AN ANALYSIS OF AFFECTIVE TEACHER/STUDENT MATHEMATICAL INTERACTIONS IN TWO URBAN MIDDLE SCHOOL CLASSROOMS

BY

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ABSTRACT OF THE DISSERTATION

An analysis of affective teacher/student mathematical interactions in two urban middle school classrooms

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Learning mathematics often evokes powerful affective reactions including uncertainty, frustration, or anxiety as well as satisfaction, joy and elation. Such emotions may contribute to or impede students' "in the moment" engagement as well as their longer-term motivation to learn mathematics. Research suggests that motivation and engagement are malleable constructs influenced by the contexts in which learning takes place. The language and actions which teachers use when interacting with their students may contribute to the development of learning environments that foster (or inhibit) student motivation and mathematical engagement.

This study examines classroom videotapes that capture "in the moment" mathematical interactions between two urban middle school teachers and their students. It describes the teachers' reflections on their own affect, and their explanations of their rationales for affective and instructional decisions, as reported during videotaped retrospective stimulated recall interviews. Three research questions are addressed:

1.) During the teacher/student mathematical interaction episodes that were analyzed, what affective language and/or actions do these two urban mathematics teachers use to foster or support (or possibly discourage or impede) students' engagement with conceptually challenging mathematics, as their students work in small groups?
2.) What can be inferred about the students' engagement with the mathematics immediately subsequent to the identified teacher/student mathematical interaction episodes?
3.) In retrospective stimulated recall interviews, what rationale do these two teachers provide for their own affective language and/or actions when interacting with students during key affective events?

To answer these questions, fifty-seven videotaped teacher/student mathematical interaction episodes that were recorded during a span of eight days of classroom periods (four days per teacher) are analyzed. From the coded video data, five themes regarding the teachers' affective language and actions emerge and are discussed: (1) Teacher interest in students' mathematical engagement, reasoning and representation, (2) Teacher validation of student mathematical reasoning, (3) Teacher support of student autonomy, (4) Teacher encouragement of discourse and (5) Teacher promotion of respect.

This study contributes to an emerging discussion about the contextual factors contributing to student engagement in mathematics classrooms. The study has implications for teacher education and teacher professional development, as teachers apply specific affective strategies to promote student engagement with mathematics. It also has implications for education researchers investigating specific teaching strategies that promote student mathematical engagement. Limitations include the small sample of teachers, the qualitative nature of the analysis, and the reliability of inferences drawn from videotaped data.

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DEDICATION

This research was inspired by the devoted and enthusiastic teachers at RBMS who live the pursuit, "Quality learning by all, whatever it takes."

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CHAPTER 1: INTRODUCTION

The purpose of this thesis is to examine the specific affective language and actions of two urban middle school teachers as they interact with their students in the context of mathematics lessons *and* to understand how the teachers' language and actions may foster and/or support student mathematical engagement. This chapter begins with a brief discussion of the issues that point to the importance of studying affective interactions in urban mathematics classrooms. Next, the data set is introduced followed by an overview of the method of analysis of the data set. A brief discussion of the main findings of this research is then presented. This chapter is concluded with a description of the limitations and suggestions of how the findings in this thesis may be applied, in context as pedagogical tools for practicing teachers.

This study considers why affect matters during mathematical interactions between teachers and their students, taking into consideration the perspectives of leading educational policy groups in mathematics as well as recent international student assessment data in mathematics. This study draws on the theoretical perspectives and empirical evidence presented in the fields of education research, education psychology, mathematics education and urban studies. The theoretical frameworks used in this study present affect, motivation and student engagement as dynamically interrelated in the contexts of teacher/student interactions.

"Effective teaching conveys a belief that each student can and is expected to understand mathematics and *that each will be supported in his or her efforts* to accomplish this goal" (NCTM, 2000, p. 17, italics inserted for emphasis). This is the position of the National Council of Teachers of Mathematics, in its document *Principles*

and Standards for School Mathematics (2000). Seven years later the Trends in International Mathematics and Science Study (TIMSS) published data indicating U.S. students in schools with higher proportions of poor students scored lower than U.S. students in schools with smaller proportions of poor students (U.S. Department of Education, 2009). Furthermore, the "effect size of the difference in mathematics scores between U.S. White and Black (eighth graders) is 1.1 times the effect size between the United States and Chinese Taipei, the country with the highest estimated score" (U.S. Department of Education, 2009, p. 28). When these data are interpreted in this way there is little doubt that the ideal of the NCTM is not being met. In the United States students living in poverty and Black students are being left behind.

According to The National Research Council, school contexts can make a difference for students living in poverty and can have a productive influence on student motivation and engagement. In engaging schools, teachers are supported in their efforts to address the psychological needs of their students. Engaging teachers show interest in their students' lives, convey high expectations and promote a sense of belonging in their classrooms (*Engaging Schools*, 2004). This corroborates the NCTM's assertion that the actions of the teacher contribute to "students'…confidence…and disposition toward mathematics" (2000, p.17, retrieved from www.nctm.org). With this in mind, it would be of value to gain knowledge about specific actions and language that teachers employ to develop engaging classroom environments, foster student confidence and promote positive dispositions towards mathematics so that other teachers may apply these strategies to improve their practice.

In a recent book, *Emotions in Education,* (Schutz & Pekrun, 2007) leading scholars from different fields including educational psychology, emotion psychology, sociology and teacher education have provided various perspectives on the interplay between emotions, learning and teaching. Two key themes that resonate throughout this book are: (1) emotions interact with the processes of learning and teaching *and* (2) emotions may influence student motivation, goals, values and achievement. According to Pekrun, Frenzel, Goetz & Perry (2007), "specific features of the classroom and social environments contribute to the development of academic emotions, and … emotions influence students' learning and achievement as well as teachers' instructional behavior" (Schutz & Pekrun, 2007, p. 4-5).

Learning mathematics often evokes powerful emotions such as excitement, satisfaction, joy, apprehension, frustration, anger, or anxiety. These emotions may influence students' motivation to become engaged in or withdraw from the mathematics. The National Mathematics Advisory Panel found evidence that students who believe that success in mathematics depends on having an innate ability are less likely to persevere when solving complex mathematical problems as compared with their peers who believe that success is related to effort. The Panel also found evidence that risk factors for "math anxiety" include fear of public embarrassment and fear of negative teacher attitudes (NMAP, 2008). These findings suggest that students' affective responses to the mathematics and to learning environments may influence student engagement.

The Trends in International Mathematics and Science Study (TIMSS, 2007) reported correlations between student attitude and achievement based on student questionnaire data and student achievement data. Fourth and eighth grade students who

were categorized as having a positive attitude toward mathematics also achieved the highest scores on the mathematics assessments as compared with their peers who fell into the medium and low attitude categories. TIMSS (2007) also revealed correlations between student self-confidence and achievement; students who reported high levels of self-confidence in mathematics had the highest average achievement, and those who reported low levels of self-confidence in mathematics had the lowest average achievement.

In the United States, minority students and minority students who live below the poverty level (indicated by those who are eligible for free or reduced-price lunch) have historically performed poorly on achievement tests as compared with white students. According to the National Assessment of Educational Progress, Black eighth graders achieved scores thirty-one percent lower than White students and *impoverished* Black students achieved scores twenty-one percent lower than impoverished White students (NAEP, 2007). In low-income urban communities many factors, including social norms may influence students' motivation and engagement in academics. Appearing weak or wrong in front of their peers may result in possibly negative consequences ranging from social ostracism to the threat of physical danger in or after school. These social influences may dictate that students maintain a level of toughness both on the street *and* in the classroom (Dance, 2002). Another potential hurdle for students in urban settings is low motivation within the school faculty. "…according to a survey of administrators, teachers at advantaged schools were more likely to have high morale and positive attitudes about their students. Teachers at disadvantaged schools were more than twice as likely to have difficulty motivating students as those at advantaged schools." (*Women, Minorities, and*

Persons with Disabilities in Science and Engineering, National Science Foundation (1994) www.nsf.gov Accessed, January 28, 2011).

Despite these difficulties, classrooms *do* exist in urban settings where teachers, through their actions and language, create supportive environments for learning. In these "emotionally safe" classrooms, each student is supported in his or her efforts with classroom norms of respect and dignity. The teachers set clear expectations that all students demonstrate respect for their peers and for the ideas presented by everyone in the classroom. Value is placed on all mathematical contributions, and students are encouraged to discuss their ideas freely without fear of ridicule or humiliation.

The purpose of this thesis is to examine the interplay between teacher language and actions and student engagement in urban settings by looking at the interactions between two urban mathematics teachers and their students. The majority of the students in these two middle school classrooms are Black and/or Hispanic minorities living in economically disadvantaged communities. These students are faced with challenges unique to minorities in urban settings, including historically low achievement, social pressures, and low morale of school faculty members. The two teachers in this study had been part of a multi-year professional development project (Schorr, Warner, Gearhart, & Samuels, 2007) at Rutgers University prior to this research. They were identified by the project team as being skilled in creating *emotionally safe learning environments* where urban students can successfully engage in conceptually challenging mathematics. The term, *conceptually challenging mathematics* refers to problem solving including reasoning and proof, multiple representations of mathematical ideas and making connections among mathematical concepts.

In this study, videos recorded as part of a larger study conducted by the MetroMath Center at Rutgers University were examined. One goal of the MetroMath study was to investigate mathematics education in urban settings through the use of videotaped classroom observations and interviews with teachers and students. Interviews were conducted with each teacher to collect information about their professional background, beliefs about mathematics and feelings about their students. Prior to each two-day lesson, the MetroMath researchers interviewed and videotaped each teacher to collect information about the forthcoming lesson, including the teachers' expectations of how the students would respond to the mathematics in the lesson. Next, the researchers videotaped classroom lessons in each of the teachers' classrooms from three different camera angles. Within two days subsequent to each two-day lesson, the researchers conducted and videotaped a stimulated recall interview during which each teacher viewed video clips, selected by the researcher, from the classroom observation data. These video clips highlighted "key affective events" during which an explicit or inferred (by the MetroMath researchers) change in affect was expressed by one student, many students or by the teacher (Schorr & Goldin, 2008). In the interviews, the teachers reflected on their own affect and made inferences about their students' affect during the selected key affective events from the lesson.

The focus of the current study centers on *teacher/student mathematical interaction episodes,* intervals of time during the classroom observation data when the teacher is interacting with one or more students in the context of conceptually challenging mathematics. During a teacher/student mathematical interaction episode, the teacher and student(s) engage in an overt verbal exchange or non-verbal communication

that pertains to mathematics. These episodes are examined for evidence of teacher and/or student emotional engagement, behavioral engagement and cognitive engagement. Emotional engagement refers to affective responses such as curiosity, boredom, happiness or anger. Inferences about student and teacher emotional responses are drawn based on language, facial expression, tone of voice and gestures. Behavioral engagement refers to the overt participation in solving the mathematical problem including writing, talking, using a calculator and thus are easily identifiable in these data. Cognitive engagement refers to the strategies or types of processing that the student uses during learning. Since these processes often are not observable, it is difficult to make claims about subjects' cognitive engagement in a video analysis study. "Defining and examining the components of engagement individually separates students' behavior, emotion, and cognition. In reality, these factors are dynamically interrelated within the individual; they are not isolated processes" (Fredricks, et al., 2004). It is not clear how behavioral, emotional and cognitive engagement interact within each individual. There is considerable disagreement about the interplay among these components, which will be discussed in the review of the literature.

The analysis of classroom observation data is presented in the Appendix as a chronological tabulation of the analyzed and coded teacher/student(s) mathematical interaction episodes is in the appendix (A.1, A.2, B.1 & B.2). Each episode is introduced with a list of the individuals involved in the interaction as well as a description of relevant developments leading up to the interaction including inferences regarding what may have prompted the interaction. Next, qualitative interpretations of teacher and student affective behaviors as they pertain to the mathematical interactions are listed

beside the transcribed dialogue including descriptive indicators for the inferred teacher and/or student emotions. A coding scheme for teacher affective language and actions, which was developed for this study, is then applied based on the qualitative interpretations. Each episode is concluded with a qualitative summary, which describes the interplay between the emotions and the students' mathematical progress within that episode. Finally, student engagement immediately subsequent to the teacher/student(s) mathematical interaction is described.

From the analysis of the teacher/student(s) mathematical interaction episodes, five common themes of teacher language and actions emerged: 1) teacher interest in the students' mathematical engagement, reasoning and representation; 2) teacher validation of student reasoning; 3) teacher support of student autonomy; 4) teacher encouragement of student discourse and 5) teacher promotion of respect among all members of the classroom. These themes represent the findings of this study and are described in detail in Chapter Four. Each theme is supported with examples from the classroom observation data and with references to all episodes in the Appendix that illustrate the specific theme.

The first theme is revealed as both teachers *express interest in their students' mathematical engagement, reasoning and representation* as they question their students and sit amongst them in small groups. In one classroom, students seem to be emotionally charged, and social conflicts sometimes have the potential of interfering with mathematical engagement. The teacher consistently shows interest in students' mathematical ideas, yet she rarely contributes her own mathematical ideas unless a social conflict appears to threaten the students' mathematical engagement. In these instances, the teacher calmly and swiftly intervenes and redirects the students to the mathematics by

posing a question relevant to the problem. Another finding in this same classroom is that most students remain engaged with mathematics after the teacher walks away and are often productive in their efforts toward solving the problem. In both classrooms, the teachers' expressions of interest appear to foster the students' engagement, as the students respond to the teacher's questioning by explaining their mathematical reasoning.

The second theme that emerged from the analysis is that both teachers *validate their students' mathematical reasoning* by using affirmations such as, "that's a good question". One of the teachers consistently validates the ideas of her students, regardless of whether their mathematical ideas were right or wrong. Often, when her students are headed down a mathematically incorrect path, the teacher does not correct them. Another finding in this same classroom is that although the students appear to remain engaged subsequent to each teacher/student(s) interaction episode, the students continue to present faulty reasoning and in two cases, the teacher eventually intervenes, presenting the solution of the problem to the whole class.

The third theme that emerged from the analysis is that both teachers make efforts to *support their students' autonomy* to solve the mathematical problems independent of teacher input. After a brief introduction to each problem, the teachers circulate throughout the classroom as the students work in small groups. In one classroom when the students need to use the formulas for area and perimeter of a rectangle, the teacher does not explicitly state the formulas. Instead, she poses questions with the inferred intent to remind the students how to derive the formulas. After a few minutes of discourse among the students and the teacher, one student becomes very animated when he recalls the formula and shares it with his group.

The fourth theme, *teacher encouragement of student discourse* is identified in both classrooms as the teachers encourage the students to collaborate with their peers to solve each problem. In one classroom, the teacher encourages the students to move about the room and discuss their ideas with students from different groups, particularly when it is inferred that she perceives the students are having difficulty with the mathematics.

The fifth theme, *teacher promotion of respect* is exhibited in both classrooms as the teachers model behaviors such as active listening when students explain their mathematical reasoning. In one classroom, the teacher explicitly states the importance of showing respect as she addresses the whole class, "I would like you to respectfully listen first…Please don't cut her sentences." [B.1.126] Both teachers place value on all mathematical contributions both explicitly through their language and actions and implicitly through their actions. One teacher, in her retrospective interview, reflects on how her students need to "listen" to each other instead of just "hear" each other.

These five themes emerge from a rigorous analysis of the language and actions of the two teachers as they interact with their students during fifty-seven teacher/student mathematical interaction episodes. Interactions between teachers and students are extremely complex and the conclusions of this research demonstrate how the teachers' language and actions are influenced by the contexts in which they take place. These contexts include features of the mathematical problems, student characteristics and teacher characteristics. *Contextual features are considered instrumental in the interpretation* of the teachers' and students' language and actions. The specific contextual features that are identified in these two classrooms are discussed in Chapter 5 including a presentation of the interplay between the contextual features and the five

themes of teacher affective language and actions.

As is the case with all qualitative research, this study is subject to certain limitations. Due to the depth at which the data are examined, only a small sample was considered, which is a main limitation of this study. Secondly, inferences by the researcher about teacher and student emotional engagement are made based on observable behaviors including facial expressions, tones of voice and gestures, and there is evidence to show that observable behaviors are not always consistent with actual emotions and/or engagement. Although every effort has been made to be objective in the qualitative descriptions, these inferences are subject to the perspective of the researcher. This study is limited in the extent to which the reliability of such inferences is checked.

The two teachers in this study demonstrate strategies for fostering student engagement that may be important for future larger-scale studies of affective interactions between students and teachers. Some questions that may be explored in future research include: When is it most appropriate to validate students' mathematical ideas and when is it better not to? When is it appropriate to correct a student and how can we attend to the emotions of a student during a mathematical intervention? When is it appropriate for a teacher to intervene with a student who is experiencing frustration with mathematics? What types of interventions may teachers use to celebrate mathematical success of their students? How might teacher language and/or actions interact with specific student engagement structures (Goldin, Epstein, Schorr & Warner, 2011)?

This research may have practical utility for pre-service and practicing teachers both in professional development and during instruction. Teachers may apply the coding scheme in the study of their own practice as they learn specific affective strategies that

they can use when interacting with their students. The five themes that emerged in the analysis may possibly have practical utility as tools for teachers to use during interactions with their students. Teachers may apply the language and actions as described in each theme during instruction as tools for fostering and supporting student engagement with conceptually challenging mathematics.

CHAPTER 2: THEORETICAL PERSPECTIVES AND LITERATURE REVIEW

"Emotions are at the heart of teaching...Emotions are dynamic parts of ourselves and whether they are positive or negative, all organizations, including schools are full of them." (Hargreaves, 1989, p.835).

2.1 Introduction

In this chapter, I discuss important topics that are relevant to teacher interactions and student engagement in mathematics classrooms. First, I address theoretical perspectives on affect with a focus on emotions in education, particularly mathematics education. I describe the different methodological approaches that have been used to research affect in the classroom. I then review empirical studies that examine the interplay between emotions, learning and teaching. Next, I consider theories on motivation and engagement, and I discuss relevant empirical studies that explore motivation and engagement in the context of mathematical learning and teaching. I conclude this chapter with a discussion about the specific challenges faced by teachers and students in urban settings.

2.2 Theoretical perspectives on affect

"Emotions are the most fundamental process, which underlies every expression of evaluation in one way or another. While a student is engaged in a mathematical activity, there is a continuous unconscious evaluation of the situation with respect to personal goals" (Hannula, 2002, p. 29). Examples of *emotions* include curiosity, interest, joy, satisfaction, and elation, as well as frustration, disappointment and anger. Emotions are considered by many in education research to play an influential role in aspects of learning including motivation, engagement and student performance. As Op 't Eynde, DeCorte

and Verschaffel state: (2007), "…affective processes like emotions and moods, in close interaction with cognitive, motivational, and volitional processes, constitute students' learning behavior" (p.187). Meyer and Turner (2006) identify "positive emotions" as supportive of instructional interactions and they associate positive classroom environments with productive student motivation and engagement. Boekarts (2007) points out, however that "not all learning activities elicit enjoyment" (p. 46). She conjectures that some mild negative emotions associated with doing academic tasks may help to stimulate engagement when experienced in the presence of the positive emotions associated with the pride of successfully completing a difficult task. Pekrun, Frenzel, Goetz and Perry (2007) suggest yet another perspective on the benefit of negative emotions in their discussion of emotions as either "activating" or "deactivating". They posit, "negative activating" emotions such as anxiety based on anticipation of failure may be beneficial in "strengthen(ing) motivation to invest effort and avoid failure" (p. 26).

Zemblyas (2007) presents three divergent theories of emotions in education and discusses the methodological implications of each. First, the *psychodynamic* view takes emotions as instantaneous, private, psychological responses and something that happens to the individual with "little interest in the details of the socio-cultural context" (Zemblyas, p. 60). Researchers taking this perspective may use self-reports and questionnaires to gather data and employ quantitative analyses of the data to study student emotions in the context of learning. In contrast, the *social constructivist* view takes emotion as a process that is largely influenced by the social culture and is regulated by the interactions that take place in social settings. In this view, emotions are part of a "dynamic, continuously fluctuating system."(p. 61) and so qualitative methods such as

the analysis of interviews and narratives may be used to study emotions in education. These methods capture the "meanings of emotional experiences" (Zemblyas 2007, p. 62) in the social context. Third, the *interactionist* perspective strikes a balance between the psychodynamic and the social constructivist approaches by considering the interaction between the individual's emotions and the social context. In this perspective "language, bodily performance and culture" interact to constitute the emotional experiences in the classroom (p. 64). Researchers taking this perspective may use the analysis of video taped interviews with teachers and students along with classroom observations to study "various sets of *discourses embedded in curricula*, *classrooms and learning environments*…(and)…show how the individual and the social may be studied *simultaneously*" (Zemblyas 2007, p. 66, italics added for emphasis).

Matthews, Zeidner and Roberts (2006) review three key psychological concepts relevant to educational contexts: dispositional constructs, mediating constructs, and educational outcomes. In their view, dispositional constructs are comprised of personality traits, motivation orientations, cognitive style and general cognitive abilities. Within the dispositional constructs five general personality traits: openness, extraversion, conscientiousness, agreeableness and neuroticism, are believed to either foster adaptive or maladaptive processes for learning. For example, students who are conscientious will exhibit adaptive processes such as effort and persistence. On the other hand, students whose temperament is neuroticism will exhibit maladaptive processes such as irritability and avoidance. Matthews et al. (2006) suggest that these traits may be treatable in social situations. "The qualities of the distress-prone child, such as hyperawareness of threat…may lead to avoidance of feared social situations. This behavior pattern, in turn,

affords fewer opportunities to develop emotion recognition skills…By contrast, traits that promote engagement with challenging situations…lead to greater opportunities for learning skills for handling exciting (but potentially risky) encounters" (Matthews et al., p. 171). Educating teachers for "developing professional skills for dealing with children of variable temperamental qualities" (p. 182) may help to promote productive engagement.

Pekrun, Goetz, Titz and Perry (2002) define academic emotions as "any emotions that are directly related to academic learning, classroom instruction and achievement" (p. 92). These emotions are experienced by students and teachers and include those related to success or failure (achievement emotions) as well as emotions related to the process of learning (activity emotions), (Pekrun, Frenzel, Goetz & Perry, 2007). According to their *control-value theory,* Pekrun and colleagues maintain that "individuals experience achievement emotions when they feel in control of, or out of control of, achievement activities and outcomes that are subjectively important to them, implying that *control appraisals* and *value appraisals* are the proximal determinants of these emotions." (p. 16). Factors that influence students' appraisals include individual achievement goals, achievement related control and values beliefs, and certain features of the classroom environment. According to Pekrun et al. (2007), achievement emotions are influenced by many factors: the cognitive quality of instruction, support of autonomy and cooperation, goal structures and expectations, feedback and consequences of achievement, treatment of emotions and support of self-regulation. Theoretically, when the cognitive task demands match students' competence and interest, students are likely to experience positive emotions. Students who perceive a sense of control (autonomy) over their

learning may experience positive emotions and adopt positive values toward learning. When expectations or goals are perceived to emphasize competition students may feel angry, anxious or hopeless in the face of frustration. In contrast settings where students may feel supported, encouraged and in control of their learning may feel happy, hopeful and interested. Control-value theory assumes that achievement emotions affect cognition, motivation, strategy use, self-regulation and achievement.

> …it can be assumed that positive or negative emotions that do not relate to an ongoing achievement activity distract attention away from the activity so that they reduce cognitive resources available for task purposes and impair performance needing such resources. For example, if a student is angry about failure, or worries about an upcoming exam, she will experience difficulty in concentrating on learning. Positive emotions relating to the activity, on the other hand, are assumed to focus attention on the activity, thus benefiting performance.
>
> (Pekrun et al. 2007, p. 26)

In the above examples it is inferred that the negative emotions are influenced by the student's values placed on performance of passing the exam rather than mastery of the material. Cognition is then impaired in the presence of negative achievement emotions in the anticipation of failure. On the other hand, positive emotions such as interest in the novelty of a task, especially when accompanied by perceptions of control and value of the task, may activate a student's interest to engage in learning.

McLeod (1989) describes three constructs of affect, which range in stability and intensity: (1) *emotions* (most intense and least stable), (2) *attitudes* (moderately intense and moderately stable), and (3) *beliefs,* (least intense and most stable). DeBellis and Goldin (1997, 2006, p. 135) proposed an additional construct, (4) *values* which refer to "deep, 'personal truths' or commitments cherished by individuals. Theoretically, these four constructs interact dynamically within the individual and are influenced by external forces including social and cultural conditions, external contextual factors as well as the emotions, attitudes, beliefs and values of others (DeBellis & Goldin, 2006). While each of these constructs are important to learning and teaching, in this review I choose to place emphasis on emotions in the classroom. In the current study, I examine the emotional engagement of teachers and their students in the contexts of classrooms.

DeBellis and Goldin (2006) describe affect as an internal representational system as they explain that "the states of emotional feeling *carry meanings for the individual*. They *encode and exchange information* in interaction with other internal systems of representation, in a way essential to mathematical understanding and problem-solving performance" (p. 133). Central to their theory is the concept of meta-affect, or feelings about one's feelings based on an individual's cognitive appraisal of a task. To illustrate the definition of meta-affect, DeBellis and Goldin(2006) give the example of riding a roller coaster, an experience that some people pay money for in order to experience the feeling of fear. During the ride, the individual might feel fear from the twists, turns and upside-down loops, yet at the same time, be thrilled by this fear, based on their cognitive appraisal that they are physically secure and the ride is most likely safe. Extrapolated to the classroom context, students may feel fear in a setting where the teacher and/or the

students do not tolerate mistakes. Alternatively, in a classroom where mistakes are accepted as a necessary part of the learning process, students will more likely experience productive meta-affect knowing that they are "safe" from embarrassment and humiliation in the face of an incorrect answer. DeBellis and Goldin (2006) explain "…the most important goals in mathematics are not to eliminate frustration, remove fear and anxiety or make mathematical activity consistently easy and fun. Rather, they are to develop meta-affect where the emotional feelings about the emotions associated with impasse or difficulty are productive of learning and accomplishment" (p.137).

Goldin (2000) conceptualizes a dynamic relationship between cognition and affect in his affective pathways model. He suggests that affective pathways are repeated experiences and are integral to the development of the enduring affective traits attitudes, beliefs, engagement and motivation (*global affect*). He presents an idealized example of his affective pathways model to expound how the changing states of emotions and feelings (*local affect*) may interact with the cognitive process of problem solving.

> [A] student approaching a problem in mathematics may initially experience *curiosity*, followed by a sense of *puzzlement* or *bewilderment* if the problem is unfamiliar or difficult. Repeated unsuccessful attempts may evoke *frustration*. Perhaps after one or several changes of strategy, the student experiences some *encouragement* as progress seems to occur, *elation* at a new insight or breakthrough, followed by *satisfaction* with having solved a difficult problem or understood a new mathematical concept. Alternatively, the student's *frustration* may lead to

> *anxiety, fear* and/or *despair,* evoking avoidance strategies and defense mechanisms – a very different pathway.
>
> (Goldin in Hannula, Evans, Philippou, & Zan, (2004), see also Goldin, 2000)

In this idealized example of Goldin's affective pathways model, the student begins with a feeling of puzzlement then frustration, which leads to two very different possible pathways. In both cases, there may be several opportunities for a skilled teacher to interact with the student in a manner that supports productive affect in the context of the mathematics. Yet, in the case where the frustration leads to "anxiety, fear and/or despair", it may be more necessary for a teacher to intervene in a manner that supports student engagement or re-engagement with the problem. This is not to say that teachers need to eliminate frustration from the problem solving process. As stated by Goldin, Epstein, Schorr, and Warner (2011), "feelings of impasse, frustration, and disappointment" may be "associated with constructive engagement" (p. 551) (see also, Boekarts, 2007).

Certain characteristics of classroom environments including teacher language and actions may influence student emotions (Op 't Eynde & Turner, 2006; Hannula, 2006; Pekrun et al., 2007). The cognitive quality of learning tasks, autonomy support, goals (Linnenbrink, 2007) and expectations and teacher feedback all contribute to the development of students' perception of what they can control and what is valued in the classroom (Pekrun et al., 2007).

> …[O]ne can claim that every emotion is situated in its instructional context by virtue of four characteristics. First, emotions are based on

> students' cognitive interpretations and appraisals of specific situations. Second, students construct representations and appraisals based on the knowledge they have and the beliefs they hold, and thus, individuals' interpretations are influenced and vary by factors such as age, personal history, and home culture. Third, emotions are contextualized because individuals create unique appraisals of events within situations. And finally, emotions are unstable because situations, and also person-in-the situation, continuously develop (Op 't Eynde & Turner, 2006, p. 368).

2.3 Methods used to measure affect in education

Questionnaires and surveys are widely used to measure specific affective variables in the classroom. For example, the Math Anxiety Questionnaire (Meece, 1981) was designed to measure the cognitive and affective components of math anxiety as defined using six dimensions of "negative reactions to mathematics for assessment: dislike, lack of confidence, discomfort, worry, fear and dread, and confusion/frustration"(Wigfield & Meece, 1998). The Mathematics Anxiety Rating Scale (Richardson & Suinn, 1976) was developed to measure anxious reactions to doing mathematics in school and in everyday life. Questionnaires have also been used to study the more stable constructs of affect beliefs and attitudes. The Mathematics Related Beliefs Questionnaire (Op't Eynde & DeCorte, 2003) was developed to assess students' beliefs about mathematics in Flanders and was then revised for a wider population including Spanish and English speaking students (Diego-Mantecon, et al., 2007). The Fennema-Sherman Attitude Scales (Fennema & Sherman, 1976) were originally developed to study gender differences in student attitudes about learning math but have

been applied in more general contexts to study student attitude toward mathematics and science. These measures generate quantitative data that is useful for hypothesis testing and for generating ideas for further research.

There are some limitations to relying solely on quantitative research in education. Matthews, Zeidner, and Roberts (2006) point out when using self-reports to assess emotions; "one may question whether individuals have accurate insight into their emotional functioning" (p. 167). These instruments are typically administered once, when students are not simultaneously engaged in learning, for example before class begins or after class ends. Although questionnaires may help to identify certain variables that could be useful in understanding trait-like affective constructs, they do not shed light on the "collective emotions" and the *interplay* between emotions and learning. Emotions are fluid, dynamic and students' emotional reactions are influenced by the social interactions taking place before, during, and after class. According to Pekrun and Schutz (2007), "there is a need for adapting observational systems of emotions such that they can be integrated into video-based classroom studies, and be used for analyzing students' and teachers' ongoing emotions in classroom discourse" (p. 324).

2.4 Empirical studies on affect in mathematics education

Data from large-scale international studies indicate a relationship between student affect and achievement in mathematics. International studies including the Third International Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (PISA) have examined student achievement as well as affective reactions to academic work. Scholars using Trends in International Mathematics and Science Study (TIMSS, 2007) examined mathematics and science

achievement for students in fourth grade (36 countries) and eighth grade (48 countries). The mathematics assessment addressed the topics of number, algebra, geometry, measures, and data and chance. The cognitive domains, knowing, applying and reasoning were also assessed at the fourth and eighth grade levels. TIMSS also includes data regarding students' attitudes and self-confidence in mathematics compiled in the Index of Students' Positive Affect Toward Mathematics (PATM). Students' attitudes toward mathematics were assessed based on responses to the following three statements: "I enjoy learning mathematics." "Mathematics is boring." "I like mathematics." Students who were categorized as having a positive attitude toward mathematics achieved the highest scores on the mathematics achievement assessments as compared to their peers who fell into the medium and low attitude categories (TIMSS, 2007, p. 174). Internationally, the students in the high PATM category had an average achievement score of 483 points, while those in the low PATM category had an average achievement score of 454. In the United States, those in the high PATM category had an average achievement of 535 and those in the low PATM category had average achievement of 517 (see Exhibit 4.8, TIMSS p. 175). While these results may indicate a correlation between negative affect and poor performance, it is impossible to identify which if either of these plays a causal role. These data do not shed light on underlying causes of negative affect and/or poor performance. Analyses of these data do not offer solutions to the problem of low achievement nor do they suggest methods for fostering positive affect toward mathematics. Also, given the diversity of these international studies, cultural values are likely to influence students' attitudes toward mathematics. It is difficult to

draw meaningful conclusions from these data about how best to promote positive affect or to improve student performance at the classroom level.

Research on math anxiety has been a popular angle for examining the interplay between emotion and achievement. As defined by Ashcraft (2002), math anxiety is "a feeling of tension, apprehension, or fear that interferes with math performance" (p. 181). Empirical evidence has linked math anxiety to cognition and math performance (e.g., Ashcraft & Faust, 1994, Faust et al., 1996,Wigfield & Meece, 1988). Wigfield and Meece (1988) found that negative affective reactions correlated negatively and significantly with math performance in a study of five hundred sixty four students in grades five - twelve, math classes. In an analysis of data collected in thirty countries worldwide, results from the 2003 Programme of International Student Assessment (PISA, 2003) confirm that anxiety in mathematics is negatively associated with mathematics achievement (Törnroos, Ingemansson, Pettersson & Kupari, 2006).

Hembree (1990) conducted a meta-analysis of one hundred fifty-one empirical studies to examine the nature, effects and relief of mathematics anxiety. Although Hembree (1990) included a wide range of grade levels, from grade three through post-secondary, approximately eighty percent of the studies (one hundred-twenty-two out of one hundred fifty-five) were conducted at the post-secondary level. One of the questions considered in the meta-analysis was whether there is a causal relationship between math anxiety and math performance. Specific student outcomes correlated with math anxiety were performance on aptitude tests, student attitude and avoidance behaviors.

Findings suggested negative and significant correlations between math anxiety and math performance ranging from ($r = -0.25$) to ($r = -0.40$). Math anxiety correlated

even more negatively with student enjoyment of mathematics (r = -0.75), self-confidence in mathematics, (r = -0.82), and motivation to do mathematics, (r = -0.64). Of particular interest in the realm of teacher education are the findings that college students enrolled in teacher education courses indicated the highest level of anxiety on the Mathematics Anxiety Rating Scale (MARS, Richardson & Suinn, 1972) even as compared to remedial students taking mathematics in college. According to Hembree (1990) these results are of "national import" because "…when otherwise capable students avoid the study of mathematics, their options regarding careers are reduced, eroding the country's resource base in science and technology" (p. 34). Some limitations of Hembree's (1990) meta-analysis must be considered when using these data to draw conclusions. Although the studies ranged in grade level, a large majority of the studies were conducted at the post-secondary level. The cognitive demands of mathematics may be much different at the post-secondary level as compared to the elementary, middle and high school levels. Also, the maturity level of the older students may influence how those students manage their emotions. Although Hembree's (1990) study may highlight the importance of attending to affective issues with regard to mathematics, research that explores the interplay between affect, cognition and behavior are most relevant to the current study.

Recent research in neuroscience demonstrates that math anxiety interferes with working memory when students engage in multi-step mathematical problems. Lyons and Beilock (2011) used studied magnetic resonance imaging to record neural activity in their study of cognitive control resources. For the study, thirty-two college students were categorized either "highly math anxious (HMA)" or "low math anxious (LMA)" according to their responses to the Short Math-Anxiety Rating Scale (SMARS, Alexander

& Martray, 1989). The subjects then completed math and reading tasks which each had two levels of difficulty, easy and hard. Neural activity was measured using functional magnetic resonance imaging (MRI) immediately prior to and during math task completion. Behavioral measures were the proportion of incorrect responses to the math tasks. The results indicated that indeed, HMA individuals produced higher math deficits than LMA individuals. MRI data revealed "neural evidence" characteristic to math anxiety is evident prior to engagement with the task. Interestingly, the data illustrated that those HMA individuals who did not present neural images of anxiety leading up to the task performed better on the task. "Thus, our data are most consistent with the notion that HMAs' math deficits are determined primarily by how well they respond to and perhaps reinterpret their anxiety response rather than by the magnitude of those anxiety responses or their math skills per se." (p. 7). These results suggest that anticipation of doing mathematics and the negative feelings leading up to the mathematics tasks may interfere with HMAs' ability to complete the task correctly. Lyons and Beilock (2011) recommend educational interventions, which focus on helping students to "marshal cognitive control resources and effectively check one's math-related anxiety response once it occurs – but before it has a chance to reduce actual math performance" (p. 8).

Op 't Eynde, DeCorte and Verschaffel (2007) review three of their empirical studies in which they examined the role of student emotions in problem solving. Their general research question for all three studies was "What is the role of emotional processes in mathematical problem solving and which student characteristics and classroom context factors determine these processes?" (p. 189). The subjects of the studies were Flemish junior high students (ages fourteen to sixteen) and for each study

the researchers implemented a different methodology to analyze all or some of the subjects' emotions about mathematics. For the first study, the investigators administered the Mathematics Related Beliefs Questionnaire (Op't Eynde & DeCorte, 2003, 2007) to three hundred sixty-five students. The results of this first study indicated that students' beliefs about mathematics can be represented by four dynamic factors: (1) "the role and functioning of the teacher" in the class context, (2) "(intrinsic) value and perceived competence in mathematics", (3) "the usefulness of mathematics in real life" and (4) "the view of mathematics as a domain of excellence" (p. 191).

In the second study, Op 't Eynde et al. (2007) took a closer look as a multiple-case study of sixteen students in problem-solving situations (Op 't Eynde, DeCorte & Verschaffel, 2006). First, researchers collected data using the Online Motivation Questionnaire (Boekarts, 2002), administered after the students read the problem, but before they started working on the problem. Students were asked to think aloud while solving a complex mathematical problem and researchers videotaped the problem solving session. Later the researchers used the video data in two ways. First, they "labeled and identified" student emotions by describing facial expressions, vocalizations and "bodily actions" as evidenced during problem solving. Next, they used the video data in stimulated recall interviews to question the students about their affect during problem solving. Op 't Eynde et al. (2006) state "analysis of students' interview data" was "core" to interpreting students' appraisals of the events captured on the problem-solving videos (p. 200). From the analysis, "six rich narratives" were used to examine the dynamic interactions between "(meta)cognitive, motivational and emotional processes" during problem solving (p. 201). The researchers concluded that three out of eight students with

negative perceptions of their ability in mathematics wanted to avoid the problem. In contrast, none of the students who were identified as having a positive profile according to the MRBQ reported avoidance goals. The researchers also noted differences in how the students coped with negative emotions during problem solving. While some students were able to stay focused on the task in the face of negative emotions, others exhibited avoidance behaviors. The researchers concluded: "Students' competence to self-regulate these unpleasant emotions in effective ways may be an important determinant of successful mathematical problem solving" (p. 199). The researchers acknowledge that time-consuming nature of this level of analysis presents the limitation of a small sample size, yet they contend this methodology is aligned with their theoretical framework and "most directly addresses our central research question" (p. 195).

In the last of the three studies, Op 't Eynde et al. (2007) examined coping strategies in three hundred ninety-three students using items adapted from COPE (Carver, Weintraub & Scheirer, 1989). Items in the COPE-questionnaire were developed to measure the dispositional and situational coping strategies in individuals (Carver, Weintraub & Scheirer, 1989). Op 't Eynde et al. (2007) asked students about their use of coping strategies in three different situations (1) when taking a difficult test, (2) when faced with difficult homework and (3) during a difficult math lesson. Results indicated that some students used active coping strategies to stay focused on the task while others activated coping strategies counterproductive to learning, including abandoning the task and denial. From the results of the three studies, Op 't Eynde, DeCorte and Verschaffel (2006) concluded "there is an individually changing flow of emotional experiences that derives from students' interpretations and appraisals of the events that occur during

mathematical problem solving… ” Students’ cognitive appraisal of the math problem was central to the students’ flow of emotions (Op ‘t Eynde, DeCorte & Verschaffel, 2006). “Experiencing the inadequacy of the cognitive strategies used is apparently as much an emotional as a (meta)cognitive process**.** Emotions then trigger students to redirect their behavior and to look for alternative (cognitive) strategies (i.e., problem solving as a coping strategy)” (Op ‘t Eynde, DeCorte & Verschaffel, 2007, p. 198).

Op ‘t Eynde et al., (2007) advise that teachers need to attend to students’ development of coping strategies when faced with difficult mathematical problems.

> “Teaching students how to solve mathematical problems then implies that we have to teach them how to cope effectively with feelings of frustration or sometimes anger…when teaching and learning mathematical problem solving, the allowance of space for negative emotions might be an educational goal from a cognitive as well as a motivational, point of view. Indeed, only when experiencing negative emotions will students have the opportunity of learning how to deal with them.”
>
> (Op’ t Eynde, DeCorte & Verschaffel, 2006, p. 204)

Malmivuori (2006) conducted a quantitative study to examine students’ beliefs about mathematics and about themselves as mathematical learners, their affective responses in math class, self-regulatory patterns, behavioral patterns and mathematical performance. Data was collected using student self-report questionnaires administered to seven hundred twenty-three students age thirteen attending public schools in a metropolitan area of Finland. Student beliefs (self-confidence) were assessed using two scales: one measured student *self-efficacy* (e.g. could surely come off more difficult

maths") and the other measured students' *low-self esteem*, (e.g. "I am not the type who succeeds in mathematics") (p.156). Negative affective responses included *fear of mathematics* (e.g., "I often feel relieved after math class") and math *test anxiety* (e.g., I am always worried about my success before math exams"). Positive affective responses included *enjoyment* (e.g., I find mathematics as enjoyable school subject") and *liking of mathematics* (e.g., "I like solving mathematics problems"). Students' self-regulatory (behavioral) patterns were measured based on *persistence* (e.g., "I complete solving math problems even though they appeared boring"), *integration* (e.g., "I try to connect new things with what I already know of mathematics") and *preference for a challenge* (risk taking) (p. 156). Students' *mathematical performance* was measured using a twenty-six item test assessing students' aptitude with numbers, word problems and mathematical patterns.

Malmivuori (2006) organized student responses to the questionnaires according to inter-correlations between variables. Among the strongest inter-correlations appeared between students' *liking mathematics* and their *preference for a challenge,* r = .71. Students' beliefs of *self-efficacy* was positively correlated with both *persistence,* r = .39 and *preference for a challenge,* r = .45. *Performance* on the math test was most positively correlated with *preference for a challenge,* r = .30, and *self-efficacy,* r = .28 which contrasted with the negative correlations between *performance* and *low self-esteem*, r = -.42 and *fear of mathematics,* r = -.31. Malmivuori (2006) concluded that her findings support the "co-constructive and dynamic nature of affect and cognition, in which the functioning of self-appraisals and self-regulation ultimately determine the role of affect in students' mathematical learning or performance processes" (p. 161).

Many other studies have highlighted the importance of attending to emotions in the mathematics classroom. Although I do not discuss these studies in depth in this review, they deserve mention for their contribution to the study of affect in mathematics education. In her thesis research, DeBellis (1996) explored the interplay between affect and cognition in the context of mathematical problem solving through the analysis of task-based interviews with elementary school students. She made inferences about students' emotions based on tone of voice, body language, facial expressions and gestures. DeBellis concluded, "the system of affect appears to be an essential component in the development of mathematical understanding and problem solving ability." (p. iv) Meyer and Turner (2007), describe emotions as helping to "define classroom experiences, providing powerful rationales for engaging in and avoiding, even abandoning, teaching and learning opportunities" (p.243). In their research on classroom motivation over the course of ten years, Meyer and Turner and their colleagues recognized emotion as ever-present in the contexts of real classrooms. When they interviewed upper elementary school students about their approaches to a mathematical project, students responded with expressions of elation, love, enjoyment and anger (Meyer & Turner, 2002, Meyer, Turner & Spencer, 1997). In a subsequent study, Turner, Thorpe and Meyer, (1998) described the motivational-affective patterns in the context of upper elementary school classrooms and concluded "emotions were central to understanding the students' goals, self-efficacy, and strategies (Meyer & Turner, 2002, p. 108). In their findings, it was evidenced that students who experienced consistent positive emotions in the context of learning reported fewer incidences of avoidance

behaviors (Turner, et al., 1998, Turner et al., 2002, Turner, Meyer, Midgley & Patrick, 2003).

The theoretical perspectives presented above suggest that affect interacts in important ways with cognition, motivation and engagement. Zemblyas' (2007) interactionist perspective of emotions provides a general framework taking the mind, body, individual and social aspects as all contributing to the study of emotions in education. This perspective shares theoretical features with the socio-cultural aspects of control-value theory (Pekrun et al., 2007) as well as the interconnectedness of cognition and emotion presented in Goldin's (2000) affective pathways model. According to Pekrun and colleagues (Pekrun, Freznel, Goetz & Perry, 2007; Pekrun, Goetz, Titz & Perry, 2002), certain characteristics of learning environments, including the cognitive and motivational qualities of instruction, influence students' and teachers' emotions. These perspectives are supported by empirical evidence that demonstrates link between affect and mathematical achievement (TIMSS, 2007; Hembree, 1990; Lyons & Beilock, 2007), problem-solving (DeBellis, 2006, Op't Eynde et al., 2006, 2007) and motivational patterns (Malmivuori, 2006, Turner et al., 1998, Turner et al., 2002).

2.5 Theoretical perspectives on motivation

"Motivation reflects the reasons behind our thoughts and actions."

(Brophy, 2010, p. 2)

In the literature definitions of motivation vary from either an enduring "trait" (Dweck, 2002) or a cognitive and affective process (Anderman & Wolters, 2006; Hannula, 2004, 2006). Hannula (2004, 2006) defines motivation as a state with "potential to direct behavior that is built into the system that controls emotion. This potential may be manifested in cognition, emotion and/or behavior" (Hannula, 2006, p. 166). Ryan and Deci, (2000) conceptualize a continuum ranging from intrinsic motivation, controlled by the self to extrinsic motivation, controlled by external forces. According to Pintrich (2003), "motivational constructs are used to explain the instigation of behavior, the direction of behavior (choice), the intensity of behavior (effort, persistence), and actual achievement or accomplishments" (p.104).

Some traditional views take student motivation as a "global" trait with two options, either "motivated" or "not motivated" (Linnenbrink & Pintrich, 2002b). In contrast, Linnenbrink and Pintrich (2002b) describe three key assumptions of the social cognitive perspective on motivation. First, motivation is *dynamic* and *multifaceted* which takes the construct as a state rather than a trait. Second, motivation is "*situated, contextual* and *domain specific*" (p. 314, italics inserted for emphasis) meaning that students could be motivated differently depending on the class or context. Third, individual's cognitive *appraisals* of their academic progress mediate motivation. "That is, students' own thoughts about their motivation and learning play a key role in mediating their engagement and subsequent achievement" (p. 314). Linnenbrink and

Pintrich (2002b) take this conceptualization of motivation as promising because it "provides hope for teachers … and suggests that the instructional efforts and the design of classrooms and schools can make a difference in motivating students for academic achievement" (p. 314).

A variety of theoretical perspectives on motivation are relevant to education research. Bandura's self-efficacy theory (Bandura, 1996, 1997) takes individuals' beliefs that they can perform a particular task or domain as central to motivation. "Students who have more positive self-efficacy beliefs…are more likely to work harder, persist, and eventually achieve at higher levels…(and) are more likely to choose to continue to take more difficult courses" (Linnenbrink & Pintrich, 2002, p.315). Contextual factors are considered mediators of self-efficacy including task difficulty, types of feedback and social norms (Pintrich, 2003). In Ryan and Deci's (2000) self-determination theory psychological needs, autonomy, relatedness and competence play a central role in motivation. Achievement goal theorists (Ames, 1992; Dweck & Leggett, 1988) study "the reasons why students choose to engage with a particular task" (Anderman & Wolters, 2006). In review of theoretical models of motivation, Pintrich (2003b) draws out the main aspects that are thought to drive student motivation and categorizes these into three components: (a) expectancy components; (b) value components; and (c) affective components. The two main categories of expectancy components are control beliefs and self-efficacy beliefs. Control beliefs involve how much an individual feels that they are in control of their behavior as compared to how much they rely on external controls to control their behavior. Theoretically there is a general consensus that "internal control results in positive outcomes (i.e., more cognitive engagement, higher achievement, higher

self-esteem), whereas sustained perceptions of external or unknown control can result in negative outcomes (lower achievement, lack of effort, passivity, anxiety)" (Pintrich, 2003b, p.106). Motivation is also supported by self-efficacy, beliefs about one's ability to accomplish a task (Bandura, 1986). The context of the learning environment may play a role in influencing students' self-efficacy.

According to Ryan and Deci (2000) *intrinsic motivation* is the "natural inclination toward assimilation, mastery, spontaneous interest, and exploration" (Ryan & Deci, p. 3). Based on empirical studies, Ryan and Deci identify three universal and context-dependent, psychological needs that foster self-motivation: autonomy, competence and relatedness. Ryan and Deci have studied the application of these needs across a variety of domains including in the workplace (Deci & Ryan, 1988), in childrearing, and in managing disease (Williams, Rodin, Ryan, Grolnick & Deci 1988). *Autonomy,* (Grolnick, Deci & Ryan, 1997, Ryan & Deci, 2000) is "the freedom to be self-directed" (Merriam-Webster, accessed December 15, 2011) which according to Ryan and Deci is supported by "choice, acknowledgment of feelings, and opportunities for self-direction" (p.70). Teachers who support autonomy (1) "create opportunities for students to take the initiative during learning activities by building instruction around students' interests, preferences, personal goals…and curiosity" (2) use "noncontrolling informative language" and (3) "acknowledge the students' perspectives and feelings" (Jang, Reeve & Deci, 2010, p. 589, see also; Reeve & Jang, 2006; Patrick, Skinner & Connell, 1993). *Competence*, an individual's belief in their own ability is supported by "…optimal challenges, effectance-promoting feedback and freedom from demeaning evaluations" (Ryan & Deci, 2010, p. 70; see also Wigfield et al., 2006 & Stipek, 2002). *Relatedness* is

"the need to feel belongingness and connectedness with others" (Ryan & Deci, 2010, p. 73). Teachers who support relatedness create a "caring and supportive environment" (Fredricks, Blumenfeld & Paris, 2004, p. 80) in which individuals feel a sense of belonging to the learning community. According to Furrer and Skinner (2003, p. 149), relatedness may promote student engagement: "Feeling special and important to key social partners is hypothesized to trigger energized behavior such as effort, persistence and participation; to promote positive emotions such as interest and enthusiasm; and to dampen negative emotions, such as anxiety and boredom." Furrer and Skinner (2003) suggest children who do not feel a sense of belonging are at greater risk for experiencing negative emotions and subsequently becoming disaffected (see also, Skinner, Furrer, Marchand & Kinderman, 2008). External influences shown to undermine intrinsic motivation may include specific actions or language of teachers such as threats, deadlines and imposed goals dictated by the teacher also may undermine intrinsic motivation (Ryan & Deci, 2000). When students learn to expect "tangible task-contingent rewards", their intrinsic motivation to learn may be diminished (Middleton & Spanias, 1999 p. 69). These external forces impose a shift in the focus from internal desire to master a new skill to an external "locus of causality" (Ryan & Deci, 2000, p. 4; Deci, Koestner & Ryan, 1999) to achieve a certain grade, or reach a deadline. Consequently, intrinsic motivation to learn for the sake of increasing competence may be diminished.

Ryan and Deci (2000) acknowledge that beyond early childhood, most of the activities that individuals do are not intrinsically motivated due to the nature of social responsibilities. Thus, most individuals are motivated extrinsically "in order to attain some separable outcome", (Ryan & Deci, 2000, p. 71). Ryan and Deci (2001) illustrate

the continuum of *self-determination*, showing amotivation, "the state of lacking the intention to act" (p. 72) at the far left. Four types of extrinsic motivation are conceptualized on a continuum from least autonomous to most autonomous: external regulation, introjected regulation, regulation through identification and integrated regulation.

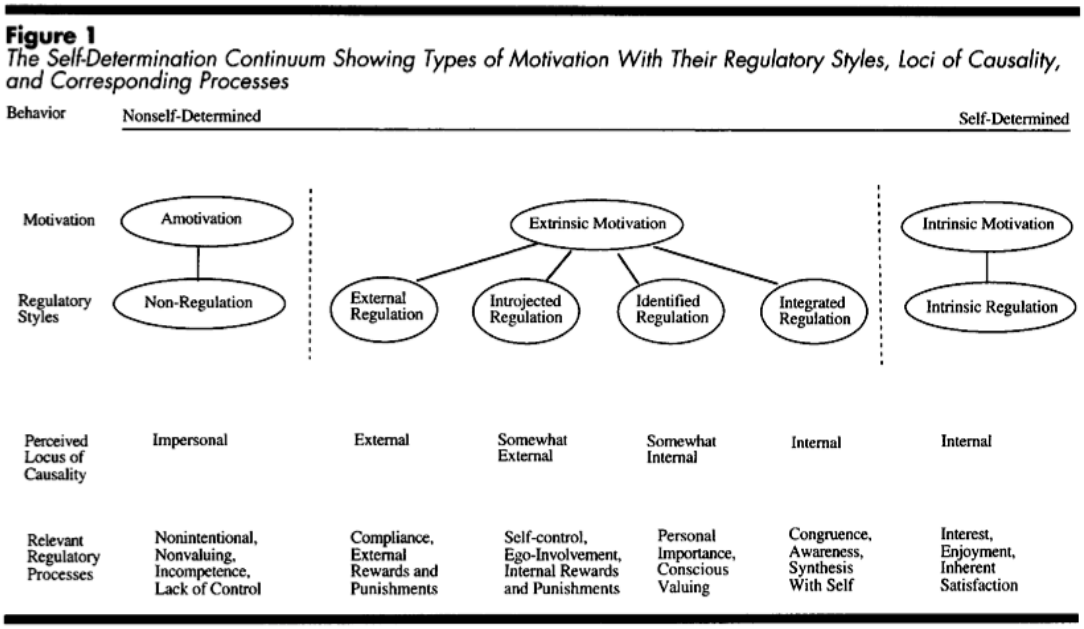

Figure 1: Ryan and Deci's (2000) model of motivation (p. 72)

Individuals motivated through *external regulation* seek to gain external rewards that typically not directly related to success with the task, for example, earning a pizza party for the highest score on a test. *Introjected regulation* is driven by the avoidance of failure and guilt, such as doing homework to avoid the teacher becoming angry that the homework is not complete. *Regulation through identification* is characterized by the value that an individual places on doing the task such as doing homework in order to prepare for an upcoming quiz. *Integrated regulation*, the closest extrinsic classification to intrinsic motivation, is characterized as the most autonomous and "fully assimilated to

the self" such as wanting to understand the assignment in order to get a better grade, go to college and attain a good job. According to Ryan and Deci (2000), environments that satisfy the three basic needs autonomy, relatedness and competence promote this highest level of extrinsic motivation.

Taking the perspective of self-determination theory, Hannula (2006) presents two hypothetical classroom contexts to illustrate how teachers' actions may either undermine or support student motivation.

> "For example, in a teacher-centered mathematics classroom that emphasizes rules and routines and individual drilling, there is little room to meet the students' need for autonomy or social belonging within the context of mathematics learning. More student-centered classrooms with a lot of teamwork going on, and where the emphasis is on meaning making, there may be many opportunities to meet different needs; such approaches, by definition…rely on students exhibiting autonomy in their social interactions." (Hannula, 2006, p. 167)

In the first classroom, the teacher is in control paying little attention to supporting student autonomy, and the nature of "individual drilling" does not appear to support the psychological need of relatedness. In the second classroom, teamwork may support relatedness and autonomy by placing emphasis on the students to interact with the mathematics and with each other. It may be inferred that the teacher who emphasizes the importance of "meaning making" may support competence. This hypothetical example may illustrate how a teacher may support the three psychological needs discussed by Ryan and Deci (2000).

Achievement goal theory is widely accepted in education research as a theoretical basis for understanding individuals' "general focus or purpose for achievement" within a particular domain or situation (Pintrich, Conley & Kempler, 2003, p.321). Two main achievement goal orientations are considered in the literature: mastery orientation and performance orientation (Ames, 1992; Ames & Archer, 1988; Meece, Anderman & Anderman, 2006; Anderman & Midgley, 1997; Middleton and Spanias, 1999; Midgley et al., 1998; Dweck, 2000; Anderman, Austin & Johnson, 2002).

Mastery orientation refers to a goal of self-improvement tendency through the understanding and mastery of tasks, independent from the success of others (Anderman, 2006; Meece, Anderman & Anderman, 2006). Characteristics of mastery goal orientation include willingness to accept a challenge, persistence in the face of difficulty, and judgment of success against one's own past performance (Anderman, 2006). Mastery-goal orientation has been associated with positive outcomes including interest, positive affect, persistence and effort as well as cognitive engagement and productive strategy use (Pintrich, 2000, Ames, 1992). *Performance orientation* refers to a goal of "looking good" in comparison to others by demonstrating ability. Performance-oriented students "striv[e] to be better than others and [use] social comparisons to make judgments of ability and performance" (Meece, Anderman & Anderman, 2006, p. 490). Performance orientation has been generally viewed as less productive than mastery orientation and has been linked to maladaptive outcomes including negative affect (Linnenbrink, 2007). One exception to this view is Dweck and Leggett's (1988) model of performance orientation goals, which assumes that students with high perceptions of their ability to

complete a task actually respond in a similar manner to mastery-oriented students (Pintrich, 2003).

Recently, mastery goal orientation and performance goal orientation have been divided into sub-categories to include approach and avoidance dimensions (Anderman & Wolters, 2006, Pintrich, 2000): mastery-*approach,* mastery-*avoid,* performance-*approach* and performance-*avoid*. Mastery-approach orientation focuses on increasing competence whereas mastery-avoid is characterized by the avoidance of lack of mastery (Wolters, 2004). Performance-approach is categorized by the desire to demonstrate ability whereas performance-avoid is characterized as the desire to avoid showing the lack of ability leading to avoidance strategies (Pintrich, 2000, Turner et al., 2002). Performance-avoid goal orientation has been associated with avoidance behaviors such as avoiding help-seeking, self-handicapping and avoiding novelty (Anderman & Wolters, 2006).

Goal orientations are considered by some to "develop in response to one's schooling experience" (Linnenbrink, 2007, p. 110). One widely held theoretical assumption is that mastery goal orientation is the most beneficial in terms of student outcomes. "The logic of the argument is that when students are focused on trying to learn and understand the material and trying to improve their performance relative to their own past performance, (mastery) orientation will help them maintain their self-efficacy in the face of failure, ward off negative affect such as anxiety, lessen the probability that they will have distracting thoughts, and free up cognitive capacity and allow for more cognitive engagement and achievement" (Pintrich, 2003b). In contrast, students who are distracted by thoughts about doing better than others may experience unproductive

negative affect including anxiety, and thus have a reduced cognitive capacity (see also, Pekrun, 1992).

Goal theory extends to the classroom in that the *goal structure* of a classroom is determined based on the types of instructional practices, tasks, assessment and discourse (Wolters, 2010; Urdan 1997, Urdan, T. & Schoenfelder, E., 2006). Ames and Archer (1988, p. 261) illustrate theoretical comparisons of climate dimensions in mastery-structured vs. performance-structured classrooms. In mastery-goal structured classrooms, success is defined as improvement and progress, value is placed on effort and learning, the teacher places value on how students are learning, and on the process of learning. In these classrooms, "trying hard is important and…all students can be successful if they work hard to learn" (Wolters, 2004; see also, Midgley et al., 1998). In performance-goal structured classrooms, success is defined as high grades, the teacher places value on high ability, and the teacher emphasizes individual student performance and provides extrinsic rewards for demonstrating ability (Wolters, 2004).

2.6 Empirical studies on motivation in education

Ryan and Patrick (2001) studied how students' perceptions of the classroom social environment relate to changes in their motivation and engagement. They hypothesized that specific characteristics of the classroom environment controlled by teacher (teacher support, promoting interaction and promoting mutual respect) may foster student motivation and engagement. They also explored how emphasis on performance goals may affect student self-efficacy and lead to disruptive behavior. They note the importance of a supportive environment in the domain of mathematics where performance is generally attributed to innate ability (Schoenfeld, 1992). "A social

environment in math class that is perceived as supportive, where students are not allowed to tease or ridicule others, where sharing ideas and opinions is fostered and valued, and where teachers do not emphasize students' relative performance is expected to be facilitative of adaptive patterns of motivation and engagement" (Ryan & Patrick, p. 443).

Ryan and Patrick (2001) administered self-report surveys to two hundred thirty three students and fifteen teachers in the Midwestern United States. Forty percent of the students in the study were eligible for free or reduced lunch, forty-five percent of the students were Caucasian and fifty-five percent were African American. Student self-reports were used to measure students' perceptions of particular classroom environment variables: teacher support, promoting interaction, promoting mutual respect and promoting performance goals. Motivation variables included students' perceptions of their own academic efficacy, social efficacy with the teacher and social efficacy with their peers. Students' perception of their own engagement was measured with questions about "self-regulated learning and disruptive behavior in math class."

Results indicated significant positive correlations between students' perceptions of teacher support and the student motivation variables, academic efficacy ($r = .35$) and social efficacy with the teacher ($r=.71$). Students' perceptions of teacher support were less strongly correlated but still significant with social efficacy with peers ($r = .17$). Perceived teacher support correlated positively with the engagement variable self-regulated learning ($r = .44$) and negatively with disruptive behavior ($r = -.41$). Perceptions that the teacher promoted interaction correlated positively with all three indicators of motivation: academic efficacy ($r = .14$), social efficacy with teacher ($r = .47$) and social efficacy with peers ($r= .15$) as well as with the engagement variable self-

regulated learning (r = .25). Ryan and Patrick (2001) also noted that the indication that promoting interaction is beneficial in classrooms "is an important finding, given that teachers may be reluctant to allow students to talk with each other during academic work because of management concerns" (p. 455).

Ryan and Patrick (2001) concluded that students' perceptions of supportive classroom social environments explained motivational and engagement patterns and "...when students believed that their teacher tried to understand them and was available to help, they engaged less in off-task and disruptive behavior in the classroom" (p. 454). They also concluded a link between students' perceptions of the classroom environment, student affect and engagement. "[B]ecause students experience less anxiety in environments that feel respectful, (they) are less likely to have their cognitive engagement undermined." (p. 455).

Wolters (2004) studied the relationships between students' perceptions of classroom goal structures and students' motivation, cognition and achievement in a study of five hundred twenty-five suburban seventh and eighth grade students. Self-report surveys were administered to measure the students' perceptions of classroom goal structures, and to classify the students' personal motivational beliefs as mastery, performance-approach or performance-avoid oriented. Motivational engagement was measured based on students' choice about enrolling in future mathematics course, students' perceived effort, persistence and procrastination in mathematics class. Students' learning strategies were measured in terms of cognitive strategies and meta-cognitive strategies. Students' grades in mathematics class and standardized test scores were used as achievement data.

Students who perceived the classroom goal structure as mastery oriented "tended to report more adaptive motivational engagement and less procrastination (a form of self-handicapping) than students who did not report their classrooms as strongly emphasizing mastery goals" (p. 246). Students' perceptions of mastery structure correlated positively and significantly with motivational engagement variables: choice, (r=.35), effort, (r=.29), persistence, (r=.29) and negative significant correlations with procrastination, (r=-.26). Students' perceptions of performance approach structure, correlations with choice, effort and persistence were not significant but the correlation with procrastination was positive and significant at (r=.16).

Wolters (2004) concluded that these results link classroom goal structures to motivational engagement. "Students who perceived their classroom as having more of an emphasis on learning and improving their ability tended to report more adaptive motivational engagement and less procrastination" (p. 246) as compared to their peers who did not perceive mastery goal structure in their classrooms. The findings indicate that students' perceptions of classroom goal structure were generally aligned with their personal goal orientations. Wolters (2004) noted that the "correlational nature of the present data, however, precludes any causal conclusions about these relations" and suggests the study of "changes in students' goal orientations after exposure to classroom environments with particular goal structures would explicate the causal links between these constructs" (p. 246).

Taking revised goal theory as a framework Pintrich (2000) hypothesized that a combined mastery and performance-approach goal orientation may predict adaptive behavior in one hundred-fifty junior high school students. Pintrich (2000) conceptualized

a 2 x 2 matrix of combined mastery and performance goal orientations (Table 1) and studied how student membership to a particular combined goal orientation may influence motivation, strategy use, affect and performance.

	High performance	Low performance
High mastery	High mastery/high performance	High mastery/low performance
Low mastery	Low mastery/low performance	Low mastery/low performance

Table 1. Goal orientation matrix (Pintrich, 2000)

Pintrich (2000) characterized individual student goal orientations based on student responses to questionnaires adapted from Midgley et al. (1998). A sample item to assess mastery goals is: "In this class, understanding the work is more important to me than the grade I get". A sample to assess approach performance goals is: "In this class, I like to show my teacher that I'm smarter than the other kids" (p. 555). Students were then characterized in four distinct groups according to their goal orientation ratings: high-mastery/low-performance approach (n = 30), high-mastery/high performance (n = 45), low-mastery/high performance (n = 35) and low-mastery/low-performance (n = 40).

Three student general outcomes (motivational beliefs, affect, strategy use) were studied based on student responses to an adapted version of the Motivated Strategies for Learning Questionnaire (Pintrich & DeGroot, 1990) and a fourth outcome, performance was measured using students' semester grades in mathematics. Outcome variables considered were motivational beliefs (self-efficacy, task value and test anxiety), affect (positive or negative), strategy use (self-handicapping, risk taking, cognitive strategy use, meta-cognitive strategy use), and classroom performance (grades in math class). Data

were collected in three waves: once at the beginning of eighth grade (T1), once at the end of eighth grade (T2) and once at the end of ninth grade (T3).

Over time, general and statistically significant declines were reported in self-efficacy and task value for students in both high-mastery groups and in the low-mastery/high-performance group. As Pintrich (2000) noted, these findings are parallel to other studies of (e.g., Eccles et al., 1993, Wigfield et al., 1996) that have demonstrated declines in adaptive outcomes in middle-school aged students. In Pintrich's (2000) study, students in the high-mastery/high-performance group reported the highest self-efficacy, task value and risk taking in each of the three waves and the lowest levels of self-handicapping in each of the three waves. The high-mastery/high-performance group had the highest levels of positive affect at T1 with a sharp decline at T2 but ended up with the highest positive affect at T3. Students in the low-mastery/low-performance group reported the lowest levels of positive affect at T1 and T2 and only slightly higher than the low-mastery/high performance group at T3. Students in the low-mastery/high-performance group reported the lowest levels of self-efficacy and task value and positive affect at T3 and both low-mastery groups reported the two highest levels of self-handicapping at T1, T2 and T3.

When comparing the adaptive inclinations of each goal orientation group, Pintrich (2000) stated the results of his study confirm that mastery goal orientation appears to be associated with some adaptive student outcomes including self-efficacy and positive affect. He notes the "debilitating" patterns exhibited by the low-mastery/high-performance group over time in all outcomes. However, he notes that there is value in adopting a high-mastery/high-performance-approach orientation.

> "[I]f mastery goal students also adopt an approach performance orientation, there seems to be little cost in terms of motivation, affect, cognition or achievement...classroom situations often engender some competition and social comparison, invariably, given their general structure, and if students are focused on 'approaching' the competition and comparison, there do not have to be detrimental effects if they also are oriented to mastery of their schoolwork"
>
> (Pintrich, 2000, p. 553).

Turner, Midgley, Meyer, Gheen, Anderman, Kang and Patrick (2002) hypothesized that there would be a negative correlation between teachers' emphasis on mastery goals and students' use of avoidance strategies in mathematics classrooms. Utilizing student self-reports and classroom observations they conducted a multi-method study to examine conducted the effects of classroom context on student motivation. Student surveys were administered to one thousand ninety-two sixth-grade students in midwestern classrooms (70% Euro-American and 30% African American) to measure (1) students' avoidance patterns and (2) students' perceptions of the classroom goal structure as either mastery or performance.

Trained observers audio taped mathematics instruction and took field notes to report teachers' and students' facial expressions, gestures and/or student behavior. The field notes were later integrated into the transcriptions of the audiotapes. Teacher discourse was then coded according to three major categories: *instructional, organizational* and *motivational*. *Instructional discourse* was coded for supportive (scaffolding) or non-supportive (non-scaffolding) moves. Scaffolding discourse was

coded for instances when the teachers promoted autonomy, held students accountable for learning and transferred responsibility of learning. Non-scaffolding discourse included instances when the teacher gave explicit instructions on how to do a problem or gave a single answer to a problem. *Organizational discourse* was noted when the teacher was either supportive of on-task behavior or when instruction was interrupted to address off-task student behaviors. *Motivational discourse* was coded according to three categories: focus on learning, positive emotions, and peer support and collaboration. *Non-supportive motivational discourse* also was coded according to three categories: focus on errorless performance, impersonal or negative affect, and individual success and failure.

The researchers described teachers' discourse patterns in four of the classrooms which students perceived as either high-avoidance or two low-avoidance. In the two high-avoidance classrooms, the teachers used motivational discourse infrequently, (8% and 6%) and in both classrooms, the teachers placed emphasis on correctness. In the two low-avoidance classrooms, the teachers used motivational discourse more frequently (23% and 20%) and also emphasized mathematical reasoning, asking the students to explain "why" their answers were correct. "Mastery messages" communicated to the students may have conveyed to students that they should not feel ashamed if they do not understand. These teachers demonstrated that "making mistakes and asking questions were natural and necessary parts of learning" (p. 103). Turner et al.'s (2002) study illustrates that in classrooms where teachers promote mastery-learning students exhibit desirable cognitive and affective outcomes. "Our data indicate that teachers conveyed mastery messages to their students in part, through explicit admonitions to students not to feel inadequate or ashamed when they did not understand. By modeling their own

thinking process, mastery-oriented teachers demonstrated that being unsure, learning from mistakes, and asking questions were natural and necessary parts of learning" (p. 103).

After the researchers analyzed the classroom observation data, they examined the results of the student surveys and they found negative and significant correlations between students' perceived emphasis on mastery goals and avoidance strategies: self-handicapping (r=-0.28), avoiding help-seeking (r=-0.29), and avoiding novelty (r=-0.19). Turner et al. (2002) acknowledge that they found no significant findings to link performance goals with avoidance behaviors.

Turner et al. (2002) conclude that their findings highlight the patterns of classroom interactions stating students' perceptions of classroom contexts do influence students' adaptive and maladaptive outcomes. Further, students' perceptions are based on more than one feature of teacher discourse. "…[A]s revealed in the instructional discourse, a mastery environment appears consistent of both cognitive and motivational or affective, components. Instructional discourse patterns that supported students both cognitively and motivationally were characteristic of the low-avoidance/high-mastery classrooms. Conversely, instructional discourse practices that emphasized cognitive aspects such as 'final answers' or sharing reasoning but did not adequately build understanding or overtly address the motivational concerns of students were typical of high-avoidance/low-mastery environments." (p. 103).

Linnenbrink (2005) also studied the relationship between classroom goal structure and student outcomes, yet her study differs from the Turner et al. (2002) study in many ways. First, Linnenbrink (2005) set the classroom goal structures as the independent

variable by assigning a specific goal structure for each teacher to model unlike Turner et al. (2002) who determined the classroom goal structure based on student self-reports and discourse analysis. Second, Linnenbrink examined the approach dimension effects of classroom goal structure and personal goal orientations on twelve student affective/cognitive outcome variables unlike Turner et al. (2002) who focused on avoidance behaviors as outcome variables. Linnenbrink (2005) collected data from two hundred thirty-seven elementary fifth and sixth grade students in ten ethnically diverse classrooms situated in a working class suburb of a midwestern city. Prior to the start of the school year, teachers' instructional styles were determined based on their self-reported responses to the teacher version of Patterns of Adaptive Learning Scales, PALS (Midgley et al., 2000). Aligned with their responses to PALS, each teacher was assigned one of the following goal structures: *mastery goal*, *performance-approach* or *combined mastery/performance-approach*. To illustrate the expected teacher behaviors, the researchers provided specific examples of teaching techniques and/or language that were characteristic of each goal structure. Students' initial goal orientations were also assessed once, at the beginning of the study, using items on the mastery and performance-approach scales from PALS. Because her study focused only on the "approach" dimension of goal orientations, Linnenbrink (2005) did not use performance-avoid scales in the data collection. Student outcome variables were organized into five themes: *motivation* (self-efficacy, confidence in ability to do mathematics, and value of mathematics), *emotional well-being* (positive affect, negative affect, and test anxiety), *help seeking* (adaptive, expedient and avoidant) and *cognitive engagement (*quality and quantity of self-regulation) and academic achievement were measured through self-report

questionnaires administered at the beginning and at the end of the study. Math achievement was measured using the textbook unit math exam, administered pre-study, post-study and as a follow-up assessment five weeks after the end of the math unit.

Positive and statistically significant correlations were evidenced in the results between student personal mastery goal orientation and the following outcomes: (pretest and post-test correlations are reported respectively) self-efficacy (r=.41 and r=.29), interest, (r=.45 and r=.31), utility (r=.21 and r=.24), positive affect (r=.42 and r=.27), adaptive help seeking (r=.28 and r=.23), and self-regulation (r=.56 and r=.43). Negative and statistically significant correlations resulted between student personal mastery goal orientation and the following outcomes: negative affect (r=-.29 and r=-.29), expedient help seeking, (r=-.35 and r=-.32), and avoidant help seeking, (r=-.20 and r=-.24). Correlations also resulted in student personal performance-approach goal orientation and compared to the following outcomes: self-efficacy (r=.18 and r=.10), interest (r=.27 and r=.18) and positive affect (r=.32 and r=.20). Correlations between personal performance-approach goals and maladaptive outcomes, negative affect, expedient help-seeking and avoidant help seeking were not significant. Over time, "students in the performance-approach condition experienced an increase in expedient help-seeking, which is a maladaptive pattern whereas those in mastery and combined conditions (mastery/performance-approach) experienced a decrease in expedient help-seeking, which is an adaptive pattern" (p. 205). Linnenbrink (2005) concludes, "[a]s personal performance-approach goals were maladaptive in the current study, it is important that the environment mainly focuses on mastery and that competition is based on groups competing rather than individual competing" (p. 210).

Danielsen, Wiium, Wilhelmsen and Wold (2010) conducted a large-scale study to examine how students' "perceived teacher support, defined as teacher provision of friendliness and fairness, perceived student autonomy, and perceived classmate support relate to student-self reported academic initiative" (p. 251). The researchers used questionnaire data that was collected in 2005 – 2006 for the Norwegian portion of the World Health Organization survey of Health Behavior in School-aged Children. The sample contained one thousand five hundred ninety-one grade 8 students (13-years-old) selected from each school.

Researchers developed scales to measure student's perceptions of academic initiative, teacher support, classmate support, and student autonomy. Perceived teacher support was measured using two items, "our teachers treat us fairly" and "most of my teachers are friendly" (p. 254). Students' perceptions of autonomy support reflected "the extent to which the students perceive they have an influence in their use of time and activities in lessons, or are provided choices and opportunities for participation" (p. 259). Researchers constructed a latent second-order factor using these two variables to study the combined effect of pedagogical caring and autonomy support (PCAS) on student outcomes. PCAS was related strongly and significantly to academic initiative (r=.86). Strong positive and significant correlations resulted between perceived teacher support and perceived student autonomy (r = .75). Danielsen et al. (2010) conclude that the results of their study "are in accordance with the abundance of research that suggests that teachers are among the most powerful influences on student learning (e.g., Hattie, 2009)". Furthermore, they state that the findings emphasize just how important emotional support in the classroom is for student learning.

Wentzel (1997) examined how "perceived pedagogical caring" may influence student motivation to learn in two hundred forty- eight, predominately Caucasian middle school students in mid-Atlantic region of the United States. Questionnaires were administered in the spring of sixth grade and eighth grade for the purpose of examining possible patterns or changes in motivation over time. Student questionnaires were used to measured students' perceptions of: (1) caring from teachers, (2) psychological distress, (3) control beliefs (4) pursuit of social goals (5) academic effort (6) irresponsible and (7) pro-social behavior. Students also were asked, "How do you know when a teacher cares about you? List three things that teachers do to show you that they care about you", "How do you know when a teacher does not care about you? List three things that teachers do to show they do not care about you" (p. 414). Students' end-of-year cumulative grade point averages were used as a measure of academic achievement. At both data collection points, (sixth grade: T1 and eighth grade, T2) results of Wentzel's (1997) study indicated significant and positive correlations between perceived teacher caring and students' academic effort, (T1, r = .13 and T2, r = .36). Alternatively, students' reports of experiencing distress correlated negatively and significantly with perceived teacher caring (T1 was not reported, T2, r = -.23). Perceived teacher support also correlated positively and significantly with internal control (T2, r = .27), pro-social behavior, (T1, r = .14), pro-social goal pursuit, (T1, r = .20, T2, r = .39). Responsibility goal pursuit correlated positively and significantly with academic effort, (T1, r = .67, T2, r = .59). Wentzel concluded "…the results of the present study provide strong evidence in support of the notion that students are more likely to engage in classroom activities if they feel supported and valued" (p. 417). Wentzel offers specific recommendations of

teacher's behaviors that may foster students' perceptions of supportive teachers include constructs discussed in the motivation literature (i.e., autonomy support and relatedness). These specific behaviors are studied further in other research in terms of teacher's provision of autonomy support and provision of structure (Jang, Reeve & Deci, 2010).

Discussion

"[I]ntrinsic motivation to learn within each individual student is significantly influenced by the culture of the mathematics classroom", which is "dependent on the participants in the classroom – the teacher and the students" (Grouws & Lembke, 1996, p. 39). The theoretical perspectives reviewed here suggest that through their language and actions, teachers influence student motivational outcomes. Offering students' choice, encouraging discourse and selecting appropriate tasks to are suggested to support student motivation. Ames (1992) suggests that one way to support students' perception of control is to offer choices about strategy and pacing while supporting students' development of self-management. Teachers who convey the importance of mastery through an emphasis on learning for understanding may influence goal orientations of their students. The studies reviewed above reveal that students have in fact been influenced by the motivational orientations of their teachers (Ryan & Patrick, 2001, Danielsen, 2010, Wolters, 2004).

> "[T]eachers help to construct the classroom social environment by creating norms and rules for student social behavior in the classroom and giving explicit messages regarding students' interactions with their classmates. The types of academic tasks teachers assign can encourage or

> dissuade cooperation and sharing of expertise. Furthermore the types of participation structures they establish and the way they publicly recognize students contribute to the social environment" (Ryan & Patrick, 2001, p. 438).

Anderman and Wolters (2006) describe three important constructs of motivation: goals, values and affect and they point out that although these three constructs are studied separately throughout the literature, they are interrelated. The research reviewed here suggests that students' motivations are mediated by their emotions. "Because the belief that occasional failure is acceptable in learning mathematics predicts mathematical self-confidence, the practice of allowing children to struggle with challenging problems, even in the elementary grades, is supported" (Middleton & Spanias, 1999, p. 70). These studies highlight the important role that teachers play in students' motivation to engage in learning through offering affective and cognitive support and by emphasizing mathematical understanding and effort. Turner et al.'s (2002) study may have particular utility for because they presented specific teacher language and behaviors in three categories: organizational, instructional and motivational. The analysis of specific teacher language and behaviors in the context of student engagement may inform practicing teachers, teacher educators and for education researchers on which types of interactions may contribute to student engagement or disaffection. Teachers who can skillfully use instructional and motivational discourse may potentially create environments where students motivated to engage in mathematics.

2.7 Theoretical perspectives on student engagement

Student engagement has been conceptualized as a psychological process that interacts dynamically with motivation (Ames, 1992; Ryan & Deci, 2001; Pekrun et al., 2007; Linnenbrink, 2007 and Skinner, Furrer, Marchand & Kinderman, 2008) by which students attend to, show interest in and put forth effort toward academics (Newman, 1992; Marks, 2000). Engagement in academic work has been predictive of academic achievement in many research studies (e.g., Klem & Connell, 2004 and references therein) and disengagement from academic work is associated with higher rates of dropping out in high school students (National Research Council, 2004). Disengagement is particularly acute for "at risk" children living in high-poverty urban areas (Wigfield, Eccles, Schiefele, Roeser & Davis-Kean, 2006). Facilitators of student engagement include teacher support (Skinner et al., 2008), high expectations and sense of community or belonging in schools and classrooms (National Research Council, 2004).

According to Newmann (1992), student engagement in academic work is the "psychological investment in effort directed toward learning, understanding, or mastering the knowledge, skills, or crafts that academic work is intended to promote" (p. 12). Newmann (1992) acknowledges, however, that this definition of engagement does not specify the depth of learning, the utility of academic tasks, nor does it specify the level of thinking required by students to be engaged with academic work. According to Newmann (1992) schoolwork is often "meaningless", "mechanistic" and "trivial" and rarely engages students to think critically. (p. 13) Newmann (1992) posits that these characteristics contribute to student disengagement from academic work. "Based on the assumption that all humans share a fundamental need to develop competence, educators

can enhance student engagement in academic work by attention to two general factors: building a sense of student membership in the school at large, and designing academic tasks to maximize the authenticity of schoolwork" (p. 33). Newmann (1992) maintains high expectations that demonstrate "clear goals, support, equity and caring" coupled with meaningful academic tasks are critical to "enhance intrinsic interest" and promote student engagement (p. 33).

In Finn's participation-identification model (Finn, 1989), student engagement is comprised of four levels of student *participation* and two levels of identification. The four levels of participation are: (1) attending class, paying attention and doing assigned work, (2) enthusiasm toward academic work including asking questions and seeking help from the teacher (3) participation in extra-curricular activities, and (4) participation in "academic goal-setting and decision making" (Finn, 1989, p. 129). Finn's (1989) *identification* construct refers to students' (1) sense of belonging in the classroom and wider school context, and (2) the value a student places on academic work and academic success. According to this perspective, participation and identification function in a cyclical process. "[P]articipation leads to academic success, which, in turn, influences identification with school (i.e., the affective dimension of engagement, such as valuing, belonging). Identification increases the likelihood of engagement" (Marks, 2000, p. 158). In this review, I attend to perspectives and research that pertain to the types of behaviors identified in the first two levels of Finn's participation construct because of the relevance to the "in the moment" types of engagement that take place during teacher/student(s) interactions. Both Newmann's (1992) and Finn's (1989) perspectives of student engagement include cognitive, behavioral and/or emotional components.

More recent literature distinguishes student engagement into three constructs: behavioral engagement, emotional engagement and cognitive engagement (Fredricks, Blumenfeld & Paris, 2004; Klem &Connell, 2004). Fredricks et al. (2004) describe student engagement as a "multifaceted" meta-construct made up of behavioral engagement, emotional engagement and cognitive engagement, which interact within the individual. According to Fredricks et al. (2004), *behavioral engagement* is: (1) involvement with school sports or clubs, (2) school attendance and (3) "involvement in learning". This definition is similar to the first three levels of Finn's (1993) participation model. Behavioral engagement has been measured using self-report surveys such as the Rochester School Assessment Package (Wellborn & Connell, 1987) and through observational measures (e.g. Patrick, Anderman, Ryan, Edelin & Midgley, 2001). *Emotional engagement* pertains to the feelings that students experience including curiosity about how to solve a problem, anger about a disagreement with a classmate, or excitement about a good grade on a test (Fredricks et al., 2004). Students' school-related emotions including specific task-based emotions and their general feelings about school have been studied using self-reports including the Rochester School Assessment Package. According to Fredricks et al., (2004) there are limitations to using questionnaires when studying emotional engagement for example "survey items do not specify the source of emotions" (p. 67). Researchers have suggested the use of qualitative methods such as classroom observation and interviews to augment the quantitative findings (Hughes, Wu & West, 2011; Fredricks, et al., 2004; Skinner et al., 2008). Inferences about emotional engagement may be made based on observations of body language, gestures, facial expressions or tone of voice (Goldin, Epstein, Schorr & Warner, 2011). *Cognitive*

engagement refers to the strategies that students use to learn in terms of how students organize new information and flexibility in problem solving. Fredricks et al., (2004) state that cognitive engagement is difficult to measure because it "is not readily observable, it must be either inferred from behavior or assessed from self-report measures" (p. 68). Inferences about cognitive engagement based on observation of behavior may not be a reliable method for interpreting the level of strategies being used. Fredricks et al. (2004) note that although self-reports have been widely used to measure student engagement, there are many limitations to these measures. First, teacher questionnaires may combine different aspects of behavior on the same scale. For example, combining "conduct, persistence and participation" on the same scale may take the assumption that students who are well-behaved are focused on the academic work and participate. Second, questionnaires are rarely task-specific or domain-specific and according to Fredricks et al. (2004), this problematic because student engagement may vary depending on the context of the subject and/or the task. Third, questionnaires are disconnected from the actual learning process, typically administered prior to or subsequent to active learning.

Fredricks et al. (2004) suggest that in the study of engagement, researchers should "unite" behavioral, emotional and cognitive components "in a meaningful way" (p. 60). This perspective may "provide a richer characterization of children than is possible in research on single components… examining antecedents and consequences of behavior, emotion and cognition simultaneously and dynamically, to test for additive or interactive effects" (p. 61). Finally, the consideration of engagement as the individual interacting with contextual features in the classroom "…promises to help us to better understand the

complexity of children's experiences in school and to design more specifically targeted and nuanced interventions" (Fredricks, Blumenfeld & Paris, 2004, p.61).

In an attempt to study the dynamics of engagement with motivation, Skinner et al., (2008) offer a model that distinguishes *indicators* from *facilitators* of engagement. Indicators of engagement are taken as processes that are internal to the individual and are either emotional or behavioral in nature. Facilitators of engagement are considered external to the individual including such features as teacher support and classroom context. Skinner et al. (2008) conceptualize indicators of student engagement in two broad categories, behavioral and emotional engagement, each with two subcategories engagement or disaffection. Indicators of behavioral engagement include: action initiation, effort, exertion, persistence, intensity, concentration, absorption and involvement. Indicators of behavioral disaffection are: passivity, giving up, withdrawal, inattentive, distracted, mentally disengaged and unprepared. Indicators of emotional engagement are: enthusiasm, satisfaction, pride, vitality and zest. Indicators of emotional disaffection are: boredom, frustration/anger, sadness, worry/anxiety, shame and self-blame.

Goldin, Epstein, Schorr and Warner (2011) propose a tool for studying complex patterns of student behavioral, emotional and cognitive engagement. They present the concept of "engagement structures" defined as "idealized, hypothetical constructs, analogous in many ways to cognitive structures. Goldin et al. (2011) describe complex "in the moment" affective and social interactions as students work on conceptually challenging mathematics" (p. 547). Each engagement structure is comprised of ten components, which are "simultaneously present and dynamically interacting" in a

"behavioral/affective/social constellation" (p. 549). In Goldin et al.'s (2011) view, engagement is activated by (1) "a goal or motivating desire" which manifest in (2) "patterns of behavior including social interactions oriented toward fulfilling the desire". (3) "Affective pathways" (Goldin, 2000) are also activated by the motivating desire and represent the interaction between emotions and cognition, while (4) "expressions from which affect can be inferred" may help to interpret the "reasons for particular behavioral patterns". (6) Meta-affect refers to the "meanings" that individuals associate with their emotions and Goldin et al. (2004) maintain that these meanings are "highly context dependent" (p. 551). In response to emotions, motivations and beliefs, students engage in (7) "self-talk" which may facilitate activation of a particular engagement structure. For example, a student who feels happy about her progress may think, "I really 'get' this problem" and subsequently engage in the "Let me teach you" structure. (8) Beliefs and values are considered critical to engagement structures because they are thought to influence students' appraisals and achievement emotions (Pekrun et al., 2007). For example, in the face of difficulty a student who believes in her own capabilities and resources may appraise impasse during difficult problems as activating. In contrast, a student who believes that the teacher holds the "key" to her success may disengage from the problem in the face of impasse. Similar to beliefs, students' (9) motivational orientations, personality traits and self-identity are thought to "influence the activation and continued in-the-moment influence of an engagement structure". Finally, Goldin et al. (2011) posit that mathematical "strategies and heuristics" interact with the strands and depending on students' motivations, beliefs, and emotions students may choose different types of strategies (e.g. exploratory vs. procedural) when solving mathematical tasks.

Goldin et al. (2011) suggest that these engagement structures can be used to examine the complex patterns that are characterized by different behavioral, cognitive or affective behaviors.

Goldin et al. (2011) present a preliminary list of specific engagement structures and I will discuss three examples here. "Get the job done" is motivated by the desire to complete the task with little investment in deep mathematical learning. In this structure, behavior is focused on getting the work done, possibly soliciting cooperative group members to help work toward task completion. "I'm really into this" is motivated by the students' intrigue about the task itself and could represent features of mastery goal orientation (e.g., Ames, 1992). Goldin et al. (2011) do not suggest specific behaviors, but examples might include, ignoring the bell that indicates the end of class, talking about the problem solving process with peers or the teacher. The engagement structure "It's not fair" is motivated by the desire to address students' perceived unfairness when students are working in a group. In this structure, involvement with the mathematics becomes secondary to "restoring fairness" within the social group. Goldin et al. (2011) suggest that the engagement structures have theoretical and empirical utility as well as practical value for considering, researching addressing affective issues in classrooms.

2.8 Empirical studies on student engagement

Stipek (2002) asks, "Why might students be more engaged in classrooms characterized by respect and social support? Perhaps students who feel supported and cared for feel less apprehensive about making mistakes; they do not need to spend their time trying to avoid looking incompetent." (p. 327). The studies reviewed in this section

highlight how teachers' actions and students' perceptions of teacher support interact with students' emotional, behavioral and/or cognitive engagement in classroom settings.

Patrick, Ryan and Kaplan (2007) hypothesized students' perceptions of teacher's emotional and academic support may facilitate students' behavioral and cognitive engagement. They tested this hypothesis in a study of six hundred two fifth graders in five schools situated in middle-class neighborhoods. Patrick et al. (2007) administered surveys to measure students' perceptions of the classroom environment, students' self-perceptions of their engagement, and students' motivational beliefs (PALS, Midgley et al., 1996). The surveys were administered once at the end of fourth grade and once at then end of fifth grade. Students' final report card grades in mathematics from fourth and fifth grade were used as achievement data. Students' perceptions of the social environment were measured with questions about teacher emotional support, peer emotional support, promoting mutual respect and task-related interaction. Cognitive engagement was measured with self-reports of self-regulation strategies, planning, monitoring and regulating cognition and behavioral engagement was measured based on perceptions of task-related interaction. Student motivation was measured using items from PALS (Midgley et al., 1996). Findings evidenced students' perceptions of teacher emotional support were correlated positively and significantly with measures of cognitive engagement (self-regulation strategies, $r = .53$), and behavioral engagement (task-related interaction, $r = .49$). Patrick et al. (2007) state that their findings are "strong evidence that the classroom social environment is important to student engagement.

Skinner, Furrer, Marchand and Kinderman (2008) studied the influence of teacher support of autonomy, relatedness and competence (Ryan & Deci, 2000) on student

behavioral and emotional engagement. The researchers purposefully chose children in the fourth, fifth, sixth and seventh grades because of the motivational decline that often takes place in the transition from elementary school to middle school (e.g., Eccles et al., 1993). Data was collected from eight hundred five late elementary and middle school students in rural-suburban upstate New York. The study was informed by self-determination theory and is depicted using the model in Figure 2. In this view, the teacher's actions are considered as a primary influence of student engagement and learning outcomes.

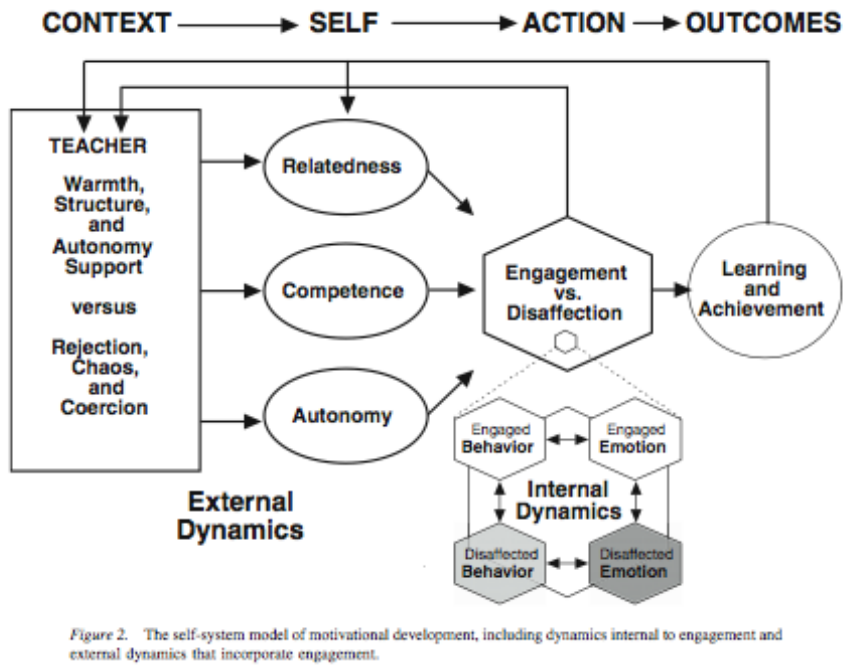

Figure 2: Skinner et al.'s (2008) theoretical model of motivational development

Researchers administered student self-reports once in the Fall and once in the Spring to gather data on students' perceived behavioral engagement (effort, attention, and persistence) or behavioral disaffection (lack of effort and withdrawal), students' perceptions of competence, control, academic autonomy and relatedness to their teacher. Teacher support was measured by student self-reports and teacher self-reports. Teachers

reported on their perceptions of the level of involvement, structure and autonomy support they provided for each of the students as individuals.

Results indicated that teacher support influences student engagement and disaffection. Student reports of teacher support, correlated positively and significantly with behavioral engagement in the fall and spring respectively (r = .48 and r = .54), and emotional engagement (r = .61 and r = .60), and negatively with behavioral disaffection (r = -.44 and - .50), and emotional disaffection (-.52 and -.51). "As expected, student reports of teacher's support predicted improvements in emotional and behavioral engagement and declines in behavioral and emotional disaffection over time" (Skinner et al, 2008, p. 774). Results indicated declines from Fall to Spring of the disaffected emotions, boredom and frustration and the researchers interpreted this as a cause and effect type relationship between teacher support and student emotional engagement. It is difficult to conclude specific teacher behaviors that may have led to this development and the investigators conclude that this is a limitation due to the quantitative nature of the study. Also, the data were collected at only two time periods, once at the beginning and once at the end of the school year. "It is more likely, for example, that the effects of student-teacher interactions on children's subsequent engagement play out over a period of weeks or months" (Skinner et al., 2008, p. 778). The researchers concluded that emotional engagement "is the strongest contributor to the …internal dynamics of engagement, bolstering behavioral engagement and staving off behavioral disaffection. (p. 778).

Jang, Reeve and Deci (2010) studied relationships between students' classroom engagement, teacher autonomy support and teacher structure in a study of one-hundred

thirty three teachers in Midwestern high schools. Trained raters using two separate seven-point scales assessed teachers' autonomy support and teachers' structure. According to Jang et al., (2010) autonomy supportive teachers "create opportunities for students to take the initiative during learning activities by building instruction around students' interests, preferences, personal goals, choice making…" (p. 589). Indicators of autonomy support were organized into three themes, which included specific indicators of teacher behaviors on both ends of each scale: (1) "relies on extrinsic sources of motivation" vs. "nurtures inner motivational resources", (2) uses "controlling language" vs. uses "informational language" and (3) "counters & tries to change students' negative affect" vs. "acknowledges & accepts students' negative affect". (p. 592). According to Jang et al., (2010) teachers who provide structure set out clear expectations about "desired educational outcomes" (p. 589 and references therein). Indicators of teachers' structure were organized into three themes: (1) "absent, unclear, ambiguous, confusing directions" vs. "clear, understandable, explicit, detailed directions", (2) "weak guidance" vs. "strong guidance" and (3) feedback that is ambiguous (or lack of feedback) vs. feedback that helps to build skills, or is instructive (p. 593). The raters also visited classrooms to observe and to rate one thousand, five hundred eighty two students' collective behavioral engagement during instruction. Six behaviors were rated including "attention, effort, verbal participation, persistence, positive emotion, and voice" (p.593). Student questionnaires were used to assess individual students' perceptions of their own engagement during the same lesson that was observed.

Results of Jang et al.'s (2010) indicate positive and significant correlations between teacher autonomy support and teacher structure, r (133) = .60; between

autonomy support and students' collective behavioral engagement, r (1582) = .76; between autonomy support and student self-reported engagement, r (1582) = .36; and between teacher structure and students' collective behavioral engagement, r (1582) = .30. Jang and colleagues (2010) state that their findings have "implications for teachers wrestling with the daily goal of supporting students' engagement during learning activities" (p.597). They conclude that teachers strike a balance between structure and autonomy support. "[T]eachers seeking engagement-fostering instructional strategies need not choose between providing autonomy support or structure but, instead, can focus their instructional energies on providing autonomy support and structure" (p. 597). Jang et al.'s (2010) study is distinctive in that is an analysis of multiple sources of data: on teacher behavior, student perceptions of teacher support, observations of teacher behavior, and student engagement (self-reported and observed).

Skinner and Belmont (1993) researched the effects of three types of teacher behavior; involvement, structure and autonomy support on their students' behavioral and emotional engagement. The study was conducted over the course of one school year with one hundred forty-four students and fourteen teachers in grades three through five. Teachers reported on their perceptions of student engagement and students reported on their own behavior and emotions in the classroom. When the students reported that they perceived the teacher to be kind and friendly, they reported feeling happy (emotional engagement). When students perceived the teacher as providing clear expectations and "strategic help", they tended to show greater levels of persistence on academic task. The researchers concluded that there was a correlation between teacher behavior and students' engagement.

Skinner and Belmont (1993) found a reciprocal relationship between student engagement and teacher engagement. The students who were perceived to show the most interest and enthusiasm for learning also were the ones who received the most attention and autonomy support from the teacher. Although the term "strategic help" was not specifically defined, one may assume that this term refers to help that would be strategic in directing the student in a productive path towards solving a problem or completing a task.

> " [A] social environment in math class that is perceived as supportive, where students are not allowed to tease or ridicule others, where sharing ideas and opinions is fostered and valued and where teachers do not emphasize students' relative performance is expected to be facilitative of adaptive patterns of motivation and engagement"
>
> (Ryan & Patrick, 2001, p. 443).

Finn (1993) conducted a large-scale study of the engagement of five thousand forty-five eighth graders who were identified as meeting the criteria for "at-risk" students. The subjects met three criteria to be considered "at-risk", (1) urban minority, (2) language minority and (3) low socio-economic status. Surveys were administered to students, parents, teachers and school administrators and student achievement data was collected from students' standardized test scores in Reading and Mathematics. Results support Finn's (1989) participation model that "being an active participant in the classroom…is a particularly important antecedent of school performance even among

high-risk students" (p. 61). Although the results indicated students' perceptions of "school warmth" was not significantly related to students' academic performance.

Discussion

"When students engage in classroom learning, there is almost always some aspect of the teacher's behavior that plays a role in the initiation and regulation of the engagement." (Jang, Reeve & Deci, 2010, p. 588)

Newmann's (1992) theoretical perspective, engagement a psychological construct that is mediated the rigor of academic work and by support that students receive (or perceive) from teachers. This perspective assumes that teachers who have high expectations and communicate this to their students in a caring manner will foster their students' engagement. Finn (1989) takes engagement as a cyclical relationship between students' participation in school activities with a sense of belonging and value (identification). In this perspective, students who actively participate in academic work will likely have academic success, which will lead them to a sense of belonging and value of academic work, which in turn will lead to sustained engagement.

Fredricks et al.'s (2004) perspective that student engagement is "multi-faceted" suggests the importance of studying student engagement in context as student engagement is likely influenced by social, emotional and cognitive aspects of the classroom (i.e., peer/teacher relationships, level of task difficulty). Goldin et al.'s (2011) engagement structures model considers contextual features as part of their "constellation" of behavioral/affective/social components. The engagement structure model may have particular utility for researchers to examine the dynamic interplay between "in the moment" student engagement and specific teacher language and actions.

The research reviewed here suggests that teacher actions and students' perceptions of teacher support interact with students' behavioral and cognitive engagement in mathematics classrooms. Skinner et al. (2008) concluded that students' perceptions of teacher support are predictive of students' behavioral engagement. Jang et al. (2010) observed and rated specific teacher behaviors against student behavioral engagement and found that autonomy supportive teacher behaviors fostered student behavioral engagement.

2.9 Teaching and learning in urban settings

According to data from the National Assessment of Educational progress, (National Center for Educational Statistics, 2009, 2011), students in both grades four and eight across income levels and racial/ethnic groups have shown gains in achievement in mathematics and reading from 1978 through 2011. This may seem like good news. Until we take a closer look at the disaggregated data for income levels and race/ethnicity. The gaps between White students and Black students, White students and Hispanic students, non-poor students and poor students have maintained approximately the same levels during this twenty-one year period. In 1990, the average scale score on the NAEP in mathematics was 270 for White eighth graders and 237 for Black students. In 2011, the average scale score on the NAEP in mathematics was 293 for White students and 262 for Black students. So, both subgroups improved achievement overall, the score gap between the two groups only narrowed by two points over the course of eleven years. NAEP data indicates that eighth graders who achieve below the twenty-fifth percentile in mathematics are somewhat equally distributed: thirty-three percent White, twenty-eight percent Black and thirty-two percent Hispanic. However, when looking at the "top

achievers", the statistics presented in the Nation's Report Card (NCES, 2011) demonstrate that White students are outperforming Black students in mathematics. Of students who achieved above the seventy-fifth percentile in mathematics, *seventy-two percent* are White, *five percent* are Black and *eleven percent* are Hispanic. And, in the group of lowest performing students, those eighth graders who achieved below the twenty-fifth percentile in mathematics, sixty-eight percent were living in poverty. For eighth graders who achieved above the seventy-fifth percentile in mathematics only twenty-percent were living in poverty. The No Child Left Behind Act (2002) was intended to focus on underserved children including low achieving students living in poverty.

These numbers prove to satisfy the "gap gazing" (Gutierrez, 2007) approach to research in education clearly documenting the disparities in achievement between White students and Black students, non-poor students and poor students. Yet according to Gutierrez (2007), "this approach points out there is a problem, but fails to offer a solution" (p. 13). Using achievement data as a lens for analysis disregards the context in which learning takes place and "it ignores successful contexts serving marginalized students that have been documented in the literature" (Gutierrez, p. 13).

"No one can go back and start a new beginning, but anyone can start today and make a new ending" (McLeod, 2011).

What do we do with all of this data? Researchers have attributed the causes of achievement gaps to lower funding pushed to urban schools (McLeod, 2011), lack of resources including qualified teachers (Flores, 2007, Peske & Haycock, 2006), high levels of teacher attrition (Luekens, Lyter & Fox, 2004). Thirty-seven of forty-eight

states provide less funding for school with ethnically diverse populations (McLeod, 2011). According to the Education Trust, students in schools serving the poor and minorities, students are held to a lower standard and high poverty districts employ twice as many teachers who are not certified in their disciplines as compared to more affluent districts. Furthermore, even students who enroll in "higher level" courses are exposed to work that is less rigorous as compared to the work in suburban schools.

The National Council of Teachers of Mathematics maintains that mathematics should be accessible to all. "All students, regardless of their personal characteristics, backgrounds or physical challenges, can learn mathematics when they have access to high-quality mathematical instruction" (NCTM, Executive Summary, p. 2).

Gutierrez (2007) asserts "context matters" when attending to issues of equity in mathematics education. By "equity", Gutierrez (2007) "means fairness not sameness". She suggests four necessary dimensions of equity: access, achievement, identity and power. Access to "quality mathematics teachers…a rigorous curriculum…(and)…a classroom environment that invites participation" (p. 2) is one aspect of access to equitable contexts. According to Gutierrez (2007), attending to students' identities promotes equity by giving students "opportunities to see themselves in the curriculum (mirror) as well as have a view onto a broader world (window)" (p.3). The power dimension of equity considers social issues in the classroom including opportunities for the students to express their voice.

Gutierrez (2007) conducted a study of nine U.S. high schools with a large proportion of minority and/or working class students. She identified successful schools based on achievement data and on course taking data. Successful schools had higher

numbers of students taking more than the districts' minimum requirements in mathematics. Gutierrez (2007) used qualitative methods including teacher questionnaires and teacher interviews to examine distinguishing characteristics of "effective math departments". Four common characteristics emerged in the schools where students showed signs of success, three of which could potentially be directly controlled by the teachers. First, successful schools implemented curricula that was rigorous and offered few low-level mathematics courses. The teachers in the successful high schools worked collaboratively and relied on their colleagues for professional development. Third, the teachers in the successful schools demonstrated commitment to their students by holding them accountable to high standards and these teachers also reported that they felt a responsibility to motivate their students to want to learn. Finally, the teachers in successful schools used innovative instructional practices to cover the curriculum, including making "mathematics relevant to students' lives…(and)…by offering choices of topics for larger projects" (p. 6).

Social and affective conditions in the context of the urban classroom have an impact on teaching and learning because there is often disconnect between the social expectations of the students on the street and the social expectations of the students in the classroom (Anderson, 1999; Dance, 2002). Students in urban areas often must adopt "street-savvy" postures in order to appear tough in social situations and they ultimately use these postures as defense mechanisms for survival on the street. The consequences of being wrong or appearing weak in front of peers in the urban classroom may be potentially dangerous (Anderson, 1999). In her book, *Toughfronts: The Impact of Street Culture on Schooling*, Dance (2002) describes the dilemma that students face when going

from the street to the classroom. As students develop a posture of hardness, maintaining a tough façade, their overt actions and defense influence become internalized. "As students become hard, they convince themselves that they cannot care about others, because others, including mainstream teachers" (Dance, 2002, p.57). When considering this perspective with Ryan and Deci's (2000) model of motivation, which takes relatedness as basic psychological need, these defense mechanisms may interfere with student motivation. Teachers in urban classrooms face particular challenges when seeking to earn student respect. Often, students do not trust teachers because the teachers represent the mainstream culture that the students may distrust. (Gomez-Chacon, 2000; Anderson, 1999; Dance, 2002). These challenges *need not be* barriers to students engaging in meaningful mathematics. We know that urban students are capable of successfully engaging in conceptually challenging mathematical tasks (Schorr, Warner, Gearhart & Samuels, 2007).

Delpit and White-Bradely (2003) describe their experiences teaching low-income students in urban schools where "packaged" reading and classroom management programs are mandated. "In schools serving poor and African American children there is typically little or no focus on developing the humanity, the integrity or the thinking and leadership capacity of the children served or the teachers who try to serve them. They are rote, robotized, and ruled by outside forces" (Delpit & White-Bradley, 2003, p. 286). One specific example that is personally relevant is their description of a management technique called the "go-around-cup". This is a cup managed by the teacher, containing craft sticks with each student's name written on one stick. The relevance to my life is that my daughter's kindergarten teacher has a cup of "magic sticks", that serves the same

purpose as the "go-around-cup". My daughter attends a middle-class school with a student body of three hundred fifty students and two are African American. In both Delpit's example and in my daughter's classroom, the sticks are used in the same way: when the teacher has a series of questions to ask, s/he picks a stick from the cup and calls on the student whose name is written on the stick. One perspective of the utility of the sticks is that it is an equitable way to manage student participation so that every student has an opportunity to speak. However, Delpit and White-Bradley (2003) offer a different perspective, suggesting that this management technique may prevent students who are excited to share their ideas from being chosen to share when the idea is "fresh". They offer a simple change in strategy in which each student holds his or her stick and chooses when he/she wants to use it. This method still promotes equity, but gives students control and supports student autonomy by offering students a choice of when to participate (Ames, 1992; Ryan & Deci, 2000).

Little research has been done to examine the impact of student-teacher relationships on academic outcomes (Decker, Dona and Christenson, 2007). Decker et al. (2007) conducted a study, which considered the perspectives of the teacher and the student with regard to the teacher-student relationship in a study of forty-four African-American students and twenty-five teachers. The students and the teachers ranged from kindergarten through sixth grade with the majority of the subjects concentrated at the kindergarten level. The students in the study had been identified by their teachers as "behaviorally at-risk" based on their exhibition of behaviors including fighting, swearing, crying, pouting bothering others, difficulty controlling anger, talking back to adults and being hyper" (p. 88). The researchers explored the following questions: "Is the quality of

the student-teacher relationship predictive of students' social, behavioral, engagement and academic outcomes? If so, for which outcomes is the relationship most important?" (p. 87). The investigators used questionnaires to measure teacher perceptions (Student-Teacher Relationship Scale; Pianta, 2001) and student perceptions (Relatedness Scale, Wellborn & Connell, 1987) of student-teacher relationships. The researchers measured social and emotional functioning using a teacher report and a student report of the social skills rating system developed by Gresham and Elliot (1990). Student behavioral and emotional engagement was measured using teacher-reports, student reports and by using observations by trained research assistants. Finally, students' academic performance was measured based on teachers' perceptions of students' academic performance as well as oral fluency and literary skills tests administered by research assistants.

The findings of Decker et al.'s (2007) study indicate students' perceptions of emotional quality correlated positively and significantly with students' perceptions of engagement (r =.38). The researchers reported when students in their study perceived "positive emotional quality in the student-teacher relationship, the amount of behavior referrals they received decreased and the amount of time they spent on-task increased" (p. 104). Decker et al. (2007) conclude that students' perspectives were related to behavioral outcomes and suggest that school psychologists may act as mediators in the student-teacher relationship. The researchers do not offer specific recommendations for what types of interventions would foster positive emotional quality between students and their teachers.

Jo Boaler and Megan Staples (2008) report on the academic success of students in one urban, ethnically diverse school. "Railside" was one of three California schools

involved in a five-year longitudinal study of seven hundred students. Each of the schools served a different demographic; one urban and ethnically diverse, one coastal and primarily White, and one rural and primarily White/Latino/a students. At the end of the second year of the longitudinal study, the researchers became aware that student achievement at "Railside" on study-designed tests, district tests and California Standards tests had improved significantly as compared to student achievement at the other two schools. This trend continued for the next two consecutive years and by the end of the fourth year of the study, achievement gaps that were noted at the beginning of the study were completely eliminated. The researchers set out to examine what conditions may have been in place when these gains were made.

