mean difference –0.25, $p < 0.002$). Still, over the course of the year, students' self-concept (mean difference –0.12, not significant) in mathematics did not decrease, "suggesting that students' *domain-specific* mathematics beliefs had not been altered" (p. 135).

Schiefele and Csikszentmihalyi (1995) set out to investigate the relationships among mathematics achievement, mathematics ability, quality of students' experience when doing mathematics, interest, and achievement motivation. Their participants, 108 first- and second-year students from two suburban high schools, completed the Experience Sampling Form when contacted via an electronic pager over one week. These students were identified by their teachers as talented in one or more of the five areas: mathematics, science, music, art, and athletics. Quality of experience, described as a “multidimensional construct that consists of emotional, motivational, and cognitive aspects of experience (Csikszentmihalyi, 1988a)” (p. 165), was analyzed using the following dimensions: potency (e.g., active-passive, alert-drowsy), affect (e.g., happy-sad, cheerful-irritable), concentration, intrinsic motivation, self-esteem, importance of the activity to individual, and skill as related to the activity. Mathematics achievement was measured by students’ grades over 5 academic years, and mathematical ability was determined by the students’ scores on the mathematics subtest of the Preliminary Scholastic Aptitude Test (PSAT). Students completed questionnaires to provide data about their interest in mathematics (e.g., “Mathematics is my favorite subject”) and their achievement motivation, which measured the willingness to invest effort as well as the level of persistence a student put forth on a task.

Schiefele and Csikszentmihalyi (1995) found that student interest and the quality of experience were significantly related (potency $r = 0.33$, $p < 0.01$; motivation $r = 0.39$, $p < 0.01$; skill $r = 0.32$, $p < 0.01$). However, ability did not predict experience (potency $r = –0.07$; motivation $r = 0.24$; skill $r = 0.16$), which “underlines the independent and significant role affective variables possibly play for learning mathematics in school”

(p. 177). The authors did not find that achievement motivation significantly predicted the measures of achievement (grades from 1985/86 $r = 0.23$; 1986/87 $r = 0.15$; 1987/88 $r = 0.09$) or ability. Because these students were specifically selected due to an identified talent in one or more areas, including mathematics, these findings may not be generalizable to a larger population of ninth- and tenth-grade students. As stated by the authors, this study did not measure perceived competence or perceived ability, but students who are identified by their teachers may be more likely to perceive themselves as high-ability than other students who may not be singled out by teachers.

Stipek, Salmon, Givven, Kazemi, Saxe, and MacGyvers (1998) looked at motivation from a mathematics education point of view. They observed 24 teachers and their 624 students from fourth-, fifth-, and sixth-grade during mathematics lessons within a single school year. In addition to classroom observations the students were given a questionnaire, presumably created by the authors, designed to measure students' beliefs, values, and goals surrounding mathematics, by including items about perceived ability, mastery and performance orientations, positive and negative emotions, and enjoyment of mathematics. The questionnaire was given shortly after the school year began and again after a fractions unit was taught. The authors found that student motivation was consistent from the first to second administration, suggesting that motivation may be not easily be changed over a short period of time ($p < 0.001$). Additionally, students whose perceived competence was high were more likely to report a mastery orientation (correlation 0.52, $p < 0.001$). These students also were more likely to report positive emotions (e.g., pride; correlation 0.53, $p < 0.001$) and more enjoyment (correlation 0.43, $p < 0.001$) than those students who reported low perceived competencies. It appears, then, that students' motivation may be correlated with how students perceive their mathematical ability and competence.

Certainly, other theories of motivation exist and can explain causes for student motivation in addition to the three discussed in more detail here: achievement goal theory, expectancy-value theory, and attribution theory. The small sampling of studies here indicate that motivation is a complex construct, with many variables influencing students. Motivational constructs are often considered long-term traits, particularly the goal orientations. Though not discussed in detail within the literature, some constructs such as the value of an activity and self-efficacy may contribution to in-the-moment motivation for students to engage in an academic task.

2.3. Affect

The affective domain has become of increasing interest in educational psychology and mathematics education in recent years. In their *Handbook of Research on Student Engagement,* Christensen, Reschly, and Wylie (2012) suggest that understanding affective connections within the academic environment was necessary when investigating student engagement and motivation (see also Wentzel & Wigfield, 2009). Much of the research on affect focused on test anxiety and attribution theory, until recently (Anderman & Wolters, 2006). Recent calls to address emotion and affect have led to more research reports, including a book on *Emotion in Education*, edited by Schutz and Pekrun (2007). In their introductory chapter, Schutz and Pekrun state that, “Emotions have surfaced as an important contributing factor to the success of students,” (p. 9).

The term affect has been used broadly to describe a range of phenomena. To some, *affect* is interchangeable with *emotion* (Mandler, 1989), though others include mood or traits within affect as well (Rosenberg, 1998). Affect has been thought of as an emotional reaction or response to a task or outcome (e.g., success or failure) or the self–worth a person feels after performance on a task (Pintrich & Schrauben, 1992). Mandler (1989) describes emotions as responses to discrepant events, which may include both errors (e.g., "My strategy did not work") and successes ("I tried a different approach and it worked"). He suggests that errors typically yield negative affective responses, such as unhappiness or despair, and that success will bring about positive affective responses, perhaps joy and satisfaction. The more unexpected the result or outcome, the more intense the emotion may be. Mandler argues that it is impossible to have learning without affect, and that "the best we can do at present is to understand how learning and affect come about, how they interact, and how their inevitable symbiosis can be put to the use of our students and our society" (p. 17).

Pekrun, Goetz, Titz, and Perry (2002) reported on several of their studies in which they determined that students experience a rich diversity of emotions in academic settings. The Academic Emotions Questionnaire was developed to help the researchers better study the emotions felt as they pertain to test-taking, being in class, and learning: enjoyment, hope, pride, relief, anger, anxiety, hopelessness, shame, and boredom. These emotions are typically described as being either positive or negative, but Pekrun and colleagues categorize these emotions using a second dimension as well. Both the positive and negative emotions can be either activating or deactivating, creating four general categories. These are: positive activation emotions (enjoyment of learning, hope for success, pride); positive deactivating emotions (relief, relaxation after success, contentment; negative activating emotions (anger, anxiety, shame); and negative deactivating emotions (boredom, hopelessness; Pekrun, 2009).

After identifying these four categories, Pekrun et al. (2002) looked at whether these emotions "trigger, sustain, or reduce academic motivation" (p. 97) in several studies. One study found that for university students (n = 230) positive activating emotions enhance academic motivation (e.g., enjoyment x study interest $r = 0.62$, $p < 0.001$; hope x effort $r = 0.49$, $p < 0.001$), and that negative deactivating emotions may diminish motivation (e.g., boredom x study interest $r = -0.63$, $p < 0.001$). The other two categories, positive deactivating and negative activating emotions, are more complex and may either motivate students or may reduce motivation, depending on other factors (e.g. how students cope with emotions such as anger).

Linnenbrink (2007) also looked at the relationships between affect and motivation, specifically investigating goal orientations alongside engagement. Rather than use the terms positive and negative, she adopted the phrases pleasant (i.e., positive) and unpleasant (i.e., negative) to describe affect, which includes both emotions and moods. Linnenbrink reported that, as a result of several studies, she and her colleagues noted that mastery goals were associated with higher levels of pleasant affect. In one such study of 237 fifth- and sixth-grade students, the pretest[6] mastery goals significantly correlated with pretest positive affect (0.42, $p < 0.05$) and posttest positive affect (0.27, $p < 0.05$; Linnenbrink, 2005). However, the associations between performance goals and affect were not consistent, leading the researchers to suggest that there is either no relationship or a positive relationship with *both* pleasant and unpleasant affect. In looking

[6] In the two weeks between time points, students worked in the same 4-person heterogeneous group during their mathematics lessons.

at the relationships between pleasant/unpleasant affect and behavioral/cognitive engagement, the only consistent finding appears to be that unpleasant affect may undermine behavioral engagement. It may be the case that unpleasant affect also undermines cognitive engagement, but Linnenbrink notes this has not been clearly shown in studies. While the author suggests there may be a positive relation between pleasant affect and behavioral engagement, the studies indicate there is possibly no relationship.

Linnenbrink-Garcia, Rogat, and Koskey (2011) conducted two studies to investigate how students' affect influenced their engagement in small groups in mathematics class (Study 1: $n = 137$ fourth- and fifth-graders, Study 2: $n = 192$ fifth- and sixth-graders). Students were assigned to either three- or four-person groups. The first study was conducted in one day, while the second study collected data at three time points over five weeks. The instrument was developed by the authors, measuring affect using items based on the work of Watson and Tellegen (1985) and Thayer (1986), and included items to measure positive group interactions as well as social loafing. The authors clarify that social loafing occurs when students reduce their effort when working in small groups. In Study 1, the analysis found that social loafing resulted from negative affect (both activated such as anger, $r = 0.38$ and deactivated such as tiredness, $r = 0.43$). At the same time, deactivated positive affect (e.g., calm) contributed to positive group interactions ($r = 0.47$) and deactivated negative affect was negatively linked to positive group interactions ($r = –0.38$). Qualitative analysis of videotaped sessions in study 2 demonstrates a "cyclical nature of the relation between quality of group interaction and affect" (Linnenbrink-Garcia et al., 2011, p. 21). These findings also demonstrate evidence that cycles of negative group interactions can be interrupted by positive affect and positive interactions. The authors encourage future research to further investigate the dynamic relations between affect and engagement in various educational settings.

In mathematics education, specifically, the discussion within the affective domain has centered on mathematics anxiety, gender-differences in student confidence and enjoyment of mathematics, and a range of attitudes toward mathematics (Fennema & Sherman, 1978; McLeod, 1992). In addition, teachers' beliefs have been studied to a greater extent in the past several decades, as indicated in Philipp's (2007) review of mathematics teachers' beliefs and affect. This topic is important, as the beliefs a teacher has and shares with her students likely influences how the students' beliefs will shape throughout their schooling.

As a result of the call in mathematics education to focus on affect, several theoretical perspectives have been and are being developed to consider the larger picture of the affective domain (e.g., Hannula, 2002; Malmivuori, 2006; Op 'T Eynde, De Corte, Verschaffel, 2006). Additionally, researchers working to develop these theoretical perspectives suggest that emotions contribute to the development of attitudes (e.g., Hannula, 2002) and beliefs, as well as motivation (e.g, Op 'T Eynde, De Corte, Verschaffel, 2006)

Not all mathematics education researchers view affect in the same way, similar to the domains of engagement and motivation. For the present study, I adopt the positions of McLeod (1992), and DeBellis and Goldin (2006). McLeod (1992) defines the affective domain in mathematics education as encompassing three domains: 1) emotions, 2) attitudes, and 3) beliefs. DeBellis and Goldin (2006) identified a fourth component: 4*)* *values, morals,* and *ethics*. *Emotions* are the rapidly-changing feelings which may be expressed as students engage in mathematical activity. Emotions tend to be relatively unstable and range from mild to intense, whereas *attitudes* are generally moderately stable, as they describe predispositions students may have towards mathematics. Individuals also hold several beliefs which contribute to the affective domain. These *beliefs*, which are highly stable, may be about mathematics, about themselves as doers of mathematics, or about teaching mathematics. Values, ethics, and morals "refer to the deep, 'personal truths' or commitments cherished by individuals" (DeBellis & Goldin, 2006, p. 135).

In their description of the four components of the affective domain, DeBellis and Goldin (2006) have developed a tetrahedral model, in which "each vertex … may be understood as interacting dynamically with the others in a particular individual" (p. 136). An individual likely holds stable values, beliefs, and attitudes towards mathematics, but may experience a variety of emotions while working on a mathematical task. For example, at the start of the task, a student may experience confusion or curiosity about the problem. The student may recognize a first step toward solving the task and may feel elation. Suppose this student then realizes that his or her first strategy did not work. Then the feeling of either bewilderment or frustration may arise, as the student adapts to try a different strategy. In one case, the student may continue throughout the problem successfully, leading him or her to feel a sense of satisfaction and joy. Another possible path may lead the student to unsuccessfully try several strategies but not yield an answer. This student may feel frustrated, leading to feelings of anxiety, anger, and finally despair. These are examples of *affective pathways* (Goldin, 2000), which can be described as recurrent sequences of emotional feelings. When a student follows a particular pathway repeatedly, related long-term structures (i.e., attitudes, beliefs) may form within the individual.

Goldin (2000) refers to a notion of *powerful affect*. Rather than speak of positive (happy or pleased) or negative (disappointment or distressed) affect as good and bad, respectively, he considers many feelings, emotions, and attitudes as being potentially powerful. In the example given above, when the student felt frustration, another strategy was used successfully and the problem was satisfactorily solved. This frustration, which may sometimes act as a mental roadblock, can also provide opportunities to try a different strategy, which if successful, may leave the student feeling contented and confident about his or her ability to solve the problem.

Goldin and his colleagues also note the importance of the interplay of affect and cognition, as this relationship may influence student motivation and engagement as well. To better investigate the relationship between affect and cognition, Gómez-Chacón (2006) looks at both the local affect and the global affect. Within the mathematics education domain, local affect is “understood as the states of change of feelings or emotional reactions during the resolution of a mathematical activity” (p. 150). Global affect describes the “results of the routes followed (in the individual) in the local affect, which continually contribute to the construction of general structures of the concept of oneself and of beliefs about mathematics and its learning” (p. 151). Gómez-Chacón illustrates an example of a case study student who claimed to feel afraid of mathematics (global affect) and often expressed negative emotional reactions, including anxiety. This student still experienced some positive emotions and feelings, such as surprise and satisfaction (local affect), but the previous negative experienced predisposed the student to the global affective response of fear.

As the study of affect continues to grow, particularly in mathematics education, researchers need to find to continue to find ways to understand and explain the complexity of this construct. As various dimensions of affect, such as emotions and beliefs, are studied, it is important to continue to consider the relationship these dimensions have with others within the affective domain. Additionally, as hinted throughout this chapter thus far, the relationships between and among affect and motivation, engagement, and cognition should continue to be investigated to better understand the influence affect has on student learning.

2.4. Algebraic thinking in middle schools

The National Council of Teachers of Mathematics (NCTM) and many others assert that some algebraic concepts should be introduced prior to high school (e.g., Moses & Cobb, 2001; Moss & Beatty, 2006; Smith, 1996). Students who have been exposed to algebraic concepts in grade eight or earlier ought to have more opportunities to succeed in subsequent math courses and take more math courses in their high school years than their counterparts. Currently, several curricula for grades six, seven and eight, such as *Connected Mathematics* (Lappan et al., 1998), *Mathematics in Context* (National Center for Research in Mathematical Sciences Education, 2003), and others such as the Scott-Foresman (Charles et al., 1999) and Glencoe/McGraw Hill (Holliday et al., 2003) middle-school texts, include several algebraic concepts, such as introducing and using a variable and representing mathematical relationships with equations.

The literature reviewed here on algebraic thinking in middle schools is a small subset of this vast topic. I choose to focus on research that somewhat relate to the present study using the following criteria. The research had to be conducted with students in the middle grades and: 1) investigated the use or concept of a variable; 2) examined interpretations of the meaning of the equality sign; 3) used the *Connected Mathematics* curriculum, the series used by the participants in the current study; or 4) focused on non-routine problems which are typically not presented in textbooks and often ask students to *think conceptually* about the problem rather than merely perform computations.

One challenge students often face when learning algebra is an emphasis on procedures to manipulate algebraic expressions. As many mathematics educators suggest, such students may lack conceptual understanding of the patterns and the mathematical processes behind such procedures (Davis 1992; Kieran, 2007). In many mathematics classrooms, algorithms used to solve equations for a given variable are taught in a straightforward manner which students are expected to memorize. These algorithms are typically based on the notion that algebra is "generalized arithmetic," a view which does not lend itself to conceptual understanding (Kaput, 2000). One component of the "generalized arithmetic" view includes using the equality sign and using variables to represent specific values, rather than the general "unknown."

Researchers tend to agree that the equality sign as a symbol of equivalence and balance should be developed early on in school, even though algebraic concepts have historically not been taught until a late-middle school or high school algebra course (Kieran, 1981; Knuth, Stephens, McNeil, & Alibali, 2006). In addition, students should have multiple experiences with variables (Schoenfeld & Arcavi, 1988). Students' first experiences may include seeing and using shapes as placeholders for numbers, particularly in elementary school. By establishing such algebraic ideas early, students will be better prepared to work with symbols in high school and will be able to use procedures with understanding (NCTM, 2000). Algebraic concepts are introduced in elementary school in other countries, such as Japan and China, a fact that should encourage us to prepare our students to live in an international society (Cai, 2004; Silver, Leung, & Cai, 1995).

Alibali et al. (2007) sampled 81 students who used the *Connected Mathematics* curriculum to investigate their understanding of the equality sign. Data were collected at four time points over three years, starting when the students entered grade 6 and ending as they completed grade 8. The authors attempted to determine if students identified equivalent equations, as in this given example.

$$2n + 15 = 31 \qquad\qquad 2n + 15 - 9 = 31 - 9$$

Only about 50% of sixth-grade students who received this item ($n = 55$) correctly stated that the two equations have the same solution, whereas over 75% of eighth-grade students made the same statement. This increase suggests that students improved in their understanding of equivalent expressions and equations. On this and similar items, student responses were categorized as, "*answer after the equal sign, solve and compare, recognize equivalence, substitution, other,* and *no response/don't know*" (p. 228). Student work was classified as "solve and compare" when it showed that they solved the equations to see if the value for the variable was the same. The more sophisticated category of "recognize equivalence" was applied when the students demonstrated that they recognized that the value of the number sentence, and therefore the value of the variable, did not change. One student whose strategy was coded this way wrote, “Yes, [the number represented by *n* is the same in the two equations] because in the first equation the [*sic*] do not subtract anything, but on the second, they subtract from both sides (the equation and the answer)” (p. 229). Though an increasing number of students used a “recognize equivalence” strategy across the four time points, the results show that 47% of students never used this straetgy.

In a related study by Knuth et al. (2005), 373 middle school (sixth- through eighth-grade) students were asked the same question about the equality sign in the number sentence 3 + 4 = 7. Responses were coded as either providing a *relational* or *operational* view of the equality sign. An example of an operational response was, “‘It means whatever is after it is the answer’ (eighth-grade student)” (p. 71). A typical relational response was, “‘The things on both sides of it are of the same value’ (7^{th} grade student)” (p. 71). Though, as in the Alibali et al. (2007) study, the percentage of students who provided a relational interpretation of the equality sign was higher for eighth-grade students than for sixth-grade students, the authors note that only 46% of eighth-grade students responded with a relational view of the equality sign. They propose that middle school mathematics teachers generally assume that because students have been working with the equality sign for so long that it is not necessary to review the meaning of this symbol. However, these results suggest the teachers’ assumption is incorrect. Even if students have been exposed to the relational view of the equality sign in elementary school, it seems as though middle school teachers must also attend to this concept as to not allow students to maintain or revert to an operational interpretation (“do something”).

In the studies reported above, students in eighth grade were more likely to adopt a relational view of the equality sign than those in earlier grades. Still, many middle school students held the operational view and several individuals vacillated on their responses over time. The researchers suggest that the classroom context at the time of the assessment may influence how a student responded. For example, students may have been working with an operational view of the equality sign, possibly encouraging that view on the assessment. Such a possibility strengthens the argument for paying attention to the relational view of the equality sign across all the grades (i.e., grades 1 – 8).

Capraro and Joffrion (2006) report on the use of a variable to represent multiple or a range of values. Recognizing the importance of being able to translate from word problems into algebraic and number sentences, this study focuses on that translation through multiple-choice pre- and post-tests taken by 668 seventh- and eighth-grade students. On the post-test, fewer than half the students (43.1%) were able to correctly respond to the following item:

> Tachi is exactly one year older than Bill.
>
> Let T stand for Tachi's age and B stand for Bill's age.
>
> Write an equation to compare Tachi's age to Bill's age.
>
> (Capraro & Joffrion, 2006, p. 164)

Students were expected to respond $T = B + 1$, or provide an equivalent equation. Yet students who gave an incorrect response made one of the following errors: 1) transposed T and B, 2) rewrote the problem in words, 3) suggested an inequality rather than an equation, 4) used other, non-relevant variables, or 5) left the question blank. The authors suggest that careful attention be paid when teaching the translation from words to algebraic expressions or equations. They claim that conceptual understanding must accompany the procedures in order for students to perform the task of translating into algebraic expressions properly.

Arslan and Altun (2007) found that students in grades seven (n = 30) and eight (n = 26) learning to merely manipulate expressions were typically unable to transfer those skills to solve a non-routine problem. These non-routine problems were intended to represent a real-life situation, such as the "Handshake Problem," or continue a pattern to find the twentieth iteration when given the first three figures (the pattern contained triangles building on one another in similar triangular patterns). In their study, non-traditional problems tended to be more accessible to students who have achieved conceptual understanding of the mathematics required, as indicated by average scores on a pre- and post-test devised by the researchers. Between the two time points, half the students were taught specific problem solving strategies such as look for a pattern, make a drawing, or make a systematic list; the other students followed their regular curriculum, thus generating a control. While there was no significant difference between the groups on the pre-test, the control group performed significantly poorer on the post-test (seventh grade $t = 2.89$, $p < 0.05$; eighth grade $t = 2.05$, $p < 0.05$). Despite the small sample, these findings suggest that problem-solving strategies can be taught and may even improve student success on non-routine problems.

Conceptual understanding of algebraic topics may allow students to understand the meaning behind the algorithms, to recreate the procedures when they are not memorized, and to apply skills already learned to different problems. In order to better gain conceptual understanding within algebra, understanding the concepts of the equality sign and variable ought to go beyond procedural thoughts such as, “What is the next step?”

2.5. Small group work and cooperative learning

Interest in collaborative and cooperative learning in mathematics education emerged primarily in the late 1970s and 1980s (Noddings, 1985). Though the terms collaborative and cooperative are often used interchangeably in the literature (see O'Donnell, 2006), collaborative learning specifically refers to groups in which the members have been assigned a specific role or task to complete. Each group member brings his or her completed task to the other members and together they create the whole product. When students are working cooperatively, roles are not assigned, nor are students necessarily provided with separate tasks. In both situations, the members are expected to work together to arrive at a solution to the given problem (Cohen, 1994; Slavin & Cooper, 1999). Johnson and Johnson (1999) emphasize that placing students in a group does not oblige students to work cooperatively since one student may do all the work or students may essentially work individually.

Early, primarily quantitative, work (e.g. Peterson & Janicki, 1979; Rosenshine, 1979) could not confirm whether small group work in mathematics promoted achievement better than traditional classroom settings. In contrast, later studies have succeeded in delineating certain conditions under which small–group learning benefits students in all grades and of all ability levels (e.g. Davidson, 1985; Kieran 2001; Slavin, 1985; Slavin, Leavey, & Madden, 1984; Webb, 1982, 1985; Webb & Mastergeorge, 2003). Such small groups may have anywhere from two to six students, and may be a homogeneous or heterogeneous group with respect to ability or academic achievement.

Webb's (1982, 1991) work with mathematics students of various ages suggests that when working in small groups students who provided their classmates with explanations about the mathematics were likely to attain higher achievement, based on pre– and post–test scores. As for students who needed or requested help, they appeared to perform better if the type of help they received matched the type they requested (Webb, 1991). Not receiving an explanation in response to a question or error led to negative achievement, likely because "receiving no response at all would leave intact the student's misunderstanding or lack of understanding" (Webb, 1982, p. 652). In Webb's (1982) study, 96 middle school students (grades 7, 8, 9) were placed into groups either of uniform ability ($n = 14$ groups) or mixed ability ($n = 21$ groups) based upon results of standardized tests. The uniform-ability students were more likely to experience a lack of response than those in mixed-ability groups ($F(1, 32) = 8.87, p < 0.01$). In addition, those who did not receive a response to a question were less likely to score well on the post-test ($r = -0.53, p < 0.01$). These findings are important, but not unexpected. Teaching a concept one has recently learned reinforces that knowledge or understanding for the instructor (O'Donnell, 2006).

Studies focused on the relationship between discourse and academic success suggest some students need to practice communicating about and within mathematics (Cohen, 1996; Webb, 1991; Webb & Farivar, 1994). Kramarski and Mevarech (2003) contend that students who do not verbally communicate their mathematical ideas may have poor mathematics communication skills. While other possible reasons should be explored, the authors claim that for such students, working with others may prove difficult in part because they are not able to or not comfortable expressing their reasoning or lack of understanding of the mathematics. The authors suggest that, “Under certain conditions students in junior high schools can learn to provide mathematical arguments to justify their ideas” (Kramarski & Mevarech, 2003, p. 305).

Kieran (2001) observed that when one student in a working partnership appears to not modify his own views in order to accept another student’s mathematical ideas, it is because of a lack of productive “public discourse” (p. 206). A student who appears to not have the tools to adequately communicate his ideas with his partner is more likely to have non-mutually productive discussions. In Kieran’s study, six pairs of 13-year-old students were observed discussing a graphing problem. For four of the six pairs, one student performed significantly better than his partner (e.g., 100% versus 57%) on follow-up tasks similar to those worked on together. Kieran explains that in these four pairs the student who earned a lower score was less likely to share his ideas and explanations. The lower-performing partner also asked fewer questions; questions that were asked were asked were either not necessarily relevant to the task at hand or were not addressed appropriately by the partner (e.g., the partner gave an answer without an explanation). These findings imply that students ought to be encouraged and taught to work with others, ask questions, and explain their reasoning to their classmates and teachers. It is not enough to ask students to talk but they need to effectively communicate their ideas (Sfard & Kieran, 2001).

Barron (2003) observed 48 sixth–grade students working on a mathematics unit. In the quantitative portion of her study, four of the twelve triads were considered significantly less successful than the other eight groups. Those groups who were considered more successful (produced written solutions that was at least 50% correct) had a mean score of 95% on the problem solving task. Less successful groups had a significantly lower mean score of 29% ($p < 0.01$). When trying to determine the reason for this difference, Barron found that “neither prior achievement of group members nor the generation of correct ideas for solution could account for between–triad differences in problem–solving outcomes” (p. 307). Looking deeper into the number of correct proposals for solutions or strategies, the author observed that the more successful groups accepted or discussed 70% of the correct proposals for solutions or strategies. The less successful triads accepted significantly fewer correct proposals (24%, $p \leq 0.01$) and ignored or rejected 76% of correct proposals.

The differences between the responses of more and less successful groups prompted Barron (2003) to investigate qualitatively using video records of student interactions. The groups of students who arrived at correct solutions were more likely to accept or discuss correct proposals and were less likely to reject or ignore those ideas. In cases when one or more students insisted that they were correct and disregarded other students’ ideas, the group did not accept a correct proposal or did not complete all parts of the problem. Groups that were considered successful wrote down correct answers to most parts of the problem, and all members of the triad discussed the correct answer and how to arrive to the solution. Barron explains, “It was not that more successful groups were immune to problems of coordination but rather that members used strategies that evoked or recruited joint focus of attention [to the task]” (p. 349).

It appears that mathematical discussion of ideas can contribute to learning when students are working in small groups. Weber, Powell, Maher, & Lee (2008) propose that the learning environment can contribute to students’ confidence to present mathematical ideas to their peers and accept disputes. Eight middle school students collaborated on a probability task in a summer session, which took place during a three–year project at an urban middle school. While working in pairs on non–traditional mathematics problems, the students were often encouraged to discuss their ideas and challenge one another mathematically. Their suggestions and answers were never validated as correct (or incorrect) by the teachers. Within this learning environment, "students were arbiters of whether or not a solution was correct and a solution was only regarded as correct if the students all agreed that 'it made sense'" (p. 250).

Weber, Powell, Maher, and Lee (2008) suggest several learning opportunities arose during one session. The task used in this session asked students to use a computer program to determine if different fictional companies were producing fair dice, meaning that each of the six sides had the same probability of occurring. Students created posters to share their findings with one another, including the data to support their conclusions. Over six episodes, two students questioned the accuracy of a student who submitted that the company 'Calibrated Cubes' was fair because three of the possibilities (1, 2, and 3 out of 6) appeared the same number of times when the die was rolled 80 times via the computer. One student questioned the frequency of the other possible numbers since this information was not displayed on the poster. A second student suspected that 80 rolls of the die were too few to make a final determination. Arguments about the minimum number of rolls required, i.e., the sample size, and whether this detail was important, generated a debate among the students. This discussion led the students to create better mathematical arguments during which students discussed their ideas with one another, listened to others, and justified their own mathematical thinking.

Leikin and Zaslavsky (1997) suggest that working in small groups and communicating with classmates may influence students' attitudes toward learning. In this study, ninth-grade students (*n* = 74) worked in small groups of four for three (*n* = 42) or six (*n* = 32) units of algebra. At the conclusion of these lessons, students completed a questionnaire asking about their attitude toward working in small groups, using a scale of 1 (negative) to 4 (positive). The 47 items on the questionnaire were divided into four general categories: 1) mathematical communication, 2) social and affective aspects of learning, 3) characteristics of the setting, and 4) the learning setting in general. Students stated their most positive attitudes were toward mathematical communication (i.e., explaining ideas to others 84.3%, posing questions 94.3%) and the characteristics of the learning setting (classroom environment 82.8%, appropriateness of the learning pace 85.7%, appropriateness of the group arrangements 85.7%). The least positive attitudes were reportedly toward the social and affective aspects (i.e., relationships with classmates 41.4%, students' self-confidence 54.3%, students' responsibility to learn 40%).The authors suggest that enjoyed communicating with classmates about mathematical problems but they were unable to make claims about the influence the social-affective aspects may have on learning.

Nichols and Miller (1994) examined the effects of cooperative learning on motivational variables for Algebra II students (*n* = 62) in eleventh- and twelfth-grade. The students were placed into heterogeneous groups of four or five students, based on Slavin's (Slavin, Leavey, and Madden, 1984) Team-Assisted Individualization program. First, the authors found that the students in cooperative learning groups (*n* = 32) performed better on the teacher's exam (*mean score* = 78.12) than the students in the traditional lecture class (n =30; *mean score* = 70.20, $F(1, 59) = 5.6$, $p < 0.05$). In addition, there were significant differences ($p < 0.01$) on the measures of learning goal orientation (*mean scores* 4.30 vs. 3.77, $F(1, 59) = 20.18$), intrinsic motivation (*mean scores* 3.59 vs. 3.15, $F(1, 59) = 17.92$), and efficacy (*mean scores* 3.37 vs. 3.15, $F(1, 59) = 13.61$). As the authors note, it is unclear which aspect of cooperative learning may have contributed to these differences, it appears that cooperative learning improved not only the test scores for these students but also their motivation to learn.

Roger and David Johnson have promoted cooperative learning for over 4 decades (see Johnson & Johnson, 1999). In a recent meta-analysis (Roseth, Johnson, & Johnson, 2008), they compared how three different goal structures – cooperative, competitive, and individualistic – impacted the achievement and social relationships among early adolescents. Cooperative goal structures are characterized by the presence of *positive interdependence* in the condition. That is, the "individuals perceive that they can reach their goals if and only if the other individuals with whom they are cooperatively linked also reach their goals" (p. 225). The competitive goal structure is identified by a negative interdependence, which "exists when individuals perceive that they can obtain their goals if and only if the other individuals with whom they are competitively linked fail to obtain their goals" (p. 225). Finally, the individualistic goal structure is distinguished by a lack of any interdependence between the students.

The authors identified 129 manuscripts detailing 148 studies from an initial sample of over 17,000 manuscripts using the following criteria: the participants were middle school students (grades 6 – 9), and the study researched the effect of social interdependence on either achievement or social relationships (Roseth, Johnson, &

Johnson, 2008). Achievement was defined as performance on task and could be measured by a multitude of assessments such as accuracy of answers on tests, quality and accuracy of problem solving, or recall or retention. Peer relationships were also measured, usually using surveys or questionnaires, rating of classmates, or observations of interactions. For middle-grade students, cooperative goal structures were associated with higher levels of achievement than either the competitive structure (effect size, or standardized mean differences = 0.46) or the individualistic structure (effect size = 0.55). Positive peer relationships were associated with cooperative goal structures at higher levels than competitive (effect size = 0.48) or individualistic goal structures (effect size = 0.42). The authors suggest that cooperative goal structures encourage a significant relationship between levels of achievement and positive peer relationships ($\beta = 0.57$, $p < 0.01$) for students at these grade levels.

The studies included here on small group work promote cooperative learning as beneficial, but recognize the limitations of placing students in a group and instructing them to work together. Several researchers including Webb (1982), Kieran (2001), and Barron (2003) recognize the communication, or verbal interaction, between students must be effective. These and other studies demonstrate that, under certain conditions, cooperative learning can improve achievement (e.g., test scores or understanding; Roseth, Johnson, & Johnson, 2008) and learning experiences (e.g., Leikin and Zaslavsky, 1997).

Much work has been done on studying engagement, motivation, affect, and small group work as independent factors. Research has demonstrated the potential impact each of these aspects – engagement, motivation, affect, and small group or cooperative learning – has on student achievement in mathematics and other subjects. However, more studies are needed which bring these together to understanding the complexity of student interactions as they work on a conceptually challenging mathematics problem. The literature review contributes to the theoretical framework which guides this study, especially as I consider ideas of affect, motivation, and engagement while learning mathematics.

2.6. Engagement structures: Theoretical framework for the present study

This study's theoretical framework is guided by the concept of engagement structure. Goldin, Epstein, and Schorr (2007) and colleagues developed the construct of engagement structure to describe an idealized, recurring highly affective pattern inferred from observed behavior (Goldin, Epstein, Schorr, & Warner, 2011; Epstein et al., 2007; Schorr, Epstein, Warner & Arias, 2010-a;b). An engagement structure, according to Goldin, Epstein, and Schorr (2007), is a behavioral/affective/social constellation which may become active in a given social context, and may exist in each individual. An engagement structure becomes *active* or *activated* when a social situation arises that stimulates a particular motivating desire, or set of motivating desires, and prompts some action intended to satisfy the motivating desire.

Engagement structures are comprised of the following strands of simultaneously-occurring, mutually-interacting components, as described by Goldin, Epstein, Schorr, and Warner (2011):

(1) a characteristic *goal* or *motivating desire*,

(2) characteristic *patterns of behavior* including social interactions oriented toward fulfilling the desire,

(3) a characteristic *affective pathway* experienced by the individual,

(4) external *expressions of affect,*

(5) *meanings* encoded by emotional feelings,

(6) *meta-affect* pertaining to emotional states

(7) characteristic *self-talk* or inner speech,

(8) interactions with *systems of beliefs and values*,

(9) interactions with longer-term *traits, characteristics,* and *orientations,* and

(10) interactions with characteristic problem-solving *strategies* and *heuristics*.

(p. 549)

Initially termed "archetypal affective structures" (Goldin, Epstein, & Schorr, 2007), the concept of engagement structure came about when reviewing video data from a previous study in which entire classes as well as small groups of middle school students were observed as they worked on conceptually challenging mathematics problems. Conceptually challenging problems are those which Schorr, Epstein, Warner & Arias (2010b) regard as non-routine and may contain conceptual or cognitive hurdles for students. Selected students from these classes were interviewed in video based stimulated recall retrospective interviews (Alston et al., 2007; Epstein et al., 2007). Upon analyzing the classroom and interview video data, the senior researchers inferred certain recurring affective dynamic patterns of behavior.

Epstein et al. (2010) identify critical aspect of engagement structures referred to as a *motivating desire*, meaning the "individual's desire, interest, sense of goal or purpose, inspiration, or aspiration to engage in or persist in an activity (see also Alderman, 2008; Eccles, Wigfield, & Schiefele, 1998; Schunk & Zimmerman, 2009)" (p. 2). The concept of the motivating desire is associated with a *need* as described by

Murray (2008) in his book *Explorations in Personality* (70th Anniversary Edition). Murray's work examined the concept of personality, focusing on a number of variables including that of *need* and *press*. An individual's *needs* may not be observable but may nonetheless prompt the individual to act in a certain way to satisfy that need. The individual's actions may also be influenced by *press*, or a stimulus situation which is part of the environment and "usually appears in the guise of *threat of harm* or *promise of benefit*" (Murray, 2008, p. 41). In a classroom situation, a student's motivating desire may be evoked by a particular set of circumstances in the social environment, such as a mathematics class (Goldin, Epstein, & Schorr, 2007).

As mentioned, these structures intend to describe in-the-moment engagement and can be influenced by a student's emotional feelings. The arousal of some of these emotions may depend on the particular situation, such as being in a mathematics class, working cooperatively with others in a small group, working on a non-routine task. A student's emotions may be influenced by whether he likes or respects his teacher, whether he and his classmates can work together well, and whether he believes he can successfully understand and find a solution to the task, among other things (Op 'T Eynde, De Corte, Verschaffel, 2006). Emotions often change as the problem solving session continues, which may contribute one engagement structure branching into another.

Nine engagement structures are identified by Goldin and colleagues (2011), and other potential structures are currently being explored by the Rutgers University research team. The structures under investigation in this study are referred to as Let Me Teach You and Look How Smart I Am. The remainder of this section first briefly introduces the other seven engagement structures before describing in more detail the structures Let Me Teach You and Look How Smart I Am.

In contrast to Let Me Teach You and Look How Smart I Am, the other engagement structures do not necessarily necessitate both mathematical problem focus and a specific social interaction. Those structures, identified by Goldin, Epstein, Schorr, and Warner (2011), are given the names: *Get the Job Done*, *Check This Out*, *I'm Really Into This*, *Don't Disrespect Me*, *Stay Out of Trouble*, *It's Not Fair*, and *Pseduo-Engagement*. A student who has activated the Get the Job Done structure has a motivating desire to complete the task assigned by the teacher. The student is likely to follow directions and attempts to answer all the questions on the task. This structure pertains to the mathematical nature of the task but does not necessarily require a social interaction. Still, the student may urge classmates to work on the task and write down answers. The structure Check This Out becomes active when a student has a motivating desire to receive some reward for completing a task. A student may recognize an intrinsic value such as enjoyment or an extrinsic or utility value such as a future "payoff" (e.g., the information could be useful at a later time). I'm Really Into This is activated for a student who has a motivating desire to understand the problem or its solution, and is associated with Csikszentmihalyi's (1990) concept of "flow." A student who has an active Check This Out or I'm Really Into This structure is likely immersed in the mathematical task and may or may not be interacting with a classmate.

The structures Don't Disrespect Me, Stay Out Of Trouble, and It's Not Fair have a stronger social interaction component compared to the mathematical component. A student who has a motivating desire to protect his sense of self-respect or dignity and acts on this desire has an active Don't Disrespect Me structure. The student may feel threatened and may become angry. Oftentimes, though the mathematical ideas were questioned, perhaps by a classmate, the student may become combative rather than defend those ideas (Epstein, Schorr, and Goldin, 2007). Alternatively, a student may activate the Stay Out Of Trouble structure if she has a motivating desire to avoid a conflict or some distress. This structure's social component is such that a student may avoid engaging with others for academic or personal reasons (e.g., not to anger others).

The engagement structure It's Not Fair is characterized with a motivating desire to rectify some unfairness as perceived by a student. The student may attempt to call attention to the inequity, rather than continue to work on the task (Goldin et al., 2011). A student who has an active Pseudo-engagement structure attempts to avoid working the problem but desires to appear to be putting forth effort. Though the student does not wish to be engaging with the task, he does not want recognition for his lack of participation. He will try to appear attentive to avoid being noticed. Unlike these seven structures, Let Me Teach You and Look How Smart I Am both involve situations in which a student is providing his or her classmate(s) with some information about the mathematical problem or situation. However, the hypothesized motivating desires and resulting actions for each of the two structures are different.

Let Me Teach You (LMTY) becomes active when one student (tutor) experiences a motivating desire to teach another person (tutee) something that he or she knows that the other person does not appear to know. The need underlying this motivating desire was described as *nurturance* by Murray (2008). In this situation, the student attempts to help his or her classmate understand the assigned problem or the mathematics, perhaps in the service of satisfying a need of the classmate. Ideally, the student would be successful in communicating the ideas to his classmate and would feel a sense of satisfaction from helping someone. However, it is also possible that the tutee may resist the help for some reason. As an illustration, consider the situation in which one student attempts to help another student who seems to be struggling with a mathematics problem. In one case, the second student may be appreciative of the help and may ask questions to further his or her own understanding of the problem and its solution. LMTY would then continue to be active for the tutor. In another potential case, the would-be tutee could be feeling frustrated because he or she is having difficulty either understanding why his or her strategy is not working, or what the tutor is saying. Upon hearing his or her classmate offer some assistance or explanation, the second student may adopt a negative attitude toward the tutor and be unreceptive to suggestions or offers for help. The tutor, in that event, may not remain in the LMTY structure (at least with the tutee during that session). Indeed, he or she may even experience unpleasant feelings such as unhappiness, disappointment, etc. (Epstein et al., 2010), which in turn may evoke a different engagement structure for the would-be tutor.

Look How Smart I Am (LHSIA) may become active when a student hypothetically says to himself or herself something akin to, “I know this but the others in my group do not,” and subsequently realizes that this may be an opportunity to let the others see how much he or she does know. The need described by Murray (2008) which we identified as underlying the motivating desire is referred to as *achievement,* specifically by boosting one’s own intellectual status. The motivating desire is to impress others or “show off,” and to have others recognize his or her mathematical ability, knowledge or intelligence. Such an individual may be assertive when expressing these ideas, as he or she may not be satisfied until others acknowledge how smart he or she is or how much he or she knows. Reactions by classmates may include acknowledging how much that student knows (e.g., "You're so smart"), but may also involve others either ignoring or rejecting that student's contribution. If the student is either ignored or rejected, she may become defensive, either of her ideas or of herself. She may feel disrespected, and another engagement structure may be activated for this student.

In the present study, I examine how both the mathematics environment and the small group setting may have influenced middle school students' motivation and engagement. This chapter has described many complex factors – motivation, affect, learning environment (small group work), and subject matter – that may influence a students' engagement in the classroom. The literature for each of these topics is vast, so the discussion has been narrowly focused to provide a background for this study. In the following chapter, I present the research questions prior to the data collection and data analysis methods.

Chapter 3: Research Questions and Methods

This dissertation uses qualitative data to examine in-the-moment student engagement in mathematics by investigating the two engagement structures, “Let Me Teach You” (LMTY) and “Look How Smart I Am” (LHSIA) in the context of urban middle school classrooms. The theoretical concept of engagement structure is still new, and therefore it warrants more empirical research to help identify and characterize each such structure in more detail (Goldin, Epstein, Schorr, & Warner, 2011).

The two structures LMTY and LHSIA have both mathematical and social components, as described in Chapter 2. They both involve situations in which a student is providing one or more classmates with some kind of information about the mathematics or the problem. Other engagement structures may require either interaction with other individuals or involvement with the mathematics, but not necessarily both. In this study, I am particularly interested how both the mathematics environment and the small group setting may influence students’ motivation and engagement. Though the structures LMTY and LHSIA are related, the motivating desires for each are different. A student for whom the LMTY structure is active is motivated to help a classmate and thus to work in the service of another. A student with an active LHSIA structure is motivated to impress upon others that he is smart, intelligent, or has mathematical ability, and in this sense to work in the service of the self (by attaining recognition). Despite these different motivating desires, some behaviors are likely to be similar for the student with an active LMTY structure and the student with an active LHSIA structure. For example, a student who is correcting a classmate may be on one occasion trying to help that person understand the error or, on another occastion, may be trying to show that he has the correct answer and is therefore smart.

In this chapter, I present the detailed research questions, and then describe the methods I used to address them.

3.1 Research questions

To learn more about the differences and similarities between the LMTY and LHSIA structures, this dissertation addresses the following research questions.

Research questions about the Let Me Teach You (LMTY) structure:

1) How can we characterize interactions in which urban middle-school students attempt to teach or explain something to one another during small group work?
    a. What observable behaviors do students exhibit which indicate they are attempting to help others?
    b. What, if any, are some of the observable cues that prompted a student to attempt to help another student?
    c. How do the other members of the class respond to the one who is trying to teach them?
    d. What mathematical ideas are expressed by the students in the group during the interactions in which one student is attempting to teach or help others?
    e. What is the relationship, if one exists, between the students' behaviors as observed in the video analysis and a set of predetermined items on a self-report questionnaire (see below)?

This first research question addresses the engagement, the behavior, and the interactions that take place for middle school mathematics students during such moments.

For this study, neither the teacher nor the students received any guidance on how students should work together, other than the teacher's usual instructions to her students (see Appendix D). Therefore, in answering this question, I use classroom evidence to characterize engagement behaviors and interactions for students who help or teach classmates within their usual environment, where there has been no specific direction to the students to engage in peer tutoring.

Research questions about the Look How Smart I Am (LHSIA) structure:

2) How can we characterize interactions in which urban middle-school students attempt to impress another student with his or her ideas about the mathematics problems, or with his or her intelligence or ability?

   a. What observable behaviors do students exhibit which indicate they are attempting to show off or impress others?

   b. What, if any, are some of the observable cues that prompted a student to attempt to show off or impress others?

   c. How do other members of the class respond to the one that is attempting to show off?

   d. What mathematical ideas are expressed by the students in the group during the interactions in which one student is attempting to show off or impress others?

   e. What is the relationship, if one exists, between the students' behaviors as observed in the video analysis and the predetermined items on the self-report questionnaire (see below)?

The second major research question addresses the matter of how students perceive themselves and how others perceive those students with regard to intelligence, mathematical ability, and knowledge. A student who is attempting to show off may be displaying his or her mathematical contribution in a conspicuous manner with the hope or goal of inviting admiration. A student who wishes to impress others may not be so conspicuous, but still tries to earn a favorable opinion of others, with respect to his or her mathematical ability or knowledge. In this study, I use classroom evidence to characterize engagement behaviors and interactions for students who try to impress others with their ideas about the mathematics problems, or with their intelligence or ability.

3) What distinctions, if any, can be made observationally between the interactions in which a student attempts to teach others and the interactions in which a student attempts to impress others, as described above?

The third major research question addresses the possibility of students exhibiting similar behaviors when either the LMTY or LHSIA engagement structure is active. By answering the first two research questions, I identified behaviors and interactions when students attempt to help one another and when students attempt to impress others. While there were many instances in which the difference in motivating desire is clear, I hypothesized that there will be instances where the difference will not be explicit. Therefore, in answering this research question, I identified moments where both LMTY and LHSIA engagement structures are potentially active for a student. I differentiated these moments from those in which only one of either LMTY or LHSIA is the active engagement structure. I analyzed those episodes in which the active engagement structure is not immediately apparent to identify cues that indicate which motivating desire may be influencing the student's behavior.

By answering these research questions, I characterized and described the engagement structures "Let Me Teach You" and "Look How Smart I Am" so they can both be better understood. This study used evidence from middle school mathematics classrooms to further develop the theoretical understanding of engagement structure. By studying observed student behavior and qualitatively analyzing the students' self-report data from the questionnaire, I provided descriptive details of these two structures.

Next I provide details of the data collection and analysis process.

3.2 Methodology

This study uses qualitative methods to analyze video and survey data, allowing me to understand the recurring, dynamical patterns of interactions between and among individuals as they work on a conceptually challenging mathematics problem. In order to characterize the engagement structures, "Let Me Teach You" (LMTY) and "Look How Smart I Am" (LHSIA), I made inferences about students' intentions based on their behavior captured on video, as well as analyze the self-report data students provided via the questionnaire administered to the students.

First I shall detail the setting and briefly describe the larger study, providing context to the participants and data sources from which I drew my analysis. Next the sample of participants is described, followed by the data collection procedures, including presentation of the task given to students. Finally, I explain how I analyzed the data in three consecutive phases.

3.2.1 Setting

My study comes out of a larger project conducted by the MetroMath research team, of which I was a member. In order to explore and study engagement structures and student engagement, the research team conducted observations in four schools situated in a large urban school district in New Jersey during the 2008 – 2009 school year. Each school housed grades PreK–8 at the time of the study, though only seventh- and eighth-grade classrooms were observed. Within this district, approximately 56% of students are African-American and approximately 35% are Hispanic[1]. These schools are in an economically depressed area, with 87% of students eligible for free or reduced-priced lunch. Families in these neighborhoods are considered to be working class and "low income." Students in the middle grades (6–8) use the *Connected Math* series (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2002), one of the *Standards*-based curricula which received funding from the National Science Foundation for its development.

Four teachers, each in a different school, were invited to participate in the larger study. For each teacher, two separate classes of students participated in the study. These teachers had worked with MetroMath researchers on other projects or in professional development. Each teacher's style allowed students to work together in groups and encouraged students to defend, argue, and justify their mathematical ideas. This teaching style promoted student interactions, therefore allowing me to investigate the two engagement structures in a meaningful way. In classrooms where a teacher may be lecturing to students or where students are expected to work independently, there may be fewer or less overt behaviors to observe. Students may continue to be engaged,

[1] Percentages of race/ethnicity are rounded to assist with confidentiality of the district, schools, and students.

particularly at the cognitive level, but few inferences could be made about the motivating desires or behaviors of those students.

The study on which this thesis reports focused on the four classes of the two regular classroom teachers (Ms. A and Ms. S). The other two teachers, who are math coaches, are not part of the study reported here. Each teacher was asked to designate one of her classes as “higher ability” and the other “lower ability,” indicating the ability of the class as a whole. The criteria were not consistent among the teachers; teachers determined the classification based upon student performance in class, the school’s criteria (e.g., one class may be considered an honors group), and/or on the results of standardized tests. This variability may contribute to students demonstrating a variety of mathematical abilities as well as a diversity of behaviors within the mathematics classroom.

3.2.2 Sample

Participants were 31 seventh-grade and 24 eighth-grade students. The students in both of Ms. A’s classes were in the seventh grade. The students in both of Ms. S’s classes were in the eighth grade. Students worked together in groups of three or four on a specific non-routine mathematical problem so that students could interact and share ideas as they worked toward solutions. All students in this study worked on the same mathematical task, to provide a measure of control for task effects.

In each class, students were placed into small groups of 3 or 4 using no prior knowledge about the individual students (Rossman, Schorr, & Warner, 2010). Prior to the first day of observation in each school, each student was required to obtain signed permission slips from parents or guardians giving consent for the child to be videotaped.

The teachers provided a list of the students' names of those who did and those who did not have parental permission to be videotaped. The students for whom we had signed permission slips were placed into the groups using random assignment via Excel. Since the researchers were unaware of any preexisting dynamics between and among students, the random assignment of groups provided opportunities for a variety of interactions within the small group.

To assign the groups, the names of those students for whom we had signed permission slips were entered alphabetically into a single column of an Excel spreadsheet. In the adjacent column, the random number generation function associated each name with a random number between zero and one. The column of randomly generated numbers was sorted from smallest to largest, rearranging the corresponding students' names. From this ordered list, the first three students were grouped together, then the second three, and so on until all students were assigned to a group. Groups of four were created, as needed, when one or two students did not fit into a group of three. The grouping continued to follow the ordered list. When the research team arrived at the classroom, some students were absent or no longer in the class, and others had brought in signed permission slips. Therefore, some students were reassigned so we could maintain groups of three or four students. Such decisions were made on the spot by the researchers who attempted to maintain the original group assignments, so that the teacher's bias or preference was less likely to influence the groupings.

In Ms. A's two classes, nine groups were observed, and Ms. S's two classes include eight groups, for a total of 55 students in my sample. Table 3-1 summarizes the number of groups and students in each class.

Table 3-1: Groups of students* in each class (Total: 17 Groups)

Ms. A – Seventh Grade		**Ms. S – Eighth Grade**	
Class 1: Higher ability		Class 1: Higher ability	
Gp #	Students	Gp #	Students
1	Abby, Julian, Samara	1	Manny, Damon, Deanna
2	Juan, Amanda, Eliot, Manuel	2	Ricardo, Carol, Nikki
3	Liza, Bridget, Jenna, James	3	Carly, Christina, Christian
		4	Leticia, Sherelyn, Monique
		5	Kevon, Georgia, Nadira
	Total: 11 students		Total: 15 students
Class 2: Lower ability		Class 2: Lower ability	
Gp #	Students	Gp #	Students
1	Talia, Jaden, Genevieve, Andrew	1	Leo, Ordena, Ta'keisha
2	Pedro, Kian, Rico	2	Nazira, Aleana, Keshia
3	Onan, Raina, Joe	3	Ta'Shawna, Carla, Tyesha
4	Danica, Shanika, Shannen		
5	Wilson, Lewis, Mitchell, Martin		
6	Larry, Janelle, Ella		
	Total: 20 students		Total: 9 students

*Student names are pseudonyms

3.2.3 Data collection

The data were collected during observations over two days in each classroom. My analysis focuses on the first day, when students begin work on the task and come up with solutions. On the second day, students continued to work and presented their solutions to the whole class. During these classroom observations, we collected: video and audio recordings of each group's activity, researcher field notes, documents created by the students, and questionnaires administered to students. The field notes and students' written work were used to support findings from the observation data, as per Creswell's (2007) suggestion that multiple sources of data be used to assist with triangulation of findings. In the remainder of this subsection, the task is described, followed by the procedure to collecting observation and questionnaire data.

The task

The Building Blocks task was selected for its likelihood of being conceptually challenging to most participating students. Stein, Smith, Henningsen, and Silver (2000) suggested that conceptually challenging tasks should promote conceptual understanding, allow for multiple solutions, and support the development of problem-solving skills. This task encourages pattern recognition, multiple representations, and multiple solutions. The task was adapted by an eighth-grade mathematics teacher who provided the task to her students during a pilot study (Epstein, Goldin, Schorr, Capraro, Capraro, & Warner, 2010). The instructions for the task and corresponding figure (3-1) depicting the one-block high tower, the two-block high tower, and the three-block high tower provided to the students for the task are as follows.

I was constructing towers as you see below. I noticed that each time I made the tower higher, I had to add more blocks on the sides to stabilize the structure. I would like to know how many cubes I will need to build a 5-block high tower and a 10-block high tower. Generalize, if you can, on how many blocks I will need for any size tower?

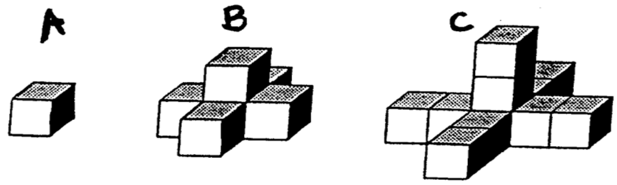


Figure 3-1: Illustration for the "Building Blocks" task

During the problem-solving session, students were encouraged to represent the structures by drawing sketches, using Snap Cubes (plastic interlocking blocks) to build towers, and/or creating tables to record their findings as they went through the task. The possibilities for modeling the towers allowed students to recognize emerging patterns, leading them toward solutions. Further details and analysis of this task can be found in chapter 4.

Classroom observations

Observations were conducted in each classroom over two school days between November 2008 and January 2009; my analysis focuses on the first day only. Each class period consists of 90 minutes, with the last 30 minutes set aside for completion of the questionnaire, which is described in more detail below. Students work in small groups to arrive at a solution to the Building Blocks task for one class period. The three or four students brought their desks together so they could easily discuss and share ideas. Each group was recorded separately on video, using a stationary camera positioned near their work space to capture all members of the group. One researcher followed the classroom teacher with a video camera to capture her interactions with the students. A digital audio recorder was placed on a desk at each group to capture the students' verbal statements while they worked on the assigned mathematics problem.

Field notes were recorded using a protocol that I and other graduate student researchers designed for the larger study (see Appendix A). During each class, each observer took notes while concentrating on one or two small groups, allowing each of us to provide more details about a small number of students. All observers, including myself, took on the role of non-participant observers and interacted with the students as little as possible. We did this with the goal of observing natural interactions between the teacher and students, as much as possible.

Data from these observations include the field notes and the audio and video recordings. Student work was also collected, and more details are given below. Transcripts were created from the audio and video recordings. I transcribed the video and audio of two groups in Ms. S's Class 2: 1) Leo, Ordena, Ta'keisha, and 2) Aleana, Keshia, Nazira. The recordings for the other fifteen groups were transcribed by hired consultants. I verified and edited all seventeen transcripts, as necessary. Video was originally recorded on MiniDV cassettes and DVDs of the recordings were created. Audio recordings were transferred into computer files and CDs of the recordings were created. The original MiniDV cassettes were labeled according to teacher, class, and date, and then stored in a secure location.

Student work

As students worked to solve the given problem, they often recorded their work on one or more sheets of paper. Some groups also wrote their solutions and ideas on a transparency sheet for the purpose of sharing their strategies with their classmates. Each day, students were instructed to write their names on each piece of paper, and the research team collected each individual's work and each group's prepared transparency sheet, when available. By examining what the students wrote, I was able to clarify some of the mathematical ideas the students expressed throughout the problem solving session. Student work was scanned and saved in electronic folders designated for each group. The work is included in Appendix D as part of the coded transcripts.

Questionnaire

The questionnaire, developed by the team of senior researchers and graduate students, asked students to report on their thoughts, feelings, and experiences while working in groups on the task. This questionnaire was field-tested in similar middle-school classrooms during a pilot study the previous school year, as well as in a course for pre-service teachers. Based on the survey results and feedback, the questionnaire was revised, producing the version used in this study. The specific items analyzed in this study are given in Appendix B.

The questionnaire administered during this study contained several types of questions: (1) 5 open-ended questions asking students to share memorable moments; (2) 42 items describing thoughts or experiences the students might have during class using a 3-point Likert scale: 0 (never), 1 (some of the time), or 2 (all of the time); (3) 22 words describing emotional feelings a student might have experienced (e.g., interested, successful, curious, discouraged, confused, frustrated) also using a 3-point Likert scale: 0 (not at all); 1 (somewhat), or 2 (very much); (4) 32 statements about the student's perceived behaviors in class on a 3-point Likert scale: 0 (hardly ever), 1 (sometimes), or 2 (often), and (5) 25 yes/no items suggesting whether the student did or did not have a corresponding thought about the class. Certain questionnaire items were intended to correspond to the engagement structures LMTY and LHSIA. Those items were analyzed and are provided in Tables 3-2 and 3-3.

Approximately thirty minutes before the end of the class period, students were asked to complete the questionnaire independently. One researcher guided the students through the instrument by reading the instructions and the first few questions or statements on each page. Students were advised to give honest responses and not share their answers with others while completing the questionnaire. Each student's questionnaire was scanned and saved in electronic folders for each class.

Table 3-2: Questionnaire items possibly indicating LMTY

Type of item	*Item*	*Possible responses*
Student thoughts or experiences during class	I wanted to teach another student something that I knew that this other student did not know	Never Some of the time All of the time
	I listened carefully to ideas of someone I was trying to help.	
	I helped someone see how to do the math.	
	Others listened carefully to my ideas.	
Behaviors	I gave helpful suggestions.	Hardly Ever Sometimes Often
	I worked cooperatively.	
Thoughts	I like teaching this person things I know.	Yes / No

Table 3-3: Questionnaire Items possibly indicating LHSIA

Type of item	*Item*	*Possible responses*
Student thoughts or experiences during class	I wanted people to think I'm smart.	Never Some of the time All of the time
	I tried to impress people with my ideas about the problem.	
	People seemed impressed with the ideas I shared about the problem.	
	People saw how good I was at the math we did today.	
	I felt smart.	
	I wanted to show someone that my way was better.	
	I was a lot better at math than others today.	
Behaviors	I was the leader.	Hardly Ever Sometimes Often
	I was bossy.	
	I wanted to show off.	
	I liked to be right.	
Thoughts	I want you to know just how smart I am.	Yes / No
	People think I'm smart.	
	I wish the teacher would call on me, so I can show how much I know.	

3.2.4 Data analysis

My analysis was conducted in three phases, as shown in Figure 3-2. In the first phase, I analyzed the video, transcripts, and student work, looking for evidence of one or more students having an active LMTY or LHSIA engagement structures and coding the transcripts. The second phase consisted of analyzing the questionnaire data and comparing these results to findings from the video data. The third research question was addressed in the third phase where I compared two engagement structures. Each phase is described in more detail in the following sections.

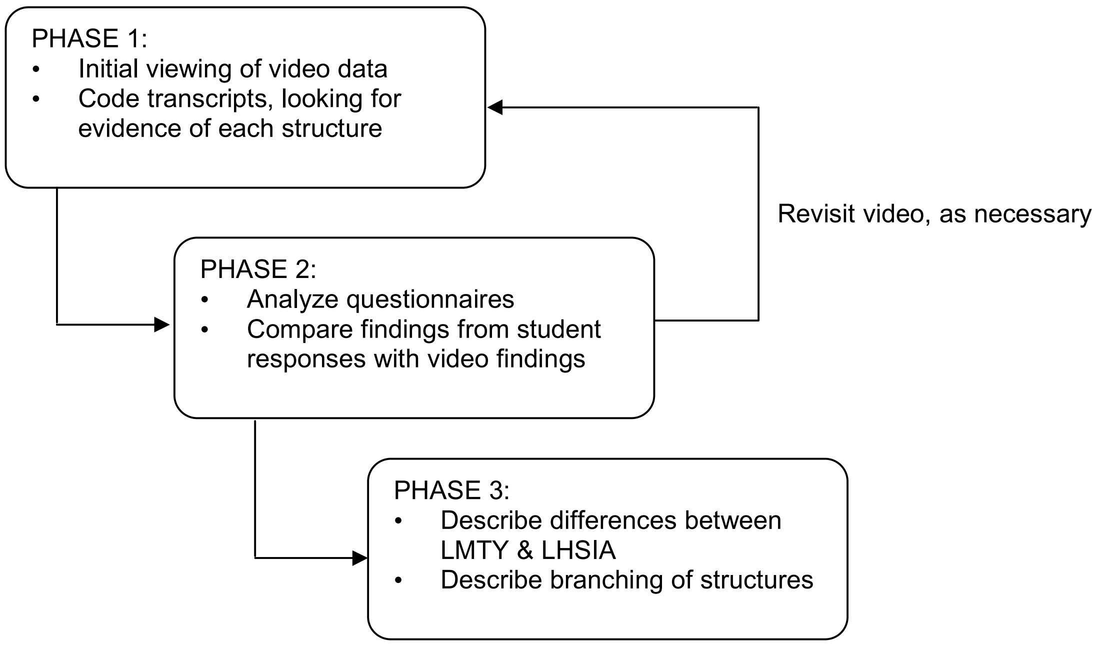


Figure 3-2: Three phases of analysis

Phase one: Video data analysis

The first step of my analysis was to watch the video recording for each of the 17 groups from the first day and look for evidence that one or both the engagement structures LMTY or LHSIA may have been activated for any individual in each small group. Each time I viewed a video, I also listened to the corresponding audio recording as I was often able to hear student utterances which were not clear from the video. After watching the first video of one small group, I created a Video Analysis Summary Sheet (see Appendix C) to help me organize my notes about which student(s) in the group may have an active LMTY or LHSIA structure. The summary sheet is organized to document the episode(s), student(s), and the observable behaviors of the student(s) that may provide evidence of an active structure. My notes on the summary sheet also describe the mathematical context of the potential episode. After creating the summary sheet, I revisited the first video in order to have one Video Analysis Summary Sheet for each small group. I returned to the summary sheets throughout the coding process to review my initial notes and to keep track of the episodes in which I believed a student exhibited an active engagement structure.

During this initial viewing of each video, I looked for observable behaviors that allowed me to infer that at least one student in the group had an active LMTY or an active LHSIA structure. Such behaviors which may indicate an active LMTY engagement structure include: (a) trying to teach a mathematical idea or procedure to a classmate; (b) trying to explain why a particular step contributes to the solution to other students; (c) trying to explain why a particular answer makes sense to a classmate; (d) trying to respond to a request made by a classmate to explain something related to the problem and follows through with such an explanation; (e) trying to inquire if other students understand something about the mathematics or the problem. The behaviors which allowed me to infer that at least one student in the group had an active LHSIA engagement structure include: (a) trying to demonstrate his or her knowledge to a classmate(s), not necessarily in the service of trying to teach others; (b) trying to show that his or her solution method is better than those suggested by other students; (c) trying to impress his or her classmates with what he or she knows; (d) trying to argue in support of his or her mathematical ideas whether or not there is disagreement from another individual. These behaviors were noted on the Video Analysis Summary sheet as I watched each video. The students' actions became the basis of the initial list of codes, which are described in more detail below.

After each video had been viewed once and the summary sheet was completed, a single transcript was created for each group from the video and audio recordings. Outside consultants transcribed all but two videos, and I verified each of the final products. The transcript documents were created using a table format. There were columns for the time in video, the person who was speaking, and the text of the speech. A separate column was reserved for comments about the significant actions and tone of voice and details about the active engagement structure, which were recorded during the coding process.

After the transcripts were created and verified, they were analyzed alongside the video, using multiple coding schemes (Miles & Huberman, 1994). The initial list of codes was developed prior to the first stage of coding. They are based upon my research questions, the literature, the questionnaire, and reports on research related to this study (for examples, see Epstein, Goldin, Schorr, Capraro, Capraro, & Warner, 2010; Schorr, Warner, Epstein, & Arias, 2010a; b). Table 3-4 displays the list of initial codes. During this stage of coding, I watched each video at least twice in order to determine the episodes in which one or more students appeared to have an active LMTY or LHSIA structure using these codes.

Table 3-4: Initial codes for LMTY and LHSIA

Code	Action	Observed Behavior	Description/Reasons
LMTY – Exp	Explanation	Student explains by doing one or more of the following: • teaching • responding to a question about the problem • explaining multiple times • providing information about the mathematics or problem	Student believes s/he can help another student understand something about the problem
LMTY – Sug	Suggestion	Student suggests would-be tutee try a particular strategy	Student believes that another student can reach the solution if a different strategy is used
LMTY – Qst	Question	• Student is asked a question such as, "How did you get that answer?" by a classmate • Student asks a classmate a question such as, "Do you understand?"	Student tries to help other students understand a solution or strategy when asked. Student wants to know if a classmate understands the concept or strategy.
LHSIA – Inf	Information shared	Student shares an answer or strategy with the intention of showing off his or her knowledge or intelligence	Student wants others (e.g., classmates or teacher) to recognize his mathematical ability, intelligence, or knowledge about the problem
LHSIA – Sug	Suggestion	• Student suggests his/her solution is better than another students' solution • Student suggests his/her strategy is better than others	Student wants others to recognize that his/her ideas are better than others because he/she is smart
LHSIA – Qst	Question	• Student is asked by classmates or teacher to share his/her ideas, strategy or solution to the problem • Student may or may not give an appropriate response to a question or request (e.g., A classmate requests an explanation, which may or may not be given)	Student is able to show off his mathematical ability or knowledge about the problem because he/she was asked to share his ideas by another. Student may believe he/she can demonstrate how smart he/she is by *not* responding to a request or question; alternatively, student may wish to demonstrate his/her intelligence by answering questions from others

After the initial codes were applied, I revisited the data to determine if additional codes emerged (Creswell, 2007; Miles & Huberman, 1994). The empirically developed codes became the second stage codes, and were developed while viewing the video data alongside the transcripts multiple times. These codes include observed student behaviors which had not yet been identified but appeared to be evidence of the activation of either the “Let Me Teach You” or “Look How Smart I Am” engagement structures. Because I was coding for behaviors, I chose to use verbs in the present tense to suggest an action or process (e.g., explains procedure, builds on idea, corrects a classmate, looks for confirmation) (Seldaña, 2009). I reviewed each group’s video and corresponding transcript three or more times using the second set of codes.

I viewed the videos in such a pattern that each video was revisited after I had seen all seventeen videos. This allowed me, for example, to identify a code in the seventeenth video and then review the other sixteen videos to determine whether the particular behavior might have been exhibited by other students. I used this circular pattern in an attempt to identify each potential episode in which a student activated either a LMTY or LHSIA structure.

After the second stage codes were applied, I revised the codes so that similar codes were combined and each code was defined to indicate its unique features (Miles & Huberman, 1994). Once a complete list of codes was created, I revisited the video, audio, and transcript data once again to ensure consistency. Throughout this process, I created documents to place the transcripts alongside the code, leaving space to give my interpretation and reason for inferring that the student had either an active LMTY or

LHSIA structure within an episode. These documents are found in Appendix D. The complete list of codes for the two structures is included in Tables 3-5 and 3-6.

After the video and transcripts of all seventeen groups were coded, I asked an independent observer who is familiar with the larger study to assist in verifying the coding scheme. I gave her the video and audio data and corresponding transcripts for five groups of students. Before she coded any data, I provided her with the overall descriptions of each structure and each code, similar to those in Tables 3-5 and 3-6. Furthermore, we discussed any questions she had. The researcher independently viewed the videos and coded the transcripts. We agreed on 80% of the codes over those five transcripts. The disagreements typically took the form of one of us coding a behavior as evidence of either LMTY or LHSIA when the other person did not. We then discussed each instance of disagreement until we agreed upon whether or not one of the two structures may have been active at a particular moment and the reasons for our decision. Approximately half the time, we agreed that the code applied to the behavior and the other times, we agreed that the code did not apply. The changes made did not seem to be notably biased toward my original determinations or toward the other researcher's direction, given that we each accepted one another's justification approximately the same number of times. Upon completion of this process, we arrived at 100% agreement of the codes. Once this process was completed, I reviewed the remaining twelve transcripts once again to strive for a consistent coding process.

Table 3-5: Codes for LMTY

Code	Action	Observed Behavior	Description/Reasons
1	Explains the instructions, procedure, or some characteristic of the problem or its solution	Student attempts to explain some aspect of the problem to a classmate(s).	Student believes s/he understands the problem and a classmate(s) does not. The explanation includes various details. Student may explain in response to questions from classmate(s).
2	Modifies explanation	Student modifies or clarifies a previous explanation.	Student has given an explanation already, but realizes that the classmate(s) is still confused about that particular thing (e.g., instruction, strategy, procedure).
3	Provides additional details	Student had given some part of an explanation and is now giving additional details to help clarify the explanation.	Student is persisting to give help to the classmate(s) by providing more details. Student may believe that the classmate(s) did not fully understand the explanation or that the classmate would benefit from more information.
4	Corrects a classmate*	Student attempts to correct a classmate, in a constructive manner. The student intends to help the classmate learn and understand the solution, answer, or strategy.	Student appears to be trying to help classmate by giving the correct answer or strategy. Student may be reminding classmate of the correct idea stated earlier.
5	Restates or interprets a classmate's or teacher's strategy or solution.	Student restates what another person has said with regard to the solution, instructions, or strategy, in his/her own words.	Student appears to believe that the strategy or solution given by another is correct but that someone does not understand it. By restating it, the student is aiming to help the classmate(s) understand.
6	Builds on another's idea	Student aims to move forward with an idea, solution, or strategy stated by someone else	A classmate may have a good start, but the student believes he can help classmate(s) to understand by building on what she said.

Table 3-5 (continued): Codes for LMTY			
7	Responds to specific questions, with short answers	Student gives answers to a classmate who asks for them, in a way intended to help the classmate learn and understand those answers.	Student has a patient tone of voice so that, along with body language, the student appears to want to help the classmate. Explanations may have already been given, and therefore, may not be necessary.
8	Questions classmate to help classmate obtain solution or clarify error	Student may ask classmate, "How did you get that answer?", or "What was your strategy?"	Student asks questions so that the classmate might recognize the error or might be able to make progress toward the solution.
9	Offers to teach or help classmate explicitly	Student may say, "I can help you with that," or asks "Do you want me to explain that to you?"	Student may want to help a classmate understand an idea, strategy, or solution.
10	Asks if classmate understands	Student has explained something already and checks if the explanation makes sense to the classmate.	Student wants to make sure that the classmate understands the explanation already given.
11	Offers compassion or reassurance	Student may recognize that a classmate is struggling and say, "I had a hard time with that too," or "If you keep trying you will understand it."	Student wants to help the classmate to continue with the problem so that the classmate will learn or understand strategy or solution.

*Note that "Corrects a classmate" is a code for both LMTY and LHSIA; the observed behavior and description/reason note the differences. Only one instance of "Correcting a classmate" is coded both LMTY and LHSIA.

Table 3-6: Codes for LHSIA

Code	Action	Observed Behavior	Description/Reasons
1	Calls out answer	Student appears to want to be the first one to give an answer as soon as a question is asked by the teacher or another student.	Student indicates he/she has a correct answer or idea by wanting to share it with the class. Student does not want to wait to be called on to answer.
2	Talks over others	Student interrupts or talks over another student.	Student indicates his/her idea is better or “correct” by disregarding what a classmate says in a small group setting or when responding to a question. Student may also be interrupting a classmate.
3	Controls math tools	Student attempts to take control of tools (e.g., computer, manipulatives, pens) to show his/her solution or calculations.	Student indicates his/her idea is better or the “correct” one by taking over the tool(s) to show classmates the answer or solution.
4	Says “I got it!”	Student calls out that he/she got the answer.	Student states it aloud enough so other students hear that I got the answer.
5	Asserts, “I will figure this out.”	Student states that he/she will be able to get the solution.	Student indicates confidence in him/herself that he/she has the ability to get the answer.
6	States, “This is easy.”	Student states the task is easy.	Student statement implies that the student believes he/she knows how to do it. Affirms from the outset that the student’s ability to solve the problem is strong.
7	Gives answer and does not listen to others’ comments	Student responds to specific question but is corrected by another student and doesn’t listen to suggestion	Student may believe that his/her answer is correct (because he/she is smart) and therefore does not have to listen to others.

Table 3-6 (continued): Codes for LHSIA			
8	Corrects a classmate*	Student may give classmate an answer, solution, or strategy that student believes is correct.	By correcting a classmate, the student suggests that he is smart because he has the correct answer or strategy. The context and tone of voice determine that the student is in service of himself rather than the classmate.
9	Agrees with classmate	Student agrees with the classmate, affirming the idea as though he/she thought of it as well.	Student suggests that if he/she has idea independently, then he/she is smart, like the classmate.
9A	Disagrees with classmate	Student disagrees with classmate's answer or suggestions and ignores it.	Student disagrees with something a classmate said and defends his/her own ideas. Student believes his/her own idea is correct.
10	Builds on classmate's ideas	Student states ideas that seem to build on previous ideas stated by a classmate(s).	Student may appear to be smart, or to have good mathematical ideas, if he/she can further the ideas stated by others.
11	Says, "I told you so."	Student suggests that the correct idea was stated earlier by him/her.	"I told you" as a classic indicating of, "I'm smart, I was right," or "I already knew that."
12	Tries to show, "I can keep up with you."	Student may repeat what others say, may offer suggestions about the mathematics or the procedure, or may confirm that what others say is correct.	Student indicates he/she is smart, intelligent, and/or has the ability to get the solution or strategy, with or without the help of others (e.g., classmates). The student may also be indicating that he/she can keep up mathematically with other students.
13	Says, "Got it?" to classmate	Student says, "Got it?" with emphasis to see if the classmate understands, but also to affirm his/her own knowledge.	Student may think, "I am smart because I understand this. My classmate should understand this as well."

Table 3-6 (continued): Codes for LHSIA			
14	Demands attention to one's own ideas	Student speaks to share ideas, suggestions, or strategies and demands the attention of his/her classmates. This includes occasions when the student is the first in the group to speak.	Student wants others to listen to him/her and to treat him/her as though she is smart. He/she wants his/her ideas to be valued.
15	Looks for confirmation from classmate	Student shares an idea or makes a suggestion and looks to classmate(s) for confirmation that the idea or suggestion is correct.	Student will feel smart if a classmate indicates that the idea or suggestion is a correct. If the group or class follows student's suggestion, the student will have demonstrated mathematical ability or intelligence.
16	Tries to "one up" a classmate	Ex. Student yells out, "I got a 98" (e.g., on a quiz/test). Another student yells out, "I got 100."	Student demonstrates that s/he has outdone another student.
17	States that someone is smart	Student says, "I'm smart," "You're smart," or "We're smart"	Student is declaring that he/she, a classmate, or the whole group is smart. This may be to encourage or to celebrate a success.
*Note that "Corrects a classmate" is a code for both LMTY and LHSIA; the observed behavior and description/reason note the differences. Only one instance of "Correcting a classmate" is coded both LMTY and LHSIA.			

Phase two: Questionnaire data analysis

During phase two of my analysis, I analyzed the students' questionnaire responses from the first day and compared these results to the findings of the video analysis conducted in the first phase. This phase of analysis took place after I completed the video data analysis, so that my findings based on video were not influenced by the questionnaire responses. Here, I analyzed the students' responses qualitatively to determine how they perceived their own experiences in that day's mathematics lesson. Analyzing the questionnaire responses was a crucial component of this study since these questionnaires provided the only source of self-report data.

It is important to note that the questionnaire was completed immediately after students had worked in the problem-solving session for approximately one hour. During that time, several engagement structures may have been active for a student and that student may have responded on the questionnaire accordingly. Therefore, a student's responses may suggest that both LMTY and LHSIA were active, but it should not be assumed that both engagement structures were active at the same time; the student may be referring to two or more different moments during class.

Analysis was conducted on the questionnaire items which may have indicated that either LMTY or LHSIA was active for a student (see Tables 3-2 and 3-3). I viewed each student's responses to these items using the scanned questionnaire documents. Each student's responses are included in a table format Appendix D as part of the coded transcripts. I determined that a student may have had an active LMTY structure based on responses to the questionnaire items described in Table 3-2 by giving a response of either

2 (all of the time) or 1 (some of the time) to each item. The same determination was applied for the LHSIA structure.

After identifying those students who gave responses indicating that either LMTY or LHSIA (or both) may have been active during the problem-solving session, I compared the results to my findings from the first phase of analysis. There were 21 students who reported on the questionnaire that they had an active LMTY structure or the motivating desire to help a classmate for whom I did not find evidence of this activation. There were 11 students who reported that the LHSIA structure was activated or there was a motivating desire to appear smart though I did not find evidence to suggest this. Additionally, there was only 1 student for whom I inferred an active LMTY and an active LHSIA structure whose questionnaire results suggest each structure was not active. For these students, I reviewed their group video at least once in an attempt to identify the discrepancy.

Phase 3: Comparison of behaviors for LMTY and LHSIA

After the completion of the video and questionnaire analyses conducted in the first two phases, I constructed a set of differences between the two structures. This component of analysis is significant because there were instances where I initially was not able to determine which structure, LMTY or LHSIA, was active. In some cases I inferred that both engagement structures were active for a student. Therefore, I identified which characteristics appeared to be similar and which may be different for the engagement structures LMTY and LHSIA. I then applied these criteria to those episodes, in which both structures appeared to be active. From this I was able to determine if I could infer that only one or the other structure was active at that time, or if both appeared to be active at the same time. Ultimately, I coded a behavior for both LMTY and LHSIA 13 times for 12 students.

As a consequence of this work, this phase also included an attempt to record instances of a student for whom one structure was active branched into the other (or a third) structure. As mentioned in the theoretical framework, several engagement structures other than LMTY and LHSIA have been identified. Though this study does not focus on those structures, I felt it important to acknowledge the likelihood that others may become active during the problem-solving session. For example, a student whose behavior indicated that he was trying to impress his classmates with his strategy for solving the problem may have been asked to explain his strategy. Thus if the student complied, offered an explanation, and took steps to determine if his classmates understood the strategy, he may have branched from being active in LHSIA to an active LMTY structure. By working through these three phases of analysis, which are depicted in Figure 3-2 earlier, I aimed to more clearly describe each of the two engagement structures Let Me Teach You and Look How Smart I Am.

Chapter 4: Analysis and Findings

This chapter is the presentation of my findings based upon my analysis of the 17 groups of students from the four classes. Prior to presenting specific results, I first give a detailed analysis of the task, and describe some of the common strategies the students used to find solutions. Following this, I present the summary of results of the coding described in chapter 3. The full results can be found in the coded transcripts in Appendix D, which also includes the teacher introductions to the task and the questionnaire results for each student.

Next I describe how these codes led to my characterization of broader features of the behaviors associated with each engagement structure, Let Me Teach You and Look How Smart I Am. Each feature is described in detail, and nine events are selected for the purpose of illustrating these features. In this chapter, I use the word "event" to refer to the classroom excerpts that are given as examples of analysis. The coded transcripts in Appendix D are segmented into episodes; the events given below typically constitute a portion of an episode.

4.1 The Building Blocks task

The seventh- and eighth-grade students in these schools had approximately 90-minute blocks of time for math class on the days we conducted our classroom observations. In each class, the teacher introduced the Building Blocks task on the first day of our observations. The teachers chose how to introduce the task and which instructions to give; the researchers did not intervene or influence either teacher's delivery. These introductions can be found in Appendix D. As the students began working on the task, the teacher visited each group. Sometimes she would listen to the

> I was constructing towers as you see below. I noticed that each time I made the tower higher, I had to add more blocks on the sides to stabilize the structure. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower?

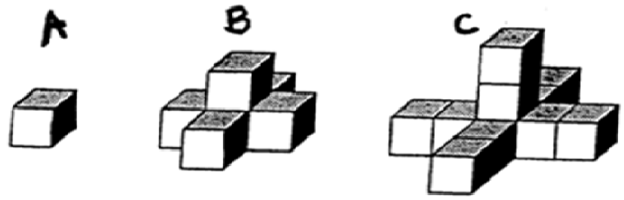


Figure 4-1: Building Blocks task

students' conversation, and other times she would ask questions to understand their strategies. In the last thirty minutes of class, the researchers administered the questionnaire to the students.

The tower labeled A is called the 1-block high tower and the total number of blocks is one. In the diagram labeled B, a total of six blocks are used because one block is added to the height and one block is added to each side of the base. This pattern can be identified recursively, since, given the value of the first term, each further term can be defined as a function of the preceding terms. A total of five blocks are added – one on each side and one to the height – to generate the next tower structure, suggesting that the pattern can be written as:

$$T_N = T_{N-1} + 5$$

where N represents the height of the tower, and T_N represents the total number of blocks in the structure. In fact, the diagrams A, B, and C represent the first three terms in the sequence, respectively: $T_1 = 1$, $T_2 = 6$, and $T_3 = 11$. Many students in this study had difficulty recognizing that the block in tower A was still present as the middle block in the subsequent towers. That is, some students counted only 5 blocks instead of 6 in the 2-block high tower, depicted in tower B.

The algebraic solution can be expressed as a linear closed-form equation as well. The height of each tower is given (e.g., height of 1, 2, 3, etc.). The total number of blocks on each of the four sides is the same, and is one less than the total number of blocks in the height. Seeing the construction of the towers in this way, the linear equation could be written as:

$$T_N = 4(N - 1) + N,$$

where N represents the height of the tower, and T_N represents the total number of blocks in the structure. Another way to view the total number of blocks is to note that each side plus the height has $N - 1$ blocks in addition to middle hidden block in the base. One may write this as:

$$T_N = 5(N - 1) + 1.$$

Both of these expressions can be simplified algebraically to:

$$T_N = 5N - 4.$$

The various ways to represent the problem and the solution allow students to choose different approaches and strategies as they work through the task to find the total number of blocks in the 5-block high tower, the 10-block high tower, and the 100-block high tower.

In this study, some students attempted to find shortcuts to the pattern that ultimately do not work. Many students tried some variation of a proportion, which does not apply to this equation. One can see that a proportion does not work with the first two iterations.

If the proposed proportional relationship is:

$$\frac{\text{Height of tower}}{\text{Total number of blocks}} = \text{constant},$$

then the first two iterations would be $\frac{1}{1}$ and $\frac{2}{6}$, which are not equal.

Despite recognizing this, some students tried to determine a proportional relationship with iterations after finding other correct answers. For example, many students correctly found that there are a total of 21 blocks in the 5-block high tower. In order to determine the total number of blocks in the 10-block high tower, some students attempted to double this total, since 10 is two times 5. That is, these students thought the relationship to be:

$$\frac{5}{21} = \frac{10}{42}$$

However, following the correct general equation, the 10-block high tower has 46 total blocks ($5\times10 - 4 = 46$) in the structure. Some of the events below demonstrate incorrect strategies such as this, as the two engagement structures are further explored.

4.2 Results of coding

As I analyzed the data, I started with an initial list of codes that had been developed prior to the first stage of coding. Upon viewing the video recordings of the students, along with the audio recordings and the transcripts, several new codes needed to be created to describe the data, leading to a revised final coding scheme. These codes label observed or inferred behaviors that provide evidence for one of the two engagement structures, and are summarized below in Tables 4-1 and 4-2.

Table 4-1
Codes for LMTY: Number of students (S) and instances of the code (C) within each class

	Ms. A's classes				**Ms. S's Classes**				**Total**	
	Class 1 (11)		Class 2 (20)		Class 1 (15)		Class 2 (9)		(55)	
Code/Action	S	C	S	C	S	C	S	C	S	C
1: Explains the instructions, procedure, or some characteristic of the problem or its solution	4	13	9	11	8	20	6	17	27	61
2: Modifies explanation	3	3	1	1	2	5	2	4	8	13
3: Provides additional details	1	3	0	0	6	10	4	5	11	18
4: Corrects a classmate	4	4	2	2	2	3	4	8	12	17
5: Restates or interprets a classmate's or teacher's strategy or solution.	1	1	0	0	1	1	1	1	3	3
6: Builds on another's idea	3	3	1	1	2	2	1	1	7	7
7: Responds to specific questions, with short answers	2	3	3	3	6	16	2	2	13	24
8: Questions classmate to help classmate obtain solution or clarify error	0	0	0	0	2	3	0	0	2	3
9: Offers to teach or help classmate explicitly	1	1	0	0	0	0	0	0	1	1
10: Asks if classmate understands	2	3	2	3	2	3	2	2	8	11

Table 4-1 (continued)

Code/Action	**Ms. A's classes** Class 1 (11)		Class 2 (20)		**Ms. S's Classes** Class 1 (15)		Class 2 (9)		**Total** (55)	
	S	C	S	C	S	C	S	C	S	C
11: Offers compassion or reassurance	0	0	0	0	0	0	0	0	0	0
Total number of students with LMTY codes (percentage of total number of students in class)	5 (45)		9 (45)		9 (60)		7 (78)		30 (55)	

*"Correcting a classmate" was a behavior that indicated either an active LMTY structure or an active LHSIA structure; See Table 3-5 for more details on differences

Table 4-2
Codes for LHSIA: Number of students (S) and instances of the code (C) within each class

	Ms. A's classes				**Ms. S's Classes**				**Total**	
	Class 1 (11)		Class 2 (20)		Class 1 (15)		Class 2 (9)		(55)	
Code/Action	S	C	S	C	S	C	S	C	S	C
1: Calls out answer	1	1	0	0	0	0	0	0	1	1
2: Talks over others	0	0	1	1	3	3	1	1	5	5
3: Controls math tools	0	0	2	2	0	0	2	2	4	4
4: Says "I got it!"	3	3	1	1	4	6	2	2	10	12
5: Asserts, "I will figure this out."	0	0	1	1	1	1	2	2	4	4
6: States, "This is easy."	2	2	1	1	2	4	2	4	7	12
7: Gives answer and does not listen to others' comments	1	1	2	3	0	0	2	3	5	7
8: Corrects a classmate*	2	3	4	6	2	3	3	6	11	18
9: Agrees with classmate	0	0	2	2	7	10	4	4	13	16
9A: Disagrees with classmate	0	0	5	5	2	2	2	7	9	14
10: Builds on classmate's ideas	4	5	1	1	4	6	3	3	12	15
11: Says, "I told you so."	0	0	0	0	1	1	3	4	4	5
12: Tries to show, "I can keep up with you."	3	4	3	3	4	11	3	5	13	23
13: Asks, "Got it?" of classmate	0	0	0	0	1	1	0	0	1	1
14: Demands attention to one's own ideas	6	6	7	15	7	11	5	9	25	41

Table 4-2 (continued)

Code/Action	Ms. A's classes				Ms. S's Classes				Total (55)	
	Class 1 (11)		Class 2 (20)		Class 1 (15)		Class 2 (9)			
	S	C	S	C	S	C	S	C	S	C
15: Looks for confirmation from classmate	0	0	5	8	3	7	5	7	13	22
16: Tries to "one up" a classmate	0	0	0	0	1	1	1	1	2	2
17: States that someone is smart	0	0	1	1	0	0	0	0	1	1
Total Number of students with LHSIA codes (percentage of total number of students in class)	8 (73)		13 (65)		12 (80)		8 (89)		41 (75)	

*"Correcting a classmate" was a behavior that indicated either an active LMTY structure or an active LHSIA structure; See Table 3-5 for more details on differences

The full results of this coding can be found in Appendix D. This appendix contains not only the teacher introductions to the task for each class, but also the transcripts of the episodes and questionnaire results for each observed group of students. In each episode, one or more students were inferred to have either an active LMTY or LHSIA engagement structure. The questionnaire results for those items which may indicate the activation of either the LMTY or LHSIA structure are given for each individual student in the group.

Tables 4-1 and 4-2 present the final codes for each of the two engagement structures in such a way as to note the number of students for whom each code was applied and the number of instances in which the code was applied for each of the four classes. An instance was coded for a student's speaking turn and behavior. The totals refer to the number of students and total number of instances where a code was applied, across all four classes. Also the total number of students who were assigned any of the codes for each of the two structures is given.

For each engagement structure, some coded behaviors were observed much more often than others. Some codes were developed in conjunction with a related research project (Sanchez-Leal, 2012). For example, for the LMTY structure (Table 4-1), the code "Offering compassion or reassurance" (no. 11) was not observed in any of the four classes in my study However, this code was hypothesized by the researcher to exist in a classroom setting as a teaching behavior. It is possible that students did offer one another compassion or reassurance and that I was not able to infer this from their behavior on those occasions. Perhaps students were unsure how to offer reassurance to a classmate or they did not find it necessary to do so. Similarly, only two students (from the same class) in this study displayed the behavior, “Questions classmate to help classmate obtain solution or clarify error” (no. 8). In each case, the student asked his classmate to restate the answer or explain the strategy to obtain the answer. The student then would follow up with an explanation as to why he thought the answer or strategy was incorrect. One reason why this behavior was not observed more often could be that students simply corrected a classmate they believed to be incorrect rather than trying overtly to understand the other’s strategy first.

Upon coding the data for possible evidence of the LHSIA structure (Table 4-2), I inferred the behavior “Demands attention to one’s own ideas” (no. 14) on 41 occasions and the behavior “Tries to show, ‘I can keep up with you,’” (no. 12) in 23 instances. Both coded behaviors indicate some aspect of wanting to appear competent or intelligent to one’s classmates. A difference between these two codes exists in that “Demands attention” suggests that it is important for others to recognize the student’s intelligence or ability. However, “I can keep up with you” indicates that one does not want to appear incompetent to one’s classmates, more so than wanting recognition for competence. These and all other coded behaviors were inferred based on the student’s actions, speech, tone of voice, and the context surrounding the behavior. For example, many students who were the first to speak or share an idea of how to approach the problem were inferred to “Demand attention,” (no. 14) including Leticia from Ms. S’s Class 1 and Juan from Ms. A’s Class 1. Nikki, in Ms’s Class 1 for example, had several behaviors that were coded as trying to keep up with her classmates (no. 12). In one such instance, she built on one of her classmate’s ideas, stating, “Because this is one, you add five, that is six. Add five, that’s 11. Add five, and so on and so on” (line 7). Nikki appeared to be demonstrating that she could follow her classmate's strategy, suggesting that she was able to "keep up" with the classmate mathematically.

Some codes which were applied only once include, "Calls out answer" (no. 1), "Asks, 'Got it?' of classmate" (no. 13), and "States that someone is smart" (no. 17). The latter two codes may have been too specific to be applied on multiple occasions, though it was difficult to attempt to broaden each of those to include similar behaviors. Each of these codes was included because the individual who made the statement was essentially comparing his ability or intelligence to that of a classmate. In a whole class setting, a student may be likely to "call out [an] answer" in order to garner recognition from others. The students in my study primarily worked in small groups, and therefore, my interpretation of this code did not often seem applicable in this environment.

Another consideration should be noted about the frequency of some of these codes. Many times, one student exhibited behavior leading to the assignment of a particular code several times while other students were assigned the same code only once or twice. One example occurred with the LMTY code, "Corrects a classmate" (no. 4). Tyesha, a student in Ms. S's Class 2, was inferred to have exhibited this behavior five times within the problem-solving session, whereas five other students in Ms. S's two classes appeared to have done so only once or twice. In Tyesha's case, she tried to explain her understanding of the construction of the towers to her classmates who often attempted to repeat Tyesha's explanations. Tyesha would disagree with their interpretation, and would then correct them along with modifying her explanation or providing additional details.

Such qualifications are noted to make clear that the frequency reports provide only a general indication of which behaviors were observed more often, and which less often.

The questionnaire responses were analyzed to compare the students' self-report data to my findings based on the video analysis. Each student's responses to specific items that may indicate either an active LMTY structure (total of 7 items) or an active LHSIA structure (total of 14 items) can be found in Appendix D. Tables 4-3 and 4-4 report the totals of responses for each item by class and for all participants. For example, the responses given in Ms. A's Class 1 to the item "I wanted to teach another student something that I knew that the other student did not know" were: 7 "All the time," 2 "Sometimes," and 1 "Never." Across all four classes, this item received the responses: 24 "All the time," 18 "Sometimes," and 12 "Never."

Table 4-3
Questionnaire results for items which may indicate LMTY structure

Item	Responses	**Ms. A's classes** Class 1[c] (11)	Class 2 (20)	**Ms. S's classes** Class 1 (15)	Class 2 (9)	**Total** (55)
Statements (All the time, Sometimes, Never)						
I wanted to teach another student something that I knew that the other student did not know.	All the time	7	7	7	3	24
	Sometimes	2	5	6	5	18
	Never	1	8	2	1	12
I listened carefully to the ideas of someone I was trying to help.	All the time	8	8	12	5	33
	Sometimes	2	10	3	3	18
	Never	0	2	0	1	3
I helped someone see how to do the math.	All the time	2	4	6	1	13
	Sometimes	7	10	6	6	29
	Never	1	6	3	2	12
Others listened carefully to my ideas.	All the time	3	9	12	4	28
	Sometimes	7	9	3	5	24
	Never	0	2	0	0	2
Behaviors (Often, Sometimes, Hardly ever)						
I gave helpful suggestions.	Often	6	8[b]	12	3	29
	Sometimes	4	9	3	6	22
	Hardly Ever	0	2	0	0	2
I worked cooperatively.	Often	6[a]	12[a]	13	7	38
	Sometimes	3	5	2	2	12
	Hardly Ever	0	2	0	0	2
Thoughts (Yes/No)						
I like teaching this person things that I know.	Yes	9	12	12	9	42
	No	1	8	3	0	12
Total number of students whose responses may indicate active LMTY structure (percentage of total number of students in class)		10[c] (100)	17 (85)	15 (100)	9 (100)	51 (90)

[a] Student(s) gave No Answer to this item.
[b] One student in this class circled both "Often" and "Sometimes"
[c] Questionnaire responses for one student are not available

Table 4-4
Questionnaire results for items which may indicate LHSIA structure

Item	**Responses**	**Ms. A's classes** Class 1[c] (11)	Class 2 (20)	**Ms. S's classes** Class 1 (15)	Class 2 (9)	**Total** (55)
Statements (All the time, Sometimes, Never)						
I wanted people to think that I'm smart.	All the time	3	7	9	6	25
	Sometimes	4	8	6	3	21
	Never	3	5	0	0	8
I tried to impress people with my ideas about the problem	All the time	2	5	6	5	18
	Sometimes	5	6	8	4	23
	Never	3	9	1	1	13
People seemed impressed with the ideas I shared about the problem.	All the time	2	6	6	1	15
	Sometimes	7	9	8	7	31
	Never	1	5	1	1	8
People saw how good I am at the math we did today.	All the time	3	9	4	3	19
	Sometimes	7	9	9	6	31
	Never	0	2	2	0	4
I felt smart.	All the time	6	12	10	5	33
	Sometimes	4	6	4	3	17
	Never	0	2	1	1	4
I wanted to show someone that my way was better.	All the time	0	2	4	2	8
	Sometimes	6	5	4	3	18
	Never	4	13	7	4	28
I was a lot better at math than others today.	All the time	1	3[a]	3	3	10
	Sometimes	6	7	10	6	29
	Never	3	9	2	0	14

Table 4-4 (continued)

		Ms. A's classes		**Ms. S's classes**		
Item	**Responses**	Class 1[c] (11)	Class 2 (20)	Class 1 (15)	Class 2 (9)	**Total** (55)
Behaviors (Often, Sometimes, Hardly ever)						
I was the leader.	Often	0	7	1	2	9
	Sometimes	6	8	12	6	31
	Hardly Ever	4	2	2	1	14
I was bossy.	Often	1	0[a, a]	0	0	1
	Sometimes	1	2	0	1	4
	Hardly Ever	8	16	15	8	47
I wanted to show off.	Often	0	0[a]	0	1	1
	Sometimes	2	3	1	1	7
	Hardly Ever	8	16	14	7	45
I liked to be right.	Often	6[a]	7[a, a, a]	11	2	26
	Sometimes	3	8	4	7	22
	Hardly Ever	0	2	0	0	2
Thoughts (Yes/No)						
I want you to know just how smart I am.	Yes	7	13	9	9	38
	No	3	7	6	0	16
People think I'm smart.	Yes	5[a, a]	15	8	5	33
	No	3	5	7	4	19
I wish the teacher would call on me, so I can show how much I know.	Yes	5	7[b]	4	7	23
	No	5	12	11	2	30
Total number of students whose responses may indicate active LMTY structure (percentage of total number of students in class)		9[c] (90)	15 (75)	15 (100)	9 (100)	48 (89)

[a] Student(s) gave No Answer to this item.
[b] Student wrote in response "Sometimes"
[c] All questionnaire responses for 1 student are omitted

I analyzed the questionnaire results to determine if a student's responses indicated a possible presence of either the LMTY or LHSIA structure at one point or another, using the items which may suggest an active structure shown in these tables. For each student, I compared the number of positive responses (i.e., All the time, Often, Sometimes, Yes) to the number of negative responses (i.e., Never, Hardly ever, No). Also, the specific items were taken into consideration to determine if a student may have activated a particular engagement structure. For example, Rico (in Ms. A's Class 2) responded Sometimes to, "I listened carefully to the ideas of someone I was trying to help," and "Others listened carefully to my ideas." He also responded, "Never/No" to the items, "I wanted to teach another student something that I knew that the other student did not know," and "I like teaching this person things that I know." I interpreted these results to indicate that Rico may believe he listened to his classmates but he either did not the motivating desire or perhaps the opportunity to teach or help his group.

