

Description: Clip 3 of 3: Generating multiple models by doubling Parent Tape: Building large models to show equivalence: A generalization Date: 1993-10-11 Location: Colts Neck Elementary School Researcher: Professor Carolyn Maher	Transcriber(s): Yankelewitz, Dina Verifier(s): Yedman, Madeline Date Transcribed: Spring 2009 Page: 1 of 8
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- 11.0.222 T/R 1: I remember last Thursday when I walked around the room then I said could you make another model and a lot of you said "oh you know I don't have enough. I don't have enough of these blocks so I said can you imagine it and I remember talking and I know Andrew actually made the model when David had a theory that he shared with um Erik and Alan and Meredith, right, David? And so he shared a theory and I remember Erik said hey wait a minute that's what Andrew built! And then Jessica said that they already built what the theory was, that's what I heard, so I'd like to hear um, David's theory again, if you don't mind, David, if you think you can remember your theory and Andrew I want you to listen very carefully and Jessica and the rest of you I want you to listen carefully to David's theory because it really has to do with if I were to make another model, is it possible do you think to make another model if we had more blocks, it is a possible thing to do? [Student says yes]. How many of you think we can [Most/all students visible raise hands]. Ok. How many of you think we can make another model? Some of you aren't sure, how many of you aren't sure? Meredith's not sure? Erik's not sure? Danielle's not sure? Audra's not sure. Ok. How many of you are sure we can make another model? [All other students raise their hands.] Ok, that looks like that's James and Alan and Andrew and Jessica and Beth and Sarah, Kelly, Graham, Brian, Michael, Caitlin, did I leave anybody out? David is sure. Ok. Let's listen to David's theory and see if we could convince those or else they have to show us our theory doesn't work.
- 11.0.223 David: Well, first, um, Meredith made um, a model which had one orange, one blue, and one black.
- 11.0.224 T/R 1: Ok, she made a model with an orange a blue and a black. That's what you told me?
- 11.0.225 David: Yeah. And then she had, um, the whites, I think they were something like
- 11.0.226 Erik: Twenty-fourths.
- 11.0.227 David: Yeah, one twenty-fourth and the reds were one twelfth and, um,
- 11.0.228 Erik: Just like the one up there.
- 11.0.229 David: Yeah.
- 11.0.230 T/R 1: So you're saying that if I had an orange, a blue and a black, that the model should look like the one up here.
- 11.0.231 Erik: Just about.
- 11.0.232 T/R 1: But it doesn't.

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- 11.0.233 Erik: well...
- 11.0.234 T/R 1: Right? See what happens?
- 11.0.235 Erik: But then, then the one, then the, the uh, um, I don't know
- 11.0.236 Alan: Then the reds couldn't be twelfths.
- 11.0.237 Erik: Yeah, then the reds couldn't be twelfths and the whites couldn't be twenty-fourths.
- 11.0.238 Alan: Right, it would either take one [inaudible]
- 11.0.239 T/R 1: Andrew, what do you think? Andrew and Jessica, what do you think?
- 11.0.240 Andrew: [Refers to twenty-four cm model on desk] Well, I made a model that had the white was one forty-eighth and the purples were twelfths and the white was, I mean the red was twenty-fourths and I took two browns as the thirds and two dark greens as the fourths and they I called them the fourths and then the whole was four oranges and two purples.
- 11.0.241 T/R 1: Now, you're telling me that you used browns, two browns to be
- 11.0.242 Jessica: One, like one, one third.
- 11.0.243 Andrew: Yeah.
- 11.0.244 T/R 1: One brown was one third, two browns was two thirds?
- 11.0.245 Andrew: No
- 11.0.246 T/R 1: Is that what you're telling me?
- 11.0.247 Erik: No
- 11.0.248 Andrew: Two browns was one third
- 11.0.249 Erik: Two browns was one third.
- 11.0.250 Andrew: I took two browns and put them together
- 11.0.251 T/R 1: Two browns to be one third!
- 11.0.252 Andrew: Yeah.
- 11.0.253 T/R 1: Oh, ok, that's not going to fit. But maybe, um, you want to come up here and do that? [Andrew and Jessica come to front of class.] Ok, here you go. Why don't you build that right here. Do it up front here, uh, why don't you come all the way around, Jessica. Ok, let's see what they're doing here because, um, it looks to me as if you need a bunch of rods to do this. [They work for about two minutes to build the model of a train of four oranges and two purples, six brown rods and eight dark green rods, and twelve purple rods, twenty-four red rods, and white rods]
- 11.0.254 Andrew: It might not be enough.
- 11.0.255 T/R 1: Now, I want all of you to see what Jessica and Andrew are building, and, now you all can't come up at one time, so I'm gonna, if it's ok with Mrs. Phillips, I'm gonna ask you in little groups to go

