

Description: How many numbers between 0 and 1? Parent Tape: Infinite Number Line Date: 1993-10-11 Location: Colts Neck Elementary School Researcher: Professor Carolyn Maher	Transcriber(s): Schmeelk, Suzanna Verifier(s): Cann, Matthew Date Transcribed: Spring 2009 Page: 1 of 5
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1.		RT1	Okay. So let's go back to what your assignment was. We were trying to figure out what sort of happens in between and just as we said these keep on going and there are infinitely many, what we are going to be learning over the next maybe six or seven years of math that even in between zero and one there are also infinitely many numbers.
2.		Students	There are?
3.		RT1	Yeah.
4.		Erik	Infinitely many?
5.		RT1	We have infinitely many. Now, I want you to think about that a little bit and that's my statement. Mathematicians claim that between zero and one there are also infinitely many numbers.
6.		Alan	Infinitely between zero and one?
7.		Students	[mumbles]
8.		RT1	They also claim that there are infinitely many fractions. I want you to think about that. I use to, when my son was your age he use to think about those things.
9.		Jessica	In fractions or just ...
10.		Andrew	One million
11.		Students	[murmuring]
12.		RT1	Infinitely many fractions. Now, that is something to think about. I don't expect you to solve that problem in your mind right now; mathematicians have worked on this for centuries.
13.		Erik	I just don't understand how there can be infinitely many numbers between zero to one.
14.		Andrew	Fractions... fractions.
15.		A student	I know why.
16.		Erik	No, infinitely many numbers from zero to one. It doesn't
17.		RT1	Alan.
18.		Erik	Make sense.
19.		Alan	You can divide that line into the smallest of fractions. You could divide it into zillionths.
20.		Students	Yeah.
21.		RT1	Did you hear what Alan said? Erik, did you hear what Alan said? You want to say that one more time, Alan?
22.		Alan	You could divide it into zillionths and there would still be space in there.
23.		Michael	If you had the longest number line in the world.
24.		Erik	If you divide it into zillionths depending on what number you want to hold it to.
25.		RT1	No. no. Michael, I'm talking about this little piece between zero and one.
26.		Alan	Yeah, but you could still divide it
27.		Michael	But if you made a number line to show zillionths, then you would have to have the longest number line in the world.
28.		Erik	Alan. Alan. That doesn't make sense.
29.		Alan	Yes it does.
30.		Erik	Even if you were to divide it into zillionths is depending on how big your one whole is. If you one whole is ten you can't divide it into zillionths.
31.		Alan	Well, from zero to one, you could. You could divide it into zillionths.
32.		Erik	If you one whole is ten, how could you divide it into zillionths?
33.		RT1	Erik, what if you one whole is a zillion?
34.		Erik	Then you could divide it into zillionths.
35.		RT1	Well, I think that is what, I think, Alan is saying.
36.		Erik	I know but I'm just
37.		RT1	What if you called your one whole a billion? What if your one whole you called a google?
38.		Erik	Hah. A google.
39.		Alan	What if you called the zero to one a billion?
40.		RT1	Anyway, these are ideas for you to think about and argue about and wonder about and that's what is fun about doing mathematics to be dealing with some of these really interesting ideas and to be able to think about this idea about infinity and it is a very very important idea to be able to think about in mathematics and we would not be out in space if weren't thinking about those problems. You are the future scientists and mathematicians and problem solvers so I want you to think about these things, okay? And think about them over a long period of time and every now and then we will come back and talk about them if you like. Okay? But, let's keep that idea that I want us, just for now, to focus on the interval between zero and one and I want to be able to place fractions, as many fractions as you can imagine, and then lets use even talk about some that you can't fit here because it gets hard to squeeze them in, but that you could imagine. You know when we talk about this little piece of a line, that we have drawn here, it is supposed to be