Similarly, I interpreted Janelle's (in Ms. A's Class 2) questionnaire responses to indicate she did not have an active LHSIA structure, though I did code three of her speaking turns as suggesting an active LHSIA structure. Though Janelle responded, "Sometimes/Yes" to four items including, "I wanted people to think I'm smart," and "I liked to be right," she responded "Never/Hardly ever/No" to the other ten items, including, "I tried to impress people with my ideas about the problem," and "I was a lot better at math than others today." After reviewing each individual's questionnaire results I determined that 51 of the 54 responding participants indicated they had an active LMTY structure or had a motivating desire to help classmates. Out of the 54 students, 48 indicated they had an active LHSIA structure or had the motivating desire to appear smart to others.

For example, Manuel from Ms. A's Class 1, gave mixed responses to items which may indicate an active LHSIA structure. He indicated "All the time/Sometimes" to 5 items, including, "I wanted people to think that I'm smart," "People seemed impressed with the ideas I shared about the problem," and "I liked to be right." However, he also indicated "Never/No" to 5 other items, such as, "I tried to impress people with my ideas about the problem," and "I want you to know just how smart I am." (The items "I was bossy" and "I wanted to show off" were not considered for reasons I discuss shortly.) Using the coding scheme described in Table 4-2, I inferred that Manuel may have had an active LHSIA structure on five occasions during the problem-solving session. Upon carefully reviewing those occasions, I inferred that Manuel wanted to contribute to the group's progress and solution but perhaps he did not need to be seen as "better" than his classmates at math.

In contrast, there were students like Pedro from Ms. A's Class 2 who responded affirmatively to all of the items which may indicate an active LMTY structure. Of the seven codes applied to the behaviors of Pedro and his two classmates, four were coded for the presence of the LMTY structure. This suggests that for Pedro and similar students there was agreement between the questionnaire results and my video analysis.

Two questionnaire items did not appear to correlate with the LHSIA structure: "I was bossy" and "I wanted to show off." Only 5 of the 52 students who gave responses answered, "All the time" or "Sometimes" to the statement, "I was bossy," and 8 of the 53 responding students gave these positive answers to "I wanted to show off." There are several potential reasons for these response rates. It is possible these statements were not interpreted by the students as indications of appearing smart, intelligent, or competent, or that the students who may have believed they were bossy or trying to show off did not want to admit these behaviors. It is likely that students did not associate being "bossy" or "showing off" as characteristics of being a leader, since 40 students replied "All the time" or "Sometimes" to, "I was the leader." If this is the case, students in middle school may associate being the leader as asserting authority, but being bossy may be perceived as being overbearing or inflexible (Kayworth & Leidner, 2002).

For 22 of the 54 students for whom I have questionnaire responses, I found that though their answers indicated either the activation of the LMTY structure or a potential motivating desire to help a classmate, I did not observe behavioral evidence of this structure. Additionally, for 10 of the same 54 students, I found that though their answers indicated the activation of the LHSIA structure or a potential desire to demonstrate their ability or knowledge, I did not observe behavioral evidence of an active LHSIA structure. These discrepancies are discussed in more detail below via the examples given below in events three, five, six, seven, and eight.

4.3 Behavioral features of the engagement structures

As I analyzed the data, it became clear that some codes indicated certain recurring behaviors, while other codes were infrequently applied to my student population. This may have occurred because the behaviors were difficult to observe (e.g., "Tries to 'one up' a classmate" no. 16) or because the particular classroom environments I observed did not provide opportunities for a behavior (e.g., "Calls out answer" no. 1). Upon reviewing the coded analysis, certain more global *features* of the behaviors associated with each engagement structure emerged as recurring characteristics throughout the episodes. These features are broader generalizations than the codes described above, and do not necessarily correspond to specific subsets of codes. The students for whom I inferred an active LMTY structure appeared to be teaching or explaining something to a classmate in such a way that similar behaviors were observed for each of the groups in all of the classes. The features of these behaviors are: 1) teaching to clarify, 2) teaching procedures, 3) teaching or explaining strategies, and 4) checking for understanding. Details of each feature are given below. The students for whom I inferred an active LHSIA structure appeared to try to demonstrate their mathematical ability or intelligence to others in the class with similar behaviors that were observed across the groups and classes. These features of these behaviors are: 1) expressing an idea, 2) stating an answer or answers, 3) correcting others, 4) stating, "I'm smart," or "We're smart," 5) keeping up with others in the group.

4.3.1 Let Me Teach You (LMTY) features

The following four features of the behaviors associated with the LMTY engagement structure were identified from the data as different behaviors of students who attempted to teach or explain some concept to a classmate: 1) teaching to clarify, 2) teaching procedures, 3) teaching or explaining strategies, 4) checking for understanding. While the engagement structures have multiple facets (e.g., goals or motivating desires, patterns of behavior, expressions of affect, and problem-solving strategies and heuristics), students who exhibit an active LMTY structure all appear to have one broad goal or motivating desire: to help a classmate understand some aspect of the problem or solution. These features are one way to differentiate the behaviors that all seem to be in service of the same goal. Here I describe in more detail what is meant by each of these features for the LMTY engagement structure.

1) Teaching to *clarify*

Many of the students in these four classes attempted to make some aspect of the problem or a strategy more clear to one or more classmates. In some cases, a student responded to a specific question, such as, “What do you mean?” The student for whom the LMTY structure was activated responded appropriately, providing the details of the construction of the towers, or explaining why a 2-block high tower was called such. Additionally, some students used one or more tools, usually the given diagrams or physical blocks, to explain a strategy that a classmate did not understand. Typically, when a student teaches or explains something with the intention of clarifying a point that had already been made, the student wants his or her classmate to understand that strategy or idea.

2) Teaching *procedures*

The students in this study sometimes helped their classmates with a procedural task, such as drawing a representation of the towers or setting up a table or chart to organize information. In doing this, they were teaching something that may be important or useful even though it may not require conceptual understanding. In the context of these classes, a task such as describing how the towers were constructed may appear to be procedural but for these students a deeper understanding was required.

3) Teaching or explaining *strategies*

Students taught or explained their strategies in various ways to their classmates. In the context of the Building Blocks task, the strategies shared included identifying the height of the tower, counting the total number of blocks in each tower, recognizing patterns, and developing equations. Some students attempted to explain a strategy in multiple ways, particularly if they persisted in explaining the approach to someone who was having difficulty understanding. In addition, several incorrect strategies were shared, some of which are included in the events below.

4) Checking for *understanding*

The students for whom I inferred the LMTY structure was active often checked with a classmate to see if he or she understood the idea or strategy being shared. For example, asking a classmate, "Do you understand?" was sometimes followed by an explanation of a strategy, a behavior often associated with an active structure. In other cases, the student asked, "Do you get it?" after explaining an idea. Some classmates stated that they did understand the idea, while others admitted when they were still confused about some aspect of the problem or its solution.

4.3.2 Look How Smart I Am (LHSIA) features

Five features of the LHSIA engagement structure were identified as different behaviors of students who wanted others to perceive them as smart or having mathematical ability: 1) expressing an idea, 2) stating an answer or answers, 3) correcting others, 4) stating, "We're smart" or "I'm smart, and 5) keeping up with other(s) in the group. The students for whom I inferred an active LHSIA structure displayed behaviors that appeared to be associated with a motivating desire to have others (e.g., classmates, teacher) acknowledge their intelligence or mathematics skills. The affect of these students varies, as some individuals are defensive of their ideas while others are friendly toward their classmates. These features, described in more detail, demonstrate the variety of behaviors of students who may have activated the LHSIA engagement structure.

1) Expressing an idea

Several students tried to impress their classmates or demonstrate their ability by expressing an idea about the problem or the solution. Some of these ideas were new while others built on the ideas that others had already stated. Some of these ideas were strategies about constructing the towers or counting the blocks in the tower. Other students described a pattern that they recognized, such as the increase of the total number of blocks by 5. The researcher inferred an active LHSIA structure for these students because they appeared to be motivated by a desire to appear smart, rather than the desire to help their classmates understand something.

2) Stating an answer or answers

Several students stated an answer that they had reached. Some answers were responses to specific questions, such as, "What did you get for the 5-block high tower?" while other answers were spontaneous announcements, for instance, "21, 26, 31, 36, 41, 46…" These answers ranged from giving brief answers without clarification, (e.g., "Eleven") to more elaborate explanations, including, "I found a pattern. One plus 5 equals 6, 6 plus 5 equals 11…" By giving these answers, the students were inferred to be motivated to let their classmates know how smart they were. These inferences were made based the student's behavior, often described either by the student's tone of voice which may have been impatient, brusque, or boastful, or by the lack of a follow-up explanation to the given answer. For at least one group, this feature was identified alongside another, Correcting others (feature no. 3), because the student stated his own answer as a way to correct his classmate.

3) Correcting others

One common way for students to attempt to assert their mathematical ideas or ability is to correct a classmate. Sometimes the student would disagree with the classmate's answer, for example, of the total number of blocks in the 100-block high tower. Other instances involve a student correcting a classmate about what the task is really asking them to do. Some of these situations indicate a disagreement between the students. Another path has been for the student to branch from an active LHSIA structure to an active LMTY structure when the student explains his or her answer or strategy.

4) Stating, "We're Smart" or "I'm Smart"

Some students did state out loud, "We're smart." This often followed determining an answer or recognizing that a strategy was useful. Also, at least one group of students made this statement along with a statement indicating they believed they were the "first group finished" with the task, possibly equating speed of completion with intelligence. Some students may have stated, "We're smart," to take partial credit for what the whole group has done, regardless of their level of contribution. Alternatively, a student who was instrumental in determining an answer or a successful strategy may be giving the rest of the group credit along with him or her by stating, "We're smart."

In contrast to students who wanted to give credit to the entire group, other students made statements such as, "I'm smart," "I get it," or, "I'll figure this out." These statements suggest that these students may have wanted individual recognition for their own contribution to the solution. For example, a student who claimed that the task was easy may be stating this out loud so that his classmates perceive him as a ‘smart student.’ A student who stated, “I’ll figure this out” may be motivated to indicate her own confidence in her own ability to arrive at a solution, with or without the assistance of her classmates.

5) Keeping up with other(s) in the group

Much of the LHSIA engagement structure is about how the student is perceived by others, particularly his or her classmates. In order to be perceived as smart or intelligent by his or her classmates, a student may make statements indicating that he or she is trying to keep up with the others, mathematically. For example, while brainstorming solution strategies, the student may express an idea or agree with the idea of a classmate. A student may also ask for confirmation of a strategy or answer from a classmate, with the tone of voice suggesting the question, “I have the correct answer, don’t I?” The student who activates a LHSIA structure with the intention of keeping up with his or her classmates may not necessarily have confidence in his or her own mathematical abilities, as we will see with the questionnaire responses.

Tables 4-5, 4-6, 4-7 and 4-8 present a summary of the features by episode for each group of students. Each table contains the summary for one class. For example, table 4-5 displays the information for Ms. A’s Class 1 which includes three groups of students. The transcript of the first group, Abby, Julian, and Samara, contains five distinct episodes of different durations. In the first episode, I observed behaviors of Abby and Julian which exhibited the following features: teaching to clarify (LMTY), teaching and explaining a strategy (LMTY), and expressing an idea (LHSIA). These features did not necessarily appear in this order and they may have been inferred from several behaviors or just one action or verbal utterance. Note that Samara did not display any behaviors which led me to infer one of the behavioral features. This corresponds with a lack of behaviors suggesting the presence of an active LMTY or LHSIA structure. The table continues to display information about a second group of students in this class: Juan, Amanda, Eliot, and Manuel. For this group, there were six episodes for which I inferred one of the two engagement structures. The remainder of this and the corresponding tables continue to give similar information for all the groups of participating students.

Across the 4 classes and across 89 episodes, the four features of the behaviors associated with the LMTY engagement structure occurred as follows: 1) teaching to clarify (38 times), 2) teaching procedures (31 times), 3) teaching or explaining strategies (6 times), 4) checking for understanding (11 times). Similarly, the five features of the behaviors associated with the LHSIA structure occurred in this way: 1) expressing an idea (30 times), 2) stating an answer or answers (20 times), 3) correcting others (21 times), 4) stating, “We’re smart” or “I’m smart” (27 times), and 5) keeping up with other(s) in the group (25 times). These frequencies confirm that these features do recur throughout the problem-solving session for different students in different classes.

Table 4-5: Features within each episode for Ms. A's Seventh Grade Class 1		
1 Abby, Julian, Samara - 5 episodes		
Episode 1 (3 minutes)	LMTY	Teaching to clarify (Abby, Julian) Teaching or explaining a strategy (Abby)
	LHSIA	Expressing an idea (Abby)
Episode 2 (3 minutes)	LMTY	Teaching to clarify (Abby, Julian) Teaching a procedure (Abby) Teaching or explaining a strategy (Abby)
Episode 3 (2 minutes)	LMTY	Teaching a procedure (Julian, Abby)
	LHSIA	Expressing an idea (Julian) Correcting others (Abby)
Episode 4 (2 minutes)	LMTY	Teaching to clarify (Abby) Teaching or explaining a strategy (Abby)
	LHSIA	Stating, "I'm smart" or "We're smart" (Abby)
Episode 5 (less than 1 minute)	LHSIA	Correcting others (Abby)
2 Juan, Amanda, Eliot, Manuel - 6 episodes		
Episode 1 (2 minutes)	LMTY	Teaching to clarify (Amanda)
	LHSIA	Expressing an idea (Manuel) Stating an answer (Juan) Keeping up with others (Eliot)
Episode 2 (3 minutes)	LMTY	Teaching to clarify (Amanda)
	LHSIA	Expressing an idea (Manuel)
Episode 3 (1 minute)	LMTY	Teaching to clarify (Manuel)
Episode 4 (2 minutes)	LMTY	Teaching a strategy (Manuel)
	LHSIA	Stating an answer (Manuel, Amanda)
Episode 5 (4 minutes)	LMTY	Teaching or explaining strategies (Manuel) Checking for understanding (Manuel)
	LHSIA	Stating "I'm smart," or "We're smart" (Amanda, Manuel) Keeping up with others (Amanda)
Episode 6 (less than 1 minute)	LMTY	Teaching or explaining strategies (Amanda)
3 Liza, Bridget, Jenna, James – 3 episodes		
Episode 1 (less than 1 minute)	LMTY	Teaching to clarify (Liza)
Episode 2 (less than 1 minute)	LHSIA	Expressing an idea (Jenna) Keeping up with others (Bridget)
Episode 3 (less than 1 minute)	LHSIA	Stating "I'm Smart" or "We're Smart" (Bridget)

Table 4-6: Features within each episode for Ms. A's Seventh Grade Class 2		
1 Talia, Jaden, Genevieve, Andrew – 2 episodes		
Episode 1 (4 minutes)	LMTY	Teaching to clarify (Jaden) Teaching or explaining a strategy (Genevieve, Jaden)
	LHSIA	Expressing an idea (Jaden) Stating an answer or answers (Talia)
Episode 2 (3 minutes)	LHSIA	Expressing an idea (Genevieve) Stating an answer or answers (Talia) Correcting others (Talia) Keeping up with others (Jaden)
2 Pedro, Kian, Rico – 3 episodes		
Episode 1 (3 minutes)	LMTY	Teaching to clarify (Pedro)
	LHSIA	Expressing an idea (Pedro, Kian)
Episode 2 (3 minutes)	LMTY	Checking for understanding (Pedro)
Episode 3 (4 minutes)	LHSIA	Expressing an idea (Kian, Pedro) Keeping up with others (Kian)
3 Onan, Raina, Joe – 6 episodes		
Episode 1 (3 minutes)	LMTY	Teaching or explaining a strategy (Onan)
	LHSIA	Expressing an idea (Onan)
Episode 2 (2 minutes)	LHSIA	Stating an answer or answers (Onan) Keeping up with others (Onan)
Episode 3 (2 minutes)	LHSIA	Expressing an idea (Onan) Stating an answer or answers (Onan) Stating, "I'm smart" or "We're smart" (Raina)
Episode 4 (1 minute)	LHSIA	Stating an answer or answers (Onan) Correcting others (Onan)
Episode 5 (1 minute)	LMTY	Teaching a procedure (Onan)
Episode 6 (less than 1 minute)	LHSIA	Stating, "I'm smart" or "We're smart" (Onan, Raina)
4 Danica, Shanika, Shannen – 4 episodes		
Episode 1 (less than 1 minute)	LMTY	Teaching to clarify (Danica)
Episode 2 (1 minute)	LMTY	Teaching to clarify (Shannen, Danica)
	LHSIA	Stating an answer or answers (Danica) Correcting others (Shannen) Keeping up with others (Danica)
Episode 3 (1 minute)	LMTY	Explaining a strategy (Shannen) Checking for understanding (Danica)
	LHSIA	Expressing an idea (Shannen)
Episode 4 (2 minutes)	LMTY	Teaching to clarify (Danica) Checking for understanding (Danica)
	LHSIA	Stating, "We're Smart" or "I'm Smart" (Danica, Shannen)

Table 4-6 (continued)		
5 Wilson, Lewis, Mitchell, Martin – 4 episodes		
Episode 1 (less than 1 minute)	LHSIA	Stating, "I'm smart," or, "We're smart." (Mitchell)
Episode 2 (1 minute)	LMTY LHSIA	Teaching or explaining a strategy (Mitchell) Keeping up with others (Lewis)
Episode 3 (3 minutes)	LHSIA	Expressing an idea (Mitchell) Stating, "I'm smart" or "We're smart" (Mitchell) Correcting others (Lewis)
Episode 4 (less than 1 minute)	LHSIA	Stating, "I'm smart" or "We're smart" (Lewis) Keeping up with others (Mitchell)
6 Larry, Janelle, Ellen – 6 episodes		
Episode 1 (1 minute)	LMTY LHSIA	Teaching to clarify (Ellen) Teaching or explaining a strategy (Larry) Expressing an idea (Larry) Correcting others (Ellen)
Episode 2 (2 minutes)	LMTY LHSIA	Teaching or explaining a strategy (Larry) Expressing an idea (Larry) Correcting others (Larry) Stating, "We're smart," or, "I'm smart" (Larry)
Episode 3 (2 minutes)	LMTY LHSIA	Teaching to clarify (Larry) Correcting others (Larry. Janelle)
Episode 4 (1 minute)	LHSIA	Correcting others (Janelle) Stating an answer or answers (Janelle)
Episode 5 (less than 1 minute)	LHSIA	Stating an answer or answers (Ellen)
Episode 6 (4 minutes)	LMTY LHSIA	Teaching or explaining a strategy (Ellen) Stating an answer or answers (Ellen) Correcting others (Larry)

Table 4-7: Features within each episode for Ms. S's Eighth Grade Class 1		
1 Manny, Damon, Deanna – 5 episodes		
Episode 1 (less than 1 minute)	LHSIA	Expressing an idea (Damon) Keeping up with others (Damon)
Episode 2 (less than 1 minute)	LMTY	Teaching to clarify (Manny) Teaching or explaining strategies (Manny)
	LHSIA	Stating an answer or answers (Damon) Keeping up with others (Damon)
Episode 3 (1 minute)	LMTY	Teaching to clarify (Manny) Teaching or explaining strategies (Damon, Manny)
	LHSIA	Expressing an idea (Damon) Keeping up with others (Damon)
Episode 4 (2 minutes)	LMTY	Teaching to clarify (Manny)
	LHSIA	Stating, "I'm smart" or "We're smart (Deanna) Keeping up with others (Damon)
Episode 5 (3 minutes)	LMTY	Teaching to clarify (Manny) Teaching procedures (Manny)
	LHSIA	Keeping up with others (Damon)
2 Ricardo, Carol, Nikki – 6 episodes		
Episode 1 (1 minute)	LMTY	Teaching to clarify (Carl)
	LHSIA	Expressing an idea (Nikki) Stating an answer (Carl) Correcting others (Carl) Stating, "I'm smart," or "We're smart"
Episode 2 (2 minutes)	LMTY	Teaching to clarify (Carl)
	LHSIA	Expressing an idea (Nikki) Correcting others (Nikki) Stating, "I'm smart" or "We're smart" (Nikki, Carl)
Episode 3 (3 minutes)	LMTY	Teaching to clarify (Carl) Checking for understanding (Carl)
	LHSIA	Stating "I'm smart" or "We're smart" (Nikki) Keeping up with others (Nikki)
Episode 4 (less than 1 minute)	LHSIA	Stating, "I'm smart" or "We're smart" (Carl)
Episode 5 (1 minute)	LHSIA	Keeping up with others (Nikki, Ricardo)
Episode 6 (4 minutes)	LHSIA	Expressing an idea (Nikki) Correcting others (Carl) Stating, "I'm smart," or "We're smart" (Carl)

Table 4-7 (continued)		
3 Carly, Christina, Christian - 7 episodes		
Episode 1 (less than 1 minute)	LMTY LHSIA	Teaching to clarify (Christian)\ Stating “We’re smart” or “I’m smart” (Christian)
Episode 2 (1 minute)	LMTY LHSIA	Teaching or explaining strategies (Christian, Carly) Expressing an idea (Carly)
Episode 3 (2 minutes)	LMTY	Teaching or explaining strategies (Carly)
Episode 4 (less than 1 minute)	LMTY	Teaching or explaining a strategy (Carly) Checking for understanding (Carly)
Episode 5 (less than 1 minute)	LMTY LHSIA	Teaching or explaining a strategy (Christian) Stating an answer or answers (Christian) Correcting others (Christian)
Episode 6 (3 minutes)	LMTY LHSIA	Teaching to clarify (Christian) Stating an answer or answers (Christian) Correcting others (Christian)
Episode 7 (less than 1 minute)	LHSIA	Stating “We’re smart” or “I’m smart” (Christian)
4 Leticia, Sherelyn, Monique – 8 episodes		
Episode 1 (3 minutes)	LMTY LHSIA	Teaching or explaining a strategy (Monique) Expressing an idea (Leticia, Monique) Keeping up with others (Sherelyn, Leticia)
Episode 2 (less than 1 minute)	LHSIA	Expressing an idea (Monique) Keeping up with others (Leticia)
Episode 3 (2 minutes)	LMTY	Teaching to clarify (Monique)
Episode 4 (1 minute)	LMTY LHSIA	Teaching to clarify (Monique) Keeping up with others (Leticia)
Episode 5 (2 minutes)	LHSIA	Expressing an idea (Sherelyn)
Episode 6 (1 minute)	LMTY	Teaching to clarify (Leticia)
Episode 7 (2 minutes)	LMTY LHSIA	Teaching to clarify (Monique) Teaching a procedure (Monique) Keeping up with others (Monique)
Episode 8 (less than 1 minute)	LMTY LHSIA	Teaching or explaining a strategy (Leticia) Expressing a idea (Leticia)

Table 4-7 (continued)		
5 Kevon, Georgia, Nadira – 5 episodes		
Episode 1 (1 minute)	LMTY	Checking for understanding (Georgia) Teaching to clarify (Georgia)
	LHSIA	Expressing an idea (Georgia)
Episode 2 (1 minute)	LMTY	Teaching to clarify (Georgia) Teaching or explaining a strategy (Georgia)
Episode 3 (3 minutes)	LMTY	Teaching or explaining a strategy (Kevon)
	LHSIA	Stating, “I’m smart” or “We’re smart” (Nadira) Expressing an idea (Nadira) Keeping up with others (Nadira)
Episode 4 (4 minutes)	LMTY	Teaching to clarify (Georgia) Teaching or explaining a strategy (Georgia)
	LHSIA	Stating, “I’m smart” or “We’re smart” (Nadira)
Episode 5 (less than 1 minute)	LMTY	Teaching a procedure (Kevon)

Table 4-8: Features within each episode for Ms. S's Eighth Grade Class 2		
1 Leo, Ordena, Ta'keisha - 10 episodes		
Episode 1 (2 minutes)	LMTY	Teaching or explaining a strategy (Leo) Checking for understanding (Leo)
	LHSIA	Stating an answer (Leo)
Episode 2 (2 minutes)	LHSIA	Expressing an idea (Leo)
Episode 3 (1 minute)	LHSIA	States an answer (Leo)
Episode 4 (1 minute)	LHSIA	Expressing an idea (Ta'keisha) States an answer (Ta'keisha) Correcting others (Ta'keisha)
Episode 5 (4 minutes)	LHSIA	Expressing an idea (Leo) Correcting others (Ta'keisha, Leo)
Episode 6 (1 minute)	LHSIA	Stating "We're smart," or "I'm smart" (Ordena, Ta'keisha, Leo)
Episode 7 (1 minute)	LMTY	Teaching to clarify (Leo)
Episode 8 (2 minutes)	LMTY	Teaching to clarify (Ta'keisha)
	LHSIA	Expressing an idea (Ta'keisha) States an answer (Ta'keisha, Leo) Correcting others (Ta'keisha, Ordena) Keep up with other(s) in group (Ordena)
Episode 9 (3 minutes)	LMTY	Teaching or explaining a strategy (Ordena)
	LHSIA	Stating an answer (Leo) Correcting others (Ordena, Ta'keisha)
Episode 10 (3 minutes)	LMTY	Teaching or explaining a strategy (Ta'keisha)
	LHSIA	Stating "We're smart" or "I'm smart" (Ta'keisha, Ordena)
2 Nazira, Aleana, Keshia – 5 episodes		
Episode 1 (1 minute)	LMTY	Teaching to clarify (Aleana)
	LHSIA	Expressing an idea (Aleana)
Episode 2 (3 minutes)	LMTY	Teaching to clarify (Nazira) Checking for understanding (Nazira)
Episode 3 (2 minutes)	LMTY	Teaching or explaining a strategy (Keshia, Aleana)
	LHSIA	Stating "We're smart" or "I'm smart" (Nazira) Keeping up with others (Nazira)
Episode 4 (2 minutes)	LMTY	Teaching to clarify (Aleana) Teaching or explaining a strategy (Aleana)
	LHSIA	Stating an answer or answers (Nazira)
Episode 5 (1 minute)	LHSIA	Correcting others (Aleana)

Table 4-8 (continued)		
3 Carla, Tyesha, Ta'Shawna – 4 episodes		
Episode 1 (9 minutes)	LMTY	Teaching to clarify (Tyesha)
		Teaching or explaining a strategy (Tyesha)
	LHSIA	Expressing an idea (Carla)
		Stating "We're smart" or "I'm smart" (Carla)
		Keeping up with others (Carla, Ta'Shawna)
Episode 2 (11 minutes)	LMTY	Teaching to clarify (Tyesha)
		Teaching or explaining a strategy (Tyesha)
		Checking for understanding (Tyesha)
	LHSIA	Expressing an idea (Carla)
		Stating "We're smart" or "I'm smart" (Carla)
		Keeping up with others (Carla, Ta'Shawna)
Episode 3 (1 minute)	LMTY	Teaching to clarify (Tyesha)
	LHSIA	Expressing an idea (Tyesha)
Episode 4 (2 minutes)	LHSIA	Expressing an idea (Tyesha)
		Correcting others (Tyesha)

4.4 Classroom events

The nine events below are illustrative of the two engagement structures LMTY and LHSIA. They were selected to depict the features of each structure as well as many of the codes used to infer these engagement structures (see Tables 4-1 and 4-2). The first event was specifically selected to illustrate the LMTY structure for one student. The second event is presented because it exemplifies how a student may have an active LHSIA structure throughout the entire problem solving session. The events that follow are less straightforward. In events three and four, two students are inferred to have active LMTY structures, while event five depicts a student for whom an active LHSIA structure was inferred even though the questionnaire results do not necessarily support this finding. Event six portrays two students who have different structures within a short period of time. A unique excerpt in event seven portrays a student, inferred to have an active LMTY structure, interpreting her teacher's explanation while the teacher is an active member of the group. None of the other events and episodes reported in this study include moments when the teacher is present, as my focus is on student interactions. The last two events, eight and nine, provide potential evidence of branching from one engagement structure to the other. While these events focus on LMTY and LHSIA, other engagement structures may be, and likely are, active for one or more students.

As noted in the literature review, much work has been done to study productive, or in some cases, unproductive groups as mathematics students work cooperatively or collaboratively. While that research focused on student learning, this analysis specifically focuses on the activation of these two engagement structures for these students. Student learning is always an ultimate goal, but here I do not categorize episodes or events as sharing correct or incorrect information, but rather focus on the in-the-moment motivating desire and engagement of the students.

As I illustrate in the classroom events below, the features of the behavior associated with one of these two engagement structures are not mutually exclusive. For example, a student who has an active LMTY structure may be teaching or explaining a strategy to a classmate, but may also be clarifying the strategy if it has been discussed already. A student who has activated the LHSIA structure may be stating an answer and correcting a classmate at the same time, as well. These features do recur throughout the analyzed transcripts, as indicated in Table 4-5.

Event 1

In this first event we meet Tyesha, Carla, and Ta'Shawna who are all girls in Ms. S's Class 2. I infer that Tyesha has an active LMTY structure, which she appears to maintain throughout this event because her classmates keep asking questions about her strategy. The features *teaching to clarify, teaching or explaining strategies,* and *checking for understanding* are all demonstrated in this event. At the beginning of the problem-solving session, Tyesha starts to explain to Carla and Ta'Shawna how she visualizes the 5-block high tower, recognizing that there should be 5 blocks for the height as well as for each side. She interprets this as the 5-block high tower having a total of 25 blocks, not yet realizing that the middle block is the base or starting point for each of the legs as well as the height. Her classmates, particularly Carla, ask her how she determined there were 25 blocks. Tyesha then tries to explain her counting strategy using the given diagrams. In the first moments of this event, she attempts to explain how the 5-block high tower was constructed, building off the existing 3-block high tower in figure C. (Line numbers refer to the annotated transcript in Appendix D.)

21 Tyesha: If we got 2, 1 each. Two, right? That and that, two come out, one each. Two, right? (likely referring to the 2-block high tower, figure B)
So that's 1, 2, 3, 4, 5, 6, 7, 8, 9, right? Then you got two coming out, then you got this one right here and these two right here, right? So you got 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, right? (referring to the 3-block high tower)
Then put this already, 3, add 2 more, boom. This is already 2, 3 more. So that makes it 5, right? This, 3 more, this, 3 more, right?
Now it's a 5-block high tower. (referring to her drawing)

22 Carla: So count the **whole** thing? (emphasizes the word "whole")

23 Tyesha: (counting the blocks, pointing with her pencil as she goes along)
One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty, twenty-one, twenty-two, twenty-three, twenty-four, twenty-five. (says "twenty-five" with satisfaction, possibly implying the question, "Do you see that?" to Carla and Ta'Shawna)
Five, ten, fifteen, twenty, twenty-five
(counting again, pointing to her drawing on her paper)
(Ta'Shawna uses her pen to point to Tyesha's paper while Tyesha is counting)

24 Carla: Oh, now I get it. (brief pause)

25 Carla: So you're trying to say, that, like, add, like five blocks to each, like, (pauses while pointing to the diagram from the task on Tyesha's paper, trying to understand Tyesha's explanation) like, this one.

26 Tyesha: (pointing to the diagrams on her paper using her pencil)
Yeah. No.
Each, like, say, each set. Say if we call this set one, call this set two, this set three, this set four, and this right here, set five.
(drawing circles on blocks on Figure C, to denote each leg and the height into what she calls sets – see Tyesha's work)
Right?
[Carla: (agreeing) Yeah.]

27 Tyesha: Each set has five blocks (holds up five fingers)
[Carla: Oh! (indicates understanding)]
There's five blocks for each set.
[Carla: Okay.]
So if there's five blocks for each set, then, add together, and you count it up, there's twenty five blocks in all.
(Tyesha's tone of voice is patient and controlled. Tyesha continuously

> makes eye contact with Carla and sometimes Ta'Shawna. When she is done speaking, Tyesha settles back into her chair.)

Earlier, Tyesha had read the problem aloud to the whole class and has already begun expressing her ideas on how the towers are constructed to her small group. As Tyesha continues to explain her counting strategy, her two classmates continue to ask questions, suggesting that they may be having difficulty visualizing what Tyesha is saying. I infer that Tyesha has an active LMTY structure because she continues to explain her ideas and attempts to modify her explanation of how she counted all the blocks. In addition, she refers to the diagram on the task paper, using this as a tool to help her teach her two classmates.

Carla, in particular, seems persistent in trying to understand Tyesha's strategy. At the same time, Tyesha continues to check for understanding, often asking, "Right?" after she explains something. In response, Carla asks questions and gives her own interpretation of Tyesha's strategies, as in line 25. In response to Carla's comments, Tyesha continues with her own explanation. Therefore, she is correcting Carla, and doing so in a way that appears to be intended as helpful, as evidenced by the modified explanation. Here she is calling each side or leg a "set" of blocks and says that, "Each set has 5 blocks." All of Tyesha's behaviors here contribute to my inference of an active LMTY structure for her. Tyesha's behavior is consistent with several LMTY features, as she first teaches her counting strategy, then teaches to further clarify this strategy, and continues to check to see if her classmates understand this strategy.

It was common in all of the observed classes for students to mistakenly believe there were 25 total blocks in the 5-block high tower. Tyesha is able to explain how she came up with this number, expecting the same number of blocks in each leg as there were in the height of the tower. About 20 minutes after this brief event, Ms. S has the students use physical blocks to recognize their counting error. All three students are then able to continue with correct answers, leading them to start recognizing some patterns in their work.

Tyesha's questionnaire responses to the items which may indicate an active LMTY structure are all positive, including, "I helped someone see how to do the math," and, "I like teaching this person things that I know." In addition, Tyesha gives positive responses to almost all the items which may indicate an active LHSIA structure, such as, "I wanted people to think that I'm smart," and, "I tried to impress people with my ideas about the problem." Throughout the problem-solving session, I inferred an active LHSIA structure for Tyesha on only two occasions, compared with thirteen inferences for an active LMTY structure. Tyesha may have felt confident in her mathematical ability which contributed to her desire to help her classmates. Alternatively, she may have taught her classmates and tried to help them understand the problem and solutions as a way to get recognition or acknowledgment for her own intelligence and mathematical contributions.

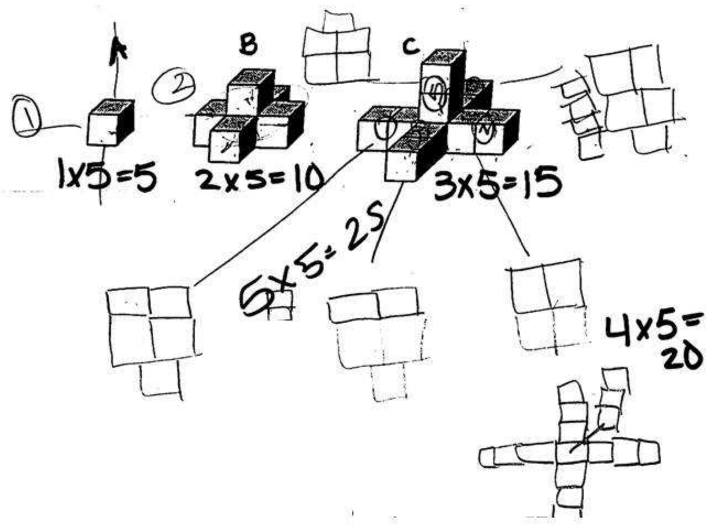


Figure 4-2: Tyesha's drawings

This event, and the entire problem-solving session, portrays a student who appears to maintains an active LMTY structure over a sustained period of time. Tyesha used the diagrams given on the task handout as well as her own drawings as tools to assist with her explanations. Tyesha’s classmates appear to have encouraged her assistance and also seemed receptive to her explanations because they continued to ask questions about the task and her strategies. Perhaps if Carla and Ta’Shawna had responded differently to their classmate’s initial explanations, Tyesha’s motivating desire may have shifted from wanting to help her classmates, depending on how either Carla or Ta’Shawna may have reacted.

Event 2

This next group, from Ms. A’s Class 2, is an example of how a student may have an active LHSIA structure throughout the problem-solving session. Onan, one of the two boys in this group, begins the session offering a strategy that his classmates Joe and Raina, the only girl, hardly ever seem to question. The session itself ran about 50 minutes long, and therefore, here I include a several excerpts from throughout the class session to demonstrate the evidence from which I infer Onan's LHSIA structure. After the teacher gave her introduction to the problem, Onan was the first to express his ideas about the problem and a strategy. Breaks in the transcript are indicated using ellipses (…) and by the line numbers from the annotated transcript (Appendix D).

1 Onan:	So what we can do is, we can find like, multiplication like we do with area, like we did with the problem yesterday. We could do that here, multiply by whatever it is. … (using his pen to count the blocks in figure C given in the task) So there's 10 right here …
2 Raina:	*inaudible, laughs*
3 Onan:	Right the way I said it?
4 Raina:	Yeah.
5 Onan:	Take it by 5 by the 10 that we had there and then we'll get 50 right? And then 10 times 10 is, what is it – 10 times 10 is 100 …
6 Joe:	But, why are you times-ing it by 10?
7 Onan:	'Cuz the last block is 10 (points to figure C on the task paper).

After this exchange among the three students, Ms. A approaches the group. She helps them realize that there are 11, not 10 blocks in the 3-block high tower, or figure C.

28 Onan:	So let's try it again we should go by, how many do we have here again? 11? So let's times all of these by 11 and let's see what we'll get.
29 Joe:	So 5 times 11.
30 Onan:	5 times 11 equals 55. (writes down as he speaks)
	…
47 Raina:	This is C. (puts the 3-block high tower she has constructed on Onan's desk)
48 Onan:	Yeah, that's C. Alright let's say, let me write this down. We could say that we think, it's gonna be… We think that 55 is the next block … 5 goes straight up and the other ones that are left over we divide them over the sides, right?
49 Raina:	Yeah.
	…

52 Joe: So we don't do that? The answers we got for like right here. (Joe points to the task paper, and seems to be referring to the total number of blocks determined for the towers in phases A, B, and C)

53 Onan: No 'cuz this doesn't have any blocks around it. …
If you take 1 times 11, you're still gonna get 11, I mean 1.
…

The students spend some time writing on their individual papers the solution, or rather the strategy they have come up with toward a solution. This consists mostly of Raina and Joe copying what Onan has written throughout the session. After they complete their work, they students discuss non-class related things and play with the cubes.

71 Onan: We're finished (holds up paper to show Ms. A)

72 Raina: We're finished, Ms. A.

73 Onan: We were the first group to finish. Haha! (smiles)

74 Raina: We're the smartest.

75 Onan: Yeah, we're the smartest in this whole class.

These brief excerpts were selected to show Onan's active LHSIA structure throughout the problem-solving session. Onan may have activated other engagement structures during class. In fact, I have inferred, at times not included above, that LMTY active as well (see Appendix D). Still, I suggest that LHSIA may have been a primary structure for Onan during this class, based on his observed behavior.

First, Onan is the first to express an idea about a strategy. His approach is based on his belief that there were 10 blocks in the tower depicted by figure C. He wants to multiply this number, ten, by the number of blocks in height of the 5-block high tower and the 10-block high tower. Though his strategy is incorrect, Onan's two classmates, Joe and Raina, do not disagree or suggest an alternative strategy. Moreover, Raina validates Onan's strategy when he asks, "Right, the way I said it?" Joe and Raina continue this pattern of behavior to not question Onan's strategies or ideas throughout the event.

Onan's behavior, including his attitude as though he is the leader of the group, lead me to infer he started with an active LHSIA structure. In line 52, when Joe asks about using the total number of blocks for the towers in figures A and B (the 1-block high tower and the 2-block high tower, respectively), Onan dismisses those answers, suggesting that they do not contribute to the answers they are looking for. Onan even seems to realize that his strategy does not apply to the first phase, with the 1-block high tower, as 11 times 1 equals 11, which is not the number of blocks in that tower.

Onan often states answers, as he does when Joe asked why they were multiplying by 10, with, "'Cuz the last block is 10" (line 7). Onan not only states an answer, but takes on the role of a leader whose ideas and strategy the group will follow. Finally, as the problem-solving session is ending, Onan says that his group was the first to finish and that they are, as a result, the smartest group in the class. Because I infer that Onan had an active LHSIA structure throughout the session, I wonder if he believes that the group, or at least each person in the group, is smart, or if he perceives only himself as smart and is feigning false humility. Onan may possibly be giving the whole group credit for their work because he believes they contributed toward their solution, or he may be giving them credit because he was part of the group.

Onan's questionnaire responses suggest my inferences about his active LHSIA structure may be correct. He gives positive responses to most items, including, "People seemed impressed with the ideas I shared about the problem," and, "I felt smart." Still, he gives some negative responses to certain items, such as, "I wanted to show someone that my way was better," and "I want you to know just how smart I am." Onan also circled, "Often," to the item, "I was the leader," which also agrees with my inferences about how he perceived his role in the group. Onan's responses to the items which may indicate an active LMTY structure were also almost all positive, with one exception, "I wanted to teach this person things that I know." Though there were only two coded instances of an active LMTY structure for Onan, he often has to explain his strategy and ideas to his classmates so that they can follow along. Though I suggest that Onan often expresses his ideas with the intention to demonstrate his mathematical ability, he may be intending to help his classmates understand his strategy as well.

Event 3

Two students may have an active LMTY structure at approximately the same time, as demonstrated by Keshia and Aleana in this event. These two girls, in Ms. S's Class 2, worked with a third classmate Nazira (also a girl). The students seem interested in determining a final solution quickly. They consider constructing a table or determining an equation, and ultimately construct the 5-block high tower using physical blocks. When Ms. S comes to the group, she helps the students realize that their variables are the height of the tower and the total number of blocks in the tower structure. After Ms. S leaves the group, the three students start discussing the total number of blocks in each tower structure. They start suggesting that there is 1 block in the 1-block high tower, 5 blocks in the 2-block high tower, and 10 blocks in the 3-block high tower.

35 Keshia:	Where you get five from, Nazira? (referring to the tower in figure B, the 2-block high tower) *Nazira works on the graphing calculator she retrieved from her bag a moment ago*
36 Aleana:	Look here. (counts the blocks in tower B, constructed from blocks in front of them)
37 Keshia:	But that, say, y'all wanna put, uh, so, you got to count that one too. (referring to the middle block that is surrounded by the other blocks)

	(Aleana picks up the top block in the 2-block tower) 'Cuz that's the one that's holding up all them.
38 Aleana:	Oh, snap! Nazira, we did something wrong. She [Ms. S] just got finished explaining, we had to find the total numbers of blocks, and it's one, six.
39 Nazira:	It's one, who? (Still working on the calculator)
40 Aleana:	It's one. And if we count all of these it's six.
//Keshia://	Six.

The three girls had been building their ideas off of Nazira's earlier suggestion that there are 5 total blocks in the 2-block high tower, a common misconception across the groups and classes. Keshia asks about this, and then points out to Aleana that there are 6 total blocks in the 2-block high tower, not 5. She explains to Aleana that the middle base block must be counted with all the others. It appears that recognizing the error activated the LMTY structure for Keshia, who teaches a counting strategy, part of the third LMTY feature, to Aleana so that all the blocks in the tower structure are properly counted.

Aleana sees this and agrees, telling Nazira that they made a mistake. Her behavior leads me to infer that she has an active LMTY structure, possibly prompted by Keshia's explanation and is also looking to teach the counting strategy. Aleana points out to Nazira that the 1-block high tower has a total of 1 block, and the 2-block high tower has a total of 6 blocks. By stating to Nazira that they made an error, rather than having Keshia explain this to Nazira herself, she may be trying to demonstrate her own understanding to both of her classmates. Aleana repeats her brief explanation when Nazira asks, "It's one, who?" Keshia also joins in on, "Six."

The questionnaire responses of both Aleana and Keshia for the items which may indicate a LMTY structure are all positive. Both girls circled, "All the time" to the statement, "I wanted to teach another student something that I knew that the other student did not know, and "Sometimes" to the statement, "I helped someone see how to do the math." In addition, both girls gave almost all positive responses to the questionnaire items which may indicate an active LHSIA structure, including, "People saw how good I am at the math we did today." Both students responded, "Never" to the item, "I wanted to show someone that my way was better." Outside of the event shared above, there were two instances in which I inferred Aleana had activated the LHSIA structure and no instances for Keshia. Though these two girls show few behaviors from which I inferred an active LHSIA structure, both may still want to be recognized as smart by their classmates.

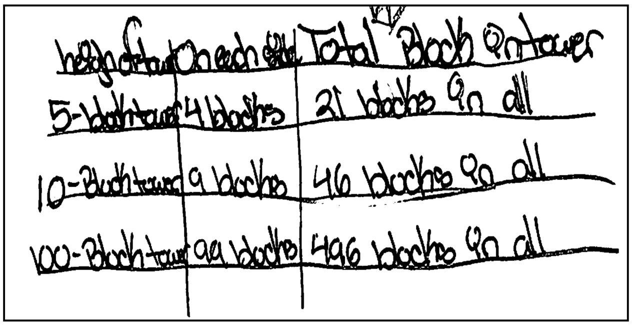
Event 4

This event, which takes place about 20 minutes into the session, introduces Julian, a student in Ms. A's Class 1 for whom I infer an active LMTY structure. Julian's explanation encourages his classmate Abby to explain an idea as well, which led me to infer that she may have an active LMTY structure as well. Julian and Abby have mostly been exchanging ideas throughout the problem-solving session. Their third classmate, Samara, does not seem to understand their ideas and strategies, and therefore she appears to not engage with the mathematics in the same way. Samara often looks around the classroom or looks at her paper without writing things down or asking her classmates for help. Toward the end of class, Samara starts to write answers on her paper, but seems to be mostly copying what Julian and Abby have, rather than trying to understand the mathematics and answers herself.

Julian and Abby have been drawing representations of their answers, but realize they will have difficulty if they try to draw the 100-block high tower. Ms. A worked with the group shortly before the event below. She encouraged the students to identify a general rule or pattern that will help them find the total number of blocks for a tower of any height. After Ms. A leaves to attend to another group, Abby describes the pattern as she sees it. Using examples, she states that there is one less block on each side than there is in the height, "If the height is 2, there is gonna be 1 on each side." Julian then proceeds to suggest how they can organize their information.

51 Julian:	We can do a T chart, a T chart right here and we can put height of the tower, how many blocks on each side, you could put the 10 block tower.
52 Abby:	Where do I put the T-chart, right here? (pointing to a space on her paper)
53 Julian:	Give me that piece of paper. (Samara gives him a piece of blank paper.) Oh, you can do it like this. (Abby and Samara look on as Julian writes on the paper just given to him.) Height, tower, like that, look, um, on each side. Alright, now the 10-block on each side is gonna be 9 blocks. You could say, and then the 12-block one, because it would be 11.
54 Abby:	Yeah, and then instead of making the T-chart we can make, we can make it like on this side it could be the total blocks, the total of the blocks of each column, we could make it like a chart.
55 Julian:	Since we can't do the 100 one, we can just do it on the chart.

Julian begins by stating they can organize their information in a T-chart, by which he means a table that organizes their information to display the height of the tower, the number of blocks on each side, and the total number of blocks in the tower (see Figure 4-3a). Julian's statement prompts Abby to ask a question about where on their paper a T-chart should go. Julian responds by drawing the chart on a paper, showing Samara and Abby how he is thinking about how to organize his information. He states that the chart should include the height of the tower and the number of blocks on each side. He includes the specific example of the 10-block tower, repeating that each of the four sides



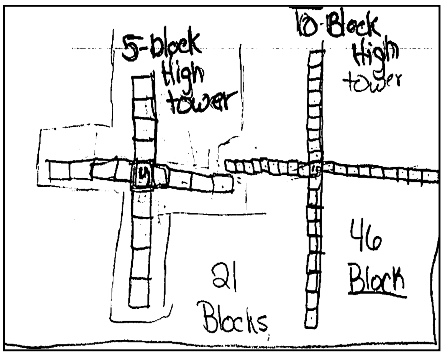


Figure 4-3a: Julian's completed T-chart
Figure 4-3b: Abby's drawing representation of the 5-block high tower and the 10-block high tower

has 9 blocks. I infer that Julian has an active LMTY structure because he has chosen to explain the procedure of making the T-chart in his quiet tone of voice. Julian's behavior is consistent with the second LMTY feature of explaining a procedure. I suggest that Julian is explaining a procedure rather than a mathematical strategy because these students have already agreed that there is one less block in each side than there is in the height of the tower.

Abby builds on Julian's idea and suggests that the chart should also include a column that depicts the total number of blocks in the tower. I inferred that Abby has activated the LMTY structure as well because she explains another part of that procedure. Julian responds by continuing with these ideas, stating that they can include the information for the 100-block high tower in their chart since they find drawing the representation to be an immense task.

On the questionnaire, both Abby and Julian give all positive responses to the items which may indicate an LMTY structure, including, "I helped someone see how to do the math," and, "I like teaching this person things that I know." I inferred for both students an active LMTY structure, on several occasions throughout the problem-solving sessions. There were also several occasions where I conjectured for each student an active LHSIA structure (though fewer than the LMTY moments). As I expected, both Abby and Julian gave almost all positive responses to the items which may indicate a LHSIA structure. Such items include, “I felt smart,” and, “People saw how good I am at the math we did today.” Abby gave negative responses to the items, “I was a lot better at math than others today,” and, “I want you know just how smart I am.” Julian circled, “No” to the item, “I wish the teacher would call on me, so I can show how much I know,” perhaps suggesting that he did not want everyone’s attention rather than he did not want to be recognized for being smart or having correct ideas in mathematics class that day.

Event 5

In this next event from Ms. S’s Class 1, we meet Carl for whom I infer has an active LMTY structure, Nikki for whom I infer has an active LHSIA structure, and Ricardo. Throughout the problem solving episode, Ricardo appears to be engaged, but it is very difficult to hear him on the video and audio recordings. Nikki and Carl carry on most of the conversation and seem to work well together. Here, Carl seems to be clarifying what a 3-block high tower is. Nikki first seems to try to correct Carl but then agrees with his explanation, leading her to claim that she understands. This exchange among the students shows how a student may respond when another is attempting to teach or explain something.

17 Nikki:	So would you consider this a 1-block tower? (pointing to Carl’s paper) Because this one is 2, that one is 1, I know.
18 Carl:	It’s 1, 2, 3. (pointing to the diagrams on his paper)
19 Ricardo:	*inaudible*
20 Nikki:	That’s not a 3-block tower! The 3-block tower goes up…

21 Carl: It is 3 from the one like this. One, two, and three would be like this. (uses his pen to point to the towers A, B, and C on his task paper) Nikki, you're smart.

22 Nikki: Oh!!! I get it. I get it. I get it.

I infer that Carl has an active LMTY structure in response to Nikki's request for an explanation of the 1-block high tower. He responds directly to her question, attempting to clarify the height of each of the towers depicted on the given task paper. When Nikki responds, she disagrees with Carl's explanation, likely referring to figure C, stating, "That's not a 3-block [high] tower!" I infer that the LHSIA structure has been activated for Nikki, who appears to be demonstrating the third LHSIA feature: correcting others. First, Nikki argues with Carl, then expresses her own interpretation, "The 3-block [high] tower goes up." Carl persists in his own explanation, using the diagrams again to support his justification, "It is 3 from the one like this." At the end, he affirms that he believes Nikki can understand this, stating, "Nikki, you're smart." Carl's support may have contributed to Nikki claiming to comprehend, "I get it." Nikki now states that she agrees with Carl and wants to convey that she is smart to her classmate. I infer that Nikki again has an active LHSIA structure, this time demonstrating the fifth LHSIA feature, because she is claiming that she "gets it," or in other words is keeping up with Carl intellectually.

My interpretations based on the video and audio of this group seem to correspond with the students' questionnaire responses. All three students gave only positive responses to the items which may indicate an active LMTY structure. For example, Carl responded, "All the time" to the item, "I helped someone see how to do the math" and "Often" to "I gave helpful suggestions." My analysis of this group suggests that they all worked together toward a strategy and solution. In addition, Nikki, Ricardo, and Carl gave mostly positive responses to the items which may indicate an active LHSIA structure. Nikki responded, “All the time” to items such as, “I wanted people to think that I’m smart,” and, “I felt smart.”

However, some responses stood out as potential discrepancies between student responses and the video analysis. Specifically, Nikki responded “No” to the items, “I want you to know how smart I am,” and “People think I’m smart.” Carl also responded “No” to the first of these, and Ricardo responded “Yes” to both items. The negative responses given by Nikki and Carl may have occurred for several reasons, such as misunderstanding or carelessness on the part of the student when responding to the item. However, if Nikki and Carl intended these as appropriate responses, it may be the case that these students were not focused on how their classmates perceived them, even if they felt confident in their own intelligence and mathematical ability as it pertained to this problem solving session.

Event 6

In this event, we meet Larry, Janelle, and Ellen who are all students in Ms. A’s Class 2. I infer that Janelle has an active LHSIA structure here as she tries to show Larry the correct way to construct a 5-block high tower with the SnapCubes. The features demonstrated in this event are *expressing an idea, stating an answer or answers,* and *correcting others.* Prior to this excerpt, Larry seemed to think that the 5-block high tower was determined by counting the visible blocks in the height which excludes the middle hidden block. Several groups in this study discussed the decision whether to include this block as part of the height, part of each side, and/or part of the total number of blocks in the tower structure.

74 Janelle:	You gotta take these out. Let me show you which ones (speaking to Larry, reaches over to his tower structure and removes a cube from one of the legs) I got 4 on the sides (puts up 4 fingers)
75 Larry:	No, you don't!
76 Janelle:	You do.
77 Larry:	How much you wanna bet it's not gonna get… (He removes some more blocks from his tower structure and shows Janelle. She takes the structure away from him and counts the total number of blocks the tower currently has to show him.)
78 Ellen:	I have 25.
79 Janelle:	(after counting) 21.
80 Larry:	Now add 1 for the 5.
81 Janelle:	I know but…
82 Larry:	Why you saying "ooooweeee" to me?
83 Janelle:	Because you're acting like I'm dumb or something.

In Janelle's first speaking turn of this event, she corrects Larry, telling him that his tower structure has too many blocks for the 5-block high tower. She also takes control of his tower construction to help explain which cubes should be removed. She does this as part of her efforts to tell him how to correctly construct a 5-block high tower, adding that there should be only 4 blocks on each of the sides, expressing her idea of the correct 5-block high tower. When Larry receives his structure back, he continues to remove some cubes and counts the total number of cubes in his structure. This researcher could not hear Larry state a total amount, though Ellen chimes in stating that she counted 25 blocks in her 5-block high tower. Janelle again takes Larry's tower structure and counts 21 total blocks in his 5-block high tower. She states this answer without seeming to acknowledge Ellen's suggestion of 25 blocks. This action of stating an answer without considering a classmate's suggestion also suggests an active LHSIA structure for Janelle. Janelle's final statement in this brief exchange also indicates that she may feel the need to prove her mathematical ability to her classmates.

Larry does not appear pleased that Janelle is correcting him and using his work as an opportunity to possibly demonstrate how smart she is. This is made clear when Larry makes the statements, “No you don’t!” and “How much you wanna bet it’s not gonna get…” Ellen hardly engages in this conversation at all, only stating the total number of blocks she believes is in the 5-block high tower.

What is striking about Janelle is that most of her questionnaire responses do not necessarily indicate an active LHSIA structure. She gave positive responses only to the items, “I wanted people to think that I’m smart,” “People saw how good I am at the math we did today,” “I liked to be right,” and “I want you to know just how smart I am.” Some of the items to which she replied “Never/No” to include, “I tried to impress people with my ideas about the problem,” and “People think I’m smart.” The positive responses alone suggest that possibly Janelle did have a motivating desire to be recognized as smart or mathematically competent by others. The negative responses may indicate that she felt she was unsuccessful at getting any acknowledgement for her ideas and contributions. Janelle’s responses to the items which may indicate an active LMTY structure were almost all negative, with one exception, “I worked cooperatively.” My findings suggest agreement in this case as I did not assign any LMTY codes to Janelle’s behaviors.

Event 7

This event depicts Leticia, a student in Ms. S’s Class 1, who is inferred to have an active LMTY structure. Here we see Leticia interpreting and clarifying the teacher’s suggestion to her two classmates, Monique and Sherelyn. This event is the only one analyzed for this study in which the teacher is engaged in conversation with the students.

Ms. S comes to this group after the students have been working for over 10 minutes on the problem. These girls seem to be recognizing a pattern of adding 5 blocks to the total, so Ms. S asks them to explain how they know to add 5 blocks. During this exchange, Leticia suggests they might use a table to organize their information and help them develop an equation. Ms. S encourages this idea and asks them to verbalize what their variables will be. Many students in this study stated that "blocks" is one of the variables without specifying whether they are referring to the height or the total number of blocks in the tower structure. Therefore, Ms. S seems to be encouraging these students to be specific in identifying the two variables.

106 Ms. S: Yeah, so you're going to label, you need to give me a heading, so I know what you're talking about… [so I could] look at the table, and know exactly what we're comparing here.

107 Leticia: (looking at Monique and Sherelyn) I think what, I think what she's trying to say is that instead of saying how many blocks we need to add on and not knowing what that next thing is, I think we should – how many blocks compared to how tall the tower is. So, say we have a 5-block high tower – like, how many blocks.

108 Monique: Yeah.

109 Sherelyn: How many blocks around it

Prior to this brief event, the three girls have suggested that a table would be helpful and shared this with their teacher. Ms. S pointed out that the word "blocks" as a heading did not provide give enough information. As Ms. S encourages them to state what the headings or variables will be, Leticia jumps in. I infer that the LMTY engagement structure has been activated for Leticia and that she is demonstrating the first LMTY feature: teaching to clarify. Leticia is clarifying her teacher's suggestion as well as building on the ideas of her classmates and Ms. S. By saying that they need to know how tall the tower is, as well as the total number of blocks needed to build that tower, Leticia is teaching both a procedural idea as well as a conceptual one. Writing the words "Height" and "Total blocks" for their table may be procedural in that it can merely involve what words to write down. However, conceptually, the students appear to be recognizing what aspects of the problem are essential for finding solutions. Identifying the variables themselves may help the students understand the problem on a deeper level than if the information was given to them.

When Leticia completed her questionnaire, her responses for the items that may indicate the LMTY structure were mixed. She circled "Never" to the items, "I wanted to teach another student something that I knew that the other student did not know," and, "I helped someone see how to do the math." However, she gave positive responses to other items, including, "I gave helpful suggestions," and, "I like teaching this person things that I know." This event is part of only one of two episodes coded as an active LMTY structure for Leticia, so this particular moment may not have stood out in her mind as a teaching moment when she filled out the questionnaire at the end of the problem solving session. In addition, she may not have intended to teach her classmates something, and therefore did not come into the problem-solving session with the desire to help her classmates. Still, her positive responses suggest that she believes she contributed to their solution strategy in some way, which I have interpreted as an active LMTY engagement structure.

Event 8

The following event from Ms. S's Class 2 demonstrates how one student, Christian is inferred to have branched from an active LHSIA structure to the LMTY structure, as he states his answer and strategy for finding the total number of blocks in the

100-block high structure. The event, starting at line 45, takes place 20 minutes after the students have begun work in their groups. Several times throughout the problem-solving session, another student, Carly, has suggested that the relationship between the height and the total number of blocks is a proportional one. About 5 minutes into the problem-solving session, the third member of this group, Cristina, has asked Carly to explain.

23 Cristina:	How is it like a proportion?
24 Carly:	Like, 1 T equals 1 C which equals, then you do, we have to figure out how much is 5, so we do 5 C, no 5 T is… how much it would be… then you have to like, cross-multiply.

Carly may be attempting to teach her proportions strategy to Cristina, in response to the request. However, her hesitant pauses in her speech suggest she might be thinking out loud, describing proportions in general and attempting to fit it into this problem, particularly since she mentions "cross-multiply."

45 Cristina:	I think it's 470.
46 Christian:	It's 496.
47 Carly:	I got 407.
48 Christian:	How'd you get 407 though?
49 Cristina:	The proportions.
50 Carly:	Where'd you get 496?
51 Christian:	I kept adding on boxes.
52 Carly:	So you just kept adding on 5?
53 Christian:	Yeah, that's how I got to 100. I even double-checked.

This event starts when Cristina states that she thinks there are 470 total blocks in the 100-block high tower. Christian corrects her, since he has a different answer. I infer that he has activated the LHSIA engagement structure based on this action, as well as his tone of voice which suggests that he believes he is correct. When Carly also gives a different answer of 407, he asked her how she got her answer. Cristina responds that she used Carly's proportion strategy, and her work (see Appendix D) supports this claim. Carly responds by asking Christian how he got his answer, prompting him to explain that he simply added 5 blocks to get the total for each successive tower. I inferred that Christian may have branched from the LHSIA structure when stating his answer to the LMTY structure because he was asked to explain his answer. When Christian stated his answer, "It's 496," his tone of voice suggested that he was correct. This is also supported by his statement, "I even double-checked," possibly giving Christian more confidence that he has the correct answer.

Though I have inferred an active LMTY structure for Christian, here and on four other occasions, Christian gives several negative responses for the questionnaire items which may indicate a LMTY structure. Specifically, he responded, "Never" or "No" to the following items: "I wanted to teach another student something that I knew that the other student did not know," "I helped someone see how to do the math," and, "I like teaching this person things that I know." Christian does, however, give positive responses to almost all the items which may indicate a LHSIA structure, including, "I wanted people to think that I'm smart" and "People saw how good I am at the math we did today." This combination of responses suggests that Christian may have been motivated by a desire to appear smart to his classmates. It is possible that his explanations to his classmates were given in service of demonstrating his mathematical ability. Carly, who I infer may have had an active LMTY structure in line 24, gave positive responses to all of the items which may indicate a LMTY structure and most of the items which may indicate the LHSIA structure. These responses correspond to my findings where I have inferred that Carly has both the LMTY and LHSIA structures active throughout the problem-solving session.

Event 9

In this event, we meet Leo, who I inferred has branched from an active LMTY structure to an active LHSIA structure over several minutes. He is working with two girls, Ta'keisha and Ordena, all students in Ms. S's Class 2. At the beginning of the problem-solving session, Leo begins by explaining his interpretation of the instructions to his two classmates, who appear to ignore Leo while he attempts to understand the problem and explain his ideas. The students seem to struggle with what is meant by a "5-block high tower." Just before Leo speaks in this event, Ta'keisha seems to recognize that there are 6 total blocks in the tower depicted in figure B, though she does not seem to realize this is the 2-block high tower. Leo's suggestion seems to indicate that he believes there should be a total of 5 blocks in the tower.

14 Leo:	What we need to do is decrease that and build it like 4 on the sides and try to balance it on each border. Or try to build it higher. You know what I'm saying?
15 Ordena:	No. (looks away from the group and glances toward the rest of the classroom)
16 Leo:	(continues talking to Ta'keisha) So if this one is set C, a 10-block… That's actually 11. So we need to take the one from the middle and tyr to balance it on 1, 2, 3, 4 sides. But the cubes have to move in a little. You keep adding more. So that's what I think it's trying to do.

Ta'keisha and Ordena do not respond to Leo, suggesting that that they are ignoring him, or at least his ideas. The two girls do mumble something inaudible to one another, and Leo decides to get a bag of cubes. All three students take some of the cubes.

It is not clear if Ordena is building a tower structure. Ta'keisha's construction with the blocks is not visible on the video, but she shares it with Leo, who responds.

22 Leo:	(to Ta'keisha) So here. This is how it would look. (starts building a tower with his own cubes)
23 Ta'keisha:	It was six blocks.
24 Leo:	This is what I was talking about. See how this is?
25 Ta'keisha:	(to Ordena; possibly ignoring Leo) Don't we gotta find an equation?
26 Leo:	This is what I was talking about. See how this is, right? In the middle. So this is what I was trying to do. Make it wide. (putting the blocks out to explain what he means; Ta'keisha sometimes glances to what Leo is doing, but she mostly focuses on Ordena) Make it wide. And then balance it on the middle. (tone of voice suggests he is getting frustrated)
27 Ordena:	It will probably…
28 Ordena:	So that's behind it… (Ta'keisha focuses her attention to Ordena)
29 Ta'keisha:	No, but look (pointing to something on Ordena's paper)
30 Ordena:	Yeah, I know.
31 Leo:	So what I'm trying to do is balance it. (focuses on his own tower construction)
32 Leo:	(to Ta'keisha) See.
33 Ta'keisha:	(to Ordena) See if you could see it in my head, those are not pushed together.
34 Leo:	Okay, now I know. (returns to working with his blocks)

When Leo first speaks, in lines 14 and 16, he tries to explain to his classmates his understanding of the problem. I infer that Leo has an active LMTY structure because he is explaining his idea and asking, "You know what I'm saying?" This behavior is consistent with the third LMTY feature, teaching a strategy, and the fourth feature, checking for understanding. Rather than listening to Leo, his classmates seem to ignore him. Ordena responds, "No," rather brusquely. Ta'keisha simply ignores him by focusing her attention to Ordena.

After he brings blocks to the table and constructs a tower according to his understanding, he says to Ta’keisha, “This is how it would look… This was what I was talking about.” This behavior suggests he is trying to command her attention, and even have one of his classmates confirm that his idea is valid. His behavior suggests his motivating desire has shifted from trying to help his classmates understand his strategy to getting their attention and agree that he has a correct idea. His tone of voice has changed, suggesting that he has become frustrated. I infer from this behavior that the active LMTY structure has branched and activated the LHSIA structure for Leo. I infer that the LHSIA structure is still activated when, in line 34, Leo states, “Now I know.” Here, though he is not revealing exactly what it is he knows, he seems to be stating that he is smart, consistent with the fourth LHSIA feature. Throughout the rest of the problem-solving session Leo appears to be vying for his classmates’ acknowledgement of his ideas, though he seems he never truly receives a satisfactory response from them.

I inferred that Leo had an active LMTY structures at four distinct instances, two of which are given above. This can be compared to the nine instances during which I inferred Leo to have activated the LHSIA structure; again two of these occurrences are given within this event. Leo’s questionnaire responses seem to support my findings based on the video and audio data of this group. He gave positive responses for items indicating LMTY, including “Sometimes” for “I gave helpful suggestions” and “Yes” for “I like teaching this person things that I know.” With respect to the items indicating the LHSIA structure, Leo’s responses for items like, “I felt smart,” “I tried to impress people with my ideas about the problem,” were all positive. Curiously, Leo circled, “All the time” to the item, “People saw how good I am at the math today,” but “No,” to “People think I’m smart." One possible reason for these thoughts is that Leo may have felt that he was good at the math this particular day, but it does not necessarily mean that he is smart, which he may view as a long-term trait.

The qualitative analysis of the coding, the features, the questionnaire results, and selected events have been included in this chapter in order to present and illustrate the major findings of this study. Prior to this study, the engagement structures Let Me Teach You and Look How Smart I Am were defined operationally by means of motivating desires, using questionnaire items. This analysis adds another broad component: the features of the behaviors associated with each engagement structure. These features are identified through qualitative analysis of the students' verbal and non-verbal behavior. In addition, reviewing the questionnaire results and comparing them to the video analysis suggests that the coding process may provide insight into the active engagement structures for many students, though there was not 100% agreement. The nine selected events were chosen to provide some insight into the complexity of the engagement structures, and the multifaceted ways they may be identified and described.

Chapter 5: Discussion

In this chapter, I address and discuss the research questions, referring briefly to examples presented in Chapter 4 and in Appendix D. I then present the limitations for the study, the theoretical implications, the implications for teaching, and suggestions for future research.

5.1 Answering the research questions

The purpose of this study was to characterize and then compare two engagement structures, “Let Me Teach You” (LMTY) and “Look How Smart I Am” (LHSIA), developed earlier as theoretical constructs by Goldin, Epstein, Schorr, and Warner (2011), as they were inferred to occur during the problem solving activity of urban middle school students working in small groups. I documented observable behaviors which led me to infer the activation of each engagement structure, as well as cues which may have prompted a student to activate the structure, and the responses of the student’s classmates. In addition, I noted the various mathematical ideas expressed during the problem solving sessions. Finally, I compared the results of my video analysis to the students’ self-report responses, to determine in each case whether there was general agreement between the two sources of findings.

Here I first present my summary for the first three sub-questions related to the LMTY structure, then for those sub-questions related to the LHSIA structure. I then discuss the fourth sub-question which asks about the mathematical ideas expressed during the episodes in which either LMTY or LHSIA is inferred. The fifth sub-question, which compares the video analysis to questionnaire results, is discussed within the larger context of the questionnaire findings.

Finally, I offer my conclusions regarding the comparison between the two structures.

1) How can we characterize interactions in which urban middle-school students attempt to teach or explain something to one another during small group work?

The first part or sub-question of this research questions asks, “What observable behaviors do students exhibit which indicate they are attempting to help others?” In Table 4-1, I presented the observed student behaviors which led me to infer that a student was attempting to help his or her classmate(s) and had an active LMTY engagement structure. Students often appeared to be explaining the instructions, the procedure, or some characteristic of the problem or its solution. Many times such students supplemented their verbal explanations by using tools such as drawings, the given diagrams in the task, or the physical blocks or SnapCubes (concrete external representations) that were available to students. Using such tools or manipulatives may have provided more opportunities for students on the giving and receiving end of help to develop a deeper understanding of the mathematical patterns described in the task (Moyer, 2001). Students were also inferred to have the intention to be helpful when they responded to a classmate’s questions with brief answers and when they asked if a classmate understood a strategy or answer.

Many of these inferences were made because of the *context* of these statements. For example, Georgia (in Ms. S’s Class 1) confirmed that the 2-block high tower had a total of six blocks, in response to Kevon’s question, “But that’s six, right?” (line 18). This was inferred to be the continuation of an active LMTY structure during which Georgia had described why the middle block was hidden, “Because you’re covering it [with other blocks]” (line 17).

The second sub-question asks, “What, if any are some of the observable cues that prompted a student to help another student?” The most common observable cue that prompted an (inferred) active LMTY structure was a specific verbal request to explain something about the problem or an answer, as seen in the example of Kevon’s question for Georgia, “How are you supposed to get 5 [blocks]”? (lines 14). But requests for help were not always verbal, as was the case for Deanna (in Ms. S’s Class 1). Her classmates, Damon and Manny, were discussing the pattern of adding five blocks to find the total number of blocks in each tower structure. Though Deanna had stated that she didn’t understand (line 22), she did not speak up again. She only received help when Manny noticed that Deanna was shaking her head to indicate that she did not understand the pattern that he described (line 30). Some students, like Tyesha (in Ms. S’s Class 2) and Christian (in Ms. S’s Class 1), seem to activate the LMTY structure when their classmates asked specific questions such as, “So you’re trying to say add five blocks to each”? (Carla to Tyesha, line 25) or “Where’d you get 496?” (Carly to Christian, line 50). Questions like these seemed to activate or maintain the LMTY structure for the recipient of these questions. Often in response, the classmate who asked the question listened to the explanation or answer and tried to understand it. Carla continuously tried to interpret Tyesha’s explanations, for example, saying, “So you have to add five to there, right? No, four?” (line 30).

Some students began explaining a concept or idea spontaneously, without an observable prompt such as a classmate request. This was the case for both Aleana (in Ms. S’s Class 2) and Leo (in Ms. S’s Class 2). Aleana recognized the pattern of the total number of blocks was to increase by five from one tower to the next and was initially ignored by her classmates who continued to work separately from one another. Aleana repeated her idea a few moments later, “I’m trying to let you know it’s going up by fives,” (line 67) again trying to describe the pattern. This time, her two classmates heard her and asked her to clarify her idea, possibly maintaining (or possibly initiating) the LMTY structure for her.

To respond to the third sub-question, “How do the other members of the class respond to the one who is trying to teach them?” I reviewed the episodes in which a student was inferred to have an active LMTY structure. Several students who accepted their classmate’s help or explanation followed up by building on those ideas. For example, in the third event presented in chapter 4, Aleana (in Ms. S’s Class 2) responded to Keshia’s explanation of the 2-block high tower having six total blocks by turning to their third classmate Nazira to explain this same concept, “Nazira, we did [something] wrong… we had to find the total numbers of blocks” (line 38). Aleana not only accepted Keshia’s ideas as correct, but also used the explanation to help another classmate. Ellen (in Ms. A’s Class 2) also built on Larry’s ideas after he explained that tower D could be constructed from tower C by adding a block to each side. Ellen added, “And going up” (line #1:53:02), suggesting agreement with Larry, that they needed to add a block to the height as well as to each side.

Not all students simply accepted their classmate’s explanations, however, and the LMTY structure, in turn, appeared to become active for other students as they corrected their classmate’s statements. For example, Julian (in Ms. A’s Class 1) disagreed with Abby’s explanation of the 5-block high tower when she interpreted that to mean the total number of blocks in the tower was only five. Julian corrected her, but also appeared to activate the LMTY structure as he explained that the diagrams given to them were the 1-block, 2-block, and 3-block high tower.

Leo (in Ms. S's Class 2) was also often ignored by his classmates when he attempted to explain his strategy to his classmates. When he tried to repeat his ideas, his classmates Ta'kesiah and Ordena often rebuffed him or dismissed him with quick responses. One such instance occurred when Leo asked Ordena, "You know what I'm saying?" (line 14). Ordena responded by saying, "No," and turning away from him. These responses appeared to prompt Leo to deactivate the incipient LMTY structure and activate the LHSIA structure, or to branch from one structure to the other. When Leo spoke to his classmates after being rebuffed in the first several minutes, he appeared to be more insistent with his ideas, making statements such as, "This is what I was talking about," (line 26) or, "But that's not all you do" (line 87).

Some students activated another structure, such as LHSIA, in response to a classmate's attempt to help or explain an idea, as in the case of Nikki and Carl (in Ms. S's Class 1). Carl was inferred to have an active LMTY structure because he tried to explain why the towers labeled A, B, and C had heights of one, two, and three. Nikki responded by arguing with Carl, "That's not a 3-block [high] tower!" (line 20). Carl persisted in explaining the pattern of the blocks to Nikki, who responded, "I get it" (line 22). I infer an active LHSIA structure for Nikki first because she disagreed with Carl about which was the 3-block high tower Nikki then claimed to agree with Carl and may have been attempting to demonstrate her own mathematical understanding.

These examples are given to demonstrate that though most students for whom an active LMTY structure was inferred were found to explain something, a wide variety of behaviors led me to infer the activation of this structure. The *features* described in chapter 4 characterized the verbal behaviors of teaching to clarify, teaching a procedure, teaching a strategy, and checking for understanding. Students' utterances were supplemented with non-verbal behaviors including, referring to drawings or tables and using the SnapCubes. In addition, while many students began to explain something because they were specifically asked to do so, others responded to non-verbal cues such as a classmate shaking her head. Still others appeared to explain or teach a concept without an observable indication that help was needed.

The responses of classmates of the student who tried to help in some way seemed to vary greatly. Some students ignored their classmate, others were inferred to activate another structure, and still others requested more details for the problem or strategy. The LMTY structure can be characterized for many interactions for these students as: 1) request for help is made, 2) explanations of ideas are given, and 3) follow up questions are asked or student builds on those ideas. However, as we have seen here and in chapter 4 many interactions did not follow this script, in part because of the dynamic actions and reactions students have with one another and with the mathematics.

2) How can we characterize interactions in which urban middle-school students attempt to impress another student with his or her ideas about the mathematics problems, or with his or her intelligence or ability?

The operative motivating desire for a student with an active LHSIA structure is to prove that he or she is smart, to display his or her mathematical ability, or demonstrate his or her mathematical competence. Such students are attempting to impress others or show off to members of the class. With this population of urban middle-school students, there appeared to be three major differentiations of this motivating desire, described here as self-talk or inner speech: 1) "I want you to know how smart I am" (*without comparing* oneself to a classmate's intelligence), 2) "I want to prove that I am *smarter than you*," or 3) "I am *at least as smart as you*. I can keep up with your mathematical ideas." The first differentiation of these motivating desires, "I want you to know how smart I am," could describe a student who is either attempting to impress others or to show off (inviting admiration in a more conspicuous manner than impressing others). The second of these desires, "I want to prove that I am *smarter than you*," is associated with to a need to show off or gain attention or admiration for one's mathematical ability or knowledge. The third differentiation is connected to a desire to impress others or to earn the favorable opinion of others in a manner that is less conspicuous than showing off.

Furthermore, a student who is observed as showing off may not necessarily have an active LHSIA structure, as a student may be looking for attention from others for a range of reasons; the student may be intent only on distracting classmates or the teacher from the task at hand. Those who are inferred to have the LHSIA structure activated in this study displayed behaviors suggesting that they particularly wanted to show off their mathematical ability or knowledge.

The first sub-question of the second research question asks, "What observable behaviors do students exhibit which indicate they are attempting to show off or impress others?" The behavior that I observed most frequently was demanding attention to one's own ideas (observed 41 times), which could correspond to any of the three differentiations of the motivating desire depending on the context in which the statement was spoken. Students demanded attention in different ways, such as being the first to state an idea at the beginning of the problem-solving session, or getting a classmate's attention in a manner similar to Abby (in Ms. A's Class 1). Abby successfully drew Julian's attention to her strategy by saying, "See what I'm doing… 'Cuz, look, Julian" (line 27), Some students simply stated, "I got it," such as Ta'Shawna (in Ms. S's Class 2) who had asked her classmate Tyesha to explain her strategy of multiplying the height by five to get the total number of blocks (line 16).

Classmates Bridget and Jenna (in Ms. A's Class 1) each built on one another's ideas (a behavior observed 15 times) as a way to establish her own mathematical competence. Bridget initiated the exchange by asking if a block was added to each side as well as to the height, to which Jenna responded, "You add one [block] to each one [side]" (line 12). Bridget's response further developed this idea, "If I had a five block high tower, it would be five blocks going up and four blocks going around" (line 13). Both students stated these ideas to their entire group, and both conveyed a confident tone of voice which suggested that they wished to demonstrate their ability to successfully work toward a solution.

The behaviors that led me to infer an active LHSIA structure for students who may have wanted to demonstrate that they were smarter than others in their group include trying to "one up" a classmate (observed 2 times) or stating, "This is easy" (observed 12 times). Nadira (in Ms. S's Class 1) is one student who may have had this motivating desire. Early in the problem solving session, she stated that she knew the answer (line 22), her claim being that the 100-block high tower to have a total of 200 blocks. Nadira's classmate Georgia responded by displaying her confusion about Nadira's answer and showing the others that there were a total of eleven blocks in the 3-block high tower.

After Nadira and her classmates agreed on the correct strategy and answers, they all worked independently to write down their answers. Nadira then made the statement, "Now I'm finished. Now all you have to do is hurry up," (line76) which suggests that she may be "one-upping" her classmates by being finished before both of them.

Damon (in Ms. S's Class 1) may also have had this particular motivating desire to try to "one-up" his classmates (lines 31 – 40). Deanna, another member of his group, requested that her group members explain how the towers were constructed. Though Manny, the third member of the group, provided most of the explanations, Damon often repeated what Manny said and chimed in with, "Yeah." Manny does not respond directly to Damon's statements and seems to focus his explanations toward Deanna. Once Deanna claimed to understand, Damon asked her, "Get it?" (line 40) with emphasis, raising his voice, and with a tone possibly suggesting that she should understand. In doing so, he appeared to want Deanna to know that he understood this concept and perhaps he was smarter than her for being able to comprehend the construction of the towers. Deanna responded to Damon by saying "Yes!" in a tone that suggests she may have felt exasperated with him or defensive because she needed further explanation.

Throughout these four classes of students, there were many instances where I inferred that a student may have had a motivating desire to demonstrate they were "as smart as" a classmate or classmates and wanted to "keep up with [others'] mathematical ideas." The behaviors which most often led me to the inference include trying to show, "I can keep up with you" (observed 23 times), looking for confirmation from a classmate (observed 22 times), and agreeing with a classmate (observed 16 times). In Ms. A's Class 1, I inferred that Amanda tried to demonstrate that she could keep up with her classmates at the beginning of the problem solving session. Amanda began the group discussion by asking, “How much did they increase it by each time?” (line 6), likely referring to the increase in total number of blocks. Two of her classmates, Juan and Manuel, responded by expressing their ideas, recognizing that the pattern was to increase the total number of blocks by five for each iteration. Amanda appeared to want to demonstrate her own understanding of their explanation, repeating what Juan stated about the height increasing by one block each time, “They’re increasing by one because, look, they started the height with one, then it’s two, then it’s three.” (line 14). I inferred that Amanda wanted her classmates to know that she was able to comprehend their ideas, suggesting the presence of an active LHSIA structure. Amanda’s classmates responded by agreeing with her statement and building on the idea of adding one block to the height and each of the sides.

One group of three students in Ms. S’s Class 1 appeared to each have this motivating desire to be “at least as smart as” one another at least once during the problem solving session. Sherelyn voiced her agreement with her classmate Leticia who attempted to describe the pattern depicted in the diagrams on the task paper. Sherelyn’s statement, “Like you said… it’s saying if you keep on going in a pattern…” (line 2), suggests that she wanted to be seen as having good ideas about the problem. By agreeing with Leticia, Sherelyn may have been trying to express that she believes she is as smart as her classmate. Shortly after this exchange, Leticia seemed to be expressing this same motivating desire. During an exchange of ideas, the third student Monique explained how she thought the towers were constructed. Leticia responded by stating she had been confused, but she now understood Monique’s explanation, “I was about to ask a question… but now I see like that block in the bottom" (line 18). With this statement, Leticia appeared to want to inform her classmates that she had the same level of understanding as they did. Several minutes later, Monique also appeared to indicate a motivating desire to demonstrate that she was perceived to be as smart as her two classmates. Throughout this problem solving session, Monique had been explaining her ideas and strategies to the others. In line 88, she expressed the idea that the 5-block high tower could be built off the 4-block high tower, "See, it's five right here, and here it's four. It's just adding one, right?" By adding the question, "Right?" Monique may be suggesting that if her classmates agree with her, they have confirmed not only her answer, but also that she is as smart as the other two students. After each of these instances, the students agreed with the classmate who appeared to want to be perceived as smart, with someone saying, "Yeah" and then continuing on with the discussion of the problem.

A wide range of behaviors were observed leading me to infer an active LHSIA structure for a student. Some students made claims to understand a concept, while others restated a strategy explained by a classmate. Some students appeared to want to be smarter or better in some way than others by stating that they finished writing or answering all the questions first, or by suggesting they understood an idea that a classmate did not seem to understand.

My analysis relating to the second sub question for LHSIA, "What, if any, are some of the observable cues that prompted a student to attempt to show off or impressing others?" yielded vastly different results for the corresponding question for LMTY. The students for whom I inferred an active LMTY structure often responded to a classmate's question, confusion, or request for additional information, likely prompting the activation of that structure. However, I found hardly any observable cues prompting the activation of the LHSIA structure. The most notable exception occurred when Leo, who appeared to have an active LMTY structure, seemed to branch into an active LHSIA structure after being ignored and rebuffed by his two classmates (lines 24 - 26). It is likely that many non-observable features of the environment encouraged the activation of this structure, including the set-up of the small group work for the day, the fact that students knew they were being observed and recorded, the classmates they were asked to work with, the normal behaviors of these students, or one of many other possible reasons.

In addressing the third sub-question, “How do other members of the class respond to the one that is attempting to show off?” I observed that the other students did not seem to react directly to these behaviors (i.e., attempting to impress others or demonstrate his or her own mathematical competence). Most students appeared to continue the discussion by expressing their own ideas, agreeing with their classmates ideas, or asking a question. In one such instance, Amanda (in Ms. A’s Class 1) was inferred to have an active LHSIA structure because she built on her classmate Juan’s idea of increasing the height, “They’re increasing by 1 because, look, they started the height with 1, then it’s 2, then it’s 3” (line 14). Manuel followed up by agreeing with her, “Oh yeah, I think they are increasing the height by 1, and then the sides” (line 15). Some students activated the LHSIA structure for themselves, particularly if they disagreed with the idea their classmate was putting forth. In one such example, Ordena (in Ms. S’s Class 2) said in response to Leo’s incorrect construction of a 5-block high tower, “I think we gotta do what **we** [Ta’keisha and I] did” (line 86). Still others asked questions, prompting their classmate to branch to an active LMTY structure, as we saw with Carly and Christian in the eighth event presented in Chapter 4.

As stated earlier, several verbal behaviors led me to infer an active LHSIA structure for many participants in this study, which have been described by the behavioral features: 1) expressing an idea, 2) stating an answer or answers, 3) correcting others, 4) stating, “I’m smart,” or “We’re smart,” 5) keeping up with others in the group. It was difficult to observe non-verbal behaviors to support these features, though it is likely that students’ facial expressions or other non-verbal actions on different occasions accompanied the activation of a LHSIA structure. In this study I was not able to discern any patterns of observable, non-verbal behavior to suggest the potential activation of the LHSIA structure for a student. As mentioned earlier, many non-observable cues likely set the stage for the activation of LHSIA for some students, such as working in small groups or being observed by several researchers and being videotaped in class. However, the data do not support or disprove this conjecture.

Once the LHSIA structure was activated for a student, there were several kinds of responses from classmates. Students seemed to mostly respond to what was said, rather than taking notice that a student was inviting recognition for his or her mathematical ability or knowledge. One exception occurred with Damon and Deanna. Damon had asked Deanna, “Get it?” (line 40) with a tone of voice suggesting that Deanna ought to understand the concept. Deanna responded, “Yes!” (line 41) possibly in a defensive tone, suggesting tacitly that she knew the exchange had to do with her ability. Non-verbal responses may have been present but they were not observable for the students.

The mathematical ideas for those students who were inferred to have an active LMTY structure were not found to be different in any obvious way from those students who were inferred to have an LHSIA structure. Therefore, I address the fourth sub-question for the two overarching research questions together, “What mathematical ideas are expressed by the students in the group during the interactions in which one student is attempting to teach or help others?” and “What mathematical ideas are expressed by the students in the group during the interactions in which one student is attempting to show off or impress others?” All the concepts and strategies, which have been discussed throughout the many examples from the data, are specific to the “Building Blocks” task, since the same task was given to all four classes.

Most of the students expressed ideas about how to construct the towers, including how the blocks were added to the previous iterations in order to continue the pattern and the fact that a total of five blocks were added each time. Another pattern that was discussed by Manuel (in Ms. A’s Class 1) and Tyesha (in Ms. S’s Class 2) was the last digit of the total number of blocks ended in either a 1 or 6.

There were several discussions about how to count the total number of blocks, particularly as students were deciding whether the block that was the 1-block high tower remained there for the 2-block high tower and those that came after it. At least one student in five different groups, including Damon (in Ms. S’s Class 1) and Amanda (in Ms. A’s Class 1), recognized that there was one less block in each side than there was in the height. Not all students were able to use this to determine the total number of blocks. But other students, including Amanda’s group, were able to build on the concept that the 100-block high tower had 99 blocks on each of the sides. The students typically recognized that they could multiply 4 and 99 to get the product of 396, and then add the 100 blocks to get a total of 496 in the 100-block high tower.

Some of the incorrect strategies used to find the total number of blocks include an incorrect formula and attempting to use proportions. One incorrect strategy students tried to use was to multiply five by the height of the tower. This can be done, if four is then subtracted from this product, due to the fact that there is one less block on each of the sides than there is in the height. Pedro, Kian, and Rico (in Ms. A's Class 2), however, seemed to recognize the "adding five" pattern and used that to determine that the number of blocks in each of the sides was the same as the number of blocks in the height. Based on these patterns, Kian suggested, "All you have to do is times it [the height] by 5" (line #1:44:03).

In additional, several students, including Carly (in Ms. S's Class 1) and Ta'keisha (in Ms. S's Class 2) attempted to use proportions. For example, Ta'keisha correctly found that there were 21 total blocks in the 5-block high tower, and suggested a proportional relationship based on these numbers. That is, she determined that since ten was twice five, she could also double the total number of blocks, meaning that there would be 42 total blocks in the 10-block high tower. When the students presented their strategies to their teachers, Ms. A or Ms. S, the teachers continued to guide them to correct strategies, and encouraged them to continue finding the generalized pattern and to write down their solutions.

There were two primary data sources used to ascertain if a student may have had an active LMTY or LHSIA structure during the problem-solving session: the video analysis and the questionnaire responses. Here I address the fifth sub-question for the first two major research questions, “What is the relationship, if one exists, between the students’ behaviors as observed in the video analysis and the predetermined items on the self-report questionnaire?” Across the 54 participants for whom we have questionnaire results, only two students who indicated not activating one of the two structures are students that I inferred to have an active structure from their observable behavior. Mitchell (in Ms. A’s Class 2) responded, “Never/Hardly ever/No” to four of the seven items that may indicate a LMTY structure as well as to ten of the fourteen LHSIA items. In addition, Janelle (in Ms. A’s Class 2) responded positively to only four of the fourteen items which may indicate an active LHSIA structure.

There were five students for whom there was agreement from the video analysis and questionnaire data that one or both structures were not active. Students such as Genevieve and Andrew (both in Ms. A’s Class 2) often worked independently and did not contribute to the group discussion very often. Other students, like Rico, were often off-task or did not want the teacher to see what he was doing.

There were 22 students whose questionnaire responses indicated an active LMTY structure though I did not infer this structure from the behavioral evidence. Likewise, there were 11 students whose questionnaire responses indicated an active LHSIA structure though the behavioral evidence did not lead me to infer this structure. Though I reviewed the video analysis for these cases, I still concluded there was a lack of behavioral evidence to suggest the activation of the structure. A student may have had a motivating desire, but no opportunity to engage in the behaviors characteristic of the structure. Alternatively, a student may believe his or her actions were indeed in service of either teaching a classmate or attempting to impress others with his or her mathematical ability, and I interpreted the behavior in a different way. Future research could attempt to address these discrepancies by interviewing the students to ask them about their motivating desires and their behaviors in a stimulated-recall context. Perhaps if the questionnaire instrument is further refined and validated, as the Rutgers University team is presently doing, the items can better ascertain whether the motivating desire was present and whether the behavior corresponded to that motivating desire.

It is interesting to note that 100% of Ms. S's students gave questionnaire responses indicating activation of both the LMTY and LHSIA structures, though there is no indication if they were active at the same time. Also, it is important to note that in these classes, some students' behavior did not lead me to infer a structure for each student. However, in Ms. A's classes, there were six students whose responses suggest that either the LMTY or LHSIA (or both) structure was not activated during the problem-solving session. Additionally, only one of these students is in Class 1, which Ms. A had indicated to be the higher ability class in comparison to Class 2, suggesting there may be a difference between these classes as well. The investigation as to why this may be so (or whether it is due to chance variation) is beyond the scope of the current study. However, future studies should anticipate the possibility of such class to class differences. One potential investigation that might yield interesting results should look at all the questionnaire data to see what additional differences there may be for the other engagement structures. In addition, a study of the teachers' practices over a period of time may provide insight. While one cannot assume the classroom teacher is causing such differences, there may be insight to be gained by examining the relationship between teacher practices and students' engagement structures.

The third research question focused on a comparison between the LMTY and LHSIA engagement structures. There were 10 students for whom I inferred that both structures appeared to be active at the same time, suggesting that these engagement structures are not mutually exclusive – both motivating desires may function simultaneously (possibly with one functioning in service of the other, as discussed below). In fact, it may be possible that most students for whom an active LMTY structure was inferred also had an active LHSIA structure. One student for whom I inferred both structures to be active is Aleana (in Ms. S's Class 2) who shared her thoughts on the problem before the others in her group spoke up. She said, "Oh, I think they wanna know... They want to know how many cubes will they need to build a 5-block high tower" (line 4). In saying, "I think they want to know," Aleana appears to suggest that she realizes what the problem is asking. By expressing this idea, she is asserting her mathematical understanding. She follows up by clarifying this, though she is repeating the instructions, "They want to know how many cubes they will they need to build a 5-block high tower." Aleana seems to be explaining the task and clarifying the instructions for her classmates. In these statements, Aleana appears to be activating the LHSIA structure as well as the LMTY structure. In another example, Onan (in Ms. A's Class 2) is explaining his understanding that the last figure on the task handout (the 3-block high tower) has a total of 10 blocks (though it is really 11 blocks). His classmate Joe asked why Onan is multiplying the height of the desired tower by 10. That is, the 5-block high tower, according to this strategy, should have 50 total blocks since five times ten is fifty. Onan says, "'Cuz the last block is 10 (points to figure C on the task paper)… So that's how we have to do it, take it by 10, it would get us the last total that we have to get" (line

7). Onan seems to be answering Joe's question and explaining his strategy suggesting an active LMTY structure. At the same time, his confident tone of voice as he says, "So that's how we have to do it," suggests that he believes his ideas are correct and that his classmates should follow his approach to these problems. Onan appears to have an active LHSIA structure and it may be the case that his explanations are in service of this structure, making the LMTY structure secondary.

Students who have a motivating desire to help a classmate may first need the confidence in their own ability, intelligence, or competence before sharing their ideas. This relationship between two structures is called "branching" by Goldin et al. (2011). A shift in a student's actions and behavior may be in service of a different engagement structure. Two examples were described in detail in events eight and nine in chapter 4. Event 8 detailed how Christian (in Ms. S's class 2) may have branched from an active LHSIA to an active LMTY when he and his group mates shared answers for the total number of blocks in the 100-block high tower. In event 9, Leo (in Ms. S's Class 2) was inferred to have branched from a LMTY structure to an active LHSIA structure after his classmates appeared not to heed him or not to give his explanations any attention. This hypothesis was difficult to investigate for two reasons. First, the questionnaire was not administered until the end of the problem solving session, so as a static instrument, it was not designed to have students inform researchers *when* they felt certain thoughts or *why* they perceived their behavior to be a certain way. Second, the motivating desire for LMTY seemed to be more overt and apparent than the motivating desire for LHSIA to an observer as it was inferred based on the behavior of a student attempting to assist a classmate. Therefore, in relying on my own observations and inferences, I can only conjecture that this relationship exists and cannot empirically confirm or refute such a conjecture.

For each of the four classes, more students were coded to have active an LHSIA structure than the LMTY structure (based on the observations of students' in-class behavior). While the data are insufficient to allow us to conclude why this may be, one possibility to consider (if the result is generalizable) is that for this population of urban middle school students, the motivating desire to appear smart, competent, or intelligent to others may have been stronger than the motivating desire to help or teach a classmate some aspect of the task or its solution. Recalling that for this study students were observed for just one day of a problem-solving session, one ought to consider that these students see each other every day in class and often work together in math class and possibly in other classes. Each student has presented himself to his classmates each day and must maintain this presentation or performance throughout the school year (Goffman, 1959). In his seminal book, *The Presentation of Self in Everyday Life*, Goffman reminds us that the audience (or in this case, the classmates) may become confused or displeased if an individual behaves in an abnormal way. In this context, a student who tends to try to garner attention from others may be expected to act as though the LHSIA structure is active for her. She may be *expected* to attempt to show off her mathematical knowledge or ability. Another student may be expected to try to help his classmates understand a particular strategy and will attempt to fulfill others' expectations.

5.2 Limitations of this study

There are numerous limitations inherent in this study. One limitation is that the study is not designed to be nearly comprehensive enough for one to generalize these findings broadly. This study took place in four urban middle school classrooms, a population unlike other environments and other age-groups or grade levels. I have drawn from a small sample, including only students from two teachers. The teachers were asked to participate in this study because of their unique characteristics. Specifically, the project researchers had previously seen these teachers encourage their students to argue and defend their ideas, urge students to justify their reasoning, and promote group work within their classrooms. The engagement behaviors of these students are thus likely to be different from those who have different teachers and teaching styles. And the problem task was a specific one; other tasks might evoke different patterns.

Another limitation lies in the reliability of the researcher's interpretations of student behavior based on the video and audio recordings. I made inferences about the middle-school students from their behaviors, words, and tone of voice. I do not know these students, nor am I familiar with their environments and everyday lives. To reduce the effects of this limitation, I did not rely only on my initial impressions of the students and their behaviors, but instead revisited the video and audio recordings multiple times. This allowed me to consider the mathematical and social contexts of what students said and did, as well as to pick up on details which may have been significant in the moment. In addition, another researcher viewed videos of five classes (selected to get a range of classes and behaviors) and independently drew inferences whether Let Me Teach You or Look How Smart I Am was active for a student. We compared and discussed our findings, initially agreeing approximately 80% of the time, and ultimately arriving at 100% agreement, about our conclusions. The codes that were used to help us identify potential LMTY or LHSIA structures were developed for this study and were therefore new to both the other researcher and me. Researchers investigating these two and other structures in the future might consider either identifying codes or particular behaviors prior to analyzing video and corresponding transcript or training verifying researchers prior to the coding process. This might lead to a higher level of initial agreement with reference to which structure may be active for a given set of students.