Videotaped interviews with the teachers and classroom observations were collected. In the analysis phase, of teacher moves and student engagement in the videotaped classroom observations were studied. A quantitative analysis was conducted to examine how teachers used the time during class including teacher talking, teacher questioning. The researchers analyzed the types of questions that teachers posed to the students and categorized the questions as, *probing*, *extending* or *orienting.*

2.10 Teacher/student relationships

I conclude with a discussion of teacher/student relationships while drawing on the perspectives on affect, motivation and engagement discussed earlier in this chapter. As discussed earlier Ryan and Deci (2000) present relatedness as one of the three basic psychological needs. According to Martin and Dowson (2009), relatedness influences students' beliefs when then directs students' behaviors. "In the academic context, for example, good relationships with a particular teacher are likely to lead students to

internalize at least some of that teacher's beliefs and values about schoolwork" (p. 330). "Relatedness in the academic domain teaches students the beliefs, orientations, and values needed to function effectively in academic environments. In turn, these beliefs (if positive and adaptive) direct behavior in the form of enhanced persistence, goal striving, and self-regulation" (p. 329). Martin and Dowson (2009) point out inconsistencies in the goal theory literature with respect to the conflicting comparisons of performance orientation outcomes and mastery orientation outcomes. They suggest that relatedness may be a "mediating process" that can help to explain these inconsistencies in the research. "In performance-oriented environments where students experience positive relationships, these environments may be perceived by students as being supportive in the path to achievement…. On the other hand, a performance-oriented environment in the context of poor relationships may be perceived as a 'dog-eat-dog' context rather than a supportive one" (p. 331). Martin and Dowson (2009) suggest that integrating relatedness in the classroom may be most beneficial to students at-risk including African American students (they cite Ladson-Billings, 1995). Martin and Dowson (2009) propose "connective instruction" as a bi-directional multi-tiered approach to support relatedness as means to student motivation, engagement and achievement. The authors take connective instruction as "bi-directional" because it may impact both students and teachers by giving teachers flexibility to adjust their teaching in response to their students' reactions. *Substantive connectiveness,* based on expectancy-value theory and mastery goal-orientation, promotes students' interest in content through engaging and relevant learning tasks. Characteristics of *interpersonal connectiveness* include "actively listening to students' views, allowing students to have input into decisions that affect

them, getting to know students, showing no favoritism but affirming all students, accepting students' individuality and having positive but attainable expectations for students" (p. 345). *Instructional connectiveness* includes elements that maximize "opportunities for students to develop competence, providing clear feedback to students, explaining things clearly and carefully, injecting variety into teaching methods, encouraging students to learn from their mistakes" (p. 346).

"In the classroom context, the concept of student motivation is used to explain the degree to which students invest attention and effort in various pursuits, which may or may not be the ones desired by their teachers. Student motivation is rooted in students' subjective experiences, especially those connected to their willingness to engage in learning activities and their reasons for doing so" (Brophy, 2010, p. 3). Is it possible that teacher interventions may influence the affective pathways, motivation and/or engagement that in mathematics classrooms? The most important part of this literature as it relates to the current study is the potential impact that teachers may have in mediating student emotions in the mathematics classroom. This research identifies specific affective behaviors that two teachers exhibit when interacting with students who experience strong affective reactions (both positive and negative) in the context of solving mathematics problems. "The challenge to the educator is to interrupt the incessant negative feelings" "The challenge to the researcher is to find ways to do this" (Goldin, 2000, p. 218).

The current study examines the affective language and action of two middle school mathematics teachers who have been identified as having the potential to create "emotionally safe" learning environments. Videotaped student/teacher interactions are

identified and analyzed based on specific affective language and actions of the two teachers in the context of their students' engagement with conceptually challenging mathematics. The method of analysis used in this study aligns most closely with the interactionist view of emotions (Zemblyas, 2007) because I simultaneously attend to the individuals' emotions and the social environment in the context of mathematical problem solving. From the analysis of qualitative data I identify *patterns* in the emotional and behavioral elements of teacher/student interactions. These patterns are complex and cannot be quantified using surveys taken "after the fact" especially when the emotions take place "in the moment". Interactions between teachers and their students are influenced by the four affective structures, attitudes, beliefs and values and emotions (McLeod, 1989; DeBellis and Goldin, 2006). Of these, emotions are subject to observation and inferences may be drawn based on these observations.

Summary

In this chapter, I review theories and research which represent the foundation of this study and I summarize with four main ideas: (1) emotions and learning are interactive processes, (2) student motivational beliefs and values influence student engagement in learning, (3) student engagement in learning may be malleable and (4) the language and actions of teachers may be influential in supporting student engagement. These two teachers navigate their students' affective, behavioral and cognitive triumphs and pitfalls. Through their language and actions they at times support and foster mathematical engagement and at other times may impede mathematical engagement.

Goldin's (2000) affective pathways model is relevant to this study because teachers interact with their students at moments of frustration. In this study, the two

teachers appear to navigate the students' affective pathways by asking thought-provoking questions, sometimes skillfully at just the right moment. At times, these interventions are followed by a change in students' problem solving strategy or a lively discussion among the students. At other times, the interventions are not so fruitful taking the students off-track in an unproductive manner.

CHAPTER 3: RESEARCH QUESTIONS AND METHODOLOGY

The purpose of this study is to understand the interplay between the affective language and actions of two urban middle school teachers and their students' engagement in the context of mathematical interactions. This is accomplished by viewing videotaped classroom observations and: (1) identifying episodes when each of the teachers interacted with one or more students in the context of a mathematics lesson in which students are working in groups, (2) identifying, describing and classifying in context the affective language and actions of each teacher during each of these episodes, (3) identifying, describing and classifying in context the mathematical engagement of the students during the interactions, and when evident (4) describing student mathematical engagement during and immediately subsequent to each episode. I enhance the classroom observation findings by viewing retrospective videotaped teacher interviews, conducted after instruction. When available in these interviews, I describe the teachers' rationale for their affective language and behavior during interactions with their students. From these descriptions, I then (1) make inferences about the source and/or intent of the teachers' language and actions as evidenced the classroom observation data, and (2) enhance inferences that I make about student and teacher affect in the descriptions of interactions during the episodes. I present the results in a descriptive analysis highlighting patterns of the teachers' affective language and actions presented as five themes of teacher affective language and actions.

In the following sections, I describe the research methodology including the source of the data, a description of the data, the procedure, and analysis.

3.1 Original Data Source

This research examines video data collected as part of a larger study that was conducted by the MetroMath Center and sponsored by the National Science Foundation at Rutgers University the years 2004 – 2011. MetroMath sponsored a multi-faceted and multi-disciplinary program focused on improving mathematics teaching and learning in urban schools through research in urban education, mathematics education, cognitive science, psychology and educational policy. One of the key interests to the Metro Math research agenda was the "role of affect and motivation in the learning of conceptually challenging mathematics" (www.goldin.rutgers.edu/vision.shtml) in urban settings. The term, "conceptually challenging mathematics" refers to non-routine problem solving deeply rooted in developing conceptual understanding. This term is defined in more detail later in this chapter.

The MetroMath research team gathered data during the 2006-2007 academic year in three urban middle school mathematics classrooms situated in two New Jersey cities. For each teacher, five classroom lessons were videotaped and the tapes were transcribed. Each lesson spanned two class periods taking place on two successive or nearly successive days. A mathematical problem was presented at the start of the first class period and developed over the course of the two days. During each lesson, three cameras were used to capture the events of the lesson: one camera was stationary, recording a broad view of the classroom, while the two other cameras were roving following four focus students. The four focus students were chosen by the MetroMath research team based on pre-study observations and are representative of ethnicity and gender of each specific class. Also part of the data set are video taped interviews with each teachers,

recorded and transcribed by MetroMath researchers. Teacher interviews were conducted before the study, before the first class period and immediately following the second class period of each cycle. The purpose of the pre-study interview was to gain information about each teacher's background as well as each teacher's perspective on students, assessment, mathematics, community involvement, parental involvement and student affect. One pre-lesson interview was conducted with each teacher at the start of each lesson to gain information about the upcoming lesson and the teachers' expected outcomes of the lesson, including the teachers' anticipated affective reactions of the students to the mathematics. Immediately following each two-day lesson, the research team identified key affective events evidenced in the classroom observation video data. Post-lesson stimulated-recall interviews were then conducted with teachers and students who were shown video clips of the key affective events and then were asked questions about their affect during the selected events (Schorr, Epstein, Warner, Arias, 2008). The teachers were also asked to speculate on student affect during some of the events. Written artifacts, including samples of student work and researcher field notes complete the data set. The current study draws on these data from two teachers, two lessons each to address the research questions.

To sum up, the data set from each lesson consists of the following:

- Classroom observation data
    - Videotapes and transcriptions from three camera angles, one stationary, two roving
    - One lesson, spanning two consecutive or nearly consecutive class periods
- Interview data
    - Videotapes and transcriptions
        - Teachers interviewed by researchers
            - Prior to first day of data collection
            - After the second class period (within one or two days)

The current study focuses exclusively on data collected during two lessons. The data used in the current study were collected between the months of September through December 2006. The rationale for looking at these two lessons is that in September it is likely that teachers establish the classroom environments. Data collected toward the middle of the school year in November and December provide evidence for how the classroom environment was sustained over time. In this thesis I refer to each lesson by the title of the mathematics lesson. For Ms. B, "Fences for Grazing" was implemented in September and "The DJ Problem" was implemented in November. For Ms. E, "Adding and Subtracting Fractions" was implemented in October and "Finding Areas" was implemented in November.

The demographics of the student populations in both schools included high numbers of minority students (Table 1) and high numbers of students from low socioeconomic status.

School	Black students	Hispanic students	White students
Ms. B's school	93 %	7 %	> 1%
Ms. E's school	27 %	71 %	2 %

Table 2. Student demographics by race
Source: National Center for Education Statistics (2008)

The specific questions for the current study are presented next, followed by the definitions of five pertinent terms:

3.2 Research Questions and Definitions of Terms

1) During teacher/student(s) mathematical interaction episodes, what affective language and/or actions do these two urban mathematics teachers use to foster or to support (or possibly discourage or impede) their students' engagement with conceptually challenging mathematics when working in small groups?
2) What can be inferred about these students' engagement with the mathematics immediately subsequent to the identified teacher/student(s) mathematical interaction episodes?
3) In stimulated recall interviews, what rationale do these two teachers provide for their own affective language and/or actions when interacting with students during key affective events?

Terms Defined

The research questions presented here include terms that require definition for use in the context of this research. They are (1) teacher/student(s) mathematical interaction episodes, (2) affective language and/or actions of the teacher, (3) conceptually challenging mathematics, (4) perceived off-task behavior, and (5) key affective events. The third and fifth terms listed here, "conceptually challenging mathematics" and "key affective events", were defined for previous analysis as part of the MetroMath research project (Schorr and Goldin 2008, p. 135).

(1) Teacher/student(s) mathematical interaction episode refers to an interval of time during which the teacher and one or more student(s) are engaged in an overt verbal exchange or non-verbal communication in the context of mathematics. The beginning of an episode is marked by a teacher's intervention with a small group of students. The intervention may be initiated (1) following a mathematical question posed by the student(s), *or* (2) immediately preceding student(s) engagement in a mathematical discussion with the teacher, *or* (3) following the teacher posing a question to the student(s) about the mathematics, *or* (4) occurring at a time when students it is inferred that the teacher perceives the students to be off-task from the mathematics. These descriptors are not mutually exclusive and they may have common characteristics accordingly, there may be some overlap with regard to the nature of the teacher intervention. For example, the teacher's efforts to re-engage students who are perceived to be off-task often is often accomplished through the initiation of a mathematical discussion or the posing of a mathematical question. The end of an episode is marked by a distinct conclusion when the overt mathematical interaction between the teacher and the

student(s) comes to a close (1) the teacher walks away, or (2) the students return to their work.

When evidenced in the data, student(s') inferred mathematical engagement and/or understanding immediately following each teacher/student(s) mathematical interaction episode is described. Examples of inferred mathematical engagement may include the student changing his strategy or the students in the group engaging in a mathematical discussion about what to do next. Alternatively, examples of student disengagement after certain episodes are also evident in the data. Once the teacher steps away from the group students sometimes engage in a social interaction, teasing one another about something that was said during the episode or the students may sit back in their chairs with apparent relief that the teacher has left their group. The intent here is to provide the most descriptive account of the context of the interaction by using every piece of data to the fullest extent. In some cases the students' engagement following an episode informs the interpretation of interactions occurring prior to or subsequent to the episode. And in other cases, the students' engagement following an episode may be relevant to the teachers' reflections during the stimulated recall interview. I do not draw cause-and-effect type conclusions about the impact of the interaction on student engagement.

(2) *Affective language and/or actions of the teacher* are utterances, gestures, expressions or moves made by (a) the teacher where the teacher's own emotions seem apparent or can be inferred, as well as (b) utterances, gestures, expressions or moves made by the teacher which makes reference (explicit or inferred) to some aspect of student emotion.

Examples of affective language and/or actions include the teacher's expression of approval and pleasure by smiling and nodding her head, gesturing "thumbs up" or "high-five", stating that she is proud of student effort or work. A teacher may demonstrate his or her satisfaction by announcing to the class that a particular student has a "good point" to share, and may encourage the whole class to listen as the student describes her mathematical idea. Conversely, the teacher may exhibit disapproval by using a stern tone of voice, or by reprimanding students. The teacher may express her dissatisfaction with a student's apparent disengagement by frowning or by shaking her head, "no", or by placing her hand on a students' shoulder. The teacher may also explicitly acknowledge student frustration or anger when the student is experiencing an impasse by offering sympathy or by providing encouragement.

(3) *Conceptually challenging mathematics* is "mathematical activity involving the need for a new conceptual understanding and/or for conceptual change in which students are likely to experience the need to 'figure something out', or to have a sense of impasse. In this domain, mathematical meanings are at least as important as mathematical procedures, and problems are likely to be non-routine for problem solvers." (Schorr & Goldin, 2008, p. 135).

(4) *Perceived off-task behavior* is an occasion when it appears to the teacher that student(s) is (are) not engaged in the mathematics during instructional time. The inference that the teacher perceives off-task behavior may derive from student actions such as shouting across the room to another student about a non-math topic, discussing weekend plans, doodling or drawing, dancing, brushing hair, sleeping, texting on a cell phone, playing with classroom supplies and so on. This does not include instances when

students need to retrieve supplies or tools to be used to complete the assigned task, such as rulers or calculators, nor does it include instances when students leave the classroom to use the restroom or to attend to other business. It is important to point out here that some students may be kinesthetically involved in something while remaining cognitively engaged. Thus, students who are observed doodling, tapping a pen on the table or fidgeting, or even staring out the window, may be perceived by the teacher as exhibiting off-task behavior but may be still be engaged with the mathematics. In each case, the teacher's perception of off-task behavior is inferred based on the evidence of the teacher's acknowledgement (verbal or non-verbal) of student engagement (or non-engagement) with the mathematics as well as the student(s') language and/or actions.

(5) *Key affective events* are defined as "an occasion in the context of doing or discussing mathematics, where significant affect or a significant change in affect (of the student or the teacher, or across the class) is expressed or can be inferred. Examples include moments of frustration, anger, or withdrawal, as well as engagement, elation, delight or satisfaction" (Schorr and Goldin, 2008, p. 135). The key affective events used in this study were previously identified by members of the MetroMath research team for use in stimulated recall interviews with the teachers and the students.

Although each teacher displays her own pedagogical style and strategies, some common contexts are present in the classroom environments. The lessons tend to be learner-centered, rather than teacher-centered. Students are often seated in small groups where they work together on a mathematical task as the teacher circulates through the room, periodically stopping to join or to observe the small group discussions. In both classrooms, the students freely move throughout the classroom, collaborating with each

other during class. Both teachers encourage mathematical collaboration and frequently invite students from different groups to move about the room to discuss their ideas. Both teachers engage their students in discussions about conceptually challenging mathematics and the teachers encourage students to listen critically to their peers. As the teachers work toward achieving the objective of each lesson, they frequently elicit student input to help guide the lessons.

One of the goals of the MetroMath project was to document the events that took place in relatively "emotionally safe" urban middle school mathematics classrooms, and to examine the events from four distinct yet interconnected lenses: teacher interactions, social interactions, mathematical learning and affect (Schorr, Epstein, Warner, & Arias, 2008). The teachers in this study had participated in after-school programs and graduate research projects with members of the MetroMath research team at Rutgers University. Both teachers in this study were purposely chosen as subjects for the MetroMath study because they demonstrated skills that could potentially contribute to the type of learning environments that were of interest to the research agenda. I chose to examine interactions between these two teachers and their students because my goal was to search for patterns or themes that may be applied to create emotionally safe environments for students to engage in mathematical learning.

3.3 Procedure

The methods used in this study "show how the individual and the social may be studied simultaneously without undermining their unity in teaching and learning" (Zembylas, 2007, p. 66). In Zembylas' (2007) interactionist perspective, emotions are experienced in the individual as well as in social relationships. He quotes Leavitt

(1996),"socialized human bodies, bodies that normally exist as groups and in interaction rather than isolated entities, have their being in recurrent situations that call forth the meaning/feeling responses we recognize as emotions" (p. 542-525).

The analysis of videotaped classroom interactions attends to the dynamics of the individual and social emotions during teacher/student(s) interactions in the context of learning. The methods in this study also attend to student behavioral engagement. Many studies have used self-report questionnaires to measure students' perceived levels of engagement. According to Wigfield and Cambria (2010), self-report measures may not "capture how engagement exists in the interaction of the individual and context rather than in just the individual"(p. 62). In the current study classroom observation video data captures the complex interactions taking place "in the moment" between the teachers, their students and the context of conceptually challenging mathematics. The retrospective teacher interviews provide insight into the teachers' rationale for their language and/or actions while interacting with their students. I report on specific teacher actions and language which may be inferred to foster or support (or possibly discourage or impede) students' emotional and behavioral engagement as observed in the classroom observations and as described in the teachers' own voices during retrospective interviews.

The first step in the procedure is to identify teacher/student(s) mathematical interactions by viewing the classroom observation videotapes and reading the transcripts for each of the eight class periods. I first view each of the three videotapes of one class period from beginning to end and make a note of the approximate time stamps on the transcripts each time the teacher is speaking with a student, or standing in close proximity to a student. I then review each videotape a second time, looking closely at each of these

noted interactions. If the interaction is mathematical in nature, I identify it as an episode for analysis and note the exact times of the beginning and the end of the episode on the transcript. If the interaction is not mathematical in nature such as when the teacher is issuing a hall pass to a student or helping a student find supplies I do not identify it as an episode for analysis. If a teacher/student(s) mathematical interaction episode is captured on more than one camera, I choose the angle that shows the best view of the teacher and the students and/or the camera that best captures the audio of the interaction. On occasion, I use multiple camera angles to analyze the same episode. For example, in some cases when the camera with the best audio is positioned behind the teacher, I view a different camera angle to observe the teacher's facial expressions and gestures. Each episode is assigned a distinct label based on the following nomenclature: first initial of the teacher's last name, lesson number, day order and episode number. For example the teacher/student(s) mathematical interaction episode (TSMIE) from Ms. E's classroom collected during the first lesson on day one, third episode is labeled, TSMIE [E.01.a.03], (teacher/student mathematical interaction episode [Ms. E, Lesson 1, Day a, episode 3].

A total of fifty-seven teacher/student(s) mathematical interaction episodes are evidenced in the classroom observation data in Ms. E's and Ms. B's classrooms, during eight days of instruction, four days per teacher. Most of the episodes average about five minutes in length with outliers ranging from thirty seconds to twenty-six minutes. In Ms. E's classroom, twenty-one episodes are identified, totaling approximately one hundred two minutes. In Ms. B's classroom, thirty-six episodes are identified, totaling approximately one hundred twenty-seven minutes. These episodes yield approximately two hundred thirty minutes in which the teachers are engaged in interactions with their

students. Comprehensive video observation data records indicating camera angles and time codes are located in Appendix C.

3.4 Analysis

For the next step of the procedure, I analyze the teachers' affective language and actions that take place during interactions with their students considering possible antecedents prior to and consequences of each interaction episode. Fredricks, Bluemenfeld, Bluemenfeld and Paris (2004), call for "…thick descriptions of classroom contexts" in order to understand certain characteristics of classroom environments which support student engagement. "…(A)lthough the research shows that teacher support is associated with engagement, we know less about what aspects of the context create those perceptions of the teacher" (p. 86). "In addition, the research has used variable-centered rather than pattern-centered analytical techniques. As a result, we have little information about interactions or synergy…[f]uture research should address the difficulties of studying how individuals and contexts interact." (p. 87) and they suggest that this could be approached through "multi-method, observational, and ethnographic studies". In this spirit, I follow video analysis model presented by Powell, Francisco and Maher (2003, p. 413) to examine the interactions between the teachers and their students in the contexts of conceptually challenging mathematics.

Powell et al.'s (2003) model involves seven stages:

(1) "viewing the data,
(2) describing the data,
(3) identifying critical events,
(4) transcribing,
(5) coding,
(6) constructing a storyline and
(7) composing a narrative."

It is important to note that this (2003) video analysis model is intended to be iterative, and the stages are not necessarily applied in a sequential manner.

Multiple viewings of the same episode are a necessary part of the procedure to describe the facial expressions, gestures, tones of voice, and utterances of all subjects. In this study it is common for many students to be interacting with the teacher at one time, thus each viewing may focus on a different subject to develop a complete account of the interaction. For example, each time I (1) view the data, (2) describe the data and (3) identify a critical event (for this study, a critical event is named: *teacher/student(s) mathematical interaction episode*), I view the video and transcripts of the episode a second, third, fourth time and so on. Upon each viewing, I gather more information about the nature of the interaction including teachers' affective language and behavior, the students' engagement and the mathematical progress. By viewing the episodes multiple times, I am able to develop the "thick" detailed descriptions of each teacher/student(s) mathematical interaction episode, which are the basis of the "pattern-centered" analysis.

For the analysis of each episode, I first view the data for several minutes leading up to the interaction and describe the students' engagement or disengagement as well as the teachers' actions when available. I then record my interpretation of what may have

prompted the interaction (i.e., the teacher questioned a student, a student posed a question to the teacher, etc.).

An excerpt of the transcripts is shown in Table 2 to provide an example of the format of the classroom observation data analysis.

Begin TSMIE [B.01.a.04] 19:43 – 22:51 (C)			
Participants: Shay, Ray, Ms. B *Ms. B initiates this interaction as she approaches the table and briefly observes the work that the students are doing by looking at their papers. She leans on the table next to Shay, and then moves the materials bucket and stands next to LaToya.*			
19:43	Ms. B	Explain how you are getting that.	Ms. B appears ***curious*** about the students' mathematical reasoning.
19:46	Shay	[Ms. B sits down on the top of the empty student desk.] Look, you do two sides and then you see what the remainder is and then you [Shay is holding a paper up to show Ms. B. He is inaudible.]	
19:58	Ms. B	Okay, so how come you have so many pieces of paper? [Ms. B points to a pile of papers that are on the desk in front of Shay.]	Shay has at least five sheets of paper in front of him and Ms. B conveys ***curiosity*** regarding the large number of papers.
19:59	Shay	Because I got a lot of answers. [Shay picks up the pile of five papers, one piece at a time.]	It is inferred that Shay is defending his process for solving the problem.
20:04	Ms. B	Oh ok. [Ms. B looks sideways at Shay]	Ms. B appears ***satisfied*** by Shay's explanation but it may be inferred that she is cautious and may ***recognize that Shay is becoming frustrated*** by the perceived enormity of the problem.

Table 3: Example of analyzed transcript

The first column represents the time stamp according to the videotape, the second column shows the name of the speaker and the third column shows the transcribed dialogue. Within the transcribed dialogue, I describe the teacher's and students' facial expressions, gestures, tone of voice and/or student written work in square brackets. In the

fourth column, alongside the transcribed dialogue, I record my interpretations of the affective actions and language of the teachers and of student engagement. In these inferences, I emphasize words or phrases that are considered affective in nature in bold italics. Inferences about student and teacher affect are based on the students' and teachers' language, facial expressions, gestures, tone of voice, language and written work. The social interactions amongst the students are also considered as part of the interpretation in as much as I report on any social transactions that occur during each episode. In an effort to verify my interpretations, once analysis was complete, I trained two independent researchers to view six episodes, three from each teacher. I provided the researchers with the affect descriptive words that I used in the fourth column, while blinding the reasons for my interpretations. I purposely changed some of the affect descriptive words throughout the analyzed transcripts to test the integrity of the researchers' work. The researchers were asked whether they strongly agree, agree, disagree or strongly disagree with each of the affective descriptive words and in the case of "strongly disagree", the researchers were also asked to make suggestions for more appropriate descriptors. Miles and Huberman (1994) recommend dividing the total number of agreements by the sum of the disagreements and agreements to calculate inter-coder reliability. When following this formula, inter-coder reliability was eighty-seven percent for one researcher and ninety-one percent for the other researcher.

I conclude each episode with a brief summary and interpretation, highlighting the key affective events and the teachers' and students' affective language and/or actions in the context of the mathematical progress. In each concluding summary, I describe evidence of student engagement in the mathematics, and/or student mathematical

understanding immediately following each episode.

Prior to data analysis, I developed a preliminary list of expected language actions to be exhibited by the teachers and the students. After viewing the data during the analysis phase, I added to the list and subsequently developed three broad categories of teacher and student language and actions, which indicate mathematical engagement, emotional engagement, and perceived off-task behavior. The complete lists including are provided in tables below, organized by category. Table 4: Examples of the teacher engagement with the mathematics with her student(s), Table 5: Examples of teacher emotional engagement, Table 6: Examples of student engagement in the mathematics, Table 7: Examples of student emotional engagement, Table 8: Examples of off-task behavior exhibited by the students that teachers might notice.

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Table 4

Examples of teacher engagement with the mathematics with her student(s)

- The teacher is standing at the front of the classroom, leading a whole class discussion about the mathematics.
- The teacher is speaking to the students, i.e. asking a question or answering a mathematical question posed by a student.
- The teacher is standing or sitting with the students, silently observing the students.

Table 5

Examples of teacher emotional engagement

- The teacher expresses pleasure by
  - Making a statement such as, "That's a great suggestion." or "I am proud of you."
  - Clapping hands or giving a "high five" to a student.
  - Nodding head in agreement.
  - Raising eyebrows and smiling at a student.
- The teacher expresses frustration by
  - Making a statement such as, "I'm trying to help you, but you are not paying attention." or, "I already explained the directions five times. Get to work!"
  - Using the "teacher stare", looking at a student with a stern facial expression.
  - Rolling his or her eyes.
- The teacher acknowledges the students' affective response by saying:
  - "That is puzzling."
  - "You look like you disagree with that."
  - "I can see that you are excited to share your idea with the class."
  - "You worked hard today. I'm proud of you."
  - "Ask the members of your group for help."

Table 6

Examples of student engagement in the mathematics

- The student asks the teacher a question about the mathematics.
- The student volunteers to answer a mathematical question posed by the teacher or by another student.
- The student is engaged in a mathematical discussion with other students or with the teacher.
- The student gestures toward another student's mathematical work to comment or make an inquiry about the math.
- The student is using mathematical tools, including a ruler, a calculator, or a pencil to attend to the task.
- The student answers a question posed by another student or by the teacher.
- The student comments on another student's work.

Table 7

Examples of student emotional engagement

- The student showing disappointment or frustration by
  - frowning or shaking her head.
  - disengaging from the task, slumping down in his seat.
  - making a statement such as "I don't understand." or "I'm just not good at this."
- The student showing excitement by
  - jumping up and down.
  - raising hand excitedly (shaking back and forth) to get the teacher's attention.
  - making a statement such as, "I got the answer! I got it!"
- The student showing pleasure by
  - breaking out into a smile.
  - nodding her head.

Table 8

Examples of off-task behavior exhibited by the students that teachers might notice

- The student is shouting across the room to another student about a non-math issue.
- The students are discussing weekend plans.
- The student is brushing her hair.
- The student is drawing a picture (non-math related).
- The student has his head on the desk with eyes closed.
- The student is texting on a cell phone.

Classification of teacher affective actions and language

For the next phase of analysis, I use the descriptions in the fourth column of the transcripts to classify the teachers' affective language and actions according categories of teacher affective language and actions. Some of the following categories were identified prior to data analysis and more were developed after viewing the video. Operational definitions of each category including a definition, an example and interpretation are presented in Table 9. The table is followed by complete definitions of each category.

Code	Category	Example(s)	Interpretation
Int	Teacher expresses interest in student's mathematical engagement, reasoning and/or representation	Engagement: "What are you working on?" Reasoning: "Six forty divided by ten, why did you do that?" Representation: "I need you to draw it out."	Teacher may be curious about the students' mathematical progress or may intend to stimulate student thinking through discourse.
Val	Teacher providing validation of a student's comment/answer	"I want you to think about it that way 'cause I like, I like that idea."	Teacher validates the student's mathematical idea by stating that she "likes" that idea.
Raa	Respectfully responding to alternate approaches	"Yeah you are absolutely correct. If there were traffic it would be longer."	Teacher responds to the student's comment that traffic would interfere with the time traveled, despite the fact that traffic is irrelevant to solving the problem.
Dis-m	Teacher explicitly expresses her disapproval of student mathematical reasoning.	"Okay, we're having a problem here. I haven't seen three fourths on this table. Can I see three-fourths on this table?"	Teacher tells a student or students that they have made a mistake.
Dis-b	Teacher expresses her disapproval of a student's behavior.	"Can you guys pay attention to me …It is very frustrating when the different things are going on. And I'm trying to give you my attention."	Teacher expresses her own frustration toward off-task student behavior and/or students all talking at once.

Table 9: Teacher Affective Language and Actions Categories

Code	Category	Example(s)	Interpretation
Ptp	Teacher encourages peer-to-peer respect	"Let's all wait, okay. How about everybody take a few seconds, Naomi started it…other people were chiming in just think about it… " "…okay, so let's listen and let's just say if we agree or disagree. Go ahead."	Teacher suggests concrete ways for the students to demonstrate respect for one another by encouraging students to wait their turn to speak and by encouraging them to 'listen' to other's ideas.
Blds	Teacher building on student ideas	So wait. Time out just to clarify you guys are here. Fifty-five, no problem. If I'm going 53 miles per hour, 'cause I am going to change up, 'cause Geraldo made a suggestion.	Teacher clarifies the mathematical answer (since more than one answer was possible). Teacher uses a student-generated suggestion to begin calculations. This speed was not part of the original problem.
Res	Teacher demonstrates respect for student	Do you want to respond to that or do you want a minute to think about it? You don't have to. I mean, if you don't feel comfortable, that's fine."	Teacher acknowledges that a student has input, while respecting the student's need to put his or her thoughts together before responding to the task. The teacher demonstrates respect to a student who does not feel comfortable sharing his idea.
Endis	Encouraging student discourse/open discourse	"Okay, Quadree? You want to add to that? Does anybody else have anything they want to share about the things they notice from yesterday?" "...maybe do you want to ask him why he did that?"	Teacher makes comments that may be intended to prompt students to respond to each other's answers. Often, this takes the form of a question asking the whole class or a particular student to agree, disagree, or respond in some other way to the original students' input.

Table 9 (continued): Teacher Affective Language and Actions Categories

Definitions of Teacher Affective Categories

Expressing interest (Int): Through her language, the teacher expresses explicit curiosity about the student(s)' mathematical engagement, mathematical reasoning and/or mathematical representation. The teacher may question the students about their general mathematical progress (engagement), or the teacher may pose a more specific question asking the students to explain their reasons for mathematical "moves" (reasoning). The teacher may question the student's understanding of the mathematical problem (reasoning). The teacher may also pose questions about mathematical representations that the students have developed or the teacher may ask the student to explain their reasoning via a representation. Teacher may also express interest by positioning herself in close proximity (either sitting or standing), leaning in to quietly observe a student or students' as they are engaged with the mathematics.

Providing validation (Val): The teacher expresses explicit praise of student mathematical reasoning, representation and/or engagement. Expressions of praise can be verbal or non-verbal. A verbal expression of praise may be "you did a really great job today, I'm proud of you", or "I like the way you are thinking!". Non-verbal expressions of praise may include a "high five" or a nod of the head.

Respectfully responding to alternate approaches (Raa): The teacher responds to or makes note of a student's alternate mathematical approach or contribution to the problem. In some cases, the student's suggestion may not

be useful to solving the problem, but the teacher respectfully acknowledges their contribution.

Expressing disapproval (Dis-m): The teacher expresses explicit disapproval of a student's mathematical ideas by stating that they have made a mistake. The nature of the teacher's expression may be interpreted as negative or they may be interpreted as positive (as in constructive criticism).

Expressing disapproval (Dis-b): The teacher expresses explicit disapproval of a student's behavior by telling the student that his/her behavior is inappropriate. The nature of the teacher's expression may be interpreted as negative in nature or they may be interpreted as positive (as in constructive criticism).

Promoting student-to-student respect (Ptp): The teacher verbalizes the importance of demonstrating respect toward all individuals in the classroom. The teacher suggests and/or models specific ways for the students to show respect for their peers such as actively listening when others are talking, waiting their turn to ask questions and by asking thoughtful questions about other students' mathematical ideas.

Teacher building (Blds): The teacher considers a student's mathematical suggestion then builds on her mathematical ideas by taking subsequent steps to solve the problem based on the student's ideas. This could include interactions when the teacher encourages students to build on each other's ideas. While this category of teacher language/behavior could be considered more cognitive in nature than affective, building on students' ideas may

support autonomy (e.g., Ryan and Deci, 2000) and self-efficacy (e.g., Bandura, 1994). I would tentatively suggest that building on a student's contribution could potentially provide motivational supports and promote emotional, behavioral and cognitive engagement.

Teacher demonstrates respect (Res): The teacher explicitly expresses respect for student (mathematical or non-mathematical) through her language and actions. This includes interactions when the teacher respectfully considers a student's mathematical idea. This also includes interactions when the teacher through her language and/or actions expresses explicit respect for a student's physical or emotional comfort.

Encouraging student discourse (Endis): The teacher asks a student to discuss her idea with students in the group, or the teacher may invite students from different groups to discuss a mathematical idea. The teacher may ask a student to present his solution to a mathematical problem to the whole class and invite students in the audience to comment or ask questions.

To support the inferences about teacher and student affect from the classroom observation data, I consider the teachers' rationale for their affective actions or language during interactions with their students. Research Question #3 attends to the teachers' voices by examining teacher responses to stimulated-recall interview questions about their own affect and that of their students during key affective events that were selected by the MetroMath research team as described earlier.

I address Research Question #3 by viewing stimulated recall interviews that were conducted with each teacher within two or three days subsequent to the completion of

each two-day lesson. Prior to the interviews, the research team collaboratively chose key affective events from the videotaped lesson that showed evidence of "significant affect or a significant change in affect" on the part of the teacher or the students or the whole class (Schorr & Goldin, 2008, p. 135). During these interviews, the researchers showed video clips of the key affective events and asked the teacher questions about the clips, which focus on mathematical, social and affective events. Detailed descriptions of the key affective events and the teachers' affective responses to each event are provided.

3.5 Results

The results of this study are a descriptive analysis, which illustrate patterns of teacher affective language and actions identified in the analyzed transcripts of classroom observations and in the videotaped teacher interviews. For each teacher, I develop a narrative organized by categories of patterns of teacher affective language and actions in the context of supporting or fostering student engagement. In some episodes it is evident that the teacher language and actions may hinder or undermine student engagement with the mathematics and I also report on those instances in the analysis. In these narratives, I discuss events that took place in each classroom, and also draw on each teacher's responses to the pre-study, pre-lesson and post-lesson interviews. I take into consideration the teachers' backgrounds, philosophies and discussions of students' affect in the context of the mathematics. I draw comparisons, highlighting the differences and similarities about how the two teachers interact with their students and the nature of student engagement with the mathematics in each classroom. Finally, I offer suggestions for future research address methodological considerations for studying teacher/student(s) interactions in classroom contexts.

CHAPTER 4 FINDINGS OF THIS RESEARCH

4.1 Introduction

In this chapter, I present the findings of the classroom observation video analysis and teacher interviews for both Ms. E and Ms. B. I describe the specific affective language and actions of Ms. E and Ms. B as they as they interact with their middle school students and working in small groups to solve conceptually challenging mathematics problems. I discuss observed and inferred consequences of teacher language and actions as evidenced by the students' mathematical engagement during and immediately subsequent to their interactions with the teacher. I describe how Ms. E and Ms. B, through their affective language and actions foster or support (or possibly discourage or impede) students' engagement with conceptually challenging mathematics. The data source consists of twenty-four videotapes of classroom observations (twelve for each teacher) from which I identify and analyze fifty-seven teacher/student(s) mathematical interaction episodes. Also included in the data set are four videotapes of two retrospective stimulated recall interviews for each teacher. These interview videotapes exhibit the teachers' general reflections on each lesson and their reflections on specific clips from the classroom observation data. Teacher interviews were viewed and described after the analysis of all teacher/student(s) mathematical interaction episodes was complete.

The analysis and coding of the teacher/student(s) interaction episodes resulted in five themes describing teacher affective language and actions that highlight specific patterns that occurred: (1) teacher interest in student mathematical engagement, reasoning and representation, (2) teacher validation of student reasoning, (3) teacher

promotion of student autonomy, (4) teacher promotion of mathematical discourse, and (5) teacher promotion of respect. In their reflections during the retrospective stimulated recall interviews, the teachers confirm the presence of these themes.

This study examines specifically the teacher's affective language and behavior during the teachers' *mathematical* interactions with their students. To set the context for the findings I first present the four mathematical problems (Section 4.2) that the teachers assigned during the eight days of observed lessons. Ms. E assigned the first two problems: "Adding and Subtracting Fractions "and" Finding Areas and Other Products. The second two problems: Ms. B assigned "Fences for Grazing "and" The DJ Problem. I discuss possible solutions and answers for each problem to assist the reader in interpreting the results. In the next section of the chapter (4.3) I discuss the five themes that emerged from the coding of the video data for Ms. E and for Ms. B. The themes are presented in the order of their prevalence, from the most frequently occurring to the least frequently occurring.

After the complete discussion of the five themes for Ms. E and Ms. B I discuss the teachers' reflections on the lessons by describing their responses to retrospective stimulated recall interview questions in Section 4.4. I discuss the instances when the teachers' reflections pertain to the five themes and I also note the teachers' reflections that confirm inferences made during the analysis of the classroom observation data. I conclude the chapter in Section 4.E with a brief synthesis, comparing the two teachers' language and actions, I highlight similarities and differences in their affective language and actions as they interact with their students.

4.2 Context of the Analysis

To set the context for the analysis of the teacher/student mathematical interactions presented in this chapter, I first discuss the four mathematical problems that the teachers assign. I describe possible solutions to each problem as well as possible cognitive hurdles that students may encounter when solving the problems. According to Goldin (1979) when teaching mathematics, it is important to have a deep understanding of the problem(s) cognitive structure. Understanding the structure of mathematical problems can help teachers to apply productive interventions when and if students encounter cognitive hurdles during problem solving.

In each of the four lessons, the teachers assign one mathematical problem, which the students work in small groups to solve. Ms. E assigns two problems directly from the students' textbooks: "Adding and Subtracting Fractions" in the first lesson and "Finding Areas and Other Products". Ms. B assigns two problems each on a single sheet of paper, which she distributes to each student. The first problem, "Fences for Grazing" was created by Ms. B as a "benchmark" problem to assess the students' understanding of area and perimeter. The second problem, "The DJ Problem" was taken from the textbook and Ms. B retyped the problem statement on a single sheet of paper. Following each of the problems I present examples of possible solution(s) in order to help the reader interpret the themes for each teacher.

4.3 The Mathematical Problems

4.3.1 Ms. E, Lesson One: "Adding and Subtracting Fractions"

Statement of the Problem

INVESTIGATION 4

Adding and Subtracting Fractions

Knowing how to combine and remove quantities is a skill that is helpful for understanding the world around you. The mathematical names for combining and removing are adding and subtracting. For example, if you owned two lots of land and you bought another half a lot, you could combine the two lots and the half lot to determine that you owned $2 + \frac{1}{2}$, or $2\frac{1}{2}$, lots of land.

The problems in this investigation require you to add and subtract fractions. As you work on these problems, use what you have learned in earlier investigations about finding equivalent fractions and rewriting fractions as decimals.

Dividing Land

When Tupelo township was founded, the land was divided into sections that could be farmed. Each *section* is a square that is 1 mile long on each edge—that is, each section is 1 square mile of land. There are 640 acres of land in a 1-square-mile section.

The diagram on the next page shows two *adjacent* sections of land, sections 18 and 19. Each section is divided among several owners. The diagram shows the part of the land each person owns.

Determine what fraction of a section each person owns. Explain your reasoning.

■ Problem 4.1 Follow-Up

Determine how many acres of land each person owns. Explain your reasoning.

Figure 3: Statement of the Problem, "Adding and Subtracting Fractions" (CMP, p. 43)

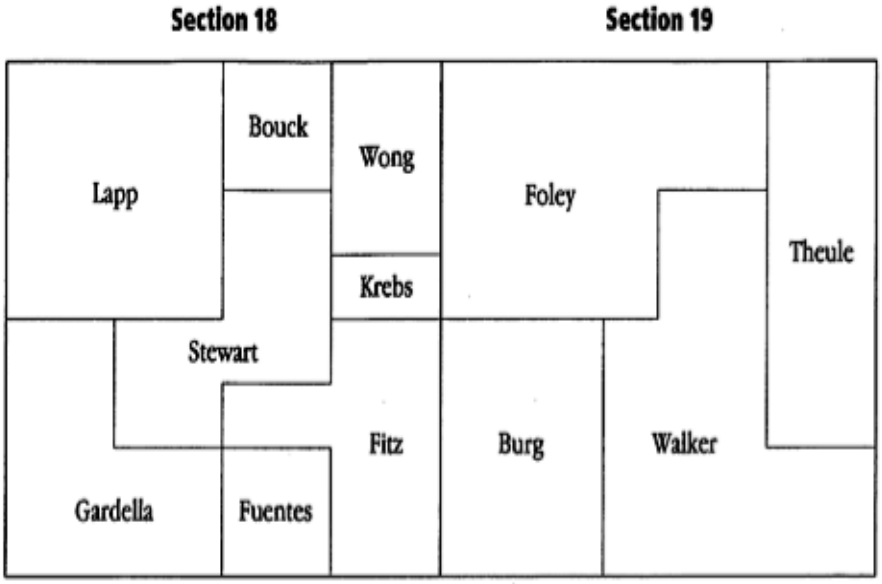

Figure 4: Map of Sections 18 and 19, "Adding and Subtracting Fractions" (CMP p. 44)

Problem Statement: Determine what fraction of a section each person owns. Explain your reasoning.

Problem Solutions: "Adding and Subtracting Fractions"

One way to successfully complete this problem is to examine Section 18 (Figure 4) and establish measurements of the linear segments bounding each property in relation to the linear measures bounding the whole Section 18. This could be accomplished either by using a ruler with metric or English units of length (centimeters or inches) or by using non-standard measuring techniques, such as marking off lengths on a piece of paper to measure and compare the linear dimensions of the properties relative to the linear dimensions of Section 18.

First, it is essential to consider that Section 18 is a square defined by four sides of equal measure and four right angles. There are eight properties within Section 18 and five of the properties are rectangular (or square) in shape. The other three properties are irregularly shaped polygons with right angles and straight edges, thus they can be considered portions of squares and/or rectangles. Knowing the measures of the sides of the rectangular properties within Section 18, one could then multiply the two dimensions, width times height, to find the area of each rectangular property. To find the areas of the irregularly shaped properties, use addition or subtraction to account for the areas of the extra or missing pieces.

In this solution, I assign the unit one whole to represent the length of one side of Section 18. Each of the dimensions of the individual landowners' properties will then be expressed as a fractional value of one whole. Since the title of this investigation is "Adding and Subtracting Fractions", this seems like a reasonable route for solving this problem.

To determine the dimensions of Lapp's property, note that Lapp's property

together with Gardella's property makes up the total measure of the height of the square. By confirming the congruence of Lapp's property to Gardella's property, and showing that together the measures of the heights of both properties combined equal the measure of the height of the square Section 18, the height of Lapp's property can be represented as one-half the length of the side of the square Section 18. The width of Lapp's property can be figured using a similar procedure by showing that the width of Lapp's property is congruent to the combined width of Bouck's property and Wong's land. Taken together, these three properties span the total width of Section 18. Thus, the width of Lapp's property can be also represented as one-half of the width of Section 18. To calculate the area of Lapp's property, multiply the height (one-half) by the width (one-half) resulting in the product one-fourth. Thus, Lapp's property covers one-fourth of the entire Section 18. Gardella's property is represented in an irregular "L" shape but can be calculated using the information we now know about Lapp's property. To calculate the area of Gardella's property, first determine the height (on the left) and width (on the bottom). Using similar procedures as explained above for Lapp's property, determine that the height on the left side of Section 18 and the horizontal dimension on the bottom of Section 18 are congruent to the height and width of Lapp's property. For a moment, we consider Gardella's land a square, congruent to Lapp's property with an area one-fourth of Section 18. However, a small square section is cut out on the upper right hand corner of Gardella's property, resulting in the "L"-shape. So, if we subtract the area of Gardella's property considered as a square (one-fourth) minus the area of the small square cut-out, the result would be the total fractional part owned by Gardella. The dimensions of the small square cut-out are each one half of the dimensions of Lapp's

land, thus, the lengths of the sides of this small square are determined by multiplying one-half times one-half to equal one-fourth. To calculate the area of the small square, multiply one-fourth times one-fourth which equals one-sixteenth. The area of the small square is one-sixteenth of Section 18. Finally, subtract the area of Gardella's land without the square missing (one-fourth) minus the area of the missing square (one-sixteenth) and arrive at three-sixteenths. Gardella owns three-sixteenths of Section 18. The fractional values of the other properties could be determined using similar procedures but will not be discussed here for two reasons. First, neither the teacher nor the students used this approach to solve the problem and secondly, the students did not complete the problem. They only got as far as finding the fractional values of Lapp's property and Gardella's property.

Key information when following this solution path is that Section 18 is a quadrilateral with four right angles and four equal sides and is therefore a square. In the text immediately above the problem statement, the students are given the extraneous information that one square mile equals six hundred forty acres. This information could be considered extraneous information because this particular task only asks for the fractional values of each property within Section 18 (Figure 1).

I now describe a second possible solution path which considers six hundred forty acres a very important piece of information. This solution path is relevant because the students attempt to solve the problem in this way and it is inferred that the reason that the students use this solution path because their teacher poses a constraint. Ms. E distributes grid paper and she instructs the students to represent Section 18 on the grid paper prior to assigning fractional values to each property. This constraint requires the coordination of

many mathematical ideas and thus poses many cognitive challenges, which will be described in this discussion.

First, it is important to recall that the given information preceding the statement of the problem states that Section 18 is square and covers exactly one square mile, equivalent to six hundred forty acres. Given its shape and the area one square mile, Section 18 can be represented using many different whole number dimensions on the grid paper. Taking the area of Section 18 as six hundred forty acres poses a cognitive challenge. Since six hundred forty is not a perfect square, it cannot easily or immediately be represented on the grid paper as such. An arithmetic manipulation must take place before Section 18 can be drawn on the grid paper. The square root of six hundred forty is *approximately* twenty-five and three tenths, not a whole number. A square with these dimensions could be difficult to precisely draw on grid paper and would be even more difficult to partition into the individual land-owners' property, especially given that twenty-five and three tenths is and approximation.

Whole number dimensions would be preferable for representing Section 18 on grid paper so that the subsequent task of calculating fractional values of each property is easier. Yet, exactly what whole number dimensions are best? Using the dimensions ten by ten, for example, the area of Section 18 equals one hundred square units. Dividing six hundred forty acres by one hundred square units would yield a value of six and four tenths acres for each small square. This would be difficult to work with when finding the fractional value of each separate plot of land. This is just one example of the many possible whole number dimensions possible for representing Section 18 on grid paper.

Drawing an eight by eight square is a very effective way to represent Section 18 on grid paper for two reasons. First, the area of an eight by eight square equals sixty-four which is equal to one tenth of six hundred forty. Thus, each small square within Section 18 would represent ten acres (a whole number!). Secondly, (by accidental good fortune) the properties are divided into fourths, eighths and sixteenths of Section 18, all fractions of or multiples of eight. It has been inferred from the classroom observation data and from the post-lesson interview with Ms. E, that the teacher fully expected the students to have a fairly easy time arriving at this representation. In the interview, she expresses surprise that the students had so much difficulty with the subtask. Ms. E's reflections are discussed in detail in Section 4.5.

One way to arrive at an eight by eight square begins with dividing six hundred forty by ten so that each small square on the grid paper is assigned the value of ten acres. Next, take the square root of sixty-four which equals assign eight as length of both the height and width of Section 18. The area of the square would be eight times eight, which equals sixty-four square units. Each square unit equals ten acres, thus the area of Section 18 as sixty-four square units times ten acres per square unit equals six hundred forty acres. Presented here, this may seem like an "easy fix" to satisfy the conditions of the subtask and move ahead to solving the original problem as stated in the mathematics book. However, this solution requires complex mathematical thinking. First, the knowledge that six hundred forty is a magnitude of ten larger than six hundred forty is unlikely obvious to the average seventh grade student. Second, recognizing that sixty four as a perfect square would yield a square with sides eight units long is also unlikely to be obvious. Finally, since the students have not really looked closely at the relative sizes

of the properties within Section 18, they may not immediately recognize the relationships in terms of halves, fourths, eighths, etc. Thus, it is very unlikely that they would recognize the utility of the number eight for the dimensions of Section 18.

Once the subtask of drawing Section 18 on grid paper is completed, each region of land within Section 18 could be drawn based on relative size to the whole square. The dimensions of Lapp's land measure one half the length of each of the sides of Section 18, thus, if the whole Section measures eight squares by eight squares, Lapp would own a four by four square, positioned in the upper left hand corner of Section 18. A four by four square has an area of sixteen square units and Section 18 is an eight by eight square which yields an area of sixty-four square units. Thus, Lapp's land has a fractional value of sixteen over sixty-four, which can be simplified to one-fourth. The fractional values for all of the properties could be determined using similar procedures.

4.3.2 Ms. E, Lesson Two: "Finding Areas and Other Products

Problem Statement

Let's look at some of the problems Paulo and Paula had to solve while they were selling brownies.

Problem 5.1

The brownies are baked in square pans, and they are sold as fractional parts of a pan. A whole pan of brownies costs $24 dollars. The cost of any fractional part of a pan is that fraction of $24.

A. One pan of brownies was $\frac{2}{3}$ full. Mr. Sims bought $\frac{1}{2}$ of what was in the pan. What fraction of a full pan did Mr. Sims buy? How much did he pay?

B. Paulo's aunt Serena asked to buy $\frac{3}{4}$ of what was left in another pan. The pan was half full. How much of a whole pan did Aunt Serena buy? How much did she pay?

Problem 5.1 Follow-Up

For A and B above, draw a picture to show what each brownie pan looked like before Mr. Sims and Aunt Serena bought part of what remained. Then draw a picture that shows how much of each pan the customer got and how much was left. Mark your drawings so that someone else can easily see what fraction of the pan each customer bought.

Model of a Brownie Pan

Use the drawings to check your computations in A and B for the fraction of the brownie pan and the price each customer paid.

Figure 5: Statement of the Problem: "Finding Areas and Other Products"

Problems 5.1A and 5.1B may be solved computationally by multiplying the fractions to determine the portions Mr. Sims and Aunt Serena purchased. To determine the amount that Mr. Sims and Aunt Serena paid for their brownies, multiply their fractional purchase by the total cost of the whole brownie pan. For example, to answer Problem 5.1A, multiply two-thirds that was left in the pan times one-half the amount that Mr. Sims wanted to purchase. To answer Problem 5.1B, multiply one-half, the amount of brownies left in the pan times three-fourths, the amount that Aunt Serena wanted to buy relative to what was left. Although Ms. E never explicitly states that the students are working on Problem 5.1 Follow-up (see Figure 3), Ms. E instructs the students to draw the models first and then to use the models to name the fraction of each brownie and to find the cost paid for each brownie as stated in Problems 5.1A and 5.1B. Examples of how models can be used to represent Problems 5.1A and 5.1B are illustrated and discussed below to guide the reader in interpreting the mathematical interactions discussed in the findings.

Problem Solutions: "Finding Areas and Other Products"

Problem 5.1 A

Begin with a model drawn of the square pan of brownies. The pan is a square divided into three equal sections, and two of the sections are shaded. This represents a brownie pan that that is 2/3 full (Figure 6). The shaded section represents the part of the pan that has brownies in it.

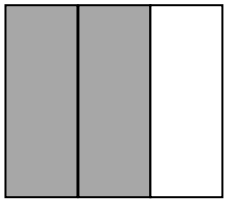

Figure 6: Brownie pan that is 2/3 full.

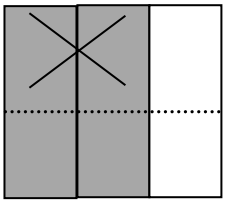

Figure 7: Marked section of brownie pan: "half of what's left"

Since Mr. Sims wants to buy half of what is left in the brownie pan, the brownie pan is cut in half horizontally (Figure 7), including the shaded and non-shaded sections. This shows the total number of pieces in relation to the whole pan. Next, one-half of the shaded section is marked to represent the "half of what's left" that Mr. Sims purchases.

To determine what fraction of the full pan Mr. Sims bought, first count the sections that are marked (two regions, Figure 7). The marked sections represent the part that Mr. Sims purchased. Next count the total number of regions (six regions, Figure 7), which represent the total number of regions in the whole pan. Finally, Mr. Sims purchased two regions out of six regions of the whole brownie pan. **Mr. Sims purchased 2/6 of the whole pan.**

Note: Another way to solve this problem using the model would be to recognize that half of two thirds equals one-third (Figure 6), so simply cross out or mark one of the two sections shaded in Figure 4, leaving one-third in the pan. This is not how Ms. E and her students approached the problem.

The question, "How much did he pay?" could be determined either conceptually or algorithmically. First recall that the cost of one whole brownie pan is twenty-four dollars. A conceptual approach would begin with the idea that one-sixth of twenty-four equals four, so two-sixths of twenty-four equals eight. To solve this problem algorithmically, multiply the number two-sixths by the number twenty-four, which equals eight dollars.

Mr. Sims spent eight dollars on the brownies.

5.1A Answer: Mr. Sims purchased 2/6 of a brownie pan
Mr. Sims spent $8.00 on the brownies.

Problem 5.1B

To answer the question, "How much of a whole pan did Aunt Serena buy?" first recall that the brownie pan was half full and Aunt Serena wanted to buy three-fourths of what was left. Begin by drawing a model of a square brownie pan, cut into two equal sections. Step (1): shade one of the sections to represent a brownie pan that is half full (Figure 8). Step (2): divide the brownie pan into four equal sections in order to represent that the half that is full is cut into fourths (Figure 9). Step (3): mark three-fourths of the shaded space in the brownie pan to represent "three-fourths of what was left" (Figure 10). It is important to draw the lines horizontally across the entire brownie pan (including the empty section) for determining the fraction of the whole that Aunt Serena buys. Step (4): name the marked space as a fraction of the whole square. Three sections are marked out of a total of eight equal sections in the whole brownie pan.

Figure 8: Problem 5.1 Step 1
Step (1): Brownie pan that is half full. The shaded section represents the part of the pan that has brownies in it.

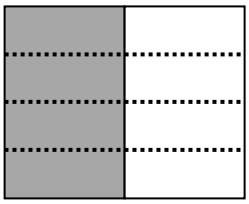

Figure 9: Problem 5.1 Step 2
Step (2): Draw three horizontal lines so that the whole square is divided into four equal horizontal sections.

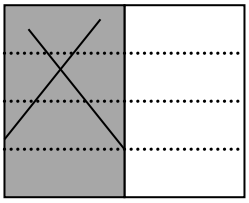

Figure 10: Problem 5.1 Step 3

Step (3): Mark a section to represent three-fourths of the shaded section. This is the part of the whole brownie pan that is purchased by Aunt Serena.

Step (4): Aunt Serena purchased three sections out of eight sections in the whole brownie pan.
Aunt Serena bought three-eighths of a whole brownie pan.

To answer the question "How much did she pay?", recall that the whole brownie pan costs twenty-four dollars. A conceptual approach would begin with the idea that one-eighth of twenty-four equals three, so three-eights of twenty-four equals nine. To solve this problem algorithmically, multiply the fraction of the whole pan that Aunt Serena purchased (three-eighths) by the cost of the total pan (twenty-four dollars) to equal nine dollars.

5.1 B Answer: Aunt Serena bought 3/8 of a whole brownie pan.
Aunt Serena spent $9.00 on the brownie.

4.3.3: Ms. B, Lesson One: Fences for Grazing

Statement of the Problem

Fences for Grazing

Farmer Joe has a cow named Bessie. He bought 100 feet of fencing. He needs you to help him create a rectangular fenced in space with the maximum area for Bessie to graze.

- Draw a diagram with the length and width to show the maximum area
- Explain how you know you found the maximum area
- How many poles would you have for this area if you need 1 pole every 5 feet?

Figure 11: Statement of the Problem: "Fences for Grazing"

Problem Solution: Ms. B, "Fences for Grazing"

One method of solving "Fences for Grazing" is to generate a listing of all possible dimensions for rectangles with perimeters equal to one hundred. For example, using ten as one dimension and forty as the other dimension would yield a perimeter of 40+ 10 + 40 + 10 = 100 feet and an area of 40 x 10 = 400 square feet. A rectangle measuring five by forty five would yield a perimeter of 45 + 5 + 45 + 5 = 100 feet and an area of 45 x 5 = 225 square feet. A rectangle measuring twenty-five by twenty-five would yield a rectangle with a perimeter of 25 + 25 + 25 + 25 = 100 feet, and an area of 25 x 25 = 625 square feet (See Figure 10).

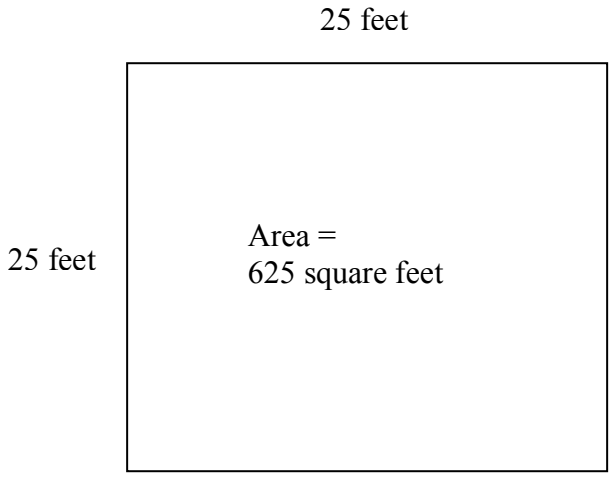

Figure 12: Representation of the fence with the "maximum area".

Answer: The rectangular region that would provide the largest area using one hundred feet of fencing is <u>a square measuring twenty-five feet on each side, with an area equal to six hundred twenty-five square feet.</u> Poles will be placed every five feet along the edge of the fence. Each side will have four poles, beginning five feet from the corner. Four

poles times four sides equals sixteen poles, plus four poles, one for each corner. Thus, there will sixteen plus four = twenty poles along the edge of the fence.

4.3.4: Ms. B, Lesson Two: " The DJ Problem"

Statement of the Problem

The DJ Problem

You are on a committee to select a DJ for a school party. The committee has obtained price quotes from three DJs:
Tom's Tunes charges $60 an hour
Solidus Sounds charges $100 plus $40 an hour
Light Plastic charges $175 plus $30 an hour

c. Which DJ would you choose? What variables might affect your decision?
d. For each DJ, write an equation you could use to calculate the total cost from the number of hours worked? Let y be the total cost and x be the number of hours worked.
e. Graph all three equations in the same window of your calculator. Make a sketch of the graph you see.
f. What information does the coefficient of x represents in each equation
g. What information does the y-intercept represent in each equation?

Use your calculator to answer a-c.

a. For what number of hours are the costs for Toms Tunes and Solidus Sounds equal? What is the cost for that time?
b. What would the cost be for each DJ if he or she worked 8 ½ hours?
c. You have $450 to spend on a DJ. How many hours could each DJ work for this price?

Figure 13: Statement of the Problem: "The DJ Problem"

Problem Solutions: "The DJ Problem"

A conceptual approach using tables of values would represent the hours starting from one hour through twelve hours listed in the left hand column. Next to each hour value, the total cost in dollars for that hour can be represented in the right hand column. To calculate the dollar amount for each DJ, multiply the hourly rate by the number of hours, then add the fixed cost, if the DJ charges a fixed cost. I have listed examples for the first three hours for each of the three DJs. This process would be continued for each successive hour and the rates for each of the DJs from one through twelve hours is shown in Figure 14.

Tom's Tunes charges a rate of $60 for each hour that he plays.

60 x 1 = $60 for one hour

60 x 2 = $120 for two hours

60 x 3 - $180 for three hours

Solidus Sounds charges a one-time flat rate of $100 plus $40 for each hour that he plays.

40 x 1 + 100 = $140 for one hour

40 x 2 + 100 = $180 for two hours

40 x 3 + 100 = $220 for three hours

Light Plastic charges a one-time flat rate of $175 plus $30 for each hour that he plays.

30 x 1 + 175 = $205 for one hour

30 x 2 + 175 =$235 for two hours

30 x 3 + 175 = $265 for three hours.

Toms Tunes			Solidus Sounds			Light Plastic	
X (hours)	Y (dollars)		X (hours)	Y (dollars)		X (hours)	Y (dollars)
1	60		1	140		1	205
2	120		2	180		2	235
3	180		3	220		3	265
4	240		4	260		4	295
5	300		5	300		5	325
6	360		6	340		6	355
7	420		7	380		7	385
8	480		8	420		8	415
9	540		9	460		9	445
10	600		10	500		10	475
11	660		11	540		11	505
12	720		12	580		12	535

Figure 14: Cost of each DJ from one hour through twelve hours

A second approach to solving this problem is to represent each DJ's rate as a linear function on a coordinate graph. To graph the cost of each DJ, each pair of values in Figure 14 can be represented as a location on the coordinate plane (hours, dollars). For each DJ, these points could be plotted on a graph and then connected to form three straight lines, each line representing the increasing cost of one DJ as dependent on the number of hours he plays.

A third approach to solving this problem is to write three algebraic equations to represent each of the DJ's costs from hours one through twelve. Each equation would then be graphed on the same coordinate grid. The point of intersection would show the hour in which all three price quotes would be equal, then after that, the line that is lowest (closest to the x-axis) would show the most economical of the three DJs.

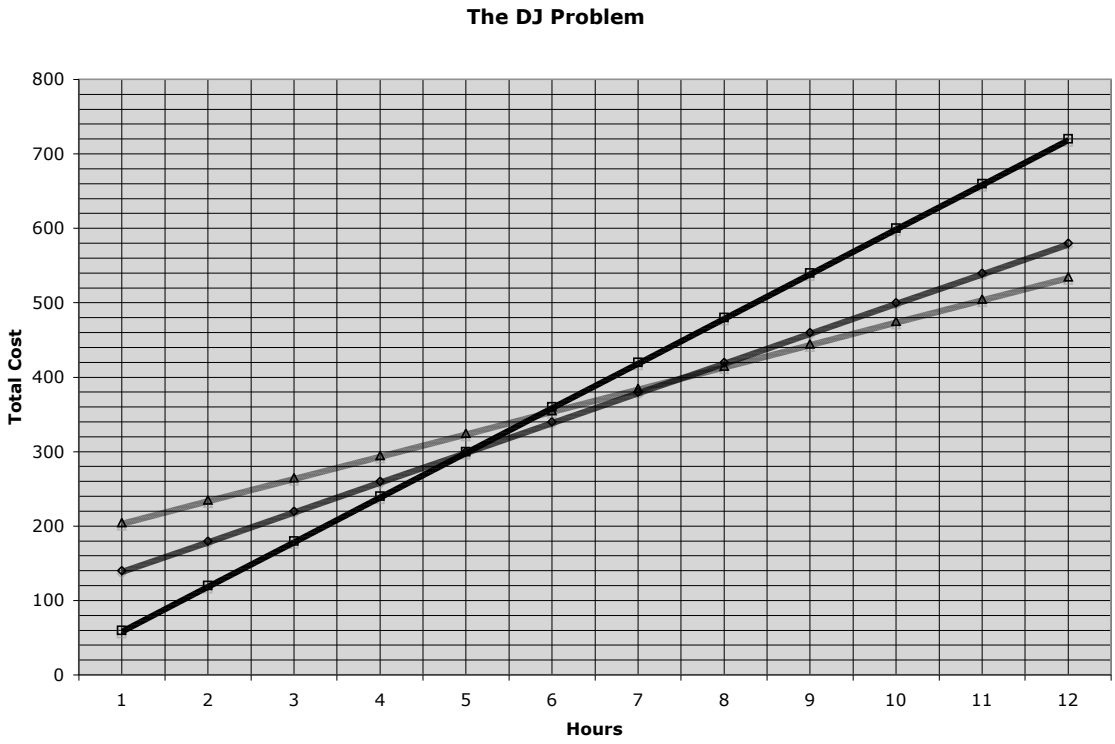

Figure 15: Graphical comparison of three DJ rates from one through twelve hours

In both classrooms, the students work in small groups of three to five students to solve the mathematical problem posed that day. As the students work on the problem, Ms. E and Ms. B circulate throughout the classroom and engage in mathematical interactions with their students. This research study examines these teacher/student(s) mathematical interactions with a particular focus the teachers' affective language and actions. Complete and chronological transcripts of the fifty-seven episodes are in the Appendix.

Table 4.1: Listing and location of chronological transcripts

Teacher	Location of Transcripts	Lesson Title	Number of Episodes
Ms. E	Appendix A.1	"Adding and Subtracting Fractions"	14
	Appendix A.2	"Finding Areas and Other Products"	7
Ms. B	Appendix B.1	"Fences for Grazing"	18
	Appendix B.2	"The DJ Problem"	17

The analysis and coding of the teachers' affective language and actions resulted in patterns of teacher moves that are inferred to foster and support (and at times impede and discourage) student mathematical engagement. These patterns are organized into five themes, which are discussed in the following section.

4.4 Themes

I have identified five themes that emerge from the coding of teacher affective language and actions during teacher/student(s) interaction episodes: (1) teacher interest in student mathematical engagement, mathematical representations, and or mathematical reasoning; (2) teacher validation of student input; (3) teacher promotion of autonomy, (4) teacher promotion of mathematical discourse and (5) teacher promotion of respect. Each theme is introduced and discussed with examples of language and/or actions drawn from the data that are representative of the specific theme. For each teacher the findings of each theme in the classroom observation data are summarized in two tables, one for each lesson. For each teacher, both tables summarize where the specific code for that theme was applied (or not) in every teacher/student(s) interaction episode. Following the summary tables for each teacher, I provide particular cases that are illustrative of the specific theme, presented as excerpts of teacher/student(s) interaction episodes. The episode number of each particular case included in the discussion is underlined in the summary table. Within the text of each excerpt I underline specific actions and/or language according to the application of the code and beneath each excerpt I note the specific code that was applied to the interactions. Teacher/student(s) mathematical interactions are complex and multifaceted and within one episode it is common for more than one category of teacher language and actions to be coded simultaneously. In the analysis of the classroom observation transcripts, I coded each teacher utterance and/or move according to my coding scheme for affective language and actions. Alongside the transcribed data, I indicated the codes and supported the codes with descriptions of my interpretations of each teacher's language and actions in the context of the social and

mathematical interaction in each instance. The complete analysis of all fifty-seven episodes is presented in the appendix (see Table 4.1) and the episodes are organized in chronological order as they occurred in each classroom. In the presentation of the findings, I choose to highlight one theme within each excerpt independent of others codes that may be noted in the complete analysis.

4.4.1 Teacher expresses interest in student(s)' mathematical engagement, reasoning and representation(s)

In the analysis of the teacher/student(s) mathematical interaction episodes, the code, "interest" is applied anytime it is explicit or it can be inferred that teacher demonstrates interest and/or curiosity in the students' mathematical engagement, mathematical reasoning and/or mathematical representations. Through their language, Ms. E and Ms. B express curiosity about student mathematical engagement with general questions for example: "What are you working on?" (Appendix A.1, E.01.a.01, 11:10), "Okay, so what's up?" (Appendix B.1, B.01.a.02, 08:59) and "What are you guys doing now?" (Appendix B.1, B.01.a.03, 12:40) Ms. E and Ms. B show interest in the students' mathematical reasoning by asking the students to explain the reasons for their mathematical "moves" for example: "Why did you do that?" (Appendix A.1, E.01.a.03, 15:10), "What do you mean, multiply the outside?" (Appendix B.1, B.01.b.03, 07:21) Similarly, both teachers express interest in students' mathematical representations and frequently ask their students to explain or to produce representations. For example, "Isn't three-eighths bigger than three-sixteenths? Can you show us that?" (Appendix A.2, E.02.b.02, 29:08), "So how did you get these pictures here?" (Appendix B.1, B.01.a.07, 35:58) "Why are you putting in those circles?" (Appendix B.1, B.01.a.03, 12:49) In these

instances, the teachers' language is interpreted to convey interest in the students' reasoning, which supports the representation. "Interest in student representation" is coded concurrently with "interest in student reasoning" in all episodes except for one (E.01.b.04).

Teacher interest is also coded based on inferences I make from the teachers' tones of voice, body language and gestures on the occasions when they lean on the table (i.e., Appendix A.1, E.01.a.03, 15:08; Appendix B.1, B.01.a.01, 05:13), or sit down at the table with the students (i.e., Appendix A.1, E.01.a.05, 22:16; Appendix B.2, B.02.a.04, 17:49), or stand in close proximity to the students (i.e., Appendix A.1, E.01.a.02, 13:33; Appendix B.1, B.01.b.11, 38:59). Both teachers frequently gesture toward the students' work either on their papers or on the calculators when questioning them about their reasoning (i.e., Appendix A.1, E.01.a.03, 15:23; Appendix B.1, B.01.a.04; 19:58). In sections 4.4.1.a, and 4.4.1.b I provide excerpts from the classroom observation data, which are illustrative of Ms. E and Ms. B, respectively showing interest through their language and actions. Underlined phrases provide the most specific basis for the coding of "interest". The titles of each episode discussed here are underlined in the theme tables.

4.4.1.a Teacher Interest: Ms. E

The coding of all teacher/student(s) mathematical interaction episodes reveals that Ms. E shows interest in students' mathematical engagement, mathematical reasoning and/or mathematical representation in twenty of the twenty-one episodes. Tables 4.2 and 4.3 summarize the coding of "interest" in all twenty episodes; evidence of interest in engagement (Y), evidence of interest in mathematical reasoning (rs), evidence of interest in mathematical representation (rp). In the following discussion, I present two examples

from the classroom observation data; the first excerpt illustrates a case when Ms. E does not overtly show interest in the students' reasoning, the second excerpt illustrates how Ms. E's expresses interest in student mathematical engagement, reasoning and/or representation.

Table 4.2: Teacher Interest, Ms. E, "Adding and Subtracting Fractions"

	E.01.a.01	E.01.a.02	E.01.a.03	E. 01.a.04	E.01.a.05	E.01.a.06	E.01.a.07	E. 01.a.08	E.01.b.01	E.01.b.02	E.01.b.03	E.01.b.04	E.01.b.05	E.01.b.06
Coding for Teacher Interest	Y rs rp	Y rs	Y rs rp	Y	Y rp rs	Y rs rp	Y rs rp	Y rs rp	NE	Y rs rp	Y rp rs	Y rp	Y rs rp	Y rs

Y = yes, rs = interest in student reasoning, rp = interest in student representation, NE = not evidenced

Table 4.3: Teacher Interest, Ms. E, "Finding Areas and Other Products"

	E.02.a.01	E.02.a.02	E.02.a.03	E.02.b.01	E.02.b.02	E.02.b.03	E.02.b.04
Coding for Teacher Interest	Y rs rp	Y rs rp	Y rs rp	Y rs	Y rs rp	Y rp	Y rs rp

Y = yes, rs = interest in student reasoning, rp = interest in student representation

Example 1: Teacher interest in student(s)' mathematical engagement, reasoning and representation(s)

Episode E.01.b.01, (Appendix A.1) (the one episode where Ms. E did not overtly show interest), takes place in the first few moments of the second day of the lesson "Adding and Subtracting Fractions". On the first day of this lesson, Ms. E had imposed a subtask to the problem by instructing the students to represent a square section of land on

grid paper given the condition that the land covered six hundred forty acres. This task poses the cognitive challenge that six hundred forty acres is not a perfect square. For a complete discussion of this subtask, see section 4.B.1.b (pg. 8). It is likely that Ms. E made the assumption that her students would be able to solve this subtask relatively easily and then move on to main problem as stated in the book, to name each property within the square section of land as a fraction of the whole section.

In episode E.01.b.01 (Appendix A.1) Ms. E begins the class by resolving the subtask for the students. Ms. E stands at the front of the classroom, next to the overhead projector and the students are seated at tables in groups of three to four. Ms. E leads a whole class presentation describing how the students should draw the square section on the grid paper. In this presentation, Ms. E imposes her own mathematical ideas to resolve the students' difficulty with the subtask. It is inferred that Ms. E's intention is to put the students on the right track to naming the fractional parts of Section 18 and she does not overtly express any interest in the mathematical reasoning (her own nor the students') that explains why six hundred forty acres can be represented as an eight by eight square on the grid paper.

2:10 Ms. E: What is there on the words to tell you that the shape we're using today is a square? I may be wrong so correct me if I am.
Read part of the problem that tells me …
[Manuel C and Noah are swatting at each other with their arms.]
Can you handle yourself? [Manuel C], [Noah]?
Page forty-three, which part of the problem tells me that section eighteen is a square? [Ms. E writes on the overhead projector.]
2:38 Manuel D: Oh I know…
2:38 Ms. E: Yes? [Ms. E nods her head toward Manuel D.]
2:38 Manuel D: Here it says, each section is a square that is one mile long.
2:43 Ms. E: Each section is a square that is one mile long. So we're going to do, for today, its going to be eight across and eight down. Now, this is eight across and eight down, how big should that Lapp's land be, how many boxes? [Eric]? How

many across, how many down? [Manuel C and Dave raise hand] Not sure? Okay, yes, [Dave?].

3:13 Dave: Sixteen.

3:15 Ms. E: Sixteen. [Ms. E writes on the overhead projector.] And how many would that be across and how many would it be down?

3:18 Dave: Four by four.

3:20 Ms. E: Four by four? Okay one, two, three, four one, two, three, four. How about Bouck?
Bouck.
[Manuel D and Dave raise their hands.]
B-o-u-c-k, yes? Two by two? All right, so we'll start with these two and then divide the land according to the people that own them. Any question? By the end of the period, which is very short, by the end of the period I should have a fraction name from each one of these.

Evidence of teacher interest in student reasoning and student representation was <u>not</u> determined.

(Appendix A.1, E.01.b.01)

In this episode, Ms. E engages students in an "initiation, reply, evaluation", (IRE) (Mehan, 1979) interaction with her students about the solution to the subtask. One interpretation could be that Ms. E is expressing interest in the students' understanding of the subtask. However I interpret this discussion as showing no evidence of teacher interest in student reasoning or representation because Ms. E does not discuss or address the reasons why she has determined that Section 18 can be drawn as an eight by eight square on the grid paper.

Example 2: Teacher interest in student(s)' mathematical engagement, reasoning and representation(s)

Episode E.01.b.02 (Appendix A.1) takes place immediately following E.01.b.01 after Ms. E has instructed the whole class to draw an eight by eight square on their grid paper. Manuel D and Dave are trying to assign fraction names to the individual properties within Section 18. It is inferred that Ms. E recognizes that Manuel D is

experiencing impasse because she sits down next to Dave and intervenes expressing interest about how Manuel D would represent and name fractional parts in an area model.

Episode E.01.b.02 (Appendix A.1) begins when Ms. B sits down at Manuel D's table and expresses interest in the students' knowledge of fractions when she asks, "Give me an example of a fraction." Manuel D responds by asking Ms. E, "This one should be, four over four?" It is inferred that Manuel D is referring to Lapp's property, which is one-fourth of Section 18. Dave points out that four over four is equal to one whole and then Manuel D responds, "Isn't this whole thing a whole?" Manuel D is likely referring to all of Section 18 because he covers his drawing with the palm of his hand. Ms. E expresses interest in Manuel D's reasoning when she asks, "what would you call this instead?"

1:48 Ms. E: So what would you call this instead?
[Ms. E raises her voice to address students at a different table.] Fraction names, we're looking for fraction names. Each portion has to have a fraction name.
[Ms. E returns her attention to Manuel D and asks,]
So what do you call this?
1:59 Manuel D: A whole.
2:00 Ms. E: Huh?
2:01 Manuel D: A whole. [Manuel D shrugs his shoulders.]
2:02 Ms. E: A whole? [Ms. E asks Dave] Can you call it a whole? Do you agree with him?

Coded for teacher interest in student mathematical reasoning and interest in student mathematical representation.

(Appendix A.1, E.01.b.02)

Next, Ms. E expresses interest in Manuel D's mathematical reasoning when she asks, "So, what is it?" Next, Manuel D exhibits frustration when he says, "damn it" and Ms. E intervenes showing interest in how Manuel D would draw a representation of any fraction.

2:40 Ms. E: So, what is it? [Ms. E is leaning on the table with her arms crossed.]
2:44 Manuel D: One, two, three, four. No, damn it! One, two, three, four.

2:54 Ms. E: Okay, umm, on the other side of this paper, can you show me how you name a fraction? Name a fraction. Any fraction.
3:01 Dave: Half.
3:02 Ms. E: Half, okay, show me half. Draw a half. Show me.
3:08 Dave: It would be point five which equals half.
[Dave is looking at his paper as he speaks.]
3:11 Ms. E: Okay…why is the numerator one?
3:14 Manuel D: Umm, because that's one out of two.
3:18 Ms. E: One out of two [Ms. E gestures with her pointed finger] Can you show me another fraction? Another fraction. [Ms. E points at Manuel D's paper] What's this?
3:26 Manuel D: Four over four.
3:28 Ms. E: Four over four? Could you show me four over four? [Manuel D draws]
and? …[Ms. E points to Manuel D's paper]
How do you know what the numerator is?
3:40 Manuel D: Because each one of them have um a cover.
3:47 Ms. E: Uh haa
3:48 Manuel D: Each one of them have this cover.
3:50 Ms. E: Shade, okay, cover, okay. And how do you know this is four?
3:53 Manuel D: Because this is four squares.
3:55 Ms. E: Can you give me another, uh, fraction with four as a denominator? [Ms. E points to Manuel D's paper]
4:00 Manuel D: What do you mean? [Manuel D furrows his brow.]
4:01 Ms. E: Another fraction, not four four but where the four is the denominator.
[Manuel D writes on his paper.]
Okay, how would two four look like?
4:08 Manuel D: Umm… [Manuel D writes on his paper]
4:12 Ms. E: Why is this a two? [Ms. E points to Manuel D's paper, she continues to point to his paper through 4:27.]
4:15 Manuel D: Umm, because two of the windows are shaded. [Manuel D raises his eyebrows]
Or this could also be half. Couldn't it?
4:22 Ms. E: Is it?
4:24 Manuel D: It is because…half of four.
4:25 Ms. E: Okay, two are shaded and why is this four?
4:27 Manuel D: Because there are four windows.
4:29 Ms. E: Okay [Ms. E turns Manuel D's paper over to show his drawing of Section 18.] so what's the name for Lapp's land?

Coded for teacher interest in student mathematical reasoning and student mathematical representation.

(Appendix A.1, E.01.b.02)

Next, Manuel D counts the grid squares within Lapp's land "This would be…

one, two, three… twelve, thirteen, fourteen, fifteen, sixteen... so if there's sixteen, four

over sixteen." He states that the fraction name for Lapp's land is "four over sixteen". Although this is numerically correct, Manuel D appears to have presented faulty mathematical reasoning because he has counted each of the sixteen squares in Lapp's land, says, "there's sixteen" and writes sixteen as the denominator. At this point, it is inferred from Manuel D's language that he does not consider Section 18 as "the whole"; he counted the sixteen squares in Lapp's property and applied sixteen as the denominator of the fraction to represent Lapp's property. Because Manuel D does not explain his reasoning, it cannot be determined from the data how Manuel D determined "four" for the numerator. It is inferred that Ms. E does not recognize Manuel D's faulty reasoning because he has presented a correct numerical answer. The camera does not show Ms. E's face, so it is possible that she was not watching Manuel D as he counted the sixteen squares. It is also possible that Ms. E watched Manuel D count the squares, but did not understand his faulty reasoning.

Ms. E then expresses interest in Manuel D's reasoning for naming Lapp's land "four over sixteen" and Manuel D begins to express doubt in his answer.

5:04 Ms. E: <u>Why sixteen?</u>
5:07 Dave: Because it's sixteen square miles.
5:09 Ms. E: Sixteen squares. Sixteen square miles.
5:11 Manuel D: But then…
5:12 Dave: Because one square equals a square mile, right?
5:16 Ms. E: <u>Okay, and why, why, why four? You said four out of sixteen.</u>
5:23 Manuel D: Now I am changing my mind, [Manuel D puts his pencil to his lips] none of them are shaded or something. It should be like zero over sixteen.
5:29 Ms. E: So shade Lapp's land. Go ahead, shade it. Shade Lapp's land. [Manuel D erases and shades] <u>Now that's shaded, what do you call it</u>?

Coded for teacher interest in student mathematical reasoning and student mathematical representation.

(Appendix A.1, E.01.b.02)

Episode E.01.b.02 (Appendix A.1) is illustrative of how Ms. E's interest in Manuel D's mathematical representation and mathematical reasoning fosters his engagement with the mathematics. Ms. E's expressions of interest in Manuel D's mathematical reasoning and representation appear to foster his engagement with the mathematics because he answers each of her questions which she poses rapidly, one after another. Also, Ms. E's expressions of interest, "why four" may have triggered a switch in Manuel D's thinking when he changes his mind about his answer (5:23). Manuel D presents faulty mathematical reasoning, which is not addressed by Ms. E. Instead, Ms. E once again expresses interest in Manuel D's reasoning when she asks him to record his work on a transparency, likely to present to the whole class at a later time.

4.4.1.b Teacher Interest: Ms. B

The code, "interest" was applied in Ms. B's classroom observation video analysis in each of the thirty-five teacher/student(s) mathematical interaction episodes) when it is inferred that Ms. B demonstrates explicit interest in student mathematical engagement (Y), mathematical reasoning (Yrs) and/or mathematical representation (Yrp) (Tables 4.4 and 4.5). The following discussion highlights excerpts from two episodes that are illustrative of Ms. B showing interest in student mathematical engagement mathematical representation and/or mathematical reasoning (B.01.a.02 and B.01.a.06). Underlined phrases provide the most specific basis for the coding of Ms. B's interest in student mathematical reasoning and Ms. B's interest in student mathematical representation.

Table 4.4a: Teacher Interest, Ms. B, "Fences for Grazing", Day 1

	B.01.a.01	B.01.a.02	B.01.a.03	B. 01.a.04	B.01.a.05	B.01.a.06	B.01.a.07	B. 01.a.08
Coding for Teacher Interest	Y	Y rs rp	Y rs rp	Y rs rp	Y rs	Y rs rp	Y rs rp	Y rs

Y = Yes, rs = interest in student reasoning, rp = interest in student representation

Table 4.4b: Teacher Interest, Ms. B, "Fences for Grazing", Day 2

	B.01.b.01	B.01.b.02	B.01.b.03	B.01.b.04	B.01.b.05	B.01.b.06	B.01.b.07	B.01.b.08	B.01.b.09	B.01.b.10	B.01.b.11
Coding for Teacher Interest	Y rs	Y rs	Y rs	Y rs	Y rs	Y rs	Y rs	Y rs	Y rs	Y rs	Y

Y = Yes, rs = interest in student reasoning, rp = interest in student representation

Table 4.5a: Teacher Interest, Ms. B, "The DJ Problem", Day 1

	B.02.a.01	B.02.a.02	B.02.a.03	B.02.a.04	B.02.a.05	B.02.a.06	B.02.a.07	B.02.a.08	B.02.a.09
Coding for Teacher Interest	Y	Y rs	Y rp rs	Y rs rp	Y rs	Y rs rp	Y rs rp	Y rs	Y

Y = Yes, rs = interest in student reasoning, rp = interest in student representation

Table 4.5b: Teacher Interest, Ms. B, "The DJ Problem", Day 2

	B.02.b.01	B.02.b.02	B.02.b.03	B.02.b.04	B.02.b.05	B.02.b.06	B.02.b.07
Coding for Teacher Interest	Y Rs rp	Y rs	Y rs	Y rs	Y rs rp	Y rs	Y rs rp

Y = Yes, rs = interest in student reasoning, rp = interest in student representation

Example 3: Teacher interest in student(s)' mathematical engagement, reasoning and representation(s)

Episode B.01.a.02 (Appendix B.1) begins approximately eight minutes in to the first class period of "Fences for Grazing". Ms. B has just finished introducing the problem to the whole class and the students have been working in their small groups for approximately four minutes. Immediately prior to the episode, both Ghee and Shawn have their hands raised for Ms. B's attention. Shawn tells Ms. B that he needs a pencil and it is inferred that Ghee may think he is finished with the problem and is raising his hand to show his solution to Ms. B. She expresses interest in the students' mathematical engagement when she asks, "yeah, you're done?" Ms. B expresses interest in the

students' reasoning when she asks, "so what do you guys have to do?" Ghee responds, "there's a hundred feet of fencing". Ms. B expresses interest in Ghee's mathematical reasoning when she asks him to explain what that means by "a hundred feet of fencing". Ms. B also expresses interest in how the students would represent one hundred feet of fencing.

8:59 Ms. B: Ok, so what's up? [Ms. B looks around at each member of the group]. Yeah you're done.
9:01 Dana: We drew a rectangle.
9:05 Ms. B: Ok, you drew a rectangle.
[Ms. B looks at each student.]
9:11 Ghee: ...maximum area...(inaudible)
9:13 Ms. B: Ok. Ok. [Ms. B looks down at a paper on the desk.]
9:19 Ms. B: Ok, ok. So what do you guys have to do?
9:20 Ghee: I know there is a hundred feet of fencing...
9:24 Ms. B: So what does a hundred feet of fencing mean? [Ghee keeps speaking too quietly for the camera. Dana also speaks, but her words are inaudible. Ms. B looks at Dana as she speaks.]
9:30 Ms. B: Okay, I'm just asking. [Ms. B's posture changes, she sits up slightly.]
[Ms. B leans in and gestures with her hand toward Ghee and asks:]
How can you show me that?

Coded for teacher interest in mathematical engagement and teacher interest in student mathematical reasoning and mathematical representation.

(Appendix B.1, B.01.a.02)

Ghee says something to Ms. B, which is inaudible, and it is inferred that Ms. B's response, expresses interest in Ghee's mathematical reasoning about his representation.

10:58 Ms. B: Can you explain to me why you did that?
[Ms. B directs this question to Ghee. She furrows her brow and cranes her neck to look at Ghee's work.]
11:00 Ghee: ... a hundred...and I found out that the sides have the same length, the top and the bottom, and the sides...the sides...[inaudible]
11:28 Ms. B: Okay…[Ms. B looks down at the paper in front of her.] Okay, and so what did you just find?
[Ms. B furrows her brow.]
11:32 Ghee: The area...(inaudible)

Coded for teacher interest in student mathematical reasoning and mathematical representation.

(Appendix B.1, B.01.a.02)

For the next few minutes of this episode, both Ghee and Dana engage in a mathematical discussion with Ms. B but they present faulty reasoning, defining perimeter as the "outside". Ms. B intervenes by reflecting back on an earlier lesson, "Do you remember when we talked about perimeters, what word …helped us out?" Ghee recalls that the word, perimeter contains the word, "rim", which helps him to recall that perimeter is the "outside" of a figure and the area is the "inside".

Ms. B's interactions with the students in this episode foster their mathematical engagement in two ways (1) she questions their reasoning about the meaning of one hundred feet of fencing which may have helped the students realize that they had not finished the problem and (2) Ms. B's expression of interest, "okay, and so what did you just find?" activates a discussion about the difference between area and perimeter. In order for the students to successfully solve "Fences for Grazing" it is imperative that they understand the definitions of area and perimeter and how to calculate the area and the perimeter of any rectangle.

Example 4: Teacher Interest in Student(s)' Mathematical Engagement, Reasoning and Representation(s)

Episode B.01.a.06 (Appendix B.1) occurs on the first day of "Fences for Grazing" in Ms. B's classroom. The students have been working in small groups for approximately twenty-five minutes. Al initiates the interaction when he summons Ms. B to his table. This is the second time that Ms. B has interacted with this group since the beginning of the class period. During an earlier episode not discussed here, (Appendix B.1, B.01.a.03), Al expressed confusion about how representations of rectangles do not need to be drawn to scale to solve this problem. The present episode (B.01.a.06) begins

approximately ten minutes after B.01.a.03 when Ms. B approaches the table and sits down in an empty desk next to Al. Ms. B expresses interest in the students' representations and reasoning about their representations when she asks them to explain why their representations could be answers and when she asks them to "draw it out".

29:32 Ms. B: Oh you found it out? What did you find?
[Ms. B sits down at an empty student desk next to Kenny.]
29:34 Al :We found umm, like it don't matter what size you make it. [Al is looking at Ms. B as he speaks. Ms. B shifts some calculators on the desk then rests her neck on her hand as she looks at Al. Ms. B's face is out of view since the camera is positioned at her back.]
You know how they drew it in the books right? It could be a square, like that big, and they put like seventy-five [Ms. B: "Ok"] or and then and they be like two, but they put like twenty. Well I figured that out, well you can make that half, but then the perimeter outside [Al traces the outline of his rectangle with his ruler.] is still gonna add up to the same thing. [Al raises his eyebrows, nods his head and smiles at Ms. B.]
30:11 Ms. B: What's the same thing?
30:13 Natasha: Like say one hundred, [Natasha points at her paper.] like say you make the box like this small [Natasha draws a small rectangle on a piece of scrap paper.] and then you do eighty and then ten. And just like you did eighty and then ten right there.
30:25 Ms. B: So what's the perimeter right here? [Ms. B points at Natasha's drawing on the scrap paper.]

Coded for teacher interest in student mathematical reasoning and teacher interest in student mathematical representation.

(Appendix B.1, B.01.a.06)

Natasha responds explaining to Ms. B that the lengths of the sides need to add up to one hundred. As she explains her reasoning to Ms. B, Al points out that Natasha has drawn a square and it is inferred that both Al and Natasha have reasoned that the square representation cannot be used because it does not meet the criteria for a rectangle.

From the video, it cannot be determined what Al has written on his paper. Ms. B expresses interest in Al's representation as she points at his paper and asks if he has a possible answer.

31:39 Ms. B: So is this a possible answer right here? [Ms. B points at Al's paper.]
31:40 Natasha and Al: Yeah!
31:43 Ms. B: Why?
31:45 Natasha:[Al- "it's outside of it"] And for me, [Al – "it's a hundred."]
I don't think there is no right or wrong answer because anyone can say anything, like they can say, ummm, [Natasha looks at Al, shakes her head, smiles then looks back at Ms. B] …they can probably say twenty plus forty or they can even say umm twenty-five, twenty five, like that, umm. Oh, no but it's like that.
[Al is leaning back in his chair, rubbing a ruler on the back of his head.]
32:07 Ms. B: What do you mean, I just need you to draw it out?
32:12 Natasha: Like what?
32:13 Ms. B:I don't know, whatever you just said, [Ms. B points at Natasha's paper] can you repeat it and draw it? You just said like twenty-five and twenty-five.
[Natasha smiles.]
32:18 Natasha: Yea, I said, let's say…let's say someone if put twenty-five and twenty-five for each side, but then I said no because…
32:28 Ms. B: Why would you put twenty-five for each side?

(Appendix B.1, B.01.a.06)

Coded for teacher interest in mathematical representation and mathematical reasoning.

Next, Natasha explains to Ms. B that twenty-five for each side is not a possible answer (even though it is the correct answer) and it is inferred that Natasha thinks that the length and width of a rectangle cannot be the same measure. Ms. B expresses interest in Natasha's reasoning but she does not pass judgment on or correct Natasha.

In this episode, Ms. B's expressions of interest are inferred to show foster Al's and Natasha's mathematical engagement when she asks what they "found out". Through her language it is inferred that Ms. B shows interest in the students' reasoning by asking Natasha and Al to defend their reasoning when she asks them to draw a representation to explain what they mean. Al and Natasha respond to Ms. B's questioning by engaging in a

mathematical discussion, which fosters their progress toward solving the mathematical problem.

4.4.2 Teacher Validation of Student Mathematical Reasoning

In the analysis of classroom observation data, the code "validation" is applied any time it is explicit or it can be inferred that the teachers agree with or show approval of a student's mathematical reasoning. Through their language, Ms. E and Ms. B validate students' mathematical ideas, questions and answers with their language for example: "It's very important what you're saying." (Appendix E.2, E.02.b.01, 8:59) "That's a good question" (Appendix B.2, B.02.a.03, 17:16).

Teacher validation is also coded based on inferences I make from the teachers' gestures, body language and tone of voice on the occasions when they nod their head or raise their voice in response to a student's mathematical idea, spoken or written (Appendix A.1, E.01.a.03, 15:47,15:55; Appendix B.2, B.02.a.03, 16:48) or point at a student in response to a student's idea and/or answer (Appendix B.2, B.02.a.01, 00:19) or at a student's work. These inferences multidimensional and are made based on the context in which the validating language and actions are inferred. For example, a teacher may nod her head for other reasons than validation, such as encouragement or nodding in agreement if a student asks for permission to take a new piece of graph paper.

4.4.2.a Teacher Validation: Ms. E

The analysis of all teacher/student(s) mathematical interaction episodes reveals that Ms. E validates her students' mathematical reasoning in thirteen out of twenty-one episodes. Tables 4.6 and 4.7 summarize the coding of "validation"; evidence of teacher validation (Y) and evidence of teacher validation of faulty reasoning (Yf) and no evidence of teacher validation (NE).

In the following discussion, I present two examples of cases from the classroom observation data. The examples illustrate, in context, how Ms. E validates her students' mathematical ideas. The first example takes place during the first day of "Adding and Subtracting Fractions" (Appendix A1, E.01.a.04). Ms. E validates N'yette's faulty reasoning. The second episode occurs during the same lesson, (Appendix A1, E.01.a.05) when Ms. E validates Juan's correct mathematical reasoning.

Table 4.6a: Teacher Validation, Ms. E,
"Adding and Subtracting Fractions", Day 1

	E.01.a.01	E.01.a.02	E.01.a.03	E. 01.a.04	E.01.a.05	E.01.a.06	E.01.a.07	E. 01.a.08
Coding for teacher validation	NE	NE	Y	Yf	Y	Y	NE	Y

Y = yes, Yf = yes, faulty, NE = not evidenced

Table 4.6b: Teacher Validation, Ms. E,
"Adding and Subtracting Fractions", Day 2

	E.01.b.01	E.01.b.02	E.01.b.03	E.01.b.04	E.01.b.05	E.01.b.06
Coding for teacher validation	Y	Yf	NE	NE	Y	NE

Y = yes, Yf = yes, faulty, NE = not evidenced

Table 4.7 Teacher Validation, Ms. E,
"Finding Areas and Other Products", Days 1 & 2

	E.02.a.01	E.02.a.02	E.02.a.03	E.02.b.01	E.02.b.02	E.02.b.03	E.02.b.04
Coding for teacher validation	Y	NE	NE	Y	Y	Y	Y

Y = yes, Yf = yes, faulty, NE = not evidenced

Example 1: Teacher validation of student reasoning

In TSMIE E.01.a.04 (Appendix E.1), N'yette and Tina are trying to name the fractional parts of Section 18, a square section of land (see "Adding and Subtracting Fractions"). Immediately prior to this interaction, Ms. E asks N'yette to choose and draw a representation of "any fraction". N'yette correctly represents one-third by drawing a rectangle with three close to equal parts (she does not use a measuring tool to make precise partitions), and she shades in one of the three parts. Ms. E validates N'yette's correct representation by nodding her head. Next, Ms. E asks N'yette to apply this reasoning to assigning fractional values to Lapp's property in Section 18. N'yette draws five equal sections within Lapp's property, shades in one of the five sections and then names Lapp's property as "five and one-half" of the total section of land. This is faulty for at least two reasons (1) N'yette did not name the fractional value of Lapp's property within Section 18 (one-fourth) and (2) N'yette shaded one-fifth of the area of Lapp's property and named it "five and one-half". Ms. E asks N'yette's partner, Tina for her opinion and Tina agrees with N'yette's faulty reasoning. It is inferred that Ms. E then validates N'yette's answer by suggesting that the girls write their answer on their papers,

pauses momentarily, and then briefly questions the girls' reasoning before walking away from their table.

19:55 Ms. E: Okay, [Ms. E makes eye contact with Tina, nods her head slightly then looks back at Tina's paper.]
So if you would name one of those parts. Turn your, turn it please, say for example Lapp, if you, you would name Lapp what would it be?
20:12 N'yette: [Ms. E flips over Tina's paper] That's one third?
20:12 Ms. E: Uhh um [agreeing].
20:14 N'yette: And you want to name that?
20:20 Ms. E: Uhh um...what do you need to do?
20:21 N'yette: Ohhh, we got to look at it like this. [N'yette gestures with her pencil an up and down motion on Tina's drawing of the section of land.]
20:23 Ms. E: What do you mean?
20:26 N'yette: You know like when you do the first one, you did that, [N'yette flips Tina's paper over and writes on the back of the paper.] instead you do it like this.
20:30 Ms. E: Could you do it on that side, please? Could you do it on this side?
[Ms. E turns Tina's paper back over to show Tina's drawing of the section of land.]
20:33 N'yette: You draw like a big box and just do…cut it in half
20:37 Ms. E: Go ahead. [Ms. E nods her head at N'yette. She remains in the same position, leaning on the table.]
20:41 N'yette:[N'yette draws what appears to be a representation of fifths on Tina's paper in the section assigned to Lapp.]
… cut it in half, like that, and then after I cut it in half you would do, shade it
20:52 Ms. E: Okay…
20:55 N'yette: And you could probably shade half of that, [N'yette shades in one fifth of the section of Lapp's land on Tina's paper.] to make it like that, or just …
20:59 Ms. E: And what would you name this [Ms. E points to the area on the section of land that represents Lapp on Tina's paper.] if you did that?
21:10 N'yette: [N'yette is counting to herself] One out of five, one half out of five
21:12 Ms. E: What is one out of five, what is one?
21:15 Tina: One half.
21:16 N'yette: One half, it would go like this five and one half.
21:19 Ms. E: Five and a half? [Ms. E looks at N'yette.]
What do you think? [Ms. E looks at Tina.]
21:21 Tina: I think the same thing.
21:22 Ms. E: So you would name this five and a half? [Ms. E gestures with her hand on Tina's paper, the section that is owned by Lapp.]
21:24 Tina: Yeah.
21:25 Ms. E: You want to write that down?
[Tina and N'yette begin to write on each of their papers.]
21:27 N'yette: See we should have…(inaudible)…I'm about to do the same thing.
21:30 Ms. E: Why is it, why is it five and a half, I don't see why it's five and a half.
[Ms. E scrunches her face.]

21:34 N'yette: Because the way we did it.
21:40 Tina: [inaudible] Pieces….they said that each person gets one square, [Tina turns back to her math book and begins reading from the task statement] there are six-hundred forty acres of land in one square mile section.
21:55 Ms. E: Uh umm.
22:57 Tina: So…that…and the reason why they get one half is because…
22:14 Ms. E: Can you think about that? I'll be right back.

Coded for teacher validation.

[Appendix A.1, TSMIE E.01.a.04]

Ms. E does not address this problem with Tina and N'yette before the end of this lesson. Shortly after this interaction Juan, who is seated at a different table, offers the correct answer to the subtask (E.01.a.05, Appendix A.1). Juan's breakthrough is evidenced in "Teacher Validation, Example 2". Ms. E invites N'yette and Tina to consider Juan's solution to the subtask when she asks them to join Juan, Dave and Manuel D and she asks Manuel D to explain his (Juan's) reasoning to N'yette and Tina. When N'yette does not agree with Manuel D's (and Juan's) reasoning, Ms. E gives N'yette permission to continue with her (faulty) reasoning by suggesting, "Maybe you could continue doing this?" [Appendix A.1, E.01.a.06, 29:40 (L)].

Example 2: Teacher Validation of Student Reasoning

During episode E.01.a.05 (Appendix A.1) Ms. E is seated next to Dave and directly across the table from Juan. This interaction takes place approximately twenty-three minutes into the class period and up until this point, there is evidence that Juan has not spoken or interacted with anyone else during this class period. Ms. E is engaged in a mathematical discussion with Dave and Manuel D about how to represent Section 18 on square grid paper (see "Adding and Subtracting Fractions"). Both Manuel D and Dave are struggling with this task and Manuel D has reasoned that if each small square is assigned a value of ten, then Section 18, which covers an area of six hundred forty acres,

can be represented on grid paper with an area of sixty-four squares. Dave has offered the idea of drawing a sixteen by four rectangle, because the product of sixteen and four equals sixty-four. However, a sixteen by four rectangle does not consider the condition of the problem that Section 18 must be represented as a square.

It is inferred that Juan suggests that Section 18 be represented as an eight by eight square but he inaudible on the videotape. Ms. E appears to recognize that Juan has made a breakthrough and has quietly offered his idea because she becomes visibly excited as she sits up straight in her chair and points toward Juan. Ms. E validates Juan's mathematical thinking starting when she asks him to repeat his idea. This is the first speaking turn that Juan has had during this lesson. Ms. E's validation of Juan's idea marks a pivotal moment in the solution of the subtask for Juan, Manuel D and Dave.

23:12 Ms. E: <u>What…what were you saying?</u>
[Ms. E is looking at Juan <u>and points her finger at Juan.</u>]
23:15 Juan: What to get like sixty-four?
[Juan is looking at Ms. E.]
23:15 Ms. E: Yea...huh.
23:17 Juan: Go eight across and eight down.
23:18 Ms. E: <u>Can, can we try that, eight across and eight down?</u> Would that give you sixty-four?

Coded for teacher validation.

(Appendix A1, E.01.a.05)

Ms. E's validation of Juan's mathematical idea has at least two important consequences for the students' mathematical engagement and their success in solving the problem. First, her validation of Juan's suggestion is pivotal because the boys have now surpassed the conceptual hurdle of drawing Section 18 on the grid paper. If the students proceed with Juan's idea, they can now engage with the "main" task, which is to name each property as a fractional part of Section 18. Second, Ms. E's validation may have

consequences for many other students' mathematical engagement because during the episode immediately subsequent to this interaction, Ms. E invites several students from other tables to join Manuel D, Dave and Juan. She encourages Manuel D to share the representation of an eight by eight square on the grid paper. Ms. E does not ask Juan to explain his reasoning behind choosing an 8 x 8 square and she does not call on Juan again to explain or give his correct answer.

4.4.2.b Ms. B: Validation

The analysis of all teacher/student(s) mathematical interaction episodes reveals that Ms. B validates her students' mathematical ideas in twenty-one out of thirty-five episodes. Tables 4.8 and 4.9 summarize the coding of "teacher validation"; evidence of teacher validation (Y) and evidence of teacher validation of faulty reasoning (Yf) and no evidence of teacher validation (NE). In the following discussion, I present two examples of cases from the classroom observation data, which illustrate, in context, how Ms. B validates her students' mathematical ideas. In one of these episodes (B.02.a.09), it is inferred that Ms. B validates Rita's faulty reasoning, and then Ms. B remains at the table with as Rita corrects her own reasoning.

Table 4.8a: Ms. B, Validation, "Fences for Grazing" Day 1

	B.01.a.01	B.01.a.02	B.01.a.03	B. 01.a.04	B.01.a.05	B.01.a.06	B.01.a.07	B. 01.a.08
Coding for teacher validation	NE	Y	Y	NE	NE	NE	Y	Y

Y = yes, NE = no evidence

Table 4.8b: Ms. B, Validation, "Fences for Grazing" Day 2

	B.01.b.01	B.01.b.02	B.01.b.03	B.01.b.04	B.01.b.05	B.01.b.06	B.01.b.07	B.01.b.08	B.01.b.09	B.01.b.10	B.01.b.11
Coding for teacher validation	Y	Y	Y	Y	Y	Y	NE	Y	NE	Y	NE

Y = yes, NE = no evidence

Table 4.9a: Ms. B; Validation, "The DJ Problem", Day 1

	B.02.a.01	B.02.a.02	B.02.a.03	B.02.a.04	B.02.a.05	B.02.a.06	B.02.a.07	B.02.a.08	B.02.a.09
Coding for teacher validation	Y	NE	Y	Y	Y	Y	NE	Y	Yf

Y = yes, Yf = yes faulty, NE = no evidence

Table 4.9b: Ms. B; Validation, "The DJ Problem", Day 2

	B.02.b.01	B.02.b.02	B.02.b.03	B.02.b.04	B.02.b.05	B.02.b.06	B.02.b.07
Coding for teacher validation	Y	NE	Y	Y	NE	Y	NE

Y = yes, NE = no evidence

Example 3: Teacher validation of student reasoning

In the minutes leading up to episode B.02.a.03 (Appendix B.2) Ms. B asks the students to justify their reasoning for stating that the "second one" is better when she asks, "How do you know the second one is better?" (Appendix B.2, B.02.a.03, 13:08) Ms. B is seated in a chair and as she poses this question, she slides her chair closer to the table and leans her right elbow on OJ's desk. Natasha and Al explain how to calculate the cost of the second DJ for an eight-hour party. Ms. B makes a comment, "Oooh, that's a long party. Okay. Eight hours…" (Appendix B.2, B.02.a.03, 13:11) Next, Al and

Natasha determine that the "third DJ" has the best price for a party that will last eight hours.

At 16:09, OJ says something that is inaudible, Ms. B leans toward him, and it is inferred that OJ suggests the idea of calculating rates for a five-hour party. At 16:21, Ms. B restates OJ's idea, "Ok, okay, so you said party for five hours?" and then publicly validates his idea, offering his suggestion as "a good idea" for the whole group to Meanwhile, Natasha and Kenny are discussing the amount of money they could save by hiring "the third DJ", Light Plastic which costs $415 for eight hours compared to hiring "the second DJ", Solidus Sounds for $420 for a period of eight hours.

16:43 Ms. B: So right now, you're working on for how many hours?
[Ms. B cranes her neck to look toward Al's paper.]
16:47 Student: Eight.
16:48 Ms. B: For eight hours. <u>But [OJ] just said lets be "geeks" '[Ms. B looks towards Natasha, and nods her head] and party for five hours [Ms. B turns and smiles at OJ].</u>
16:55 OJ: She, she…wanted to bring up we should party for less longer. [OJ looks at Natasha and smiles, his head is cocked to the side.]
Am I right?
17:00 Ms. B: So you think we should party longer than eight hours?
[Ms. B sits up in her seat and points towards Natasha.]
17:01 Natasha: No, it don't matter, I'm sayin' that um since it's um since the party's not so high, it don't matter how many hours you gonna party [inaudible]….
[Natasha looks down a her calculator as she speaks, Ms. B is seated with her hands folded, she is looking at Natasha as she speaks.]
17:09 OJ: That long.
17:10 Natasha: A lot. [Natasha looks up at OJ, then looks back at her calculator]
Yeah.
[Natasha raises her voice, she continues to look at her calculator, then looks up at
Ms. B when she finishes speaking.]
I was trying to say like say if he was partyin' for a little bit.
17:14 Al: Wait, wait, Wait!
17:16 Ms. B: <u>So that's a good question.</u>

Coded for teacher validation

(Appendix B.2, B.02.a.03)

Ms. B's validation of OJ's suggestion appears foster student engagement by prompting Natasha and Al to consider calculating DJ rates for hours other than eight hours. This is evidenced in the moments immediately subsequent to episode B.02.a.03 (Appendix B.2) after Ms. B walks away from this group. Natasha, Al, Kenny and OJ discuss the number of hours they want to use to compare DJ rates. Also, Ms. B's next interaction with this group takes place on the second day of the lesson (B.2.b.02, Appendix B.2) and at that time, Natasha, Kenny, Al, Jay and OJ calculate the DJ rates for yet a third time frame (six hours).

Example 4: Teacher validation of student reasoning

Episode B.02.a.06 takes place during the second day of "The DJ Problem" in Ms. B's classroom. Immediately prior to the interaction here, Ghee tells Ms. B "Yup, uhhh, Solidus. Solidus." (Appendix B.2, B.02.a.06, 24:33). Ms. B asks Ghee to justify his reasoning for choosing Solidus Sounds, he begins to explain his reasoning to her. Devin interrupts asking, "Why can't it be ten hours?" and Ms. B validates Devin's suggestion, which appears to have consequences for Ghee's reasoning.

26:07 Devin: [Devin looks at Ghee and says:]
Why couldn't it be like ten hours? Instead of six?
[Ms. B straightens in her chair and turns to glance towards Devin. Ms. B then looks around the table.]
26:09 Shawn: She wanted to, she wanted to do this one, but that's when we found out that.
26:13 Ms. B: Wait real quick, cause [Devin] just whispered something. Let him whisper it again.
[Ms. B puts her hand up to her face as if she is telling a secret as she speaks to Ghee and Lenny. She smiles at Devin and lowers her voice to a near whisper. Devin smirks.]
26:18 Ghee: He said, why couldn't it be ten.
[Ghee looks at Ms. B and begins pushing buttons on a calculator.]
26:24 Ms. B: Did you guys hear what [Devin] just said?
[Ghee, Dana, Shawn and Lenny are all looking at their papers or their calculators.]
[Ms. B smiles as she leans in towards Dana and Shawn. She is smiling.] [Dana], [Shawn]?
[Devin is looking down at his paper and glances up as Ms. B says each student's name.]
26:28 Shawn: Nope.
26:29 Ms. B: [Dana]?
26:29 Dana: I heard him.
[Dana is writing on her paper, her head is turned away from Ms. B.]
26:29 Ms. B: What did he say?
26:30 Dana: Why can't it be ten hours instead of six?
[Dana continues to write on her paper as she answers Ms. B. Her tone of voice has no affect.]
26:34 Ms. B: That is a good question! [Ms. B smirks and smiles.]
Why *can't* it be ten hours instead of six hours?
[Ms. B glances around the table. Dana, Ghee and Shawn are writing on their papers.]

Coded for teacher validation.

(Appendix B.2, B.02.a.06)

Ghee and Lenny challenge the idea of a ten-hour party and argue that it is too expensive and too long to have a party for that number of hours. When Shawn asks who would party for that long, Ms. B becomes very animated, still validating Devin's suggestion.