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		made up of lots and lots and lots and lots of infinitely many tiny little points and I can't put them all in so I just draw this line. [Andrew raises hand.] What do you think Alan?
41.	Alan	Well, as I was saying before about the zillionths, you could have a line the size of a dust particle and you could put that on there a zillion times. You would have zillionths.
42.	Michael	If you had a pin that was smaller than a dust particle, then ...
43.	Erik	Something that is smaller than a dust particle ... a dust bug ... a hundred dust bug can fit into a dust particle ...
44.	Alan	I wasn't talking about insects.
45.	RT1	Andrew?
46.	Andrew	Well, if you made a number line and you took a magnifying glass or a microscope, and put your number line under it, you would see that you have a lot of room left to put the one-hundredth, one-thousandth and the one-millionth.
47.	Alan	[inaudible]
48.	RT1	Did you all hear what Andrew said?
49.	Alan	Yes. If you did put it under a microscope it would look like you had enough room to put another zero to one in there. It would look like that. You could have it enlarged so that the line from the zero would be this big [raises hands and makes space between them about a foot]. And you would still have room there to put more.
50.	RT1	What happens when scientists discover more and more powerful telescopes?
51.	Michael	Than, the more numbers you could fit onto one number line.
52.	RT1	What do they see in the sky when they look with more and more powerful telescopes? Okay, they use to think they knew all the stars and then they invented more powerful instruments, right? What did they find?
53.	Students	More stars.
54.	RT1	So they invented more powerful instruments and what did they find?
55.	Students	More stars.
56.	RT1	So, is it that the stars aren't there or is it that we don't have the instrument?
57.	Students	We don't have the instrument?
58.	RT1	Okay. So, it is sort of like that on the number line. Like Andrew's example
59.	Students Space
60.	Alan	Right.
61.	Andrew	If we could make more powerful microscopes, then we would see more space every time we looked through a different one.
62.	RT1	Brian?
63.	Brian	So, like what Alan said, you can put zillionths in.
64.	RT1	So you think you can put zillionths in? You are changing your mind? So you are sort of agreeing that there are lots and lots of fractions between zero and one if you had this ...
65.	Erik	Yeah...
66.	Alan	Even like you could even make it smaller than that and make smaller pieces to put in there.
67.	Erik	What I don't understand is that if you are using a microscope to get more space, in actual reality you are not getting more space.
68.	RT1	That is an interesting idea, isn't it Erik?
69.	Erik	You see, when you are using the microscope it looks like you are getting more space, but in actuality you're not getting any more—it just looks like it; but, you are not.
70.	RT1	What do you have to say about it?
71.	Andrew	Well, actually you are because the human eye can't see that, but
72.	Alan	When you enlarge it you can see how much space you have left between the zillionth and the zero.
73.	Erik	Yeah, but Alan what you were saying before you were saying that when you use the microscope you get more space in that number line. That is what you were saying before.

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74.	Alan	No. That's not what I was saying.
75.	Erik	From what I understood, you were saying that if you use a microscope you get more space on the number line. It is not true.
76.	Alan	What I mean is, look, if you had some really small pen, if they come-up with it, you could draw a small line in the space you; but you don't really know how much space you have left between the zillionth and the zero. You don't really know that because you can't see it so you look at it under a microscope you could see how much space you have left.
77.	RT1	Yeah, it might be, Erik, when you were thinking more space you were thinking of extending it ...
78.	Erik	Yeah, the first time the way he said it that's why
79.	RT1	That's not what I heard Erik, he drew a picture yesterday and what I heard yesterday, Alan saying is that he was [Alan nodding head yes] still talking about that same space. Both of you had a different picture in your heads about the kind of space; and, you were kind of talking about the picture in your head and Alan was talking about the picture in his head and I think Andrew's picture matches Alan's picture and Jessica's picture, but I'm not sure.
80.	Alan	Like what I'm saying if you looked at it under a microscope...
81.	Brian	Like the human eye...
82.	Alan	Yeah, like the human eye...
83.	Alan and Brian	You can't see it like a dust particle.
84.	Brain	Like the zillionths and trillionth
85.	Alan	Yeah, you just have to magnify it.
86.	RT1	David.
87.	David	I think that you can't really see it to well, but if you use a microscope then you are seeing closer and it looks like you are seeing more, but you're really not you're just looking closer than before.
88.	Students	[Multiple exclamations simultaneously.]
89.	Alan	Yeah, because there is more space even between that.
90.	RT1	Okay. I am interested in how some other people are thinking about it. I have not heard some of the girls thinking about this. Laura, what do you think? You are listening to this very carefully what is your opinion on this?
91.	Laura	[Smiles and shakes her head.]
92.	RT1	Do you have an opinion?
93.	Erik	Are you a little bit lost?
94.	RT1	I don't think so she is listening very carefully to both ideas. What do you think?
95.	Laura	[Laura shakes here head left to right and mumbles.]
96.	RT1	What do you think between zero and one, here? What kind of numbers do you think you are seeing there? Any idea? No?
97.	Laura	[Laura shakes here head left to right and opens her mouth as if to speak but remains quite.]
98.	RT1	Any idea? I want to hear from some other people. What are your ideas about the numbers between zero and one? Okay, Audra, and then Jessica and then Mark. Okay, Audra?
99.	Audra	I really do agree with them because
100.	RT1	With whom?
101.	Audra	With Andrew and Alan because the human eye can't see it if you are making it that small so if you put it under a microscope you really could see more ...
102.	Students	It would be really tiny but you could still see it ...
103.	RT1	Jessica?
104.	Jessica	Well, I think I agree with Alan and Andrew because you really can't see if there are any little spaces; but when you put it under a microscope there might be a huge space that you could fit a lot of spaces
105.	Beth	He is not saying that it is getting bigger, he is just saying that it is not going to stop ...
106.	Michael	Oh, it is sort of like the more you see the more space you have
107.	Alan	Yeah.
108.	RT1	What about that? Michael said that it is sort of like the more you see the more space you have? That is Michael's question; what do you think?
109.	Beth	It is hard to explain.
110.	RT1	It sure is.. It sure is hard to explain. Kelly, were you going to say something?
111.	Kelly	[Nods head]
112.	RT1	No? Okay. Mark and then we'll hear from Jen.