A third potential limitation lies in the students' questionnaire responses. The students were administered the questionnaire and were asked to respond regarding their experiences in class that same day. When I compare my findings to the students' responses, it is important to remember that the students and I may have interpreted classroom events in different ways. There may be inconsistencies between my findings based on video analysis and the questionnaire responses, though I only identified inconsistencies with 2 of the 54 students. Throughout the problem-solving sessions, student behaviors or motivating desires may have been interpreted in one way while they were intended to come across in a different way. For example, I may have misconstrued an effort to help a classmate as a student trying to impress the classmate with her mathematical ability. For example, I suggested that Georgia (in Ms. S's Class 1) was explaining to clarify how the first four towers were constructed at the beginning of the problem-solving session (lines 4 – 12). However, she may have had a motivating desire to impress her classmates with her level of understanding instead of trying to help them.

The questionnaire is a static instrument intended to measure dynamic interactions. Therefore, the students may have responded only to some classroom moments, such as those that stand out for the student. I consider the case of Christian (in Ms. S's Class 1), who is mentioned earlier. Though I have identified several behaviors from which I identified an active LMTY structure, most of his corresponding questionnaire responses suggest he did not activate the LMTY structure. Perhaps he did not recall moments at the beginning of the problem-solving session during which I observe he is explaining how the towers are constructed.

It is also possible that some students did not interpret some questionnaire items in the way they were intended, or that other students were not motivated to respond honestly to all items. I speculate if this may be the case for Mitchell (in Ms. A's Class 1) who gave mostly negative responses to the items which may indicate either an active LMTY or LHSIA structure. I inferred from his behavior that each of these structures was active or present at least once for Mitchell. Any discrepancies between the questionnaire responses and video analysis findings may be the result of the students' interpretation, rather than my misinterpretation of behavior.

5.3 Theoretical implications of the study

My research builds on the theoretical work of Goldin, Epstein, Schorr, and Warner (2011) who describe the psychological construct they called an *engagement structure*. My focus on two such structures, Let Me Teach You and Look How Smart I Am, is an initial attempt to systematically investigate empirically these patterns of engagement, and possibly to lay the groundwork for validating the construct. By recording observed behaviors of urban middle-school students, I presented explicitly the evidence I used to infer the activation of these two engagement structures.

The features presented in chapter 4 are generalized recurring behaviors for each engagement structure. While they emerged from the data, I see them as potentially satisfying some of the proposed strands of engagement structure as described by Goldin et al. (2011). Specifically, they seem to describe the "(1) characteristic *goal* or *motivating desire*" and the "(2) characteristic *patterns of behavior* including social interactions oriented toward fulfilling the desire" (p. 549). Future research should attempt to address the other strands, particularly those addressing *affective pathways*, *expressions of affect*, and *meta-affect*. To fully understand how all the proposed strands interact for each engagement structure, additional work needs to address each strand. This study addressed "in-the-moment" engagement, but as Goldin et al. (2011) note other strands that refer to longer-term traits (e.g., *beliefs and values*) impact the engagement structures as well and should be addressed in future research. For mathematics educators, understanding these engagement structures may provide a lens through which they can better grasp how behavioral, cognitive, and affective engagement are intertwined.

5.4 Implications for teaching

Classroom teachers of mathematics who may be looking for ways to increase student engagement for their students may benefit from learning more about these engagement structures. These structures may provide teachers with the language (based on research evidence) to describe in-the-moment motivating desires and engagement, as well as accompanying behaviors. Recognizing the potential impact the engagement structures may have on student learning can help teachers make decisions about how to support productive interactions between and among students who are working on mathematical problems. This work aims to contribute to the goal of having a way to document in-the-moment engagement and motivation, either via observation or class surveys. Teachers may find this appealing in that they can immediately employ pedagogical strategies to encourage productive interactions between and among students who are working together.

5.5 Future research

As mentioned above, this study included the participants from a small and specific population. I think it is important to continue research on LMTY and LHSIA with students from other backgrounds and at other grade levels to determine if similar patterns of behavior are observed, and what differences may exist. Some directions for future research were mentioned earlier. Interviewing students after an observed problem-solving session during which the student completes a questionnaire may assist in finding a methodological manner to have reliability between the questionnaire results and the video analysis. All researchers involved in the study ought to have a clearly defined coding scheme and process for applying the codes, when possible. Identifying the teacher's influence on student engagement and which structures become active may help future teachers when the research can be presented in a useful way.

For this study, I focused on only two of the ten strands, though they all contribute to the description of engagement structures. Both observational and questionnaire data ought eventually to address strands regarding affective pathways, beliefs and values, and all the other strands. A questionnaire that has an item for each strand for each proposed engagement structure may be overwhelming for a study participant, but could help build a more complete picture of these engagement structures.

One potential study might ask students to complete a pre-observation questionnaire about beliefs, values, and other longer-term traits. Consider the hypothetical example of a student who claims that completing a mathematical problem quickly is evidence of being smart. This student may hurry through the work during a problem-solving session and may announce to others when he believes he is done with the work. This student's behaviors alone might lead researchers to infer that he has an active Get the Job Done structure; but for this student, the behaviors may be directly related his belief that "quick" equals "smart," suggesting the LHSIA structure is active. For another student, Get the Job Done may be active in service of a desire to please the teacher or the desire to spend remaining time on another activity. Such interrelationships may prove to be important in understanding how each strand contributes to the engagement structures.

This study has focused solely on two engagement structures, allowing me to develop an understanding of each more deeply. A complete picture can be constructed after other structures are investigated in ways similar to the investigation of LMTY and LHSIA. However, by looking so intently for evidence of the activation of LMTY and LHSIA , I may have inferred one of these two structures when another structure may have been more appropriate. For example, a student for whom I inferred an active LHSIA structure may have merely been looking for attention in any way she can. Though she may have chosen the mathematical environment to do so, perhaps her behavior and motivating desire can be better identified by a not-yet-described structure such as "Focus on Me" or "Value Me." Future studies ought to put into place additional reliability measures, in order to better distinguish among the active engagement structures.

This study utilized both video data and questionnaire data. Surveys and questionnaires have been and will continue to be essential in understanding teachers' and students' perceptions of the classroom. The questionnaire instrument used for this study should be further developed and tested for internal reliability and validity, and revised as necessary. In addition, observations, interviews with students and teachers, and other qualitative data (e.g., student work) may help us better understand all that contributes to student engagement.

The future directions of research on engagement structures may build on the current study, or may travel a very different course. The possibilities of study populations and specific foci (e.g., various strands, different structures, different student populations, teacher influences) are wide-ranging and exciting. Future research will contribute to a more comprehensive understanding of engagement structures and how they can contribute to the field of mathematics education. As researchers continue to build on this work to understand the benefits of student engagement, it becomes increasingly important to better understand the complexity of students' "in the moment" engagement in the mathematics classroom.

Appendix A: Field Note Protocol

Date: ________________ School: _______________________
Teacher's name: _______________________ Class #: _________
Observer's name: __________________

NOTE TO OBSERVERS: Below is a list of categories of things to be aware of and note while you are observing your group or group(s) of students. Please take the time to read this list prior to the observation to familiarize yourself with things of importance to write about.

Possible Roles students may take on in a group (you may observe others, which is good, so do not limit yourselves to these alone):

- Group leader;
- Copycat (student who appears to be copying down whatever their group mates are writing—possibly with or without thought);
- Task-master (makes sure every student has a task to complete and that they are completing those—may or may not be different than group leader);
- Teacher (explains procedures or concepts related to the mathematical task to another group member or member(s));
- Time-keeper (keep track of the amount of time the group may have to complete a certain task or task(s) during the class period);
- Loner (rarely or never interacts with other group members; barely speaks to other group members);
- Reporter (student who appears to be the spokesperson for the rest of the group—i.e. explains group's ideas when the teacher arrives at the group or when the group is questioned by someone…could also be the student that presents the group's idea to the rest of the class during whole-class group presentations);

Criteria of "interesting" groups:

1. Talkative group; Members are verbally communicating
2. Communicated disagreement w.r.t. the problem, strategy, and/or solution
3. Obvious teamwork: idea sharing, building off one another
4. Multiple types of group dynamics within a session (e.g. work alone in parallel, then share, then build on their ideas together)
5. One (or more) students trying to explain or teach a group mate
6. One in the group takes charge: a leader, task-master
7. Change in the "leader" of the group; changes in any of the other roles (see possible list above);
8. Express good/unique/different strategies to approach or solve the problem
9. Taking the task to another level (i.e. posing a "What if…?" scenario and/or raising the cognitive demand of the task)

Events where teacher is involved in student difficulty

**Student difficulty includes any time the teacher perceives that a student has trouble with a problem, does not necessarily involve a student error

Specifically:

- Time at which teacher becomes involved
- Description of student's difficulty, including how difficulty came about (i.e. did a student ask the teacher a question about something he/she did not understand, did the teacher/another student ask a student a question that he/she could not answer)?
- Teacher response (ex: probing the student, opening the question to the rest of the class, giving the student time to think about the difficulty)
- Description of behavior related to student's affective engagement following teacher response
- Inference of student's emotions based on affective behaviors following teacher response
- Description of student's cognitive engagement before/after difficulty

Please write the names of the Students and where they were seated at the group tables you observed.

FRONT OF ROOM

Group #:_____

Group #:_______

Table #	Time	Structure	Interpretative comments	Descriptive Details

Appendix B: Questionnaire Items for Inferring Engagement Structures

Print Your Name:__________________________ Today's Date:___________________
Your Class:____________________ People in your group:________________________

We will read the first few questions to you and then you will read them yourself. After each question, please indicate your answer. For each question, please circle one of the 3 answer choices. The 3 choices are:

0: "It was **never** this way for me in this class **during this lesson.**"
1: "It was like this for me **some of the time** in this class **today.**"
2: "It was like this for me **all of the time** in this class **today.**"

11. I wanted people to think that I'm smart.

0 never **1** some of the time **2** all of the time

12. I tried to impress people with my ideas about the problem.

0 never **1** some of the time **2** all of the time

13. People seemed impressed with the ideas I shared about the problem.

0 never **1** some of the time **2** all of the time

14. People saw how good I was at the math we did today.

0 never **1** some of the time **2** all of the time

15. I felt smart.

0 never **1** some of the time **2** all of the time

16. I wanted to teach another student something that I knew that this other student did not know.

0 never **1** some of the time **2** all of the time

17. I listened carefully to the ideas of someone I was trying to help.

0 never **1** some of the time **2** all of the time

18. I helped someone see how to do the math.

0 never **1** some of the time **2** all of the time

19. Others listened carefully to my ideas

0 never **1** some of the time **2** all of the time

20. I wanted to show someone that my way was better.

0 never	**1** some of the time	**2** all of the time

42. I was a lot better at math than others today.

0 never	**1** some of the time	**2** all of the time

Here are some ways people behaved in their group in class today.. Think about how you behaved. Then tell us how you think you behaved in your group today.

0: "I **hardly ever** behaved this way in my group **today.**"
1: "I **sometimes** behaved this way in my group **today.**"
2: "I **often** behaved this way in my group **today**."

In my group today:

2. I was the leader ………………………	**0** hardly ever	**1** sometimes	**2** often
5. I gave helpful suggestions ………………	**0** hardly ever	**1** sometimes	**2** often
7. I was bossy ………………………	**0** hardly ever	**1** sometimes	**2** often
9. I wanted to show off ……………………	**0** hardly ever	**1** sometimes	**2** often
14. I worked cooperatively ………………	**0** hardly ever	**1** sometimes	**2** often
16. I liked to be right ……………………	**0** hardly ever	**1** sometimes	**2** often

Students have told us about some of the thoughts they have had while working in their math class. Read each of the thoughts they described to us. Circle **yes** next to each thought that you had in class today and circle **no** if you did not have this thought today.

6.	yes no	I want you to know just how smart I am.
11.	yes no	People think I'm smart.
12.	yes no	This stuff is really boring.
14.	yes no	I wish the teacher would call on me, so I can show how much I know.
25.	yes no	I like teaching this person things that I know.

Appendix C: Video Analysis Summary Sheet

Video: (Title/Name, with Teacher, Class, Date, Time); Audio: (File name, Time)

Evidence of: LMTY _____ LHSIA _____ Both ______ None ____

Let Me Teach You

Student Name / Time in Video	Evidence	Yes/No
	Trying to teach or explain a mathematical idea or procedure to a classmate	
	Trying to explain why a particular step contributes to the solution to other students	
	Trying to explain why a particular answer makes sense to a classmate	
	Trying to respond to a request made by a classmate to explain something related to the problem and follows through with such an explanation	
	Trying to inquire if other students understand something about the mathematics or the problem	
	(*Other*)	

Look How Smart I Am

Student Name / Time in Video	Evidence	Yes/No
	Trying to demonstrate his or her knowledge to a classmate(s) or teacher, not necessarily in the service of trying to teach others	
	Trying to show that his or her solution method is better than those suggested by other students	
	Trying to impress classmates with what he or she knows (ability, knowledge, or intelligence)	
	Trying to argue in support of his or her mathematical ideas whether or not there is disagreement from another individual	
	(*Other*)	

Ms. A Class 1 Introduction to Building Blocks Task

This is the introduction Ms. A. gave to her grade seven Class 1.

T: Ms.S / Teacher
St(s): Unknown Student(s)

T: **Okay class let's start. Building block dilemma. I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5 block high tower, a 10 block high tower, and a 100 block high tower. Generalize if you can on how many blocks I will need for any size tower. So how many stages do we have here?**

Sts: Three

T: **So you need to come up with your 5 block tower. The first one is how many blocks?**

Sts: One

T: The second one?

Sts: Five. Six.

T: **Let's be more specific, if I am talking about the height of the tower. Let's go by the height of the tower, so the first one?**

Sts: One.

T: The second one?

Sts: Two. Six.

T: The third one?

St: Three.

T: We're talking about height. Okay I want you to start a new *inaudible* now, and I'm going to be going around.

Samara, Julian, Abby

This is Group 1 from Ms. A's grade seven Class 1.

Verbal emphasis indicated by <u>underline</u>
Actions included either in *italics* or (parentheses) as appropriate.
Pauses in speech are indicated by …
Words inserted to help clarify a student's words are included in [brackets].
When students voices overlap, one or both sets of words are included in //double slash marks//.

The number included next to either LMTY or LHSIA refers to the code, included in the table of codes.

Samara, Julian, Abby			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
	The teacher has just finished her introduction to the problem and given instructions.		
1	Julian (J)	Alright, to make 5 blocks, we have to keep on adding blocks on the side.	**EPISODE 1** Students appear to want to start working toward a solution.
2	Abby (A)	Yeah, because this one only has one.	
3	J	If you add one for each side and one on the top, yeah.	
4	A	It's just going to keep going wider and higher.	
5	J	So we have to draw it?	
6	A	Yeah I think so.	
7	A	We need to add more blocks on the side. It says I would like to know how many cubes I need to build a 5 block high tower. It says, look, it says we need.	
8	Samara (S)	Okay. *Students are given markers to write with.*	
9	A	It says we will need to build a 5 block high tower, a 10 block high tower, and a 100 block high tower. It says to generalize if you can on how many blocks we will need for any size tower.	

Samara, Julian, Abby			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
10	S	What does generalize mean?	
11	A	I don't know. *Students look around for a minute*. Alright, look. We need scratch paper. (Sounds like student mumbling in response.) So, we need to construct, do we need real blocks? (No response from two classmates) Let me ask a question. If we need a high block tower, we need to make it like this.	
12	S	It's the height?	
13	A	Two, 3, 4, and then 5. (drawing on her paper) Look, you see how it's like this? (shows Julian her paper)	
14	S	What's holding it?	
15	A	What?	
16	S	What's holding it?	
17	A	It's too (inaudible) If they saying you need to make a 5 block high tower, you need a flat top, because you only have 5 blocks, 1, 2, 3, 4, 5. Look if…	**LMTY (1)** Abby responds to Samara's question. She explains her understanding of what a 5-block high tower is.
18	J	No, no, no, listen, look, see how its, alright, this is a 1-block and this is a 2-block high tower, (pointing to the corresponding diagrams on the task paper) so you just adding 1 more to each side, like that, look, and this is a 3-block high tower. And add two more.	**LMTY (1, 4)** Julian corrects Abby, suggesting that her understanding of a 5-block high tower is incorrect. He explains his understanding, pointing out on the diagram the 1-, 2-, and 3-block high towers.
Interpretation: As Abby starts explaining how she is thinking about the 5-block high tower, Samara asks, "What's holding it?" though I am not sure what she meant by that. Abby then responds to the question, explaining her understanding of the 5-block high tower. Since she says, "you need a flat top, because you only have 5 blocks" and given Julian's response, it seems that Abby thought that the entire tower structure only had 5			

Samara, Julian, Abby			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
blocks. This is different than having a height of 5 blocks, which is what Justin says in the next turn. Justin seems to have heard Abby's explanation, and corrects her, saying "No, no." He refers to the task paper, pointing to figure A and calling that a 1-block high tower. He points to figure B when he says it is a 2-block high tower. When he points, he appears to be trying to indicate that there are two blocks in the middle, making up the height. By correcting Abby and pointing to the diagram papers, he seems to be explaining that the 5-block high tower has a height of 5 blocks, and does not have a total of 5 blocks.			
19	S	We need a 5.	
20	A	You need a 5.	
21 10:38 :35	J	Okay look, so it's going to be like this. 2, 3, 4, 5, 6.	
22	A	Then we need to make, we need to make the 4 block tower, and the 5 block tower.	
23	S	5,10,100.	
24	A	So, but we need to make the 5 block tower.	
25	S	But we need 4 blocks.	
26	A	Yeah, but to see, to see how to do 5 block tower, we need to do 4 block tower. So four block, four, four. (Counting)	
27	J	Look see, look, I think this is the one right here. (Counting)	
28	A	And then just add the number on. Three. Alright, then that's a high tower. (counting) Now you see what I'm doing. And then 1, 2, 3 right here. And then 3 here is right here. In the middle? There's 5 [blocks]. There's 5 going up, so on each side there going to be 4. And then in the middle, going up, there's going to be 5. Cause, look Julian, cause look, cause on each one you have to add one more to each side. (putting her paper in front of Julian's paper) Yeah, so look, so if that's 3, and that's 2, 5 and then 4 on each side, 4 on each side, 4 on each side and then 5 going up.	**LHSIA (14)** **LMTY (1)** Abby first seems to talk out loud as she is working. She then demands the attention of her classmates when she tells them to "look." She continues by explaining how she thinks the towers are constructed, using the figures on the task paper as examples.

<table>
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<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>29</td><td>J</td><td>So this is 5?</td><td></td></tr>
<tr><td>30</td><td>A</td><td>Yeah, 5 going up.</td><td></td></tr>
<tr><td>31</td><td>J</td><td>Okay.</td><td></td></tr>
<tr><td colspan="4">Interpretation: Abby starts by speaking out loud as she appears to be figuring out some of the details about the towers. She starts to explain that there are 5 blocks “going up” for the height, and there are 4 blocks on each side. She does not seem to have the attention of her classmates, so she tells them to “look,” demanding their attention. Abby also puts in her paper in front of Julian in order to get his attention. She continues to explain her idea, which seems to be that for each tower, there is 1 less block on each side than there is in the height, “if that’s t3 and that’s 2, 5 and then 4 on each side.”</td></tr>
<tr><td>32</td><td>S</td><td>I don't get it. (to Abby who does not respond)

Students are all writing or drawing on their papers. Sometimes we hear them counting out loud.</td><td>EPISODE 2
Samara indicates that she does not understand what her classmates are doing, how they are thinking about the problem. Both Abby and Justin (eventually) respond to her questions.</td></tr>
<tr><td>33</td><td>A</td><td>It’s going to look like a cross, you see? (counting)</td><td></td></tr>
<tr><td>34</td><td>S</td><td>What are you doing?</td><td></td></tr>
<tr><td>35</td><td>A</td><td>No, I'm not doing nine, I'm doing the 5 high block tower. If you’re doing five, if you’re doing five it’s going to have to be 4 on each side. Like four on each side.
//J: Four on each side//
Yeah, and then there’s going to be five in the middle. See?</td><td>LMTY (1, 7)
Abby first responds to Samara’s question, “What are you doing?” by stating that she is “doing the 5 high block tower.” She then goes on to explain how she is thinking about the 5-block high tower, essentially repeating her</td></tr>
</table>

Samara, Julian, Abby			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
			explanation from earlier.
36	J	Cause the middle one is the fifth one.	
37	A	Yeah, so look. (puts her paper in front of Samara) On each side there's going to be: 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4 (counting) The middle block is the fifth one, so, and the middle block you need a stack of 5, since its only 2 dimensional, I can only put the number 5 there. But I know I need to stack 4 more. There.	**LMTY (2)** Abby modifies her explanation to Samara by showing her what she has drawn already and counting out the number of blocks on each side. She also explains that she is drawing in 2 dimensions a 3 dimensional structure.
38	J	(to Samara) Look, look, listen, you see, look at, you see, um, the middle one is the third one for each side. See A? The first one is in the middle.	**LMTY (2, 6)** Justin builds on what Abby is saying and also modifies the explanation for the construction of the 5-blcok high tower. He refers to the first stage, figure A, to indicate where the middle block has come from.
Interpretation: This episode which includes an active LMTY structure for both Abby and Julian seems to be brought on by Samara's question, "What are you doing?" and may also be brought on by Samara's statement, "I don't get it" a few moments earlier which seemed to be ignored at the time. Abby is the first to respond to the question, and essentially repeats her explanation earlier that there are 5 blocks in the middle (height) and 4 blocks on each of the sides. The students do not seem to count up the total number of blocks yet but are focused on the construction of the towers. Abby then appears to modify her explanation a little, by putting her paper in front of Samara so she can use what she has done so far to help. She also counts the individual squares, which represent blocks, for each of the sides. Because she is drawing this 2-dimensionally (as if a birds			

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<tr><td colspan="4">eye view), Abby explains that she cannot draw the 5 blocks that make up the height, but she does put the number 5 in the middle to indicate they are there (see student work).</td></tr>
<tr><td>39</td><td>A</td><td>Yeah, and then each one, you have to add one more to each side. That’s why this one is two, this one is three, there must be another four, and then one more is five.</td><td></td></tr>
<tr><td>40</td><td>J</td><td>I really don't get it, do we have to draw it?</td><td></td></tr>
<tr><td>41</td><td>A</td><td>Uh, yeah, I think we have to draw it.</td><td></td></tr>
<tr><td>42</td><td>S</td><td>Permanent marker? (waves marker)</td><td></td></tr>
<tr><td>43</td><td>A</td><td>I guess so.
(looks around the room, perhaps as if to see if she can ask the teacher)</td><td></td></tr>
<tr><td>44</td><td>A</td><td>Cause that would make it like a little bit more easier so we can like.</td><td></td></tr>
<tr><td>45</td><td>S</td><td>I don't know how to draw.</td><td></td></tr>
<tr><td>46</td><td>A</td><td>(leans toward Samara, using her pencil to simulate drawing)
Look, like draw two, I mean four on each side, and then, and then another one in the middle, and then a four on each side, and then a five in the middle because, don't draw it like that, just draw, let me draw.
(returning to her own paper) Now this is a ten block high tower. And a nine.</td><td>LMTY (1)
Responding to Samara’s statement, I don’t know how to draw,” Abby gives a suggestion for how to draw the 5-block high tower on her paper.</td></tr>
<tr><td colspan="4">Interpretation: Abby and Julian had been discussing a strategy for the construction of the tower structures. They both suggest that they must draw representations of the tower structures. Samara, who has expressed that she did not understand the strategy, says, “I don’t know how to draw.” This seems to prompt Abby into suggesting a procedural method for drawing the tower representations. Abby’s actions suggest that she wants to help Abby, since she leans in toward Samara and uses her pencil to simulate what the drawing might look like.</td></tr>
<tr><td>47</td><td colspan="2">These students spend the next 7 minutes mostly drawing and writing on their own papers. Abby and Julian, in particular seem comfortable drawing their 2 dimensional representations of the 5-block high tower and the 10-block high tower. Abby reminds Samara that for the 10-block high tower there are 9 blocks on each of the four sides. This leads her to say that there are 99 blocks on each of the four sides for the 100-block high tower. The students discuss whether that is something they can draw. Someone (not sure who) suggests that they should explain</td><td></td></tr>
</table>

Samara, Julian, Abby			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
	that in words, rather than try to draw it out. Much time is spent without discussion, while the students continue to work independently on their own papers.		
48	*The teacher, Ms. A, comes to the group. Abby starts by explaining how she and the others have drawn the 5-block high and the 10-block high towers. She suggests that the 100-block high tower is going to be "a lot of boxes." It seems that at first Ms. A focuses on Abby, who brings Julian into the conversation by glancing at him, perhaps in a way that asks him to contribute his ideas as well.* *Ms. A says what they have done so far is "interesting" and asks them how many blocks are in the 5-block high tower. The students had not yet determined this result, and determine that there are 21 blocks in the 5-block high tower. Ms. A asks them to explain that as well as explain their answers for the 10-block high tower. During this conversation, Samara does not seem to say anything. Yet, she does seem to write down answers, such as a total of 21 blocks, as the others discuss it.* *When Ms. A asks about the 100-block high tower, Julian chimes in saying that they could multiply 4 times 99, which is the number of blocks on each side, and then add 100 blocks for the height.* *Ms. A then encourages them to come up with a general rule that could be used for a tower with any height, before moving on to another group. She has spent about 4 minutes with this group.*		
49	J	We have to develop the rule. The rule is… alright. See, this a two block high tower, this one, all you have to do is add. *Talking over one another*	**EPISODE 3** **LHSIA (14)** Julian demands attention to his idea about what the rule is. He states, "The rule is…" but then is unable to complete his thought.
50	A	No, negative, yeah. Like, no, subtract the 1 from each side if the height is 2. If the height is 2, there is gonna be 1 on each side. If the height is 3 there is gonna be 2 on each side, because you need to subtract 1.	**LHSIA (8)** Abby corrects Julian who suggested adding a number. She states that they need to

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<tr><td></td><td></td><td></td><td>“subtract the one from each side.”</td></tr>
<tr><td colspan="4">Interpretation: After Ms. A leaves the group, Julian jumps in to say that they “have to develop the rule,” and he tries to express what he thinks the rule is. He may be trying to demonstrate to the others his mathematical ability by suggesting he knows what the general rule is.
Abby listens to him and disagrees with a way that he starts to explain, “add a negative…” She corrects him saying that they “subtract the 1 from each side.” She corrects him and continues on with her explanation, giving specific examples of how many blocks will be on each side for certain heights. In doing this, Abby may be trying to demonstrate her own mathematical ability to her classmates.</td></tr>
<tr><td>51</td><td>J</td><td>We can do a T chart, a T chart right here and we can put height of the tower, how many blocks on each side, you could put the 10 block tower.</td><td></td></tr>
<tr><td>52</td><td>A</td><td>Where do I put the T-chart, right here? (indicating to a space on her paper)</td><td></td></tr>
<tr><td>53</td><td>J</td><td>Give me that piece of paper.
(Samara gives him a piece of blank paper.)
Oh, you can do it like this. (Abby and Samara look on as Julian writes on the paper just given to him.)
Height, tower, like that, look, um, on each side. Alright, now the 10 block on each side is gonna be 9 blocks. You could say, and then the twelve block one, because it would be 11.</td><td>LMTY (1)
Julian suggested that they could organize their information in a T-chart. He responds to Abby’s question by drawing a T-chart as an example, including the height of the tower and the number of blocks on each side.</td></tr>
<tr><td>54</td><td>A</td><td>Yeah, and then instead of making the T-chart we can make, we can make it like on this side it could be the total blocks, the total of the blocks of each column, we could make it like a chart.</td><td>LMTY (6)
Abby builds on Julian’s idea of the chart, suggesting another column to include the total number of blocks in the tower.</td></tr>
<tr><td colspan="4">Interpretation: Since Abby and Julian seem to agree on the general idea, Julian suggests that they organize their information in a T-chart, which would include the height of the tower and the number of blocks on each side. (See student work.) Though Abby asks</td></tr>
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Samara, Julian, Abby			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
about where they should write down their T-chart, Julian responds by describing and writing what the T-chart might look like. He includes the specific example of the 10-block high tower, including the fact that each of the four sides has 9 blocks. His tone of voice is quiet, which is typical for him during this problem solving session, suggesting he wants to share his idea with his classmates. Abby responds by building on his idea, suggesting that they add a column which gives the total number of blocks in the tower structure.			
55	J	Since we can't do the 100 one, we can just do it on the chart.	
56	A	You mean like what like what we did with these, the rest of em (inaudible) Heights. (Inaudible)	
57		*For the next 4 minutes:* *Abby and Julian continue to write on their papers while Samara seems to be looking on and re-reading the original problem. After a brief discussion about writing in marker (as the researchers and teacher asked) or in pencil, Samara starts to write on her paper. She often looks to Abby's paper as well. It is possible she is just copying down information, and may not understand the strategies or information that Abby and Julian have discussed so far. Up until this point, Samara has not contributed to the conversation, except to sometimes ask questions, and has not demonstrated clearly that she understands the problem or any solutions.*	
58	S	It's 4 on each side, right?	**EPISODE 4**
59	A	Yeah, for the fifth one? There's four on each side. And then in total you just need to add the four sides, which is four blocks on each side and then the middle which is 5. And it'll give you 21.	**LMTY (1, 3)** Abby responds to Samara's question, regarding the 5-block high tower. She confirms that there are 4 blocks on each of the four sides. She adds that they can find the total number of blocks by adding up the number of

Samara, Julian, Abby			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
			blocks on the four sides plus the number of blocks in the height.
60	S	Alright. Are you sure its 21? (Abby seems distracted by activity going on in another part of the room – off camera. In order to get her attention, Samara kicks at her under their desks.)	
61	A	Yeah, because if you see this one, it's three. (uses her pencil to point to Samara's paper) You, but you can't see the (inaudible) block. This one, it's 3. (inaudible)	
62	S	(Counting, to 12)	
63	A	You can't count this more than once. You counted it like 3 times. Look. I'm trying to make sense. Look. If that's 3, right? 3 plus 2 plus 2 plus 2 plus 2 (enters into calculator) ... equals 11. See, so this equals 11 blocks.	**LMTY (3, 4)** Abby explains how to count the total number of blocks in a tower structure, using the 3-block high tower as an example.
64	S	11.	
65	A	Yeah, it's 11 blocks. C is 11 blocks.	
66	J	C, where did you get? Oh.	
67	A	C is, because I was trying to explain to Samara, she's telling me how this five block high tower has 4 sides. I have 4 blocks on each side, what I explained to her with number 3 is that in number 3 you see, you can't see the bottom one, but there is 1 in the bottom. So that's three right there, you just need to subtract one. One from each. If you're going to do three on each side, then the high block tower has to be 4. That's all I was explaining to her and showed her in the calculator.	**LMTY (3, 5)** Abby responds to Julian's question about the tower in figure C. She restates some of her earlier explanations as well as provides additional details about the middle hidden block.
Interpretation: Samara begins this episode by confirming with Abby that the sides in the 5-block high tower each has 4 blocks. Abby confirms this, and confirms this by			

Samara, Julian, Abby			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
providing more details, particularly the total number of blocks (21) that the 5-block high tower has. Samara continues to ask questions, wondering, "Are you sure it's 21?" Abby responds yes, but uses the 3-block high tower to explain why. It is possible she used the 3-block high tower because the diagram was provided on the task paper. When Samara incorrectly counts 12 blocks in the 3-block high tower, Abby corrects her, letting her know she can't count the middle block multiple times. She demonstrates the correct result by showing her which numbers to add, "3 plus 2 plus 2 plus 2 plus 2." By providing this new level of detail, Abby is attempting to show Samara the strategy for determining the total number of blocks. Julian had not been part of the conversation so far and asks a question about the total number of blocks in the 3-block high tower. Abby lets him know that she explaining to Samara the rationale behind adding "3 plus 2 plus 2 plus 2 plus 2." She added that if there were 3 blocks on each of the four sides, then the height is supposed to be 4 blocks. Abby has restated her own explanation to Samara for Julian, so that he is able to understand as well.			
68		*The students write on their own papers and make comments that are off-task.*	
69	A	Alright, for the ten block tower, for the ten block tower, you would get, nine on each side. Um (counting) And the total blocks, of blocks, is 46 blocks. Oh, and now for the 100 blocks towers. (She's talking out loud, but it's not clear whether Julian or Samara is listening – Samara responds to Abby)	
70	S	Excuse you.	
71	A	You just need a, 99 plus 99 plus 99 plus 99 plus 100 equals 496. (using calculator)	
72	S	How much?	
73	A	496. 99 (talking to herself) 496 for.. Now we could explain, how we did each one on the back.	
74	J	I'm on this one. (Julian has been drawing what appears to be the 10-block high tower on his paper)	
75	A	What is that? The 100?	
76	J	No, the ten. But, I need that. We need that first.	
77	A	Oh the chart? (Gives her paper to Julian, who seems to copy down what Abby has written). This is easy.	**LHSIA (6)** Abby has been speaking out loud

Samara, Julian, Abby			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
			as she writes down her answers. She claims that the task is easy.
78	S	Alright.	
79	A	(explaining to Julian what she has written, leaning over and pointing at her paper) The first is the height of the tower, the second is on each side. And then on each side. And then total of blocks in tower.	
Interpretation: Abby states, "This is easy," likely referring to the task they have been working on. She has been talking and sharing her thoughts and she continues to work on getting the answers and writing them down. While she seems willing to share her ideas, this may be so that she can demonstrate how smart she is to her two classmates. Both Samara and Julian seem to turn to her with questions. They also seem to be writing down exactly what she says or writes, whether or not they understand the ideas themselves.			
80		*For about 10 minutes:* *Abby and Samara briefly discuss that to "generalize" (from the instructions) might mean to find a conclusion.* *While Abby and Julian continue to write on their own papers, Samara mostly just sits and looks around but does not write on her own paper.* *Students have brief discussions, such as checking with one another about having 46 total blocks in the 10-block high tower. Some of these discussions appear to be off-task.* *Samara eventually resumes writing on her paper again.*	
81	J	The hundred is 492, right?	**EPISODE 5**
82	A	Yeah, cause you just multiply it. 496! Because you multiply each side which is…	**LHSIA (8)** Abby corrects Julian's suggested answer of 492 blocks for the 100-block high tower.
83	J	99 times 5 plus 1	
84	A	99. (starts entering into calculator) (looks up as if shocked that Julian said 5). No, no, not five. 99 plus 99 plus...	

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<tr><td>85</td><td>J</td><td>(takes calculator away from Abby) No. It’s not… Listen to me.</td><td></td></tr>
<tr><td>86</td><td>A</td><td>Its four sides. (waving her four fingers at Julian)
1,2,3,4. (points to one of the diagrams on her paper)</td><td></td></tr>
<tr><td>87</td><td>J</td><td>Look. 1, 2, 3, 4, 5 sides.
I'm not talking about that, I'm just saying at all.</td><td></td></tr>
<tr><td>88</td><td>A</td><td>Oh, okay, don't multiply by five because then you will get the wrong answer, because this is not 99. (pointing at Julian’s paper)</td><td>LHSIA (7)
Abby tells Julian that he should not use a strategy of multiplying by 99 (the number of blocks on each side) by 5 since the height is not 99 blocks, but rather 100 blocks.</td></tr>
<tr><td>89</td><td>J</td><td>I add the middle after one.</td><td></td></tr>
<tr><td colspan="4">Interpretation: When Julian asked a question to confirm the total number of blocks in the 100-block high tower, Abby took the opportunity to correct him (the answer is 496, not 492). Her tone of voice, which is raised slightly, and her action to start explaining why suggest she may be trying to correct his reasoning (or teach him). However, perhaps because of Julian’s interruption, Abby seems to activate a LHSIA structure. Julian starts to explain his strategy of determining the total number of blocks in the tower, which is a little different than Abby’s (though both appear to be correct). She does not appear to accept his explanation though, telling him, “Don’t multiply by 5 because then you will get the wrong answer.” Abby appears focused on her own strategy, which based on what she has said so far, seems to be based on adding the number of blocks, rather than multiplying. By not listening to or considering Julian’s strategy, Abby seems to have an active LHSIA structure, because she seems to believe her strategy is the correct one.</td></tr>
<tr><td>90</td><td>A</td><td>After saying something inaudible to Samara, Abby raises her hand, probably to get the teacher’s attention. She seems to ask the teacher, who is off-camera if she can get blocks. She then gets up and gets a set of SnapCubes.</td><td></td></tr>
<tr><td>91</td><td>A</td><td>Let’s make a demonstration of the, demonstration of a 5 block.
Abby starts putting together a tower structure, specifically the 5-block high tower.</td><td></td></tr>
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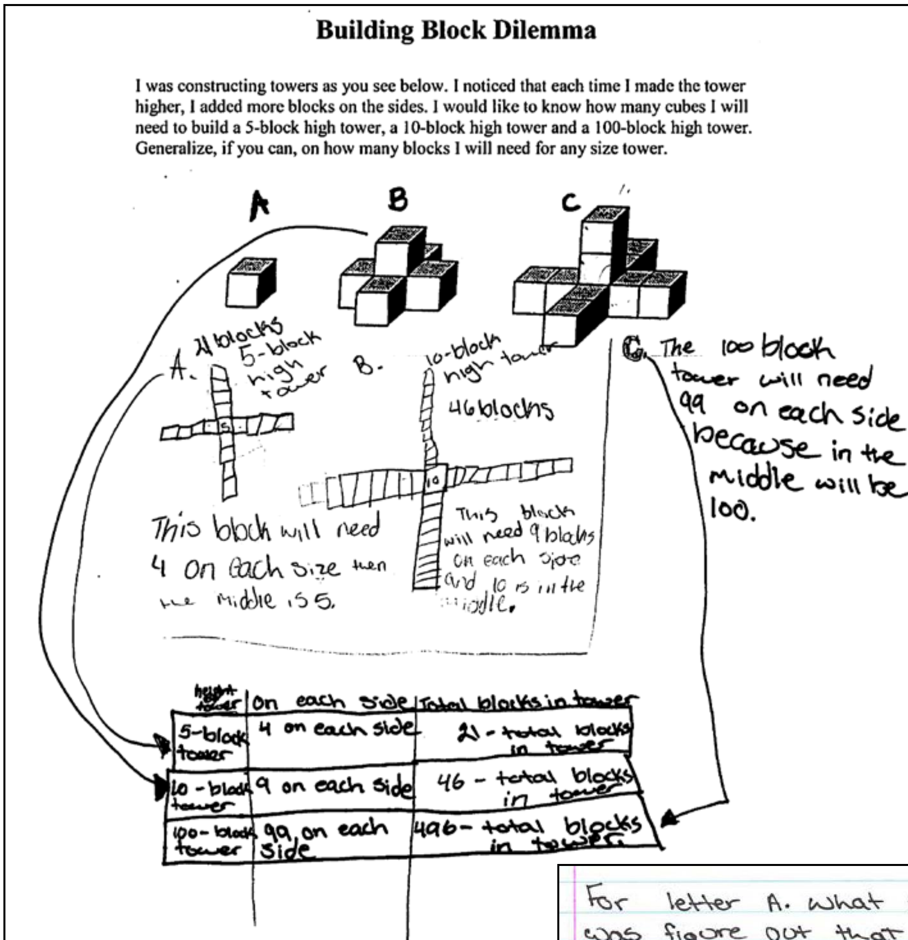
Samara, Julian, Abby			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
		Julian and Samara continue writing on their own papers and don't seem engaged with what Abby is doing.	
92	A	Look this is a 5 block. (showing Julian the structure constructed with the cubes) This is a 5 high block tower. (counting) *Julian nods in acknowledgment.* Do the ten, do the ten block high tower. And then let's do the 100 block tower. (*Now Julian has taken some cubes as well and is putting them together. He and Abby seem to be working together on the 10-block high tower. Samara is not involved at this point.*)	
93	A	Remember, there is 9 on each side.	
94	A	Tell me how many you want altogether. Put nine together?	
95		*Abby helps Julian put together the 10-block high tower using the cubes. As they continue to write, they look at the two towers they have constructed – the 5-block high tower and the 10-block high tower. What they say about these towers is inaudible.* *Samara seems to continue writing, and sometimes is distracted from the work. She does not appear to pay attention to the tower structures at all, nor do Julian or Abby include her in their discussions.* *The three students briefly discuss finishing their work. Other discussions are off-task until the teacher tells them to put away all their materials to get ready for the questionnaire.*	

Samara's Work, in Nov. 2008



Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.

A B C

A. 21 blocks 5-block high tower

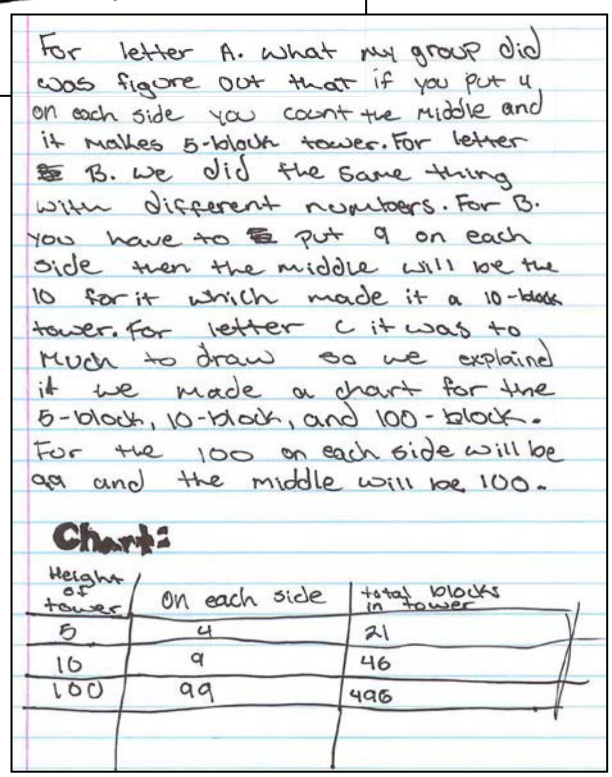
This block will need 4 on each size then the middle is 5.

B. 10-block high tower 46 blocks

This block will need 9 blocks on each side and 10 is in the middle.

C. The 100 block tower will need 99 on each side because in the middle will be 100.

height tower	On each side	Total blocks in tower
5-block tower	4 on each side	21 - total blocks in tower
10 - block tower	9 on each side	46 - total blocks in tower
100 - block tower	99, on each side	496 - total blocks in tower



For letter A. what my group did was figure out that if you put 4 on each side you count the middle and it makes 5-block tower. For letter B. we did the same thing with different numbers. For B. you have to put 9 on each side then the middle will be the 10 for it which made it a 10-block tower. For letter C it was to much to draw so we explained it we made a chart for the 5-block, 10-block, and 100-block. For the 100 on each side will be 99 and the middle will be 100.

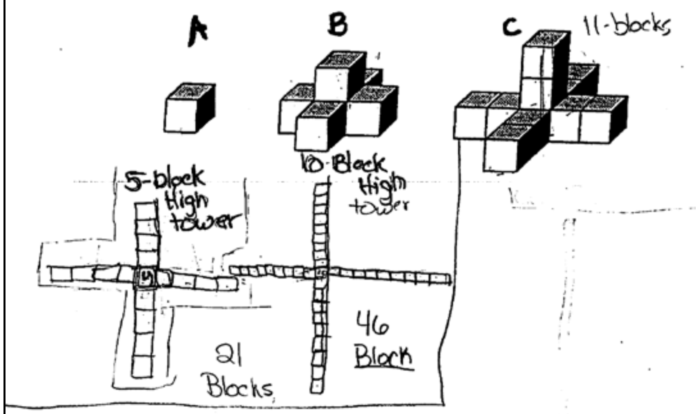
Chart:

Height of tower	On each side	total blocks in tower
5	4	21
10	9	46
100	99	496

Abby's Work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.

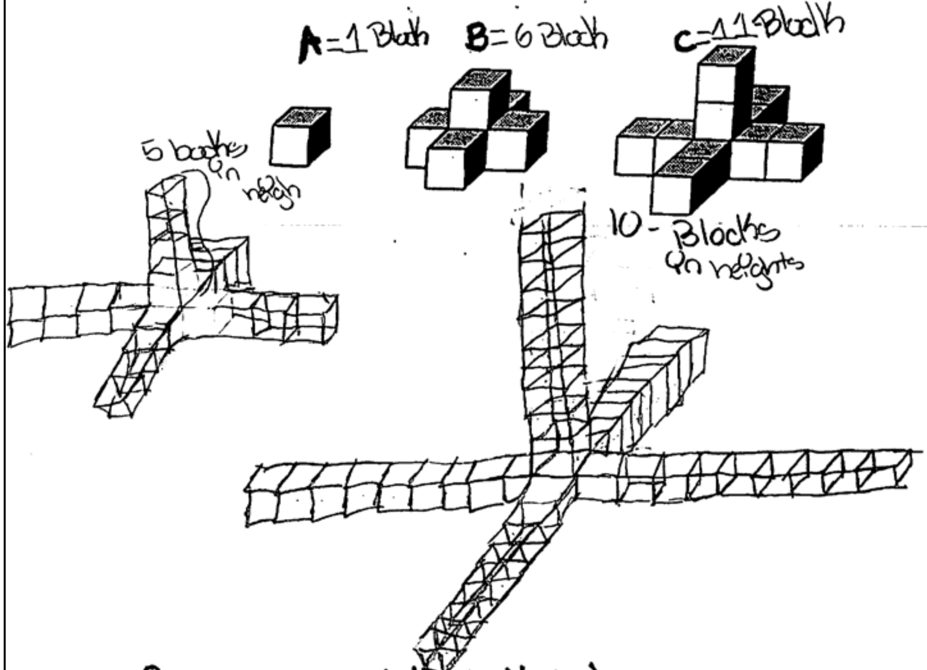


height of tower	On each side	Total Blocks in tower
5-block high tower	4 Blocks on each side	21 blocks
10-Block High tower	9-on each side	46 Blocks
100-Block tower	99 on each side	496 Blocks

(1) In the 5-high block has 4 on each side and we concucled that to find the amount of blocks on each number high tower is by just subtracting one from each 4 sides and the high block tower

1. In the 5-high block tower has 4 blocks on each side and theirs 4 sides to find the amout you just have to subtract 1 from each side so its has 4 on each side and 5 in the height. so the total amout of Block their is 21 Blocks.
2. In the 10-high Block tower has 9 blocks on each side and theirs 4 sides to find that amout we repeart the

Julian's Work, in Nov. 2008



Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.

height of tower	On each side	Total Block in tower
5-block tower	4 blocks	21 blocks in all
10-Block tower	9 blocks	46 blocks in all
100-Block tower	99 blocks	496 blocks in all

To figure out the problem we look at the three figures on the page. Figure A is the middle block that does not show in figure B and C. We figure out the probem I thought that we need to add one less than the heigh to each side. For ex: 5 block height tower will equal to 4 blocks on the other side.

Questionnaire Responses for Samara, Julian, Abby

Questionnaire items which may indicate LMTY structure

Questionnaire Items	**Samara**	**Abby**	**Julian**
Statements (All the time, Sometimes, Never)			
I wanted to teach another student something that I knew that the other student did not know.	Sometimes	All the time	All the time
I listened carefully to the ideas of someone I was trying to help.	All the time	All the time	All the time
I helped someone see how to do the math.	Sometimes	Sometimes	Sometimes
Others listened carefully to my ideas.	Sometimes	All the time	Sometimes
Behaviors (Often, Sometimes, Hardly ever)			
I gave helpful suggestions.	Sometimes	Often	Often
I worked cooperatively.	No response	Often	Often
Thoughts (Yes/No)			
I like teaching this person things that I know.	No	Yes	Yes

Questionnaire Responses for Samara, Julian, Abby

Questionnaire Items Which May Indicate LHSIA structure

Questionnaire Items – Statements	**Samara**	**Abby**	**Julian**
Statements (All the time, Sometimes, Never)			
I wanted people to think that I'm smart.	Sometimes	Sometimes	Sometimes
I tried to impress people with my ideas about the problem.	Sometimes	Sometimes	All the time
People seemed impressed with the ideas I shared about the problem.	Sometimes	Sometimes	Sometimes
People saw how good I am at the math we did today.	Sometimes	Sometimes	Sometimes
I felt smart.	Sometimes	All the time	Sometimes
I wanted to show someone that my way was better.	Sometimes	Sometimes	Sometimes
I was a lot better at math than others today.	Sometimes	Never	Sometimes
Behaviors (Often, Sometimes, Hardly ever)			
I was the leader.	Hardly ever	Sometimes	Sometimes
I was bossy.	Hardly ever	Hardly ever	Hardly ever
I wanted to show off.	Hardly ever	Hardly ever	Hardly ever
I liked to be right.	No response	Often	Sometimes
Thoughts (Yes/No)			
I want you to know just how smart I am.	No	No	Yes
People think I'm smart.	No	No	Yes
I wish the teacher would call on me, so I can show how much I know.	No	Yes	No

Juan, Eliot, Amanda, Manuel

This is Group 2 from Ms. A's grade seven Class 1.

Verbal emphasis indicated by underline
Actions included either in *italics* or (parentheses) as appropriate.
Pauses in speech are indicated by …
Words inserted to help clarify a student's words are included in [brackets].
When students voices overlap, one or both sets of words are included in //double slash marks//.
Sts: Multiple students/unsure which are speaking
U: Unknown single student.

The number included next to either LMTY or LHSIA refers to the code, included in the table of codes.

Juan, Eliot, Amanda, Manuel			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
		The teacher has just introducing the problem and has instructed the students to work in their groups. This has taken a little more than one minute.	
1	Manuel (M)	Okay	**EPISODE 1**
2	Eliot (E)	Okay	
3	Juan (J)	Okay	
4	Amanda (A)	Okay	
5	M	So each time they….	
6	A	How much did they increase it by each time?	
7	M	They aren't…(counting)	
8	J	They increased it by 5.	**LHSIA (14)** Juan answers Amanda's question, calling attention to his idea that the number of blocks increases by 5.
9	M	They increased it by 5, each time they add 1 on top of it. *One of the researchers gives them markers to write with.*	**LHSIA (10)** Manuel builds on Juan's idea, adding that there is a block added to the height each time.

Juan, Eliot, Amanda, Manuel			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
10	A	So this is block 1 right?	
11	M	Yeah and block B they add 1	
12	E	And block B the height is 2	**LHSIA (12)** Eliot contributes, stating that figure B on their task paper has a height of 2, which was discussed during the teacher's introduction.
13	J	The height is increasing by 1	
14	A	They're increasing by 1 because look they started the height with 1 then it's 2 then it's 3	**LHSIA (10, 12)** Amanda contributes her first non-question by building on Juan's idea tha the height is increasing by 1 block with specific examples.
15	M	Oh yeah I think they're increasing the height by 1 and then the sides	
16	Sts	And then the sides by 1..yeah *Ms. A approaches the group and listens in but does not say anything*.	
17	A	So they're both increasing by 1 the height and the sides	
18	E	The sides…yeah	
19	A	So if we need to make up a 5 block high tower…	
20	M	5 block high tower…	
21	J	The height's increasing by 1 then the sides need to increase by 1	
Interpretation: All four students jump into the conversation with their ideas immediately. This may show good collaboration to work on the problem. It may also demonstrate that each student wanted to keep up, intellectually, with the others in the group. By answering Amanda's question, Juan seems to share the first concrete idea, "They increased it by 5 [blocks]." Manuel jumps in, repeating Juan's statement and adding that the height increases by 1 block. After Manuel states that another block is added to the height to create figure B, Eliot also jumps in, perhaps showing that he is able to keep up with the others, with respect to the mathematical task. He does this by stating that the height of the tower depicted in figure B has a height of 2 blocks, a statement that was discussed during			

<table>
<tr><th colspan="4">Juan, Eliot, Amanda, Manuel</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td colspan="4">the teacher’s introduction to the problem.
Amanda had posed a couple of questions, which contributed to the conversation itself. She then builds on what Juan says about the height increasing by 1 block by giving the specific examples, “1 then it’s 2 then it’s 3,” likely referring to the heights of the towers depicted by figures A, B, and C.</td></tr>
<tr><td>22</td><td>M</td><td>A 4, a 4 one would be um, a 5 block tower would be…I guess they’ll be 4 blocks in each side cuz if here there’s 2 um for number 3…
(He leans over to Amanda’s paper to point to the diagrams)</td><td></td></tr>
<tr><td>23</td><td>A</td><td>Wait wait wait if it would be a 4 block tower (shows 4 fingers on her hand)
it would be 3 blocks and if it’s 5 block tower then it would be 4 blocks</td><td>LMTY (6)
Amanda explains the number of blocks that are in each side for the 4-block high tower and the 5-block high tower.</td></tr>
<tr><td>24</td><td>E</td><td>Yeah it would be 4 blocks</td><td></td></tr>
<tr><td colspan="4">Interpretation: Amanda seems to build on Manuel’s idea, which he seemed to have some difficulty expressing. She builds on this, restating his idea that a 4-block high tower has 3 blocks on each of the sides, and the 5-block high tower would have 4 blocks on each side. This idea seems to be accepted since Eliot verbally agrees with her, and the others do not disagree.</td></tr>
<tr><td>25</td><td>A</td><td>Let’s just do, hold on let’s just draw the D one, let’s do D, let’s do 4. This would be 4
Students start writing on their papers.</td><td></td></tr>
<tr><td>26</td><td>M</td><td>Wait but, is it the height? When they’re talking about 5 block, is it the height?</td><td></td></tr>
<tr><td>27</td><td>A</td><td>If it’s a high tower that means it’s the height (gestures height with her hand)
but we’re doing the sides and the height we’re just visualizing first.</td><td></td></tr>
<tr><td>28</td><td>M</td><td>So they’ll be…5….this is a 4…</td><td></td></tr>
<tr><td>29</td><td>E</td><td>It’s all going by 1</td><td></td></tr>
<tr><td>30</td><td>A</td><td>It’s all going by 1…</td><td></td></tr>
<tr><td>31</td><td>M</td><td>Yeah ok, I get it</td><td></td></tr>
<tr><td>32</td><td>Sts</td><td>(working on the problem)
Teacher leaves and goes to another group</td><td></td></tr>
<tr><td>33</td><td></td><td>For about 2 minutes:
The students continue to write on their papers and discuss how to draw it. Manuel suggests drawing it 3-dimensional, similar to</td><td></td></tr>
</table>

Juan, Eliot, Amanda, Manuel			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
		the given diagrams. Amanda seems to try that and finds it more challenging than she thought it would be. They all continue to discuss drawing a 5-block high tower, as they write or draw.	
34	E	What was it? 4, it went up by 1, right?	**EPISODE 2** The students continue discuss the drawing of the towers, and how the towers are constructed.
35	A	Yeah but since the side is 2 so it went up 1 side.	
36	E	So this should be 3 in total because there's 1 down here?	
37	A	Yeah the sides would be 3 and then the height would be 4 cuz then it'll be 4, 3 and 5,4, get it?	**LMTY (1, 10)** Amanda responds to Eliot's comments by providing him with the number of blocks that should be on each side for a specific height. She checks that he understands by asking, "get it?"
38	E	Yeah.	
Interpretation: As the students have been trying to draw representations of the 4-block high tower, Eliot asks about the number of blocks in the structure. He is not talking about the total, it seems, but I'm not sure whether he initially refers to the number of blocks in the height or in one of the sides. Though Amanda initially gives him a quick response (line #35) Eliot follows up, by asking about what appears to be the 4-block high tower. Amanda answers this question with the details that the 4-block high tower has 3 blocks on each of the sides. She also checks to see if Eliot understands by asking him, "Get it?" to which he responds, "Yeah."			
39	J	It's hard to draw the squares	
40	A	Alright wait…	
41	J	Or cubes…cubes I mean	
42	A	Just don't worry about the drawing right now, let's worry about the drawing later. Anyways…now we gotta build a 5 block high tower	

Juan, Eliot, Amanda, Manuel			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
43	M	High tower…yeah	
44	A	So if 4 was…wait, 4 was 3 then 5 would be 4 so the height would be 5…	
45	J	Exactly it would be 4	
46	A	But you see how the height they really don't include the first one they start from the second. You can't see the first	
47	J	But you know it's there	
48	M	Yeah but you know it's in there I guess it's flat	
49	E	Right here pretty much I built a 5 block high tower right here, there's 1 down here	
50	M	Yeah but that can't be, that can't be number D cuz number D is going to be a 4 block high tower	
51	E	Yeah, alright. So at this point I'm gonna start here	
52	A	I know how to draw a cube	
53	M	That's the hard part drawing cubes anyway um	
54	E	For number D you have to draw a 4 block high tower	
55	J	D? It's a 5 block high tower	
56	E	With how many over?	
57	M	I have a good idea how…look like in the middle you could put 4, 4, 4 is behind. See? That's how I did it, and you know plus 4, I'm gonna write the 4 is the height. (brief pause to write/draw) See? I just did it like that and I'm gonna put 4	**LHSIA (4, 14)** Manuel says he has a "good idea" of how to draw a representation of the 5-block high tower. He explains his strategy to his group as he is drawing.
58	E	It's the height, yeah	
59	M	Right? Cuz you can't draw the height I'm just gonna leave it like that	
60	A	That could work. But we have to make a 5 block tower and a 10 block high tower and 100 block high tower	
Interpretation: Manuel had earlier expressed difficulty with drawing representations of the tower structures. Here, he lets his classmates know that he has figured out a way to represent the towers in a way that does not require an exact representation of the height,			

<table>
<tr><th colspan="4">Juan, Eliot, Amanda, Manuel</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td colspan="4">“Cuz you can’t draw the height.” Based on his student work, it seems he has drawn a top-down view of the towers in which the four sides are depicted and the number of blocks in the height is written as a numeral in the middle of the drawing. (See student work.)</td></tr>
<tr><td>61</td><td></td><td>The students continue to briefly discuss the drawing of the tower structures. Manuel appears to start counting the total number of the blocks in the 5-block high tower, when he is interrupted by Amanda.
Amanda suggests that now that they have the information needed for the 5-block high tower – that each of the four sides has 4 blocks – they could double these numbers to get the necessary information for the 10-block high tower. That is, since 10 is two times 5, which is the number of blocks in the height, then you can also multiply 4 by 2 to get 8 blocks on each of the four sides.
Manuel seems to misunderstand her at first, thinking that she means to find the total number of blocks in the 5-block high tower by adding 10 and 8. This leads Eliot to suggesting that there are 18 total blocks in the 5-block high tower.</td><td></td></tr>
<tr><td>62</td><td></td><td>Amanda goes through each stage between the 5-block high tower and the 10-block high tower, and realizes that it is the 9-block high tower that has 8 blocks on each of the sides and that the 10-block high tower actually has 9 blocks on each side. She realizes her suggestion is actually incorrect which she shares with her classmates.
Manuel returns to trying to determine the total number of blocks in the 5-block high tower, initially suggesting 23 blocks and then stating that it is incorrect. He reminds Amanda and the others that they are supposed to find the total number of blocks for the towers of various heights.</td><td></td></tr>
<tr><td>63</td><td>M</td><td>Generalize if you can on how many…</td><td>EPISODE 3
The students discuss the total number of blocks in the tower structures.</td></tr>
</table>

Juan, Eliot, Amanda, Manuel			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
64	J	(counting) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11	
65	A	(counting) 11 and 4 then…	
66	E	This one was 1	
67	M	Let's just add a cube here and this would be 4 side or height then 3 add another block add another block add another block would be (counting) it would be 16	
68	M	For B it would…	
69	A	D would be 16 blocks	
70	M	(Counting)	
71	A	Then you add another one 4,8,12,16	
72	E	So how many blocks?	
73	A	(counting) E would be 21 blocks	
74	M	E?	
75	A	Yeah E would be 21 blocks…	
76	J	I'm confused	
77	M	What are you confused about? What are you confused on? We'll help you it's a group!	**LMTY (9)** Manuel explicitly offers to help clarify whatever has confused Juan.
78	A	Let me see something I'm just gonna do the 6	
79	M	Cuz if it's…the next one is…hmm..no	
80	J	How much was for D?	
81	M	Huh? How many was for D?	
82	J	Yeah	
83	M	Uh… it was 16 blocks	
84	J	How many are there on each side?	
85	M	Oh, on each side it's 3.	**LMTY (7)** Manuel answers Juan's questions about why there are 16 blocks in the 4-block high tower, which they call figure D.
86	J	What about up and down?	
87	M	Up and down is 3	
Interpretation: The students are all discussing the total number of blocks in various tower structures, which they seem to call by their letter following the pattern started on their task paper. Juan admits that he is confused. This prompts Manuel to offer to help clarify this confusion. Juan replies by asking specific questions, "How much for D?" which seems to			

Juan, Eliot, Amanda, Manuel			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
be asking how many total blocks are in the 4-block high tower. Manuel responds to these questions, following through with the promise to help Juan.			
88	E	There's 16 blocks for D right?	
89	M	D? it's………can't do it. *He may be referring to drawing a 3-dimensional representation of the 4-block high tower.*	
90	J	Yeah you can.	
91	M	No you can't I'll tell you you just add 3 then put 4 in the middle and that represents the height that's how I see it	
92	A	This would be, uhh, 6, 5.	
93	M	Ok.	
94	A	Then 7,6 then 8…	
95	M	What are you doing? The 10?	
96	A	Then 9 would be…	
97	M	Cuz look that would take long doing.	
98	A	Wait wait wait hold on cuz you still have to figure how much blocks.	
99	M	Oh yeah… It would be for 10…	
100	A	Wait, let's see if we can find a pattern it went from 1 to 6 then 6 to 16 then 16 to 21 any pattern?	
101	J	1, 5…	
102	Sts	(working on the problem)	
103	M	Oh I found a pattern look look look, 1 plus 5 equals 6, 6 plus 5 equals 11, 1 plus 5 equals 6 plus 5 equals 11 plus 5 equals 16 plus 5 equals 21.	**EPISODE 4** **LHSIA(4)** Manuel claims he has 'got it' by finding a pattern. He describes his pattern before stating to his classmates, "It's going by fives."
104	Sts	Plus 5 equals 21.	
105	M	So we found a pattern it's going by fives.	
106	A	Ohh! Okay, wait, I just figured out something. This would be 5, 4 the other one would be 6, 5 the other one would be…	**LHSIA (10, 14)** Amanda calls attention to her own idea, in which she discusses the

Juan, Eliot, Amanda, Manuel			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
			relationship between the height of the tower and the total number of blocks in each of the four sides.
107	M	5, 4?	
108	A	Look, look, watch. It would be the 10 height would be 10, 9 not 10, 8 so we can't double it because look this would be 5,4 then the next one would be 6,5 then the next one would be 7,6, then the next one would be 8,7 then the next one 9,8 and then the next one would be 10,9 not 10, 8.	**LHSIA (8)** Amanda corrects her earlier statement about doubling the numbers in order to help find the total number of blocks in the tower structures.
109	M	Oh! I see where you, ohhh.	
110	A	So we can't just double this one.	
Interpretation: Amanda (line #100) suggests that they should start looking for a pattern to help them determine the total number of blocks in each tower structure. Manuel is the first to suggest that he found a pattern, which is that the total number of blocks increases by 5 when the height is increased by 1. He even explains with specific examples, starting from the first towers given on the task paper, "One plus 5 equals 6, 6 plus 5 equals 11." Amanda may or may not have heard him, as she chimes in with a different pattern to describe. Building off what she stated earlier, she describes the relationship between the number of blocks in the height and the number of blocks on one of the four sides, which is one less than the number of blocks in the height. She continues to discuss this idea, correcting an earlier idea she had. Previously she stated that since 10 was twice 5, perhaps they could double both the height and the number of blocks on each side. She realizes now that this will not work since the number on each side must be one less than the number in the height. She states this as, "[The 10-block high tower] would be 10, 9 not 10, 8."			
111	M	I see 5,4 6,5 7,6 8,7 9,8 10,9	
112	A	Because all you're doing is adding 1 here but then taking….	
113	E	It's going by fives?	
114	A	Huh?	
115	E	It's going by fives.	
116	A	Yeah.	
117	M	Yeah.	
118	A	So this one should be…	
119	J	By 5s?	

<table>
<tr><th colspan="4">Juan, Eliot, Amanda, Manuel</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td>120</td><td>M</td><td>No, no, no. Yeah, the total is going by 5s the total block are increasing by 5.
(using hand gestures to help explain)</td><td>LMTY (1, 7)
Eliot and Juan have both asked about the ‘going by fives’ pattern. Manuel explains that it is the total that is increasing by 5 blocks.</td></tr>
<tr><td>121</td><td>A</td><td>How we get 21 blocks…oh and then you add 1 here and then here and then add 1 here and then………so 1 2 3..</td><td></td></tr>
<tr><td colspan="4">Interpretation: Manuel responds to the question that both Eliot (line #113) and Juan (line #119) have posed about the pattern going by fives. He clarifies that the total number of blocks is what is increasing by 5, not some other aspect of the tower.</td></tr>
<tr><td>122</td><td colspan="2">For about 4 minutes:
Manuel asks his classmates if they think they need the SnapCubes that other students have gotten. Amanda claims they do not need them, because they already found a pattern, to add 5 blocks to the previous total. They start discussing how to find the total number of blocks in the 10-block high tower by following this pattern, when the teacher comes to their table.</td><td></td></tr>
<tr><td>123</td><td colspan="2">The teacher is at the table for about 7 minutes:
When the teacher approaches their table, they ask her about whether they have to draw representations of each of the towers. Amanda once again brings up her idea of doubling the numbers for the height and the sides. Ms. A points to Amanda’s paper and asks about the 5, 4 pattern she has written. The students consider that continuing to add 5 might be the way to find the total number of blocks in the 100-block high tower, but Amanda says she thinks there is another method. Manuel starts brainstorming another strategy involving finding a pattern for the sides, so that they might be able to determine how many blocks are on the sides of the 100-block high tower. Ms. A suggests they use the first few stages to see if it works.
As the students start to discuss patterns, they mention that the total number of blocks increases by 5, and Ms. A asks them to explain that further. She then asks how this relates to an increase of 4 blocks on the sides. She leaves them to think about it while she visits another</td><td></td></tr>
</table>

Juan, Eliot, Amanda, Manuel			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
	group.		
124	M	4 times 2 is 8….	**EPISODE 5**
125	Sts	Wait, wait, wait!	
126	A	Wait a minute… My idea! (laughing)	
127	M	I don't know if I have the same idea. Cuz we already know the height is 100 so and if this is 0, 1, 2 (counting and pointing to Amanda's paper)	
128	J	So wait a second.	
129	M	So if we do 4 times 9?	
130	J	This adds up to 100?	
131	M	4 times 9	
132	E	All these blocks together?	
133	M	I think it will be 136 cause… Look, this is block number 1 and this 4 times 0 is 0. Block number 2, 4 times 1 equals…	**LMTY (1)** Manuel explains why he thinks that they should multiply 4 times the number of blocks on the side.
134	E	4	
135	M	Yeah this is block number 3 4 times 2 equals	
136	E	8	
137	M	8 this is block number…	
138	A	Ohhhhh!	
139	M	Get it? So that means if we probably do 4 times 9 we'll get the answer.	**LMTY (10)** Manuel asks if Amanda and his other classmates understands and then continues to suggest a strategy to give them a result.
140	A	Wait, wait, wait. I understood it.	
Interpretation: While speaking with the teacher, Amanda had suggested that there might be another way to find the total number of blocks for the 100-block high tower other than adding 5. Manuel comes up with the idea that they can multiply 4 times the total number of blocks on one of the sides. He starts giving specific examples, stating that there is 1 block on the side of the 2-block high tower and 4 times 1 is 4 which are all of the blocks in the tower except the 2 blocks in the height. As he explaining his classmates jump in supplying answers, suggesting that they believe they understand Manuel's strategy. This appears to prompt Manuel to ask, "Get it?" which he follows with, "If we do 4 times 9			

Juan, Eliot, Amanda, Manuel			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
we'll get the answer," where 9 is the number of blocks on one side of the 10-block high tower.			
141	M	Not 4 times 9. Sorry, 4 times, 4 times…	
142	A	I understand what you're saying because you're doing 4 times…but at the same time	
143	M	But at the same time I'm not sure	
144	E	Here we have 8	
145	A	But look, look, 4	
146	M	But I'm not sure	
147	A	4 times 1 is 4, 4 times 2 is I think it's…	
148	E	4 times 3, 4 times 2, 4 times…like that?	
149	A	Wait I think it's like, is this the first figure with numbers the second, the third this would be the 4^{th}, 5^{th}, 6^{th}, 7^{th}, 8^{th}, 9^{th} so you're off by 1.	
150	M	I know but I still…	
151	A	I think it would be 4 times 99	
152	M	Yeah, yeah, I just said that cuz I was thinking about 10 blocks. I'm sorry.	
153	A/M	4 times 99 will be	
154	E	396.	
155	M	100 plus 396…	
156	A	You gotta make sure you're (inaudible)	
157	E	396	
158	M	It'll be 496.	
159	E	But how?	
160	M	Because 100 because we already know because that's the side blocks 396 side blocks plus your 100 height	**LMTY (1)** Manuel responds to Eliot's question of how the 100-block high tower has a total of 496 blocks. He explains that there are 396 blocks on all four sides.
161	A	It would be 496	
162	E	Yeah…	
163	M	Ohhh snap	
164	J	Yeah what was it?	
165	E	That times 99	
166	M	(to Juan) Get it? Get it?	**LMTY (10)**

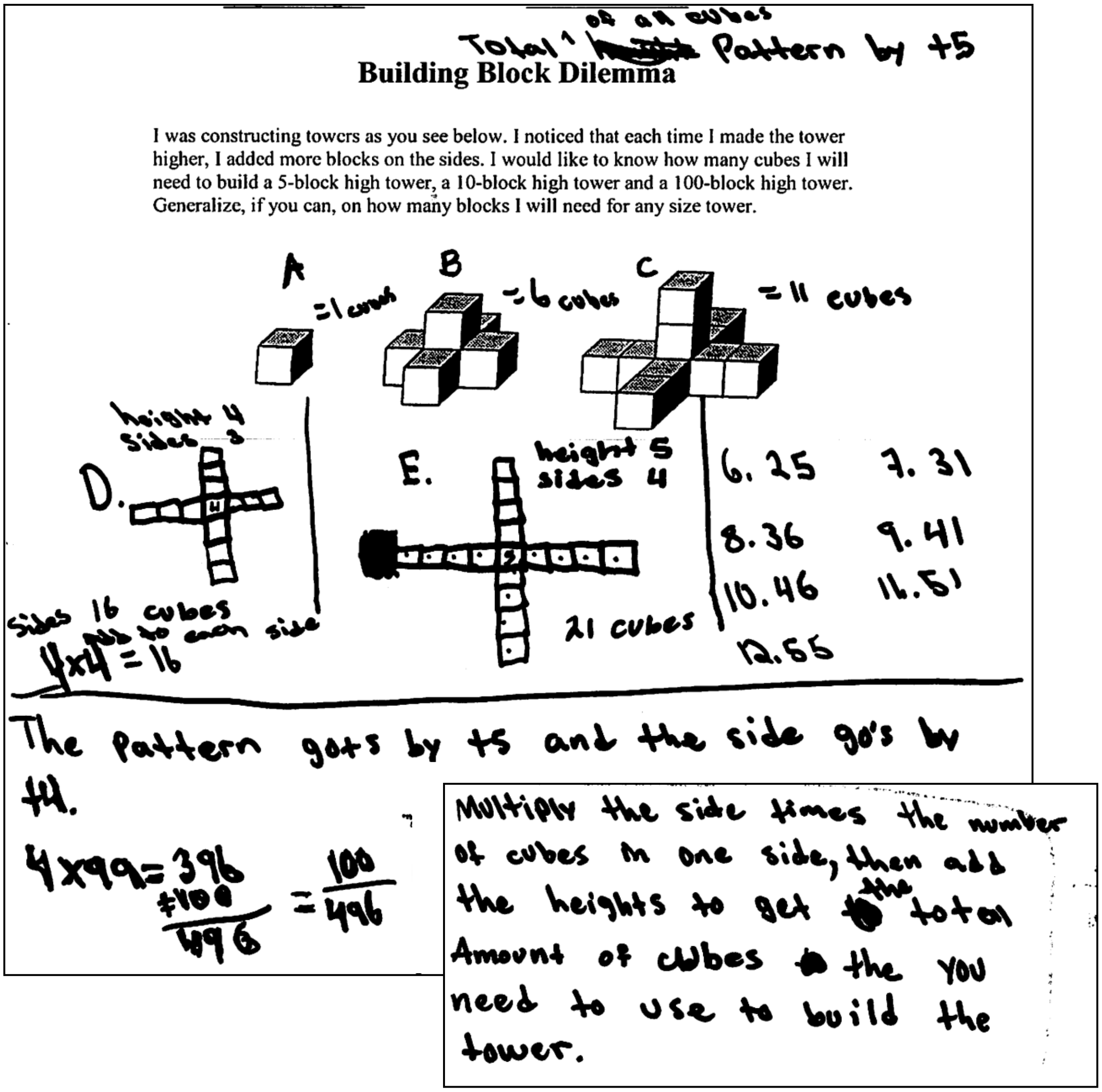
<table>
<tr><th colspan="4">Juan, Eliot, Amanda, Manuel</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td></td><td></td><td>(Juan shakes his head, no)
No? Cuz look
Because the 5 block was higher by 4, right?
(points to Juan's paper to explain)</td><td>Manuel asks if Juan understands the result they just got.</td></tr>
<tr><td colspan="4">Interpretation: Manuel, Amanda, and Eliot all work together to determine that there are 496 total blocks in the 100-block high tower. Eliot seemed to understand that they multiplied 4 and 99 to get 396 blocks on all sides for the 100-block high tower. But he asks, "How?" when Manuel states that there are 496 total blocks. Manuel responds by explaining that they need to add the 100 blocks in the height to the 396 blocks on all the sides.
Manuel then asks Juan who has been the most quiet if he understands, "Get it?" When Juan shakes his head no, Manuel starts to explain something, but it's not clear that the explanation was sufficient for Juan.</td></tr>
<tr><td>167</td><td>E</td><td>Equals 396</td><td></td></tr>
<tr><td>168</td><td>J</td><td>What was the pattern?</td><td></td></tr>
<tr><td>169</td><td>E</td><td>100 and then 6, 396</td><td></td></tr>
<tr><td>170</td><td>A</td><td>I was thinking the same thing cuz I was thinking but except you got with the thing</td><td>LHSIA (12)
Amanda states she had the same idea as Manuel.</td></tr>
<tr><td colspan="4">Interpretation: Amanda states, "I was thinking the same thing," implying that she had the same idea as Manuel about how to come up with a pattern that would allow them to find the total number of blocks in the 100-block high tower. Since Manuel has been suggesting several ideas, it is possible that Amanda wants her classmates to know of her ideas as well.</td></tr>
<tr><td>171</td><td>M</td><td>Yeah I know cuz I was thinking about 10</td><td></td></tr>
<tr><td>172</td><td>A</td><td>Cuz you were thinking about the 10</td><td></td></tr>
<tr><td>173</td><td>M</td><td>Yeah 10 block tower</td><td></td></tr>
<tr><td>174</td><td>A</td><td>The 100</td><td></td></tr>
<tr><td>175</td><td>M</td><td>I forgot about that we figured out…</td><td></td></tr>
<tr><td>176</td><td>A</td><td>Yeah, we were the first group that got it.
(tone of voice is exited, higher pitch than normal)
Give me a high 5. (High-fives Manuel)</td><td>LHSIA (4)
Amanda is excited that the group came up with an answer.</td></tr>
<tr><td colspan="4">Interpretation: Amanda expresses her excitement that her group came up with the answer. Since she claims they are the "first group that got it," she may be expressing some excitement that she and her classmates are smart, and were able to come up with a strategy. She seems to acknowledge that they worked together to arrive at some answers by the use of the word "we" and 'high-fiving' Manuel.</td></tr>
<tr><td>177</td><td>E</td><td>So 4 time 99 gives you 396</td><td></td></tr>
<tr><td>178</td><td>M</td><td>Wait, let's test it out, 13 plus 10.</td><td></td></tr>
<tr><td>179</td><td>A</td><td>Wait, wait. You're doing something wrong.</td><td></td></tr>
<tr><td>180</td><td>M</td><td>Whoa what am I doing?</td><td></td></tr>
</table>

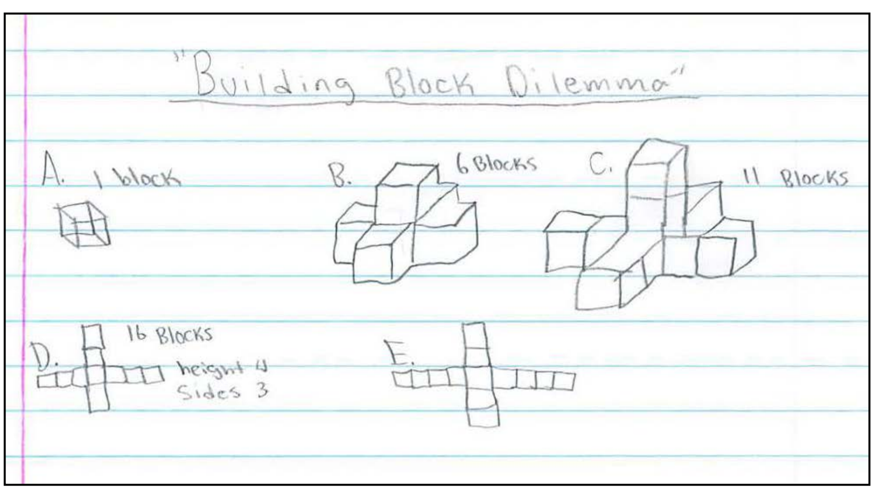
Juan, Eliot, Amanda, Manuel			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
181	E	Equals 23	
182	A	4 times 1 is 8 4 times 2 4 times 1 is 4 4 times 2	
183	M	What you said? What did you say the answer was for 4 and 10?	
184	A	46	
185	M	Yeah, look. When I used the side pattern way I got 36, 4 times 9, plus the 6.	
186	E	But Manuel where did you get 13 plus 10?	
187	A	No, he didn't say nothing about 13	
188	M	Yeah I'm always doing mistakes, sometimes. So don't listen to the first thing I said	
189	E	So what did you say then?	
190	A	Because you know if we follow the…	
191	M	Because look you understand what I said before right? That the side pattern is increasing by 4 cuz look there's no sides here so it's 0	**LMTY (2)** Manuel answers Edwin's question by modifying his earlier statement, adding, "The side pattern is increasing by 4."
192	E	0?	
193	M	Ok, here is block number 2 and it's 1 and 4 times 1 is 4 so 4 blocks	
194	E	Here's 8 and here's…	
195	M	So if it's	
196	E	4 sides?	
Interpretation: Eliot gets confused by something Manuel said earlier, and asks him to clarify. Though Amanda starts to explain, Manuel jumps in to explain how the number of blocks on all four sides increases by 4 for each successive tower.			
197	A	Yay!	
198	M	We're smart	**LHSIA (17)**
		For about 10 minutes: The students call their teacher over to the group again so they can explain their ideas. Mostly Manuel and Alicia are explaining the group's strategy, though Eliot tries to chime in as well. Ms. A challenges their strategy, asking if it works for a tower of any height. The students determine that it does work, by comparing their strategy to known results. Ms. A then asks them to come up with a	

Juan, Eliot, Amanda, Manuel			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
		general rule, and describe, in words, the numbers in the given specific examples.	
		The students discuss how they might explain, in words, the relationships that they have given with the specific examples of the 5-, 10-, and 100-block high towers. Amanda comes up with a description that she writes down. She then tells the other students her explanation so they can write it down as well.	
199	A	All we have to do is multiply the sides by the number of cubes in one side then we have to add the height to get the total amount of blocks you used to build your tower.	**EPISODE 6** **LMTY (1)** Amanda has found a way to describe in words their strategy. She is reciting what she wrote down to her classmates.
200	E	Yeah that sounds good.	
201	M	Wait what did you say again? So let's write the explanation in the bottom	
202	A	Multiply the side times the number of cubes in one side.	
203	E	Wait calm down.	
204	M	She said like two words and you're like calm down…I write fast.	
205	A	The number… of cubes …in one side. (says slowly so that her classmates can write it down) Then … add… the height… to get… the total amount… of cubes, the cubes That you used … to build the tower. That you used to build the tower. //Sts repeat what she says as they write//	
Interpretation: The students had struggled, both with the teacher and after she left the group, to explain in words their strategy for finding the total number of blocks in the towers of various heights. Amanda came up with a way, and was willing to share her description with her classmates. After she wrote it down for herself, at her classmates' request, she recited her written work for the other three students.			
	The students continue to write down their answers and strategies. When they speak, their discussion is usually		

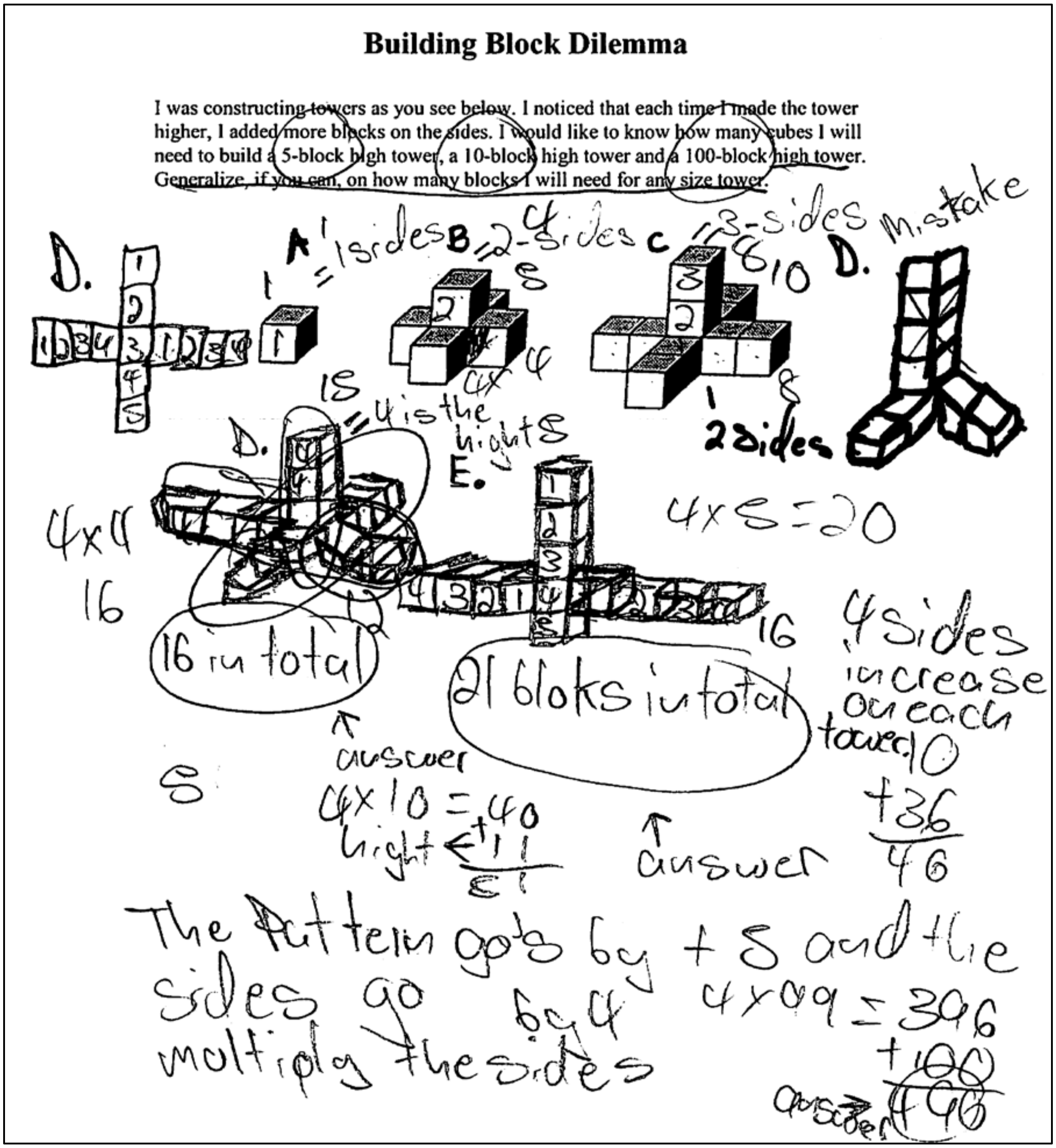
Juan, Eliot, Amanda, Manuel			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
	about what they are writing, and continuing their previous conversations.		

Juan's work, in Nov. 2008

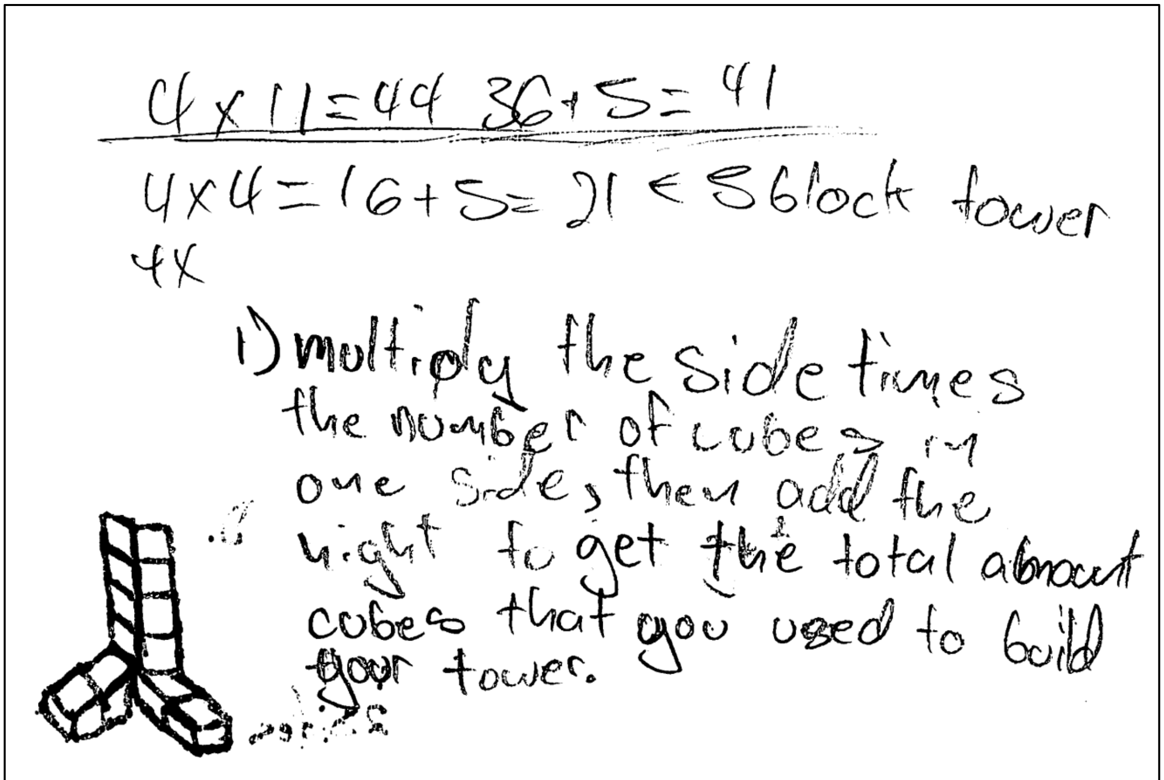


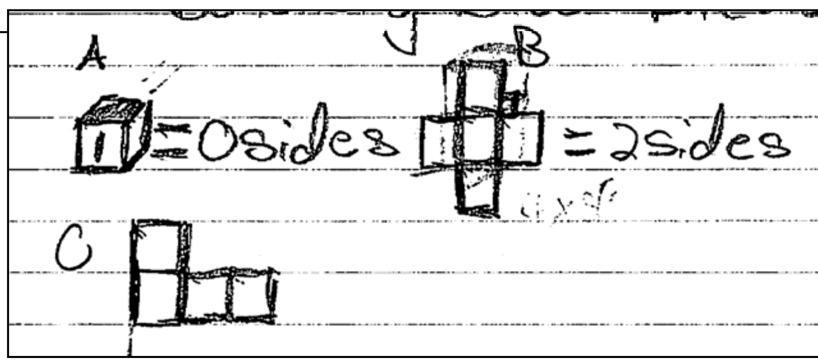


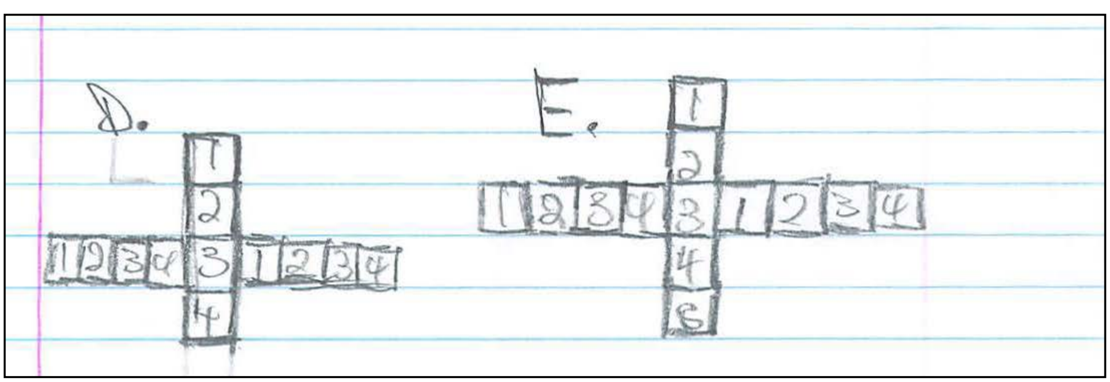
Eliot's work, in Nov. 2008



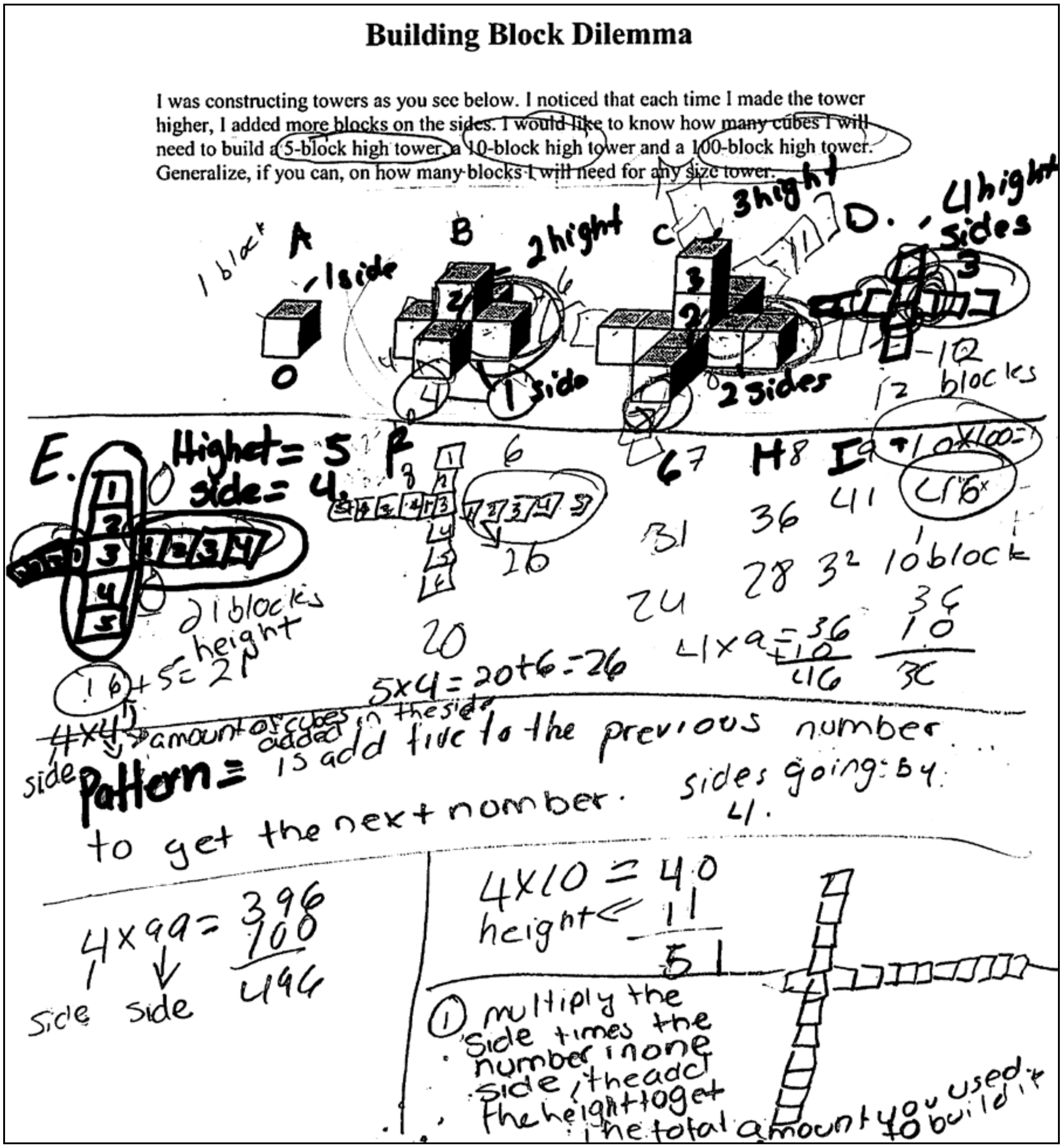
Eliot's work, in Nov. 2008







Amanda's work, in Nov. 2008



Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.

A 1 block 1 side 0

B 2 hight 1 side 4 6

C 3 hight 2 sides

D. 4 hight sides 3 12 blocks

E. Highet = 5 side = 4. 21 blocks height

16 + 5 = 21

4x4 amount of cubes in the side

F 26 20

5x4 = 20 + 6 = 26

G 7 31 24

H 8 36 28 3^2

I 9 41 10 block 416

4x9 = 36 + 10 = 46

36 + 10 = 36

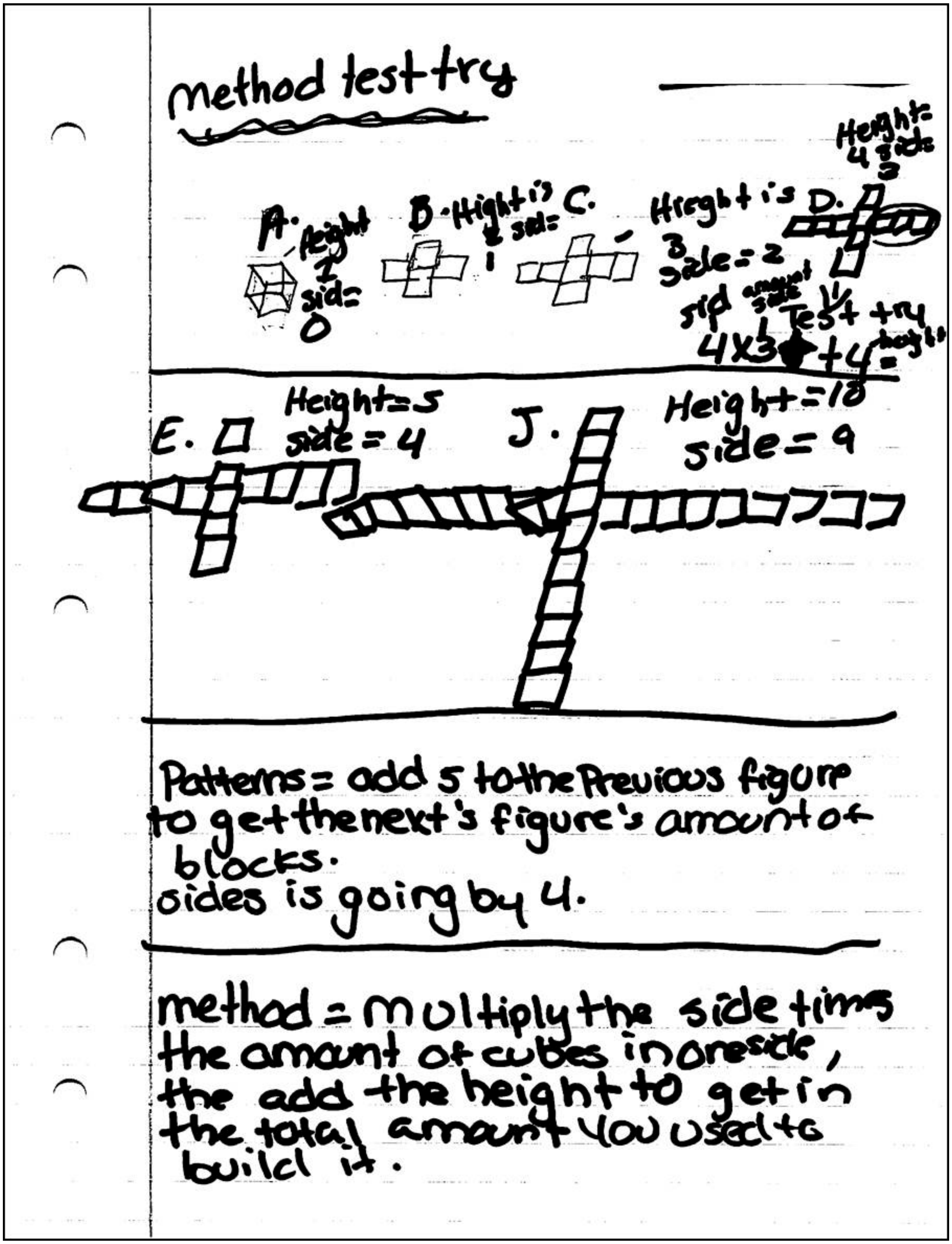
Pattern = added is add five to the previous number to get the next nomber. sides going by 4.