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- up there and take a look at their model and um so we can be able to talk about it and then some of you maybe can look at it from where you're sitting. I know that Gregory and Danielle are very fortunate - they have front row seats. I think, can you see Alan and Erik?
- 11.0.256 Erik: Not really.
- 11.0.257 T/R 1: Not really. So some of you may have to go up in a minute to see what they're doing.
- 11.0.290 Jessica: Well, what we did was we made a model and we counted um
- 11.0.291 Andrew: Four oranges
- 11.0.292 Jessica: four oranges and two purples as our whole and for our thirds we counted, we counted two oranges as one, I mean two browns as one. [holds up two brown rods end to end] And we had
- 11.0.293 Andrew: that was our third.
- 11.0.294 Jessica: That was our thirds, and for our fourths we counted two greens as one [holds up a train of two dark green rods], two dark greens as one.
- 11.0.295 Andrew: Purples were our twelfths, the reds were the twenty-fourths [Jessica says twenty fourths] and the whites were forty-eighths.
- 11.0.296 Jessica: Forty-eighths. And we think that, we think that, three um, fourths are bigger than two thirds by either, um, one forty- I mean four forty-eighths um, two twelfths, or, um
- 11.0.297 Andrew: No, two twenty-fourths.
- 11.0.298 Jessica: Two twenty-fourth or what's that? One twelfth.
- 11.0.299 T/R 1: What do you think about that, Michael?
- 11.0.300 Michael: I guess I agree with it, it's what I came up with.
- 11.0.301 T/R 1: You came up with the same model, didn't you?
- 11.0.302 Michael: Yeah
- 11.0.303 T/R 1: Did anyone else come up with that same model? That's very lovely. Thank you so much, Andrew, and does anybody have a question to ask Andrew and Jessica before they're finished? Does anybody have a question? Does anybody have a comment? You sure you don't want to ask them any of that? Sarah what do you think?[Sarah says no.] Is that interesting [Sarah says mmm hmmm]? It's very interesting Ok, um, I'm going to ask you to sit down and I want to thank you very much for making that model for us. But I guess I'm asking the question, uh, to Meredith and James and to Erik and Alan right now, uh, does this have anything to do with your theory and the theory you tested? Meredith and David and Erik and Alan - does this model have anything to do with the theory you tested, David?

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- 11.0.304 David: Uh, yes because we thought that the ones would be one forty-eighth
- 11.0.305 Erik: And and the
- 11.0.306 David: And then the reds would be, um, one
- 11.0.307 Erik: Twenty-fourth and the purple, well originally, we thought that the light greens would be, well David thought that the light greens would be twelfths, but then we tried it and they would become the sixteenths, so then we tried the purple, yeah the sixteenths and we tried the purple and then that was the twelfths.
- 11.0.308 Alan: Since whites are doubles, they're forty-eighths
- 11.0.309 Erik: So, in other words we doubled everything.
- 11.0.310 Alan: Yeah. You basically just added, like, there originally were just two oranges, now there are four oranges and an extra purple. Now there are six, there are six browns.
- 11.0.311 T/R 1: So let's see, on this model here we had an orange and a red, and then on that model there we have two orange and a purple and in this orange here we have four orange and two purple. All of these represent one, is that a surprise?
- 11.0.312 Alan: It could have been two purples changing into a brown.
- 11.0.313 T/R 1: It could have been two purples changing into a brown
- 11.0.314 Alan: Yeah.
- 11.0.315 T/R 1: That's true.
- 11.0.316 Alan: And
- 11.0.317 T/R 1: I guess my question is what you called one in each of these models? Are they related in any way? The lengths? All of these you called one, are the lengths in any way related to each other, if you study each of the models you built. You see this one here you called the orange and red one, isn't that right, and here you called one two orange and purple, right?
- 11.0.318 Alan: So basically it's just doubled. That's
- 11.0.319 T/R 1: What do you mean by that "basically it's doubled", Alan? That's an interesting idea. In what way is it doubled?
- 11.0.320 Alan: Um, ok, it's doubled because it now it has four oranges and two purples or a brown, so
- 11.0.321 T/R 1: But the first one doesn't have any purples.
- 11.0.322 Alan: Well, that's because this had nothing to do with the first problem because of the first question, but
- 11.0.323 T/R 1: I'm not sure I understand what you're saying.
- 11.0.324 Alan: Had there have been sixths.
- 11.0.325 Erik: I know.