26:48 Shawn: How, how, how you gonna go for that long?
26:48 Ms. B: Oh I like to party.
[Ms. B opens her eyes wide, smiles and gestures with her hand.]
I'll party for fifteen hours, I'll be there. Fifteen hours!
[Ms. B smiles and shakes her head.]
26:53 Shawn: Daaaag…..it would be super jiving.
[Shawn is smiling]
26:54 Ghee: Yea, I know right.
[Ghee sits back in his chair. He is smiling.]
26:55 Ms. B: That is a good question!
[Ms. B shrugs her shoulders and folds her arms, and raises her eyebrows as she looks at Lenny. Her tone of voice expresses excitement.]

Coded for teacher validation.

(Appendix B.2, B.02.a.06)

It is inferred that Ms. B's validation of Devin's suggestion fosters Ghee's, Lenny's and Shawn's mathematical engagement because it opens a window to consider DJ rates for a different time frame other than six hours. At the beginning of the episode, Ghee, Larry and Shawn had only considered and calculated DJ costs for a six-hour party. When Ms. B validates Devin's idea of considering a ten-hour party, she builds on Devin's thinking and offers another idea for considering how many hours to compare the DJs' rates. Subsequent to this interaction, Ghee changes his original answer and reasons that Light Plastic would be the best DJ as compared to Solidus Sounds, "'Cause, you you

party longer for cheap. You party longer for twenty-five dollars less." (Appendix B.2, B.02.a.06, 28:47).

Example 5: Teacher validation of student reasoning

Episode B.02.a.05 (Appendix B.2) takes place during the first day of "The DJ Problem" in Ms. B's classroom when Ms. B validates Frank's emotions. The coding reveals Ms. B's validation of students' feelings/ emotions during class may foster student engagement with the mathematics. In the first example, (B.02.a.05, Appendix B.2), Ms. B validates Frank's will to not talk because his peers are making fun of the way he speaks. After she validates Frank's feelings, she asks the students to come up with a different way to communicate about the mathematics.

20:46 Ms. B: You said, [a student hands a crumpled paper to Ms. B and Ms. B reads from the paper, she is smiling]
"you not going to talk?" He says no. "They want, they want to make fun of the way I talk?"
[Ms. B pauses and looks across at Frank. Her tone of voice is gentle as she speaks to him.]
<u>So you not going to talk because they make fun of the way you talk?</u>
<u>[Ms. B's tone of voice changes to more lighthearted.]</u>
So, okay, is there any other way we can communicate and get the math done? How else can we communicate? <u>That's fine if you don't want to talk to me, that's fine. You don't have to! [Ms. B has her eyebrows raised and she is smiling at Frank].</u>
21:12 Student: [Frank]!
21:15 Ms. B: Is there another way to get it done? What are your writing thoughts?
[Frank shrugs]

Coded for teacher validation.

(Appendix B.2, B.02.a.05)

Subsequent to this interaction after Ms. B walks away from the table, Frank and Jen engage in an argument possibly about Frank not wanting to speak. It appears that Ms. B's validation did not support the students' engagement, but it may have helped to

set an expectation that the students communicate in any way possible to solve the mathematical problem.

4.4.3 Teacher Promotion of Student Autonomy

Ms. E and Ms. B through their language and actions, encourage their students to solve mathematical problems independent of teacher input. In the analysis of classroom observation data, autonomy support "Promotion of Autonomy" is identified when the teachers' language and/or actions encourage students to solve the mathematical problems independent of the teachers' mathematical input. Autonomy is also coded when the teachers acknowledge or recognize the "students' perspectives and feelings" (Jang, Reeve and Deci, 2010). "Promotion of Autonomy" was not part of the original coding scheme for affective language and actions, however it emerged inductively during the analysis phase as I began to recognize that the teachers repeatedly through their actions and language placed emphasis on student input to make mathematical progress. Thus, support of student autonomy emerged as an important theme for supporting and fostering student engagement in these two middle school classrooms.

Jang, Reeve and Deci (2010) developed and applied a Likert-type scale for evaluating teacher language and behavior in an observational study of high school teachers in English, math, science and social studies classes. Their scale lists three categories of teacher language and behavior that is supportive of student autonomy followed by specific indicators of each category:

- nurtures inner motivational resources:
    - interest, enjoyment, sense of challenge, creates opportunities for initiative;
- informational language:

  - informational, flexible, provides choices, options, identifies value, meaning, use, benefit, importance of requests
- acknowledges & accepts students' negative affect:
  - listens carefully, openly, understandingly, accepts negative affect, complaints are OK. (p.592).

In the analysis of classroom observation data, I use Jang et al.'s (2010) descriptors of the teachers' language and actions to identify the teachers' autonomy supportive language and actions.

During mathematical interactions with their students, Ms. E and Ms. B promote their students' autonomy include: In "Adding and Subtracting Fractions", when Felipe asks Ms. E whether he should use inches or centimeters to measure Lapp's property. Ms. E offers Felipe a choice, "It doesn't matter, which one do you want to use?" (Appendix A.1, E.01.a.02). In "Fences for Grazing", Shay asks Ms. B if each group will have a different answer and Ms. B provides a sense of challenge when she replies, "That's what you're gonna decide and see...I'm not gonna tell you what the other groups have but when you go and visit the other groups, you'll be able to see what they have...and you'll answer your own question." (Appendix B.1, B.01.b.02, 4:05). Underlined phrases provide the most specific basis for the coding of promotion of autonomy. Underlined and bolded phrases provide the most specific basis for the coding of obstruction of autonomy.

4.4.3.a Ms. E: Promotion of student autonomy

In Ms. E's classroom, "Promotion of Student Autonomy" was coded in sixteen out of twenty-one episodes as summarized in Tables 4.9 and 4.10. However, Ms. E's use of autonomy supportive language and actions was found to be inconsistent within many

episodes as Ms. E starts off following the students' initiative but when the student presents faulty reasoning and/or exhibits negative affect, Ms. E intervenes thus diminishing student autonomy.

Table 4.10a: Teacher Promotion of Student Autonomy, Ms. E, "Adding and Subtracting Fractions", Day 1

	E.01.a.01	E.01.a.02	E.01.a.03	E. 01.a.04	E.01.a.05	E.01.a.06	E.01.a.07	E. 01.a.08
Coding for teacher promotion of student autonomy	Y	Y	Y	Y	Y	Y	Y	Y

Y = yes, NE = no evidence

Table 4.10b: **Teacher Promotion of Student Autonomy, Ms. E, "Adding and Subtracting Fractions", Day 2**

	E.01.b.01	E.01.b.02	E.01.b.03	E.01.b.04	E.01.b.05	E.01.b.06
Coding for teacher promotion of student autonomy	NE	Y	Y	Y	Y	Y

Y = yes, NE = no evidence

Table 4.11: Teacher Promotion of Student Autonomy, Ms. E, "Finding Areas and Other Products", Days 1 & 2

	E.02.a.01	E.02.a.02	E.02.a.03	E.02.b.01	E.02.b.02	E.02.b.03	E.02.b.04
Coding for teacher promotion of student autonomy	Y	NE	NE	Y	NE	Y	Y

Y = yes, NE = no evidence

As illustrated in the two examples that follow, Ms. E creates opportunities for students to take initiative in solving the problem, but then when the students appear to experience impasse and/or negative affect Ms. E intervenes, switching to a role that may not be considered supportive of student autonomy but rather controlling. Underlined phrases provide the most specific basis for the coding of teacher promotion of autonomy. Bold underlined phrases illustrate instances when the teacher language and/or action(s) diminish student autonomy. *Example 1: Promotion of Student Autonomy*

Episode E.02.a.01 (Appendix A.2) is the first teacher/student(s) mathematical interaction episode recorded in Ms. E's classroom during the second day of the lesson, "Finding Areas and Other Products". N'yette is standing at the overhead projector and Ms. E is seated at a student desk near the back of the classroom. It appears that N'yette is attempting to present her solution to Problem 5.1A. Immediately prior to this episode, N'yette reads the problem from the book and realizes that she had misinterpreted the conditions of the problem. She draws a square on the overhead projector and then draws two vertical lines to divide the square into three approximately equal sections. N'yette shades two of the three sections.

9:45 Ms. E: <u>What if you *did* cut it in half?</u>
[N'yette has drawn a model of a square brownie pan, cut into thirds with two sections shaded, showing a representation of two-thirds.]
9:49: N'yette: Like this? [N'yette is off camera]
9:51 Ms. E: <u>Yes, like you originally said.</u>
9:52 Dave: You would have to pay more, he would have to pay more, it will be it will be. [Dave and Manuel D turn around in their seats and answer Ms. E in unison.]
9:54 Manuel D: No he would pay less, no he would pay less.
9:55 N'yette: It would be, it would be four out of….
9:57 Manuel D: He would pay, half, he would pay four dollars.
10:00 Dave: Yea but he'll pay less…cause the bigger brownies
10:01 Manuel D: He will pay less.
10:03 N'yette: So the fract…so the fraction if I cut, so the fraction if I cut it in half would be four-sixths.

10:05 Dave: If you cut it into more pieces then the pieces are going to be bigger, and he gonna pay less, but he want more brownies.
10:11 N'yette: That's….That's what I'm saying but [Ms. E] said what if I cut it in that
10:13 Ms. E: [Ms. E is seated in a student desk in the middle of the classroom. She raises her voice and addresses the class.]
But what if I… leave it like that for a second, *what if* I wanted to buy half of that. I cut it in half this way.
10:21 N'yette: Half of this?
10:21 Ms. E: Yeah!
10:24 Ms. E: **Couldn't I do it that way?**
[Manuel D and Dave both say "Yes".]
I'm not buying the whole thing, I just want half of it.
10:29 N'yette: So this is what was bought, so in reality he did buy two out of six!

In response to Ms. E's intervention, Nyette correctly answers the question.

10:31 Manuel D: So you want half….So isn't that, lemme get, he want half of a half….
10:36 Ms. E: Could you explain that?
10:37 Dave: That's four out of six, right there.
10:39 N'yette: Because if he is buying half, I say in reality he is buying, [N'yette waves her pen at Dave as she speaks] one two out of six, because if I look at it this part is bought, right, unless I split it, so [Ms. E] said you have to split it, well what if I split it,
[The camera pans to show the projected image of N'yette's model of the brownie pan, showing two vertical sections shaded in to represent two-thirds, and she has drawn a line horizontally across the middle of the square.]
so I split it, its four out of six, if I split it the fraction is four out of six, BOOM. And remember, this whole side was already bought, this side is already bought, and if I split it, it's two out of six.
11:04 Ms. E: So would I be paying more if I split it in half this way?
11:05 Manuel D: It would be the same...but I don't get, I don't get.
11:06 Dave: You said that like three times already, that's three right there, that's right there. One two three four five six, that's four out of six. Why cut it in half, you don't have to cut it in half.
11:15 N'yette: Because [Ms. E] said "What if you cut it in half?".

N'yette's reasoning is based on what Ms. E told her to do.

11:18:Manuel D: But I don't, but I don't….but look I don't get, I don't get the question that [Ms. E] just said. Wait I don't get the question that [Ms. E] said.
[Ms. E approaches the front of the classroom.]

It is inferred that Ms. E approaches the front of the classroom in response to Manuel D's impasse.

11:18 Manuel D: Does she want half of the half, or yea it's like?

11:18 Dave: That's would of….would of.
11:27 N'yette**: [Ms. E is standing next to N'yette]**
No she just said cut it in ha- just split it like this so it would be the….
11:30 Manuel D: No she said what if you cut it what would I have to pay? [Ms. E looks at the overhead projector as Manuel D speaks.]
11:34 Ms. E: Right, [Ms. E points and nods towards Manuel D] yea, what if, if I cut it like that, what would I have to pay?
11:38 Manuel D: The question is, my question is, what half do you want, like do you want this half or half of that half?
11:46 Ms. E: **Doesn't matter I can take this half, that half, that half, [Ms. E gestures towards the overhead projector] doesn't matter.**
11:48 Manuel D:I mean, we want one….
11:51 N'yette: 'Cause she still take….
11:52 Ms. E: If I want one piece only, would that be the same as saying that I want one half of this?
11:56 N'yette and Manuel D: No.
11:57 Manuel D: No, that would be four dollars.
11:58 Ms. E :So, I'm….I'm gonna say I want half of that.
[Ms. E gestures towards the overhead projector with both hands.]
It doesn't matter which half…
[Ms. E raises her voice to address Betty]
[Betty]! **I want half**
[Ms. E forcefully gestures toward the overhead projector] of this, it doesn't matter which half, it doesn't matter how you cut it. I just want half of that.

Ms. E appears to be expressing frustration when she speaks and gestures forcefully toward the overhead projector.

12:02 Manuel D: Yes.
12:04 Ms. E: Would that be the same as the other one? How much would I be paying if I get this lower half? [Ms. E is standing in front of the class asking the questions, then walks towards the back of the room]
12:15 Dave: You would pay like, you would pay about….
12:16 N'yette: Same thing.
12:17 Manuel D: Eight dollars, it's the same as the other one.
12:19 Ms. E: How come it's the same thing?
12:21 Dave: It gotta be lower, it gotta be lower…its gotta be lower number.
12:23 N'yette: That's what I said…that's what…
12:23 Manuel D: He's gonna pay the same thing because you're breaking one third.
12:26 N'yette: I, I pay eight dollars…
12:30 Ms. E: Well could you show me that I'll pay the same?
[Ms. E walks away from the overhead projector toward the back of the classroom.]
12:26 Dave: So she'll be paying about four…twenty-five, four twenty-five.
12:30 N'yette: I said that, he said that….
12:32 Manuel D: Look, now I agree with [N'yette], Because look, look, now I'm a do,

look…
12:35 Ms. E: They said they're saying it's the same thing,
[Ms. E is seated in the back of the classroom and she is talking to the students seated near her.] [N'yette's] saying it's the same thing…[N'yette], what...what are you saying are the same thing?
12:43 N'yette: I'm saying either way you look at it, even though I looked at it, I …I said [Dave's] was right because…
12:48 Ms. E: Wait wait wait…there's two, there are two things. [Manuel D is standing in front of his desk.]
12:50 N'yette: Okay, I said [Dave's] was right because if did, if I just did mine the way I did mine but I still had to do it the same way, and I looked at it, I'm still getting the same questions as I did before just in a different way.
12:52 Manuel D: You he is paying the same thing. Ok so we paying the same thing.
13:06 Ms. E: Okay so….
13:07 N'yette: And we, we paying the same thing for either or.
13:11 Ms. E: Okay, so could you show us that we paying the same thing?

Coded for promotion of student autonomy and ***teacher intervention in student autonomy.***

(Appendix A.2, E.02.a.01)

Ms. E's interactions in episode E.02.a.01 (Appendix A.2) illustrate how Ms. E initially appears to support student autonomy through her language and actions. Ms. E sits in the back of the classroom amongst the other students while N'yette to shares her solution to the problem. Ms. E asks N'yette to determine the value of half of the two thirds that she has drawn on the overhead projector and Ms. E builds on N'yette's earlier reasoning by asking, "like you originally said". Ms. E then silently observes an exchange between Dave, N'yette and Manuel D. Ms. E then intervenes, asking the students for the fractional value if she buys "half of it." After this intervention, N'yette correctly reasons, "in reality, he did buy two out of six!" Manuel D expresses confusion about N'yette's solution and it is inferred that Ms. E responds to Manuel D's confusion by approaching the front of the classroom. This action by Ms. E is inferred to contradict her original support of student autonomy.

Example 2: Promotion of student autonomy

Episode E.01.a.06 takes place on the first day of "Adding and Subtracting Fractions" in Ms. E's classroom. Ms. E is seated at Manuel D's table and she has invited Felipe, Brielle, N'yette and Tina from different tables to discuss the solution to the subtask with Manuel D. In the episode immediately prior to this (E.01.a.05), not discussed here, Ms. E validates Juan's mathematical breakthrough when he expresses that Section 18 can be represented as an eight by eight square on grid paper. It is inferred that Ms. E wants to share Juan's idea with many students in the class and she does so by asking Manuel D to evaluate Felipe's representation and N'yette's representation. Felipe and N'yette are assigned to different groups but they have each drawn a rectangle to represent Section 18, which is faulty. Instead, Section 18, which covers six-hundred forty acres, can be represented as an eight by eight square on grid paper with each small grid square representing ten acres. See "Adding and Subtracting Fractions" for a complete discussion and solution of this subtask.

In this excerpt, Ms. E appears to be hopeful that Manuel D will disagree with N'yette and Felipe's faulty reasoning and then possibly send them on the correct path to solving the subtask. I discuss N'yette's reaction to this here and Ms. E's inferred support of N'yette's autonomy as N'yette stands firm that her representation is correct.

27:26 N'yette: Ms. E, I got a question.
27:28 Ms. E: **Hold on my dear [Ms. E speaks to N'yette], let's go back to the problem to see if we're correct so far. Each section is a square that is one mile long, on...on each edge, that is each section is one square mile of land.**
[Ms. E looks at Juan, shakes the paper and asks:]
So could they continue doing this? [Ms. E's facial expression is serious. She is not smiling.]
[Ms. E holds paper to N'yette and asks:]
Do you think this is okay?
27:47 Dave: They can but they gonna have to make like three more grids.

27:49 Ms. E: What do you mean "three more grids?"….[inaudible]
27:51 Dave: I mean not grids but they gonna have to make, make three more of these [Dave points to N'yette's paper with his pencil], because to make it bigger cause one square mile is…
27:55 N'yette: To make it bigger? [N'yette is asking Dave]
27:59 Ms. E: Is there any problem with this one?
28:00 Dave: No that equals six forty.
28:03 Ms. E: Mm hum.
28:09 Manuel D: They, they trying to do, like the same exact thing too, and I'm sure it is...
28:13 Ms. E: Uhh um, what is that, What is that same exact thing that you trying to do? [Ms. E taps her finger on Juan's book.]
You, you said you are trying to do [Ms. E points at Manuel's paper] the same exact thing, [Ms. E points at Juan's book] here what is that?
28:20 Dave: And they will equal six hundred forty acres, six hundred forty acres, you could do it in a different way too.
28:22 Manuel D: Right, I'm say…we like when you add them all, that they equal sixty four
28:29 N'yette: We didn't, we didn't,
[N'yette takes the paper out of Ms. E's hands and points to her paper as she speaks] we didn't do it all in like ten, four like ten four, but we did it all different, like ten, four, four, four, is each box is something different but its always gonna be ten and four.
[Ms. E reaches for a ruler and holds it in her hand.]
28:44 Manuel D: What fraction is each one of them?
28:45 N'yette: 'Cause if you did, we didn't do that, we used this ….so we did Lapp, okay, it's one two three, its five and one half here. One, two, three, four, one, two, three, four, ok. One, two, three ,four ,five, we did across five, right here, okay, this two, one, two, one, two, four, one, two, three , one, two, three, one, two, six!
28:50 Ms. E: <u>Do you want to continue….</u>[Ms. E speaks to N'yette.]
28:54 N'yette: See we did all different shapes its, but its all, but its, its always, it always has something to do with four something. [N'yette looks up at Manuel D.]
29:27 Ms. E: <u>Do you want to continue doing this?</u>
[Ms. E asks N'yette, taps on N'yette's paper with a ruler and nods her head.]
29:28 Manuel D: [Ms. E looks at Manuel D as he speaks.]
So we, we can do the same as them but in a different way, instead of, instead of doing four, (Ms. E: Um-hum) we do, we do two (Ms. E: Um-hum), instead of doing five right? [Manuel D asks N'yette]
29:39 N'yette: Yea five and one half.
29:40 Ms. E: <u>How would you do it here then?</u>
[Ms. E points to Manuel D's paper with the ruler],
[Ms. E speaks to N'yette without looking at her]
<u>Maybe you could continue doing this?</u>

Coded for teacher support of student autonomy.

(E.01.a.06, Appendix A.1)

In this excerpt, it is inferred that Ms. E begins by supporting Manuel D's autonomy by asking for his evaluation of other students' work. It is also inferred that Ms. E initiated this interaction with the expectation that Manuel D would enlighten many other students to the idea that Section 18 is best represented (in her opinion) as a square with dimensions eight by eight. When Manuel D agrees with N'yette's representation, Ms. E does not negate his reasoning. Also, N'yette defends her faulty reasoning and it is inferred that she feels strongly that her representation is correct as she exclaims her reasoning. Ms. E then asks N'yette three times if she wants to proceed with her solution as is. Ms. E does not directly address N'yette's faulty reasoning. Ms. E's support of N'yette's autonomy in this example may support N'yette's immediate engagement with the mathematics because N'yette expresses clear enthusiasm and pride in her work. It is possible that Ms. E has decided not to argue with N'yette because she recognizes this. However, the consequences of N'yette continuing with faulty reasoning at this stage in this problem may possibly have detrimental consequences to her future engagement when she is unable to position the individual properties within Section 18 if it is represented as a rectangle.

4.3.3.b Ms. B: Promotion of Student Autonomy

Promotion of student autonomy is coded in thirty-three out of thirty five episodes in Ms. B's classroom. Ms. B imposes her own mathematical ideas very seldom, if ever when her students are engaged with the mathematics. By design, both problems, "Fences for Grazing" and "The DJ Problem" are supportive of student autonomy because students

are given a choice about how to solve them. Two examples follow which illustrate, in context, how Ms. B supports her students' autonomy by accepting student complaints (B.01.b.04) and when she creates opportunities for student initiative (B.02.a.03).

Table 4.12a: Promotion of Student Autonomy, Ms. B, "Fences for Grazing", Day 1

	B.01.a.01	B.01.a.02	B.01.a.03	B. 01.a.04	B.01.a.05	B.01.a.06	B.01.a.07	B. 01.a.08
Coding for teacher promotion of student autonomy	NE	Y	Y	Y	Y	Y	Y	Y

Y=yes, NE=not evidenced

Table 4.12b: Promotion of Student Autonomy, Ms. B, "Fences for Grazing", Day 2

	B.01.b.01	B.01.b.02	B.01.b.03	B.01.b.04	B.01.b.05	B.01.b.06	B.01.b.07	B.01.b.08	B.01.b.09	B.01.b.10	B.01.b.11
Coding for teacher promotion of student autonomy	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

Y=yes, NE=not evidenced

Table 4.13a: Promotion of Student Autonomy, Ms. B, "The DJ Problem", Day 1

	B.02.a.01	B.02.a.02	B.02.a.03	B.02.a.04	B.02.a.05	B.02.a.06	B.02.a.07	B.02.a.08
Coding for teacher promotion of student autonomy	Y	Y	Y	Y	Y	Y	Y	Y

Y=yes, NE= not evidenced

Table 4.13b: Promotion of Student Autonomy, Ms. B, "The DJ Problem", Day 2

	B.02.a.09	B.02.b.01	B.02.b.02	B.02.b.03	B.02.b.04	B.02.b.05	B.02.b.06	B.02.b.07
Coding for teacher promotion of student autonomy	Y	Y	Y	Y	Y	Y	Y	Y

Y=yes, NE = not evidenced

Example 3: Promotion of student autonomy

Episode B.01.b.04 (Appendix B.1) occurs on the second day of "Fences for Grazing". Ms. B is seated in a chair between Kenny and Al and she asks the students to put their ideas down on chart paper. Natasha then tells Ms. B that she disagrees with Kenny's idea.

15:22 Ms. B: [Ms. B is seated on a student chair, and leans in between Kenny and Al]. Since you don't have your work, why don't you go over your thoughts, ok? So do that now, on the um chart paper…
15:28 Natasha: I disagree.
15:29 Ms. B: You disagree? <u>It's okay to disagree.</u>
[Ms. B looks at Natasha, raises her eyebrows and nods her head.]
<u>I disagree with people all the time, you can disagree</u>, but you have to have reasons as well, right? You still can try and convince them.
<u>Did you show 'em…did he listen?</u>

Coded for teacher promotion of student autonomy. (Appendix B.1, B.01.b.04)

Ms. B uses autonomy supportive language as she acknowledges and validates Natasha's emotional feelings (disagreement). Ms. B stresses the importance of using reasons to make convincing arguments about mathematical ideas/answers. Ms. B also points out the importance of communication from both perspectives (the speaker and the listener) when she asks, "Did you show 'em…did he listen?". Immediately subsequent to

this episode, Natasha and Kenny sit silent and motionless, not writing, not speaking. Thus, the immediate impact of Ms. B's intervention on the students' engagement cannot be determined from the video data.

Example 4: Promotion of student autonomy

Episode B.02.a.03 (Appendix B.2) occurs on the first day of "The DJ Problem" in Ms. B's classroom. Immediately prior to this episode, Ms. B is seated in a chair at Dana's table. She leans back to face Natasha's table and then slides her chair closer to Natasha's table. Ms. B listens to a discussion between Natasha, OJ and Al and then asks them to explain why "the second one", Solidus Sounds is a better choice for a DJ. She promotes the students' autonomy by allowing the students to lead her through the calculations as if she is the student and they are the teachers.

13:08: Ms. B: <u>How do you know</u> the second one is better?
[Ms. B is seated in a chair, she slides her chair from the adjacent table and leans her right elbow on boy's desk. She is smiling and looking at Al.]
13:08 Al: Because we said we gonna party for eight hours, right?
[Al is looking at Ms. B.]
13:11 Ms. B: Oooh, that's a long party. Okay. Eight hours…
13:15 Al: Right? And then, its forty dollars. [Al points at his paper, he is looking at Ms. B.]
Wait.
[Al moves his calculator and looks at the problem sheet.]
13:18 Natasha: Mmm hmm
13:19 Al: Forty dollars an hour right?
[Al looks at Ms. B and nods his head. Ms. B is out of camera range.]
13:20 Ms. B: Okay.
13:22 Students: Sixty. Sixty.
13:25 Al: The DJ is charging a hundred. Keep in mind, [Al points toward Ms. B, and looks at her, tilting his head to the side, emphasizing his words hitting his hand on the desk] the DJ is charging one hundred, just for a day.
13:30 Natasha: See he is charging a hundred dollars for. [Natasha points at Ms. B. <u>Ms. B is seated at a student desk, with a calculator in front of her. She is looking at Natasha.</u>] Don't put, don't put a hundred in, just put…
<u>[Ms. B's facial expression is serious as she looks at Natasha and listens to and follows their instructions.]</u>
13:35 Al: Forty times eight.

13:37 Ms. B: Forty times eight.
[Ms. B is pushing buttons on a calculator.]
13:38 Natasha: Mmm hmm.
13:38 OJ: [Ms. B looks up at Natasha.]
We could do what's it's name...We could do the third one.
13:40 Natasha: We could do a hundred.
13:41 Ms. B: Why am I adding a hundred?
[Ms. B looks at Natasha and then at OJ.]
13:42 Natasha: Because, for the down payment is, it's more like a down payment is a hundred dollars.
[Natasha is looking at Ms. B, gesturing with her hands as she speaks.]
13:45 Ms. B: Okay.
[Ms. B looks at the calculator and pushes buttons.]
13:46 Natasha: So you could do plus one hundred.
13:48 Al: So you could get four twenty.
[Ms. B looks up at Al, her eyebrows are raised and her tone of voice is calm.]
13:52 Ms. B: So you pay four hundred twenty dollars.
13:54 Students: Four hundred twenty dollars.
13:57 Al: Yeah, like instead of paying like um, a thousand dollars, and stuff like that.
[Al nods his head as if to emphasize his words.]
13:58 Natasha: Thirty times eight.
13:59 Ms. B: And how many hours is that for?
[Ms. B looks at Natasha.]
14:00 Student: Eight.
14:00 Ms. B: Eight hours. So do you guys want to keep track of what you do. [Ms. B looks around at all of the students, her tone of voice is calm and tentative.]
So just in case you know what's going on.

Coded for teacher support of student autonomy.

(B.02.a.03, Appendix B.2)

It is inferred that Ms. E's support of student autonomy supported Natasha and Al's mathematical engagement because as she takes on the role of the student, they take the initiative of leading her through the solution for calculating the DJ rates for a period of eight hours. Immediately subsequent to this interaction, Natasha, OJ and Kenny engage in a mathematical discussion about which DJ is "the cheapest" and Ms. B occasionally asks a question about their mathematical reasoning. Ms. B remains at Natasha's table for approximately three and one-half minutes until 17:35. After Ms. B

leaves this group, Natasha and Al continue to discuss the number of hours they should use to calculate the DJ rates.

4.4.4 Teacher Promotion of Mathematical Discourse

In the analysis of the teacher/student(s) mathematical interaction episodes, the code, "promotion of mathematical discourse" is applied anytime it is explicit through her language and/or actions that the teacher encourages students to engage in discussion that pertains to the mathematics. Through their language, Ms. E and Ms. B encourage the students to discuss the mathematics with their peers. For example: "Could you explain to Felipe what you trying to do?" (E.1.a.06, 25:17), "Did you guys hear what [Devin] just said?" (B.2.a.06, 26:24)

4.4.4.a Ms. E: Promotion of mathematical discourse

During Ms. E's first lesson, "Adding and Subtracting Fractions" the students are in small groups at tables. Ms. E's second lesson, "Finding Areas and Other Products", appears to be situated in a different classroom from the first lesson because the furniture and the physical space appears different. In the second classroom, the students are seated at individual desks and which are arranged in rows facing the front of the classroom. During each of the four class periods in Ms. E's class, the students move freely throughout the classroom discussing their mathematical ideas with each other. Ms. E invites students to move around the classroom to share their mathematical ideas and to help other students who are experiencing impasse with the mathematics.

Table 4.14a, Teacher Promotion of Mathematical Discourse, Ms. E, "Adding and Subtracting Fractions", Day 1

	E.01.a.01	E.01.a.02	E.01.a.03	E. 01.a.04	E.01.a.05	E.01.a.06	E.01.a.07	E. 01.a.08
Coding for teacher promotion of mathematical discourse	NE	NE	Y	Y	Y	Y	Y	Y

Y=yes, NE=no evidence

Table 4.14b, Teacher Promotion of Mathematical Discourse, Ms. E, "Adding and Subtracting Fractions", Day 2

	E.01.b.01	E.01.b.02	E.01.b.03	E.01.b.04	E.01.b.05	E.01.b.06
Coding for teacher promotion of mathematical discourse	Y	Y	Y	Y	Y	Y

Y=yes, NE=no evidence

Table 4.15, Teacher Promotion of Mathematical Discourse, Ms. E, "Finding Areas and Other Products", Days 1 & 2

	E.02.a.01	E.02.a.02	E.02.a.03	E.02.b.01	E.02.b.02	E.02.b.03	E.02.b.04
Coding for teacher promotion of mathematical discourse	Y	NE	NE	Y	Y	NE	NE

Y=yes, NE=no evidence

Example 1: Promotion of mathematical discourse

In episode E.01.b.03 (Appendix A.1) takes place during the second day of "Adding and Subtracting Fractions" in Ms. E's classroom. Ms. E is standing next to Felipe and immediately prior to this interaction Ms. E asks Felipe to draw a representation of three-fourths. Felipe responds, "It's impossible". (E.01.b.03, 11:00) Ms. E then poses the question to Eric who is seated across the table from Felipe. It is inferred that Ms. E is not satisfied with Eric's representation because she raises her voice and broadcasts a general plea for help.

11:50 Ms. E: Could somebody help us out? [Ms. E turns to the table that is adjacent to Felipe's table where Noah and Manuel C are seated.] Could somebody show us three-fourths on this table? Could somebody help us please?
11:54 N'yette: Oh, I could help you.
[N'yette approaches Felipe's table from a different table.]
11:55 Ms. E: Could you please? Could you draw three fourths over here? [N'yette leans on Felipe's table and draws.]
12:13 Ms. E:[Ms. E asks N'yette]
So how does three fourths look like?
[N'yette draws a rectangle with three of four sections shaded.]
[Ms. E puts her hand on Felipe's shoulder and addresses him as she points to N'yette's paper.]
[Felipe]. Could you write that, three fourths please.
12:16 Felipe: Oh yeah, I forgot that…
12:18 Ms. E: Does that mean three fourths? [Ms. E is looking at the table or N'yette's paper. N'yette remains at the table.]
12:19 Felipe: Yes.
[Felipe does not look up from his work when he responds to Ms. E.]
12:20 Ms. E: All right, so how would, say for example, umm…[Ms. E hesitates as she glances around the table at the students' papers] three-eighths. How would three-eighths look like? [Ms. E stands back and folds her arms.]
12:2 Manuel C: Look. That's three fourths.
[Manuel C is seated at the adjacent table.]
12:29 Ms. E: Let me see. Ah - Where is three fourths here? Three fourths. [Ms. E repeats "three-fourths" with emphasis.]
12:37 Felipe: Here you go, three [Felipe has correctly drawn a representation of three-eighths. He shows Ms. E who briefly leans in and glances at his work, but does not respond.]
12:39 N'yette: You said three-eighths?

12:40 Ms. E: Three-eighths. I see, do you agree that this is three-fourths? [Ms. E shows Manuel C's work to N'yette.]

Coded for teacher promotion of mathematical discourse

(Appendix A.1, E.01.b.03)

This excerpt illustrates how Ms. E relies on student discourse to help students correct their faulty mathematical reasoning. When Felipe announces that it is impossible to express three-fourths because it is not an accurate fraction, Ms. E appears to become agitated as she broadcasts for help. In this example N'yette's intervention seems to help jog Felipe's memory, but N'yette and Felipe do not actually engage in discourse with each other.

Example 2: Promotion of mathematical discourse

Episode E.02.b.02 (Appendix A.2) takes place on the second day of "Finding Areas and Other Products" in Ms. E's classroom. Immediately prior to this episode, Manuel D is standing at the front of the classroom presenting his solution of Problem 5.1A to the whole class. Manuel D begins to explain his solution it appears that Ms. E perceives that Manuel D is struggling. She encourages students from the audience and then calls specifically on N'yette to approach the overhead to help Manuel D.

12:39 Ms. E: [Ms. E addresses Tora and Dave who are talking to each other.]
Here you go again…Enough!
Oh [Jackie] do me a favor… could you get those two new students uh books from the closet…
[Ms. E speaks to Manuel D]…Walk us through what you're doing?
Shhh…Shhh…
Tell us what you're doing [Manuel D is writing on the overhead transparency and Ms. E is looking at the projected image on the wall as she speaks.]
13:28 Ms. E: And speak up please, so everyone in the room can hear you.
[Ms. E looks out toward the audience as she speaks. Ms. E is seated on the desktop and is holding the sides of the desk with her hands.]
13:38 Manuel D: So…that…um…according to the question [Manuel D looks back at the overhead projector screen]…this
13:44 Ms. E: Speak up!

[Ms. E speaks to Manuel D, she momentarily leaves her seat to adjust something on the overhead projector.] Now is the time to speak up. Yes, I heard you a lot earlier.

From Ms. E's language in her next two speaking turns, it is inferred that Ms. E thinks that Manuel D is experiencing impasse as she encourages students seated in the audience to help Manuel D.

14:35 Ms. E: Help, help help…(students talking)
15:14 Ms. E: [N'yette], can you come and help.
[Dave, Juan, and N'yette, now surround Manuel D at the overhead projector. All four students appear to be working cooperatively to solve the problem.]
15:22 Ms. E: Shh… [Felipe].
[N'yette, Manuel D and Dave are all standing at the overhead discussing the problem. The specific discussion is inaudible. Felipe and another male student approach the overhead projector.]
15:47 Ms. E: That, that one you're cross- …[N'yette], the one you just shaded, what's that?
[Ms. E is looking at the overhead projector.]
15:55 N'yette: This one is the half that's already gone.
15:57 Ms. E: That part is already gone.
[Ms. E's eyebrows are raised and she is leaning towards the students in the audience.]
15:59 N'yette: Yeah.
16:00 Ms. E: You see that? [Ms. E looks at the students in the audience.]
That part they're saying is already gone. So now…

Coded for teacher promotion of mathematical discourse.
(Appendix A.2, E.02.b.02)

This excerpt illustrates how Ms. E encourages students to help one another when they experience impasse with the mathematics. In this case, it is inferred that Manuel D's engagement with the mathematics is supported when he gets help from N'yette because he has a mathematical breakthrough after listening to N'yette's reasoning.

4.4.4.b Ms. B: Promotion of mathematical discourse

In Ms. B's room, the students are seated in small clusters of desks in rectangular configurations for both "Fences for Grazing" and "The DJ Problem". When the students are working to solve the problems, they remain seated with their group. At the end of each lesson the students share their solutions with students from different groups. At the end of "Fences for Grazing", the students circulate with their working groups throughout the room and make comments on Post-it notes about the other students' work. At the end of "The DJ Problem", Ms. B sits in a student chair at Natasha's table and invites all of her students to assemble around the table to take turns presenting their solutions to the problem.

Table 4.16a, Teacher Promotion of Mathematical Discourse, Ms. B, "Fences for Grazing", Day 1

	B.01.a.01	B.01.a.02	B.01.a.03	B. 01.a.04	B.01.a.05	B.01.a.06	B.01.a.07	B. 01.a.08
Coding for promotion of mathematical discourse	Y	Y	Y	NE	Y	Y	Y	Y

Y = yes, NE = not evidenced

Table 4.16b, Teacher Promotion of Mathematical Discourse, Ms. B, "Fences for Grazing", Day 2

	B.01.b.01	B.01.b.02	B.01.b.03	B.01.b.04	B.01.b.05	B.01.b.06	B.01.b.07	B.01.b.08	B.01.b.09	B.01.b.10	B.01.b.11
Coding for promotion of mathematical discourse	Y	Y	Y	Y	NE	Y	NE	Y	Y	Y	Y

Y=yes, NE = not evidenced

Table 4.17a, Teacher Promotion of Mathematical Discourse, Ms. B, "The DJ Problem", Day 1

	B.02.a.01	B.02.a.02	B.02.a.03	B.02.a.04	B.02.a.05	B.02.a.06	B.02.a.07	B.02.a.08	B.02.a.09
Coding for promotion of mathematical discourse	Y	Y	Y	Y	Y	Y	Y	Y	Y

Y = yes, NE = not evidenced

Table 4.17b, Teacher Promotion of Mathematical Discourse, Ms. B, "The DJ Problem", Day 2

	B.02.b.01	B.02.b.02	B.02.b.03	B.02.b.04	B.02.b.05	B.02.b.06	B.02.b.07
Coding for promotion of mathematical discourse	Y	Y	Y	NE	Y	Y	Y

Y = yes, NE = not evidenced

Example 3: Promotion of mathematical discourse

Episode B.01.a.02 (Appendix B.1) takes place in Ms. B's class during the first day of "Fences for Grazing" with Ghee, Dana, Shawn, Devin and Emmy. Ms. B is seated next to Devin. Immediately prior to this interaction, Ms. B redirects the students to read the problem, "so let's go back to what you have to do…". It is inferred that Ms. B recognizes that the students are not sure how to proceed with solving the problem. When Dana asks Ms. B if they should find numbers that have a sum of one hundred, Ms. B responds by encouraging her to ask the members of her group.

10:51 Ms. B: Do you guys agree with what [Ghee] just said?
10:58 Ms. B: Can you explain to me why you did that?
[Ms. B directs this question to Ghee. She furrows her brow and cranes her neck to look at Ghee's work.]
11:00 Ghee:... a hundred...and I found out that the sides have the same length, the top and the bottom, and the sides...the sides...[inaudible]
11:28 Ms. B: Okay…[Ms. B looks down at the paper in front of her.] Okay, and so what did you just find?
[Ms. B furrows her brow.]
11:32 Ghee: The area...(inaudible)
11:46 Ms. B:[Devin], [Shawn], [Dana]…[Ms. B looks at each student as she says their names and smiles at Dana]
So what do you think he just found?
11:52 Dana: The maximum area. The maximum area. [Dana looks at Ms. B. Ms. B is looking down at the paper in front of her.]
11:54 Ms. B: Did you guys talk about what area and perimeter is?
[Ms. B has her arms folded and is leaning forward on the desk.]
11:59 Ms. B: [Ms. B bows her head down then poses a question to the whole group.]
What's the area and what's the perimeter. How would you define the perimeter?
[Ms. B looks from one student to the other]
12:12 Ghee: The outside.
12:14 Ms. B: Okay. Right? Right? Right? [Ms. B looks around at each student in the group.]
[Ms. B shrugs her shoulders and asks:] Wrong?

Coded for teacher promotion of mathematical discourse
(B.01.a.02, Appendix B.1)

The students do not respond so Ms. B then asks Ghee to explain his reasoning to her and then she asks Devin, Shawn and Dana to respond to Ghee's explanation.

It is inferred that Ms. B recognizes that all students in this group are not engaged with the mathematics when she calls three of the students by name. It is possible that she tries to foster the students' mathematical engagement by asking them to identify or name what Ghee has found. Ms. B's intervention appears to successfully but temporarily foster Ghee's and Dana's mathematical engagement as they discuss their ideas with her but not with one another. Devin and Emmy do not engage in discourse during this episode and Shawn only offers one idea, "the area's the outside". Ghee quickly responds to Shawn's faulty reasoning, "Don't you see 'rim' here?" It is inferred that Ghee is referring to the word, 'rim' in 'perimeter'.

Example 4: Promotion of mathematical discourse

Episode B.01.a.08 (Appendix B.1) takes place on the first day of "Fences for Grazing" in Ms. B's classroom. Dana initiates the interaction when she complains to Ms. B, "But they are not even working!" It is evidenced by Dana's language that she is angry because her group members are not engaged with solving the problem. It is inferred that Dana is also angry because she has reached a point of impasse with the mathematics and it is possible that her anger tied to the frustration with the mathematics is directed at her peers. Ms. B responds to Dana's complaint by suggesting that it is Dana's responsibility to engage in a discussion with her peers.

36:41 Shawn: [Ms. B] [Shawn calls out to Ms. B and turns toward the camera]….she's [pointing toward Emmy] not coming up with no kinds of ideas and I….she's not doing nothing.
[Emmy is leaning her chin on her fist.]
36:48 Dana: But they are not even working!!! [Dana is looking at Ms. B]
36:50 Ms. B: <u>So help them work!</u>

36:51 Shawn: We trying to help them…
36:52 Ghee: It's all she's been doing. [Ghee is looking down at his paper.]
36:53 Dana: There's only three of us in this group that's helping and everyone else's just sitting there trying get this!
[Dana is looking at Ms. B, her voice is raised.]
37:00 Ms. B: Well don't give 'em the answer, make them give you the answer.
[Ms. B is out of view, but her arm can be seen, she is sitting at the desk next to Devin.]
37:02 Ghee: How we supposed to make them...we've been talking to them, they been talking 'bout something other.
37:09 Ms. B: So ask her, well lemme see, ask her a question.
37:14 Dana: Emmy, what's the maximum area?
37:15 Ghee: Look. [Ghee points at Emmy]
37:16 Ms. B: I'm asking what's the maximum area?
37:18 Dana: We don't know cause we already said our…
37:20 Ms. B: What's the maximum area [Ms. B looks toward Emmy], what's the maximum area [Ms. B looks toward Shawn], what's the maximum area [Ms. B looks toward Ghee].

Coded for teacher promotion of mathematical discourse.

(Appendix B.1, B.01.a.08)

Ms. B's language and actions when she asks each student to tell her the maximum area is interpreted as defense of Emmy when she looks at each student and asks, "what's the maximum area", proving that Emmy is not the only student who does not have an answer at this point. Immediately subsequent to this interaction both Ghee and Dana become engaged in a mathematical discussion facilitated by Ms. B.

Example 5: Promotion of mathematical discourse

Episode B.02.a.06 (Appendix B.2) takes place in Ms. B's class about halfway through the first day of "The DJ Problem" when Devin quietly asks, "Why couldn't it be like ten hours instead of six?" At this point in the lesson Ghee had decided that Solidus Sounds was the best DJ for the school dance based on his cost based on a six-hour party. He has not considered any other options with reference to time. When Ms. B questions the students about why they chose Solidus, Devin poses his question and Ms. B draws

attention to his question. Devin is a quiet student who rarely participates with the group discussions.

26:13 Ms. B: Wait real quick, cause [Devin] just whispered something. Let him whisper it again.
[Ms. B puts her hand up to her face as if she is telling a secret as she speaks to Ghee and Lenny. She smiles at Devin and lowers her voice to a near whisper. Devin smirks.]
26:18 Ghee: He said, why couldn't it be ten.
[Ghee looks at Ms. B and begins pushing buttons on a calculator.]
26:24 Ms. B: Did you guys hear what [Devin] just said?
[Ghee, Dana, Shawn and Lenny are all looking at their papers or their calculators.]
[Ms. B smiles as she leans in towards Dana and Shawn. She is smiling.] [Dana], [Shawn]?

[Devin is looking down at his paper and glances up as Ms. B says each student's name.]
26:28 Shawn: Nope.
26:29 Ms. B: [Dana]?
26:29 Dana: I heard him.
[Dana is writing on her paper, her head is turned away from Ms. B.]
26:29 Ms. B: What did he say?

Ms. B remains seated as the students explore other options for the amount of time that the class would hire the DJ. Ghee calculates the rates for all three DJs and determines that Light Plastic would be the best choice of DJ for a ten-hour party. This exchange spans approximately four minutes when Ms. B takes notice of a question Larry asks.

30:36 Ms. B: What did you just say, Lenny? Hold on, let him, Lenny just said something. We didn't hear him. [Ms. B leans in and smiles] We talked right over him. What did you say Lenny?
30:45 Ghee: He said why ain't we just figured out all of them?
30:47 Ms. B: So does anyone want to answer his question?
[Ms. B is shuffling papers on the desk in front of her. She does not look at the students when she speaks.]

Coded for teacher promotion of mathematical discourse.

(Appendix B.2, B.02.a.06)

This interaction is followed by a humorous exchange between Ms. B and Ghee when Ghee admits that they didn't want to work so hard to find out all of the hours. Ms. B's tone of voice and gestures convey surprise and shock at Ghee's suggestion and she expresses that she is personally offended by Ghee's actions, "In my class, you didn't want to work?" Ms. B is animated, with hand gestures emphasizing each word as she tells the boys, "when it comes to money, don't you want to give the…You want to see exactly what you doing!"

Example 6: Promotion of mathematical discourse

Episode B.01.b.11 takes place during the final few minutes of the second day of "The DJ Problem" in Ms. B's class. Immediately prior to this episode, Shay expresses that he thinks Dana's group has computed the area of their rectangle incorrectly. Instead of multiplying length times width, the students added both lengths (40+40) and added both widths (10+10), then multiplied the sums together. They presented their answer as the product of eighty and twenty ("sixteen hundred"). Dana is standing at a table adjacent to Shay and she appears to overhear Shay's comments. Dana raises her voice in inferred defense of her answer.

38:59 Dana: [Ms. B approaches Dana and Shay, standing in between them, but distanced about one foot away so that the two students can make direct eye contact.]
Alright, but when y'all add all y'all, but when we add all our…so what? Eighty plus twenty equals a hundred. [Dana leans in and points to the chart paper.]
That's how we get ours...just like everybody else's.
[Ms. B looks at Shay.]
39:11 Ms. B: <u>What? I'm just listening</u>.
[Ms. B looks at Shay.]
39:14 Dana: Y'all tellin' us that we're wrong but we're not wrong!

(Appendix B.1, B.01.b.11)

Coded for teacher support of student discourse.

This episode is illustrative of how Ms. B promotes student discourse through her actions. Ms. B does not intervene. She stands in close proximity to Dana and Shay with her arms folded, glancing back and forth between them. They shout loudly and become physically engaged, leaning in to each other as they each attempt to defend themselves (or their mathematical ideas). Ms. B's facial expression is inferred to show that she is pleased because she smiles slightly. The students continued to argue about Dana's answer until 40:46 until Ms. B glances at the clock and announces that the class period is over.

4.4.5. Teacher Promotion of Respect

The theme, "promotion of respect" is derived from the coding of peer-to-peer respect as evidenced in the classroom observation data. In the analysis of the classroom observation data, the code "peer-to-peer respect" is applied each time the teacher explicitly, through their language and/or actions promote respect amongst the students. For example, promotion of respect is coded when the teacher encourages students to wait their turn to speak and to 'listen' to other's ideas. "Go ahead Devin continue. Listen to what Devin is saying." (B.01.a.08, 42:36)

4.4.5.a Teacher promotion of respect: Ms. E

In Ms. E's classroom observation data, evidence of Ms. E promoting respect is coded in three out of twenty-one episodes. In these three episodes, Ms. E explicitly tells the students to respect each other and to listen to each other. In each of these three cases, one student is trying to explain his or her reasoning to many students and many of the

students in the "audience" are talking out of turn, or it is inferred that Ms. E perceives these students to not be paying attention. In each of these three cases, Ms. E interrupts the flow of the mathematical discussion to address the perceived disrespectful behavior. As Tora begins to present her solution to Problem 5.1B, Ms. E says, "I want you to follow what [Tora] is going to say. [Ms. E glances sideways at the students in the audience.] Shhh…shhh. [Eli], you have to be very respectful." In the same interaction, Ms. E also encourages respect for Tora's approach to solving the mathematical problem. "If [Tora] needs to do that, let her do it." (E.02.b.01, 3:41, Appendix A.2)

Table 4.18a, Teacher Promotion of Respect
Ms. E, "Adding and Subtracting Fractions", Day 1

	E.01.a.01	E.01.a.02	E.01.a.03	E. 01.a.04	E.01.a.05	E.01.a.06	E.01.a.07	E. 01.a.08	E.01.b.01	E.01.b.02	E.01.b.03	E.01.b.04	E.01.b.05	E.01.b.06
Coding for teacher promotion of respect	NE	NE	NE	NE	NE	NE	NE	NE	NE	NE	NE	NE	NE	Y

Y=yes, NE = not evidenced

Table 4.18b, Teacher Promotion of Respect,
Ms. E, "Adding and Subtracting Fractions", Day 2

	E.01.b.01	E.01.b.02	E.01.b.03	E.01.b.04	E.01.b.05	E.01.b.06
Coding for teacher promotion of respect	NE	NE	NE	NE	NE	Y

Y=yes, NE = not evidenced

Table 4.19, Teacher encourages respect, Ms. E, "Finding Areas and Other Products", Days 1 & 2

	E.02.a.01	E.02.a.02	E.02.a.03	E.02.b.01	E.02.b.02	E.02.b.03	E.02.b.04
Coding for teacher promotion of respect	NE	NE	NE	Y	Y	NE	NE

Y=yes, NE = not evidenced

Example 1: Promotion of respect

Episode E.02.b.01 (Appendix A.2) occurs on the second day of "Finding Areas and Other Products" in Ms. E's classroom. At the start of this episode, Ms. E invites Tora to the overhead projector (3:11) to share her solution to Problem 5.1B with the whole class. After approximately nine minutes of Tora's presentation, the students in the classroom begin to carry on side conversation and the noise level in the classroom increases. It is inferred that many students in the audience are not listening to Tora's presentation. Meanwhile, Tora is speaking very quietly and Dave is badgering Tora for speaking too quietly.

12:04: Ms. E: Shh…Gentlemen, [to Manuel D and Dave] I'm sure that when you get up there, you'll wanna be listened to, right? Shh…
12:15 Manuel D: Well, can I get up there already?
12:19 Ms. E: Shhh…We're about to be done with [Tora's] explanation.
[Ms. E looks at the projected image of Tora's work and reads,]
Aunt Serena bought nine dollars of the pan. And how much, um, how much in fraction is it?
She bought …[Tora gestures towards the transparency and Ms. E reads her answer] three-eighths of the pan.
12:35 Lorissa: Here, its from [Jenny] [student gives paper to Tora].
12:38 Dave: You ain't gonna be able to see it on the transparency.
12:39 Ms. E: And, could you read what she has there?

12:44 Dave: There's no transparency.
12:45 Tora: Half of twelve is two.
12:48 Ms. E: Half of twelve is two. Do you agree?
12:52 Dave: Half of twelve is two? Half of twelve is six! [Manuel D looks at Dave and laughs.]
12:56 Ms. E: Half of twelve is…
12:58 Tora: six!
12:59 Ms. E: You can definitely, you can definitely ask, but laughing at somebody's idea is not a respectful thing to do.
13:07 Manuel D:… being respectful
13:08 Ms. E: Well, I want you to be respectful in my class.

Coded for teacher promotion of respect.

(Appendix A.2, E.02.b.01)

Episode E.02.b.01 (Appendix A.2) is illustrative of Ms. E's value of student respect in her classroom. She explicitly tells the students to be respectful of others. It is not clear whether or not Ms. E's pleas for respect are effective in fostering or supporting student engagement or whether or not the students actually respect one another after her requests.

4.4.5.b Ms. B: Teacher promotion of respect

In Ms. B's classroom observation data, evidence of Ms. B promoting respect is coded in five out of thirty-five episodes. Ms. B promotes respect in her classroom through her language when she tells students to listen to each other's ideas. Ms. B also promotes respect amongst the students through her actions, using eye contact or gestures to encourage students to listen to each other. At the end of the second day of each lesson, Ms. B provides time for the students to share their answers. However rather than have one (or a few) students stand at the front of the classroom and present, she uses a different strategy for each class which may be interpreted to promote respect. On the second day of "Fences for Grazing", Ms. B instructs the students to circulate with their

working groups to visit each of the other tables (she controls the time at each table). The students then write their comments on post-it notes and stick the notes to their classmates' work. On the second day of "The DJ Problem", Ms. B invites all of the students to gather around Natasha's table and students take turns sharing their solutions.

Table 4.20a, Teacher Promotion of Respect, Ms. B, "Fences for Grazing", Day 1

	B.01.a.01	B.01.a.02	B.01.a.03	B. 01.a.04	B.01.a.05	B.01.a.06	B.01.a.07	B. 01.a.08
Coding for teacher promotion of respect	NE	NE	NE	NE	NE	NE	NE	Y

Y=yes, NE = not evidenced

Table 4.20b, Teacher Promotion of Respect, Ms. B, "Fences for Grazing", Day 2

	B.01.b.01	B.01.b.02	B.01.b.03	B.01.b.04	B.01.b.05	B.01.b.06	B.01.b.07	B.01.b.08	B.01.b.09	B.01.b.10	B.01.b.11
Coding for teacher promotion of respect	NE	NE	NE	Y	NE	NE	NE	NE	NE	NE	Y

Y = yes, NE = not evidenced

Table 4.21a, Teacher Promotion of Respect, Ms. B, "The DJ Problem", Day 1

	B.02.a.01	B.02.a.02	B.02.a.03	B.02.a.04	B.02.a.05	B.02.a.06	B.02.a.07	B.02.a.08	B.02.a.09
Teacher promotes respect	Y	NE	NE	NE	NE	NE	NE	Y	NE

Y = yes, NE = not evidenced

Table 4.21b, Teacher Promotion of Respect, Ms. B, "The DJ Problem", Day 2

	B.02.b.01	B.02.b.02	B.02.b.03	B.02.b.04	B.02.b.05	B.02.b.06	B.02.b.07
Coding for teacher promotion of respect	Y	NE	NE	NE	NE	NE	Y

Y = yes, NE = not evidenced

Example 2: Promotion of respect

Episode B.2.a.08 (Appendix B.2) occurs on the first day of "The DJ Problem". It is important to consider Dana's mood during this class period when considering this example. Dana is typically an outspoken and dominant female student in this classroom. Yet, on this day, Dana has not been interacting with her group, she has been engaged, quietly in solving the problem her own way. It appears that Ms. B recognizes a difference in Dana's mood and that she encourages Dana to share her idea with the group as a way to engage her in mathematical discourse with the others at the table. Immediately prior to this interaction, Dana expresses that she used a different variable when writing the equation to represent Light Plastic's rates.

41:42: Dana: And they said use any variables, so I used J.
[Dana looks at her paper as she speaks. Ghee is resting his head on the desk, he is looking away from Dana.]
41:46 Lenny: Now it's confusing.
41:49 Ghee: Whoa, how do you party for ten hours?
[Ghee is resting his chin on the desk. He is smiling at Lenny.]
41:53 Ms. B: Okay, so can you explain the equation to the group?
41:55 Lenny: Come on, do you want to do that all day?
41:58 Ms. B: <u>Did you have a different way?</u> [Ms. B's facial expression is off camera.] <u>Let's listen to what [Dana] is going to say. [Ms. B puts the palm of her hand on Ghee's desk, her voice is raised over the students talking.] Cause [Dana] has a thought, two different thoughts.</u>

42:03 Dana: Yea I don't know if I'm right.
[Ghee, Shawn and Lenny are all talking at once.]
42:06 Ms. B: Are we listening? [Ms. B leans in toward Lenny and Shawn and furrows her eyebrows.] Go ahead Dana, so there is two different thoughts. [Ms. B gestures towards Ghee and Lenny.]

Coded for teacher promotion of respect.

(Appendix B.2, B.02.a.08)

This episode is illustrative of Ms. B's promotion of respect for Dana's different mathematical ideas. Through her language and actions it is inferred that Ms. B encourages Ghee, Lenny and Shawn to be respectful of Dana's alternate idea.

4.5 Discussion of Retrospective Stimulated Recall Interviews:

The teachers' perspectives

In this section, I describe each teacher's perspectives on the students affect and on their own affect during the eight days of instruction in her classroom. I highlight the teachers' rationale for their own affective language and actions when interacting with students during key affective events. I also discuss the teachers' affect (explicit and inferred) as evidenced during the retrospective interviews. During these four interviews (one interview was conducted after each of the four lessons) Ms. E and Ms. B first were asked to reflect on their general perceptions of the success of each lesson. Next, they viewed select video clips of key affective events (KAE) from the classroom observation data. Specific interview questions with reference to each KAE were designed so the teachers could articulate their reflections on (1) student mathematical engagement, (2) their inferences about students' affect as it pertains to the mathematical problem solving, (3) their own affect, and (4) their rationale for specific instructional or classroom management.

All analysis of the classroom observation data was completed prior to my viewing of the retrospective interviews. The teachers' reflections as evidenced in these interviews strengthen the inferences made in the analysis of teacher/student(s) mathematical interaction episodes. When relevant, I make connections between the teachers' reflections during the interviews as they pertain to the themes that emerged from the coding of the classroom observation data. Complete transcripts of the four interviews are as well as the protocol for the interviews are included in the appendix (E.1, E.2, F.1, F.2).

4.5a Ms. E's responses to retrospective stimulated-recall interviews

In this section I describe Ms. E's responses to the stimulated-recall interviews, conducted during "Adding and Subtracting Fractions" and "Finding Areas". Ms. E discusses her perspective on each lesson in general and responds to four key affective events in "Adding and Subtracting Fractions and three key affective events in "Finding Areas and Other Products" ("Fractions" KAE 1, "Fractions" KAE 2, "Fractions" KAE 3, "Fractions" KAE 4 and "Areas" KAE 1, "Areas" KAE 2, "Areas" KAE 3).

Ms. E's reflections: "Adding and Subtracting Fractions"

The researcher begins the interview: "So first could you share some of your thoughts about the lessons we observed?" Ms. E describes that she felt "uneasy" because the students appeared to struggle to represent a six hundred forty-acre section of land as a square on the grid paper that she provided. Recall that this was a subtask assigned by Ms. E and was not part of the original problem as stated in the textbook. The students spent the entire first period of "Adding and Subtracting Fractions" trying to solve this subtask. One correct representation of six hundred forty acres would be a square with dimensions eight units by eight units.

Ms. E recounts that although she was "uneasy" because the students were struggling with the subtask, she made a conscious decision not to impose her own mathematical ideas. "I wanted them to start with sixty-four because I thought it would have been easy but also didn't want to suggest my own thinking at that point I wanted them to just come up with something and see how it went."

(Appendix E.1, 00:41) This is consistent with the events documented during the classroom observations as Ms. E repeatedly followed the students' thinking regardless of whether it was right or wrong (i.e., Appendix A.1: E.01.a.04, E.01.a.06). Ms. E also describes that she was "uneasy" because she did not expect the students to struggle so much on the subtask especially because students in her other classes had successfully completed the subtask without her guidance. Ms. E adds that the amount of time spent on this subtask was excessive compared to what she had anticipated when planning the lesson. "Wednesday and Thursday no, even up to today I didn't accomplish the things I wanted to accomplish with that group." (Appendix E.1, 7:53)

"Adding and Subtracting Fractions", Key affective event 1

"Fractions" KAE 1 was documented on the first day of "Adding and Subtracting Fractions" and coincides with episode E.01.a.06 (Appendix A.1) from the classroom observation data. Ms. E is seated at Manuel D's table and she invites N'yette and others to the table. Juan has just correctly solved the subtask and Ms. E is asking Manuel D to explain Juan's idea to many students who are now standing in close proximity to the table. It is inferred from the classroom observation data that Ms. E recognizes that N'yette has incorrectly represented Section 18 on her grid paper. The problem clearly stated that Section 18 must be a square, however N'yette has drawn a rectangle on her grid paper to represent six hundred forty acres of land. Ms. E's response during the interview confirms this inference. "I really wanted her to see that it's specified that it's one square mile so anything of the model that doesn't fit the square shouldn't be used at that point." (Appendix E.1, 15:36)

In the classroom observation data, Ms. E does not negate N'yette's reasoning but she asks Manuel D to evaluate N'yette's representation. Ms. E's reflections support inferences made during the analysis that Ms. E encourages student mathematical discourse and in this case, her motive is so that students can help their peers by providing "inspiration" of different mathematical ideas.

After viewing "Fractions KAE 1", Ms. E reflects that she was interested in the students' reasoning about their representations. "I was trying to understand and follow everyone's thinking. " (Appendix E.1, 15:36) Ms. E describes that at that moment, she was trying to encourage the students to engage in a mathematical discussion, and that she had made a purposeful decision to not provide her own input. Ms. E appears to promote student mathematical discourse as an intervention to help N'yette correct her faulty reasoning when she questions Manuel D's about N'yette's representation. Ms. E redirects the students to read the problem again and she reflects that she was hopeful that reviewing the problem statement would help the students to recognize that N'yette had represented Section 18 as a rectangle instead of as a square. It is inferred that Ms. E recognized that N'yette's representation of Section 18 as a non-regular rectangle, would lead her (and possibly other students) into further mathematical trouble. Nonetheless, rather than impose her own mathematical ideas, or explicitly *tell* the students what N'yette did incorrectly, it appears that she made a purposeful decision to act as facilitator rather than leader. Ms. E's decision not to impose her own mathematical ideas is interpreted to be supportive of student autonomy. However, Ms. E contradicts this inference during the interview when she expresses frustration about not being able to influence her students' reasoning. "...no matter how much I try to sway them on

something that I want to think of when they are not ready to listen to it or when they are not ready to focus on what I want them to focus on they will just keep on going on with what they want to say and it doesn't matter what the teacher says and it's not surprising to me." (Appendix E.1, 16:22)

"Adding and Subtracting Fractions", Key affective event 2

"Fractions" KAE 2 was documented during the first few minutes of the second day of the lesson, "Adding and Subtracting Fractions". Ms. E addressed the whole class as she modeled how to draw Section 18 on the grid paper as an eight by eight square. In the interview, Ms. E describes her pedagogical rationale for this decision, explaining that the students were struggling so much on the subtask and she states "I was trying to set some parameters saying that okay…do a square with sixty-four inside which was my original plan" (18:31). Ms. E does not explicitly describe what emotions she was feeling during KAE 2.

Ms. E recalls that during the previous day she recognized that many students had struggled with finding dimensions of a square that could represent six hundred forty acres on the grid paper. Ms. E recognized that without a correct answer to the subtask, that many students may have encountered more difficulties when trying to complete the next stages of the problem. It is inferred that Ms. E's decision to intervene may be based on her recognition of student impasse and her intention was for students to re-engage with the mathematics of naming fractional parts of Section 18. "I see that people are just struggling with the drawing and its becoming the drawing class and not the math class so I said okay let me set some parameters and see where it goes."

"Adding and Subtracting Fractions", Key affective event 3

"Fractions" KAE 3 was documented on the second day of "Adding and Subtracting Fractions". Ms. E was not present in this key affective event, and although the current study focuses exclusively on classroom observation data when the teacher is present Ms. E's comments during this segment of the interview are relevant to the inferences made in the analysis.

Immediately preceding "Fractions" KAE 3, Ms. E had recognized that Felipe had presented faulty reasoning when drawing a representation of three-fourths. Ms. E called out a general plea for help, to which N'yette responded by walking over to Felipe's table. N'yette also disagreed with Felipe's reasoning and subsequently began to explain a correct representation of three-fourths. During the interview Ms. E reflects that she asked for N'yette's opinion of Felipe's work possibly in hopes that N'yette would recognize the error and then help to correct his reasoning. Ms. E infers that N'yette may have felt empowered at that moment because she was being asked to provide her opinion on the other student's work. It is inferred that Ms. E used this strategy as a means to foster student engagement in peer-to-peer mathematical discussions and to promote student self-efficacy.

Ms. E reflects on the lesson, "Adding and Subtracting Fractions"

Ms. E describes that although she was disappointed by the mathematical progress that was made during this lesson, that she sees "another layer" of success. Ms. E describes feeling encouraged by the level of mathematical engagement and student discourse during this two-day lesson. She reflects that some students were engaged in valuable mathematical discussions, and presented

progress with regards to defending their own mathematical reasoning in verbal exchanges. "Which is … sharing their ideas and defending their answers and if anything these two days did that for some of the kids" (Appendix E.1, 25:06).

Ms. E explains that at the beginning of each school year, she strives to establish an environment where the students feel respected by her and by their peers. In this environment, Ms. E feels that students should be able to discuss their mathematical ideas freely regardless of whether their ideas are right or wrong. Ms. E asserts that it is important for the students to be able to discuss mathematical ideas without the teacher being present, which is interpreted to mean that Ms. E values student autonomy.

Ms. E's reflections: "Finding Areas"

During the retrospective interview that was conducted after the lesson "Finding Areas and Other Products", Ms. E exhibits body language that is inferred to express that she is extremely tired and disillusioned about teaching in general. Throughout the interview, Ms. E looks down at her hands and leans on the table. She appears exhausted as she leans her head in her hands and rubs her face. When speaking about the students' mathematical progress, Ms. E expresses disappointment stating that she was working hard to finish something by the end of the week, but that she did not achieve this goal.

"Finding Areas", Key affective event 1

As Ms. E reflects on the students' behavior during "Areas" KAE 1, she discusses the importance of student-to-student respect in her classroom. During "Areas" KAE 1, Tora was standing at the overhead projector, attempting to present her solution to the mathematical problem (Appendix E.2, E.02.b.01, 9:00). Tora was speaking too quietly for the audience to hear and many students as well as Ms. E asked her to raise her voice.

Ms. E said, “Talk like you always talk”. Many students, particularly Manuel D and Dave repeatedly interrupted Tora, either peppering her with mathematical questions, or berating Tora for speaking so softly. Ms. E interrupted the mathematical discussion to address this student behavior. “I would like for you to respectfully listen first to what [Tora] is saying, interrupting only when it’s necessary. Please don’t cut her sentences. If you have any question, and she’s not talking, you can ask her a question.” (Appendix E.2, E.02.b.01)

During the interview, Ms. E recalls feeling disappointed by the boys’ behavior. While she accepted that the boys wanted to argue their mathematical reasoning, she felt disappointed by their disrespectful behavior.

> “So they were actually questioning the math but they were interrupting her a lot so I think it was very important for me to point out that they have to be respectful those are things that I find I always find that I have to say things like that in the beginning. Since it’s not really a bad thing they are doing they are listening to probably not following [Tora] they were interrupting her also they were talking about the math but I think to create an environment where questioning and justification is the norm everyone should be respectful it is very important that people are respectful of each other.” [Appendix E.2, 12:44]

When the interviewer asks, “… so in this particular instance what prompted you to stop and adjust this?” (Appendix E.2, 14:15) Ms. E responds by explaining that she thinks it is important to address the issue of respect especially when setting up a

respectful environment at the beginning of the school year. "[W]hen they are talking to each other they are talking like that they don't even allow another person to talk let alone finish a sentence so that's just how they are and um its very hard to create an environment where people would be encouraged to talk ... it's very important." (Appendix E.2, 14:24) This evidences that Ms. E's commitment to establishing an emotionally safe environment "where people would be encouraged to talk" is aimed at supporting students' engagement in mathematical discourse.

"Finding Areas", Key affective event 2

As Ms. E reflects on the students' behavior during "Areas" KAE 2, she describes her frustration with the students' faulty mathematical reasoning. During "Finding Areas" KAE 2, Ms. E questioned the students about the fraction name for a visual representation of three-sixteenths (Appendix A.2, E.02.b.02). Many students believed that the representation should have a fraction name three-eighths, which is incorrect. During the interview, Ms. E reflects on her feelings during "Areas" KAE 2, "I'm getting very frustrated with the idea that they think that three eights and three sixteenths could be the same." Ms. E also recalls feeling frustrated because she perceived some students to be off-task at that time during the lesson. In particular, she expresses her frustration with N'yette.

> [N'yette] wasn't paying attention and she's the one that I wanted to address mostly because she's the one that was who had strong feelings about this three eights and three sixteenths being the same… but she wasn't latching on so um I was my

> voice was so high I must have been very frustrated at that point. (Appendix "E.2, 26:36)

Ms. E also recalls feeling frustrated with the pacing of the lesson during "Areas" KAE 2. "Maybe I wanted to finish that lesson and I couldn't even get there I was getting very frustrated that I couldn't so that was another reason why I was uh my voice was so high eek didn't' sound good." (Appendix E.2, 28:15)

Ms. E notes three different reasons why she felt frustrated during this key affective event. First, she was frustrated because students named the representation of three-sixteenths as the fraction, three-eighths. Second, Ms. E was frustrated with the pacing of the lesson, possibly because the class was spending too much time discussing the differences between three-eighths and three-sixteenths. Third, Ms. E was frustrated because many students became disengaged as this interaction unfolded and many side conversations going on.

"Finding areas", Key affective event 3

During "Areas" KAE3, Ms. E made the decision to end the argument about three-eighths and three-sixteenths. She reflects that her decision was based on her perception that most students were not engaged with the mathematics at this point. "Students were giving any answers, any fraction that they could come up with." (Appendix E.2, 33:10) In hindsight, Ms. E reflects that she could have abandoned the discussion earlier, but seems to have gotten caught up in the moment. When asked how she felt when she decided to move on to a different task, Ms. E reflects that she was feeling tired and she reflects that she thought the students also were tired. "Tired. I was very tired I was very tired and they all

looked tired we were all tired actually it wasn't just me they all looked very tired to me" (36:12).

Table 4.22: Summary of Key Affective Events, Ms. E

Interview topic	**Teacher's reflections of her own emotion(s)**	**Teacher inferences of student(s)' emotion(s)**
"Adding and Subtracting Fractions":		
"Fractions" General overview	Uneasy at the students' impasse	Students struggled with the subtask
"Fractions": KAE 1	Curiosity, confusion, hopeful	N/A
"Fractions": KAE 2	N/A Ms. E describes her pedagogical rationale	N/A
"Fractions": KAE 3	N/A	Empowered, proud
General conclusions about "Fractions"	Disappointed by mathematical progress, yet pleased with level of mathematical engagement	N/A
"Finding Areas and Other Products"		
"Areas" General overview	Exhaustion, disappointment about the mathematical progress, sadness	Confusion
"Areas": KAE 1	Disappointed by student disrespect toward peers	Disrespectful toward peers
"Areas": KAE 2	Frustration with the students' mathematical reasoning; Frustrated with the pacing of the lesson; Frustrated by student disengagement	Student impasse, students were disengaged from the mathematics
"Areas": KAE 3	Tired, frustration, stressed	Student impasse, tired, disengaged

4.5b Ms. B's responses to the retrospective stimulated recall interviews

In this section I describe Ms. B's responses to the stimulated-recall interviews, conducted during "Fences for Grazing" and "The DJ Problem". Ms. B discusses her perspective on the lesson in general and responds the key affective events for each of the two lessons ("Fences" KAE 1, "Fences" KAE 2, "Fences" KAE 3, "Fences" KAE 4, and "DJ" KAE 1, "DJ" KAE 2, "DJ" KAE 3 and "DJ" KAE 4) that occurred over the course of each two-day lesson.

Ms. B: "Fences for Grazing"

I describe Ms. B's responses to the interview questions conducted following the two-day lesson titled "Fences for Grazing". Ms. B discusses her perspective on the lesson in general and responds to four key affective events ("Fences" KAE 1, "Fences" KAE 2, "Fences" KAE 3 and "Fences" KAE 4) that occurred over the course of the two days.

The interviewer begins by asking for Ms. B to reflect on the lesson in general across the two class periods. Ms. B expresses that she was pleased with the lesson even though the students were confused about how to calculate area and perimeter. Ms. B indicates that she had anticipated that the students would "have a little bit of a problem" with recalling how to calculate area and perimeter [Appendix F.1, 00:34]. The interviewer asks Ms. B to explain her rationale for choosing a problem that was not taken from the Connected Mathematics book, which is the mathematics program purchased by the school district. Ms. B explains that the problem, "Fences for Grazing", is being used as a "baseline" to assess whether or not the students know area and perimeter without explicitly being told or taught about these concepts. Later in the interview, Ms. B notes

that in the weeks prior to this lesson, the students had been working on open-ended problems that involve either area *or* perimeter but that this was the first time the students had encountered both area *and* perimeter in one problem. Ms. B notes that the students were confused about "what area is and what perimeter is" which "put them at a disadvantage on answering the problem" (Appendix F1, 02:17). Because of this "confusion", Ms. B states that the students had to "waste time on figuring out how to find the area, how to find the perimeter." (Appendix F1, 02:17).

Ms. B expresses pride in her students' engagement with the mathematics across the two days and her responses to the key affective events are described here. She specifically remarks that she is proud of her students despite the fact that they did not completely solve the assigned problem. "They didn't get the answer, it's okay. You're working towards an answer, and they need to know that they're doing a good job. They are, I'm proud of them." (Appendix F.1, 16:28).

"Fences" Key affective event 1

"Fences" KAE 1 was recorded during the first day of "Fences for Grazing" in Ms. B's classroom. Dana asked Ms. B for help with the mathematics, "Ms. B, what do we do?" Ms. B reflects on her interaction with Dana and interprets Dana's behavior in that moment by describing her general character. Ms. B describes Dana as a student who frequently seeks help from the teacher when she experiences impasse with the mathematics. Ms. B explains that Dana does not typically engage with her group when she has difficulty with a problem. "I just think [Dana] wanted an answer at this point. She didn't want to try to think about how to get it, she just wanted to know, 'okay, what's the formula for it so I can plug in numbers and get the answer.'" (Appendix F.1, 3:35).

"Fences" Key affective event 2

In "Fences" KAE 2 Ms. B is asked to reflect on classroom observation data that coincides with episode B.01.b.11 (Appendix B.1). In the moments that precede this key affective event, Shay was reviewing Dana's group's solution and made a comment that the solution was wrong. Meanwhile, Dana was standing at an adjacent table and apparently overheard Shay's comment. Dana became visibly angry, and approached Shay, loudly defending her group's solution. Ms. B describes that she heard the commotion from across the room and approached the students. Ms. B stood about one foot away from the students, centered between Dana and Shay and silently observed their interaction. After viewing this clip Ms. B smiles and reflects that her silent presence was a purposeful decision during this interaction between Dana and Shay. She describes how she encourages students to learn to listen to each other and it is inferred from her interview response that Ms. B saw this interaction as a learning opportunity for Shay and Dana. "We learned how to listen to each other, but just the fact we were talking about it, and I wasn't even over there. I wasn't talking at all. Um, usually that's what happens in class. Like, I won't intervene, as long as they're not fighting". Ms. B explains that sometimes the students speak loudly in her classroom but she indicates that she is accepting of that, "I don't' mind talking in class" (Appendix F.1, 07:08).

When the interviewer asks Ms. B if anything in "Fences" KAE 2 that surprised her. Ms. E describes how she was not particularly surprised that the two groups spontaneously joined together, but indicates that she was pleased because her students were engaged and independent, "my students really are working without me". It is inferred that Ms. E allows students to move freely about the classroom if students are

engaged with the mathematics. "Even though they were suppose to be in their own quadrants, they did move over so they were talking about the problem, so it's okay. It's not that they were off task. So, I'm not going to say surprised, but that stood out, and I like that" (Appendix F.1, 08:52).

"Fences", Key affective event 3

"Fences" KAE 3 occurred after students had circulated through the classroom and written comments on post-it notes about each other's solutions to "Fences for Grazing". Students were returning to their home tables to get ready for dismissal and Ms. B recalls that at that time during the class period she recognized that the students were reading the comments about their work that were written by the other students. Ms. B reflects that she recognized that students were curious about the comments that their classmates had written and that some students were eager to respond to the comments.

> "[T]hey wanted to see what the comments, what comments were written for their group. And that's understandable, but now that just brings a whole 'nother conversation because now they want to see why you wrote that, now they want to explain why they did what they did, but there was no time for." (Appendix F.1, 12:03)

Ms. B describes that the students were so engrossed with looking at the comments that they were not listening to her instructions as she tried to conclude the lesson.

> "They weren't listening to me at that point, because they were still wrapped up in their problems, so I pulled them away from their work, and thinking fast, in the middle, there was nothing there. We had to meet in the middle of

the grid, and so I can give them instructions so we can end the class."

(Appendix F.1, 12:03)

Ms. B's reflections on "Fences" KAE 3 indicate that she is aware that her students' emotions interact with their behavioral engagement. Ms. B recognized that the students were curious and possibly angry about what others had written about their work. She responded to this very quickly concluding that the students' powerful affective responses could either (1.) interfere with the students listening to her instructions as she concluded the class and/or (2.) prompt students to crumple or destroy the post-it notes in anger. She used her interpretations of the students' affect to drive her intervention.

"Fences": Key affective event 4

During "Fences" KAE 4, Ms. B publicly validated Shay for his hard work during that class period. She announces to the class, "When [Shay] works, he works" (Ms. B, C1,a, 40:32 (L)). In the interview Ms. B explains her rationale for making this announcement to the whole class, "I just wanted to make the comments…because someone [another student] did say, 'That was [Shay's] group.' Because [Shay] doesn't always work. You know, [Shay] would walk around in the hallway, and just draggin', not doing anything." (Appendix F.1, 14:33) Ms. B then reflects on Shay's emotions and his engagement with the mathematics during the two class periods. "But when [Shay] works, he works. He knows what he's doing, and you can tell he was actually enjoying himself because he was actually on the floor getting work done, and you know, when you start seeing their behaviors and noticing the things that they do, I was proud of him" (Appendix F.1, 14:33).

During the interview, Ms. B remarks that her interactions with Shay can be difficult at times "…[we] battle, him and I go at it, and it's okay. Because I see what he can do. He's capable of, so I won't back down on him" (Appendix F.1, 15:41). In the interview, Ms. B's pride shines through in her facial expressions and her language as she smiles when she speaks about Shay's engagement and his emotions during these two days. Ms. B reflects that Shay expressed pride in his own engagement with "Fences for Grazing". "[H]e even said after day one, 'I did a good job, didn't I?' And I said, 'Yes, you did a good job.' And he said, 'Well, I'm going to have another good day.' So, for him to say that, and he said, 'And, I'm going to come to class on time.' So, he felt success, and he felt happy" (Appendix F.1, 15:41). It is inferred that Shay's pride appears to foster his anticipated engagement with the mathematics for the following day.

When speaking in general about the importance of praise Ms. B states, "Because we all want to know when we do a good job. Adults, children, anyone. When you do a good job, you want to be recognized at times, so I think it's needed so that they know they are doing a good job"(Appendix F.1, 16:28). Ms. B praises the students for being engaged with the mathematics even if they do not arrive at an answer by the end of the class period. "They didn't get the answer, it's okay. You're working towards (sic) an answer, and they need to know that they're doing a good job. They are, I'm proud of them." Ms. B invites the students to give themselves a round of applause at the end of the class period as she congratulates them on a job well done.

When the interviewer asks Ms. B how the class responded to the lesson, "Fences for Grazing", Ms. B again expresses her happiness and pride with the students' engagement. She recalls that the students came to class on the third day, wanting to

continue discussing solutions to the problem. **"**Well, the next day they wanted to go back into it again because they wanted to keep working on it, so like that is a good feeling for them, not to just dismiss it, and it's like, 'Okay. There's something to do.' They're trying to figure it out" (Appendix F.1, 17:08).

Ms. B also describes how the students sometimes become disengaged from the mathematics especially when the mathematical problem requires many steps and/or the problem can be solved in multiple ways. For example with "Fences for Grazing" both Dana's group and Natasha's group arrived at one answer and then indicated that they were "finished" with the problem. "Yes, they were engaged but I think once they get an answer, and this is a problem across the board, once they get an answer, they think they're done. So a couple, then I said, 'Well how do you know that that's the maximum area?' 'Well', they said, 'Well, just because we did this'" (Appendix F.1, 18:04).

Ms. B's reflections on "The DJ Problem"

In this section, I describe Ms. B's responses to the interview questions conducted during the retrospective interview following "The DJ Problem". Ms. B was asked to reflect on four key affective events, "DJ" KAE 1, "DJ" KAE 2, "DJ" KAE 3 and "DJ" KAE 4. When relevant, I discuss Ms. B's responses as they pertain to the themes discussed in the findings of the classroom observation data analysis.

Ms. B reflects in general about "The DJ Problem" stating, "Um, I think it went okay. They didn't um, get to the level that I wanted them to get to…" (Appendix F.2, 00:41). When the interviewer asks Ms. B if she was "pleased with the level of engagement", Ms. B reflects that she felt happy about the students' engagement with the

problem and she reflects, "Yes. Because the more they talked about it through the whole class, I did notice that, it did keep their attention, um, I think they liked it, because it did relate. We are going to be having an 8th grade dance and we need a DJ, so this is real. We are trying to raise some money." (Appendix F.2, 5:17). Ms. B described earlier in the interview that she had taken the DJ problem from the book and retyped it on one sheet of paper. While she does not explicitly explain her rationale for presenting the problem this way it is inferred that this presentation made the problem more relevant to the students as compared to a problem that they read directly from the mathematics textbook.

> "…when their lessons where they could relate to every day life or things that are going on in our own personal life, then it makes sense. So this made sense to them because they have to raise this money, so money, when we talk about money, they have to raise this money, they're into it, because it's coming from them, it has something to do with them. So yes, when it has something to do with every day life, they're more into it, they're more in tune to figuring out why." (Appendix F.2, 5:49).

"DJ" Key affective event 1

"DJ" KAE 1 occurred during the first day of "The DJ Problem" when Ray, Rita and Shay were creating tables to compare the different rates for the three DJs (Appendix B.2 B.02.a.04, 18:40). Immediately prior to "DJ" KAE 1, Ray was seated at a table with Rita and Shay. Ray was quietly engaged with the mathematics and he was working independently. Rita and Shay had asked Ms. B for help with graphing the equations for the DJs and Ms. B responded to their question by asking Ray for his input. Ms. B asked Ray "what do you have to say about that, what do you think?" Ms. B's reflection ay

during "DJ" KAE 1 confirms the inferences made during the analysis of episode B.02.a.04 that she recognized that she caught Ray off guard.

> "Um, [Ray] when I asked him what does he think about that, I realized that I broke his train of thought. He was actually um working through it, he didn't get as far as the other group but he was on the right track but I kind of just broke his train of thought. That's why I just let him go back because he didn't hear what was going on." (Appendix F.2, 9:06)

Ms. B acknowledges that she realized her mistake and she makes the inference that Ray may have felt bad because he thought that Ms. B perceived him to not be paying attention. She suggests that she did not take the time to assess Ray's engagement before speaking to him.

> Because I asked him what does he think about that so now put him on the spot of what he thinks about that, but he wasn't really paying attention because he had his own thought going on, so he couldn't have done both. So, that just probably not the best thing on my part, to just jump right in and ask him a question, but not realizing he wasn't working either. I didn't realize it until I after I asked him and he looked up and back down like, he was working. (Appendix F.2, 9:37)

The interviewer asks Ms. B to reflect on the students' mathematical thinking and mathematical behavior during "DJ" KAE 1. Ms. B smiles as she reflects that she was impressed by Shay's mathematical engagement during the lesson. "Um, he was into it. um, he was focused, um, [Shay] is very, very active but just to see how he settled down

as soon as he came in, got right to working on that problem, I was impressed because he really started right away. And he worked through it, the whole entire problem. So, he was into it" (Appendix F.2, 11:31).

"DJ" Key affective event 2

"DJ" KAE 2 took place during the first day of "The DJ Problem" in Ms. B's class when Ms. B was standing (or seated) at Dana's table. She also comments on the students' engagement as compared to "the first group", which I interpret to mean Shay's group as discussed in "DJ" KAE 1. "I'm not gonna say into it, but they weren't into it as much as the first group was. They were kind just like, talking about it, they were just saying six hours and [Ghee] said they put 6 hours because that was the easy way out. So, um, I could've asked him maybe, what do you mean was the easy way out?" (Appendix F.2, 14:22)

Ms. B reflects that Dana, Devin, Ghee, Lenny and Shawn were all using different methods to solve the problem and she comments the students were not keeping track of their calculations. "I don't think they were all on the same page. They were all kind of in their own world." (Appendix F.2, 15:40)

When the interviewer asks Ms. B to reflect on the students emotions during "DJ" KAE 2, Ms. B reflects that she thought the students were feeling confused about what they needed to do to solve the problem. "Confusion. I think Dana was confused a little bit, I think she knew she had to do something else, I don't think she was satisfied with her answer, but she didn't know exactly which way to go, at that point. I don't think she was done. But she didn't know where to go." (Appendix F.2, 16:25) Ms. B considers uncertainty as a necessary process when solving mathematical problems, especially when

students are in the early stages of their solution. She describes a sort of brainstorming step to solving "The DJ Problem" as students test out different ideas.

> Being that it was day one, it was ok…that's when the questioning comes about, like when they were saying 'let's party for ten hours' but then someone said, um 'no, that's too long', so then they tried a different number, the next, when they keep on, when they kept throwing out different numbers, then that's when they'll realize, maybe it is cheaper somewhere else. Just them throwing out numbers to each other was ok. For right now.
>
> (Appendix F.2, 16:51)

The interviewer asks Ms. B to reflect on "what was going on" by the end of the third day of "The DJ Problem". Classroom observation data was not collected on the third day of the lesson, but the students did not complete the problem in the first two days so they continued working on the problem. Ms. B offers an interesting perspective on the importance of student collaboration during mathematics class. Shay was absent during the second day of "The DJ Problem" when the class convened to speak about the problem. Ms. B reflects that in Shay's absence, Rita, who had been working with Shay on the first day of "The DJ Problem" experienced difficulty in articulating her mathematical ideas to the group. "…'cause she didn't have that support from her partner that she worked with. But she realized that she was right, so in the discussion, when we have today, she'll be able to express it. So, when [Shay] is with her, I think the two working together is good." (Appendix F.2, 17:32)

When the interviewer asks Ms. B, "She [Rita] was working. How do you think she felt about that?" Ms. B replied,

> "She still understood what she was doing, but when she came against the whole group, she wasn't, she's not very vocal. She just was, she was just saying to herself, 'I still don't understand, I still don't agree, I still don't agree with them.' Whereas if she was loud and outspoken, where [Shay] would be like "No, you're wrong, it's this way." So she kinda just, she just didn't agree. You know, she didn't argue it down." (Appendix F.2, 18:45)

The interviewer asks Ms. B if the students will have another opportunity to share their solutions to "The DJ Problem". Ms. B indicates that they will have an opportunity to share their ideas and she describes that her students enjoy sharing their work with one other. She highlights the value of student's sharing their mathematical solutions as it sometimes helps to enlighten other students to about different problem solving methods. "They like to share their work, I think when they share their work, those who didn't get it, they're gonna realize 'Oh, that's all we had to do?'" (Appendix F.2, 20:44)

Ms. B is asked to reflect on what "stood out" to her during the two days of "The DJ Problem".

> "Um, [Shay] stood out, [Shay] stood out to me just because um, he worked through it very quickly and he is a worker, the class knows him to joke around a lot, and not really do work but lately he has been doing all of his work and he's been engaged, he's been answering questions, so he's really standing out to me at this point because of the way he's working, the way he's working, he seems

> like he's excited to come to the class and…"
>
> (Appendix F.2, 23:09)

"Coming on time to class on time, he would come late to class and just, whatever, like if he wants to come in the beginning, or he'll come in ready to joke around but now he's coming in on time, and ready to work." (Appendix F.2, 23:41)

The interviewer asks Ms. B to reflect on reasons for Shay's change in behavior and mathematical engagement.

> I just think [Shay] is starting to really like the math class. [Ruth] also wasn't really working, wasn't really working in math class. Over just the last couple of weeks, she started working a little more. So just seeing her really involved and engaged too. So two of them who aren't really doing anything now working together and they're getting through the tasks. That stood out to me. (Appendix F.2, 24:07)

Ms. B does not offer any rationale for Shay's and Rita's increased mathematical engagement.

"DJ" Key affective event 3

"DJ" KAE 3 occurred at the end of the class period and it coincides with episode B.02.a.08. Ms. B is asked to reflect on Dana's behavior on the first day of "The DJ Problem".

> She looked upset. Um, sad, something was going on because I did ask her, um "Is everything ok?" And she said she doesn't wanna talk about it, ask at the end of class. And I said, cause I know you weren't asking (acting)

> your same way. That was um, cute. [Ghee] put his arm around her cause he was trying to, I guess make her feel better, asking what was wrong. I think he blew in her ear though. So, but, um, you can definitely tell. (Appendix F.2, 29:18)
>
> She was trying to be focused and on task, but I could clearly see that there was something else going on. So you know, when we say we want them to be focused and to do what they're supposed to be doing, we don't know if something is going on. Cause we all have days sometimes, some of us could focus better than others, but I could tell she was trying, but she wasn't there. (Appendix F.2, 30:20)

After Ms. B views "DJ" KAE 4, she reflects that in the moment of the lesson she may not have realized how upset Dana was during class. She describes one of the struggles of trying to meet the needs of all student in the class is that "you don't see everything that's going on" (Appendix F.2, 32:16). She describes that reflecting on the students' affect during the lesson helped her to "step back for a minute and just be a little bit more sensitive, I think" (Appendix F.2, 32:16).

Table 4.23: Summary of Key affective events, Ms. B

Interview topic	Teacher's reflections of her own emotion(s)	Teacher inferences of student(s)' emotion(s)
"Fences for Grazing"		
"Fences" general overview	Proud of students' mathematical engagement	Frustration, confusion about the formulas for area and perimeter
"Fences" KAE 1	Understanding of Dana's needs	Frustration (Dana)
"Fences" KAE 2	Satisfied with student engagement, happy	Engaged with the mathematics
"Fences" KAE 3	Cognizant of strong student emotions	Curiosity, anger about other students' feedback on their work
"Fences" KAE 4	Pride in Shay's engagement	Pride in his accomplishments. Excitement about math class the next day
"The DJ Problem"		
"DJ" general overview	Students did not meet Ms. B's expectations in terms of mathematical progress. Ms. B was pleased with the level of students' mathematical engagement.	Students were engaged with the mathematics because they can relate to planning a party and hiring a DJ.
"DJ" KAE 1	Shay, Rita and Ray were immediately engaged with solving the problem.	Worried that Ms. B thought he was not paying attention. "Caught off guard" by her questioning
"DJ" KAE 2	"I was just surprised that no one was really keeping track."	Absence of student discourse, they were not "on the same page". Dana was experiencing "Confusion".
"DJ" KAE 3	Cognizant of student confusion about the assigning of variables.	Dana was not engaged in discourse, "she wasn't as outspoken as she usually was throughout the class, she was a lot quieter".
"DJ" KAE 4	Happy that Ghee was trying to comfort Dana.	Dana appeared sad, "She looked upset. Um, sad, something was going on because I did ask her, um "Is everything ok?"

Summary

In this chapter, I presented the findings of the analysis of fifty-seven teacher/student(s) mathematical interaction episodes that were evidenced during eight days of instruction in two urban middle school classrooms. I organized and presented the findings according to the five themes of affective teacher language and actions that emerged from the coding of the episodes: (1) teacher interest in student mathematical engagement, student mathematical reasoning and student mathematical representation, (2) teacher validation of student reasoning, (3) teacher promotion of student autonomy, (4) teacher promotion of mathematical discourse, and (5) teacher promotion of respect. For each theme, I included four summary tables (one table for each lesson) to indicate the presence of each theme as it was coded in the teacher/student(s) mathematical interaction episodes. Following the tables, I presented excerpts from the transcriptions of the classroom observation video data that were illustrative of each of the five themes. For each excerpt, I discussed possible consequences of the teachers' affective language and/or actions with respect to student mathematical engagement.

Taking the teachers' perspectives into consideration, I presented descriptions of Ms. E's and Ms. B' reflections as evidenced during retrospective stimulated recall interviews conducted shortly after the second day of each lesson. The teachers reflected on their students' affect and mathematical engagement "in the moment". They also reflected on their own affect and provide rationale for instructional decisions made "in the moment". The teachers' reflections during these interviews in most cases confirm the inferences made during the analysis of the teacher/student(s) mathematical interaction episodes. In other cases, the teachers' reflections provided an in-depth look at the

teachers' rationale that conflicted with some inferences made during the classroom observation analysis. In conclusion, the inclusion of the teachers' interviews emphasizes the importance of attending to context in the analysis and interpretation of teacher-student interactions. In the following and final chapter of this thesis, I discuss the significance of contextual features that influence teacher language and actions as they interact with their students.

CHAPTER 5: CONCLUSIONS, LIMITATIONS AND CONSIDERATIONS FOR FUTURE RESEARCH

A general purpose of this research has been to gain a better understanding of how affective teacher/student interactions may impact "in the moment" student mathematical engagement. In this chapter, I first provide a brief overview of the data set and method of analysis used in this study. Next, I describe my experiences as a math teacher in a high minority, high poverty middle school, which inspired the study of teacher/student interactions and informed my research perspective. In the third section of this chapter, I discuss specific contextual features that were identified in this study as important in the interpretation of the teachers' affective language and actions during mathematical interactions with their students. I provide examples from the data to illustrate how teacher language and actions can be influenced by these contexts in which they take place. In the fourth section of this chapter I revisit each of the three research questions and I discuss the findings of each supported by examples from the data analysis. Next I describe the limitations of this study. Lastly, I conclude the chapter with a discussion of the implications and considerations for future research.

5.1 Overview of the Study

In this research I studied the language and actions of two urban middle school teachers during mathematical interactions with their students. Ms. E is a seventh grade mathematics teacher and Ms. B is an eighth grade mathematics teacher and they teach in two different schools within the same large urban school district. The majority of the students in these classrooms are African American and/or Hispanic minorities in low-income families.

The data set includes video of four lessons, each spanning two consecutive days, two in Ms. E's classroom and two in Ms. B's classroom. Each lesson is recorded from three different camera angles: one stationary, which captures a wide view of the whole classroom, and two roving cameras, each of which capture interactions at close range. To begin the analysis, I first viewed the video data from each of the three camera angles for the eight days of instruction, totaling twenty-four videos. From the classroom observation video data, I identified every *teacher/student mathematical interaction episode* as evidenced by the teacher engaging in a mathematical interaction with one or more students. All video data were transcribed prior to this study and before the analysis. Next, I verified and analyzed the transcriptions of each identified episode, which total fifty-seven teacher/student mathematical interaction episodes (twenty-one from Ms. E's classroom and thirty-six from Ms. B's classroom). In the analysis of each episode I first recorded descriptions of the teachers' and students' tones of voice, gestures and facial expressions within the transcriptions. I then developed interpretations of the teachers' affective language and actions, which included my inferences about the teachers' emotions and/or rationale for their language and actions based on the descriptions mentioned above. I noted these interpretations alongside the transcriptions for each episode.

The analysis is informed by the events that take place before, during and subsequent to each teacher/student interaction. Fredricks et al. (2004) suggest the importance of considering "antecedents and consequences of behavior, emotion and cognition simultaneously and dynamically" (p. 60) when studying the potential effects that may influence student engagement. In this spirit, I report on the events that transpire

prior to and immediately subsequent to each teacher/student mathematical interaction episode. The nature of these events may be either mathematical or social and may be interpreted as relevant to the immediate episode or may be referenced as influential to a later episode.

Next, based on the interpretive descriptions, I applied a novel coding scheme that I developed for this research (see Chapter Three) for the teachers' affective language and actions. The codes are often applied simultaneously and repeatedly within the same interaction episode. For example the teacher may *show her interest* in a students' reasoning in the same moment when she *validates* the student's mathematical reasoning. Similarly, the teacher may *encourage students to engage in mathematical discourse* at the same time that she reminds the students to listen to each other's ideas, thus *promoting respect*. The simultaneous exhibition of codes is evidenced in the analyzed transcripts of all episodes. The chronological record of all fifty-seven teacher/student mathematical interaction episodes is included in Appendixes A.1, A.2, A.3 and A.4.

After completing the analysis of all teacher/student(s) mathematical interactions I viewed retrospective, stimulated recall interviews conducted with each teacher, which were recorded within two days after each two-day lesson. Each interviews focus on two main areas: (1) the teachers' reflections of the success of each lesson in general and (2) the teachers' reflections on specifically selected video clips, which highlight key affective events (Schorr & Goldin, 2008). In these interviews, the teachers describe their inferences about their students' affect as well as their rationale for their own specific language and actions during the key affective events. The teachers' reflections helped to

inform and, in many cases, confirm inferences that pertain to teacher language and actions described during the analysis of the classroom observation data.

The interpretations of teachers' language and actions were informed by the contexts in which they take place. I consider these contexts as *instrumental* in understanding the reasons why the teachers act in certain ways when interacting with their students. Furthermore, these contexts inform the conclusions about the impact of teacher language and actions on the students' mathematical engagement.

In the following section, I explain my perspective as a former middle school mathematics teacher, which informs my interpretations of the contextual nature of teacher/student interactions.

5.2 The researcher's perspective

I embarked on this study with the hope of connecting theory and research with practice by developing a set of pedagogical tools for teachers to utilize as they interact with their students in mathematics classrooms. As a middle school mathematics teacher attending graduate school part-time I developed an interest in many of the theoretical ideas discussed during graduate seminars. I often wondered how these ideas could be made practical for application in the classroom. Prior to embarking on this research, I taught elementary and middle school mathematics in three very different schools, one high-income parochial school, one middle-income public school and one low-income public school. When I accepted the position to teach eighth grade mathematics in a low-income, high-minority school, I felt quite confident that my eight years of experience teaching in the two other schools had prepared me for this new position. Contrary to my expectations, the challenges that I faced during that school year proved greater than I had

ever imagined. These experiences, which I briefly describe in this section, are the inspiration for this study.

In the weeks leading up to the first day of school, I participated the district's professional development program, which was attended by new teachers and veteran teachers. Veteran teachers introduced themselves and recounted stories about the students who were about to walk into my classroom. The seventh grade mathematics teacher who was assigned as my "mentor" described the incoming eighth graders as a single entity, "unmotivated", "unteachable" and "tough" with a reputation of being "non-learners" since Kindergarten. She sympathized, "… just get them through the year…next year's class is much better."

On the first day of school (in the first moments of the first class period), the students were vocal about our obvious differences, remarking on where they thought I was from naming a wealthy neighboring town "She from [Riverdale]", they commented on my clothing and my jewelry, "Ms. Brett, that's some *nice ice*". I recognized that these comments were not compliments, rather outright declarations that we were not from the same world. My students' judgments of me stung on a superficial personal level and on a profound professional level. I wondered if my students harbored such intense distain for me without even knowing me, how could I possibly help them to learn mathematics?

During the second week of school (things had not improved) I dragged my exhausted self into my first day of graduate school at Rutgers University. The course was *Introduction to Mathematics Education* and Dr. Alice Alston began each session by presenting conceptually challenging problems for us to solve. I enjoyed the mathematical discourse with my fellow students and with Dr. Alston yet I left each class dismayed

wondering how I could engage my students with these types of problems. I was beginning to agree with my colleagues, as I too doubted my students' abilities to solve the conceptually challenging problems. Dr. Alston thoughtfully encouraged me to maintain high expectations for my students despite the daily difficulties of getting my students to engage in any type of academic work.

The following day at school, Rubin refused to write anything on his paper because he said, "whatever I put down will be wrong". In the same class period, Taniqua fell asleep at her desk with her hood pulled tight around her head. Later that day, Chantal screamed at me when I suggested that she come for extra help, "I ain't comin' and you can't make me!" Each day presented similar challenges and I struggled to maintain high expectations for myself and for my students. I began each lesson with a conceptually challenging "Problem of the Day" which had what I called "multiple entry points", opportunities for students to engage with the mathematics on a variety of levels. Most days, I went home thinking that I had reached only a small percentage of the students, those brave enough to engage in mathematical discussions with me.

Slowly, over a period of many weeks, more students began to engage with the mathematics. One day in October, Taniqua took her hood off and made eye contact with me. In December, as the date for the state assessments was looming, Chantal smiled at me and said, "Ms. Brett, you stressed? Whhhhyyy?" I was stressed because our school was "at risk" for state intervention after a failure to meet "Adequate Yearly Progress" on the state assessments in previous years. As the only eighth grade mathematics teacher, I had the sole responsibility of "making the grade" by raising the percentage of "Proficient"/"Advanced Proficient" scores by at least 19.1% from the prior school year.

Meanwhile, Chantal who was now coming willingly for extra help on a weekly basis, was improving slowly but was still puzzled when I asked her to name an equivalent fraction for the number, one-half.

The administrators began to take note of the changes in my students and made more frequent visits to my classroom. The superintendent encouraged other teachers who were struggling to engage the same students to observe my teaching strategies. In April, during an administrative walk-through, the superintendent, Dr. K observed twins Paula and Cheryl having a disagreement about their mathematical solution, Paula insisted the answer was three-fourths and Cheryl insisted the answer was six-eighths. I stood close by, watching and waiting as the girls each explained their reasoning, yelling and pointing at each other. After several moments passed, both girls burst out in hysterical laughter (as did the other students) when they realized that they both had the same answer represented in two different ways. Dr. K. smiled, winked and then left the room. In June, four students (Taniqua as the ring leader) voluntarily stayed after school for "extra math" and asked me to write complicated algebraic expressions on the white board for them to simplify. At the eighth grade graduation ceremony, the entire eighth grade class chanted "Brett! Brett! Brett!" as the superintendent announced a thirty-percent gain in the eighth grade mathematics state assessment. He also proudly noted a historical accomplishment: Raoul was the first special education student in the history of this school to achieve "Advanced Proficient" in mathematics.

What happened over the course of those one hundred eighty days in my classroom? Why did those students eventually engage in Mathematics class and but not in Language Arts or Robotics? Why after eight years of "failure" and being passed along

from one grade to the next, did many of these students prove themselves to be capable learners? When I asked Chantal why she "worked" in my class but not in others, she replied, "You keep it real, Ms. Brett." It was this experience with these "losers" turned "learners" that provoked my curiosity about the nature of teacher/student interactions. As I reflect on my experiences, I believe that my *attention to the contexts* in which interactions with my students took place was pivotal to engaging my students. To give just one example, a major context at the beginning of the school year was the students' mathematical dispositions. When I questioned the students' mathematical reasoning as a strategy to stimulate discussion, they reacted with anger, resentment and fear. They complained to the principal that when they answered my questions, I asked them even more questions. They reported that I wasn't teaching them because I was "making them do all the work". They were not accustomed to being asked to explain their reasons for their answers. I responded to this particular context with *relentless* high expectations for student participation in class. I gave pep talks and straight talks and explicitly expressed my belief on a daily basis in my students' capabilities to be strong and confident mathematical problem solvers. In the design and delivery of each lesson, I included many different levels and opportunities for students to engage in mathematical discussions so that they would begin to develop a sense of mathematical self-confidence. Also, I intentionally allowed students to give incorrect answers, which I used as opportunities to build mathematical discussions around. In these discussions, I often explicitly pointed out that making mistakes are an important part of the learning process. This may have contributed to the students' emergent confidence because I created an environment where it was acceptable to make mistakes without fear of criticism.

At that time I did not have the energy or resources to carefully examine the specific "in the moment" interactions with my students that may have fostered the students' engagement with mathematics. In this study I have had the opportunity to closely examine the language and actions of these two teachers and have been able to identify specific contexts, which appear to influence the teachers' language and actions as they interact with their students.

5.3 The importance of context

In this section I will discuss specific contextual features that were identified as important in the analysis of the teacher/student interactions. I will provide reasons why each contextual feature emerged as an important factor in interpreting the teacher/student interactions and I will include two examples from the data, which illustrate the interplay between context, teacher language and behavior and student mathematical engagement.

Merriam-Webster defines the word "context" to mean: "the interrelated conditions in which something exists or occurs" (http://www.merriam-webster.com/dictionary/context accessed June 18, 2012). In each of these classrooms, the specific contexts are organized in three main categories, each with two or more sub-categories: (1) the four mathematical problems, their design and their delivery; (2) the students; their personalities and dispositions, their social place in the class, their "in the moment" mathematical engagement (or disengagement) and their mathematical understanding; (3) the teacher, her position of authority, her mathematical understanding of these problems, her pedagogical approaches, her perceptions (inferred or explicit) of the students' dispositions, her perceptions (inferred or explicit) of the social and mathematical interactions in the classroom, her perceptions (inferred or explicit) of

student mathematical engagement and her inferred perceptions and/or knowledge of her students' mathematical abilities.

The four mathematical problems in Ms. B and Ms. E's lessons are designed to be conceptually challenging. That is, solving each problem requires the students to think deeply about the mathematics possibly by following multiple solution paths to arrive at the same answer. In this context, the teacher may provide the students time to discuss their mathematical ideas with her and with one another. The teacher may also possibly allow her students to experience a certain degree of frustration without intervention. The delivery of the problems refers to how the teachers introduce the problems to the students. In these data, one teacher introduces each problem by telling the students to open their textbook to a particular page then she calls on a volunteer to read the problem aloud. The other teacher describes a "real-life" scenario as a means for introducing the problem. These different delivery styles can and as evidenced in these data do have consequences for student mathematical engagement. When the mathematical problem is relevant to the students' lives in these data, Ms. B continually revisits the "purpose" of the problem, which she asserts are relevant to the students' real life.

To give one example on the importance of context, I now describe an interaction that takes place in Ms. B's class. Shay is a boy with a "tough" disposition who Ms. B describes as a student whom she "battles with". Nonetheless, she maintains high expectations for him because "she can see the work that he is capable of". In this interaction, Shay is engaged with the mathematical task and when Ms. B approaches his table, he expresses his frustration about the enormity of the solution. Shay has many pieces of paper with his own mathematical representations scattered in front of him. Ms.

B glances at Shay's papers and appears to consider his frustration but does not offer explicit sympathy, nor does she intervene or impose her own mathematical ideas. Ms. B listens to his complaints, encourages Shay to remain engaged until he finds the answer and then she walks away.

In this case, it is inferred that Ms. B's *perception of Shay's mathematical ability* may influence her decision not to intervene. It is possible that when Ms. B glances at Shay's representations, she sees evidence that he does not need a mathematical intervention at this point. Here, the context of Ms. B's pedagogical approach is to allow her student to experience some frustration but hopefully come to the correct answer on his own in this *conceptually challenging* context, which may possibly support his self-confidence as a problem solver.

Later in the same lesson, many other students express surprise, impressed by Shay's presentation of the solution. Upon hearing this Ms. B raises her voice to address the whole class and publicly praises Shay for his sustained engagement with the mathematics. Here, it is inferred that Ms. B recognizes that Shay's social status as the tough boy who does not typically engage has come into question. She loudly exclaims, "When Shay works, he WORKS!"

To sum up, the conceptually challenging nature of the problem presents a challenge for Shay, who exhibits frustration with the challenge. Ms. B appears to recognize Shay's frustration but it is inferred that she makes a deliberate decision not to remove his frustration when she observes the work that he has done. Ms. B maintains high expectations for Shay with the belief that he has the ability to solve the problem. When Shay presents a near complete solution Ms. B publicly praises his mathematical

engagement and perseverance with this problem, which is informed by her perception of Shay's social status in her classroom. As evidenced in the classroom observation data and in the interview data, Ms. B's choices are dependent on her assessment of Shay's disposition and mathematical abilities. Ms. B's language and actions in these interactions have consequence on Shay's engagement as evidenced in the data when Shay says to Ms. B, "I did a good job today, didn't I? And I'm going to do a good job again tomorrow…And I'm gonna come to class on time."

The second example highlights the importance of the context of *the teacher's mathematical understanding*. The mathematics may include potential cognitive hurdles, misconceptions and alternate conceptions, which are important for teachers to understand (Goldin, 2003) because they may emerge as the students solve the problems. When students encounter difficulties in solving conceptually challenging mathematics problems, the teachers' understanding of potential misconceptions and alternate conceptions informs the teachers' interventions and, as evidenced in these data, has consequence for student mathematical engagement. In this example, Ms. E poses a subtask in the delivery of the problem, "Adding and Subtracting Fractions". Prior to assigning fractional values to properties of land within a square section, which covers six hundred forty acres, Ms. E distributes grid paper and instructs the students to represent the whole section of land on the grid paper. Mathematically, this subtask poses many conceptual hurdles including the understanding that six hundred forty is ten times a perfect square integer. Contrary to Ms. E's expectations, the students struggle with the subtask for the entire first period of this two-day lesson and it is inferred that Ms. E does not understand the reasons for the students' mathematical difficulties.

As the students struggle, Ms. E expresses interest in their mathematical ideas and validates their mathematical reasoning even when their ideas are faulty. Taking Ms. E's mathematical understanding of this problem as a context is crucial when interpreting her affective language and actions as she interacts with her students through the conceptual hurdles of this subtask. In the beginning of the second day of this lesson she solves the subtask for the students, telling them to draw an eight by eight square on grid paper but she does not address the reasons for the specific dimensions.

During the retrospective interview following this lesson, Ms. E expresses her own surprise and bewilderment about why the students had such difficulty with the subtask. She explains her rationale for modeling the solution to the subtask on the second day of the lesson was so that her students could then proceed with the intended objective of the lesson, which was to assign fractional values to each property of land with in the square. In sum, the teacher's understanding of the mathematics has particular consequence in the context of conceptually challenging mathematics where students experience conceptual difficulties. Without deep understanding of the potential misconceptions and alternate conceptions the teacher may not have the knowledge of how to apply effective interventions to help their students to get back on the right track. In this example, when the teacher intervenes, she does not address the meaning of the mathematics, and this may have long term consequences of student mathematical engagement in that the students come to expect the teacher to rescue them any time they experience impasse with the mathematics.

Attending to the contexts identified here, the research findings include rich descriptions of the interactions between these two teachers and their students as they

engage in conceptually challenging mathematics. In the next section, I discuss my interpretations of the findings as I consider the contexts in which the interactions take place as critical.