4x99 = 396 + 100 = 496 side side

4x10 = 40 + 11 = 51 height

① multiply the side times the number in one side, the add the height to get the total amount you used to build it

Amanda's work, in Nov. 2008



method test try

A. Height 1 side = 0

B. Hight is 2 side = 1

C. Hieght is 3 side = 2

D. Height = 4 side = 3

Test try 4x3 + 4 = height

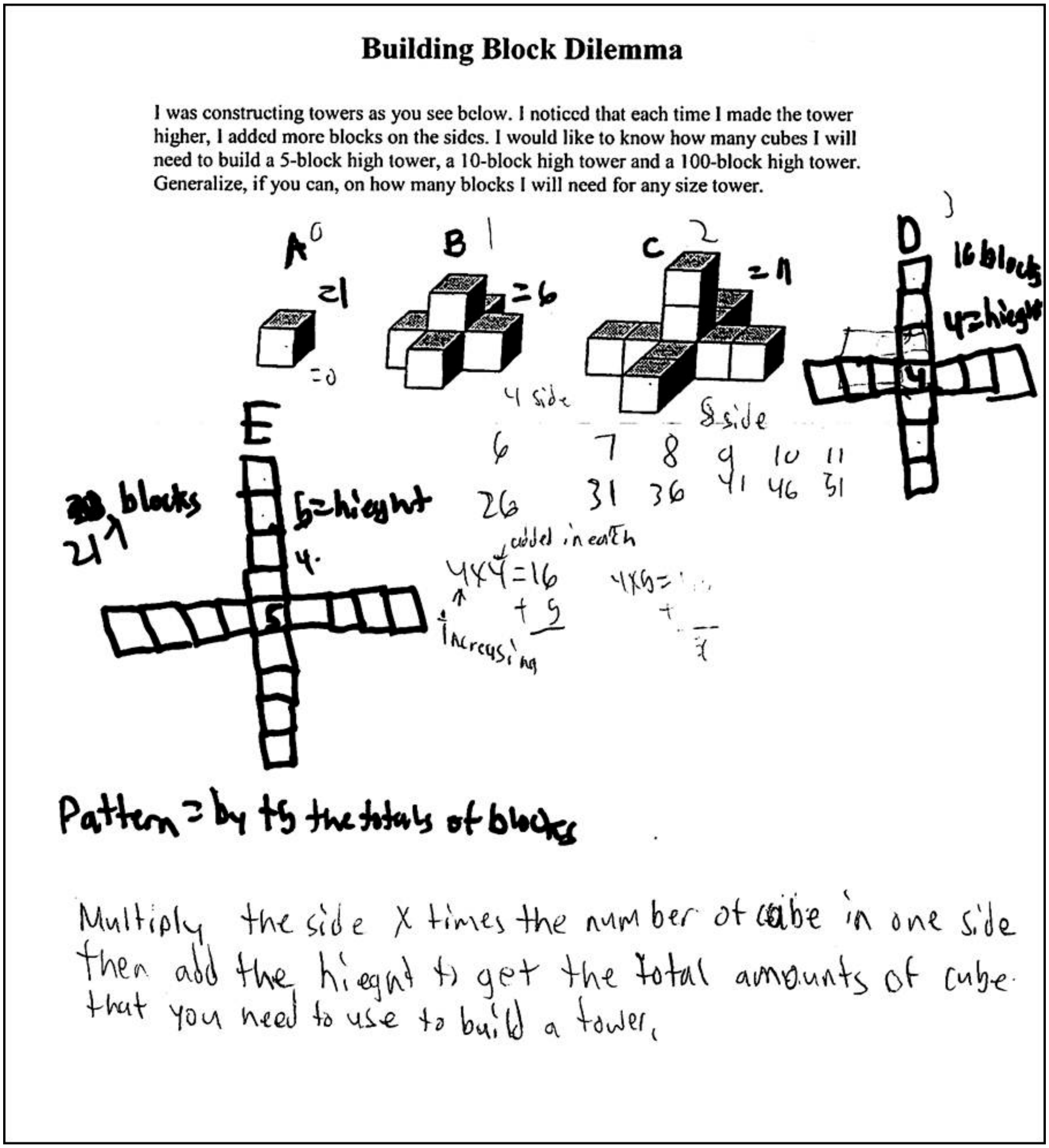
E. Height = 5 side = 4

J. Height = 10 side = 9

Patterns = add 5 to the Previous figure to get the next's figure's amount of blocks.
sides is going by 4.

method = multiply the side times the amount of cubes in oneside, the add the height to get in the total amount you used to build it.

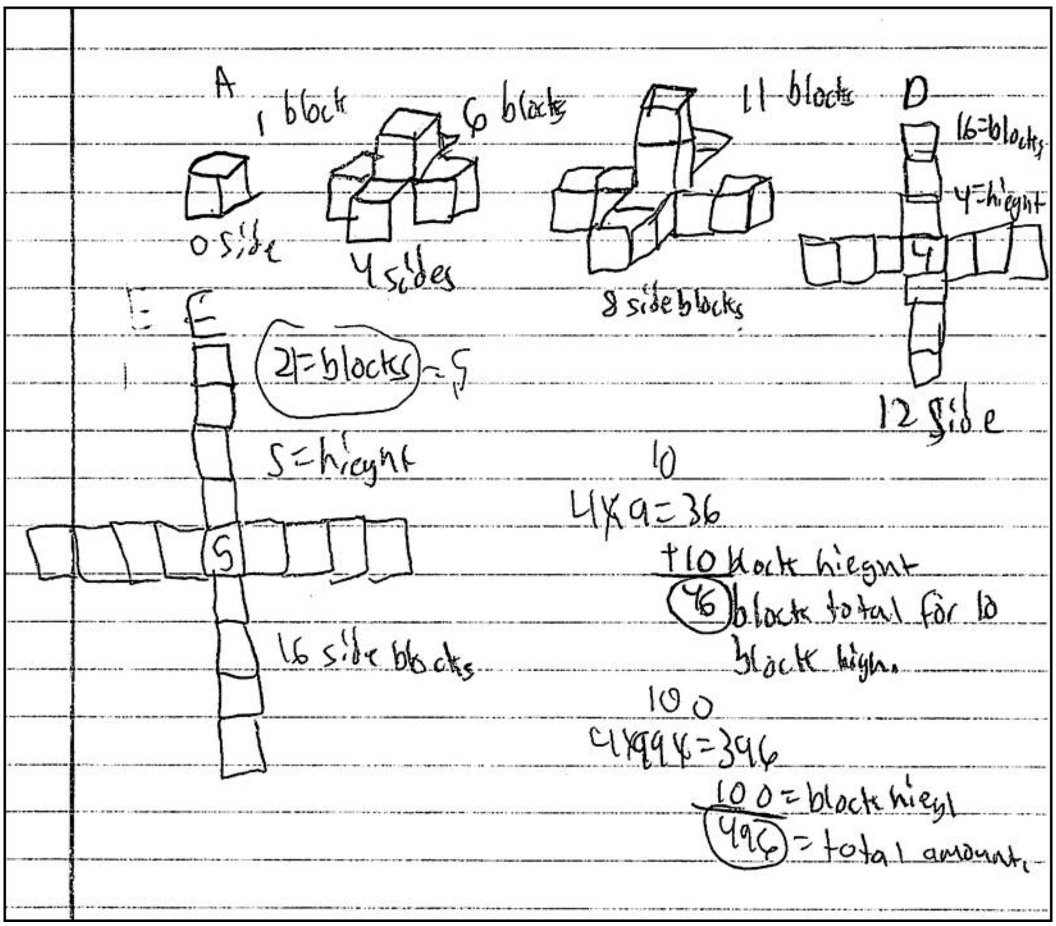
Manuel's work, in Nov. 2008



Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.

Multiply the side X times the number of cube in one side then add the hiegnt to get the total amounts of cube that you need to use to build a tower, Manuel's work, in Nov. 2008



Questionnaire Responses for Juan, Eliot*, Amanda, Manuel

Questionnaire items which may indicate LMTY structure

Questionnaire Items	Juan	Amanda	Manuel
Statements (All the time, Sometimes, Never)			
I wanted to teach another student something that I knew that the other student did not know.	All the time	All the time	All the time
I listened carefully to the ideas of someone I was trying to help.	All the time	All the time	All the time
I helped someone see how to do the math.	Sometimes	Sometimes	All the time
Others listened carefully to my ideas.	Sometimes	All the time	All the time
Behaviors (Often, Sometimes, Hardly ever)			
I gave helpful suggestions.	Often	Often	Often
I worked cooperatively.	Sometimes	Often	Often
Thoughts (Yes/No)			
I like teaching this person things that I know.	Yes	Yes	Yes

*Questionnaire data for Eliot are not available

Questionnaire Responses for Juan, Eliot*, Amanda, Manuel

Questionnaire Items Which May Indicate LHSIA structure

Questionnaire Items – Statements	**Juan**	**Amanda**	**Manuel**
Statements (All the time, Sometimes, Never)			
I wanted people to think that I'm smart.	Sometimes	All the time	All the time
I tried to impress people with my ideas about the problem.	Sometimes	All the time	Never
People seemed impressed with the ideas I shared about the problem.	Sometimes	All the time	All the time
People saw how good I am at the math we did today.	All the time	All the time	All the time
I felt smart.	Sometimes	All the time	All the time
I wanted to show someone that my way was better.	Never	Sometimes	Never
I was a lot better at math than others today.	Sometimes	Sometimes	Never
Behaviors (Often, Sometimes, Hardly ever)			
I was the leader.	Hardly ever	Sometimes	Hardly ever
I was bossy.	Hardly ever	Hardly ever	Hardly ever
I wanted to show off.	Hardly ever	Hardly ever	Hardly ever
I liked to be right.	Often	Often	Sometimes
Thoughts (Yes/No)			
I want you to know just how smart I am.	Yes	Yes	No
People think I'm smart.	Yes	Yes	No response
I wish the teacher would call on me, so I can show how much I know.	No	Yes	No

*Questionnaire data for Eliot are not available

Bridget, Liza, James, Jenna

This is Group 3 from Ms. A's grade seven Class 1.

Verbal emphasis indicated by underline
Actions included either in *italics* or (parentheses) as appropriate.
Pauses in speech are indicated by …
Words inserted to help clarify a student's words are included in [brackets].
When students voices overlap, one or both sets of words are included in //double slash marks//.

The number included next to either LMTY or LHSIA refers to the code, included in the table of codes.

<table>
<tr><th colspan="4">Bridget, Liza, James, Jenna</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td></td><td colspan="2">The teacher has just finished her introduction to the problem and given instructions.</td><td></td></tr>
<tr><td></td><td colspan="2">The students start by talking about non-task related things. Then they start to work independently. After a short while, Bridget, Liza and James start discussing what the problem is asking them to find.
James and Jenna appear to argue over Jenna's participating in the group.</td><td></td></tr>
<tr><td>1</td><td>Liza (L)</td><td>Oh, look, look, look!
(James and Bridge look at her paper. Jenna continues to work alone) Every time it goes…

//B: They add a block here here and here and here. (points to the diagram on Liza's paper)//

No, they add one here, but also add another one but it's one right here. So this is three blocks going up, but this is two around.</td><td>EPISODE 1
LMTY (1, 4)
Liza explains her understanding of the tower construction "three blocks going up, but …two around" especially after she 'corrects' Bridget's contribution about where blocks are added</td></tr>
<tr><td>2</td><td>Bridget (B)</td><td>So it would be five blocks going up and four going around.</td><td></td></tr>
<tr><td colspan="4">Interpretation: The students had mostly been working independently up until this point. First Liza appears excited to share her observation by stating, "Look, look!" She started to explain her understanding, "Every time it goes" but then Bridget interrupted her, stating that blocks were added in several places. Because Liza's seat is far from the camera, it is difficult to know whether Bridget's statement could be considered correct or not.</td></tr>
</table>

Bridget, Liza, James, Jenna			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
Liza appears to make a different observation, saying, “No,” and is essentially correcting Bridget, as inferred by her statement, “They add one here, but also add another one but it’s one right here.” It is not clear to the observer where “here” for any of these comments. Liza then tries to explain to her classmates, particularly James and Bridget who are paying attention, about the relationship between the number of blocks in the height and the number of blocks on each side, “This is three blocks going up, but this is two around.” Bridget seems to grasp what Liza is saying by her response, “So it would be five blocks going up and four going around.” Liza seems to have successfully explained her interpretation about how the towers are constructed, at least to Bridget.			

3	L	Yeah.
		OK. I’m going (to ask) (raises hand)
		Liza, James: No, ‘cause she can’t help us.
		//She can’t help us.//
		//B: you’re lying.//
		//She told us.//
		So then.	

4	James (Ja)	It’s three going up because the base, you start with one. (overlapping voices)	**EPISODE 2** The discussion of the construction of the towers continues, with Jenna’s participation.
5	Jenna (Je)	It’s like this .. it’s two going up.	**LHSIA (14)** This is the first time Jenna seems to contribute to the group discussion about the tower structures. Therefore, she appears to be demanding attention to her own ideas.
6	B	No, I get it, so it would be like five going up and four going around.	**LHSIA (12)** Bridget seems to be saying to Jenna that she can keep up with her and her ideas.

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<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td colspan="4">Interpretation: Jenna participates in a way that shares her own ideas for the first time during the problem solving session. Because she starts with “It’s like this,” it is not clear to me which tower structure she is talking about, though likely she is referring to the 2-block high tower, since she states, “It’s two going up.” By stating this and expressing her own idea, she appears to be demanding the attention of her classmates.
In response, Bridget claims, “No, I get it,” suggesting to Jenna that she understands – though it is not clear to the observer if she understands what Jenna was saying or how the towers are constructed. To further her claim that she understands and is keeping up with her classmates, mathematically, she explains it in her own words, “It would be like five going up and four going around.”</td></tr>
<tr><td>7</td><td>Je</td><td>So it would be one less on the bottom every time.</td><td></td></tr>
<tr><td>8</td><td>B</td><td>No, one more each time</td><td></td></tr>
<tr><td>9</td><td>Je</td><td>One less each time, on the bottom I’m talking about right now.</td><td></td></tr>
<tr><td>10</td><td>Ja</td><td>On the bottom it’s going to be four, and on the top it’s going to be five.</td><td></td></tr>
<tr><td>11</td><td>B</td><td>You add one to each and every side though, every time you go higher and higher?</td><td></td></tr>
<tr><td>12</td><td>Je</td><td>Yeah. You add one to this too. You add one to each one.</td><td>LHSIA (10)
Jenna builds on the idea that Bridget just asked about, which appears to be that a block is added to each side as well as to the height for each next iteration.</td></tr>
<tr><td colspan="4">Interpretation: Jenna and Bridget continued to exchange ideas and comments for several turns. It seems that Jenna is expressing the idea that each side has one less block than the height, but Bridget is stating that the height has one more block than each side. Though they are basically the same statement, these two girls do not seem to realize that right away. Bridget concludes this part of the discussion by stating and asking if a block is added to each side when a block is added to the height.
Jenna confirms and builds on Bridget’s idea, stating again that, “You add one [block] to each one [side].” By doing so, especially since she seems to speak up only when she wishes to express her ideas about the problem, I infer that Jenna has maintains a LHSIA engagement structure for several speaking turns.</td></tr>
<tr><td>13</td><td>B</td><td>Alright so if I had a five block high tower, it would be five blocks going up and four blocks going around. And then for a 100 block tower, it would be 100 going up and 99 going around.</td><td>LHSIA (10, 14)
Bridget also continues to build on the ideas she and</td></tr>
</table>

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<tr><td></td><td></td><td>And then if we had a 10 block, it would be 10 block high and 9 blocks around.
//Je: Yeah, alright.//</td><td>Jenna were expressing. By giving more than one example, she appears to be demanding her classmates' attention.</td></tr>
<tr><td colspan="4">Interpretation: Bridget builds on the ideas that she and Jenna were expressing, by giving specific examples of the way the towers are constructed. Specifically, Bridget is giving the tower construction of the towers mentioned in the task problem: the 5-block high tower, the 10-block high tower, and the 100-block high tower. Her tone of voice conveys confidence in her statements and she speaks loudly enough to demand the attention of her classmates while she speaks.</td></tr>
<tr><td></td><td colspan="2">For about 4 minutes:
The students then discuss that they need to write the information down on their papers. They continue to work individually.</td><td></td></tr>
<tr><td>14</td><td>B</td><td>This was easy, I'm done.</td><td>EPISODE 3
LHSIA (6)
Bridget "That was easy." She resists Liza's suggestion that they have to "explain it."</td></tr>
<tr><td colspan="4">Interpretation: Bridget decides to share with her classmates that she believes she is "done," by which she appears to mean she is finished writing down her statements about how the towers are constructed. According to her student work, Bridget has not yet discussed the total number of blocks needed for each tower structure. In stating, "That was easy," Bridget appears to be expressing that she is smart since she, first believes the task was easy, and second believes that she has completed the work. In her next speaking turn below, she does not appear to want to do additional work, particularly if the work is not required.</td></tr>
<tr><td>15</td><td>L</td><td>You gotta explain it. I'm gonna explain it.</td><td></td></tr>
<tr><td>16</td><td>B</td><td>No we don't.</td><td></td></tr>
<tr><td>17</td><td>L</td><td>I'm gonna explain it.
(Bridget raises her hand))</td><td></td></tr>
<tr><td>18</td><td>L/Ja</td><td>She can't help us</td><td></td></tr>
<tr><td>19</td><td>B</td><td>I know, I'm gonna ask if we have to explain each and every part.</td><td></td></tr>
</table>

<table>
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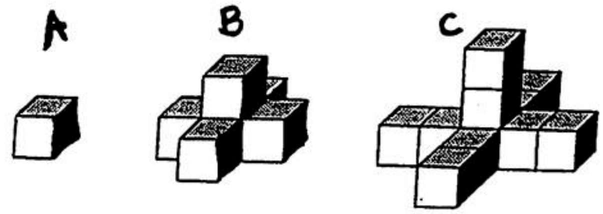
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td></td><td></td><td>//No, she can’t help us.//</td><td></td></tr>
<tr><td>20</td><td>B</td><td>There’s really nothing to explain. You just added one to each and every one.</td><td></td></tr>
<tr><td>21</td><td>Ja</td><td>Well, that’s how you explain it. Like that.
(Students work to write independently. Sometimes small utterances are heard but can’t make out words.)
(In about a minute the audio recorder falls. Students argue as researcher picks up materials.)</td><td></td></tr>
<tr><td></td><td colspan="2">For about 6 minutes:
The teacher approaches the group and she has the students explain how they determined their pattern – noticing that each leg has one less block than the height. Liza does most of the explaining, though Bridget was the first to speak to Ms. A, saying, “I’m done.” Bridget has blocks in front of her, and Ms. A encourages her to build the towers using those blocks. Ms. A also encourages the students to agree on the total number of blocks each tower has. So far they hadn’t discussed the total number of blocks in the tower structures. Jenna is the only student who wrote the totals on her paper by the end of class. Before she leaves the group, she encourages them to keep working. She reminds the students, particularly Bridget, that they want to find a generalization to apply to the 100-block high tower, the 1000-block high tower.</td><td></td></tr>
<tr><td></td><td colspan="2">For about 25 minutes:
After the teacher leaves the group, the students use the blocks a little more, but it is not clear what they are doing. It appears that James becomes upset with Bridget, but this seems to blow over. James and Jenna argue, though the other two in the group suggests that they just ignore one another. They even ask the teacher to intervene, and she speaks to James privately.
The students discus other things, with Bridget even stating, “We ain’t even talking about math.”</td><td></td></tr>
<tr><td></td><td colspan="2">For about 11 minutes:
Ms. A returns to the group to see if they have made any progress. Bridget repeats that they are done and have been for a while. The teacher asks the students specific questions, about how many total blocks are in the 5-block high tower, and goes back to the 2-block high tower and the 3-block high tower. To help them answer</td><td></td></tr>
</table>

Bridget, Liza, James, Jenna			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
	these questions, Bridget picks up a bag of SnapCubes that the students use to build the tower structures. While the students answer their questions (i.e., state that there are 11 blocks in the 3-block high tower and there are 21 blocks in the 5-block high tower), they never seemed to write this down on the papers they handed in. It appears that any of this information was written in the students' notebooks, but the students did not hand in those pages. Ms. A also asks them to recognize a pattern, which they state is adding 5 blocks for each successive tower.		
22	B	(After the teacher walks away from the group) It's 46! You heard me? It's 46.	
23	Ja	I got 18…again…for the 5th one…	
		The students seem to stop working on the task and are having discussions off-task. They play with the structures created by the blocks (SnapCubes), and create other structures using the blocks.	

Bridget's work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



5 block high tower:- 5 blocks going up and 4 going around each side.

10 block high tower: 10 blocks going up and 9 going around each side.

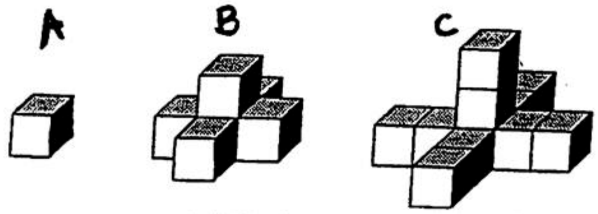
100 block high tower: 100 blocks going up and 99 blocks going around each side.

So you are + 5 each time your ~~making~~ building blocks again.

Liza's work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



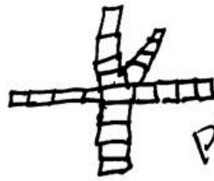
D. 4 Blocks going up and 3 going ~~around~~ across

5Block tower E 5 Blocks going up and 4 going across

10 Block high tower – 10 blocks going up and 9 across

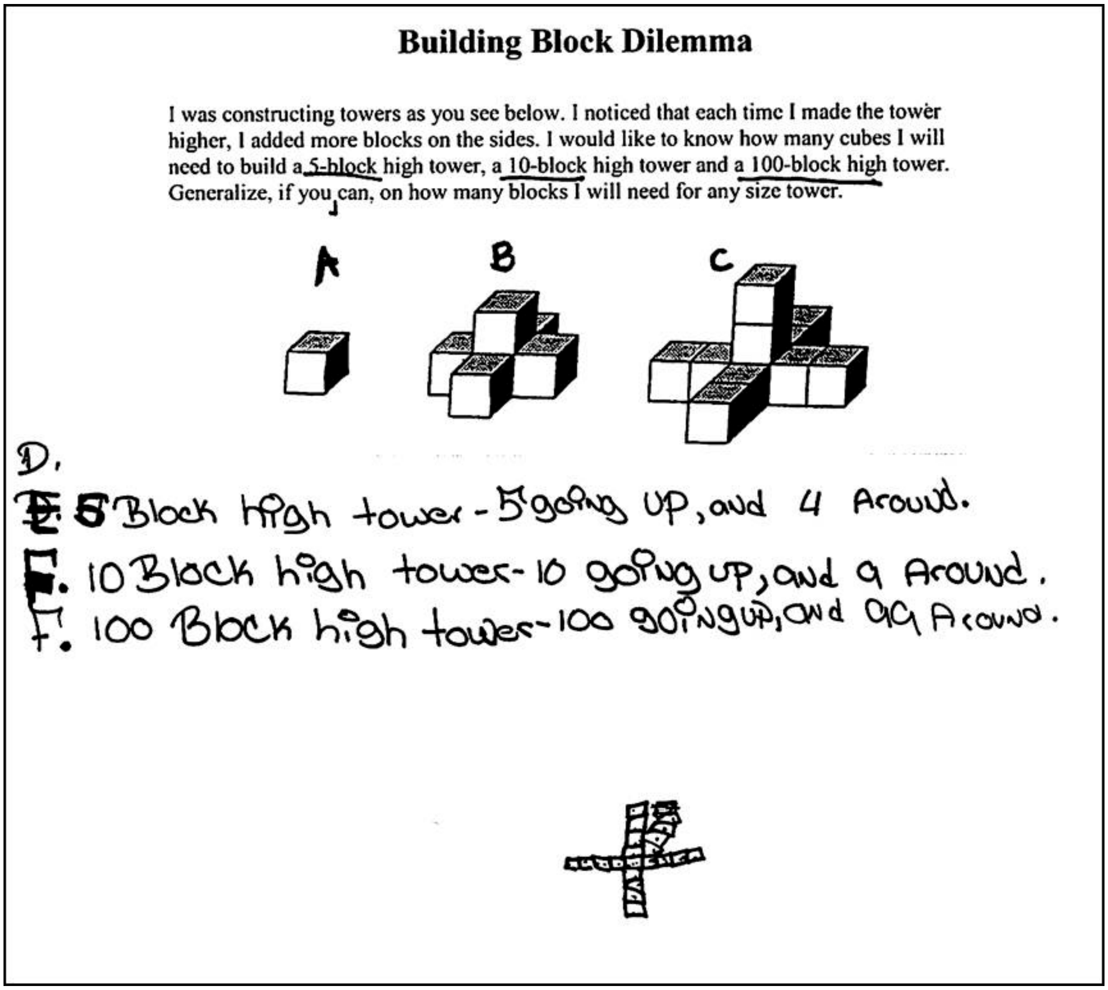
100 Block high tower – 100 blocks going up and 99 accoss.

I got all of my answers because I saw a pattern when the towers get higher the number going up is bigger by 1 to the one's going across. And for each Phase we are adding 5

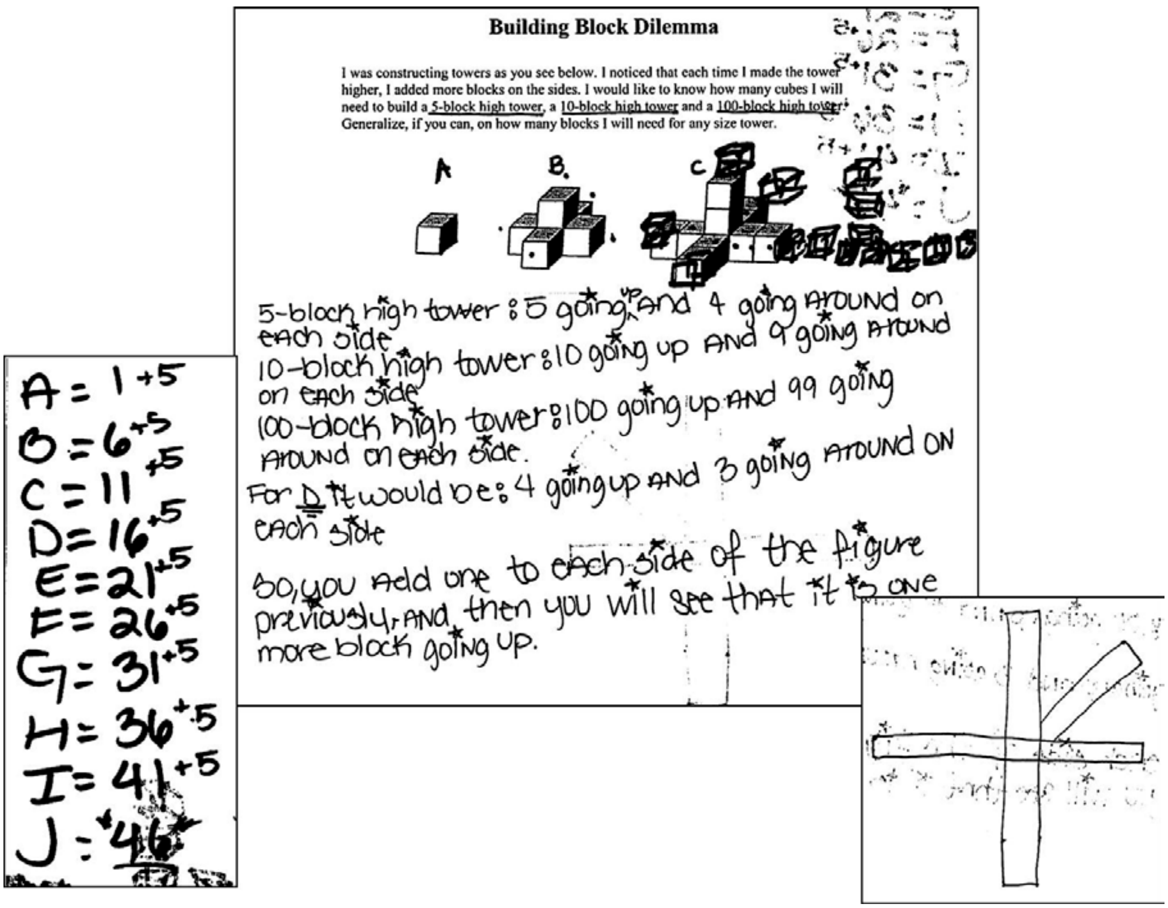


Phase 5

James's work, in Nov. 2008



Jenna's work, in Nov. 2008



Questionnaire Responses for Bridget, Liza, James, Jenna

Questionnaire items which may indicate LMTY structure

Questionnaire Items	**Bridget**	**Liza**	**James**	**Jenna**
Statements (All the time, Sometimes, Never)				
I wanted to teach another student something that I knew that the other student did not know.	Sometimes	All the time	Never	All the time
I listened carefully to the ideas of someone I was trying to help.	Sometimes	Sometimes	All the time	All the time
I helped someone see how to do the math.	Sometimes	Never	Sometimes	All the time
Others listened carefully to my ideas.	All the time	Sometimes	Sometimes	Sometimes
Behaviors (Often, Sometimes, Hardly ever)				
I gave helpful suggestions.	Sometimes	Sometimes	Often	Sometimes
I worked cooperatively.	Often	Sometimes	Sometimes	Often
Thoughts (Yes/No)				
I like teaching this person things that I know.	Yes	Yes	Yes	Yes

Questionnaire Responses for Bridget, Liza, James, Jenna

Questionnaire Items Which May Indicate LHSIA structure

Questionnaire Items – Statements	Bridget	Liza	James	Jenna
Statements (All the time, Sometimes, Never)				
I wanted people to think that I'm smart.	Never	All the time	Never	Never
I tried to impress people with my ideas about the problem.	Sometimes	Sometimes	Never	Never
People seemed impressed with the ideas I shared about the problem.	Sometimes	Sometimes	Never	Sometimes
People saw how good I am at the math we did today.	Sometimes	Sometimes	Sometimes	Sometimes
I felt smart.	Sometimes	All the time	All the time	All the time
I wanted to show someone that my way was better.	Never	Sometimes	Never	Sometimes
I was a lot better at math than others today.	All the time	Sometimes	Never	Sometimes
Behaviors (Often, Sometimes, Hardly ever)				
I was the leader.	Sometimes	Sometimes	Hardly ever	Sometimes
I was bossy.	Often	Sometimes	Hardly ever	Hardly ever
I wanted to show off.	Sometimes	Sometimes	Hardly ever	Hardly ever
I liked to be right.	Often	Often	Sometimes	Often
Thoughts (Yes/No)				
I want you to know just how smart I am.	Yes	Yes	Yes	Yes
People think I'm smart.	Yes	No response	No	Yes
I wish the teacher would call on me, so I can show how much I know.	Yes	Yes	Yes	No

Ms. A Class 2 Introduction to Building Blocks Task

This is the introduction Ms. A gave to her grade seven Class 2.

T: Ms. A / Teacher
St(s): Unknown Student(s)
J: Jaden

T: Ok class I want you to write the name, date, your math teacher name, and class, your homeroom number on the paper.

Ok let's read the test together. Pay attention while we're reading. Building block dilemma. I was construction towers as you see below, I noticed that each time I made the tower higher, I added more blocks on the sides. Where are the sides?

T: Can anybody tell me which part of B is representing the sides?
Ok, how many sides do you have?

Sts: (Mumble) Six, four

T: Do you all agree with that?

Sts: Yeah

T: 6 or 4, some say 6 I heard 4

J: Four, four

St: Five

T: So if I tell you that who is the person to your side?

St: Two

T: Ok so you have how many, how many sides do you have?

Sts: Two

T: Ok so you have one on this side and one on this side, you can have one facing you and one behind you. Ok. So when I talk about sides, I'm talking about lines horizontally. So if you look at B, what sides do you have?

St: Four

T: Do you all agree with that? Anybody can't see that?

J: I think its 8

T: Now so I added more blocks on the sides, so you know which sides we are talking about. Ok now what about the vertical. Are you adding to when you are comparing A to B? Are you adding to it?

Sts: No, yes

T: Ok so we are adding on the sides horizontally and you're adding vertically. Ok? I would like to know how many cubes I would need to build a 5 block high tower, a 10 block high tower, and a 100 block high tower. Generalize if you can on how many blocks I would need for any sides tower. So basically, what you're looking for is the number of cubes in the whole tower if you have 5 blocks in height, 10 blocks in height, 100 blocks in height. Ok? Anything in between you can find if you want. Ok get started in your work.

Talia, Jaden, Genevieve, Andrew

This is Group 1 from Ms. A's grade seven Class 2.

Verbal emphasis indicated by underline
Actions included either in *italics* or (parentheses) as appropriate.
Pauses in speech are indicated by …
Words inserted to help clarify a student's words are included in [brackets].
When students voices overlap, one or both sets of words are included in //double slash marks//.

The number included next to either LMTY or LHSIA refers to the code, included in the table of codes.

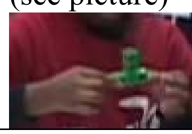
Talia, Jaden, Genevieve, Andrew			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
1	Ms. A	*The teacher has just finished giving her introduction to the problem and instructions. This takes a little more than 3 minutes.*	
2		(Teacher walks over and continues on)	
3	Talia (T)	Ok let's start. (Some other things said before they start talking about the problem.)	**EPISODE 1**
4	Genevieve (G)	You see that its one block, and then they add one there, on there, one there	**LMTY (1)** Genevieve explains her interpretation of the problem (how the blocks are added to construct the towers)
5	Talia (T)	1,2,3,4	
6	G	No, no, no they add block, and they then add a block here, and there and	
7	Jaden (J)	there. I don't get this problem.	
8	G	And then 1,2,3,4,5	

Talia, Jaden, Genevieve, Andrew			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
9	J	They're adding a block, Talia. Ok, can I read the question? I was constructing towers as you see below. I noticed as each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5 block high tower, and a 10 block high tower, and a 100 block high tower. Generalize if you can on how many blocks will I need for any size tower. 'Kay so I think this is the 5 block	
10	T	Well count 1,2,3,4,5 but there should be a tower coming out, I mean a block coming out so 6. (She points to Genevieve's paper to explain her counting strategy.)	
11	J	Ok so this could be the 10 (keeps tapping Talia to get her attention) It can't be at the bottom because look at this, its 2 on each side so this is... you could count by 2... well, actually, you can't count by 2…	**LMTY (1)** Jaden believes he understands what the problem is looking for, so he tries to explain it
12	T	So just count them 1,2,3,4,5,6,7,8,9,10, cause the total is 10 blocks (smiles and moves her head as she says "blocks")	**LHSIA (7)** Talia gives her answer to 'just count' all the blocks. Her tone on 'ten blocks' is higher in pitch and said with slightly more emphasis.
13	J	No, y'all not listening to what I'm sayin'. You see there are 2 sides on each side of this thing. (pointing on his paper with his finger while counting out loud) 1, 2, and then 3,4,5,6,7,8,9,10,11,12...there were 14 sides all together. And C, And then C is 14, and C is 14 all together.	**LMTY (2) LHSIA (14)** Jaden first tells his classmates that they should listen to him. He then explains his thinking about the problem: that the problem is asking for the number of sides (*I am not*

<table>
<tr><th colspan="4">Talia, Jaden, Genevieve, Andrew</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td></td><td></td><td></td><td>completely sure what that means – I think the faces of the blocks, similar to surface area)</td></tr>
<tr><td colspan="4">Interpretation: After re-reading the problem (line#), Jaden seems to think he knows how to approach the problem, and tries to explain to Talia and his other classmates his strategy, “you could count by 2.” Jaden wants to share his strategy with his classmates, leading me to infer an active LMTY structure.
Talia responds to Jaden, telling him to count the blocks. She gives her own answer, without appearing to consider what Jaden just said. Her response suggests she has an active LHSIA structure as she gives her own answer with minimal explanation.
Jaden in return, appears to have an active LHSIA structure by stating, “y’all are not listening to what I’m sayin’.” At the same time, he continues to try to explain his strategy of counting by 2. Although it is not very clear, it appears that Jaden is trying to count the number faces of blocks that are visible in the tower structure. (Note that if this was the case, 14 would not be the correct answer for the tower in figure C.) Because Jaden persists with his earlier strategy, and attempts to modify his explanation, I infer that he still has an active LMTY structure in addition to the active LHSIA structure in line #.</td></tr>
<tr><td>14
15</td><td>G</td><td>It could be like this, too. (drawing on her paper)
Hmm-hmm.
And it’s 5. 1, 2,3,4,5. It could be like that. It could be one there but you don’t need…</td><td></td></tr>
<tr><td>16</td><td>T</td><td>It could be holding it. It could be holding it but it could also be like that.</td><td>I think they are suggesting that the middle hidden block might not actually be there anymore, in figures B and C.</td></tr>
<tr><td>17</td><td>J</td><td>Do you get this at all? Do you get this?
(Andrew shakes his head and basically refuses to speak)
//Genevieve: Let him talk then!//
Jaden and Talia point the audio recorder toward Andrew. He still refuses to speak and they all fiddle with the recorder for a few seconds. They all look at Andrew waiting for him to say anything.
You don’t get the concept?
(Talia asks Andrew to explain what you</td><td></td></tr>
</table>

Talia, Jaden, Genevieve, Andrew			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
		know. He refuses to speak.) I already did, I already told her.	
18	G	Look at what is says, 5 blocks, 1, 2,3,4,5. Five. It could be holding one down, or it could just be like that, too. Nah.	
	The students' discussion moves to the direction of whether or not they understand the problem or not. Jaden tries to continue to explain his understanding, which he stated earlier. However, Talia and Genevieve continue to talk over him, so it is difficult to know exactly what he is saying. Talia counts to 10, perhaps in a way of counting the number of blocks she believes is in figure C, representing the 3-block high tower. *They discuss asking for help, particularly after the researcher says they should ask their teacher for assistance.*		
	Jaden calls over one of the teacher's assistants and asks for her help. The teacher assistant, Ms. G, asks the students to verbalize what the problem is asking. She also tries to help them recognize how the towers are constructed, using a pattern. She gets then a bag of SnapCubes, and then leaves them to work on their own for a while.		
19	Sts	*Genevieve, Talia, and Jaden all use blocks to construct their own interpretations of the towers. Andrew looks on.*	**EPISODE 2** The students try to determine how the towers are supposed to be constructed.
20	T	(to Genevieve) I know what you're trying to say. (Talia is using the SnapCubes to lay down a base for what appears to be the 2-block high tower) But what **I'm** trying to say is it could be like that (Based on the video, I think Talia believes that the middle block remains as part of the tower structure, even though it is hidden in the 2-block high tower) (to Andrew) Which way do you think it is?	**LHSIA (9A, 15)** Talia disagrees with Genevieve's idea (I believe Genevieve thinks that the middle block is no longer there for the 2-block high tower) Talia uses the blocks to help

Talia, Jaden, Genevieve, Andrew			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
			state her strategy for constructing the towers.
21	A	Like that (points to the structure that Talia has just put together)	
	T	Thank you.	
Interpretation: Now that the students have SnapCubes to work with, they are working toward determining the construction of the towers, along with the total number of blocks in a tower structure. I believe that Genevieve has suggested that the middle block from the first tower is no longer there in the 2-block high tower and 3-block high tower. Based on what Talia does with these blocks, she seems to disagree with Genevieve's idea, suggesting that the middle block is still there. I infer that she is trying to demonstrate that her idea is correct, leading to an active LHSIA structure. Talia also tries to get confirmation from Andrew that her idea is correct by asking him, "Which way do you think it is?"			
22	J	I got mines like that (displays a structure he constructed – not a complete tower as per the task.) It could be like this too. (Jaden's classmates seem to ignore his idea)	**LHSIA (14)** Jaden is trying to get his classmates' attention by showing them his construction using the SnapCubes
Interpretation: Jaden tries to get his classmates' attention to the tower he constructed using the SnapCubes as well. He makes several efforts to get their attention, as demonstrated so far in the problem-solving session. However, most of the time, his classmates seem to ignore his ideas and strategies. Perhaps because they continue to ignore him, Jaden may be more motivated to be acknowledged by them as having some mathematical ability.			
23	T	I just think it would fall. (perhaps referring to the construction that Genevieve suggests)	
24	G	It wouldn't fall.	
	T	*Inaudible; indicates with her hands something*	

Talia, Jaden, Genevieve, Andrew			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
		that about there not being something in the middle to hold up the other blocks	
25	J	I did this, I did this (picks up his tower structure and shows it to his three classmates)	
26	T	(to Jaden) Is that there? (likely referring to the diagrams on the task paper)	
27 1	J	No.	
28	T	Exactly.	
29	J	But does it say it has to be here?	
30	T	Yeah, it has to be because you're doing the problem.	
31	J	But still you tryin' to figure out for the 5 tower, 5 high tower block, so look. 1,2,… (It is possible that Jaden thinks that the total number of blocks is supposed to be 5, since that is what he has in front of him (see picture) 	
32	T	This is the second one (She displays the correct tower B) B is this one.	
33	J	3, 4, 5, 6. I think this is 6.	
34	U	(Announcement)	
35	T	That could be that one. The 10 one.	
36	Research er	(Hands more paper)	
37	A	I know how to do this. Watch this. (Grabs bag of blocks) //Genevieve: Into audio recorder apologizing about something.//	
38	T	Look at this one. This one is this one (taps the task paper with her tower construction). It's big. Really big. Okay. 1,2,3,4,5,6,7,8,9,10,11 (points at the structure she constructed – the 3-block high tower). So	**LHSIA (7)** Talia is giving her answers – the total number of blocks in the

Talia, Jaden, Genevieve, Andrew			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
		this one is 11. This one is 6. Let's just say 6 for now and this one is 1.	3-block high tower, the 2-block high tower, and the 1-block high tower, respectively. not really going out to "impress others" but she's not really teaching, because she hasn't sought to explain. Though later she does clarify that she was counting the cubes and not the sides (yet). Alternatively, all students seem to have an active GTJD structure.
39	G	That's 2, I mean 4	
40	T	Yeah but its 1 cube, I'm counting the cubes. I'm not counting the sides.	
Interpretation: Talia continues to build tower structures using the cubes, particularly of the towers depicted in figures B and C on the task paper. She seems to realize that the goal is to determine the total number of blocks in the tower, so she counts the number of blocks in her 3-block high tower. Andrew may have started to suggest an idea, as he reached for the bag of cubes, but he does not immediately suggest a strategy or answer. After Talia's statement that there is 1 block in the 1-block high tower, Genevieve makes a statement to disagree, but Talia does not consider her contribution. Talia clarifies that she is counting the cubes rather than the sides. These behaviors and statements altogether lead me to infer an active LHSIA structure for Talia.			
41	J	This is what I got for 10.	
42	G	You got on here one over here	
43	T	No, I'm only counting. (overlapping with Genevieve)	
44	G	I know.	**LHSIA (9A)**

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<tr><td></td><td></td><td>(uses her pen to point to the diagram on Talia’s paper)
1 square right there, 1 square right there, 1 square there and 1 square on top</td><td>Genevieve disagrees with Talia’s counting strategy</td></tr>
<tr><td colspan="4">Interpretation: Genevieve is responding to Talia’s statements about the total number of blocks in the towers. Talia first stated that there are 11 blocks in the 3-block high tower, 6 blocks in the 2-block high tower and 1 block in the 1-block high tower. Genevieve started to disagree at that point, stating, “That’s 2, I mean 4.” Talia attempted to note a difference between what she was counting, the number of cubes, and what Genevieve may have been counting, which is the number of sides on the tower or cube. Genevieve continues with her disagreement, explaining where she obtained her answer of 4 from. By persisting with her position, I infer that she has an active LHSIA structure and wants her ideas to be acknowledged by Talia.</td></tr>
<tr><td>45</td><td>T</td><td>I’m not counting that one yet,
//G: inaudible//
Ok, but I’m not counting that yet, I’m only counting this.
//G: Look. One square right there. One square right there. (writing on her paper as she speaks)//
I know, but I’m just counting the cube.
//J: inaudible, gestures with his head toward Andrew
I don’t know (in response to Jaden)
(to Andrew) What are you doing?
Andrew appears to be connecting several interlocking blocks together to create some structure. He does not respond verbally.</td><td></td></tr>
<tr><td>46</td><td>J</td><td>This is what I got for 10 cubes.
(Jaden shows a tower structure that has 10 cubes in it, which is different than the one asked for in the task.)
(I think that the others are ignoring Jaden’s structure, since it is different from those asked about in the task.)</td><td></td></tr>
<tr><td></td><td></td><td>Talia seems to make a comment and laugh about something going on off camera. Jaden laughs with her.
Genevieve says something inaudible. Talia responds by counting to 5, but she is holding the 2-block high tower that she has constructed.</td><td></td></tr>
<tr><td>47</td><td>A</td><td>(with the 5-block high tower in his hand</td><td></td></tr>
</table>

Talia, Jaden, Genevieve, Andrew			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
		that he has constructed, counting the number of blocks on a single side) 1, 2, 3, 4, 5. (repeats counting to 5 with the other sides, seems to have 5 additional blocks on each side, rather than just 4 coming off the middle block)	
48	G	(to Andrew) There you go, thank you. That's what we were, I was trying to… Now you just gotta count them all (looking at Andrew's model)	
49	T	Now, this block (showing Jaden the 2-block high tower) This is 11, this is 11.	
50	J	This is 10. (showing his tower structure that does not follow the pattern given in the task)	
51	G	This is 11 but that's from the 10 thing.	
52	J	I have a, I have a, I have a box, this is what I did with the 10-block. (draws or writes something on his own paper)	
53	G	Somebody do the 10 block while he's [Andrew] doing the 5.	
54	T	I want to do the one block.	
55	G	It's only one!	
56	T	I know!	
57	G	(Takes bag of blocks from Andrew, who indicates he wants more blocks.) He's doing the 5-block. (to Andrew) Count all of that to see.	
58	Talia	This is the 10 block (she has taken 1 block off the structure, from the height; this structure has a total of 10 blocks, but does not follow the given pattern)	
59	A	There are 25 in here.	
60	G	Every time I want to do something… (*rest is inaudible*)	
61	Sts	*Overlapping discussion, difficult to*	

Talia, Jaden, Genevieve, Andrew			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
		understand	
62	T	He did the 10-block (referring to Jaden)	
63	J	This is the 10 block; I did the 10 block already. I'm counting it by 2's.	
64	G	It's not like that, you gotta do it like that! (getting animated as she points at Andrew's, repeatedly; tapping hand to head indicating frustration, I think with Jaden)	
65	J	No I don't. You could make any tower! (emphasizing "any tower" to indicate that he is correct)	
66	T	That's a 5?	
67	G	No its gotta be like this. (using the blocks in her hand to point to the task paper in front of her) You gotta add (likely referring to add to the tower structures that were given on the problem task) (She continues to put blocks together, using the SnapCubes from the bag in front of her)	
68	J	Yes, it is. It's any tower. (Andrew and Talia seem to be staying out of the conversation between Genevieve and Jaden)	
69	G	I'm doing my own thing then (as she continues to work with some blocks in her hand)	
70	Talia	So, as I was saying, this is the 6. (referring again to the 2-block high tower)	
	The teacher assistant, Ms. G, returns to the group, and spends the remaining 30 minutes with these students. Jaden continues to make his point that he has already constructed a 10-block tower, and she responds by asking him if it follows the same pattern as in the diagrams. Ms. G also focuses on Andrew's construction of the 5-block high tower, asking him how many blocks are in that tower structure. We see that both Talia and Jaden have each started putting together a different kind of structure with the SnapCubes. *Genevieve also appears to be putting together a 10-block high tower, which she shares with Andrew. She is*		

Talia, Jaden, Genevieve, Andrew			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
	counting the number of blocks in each leg, starting with the middle base block each time. The only difference is that for two opposite legs, there are not 10 blocks on each side (this may be because of a shortage of cubes available because other students are also working). *Ms. G seems to be guiding Talia and Jaden through each tower structure, as we see a 2-block high tower that has been constructed. Jaden counts that there are 6 blocks in this structure, and he refers to the tower has having a height of 2. Talia is constructing a different structure that may or may not be related to the task – she then takes it apart.* *As Jaden holds what appears to be a 4-block high tower that he has put together, Ms. G asks him to identify how many blocks have been added to one tower to get the next tower. To help him see there are 5 blocks, she takes away cubes to make the tower a 3-block high tower and has him put the necessary blocks on that structure to make it a 4-block high tower. Ms. G seems to spend most of her attention on Jaden.* *Talia seems to shut down, as she is not in the conversation with Ms. G and Jaden. Genevieve and Anthony mostly seem to work on their own, sometimes discussing the problem together.* *Ms. G then refers to the tower that Andrew has constructed, and asks Jaden and the others questions to encourage them to recognize the plus 5 pattern. They realize, and appear to agree, that there are 21 blocks in the 5-block high tower. However, then Jaden suggests there are 26 blocks in the 10-block high tower (rather than in the 6-block high tower). Ms. G then encourages the students to record the information, in a table, which is seen in the student work.* *Ms. G then encourages the students to think about a formula that might help them determine the total number of blocks in the 100-block high tower more quickly than continuing to add 5. Jaden is the most verbal student during this exchange, as Andrew, Genevieve, and Talia do not seem to be making suggestions as to an equation.* *Talia and Genevieve are playing the cubes, just putting them together in a non-task related way. Genevieve defends her actions, suggesting that it contributes to an*		

Talia, Jaden, Genevieve, Andrew			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
	answer. *Jaden accuses the others of not working, like he is. From here until the end of the session (about 3 minutes), Ms. G focuses on helping Jaden recognize a formula or equation. Genevieve breaks apart the SnapCubes and puts them away. Talia and Andrew do not seem engaged, though they may be listening to the conversation between Jaden and Ms. G.*		
	Ms. A gives entire class instructions to clean up and gather their work. The problem-solving session is over and the students are administered the questionnaire for the remainder of the class period.		

Talia’s work, Nov. 2008

Not included in data

Andrew’s Work, Nov. 2008

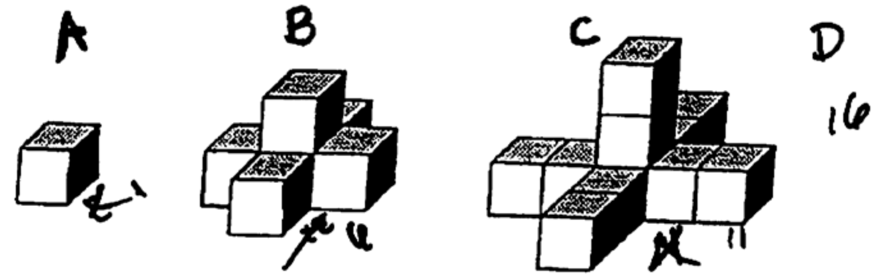
Not included in data

Talia’s and Andrew’s student work is not available. The reason for this is not known.

Jaden’s Work, Nov. 2008

Building Block Dilemma

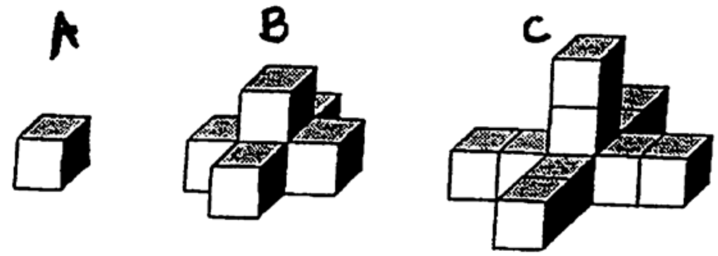
I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.

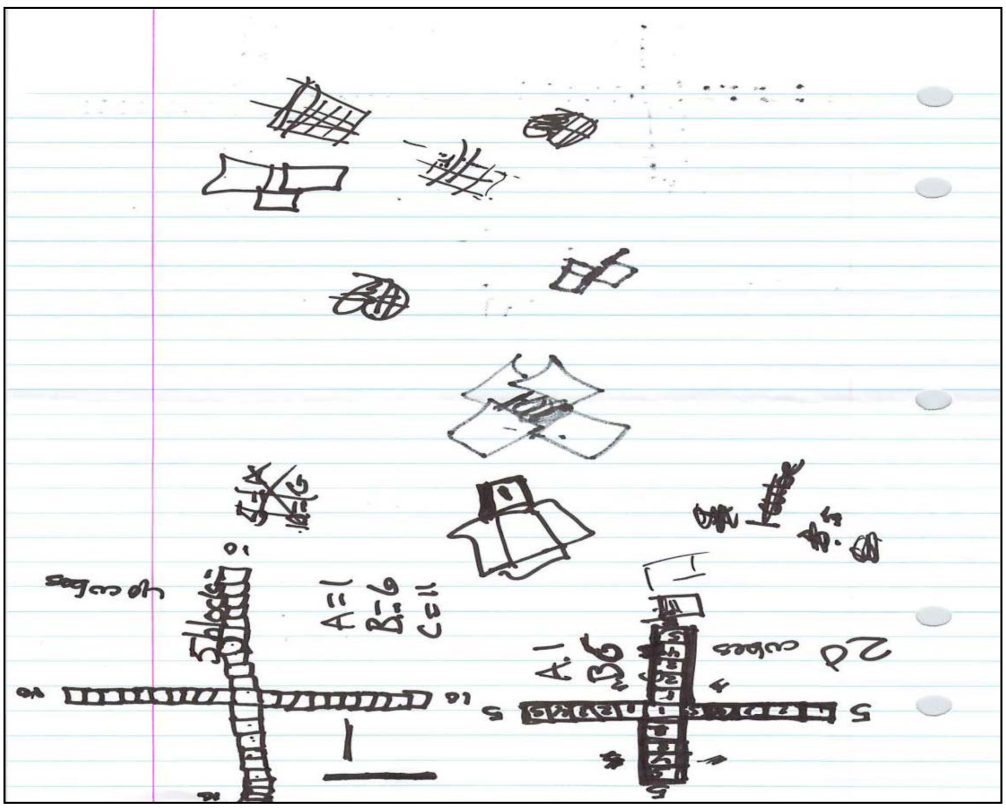


Genevieve's Work, Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.





Questionnaire Responses for Talia, Jaden, Genevieve, Andrew

Questionnaire Items Which may indicate LMTY structure

Questionnaire Items	Talia	Jaden	Genevieve	Andrew
Statements (All the time, Sometimes, Never)				
I wanted to teach another student something that I knew that the other student did not know.	Never	All the time	All the time	Never
I listened carefully to the ideas of someone I was trying to help.	Sometimes	All the time	All the time	All the time
I helped someone see how to do the math.	All the time	Never	Sometimes	Sometimes
Others listened carefully to my ideas.	All the time	Sometimes	Sometimes	All the time
Behaviors (Often, Sometimes, Hardly ever)				
I gave helpful suggestions.	Sometimes/ Often*	Often	Sometimes	Often
I worked cooperatively.	Often	Often	Sometimes	Often
Thoughts (Yes/No)				
I like teaching this person things that I know.	Yes	Yes	Yes	Yes

*Both answers were circled.

Questionnaire Responses for Talia, Jaden, Genevieve, Andrew

Questionnaire Items Which May Indicate LHSIA structure

Questionnaire Items – Statements	**Talia**	**Jaden**	**Genevieve**	**Andrew**
Statements (All the time, Sometimes, Never)				
I wanted people to think that I'm smart.	Sometimes	All the time	Never	Never
I tried to impress people with my ideas about the problem.	Never	All the time	Never	Never
People seemed impressed with the ideas I shared about the problem.	Sometimes	Sometimes	Never	Sometimes
People saw how good I am at the math we did today.	Sometimes	Sometimes	Sometimes	Sometimes
I felt smart.	Never	All the time	Sometimes	All the time
I wanted to show someone that my way was better.	All the time	Never	Sometimes	Never
I was a lot better at math than others today.	Never	Sometimes	Never	Never
Behaviors (Often, Sometimes, Hardly ever)				
I was the leader.	Hardly ever	Often	Hardly ever	Sometimes
I was bossy.	Hardly ever	Sometimes	Hardly ever	Hardly ever
I wanted to show off.	Hardly ever	Hardly ever	Hardly ever	Hardly ever
I liked to be right.	Often	Hardly ever	Hardly ever	Often
Thoughts (Yes/No)				
I want you to know just how smart I am.	Yes	Yes	No	No
People think I'm smart.	Yes	No	Yes	Yes
I wish the teacher would call on me, so I can show how much I know.	No	Yes	No	No

Rico, Pedro, Kian

This is Group 2 from Ms. A's grade seven Class 2

Verbal emphasis indicated by underline
Actions included either in *italics* or (parentheses) as appropriate.
Pauses in speech are indicated by …
Words inserted to help clarify a student's words are included in [brackets].
When students voices overlap, one or both sets of words are included in //double slash marks//.

The number included next to either LMTY or LHSIA refers to the code, included in the table of codes.

Rico, Pedro, Kian			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
	Ms. A	*The teacher has just finished giving her introduction to the problem and instructions. This takes a little more than 3 minutes.*	
1	Rico (R)	Ok this is what's gonna happen.	**EPISODE 1** The students quickly determine a strategy, and arrive at what they think is the answer.
2	Kian (K)	(leans over to Pedro's paper, uses his pencil to point to the figures) That's 1 that's 2 that's 3 and after 3 its…	
3	Pedro (P)	It's followed by 5 so…	
4	K	Where you see 5 at?	
	R	//(pointing to figure B on Pedro's paper as Pedro is speaking; counts) 1, 2, 3, 4, 5//	
5	P	(gestures to the figures on his paper as he speaks) Right here, they add another 5 so when they get up to 100, it's gonna be like 100 up here, 100 over there, 100 over here, 100 over there. So it's just like 100 times 5. That's for the 100 block tower. A 5 block tower is gonna be 5, it's gonna be um, 5 up, 5 this way 5 that way 5, 25 in total.	**LMTY (1, 7)** **LHSIA (14)** Pedro responds to Kian's question, and expands on his own answer by giving what he thinks is the answer for the 100-block high tower.

<table>
<tr><th colspan="4">Rico, Pedro, Kian</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td colspan="4">Interpretation: Pedro seems to have the most confidence, perhaps, of the students in this group when he states, “It’s followed by 5,” at line #. I infer that Pedro may have an active LHSIA structure because he is willing to state his first ideas. Kian asks Pedro about this, “Where you see 5 at?” which encourages Pedro to explain his ideas further. Pedro’s explanation leads me to infer that the LMTY structure has been activated for him as well. Though Pedro seems to jump to the 100-block high tower, suggesting that the four legs and the height each have 100 blocks, he uses this explanation to respond to Kian. Pedro follows up by returning to the 5-block high tower, and even states an answer, that there would be 25 total blocks in the 5-block high tower.</td></tr>
<tr><td>6</td><td>K</td><td>Let’s say you get to 100 .</td><td></td></tr>
<tr><td>7</td><td>P</td><td>Yeah so it’s gonna be….</td><td></td></tr>
<tr><td>8</td><td>K</td><td>So it’ll be like 5 times 100</td><td></td></tr>
<tr><td>9</td><td>P</td><td>It’s gonna be 5 times 100</td><td></td></tr>
<tr><td>10</td><td></td><td>Students working on multiplication on graphing calculators that they have at their desks.</td><td></td></tr>
<tr><td>11</td><td>Sts</td><td>It’s gonna be 25</td><td></td></tr>
<tr><td>12</td><td>K</td><td>It’s gonna be…</td><td></td></tr>
<tr><td>13</td><td>R</td><td>500</td><td></td></tr>
<tr><td>14</td><td>P</td><td>(writing as he speaks) 5 block…let me try…10 block.
//Rico and Kian interrupt, inaudible//
A 10 block tower will be</td><td></td></tr>
<tr><td>15</td><td>K</td><td>10 times 5, 10 times 5.</td><td></td></tr>
<tr><td>16</td><td>P</td><td>10 times 5? Yeah, you’re right, 10 times 5… equals 50 blocks. So, with 100, it’s 100 times 5 equals 500.
[brief pause]
So the 100 is going to be 100 going up, so it’s going to be 100 times 5 sides</td><td></td></tr>
<tr><td>17</td><td>K</td><td>We already did this one.</td><td></td></tr>
<tr><td>18</td><td>R</td><td>That’s 500.</td><td></td></tr>
<tr><td>19</td><td>P</td><td>Yeah so 100 times 1, 2, 3, 4, 5.</td><td></td></tr>
<tr><td>20</td><td>K</td><td>It’s 500.</td><td></td></tr>
<tr><td>21</td><td>P</td><td>So it’s 500.</td><td></td></tr>
<tr><td>22</td><td>R</td><td>500 blocks, tower blocks.</td><td></td></tr>
<tr><td>23</td><td>K</td><td>All you have to do is times it by 5.</td><td>LHSIA (10, 12)
Kian is stating in words, “times it by 5,” the strategy that Pedro has basically been stating.</td></tr>
<tr><td>24</td><td>P</td><td>Um-huh. (still writing on his own paper)</td><td></td></tr>
</table>

Rico, Pedro, Kian			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
		The students glance at their papers, and look around. They seem to believe they are done with the problem.	
Interpretation: Kian builds on what Pedro has been saying so far, and puts the strategy in general terms, "All you have to do is times it by 5." Because this has been Pedro's strategy, Kian might be stating this to indicate that he understands the strategy. Though Kian did not ask for confirmation, Pedro provides it, affirming with, "Um-huh." Kian's behaviors lead me to infer a LHSIA structure, as he expresses the strategy, which seems to be adopted as the group strategy.			
25	K	Which one you gonna do?	
26	P	I'm about to write something here.	
27	K	5 block…what the hell's that? Pedro, what's that? (*Kian keeps trying to whisper to Pedro, possibly about off-task things*)	
28	P	I'm writing 500	
29	P	So um, you got this Rico? You get me?	**EPISODE 2** **LMTY (10)** Pedro asks if Rico understands his strategy. Rico has not written much down, and has not said much during the earlier conversation
30	R	Don't write on the paper, she's walking around and she's going to see it.	
Interpretation: Pedro, who has been guiding the strategy of multiply the desired height by 5, asks Rico if he understands this idea. Pedro may be doing this because Rico has not written much, if anything, down, and has not said much during the conversation so far. Rico does not respond with a yes or no, as he seems more interested in not being noticed by someone, possibly the teacher (or another "she.") By asking Rico this question, I infer that Pedro may still have an active LMTY structure from earlier. He appears to want his classmates to understand the strategy and understand. In fact, Pedro follows up with this question in 2 minutes, at line #.			
31	Sts	*The students seem to believe they are done with the task, though they have not yet written anything down, except in their notebooks. Sometimes they appear to be speaking with one another, but do not want*	

Rico, Pedro, Kian			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
		to be heard by the recordings.	
32	K	So, Pedro what's that? Homework? (pointing at the notebook)	
33	P	Towers, that's the 100 towers	
34	K	(inaudible)	
35	R	Just circle 500 'cuz 500 is the total.	
36	P	So, uh, so we already know what's the 5 block, what's the 10 block, what's the 100. You got it, Rico?	**LMTY (10)** Similar to earlier, Pedro asks both Rico and Kian if they have the answer and if they understand.
37	R	Yeah.	
38	P	You understand, Kian?	
39	K	I know.	
Interpretation: Pedro seems to be following up with the earlier conversation, at line #. He recaps that they know the answer, the total number of blocks in the 5-block high tower, the 10-block high tower, and the 100-block high tower. He asks Rico and Kian if they both understand, presumably the strategy that led them to these answers. Note that 500 is the only answer they state out loud during this exchange. Because Pedro is again checking if his classmates understand, I infer that he still has an active LMTY structure.			
40	R	50 times 10 is 500. (This seems to be said in isolation. No discussion about 10 or 50 is mentioned earlier, and this is not followed up with more talk about the task.)	
41	Sts	*Kian is copying something down into his notebook. The students do not appear to do more task-related work. Their conversations are off-task.*	
42	K	(asks teacher) We need those? (gets up to get blocks)	
43	R	Why do you need that for? You just wanna play with it.	
44	P	Shhhhhh…..	
	Kian gets out several blocks, and gives some to Pedro. Rico also takes some. Pedro and Rico start putting the cubes together. There is discussion, but all inaudible. The structures the boys are putting together appear to follow the pattern given in the task.		
	For about 5 minutes: *The three boys use the blocks to put together various*		

Rico, Pedro, Kian			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
	tower structures. They appear to be doing this to just have something to do, to "play" as Rico stated earlier, though they are creating structures mimicking those given in the task. Sometimes they count the blocks they have in the structure, but they appear to confirm the pattern they have already discovered: 5 times the number of blocks in the height. For example, Pedro claims to build the 5-block high tower and counted 25 total blocks in the structure. The boys have other discussion that is off-task as well.		
45	Sts	*Fooling around and laughing*	
46	Pedro	(to Rico) Can you get that for me?	
47	Rico	No! Get it yourself. You're lazy	
48	Pedro	I'll get it	
49	Kian	Pedro, you know what I realized, see how it's 5? Since it's 5 on each side, it's 10 times 5. (as he speaks, he leans towards Pedro's desk with his block tower structure, which has a height of 9 but does not appear to have all the blocks on the sides at this time) You get it?	**EPISODE 3** **LHSIA (14, 15)** Kian demands Pedro's attention as he shares his thoughts about multiplying 10 times 5. He also looks for confirmation by asking, "You get it?"
Interpretation: Kian appears to have made a realization about how to determine the total number of blocks in each tower structure, now that he has been 'playing' with the blocks. He suggests that they should multiply 10 and 5. I believe, based on the structure he has put together using the blocks that he is referring to the fact that there are 5 "sides" (including the height and the other four legs) and that he has 10 blocks in the height. His structure as it exists at this point in time does not appea to have all the blocks needed on each side. As Kian comes to this understanding of the (incorrect) relationship between the height and the total number of blocks, he shares it with Pedro, demanding his immediate attention. His tone of voice sounds excited. He even seems to want Pedro to confirm his idea when he asks, "You get it?"			
50	Pedro	It's 10, 10, 10, 10, 10	
51	Kian	No, look…	
52	Rico	(counting by 1) it's 35. I got 35	
53	Kian	See how it's 10 right? And we multiplied it by 5. 10 times 5 'cuz of the surrounding	
54	Rico	Why you doing 5? Ya'll doing 5? Ya'll doing 10?	

Rico, Pedro, Kian			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
55	Pedro	I was trying to do 100.	
56	Kian	This is 10 with the 5 surrounding it. 'cuz you know how we got the 10 times 5 'cuz of this look. Look 10 and then we get 10 times 5	
57	Kian	What you doing? (to Pedro)	
58	Pedro	I'm gonna do the 10.	
59	Rico	The what?	
60	Pedro	I'm about to do 10. I'm about to put those back over here.	
61	Kian	You trying to make us work?	
62	Pedro	No.	
63	Rico	(to Kian) You know how much you got here?	
64	Kian	5	
	For almost 1 minute: Though the boys are using the blocks to construct tower structures, they think of their activity as play, rather than work (based on their statements earlier, line #(1:52:51). They continue this discussion and state that they cannot build the 100-block high tower because it will not stay together. The three have other off-task discussion as well.		
65	Rico	(to Kian) Stop playing	
66	Pedro	Put 10 on these sides, you gotta put 10 on all these sides	
67	Kian	Put 5 more?	
68	Pedro	Yeah 5 more on every side	
69	Kian	Why?	
70	Pedro	'Cuz it's 10 by 10, there's 10 on every side. Oh snap I think I made an…	**LMTY (1)** Pedro has been telling Kian that his 10-block high tower isn't finished because the structure should have 10 blocks on each side. Currently each leg has only 5 blocks beyond the middle.
71	Kian	But look it, we got 10 times 5 'cuz it's 10 and then 5	

Rico, Pedro, Kian			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
72	Pedro	Yeah but I said you gotta do 10 more on every side, if you add 1 on this side you gotta add 1 on every side but they're all even	
73	Kian	So add 5 more? (He begins to take blocks to add them to his existing tower structure)	
74	Pedro	Yeah, add 5 more on all of them.	
Interpretation: Pedro has been stating since the beginning of class that the total number of blocks can be found by multiplying 5 by the height (i.e., 5, 10, etc.). Kian had been trying to draw some conclusion about the problem using the tower structure he created. However, this structure was not correct according to the pattern. Though the pattern they are using is also incorrect, Pedro tells Kian that he needs more blocks on each of the sides, since there are only 5 coming off the middle block, rather than 10 blocks. Pedro appears to have an active LMTY structure, in part, because he is responding to Kian's questions patiently. Pedro does not appear to be trying to show off or demonstrate his own intelligence or ability.			
75	Kian	What?	
76	Pedro	On all of them.	
77	Rico	This is 50 right here.	
78	Pedro	Well he got 50.	
79	Rico	I got 50.	
80	Kian	We all should just do..I do the second one and you do the first one.	
81	Rico	I could do another 10 right here and another 10 right here.	
82	Pedro	Let's see who could finish first. (referring to the assignment of creating the other tower structures mentioned in the problem)	
83	Rico	…another 10 right here, another 10 right here, another 10 right here and I'd have 100.	
84	Pedro	I got the first one.	
85	Rico	Yo you heard? Pedro, I could do another 10 right here, another 10 right here, another 10 right here, another 10 right here and I'll get 100.	
86	Pedro	Another 10 right there, another 10 right there, another 10 right there and that's 100? 10 times 5 I mean 20 times…oh, yeah.	
87	Kian	He has 50 if he puts 10 everywhere he'll get 100….so what are we gonna do now?	
88	Pedro	This looks like a streetlight. Imagine street lights have all these little bulbs?	

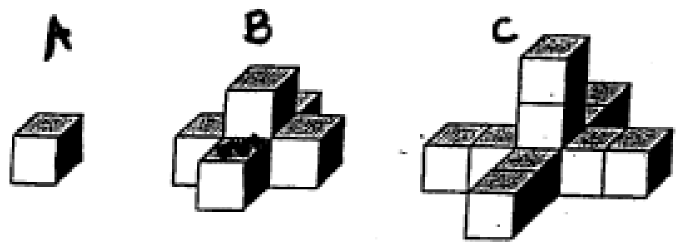
Rico, Pedro, Kian			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
89	Rico	I'm actually gonna do 100…why not?	
90	Kian	Start putting it on the floor.	
	For about 6 minutes: *The students tell Rico to "put away" the blocks, perhaps because he was going to try to put together the 100-block high tower. They do not seem to want the teacher to watch them as they work.* *Kian reads the task instructions out loud. Rico wants to throw away a paper, but a researcher tells them that they want to keep all the papers the students write on. She takes the paper away from Rico.* *Kian tries to encourage his classmates to continue working on the problem. Pedro claims that they need "assistance" perhaps by the teacher. Kian seems to want to appear to the teacher that they are working, and mentions writing down their explanation. In addition, shortly after Rico puts the bag of blocks back on the shelf, Kian retrieves the bag, so that they can keep working (or playing) with the blocks. As Rico puts together a tower structure, Pedro tries to instruct him how to construct the 10-block high tower. The two boys are disagreeing about the procedure when Ms. A comes to the group.*		
	For almost 16 minutes: *Ms. A asks the students to describe the pattern and show her, using the blocks. She also encourages them to find a way to organize their information, as they write it down. When the students disagree, such as whether there are 11 or 12 total blocks in the 3-block high tower (shown in figure C), she asks them to explain why they think the answer is 11 or 12, which helps them realize why there are 11 blocks.* *Though it does not follow the pattern they suggested earlier (5 times the height), the students are stating the correct total number of blocks for the towers of heights 1, 2, 3, 4, and 5, when prompted by their teacher. Pedro has taken the role of instructing Kian anod Rico what to do with respect to building the towers using the blocks. In addition, Pedro is being instructed by Ms. A to write down the information they agree on.* *After much discussion, the students realize that for the 5-block high tower, there are 5 blocks in the height, but only 4 blocks on each side. Each side would have 5 blocks if you include the middle block each time; they realize that*		

Rico, Pedro, Kian			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
	the middle block should not be counted 5 times. Shortly after this, the teacher stops them, and asks the entire class to start packing up their work. All the students in the class complete the questionnaire after this.		

Rico's work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



D
16 Block

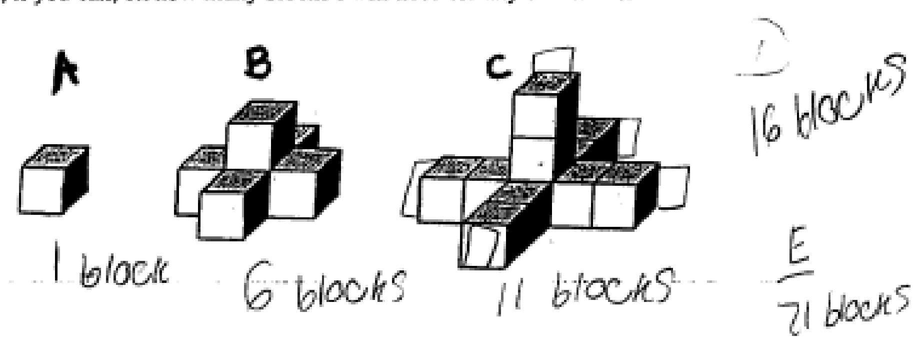
1block 6 block 12 block

E
21 Block.

Pedro’s work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



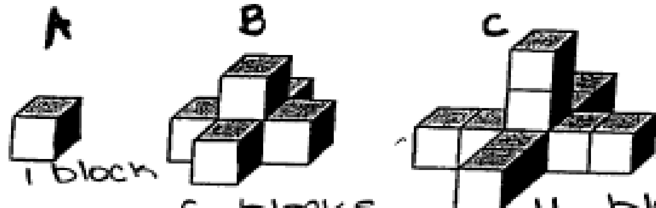
5- block high towe

Kian's work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.

A B C



1 block

6 blocks

11 blocks

D
16 blocks

E
21 blocks

5 blocks high
1 on each side

Questionnaire Responses for Rico, Pedro, Kian

Questionnaire Items Which May Indicate LMTY structure

Questionnaire Items	Rico	Pedro	Kian
Statements (All the time, Sometimes, Never)			
I wanted to teach another student something that I knew that the other student did not know.	Never	All the time	All the time
I listened carefully to the ideas of someone I was trying to help.	Sometimes	All the time	Sometimes
I helped someone see how to do the math.	Never	Sometimes	Sometimes
Others listened carefully to my ideas.	Sometimes	All the time	All the time
Behaviors (Often, Sometimes, Hardly ever)			
I gave helpful suggestions.	Hardly ever	Sometimes	Often
I worked cooperatively.	Sometimes	Often	Often
Thoughts (Yes/No)			
I like teaching this person things that I know.	No	Yes	Yes

Questionnaire Responses for Rico, Pedro, Kian

Questionnaire Items Which May Indicate LHSIA structure

Questionnaire Items – Statements	**Rico**	**Pedro**	**Kian**
Statements (All the time, Sometimes, Never)			
I wanted people to think that I'm smart.	Sometimes	Sometimes	All the time
I tried to impress people with my ideas about the problem.	Never	Sometimes	Sometimes
People seemed impressed with the ideas I shared about the problem.	Never	Sometimes	All the time
People saw how good I am at the math we did today.	Sometimes	All the time	All the time
I felt smart.	All the time	All the time	All the time
I wanted to show someone that my way was better.	Never	Sometimes	Never
I was a lot better at math than others today.	Never	All the time	Sometimes
Behaviors (Often, Sometimes, Hardly ever)			
I was the leader.	Hardly ever	Often	Sometimes
I was bossy.	Hardly ever	Hardly ever	Hardly ever
I wanted to show off.	Hardly ever	Sometimes	Hardly ever
I liked to be right.	Sometimes	Sometimes	Often
Thoughts (Yes/No)			
I want you to know just how smart I am.	No	Yes	Yes
People think I'm smart.	No	Yes	Yes
I wish the teacher would call on me, so I can show how much I know.	No	Yes	No

Onan, Joe, Raina

This is Group 3 from Ms. A's grade seven Class 2.

Verbal emphasis indicated by <u>underline</u>
Actions included either in *italics* or (parentheses) as appropriate.
Pauses in speech are indicated by …
Words inserted to help clarify a student's words are included in [brackets].
When students voices overlap, one or both sets of words are included in //double slash marks//.

The number included next to either LMTY or LHSIA refers to the code, included in the table of codes.

Onan, Joe, Raina			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
	The teacher, Ms. A, has just finished her introduction. She gives an interactive introduction, having the students respond to questions like "How many sides are there?" meaning how many sides can we add blocks to (not including the height).		
1	Onan (O)	So what we can do is, we can find like, multiplication like we do with area, like we did with the problem yesterday. We could do that here, multiply by whatever it is. We're gonna need calculators (takes the calculator from Joe's desk). Alright let's try, each side, whatever there is – there's 1, 2 (using his pen to count the blocks in one of the figures given in the task). So there's 10 right here (Joe points at something on Onan's paper). So we need another 10 in height. So what you think, what's your take on this? (seems directed at Raina. She waves her hand in response, possibly to indicate that she does not know how to respond, and takes the paper from Onan to read it to herself.) Joe you gotta think about something too aight?	**EPISODE 1** **LHSIA (14, 15)** Onan first to speak, has an idea relating to previous work; Seems to think his idea is good: "Right, the way I said it?" Looks for validation
2	Raina (R)	(inaudible, laughs and makes hand gesture similar to earlier. It's unclear what the hand waving means.)	

<table>
<tr><th colspan="4">Onan, Joe, Raina</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td>3</td><td>O</td><td>Right the way I said it?</td><td></td></tr>
<tr><td>4</td><td>R</td><td>Yeah</td><td>(validates his idea, so he continues below)</td></tr>
<tr><td>5</td><td>O</td><td>Take it by 5 by the 10 that we had there and then we'll get 50 right? And then 10 times 10 is, what is it…10 times 10 is 100, 100 by 10, 200 right? Or something like that? (checks his arithmetic on the calculator)
Yeah 1000. Alright, so, um, we could do it like that start with 5 times 10 equals 50 then 10 by 10 and then 100 by 10 we should try it like that let's see what we get.</td><td></td></tr>
<tr><td>6</td><td>Joe (J)</td><td>But why are you times-ing it by 10?</td><td></td></tr>
<tr><td>7</td><td>O</td><td>Cuz the last block is 10 (points to figure C on the task paper).
The last set of blocks counted all in total is 10, go ahead, try it, see? (Joe takes the paper, from Onan, who got it back from Raina.)
So that's how we have to do it, take it by 10, it would get us the last total that we have to get. Wait, let me see something let me get this paper…
(Joe takes a blank piece of paper. Raina takes the task paper again and reads it. Joe is looking at the task paper as well.)</td><td>LMTY (1) LHSIA (14)
Onan is explaining his idea in response to Joe's question. His tone of voice, as well as the fact that he is continuing with his strategy, suggests that he is confident in his idea; leads me to infer Onan is still in LHSIA from earlier.</td></tr>
<tr><td colspan="4">Interpretation: Onan is the first to speak, and suggests that they try a strategy similar to a problem they worked on the previous day. These behaviors lead me to infer that Onan starts with an active LHSIA strategy. He looks for validation from his classmates, when he asks, "So what do you think? and, "Right, the way I said it?" His classmates appear to agree that the group should continue with Onan's idea. This agreement appears to encourage Onan to continue discussing his strategy, where he may be thinking out loud. Joe, who has agreed with Onan so far, asks why they are multiplying various numbers by 10.
Onan responds to Joe's question by explaining his rationale. This explanation seems to be based on the fact that Onan thinks that there are 10 blocks in the tower represented by figure C. I infer that the question has activated an active LMTY structure for Onan. He continues with his strategy after his explanation, which leads me to infer that the LHSIA structure is still active.</td></tr>
</table>

Onan, Joe, Raina			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
8	J	Wait, hold on. Think there's a block under this? (points to figure C on the task paper that Raina is holding)	
9	O	Huh? (Looks up from what he was writing on his own paper)	
10	J	Is there a block under this? (Joe seems to be referring to the middle block which is hidden by the other blocks; Onan so far has not counted this block)	
11	O	No. (Looks over at the paper Raina has in front of her)	
12	*For about 4 minutes:* *At this point, Ms. A comes over to the group and tries to figure out what they are discussing so far. She helps them realize they were counting the blocks wrong – there is a block hidden in the middle of the towers so the total number is 1, 6, 11, for the towers represented by figures A, B, and C, respectively. Ms. A tells the students to get a bag of SnapCubes, but they do not use them until after she leaves. The group continues with Onan's idea from above after she leaves.*		
13	O	So, how many blocks do we need?	Onan seems to continue his role as leader of the group, and makes sure to include his classmates as having roles.
14	R	I don't know. *Onan and Raina both take cubes from the bag.*	
15	J	You need 11. (Takes some of the blocks from Raina. Raina starts taking apart the blocks she took from the bag.)	
16	O	Count how many blocks you have there.	
17	R	How many I have where?	
18	O	Of those (points to the cubes she has just put down)	
19	R	You mean like this? (pulls apart two cubes and puts them up to show Onan)	
20	O	Yeah, like that.	
21	R	(Counts) I got 8.	

Onan, Joe, Raina			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
22	O	(to Raina) You got 8? (counts the blocks that Joe has on his desk) 9, 10, 11 (picks up some blocks from Joe's desk, but gives them back to Joe who appears to be taking them apart) (to Joe) Oh yeah, you were right!	
23		*Joe seems to be putting together a 2-block high tower with the SnapCubes. Raina is also putting together a tower structure, though it's not clear which one.* *Onan puts the audio recorder on top of one of the task papers. He and Joe move things on their desks to get settled.*	
24	O	Alight now let's try it again let's count how many we have.	**EPISODE 2** **LHSIA (14)** Onan's tone of voice suggests he is speaking with authority. He keeps giving directions or asking questions of his group as though he is the leader. (Demands attention)
25	J	I got 5	
26	O	Aight you have 5 how many do you have?	
27	R	Me? (counts) 11	
28	O	So let's try it again we should go by, how many do we have here again? 11? So let's times all of these by 11 and let's see what we'll get.	**LHSIA (2, 3)** Onan is giving instructions, collecting information from others, and deciding what to do with it (multiply by 5)
29	J	So 5 times 11…	
30	O	5 times 11 equals 55. (writes down as he	

Onan, Joe, Raina			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
		speaks)	
31	R	Look it's like this…	
32	O	Yeah, yeah, yeah, yeah, yeah. 55 and that has to be 110 that times 11 equals 110	
33	R	We're supposed to write that down?	
34	O	No that's just scrap paper and	
35	R	11 times 100	
36	O	Yeah times 11	
37	J	11,000 I mean…	
38	O	1,100 right?	
39	J	Just say eleven hundred	
40	O	Alright now let's see if we have this right and then after we have the 55 it'll double again like 55 210 and 2100 right? It'll keep on doubling	**LHSIA (15)** Onan is looking for validation of his strategy from others, asking "right?"
41	R	Yeah, it will.	
42	O	Yeah, we should do it like that then. Now we have all of them done so we don't need anything.	
Interpretation: In the above episode, I suggest that Onan (still?) has an active LHSIA structure, despite the teacher's interruption. He continues with his strategy, and gives directions to his classmates. Onan collects information from his group mates and decides what to do with it, suggesting that he believes it to be his rule to guide the group through the problem. His tone of voice is authoritative, as though he is the leader of the group. When he asks, "right?" he appears to be looking for validation from others in his group. Raina seems to be the primary person giving him that validation. She responds affirmatively when he asks, "right?"			
43	*For about 4 minutes:* *Raina suggests she wants to try something, and puts together the blocks to show the towers depicted in figures A, B and C from the diagram on their paper. Onan and Raina briefly discuss if their strategy is correct and agree it is. While this is going on, and as Onan and Raina continue their discussion, Joe has been quietly putting together several tower structures as well. They appear to be the same towers depicted in the task as well.*		
44	O	Alright, now we have to explain how do we, why do we think that the amount we have is supposed to go up on the blocks that she has set.	**EPISODE 3** **LHSIA (14)** Onan shares his idea/strategy with

<table>
<tr><th colspan="4">Onan, Joe, Raina</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td></td><td></td><td></td><td>the group.</td></tr>
<tr><td>45</td><td>R</td><td>That’s what I’m trying to tell you.</td><td>LHSIA (11)
Raina tells Onan that she has been trying to make this same suggestion</td></tr>
<tr><td colspan="4">Interpretation: Onan, whom I infer has had an active LHSIA structure for the entire problem-solving session so far, offers another strategy, “why do we think that the amount we got has to go up…” Onan continues to demand attention for his ideas, again, as though he is the leader of this group.
Raina, who had been validating his ideas previously, suggests that she gave him this idea. By stating, “That’s what I’m trying to tell you,” I infer that Raina also has activated the LHSIA structure to defend her own ideas as well.</td></tr>
<tr><td>46</td><td>O</td><td>Why didn’t you say that then? Alright now we explain it.</td><td></td></tr>
<tr><td>47</td><td>R</td><td>This is C. (puts her 3-block high tower structure on Onan’s desk)</td><td></td></tr>
<tr><td>48</td><td>O</td><td>Yeah, that’s C. Alright let’s say, let me write this down. We could say that we think, it’s gonna be… we think that 55 is the next block, set of blocks that’s gonna go up for the next amount of blocks have, and that’s 5 blocks. And the other 10…
Wait, (pauses for a moment, as though he’s thinking) scratch that, scratch that. (continues to think for a few moments)
Alright let’s say 5 goes straight up, (gestures “up” with his pen”) right, Raina? (he is getting Raina’s attention who was continuing to work with the blocks)
5 goes straight up and the other ones that are left over we divide them over the sides right?</td><td>LHSIA (14, 15)
Onan takes on the role as the one who is going to state and write down the strategy for the group. He demands Raina’s attention as well as asks for validation, “right?”</td></tr>
<tr><td>49</td><td>R</td><td>Yeah.</td><td></td></tr>
<tr><td colspan="4">Interpretation: Onan appears to have taken on the role of the “smart” one in the group, as he offers his ideas whereas the other two do not seem to contribute their own ideas for a strategy. Onan offers to write down his strategy for the group. (Later the whole group will copy off his “draft” of the group strategy.) Raina returns to validating his idea, when Onan asks, “right?” and she responds, “Yeah.”</td></tr>
<tr><td>50</td><td colspan="2">The students discuss writing down the strategy, deciding who will do what. Onan starts writing information down, while Joe and Raina still work with the SnapCubes. Raina asks Onan about a structure she</td><td></td></tr>
</table>

Onan, Joe, Raina			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
	has put together that does not follow the pattern given in the task. Onan responds that it has to follow that pattern.		
51	O	We divide all the left over blocks. (Onan is writing again and Joe looks on while he does this.)	**EPISODE 4**
52	J	So we don't do that? The answers we got for like right here. (Joe seems to be referring to the total number of blocks determined for the towers in phases A, B, and C)	
53	O	No cuz this doesn't have any blocks around it.	**LHSIA (9A)** Onan has been sticking to his strategy (to multiply the height (e.g., 5) by the total of a given number of blocks. When Joe or Raina asks questions, he maintains that his strategy is correct.
54	J	No, look, this is um.	
55	O	If you take 1 times 11, you're still gonna get 11, I mean 1.	
56	J	What about if you add, I mean, you times this to 11 the answers to 11?	
57	O	How do you mean like 5, 100, 10 and 1100 times 11?	
58	J	Yeah like double them up again?	
59	O	But it'll keep giving us the same answers.	

Interpretation: Onan defends his strategy when others asks questions. Perhaps he thinks others are questioning whether his strategy is correct. His two classmates, however, are not suggesting another strategy or method. Joe just seems to be suggesting that they already have some answers. Onan appears to continually have an active LHSIA structure, as in this case, when Joe appears to question him. Joe seems to be referring to the total number of blocks they got for the 1-block high tower and the 2-block high tower. Onan's response of, "No," appears to be his way of disagreeing with Joe's suggestion that those answers might be helpful.

Alternative Interpretation: Perhaps Onan has active LMTY as well here. He has been asked about his strategy and he gives an explanation to multiply 1 and 11. (It appears that Onan has combined his earlier strategy to multiply the requested height (e.g., 5, 10) by the total number of blocks depicted in C, with the correction of how many blocks are indeed in this tower structure. Earlier the group thought there were only 10, rather than 11 blocks in the 3-block high tower.)

Onan, Joe, Raina			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
60	*The students start discussing what they should write on their papers, the ones they are going to hand in. Onan tells Raina to copy down what he has already written on his "scrap paper." While Joe looks on, Raina starts writing on her paper. Onan sometimes tells her what to write, and writes on his own paper as well.*		
61	J	So how we gonna do a hundred? (Joe seems to ask this in responses to Onan's statement about the 100-block high tower as he writes on his paper)	**EPISODE 5**
62	O	The same way we did 5 and 10. (pointing to his paper as he says "5" and "10")	**LMTY (7)** Onan is responding to a specific question from Joe about the finding the solution for the 100-block high tower.
63	J	Like what?	
64	O	Like we go up the 10, right? And then the leftovers we divide it on the four sides (Raina is writing on her paper – likely her solution – refer to the student work)	
65	J	What do we divide it by?	
66	O	You're not gonna divide it by anything. You're gonna *split* it between 4 sides…what's half of 100? 25?	*Continuing from above:* Onan is explaining his strategy, in response to Joe's question.
67	J	50	
68	O	50, 75 and 100 right? That's it! And the leftovers from 100 (writing) will also do the same thing for 100 blocks. (Both Raina and Onan write the solution on their papers.)	
69	R	We'll do the same thing for 100 blocks?	
70	O	100 blocks, yes. Period. Alright there we go now we can start transferring all the things that we did on this paper. Alright?	
Interpretation: Onan appears to have an active LMTY structure, perhaps activated when Joe asked a question about the strategy and how they will find the solution for the 100-			

<table>
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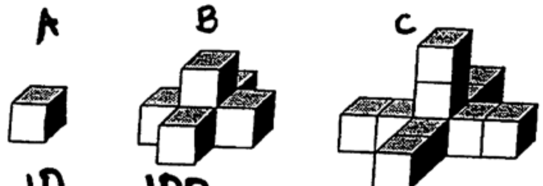
<tr><td colspan="4">block high tower. First, Onan responded that they get the answer for the 100-blocks the same way they obtained the answer for the 5- and 10-block towers. Then when Joe expressed a misunderstanding about dividing, Onan clarified that they split the blocks among the 4 sides once they have the correct number of blocks in the height. Then when Raina asks, “We’ll do the same thing?” she appears to be asking for clarification, which Onan gives, answering affirmatively.</td></tr>
<tr><td>71</td><td colspan="2">For approximately 15 minutes:
All three students focus on writing up their solution, discussing it from time to time. Some of these discussions are procedural (e.g., how to spell “vertical”) but do not appear to activate a LMTY structure for any of these students. They “finish” their work and then turn to discussing things not related to the task. After a few minutes, the teacher announces that they should start to wrap up their work. She is near their desk so Raina and Onan both tell her that they are finished.</td><td></td></tr>
<tr><td>72</td><td>R</td><td>(Raina and Joe are breaking apart and putting together the linking cubes they used.)
We finished, Ms. A.</td><td>EPISODE 6</td></tr>
<tr><td>73</td><td>O</td><td>We were the first group to finish. Haha! (smiles).</td><td>LHSIA (17)
Onan, along with his next statement is stating that the group is smart, because they were the first to finish the task</td></tr>
<tr><td>74</td><td>R</td><td>We’re the smartest.</td><td>LHSIA (17)
Raina
Both students announce they were the first group to finish and that they are the “smartest.”</td></tr>
<tr><td>75</td><td>O</td><td>Yeah, we’re the smartest in this whole class (serious face).</td><td></td></tr>
<tr><td colspan="4">Interpretation: Both Raina and Onan make statements about being the smartest group. Joe does not contribute to this part of the conversation. I wonder if the, “We’re the smartest,” is a group mentality, however. They seem to equate “finishing first” with being the “smartest.” When Onan adds, “Yeah, we’re the smartest,” I wonder if he believes that the group, or at least each person in the group, is smart, or if he perceives</td></tr>
</table>

Onan, Joe, Raina			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
only himself as smart. Onan was the student to direct which the group's strategy and did not often encourage Raina or Joe to offer different ideas. The students believe they have completed their solution (even though it is incorrect, they do not know this). Perhaps because the group followed Onan's initial suggested strategy, he feels that he contributed a lot and he may believe that others (Raina and Joe) see him as smart. When Onan is smiling he seems to be enjoying the group status as finishing first and being smart.			

Onan's Work, in Nov. 2008

Building Block Dilemma

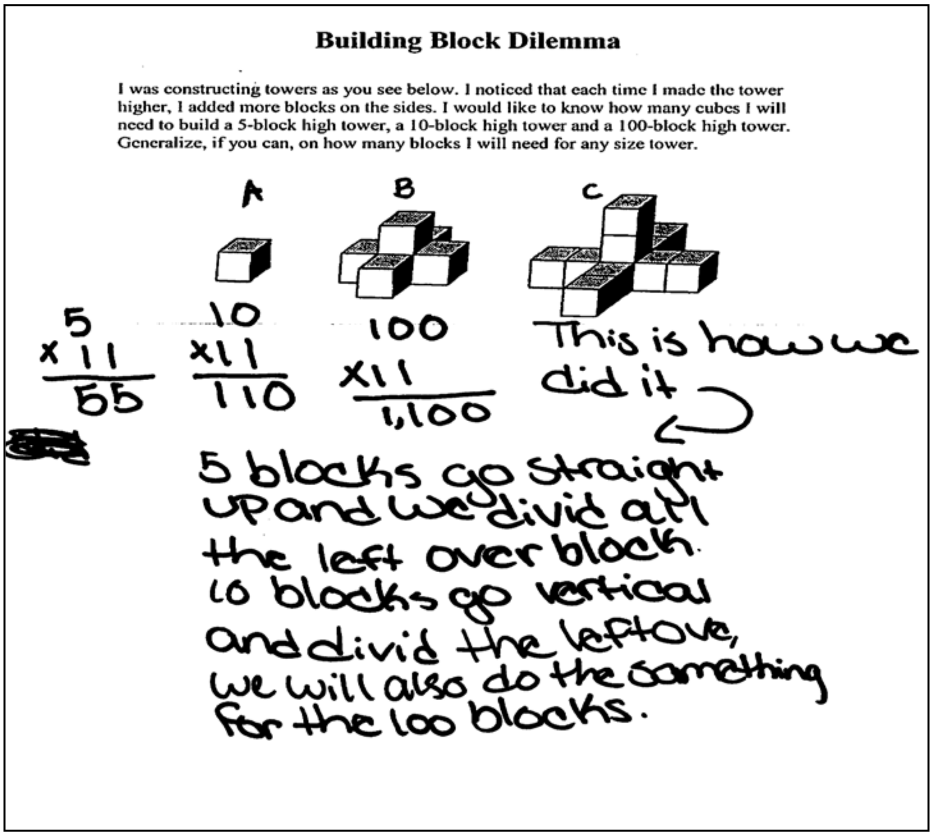
I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



$11 \times 5 = 55$ $10 \times 11 = 110$ $100 \times 11 = 1{,}100$

What we will do is take 5 block to go straight up and then we divide all the left over blocks. We will take 10 blocks vertical and then divide the left over blocks. We will also do the same thing for the 100 blocks.

Raina’s Work, in Nov. 2008



Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.

A B C

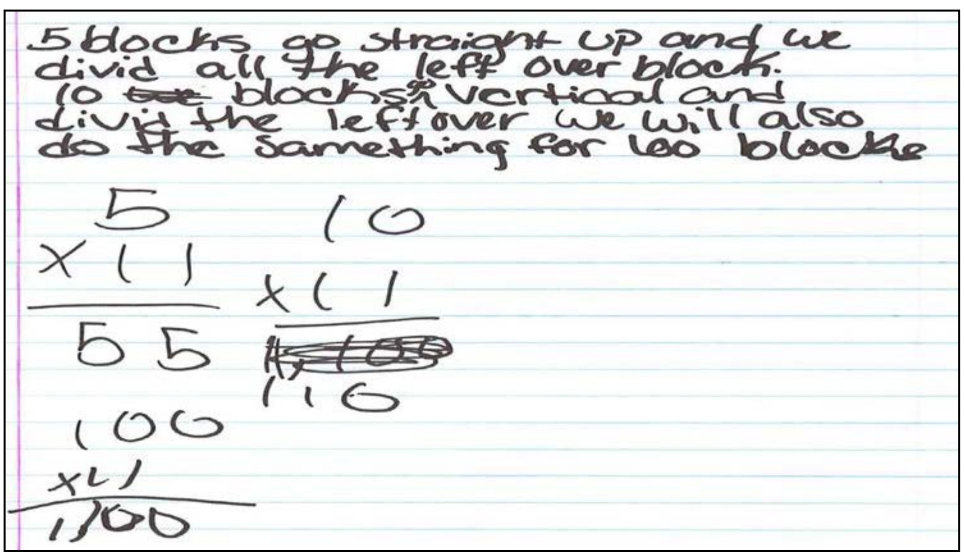
5
x 11
55

10
x11
110

100
x11
1,100

This is how we did it

5 blocks go straight up and we divid all the left over block.
10 blocks go vertical and divid the leftove, we will also do the samething for the 100 blocks.