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- 11.0.326 T/R 1: We didn't have sixths, we had twelfths here.
- 11.0.327 Alan: Mmm hmm.
- 11.0.328 Erik: I think I know what he's saying.
- 11.0.329 Alan: Right, there you have twenty-fourths and the whites are forty-eighths this time. Now, up there, there are no purples, because they weren't put on. But had they have been, on the bottom, which they are, they are twelfths, because
- 11.0.330 Erik: Purples? In that
- 11.0.331 Alan: Purples are twelfths.
- 11.0.332 Erik: In that model they became twelfths, but over there they would be the sixths. Like Amy said, if
- 11.0.333 Alan: Right, because if you double each of them, it would come out to twice the number.
- 11.0.334 Erik: Exactly!
- 11.0.335 T/R 1: James?
- 11.0.336 James: Uh, I think um, that um, because there are two oranges and two purples I agree with Alan that it's double but why the red's there, it's two reds make a purple and that, that means the two oranges and the red make two oranges and a purple.
- 11.0.337 Alan: Yeah, cuz if you took the two oranges out of that model and a purple, and then two more oranges and a purple, and you put them on top of each other, they'd be equal. But if you put em side to side you'd have four oranges and two purples, or the two purples could be a brown. So it's basically doubled, each of the length is doubled.
- 11.0.338 T/R 1: I wonder if the rest of you see this, I'm saying, this is an orange and it's not a purple, it's an orange and a red, right? Now, how does this get doubled to be this? I see there are two oranges, instead of one orange, I see the one orange length got doubled, instead of one orange there's two, right? Isn't that true? But how did the red get doubled?
- 11.0.339 Alan: The red-
- 11.0.340 T/R 1: I'm confused, how did the red get doubled here?
- 11.0.341 Alan: The red
- 11.0.342 T/R 1: I see the orange got doubled here because there are two oranges, right? From one orange to two oranges, I don't know how did the red get doubled? I don't see that. Jessica? Kimberly.
- 11.0.343 Kimberly: Well, they used a purple and the red, two reds make a purple, so now if they have a purple, they doubled the red.
- 11.0.344 T/R 1: Is that what you were going to say?
- 11.0.345 Jessica: Yeah.

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- 11.0.346 Alan: I was going to say something different
- 11.0.347 T/R 1: So you're telling me that instead of the one orange and one red, we have two oranges and two reds in this model. But they just called it a purple rather than two reds. Do the rest of you see that? [mmm hmm] Ok, so this model is doubled of this, now you have to convince me that this model is double of this, so instead of two oranges and a purple, what should we have now if it's doubled? Don't look. What would you expect we would have then if it's doubled? Danielle.
- 11.0.348 Danielle: Um, four oranges and two purples.
- 11.0.349 T/R 1: Let's see. Do we have four oranges and two purples?
- 11.0.350 Erik: One, two, three, four, yup, or four oranges and one brown.
- 11.0.351 T/R 1: Or four oranges and one brown.
- 11.0.352 Alan: Yep
- 11.0.353 T/R 1: Ok, this is the question I ask you. If I were to make another model, Andrew's hand is up, Andrew knows my question, what do you think my question is, Andrew?
- 11.0.354 Andrew: If you were gonna make another model, what, um, the doubles be?
- 11.0.355 T/R 1: Ok, what would my one look like in terms of rods? Brian!
- 11.0.356 Brian: Um, forty-eight.
- 11.0.357 T/R 1: What would I call one? Imagine in your head what I would call one?
- 11.0.358 Brian: Forty-eight? Cuz there would be, well, cuz there would be forty-eight whites equal up to one and then.
- 11.0.359 T/R 1: Well, we have forty-eight whites going up to one here, don't we?
- 11.0.360 Brian: Oh!
- 11.0.361 T/R 1: In this model.
- 11.0.362 Erik: So we have to double that?
- 11.0.363 Alan: But, no!
- 11.0.364 T/R 1: I don't know, I'm asking you, that's my question, Andrew what do you think?
- 11.0.365 Erik: Well you're saying what-
- 11.0.366 Alan: No, it can't
- 11.0.367 Andrew: Well, the whole would be eight orange rods and
- 11.0.368 Alan: It can't be done
- 11.0.369 T/R 1: Eight orange rods, I'm listening.
- 11.0.370 Erik: Eight orange rods and two browns
- 11.0.371 Andrew: And two browns.
- 11.0.372 T/R 1: And two brown rods.