5.4 Discussion of the research findings

In this section, I address the three research questions by reintroducing each question followed by interpretations of the findings as they pertain to each question. I support the interpretations with examples from the data and I discuss how the specific contexts described earlier may inform the interpretations of the data.

5.4.1 Research Question #1

During teacher/student(s) mathematical interaction episodes, what affective language and/or actions do these two urban mathematics teachers use to foster or to support (or possibly discourage or impede) these students' engagement with conceptually challenging mathematics when working in small groups?

This question was addressed through the analysis of the classroom observation data including qualitative descriptions and coding of teacher language and actions as described earlier. From the analysis repeated patterns of teacher language and actions emerged and in the findings these patterns were organized according to five themes:

(1) Teacher interest in student mathematical engagement, reasoning and representation,
(2) Teacher validation of student reasoning,
(3) Teacher promotion of student autonomy,
(4) Teacher promotion of mathematical discourse,
(5) Teacher promotion of respect.

Themes (1), (2), (4) and (5) were drawn directly from the coding scheme and theme (3) emerged from the analysis as a new theme. In Chapter Four each theme is discussed in detail and is supported with illustrative examples from the data analysis. I now describe how each theme of teacher affective language and actions interacts with student engagement and I emphasize the importance of the previously discussed contexts in the interpretations of the data.

The first theme, teacher interest in student mathematical engagement, reasoning and representation emerged dynamically in the analysis from repeated viewings of the classroom observation data. In the early phases of the classroom observation analysis, the code "interest" was applied when it was inferred or evidenced that Ms. E and Ms. B through their language and actions exhibit curiosity, confusion, attentiveness and concern toward their students. In subsequent phases of the analysis, however it became evident that the code "interest" was being applied in every single episode and in most cases multiple times within each episode. As a result, each episode was reviewed many times and examined more closely in terms of the subject(s) of the teachers' expressions of interest. Three sub-codes of teacher interest emerged: (1) interest in mathematical engagement, (2) interest in mathematical reasoning and (3) interest in mathematical representation. Thus, the theme, "teacher interest in student mathematical engagement, mathematical reasoning and mathematical representation" emerged.

Ms. E and Ms. B exhibit interest through their actions when they sit down amongst the students, lean in toward the students, lean in toward the students' work and/or gesture toward the students' work. At times, the teachers inquire about the students' mathematical reasoning and/or representations in a manner that feigns

confusion. Other times, the teachers sit in close proximity to the students and express interest without direct intervention. The teachers' observed and inferred expressions of interest appear to foster and support their students' engagement when the students respond to the teachers' questions and they explain their mathematical reasoning and representations. As evidenced in the data, the teachers' expressions of interest at times interfere with student engagement by derailing students' mathematical thinking or by unintentionally encouraging a student's faulty reasoning.

Ms. B's expressions of interest include questioning about the students' mathematical reasoning and representation. In most cases Ms. B observes her students before she engages in a discussion with them and it appears that she considers all aspects of the experience including the mathematical interactions and the social interactions. Ms. B's expressions of interest appear to foster the engagement of shy students like Devin, who hides his good ideas until Ms. B expresses interest in his ideas. When Ms. B expresses curiosity about why Devin crumpled up his paper, he reveals his reasoning, which then fosters the mathematical engagement of the other students in his group.

Ms. B's expressions of interest are inferred to motivate students like outspoken and jovial Al, who is proud to share his ideas when the teacher takes on the role of the student while he "teaches" her how to calculate rates in "The DJ Problem". Ms. B expresses interest as she follows Al's reasoning by punching buttons on a graphing calculator as he leads her through a series of calculations. In both of these examples, Ms. B appears to consider the contexts of the students' dispositions when interacting with her students. Her expression of interest in shy Devin's idea is inferred to give him the courage to share his discarded idea. When interacting with Al, Ms. B attends to his

expressive disposition by assuming the role of student while he gregariously explains to her how to solve the problem.

Ms. E expresses her interest in the students' reasoning and representation by asking her students to explain their ideas and their drawings. In "Adding and Subtracting Fractions", Ms. E imposes a subtask which the students must complete before successfully solving the original problem. This subtask poses a cognitive hurdle for her students and many students present faulty reasoning which will be detrimental to the students' success at solving the original problem. Ms. E expresses interest, asking the students to explain their (faulty) mathematical reasoning.

In response to Ms. E's expressions of interest, her students engage in mathematical discourse and defend their reasoning and in these cases, Ms. E's expressions of interest are initially supportive of student "in the moment" mathematical engagement. However, when Ms. E expresses interest in the students' faulty reasoning, they often continue with the faulty reasoning, which may result in detrimental consequences when the students encounter difficulty in the later stages of problem solving. Furthermore, Ms. E does not intervene when students proceed with faulty reasoning even after her explicit expressions of interest. This is an important finding because it illustrates how the context of Ms. E's mathematical understanding may influence her interest in students' reasoning. Her expressions of confusion may be genuine as she considers how to intervene with the students' mathematical misconceptions. This inference is confirmed in Ms. E's retrospective interviews when she reflects on her confusion about the students' mathematical difficulties during the

lesson and she expresses continued bewilderment about how she could have intervened to resolve their impasse.

The teachers' expressions of interest at times appear to interfere with their students' engagement in at least two ways. In some cases the teachers' expressions of interest appear to interrupt the students' train of thought and in other cases, their expressions of interest appear to encourage student engagement in faulty mathematical reasoning. To give an example of when the teachers' expression of interest is disruptive to the students' mathematical engagement, I describe an interaction from Ms. B's classroom. Shay, Rita and Ray are seated together when Shay and Rita experience impasse with the mathematics and summons Ms. B. In response to Rita's request for help, Ms. B calls on Ray and asks him to comment on Rita's question. Ray stops writing and looks up at Ms. B with an expression that is interpreted as if he feels he has been "caught" being disengaged. During her retrospective interview, Ms. B is shown a clip of this interaction and she reflects that as soon as Ray looked up at her she recognized that she had "interrupted his thought process". Ms. B reflects with regret that she was too late in recognizing that Ray had been deeply engaged with solving the mathematics problem on his own and he was not aware of the difficulties that Shay and Rita had encountered. She reflects that her questioning impeded Ray's mathematical engagement and possibly caused him to feel threatened at the same time. Ms. B speculates that when Ray could not provide an immediate response to her question he may have felt that she thought he wasn't paying attention. Taking Ms. B's inferred perception of Ray's mathematical engagement as a context in this interaction, it appears that she recognizes the importance of attending to this context "in the moment". In this case (regrettably so) her failure to

take his engagement into consideration prior to expressing her interest may have actually interrupted Ray's engagement.

Contexts that appear to influence the teachers' expressions of interest include the teachers' inferred perceptions of the students' dispositions and their inferred perceptions and/or knowledge of the students' mathematical abilities. These contexts became apparent after repeated viewings of the classroom observation data, which evidence the teachers express interest toward different students in different ways. For example Ms. E's inferred perceptions of her students' dispositions and mathematical abilities may influence her interactions with Manuel D, Dave and Juan. It is inferred that Ms. E perceives Manuel D as an outspoken and mathematically proficient student as she expresses keen interest in Manuel D's reasoning each time she interacts with him. Her interactions with Manuel D are lengthy, sometimes lasting several minutes. When Manuel D is seated at a table with Dave and Juan, Ms. E dismisses the ideas offered by Dave; she glances sideways at him and questions his reasoning abruptly. In the same episode, Ms. E expresses interest in Manuel D's ideas as she leans toward him, looks at his work and asks him to evaluate the work of other students.

Juan makes a pivotal discovery during the first day of "Adding and Subtracting Fractions" when he quietly offer the correct answer to the subtask by suggesting that six hundred forty acres be presented as an eight by eight square. Initially, Ms. E expresses visible excitement toward Juan as she points at him and asks him to repeat what he said. Then she turns to Manuel D and asks him to explain Juan's discovery to many other students in the class as she summons other students to Manuel D and Juan's table. Ms. E

does not publicly credit Juan for his discovery and in fact, there is no evidence in this or subsequent episodes that Ms. E engages in further discussion with Juan.

In the retrospective interviews Ms. E describes her perceptions of Manuel D and Juan's dispositions. She refers to Manuel D as "interesting" and describes Juan as shy and she notes that he does not like to talk in front of others. Taking the observation data and interview data together, at least two interpretations could be drawn about Ms. E's rationale for her language and actions. In one interpretation, Ms. E may recognize that Juan is shy and she uses Manuel D's voice as a vehicle for sharing the idea without putting Juan on the spot, in a sense protecting Juan. Another interpretation is that Ms. E's excitement about Juan's mathematical breakthrough overshadows her consideration of Juan's ego and she has more confidence that Manuel D could effectively broadcast the discovery so that other students could benefit from the idea. In both interpretations, the context of the teachers' perceptions of her students' social and academic capabilities are possibly influential of her language and actions.

The second theme that emerged from the analysis is <u>teacher validation of student mathematical reasoning,</u> when the teacher explicitly acknowledges student responses with expressions such as, "that's a good question" or "what you are saying is very important". Interestingly, when students present correct reasoning, Ms. E and Ms. B *do not always* overtly validate the students' responses. One interpretation of the intention of withholding direct validation is to promote deep mathematical engagement as the students develop arguments to defend their own reasoning. This strategy may have potentially negative results when a student interprets the teachers' non-validation as an expression that her answer is incorrect.

For example in “Fences for Grazing” Natasha expresses the correct answer stating that the largest area with a perimeter of one hundred feet is a square measuring twenty-five feet by twenty-five feet. Ms. B questions Natasha’s mathematical reasoning while showing no change in emotion in her tone of voice or facial expressions that could be inferred to indicate validation or negation of Natasha’s answer. Natasha responds by changing her answer, stating that a square is not a rectangle (in her opinion), and then reasons that the region must be a rectangle with non-equal sides. Ms. B does not validate nor negate Natasha’s misconception. Ms. B’s decision not to overtly validate Natasha’s original answer appears deliberate and is interpreted as a strategy to support Natasha’s mathematical engagement. It is possible that Ms. B’s language and actions in this interaction are influenced by her perception of Natasha’s disposition. Throughout the classroom observation data, Natasha exhibits off-task behavior (drawing and socializing) at times when Ms. B is present at her table. It is possible that Ms. B knows this and her decision not to validate Natasha’s answer is an effort to promote further mathematical engagement by questioning Natasha about her reasoning.

Another example where it is inferred that Ms. B’s perception of student disposition is a context for her language and actions is in her direct validation of Devin’s ideas. As evidenced in the classroom observation data Devin, a shy immigrant boy who is small in stature is seated with students who may be considered more dominant, as they are physically larger than and more vocal than Devin. During one episode, the students were comparing the rates for three DJs for a party and thee number of hours had not been given. The intent was for the students to compare the DJ rates for many different sets of hours (i.e., from one hour through ten hours) to consider the best rate for a variety of

scenarios. The other students in Devin's group had determined that the party would last six hours and they calculated the rates for all three DJs for a period of six hours. When Ms. B sits down at an empty desk next to Devin, a gregarious boy named Ghee announces that they have determined the best DJ to be Light Plastic. Meanwhile, Devin had calculated the rates for a ten-hour party and concluded that the best DJ was Solidus Sounds but he has not shared his idea with the other students. In Ms. B's presence, Devin quietly asks, "Why can't it be ten hours?" Ms. B validates Devin's question stating, "that's a good question" and then she asks the other students in the group to answer Devin's question. Prior to this interaction, the other students in the group were disengaged from the mathematics because they appeared to think they had solved the problem. It appears that Ms. B's validation of Devin's "good question" promotes the students' engagement because the other four students subsequently become reengaged with the mathematics as they calculate the rates for ten hours, which gives them a different answer. The students' curiosity then appears to be activated because they then calculate the rates for all three DJs from one to ten hours. In this interaction Ms. B's validation is aimed at Devin's reasoning, but not a specific mathematical answer when she asks, "Why can't it be ten hours?" Ms. B's validation of Devin's suggestion may be influenced by the context of the other students' disengagement, as they perceive that they have solved the problem. Also, Ms. B's interpretation of the social dynamics in this group appears to influence her marked enthusiasm about Devin's suggestion as she places great value on his suggestions. It is possible that the other students may have otherwise dismissed Devin's ideas because he is soft-spoken and he does appear to engage in social (off-task) interactions with the other students.

The third theme that emerged from the analysis is the teacher promotion of student autonomy. Ryan and Deci (2000) characterize autonomy support as providing "choice, acknowledgement of feelings, and opportunities for self-direction" and they suggest that autonomy support is an "innate psychological need" (p. 68). According to Jang, Reeve and Deci (2010) teacher autonomy support includes acceptance of students' negative affect and frustration during learning. Both Ms. E and Ms. B appear to have intentions of providing student choice in their assignments of conceptually challenging problems that can be solved using a variety of techniques. Both teachers offer opportunities for their students' self-direction in solving the mathematical problems by encouraging the students to take initiative in solving problems. Both teachers offer very little mathematical instruction when they introduce the problems. As the students work on the problems in small groups, the teachers circulate throughout the classroom, stopping to interact with one group at a time. When faced with students' inferred or explicit frustration, Ms. B and Ms. E exhibit language and actions that are quite different from each other.

Ms. E appears to support student autonomy through her language and actions during the first several minutes after she introduces the problem on the first day of each lesson. She circulates through the classroom as the students work in small groups to solve the problem. Ms. E stops at the tables and she asks general questions that pertain to the students' behavioral engagement with the problems such as "How is it going?" and "What are you working on?" When the students respond to Ms. E, she follows their mathematical initiative until it is inferred that she perceives (or it is explicit) that the students experience impasse or frustration with the mathematics.

In these instances, Ms. E intervenes in one of four ways: (1) she provides mathematical instruction not directly related to the problem but possibly intended to support the student's mathematical thinking; (2) she redirects the students to re-read the statement of the problem; (3) she asks other students to offer their opinion about the correctness of the troubled student's work, sometimes by handing the student's work over and (4) she imposes her own mathematical ideas in a whole class instruction. It is inferred her language and actions, particularly in the first, third and fourth intervention style may not support student autonomy because it appears that she is not accepting of her students' negative affect and/or their mathematical impasse. Furthermore, in her first and fourth intervention style, Ms. E sometimes dismisses the students' reasoning and imposes her own mathematical ideas, which is not supportive of student autonomy. It is possible that this intervention style may convey to the students that their mathematical success cannot include impasse and that they are dependent on her intervention.

The immediate consequence of Ms. E's intervention in "Adding and Subtracting Fractions" is that the students defend their faulty reasoning to Ms. E. They then argue with Ms. E until she concedes and the students proceed with their faulty mathematical ideas. While this may appear to be supportive of student self-direction in the moment, the consequence of allowing students pursue faulty reasoning may be detrimental to future mathematical success in solving the problem. Furthermore, after additional investment in the solution of the problem, the students may adopt misconceptions that may be difficult to resolve. In "Finding Areas and Other Products", the immediate consequence of Ms. E's intervention is an argument between her and many of her students, each side standing strong by their reasoning. With raised voices, both Ms. E

and the students attempt to defend their reasoning and the end result is that Ms. E makes the decision to abandon the problem and promises to return to it another day.

The classroom observation data and the interview data evidence how Ms. E's actions and language are influenced by the contexts in which they take place. When the students become disengaged and many students are speaking at once, Ms. E's frustration mounts and it becomes clear that her frustration with the students' disengagement may influence her instructional effectiveness. As evidenced in the interview data Ms. E explains her rationale for intervening when students experience frustration to be informed by two main contexts: the students' mathematical engagement, and her perceptions of the students' mathematical understanding. When the students appear disengaged from the mathematics, Ms. E expresses frustration and she becomes more vocal in the interactions. When her students appear to experience impasse Ms. E expresses confusion about the students' faulty reasoning. She does not articulate her understanding of their misconceptions and alternate conceptions. Thus, Ms. E's own mathematical understanding may have contributed to her difficulties in supporting her students' autonomy.

Ms. B nurtures her students' "inner motivational resources" (Jang et al., 2010, p. 589) in her delivery of the mathematical problems as both "Fences for Grazing" and "The DJ Problem" were assigned as "real-life" problems. "Fences for Grazing" was designed as a "benchmark" to assess the students' understanding of area and perimeter. Although Ms. B obtained "The DJ Problem", from the textbook, she typed out the problem on a single sheet of paper and distributed one copy to each student. Ms. B's presentation of "The DJ Problem" begins with a brainstorming session about party supplies for the

upcoming eighth grade party. The students' excitement is palpable as they yell out ideas for the party supplies. When Rita calls out "music", Ms. B replies, "Oh you want a DJ? Well, it costs money. Costs money for a DJ, doesn't it?" She then directs the students to read the task and she announces that the students are responsible for choosing one of three DJs based on the different price quotes for each. Taking Ms. B's understanding of the students' social nature as a context for interpreting Ms. B's actions it may be concluded that she capitalizes on the social nature of eighth graders in her presentation of the DJ problem. Throughout the lesson, especially during moments of frustration or impasse, Ms. B refers the students back to the purpose of solving the problem, which is to choose the DJ that would be the best value for the eighth grade party. Each time she reminds the students of their "important" role in the decision making process, they become reengaged with the mathematics. In the retrospective interview following the "DJ Problem" lesson, Ms. B conjectures that her students' sustained engagement may have been a consequence of the relevance of the mathematics to their lives. The emphasis on the students' informed choice is interpreted as supportive of student engagement.

When her students are working in small groups to solve the mathematical problems Ms. B supports their autonomy in her actions as she sits quietly observing and it is inferred that her actions convey to her students that she expects them to take the initiative to solve the mathematical problems. In one interaction, Rita summons Ms. B to her table and asks Ms. B for help. Ms. B approaches Rita's table and reacts with feigned surprise, "You need *my* help?" Ms. B's language and actions in this interaction are interpreted as theatrical because she acts as if she may not be able to answer Rita's

question. In this interaction, Ms. B expresses a subtle teasing which imply that Rita is has asked the "wrong person" to help her with a mathematical problem. Ms. B sets the expectation that her role as teacher is not to "rescue" the students when they experience impasse with the mathematics.

Ms. B appears to expect and to accept students' negative affect when solving mathematical problems. When a student expresses negative emotions including embarrassment, anger and frustration, Ms. B acknowledges the student's feelings. In some cases, Ms. B then walks away from the student and the student becomes reengaged with the mathematics. In other cases, Ms. B poses a mathematical question, redirecting the students to the problem. Ms. B allows students to engage in mathematical arguments without imposing her own ideas. When students disagree with one another's mathematical ideas, Ms. B expresses acceptance, telling the students "it's okay to disagree" and she acknowledges that she disagrees with people all the time. She stresses the importance of listening to others' ideas.

This brings us to the fourth theme, <u>teacher promotion of mathematical discourse.</u>

Ms. E and Ms. B foster discourse by providing opportunities for students to share their mathematical ideas within their small groups as well as with the whole class. Both teachers exhibit language and actions that convey their value of mathematical discourse as an important ingredient for successful mathematical problem solving. Ms. E and Ms. B acknowledge most student contributions to class discussions regardless of their correctness.

When students experience impasse, Ms. E and Ms. B invite students to ask their peers for help. Often, when Ms. B is summoned to a table by a student, her first question

is, “did you ask your group?” Ms. E attempts to promote mathematical discourse by asking students to comment on each other’s mathematical ideas as she frequently asks, “Do you agree?”

As evidenced in the classroom observation data during both “Adding and Subtracting Fractions” and “Finding Areas and Other Products”, Ms. E calls on a select few students to engage in mathematical discourse in both whole group and small group interactions. Manuel D and N’yette often are quite vocal in the class and it appears that the context of their social dispositions may influence Ms. E’s tendency to call on Manuel D and N’yette for help when other students are experiencing difficulty with the mathematics. Ms. E’s students respond by considering others’ mathematical ideas and as Ms. E reflects in her interviews, the students sometimes “get inspired” by looking at others’ work. When speaking about why she asks students to help their classmates who are struggling, Ms. E reflects on N’yette in particular, explaining that N’yette feels “empowered” when she is asked to help her classmates solve the problems.

Ms. E promotes mathematical discourse as she invites students to present their solutions to the mathematical problems in whole class presentations from the overhead projector. While Ms. E’s intention may be to engage many students during these presentations, in both lessons many students are observed as being disengaged from the presentation as they slouch in their seats, talk to their neighbor or turn away from the front of the classroom. Taking Ms. E’s interpretation of the students’ dispositions and mathematical abilities as contexts here may help to explain why she only calls on select students to help their classmates, namely Manuel D and N’yette.

Ms. B promotes mathematical discourse when she sits or stands in close proximity of the students and silently observes students' interactions. When students express frustration with the mathematics, Ms. B responds by asking, "did you ask your group?" Ms. B expresses her expectation that the students are responsible for their group members' learning as much as they are responsible for their own learning. When Dana expresses anger toward Emmy and Devin for not helping her with solving the problem Ms. B does not place blame on the accused. Rather, she offers specific suggestions for questions Dana may pose to Emmy and Devin to engage them in a conversation. Taking the students' dispositions and mathematical abilities as contexts, it is possible that Ms. B is sensitive to Emmy's quiet nature and mathematical uncertainty, as she appears to deflect Dana's accusations with a constructive suggestion.

Ms. B promotes mathematical discourse through her language and actions by asking students to present their work on large chart paper on the second day of each lesson. When some students complain that they are not finished Ms. B responds by telling them that they should present their ideas that they have "so far". On the second day of "Fences for Grazing", Ms. B directs the students to circulate with their groups to visit other tables and comment on the other students' solutions. She provides post-it notes for the students to write their questions and prior to sending the students to the other groups she reviews her expectations that the comments be specific and related to the mathematical solution as presented on the chart paper. Ms. B reminds the students that general comments such as "good job" are not acceptable because they lack specificity. During this exercise, Ms. B circulates through the classroom and interacts with the students helping them to articulate their questions and comments. At the end of

the second class in "Fences for Grazing" Shay concludes that Dana's calculations are incorrect and Dana becomes visibly angry at Shay for his comments. As the two students yell back and forth at each other Ms. B stands in close proximity but she does not intervene.

In the retrospective interview following "Fences for Grazing" Ms. B reflects on the interaction between Dana and Shay and she explains her rationale for not intervening. She says, "it's okay to get loud" as long as the students are talking about the mathematics. Further, she maintains that her reason for not intervening was because she was happy to see that the students were "really into it". She expresses pride in her students' enthusiasm about the mathematics and explains that they need practice in learning to listen to one another's mathematical ideas. Listening to the ideas of others is a characteristic of respectful behavior and this brings us to the next theme.

The fifth theme, promotion of respect is evidenced in both classrooms as Ms. B and Ms. E place value on the mathematical contributions of all members of the class. Both teachers emphasize the importance of respect as they encourage their students to consider the mathematical ideas of their peers. The teachers' promotion of respect is evidenced as supportive of student mathematical engagement when students interact in mathematical discussions where disagreements sometimes take place. Both teachers acknowledge that it is appropriate to disagree and they model how to do so using respectful language and actions.

Ms. B explicitly promotes respect by modeling and explaining specific strategies of showing respect. When the students at Frank's table tease him for the way he speaks, Ms. B does not reprimand the other students. Rather, she focuses on Frank who refuses

to discuss his ideas as a result of his peers' teasing. Ms. B encourages Frank to find alternate means of communication so that he can remain engaged with the mathematics. In this interaction Ms. B does not give attention to negative behavior, rather she attends to target of the disrespectful behavior and teaches him constructive ways to defend himself. When students are working in small groups Ms. B promotes respect through her actions when she taps on a students' desk to get their attention and points to a student who is speaking. The subtlety of this type of interaction may foster student engagement with the mathematics, as it is unobtrusive to the mathematical interaction.

Ms. E overtly expresses the importance of respect when she publicly reprimands students for their disrespectful behavior. For example, in "Finding Areas and Other Products" Tora stands at the overhead projector and struggles to explain her mathematical solution to the whole class. Manuel D and Dave complain loudly that they cannot hear Tora and they tease her for her shyness at the overhead projector. Ms. E looks directly at Dave and Manuel D and reprimands them for taunting Tora, "you have to be very respectful". In Ms. E's case, most of her explicit declarations of the importance of respect take place during whole class interactions when a student stands at the front of the classroom to explain their reasoning to the whole class.

In Ms. B's classroom observation data, the one interaction that involved the whole class took place as the students all gathered around one table with the presenter seated and the onlookers standing. It is quite possible that the nature of the presentation style may provide a sense of comfort to the presenter because he or she is not literally "on the spot" in the front of the classroom.

The contexts that appear to interact with the teachers' promotion of respect include the teachers' inferred perceptions of students' dispositions, social interactions and mathematical engagement (or disengagement). As evidenced in the data, the teachers' language and actions in their promotion of respect vary from student to student and also vary depending on the students' in the moment engagement.

5.4.2 Research Question #2

> What can be inferred about students' engagement with the mathematics immediately subsequent to the identified teacher/student(s) mathematical interaction episodes?

This question considers whether or not the students remain engaged with the mathematics subsequent to each interaction with their teacher. Subsequent to the episodes recorded in Ms. E's classroom, the breakdown of student engagement is as follows: engaged, thirteen episodes; not engaged, five episodes (two of these marked the end of the class period); could not be determined, three episodes. Subsequent to the episodes recorded in Ms. B's class, the breakdown of student engagement is as follows: engaged, nineteen episodes; not engaged, nine episodes (two of these episodes marked the end of the class period); could not be determined, eight episodes.

As in the conclusions regarding teacher language and actions during classroom observation data, these observations are highly contextual and it does not suffice to record simply "engaged" or "not engaged". It is implicit that "not engaged" has a negative connotation suggesting that the teacher's presence contributes to the students' engagement and once she walks away the students

become off-task. However, it is possible that student disengagement subsequent to an interaction with the teacher is a desirable outcome when it is a result of the student solving the problem. While this type of outcome is very likely to occur in mathematics classrooms, the mathematical problems presented in these four lessons are conceptually challenging meaning that they involve substantial reasoning and are time consuming to solve. Moreover, there is no evidence from these data that the students solved the problems during or subsequent to any teacher interactions.

Due to the complex nature of teacher/student interactions, it is difficult to generalize reasons why students remain engaged or become disengaged from the mathematics. In some cases, the reasons for disengagement appear to be social in nature as the students begin a social conversation in the teachers' absence. Or, the students sometimes exhibit inferred relief as they sit back in their chairs and exhale loudly when the teacher has moved away from their table. In other cases, the students appear disengaged possibly because they believe that they have completed the problem. Finally, there are cases when students appear to become disengaged following the interaction episode because they experience impasse with the mathematics.

In short, surface characteristics that may be interpreted as evidence of student engagement or student disengagement are not strong enough to allow one to draw any solid conclusions. The method of analysis that was used in this study, despite its detailed and close-in nature, did not enable me to draw any reliable or measurable conclusions about student engagement subsequent to the interactions

with the teachers. Inferences about what may influence student engagement following an interaction with the teacher are very difficult to make given the complexities of contextual features at play. In future studies, analysis of the students' specific activated engagement structures subsequent to each teacher/student interaction may be useful for understanding the consequence of specific types of teacher language and actions.

5.4.3 Research Question #3

> In retrospective stimulated recall interviews, what rationale do teachers provide for their own affective language and/or actions when interacting with students during *key affective events*?

Both teachers were interviewed within one or two days following the completion of each two-day lesson. During these interviews, the teachers viewed video clips from the observation data and reflected on their rationale for their language and/or actions. The teachers' responses in these interviews confirm many of the inferences made in the analysis of the classroom observation data.

In the retrospective interviews following "Adding and Subtracting Fractions" and "Finding Areas and Other Products", Ms. E expresses disappointment about the students' mathematical progress during each lesson. She reflects that she felt "uneasy" when the students experienced impasse with the mathematics because she explains that she was unsure of how to help her students to resolve their difficulties. She also reflects on her frustration with her the students when they exhibited off-task behavior and she cringes at the sound of her own voice as she views herself raising her voice to address the students in the final moments of "Finding Areas and Other Products". These reflections confirm

the inferences made about Ms. E's tolerance of student frustration and/or mathematical impasse. Ms. E's reflections confirm the conclusions about her conflicting support of student autonomy when she describes the difficulties of getting students to understand the mathematics from her perspective. Ms. E describes how her students do not listen to her mathematical ideas, "no matter how much you try to sway them…" Ms. E speaks about the importance of student discourse as a method for broadening students' mathematical ideas. Ms. E emphasizes the importance of peer-to-peer respect in her classroom and describes that she feels annoyed when students are disrespectful of one another. Ms. E reflects on the interaction described earlier when Manuel D and Dave tease Tora during her presentation. Ms. E describes the interaction and explains, "So they were actually questioning the math but they were interrupting her a lot so I think it was very important for me to point out that they have to be respectful… I think to create an environment where questioning and justification is the norm everyone should be respectful it is very important that people are respectful of each other".

Ms. E expresses that she feels aggravated when she perceives students to be disengaged from the mathematics. "I can see people who are not paying attention um that aggravates me a lot but then I have to think also they may also be listening without me knowing that they are." When she reflects that she perceived that N'yette was not paying attention, Ms. E explicitly states that she was frustrated with N'yette's disengagement. "I was trying to pull her into this conversation but she wasn't latching on so um I was my voice was so high I must have been very frustrated at that point."

In the retrospective interviews following "Fences for Grazing" and "The DJ Problem" Ms. B reflects that she anticipated that the students would struggle with

specific parts of each problem. For example she acknowledges she anticipated her students' confusion about how to calculate area and perimeter in "Fences for Grazing". Ms. B's interview responses support the inferences made about her support of students' autonomy when she explains that the students know that sometimes she will not answer their questions. She reflects that some students went home and looked up the definitions for area and perimeter after the first day of "Fences for Grazing". "…[A] couple students did go home to find out what area and perimeter were, so when they came back, they were able to have a little bit of clarity because they knew I wasn't going to answer them."

In this example, it can be concluded that Ms. B's decision not to answer certain questions is driven by her belief that the students can be independent problem solvers when put to the task. Ms. B expresses pride in her students' mathematical engagement and she emphasizes the importance of engagement over problem completion. "Did they finish the problem? No. Did they work? Yes they did."

In particular, in both interviews, Ms. B speaks with pride about Shay's engagement. She describes Shay as a student who is typically disengaged from schoolwork as he comes to class late if at all. Yet in her class Shay engages with the mathematics and she publicly applauds his engagement. When asked how she thought Shay felt after her public affirmation, Ms. B replies, "He probably felt good. He sees, because we battle, him and I go at it, and it's okay. Because I see what he can do. He's capable of, so I won't back down on him, so I think he felt good, and he even said after day one, 'I did a good job, didn't I?' And I said, 'Yes, you did a good job.' And he said, 'Well, I'm going to have another good day.' So, for him to say that, and he said, 'And, I'm going to come to class on time.' So, he felt success, and he felt happy." This

illustrates strongly that Shay's experiences in Ms. B's classroom motivate him and that his engagement is sustained beyond the constraints of the class period.

In both teachers' retrospective interviews, they reflect on the contexts of the mathematical problems and the individual students' dispositions as influential of their instructional decisions. Students engaged with conceptually challenging mathematics are likely to experience impasse during problem solving. A certain level of skill is required for the teacher to help students to navigate these feelings while supporting sustained engagement with the mathematics. Furthermore, a deep understanding of the mathematics including potential misconceptions and alternate conceptions is necessary for knowing if, when or how to intervene when students experience difficulty. Taking students' individual dispositions as a context *within* the context of the mathematics, this presents a complex web of considerations teachers need to make when interacting with their students.

5.5 Limitations

One limitation of this study is the small sample. This study examines four lessons given by two teachers and thus the results may not be generalized to a wider population of middle school mathematics teachers nor to a wider range of communities including suburban and rural areas. Another limitation of this study is that inferences have been made about teacher and student affect based on teachers' and students' facial expressions, tone of voice and gestures. Inferences about student mathematical engagement are drawn from the researcher's interpretation of students' utterances, facial expressions and tones of voice, as evidenced in the classroom observation data. These inferences are subject to the researcher's perspective and we don't know how strongly we can rely on these

inferences. The validity of the findings is subject to the inferences drawn by the researcher and the limited reliability check. Nonetheless, this work is not intended to be definitive, rather descriptive and possibly thought provoking as we deepen our understanding of the interplay between affective teacher interactions and student mathematical engagement. Finally, is difficult to make generalizations about "in the moment" classroom interactions, as every classroom context is different and over time, every classroom environment changes (Franke et al., 2007).

5.6 Implications and considerations for future research

These research findings and limitations suggest questions for further research about the nature and consequences of affective teacher/student interactions in mathematics classrooms as they pertain to student engagement. The purpose of this study was to examine teachers' affective language and actions as they interact with their middle school students in the context of conceptually challenging mathematics. A more general purpose of this study was to contribute the growing body of literature on student engagement taking the lens of teacher moves that may support and/or foster student mathematical engagement.

One direction for future study would be to follow the flow of teacher affective language and actions over a course of one lesson. Also, it may be of value to investigate how affective teacher/student interactions interact with student engagement over the course of a full school year in one classroom. Building on the work done by Goldin, Epstein, Schorr & Warner, (2011) and Sanchez-Leal (in press), the examination of teacher affective language and actions simultaneously with the analysis of student

engagement structures may reveal specific patterns of teacher/student interactions. It is possible that specific teacher language and/or actions contribute to the activation of specific engagement structures and this would be valuable to know in designing effective interventions for classroom teaching.

Meyer and Turner (2006) suggest that results of empirical studies of emotions in classroom contexts may be useful to educators when presented as "rich descriptions of contexts and specific examples of strategies within these contexts that may have the potential for maximizing learning in their classrooms" (p. 387). Teachers may apply the coding scheme developed for this research as a professional development tool as they reflect on patterns of their own teaching practices. Teacher educators may use analysis of video taped classroom observations as a teaching tool for pre-service or in-service teachers to illustrate and interpret the teacher/student interactions in context of real classrooms. As I set out on this research with the goal of helping connect research with practice, the five themes that resulted from the analysis may be used as pedagogical tools for fostering and supporting student mathematical engagement.

The conclusions of this dissertation suggest implications for research and practice. From a research perspective, the analysis of this small set of data was very complex and developed over a course of two years. During the first stages of the analysis of teacher language and actions, I recognized the importance of attending to social and mathematical interactions that took place before, during and after each episode in interpreting the teachers' language and actions. In the qualitative descriptions, I considered the language and actions of the students to inform my interpretations of the

teachers' language and actions. Because most episodes involved the teacher interacting with more than one student, the analysis was very complex. Through the repeated viewings of classroom data and I came to know the students' individual dispositions and I also made inferences about the social relationships amongst the students. These features are of critical importance when studying the effects of teacher/student interactions because students act in certain ways with their peers and in other ways with their teacher. Knowing how the students relate to one another may inform the teachers' language and actions during small group interactions. From a research perspective, the teacher interview data were crucial in confirming my original inferences and constructing my interpretations of how the contextual features influenced each interaction in these classrooms. By attending to the teachers' own voices, I was able to draw conclusions about why they interact with different students differently and why on certain days, they may approach some students in different ways.

The practical conclusions of this thesis suggest five themes of teacher language and actions that may be applied as pedagogical tools in classrooms for fostering and supporting student mathematical engagement. As discussed throughout this chapter, I caution that these themes must be applied in the contexts of each individual classroom. Although the contexts identified in these classrooms may be generalized to many classrooms, some may have more impact than others in different classrooms. Contexts may vary from classroom to classroom and from year to year and may depend on many factors, a discussion that is of course of great importance, but beyond the scope of this dissertation.

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APPENDIXES

Descriptive information for Appendices A and B

The transcripts of the teacher/student(s) mathematical interaction episodes have been arranged in chronological order as they occurred in each classroom and this was confirmed by observing the video data from each of the three camera angles. The time codes for the episodes are not always consistent because the start time for all three cameras was not synchronized. The cameraperson's first initial of their first name follows each time code listed at the start of each episode.

Each episode is preceded by a brief introduction listing the names of the participating students and the teacher as well as a brief description of the inferred source of the interaction. Each episode is followed by a brief discussion of the events that transpired during the episode with regard to teacher and/or student affect as it pertains to the students' mathematical engagement and/or progress during the episode. Within the descriptions certain words are italicized and bolded. These words represent the inferred teachers' affective language and actions. Codes for teacher affective language and/or actions are presented in square brackets. The complete coding scheme is presented in Chapter Three.

APPENDIX A.1

Ms. E: Classroom observation analysis, "Adding and Subtracting Fractions"

Day One

Eight *teacher/student(s) mathematical interaction episodes (TSMIEs)* were identified in the video data collected in Ms. E's classroom on October 11, 2006, Day One of "Adding and Subtracting Fractions" during a forty-five minute class period.

Time	Speaker	Transcript	Descriptions / Codes
Begin TSMIE [E.01.a.01] ***11:10 – 12:30 (L)***			
Participants: Ms. E, Manuel D, Dave Ms. E initiates this episode by asking a general question about Manuel D and Dave's progress.			
11:10	Ms. E	What are you working on?	Ms. E poses a general question to Manuel D, Dave and Juan when she asks, "what are you working on?" Her tone of voice and her language convey ***curiosity*** about the boys' progress yet she is ***not specific about the mathematics.*** It may be inferred that she conveys ***interest*** in the boys' work when she leans on the table in between Manuel and Dave, points to students' papers and books and asks questions. Ms. E's facial expression(s) cannot be described in this episode because she faces away from the camera. **[Int]**
11:14	Manuel D	[Replies to Ms. E with an inaudible response.]	
11:22	Dave	Seven…Seven, it can be.	Dave answers Ms. E but it is not clear what he means by "seven".
11:28	Ms. E	Can we see it here? Can you use any other numbers except for seven or it's seven. [Ms. E points to	Ms. E's tone of voice is ***gentle*** and ***inquisitive***. She demonstrates ***curiosity*** about other possible answers and it may be inferred that she is asking Dave to represent his idea (seven) on paper.

		Dave's paper.]	**[Int, rp, rs]**
11:30	Manuel D	Umm it's a…	
11:35	Ms. E	Does it have to be seven?	Ms. E's tone of voice remains ***gentle*** as she challenges Dave to defend his answer. It may be inferred that Ms. E is ***curious*** about Dave's reasoning for choosing the number seven. **[Int, rs]**
11:36	Manuel D	No it could be a number from seven and above seven and above.	Dave has not responded to Ms. E's questioning. Manuel D suggests that the number could be equal to or greater than seven.
11:37	Ms. E	Seven and above.	Ms. E repeats Manuel D's suggestion "seven and above". Her tone of voice is ***calm*** and does not indicate whether she agrees or disagrees with Manuel D's answer "seven and above".
11:38	Dave	It could be six.	Dave offers the number "six" that contradicts Manuel D's suggestion, "seven and above".
11:38	Ms. E	It could be six.	Ms. E acknowledges Dave's response, but shows ***no emotion.*** She does not draw attention to the fact that using six would be contradictory to Manuel's suggestion, "seven and above".
11:42	Manuel D	It can't be six, cause six will give you over a hundred. Let me check that…eight.	Manuel D disagrees with Dave and appears to be trying to disprove Dave's idea by doing computations on his calculator.
11:43	Dave	But six will give you like one hundred and three.	Comment: to get "like one hundred and three" it may be conjectured Manuel divided 620 by 6, intending to enter 640.
11:44	Manuel D	It gave me perfectly eighty. [Manuel D holds the calculator with the screen toward Dave.]	It may be inferred that Manuel D has divided six hundred forty by eight, which would result in a quotient of eighty. And it appears that Manuel D is showing the quotient on the screen of his calculator to "prove" his answer to Dave.
11:44	Ms. E	Eighty what do you mean eighty?	It appears that Ms. E is ***interested*** in Manuel's mathematical reasoning supporting his statement, "It gave me

			perfectly eighty" because her tone of voice conveys ***curiosity.*** **[Int, rs]**
11:49	Dave	[Dave questions Manuel.] Eighty so what?	
11:50	Ms. E	So eight what?	Ms. E ***restates Dave's question***.
11:52	Manuel D	So each one is one supposed to represent eight.	
11:53	Dave	It was eighty out of a hundred	
11:54	Ms. E	Hmmm, so how would that be here? [Ms. E places palm of her hand on Manuel's drawing and turns the paper, gesturing to Manuel's drawing]	Ms. E ***shows interest*** in Manuel's representation of the section of land when she puts her hand on his paper and rotates the paper toward Manuel. It may be inferred that Ms. E wants Manuel D to represent his mathematical idea on the grid paper. **[Int, rp]** Manuel D squirms in his chair and pulls his chair closer to the table. It could be inferred that Ms. E's questioning about his drawing may be ***making Manuel D feel uncomfortable***. His body language may be interpreted to show that his is engaged with the mathematics even in the presence of discomfort. Manuel D appears motivated to please Ms. E because he answers her questions quickly and often makes direct eye contact with her.
11:57	Manuel D	So each square is like representing a hundred.	Manuel D suggests a numerical value for each square.
11:58	Ms. E	[Speaking to the whole class] How many across how many down, have we decided on what to	Ms. E does not respond to Manuel D's suggestion when she raises her voice to address the class. Ms. E's tone of voice when she addresses the whole class may be

		do?	inferred to be show that she is feeling ***annoyed*** that the students have not completed the subtask when she asks, "How many across, how many down?" **[Dis-m]** Ms. E's language and tone of voice may be inferred to convey ***impatience*** with the class when she asks, "have we decided on what to do?" At this point Manuel D and Dave are experiencing ***impasse*** on the subtask. They have not been able to successfully determine appropriate dimensions for representing Section18 on grid paper.
12:04	Dave	We can't start drawing unless we figure out how many…	
12:07	Ms. E	Can't start drawing? Um…hum.	Ms. E's tone of voice is ***quiet, gentle, understanding.*** It may be inferred that she agrees with Dave that they cannot start drawing without knowing dimensions for Section 18.
12:15	Manuel D	Can we use one…paper or something.(inaudible)	
12:20	Ms. E	Which one?	Ms. E appears ***curious.*** **[Int]**
12:30	Manuel D	This one, so we can…so each square is representing eight.	
12:30	Ms. E	Yeah! [Ms. E is responding to Manuel D's request for paper.]	Ms. E's language and tone of voice suggests ***affirmation***. She sounds ***enthusiastic*** as she responds to Manuel D's request for paper as she walks away from the table.
12:30	Manuel D	….so each square is representing eight.	
END TSMIE [E.01.a.01]			

Discussion of TSMIE [E.01.a.01]

Ms. E initiates this episode when she expresses general interest in the students' progress. She asks, "What are you working on?" Throughout the episode, the mathematical ideas and mathematical questions posed by Ms. E are all based on the student-generated ideas. Both Manuel D and Dave appear to remain engaged throughout the episode by posing ideas for what numerical value can be placed on each square of the grid paper. When Manuel D and Dave present ideas about the dimensions of the section of land, Ms. E does not offer mathematical guidance or affirmation. Rather, she asks a leading question, "Does it have to be seven?" Manuel D squirms in his chair and then pulls his chair in closer to the table. He appears to feel discomfort, yet he remains engaged in solving the subtask.

At the end of the episode, Dave states that they "can't start drawing" until they determine the dimensions of the section of land. Ms. E acknowledges this statement by repeating Dave's words and then walks away from the table. Might there be a missed opportunity here to encourage Dave to go ahead with his idea that they determine the dimensions?

When the episode ends, Manuel D and Dave have not established an appropriate scale to represent six hundred forty acres on the grid paper. It does not appear obvious to the students that using one square on the grid paper to represent ten acres would yield sixty-four, a perfect square. This representation would allow them to draw the figure as a square with dimensions eight by eight. Perhaps Ms. E expected this to be obvious to them, which could account for her (suppressed) annoyance?

The episode is concluded when the Ms. E addresses the whole class, "have we

decided on what to do?" and then walks away from Manuel D's table. Ms. E's tone of voice when addressing the class conveys that she may be annoyed and/or surprised that the students are struggling with the subtask.

Immediately subsequent to this episode, Manuel D and Dave appear to remain engaged with attempting to find the correct dimensions for drawing Section 18 on grid paper because they are leaning over their work and writing on their papers.

Begin TSMIE [E.01.a.02] 13:30 – 14:53 (C)			
Participants: Ms E, Eric and Felipe In this episode, Ms. E approaches Eric's table as Eric and Felipe are working independently. She initiates a conversation with Eric by asking, "So what do you think?"			
13:33	Ms. E	[Students are working independently when Ms. E approaches Eric and stands in close proximity to the table.] So what do you think?	Ms. E asks a general question to Eric in a somewhat vague and quiet tone of voice. ***She is not smiling***, and after posing the question to Eric, she momentarily looks behind her at another group of students before returning her attention back to Eric. **[Int, rs]**
13:39	Eric	I think that Gardella has the most. [Eric points to his paper with his pencil.]	
13:41	Ms. E	Hmmm? You don't have to whisper, you can talk to me. [Ms. E smiles and rubs Eric on the back as she looks up at the camera. Ms. E pushes her hair back behind her ear.]	It may be inferred that Eric is ***shy*** in front of the cameras, or ***shy*** toward Ms. E. She smiles, rubs him on his back, possibly attempting to ***comfort*** Eric and says, "you can talk to me", which may be inferred that Ms. E is demonstrating ***kindness*** and ***acceptance*** to Eric.
13:44	Eric	[Eric smiles and points at his throat with one hand and points at his work with the other hand.] I think Gardella has the most space.	
13:47	Ms. E	Yea you think so? Did you measure these? [Ms. E gestures toward the properties of land on Eric's book as if using her fingers to compare the size of two different properties.]	Ms. E is leaning on the table with her hands, looking down intently at Eric's work. She is ***not smiling*** yet conveys ***curiosity*** about Eric's reasoning when she asks, "Yea you think so? Did you measure these?" **[Int, rs]**
13:50	Eric	No, oh wait.	Eric appears to be ***uncertain*** about his answer, possibly in response to Ms. E's question.
13:56	Ms. E	[Ms. E leans over the table toward Felipe and questions Felipe.] Did you measure, did you measure these, lets see. How many boxes do you have here? [Felipe is working with a ruler.]	Ms. E ***conveys curiosity*** about whether or not Felipe measured two of the properties of land. **[Int, rs, rp]**
14:00	Felipe	One, two three, four, five, six,	

		seven.	
14:02	Ms. E	Okay, did you measure Lapp and Gardella? Measure it. If you measure this one, measure that.	Ms. E appears ***interested*** the boys reasoning, that Gardella's land is the biggest. She tells Felipe to measure the segments bounding each region. **[Int, rs]**
14:15	Felipe	Ummm, centimeters or inches.	Felipe appears ***uncertain*** about which units of measure to use.
14:17	Ms. E	It doesn't matter, which one do you want to use?	Ms. E's tone and language convey ***indifference*** about which units of length he should use to measure the properties of land owned by Lapp and Gardella. This may be a move to promote student autonomy by letting Felipe choose the units. She waits for Felipe to answer her. **[Blds]**
14:23	Felipe	[Felipe begins to measure using the ruler] It's seven!	Felipe does not indicate the units, but appears ***excited*** when he announces the length.
14:24	Ms. E	Seven? Yea but how about just Lapp's?	It may be inferred that when Felipe measured "seven", he measured the vertical dimensions the entire Section 18.
14:29	Felipe	Just Lapp's?	
14:29	Ms. E	Uh um.	Ms. E ***encourages*** Felipe to measure Lapp's land then Gardella's land. **[Int]**
14:30	Felipe	Three.	
14:30	Ms. E	How about Gardella, Gardella.	
14:34	Felipe	Ummm.	
14:36	Ms. E	Use zero to three here. Take zero down. [Ms. E guides Felipe's ruler down the page.]	It appears that Felipe is using the ruler incorrectly because he does not reposition the ruler to start from zero when measuring Gardella's land, which is located adjacent to Lapp's land. Ms. E is ***calm*** and ***gentle*** as she instructs Felipe on how to properly measure with a ruler. Ms. E is ***not smiling***. Her facial expression appears ***serious*** and ***intent*** on Felipe reading the correct measurements.
14:41	Felipe	Take what? [Felipe looks up at Ms. E.]	Felipe appears ***confused*** about what Ms. E means by "take zero down".
14:42	Ms. E	Zero! Start from zero.	Ms. E appears ***impatient*** with Felipe when she says, "zero!" but then her tone changes to being ***calm*** and ***gentle*** when she says, "Start from zero". **[Dis-m]**
14:44	Felipe	Three.	

14:45	Ms. E	Three.	Ms. E repeats Felipe's statement, "three".
14:46	Felipe	I used…[inaudible]	It may be inferred that Felipe begins to explain his reasoning. Ms. B is distracted away from Felipe when another student asks her for a ruler.
14:47	Ms. E	Right okay, so…[Issac asks a question, and Ms. E looks at him.] Hmmmm?	
14:52	Issac	Ruler.	
14:53	Ms. E	Ruler, can you just share rulers please? [Ms. E turns around and addresses the students who are sitting at a group behind her.] Do you need your ruler, can I borrow your ruler for a second? [Ms. E obtains a ruler and tosses the ruler to Issac.] So you decided to use….. [Ms. E begins to walk away from the table to address the whole class.] Is there anything in six hundred forty that tells you ummm, six hundred forty acres of land?	Ms. E provides a ruler for Issac, which she takes from another student who is with a different group. She addresses Felipe, but is not specific and does not finish her question, "So you decided to use…" Felipe appears to be using the ruler to measure something on his paper and he does not look up at Ms. E. It may be inferred that Ms. E recognizes and is ***satisfied*** with Felipe's level of engagement so she does not finish her question. **[Sat]** She then walks away and addresses the whole class, asking, " Is there anything in six hundred forty..."
End TSMIE [E.01.a.02]			

Discussion of TSMIE [E.01.a.02]

Ms. E initiates this episode when she asks Eric a non-specific, non-mathematical question, "So what do you think?" Ms. E's affective language and behavior throughout the majority of this episode suggest that she is conveying a calm, gentle manner as she encourages the boys to convince her of Eric's claim, "I think that Gardella has the most". Ms. E appears to support student autonomy in her questioning style and in her offer of student choice. When she questions the students, she inquires about their thinking as opposed to asking them direct mathematical questions (such as "What is the square root of sixty-four?). She appears curious about the reasoning behind Eric's claim that Gardella has the most space and encourages the boys to prove the claim with actual

measurements. Ms. E supports student choice when Felipe asks which units of measure he should use, and she answers, “it doesn’t matter”.

Ms. E exhibits a brief moment of impatience with Felipe when he appears to use the ruler incorrectly yet within seconds, she returns to a gentle tone of voice and she teaches Felipe how to correctly use the ruler. This intervention appears to be useful when Felipe subsequently recognizes that Gardella’s land and Lapp’s land each have one dimension that measures three units. This revelation also suggests that Felipe was using the ruler incorrectly in the beginning of the episode when he measured Section 18 to be seven units long.

Near the end of the episode, Ms. E begins to ask Felipe a question but he does not respond. It is inferred that Ms. E is recognizes and is satisfied with his level of engagement because she does not finish asking her question. Ms. E walks away to address the whole class asking if they notice anything about the six hundred forty acres of land. It may be inferred that Ms. E is trying to lead the students to see the number six hundred forty as a “clue” and it is possible that she does not understand why the students do not recognize that Section 18 can be represented as an eight by eight square on the grid paper.

Eric’s original claim, “Gardella has the most space” which was questioned by Ms. E, “Did you measure these?” does not appear to have been resolved during this episode. After Ms. E walks away, both Felipe and Eric appear to be engaged in the mathematics because they are leaning into their work, reading, writing and measuring with a ruler.

BEGIN TSMIE [E.01.a.03] 15:08-17:11 (L)			
Participants: Ms. E, Manuel D, Dave In this episode, the Ms. E initiates a mathematical discussion with Manuel D when she approaches the table after Manuel D asks Dave, "How about six forty divided by ten?" Ms. E leans across Juan's place, leaning her upper body, elbows and forearms on the table and asks Manuel D why he divided six hundred forty by ten. Ms. E's body language, tone of voice and questions seem to convey that she is interested in Manuel's mathematical thinking.			
15:08	Ms. E	Six forty divided by ten, why did you do that? [Ms. E leans on the table in front of Juan, and reaches across to point to Manuel D's paper. She rubs her hands back and forth as she speaks.]	Ms. E's language and tone of voice suggest that she is ***curious*** about Manuel D's reasoning for divided six hundred forty by ten. She leans on the table and looks intently at Manuel D and Dave's papers showing ***interest***. Ms. E appears to feel ***serious***. She is not smiling. **[Int, rs]**
15:09	Manuel D	What?	
15:10	Ms. E	Why did you do that?	Ms. E's tone of voice is ***inquisitive*** and her words convey that she is looking for Manuel D to support his reasoning for dividing six forty by ten. **[Int, rs]**
15:12	Manuel D	Because uhh...each one [Manuel D points to his paper with his pencil.]	
15:13	Dave	Ok hold on let me see.	
15:15	Manuel D	Each square	
	Ms. E	Um-hum. [Ms. E looks at Dave's paper. Manuel D stops talking and also looks at Dave's paper.]	Ms. E pulls her attention away from Manuel D momentarily to look at Dave's work. She says "um-hum", but her tone does not indicate that she is interested in Dave's work. Rather, it seems like she is entertaining Dave's idea, but she may not be genuinely interested in his thinking at this time.
15:23	Ms. E	Okay so if you drew [Ms. E looks away from Dave's paper turns to Manuel D and points to Manuel D's paper.] a bo- a square with sixty-four squares inside, how would that look like?	It appears that Ms. E has ***lost interest*** in what Dave is working on, but is ***curious*** about Manuel's thinking because she points at his work and asks him to draw a square with sixty-four squares on

			the grid paper. She seems to be ***encouraging*** Manuel D to complete the subtask of drawing the section of land. **[Int, Enc]**
15:28	Dave	You could multiply too…(inaudible)	
15:30	Manuel D	A normal square. [Manuel D is shrugging his shoulders.]	
15:31	Ms. E	Let's see. [Ms. E is nodding her head.]	Ms. E ***encourages*** Manuel D to draw a diagram based on his "normal square" by nodding her head in possible agreement and saying, "let's see". Thorough her tone of voice and facial expression and gestures, Ms. E appears to ***express hope*** that Dave will be able to produce the correct solution to the subtask of drawing a square region with an area of sixty-four. (To represent six hundred forty acres.) **[Int, rp]**
		[Ms. E straightens up to standing, keeps her hands on the table.]	
15:44	Manuel D	Yea, I could do ten of these and hook it up.	It is unclear what Manuel D means by "ten of these and hook it up".
	Ms. E	[Ms. E is nodding her head.] Sixty-four? Sixty-four boxes inside. [Ms. E turns her head to look toward Eric seated at a different table. He is raising his hand but then puts his hand down and Ms. E turns her head back toward Manuel D.]	Ms. E is ***providing encouragement*** by nodding her head also by remaining at Manuel D's table despite the fact that Eric is summoning her attention from another table. **[Enc]**
15:45	Manuel D	Um..hum.. and each box is going to represent ten. [Manuel D is looking at Ms. E]	
15:47	Ms. E	Ten, [Ms. E raises her eyebrows and nods her head.] uhh um..Let's see that….	Ms. E ***expresses enthusiasm*** with her facial expression, gesture of nodding her head and her tone of voice, which seems to convey ***excitement*** and ***anticipation*** that Manuel D has correctly articulated "each box is going to represent ten". **[Int, rp, Val]**

15:48	Manuel D	Or you could just do ten representing sixty-four.	
15:50	Ms. E	Ok, which one do you want to use? How about that um…	Ms. E asks for Manuel to decide whether each box is going to represent ten or "ten representing sixty-four", both suggestions that Manuel D has recently offered. This may ***provide encouragement*** to Manuel because it may demonstrate to the student that ***Ms. E has confidence in him (promoting autonomy)*** that he is capable of making the correct choice. **[Int]**
15:54	Manuel D	sixty-four?	
15:55	Ms. E	Okay! [Ms. E nods her head.]	Ms. E seems ***happy*** about Manuel D's decision to use "sixty-four" because she is nodding her head and her ***tone of voice brightens***. However, it is unclear what Manuel means by "sixty-four". **[Val, Enc]**
15:55	Dave	Look look look, its almost around here, so it gotta be…	
15:59	Ms. E	What are you trying to do? Dave? [Ms. E furrows her eyebrows and tilts her head inquisitively.]	Ms. E's tone may be inferred to exhibit ***annoyance toward Dave***. Ms. E's tone when she speaks to Dave is distinctly different than the tone that she uses with Manuel D. When she asks mathematically specific questions, she generally directs them to Manuel D and appears to convey genuine interest in Manuel D's mathematical ideas. Conversely, when Ms. E speaks to Dave, her tone is somewhat dismissive and her language tends not to be mathematically specific. **[Int, rs, Dis-m]**
16:03	Dave	We trying to umm, get [Ms. E: "uh-huh"] this, because whatever number you get will get you, you multiply that by the number of squares, [Ms.	It may be inferred that Ms. E is ***not genuinely interested*** in Dave's suggestion because she utters, "uh, huh, uh huh" as if she

		E: "uh-huh"] and that will give you six-forty so.	is hearing his words, but not particularly interested in what he is saying. Also, ***she directs her attention back to Manuel D promptly without providing any feedback to Dave.***
16:16	Ms. E	Ohh..[Ms. E turns her attention back to Manuel D, raises eyebrows, nods vigorously and asks Manuel D:] Do you want to go with your sixty-four, let's see your sixty-four? How would a square with sixty-four inside look like? [Ms. E purses her lips]	Ms. E's facial expression and nodding gesture may be inferred to ***show approval*** of Manuel D's idea. **[Val]** Now, it is more clear that Manuel D's earlier suggestion, "sixty-four" meant that Manuel determined that the area of the region on the grid paper should be sixty-four. Ms. E ***promotes Manuel D's autonomy*** when she says, "Do you want to go with your sixty-four?"
16:24	Manuel D	A square with sixty-four…	
16:27	Ms. E	How would this look like? [Ms. E is pointing to Manuel's paper.]	Her mathematical language is not specific, but it may be inferred that she is ***curious*** and ***hopeful*** that Manuel D will the represent the square with an area of sixty-four units. **[Int, rp]**
16:30	Manuel D	This?	
16:33	Ms. E	Umhum…Can, can I see it? [Ms. E is nodding her head.] Draw me the lot. [Ms. E is leaning over and pointing to Manuel D's calculator display.] that has sixty-four inside.	Ms. E may be ***hopeful*** in anticipation of Manuel D representing Section 18 on grid paper with an area of sixty-four units. Although we cannot see the display on the calculator, it may be inferred that Ms. E sees something that is mathematically correct on the calculator display because she points to his calculator and her voice sounds ***enthusiastic.*** **[Int, rp]**
16:40	Manuel D	I need to find out…What times sixty-	Manuel D is still struggling to

		four…what times what is sixty-four. What times what equals sixty-four.	find two factors that will yield a product of sixty-four.
16:48	Dave	Look. [Dave turns his calculator to show something to Manuel D, points at the display with his pencil.]	
16:49	Ms. E	Answer that question. Do you know? [Ms. E asks Juan and points her finger at Juan.] You can ask him. [Ms. E speaks to Dave and Manuel and points to Juan again]. What was your question? [Ms. E leans in toward Juan and Manuel D.]	It may be inferred that Ms. E is attempting to encourage student discourse and to engage Juan with the mathematics. She asks Manuel D to repeat his question likely so that Juan can have an opportunity to answer the question.
16:55	Juan and Manuel D	What equals sixty-four?	
16:59	Ms. E	What? What's your question? No but you are trying to find?	Ms. E encourages Manuel D to collaborate with Juan by asking Manuel D to repeat his question to Juan. **[Endis]**
17:00	Manuel D	What equals sixty-four?	
17:01	Ms. E	What equals sixty-four?	Ms. E repeats Manuel D's question for Juan to hear.
17:02	Manuel D	Yea so that could be my grid here?	
17:04	Dave	That could be what equals six forty? [Manuel slaps his own forehead, shakes his head, "no", he smiles, glances at Ms. E and then takes a calculator.]	Manuel D may be slapping his head as if to express ***exasperation*** toward Dave. Manuel D has already determined that the representation of six hundred forty acres will be a square with sixty-four units. It appears as though Dave has not yet grasped this idea and is still stuck on finding two factors with a product of six hundred forty.
17:10	Manuel D	Let me see.	
17:11	Dave	See look, cause look. (inaudible)…It's what its gotta be. No its not, what equals sixty-four is	It appears that Dave has missed the point in the discussion when Manuel D recognized that each

		what equals the six hundred forty.	square would represent ten acres. As was noted earlier, Ms. E did not show interest in Dave's thinking and it might be inferred that she recognized at some point that he was not engaged at the same level or with the same problem as Manuel D.
END TSMIE [E.01.a.03]			

Discussion of TSMIE [E.01.a.03]

Ms. E initiates this episode when she asks Manuel D why he divided six hundred forty by ten. She appears to be excited about something that Manuel D has produced on his calculator, yet she does not use specific mathematical language when she asks Manuel, "How would this look like?" Manuel D expresses that Seciton 18 could be represented as "a normal square" and he also makes a conceptual leap when he says ".. and each box is going to represent ten". Manuel D appears to understand that six hundred forty acres can be represented as a region sixty-four squares on the grid paper if each square represents ten acres. He now struggles to find two factors whose product would yield sixty-four.

Ms. E encourages student discourse when she prompts Manuel D to pose a question to Juan about the mathematical task. Prior to this point in the lesson, Juan had not spoken.

In Ms. E's interactions with Dave she exhibits what is inferred to be annoyance. Her tone of voice and discourse toward Dave are dismissive. It is possible that Ms. E treats Dave differently because she recognizes that he is not engaged at the same deep level as Manuel D. She is clearly excited by what Manuel D has to say yet when she speaks to Dave, her utterances are "uh-huh" and "hmmm". At the end of the episode,

even Manuel D appears exasperated with Dave because he Dave seems still stuck on finding two factors that yield a product of six hundred forty. It is possible that Dave was not hearing Manuel D's discussion with Ms. E about representing each grid square as ten acres.

After Ms. E walks away from the table, the boys appear to remain engaged in the mathematics and Juan enters into the conversation with Manuel and Dave for the first time since the beginning of class. Juan: "What equals sixty-four?" Dave: "No, it's not what equals sixty-four, it's what equals six-forty." Manuel D: "Trying to find out sixty-four so that could be like my design, you are gonna see what I'm trying to do."

Begin TSMIE [E.01.a.04] 18:45 – 22:14 (C)			
Participants: Ms. E, N'yette, Tina Ms. E initiates this episode with Tina and N'yette who are seated at the same table. Ms. E instructs N'yette to only "concentrate on Section 18 for now". She remains at their table for approximately four minutes, questioning the girls about how to determine fractional parts of a whole.			
18:45	Ms. E	Oh, just concentrate on eighteen for now. [Ms. E shakes her head, 'no' and N'yette erases on her paper.] So what would you call Lapp? [Ms. E points at Tina's paper.]	Ms. E is giving instructions to N'yette about which part of the task she is to complete. The diagram in the book shows two adjacent sections of land, sections 18 and 19 and N'yette has apparently drawn both sections on her paper. Ms. E appears to convey ***no emotion*** when she then questions Tina. **[Int]**
18:53	Tina	You mean…(inaudible)	
18:55	Ms. E	Look back, [Ms. E reaches across the table and turns the page in Tina's book] look, it asks determine what fraction…	Ms. E directs Tina to look back at the task statement in the book, perhaps to be clarify her question "So what would you call Lapp?" Ms. E's language is directive, and her tone of voice is ***matter-of-fact***. She is ***not smiling*** and she leans on the table next to N'yette.
19:01	student from a differe nt table	Ms. E.	
19:02	Ms. E	Yes. [Ms. E replies to the student from the other table then turns her attention back to Tina and N'yette.]	
19:05	N'yette	I don't get it….[N'yette shows Ms. E the paper then her words are inaudible.]	N'yette expresses that she does not understand.
19:05 - 19:28		[N'yette and Tina to work independently as Ms. E looks on.]	Ms. E stands silently, her face is off camera, but she is leaning on the table, drumming her fingers on the table.
19:29	Ms. E	What, how would you find the fraction… [Ms. E leans on the table, looking at Tina's paper.] Uhh…. the name of the fraction,	Ms. E asks N'yette to name a fraction. Ms. E is ***not smiling***, she has her hands folded and she is leaning on the table.

		give me a fraction.	**[Int]**
19:33	Tina	One third.	
19:33	Ms. E	One third. Could you show me, flip your paper over [Ms. E gestures with her hand.] and show me one-third on the other side of that paper [Ms. E points at Tina's paper], show me one third...How does one third look like? [Tina looks up at Ms. E.] If you drew one third, how would that look like?	It may be inferred that Ms. E recognizes that N'yette's and Tina are having difficulty representing fractions. She directs them away from the assigned task to a different task of representing any fraction. It appears that Ms. E may be drawing on the girls' prior knowledge of representing fractions when she asks N'yette to choose a fraction to represent. **[Int, rp, Rsi]**
19:46	Tina	One third?	
19:47	Ms. E	Uhh um.. [Ms. E looks on quietly and intently, pursing her lips as Tina draws a representation of one third on her paper.]	Ms. E ***encourages*** Tina to proceed and waits to see Tina's representation of one-third. **[Enc]**
19:55	Ms. E	Okay, [Ms. E makes eye contact with Tina, nods her head slightly then looks back at Tina's paper.] So if you would name one of those parts. Turn your, turn it please, say for example Lapp, if you, you would name Lapp what would it be?	It may be inferred that Ms. E is ***pleased*** with Tina's representation when she nods her head in agreement. **[Blds]**
20:12	N'yette	[Ms. E flips over Tina's paper] That's one third?	
20:12	Ms. E	Uhh um [agreeing].	
20:14	N'yette	And you want to name that?	
20:20	Ms. E	Uhh um...what do you need to do?	Ms. E continues to lean on the table looking at the girls' papers as N'yette takes control of the conversation and Tina sits quietly looking on. Ms. E's question conveys ***curiosity.*** Her tone of voice is ***calm***. Her facial expression is ***serious***. **[Int,rs]**
20:21	N'yette	Ohhh, we got to look at it like this. [N'yette gestures with her pencil an up and down motion on Tina's drawing of the section of land.]	
20:23	Ms. E	What do you mean?	
20:26	N'yette	You know like when you do the first one, you did that, [N'yette flips Tina's paper over and writes on the back of the paper.] instead you do it like this.	

20:30	Ms. E	Could you do it on that side, please? Could you do it on this side? [Ms. E turns Tina's paper back over to show Tina's drawing of the section of land.]	**[Int, rp]**
20:33	N'yette	You draw like a big box and just do…cut it in half	
20:37	Ms. E	Go ahead. [Ms. E nods her head at N'yette. She remains in the same position, leaning on the table.]	It may be inferred Ms. E is trying to be ***encouraging*** through her language and tone of voice when she says, "Go ahead" to N'yette. **[Enc]**
20:41	N'yette	[N'yette draws what appears to be a representation of fifths on Tina's paper in the section assigned to Lapp.] … cut it in half, like that, and then after I cut it in half you would do, shade it	It appears that N'yette is presenting faulty mathematical reasoning.
20:52	Ms. E	Okay…	It appears that Ms. E is ***emotionless*** when N'yette presents incorrect mathematical reasoning. She is not smiling and she is looking at N'yette's work on the paper.
20:55	N'yette	And you could probably shade half of that, [N'yette shades in one fifth of the section of Lapp's land on Tina's paper.] to make it like that, or just …	N'yette's incorrect mathematical reasoning is confirmed here when she refers to one fifth that she shades in as "half".
20:59	Ms. E	And what would you name this [Ms. E points to the area on the section of land that represents Lapp on Tina's paper.] if you did that?	Ms. E conveys ***curiosity*** about the fractional name that N'yette would give to Lapp's land. **[Int, rs]**
21:10	N'yette	[N'yette is counting to herself] One out of five, one half out of five	N'yette exhibits incorrect mathematical reasoning and Ms. E responds by asking N'yette to be more specific about her mathematical reasoning.
21:12	Ms. E	What is one out of five, what is one?	
21:15	Tina	One half.	
21:16	N'yette	One half, it would go like this five and one half.	N'yette continues to present incorrect mathematical when she names the region that she previously called "half", now "five and one half".

21:19	Ms. E	Five and a half? [Ms. E looks at N'yette.] What do you think? [Ms. E looks at Tina.]	It appears that Ms. E recognizes that N'yette has presented faulty reasoning, yet she does not negate nor affirm her answer. Instead, Ms. E repeats N'yette's statement, "five and one half" as a question and then asks for Tina's opinion.
21:21	Tina	I think the same thing.	Tina agrees with N'yette and once again, Ms. E does not overtly validate nor negate the girls' mathematical reasoning, despite the fact that it is faulty.
21:22	Ms. E	So you would name this five and a half? [Ms. E gestures with her hand on Tina's paper, the section that is owned by Lapp.]	
21:24	Tina	Yeah.	
21:25	Ms. E	You want to write that down? [Tina and N'yette begin to write on each of their papers.]	***This is a key moment because the Ms. E seems to be conveying to the girls that their faulty reasoning is correct by asking them to "write that down". It appears that the girls do not have a basic understanding of how to represent fractional parts of a whole. ***
21:27	N'yette	See we should have…(inaudible)…I'm about to do the same thing.	N'yette appears to feel confident about her reasoning.
21:30	Ms. E	Why is it, why is it five and a half, I don't see why it's five and a half. [Ms. E scrunches her face.]	At this point, Ms. E's appears ***confused*** about expressing Lapp's land as "five and a half". **[Con]**
21:34	N'yette	Because the way we did it.	
21:40	Tina	[inaudible] Pieces….they said that each person gets one square, [Tina turns back to her math book and begins reading from the task statement] there are six-hundred forty acres of land in one square mile section.	Tina refers back to reading the task statement in her math book, a strategy that Ms. E used earlier when there was a question about how to complete the task. However, the information presented in the task statement is unlikely to help Tina resolve the current impasse that involves finding fraction names for part of a whole.
21:55	Ms. E	Uh umm.	
22:57	Tina	So…that…and the reason why they get one half is because…	As Tina talks, Ms. E looks toward the door of the classroom, raises her eyebrows and then scratches her face. It may be inferred that Ms. E is distracted and concerned about the

			time because she leaves Tina and N'yette to address the whole class. The faulty reasoning regarding the fraction name for Lapp's land is not resolved.
22:14	Ms. E	Can you think about that, I'll be right back.	Ms. E leaves the table, indicating that she will return to hear Tina's explanation, yet she does not return before the end of the class period.
End TSMIE [E.01.a.04]			

Discussion of TSMIE [E.01.a.04]

Ms. E initiates this episode when she gives directions to N'yette about the assignment. It appears that Ms. E intervenes because she recognizes that Tina and N'yette students may be struggling to assign a fraction name to Lapp's land. She draws on the students' prior knowledge when she asks Tina and N'yette to draw a representation of "any fraction".

Ms. E expresses her approval of Tina's mathematical understanding when Tina correctly represents the fraction, one-third.

N'yette then divides Lapp's land into five sections and she shades one. N'yette's reasoning is not given. When Tina and N'yette concur that the land owned by Lapp should be called "five and one half", Ms. E does not overtly validate their thinking but it is inferred that she provides some validation when she calmly asks, "You want to write that down?" despite the fact that this claim presents faulty mathematical reasoning. About five seconds later, after the girls have begun to write five and one half on their papers, Ms. E expresses confusion about their answer.

The girls' incorrect faulty reasoning is not resolved and the episode ends when Ms. E asks the girls to put more thought into why Lapp's section would represent "five

and one half". Ms. E tells Tina and N'yette that she'll "be right back" but she does not return to their table before the end of the class period.

Neither N'yette nor Tina makes mathematical progress toward solving the problem. The girls appear to remain engaged after this episode as they continue to write on their papers, yet despite their engagement, it is questionable that they are making mathematical progress.

BEGIN TSMIE [E.01.a.05] **22:16 - 23:39 (L)**			
Participants: Ms. E, Manuel D, Dave, Juan In this episode, Ms. E approaches the table, smiling and says, "what happened to me?" Approximately five minutes have passed since the last time Ms. E interacted with this group of students. Ms. E appears relaxed and casual as she sits down next to Dave and initiates a mathematical discussion with the students seated at this table. Manuel D and Dave each have each drawn a rectangle on their papers instead of a square. Each figure has one dimension visibly and significantly shorter than the other. Their figures do not satisfy the condition that Section 18 is a square. Also, earlier in the class period, during TSMIE [E.01.a.03] Manuel D states that Section 18 should be represented as "a normal square" (15:30). Ms. E questions Manuel D and Dave, asking them how the rectangles that they drew are the same as the figure shown in their math textbook.			
22:16	Ms. E	Okay, I should see a fraction for each section [addressing the whole class]. [Ms. E approaches Manuel D's table smiling and sits down in a chair next to Dave, across from Juan.] So what happened to, what happened to me [Smiling]. All right, what happened? (Dave "Times ten") Okay, so what are you gonna do with this [Pointing her finger at Dave's paper]?	Ms. E approaches table ***smiling,*** her facial expression, body language, tone of voice and language suggest that she is ***relaxed.*** She sits down at a chair next to Dave, across from Juan. It may be inferred here that Ms. E's comment, "what happened to me?" may suggest Ms. E's curiosity about the mathematical progress that was made in her absence.

22:25	Manuel D	We're gonna place, each [Manuel D points his pencil at Dave's book, then at his own paper.],	
22:26	Ms. E	Okay….	Ms. E appears to be ***interested*** in hearing what Manuel D is about to say when she leans in and says "okay". Manuel D and Dave have each drawn a rectangle with dimensions sixteen by four to represent the section of land with an area of six hundred forty square acres. The boys have made the conceptual leap that each square on the grid paper can represent ten acres. However, they have drawn a rectangle instead of a square, which will interfere with their ability to correctly draw the properties of land assigned to each landowner. **[Int]**
22:27	Manuel D	…each one of the properties [Ms. E: "um..hum"]	
22:30	Ms. E	Okay, each one of these… So how are you gonna put Lapp how does that, how does that work?	Ms. E does not draw attention to the fact that the section of land is drawn incorrectly (rectangular instead of square). Instead, she asks the boys how they will represent Lapp's land. Lapp is one of the landowners indicated in figure that accompanies the statement of the task. It appears that Ms. E is ***curious*** about how the boys are going to divide the rectangle to accommodate the individual properties in the rectangular Section 18. **[Int, rp]**
22:34	Manuel D	So we are gonna have to divide. Lapp times something. Lapp times sixty-four, right? [Manuel D looks at Dave.]	It may be inferred that Manuel D is asking Dave for affirmation.
22:41	Ms. E	But how is this this, I don't see why this is this shape.	Ms. E appears to become ***anxious*** as she speaks more quickly than she did in

		This…this figure…	previous episodes. She seems to be ***challenging*** the boys about how the rectangular section that they drew is the same figure as the figure that accompanies the statement of the task. She appears hopeful that her questioning will lead the boys to recognize that the figure is drawn incorrectly. **[Con]**
22:47	Manuel D	It don't have to be the same figure.	

22:50	Ms. E	…doesn't have to be the same figure.	Ms. E repeats Manuel D's claim that the shape does not have to be the same figure. Her tone of voice that may be perceived to convey ***approval*** of Manuel's claim. **[Val]**
22:51	Dave	Because look, sixteen across…	Dave appears to be proving to Ms. E that his rectangle has the dimensions to yield an area of sixty-four square units.
	Ms. E	Okay…	
22:52	Dave	four down.	
22:53	Ms. E	And it gives you	
22:55	Dave	Sixteen times four….sixty-four	
22:55	Ms. E	Is it the only way you can draw a square with sixty-four inside, a figure [raises eyebrows and nods head to the side] with sixty-four inside?	Ms. E appears ***curious*** when she questions Manuel D and Dave about the shape of a figure with an area of sixty-four. It is likely that Ms. E ***hopes*** for Manuel D and Dave to discover that Section 18 must be a square. **[Int, rs, rp]** Ms. E corrects herself when she states, "a square with sixty-four inside, a figure with sixty-four inside." This is interesting because the statement of the problem clearly states that the section of land must be a square, this is not ambiguous. Yet, ***Ms. E is ambiguous in her language that the section of land must be a square.*** She first says, "square", then restates the question, using the word, "figure".
23:02	Dave	Sixty-four times ten is six hundred forty.	
23:05	Ms. E	Oh, [nodding head toward Dave] I see that.	Ms. E's tone of voice sounds as if she is in agreement of Dave's statement that "sixty four times ten is six hundred forty". She nods her head, possibly affirming Dave's answer. However, Dave's statement really does not have anything to do with finding a figure with sixty-four inside. *** After Ms. E replies to Dave, Ms. E

			looks across the table at Juan, and her face looks ***surprised and excited.*** It appears that Ms. E is ***interested*** in something that Juan said. *** **[Int]**
23:07	Manuel D	You can do eighty times ten is four [Ms. E's posture straightens, and she looks across the table at Juan].	It may be inferred that Ms. E did not hear Manuel D because she was listening to Juan.
23:12	Ms. E	What…what were you saying? [Ms. E is looking at Juan and she points finger at Juan]	It appears that Ms. E is very interested in Juan's mathematical idea. Her voice becomes ***excited*** and she speaks quickly almost stumbling over her words, leaning in toward Juan. **[Int, Endis]**
23:15	Juan	What to get like sixty-four? [Juan is looking at Ms. E.]	
23:15	Ms. E	Yea..huh.	Ms. E's tone of voice conveys ***excitement*** about how Juan got the number sixty-four. It may be inferred that she is excited because Juan's idea is a turning point toward a productive solution path. **[Int, rs, Val]**
23:17	Juan	Go eight across and eight down.	Juan has presented correct reasoning for drawing a square with an area of sixty-four square units.
23:18	Ms. E	Can, can we try that, eight across and eight down? Would that give you sixty-four?	Ms. E's tone of voice appears to show ***encouragement*** and ***excitement*** about Juan's mathematical idea to "go eight across and eight down". Although Ms. E does not explicitly use words that acknowledge Juan has the right answer, she ***affirms*** Juan's idea by asking Dave and Manuel D "can we try that"? **[Int, rs, Val]**
23:21	Dave	Yeah.	
23:23	Manuel D	[Manuel D punches keys on his calculator.] Eight times eight	
23:27	Ms. E	And what are you doing when you do that?	Ms. E is not specific in her questioning but it may be inferred that she is ***curious*** about why Manuel D is multiplying eight by eight. **[Int, rs]**
23:28	Dave	Multiplying.	

23:29	Ms. E	Multiplying what?	Ms. E appears ***curious.*** **[Int]**
23:31	Dave	Eight times eight.	
23:33	Ms. E	All right let's see, umm how would that look like, if you have umm, an eight by eight?	Ms. E conveys ***curiosity*** about how "an eight by eight" (square) would be represented on the grid paper. She specifically asks the boys "how would that look like…eight by eight". **[Int, rp]**
23:39	Dave	But we have to do it in the box.	
End TSMIE [E.01.a.05]			

Discussion of TSMIE [E.01.a.05]

Ms. E initiates this interaction when she shows interest in the representations of Section 18 that Manuel D and Dave have drawn on their grid papers. The boys have each drawn a rectangle so their representations contradict the condition that Section 18 is a square. Ms. E does not correct the boys, but instead asks the boys how they would place Lapp's property in their representations. Next, she questions their representation saying that she does not understand their figure. Dave defends his representation by stating that the dimensions of the rectangle (sixteen by four) produce an area of sixty-four square units. Ms. E then asks the boys to name other possible dimensions for a "figure" with an area of sixty-four. Despite the fact that the problem specifically states that Section 18 is square, Ms. E is ambiguous in her language which may cause some confusion.

Meanwhile, it is inferred that Juan is working independently to solve the problem and he may in fact have the answer. Ms. E appears to notice his breakthrough and becomes excited. Juan quietly stated dimensions for a square section of land with an area of sixty-four as "eight by eight". Ms. E eagerly asks Juan to repeat his idea then she

encourages the three boys to draw this representation on their grid paper.

Juan's suggestion has prompted mathematical progress toward a solving the subtask. Ms. E's intervention may have encouraged Juan to share this idea with Manuel D and Dave because Juan appears to be a quiet student, possibly reluctant to share his ideas. In this episode, Ms. E's expression of excitement, coupled with her mathematical questioning and encouragement for Juan to share his idea all appear to contribute to the mathematical progress of the group.

Subsequent to this episode, Juan, Manuel D and Dave continue to remain engaged with the mathematics as they draw the eight by eight square on their grid paper. In the next episode Ms. E invites students from other tables to collaborate with Juan, Manuel D and Dave.

Begin TSMIE [E.01.a.06] 23:40 - 30:26 (L)			
Participants: Ms. E, N'yette, Tina, Jennifer, Brielle, Manuel D, Felipe, Dave, Juan, Noah In this episode, Ms. E remains seated at Manuel D's table. Juan has recently presented the idea that the dimensions of the Section 18 should be eight by eight on the grid paper. Ms. E appears excited about this idea and she invites students from other groups to engage in a discussion with Manuel D, Juan, and Dave.			
23:40	Ms. E	[N'yette] come here. And ….	Ms. E calls students from another table to come to Manuel D's table.
23:42	N'yette	[Tina]	
23:46		[Tina] [Tina and N'yette get up and join Manuel D at the table with the Ms. E] Look. Can you explain what you are doing as you are doing it [Manuel]?	Ms. E asks Manuel D to explain his process of solving the task, which shows that she is ***pleased*** with his mathematical ideas and finds value in having Manuel D sharing his reasoning with his peers. **[Val]**
23:47	Manuel D	One..two..three..four..five..six..seven..eight. [Manuel D is counting aloud, but to himself.]	
23:50	Ms. E	[Felipe] come here for a second?	

		[Felipe approaches Manuel D's table.]	
23:56	Manuel D	It would look almost exactly as this. [Manuel D points to the figure in his book].	It may be inferred that Manuel D has recognized that the eight by eight figure is similar in shape to the original figure presented in the problem. He has satisfied one condition of the problem.
23:58	Ms. E	Ahha, Oh yea, so what do you think Juan?	Ms. E's tone of voice sounds like she is ***happy***, with Manuel D's mathematical reasoning that Section 18 must be represented as a square. It may be inferred Ms. E encourages Juan to engage in the discussion especially since this was originally his idea. Ms. E's tone with Juan is ***gentle*** and her language is ***encouraging.*** **[Val, Endis]**
24:03	Felipe	What?	
24:07	Ms. E	[Jennifer], you want to see this? [Brielle], could you look over, [Brielle]. [Manuel D] could you explain what you are doing please?	Ms. E asks Manuel to explain what he is doing for the new students who have joined the group. The video shows that Manuel D had just drawn an eight by eight square on his grid paper. Ms. E's tone of voice is ***gentle*** when she calls Jennifer to the table. Brielle is the fifth student that Ms. E has invited to look at Manuel's work. Ms. E appears ***pleased*** with Manuel D's representation. Ms. E's motive for inviting so many other students to hear Manuel D's reasoning may be to help more students. It may be inferred that the other students are still struggling to overcome the conceptual hurdle that has been imposed by the subtask.

24:19	Manuel D	I'm doing cause….each box is gonna represent ten right, because, because sixty-four times ten equals six-forty…	Here, Manuel D describes arithmetically how he came to sixty-four. He does not explain why he assigned a value of ten to each box.
24:22	Dave	Six-hundred forty.	
24:27	Manuel	Six-hundred forty.	
24:29	Ms. E	So you are thinking what, so you are thinking…	Ms. E gestures toward Manuel, possibly to ***encourage*** him to be clear in his explanation for his classmates. **[Endis]**
24:30	Manuel	[Manuel interrupts Ms. E.] You gotta find, what times what equals sixty-four so that would be your shading.	
24:36	Ms. E	Okay let's see, and let's see what you have. [Ms. E asks Felipe]	Ms. E appears ***curious*** about Felipe's mathematical progress. **[Int, rp]**
24:38	Manuel D	We need sixteen times…	
24:42	Ms. E	How many, what what did you do here [Ms. E points to Felipe's paper.]?	Ms. E's language and tone of voice is ***inquisitive*** implying that she wants to understand what Felipe has done on his paper. **[Int, rs]**
24:44	Felipe	Umm… I did all this. [Felipe holds up his paper.]	
24:46	Ms. E	Let's see. Let's see. Is this right, something that you are thinking about? [Ms. E passes Felipe's paper to Manuel D and asks Manuel D's opinion of Felipe's work.]	Ms. E asks Manuel D if he thinks Felipe's work is correct. This could be ***inferred that she respects*** Manuel D's opinion.
24:52	Manuel D	[Manuel D counts across on Felipe's paper] One, two ,three, four, five, six, seven , eight, nine, ten…[counting down on Felipe's paper] One, two, three, four, five, six, seven , eight, nine [Manuel D looks up from Felipe's paper at Ms. E.]	
24:52	N'yette	[Speaking to Juan]	

		You said each box is what, ten….so you do the first ten.	
24:56	Juan	If each box is ten, you gotta get sixty-four.	
25:00	Ms. E	Do you agree with him? [Ms. E asks Manuel D about Felipe's work and scratches her eyebrow.]	This is the second time that Ms. E has asked for Manuel D's opinion of Felipe's work. Ms. E's tone of voice appears to convey ***doubt*** regarding Felipe's work. **[Int,rs, Endis]**
25:02	Manuel D	[Manuel D shakes his head 'no' and gestures with hands up.]	
25:03	Ms. E	Why, what are you thinking?	Ms. E appears ***curious*** about why Manuel D disagrees with Felipe's work. **[Int, rs]**
25:05	Manuel D	Because, its like ten times ninety equals…equals...ten times nine equals ninety…but I don't see what ninety equals, goes into sixty-four. [Manuel D is speaking to Felipe.]	Based on Manuel D's language, it appears that Felipe has drawn a rectangle that measures ten units by nine units.
25:17	Ms. E	And what are you thinking to do? [Ms. E moves Felipe's paper and points toward Manuel D's paper with her finger.] Could you explain to Felipe what you trying to do? [Ms. E looks away toward another table.]	Ms. E demonstrates ***trust*** in Manuel D's mathematical ideas by suggesting that he explain his reasoning to Felipe. This action ***affirms*** yet again that Manuel D's thinking is correct and values his representation. **[Endis, Val]**
25:22	Manuel D	[Manuel D shows his paper to Felipe.] Here, what you can do is, find something that can be, that can represent each part, sixty-four….you can do sixty-four….	While Manuel is talking to Felipe, Ms. E has her hand on Felipe's paper and is sitting quietly listening to Manuel D, ***conveying interest*** in Manuel's ideas. **[Int]**
25:35	Ms. E	Could you speak up so [Brielle] can hear you?	**[Endis]**
25:36	Manuel D	So you, so you fit, into each one of those boxes fit ten, and out of ten times sixty-four will equal six, six forty. Now you got, now you gotta, now you gotta find what times, what times what equals	Manuel D is able to articulate why he used using sixty-four "boxes" to represent six hundred forty acres.

		sixty-four so you - that will be your shape.	
25:53	Ms. E	So your shape is going to be what?	Ms. E's tone of voice demonstrates ***anticipation***. She seems to expect Manuel D to tell her the dimensions of his "shape" and possibly to state that the shape will be a square. **[Int, rp]**
25:55	Manuel D	Uhh eight times eight.	Manuel D presents the correct answer to the subtask.
25:57	N'yette	Wait can I say something [N'yette points and smiles at Manuel D]?	
25:58	Dave	Yes 'cause eight times eight equals sixty-four.	Dave confirms Manuel D's statement by stating the product.
26:01	Ms. E	[Speaking to Felipe, and Felipe leaves] Do you want to try that?	It may be inferred that Ms. E is asking Felipe to "try that" because she is ***pleased*** with Manuel D's idea to use eight by eight as the dimensions for Section 18. **[Sat, Blds]**
26:01	N'yette	I see, I see, I see what ya'll saying but ours is similar to yours but its some different, we did it the exact way of the chart, but at the same time we did it ten down and four across. So what ya'll did was sixteen we did ten.	
26:21	Manuel D	[Manuel D takes a calculator and begins doing calculations] Lemme see..one, two, three, four, five, six	
26:27	Ms. E	[Noah], [Noah] come here	
26:30	Ms. E	Check this out.	Ms. E's language, "Check this out" implies that she is ***enthusiastic*** about the work being done at Manuel D's table, possibly providing affirmation for Manuel D, Dave and Juan. Ms. E's tone of voice and language seems ***casual*** as she says, "check this out"**.** **[Endis, Blds]**
26:33	Manuel D	Six. What? Six forty right, Six	

		forty, Six forty divided by twenty right? [Manuel D looks at N'yette]	
	N'yette	We did it times forty. [N'yette shakes her head 'no'.]	
26:35	Dave	We did ten.	
26:40	Manuel D	[Ms. E walks away from the table.] You did it ten times…forty…forty…hey we could do that too look, look you can do forty times sixteen equals six, six forty. So in a confusion way which they don't know they are right. [Manuel D holds the calculator and taps it on the table as he speaks to Dave].	Manuel D states that N'yette and Tina are correct despite the fact that they did not use a square with dimensions eight by eight.
26:50	Dave	[Dave grabs N'yette's paper]	
27:05	Manuel D	[Ms. E returns to the table, standing, but leaning on the table, looking at Manuel D's paper.] Look, they are right. [Manuel D looks up at Ms. E.]	
27:07	Ms. E	Why?	
27:08 27:10	Manuel D Dave	Because, umm, you, you divide (Ms. E: Um-hum) six forty (Ms. E: Um-hum) times forty (Ms. E: Um-hum), would equal sixteen (Ms. E: Um-hum), so then they….	
27:20	Ms. E	[Ms. E interrupts Dave] So can they use this? [Ms. E takes N'yette's paper out of Dave's hands and Ms. E holds the paper toward Manuel D, and looks up at him.]	Once again, Ms. E is demonstrating that she has ***confidence*** in Manuel D's mathematical ideas because she takes the paper out of Dave's hands to ask Manuel D's opinion. **[Blds, Int, Endis]**
27:21	Manuel D	Yes.	Manuel D is incorrect in agreeing that N'yette and Tina can use the figure that they have drawn on their paper.
27:21	Dave	Yes they do.	
27:22	Ms. E	<u>They can use this, so let's look</u>	It may be inferred that Ms. E is

		back at the problem and see if we're okay still. [Ms. E turns a page in Dave's book and runs finger along text as she reads the task statement from the book].	***dissatisfied*** with Manuel D's reasoning when she directs the students to review the problem statement in the book. **[Dis-m]**
27:26	N'yette	Ms. E, I got a question.	
27:28	Ms. E	Hold on my dear [Ms. E speaks to N'yette], let's go back to the problem to see if we're correct so far. Each section is a square that is one mile long, on...on each edge that is each section is one square mile of land. [Ms. E looks at Juan, shakes the paper and asks:] So could they continue doing this? [Ms. E's facial expression is serious. She is not smiling.] [Ms. E holds paper to N'yette and asks:] Do you think this is okay?	Ms. E's language is maternal "my dear" and her tone is ***gentle*** when speaking to N'yette. When Ms. E asks Juan if the girls could continue with their drawing, her tone of voice is ***doubtful, challenging.*** Juan does not offer a response.
27:47	Dave	They can but they gonna have to make like three more grids.	
27:49	Ms. E	What do you mean "three more grids?"….[inaudible]	Ms. E asks for clarification from Dave about "three more grids". Her tone of voice sounds ***impatient.*** **[Dis-m]**
27:51	Dave	I mean not grids but they gonna have to make, make three more of these [Dave points to N'yette's paper with his pencil], because to make it bigger cause one square mile is…	
27:55	N'yette	To make it bigger? [N'yette is asking Dave]	
27:59	Ms. E	Is there any problem with this one?	
28:00	Dave	No that equals six forty.	
28:03	Ms. E	Mm hum.	
28:09	Manuel D	They, they trying to do, like the same exact thing too, and I'm sure it is...	
28:13	Ms. E	Uhh um, what is that, What is that same exact thing that you trying	Ms. E's body language conveys ***interest*** as she is leaning on the

		to do? [Ms. E taps her finger on Juan's book.] You, you said you are trying to do [Ms. E points at Manuel's paper] the same exact thing, [Ms. E points at Juan's book] here what is that?	table with her elbows, leaning in toward Manuel D and Juan. She appears ***anxious*** as she questions Manuel D. **[Int, Endis]**
28:20	Dave	And they will equal six hundred forty acres, six hundred forty acres, you could do it in a different way too.	
28:22	Manuel D	Right, I'm say…we like when you add them all, that they equal sixty four	Manuel D appears to believe that the shape of the section is not important as long as the area of the section equals sixty-four. Thus, he finds no problem with the fact that Tina and N'yette have not used a square to represent the section of land.
28:29	N'yette	We didn't, we didn't, [N'yette takes the paper out of Ms. E's hands and points to her paper as she speaks] we didn't do it all in like ten, four like ten four, but we did it all different, like ten, four, four, four, is each box is something different but its always gonna be ten and four. [Ms. E reaches for a ruler and holds it in her hand.]	
28:44	Manuel D	What fraction is each one of them?	
28:45	N'yette	'Cause if you did, we didn't do that, we used this ….so we did Lapp, okay, it's one two three, its five and one half here. One, two, three, four, one, two, three, four, ok. One, two, three ,four ,five, we did across five, right here, okay, this two, one, two, one, two, four, one, two, three , one, two, three, one, two, six!	Ms. E maintains her stance, leaning on the table, not speaking, but looking at N'yette's paper as N'yette speaks. Her facial expression is ***serious*** as she glances back and forth from N'yette's paper to Manuel's work. N'yette once again states the faulty reasoning that Lapp "is five and one half" and Ms. E once again, does not attempt to

			correct this faulty reasoning. **[Int]**
28:50	Ms. E	Do you want to continue…[Ms. E speaks to N'yette.]	
28:54	N'yette	See we did all different shapes its, but its all, but its, its always, it always has something to do with four something. [N'yette looks up at Manuel D.]	
29:27	Ms. E	Do you want to continue doing this? [Ms. E asks N'yette, taps on N'yette's paper with a ruler and nods her head.]	Ms. E's tone of voice is ***not excited***, nor ***enthusiastic***. She is speaking softly, leaning on the table, tapping a ruler on N'yette's paper. It appears that Ms. E has abandoned all efforts to redirect N'yette toward the correct answer.
29:28	Manuel D	[Ms. E looks at Manuel D as he speaks.] So we, we can do the same as them but in a different way, instead of, instead of doing four, (Ms. E: Um-hum) we do, we do two (Ms. E: Um-hum), instead of doing five right? [Manuel D asks N'yette]	
29:39	N'yette	Yea five and one half..	
29:40	Ms. E	How would you do it here then? [Ms. E points to Manuel D's paper with the ruler], [Ms. E speaks to N'yette without looking at her] Maybe you could continue doing this?	It may be inferred that N'yette and Tina are proceeding incorrectly with the subtask as well as the main task, yet Ms. E allows them (even encourages them) to proceed in their own way. N'yette and Tina return to their table. It appears that Ms. E does not want to stand in the way of Tina and N'yette's mathematical thinking, despite the fact that it is faulty.
29:44	N'yette	Can I have another grid? [Tina goes back to her table and N'yette goes to get another piece	

		of paper.]	
29:45	Ms. E	Sure. [Ms. E stands up and momentarily walks toward the front of the classroom.]	Ms. E's language and tone of voice is ***casual*** and ***relaxed***.
29:44	Manuel D	[Speaking to Ms. E who is standing and leans on the table across from Juan.] You can do, two of them, cause you are taking two out of ten for the eight, and then you are doing…you get what I'm trying to do?	
29:59	Ms. E	Not really. If you can show me there on the paper [Ms. E points to Manuel D's paper.] maybe then I didn't understand	Ms. E's tone of voice is ***inquisitive*** and ***curious***. She appears ***gentle*** and ***patient*** with Manuel D as she waits for him to "show (her) there on the paper". **[Int, rs, rp]**
30:01	Manuel D	It would have to be three like that.	
30:03	Ms. E	So where is that going to be in your drawing, where is that, Lapp's land? [Ms. E moves to stand next to Juan and points at his book with her index finger.] If you drew Lapp's land on your [pointing at Manuel D's] paper, where would it be?	Ms. E's tone of voice is ***inquisitive*** as she redirects Manuel D to assign Lapp's land in his drawing of Section 18. **[Int, rp]**
30:26	Manuel D	[pause] You can draw it like this too.	
		Okay…[nods and scratches her cheek] [Ms. E walks away from Manuel D's table toward Eric's table]	It does not appear that Manuel has answered Ms. E's question, "where is that, Lapp's land?" Ms. E walks away from the table and addresses the whole class.
End TSMIE [E.01.a.06]			

Discussion of TSMIE [E.01.a.06]

In this episode, Ms. E expresses excitement about Juan's representation of Section 18 as an eight by eight square by inviting many students to join Manuel D, Dave and Juan at their table. It appears that Felipe's group and N'yette's group have not used a square

to represent Section 18 and may be inferred that Ms. E is trying to use Juan's representation as an example of a correct solution. Ms. E relies on Manuel D as the primary speaker as she questions his opinion of Felipe's work and of N'yette and Tina's work. Once, Ms. E asks for Juan's opinion of N'yette's work, but he does not respond to her question.

After examining their work, Manuel D concludes that the other students' representations are 'correct' because they each have an area of sixty-four square units. However, both N'yette and Felipe have drawn rectangles instead of squares. It is inferred that Ms. E recognizes these errors and tries to redirect the students by re-reading the statement of the problem, which clearly states one condition that Section 18 must be a square.

It appears that these issues remain unresolved and Ms. E then asks Tina and N'yette if they would like to proceed with work, they agree and return to their seats.

Subsequent to this episode, Ms. E invites Manuel D to share his group's solution with the whole class. Although she does not affirm or disapprove of this solution, it may be inferred that Ms. E is offering the class an example of the correct way to draw the region of land.

Begin TSMIE [E.01.a.07] 33:43 – 35:04 (L)			
Participants: Ms. E, Manuel D, Dave Ms. E has just finished addressing the class to provide clarification that the region of land must in fact be square in shape and she offered for students to "start over" if their representation does not satisfy this condition. Ms. E returns to Manuel D's table and continues to engage in a dialogue with Dave and Manuel about their solution. Once again, Juan looks on quietly, but does not engage in the conversation.			
33:43	Ms. E	Okay let's measure this, measure this, this side. [Ms. E is speaking to Manuel D and is holding a ruler to measure something on his paper.]	Ms. E's tone of voice and language ("let's measure this") may suggest that Ms. E is working collaboratively with Manuel D.
33:48	Manuel D	This one is like same, but you're already taking a piece out of it. (Ms. E: Um-hum)	
33:54	Dave	Do we measure it with the centimeters or the inches?	
33:55	Ms. E	It does not matter. [Ms. E looks at Dave's work.]	Ms. E offers Dave choice about which units of measure to use.
34:02	Ms. E	Okay, [Ms. E addresses Dave] what can you say about Lapp's land and Gardella's land in terms of the length….inch.	
34:12	Manuel D	Ah…Lapp's land is totally bigger.	
34:14	Ms. E	Is Lapp's, Lapp's land bigger than Gardella's? [Ms. E looks at Manuel's work]	Ms. E responds to Manuel's question by repeating his question and looking at his paper. She demonstrates that she is ***interested*** but perhaps not convinced by Manuel's claim "Lapp's land is totally bigger". **[Int]**
34:17	Manuel D	Yea because it….	
34:18	Ms. E	In terms of the length.	Ms. E speaks as though she is looking for Manuel D to provide a convincing argument for "Lapp's land is totally bigger".

34:21	Manuel D	But as you can *see*, [pointing at his book] that, you can see that Stewart is taking a piece of Gardella. [Manuel D looks up at Dave]	Manuel D recognizes that Gardella' land is not congruent to Lapp's land because Stewart's property is located in the upper right hand corner of Gardella's land.
34:27	Ms. E	What do you think Dave?	Ms. E's tone of voice is ***inquisitive***, ***curious*** about Dave's mathematical opinion regarding the size of Lapp's land as compared with Gardella's land. **[Int]**
34:31	Dave	[pause] I think that they both the same.	
34:33	Ms. E	In terms of the length?	Ms. E seems ***curious*** about what Dave means by "the same". **[Int]**
34:35	Dave	Yeah. [Ms. E leans on the table in between Manuel D and Dave] But in width, the width is probably… (Ms. E: "Um-humm")	
34:37	Manuel D	Cause, cause land going, La Lapp going down is like smaller	
34:41	Ms. E	So if you would do this [Ms. E points to Dave's book] on this paper … [Ms. E points to Dave's paper then holds her finger to her lips as she watches Dave measure something on his book]	It may be inferred that Ms. E recognizes that the boys are not taking the dimensions of the properties into consideration. Yet, in order to finish drawing Section 18 on grid paper, it is necessary for the boys to attend to the dimensions of each property. Ms. E seems ***anxious*** for the boys to represent the properties in the eight by eight square on their papers.
34:51	Dave	It would go…	
34:52	Manuel D	It would be like 5 percent of ….[Ms. E leans back from the table.]	
34:53	Dave	It would be about….one and a fourth, its one and a fourth.	
34:54	Ms. E	What?	Ms. E expresses ***confusion*** about Dave's answer. **[Con]**
34:57	Dave	It's one hundred forty.	
34:58	Ms. E	So what would it be here?	Ms. E appears ***curious*** about Dave's

		[Ms. E taps Dave's paper.]	representation of Gardella's land on the square section of land drawn on his paper. **[Int]**
34:59	Dave	Two times forty.	
35:01	Ms. E	Okay, just this and this would be Gardella's?	Ms. E looks for clarification about the size of Gardella's land as she gestures toward Dave's paper.
35:04	Manuel D	For Gardella's, I think that for Gardella's instead of doing four we should do five.	
		[Ms. E walks away]	
End TSMIE [E.01.a.07]			

Discussion of TSMIE [E.01.a.07]

Ms. E questions the boys about the size and placement of Gardella's land. It appears that Manuel D recognizes that Gardella and Lapp's properties are not congruent when he states that Stewart's property is "taking a piece" of Gardella. Ms. E suggests that the boys compare the lengths of the two properties. She follows the boys' lead and does not offer judgment on their mathematical ideas. She supports student choice when Dave asks her which units he should use to measure the properties in Section 18. <u>She does not correct faulty reasoning when the boys present incorrect ideas. It does not appear that the boys have made mathematical progress in this episode.</u>

After Ms. E walks away from the table, Manuel D and Dave remain engaged with the mathematics as they discuss the dimensions of Gardella's land. Dave: "Five times what?", Manuel: "Five and three, let me see.", Dave: "Yeah, fifteen.", Manuel: "Fifteen, but that would never come to sixty-four.", Dave: "Fifteen times fifteen, yes it will."…."Manuel D: "It's working my head yo! Let see. What are they doing over there."

This episode evidences Ms. E continuing to guide the boys by asking them to represent their ideas on paper. She maintains a gentle tone of voice when addressing the boys and shows interest through her body language as she leans into the table as well as through her language, restating the boys' ideas and asking them questions.

Begin TSMIE [E.01.a.08] 38:10 – 40:59 (L)			
Participants: Ms. E, Dave, Manuel D In this episode, the Ms. E initiates the interaction by approaching Dave, Manuel D and Juan's table and asking, "So what did you get?" She stands behind Manuel D and Juan.			
38:10	Ms. E	So what did you get? [Ms. E is rubbing her cheek with her hand]	Three minutes have passed since Ms. E's previous interaction with Manuel D and Dave. It appears that she is ***interested*** in the boys' progress in identifying the dimensions for Gardella's land. **[Int]**
38:12	Dave	We think, we thought, that what we did for Lapp was too small, so it's got to be bigger.	
	Ms. E	Okay.	
38:16	Manuel D	Cause for Lapp it was like up to here, and	
	Ms. E	Um hum	Ms. E is ***showing interest*** in Manuel D's description of where Lapp's land is in his drawing. **[Int]**
38:20	Manuel D	Then Gardella will have all this space,	
38:21	Ms. E	Uh huh, so?	
38:23	Dave	And they are the both same size, so it can't…	
38:24	Ms. E	So where would Lapp's land end and where would Gardella's land begin?	She appears to convey ***no emotion*** as she questions the boys about the orientation of the different landowners' property.

38:27	Manuel D	No not really, not the same exact….beginning, beginning in the bottom.	
38:29	Ms. E	Right…	
38:30	Manuel D	Lapp on the top.	
38:32	Ms. E	Right, okay.	
38:35	Manuel D	So that should be some more around here.	
38:40	Ms. E	Do you agree? [Ms. E taps Juan on the shoulder] Did you measure it? [Ms. E asks Manuel D, and scratches her nose.]	Juan is sitting back in his chair and appears to be disengaged from the mathematics. He has not spoken since offering the correct dimensions of the section of land at 23:17, more than ten minutes ago. When Ms. B asks Juan, "do you agree?" it is unclear whether she is referring to Dave's claim that Lapp's property and Gardella's property are the "same size" or if she is referring Manuel D's claim that they are "not really" the same size. Ms. E is inquisitive about whether or not the boys measured "it". Perhaps she is referring to Lapp's land and/or Gardella's land. **[Endis]**
38:48-39:19	Manuel D	But if they are gonna be the same, Lapp has got to start at four, it's got to end at four. Don't you think? If they are gonna be the same, right? Then Lapp has got to end at four just like Gardella. Do you understand? [Manuel is speaking to Dave] Like this, if they are the same like you say going up and down, then Lapp is gonna have um like half.	Ms. E answers a question from a student off camera then returns her attention to Manuel D and Dave.

39:01	Ms. E	Yes…..Then find where Lapp's land is [Ms. E gestures with her hand] and the rest of the land. [Ms. E is speaking to a student off camera at a different table – the student is behind Dave.]	Ms. E is giving direction to a student from a different group.
39:18	Dave	So it would be four going down, saying both get four going on the length and what would be the width, cause hers is…	Ms. E is looking at Dave as he speaks. **[Int]**
39:25	Manuel D	But I think, but I think, they have the same width though except that, except, except that Stewart is getting some of Gardella's right here.	
39:36	Ms. E	So what would it be then, here? [Ms. E leans in between Manuel D and Juan and takes a paper from Dave and taps on the paper.] [Ms. E turns to Juan, and speaks to him in a higher pitched voice] What do you think?	Ms. E's tone with Juan is ***gentle***, as if to attempt to gently include him in the conversation.
39:40	Manuel D	I don't really think that Gardella is the same so..	
39:43	Ms. E	Did you measure it?	Ms. E's tone of voice indicates ***curiosity*** about whether or not the boys measured the plot of land. **[Int]**
39:46	Manuel D	Centimeters right?	
39:47	Ms. E	Doesn't matter, what do you want to use?	Ms. E appears ambivalent about the units of measure. She supports Manuel D's thinking offering him the ***choice*** of which units to use. **[Blds]**
39:52	Manuel D	Let's go with centimeters, [Manuel D leans in to measure in his book] well Lapp is about three.	
39:54	Ms. E	Lapp is about three ok…[Ms. E looks on as Manuel D measures]	

39:58	Manuel D	And Gardella's about, the same	
40:01	Ms. E	Okay, across, how about across?	Ms. E ***encourages*** Manuel D to tell her the width of Gardella's land. She is leading him through the process. **[Int]**
40:04	Manuel D	Across, across I know the distance, hold on hold on hold on.	
40:08	Ms. E	[Ms. E turns toward Juan and says something inaudible, then returns to look at what Manuel D is drawing].	It may be inferred that since Juan did not enter the conversation after Ms. E's prompting at 38:40 she may be saying something to him to ***encourage engagement*** in the mathematics. **[Endis]**
40:15	Manuel D	It's about the same as…	
40:16	Ms. E	Okay [Ms. E nods her head] and…	
40:23	Manuel D	So they are the same on everything.	
40:20	Ms. E	Okay so what do you think we should do on this one?	Ms. E asks Manuel D's input on what they should do on the next one. Again, she uses language, "what do you think ***we*** should do?" that indicates that she is working with the students and considers herself a part of the group. At the same time, her language is very general and she does not specifically indicate what she means by "this one".
40:27	Manuel D	So it means like, four, (Ms. E: Um-hum), four (Ms. E: Um-hum), and the other one is like four, four, and then and then Stewart is taking a little two over four. [Ms. E nods her head]	It may be inferred that Ms. E is ***pleased*** with Manuel D's mathematical ideas as she nods her head in ***agreement*** with Manuel D's answer. **[Val]**

40:35	Ms. E	Okay, [Ms. E nods her head and points her finger at Manuel D's paper] let's see Lapp's, where would Lapp's be?	Ms. E appears ***curious*** about where Manuel D would place Lapp's land in Section 18. **[Int]**
40:37	Manuel D	[Bell rings] So Lapp's would be one, two, three, four	Manuel D and Ms. E appear to be deeply engaged with the mathematics as they do not acknowledge the bell,which may signify the end of the class period.
40:41	Ms. E	Okay. And up to where? [Ms. E scratches her nose then moves toward the front of the classroom, stopping to stand behind Manuel D.]	Ms. E demonstrates ***curiosity*** about the position of Lapp's property. **[Int]**
40:44	Manuel D	Up to there and then Stewart would come to here and take this little four……there you go…that the four…..we are.	
40:49	Ms. E	All right, [pause, Ms. E is out of camera view now] Dave what do you think?	**[Endis, Int]**
40:59	Dave	I agree.	
End TSMIE [E.01.a.08]			

Discussion of TSMIE [E.01.a.08]

In this episode, Ms. E conveys interest in Manuel D, Dave and Juan's progress toward representing the individual properties within Section 18 on the grid paper. Throughout the episode, Ms. E asks questions about the boys' reasoning, "did you measure it?", and she leans in, observing as Manuel D and Dave describe their thinking. It appears that Ms. E tries to engage Juan with the mathematical discussion when she speaks to him twice in this episode. First she asks Juan directly, "What do you think?" The second utterance is not audible, but it appears that Ms. E is making an attempt to foster Juan's engagement with the mathematics as she asks for his opinion. Despite these interactions, Juan does not participate in the discussion.

The bell rings at 40:37, indicating that the class period has ended. When Ms. E announces that it is time to go to lunch, Manuel D's response is "We are?" as if he is surprised or possibly disappointed that the work session has come to an end.