5 blocks go straight up and we divid all the left over block.
10 blocks go vertical and divid the leftover we will also do the samething for 100 blocks

5
x 11
55

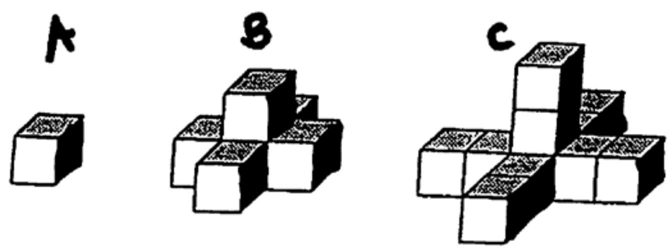
10
x 11
110

100
x11
1,100

Joe's Work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



$$\begin{array}{r} 5 \\ \times 11 \\ \hline 55 \end{array} \quad \begin{array}{r} 10 \\ \times 11 \\ \hline 110 \end{array} \quad \begin{array}{r} 100 \\ \times 11 \\ \hline 1{,}100 \end{array}$$

We would do
5 blocks go straight up and then we divide all the lefted overs.
10 blocks go vertical and then We will divide the lefted overs.
We will do the samething to the 100 blocks.

Questionnaire Responses for Onan, Raina, Joe

Questionnaire Items Which May Indicate LMTY Structure

Questionnaire Items	**Onan**	**Raina**	**Joe**
Statements (All the time, Sometimes, Never)			
I wanted to teach another student something that I knew that the other student did not know.	All the time	Sometimes	Sometimes
I listened carefully to the ideas of someone I was trying to help.	Sometimes	All the time	Sometimes
I helped someone see how to do the math.	All the time	Sometimes	Sometimes
Others listened carefully to my ideas.	All the time	Sometimes	All the time
Behaviors (Often, Sometimes, Hardly ever)			
I gave helpful suggestions.	Sometimes	Sometimes	Often
I worked cooperatively.	Often	Sometimes	Often
Thoughts (Yes/No)			
I like teaching this person things that I know.	No	Yes	Yes

Questionnaire Responses for Onan, Raina, Joe

Questionnaire Items Which May Indicate LHSIA structure

Questionnaire Items – Statements	**Onan**	**Raina**	**Joe**
Statements (All the time, Sometimes, Never)			
I wanted people to think that I'm smart.	Sometimes	All the time	All the time
I tried to impress people with my ideas about the problem.	All the time	All the time	All the time
People seemed impressed with the ideas I shared about the problem.	All the time	Sometimes	All the time
People saw how good I am at the math we did today.	All the time	Sometimes	All the time
I felt smart.	All the time	Sometimes	All the time
I wanted to show someone that my way was better.	Never	Never	Sometimes
I was a lot better at math than others today.	All the time	Sometimes	All the time
Behaviors (Often, Sometimes, Hardly ever)			
I was the leader.	Often	Hardly ever	Often
I was bossy.	Hardly ever	Hardly ever	No Answer
I wanted to show off.	Hardly ever	Hardly ever	Hardly ever
I liked to be right.	Sometimes	Sometimes	No Answer
Thoughts (Yes/No)			
I want you to know just how smart I am.	No	Yes	Yes
People think I'm smart.	Yes	Yes	Yes
I wish the teacher would call on me, so I can show how much I know.	No	No	Yes

Danica, Shanika, Shannen

This is Group 4 from Ms. A's grade seven Class 2.

Verbal emphasis indicated by underline
Actions included either in *italics* or (parentheses) as appropriate.
Pauses in speech are indicated by …
Words inserted to help clarify a student's words are included in [brackets].
When students voices overlap, one or both sets of words are included in //double slash marks//.

The number included next to either LMTY or LHSIA refers to the code, included in the table of codes.

<table>
<tr><th colspan="4">Danica, Shanika, Shannen</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td colspan="4">Prior to this point, after the teacher (Ms. A) stopped giving her introduction, the three students start to discuss how to approach the problem. They appear to be suggesting that the 5-block high tower can be built off the towers depicted in the diagram, by adding two more blocks to the height (Tower C in the figure is a 3-block high tower, Tower B is a 2-block high tower and Tower A is a 1-block high tower).</td></tr>
<tr><td>1</td><td>Shanika (Sa)</td><td>Wait, can y'all explain that to me?</td><td>EPISODE 1
Shanika asks for an explanation of what the two girls are saying, up until this point.</td></tr>
<tr><td>2</td><td>Danica (D)</td><td>Okay first, like if you go up high, it's 3 right?
//Sa: Yeah//
And you want to add 2 to get to 5. And you always add 1 more. So it gets higher.
//Sn: Add one more. That's 3, that's 5.//</td><td>LMTY (1)
Danica explains how to construct the 5-block high tower.
It sounds as though Shannen is speaking as well.</td></tr>
<tr><td colspan="4">Interpretation: Danica responds to Shanika's request to explain what she and Shannen have been discussing up until this point. Danica seems to be trying to build the 5-block high tower from figure C, which is 3 blocks high. She suggests adding 2 more blocks to the height. I infer that Danica has an active LMTY structure because she responds to an explicit request for an explanation, appearing to follow up on what she stated earlier.</td></tr>
<tr><td>3</td><td>Sa</td><td>Ohhhhh…So it's 5, then it's 3, 3, 3…
Okay. But look, it says I would like to know how many cubes I will need to build a 5 block high tower, a 10 block high tower…
(reading task instructions)</td><td></td></tr>
</table>

Danica, Shanika, Shannen			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
4	D	Okay we did the 5.	
5	Sa	So, how do we write that?	
6	D	2... another hundred right here.	
7	D	5… (Students all write on their own papers, quietly, for about 30 seconds. Shannen and Shanika appear to be looking at what Danica is writing before writing on their own papers.)	
8	D	We need 5 more, right?	**EPISODE 2** **LHSIA (15)** Danica expresses her idea about how to continue, and asks, "right?" suggesting she is looking for confirmation from Shannen.
9	Shannen (Sn)	Four.	
10	D	Four?	
11	Sn	Two… Two for the sides and two for the high.	**LMTY (1, 7)** Shannen is explaining why she gave an answer of "Four."
12	D	Wait, you're including this one and this one.	
13	Sn	No, she said the sides are not supposed to write.	**LHSIA (8)** Shannen corrects Danica, likely referring to her interpretation of the teacher's instructions. These seem to be the reasons for not including "this one and this one."
Interpretation: Danica first appears to have an active LHSIA structure when she expresses her idea that they need "5 more" blocks, I presume, for the next tower structure. It is not clear whether this refers to the 6-block high tower, building off the 5-block high tower, or if it refers to the 10-block high tower, which is included in the task instructions. She checks for confirmation from Shannen that her strategy is correct. I infer that Shannen has an active LMTY structure because Shannen explains that there are two blocks on the sides and two blocks in the height, when Danica does			

<table>
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<tr><td colspan="4">not seem to understand why they need 4 more blocks. (It is not clear to the researcher which tower structure they are referring to exactly.)
I infer that Shannen then activates the LHSIA structure because she corrects Danica's understanding of her explanation. Shannen gives information that she appears to have understood from the teacher (it appears that the "she" above refers to the teacher, Ms A).</td></tr>
<tr><td>14</td><td>D</td><td>And you're going horizontal.
(uses hand gestures to help make her point)</td><td></td></tr>
<tr><td>15</td><td>Sn</td><td>That's 1, 2, 3, 4, 5, 6.
(uses pen to point to one of the figures on the task paper)</td><td></td></tr>
<tr><td>16</td><td>Sa</td><td>You will need 2 more blocks and then how many… how many on that side?</td><td></td></tr>
<tr><td>17</td><td>D</td><td>There's 2, 4, 6, 8, 9, 10.</td><td>LMTY (3, 7) LHSIA (14)
Danica responds to a specific question from Shanika, who asks, "How many?" Danica counts out loud to 10, pointing to the blocks drawn in the diagram corresponding to figure C.</td></tr>
<tr><td colspan="4">Interpretation: Danica appears to have both the LMTY and LHSIA structures activated here. She has been stating her ideas about how to get the answers to the problem. In line # she may be responding to Shanika's question, "How many on that side?" with the answer 2. However, she also seems to be giving the answer to the total number of blocks in the 3-block high tower, perhaps because she just counted the total number of blocks in the 2-block high tower.
Danica appears to have two motivating desires. She appears to be motivated to help Shanika to get the answers. Danica may be trying to explain her strategy for counting the total number of blocks, by pointing to the blocks on the paper to support her answer. Also, Danica appears to be motivated to impress her two classmates with her understanding of the problem, because she answers Shanika's question and adds additional information.

Alternatively, Shannen may have only one of these motivating desires, even though I infer both from her actions.</td></tr>
<tr><td>18</td><td>Sa</td><td>Wait, but I'm saying for the 5, do we need to add like, 2 more? Because it's going to be 4 anyway because it's 2 and</td><td></td></tr>
</table>

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<tr><th colspan="4">Danica, Shanika, Shannen</th></tr>
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<tr><td></td><td></td><td>after 2 it’s going to be 4 but don’t we need 5? I don’t… Because…</td><td></td></tr>
<tr><td>19</td><td>D</td><td>Do you see the pattern?</td><td>EPISODE 3
LMTY (10)
Danica asks Shanika if she “sees the pattern?”</td></tr>
<tr><td>20</td><td>Sa</td><td>It started out with one…</td><td></td></tr>
<tr><td colspan="4">Interpretation: Danica appears to have an active LMTY structure because Shanika continues to express some confusion and Danica asks if she understandSHs, or “sees the pattern?” Shanika starts to share what she does know or understand. In the next speaking turn, Shannen begins to assist with the explanation.</td></tr>
<tr><td>21</td><td>Sn</td><td>With two blocks, there’s 1 on each side. With 3 blocks, there’s 2 on each side. So with 5, there’s was only 1 left.</td><td>LMTY (1) LHSIA (2)
Shannen seems to speak over both Shanika who is expressing confusion and Danica. Shanika states her understanding of the relationship between the number blocks in height and number of blocks on the sides.</td></tr>
<tr><td>22</td><td>Sa</td><td>Oh…okay.</td><td></td></tr>
<tr><td colspan="4">Interpretation: Shannen now appears to have both the LMTY and LHSIA structures activated in line #. Shanika continues to ask questions to clarify her understanding of the task, asking if they “need 5?” It is not clear to me if they are referring to 5 blocks in the height, if they mean 5 blocks in the height, or some other use of the 5 blocks.
Perhaps building off Danica’s question, “Do you see the pattern?” in line #, Shannen starts explaining her understanding of the pattern using the first two tower structures as examples. I infer an active LMTY structure because Shannen is explaining the strategy to her classmates. I also infer an active LHSIA structure because she speaks over Danica and Shanika.</td></tr>
<tr><td>23</td><td>D</td><td>Get it?
(Shanika shakes her head no.)</td><td></td></tr>
<tr><td>24</td><td>D</td><td>Write down… do you wanna write down?</td><td></td></tr>
<tr><td>25</td><td>Sa</td><td>Whatever, do you wanna show me? (asks Shannen)</td><td></td></tr>
<tr><td>26</td><td>Sn</td><td>Okay, there’s 3 blocks here, there’s only 2 on each side. Just like there’s 2 blocks with 1, so the sides will always be less than the tower.</td><td>LMTY (1)
Shannen explains the strategy, stating that there is one less block on each side than there is in the</td></tr>
</table>

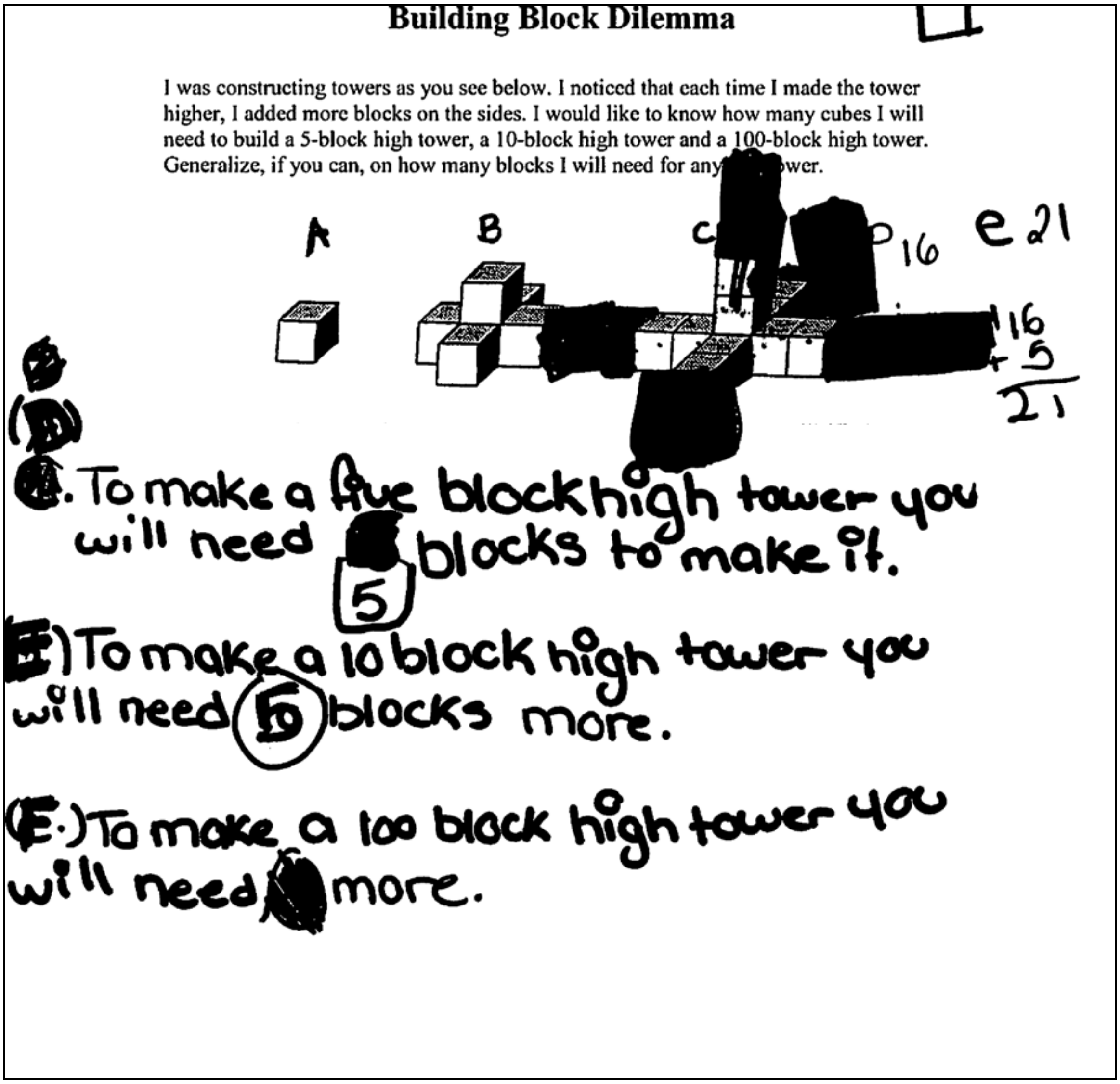
Danica, Shanika, Shannen			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
			height.
Interpretation: Shannen responds to Shanika's request to "show" her the strategy, leading me to infer that she has an active LMTY structure. Shannen appears to build on what she was saying earlier, and explains the strategy using examples that are given in the task (i.e., the 3-block high tower and the 2-block high tower).			
27	Sa	But look. I saw this. It's 4, right? You see 4? And 4, and 4, but it's another one. So that will be 3… no…	
28	D	There's only 3…	
29	Sa	No, I'm saying…	
30	D	No, I get what you're saying.	
31	Sa	Look, it's 1 block right? And it's 2 under to make this bigger cause it was 1.	
32	D	No there's 1 under.	
33	Sa	Yeah it's 1 under that's what I said.	
34	D	And then here it's 2.	
35	Sa	It's 3 under… I mean it's 1 under to make a 3.	
36	Sn	Yeah.	
37	Sa	Now listen to what I'm saying, it's another one under here. So we have 2 right here and 2 right here, that's 4, and then plus that one that could be 5, but it's 3… it would be like…	
38	D	Oh, I get what you're trying to say.	
39	Sa	Yeah, but we're only adding 1…	
40	Sn	We're only adding once to get what we need.	
41	D	So how many do we need? 1, 2, 3, 4, 5, 6, 7, 8, 10, right?	**EPISODE 4**
42	Sn	So it's 1, 2,….	
43	D	Oh I get it! 10? We need 10 blocks for the 5, then we're going to need 20!	**LHSIA (4)** Danica claims to "get it," or understand the total number of blocks that will be needed for the 5-block high tower and the 10-block high tower.

44	Sn	Oh that's the pattern!
		//D: Yeah.
		//Sa: Okay.
			LHSIA (4)
			Shannen chimes in to indicate that she recognizes the pattern as well.

<table>
<tr><th colspan="4">Danica, Shanika, Shannen</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td>45</td><td>Sn</td><td>10?</td><td></td></tr>
<tr><td>46</td><td>Sa</td><td>10 times 5?</td><td></td></tr>
<tr><td>47</td><td>D</td><td>No, like it’s going in a pattern. This is 5, now we need 10… for 10 we need 20. For the 100 we need 200. Get it?</td><td>LMTY (1, 7, 10)
Danica responds to Shanika’s question if they are multiplying 10 and 5. She then explains that there is a pattern, giving the answers rather than describing the pattern. When she asks, “Get it?” Danica appears to be asking Shanika if she understands the explanation.
Suggesting that the pattern is needing twice as many blocks for the whole tower as the height… so for 5 blocks high, they only need 5 more blocks, for a total of 10, etc.</td></tr>
<tr><td>48</td><td>SR</td><td>Yeah, 5, 10. . .</td><td></td></tr>
<tr><td colspan="4">Interpretation: Danica began this episode by counting to get an answer for the total number of blocks needed for one of the tower structures. (It is not clear to me which tower she was referring to.) She then exclaims that they need 10 total blocks for the 5-block high tower and 20 total blocks for the 10-block high tower. She says, “I get it!” presumably to let her classmates know that she has understood something new. By stating her answers after this, I infer that she activates the LHSIA structure by letting her classmates know that she has gotten an answer.
Shannen follows up stating, “Oh, that’s the pattern!” She also indicates that she understands what Danica has just said by saying, “Oh!” I infer that she also has an active LHSIA structure because her tone of voice suggests a level of understanding and confidence, as though she wants her two classmates to know she recognizes the pattern. Shanika continues by asking if they multiply 10 and 5, which is not the pattern suggested by Danica and Shannen. Danica responds to the question, and appears to try to explain her answer to Shanika, expanding on her earlier statement by adding, “For the 100 [block high tower], we need 200 [blocks].” Because she appears to be trying to explain the idea, I infer that Danica has activated the LMTY structure. Danica continues to follow-up her explanation by asking if Shanika understands, “Get it?” in a supportive tone of voice.</td></tr>
<tr><td colspan="4">From here until end of class, approximately 37 minutes:</td></tr>
</table>

<table>
<tr><th colspan="4">Danica, Shanika, Shannen</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td colspan="4">At this point, the students start writing down their solutions and discuss how to label everything. For a short while, they return to their discussion of their solution, suggesting that they need 20 blocks to build a 10-block high tower. They discuss non-task related things as well.
After about 10 minutes, they ask for a set of interlocking blocks, which an observer brings them. They start using the blocks immediately, though there does not appear to be a consensus about which tower structure each student is constructing. Ms. A (the teacher) comes over to their desks just as they start discussing what the construction should look like. The students are guided to recognize that they had not been counting the total number of blocks in each tower (A, B, C, etc.) and come to see that they have to add 5 blocks to the tower structure in order to get to the next tower in the sequence. Ms. A is with the students for about 9 minutes.
After Ms. A leaves the group, the students continue to try to find the total number of blocks in the 10-block high tower (they had done the 5-block with their teacher). Ultimately, they Danica, who seems to be trying to finish the task, states that stage D has 16 blocks, and stage E has 21 blocks. It is not clear if she recognizes these as the 4-block high tower and the 5-block high tower, respectively. However, there do not seem to be any more incidents of LHSIA or LMTY. If anything, Get The Job Done seems to be at play here. The teacher asks the class to clean up, before the students take the questionnaire.</td></tr>
</table>

Danica's work, in Nov. 2008



Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any [redacted]wer.

A B C [redacted] D 16 e 21

$$\begin{array}{r} 16 \\ +\ 5 \\ \hline 21 \end{array}$$

[redacted]. To make a five block high tower you will need [redacted] blocks to make it. $\boxed{5}$

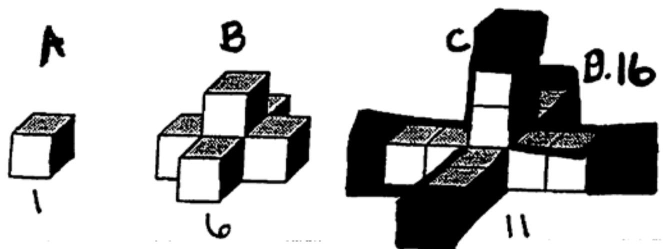
E) To make a 10 block high tower you will need 5 blocks more.

(E.) To make a 100 block high tower you will need [redacted] more.

Shanika' s work, Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



D. 16 E. 16 + 5 = 21

D. To make a five-block high tower you will need ~~18~~ 5-blocks to make it.

E. To make a 10-block high tower you need 10-blocks more. ~~to~~

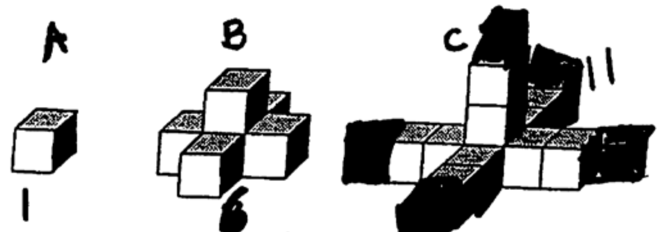
F. To make a 100-block ~~tower~~ high tower you will need 5

Shannen's work, Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.

D 16 e 21



D To make a five-block high tower you will need [illegible] blocks to mak it.

E To make a 5 block high tower you need to add 10 more blocks.

F To make a 100 block towrer you would need [illegible]

Questionnaire Responses for Danica, Shanika, Shannen

Questionnaire items Which may indicate LMTY structure

Questionnaire Items	**Danica**	**Shanika**	**Shannen**
Statements (All the time, Sometimes, Never)			
I wanted to teach another student something that I knew that the other student did not know.	Never	All the time	Never
I listened carefully to the ideas of someone I was trying to help.	Sometimes	All the time	Sometimes
I helped someone see how to do the math.	All the time	Never	Sometimes
Others listened carefully to my ideas.	Sometimes	All the time	Sometimes
Behaviors (Often, Sometimes, Hardly ever)			
I gave helpful suggestions.	Often	Sometimes	Often
I worked cooperatively.	Often	Often	Often
Thoughts (Yes/No)			
I like teaching this person things that I know.	No	Yes	No

Questionnaire Responses for Danica, Shanika, Shannen

Questionnaire Items Which May Indicate LHSIA structure

Questionnaire Items – Statements	**Danica**	**Shanika**	**Shannen**
Statements (All the time, Sometimes, Never)			
I wanted people to think that I'm smart.	Never	Never	Sometimes
I tried to impress people with my ideas about the problem.	All the time	Never	Never
People seemed impressed with the ideas I shared about the problem.	All the time	Sometimes	Never
People saw how good I am at the math we did today.	All the time	All the time	Sometimes
I felt smart.	All the time	All the time	Sometimes
I wanted to show someone that my way was better.	Never	Never	Never
I was a lot better at math than others today.	Sometimes	Never	Never
Behaviors (Often, Sometimes, Hardly ever)			
I was the leader.	Sometimes	Hardly ever	Sometimes
I was bossy.	Hardly ever	Hardly ever	Hardly ever
I wanted to show off.	Hardly ever	Hardly ever	Sometimes
I liked to be right.	Sometimes	Sometimes	Often
Thoughts (Yes/No)			
I want you to know just how smart I am.	No	Yes	No
People think I'm smart.	Yes	Yes	Yes
I wish the teacher would call on me, so I can show how much I know.	No	Yes	No

Wilson, Mitchell, Martin, Lewis

This is Group 5 from Ms. A's grade seven Class 2.

Verbal emphasis indicated by underline
Actions included either in *italics* or (parentheses) as appropriate.
Pauses in speech are indicated by …
Words inserted to help clarify a student's words are included in [brackets].
When students voices overlap, one or both sets of words are included in //double slash marks//.

The number included next to either LMTY or LHSIA refers to the code, included in the table of codes

<table>
<tr><th colspan="4">Wilson, Mitchell, Martin, Lewis</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td></td><td>Ms. A</td><td>The teacher has just finished giving her introduction to the problem and instructions. This takes a little more than 3 minutes.</td><td></td></tr>
<tr><td>1</td><td>Mitchell (Mi)</td><td>I think I got the answer for the 5 block tower</td><td>EPISODE 1
LHSIA (14)
Mitchell is the first to share his thoughts after the teacher's introduction to the problem. He claims that he has the answer already.</td></tr>
<tr><td>2</td><td>Lewis (L)</td><td>Huh?</td><td></td></tr>
<tr><td>3</td><td>Mi</td><td>I got the answer for the 5 block tower.</td><td></td></tr>
<tr><td>4</td><td>L</td><td>Oh I think they're gonna pass out blocks, look.</td><td></td></tr>
<tr><td>5</td><td>Mi</td><td>I'm not gonna wait for them to pass out blocks.</td><td></td></tr>
<tr><td colspan="4">Interpretation: Mitchell is not only the first in the group to speak, he expressly states that he has an answer "for the 5 block [high] tower." He does not provide this answer, even though he seems to want his classmates to know that he has an answer. I infer that he has an active LHSIA engagement structure because of his declaration of having an answer, as well as, the fact that he is the first to speak up in his group.</td></tr>
<tr><td>6</td><td>L</td><td>That's what everyone's doing. You know, right here?</td><td></td></tr>
<tr><td>7</td><td>Mi</td><td>I got the answer for it though for the first</td><td></td></tr>
</table>

Wilson, Mitchell, Martin, Lewis			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
		one.	
8	L	I know but you gotta solve it before you get the answer. We gotta solve it first then we gonna get an answer.	
	For a little more than 2 minutes: *The teacher approaches this group immediately and asks them what they started discussing. Lewis tells her that Mitchell claimed to have the answer but he (Lewis) thinks they should "all come up with the same answer." Ms. A tries to encourage all the students to participate by starting with the first three phases of the towers as given in the task. Ms. A tells them to record the information that they determine as they keep working. The students, especially Mitchell and Lewis, are able to provide correct answers for the total number of blocks for the 1-block, 2-block, and 3-block high towers. Ms. A asks them if they notice a pattern, to which Mitchell responds that the totals are increasing by 5 blocks each time. She encourages the group to keep working and building on those ideas.*		
9	L	Aight, so we gotta try to get 100 right?	
10	Mi	We gotta try to get the 100 one.	
11	L	Get your hand off your mouth.	
12	Mi	Let me see your calculator, Martin. (Martin gives over the calculator.)	
13	L	I need a calculator (to one of the researchers).	
14	Mi	I don't know how to do that really I can just do everything in my head. I can't really put everything down on paper.	
15	L	What's the next block?	
16	Mi	Oh, wait! I got it now. I'm gonna make one of them charts that they be having on the tests.	
17	Martin (Ma)	It really isn't that hard.	
18	Mi	You know that test? They be having a chart like this like a rectangle. I'm gonna put how high it is and how many.	
19	Ma	You're gonna put like the number of blocks.	
20	Mi	Yeah, like the number of blocks that's on top of it and then number, how many. That's what I'm planning on doing.	

Wilson, Mitchell, Martin, Lewis			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
21	L	So now we just gotta um…	
22	Ma	We just gotta work with each other. On one side, it's the number. On the other side it's the number of blocks.	
23	Mi	But the height gonna be whatchamacallit? I think this is horizontal.	
24	L	We gotta make a T-chart.	
25	Mi	That's not what I'm doing. I'm doing something different. (Mitchell looks over at Martin's paper as they both write down something. Neither Lewis nor Wilson are writing anything down.)	
26	L	(inaudible; starts writing on his paper)	
27	Mi	Damn, that's messed up, everybody doing something different than me.	Mitchell may be referring to the fact that, according to his work, his chart is written with the height and total number of blocks in a horizontal format rather than the vertical format his classmates' work is in.
		Ms. G (the teacher aide) approaches the group to see what they are doing at this point. She repeats their plan, to record their information in a chart and moves to another group.	
28	L	So number 3 …11 so what's the main number 100 we gotta get?	
29	Ma	Uh-huh.	
30	L	So it's gonna be right here then. See that's the main number we gotta get	
31	Mi	11, 16 and 21.	
32	L	Oh! I know what he doing, he doing something like addition	
33	Mi	It's the tiniest difference.	
34	L	No, it's the same thing you see?	
35	Mi	No, I just wrote mine wrong. Okay, you did	

Wilson, Mitchell, Martin, Lewis			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
		yours like….nah never mind.	
36	L	That's crazy.	
37	Mi	I'm gonna do another one for 10.	
38	L	What number you doing?	
39	Mi	(mumbled)	
40	L	Huh?	
41	Mi	I'm gonna make another one of these with 10.	
42	L	Oh no, wait, she coming. (talking to self: 11, 16…)	
		For 4 minutes: *The boys spend a few minutes writing in their own papers independently.*	
43	Mi	Ok you know how they all go up by 1?	**EPISODE 2**
44	L	Oh I see it too!	
45	Mi	It change to 6 (*describing figure B as having 6 blocks*)	**LMTY (1)** Mitchell tries to explain that when the height increased from 1 block to two, the total number of blocks increased to 6.
46	L	Yeah, 6.	**LHSIA (9, 12)** Lewis agrees with Mitchell's statement, and by repeating 6, seems to be indicating that he may have had that answer already.
47	Mi	I need to write this on the back	
48	Ma	I don't have enough room.	
49	L	I got it. (*as in "I understand what you're saying"*)	

Interpretation: Mitchell seems to want to help his classmates recognize a pattern that he appears to have recognized. I infer that Mitchell has an active LMTY structure as he shares that the height is increasing by 1 block each time, and that as the height went from 1 to 2 blocks, the total number of blocks increased to 6. Though these ideas were discussed with Ms. A when she first visited the group, perhaps Mitchell did not believe that each of his classmates realized this idea already.

Lewis is listening to Mitchell's ideas and agrees with him, "Yeah, 6." I infer that Lewis

Wilson, Mitchell, Martin, Lewis			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
has an active LHSIA structure since he not only agrees by saying, "Yeah" but repeats the answer, "6" suggesting that he may want his classmates to know that he is able to keep up, mathematically.			
		For about 4 minutes: *Ms. A returns to the group. She tries to help them see a relationship between the height of the tower and the total number of blocks in that tower. Just before she leaves she asks them if they are going to continue to use the same strategy, adding 5 blocks, to find the total number of blocks in the 100-block high tower.*	
50	Mi	I just noticed this…the answer for the 100 one is whatchamacallit 5 times 99 plus 1 I just noticed that, it's not just as simple as adding…I'm-a wait and see if ya'll get the same answer as me	**EPISODE 3** **LHSIA (14, 15)** Mitchell demands attention from his classmates as he tells them the strategy that he "just noticed." He then states he will wait for confirmation if they have the same answer that he has.
Interpretation: Mitchell is stating his strategy but does not explain it in further detail. Following this, his classmates do not ask for further explanation at this time. I infer that Mitchell has an active LHSIA structure here because he states his strategy, "5 times 99 plus 1," and says, "I just noticed," perhaps to indicate that he is expressing an idea to his classmates. Mitchell then states that he wants to see if the others get the same answer for the 100-block high tower, suggesting he will wait for confirmation from his classmates. This may be why he did not explain his strategy any further, particularly if he believes his classmates are using a different strategy (e.g., adding 5 blocks each time).			
51	Mi	You gotta finish it on the back (it's not clear to me if he is speaking to a specific classmate or just speaking in general)	
		Another teacher aide (OT) comes and sits with the group. She seems to be mainly focused on Wilson, but asks questions about the task to the other three students as well.	
52	M	You gotta figure out the height too?	
53	L	You gotta figure out the whole thing…	
54	Mi	I don't get that. The height is going to be	

Wilson, Mitchell, Martin, Lewis			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
		100.	
55	OT	It started with 1, right?	
56	Mi	Yeah, but I don't get that.	
57	Ma	In a 100 block high tower.	
58	La	Wait, wait, wait, wait…the height….it's 21!	
59	Mi	Yeah, it's 1 and 1. (seems to be talking about the 1-block high tower) I got 101 for 21.	
60	Ma	I got 24.	
61	Wilson (W)	4 block 5 high?	
62	Mi	But I didn't do what was on the paper. Go to 10, 10 is 46, right? Just keep multiplying by 10 and then add it up.	**LHSIA (14)** Mitchell shares a strategy that he claims is different than "what was on the paper."
63	Mi	But check this out. Lewis, look this is my answer, look this is the easiest way to get it. I swear to god, look. (gives his paper to Lewis and shows him the calculator screen; each student has a graphing calculator to work with)	**LHSIA (6)** Mitchell claims that his way is the "easiest way."
64	L	But what if we don't agree with what you have…	**LHSIA (9A)** Lewis suggests that he might not agree with Mitchell's answer or strategy.
65	Mi	It's gonna be so simple, once you get it.	
66	L	This is long.	
67	Mi	Are you serious? (says as with disbelief, in his tone of voice)	

Interpretation: Mitchell appears to have an active LHSIA engagement structure throughout the exchange above. He first tells his classmates that his strategy is different than the one suggested by the task. He seems to be demanding their attention so he can share his ideas, as he starts to do by saying, "Keep multiplying by 10 then add it up." Mitchell also makes a point to share his paper and his calculator screen with Lewis, stating that his strategy is the "easiest way." His wording, coupled with his next speaking turn, "It's gonna be so simple," suggests that he believes the task is easy.

Lewis responds by disagreeing with Mitchell, or rather saying that his answer might be different than with Mitchell's answer is, once Lewis arrives at his own answer. By voicing

Wilson, Mitchell, Martin, Lewis			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
this opinion, I infer that Lewis also has an active LHSIA structure, as his statement appears to indicate that he believes he is smart enough to come up with his own answer to the problem.			
		The teacher aide brings over a bag of SnapCubes. All the students in the group work with these cubes. The teacher aide continues to mostly work with Wilson.	
68	L	Now I'm-a show you, I made 21 of these (using the SnapCubes to begin constructing the tower structures, Mitchell looks on; Martin also begins to construct something with the blocks)	
69	L	(Counting by 2s)	
70	L	This is the height (*displaying a single tower of blocks he has put together, there are no blocks on the sides*)	
71	Mi	Of what?	
72	L	The, uh, the 100.	
73	Mi	I did mines by 10 though.	
74	L	We gotta figure out the whole problem. (emphasizes the word "whole" when speaking)	
75	Mi	Go ahead we gotta figure out the whole problem if we got to… (waves his hand as if he is impatiently waiting for Lewis to get an answer)	
76	L	Aight there's 1 right	
77	Mi	And then you have 5…	
78	L	(*Counting*)….5 and then you gotta another one	
79 2	Mi	Let's try the calculator and see what you get….they're uneven (likely referring to an aspect of the towers that one of his classmates has constructed).	
80	L	I know….Hold, hold up now! Look. First it's this, it's just 1, 2, 3, 4. It's just this one by itself. Then you add 5 to it to this and this. Then you add another one then you gonna get these on the other side. Then you add another one and you gonna get these – on the outside 1 more, 1 on the side.	
81	Ma	24. I already did it	

Wilson, Mitchell, Martin, Lewis			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
82	Mi	But they need 1 on each side for every single one.	
83	Ma	You gotta do this on every side.	
84	Mi	You like doing that don't you?	
85	Ma	Nah that's the 24 one, because when I was doing my chart, the 100 one would end up being 24 and then 21 ended up being 100.	
	The teacher aide asks all the boys in the group about the video recording, if they will be on television. They state that they will not be television, but the recording was to "see how smart" they are (according to Mitchell). *The aide continues to work with Wilson, though sometimes Lewis joins in their conversation as well.*		
	For about 4 minutes: *The students discuss things other than the task (e.g., favorite colors, television shows).*		
86	L	I'm done.	**EPISODE 4** **LHSIA (4)** Lewis tells his group members that he has finished getting the answers to the problem.
87	Mi	What did you get for 100?	**LHSIA (15)** Mitchell asks Lewis what his answer is for the 100-block high tower, possibly looking for confirmation of his earlier answer.
88 2:07:05	L	21	
Interpretation: Lewis states that he is "done," suggesting that he has all the solutions he believes he needs. I infer that Lewis has an active LHSIA engagement structure, as evidenced by the fact that he wanted to tell his classmates that he was finished with the task. This announcement seems to prompt Mitchell to ask Lewis what his answer was for the 100-block high tower, to which Lewis replies 21. This answer is the same was what Mitchell had earlier. I infer Mitchell also has an active LHSIA structure because he appears to be looking for confirmation of the answer he gave earlier, which was to correct			

Wilson, Mitchell, Martin, Lewis			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
21 with the “100 block tower.” Though the group seemed to get correct answers, the total number of blocks for the 5-block high tower and the 10-block high tower (as evidenced by their student work), they do not follow their pattern for the 100-block high tower. Rather than determine the total number of blocks, they find the height of the tower for the structure that has close to 100 blocks in it. The 21-block high tower has a total of 101 blocks, but the students treat this particular structure as though there are 100 blocks. They do not seem to realize that they are no longer following their earlier correct pattern.			
	For the remainder of class, approximately 15 minutes: *Once or twice, Lewis tries to explain a concept to Wilson, at the request of the teacher aide. Otherwise, the students do not talk about the task or its solutions anymore. They have many off-task conversations and play with the SnapCubes some. At one point, Mitchell says that they are finished, and therefore they have nothing (class-related) to do. They start cleaning up their blocks when the Ms. A asks them to do so.*		

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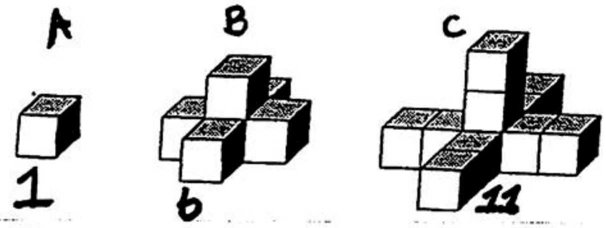
Wilson's work, Nov. 2008

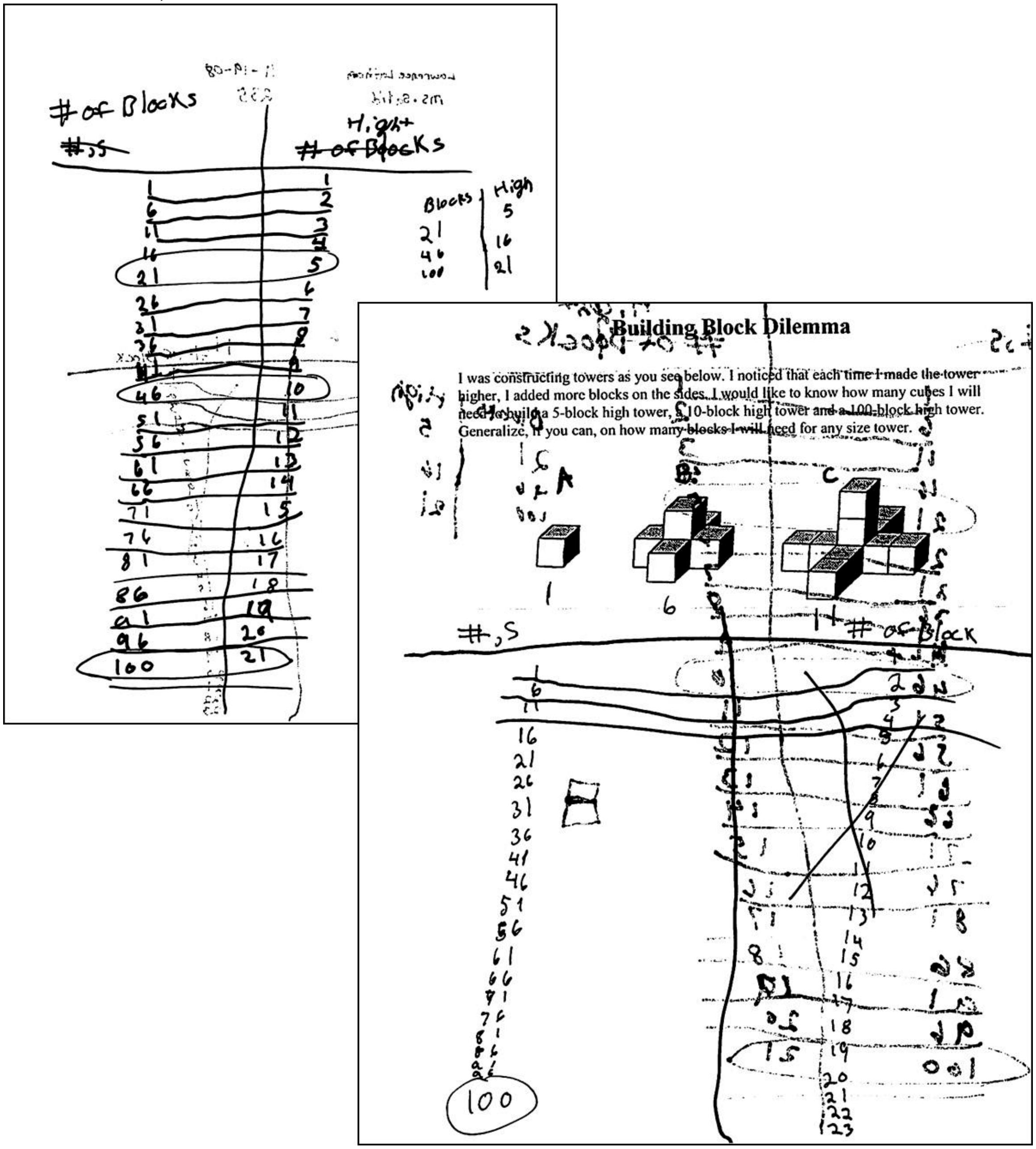
Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



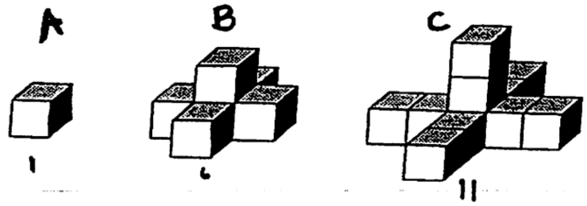
# of Blocks	Hight
1	1 ~~6~~ cubes
2	6 8
3	11
4	16
5	24
6	26
7	31
8	36
9	41
10	
11	
12	
13	
14	
15	
16	
17	
18	

Lewis's work, Nov. 2008



Mitchell's work, Nov. 2008

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.

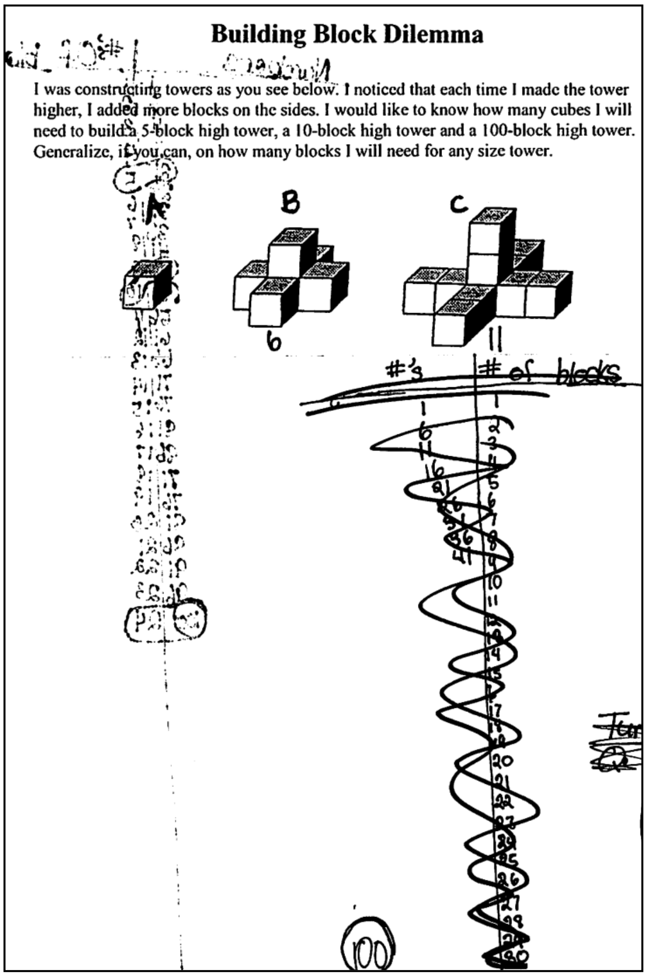


#'s	1	2	3	4	5
# of blocks	1	6	11	16	21

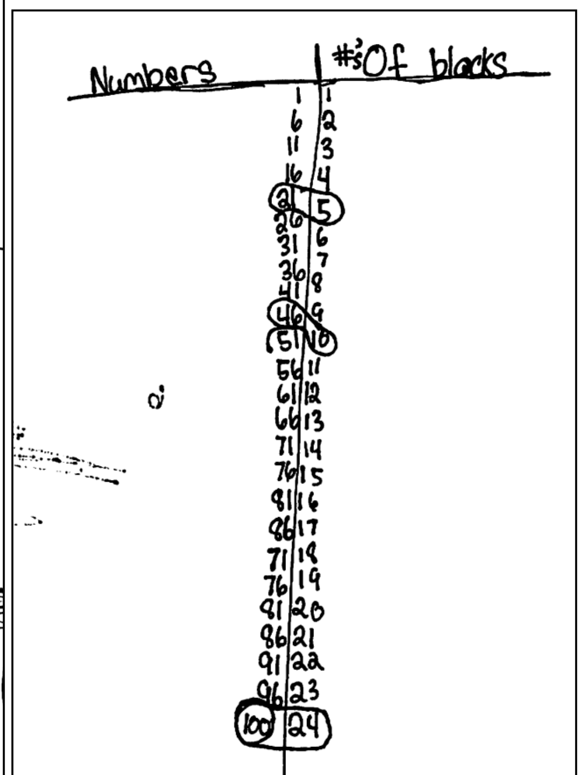
#'s	6	7	8	9	10	11
# of blocks	26	31	36	41	46	51

#	10	20	30	40	50	60	70	80	90	100
# □	46	96	146	196	246	296	346	396	446	496

Martin's work, Nov. 2008


Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



Questionnaire Responses for Wilson, Lewis, Mitchell, Martin

Questionnaire Items Which may indicate LMTY structure

Questionnaire Items	Wilson	Lewis	Mitchell	Martin
Statements (All the time, Sometimes, Never)				
I wanted to teach another student something that I knew that the other student did not know.	Sometimes	All the time	Never	Sometimes
I listened carefully to the ideas of someone I was trying to help.	Sometimes	All the time	Sometimes	Never
I helped someone see how to do the math.	Sometimes	All the time	Never	Never
Others listened carefully to my ideas.	Sometimes	All the time	Sometimes	All the time
Behaviors (Often, Sometimes, Hardly ever)				
I gave helpful suggestions.	Often	Sometimes	Sometimes	Sometimes
I worked cooperatively.	Sometimes	No answer	Hardly ever	Often
Thoughts (Yes/No)				
I like teaching this person things that I know.	Yes	Yes	No	No

Questionnaire Responses for Wilson, Lewis, Mitchell, Martin

Questionnaire Items Which May Indicate LHSIA structure

Questionnaire Items – Statements	**Wilson**	**Lewis**	**Mitchell**	**Martin**
Statements (All the time, Sometimes, Never)				
I wanted people to think that I'm smart.	Sometimes	All the time	Sometimes	All the time
I tried to impress people with my ideas about the problem.	Sometimes	Sometimes	Never	Never
People seemed impressed with the ideas I shared about the problem.	All the time	Sometimes	Never	All the time
People saw how good I am at the math we did today.	All the time	All the time	Never	All the time
I felt smart.	Sometimes	All the time	Never	All the time
I wanted to show someone that my way was better.	Sometimes	Sometimes	Never	All the time
I was a lot better at math than others today.	Sometimes	No answer	Never	Never
Behaviors (Often, Sometimes, Hardly ever)				
I was the leader.	Often	Sometimes	Hardly ever	Often
I was bossy.	Sometimes	No answer	Hardly ever	Hardly ever
I wanted to show off.	Sometimes	No answer	Hardly ever	Hardly ever
I liked to be right.	Often	No answer	No answer	Often
Thoughts (Yes/No)				
I want you to know just how smart I am.	Yes	Yes	Yes	Yes
People think I'm smart.	Yes	Yes	Yes	Yes
I wish the teacher would call on me, so I can show how much I know.	Yes	Yes	No	No

Ellen, Larry, Janelle

This is Group 6 from Ms. A's grade seven Class 2.

Verbal emphasis indicated by underline
Actions included either in *italics* or (parentheses) as appropriate.
Pauses in speech are indicated by …
Words inserted to help clarify a student's words are included in [brackets].
When students voices overlap, one or both sets of words are included in //double slash marks//.

The number included next to either LMTY or LHSIA refers to the code, included in the table of codes.

Ellen, Larry, Janelle			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
	Ms. A	*The teacher has just finished giving her introduction to the problem and instructions. This takes a little more than 3 minutes.*	
1	For about 4 minutes: *The students start by reading the problem to themselves, sometimes speaking aloud. One of the teacher aides, Ms. G, approaches the group and asks them about the task. When they tell her they cannot draw the tower structures, Ms. G brings them SnapCubes so they can construct the towers with the manipulatives. Though all three students take out blocks to work with, they seem to be working independently and do not seem to have assigned a specific task to each person. Ms. G appears to leave the group while they work with the blocks.*		
2	For almost 3 minutes: *Students working to build blocks. Talking amongst themselves so quietly, only mumbling heard on audio/video.*		
3	For about x minutes: *Ms. A approaches the group and asks what they are currently working on. Larry answers that they are trying to determine phase D based on phase C, which is given on the handout. Larry realizes how the towers are built one after the other. Ms. A prompts him to explain that phase B is built off of phase A by adding one block to each of the sides (so four in total) and another block to the height. Larry and Janelle work to count the total number of blocks in phase C, or the 3-block high tower. Ms. A encourages them to keep a record of their findings as they continue to*		

Ellen, Larry, Janelle			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
	work – both the height and the total number of blocks.		
4	Janelle (J)	What are you doing?	**EPISODE 1** The students are working on their own, without any teacher assistance.
5	Larry (L)	Which one is this one? (referring to the tower structure he constructed) This one's C right?	
6	J	(inaudible – counting)	
7	L	Look and then after that we gotta draw it. We gotta draw D	
8	Ellen (E)	Alright so what are we doing now?	
9	L	We doing D until we get to 10 high or 100	**LHSIA (14)** Larry responds to Ellen's question. He gives his suggestion of what the group should be doing, which seems to be find the solutions for the 10-block high tower and 100-block high tower.
10	E	It gotta go 5 vertical	**LHSIA (8)** Ellen appears to correct Larry's when she suggests that they need the 5-block high tower.
Interpretation: Larry had been participating a lot when either Ms. G or Ms. A were with the group. Now that the students are working on their own, he still seems interesting in expressing his ideas, as he did in line 7 when he stated that they had to draw the tower structure represented by phase D. Here in line 9, Larry says that they are working on determining the next phase (D) and that they will keep going until they get to the 10-			

Ellen, Larry, Janelle			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
block and 100-block high towers. His tone of voice suggests an air of authority and confidence in his ideas. His tone of voice and statement that they should continue working until they have solutions to the 10-block and 100-block high towers together suggest an active LHSIA structure, since he seems confident that the group will successfully arrive at the necessary answers. Ellen responds to Larry's suggestion in line 10. She seems to be correcting his suggesting of finding the solutions for the10-block and the 100-block high towers by stating that they need the 5-block high tower. Her tone of voice also carries an air of authority. I infer that the LHSIA structure has been activated for her as she seems to assert that she knows what to do, despite asking the question, "So what are we doing now?"			
11	L	5 vertical height, right? 5 D	
12	L	But all we gotta do is add 1 to the sides (points to the 4 sides of the tower structure constructed in front of him as he speaks)	**LMTY (1)** Larry shares his strategy for constructing the towers, "add 1 to the sides."
Interpretation: Larry seems to have agreed with Ellen's statement earlier that they need the solution for the 5-block high tower. He then offers his strategy for how they can determine the 5-block high tower, stating that they just have to add 1 block to each side as they go to the next phase or height for the tower structures. I infer that Larry has an active LMTY structure here because he appears to be trying to explain this strategy, as evidenced by the fact that he points to the sides of the 3-block high tower structure he created using the SnapCubes.			
13	J	I have mine here so let's….	
14	E	So how are we going to D now?	
15	E	How we have it now but just put 1 more, right?	
16	J	Yeah	
17	L	Put 1 on each side	**LMTY(7)** Larry responds to Ellen's question, "Put 1 more [block]?"
18	E	And going up	**LMTY (4, 6)** Ellen builds on Larry's response, adding that a block must be added to the height as well, when they add a

<table>
<tr><th colspan="4">Ellen, Larry, Janelle</th></tr>
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<tr><td></td><td></td><td></td><td>block to each side.</td></tr>
<tr><td>19</td><td>L</td><td>Yeah 1 up…alright</td><td></td></tr>
<tr><td colspan="4">Interpretation: Larry appears to still have an active LMTY structure from line #(1:52:38). In response to Ellen’s question about how to get the tower that would be represented in phase D, Larry basically repeats what he stated earlier about adding one block to each side. He responds with a short answer, perhaps because he does not believe that a longer explanation is needed at this time.

Ellen again jumps in, partly building on Larry’s response and also possibly correcting him. She adds to his statement about adding a block to each side that they also need to add one block to the height, in order to build phase D off of phase C. Because she was the one who asked the question, “How we have it now but just put one more?” she may be stating that a block must be added to the height in order to explain this to Larry. Larry’s response, “Yeah, 1 up,” indicates that he listened to and agreed with Ellen’s suggestion. These statements suggest that the LMTY structure was activated for Ellen as well as for Larry.</td></tr>
<tr><td>20</td><td>J</td><td>So how we gonna do that</td><td></td></tr>
<tr><td>21</td><td>E</td><td>I feel progress
//L: counting//
And then 2 up right? 3 up right?</td><td></td></tr>
<tr><td>22</td><td>L</td><td>16</td><td></td></tr>
<tr><td>23</td><td>E</td><td>What?</td><td></td></tr>
<tr><td>24</td><td>L</td><td>16</td><td></td></tr>
<tr><td>25</td><td>E</td><td>I got 16</td><td></td></tr>
<tr><td>26</td><td></td><td>Janelle and Ellen discuss the fact that they do not know how to draw the examples of the towers. Larry suggests that they do not have to create drawings.</td><td>EPISODE 2
The students discuss strategies to find the total number of blocks.</td></tr>
<tr><td>27</td><td>L</td><td>Oh I know man! I know, look! (gets Janelle’s and Ellen’s attention) All you’re doing is adding 5s (says with excited tone of voice)</td><td>LHSIA (14)
Larry excitedly states his idea to his two classmates.</td></tr>
<tr><td>28</td><td>J</td><td>No you’re adding more
//Sts: several voices overlapping//</td><td></td></tr>
<tr><td>29</td><td>L</td><td>I know that but all you’re doing is 1 plus 5 is 6 and 6 plus 5 is 11, 7 plus 5 is 16. I mean, 16 plus 5 is 21 and 21 plus 5 is 26</td><td>LHSIA (9); LMTY (1)
Larry first states, “I know that,” in response to his</td></tr>
</table>

<table>
<tr><th colspan="4">Ellen, Larry, Janelle</th></tr>
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<tr><td></td><td></td><td></td><td>classmates disagreeing with his original statement. Then he explains his reason for suggesting that 5 blocks are added each time.</td></tr>
<tr><td>30</td><td>E</td><td>Yeah? You’re just adding by 5s</td><td></td></tr>
<tr><td>31</td><td>L</td><td>Yeah</td><td></td></tr>
<tr><td colspan="4">Interpretation: Larry appears to again have an active LHSIA structure as he has made a realization about a pattern with respect to the total number of blocks in the tower structures. He realizes, due to the pattern perhaps as written on his paper (see student work) that the totals increase by 5 from phase to phase. In noticing this pattern, he demands the attention of his two classmates and excitedly shares this new observation. Larry’s statement prompts a brief but lively debate within his group; unfortunately most of what was said was not clear enough for transcription. It seems that Janelle stated that more blocks were added each time, but it is possible that she is referring something different than what Larry means.
First Larry agrees with his classmate’s contributions as evidenced by stating, “I know.” He may have said this to not only agree but also to indicate that he is smart enough to understand their arguments. Larry then continues by explaining his reasoning for suggesting that the totals increase by 5 – noting sum of 5 and the total from the previous iteration. Ellen appears to accept this explanation by stating, “You’re just adding by fives.”</td></tr>
<tr><td>32</td><td>J</td><td>I don’t wanna scribble it out, the paper is gonna look ugly.</td><td></td></tr>
<tr><td>33</td><td>L</td><td>I’ll scribble and scrabble.</td><td></td></tr>
<tr><td>34</td><td>J</td><td>It looks ugly.</td><td></td></tr>
<tr><td>35</td><td>L</td><td>We’re the only ones that got orange [blocks] and…</td><td></td></tr>
<tr><td>36</td><td>J</td><td>I know I wanted the pink and…</td><td></td></tr>
<tr><td>37</td><td>E</td><td>Get pink.</td><td></td></tr>
<tr><td>38</td><td>J</td><td>The black and red.</td><td></td></tr>
<tr><td>39</td><td>E</td><td>Let me see! How much do you have on top? 5 on top right?</td><td></td></tr>
<tr><td>40</td><td>L</td><td>No I got 6. Count it from this one (picks up the tower structure and points to the bottom block that is usually hidden)</td><td>LHSIA (8)
Larry corrects Ellen when she suggests that his tower structure has 5 blocks in</td></tr>
</table>

Ellen, Larry, Janelle			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
			the height. The structure he has in front of him at the moment has a total of 6 blocks, like he says.
41	E	Oh yeah	
42	L	(counting) oh no this one's 6…	
43	E	Janelle's looks wrong. What'd you do?	
44	L	E is 21 and then F is 22 because 1… (counts the total number of blocks in his tower structure)	
45	E	Let me make sure.	
46	L	Are you doing the….	
47	E	Alright so what are we doing for E? How do we build that? What are you doing?	
48	L	A atom…	
49	E	Ohhh! How do we, what do we do for E now?	
50	L	What we do for E? Didn't you do? Aight let's see and then add 5 out of 6 out of 6 E	
51	E	21…	
52	L	And now doing it we don't have to build it like that? (counting) You already got it in your thing.	
53	E	And for F?	
54	L	All you're doing is the same thing	
55	E	Up to how many letters do we have to go up anyway?	
56	L	I'm doing it up to 10, 10 high.	
57	J	How much is that?	**EPISODE 3** The students discuss which structure is a 5-block high tower.
58	L	5, 5 up, 5 block high high tower	
59	J	(picks up her tower structure & Larry's structure to compare) This is 5 blocks too.	
60	L	No, you gotta do 5 here (points to the height of his structure, counting from the first *visible*	**LMTY (4)** After answering

Ellen, Larry, Janelle			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
		block in the height)	Janelle's question about which tower he had constructed in front of him, Larry corrects Janelle's idea of which is the 5-block high tower.
61	J	I did.	
62	L	No. (counting the height of the tower structure he has in front of him, which appears to be the 6-block high tower)	**LHSIA (8, 9A)** Larry continues to disagree with Janelle, stating that he has the correct 5-block high tower.
63	J	(counts the height for the tower structure that she has in front of her – it appears to be the 5-block high tower)	**LHSIA (9A)** Janelle counts the number of blocks in the height of her tower structure, since she continues to disagree with Larry.
64	L	No, not there, right here! (counts the number of blocks in Janelle's tower, starting with the first visible block, so he counts 4 blocks)	
65	J	No, you count this one too.	
66	L	Yes you do.	
67	E	Alright so for G what we're doing is…	
68	J	It's not invisible!	
Interpretation: Janelle begins this exchange by asking Larry which tower structure he has in front of him, prompting him to respond that he has the 5-block high tower. When she claims to have the same tower (though the two structures have different heights), Larry seems to try to explain why his is the correct 5-block high tower, even though it has a total of 6 blocks in its height. Though Larry is correcting Janelle, he seems to have an active LMTY structure because he tries to explain his strategy – to count the height starting from the first visible block rather than from the middle block which is hidden by the other blocks that extend off of it.			

<table>
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<tr><td colspan="4">Janelle claims that she indeed has the 5-block high tower, prompting Larry to explain by counting how he determines the 5 blocks for the height. This disagreement between the two students suggests that the LHSIA structure is activated for both Larry and Janelle, who both continue to argue that their position is the correct one. Janelle responds to Larry by counting the blocks in the height of her own tower structure (which is 5, including the middle block). Larry’s response is to also count the same blocks, but he does not include that middle block. Janelle specifically claims that the middle block should be counted, “No, you count this one too,” and adds, “It’s not invisible.”

As we see in the next several speaking turns, Larry and Janelle do not seem to come to an agreement, and just let go of their disagreement.</td></tr>
<tr><td>69</td><td>L</td><td>Alright I’m doing it up to 10</td><td></td></tr>
<tr><td>70</td><td>E</td><td>So all that we’re basically doing is adding up and the sides. We add on the sides too right?</td><td>LHSIA (12, 15) Ellen contributes her strategy, “adding up and the sides,” asking for confirmation about adding the sides.</td></tr>
<tr><td>71</td><td>L</td><td>Yeah</td><td></td></tr>
<tr><td>72</td><td>E</td><td>I got my blocks already here</td><td></td></tr>
<tr><td colspan="4">Interpretation: Ellen had not participated in the earlier conversation between Larry and Janelle. Here she contributes her strategy for finding the solutions, stating that they need to add the blocks that are in the height as well as the blocks that are included in all of the sides. Because of her lack of participation earlier, she may be trying to indicate that she can keep up with her classmates, mathematically, by expressing these ideas, suggesting the activation of the LHSIA structure. In addition, she asks for confirmation from Larry that they are supposed to be adding the blocks from the sides as well.</td></tr>
<tr><td>73</td><td colspan="2">For about 1 minute:
The students continue to work, mostly independently on constructing towers, and determining the total number of blocks in the different towers. When they do speak to one another, they discuss what information to write down.</td><td></td></tr>
<tr><td>74</td><td>J</td><td>You gotta take these out. Let me show you which ones (speaking to Larry, reaches over to his tower structure and removes a cube from one of the legs)
I got 4 on the sides (puts up 4 fingers)</td><td>LHSIA (3, 8) Janelle corrects Larry by telling him that he has an incorrect number of blocks in his</td></tr>
</table>

Ellen, Larry, Janelle			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
			tower structure. She takes control of the math tools by removing a cube from his structure.
75	L	No, you don't!	
76	J	You do.	
77	L	How much you wanna bet it's not gonna get… (He removes some more blocks from his tower structure and shows Janelle. She takes the structure away from him and counts the total number of blocks the tower currently has to show him.)	
78	E	I have 25.	
79	J	(after counting) 21	**LHSIA (7)** Janelle gives her answer that there are 21 blocks in the 5-block high tower.
80	L	Now add 1 for the 5.	
81	J	I know but…	
82	L	Why you saying "ooooweeee" to me?	
83	J	Because you're acting like I'm dumb or something.	
Interpretation: Janelle seems to have an active LHSIA structure here as she tries to tell Larry that his tower structure is incorrect, if he wants to have the 5-block high tower. First she tells him that he has too many blocks within the physical structure, then she takes the structure away from him to remove a block, taking control of the mathematical tools Larry is using. When Larry receives his structure back, he continues to remove some cubes and counts the total number of cubes in his structure. This researcher could not hear Larry state a total amount, though Ellen chimes in stating that she counted 25 blocks in her 5-block high tower. Janelle again takes Larry's tower structure and counts 21 total blocks in his 5-block high tower. She gives this answer without seeming to acknowledge Ellen's suggestion of 25 blocks. This action of stating an answer without considering a classmate's suggestion also suggests an active LHSIA structure for Janelle.			
84		For 5 minutes*:* *The three students seem confused about what to do next. Larry suggests they need to wait to talk to Ms. S. Though they were able to*	

Ellen, Larry, Janelle			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
		determine the answer for the 5-block high tower, they do not seem to know how to move forward for the 10-block high tower. The three talk to other students. Most of what is said among the students is off-task and/or inaudible. Ellen encourages the group to focus on the task again.	
85	E	Ok let's figure out what we were doing before this. F right?	
86	L	We, no, we was on G!	
87	E	So G must be 31…G is 31	
88	J	(counts on her hand) Yeah, 31.	
89	E	So everybody write down 31.	**LHSIA (14)** Ellen tells her classmates to use the answer that she has suggested, that there are 31 total blocks in phase G.
90	L	No, esperate.	
91	E	No, that's not it, because if we add…	
92	L	(counting) 35. You was wrong.	
93	E	I was wrong?	
94	L	Yeah. (The three laugh at this exchange.)	
Interpretation: Ellen states that phase G has a total of 31 blocks. I infer that the LHSIA structure was activated for Ellen because she demands attention to this idea when she tells her two classmates to write down this answer. When Larry counts to 35 on his 7-block high tower and tells Ellen she was incorrect, she does not seem upset. This may be because she already had changed her mind as demonstrated in line # (2:12:55).			
95		*Larry gets up to pick up a paper that drops several times. The girls do not really speak while they wait for him. Janelle continues to take cubes out of the bag, but does not do anything else with them. The students are off-task for a short while until Ellen brings them back to the task again. Larry suggests that they should wait for Ms. S but Ellen says they do not need her assistance.*	
96	E	Alright all we're doing is adding by 5s right	**LHSIA (14)**

<table>
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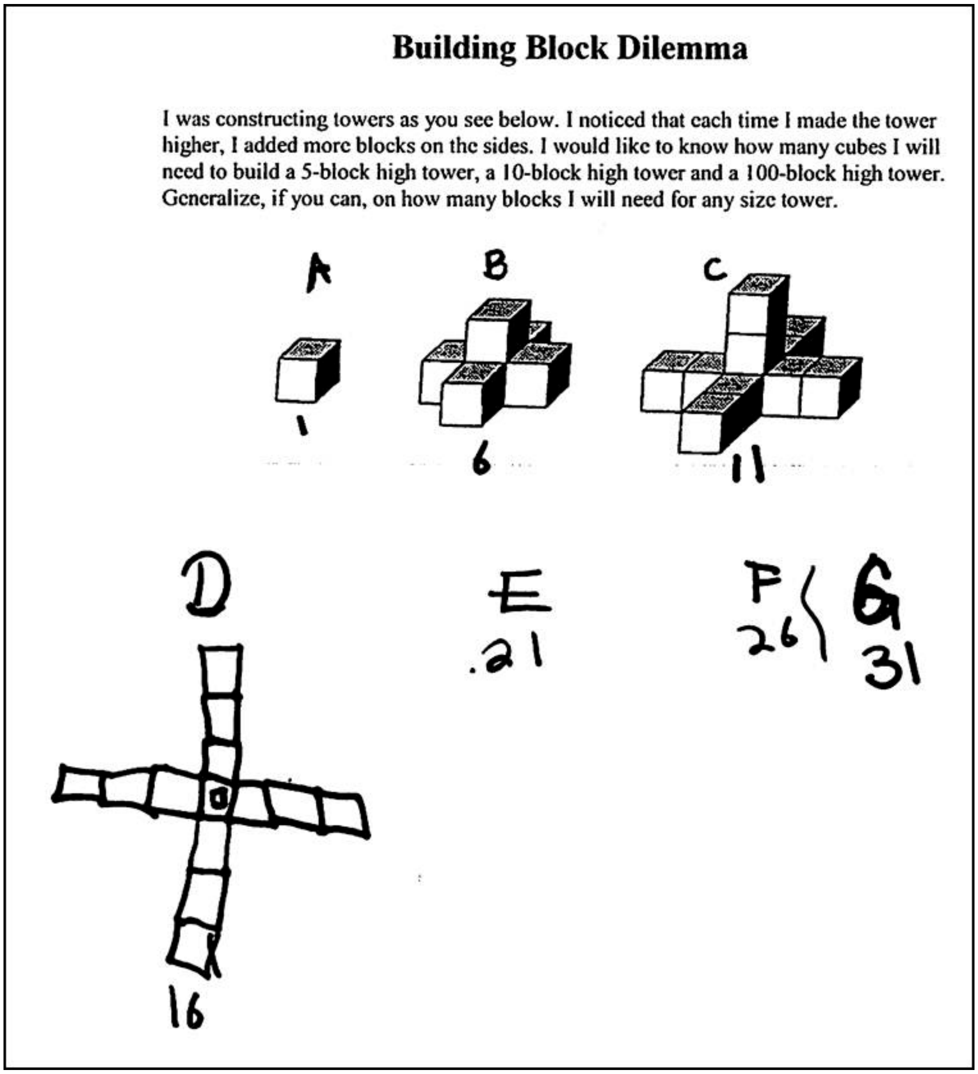
<tr><td></td><td></td><td>so G must be 31.</td><td>Ellen is stating an answer – the total number of blocks in the tower for phase G.</td></tr>
<tr><td colspan="4">Interpretation: Ellen not only has gotten the group to pay attention to the task again but also states her answer for the tower represented by phase G (the 7-block high tower). Ellen appears to have an active LHSIA structure here, as she again demands the attention of her classmates to this answer. In addition, she gives a reason for her answer, “adding by fives.” Ellen continues to assert her answer in her next speaking turn, and Larry agrees.</td></tr>
<tr><td>97</td><td>L</td><td>But we already got 5, the 5-block high tower so now we gotta mark it.</td><td></td></tr>
<tr><td>98</td><td>E</td><td>I’m sure, it’s pretty much 31.</td><td></td></tr>
<tr><td>99</td><td>L</td><td>Alright, I know that’s G is 31 but we gotta wait.</td><td></td></tr>
<tr><td>100</td><td>E</td><td>Wait for what?</td><td></td></tr>
<tr><td>101</td><td>E</td><td>Alright, I’m doing G. I’m sorry. (starts adding blocks to the tower structure constructed in front of her.</td><td></td></tr>
<tr><td>102</td><td>L</td><td>But we gotta wait. We need Ms. A.</td><td></td></tr>
<tr><td>103</td><td>E</td><td>No we don’t we can do the work.</td><td>LHSIA (5)
Ellen state that they can do the work, or figure out the answer, without the help of Ms. A.</td></tr>
<tr><td colspan="4">Interpretation: Ellen has repeatedly stated that they do not need Ms. A in order to continue finding the solutions as instructed by the task. By stating, “We can do the work,” she seems to be saying that the three can work to figure out the answers. I infer that Ellen has an active LHSIA structure because of her words and apparent confidence in their mathematical ability.</td></tr>
<tr><td>104</td><td>J</td><td>(counting the total number of blocks in the 7-block high tower that she has constructed)
I got 30.</td><td></td></tr>
<tr><td>105</td><td>E</td><td>Alright, so it’s 31.</td><td></td></tr>
<tr><td>106</td><td>J</td><td>(after counting again)
30
(puts her head in her hands and frowns; perhaps she is frustrated that she is getting a different answer than her classmates for the 7-</td><td></td></tr>
</table>

<table>
<tr><th colspan="4">Ellen, Larry, Janelle</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td></td><td></td><td>block high tower)</td><td></td></tr>
<tr><td>107</td><td>E</td><td>1 plus 5 is 6</td><td></td></tr>
<tr><td>108</td><td>L</td><td>Let me see yours (takes Janelle’s tower structure and starts counting the total number of blocks)</td><td></td></tr>
<tr><td>109</td><td>E</td><td>6 plus 5 is 11, 11 plus 5 is 16, 16, 21, 26, 31.

(The students look around the room at their other classmates and are off-task for about a minute. Larry is holding up his hand, but no teachers come to address his question.)</td><td>LMTY (1)
Ellen uses the pattern the group discussed earlier – “adding by 5s” – to explain why the tower represented in phase G/the 7-block high tower has a total of 31 blocks.</td></tr>
<tr><td colspan="4">Interpretation: Ellen seems to insist that there is a total of 31 blocks in the 7-block high tower (though they are not referring to the heights of the various towers), though Janelle keeps counting 30 blocks instead of 31. Ellen explains that 31 is the correct answer by using the pattern of increasing the total by 5 each time. She starts off explaining in detail, “one plus five is 6,”… and then just counts by five, “21, 26, 31.” I infer that Ellen has an active LMTY structure because she is trying to explain the strategy to Janelle. Janelle’s question below, “How come I got 30?” indicates that she may not have been convinced by Ellen’s explanation.</td></tr>
<tr><td>110</td><td>E</td><td>(again counting the tower structure in front of her)
I got 29! Oh yeah 30. (as she realizes she is holding a cube that needs to be added to the height)</td><td></td></tr>
<tr><td>111</td><td>L</td><td>Alright G is what, 31?</td><td></td></tr>
<tr><td>112</td><td>J</td><td>How come I got 30?</td><td></td></tr>
<tr><td>113</td><td>L</td><td>‘Cuz you’re wrong!</td><td>LHSIA (8)
Larry insists that Janelle’s tower is incorrect in some way.</td></tr>
<tr><td>114</td><td>J</td><td>No I’m not</td><td></td></tr>
<tr><td>115</td><td>L</td><td>Mira mira watch look.
(He switches tower structures with Janelle and they each count one another’s’ tower structures.)</td><td></td></tr>
<tr><td colspan="4">Interpretation: Larry tells Janelle that her answer of 30 total blocks is incorrect,</td></tr>
</table>

Ellen, Larry, Janelle			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
essentially trying to correct her. I infer an active LHSIA structure because he continues to state that his answer, 31 blocks is correct. Janelle responds by disagreeing, "No I'm not." Larry then assertively takes her tower and gives her his tower so they can count the total number of blocks for each tower, presumably to see if they arrive at the same results.			
116	E	Never mind I got all of mine	
117	J	You got 7 [blocks] going up	
118	E	So do I	
119	J	(Larry points to the bottom block of Janelle's tower) Gimme! (Larry gives Janelle her structure and takes back his own structure)	
120	J	I'm staying at 26. (removes some cubes from her tower structure)	
121	E	I'm gonna check myself. Alright I got 31.	
122	J	I'm-a go back all the way to 1. (She takes apart her tower structure and builds the structures depicted in figures A, B, and C. She counts the total number of blocks for each as she continues along.)	
123	Ms. A	Ok class another couple of minutes and we're going to wrap it up so wrap whatever you have…	
124	E	Come on, let's just build it real quick.	
125	L	We need help	
126	E	We don't need help. Come on just 2 on each side.	
127	E	If we do 2 on each side look 2,4,6,8	
128	L	You got mad different colors lady damn you took my red yo.	
129	Sts	(speaking in Spanish)	
130	J	I keep on getting different numbers now.	
131	E	What if we keep on going this way?	
132	L	No we can't do that. (It is not clear what they "can't do" to the observer)	
133	E	Yes we can. Anything is possible.	
134	Ms. A	I want you to stop now. (Ms. A continues to give instructions on packing up their work and preparing the students to take the questionnaire)	

Ellen's work, Nov. 2008



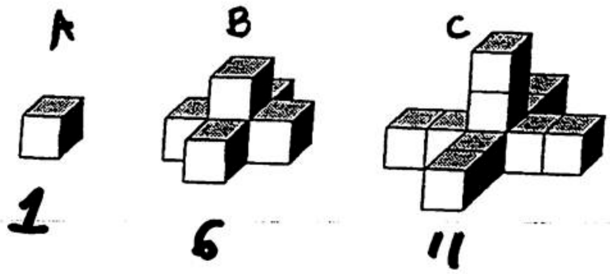
Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.

Larry’s work, Nov. 2008

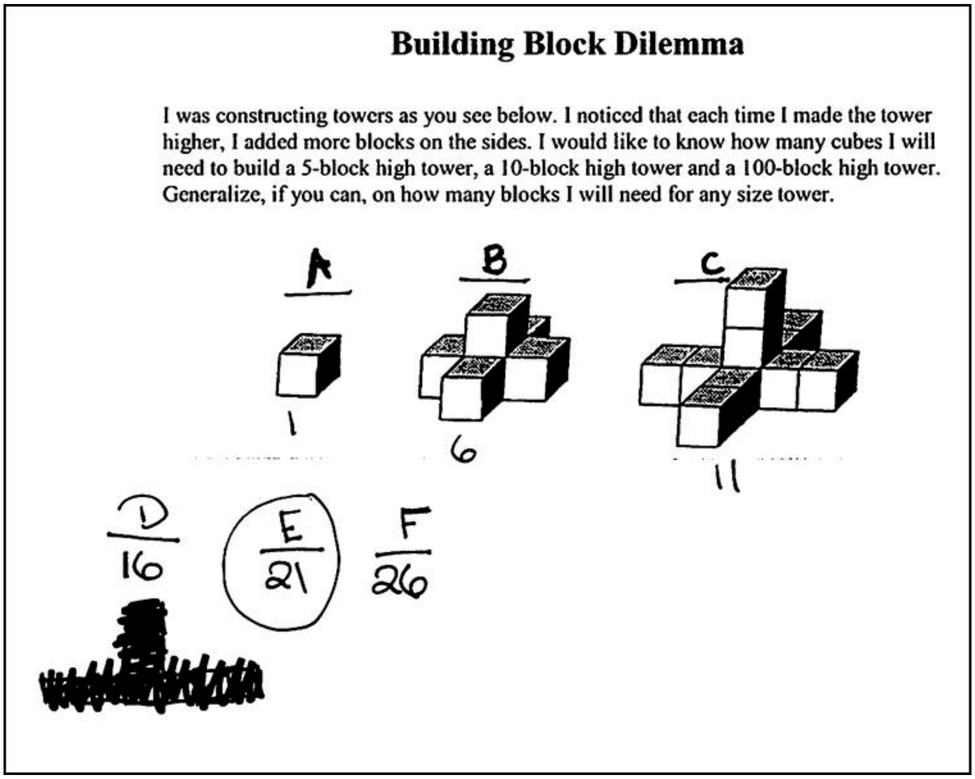
Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



D.16 E.21 F.26 g.31

Janelle's work, Nov. 2008



Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.

Questionnaire Responses for Ellen, Larry, Janelle

Questionnaire Items Which May Indicate LMTY Structure

Questionnaire Items	Ellen	Larry	Janelle
Statements (All the time, Sometimes, Never)			
I wanted to teach another student something that I knew that the other student did not know.	Never	Sometimes	Never
I listened carefully to the ideas of someone I was trying to help.	All the time	Sometimes	Never
I helped someone see how to do the math.	Sometimes	Sometimes	Never
Others listened carefully to my ideas.	Sometimes	Never	Never
Behaviors (Often, Sometimes, Hardly ever)			
I gave helpful suggestions.	Sometimes	Often	Hardly ever
I worked cooperatively.	Often	Hardly ever	Sometimes
Thoughts (Yes/No)			
I like teaching this person things that I know.	Yes	No	No

Questionnaire Responses for Ellen, Larry, Janelle

Questionnaire Items Which May Indicate LHSIA structure

Questionnaire Items – Statements	**Ellen**	**Larry**	**Janelle**
Statements (All the time, Sometimes, Never)			
I wanted people to think that I'm smart.	Never	All the time	Sometimes
I tried to impress people with my ideas about the problem.	Sometimes	Sometimes	Never
People seemed impressed with the ideas I shared about the problem.	Sometimes	Sometimes	Never
People saw how good I am at the math we did today.	Sometimes	Sometimes	Never
I felt smart.	All the time	Sometimes	Sometimes
I wanted to show someone that my way was better.	Never	Never	Never
I was a lot better at math than others today.	Sometimes	Sometimes	Never
Behaviors (Often, Sometimes, Hardly ever)			
I was the leader.	Sometimes	Sometimes	Hardly ever
I was bossy.	Hardly ever	Hardly ever	Hardly ever
I wanted to show off.	Hardly ever	Hardly ever	Hardly ever
I liked to be right.	Often	Sometimes	Sometimes
Thoughts (Yes/No)			
I want you to know just how smart I am.	Yes	No	Yes
People think I'm smart.	Yes	No	No
I wish the teacher would call on me, so I can show how much I know.	Yes*	Yes	No

*Student wrote in response: Sometimes

Ms. S Class 1 Introduction to Building Blocks Task

This is the introduction Ms. S. gave to her grade eight Class 1.

T: Ms.S / Teacher
St(s): Unknown Student(s)
L: Leticia
M: Manny

T: All right guys, so, everybody should have paper in front of them. What we're going to ask that people to do is write with, well I'm sure that they told you this in your individual groups, but just so that we're all on the same page, please write with the markers, because sometimes with pencils you write very light, or it smudges, and it gets messed up or whatever the case may be. So, write with the permanent markers that they provided for you and I would suggest using the skinny size of the permanent markers because that will be closest to mimicking your pencil. On top of that, please make sure that on any sheet of paper that you work on during this period, you write your name, the date, your homeroom, which is me, [number given], Ms. S., and um, I believe that is it. So make sure you put that on all of your papers. By all your papers, I do mean every single piece of paper you work on so if you start an idea, you realize it's not the idea you want, and your intentions is to crumple up the paper, please do not do that. Keep that paper there, you can cross it out or do whatever you want, and then move forward from there. Um, so, we're going to be working with kind of exploring this problem you have in front of you which is called the building block dilemma it shouldn't seem too foreign because it does fit right in with what we've been doing here, um, what we've been doing during these past couple of days. So, I need someone to read the problem for me.

L: Building Black Dilemma. I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks to the sides. I would like to know how many blocks, how many cubes you need to build a 5 block high tower, a 10 block high tower, and a 100 block high tower. Generalize if you can on how many blocks I would need on any size tower.

T: Okay, so the first thing that I'm going to ask you as guys kind of a as whole group, is after reading that problem, someone in your own words just kind of think about and tell me what it is the problem is asking, what it is you're looking for, what are you trying to figure out?

St: How many blocks you need for any sized tower.

T: For any sized tower. Do they give you any specifics?

St: No…yeah

T: Do they give you any examples of some they did? That they want you to figure out?

Sts: 5, 10 and 100.

T: So you said any block tower. How do I know what that means? Like, what does a 2 block tower look like? How do I know what that means? Does it only have 2 blocks in it?

M: It’s 2 blocks high

T: It’s 2 blocks high. Okay so, one of the things I want you to kind of spend the first couple of minutes kind of discussing in your group before you move forward, is, and I’m not saying that was the right assessment. I want you to discuss in your groups and come up with a consensus on what the problem is asking. You need to figure out, they give you a picture. How is that picture useful? Why is it there in front of you? Does it…What does it do to help you solve the problem? And from there, you can then move forward. Another key point in the problem is where it says, “Generalize if you can” on how many blocks you need for any size tower. By generalize it means you need to come up in your group with what that means. So does it mean to generalize something, how can you represent that. So the key I said a little earlier that we came up with earlier was to finding it for any size tower, what would you do to represent that on your paper, how could you do that or some other way? If you can’t do that on your paper, maybe you can do it physically. You are allowed you use any of the materials in the room, which means if you want to use a calculator, feel free. If you’d like to use blocks you’ve got a great assortment of them here for you to use. You can use connectables or not, if you want to use tape, there’s tape, if you want graph paper, you know where graph paper is, get up and get it. Uh. . . But nobody should be really going for materials in the first five minutes because in the first 5 minutes, you’re scoring through the problem, figuring out the information you have in front of you, and really figuring out what that information is asking you. Okay? So… And if you’re in your group and you’re not convinced of what someone is saying or what they’re saying, do not just go, “Uh huh, uh huh, uh huh,” because you think they have an answer and you don’t have any answer, if it doesn’t make sense to you, question them on it so that they’re forced to explain it to you. If their answer is right or its logical or its fake or something, then at some point it should start to make sense to you. They should be able to justify it. Does that make sense? Okay!

Deanna, Manny, Damon

This is Group 1 from Ms. S's grade eight Class 1.

Verbal emphasis indicated by underline
Actions included either in *italics* or (parentheses) as appropriate.
Pauses in speech are indicated by …
Words inserted to help clarify a student's words are included in [brackets].
When students voices overlap, one or both sets of words are included in //double slash marks//.

The number included next to either LMTY or LHSIA refers to the code, included in the table of codes.

<table>
<tr><th colspan="4">Deanna, Manny, Damon</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td></td><td colspan="2">The teacher has just finished her introduction to the problem and given instructions. This has taken a little more than three minutes. The three students are sitting at desks which are situated to all face one another.
When the teacher finishes, the students all lean in toward each other.</td><td></td></tr>
<tr><td>1</td><td>Damon (D)</td><td>I think it mean … I think it mean that…
I think it means that how many more blocks do we need for 5. I think we need four more blocks.
(As he says this he turns to Manny for verification – he seems to have excluded Deanna.)</td><td>EPISODE 1
LHSIA (14; 15): Damon, first student sharing his idea and interpretation of the problem, what the task is asking for and how to get the answer.</td></tr>
<tr><td colspan="4">Interpretation: It seems Damon is trying to impress others with his idea. His tone of voice is matter-of-fact and calm. He points to his paper with his pen as he speaks and often looks up at Manny. By saying "I think it means…" and looking at Manny, he appears to want Manny to agree with his interpretation of the problem.</td></tr>
<tr><td>2</td><td>Manny (M)</td><td>(responds to Damon)
No, no, no, it says how many for a hundred blocks.
(slight emphasis on "a hundred")
Manny is looking at his paper referencing what it says there.
Deanna is sitting quietly listening and looking at the two boys as they discuss this.</td><td></td></tr>
<tr><td>3</td><td>M</td><td>(looks at Damon) It says for 100 blocks.</td><td></td></tr>
</table>

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<tr><td>4</td><td>D</td><td>(to Manny) How many for 100 blocks?</td><td></td></tr>
<tr><td>5</td><td>M</td><td>Yeah. So when it’s 100 blocks high how many are you going to have?</td><td></td></tr>
<tr><td>6</td><td>D</td><td>What you mean?</td><td></td></tr>
<tr><td>7</td><td>M</td><td>How many blocks is going to be needed to make the 100?</td><td></td></tr>
<tr><td>8</td><td>Deanna (A)</td><td>Oh.
(looks back at her paper)</td><td></td></tr>
<tr><td>9</td><td>D</td><td>How you know that one? How you know that?</td><td></td></tr>
<tr><td>10</td><td>M</td><td>I went like, wait, let me see.
(looks back at his paper.)</td><td></td></tr>
<tr><td>11</td><td>D</td><td>I’m not convinced by your statement.</td><td rowspan="2">LHSIA(14): Damon states that he is “not convinced” by Manny’s different interpretation of the problem. He is also laughing.</td></tr>
<tr><td></td><td>D</td><td>Laughs now that he has expressed disagreement with Manny’s statement. Manny is also laughing and Deanna is smiling.</td></tr>
<tr><td colspan="4">Interpretation: It seems Damon is continuing to impress others by suggesting that he is “not convinced by [Manny’s] statement.” Manny had a different interpretation of the problem than Damon and perhaps Damon is convinced his interpretation is correct, not Manny’s. It is possible that Damon’s laughter is meant to break any tension that may exist, which might be considered successful since Manny is also laughing and Deanna is smiling.
Alternate interpretation: Damon’s and Manny’s laughter may contribute to an interpretation other than LHSIA. Damon may not understand Manny’s interpretation, as Manny did not give an explanation. Therefore, Damon may be making a joke to cover up his confusion.
Alt 2. Damon repeated words that Ms. S stated in her introduction (making sure that you are convinced by others ideas). He may be joking by repeating her statements.</td></tr>
<tr><td>12</td><td>D</td><td>(to Manny)
What you say? Huh?
Manny is looking at his paper again, possibly re-reading the problem.</td><td>EPISODE 2
Damon asks Manny for more information.</td></tr>
<tr><td>13</td><td>M</td><td>These are two blocks high.
(points with his pen to the blocks depicted in figure B on the paper)</td><td>LMTY(1): Manny is pointing to the given diagrams to clarify that tower B is the 2-block high tower.</td></tr>
<tr><td colspan="4">Interpretation: Manny seems to be responding to Damon’s repeated questions to explain his ideas.</td></tr>
</table>

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<tr><td>14</td><td>D</td><td>Two block high. Yeah.
(still looking at Manny and Manny’s paper)</td><td>LHSIA(9;12): Damon is agreeing with Manny, repeating what Manny just said.</td></tr>
<tr><td colspan="4">Interpretation: Damon says, “Yeah,” agreeing with Manny as though he had the same understanding of the problem as Manny. If he has the same idea, perhaps he believes Manny and perhaps Deanna will see him as “smart.” Damon is trying to demonstrate both his mathematical competence and academic/social status.</td></tr>
<tr><td>15</td><td>D</td><td>(looks down at his paper briefly before looking up at Manny)
B, so B is 2 blocks high. So C is a 3 block high. Write it down?</td><td></td></tr>
<tr><td>16</td><td>M</td><td>(looking down at his paper)
When it goes up one, it adds one on each side.
(point to the diagram on his paper with his pen as he speaks; Manny then looks back up at Damon; Deanna is looking at Manny while he talks)</td><td>LMTY(1): Manny is explaining the construction of the towers, how blocks are added to build the next tower.</td></tr>
<tr><td>17</td><td>D</td><td>Yeah.</td><td></td></tr>
<tr><td>18</td><td>M</td><td>(to Damon)
So if its 30 then you add 1, that’s 4 (showing 4 fingers), that’s 5 (showing 5 fingers) more.</td><td>LMTY(3): Manny is providing specific example, about how to construct each successive tower, adding 5 blocks.</td></tr>
<tr><td colspan="4">Interpretation: Manny begins to explain his understanding of how the towers are constructed. He appears to be thinking out loud and sharing his ideas to his two classmates. He gives an example to support his explanation about adding 5 blocks to obtain the next tower.</td></tr>
<tr><td>19</td><td>D</td><td>So for a 100 it would be 99 on each side?
(Damon points and gestures toward Manny’s paper)</td><td>LHSIA(10; 15): Damon is sharing a new idea, building on Manny’s idea. He says this with a question in his voice.</td></tr>
<tr><td>20</td><td>M</td><td>(to Damon)
Wait up. Wait up.
(says quickly)</td><td></td></tr>
</table>

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<tr><td>21</td><td>D</td><td>For 100, there would be 99 on each side!
(Damon says this emphatically)</td><td>LHSIA(2; 14): Damon is repeating his statement, with emphasis.</td></tr>
<tr><td colspan="4">Interpretation: For the past two utterances, Damon has suggested that for a 100-block high tower, there are 99 blocks on each side, a mathematically correct idea. He seems to be building on Manny's idea where he demonstrates that 5 blocks are added each time. Perhaps Damon recognized the relationship between the height and the length of each side based on Manny's explanation. The first time Damon says this, he has a question in his voice, suggesting that he is looking for agreement and wanting recognition for his idea. When Manny does not immediately agree ("Wait up"), he repeats his idea (same exact words) more forcefully, using a more insistent tone of voice. Perhaps he thinks this will get his point across better. One difficulty for Damon is that he has reached an idea that the group is not prepared to grasp. Manny is still looking at the three diagrams in front of him and, in the next line, Deanna admits she does not understand what the two boys have said so far.</td></tr>
<tr><td>22</td><td>A</td><td>I don't get it.</td><td>EPISODE 3
Deanna states that she is confused.</td></tr>
<tr><td>23</td><td>D</td><td>You don't get what I'm saying?
(Deanna shakes her head indicating, "no" and is smiling)
Damon points to Manny's diagram trying to clarify his analysis.</td><td></td></tr>
<tr><td>24</td><td>M</td><td>Wait up, Wait up. (says quickly)</td><td></td></tr>
<tr><td>25</td><td>D</td><td>(using the tip of his pen to point to the diagrams on Manny's paper)
If you're saying that if it's two blocks high and one on each side every time I go up one this will go up one. So that's what I'm saying it's going to be. 100 on 99. A hundred.</td><td>LMTY(1): Damon is quickly explaining how each tower is built from the previous one.
LHSIA(14): Damon is referring to his idea from earlier (99 blocks on each side when the height is 100)</td></tr>
<tr><td colspan="4">Interpretation: Deanna indicates she is confused. Damon, who has likely been trying to impress his classmates with his mathematical ability, attempts to explain his idea again. He appears to want to teach or help Deanna by giving this explanation. He seems to be trying to make a connection between putting 5 blocks on a structure in order to build the next tower and his thinking of the 100-block high tower (which has 99 blocks on each of the sides or legs). By referring to his earlier idea, I believe he is still trying to</td></tr>
</table>

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<tr><td colspan="4">demonstrate his mathematical ability and demand his classmates' attention while also trying to explain this idea to Deanna.</td></tr>
<tr><td>26</td><td>M</td><td>So it's like when you add 1 you add 4.</td><td></td></tr>
<tr><td>27</td><td>D</td><td>Yeah.</td><td></td></tr>
<tr><td>28</td><td>M</td><td>So you add 5 at a time.
(looks at Deanna, including her in the conversation)</td><td></td></tr>
<tr><td>29</td><td>D</td><td>Yeah.</td><td></td></tr>
<tr><td>30</td><td>M</td><td>Alright. So let's do that.
One more is 5 more, which is…
(looks over to Deanna while he speaks; she smiles and shakes her head, indicating that she does not understand their strategy)
Deanna doesn't understand.</td><td></td></tr>
<tr><td colspan="4">Interpretation: Manny appears to have followed Damon's explanation and provides his own interpretation, "when you add 1 you add 4… you add 5 [blocks] at a time." By giving this interpretation, Manny could be trying to clarify Damon's explanation for Deanna. When he checks whether she is following, though, she admits she still does not understand how the towers are being built.</td></tr>
<tr><td>31</td><td>D</td><td>She knows what I'm saying.</td><td></td></tr>
<tr><td>32</td><td>M</td><td>(Manny reaches over to Deanna's desk and moves Deanna's paper closer to him. He uses his pen to point to her paper; she looks at her paper, presumably at the part he is pointing to while he speaks)
Okay, one block you got zero (pointing to figure A).
Then he added one, he added one to each side of it (pointing to figure B).</td><td>LMTY(1; 2): Manny explains and expands on his explanation to Deanna why you add 5 blocks to each tower. He often points to the diagrams on the paper to help demonstrate his point.</td></tr>
<tr><td>33</td><td>D</td><td>Yeah.</td><td></td></tr>
<tr><td>34</td><td>M</td><td>Then he added one more, one more to each side
(pointing to figure C; Deanna nods her head slightly).</td><td></td></tr>
<tr><td>35</td><td>D</td><td>Every time you add one to the to //the height.//
(Damon points to Deanna's paper, along with Manny)</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>36</td><td>M</td><td>//Every time// you make it one higher it adds 5 blocks.</td><td></td></tr>
</table>

Deanna, Manny, Damon			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
Interpretation: Deanna prompts Manny into LMTY by expression her confusion. Manny begins his explanation by describing the three towers given on the task paper. He is explaining how each tower is constructed, building on the previous tower. He also explains that 5 blocks are added to one tower to get the next tower. Manny is patiently explaining this to Deanna, referring to and pointing to the diagrams on the task paper.			
37	D	Yeah.	**LHSIA(9; 12)**: Damon agrees with his classmate, indicating his own understanding.
38	M	One, two, three, four, and five. (Manny pointing to Deanna's paper as he counts, to make his point)	
39	A	Oh. (smiling; brings her paper closer to her, as if she is going to write something. She doesn't.)	
40	D	Get it? (He says this emphatically.)	**LHSIA(13)**: Damon is suggesting that Deanna should understand this.
41	A	Yes!!! (she says with an exasperated tone of voice) (Both boys retreat off her desk back to their own spaces)	
Interpretation: Manny continues to help Deanna understand the initial construction of the blocks, by counting as he points to her paper where the 5 added blocks are. Meanwhile, Damon continues to interject with, "Yeah," suggesting that he agrees with Manny's explanations and that he wants to demonstrate that he already understands the problem. As Deanna indicates that she understands what Manny has been explaining, Damon says, "Get it?" in a way that could be taken as rude. He has raised his voice and says this with emphasis, possibly suggesting that she should understand this by now. Though Deanna says that she understands, she has not provided any other evidence that she does indeed understand how the towers are constructed at this time. Perhaps, because of the way Damon questioned her, she would not have said she was still confused. In addition, Damon may have wanted to make sure that Deanna understood the problem thus far so that they could move on, or Get the Job Done.			
		The boys turn their attention to Nikki's table where Ms. S is working with the students. They seem interested in observing how she interacts with those students.	
42	M	(They sit quietly for a few seconds and then Manny starts to review where they stand.)	**EPISODE 4** After a brief

Deanna, Manny, Damon			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
		All right now… So we add 5 every time we go up one.	distraction, the group returns to the problem.
43	D	So we but we're trying this one. (points to figure A on Manny's paper, asking which diagram Manny is referring to)	
44	M	Wait, wait.	
45	D	That one we don't know what we're doing right there.	
46	M	OK. This plus 5 equals… This one has 6. This one has… (moving his pen to the next tower)	**LMTY(1):** Manny is explaining that tower A has one block and by adding 5 blocks, tower B has 6 blocks.
47	D	Yeah. How you get 6 for this one? (points to tower B on Manny's paper)	
48	M	(counts out loud, using his pen to assist him, as he points to tower C on his paper) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11. Yeah, so you keep adding 5. So it's five.	Manny continues to count in order to get the total number of blocks in tower C.
49	D	This 11 blocks, it's 11? (referring to tower C)	
50	M	Yeah. 1 to the arm in there equal 5, 6, 7, 8, 9, 10, wait, 1, 2, 3, 4 (Both Deanna and Damon point to the diagram on Manny's paper, to assist.)	
Interpretation: Damon states that "We don't know what we're doing," possibly meaning that he does not know what the next step is. Manny responds by explaining that he has added 5 blocks (from the explanation earlier) to tower A to get 6 blocks in tower B. He continues to count and respond to Damon. He confirms for himself that 5 blocks are added each time by counting the number of blocks in tower C. He notes that the 5 blocks have been added to tower B in order to get 11 blocks in tower C.			
51	D	It's 6. It's 8 altogether on the outside. It's 11. (As Damon makes this comment, he is pointing to a place on m's page and Deanna is also pointing there at the same time.)	
52	M	It's 11.	
53	D	It's 11. It's 11 blocks	
54	M	This one got six.	
55	D	This one got 1.	
56	M	Alright so five block would be, this plus 10.	