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113.		Mark	I think that I agree with Alan and Andrew because they are right you can't see the thing but if you put it under a ...
244		Mark	microscope and if it is a really powerful one you would have a huge space there.
245		RT1	Okay. Amy?
246		Amy	Not sure.
247			Okay, you are not sure about that you are going to think about it some more. Okay. David?
248		David	I think that you can take the little smallest thing and then put it under a microscope and you will have a lot more space but you don't. It looks like a lot more space but it really isn't. You are just magnifying it.
249		Students	Yeah. So it looks like you have more space but you really don't.
250		Michael	It looks like you have more space and humans take advantage of it and take that really big space and mark these really really little lines on it that you really just can't see on it.
251		RT1	Okay. We are going to give number names to all those really really little lines. Won't that be fun?
252		Students	[Sigh/laugh]
253	35:51	RT1	Alan?
254		Alan	Up here [walks up to overhead]. If like this could be the zero and this could be the size of a bar and there could be your line. Now, if you had the hundredths which would probably go somewhere in here it would look say if it were right here. And, then you would have all that space in there. It looks like it, but you really don't have that much space. It's just that if you had it really big that is how much space you would think you could see.
255		RT1	So,
256		Alan	So that means you could divide this into halves and thirds and fourths and fifths and all of that.
257		RT1	So, you are telling me, let me see if I understand this. The rest of you will you help me with this? You are telling me that this bar over here that is marking zero, right? Okay, Michael is making it over there, but this bar that is marking zero—you've magnified it because you got a very powerful microscope. And, so, what you are telling me now, is that it would be really hard to place—what did you say? One-hundredths? Or one-thousandths? Okay. One-hundredths. It, would be really hard to see the placement of it but it may end up so close to zero you can't even mark it, but once you magnify it you will have all this extra space in between. That's interesting.
258	35:58	Alan	Yeah, because it looks like you got a lot of space, but you only really have the tincy-wincy bit of space in between there. I mean you could take like a really small pen and you could divide this up into all those pieces, but if you look at it with your regular eye you couldn't see that so you would have to make it bigger.
259	36:24	RT1	Laura, does that help? Does that make sense? Be sure Laura understands what you are saying.
260	36:39	Alan	Yeah, now that's their [inaudible] you could divide that space up into all little lines. Now if you magnified those spaces—and here would be the little bars—y you could divide this space up into little tiny pieces and that you could divide up into little spaces.
261	36:56	RT1	Okay. Does anyone have a question to ask Alan about this? Okay. We have a couple comments. David and Brian have a couple comments. Thank you very much, Alan. Do you have a question for him?
262	38:48	David	I don't really have a question.
263		RT1	Comment.
264		David	On my paper I had a ruler that I put up to it that I was using and I think it was millimeters or something. I had a ten inch number line so I just put it after one millimeter that was one one-hundredth.
265		RT1	So that is how you placed one one-hundredth
266		David	Yeah.
267		RT1	Okay. Brian.
268		Brian	I have a comment about what Alan and Andrew said. You see humans don't have powerful enough eyes to see where the zillionths are so there really is a lot of room but you don't see it because the human eye is not as powerful as a microscope.
269		Michael	Oh. I get it so there is a lot of room that you can't see.

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270		Alan	Say in the future that you come up with this really high powered microscope you could make that zero bar from the floor to the ceiling that would maybe let you see it being that big. You could divide it up into such small pieces that when you took off the microscope you wouldn't see anything it would be so tiny and so small that you couldn't see it but there really is space there and if you magnify those really tiny pieces you could divide those up into spaces.
271		David	Then you would probably need something with a really small point to write that small.
272		RT1	So it sounds like the instruments get in the way, right, not the numbers. What do you think James? You are so quite back there James, Amy and Jakki what are you thinking through all this discussion?
273		James	It really does make sense that there is more space in between the zillionths etc. etc.. So I agree with Alan mostly.
274		Alan	The biggest number you could think of you could make one and so on; you could go on forever with this... I mean you could keep in magnifying it and magnifying it and magnifying it, dividing it, magnifying it dividing it.
275	41:02	Brian	You could take the number line that has so much little space between it and if you look at it with a very powerful microscope then you would be able to put billions in it; so, it doesn't matter how big it is it could be as small as a germ and you could still put germs in it.
276		RT1	David.
277		David	I was going to say what Brain said that it could be as big as a dust bug and just ...
278		RT1	Gregory, what do you think about all this dust bugs and things that big? Do you have any editorial comments on this discussion?
279		Gregory	No.
280		RT1	Meredith.
281		Meredith	I think what he is trying to say is that if you look at it through the microscope then there is a lot of space but if you just look at it through the human eye then there isn't very much space in there.
282		RT1	That is a good synthesis.