Ms. E concludes the class period by assigning homework,

> Okay, so this is your homework for tonight, I want you to … Finish this, by tomorrow I should see what you say about each part, what are the fractional parts, … Give me a fraction. Is it one fourth, is it one third, is it, and all of the different parts. If you have to guess, please do, but I should see and, a label for each one.
>
> ["Adding and Subtracting Fractions, Day One" 43:38(C)]

In order to correctly complete the homework assignment, it would be necessary for the students to have completed the subtask of drawing Section 18 as an eight by eight square on the grid paper. It is evidenced in the video data that many students did not complete the subtask and it is not clear whether or not Ms. E recognizes this when assigning the homework.

At the beginning of the class period, Ms. E had introduced the lesson by stating that the students would be working on adding and subtracting fractions. However, there is no evidence showing the students adding or subtracting fractions during this classroom lesson.

Ms. E: Classroom observation analysis, "Adding and Subtracting Fractions"

Day Two

This class period takes place on the second day of "Adding and Subtracting Fractions" in Ms. E's classroom. The students continue working on the problem that was assigned on the previous day, assigning fractional values to the properties within a one square mile section of land (See Figure 1). On the first day of the lesson, Ms. E had imposed a subtask, which required the students to draw "Section 18", a square plot of land, on grid paper. As was described in the previous section, many students experienced difficulty with the subtask and spent most of the first class period trying to identify appropriate dimensions for drawing the square section on the grid paper. N'yette and Felipe had represented Section 18 as a rectangle instead of a square on the first day of "Adding and Subtracting Fractions".

Ms. E begins the second day of instruction by modeling a solution of the subtask at the overhead projector. It is inferred that Ms. E recognizes that most or all of the students were not successful in completing the subtask so she intervenes by solving the subtask for them.

At the beginning of the class period, Ms. E is standing at the front of the room, next to the overhead projector. She draws a square on the overhead projector and announces to the class, "in the interest of time, I'm going to set some parameters for this one…I want you to make an eight by eight…square." She then instructs students to begin a new drawing and offers new pieces of grid paper to the students.

Begin TSMIE [E.01.b.01] 1:37 – 4:23 (C)			
Participants: Ms. E, Dave, Eric, Female student, whole class Ms. E launches the lesson with a whole class discussion about the subtask, drawing Section 18 on grid paper. First, she asks student volunteers to identify key information in the statement of the problem that would indicate that Section 18 is a square shape. She then models on the overhead projector how do draw Section 18 as a square with sides eight units in length on the grid paper. Next, Ms. E calls on student volunteers to identify the dimensions of two individual properties within Section 18 and models how to draw these two properties inside the eight by eight square on the overhead projector.			
1:37	Ms. E	Can I have everyone's attention now? [Manuel C]. Okay we're going to do an eight by eight… [Ms. E draws on projector] square. [Ms. E looks up at students.] Which part of your problem tells you that a square is a shape that we're going to use to represent this problem? [Ms. E opens her textbook as she is speaking to the students.]	Ms. E is standing next to the overhead projector at the front of the classroom. She has one hand on the overhead projector and one hand on her hip. Ms. E appears to recognize that some students are exhibiting off-task behavior. Many students can be seen shuffling papers, opening and closing books and many students can be heard talking. Ms. E's tone of voice may be inferred to show ***displeasure*** with the students' off –task behavior when she says, "can I have everyone's attention now?" She does not provide a mathematical reason for using the dimensions eight by eight. **[Dis-b]**
2:03	Many students	The picture.	
2:05	Ms. E	One the picture, how about what's on the words? [Ms. E shifts her stance and stands with her right hand on her hip as she speaks to the students.]	Ms. E's body language, tone of voice and language convey that she is ***focused*** and ***serious*** about the students identifying the key information in the statement of the task that would indicate that Section 18 should be represented as a square shape.
2:09	Female student	The diagram.	

2:10	Ms. E	What is there on the words to tell you that the shape we're using today is a square? I may be wrong so correct me if I am. Read part of the problem that tells me … [Manuel C and Noah are swatting at each other with their arms.] Can you handle yourself? [Manuel C], [Noah]? Page forty-three, which part of the problem tells me that section eighteen is a square? [Ms. E writes on the overhead projector.]	Ms. E does not acknowledge the response, "the diagram." Ms. E gives specific directions for the students to identify the words in the statement of the task that indicate that Section 18 is as square. Ms. E may possibly be ***trying to motivate*** students to become engaged by suggesting that her own understanding of the task may be incorrect. "I may be wrong, so correct me if I am." **[Endis]** Ms. E then states the page number and repeats her question for the third time, encouraging students to find the information in the statement of the task by saying, "Read the part of the problem that tells me..." She is interrupted briefly possibly by off-task behavior exhibited by Manuel C and Noah, which she quickly addresses. Her she lowers her voice to address the boys by name, but speaks with a tone that conveys ***annoyance*** toward the boys' off-task behavior. Her language, "can you handle yourself [Manuel],[Noah]?" may convey to the boys that she expects them to improve their behavior. **[Dis-b]** Ms. E is ***not smiling*** and is standing still at the front of the classroom with one hand on her hip and appears to ***anxiously*** await appropriate student responses.
2:38	Manuel D	Oh I know…	
2:38	Ms. E	Yes? [Ms. E nods her head toward Manuel D.]	Ms. E acknowledges Manuel D's response.
2:38	Manuel D	Here it says, each section is a square that is one mile long.	
2:43	Ms. E	Each section is a square that is one mile long. So we're going to do, for today, its going to be eight across and eight down. Now, this is eight across	Ms. E repeats Manuel's answer but she ***does not explicitly validate nor does she negate*** Manuel D's answer. **[Endis]** Again, Ms. E tells the students to make a square with "eight across and eight down", but she does not give the mathematical reason for this.

		and eight down, how big should that Lapp's land be, how many boxes? [Eric]? How many across, how many down? [Manuel C and Dave raise hand] Not sure? Okay, yes, [Dave?].	Ms. E poses a question to the class about the dimensions of Lapp's land within Section 18. She calls on Eric who has his hand raised. Eric looks down at his paper for a few seconds, ***hesitating*** he seems ***unsure***. It may be inferred that Ms. E conveys ***sympathy*** when she says, "not sure?" Eric shakes his head, "no" Ms. E seems to ***excuse*** Eric from answering and calls on Dave who has his hand raised.
3:13	Dave	Sixteen.	
3:15	Ms. E	Sixteen. [Ms. E writes on the overhead projector.] And how many would that be across and how many would it be down?	Ms. E repeats Dave's answer and then poses another question about the dimensions of Lapp's land. Ms. E appears to be ***validating*** Dave's ideas by writing his answers on the overhead. **[Val]** Dave has not answered the second part of her question, "how many across, how many down?" She asks for the dimensions a second time.
3:18	Dave	Four by four.	Dave presents correct reasoning.
3:20	Ms. E	Four by four? Okay one, two, three, four one, two, three, four. How about Bouck? Bouck [Manuel D and Dave raise their hands.] B-o-u-c-k, yes? Two by two? All right, so we'll start with these two and then divide the land according to the people that own them. Any question? By the end of the period, which is very short, by the end of the period I should have a fraction name from each one of these.	Ms. E repeats Dave's answer, "four by four" and then ***validates*** his answer as she draws this representation in Section 18 on the overhead projector, which may convey that she agrees with Dave's answer. **[Blds, Val]** Ms. E then poses another question about the dimensions of Bouck's land. It appears that a student answered "two by two" off camera, which Ms. E subsequently writes on the overhead. Ms. E then states with emphasis in a ***serious*** and ***business-like*** tone of voice that the class period is "very short", implying a sense of ***urgency*** and an ***expectation*** that the students work quickly. She then poses the task for the period to find a fraction name for each landowner.
3:54	Student	I can't see this one.	

4:00	Ms. E	I'm sorry…[Ms. E moves the image on the overhead projector.] The whole thing is an eight by eight, then Lapp's is a four by four and then two by two. We'll only have a couple rulers at each table so everyone can have access to one. [Ms. E distributes rulers to each table.]	It appears that in response to a student's complaint that he or she could not see the image on the overhead projector. Ms. E responds by expressing an ***apology*** and she adjusts the image. Ms. E summarizes the progress made so far with regards to the dimensions of Section 18 and two of the landowners within Section 18.
4:23	Ms. E	[Felipe walks to the front of the classroom.] We don't have much time so hurry up, let's do the land. [Ms. E turns on the lights.]	Ms. E tells the students to "hurry up" since the period is shortened.
End TSMIE [E.01.b.01]			

Discussion of TSMIE [E.01.b.01]

In this episode, Ms. E launches the lesson by resolving the conceptual barrier that most of the class struggled with during the previous class period.

In the first minute of this episode, Ms. E addresses off-task behavior exhibited by Manuel C and Noah by lowering her voice and asking sternly, "Can you handle yourself?" The boys seem to discontinue the off-task behavior, it is not clear, however whether they then become engaged with the mathematics as Manuel C can be seen smiling to himself.

Throughout this episode, Ms. E's serious tone of voice, utterances and body language convey her commitment to engagement with the mathematics. Ms. E encourages student engagement by eliciting information from the students about the

conditions of the problem. After she poses the question twice, it appears that Ms. E is not satisfied with the level and/or quality of student responses. She asks the question a third time suggesting that she may be wrong, offering the students an opportunity to correct her. Ms. E then models the placement of two properties using the students' input to develop the representation on the overhead projector.

Subsequent to this episode the majority of the students appear to be engaged with drawing an eight by eight square on grid paper because they are writing, using rulers and writing on the grid paper.

Begin TSMIE [E.01.b.02] 1:34 – 6:29 (L)			
Participants: Ms. E, Manuel D, Dave, Jerry (Jerry was not in this group on Day One, he is seated in Juan's chair.) This episode begins when Ms. E sits down at the table next to Jerry, and across from Dave and Manuel D. Ms. E. She poses a question, "give me an example of a fraction." Throughout this episode, it appears that Ms. E communicates primarily with Manuel D and occasionally with Dave. Jerry does not participate in the discussion. During the first minute of this episode, Ms. E also interacts with students from three other groups while remaining seated at Manuel D's table.			
1:34	Ms. E	Give me an example of a fraction. [Ms. E is sitting next to Jerry and she has her hand on her left cheek with her elbow leaning on the table. Ms. E is not smiling.]	Ms. E's question is not directly related to the problem. **[Rsi, Endis, Int, rp]**
1:35	Manuel D	This one should be, four over four?	
1:37	Ms. E	Four over four? You call it four over four?	Manuel D has presented faulty reasoning. Ms. E responds by repeating his answer.
1:38	Dave	So then it would be a whole.	Dave seems to recognize that Manuel D has presented faulty reasoning.
1:39	Ms. E	That would be a whole? So can you call it a whole?	**[Int, rs]**
1:41	Manuel D	Isn't this whole thing a whole? [Manuel D indicates that he is referring to Section 18 on his paper using his whole hand.]	Based on Manuel D's gestures and language, it appears that he understands that Section 18 represents a "whole".
1:43	Dave	Yeah, but you said four out of four and four out of four is a whole.	Dave may be trying to convey that if Section 18 represents a whole as recently suggested by Manuel D, ("Isn't this whole thing a whole?") then "four over four", which represents one whole, cannot be used to name a fractional piece *within* the whole.

1:48	Ms. E	So what would you call this instead? [Ms. E raises her voice to address students at a different table.] Fraction names, we're looking for fraction names. Each portion has to have a fraction name. [Ms. E returns her attention to Manuel D and asks,] So what do you call this?	It appears that Ms. E is ***hopeful*** that the boys will correct Manuel D's faulty reasoning. Ms. E is distracted for a moment while she clarifies the direction for students at a different table. Ms. E immediately returns her ***interest*** to Manuel D and rephrases her question asking, "So what do you call this?" **[Int, rs, rp]**
1:59	Manuel D	A whole.	
2:00	Ms. E	Huh?	Ms. E's tone of voice may be conveyed to express ***confusion*** and ***surprise*** at Manuel's reasoning. **[Con]**
2:01	Manuel D	A whole. [Manuel D shrugs his shoulders.]	Manuel's tone of voice and gesture of shrugging his shoulders may be inferred to show that either he feels ***frustrated*** by Ms. E's repeated question, or he is ***defensive*** of his answer.
2:02	Ms. E	A whole? [Ms. E asks Dave] Can you call it a whole? Do you agree with him?	It appears that Ms. E ***disapproves*** of Manuel D's answer because her tone of voice appears to convey ***doubt*** as she calls on Dave for his opinion. **[Int, rs, Dis-m, Con]**
2:07	Ms. E	[Ms. E raises her voice and addresses students at a different table.] Ladies at the other table, you should have at least a drawing by the time I get to your desk. No, we're starting a new one, get a new graph paper. Oh my gosh.	Ms. E poses a threat to the girls. "Ladies, you should have at least a drawing by the time I get to your desk." Ms. E conveys ***anger*** toward the girls possibly because they are disengaged with the mathematics. **[Dis-b]**
2:15	Dave	What's today's date? What's today's date? What's today's date? [Dave elbows Manuel D]	Dave is temporarily off task when he asks Manuel D "what's today's date?" Manuel D quickly answers Dave then it appears that Manuel D becomes reengaged when he says, "….so there should be…"
2:21	Manuel D	It's the twelfth. [Manuel D speaks to Ms. E] umm, so there should be…one, two, three, four [Manuel D is counting with his pencil on	

		his paper.]	
2:28	Ms. E	Okay.	Ms. E appears to demonstrate ***interest*** in Manuel D's idea by saying, "okay". **[Int]**
2:29	Manuel D	Two over four.	
2:31	Tora	You got to copy eighteen and nineteen. [Tora is speaking to Ms. E.]	Tora, who is sitting at a different table, is discussing the assignment with her group but can be heard off camera.
2:33	Ms. E	[Ms. E raises her voice to address the class. Lets focus on eighteen, just eighteen for now.	It is apparent that Ms. E has overheard Tora speaking to her group, "you got to copy eighteen and nineteen" because she corrects her and reminds the whole class that they should only "focus on eighteen".
2:40	Ms. E	So, what is it? [Ms. E is leaning on the table with her arms crossed.]	Ms. E returns her attention to Manuel D, conveying ***interest*** as she leans forward on the table. Her tone of voice, facial expression and posture convey that she is ***serious*** and ***business like*** and ***hopeful*** that Manuel D will produce a correct answer. **[Int, rs]**
2:44	Manuel D	One, two, three, four. No, damn it! One, two, three, four.	Manuel D expresses ***anger*** when he says, "No, damn it!"
2:54	Ms. E	Okay, umm, on the other side of this paper, can you show me how you name a fraction? Name a fraction. Any fraction.	It appears that Ms. E responds to Manuel D's anger by intervening with a task that presents a lower level of cognitive demand. She hesitates and her tone of voice becomes ***reassuring***. It may be inferred that Ms. E attempts to ***redirect Manuel D's anger*** by engaging him in a different problem. The open-ended question, "show me how you name a fraction….any fraction.". **[Enc]**
3:01	Dave	Half.	Before Manuel has a chance to answer, Dave quickly answers, "half".

3:02	Ms. E	Half, okay, show me half. Draw a half. Show me.	Ms. E repeats Dave's suggestion and asks Manuel D to draw a half. It may be inferred that Manuel D hesitates, because Ms. E repeats her question "Draw a half" putting emphasis on each word. **[Int, rp Enc]**
3:08	Dave	It would be point five which equals half. [Dave is looking at his paper as he speaks.]	Dave appears to be talking to himself.
3:11	Ms. E	Okay…why is the numerator one?	Ms. E does not respond to Dave but questions Manuel D about his symbolic representation of one-half. **[Int, rs]**
3:14	Manuel D	Umm, because that's one out of two.	Manuel's tone of voice conveys that he is ***unsure*** of his answer, "one out of two".
3:18	Ms. E	One out of two [Ms. E gestures with her pointed finger] Can you show me another fraction? Another fraction. [Ms. E points at Manuel D's paper] What's this?	Ms. E appears ***pleased*** with Manuel D's answer as she repeats Manuel D's statement. She does not overtly validate or negate Manuel D's answer. Ms. E continues to demonstrate ***interest*** in Manuel D's thinking as she asks him to produce another fractional representation. **[Sat, Endis, Int]**
3:26	Manuel D	Four over four.	
3:28	Ms. E	Four over four? Could you show me four over four? [Manuel D draws] and? …[Ms. E points to Manuel D's paper] How do you know what the numerator is?	Ms. E asks Manuel D to draw a representation of "four over four". Ms. E shows ***interest*** in Manuel D's mathematical thinking, asking him to explain "what the numerator is." **[Int, rp, rs, Endis]**
3:40	Manuel D	Because each one of them have um a cover.	Manuel D is describing that each section in his picture has been shaded.
3:47	Ms. E	Uh haa	Ms. E's language may ***validate*** Manuel's idea, indicating that she ***agrees*** with or understands Manuel D's reasoning. **[Val]**

3:48	Manuel D	Each one of them have this cover.	
3:50	Ms. E	Shade, okay, cover, okay. And how do you know this is four?	Ms. E's tone of voice and language, "okay" conveys that she is ***convinced*** of Manuel D's explanation up to this point. Ms. E continues to show ***interest*** in Manuel D's mathematical thinking, when she points at Manuel D's paper and asks, "how do you know this is four?" Ms. E's tone of voice appears ***gentle*** and ***encouraging***. **[Int, rs, rp, Endis]**
3:53	Manuel D	Because this is four squares.	Manuel D presents correct reasoning as he describes his picture as having four squares. Manuel D's tone of voice and language convey ***confidence*** in his answer.
3:55	Ms. E	Can you give me another, uh, fraction with four as a denominator? [Ms. E points to Manuel D's paper]	Ms. E ***does not validate nor does she negate*** Manuel D's response. Once again, she asks him to show her another fraction, yet this time, she sets a condition by naming the denominator. **[Endis, Int, rp]**
4:00	Manuel D	What do you mean? [Manuel D furrows his brow.]	Manuel D seems to be ***perplexed*** because he furrows his brow, looks at Ms. E and asks, "What do you mean?".
4:01	Ms. E	Another fraction, not four four but where the four is the denominator. [Manuel D writes on his paper.] Okay, how would two four look like?	**[Rsi]** Ms. E provides clarification. It may be inferred that Manuel writes the symbolic representation for two fourths on his paper. **[Int, rp]**
4:08	Manuel D	Umm… [Manuel D writes on his paper]	

4:12	Ms. E	Why is this a two? [Ms. E points to Manuel D's paper, she continues to point to his paper through 4:27.]	Ms. E immediately points at Manuel D's paper and asks, "why is this a two?" with an ***inquisitive*** tone of voice. Ms. E shows ***interest*** in Manuel D's mathematical thinking because she continues to question him and also because she points to his paper. **[Int, rs]**
4:15	Manuel D	Umm, because two of the windows are shaded. [Manuel D raises his eyebrows] Or this could also be half. Couldn't it?	It is possible that as Manuel D is speaking, "two of the windows are shaded", he raises his eyebrows because he correctly ***recognizes*** that his representation of two-fourths can also represent one-half. Yet, Manuel D expresses ***doubt*** when he looks up at Ms. E and asks, "Couldn't it?"
4:22	Ms. E	Is it?	Ms. E ***does not validate*** Manuel D's discovery about one-half, but poses a question, "is it?" back to Manuel D. **[Int, rs]**
4:24	Manuel D	It is because…half of four.	Manuel D provides a brief explanation to support his reasoning.
4:25	Ms. E	Okay, two are shaded and why is this four?	Ms. E disregards Manuel D's explanation for one-half, yet appears ***eager*** as she persists in questioning Manuel about the representation equaling two out of four. It appears that she is ***interested*** in his reasoning why the denominator is four. **[Int, rs]**
4:27	Manuel D	Because there are four windows.	Manuel D correctly responds to Ms. E's question.
4:29	Ms. E	Okay [Ms. E turns Manuel D's paper over to show his drawing of Section 18.] so what's the name for Lapp's land?	Ms. E may convey that she is ***satisfied*** with Manuel D's understanding of fractional representations because she turns Manuel D's paper over and directs him to the original task, "so what's the name for Lapp's land?" **[Sat]**
4:35	Manuel D	This would be… one, two,	Manuel D counts the grid squares

		three… twelve, thirteen, fourteen, fifteen, sixteen... so if there's sixteen, four over sixteen.	within Lapp's land, then states that the fraction name for Lapp's land would be "four over sixteen". Although this is numerically correct, it appears that Manuel D has presented faulty reasoning. The "sixteen" which is the denominator in Manuel D's answer seems to have come from the sixteen squares with in Lapp's land because he counts each one and says, "there's sixteen". It is unclear how Manuel D determined "four" for the numerator. Manuel D expressed understanding in the beginning of this episode that Section 18 would represent the whole when naming each property as a fraction. Yet he appears to have abandoned that idea now.
4:45	Ms. E	Is it four over sixteen?	Ms. E asks Manuel to tell her if he thinks the answer is four over sixteen.
4:47	Manuel D	I guess…	Manuel's appears ***uncertain.***
4:48	Ms. E	So write four over sixteen. [Manuel D erases and writes on paper.] [Dave], [Manuel D] is naming it four over sixteen do you agree?	Ms. E ***validatets*** of Manuel's answer by telling him to write "four over sixteen". **[Val]** It is possible that Ms. E does not recognize Manuel D's faulty reasoning because he has presented a correct numerical answer but the process that lead him to the answer is faulty. The camera does not show Ms. E's face, so it is possible that she was not watching Manuel D as he counted the sixteen squares. It is also possible that Ms. E watched Manuel D count the squares, but did not recognize the mistake. Ms. E looks to Dave to express his opinion of Manuel's answer.

4:54	Dave	Where? For Lapp? Hold on, lemme, I gotta.	It is possible that Dave may be disengaged because he appears to stammer and stalls, saying, "hold on, I gotta…"
5:03	Manuel D	I wrote four over sixteen because I mean..	
5:04	Ms. E	Why sixteen?	
5:07	Dave	Because it's sixteen square miles.	Dave expresses faulty reasoning.
5:09	Ms. E	Sixteen squares. Sixteen square miles.	Ms. E repeats but does not correct Dave's idea.
5:11	Manuel D	But then…	
5:12	Dave	Because one square equals a square mile, right?	All of Section 18 equals one square mile.
5:16	Ms. E	Okay, and why, why, why four? You said four out of sixteen.	Ms. E does not appear to negate Dave's faulty reasoning. Ms. E expresses ***curiosity*** about the reason why the numerator is four in Manuel D's answer. **[Int]**
5:23	Manuel D	Now I am changing my mind, [Manuel D puts his pencil to his lips] none of them are shaded or something. It should be like zero over sixteen.	Manuel D appears ***uncertain*** about his answer, "four out of sixteen" because he states that none of the squares in Lapp's land are shaded.
5:29	Ms. E	So shade Lapp's land. Go ahead, shade it. Shade Lapp's land. [Manuel D erases and shades] Now that's shaded, what do you call it?	Ms. E appears to recognize Manuel D's conceptual barrier. **[Rsi]** Ms. E's tone of voice is initially ***abrupt*** when she says, "So shade Lapp's land." Manuel D hesitates then Ms. E's tone becomes ***gentle*** when she repeats her suggestion, ***encouraging*** Manuel D by saying, "Go ahead, shade it." Ms. E waits for Manuel D to shade Lapp's land then asks him to tell her the fraction name for Lapp's land. **[Enc]**
5:41	Manuel D	Umm, a whole? [Manuel D shrugs his shoulders.] So it would be like sixteen over sixteen?	Manuel D appears ***uncertain*** about his answer.

5:46	Ms. E	Sixteen over sixteen, do you agree this is sixteen out of sixteen? [Ms. E asks Dave.]	Ms. E's face is off camera, but it appears that she is ***dissatisfied*** with Manuel D's answer because she asks Dave, "do you agree?" **[Dis-m]**
5:49	Dave	That's a whole, yep. [Dave returns to his work.]	Dave's tone is dismissive, he appears to be busy drawing, and possibly answers the question quickly, with little thought.
5:50	Ms. E	It's a whole, okay so sixteen out of sixteen, you're changing it, not four out of sixteen anymore?	Ms. E appears to ***accept*** Dave's affirmation of Manuel D's thinking. She does draw attention to the fact that Manuel D is changing his answer from his original idea, "four out of sixteen." Ms. E appears to be ***seeking understanding*** of Manuel D's answer.
5:54	Manuel D	No. [Manuel D shakes his head, 'no']	
5:55	Ms. E	So what would this be? [Ms. E points at Manuel D's paper.]	It may be inferred that Ms. E is pointing at a different property within Section 18. **[Int, rs]**
6:01	Manuel D	I guess, four over four?	Manuel D appears ***doubtful*** as he assigns a fractional value to another property within Section 18, using the same faulty reasoning.
6:03	Ms. E	So write four over four… [Manuel D writes] and why is it four over four?	Ms. E does not correct Manuel D and despite the fact that it is incorrect she says, "So write four over four." Her tone of voice conveys ***acceptance*** and is ***gentle*** and ***calm***. After she tells him to write "four over four" on his paper, sounds ***inquisitive*** as she questions his reasoning. It is inferred that Ms. E validates Manuel D's reasoning when she tells him to write it down. **[Val(f), Blds, Con]**
6:08	Manuel D	Because four boxes and each one of them are like shaded. One's shaded.	Manuel D presents correct reasoning for naming the property four over four if the region stood alone, yet not within the context of being a fractional part of Section18.
6:13	Ms. E	Okay, and Wong would be?	Ms. E does not correct Manuel D's

			faulty interpretation. She then asks Manuel to give a fractional name to Wong's property. \
6:15	Manuel D	Six over six.	Manuel D's reasoning (although incorrect) is consistent. He is naming each property as a whole based on the area or number of grid squares within each property.
6:16	Ms. E	Six over six, okay.	Ms. E does not correct Manuel D. She maintains a tone of voice that conveys ***interest***. Although his reasoning is faulty, she does not intervene. **[Int]**
6:19	Manuel D	That [Krebs] would be two over two. [Manuel D writes on paper] Fitz would be eight over eight.	
6:29	Ms. E	Could you put that on your transparency please? [Ms. E places a piece of transparency paper on Manuel's table and walks away.]	Ms. E asks Manuel D to put his work on a transparency to be used later in the class period for presentation to the whole class. This action may possibly ***convey validation*** of Manuel D's faulty ideas. **[Val(f)]** By asking Manuel D to write his (faulty) solution on a transparency, it may be inferred that Ms. E intends for Manuel D to share his ideas with the whole class.
End TSMIE [E.01.b.02]			

Discussion of TSMIE [E.01.b.02]

It is inferred that Ms. E initiates this episode with the boys because she recognizes that they are having difficulty getting started with the task of assigning fractional values to the properties within Section 18.

Approximately one minute into this episode, Manuel D expresses anger when trying to assign fractional values to the properties within Section 18 [2:44]. Ms. E intervenes and redirects Manuel D to draw and name an example of "any fraction". It is

inferred that Ms. E intends for this intervention to redirect Manuel D's anger and keep him engaged with the mathematics. She offers him choice by asking him to name "any fraction". It is likely that Ms. E's motive is to lead Manuel D toward applying prior mathematical knowledge to assigning fractional values to the properties within Section 18.

This intervention, however, did not translate to success with the task of assigning fractional values to the properties within Section 18. First, Manuel D presents faulty reasoning when assigning fraction names to the properties, by naming Lapp's land "four over sixteen". Although this may appear to be a mathematically correct answer, it is based on Manuel D's faulty idea that the sixteen in the denominator represents the sixteen squares within Lapp's four by four region of land. He later changes his mind and then incorrectly names Lapp's land "sixteen over sixteen".

Ms. E does not attempt to correct Manuel D in either instance of demonstrating faulty reasoning. Once, she calls on Dave to consider whether or not Manuel D's answer "sixteen over sixteen" is correct and Dave responds with affirmation. Subsequently, Manuel D identifies the fraction name for each property as the "area" over "area", which is faulty reasoning.

Before departing from the table, Ms. E hands Manuel D a piece of transparency paper and asks him to write his (faulty) solution on the transparency.

Immediately subsequent to this episode Manuel D and Dave appear to remain engaged with the mathematics however it appears that despite their engagement, their thinking is incorrect.

Begin TSMIE [E.01.b.03] 10:15 – 12:40 (C)			
Participants: Ms. E, Felipe, Eric Ms. E initiates this episode when she approaches a table of four boys and asks them to show her a mathematical representation of "four over four".			
10:15	Ms. E	Show me four over four. [Ms. E is off camera.]	Ms. E initiates a mathematical interaction by asking the boys at Felipe's table to show her a representation of "four over four". **[Endis, Int, rp]**
10:22	Felipe	Four over four? [Felipe's table comes into camera view.]	
10:23	Ms. E	Um-hmm. [Ms. E is leaning on Felipe's table, her lips are pursed and her facial expression is serious.]	Ms. E appears to be ***curious*** about the boys' mathematical understanding of representing the fraction "four over four". Ms. E's question, body language and tone of voice may indicate that she is ***serious*** and ***curious*** about Felipe's understanding of representing fractions. **[Int, rp]**
10:25	Felipe	Four slash four. [Felipe draws a representation showing two squares each partitioned into four equal sections, separated by a fraction bar.]	Felipe appears confident about his answer, a literal translation of "four over four".
10:25	Ms. E	Okay. Does that mean four over four?	Ms. E's tone of voice may indicate that she is ***tentative*** about Felipe's understanding. It is possible that she is feeling ***surprised*** by Felipe's answer. She is not smiling. **[Int, rs, rp]**
	Felipe	That's four boxes over another four boxes.	
10:29	Eric	Yes it is, yes it is.	
10:32	Ms. E	Okay. Could you show me… three-fourths? [Ms. E leans in and taps her hand on the paper where Felipe has drawn his representation for four over four.]	"Okay….' is drawn out as if Ms. E is thinking about how to respond to the boys' mathematical thinking. **[Int, rp]**

10:37	Felipe	Three fourths? [Felipe writes on paper.] That ehh wasn't the thing I was hoping for but… okay, like this and like this.	Felipe seems to be struggling with creating a representation of three-fourths. He draws an L-shaped figure divided into three parts rather than four parts.
10:47	Ms. E	What would you shade if it's three-fourths. Would you shade anything?	Ms. E's tone of voice may be inferred to suggest that she is ***doubtful*** about Felipe's representation. **[Int, rs, rp, Blds]**
10:51	Felipe	No, [Felipe looks up at Ms. E] cause it's an improper fraction.	Felipe appears ***confident*** in his reasoning.
10:53	Ms. E	Improper fraction, what does that mean? [Ms. E looks from one student to the other.]	Ms. E is seeking information her tone of voice is ***inquisitive***. **[Int, rs]**
10:55	Felipe	It doesn't go.	
10:55	Eric	It's not an accurate fraction.	
10:57	Ms. E	Could you show me three-fourths?	Ms. E is off camera because the camera is focused on Eric at this point, but it appears that she posed this question to Eric because he begins to shuffle his materials around as if he is getting ready to draw something on his paper. **[Int, rs]**
11:00	Felipe	It's impossible.	
11:03	Ms. E	It's impossible to show me three-fourths?	Ms. E appears ***surprised*** by Felipe's claim that it is impossible to "show" three-fourths. She does not correct Felipe.
11:04	Felipe	Yes, because if you're doing three fourths the three would be over four.	
11:07	Ms. E	Could you show me, umm draw me a whole thing. One whole thing. [Eric draws] How would you show me three-fourths in that square? [Eric draws]	Ms. E leans in toward Eric and gestures with her hand toward his paper. It appears that Ms. E is ***curious*** about Eric's representation of three-fourths. **[Int, rp]**

11:29	Felipe	Can I draw nineteen?	Felipe is asking permission to draw Section 19, which is similar to Section 18. Felipe appears to have abandoned all efforts to consider the correct representation of three-fourths.
11:31	Ms. E	Yes, you can draw nineteen. What do you mean draw nineteen? On what?	Initially, Ms. E responds with a ***gentle*** and ***dismissive*** tone of voice and gives Felipe permission to draw "nineteen". It appears that Ms. E did not listen to what Felipe was asking before answering him. Felipe's inquiry appears to register with Ms. E when she questions him about his intentions.
11:35	Felipe	On the, like over here.	
11:36	Ms. E	Okay we're having a problem here. I haven't seen three fourths on this table. Can I see three-fourths on this table? [Ms. E taps the table with her open palm. Ms. E then looks from one student to another.]	Ms. E ignores Felipe's answer about Section 19. She raises her voice and appears ***to express anger*** with the Felipe's level of mathematical engagement to the task at hand. **[Rsi, Dis-m, Dis-b]** Ms. E also appears ***angry*** about the lack of mathematical progress when she taps on the table and says, "we're having a problem here". Next, she lowers her voice to a more ***gentle*** tone. It appears that she is hopeful that at least one student will be able to produce a correct representation of three-fourths.
11:47	Eric	Three-fourths.	

11:50	Ms. E	Could somebody help us out? [Ms. E turns to the table that is adjacent to Felipe's table where Noah and Manuel C are seated.] Could somebody show us three-fourths on this table? Could somebody help us please?	Ms. E's tone of voice and action may be inferred to convey ***annoyance*** and ***frustration*** with the lack of mathematical progress at Eric and Felipe's table. She turns around and appeals to the students at an adjacent table for help. This act may suggest that Ms. E feels a sense of ***anxiety*** about this issue. **[Dis-m, Endis]**
11:54	N'yette	Oh, I could help you. [N'yette approaches Felipe's table from a different table.]	N'yette approaches Ms. E from a different table and conveys ***confidence*** when she says "I could help you."
11:55	Ms. E	Could you please? Could you draw three fourths over here? [N'yette leans on Felipe's table and draws.]	Ms. E appears ***relieved*** as she leans back from the table and watches as N'yette draws a representation of three-fourths. **[Int, rp]**
12:01	Felipe	I'm gonna draw nineteen. [Ms. E briefly glances at Felipe, then turns her attention back to N'yette who is drawing a rectangle on her paper, divided into four seemingly equal parts.]	Felipe states once again that he is going to draw Section 19. Ms. E does not redirect Felipe and appears indifferent that Felipe is veering off-task.
12:13	Ms. E	[Ms. E asks N'yette] So how does three fourths look like? [N'yette draws a rectangle with three of four sections shaded.] [Ms. E puts her hand on Felipe's shoulder and addresses him as she points to N'yette's paper.] [Felipe]. Could you write that, three fourths please.	N'yette's representation correctly depicts three fourths. Ms. E puts her hand on Felipe's shoulder, possibly to get his attention and to draw him back into the task. It may be conveyed that Ms. E ***is pleased*** with N'yette's drawing and is using it to help Felipe get back on task. **[Int, rp]**
12:16	Felipe	Oh yeah, I forgot that…	Felipe seems to acknowledge that he understands N'yette's drawing when he says with ***enthusiasm***, "Oh yeah, I forgot that."
12:18	Ms. E	Does that mean three fourths? [Ms. E is looking at the table or N'yette's paper. N'yette remains at the table.]	**[Int, rs, rp]**
12:19	Felipe	Yes.	It appears that Felipe continues to

		[Felipe does not look up from his work when he responds to Ms. E.]	draw Section 19 because he is using his ruler and concentrating on his paper.
12:20	Ms. E	All right, so how would, say for example, umm…[Ms. E hesitates as she glances around the table at the students' papers] three-eighths. How would three-eighths look like? [Ms. E stands back and folds her arms.]	It may be inferred that Ms. E is asking Felipe to engage in representing a different fraction to apply his "renewed" knowledge. **[Int, rp]**
12:28	Manuel C	Look. That's three fourths. [Manuel C is seated at the adjacent table.]	
12:29	Ms. E	Let me see. Ah - Where is three fourths here? Three fourths. [Ms. E repeats "three-fourths" with emphasis.]	Ms. E shifts her attention from Felipe's table to Manuel C's table and looks at Manuel C's work. It may be inferred that Manuel C has presented faulty reasoning because after Ms. E looks at his work, she repeats "three-fourths" with emphasis on "three" and "fourths". Her ***annoyed*** tone of voice suggests that she believes that Manuel C either didn't hear her or wasn't listening. **[Int, rp, Dis-b]**
12:37	Felipe	Here you go, three [Felipe has correctly drawn a representation of three-eighths. He shows Ms. E who briefly leans in and glances at his work, but does not respond.]	
12:39	N'yette	You said three-eighths?	
12:40	Ms. E	Three-eighths. I see, do you agree that this is three-fourths? [Ms. E shows Manuel C's work to N'yette.]	It may be inferred that Manuel C has presented faulty reasoning and Ms. E is asking N'yette her opinion of Manuel C's representation of three-fourths. **[Int, rs, rp, Endis]**
End TSMIE [E.01.b.03]			

Discussion of TSMIE [E.01.b.03]

Ms. E initiates this episode when she asks Felipe and Eric to show her "four over four". Felipe is able to represent "four over four" showing a picture of four squares

literally "over" four squares, separated by what appears to be a fraction bar. When Ms. E asks the boys to show her a representation of three-fourths, Felipe confidently yet incorrectly states that it is impossible to do so. Ms. E's mathematical language is inconsistent. She first asks the boys to represent "four over four" rather than "four fourths". Next, she asks the boys to represent "three-fourths" rather than "three over four". It is not clear whether this shift made a difference in the boys' comprehension, but it is interesting that Felipe was able to provide an answer rather quickly to the first question, "four over four" but struggled when asked to represent "three-fourths".

When Felipe states that it is impossible to draw a picture of three fourths because it is an improper fraction, Ms. E appears surprised. It is inferred that Felipe has become disinterested with the task of naming fractional parts when he asks if he can move on to drawing Section 19. He appears to want to avoid the task of representing three-fourths. At first, Ms. E answers "yes", but it appears that she was not listening to Felipe's question. Then, Ms. E appears to become annoyed with Felipe's level of engagement and expresses disappointment with the boys' mathematical progress. She appeals to students at an adjacent table for help. Subsequently, N'yette approaches from a different table and draws a correct representation of three fourths. Ms. E draws Felipe's attention to N'yette's correct representation and Felipe enthusiastically acknowledges that he understands. Ms. E checks Felipe's understanding by asking him to represent three-eighths, which he is able to do correctly.

Immediately subsequent to this episode, Felipe is engaged with drawing Section 19, rather than the assigned task. Ms. E does not redirect Felipe, perhaps she does not

hear him, or perhaps she is distracted by the events that transpire at the adjacent table that will be discussed in the next episode.

It appears that Felipe's mathematical understanding of representing fractions was corrected. However, there is no evidence indicating the status of Eric's understanding at the end of this episode.

This episode ends when Ms. E shifts her attention to Manuel C's table to discuss the representation of three-fourths.

Begin TSMIE [E.01.b.04] 9:30 – 15:03 (L)			
Participants: Ms. E, N'yette, Manuel C, E, Noah Immediately preceding this episode, Ms. E was interacting with the students at Felipe's table, which is adjacent to Manuel C's table. It appears that Ms. E recognized that students at Felipe's table and Manuel C's table were having difficulty assigning fractional values to the properties in Section 18. In TSMIE [E.01.b.03], Ms. E had asked the students at Felipe's table to write a representation for three-fourths and when Felipe declared that it was impossible to draw three-fourths because it is an improper fraction, Ms. E appealed to the students at Manuel C's table for help. Ms. E initiates this episode when she appears to notice some faulty reasoning presented by one or more students at Manuel C's table. She shows Manuel C's paper to N'yette poses the question, "…do you agree that this is three fourths?"			
9:30	Ms. E	I see, do you agree that is the three-fourths? [Ms. E shows Manuel C's work to N'yette]	Ms. E's tone of voice is ***inquisitive*** and ***doubtful*** as she asks N'yette to assess Manuel C's representation of three-fourths. **[Int, rs, Endis]**
9:36	N'yette	Three out of four? [N'yette looks up at Ms. E.]	
9:37	Ms. E	Three out of four. [Ms. E is standing, facing N'yette, holding Manuel C's paper.]	Ms. E repeats N'yette. She is not smiling.
9:40	Manuel C	Three going down and four going across.	Manuel C seems present faulty reasoning when he attempts to interpret his drawing for Ms. E and N'yette.
9:41	N'yette	No. [N'yette shakes her head.]	N'yette indicates that she disagrees with Manuel C's representation.
9:42	Ms. E	Can you show me? Can you show them three-fourths? [Ms. E puts Manuel C's paper down on the table in front of Manuel C.]	Ms. E appears ***curious*** about how N'yette would represent three-fourths. **[Int, rp]**
9:45	Manuel C	It's three going down and four going across or over four?	
9:46	N'yette	I disagree because this is how, it has to be four, basically like a box.	I appears that N'yette's description may have helped Manuel C to understand how to represent a fraction.

9:50	Manuel C	Oh, I get it. [Noah is standing up watching N'yette] Like a box with the lines and you shade three out of four.	
9:55	N'yette	Yeah, that's three-fourths. [N'yette nods her head]	
9:59	Ms. E	So what would three-eighths be?	Ms. E's tone is ***inquisitive*** as appears ***curious*** and checks for student understanding by asking them to represent three-eighths. Ms. E used this same strategy in the previous episode with Felipe. **[Int, rp]**
10:01	Noah	Three-eighths would be like have eight boxes and shade four.	
10:03	Ms. E	All right, so what do you call this then? [Ms. E points to Noah's paper] if you shade it.	Ms. E does not correct Felipe's faulty reasoning. It may be inferred that Ms. E is pointing to one of the properties in Section 18 on Noah's paper. **[Int, rs, rp]**
10:09	N'yette	They didn't shade it yet.	
10:10	Ms. E	So how how, what would that be if you shade it? Lapp, Lapp's land. [N'yette raises her hand] what would you call that? [Ms. E points to Noah's paper]	Ms. E intervenes and ***encourages*** the students to imagine that Lapp's land is shaded. It may be inferred that Ms. E is becoming ***impatient*** for the students to provide a fraction name for Lapp's land. Ms. E is leaning into the table and frequently points at the students' papers. **[Endis, Int, rp Dis-b]**
10:16	N'yette	Oh, I could do it, I could do it, I could do it. This not my group, I can't do it.	Initially, N'yette appears ***excited*** to share her mathematical ideas. Then, she shows restraint when she states that "this is not my group", suggesting that she should wait for those students to respond to Ms. E first.
10:20	Ms. E	That's okay.	Ms. E gives permission for N'yette to speak even though it is not her group. **[Enc]**

10:22	N'yette	Okay, we would do [Noah stands up to look at N'yette] first we would count one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen. Sixteen. How much you want to shade? Two? One fourth? How much you want to shade?	N'yette is asking her peers to decide how many squares they should shade in Lapp's land.
10:34	Ms. E	What if you shade a whole thing? That's all of Lapp's land right?	Ms. E intervenes by suggesting "What if you shade a whole thing? That's all of Lapp's land right?" **[Int, rp]**
10:38	N'yette	Sixteen over sixteen…that's what it would be if you shaded the whole thing.	
10:41	Ms. E	So are you going to call this sixteen by sixteen then?	**[Int, rs]**
10:44	N'yette	Ya'll gonna shade the whole thing, or ya'll gonna leave one half or two halves? What? [Ms. E looks at Ed.]	
10:50	Ms. E	What's your one whole?	**[Int, rs, rp]**
10:51	Ed	[Ed points to paper] two-fourths and two-fourths here.	
10:53	Ms. E	Show me one whole, what's a one whole here? [Ms. E directs this question to Ed.]	Ms. E repeats her question to Ed. **[Int, rs, rp, Endis]**
10:55	N'yette	One whole is…shading the whole thing, sixteen by sixteen.	
10:55	Noah	Shading the whole thing.	
10:58	Ms. E	Sixteen by sixteen.	
10:59	N'yette	That's one whole.	
11:00	Ms. E	That's one whole.	Ms. E repeats the students' suggestion, but her tone of voice may be inferred to express ***doubt***.
11:01	Manuel C	Do eight.	Ms. E is quietly looking on as the students discuss the mathematics. It appears that all students at this table are engaged with the mathematics, yet are not making progress.
11:01	N'yette	Ohhhhh	
11:02	Manuel C	Shade half of it.	
11:08	N'yette	Do eight and shade half of this? That'll be.. hmmm…	
11:10	Manuel C	Yeah, eight out of….	
11:13	N'yette	That'll be eight, wait, eight	

		halves over sixteen right here, it'll be like this. [Ms. E looks on and fills her mouth full of air, pursing her lips. She very subtly shakes her head, "no".]	Ms. E's facial expressions and gesture [shaking her head 'no'] may appear to convey ***disappointment*** with the students' discussion. **[Int, Dis-m]**
11:18	Ed	Over sixteen.	
11:19	N'yette	It'll be like this [N'yette takes sheet of paper] sixteen, eight, because you shaded eight and then you still have half, you shaded half of [pointing], so it would be just like this.	
11:35	Ms. E	All right, [Ms. E scratches her face and then points to the statement of the task in a students' book] let me draw your attention on this one. Each section is one square mile of land. If we call this a section, can we call this one whole?	Ms. E intervenes by reading a portion of the problem from the book. It appears that by re-reading the problem, Ms. E may be hopeful that the students recognize Section 18 as the "one whole". **[Rsi]** Ms. E appears to convey ***no emotion.***
11:42	Ed	Yeah.	
11:43	Ms. E	So if we call this one whole then what do you call this?	Ms. E builds on the students' ideas, and it is likely that "this" that Ms. E is referring to is Lapp's property. **[Blds, Int, rs, rp, Endis]**
11:47	Manuel C	One fourth.	Manuel C appears to have given the correct fraction name for Lapp's land.
11:48	Ms. E	Why is it one fourth?	Ms. E does not validate Manuel C's answer. She expresses ***no change in emotion***. **[Int, rs]**
11:49	Manuel C	Because that has taken …	
11:50	N'yette	That's only half of it.	
11:51	Manuel C	Like if you cut this in half, then you just take half of the half.	Half of a half equals one-fourth.
11:53	Ms. E	Uh huh, okay.	
11:56	N'yette	Each of them would be a box, and if this wasn't here, it would still be, it's like four boxes.	N'yette correctly explains why Lapp's land represents one-fourth of Section 18. She shows that Section 18 could be portioned

			into four equal parts, each congruent to Lapp's property.
12:00	Ms. E	All right, so can you write that there? What is it again?	Ms. E asks Manuel C to reiterate the answer. **[Blds, Int]**
12:03	Manuel C	One fourth.	
12:04	Ms. E	Write it please.	Ms. E politely asks the students to write the answer on the paper. Yet, Ms. E ***does not smile*** nor does she seem to celebrate this correct reasoning. But it is inferred that she validates Manuel C's answer by telling him to "write it".
12:06	N'yette	They're right. If it is four. one, two, three, four. [N'yette claps her hands.]	N'yette appears to exhibit ***joy*** about the mathematical progress.
12:08	Ms. E	Are they right?	**[Endis, Int, rs]**
12:09	N'yette	Yeah.	
12:10	Ms. E	All right, now if this is one-fourth, what do you call this?	Ms. E proceeds with the task of identifying the fraction name for another section.
12:13	N'yette	One half	It may be inferred that the students have presented faulty reasoning here because there is no property that represents one-half of Section 18.
12:14	Manuel C	One fourth	
12:14	N'yette	One-fourth, No, one-half.	
12:17	N'yette	Cause if you put another one, if you put another one there	
12:18	Manuel C	Yeah because yeah. No, its one-fourth.	
12:19	Ms. E	One-fourth?	
12:20	Manuel C	Yes because it's cutting the big block into an eight.	
12:23	Ms. E	So this is one-fourth and this is one-fourth.	
12:24	Manuel C	Yeah	
12:25	N'yette	I don't think that's one-fourth.	N'yette disagrees with Manuel C's reasoning.
12:25	Ms. E	So you want to write one-fourth, why not? [Ms. E rubs her nose with her hand. She is looking down at Manuel C's paper.]	**[Int, rs, Raa, Endis]**

12:26	N'yette	Cause if this is one-fourth, well how can this be one-fourth?	N'yette is comparing two different properties that are different sizes. It appears that she understands that if the two properties are different sizes then it they cannot have the same fraction name.
12:29	Manuel C	'Cause look (N'yette turns around), look [N'yette].	
12:34	Ed	What you said…	
12:36	Manuel C	There's a big thing. There's one block here and one block here.	
12:38	N'yette	Ohh, I see what you saying. Go one, two, three, four [N'yette laughs] five.	
12:44	Ms. E	But you had a question to him about the one-fourth.	Ms. E redirects N'yette to her disagreement. She appears to be ***encouraging*** N'yette to explain why she disagrees with Manuel C's reasoning. **[Endis, Int]**
12:46	N'yette	Yeah.	
12:47	Ms. E	What's your question?	Ms. E is ***encouraging*** student discourse. **[Endis]**
12:48	N'yette	I said, how could this be one-fourth and this be one-fourth?	
12:54	Manuel C	Cause this is like that and there's another piece that would be one, two, three, four.	
12:56	N'yette	No.	N'yette disagrees with Manuel C's reasoning.
12:57	Ms. E	Why is that a problem? [Ms. E touches N'yette on the back]	Ms. E ***encourages*** N'yette to articulate the mathematical reason why it is a problem for both properties to be named one-fourth. **[Int, Endis]**
12:57	N'yette	That's a problem, that's a problem.	
12:58	Noah	That's not one-fourth	
13:00	N'yette	Why?	
13:01	Noah	Cause is this is one-fourth…	

13:01	N'yette	If you want us to do it like that, this just can't be one-fourth, know why?	
13:05	Noah	If this is one-fourth, then why is this part smaller?	
13:06	N'yette	No listen, no listen, if this is one-fourth then how could this be one-fourth because even though these four, what about the rest of the boxes? [N'yette looks at Ms. E]	
13:16	Ms. E	So can you call both of them one-fourth? [Ms. E points at Manuel C's paper, specifically at the properties that represent Lapp's property and Bouck's property.]	Ms. E intervenes by asking a second time whether both properties can be named one-fourth. **[Int, rs, Endis]**
13:19	Manuel C	No.	
13:20	Ms. E	So which one is correct then?	**[Int]**
13:22	N'yette	This one [N'yette points with pencil]	
13:22	Ms. E	This is one-fourth? If this is one-fourth then what do you call this?	**[Int]**
13:24	N'yette	This is one-half.	N'yette's answer is incorrect. Bouck's land represents one-sixteenth of Section 18.
13:25	Manuel C	This is one-half.	
13:26	Ms. E	One-half….okay, so you want to call that one-half? And what do you think? [Ms. E directs her question to Jenny] [Manuel C is erasing on his paper.]	Ms. E repeats N'yette's answer, but her voice may express ***doubt.*** Jenny has been silent thorough this discussion, it appears that Ms. E is attempting to engage her in the discussion by asking, "what do you think?"

13:35	Ms. E	[Ms. E is leaning down, looking at Jenny as she speaks.] How could it be? What's your question? How could it be…? How could it be one-half? Which one? [Ms. E raises her voice] [Jenny] has a question, how could this be…[Ms. E touches Jenny's hand and leans in as she speaks to her] Say your question again.	Ms. E appears to be ***encouraging*** Jenny to engage in the discussion. **[Int, rs, Endis]** Ms. E repeats Jenny's question and it may be inferred that Ms. E is validating Jenny's ideas in an effort to support Jenny's contribution to the discussion.
13:53	Ed	It's kind of four pieces, it's four pieces into one block.	
13:56	Ms. E	But what are you naming this again? [Ms. E is now kneeling next to the table, pointing to Manuel C's paper.]	Ms. E kneels next to the table conveying ***interest*** in the students' discussion of this task. Her facial expression is still ***serious*** and she is ***intently*** listening to the students' ideas. **[Int]**
13:57	Manuel C	One-fourth.	
13:58	Ms. E	One-fourth.	
13:59	Ed	That's what it's supposed to be.	
14:07	P.A. system	Please excuse the interruption, is [Ms. E] up there?	An announcement from the P.A. system indicates that a visitor is en route to the classroom.
14:09	Ms. E	Yes? [Ms. E maintains her gaze at the students' work as she responds to the loudspeaker]	
14:11	P.A. system	Okay, I'm sending up Ms. Schorr	
14:14	Ms. E	Who? [Ms. E looks puzzled as she looks up at the loudspeaker]	
14:15	P.A. system	[Ms. S].	
14:16	Ms. E	Okay, [Ms. E raises her eyebrows and smiles] [Dr. S] , all right!	
14:19	P.A. system	Thank you.	Ms. E looks on as Manuel C explains his mathematical thinking. **[Int]**
14:19	Manuel C	This is one-fourth.	
14:20	Ms. E	Right.	
14:21	Manuel C	This is one-fourth.	
14:22	Ms. E	Uh huh.	
14:23	Manuel C	And one-fourth.	
14:24	Ms. E	One-fourth.	

14:26	Manuel C	But now this is split into four pieces	
14:28	Ms. E	Okay…	
14:29	Manuel C	So this is one-fourth, cause this is the piece you're going to take out and there's three left	
14:42	Ms. E	Okay… so you…	
14:46	N'yette	So that would be… no	
14:47	Ms. E	One-fourth of what?	
14:48	Manuel C	Of this block right here	
14:50	Ms. E	And what do you call this whole block right here? [Ms. E traces Lapp's land with her finger on Manuel C's paper.]	Ms. E appears ***curious*** about Manuel C's fraction name for Lapp's land. **[Int, Endis]**
14:51	N'yette	One-fourth.	
14:52	F	So it's one-fourth of what? [Ms. E's eyebrows are raised.]	**[Int, rs]**
14:52	N'yette	One-fourth of one-fourth, that makes no sense.	
14:56	Ms. E	One-fourth of…?	
14:57	Manuel C	One-half	
14:59	Ms. E	Is it one-fourth of one-half? Is it one-fourth of one-fourth? Is it one-fourth? Is it one-half? What is it? [Ms. E is gesturing with her hand.]	Ms. E's tone of voice conveys ***frustration*** as she restates all of the suggestions that the students in this group have mentioned. **[Dis-m, Int, Endis]**
15:03	N'yette	I say that this box is still one-half and this is still one-fourth and if you try to take, if you try to do this inside the box that would be [Ms. E walks away] one-half, that would be, hold up…haha, let me see, this whole box. one, two, three, four, five, six, seven, eight.	
End TSMIE [E.01.b.04]			

Discussion of TSMIE [E.01.b.04]

This episode begins when Ms. E appears to recognize that students at Manuel C's table have presented faulty reasoning when drawing representations of three-fourths. She appears frustrated possibly because the students have not been able to draw a correct

representation for the fraction three-fourths. Ms. E appeals to N'yette for help in showing the group how to correctly represent three-fourths.

Ms. E remains with this group of students as they proceed to assign a fractional value for Lapp's land. Ms. E's tone of voice, posture and gestures convey a sense of urgency as she asks the students, "So how how, what would that be if you shade it? Lapp, Lapp's land."

Ms. E intervenes when the students appear to struggle to name Lapp's land by asking the students what part of the picture represents one whole. The students struggle with this for a few minutes before they recognize that Section 18 represents the "whole" and then Lapp's land represents one-fourth.

Once the students are able to correctly name Lapp's land one-fourth, Ms. E questions the students about the fraction name for Bouck's land, which represents one-sixteenth of Section 18. The students have difficulty with this task because it appears that they are now thinking of Bouck's property as it compares to Lapp's property. Manuel C names Bouck's land one-fourth and N'yette agrees with him. Ms. E calls attention to the idea that the properties are different sizes and N'yette subsequently recognizes that both properties cannot then have the same fraction name within Section 18.

Manuel C presents a series of incorrect mathematical ideas, naming Bouck's land one-fourth, then one-half. When Ms. E asks Jenny's opinion of Manuel C's answer, Jenny disagrees. Manuel C then claims again that Bouck's land is one-fourth.

Ms. E intently observes Manuel C as he works on his paper. It appears that his drawing has helped to show that Bouck's land is one-fourth of one-fourth which N'yette is able to correctly articulate, but claims, "that makes no sense".

Subsequent to this episode, the students remain engaged in finding the fraction names for each property. They continue to present faulty reasoning when they assign fraction names to each of the other properties. N'yette remains engaged with the students at this table rather then returning to her assigned group.

Begin TSMIE [E.01.b.05] 16:44 – 19:28 (L)			
Participants: Ms. E, Brielle, Jahay, Betty Ms. E initiates this episode when she poses a mathematical question to the girls at Brielle's table, "why four-eighths?" It appears that one or more of the girls have named one of the properties four-eighths and Ms. E is curious about their reasoning. Ms. E expresses curiosity by leaning on the table and by asking students to explain their reasoning.			
16:44	Ms. E	Why four-eighths?	Ms. E's tone of voice conveys a sense of ***urgency*** and ***curiosity*** when she asks the girls "why four-eighths" which apparently is written on one of their papers. **[Int]**
16:46	Brielle	Because...four out of the whole thing, eight.	
16:54	Ms. E	Show me four-eighths in the back of this paper, show me four-eighths [Ms. E directs this question to Brielle.]	Ms. E asks Brielle to divert from the task to draw a representation of four-eighths on the back of a piece of paper.
16:58	Brielle	Four-eighths.	
16:59	Ms. E	I see the number, if you drew that. [Ms. E points to the paper.] Show me four-eighths. [Brielle draws on paper] How do you know this is four-eighths?	Ms. E demonstrates ***curiosity*** in Brielle's mathematical thinking. It is apparent that Brielle has drawn the number 4/8 on the paper, yet Ms. E's tone of voice conveys ***annoyance*** when she repeats her request, "show me four-eighths". Ms. E asks Brielle to provide an illustration of four-eighths. **[Int, Dis-m]**
17:14	Brielle	'Cause you shade four out of eight.	
17:19	Ms. E	Okay, so you should shade four out of eight……[Brielle finishes her drawing of a rectangle portioned into eight equal sections, with four sections shaded and Ms. E points to the drawing of Section 18] how is this four-eighths?	**[Dis-m]** Ms. E apparently recognizes that Jahay has made an error by naming one of the properties four-eighths because she asks, "how is this four-eighths?" while pointing to Jahay's paper. Her tone of voice conveys ***doubt***.
17:20	Brielle	That's not it, four-eighths.	

17:21	Ms. E	What is this then?	There is a sense of ***urgency*** when she asks Brielle to give the correct fraction name for Lapp's land.
17:23	Brielle	That's umm… sixteen out of… [Brielle puts her hand on her head.]	Brielle appears to be struggling to name the denominator.
17:28	Ms. E	What's the whole thing?	Ms. E intervenes by asking Brielle to name the "whole thing". Her tone of voice conveys ***patience***. **[Enc, Int]**
17:35	Jahay	Sixteen out of umm… [Brielle writes on paper]	
17:43	Ms. E	There's a calculator if you want to use it….	Ms. E offers support by suggesting that the students use a calculator. Her tone of voice is ***gentle*** and ***calm***. **[Enc]**
17:44	Brielle	[Brielle says to Jahay] go get a calculator [Jahay leaves the table to get a calculator].	
17:46	Ms. E	Can you write sixteen? So you say sixteen?	Ms. E sounds ***encouraging*** when she suggests that Brielle write sixteen on her paper. **[Enc]**
17:49	Brielle	Right here?	
17:49	Jahay	Sixteen out of…. Sixty-four. [Jahay has obtained a calculator]	It is likely that Ja has used the calculator to multiply eight times eight.
17:55	Ms. E	Do you agree? Why is it?	Ms. E does not validate this correct answer, but she asks Brielle if she agrees. **[Endis, Int]**
17:57	Jahay	Because there's sixty-four boxes.	
17:58	Brielle	Because eight times eight is sixty-four.	Brielle's facial expression and gestures indicate that she is confident in her answer.
18:00	Ms. E	Where is the sixty-four?	Ms. E demonstrates ***curiosity*** in Brielle and Jahay's understanding of the denominator, sixty-four. **[Int]**
18:02	Jahay	The whole thing is sixty-four right here.	

18:04	Brielle	'Cause see if you count all these little boxes there's sixty-four.	
18:08	Ms. E	Explain this to [Betty]. [Ms. E raises her voice and calls students from a different table to come to Brielle's table.] Ladies, ladies, ladies come here, once you have your drawings, umm right. [Brielle], can you explain why you call this sixteen out of sixty-four	It appears that Ms. E may be ***pleased*** with Jahay and Brielle's mathematical thinking. Ms. E sounds ***excited*** when she summons the girls from the other table. **[Int, Endis]**
18:21	Brielle	Because eight times, this is eight by eight, the whole this is eight by eight, so eight times eight is sixty-four	Ms. E looks on as Brielle explains her reasoning to the other students.
18:26	Ms. E	Where is sixty-four there?	
18:28	Brielle	Sixty-four of these small boxes.	
18:29	Ms. E	Want to count and find out?	Ms. E encourages Brielle to count all of the boxes rather than to multiply eight times eight to prove that there are sixty-four "small boxes". **[Int]**
18:30	Brielle	One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen… [Brielle continues counting all sixty-four squares in Section 18 on her paper]	
18:55	Dave	Ms. E, Ms. E, you want me to put that on a transparency? [Dave is seated at a different table]	
18:58	Ms. E	Yes.	Ms. E responds to Dave's question.
19:03	Ms. E	[Brielle finishes counting] So why do you call this sixteen out of sixty-four?	Ms. E asks Brielle once again why she named Lapp's land "sixteen out of sixty-four". It is likely that Ms. E is asking Brielle to reiterate this for the other students at the table. **[Int]**
19:10	Brielle	Because if you count these boxes it's sixteen.	Brielle is referring to the sixteen "boxes" in Lapp's property.
19:12	Ms. E	Can you put this on a transparency? [Brielle agrees] Just this.	Ms. E may be showing her ***approval*** of Brielle's answer by asking her to share her answer

			with the whole class. **[Val]**
19:14	Brielle	Okay. [Ms. E walks away from the table and Brielle keeps working]	
19:25	Ms. E	[Ms. E hands a transparency to Brielle] Just this section right here.	It may be inferred that Ms. E wants Brielle to show the fractional value of Lapp's property only.
19:27	Brielle	I need a marker.	
19:28	Ms. E	You can draw this in this Lapp and just show this.	Ms. E gives directions for how Brielle should present her answer.
End TSMIE [E.01.b.05]			

Discussion of TSMIE [E.01.b.05]

At the beginning of this episode, it appears that Ms. E notices an incorrect answer on Jahay's paper because she asks Jahay and Brielle, who are working together, "Why four-eighths?" After Ms. E questions Brielle and Jahay, it appears that the girls rethink their original answer and subsequently present correct reasoning, naming Lapp's land sixteen over sixty-four. Ms. E appears pleased with Jahay and Brielle's mathematical thinking when she asks them to explain their reasoning to students from adjacent tables. Ms. E also asks Brielle to write her answer on a transparency for a whole class presentation later in the period.

This episode ends when Ms. E walks away from the table. Brielle and Jahay appear remain engaged in drawing their solution on the transparency after Ms. E has left their table.

Begin TSMIE [E.01.b.06] **25:56 – 34:10 (C)**			
Participants: Ms. E, Manuel D, Felipe, Eric, Dave, N'yette, Brielle In this episode, Ms. E brings the small groups together for a whole class to discuss the fraction names for Lapp's property relative to Section 18. Most of the students in the class have spent the majority of this class period working in cooperative groups to solve this part of the task. Ms. E invites individual students to present their work at the overhead projector.			
25:56	Ms. E	All right class, I'm sorry to do this. [Ms. E places her palms on her cheeks.] We're just going to talk about Lapp's so if you have a name for a Lapp you're good for now. [Ms. E sits down on the table next to Manuel C.] [Brielle], you have a name for Lapp, right? [Ms. E puts her hand on Felipe's shoulder.] Lapp? Just Lapp? You have a fraction name for Lapp? Okay, umm, Sindy, could you? One second, [Ms. E holds up one finger then points to the overhead projector] right here. [Felipe is standing at the overhead projector.]	Ms. E announces that students will present their answers for the fraction name for Lapp's land. This announcement is at least the fourth time that Ms. E has addressed the class, asking the students to direct their attention to the front of the room for presentations. Her tone of voice and language convey that she respects the students' engagement and she is ***apologetic*** for interrupting their work. It may be inferred that Ms. E is ***attempting to reassure*** students and to ***reduce their stress*** level before presentations when she says, "We're just going to talk about Lapp's so if you have a name for a Lapp you're good for now." **[Res]** Ms. E speaks directly to Brielle who she worked with in the previous episode, ***reassuring*** her that she had already found a fraction name for Lapp when she says, "[Brielle], you have a name for Lapp, right?" Ms. E points to the overhead projector.
26:30	Ms. E	[Felipe is standing at the overhead projector, Brielle approaches the projector] Okay, can I have everyone's attention to the middle please [Ms. E's voice is raised.] five,	Ms. E appears to be trying to get the students' attention as she counts down from five to one. Ms. E sounds ***surprised*** when she notices that Felipe is not standing at the overhead projector.

		four, turn around, three [Felipe walks behind Ms. E, back to his seat], presentation please two, no we'll start with [Felipe], we'll start with you.	
26:47	Felipe	I want to sit down [Felipe is smiling].	Felipe has already drawn his answer (four-fourths) on the overhead projector, but it appears that he is feeling too ***shy*** to stand in front of his peers and present his answer.
24:48	Ms. E	[Felipe]... come on.	Ms. E tries to ***encourage*** Felipe to discuss his answer with the class. **[Enc]**
26:50	Felipe	I don't want to do it. [Felipe shakes his head, 'no']	Felipe resists Ms. E's request.
26:51	Ms. E	Umm, [Manuel D]? [Ms. E is smiling.]	Ms. E abandons her efforts with Felipe and asks Manuel D to present. Ms. E's tone of voice is ***friendly*** and ***encouraging***. **[Enc]**
26:53	Dave	No, not yet, we're not done.	Dave is seated at Manuel D's table and resists Ms. E's request but Manuel D gets up from his seat and approaches the overhead projector. Dave appears to reluctant to share.
26:54	Ms. E	No just this one, just explain your answer there, just Lapp, Lapp, nothing else, just Lapp [Manuel D and Dave go to projector].	Ms. E appears to be trying to ***reassure*** Manuel D by telling him that he only had to present his fractional name for Lapp's land. "No just this one, just explain your answer there, just Lapp, Lapp, nothing else, just Lapp." **[Enc, Int]**
27:00	Felipe	I don't know, I froze up, figure of speech. [Felipe is speaking to the boys who are seated at his table.]	
27:06	Ms. E	You don't agree with [Felipe] that it's four by four?	Ms. E questions Manuel D about his answer, asking him if he agrees with Felipe. Her tone of voice is ***inquisitive***. **[Int]**
27:09	Eric	It is four by four.	
27:14	Manuel D	We did sixteen.	

27:15	Felipe	Sixteen?	
27:15	Ms. E	What's your name for Lapp? Fraction name.	
27:19	Dave	Sixteen over sixteen.	
27:20	Ms. E	Sixteen over sixteen.	Ms. E flatly repeats Dave's answer. It is incorrect, but Ms. E does not correct the answer. Recall that in an earlier episode, Ms. E asked Manuel D to record his (faulty) answer on a transparency.
27:21	Manuel D	Or a whole.	
27:22	Dave	Or a whole.	
27:23	Ms. E	Or a whole, do you guys agree? Could you explain why?	Ms. E flatly repeats Manuel D's and Dave's answer then poses a question to the class, "Do you guys agree?" Her language is casual and her tone of voice is ***inquisitive*** as she possibly ***hopes*** for students to disagree with Manuel D's and Dave's answer. Then, she gives Manuel D an opportunity to explain his reasoning by asking, "Could you explain why?" **[Endis, Int]**
27:29	Manuel D	Because there's sixteen in all and none of them are shaded or anything so it should be sixteen cause none of them are shaded.	It may be inferred that Manuel D still does not recognize that Section 18 represents one whole. This faulty reasoning was initially presented during an interaction with Ms. E in [E.01.b.01].
27:44	Ms. E	Do you agree? Do you agree? [Ms. E looks directly at student groups around the classroom when she repeats the question.]	Ms. E does not correct Manuel D's faulty reasoning. She poses a question to the class asking the students if they agree with Manuel D's reasoning. **[Endis]**
27:48	N'yette	No.	
27:49	Ms. E	Can you talk to them? [Ms. E points toward where Manuel D and Dave are standing.]	When N'yette ***disagrees*** with Manuel D's answer, Ms. E asks N'yette to "talk to them". Ms. E may be ***hopeful*** that N'yette will be able to help Manuel D and

			Dave correct their faulty reasoning. **[Endis]**
27:52	N'yette	[N'yette approaches the overhead projector] I don't agree because…	
27:54	Ms. E	[Felipe], sixteen over sixteen? Do you agree?	Ms. E questions Felipe about Manuel D's answer. **[Endis]**
27:59	N'yette	How would it be sixteen over sixteen if this whole thing is sixteen right?	
28:03	Ms. E	Wait, lets just move this, wait. [Ms. E adjusts the transparency on the overhead and then returns to sit on Manuel C's table.]	
28:05	Manuel D	None of them are shaded.	
28:07	N'yette	None of them are shaded okay. So, none of them is shaded but what we did was, we took, is rather that you four, four, four and this is one, so we shaded one whole box out of one. I know but… [N'yette traces each section with her pen that would represent one fourth of Section 18.]	N'yette appears to be showing Manuel and Dave that Lapp's land may be considered one-fourth of Section 18 because there are four equal sections congruent to Lapp's land within Section 18.
28:31	Dave	So that is one whole.	
28:33	Manuel D	This is like a fourth of the whole thing.	It appears that Manuel D is beginning to understand N'yette's reasoning why Lapp's property can be named 'one-fourth'.
28:42	N'yette	[N'yette writes on the transparency] Yeah, yeah but…	
28:44	Manuel D	We're talking inside the box, not the whole thing.	Manuel D then returns to his claim that Lapp's land is somehow it's own entity aside from Section 18.
28:46	N'yette	But if this is sixteen.	
28:50	Dave	This is like this [Dave puts his hands on the projector.]	

28:52	N'yette	Okay, if we're focusing on Lapp, if Lapp is sixteen over sixteen without shading.	
29:09	N'yette	One whole of the whole eight by eight	
29:16	Ms. E	[Manuel C], did you want to explain what you were saying?	Ms. E's tone of voice is ***gentle***. It may be inferred that she is trying to ***encourage*** Manuel C to join the discussion by sharing his ideas. **[Enc]**
29:18	N'yette	One whole, out of eight by eight, and if you take…	N'yette is gesturing on the diagram that Lapp's land is one whole out of the eight by eight square.
29:23	Dave	But we're not doing this, we're not doing this part of that.	
29:27	N'yette	One, two, three, four. One, two, three, four. We got eight.	N'yette is counting the units on the edges of Lapp's land. It is unclear why she is counting the edges.
29:23	Dave	What?	
29:34	N'yette	Four down, four up. Eight. right? No, that's not what I'm talking about.	N'yette repeats counting the edges of the squares.
29:41	Dave	Yes, it is.	
29:42	N'yette	One, two, three, four. One, two, three, four. Because, okay [Manuel D walks in circle and laughs.]	Manuel appears to recognize that N'yette's reasoning is correct as he appears to laugh to himself.
29:47	Ms. E	All right, umm, I can't quite get everyone here, and I understand you're still working on something and I apologize that I'm interrupting, but could you just look one second and tell me if you agree that Lapp's land is… what are you claiming Lapp's land is?	Ms. E raises her voice in an attempt to gain the attention of the whole class. Her language indicates that she is ***apologetic*** for interrupting the students, yet her tone of voice may be inferred to indicate that she is ***annoyed*** at the class for not paying attention to the discussion that is taking place at the overhead projector. **[Res, Dis-b]**
30:13	Manuel D	Sixteen over sixteen.	Manuel D presents faulty reasoning.

30:14	Ms. E	Sixteen over sixteen, so I'm going to say that I'm going to base everyone's grade based on the answer that's on the projector right now. And I'm going to say everyone will be graded based on sixteen over sixteen. All right, so could we hear some…	Ms. E raises her voice and restates Manuel D's answer, "sixteen over sixteen". It may be inferred that Ms. E is ***annoyed and frustrated*** that students have not disagreed with M's faulty reasoning. Her arms are folded and her tone of voice is ***serious***. It appears that she is trying to get the students' attention when she poses a threat: "And I'm going to say everyone will be graded based on sixteen over sixteen." **[Endis, Threat]**
30:29	Manuel D	We realize, we realize…	
30:31	Ms. E	Yes?	Ms. E's tone of voice conveys ***expectation*** of a valid argument. **[Int]**
30:33	Dave	One, four, four, four, eight, twelve, sixteen.	
30:38	Manuel D	What we did was we multiplied four by four.	
30:43	Ms. E	So what are we calling this now?	**[Int]**
30:46	Manuel D	One-fourth.	It appears that Manuel D has changed his answer from sixteen over sixteen to one-fourth.
30:47	Ms. E	And why is it one-fourth?	
30:49	Manuel D	Because umm, four of Lapp's land could make the whole land.	Manuel D presents a correct explanation for naming Lapp's land one-fourth.
30:57	Ms. E	[Felipe] do you agree its one fourth?	**[Int, Endis]**
31:00	Felipe	I wasn't paying attention.	
31:02	Ms. E	You weren't paying attention?	Ms. E does not reprimand Felipe for not paying attention.
31:03	Eric	[Eric raises his hand] I agree.	
31:04	Ms. E	You agree? Why do you agree?	Ms. E expresses ***interest*** in Eric's thinking. **[Int]**
31:07	Eric	Because they really did make a mistake and it's really four over four	Eric presents faulty reasoning.

31:11	Ms. E	Is it four over four?	Ms. E questions the class, repeating Eric's answer, "is it four over four". Her tone of voice and questioning technique may convey ***doubt*** in Eric's answer.
31:13	Eric	It's four by four.	Eric states the dimensions of Lapp's property.
31:15	Ms. E	So what fraction will you give it, [Eric]?	**[Int]**
31:18	Eric	Four slash four.	Eric offers faulty reasoning.
	Ms. E	Could you come write it? [Ms. E hands Eric an overhead projector pen.]	Ms. E does not correct Eric and responds by handing an overhead projector pen to Eric and invites him to the overhead.
31:21	N'yette	And I said this would be half because this is half of one fourth. What is half of one fourth? One fourth [Eric goes to the overhead projector and Felipe gives him marker.]	
31:33	Manuel D	It's not really, the little box is one fourth.	
31:34	Ms. E	I believe you have a different answer but you seem like you are talking. [Noah], don't want to get involved in this one? [Ms. E is sitting on the table in front of Noah.]	Ms. E speaks quietly to Noah, ***gently*** suggesting that he has a different answer, but does not force him to get involved with the discussion. Noah leans back in his chair and looks toward the overhead projector. It is not clear whether or not he answered Ms. E. Ms. E returns her attention to the discussion at the overhead projector. **[Enc, Endis]**
31:38	N'yette	That's half of the box.	
31:49	Manuel D	That little box right there is a fourth of that box . [Eric asks Ms. E something inaudible, Ms. E nods, then Eric writes on the overhead projector.]	It may be inferred that Eric is asking Ms. E where he should write his answer, four over four.
31:51	Ms. E	[Eric finishes writing and steps away from the overhead projector]. So [Eric] is calling it four fourths.	Ms. E raises her voice and announces to the class, "Richard is calling it four fourths."
31:53	Manuel D	I don't agree.	Manuel D does not explain his

			reason for disagreeing with Eric.
31:56	Dave	It could be, it could be either of the two because four over four would give sixteen…	
32:03	Ms. E	Ladies, can you see this? Do you agree that it's four fourths?	Ms. E appears to try to elicit student engagement by asking, "can you see this?" **[Int]**
32:07	N'yette	[N'yette gets up from her chair] Excuse me, [Manuel], [Manuel], [Manuel], you're right, you're right, this is one-fourth and this is one-half.	
32:18	Manuel D	That would be one-fourth of that.	
32:38	Ms. E	Okay, just one more, [Ms. E raises her hand to gesture, "one" and raises her voice] just one more idea today, have a seat, have a seat, just one more idea today. So we have the name sixteen over sixteen, four over four, one fourth, and one more. [Ms. E puts Brielle's transparency on the overhead projector.] Ready for one more? Just one more. Could I have your attention please?	Ms. E raises her voice probably to get the students' attention then she reviews the answers that have been presented on the overhead projector. Ms. E uses ***respectful*** language "please" as she requests the students' attention. Ms. E reviews the answers that have been presented thus far without offering judgment on which ones are correct or incorrect. **[Res, Endis, Dis-b]**
33:05	Ms. E	Sindy's group. Just one more thing, okay, this is a different idea so I'd like for you to look at it. I can't I can't, Umm, can I interrupt you for a second? May I, may I, may I? Please look, one second, just one last idea today, one last idea. [Ms. E points toward Brielle who is standing at the overhead projector. The bell rings.] Go ahead. [Ms. E gives Brielle permission to begin her presentation.]	Ms. E calls on Sindy's group to get their attention. Her tone of voice is gentle and her language stresses the importance of the "one more idea". Ms. E is ***polite*** when she addresses the class despite the noise level, which is still loud. Her tone of voice and body language suggest that she is struggling to get the students' attention. Ms. E's language, "May I, may I, may I? Please look, one second…" suggests that she is trying to ***show respect*** for the discussions that are taking place at

			the individual tables, yet she wants the class to take notice of Brielle's presentation. **[Res, Ptp]**
33:36	Brielle	You can't see anything.	
33:38	Ms. E	Oh, we're having a hard time with this. [Ms. E adjusts the transparency.] Can you see it?	
33:48	Manuel C	Let me see that one right there [toward Felipe's table].	
33:51	Ms. E	Okay, one last idea today.	
33:54	Brielle	What I did was that eight by eight, [Ms. E is looking toward Manuel C and gestures for Manuel C to look toward the overhead.] I multiplied eight by eight, which gave me sixty-four, and the only thing for Lapp's that I did was sixteen so what I did, I put sixteen out of sixty-four. That was my fraction.	Brielle presents a correct method for identifying the correct fraction name for Lapp's land. Ms. E may be responding to off-task behavior exhibited by Manuel C when she looks at him and gestures to the overhead. **[Dis-b]**
34:10	Ms. E	All right, so for today, we have these answers. Thank you. We have sixteen over sixteen, four over four, one fourth, and sixteen out of sixty-four. So far we don't have a consensus or a name for Lapp, so we'll work on that tomorrow but I want you to finish your names for all of the sections, eighteen and nineteen. All right?	Ms. E does not comment on Brielle's answer but thanks her. Ms. E restates all of the answers that were presented but does not pass judgment on any of the answers. Ms. E assigns homework for the students to identify the fraction names for all of the sections. The class period ends.
End TSMIE [E.01.b.06]			

Discussion of TSMIE [E.01.b.06]

Ms. E initiates this interaction calling all small groups together to present their fractional answer for Lapp's property. Many students are talking during her announcement and she makes at least three attempts to get the attention of the whole class. Some students appear to protest, claiming that they are not finished with their

work and presumably not ready to present their ideas to the whole class. Ms. E responds by offering assurance that they only need to report on the fractional name for Lapp's property. Evidence from prior episodes shows that at least three groups had documented answers for Lapp's property while Ms. E was at their tables.

Throughout this episode, the noise level in the classroom is loud with students talking. Ms. E raises her voice five times to get the attention of the whole class. At 29:47 Ms. E attempts to get the students' attention while apologetically acknowledging that she is interrupting their work. "All right, umm, I can't quite get everyone here, and I understand you're still working on something and I apologize that I'm interrupting, but could you just look one second and tell me if you agree that Lapp's land is… what are you claiming Lapp's land is?"

The first two groups to present their answers, (Felipe and Manuel D/Dave) present faulty reasoning, naming the Lapp's land four fourths and sixteen sixteenths respectively. Ms. E does not make any moves to correct them. She poses a question to the class, asking, "Do you agree? Do you agree?". Most students appear disengaged from the discussion as they continue to write on their papers or talk to their peers. However, N'yette expresses that she disagrees and Ms. E invites her to the overhead projector to "talk to them". Ms. E looks on silently while N'yette attempts to explain the reason why the fraction name for Lapp's land should be one-fourth. Manuel D is not convinced, but when Ms. E announces to the class that she is going to base everyone's grade on Manuel D's answer, sixteen over sixteen, he appears to consider N'yette's reasoning. Manuel D demonstrates a shift in his reasoning and he is correctly able to describe why the fractional name for Lapp's land is one-fourth.

In the last minute of this episode, Ms. E invites Brielle to present her answer, which is correct, sixteen sixty-fourths. Brielle presents a brief explanation by showing that Lapp's property covers sixteen units and the whole Section 18 covers sixty-four units. Ms. E restates each of the four different answers that were presented, but she does not pass judgment on any of them. Ms. E sums up the lesson and assigns homework to conclude this episode and the class period. "So far we don't have a consensus or a name for Lapp, so we'll work on that tomorrow but I want you to finish your names for all of the Sections, 18 and 19."

The class period ends shortly after the conclusion of the last episode (TSMIE [E.01.b.06]) during which four different student groups presented four different answers, two of which are correct. Ms. E does not pass judgment on the answers, yet encourages the students' engagement as she asks students to tell whether they agree or disagree with some of the answers. Ms. E concludes the lesson by restating each of the four answers, "All right, so for today, we have these answers… sixteen over sixteen, four over four, one fourth, and sixteen out of sixty-four. So far we don't have a consensus or a name for Lapp, so we'll work on that tomorrow…" Ms. E then assigns the students homework to finish assigning fractional values to all of the properties within Section 18 and Section 19.

APPENDIX A.2

Ms. E: Classroom observation analysis, "Finding Areas and Other Products"

Day One

The first day of "Finding Areas and Other Products" in Ms. E's class begins with Ms. E posing problem 5.1A to the whole class and asking for a volunteer to present a solution to the problem from the overhead projector. Manuel D volunteers first, followed by Dave and then N'yette each taking a turn explaining their solutions to Problem 5.1 A. Approximately three minutes into the footage, Ms. E stands up and circulates around the classroom, briefly speaking to individual students seated at their desks. It appears that she is attempting to encourage student engagement with the presentations at the front of the room because she can be heard saying, "at least open your notebook" and "do you agree with them [Manuel D and N'yette]? At 4:50 (S) Ms. E makes a brief appearance at the front of the room, and addresses the whole class, "I need to ask you…umm…do you agree with this?" It is inferred that Ms. E is asking for students' feedback on work presented by Manuel D and N'yette. She rearranges the transparencies on the overhead projector and she looks around the classroom as students respond to her question. Student responses are varied, some say they agree while others disagree, yet the responses are not supported by students' reasoning. Ms. E then asks for a show of hands in agreement with the solution and four students raise their hand. Ms. E then returns to the desk amongst the students in the audience, leaving N'yette to explain her solution to the class.

Immediately prior to this episode, N'yette, Tina, Dave and Manuel D are engaged in a disagreement pertaining to the conditions of the mathematical problem, 5.1A. N'yette seems to have misinterpreted the question, thinking that Mr. Sims bought one-third of two-thirds of the brownie pan. Tina, Dave and Manuel D argue with her, apparently trying to correct N'yette. Ms. E is seated in a student desk near the back of the classroom, looking on silently as the students argue in front of the class. At 7:48, N'yette reads the problem from the textbook and then realizes that she has misinterpreted the problem. She starts a new solution by drawing a square, dividing it into three sections using two vertical lines and then she shades two of the three sections.

Begin TSMIE [E.02.a.01] 9:45 – 18:33 (C)			
Participants: Ms. E, Manuel D, Dave, N'yette Ms. E initiates this episode by posing a mathematical question to N'yette about her brownie pan model.			
Time	**Speaker**	**Transcript**	**Descriptions / Comments**
09:45	Ms. E	What if you *did* cut it in half? [N'yette has drawn a model of a square brownie pan, cut into thirds with two sections shaded, showing a representation of two-thirds.]	Ms. E's tone of voice may be inferred to convey ***curiosity about the students' thinking.*** It is not clear exactly what part of the brownie pan she is referring to. It may be inferred that she is asking what would happen if N'yette cut the whole brownie pan in half horizontally, which is one way to solve this problem. **[Int]**
09:49	N'yette	Like this? [N'yette is off camera]	
09:51	Ms. E	Yes, like you originally said.	Ms. E ***expresses no emotion as she builds on N'yette's earlier mathematical idea\.*** **[Blds]**
09:52	Dave	You would have to pay more, he would have to pay more, it will be it will be. [Dave and Manuel D turn around	Many students answer Ms. E's question at once, raising their voices over each other. They sound ***excited*** as they

		in their seats and answer Ms. E in unison.]	emphasize key words "he would pay *less*" or "he would pay *more*", expressing opposing ideas.
09:54	Manuel D	No he would pay less, no he would pay less.	Manuel D, Dave, Lorissa and Tora all turn around to address Ms. E. They appear ***excited*** about their mathematical ideas as their voices are raised and they are all talking at once.
09:55	N'yette	It would be, it would be four out of….	
09:57	Manuel D	He would pay, half, he would pay four dollars.	
10:00	Dave	Yea but he'll pay less…cause the bigger brownies	
10:01	Manuel D	He will pay less.	
10:03	N'yette	So the fract…so the fraction if I cut, so the fraction if I cut it in half would be four-sixths.	
10:05	Dave	If you cut it into more pieces then the pieces are going to be bigger, and he gonna pay less, but he want more brownies.	
10:11	N'yette	That's….That's what I'm saying but [Ms. E] said what if I cut it in that ….	
10:13	Ms. E	[Ms. E is seated in a student desk in the middle of the classroom. She raises her voice and addresses the class.] But what if I… leave it like that for a second, *what if* I wanted to buy half of that. I cut it in half this way.	When Ms. E poses a hypothetical question to N'yette it may be inferred that she expresses ***curiosity*** about N'yette's reasoning "if" she cuts the brownie pan in half "this way". **[Int]**
10:21	N'yette	Half of this?	
10:21	Ms. E	Yeah!	
10:24	Ms. E	Couldn't I do it that way? [Manuel D and Dave both say "Yes".] I'm not buying the whole thing, I just want half of it.	Ms. E expresses ***curiosity*** about the fractional value of half of "it". The students seated in front of her block Ms. E's gestures and her language is not clear about which way she wants to cut the pan in half. It may be inferred that Ms. E is asking N'yette to cut the brownie pan in half horizontally; so that the line that cuts the pan in half is perpendicular to the lines that

			cut the pan in thirds. **[Int]**
10:29	N'yette	So this is what was bought, so in reality he did buy two out of six!	N'yette presents correct reasoning after Ms. E's intervention. She appears ***confident*** about her answer, "two out of six" because she sits up and gestures toward the overhead projector and then writes what may be inferred to be her answer on the overhead projector.
10:31	Manuel D	So you want half….So isn't that, lemme get, he want half of a half….	
10:36	Ms. E	Could you explain that?	Ms. E's tone of voice is ***calm.***
10:37	Dave	That's four out of six, right there.	
10:39	N'yette	Because if he is buying half, I say in reality he is buying, [N'yette waves her pen at Dave as she speaks] one two out of six, because if I look at it this part is bought, right, unless I split it, so [Ms. E] said you have to split it, well what if I split it, [The camera pans to show the projected image of N'yette's model of the brownie pan, showing two vertical sections shaded in to represent two-thirds, and she has drawn a line horizontally across the middle of the square.] so I split it, its four out of six, if I split it the fraction is four out of six, BOOM. And remember, this whole side was already bought, this side is already bought, and if I split it, it's two out of six.	N'yette indicates that she came to her answer with help from Ms. E who "said you have to split it" which N'yette clearly interpreted as cut horizontally in half. N'yette expresses ***enthusiasm*** that she is ***confident*** about her reasoning. N'yette expresses ***excitement*** in her expression, "BOOM".
11:04	Ms. E	So would I be paying more if I split it in half this way?	Ms. E's tone of voice and language suggest that she is ***curious*** about a possible different answer if the square was split it in half a different way. **[Int]**

11:05	Manuel D	It would be the same….but I don't get, I don't get.	Manuel seems ***puzzled*** because he says, "I don't get, I don't get." **.**
11:06	Dave	You said that like three times already, that's three right there, that's right there. One two three four five six, that's four out of six. Why cut it in half, you don't have to cut it in half.	Dave appears ***impatient*** with Manuel D when he says, "you said that like three times already".
11:15	N'yette	Because [Ms. E] said "What if you cut it in half?".	Dave has questioned why N'yette cut the brownie pan in half and N'yette continues to convey ***confidence*** about her answer, citing Ms. E's intervention to support her reasoning.
11:18	Manuel D	But I don't, but I don't….but look I don't get, I don't get the question that [Ms. E] just said. Wait I don't get the question that [Ms. E] said. [Ms. E approaches the front of the classroom.] Does she want half of the half, or yea it's like?	Manuel D appears ***anxious*** because he does not understand Ms. E's question as he stutters over his words. It may be inferred that Ms. E ***recognizes Manuel D's impasse and mounting frustration*** and is approaching the front of the room in order to intervene and help him. **[Rsi]**
11:18	Dave	That's would of….would of.	
11:27	N'yette	[Ms. E is standing next to N'yette] No she just said cut it in ha- just split it like this so it would be the….	
11:30	Manuel D	No she said what if you cut it what would I have to pay? [Ms. E looks at the overhead projector as Manuel D speaks.]	
11:34	Ms. E	Right, [Ms. E points and nods towards Manuel D] yea, what if, if I cut it like that, what would I have to pay?	Ms. E ***acknowledges and affirms Manuel D's mathematical question*** because she nods and says, "right". **[Val]**

11:38	Manuel D	The question is, my question is, what half do you want, like do you want this half or half of that half.	
11:46	Ms. E	Doesn't matter I can take this half, that half, that half, [Ms. E gestures towards the overhead projector] doesn't matter.	Ms. E's tone of voice is ***reassuring.***
11:48	Manuel D	I mean, we want one….	
11:51	N'yette	'Cause she still take….	
11:52	Ms. E	If I want one piece only, would that be the same as saying that I want one half of this?	It may be inferred that Ms. E intervenes in an effort to clarify the mathematical question and possible alleviate Manuel D's mounting ***frustration.*** **[Rsi]**
11:56	N'yette and Manuel D	No.	
11:57	Manuel D	No, that would be four dollars.	
11:58	Ms. E	So, I'm….I'm gonna say I want half of that. [Ms. E gestures towards the overhead projector with both hands.] It doesn't matter which half… [Ms. E raises her voice to address Betty] [Betty]! I want half [Ms. E gestures towards the overhead projector, more forcefully] of this, it doesn't matter which half, it doesn't matter how you cut it. I just want half of that.	It may be inferred that Ms. E perceives Betty's behavior as off-task and she is trying to re-engage Betty because Ms. E raises her voice, calls Betty's name, then repeats the mathematical question. **[Dis-b]** Ms. E's gestures are exaggerated and her voice is raised which may be inferred to show that she is ***angry.*** She may be ***dissatisfied*** with Betty's off-task behavior, or she may be ***dissatisfied*** with Manuel D because he does not concede to her mathematical reasoning. **[Dis-m]**
	Manuel D	Yes.	

	Ms. E	Would that be the same as the other one? How much would I be paying if I get this lower half? [Ms. E is standing in front of the class asking the questions, then walks towards the back of the room]	
12:15	Dave	You would pay like, you would pay about….	The students are all talking over one another and the tone in the room is collective ***excitement.*** It appears that each student feels strongly about his or her own opinion and that the students are not listening to each other. Ms. E expresses curiosity about N'yette's reasoning: "it's the same as the other one". **[Int]**
12:16	N'yette	Same thing.	
12:17	Manuel D	Eight dollars, it's the same as the other one.	
12:19	Ms. E	How come it's the same thing?	
12:21	Dave	It gotta be lower, it gotta be lower…its gotta be lower number.	
12:23	N'yette	That's what I said…that's what…	
12:23	Manuel D	He's gonna pay the same thing because you're breaking one third.	
12:26	N'yette	I, I pay eight dollars…	
12:30	Ms. E	Well could you show me that I'll pay the same? [Ms. E walks away from the overhead projector toward the back of the classroom.]	Ms. E appears to be challenging the students to present a valid proof that each half of the remaining brownies would cost the same amount of money.
12:26	Dave	So she'll be paying about four…twenty-five, four twenty-five.	
12:30	N'yette	I said that, he said that….	
12:32	Manuel D	Look, now I agree with [N'yette], Because look, look, now I'm a do, look…	
12:35	Ms. E	They said they're saying it's the same thing, [Ms. E is seated in the back of the classroom and she is talking to the students seated near her.] [N'yette's] saying it's the same thing…[N'yette], what...what are you saying are the same thing?	

12:43	N'yette	I'm saying either way you look at it, even though I looked at it, I …I said [Dave's] was right because…	
12:48	Ms. E	Wait wait wait…there's two, there are two things. [Manuel D is standing in front of his desk.]	
12:50	N'yette	Okay, I said [Dave's] was right because if did, if I just did mine the way I did mine but I still had to do it the same way, and I looked at it, I'm still getting the same questions as I did before just in a different way.	
12:52	Manuel D	You he is paying the same thing. Ok so we paying the same thing.	
13:06	Ms. E	Okay so….	
13:07	N'yette	And we, we paying the same thing for either or.	N'yette shows that there can be more than one way to solve a problem. Both her solution and Dave's solution are the same despite the fact that they got the answers in a "different way".
13:11	Ms. E	Okay, so could you show us that we paying the same thing?	Ms. E exhibits ***curiosity*** as she redirects N'yette to answer her question. **[Int]**
13:14	Dave	So one, one, one-half of two-thirds equals to one third [Manuel D approaches the overhead projector and N'yette sits in Manuel D's seat.]	
13:19	Ms. E	Noah what do you think?	Ms. E expresses ***interest*** in Noah's opinion. It is inferred that Ms. E's intent is to engage Noah engage with the mathematical discussion because he has not spoken yet during this interaction. **[Int, Endis]**
13:19	Noah	They are the same.	
13:20	Dave	I don't get where the heck she get the two-sixths from.	
13:22	N'yette	'cause I broke it down.	
13:26	Dave	I know where she gets the two-	

		sixths from because she cut it in half….but he didn't say nothing about buying.	
13:29	Manuel D	Like, like, like there's no Mr…..[inaudible] and this is what someone else got right?	
13:40		Loudspeaker interruption.	
13:45	Manuel D	[Inaudible due to loudspeaker] And it ain't gotta be the same….'cause like half of that eight dollars would be four dollars. Cause you're still paying the same, cause it's two brownies. So it would be like eight dollars.	
13:52	Tina	Ohhhh I see now!	
13:55	Ms. E	Where is the four dollars, tell us again?	Ms. E conveys interest as she is asking for clarification. **[Int]**
13:57	Manuel D	Like right here.	Dave appears to defend himself "I said four dollars" and then engages in a brief disagreement with Manuel D.
14:00	Dave	I said four dollars.	
14:02	Manuel D	No you said that it would be smaller.	
14:04	Dave	Yea four dollars.	
14:06	Manuel D	No you just said [inaudible]…like that	
14:08	Dave	[Dave turns around in his seat and asks Eli.] Didn't I say four dollars?	
14:10	Ms. E	What's four dollars?	Ms. E either does not hear Dave and Manuel D's disagreement or she chooses to ignore it. Conveys ***curiosity*** about what "four dollars" represents. **[Int]**
14:12	N'yette	I agree with him!!	
14:13	Dave	Twenty-four, eight, twelve, sixteen, twenty, twenty-four .	
14:17	Ms. E	You saying you see something, you see the four dollars…[Tora and Jackie are in the background of this camera view, hitting one another and smiling.]	Ms. E appears ***curious*** about the student's reasoning, but it is not clear to whom she is directing the question. **[Int]**

14:24	Manuel D	[Manuel D is seated in front of the overhead projector and Felipe is standing next to Manuel D.] He's saying that one brownie is four because four times six is…is twenty-four.	Manuel D interprets Dave's reasoning.
14:29	Ms. E	Could you write that down? [Tora], [Jackie], could you move away from each other?	Ms. E appears to be ***calm*** as she asks Manuel D to "write that down". Ms. E ***calmly*** addresses the off-task behavior of Jackie and Tora by asking them to move away from one another. **[Dis-b]**
14:30	Dave	[Dave approaches the overhead projector, stands in front of Felipe and gestures to the overhead.] Exactly….. look here, look, look, four, sixteen, twelve, four, eight, twelve, uhh, sixteen, twenty, twenty-four.	Dave appears to want to argue with Manuel D. Manuel D seems to be presenting a valid proof, explaining why Mr. Sims would spend eight dollars on brownies. Despite Manuel D's clear explanation, Dave questions why Mr. Sims would pay eight dollars. Meanwhile, Ms. E is seen in the background, squatting down next to Tora's desk, possibly addressing the recent off-task behavior exhibited by Tora and Jackie. **[Dis-b]**
14:44	Manuel D	Hold up, that's what I did, so he's buying, you're saying that one, that four dollars would be one sixth, but Mr., Mr. Sims is buying two-sixths, ummm…two-sixths.	
14:58	Dave	Exactly! He's buying two-sixths, he's buying two-sixths, then how much he gotta if he's buying two-sixths.	
15:03	Manuel D	He's will be paying eight dollars, cause…	
15:05	Dave	How eight dollars?	
15:07	Manuel D	Because look look look look…..	
15:07	Dave	Look, Look look, eight, sixteen, twenty four..	
15:09	Manuel D	Because if one sixth, because cause if four, look, the four right there that's one, and six, lets just say it's six, so that's one brownie, four is one brownie.	
15:23	N'yette	Either way you look at it…these two…either way you look at it	

		the fraction would be [N'yette is now standing a the projector].	
15:27	Dave	If four is one brownie, one brownie costs four dollars!	
15:30	Manuel D	Exactly, but he's buying two brownies because, he's buying ummm a third….THIS!	Manuel D is ***excited*** as he defends his mathematical reasoning.
15:36	Dave	The pieces are bigger, that's why he is only buying two brownies.	
15:38	Manuel D	Look, this is a third, he is buying two brownies.	
15:40	N'yette	…Even if the pieces is small….	
15:41	Manuel D	He wants look…	
15:43	Dave	No if the pieces are smaller it would be one third.	
15:44	Manuel D	Look then he's buying this…this is what he's buying right, and she broke it down like this to pay four dollars, for each brownie. So at least you buying the same amount, like he's buying one third, or two sixths, either way	Manuel D demonstrates that he understands that the fractional amount purchased is one-third, which is equivalent to two-sixths.
16:00	Ms. E	Okay so there is a lot of different conversations going on in the front. [Ms. E cannot be seen from this camera angle, but her voice can be heard.]	Ms. E's tone of voice and language indicate that she ***disapproves of*** students' behavior as they all talk at once. **[Dis-b]**
16:05		BEEEEEEEEEEEPPPP	
16:08	Dave	But where do you get the four-sixths from? [Manuel D – "four sixths?", Felipe walks up to the projector.]	
16:10	Dave	Look…..	
16:13	Felipe	Yea where do you get four-sixths from?	
16:18	Manuel D	Four-sixths. Cause four, cause the four represents one…	
16:22	Dave	So the same thing as two sixths equals one third.	Dave identifies two equivalent fractions.
16:22	Ms. E	[Mrs. E can be heard off camera, speaking to students who are seated out of visual range.] So what do you think about what they are saying? Have you	Ms. E appears to be trying to engage some students in the discussion at the front of the room. **[Endis]**

		followed it enough to get an opinion?	
16:25	N'yette	Basically….	
16:32	Manuel D	Okay, we got it… [Manuel D pushes back from the overhead projector and stands up.]	Manuel D expresses ***confidence*** in their solution.
16:40	Ms. E	I think we have questions from the back…	
16:46	Manuel D	What's your question?	
16:48	Ms. E	[Felipe], [Felipe] has a question….what's your question?	Ms. E's tone of voice is raised initially. She tries to ***encourage student discourse*** by quietly asking Felipe to pose his question. It appears that Felipe does not have a question, or may be too shy to ask his question aloud. **[Endis]**
16:53	Ms. E	Oh I thought you had a question. [Ms. E approaches the overhead projector.] So what did we agree on, what did we disagree on.	Ms. E appears to feel ***surprised*** that Felipe does not have a question.
16:57	Dave	Two sixths equals one third.	
16:58	Ms. E	Say that again. [Ms. E is standing at the overhead projector.]	
16:59	Many students	Two-sixths equals one third.	
17:05	Ms. E	Yes? No? YES! Okay, let's go back to what we were doing earlier…So and I want to see this on your notebook. [Ms. E is seated at the overhead projector.]	Ms. E ***does not appear to convey any emotion*** as she appears to attempt to get a sense of a class consensus.
17:23	Manuel D	I got that on my notebook…	
17:25	Ms. E	HEY! We have in five, and four…ok two thirds of a pan, Mr. Sims bought go ahead [Dave]…Yes.	It appears that Ms. E is using a countdown from five to get the students' attention because some students begin shushing each other after Ms. E starts counting.
17:40	Dave	A third.	

17:41	Ms. E	One third YES?…and he is going to pay.	Ms. E appears to be making efforts to provide closure to this discussion by asking for the class to agree that Mr. Sims bought one-third of the brownie pan.
17:44	Dave	Eight dollars.	
17:50	Ms. E	…what proves that he is paying eight dollars, can you give me some number sentences that prove this?	Ms. E asks for students' reasoning that Mr. Sims pays eight dollars for one-third of the whole brownie pan.
17:57	Dave	'Cause there's two-thirds left.	
17:57	Ms. E	Okay, go ahead…[Dave]	Ms. E encourages Dave to speak, but Dave concedes to Manuel D. **[Endis]**
17:57	Dave	Let him go. [Dave points to Manuel D]	
18:00	Manuel D	[Manuel D has his hand raised.] How about this…that umm since it's you take the three the numbers and you multiply by eight and we get twenty-four, so you're paying eight because eight is one fourth.	
18:15	Ms. E	Uhhh uh…..eight dollars times three dollars…	
18:16	Manuel D	Time three….eight dollars is like a third of twenty -four….a third of twenty-four.	
18:22	Ms. E	Eight dollars is…a third of twenty-four…do we agree do we disagree are we done with this one? [Ms. E writes on the overhead transparency as she speaks.]	Ms. E asks for the students' approval of Manuel D's answer.
18:33	Class	YESSS!!! [class agreeing]	Many students answer in unison.
End TSMIE [E.02.a.01]			

Discussion of TSMIE [E.02.a.01]

At the beginning of this episode, Ms. E appears to recognize that N'yette has presented faulty reasoning and is struggling to understand how to interpret the conditions of the problem. Ms. E intervenes, building on N'yette's mathematical idea asking asking

what would happen if she cut the brownie pan in half. Throughout the episode, Ms. E's tone is calm and she does not pass judgment about the students' path of mathematical thinking. However when the noise level in the classroom becomes loud, Ms. E appears to recognize that students may be off-task. At this point, she becomes agitated stating that there are "a lot of conversations going on".

In this episode, N'yette, Dave and Manuel D appear to be engaged in the mathematical discussion. Since these students are standing at the front of the classroom, it may be inferred that Ms. E expects the students who are seated to attend to the discussion (or to participate in the discussion). However, when Ms. E calls on Noah and Felipe to express their opinion or question, Noah gives a two-word answer, "I agree" and Felipe withdraws his question (to Ms. E's surprise).

At the end of the episode, Ms. E appears to act without emotion as she writes down the students' final answers to the task on the overhead projector. The episode is concluded when Ms. E asks the class, "are we done with this one?" and the class responds with a collective and enthusiastic, "Yes!"

Immediately prior to this next episode, Ms. E is walking between two rows of student desks, looking at the students' notebooks. She makes brief inaudible comments to each individual student while looking at their notebooks. It is inferred that Ms. E is checking to see if the students have written the answers to the previous problem. Tora and Manuel D each present a different incorrect answer to Problem 5.1B: "Paulo's aunt Serena asked to buy ¾ of what was left in another pan. The pan was half full. How much of a whole pan did Aunt Serena buy? How much did she pay?"

At 20:00, Lorissa is seated at the overhead projector. She has drawn a model of a

square brownie pan showing the fraction, three fourths. Ms. E asked the whole class, "Do you agree?" Some students state that they "totally disagree". Lorissa then erases her model.

At 21:32, Tora draws a model of a square brownie pan showing the fraction three-sixths. Ms. E looks at Tora's model then she reiterates the problem "I want to buy three fourths of half a pan".

At 22:45, Manuel D asks if he can "go up" to the overhead projector. She responds, "Yeah, of course!". Manuel D draws a square divided into thirds. He labels two of the three sections, "bought", and, "Mr. Sims", respectively.

Ms. E initiates this interaction because she is recognizes that the students are having difficulty representing this three-fourths of one half. Ms. E had been acting as a bystander, witnessing each student's attempt to solve the task. She intervenes after she asks the students "do you agree with this?" and very few students respond to her question. Ms. B approaches the overhead projector and addresses the whole class.

Begin TSMIE [E.02.a.02] [23:00 - 25:42, (C)]			
Participants: Ms. E, whole class			
Time	**Speaker**	**Transcript**	**Descriptions / Comments**
23:00	Ms. E	[Manuel D is seated at the overhead projector. Eli and Tora are on either side of Manuel D, looking at the transparency.] All right so do you agree with this? [Ms. E is standing next to the overhead projector with her hands on her hips. She addresses the whole class when she speaks.]	It appears that Manuel D, Eli and Tora are engaged with the mathematics because they are looking at the transparency, writing and discussing. It appears that Ms. E is ***dissatisfied*** with the faulty reasoning presented by Tora and by Manuel D and has initiated an intervention by asking the class, "do you agree with this?" Ms. E appears to be ***concerned*** that the students have presented faulty

		What's going on? Do you agree with this? [Ms. E points at the overhead projector.] Huh? We haven't decided on one yet. [Ms. E stands silently with her hands on her hips, looking at the overhead projector.]	reasoning.
23:19	Dave	Balling..	
23:24	Ms. E	All right, so we have Mr. Sims in the particular pan. Is it the same as what you did? [Ms. E raises her hand and raises her voice to address the class.] How many were able to finish B?	Ms. E appears to express ***no emotion*** at this point, and it may be inferred that she is unsure about how to proceed in guiding the students towards recognizing the correct answer.
23:30	Manuel D	I was…	
23:35	Ms. E	All right, so I want you to look at what's going on here.	Ms. E's tone of voice appears to convey ***no emotion.***
23:40	Dave	She paid nine bucks.	Dave's comment goes unnoticed, but he does present a correct answer for the amount of money Aunt Serena will spend on brownies.
23:43	Ms. E	Mr. Sims and Serena in the same pan. [Ms. E rubs the palms of her hands together.]	Ms. E appears to be trying to provide hints to the students by pointing out that Mr. Sims and Aunt Serena are represented in the same pan according to Manuel D's model.
23:44	Dave	She paid nine dollars.	Once again, Dave's comment goes unnoticed.
23:50	Ms. E	[Ms. E claps her hands together.] Ok…let's look at the problem again. [Ms. E picks up a textbook from the desk next to the overhead projector and looks out at the whole class.] Anybody read the	It may be inferred that Ms. E is asking the students to revisit the problem because she is ***disappointed*** with the students' mathematical progress at this point. Ms. E's tone of voice and facial expressions appear to convey no emotion, yet her actions of sending students back to their desks and

		problem for me please?	essentially taking control of the conversation that may lead one to infer her ***disappointment.*** It appears that Ms. E wants the students to be able to recognize that they have misinterpreted the problem and she may be ***hopeful*** that reading the statement of the task will help the students towards mathematical success.
23:55	Tina	Yes he did pay nine dollars. [Tina nods her head and looks toward Dave and Manuel D.] Oh…um..I can read it. [Tina leans forward to get her notebook.]	Tina agrees with Dave.
23:57	Ms. E	Hold on, stop. Hold on hold on hold on…have a seat first. [Ms. E walks to the light switch and turns on the light.] [Lorissa] have a seat.	Ms. E interrupts Tina and waits for all students to be seated and quiet before Tina continues reading the problem.
24:09	Dave	Turn off the light….Balling!	
24:11	Ms. E	Have…a…seat, have a seat. All right, letter B, someone please read the problem.	When Ms. E repeats "have a seat" for the third time, she emphasizes each word and her tone of voice conveys ***anger*** at the students who are not heeding her request.
24:21	Tina	[Ms. E stands in front of Tina's desk as Tina reads the statement of the task from the math textbook.] Paulo's aunt Serena asked to buy three-fourths of what was left in another pan. The pan was half full. How much of the whole pan did aunt Serena buy? How	

		much did she pay? [Tina looks up at Ms. E.]	
24:32	Ms. E	Ok. [Ms. E is leaning on the water bottle next to the overhead projector, holding a math book in her hands. She looks at the students, pauses and then asks:] What's going on in this problem? Yes. [Ms. E responds to Dave who has his hand raised.]	Ms. E's tone of voice is ***calm*** when she addresses the class. She appears to feel ***relaxed.***
24:37	Dave	They want to know that…Ummm… [Dave looks at his book] that she had to buy three fourths of what was left in the other pan so she bought, she bought three eighths and she paid nine dollars. [Dave looks up towards Ms. E.]	Dave presents part of the key information stated in the task. He also presents the correct answer here both parts of the mathematical task. Ms. E does not acknowledge Dave's answers to the task.
24:53	Ms. E	Ok, three fourths of what was left in another pan. My big question is this. Is this the same pan as Mr. Sims pan? [Ms. E is leaning on the overhead projector and taps her hand on the projector. She looks around the classroom.]	It may be inferred that Ms. E recognized that some students were using the same model used in Problem 5.1A. Instead, they should be using a new model to satisfy the conditions of Problem 5.1B. Ms. E is ***not smiling*** as she looks around the classroom from where she is standing. Ms. E appears to be ***serious*** about getting all of the students to understand that Aunt Serena is buying brownies from a *different* pan than Mr. Sims.