Deanna, Manny, Damon			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
57	D	That plus 10. So that'd be 21?	
58	M	Yeah.	
59	D	21 blocks?	
60	M	Cause look. If you go to 4 you have 5 more that's…	**LMTY(1)**: Manny is answering specific questions, explaining that 5 blocks is added to create the next tower. He does this to explain how he determined that the 5-block high tower has 21 blocks.
Interpretation: For a few exchanges, Damon and Manny are discussing their answers and their strategies, somewhat. (Examples: Damon, "It's 6. It's 8 altogether on the outside. It's 11." Manny, "So five block would be this plus 10.") When Damon questions the answer 21, Manny is again explaining that five blocks are added to each tower, suggesting the move from the 4-block high tower to the 5-block high tower.			
61	D	Yeah 21. Yeah. (all three students start to write on their papers) So 5, so the 5 block right? Which one are you doing?	
62	M	5 block.	
63	D	That's the C though (pointing on Manny's paper to tower C). This one is 5 blocks?	
64	M	No. When you've got 5 blocks high it's um, you got 20 blocks 21 blocks in all.	
65	D	When you've got 5 blocks high is 21. (with a doubtful tone of voice, indicating he does not understand)	
66	M	Five blocks high, Cause look right now it's 3. (referring to the height of tower C) When you add 1 you add 4 more. So that's 5. Add 1 more. (asking Damon to do this.) 4	
67	D	4 and then that's	
68	M	That adds 5 so that's 16. Then 5 more. 21. (They all start reflecting on this and writing things on their papers.)	**LMTY(2; 3)**: Manny elaborates on previous answer, explaining that 5 blocks are added to get the next tower.

Deanna, Manny, Damon			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
Interpretation: Damon expresses confusion. He seems to indicate he thought the 5-block high tower was represented by tower C in the figure given on the task paper, though that is really the 3-block high tower. He expressed doubt about the 5-block high tower having 21 blocks, suggesting that he either does not understand why or that he might not agree with that. Manny continues to explain that tower C is the 3-block high tower and that when 5 blocks are added, he has the 4-block high tower. Again, 5 blocks can be added to the 4-block high tower to get the 5-block high tower which has 21 blocks in total. (Note that since they do not have blocks, they appear to just be thinking about it. None of these three have drawn any representations or created any tables as part of their work yet.)			
69	D	So a five block.	
70	M	No, that's not the question. You've got to answer the ones up there. (M points to the place on the paper where the questions that need to be answered are located.)	Get the Job Done: Manny reminds Damon of the original instructions, which include which questions to answer.
71	D	For a 5 block high tower. (All three students resume writing on their papers, presumably writing the answer to the question.)	
72	D	A five block high tower would need 21 blocks.	
73	D	(points to Manny's paper) Now do you know? Are these all the same ones? It's just increasing? (pointing to the figures on Manny's paper)	
74	M	Yeah, look.	
75	D	Oh.	
76	M	This one is…	
77	D	Is that increasing? That increased by 5	
78	M	But then we added 1 and we added the ones around. (using his pen to point to tower B in the figure on his paper)	
79	D	Yeah.	
80	M	Right here he added another one and the ones around. (using his pen to point to tower C on his paper)	
81	D	Yeah.	**LHSIA(9; 12)**: Damon switches from asking

Deanna, Manny, Damon			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
			questions to saying, "Yeah," in response to Manny's explanations.
Interpretation: Damon asks Manny how he knows that the 5-block high tower has 21 blocks, indicating he does not understand or does not see the pattern yet. His questions suggest he possibly did not realize that the number of blocks added each time is the same, 5 blocks. Manny then continues to explain, though he seems to be saying the same thing he said earlier. When one block is added (presumably to the height), then 4 more blocks are added around the sides. He points to the drawn figures on his paper to help support his explanation.			
82	A	Oh. I got that one. I got that one. (She is smiling.)	**LHSIA(4; 9)**: Deanna is verbalizing that she understood the answer Manny was giving to Damon.
83		*The group continues working quietly reviewing what they need to write down to get the job done.*	
84	M	Now let's figure out 100.	
85	D	We need to do 2. And let's do 10.	GTJD: Damon appears to be more interested in answering each part of the problem before jumping ahead.
	For 2 1/2 minutes: *Manny suggests they get an equation. Damon and Deanna review the information they have just been discussing. There is no evidence that any one student has an active LMTY or LHSIA structure during this time. Someone suggests using the letter* n *in their equation. The group continues to share the answers to the total number of blocks. Damon suggests that the 10-block high tower should have 42 blocks, double the total number of blocks for the 5-block high tower. Manny gets an answer of 46 blocks. Damon asks how he gets that, and Manny responds with his strategy from earlier, "Keep adding 5 [blocks]."*		
	For 5 minutes: *The students work on their own papers for a little bit.*		

Deanna, Manny, Damon			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
	Damon suggests they might need the linking cubes, but Manny says they already have the answer. The group discusses how to get the equation. They also discuss creating a table on their papers, and Damon asks a lot of questions about how to set up the table.		
86	D	What's the variable? Height of tower?	**EPISODE 5** In this episode, the students are each creating a table on their papers, to demonstrate the relationship between the height of the tower and the total number of blocks.
87	M	The number of blocks	**LMTY(7)**: Manny is responding to a specific question about the table.
88	D	The number of blocks?	
89	A	The number of blocks needed.	
90	M	Start off with 0, 0.	
91	D	0, 0. So what's the first one?	
92	M	Then for 1 it's 1	
93	D	How you know for 1 it's 1?	
94	M	Because it's right there. (pointing to the diagram of tower A on his paper). There's a one and it's one.	**LMTY(7)**: Manny is responding to specific question, using the diagram on the task paper to give the answer. Damon has asked how he knows to write 1 in both the "Height" column and "Number of blocks needed" column.
95	D	One. (writing the number down on his paper)	
96	A	And with the other one.	
97	D	If it's 2, for two of these… (pointing to tower B on Manny's paper)	

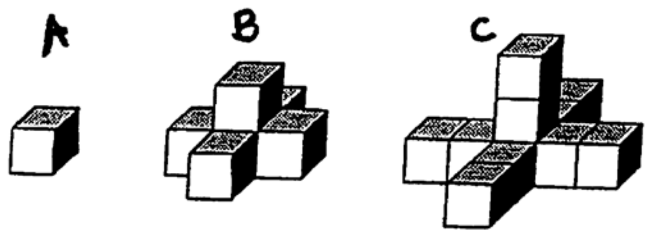
Deanna, Manny, Damon			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
98	M	Two is 6.	
99	D	Yup! For two is 6. For 3 is… (stops writing and looks over at Manny)	
100	M	11.	
101	D	How you know it's 11?	
102	M	It's this one. (pointing to the diagram of tower C on his paper.) It's this one right here. (Manny then points to where it is on Damon's paper.) *Students resume writing on their own papers quietly.*	**LMTY(7)**: Manny is responding to a specific question, using the diagram on the task paper to give the answer. Damon has been asking for the total number of blocks for the first three tower heights.
103	M	How much is 4?	
Interpretation: As the students are creating and filling in their table, Damon continues to ask specific questions so that someone, namely Manny, will give him the answers. For example, at the beginning of this episode, Damon is asking how they should label their table. This seems to prompt Manny into an active LMTY structure again as he explains that they need to record the height of the tower and the corresponding total number of blocks needed for that tower. This pattern is repeated for several speaking turns.			
104	D	Look, it's just going up 5! (Damon sounds excited)	Possible a-ha moment for Damon.
105	M	I know. (says with matter-of-fact tone of voice)	
106	D	(inaudible) (Manny looks over at Damon and pauses for a moment)	
107	D	So what would be 4?	
108	M	4 is 16.	
109	D	And then 5 will be like 21. Because we did that one.	
110	D	(inaudible) on that for 10?	
111	M	Yeah	
112	D	Six would be	
113	M	26	
114	D	7 would be 31, right	
115	M	Yeah	
116	D	What would 8 be?	
117	M	36	
118	D	Oh yeah?	

Deanna, Manny, Damon			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
119	D	And I think 9 would be 41, right?	**LHSIA(15)**: Damon is checking with Manny that he has the correct answer.
120	M	Yeah.	
121	D	And 10 would be 46.	
122	M	Yeah.	
123	D	We're done, right?	
124	M	Yeah.	
Interpretation: Since the possible "a-ha" moment, Damon continues to ask Manny for the answers, specifically the total number of blocks needed for various heights. As all three students record this information in the table, Damon may be trying to maintain his active LHSIA by, at times, repeating information already given or suggesting answers and checking them with Manny (e.g., "I think 9 would be 41, right?") Even though Damon seemed to see the pattern of adding 5 blocks for each successive tower, he may be checking his answers with Manny out of habit. Another possibility is that Damon would prefer that Manny give him answers, rather than suggest an incorrect answer which may result in Damon no longer looking "smart." For all three students, it is possible that they each have an active Get The Job Done structure, as they are working independently, rather than continuing to discuss their answers.			
Between the end of this conversation and the end of class (about 10 minutes) the students resume a conversation trying to determine an equation to help them generalize, as stated in the instructions. Again mostly Damon and Manny are involved in this conversation, though Deanna is looking on and appears to be listening to them. They use calculators to try to help them determine an accurate equation that works for all the values in their tables.			

Deanna’s Work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



A 5-block high tower would need 21 blocks.
A 10-block high tower would need 46 blocks.

Title: Building Block Dilemma

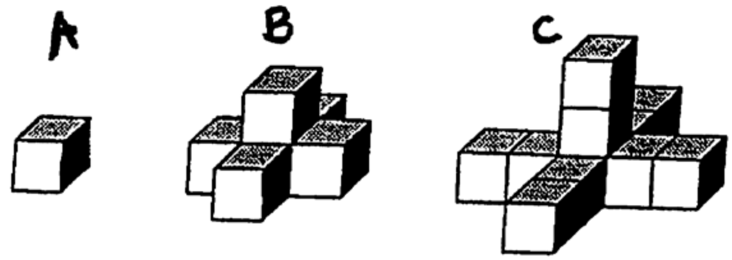
height of tower	0	1	2	3	4	5	6	7	8	9	10
# of blocks needed	0	1	6	11	16	21	26	31	36	41	46

A 100-block high tower would need 496 blocks.

Manny's Work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



a 5 block high tower would need 21 blocks
a 10 block high tower would need 46 blocks
a 100 block high Tower would need 496 blocks

equation:

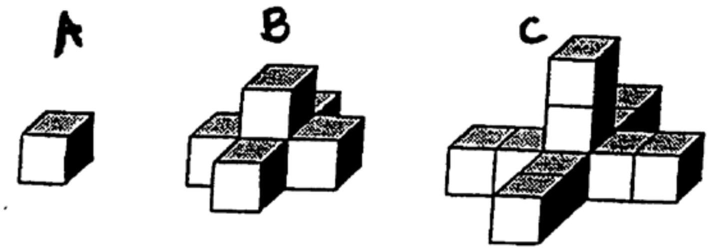
"Building block dilemma"

height of Tower	number of blocks needed
0	0
1	1
2	6
3	11
4	16
5	21
6	26
7	31
8	36
9	41
10	46

Damon's Work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



a 5 block high tower would need 21 blocks
a 10 block high tower would need 46 blocks
a 100 block high tower would need 496 blocks

equation:

Building Block Dilemma

height of Tower	# of blocks needed
0	0
1	1
2	6
3	11
4	16
5	21
6	26
7	31
8	36
9	41
10	46

Questionnaire Responses for Deanna, Manny, Damon

Questionnaire items which may indicate LMTY structure

Questionnaire Items	**Deanna**	**Manny**	**Damon**
Statements (All the time, Sometimes, Never)			
I wanted to teach another student something that I knew that the other student did not know.	Sometimes	Sometimes	All the time
I listened carefully to the ideas of someone I was trying to help.	All the time	Sometimes	All the time
I helped someone see how to do the math.	Sometimes	Sometimes	Sometimes
Others listened carefully to my ideas.	Sometimes	All the time	All the time
Behaviors (Often, Sometimes, Hardly ever)			
I gave helpful suggestions.	Often	Often	Often
I worked cooperatively.	Often	Often	Sometimes
Thoughts (Yes/No)			
I like teaching this person things that I know.	Yes	Yes	Yes

Questionnaire Responses for Deanna, Manny, Damon

Questionnaire Items Which May Indicate LHSIA structure

Questionnaire Items – Statements	Deanna	Manny	Damon
Statements (All the time, Sometimes, Never)			
I wanted people to think that I'm smart.	All the time	All the time	Sometimes
I tried to impress people with my ideas about the problem.	All the time	Sometimes	All the time
People seemed impressed with the ideas I shared about the problem.	Sometimes	Never	Sometimes
People saw how good I am at the math we did today.	All the time	Never	Sometimes
I felt smart.	All the time	All the time	All the time
I wanted to show someone that my way was better.	Never	Never	All the time
I was a lot better at math than others today.	Sometimes	Sometimes	Never
Behaviors (Often, Sometimes, Hardly ever)			
I was the leader.	Sometimes	Sometimes	Sometimes
I was bossy.	Hardly ever	Hardly ever	Hardly ever
I wanted to show off.	Hardly ever	Hardly ever	Hardly ever
I liked to be right.	Often	Sometimes	Sometimes
Thoughts (Yes/No)			
I want you to know just how smart I am.	Yes	No	Yes
People think I'm smart.	Yes	Yes	No
I wish the teacher would call on me, so I can show how much I know.	Yes	No	No

Nikki, Ricardo, Carl

This is Group 2 from Ms. S's grade eight Class 1.

Verbal emphasis indicated by <u>underline</u>
Actions included either in *italics* or (parentheses) as appropriate.
Pauses in speech are indicated by …
Words inserted to help clarify a student's words are included in [brackets].
When students voices overlap, one or both sets of words are included in //double slash marks//.

The number included next to either LMTY or LHSIA refers to the code, included in the table of codes.

Nikki, Ricardo, Carl			
Line No.	**Spkr**	**Transcript/Comments**	**Structure; Observations**
	The teacher has just finished her introduction to the problem and given instructions. This has taken a little more than three minutes. The three students are sitting at desks which are situated to all face one another.		
1	Nikki (N)	See how this one varies? And this one? (pointing to the figures on her paper)	**EPISODE 1** **LHSIA (14):** Nikki is the first to speak and start to share her ideas about the problem.
2	Ricardo (R)	This one?	
3	N	This is the one under that one. Under this one there is that one. Just add one more. (looking mostly at Carl who is sitting across from her as she speaks)	
4	Carl (C)	I know.	
5	N	You add both of them.	
6	C	You add five.	**LHSIA (9; 10):** Carl indicates he already knows the information Nikki is sharing. He builds on her idea of how to add the blocks by suggesting that five

Nikki, Ricardo, Carl			
Line No.	**Spkr**	**Transcript/Comments**	**Structure; Observations**
			blocks are added.
7	N	Yes. Because this is one, you add five that is six. Add five that's 11. Add five, and so on and so on.	**LHSIA (10; 12):** Nikki continues to build on Carl's ideas by stating the total number of blocks in the given structures A, B, and C in the task.
Interpretation: Nikki is the first to share her ideas, demanding the attention of her classmates. She may believe her initial ideas about the problem are correct. Carl responds with, "I know," suggesting that he also wants to demonstrate that he has the correct ideas about the problem. He builds on what she was saying, indicating that five blocks are added to each structure. Nikki continues to demonstrate that she understands the problem by building on what Carl said, stating the total number of blocks needed in the one-block, two-block, and three-block high tower, the first three examples given on the task paper. In addition to showing that they each have correct ideas, it appears that both Carl and Nikki want the other (and possibly Ricardo) to know that each can keep up with the other mathematically.			
8	R	(inaudible)	
9	C	You can't do that because if you do five times one for the first one. (using his pen on Ricardo's paper to explain)	**LMTY (4):** Ricardo suggests something that Carl decides is not correct. Carl says, "You can't do that," correcting Ricardo and starts to explain why his suggestion would not work.
10	N	You go to five. You could, you could. Wait, wait, wait. You could do it if you want to. (smiles as she says this)	Nikki seems to try to validate Ricardo's contribution.
	R	(smiling at what Nikki is saying)	
11	N	Five times one plus 1 is what?	Nikki tries to use Ricardo's suggestion to see if it can work, as she mentioned earlier.
12	C	Six. And the first block each time.	**LMTY (4)** **LHSIA (8):**

<table>
<tr><th colspan="4">Nikki, Ricardo, Carl</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript/Comments</th><th>Structure; Observations</th></tr>
<tr><td></td><td></td><td></td><td>Carl continues to help Nikki and Ricardo see why Ricardo’s suggestion does not work in this case.</td></tr>
<tr><td>13</td><td>N</td><td>Oh, yeah. You’re right, you’re right.
(conceding Carl’s point)</td><td></td></tr>
<tr><td>14</td><td>C</td><td>And who is the smart one in this group?</td><td>LHSIA (17):
Carl is suggesting that Nikki is “the smart one,” said with respect.</td></tr>
<tr><td colspan="4">Interpretation: Though it was not captured on the audio or video, it seems that Ricardo made a suggestion about multiplying five times a number, perhaps the height of the tower, in order to get the total number of blocks needed in the tower. Carl is the first to suggest that the idea is incorrect. He appears to want to help Ricardo understand why the idea would not work, using the first tower as an example. I believe Carl is stating that if you multiply 5 times the height, in this case 1, you would get 5, rather than 1. The total number of blocks in the one-block high tower is 1, a single block. When Nikki tries to validate Ricardo’s idea and see if it is a possibly correct idea, Carl corrects both Ricardo and Nikki by stating that 5 times 1 plus 1 is 6, which is the total number of blocks in tower B, rather than tower A. Therefore, it seems that Carl was both trying to show he was smart for recognizing the incorrect idea and trying to help his classmates to see why that idea was incorrect.</td></tr>
<tr><td>15</td><td>N</td><td>For a 10 block tower. So this one is a one.</td><td></td></tr>
<tr><td>16</td><td>R</td><td>(inaudible)</td><td></td></tr>
<tr><td>17</td><td>N</td><td>So would you consider this a one block tower? (pointing on Carl’s paper)
Because this one is two, that one is one I know.</td><td>EPIOSDE 2
Nikki asks Carl a question that indicates she is asking for help.</td></tr>
<tr><td>18</td><td>C</td><td>It's one, two, three.
(pointing to the diagrams on his paper)</td><td>LMTY (7):
Carl responding to Nikki’s question about the height of the towers given as examples on the task paper.</td></tr>
<tr><td>19</td><td>R</td><td>(inaudible)</td><td></td></tr>
<tr><td>20</td><td>N</td><td>That’s not a three block tower!
The three-block tower goes up…</td><td>LHSIA (9A):
Nikki disagrees that what her classmate</td></tr>
</table>

<table>
<tr><th colspan="4">Nikki, Ricardo, Carl</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript/Comments</th><th>Structure; Observations</th></tr>
<tr><td></td><td></td><td></td><td>called the three-block high tower is indeed that tower. She gives her understanding of what a three-block high tower is.</td></tr>
<tr><td>21</td><td>C</td><td>It is three from the one like this. One, two, and three would be like this. (using his pen to point at the towers A, B, and C on his task paper)
Nikki, you’re smart.</td><td>LMTY (1):
Carl explains that the first three towers given on the task paper have respective heights one, two, and three.
LHSIA (17):
“You’re smart” affirming what he said earlier.</td></tr>
<tr><td>22</td><td>N</td><td>Oh!!! I get it. I get it. I get it.</td><td>LHSIA (4, 9):
Nikki indicates that she understands and now agrees with Carl’s explanation by saying, “I get it.”</td></tr>
<tr><td colspan="4">Interpretation: In the exchange above, Carl seems to activate a LMTY structure when Nikki asks him a question about the names of the towers included in the task paper. He responds that the heights of towers A, B, and C are 1, 2, and 3, respectively.
However, Nikki seems to disagree that tower C is a 3-block high tower. Though she asked Carl for his help, she still seems to be putting forth her understanding of the problem. Because Carl still realizes that the 3-block high tower is represented in tower C, he continues to explain why this is the case, almost admonishing Nikki for not seeing it herself: “you’re smart.”
Nikki then claims to understand Carl’s explanation, “Oh! I get it,” now agreeing with him about the 3-block high tower.</td></tr>
<tr><td>23</td><td>R</td><td>(inaudible, as he points to the diagrams on C’s paper)</td><td></td></tr>
<tr><td>24</td><td>C</td><td>I’m gonna draw it. I’m going to do the whole thing now.</td><td></td></tr>
<tr><td>25</td><td>N</td><td>I wanna draw it.
For the 5 or 10 block tower</td><td></td></tr>
<tr><td>26</td><td>C</td><td>OK. Everybody think to themselves. I have to think.</td><td>LHSIA (5):
Carl is asserting</td></tr>
</table>

<table>
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<tr><th>Line No.</th><th>Spkr</th><th>Transcript/Comments</th><th>Structure; Observations</th></tr>
<tr><td></td><td></td><td>(to teacher who has just walked up to the group)
I got this myself.</td><td>that he can arrive at the solution, but he must think for himself first.</td></tr>
<tr><td colspan="4">Interpretation: Carl suggests that he can get the answer, “I got this myself” but first he needs to think. His statement indicates he is confident in his ability to get the answer. However, he seems to want to work independently for a while rather than continue to discuss this with his classmates.</td></tr>
<tr><td>27</td><td>

C</td><td>Teacher walks toward group and they notice her.
I got this.</td><td></td></tr>
<tr><td></td><td colspan="2">The teacher comes over for a few minutes and asks the students to explain what they are doing. They tell her they figured out that you can add 5 blocks to a tower to get the total number of blocks in the next tower. Carl also suggests another strategy, which seems to be multiply 5 to the number one less than the height and add one more block (e.g., if n = height, 5(n -1) + 1). Ricardo does not appear comfortable with this strategy so he says he is going to continue adding 5 blocks. The teacher encourages the students to try both strategies and compare their solutions.</td><td></td></tr>
<tr><td>28</td><td>C</td><td>Where’s she [the teacher] going?
(The group starts working quietly and each student is working independently.)</td><td></td></tr>
<tr><td>29</td><td>N</td><td>I missed some one.</td><td></td></tr>
<tr><td>30</td><td></td><td>Group continues individual work without conversation for several minutes. Sometimes they mumble, but it is difficult to hear exactly what they are saying.</td><td></td></tr>
<tr><td>31</td><td>N</td><td>I got my five-block tower.
(in a sing-song voice)</td><td>EPIOSDE 3
LHSIA (4):
Nikki lets her classmates know that she has an answer for the 5-block high tower.</td></tr>
<tr><td colspan="4">Interpretation: Nikki’s use of a sing-song voice indicates she is excited that she has an answer for the 5-block high tower. Alternatively, she may have just completed her drawing, rather than come up with a numerical answer for the total number of blocks. Either way, she appears to want her two classmates to know that she has come up with the answer (or drawing).</td></tr>
</table>

<table>
<tr><th colspan="4">Nikki, Ricardo, Carl</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript/Comments</th><th>Structure; Observations</th></tr>
<tr><td>32</td><td>C</td><td>(smiles at N and says something inaudible).</td><td></td></tr>
<tr><td>33</td><td>N</td><td>(to C) What did you do for your 4 block tower?</td><td></td></tr>
<tr><td>34</td><td>C</td><td>(Something about getting her own answer and not asking him for it.)</td><td></td></tr>
<tr><td>35</td><td>N</td><td>No, I just did it.</td><td></td></tr>
<tr><td>36</td><td>R</td><td>Yeah, she did it.</td><td></td></tr>
<tr><td>37</td><td>C</td><td>4 is 16.</td><td></td></tr>
<tr><td>38</td><td>N</td><td>Huh?</td><td></td></tr>
<tr><td>39</td><td>C</td><td>4 is 16</td><td></td></tr>
<tr><td>40</td><td>N</td><td>How did you do it?
(She reaches over as if to grab or look at his paper.)</td><td></td></tr>
<tr><td>41</td><td>C</td><td>Wait, wait.
(Does not offer his paper to Nikki and keeps his hands over his paper, preventing her from taking it)</td><td></td></tr>
<tr><td>42</td><td>N</td><td>Yeah?</td><td></td></tr>
<tr><td>43</td><td>C</td><td>What'd you did, 4? 16 – OK.</td><td></td></tr>
<tr><td>44</td><td>N</td><td>Uh, huh. But how did you get it?
(sounds accusatory, as though he ought to be giving her the answer)</td><td></td></tr>
<tr><td>45</td><td>C</td><td>I did the right (unintelligible) first. It was, um, um, I added 5.</td><td></td></tr>
<tr><td>46</td><td>N</td><td>Oh!!! OK. (looking down at her own paper) I get it. I get it. You use the number of blocks that you put on the previous one before to get the one that comes after it. So say if I wanted to my 5 block tower I would do 4 times 5. I get it now.</td><td>LHSIA (4, 12, 15): Nikki first states that she understands the procedure to get the total number of blocks. Then she proceeds to explain it her own words.</td></tr>
<tr><td>47</td><td>C</td><td>Yeah.</td><td></td></tr>
<tr><td>48</td><td>N</td><td>I get it.</td><td></td></tr>
<tr><td colspan="4">Interpretation: Nikki wanted Carl to explain his method for getting his answers. First he refuses to do so because he wants her to get her own answer, but then says that he added 5 blocks. Nikki then thinks she understands his method to obtain a solution and says, "I get it." She then proceeds to explain her understanding of Carl's method. She looks for his confirmation of her explanation, which he gives, "Yeah."</td></tr>
<tr><td>49</td><td>R</td><td>For the first time.</td><td></td></tr>
</table>

Nikki, Ricardo, Carl			
Line No.	**Spkr**	**Transcript/Comments**	**Structure; Observations**
		(not sure what he is referencing)	
50	N	So the 6 now that's 21. Seven, you would be 27.	
51	C	What did you say? Which one is…?	**LMTY (8):** Carl questions Nikki's stated answers in order to better understand what she is saying.
52	N	The 6 block tower.	
53	C	5 times 5 plus 1.	**LMTY (1; 4):** Carl explains how he got the total number of blocks in a tower. He is correcting Nikki's statements that a 6-block high tower has 21 blocks and the 7-block high tower has 27 blocks, by giving her his explanation.
54	N	But it's…wait.	
55	C	I got 26.	
56	N	Wait, 1, 2, 3, 4. (continues counting) Oh yeah, I was counting wrong.	
57	N	I get it now! (in a triumphant sing-song voice)	**LHSIA (4):** Nikki is letting her two classmates know that after her conversation with Carl she understands part of the solution.
Interpretation: When Nikki gives her answers for the total number of blocks in the 6- and 7-block high towers (21 and 27, respectively), Carl seems to recognize these numbers as incorrect and asks her about her answers. When he replies with his own answer and a brief explanation, "5 times 5 plus 1," Carl appears to be helping Nikki understand what he believes is the correct solution. When Nikki claims that she was counting wrong and that she gets it now, she is demonstrating her understanding of the solution.			
58	R	(inaudible)	

Nikki, Ricardo, Carl			
Line No.	**Spkr**	**Transcript/Comments**	**Structure; Observations**
59	N	So we should put our ideas together. Well, me and Ricardo had the same idea. So. We just take forever to write things down on paper. I'm (unintelligible) multiplication. To get the 10 block it would be… (thinks to herself).	**LHSIA (12):** Nikki is suggesting that her ideas are good and should be included with Carl's ideas.
Interpretation: Nikki has said multiple times that she "gets it" indicating she understands the task and some part of the solution. She may be trying to let her two classmates know that she understands so that they believe she is smart. Nikki says that they "should put our ideas together" indicating that each of them has a good idea that should be valued and included. She suggests, when she said, "we take forever to write things down," that she feels speed in mathematics tasks is equivalent to or an indication of going good at math or being smart. By stating why they are 'taking forever' she may be trying to show that she is still smart and it is the writing part of the work that is slowing her down.			
60		*The group is quietly working independently for a while.*	
61	R	(to C) (unintelligible)	
62	C	You don't need a calculator.	
63	N	For a 4 block, um.	
64	C	(to observer standing near their table) I say get another worksheet? You get another one? (observer is confused)	**EPISODE 4** **LHSIA (6):** Carl asks for another problem because he sees himself as done with this problem – as though this problem is easy.
65	C	Another, like another problem?	
66	Obsr	Another problem? (I think C finds the problem to be insufficiently challenging and is looking to try another problem since they have solved this one already. It has taken them less than 15 minutes.)	
67	C	Another, like another problem?	
68		*The observer explains that they are only working on this problem for the class period. They all work quietly independently for a*	

<table>
<tr><th colspan="4">Nikki, Ricardo, Carl</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript/Comments</th><th>Structure; Observations</th></tr>
<tr><td></td><td></td><td>while.</td><td></td></tr>
<tr><td colspan="4">Interpretation: Carl is asking one of the researchers/observers if they are going to get another problem to work on. Carl has answered many of the questions given in the task quickly and appears to think this problem is not very challenging. Carl seems to have finished writing his solution, therefore, he may have found the problem to be easy. Many students believe that ability (rather than effort) leads to being successful in mathematics (relating to finding the problem easy).
Carl may have finished writing information on his paper, which at the end of the period includes solutions for the 5-block, 10-block, and 100-block high towers. Also, he has written in words the generalized solution, but not in algebraic form. The words alone do not clearly give the generalized solution and rely on the examples for clarification (see student work at end of document). It is not clear whether Carl has not expressed any concern whether his two classmates understand the solution and have finished writing it on their own papers.</td></tr>
<tr><td>69</td><td></td><td>The group appears to get distracted or off-task at times.</td><td></td></tr>
<tr><td>70</td><td>N</td><td>5 block. 5 times 4. Plus 1 …that’s 21.
(Says this out loud, but does not seem to be speaking to anyone in particular)</td><td></td></tr>
<tr><td>71</td><td>N</td><td>The 100 block.</td><td></td></tr>
<tr><td>72</td><td>N</td><td>Wouldn’t it be 5 X 96…no, 99</td><td></td></tr>
<tr><td>73</td><td>C</td><td>Yes, Nikki [says student’s first and last name]</td><td></td></tr>
<tr><td>74</td><td>R</td><td>(talks to C but it cannot be heard. R and C have stopped working. N continues to work)</td><td></td></tr>
<tr><td>75</td><td>N</td><td>What is 5 times 99?
(No one in this group has gotten blocks or calculators, or any tools to assist them)</td><td></td></tr>
<tr><td>76</td><td>C</td><td>OK, if it’s 100 and then just going by 4 say.</td><td></td></tr>
<tr><td>77</td><td>N</td><td>Oh you mean (unintelligible)</td><td></td></tr>
<tr><td>78</td><td>C</td><td>Five times (unintelligible) minus 5</td><td>EPISODE 5</td></tr>
<tr><td>79</td><td>N</td><td>I know!! I was just telling you was wrong but you was right
Carl and Nikki both laugh.</td><td>LHSIA (9; 12)
Nikki has been making a lot of statements and asking questions. Carl has responded to many of them. It appears that he disagreed with Nikki on something, because she says here that</td></tr>
</table>

<table>
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<tr><th>Line No.</th><th>Spkr</th><th>Transcript/Comments</th><th>Structure; Observations</th></tr>
<tr><td></td><td></td><td></td><td>she knows that he is correct.</td></tr>
<tr><td>80</td><td>R</td><td>Was trying to make you feel better.
Not that I didn’t know it.</td><td>LHSIA (4)
Ricardo says he was trying to make Nikki feel better about asking questions. He indicates that he knew the answer to the questions she was asking.</td></tr>
<tr><td colspan="4">Interpretation: Here, Nikki almost sounds defensive when she says, “I know!!!” suggesting that she believes she knows her classmate is correct and that she agrees with him now, even if she did not just a few moments go. By agreeing with Carl, Nikki may be indicating that she understands the solution strategy Carl is suggesting, thereby indicating that she is capable of keeping up with him mathematically.
Ricardo may also have a LHSIA structure active here. His words, “Not that I didn’t know it” indicate that he may have felt smart as well because he knew the answer. He was trying to “make [Nikki] feel better” somehow and perhaps he was doing this by either helping or pretending to not know the answer. Note that it has been difficult to understand what he is saying throughout most of this class session.</td></tr>
<tr><td>81</td><td colspan="2">For the rest of class (about 12 minutes), the students talk about things other than the problem. They appear to be writing on their papers still, especially Nikki. At one point Carl makes a comment about having his head down, perhaps indicating that he wants to look like he is still working to others (e.g., observers, teacher). From time to time Nikki brings the problem back up. When she does this, Carl and/or Ricardo sometimes respond to her to answer a question or say something about the mathematics.</td><td></td></tr>
<tr><td>82</td><td>N</td><td>Five times nine, forty-five, forty-six.</td><td>EPISODE 6</td></tr>
<tr><td>83</td><td>C</td><td>You people are slow. I’m going to have to ask if I can use the bathroom</td><td>LHSIA (6)
Carl is stating that his two classmates are “slow” to finish writing up the solution.</td></tr>
<tr><td colspan="4">Interpretation: Carl accuses his two classmates of being slow. He states he might go to the bathroom, perhaps indicating that he is bored waiting for them. Perhaps Carl believes that being quick or fast in math correlates to being smart or intelligent. Since he</td></tr>
</table>

Nikki, Ricardo, Carl			
Line No.	**Spkr**	**Transcript/Comments**	**Structure; Observations**
believes he is done with his work, he may be indicating that because he was quicker perhaps he is smarter than the other two.			
84	N	Basically, what you're really doing is… You know how, like, the 100 blocks you subtracting that one but you still add that one to the (inaudible) (animatedly using hands as she speaks to help illustrate her point)	
85	C	Subtracting that one, what you mean?	
86	N	You know how you add the one but you use 99? (seems to primarily be speaking to Carl)	
87	C	Add the 1? Yeah. No. That's not. I know what you're saying. You still going to put that one. But that one right here that's equal to 5. That one is equal to one. You see?	
88	N	I'm just saying you're basically just subtracting that one and basically just putting it right back in. (seems almost defensive in her speech; resumes writing on her own paper as Carl leans back in his chair)	**LHSIA (10, 15)** Nikki says this as though she may be looking for confirmation, as she builds on the ideas stated earlier about multiplying 5 times a number one less than the height and adding one block (for the middle hidden block).
Interpretation: Nikki seems to be restating the strategy in other words, building on the ideas that were stated about the problem earlier. She speaks about the fact that to get the total number of blocks, their strategy was to multiply five by a number one less than the height and add a block (for the middle hidden block). The way she states this may be confusing Carl, indicated by his response, "Add the one? Yeah. No…" By sharing her interpretation of the ideas, Nikki may be looking for confirmation from Carl that her ideas are correct			
89	*Nikki mentions that they need to label their work (e.g., a 5-block high tower has 21 blocks, rather than 21 "hot dogs." After this brief exchange about units, the students continue to have conversation, mostly not related to class, until the teacher instructs them to begin cleaning up their desks so we can administer the questionnaire.*		

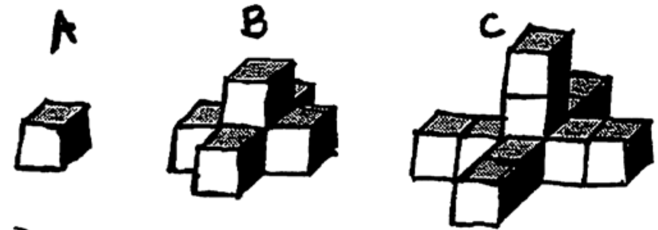
Nikki, Ricardo, Carl			
Line No.	**Spkr**	**Transcript/Comments**	**Structure; Observations**
90	C	Can't do it that way. You can't do it for *x*. I already tried that.	
91	N	You could!	
92	C	No, you can't.	
93	N	Yes, you could. $y=mx+b$ (Nikki smiles broadly at C)	
94	C	Oh, wait.	
95	N	You could. Because your independent variable is 5 and the rate of change…	**LHSIA (2; 10)** The students had mentioned an equation earlier in the class period. Nikki seems to still be thinking about this, perhaps in order to answer the question, "generalize for any size tower." She seems to be defending her idea of using the $y=mx+b$ equation by making suggestions about the "parts" of that form.
Interpretation: Nikki is revisiting and building on the idea of using an equation, perhaps to answer the question "Generalize for any size tower." She seems to be defending her idea of using the $y=mx+b$ equation. As part of her statement, she is talking over Carl, who is disagreeing with her over the use of this particular form of equation. Her defense of her ideas suggests she believes she is correct and wishes to have her classmates recognize this as well.			
96	C	But if you do…if you do… what are you doing?	
97	N	The equation is saying that if (inaudible)	
98	C	Five, five, I mean, y equals. Five plus… Oh, five times five, twenty five. Plus one is twenty-six. (inaudible)	**LHSIA (9A; 11)** Carl is working through the answer as if he obtained the answer in the way Nikki suggested. He

Nikki, Ricardo, Carl			
Line No.	**Spkr**	**Transcript/Comments**	**Structure; Observations**
			indicates that he would get the number twenty-six, which is not the total for the 5-block high tower
Interpretation: Carl disagrees with Nikki about the equation that they might be able to use to "generalize for any size block tower." He works through his interpretation of the equation she has suggested and seems to get 26, which is the total number of blocks needed for the 6-block high tower, rather than the 5-block high tower. Because of his "proof by contradiction" he has shown that he is somewhat correct. The students have not yet discussed that the *x*-variable could represent one less than then height (or perhaps x = height, and have $y = 5(x -1) +1$.).			
99	T	*The teacher gives instructions for the students to collect all their written work and other materials so we can collect them. The students are asked to move their seats for the questionnaire. Also, the teacher says they will have more time to work on the task the following day. Nikki and Carl display disappointment that they will be asked to work on the same problem rather than a different one. The students continue to pack up their materials.*	

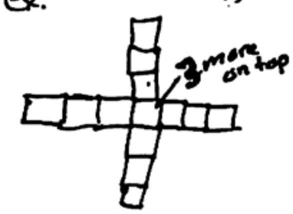
Nikki's Work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



ex. (4 block)



3 * 5+1 = 16 cubes/blocks

(5 block)

5x4+1=21 cubes / blocks

(100block)

5 * 99 = 495 + 1 = 496 cub blocks

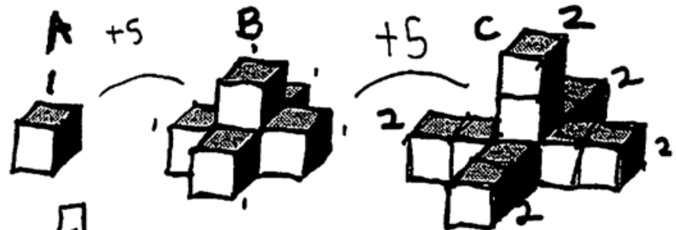
(10 blocks)

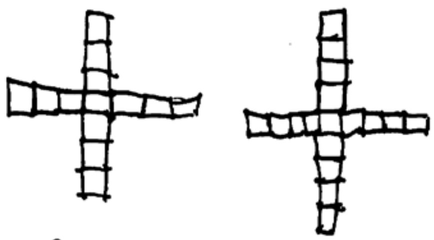
5x9+1=46 cubes/blocks

Ricardo's Work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.





4 top
3 side

5 top
4 side

5-block = 4x5 = 20+1 = 21 blocks 10-block = 5x9 = 45+1 = 46 blocks

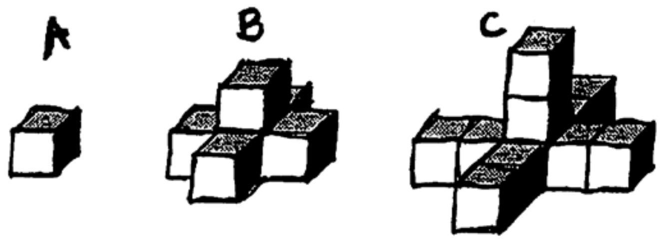
100-block = 99x5 = 495+1 = 496 blocks

* I took the # of cube's before I did the next tower EX: Tower 3
2x5=10+1=11

Carl's Work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



add +5 each tower and you started with one block (+1)

11 (5 block)
+ 5
16 blocks for the 4-block tower
+ 5
21 blocks - 5 block tower

(Another 5 block)
× 5
4
20 + 1 = 21 blocks

(10 block)
5
* 9
45 + 1 = 46 blocks

* (100)
99
× 5
495 + 1 = 496 blocks

* Multiply the perious block because when it was at the one block tower it was only 1 block. So for each you just multiply by the number before it and add that one block.

Questionnaire Responses for Nikki, Ricardo, Carl

Questionnaire items which may indicate LMTY structure

Questionnaire Items	**Nikki**	**Ricardo**	**Carl**
Statements (All the time, Sometimes, Never)			
I wanted to teach another student something that I knew that the other student did not know.	All the time	All the time	All the time
I listened carefully to the ideas of someone I was trying to help.	All the time	All the time	All the time
I helped someone see how to do the math.	All the time	Sometimes	All the time
Others listened carefully to my ideas.	All the time	All the time	All the time
Behaviors (Often, Sometimes, Hardly ever)			
I gave helpful suggestions.	Often	Often	Often
I worked cooperatively.	Often	Often	Often
Thoughts (Yes/No)			
I like teaching this person things that I know.	No	Yes	Yes

Questionnaire Responses for Nikki, Ricardo, Carl

Questionnaire Items Which May Indicate LHSIA structure

Questionnaire Items – Statements	**Nikki**	**Ricardo**	**Carl**
Statements (All the time, Sometimes, Never)			
I wanted people to think that I'm smart.	All the time	All the time	All the time
I tried to impress people with my ideas about the problem.	All the time	All the time	Sometimes
People seemed impressed with the ideas I shared about the problem.	Sometimes	All the time	All the time
People saw how good I am at the math we did today.	All the time	All the time	All the time
I felt smart.	All the time	All the time	All the time
I wanted to show someone that my way was better.	Never	All the time	All the time
I was a lot better at math than others today.	Sometimes	Sometimes	All the time
Behaviors (Often, Sometimes, Hardly ever)			
I was the leader.	Sometimes	Sometimes	Often
I was bossy.	Hardly ever	Hardly ever	Hardly ever
I wanted to show off.	Hardly ever	Hardly ever	Hardly ever
I liked to be right.	Often	Often	Sometimes
Thoughts (Yes/No)			
I want you to know just how smart I am.	No	Yes	No
People think I'm smart.	No	Yes	Yes
I wish the teacher would call on me, so I can show how much I know.	No	Yes	No

Cristina, Christian, Carly

This is Group 3 from Ms. S's grade eight Class 1.

Verbal emphasis indicated by <u>underline</u>
Actions included either in *italics* or (parentheses) as appropriate.
Breaks in speech are indicated by …
Words inserted to help clarify a student's words are included in [brackets].
When students voices overlap, one or both sets of words are included in //double slash marks//.

The number included next to either LMTY or LHSIA refers to the code, included in the table of codes.

Cristina, Christian, Carly			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
		Students introduce themselves to the audio and video recorder.	
1	Cristina (Cr)	Cristina [Last Name]	
2	Carly (Ca)	Carly [Last Name]	
3	Christian (Ch)	Christian [Last Name]	
4	Sts	*Students quietly reading the problem.*	
5	Ch	Oh, this is kind of easy.	**EPISODE 1** **LHSIA (6):** Christian states that the task appears to be easy after looking at it briefly.
6	Ca	What are they?	
7	Ca	This one has…	
8	Ch	This one is in blocks of towers, and you put blocks on the sides, see?	**LMTY (1):** Christian responds to Carly's question, explaining how the towers are constructed, by putting blocks on each side.
Interpretation: The students start looking at the task before the teacher orients them to the problem. Before Ms. S starts speaking, Christian suggests that the task will be easy. When Carly has a question about what the task is about, Christian responds by explaining that the towers are constructed by putting blocks on each side. It appears as though the conversation may have continued except the teacher began her introduction.			
	The teacher gives her introduction to the problem and		

Cristina, Christian, Carly			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
	instructions. This takes a little more than three minutes. The three students are sitting at desks which are situated to all face one another. *When the teacher finishes, the students all lean in toward each other.*		
9	Ca	Okay	
10	Ch	Okay	
11	Ca	Okay, um . . .	
13	Ch	(Pointing with his pen on Carly's paper) Okay so you build it with blocks right there, blocks on the side…	**EPISODE 2** **LMTY (1)** Christian is explaining the construction of the towers to Carly, by pointing to the figures on her paper.
14	Ca	This one? I'm beat. How much could you think you …?	
15	Ch	That's it.	
16	Ca	Exactly. She has 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11. (using her pen to count the total number of blocks in tower C, the 3-block high tower) … Because of the one in the middle. Because … every extra tower is like 5 extra cubes.	**LMTY (3; 6):** Carly is building on what Christian says about adding blocks to each side, counting the blocks in tower B on the task paper. She then goes on to explain that there is a block in the middle that is hidden from view. She adds that in order to get the next tower, you add 5 blocks to the total.
17	Ch	We don't have to solve this right now.	
18	Ca	It's like proportion.	**LHSIA (14)** Carly offers her own idea about the problem, stating that that it is like a proportion.
19	Ch	It is, let's solve this right now.	
Interpretation: The students, particularly Christian, seem to pick up their conversation right where they left off when the teacher gave her introduction to the problem. Christian starts (or continues) to explain the construction of the towers. Carly then builds on Christian's explanation, counting out loud the total number of blocks for the tower shown in figure B. She provides more details as she states that there is a block "in the middle" suggesting that she wants her classmates to understand why there are 11 blocks, rather than 10 perhaps.			

<table>
<tr><th colspan="4">Cristina, Christian, Carly</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td colspan="4">Carly then suggests that the solution might be obtained by using a proportion. She may be trying to demonstrate her mathematical ability by making this observation.</td></tr>
<tr><td>20</td><td colspan="2">The students work quietly as they write on their own papers for several seconds.</td><td></td></tr>
<tr><td>21</td><td>Cr</td><td>How is it like a proportion?</td><td></td></tr>
<tr><td>22</td><td>Ca</td><td>Huh?</td><td></td></tr>
<tr><td>23</td><td>Cr</td><td>How is it like a proportion?</td><td></td></tr>
<tr><td>24</td><td>Ca</td><td>Like, 1 T equals 1 C which equals, then you do, we have to figure out how much is 5, so we do 5 C.. no 5 T is. . . how much it would be… then you have to like, cross multiply.
(Cristina appears to be looking on at Carly’s paper as she speaks and writes.)</td><td>LMTY (3; 7):
Cristina asks Carly to explain how the problem is like a proportion, so she can understand Carly’s strategy.
Carly responds, explaining as she is thinking through how to set up the proportion.</td></tr>
<tr><td>25</td><td>Cr</td><td>Okay. . .</td><td></td></tr>
<tr><td colspan="4">Interpretation: Cristina asks Carly to explain the proportion that she has suggested. Carly starts her explanation, appearing to make a proportion of 1T to 1C. (Likely, T refers to towers or the height of the tower, and C refers to cubes or the total number of cubes or blocks it takes to create the structure.) Carly mentions that cross-multiplying, a common strategy students might use when working with proportions.</td></tr>
<tr><td>26</td><td colspan="2">The students continue to work quietly and independently. They then start discussing different strategies they might use to determine the answers. For example, Carly suggests making a chart, and Christian says he was trying to make a diagram. They are discussing this when the teacher comes to the group.</td><td></td></tr>
<tr><td>27</td><td colspan="2">When she arrives at the table, the teacher asks for an update as to what they are doing and what they have done already. As Christian explains he is about to make a chart depicting the height of the tower and the total number of blocks in the tower, Carly and perhaps Cristina, continue to write on their own papers. They do not seem to talk to her during this exchange. The teacher encourages the students to discuss their strategies, and then lets them continue to work.</td><td></td></tr>
<tr><td>28</td><td>Ch</td><td>I think we got it now.</td><td>EPISODE 3</td></tr>
<tr><td>29</td><td>Cr</td><td>Where’d you get the four?
(question seems directed at Carly)</td><td rowspan="5">LMTY (1; 7):
Carly responds to Cristina question about the 4-block high tower. She explains that he could get the</td></tr>
<tr><td>30</td><td>Ca</td><td>Four? What do you mean?</td></tr>
<tr><td>31</td><td>Cr</td><td>Four block tower?</td></tr>
<tr><td>32</td><td>Ch</td><td>16.</td></tr>
<tr><td>33</td><td>Ca</td><td>16.</td></tr>
</table>

<table>
<tr><th colspan="4">Cristina, Christian, Carly</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td>34</td><td>Ca</td><td>Because it’s plus 5.</td><td>total number of blocks to be 16 by adding 5 blocks to get the next tower.</td></tr>
<tr><td>35</td><td>Cr</td><td>Oh. . . I know that’s what I was getting right now. I was getting frustrated.
(Inaudible comments)</td><td></td></tr>
<tr><td colspan="4">Interpretation: Cristina asks Carly how many total blocks she got for the 4-block high tower, and how she got that answer. First Carly responds with a quick answer, “16.” She then explained how she got her answer, by adding 5 to each iteration of the tower. Her actions, particularly the explanation, suggest she was motivated to help Cristina.</td></tr>
<tr><td>36</td><td></td><td>The students continue to work independently and quietly. If they speak, it is inaudible to the listener.</td><td></td></tr>
<tr><td>37</td><td>Ch</td><td>Is there a quick way to get to 100?</td><td></td></tr>
<tr><td>38</td><td>Ca</td><td>What?</td><td></td></tr>
<tr><td>39</td><td>Ch</td><td>Is there a quick way to get to 100?</td><td>EPISODE 4
Here, Christian opens a dialogue, but Carly does not explain her strategy in detail.</td></tr>
<tr><td>40</td><td>Ca</td><td>Yeah you gotta. . . you have to do like, proportions. Like (inaudible comments). Get it?</td><td>LMTY (7; 10)
Carly responds to a specific question from Christian, suggesting her proportions strategy from earlier. Christian specifically asked a question that Carly has answered… however, she is still indicating this proportions idea. I wonder what her work had shown up to this point.</td></tr>
<tr><td colspan="4">Interpretation: Christian asks an important question about a quick way to get the answer of the total number of blocks for the 100-block high tower. Carly responds that there is a quick way and once again suggests her proportions strategy that she mentioned earlier in the class. Either she does not really explain it or her explanation is inaudible to the listener. Still, she asks if he “Get[s] it?” in a way that indicates that that she wants him to understand and go further with the problem.</td></tr>
<tr><td>41</td><td colspan="2">The students discuss whether they are allowed to use calculators. Two students get up to get the calculators and the blocks that perhaps other groups are using. Both Cristina and Christian reach out to the blocks as though to do something with them, however, both leave the blocks in</td><td></td></tr>
</table>

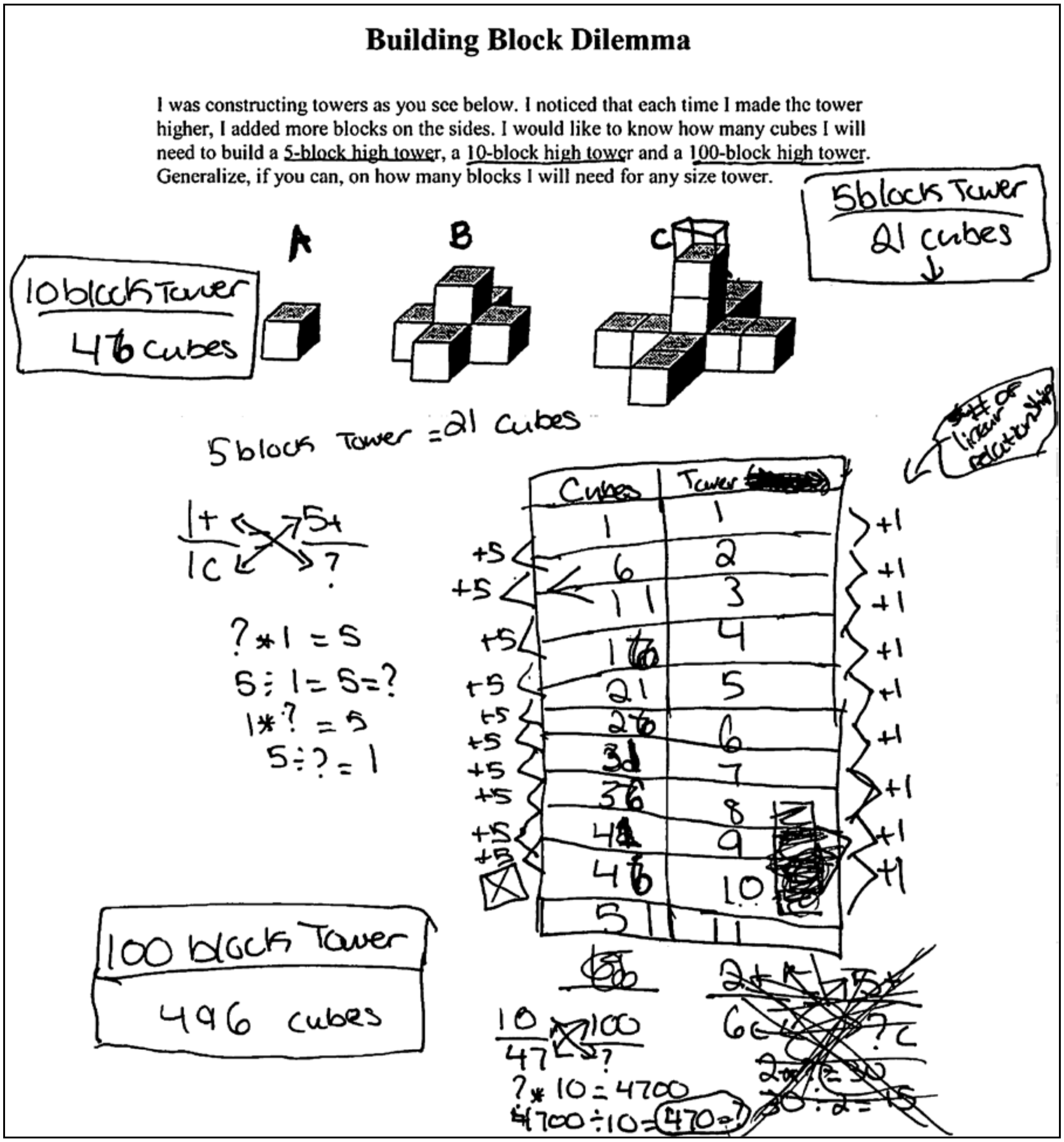
Cristina, Christian, Carly			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
	the bin. The students do not use them.		
42	*Carly and Cristina briefly discuss whether the proportions strategy will work or not. They suggest that it will not, noting that a 2-block high tower has 6 blocks and Cristina says that for the 10-block high tower she arrived at an answer of 47 blocks. Though this number is incorrect, the proportions strategy does not work (with 46 or 47 blocks in the 10-block high tower).*		
43	Ch	I need help with this last one.	
44	Ca	Okay, okay, okay. . .	
45	Cr	I think it's 470.	**EPISODE 5** The students discuss their answer for the 100-block high tower.
46	Ch	It's 496.	**LHSIA (8)** When Cristina states that she believes the total number of blocks for the 100-block high tower is 470, Christian corrects her with his answer of 496.
47	Ca	I got 407	
48	Ch	How'd you get 407 though?	
49	Cr	The proportions.	**LMTY (1)** Christian responds to Carly's question about how he got an answer of 496 blocks for the 100-block high tower by explaining that he continued to add 5 blocks to each tower.
50	Ca	Where'd you get 496?	
51	Ch	I kept adding on boxes.	
52	Ca	So you just kept adding on 5?	
53	Ch	Yeah, that's how I got to 100. I even double-checked. I just don't get this last question. (He may be referring to the last instruction: "Generalize if you can for any size tower.")	
Interpretation: Cristina states her answer for the 100-block high tower is 470 total blocks. Christian corrects her, giving his answer of 496. He states this with a tone of voice that suggests he believes he is correct even though his two classmates have different answers. Carly also states her answer, which is different (407). Christian asks Carly how she got her answer, prompting her to say she used proportions, a strategy that earlier she said she did not think worked. Carly then asks Christian how he got his answer, prompting him to explain that he simply added 5 blocks to get the total for each successive tower. He may have branched from the LHSIA structure when stating his answer to the LMTY structure because he was asked to explain his answer. Christian's tone of voice earlier may be supported here by his statement, "I even double-checked," possibly giving Christian more confidence that he has the correct answer.			
54	Group	*Actively working independently*	
55	Ch	I still ain't found out the answer for the last one. Dang. . . I still ain't getting it.	

Cristina, Christian, Carly			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
56	Group	*Actively working*	
57	Cr	I got 492.	**EPISODE 6**
56	Ch	496. Are any of you on the last problem yet?	**LHSIA (8):** Christian corrects Cristina's answer for the 100-block high tower, and initially dismisses her answer.
57	Ca	494.	
58	Ch	I got 496… how did you get that?	**LMTY (4; 8):** Christian repeats his answer, which is different from those of his classmates. He then questions them to see why they have different answers.
59	Cr	I kept on adding 5…	
60	Ch	I did 6… look at me too. . . I got 496.	
61	Ca	I have no idea.	
62	Ch	For which one? The last question?	
63	Cr	Huh?	
64	Ch	The last question?	
65	Ca	Proportions don't work.	
66	Cr	I know because I did it and it doesn't work. I don't know.	
67	Ch	For which one? There are 100 blocks, right?	
68		(inaudible conversation; Carly gets up briefly, returns in approximately 2.5 minutes)	
69	Cr	First?	
70	Ch	Huh?	
71	Cr	First?	
72	Ch	I think so. . .	
73	Ch	Hey wait, Cristina. . .How did you get 497?	**LMTY (4; 8):** Christian asks Carly how she got her answer for the total number of blocks. When she indicates that she added 5 to 47, he realizes that she made an error. He corrects her, telling her that a 10-block
74	Cr	Because I kept on adding 5. I did 47+5+5+5. . .	
75	Ch	47+5?	
76	Cr	Yeah because that equals 10.	
77	Ch	Where'd you get 47 from?	
78	Cr	10! 10 in a tower and then. . .	
79	Ch	10 equals 46 cubes.	
80	Cr	47!	
81	Ch	10 equals 46 cubes!	
82	Cr	47!	
83	Ch	Well…	

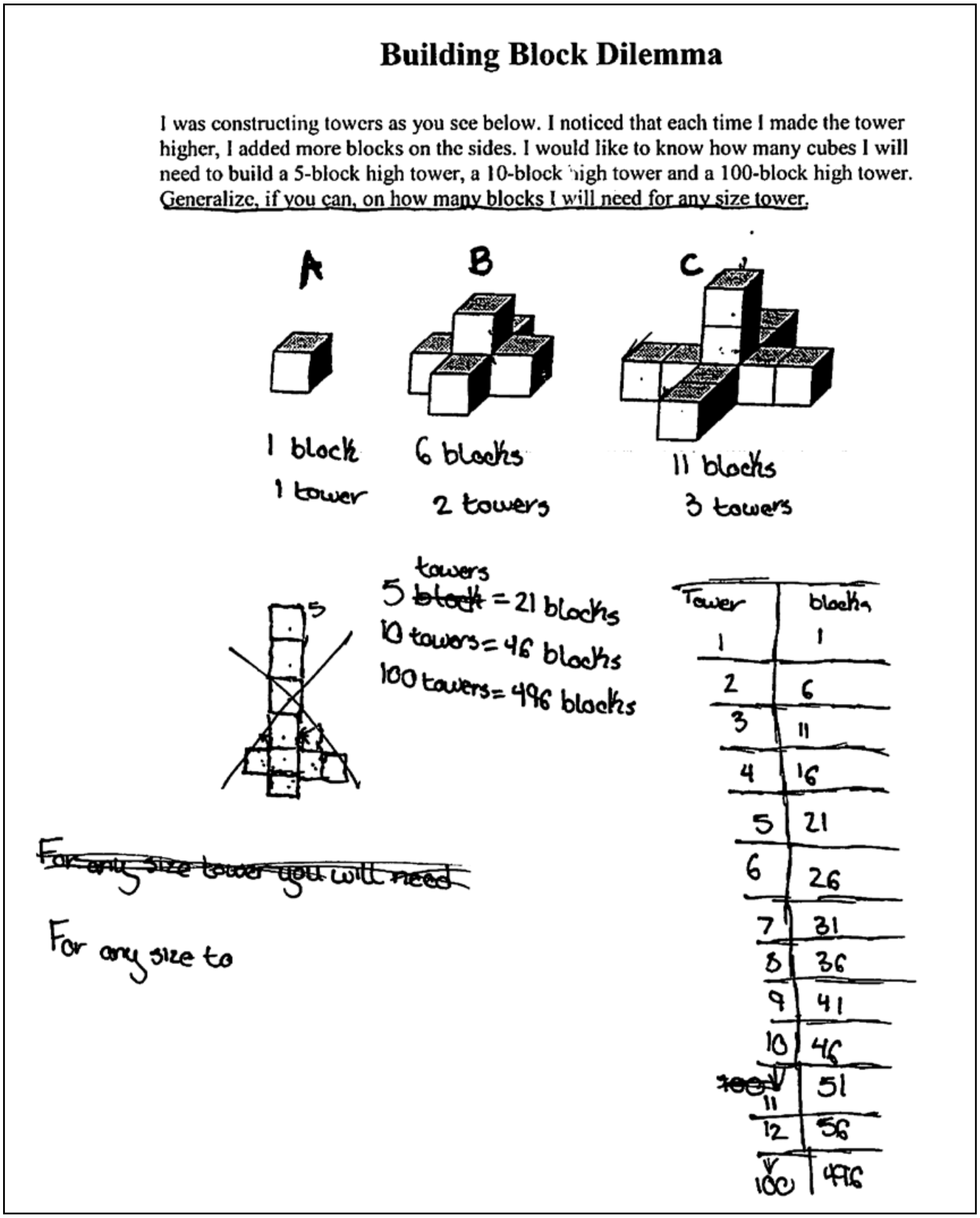
Cristina, Christian, Carly			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
84	Cr	What's in that. . . 1+6, I mean 1+5+5 is 11, plus 5 is 16, plus 5 is 21. . . Ohh….	high tower has 46, not 47 cubes. Cristina goes over her work and recognizes her error.
85	Ch	(looking on Cristina's work) Yeah!	
86	Cr	I messed up right here! 26+5 is 31+5 is 36+5 is 41+5 is 46.	
87	Ch	Yeah exactly! Uh-huh!	
88	Cr	Shut up.	
89	Ch	That's why I was like 496…	
Interpretation: Cristina and Carly continue to share answers for the total number of blocks in a 100-block high tower that are incorrect. Christian initially just says, "496," dismissing the other suggestions possibly because he is confident in his own answer. After the second incorrect suggestion, Christian asks the others how they got their answers. They repeat the earlier comment that the proportions strategy suggested at the beginning of class does not work. Christian again asks Cristina how he got the answer 497, to which she replies that she added 5 repeatedly. She says she started adding 5 to 47, the answer she has for the total number of blocks for the 10-block high tower. Christian corrects her, stating this is supposed to be "46 cubes" instead. Cristina realizes her arithmetic error.			
90	*The students continue to discuss their answers. Christian asks how they could get the answer for the 100-block high tower without having to add 5 blocks each time, "in a quicker way." The students do not have an answer for that. Both Carly and Cristina realize, from the tables they created on their paper, that there is a "1, 6, 1, 6…" pattern in the ones digit of the total number of blocks column. After this brief discussion, the teacher begins to tell them to put their work and materials away, for the questionnaire.*		
91	Ch	(after packing up his work per instructions) I found out the last . . .	
92	St	That doesn't make sense.	
93	Ch	But how many boxes it takes to make the whole chart.	
94	T	Pack all the cube packs that you used, or if you used calculators, or whatever, put those back. Oh, also, put your sharpie markers just in the middle with your stuff. . . oh, just joking, hold onto your sharpie markers.	
95	Ch	That was actually kinda easy.	**LHSIA (6):** Christian confirms that the task was easy, a statement he made at the beginning of the class period.
Interpretation: Christian started the class period stating that the task appeared to be easy. Here, he confirms that he found the problem to be "kinda easy." By stating this, he may confirming that			

Cristina, Christian, Carly			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
he always believed he had the ability to solve the problem, though the group experienced some difficulty.			

Cristina's work, in Nov. 2008



Christian's Work, in Nov. 2008



Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.

A — 1 block, 1 tower

B — 6 blocks, 2 towers

C — 11 blocks, 3 towers

5 ~~block~~ towers = 21 blocks

10 towers = 46 blocks

100 towers = 496 blocks

Tower	blocks
1	1
2	6
3	11
4	16
5	21
6	26
7	31
8	36
9	41
10	46
~~100~~ 11	51
12	56
100	496

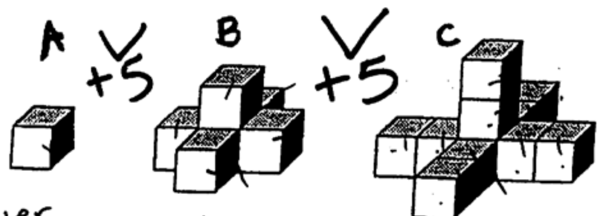
~~For any size tower you will need~~

For any size to

Carly's Work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



1 tower = 1 cube

2 towers = 6 cubes

6-1=5

3 towers = 11 cubes

11-6=5

Every Extra ~~position~~ tower = 5 extra cubes added to the figure

t= tower
c= total cubes

[redacted]

	# of cubes in tower	Total # of cubes	
	1	1	
+1	2	6	+5
+1	3	11	+5
+1	4	16	+5
+1	5	21	+5
+1	6	26	+5
+1	7	31	+5
+1	8	36	+5
+1	9	41	+5
+1	10	46	+5

Questionnaire Responses for Cristina, Christian, Carly

Questionnaire items which may indicate LMTY structure

Questionnaire Items	Cristina	Christian	Carly
Statements (All the time, Sometimes, Never)			
I wanted to teach another student something that I knew that the other student did not know.	Sometimes	Never	Sometimes
I listened carefully to the ideas of someone I was trying to help.	Sometimes	All the time	All the time
I helped someone see how to do the math.	Sometimes	Never	All the time
Others listened carefully to my ideas.	All the time	All the time	All the time
Behaviors (Often, Sometimes, Hardly ever)			
I gave helpful suggestions.	Sometimes	Often	Often
I worked cooperatively.	Often	Often	Often
Thoughts (Yes/No)			
I like teaching this person things that I know.	Yes	No	Yes

Questionnaire Responses for Cristina, Christian, Carly

Questionnaire Items Which May Indicate LHSIA structure

Questionnaire Items – Statements	**Cristina**	**Christian**	**Carly**
Statements (All the time, Sometimes, Never)			
I wanted people to think that I'm smart.	All the time	All the time	All the time
I tried to impress people with my ideas about the problem.	All the time	All the time	Sometimes
People seemed impressed with the ideas I shared about the problem.	Sometimes	Sometimes	Sometimes
People saw how good I am at the math we did today.	Sometimes	Sometimes	Sometimes
I felt smart.	Sometimes	All the time	Sometimes
I wanted to show someone that my way was better.	All the time	Sometimes	Never
I was a lot better at math than others today.	Sometimes	Sometimes	Sometimes
Behaviors (Often, Sometimes, Hardly ever)			
I was the leader.	Sometimes	Hardly ever	Sometimes
I was bossy.	Hardly ever	Hardly ever	Hardly ever
I wanted to show off.	Hardly ever	Hardly ever	Hardly ever
I liked to be right.	Often	Often	Often
Thoughts (Yes/No)			
I want you to know just how smart I am.	Yes	No	Yes
People think I'm smart.	Yes	No	Yes
I wish the teacher would call on me, so I can show how much I know.	No	No	No

Monique, Leticia, Sherelyn

This is Group 4 from Ms. S's grade eight Class 1.

Verbal emphasis indicated by underline
Actions included either in *italics* or (parentheses) as appropriate.
Breaks in speech, such as pauses or trailing off , are indicated by …
Words inserted to help clarify a student's words are included in [brackets].
When students voices overlap, one or both sets of words are included in //double slash marks//.

The number included next to either LMTY or LHSIA refers to the code, included in the table of codes.

Monique, Leticia, Sherelyn			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
	The teacher gives her introduction and Leticia from this group reads the problem to the class. The teacher continues her discussion of the problem and gives instructions. This has taken a little more than three minutes. The three students are sitting at desks which are situated to all face one another.		
1	Leticia (L)	(*talking very quietly, some is inaudible*) Isn't it… It says generalize if you can on how many blocks I will need on any size tower. What I think that picture is telling us… it's showing us a generalization, so, this is a 10 block high tower, … 2 block high…. So…	**EPISODE 1** **LHSIA (14):** Leticia is sharing her ideas, letting her two classmates know what she "think[s] the picture is telling us."
2	Sherelyn (S)	Like you said, right here, like you if… like it's saying if you keep on going in a pattern, how much will it give you like, around it? (uses her pen to point at the diagram on Leticia's paper) (Monique is writing on her paper)	**LHSIA (9):** Sherelyn is agreeing with Leticia, "Like you said," indicating that she agrees with her.
Interpretation: It seems that Leticia wants her classmates to listen to her ideas, as she is the first one to share her ideas. She gives her interpretation of the problem, suggesting that she thinks her ideas should be valued by her two classmates. Sherelyn agrees with Leticia's ideas, suggesting two things: 1) Sherelyn believes that Leticia's ideas are good, she is recognizing that Leticia had valuable knowledge of the problem, and 2) Sherelyn wants to be seen as also having good ideas about the problem. Sherelyn does ask a question, which I believe she is asking about the blocks going around (and possibly on top of) the middle block from figure A. This suggests that Sherelyn may not feel she completely understands the problem, and that she believes Leticia can offer more ideas.			
3	L	Yeah… so it's like… it's saying like… from what I think it's saying… it's like… a 5 block high tower, so it's like you actually like stack	

Monique, Leticia, Sherelyn			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
		the blocks, so it's like 1, 2, 3, 4, 5.	
4	S	And then we have to find… (uses her pen to point to Leticia's paper)	
5	L	Yeah	
6	Monique (M)	Yeah whatever goes… (points to Leticia's paper)	
7	L	Yeah	
8	M	Whatever goes up here is going, whatever is up here is going, but we also have to count…	
9	L	And then 5.	
10	M	Yeah, we also have to count the ones in the middle. 'Cuz then it seems like…So this one (perhaps pointing to figure B in the task) might be the 2 block high tower, because it's that one in the middle that's making this one high, and it's that one in the middle that's making that one 3 (perhaps pointing to figure C) and it will all go out. So just put 2 on the outside, so if you wanna do 4, if you wanna do 4 you would just add another one and that will be 4 because that's the one in the middle. (Leticia nods her head, "yes," as though in agreement or understanding with Monique)	**LHSIA (9, 10):** Monique is agreeing with Leticia and Sherelyn, "Yeah," and builds on their ideas to demonstrate her own understanding of the problem.
Interpretation: Monique makes her first major contribution by both agreeing with the ideas her two classmates have expressed, "Yeah," suggesting that she understands Leticia's and Sherelyn's ideas. Up to this point, Leticia and Sherelyn seemed to discuss how they might generalize the problem and how they construct the towers. Leticia says in line 3, "you actually stack the blocks". Here, Monique assigns the figures on the task paper to specific tower heights. Based on what she says, I believe she means that figure B is the 2-block high tower, although we cannot see this for certain in the video. Therefore, Monique can be said to be building on her classmates ideas by using their suggestion to "stack the blocks" to explain that the figures drawn in the task correspond to specific heights (e.g., figure B is the 2 block high tower). By stating this, she is explaining her understanding of the task and demonstrating her own knowledge			
11	S	Like there… like here there's nothing, and then around here there's 4, and then 4, and then 4 more around it, and it keeps on going in a pattern.	
12	M	Yeah	
13	L	Yeah it keeps on just adding that	
14	M	Yeah, it just… this one is always… you put that one right there, and how would that be 2 blocks? Because they just added… you can put one on top of it and one around it.	**LMTY (1):** Monique is explaining the construction of the towers, by recognizing that the middle block will stay put and will contribute to the height.

<table>
<tr><th colspan="4">Monique, Leticia, Sherelyn</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td>15</td><td>S</td><td>So we could do that for a 5 block too. Since this 3 would do 4… then 5…</td><td></td></tr>
<tr><td>16</td><td>L</td><td>Yeah!</td><td></td></tr>
<tr><td>17</td><td>M</td><td>Yeah because this one you would only get 3 blocks, this right here, so you just add, that’s the one right there, to get 4… then add to get 5.</td><td>LMTY (3): Monique is providing additional details, explaining the height of the 3-block and 4-block high towers, building on her explanation of the 2-block high tower</td></tr>
<tr><td colspan="4">Interpretation: Monique had given an explanation that seemed to be building on the ideas of her classmates, regarding how to properly count the height of the tower (i.e., including the middle block). It seems she continued to explain her idea, using the example of the two block high tower. Perhaps Monique believed that Leticia was confused (or perhaps Monique disagreed with the way Leticia was describing the problem), and this may be why Monique continued to explain how she believed the towers were constructed. Leticia’s next line, “I really didn’t kind of get it,” suggests that it is possible Monique picked up on this, though Leticia’s confusion was not obvious to me, as an observer, until she says this.</td></tr>
<tr><td>18</td><td>L</td><td>Yeah, see, I was about to ask a question because I really didn’t kind of get it, but now I see like that block in the bottom…</td><td>LHSIA (12): Leticia is letting her classmates know that she was confused, but now understands what they were saying about the middle hidden block and the construction of the towers.</td></tr>
<tr><td colspan="4">Interpretation: Leticia was the first in the group to share her ideas at 1:58:48. As her classmates begin explaining ideas about how to construct the towers, Leticia agrees, saying, “Yeah” a few times. Here, Leticia seems to want to make sure that her classmates know she understood what they were saying by explaining that she had been confused but no longer is, saying, “now I see like that block in the bottom.” Therefore, she lets them know that she has the same level of understanding or knowledge as the other two, by mentioning “that block in the bottom.”</td></tr>
<tr><td>19</td><td>M</td><td>Yeah</td><td></td></tr>
<tr><td>20</td><td>S</td><td>Is this block.
(indicating which block is the bottom)</td><td></td></tr>
<tr><td>21</td><td>L</td><td>Yeah, it’s just x, and it’s like 1, 2, 3…</td><td></td></tr>
<tr><td>22</td><td>M</td><td>Yeah</td><td></td></tr>
<tr><td>23</td><td>S</td><td>Yeah because you can’t see it, these two are stacked…</td><td></td></tr>
<tr><td>24</td><td>M</td><td>You just add it</td><td></td></tr>
<tr><td>25</td><td>L</td><td>Because you can’t see it…</td><td></td></tr>
<tr><td>26</td><td>M</td><td>Yeah.</td><td></td></tr>
<tr><td>27</td><td>S</td><td>If you do it with the blocks that you see.</td><td></td></tr>
<tr><td>28</td><td>L</td><td>Yeah</td><td></td></tr>
<tr><td>29</td><td>L</td><td>You want to, um…</td><td></td></tr>
</table>

Monique, Leticia, Sherelyn			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
30	M	Wait, what happened?	
31	L	The question is asking us that, I think, that all the question is asking us is like, asking us for these stacked towers, how many blocks, how many…	**EPISODE 2** Monique asks (Line 30), "What happened?" Leticia and Sherelyn respond with their interpretation of what they are supposed to do, find the total number of blocks, possibly an equation.
32	S	Yeah, how many… yeah, like how many… if we can determine it, or like, um… if there's an equation to tell us without us having to stack the blocks…	
33	L	Yeah.	
34	M	(says with a confident tone of voice) Yeah if they say we want a 5 block tower, it would… it would really not be 5…1, 2, 3… a 5 showing probably, but it wouldn't be… that extra one in the middle one…. That makes it…It's just 5. (points at the diagrams on Leticia's paper)	**LHSIA (12; 14):** Monique first asked a question and now is asserting her ideas about the 5-block high tower. Her tone of voice and the fact she is pointing at Leticia's paper indicates she is more confident about her ideas.
35	L	Yeah, it would be the one that adds…so it's like… this could be, this 3 could be…	
36	L	Yeah, this one…(talking over each other)…it's like… (talking over each other)… yeah… it's all on the sides. I get it.	**LHSIA (2; 9; 12):** Leticia and her classmates are talking over one another to share their ideas. Leticia had indicated confusion earlier, and is now stating that she understands. Content of discussion is not clear.
37	M	So to add 5 to this one, it would… it would be… all you do is add 2 more…	
38	L	2 more!	
39	M	2 more to this one, 2 more to that one, because it's that one in the middle	
40	L	Exactly	
Interpretation: Monique and Leticia are both sharing their ideas, with Sherelyn trying to share hers as well. All three students talk over one another, with excitement to share their ideas. They do not seem to want to stop one another from speaking. They each appear to want to all share			

<table>
<tr><th colspan="4">Monique, Leticia, Sherelyn</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td colspan="4">their ideas, and impress upon the others in the group that they have an understanding of the task. Each of them seems to want to make sure the other two girls in her group realizes that she has the same level of understanding as the others. Leticia says, “Yeah” often, to agree with the others, and they each seem to be trying to express similar ideas.</td></tr>
<tr><td>41</td><td>M</td><td>And that adds 5</td><td>EPISODE 3
The three girls discuss how to construct one tower based off the previous one – where to add the blocks. Monique offers advice to keep track of their work.</td></tr>
<tr><td>42</td><td>L</td><td>The thing is… the this is we have to even it out. Because if we don’t… then it’s like…</td><td></td></tr>
<tr><td>43</td><td>M</td><td>It would look like that</td><td></td></tr>
<tr><td>44</td><td>L</td><td>Because it would be like, say 4 on this side, then like 5 on the other side.</td><td></td></tr>
<tr><td>45</td><td>M</td><td>Then… okay, so if we put one in the middle, then we have to be sure we put 4 right there or right there… but we also have to count, like, instead of going 1, 2, 3, 4, and messing up, we gotta always remember to count that one block in the middle –</td><td>LMTY (1): Monique is addressing Leticia’s concern of not having the same number of blocks on each side, saying “we have to make sure we put 4 right there or right there.”</td></tr>
<tr><td>46</td><td>L</td><td>Yeah.</td><td></td></tr>
<tr><td>47</td><td>M</td><td>1, 2, 3, 4… To make sure all those are right there… Same thing for 10 and 100. Just keep going. All you do is put…</td><td></td></tr>
<tr><td>48</td><td>L</td><td>Yeah you just keep…</td><td></td></tr>
<tr><td>49</td><td>M</td><td>You just keep adding onto it.</td><td></td></tr>
<tr><td>50</td><td>L</td><td>Yeah…</td><td></td></tr>
<tr><td colspan="4">Interpretation: The students were all contributing their ideas about constructing the towers, by building on the previous tower. When Leticia raised a concern about having the same number of blocks on each side of the tower, “say 4 on this side and like 5 on the other,” Monique addressed this, saying they have to be careful when they count, and recommends an approach, “we also have to count, … we gotta always remember to count that one block in the middle.” By suggesting the way they should count, she is explaining a method to keep track of information that will help lead the group to a solution.</td></tr>
<tr><td>51</td><td>S</td><td>So do you want to, um, keep doing, um, the pattern? And then like…</td><td></td></tr>
<tr><td>52</td><td>L</td><td>Yeah I think like…</td><td></td></tr>
<tr><td>53</td><td>M</td><td>For these it would be 4….</td><td></td></tr>
</table>

Monique, Leticia, Sherelyn			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
54	S	Yeah I think we should finish	
55	L	For these it would be 4… Finish the pattern for 5 and 10.	
56	M	(inaudible) Because if we draw it… it we draw it it's going to be hard because we gotta make sure that block right there…	
57	S	Yeah… our own visualization	
58	M	(inaudible) With the blocks… and then we could do a table for it	
59	S	Yeah.	
60	L	We could also draw it too.	
61	M	Yeah.	
62	S	Like if they don't get the same…	
63	M	Then we… yeah, if that's the case, if they don't get the one we want to block… we can draw it… but if we draw it, we gotta make sure that we notice that… that we add one. The blocks is right there.	
64	L	But I think that's probably like why… that's why we put A right there.	
65	M	Yeah, A right there, to show that one (off camera Sherelyn gets a bucket of blocks to use)	
66	L	I didn't get that at first and I'm like, what is that?	**EPISODE 4**
67	M	That's…	
68	L	What is that one doing there? But then I noticed…	**LHSIA (12):** Leticia again lets the others know she had been confused about something but now she understands, "I noticed."
69	M	But also… look, that ones in the middle because.. if it wasn't stacked, then you would just notice that… it would be all connected.	**LMTY (3):** Monique gives additional details about the construction of the towers, "If it wasn't stacked…"
70	L	It would kind of be a flat pattern.	
71	M	Yeah.	
72	S	If you took that one off…	
73	M	And then it would just show the four… It would just be like, if you were to include…. (inaudible) Right… the one right here	
74	L	Yeah.	
75	M	Yeah, that one on top… then you just keep going around.	

<table>
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<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td colspan="4">Interpretation: Similar to earlier (line 18), Leticia says to her classmates that she was confused about something but before asking the question says that she figured it out, “I noticed”. By saying this to Monique and Sherelyn, she seems to want them to know that she has clarified her misconception on her own, indicating that she was smart enough to do that. She appears to want her classmates to know that she is staying with them, mathematically, throughout this session. Though Leticia tries to indicate that she understands, Monique still tries to explain the idea she was sharing earlier, which seems to be that the middle block is still there, even if it is hidden within the structure. Monique gives additional details saying, “If it wasn’t stacked… it would be all connected,” suggesting that the construction of the towers would be different. Monique appears to want to help Leticia understand the ideas she was stating moments earlier. Leticia responds in her own words, “It would be kind of a flat pattern,” suggesting that she did understanding. Both Monique and Leticia say, “Yeah” indicating that each understands, possibly leading Monique to feel that she was able to help Leticia.</td></tr>
<tr><td>76</td><td>S</td><td>Like this right here? Like, um… when it goes high you keep adding one, and then when it’s going around, it adds 4.
(she uses her pen to point at the diagrams on Monique’s paper)</td><td>EPISODE 5
LHSIA (14): Sherelyn shares her strategy for building the towers that recognizes that when you add one block to the height, you add 4 more blocks, which is 1 on each side.</td></tr>
<tr><td>77</td><td>M</td><td>Yeah.</td><td></td></tr>
<tr><td>78</td><td>S</td><td>Yeah… (talking over one another)</td><td></td></tr>
<tr><td colspan="4">Interpretation: Sherelyn has been contributing to the conversation, often starting to say something, which becomes inaudible because the others jump in. She also often says, ‘Yeah’ to indicate agreement with her two classmates. Here, the girls had been discussing</td></tr>
<tr><td>79</td><td>M</td><td>So like, 4, then… here is 8….
(the girls take blocks out of the bin and start to construct something; blocked from camera view)</td><td></td></tr>
<tr><td>80</td><td>L</td><td>It would be a 3 block… so that would be a 3 block… that would be a 3 block tower high.</td><td></td></tr>
<tr><td>81</td><td>M</td><td>No, it would be, 1, 2, 3, 1,2,3, 1,2,3 …</td><td></td></tr>
<tr><td>82</td><td>L</td><td>Which would be 6…</td><td></td></tr>
<tr><td>83</td><td>M</td><td>Yeah</td><td></td></tr>
<tr><td>84</td><td>S</td><td>Then to add onto the 5 block high tower, you just keep adding on.</td><td></td></tr>
<tr><td>85</td><td>M</td><td>Yeah</td><td></td></tr>
<tr><td>86</td><td>L</td><td>You know what I think we’re going to have problems on though? I think we’re going to have problems on when the problems done… because of that blue problem.</td><td></td></tr>
<tr><td>87</td><td>S</td><td>Yeah!</td><td></td></tr>
<tr><td>88</td><td>M</td><td>So, 1, 2, 3, 4, 5</td><td></td></tr>
<tr><td>89</td><td>M</td><td>Like, here, see look, see 5 right here, and here</td><td>LHSIA (15)</td></tr>
</table>

Monique, Leticia, Sherelyn			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
		is 4. It's just adding one, right?	Monique begins to explain the tower construction, though no one asked for an explanation. She then asks for confirmation about adding one.
90	L	Yeah	
91	S	The thing is if we was… like I don't think this would work…	
92	L	If we went for the 10 block high tower,	
93	S	5 here…	
94	L	So, 1, 2, 3, 4, 5, 1, 2, 3, 4, 5, so you would use… 6? (They seem to be working together to build a single structure with blocks)	
95	S	Oh…	
96	L	5…	
97	S	Then 1, 2, 3…	
98	L	3	
99	S	3, 4, 5, 6…7…oh wait, um…	
100	At this point the teacher comes to the group and asks the students to fill her on what they have been doing up to this point. They explain their understanding of the construction of the tower. Monique says, "Every time we get one tower high, it adds like, it adds 4 around it." When asked to clarify what a "tower" is, they describe how the blocks are added and state that figure C in the diagram is a 3-block high tower. Ms. S asks the students to explain to her how they know that 5 blocks are added to the tower to create the next structure. Monique also explains the difference between a 5-block high tower and "showing" 5 blocks in the height (because of the middle hidden block). At the teacher's suggestion the students say they should make a table, as that might be helpful for individuals who do not understand how the towers are constructed based on the picture. Leticia also suggests that the table might help her and her classmates come up with an equation. The equation would answer the question to "generalize" and help them come up with the total number of blocks for the 100-block high tower without having to build it with physical cubes. Ms. S then asks the girls to explain their plan to her. They state they are going to make a table that goes up to 100 (for the 100-block high tower). Monique suggests they go		

<table>
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<tr><td></td><td colspan="2">from 1 to 100 in increments of 1, to which Sherelyn suggests they go in increments of 5. Leticia responds suggesting that they could try to get the equation from the first five entries in the table (heights 1, 2, 3, 4, 5). Ms. S responds that they have different options they can pursue at this point, and then asks them how they are going to set up their table. She asks them what the variables are.</td><td></td></tr>
<tr><td>101</td><td>Ms. S</td><td>Sounds good to me. What are your variables here? Because in a table normally… how many, what are you comparing?</td><td>EPISODE 6
The teacher is still with the group, specifically discussing what the variables are for this task.</td></tr>
<tr><td>102</td><td>S</td><td>Blocks and…</td><td></td></tr>
<tr><td>103</td><td>M</td><td>Blocks and how many we’re adding on.</td><td></td></tr>
<tr><td>104</td><td>Ms. S</td><td>Okay think about how you want to articulate that. Because you’re going to need… what do you need in a table? What do you need at the top of the table?</td><td></td></tr>
<tr><td>105</td><td>M</td><td>Two</td><td></td></tr>
<tr><td>106</td><td>Ms. S</td><td>Yeah, so you’re going to label, you need to give me a heading, so I know what you’re talking about. Even though you might know what you’re talking about, say blocks or whatever, it needs to be very clear, so that if I, you know, brought your little brother that’s in 5th grade in here, he would need to be able to look at the problem, and look at the table, and know exactly what we’re comparing here.</td><td></td></tr>
<tr><td>107</td><td>L</td><td>(looking at Monique and Sherelyn when saying this) I think what… I think what she’s trying to say is that instead of saying how many blocks we need to add on and not knowing what that next thing is, I think we should… how many blocks compared to how tall the tower is. So, say we have a 5 blocks high tower… like, how many blocks…</td><td>LMTY (5): Leticia provides her two classmates with her interpretation of the teacher’s suggestion, that they should compare the height to the total number of blocks (in a table).</td></tr>
<tr><td>108</td><td>M</td><td>Yeah</td><td></td></tr>
<tr><td>109</td><td>S</td><td>How many blocks around it</td><td></td></tr>
<tr><td colspan="4">Interpretation: Throughout this episode, the students have answered the teacher’s questions about how they thought about the construction of the towers and how they plan to move forward. The three students agree on making a table, and the teacher helps them think about what they need to properly construct a table so others looking at it will understand. Ms. S explains in general terms that they need to label the table and each of the variables, or columns, represented in the table. Leticia interprets this guidance from her teacher and gives her interpretation to her two classmates.
Leticia specifically states that she thinks that the table should include both the height of the tower, “how tall the tower is” with “how many blocks” in total in the tower structure. This is different</td></tr>
</table>

Monique, Leticia, Sherelyn			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
from Monique's earlier suggestion of, "Blocks and how many we're adding on." Ms. S pointed out that "blocks" did not give specific information (e.g., the height of the tower or the total number of blocks in the structure?), and Leticia builds on this idea saying that they need to know how tall the tower is. She also suggests that if they write how many blocks they need to add (which should be constant, 5 blocks are added to build the next tower structure) they will not know the total number of blocks in tower structure. This is a unique interaction in that a student's interpretation of the teacher's instruction is given while the teacher is still present with the group.			
110	The teacher then moves onto asking the students what they will do when writing an equation to represent the relationship between the height of the tower and the total number of blocks needed to build the tower. Independent and dependent variables are identified by the students, particularly the fact that the height of the tower should be the independent variable, making the total number of blocks the dependent variable. The teacher then leaves and moves onto another group.		
111	M	Okay, so, instead of doing… instead of finishing it up, I joined it because it would be hard to keep the blocks.. um… (is pointing with her pen at her paper)	*She may be referring to the table (see student work); it is not clear whether she started to construct the one on her paper yet.*
112	L	Yeah	
113	S	So make a table	
114	M	Yeah, a table… but …the height, so that's…	
115	S	We could put height and then in parentheses, blocks	
116	M	And then for the other one, we're just going to put how many blocks, and put numbers…	
117	L	I think we should say how tall the towers going to be because we're actually doing the height for the tower.	
118	M	Yeah that's for the height, but for the, um…	
119	S	Around it.	
120	L	I just think we should write a note to keep in mind, so the independent variable is the height. (Each student takes a moment to write independently on their own papers.)	
121	M	It's the… it's the 2… the blocks (inaudible)	**EPISODE 7**
122	S	I think, um…	
123	L	What did you say?	
124	M	That the table is going to have two columns.	**LMTY (1):** Monique is explaining her approach to set up the table so they can record their

Monique, Leticia, Sherelyn			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
			information properly.
125	S	I think that, um… around it…	
126	M	How many blocks there is, right?	**LHSIA (15):** Monique asks, "right?" to have her classmates confirm that her suggestion to label one of the columns as "how many blocks."
127	S	Then for the height of the tower	
128	S	So it's the total number of blocks?	
129	M	The total number in all, so, it's how many blocks in all that are there. So for the two… its…	**LMTY (7):** Monique responds to Sherelyn's question, confirming again that the headings for the table should be the total number of blocks, as well as the height of the tower.
Interpretation: It appears that Monique may have activated both the LMTY and LHSIA structure in this brief episode. The three girls have been discussing the table to create to organize their information. First, Monique appears to be trying to help the others by suggesting that there are two columns in the table. Leticia asked Monique to repeat this information, as she may not have heard the first time. (It was almost impossible to hear that on the audio/video recordings.) Then, Monique suggests that one of the variables in the table should be labeled the height of the blocks, but she asks for confirmation from her classmates. This suggests that she believes she is correct, and wants to demonstrate this correct idea by having her classmates agree with her. At this point, Sherelyn asks Monique to confirm that they are talking about the total number of blocks, which Monique does. She explains, "the total number in all, so it's how many blocks in all that there are." So it appears that Monique was willing to help her classmates, by explaining her ideas and answering their questions. At the same time, she appeared to want her classmates to recognize her as smart or knowledgeable by having them confirm her ideas.			
130	S	I think, um, that tower adds by one and around it has 4.. like, um, if we wanted to get 100 blocks, it would be 100, and then… it would be… let me see, um….	
131	Unknown	Hold on	
132	S	Like, um…	
133	M	We're actually adding on 5	
134	S	No…	
135	L	4…	
136	M	No 4	
137	S	No, 5, 1, 2, 3, 4, 5.	
138	M	Because that block in the middle is acting as another block and then that's all of them, so…	
139	L	Oh… ohhh! (in an excited voice, smiles)	**EPISODE 8**
140	M	If we got 100, it would be 100 plus 5 right?	
141	L	Hold on, I think I got an idea. So it's like,	**LHSIA (14):** Leticia

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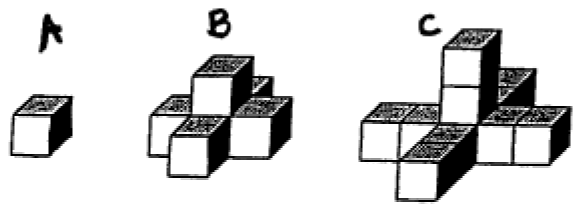
<tr><td></td><td></td><td>how many, how high you want it, is like, say you go with 1, it’s 5 blocks. If you wanted, um a 2 block high tower, it would be 10 blocks.</td><td>suggests a strategy that correlates a height of 1 with 5 and a height of 2 with 10.</td></tr>
<tr><td>142</td><td>S</td><td>Huh?</td><td></td></tr>
<tr><td>143</td><td>M</td><td>Wait, explain that over again?</td><td></td></tr>
<tr><td>144</td><td>L</td><td>You see how every time, so every time we have one block, right? And one tower? It has 5 all around it? Yeah, so, um, so for every… for every one we add on, it would be 5 added <u>on</u>. So in the table it would be like 1, 2, 3, 4, 5, 5, it would be like, 10, 15, 20, 25.</td><td>LMTY (1, 3): Leticia explains her strategy, using better examples: If you are adding 5 blocks to the total each time, to go from tower of height 1 to tower of tower 2, add 5 blocks, height of 1 to height of 3, add 10 blocks.</td></tr>
<tr><td colspan="4">Interpretation: Leticia appeared to be excited as she gained an insight into the problem. She demanded her classmates’ attention as she tried to explain that when adding one block to the height, you add five blocks. She then said that if you want a 2-block high tower, “it would be 10 blocks” which appeared to confuse her classmates, as they asked her to explain it again. Leticia then seemed to branch from an active LHSIA structure into an active LMTY structure. She reminds her classmates that to build a tower, they add one block to the height, and add a total of 5 blocks (4 blocks on the sides). She then continues her explanation, using specific examples of different tower heights. She appears to be suggesting that since they add five blocks every time a height is increased by 1, they can skip going one at a time, and can add a total of 10 blocks when increasing the height by 2, or a total of 15 blocks when increasing the height by 3.
Alternative: Leticia may be suggesting that the total number of blocks is obtained by multiplying 5 by the height. However, this is not explicitly stated (the way other students in this and other classes have stated it) and she does not seem to jump to the idea that a 10-block high tower has a total of 50 blocks.</td></tr>
<tr><td colspan="4">Between the end of this conversation and the end of class (~7 mins), the three students Monique, Leticia, and Sherelyn discuss how to record the information in the table. They seem to be determining their method to find the total number of blocks for the requested towers (of heights 5, 10, and 100), as well as how to best set up their table. They discuss whether to set it up to enter information horizontally or vertically, and they all suggest whether they should record the height as increasing by 1, or by a different number.
The students then discuss what they should enter into their tables. They briefly discuss the idea that the total number of blocks is 1 when the height is one, as Leticia suggests the 1-block high tower is included on their papers as a placeholder or reminder.
The girls then spend some time writing on their papers quietly. They are still writing, with the occasional question to one another when Ms. S instructs the class to start packing up their work.</td></tr>
</table>

Monique's Work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



① Independent: the height of the tower

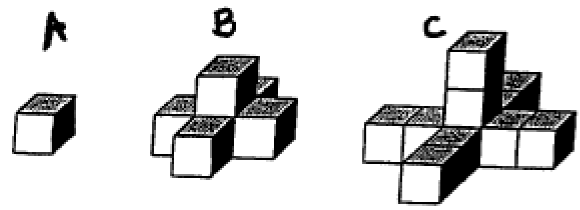
(blocks) Height	0	1	2	3	4	5	6	7	8	9	10
Blocks total	0										

② Dependent: How Many Blocks in All

Leticia's Work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



Independant variable = height of the tower

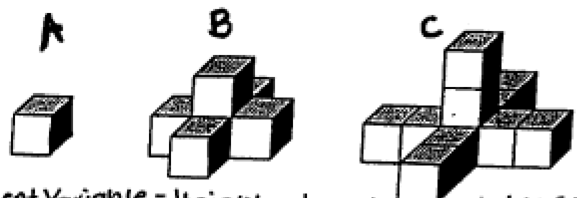
Dependant variable = how many blocks in all (total number)

Height of tower (blocks)	0	1	2	3	4	5	6	7	8	9	10
Total blocks	0	1	6								

Sherelyn's Work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



Independent Variable = Height dependent variable = Total blocks in all

Height(blocks)	0	1	2	3	4	5	6	7	8	9	10	
Total blocks	0	1	6	11								

Questionnaire Responses for Monique, Leticia, Sherelyn

Questionnaire items which may indicate LMTY structure

Questionnaire Items	Monique	Leticia	Sherelyn
Statements (All the time, Sometimes, Never)			
I wanted to teach another student something that I knew that the other student did not know.	All the time	Never	All the time
I listened carefully to the ideas of someone I was trying to help.	All the time	All the time	All the time
I helped someone see how to do the math.	All the time	Never	All the time
Others listened carefully to my ideas.	Sometimes	All the time	All the time
Behaviors (Often, Sometimes, Hardly ever)			
I gave helpful suggestions.	Often	Often	Often
I worked cooperatively.	Often	Often	Often
Thoughts (Yes/No)			
I like teaching this person things that I know.	Yes	Yes	Yes

Questionnaire Responses for Monique, Leticia, Sherelyn

Questionnaire Items Which May Indicate LHSIA structure

Questionnaire Items – Statements	Monique	Leticia	Sherelyn
Statements (All the time, Sometimes, Never)			
I wanted people to think that I'm smart.	Sometimes	Sometimes	Sometimes
I tried to impress people with my ideas about the problem.	Sometimes	Sometimes	Sometimes
People seemed impressed with the ideas I shared about the problem.	All the time	All the time	All the time
People saw how good I was at the math we did today.	Sometimes	Sometimes	Sometimes
I felt smart.	All the time	All the time	All the time
I wanted to show someone that my way was better.	Never	Never	Sometimes
I was a lot better at math than others today.	Sometimes	Never	All the time
Behaviors (Often, Sometimes, Hardly ever)			
I was the leader.	Sometimes	Sometimes	Sometimes
I was bossy.	Hardly ever	Hardly ever	Hardly ever
I wanted to show off.	Hardly ever	Hardly ever	Sometimes
I liked to be right.	Sometimes	Sometimes	Sometimes
Thoughts (Yes/No)			
I want you to know just how smart I am.	Yes	Yes	No
People think I'm smart.	Yes	No	Yes
I wish the teacher would call on me, so I can show how much I know.	Yes	No	Yes

Nadira, Kevon, Georgia

This is Group 5 from Ms. S's grade eight Class 1.