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- 11.0.373 Alan: You can't double that. You can't double that model because if you did, then you wouldn't be able to third it.
- 11.0.374 Erik: You wanna make a bet - all you had to do is train it - you just train it!
- 11.0.375 Alan: Right because if you doubled that it would be eight oranges and two browns, now is there any rod that could third that?
- 11.0.376 Erik: Well if you use a train
- 11.0.377 Andrew: Yeah
- 11.0.378 Erik: If you use a train, just like in Andrew's theory.
- 11.0.379 Alan: Well, if you train the rod, but that would make it not equal.
- 11.0.380 Andrew: It would probably be-
- 11.0.381 Alan: Up there, it's just plain, except for the whole.
- 11.0.382 Andrew: It would probably be three browns would be the thirds and three dark greens would be the fourths.
- 11.0.383 Alan: Right, but that would be using more than one rod to make another rod to fit, fit the same thing.
- 11.0.384 Erik: Yeah, so you can do that! Just like, you, Andrew said, you can use a train to make a third and a fourth. Cuz he, like, I, I overheard, they said that if you can use a train to make a whole why can't you use it to make a third and a fourth?
- 11.0.385 Andrew: Yeah.
- 11.0.386 T/R 1: David?
- 11.0.387 Andrew: And a half
- 11.0.388 Alan: But then it wouldn't be equal.
- 11.0.389 Erik: Yeah they would! Cuz the third could be, like in that model, Andrew used the two browns, that's equal!
- 11.0.390 Alan: But in that model, the three browns don't have anything attached on so it's totally equal
- 11.0.391 Erik: So? They just doubled it!
- 11.0.392 Alan: But if you added something on
- 11.0.393 Erik: We just doubled, we doubled that model to equal that model.
- 11.0.394 Andrew: Yeah, and I doubled the brown - two browns,
- 11.0.395 Erik: Yeah, exactly.
- 11.0.396 Andrew: So in the next model
- 11.0.397 T/R 1: David, what do you think? Did you want to say something?
- 11.0.398 David: Um, I agree with Erik
- 11.0.399 T/R 1: What part of what Erik said?
- 11.0.400 David: Well, Alan didn't think that you could uh third it, but like Erik said that you can train it and put the other blocks onto the other one
- 11.0.401 Alan: What I meant, what I meant is, you can't third it just using one rod.

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- 11.0.402 T/R 1: Ok, Alan.
- 11.0.403 Erik: Exactly. You can't third it using one rod, but you can third it using trains.
- 11.0.404 T/R 1: Ok, so
- 11.0.405 Alan: You could double that, but you would have to use two rods to make it
- 11.0.406 T/R 1: Ok, so you think you can double it and you think you can imagine - can you make one bigger than that?
- 11.0.407 Erik: If you doubled that, it would be sixteen oranges [laughter] and, sixteen oranges and four browns!
- 11.0.408 T/R 1: Ok, the question I want to leave you all to think about, I'd like you to uh, first I'd like to thank you for the wonderful models you built, but the question I'd like you to think about is, uh, is there, is there a biggest model?
- 11.0.409 Erik: Thirty-two oranges! [laughs]
- 11.0.410 T/R 1: Is there a biggest model? And if you don't have enough, uh, rods, you could imagine, we could write to Cuisenaire and we can have them ship us buckets and buckets and buckets and buckets
- 11.0.411 Erik: Or we could combine all our stuff.
- 11.0.412 We could start by that but my question to all of you is there a biggest model? Why or why not? And I'd like you to write to me about, about that. Would you do that? Would you write to me? Maybe Mrs. Phillips can let you combine and build together, that might take a little while.