25:04	Many students	No.	Ms. E's expresses ***confusion*** in her tone of voice and language when she does not understand the students' models on the overhead projector. Ms. E's tone appears to become argumentative after N'yette indicates that she agrees with the incorrect drawing, "a little bit".
25:06	Ms. E	Then….[Ms. E looks at the overhead projector] I don't understand this. [Ms. E looks at the students. There is a pause, students do not respond to her question.] So what's going on here? Do you agree with this?	
25:18	N'yette	A little bit.	
25:18	Ms. E	No! Because what you're saying here…what is this drawing telling us?	
25:22	Manuel D	That she is trying to buy three fourths of what's left in the pan Mr. Sims bought,	Manuel D shows that he correctly understands the statement of the task that Aunt Serena is buying brownies from a different pan than Mr. Sims. Ms. E's language conveys ***curiosity*** while her tone of voice and facial expression remain emotionless.
25:28	Ms. E	But is there a problem there?	
25:30	Manuel D	Yes, because it's saying that half of another pan.	
25:34	Ms. E	Ooooh, ok. Does that change the problem at all? Does that change the problem?	Ms. E appears enlightented by Manuel D's acknowledges that she understands Manuel D's explanation. Ms. E then conveys ***curiosity*** about the details of the task when she asks, "does that change the problem?"
25:42	Manuel D	Can I go now… [Manuel D has his hand raised, then he throws his hand down.]	Manuel D appears ***eager*** to present his ideas and it may be inferred that his is getting ***impatient*** when he throws his arm down.
25:42	Tina	Yes.	
25:42	Ms. E	All right so lemme see letter B. How many people were able to do letter B yesterday? Lemme see. [Approximately six students raise their hands] So not, not a lot then. So lets do letter B then. Let's do letter B. And this is another pan. This is *another* pan.	Ms. E assigns the task for the students to complete independently at their seats. She begins to circulate through the classroom. Many students begin shuffling papers as if they are getting organized to begin letter B.

25:44	Dave	[Ms. E] can I use the bathroom?	
25:45	Ms. E	Okay, let's do letter B. It's a different pan, it's a different pan all together. So let's see a different pan. [Dave leaves the classroom.]	
End TSMIE [E.02.a.02]			

Discussion of TSMIE [E.02.a.02]

Immediately prior to this episode, three students had presented faulty reasoning when attempting to begin solving the mathematical task at the overhead projector. When this episode begins, Ms. E does not engage in a discussion with each individual student about their faulty reasoning. Instead, she states that she is confused and calls on Tina who volunteers to re-read the problem in the book. Ms. E then leads a discussion from the overhead projector and it appears that only a handful of students engage. Dave, Manuel D and Tina each evidence correct understanding of the problem even before Tina reads the problem from the book. For these three students, it does not appear necessary to read the problem one more time. The other students in the classroom look on silently and do not engage in the conversation.

This episode is an example of a teacher trying to address many students with a whole class discussion, yet evidences low levels of student engagement. It is included in the analysis because it evidences a distinct shift in student engagement when the teacher is trying to lead a discussion. In the first moments of this episode three students demonstrate that they understand the problem and have presented a mathematically correct answer. Nonetheless, Ms. E assigns the problem to the whole class as if it is new to all. During the final moments of this episode, Manuel D exhibits frustration possibly

because he has not had an opportunity to share his answer with the class. Also after Ms. E says, "let's do letter B", Dave asks to go to the bathroom and leaves the room.

Immediately subsequent to this episode, it appears that less than half of the students in the room are engaged in solving the problem. Many students are looking around the classroom, sitting back in their chairs, requesting to go to the bathroom, and walking about the classroom.

Ms. E is now seated with a group of five girls when the camera comes to focus on this interaction. Prior to this episode, at 25:42, Ms. E indicates that the students were to work on "letter B" and she circulated throughout the room, briefly speaking with small groups one at a time. Ms. E has come to sit at a student desk with this group of girls. It appears that some of the girls are disengaged with the mathematics and Ms. E is determined to reengage them with the mathematics by asking them questions about the conditions of the mathematical problem.

Begin TSMIE [E.02.a.03] 30:28- 32:35 (C)			
Participants: Ms. E, Tora, three female students			
Time	**Speaker**	**Transcript**	**Descriptions / Comments**
30:28	Ms. E	So what is still on the pan?	
30:39	Ms. E	[Ms. E is seated at a student desk, leaning toward Tora. The camera is positioned behind Ms. E, so her facial expressions are not always visible during this episode.] The pan is….read the problem again, read the problem again…read the problem one more time, read the problem one more time. Tell me …what is on the pan.	Ms. E's tone of voice is ***gentle,*** yet she appears to be ***anxiously*** trying to engage the students with the mathematics. She repeatedly asks the girls to read the problem again. The students do not show evidence that they are reading the problem again despite Ms. E's repeated requests.

30:47	Student wearing headband	The pan…The pan was three fourths when it was half full….	
30:50	Ms. E	So is it three fourths or a half full? [Ms. E waves her hand back and forth.]	Ms. E's tone of voice may be inferred to convey that she is ***impatient*** with the students' behavioral engagement. [Dis-b]
30:52	Student wearing headband	Three-fourths.	
30:53	Ms. E	Three-fourths.	Ms. E does not validate nor negate the students' idea when she repeats the student's claim.
30:54	(someone)	Three fourths was left of the other pan…[The camera pans to show a different group of students working at the overhead projector.]	
31:00	Ms. E	So is it three fourths or *half*? What was on the pan?	It may be inferred that Ms. E recognizes the students' faulty answer and is ***dissatisfied*** with the student's answer, "three-fourths". She repeats her question, emphasizing "half". **[Dis-m]** Ms. E's tone of voice may be inferred to show that she is still ***anxious*** that the students are not engaged with the mathematics. **[Dis-b]**
31:00	Student	Half.	
31:03	Ms. E	What was on the pan.	
31:05	Student	The…what's on the pan is…(inaudible)	
31:08	Ms. E	Let's read it again …Paulo's aunt Serena [student yells looking at Tora] asked to buy…[Ms. E raises her eyebrows] [Ms. E looks back and forth between the students as she reads. She points her finger at the text while she reads	After repeated (and failed) requests, Ms. E begins to read the problem with a raised voice. It appears that these students are not engaged with the mathematics because they are looking around, smirking, talking out of turn. Ms. E appears ***angered*** by the lack of student engagement with the mathematics and she reads the

		the problem. Ms. E looks at Tora and says:] [Tora] …asked to buy three fourths of what was left on the pan. [Tora furrows her brow. Headband girl smirks.] The pan was half full. So show me a pan that's half full. Show me a pan that is half full….	problem aloud. **[Dis-b]** After Ms. E reads the problem, she tells the students the answer to her earlier question, "So what is still on the pan?" [30:28]
31:30		[Three students begin to write in their notebooks.]	
31:38	Ms. E	[Ms. E folds her arms on the desk and looks from one student to the next. The student seated next to Ms. E is looking toward the front of the classroom.] Show me a pan that is half full…[Ms. E gestures towards the student sitting next to her and the student starts writing.]	
31:38	Tora	I could do it, I could do it, I could do it, ……	Tora's tone of voice indicates ***excitement*** that she can show Ms. E a pan that is "half full"**.**
31:39	Ms. E	Lemme see? A pan that is half full?	
31:41	Tora	Shut up, [Jackie].	
31:42	Ms. E	Lemme see it? A pan that is half full. Lemme see?	Ms. E appears ***curious*** to see Tora's representation of "a pan that is half full". **[Int]**
31:48	Tora	Right here right here right here.	
31:49	Ms. E	Okay, Do you agree with this? [Ms. E looks at the student who is leaning on the desk where Ms. E is seated.]	
31:51	Tora	Yes!	Tora exhibits ***confidence*** in her answer as she sits up straight and loudly declares "yes!" to Ms. E's question.

31:52	Ms. E	Half full?	
31:53	Tora	Uh um.	
31:53	Ms. E	Could you mark it with a fraction name?	
31:56	Tora	[Tora looks down at her notebook, then looks up at Ms. E.] A half!	
31:59	Ms. E	All right, and and Aunt Serena came and said what? Aunt Serena came and said what? I want to buy… [Ms. E waves her hand, she is leaning on the desktop with both elbows, looking at the students.]	It may be inferred that Ms. E is becoming ***impatient*** with the girls' level of mathematical engagement and the their level of understanding of the conditions of the task. **[Dis-b]**
32:08	Tora	I want to buy...[Tora hesitates then Ms. E taps many times on a student's textbook. Tora looks at the textbook then answers Ms. E's question.] Three fourths!	Ms. E's gesture of rapidly tapping on the student's textbook may be inferred to show Ms. E's increasing level of ***impatience*** as the girls make slow progress towards completing this task. **[Dis-b]**
32:12	Ms. E	Three fourths of …	
32:15	Tora	A half pan.	

32:17	Ms. E	Alright, so how would you show me that? I want to buy three fourths - of - half.	Ms. E conveys ***satisfaction*** when she acknowledges that Tora has correctly described the conditions of the task. **[Val]**
32:19	Tora	This is a fourth, this is a fourth right? [Tora has her notebook on her lap and looks up at Ms. E.]	
32:21	Ms. E	But this is gone! I want to buy three fourths of this! [Ms. E leans in and taps on Tora's notebook, pointing to Tora's model of the brownie pan.]	It appears that Ms. E and Tora disagree about Tora's model of the brownie pan. Initially, Ms. E argues with Tora about which side of the pan represents the brownies that have been purchased. Ms. E then asks Tora to interpret her model for her, asking which side of the pan represents what is left. Ms. E appears ***curious*** about Tora's understanding of "what's left" in the brownie pan. **[Int]**
32:25	Tora	Oh, this is gone? No this is left, this is left. [Tora points to the model of a "half full" brownie pan that she has drawn in her notebook.]	
32:35	Ms. E	This is what's left, right, what's left, this is left? [The camera pans to a different group of students.]	
End TSMIE [E.02.a.03]			

Discussion of TSMIE [E.02.a.03]

Ms. E joins this group of five girls approximately five minutes after assigning Problem 5.1B for the students to work on in their small groups. It appears that these students have made no mathematical progress in the past five minutes because the students have nothing written in their notebooks.

During the first moments of this episode, Ms. E exhibits patience and appears ready to help the students get started on the problem. Yet, she becomes increasingly angered by the lack of student engagement. The girls in this group are talking out of turn, yelling across the room, and looking around. Ms. E asks the girls to identify how full the brownie pan was by asking them to read the problem again. Ms. E then appears to become angry as she struggles to engage the students in a discussion about the mathematical problem. Ms. E repeats the same questions multiple times. At [32:08] Ms. E taps rapidly on a student's math book, likely as a reminder for Tora to re-read the problem.

At 21:10, approximately ten minutes prior to this episode, Ms. E indicated that this problem had been assigned the day before. The fact that the students appear so unfamiliar with the problem may prompt Ms. E to feel anger.

It may be inferred that Tora is the most engaged with the mathematics because she is speaking to Ms. E and writing in her notebook. The four other girls' do not appear to be engaged because throughout the episode, they look around, and they do not respond to Ms. E's questions. This episode is concluded when the camera pans away to focus on a different group of students. Therefore, student engagement immediately subsequent to this episode cannot be determined because it is not evidenced on any video data.

Discussion: Ms. E Cycle Two, Day One

The first day of "Finding Areas and Other Products" in Ms. E's class begins with N'yette, Manuel D and Dave at the front of the room as they present their solutions for Problem 5.1 A on the overhead projector. Ms. E demonstrates curiosity and / or interest in the students' ideas during the lesson and acts as a mediator, asking leading questions as the students present their ideas. Ms. E intervenes when students disagree by posing questions about the solution to the problem. Throughout the class period, many students who are seated in the audience appear to be disengaged because they have side conversations with their neighbors and the researcher observed the teacher circulating through the classroom addressing perceived off-task behavior. Ms. E appears to attempt to address student disengagement by circulating among the students in the audience, posing questions such as, "are you writing this down?" or "what do you think about their solution?" Meanwhile, the same students; N'yette, Manuel D, Dave contribute the majority of ideas to mathematical discussions. Ms. E exhibits annoyance, agitation or frustration with student engagement approximately eleven times during the three episodes described above.

Ms. E: Classroom observation analysis, "Finding Areas and Other Products"

Day Two

Four teacher/student(s) mathematical interaction episodes (TSMIE), totaling approximately forty-one minutes were evidenced in the video data collected on Day Two of "Finding Areas and Other Products" during a 45-minute class period in Ms. E's classroom.

The Problem / Materials / Seating Arrangement

At the beginning of this class period, Ms. E stands at the front of the classroom next to the overhead projector. The students are seated facing the front of the classroom in individual desks, arranged in rows.

Ms. E begins the lesson by telling the students to reconstruct their mathematical ideas from memory based on what they recall from the previous day's lesson. She specifically instructs the students not to look back at their work from the previous day's lesson. This is significant because later on in the lesson, Ms. E intervenes when Tora experiences impasse and explicitly tells Tora to refer to her notes from the previous day.

> "Um, I need you to reconstruct your idea from yesterday from memory.
> [Ms. E points to her own head]
> We don't have um, I don't want you to look at what you did yesterday, but I want you to try to remember how, what you drew and as you're drawing them, could you talk to us about what you drew, yesterday."
> ["Finding Areas and Other Products" Day 2, 0:23, C]

Ms. E then calls on Dave who volunteers to read the statement of the task aloud to the whole class.

In this episode, Ms. E asks Tora to reconstruct her solution to the mathematical problem, which the students worked on during the previous lesson. The task involved naming fractional pieces of a square brownie from which portions had been sold during a bake sale. The complete task is provided in Chapter Four, Section 4.2.2.

Tora presents her solution from the overhead projector as Ms. E sits close by on the top of an empty student desk. Ms. E facilitates student discourse as she asks for input from Tora's peers. When Tora appears to struggle with her explanation, Ms. E asks for the other students to help her. Although some of Tora's peers attempt to help, others exhibit off-task behavior, which Ms. E explicitly addresses. Ms. E emphasizes the importance of peer-to-peer respect when addressing students who appear to be off-task throughout this episode.

BEGIN TSMIE [E.02.b.01] 3:11 – 13:19(C)			
Participants: Ms. E, Tora, Jenny, Dave, Whole class			
Time	**Speaker**	**Transcript**	**Descriptions / Comments**
3:11	Ms. E	Ok so, we are trying to answer letter B. [Ms. E is reading the details of the task from Manuel D's book, which is positioned on his desk.] We're trying to answer letter B, which says Paolo and Paula, no, Paolo's Aunt Serena asked to buy three-fourths of what was left in the pan. [Tora], could you start that? [Tora has her head down on the desk.] Could you start your answer from yesterday? [Tora stands	Ms. E tells the whole class that they will focus on "letter B" then reiterates some of the details from the statement of the task. Her tone of voice is authoritative as she introduces the lesson. Ms. E then expresses ***interest*** in Tora's mathematical reasoning when she asks Tora to start her presentation. **[Int]**

		up and walks to the overhead projector.]	
3:28	Dave	She couldn't even…	
3:31	Ms. E	See, she's going to walk us through what she did yesterday. [Ms. E sits on the top of an empty student desk positioned at the front of the room.]	Ms. E appears to model ***respect*** for Tora as she explains to the class that Tora will explain her reasoning to the class. **[Res, Ptp]**
3:35	Dave	She's gonna draw and then be like "I don't know what to say."	Dave makes a comment that may be considered disrespectful towards Tora, teasing her about being shy in front of the class.
3:37	Tora	I don't know.	Tora expresses doubt about the mathematics.
3:41	Ms. E	[Ms. E swings her legs as she sits on the desktop. Her body is turned to face Tora at the overhead projector.] I want you to follow what [Tora] is going to say. [Ms. E glances sideways at the students in the audience.] Shhh…shhh. [Eli], you have to be very respectful. If [Tora] needs to do that, let her do it. [Tora draws a square and divides it into six equal pieces.]	Ms. E's body language may suggest that she is ***relaxed*** as she swings her legs. It may be inferred that Ms. E has ***confidence*** in Tora's mathematical ideas as she asks the class to follow Tora's explanation. Ms. E speaks quietly when she speaks directly to Eli about peer-to-peer respect "you have to be very respectful." Ms. E ***respectfully responds to alternative approaches to solving the mathematical problem*** when she says, "If [Tora] needs to do that, let her do it." **[Raa, Res, Ptp]**
4:29	Ms. E	Push it up. [Ms. E gestures for Tora to move the transparency up.] [Tora moves transparency up.] No, the paper. So the pan was half full. [Tora sits and looks at her drawing.]	It appears that Tora may be hesitant to proceed because after completing drawing the model of the brownie pan she looks at her drawing and then rolls her eyes upward. It may be inferred that Ms. E thinks that Tora is experiencing difficulty continuing with the mathematics because she reads part of the task aloud, "So the pan was half full".
4:54	Tora	I don't know. I don't know. [Tora writes on the transparency, then looks out at the class, then at Ms. E.]	Tora appears to have reached a point of impasse.
4:55	Ms. E	Okay… and Aunt Serena	Ms. E's tone of voice is ***calm*** as she

		wants to buy, how much of that? [Ms. E leans towards Manuel D's book from her seat atop the desk.]	expresses ***curiosity*** about how much Aunt Serena wants to buy. **[Int]**
5:01	Tora	Three-fourths. [Tora mouths, "oh" and then writes on the transparency.]	
5:02	Ms. E	Three-fourths of what was on the pan. [Tora draws a new square on the transparency underneath her original square. The original square has been shaded to show three-sixths.]	Ms. E reiterates Tora's words and reminds students of the context of the task when she adds, "of what was on the pan".
5:08	Tora	[Jenny] you have it on your notebook? [Jenny approaches the overhead projector. Jenny divides Tora's new square into four equal sections then into eight equal sections. Jenny then shades in three of the eight sections then returns to her seat.]	It appears that Jenny is coming to help Tora, possibly because they are in the same group.
5:20	Ms. E	Do you have it in a notebook, have it in a notebook, right? So you're saying…	Ms. E appears to recognize Tora's impasse. Contradictory to her original instructions for the students to construct their ideas from memory, Ms. E's suggests for Tora and Jenny to look back in the notes. Ms. E's tone of voice is ***gentle and calm*** as she encourages Tora to proceed. **[Endis]**
5:38	Dave	We can't hear you!	Dave sounds ***impatient*** when he complains that he and other classmates cannot hear her.
5:39	Ms. E	Speak up, speak up. Your normal, just use your normal voice.	Ms. E encourages Tora to use her "normal" voice so that the other students can hear her. Initially, Ms. E raises her voice when she says, "Your normal," and then she sounds ***calm,*** saying, "use your normal voice."
5:44	Tora	(inaudible-explaining what she has on transparency) … Three-fourths and that was	

		four fourths…	
5:51	Ms. E	Ok… So what's your answer? How much Aunt Serena buying? Write it down.	Ms. E's tone of voice conveys ***curiosity*** about Tora's answer. It is unclear what Ms. E is asking for when she says "how much?" **[Int]** Her tone of voice changes to sound ***straightforward and blunt*** when she says, "write it down".
5:55	Tora	Eight dollars?	Tora states that she thinks that Aunt Serena spends eight dollars on the brownies, which is incorrect. The correct answer is nine dollars. Tora does ***not appear confident*** in her answer because she presents it in the form of a question.
5:57	Ms. E	Write it down. What is the fractional part? What do you call that part that she's buying? [Ms. E approaches the chalkboard and erases something in the space where the overhead is projecting.]	Ms. E does not validate nor does she negate Tora's incorrect answer. Ms. E bluntly repeats "write it down". Ms. E then inquires with a ***curious*** tone, "what is the fractional part? What do you call that part that she is buying?" **[Int]** This may lead to the inference that Ms. E's previous question, at 5:51, "How much?" was intended to mean what fraction of the pan.
6:02	Tora	Three out of four. [Tora speaks as she writes on the transparency.]	Tora responds to Ms. E's question and her answer is incorrect. The correct answer for the fractional part that Aunt Serena is buying is three-eighths.
6:03	Dave	She could talk all loud when she talking to her friends, but she not talkin' loud now.	Dave criticizes Tora for speaking quietly during her presentation.
6:07	Tora	Be quiet. [Tora looks toward Dave when she speaks.]	Tora responds to Dave's criticism.
6:10	Manuel D	[Tora], why is it…	
6:11	Tora	Three…He already…But this was already gone. [Ms. E returns to her seat on top of the desk near Manuel D and Dave.]	
6:18	Ms. E	Shhh…	Ms. E has not responded to Tora's

		[Ms. E looks out towards the class, her brows are furrowed.] [Jenny approaches the overhead projector and consults with Tora.] Ok why did you change your answer? How is it now three eighths? [Jenny] empty your mouth. [Jenny is standing next to Tora, writing on the transparency. Tora and Jenny are having a conversation, which is inaudible.]	incorrect answer, "three out of four". Ms. E's utterance of "shhh..", her facial expression and body language suggest that she is ***annoyed*** with the students in the audience for being noisy. **[Dis-b]** Tora has changed her answer from three-fourths to three-eighths and Ms. E appears to be ***curious*** about why Tora chose to change her answer when she asks, "Ok why did you change your answer? How is it now three eighths?" **[Int]** It may be inferred that Tora's discussion with Jenny influenced Tora's decision to change her answer.
6:37	Lorissa	'Cause it was eight pieces.	
6:40	Ms. E	That' okay, go ahead. That's okay. That's okay, [Tora] go ahead. [Ms. E is nodding her head.]	Ms. E's tone of voice and language is ***encouraging*** and ***supportive***. Ms. E appears to be encouraging Tora about something, when she says, "That's okay [Tora], go ahead." **[Endis]**
6:44	Tora	You're in my group. This is eight pieces and…	It may be inferred that Tora has divided the whole brownie pan into eight pieces as a consequence of help from Jenny and Lorissa.
6:47	Ms. E	And how much did you say she's going to pay?	Ms. E appears ***curious*** about the amount of money Tora thinks that Aunt Serena will pay for the brownies. **[Int]**
6:49	Tora	Eight dollars.	Tora offers an incorrect answer for the amount of money Aunt Serena is going to pay for three-eighths of the brownie. The whole pan is worth twenty-four dollars and three-eighths of twenty-four dollars is nine dollars.
6:50	Ms. E	How is it eight dollars? [Lorissa moves to sit directly in the desk in front of the overhead projector, facing Tora.]	Ms. E questions Tora, appearing ***curious*** about her answer eight dollars. **[Int]** Ms. E's tone of voice is ***emotionless,*** she does not validate nor negate Tora's answer. It may be inferred that Lorissa is moving closer to Tora because they are in the

			same group and Lorissa is hoping to help Tora.
6:52	Tora	Because like…	
6:56	Manuel D	How is it eight dollars, [Tora]? [Manuel D is leaning on his desk with his head resting on his right hand.]	Manuel exhibits ***curiosity*** because he is leaning forwards and he asks Tora why the answer is "eight dollars".
6:57	Dave	It's not eight dollars. You have the fractional part right here…	
7:01	Manuel D	How is it eight dollars?	
7:06	Tora	Twenty-four…Half of it is twelve, right here, this is twelve dollars. Divide twelve by six dollars. [Ms. E looks toward the students in the audience as Tora speaks.]	Students in the audience can be heard talking which may be perceived as off-task behavior. It is possible that Ms. E looks toward the audience because the chatter distracts her.
7:19	Ms. E	Speak up little bit more, try again, sorry. [Ms. E looks at the projected image of Tora's model.]	Ms. E asks Tora to raise her voice and to start over so that all students in the class may hear her explanation. Ms. E ***demonstrates respect*** for Tora apologizing for interrupting. **[Ptp]**
7:21	Dave	Talk louder!	Dave asks Tora to speak louder for the third time since the start of this episode.
7:23	Ms. E	Would you mark them please?	Ms. E's tone of voice is ***inquisitive*** when she asks Tora to mark the sections on the brownie pan model, which is projected on the wall. **[Int]**
7:26	Tora	This whole thing is twenty-four dollars, [Tora gestures to the whole brownie pan.]	
7:27	Ms. E	Ok…	
7:31	Tora	This is twelve dollars, this is twelve dollars, plus two.	
7:36	Ms. E	Where is the two?	Ms. E's tone of voice is blunt and her language suggests that she is ***curious*** about where the two is represented on the overhead. **[Int]**
7:36	Tora	Right here,	It is possible that Ms. E recognizes and is

		[Ms. E's posture changes as she slumps back and then looks away from Tora towards the students in the audience.]	***disappointed*** that Tora is presenting incorrect reasoning. **[Dis-m]**
7:39	Manuel D	What number would you say is right here?	
7:41	Lorissa	She says that's twelve, then that's six…	
7:42	Manuel D	I don't get it.	
	Lorissa	And then that's two.	
7:46	Ms. E	So that part is twelve, and which part …are you naming six? [Ms. E looks at the projected image as Tora speaks.]	Ms. E may be trying to understand Tora's reasoning by reiterating "so that part is twelve". She conveys ***curiosity*** when she asks, "which part …are you naming six?" **[Int]**
7:50	Tora	This, those two, this whole thing over here is twelve, these two…[Tora points to transparency with the pen.]	Tora appears to be referring to each half of the brownie pan when she says, "this whole thing over here is twelve, these two."
7:58	Ms. E	So what do you think? [Ms. E looks at the students in the audience and pauses after she asks the question.]	It appears that Ms. E is asking the students in the audience for their opinion of Tora's answer. Ms. E's tone of voice is ***calm*** and she speaks quietly.
7:59	Tora	Eight dollars.	
8:05	Ms. E	And how is it eight dollars again? Where is…[Ms. E looks at the projected image on the wall.]	Ms. E asks Tora to explain her answer again. Although Tora has presented faulty reasoning, Ms. E has not given any feedback indicating her own opinion of Tora's answer. Her tone of voice conveys ***curiosity,*** but also ***lacks enthusiasm.*** **[Int]**
8:08	Tora	Six plus two…is si-…is eight. [Tora looks at Ms. E.]	
8:10	Tora	There's two there	
8:12	Ms. E	Six and two…circle the six and the two that you're adding please. That's six and that's two.	Ms. E asks Tora to circle the six and the two that she is adding, possibly to provide clarification. It appears that Ms. E may be attempting to stage Tora's work so that either Tora or another classmate will recognize Tora's faulty reasoning.
8:15	Dave	Why are you adding them?	

8:21	Ms. E	Good question, why are you…that's a good question, why are you adding them?	Ms. E ***validates*** Dave's question by saying, "good question" and repeating Dave's question to Tora, "why are you adding them?" Ms. E appears to be ***excited by*** Dave's question. **[Val]**
8:23	Tora	Cause I'm not gonna multiply them…or divide it.	It appears that Tora may be mocking the question, "why are you adding them [six plus two]?" Tora has not provided a valid reason for adding six plus two.
8:25	Ms. E	Huh? But why are you adding? That's a good question.	Ms. E ***validates*** Dave's question again as she repeats it and says, "that's a good question." **[Val]**
8:28	Tora	Cause you gotta ask for the money, you're not gonna subtract them. [Tora looks at Ms. E as she speaks.]	It appears that Tora has not provided a mathematically sound argument.
8:37	Ms. E	Do you agree? Do you agree? [Ms. E leans towards the class.]	Ms. E sounds ***curious*** about the opinions of the other classmates when she asks, "Do you agree? Do you agree?" **[Int, Endis]** It appears that Ms. E may be directing her question at a particular student, but it is not clear from this camera angle specifically where she is looking.
8:39	Manuel D	No I don't agree. [Manuel has his arms folded and his head is resting on his arms.]	
8:41	Ms. E	Ok so…	
8:43	Dave	How come you can't talk like that now? [Dave points towards Tora with his hand.]	Dave appears to be ***frustrated*** with Tora for not speaking loudly during her presentation. This is at least the third time that Dave has commented about Tora being too quiet.
8:47	Tora	I am talkin' like that.	
8:50	Ms. E	I'm getting very impatient. [Ms. E is leaning forward with her forearms resting on her legs. She pauses and shakes her head "no" as she looks out towards the class.]	Ms. E clearly but quietly states that she is getting ***impatient.*** "I am getting very impatient." Her tone of voice sounds emotionless and it does not appear that she is speaking to anyone in particular. It may be inferred that she is ***angry*** because it many students are talking at once. Students are not either not

			engaged with the mathematics or they are not listening to each other (or both). **[Dis-b, Dis-m]**
8:54	Tora	Be quiet. [Tora is looking at Dave.]	
8:56	Ms. E	The people in the back cannot hear you. [Ms. E points to the back of the classroom with her hand. Her voice is raised and her brow is furrowed as she shakes her head.]	Ms. E's tone of voice and furrowed brow may be inferred to show her ***frustration.*** Ms. E tells Tora that she needs to speak loudly so that the people in the back of the classroom can hear her. **[Dis-b]**

8:57	Manuel D	Talk loud!	
8:59	Ms. E	[Ms. E looks directly at Tora and nods as she says,] Talk like you always talk. [Leon] has to hear what you're saying. [Ms. E points and leans towards the back of the classroom.] It's very important what you're saying.	Ms. E speaks loudly as she appears to coach Tora to speak loud enough for all to hear her. Many voices of the students can be heard, but their words are inaudible. Possible off-task behavior as students may be losing interest in the presentation. Ms. E points towards the back of the classroom where Leon is sitting, indicating that he needs to be able to hear Tora's voice. Ms. E ***validates*** Tora's reasoning and reiterates to Tora that all students in the class must be able to hear her. Ms. E places emphasis on "very important" she says, "It's very important what you're saying." **[Val]**
9:08	Tora	There's twenty-four in the whole, twenty-four dollars in the whole thing. There's twelve right here, so twelve plus twelve plus twelve equals twenty-four. [As Tora explains her reasoning, Ms. E looks out toward the class.]	
9:15	Manuel D	Can you do the math so we can see you?	
9:16	Dave	Twelve plus twelve plus twelve.	Dave appears to be mocking Tora for incorrectly stating an extra twelve when she said, "so twelve plus twelve plus twelve".
9:19	Tora	I said twelve plus twelve.	Tora appears to be defending herself in response to Dave's comment.
		(kids laugh)	
9:22	Ms. E	Alright, uh, [Ms. E looks at Dave then towards the students sitting in the audience. As she speaks, she emphasizes each word with a nod of the head.] I would like for you to	Ms. E expresses ***disapproval of*** student off-task behavior as she responds to the laughter and mocking that is coming from students in the audience. Ms. E's tone of voice and language is ***firm.*** She speaks slowly and loudly as she

		respectfully listen first to what Tora is saying, interrupting only when it's necessary. Please don't cut her sentences. If you have any question, and she's not talking, you can ask her a question. [Ms. E is shaking her head, no.] No mocking, no mocking [Ms. E states this with emphasis.]. And…um, wait for her to finish her sentences. [Ms. E glances at Dave and then returns her attention to Tora.]	explains her expectations of peer-to-peer respect to the class. Ms. E provides specific examples when she tells the students that she does not want them to interrupt Tora when she says, "Please don't cut her sentences." Ms. E also indicates that questions from the audience are welcome only "necessary" and when "she's not talking". **[Dis-b]** Ms. E's intervention here is focused on the social interactions. Dave has made Tora repeated comments about Tora's presentation style since the beginning of this episode.
9:46	Tora	There's twenty-four things, twenty-four dollars in the whole pan, [Ms. E looks at the students in the audience and gestures from her own ear toward a student.] in the whole pan. There's twelve plus twelve equals twenty-four, and this was already bought, but there was none, half of the pan was bought and since the...there was…wait… [Tora hesitates and looks at the model on the transparency.]	Ms. E may be gesturing to a student who may be ***exhibiting off-task behavior*** to listen to Tora. **[Dis-b]**
10:02	Ms. E	So there's six on the top… and where's the other six?	Ms. E appears to be ***curious*** about Tora's reasoning when she reiterates part of Tora's reasoning. Ms. E then inquires about where the other six appears in the diagram. **[Int]**
10:07	Lorissa	This part is ten [Lorissa points to the transparency with her pen.], it's supposed to equal twelve.	
10:13	Ms. E	What's going on? [Ms. E scans the audience with her eyes.]	Ms. E asks a non-specific question. It is unclear whether her question refers to the mathematics or to the student chatter that can be heard in

			the background. Lorissa intervenes pointing towards Tora's model. She is leaning forward across her desk, pointing to the overhead transparency. Ms. E asks Lorissa and Tora to explain to her what they are discussing.
10:14	Lorissa	She add the six here when there's ten here…nine dollars [Ms. E is looking at the overhead projector. Her facial expression is emotionless.]	
10:18	Manuel D	Tora, couldn't it have been eight dollars because, because three, because there's three pieces in each one	Manuel D disagrees with Tora.
10:24	Ms. E	So what's going on? [Ms. E sits up straight.]	For the second time in 11 seconds, Ms. E asks the non-specific question, "what's going on?" She appears ***curious*** about Tora and Manuel D's reasoning. **[Int]**
Cecilia's camera pans to show the students sitting in the audience from 10:23 – 10:49 Manuel C and Noah exhibit off-task behavior, discussing the whereabouts of a lollipop.			
10:45	Ms. E	Alright, so now it's what?	Ms. E is ***curious*** about Tora and Manuel D's answer. **[Int]**
10:48	Tora	Nine.	Tora has changed her answer from eight dollars to nine dollars.
10:48	Ms. E	Nine dollars…Aunt Serena is paying nine dollars. Could you repeat how it became nine, because earlier it was eight. [Ms. E is speaking to Tora as she looks at the students seated in the audience. Ms. E leans forward and grips the edge of the desk.]	Ms. E repeats Tora's answer, "nine dollars". Ms. E then raises her voice and asks Tora to repeat her reasoning for changing her answer to nine dollars instead of eight dollars. Ms. E seems ***curious*** about why Tora changed her answer from eight dollars to nine dollars. **[Int]**
10:55	Manuel D	Can we just explain?	Manuel D appears to be ***impatient*** with the mathematical progress and appeals to Ms. E asking "Can we just explain?". It is unclear whether or not Ms. E answers him, but he

			slams his hand down on the desk, possibly because he is ***frustrated*** that he could not have an opportunity to explain.
10:58	Tora	You come here. [Tora is looking towards Jenny.]	It appears that Tora has asked Jenny to join her at the overhead. It is likely that Tora is feeling pressured and/or uncertain about why the answer is nine dollars. It is likely that she is appealing to Jenny for help to explain why the answer is nine dollars.
10:59	Ms. E	[Jenny], could you come please. [Ms. E gestures towards the front of the room.] It's very important to hear what you say. [Ms. E is smiling.] [Jenny does not come to the overhead.] Students in the audience can be heard saying "come on, go".]	It appears that Ms. E recognizes that Tora is experiencing impasse. Ms. E intervenes by ***encouraging student discourse***. **[Endis]** Ms. E then ***validates*** Jenny by saying, "It's very important to hear what you say." Ms. E is smiling when she says this and this is the first time during this class period that Ms. E is captured smiling. **[Val]** It appears that some students in the audience may be becoming impatient as they say, "come on, go.", etc.]
11:09	Tora	If its twenty-four dollars in the whole pan, [Tora looks at Ms. E] half a pan is twelve, and since twelve plus twelve is twenty-four, you take this twelve right here, … half of twelve is six, three plus three equals .. six. and six plus three equals nine. [Tora points to the model she has drawn as she speaks and looks at Ms. E after stating each operation. When she is finished with her explanation, Tora sits back in her seat.]	Tora appears to portion the brownie pan into sections, naming each section with a dollar value. Tora is able to provide correct reasoning for the cost of the portion of Aunt Serena's brownie.
11:28	Ms. E	Alright, so if you have to write your answers in sentence form, what would it be? The questions	Ms. E does not provide overt validation of Tora's explanation. But she ***encourages*** Tora to present

		are… [reading from Manuel D's book] "How much of a whole pan did Aunt Serena buy?" What's your answer?	the answer in "in sentence form". Ms. E's tone of voice is ***blunt.*** **[Endis]**
11:40	Tora	How do you spell Serena	
11:42	Ms. E	S-e-r-e-n-a. Please copy this in your notebook if you don't have your notebook… I should see this on your notebook, if you don't have it yet.	Ms. E provides the correct spelling for "Serena" and then raises her voice and announces loudly to the class that the students should copy Tora's answer into their notebook. This action may ***validate*** Tora's answer of nine dollars. **[Val]**
11:50	Tora	S-e…. [looks at Ms. E]	
11:52	Ms. E	r-e-n-a. Shhh. [Ms. E's brows are furrowed.] I'm trying to figure out where the noise is coming from, now I see that it's from you [Ms. E looks in Noah's direction and points at him].	Ms. E shushes the class. She explicitly conveys ***disapproval of*** Noah's off task behavior when she says, "I'm trying to figure out where the noise is coming from, now I see that it's from you." **[Dis-b]**
		[Manuel D and Dave laugh at Noah.]	
12:04	Ms. E	Open your notebook. Open your notebook. And if anything, copy the answers into your notebook. Shh…Gentlemen, [to Manuel D and Dave] I'm sure that when you get up there, you'll wanna be listened to, right? Shh…	Ms. E tells Noah to open his notebook and copy Tora's answers. She may be indicating that she recognizes that Noah is not engaged. While she is asking Noah to become engaged, her expectations appear to be low when she suggests that he copy Tora's answer "if anything". Ms. E addresses Manuel D and Dave. "I'm sure that when you get up there, you'll wanna be listened to, right?" She expresses ***disappointment*** in the boys' behavior while ***promoting peer-to-peer respect.*** **[Dis-b, Ptp]**
12:15	Manuel D	Well, can I get up there already?	It appears that Manuel D is anxious to participate.
12:19	Ms. E	Shhh…We're about to be done with [Tora's] explanation. [Ms. E looks at the projected	Ms. E shushes the students in the audience ***encouraging peer-to-peer respect.*** Many students can be heard

		image of Tora's work and reads,] Aunt Serena bought nine dollars of the pan. And how much, um, how much in fraction is it? She bought …[Tora gestures towards the transparency and Ms. E reads her answer] three-eighths of the pan.	speaking off camera. **[Ptp]** Ms. E speaks quickly, when she says, "We're about to be done with [Tora's] explanation." Ms. E appears to be redirecting the students' attention to Tora's solution by reading what Tora has written on the overhead projector. "Aunt Serena bought nine dollars of the pan." Ms. E conveys ***curiosity*** about the fraction name for the amount that Aunt Serena purchased. **[Int]**
12:35	Lorissa	Here, its from [Jenny] [student gives paper to Tora].	
12:38	Dave	You ain't gonna be able to see it on the transparency.	
12:39	Ms. E	And, could you read what she has there?	
12:44	Dave	There's no transparency.	
12:45	Tora	Half of twelve is two.	Tora presents faulty reasoning.
12:48	Ms. E	Half of twelve is two. Do you agree?	Ms. E's facial expression shows ***no emotion*** and she does not agree with nor negate Tora's answer. Ms. E reiterates Tora's incorrect statement then directs a question toward the class, "Do you agree?"
12:52	Dave	Half of twelve is two? Half of twelve is six! [Manuel D looks at Dave and laughs.]	Dave's language and tone of voice appears to ***mock*** Tora's idea. Manuel D is also laughing at Tora's statement, "Half of twelve is two."
12:56	Ms. E	Half of twelve is…	
12:58	Tora	six!	
12:59	Ms. E	You can definitely, you can definitely ask, but laughing at somebody's idea is not a respectful thing to do.	Ms. E ***disapproves of*** Dave's and Manuel D's laughter in response to Tora's statement, "Half of twelve is two". She reprimands the boys and ***encourages peer-to-peer respect***. **[Dis-b, Ptp]**
13:07	Manuel D	… being respectful	
13:08	Ms. E	Well, I want you to be respectful in my class.	Ms. E's tone of voice is firm and her delivery of the statement, "I want

		Alright, could somebody explain their um, ideas from yesterday? Would you like? Oh, [Felipe], would you like to go up?	you to be respectful in my class" is delivered with complete certainty. Ms. E is making a clear statement that ***she expects students to respect one another.*** **[Dis-b, Ptp]**
End TSMIE [E.02.b.01]			

Discussion of TSMIE [E.02.b.01]

Ms. E invites Tora to present her solution to the problem that the students worked on during the previous day's class. Tora is charged with answering two questions; "How much of a whole pan did Aunt Serena buy?" and "How much did she pay?" Tora's head was down on her desk before Ms. E called on her. Because Tora did not willingly volunteer to present her solution and because Tora struggles in her explanation, it may be inferred that Tora was not prepared to discuss her ideas. Ms. E sits on the top of an empty student desk next to the overhead projector and remains seated there while Tora presents her solution.

Initially, Tora presents incorrect answers to each of the two questions posed in the statement of the task. First, she states that the fractional part of the pan purchased by Aunt Serena is "three out of four" [6:02, C]. The correct answer is three-eighths. Ms. E does not respond to Tora's answer, but students in the audience, including Manuel D question her answer. Jenny and Lorissa, both members of Tora's group intervene and Tora subsequently changes her answer to three-eighths. At 6:18 [C], Ms. E questions Tora about the change, "Ok why did you change your answer? How is it now three eighths?" Tora's answer provides an acceptable answer because Ms. E moves onto the next question, how much would Aunt Serena pay for the brownies.

Tora offers an incorrect answer for the second part of the problem, stating that Aunt Serena would spend "Eight dollars" [6:49, C] on the brownies. The correct answer is nine dollars. Ms. E questions Tora, "How is it eight dollars?" [6:50, C] Ms. E follows Tora's explanation as Tora write on the diagram, which unfortunately cannot be seen on any of the video.

Once again, Lorissa intervenes which appears to prompt Tora to change her answer to nine dollars, which is the correct answer. Ms. E once again does not pass judgment on Tora's answer, but asks her to explain her reasoning. Tora appeals to Jenny for help, but Jenny refuses to come to the overhead even after an additional request made by Ms. E. Tora is able to explain correct reasoning without assistance. "If it's twenty-four dollars in the whole pan, half a pan is twelve, and since twelve plus twelve is twenty-four, you take this twelve right here, … half of twelve is six, three plus three equals .. six. and six plus three equals nine. [11:09, C].

Ms. E does not provide overt validation of Tora's final answer, however she does prompt Tora to write her answer in sentence form, suggesting that her answer is complete. Ms. E also suggests that the students in the audience write down Tora's answer in their notebooks, which may be inferred as further validation. "Alright, so if you have to write your answers in sentence form, what would it be?" [11:28, C]. It is likely that Tora made mathematical progress during this episode because she had the support of her peers (Jenny and Lorissa).

Aside from the mathematics, this episode indicates evidence of the teacher supporting Tora's efforts to present her mathematical ideas to the whole class. Many times during this episode, Tora appears timid, speaking too quietly for the students in the

audience to hear. Dave repeatedly mocks Tora throughout the episode, suggesting that Tora can talk loud when she is in a social setting with her friends but that she is too quiet when presenting to the class. Tora exhibits low mathematical self-efficacy as she frequently looks at Ms. E for affirmation. It is also possible that she is feeling intimidated being in the spotlight, especially considering the frequent taunting by Dave. Ms. E attempts to provide encouragement, repeatedly telling Tora that her (mathematical) ideas are important.

Throughout the episode, Ms. E makes repeated attempts to address off-task behavior exhibited by the students in the audience. Through her tone of voice and her language, Ms. E conveys that she expects peer-to-peer respect in her classroom. When Eli, Dave and Manuel D interrupt and/or mock Tora's presentation Ms. E comments about the importance of respect. "you have to be very respectful."(3:41)

Immediately subsequent to this episode, Ms. E continues to facilitate a whole class discussion lead by Manuel D about the next part of the task. Some students remain engaged with the mathematics.

This episode begins when Manuel D approaches the overhead projector to describe his group's solution to the following mathematical problem: "Paulo's aunt Serena asked to buy three-fourths of what was left in another [square brownie] pan. The pan was half full. How much of a whole pan did Aunt Serena buy?"

Manuel D sits on a stool in front of the class as he begins to present the solution on the overhead projector. Ms. E is seated on the desk top of a student desk, adjacent to the overhead projector. During the episode, Manuel D appears to struggle with the mathematics and is joined by members of his group including Dave and N'yette.

Begin TSMIE [E.02.b.02] 12:39- 38:51 (L and C)			
Participants: Manuel D, N'yette, Ms. E, Dave			
Time	**Speaker**	**Transcript**	**Descriptions / Comments**
12:39 (L)	Ms. E	[Ms. E addresses Tora and Dave who are talking to each other.] Here you go again…Enough! Oh [Jackie] do me a favor… could you get those two new students uh books from the closet… [Ms. E speaks to Manuel D]…Walk us through what you're doing? Shhh…Shhh… Tell us what you're doing [Manuel D is writing on the overhead transparency and Ms. E is looking at the projected image on the wall as she speaks.]	Ms. E intervenes, saying, "Here you go again…Enough…" to Tora and Dave. It may be inferred that Tora and Dave are exhibiting behavior that is perceived to be ***off-task*** by Ms. E. Ms. E expresses ***disappointment*** in Tora and Dave's behavior when she says "Enough!" The new students that Ms. E refers to are not identified. Ms. E expresses ***interest and curiosity*** in Manuel D's mathematical thinking when she says "Walk us through what you're doing." Manuel has been writing on the overhead projector for approximately one minute.

			He is not speaking. Students in the audience can be heard talking and Ms. E addresses shushes them.
13:25	Student	Show it, show it, show it!	
13:28	Ms. E	And speak up please, so everyone in the room can hear you. [Ms. E looks out towards the audience as she speaks. Ms. E is seated on the desktop and is holding the sides of the desk with her hands.]	Ms. E's tone of voice is ***direct.*** She is not smiling. She is leaning in and it may be inferred that Ms. E is waiting for Manuel D to describe his mathematical solution to the task. She ***encourages open discourse*** when she says, "speak up please, so everyone in the room can hear you".
13:38	Manuel D	So…that…um…according to the question [Manuel D looks back at the overhead projector screen]…this	It may be inferred that Manuel is feeling unsure because he hesitates.
13:44	Ms. E	Speak up! [Ms. E speaks to Manuel D, she momentarily leaves her seat to adjust something on the overhead projector.] Now is the time to speak up. Yes, I heard you a lot earlier.	Ms. E's tone of voice and language may be inferred to convey that she is ***angry*** as she speaks loudly and reminds Manuel D for the third time to "speak up" as he presents his answer. Ms. E uses sarcasm when she says, "I heard you a lot earlier." Ms. E is likely referring to the episode when Manuel [and Dave] badgered Tora for speaking too quietly throughout her presentation. Students can be heard in the background complaining that Manuel D is not speaking loud enough.
13:48	Tora	We can't hear you…that's what you did to me…that's what you did to me.	Tora appears to be expressing ***anger*** towards Manuel D for his

			disrespectful behavior during her presentation.
13:50	Ms. E	[Ms. E speaks to the students in the audience] It does not mean that you can do it to him now. I want you to show him respect. Shh…[Tora], I would like you to… [Many students are talking at once.]	Ms. E then lowers her voice and sounds ***stern*** when she speaks to Tora, "It does not mean that you can do it to him now." Ms. E is not smiling and her expression is ***serious.*** Ms. E states her expectation of ***peer-to-peer respect***.
14:00	Male student seated behind Tora	Shut up! [Slams fist on the table.]	The student sounds very ***impatient*** and ***angry.***
14:01	Mrs. E	That's enough…[Ms. E points her hand towards the audience.]	Ms. E expresses ***disapproval of students' off-task behavior.*** Ms. E appears to be trying to gain control of the class.
14:03	Ms. E	The more you say something to somebody, the more we can't finish. [Ms. E crosses her leg and crosses her arms.]	Ms. E appears to feel ***frustrated*** with student behavior when she indicates that their off-task talking is preventing the class from making mathematical progress.
14:06	Manuel D	The question says that this, this fraction over here… [Manuel D looks at the overhead projector as he speaks.]	
14:10	Ms. E	Can you hear, [Leon]? [Ms. E looks towards the students seated in the audience.]	Ms. E ***encourages open discourse*** and ***peer-to-peer respect*** when she asks Leon, who is seated in the back of the classroom if he can hear what Manuel D is saying. **[Ptp]**
14:13	Leon	No	
14:15	Ms. E	[Jerry] said he can't hear, [Jerry] said he can't hear	Ms. E first quietly states, "[Jerry] said he can't hear" directly to Manuel D. She quickly repeats herself with a tone, which conveys

			annoyance when she repeats, "[Jerry] cannot hear."
14:17	Manuel D	According to the question….this fraction was…[Manuel D looks up at the audience, he is smiling and his eyebrows are raised when he speaks directly to Jerry.] Ms. E: "Good." Can you hear me now? [Manuel D has drawn a square cut into sixteen equal pieces and he has shaded eight of the pieces.]	Manuel D's tone of voice and facial expression convey ***annoyance*** and ***sarcasm*** when he asks his classmates whether they can hear him. Ms. E ***affirms*** Manuel D's presentation volume.
14:28	Ms. E	Half full… [The projected image on the wall shows Manuel D's models. He has drawn two squares, each divided into sixteen relatively equal pieces in a four by four grid. One of the squares has a curvy line drawn through it. The other square, has eight sections shaded in.]	Ms. E restates part of the task statement.
14:29	Manuel D	So, so… [Manuel D looks up towards the projected image on the wall, then looks back at the overhead projector.]	It appears that Manuel D struggling to explain the solution to the task.
14:35	Ms. E	Help, help help…(students talking)	Ms. E ***encourages mathematical discourse***. It may be inferred that Ms. E recognizes that Manuel D has reached a moment of impasse when she says, "help, help, help". **[Endis]**
14:36	Dave	It's three eighths	
14:37	Manuel D	I forgot what is three eighths …[Manuel D looks at Dave, smiling and laughing.]	Manuel appears to be appealing to Dave for help. Dave is in Manuel D's group.

14:38	Dave	You said you forgot what? [Dave approaches the overhead projector then briefly returns to his seat to get a piece of paper.]	Dave appears to be asking for clarification from Manuel D and then returns to his seat to get a piece of his work.
14:57	Ms. E	Turn the light on…the light. From whatever was left, Aunt Serena is going to buy three-fourths… three-fourths of what was left.	It may be inferred that Ms. E recognizes that Manuel D is struggling with the mathematics. Ms. E intervenes by restating part of the task statement, placing emphasis on the words, "three-fourths". Her tone of voice is ***direct.***
15:12	N'yette	So you cross out three.	N'yette can be heard off-camera.
15:14	Ms. E	[N'yette], can you come and help. [Dave, Male student, and N'yette, now surround Manuel D at the overhead projector. All four students appear to be working cooperatively on the mathematical task.]	Ms. E responds to N'yette's comment and she ***encourages student mathematical discourse*** as she suggests that N'yette join Manuel and Dave at the overhead projector. **[Endis]**
15:22	Ms. E	Shh… [Felipe].	Ms. E appears to be addressing general off-task behavior when she shushes the class, then she specifically calls on Felipe who may be talking when Ms. E does not want him to. Her tone of voice conveys a ***warning.***
		[N'yette, Manuel D and Dave are all standing at the overhead discussing the problem. The specific discussion is inaudible. Felipe and another male student approach the overhead projector.]	
15:47	Ms. E	That, that one you're cross-…[N'yette], the one you just shaded, what's that?	Ms. E's voice is raised and her language conveys ***curiosity***, prompting N'yette

		[Ms. E is looking at the overhead projector.]	to explain her reasoning for shading a portion of the model.
15:55	N'yette	This one is the half that's already gone.	N'yette answers Ms. E stating "This one is the half that's already gone [purchased]."
15:57	Ms. E	That part is already gone. [Ms. E's eyebrows are raised and she is leaning towards the students in the audience.]	Ms. E repeats N'yette's statement loudly for the students seated in the audience. Ms. E appears ***serious.***
15:59	N'yette	Yeah.	
16:00	Ms. E	You see that? [Ms. E looks at the students in the audience.] That part they're saying is already gone. So now…	Ms. E appears to be interpreting the model that has been drawn at the overhead projector for the students seated in the audience. She expresses ***interest*** in the model possibly to ***promote student engagement*** for those who are not directly involved in the presentation of the solution of the task. **[Int, rp]**
16:04	N'yette	And these three…	
16:06	Ms. E	Where's the three? [Felipe walks back to his seat.]	Ms. E expresses ***interest,*** asking N'yette for clarification of part of her solution. **[Int]**
16:07	N'yette	Aunt Serena bought these three.	
16:09	Student	Yes.	
16:10	N'yette	And then…and that's the part that was already…yeah this was left…no, this…	
16:19	Manuel D	This is what was left. This is the part that was already bought. [Manuel D points at the overhead projector. He is speaking to N'yette.]	
16:28	Ms. E	[Dan returns to his seat.] So how much did Aunt Serena buy? [Ms. E is scratching the side of	Ms. E's voice is raised. She is not smiling. It may be inferred that she is feeling ***confused*** and ***disappointed***

		her face as she looks up at the projected image of the students' work. The students have drawn a model that represents three-sixteenths.]	because the students have produced a model that represents three-sixteenths, yet the correct answer is three-eighths.
16:35	Student	Three…out of eight	
16:39	Ms. E	Three out of eight…Wait a minute. Class, do you agree that… [Ms. E begins to approach the overhead projector.]	Ms. E repeats the given answer then she expresses ***doubt*** when she asks the class if they agree with the answer, "three out of eight" as the fractional piece of the brownie purchased by Aunt Serena.
16:45	Manuel D	[Ms. E]? [Manuel D has his hand raised.]	
16:46	Ms. E	Yes? [Ms. E approaches the overhead projector and has her hand on her hip.]	Ms. E is responding to Manuel D's request for her attention. She does not look at Manuel D when he speaks.
16:49	Manuel D	I was having second thoughts…I thought it was …sixteenths.	Manuel D expresses that he has ***doubts*** about the answer, stating that he "thought it was sixteenths." It may be inferred that Manuel's doubts are a consequence of the incorrect representation on the overhead projector coupled with Ms. E's expression of doubt.
16:51	Ms. E	Look, one, two, three…one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen…[Ms. E counts the individual squares drawn in the students' model of the brownie pan on the transparency. The model shows a square partitioned into sixteen equal sections. Eight of the sections on the left side of the model are shaded as well as three of the sections on the right side.]	Ms. E does not acknowledge Manuel D's "second thoughts". She appears ***determined*** to engage the students in understanding why their model does not align with their answer. She counts the three shaded squares in the model of the brownie pan and then counts all sixteen squares, which represent the number of sections in the whole

			brownie pan model.
17:01	N'yette	We can't count that cause they're already gone. [N'yette is shaking her head no gesturing towards the model on the transparency.]	N'yette passionately defends her answer as she leans in toward Ms. E and emphasizes "gone".
17:04	Ms. E	But… [Ms. E returns to her seat on the empty student desk top] …that's part of a whole pan.	Ms. E's tone of voice is ***calm*** and she hesitates as she responds to N'yette. It appears that ***Ms. E disagrees*** with N'yette's reasoning.
17:11	N'yette	Yeah, that's part of a whole pan, but it's not in the pan, it's already gone.	N'yette remains ***steadfast*** in defending her claim that the missing half should not be considered when naming the fractional part purchased by Aunt Serena.
17:17	Ms. E	Alright, could you draw the same drawing below that? What's going on? Can you draw the same drawing below that without shading that first half? [Ms. E raises her eyebrows and is looking at the overhead projector as she speaks. Draw the same drawing. Hold on. [Ms. E puts her hands on the shoulders of two boys who are standing behind Manuel D] …draw the same drawing that you have.	Ms. E does not affirm nor negate N'yette's explanation. It may be inferred that Ms. E intervenes because she is ***displeased*** with N'yette's faulty reasoning. Ms. E intervenes, asking N'yette to draw a new model and she gives N'yette ***explicit instructions*** for how to draw the model. The two boys standing behind Manuel D appear to be having a mathematical discussion because they are looking at a notebook. Ms. E subtly gets their attention by quietly saying, "Hold on" and briefly putting her hands on each of their shoulders.
17:34	Manuel D	[Ms. E],	Manuel appears to be trying

		[Manuel D leans in toward Ms. E] I'm starting to have second thoughts…it's sixteenths	to get Ms. E's attention about his "second thoughts".
17:38	Ms. E	No, hold on I'm still trying to figure out what you're saying with this one. [The camera pans to the projected image of N'yette's new drawing. She draws a square with sixteen "equal" sections.]	Ms. E's appears to ignore Manuel D's suggestion because she interrupts him. Her voice is raised and she sounds ***anxious and curious*** to understand the students' mathematical reasoning about the model showing three-sixteenths that they have named three-eighths.
17:46	N'yette	Ok so, she said without shading.	
17:50	Ms. E	[Dan gets out of his seat and approaches the overhead.] [N'yette has drawn a square divided in to sixteen equal sections.] Alright, alright, now cross out what Aunt Serena bought. [Ms. E is speaking to the students at the overhead, Manuel and N'yette.] No, don't shade out anything,[Ms. E is shaking her head, "no".] just cross out what Aunt Serena bought. [Lina's camera shows that N'yette makes an "x" through three sections of the brownie pan model.] Ok, [Ms. E sits back and looks towards the students in the audience and asks,] what do you say that part is, that Aunt Serena bought? That they are saying.	Ms. E ***imposes her own mathematical ideas*** giving specific instructions to N'yette about how she wants her to draw the new brownie pan model. "Now cross out what Aunt Serena bought. No, don't shade out anything, just cross out what Aunt Serena bought." N'yette presents faulty reasoning when she shades in three of the sections. While the task statement indicates that Aunt Serena bought "three fourths of what was left" (after one half of the pan had already been purchased). The correct answer is three-eighths, but the N'yette's model shows three-sixteenths. It may be inferred that Ms. E is ***satisfied*** with the model because she poses a question

			to the class. Ms. E conveys ***interest*** when she poses a mathematical question to the class, asking for the fractional name of the part of the brownie that Aunt Serena bought. It appears that Ms. E has purposefully asked N'yette to shade *only* the section that Aunt Serena bought in hopes that these students would recognize that the three crossed out sections represent three-sixteenths of the brownie pan instead of three-eighths.
18:06	Dave and other students	Three-eighths.	
18:07	Ms. E	No [Ms. E shakes her head, "no"] I'm not convinced that's three eighths.	Ms. E expresses ***doubt*** about the students' answer of three-eighths thorough her gesture, shaking her head "no" and her language, "I'm not convinced that's three eighths."
18:11	Manuel D	I think it's six...sixteenths.	Manuel tentatively offers a different answer. It may be inferred that he has found an answer equivalent to three-eighths, using sixteenths as the denominator to agree with the model.
18:12	N'yette	No, she said do the three out of eight because in the pan, in the story it says, [Ms. E looks at a student in the audience momentarily and shakes her head, "no".] she bought what was left, in this what…[N'yette is gesturing to the diagram of the pan and	N'yette references the statement of the task in an attempt to support the solution, "three out of eight".

		looking at Ms. E] half was left…	
18:21	Ms. E	[Ms. E is speaking to a student off camera] Somebody's out… But, let's pretend the other half is not left [Ms. E waves her hand side to side and shakes her head, "no"] and that's what Aunt Serena bought.	Ms. E may be responding to a student's request to leave the classroom to go to the restroom when she says, "somebody's out". This distraction does not interfere with the mathematical discussion because she quickly returns to address N'yette. Ms. E expresses N'yette's ***disapproval*** of N'yette's faulty reasoning with her gestures (shaking her head "no") and language. Ms. E's tone of voice is ***argumentative.***
18:25	N'yette	So if we pretend that Aunt Serena bought that half, three out of…	
18:27	Manuel D	[Ms. E]?	This is the third time in the past minute that Manuel D has tried to get Ms. E's attention.
18:30	Male student	Or you could shade something…take three rows…[Male student standing behind Manuel D leans in and points at the overhead projector.]	
18:35	Ms. E	Could you, could you understand what I'm asking then?	Ms. E addresses the students in the audience possibly in an effort to engage more students in this mathematical discussion. She may be exhibiting ***frustration*** as she makes a possible and subtle appeal for more students to recognize the faulty

			reasoning.
18:37	N'yette	Yeah, three out of sixteen then.	N'yette presents correct reasoning.
18:40	Ms. E	Right that's three out of sixteen. But [Tora], [Tora], your answer earlier was three out of eight. [Jenny], your answer earlier was three out of eight. They're saying it's three out of sixteen.	Ms. E ***validates*** N'yette's answer that the crossed out section in her model represents three-sixteenths. It may be inferred that Ms. E is attempting to ***encourage student mathematical discourse*** by pointing out a discrepancy between the students' answers.
18:50	N'yette	I was just…[N'yette is tapping her pen on the overhead projector.] We had three out of eight.	N'yette defends her group by stating that her group presented the answer three out of eight.
18:54	Manuel D	Three-eighths.	
18:56	Ms. E	But, you're saying the one you crossed out is three out of sixteen. [Ms. E is pointing at the overhead projector.]	Ms. E's tone of voice may be inferred as ***argumentative and she expresses disapproval of N'yette's mathematical reasoning.*** Ms. E's facial expression shows confusion due to the group's wavering between the answers three out of sixteen and three out of eight. Ms. E points out that N'yette's group named the model on the overhead was named three out of sixteen. Ms. E's tone of voice conveys ***doubt***. N'yette's facial expression may convey confusion.
18:57	Dave	But we think it's three out of eight.	Dave sounds insistent when he says, "we think it's three out of eight."
18:58	N'yette	If, if we had this extra part, it would be…	N'yette appears to ***defend*** her reasoning. Ms. E looks back and forth

			from the overhead projector to the class. Her facial expression is serious.
19:01	Dave	You ain't know that	
19:03	N'yette	…but it's gone so its three out of eight [N'yette returns to her seat and Ms. E approaches the overhead projector.]	N'yette presents faulty reasoning when she says, "If we had this extra part, it would be [three out of sixteen]…but it's gone so its three out of eight" It appears that N'yette sees the remaining half of the brownie pan, which is cut into eight pieces, as the whole.
19:06	Ms. E	Listen, class…Ok… [Ms. E stands up and approaches the overhead projector as N'yette returns to her seat.] [Manuel can be heard speaking to Ms. E, The camera pans to show students in the audience, zooming in on N'yette who is drawing in her notebook.]	It may be inferred that Ms. E recognizes the faulty reasoning and is ***unhappy*** with the progress of the discussion as she intervenes as she approaches the overhead projector. Ms. E is attempting to take control of the discussion based on her body language and utterances.
19:11	Manuel D	Let's say that it was six-sixteenths, right? [Manuel D looks up at Ms. E]	
19:14	Ms. E	Okay...[Ms. E is looking at the overhead projector.]	Ms. E seems to convey ***interest*** in Manuel D's explanation because she is looking at the work that he has done on the overhead projector and she is ***actively listening*** when she says, "okay..".
19:15	Manuel D	So, break it down. What you do for three-eighths, you break it down….	
19:22	Ms. E	Can you prove that this is now three-eighths?	Ms. E's tone of voice conveys ***curiosity*** about the

			students' reasoning why the model represents three-eighths.
19:24	Manuel D	Yeah because, you break down. [Ms. E returns to sit on top of the empty student desk.] ...six-sixteenths. [Manuel is writing on the transparency as he speaks.]	
From Cecilia's camera:			
22:02	Ms. E	Shh, you're not listening to your own presentation. [Ms. E points towards students seated in the audience.] Gentlemen, gentlemen [Ms. E leans in towards the students seated in the audience], you're not listening to your own presentation. [Lina's camera pans to the image projected on the wall showing Manuel's drawing of the brownie pan with six-sixteenths crossed out. Next to the model of the pan, Manuel has written 3/8 and underneath the model, Manuel has written 6/16 ÷ 2/2 = 3/8.]	Ms. E's voice is raised and firm which may convey ***anger*** with some male students for not listening to the discussion despite the fact that Manuel is one of his group members. It may be inferred that Mrs. E recognizes that Manuel D has presented correct reasoning and that she is eager for the students in the audience to attend to Manuel D's ideas. Manuel has presented correct reasoning in the model and in the numerical representation below the model, showing the proper arithmetic for renaming six-sixteenths as three-eighths by dividing by [two over two] one.
22:10	Felipe	Why?	
22:11	Ms. E	Huh?	
22:14	Felipe	Why?	
22:15	Ms. E	What do you need? A ruler? Uh…six…eighteen?... Alright class, could you look at	Ms. E is responding to a request for something by someone off camera. Ms. E appears to be reading Manuel D's writing from the

		this first, because earlier… [Ms. E looks at Tora and Lorissa and leans in towards them.] [Tora] and [Lorissa], when you were presenting here, they were listening to you. In fact, they were asking questions, I expect the same thing from you now. [Ms. E raises her voice and addresses the whole class.] You're saying this is, [N'yette], [N'yette]. I'm asking you about this part that you said Aunt Serena bought. N'yette: Uh-huh That, you're saying its three-eighths. Class, [Ms. E raises her hand gesturing towards the students in the audience.] do you agree with that? That this is three-eighths.	overhead projector. He has written 6/16 ÷ 2/2 = 3/8 but his penmanship is sloppy, so the 16 appears as 18. It may be inferred that Ms. E perceives that the current behavior exhibited by Tora and Lorissa to be off-task because she expresses her ***displeasure*** when she points out the fact that the girls are speaking out of turn. Ms. E is clearly encouraging ***peer-to-peer respect*** when she speaks to Tora and Lorissa, "I expect the same thing from you now." Ms. E then attempts to redirect the students' attention towards the model drawn on the overhead projector, asking them if they agree. First, she directs a question to N'yette. "I'm asking you about this part that you said Aunt Serena bought. That, you're saying its three-eighths." "Class, do you agree with that? That this is three-eighths." Ms. E's tone of voice and questioning convey ***doubt.***
22:53	Students	Yeah	
22:54	Ms. E	Three-eighths right here?	
22:54	Students	Yeah.	Students sound ***uninterested.***
22:55	Ms. E	Then how come there are sixteen parts there? [Ms. E gestures towards the model of the brownie pan written on the overhead projector.]	Ms. E is not smiling as she ***questions the students' mathematical reasoning***.
22:58	N'yette	Because the other half is gone!	N'yette sounds ***frustrated*** as

			if she feels like she has already explained that "the other half is gone" .
23:59	Ms. E	Does it matter if the other part is gone or not? If I start from the beginning…	Ms. E's voice is raised and she appears to convey ***frustration.***
	Manuel D	No! No! No!	Manuel appears ***angry.***
23:00	Ms. E	Listen, [Ms. E puts her hand on Manuel D's shoulder briefly.] …if I start from the beginning and I cut the brownies [Ms. E gestures with her arm as if she is cutting something], I'm going to cut it in sixteen pieces. And this part that she's buying is three out of that. So how can you call that a three-eighths? When I started [Ms. E gestures a circle with her arm and then gestures cutting the brownie with her arm.] and I cut the pieces, I cut eighteen pieces, uh sixteen pieces. How can you now call…	It is possible that Ms. E is ***agitated*** and Manuel D because she pats him on the shoulder and says, "Listen". Ms. E appears to feel ***frustrated*** about the lack of progress of the mathematical discussion. Ms. E's gestures are exaggerated when she acts out cutting the brownie pan into sixteen pieces. She appears ***angry*** now as she exaggerates her gestures of cutting the brownies.
23:29	Tora	But isn't that already not gone?	
23:32	Ms. E	But what if it's not gone? [Ms. E shrugs her shoulders and points to the image on the overhead projector.] What if it's still there?	Ms. E's tone of voice is ***argumentative*** as she is challenging the students' reasoning.
23:36	Tora	But they didn't mention that.	
23:37	Ms. E	So my question is, does it make a difference if it's gone and if it's not there and if it's there? [Ms. E is gesturing up and down with her hands as she speaks.] Does it change the fact that now it's three-eighths? Can you	Ms. E restates her question to the class, conveying ***curiosity*** about whether or not it matters if the half of the brownie pan has been purchased or not. Her tone of voice remains ***argumentative.*** Ms. E's tone of voice

		prove me that because I'm not convinced that if somebody bought *this part* then *this* three-sixteenths all of a sudden becomes three-eights. [Lina's camera pans to show eleven students in part of the audience. Five students are looking down at their books / desks. Some of these students are writing in their notebooks. Four students are talking to the person seated next to them. Two students are looking toward the front of the classroom.] [The camera pans to Ms. E who is still standing at the overhead projector. Her hands are raised, pointing towards her face.] Ok, I don't think I'm getting all of you, I want you to look at my eyes right now. Every single person in the room. [Ms. E gestures a circle with two hands.] Look, look, look at me. Every single person in the room I want to see your eyes looking at me right now. [Ms. E leans in towards the class and points towards her own eyes.] I still don't see some eyes. Shh. Would you please relax, I'm getting so agitated here. You're talking too much I don't even realize I'm getting agitated, that's because of you're talking on my side.	conveys ***displeasure*** and ***impatience*** with the students' mathematical reasoning. Ms. E appears to recognize that many of the students are disengaged from the conversation. She is speaking quickly and loudly. Ms. E's tone of voice conveys that she is ***agitated*** and her language indicates that she is ***angry*** with the students because they are not engaged with the mathematics. Ms. E is getting visibly ***frustrated*** and appears to be attempting to get students engaged when she says, "I want to see your eyes looking at me right now." Ms. E is talking to an individual student, likely Dave or Manuel D, indicating that the student is talking too much while she is addressing the class. Ms. E is clearly ***agitated*** because she says, "I'm getting so ***agitated*** here. You're

		[Jenny is speaking to the student next to her.] [Jenny] I want you to look at me. I have a question for you: This part, if I cut the whole pan I'm going to cut the whole pan this way, it's going to be sixteen pieces. This part, right here, what do you call that?	talking too much I don't even realize I'm getting agitated, that's because of you're talking on my side." Ms. E redirects the students to the mathematics and her tone of voice softens.
24:33	Students	Three.	Ten out of twelve students are now looking toward the front of the classroom where Ms. E is standing. It may be inferred that they have become engaged or pseudo-engaged after Ms. E's behavior intervention.
24:34	Ms. E	Three out of what?	The students have not satisfied Ms. E's request, so she presses them to be more specific when she asks, "Three out of what?"
24:35	Students	Sixteen.	
24:36	Ms. E	Three out of sixteen. Now, you said this part is sold, [pointing to the overhead projector] has been sold. [Ms. E is covering part of the diagram of the brownie pan with her hand.] How come the name change into three-eighths? I'm not convinced [Ms. E is shaking her head now] I have to change the name to three-eighths. [Ms. E is pointing at the overhead	Ms. E's tone of voice and language indicate that she validates the students' answer, "three out of sixteen". Ms. E appears ***puzzled*** by the students' claim that selling half of the brownie pan changes the fraction name for Aunt Serena's brownie portion from three out of sixteen to three out of eight.

		projector] But to *begin with* [stated with emphasis, Ms. E steps away from the overhead projector], I cut it into sixteen parts. [Eric has his hand raised]	
24:57	Manuel C	The rest is sold	
24:58	Ms. E	[Eric], [Eric],…	Ms. E calls on Eric who had his hand raised.
25:01	Eric	The rest is covered, so…	
25:04	Ms. E	So you changed the name? Now all of a sudden its three-eighths? [Eric nods and puts his finger in his mouth.] Alright, so let's see this part. Hey, this part is one half, right?	Ms. E expresses ***doubt*** as she restates the students' faulty reasoning. Ms. E intervenes by presenting a hypothetical situation to the students using a simpler problem to address the cognitive hurdle that the class seems to be experiencing.
25:12	Students	Yes.	
25:15	Ms. E	That gets sold, [Ms. E is writing on the overhead projector and then gestures as if she is pushing half of the brownies away] so then can we now then call this part one pan? Only because this was sold? [Ms. E is leaning on the overhead projector, gesturing to the screen and alternating looking down at the screen and up at the students]	Ms. E appears ***excited*** as she intervenes with a simpler problem.
25:18	Dave	No, you gotta call it a half a pan.	
25:20	Ms. E	No, that's what I'm *saying.* [Ms E states this with emphasis and throws her hands up.] Why, why can't, what do you call this then? Half. If I sell this, can I now call this a one whole pan?	Ms. E appears ***relieved*** when many students appear to demonstrate correct reasoning about the simpler example. Ms. E's comment, "That's what I'm saying" can be inferred to mean that

			she is in ***agreement*** with her students.
25:20	Manuel D	Half.	
25:21	Ms. E	Half. If I sell this, can I now call this a one whole pan?	Ms. E expresses ***curiosity*** about the students' reasoning about the simpler example.
25:31	Class	No…	
25:32	Ms. E	So what do you call this?	
25:33	Student	A half.	
25:35	Ms. E	This is one half regardless of whether or not I sold, sell this part, right?	Ms. E affirms her students' reasoning and it may be inferred that Ms. E feels ***pleased*** that the students are presenting correct reasoning.
25:36	Student	Yes.	
25:37	Ms. E	So why can we change the name of this one and not on this one?	Ms. E's tone of voice and language convey ***curiosity*** about the students' reasoning for changing the fraction names on the original task. Ms. E appears to be trying to build a connection between the simpler example and the original task.
25:41	Tora	I don't know.	
25:47	Ms. E	See, you're saying if I sell this part, I can change the name of this part.	Ms. E challenges students to make the connection between the simpler example and Aunt Serena's brownies. Ms. E's tone of voice is argumentative.
25:50	Student	No, you can't.	
25:51	Ms. E	But that's what you said here. [N'yette]. You sold this part and all of a sudden, this has a different name.	Ms. E expresses ***frustration*** with the students' earlier reasoning.
25:57	Eric	Cause I thought it was…um left over after you sold it.	
26:01	Ms. E	Well I'm just saying, why do they have two names? Why can it be three-eighths and	Ms. E's tone of voice is ***argumentative.*** She seems to be challenging the

		then all of a sudden be three-sixteenths? What is it, is it three-eighths or three-sixteenths or can it have both names? [Ms. E shrugs her shoulders and gestures with her palms up.]	students to present a valid argument for being able to name the same fraction model with two different names. Ms. E appears ***angry*** that some students ***maintain*** their stance that the same fraction can have two different names.
26:15	Dave	You gotta find the one that's equivalent to both of them.	
26:17	Ms. E	Ok, you need to find one that's equivalent. Is three-eighths and three-sixteenths equivalent three-sixteenths equivalent to each one? To each other? [Eric]. [Ms. E gestures her hand towards Eric.]	Ms. E appears to agree with Dave's suggestion of equivalence. Ms. E appears ***curious*** about her students' reasoning about fractional equivalence.
26:26	Eric	I think you could have both of them.	Eric presents faulty reasoning.
26:27	Ms. E	You can have both names. [Ms. E approaches the overhead projector and points to the diagram.] So we could call this part either three-eighths or three-sixteenths? [Ms. E throws her arms out to the side.]	Ms. E does not validate nor negate Eric's faulty claim when she restates Eric's reasoning. Ms. E appears to demonstrate displeasure about Eric's suggestion as she throws her harms out and shrugs her shoulders.
26:35	Dave	Yeah you can because eight is equivalent to sixteen.	Dave presents faulty reasoning stating that eight is equivalent to sixteen.
26:38	Ms. E	Eight is equivalent to sixteen. [Ms. E wrinkles her face and pushes her hair off of her shoulders and neck as she speaks.] What do you mean by that? If I have eight dollars, and you have sixteen dollars, [gesturing towards Dave] do we have the same amount of money? [Ms. E has her hands raised towards	Ms. E's tone of voice and facial expression convey ***disbelief*** when she questions Dave's faulty claim. Ms. E's tone of voice is ***argumentative*** when she questions Dave about the equivalence between eight and sixteen. Ms. E interrupts before Dave has time to respond to

		her shoulders.]	her question, "What do you mean by that?" She then offers a hypothetical example, comparing eight dollars to sixteen dollars. Ms. E's tone of voice exhibits ***frustration*** and ***disbelief*** in Dave's claim.
26:46	Dave	No, but, it's half of sixteen. [Dave turns the page in his book and looks down.]	
26:49	Eric	No, it could only be three-sixteenths because…	Eric states the answer correctly.
26:50	Ms. E	[Eric]? What was that?	Ms. E asks Eric to repeat his statement.
26:53	Eric	It could only be three-sixteenths cause you um…	Eric repeats his claim, but does not finish stating his reasoning because Ms. E interrupts him.
26:55	Ms. E	This could only be three-sixteenths? What if I sell half of it?	Ms. E challenges Eric's reasoning. Ms. E expresses ***curiosity*** about Eric's reasoning once half of the brownie pan is sold.
26:58	Eric	It's still the whole square.	
27:00	Ms. E	[N'yette], do you agree with that?	Ms. E encourages student discourse by asking N'yette to comment on Eric's mathematical claim.
27:01	Dave	If you sell half of it.	
27:03	N'yette	What happened? What'd you say?	N'yette may have been off-task or not listening to the conversation because she asks Ms. E to repeat the question.
27:04	Ms. E	Could you follow that idea, [Eric] could you say that again?	Ms. E encourages student discourse by asking Eric to repeat his claim for N'yette to hear.
27:05	Dave	It would be three-eighths and then…	
27:07	Eric	It's still the whole square.	
27:08	Ms. E	Huh?	
27:08	Eric	It's still the whole square.	
27:10	Ms. E	It's still gonna be the whole	Ms. E appears ***excited*** as she

		square, so what do you call this? What do you call this now? [Tora] could you follow with me please? What do you call this now? Without selling anything now, what do you call this?	repeats Eric's claim and then appears to be posing a question to the whole class. Ms. E is speaking quickly and her tone of voice may convey that she is feeling ***tense*** about the students' response to her question. Ms. E lowers her voice and addresses Tora directly. It may be inferred that Ms. E recognized that Tora was off-task. When she speaks to Tora, Ms. E's tone of voice and language may be inferred to show that she is ***annoyed*** with Tora's level of engagement in the mathematical discussion. Ms. E's voice is raised, she appears to be ***agitated*** and ***anxious*** for students to provide her with an answer to her question, "Without selling anything now, what do you call this?"
27:23	Eric	Three-sixteenths . [Biting his nails]	
27:24	Ms. E	Three-sixteenths. What if I sell the other half? Do you change the name?	Ms. E appears ***satisfied*** with the students' answer, "three-sixteenths" because she quickly repeats the answer before posing another question.
27:27	Students	Yes.	
27:29	Ms. E	To what?	
27:29	Dave	Three-eighths	Dave appears to feel ***confident*** about his answer because he states it without

			hesitation. He is sitting up straight and looking directly at Ms. E.
27:30	Ms. E	So, without changing its size, without changing anything, just selling half, all of a sudden you can change the name?	Ms. E appears ***discouraged*** when she challenges Dave about his answer asking, "all of a sudden you can just change the name?"
27:37	Dave	Yes…Cause you took, cause look. If you take away like if sixteenths the whole thing and you take away half of it, half of sixteen is eight and then it'll turn into three-eighths. So that's changing the name.	Dave appears to present faulty reasoning that if half is taken away then the eight pieces that are left represent the whole.
27:49	Ms. E	Could somebody please convince me because I'm not convinced, that one name…[Eric]. N'yette: "We just did!" No, I'm not convinced that, because look, [Ms. E jumps down from the desk she is sitting on and approaches the overhead and points to the diagram on the overhead projector] I told you about this half. I sell the half, can I change the name?	Ms. E appears ***confused*** and ***frustrated*** and she is appealing to the students in the class to "convince" her that the name should change. Ms. E expresses ***doubt*** in the students' claim and revisits the simpler example that she posed earlier.
28:03	N'yette	No.	
28:04	Ms. E	So, the answer here is no. [Pointing to the diagram, then Ms. E steps backwards and folds her arms.]	It may be inferred that Ms. E may feel ***frustrated*** with the students' faulty reasoning.
28:04	Students	Yes.	
28:06	Dave	But you can change the name!	Dave stands by his faulty reasoning.
28:07	Manuel D	Ms. Endaya, I got a question.	
28:08	Student	Somebody got a question	
28:08	N'yette	It depends what kind of fraction it is that you wanna…	N'yette appears ***agitated*** because she her voice is raised.
28:12	Ms. E	So sometimes you can change the names? Sometimes you	Ms. E's tone of voice is ***argumentative*** and ***sarcastic***

		can't?	as she reiterates N'yettes' reasoning, "So sometimes you can change the names? Sometimes you can't?"
28:14	N'yette	Yeah.	
28:15	Ms. E	So can you show me when you can change the name, and when you cannot change the name?	Ms. E appears ***agitated*** and ***curious*** about the students' ideas about being able to change the mathematical principles with regards to naming fractional parts of a whole.
28:18	N'yette	Right here, and right there, it's a different one.	
28:21	Ms. E	[Eric], what do you think? What do you think?	Ms. E appeals to Eric for his opinion possibly because Eric had presented correct reasoning earlier during this discussion.
28:25	Eric	You could still change the answer . so…	
28:27	Ms. E	You can change it?	
28:28	Eric	Yeah. [Eric is biting his nails.]	
28:29	Ms. E	So can I also call that three twenty-fourths, let's just call it three twenty-fourths.	Ms. E offers an arbitrary name for the fraction, "three-twenty fourths". She appears to convey ***frustration*** by suggesting an arbitrary fraction.
28:31	Students	No, no	
28:32	Manuel D	Ms. Endaya, Ms. Endaya! [Manuel D slams his hand down on the table.]	It may be inferred that Manuel D slams his hand down on the table because he is ***frustrated*** that he is not getting Ms. E's attention.
28:34	Ms. E	I feel like calling it three twenty-fourths.	Ms. E expresses sarcasm when she states that she is going to call the amount Aunt Serena bought "three twenty-fourths" because she feels like it.
28:37	Student	Three twenty-fourths?	
28:38	N'yette	Not just cause you feel like it, you can't	
28:40	Ms. E	So how is it? How do you name it then? [Ms. E has her arms	Ms. E has her arms crossed in possible ***annoyance*** that

		crossed.]	the students remain "stuck" on the idea that the fraction name may change. Ms. E sounds like she is ***begging*** someone in the class to provide a valid mathematical reason for whether the answer is three-eighths or three-sixteenths.
28:45	N'yette	If it's possible… can you change it.	
28:47	Ms. E	[Ms. E sits back down on the top of the student desk. Her voice is raised when she addresses the class.] But what if you don't sell the other half? What if you don't sell the other half? [Ms. E put her finger on Manuel D's arm.] Alright, what's your question. What's your question?	Ms. E's tone of voice and body language convey ***frustration.*** She appears to be ***arguing*** with the students she she asks, "But what if you don't sell the other half?" Ms. E addresses Manuel D, her tone of voice still may be inferred to show her ***frustration*** with the students' incorrect mathematical reasoning.
28:58	Manuel D	That…	Manuel is slumped down in his chair and appears to convey ***defeat*** in his body language and ***frustration*** in his tone of voice.
	Ms. E	That….	Ms. E ***encourages*** Manuel D to complete the statement of his idea.
	Manuel D	That, if you turn three-sixteenths into three-eighths…or, is three-eighths bigger than three-sixteenths?	Manuel states correct reasoning that three-eighths is larger than three-sixteenths. It appears that Manuel understands the idea that the fraction names are not interchangeable because they are not equivalent.
29:08	Ms. E	Isn't three-eighths bigger than three-sixteenths? Can you show us that? [Ms. E purses her lips and gestures with her hand	Ms. E repeats Manuel D's claim and then conveys ***curiosity*** by asking him to provide an example for the

		towards the overhead projector. Manuel D stands up.]	class. It appears that Ms. E ***values*** Manuel D's mathematical reasoning because she asks him to share his idea with the class.

29:13	Dave	Cause look, this is three-sixteenths and you take away half of it, you can't call it three-sixteenths no more. [Manuel D sits down. Dave is sitting at the overhead projector. Ms. E is pointing towards Manuel D and shaking her hand as Dave speaks.]	
29:18	Ms. E	[Ms. E is pointing at Manuel D] But I wanna hear, yeah he's saying, [Manuel D] is saying three-sixteenths is bigger than three-sixteenths, can you show us that? [Manuel stands up an approaches the overhead projector.]	Ms. E demonstrates that she values Manuel D's reasoning when she says, "But I wanna hear, yeah he's saying."
29:25	Dave	It is bigger.	
29:28	Manuel D	Because the pieces are gonna be bigger, because of right here. [Manuel D points at the diagram on the overhead projector.]	
29:30	Ms. E	Yes and I want, I'd like, especially the ones who were in this group, [N'yette] I'm calling your name again, um, [Tora], you have a different idea than this. [Manuel D sits down] Would you go and show us [Ms. E gestures towards the overhead projector] how three-eighths and three-sixteenths three-sixteenths look like? [Manuel D stands up and approaches the overhead projector where Dave is still sitting.]	Ms. E appears to be ***displeased*** with the perceived off-task behavior exhibited by N'yette and Tora. Ms. E's tone of voice and language may be inferred to express ***frustration*** because it is at least the second time she has had to call for N'yette's attention. Ms. E's tone of voice and persistence appear to convey her ***eagerness*** for the students in the audience to hear Manuel's explanation.
29:42	Tora	I have a different idea, it's the same thing.	
29:44	Ms. E	No, three-sixteenths hasn't	
29:46	Tora	Oh yeah, oh yeah.	
29:47	Ms. E	…hasn't come up in your drawing. I want the same size, could you show us three-	Ms. E seems to have relaxed now and she appears ***hopeful*** that Manuel D's reasoning may

		sixteenths and three-eighths. [Ms. E is speaking to Manuel D] [Ms. E is now speaking to the students in the audience] Then maybe I can call it three twenty-fourths or three-seventy fives.	help correct faulty student reasoning. Ms. E appears to be ***cynical*** when she makes an apparently facetious claim, "Then maybe I can call it three twenty-fourths or three-seventy fives."
30:00	Student	No!	
30:01	Tora	You can't do that.	
30:03	Ms. E	But you just did it! It's three-eighths and then it's…	Ms. E is now ***mocking*** the faulty reasoning presented earlier**.**
30:04	Dave	But we didn't!	Dave's tone is ***argumentative.***
	Manuel D	Yes you did!	
	Dave	You just said if we buyin' half.	
30:07	Ms. E	Alright, go ahead.	
30:07	N'yette	At certain times you could, cause it's…	
30:10	Ms. E	So show me the certain times.	Ms. E expresses interest in the "certain times' that N'yette claims that a fraction name can be changed. Her tone of voice suggests that she is ***displeased*** with the students' faulty reasoning.
30:12	N'yette	…but that doesn't mean, [Ms. E], but that doesn't mean just because you feel like changing it, [waving her arms in the air] you gotta change it. You have to change it when it's …[hold her arms up an still and pauses]	N'yette seems ***excited*** to express her ideas because her voice is raised. She affirms that the fraction name cannot be arbitrarily changed.
30:20	Student	Right there	
30:20	Ms. E	Yes	
30:22	N'yette	When it's…when it's time to. [sits back in her chair and looks down at her book]	N'yette is ***adamant*** that it is possible to change a fraction name, "when it's time to".
30:25	Ms. E	What's the time to then? When is it time to then?	Ms. E questions N'yette's apparent invention of mathematical principles.
30:31	Dave	When they decide to buy a half.	

30:35	N'yette	When you have half left. [N'yette approaches the overhead and stands next to Manuel D]	N'yette's tone of voice is ***argumentative.***
30:38	Student	Shut up, please.	
30:38	Ms. E	When you have half left, then…	Ms. E repeats N'yette's statement.
30:41	N'yette	Not half of a half, [pointing at the overhead projector] but I'm saying half of a sixteenth, or half of something [shakes hands in the air and shakes her head]	N'yette attempts to provide an argument for changing the name but is unsuccessful. She throws her hands up in possible ***defeat.***
30:47	Ms. E	So tell me when, what's the rule in changing names? When can I decide to change names?	Ms. E persists and maintains her ***curiosity*** about the students' perception of the rules for changing fraction names.
30:52	Dave	When somebody decides to…um, buy the other half.	Dave repeats his claim for at least the third time.
30:57	N'yette	Like if somebody decides that they take the half of the half.	
	Mrs. E	Ok, ok, hold on…	
31:01	Dave	When somebody decides to buy three-sixteenths…	
31:03	Ms. E	Then I can change the name of three-eighths to three-sixteenths?	
31:06	Dave	Then it'll turn into three-eighths.	
31:08	Ms. E	Hmm.	Ms. E's tone of "hmm" sounds like she is ***puzzled.*** She is calm as if possibly considering which direction she should steer the class at this point.
31:08	N'yette	If this...aight, it's the whole thing, [Ms. E]. [Ms. E]. [Many students in the classroom are talking at the same time.]	It appears that many students are not engaged with the discussion at the overhead projector because many students can be heard talking at the same time.
31:12	Ms. E	Gentlemen...[Ms. E looks towards the students in the audience, snaps her fingers three times then returns to look at N'yette] Yes, yes go ahead.	Ms. E appears to address the perceived off-task behavior of some boys when she says, "gentlemen" and snaps her fingers. It may be inferred that she is unhappy with the boys' behavior.

			Ms. E ***encourages*** N'yette to explain her reasoning when she says, "Yes, yes go ahead."
31:14	N'yette	This is when you change it. [N'yette is standing at the board reaching up to point at the image projected on the chalkboard] Ok, this whole thing, it's a whole cost of four dollars right? [N'yette looks back at Ms. E.] So if somebody. Alright, say somebody buy this whole side right here. [Ms. E snaps her fingers towards the students sitting down in the audience then looks back towards N'yette.]	Ms. E snaps her fingers expressing disapproval of student behavior that she perceives to be off-task.
31:26	Ms. E	Un-ha	Ms. E conveys to N'yette that she is listening to her explanation by saying, "uh-huh".
31:28	N'yette	Boom! Then another person comes	
31:29	Ms. E	Uh-huh	
31:30	N'yette	They be like "Oh, that's only three-eighths, because they took the other half so they could call it three-eighths.	
31:35	Ms. E	Three-eighths.	
31:36	N'yette	You see what I'm saying? That's when you can change it.	
31:39	Ms. E	I hear what you're saying. [Ms. E is looking at Dave as she speaks to N'yette.]	Ms. E states that she "hears" what N'yette is saying, yet she does not appear to be listening to N'yette. Ms. E appears distracted by the chatter that is going on around the room.
31:41	N'yette	When you have a equal amount.	
31:43	Dave	She's trying to tell you	
31:48	Manuel D	[Ms. E]	
31:49	Ms. E	Yes?	
31:50	Manuel D	Where am I?	
31:51	Ms. E	Five in four. Can we have one conversation here? Shh. three, in two…and one.	Ms. E appears ***calm*** as she demonstrates that she is ***displeased*** with the level of

		[Ms. E asks Manuel D who has returned from his seat to the overhead projector] Go ahead. What were you trying to explain here?	student engagement in the mathematical discussion. It may be in inferred that she is counting down to get the students' attention.
32:10	Manuel D	That this is what three-eighths look like.	
32:13	Ms. E	Could you mark it please? [Eric, Eric] When you were talking earlier, people were listening to you.	Ms. E asks Manuel D for clarification of his model when she asks, "Could you mark it please?" Ms. E then ***calmly*** indicates her ***displeasure*** regarding Eric's perceived off-task behavior by reminding him, "When you were talking earlier, people were listening to you."
32:18	Manuel D	This here would be three-eighths	
32:20	Ms. E	Ok, [N'yette], when you were talking earlier up here, I was listening to you. Look here, look three-eighths and…	Ms. E ***calmly*** indicates her ***displeasure*** regarding N'yette's perceived off-task behavior.
32:28	Dave	Three-sixteenths.	
32:31	Manuel D	It looks way different because three-sixteenths got smaller pieces and three-eighths got bigger pieces.	Manuel appears to be providing an explanation to support the models he has drawn of three-sixteenths and three-eighths.
32:36	Ms. E	So what are you saying?	There is a sense of ***urgency*** in Ms. E's voice as she conveys ***curiosity*** about understanding Manuel's explanation.
32:36	Manuel D	That instead of…it could be eighths or probably six-sixteenths but it's like …	
32:41	Ms. E	So do you agree that you can change names from three-eighths to three-sixteenths?	Ms. E interrupts Manuel D. Her tone of voice is ***calm***.
32:45	Manuel D	No. Cause that's gonna be like…	
32:49	Dave	Yes you can, you could because what if they decide to buy half of the brownie pan? What is it?	
32:53	Manuel D	Aight.	
32:54	Dave	What is it? Three-eighths.	
32:55	Manuel D	No it can't be. Aight if it's	

		three-eighths then what it's gonna be there? It can't be three-sixteenths.	
33:00	Dave	If this, look..	
33:02	Manuel D	Alright come on, like, to be three-eighths, right? Right here to be three-eighths it's gotta be this whole thing right here. That's three-eighths right there.	
33:13	Dave	But if they wanna buy half of the pan then what is it?	
33:16	Ms. E	[Ms. E looks towards the students in the audience.] Do you hear what that, do you hear that? Um, could you say that again? Alright, raise your hands. Raise your hands.	Ms. E's tone of voice is ***calm***. She asks the students in the audience if they can hear the discussion between Manuel D and Dave. Ms. E intervenes, suggesting that Manuel D use his most recent models to show that half the pan was purchased.
33:22	Dave	I said half a pan, I didn't say a whole pan.	
33:32	Manuel D	But we're not doing "if" [Manuel D gestures with his hand and sits up straight.] half of the brownie. We're doing…	
33:34	Ms. E	Alright, let's just say you bought half of the pan, could you shade that part that was bought, in those two boxes? Those two pans.	Ms. E wants Manuel D to change his two models to show that half of each pan has been purchased. It may be inferred that she is attempting to have Manuel D's models demonstrate for the class that shading one half does not change the fractional value of the original fractions.
33:40	Manuel D	I don't even know…[Manuel D looks at Ms. E]	Manuel D appears ***confused*** by Ms. E's request.
33:41	Ms. E	If, if [Ms. E stands up and walks towards the overhead projector, pointing at the diagrams of the brownie pans] I buy half here could you shade that please?	Ms. E provides clarification of her request to Manuel D. Ms. E ***imposes her own mathematical ideas.***
33:45	Manuel D	Half of it?	

33:46	Ms. E	Half, half here, half of this.	
33:48	Manuel D	This one?	
33:49	Ms. E	Yes.	
33:50	Manuel D	So if this one right here…	Manuel D points to the half that has already been partially shaded.
33:53	Ms. E	Could I also buy the lower half? If I want the lower half?	Ms. E corrects Manuel D by posing the idea that she buys the lower half, which has not been shaded at all.
33:56	Dave	Yeah but then you're gonna have to pay for the whole thing	
33:59	Ms. E	No let's say there's somebody who came earlier and bought the lower half.	Ms. E imposes her own mathematical ideas as she leads Manuel D and Dave shading the model.
34:03	Manuel D	Then…	
34:04	Ms. E	Shade that.	
34:05	Dave	Bought the whole lower half?	
34:06	Ms. E	Lower half.	
34:08	Dave	Then you won't have none to eat!	
34:10	Manuel D	Then you gonna have three-fourths.	
34:11	Ms. E	Let's just say somebody bought this [Ms. E points to half of the model of the brownie pan drawn on the transparency.] Would this be half? Would this be a half?	Ms. E poses a mathematical question.
34:16	Dave	No.	
34:17	Ms. E	Would this be a half? [Ms. E addresses the students seated in the audience.] Guys, I need you, I need you with me. I really need you with me here. It's very important. I'm not gonna invest time on this, just so we could scream at each other here. This is absolutely very important. If you're losing, um, losing attention right now, [Ms. E puts her hand on Tora's shoulder momentarily. Tora looks down at her shoulder after Ms. E removes her hand.]	It appears that Ms. E is ***displeased*** by the lack of student engagement. She interrupts the mathematical discussion to address the students and exhibits ***frustration*** with the students' level of engagement, stressing the importance of the mathematical discussion. Ms. E expresses her feeling that this discussion is important especially since some students still do not understand.

		I want you to try a little bit harder for just a few minutes before I start a new one. So, that's why we're still talking about it. People still don't get it. So if I'm saying somebody came and bought this half…what do you call this part?	Ms. E pleads for students to try to pay attention for "just a few minutes" with the promise that they will move on to a different problem soon.
34:51	Ms. E	How could it change now? When the pan didn't change?	
34:55	N'yette	Cause the other half is gone!	
34:58	Ms. E	I-I don't get that. I don't get why the name changes. This is three-fourths.	It may be inferred that Ms. E is ***disappointed*** by the students' unyielding faulty reasoning.
35:03	Student	Here, this part right here is gone.	
35:05	Ms. E	This is three-fourths of a half, right? This is three-fourths of a half. Does that mean its also three-fourths? Because when you say three-fourths, you mean three-fourths of a whole. How can of a whole and three-fourths of a half be the same?	Ms. E appears to express ***anger*** because her voice is raised with the students about the possibility of equivalence between three-fourths of a half and three-fourths of a whole.
35:21	Dave	It's not the same.	
35:22	Ms. E	That's what I'm trying to say. How can this three-eighths right here, three-eighths right here, you're calling this three-eighths is it three-eighths of three-eighths of a half.	Ms. E expresses ***relief*** after Dave states that three-fourths of a whole and three-fourths of a half are not equivalent.
35:36	Jenny	Three-eighths of a half.	
35:38	Ms. E	[Jenny]? [Jenny sinks her head down.] Do you, who thinks its three-eighths of a whole? [No students respond] Or who thinks its three-eighths of a half? [Many students respond by calling out.]	It appears that Jenny may be ***embarrassed*** that Ms. E recognized her answer. Ms. E demonstrates ***curiosity*** about student reasoning.
35:44	Dave	It's three-eighths of a half. So if it was three-sixteenths, it would be three-sixteenths of a whole.	Dave presents correct mathematical reasoning.

35:50	Manuel D	No, it's three-fourths of a half…	
35:52	Ms. E	Alright, so three-eighths of a…you're calling this three-eighths of a what?	
35:54	Students	Whole.	Ms. E is leading the students through the process of naming the fractional parts of a model, comparing halves to wholes. It may be inferred that she is offering ***encouragement*** to the students as she lead them through the process.
35:58	Ms. E	Of a whole. Three-eighths of a whole	
36:01	Manuel D	Um, three-fourths of a half.	
36:03	Ms. E	Is it three-eighths of a whole? It's three out of how many?	
36:06	Dave	Three-eighths of a half	
36:08	Ms. E	Three-eighths out of a half and three-sixteenths of…?	
36:11	Students	A whole.	
36:12	Ms. E	Of a whole. Now, do you agree with that?	Next to the original model that shows three-sixteeths, Ms. E has written three-eighths of a half and three-sixteenths of a whole on the overhead projector.
36:15	Dave	Yes.	
36:15	Manuel D	No.	
36:18	Dave	Yeah…why not? Why not?	
36:18	Ms. E	Ok, go ahead, go ahead, go ahead.	
36:19	Dave	Because sixteen, eight is half of sixteen.	
36:22	Manuel D	But wouldn't the three change?	Manuel D presents faulty reasoning as he expresses ***doubt.***
36:23	Dave	No. Why?	
36:35	Manuel D	Why's it gonna stay the same? Because the pieces are smaller than…The pieces are gonna be smaller, it's either gonna have to be six-sixteenths or, cause…I don't know, I don't think it can be, I just don't think it's three-sixteenths.	It is likely that Manuel is still thinking that the answer to the original task should be three-eighths which can be expressed as six-sixteenths.
36:39	Ms. E	Ok, start from the top. One more time. Hopefully we can get something here. One, two, three. What do we call this?	Ms. E appears to recognize that Manuel D has reached a conceptual barrier. Her tone of voice may be inferred to convey ***confidence*** that the

			students are close to developing understanding. She expresses ***hope*** for the students' conceptual understanding. Ms. E has drawn two identical brownie pans on the overhead projector. Both pans are divided into sixteen pieces and both show a representation of three-sixteenths.
36:56	Students	Three-sixteenths.	
36:58	Ms. E	Three-sixteenths of a what?	Ms. E's tone of voice is ***calm*** and controlled as she takes charge of the conversation, leading the whole class discussion.
36:59	Students	A whole.	
37:01	Ms. E	A whole.	
37:01	Dave	three-eighths of a half.	
37:06	Ms. E	Ok, hold on. (teacher starts writing on transparency) This is gone. What do we call this?	
37:12	Students	Three-eighths	
37:13	Ms. E	Three-eighths of…?	
37:14	Dave	Half.	
37:16	Ms. E	Of a half. Yes? No?	
37:20	Dave	Yes, yes.	Dave affirms his own answer.
37:22	Ms. E	Now, if it's three-eighths …	
37:23	Dave	How you get it?	
37:24	Ms. E	Yes? No? Who says yes? three-eighths of a half? [All students in the audience raise their hands] So if its three-eighths of a half, what is it then…?	Ms. E conveys ***curiosity*** about students' opinions.
37:33	Dave	If it's three-eighths of a half?	
37:35	Ms. E	Yes.	
37:46	Dave	Then its…	
37:39	Ms. E	If it's three-eighths of a half, what is it then?	It appears that Ms. E is trying to lead the students to name "three-eighths of a half" in terms of "a whole" which would be read, three-sixteenths.
37:40	Dave	What you mean, what you mean?	Dave expresses confusion and appears to seek clarification about Ms. E's question.
37:42	Ms. E	What can you call it? Because we always say three-fourths.	Ms. E's tone of voice suggests that she is ***angry*** that the

			students are responding to her question.
37:48	Manuel D	I ain't say it's three-sixteenths.	
37:50	Ms. E	Shh, [Tora], I'm getting distracted	Ms. E expresses ***disapproval of Tora's behavior*** indicating that her talking is distracting.
37:52	Manuel D	I really don't get it why…	
37:53	Ms. E	Because we never say, let's say this one. What do we call this?	
37:58	Dave	Three-fourths.	Ms. E is encouraging students to give fraction names in terms of "a whole", leading up to the point when she asks for the students to express three eighths of a half.
37:59	Ms. E	Three-fourths of a what?	
38:00	Students	Of a whole	
38:01	Ms. E	It's always of a whole. We don't even say the word "of a whole." How about…this?	
38:09	Students	A half.	
38:10	Ms. E	A half of?	
38:13	Students	A whole	
38:14	Ms. E	Ok, a whole. So then what do we call three-eighths of a half? Huh? Three and a half eighths?	
38:28	Students	Three and a half.	
38:30	Ms. E	Ok three-eighths of a half would be what fraction? We can't just say three-eighths and say three-eighths.	Ms. E conveys ***curiosity*** and ***urgency*** in finding a simple fraction name for three-eighths of a half.
38:34	Manuel D	Three-eighths of a half is the same thing as…	
38:36	Ms. E	So what is three-eighths of a half?	
38:38	Dave	Three-eighths of a half?	
38:40	Ms. E	What is three-eighths of a half? What fraction is that?	
38:43	Dave	one, one, one, one…third.	Dave presents faulty reasoning.
38:47	Ms. E	Hmm, good question.	
38:50	Dave	One-fifth?	It is inferred that Dave is making guesses at this point.

38:51	Ms. E	Alright, so I'm gonna ask you this again. I'm gonna put this transparency back, there's still a question on this one. We're gonna go back to it another time.	Ms. E abandons the problem. She ***calmly*** ends the episode by telling the students that she is going to put the transparency away for a future discussion.
End TSMIE [E.02.b.02]			

Discussion of TSMIE [E.02.b.02]

In this episode, many students are confused, there are moments when the students and Ms. E are yelling at each other and mathematical progress is not made.

At the start of this episode Manuel D is seated next to the overhead projector and Ms. E is sitting on top of an empty student desk, adjacent to the overhead projector. As Manuel D begins to present his solution to a mathematical task he speaks quietly. Ms. E and many students complain that Manuel D is not speaking loud enough for all to hear. Ms. E states with a tone of annoyance, "Speak up! Now is the time to speak up. Yes, I heard you a lot earlier" [13:44, L], recalling how Manuel D had been talking out of turn during Tora's presentation.

Tora retorts, "We can't hear you…that's what you did to me…that's what you did to me" and Ms. E intervenes, sternly expressing her disapproval of Tora's taunting, "It does not mean that you can do it to him now". Ms. E reiterates her expectation of peer-to-peer respect, "I want you to show him respect." [13:50, L]

Even after Manuel D raises his voice he appears to struggle with the mathematics, as he hesitates and then looks at Dave in an apparent plea for help. Ms. E intervenes, and encourages students in the audience to "Help, help, help." [14:35, L]

Dave and N'yette join Manuel D at the overhead projector in an apparent attempt to help him with the mathematics. After some collaboration, which is too quiet to be heard on the audio, N'yette takes over the presentation and becomes the primary speaker for the

group. Throughout the episode as many as four students at a time come to support Manuel D at the overhead projector. As these students collaborate among themselves, Ms. E occasionally intervenes asking questions about the students' mathematical thinking.

Approximately four minutes into the episode, N'yette, Dave and Manuel D present their answer, which prompts a discussion that will carry on for more than twenty minutes. Their answer, "three-out of eight", is accompanied by a model showing the fractional value, three-sixteenths. While three out of eight is the mathematically correct answer to the question in the problem, the model does not agree with the numerical representation. It may be inferred that Ms. E is displeased with the students' reasoning because she says, "Three out of eight…Wait a minute. Class, do you agree that…" [16:39, L] and then approaches the overhead projector.

She counts three shaded squares and sixteen squares in the whole brownie pan on the students' model in an inferred attempt to point out the discrepancy between the visual representation and the numerical answer.

N'yette expresses faulty reasoning, asserting that if half of the brownie pan has already been purchased, as was written in the problem, then when naming the shaded section in the model it is not necessary to consider the "missing" piece. "We can't count that cause they're already gone." [17:01, L] This is contradictory to N'yette's understanding during the lesson on the previous day when she demonstrated correct reasoning that the whole pan should be considered when naming the whole.

Ms. E intervenes beginning 17:17, in an apparent attempt to help the students recognize that the entire brownie pan should represent the whole when naming fractional

parts, regardless of whether or not half is sold. “Can you draw the same drawing below that without shading that first half?” and at 17:50, Ms. E instructs, “just cross out what Aunt Serena bought.” She proceeds to lead the students through the process of naming the fractional value of what Aunt Serena bought.

The students again draw a model, which represents three out of sixteen, yet they once again name the shaded section three out of eight.

For approximately the next twenty-one minutes, Ms. E engages in a whole class discussion with the students, in an apparent attempt to help students overcome this conceptual hurdle. Throughout this discussion, it may be inferred that Ms. E exhibits displeasure with student reasoning. For example, “No [Ms. E shakes her head, “no”] I’m not convinced that’s three eighths.” [18:07, L]. Also, throughout this episode, many students engage in side conversations and it is not clear whether or not most of the students are engaged with the mathematics.

Ms. E begins to exhibit signs of frustration with the level of student engagement about five minutes into this discussion when she speaks directly to Tora and Lorissa, “You are not listening to your own presentation” [22:02, L]. And at 23:37, Ms. E clearly states her displeasure with student engagement when she says, “I’m getting so agitated here.”

The discussion escalates to a point where the students and Ms. E have raised voices, each party apparently steadfast on their interpretation of the mathematical model.

“Ok, I don’t think I’m getting all of you, I want you to look at my eyes right now. Every single person in the room.” [23:37, L]

In an apparent attempt to help the students understand why the purchased section must be considered part of the whole, Ms. E intervenes by presenting a simpler example. She draws square that is cut into two halves, "Alright, so let's see this part. Hey, this part is one half, right?... That gets sold, [Ms. E then gestures as if she is pushing half of the brownies away] so then can we now then call this part one pan? Only because this was sold?" [25:04 – 25:15, L]

The students appear to understand Ms. E's reasoning and for a moment, this intervention seems like it might be working. Ms. E exhibits relief, "that's what I'm saying". [25:20, L] However, this moment is fleeting, and it is apparent that students do not make the conceptual leap to apply this simpler problem to the original problem.

Ms. E continues to exhibit frustration and displeasure with the students' mathematical reasoning as the argument escalates and the students and teacher are shouting at each other. Some students claim that the fractional name can change depending on the situation, yet they do not provide a valid argument with this claim. Ms. E exhibits surprise and uses sarcasm remarking, "I feel like calling it three twenty-fourths." [28:34, L]

It appears that Manuel D has a mathematically valid reason to explain three eighths and three sixteenths are not equivalent fractions. Ms. E invites him to the overhead projector and he demonstrates that the two fractions are different sizes.

Ms. E pleads with the students to become re-engaged with the mathematics. "Guys, I need you, I need you with me. I really need you with me here. It's very important. I'm not gonna invest time on this, just so we could scream at each other here. This is absolutely very important." [34:17, L]

In a final attempt to correct student reasoning, Ms. E says, "Ok, start from the top. One more time. Hopefully we can get something here." [36:39, L]. During this intervention, Ms. E attempts to help students rename three-eighths of a half as three-sixteenths of a whole. Her intervention is unsuccessful when the only student response to her question, "So what is three-eighths of a half?" are Dave's answers which are inferred to be guesses, "one, one, one, one…third. One-fifth?" [38:40 – 38:50, L]

Immediately following Dave's responses, Ms. E abandons the discussion. "Alright, so I'm gonna ask you this again. I'm gonna put this transparency back, there's still a question on this one. We're gonna go back to it another time." [38:51, L] It may be inferred that Ms. E recognizes that she has exhausted her efforts to help her students with this concept and perhaps she has taken a cue from the lack of student engagement, choosing to continue this discussion at a later date.

Beginning at 39:33, N'yette reads the statement of a new mathematical problem aloud from the textbook. "The school fair was almost over. Paulo and Paula wanted to sell all the remaining brownies in a hurry, so they decided to offer a discount of 20% on all sales. They had 2 ¼ pans of brownies left. Remember, they originally sold a pan of brownies for $24." Ms. E then poses specific questions about the conditions of the problem to the whole class.

Ms. E tells the students to begin working on the task and as she walks around the classroom looking at the students' notebooks. Some students demonstrate evidence of engagement with the task as they write in their notebooks. It may be inferred that Ms. E is dissatisfied with student engagement because as she circulates throughout the room, she makes the following requests.

"[Jackie], open your notebook." [41:41, L] **[Dis-b]**

"Ladies, let me see…I don't see anything. Let me see what you have." [42:28, L] **[Dis-b]**

"I'm back and there's still nothing on the notebook. Let's go. Let's go. Getting somewhere?" [44:37, L] **[Dis-b]**

Immediately prior to the start of this episode, Ms. E walks past N'yette's desk. N'yette appears to be engaged with the mathematics because she is drawing a model of three square brownie pans in her notebook.

Ms. E's interaction with N'yette is different, and she appears satisfied with N'yette's progress. Ms. E leans in toward N'yette and quietly inquires, "So far, so good?" [45:07, L] N'yette responds to Ms. E by asking a question about the conditions of the problem. Ms. E's face is not visible from this camera angle, so facial expressions cannot be described.

Begin TSMIE [E.02.b.03] [45:07 – 46:38, L]			
Participants: Ms. E, N'yette			
Time	**Speaker**	**Transcript**	**Descriptions / Comments**
45:07	Ms. E	Are we getting anywhere? So far, so good?	Ms. E appears to be ***curious*** about the students' mathematical progress when she leans towards N'yette's desk and asks, "Are we getting anywhere? So far, so good?" Her arms are folded, and her tone of voice is ***gentle*** and it may be inferred that Ms. E is satisfied with N'yette's mathematical progress. **[Int]**
45:16	N'yette	So he, it says he…about a half, so that mean he buy whole pan…half of that.. and then that's half. [N'yette has drawn three squares in her notebook. She gestures towards her model as she speaks. Ms. E stands next to N'yette's desk with her arms folded as	It appears that N'yette may be ***lacking in confidence*** of her interpretation of the task and it may be inferred that she is ***seeking validation*** from Ms. E.