Verbal emphasis indicated by <u>underline</u>
Actions included either in *italics* or (parentheses) as appropriate.
Pauses in speech are indicated by …
Words inserted to help clarify a student's words are included in [brackets].
When students voices overlap, one or both sets of words are included in //double slash marks//.

The number included next to either LMTY or LHSIA refers to the code, included in the table of codes.

T: Teacher
U: Unknown
Sts: Several Students

Nadira, Kevon, Georgia			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
	The teacher has just finished her introduction to the problem and given instructions. This has taken a little more than three minutes. The three students are sitting at desks which are situated to all face one another. *The students look at their papers and start discussing the problem.*		
1	Georgia (G)	I read the problem. Look, it's like when they have only one thing, there's nothing, you know, ok when there's two blocks high, they only add one because it only shows one on the top, right? But you know that there's two, so then they add one, so if there's two only showing then they're saying only add two, but then the three…..you know what I mean?	**EPISODE 1** **LHSIA (14):** Georgia is the first to share her ideas about the problem, "Look, I read the problem," and then goes to explain her understanding of the problem. **LMTY (10):** Georgia asks if her two classmates understand her explanation of how the towers are constructed.
Interpretation: Georgia was the first student to speak, perhaps suggesting that she felt confident with her ideas about the problem. She seems to be explaining that the towers are constructed so that the middle base block is still there (from tower A) even though it is hidden in towers B and C. Georgia ends her statement asking, "You know what I mean?" in a tone of voice that suggests she sincerely wants to know if her classmates understand her interpretation of the task.			
2	Nadira (N)	I understand what you're saying, but….	

<table>
<tr><th colspan="4">Nadira, Kevon, Georgia</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td>3</td><td>Kevon (K)</td><td>Like if there was three then all sides would be three</td><td></td></tr>
<tr><td>4</td><td>G</td><td>Yeah, cuz the only ones that they show
(using her pen to point to the figures given on the task paper)</td><td>LMTY (1): Georgia explains construction of the towers, about the number of blocks on the side of each tower of a certain height. (Continues for several speaking turns)</td></tr>
<tr><td>5</td><td>N</td><td>Zero, one, two.</td><td></td></tr>
<tr><td>6</td><td>G</td><td>Yeah</td><td></td></tr>
<tr><td>7</td><td>N</td><td>For the 3 block</td><td></td></tr>
<tr><td>8</td><td>G</td><td>This one is actually two but they're only showing one block on the top</td><td></td></tr>
<tr><td>9</td><td>N</td><td>And this one is 3</td><td></td></tr>
<tr><td>10</td><td>G</td><td>Yeah this one is 3, but they're only showing 2 so there's two around</td><td>LMTY (2; 6): Georgia modifies her earlier explanation in response to Nadira's statement, "This one is 3," likely meaning the 3-block high tower. Georgia also builds on this idea by stating, "They're only showing 2 [blocks] so there's 2 around." They are likely referring to figure C.</td></tr>
<tr><td>11</td><td>N</td><td>So for every 1 block</td><td></td></tr>
<tr><td>12</td><td>G</td><td>So if there was 4 blocks high they only show 3, and they mean 3 on each other side.</td><td>LMTY (3): Georgia provides additional details to demonstrate what she thinks the 4-block high tower looks like, as it is not shown on the task page.</td></tr>
<tr><td colspan="4">Interpretation: Georgia's two classmates responded to her initial explanation of the problem, with statements interpretation what she said. Georgia appears to take these statements as indications that she should continue to explain her understanding of the problem, so that her two classmates also understand. She continues to build on what her classmates say, to explain that for the 3-block high tower there are two blocks visible on each side as well as in the height. She provides additional details stating that 4-block high tower similarly has three blocks visible on each side and in the height. Georgia's two classmates appear to be receptive to the information given, as they restate and try to build on her explanations.</td></tr>
<tr><td>13</td><td colspan="2">At this point the students look as though they are about to</td><td></td></tr>
</table>

<table>
<tr><th colspan="4">Nadira, Kevon, Georgia</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observations</th></tr>
<tr><td></td><td colspan="2">write independently on their papers. The teacher comes to the group and asks what they have decided so far. They restate the information they discussed among themselves. Ms. S asks if they know what question(s) they are trying to answer, and they refer to the task paper, saying the 5-block high tower and the 10-block high tower. Ms. S then leaves them to do their work.</td><td></td></tr>
<tr><td>14</td><td>K</td><td>But how are you supposed to get all the other... look, you see how there’s 2, you’re saying there’s 2 and there’s 1 on the bottom that you can’t see, right? How are you supposed to get 5?</td><td>EPISODE 2</td></tr>
<tr><td>15</td><td>G</td><td>Yeah this is a 2 block high, not the one’s around it, like…</td><td></td></tr>
<tr><td>16</td><td>K</td><td>But no, I’m saying like, if you move that block under the top one there should be another one or there’s not?
(pints to the diagram on Georgia’s paper)</td><td></td></tr>
<tr><td>17</td><td>G</td><td>No, because if it’s two blocks high already….then they just….like the squares…(laughs, tries to gesture with her hands) haha, let me see…like whatever…there’s one on the top and one on the bottom but you can’t see it because you’re covering it</td><td>LMTY(1, 7): Georgia is responding to Kevon’s question, which appears to be about getting a total of 5 blocks in a tower structure (rather than constructing a tower with height of 5). She struggles with her answer, as she tries to use her hands to help explain what she means by a 2-block high tower. She explains that middle base block is still there even though it is no longer visible “because you’re covering it.”</td></tr>
<tr><td colspan="4">Interpretation: After the teacher leaves the group, Kevon returns to the ideas to ask a question about how you “get 5?” It seems (from this question and later turns) that he believes the goal is to get a tower with a total of 5 blocks, which is different than the actual goal to get a tower with a height of 5 blocks. Georgia responds to Kevon’s question, though it is not clear if she understands what his question is. She struggles to explain, and appears to try to help him understand that the block is still there though it is no longer visible since the other blocks build off of it.</td></tr>
<tr><td>18</td><td>K</td><td>Yeah but that’s 6 right? That’s all blocks around it…</td><td></td></tr>
<tr><td>19</td><td>G</td><td>1,2,3,4,5,6…yeah…</td><td>LMTY (7): Georgia</td></tr>
</table>

Nadira, Kevon, Georgia			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
			responds to Kevon's question, confirming that there is a total of 6 blocks in the 2-block high tower.
20	K	That's 6, that's 5?	
21	G	No, 5 is high tower, not like… this one is 1, this one is 6, (counting) this one is 11, you don't have to be like an exact limit of 5	**LMTY (3, 7):** In response to Kevon's question, Georgia continues with the total number of blocks in the three towers given on the task paper (figures A, B, C).
Interpretation: Georgia continues to respond to Kevon's question about there being 6 blocks in the 5-block high tower. She counts, probably to confirm this for herself, but lets Kevon (and Nadira) know that 6 is her answer as well. As Kevon seems confused about the 6 blocks and seems to think there should be 5 blocks in that tower structure. Georgia appears to recognize this confusion and tries to address it. She states the number of blocks in the 2-block high tower (as a total of 6 blocks) and the 3-block high tower (a total of 11 blocks), which is depicted in figure C on the task.			
22	N	I know it. I know the answer	**EPISODE 3** **LHSIA (4):** Nadira claims she knows the answer.
Interpretation: Nadira had been working on her own paper for a few moments, quietly. She then lets her classmates know that she has an answer. She appears to want them to know that she has the ability to determine the answer, and in the turns below, she tells them what her answer for the total number of blocks in the 100-block high tower.			
23	G	For 100 it'll be 99…	
24	N	200 blocks.	
25	G	What?	
26	N	200 blocks, right?	
27	K	Yeah, times 10.	
28	N	See, yeah!	
29	G	What? (sounds surprised, as though she does not understand what her two classmates are saying)	
30	N	You just said it, you don't get it.	
31	K	2 times 10 is 20, 2, 4, 6, 8, 10….20, 40, 60, 80.	
32	G	2, 4, 6, 8, 10, and there's one down there, there's 11.	
33	K	You know if you move that to keep that up in the middle, you have to have 1 in the bottom. (looking at Nadira as he talks)	**LMTY (4, 5):** Kevon corrects the earlier idea that there are 10 blocks in the 3-block high tower, and builds on the

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<tr><td></td><td></td><td></td><td>Georgia’s contribution about the “1 in the bottom.”</td></tr>
<tr><td colspan="4">Interpretation: Kevon seems to have shifted from being confused and asking questions to explaining the construction of the towers to Nadira who appears to still be confused. He corrects an earlier idea, which he stated, about multiplying by 10. He now appears to agree with Georgia’s (correct) interpretation, which includes the fact that the middle base block is still there, though it is not visible. He seems to not only agree, but also understand, as is demonstrated by this explanation. He seems to be trying to help Nadira understand this idea, since he looks at her while speaking.</td></tr>
<tr><td>34</td><td>G</td><td>So it’ll be 11.</td><td></td></tr>
<tr><td>35</td><td>N</td><td>No, no wait, you’re confusing me now.</td><td></td></tr>
<tr><td>36</td><td>G</td><td>Ok. (laughs)
Um, you know, can we get up and get a box of blocks?</td><td></td></tr>
<tr><td>37</td><td>N</td><td>How many? 80, 90, 10?</td><td></td></tr>
<tr><td>38</td><td>G</td><td>No, 11, 12, 12.</td><td></td></tr>
<tr><td>39</td><td>N</td><td>Nadira gets up to get blocks. The other two students do not speak while Nadira gets blocks.</td><td></td></tr>
<tr><td>40</td><td>G</td><td>This one? Doing this one right?
(points to one of the figures given in the diagram)
Look, Nadira for this one, there has to be 3 but they cover it with these so you don’t see it.
(As she speaks, puts together the 3-block high tower with actual blocks)</td><td></td></tr>
<tr><td>41</td><td>K</td><td>You see that, right?</td><td></td></tr>
<tr><td>42</td><td>N</td><td>Yeah I know that part, but what I was saying was like whatever how many blocks is stacked up the tower times 2, that’s what I was saying, that’s how I got 200, see 3 times whatever, times 2 is 2, 4, 6, right? Or no? Or yes?</td><td>LHSIA (12, 15): Nadira explains her idea, which involves multiplying the height by 2. She ends her turn by asking if her two classmates agree with her or not.</td></tr>
<tr><td>43</td><td>K</td><td>I get you though, look cuz I see this, 2 right there times 2 is 4.</td><td></td></tr>
<tr><td>44</td><td>N</td><td>Yeah that’s what I was saying.</td><td></td></tr>
<tr><td>45</td><td>G</td><td>Ohhh.</td><td></td></tr>
<tr><td>46</td><td>N</td><td>But now I see it doesn’t work cuz of that last part right there….I’m sorry…</td><td>LHSIA (12): Nadira recognizes, with Kevon’s help, that her idea does not work out.</td></tr>
<tr><td colspan="4">Interpretation: Nadira has been on the ‘tutee’ end of Georgia’s explanations earlier, and expressed confusion a few moments ago. She appears to want to let her classmates know what her ideas are. She may be doing this in order to demonstrate to them her mathematical knowledge or ability. When Kevon explains why her idea, to multiply by 2, does not work, she admits this herself. Nadira may be saying this in order to have others believe that she understands the ideas they shared, or that she can keep up with them mathematically.</td></tr>
</table>

Nadira, Kevon, Georgia			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
47	K	How to get 100?	**EPISODE 4**
48	G	No, it would be the same thing like you just keep going on if it's 4 blocks high then you just add….	
49	K	Multiply by 10…you just multiply by 10 then...10, 30, 20, 20,..no wait	
50	G	Huh?	
51 **52**	N	Why multiply by 10? //Yeah, that's not making any sense.//	
53	K	//No, wait.//	
54	G	See look, you know what I did before? When they only show 1 it's 1 of those, so when they only show 2, it's one of those, and when they only show 3, it's one less, so when they show 100 but then they only show 99…so it would be 99 on all sides…get it?	**LMTY (1, 2, 10):** Georgia explains construction of towers, giving additional details for the 100-block high tower. She explains that there is one less block in each leg than there is in the height. She ends by asking if Kevon understands this explanation.
55	N	I like that way…	
56	K	What way?	
57	N	Her way.	
58	G	You don't get it? (to Kevon) Yes, you do….Then what? (Kevon shakes his head)	
59	N	Ok now I get it, I get it.	
60	G	(to Kevon) If you don't get it just tell me.	
61	K	I'm just trying to say, how do you get 100?	
62	G	That's what I said, how do you get 100? Ok, do, go up to 5.	
63	K	Ok so how you supposed to get 5 if there's a block under? (points to diagrams on Nadira's paper as he asks) How you supposed to get 5?	
64	N	You go like…..we need more blocks though, oh no, we don't.	
65	N	So if it's 5 there should be 4 all around. If it's 5 there should be 4 all around, right?	
66	G	If it's 5 then only 4 should be showing (constructing partial tower with the few blocks they have – not a whole set – to demonstrate the height of 5 and 4 on two of the four sides)	**LMTY (1):** Georgia is again responding to Kevon's apparent confusion about what the 5 tower means. She tries to demonstrate, using the cubes this time, that the

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<tr><td></td><td></td><td></td><td>5-block high tower has 4 blocks on each of the sides.</td></tr>
<tr><td colspan="4">Interpretation: Georgia explains to her two classmates that for a 2-block high tower there is only one block on each side, and again it’s “one less” on each side for the 3-block high tower. She uses the example of the 100–block high tower, saying that there will be 99 blocks on each side. She first explains to her two classmates and then modifies the explanation, likely because Kevon was suggesting that they can multiply by 10 in order to get the total number of blocks in a different tower. Nadira claims to understand this explanation, but Kevon admits he is still confused.
At turn #, it appears that Kevon still believes that there should be a total of 5 blocks in a tower structure, rather than a tower with a height of 5 blocks. With this, Georgia continues to explain, using the manipultatives, blocks, to assist with the explanation. Her persistence suggests that she wants her classmates to understand the concept of how the towers are constructed.</td></tr>
<tr><td>67</td><td>N</td><td>Four <u>around.</u></td><td></td></tr>
<tr><td>68</td><td>G</td><td>4, no, yeah, 4 around</td><td></td></tr>
<tr><td>69</td><td>N</td><td>So 4, 8, 12, 16, so 5 would be 16</td><td></td></tr>
<tr><td>70</td><td>N</td><td>4 times 4 is 16. Is it not 16?</td><td></td></tr>
<tr><td>71</td><td>G</td><td>4 times 4 is 16, yeah because that’s on the other side so then how about the 5 standing up, look 16 plus 5 is 21. (laughs) You get it Kevon?</td><td></td></tr>
<tr><td>72</td><td>K</td><td>Ya’ll was talking to each other.</td><td></td></tr>
<tr><td>73</td><td>G</td><td>I’m sorry, ok so 5 only 4 will be showing because the ones on the side will be covering but actually it’ll 5 tall but it only show 4 so it’ll be 4 around, get it?
(she modifies the partial construction as she speaks)</td><td>LMTY (2): Georgia responds to Kevon, modifying her explanation to demonstrate that the 5-block high tower has 4 blocks on each side.</td></tr>
<tr><td colspan="4">Interpretation: Georgia and Nadira had discussed the total number of blocks in the 5-block high tower. At the conclusion of this discussion, Georgia asks Kevin if he understands their strategy. He claims they were “talking to each other” and not giving him the necessary explanation. This prompts Georgia to modify her explanation for him. As she speaks she moves the blocks to help demonstrate the 4 blocks on each side of the 5-block high tower. Kevon seems to understand at this point, and agrees with her interpretation of the problem.</td></tr>
<tr><td>74</td><td>K</td><td>(agrees)</td><td></td></tr>
<tr><td>75</td><td></td><td>Students are working on the problem on the paper for about two minutes.</td><td></td></tr>
<tr><td>76</td><td>N</td><td>Ok, now I’m finished, now all you have to do is hurry up</td><td>LHSIA: (16)
Nadira tells the others that she has finished writing on her paper and that she wants the others to finish as well.
This also suggests GTJD.</td></tr>
<tr><td colspan="4">Interpretation: The three students in this group concluded their discussion of the problem and then each wrote down their responses individually on their own papers. Nadira breaks this</td></tr>
</table>

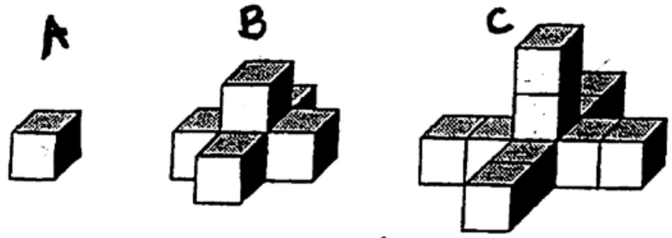
Nadira, Kevon, Georgia			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
independent work time by stating that she has finished, and that the others need to hurry up. This suggests that Get The Job Done was one active structure, in that she seems to want the work to be complete. Nadira seems to be instructing her two classmates to finish writing. Also, by telling her two classmates she is finished, she is suggesting that she is "one-upping" her classmates by being finished before either of them. She may be looking to "out-do" one or both of them, especially if she feels she has not been one of the "smart ones" in the group.			
77	G	Huh?	
78	N	All you have to do is hurry up	
79		*The students discuss briefly whether they should draw representations on their papers.* *Students work independently on their own papers.*	
80	N	What's 36 plus 5?	**EPISODE 5**
81	K	Huh?	
82	N	What's 36 plus 5?	
83	K	41	**LMTY (7):** Kevon responds to Nadira's addition question.
84	N	Thank you	
Interpretation: The students had been working independently on their own papers for several (about 5) minutes. Nadira, without any other explanation, asks her classmates for the sum of 36 and 5. Kevon responds to this question briefly, with a correct answer. By giving the response, Kevon may have been trying to help Nadira move on with the problem.			
85	*The three students continue to work quietly on their paper for a few moments. Nadira asks another arithmetic question: 99 x 4. After a brief discussion, Georgia first tells her to add the numbers together and then replies the answer is 396.*		
86	*Ms. S comes to this group while they are working and asks for them to explain what they have done since she last visited this group.* *Georgia starts to explain her strategy that there is one less block on each of the sides than there is in the height. While she starts to work with the blocks to demonstrate her thinking, the teacher has Nadira share the equation that she has come up with on her own. Kevon looks on as Nadira and Ms. S talk and s Georgia works with the blocks. Ms. S helps Nadira realize that her equation is written incorrectly, due to the nature of the order of operations. (She had obtained an answer of -2 blocks in the total tower structure, which was obviously not correct.) Ms. S encourages the students to continue to work together to combine the ideas they have shared. She also suggests they create a table or graph to represent the information they already have.* *After the teacher finishes her conversation with these students, she instructs them to begin packing up their materials.*		

Nadira, Kevon, Georgia			
Line No.	**Spkr**	**Transcript**	**Structure; Observations**
	It appears that Georgia, Nadira, and Kevon continue to share ideas, but it is inaudible. They then pack up, as instructed.		

Nadira's work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



for any # of blocks in the tower plus -1 would give you the number of blocks that will be put on each 4 sides. Then you add in the # of blocks in the tower and you get the number of blocks for each tower.

# of blocks in tower	Total # of blocks
5	2
10	45
100	401

# of block in tower	Total # of blocks

5+-1=4*4=16+5=2
10+-1=9*4=36+5=
100+-1=99*4=396+5
401

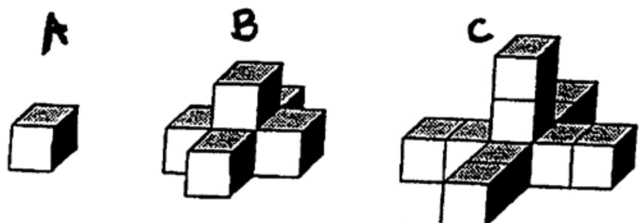
b+-1 4 1*4

$$\begin{array}{r} 99 \\ \times\ 4 \\ \hline 396 \end{array}$$

Kevon's work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



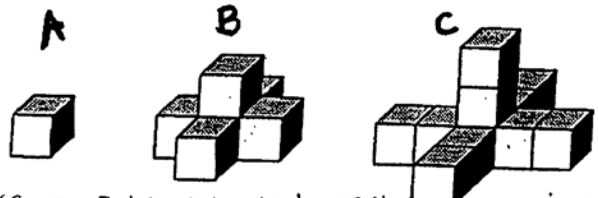
In order to be able to get any size tower you would notice that all the blocks around the tower cover another block. To make a 5-Block high tower you would need to put 5-blocks stacked up and one less around it, so you would put 4-blocks around it. So for a 5-Block Tower You would need 21-Blocks.

For a 10-Block High Tower you would do the same as what you did for a 5-Block High Tower but you just use more blocks. You stack 10-Blocks and put 9-Blocks around it

Georgia's work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



You can make a 5-block high tower by following the pattern. the pattern I see is for figure B. If the tower is 2 blocks high, it only shows 1 block on the top because it is covered by the blocks on the side. So if only 1 block is showing, one block should be on each side. In conclusion if I have a 5 block high tower, only 4 blocks will show so that means four on each side so you have to do 4*4 (for 5-block high tower) (4 = # of sides and other 4 = # of blocks on each side.) 4*4 = 16 then 16 + 5 = 21. I added 5 because it is a 5 block high tower.

- You do the same for 10 block high tower. 10 blocks stacked, only 9 showing, so 9 on each side. (9*4 = 36 36 + 9 = 45 # of blocks
- Same for 100 block high tower. 100 blocks stacked, only 99 showing. So 99 on each side. (99*4 = 396 + 100 = 496 — # of blocks

396

Questionnaire Responses for Nadira, Kevon, Georgia

Questionnaire items which may indicate LMTY structure

Questionnaire Items	**Nadira**	**Kevon**	**Georgia**
Statements (All the time, Sometimes, Never)			
I wanted to teach another student something that I knew that the other student did not know.	Sometimes	Sometimes	All the time
I listened carefully to the ideas of someone I was trying to help.	Sometimes	All the time	All the time
I helped someone see how to do the math.	Never	Sometimes	All the time
Others listened carefully to my ideas.	Sometimes	All the time	All the time
Behaviors (Often, Sometimes, Hardly ever)			
I gave helpful suggestions.	Often	Sometimes	Sometimes
I worked cooperatively.	Sometimes	Often	Often
Thoughts (Yes/No)			
I like teaching this person things that I know.	No	Yes	Yes

Questionnaire Responses for Nadira, Kevon, Georgia

Questionnaire Items Which May Indicate LHSIA structure

Questionnaire Items – Statements	**Nadira**	**Kevon**	**Georgia**
Statements (All the time, Sometimes, Never)			
I wanted people to think that I'm smart.	Sometimes	All the time	Sometimes
I tried to impress people with my ideas about the problem.	Never	Sometimes	Sometimes
People seemed impressed with the ideas I shared about the problem.	Sometimes	Sometimes	All the time
People saw how good I am at the math we did today.	Never	Sometimes	Sometimes
I felt smart.	Sometimes	Sometimes	All the time
I wanted to show someone that my way was better.	Sometimes	Never	Never
I was a lot better at math than others today.	Sometimes	Sometimes	All the time
Behaviors (Often, Sometimes, Hardly ever)			
I was the leader.	Hardly ever	Sometimes	Sometimes
I was bossy.	Hardly ever	Hardly ever	Hardly ever
I wanted to show off.	Hardly ever	Hardly ever	Hardly ever
I liked to be right.	Often	Sometimes	Often
Thoughts (Yes/No)			
I want you to know just how smart I am.	Yes	Yes	No
People think I'm smart.	No	No	No
I wish the teacher would call on me, so I can show how much I know.	No	No	No

Ms. S Class 2 Introduction to Building Blocks Task

This is the introduction Ms. S. gave to her grade eight Class 2.

T: Ms.S / Teacher
St(s): Unknown Student(s)
Ty: Tyesha

T: Okay, so, today we are actually going to depart from the normal textbooks for a minute, but these problems are probably going to be very similar to problems you have already been working on. The problems you have in front of you are called a Building Block dilemma. (static) stack up .. distribute them to each other. And I need someone to read the problem for me. Tyesha go for it.

Ty: Start now Miss (inaudible)?

T: Uh, can you hold on for a minute? Now remember as Tyshonnah is reading through the objectives, I mean is reading through the problem, you need to make sure you are reading it well because we are kind of going to have to work through and figure out what it means, what its asking you (inaudible). Tyshonna, go ahead.

Ty: Building block dilemma. I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes i would need to build a five block high tower, and a 10 block high tower, and a 100 block high tower. Generalize if you can on how many blocks I will need for any size tower.

T: Okay, so somebody kind of, in your own words, right, tell me what is going on here, what the scenario is, what's the situation?

L: Trying to build a 5 block tower, a ten block tower and a 100 block tower.

Ty: You're trying to find the pattern like if, like on the paper for a there is one block, for B there is 5 blocks, for c there is 10 blocks. You try keep on, um, in a steady pattern until you figure out how many blocks it is in 100 blocks, like your trying to go all the way to one hundred, build the blocks til you get to 100.

T: Does that sound right? Okay, so my question is, it talks about, Luis you just said something about a 5 block tower, and then Tyesha, you just said in b there are 5 blocks, right? Is that true? Is that what you said?

Ty: Mhm.

T: Is that the 5 block tower? (Students say yes, shake head yes.) Okay, so read that last sentence, read that I will need to build, okay, go ahead, read from there for me.

Ty: I will need, umm wait. I will need to build a 5 block high tower.
T: Stop there. Okay, go ahead, one more time?

Ty: I will need to build a 5 block high tower.

T: Is letter B a 5 block high tower? (Class: 'no') Okay, why not? You just said it was. Ordena go ahead
(Ordena does not respond.)

Ty: Maybe cause the height, like, in the middle of the height, it has to be five?

T: Is that true? These are the types of questions, go ahead.

St: I think I know what it's saying, um. I think what she meant is we need, um, we have to like figure out how any cubes are made out of 5 blocks.

(Another student in another group responds.)

T: Okay so here is the scenario.
Okay, the reason I asked you that question, the reason I kind of pointed out that ambiguity to you is one of the things you are going to need to do before you even start doing the question is you need to figure out what that question is asking you. So as a group, you guys are working in groups obviously, today, as a group, you guys are going to need to have a discussion about, before you even start trying to solve the problem, coming up with unanimously or agreeing upon what is the question asking you. Once you figure out what the question is asking you, you should be able to justify that to me. So if you come over and you say well yes Miss (Samuels) this is the 5 block tower, something in the question, something in either your past math knowledge, something in the picture needs to kind of tell me that you know, can support what your saying, that you can explain your answer. Does that make sense? (Class: "Yes") Okay, so you wanna go through, and what I would suggest is you wanna start thinking, before you start thinking about strategizing on how to solve the problem, come up with your group on a consensus on what the problem is. Once you come up with what the problem is, then you can go ahead and kind of start generating your data or generating different ways of solving the problem. And keep in mind previous strategies we've used for organizing a bunch of information when we have a very open problem like this. Okay?

Students begin working on the problem in their small groups.

T: And remember, if you are in a group, and somebody is saying something to you, saying the problem means this or the problem means that, and you are not convinced that thats what the problem means, you need to voice that opinion.
Oh and also, feel free to use any of the resources in the classroom. If you need graph paper, you know where the graph paper is. If you need (inaudible) of any

kind, we have plenty up here. If you need calculators, or like whatever it is you need in the room, they are available to you. But before you start doing that, you need to first figure out what the problem is asking you before you start looking.

Leo Ta'keisha Ordena

This is Group 1 from Ms. S's grade eight Class 2.

Verbal emphasis indicated by <u>underline</u>.
Actions included either in *italics* or (parentheses) as appropriate.
Pauses in speech are indicated by …
Words inserted to help clarify a student's words are included in [brackets].
When students voices overlap, one or both sets of words are included in //double slash marks//.

The number included next to either LMTY or LHSIA refers to the code, included in the table of codes.

<table>
<tr><th colspan="4">Leo Ta'keisha Ordena</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observation</th></tr>
<tr><td></td><td colspan="2">The teacher has just finished her introduction to the problem and given instructions. This has taken about 6 minutes, with students (including some from this group) reading the problem and responding to questions. The teacher let them start, and then briefly interrupted to remind her students they may use different materials in the room.</td><td></td></tr>
<tr><td>1</td><td>Leo (L)</td><td>Starts reading problem out loud (inaudible)</td><td></td></tr>
<tr><td>2</td><td>Ta'keisha (K)</td><td>Now K starts reading problem out loud.
I was constructing towers as you see below. I noticed that each time I made the tower higher…
(Leo and Ordena watch and listen to Ta'keisha)</td><td></td></tr>
<tr><td>3</td><td>L</td><td>I need more blocks on the sides. So look, we need to try to build a five block tower. But high, not low like this one.
(L is picking up on the fact that teacher emphasized the word "high" and pointed out to the students that there is a difference between a five block "high" tower and a 5 block tower.)</td><td>EPISODE 1
LMTY (1): Leo explains his interpretation of the problem to those in his group.</td></tr>
<tr><td colspan="4">Interpretation: Leo had been reading the problem and explains his interpretation of the instructions to his two classmates. He may have picked up on the fact that his teacher emphasized the word "high." He seems to claim that the five block tower will be taller or higher than the towers depicted in the given diagram. In the next several talking turns, it appears that Ta'keisha and Ordena are not taking into consideration Leo's ideas and suggestions.</td></tr>
<tr><td>4</td><td>K</td><td>How about there's three blocks in the middle</td><td></td></tr>
</table>

Leo Ta'keisha Ordena			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
		and 2 on the side? (While talking, Ta'keisha looks either down at her paper or at Ordena. She does not seem to glance toward Leo at all.)	
5	L	Try to draw it.	
6	K	*Counts the blocks on the task paper, pointing with her pen.*	
7	T	[Tells students to feel free to use any materials they need.]	
8	Ordena (O)	(turns to Ta'Keisha) This is what it's trying to say, right? *They notice an observer is watching them. They look at her and laugh, and she smiles back. Perhaps they are acknowledging that they are being observed while they work.*	
9	O	(seems to be talking to Ta'keisha, and not necessarily Leo) She say…	
10	L	Every time she added a block on the top she needed to add a block on the sides. So that's not a five block high tower.	
11	K	That's not… and what is that on?	
12	L	That's on the 4.	**LHSIA (7):** Leo answers Ta'keisha's question, "What is that on?" without giving an explanation or seeming to take into account her suggestions.
Interpretation: Leo has been arguing that the five-block high tower is not depicted on the paper, and Ta'keisha questions him on this. She has a slight attitude in her voice, as though she may be getting defensive. Up until this point, Ta'keisha and Ordena have appeared to ignore Leo's ideas. Leo's response to her question, "That's on the 4" does not include an immediate explanation and does not appear to take into consideration Ta'keisha's ideas stated up until this point.			
13	K	That's six. Six.	
14	L	What we need to do is decrease that and build it like 4 on the sides and try to balance it on each border. Or try to build it higher. You	**LMTY (1, 4):** Leo is explaining his idea for a strategy of

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<tr><td></td><td></td><td>know what I’m saying?</td><td>what to do. He is restating his statement of “four,” correcting Ta’keisha’s statement of “six.”</td></tr>
<tr><td>15</td><td>O</td><td>No.
(looks away from the group and glances toward the rest of the classroom)</td><td></td></tr>
<tr><td>16</td><td>L</td><td>(continues talking to Ta’keisha)
So if this one is set C, a ten block…That’s actually 11. So we need to take the one from the middle and try to balance it on one, two, three, four, sides. But the cubes have to move in a little. You keep adding more. So that’s what I think it’s trying to do.</td><td>LMTY (3): Leo continues his explanation, trying to explain his understanding of the problem. He states that the tower labeled C can be transformed into the 10-block high tower by removing the middle base block from the tower.</td></tr>
<tr><td colspan="4">Interpretation: In the past two speaking turns, Leo tries to explain to his classmates his understanding of the problem. He says, “You know what I’m saying?” indicating that he would like them to understand and possibly agree with his interpretation. His interpretation, which is incorrect, is that he thinks you can take the middle base block away from the existing tower labeled B and the tower labeled C. Since tower B has 6 total blocks, he seems to think he can take away a block to have only blocks. In this way, he seems to be misunderstanding what it means to have a 5-block high tower. His two classmates appear to not listen to him and his ideas. They do not seem to be doing this because they don’t agree with his idea, but rather they do not even seem to be trying to listen or understand his idea. Though Ordena seemingly brushed Leo off when he asked, “You know what I’m saying?” Leo continued to give his explanation, providing additional details and making the case for transforming tower C into a 10-block high tower.</td></tr>
<tr><td>17</td><td>K</td><td>In response to what Leo says, Ta’keisha turns to Ordena as though she is trying to ignore Leo.</td><td></td></tr>
<tr><td>18</td><td>O</td><td>Says something inaudible.</td><td></td></tr>
<tr><td>19</td><td>L</td><td>So that’s what I’m going to try to do. Take away the middle</td><td></td></tr>
<tr><td>20</td><td colspan="2">Ta’keisha and Leo begin writing on their papers. Ordena is just sitting looking around but does not write</td><td></td></tr>
</table>

Leo Ta'keisha Ordena			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
	anything. *Leo decides to get some Unifix cubes. When he returns, both he and Ta'keisha take out several blocks. He moves to bag away from the others, prompting Ta'keisha to tell him she needs two more blocks as if she is annoyed with him. Leo also offers cubes to Ordena, who takes several.*		
21	K	*Builds a tower. View obstructed by name tag. To Leo:* This is how it looks.	**EPISODE 2** Ta'keisha and Leo both build tower structures to determine the next step.
22	L	*To Ta'keisha*: So here. This is how it would look.	
23	K	6 blocks.	
24	L	This was what I was talking about. See how this is?	**LHSIA (14, 15):** Leo uses the blocks to try to support his point earlier. He demands their attention, saying "This was what I was talking about." He asks if they see what he means, likely trying to get his classmates to confirm his idea.
25	K	Don't we gotta find an equation?	
26	L	This is what I was talking about. See how this is? … in the middle. So this is what to do. Make it wide. (*putting the blocks out to explain what he means)* Make it wide. And then balance it on the middle.	
Interpretation: Leo was previously trying to share his ideas with his classmates by explaining his understanding of the problem. His classmates did not take his suggestion and seemed to mostly ignore him. Leo may have **branched** into an active LHSIA structure from the active LMTY structure because he was being ignored by his two classmates. Here, he tries to explain his idea again, using blocks that he did not previously have with him. His tone of voice suggests he is getting frustrated perhaps because his classmates are not listening to his ideas.			
27	O	It will probably	
28	O	So that's behind it	
29	K	No, but look (pointing to something on the paper)	
30	O	Yeah, I know.	

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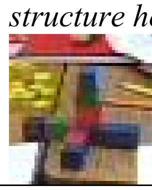
<tr><td>31</td><td>L</td><td>So what I’m trying to do is to balance it.</td><td></td></tr>
<tr><td>32</td><td>L</td><td>To Ta’keisha: See</td><td></td></tr>
<tr><td>33</td><td>K</td><td>See if you could see it in my head those are not pushed together
(She is talking about what she is visualizing but pointing to the picture on the Ordena’s page. I think she is trying to contrast the picture in her head with the representation on the page. Also she is really just talking to Ordena rather than Leo)</td><td></td></tr>
<tr><td>34</td><td>L</td><td>OK, now I know.
(L is working with his blocks, K seems to be thinking while looking at her paper, and O is just sitting back.)</td><td>LHSIA (5): Leo continues to share his knowledge with his classmates. Perhaps because they have not heeded his suggestions earlier, he is now stating that he knows, presumably the answer or a strategy.</td></tr>
<tr><td colspan="4">Interpretation: Leo has constructed his first tower structure using actual blocks, in such a way to support his idea that the middle (often hidden) block should be taken away (line 19, “take away the middle”). When Leo says, “OK, now I know,” he may be trying to tell his classmates that he thinks he knows the answer. This may be a continuation of the earlier active LHSIA structure, in which he may be saying to his classmates that he can figure out the answer without their help. In the next line, Leo asks the teacher about the middle block, suggesting that he wants confirmation from her that he is correct.</td></tr>
<tr><td>35</td><td>L</td><td>raises hand (to call over the teacher.)
Ms. S. I want to know.
(to teacher) You see I have this one.</td><td></td></tr>
<tr><td>36</td><td colspan="2">For about 2 minutes:
The group has called Ms. S over to their table. Leo brings up the issue of whether the middle base block is supposed to be there and agrees with the teacher that the blocks should stay there. When he goes to get a different set of blocks, Ta’keisha and Ordena suggest to Ms. S that they should look for an equation. Through discussion, Ms. S suggests that the students organize their information, perhaps in a table format. She tells them to use a new strategy and then leaves them.</td><td></td></tr>
</table>

Leo Ta'keisha Ordena			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
37	L	So what are we trying to do?	**EPISODE 3**
38	K	Are we going to make a table?	
39		*Ordena takes some blocks to work with*	
40	K	Do you want to make a table?	
41	O	Make a table. (says this in way conveying if you want to make a table, go ahead and make the table)	
42	L	How you going to make a table?	Leo seems to want to engage in a conversation, which could lead to LMTY or some other structure, but Ta'keisha (and to some extent Ordena) keep ignoring him and giving him 'rude' answers.
43	K	It's called paper. (Ta'keisha looks at Leo and then Ordena, suggesting that a sarcastic attitude toward Leo)	
44		How, how, how, how? (suggesting that he knows to write it down, but he wants to know by what procedure will they go about making a table?) *The three students take blocks out of the bucket.*	
45	L	How we gonna make it?	
46	K	We got blocks. (Note that at this moment Ordena and Leo are using the blocks to build a tower and Ta'keisha is not) I just wanna make a table.	
47	L	I built one five block. (Leo receives no response to this statement.)	**LHSIA (14)** – Leo calls attention to fact that he has a "five block" tower from earlier. This is a tower that contains 5 blocks, not a tower with a height of 5 blocks.

Leo Ta'keisha Ordena			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
Interpretation: Leo has tried throughout the first 10-15 minutes of the problem-solving session to engage in conversation with his two classmates, to further through the problem. His two classmates seem to continuously ignore him and his ideas and do not share their ideas with him. Leo continued to work with the blocks to construct the towers. He lets his two classmates know that he has a representation of what he calls a "five block." He appears to want their attention, and perhaps recognition for his ideas, but does not receive it from either of the two girls.			
48		*Leo picks up the bucket and is taking more blocks. Ordena is also building and Ta'keisha is writing something on her paper.* *At this point Leo has a large pile of blocks in front of him and he has put the bucket of blocks on the floor.*	
50	L	Look. (He says to Ta'keisha. She looks up but does not verbally respond to him; it seems he built a tower with 5 blocks, so he did not put the block the middle, despite the conversation with Ms. S)	
50		*Ta'keisha smiles and says something inaudible to Ordena (but seems to be neglecting Leo). He keeps working on his own constructing a bigger and bigger tower.*	
51	O	Excuse me. I need (she is saying something to an observer about something she needs, it seems related to the marker and the fact that it has 2 writing sides.) *She seems unsure about which side to use. She tries one side and the observer asks if it works and she says yes. Ta'keisha smiles and laughs (in a friendly way) about this.*	
52	L	Look what I constructed. (see picture) (Nobody responds to his statement). *He continues to add blocks to the tower structure he is working on.* 	
53	K	It says every time you add one to the top you've got to add one to the sides, right?	**EPISODE 4** **LHSIA (15):** Ta'keisha looks for

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<tr><td></td><td></td><td></td><td>confirmation of her understanding of the instructions about how to construct the towers.</td></tr>
<tr><td>54</td><td>L</td><td>So 4?</td><td></td></tr>
<tr><td>55
1
1:42:55</td><td>K</td><td>That’s 5, 1, 2, 3, 4, 5.
(She seems to be referring to a construction of blocks that we can’t see because papers are blocking the view; alternatively she may be counting off the task paper.)</td><td>LHSIA (8, 9A): Ta’keisha disagrees with Leo and corrects him that there are 5 blocks, rather than 4 added to the tower structure each time.</td></tr>
<tr><td colspan="4">Interpretation: Ta’keisha had been writing on her own paper, and now states part of the instructions regarding the construction of the towers. She appears to ask for confirmation of her understanding by asking, “Right?” Her tone of voice suggests she already believes she is correct and wants her classmates to confirm or agree with her. When Leo asks, “So 4?” he may be confirming her idea by making a suggestion to further her idea. Ta’keisha does not agree with his suggestion of adding 4 more blocks, and corrects him saying that they add 5 blocks instead. She appears to want to be correct and wants her classmates to think of her as knowledgeable about the problem.</td></tr>
<tr><td>56
11:43:0
2</td><td></td><td>Ta’keisha takes a bunch of yellow blocks from pile in front of Leo. They both continue to work constructing their own tower structures. Ta’keisha’s is not visible because there are papers in the way. Ordena is writing something on her paper.</td><td></td></tr>
<tr><td>57</td><td>L</td><td>Hold on. Let’s try to get this do this.</td><td></td></tr>
<tr><td>58</td><td>K</td><td>Now I don’t know what it is!</td><td></td></tr>
<tr><td>59</td><td>K</td><td>(to Ordena) This is how I think it’s done.
(Ta’keisha is building a tower)
Cause you know how I say I made a 5?</td><td></td></tr>
<tr><td>60</td><td>L</td><td>A 5 tower?</td><td></td></tr>
<tr><td>61</td><td>O</td><td>(turning toward Ta’keisha, giving her her full attention)
Yeah.</td><td></td></tr>
<tr><td>62</td><td>K</td><td>Two on top, I mean…three on the top and two on the bottom.
(Ta’keisha has constructed a tower which she is calling a 5 block tower because it is 3 stories</td><td></td></tr>
</table>

Leo Ta'keisha Ordena			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
		high and has 2 blocks for a side – but seems to only have one side. In other words, she has constructed a tower out of 5 blocks.)	
63	K	Then for 10. (Ta'keisha starts counting the blocks on her structure). 1,2,3,4,5. 6 on the top and 4 on the sides.	
64	L	Did you count the pieces (inaudible) on this?	
65	K	No. (she says sharply to Leo) We are not looking for 11. Five, ten, and 100. (She points to where it asks for each of these on the assignment paper.) *This seems to be a difference of opinion between Leo and Ta'keisha. Ordena looks disinterested in what they are talking about and looks off into the distance.*	**LHSIA (9A):** Ta'keisha disagrees with Leo's suggestion, which may have been something about 11 blocks. She refers to the directions to say they are looking for the 5-, 10-, and 100-block high tower. It is not clear if the students are both talking about the height of the tower or the total number of blocks in the tower.
Interpretation: Note that Ta'keisha does seem engaged with the problem throughout the problem-solving session so far (almost 20 minutes). However, she once again disagrees with Leo's suggestion and sharply tells him, "No." She may be disagreeing with him to indicate that she is the one who is smart and correct, and that he is not correct. In this exchange, it appears that Leo said something about 11 blocks, which is the correct number of blocks for the 3-block high tower. It is not clear if this is what he was referring to or not. When Ta'keisha refers to the instructions and says, "Five, ten, a hundred," it is not clear if she realizes that these numbers are supposed to refer to the height of the tower rather than the total number of blocks. Alternatively, it appears that Ta'keisha and Leo may not get along socially, so her responses to him may be related to a personal relationship, rather than an intellectual or academic one.			

Leo Ta'keisha Ordena			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
66	*For about five minutes:* *Leo starts taking apart his tower. When he is finished he writes on his own paper. Ta'keisha and Ordena appear to be writing on their own papers. Sometimes the two girls talk. This was inaudible to the listening, and may be off-topic.* The students continue to work, sometimes independently. At times, someone will ask a question, such as "How do we find 100?" but these questions are often not answered and a conversation does not follow. Other statements include, "Let's make a table", but discussions about making a table do not include what information needs to go into the table.		
67	L	So this time it's like this, right? It's like this, right? (shows Ta'keisha his paper)	
68	K	I don't know if she (inaudible) 5 blocks high.	**EPISODE 5** The students again try to build the tower structures using the blocks.
69	L	So you want the other ones?	
70	K	4, 5 (showing a stack of 5 blocks)	
71	L	I put 5	
72	K	That's	
73	L	But let's (Leo places a little block tower in front of Ta'keisha)	
74	K	(inaudible) (Ta'keisha is working with her own block tower)	
75	O	That is C.	
76	K	This one? (Ta'keisha smiles) 5 blocks high. (showing her tower)	
77	O	I don't know if it's right.	
78	K	I think we did it right.	
79	L	No (adding blocks to Ta'keisha's block model; Ta'keisha removes them as he is adding)	**LHSIA (3)** Leo is trying to add blocks to

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<tr><td></td><td></td><td>Add 1 more on that side.</td><td>Ta’keisha’s model because he thinks more are needed.</td></tr>
<tr><td>80</td><td>K</td><td>No. No. No.
(Ta’keisha laughs, shakes her head, and turns towards Ordena.)</td><td>LHSIA (9A)
Ta’keisha disagrees with Leo’s suggestion to add more blocks, saying, “No,” and removing some blocks.</td></tr>
<tr><td colspan="4">Interpretation: Both Leo and Ta’keisha are trying to suggest their way of thinking about the towers is correct. Even though they started out with their own tower structures, Leo tried to add blocks to Ta’keisha’s model. He did this because he didn’t think she had enough blocks, “Add 1 more.” She disagreed with him, saying, “No.” She even took those same blocks off to express her disagreement. Both Leo and Ta’keisha tried to demonstrate their own knowledge, but they disagreed with one another. They did not continue this conversation.</td></tr>
<tr><td>81</td><td>L</td><td>Leo takes some blocks and starts constructing something.
That’s what you get</td><td></td></tr>
<tr><td>82</td><td>K</td><td>Counting the blocks on her model. Says 1, 2, 3, 4, 5. While Leo and Ta’keisha are working, Ordena is sitting and playing with her hair.</td><td></td></tr>
<tr><td>83</td><td>O</td><td>Oh, I think…. Oh, a variable one of our variables could be height and sides…I think</td><td></td></tr>
<tr><td>84</td><td>K</td><td>You doing the five thing towers high? Is it five towers high?</td><td></td></tr>
<tr><td>85</td><td>L</td><td>Yes.
//K: That’s wrong//
No, it isn’t.</td><td></td></tr>
<tr><td>86</td><td>O</td><td>I think we gotta do what we did (meaning what she and Ta’keisha did)</td><td>LHSIA (5): Ordena tells Leo that she believes she and Ta’keisha are on the right path to the solution. By saying, “We gotta do what we did” she is telling Leo that she and Ta’keisha are smart for already figuring out the correct strategy.</td></tr>
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<tr><td colspan="4">Interpretation: Ordena appears to be disagreeing with Leo’s strategy of construcing the 5-block high towers. Ta’keisha also indicates that she disagrees, as she tells him that the 5-block high tower he constructed using blocks is “wrong.” Ordena states that she believes she and Ta’keisha are on the right path to finding the solution, suggesting that she believes they will figure out the answer. If she and her partner have the right answer, then they will be the “smart ones” in this group.</td></tr>
<tr><td>87</td><td>L</td><td>But that’s not what you all do (insistence in his voice)</td><td>LHSIA (8): Leo is correcting Ordena by stating that her strategy is “not what you all do.” He continues to try to make his point, using the physical blocks in front of him.</td></tr>
<tr><td>88</td><td>L</td><td>So if I keep this</td><td></td></tr>
<tr><td>89</td><td>O</td><td>You have to do that using 6 blocks, well, 5. Five.</td><td></td></tr>
<tr><td>90</td><td>L</td><td>So, 2, 4, 6
No, five, take 1 of these (he is moving blocks around),
So look, I just built 5.
So I’m going to build this one. (pointing to a diagram on his paper)</td><td></td></tr>
<tr><td colspan="4">Interpretation: Leo appears to be trying to correct Ordena’s assertion that she and Ta’keisha have the correct solution strategy by saying, “That’s not what all you do.” The insistent tone in his voice suggests that he still wants Ta’keisha and Ordena to value his strategy and ideas. He appears to want to be seen as smart by his two classmates.</td></tr>
<tr><td>91</td><td></td><td>L looks at the diagram on his paper and counts the cubes in one of the diagrams pointing to each as he counts to himself. Then he takes blocks and starts building.
Ordena is speaking – sounds like she is talking out loud as she sets up the table on her paper. Then Leo starts taking blocks off the table and putting them back in the bucket on the floor.</td><td></td></tr>
<tr><td>92</td><td>K</td><td>If we do an equation, we use blocks. What else we gonna use?</td><td></td></tr>
<tr><td>93</td><td>L</td><td>We need block. She used towers. And the letter that you built and the number.
B block tower.
Calls teacher to come to their table.</td><td></td></tr>
</table>

Leo Ta’keisha Ordena			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
94	*When Ms. S comes over to the group, the students ask her if the tower has to be “5 blocks high.” She refers the students to the original wording in the task. She then asks Ta’keisha and Leo to describe the different interpretations they had. She takes them through the task instructions again, having them build the first few towers to see the pattern. After Ms. S leaves the group the students all start working with blocks.*		
95	K	*Laughs, smiles, and says something inaudible to Ordena.*	**EPISODE 6** The students start constructing the towers correctly, based on their discussion with Ms. S.
96	O	She made it sound so easy.	**LHSIA (6)** Ordena suggests that the teacher made the problem seem easy, perhaps suggesting that she now understands the problem better.
97		*Now all 3 students are constructing block towers. Leo and Ordena seem really into this activity.*	
98	L	And we all got it complicated.	
99	L	I was gonna write like an x and an um…	
100	K	It should be 4 , 4 going (pointing with her pen all around her tower structure, suggesting 4 blocks in each direction)	
101	O	(inaudible) *Constructing her own tower*	
102		That’s what I just did. (smiles at Ordena; may be referring to the way Ordena constructed her tower structure)	**LHSIA (9)** Ta’keisha says to Ordena that she did the same thing, likely referring to the way they each constructed their towers. She seems to be agreeing with Ordena’s strategy.

Leo Ta'keisha Ordena			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
103	O	Oh!!! This is easy.	**LHSIA (6)** Similar to her statement earlier, Ordena claims the task is easy now that she is constructing her own tower.
104	L	It's easy. (agreeing with Ordena)	**LHSIA (6)** Leo agrees with Ordena that the task is easy. He is also putting together his own tower.
Interpretation: Each of the students in this group appears to want to be seen as smart, as if they all understand the task properly now that the teacher has helped them. Ordena and Leo both claim that the task is easy, as they construct their tower structures. Leo states, "It's easy," just after Ordena, appearing to agree with her. Ta'keisha seems to want recognition for her strategy, which she claims is the same as Ordena's strategy, "That's what I just did." Ta'keisha's tone of voice was friendly, and she smiled as she said this. These behaviors suggest she was being friendly toward Ordena, rather than being annoyed that they did they same thing.			
105	L	So this is a 5 block high tower, right?	**EPISODE 7**
106	K	How many blocks do you have? 5 blocks?	
107	O	(points to the diagram on Ta'keisha's paper) We're supposed to have 5 on each side. Five on a side.	
108	K	I need more blocks (says impatiently) *Leo gives Ta'keisha more blocks*	
109	K	You're not supposed to have 5. //O: Uh-huh// How're you supposed to have 5?	
110	L	Keep adding. (acts to put another block on Ta'keisha's tower)	
111	K	No, that's 5!	
112		(Points to Ta'keisha's model and counts the blocks going up) 1, 2, 3, 4.	
113	K	And I have to take 1 and put it right here. (taking a block and adding it to one of the sides. Then she adds one more block to the height.)	
114	K	It has 5, 10 (points to the sides as she counts)	

<table>
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<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observation</th></tr>
<tr><td>115</td><td>O</td><td>You still need one more.</td><td></td></tr>
<tr><td>116</td><td>K</td><td>25 blocks</td><td></td></tr>
<tr><td>117</td><td>L</td><td>There’s one under. 1, 2, 3, 4.</td><td></td></tr>
<tr><td>118</td><td>O</td><td>5 up</td><td></td></tr>
<tr><td>119</td><td>K</td><td>Cause the middle one</td><td></td></tr>
<tr><td>120</td><td>L</td><td>Don’t you understand it? (to Ordena)
We’ll read them. 1 block high. 2 blocks. 3 blocks.
(as he says this, l points to diagrams A, B, and C, on the paper.)
They add it.</td><td>LMTY (1)
Leo uses the task paper to help explain that the figures on the page A, B, C are the towers corresponding to heights 1, 2, 3-blocks high.</td></tr>
<tr><td colspan="4">Interpretation: Leo and Ta’keisha had been discussing how to properly construct the 5-block high tower, with 5 blocks in the height, including the middle block. Ordena’s statements suggested that she thought there should be 5 blocks on top of the middle block (rather than only 4 on top of that block). When Ordena disagrees with Ta’keisha, Leo asks, “Don’t you understand it?” He seems to take her answer as no, and then uses the figures given on the task paper to explain that the figures A, B, and C, respond to towers with heights 1, 2, and 3 blocks, respectively. He appears to be going back to his tone of voice from earlier in the problem-solving session and trying to help Ordena understand, rather than just show that he understands the problem.</td></tr>
<tr><td>121</td><td></td><td>Leo and Ordena keep taking blocks from the bucket, and adding the blocks to their tower structures. Ta’keisha does not and keeps working with what she has.</td><td></td></tr>
<tr><td>122</td><td>O</td><td>No, that’s a 5 block high tower.
(she takes her pen and points to each block on the base and counts each blocks one at a time, seems to be to herself)
1, 2, 3, 4, 5.</td><td>EPISODE 8
The students discuss the total number of blocks in the 5-block high tower.</td></tr>
<tr><td>123</td><td>K</td><td>8, 12, 16 (moving the blocks around as she counts)
It’s 21 blocks. (announces to classmates)
Ta’keisha and Ordena start writing while Leo is taking out blocks.</td><td>LHSIA (7; 14):
Ta’keisha gets an answer to the total number of blocks in the 5-block high tower and shares it with her classmates. She does not give</td></tr>
</table>

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<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observation</th></tr>
<tr><td></td><td></td><td></td><td>an explanation for her answer of 21 blocks.</td></tr>
<tr><td>124</td><td>O</td><td>(Counting the tower structure in front of her) 5, 10, 15, 20, 25. It’s 25 blocks, right?</td><td>LHSIA (15): Ordena may not have heard Ta’keisha suggest the total number of blocks in the 5-block high tower was 21, as she asks, “Right?” after suggesting that there are 25 blocks in the tower.</td></tr>
<tr><td>125</td><td>K</td><td>21.</td><td>LHSIA (9A): Ta’keisha repeats her earlier answer. Her tone of voice is insistent, indicating that she is disagreeing with Ordena’s answer of 25.</td></tr>
<tr><td>126</td><td>O</td><td>(to Ta’Keisha) It’s 25.</td><td></td></tr>
<tr><td>127</td><td>K</td><td>21</td><td></td></tr>
<tr><td>128</td><td>O</td><td>Counts again. 25</td><td></td></tr>
<tr><td>129</td><td>K</td><td>No it’s not. (insistent in tone of voice)</td><td></td></tr>
<tr><td>130</td><td>O</td><td>Yes it is. (responds with an insistent tone of voice)</td><td></td></tr>
<tr><td colspan="4">Interpretation: Ta’keisha counts the number of blocks she has in her 5-block high tower and shares that answer with her classmates. She seems to be demanding their attention to her suggested answer by making the announcement. Though it is almost a minute later, Ordena responds with her answer for the total number of blocks, 25, which is different than Ta’keisha’s answer. Ordena, however, seems to be interested in receiving confirmation from her classmates that her answer is correct, by asking, “right?” The way Ordena shared her number 25, it seemed as though she may not have heard Ta’keisha announce that the answer was 25 moments earlier. When Ta’keisha responds to Ordena, with 21, she appears to be disagreeing with her classmate. Though both girls counted out loud, neither is giving an explanation as to how they got their numbers. At this point, both girls appear to believe they are right, and hence, each is smart.</td></tr>
<tr><td>131</td><td>L</td><td>(counting Ordena’s blocks)</td><td>LHSIA (9, 12):</td></tr>
</table>

Leo Ta'keisha Ordena			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
		5, 10, 15, 20, 25.	Leo counts the number of blocks in Ordena's 5-block high tower structure and agrees with her that there are 25 total blocks.
132	K	(To Leo.) OK, y'all are counting the middle block 4 times.	**LHSIA (8; 14)**: Ta'keisha corrects Leo's counting strategy, telling him he is counting the same middle base block multiple times.
133	L	So it's 24.	
134	O	We're supposed to count the middle block.	**LHSIA (9A):** Ordena disagrees with Ta'keisha about counting the middle block.
135	K	No, you're not.	
136		So it's 24.	
Interpretation: Leo counts Ordena's blocks and appears to be supporting her answer of 25 total blocks in the 5-block high tower. He also has wanted to be seen as smart by his classmates several times throughout this problem solving session. Therefore, he may also be agreeing with her to suggest to his classmates that he is as smart as them. Ta'keisha corrects Leo and Ordena's counting strategy, as they were indeed counting the middle base block each time they counted the height and each of the 4 sides. She appears to want to correct them and inform them why they are wrong ("counting the middle block 4 times"). When Leo adjusts his answer to "24" he may still be trying to show that he is as smart as his classmates. Ordena jumps in stating that they are "supposed to count the middle block" perhaps realizing that it is part of the structure, but not realizing that she has counted the same block multiple times. Ta'keisha replies simply, "No you're not," disagreeing and correcting her again.			
137	K	Look,… (pulls apart each of the sides so that they are not attached to the center stack of blocks.)	**LHSIA (3)** Ta'keisha uses the tower that Ordena has constructed to try to make her point.
138	L	Because you're supposed to count the middle	

Leo Ta'keisha Ordena			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
139	K	You're supposed to count the middle 4 times? (challenging them)	
140	L	5, 10, 15, 20,	
141	K	You counted it 4 times.	
142	L	I don't know what you're trying to say.	
143	O	8, 16	
144	K	What I'm trying to say is… Okay (Ta'keisha points to each of the blocks on each of the 4 sides) 1, 2, 3, 4, 5. 1, 2, 3, 4, 5. 1, 2, 3, 4, 5. 1, 2, 3, 4, 5. (Each time she says "5" she is pointing to the same middle base block)	**LMTY (7; 8)** Ta'keisha responds to Leo's statement by trying to explain what the error is. She questions Leo and Ordena on their strategy to count the same block multiple times.
145	K	You counted the middle block 4 times.	
146	K	Then you gotta count these the same way. (points generally toward Ordena's tower structure) *Each student starts to write on his or her own paper.*	**LMTY (3); LHSIA (11):** Ta'keisha states that the blocks have to each be counted once.
147	K	You would need 21 blocks.	
Interpretation: Ta'keisha first seems to be trying to demonstrate her point that the other two are counting the same block multiple times by demonstrating their counting strategy, after pulling the blocks apart (to make her point more clear). She is trying to demonstrate that while the middle base block does need to be counted, it should only be counted once (not 4 or 5 times). As Ta'keisha does this, she seems to branch from LHSIA, where she is demonstrating that she has the correct answer and the others do not, to LMTY, where she puts more effort into explaining to the others their error. She counts the blocks, thereby giving Leo and Ordena more information. She physically demonstrates that they counted the base block 4 times. She seems to do this in response to Leo stating, "I don't know what you are trying to say." Leo was expressing his confusion, perhaps giving Ta'keisha an opportunity to branch into LMTY. As she provides her classmates with more instruction, "You gotta count these the same way," Ta'keisha appears to be suggesting, "I told you so" at the same time as suggesting the correct way to count the total number of blocks in the tower structure.			
148	L	(reading aloud what he is writing) You would need 21 blocks, to build…. (to Ta'keisha) OK, so how about the 10 block high tower? What is it? 42 (using his fingers; answering his own	**EPISODE 9** The students try to use the fact that the 5-block high tower has 21 blocks to

Leo Ta'keisha Ordena			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
		question)	determine the total number of blocks needed in the 10-block high tower.
149	O	So for the twenty, for the ten block high you gotta 42.	
150	L	42	
151	K	It's definitely 42.	
152	O	What, did I just add up? (They return to writing on their own papers.)	
		They seem to reach the erroneous conclusion that once you have found the number of blocks for a 5 block high tower, which they calculated correctly to be 21 blocks, you just add another 21 blocks for each level that is 5 blocks higher. So that a 10 block high tower will be 42 blocks and a 15 block high tower will be 63 blocks, and a 20 block high tower will be 84 blocks. Their argument, at one point, was whether the 15 block high tower was 63 or 64 blocks (one student added incorrectly.)	
153	O	15 block high tower is 63	
154	L	You mean 64. *The students write for about 30 seconds before continuing their conversation.*	
155	L	So look the pattern is 21. The pattern is 21, right? So keep adding 21 so for a 100 block pattern is going to be 64.	**LHSIA (14)** Leo is asserting that he has the pattern, and his tone of voice suggests that he is demanding the attention of his classmates, particularly Ordena.
156	O	Na uh	
157	L	Yes it is.	
158	O	No its not.	
159	L	Keep going by 21.	
160	O	No, it's not going to be 64. Cause 20, wait. A 20 block's gonna be 2,4,6,	
161	L	64	
162	O	63 a 20 block's gonna be 63 *While Leo and Ordena are arguing, Ta'keisha*	**LHSIA (9A)** Ordena and Leo

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<tr><td></td><td></td><td>is just writing and not involved in the dispute.</td><td>have disagreed whether the next iteration is going to be 63 or 64 total blocks, due to a calculation error. The other issue is for what height is 63/64 the total of? (For 100-block high tower, 20 block high tower, 15 block high tower?)</td></tr>
<tr><td>163</td><td>L</td><td>64!</td><td></td></tr>
<tr><td>164</td><td>O</td><td>It's gonna be 63!</td><td></td></tr>
<tr><td>165</td><td>L</td><td>64</td><td></td></tr>
<tr><td>166</td><td>O</td><td>63
That's for 20 blocks.
No that's for 15 blocks (correcting herself)</td><td></td></tr>
<tr><td colspan="4">Interpretation: The students tried to use the fact that there were 21 total blocks in the 5-block high tower to determine the number of blocks in the 10-block high tower. They all (incorrectly) agree that there are 42 blocks in the 10-block high tower (there are 46). Leo and Ordena discuss the next part of the pattern and disagree as to whether the next total will have 63 or 64 blocks. Leo says (line 156) that there is a pattern of increasing by 21 blocks. Perhaps because the instructions ask for the total number of blocks in the 5-, 10-, and 100-block high towers, Leo seems to think that there is an increase of 21 blocks for each answer. That is, there are 21 blocks in the 5-block high tower, 42 in the 10-block high tower and then 64 (an incorrect suggestion of the next iteration) in the 100-block high tower. By pointing out the pattern, he seems to be trying to demonstrate his knowledge and understanding of the problem.
Ordena, however, disagrees with him about the total number of blocks in the next iteration, suggesting that it is 63, not 64, blocks. She also suggests that there are a total of 63 blocks in the 20-block high tower, rather than the 100-block high tower. The two continue to disagree about the 63 or 64 blocks for several speaking turns. Ordena then corrects herself and states that the height will be 15 blocks, not 20 blocks as she suggested a moment ago.</td></tr>
<tr><td>167</td><td>L</td><td>15 blocks?</td><td></td></tr>
<tr><td>168</td><td>O</td><td>Yeah, that's for 15.</td><td></td></tr>
<tr><td>169</td><td>L</td><td>How come 15 blocks?</td><td></td></tr>
<tr><td>170</td><td>O</td><td>Cause you're going by 5. 5, 10, 15.
And then, that's a 15. And then a 20 block (she is doing calculations on her paper and Leo is watching.)</td><td>LMTY (7)
Ordena explains to Leo that there are 63 total blocks in</td></tr>
</table>

Leo Ta'keisha Ordena			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
			the 15-block high tower because the 21 total block increase occurs for an increase of 5 blocks in the height. She is suggesting a(n) (incorrect) pattern here.
171	L	64	
172	O	84	
Interpretation: Leo and Ordena had been disagreeing about the total number of blocks (63 or 64) in the next iteration for the total. Ordena had argued there are 63 total blocks in the 20-block high tower. However, she corrects herself, saying that there are 63 blocks in the 15-block high tower. Leo does not understand why this is for the 15-block high tower and asks, "How come?" prompting Ordena to briefly explain her answer, "Cause you're going by 5." It seems, that to her, the pattern is as you increase the height by 5, you increase the total number of blocks by 21.			
173	O	But then 25 block's gonna be	
174	L	Keep going by 21	
175	O	2, 4, 6, 8, 10, a hundred and, a hundred and 5.	
176	O	And then	
177	L	It's too long	
178	K	Multiply, just multiply. (Ta'keisha says this as an aside. She has heard them adding 21 and she is suggesting that it is quicker to multiply. She speaks to them while she has her head down working on her paper.)	
179	O	Hold on	
180	K	Just multiply 21 by 100	
181	O	21 times 100 is gonna be…	
182	K	How we gonna do 100?	
183	K	Shouldn't we do times 10?	
184	O	21 times 100.	
185	K	1, 2, 3, 4, 5, 6, 7, 8 (trails off)	
186	O	If it's	
187	K	No, that's not… cause you just said 15 is 63 that's, um, 3 times 21.	**LHSIA (8; 9A)** Ta'keisha tells Ordena that they should not multiply 21 and 100 in order to determine the

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<tr><td></td><td></td><td></td><td>total number of blocks in the 100-block high tower.</td></tr>
<tr><td>188</td><td>O</td><td>You have to do 3 times 100</td><td></td></tr>
<tr><td>189</td><td>K</td><td>I said to do 21 times 100</td><td></td></tr>
<tr><td>190</td><td>L</td><td>21 times 10</td><td></td></tr>
<tr><td colspan="4">Interpretation: Ta’keisha joins the conversation Leo and Ordena were having about continuing the pattern of 21 more blocks (for the total number of blocks) when she tells them to multiply. She may have told them to multiply because they may have been adding and she may have been suggesting a quicker strategy. Here, Ordena took up Ta’keisha’s suggestion to multiply 21 by 100 in order to get the total number of blocks in the 100-block high tower. Ta’keisha then says, “No” because the pattern isn’t to multiply the height by 21. Instead, she refers to the 15-block high tower (that they agreed on previously), saying that that 15-block high tower has 63 total blocks, which is 21 time 3 and not 21 times 15. The students then have to figure out what number to multiply by 21 in order to get the total number of blocks in the 100-block high tower, according to their pattern.</td></tr>
<tr><td>191</td><td colspan="2">For about 4 minutes:
The three students try to continue the same pattern, adding 21 blocks to the previous total. They determine what numbers to multiply 21 by in order to get the same result. For example, they multiply 21 by 5 to get a result of 105, their answer for the total number of blocks in the 25-block high tower. Ta’keisha tells them they are not supposed to multiply 21 by 100 in order to get the result for the 100-block high tower. Instead, she works independently continuing with the pattern, until she announces that they need to multiply 21 by 20 in order to determine the answer. Ordena starts to write the pattern down on her own paper (see student work). Leo switches between writing on his own paper and sharing answers he got on the calculator with Ordena. We can hear Ordena speaking out loud what she is presumably writing down.</td><td></td></tr>
<tr><td>192</td><td>K</td><td>I’m done.
(Ta’keisha seems to be proud that using her strategy she finished sooner than O.)</td><td>EPISODE 10
LHSIA(16)
Ta’keisha announces that she has finished writing on her paper, suggesting that she may be trying to “one up” the others</td></tr>
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<tr><td></td><td></td><td></td><td>in her group.</td></tr>
<tr><td>193</td><td>O</td><td>This is 70 blocks.
This is 75 blocks.
80 blocks.
85 blocks.
90 blocks.
95 blocks
100 blocks.</td><td></td></tr>
<tr><td>194</td><td>K</td><td>I told you (smiles and laughs)</td><td>LHSIA (11): Ta’keisha says to Ordena, “I told you” referring to the answer that Ordena just stated, “100 blocks.”</td></tr>
<tr><td colspan="4">Interpretation: Ta’keisha has been working independently and had figured out an answer. (It is not clear to me which tower she was interested in). She previously stated that you multiply 21 by some number in order to get the total number of blocks in the tower of a given height. For example, if the 5-block high tower (accurately) has 21 blocks, then the 10-block high tower has a total of 21 times 2 or 42 blocks. Ta’keisha announces to the others when she is “done,” suggesting that she may be trying to “one up” the others or outdo them by finishing earlier than them. As Ordena approaches an answer, we hear her saying “85 blocks, 90 blocks, 95 blocks, 100 blocks.” Ta’keisha stops what she is doing, looks up at Ordena, and tells her, “I told you.” By saying this, she seems to be suggesting that because she had what she believed to be the right answer earlier, she is smart. Her smile and laughter might suggest that she expected Ordena to get the same answer Ta’keisha had already obtained.</td></tr>
<tr><td>195</td><td>L</td><td>So, for the total is what?</td><td></td></tr>
<tr><td>196</td><td>O</td><td>420 blocks to get a 100 block tower.</td><td></td></tr>
<tr><td>197</td><td>K</td><td>You gonna put how we got that? (Ta’keisha and Ordena laugh at that statement)</td><td></td></tr>
<tr><td>198</td><td>O</td><td>(to observer) Can I get another piece of paper?</td><td></td></tr>
<tr><td>199</td><td>O</td><td>This was so easy. (smiles)</td><td>LHSIA (6)
Ordena had stated previously that the task seemed easy (lines 96, 103). Now that the group believes they have found the solution, she confirms that she thought the task was easy.</td></tr>
</table>

Leo Ta’keisha Ordena			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
199		*They all start writing. Ordena or someone else talks out loud as she writes, but it’s not clear what the person says.*	
200	K	I put 5 times each time goes to make a 5 block high tower it takes 21 blocks. (She looks only at Ordena as she speaks, and shows her her paper)	The discussion returns to the total number of blocks in various towers, including those on the task paper.
201	O	Twenty five blocks!	
202	K	It’s not 25 blocks.	
203	K	OK, look at B, right? (diagram B on the paper)	**LMTY (1, 4)**: Ta’keisha corrects Ordena’s answer that the 5-block high tower has 25 blocks. She starts to explain her strategy. She starts by explaining that there are 6, not 10 blocks in the 2-block high tower.
204	O	B is the five-block tower.	
205	O	You is 25 because you still have to count the middle.	
206	K	Look: 1,2,3,4,5,6. (She illustrates with her blocks she takes the block model apart.) //Ordena: 5, 6//	
207		*Ordena says something to insist that there are 25 blocks total.* Count the middle!	
208	K	Look! Look at this one (referring to the2-block high tower constructed from the blocks in front of her). It’s 6. You don’t do 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. It’s not 10.	
209	L	(points to a diagram on Ta’keisha’s paper and addresses his comments to Ordena.) This is a ten block high tower. 2, 4, 6, 8, 10.	
210	K	Exactly.	
211	L	(To Ordena)You’re not counting the middle 4 times, actually 5. (To Ta’keisha) So what would you….	
212	K	Look at this one (moves the blocks around as she counts) 1, 2, 3, 4, 5, 6	
213	L	Each block, keep, when you add 1 to the tower you have to add 1 to the side.	

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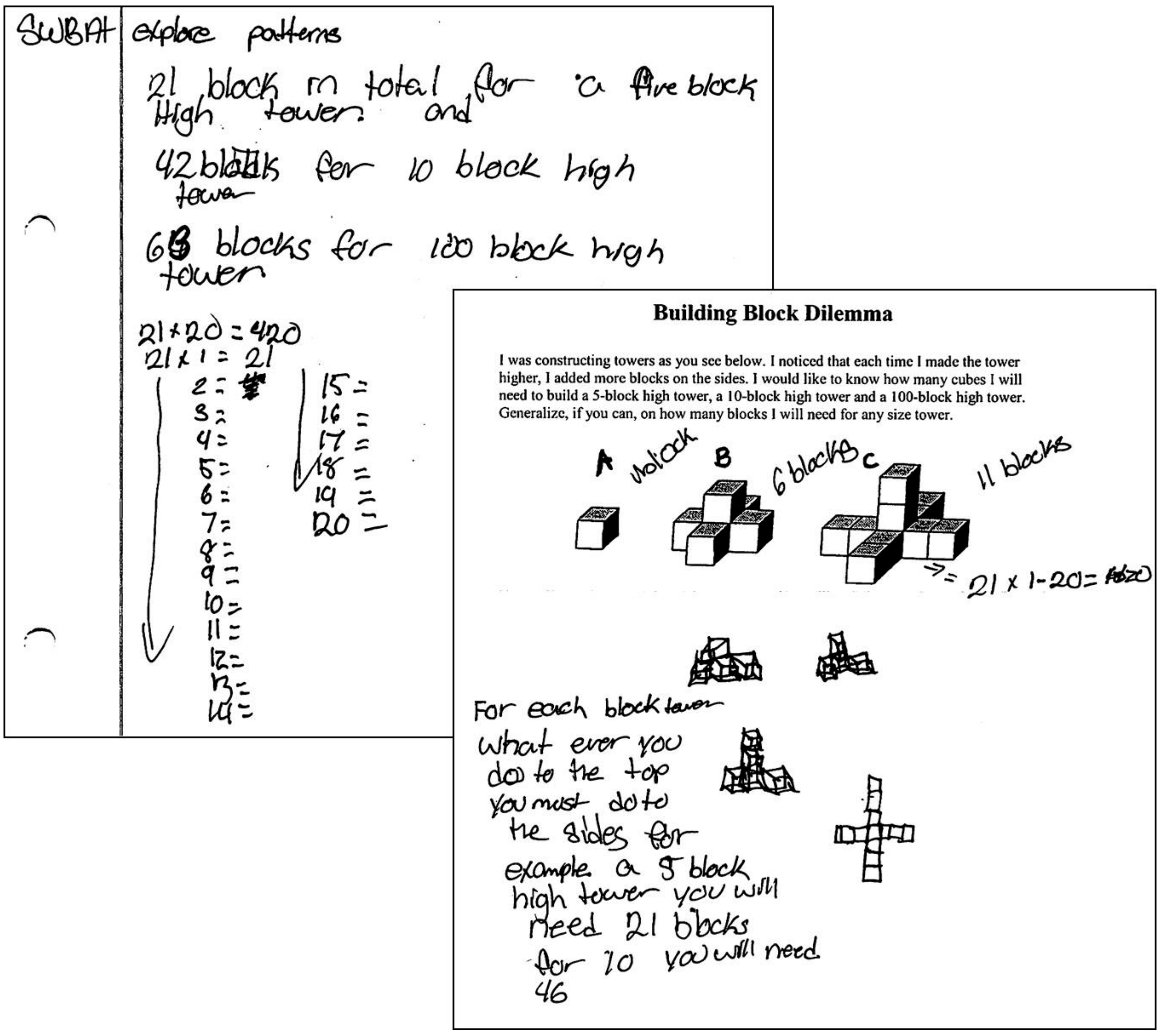
<tr><td colspan="4">Interpretation: Ta'keisha had discussed the total number of blocks in the 5-block high tower earlier, explaining that there are 21, rather than 25 blocks in this tower. At this time, when Ordena states that there are 25 blocks, Ta'keisha first corrects her, saying, "It's not 25 blocks." Ta'keisha continues to demonstrate the error of counting the middle base block multiple times, by using the 2-block high tower as an example. She calls Ordena's attention to tower B in the diagram as well as the blocks she has set up in front of her. She counts the 6 blocks out loud and repeats the fact that there are 6 blocks, not 10 in the 2-block high tower. Leo appears to try to take in this information, by repeating the fact that, "You're not counting the middle 4 times." He also repeats the instructions briefly, "When you add 1 [block] to the [height of the]tower you have to add 1 [block] to the side."</td></tr>
<tr><td>214</td><td></td><td>Students pause the conversation and all start to write on their own papers.</td><td></td></tr>
<tr><td>215</td><td>O</td><td>So for 10 we're going to need 50 cause you're not counting the middle. So you really need 49.</td><td></td></tr>
<tr><td>216</td><td>K</td><td>Huh? (asked as though she didn't hear Ordena's statement)</td><td></td></tr>
<tr><td>217</td><td>O</td><td>Uh, for…</td><td></td></tr>
<tr><td>218</td><td>K</td><td>No, we did it so we can know what was in the middle.</td><td></td></tr>
<tr><td>219</td><td>O</td><td>For 25 blocks,… No for 10 blocks you need 49 blocks.</td><td></td></tr>
<tr><td>220</td><td>K</td><td>No (shakes her head)
You don't get it. OK. (takes her blocks and starts to demonstrate.) Look. Watch this.</td><td></td></tr>
<tr><td>221</td><td>O</td><td>No, no, no. I get it.</td><td></td></tr>
<tr><td>222</td><td>K</td><td>You don't supposed to count. You only count the middle once.</td><td></td></tr>
<tr><td>223</td><td>K</td><td>You're gonna build a ten. Wanna build a ten?</td><td></td></tr>
<tr><td>224</td><td>O</td><td>Yeah, OK.</td><td></td></tr>
<tr><td>225</td><td></td><td>Ta'keisha starts building. Ordena watches. Leo continues writing on his paper.</td><td></td></tr>
<tr><td>226</td><td>L</td><td>(to Ordena) How much you say you would need for 10 blocks? I mean a ten block high tower?</td><td></td></tr>
<tr><td>227</td><td>O</td><td>(to Leo)I don't know. 49? I don't know (she hands a bunch of blocks to Ta'keisha)</td><td></td></tr>
<tr><td>228</td><td>O</td><td>(to Ta'keisha) You already got one in the middle so that's…</td><td></td></tr>
<tr><td>229</td><td>K</td><td>I know</td><td></td></tr>
<tr><td>230</td><td colspan="2">Both Ordena and Ta'keisha together count the stack of blocks in the middle of the tower. Leo starts putting blocks on the sides. He counts as he puts them there. The tower Ta'keisha has built is inverted to shaped. It</td><td></td></tr>
</table>

<table>
<tr><th colspan="4">Leo Ta’keisha Ordena</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observation</th></tr>
<tr><td></td><td colspan="2">has “support” blocks on only 2 of the 4 sides. This may be because of the available space on her desk.</td><td></td></tr>
<tr><td>231</td><td colspan="2">Ta’keisha counts the total number of blocks in her 10-block high tower. There seems to be confusion about whether there are 9 or 10 blocks in the height. The students start to realize there are 9 blocks on each side and 10 blocks in the height, for a total of 46 blocks. The tower had been built with blocks only on two sides, rather than all four, so Ta’keisha decides to start over. For at least one of the sides, they count 10 blocks, not including the middle block, leading Ordena to suggest there are 50 blocks in the 10-block high tower. They recount, leading Leo and Ordena to agree that there are 46 blocks in the 10-block high tower.
Ta’keisha decides to rebuild the 5-block high tower, and has taken apart the 10-block high tower. Ordena accuses Ta’keisha of “messing her up,” but it’s not entirely clear why. (Perhaps this is because the results for the total number of blocks has not been consistent for the 10-block high tower.) Ta’keisha confirms that there are 21 blocks in the 5-block high tower and demonstrates this to Ordena. Ordena brings up her argument from earlier, “We have to count the middle.” At this point, the teacher has asked the students to start cleaning up their materials and gives instructions to hand in their work. Ta’keisha tries one last time to explain to Ordena that the 2-block high tower has a total of 6 blocks. Ordena continues to write on her paper, and uses the calculator while Leo and Ta’keisha start cleaning up as per the teacher’s instructions.</td><td></td></tr>
</table>

Leo's Work, in Nov. 2008



SWBAT explore patterns

21 block in total for a five block High tower. and

42 blocks for 10 block high tower

6B blocks for 100 block high tower

21×20 = 420
21×1 = 21
2 =
3 =
4 =
5 =
6 =
7 =
8 =
9 =
10 =
11 =
12 =
13 =
14 =
15 =
16 =
17 =
18 =
19 =
20 =

Building Block Dilemma

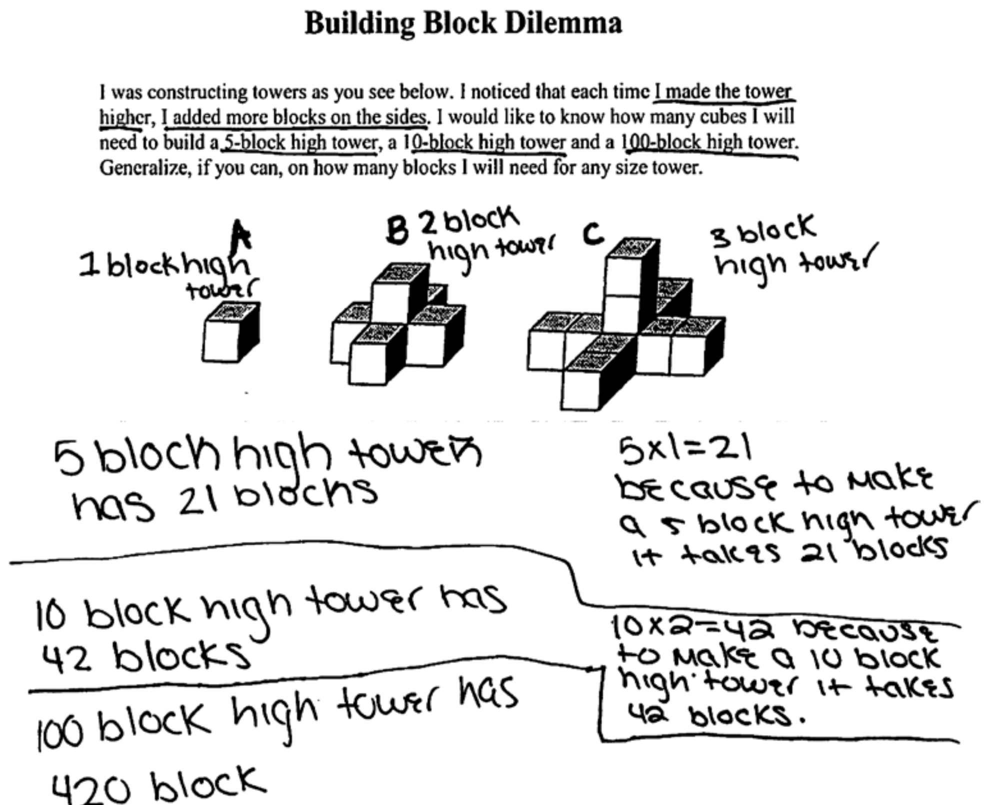
I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.

A 1 block B 6 blocks C 11 blocks

=> = 21 x 1-20 = 420

For each block tower what ever you do to the top you must do to the sides for example a 5 block high tower you will need 21 blocks for 10 you will need 46

Ta’keisha’s Work, in Nov. 2008



Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.

A 1 block high tower

B 2 block high tower

C 3 block high tower

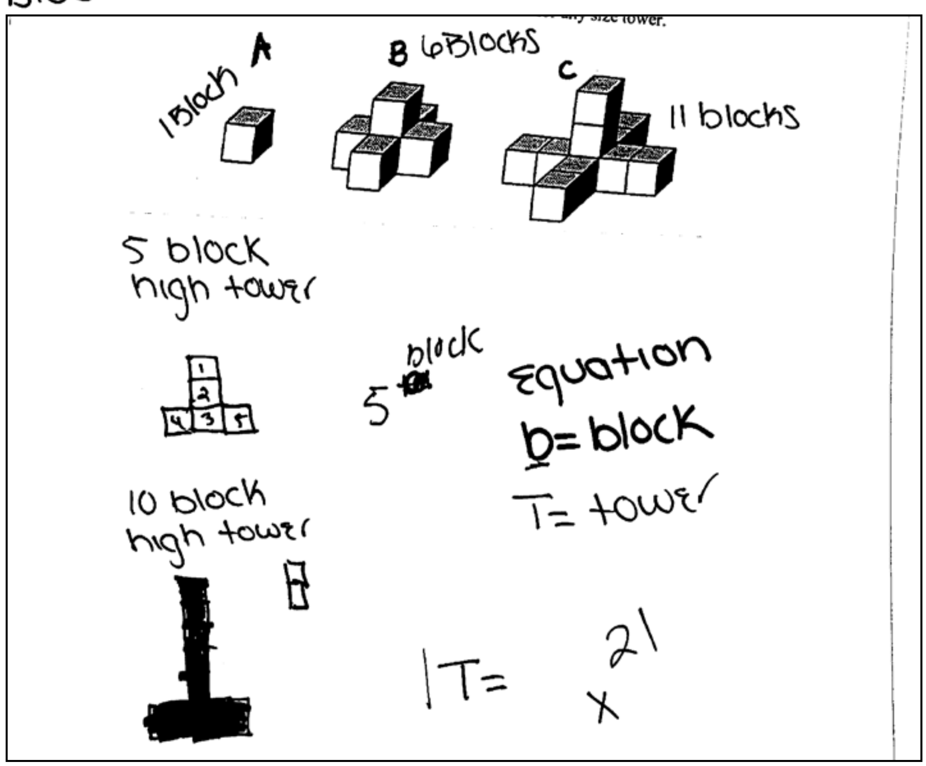
5 bloch high tower has 21 blochs

5x1=21 because to make a 5 block high tower it takes 21 blocks

10 block high tower has 42 blocks

10x2=42 because to make a 10 block high tower it takes 42 blocks.

100 block high tower has 420 block



A 1Bloch

B 6Blochs

C 11 blochs

5 block high tower

1 2 4 3 5

5 block

Equation

b= block

T= tower

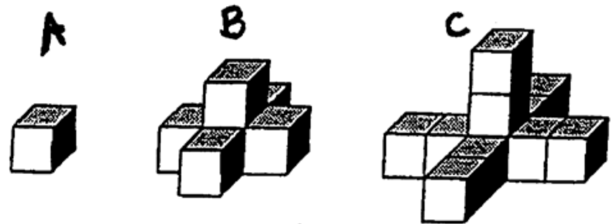
10 block high tower

1T= x 21

Ordena's Work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



1. you will need 21 cubes to build a 5block high tower

2.

building blocks.

height	side

-15 block
21X3=63 -20 block
21X4=84 -25
21X5=105 -30 blocks
21X6=126 -35 blocks
21X7=147 -40 blocks
21X8=168 -45 blocks
21X9=189 -50 block
21X10=210 -55 blocks
21X11=231 -60 block
21X12=252 -65 blocks
21X13=273 -70 blocks
21X14=294 -75 block
21X15=315 -80 blocks
21X16=336 -85 blocks
21X17=357 -90 block
21X18=378 -95 blocks
21X19=399 -100 blocks
21X20=420

Questionnaire Responses for Leo, Ta'keisha, Ordena

Questionnaire items which may indicate LMTY structure

Questionnaire Items	**Leo**	**Ta'keisha**	**Ordena**
Statements (All the time, Sometimes, Never)			
I wanted to teach another student something that I knew that the other student did not know.	Sometimes	Sometimes	Sometimes
I listened carefully to the ideas of someone I was trying to help.	Sometimes	Sometimes	All the time
I helped someone see how to do the math.	Sometimes	Sometimes	Sometimes
Others listened carefully to my ideas.	All the time	Sometimes	Sometimes
Behaviors (Often, Sometimes, Hardly ever)			
I gave helpful suggestions.	Sometimes	Sometimes	Sometimes
I worked cooperatively.	Sometimes	Sometimes	Often
Thoughts (Yes/No)			
I like teaching this person things that I know.	Yes	Yes	Yes

Questionnaire Responses for Leo, Ta'keisha, Ordena

Questionnaire Items Which May Indicate LHSIA structure

Questionnaire Items – Statements	**Leo**	**Ta'keisha**	**Ordena**
Statements (All the time, Sometimes, Never)			
I wanted people to think that I'm smart.	Sometimes	All the time	All the time
I tried to impress people with my ideas about the problem.	Sometimes	All the time	All the time
People seemed impressed with the ideas I shared about the problem.	Sometimes	Never	Sometimes
People saw how good I am at the math we did today.	All the time	Sometimes	All the time
I felt smart.	Sometimes	All the time	Sometimes
I wanted to show someone that my way was better.	Sometimes	All the time	All the time
I was a lot better at math than others today.	All the time	Sometimes	Sometimes
Behaviors (Often, Sometimes, Hardly ever)			
I was the leader.	Sometimes	Often	Sometimes
I was bossy.	Hardly ever	Hardly ever	Sometimes
I wanted to show off.	Hardly ever	Often	Hardly ever
I liked to be right.	Sometimes	Often	Sometimes
Thoughts (Yes/No)			
I want you to know just how smart I am.	Yes	Yes	Yes
People think I'm smart.	No	Yes	Yes
I wish the teacher would call on me, so I can show how much I know.	Yes	Yes	Yes

Nazira, Aleana, Keshia

This is Group 2 from Ms. S's grade eight Class 2.

Verbal emphasis indicated by underline
Actions included either in italics or (parentheses) as appropriate.
Pauses in speech are indicated by …
Words inserted to help clarify a student's words are included in [brackets].
When students voices overlap, one or both sets of words are included in //double slash marks//.

The number included next to either LMTY or LHSIA refers to the code, included in the table of codes.

T: Teacher
U: Unknown
Sts: Several Students

Nazira, Aleana, Keshia			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
	The teacher has just finished her introduction to the problem and given instructions. This has taken about 6 minutes, with students (including some from this group) reading the problem and responding to questions. The teacher let them start, and then briefly interrupted to remind her students they may use different materials in the room.		
1	Aleana (A)	(Starts reading the problem out loud.) I was constructing a tower as you can see below. I noticed that each time	**EPISODE 1** Aleana starts talking to self/group; Group discusses what the problem is asking for
2	T	And remember if you're in a group and somebody is saying something, saying that the problem means this or the problem means that and you're not convinced that that's what the problem means, you need to voice that opinion.	
3		*While the teacher is making this announcement Aleana continues to read the problem so she may not have heard the advice the teacher was giving.*	
4	A	(to her group; tone of voice is excited yet patient, as though she believes she	**LHSIA (14); LMTY (1)**

<table>
<tr><th colspan="4">Nazira, Aleana, Keshia</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observation</th></tr>
<tr><td></td><td></td><td>understands the task)
Oh, I think they wanna know. They wanna know, they wanna know how many cubes will they need to build a 5-block high tower, a 10-block high tower, and a 100 block high tower.
(Both Nikyah and Keyahna are looking at Aleana & thus seem to be paying attention to her.)</td><td>Aleana is the first to share her idea with her classmates, and she seems to have their attention. She is saying, I think I know what to do, and explains this to her two classmates (though it’s the same as the instructions).</td></tr>
<tr><td colspan="4">Interpretation: Aleana has shared an idea with the whole class during the task introduction. Here, she says, “Oh!” in a tone of voice that indicates she believes she understands the task. She appears to be demanding attention, as the one in the group who understood the instructions first (LHSIA). She then goes on to explain her understanding of the task. Though she is re-stating the instructions, her tone of voice indicates she wants to explain this to her two classmates who have not yet indicated whether they understand the task or not (LMTY).</td></tr>
<tr><td>5</td><td>T</td><td>Class announcement: Students can use materials in room.</td><td></td></tr>
<tr><td>6</td><td>Nazira (N)</td><td>(talks while Ms. S makes class announcement)
Oh, this is like…
(tapping her pen on the paper)</td><td></td></tr>
<tr><td>7</td><td>N</td><td>(pointing to her paper)
It says right here that each time I made the tower higher I added more blocks to the side. Make a *inaudible* height of the tower.</td><td></td></tr>
<tr><td>8</td><td colspan="2">Ms. S, the teacher, comes over to the group and observes, asking them to explain to her their interpretation of the problem. Aleana starts, sharing the same thing she said earlier. Ms. S asks questions getting them to clarify the problem in their own words. She also tries to encourage the other students to share their ideas as well. They realize there is some pattern of 5s. They discuss ideas such as making a table to determine the pattern they have. The students say they are looking for the answer for the 100-block high tower and then the teacher leaves the group. She has worked with them for approximately 4 minutes.</td><td></td></tr>
<tr><td>9</td><td colspan="2">After she leaves students work quietly on their own.</td><td></td></tr>
</table>

Nazira, Aleana, Keshia			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
	Aleana is writing things down on her paper but Nazira and Keshia are just staring off looking at the teacher as she walks over to work with another group.		
10	N	(inaudible; to Aleana)	
11	A	Maybe it would be like 10 times 1 equals 10. Ten plus 5, 15.	
12	N	Look, 1, 2… Wow. Look, see what they're doing now? Look it… (Aleana has gotten up, pauses to look at what Nazira is doing, and then walks away from the table. Keshia looks in Aleana's direction as she walks away)	
13	Keshia (K)	Where is she going? (*Sits as though she is waiting for Aleana to return to table*)	
14	N	(after writing on her own paper, Nazira reaches over to Keshia's paper and points to A, B, C respectively) One, two, three.	**EPISODE 2** Nazira shares her understanding regarding the heights of the tower structures
15	K	What did you say, though?	
16	N	Look… One, Two, Three.	**LMTY (1)** Nazira is showing Keshia and later Aleana how these tower structures are labeled. She appears to be showing them that the three diagramed on their papers (A, B, C) are the towers with height 1, 2, and 3, respectively.
17	A	*Aleana returns with a bucket of blocks, but does not take any blocks out.*	
18	N	*(Still speaking to Keshia)* You should... I don't know how to say it. Look. It started with one. It had two right here, it got one. It started with one. Go to one, then that's two. One. One, one, two.	**LMTY (1, 2)** Though Nazira is mostly repeating the words, "One two three" she is also

<table>
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<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observation</th></tr>
<tr><td></td><td></td><td>Then this one might be three.</td><td>pointing to the diagrams on her Keshia's paper, presumably to help with the explanation.</td></tr>
<tr><td colspan="4">Interpretation: Nazira appears to have realized that the towers given in the diagram, labeled A, B, and C, are the towers with heights one, two, and three, respectively. She shares this with Keshia who may have indicated that she did not understand with Nazira said at first. Nazira then modifies her explanation, saying, "It started with one." She appears to be explaining how to properly label the given figures – rather than call them A, B, C, they could be called by the height of each tower – 1, 2, and 3.Nazira appears to be implying that the base middle block is still there in towers B and C. I suggest she is trying to help her classmates understand, in part, because she is pointing on their papers, rather than simply stating the towers' heights.</td></tr>
<tr><td>19</td><td>K</td><td>So you're saying it'd be like this? (starts writing something on her paper)</td><td></td></tr>
<tr><td>20</td><td>N</td><td>(Nazira has her pen on Keshia's paper)
It would be 3, like add another cube, right here.
Aleana sits quietly just watching the interaction between Nazira and Keshia.</td><td></td></tr>
<tr><td>21</td><td>N</td><td>Wait, wait…
We just add it.
(talking to A)You see it?
(Aleana indicates no, shakes her head)
(Nazira leans over and points to A's paper)
Like one, one… doesn't matter right now. 1, 2, 3
(she seems to be pointing to diagrams A, B, and C)</td><td>LMTY (2, 10)
Nazira asks Aleana if she understands her explanation. Aleana shakes her head, no. Nazira modifies her explanation by pointing on Aleana's paper this time, pointing out that towers A, B, and C have heights of one, two, and three blocks.</td></tr>
<tr><td>22</td><td>A</td><td>Oh!!! Right here! (sounds like the sound of recognition.)
I get it. One, two three. (moving her finger across the towers on her paper, possibly to indicate height)</td><td></td></tr>
<tr><td colspan="4">Interpretation: Nazira continues her explanation about the towers A, B, and C having</td></tr>
</table>

Nazira, Aleana, Keshia			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
heights, one, two and three, and asks her classmate Aleana if she understands the explanation, “You see it?” When Aleana responds no, Nazira leans over to her paper to point out the connection between the towers and their respective heights. Aleana says, “Oh!” likely indicating that she understands Nazira’s explanation now, and she repeats, “One, two, three” to support that.			
23	N	*(says something inaudible)* *Reaches into the blocks bucket and starts taking some blocks.* *Students work on their own papers for a brief period, and work with the blocks. The bucket is blocking the construction taking place on Aleana’s desk.* *The students speak some as they work, but it is unclear as to what they say.*	
24	K	*Is building something on Aleana’s desk*	
25	N	That’s 3.	
26	A	But then it won’t make sense because the one she builded up the one is gonna come out. (gesturing with her hands up, then out)	
27	N	*Explaining to A*. That’s still going to be going up. Like it’s going forward. You got different floors.	**LMTY (1, 7)** Nazira responds to Aleana’s statement. Aleana seemed to indicate that she doesn’t understand something. Nazira provides an explanation, stating that the tower goes up and goes forward.
Interpretation: Nazira responded to Aleana who appeared to indicate that she didn’t understand something, saying “But then it won’t make sense.” There seems to be some confusion about how the blocks are constructed, as they appear to only have discussed the height of the various blocks. (Though there were block son the desk, the view was obstructed from the camera by the bucket holding the blocks.) Possibly Aleana was not sure about how the sides of the block were to be constructed. Nazira appears to explain that the tower does go both up in height and out on the sides. Based on Aleana’s next comment, perhaps she is trying to connect this task with a task the class has worked on previously.			
28	A	Oh!!! I just… You know how, like make tower (inaudible) height.	