		N'yette speaks.]	
45:26	Ms. E	[Ms. E moves N'yette's notebook down on her desk.] Half of what's left. [Ms. E is standing next to N'yette's desk and her arms are crossed.]	It may be inferred that Ms. E moves N'y notebook in order to read the problem in
45:30	N'yette	Mm…hm…Wait, I dinn't know. [N'yette looks away from Ms. E and covers her mouth with her hand. She is smiling.]	It may be inferred that N'yette feels ***puzzled*** and has encountered a moment of impasse because she looks away from her work and from Ms. E.
45:35	Ms. E	If I go to the booth, [Ms. E taps her foot on the floor] what would I see?	Ms. E appears to recognize that N'yette has reached a point of impasse. She ***gently*** redirects N'yette to the task by asking her a question about the context of the problem. **[Rsi, Int]**
45:37	N'yette	You would see one full pan, one full pan, and a half…fourth of a full pan. [N'yette gestures on her desk showing three pans.]	
45:42	Ms. E	Where is that here? [Ms. E gestures towards N'yette's drawing on her notebook.]	Mrs. E demonstrates ***curiosity*** and appears focused on N'yette's representation of the brownie pans. **[Int]**
45:44	N'yette	This is one whole pan, one whole pan and half. [N'yette gestures toward each part of her drawing as she speaks.]	
45:46	Ms. E	Ok. And I say to you I'm buying half of that.	Ms. E ***validates*** N'yette's drawing and her mathematical understanding when she proceeds to ask N'yette a follow-up question. **[Val]**
45:51	N'yette	Half of what?	N'yette seeks clarification.
45:52	Ms. E	What…everything. Half of everything.	Ms. E is speaking quickly and her tone of voice may suggest that Ms. E is becoming ***impatient*** with N'yette's mathematical understanding.
45:58	N'yette	So…so… [N'yette is erasing on her paper.]	N'yette appears to be considering her response to Ms. E's question.

46:02	Ms. E	[Lorissa], did you get it?	Ms. E speaks to a student who is off-camera.
46:04	Lorissa	She got my eraser.	Many students are perceived to be off-task because they are talking about an eraser.
46:06	Student	Man y'all always playin'	
46:07	Student	Shut up.	
	Ms. E	Shhh…	It may be inferred that Ms. E is responding to the off-task behavior by shushing the students. **[Dis-m]**
46:12	N'yette	So wait, Ms. Endaya.	
46:12	Ms. E	Yes?	
46:14	N'yette	Ok. If we buy a half of everything, you got here…	
46:17	Ms. E	Mm-hmm	
46:21	N'yette	He'll buy half of one whole, half of this… [Ms. E "Mmm hmm"] and half of this. [N'yette shades a portion of each of the three squares on her notebook.] It's half of everything we got. [N'yette sits back in her chair.]	In two of the three squares that N'yette has drawn, she has shaded in six out of nine sections. N'yette appears to demonstrate ***satisfaction*** with her mathematical solution as she sits back in her chair.
46:30	Ms. E	So what would the fraction be? And how much do you think I should pay? [Ms. E walks away at 46:38.]	Ms. E expresses curiosity about the answer "what would the fraction be?" She does not address the potential error in N'yette's drawing. Ms. E appears ***satisfied*** with N'yette's interpretation of the problem because she poses the question, "So what would the fraction be?" Her tone of voice is ***emotionless***. Ms. E asks the follow-up question, demonstrating interest in N'yette's understanding, "And how much do you think I should pay?" Ms. E does not wait for N'yette to answer these two questions she walks away from N'yette's desk to the opposite side of the classroom. **[Sat]**
End TSMIE [E.02.b.03]			

Discussion of TSMIE [E.02.b.03]

Ms. E initates this episode when she passes by N'yette's desk, leans toward N'yette and asks, "So far, so good?" N'yette asks for Ms. E's help in interpreting the problem. Ms. E appears to recognize that N'yette may be misinterpreting the conditions of the problem, and Ms. E asks her to visualize what she would see "if I go to the booth".

N'yette appears to exhibit confusion about the task, when she covers her mouth with her hand and looks away from Ms. E and from her notebook. Ms. E intervenes, asking N'yette to visualize the brownie pans at the booth. "If I go to the booth, what would I see?" [45:35, L]. It appears that Ms. E is encouraging N'yette to think independently about the task because she does not directly tell N'yette how to draw the model, nor does she tell her how to think about the solution to the task.

After N'yette speaks correctly about the process to solve the task, yet shows faulty reasoning in her interpretation of her drawing, Ms. E appears satisfied with N'yette's progress because she poses two follow-up questions and then walks away from N'yette's desk. N'yette appears to remain engaged with the mathematics after Ms. E leaves because she is looking at her notebook and then she follows Ms. E across the room and shows her something in her notebook.

This intervention appears to foster N'yette's engagement and understanding of the problem because she is able to accurately describe two and one-fourth pans, which she has already drawn in her notebook.

Noah initiates this episode when he asks Ms. E a question about the conditions of the mathematical task. Ms. E is standing in front of Eli's and Noah's desks with her arms folded. She is not smiling. Prior to this episode, Ms. E was standing in front of Eli and Noah's desks, observing them as they worked.

Begin TSMIE [E.02.b.04] [47:07 -50:44 (L)]			
Participants: *Noah, Eli, Ms. E, Manuel D and Dave*			
Time	**Speaker**	**Transcript**	**Descriptions / Comments**
47:07	Noah	How much does one pan have? [Ms. E is standing in front of Noah and Eli.]	It may be inferred that Noah perceives the number of pieces in a brownie pan as important for solving this problem. Ms. E sounds ***surprised*** about Noah's question when she responds, "A whole pan?" Ms. E implies that the number of pieces in a whole brownie pan is irrelevant when she poses a question back to Eli and Noah. Ms. E's tone of voice may be interpreted to show that she is ***irritated*** by the boys' question. **[Int, Dis-m]**
47:10	Ms. E	A whole pan?	
47:11	Noah	Yeah but like how many brownies does one pan have? [Ms. E crosses her arms.]	
47:14	Ms. E	How many brownies do you think it has? And does it matter if there are sixteen or twenty-four pieces in there? I'm buying a whole pan.	
47:25	Noah	You're buying two whole pans, right?	
47:27	Ms. E	Well there are two whole pans and a fourth. I'm buying *half* of everything that's on the booth. [Ms. E's arms are crossed and she purses her lips.]	Ms. E conveys information about the statement of the task. Her body language, crossed arms and pursed lips may be inferred to indicate that she is ***anxious*** for the boys to make mathematical progress with the given information.
47:33	Eli	You're buying one fourth and two pans?	
47:36	Ms. E	Two and one fourth pans.	
47:39	Noah	Ok. You said that he's buying half of…	
47:42	Ms. E	Half of everything.	

47:42	Noah	Half of everything?	
47:43	Ms. E	Mm-hmm.	Ms. E's tone of voice conveys ***encouragement*** regarding Noah's understanding of the conditions of the task. **[Enc]**
47:44	Noah	So half, so we find out what…so we find out what, um, what, what he buys? [Noah looks at Ms. E and raises his eyebrows.]	
47:52	Ms. E	What he's buying, yeah.	Ms. E validates Noah's question. **[Val]**
47:55	Noah	And all of the half?	
47:57	Ms. E	It's right here. Only for math.	
48:04	Dave	I know right. [Ms. E], what's half of one-fourth. [Dave is asking this question off-camera.]	
48:07	Ms. E	What's half of one-fourth? Good question. What is half of one-fourth?	Ms. E validates Dave's question, then she replies by asking Dave to figure out the answer to the question. Ms. E appears ***optimistic*** that Dave is capable of determining what is half of one-fourth. **[Val]**
48:09	Dave	That's what I'm asking you. [Dave smiles.]	
48:13	Eli	That's easy. [Eli shrugs his shoulders and leans towards Dave.]	Based on Eli's tone of voice, it may be inferred that Eli's initial reaction to Dave's question is criticism. Ms. E reacts by immediately posing the question to Eli. Her tone of voice is ***sharp,*** and it may be inferred that she intends to put Eli on the spot because she may be ***angered*** by his critical remark towards Dave. **[Dis-b]** Eli exhibits ***surprise*** that Ms. E has put him to the task of finding the answer. It may be inferred that Eli is uneasy with this task because he hesitates before he begins to draw a model in is notebook.
48:14	Ms. E	What is it?	
48:15	Eli	What, one fourth?	
48:16	Ms. E	Half of one fourth. [Eli scratches his forehead then looks down at his notebook.]	
		[Eli draws a square in	

		his notebook and divides the square into four equal parts.]	
48:27	Ms. E	Half of one-fourth is one half. Could you show me that half of one-fourth is half. [Ms. E is talking to a student who is out of camera view.]	It appears that Ms. E is repeating a student's claim that was made off-camera because she asks the student to provide her with proof that half of one-fourth is one-half. Ms. E's tone of voice conveys ***doubt*** about this fact. **[Con]**
48:31	N'yette	I'm sayin' it's half.	
48:37	Ms. E	What is half of one fourth? [Eli shades in one fourth of the square.]	Ms. E's tone of voice conveys ***curiosity*** about the students' understanding of half of one-fourth**.** **[Int]**
48:40	Eli	Like that? [Eli has drawn a square and partitioned it into four relatively equal sections. He shades in one of the four sections to represent fourth of the square.]	It may be inferred that Eli looks for Ms. E's approval when he sits back in his chair and asks, "Like that?"
48:41	Ms. E	Half of that.	Ms. E shows ***no emotion*** when she puts Eli to the task of finding half of the fourth that he has drawn in his notebook.
48:42	Eli	Half of that?	
48:44	Ms. E	Do you agree with [Noah]? That half of one fourth is half?	
48:46	Student	Yes.	
48:47	Ms. E	Half of one fourth is half. Could you prove that? [Noah], [Tora] is saying a fourth is smaller than… [Ms. E is sitting on the desk next to Tora.]	Ms. E appears to be using Tora's idea to stimulate Noah's thinking. **[Blds]**
48:53	Tora	A half.	
49:04	Ms. E	Than a half. [Eli changes his model to shade one half of the square.] …so what's half of one-fourth, [Eli]?	Ms. E demonstrates ***curiosity*** when posing this question to Eli. **[Int]**

49:06	Eli	Half of one fourth?	It appears that Eli is ***puzzled*** about how to find one-half of one-fourth because he continues to repeat the question and has not shown evidence of mathematical progress towards the answer. Ms. E is exhibiting ***patience*** with Eli as she continues to express ***curiosity*** as she repeats the question and waits for Eli to produce an answer. **[Int]** Eli is presenting faulty reasoning as he has shaded in one half of the square to represent one-half of one-fourth. Although he verbally defends his thinking, Eli's facial expression may be conveyed to express ***uncertainty***.
49:07	Ms. E	Mm-hmm.	
49:10	Eli	Takin' a half a part of a fourth.	
49:14	Ms. E	Takin', yeah…Could you do that? Taking half of a fourth?	
49:17	Eli	That's what I did. [Eli looks at his notebook, which shows two fourths shaded in the square. Eli looks up at Ms. E.]	
49:19	Noah	Like this? [Noah shows his notebook to Ms. E. Eli looks at the notebook.]	
49:20	Ms. E	Show me? This is… [Ms. E moves Noah's notebook towards Eli.] where's the one fourth?	It may be inferred that Ms. E is ***displeased*** with Eli's reasoning, yet she does not argue with his reasoning. She intervenes, asking Eli to interpret his model. **[Int]**
		[Noah shows the drawing in his notebook to Ms. E.]	
49:25	Eli	Oh, one fourth.	Eli's tone of voice may be inferred to convey ***surprise*** as if this is new information. It may be inferred that Eli is experiencing impasse and may be feeling ***stressed*** because he has been unable to produce an answer to the question that he claimed was "easy" moments earlier.
49:28	Ms. E	Ok, we start with one fourth. Where's the one fourth? [Noah's drawing	Ms. E raises her tone of voice and sounds ***impatient*** when she says, "I want half of that." Ms. E's gesture is quick when she points to Eli's new drawing.

		shows a representation of one half, yet he has labeled it ¼.] [Eli draws a new model and shades in one-fourth of the square.] I want half of that. [Ms. E points to Eli's new model.] What would you give me?	**[Dis-m]**
49:34	Noah	You shaded two boxes? [Eli shades in two fourths, making his new drawing identical to the original square, with a representation of one-half. Noah looks at Eli's notebook as Eli draws and then he jiggles his leg as he looks up at Ms. E.]	Based on Noah's body language, it may be inferred that he is feeling ***anxious.***
49:38	Ms. E	Ok. So if I want half of this, you're gonna give me these two. [Ms. E points to the model in Eli's notebook that shows a representation of one-half. Eli looks up at Ms. E. Noah looks at Eli after Ms. E speaks.]	Ms. E shows ***no emotion*** despite the fact that Eli has presented faulty reasoning.
49:45	Noah and Eli	Yeah. Yes.	
49:46	Ms. E	If I wanted…ok, [Ms. E turns around an speaks to someone who is out of camera view.] Let me ask you a question. I want this part, right here. There are four parts. One,	It may be inferred that Ms. E is ***displeased*** with Eli's reasoning, yet she still does not correct him. It appears that Ms. E is attempting to correct the boys without telling them that they are wrong and without telling them the correct answer. **[Dis-m]** It appears that Ms. E is appealing to Manuel D to help Noah and Eli past this

		two, three, four. I say I want half of this.	point of impasse. She interprets Eli's second drawing for Manuel D, then, Ms. E poses a question to Manuel D.
49:56	Manuel D	I don't get you. [Manuel D is standing next to Ms. E, leaning on Noah and Eli's desks. He wrinkles his eyebrows.]	Manuel expresses ***confusion*** about Ms. E's question and Ms. E responds by telling Eli to construct another drawing to represent one-fourth. It may be inferred that Ms. E recognizes that the drawings with the representations of one-half were confusing to Manuel D. Ms. E's tone of voice may convey that she is ***anxious*** about the mathematical progress because she raises her voice slightly and begins to speak quickly. **[Dis-m]**
49:57	Ms. E	Again, start again. Start your drawing. Shade one part.	
50:00	Noah	She said one…	
50:02	Manuel D	Here, use a pencil. [Manuel D hands a pencil to Eli.]	
	Ms. E	It's alright.	
50:14	Student	She said what's…	
50:16	Student	[Eli uses Manuel D's pencil to draw a square in his notebook with four approximately equal pieces then he shades in one half of one-fourth.] Ms. Endaya?	Although Eli's model correctly shows the literal representation of one-half of one-fourth, it is unlikely that his model will lead the students to the correct answer because it does not show the entire model portioned into eighths.
50:17	Ms. E	Yes?	
50:19	Manuel D	We were thinking…	
50:22	Ms. E	Half of.	
50:23	Student	One fourth.	
50:24	Ms. E	Half of one fourth. What do you call that then?	
50:26	Dave	He has to get a half and a whole because you can't split one fourth and half. [Dave has entered the converation and stands behind Eli and Noah.]	

50:32	Ms. E	A whole and a half.	
50:33	Manuel D	…because he's getting a whole pan and a half	
50:37	Ms. E	What do you call that? [Ms. E leans towards Eli and points at his notebook.]	Ms. E appears ***curious*** about the fraction name for Eli's representation. **[Int]**
50:36	Dave	It's the only way you could split…one fourth…you forgot that this counted too.	
50:39	Manuel D	What do we call what?	
50:42	Ms. E	Find out for me what that's called.	
50:43	Manuel D	How? [Manuel D stomps his foot on the floor.]	Manuel D expresses ***frustration*** by stomping his foot on the floor.
With the end of the class period approaching, Ms. E walks away from Noah and Eli and calls for the attention of the whole class so that she can assign homework and dismiss the class. No resolution was made for the question, "what is half of one fourth" during this episode. However, both Eli and Noah show evidence of mathematical engagement after Ms. E walks away. Eli is seated, with his head resting on one hand, looking at his notebook. Noah looks at Eli and asks, "Did you get it?" Eli shakes his head, "no" and bangs his pen on his desk. Both Eli's facial expression and his gestures illustrate his likely ***frustration*** and ***sadness*** that he was unable to answer the "easy" question. As Ms. E assigns the homework to the whole class, the camera shows evidence that Eli may be still pondering the question as he looks at his notebook. Manuel D expresses ***frustration*** by stomping his foot on the floor			
End TSMIE [E.02.b.04]			

Discussion of TSMIE [E.02.b.04]

This episode begins when Noah poses a question to Ms. E about the conditions of the mathematical task. Ms. E appears to be surprised by Noah's question possibly because it suggests that Noah and Eli could not have made progress in the seven minutes since the task was assigned.

The focus of the mathematical interaction takes a turn when Dave, who is seated off-camera, asks Ms. E, "Ms. Endaya, what's half of one fourth?" [48:04, L] Eli

responds to Dave's question quickly and confidently stating, "That's easy." [48:13, L] Ms. E responds by asking Eli to answer Dave's question, but her tone of voice may be inferred to suggest sarcasm and Ms. E seems to put Eli on the spot to answer the "easy" question.

Eli appears to struggle with this idea, initially drawing two different, yet similar models that both represent one-half. Ms. E exhibits displeasure with Eli's reasoning and appeals to Manuel D for help. When Manuel D presents confusion about Eli's drawings, Ms. E appears impatient, as she instructs Eli, "Again, start again. Start your drawing. Shade one part." [49:57, L]. Eli's third drawing correctly represents one-half of one-fourth of a square, yet it does not help him articulate the correct answer because he has only shaded one-half of one of the fourths. In this representation, it is not obvious that the shaded section represents one-eighth of the whole square.

The episode concludes without any resolution of the mathematical question. Both Eli and Noah appear dismayed by the fact that they were unable to present a correct solution. Manuel D stomps his foot in apparent frustration as Ms. E walks away, concluding the episode by bluntly saying, "Find out for me what that is called." [50:42, L]

APPENDIX B

Ms. B: Classroom observation analysis, "Fences for Grazing"

Day One

Eight teacher/student(s) mathematical interaction episodes (TSMIEs) are identified in the video data collected in Ms. B's classroom on October 18, 2006, Day One of Cycle One during a forty-five minute class period.

On the first of a two-day lesson titled, "Fences for Grazing", Ms. B stands near the front of the room to introduce the lesson. The students' desks are grouped in four sections, each representing one of the four quadrants of the Cartesian plane. Students are seated in groups ranging from three students to five students at each cluster of student desks. Ms. B announces that she will read the problem from the paper that she is holding as the students read along with her. Each student has a copy of the problem on his or her desk.

> "I'm going to read it out-loud, you're going to read it silently. It says Farmer Joe has a cow named Bessie. He bought 100 feet of fencing. He needs you to help him create a rectangular fenced in space with a maximum area for Bessie to graze. Draw a diagram with the length and width to show the maximum area. [Ms. E walks over to a male student and puts her hand on his desk.] Explain how you found the maximum area. How many poles would you have for this area if you need one pole for every five feet? [Ms. E looks up at the student and looks around the room at each table as she speaks.] How many bullets do you see?

Time	Speaker	Transcript	Descriptions / Comments
Begin TSMIE [B.01.a.01] 3:35 – 5:51(S)			
Participants: Whole class, Ghee, Shawn This episode begins when Ms. E introduces the problem to the students by reading aloud.			
3:35	Ms. B	I'm going to read it out-loud, you're going to read it silently. It says Farmer Joe has a cow named Bessie. He bought one hundred feet of fencing. He needs you to help him create a rectangular fenced in space with a maximum area for Bessie to graze. Draw a diagram with the length and width to show the maximum area. [Ms. E walks over to a male student and puts her hand on his desk.] Explain how you found the maximum area. How many poles would you have for this area if you need one pole for every five feet? [Ms. E looks up at the student and looks around the room at each table as she speaks.] How many bullets do you see?	Ms. B reads the problem aloud to the students as they follow along silently. When Ms. B reads the task, she annunciates each individual word and speaks with an ***authoritative*** tone in her voice. It may be inferred that her tone of voice implies that she ***values*** the attention to the problem statement. She moves through the room as she reads, putting herself in close proximity with the students. It appears that Ms. B ***may perceive this boy's behavior to be off-task***, because she glances at him briefly as she reads the task aloud. Ms. B's facial expression is ***serious*** when she looks at this student, possibly sending a message that she is serious about having his attention as she reads the task. It may be inferred that Ms. B asks these two questions to check for student understanding of the conditions of the mathematical task.
04:17	Class	Three.	
04:18	Ms. B	Three, what does that tell us?	
04:20	Class	You got three questions.	
04:21	Ms. B	You are going to have three different focus areas, you need to have three different answers, at least. You guys may begin, you are going to work in your small groups together.	Ms B is apologetic when it appears she realizes that students do not have the materials that they need.

		You can have…. Oh I'm sorry you know what [Al] pass out the buckets [Al stands up] , the buckets are now over here. So at the end of the class just put the bucket back over there. Your containers are over there.	
04:51		[There is a knock at the door and Ms. B walks to the door, nodding her head and smiling.]	Ms. B appears to be ***smiling*** at the person(s) who are at the classroom door.
04:56	Ghee	Yea he's supposed to be here. He's supposed to be in here.	
05:07		[Shay enters the room and walks to a table at the back of the classroom.]	
05:07	Student	Do you have any questions?	
05:13	Ms. B	[Ms. B approaches the table and leans on a desk at Dana's group. Dana says something that is inaudible and Ms. B looks at her.] Do you guys have any questions?	Ms. B appears ***curious*** about the students' mathematical progress when she leans on the desk and asks, "Do you guys have any questions?" Her tone of voice and language is ***casual and friendly*** as she addresses her students, "you guys".
05:14	Ms. B	[Devin speaks something inaudible. [Al] is passing out the buckets]	
05:23	Ms. B	[Ms. B sits on a chair at the empty desk across from Dana at this table.] Draw a rectangle.	
05:25	Shawn	That's what it says.	
05:29	Ms. B	Ok, I'm just asking. [Students take rulers from the bucket in the center of the table. Ms. E moves a pocketbook from the desktop and places it inside of the desk. Ms. B looks around the classroom, occasionally glancing at these students' papers.	Ms. B appears defensive in response to Shawn's comment.
05:51	Ms. B	[Ms. B stands up and walks away from this group.]	It may be inferred that Ms. B is ***satisfied*** with the students' level of engagement.
End TSMIE [B.01.a.01]			

Discussion of TSMIE [B.01.a.01]

In this episode, Ms. B introduces the problem and checks for student understanding. She appears serious about the task when presenting it to the whole class. It may be inferred that Ms. B is curious about the students' understanding of the requirements of the problem when she asks the students to tell her how many bullets they need to address. Ms. B briefly joins Dana's table, apparently checking to see if the students have any questions as they begin the task. It appears that all four students are engaged in drawing a rectangle with pencils and rulers. It may be inferred that Ms. B feels satisfied with the level of student engagement as she walks away from this table. Note: Ms. B is out of camera audio range until 6:30. She can be seen standing next to Shay at 6:34 from the still camera view and it appears that she is speaking to him, but it is inaudible from the still camera, and she is out of camera range for the two roving cameras.

Begin TSMIE [B.01.a.02] 8:35 – 13:48 (S)			
Participants: Ghee, Shawn, Devin, Dana and Emmy, Ms. B At 7:48, Ghee raises his hand and at 8:47, Shawn raises his hand, and now both boys have their hands raised. Ms. B approaches the table, likely in response to the boys. This is the second time since the beginning of the class period that Ms. B has interacted with the students in this small group.			
08:35	Shawn	[Flashes a middle finger at the camera, Ghee and Dana both look over.] See that! [Shawn smiles, then raises his hand.]	
08:54	Shawn	[Mrs. B] …I need a pencil, I need a pencil…[inaudible]	Shawn asks Ms. B if he can have a pencil and she hands the pencil to Shawn without any comment. Ms. B sits down at the empty desk across from Dana.
08:59	Ms. B	Ok, so what's up? [Ms. B looks around at each member of the	Ms. B's facial expression appears ***relaxed*** and ***calm*** as she looks

		group].... yeah you're done.	around at each student seated at this group. Her language "what's up?" suggests that she is ***interested*** in the students' mathematical progress. **[Int]**
09:01	Dana	We drew a rectangle.	
09:05	Ms. B	Ok, you drew a rectangle. [Ms. B looks at each student.]	Ms. B repeats Dana's words and looks around at each student again.
09:11	Ghee	....maximum area....(inaudible)	
09:13	Ms. B	Ok. Ok. [Ms. B looks down at a paper on the desk.]	Ms. B looks at Ghee as he talks, and she appears ***interested*** in what he is saying. **[Int]**
09:19	Ms. B	Ok, ok. So what do you guys have to do?	Ms. B's tone of voice is ***quiet*** as she appears ***curious*** about their next steps for solving the problem. This intervention may be aimed at promoting the students' autonomy when Ms. B asks the students what they need to do next. **[Int]**
09:20	Ghee	I know there is a hundred feet of fencing...	
09:24	Ms. B	So what does a hundred feet of fencing mean? [Ghee keeps speaking too quietly for the camera. Dana also speaks, but her words are inaudible. Ms. B looks at Dana as she speaks.]	Ms. B's tone of voice is ***calm***, and she appears recognize that the students may be having difficulty getting started on the problem. She questions the students, appearing ***curious*** about their understanding of "one hundred feet of fencing". **[Rsi, Int, Blds]**
09:30	Ms. B	Okay, I'm just asking. [Ms. B's posture changes, she sits up slightly.] [Ms. B leans in and gestures with her hand toward Ghee and asks:] How can you show me that?	Ms. B appears to become ***defensive*** as she straightens her posture and speaks to Dana with a ***defensive*** tone of voice. Ms. B proceeds to show ***interest*** in Ghee's representation of one hundred feet of fencing. **[Int]**
09:35	Ghee	[Ghee points with his pencil at Dana's paper.]	

		Oh right. [Ghee erases something on his paper.]	
09:36	Devin	...two sides have to be...[inaudible] the length...and the width...a hundred feet.	
10:00	Dana	[Dana looks at Ms. B] So we gotta see, what adds up to the.... the length and the width?	It appears that Dana may be confused about the conditions of the problem and may be inferred that ***Ms. B recognizes her confusion***. Ms. B intervenes by redirecting the students to the problem statement. **[Rsi]**
10:04	Ms. B	I don't know, let's read it again and see what it says.	
10:09	Dana	Draw a diagram with the length and the width to show the maximum area.	
10:17	Ms. B	So let's go back to what you have to...[Ms. B gestures with her hand, and looks at Dana as she speaks.]	Ms. E expresses ***interest*** in the next steps that the students need to make toward solving the problem. For the second time in this episode she appears to support the students' autonomy, "what you have to do…" **[Int]**
10:28	Dana	Well, ummm, do we gotta like umm...like figure out numbers that go to make it to a hundred?	Dana appears uncertain about the question and may be looking for Ms. B's help in interpreting the problem. ***Ms. B appears to recognize Dana's impasse*** and responds by encouraging student discourse. She points to Ghee, Shawn and Devin and says, "Ask your group". **[Rsi, Endis, Int]**
10:29	Ms. B	[Ms. B raises her eyebrows and points to the other students in the group as she responds to Dana.] Ask your group.	
10:38	Ghee	[Says something inaudible.]	
10:51	Ms. B	Do you guys agree with what [Ghee] just said?	It appears that Ms. B is trying to ***encourage student discourse*** to help Dana and Ghee understand the problem. **[Endis]**
10:58	Ms. B	Can you explain to me why you did that? [Ms. B directs this question to Ghee. She furrows her brow and cranes her neck to look at Ghee's work.]	When the other students do not respond, Ms. B appears to try a different intervention. Ms. B expresses ***curiosity*** about Ghee's mathematical reasoning when she asks him to "explain why

			you did that”. **[Int]**
11:00	Ghee	... a hundred...and I found out that the sides have the same length, the top and the bottom, and the sides...the sides...[inaudible]	It appears that Ghee is describing some of the attributes of a rectangle that opposite sides have the same length.
11:28	Ms. B	Okay…[Ms. B looks down at the paper in front of her.] Okay, and so what did you just find? [Ms. B furrows her brow.]	Ms. B appears ***curious*** about Ghee’s representation and it may be inferred that Ms. B is asking Ghee to name either the area or perimeter of the rectangle. **[Int]**
11:32	Ghee	The area...(inaudible)	
11:46	Ms. B	[Devin], [Shawn], [Dana]…[Ms. B looks at each student as she says their names and smiles at Dana] So what do you think he just found?	Ms. B expresses ***curiosity*** about the other students’ opinions thoughts about Ghee’s comment. It may be inferred that she asks this question in an effort to engage Devin, Shawn and Dana by asking them to identify what Ghee “just found”. At this point, Ghee and Dana have carried the majority of the discussion. **[Int, Endis]**

11:52	Dana	The maximum area. The maximum area. [Dana looks at Ms. B. Ms. B is looking down at the paper in front of her.]	Dana appears to think that Ghee solved the problem.
11:54	Ms. B	Did you guys talk about what area and perimeter is? [Ms. B has her arms folded and is leaning forward on the desk.]	It may be inferred that Ms. B ***recognizes that the students are confused*** about the difference between area and perimeter. **[Rsi]** She intervenes by naming the mathematical terms and asking the students to recall the definitions of each. **[Endis, Int]** When she bows her head down, it may be inferred that Ms. B may be disappointed with the level of student behavioral engagement. **[Dis-b]**
11:59	Ms. B	[Ms. B bows her head down then poses a question to the whole group.] What's the area and what's the perimeter. How would you define the perimeter? [Ms. B looks from one student to the other]	
12:12	Ghee	The outside.	
12:14	Ms. B	Okay. Right? Right? Right? [Ms. B looks around at each student in the group.] [Ms. B shrugs her shoulders and asks:] Wrong?	Ms. B appears to be checking for understanding while trying to engage the other students in the discussion. **[Endis]**
12:22	Ghee	It's wrong? [Ghee raises his eyebrows and looks at Dana.]	Ghee appears ***surprised*** by Ms. B's suggestion that his reasoning is wrong.
12:22	Dana	I'm saying right? Wrong?	Dana appears to be ***defensive*** when she responds to Ghee.
12:29	Ms. B	So what's the area?	Ms. B intervenes asking a mathematical question. **[Int]**
12:31	Dana	I think the area's the outside.	Ms. B does not respond to Dana's incorrect answer.
12:32	Ghee	The area is the inside, the outside is the perimeter [Shawn says something that is inaudible and Ghee responds saying:] No, the perimeter's the outside. No, haven't you heard of the perimeter… the outside [Ghee circles his arm above his head as he speaks]...the area's the inside.	Ghee's tone of voice may be inferred to convey ***confidence*** in his reasoning. Ms. B does not respond to Ghee's correct statements and it may be inferred that she is waiting for each of the students to express their ideas before affirming the idea of just one

			student.
12:54	Ms. B	Do you remember when we talked about perimeters, what word …helped us out. [Ms. B looks at Devin, Shawn and Ghee as she speaks.]	Ms. B refers to an earlier lesson in a possible attempt to stimulate the students' prior knowledge and to encourage them to apply prior knowledge to the solution of the current task.
13:02	Ghee	Perimeter!	
13:03	Ms. B	What word in perimeter helped us out? [Ms. B smiles] [Dana and Emmy are writing on their papers.]	Ms. B waits silently.
13:20	Ghee	Meter, meeter, peri-meter. I don't know [Ghee is laughing] ....rim...rim...rim, yes rim, it's the outside. That's what I said [Ghee and Ms. B are smiling]....the area's the inside.	Ghee expresses ***pride*** about his recall. Ms. B is also ***smiling*** with may be inferred to show that she also feels ***proud*** of Ghee for recalling the information. **[Val]**
13:27	Shawn	The area's the outside.	
13:38	Ghee	Don't you see rim here? [Ghee leans over toward Shawn and holds up his paper for Ghee to see.]	Ghee defends his answer to Shawn. Ms. B silently observes the exchange between Ghee and Shawn. Her facial expression may be inferred to show ***satisfaction*** with the exchange between Ghee and Shawn.
13:45	Ms. B	Okay, let's continue, do we each know what we're doing?	Ms. B appears ***satisfied*** with the level of student progress at this point when she asks, "do we know what we're doing?"
13:48	Ghee	Yes, now I do. [Ms. B sits and silently observes for approximately fifteen seconds before standing up and leaving the table.]	Ghee's affirmation may be inferred to be an expression of thanks toward Ms. B for her help.
End TSMIE [B.01.a.02]			

Discussion of TSMIE [B.01.a.02]

This episode begins when Ms. B responds to Ghee and Shawn who have their hands raised. It is inferred that Ms. B recognizes that the students may be confused about how to begin solving the problem. Ms. B intervenes by posing questions about the

conditions of the problem. Each time a student responds, Ms. B responds back with another question: "What does a hundred feet of fencing mean?", "Let's read it again.", "Did you guys talk about what area and perimeter is?" Ms. B also encourages student discourse as a possible strategy to foster the students' engagement. "Ask your group.", "Do you guys agree with what [Ghee] just said?"

Approximately four minutes into the episode, Ms. B appears to recognize that the students are confused about how to define area and perimeter. She intervenes by asking the students to recall information from a previous lesson, "Do you remember when we talked about perimeters, what word …helped us out?" This intervention appears successful as Ghee proudly responds "....rim...rim...rim, yes rim, it's the outside. That's what I said." Both Ghee and Ms. B smile in inferred pride of Ghee's illumination. Ghee's sense of accomplishment may foster his subsequent engagement with the mathematics. Before Ms. B leaves the group, she asks, "…do we each know what we're doing?" Ghee expresses satisfaction and self-efficacy, "now I do (know what to do)".

Ms. B silently observes the students for approximately thirty seconds before leaving the table. The students appear to remain engaged after Ms. B leaves their table because they engage in a brief discussion, then they begin writing on their papers.

BEGIN TSMIE [B.01.a.03] 12:40 – 19:14 (C)			
Participants: Al, Natasha, Kenny, Ms. B In the moments immediately preceding this episode, Natasha uses a calculator to divide one hundred by four to equal twenty-five. It appears that she has determined that the length of each side of the fence should be twenty-five. At 12:30, she says, "and the area gonna be...a hundred!". Kenny meanwhile is taking batteries out of a calculator. Ms. B approaches the table, stands next to Kenny and takes two calculators off of his desk and replaces the batteries as she speaks to the students.			
12:40	Ms. B	[Ms. B is standing next to Kenny's desk, looking at the calculators, as	Ms. B's tone of voice is ***calm*** and ***quiet*** as she appears

		she replaces the batteries. She does not look at the students when she asks:] What are you guys doing now?	***interested*** in the students' engagement. **[Int]**
12:48	Kenny	….we right…	
12:49	Ms. B	[Ms. B is looking at the calculators when she directs a question to Kenny.] Why you putting in those circles?	Ms. B expresses ***curiosity*** about Kenny's representation of circles. **[Int]**
12:53	Kenny	Poles!	
12:54	Ms. B	Poles? Okay...and you just put the circles on there? [Kenny is speaking, but is inaudible.] I don't know I'm just asking. Why did you put the circles *there*? [Ms. B sits down next to Kenny and looks at his paper as he speaks.]	Ms. B's tone of voice and language may be inferred to convey ***curiosity*** in Kenny's placement of the "poles" on the fence. It also may be inferred that she interprets Kenny's response about the placement of the poles as arbitrary. **[Int]**
13:05	Kenny	[Inaudible]	
13:15	Natasha	Height and length, I mean height and width are the same thing, right? [Natasha looks at Ms. B and gestures height and width with her hands.]	
13:19	Al	No. [Al's brows are furrowed as he looks at Ms. B and responds to Natasha's answer]	
13:22	Natasha	I know. It's the same.	
13:24	Al	No it's not. Nah! [Al is erasing on his paper]	
13:36	Ms. B	Height, no…She asked a question. [Ms. B quietly knocks her knuckles on the table.]	Ms. B appears to show no emotion.
13:39	Natasha	[Natasha is drawing on her paper]. Yes it is.	
13:40	Ms. B	Ask the question again, [Natasha].	Ms. B encourages student discourse. **[Endis]**
13:42	Natasha	I said, umm, [Natasha is looking at her paper] is height and length the same thing? [Natasha looks up at Ms. B and at Kenny] [Ms. B cranes her neck to look at Al's paper, he is seated diagonally from Ms. B.]	Natasha appears confused about the terminology for the sides of the rectangles.

13:55	Ms. B	So what are you drawing? [Ms. B asks Al]	Ms. B appears ***curious*** about Al's drawing. [Int, rp]
13:57	Al	I'm about to figure it out.	
14:10	Kenny	Width and height. [Kenny points to the two dimensions of the rectangle that he has drawn on his paper.]	
14:17	Ms. B	So how about labeling that, if that's what you've got to do. Length times width. [Ms. B shrugs her shoulders as she speaks to Kenny] Label the length and label the width. [Ms. B folds her arms, leans forward on the desk and looks around at each students' paper. The three students write on their papers as Ms. B sits silently, occasionally looking at each student's paper.]	Ms. B expresses ***interest*** in how students would name the length of each side of the rectangle. **[Int, rp]** Ms. B sits silently, looking from one student to another. Her facial expression appears ***emotionless*** as she observes the discussion.
14:48	Al	My rectangle, my rectangle….you measure, you get three, and down here it's not the same thing, it's longer then the one on top.	
14:58	Natasha	I'm saying it's length like this or like that. [Natasha traces the two dimensions on Al's paper with her finger.]	
15:02	Al	Like that.	
15:03	Natasha	Which one?	
15:05	Al	[Al picks up the calculator, using it like a prop to demonstrate length and width.] Like that [Al points to the width of the calculator] Is, is that right [Mrs. B]?	Al and Natasha appear confused about the names of each dimension of the rectangle.
15:14	Ms. B	[Ms. B raises her eyebrows and turns her hands up] I don't know, which is length to you?	Ms. B does not impose her own mathematical ideas and redirects the question to the students. **[Int]**
15:20	Natasha	No length is like that. [Natasha leans over to Al's paper and traces the two dimensions with her pencil] The width is like that.	
15:32	Al	Length and height, length and height is the same [Al looks at Ms. B and nods his head] uhhh…is the same answer…	Al makes this statement while looking at Ms. B and it may be inferred by his facial expression that he lacks confidence in his

			answer, and is looking at Ms. B for approval.
15:37	Natasha	No it's not!	Natasha expresses ***anger*** about Al's answer.
15:40	Ms. B	Ok, so does that help you out now? Okay. [Al and Natasha nod their heads]	Ms. B's tone of voice is ***calm*** as she confirms student understanding of "length". It may be inferred that Ms. B is ***curious*** about whether or not Natasha's confusion has been resolved. **[Int]**
15:43	Ms. B	How?	Ms. B appears to be curious about the students' reasoning. **[Int]**
15:45	Natasha	Because they said umm, draw a diagram of the length and the width. Like, I wanted to know if that was the length. [Natasha traces the longer side of the rectangle, which is drawn in a vertical direction on her paper with her pencil]. Because…height is that. [Natasha traces the longer side of her rectangle again.] That could be the same as length. [Natasha erases some writing on her paper.]	Natasha still appears confused about which dimensions should be named "length" and "height". She acknowledges that the problem uses the words "length" and "width" instead of using the word "height". It is possible that Natasha recalls the word, "height" from a different problem or context as it is not used in the current problem.
16:01	Ms. B	Okay.	
16:05	Ms. B	Okay, so how come you made you're box that size [Ms. B points at Al], and you made you're [Ms. B points at Kenny] box that size?	Ms. B's language may be inferred to convey ***curiosity*** about the reason why the three students have drawn three different-sized rectangles on their papers. Ms. B's tone of voice is calm, and her face is ***emotionless.*** **[Int]**
16:11	Al	You ain't got no box.	Al appears to correct Ms. B's use of the word "box".
16:12	Ms. B	Your rectangle.	Ms. B corrects herself in response to Al's comment.
16:16	Al	They both rectangles.	
16:18	Al	So which, which, ummm….	
16:20	Ms. B	So, is this right? [Ms. B draws a rectangle on a piece of paper. The	

		rectangle is smaller than all of the rectangles drawn by the students.] Is my rectangle a rectangle? [Ms. B looks at each of the students.]	
16:24	Al	No, it's a little too small.	
16:26	Natasha	No, no I don't think it matter what size because they gotta have four sides, if they too small, umm, and then you just put the numbers. [Natasha draws on her paper as she speaks.]	It may be inferred that Natasha is referring to the numerical labels that can define the lengths of the sides.
16:39	Ms. B	So what numbers? Any numbers?	
16:42	Natasha	No, twenty-five.	It may be inferred that Natasha has correctly solved the task, stating that each side of the rectangle should be twenty-five feet in length. Ms. B does not acknowledge her answer.
16:45	Al	Huh? Noooo. It say it's gotta be about five feet, right? The poles gotta be five feet.	
16:55	Ms. B	The poles have to be five feet.	Ms. B's tone of voice is quiet and ***emotionless*** as she repeats what Al just said.
17:01	Ms. B	So when you said numbers, what numbers are you talking about?	It appears that Ms. B is ***curious*** about what Natasha means when she said, "you just put the numbers". Natasha states that she was "confused" when she labeled the dimensions each twenty-five feet and she retracts her answer. Ms. B does not comment on Natasha's reasoning. **[Int]**
17:04	Natasha	I was going to say, I was confused, I was gonna say twenty-five, twenty-five, twenty-five, twenty five, like that, but now… [Natasha points to each side of the rectangle with her ruler as she speaks.]	
17:22	Al	…I'm going to…(inaudible)	
		[Ten-second pause, all three students are looking at their papers and writing. Ms. B is sitting still.]	
17:25	Ms. B	Okay, so [Kenny], what do you think? [Ms. B scratches her face and looks at Kenny] [The camera zooms in and shows Kenny's paper with a rectangle with two short sides and two long sides. He has also drawn	It may be inferred that Ms. B is attempting to reengage Kenny with the mathematical discussion. He has had only four speaking turns since the beginning of this

		dots along the edges of the rectangle, possibly to represent the poles as mentioned earlier.]	interaction episode. **[Endis]**
17:30	Kenny	[Inaudible]	Kenny mumbles something, but it is brief and inaudible. Ms. B looks back to Natasha.
17:37	Natasha	But this could be eighty, the sides could be eighty and this could be ten, right. Or that could be seventy and that could be….[Natasha is looking at her rectangle as she speaks]	Natasha presents faulty reasoning.
17:37	Al	I got five and a half….I got six, I got regular six inches.	
17:50	Ms. B	Where are you picking you're numbers from? [Ms. B asks Natasha] Why don't you share, [Ms. B looks and points at Al and Kenny] how you are getting the numbers that you are choosing? What numbers did you just say again? [Ms. B leans in toward Natasha and furrows her brow.]	Ms. B's tone of voice is ***calm***. She does not offer her opinion on Natasha's answers; rather she invites her to share her mathematical reasoning with Kenney and Al. **[Int, Endis]** Ms. B leans in toward Natasha and her tone of voice appears to become ***firm***. It is inferred that Natasha's numbers may confuse Ms. B. **[Con, Int]**
18:01	Natasha	Ummm, seventy and thirty and eighty and ten. I mean, yea, eighty and ten.	
18:09	Ms. B	So how are you picking those numbers? [As Ms. B speaks, she is facing Natasha, but her eyes are directed sideways at Al who has taken his nametag down and is writing on it. Ms. B reaches over, takes it from him and places it, standing up, on his desk.]	Ms. B expresses curiosity about how Natasha is choosing the dimensions of the rectangle. **[Int]** It appears that Ms. B perceives Al's behavior to be off-task because she reaches over and takes the nametag away from him. She does this without speaking and Al appears to become reengaged with the mathematics. Prior to this moment, he was looking around

			the room, and appeared to be disengaged with the mathematics. Ms. B's intervention with Al's off-task behavior, does not interrupt or impede Natasha's mathematical progress. In fact, it appears that neither Natasha nor Kenny take notice of the intervention. **[Dis-b]**
18:11	Natasha	Because they said the outside's supposed to be a hundred.	
18:14	Ms. B	Okay.	Ms. B validates Natasha's reasoning. **[Val]**
18:18	Natasha	And then I said like these could be like umm, forty forty, and ten and ten. [Natasha points to the sides of her rectangle with her ruler as she speaks.]	Natasha identifies dimensions that would correctly add up to a perimeter of one hundred, but would not provide a maximum area. It may be inferred that Ms. B recognizes Al's incorrect reasoning. At 17:37 he used a ruler to determine the dimensions of the rectangle: "I got five and a half….I got six, I got regular six inches." **[Dis-m]** Ms. B fosters student discourse asking Natasha to "show him [Al]" likely in hopes of correcting Al's reasoning. **[Endis]**
18:24	Ms. B	Okay, so now show him [Ms. B gestures toward Al].	
18:26	Natasha	Like umm, the height could be forty, and then this could be ten. I don't even know.	
18:43	Ms. B	[Ms. B leans across the desk and points at Natasha's paper with her pencil.] I just don't understand how you get those numbers.	Ms. B appears confused about why Natasha is using forty and ten as dimensions of the rectangle. It may be inferred that she is feigning confusion as a method to encourage Natasha to justify her reasoning. **[Int, Con, Endis]**
18:47	Natasha	Because the outside is supposed to be a hundred so don't you like add the numbers together or you…. [Natasha traces the perimeter of her rectangle with a ruler. The camera	

		zooms in to show that Natasha has labeled the longer sides, forty and the shorter sides ten.]	
18:56	Kenny	…five feet, and poles..[inaudible]	
19:10	Ms. B	I'll be right back. [Ms. B stands up and walks behind Kenny toward Al's desk.]	
19:14	Ms. B	[Ms. B stands next to Al's desk, and points at his work] Is that the only way? Does that thing work? [Ms. B points to the calculator that Kenny is holding, then walks away.]	Ms. B expresses ***curiosity*** about other possible solutions to the problem. **[Int]**
End TSMIE [B.01.a.03]			

Discussion of TSMIE [B.01.a.03]

Ms. B initiates this episode when she approaches the table and inquires about the students' progress with the mathematics when she asks, "what are you guys doing now?" It is inferred that Ms. B recognized that Kenny may be distracted from the mathematics by spending time replacing the batteries in the calculator.

Natasha appears confused about the definitions of the terms length, height and width and her confusion appears to be blocking her progress with solving the problem. Ms. B appears to recognize Natasha's impasse and encourages student discourse saying, "Ask the question again, [Natasha]" and approximately two minutes later, Ms. B asks, "Which is length to you?" Natasha and Al come to the conclusion that length and height could be used interchangeably when referring to one dimension of rectangles.

Then, Natasha briefly offers a correct answer to the first part of the problem, suggesting that the length of each side could be twenty-five, but quickly she retracts the answer, changing the dimensions of the rectangle to forty and ten. Ms. B does not offer her opinion on either of these answers.

At 17:37, Al presents faulty reasoning when he measures the length and width of his rectangle using inches on a ruler. It may be inferred that Ms. B recognizes his faulty reasoning and attempts to intervene by encouraging student discourse about Natasha's reasoning. Ms. B asks Natasha, "Where are you picking you're numbers from? Why don't you share? [Ms. B looks and points at Al and Kenny.] How you are getting the numbers that you are choosing?"

It is inferred that Kenny has not yet labeled the length and width of his representation and he has very limited interaction during this episode. In an inferred attempt to engage Kenny with the discussion, Ms. B asks, "Okay, so [Kenny], what do you think? [Ms. B scratches her face and looks at Kenny]". Kenny mumbles an inaudible response.

Ms. B addresses three instances of perceived student off-task behavior in a very subtle manner, never using words, only actions. Her interventions appear effective since in each case, the students cease the behavior and often become reengaged in the task. This episode ends when Ms. B tells the students "I'll be right back" and walks away from the table.

Outcomes of this TSMIE [B.01.a.03]: 1.) Natasha's confusion about the terms, length, width and height has been resolved as a consequence of her discussion with Al. 2.) Natasha recognizes that the sum of the dimensions must equal one hundred.

Immediately subsequent to this episode, it appears that Natasha becomes disengaged with the mathematics because she puts her nametag down on the desk and begins to erase on it. Al also appears disengaged as he looks on as Natasha erases, hitting his shoulder with his ruler, Kenny is out of camera range.

Begin TSMIE [B.01.a.04] 19:43 – 22:51 (C)			
Participants: Shay, Ray, Ms. B Ms. B initiates this episode when she approaches the table and briefly observes the work that the students are doing by looking at their papers. She leans on the table next to Shay, and then moves the materials bucket and stands next to LaTora.			
19:43	Ms. B	Explain how you are getting that.	It may be inferred that Ms. B is ***curious*** about the students' mathematical reasoning. **[Int]**
19:46	Shay	[Ms. B sits down on the top of the empty student desk.] Look, you do two sides and then you see what the remainder is and then you [Shay is holding a paper up to show Ms. B. He is inaudible.]	
19:58	Ms. B	Okay, so how come you have so many pieces of paper? [Ms. B points to a pile of papers that are on the desk in front of Shay.]	Shay has at least five sheets of paper in front of him and Ms. B conveys ***curiosity*** regarding the large number of papers. **[Int]**
19:59	Shay	Because I got a lot of answers. [Shay picks up the pile of five papers, one piece at a time.]	
20:04	Ms. B	Oh ok. [Ms. B looks sideways at Shay.]	It may be inferred that Ms. B recognizes that Shay may be frustrated that there are "a lot of answers". **[Rsi]**
20:14	Ms. B	[inaudible]	
20:19	Ms. B	You're working together so. Do you have, it says to answer the question for three. Did you find that out yet?	Ms. E expresses ***interest*** in the boy's mathematical progress. **[Int]**
20:30	Shay	Twenty-four, grab twenty-four. This one.	Ms. B expresses ***curiosity*** about the boys' reasoning for "this one". It may be inferred that because Shay says, "grab twenty-four" that one of their possible solutions is a rectangle with one dimension twenty-four. Ms. B ***does not validate*** nor does she negate the boys' answers. **[Int]**
20:42	Ms. B	What made you think that?	
20:45	Shay	I was just…or maybe this one. [Shay takes out one paper.]	
20:55	Frank	[inaudible]	

21:03	Shay	But if we do, we'd have to go all day…cause there is more than one way.	It may be inferred that Shay is ***frustrated*** with the potential enormity of the task. It appears that Shay is in the process of drawing every rectangle (with whole number dimensions) that has a perimeter of one hundred feet. Ms. B may ***recognize Shay's frustration*** but she does not comment on it. Rather, she expresses ***curiosity***, encouraging the students' autonomy asking, "So what do you guys have to do?" **[Rsi, Int]**
21:15	Ms. B	So what do you guys have to do?	
21:17	Frank	We got to find the area. You've got to find the maximum area. I'll do it. [Frank reaches for the paper and begins to calculate.]	
21:46	Shay	[Shay looks into the camera.]	
21:48	Ms. B	So did you guys determine how you find the area?	Ms. B appears ***curious*** about the area of the rectangle(s). **[Int]**
21:56	Shay	By adding up all the…around.	Shay incorrectly states that the area is figured by adding up the [sides].
22:03	Ray	Multiply the two sides…[Inaudible. Ray is pointing to the sides of the rectangle with his ruler.]	Ray states the correct formula for finding the area of a rectangle. Ms. B does not acknowledge either of the boys' answers.
22:14	Ms. B	So I heard three different things? How you find the area? [Ms. B directs this question to Shay]	Ms. B acknowledges both suggestions then expresses ***curiosity*** about the boys' understanding of the formula for area of a rectangle, yet she remains distanced from the conversation. **[Int]**
22:25	Shay	Add up.	
22:26	Ms. B	Hmmm?	
22:41	Ms. B	I know you guys need to discuss how to find the area, so how how do you find the, the perimeter of the rectangle?	
22.51	Shay	Length….length times height….[Ms. B walks away]	
End TSMIE [B.01.a.04]			

Discussion of TSMIE [B.01.a.04]

This episode was initiated by when it appears that Ms. B is circulating around the classroom, and Ray turns around and summons her to his table.

Shay has become frustrated because he appears to think that there may be an infinite (or enormous) number of possible rectangles that have a perimeter of one hundred feet. He says to Ms. B, “you could go all day”.

It may be inferred that Ms. B recognizes Shay’s frustration but she does not confirm nor negate his comment. She looks sideways at him a couple of times, but does not engage him in a discussion about all of the possible ways to draw the rectangle. Ms. B redirects the students and asks about the method for finding the area of a rectangle. When the Shay and Ray offer different formulas, Shay is incorrect and Ray is correct, Ms. B does not pass judgment on either suggestion. Throughout this episode, Ms. B does not smile, and speaks very quietly when addressing the students. She appears very calm while remaining focused on the students’ engagement with the mathematics.

This episode ends when Ms. B walks away from the students. The three boys appear to continue to be engaged with the mathematics, although Shay remains distracted by the frustration that it could take “all day” to find all of the rectangles with a perimeter of one hundred. He sits back in his chair, at first appearing defeated then quickly becomes reengaged in the problem, sitting up and leaning forward and looks at his paper.

Begin TSMIE [B.01.a.05] 21:41 – 27:40 (L)			
Participants: Dana, Emmy, Devin, Shawn, Ghee, Ms. B Ms. B initiates this episode when she asks the question, "what happened over here now?" This is the third time that Ms. B has interacted with this group since the beginning of the class period. Immediately prior to Ms. B joining this group, Dana is leading a group discussion, apparently checking that each of her group members has the same answers and that everyone has an answer for each of the three bullets. When Ms. B arrives at the table, Ghee proudly declares that the group has completed the problem. The students have each drawn a forty-by-ten rectangle, but it appears that the students have not calculated the area of the rectangle.			
21:41	Ms. B	So what happened over here now? [Ms. B is off camera]	
21:41	Ghee	We finished! [Ms. B sits down at an empty desk next to Devin.]	Ghee exhibits ***pride*** when he announces that his group is "finished". Ms. B acknowledges Ghee's claim that they have completed the solution to the problem.
21:43	Ms. B	You finished? [Ms. B nods her head.]	
21:44	Ghee	It's cause we didn't…ummm.	
21:47	Shawn	I told ya we didn't [inaudible]	
21:48	Ms. B	I just…	
21:51	Shawn	I just said that you talking about…be quiet!	
21:52	Ms. B	It's if you wanted to use it [Ms. B raises her eyebrows. The view of the lower part of Ms. B's face is blocked by Emmy's head]… So what did you come up with? [Ms. B furrows her brow and looks at Ghee.]	Ms. B expresses ***interest*** in the students' answer. **[Int]**
21:57	Dana	Well for this…ummm bullet two, we got a maximum, [Dana looks at Ms. B] but we already told you our maximum…	Dana initially appears ***angered*** by Ms. B's request for an explanation of the solution and she defends her group "we already told you". Ms. B points out that she was absent from the table for a while, which seems to diffuse Dana's anger as she proceeds with the explanation.
22:02	Ms. B	But I left.	
22:04	Dana	Oh, we put forty feet on both of the lengths….and ten feet on both of the widths.	
22:12	Ms. B	Why?	It may be inferred that Ms. B is ***curious*** about Dana's reasoning for choosing a forty-by-ten

			rectangle. **[Int, rs]**
22:14	Dana	Because we was trying to get to a hundred maximum.	
22:17	Ghee	… really really really stupid. Why don't we just put eighty for the length and twenty for the width….Eighty. [Ghee is speaking quietly, looking at his paper as he speaks.]	Ghee appears to "second guess" his thinking.
22:26	Dana	So we gonna put, so, if you hafta explain it cause then they think we put eighty on both…I mean eighty on here and here. That's why I put….	
22:38	Ms. B	So for the first bullet, what would your answer be? [Ms. B points to Ghee's paper and then scratches her head.]	Ms. B's tone of voice is ***emotionless***, yet her language indicates that she may be ***curious*** about the students' answer to the first bullet in the problem statement. **[Int]**
22:42	Dana	Forty feet…	
22:43	Ms. B	Well does someone want to read again what you have to do? [Ms. B motions toward all of the students, drawing an imaginary circle in the air with her pencil. She is looking down at the desk as she speaks. After Ms. B asks the question, she rests her chin on her fist.]	It appears that Ms. B may disapprove of the students' understanding of the problem. **[Dis-m]** She expresses ***confusion about the students' answers*** and she asks for a volunteer to read the problem again. **[Con, Int, Endis]**
22:46	Dana	What do we have to do?	
22:52	Shawn	[Ms. B looks at Shawn as he answers] Draw a….Draw a diagram….Draw a diagram with length and the width until the maximum…the maximum area.	
23:00	Ms. B	Okay. [Ms. B nods her head]	Ms. B appears to feel ***satisfied*** with Shawn's level of understanding of the problem because she says, "okay" and nods her head.
23:02	Shawn	And read what we got over there.	
23:05	Ms. B	So that's you're maximum area?	Ms. B expresses ***curiosity*** about

		[Ms. B looks at Dana when she asks the question and points toward Shawn's paper.]	the students' answer. **[Int]**
23:06	Students in unison	YEEES!!!	
23:10	Devin	Explain how …[inaudible]…..that's how we did it.	
23:14	Ms. B	You did? How? [Ms. B looks at Devin]	Ms. B is ***curious*** about Devin's mathematical thinking. **[Int, rs]**
23:18	Dana	I mean…oh.. we didn't explain bullet two. No we didn't.	At first, Dana appears to recognize that her group did not address the question in bullet two, then she is corrected by Shawn and retracts her idea. She appears satisfied that the group did answer bullet two.
23:24	Shawn	We just told him what we did.	
23:28	Dana	Yea, yea we did explain it.	
23:31	Ms. B	So I have a question, [Ms. B straightens her posture] how do you know that's the maximum? [Ms. B points at Ghee's paper, then looks at Dana. Ten seconds of silence pass before Ghee speaks.]	It may be inferred from the students' silence after Ms. B's question, that the students are confused about how to find the maximum area. It may be inferred that Ms. B ***recognizes the students' impasse.*** **[Int, rs; Rsi]**
23:40	Ghee	That's not the maximum, the maximum could be…it could be forty nine, and one [Ghee is smiling and gesturing with his hand]	Ghee appears to misinterpret the question here because he is using the maximum length instead of maximum area.
23:54	Dana	All right! What you trying to say, but we didn't do it like that, you wouldn't even say do it like that.	
24:00	Dana	(inaudible but in response to Ghee)	
24:04	Ghee	Yea and that was all leading up to one hundred…forty-nine and one, forty-nine and one.	Ghee appears ***proud*** of his discovery to use other numbers besides ten and forty.
24:11	Dana	[inaudible]	
24:14	Ms. B	So, do we have a maximum?	Ms. B appears to be trying to reengage the students with the mathematics.
24:17	Dana	Yeah.	
24:21	Ms. B	Do we have a maximum, [Ms. B's eyebrows are raised] are we all in agree-ance with that, so I asked you how do you know that's the	It may be inferred that Ms. B is ***curious*** about the students' reasoning behind their claim that the forty-by-ten rectangle is

		maximum? [Ms. B looks around at each of the students in the group. Her eyebrows are raised.]	the "maximum". **[Int, Endis]**
24:28	Ghee	That's not the maximum, like I was saying, forty-nine feet for each length and one for each width, but it was…[Ghee emphasizes his words and gestures with his hands in a bicycle pedal motion.]	Ghee presents faulty reasoning, sticking with his claim that the maximum length is the right answer.
24:42	Dana	Because its not width [inaudible]	
24:44	Ghee	It could be! [Ghee raises his voice when he speaks to Dana]	Ms. B's facial expression does not change when Ghee raises his voice toward Dana.
24:47	Ms. B	What do you mean it could be?	Ms. B asks Ghee to clarify his reasoning. **[Int, rs]**
24:50	Ghee	It could be the width, if you put a small one. But it wouldn't go with the ["the fencing" Dana adds in]….Bullet three. [Ghee gestures 'three' with his hand.]	
25:03	Ms. B	How do you know?	Ms. B expresses ***curiosity*** in the students' reasoning through her language. **[Int, rs]**
25:04	Dana	Because when they say they need one pole every five feet…[Shawn adds in "that's only one feet"] that's only one feet if you do that….ehh that's not even gonna work.	
25:15	Dana	you said one feet	
25:21	Ghee	It's one pole for every… one…I mean every five feet.	
25:30	Ms. B	Ok, so how can you *prove* that this is not a maximum? How can you *prove* that this is the maximum…area?	Ms. B's language conveys that she is ***curious*** about how the students can "prove" their reasoning. **[Int, rs]**
25:41	Devin	You can try different numbers and see if it works? [Ms. B shrugs her shoulders as she looks at Devin.]	
25:45	Ms. B	Ok, how would you try different numbers?	Ms. B asks for Devin to elaborate his idea. **[Int, rs]**

25:57	Ms. B	[Ms. B looks at Devin and then pushes back from the desk, smiling.] Do you want to try that and I'll be back?	Ms. B appears to be ***satisfied*** with the student's level of understanding and engagement with the mathematics when she indicates that she will leave the group and come back. **[Sat]**
25:58	Ghee	Yes, yes. [Ms. B stands up.]	
26:00	Dana	[Ms. B], why we can't just leave our maximum length like this?	Dana appears ***confused*** about the need to "try different numbers" as she defends her group's answer.
26:03	Ms. B	I didn't say you couldn't leave it like that, [Ms. B shrugs her shoulders] I was just asking do you kn…how do you know that's the maximum length?	***Autonomy support*** ***[Int, rs]***
26:09	Shawn	Cause the..	
26:10	Dana	Do all the maximum lengths have to be the same, as other people…as other groups?	Dana expresses ***curiosity*** about the answers that other student groups.
26:17	Ms. B	No I just want to know *your* maximum. [Ms. B is leaning on the table, looking at Dana. Dana looks at Ms. B, then looks down at her paper, and rests her cheek in her hand.]	
26:19	Ghee	Like I said, let's do than forty-five and five.	
26:30	Ms. B	[Ms. B looks at Dana, then looks at Devin's paper. Devin is looking away.] So what are you doing here? [Ms. B looks and points at Devin's paper.]	Devin is engaged with the mathematics likely trying different numbers as he writes on his paper. Ms. B conveys ***interest*** in Devin's work, when she asks him, "what are you doing here?" **[Int, rp]** Devin answers Ms. B but is inaudible, then looks away from the table. Ms. B looks back at the other

			students in the group. It may be inferred that the students in this group are feeling challenged by Ms. B's question, "how do you know that's the maximum length?" because they do not answer her, and they avoid eye contact with Ms. B. It may be inferred that Ms. B ***recognizes that the students are experiencing impasse*** as she silently observes the group. **[Rsi]** Dana appears to be doodling on her paper and is quiet. Ghee is sitting back in his chair, looking at his paper.
26:31		[Shawn is saying something inaudible]	
26:53	Ms. B	What? [Ghee is looking at Ms. B]	
26:55	Ms. B	What are you thinking? [Ms. B is standing and leaning on the desk, toward Ghee.]	**[Int, rs]**
27:00	Ghee	I was thinking about, we have to do this over. [Ghee is mumbling, sitting back in his chair and looking at his paper. He smiles as he talks, looking at Shawn.]	It may be inferred that Ghee is feeling ***sad*** because he states that his group has to start the problem over.
27:03	Ms. B	What do you mean you have to do it over?	**[Int, rs]**
27:05	Ghee	'Cause you want us to make, like, another big maximum. [Ghee does not look at Ms. B as he speaks to her.]	It may be inferred that Ghee is feeling ***sad*** and ***frustrated.*** It is possible that he is feeling badly that his group needs to find another "big maximum".
27:09	Shawn	Well what ever we do for the length and the width we gotta do every five feet.	
27:15	Ghee	Uh uh, it has to be at five, it has to be the minimum has to be at five.	

27:19	Dana	So why would you put, why would you put one pole right here? [Dana is gesturing with her pencil to her paper, scowling at Ghee.]	Dana appears to be ***arguing*** with Ghee's reasoning. She looks ***mad.***
27:27	Ghee	'Cause it could still be, it could still be more close there.	
27:29	Dana	Whatever! [Dana rests her cheek in her hand and looks down at her paper.] [Ms. B looks at Dana, Shawn and Ghee, but does not speak.]	It appears that ***Ms. B recognizes Dana's impasse,*** but she does not acknowledge it before walking away. Dana may be at a point where her ***anger*** is causing her to become ***disengaged*** with the mathematics because she is ***frustrated.***
27:33	Shawn	You could put ten there….that's two poles.	
27:40	Ghee	She caught me on the camera! [Ghee is smiling] [Shawn is inaudible and Ms. B walks away]	
End TSMIE [B.01.a.05]			

Discussion of TSMIE [B.01.a.05]

Ghee declares that his group has "finished" and when Ms. B asks them to explain their solution, Dana becomes defensive, "we already told you." Ms. B appears to recognize Dana's anger, and diffuses it by pointing out, "But I left."

It appears that Ms. B recognizes the students' confusion and redirects them to read the statement of the problem. When Ms. B questions the students about how they could prove that their answer is the maximum, Ghee suggests that the maximum length should be forty-nine. Ghee appears confused about what the word, "maximum" refers to and it may be inferred that he thinks that "maximum" refers to the length of the rectangle. Forty-nine is the largest whole number that could represent one dimension of a rectangle with a perimeter of one hundred.

Dana disagrees with Ghee because she points out that a forty-nine by one rectangle will not accommodate the condition that one fence post is placed every five feet. When Devin proposes that the group tries "different numbers", Ms. B stands up and begins to leave the group at 26:00.

However, it may be inferred that Ms. B senses that Dana and Ghee are frustrated because she lingers while standing next to Devin. Dana defends her group's answer when she asks Ms. B why they can't just go with their original answer. Ms. B acknowledges that she does not necessarily disagree with the student's answer, but that she needs more proof that their answer is the "maximum". Ghee expresses anger toward Ms. B, "Cause you want us to make, like, another big maximum." Ms. B does not respond to Ghee's comment. She lingers for about forty more seconds, then walks away.

The students appear to understand that there is more than one possible way to draw a rectangle using one hundred feet of fencing. However, a key piece of information that has not been discussed is exactly how one would determine whether or not the rectangle has a "maximum area". The students did not articulate that they had found the area of their rectangle.

Outcomes of TSMIE [B.01.a.05]: 1.) The students appear to understand that they may not have arrived at a correct answer as a result of Ms. B's questioning. 2.) Ghee and Dana express anger toward Ms. B for questioning their answer. 3.) Devin suggests that they "try different numbers". 4.) Dana expresses anger toward Emmy and Devin, "Ya'll aren't doin' nothin'".

After Ms. B walks away from the table, it appears that Shawn, Dana and Ghee are not engaged with the mathematics because they are looking down, silent and not writing.

Begin TSMIE [B.01.a.06] 29:20 – 34:06 (C)			
Participants: Natasha, Kenny, Ms. B This episode begins when Al summons Ms. B to tell her the answer. This is the second time that Ms. B has interacted with this group since the beginning of the class period.			
29:20	Al	It's forty, twenty twenty and forty. Yea we gave…[Mrs. B] we founded it out. [Ms. B approaches Natasha's group after leaving Dana's group.]	
29:32	Ms. B	Oh you found it out? What did you find? [Ms. B sits down at an empty student desk next to Kenny.]	It may be inferred that Ms. B's is showing ***interest*** in the group's ideas. **[Int]**
29:34	Al	We found umm, like it don't matter what size you make it. [Al is looking at Ms. B as he speaks. Ms. B shifts some calculators on the desk then rests her neck on her hand as she looks at Al. Ms. B's face is out of view since the camera is positioned at her back.] You know how they drew it in the books right? It could be a square, like that big, and they put like seventy-five [Ms. B: "Ok"] or and then and they be like two, but they put like twenty. Well I figured that out, well you can make that half, but then the perimeter outside [Al traces the outline of his rectangle with his ruler.] is still gonna add up to the same thing. [Al raises his eyebrows, nods his head and smiles at Ms. B.]	It may be inferred that Al is describing the idea that any rectangle can be labeled with measurements despite the actual size of the rectangle. Ms. B had discussed this idea with these students during TSMIE [B.01.a.03]. It may be inferred that Al feels ***confident*** and ***proud*** of this conceptual leap because he is smiling and nodding. Ms. B is leaning on the table with her elbow leaning into her hand, resting her hand on her neck. Her posture may be inferred show that ***she is interested*** in Al's ideas. **[Int]**
30:11	Ms. B	What's the same thing?	Ms. B appears ***curious*** about Al's comment, "is still gonna add up to the same thing." **[Int]**
30:13	Natasha	Like say one hundred, [Natasha points at her paper.] like say you make the box like this small [Natasha draws a small rectangle on a piece of scrap paper.] and then you do eighty and then ten. And	Natasha attempts to address Ms. B's question by sketching and labeling a rectangle on scrap paper. Throughout most of this episode, when Natasha talks

		just like you did eighty and then ten right there.	about the mathematics, her facial expression appears ***serious***.
30:25	Ms. B	So what's the perimeter right here? [Ms. B points at Natasha's drawing on the scrap paper.]	Ms. B's facial expression is ***serious***, it may be inferred that Ms. B is ***curious*** about Natasha's understanding of perimeter. **[Int]**
30:27	Natasha	A hundred.	
30:30	Ms. B	Show your group. [Ms. B points her finger toward Kenny and Al. Al is nodding his head and rubbing a ruler against his chin.]	Ms. B appears to be ***encouraging student discourse*** when she asks Natasha to show her drawing to Kenny and Al. **[Endis]**
30:32	Al	I know, I...I know.	
30:33	Natasha	That's what we talking about right there, like say they put twenty-five right like this. [Natasha writes on Al's paper but what she writes is not captured on camera.] [Al looks at where Natasha has just drawn, he nods his head, and writes, pauses and then writes again. His eyebrows are raised. The camera then zooms in to show a square that is labeled on all four sides with the number twenty-five.]	It may be inferred that Natasha has drawn a square on Al's paper and has labeled one side, twenty-five. This mathematical idea could lead the students to the correct answer because the maximum area is achieved by making a square region.
30:50	Kenny	That's a square. I thought all the…ummm square had the same sides. [Kenny is sitting back in his chair, with his face resting in his hand. He shakes his head "no".]	Kenny appears shy. It may be inferred that Kenny thinks that a square is not a rectangle. The problem states that the fence must enclose a "rectangular region". It is possible that Kenny thinks that the square cannot be used because he thinks a rectangle must have different length sides.
30:58	Natasha	Oh yeah. [Natasha and Al laugh.]	It may be inferred by Al and Natasha's reaction to Kenny's comment that they realize that the region must not be in the shape of a square.
31:00	Al	All right, I'm 'a do it over.	
31:02	Natasha	Kenny, you got that? Yea except it's a rectangle, the top is….[Natasha writes on Al's paper, then starts	

		playing with her earring.] [Ms. B looks on, silently, her hands are folded on the desk in front of her and she is leaning in toward Al. Al is drawing on his paper.]	
31:25	Ms. B	Ok so, now we're working on this rectangle right here. [Ms. B points toward Al's paper.]	
31:29	Kenny	That's a two? [Kenny points to Al's paper with his pencil, then he buries his face in his hand. Kenny furrows his brow.]	
31:39	Ms. B	So is this a possible answer right here? [Ms. B points at Al's paper.]	Ms. B appears ***curious*** about the figure drawn on Al's paper. It is not clear what Al has drawn on his paper.
31:40	Natasha and Al	Yeah!	
31:43	Ms. B	Why?	Ms. B appears ***curious*** about the "possible answer". **[Int]**
31:45	Natasha	[Al- "it's outside of it"] And for me, [Al – "it's a hundred."] I don't think there is no right or wrong answer because anyone can say anything, like they can say, ummm, [Natasha looks at Al, shakes her head, smiles then looks back to Ms. B] … they can probably say twenty plus forty or they can even say umm twenty-five, twenty five, like that, umm. Oh, no but it's like that. [Al is leaning back in his chair, rubbing a ruler on the back of his head.]	Natasha is describing that she understands there are many ways to draw a rectangle with the perimeter of one hundred. It is possible that she has not taken the idea of area into consideration OR that she does not understand the concept that rectangles with different dimensions will yield different areas.
32:07	Ms. B	What do you mean, I just need you to draw it out.	It may be inferred that Ms. B recognizes that Natasha has presented a correct idea, using a square with side length twenty-five feet. By asking Natasha to "draw it out", she may be trying to guide Natasha toward recognizing that her answer is the correct one.
32:12	Natasha	Like what?	
32:13	Ms. B	I don't know, whatever you just said, [Ms. B points at Natasha's paper] can you repeat it and draw it? You just said like twenty-five and twenty-five. [Natasha smiles.]	

			[Int] It may be inferred that Natasha feels ***happy*** that Ms. B is showing interest in her mathematical idea because she is ***smiling***.
32:18	Natasha	Yea, I said, let's say…let's say someone if put twenty-five and twenty-five for each side, but then I said no because…	
32:28	Ms. B	Why would you put twenty-five for each side?	Ms. B does not acknowledge why Natasha would disregard the square with twenty-five on each side. Rather, she appears curious about Natasha's reasoning for labeling each side twenty-five.
32:31	Natasha	To equal a hundred, but then I said that you can't do that because the length is more, bigger than, the width….the square…	It appears that Natasha begins to describe the idea that in a rectangle, one side must be longer than the other, but Ms. B cuts her off. Although Ms. B's tone of voice remains calm, her language suggests that she is ***eager*** for Natasha to proceed with the idea of using a twenty-five foot by twenty-five foot square, because she interrupts Natasha's idea, "the length is more, bigger than, the width..." Also, Ms. B's posture may be inferred to convey ***interest*** because she is leaning in toward Natasha, apparently giving Natasha her full attention. **[Int]**
32:42	Ms. B	Okay, so you're saying that the perimeter has to equal what? [Ms. B has rested her cheek on her fist and is leaning in toward Natasha.]	
32:47	Al	No matter how long you make it, how fat you make it, the area [Al is looking down at his paper.] the area…the length…the height…is gonna be the same.	
32:58	Natasha	It's like the width had to be, ummm, equal to the top and bottom and, ummm, the length had to be equal to	

		each other. [Natasha erases something on her paper.]	
33:08	Ms. B	Ok…So what's the… maximum area? [Ms. B still has her cheek resting on her hand.]	
33:11	Natasha	The area? [Natasha looks at Ms. B and traces the a rectangle shape in the air with her pencil.]	
33:13	Ms. B	The *maximum* area, [Ms. B moves Natasha's paper to show the paper with the problem written on it.] See you supposed to draw the diagram with the length and the width to show the *maximum area.* [Natasha looks at Ms. B as Ms. B speaks. Natasha is not smiling.]	It may be inferred that Al is computing the perimeter of the rectangle, which is drawn on Natasha's paper because he looks at her paper then at the calculator. Ms. B redirects Natasha to the conditions of the problem. It may be inferred that ***Ms. B recognizes Natasha's impasse.*** **[Rsi]** Natasha's facial expression may be inferred to show ***confusion***, she appears unsure of how to proceed. However, these feelings appear to be hidden behind Natasha's "tough" face.
33:22	Al	Lemme see this [Al picks up a calculator and starts pressing buttons]. What did you say the length was?	
33:24	Ms. B	I was just asking what was the maximum area is. [Ms. B pauses and looks around the table.] What are you doing with the calculator? [Ms. B asks Al.]	Ms. B's tone of voice is ***emotionless*** as she expresses ***interest*** in Al's calculations**.** **[Int]**
33:35	Al	Addin' it up. [Al raises his eyebrows then gestures with his hand toward Natasha's drawing.] The fence is going to be a hundred.	
33:40	Ms. B	You said the area, I mean the perimeter, how do you find the perimeter?	It may be inferred that ***Ms. B recognizes that the students are confused*** about the difference between area and perimeter.
33:46	Al	By the outside, wait. Yeah, [Al has his chin resting on the top edge of	Al appears ***confused*** about the correct way to calculate area and

		the calculator, he gestures toward Natasha's paper] …no the inside.	perimeter.
33:49	Natasha	No don't you add the numbers from the outside to find the perimeter [Natasha looks at Ms. B] and you multiply them to find the area? [Ms. B folds her arms.]	
33:57	Ms. B	You asking your group?	Ms. B redirects the question about how to calculate area and perimeter by ***encouraging student discourse.*** **[Endis]**
34:00	Natasha	No I'm asking you. [Natasha smiles and points with her pencil at Ms. B.]	
34:00	Ms. B	Ask your group.	For the second time, Ms. B redirects the question. **[Endis]**
34:01	Natasha	You. [Natasha smiles, looks at Ms. B, then puts her head down, laughing.]	It may be inferred that although Natasha recognizes that Ms. B would not answer her question, she was ***hopeful*** and asked the question anyway.
34:04	Ms. B	[Kenny] asked what the question was. [Ms. B is smiling and looking at Natasha.]	Ms. B remains steadfast as she smiles at Natasha. She may intend to communicate her confidence in Natasha's (and the group's) ability to answer the question.
34:06	Al	How do you find the per…don't you find the perimeter [Ms. B walks away from the table.]	
End TSMIE [B.01.a.06]			

Discussion of TSMIE [B.01.a.06]

Ms. B approaches his table and As says, "Mrs. B, we founded it out." Ms. B responds with curiosity when she sits down in a chair next to Kenny. Through most of the episode, Ms. B's facial expressions cannot be seen due to the camera angle.

It appears that Al now recognizes that the rectangles do not need to be drawn to scale, "it don't matter what size you make it".

When Natasha presents the idea to draw a square, measuring twenty-five units on a side, Ms. B does not pass judgment whether her answer is right or wrong. Kenny points out that Natasha has drawn a square and it may be inferred that Natasha and Al both recognize this as an error and subsequently work to correct this "mistake". Ms. B does not intervene.

It is inferred that Ms. B recognizes that the students have not yet calculated the area of their rectangles when she asks, "So what's the… maximum area?" Natasha appears confused by this question and Ms. B clarifies, referring back to the statement of the problem. When both Natasha and Al seem confused about the difference between area and perimeter, Natasha appeals to Ms. B for help. Ms. B encourages student discourse and student autonomy by redirecting Natasha to ask Al and Kenny. Natasha finds this funny, as she laughs and Ms. B walks away from the students.

Outcomes of TSMIE [B.01.a.06]: 1.) Al explains to Ms. B that he understands that the dimensions of the rectangles are defined not by inches but by the labels that can be assigned by the problem solver. 2.) Natasha expresses confusion about how to calculate area and perimeter.

At the end of this episode, Ms. B walks away from the table and the students appear to remain engaged in the problem. As soon as Ms. B is out of camera range, the students look directly at the camera and ask the videographer how to calculate the area and perimeter of a rectangle. The videographer does not answer the question.

Begin TSMIE [B.01.a.07] 35:01 – 37:59 (C)			
Participants: Shay, Frank. Ms. B initiates this episode when she sits down at an empty desk at the table. Shay appears frustrated when he says, "Can't find the height." Ms. B appears to recognize Shay's frustration but she does not impose her own mathematical ideas. Ms. B provides encouragement for the students to keep working until they find the "maximum area".			
35:01		[The camera pans to Shay and Ms. B speaking at his table.]	
35:06	Shay	Can't find the height.	Shay expresses that he is ***experiencing impasse.***
35:07	Ms. B	Why not?	Ms. B expresses ***interest*** in Shay's impasse. **[Rsi, Int]**
35:10	Shay	Because, if you trying to find the height…you pick two heights and it won't work for the other one.	
35:18	Ms. B	So can you show me what you have already, 'cause I left when you were working, explain what you have here. Why you did what you did.	Ms. B appears to show ***interest*** in Shay's idea as she asks him to show her evidence of his work so that she may better understand his question. **[Int]**
35:31	Shay	I'm trying to find what the maximum area is…[Shay is writing on his paper]	
35:53	Ms. B	So what is this right here that you're writing?	Ms. B demonstrates ***curiosity*** about Shay's representations**.**
35:56	Shay	The umm…the area.	
35:58	Ms. B	Okay, so that's the area. So how did you get these pictures here? I'm just…cause you have a lot of pictures. How did you get all of these pictures?	Ms. B ***validates*** Shay's ideas and it may be inferred that Ms. B is ***curious*** about Shay's reasoning for drawing "a lot of" representations. **[Val, Int, reasoning]**
36:04	Shay	We drew it and then found out which one of these we put on inside and it add up to a hundred….	
36:09	Ms. B	Okay so you wanted it to add up to a hundred. Are you guys agreeing with what he is saying? [Ms. B leans back in her chair and looks toward a different group of students] Larry…focus, focus, no one's mother is here so don't talk about	Ms. B encourages student discourse asking the other students in this group if they agree with Shay. **[Endis]** Ms. B addresses off-task behavior at a different table, expressing ***disappointment*** that

		them, Larry…	Larry is not engaged with the mathematics. She leans back in her chair and raises her voice to address [Larry]. She uses sarcasm to address Larry's comments about somebody's mother. **[Dis-b]**
36:29	Ms. B	So how did you get it? [Ms. B returns her attention to the students at Shay's table.]	Ms. B expresses ***curiosity*** about how Shay arrived at his answer. **[Int]**
36:30	Shay	It's a hundred eighty four.	
36:33	Ms. B	So wait I don't understand the pictures, how come you drew so many pictures, what are they showing us?	Ms. B continues to ***appear confused*** and ***curious*** about why Shay drew so many representations. **[Con, Int, rp, rs]**
36:38	Frank	They show all the possible ways to a hundred, we trying to find the maximum area...[inaudible]	
36:47	Shay	This can't be right.	
36:54	Ms. B	So how did you determine what came up to a hundred?	
36:58	Frank	[inaudible]….is equal to a hundred	
37:09	Ms. B	Okay.	
37:16	Shawn	Mrs. B!....Mrs. B! [Shawn is calling from a different table.]	
37:21	Ms. B	Okay, so can you keep going or are you done, you found it?	
37:24	Shay	No you can keep going but it can go all day, we're trying to find the….	Shay appears to exhibit ***frustration*** as he maintains that the task will take "all day".
37:28	Ms. B	Well I need to know the maximum area… [Ms. B smiles]	It may be inferred that Ms. B recognizes Shay's ***frustrated,*** yet she remains ***calm***, as she states "I need to know…" **[Rsi]**
37:32	Frank	Don't you multiply for the area, right?	
37:37	Ms. B	So multiply, add, subtract, and divide to get the area.	Ms. B recognizes that the students are confused about how to find the area. She does not answer Frank's question. **[Rsi, Endis]**
37:44	Ms. B	How do you find the area?	
37:46	Shay	By multiplying the length times width.	
37:48	Ms. B	That's what Frank is asking you.	

		[Ms. B is looking at Shay.]	
37:52	Shay	Frank know that already.	Ms. B does not reply to Shay's comment that Frank already knows how to find the area.
37:55	Ms. B	So can you keep on going and I'll be back to see what you come up with?	Ms. B encourages the students' autonomy.
37:58	Shay	Well we can't do that.	It may be inferred that Shay's ***frustration*** could possibly impede further mathematical progress. He appears ready to abandon all efforts to solve the problem.
37:59	Ms. B	Listen, I need to know what the largest possible area…the maximum is...cause I'm the farmer, I'm the farmer and I need the best way, I need the most grass. So I've been doing all the work and all you guys figure this out for me so I can just graze this cow here...and then I need to get some fences…hmmm…So can you figure that out for me? Yes, okay.	Ms. B appears to ***recognize Shay's frustration*** as she takes on the persona of the farmer. This intervention may be an attempt to motivate the boys to remain engaged with the mathematics. She expresses a ***sense of urgency*** "I've been doing all the work…I need to get some fences". She provides encouragement, ***"can you figure that out for me? Yes, okay."***
End TSMIE [B.01.a.07]			

Discussion of TSMIE [B.01.a.07]

In this episode, Ms. B appears to try to support Shay's engagement with the problem despite his inferred frustration, which it appears that she recognizes. Ms. B appears curious and seems to convey confusion about why Shay has drawn so many rectangles. Shay expresses frustration that so many rectangles can be drawn using one hundred feet of fencing, "No you can keep going but it can go all day, we're trying to find the…". When Shay and Frank express confusion about how to find the area of the rectangle, Ms. B redirects the question to the boys. Ms. B concludes the episode by taking on the persona of the farmer and she encourages to the boys to help her figure out the dimensions for the "most grass" for her cow.

Begin TSMIE [B.01.a.08] 36:40 – 39:58 (L)			
Participants: Dana, Shawn, Ghee, Emmy, Devin, Ms. B This episode is initiated when Shawn summons Ms. B to his group. Dana, Shawn, and Ghee are angry because they claim that Emmy has not helped to solve the problem. Emmy is seated to Shawn's right side and sits with her chin in her hand.			
36:40	Dana	[Ms. B]!	
36:41	Shawn	[Ms. B] [Shawn calls out to Ms. B and turns toward the camera]….she's [pointing toward Emmy] not coming up with no kinds of ideas and I….she's not doing nothing. [Emmy is leaning her chin on her fist.]	Shawn expresses ***anger*** toward Emmy.
36:48	Dana	But they are not even working!!! [Dana is looking at Ms. B]	Dana's is whining as she complains to Ms. B that Devin and Emmy are not helping to solve the problem. It also may be inferred that Dana, Shawn and Ghee, are experiencing impasse with the mathematics, which is likely contributing to their ***anger.***
36:50	Ms. B	So help them work!	Ms. B puts the responsibility back on the "engaged" members of the group. She does not appear sympathetic to the complaints of Dana, Shawn and Ghee.
36:51	Shawn	We trying to help them…	
36:52	Ghee	It's all she's been doing. [Ghee is looking down at his paper.]	
36:53	Dana	There's only three of us in this group that's helping and everyone else's just sitting there trying get this! [Dana is looking at Ms. B, her voice is raised.]	There are five students seated at this table. It may be inferred that Dana is ***angry*** because only three of the five students in her group are engaged with the mathematics. She may also feel angry because she has not been able to solve the problem.
37:00	Ms. B	Well don't give 'em the answer, make them give you the answer. [Ms. B is out of view, but her arm can be seen, she is sitting at the desk next to Devin.]	Ms. B responds by suggesting Dana, Shawn and Ghee hold Emmy and Devin accountable for the answers. **[Endis]**

37:02	Ghee	How we supposed to make them...we've been talking to them, they been talking 'bout something other.	
37:09	Ms. B	So ask her, well lemme see, ask her a question.	Ms. B's tone of voice is ***argumentative.*** She conveys to the students that it is everyone's responsibility to engage Emmy. **[Endis]**
37:14	Dana	Emmy, what's the maximum area?	
37:15	Ghee	Look. [Ghee points at Emmy]	
37:16	Ms. B	I'm asking what's the maximum area?	The students and Ms. B appear to be ***arguing*** because their voices are raised and their tone of voice is ***confrontational.*** By looking at each student and restating the question, Ms. B may be trying to express that each individual in the group is responsible for answering the question.
37:18	Dana	We don't know cause we already said our….	
37:20	Ms. B	What's the maximum area [Ms. B looks toward Emmy], what's the maximum area [Ms. B looks toward Shawn], what's the maximum area [Ms. B looks toward Ghee].	
37:24	Ghee	What's the maximum area? [Ghee is looking directly at Emmy.]	Emmy does not respond.
37:25	Ms. B	Do any of us know yet? [Ms. B is smiling and leaning in toward the group.]	It may be inferred that Ms. B is implying that none of the students know the answer, possibly suggesting that they all have a responsibility to answer the question.
37:27	Ghee	Ask me…ask me….ask me! [Ghee is smiling]	Ghee appears to be ***excited*** that he would be able to give the answer to the question.
37:28	Dana	[Dana is speaking over Ghee's voice] The maximum area is ten, forty, twenty. [Ghee leans toward Ms. B with a very large, emphasized smile.]	
37:33	Ms. B	Let's ask the question on the step that you're on….Let's ask a question on the step that you're on… Okay, so the final answer is	Ms. B's voice becomes ***calm***. It may be inferred that she recognizes the students' impasse and is trying to re-engage the students with the mathematics by breaking the problem down into manageable

		going to be the maximum area, right? That's the answer you're trying to find, but you don't know that yet, so what question are we trying to find….	"steps". **[Rsi]** Ms. B's tone of voice becomes more ***upbeat*** when she begins to talk about specific mathematics.
37:45	Dana	So a hundred is not our maximum area?	Dana appears ***surprised*** to learn that one hundred is not the answer to the problem.
37:47	Ghee	Yes.	
37:50	Ms. B	So a hundred is what? [Ms. B folds her arms in front of her.]	
37:51	Ghee	Our area, that's the feet, it's the feet. Length times width times height.	Ghee's tone of voice is ***defensive*** as he presents faulty reasoning.
37:58	Ms. B	[Ms. B looks at each student in the group.] What is a hundred?	**[Rsi]** Ms. B looks around at each student before asking the students "what is a hundred?" It may be inferred that she is trying to engage the students with the mathematics. More than thirty-seven minutes have been spent on this problem and it may be inferred that the students are confused about what one hundred feet of fencing represents.
38:00	Ghee	Hundred is the feet, hundred is the the the is the….FENCING!.	Ghee appears ***excited.***
38:05	Dana	Is the number that we're trying to umm….	
38:08	Ms. B	So a hundred feet is the fencing, so what does that mean? The fencing, what does that mean?	Ms. B inquires about the students' understanding of what the fencing represents (area or perimeter). **[Int, reasoning]**
38:12	Dana	Well we need a hundred feet of fencing around the...to make the….	
38:15	Ghee	It's about time.	
38:20	Devin	Hundred feet is the fencing and around the fencing we need to get, ummm	
38:24	Ms. B	Where does the fencing go?	**[Int, reasoning]**
38:25	Devin	Around it, around.	
38:30	Ms. B	Do you agree [Emmy]?	When Ms. B appears to try to engage Emmy, her tone of voice is

		[Emmy shrugs her shoulders as she looks at Ms. B] Does it go around [Ms. B gestures around the rectangle] or does it go right in the middle [Ms. B gestures inside the rectangle]?	***gentle.*** It may be inferred that Ms. B recognizes that Emmy may be having difficulty understanding the mathematics because she uses gestures and distinct words to restate the question to Emmy. **[Int, reasoning]**
38:36	Ghee / Shawn	[Ghee raises his hand] It goes around, it goes aroooooounnd!	Before Emmy has an opportunity to answer, Ghee and Shawn both answer aloud.
38:38	Ms. B	Ok, so the fencing goes around…. …so….so what are you doing right now?	Ms. B's tone of voice may be inferred to express ***disappointment*** in the students' behavioral engagement. **[Dis-b]**
38:51	Dana	We trying to find the maximum area. [Dana looks at Ms. B]	
38:54	Ms. B	And how are you doing that? [Ms. B looks at Dana, leaning in toward her.]	Ms. B expresses ***curiosity*** about the procedure that Dana is following to find the maximum area. **[Int]**
38:55	Dana	By trying to figure out.	
39:01	Ms. B	So what are you trying to do, [Shawn]? [Ms. B points at Shawn with her pencil.]	Ms. B's voice is raised when she addresses Shawn. It may be inferred that she is ***excited*** by something that Shawn did. **[Int]**
39:07	Dana	We're trying to figure out the, ummm….[Dana looks down at her paper]…well I know that we're trying to figure out the length and the width cause you said that.	It appears that Dana is faltering here because she hesitates and her facial expression shows ***uncertainty.***
39:19	Devin	You said you could do different numbers instead of these. [Ghee has his forehead down on the desk.]	
39:21	Ms. B	Who said that? Ghee said that. [Ms. B points toward Ghee, her eyebrows are raised.]	It may be inferred that Ms. B wants Ghee to be recognized for his idea to use different numbers for the

			dimensions of the rectangle. Ms. B appears ***anxious*** because her eyebrows are raised and her tone of voice is somewhat ***argumentative.***
39:23	Ghee	No I said that. But she said we can make a uhh uhh, that we need to find the, uhh, area.	
39:31	Ms. B	I did say that. [Ms. B nods her head.]	Ms. B validates Devin's comment.
39:33	Dana	There maximum area just means that…	
39:36	Ms. B	So Devin just said that, what did you say? [Ms. B asks Devin]	**[Int]** Ms. B expresses ***interest*** in Devin's idea, asking him to share his drawing with his group. **[Raa]**
39:39	Ghee	The area's the inside of the box.	
39:43	Ms. B	[Devin says something that is inaudible, it appears that he stated dimensions of a rectangle.] So can you draw that and show them? Turn your paper over and then just can show them.	
39:46	Ghee	He did do it.	Ghee, Dana and Shawn appear to express ***anger*** toward Devin for crumpling his paper.
39:46	Dana	He drew it.	
39:48	Ghee	He drew it already	
39:50	Shawn	He balled it up! [Devin unfolds the "balled up" piece of paper in his hand and lays it down on his desk.]	
39:58	Ms. B	Why did you ball it up? [Devin smoothes out the paper on his desk.] Ok, so can you bring it over into the group and see what happens. [Devin begins to show the paper to his group, then takes a new piece of paper out and begins to draw a new rectangle.] [Ms. B raises her voice to announce to the class] – Guys in about three minutes we have to start wrapping it up.	When Ms. B speaks to Devin, her tone of voice is ***calm*** and ***gentle***. It may be inferred that Ms. B recognizes that Devin ***lacks confidence*** in his idea, which may be why he crumpled his paper. Ms. B waits while Devin smoothes out the paper on his desk and encourages him to share his ideas with the group. **[Endis]** It may be inferred that Devin is ***dissatisfied*** with the condition of his paper, so he chooses to draw a new rectangle to share with his group.

			Ms. B does not interfere with this decision, and waits patiently for Devin to complete his drawing.
40:17	Ms. B	You said the thing was a hundred and forty, what's the thing? [Ms. B asks Shawn. Shawn smiles and sits back in his chair.]	Ms. B expresses ***interest*** in Shawn's idea, encouraging him to use mathematical language. **[Endis]**
40:23	Ghee	Who's talking about this?	
40:24	Shawn	[Shawn smiles at Ms. B] I was talking 'bout this, but...it wasn't about what all ya'll was talking about.	
40:40	Dana	So what do we have to do to find the maximum area?	Dana expresses ***confusion*** about how to find the maximum area.
40:43	Ms. B	That's the question I'm asking you. [Ms. B smiles at Dana. Ghee sucks his teeth and puts his head down on his desk.]	Ms. B appears to ***recognize Dana's impasse*** when she responds in a ***friendly manner***. She smiles at Dana and says, "that's the question I'm asking you." **[Rsi, Enc]** Ghee appears to be ***defeated by bewilderment*** when he shakes his head and puts his head down on his desk.
40:46	Devin	You could…	
40:47	Ms. B	You could what? [Ms. B looks at Devin. She leans in toward him, waiting for him to respond.]	It appears that Ms. B's ***interest*** is peaked when she quickly turns her attention to Devin. **[Int]**
40:54	Dana	Can't we…mult…I don't know….divide?	
40:58	Ms. B	So go ahead, explain what you were saying. [Ms. B is looking at Devin.]	Ms. B does not acknowledge Dana's comments, and it appears that Ms. B is ***excited*** about Devin's suggestion because she encourages him to explain his idea. **[Int, Raa]**
41:00	Devin	[Ghee sits up and puts his hood on] Multiply the length times the width! Find out the area. [Devin writes as he speaks, Ms. B looks at him as he speaks.]	
41:05	Ghee	You can find out the area….did you hear me…..did you hear me? [Ghee is asking Ms. B]	
41:12	Ms. B	What did you say? [Ms. B looks	

		at Ghee and leans in toward Ghee]	
41:14	Ghee	I said that…uhhh…	
41:16	Dana and Ghee together:	You can multiply the length times the width.	
41:18	Ms. B	And why would you do that?	Ms. B's tone of voice is ***calm*** as she responds to Ghee, Dana and Shawn who appear to have just made a major breakthrough toward solving the problem. **[Int]**
41:19	Dana	To see what's the area! To find the area!	Dana appears annoyed with Ms. B for asking "why would you do that".
41:21	Ghee	To figure out the maximum…area. [Ghee speaks with his hand over his mouth]	
41:25	Ms. B	Okay…	
41:27	Dana	So then we need a calculator, get a calculator.	Dana expresses ***excitement.***
41:30	Ms. B	Ok, then get a calculator!	
41:31		[Dana stands up to go get the calculator.]	
41:32	Ghee	Multiplication is uhh, uhhh, eighty times twenty	It may be inferred that Ghee has made a mistake as he added the lengths and the widths of the rectangle before multiplying.
41:37	Shawn	Eighty times twenty?	
41:38	Ms. B	So what's eighty times twenty?	
41:50	Ghee	One hundred and sixty, no…fifty-nine times twenty. [Ghee begins to write on his paper.]	
41:57	Dana	Eighty times what? Eighty times twenty??	
42:01	Shawn	One thousand six hundred.	
42:12	Ghee	Yea, one thousand, six hundred. Sixteen hundred, yea sixteen hundred. [Ghee sits back in his chair and nods his head.]	It may be inferred that Ghee is feeling ***satisfied*** with his calculation. Ghee has presented faulty reasoning. It appears that he has added the lengths and added the widths then multiplied the sums. (40 + 40) x (10 + 10) = 1600
42:23	Dana	You came up with sixteen	

		hundred? So the maximum area is sixteen hundred?	
42:25	Ghee	Yes.	
42:30	Ms. B	Do you agree? [Ms. B is looking at Devin, posing this question to him.]	Ms. B does not address Ghee's faulty reasoning, but she appears ***curious*** about whether or not Devin agrees with Ghee. It may be inferred that Ms. B is ***hopeful*** that Devin will disagree with Ghee and thus stimulate a conversation about how the maximum area is calculated. **[Int]**
42:30	Dana	Is the maximum area sixteen hundred?	
42:32	Shawn	Do you agree? [All students in the group break out in laughter. Ghee leans across his desk toward Emmy, smiling and waving his hands.]	It may be inferred that the students are now teasing Emmy for not being engaged.
42:36	Ms. B	What do you say about, what do you have to say about what Devin said? Go ahead Devin continue. Listen to what Devin is saying.	It may be inferred that Ms. B ***validates*** Devin's contribution to solving the problem. She expresses ***interest*** in Devin's mathematical idea as she encourages the other students to listen to him…Ms. B's tone of voice and language is serious. It may be inferred that Ms. B recognizes that the students are at a critical point and could either become disengaged due to off-task behavior, joking around or that they could progress toward the solution to the task. She intervenes, telling the group to listen to Devin. **[Val, Endis, Ptp, Int]**
42:47	Devin	You can get….[inaudible]…[Devin is resting his head on his arm].	
43:10	Ms. B	So…[Ms. B points to Devin's paper] [Ms. B raises her voice to	Ms. B responds to Devin's alternate approach, but he does not have an opportunity to answer as the class period is almost over and Ms. B calls the class together. **[Raa]**

		address the class.] All right, guys at this point, heads up!	
43:26	Dana	Forty times what? [Dana is talking to Ghee who is writing on his paper.]	
43:30	Ghee	Forty times sixty…that's a hundred.	
43:35	Dana	Forty times sixty is not one hundred. You added.	
43:42	Ms. B	Ok I'm still waiting, my hand is up for way too long at this point. [Ms. B is addressing the class.]	
End TSMIE [B.01.a.08]			

Discussion of TSMIE [B.01.a.08]

When this episode begins, Shawn and Dana are visibly and audibly angry and they complain to Ms. B that Devin and Emmy not engaged in helping the group to solve the problem. It is inferred that Shawn and Dana are experiencing impasse with the problem and their frustration has escalated to anger at Devin and Emmy. Ms. B does not admonish Devin and Emmy rather she explains that it is Dana and Shawn's responsibility to engage their group members. "Well don't give 'em the answer, make them give you the answer." Ms. B models some ways for the students to question Emmy about her thinking.

Ms. B appears to express anger toward Shawn and Ghee when she asks them "what's the maximum area" and it may be inferred that she is suggesting that they don't have the answer either, so they are in no position to ridicule Emmy. "Do any of us know yet? [Ms. B is smiling and leaning in toward the group.]"

Ms. B attempts to facilitate a discussion about the problem and it appears that the students are confused about how to find area. She poses questions to the students, yet

their answers do not indicate that they are cognitively engaged with the mathematics, "hundred is the feet…FENCING!", "we're trying to figure it out", Emmy shrugs her shoulders. But when Devin suggests, "you could do different numbers", Ms. B encourages him to share his ideas with the group even though he had crumpled his paper. It may be inferred that Ms. B's validation of Devin's work may encourage his engagement.

Ghee's anger appears to be relieved when he realizes (due to Devin's contribution) that area of a rectangle is calculated by multiplying length time width.

Ghee presents faulty reasoning when he attempts to calculate the area of a rectangle with dimensions forty-by-ten. Rather than multiply forty times ten, he adds the sides then multiplies the sums. (40 + 40) x (10 + 10) = 1600 and announces his answer with confidence. Ms. B does not correct Ghee's faulty reasoning, but attempts to engage Devin in a discussion, possibly hopeful that Devin may correct Ghee's mistake. Devin does not make his point, and the episode ends when Ms. B addresses the whole class.

Outcomes of this episode: 1.) Emmy remains silent and does not engage with the group discussion. 2.) Devin presents his mathematical ideas despite the fact that he had earlier crumpled his paper, possibly thinking that he was wrong. 3.) Ghee's anger is relieved as a possible consequence of Devin's contribution, "Multiply the length times the width! Find out the area." 4.) Ghee becomes engaged with finding the area of the forty-by-ten rectangle.

Immediately subsequent to this episode, Ghee continues to write on his paper, appearing to be very engaged and interested in solving the problem. However, he

exhibits frustration when he puts his pencil down in response to Ms. B's request to stop working.

Ms. E: Classroom observation analysis, "Fences for Grazing"

Day Two

Ten *teacher/student(s) mathematical interaction episodes (TSMIE)* were identified in the video data collected in Ms. B's classroom on Day Two of "Fences for Grazing" during a forty-five minute class period.

When the class period begins the students are seated in groups of three to five as they were on Day One of "Fences for Grazing". The students appear engaged in continuing their work on the problem, "Fences for Grazing" that they started during the previous mathematics class period. Ms. B circulates throughout the classroom the students work in their small groups. In the first ten minutes of the class period, it appears that Ms. B interacts with each group to determine whether or not they are prepared to present their solutions on chart paper.

Ms. B initiates this episode when she approaches the group and sits down at an empty student desk between Ghee and Devin. She expresses curiosity about the students' current status with the problem.

Begin TSMIE [B.01.b.01] 1:38 – 3:34 (S)			
Participants: Ms. B, Dana, Lenny, Ghee			
1:38	Ms. B	Ok, so where are you guys?	Ms. B expresses ***curiosity*** about the students' current status with their solution to the problem.
1:50	Ms. B	[Devin] said no. [Ms. B is looking at Devin.] Go ahead, say it. [Devin is playing with something in his hand and Ms. B reaches over	It appears that Ms. B recognizes that Devin is not in has an alternate approach to solving the problem. **[Raa]** Ms. B encourages Devin to share his

		and takes it away.]	idea with the other students in his group. **[Endis]**
2:00	Devin	…times the area. The length times the width.	
2:03	Ghee	Length times the width. [Ms. B turns around to speak to the student seated behind her.]	
2:05	Devin	We times the length times the width to get the area.	It may be inferred that Devin is responding to an error that was made at the end of yesterday's class period. At that time, Ghee had added the lengths and the widths then multiplied the sums to calculate area.
2:10	Ms. B	So, how did you figure that out? [pause] 'Cause yesterday you guys were confused on the area and perimeter…[Ms. B is looking directly at Dana, her facial expression is serious, her tone of voice is calm. Larry hands Ms. B a book, which she looks at for a second then puts under the desk where she is seated.] So, how come you just walked in today and you just know area?	It may be inferred that Ms. B is ***curious*** about the students' understanding of area. Her tone of voice is calm and she appears to be listening to the students' ideas as she pauses between her questions. **[Int]** Ms. B expresses curiosity about why the students changed their approach for calculating the area of the rectangle. **[Int]**
2:28	Ghee and Dana	'Cause we multiply… 'Cause we did it yesterday .	
2:29	Dana	Yeah we did it yesterday, but we didn't finish in a lot of time…[inaudible]	
2:37	Ms. B	Okay, okay [Ms. B nods her head] So now, what do you have to find out?	It appears that Ms. B expresses understanding when she nods her head. Ms. B appears to express ***curiosity*** about the students' next step.
2:40	Dana	The area! The maximum area!	Dana appears to be annoyed because her tone of voice may be inferred to

			convey exasperation for being asked this question again.
2:41	Ms. B	The maximum area. [Ms. B nods her head.] Okay. And did you come up with an answer?	Ms. B expresses curiosity. **[Int]**
2:45	Dana	Yeah!	
2:46	Ms. B	What was it?	
2:47	Dana	Sixteen hundred.	
2:48	Ms. B	Sixteen hundred what?	Ms. B ***does not validate nor negate*** Dana's answer, yet it may be inferred that she wants the students to express their answer in square feet. **[Int, reasoning]**
2:50	Lenny	For the um, for the area.	
2:53	Dana	Um...[Dana looks at her paper then back at Ms. B. Ms. B is making eye contact with Dana.] Sixteen hundred square feet?	
2:54	Ms. B	Sixteen hundred square feet. [Ms. B nods her head.] Okay. [Ms. B looks at Devin] So you said no, and then you said never mind…Say again…. [Ms. B looks at Dana and Larry.] He thought it was a hundred.	Ms. B appears to validate Dana's answer. **[Val]** Ms. B addresses Devin and points out that he disagreed then withdrew his comment. **[Endis, Int]** It may be inferred that Ms. B is trying to engage the other students in a discussion about Devin's reasoning when she looks at Dana and Larry, stating "He thought it was a hundred."
3:08	Lenny	He thought the area of the boxes was a hundred.	
3:10	Dana	No, the perimeter is the a hundred…No the perimeter is the a hundred.	Dana presents correct reasoning.
3:17	Shawn	[Inaudible.]	
3:18	Dana	What you talkin' bout? [Dana speaks to Shawn.]	Dana's tone of voice is argumentative.
3:20	Ms. B	So you guys, so at this point, you	Ms. B does not validate nor negate

		have your answer? [Ms. B looks directly at Dana.]	Dana's correction of Devin's faulty reasoning. It may be inferred that Ms. B is attempting to engage Dana in the mathematics, preempting a possible social conflict between Dana and Shawn.
3:22	Dana	Yeah.	
3:23	Ms. B	Ok so if you have your answer then I need for you to get another chart paper [Ms. B looks at Larry and he gets up from his chair] and show exactly what you did answer those questions, ok? [Ms. B stands up and walks away from the table.]	Ms. B gives directions for students to present their answer on chart paper. This can be inferred as a move to encourage student discourse as the students prepare to share their solution with the class. **[Endis]**
End TSMIE [B.01.b.01]			

Discussion of TSMIE [B.01.b.01]

Ms. B initiates this episode by asking the students about the current status of their solution. She expresses curiosity about how they know that the area is calculated by multiplying the length times the width recalling that yesterday they had difficulty with this concept. Dana presents her group's answer as "sixteen hundred" and when prompted by Ms. B, she says, "Sixteen hundred square feet?" Ms. B nods her head, possibly validating that Dana used the correct units.

It is inferred that the students are following the solution that Ghee presented from class on the previous day (TSMIE [B.01.a.08]) when he added both lengths then both widths then multiplied the two sums to get sixteen hundred. Ms. B does not comment on the erroneous numerical answer, nor does she ask the students to explain how they arrived at the number, sixteen hundred.

Ms. B appears satisfied with the students' progress when she directs them to get a piece of chart paper and begin recording their solution to the problem.

Outcomes of this episode: 1.) Dana has an opportunity to report the answer for her group 2.) The group has presented an incorrect answer, but Ms. B does not correct the students 3.) Ms. B appears satisfied and assigns the students the task of recording their solution on chart paper.

Immediately subsequent to Ms. B's departure from the group, the students argue about who will be the recorder of the group. They are not engaged with the mathematics.

Begin TSMIE [B.01.b.02] 3:09-4:49 (C)			
Participants: Shay, Ms. B Ms. B initiates this episode when she approaches the group and asks "Do what?" It may be inferred that Ms. B's motivation for interacting with this group is that she is circulating among each group to assess the students' progress.			
3:02	Ms. B	Do what? [Ms. B has her hands placed on an empty desk next to Shay, she is standing and leaning in toward the students.]	
3:04	Student	…maximum area	
3:05	Ms. B	Tryin' to do what? [Ms. B looks at boy to her left.]	
3:06	Student	Find the maximum area.	
3:07	Ms. B	Find the maximum area.	
03:09	Ms. B	Did you come up with anything as of yet? [Ms. B is leaning in toward the students, looking at the papers on the table in front of Shay.]	It may be inferred that Ms. B is ***curious*** about the students' mathematical progress. **[Int]**
03:11	Shay	No, the only stuff I got to…the highest was six hundred twenty-four. [Shay shuffles his papers and looks up at Ms. B. Ms. B is looking at Shay's	Shay expresses ***uncertainty*** about his answer and his tone of voice may be inferred to convey disappointment about his answer.

		papers.]	
03:14	Ms. B	The highest you got was six hundred twenty-four [Ms. B pauses, then cocks her head to the side, glances briefly at female student]..ok…is there a way that you could find out if there is a larger one? [Ms. B shrugs her shoulders and raises her eyebrows while looking at the papers on the desk]	Ms. B repeats Shay's answer, 'six hundred twenty-four'. It may be inferred that Ms. B exhibits ***curiosity*** about the possibility of a larger answer. **[Int, reasoning]**
03:30	Shay	Try and find more.	
03:32	Ms. B	Okay. [Ms. B starts to move away from the table.] You guys can get your chart papers [Ms. B points to the students and then gestures toward the papers in front of Shay] ..to put your thoughts on the chart papers as well…	
3:36	Shay	We'll never find it!	Shay expresses ***anger*** and it may be inferred that he is feeling overwhelmed by the potential enormity of the problem.
3:38	Ms. B	From what you have so far, I wanna see what's on there [Ms. B points to Shay's paper] and if you have anything else you wanna put on there you can do that as well, ok? [Shay nods his head] Make sure you explain how you found the maximum area, ok? Use words to explain it.	Ms. B encourages Shay to record his solution even though he thinks it is not complete. **[Int]** Ms. B reminds Shay to record his method of finding the "maximum area" likely because he will be sharing his work with the whole class. **[Endis]**
04:03	Shay	So, everybody gon have a different maximum area?	Shay appears to be ***curious*** about the answers determined by other groups.