Nazira, Aleana, Keshia			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
		(Nazira nods in response) It's almost something similar.	
29	N	*Says something inaudible and all 3 girls smile.* Doesn't look like the other one (may be referring to their construction) You got brains.	
30		*For about 3 minutes:* *The students continue to work, sometimes quietly. They discuss making a table and getting an equation, but they do not seem to make much progress toward either option. Nazira appears to guess some possible equations, using a trial and error method. They also briefly discuss finding an answer for a 100-block high tower.* *As the students continue to work, Nazira believes that the number 50 is the key to the answer they are looking for. When Aleana asks where 50 'comes from,' Nazira admits she cannot explain it.*	
31		*Researcher asks them to place a notebook under the audio recorder.* *At this time, the students also move the bucket so that the tower structure they constructed earlier is revealed to the camera. It is difficult to tell, but it appears that they created the 5-block high tower. There has been no indication that they counted the total number of blocks in this structure, however.*	
32	A	*Raises her hand to get the teacher's attention. Just before Ms. S comes over to the group, Aleana and Nazira suggest one more equation to try.*	
33		*For about 6 minutes:* *When Ms. S. arrives at the group, they explain that they are trying to come up with an equation and a table. When they start explaining their pattern, Ms. S asks them to physically construct the towers, using the blocks they have. She also asks them to recall information such as what they need in*	

Nazira, Aleana, Keshia			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
		order to come up with an equation, such as the variables. The students struggle to verbalize that they need to identify the height and the total number of blocks of the structure as the variables for their table. After they do so, the teacher leaves this group.	
34		*Students start writing on their own papers. Occasionally they look around at one another.* *They discuss the total number of blocks they think are in each tower structure. They seem to say that tower A has 1 total block, B has 5 total blocks and C has 10 total blocks. Nazira pulls a graphing calculator out of her personal bag and it seems as though she is trying to see what the equation might be.*	
35	K	(Counting the blocks for tower structure B) Where you get five from, Nazira?	**EPISODE 3** The students start to recognize the correct number of blocks in each tower structure.
36	A	Look here. (counts the blocks in tower B, constructed from blocks in front of them) *Nazira continues to work with her calculator.*	
37	K	But that, say, y'all wanna put uh, so, you got to count that one too. (Aleana picks up the top block in tower structure B) 'Cause that's the one that's holding up all them.	**LMTY (1, 4)** Keshia tells Aleana that they need to count all the blocks. It seems they did not count the middle base block.
38	A	Oh, snap! Nazira, we did suttim' (something) wrong. She just got finished explaining, we had to find the total numbers of blocks, and it's one, six.	**LHSIA (9) LMTY (6)** Aleana agrees with Keshia's explanation about counting all the blocks. She chimes in to help explain to Nazira that there are

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<tr><td></td><td></td><td></td><td>six, not five blocks in tower B.</td></tr>
<tr><td>39</td><td>N</td><td>It’s one, who? (Still working on calculator)</td><td></td></tr>
<tr><td>40</td><td>A</td><td>It’s one. And if we count all of these, it’s six. //K: Six//</td><td>LMTY (3)
Aleana repeats that the two-block high tower, or tower B, has six blocks. Keshia joins in for this explanation.</td></tr>
<tr><td colspan="4">Interpretation: The three girls had been working off Nazira’s earlier contribution that there are 5 total blocks in the 2-block high tower. Keshia asks about this, and then point out to Aleana that there are 6 total blocks in the 2-block high tower, not 5. She explains to Aleana that the middle base block must be counted with all the others, thereby teaching her about how to properly count the total number of blocks in the tower structure.
Aleana sees this and agrees, telling Nazira that they made a mistake. She points out to Nazira that the 1-block high tower has a total of 1 block, and the 2-block high tower has a total of 6 blocks. By stating to Nazira that they made an error, rather than having Keshia explain this to Nazira herself, she may be trying to demonstrate her own understanding to both of her classmates. Aleana repeats her brief explanation when Nazira asks, “It’s one, who?” Keshia also joins in on, “Six.”
This scenario is interesting in that it appears that Keshia had an active LMTY structure first, after asking about the total number of blocks, and then Aleana activates an LMTY structure, building on Keshia.</td></tr>
<tr><td>41</td><td>N</td><td>That’s what I was saying. (looks up from her calculator)</td><td>LHSIA (9, 11)
Nazira says she already agrees (perhaps she does). In her next speaking turn, she says that there was an issue with the ‘top block.’</td></tr>
<tr><td>42</td><td>A</td><td>You put five.</td><td></td></tr>
<tr><td>42</td><td></td><td>That’s what I was saying. Otherwise, like…the top block was messing on up. (removes the ‘top block’ from the 2-block high tower)
‘Cause we don’t know that…This stuff is on the bottom on that.</td><td></td></tr>
<tr><td>44</td><td>K</td><td>But it is.</td><td></td></tr>
<tr><td colspan="4">Interpretation: Nazira agrees with what Aleana and Keshia explain about how to count the total number of blocks in the tower. However, by saying, “That’s what I was saying,”</td></tr>
</table>

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<tr><td colspan="4">she is suggesting that she stated earlier that the 2-block high tower should have 6 blocks. I think she may have been confused about whether to count that middle base block, even though they keep talking about the ‘top block.’</td></tr>
<tr><td>45</td><td></td><td>Students seem to go back to working independently. Nazira is still working on the calculator and Keshia counts the number of blocks in the 3-block high tower structure.</td><td></td></tr>
<tr><td>46</td><td>K</td><td>Though the calculator was supposed to tell us everything.</td><td></td></tr>
<tr><td>47</td><td>N</td><td>Trying to find an equation</td><td></td></tr>
<tr><td>48</td><td>K
N</td><td>You got charge that?
Batteries.</td><td></td></tr>
<tr><td>49</td><td>N</td><td>So it fits?</td><td></td></tr>
<tr><td>50</td><td>K</td><td>Yes. (reaches into the bin to get more blocks)</td><td></td></tr>
<tr><td>51</td><td>K</td><td>The next one is 11.</td><td></td></tr>
<tr><td>52</td><td>A</td><td>It’s… it’s.. It’s like going by odds and evens.</td><td></td></tr>
<tr><td>53</td><td>A</td><td>Ten, eleven. Six is an even number, 11 is an odd number.</td><td></td></tr>
<tr><td>54</td><td>N</td><td>(working independently with the calculator) Alright, what’s up with … Once I figure it out I’ll have it down pat.</td><td>LHSIA (5)
Nazira states that she will “figure it out.”</td></tr>
<tr><td colspan="4">Interpretation: Nazira has been working on her calculator, presumably to determine an equation that will allow them to generalize the total number of blocks for a given height. Though her classmates are trying to determine a pattern with the total number of blocks (as indicated by statements like, “going by odds and evens”), she is still working with her calculator. Here she states that she will “figure it out” and “have it down pat.” By stating this she seems to be letting her classmates know that she is smart enough to figure this out. Nazira may also be trying to make up for the mistake earlier, stating that there were 5 blocks in the 2-block high tower, rather than 6 blocks.</td></tr>
<tr><td>55</td><td>N</td><td>Two is six.</td><td></td></tr>
<tr><td>56</td><td>K</td><td>Two is six. Three is eleven.</td><td></td></tr>
<tr><td>57</td><td></td><td>The students continue working, with Nazira mostly working on her calculator. Keshia and Aleana continue to contribute ideas. However, none of the three appears to be trying to teach each other or demonstrate that she is smart to the others. They may be trying to write up a table, but this is not included on the work they hand in (except for Nazira).
As they work, Aleana notices that five blocks</td><td>EPISODE 4
Aleana recognizes a pattern to the total number of blocks in the tower structures, that each structure requires 5 more blocks.</td></tr>
</table>

Nazira, Aleana, Keshia			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
		are added to each successive tower, even though the total numbers of blocks are not multiples of five. *Keshia is working with the blocks to create one of the tower structures.*	*When Aleana first notices that the total increases by 5, this could have led into an LMTY or LHSIA episode but her two classmates appear to ignore her at first. She repeats this, prompting a potential active LMTY structure.*
58	A	(possibly talking to herself at first) It is 11, no 3 is 11. (to her classmates) It's still going up by 5. 'Cause look, one plus five, one plus five equals 6. Six plus eleven.	**LMTY (1)** Aleana is explaining, possibly to Nazira who sounds confused, why she sees a pattern of 'going up by 5.'
59	K	*She has finished creating her tower structure and starts to count the blocks, by one. Aleana looks on. Keshia separates the legs after she counts them, and then counts the number of blocks in the tower*	
60	A	It's not…	
61	K	It's 21. (referring to the 5-block high tower she constructed)	
62	N	It's six.	
63	K	For what?	
64	N	For that? (Nikyah points her pencil towards Keyahna's 'taken-apart' block structure.)	
65	K	For five?	
66	N	Yeah, this…	
67	A	I'm tryin' to let you know it's going up by 5's.	
68	N	Alright, 5x plus?	
69	A	You can tell, 'cause look. One plus 5 is 6. 6 and 5 is 11. 11 and 5, 21 //K: is 16.//	**LMTY (1)** Aleana repeats the explanation she gave earlier about adding 5 blocks to each

<table>
<tr><th colspan="4">Nazira, Aleana, Keshia</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observation</th></tr>
<tr><td></td><td></td><td></td><td>tower structure.</td></tr>
<tr><td colspan="4">Interpretation: Aleana is trying to explain to Nazira and possibly Keshia that there is a pattern of ‘going up by 5’s’. She does the addition, 1 + 5 = 6, etc. to support her explanation. Nazira seems to take this and try to input the information (in an unknown way) into her calculator, suggesting, “5x plus”? Aleana responds by continuing the pattern of adding 5 blocks, despite her arithmetic mistake of saying 11 + 5 = 16. Alternatively, she may have just misspoken, rather than made an actual error with the addition.</td></tr>
<tr><td>70</td><td>K</td><td>11 and 4 would be…
//A: Yeah.//</td><td></td></tr>
<tr><td>71</td><td>N</td><td>(after doing some work on her calculator) 21, 26, 31, 36, 41, 46, (trails off).</td><td>LHSIA (10)
Nazira picks up on the ‘plus 5’ idea that Aleana and Keshia shared a moment ago, and she starts to list the sequence that follows for the total number of blocks in the tower structures, starting with that of the 5-block high tower.</td></tr>
<tr><td colspan="4">Interpretation: Nazira was listening to Aleana and Keshia explain that the pattern to find the total number of blocks in the tower structure was ‘plus 5.’ She appeared to enter something into her calculator and then picked up where Aleana and Keshia left off earlier, when they were listing the total number of blocks in the tower structures, “One plus five is six, six plus five is eleven.” When Nazira starts to list the rest of the numbers, “21, 26, 31…” she appears to be building on her classmates’ idea and states these numbers. She may have been indicating that she was smart enough to pick up on their idea and see it as correct.</td></tr>
<tr><td>72</td><td colspan="2">For approximately 4 minutes:
The students work independently, with interruptions of conversation between the three of them. Nazira shares her calculator work with Keshia. They appear to try to go past the point of the 5-block high tower, as evidenced by determining the total number of blocks in the 6-block high tower.
Aleana recognizes that the pattern for the total number of blocks is still ‘going by fives.’ Aleana and Keshia continue to write in their notebooks. (Note that this work was not given to the research team.) Nazira continues to work with her calculator and appears</td><td></td></tr>
</table>

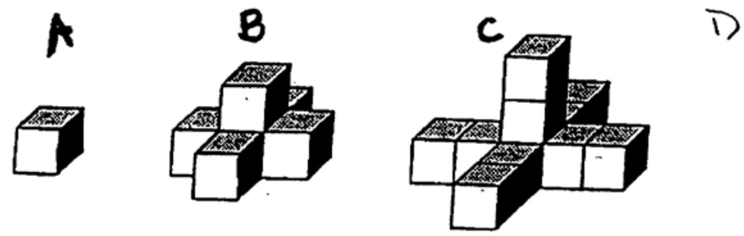
Nazira, Aleana, Keshia			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
	frustrated that she is not getting the equation to describe the pattern.		
73	N	It don't go to a hundred, it go to 101. Your last one would be 19, 18. Your last one --	**EPISODE 5** The students discuss the difference between the height of the tower and the total number of blocks in the tower.
74	A	They're not saying they want the last… I mean they don't want your total blocks. They want to get the height of 100.	**LHSIA (2, 8)** Aleana interrupts Nazira to correct her, and tell her that she is thinking about the 100 incorrectly. Nazira seems to think that the 100 refers to the total number of blocks. Aleana corrects her, saying the height is supposed to be 100 blocks, not the total number of blocks.
Interpretation: Aleana appears to be demonstrating that her idea is correct by doing two things: 1) interrupting Nazira and 2) correcting her suggestion. Nazira, in the turn just before, appeared to be telling Keshia that she was not supposed to get 100, but rather 101, which will be the total number of blocks for the 21-block high tower. Nazira incorrectly states this will be the total for the 18-block high tower. Aleana hears Nazira's statement and recognizes that the second girl is referring to the total number of blocks, rather than the height. Aleana tells her this, saying, "They want to get the height of 100." Aleana appears to have an active LHSIA structure rather than a LMTY structure because her tone of voice sounds impatient and she looks at Nazira in what might be an annoyed facial expression. In addition, she interrupted Nazira to get her point across.			
75	*For about 11 minutes (until end of problem solving session): The students resume working independently for a brief period. They also seem to complain someone, stating that, 'This is like kindergarten.' They discuss other non-task related things as well. Again, Nazira continues to work on her calculator. They alternate between conversation and working quietly, sometimes mumbling to themselves. Though they seem to revisit*		

Nazira, Aleana, Keshia			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
	the issue of the total number of blocks for the 100-block high tower, no answer or solution is discussed. They do seem to agree that the 5-block high tower has 21 total blocks. *After several minutes, Aleana shuts her notebook. A few moments later she reads the problem again and starts to write on her task paper. Nazira writes on her paper as well, though she still looks at her calculator. Keshia continues to work on her paper. It appears she is working on her task paper, and by reviewing that, it appears she was drawing 2-dimensional representations of the tower structures.*		
76	*After Ms. S tells them to put their names on their papers to collect them, the girls put the blocks away. The girls only hand in their task papers, not the papers from their notebooks they appeared to be writing in throughout the class period.*		

Nazira's work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



100 = b×1

1b = ×5

5b = b × 2

10b = b × 10

height	blocks
1	1
2	5
3	10
4	15
5	20
6	25
7	30
8	35
9	45
10	56
11	55
50	100

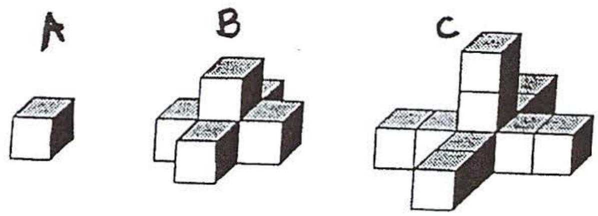
$y = mx+b$

$y = 2x+2$

Aleana’s work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower ~~and a 100-block high~~ tower. Generalize, if you can, on how many blocks I will need for any size tower.

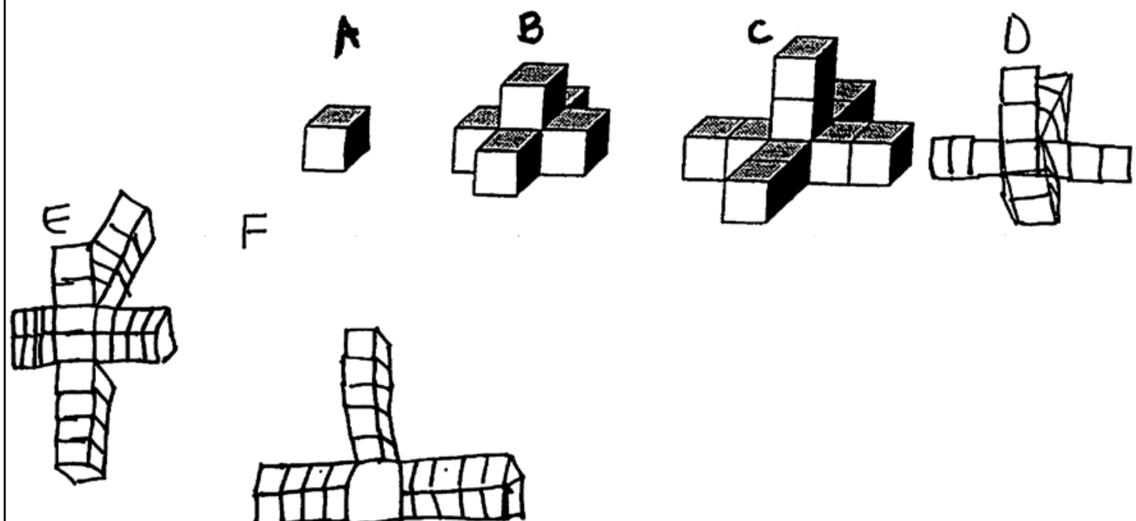


For A five block high tower you would need

Keshia's work, in Nov. 2008

Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.



Questionnaire Responses for Nazira, Aleana, Keshia

Questionnaire items which may indicate LMTY structure

Questionnaire Items	**Nazira**	**Aleana**	**Keshia**
Statements (All the time, Sometimes, Never)			
I wanted to teach another student something that I knew that the other student did not know.	Sometimes	All the time	All the time
I listened carefully to the ideas of someone I was trying to help.	All the time	All the time	Sometimes
I helped someone see how to do the math.	Never	Sometimes	Sometimes
Others listened carefully to my ideas.	All the time	Sometimes	Sometimes
Behaviors (Often, Sometimes, Hardly ever)			
I gave helpful suggestions.	Sometimes	Often	Sometimes
I worked cooperatively.	Often	Often	Often
Thoughts (Yes/No)			
I like teaching this person things that I know.	Yes	Yes	Yes

Questionnaire Responses for Nazira, Aleana, Keshia

Questionnaire Items Which May Indicate LHSIA structure

Questionnaire Items – Statements	Nazira	Aleana	Keshia
Statements (All the time, Sometimes, Never)			
I wanted people to think that I'm smart.	Sometimes	All the time	All the time
I tried to impress people with my ideas about the problem.	All the time	Sometimes	All the time
People seemed impressed with the ideas I shared about the problem.	Sometimes	Sometimes	Sometimes
People saw how good I am at the math we did today.	Sometimes	Sometimes	All the time
I felt smart.	Sometimes	All the time	All the time
I wanted to show someone that my way was better.	Sometimes	Never	Never
I was a lot better at math than others today.	Sometimes	Sometimes	Sometimes
Behaviors (Often, Sometimes, Hardly ever)			
I was the leader.	Sometimes	Sometimes	Sometimes
I was bossy.	Hardly ever	Hardly ever	Hardly ever
I wanted to show off.	Hardly ever	Sometimes	Hardly ever
I liked to be right.	Sometimes	Often	Sometimes
Thoughts (Yes/No)			
I want you to know just how smart I am.	Yes	Yes	Yes
People think I'm smart.	No	Yes	Yes
I wish the teacher would call on me, so I can show how much I know.	No	Yes	No

Ta'Shawna Tyesha, Carla

This is Group 3 from Ms. S's eighth grade Class 2.
Verbal emphasis indicated by underline.
Actions included either in *italics* or (parentheses) as appropriate.
Pauses in speech are indicated by …
Words inserted to help clarify a student's words are included in [brackets].
When students voices overlap, one or both sets of words are included in //double slash marks//.

The number included next to either LMTY or LHSIA refers to the code, included in the table of codes.

Ta'Shawna Tyesha, Carla			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
	The teacher has just finished her introduction to the problem and given instructions. This has taken about 6 minutes, with students (including some from this group) reading the problem and responding to questions. The teacher let them start, and then briefly interrupted to remind her students they may use different materials in the room.		
1	Carla (C)	Um, this is what I think. You see how they say that, I notice that each time the tower gets higher, I add in more blocks on the side. I would like to know how many cubes I would need to build a 5 block high tower. You have to find, like how many cubes we need to make a 5 block like front.	**EPISODE 1** **LHSIA (14)** Carla is the first to share an idea about the problem to the group, "You have to find how many cubes we need." She continues to read aloud the directions after the brief teacher interruption.
2	Ta'Shawna (A)	I like how you said it, because its 5 blocks. So, those don't look like the towers to me.	
3	Ms S.	*Class announcement: Students can use materials in room.*	
4	C	It says how many cubes I would need to build a 5 block high tower, a 10 block high tower, and a 100 block high tower. Generalize if you can on how many blocks I would need to build any size of tower.	

<table>
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<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observation</th></tr>
<tr><td></td><td></td><td>(reading the problem aloud to group)</td><td></td></tr>
<tr><td>5</td><td>Tyesha (T)</td><td>That's good, so we got 5 blocks, we got one block, so we got 5 blocks so far.</td><td></td></tr>
<tr><td>6</td><td>T</td><td>Oh wait, so wait, 1, 2, 3, 4, 5
(counting one of the towers in the diagram on the given task paper)
That’s not a 5-block high tower, but it’s gotta be a 5 block high tower, (gestures with her hands parallel to one another) so that means, so like each of these gotta be 5, right?</td><td>LHSIA (14, 15): Tyesha explains her understanding of the difference between the tower given on her task paper and what the 5-block high tower should look like. She asks her classmates to confirm her understanding.</td></tr>
<tr><td>7</td><td>C</td><td>Mhm.</td><td></td></tr>
<tr><td>8</td><td>A</td><td>Oh, I see.</td><td></td></tr>
<tr><td>9</td><td>T</td><td>So that means like, if this is 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, this is 10 blocks, right? (counting and pointing to figure C)
[ST: This is ten, yeah.]
//Five.//
So another 5 right here, right? Another 5 right there. Another 5 right there, and that would be 1, 2, 3, 4, 5.
[C: Mm-hmm, shaking her head in agreement]
6, 7, 8, wait, 6, 7, 8, 9, 10, 11, 12.</td><td>LMTY (1): Tyesha is explaining her understanding about how many blocks are in each structure and how to construct the 5-block high tower.</td></tr>
<tr><td>10</td><td>C</td><td>Wait, what you trying to do?</td><td></td></tr>
<tr><td colspan="4">Interpretation: Tyesha, who had read the problem aloud to the class, starts to share her ideas on how the towers are constructed, and which tower is (or is not) the 5-block high tower. This suggests an active LHSIA structure, since she seems to be demanding the attention of her two classmates. She asks for confirmation that the height has 5 blocks, “Each of these gotta be 5, right?” Her classmates appear to agree with her and listen to her as she suggests that a 5-block high tower has 5 blocks in the height (indicated by her hands). She then appears to branch into an active LMTY structure. It is possible Tyesha believed her classmates did not fully understand the construction of the towers, as she previously explained. Tyesha went from sharing her own knowledge of the problem to explaining it to her two classmates.</td></tr>
<tr><td>11</td><td>A</td><td>Look, like if we add 5 to each one.</td><td></td></tr>
<tr><td>12</td><td>C</td><td>Mm-hmm.</td><td></td></tr>
<tr><td>13</td><td>T</td><td>T: Say this is one, right? Add 5 to each one. This is this side.
[C: Yeah)]</td><td></td></tr>
</table>

Ta'Shawna Tyesha, Carla			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
		And this is that side. So if this is this side, then that is that side, and this is this side. (Drawing on her paper as she talks.) And the top part, this part, is gonna be like that right, and each has got to add to 5, right? So we have 2, 4, … [C: Mm-hmm]	
14	C	So do it over, everything beyond there, that's how it's gonna come out to be.	
15	T	1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 10, 25, 10, 15, 20, 25. So to build a 5-block, a 5 block high tower, you need 25 blocks, 25, right? To build a 10 high block you need 25 times ...	
16	A	So, how did you…so how did you get 25?	
17	T	Cause look. (draws on her paper as she talks) Say if you drew it from right, if you drew it, I don't know how I drew it, but this is the first block, and that is middle block right? Then this is … that, that.	**LMTY (1, 3)**: Tyesha continues with her explanation, responding to Ta'Shawna's request. She is trying to explain how she determined there are 25 blocks in the 5-block high tower.
18	A	So you showing this one, right? (pointing to figure C)	
19	T	Yeah I'm showing the bottom one for you. So all of this right here. 1, 2, 3, 4, hold on, 1, 2, 3, 4, 5.	
20	S	If you could (inaudible)	
21	T	All this right there right? If we got two, one each. Two, right? That and that, two come out, one each. Two, right? [C: Mm-hmm] So that's, 1, 2, 3, 4, 5, 6, 7, 8, 9 right? Then you got two coming out, then you got this one right here and these two right here, right? So, you got 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, right? Then put this	**LMTY (1):** Tyesha is explaining by counting out loud how she determined the 25 blocks. She is stating where blocks are added to get the full tower structure.

Ta'Shawna Tyesha, Carla			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
		already, 3, add two more, boom. This already two, three more. So that makes it 5, right? This, three more, this, three more, right? <u>Now,</u> it's a five block tower.	
Interpretation: Tyesha had given an answer of 25 blocks as the total number of blocks for the 5-block high tower. Though she had counted out loud, her classmate Ta'Shawna still seemed confused about how she got this answer, and asked her for an explanation. Over several speaking turns (and this continues below), Tyesha tries to explain how she constructed the 5-block high tower in order to count the total number of blocks. Her classmates still seem confused that one of the given diagrams on the task paper does not include the 5-block high tower, so she uses the diagram and builds on it ("add two more") to indicate how she determined which would be the 5-block high tower.			
22	C	So count the <u>whole</u> thing?	
23	T	(*counting the blocks, pointing with her pencil as she goes along*) One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty, twenty-one, twenty-two, twenty-three, twenty-four, twenty-five. (*says "twenty-five" with satisfaction, possibly implying the question, "Do you see that?" to C and A*) Five, ten, fifteen, twenty, twenty-five (*counting again, pointing to her drawing on her paper*)	
24	C	Oh! Now I get it.	
25	C	So you trying to say that, like, so, like, add 5 blocks to each, like this one is.	
26	T	[pointing to the diagrams on her paper using her pencil] Yeah. No. Each, like, say, each set. Say if we call this set one, call this set two, this set three, this set four, and this right here, set five. (drawing circles on blocks on Figure C, to denote each leg and the height into what she calls sets – see T's work)	**LMTY (2, 4):** Tyesha modifies her explanation, addressing Carla's apparent misunderstanding of the previous explanation.

Ta'Shawna Tyesha, Carla			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
		Right? [C: (agreeing) Yeah.]	
27	T	Each set has five blocks (holds up five fingers). [C: Oh! (indicates understanding)] There's five blocks for each set. [C: Okay.] So if there's five blocks for each set, then, add together, and you count it up, there's twenty five blocks in all. (T's tone is patient and controlled. T continuously makes eye contact with C and sometimes A. T settles back into her chair.)	
Interpretation: Carla appears persistent in trying to understand Tyesha's strategy. She asks questions and restates her own interpretation. Tyesha appears to think that Carla is still mistaken and continues with her own explanation. Therefore, she is correcting Carla, and doing so in a way that is intended to be helpful, as evidenced by the modified explanation. Here she is calling each side or leg a "set" of blocks and says that, "Each set has 5 blocks."			
28	C/A	So this if for this set, this if for this. (both Carla and Ta'Shawna speaking, overlapping)	
29	T	No. This is one. (*with hesitation in her voice, perhaps she is not sure how to answer their questions. T leans forward in her chair*)	
30	C	Yeah, this is the one. So you have to add five to there, right? [A: Yeah. (softly)] No, four?	
31	T	Okay. So this is one (*referring with her pencil to figure A*). Then, this is the two-block. (*referring to figure B and making a note on her paper*) You see the bottom one, you're basically making the two-block, right? This is three-block, bottom, dum, dum (*"dum" refers to the two blocks on top of the middle bottom block*). That's the three-block. (*pointing with her*	**LMTY (1, 3):** Tyesha explains how to call the blocks given on their task paper, that is A is the 1-block high tower, 2 is the 2-block high tower and so on. She builds on this explanation to

Ta'Shawna Tyesha, Carla			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
		pencil to figure C) If this was the four-block, it should be three squares going up like that. (*T keeps her eyes on her paper, as she writes her notes*.) That should be a four-block. There should be four on each side. (*T seems to be explaining the construction of the towers.*)	describe what the 4-block high tower would look like.
32	C	Oh! (*drawn out, implying understanding*) Now, I get it.	
33	T	Five blocks.	
34	C	So this one is five-block, right? This one, for number C? (C says with question in her voice, as though she is looking for validation)	**LHSIA (10, 12)**: Carla is building on what Tyesha is saying, trying to indicate that she understands her classmate's explanation.
35	T	Well, no. This is one. This is the one-block, right. (T looks at C, and displays her forefinger to indicate one) See, it's the one-block because you see how there's only one block in it, right. One block tall, right? This is a two-block, because the bottom – don't forget about the bottom part. (pointing at figure B, see task) That's another block right there. The bottom part added this one makes this two, and the bottom makes this two, the bottom makes this two, makes this two, right? (T is pointing at figure B with both her pencil and her finger, indicating that the height and each leg has a length of two if you include the hidden block. She then puts up two fingers.)	**LMTY (2, 4):** Tyesha modifies her explanation to demonstrate to her classmates that figure A from the task is the 1-block high tower. When she discusses the 2-block high tower, she reminds her classmates there is a middle hidden block that is there and contributes to the height of the tower (thus making B the 2-block high tower rather than the 1-block high tower).
36	T	That means there's two blocks tall, two block high tower. Right? And, this is	

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<tr><td></td><td></td><td>only a three-block high-tower
(puts up three fingers)
because the bottom
(T and C point to figure C)
and the two, make it three high tall
(uses her two hands to indicate height).
(A asks a question, inaudible, and points to the diagrams on T’s paper)
One two, three. Then the four block would be the same way.
(drawing on her paper to help make her point)
The four block would be the same way. You have that one. You had a one, two, three. Three, three, three.
[C: Oh.]
And then you had the two on the top. That’s what makes it a, uh, four-block. Make it a four block tower. So a five-block is gonna look like this. Five blocks tall, one two three four five.</td><td></td></tr>
<tr><td colspan="4">Interpretation: As Tyesha continued to give explanations about how she determined the 5-block high tower has 25 blocks, Carla and Ta’Shawna made efforts to try to understand, by asking questions and sometimes giving their interpretation of what was said. Tyesha realized they were having difficulty understanding how she constructed the towers, so she went back to the diagrams given on the task paper and explained that figures A, B, and C were the 1-block high tower, 2-block high tower, and 3-block high tower, respectively.
Carla still seems to misunderstand, as she asks if figure C is the 5-block high tower. Tyesha then modifies her explanation to include the fact that the initial block from A is still there for figures B and C, it’s a middle hidden block that is a “bottom part [which] makes this [B] two.” Tyesha continues with each tower, until she reaches the 5-block high tower, indicating that there are a total of 5 blocks in the height including what she called “the bottom part,” or the middle hidden block.</td></tr>
<tr><td>37</td><td>C</td><td>So for this one,
(pointing to figure A)
right, add 5 more, this side? That’s what I'm trying to ask.
[T: Huh?]
//Add 5 on each side right? 5 going up?//</td><td></td></tr>
<tr><td>38</td><td>T</td><td>//Yeah, on each side, 5 going up. Five going that way//
(indicating one of the legs),</td><td></td></tr>
</table>

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<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observation</th></tr>
<tr><td></td><td></td><td>five going that way (indicating with her pencil a different leg).</td><td></td></tr>
<tr><td>39</td><td>C</td><td>Alright, let me try that. Let me see. Okay. 1,2,3,4. Let me see, 5, 1,2,3.</td><td></td></tr>
<tr><td>40</td><td>C</td><td>Yeah
1,2,3.</td><td></td></tr>
<tr><td>41</td><td></td><td>Students work on their own papers for about 30 seconds before Carla speaks up again.</td><td></td></tr>
<tr><td>42</td><td>C</td><td>I'm gonna start with a one block first. 1, 2, 3, 4, 5. This is a one block, okay? This is a two block. 1, 2, 3, 4, 5, okay, got it. Oh now I get it! Look, look, and then the next one is 10, right?</td><td>LHSIA (4, 15): Carla is letting her classmates know that she ‘gets it’ or understands a part of the problem. She continues by giving what she thinks is the answer to the next one (perhaps meaning the 6-block high tower) and looks to her classmates for confirmation.</td></tr>
<tr><td>43</td><td>T</td><td>So you add 10 to it.</td><td></td></tr>
<tr><td>44</td><td>T</td><td>So you add 5 more to the tower right?</td><td></td></tr>
<tr><td>45</td><td>C</td><td>This is 10, so all you do is times 10, 60 all together.</td><td></td></tr>
<tr><td colspan="4">Interpretation: Throughout the episode above, Carla has made efforts to understand Tyesha’s explanations about the construction of the towers and how to properly determine the total number of blocks in the 5-block high tower structure. Here, Carla seems to want her classmates to know that she “gets it,” to make sure they know that she is smart enough to understand the problem and the solution strategy. She seems to want to move on to the next part, asking, “the next one is 10, right? She is looking for confirmation of her contribution, when she asks, “right?” It is not clear to me whether she is referring to the next size tower (6-block high tower) has 10 blocks in total, or if she is referring to the next part of the question, which asks for the total number of blocks in the 10-block high tower.</td></tr>
<tr><td>46</td><td>T</td><td>No, but, you wanna make it a 5 block right, so 5 times 5 is 25.</td><td>LMTY (4): Tyesha corrects Carla’s suggestion that there are 60 blocks, reminding her why</td></tr>
</table>

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<tr><td></td><td></td><td></td><td>there are 25 blocks in the 5-block high tower.</td></tr>
<tr><td>47</td><td>A</td><td>Yeah, and, 5 times 10 is um, 50, so.</td><td></td></tr>
<tr><td>48</td><td>T</td><td>50 blocks, yeah.
5 times 10 is 50. If they want 11 blocks, 5 times 11 is 55. 5 times 12, 60 blocks.
(Ta’Shawna and Carla nod in agreement with Tyesha)</td><td></td></tr>
<tr><td>49</td><td>A</td><td>Yeah, yeah. Adding five.</td><td>LHSIA (9, 12): Ta’Shawna agrees with Tyesha as indicated by nodding her head and saying, “Yeah.” She also adds her thought, “Adding five.”</td></tr>
<tr><td colspan="4">Interpretation: After Carla asks about the next one being 10 and suggests there are 60 blocks, Tyesha corrects her saying, “No,” adding “you wanna make it a 5 block [tower],” possibly suggesting that Tyesha thought Carla was still talking about the 5-block high tower.
Tyesha continues to explain, after Ta’Shawna’s suggesting that 5 times 10 is 50, so there are 50 blocks in the 10-block high tower. Ta’Shawna nods her head in agreement, along with Carla, when Tyesha continues with the ‘times 5’ rule, and adds “Yeah, yeah.” Therefore, she is not only demonstrating that she agrees with Tyesha, but perhaps she wants to be recognized by her classmates for doing so. She continues with the statement, “Adding five” to contribute more to the conversation. By doing this, I infer that she wants Tyesha and Carla to recognize that she, too, has worthy ideas and can keep up with the others.</td></tr>
<tr><td>50</td><td colspan="2">For about 3 minutes:
The three students start to write independently on their own papers. Tyesha suggests they should all make a chart to organize their information. They briefly discuss what information should be included, like the letter corresponding to the stage, the number of blocks in the height, and the total number of blocks in the tower structure. The students seem to be puzzled over how to properly label their columns. It seems they call the height of the tower, “Number of block” and the total number of blocks in the structure, “How many cubes.” The puzzlement leads to a discussion as to how many Cubes are in the first tower, the 1-block high tower.</td><td></td></tr>
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<tr><td>51</td><td>T</td><td>If it’s one block then it’s one cube, if its two blocks, then how many cubes is it? So this is one right? Two, so, like this… So 2, 4, 6, 8, 10.
(using her pencil to point at figure B on her task paper as she counts)
So, if its two block, then its 10 cubes. If it’s 3 blocks then how many cubes is it? 3,6,9,12,15, its 15 cubes. If it’s 4 blocks, then its 4, 8, 12, 16, 12. (says numbers slowly, as though she needs to check her math)</td><td>EPISODE 2
LMTY (1): Tyesha explains how to determine the total number of blocks in each tower structure. She (correctly) states that the 1-block high tower. Then she counts to demonstrate that she (incorrectly) sees 10 blocks in the 2-block high tower. She may be counting that middle base block each time she counts the height and each leg.</td></tr>
<tr><td colspan="4">Interpretation: Tyesha has been taking on the role of explaining and helping her two classmates understand her interpretation of the problem since they began their group discussion. The three students were trying to organize their information in a written table, and each student was writing on her own paper. While writing, they discussed what to include, and how to label the columns in the table. There was some inaudible discussion but it seemed that a student (possibly Ta’Shawna) thought that the 1-block high tower might have 5 blocks rather than just the 1 block in the total structure. Here, Tyesha explains, using the diagram that there is only one cube in the one-block high tower. (The students use the term cubes when referring to the blocks, as in the total number of blocks for the structure; the term blocks is used when referring to the blocks that made up the height of the structure.)
Tyesha then continues beyond the first diagram, and counts to demonstrate her explanation of the 2-block high tower having 10 blocks. She continues to count the total number of blocks in the 4-block high tower.</td></tr>
<tr><td>52</td><td>A</td><td>So what you are trying to say? Adding what? So how many are added?
… came up with 16?</td><td></td></tr>
<tr><td>53</td><td>T</td><td>If this the four block, right?
(draws a 2D image of the 4-block high tower on her paper, as Carla and Ta’Shawna look on)</td><td></td></tr>
<tr><td>54</td><td>C</td><td>The pattern is (inaudible) right? So how many cubes are added to it?</td><td></td></tr>
<tr><td>55</td><td>T</td><td>No, wait.</td><td></td></tr>
</table>

Ta'Shawna Tyesha, Carla			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
56	A	Understand, that's why, that's where the strategy comes from.	
57	T	Four blocks, so if this is four blocks, this is the middle right? (starts counting, based on the drawings on her paper) 1,2,3,4,5,6,7,8,wait,1,2,3,4,5,6,7. Aight, 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18, 19,20,21, so the 4 block is 21 cubes. If its 5 blocks its 25 cubes.	
58	A	How is it 21?	
59	T	Huh?	
60	A	Cause most of them, always to the number.	
61	T	Yeah, if the 4 block is //(voices overlapping, inaudible)// (Counting) Yeah, wait. (Counting) Oh! (inaudible) It should be 16, because 1 times 1 is 1. 2 times 2, it can't be times itself because, this one, 2 times 2 is not 4. 2 times 2 is 4, right? But (counting) it's the two block and its ten cubes in all.	**LMTY (4):** Tyesha is still trying to explain to her classmates the total number of blocks in the 4-block high tower. She is correcting Ta'Shawna (and ultimately herself) who asks about having 21 total blocks in the 4-block high tower. She now suggests there are 16 total blocks, and with Ta'Shawna's assistance she will (incorrectly) agree that there are 20 blocks in the 4-block high tower.
Interpretation: Tyesha continues in her role of explaining her strategy to her two classmates. Ta'Shawna in particular was asking questions about how she got the numbers 16 and then 21 for the total number of cubes in the 4-block high tower. Tyesha corrects her earlier idea stating that the 4-block high tower has 16 blocks. She then works through the problem again, then explains her strategy to Carla and Ta'Shawna, concluding that the two-block high tower has 10 blocks. As the conversation continues below, Ta'Shawna continues to question the strategy, possibly building on what she believed Tyesha said earlier. Tyesha then agrees that the 4-block			

Ta'Shawna Tyesha, Carla			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
high tower has 20 blocks, rather than 16 or 21.			
62	A	I know. (takes Tyesha's paper and starts pointing or writing on it) That's why I don't understand, you came up with this. Say like, if you have to add five, it would be 1 times 5 plus 5, and then if you did 2 times 5, and its 10. That how I understand it.	
63	T	Yeah, and then put 3 times 5 is 15. 4 times 5 is. (Ta'Shawna nods her head as Tyesha speaks)	
64	A	Is 20.	
65	T	Yeah. 5 times 5 is 25, 10 times 5 is 50.	
66	A	Oh! Alright, now I get it.	**LHSIA (4)** Ta'Shawna had been asking questions to understand the answers Tyesha was getting. She stated (line 62) that she had a particular way of understanding. Tyesha agrees, "Yeah." Ta'Shawna states, "I get it" to confirm that she does understand Tyesha's strategy.
67	T	Wait, but 10 times 5 is 50. If 10 times 5 is 50, then.	
68	C	What about?	
69	T	What'd we say, a 10 block, is gonna be a, 10 block.	
70	A	What about ten?	
71	A	No this was the five block. Oh, do.	
72	T	Add five more. Equals ten. We'll see how many blocks… If it's 50, then its right.	
73	A	On both sides?	
74	T	Yeah, put then on both side, so its 5	

Ta'Shawna Tyesha, Carla			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
		already.	
75	C	I think it'll be like, 2, 3, 4, 50.	
76	T	Now count. You add five to this. Now count it.	
77	C	It's 10 right? 10 *counting* Yeah, ten right?	
78	T	(counting) Erase one, cause… oh no, that's right, that's right, that's right. So this one is 20, 30, 40, 50, see?	
79	A	See, that's how I did it. So a 10 block high tower, multiply by 5.	**LHSIA (12)** Ta'Shawna lets her classmates know how she did the problem, following Tyesha's strategy
Interpretation: Ta'Shawna has been questioning Tyesha's explanation of how to count to the total number of blocks in the 4-block high tower. After a brief discussion between the two, Ta'Shawna states the she understands Tyesha's strategy, stating, "Now I get it." They continue their discussion, leading them to consider the 10-block high tower. Ta'Shawna states, "That's how I did it," agreeing with Tyesha's strategy and possibly implying that she feels she is able to keep up mathematically with her classmate.			
80	C	Yeah, multiply. So it said 5 block, right? 1 times 5 equals 5 cube, right? 5 right? 2 times 5. What about a 10 block, 1 times 10 which is 10. 2 times 10, let's see, 20. 3 times 10. This would be 30. Guess what? Look at this. It would go all the way to M (referring to the letter of the stage, if continuing from the given A, B, C). Like this right?	
81	A	No just keep this one, just keep that one, this one (inaudible) There not asking us for 20.	Get the Job Done (only do what they asked us for; they're not asking us for 20)
82	C	It said 10 block, 100 block. It did ask for 10 blocks, and then 100 blocks.	
83	A	I feel like (inaudible)	

Ta'Shawna Tyesha, Carla			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
84	C	Okay right? You can do 100 x 1 give 100 cubes in these, for this one you can do 2x100, 200 cubes in these. This one 3 x 100, 300 cubes in these. And just like how you said three time 5 equal to 15, this would be 15.	
85	T	But wait, this one, but wait, 1 times, for the 1, it's gotta be a pattern. It's gotta be 1,2,3, its always gotta be 1, like it's got, the heights got to be one, so the height is one, and over here the height is two, and over here the height is three, right? The fourths, the fourths, the heights gotta be four. For five, the height's gotta be five.	
86	A	We're just talking about if you added five. But I know, this one.	
87	T	Oh, so, for A, what you gonna do? One times one? 1 times 1 equals 1, for 2, 2 times 5 equals 10. Yeah, two times 5 equals 10, and for three, 3 times 5 gonna be 15?	
88	A	Yeah. And 3 times 1 is supposed to be, it's supposed to be, you know the height, it's supposed to be like this.	
89	C	So, you're trying to say, one, we're doing it wrong?	
90	T	Like, no, no, no, look. Like say this, if this is one, right? (uses her pencil to point to the diagrams on Ta'Shawna's paper, explaining to both Ta'Shawna and Carla) So when the two, you just add one. For three, you add two, four you add 3, for five your add 4. You don't get what I'm saying?	**LMTY (1, 10):** Tyesha is explaining that to get to a 2-block high tower, you add 1 block to the middle block in order to get a height of 2. She continues to get a height of 3, you only need to add 2 blocks, to the middle base block. In addition to this explanation, she checks with her

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<tr><td></td><td></td><td></td><td>classmates if they understand her explanation.</td></tr>
<tr><td>91</td><td>C</td><td>No (shaking her head)
(Ta’Shawna might be shaking her head slightly to indicate no)</td><td></td></tr>
<tr><td>92</td><td>T</td><td>You see, like, if this is just one cube high, for two right? It’s two cubes high, so this, one is just going on top of there. Then you take this two and put it on top of here, you take this three and put it on top of that four.
(points at the figures on Ta’Shawna’s paper as she explains)</td><td>LMTY (2): Tyesha modifies her explanation regarding how many blocks to ‘add’ to the base block to get a 2-block high tower, and so on. This is because when she asked if they understood, they both claimed they did not.</td></tr>
<tr><td>93</td><td>A</td><td>Oh, then you get 5.</td><td></td></tr>
<tr><td>94</td><td>T</td><td>Yeah.</td><td></td></tr>
<tr><td colspan="4">Interpretation: The students started talking about the number of blocks in the height each tower (line 85). (It is not clear to me why, perhaps to discuss how the towers are constructed.) Carla asks if they are doing something wrong. Tyesha says no but then explains that for the 2-block high tower, only 1 block is added to the height of the single block in figure A (see student work below for the figures). She continues her explanation as if the 3-block high tower would be built off that single block still (rather than the 2-block high tower). Her classmates claim to not understand what Tyesha is saying, so she tries to modify her explanation by stating that the 3-block high tower is constructed by placing 2 blocks on top of the other block, and so on.</td></tr>
<tr><td>95</td><td>A</td><td>Oh.</td><td></td></tr>
<tr><td>96</td><td>C</td><td>So, how we doing this one?</td><td></td></tr>
<tr><td>97</td><td>A</td><td>I just don't want you going (inaudible) so I don't get confused.</td><td></td></tr>
<tr><td>98</td><td>C</td><td>So how we do this?</td><td></td></tr>
<tr><td>99</td><td>T</td><td>A’ight. We’ll do… 1 times 5 is 5, 1 times 10 is 10, 1 times 100 is 100, right? So, for this one is that, right?
Yeah, so 2 times 5 is for this one right?
A: Yeah.
If the 3 times 5 is for this one right, so 4 times 5 is for 4, 5 times 5 is for 5. I guess that’s what we're doing. So for A.
//(voices overlapping, inaudible)//</td><td></td></tr>
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Ta'Shawna Tyesha, Carla			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
100	C	Oh, so like 100, two would be like 4 times.	
101	T	You can't do (inaudible) 100. No, like, (inaudible) have 100.	
102	C	But it sounds like, it says.	
103	T	Number of blocks, one right? So for two how many blocks is it in two? It's one block in one, its ten blocks in two, right?	
104	C	Yeah. Oh, no, it's two right?	
105	T	Oh yeah, its. This is two blocks, and this is 3, two blocks, and three, this is three. 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, this should go all the way to thirteen and we should go all the way to thirteen. So how many blocks, in one it's two, in two it's ten, in three it's 15, right?	
106	C	No this is 5.	
107	A	No, remember, 5.	
108	T	Oh! If we add five. 5, 10, 15.	
109	C	I think it's a pattern going. Five, see, it's going by fives, see? Get it?	
110	T	5, 10, 40, 5, 50 *counting* 65. So at M… At M it's gonna be 13 blocks high. (using her hands to indicate high – hands are parallel, facing each other, moving up and down) It's gonna be 13 blocks high for M and 65 cubes in all. *Tyesha leaves table. She returns in a moment with Unifix cubes.*	
111	A	Oh it's the pattern going, going by fives.	
112		*Students work independently on their paper for a short time, and Tyesha starts putting together a tower structure with blocks.*	
113	T	Aight, I'm gonna try it out. (Uses cubes to demonstrate her point, creates tower structure) So, let's see. One, that's the one right? And if we add five to it, well, this is 5. All. (counting, checking each leg has 5 blocks)	

Ta'Shawna Tyesha, Carla			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
114	*Ms. S, the teacher, approaches group and looks on. Carla starts explaining to her what they have been doing so far, and Ms. S asks her to explain why they are multiplying. When asked, Tyesha uses a single cube to demonstrate the 1-block high tower. Tyesha then uses the cubes to build other structures, starting with the 2-block high tower. She uses 10 cubes, putting 2 blocks in the height and 2 additional cubes for each side or leg. Ta'Shawna and Carla agree with Tyesha's depiction of the tower structures.* *In order to demonstrate to this group that they made an error, Ms. S asks the students to build off stage 1 (diagram A, the 1-block high tower) in order to create stage 2 (or the proper 2-block high tower). They re-read the problem and revisit the instructions. They each seem to recognize the difference between the two versions of the structures, and Tyesha puts her head in her hands upon this realization. She may have been embarrassed that they had an incorrect pattern. Ms. S continues to make sure they see the error by having them count the individual cubes, which they could not do before since they did not have the cubes. The students had been skip-counting by 2, 3, or 5 as appropriate for the particular structure.* *Ms. S tells them they had a good strategy and to continue with that, but with the correct pattern, and then leaves the group.*		
115	*When the teacher leaves the group, the students discuss the new answers they have, reviewing that the 2-block high tower has 6 cubes. They use the tower structures they constructed with the towers to count again. They write their new (correct) answers on their papers, independently.*		
116	T	Actually, one block right? So, one is one. (inaudible)	
117	C	No, she said its fine.	
118	T	No, but one block had five cubes.	
119	A	It's 11 right?	
120	T	In one block we had one cube.	
121	C	And then two blocks would be like, okay look, would be like this way, see, two,	

Ta'Shawna Tyesha, Carla			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
		like that right, 1, 2, 3, 4, 5, 6. (using the cubes to look at the 2-block high tower and count the total number of cubes)	
122	T	Yeah two bocks would have 6.	
123	C	Then 3 blocks would be, that. Oh, we did that. (counting) Add in four block on each side like that. When its 5 blocks, add 5 blocks on each side, like that. So then you just do (inaudible). (Students spend a few moments just writing on their own papers.)	
124	C	So for example, if it was 20 blocks right? 20 would be added to each, how many would be.	
125	T	Cubes in all right?	
126	A	Mhm.	
127	C	Number of cubes.	
128	T	Alright.	
130	C	Letter of blocks. Right?	
131	T	How many cubes are total number?	
132	C	Yeah, the last one.	
133	T	Total number of cubes. (appears to be writing this down on her paper)	
134	C	And then the number of blocks, right? No, that's it.	
135	T	It's just gonna be two of them. [possibly two columns in their table]	
136	A	You sure?	
137	T	Yeah.	
138	A	Alright.	
139	T	Just two, so, A ,B, C, D, E, F, G, H, I, J, K, M. So when its, so its, so A, wait, so divide it like, A,B,C, 1,2,3,4,5,6,7,8,9,10,11,12,13. Alright, so total cubes in one block, it should be, 1 cube, right? So one block is one cube, right? (referring to the cubes and built tower	**EPISODE 3** **LHSIA (10, 15):** Tyesha is building on the ideas discussed with the teacher to write down the answers in her table. She checks with her classmates for

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<tr><td></td><td></td><td>structures in front of her)
And two blocks, is, is 2 blocks, right, and 2 blocks is how many cubes?
1, 2, 3, 4, 5, 6, right?
(Carla is also counting the tower structure with Tyesha)
And 2 blocks is 6 cubes, right?
And 3 blocks.</td><td>confirmation of the answers, "6 cubes, right?"</td></tr>
<tr><td colspan="4">Interpretation: Tyesha is now sharing her ideas out loud with her classmates. She is building on the ideas discussed with the teacher and the moments immediately after the teacher left. She is writing her table on her paper, and seems to be inputting information, including the letter of the stage or height of the tower. She may be speaking out loud because she had given her two classmates somewhat erroneous information earlier (e.g., the 5-block high tower has 25 blocks). Therefore, she may want her two classmates to know that, with the correct pattern, that she understands the answers and solution strategies. She also asks, "right?" indicating that she wants her classmates to confirm her answers. Though I did not hear a verbal confirmation, her classmates did not tell her she was incorrect, which may have been taken as implicit agreement. Based on Tyesha's behaviors, she likely wants to look smart to her two classmates, possibly because of the error made earlier.</td></tr>
<tr><td>140</td><td>C</td><td>1 cube, 6, wait that's (inaudible) on it, right? So it's, 1,2,3, three here, put one more on top, put one more on top. Then …three…
So that would be, 1,2.</td><td></td></tr>
<tr><td>141</td><td>T</td><td>Wait! 2, 4, 6, 8, 10. Wait, 2, 4, 6, 8, 9, 10, 11. 11 cubes in the 3 block, and the 4 block you just add one here.
(continues building the tower on the previous one, using the cubes)</td><td></td></tr>
<tr><td>142</td><td>C</td><td>Then add another here, one here, add one here.</td><td></td></tr>
<tr><td>143</td><td>T</td><td>That's a four block right? How many cubes is it?</td><td></td></tr>
<tr><td>144</td><td>C</td><td>4, 8, 12, 16.</td><td></td></tr>
<tr><td>145</td><td>T</td><td>No, see, no, you just counted like, you just counted like its 4, 8, 3, 6, 9, 12, 13, 14, 15, 16.
(uses her pencil to point to different parts of the tower structure as she counts)</td><td>LMTY (4): Tyesha corrects the way that Carla counts the 4-block high tower, presumably so that she didn't make the same mistake they made earlier. Carla</td></tr>
</table>

Ta'Shawna Tyesha, Carla			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
			seemed to accept this as correct.
146	C	Oh, right.	
147	T	16 cubes.	
148	C	Oh, oh right.	
Interpretation: Carla and Tyesha are both sharing their ideas out loud, and are counting the total number of blocks in each tower structure. They have to repeat this, since earlier they added blocks, following a different pattern than the one given in the problem. When Carla counts, "four, eight, twelve, sixteen," Tyesha seems to think that she is reverting to the 'old' way of counting the total number of blocks in the structure and corrects her. It appears that Tyesha is correcting Carla so that she does not make the same mistake they made earlier. Tyesha does not just say, "No" but also demonstrates how she counts the total number of blocks in the 3-block high tower, "three, six, nine, twelve, …" Therefore, Tyesha seems to want to help Carla in this case.			
149	T	I think I see a pattern.	**EPISODE 4** The students all work together to get the new (correct) totals for the number of cubes in the tower structure, for the given heights.
150	C	Me too.	
151	T	Wait look. You see how its 1, 6. Oh look. What you doing? *(seemed to ask what Ta'Shawna was writing for a moment, but then dropped it and went back to her own thing)* 1, 6, 11, 16 and the next will be like… something something with a 1, the next thing would be 6, something with a 6. So that's a 5 block cube. Add one here. One here. (*Tyesha and Carla are building on a previous tower, rather than creating new ones to demonstrate with the blocks the 5-block high tower)*	
152	C	And one here.	
153	T	And one here. How many cubes is it?	
154	T/C/A	4, 8, 12, 16, 17, 18, 19, 20, 21. (All students are counting together. Their counting strategy is to count by the multiples along the legs and then count	

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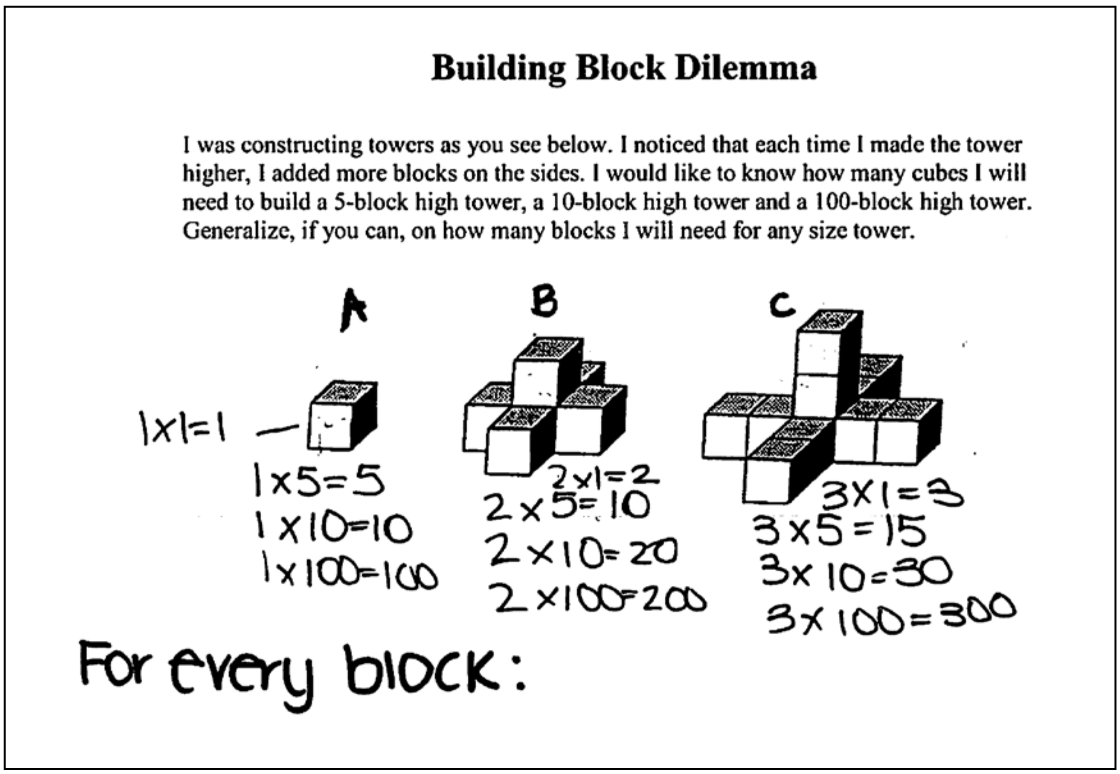
<tr><th colspan="4">Ta’Shawna Tyesha, Carla</th></tr>
<tr><th>Line No.</th><th>Spkr</th><th>Transcript</th><th>Structure; Observation</th></tr>
<tr><td></td><td></td><td>up one at a time the blocks in the height.)</td><td></td></tr>
<tr><td>155</td><td>C</td><td>21 cubes.
Let’s found the intervals and see whether it’s...
From here, it’s six.
Right.
6, 7, 8, 9, 10, that’s five.</td><td></td></tr>
<tr><td>156</td><td>T</td><td>One here. One here. One here. Add ‘em all together.</td><td></td></tr>
<tr><td>157</td><td>C</td><td>Alright
(counting)
26.</td><td></td></tr>
<tr><td>158</td><td>T</td><td>See Ta’Shawna, I told you. I see a pattern. 1, 6, 1, 6, 1, 6, and the next one gotta be something with a one. It’s gotta be like 31. Who's the, what’s her name? (inaudible)</td><td>LHSIA (11, 14): Tyesha says, “I told you” to Ta’Shawna, because she is repeating that she sees the patterns from the ones place alternating 1, 6, 1, 6. She adds what the next answer is, saying, “It’s gotta be…”</td></tr>
<tr><td>159</td><td>C</td><td>Alright, then add one. Now add one here. One here. One here. One here. One here.
(adding more blocks to the existing tower structure; Ta’Shawna helps count)
(counting)
30, 31.</td><td></td></tr>
<tr><td>160</td><td>T</td><td>It’s 31, right?</td><td></td></tr>
<tr><td colspan="4">Interpretation: Tyesha appears to have want to make sure her classmates see her as smart or knowledgeable as she says, “I told you” to Ta’Shawna. Earlier Tyesha had tried to show Ta’Shawna the pattern she sees: the ones digit for the total number of cubes alternates between 1, 6, 1, 6, which makes sense since 5 blocks are added each time. (They have not yet verbalized this part of the pattern yet, though Ta’Shawna mentioned it earlier when they were multiplying the height of the structure by 5.) Tyesha also appears to demand her two classmates’ attention by telling them what the ‘next’ answer must be based on this particular pattern.</td></tr>
<tr><td>161</td><td>C</td><td>No it’s 30, you want to count it?</td><td></td></tr>
<tr><td>162</td><td>T</td><td>Wait, for what, G?
//C: Hmm-mmm.//</td><td></td></tr>
</table>

Ta'Shawna Tyesha, Carla			
Line No.	**Spkr**	**Transcript**	**Structure; Observation**
		(counting) 30	
163	A	So what happened to the 1?	
164	*The three students continue to count tower structures and fill in answers to their individual tables. Though they are building on a 'correct' tower, they somehow see only 30 blocks for the 7-block high tower that should have 31 blocks. They continue to add 5 blocks to the previous structure, again obtaining an answer of 50 blocks for the 10-block high tower. They appear to trust their counting system rather than the pattern, which they seem to abandon. They continue to discuss the possibility of the patterns when the teacher asks them to start packing up for the day.*		

Ta'Shawna's work, in Nov. 2008



Building Block Dilemma

I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5-block high tower, a 10-block high tower and a 100-block high tower. Generalize, if you can, on how many blocks I will need for any size tower.

A

$1 \times 1 = 1$

$1 \times 5 = 5$

$1 \times 10 = 10$

$1 \times 100 = 100$

B

$2 \times 1 = 2$

$2 \times 5 = 10$

$2 \times 10 = 20$

$2 \times 100 = 200$

C

$3 \times 1 = 3$

$3 \times 5 = 15$

$3 \times 10 = 30$

$3 \times 100 = 300$

For every block:

Ta'Shawna's work, in Nov. 2008

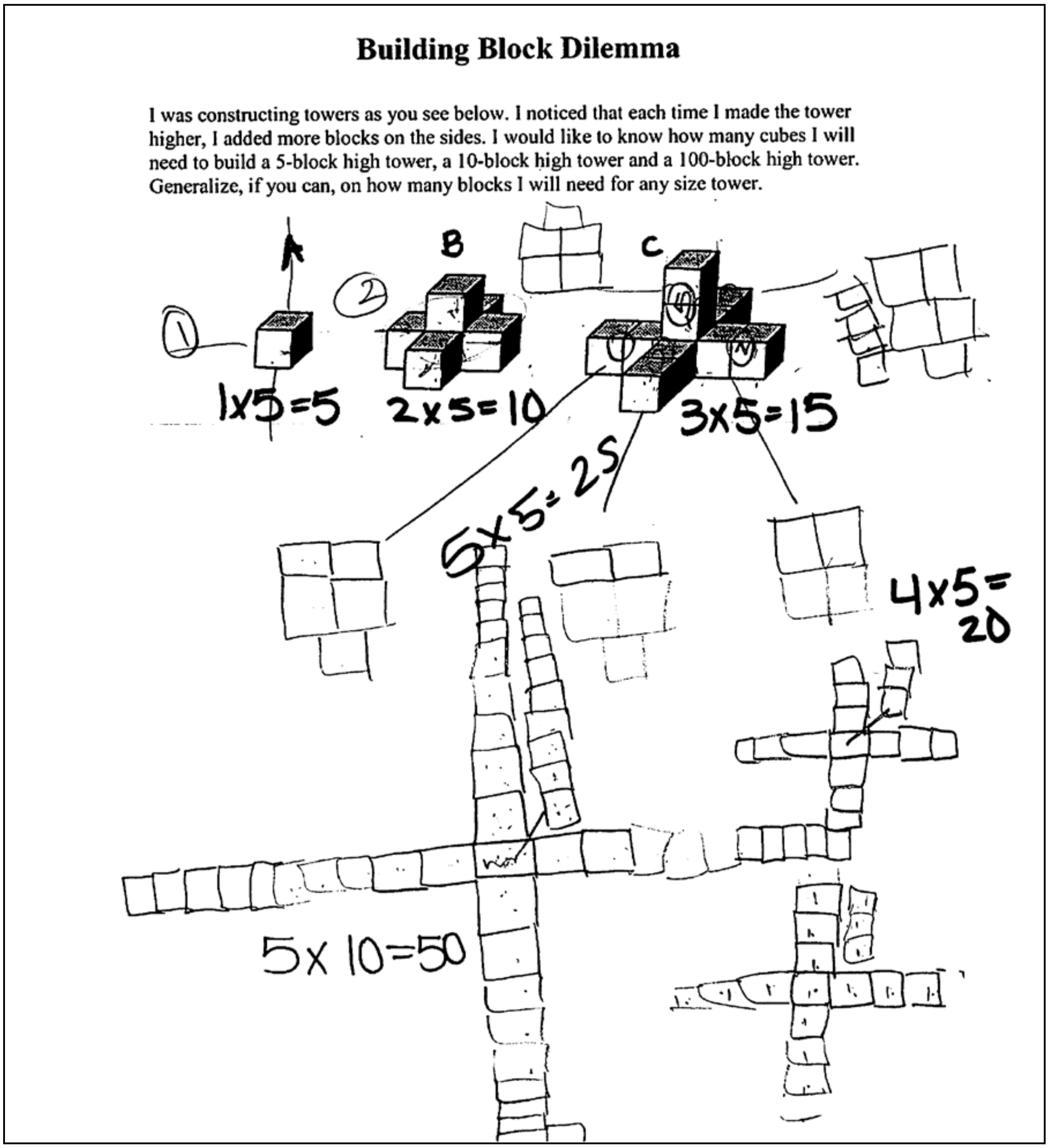
"Building Block Dilemma"

Letter of Blocks	Number of Blocks	How many cubes
A	1	5
B	2	10
C	3	15
D	4	20
E	5	25
F	6	30
G	7	35
H	8	40
I	9	45
J	10	50
K	11	55
L	12	60
M	13	65

Building Block Dilemma

Letter of Blocks	Total Number # cubes	
A (1)	1	1
B (2)	6	6
C (3)	11 cubes	11
D (4)	16 cubes	16
E (5)	21 cubes	21
F (6)	30 cubes	26
G (7)	35 cubes	30
H (8)	40 cubes	35
I (9)	45 cubes	40
J (10)	50 cubes	45
K (11)	55 cubes	50
L (12)	60 cubes	55
M (13)	65	60
		65

Tyesha's work, in Nov. 2008



Tyesha's work, in Nov. 2008

Letter of block	number of block	How many cubes?
A	1	5
B	2	10
C	3	15
D	4	20
E	5	25
F	6	30
G	7	35
H	8	40
I	9	45
J	10	50
K	11	55
L	12	60
M	13	65

Letter of block(s)	Total # of cubes
A (1)	1 cube
B (2)	6 cubes
C (3)	11 cubes
D (4)	16 cubes
E (5)	21 cubes
F (6)	26 cubes
G (7)	30 cubes
H (8)	35 cubes
I (9)	40 cubes
J (10)	45 cubes
K (11)	50 cubes
L (12)	55 cubes
M (13)	65 cubes

Carla's work, in Nov. 2008

Table

"Building Block Dilemma"

Letter Of Block	Number of Block	How many cubes
A	1	5
B	2	10
C	3	15
d	4	20
e	5	25
f	6	30
g	7	35
h	8	40
I	9	45
J	10	50
K	11	65
L	12	60
m	13	65

Questionnaire Responses for Ta'Shawna, Tyesha, Carla

Questionnaire items which may indicate LMTY structure

Questionnaire Items	Ta'Shawna	Tyesha	Carla
Statements (All the time, Sometimes, Never)			
I wanted to teach another student something that I knew that the other student did not know.	Never	All the time	Sometimes
I listened carefully to the ideas of someone I was trying to help.	All the time	All the time	Never
I helped someone see how to do the math.	Sometimes	All the time	Never
Others listened carefully to my ideas.	Sometimes	All the time	All the time
Behaviors (Often, Sometimes, Hardly ever)			
I gave helpful suggestions.	Sometimes	Often	Often
I worked cooperatively.	Often	Often	Often
Thoughts (Yes/No)			
I like teaching this person things that I know.	Yes	Yes	Yes

Questionnaire Responses for Ta'Shawna, Tyesha, Carla

Questionnaire Items Which May Indicate LHSIA structure

Questionnaire Items – Statements	Ta'Shawna	Tyesha	Carla
Statements (All the time, Sometimes, Never)			
I wanted people to think that I'm smart.	Sometimes	All the time	All the time
I tried to impress people with my ideas about the problem.	Sometimes	All the time	Sometimes
People seemed impressed with the ideas I shared about the problem.	Sometimes	Sometimes	Sometimes
People saw how good I am at the math we did today.	Sometimes	Sometimes	Sometimes
I felt smart.	All the time	All the time	Never
I wanted to show someone that my way was better.	Never	Sometimes	Never
I was a lot better at math than others today.	Sometimes	All the time	All the time
Behaviors (Often, Sometimes, Hardly ever)			
I was the leader.	Hardly ever	Often	Sometimes
I was bossy.	Hardly ever	Hardly ever	Hardly ever
I wanted to show off.	Hardly ever	Hardly ever	Hardly ever
I liked to be right.	Sometimes	Sometimes	Sometimes
Thoughts (Yes/No)			
I want you to know just how smart I am.	Yes	Yes	Yes
People think I'm smart.	No	Yes	Yes
I wish the teacher would call on me, so I can show how much I know.	Yes	Yes	Yes

References

Alibali, M. W., Knuth, E. J., Hattikudur, S., McNeil, N. M., & Stephens, A. C. (2007). A longitudinal examination of middle school students' understanding of the equal sign and equivalent equations. *Mathematical Thinking and Learning, 9*(3), 221-247.

Alston, A., Goldin, G. A., Jones, J., McCulloch, A., Rossman, C., & Schmeelk, S. (2007). The complexity of affect in an urban mathematics classroom. In T. Lamberg, & L. R. Wiest (Eds.) *Exploring Mathematics Education in Context: Proceedings of the 29th Annual Conference of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 326–333). Stateline (Lake Tahoe): University of Nevada Press.

Ames, C. (1992). Classrooms: Goals, structures, and student motivation. *Journal of Educational Psychology, 84*(3), 261-271.

Anderman, E. M., Austin, C. C., & Johnson, D. M. (2002). The development of goal orientation. In A. Wigfield & J. S. Eccles (Eds.), *Developmental perspectives on achievement motivation* (pp. 197-220). San Diego, CA: Academic Press.

Anderman, E. M., & Wolters, C. A. (2006). Goals, values, and affect: Influences on student motivation. In P. Alexander & P. Winne (Eds.), *Handbook of educational psychology* (pp. 369-389). Mahwah, NJ: Erlbaum.

Arslan, Ç., & Altun, M. (2007). Learning to solve non-routine mathematical problems. *Elementary Education Online, 6*(1), 50-61.

Ashcraft, M. H. & Moore, A. M. (2009). Mathematics anxiety and the affective drop in performance. *Journal of Psychoeducational Assessment, 27*, 197-205.

Aud, S., Hussar, W., Johnson, F., Kena, G., Roth, E., Manning, E., Wang, X., and Zhang, J. (2012). *The condition of education 2012* (NCES 2012-045). U.S. Department of Education, National Center for Education Statistics. Washington, DC. Retrieved July 11, 2012 from http://nces.ed.gov/pubsearch.

Bandura, A. (1994). Self-Efficacy. In V. S. Ramachaudran (Ed.), Encyclopedia of human behavior (Vol. 4, pp. 71-81). New York: Academic Press.

Barron, B. (2003). When smart groups fail. *The Journal of the Learning Sciences, 12*(3), 307-359.

Betz, N. E. (1978). Prevalence, distribution, and correlates of math anxiety in college students. *Journal of Counseling Psychology*, 25(5), 441-448.

Brahier, D. B. (Ed.) (2011) *Motivation and disposition: Pathways to learning mathematics*. 2011 Yearbook of the National Council of Teachers of Mathematics. Reston, VA: National Council of Teachers of Mathematics.

Bransford, J. D., Brown, A. L., & Cocking, R. R. (1999). *How people learn: Brain, mind, experience, and school.* Washington, DC: National Academy Press.

Brophy, J. (1983). Conceptualizing Student Motivation. *Educational Psychologist, 18*(3), 200-215.

Brown, T. T. (2009). An Exploratory Study of Mathematics Engagement of Secondary Students. *Middle-Secondary Education and Instructional Technology Dissertations*, 47.

Cai, J. (2004). Why do US and Chinese students think differently in mathematical problem solving?: Impact of early algebra learning and teachers' beliefs. *The Journal of Mathematical Behavior, 23*(2), 135-167.

Capraro, M. M., & Joffrion, H. (2006). Algebraic Equations: Can Middle-School Students Meaningfully Translate from Words to Mathematical Symbols? *Reading Psychology, 27*(2-3), 147-164.

Charles, R.I., Dossey, J.A., Leinwand, S.J., Seeley, C.L., Vonder Embse, C.B. (1999). *Middle School Math*. Menlo Park, CA: Scott Foresman-Addison Wesley.

Christenson, S. L., Reschly, A. L., & Wylie, C. (Eds.). (2012). *Handbook of research on student engagement*. New York: Springer.

Cohen, E. G. (1994). Restructuring the classroom: Conditions for productive small groups. *Review of Educational Research, 64*(1), 1-35.

Cohen, E. G. (1996). *A sociologist looks at talking and working together in the mathematics classroom.* Paper presented at the annual meeting of the American Educational Research Association, New York.

Combs, J. P., Clark, D., Moore, G. W., Onwuegbuzie, A. J., Edmonson, S. L., & Slate, J. R. (2011). Academic achievement for fifth-grade students in elementary and intermediate school settings: Grade span configurations. *Current Issues in Education*, 14(1). Retrieved from http://cie.asu.edu/.

Connell, J. P., Spencer, M. B., & Aber, J. L. (1994). Educational risk and resilience in African-American youth: Context, self, action, and outcomes in school. *Child Development, 65*, 493-506.

Creswell, J. W. (2007). *Qualitative inquiry and research design: Choosing among five approaches* (2nd ed.). Thousand Oaks, CA: Sage.

Csikszentmihalyi, M. (1990). *Flow: The psychology of optimal experience*. New York: Harper & Row.

Davidson, N. (1985). Small-group learning and teaching in mathematics: A selective review of the research. In R. Slavin, S. Sharan, S. Kagan, R. H. Lazarotwitz, C. Webb & R. Schmuck (Eds.), *Learning to Cooperate, Cooperating to Learn* (pp. 211-230). New York: Plenum Press.

Davis, R. B. (1992). Reflections on where mathematics education now stands and on where it may be going. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 724-734). New York: Macmillan.

DeBellis, V. A., & Goldin, G. A. (2006). Affect and meta-affect in mathematical problem solving: A representational perspective. *Educational Studies in Mathematics, 63*(2), 131-147.

Deci, E. L., Vallerand, R. J., Pelletier, L. G., & Ryan, R. M. (1991). Motivation and education: The self-determination perspective. *Educational Psychologist, 26*(3&4), 325-346.

Dweck, C. S., & Leggett, E. L. (1988). A social-cognitive approach to motivation and personality. *Psychological Review, 95*, 256-273.

Eccles, J. S. (1983). Expectancies values and academic behaviors. In J. T. Spence (Ed.), *Achievement and achievement motives: Psychological and sociological approaches* (pp. 75 - 146). San Francisco: Freeman.

Eccles, J. S., & Wigfield, A. (2002). Motivational beliefs, values, and goals. *Annual review of psychology, 53*(1), 109-132.

Eccles, J. S., Wigfield, A., Midgley, C., Reuman, D., Mac Iver, D., & Feldlaufer, H. (1993). Negative effects of traditional middle schools on students' motivation. The Elementary School Journal, 93(5), 553-574. Finn, J. D. (1993). *School engagement and students at risk*. Washington, DC: National Center for Education Statistics.

Epstein, Y. M., Goldin, G. A., Schorr, R. Y., Capraro, R., Capraro, M. M., & Warner, L. B. (2010). *Measuring engagement structures in middle-grades urban mathematics classrooms*. Paper presented at the 2010 annual meeting of AERA, Denver, CO.

Epstein, Y., Schorr, R. Y., Goldin, G. A., Warner, L. B., Arias, C., Sanchez, L., et al. (2007). Studying the affective/social dimension of an inner-city mathematics class. In T. Lamberg, & L. R. Wiest (Eds.) *Exploring mathematics education in context: Proceedings of the 29th Annual Conference of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 649–656). Stateline (Lake Tahoe): University of Nevada Press.

Fennema, E., & Sherman, J. (1978). Sex-related differences in mathematics achievement and related factors: A further study. *Journal for Research in Mathematics Education, 9*(3), 189-203.

Fredricks, J. A., Blumenfeld, P. C., & Paris, A. H. (2004). School engagement: Potential of the concept, state of the evidence. *Review of Educational Research, 74*(1), 59-109.

Furrer, C., & Skinner, E. (2003). Sense of relatedness as a factor in children's academic engagement and performance. *Journal of Educational Psychology, 95*(1), 148-162.

Goffman, E. (1959). *The presentation of self in everyday life*. Garden City, NY: Doubleday.

Goldin, G. A. (2000). Affective pathways and representation in mathematical problem solving. *Mathematical Thinking and Learning, 2*(3), 209-219.

Goldin, G. A., Epstein, Y. M., & Schorr, R. Y. (2007). Affective pathways and structures in urban students' mathematics learning. In D. K. Pugalee, A. Rogerson, & A. Schinck (Eds.), *Mathematics education in a global community: Proceedings of the 9th international conference of the mathematics education into the 21st century project* (pp. 260–265). Charlotte, NC: University of North Carolina.

Goldin, G. A., Epstein, Y. M., Schorr, R. Y., and Warner, L. B. (2011). Beliefs and engagement structures: Behind the affective dimension of mathematical learning. *ZDM – The International Journal on Mathematics Education 43* (4), 547-560.

Gómez-Chacón, I. M. (2006). Affective influences in the knowledge of mathematics. *Educational Studies in Mathematics*, *43*, 149-168.

Goodnow, C. (1993). Classroom belonging among early adolescent students relationships to motivation and achievement. *The Journal of Early Adolescence, 13*(1), 21-43.

Hannula, M. S. (2002). Attitude towards mathematics: Emotions, expectations and values. *Educational Studies in Mathematics, 49*(1), 25-46.

Harackiewicz, J. M., Barron, K. E., & Elliot, A. J. (1998). Rethinking achievement goals: When are they adaptive for college students and why? *Educational Psychologist*, *33*(1), 1-21.

Helme, S., & Clarke, D. (2001). Identifying Cognitive Engagement in the Mathematics Classroom. *Mathematics Education Research Journal, 13*(2), 133-153.

Holliday, B., Cuevas, G. J., Moore-Harris, B., Carter, J. A., Marks, D., Casey, R. M. et al. (2003). *Algebra 1*. New York: Glencoe McGraw-Hill.

Jimerson, S. R., Campos, E., & Greif, J. L. (2003). Toward an understanding of definitions and measures of school engagement and related terms. *The California School Psychologist, 8*, 7-27.

Johnson, D. W., & Johnson, R. T. (1999). Making cooperative learning work. *Theory into practice*, *38*(2), 67-73.

Kaput, J. J. (2000) *Transforming algebra from an engine of inequity to an engine of mathematical power by 'algebrafying' the K-12 curriculum*. Dartmouth, MA: National Center for Improving Student Learning and Achievement in Mathematics and Science.

Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics, 12*(3), 317-326.

Kieran, C. (2001). The mathematical discourse of 13-year-old partnered problem solving and its relation to the mathematics that emerges. *Educational Studies in Mathematics, 46*, 187-228.

Kieran, C. (2007). Learning and teaching algebra at the middle school through college levels: Building meaning for symbols and their manipulation. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 707–762). Reston, VA: National Council of Teachers of Mathematics.

Kilpatrick, J., & Swafford, J. (Eds.). (2002). *Helping Children Learn Mathematics*. Washington, DC: National Academy Press.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Klem, A. M., & Connell, J. P. (2004). Relationships matter: Linking teacher support to student engagement and achievement. *Journal of School Health, 74*(7), 262-273.

Knuth, E. J., Alibali, M. W., McNeil, N. M., Weinberg, A., & Stephens, A. C. (2005). Middle school students' understanding of core algebraic concepts: Equivalence & Variable1. *Zentralblatt für Didaktik der Mathematik, 37*(1), 68-76.

Knuth, E. J., Stephens, A. C., McNeil, N. M., & Alibali, M. W. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal for Research in Mathematics Education*, 297-312.

Kramarski, B., & Mevarech, Z. R. (2003). Enhancing mathematical reasoning in the classroom: The effects of cooperative learning and metacognitive training. *American Educational Research Journal, 40*(1), 281-310.

Lappan, G., Fey, J.T., Fitzgerald, W.M., Friel, S.N., Phillips, E.D. (2002). *Connected Mathematics.* Menlo Park, CA: Dale Seymour Publications.

Leikin, R., & Zaslavsky, O. (1997). Facilitating student interactions in mathematics in a cooperative learning setting. *Journal for Research in Mathematics Education, 28*(3), 331-354.

Linnenbrink, E. A. (2005). The dilemma of performance-approach goals: The use of multiple goal contexts to promote students' motivation and learning. *Journal of Educational Psychology, 97*(2), 197-213.

Linnenbrink, E. A. (2007). The Role of Affect in Student Learning: A Multi-Dimensional Approach to Considering the Interaction of Affect, Motivation, and Engagement. In P. A. Schutz & R. Pekrun (Eds.), *Emotion in Education* (pp. 107-124). Burlington, MA: Academic Press.

Linnenbrink, E. A., & Pintrich, P. R. (2002). Achievement Goal Theory and Affect: An Asymmetrical Bidirectional Model. *Educational Psychologist, 37* (2), 69-78.

Linnenbrink-Garcia, L., Rogat, T. K., & Koskey, K. L. (2011). Affect and engagement during small group instruction. *Contemporary Educational Psychology, 36*(1), 13-24.

Malmivuori, M. L. (2006). Affect and self-regulation. *Educational Studies in Mathematics, 63*(2), 149-164.

Mandler, G. (1989). Affect and learning: Causes and consequences of emotional interactions. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving* (pp. 3-19). New York Springer.

Marks, H. M. (2000). Student engagement in instructional activity: Patterns in the elementary, middle, and high school years. *American Educational Research Journal, 37*(1), 153-184.

Marsh, H. W. (1990). The structure of academic self-concept: The Marsh-Shavelson Model. *Journal of Educational Psychology*, *82*(4), 623–636.

McIntyre, D. J., Copenhaver, R. W., Byrd, D. M., & Norris, W. R. (1983). A study of engaged student bheavior within classroom activities during mathematics class. *Journal of Educational Research, 771*(1), 55-59.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575-596). New York: Macmillan.

Meece, J. L., Wigfield, A., & Eccles, J. S. (1990). Predictors of math anxiety and its influence on young adolescents' course enrollment intentions and performance in mathematics. *Journal of Educational Psychology, 82*(1), 60-70.

Middleton, J. A., & Jansen, A. (2011). *Motivation matters and interest counts: Fostering engagement in mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Middleton, J. A., & Spanias, P. A. (1999). Motivation for achievement in mathematics: Findings, generalizations, and criticisms of the research. *Journal for Research in Mathematics Education, 30*(1), 65-88.

Middleton, M. J., & Midgley, C. (1997). Avoiding the demonstration of lack of ability: An underexplored aspect of goal theory. Journal of Educational Psychology, 89(4), 710-718.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative Data Analysis* (Second ed.) Thousand Oaks, CA: Sage Publications.

Miller, R. B., Greene, B. A., Montalvo, G. P., Ravindran, B., & Nichols, J. D. (1996). Engagement in academic work: The role of learning goals, future consequences, pleasing others, and perceived ability. Contemporary Educational Psychology, 21, 388-422.

Moses, R., & Cobb, C. E. (2001). *Radical equations: Civil rights from Mississippi to the Algebra Project*. Boston: Beacon Press.

Moss, J., & Beatty, R. (2006). Knowledge building in mathematics: Supporting collaborative learning in pattern problems. *International Journal of Computer-Supported Collaborative Learning, 1*(4), 441-465.

Moyer, P. S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics, 47*(2), 175-197.

Murray, H. A. (2008). *70th Anniversary edition, explorations in personality*. NY: Oxford University Press.

National Center for Research in Mathematical Sciences Education. (2003). *Mathematics in Context*. Chicago: Encyclopedia Britannica.

National Council of Teachers of Mathematics. [NCTM] (1991). *Professional standards for teaching mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards of school mathematics*. Reston, VA: Author.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.

National Science Board. (2007). *National Action Plan for Addressing the Critical Needs of the U.S. Science, Technology, Engineering, and Mathematics Education System*. Retrieved from http://www.nsf.gov/nsb/stem/index.jsp.

National Science Board (2010) *Preparing the next generation of STEM innovators: Identifying and developing our nation's human capital*. Retrieved April 14, 2011, from http://www.nsf.gov/nsb/publications/2010/nsb1033.pdf

Newmann, F. M. (Ed.). (1992). *Student engagement and achievement in American secondary schools*. New York: Teachers College Press.

Newmann, F. M., Wehlage, G. G., & Lamborn, S. D. (1992). The significance and sources of student engagement. In F. M. Newmann (Ed.), *Student engagement and achievement in American secondary schools* (pp. 11-39). New York: Teachers College Press.

Nichols, J. D., & Miller, R. B. (1994). Cooperative learning and student motivation. *Contemporary Educational Psychology, 19*, 167-178.

Noddings, N. (1985). Small groups as a setting for research on mathematical problem solving. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving* (pp. 345-359). Mahway, NJ: Lawrence Erlbaum Associates.

O'Donnell, A. M. (2006). The role of peers and group learning. In P. A. Alexander & P. H. Winne (Eds.), *Handbook of educational psychology* (2nd ed., pp. 781-802). Mahwah, NJ: Erlbaum.

Op't Eynde, P., De Corte, E., & Verschaffel, L. (2006). Accepting emotional complexity: A socio-constructivist perspective on the role of emotions in the mathematics classroom. *Educational Studies in Mathematics, 63*(2), 193-207.

Pajares, F. (1996). Self-efficacy beliefs and mathematical problem-solving of gifted students. *Contemporary Educational Psychology, 21*, 325-344.

Pajares, F., & Graham, L. (1999). Self-Efficacy, Motivation Constructs, and Mathematics Performance of Entering Middle School Students. *Contemporary Educational Psychology, 24*, 124-130.

Park, S. Y. (2005). Student engagement and classroom variables in improving mathematics achievement. *Asia Pacific Education Review, 6*(1), 87-97.

Pekrun, R. (2009). Emotions at school. In K. R. Wentzel & A. Wigfield (Eds.), *Handbook of motivation at school* (pp. 575-605). New York: Routledge Taylor.

Pekrun, R., Goetz, T., Titz, W., & Perry, R. P. (2002). Academic Emotions in Students' Self-Regulated Learning and Achievement: A Program of Qualitative and Quantitative Research. *Educational Psychologist, 37*(2), 91-105.

Peterson, P. L., & Janicki, T. C. (1979). Individual characteristics and children's learning in large-group and small-group approaches. *Journal of Educational Psychology, 71*(5), 677-687.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. J. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning.* (pp. 257-315). Reston, VA: National Council of Teachers of Mathematics.

Pintrich, P. R. (2000). Multiple goals, multiple pathways: The role of goal orientation in learning and achievement. *Journal of Educational Psychology*, 92(3), 544-555.

Pintrich, P. R., & DeGroot, E. V. (1990). Motivational and self-regulated learning components of classroom academic performance. Journal of Educational Psychology, 82(1), 33-40.

Pintrich, P. R., & Schrauben, B. (1992). Students' motivational beliefs and their cognitive engagmeent in classroom academic tasks. In D. H. Schunk & J. L. Meece (Eds.), *Student perceptions in the classroom* (pp. 149-183). Hillsdale, NJ: Lawrence Erlbaum.

Planty, M., Provasnik, S., & Daniel, B. (2007). *High school coursetaking: Findings from the condition of education 2007* (NCES 2007-065). U.S. Department of Education. Washington, DC: National Center for Education Statistics.

Rosenberg, E. L. (1998). Levels of analysis and the organization of affect. *Review of General Psychology, 2*(3), 247-270.

Rosenshine, B. V. (1979). Content, time and direct instruction. In P. L. Peterson & H. J. Walberg (Eds.), *Research on teaching: Concepts, findings, and implications*. Berkeley, CA: McCutchan.

Roseth, C. J., Johnson, D. W., & Johnson, R. T. (2008). Promoting early adolescents' achievement and peer relationships: The effects of cooperative, competitive, and individualistic goal structures. *Psychological bulletin, 134*(2), 223.

Rossman, C. F., Schorr, R. Y., Warner, L. B. (2010) *Working with a Partner: An Investigation of Student Engagement in a Middle School Math Classroom*. Paper presented at the 2010 annual meeting of AERA, Denver, CO.

Ryan, R. M., & Deci, E. L. (2000). Intrinsic and extrinsic motivations: Classic definitions and new directions. *Contemporary Educational Psychology, 25*, 54-67.

Ryan, R. M., Stiller, J. D., & Lynch, J. H. (1994). Representations of relationships to teachers, parents, and friends as predictors of academic motivation and self-esteem. *Journal of Early Adolescence, 14*(2), 226-249.

Saldaña, J. (2009). *The Coding Manual for Qualitative Researchers*. Thousand Oakes, CA: Sage Publications.

Sanchez-Leal, S. (2012) *Characterizing middle school students' engagement in a technology-rich mathematics class*. (Unpublished doctoral dissertation). Rutgers University, New Brunswick, NJ.

Schiefele, U., & Csikszentmihalyi, M. (1995). Motivation and ability as factors in mathematics experience and achievement. *Journal for Research in Mathematics Education, 26*(2), 163-181.

Schoenfeld, A. H., & Arcavi, A. (1988). On the meaning of variable. *The mathematics teacher, 81*(6), 420-427.

Schorr, R. Y., Epstein, Y. M., Warner, L. B., & Arias, C. C. (2010a). Don't disrespect me: Affect in an urban math class. In R. Lesh, P. L. Galbraith, C. R. Haines, & A. Hurford (Eds.), *Modeling students' mathematical modeling competencies: ICTMA 13* (pp. 313–325). NY: Springer.

Schorr, R. Y., Epstein, Y. M., Warner, L. B., & Arias, C. C. (2010b). Mathematical truth and social consequences: The intersection of affect and cognition in a middle school classroom. *Mediterranean Journal for Research in Mathematics Education, 9,* 107–134.

Schunk, D. H. (1991). Self-efficacy and academic motivation. *Educational Psychologist, 26*, 207-231.

Schunk, D. H., Pintrich, P. R., & Meece, J. L. (2008). Motivation in education: Theory, research, and applications (3rd ed.). Upper Saddle River, NJ: Pearson.

Schutz, P. A., & Pekrun, R. (2007). *Emotion in Education*. Burlington, MA: Academic Press.

Shernoff, D. J., Csikszentmihalyi, M., Schneider, B., & Shernoff, E. S. (2003). Student engagement in high school classrooms from the perspective of flow theory. *School Psychology Quarterly, 18*(2), 158-176.

Silver, E. A., Leung, S. S., & Cai, J. (1995). Generating multiple solutions for a problem: A comparison of the responses of US and Japanese students. *Educational Studies in Mathematics, 28*(1), 35-54.

Skinner, E. A., & Belmont, M. J. (1993). Motivation in the classroom: Reciprocal effects of teacher behavior and student engagement across the school year. *Journal of Educational Psychology, 85*(4), 571-581.

Slavin, R. E. (1985). Team-assisted individualization: Combining cooperative learning and individualized instruction in mathematics. In R. Slavin, S. Sharan, S. Kagan, R. H. Lazarotwitz, C. Webb & R. Schmuck (Eds.), *Learning to Cooperate, Cooperating to Learn* (pp. 177-209). New York: Plenum Press.

Slavin, R. E., & Cooper, R. (1999). Improving intergroup relations: Lessons learned from cooperative learning programs. *Journal of Social Issues, 55*(4), 647-663.

Slavin, R. E., Leavey, M. B., & Madden, N. A. (1984). Combining cooperative learning and individualized instruction: Effects on student mathematics achievement, attitudes, and behaviors. *The Elementary School Journal, 84*(4), 408-422.

Smith, J. B. (1996). Does an extra year make any difference? The impact of early access to algebra on long-term gains in mathematics attainment. *Educational Evaluation and Policy Analysis, 18*(2), 141-153.

Steinberg, L., Brown, B. B., & Dornbusch, S. M. (1996). *Beyond the classroom: Why school reform has failed and what parents need to do*. New York: Simon and Schuster.

Stipek, D. (2002). Good instruction is motivating. In A. Wigfield & J. S. Eccles (Eds.), *The Development of Achievement Motivation* (pp. 309-332). San Diego, CA: Academic Press.

Stipek, D., Salmon, J. M., Givvin, K. B., Kazemi, E., Saxe, G., & MacGyvers, V. L. (1998). The Value (And Convergence) of Practices Suggested by Motivation Research and Promoted by Mathematics Education Reformers. *Journal for Research in Mathematics Education, 29*(4), 465-488.

Thayer, R. E. (1986). Activation-deactivation adjective check list: Current overview and structural analysis. *Psychological Reports, 58*(2), 607-614.

Turner, J. C., & Meyer, D. K. (2009). Understanding motivation in mathematics: What is happening in classrooms? In K. R. Wentzel & A. Wigfield (Eds.), *Handbook of Motivation at School* (pp. 527-552). New York: Routledge.

Turner, J. C., Meyer, D. K., Cox, K. E., Logan, C., DiCintio, M., & Thomas, C. T. (1998). Creating contexts for involvement in mathematics. *Journal of Educational Psychology, 90*(4), 730-745.

Watson, D., & Tellegen, A. (1985). Toward a consensual structure of mood. *Psychological bulletin, 98*(2), 219.

Webb, N. M. (1982). Peer interaction and learning in cooperative small groups. *Journal of Educational Psychology, 74*(5), 642-655.

Webb, N. M. (1985). Verbal interaction and learning in peer-directed groups. *Theory Into Practice, 24*, 32-39.

Webb, N. M. (1991). Task-related verbal interaction and mathematics learning in small groups. *Journal for Research in Mathematics Education, 22*(5), 366-389.

Webb, N. M., & Farivar, S. (1994). Promoting helping behavior in cooperative small groups in middle school mathematics. *American Educational Research Journal, 31*(2), 369-395.

Webb, N. M., & Mastergeorge, A. (2003). Promoting effective helping behavior in peer-directed groups. *International Journal of Educational Research, 39*, 73-97.

Weber, K., Maher, C., Powell, A., & Lee, H. S. (2008). Learning opportunities from group discussions: warrants become the objects of debate. *Educational Studies in Mathematics, 68*(3), 247-261.

Weiner, B. (1985). An attributional theory of achievement motivation and emotion. *Psychological Review*, 92(4), 548-573.

Wentzel, K. R. (1997). Student motivation in middle school: The role of perceived pedagogical caring. *Journal of Educational Psychology, 89*, 411-419.

Wentzel, K. R., & Wigfield, A. (Eds.). (2009). *Handbook on motivation at school*. New York: Routledge.

Wigfield, A., & Eccles, J. S. (2000). Expectancy–value theory of achievement motivation. *Contemporary Educational Psychology*, 25(1), 68-81.

Wigfield, A., & Eccles, J. S. (Eds.). (2002). Development of achievement motivation. San Diego, CA: Academic Press.

Zimmerman, B. J., Bandura, A., & Martinez-Pons, M. (1992). Self-motivation for academic attainment: The role of self-efficacy beliefs and personal goal setting. *American Educational Research Journal*, *29*(3), 663-676.