04:05	Ms. B	That's what you're gonna decide and see. [Ms. B looks at Shay then looks down at the papers.] Alright. I'm not gonna tell you what the other groups have but when you go and visit the other groups, you'll be able to see what they have...and you'll answer your own question. [Ms. B looks sideways at Shay]	Ms. B is supporting Shay's autonomy "that's what you're gonna decide…and you'll be able to answer your own question." She encourages student discourse by indicating that Shay will "visit the other groups". **[Int]**
04:15	Shay	So that mean we could put that what we have, there is no maximum area…[Shay mumbles and is inaudible]	
04:22	Ms. B	So there is no maximum area or do you have a maximum area? [Ms. B looks at the papers as she speaks to Shay.] What is maximum? [Ms. B looks at the other students in this group.]	It may be inferred that Ms. B recognizes that Shay is ***dissatisfied*** with his "maximum area". Yet, she expresses curiosity about whether or not he has found "a maximum area". After posing the question, "do you have a maximum area?" Ms. B pauses and the students do not respond. It may be inferred that Ms. B recognizes that the students may be having difficulty understanding her question, so she rephrases her question with a simpler question. "What is maximum?" **[Int]**
04:30	Frank	The highest.	
04:31	Ms. B	The highest. So as of right now… [Ms. B looks at Shay's paper.]	Ms. B ***validates*** Frank's answer. **[Val]**
04:33	Shay	The highest that we got is six hundred twenty-four.	
04:35	Ms. B	So is that a maximum area? Or that is not a maximum area [Ms. B shakes her head	Ms. B ***encourages Shay's autonomy*** by asking him to describe his answer. **[Int]**

		'no']…for what you have so far? [Ms. B gestures toward Shay's paper]	
04:38	Shay	It is, yeah it's our highest maximum area.	
04:40	Ms. B	That's your maximum area, [Ms. B nods her head and points toward Shay] so I don't wanna hear that you don't have an answer, [Ms. B looks directly at Shay and pauses]…okay? This is your answer so far…[Ms. B points at Shay's paper] Does it mean that you're right?	Ms. B ***validates*** Shay's answer by nodding her head in agreement. She points out that their answer may not be correct. Her tone of voice may be inferred to convey ***confidence*** in the students' mathematical progress "so far". **[Val, Enc]** Ms. B is ***abrupt*** when she addresses Shay's suggestion that his group does not have a satisfactory answer. It appears that she recognizes Shay's impasse. **[Rsi]** Ms. B makes direct eye contact with Shay to emphasizing the value of their mathematical progress "so far".
04:46	Frank	No.	
04:47	Ms. B	No. Could there be a higher one? [Ms. B shrugs her shoulders and raises her eyebrows as she looks at Shay]	
04:48	Shay	Yeah.	
04:49	Ms. B	Possibly, that might be the highest one, we don't know. Ok so you can start putting what you have so far and then you'll see what other groups have put, cause we just wanna start sharing. [Ms. B walks away from the group.]	It appears that Ms. B is emphasizing the importance of sharing mathematical ideas with peers because she appears ***eager*** for the students to begin sharing their ideas with others. **[Endis]**
End TSMIE [B.01.b.02]			

Discussion of TSMIE [B.01.b.02]

Ms. B initiates this episode when she appears to be checking in on the student's mathematical progress to see whether or not the students in each group are ready to record their answers on chart paper.

Shay expresses frustration and uncertainty about his answer, six hundred twenty-four. Ms. B appears to recognize Shay's uncertainty and validates Shay's suggestion, "Try and find more". She begins to walk away from the table, but it appears that she is drawn back by Shay's angry outburst, "We'll never find it!"

Ms. B recognizes Shay's anger and intervenes by asking him to articulate his groups' answer and she responds, "…so I don't wanna hear that you don't have an answer, [Ms. B looks directly at Shay and pauses]…okay? This is your answer so far…[Ms. B points at Shay's paper]".

Outcomes of this episode: 1.) Ms. B articulates acceptance of Shay's answer even if it may not be correct. 1.) Shay acknowledges that his answer may not be correct and that there may be a "higher" maximum area.

Immediately subsequent to this episode, Shay looks around, then pulls his seat in and looks at his papers.

Begin TSMIE [B.01.b.03] 6:40 – 11:21 (C)			
Participants: Natasha, Ms. B, Kenny, Al (Emmy is present*) Ms. B initiates this episode when she sits down next to Natasha and tells her to stop coloring her nametag. *Emmy was seated at Dana's table during the previous day's lesson. Emmy does not speak during this episode.			
06:40	Ms. B	What did you come up with now? [Ms. B speaks to Natasha, and points at Natasha's nametag, which appears to have been colored with a	It appears that Ms. B perceives Natasha's behavior to be off-task as she acknowledges Natasha's coloring of her nametag. Ms. B's comment, "this looks good" may be inferred to convey sarcasm about Natasha's coloring which

		pencil.] This looks good, but you don't need to color anymore. Where are you now? [Natasha smiles, Ms. B smiles and looks at Natasha.]	is clearly not the objective of the lesson. Both Ms. B and Natasha smile. The interaction is subtle and attention is quickly returned to the mathematics. It appears that Ms. B has a pleasant rapport with Natasha as she holds Natasha accountable for her mathematical engagement.
06:47	Natasha	Basically…I still didn't find that out. [Natasha points to her paper. Ms. B looks at Natasha's paper.]	Natasha's facial expression may be inferred to convey ***disappointment*** that she "still didn't find that out".
06:50	Ms. B	You didn't find what out? So how did you find the area? Before I left, you told me. [Ms. B purses her lips.] Hmm?? [Ms. B raises her eyebrows and looks at Natasha.] How do you find the area?	Ms. B expresses ***curiosity*** about Natasha's mathematical progress and ***curiosity*** about Natasha's knowledge, ***encouraging*** Natasha by recalling "Before I left, you told me". Ms. B acknowledges that Natasha had correctly identified the correct method for finding the area of a rectangle during a previous interaction. **[Enc, Int]**
	Natasha	Ummhum. [Natasha looks down at her desk.]	It appears that Natasha is avoiding eye contact with Ms. B as she mumbles, "I don't know".
	Ms. B	Which is the area, inside or outside? That's what he wants to know.	Ms. B appears to recognize Natasha's impasse. **[Rsi]** Ms. B's tone of voice is gentle when she addresses Natasha. It may be inferred that Ms. B is trying to encourage student discourse, "that's what he wants to know". **[Endis]**
07:06	Kenny	Inside. [Ms. B shrugs her shoulders.]	
07:08	Ms. B	Inside? [Ms. B nods her head.] So if area's inside, how do	Ms. B ***validates*** Kenny's contribution as she nods her head and she expresses ***curiosity*** when she ***builds*** on Kenny's

		you find the inside?	idea, "how do you find the inside"? **[Val, Int, Blds]**
07:18	Natasha	Multiply the outside. [Natasha adjusts her earring as she speaks. She does not make eye contact with Ms. B.]	
07:21	Ms. B	What do you mean, multiply the outside? [Ms. B looks down at the desk.]	It may be inferred that Ms. B is ***interested*** in Natasha being more specific about her explanation of how to find the area. **[Int, Blds]**
07:26	Natasha	The length…times the width.	
07:28	Ms. B	Ok, so… [Ms. B looks at Natasha] So what? [Ms. B looks at Kenny, leaning in toward him]	It maybe inferred that Ms. B appears ***hopeful*** that Natasha will proceed with an answer. Ms. B expresses ***interest*** in Kenny's reasoning as she leans toward him. **[Int]**
07:34	Kenny	I counted um, all the numbers…I got twenty-one…then I multiplied. [Kenny speaks softly and rests his cheek in his hand as he looks down at his paper.]	Kenny appears to lack confidence in his solution as he speaks softly and has his cheek in his hand.
07:50	Ms. B	[Ms. B turns around in her seat to address students at another table.] Focus yourself…and go work on the project now! [Ms. B turns her attention back to Kenny.] Go ahead.	It may be inferred that Ms. B expresses ***anger*** toward the perceived off-task behavior of students at a different table. **[Dis-b]**
08:11	Kenny	[Kenny sits back in his chair and speaks very softly as he points to his paper.] I multiply these numbers times these numbers and I got two hundred fifty-two.	It may be inferred that Kenny is ***not confident*** about his answer, and is sharing his ideas with ***reluctance.*** Because he is slumped down in his seat, looking down and speaking softly. Also, Kenny appears to be distracted by

		Inside.	noise from other groups as he looks around the classroom. It appears that Kenny has drawn a rectangle to scale on grid paper and is counting the number of squares inside the rectangle by multiplying the length times the width (twenty one times twelve equals two hundred-fifty two).
08:21	Ms. B	Do you agree? [Ms. B stands up so that Al can sit at the desk next to Natasha. Ms. B is now standing next to Kenny. Ms. B is looking at Natasha.]	Ms. B ***encourages student discourse*** as she expresses ***interest*** in Natasha's opinion of Kenny's idea. **[Endis, Int]**
08:26	Natasha	Because you don't know if that's right.	
08:28	Ms. B	How come you don't know if it's right?	Ms. B's tone of voice is emotionless as she expresses ***curiosity*** about Natasha's suggestion. **[Int]**
08:29	Natasha	Because he made a box of his own size and then he countin', um, multiplying the numbers on both sides. And what it say if I made a box this small, [Natasha draws a rectangle on her paper] I could do the same thing and then just count the numbers on the inside and just say any number. But if he say forty for the length and ten for the width, then don't you gotta multiply forty times ten? Hmm?	It appears that Natasha ***disagrees*** with the method that Kenny has used to find the area of the rectangle. Natasha's tone of voice is ***argumentative.***
08:56	Ms. B	You're trying to find the area, do you remember how to find the area of a rectangle? [Ms. B speaks to Al]	Ms. B redirects the students to the main problem and conveys ***curiosity*** in Al's understanding of how to find the area of a rectangle. **[Int]**

09:02	Al	I think I do.	Al appears confidence in his ability to find the area.
09:03	Ms. B	Okay. [Natasha], so go ahead, repeat what you just said.	Ms. B encourages student discourse. **[Endis]**
09:07	Natasha	I don't even know what I just said, um…	Natasha appears ***confused*** and disengaged from the mathematics.
09:13	Ms. B	What are you multiplying? What are you putting in the calculator? Repeat what you just said again. [Natasha looks up at Ms. B] About the boxes.	It may be inferred that Ms. B ***recognizes Natasha's impasse*** and is trying to keep the students engaged with the mathematics despite the frustration. **[Rsi, Int, Endis]** It appears that Natasha is reluctant to participate, yet Ms. B is persistent. She remains emotionless as she poses questions to stimulate Natasha's engagement. **[Int]**
09:26	Natasha	Ok, um, he sayin' that twenty-one times twelve or something like that, and that's how he got the area, but he just countin' the numbers inside the box, [Natasha reaches across the table pointing at Kenny's paper] inside his box, but I say, what if I make a box this small, [Natasha points at the small rectangle that she has drawn.] I could do the same thing and count those boxes goin' across too and just say any number. But you can't do it that way. So that's why I said if you gon' have forty for the length and ten for the width, it should be forty times ten.	Natasha ***disagrees*** with Kenny's method for finding the area of a rectangle. She provides an example to support her reasoning for disagreeing with Kenny.
10:07	Ms. B	Do you guys remember what the, um, what the fencing was? What fencing are you working with?	**[Rsi]** It appears that Ms. B recognizes that Natasha is not considering whether or not the dimensions of Kenny's rectangle

		Fencing, what number, how many feet?	yield a perimeter of one hundred. It may be inferred that Ms. B is using this question to intervene and possibly redirect the students to reconsider Kenny's idea. Ms. B leans on Al's desk, toward Natasha. Her language is abrupt and it may be inferred that Ms. B is becoming ***anxious*** about the level of student mathematical engagement. Her facial expression is serious.
10:18	Al	Five.	
10:19	Ms. B	Five feet?	Ms. B appears confused about Al's reasoning. **[Con]**
10:22	Al	I did some research.	
10:23	Ms. B	You did some research. Ok so what did you do?	Ms. B ***validates*** and expresses ***curiosity*** about Al's claim that he did some "research". **[Val, Int]**
10:28	Al	[Al rearranges his papers, Ms. B looks on, leaning down toward Al.] Like, for this one, I was just thinking of, it just came to my head. I did these on any size, but I ain't get that, so I did it this way. [Al shows Ms. B examples of other rectangles that he drew.] So what I did was, I took a ruler [Al looks up at Ms. B] Ms. B: "Okay." …and I put the line on through…	Ms. B looks at Al's paper as he attempts to explain his "research".
10:53	Ms. B	So, this number tells me what? [Ms. B traces along one edge of a rectangle.]	It may be inferred that Ms. B expresses ***curiosity*** about Al's mathematical reasoning as she asks him to clarify the meanings of the numbers on the rectangle.
10:58	Al	Which number?	

10:59	Ms. B	[Ms. B points at Al's paper.] The number forty, what does forty tell us?	**[Int, reasoning]**
11:01	Natasha	[Natasha points at Al's paper] No, forty is supposed to be here.	
11:04	Al	No, no. [Al shakes his head 'no'.]	
11:05	Ms. B	Why?	
11:07	Kenny	[Kenny points at Al's paper with his pencil] Because…this is too long…for it to be.	
11:09	Al	Because…[Inaudible conversation between Al and Natasha, mumbling.]	
11:21	Ms. B	I'll be back. [Ms. B walks away from the table.]	Ms. B does not wait for the students to respond to her question about why "forty is supposed to go here".
End TSMIE [B.01.b.03]			

Discussion of TSMIE [B.01.b.03]

Ms. B addresses Natasha's perceived off-task behavior, with an inferred sarcastic complement of Natasha's coloring on her nametag. It may be inferred that Ms. B has a positive rapport with Natasha because they both smile in response to Ms. B's comment.

Ms. B appears to recognize Natasha's impasse and encourages Natasha by expressing curiosity about how she found the area of the rectangle. Natasha responds to Ms. B's questions with very brief answers and she does not appear to be engaged. Meanwhile, it may be inferred that Kenny used grid paper to draw a rectangle and is counting the square units within his rectangle to find the area. Al has reverted to using a ruler to measure the lengths of the sides.

Kenny presents correct reasoning in finding the area of a rectangle on grid paper by counting squares within the rectangle. However, the dimensions of his rectangle (twenty-one and twelve) do not yield a perimeter of one hundred. Natasha disagrees with Kenny's method and explains to Ms. B that the only way to find the area of a rectangle is to multiply the length times the width. Al appears to be pseudo-engaged with the mathematics when tells Ms. B "I did some research" but he is unable to articulate a clear mathematical idea.

This episode is concluded when Ms. B says, "I'll be back" and walks away.

Outcomes of this episode: 1.) Natasha articulates the correct method for calculating the area of a rectangle. 2.) Kenny articulates a different method for finding the area of a rectangle by counting the squares on a grid within the rectangle. 3.) Al is pseudo-engaged with the mathematics.

Immediately subsequent to this episode it appears that Natasha is disengaged from the mathematics because she is laughing and joking around with Al. After about one minute, she becomes reengaged when she asks Kenny if she can use the calculator and begins to make calculations.

Begin TSMIE [B.01.b.04] 15:22 – 16:39 (C)			
Participants: Ms. B, Kenny, Al, Natasha Ms. B initiates this episode when she tells the students to begin making their presentation on the chart paper.			
15:22	Ms. B	[Ms. B is seated on a student chair, and leans in between Kenny and Al]. Since you don't have your work, why don't you go over your thoughts, ok? So do that now, on the um chart paper…	Ms. B's tone of voice and language may be interpreted show that Ms. B is ***eager*** for the students to begin work on their presentation even though they have not yet agreed on a rectangle that satisfies the conditions of the problem.
15:28	Natasha	I disagree.	Natasha expresses her displeasure with Kenny's ideas.
15:29	Ms. B	You disagree? It's okay to disagree. [Ms. B looks at Natasha, raises her eyebrows and nods her head.] I disagree with people all the time, you can disagree, but you have to have reasons as well, right? You still can try and convince them. Did you show 'em…did he listen?	Ms. B's tone of voice is gentle and affirming as she sympathizes with Natasha's feelings. Ms. B ***validates Natasha's right to disagree*** by stating that she also disagrees with people "all the time". Then, Ms. B ***encourages student discourse*** as she explains the importance of supporting disagreements with "reasons". **[Val, Endis, Ptp]**
15:46	Natasha	Mmm, no.	
15:48	Ms. B	Did you listen? [Ms. B looks at Kenny and smiles.]	Ms. B encourages peer-to-peer respect when she asks, "Did you listen?" **[Ptp, Endis]**
15:49	Kenny	Um, I tried listening. [Kenny smiles and looks down.]	Kenny appears embarrassed.
15:50	Ms. B	You're trying to understand it? [Ms. B smiles at Kenny.] So did you have a question about some things that you didn't understand?	It may be inferred that Ms. B ***recognizes Kenny's impasse*** in understanding Natasha's solution. Ms. B expresses ***interest*** in Kenny's confusion.

			[Rsi; Int,reasoning]
15:58	Natasha	And mine is the same thing as yours [Natasha looks at Al]… Oh, no it ain't.	Natasha's tone of voice is initially argumentative. Natasha acknowledges that her answer is not the same as Al's.
16:04	Ms. B	Can you guys get your chart papers so that you can start putting your thoughts down?	Ms. B encourages student discourse when she asks the students to write their "thoughts" about how to solve the problem on chart paper.
16:08	Al	So we put both on, both our answers?	
16:09	Ms. B	Whatever you wanna put up there, ok? And then we're gonna start going in groups and sharing our thoughts, but I need you to sit down because we're gonna start in five minutes.	Ms. B ***encourages student autonomy*** by expressing ***interest*** in any ideas the students might have. "Whatever you want to put up there". **[Int]**
16:16	Kenny	Does the inside have to be the same as the outside? [Kenny traces a rectangle on his paper with his pencil, and looks at Ms. B.]	Kenny still appears confused.
16:22	Ms. B	Does the inside have to be the same as the outside? [Ms. B looks at the other students.]	Ms. B does not answer Kenny's question, but redirects his question back to the other students in the group. **[Endis]**
16:25	Natasha	No, it's not even supposed to be the same. [Natasha resumes coloring her nametag.]	
16:27	Ms. B	Why not? [Ms. B looks at Natasha, and looks at Natasha's coloring on the name tag.]	Ms. B expresses ***curiosity*** in Natasha's reasoning that the area and perimeter do not have to be equal. **[Int, reasoning]** Episode B.01.b.03 began with Ms. B showing disapproval of Natasha's coloring. Now, Ms. B appears to make a

			conscious decision not to acknowledge the behavior.
16:29	Natasha	Because the perimeter is not supposed to be the same as the area. [Natasha colors as she speaks and she rests her head in her hand.]	
16:33	Ms. B	Why not?	Ms. B persists, asking Natasha to explain her reasoning that the area and perimeter do not have to be the same. **[Int, reasoning]**
16:35	Natasha	'Cause the area is bigger than the perimeter. [Natasha continues to color, she looks at her nametag as she speaks.]	
16:39	Ms. B	Then what's the point...[Ms. B walks away to the door.]	**[Int, reasoning]**
End TSMIE [B.01.b.04]			

Discussion of TSMIE [B.01.b.04]

This episode begins when Ms. B encourages the students in this group to write their solutions on chart paper because the class will begin to share their ideas in five minutes. It appears that the students are experiencing impasse because Natasha and Kenny disagree about their solutions.

Ms. B appears to recognize Natasha's discontent over disagreeing with Kenny's ideas and Ms. B validates Natasha's feelings. Ms. B states that she also disagrees with people all of the time. Yet, Ms. B stresses the importance of backing up a disagreement with reasons. She also encourages peer-to-peer respect by stressing the importance of listening to the ideas of others' despite disagreement.

Natasha appears to remain engaged only when Ms. B is present. And when Ms. B is present, Natasha appears to engage with reluctance, answering Ms. B's questions, but offering little more.

When Natasha resumes coloring her nametag, a behavior that was addressed by Ms. B in the previous episode, Ms. B appears to choose to ignore the behavior as she engages in a mathematical discussion with Natasha.

It appears that the students do not make mathematical progress during this episode because when Ms. B walks away the students seem confused about which answer they should record on the chart paper.

Outcomes of this episode: 1.) Ms. B validates Natasha's right to disagree with another student. 2.) Kenny expresses confusion "does the outside have to be the same as the inside?"

Immediately subsequent to this episode, all four students remain seated and silent after Ms. B walks away from the group.

Immediately prior to the next episode, TSMIE [B.01.b.05], Dana is engaged in examining the work presented by Shay's group. She appears puzzled. Ghee, Shawn, Devin and Larry appear to be pseudo-engaged, waffling back and forth between engagement and disengagement. In this episode, Dana and her group are looking at the work presented on chart paper by Shay's group.

Begin TSMIE [B.01.b.05] 24:00 – 25:45 (L)			
Participants: Dana, Ghee, Shawn, Devin, Larry, Ms. B Ms. E initiates this interaction when she inquires about the student's thoughts regarding the work presented on the chart paper.			
24:00	Ms. B	So, what do you guys think about this? [Ms. B is standing, leaning on the table next to Larry. Her facial expression is serious.]	Ms. B expresses ***curiosity*** about the students' opinions of Shay's work. **[Int]**
24:01	Dana	Oh, we…	
24:02	Ms. B	Push your chairs in. Stand up.	
24:09	Ms. B	So… is it the same or is it different from what you did? [Ms. B looks at Dana, then at the paper.]	Ms. B expresses ***curiosity,*** leading the students to compare Shay's work with their own. **[Int]**
24:11	Dana	They, they, um…Their perimeter is right because it equals up to a hundred. [Dana is holding a calculator, she points with her finger as she looks at Ms. B. Ms. B looks at Dana as she speaks.]	
24:12	Ms. B	Ok. [Ms. B nods her head and looks at the chart paper.]	It appears that Ms. B is satisfied with Dana's understanding that the solution on chart paper shows a rectangle with a perimeter of one hundred.
24:15	Ghee	Where is their answers? Where are their answers?	
24:18	Dana	They don't got they, um…they answered it right here! [Dana points at the chart paper.]	It may be inferred that Dana is annoyed at Ghee because her tone of voice sounds impatient.
24:22	Lenny	Their maximum area is…[Lenny reads the chart paper to himself, he is mumbling, inaudible. Ms. B looks at the chart paper as Larry reads.]	
24:43	Ms. B	[Ms. B glances at Larry briefly then asks.] What did they do? [Ms. B does not look at Larry. Her tone of voice is quiet and her facial expression is serious.] [Ghee is leaning his elbows on the table and moving his mouth	Ms. B expresses ***curiosity*** about the students' interpretation of mathematical process as presented on Shay's paper. It may be inferred that Ghee

		silently.]	may be off-task as he is mouthing words to a student at a different table.
24:44	Ms. B	What did they do?! [Ms. B leans in toward Ghee and looks directly at him.]	It may be inferred that Ms. B is ***angry*** at Ghee for being off task because she leans in quickly and raises her voice when she asks him, "What did they do?" **[Dis-b]**
24:47	Ghee	[Ghee grins and fidgets, his whole body is moving.] They, um…they multiplied…the things.	
24:50	Lenny	They multiplied…these two. [Larry points at the chart paper.]	
24:52	Dana	They multiplied they multiplied the length times the width. [Dana traces the rectangle with her finger. Ms. B glances briefly at Dana.]	
24:56	Devin	But when I added…it came up to a hundred too.	Devin's tone of voice sounds like he is tentative about his contribution.
24:59	Dana	Alright!	Dana looks at Devin. Her tone of voice may be inferred to convey annoyance at Devin's utterance.
24:59	Ms. B	So, what does that mean? [Ms. B looks at Devin. She is leaning on the desk, toward Devin, her eyebrows are furrowed.]	Ms. B appears to be ***interested*** in Devin's idea because she looks directly at Devin and furrows her eyebrows. It is inferred that Ms. B recognizes Dana's anger directed at Devin and she intervenes as a move to validate Devin's reasoning. **[Int, Val]**
25:02	Dana	That they, um…	
25:02	Ghee	Everything, everybody just…	
25:05	Dana	It had to equal up to a hundred. [Dana is leaning on the desk, writing. Ghee and Shawn are talking quietly to each other. Ms. B is looking at Dana.]	
25:08	Ms. B	What had to equal up to a hundred? [Ms. B's facial expression is	Ms. B appears ***interested*** in Dana's reasoning and it may be

		emotionless. She is leaning in looking at Dana as Dana writes on a piece of paper.]	inferred that she thinks that by articulating what equals up to a hundred may help Dana toward understanding this solution. **[Int]**
25:10	Dana	The [permeeter].	
25:11	Ms. B	The perimeter?	Ms. B corrects Dana's pronunciation of "perimeter".
25:12	Dan	The perimeter. [Dana shakes her head. Ms. B smiles.]	
25:13	Ghee	The "permeeter".	Ghee appears to be mocking Dana's pronunciation of perimeter.
25:14	Dana	Shut up, mind your business.	Dana expresses ***anger*** toward Ghee for mocking her pronunciation.
25:16	Ghee	Not on camera! [Ghee looks up at the camera and shakes his head.]	It may be inferred that Ghee is referring to Dana's use of inappropriate language when she said "shut up" to him.
25:18	Ms. B	So what are you doing now?	Ms. B does not acknowledge the interaction between Ghee and Dana. Ms. B's tone of voice is calm, and it may be inferred that she tries to re-engage the students with the mathematics by asking them "what are you doing?"
25:20	Ghee	You don't have to be the ignorants. Why are you acting so ignorant? [Ghee looks up at Dana. He is leaning on the desk with his elbows.]	
25:24	Ms. B	What are you doing right now?	
25:27	Dana	Oh, 'cause how he did [inaudible], he times'd this or I added, um, the width and the length…	
25:38	Ms. B	So what comment are you guys going to say to them? [Ms. B points toward Shawn and Ghee. Then, she leans on the desk speaking directly to Shawn.] Shawn? Focus. Focus. [Ms. B walks away.]	Ms. B's tone of voice may be inferred to convey annoyance at Shawn's disengagement with the mathematics. **[Int, Dis-b]**
End TSMIE [B.01.b.05]			

Discussion of TSMIE [B.01.b.05]

Ms. B expects the students to critically examine the solutions written by each of the other groups in the class. She asks these students, "what do you think about this?" Dana responds to Ms. B's questions and appears to recognize that although Shay's rectangle has the same perimeter, the area of Shay's rectangle was different from Dana's group. Devin makes an attempt to engage in the discussion, but Dana lashes out at him, "Alright!" Ms. B does not acknowledge Dana's outburst at Devin.

Ms. B looks on silently as Dana appears to try and understand Shay's solution. Twice during this episode, Ms. B addresses Ghee and Shawn expressing disappointment in their perceived off-task behavior. In both instances, it appears that Ms. B is angered by the students' disengagement when she sharply tells them to "focus".

Begin TSMIE [B.01.b.06] 26:37 – 28:05 (C)			
Participants: Shay, Ms. B, Frank This episode begins when the camera captures Ms. B at Natasha's table. Shay, Frank and another student are standing around the table, looking at the chart paper produced by Natasha's group. Ms. B is asking Frank to explain his opinion of the groups' work.			
26:37	Ms. B	[Off camera, Ms. B can be heard asking,] So what does that mean? [Ms. B is standing, leaning on the desk, looking at Frank] They didn't finish it, but what can you say about what they did already?	It may be inferred that Ms. B is ***interested*** in Frank's interpretation of the students' mathematical reasoning. **[Int, reasoning]**
26:38	Frank	It equals a hundred.	
26:38	Ms. B	Huh? [Ms. B looks up at Frank. Her eyebrows are raised. Her facial	Ms. B appears to express ***interest*** in Frank's understanding of the work on the chart paper. She listens as Frank explains

		expression is serious.]	what is written on the chart.
26:38	Frank	It equals a hundred.	**[Int]**
26:39	Ms. B	What equals a hundred?	
26:41	Frank	The length plus the width, forty plus forty plus ten plus ten equals a hundred.	
26:47	Shay	No, the perimeter is one hundred, the area is four hundred.	
26:55	Ms. B	So what can you say to them?	It may be inferred that Ms. B is ***eager*** for the students to articulate their interpretations of the mathematics on the chart paper.
26:59	Shay	Tell us what it is.	
27:00	Ms. B	Okay, what else can you say?	
27:02	Frank	He put…	
27:04	Ms. B	So would you say, if you were grading it from a one to a three, what would you give them right now?	Ms. B appears dissatisfied with the students' responses as she reframes her question to inquire about how the students would evaluate the mathematical work.
27:10	Student	A one.	
27:10	Ms. B	Why is it a one?	Ms. B expresses ***curiosity*** about the student's choice to value the work as a "one". **[Int, reasoning]**
27:12	Frank	Cause its not finished yet. When they finish it…	
27:16	Shay	It needs more work.	

27:17	Frank	Yeah!	
27:18	Ms. B	Are they completely wrong, are they off to a good start?	It appears that Ms. B is still not satisfied with the detail of the students' description. She leads the students with specific questions about the quality of the mathematical work. **[Dis-m, Int]**
27:22	Shay	Off to a good start.	
27:22	Ms. B	So maybe you might wanna say that you're off to a good start, there's some things that you're off to a good start…	Ms. B **validates** Shay's comment, "off to a good start". **[Val, Endis]**
27:27	Frank	They need to finish it…	
27:28	Ms. B	But, finishing it...What does finishing it mean? [Ms. B's eyebrows are raised and her facial expression is serious.]	Ms. B continues to express that she is ***curious about the students' reasoning*** when the students provide feedback to their peers. **[Int, reasoning]**
27:30	Shay	Adding more…completing it	
27:33	Ms. B	Completing it? What do they have to complete?	Ms. B is ***curious*** about what Shay means by "completing it". **[Int]**
27:35	Frank	They have to complete the, um, other parts of the rest of it.	
27:37	Ms. B	What are the other parts? Cause I think this is done. What's missing?	Ms. B's tone of voice becomes argumentative. It may be inferred that Ms. B is becoming ***frustrated with the general nature of the boys' feedback.***
27:40	Ray	The words.	
27:41	Shay	A lot.	
27:42	Ms. B	You guys told me, what was missing?	Ms. B reminds the boys that they had told her previously what was missing from the solution.

27:44	Frank	The maximum area.	
27:45	Ms. B	The maximum area. The answer. The area? There is no area. You guys did say the perimeter did equal one hundred, so that was good, and then, but you guys didn't get to the poles either, but how many poles are needed? Did they establish that?	Ms. B asks leading questions as she expresses interest in the students' interpretation of the mathematics on the chart paper. **[Int, Endis]**
27:59	Shay	No.	
28:00	Frank	They said one pole for every five…	
28:02	Ms. B	So you might want to put that down as well.	
28:05	Frank	Put what down?	Frank appears ***confused*** about what to write on the chart paper.
End TSMIE [B.01.b.06]			

Discussion of TSMIE [B.01.b.06]

In this episode, Shay and his group are standing at Natasha's table, looking at the work that Natasha and her group recorded on chart paper. While Shay and Frank express that the mathematical work is not complete, it appears that Ms. B is not satisfied with their level of specificity. She questions the boys, suggesting that their language is not specific enough to provide constructive feedback. Ms. B asks the students to be more specific in different ways, approximately ten times over a span of about thirty seconds. It appears that Ms. B is dissatisfied with the students' level of engagement. She begins to

argue with the boys, "I think this is finished". The boys respond to Ms. B's argumentative tone by articulating that the maximum area is missing from the solution.

The episode ends when Ms. B leaves the group to go speak with a different group of students. The camera follows Ms. B to the next group, so student engagement with the present group of students cannot be determined subsequent to the end of the episode.

Begin TSMIE [B.01.b.07] 28:34 – 30:11 (L)			
Participants: Dana, Ghee, Larry, Devin, Ms. B In this episode, Ms. B continues an earlier conversation that she had with this group. Ms. B stepped away from the group for approximately three minutes. She begins the interaction by asking the students to compare Shay's group's solution to their own solution. Immediately prior to this episode, at 26:32, Dana expresses curiosity about Shay's solution, when she says, "I wanna know how they get this." Meanwhile, the boys in the group appear to be either disengaged or pseudo-engaged as they walk back and forth, and engage in discussions not related to the mathematics. Then, Dana appears to make sense of the solution, stating that the group multiplied the length times the width.			
28:34	Ms. B	So is this the same as yours or different from yours? [Ms. B stands next to Dana. Her facial expression cannot be determined because Dana is blocking the view. Ms. B is speaking quickly.]	Ms. B expresses ***curiosity*** about how the students interpret the current solution with their own solution. **[Int]**
28:36	Dana	They are different.	
28:37	Ms. B	It's different? [Ms. B looks at Devin who is standing to her left, writing on a post-it note.]	
28:38	Dana	Yeah, 'cause we multiplied…[inaudible]	
28:54	Ms. B	So what do you guys have to say about what you see?	
29:01	Dana	That they was right! [Ms. B looks down at the paper and purses her lips.]	Dana exclaims that Shay's group produced a correct answer.
29:06	Ms. B	So if you don't get how they did that, what do you need to do? [Ms. B looks at Dana.]	
29:09	Dana	[Aks] them.	
29:10	Ms. B	How did you get this table? [Ms. B points at the chart paper and looks at Dana.] Ask them. If that's a question that you're asking, you should ask it. [Ms. B turns to look at the boys.]	It may be inferred that Ms. B is ***annoyed*** or ***anxious*** about student progress because her eyes are widened and she is looking stern as she speaks directly to Dana. Ms. B's tone of voice becomes gentler as she offers suggestions to Dana about how to give feedback to Shay's group.

		So write that down. Write it down. You know, Shawn, you're not totally focused. [Shawn is leaning on the table, he mumbles a response to Ms. B, which is inaudible.] You have to work with the group, okay? [Ms. B looks back at Dana as she writes on a post-it.] [Ms. B steps away from the table and raises her voice to addresses the class.] About 30 seconds, get your last comment down and you have to move! [Ms. B returns to the group, standing next to Dana. She leans across the table toward Shawn and Ghee.] Remember I said? "You did a good job", [Ms. B makes direct eye contact with Ghee and shakes her head, 'no'] What was it a good job about it?	Ms. B looks at the boys. Her tone of voice becomes more direct when telling the boys to write down Dana's question. It may be inferred that Ms. B is ***annoyed*** by Shawn's off-task behavior as she hands a post-it note to Dana and says quickly, "Write it down". It appears that Ms. B is ***eager*** for the students to reengage with the mathematics. **[Dis-b]** It may be inferred that Ms. B is ***dissatisfied*** with the comments that the boys are writing on post-it notes because the comments are general rather than specific. **[Dis-m]**
29:46	Shawn	Good job making it confusing.	
29:47	Ms. B	That was a good job? Because it was confusing? [Ms. B shrugs her shoulders and shakes her head.]	Ms. B appears ***surprised*** by Shawn's comment and she likely disapproves of his suggestion. **[Dis-m]**
29:48	Shawn	No.	
29:50	Ms. B	Why was it a good job?	It may be inferred that Ms. B is becoming increasingly ***anxious*** about the boys' feedback that they are giving
29:52	Shawn	No. It was a good job but somewhat was hard to comprehend.	
29:57	Ms. B	But *what* [Ms. B accentuates her	

		words by shaking her pen in the air and raising her eyebrows] was a good job?	to Shay's group. Her facial expression is serious and she is leaning in toward the boys. Her tone of voice becomes increasingly urgent as she waits expectantly for the boys to provide specific language for "what was so good" about Shay's solution. **[Int]**
30:02	Devin	The maximum area? [Devin is standing next to Larry, after he offers his idea, he steps back and leans on a desk away from the group.]	
30:03	Ms. B	That is something you, you saw that was good. *What* was so good? [Ms. B accentuates her words as she waves her pen in the air again.]	
30:07	Devin	Maximum.	It may be inferred that Devin recognizes Ms. B's frustration as quietly and tentatively offers some suggestions.
30:09	Ms. B	Hmm? [Ms. B looks at Devin and raises her eyebrows.]	Ms. B expresses ***curiosity*** about Devin's suggestion. **[Int]**
30:10	Devin	Maximum area.	
30:11	Ms. B	So, maybe you might want to put that down. [Ms. B points toward Ghee and Shawn with her pen.] [Ms. B walks away from the table and addresses the class.] Alright, move! Move!	Ms. B appears to express ***satisfaction*** with Devin's idea as she suggests that they write it down.
End TSMIE [B.01.b.07]			

Discussion of TSMIE [B.01.b.07]

Ms. B initiates this episode when she returns to work with these students for a second time, expressing curiosity about the comparison between Dana's solution and Shay's solution. At the beginning of the episode, Ms. B appears to be satisfied with Dana's level of engagement with the mathematics. Meanwhile, the boys appear to be either disengaged or pseudo-engaged with the mathematics. Ms. B appears to recognize that Shawn is off-task, and hands a post-it note to Dana, as she addresses Shawn's off-task behavior, commenting on the fact that he is not totally "focused".

Toward the end of the episode, Dana is sitting down, writing on a post-it note as Ms. B exhibits evidence of frustration with the boys as they struggle to articulate, “what was a good job”. This episode ends when Ms. B announces to the whole class that it is time to rotate to the next table. The students appear to remain engaged despite the fact that Ms. B is announcing loudly and repeatedly that they have to “move”.

Begin TSMIE [B.01.b.08] 31:04 – 31:31 (L)			
Participants: Dana, Ghee, Larry, Shawn, Devin and Ms. B Ms. B initiates this episode when she asks the group, "why do you think it's off to a good start?" Ms. B is off camera at the beginning of this episode. This group of students has just arrived at a new table, looking at Natasha's group's chart paper.			
31:04	Ms. B	So why do you think it's off to a good start?	
31:06	Ghee	They copied off of us.	Ghee accuses Natasha's group of cheating.
31:08	Ms. B	The set-up here?	
31:09	Lenny	Yeah…	
31:13	Lenny	[Ms. B stands to Larry's left and looks at him as he answers her question.] The box…it…it's all correct. [Larry shrugs his shoulders and gestures toward the chart paper.]	
31:17	Ms. B	What's correct?	Ms. B expresses ***curiosity*** about what Lenny means by "it's all correct". **[Int]**
31:18	Devin	Only bullet one.	
31:22	Ms. B	But what's correct? [Ms. B looks at Ghee and Shawn as she speaks. Her facial expression is serious.]	
31:23	Shawn	[Shawn writes on a post-it note] You were copying…	
31:24	Ms. B	No, no! [Ms. B leans in toward Shawn and shakes her head. Her eyebrows are raised.] Shawn! I don't want to hear that they were copying. They weren't copying.	It appears that Ms. B is upset by Shawn's accusation that the group copied his group's answer. **[Dis-b]**
31:26	Dana	'Cause the perimeter…'cause the perimeter equals up…equals up to a hundred. [Dana rotates her hands as she speaks.]	
31:31	Ms. B	Ok, so that's what you need to put. That's where you can start.	Ms. B's facial expression cannot be seen because Larry blocks the view of her face. However, she is nodding her head as she validates Dana's interpretation of the mathematics on Natasha's

			paper. **[Val, Endis]**
End TSMIE [B.01.b.08]			

Discussion of TSMIE [B.01.b.08]

This episode begins when Ms. B follows this group of students to a new table as they provide feedback on Natasha's group's solution to the problem. She begins by asking the students to provide specific feedback about the solution. When Shawn declares that the group copied off his group's solution, Ms. B becomes visibly annoyed, expressing disapproval of Shawn's comment.

When Dana provides a specific comment, that the solution shows a rectangle with a perimeter of "a hundred", Ms. B appears satisfied, tells the group to write down Dana's comment and then walks away.

Outcomes of TSMIE [B.01.b.08]: 1.) Dana was able to articulate that the rectangle has a perimeter of one hundred.

Immediately subsequent to Ms. B's departure from the group, the students appear engaged because they are leaning on the desk, writing or looking on as their peers write.

Begin TSMIE [B.01.b.09] 32:06 – 33:13 (C)			
Participants: Shay, Frank, Ms. B This episode takes place at Dana's table with Shay's group looking at the chart paper presented by Dana's group.			
32:06	Ms. B	Okay, so, what do you think about this? [Ms. B is standing next to Shay, leaning on the table]... [Twenty seconds pass as the student look at the chart paper] Compare them to yours,	Ms. B expresses ***curiosity*** about the students' interpretation of Dana's mathematical solution. **[Int]**

		cause you guys had some thoughts. Is it the same thing, is it different?	
32:33	Frank	It's different to us, we got different…we got different, um, different area. [Frank is leaning his elbows on the table. He points to the chart paper with his calculator.]	
32:39	Shay	Theirs is completed, but theirs is a higher number.	Shay appears to be studying the work very carefully. It may be inferred that he is deeply engaged with interpreting Dana's solution. He speaks because he speaks slowly and does not look at Ms. B.
32:48	Ms. B	So you might wanna share that with this group?	
32:50	Shay	That it's a higher area?	
32:54		[Kids in group say "our group found a higher area"…] [Ms. B walks away.]	
33:13	Frank	They did the easy one.	
End TSMIE [B.01.b.09]			

Discussion of TSMIE [B.01.b.09]

Ms. B initiates this episode and when she expresses interest in the students' feedback on Dana's group solution. Shay expresses to Ms. B that Dana's group found a "higher number". Ms. B does not pass judgment on Shay's comment and she walks away. Outcomes of this episode: 1.) Shay indicates that the group got a "higher number". 2.) Frank judges the work, "they did the easy one".

Begin TSMIE [B.01.b.10] **33:40 – 35:54**			
Participants: Kenny, Natasha, Al, Emmy, Ms. B Ms. B initiates this interaction as she asks the students to express their opinions of the work completed by Shay's group. Ms. B is standing next to Natasha, her back is to the camera, so facial expressions cannot be determined.			
33:40	Ms. B	So now you have to find an example…show it in a different way. But…[Kenny is leaning over the chart paper, using a ruler to measure.] What can you say about this? What were you doing? [Ms. B directs her question to Kenny.]	Ms. B expresses her ***curiosity*** about what Kenny was doing with the ruler.
33:45	Kenny	I was lookin' at one of the sides, this is eight inches and this seven and half inches. [Kenny smiles as he speaks, and points to the chart paper with his ruler.]	Kenny indicates that he measured a rectangle on the chart paper.
33:48	Ms. B	Well so they didn't…was what though? [Ms. B leans in and points at the chart paper.]	It appears that Shay's group drew a rectangle to represent a twenty-six foot by twenty-four foot region for the cow to graze. Kenny uses a ruler to measure and recognizes that the figure on the chart paper contradicts the true measures.
33:49	Kenny	Twenty-six.	
33:50	Ms. B	Twenty-six. They didn't use a ruler, they didn't use inches, they didn't use centimeters, they just, they were…this is…what [Natasha] was trying to explain to you. She was was trying to show you, it doesn't matter about the size of the boxes, [Ms. B leans in and points to the chart paper] it's how you what? It's how you label	Ms. B interprets Shay's alternate approach to solving the problem. She uses this opportunity to show Kenny an example of a rectangle that may not be drawn to scale but has been labeled with the intended measures. **[Val, Raa]** Ms. B appears satisfied that she is able to demonstrate the idea that the label is more important than the scale of the figure.

		it…. That's what she's trying to show you, so she's showing you. Clearly, this is not twenty-six inches, right?	
34:09	Al	But they want you to figure it out?	Al appears confused because the drawing is not drawn to scale.
34:11	Ms. B	They're just showing you…	Ms. B helps Al to interpret Shay's representation.
34:13	Al	I mean, a example? [Ms. B nods her head and looks at Al.]	Ms. B validates Al's contribution. **[Val]**
34:14	Ms. B	An example, ok? So that's where…exactly, they want you to show it like this. But, right here, the width, [Ms. B leans over and points at the chart paper.] How many boxes does it go across? How many go this way?	Ms. B expresses ***interest*** in the students' understanding of the dimensions of the rectangle. She ***builds on Shay's idea*** to help Natasha's group to interpret the solution to the problem. **[Int, reasoning; Blds]**
34:31	Natasha	Twenty-six.	
34:31	Ms. B	Twenty-six, so there's gonna be twenty-six boxes that go across, how many go [Ms. B points to height of the rectangle].	
34:38	Al	Twenty-four.	
34:38	Ms. B	Twenty-four. [Ms. B looks at Al and nods.]	Ms. B ***validates*** Al's answer. **[Val]**
34:39	Kenny	And then in each box…	
34:42	Ms. B	So if you do twenty-six by twenty-four, what would that give you?	Ms. B ***builds on Shay's idea*** and expresses ***curiosity*** about the area of the twenty-six by twenty-four rectangle. **[Blds, Int]**
34:50		[Emmy and Al use calculators. Kenny paces back and forth.]	

34:56	Al	Six twenty four.	
34:59	Natasha	See, and that's what I was saying, [Natasha points her pencil at Al] until you did…what'd you have? Oh, forty times ten, you can get the whole area.	It may be inferred that Natasha is ***excited*** because she raises her voice and leans in toward the group as she speaks. It appears that she now is convinced that the area is calculated by multiplying length times width. This is something that was not resolved in her group when they were solving the problem.
35:06	Ms. B	What area will that be?	
35:08	Natasha	I don't even know.	Natasha appears confused and frustrated.
35:11	Ms. B	Well you can tell ten…[inaudible]	
35:19	Natasha	Oh, four hundred….It's wrong because the area is…	
35:31	Ms. B	What's wrong? [Ms. B's facial expression is emotionless.]	Ms. B appears ***confused.***
35:31	Natasha	The area. I think everybody's area is wrong because everybody's is different except for that group. [Natasha points behind her.] This group's area is different because, its like, the area is supposed to be a certain size and that's what we're trying to figure out. Those numbers on the side, when we multiply them, they equal different things, different numbers.	Natasha seems to have figured out that each rectangle with different dimensions will produce a different area.
35:54	Ms. B	What do you wanna say to this group? What comment do you wanna leave for them? For this group.	Ms. B does not address Natasha's interpretation but she redirects the students to writing a comment on the post-it note.

		[Ms. B walks away.]	
End TSMIE [B.01.B.010]			

Discussion of TSMIE [B.01.B.10]

Ms. B appears excited when she finds an opportunity to address the disagreement that transpired earlier during the class period between Kenny and Natasha. By looking at other students' work, Natasha and Ms. B are able to demonstrate to Kenny that the rectangles do not necessarily need to be drawn with exact dimensions. Natasha appears relieved when her idea to label the length and width of any rectangle (not necessarily drawn to scale) is validated. This episode ends when Ms. B reminds the students to leave a comment on the chart paper before she walks away from the group. Student engagement subsequent to this episode is not determined.

Begin TSMIE [B.01.B.11] **38:59 – 40:46 (S)**			
Participants: Dana, Shay, Ghee, Ms. B Immediately prior to this episode, Shay and his group are commenting on the solution presented by Dana and her group. Dana and Shay exchange words about Dana's answer, and Shay indicates that Dana's work is incorrect. Dana appears to disagree with Shay.			
38:59	Dana	[Ms. B approaches Dana and Shay, standing in between them, but distanced so that the two students can make direct eye contact.] Alright, but when y'all add all y'all, but when we add all our…so what? Eighty plus twenty equals a hundred. [Dana leans in and points to the chart paper.] That's how we get ours….just like everybody else's. [Ms. B looks at Shay.]	It is inferred that Ms. B is ***interested*** in the dispute between Dana and Shay as she stands in close proximity between the students. **[Int]**
39:11	Ms. B	What? I'm just listening. [Ms. B looks at Shay.]	Ms. B encourages student discourse when she says, "I'm just listening." **[Endis]**
39:14	Dana	Y'all tellin' us that we're wrong but	

		we're not wrong!	
39:16	Shay	'Cause when you're um, finding the area, you times the width times the length…	
39:20	Dana	Alright, but we times'd all that up! [Dana gestures a rectangle in the air with her hand.]	
39:21	Shay	But your not supposed to.	
39:22	Dana	Alright, but we did it though.	
39:23	Shay	But you not supposed to, so its wrong	
39:24	Dana	No its not wrong, actually, no it's not wrong, actually	
39:26	Shay	It's wrong, it's wrong	
39:27	Dana	No it's not wrong! No it's not!	
39:34	Shay	Because look at that, that's what y'all got, sixteen hundred? [Shay holds up calculator toward Dana]	
39:35	Dana	Yes.	
39:35	Shay	That's what y'all got?	
39:36	Dana	Yes.	
39:37	Shay	So it's the length times the, um, forty	
39:37	Dana	Well we didn't do that and it equal four hundred, but we didn't do it!	
39:40	Shay	Yeah, so that's how you know it's forty times ten.	
39:41	Dana	But we didn't do it, so oh well.	
39:42	Ghee	Forty times ten? [Ghee has his eyebrows furrowed.]	
39:43	Shay	Yes, that's the same thing they got too	
39:46	Dana	So if forty times ten	
39:48	Ghee	Forty times ten? [Ghee has his hands on his hips.]	
39:50	Shay	Forty times ten equals four hundred.	
39:50	Dana	Oh well, [Dana shakes her head in disagreement.] We already know forty times ten!	
39:54	Shay	Ok that's what your area is, you put all of that. [Shay gestures toward the chart paper.]	
39:57	Ghee	Forty times ten? [Ghee leans on the	

		table with one hand on his hip.] You said forty times ten? Forty times ten, forty and ten? None of that add, uh, up to, uh, to a hundred. [Ghee shakes his head 'no'.]	
40:04	Shay	You add, you add this, that's forty. [Shay begins to speak loudly.] That's eighty right there and that's a hundred. Yeah, so, you don't know what you talking about. That's the perimeter, that's the perimeter. And the area is four hundred.	
40:12	Ghee	No	
40:16	Shay	Yeah	
40:18	Ghee	Eighty times ten is exactly what we did [punches numbers on calculator.]	
40:21	Dana	Eighty times twenty is not WRONG from a hundred! [Dana is yelling.]	
40:23	Frank	I think you put…	
40:25	Dana	Thank you! Thank you, eighty times, I mean, eighty plus ten, I mean twenty is a hundred…shut up! [Dana walks away from the table.]	
	Ms. B	So wait, eighty.. how…[Ms. B looks at Shay]	Ms. B tries to intervene but Shay interrupts her.
40:35	Shay	Exactly, tell 'em then you gotta multiply the length times the width, not what y'all did, and that's the maximum area	
40:37	Dana and Ghee	Alright, and that's what we did!	
40:39	Shay	And that's the maximum area	
40:41	Dana	That's what we did! [Dana is yelling at Shay from Natasha's table. Ms. B is smiling as she looks back and forth between Shay and Dana.]	
40:42	Ghee	That's exactly what we did	
40:46	Ms. B	[Ms. B looks up toward the clock on the wall.] Now, at this time I need for you to put your last comments on and move one more time, move one more time	
End TSMIE [B.01.B.11]			

Discussion of TSMIE [B.01.B.11]

This episode begins when Dana and Shay disagree about the solution to the problem as presented by Dana's group. Shay points out an error in Dana's solution, as he recognized that the area of a forty-by-ten rectangle is not sixteen hundred. Dana defends her group's solution by indicating that they multiplied all of the sides, (meaning eighty by twenty) which is how they arrived at an answer, sixteen hundred.

Ms. B only has one speaking turn during this episode, approximately twenty seconds into the episode, when she says, "Go ahead, I'm just listening." Again, toward the end of the episode, it appears that she has a question, but she does not complete the question. Most interesting in this episode are Ms. B's body language and her facial expression. She appears to suppress a smile as she participates as a silent observer, standing on the sidelines, in between Shay and Dana. It may be inferred that Ms. B is pleased with the display of student engagement because the students are clearly emotionally invested in defending their ideas (regardless of correctness). The episode ends when Ms. B asks the groups to move to the next station, which happens to be their home groups. Subsequently, the students begin to read the comments left by their classmates, and are ignoring Ms. B's request for their attention. Hence, she calls all of the students to stand near her, away from their work, in order to get the attention of the students. She acknowledges the students' strong emotions, recognizing that some students may not be happy with the comments made by their classmates and asks them not to "ball up" the post-it notes.

APPENDIX B.2

Ms. B: Classroom observation analysis, "The DJ Problem"

Day One

Eight *teacher/student(s) mathematical interaction episodes* were identified in the video data collected in Ms. B's classroom on December 6, 2006, Day One of "The DJ Problem".

Ms. B stands near the front of the classroom and paces forward and backward as she introduces the problem in the first episode. Students are seated in groups of four to five students. Each student has a copy of the problem and a graphing calculator on their desk.

Time	Speaker	Transcript	Descriptions / Comments
Begin TSMIE [B.02.a.01] 00:00 – 7:15 (L)			
Participants: Ms. B, Frank, Lenny, Shay, Rita, Dana, Ms. B initiates this episode when she poses a question to the whole class about creating a budget for the 8th grade fundraiser.			
00:00	Ms. B	[Ms. B is standing at the front of the classroom, holding a piece of paper in her hand.] Now before we begin….[Ms. B furrows her brow as she looks at the paper] our task today, the executive board [Ms. B looks up at the class, steps forward and gestures both hands towards the students] for umm 8th grade, they were talking about different fund raisers, and they were taking about a budget. What does a budget mean? [Ms. B walks backwards, looking at the students. Some students begin to answer, but their exact words are inaudible.]	Ms. B introduces a topic that may be of interest to the students: a school dance. It may be inferred that Ms. B has used this strategy to stimulate student interest in the problem. It may be inferred that Ms. B is ***curious*** about the students' prior understanding of the term, "budget" when she poses the question, "what does a budget

			mean?”. **[Int]** Ms. B’s body language (stepping back) may be inferred to imply a move to ***encourage student discourse.*** **[Endis]**
00:18	Frank	A certain amount. [Frank is off camera.]	Frank speaks without being called on.
00:19	Ms. B	The what? [Ms. B turns towards Frank and points her paper at him.]	It may be inferred that Ms. B ***is happy with*** Frank’ answer and she validates his comment by asking him to repeat it for the whole class to hear. **[Val]**
00:20	Frank	A certain amount you gotta make or they have.	
00:22	Ms. B	A certain amount that….[Ms. B nods her head and gestures, pointing her finger.]	Ms. B appears to be ***encouraging*** Frank to develop the idea. **[Enc]**
00:23	Frank	They have.	
00:25	Ms. B	They have. So when we say “they,” we’re talking about 8th grade. [Ms. B walks forward into the center of the classroom.] Certain amount of money that we have…[intercom interruption]	Ms. B includes herself in the eighth grade group when she says, “we”. Ms. B also contextualizes the problem as relevant to her students as they are eighth graders.
00:31	Ms. B	[Ms. B raises her voice over the loudspeaker.] Certain amount of money we have in our, [Ms. B gestures with her hands and nods her head towards the students, she walks backwards] in our budget. So… how much money, well we start off with zero dollars. [Ms. B gestures her hands in a horizontal scissor motion] Okay, we start off with no money in the budget. Anytime we raise money, [Ms. B gestures her hand in a circular motion, moving upwards, then moves her arm forward] it will go into our budget. So, when we want things, like I know they’re	Ms. B ignores the loudspeaker as she raises her voice and continues to describe the concept of a “budget”. It is inferred that Ms. B aims to stimulate the students’ interest when she asks “how many of you guys like to have a party, here?”

		talking about having a party, how many of you guys like to have a party, here?	A party is something that likely appeals to middle school students.
00:53		[some students raise their hands]	
00:53	Ms. B	Yeah. [Ms. B nods her head] And then we can invite people in. Well what kind of things might you need at a party?	
00:56	Lenny	Chips…soda….paper plates	
00:57	Ms. B	Chips. Soda. [As the students list the things that would cost money at a party, Ms. B keeps track by "counting" with her fingers.]	Ms. B repeats some of the suggestions that the students are calling out.
00:58-01:02		[students are yelling out suggestions] Soda!....decoration!…decoration!....[Rita in her seat to face Ms. B and say: "music"]	It may be inferred that the students are ***excited*** about planning a party because they are yelling out suggestions for the party, and the noise level in the classroom is elevated.
01:01	Ms. B	Paper plates… Music! [Ms. B quickly takes a step back as she turns toward Rita to her right, points her hand at Rita and raises her voice.] So what kind of music? What do you want, [Ms. B turns to the other students, pointing towards them] you want a radio? [Many students are talking at once.]	Ms. B appears ***excited*** by Rita's suggestion, "music" because she raises her voice and quickly turns toward Rita. **[Int, Val]** Then, when Ms. B addresses the class, her voice is raised and conveys ***enthusiasm.*** Rita is ***smiling*** and covers her face with her arm. It appears that many students are ***excited*** because the noise level in the room has become louder and many students are talking at once.
01:03	Dana	Hip hop!	
01:05	Shay	A DJ! [Shay is off-camera]	Shay yells out without waiting to be called

			on.
01:06	Ms. B	Oh you want a DJ? [Ms. B points to Shay and then pauses. Ms. B walks towards the students again.] Well, it costs money. Costs money for a DJ, doesn't it? [Ms. B lowers her voice and looks around the room. She is smiling.]	It may be inferred that Ms. B is ***happy*** with Rita's suggestion of music and Shay's suggestion of a DJ because their suggestions tie in to the problem that she is about to assign for the students to solve. **[Val, Int]**
01:11	Dana	Yeah.	
01:11	Ms. B	How much do you think it costs for a DJ? [Ms. B walks backwards and furrows her brow.]	**[Int]** It is inferred that Ms. B has reached the point that she was hoping for to introduce the lesson. Now she questions the students about the cost of a DJ.
01:13	Dana	Like probably like three hundred	
01:15	Shay	A hundred.	
01:16 01:17	Ms. B Dana	A hundred? For what? [Ms. B looks at Shay and furrows her brow.] A hundred… Three hundred.	Ms. B appears ***curious*** about what "a hundred" would pay for in terms of a DJ. **[Int]**
01:19	Shay	A hundred for like a hour.	
01:21	Ms. B	A hundred for like an hour? [Ms. B looks around at the students.]	Ms. B appears to be ***curious*** about other students' thoughts on Shay's suggestion. **[Int]**
01:23	Rita	Nah, for like five hours! [Rita turns in her seat towards Ms. B. and her facial expression is serious. Ms. B looks at Rita and her brow is furrowed.]	
01:24	Ms. B	For like five hours?	
01:25	Rita	DJ's be…DJ's be stayin' at parties for, uh, a long time. [Rita is looking at Shay who is seated at the same table, to her left].	The students engage in a discussion about how DJ's charge for their services as Ms. B looks back and forth between the students.
01:27	Shay	…(inaudible) stuff, but they pay like five hundred dollars though. [Shay is sitting back in his seat, speaking low	

		when he responds to Rita.]	
01:28	Dana	But you still gotta pay though, but they pay though. Each hour.	
01:34	Ms. B	Oh they pay each hour?	
01:35	Shay	Yeah!	
01:35	Ms. B	Okay. When, when an executive board they were taking notes, [Ms. B walks towards the center of the room.] and they were trying to list everything that we just said, um, to include with the party, so DJ [Ms. B walks backwards towards the front of the classroom.] was one of the major things because that's expensive, it comes out of our budget. [pause] [Ms. B points towards herself, her eyes widen an she scans the class while standing still at the front of the room.] If we have no money, [Ms. B walks slowly towards the center of the room and speaks slowly], we might have a few hundred dollars right now but if a DJ costs between five hundred dollars [Ms. B gestures as she presents a range of the cost for a DJ] and a thousand dollars, which one would you wanna pay [Ms. B furrows her eyebrows]?	Ms. B lowers her voice and emphasizes that a DJ is "expensive" and comes out of "our budget". It may be inferred that Ms. B's intent here is to ***stimulate student interest*** with something that is personal and tangible to the students. Ms. B offers students a choice of DJ based on the price, appearing ***curious*** and ***interested*** in their preference. **[Int]**
02:05	Students	A thousand	
02:05	Shay	Five hundred!	
02:05	Ms. B	Five hundred dollars? Why five hundred. [OJ]?	
02:08	OJ	It's less money!	OJ's tone of voice may be inferred to suggest that he thinks it is obvious to want to spend "less money".
02:09	Ms. B	Oh, it's less money. A thousand?	
02:10	Lenny	A thousand. Yea, because you can get the best DJ.	Ms. B appears to take Lenny's suggestion into consideration.
02:13	Dana	No, but…you are not gonna have enough money though to do what we wanna do.	
02:18	Ms. B	You can get the best DJ. How do you know you	It may be inferred that

		can get the best DJ, [OJ]? [Ms. B shakes her head, then leans over to her left, looking at Lenny's face.] I mean, Lenny, how do you know you can get the best DJ? [Pause]	Ms. B is ***addressing behavior that she perceives to be off task*** when she leans over and looks at Lenny. It appears that she is trying to engage Lenny in the discussion by asking him "how do you know you can get the best DJ?" **[Dis-b]**
02:25	Lenny	I don't know.	
02:28	Ms. B	[Shay?] [Ms. B gestures towards Shay with her hand.]	
02:29	Shay	You want, you want the, um, five hundred because…	
02:32	Ms. B	Let's listen! [student laughter] Let's listen!	Ms. B ***validates*** Shay's contribution and ***encourages peer-to-peer respect*** and expresses ***interest in Shay's reasoning*** when she tells the class to "Listen". **[Val, Ptp, Int]**
02:34	Shay	You want the five hundred 'cause you could still have some food. Or you could buy other supplies with food with the other five hundred. [Shay is seated, looking at Ms. B, leaning back in his chair.]	
02:40	Ms. B	Ok so, you want the lower price so that we can have more money to, more money left over to buy things.	Ms. B restates Shay's suggestion, projecting her voice so that all students can hear. **[Blds]**
02:47	Dana	Yes.	
02:49	Ms. B	Ok so now, what the executive board is doing, they're doing their research. They're looking and finding out different DJs, [pause] in the area, and they're trying to get the best deal. So when we talk about the best deal, what does that mean?	Ms. B appears ***satisfied*** with the level of student interest at this point as she moves forward to introducing the problem.

			[Sat] She continues to keep the problem interesting by telling the students that the executive board of the 8th grade is looking into local DJs for the best price to play at the party.
03:04	Dana	The best price we got….The best price.	
03:05	Ms. B	What is it?	
03:09	Dana	Or the lowest price.	
03:09	Ms. B	Lowest price? Just the lowest price?	It may be inferred that Ms. B is not satisfied with "the lowest price", but that she wants students to consider the length of time that the DJ will play. **[Blds, Int]**
03:14	Dana	The best.	
03:15	Ms. B	Well what's the best?	It may be inferred that Ms. B is ***curious*** about the students' understanding of "the best" offer. **[Int, reasoning]**
03:16	Dana	The best…	
03:16	Rita	The best offer.	
03:17	Dana	Not high, it's like not high.	
03:19	Ms. B	What do you mean by the best offer?	Ms. B appears ***curious*** about the meaning of "best offer". It may be inferred that she wants the students to clearly define "best offer". **[Blds, Int, reasoning]**
03:21	Rita	Not high, not low, but like…yeah.	
03:23	Dana	Not high and not low….in the middle.	
03:26	Ghee	So that you have good quality, so that you have…	
03:28	Shay	Best offer.	
03:32	Ms. B	Oh, lowest offer.	

03:32	Rita	But it's the best offer.	
03:33	Ghee	Best offer could be like, more quality.	
03:36	Ms. B	So someone can come in, best quality, for one hour. One hour…	
03:43	Shay	Seven thousand dollars.	
03:44	Ms. B	Seven thousand dollars. [Ms. B looks at Shay and smiles.] Best quality though.	Ms. B appears pleased by Shay's suggestion of paying seven thousand dollars for a DJ to play for one hour. **[Blds]**
03:48	Shay	Too much money, though.	It may be inferred that Shay's earlier suggestion of paying seven thousand dollars for a DJ was intentionally high to argue against "best quality" if it is at an exorbitant price.
03:52	Ghee	I'll be like "Yeah!"	During this exchange, Ms. B appears to want the students to identify time as a factor in considering the best DJ.
03:53	Ms. B	You want that person?	
03:53	Dana	No…	
03:53	Ms. B	No, so what are you trying to figure out?	
03:55	Shay	The lowest.	
03:55	Dana	The lowest price.	
03:55	Ms. B	The lowest price for what else…	
03:56	Shay	For good quality	
03:57	Ms. B	Oh, the lowest price for what?	
03:59	Rita	For more time.	
04:00	Ms. B	More time, cause when we come to party, you don't wanna just come for an hour, and its time to go, cause most of you won't be partying and dancing in an hour. It's gonna take you an hour just to get you up. So now, this is something they are going to go through. Let's read this because they're going to bring it back. What happens is the executive board, they meet, they meet the same time as us advisors... [Ms. B walks backwards, out of camera range. Students are sitting quietly, some are looking at Ms. B, some are looking at their paper, some are looking around the room] they're gonna meet with us, and they come up	Ms. B appears ***pleased*** with Rita's suggestion of "more time" when she repeats Rita's answer. **[Blds, Sat]** It appear that Ms. B understands the social patterns of eighth grade students when she says, "It's gonna take you an hour just to get you up." It

		with ideas and then they bring it back to you. So you need to understand what they're talking about. When they come up with a decision, you need to see, "Does that really make sense? Is that the *best*?" Because it's not just their money, whose money is it? [some students respond quietly, but the responses are inaudible for transcription] It's everyone's money in the eighth grade right? We're all in this together. So, everyone should have this task in front of you. [Ms. B raises her voice and steps back into camera range.] And it says, "You are on a committee, to select a DJ for a school party. The committee has committed, I'm sorry, the committee has obtained price quotes, quotes from three DJs. Tom's Tunes charged, charges sixty dollars an hour. [Ms. B puts her left hand on Shawn's shoulder. Shawn looks over his shoulder then looks down at his paper.] Solidus Sounds charges a hundred dollars plus forty dollars an hour."	may be inferred that Ms. B is referring to the shy behaviors middle school children exhibit when in the setting of a school dance. Ms. B begins to introduce the problem as she encourages the class to make informed decisions. Ms. B is ***encouraging student autonomy*** by giving the students choices. **[Auto]** It may be inferred that Ms. B is addressing behavior that she perceives to be off-task when she touches Shawn's shoulder. She continues reading the task. **[Dis-b]**
05:15	Ghee	Oh yes! [Ghee's chin is resting on the desk] [other students are clapping]	Ghee expresses ***enthusiasm*** as he exclaims "oh yes!" He

			is looking back and forth between Ms. B and his paper. He appears to be engaged.
05:16	Ms. B	Light Plastic charges a one hundred seventy five dollars plus thirty dollars an hour. [Ms. B looks up and pauses.] Which DJ would you choose? [Pause] What variables might affect your decision? [Pause] Letter B says for each DJ, write an equation you could use to calculate the total cost from the number of hours worked. Let, [Ms. B poses a question to the students] what variable are they saying?	During this exchange, it may be inferred that Ms. B is ***interested*** in knowing how well the students understand the problem. Ms. B is ***encouraging student discourse*** as she checks for understanding. **[Int, Endis, Blds]** The students respond correctly and Ms. B appears ***satisfied*** with their responses.
05:49	Many students	Y	
05:49	Ms. B	Let Y be the total cost and?	
05:52	Many students	X	
05:53	Ms. B	Let X be the number of hours worked. Oh, there's a typo there. [pause] It should be hours. It says, "Graph all three equations in the" what?	
06:05	Rita	In the same window.	
06:05	Ms. B	In the same window of your calculator. So how many different grids are you going to be using?	Ms. B checks for understanding.
06:13	Rita	Three.	Rita is incorrect. All three equations should be graphed on one grid. (graph)
06:15	Ms. B	How many grids? How many different graphs?	Ms. B does not directly correct Rita's error. Rather, it appears that Ms. B pauses then changes the language from "grid" to "graph" when she poses the question to the whole class again.
06:16	Rita	Three!	Rita's tone of voice may be inferred to express ***frustration*** as this answer has

			already been given.
06:20	Ms. B	How many different equations?	Again, Ms. B does not directly comment on Rita's incorrect suggestion. Rather, she reframes her questioning. **[Dis-m]**
06:22	Lenny	Three.	
06:22	Ms. B	[Ms. B raises her voice.] Three equations! [pause] [Ms. B lowers her voice] And how many graphs?	Ms. B's tone of voice and language may be inferred to express ***validation*** of Lenny's answer, "three equations". **[Val]** Ms. B poses this question for the third time.
06:27	Lenny	Three.	Lenny's answer is not acknowledged.
06:28	Many students	One! One graph! [many students are yelling at one time]	The students appear ***excited*** by this revelation because many of them yell the answer at once.
06:30	Ms. B	Because…[Ms. B raises her voice over the students] Well, how do you know it's that one graph? How do you know it's that one, how do you know they want one graph? [Noise level in the classroom has gotten quieter, Ms. B lowers her voice] What word?	Ms. B appears to approve of the students' answer, "One graph" because her voice is raised, she is speaking quickly and she is asking for students to defend their reasoning "how do you know they want one graph?" **[Int]**
06:40	Ghee	It just say…sketch [Ghee is looking at his paper, holding his graphing calculator in his hand.]	
06:43	Rita	All three equations in the same graph. [Rita is off camera.]	Rita now presents correct reasoning.
06:45	Ms. B	In the *same.* In the *same.* Okay?	Ms. B appears ***pleased*** with Rita's suggestion as she ***validates*** Rita's

		So Letter D says, "what information does the *coefficient* of x represent in each equation? And what information does the *y-intercept* represent in each equation?" We're going to stop there. [pause] Don't move on any further because we're not even, we're not gonna have enough time to probably get through all of it. So I'm just reading the first part to you. Are you gonna have to go back and read this over again back in your group?	answer by repeating it, and emphasizing the word "same". **[Val, Blds]** Here, it may be inferred that Ms. B is teaching the students how to interpret a word problem by emphasizing a key word "same" in the problem which should cue the students to use one graph to represent all three equations. Ms. B continues to read the problem to the students, emphasizing the key mathematical terms: "coefficient" and "y-intercept". It may be inferred that Ms. B has thought carefully about the amount of work that is reasonable for the students to complete in one class period. Ms. B ***encourages student autonomy*** and peer-to-peer interaction when she reminds the students to "go back and read this over again your group."
07:13	Students	Yeah	

07:13	Ms. B	Yeah. Yes you are. You guys may begin.	
End TSMIE [B.02.a.01]			

Discussion of TSMIE [B.02.a.01]

In this episode, Ms. B stimulates student interest in a problem that requires a high level of cognitive demand; modeling algebraic relationships as linear equations and in graphical representations. She makes the problem relevant to her students when she initiates a conversation about organizing an eighth grade dance. When Ms. B asks the students to brainstorm about the things that they would need for the event, the students exhibit excitement, shouting out ideas for the things they will need for the party.

Ms. B's approach as she introduces the problem appears to encourage student engagement in two ways; first, she stimulates student interest in a problem that is relevant to them and second, she offers the students choice, empowering them to make an informed decision. Her language and actions appear effective in supporting and fostering student behavioral and emotional engagement as the students respond to her questions in a lively and quick manner.

After approximately one minute of brainstorming, Rita suggests that they need music at the party. Ms. B expresses excitement when she raises her voice and points at Rita. It is likely that Ms. B is excited because Rita's suggestion will lead to the introduction of the DJ problem. Ms. B's approach of using a high interest topic to introduce the lesson appears effective in a very short period of time.

After Ms. B reads the problem, she checks for understanding, asking the students how many graphs they should use to represent the three equations. Initially, some students respond incorrectly, stating that three graphs are necessary instead of just one. Rather than correct the students or tell them that they are wrong, Ms. B reframes her

question. She uses specific and accurate mathematical terminology: “how many equations?”. She follows this up with “how many graphs?” The students respond quickly and with enthusiasm, stating the correct answer, “one graph”.

This episode illustrates how Ms. B gets students engaged in something that is personally important to them; a school dance and this approach may contribute to sustained interest in a problem that requires high cognitive demand. Ms. B has framed the problem so that it is in the students’ best interest to complete the problem in order to make an informed decision about how their class monies should be spent.

Immediately subsequent to this episode, the students appear to be engaged with solving the problem because they are using their calculators and talking to each other about the problem.

Ms. B stands behind Shawn, silently observing the interaction in his group.

Begin TSMIE [B.02.a.02] 10:25 – 12:32 (L/S*)			
Participants: Dana, Lenny, Ghee, Shawn, Ms. B Immediately prior to this episode, Dana, Ghee, Shawn and Lenny are engaged in a debate about which DJ to choose. It appears that Dana has made a definitive stance against using Solidus Sounds because he charges $100 flat rate before he plays any music. *In this episode, the dialogue is most audible from Lina's camera and Ms. B's facial expression is most visible from the still camera. Time codes and transcriptions of dialogue are taken from Lina's camera while the descriptions of Ms. B's facial expressions and gestures are as evidenced on the still camera data.			
10:25	Dana	[Dana is looking at her paper. Ms. B sits down in an empty student desk next to Devin. She silently observes, her hands are folded on the desk in front of her. Her facial expression shows no emotion as she glances from student to student.] Before you even start, I wouldn't.	Ms. B sits silently looking on as the students engage in a debate about how to calculate the rates for each of the different DJs. **[Int]**
10:28	Lenny	No, after, after the party over, you pay, you pay your money, you don't pay your money on…	
10:34	Ghee	Yes you do, you pay for the DJ first.	
10:37	Shawn	So for the first hour, how much would you pay? For the first hour, for the first hour how much do you pay?	
10:40	Lenny	If the thing starts skipping, you get like four hundred dollars back.	
10:44	Shawn	One hundred and forty.	
10:49 10:54	Dana Ghee	For the first hour, he gonna play, for the first hour they both gonna pay forty dollars. No, you pay sixty for the first hour.	
10:58 11:06	Shawn Ghee	Cause one seventy-five, one hundred is the charge. One hundred is the charge, one hundred is the charge for him to play anything. he wants forty dollars an hour. It's two of them man. I told you gonna go with the first one. [Ghee shakes his head, "no" as he looks down at his paper.] [Dana is pressing buttons on her calculator.]	
11:10	Dana	Three forty if we do six hours. [Dana looks up toward Ms. B. Both	It appears that Dana has just recognized Ms. B's

		Ms. B and Dana smile.] Oh snap! I didn't even know she was sitting here.	presence at the table.
11:15	Lenny	She, she look like, like, like Shakira.	
11:16	Ghee	[Ghee raises his eyebrows, leans towards Dana and asks:] If we do six hours with him how much is it?	
11:17	Dana	If we do six hours, then it's gonna be three forty. No. What? [Dana looks at Ghee and furrows her brow.] No, this three forty. [Dana is looking at her paper, pointing to the words.]	
11:25	Lenny	I go with [Dana] [Lenny leans back in his chair and looks at Devin. Ms. B looks at Devin.] Who you go with [Devin]?	
11:26	Ghee	And look if you do six hours with him, how much is it? Now do times, now do times.	
11:27	Dana	I did! Forty times six,which is two forty, and I added a hundred and it gave me three hundred forty. That's for him [Dana points at her paper], and then I did [Dana presses buttons on her calculator as Ghee looks on] sixty times six equals three hundred sixty. [Dana lowers her voice and looks down resting her forehead in her hand.]	
11:39	Ghee	Thank you! Equals three hundred and sixty! [Ghee sits back in his chair, smiling.] Did you hear that? Thank you!	
11:44	Ms. B	So what happened with him?	
11:45	Lenny	So we all picking, ummm….	
11:47	Shawn	Number two.	
11:49	Ghee	We doing, umm…	
11:51	Ms. B	How did you pick it, you just pointed your finger and said that one? [Ms. B points her finger to the problem on Lenny's desk, and looks at Ghee. Ms. B then B puts her chin on her hand.]	It may be inferred that Ms. B is using sarcasm when she asks this questions. Ms. B's facial expression shows ***no emotion.*** **[Int]**

11:54	Ghee	No! We put in. [Ghee takes a calculator and begins pressing buttons.] We went how many hour we were gonna do it right? [It appears that the disc skips briefly] ..so we did, for for her we did, six hundred, I meant sixty times six. [Ms. B moves papers on Ghee's desk and stretches her neck, looking at his work.] Which came up with three hundred sixty right? [Ghee looks at Ms. B]	Ghee responds to Ms. B's suggestion by presenting a mathematical argument for choosing Tom's Tunes.
12:08	Ms. B	Uh um. [Ms. B rests her chin in her hand.]	Ms. B responds to Ghee, it is unclear whether or not she is in agreement with his answer or if she is simply acknowledging that she is following his reasoning. Ms. B shows ***no emotion*** in her facial expression.
12:10	Ghee	So then when we did a hundred, and we did umm, [Camera briefly shows Ms. B, seated next to Devin, chin in her hand, looking at Ghee] …forty, forty times six came out to two hundred forty, [Ghee looks up at Ms. B] then when we added the hundred.	
12:27	Shawn	Three forty.	
12:29	Ghee	It came up to, three forty. [Ghee throws his hand up, sits back in his seat and smiles when looks up at Ms. B]	During this exchange, it appears that Ghee and Dana are competing to be correct. Both students are the most vocal and engaged in this group (this was true in the previous cycle as well)
12:30	Lenny	So what her's came out to? [Lenny points towards Dana. Ms. B folds her hands in front of her and looks from Lenny to Ghee.]	
12:32	Ghee	[Ghee looks at Lenny, his eyebrows are raised]Three sixty…twenty dollars over. And I didn't know that. [Ghee is looking at Shawn.]	
12:36	Dana	No you didn't cause I'm the one who figured that out. [Dana looks at Ghee and points at him]	

12:38	Ghee	I didn't even know that but I picked Solidia, right? [Ghee looks at Shawn, nods his head and raises his eyebrows] Didn't I pick…	
12:40	Ms. B	[Ms. B looks at Devin, her hands are folded on the table.] So, Devin, what do you think?	Ms. B does not acknowledge the debate between Dana and Ghee. She instead, turns her attention to Devin. It may be inferred that Ms. B is ***curious*** about Devin's thinking and that she may be ***trying to engage*** Devin with the discussion by asking his opinion. **[Int, Endis]** Ms. B's tone of voice is ***calm*** and she shows ***no emotion*** in her facial expression.
12:42	Lenny	Devin don't want no party. [Ms. B looks at Lenny.]	
12:44	Devin	How many hours is this party? [Devin has his hand on his forehead, he looks at Ghee]	
12:46	Lenny or Shawn	Hum?	
12:47	Devin	How many hours you say?	
12:49	Ghee	[Ghee looks at Devin] Six for Tom and six for that, what's her name, if you put in six. [Ms. B turns around in her seat to look at the group behind her. She briefly turns back to Dana's group and leans in towards Shawn.]	
12:53	Dana	How much for the second one? [Dana begins to calculate, Ghee looks at Dana's paper. Ms. B turns around and walks away]	
End TSMIE [B.02.a.02]			

Discussion of TSMIE [B.02.a.02]

In this episode, Ms. B silently observes as the Dana, Ghee and Shawn engage in a mathematical discussion. When the episode begins, it appears that Dana and Ghee disagree over which DJ to choose, because Dana thinks that two DJs would both cost three hundred forty dollars. Dana then calculates the second DJ with her calculator as Ghee looks on. When she gets an answer of three hundred sixty for Tom's Tunes, Dana appears embarrassed as Ghee celebrates the fact that he has been proven right. Ms. B intervenes posing making a sarcastic remark about the students' method for choosing the best deal. "How did you pick it, you just pointed your finger and said that one?" It may be inferred that Ms. B is trying to engage the other students in the conversation, when she poses this question to Ghee. She also asks Devin for his opinion. Devin appears hesitant to answer as he quietly asks Ghee, "How many hours you say?"

Dana makes mathematical progress during this episode because she is able to calculate the total cost for six hours of two different DJs. Tom's Tunes would cost three hundred sixty dollars and Solidus Sounds would cost three hundred forty dollars. This episode is concluded when Ms. B moves away from the table to work with a different group.

Immediately subsequent to this episode, it is not clear if the students are engaged with solving the task. Dana is working on her calculator, and appears to be unhappy because she is silent and is looking at her paper wearing a frown (pouting). She does not answer Lenny when he asks, "who you goin' with [Dana]?" Shawn says to Lenny, "I don't want to argue" and Lenny agrees. It is inferred that the boys recognize that Dana is grumpy and they wish to avoid a confrontation with Dana.

Ghee is looking around, then, he leans down to his desk. After approximately two minutes, the students begin to discuss the problem again.

Begin TSMIE [B.02.a.03] 12:32 – 17:35 I			
Participants: Natasha, OJ, Al, Kenny, Ms. B Immediately prior to this interaction, Ms. B is seated at Dana's table, which is adjacent to this group's table. Meanwhile, Natasha, Al, OJ and Kenny are engaged in a discussion about how to correctly compute the cost of DJ Solidus Sounds for eight hours. While Natasha is presenting correct reasoning stating that the DJ charges a one-time fee of one hundred dollars that should be added after the number of hours is multiplied by the hourly rate. Initially, Al disagrees with her, adding the hourly rate forty, to one hundred, then multiplies one hundred forty by eight hours. As Ms. B approaches the group, Al, Kenny and Natasha state, "the second one is better".			
12:35	Natasha	Hmm?	
12:36	OJ	Eight times forty an hour right?	
12:41	Student	Forty dollars an hour right?	
12:42	Student	You just pay… (inaudible)	
12:43	Al	That's three hundred twenty right there. That's three hundred twenty. Then you say…	
12:46	Natasha	That's what I'm talking about! [Natasha strikes the table with her right hand, her voice is raised.] You pay one hundred dollars down and then you got to keep paying forty every hour [Natasha hits the table repeatedly with her right hand]. That's what I'm trying to say. [Natasha is looking at Kenny.]	It may be in inferred that Natasha is ***expressing anger toward*** the boy she is speaking to, as she raises her voice.
12:52	Al	And then you add a hundred, that's four hundred twenty.	
12:55	Natasha	Yeah, that's what I just said [Al]! [Natasha looks at Al, points toward him with her hand and smiles.]	It appears that Natasha is ***surprised*** that Al is repeating her same thought.
12:58	Al	But I was telling the truth the whole time.	
12:59	Natasha	I was too.	
13:02	OJ	This one is better [OJ points to Natasha's paper.]	
13:03	Natasha	Which one, the first one?	

		[Natasha looks at OJ.]	
13:04	OJ	The second one here. [OJ points to the paper in front of Natasha.]	
	Natasha	The second one!	
	OJ	The second one is better. [OJ points to the paper in front of Natasha a second time.]	
13:08	Ms. B	How do you know the second one is better? [Ms. B is seated in a chair, she slides her chair from the adjacent table and leans her right elbow on boy's desk. She is smiling and looking at Al.]	Ms. B appears ***happy*** because she is smiling and it may be inferred that she is ***pleased*** about the students' engagement with the mathematics. **[Int]**
13:08	Al	Because we said we gonna party for eight hours, right? [Al is looking at Ms. B.]	
13:11	Ms. B	Oooh, that's a long party. Okay. Eight hours…	Ms. B exhibits active listening. **[Int]**
13:15	Al	Right? And then, its forty dollars. [Al points at his paper, he is looking at Ms. B.] Wait. [Al moves his calculator and looks at the problem sheet.]	
13:18	Natasha	Mmm hmm	Natasha affirms Al's statement "forty dollars".
13:19	Al	Forty dollars an hour right? [Al looks at Ms. B and nods his head. Ms. B is out of camera range.]	
13:20	Ms. B	Okay.	
13:22	Students	Sixty. Sixty.	
13:25	Al	The DJ is charging a hundred. Keep in mind, [Al points toward Ms. B, and looks at her, tilting his head to the side, emphasizing his words	It appears that Al is "teaching" Ms. B because he is giving all of the necessary information as she actively listens.

		hitting his hand on the desk] the DJ is charging one hundred, just for a day.	
13:30	Natasha	See he is charging a hundred dollars for. [Natasha points at Ms. B. Ms. B is seated at a student desk, with a calculator in front of her. She is looking at Natasha.] Don't put, don't put a hundred in, just put… [Ms. B's facial expression is serious as she looks at Natasha and listens to and follows their instructions.]	It appears that Natasha is "teaching" Ms. B how to calculate the total cost of the DJ for eight hours, as she gives instructions for Ms. B to input the numbers in the calculator. Ms. B's facial expression may convey that she is interested in Natasha's mathematical ideas. Ms. B appears to have assumed the role of 'student' and Natasha and Al are 'teaching' her. **[Int]**
13:35	Al	Forty times eight.	
13:37	Ms. B	Forty times eight. [Ms. B is pushing buttons on a calculator.]	
13:38	Natasha	Mmm hmm.	
13:38	OJ	[Ms. B looks up at Natasha.] We could do what's it's name...We could do the third one.	
13:40	Natasha	We could do a hundred.	
13:41	Ms. B	Why am I adding a hundred? [Ms. B looks at Natasha and then at OJ.]	Ms. B appears ***curious*** about why she should add "a hundred". **[Int, reasoning]**
13:42	Natasha	Because, for the down payment is, it's more like a down payment is a hundred dollars. [Natasha is looking at Ms. B, gesturing with her hands as she speaks.]	
13:45	Ms. B	Okay. [Ms. B looks at the calculator and pushes buttons.]	Ms. B appears to ***accept*** Natasha's explanation, and she is still assuming the role of student. **[Int]**
13:46	Natasha	So you could do plus one hundred.	
13:48	Al	So you could get four	

		twenty. [Ms. B looks up at Al, her eyebrows are raised and her tone of voice is calm.]	
13:52	Ms. B	So you pay four hundred twenty dollars.	
13:54	Students	Four hundred twenty dollars.	
13:57	Al	Yeah, like instead of paying like um, a thousand dollars, and stuff like that. [Al nods his head as if to emphasize his words.]	
13:58	Natasha	Thirty times eight.	
13:59	Ms. B	And how many hours is that for? [Ms. B looks at Natasha.]	Ms. B appears ***curious*** about the students' understanding of how many hours four hundred and twenty dollars would pay for. **[Int]**
14:00	Student	Eight.	
14:00	Ms. B	Eight hours. So do you guys want to keep track of what you do. [Ms. B looks around at all of the students, her tone of voice is calm and tentative.] So just in case you know what's going on.	Ms. B appears to ***affirm*** the students' answer, "eight hours" when she repeats it and than encourages them to keep a written record of their work. **[Val]**
14:05	Al	I was trying to tell these kids but they wouldn't listen.	
14:07	Ms. B	Ok. So how can I prove that that you said, the second one, [Ms. B looks down at the problem paper. Her brows are furrowed as she speaks] you said it's Solidius Sounds? [Ms. B looks up at Natasha then at Al.]	Ms. B expresses ***curiosity*** about how the students can confirm that "the second one" (Solidus Sounds) is the best DJ to choose. **[Int, reasoning]**
14:18	Student	Yeah.	
14:19	Ms. B	So how do you just know that second one is the best one? [Ms. B gestures with her hands and looks around	

		the table. Natasha is punching buttons on her calculator.]	
14:21	Student	That's two hundred forty.	
14:24	Al	Look, I did forty.	
14:25	Ms. B	[Kenny] just said the third one could be the best one. [Ms. B cranes her neck and her voice exhibits no emotion.]	Ms. B encourages student discourse when she makes note that Kenny votes for the third DJ (Light Plastic) as "the best one". **[Endis, Blds]**
14:29	Natasha	Yeah, the third one is the best one, the third one. [Natasha looks up at OJ, then at Ms. B.]	Natasha agrees with Kenny but does not provide reasoning.
14:31	Ms. B	Oh you guys are saying the third one, I'm sorry. [Ms. B looks down at the problem paper again.]	
14:34	Natasha	[Natasha looks at Ms. B, and leans towards Ms. B] No, first we was saying the second one. [Natasha points towards Al's paper.] [Al], the um, the third one is the best one because um, it was only four hundred fifteen [Natasha points at her calculator, then looks up at Al and laughs, he has said something that is inaudible. Ms. B is smiling and glancing back and forth between Al and Natasha.]. But for real! We shoulda did, we shoulda did the math before we started.	Natasha reasons that Light Plastic is "the best one" because he costs four hundred fifteen dollars for eight hours. Natasha recognizes the importance of "doing the math" before making a decision.
14:46	Al	So wait, wait. Let's do all of them to see which one is the correct one. [Al gestures with his hands as he speaks.] Aiite. We do the last one. We do the first one, last. That way we go	Al builds on Natasha's comment suggesting that they calculate the rates for all three DJs.

		by…(inaudible) [Ms. B is smiling as she looks at Al and Natasha.]	
15:02	Ms. B	So you still not sure? So you still working on it? [Ms. B looks at Natasha. Her tone of voice is tentative.]	Ms. B expresses ***curiosity*** about the students' decision. **[Int]**
15:03	Natasha	No we sure, we sure. The third one. He not sure. I'm sure. [Natasha looks at her paper and begins to write.]	Natasha conveys ***confidence*** in her answer that Light Plastic is the best DJ.
15:07	Al	I'm sure. But is the group sure?	
15:09	Ms. B	You sure? [Ms. B asks Al]	
15:09	OJ	Yes.	
15:11	Al	Not everybody. [Natasha puts her head down and laughs.]	
15:13	Kenny	[Inaudible]	
15:15	Al	How many hours?	
15:16	Natasha	That's what I'm saying…fifteen.	
15:17	Al	We gonna party for the same hours, only thirty right? [Al is holding his calculator with his right hand, gesturing with his left hand.]	
15:21	Natasha	So thirty times eight.	Light Plastic charges thirty dollars per hour so it may be inferred that Natasha is calculating the cost of Light Plastic for an eight-hour party.
15:25	OJ	So that's the best one then.	
15:26	Natasha	Two hundred forty. Then you add one hundred seventy five. And its four hundred fifteen. [Natasha smiles and looks down.] We on a budget, we on a budget. Cuz' we on a budget. We don't need to party for eight hours.	It appears that Natasha may be suppressing laughter.
15:46	OJ	And then we can party	

		longer.	
15:47	Ms. B	Oh, so what if you do party for eight hours? [Ms. B furrows her eyebrows, and looks at OJ, then to Al and Natasha.]	It may be inferred that Ms. B recognizes Natasha is potentially headed off task. She expresses ***interest*** in OJ's idea that "we can party for longer".
15:49	Al	It's two o seven. So, that one cheaper.	
15:53	Natasha	It's not two o seven. [Natasha is laughing. Al picks up his calculator.]	
15:57	Al	Wait, I think I did it. [Al is punching buttons on his calculator.] Oh yeah, I did it wrong.	
15:58	Natasha	He said two hundred seven.	
16:03	Al	So, forty times…	
16:04	Natasha	Two hundred five.	
16:04	Al	eight.	
16:09	OJ	[Inaudible] [Ms. B is leaning toward OJ, her hands are folded on the desk in front of her.]	
16:21	Ms. B	Ok, okay, so you said party for five hours? [Ms. B speaks to OJ.]	
16:23	Al	That's four fifteen.	
16:25	Ms. B	…party for five hours.	
16:26	Al	So you still you losing five dollars. I mean, you're saving five dollars.	
16:29	Natasha	[Natasha is nodding and smiling at Al. Ms. B is looking at Natasha.] Mmm, you save five dollars. Oooh, that's good. That can go to other stuff.	
16:35	Kenny	(Inaudible)[Ms. B looks at Kennedy and nods her head.]	
16:37	Natasha	You can buy some stuff, go to the dollar store and buy some cups and some plates and forks and some spoons and napkins.	

		[Natasha is smiling and leaning towards Al, emphasizing each word with a tap on the table. She laughs and looks towards Ms. B.]	
16:43	Ms. B	So right now, you're working on for how many hours? [Ms. B cranes her neck to look towards Al's paper.]	Ms. B appears to be ***curious*** about the cost for a five-hour party and it is inferred that she intervenes in the conversation to foster Natasha's engagement with the DJ problem after Natasha talks about the "stuff" for the party. **[Int]**
16:47	Student	Eight.	
16:48	Ms. B	For eight hours. But [OJ] just said lets be "geeks" '[Ms. B looks towards Natasha, and nods her head] and party for five hours [Ms. B turns and smiles at OJ].	Ms. B ***validates*** and suggests building on OJ's idea when she poses his idea to the whole group. **[Blds, Val]** It is inferred that Ms. B is trying to lead the students to compare DJ costs for a variety of hours.
16:55	OJ	She, she…wanted to bring up we should party for less longer. [OJ looks at Natasha and smiles, his head is cocked to the side.] Am I right?	
17:00	Ms. B	So you think we should party longer than eight hours? [Ms. B sits up in her seat and points towards Natasha.]	
17:01	Natasha	No, it don't matter, I'm sayin' that um since it's um since the party's not so high, it don't matter how many hours you gonna party [inaudible]…. [Natasha looks down a her calculator as she speaks, Ms. B is seated with her hands folded, she is looking at Natasha as she speaks.]	

17:09	OJ	That long.	
17:10	Natasha	A lot. [Natasha looks up at OJ, then back at her calculator] Yeah. [Natasha raises her voice, she continues to look at her calculator, then looks up at Ms. B when she finishes speaking.] I was trying to say like say if he was partyin' for a little bit.	
17:14	Al	Wait, wait, Wait!	
17:16	Ms. B	So that's a good question.	Ms. B validates Natasha's suggestion "partyin' for a little bit." **[Val]**
17:18	Student (possibly Kenny)	What if you did two hours?	
17:20	Natasha	It wouldn't be um that much. It would be less. [Natasha shrugs her shoulders. Ms. B looks at Natasha and nods her head.]	
17:22	Ms. B	So do we really know how many hours we are partying for? [Ms. B raises her eyebrows and looks at Natasha] Pause. [Ms. B turns around towards Dana's table, gestures with one hand and says, "I'll be right with you".]	Again, Ms. B poses a question to the students, expressing ***doubt*** that the students have completed the work. **[Con, Int]**
17:31	Al	See, if you do number one it would be five hundred forty dollars. [Camera pans to show Natasha and Al.]	

17:32	Natasha	Mmmhmm.. [Natasha furrows her brows and shakes her head.] Ain't nobody paying almost a thousand dollars!	
17:33	Al	That's too much.	
17:35	Natasha	That's almost that's almost a thousand dollars, round that off! [Natasha voice is raised, she smiles and nods her head.] [Ms. B walks away from this group.]	
End TSMIE [B.02.a.03]			

Discussion of TSMIE [B.02.a.03]

Ms. B supports the students' autonomy as she encourages Natasha and Al to lead her in a mathematical discussion and she assumes the role of a student.

When the episode begins, Natasha and Al appear to be in "let me teach you" engagement as they lead Ms. B through the process of calculating the cost of hiring Solidus Sounds for eight hours. As the students give Ms. B instructions, she adopts the role of student, punching numbers in the calculator as they speak, it may be inferred that Ms. B is pleased with the level of student engagement. Ms. B provides validation for the students' mathematical ideas in this episode when she suggests that the students keep a record of their calculations so that they may "keep track" of their work. The students make mathematical progress during this episode because they begin to calculate all three DJ rates for a set number of hours.

This episode highlights a playful moment when Ms. B smiles as she repeats and validates OJ's suggestion to "be geeks" and party for less time. Here, Ms. B is

encouraging students to consider alternative solutions and to consider OJ's mathematical idea.

In the final moments of the episode, Ms. B poses a question to the students, "so do we really know how many hours we are partying for?" Ms. B pauses for a minute, observing the students' discussion and then walks away.

Al, Natasha, OJ and Kenny remain engaged with the mathematics subsequent to Ms. B's departure as they discuss the number of hours the DJ should be hired.

Begin TSMIE [B.02.a.04] 17:49 – 18:57 (L)			
Participants: Ms. B, Shay, Rita Immediately prior to the start of this episode, Shay appears to have solved the problem. "…for one to five hours right, you would want to choose Time's, and then if it was the five to eight hours you would choose Solidias, and then, umm, eight hours or longer you would want to chose light plastic." It may be inferred that Shay believes that the conditions of the problem are designed to intentionally trick the reader when he tells Ruth, "And then like see how you got at forty dollars each hour and thirty dollars each hour, that would throw you off a little bit. You gotta add forty dollars each hour and thirty dollars each hour, that would throw you off, they be trying to throw you off." This episode begins when Ms. B sits down in a chair between Shay and Rita and asks them "what did you come up with over here?"			
17:49	Ms. B	Okay, so what did you come up with over here? [Ms. B sits down in a chair between Shay and Rita.]	Ms. B's tone of voice is ***serious*** and it may be inferred that she is ***curious*** about the students' progress at Shay's table. **[Int]**
17:53	Shay	Like, one to five hours, you would choose Tom's. [Shay looks at Ms. B and raises his eyebrows as he speaks.]	
17:57	Ms. B	Okay. [Ms. B looks at Shay's paper and nods.] Why?	Ms. B is ***curious*** about how Shay decided that Tom's would be the best choice for one to five hours. **[Int]**
17:57	Shay	Because that's how he is the cheapest.	
18:00	Ms. B	How do you now? [Ms. B looks at Shay's paper and points her finger toward Rita.]	
18:02	Rita	Because that how [inaudible] [Ms. B looks at Rita.]	
18:05	Ms. B	Okay, so you all, okay both did it. [Ms. B nods her head and raises her eyebrows as she points to both students' papers. Ms. B nods and purses her lips, then folds her arms] Okay, leave it here so you guys can explain it to me, [Ms. B raises her voice and reaches for Rita's paper and puts it on the desk in front of her] okay so what are you showing me?	Ms. B ***validates*** the students' mathematical engagement. **[Val]** Ms. B expresses ***curiosity*** about Rita's work written on the paper. **[Int, rp]**

		[Ms. B points to Rita's paper.]	
18:12	Shay	[Ms. B and Shay look at Rita's paper. Rita has drawn three tables listing the hours and total cost for each of the three DJs from one to twelve hours. She has completed two of the tables for hours one through twelve and the third table for hours one through five, see below for an example of each table through six hours.] Like from one to five right, out of all, all three of them, yea he got the less, the least price. [Shay points to Rita's paper and slides his hand down the paper as he looks at Ms. B.]	
18:16	Ms. B	Okay. [Ms. B looks on and nods her head as Shay explains. Her arms are folded and her brow is furrowed.]	It may be inferred that Ms. B is ***very interested*** in what Shay is saying because she is nodding her head and looking carefully at Rita's paper. Her eyebrows are furrowed as if she is trying to understand Shay's explanation. **[Int]**

18:19	Shay	And then from five to eight, out of all three of them, they got the least price right there, on Sounds. [The camera pans in to show Rita's paper. Rita has drawn three tables of values. For Tom's Tunes, her table is correct, but Solidus and Light Plastic are incorrect.]	

Toms Tunes		**Solidus Sounds**		**Light Plastic**	
X (hours)	Y (dollars)	X (hours)	Y (dollars)	X (hours)	Y (dollars
1	60	1	140	1	205
2	120	2	280	2	410
3	180	3	420	3	615
4	240	4	560	4	820
5	300	5	700	5	1025
6	360	6	840	6	

18:26	Ms. B	Oh "Sounds" you said has the least price. [Ms. B points to the column that shows Solidus Sounds.]	
18:28	Shay	Yeah.	Shay presents faulty reasoning.
18:28	Ms. B	Okay.	Ms. B does not address Shay's faulty reasoning. Her tone of voice continues to convey ***interest*** in his reasoning. **[Int]**
18:30	Rita	Five to eight, eight on. [Rita taps the paper with her pen.]	
18:31	Shay	And eight on, [Shay points at Rita's paper] it would be on Light Plastic. [Shay rubs his eye with his finger as he looks at Ms. B. Ms. B still has her arms folded and her facial expression is serious.]	Ms. B does not validate nor negate Shay's comments.
18:40	Ms. B	Raheem, what do you have to say about that, what do you think? [Ms. B has her arms folded and she leans in towards Raheem. Raheem had been writing on his paper before Ms. B questioned him. Now, he sits back in his chair and puts his arms down in his lap.	Ms. B appears ***interested*** in Raheem's opinion of Shay and Rita's solution. It is also possible that Ms. B is trying to engage Raheem in the conversation as he has not participated in this

		He looks at Ms. B and then looks down. He does not speak.] Raheeem. You still working on it, [Raheem nods] Go ahead, go ahead. [Raheem leans forward and resumes writing on his paper. Ms. B nods towards Raheem.] I just broke your thought. Go ahead. Ms. B gestures towards Shay and Rita with her hands.] So are you finished?	interaction. **[Int, Endis]** It appears that Ms. B quickly recognizes that Raheem is not ready to share his ideas and demonstrates respect for him as she says, "go ahead, I just broke your thought. Go ahead." It may be inferred that Ms. B is pleased with Raheem's level of engagement with the mathematics as she ***encourages*** him to keep working. **[Val, Res, Enc]** Ms. B then directs her attention back to Shay and Rita. It may be inferred that she is ***curious*** to find out if these two think they are finished with their work. **[Int]**
18:50	Shay	No.	
18:51	Rita	No.we're not finished answering it yet.	
18:54	Ms. B	Okay, so you just have your thoughts together, [Ms. B looks at Rita and nods her head] so I will let you keep going, [While the camera pans to Dana's table, Ms. B can be heard off camera saying to Shay and Rita:] I like how organized…[inaudible].	Ms. B appears satisfied with Shay and Rita's progress so far. The episode is concluded when she says, "I will let you keep going". Ms. B issues a complement prior to walking away from their table. It is inferred that Ms. B likes how Rita set up the data in a table, but this cannot be confirmed. Rita is the only student who has shown this method of organizing the rates for each DJ.
End TSMIE [B.02.a.04]			

Discussion of TSMIE [B.02.a.04]

It appears that Shay feels that he has figured out which DJ would be best for a certain range of hours and he is beginning to develop an argument to support his solution. Ms. B observes the work that Shay and Rita have completed, which although it is well organized in three clear tables of values, the calculations for Solidus Sounds and Light Plastic are incorrect. Ms. B does not comment on this error and it is not clear whether or not she recognizes the error. When Ms. B asks Raheem for his opinion, it appears that she has interrupted him mid thought. He looks defeated when he sits back in his chair and looks sad. Ms. B appears to recognize Raheem's emotion, she immediately offers him more time "You still working on it, [Raheem nods]
Go ahead, go ahead." to finish his thought. The students are out of camera range after Ms. B walks away from Shay and Rita, so student engagement immediately subsequent to this episode is unknown.

Begin TSMIE [B.02.a.05] 20:04 – 23:21 (C)			
Participants: Ms. B, Mark, Frank, Jen, Quanee, John Ms. B is standing at this table when the video footage begins, so inferences cannot be made about the source of this interaction.			
20:33		[The camera pans to Frank' table. Ms. B is standing between two boys and she leans on their desks as she speaks Frank who is seated diagonally across from where she is standing.]	
20:34	Ms. B	So don't, know.. Let me see, [Ms. B reaches her arm across the table towards Frank, Ms. B is smiling] let me see this again?	Ms. B appears ***friendly*** because she is smiling and it may be inferred that she is ***curious*** about the item that she is reaching for. **[Int]**
20:41	Mark	You should get an o for that. [Boy points to other boy, then looks down at his desk]	
20:44	Frank	No….you crazy!	
20:46	Ms. B	You said, [a student hands a crumpled paper to Ms. B and Ms. B reads from the paper, she is smiling] "you not going to talk?" He says no. "They want, they want to make fun of the way I talk?" [Ms. B pauses and looks across at Frank. Her tone of voice is gentle as she speaks to him.] So you not going to talk because they make fun of the way you talk? [Ms. B's tone of voice changes to more lighthearted.] So, okay, is there any other way we can communicate and get the math done? How else can we communicate? That's fine if you don't want to talk to me, that's fine. You don't have to! [Ms. B has her eyebrows raised and she is smiling at Frank].	It is inferred that the crumpled paper is a note written by Frank stating that he will not talk. Ms. B ***sympathizes*** with Frank and ***validates his feelings*** then suggests that the students find an alternate way to communicate so that they can "get the math done". **[Kindness, Val, Endis]**
21:12	Student	[Frank]!	
21:15	Ms. B	Is there another way to get it done? What are your writing thoughts? [Frank shrugs]	Ms. B repeats the question, and it may be inferred that she is ***eager*** for the students to engage with the mathematics because she
21:23	Jen	They trying to blame it on me, it was him.	

		[Girl smiles and points towards boy sitting next to her]	asks about the students' thoughts three times.
21:27	Ms. B	So what thoughts do you have so far? [Ms. B gestures towards a paper on the desk and she is not smiling. Boy to Ms. B's left looks up at her, then pulls his seat in and writes on his paper.] [Pause] What did you guys come up with so far?	Jen appears to be defending herself about Frank' claim that the group was making fun of him. Ms. B does not acknowledge the girl's complaint. Ms. B expresses ***interest*** in the students' progress as an inferred attempt to reengage the students with the mathematics. **[Int]**
21:34	Students	DJ Light Plastic	
21:36	Quanee	And we do seven hours, and the rest [inaudible]	
21:49	John	For seven hours all day they gonna perform [John looks up at Ms. B and rolls his pen between his fingers.]	
21:50	Ms. B	Oh really, they gonna look the same at seven hours? [Ms. B is not smiling and her eyebrows are raised] Okay, so… that's for how many hours? Seven hours. Didn't you say you were partying for seven hours?	It may be inferred that Ms. B is ***not satisfied*** with the students' solution, but she does not tell them that they are wrong. **[Dis-m]**
22:07		[Frank shakes head "no"]	
22:11	Mark	Not me, they just went along with what I said.	
22:13	Ms. B	And they just went along with what you said? What if it's because um, the executive board, they were talking about it and they were coming up with all different kinds, [Ms. B gestures with her hand] all different types of numbers. Maybe if they started right after school and then started at three o'clock then maybe they could party until nine o'clock p.m. They were saying until nine o'clock, party for six hours. But then they were saying since we are starting so early maybe we could even go longer since the school will be open until three in the morning. So they were saying let's	It may be inferred that Ms. B is trying to stimulate student engagement by offering different options for the numbers of hours the DJ could be hired for. **[Int]** It may be inferred that Ms.

		start it a three o'clock in the afternoon and end it at three o'clock in the morning. [Ms. B accentuates her words by hitting her hand on the table.] So how many hours would they be partying for? [Ms. B looks at Frank and shrugs her shoulders.] [student responds "Twelve hours"] Twelve hours, that was a thought. [Ms. B furrows her brow, gestures in the air with her hand and nods her head.] They came up with this and they were saying that they would have food all night and everything. Then someone said that would be too long, twelve hours is too long. So now another one from the executive board, another person from the executive board said "let's just only party for three hours". [Ms. B leans in and shakes her hand.] So I heard numbers like three, I heard seven, I had I heard four and half hours, and twelve hours, ten hours. [Ms. B gestures with her hand as she recalls each number.] So I'm not sure, they're not sure exactly how many hours they want to party at. [Ms. B purses her lips and pauses.] So, if they decide on a number, which one would be best? [Ms. B stands silently for approximately eighteen seconds, she is not smiling, and she looks from one student to the next. The students are also silent, some are punching numbers in their calculators. Then, she walks away.]	B is curious about the students' understanding and that she is trying to ***encourage student discourse*** through her questioning by asking a mathematical question. **[Endis]** Ms. B ***validates*** the student's answer. **[Val]** Ms. B appears to convey ***curiosity*** about which DJ would be the best if the executive board decides on a timeframe for the party. **[Int]**
End TSMIE [B.02.a.05]			

Discussion of TSMIE [B.02.a.05]

Ms. B recognizes that there is a social conflict, which she appears recognize as interfering with the students' engagement with the mathematics. She acknowledges Frank' feelings when she says, "So you not going to talk because they make fun of the way you talk? ... That's fine if you don't want to talk to me, that's fine. You don't have to!" Ms. B intervenes in an inferred attempt to foster student engagement despite the communication obstacle. "So, okay, is there any other way we can communicate and get the math done? How else can we communicate?" When the students do not provide an answer, Ms. B suggests that they write their ideas on paper. It may be inferred that Ms. B recognizes that the students may think they have satisfied the solution of the problem by figuring the costs of all three DJs for seven hours. She intervenes by presenting some different scenarios for the amount of time the DJ could be hired. Ms. B poses a question to the students, "So, if they (the executive board) decide on a number, which one would be best?

Immediately subsequent to Ms. B's departure, it appears that Mark and Quanee become engaged with the mathematics because they are using their calculators and discussing different numbers with each other. Meanwhile, Frank and Jen appear to be arguing about the issue of Frank not wanting to talk. They are not engaged with the mathematics.

Begin TSMIE [B.02.a.06] **23:51 – 32:22 (L)**			
Participants: Ms. B, Ghee, Lenny, Shawn, Devin, Dana Immediately prior to this episode, Ghee and Lenny argue about how to write the equation for Solidus Sounds. Lenny thinks that the equation should read, y = 100 + 40x. Ghee disagrees and Lenny calls out "Ms. [B]!" Ghee takes his notebook out of his desk. After Lenny looks at Ghee's notebook, he appears convinced that Ghee is correct.			
23:53	Ms. B	[Ms. B sits down at an empty desk next to Devin. Ghee is laughing, looking at Lenny. Ghee puts his notebook back in his desk. Lenny points towards Ghee's notebook, smiles and nods his head.] What happened?! [Ms. B's tone of voice is slightly raised, she is smiling as she looks at Ghee.] You pulled out that notebook, what was in there?	Her tone of voice and questions may be inferred to show that she is ***interested*** and genuinely ***curious*** about what Ghee was showing Lenny in his notebook. It is possible that Ms. B is ***excited*** about the fact that Ghee is referencing his notebook to answer a question. **[Int]**
23:59	Ghee	My stuff.	
24:01	Ms. B	[Ms. B leans her elbows on the table and folds her hands, resting her chin on her hands.] Okay, and have you found the mistake? [Ms. B nods]	**[Int]**
24:01	Ghee	[Ghee nods his head and looks down at his desk as he pulls his chair in.]	
24:02	Ms. B	Okay, so what happened over here? [Ms. B looks from Lenny's paper to Ghee's paper.]	Ms. B appears ***interested*** in her students' progress. Her question is not mathematically specific. **[Int]**

24:05	Lenny	Nah, cause we put the forty, [Lenny reaches across the table to point at Ghee's paper and Ghee begins to write on his paper.] instead of the hundred, times, instead of the hundred first. Cause it's a hundred dollars. When, when you first come in, then it's forty added on to it. [Lenny gestures moving his arm around and around in a circular motion.] That is what he messed up on.	Lenny describes a mathematical error made by Ghee.
24:17	Ms. B	Okay. [Ms. B has her arms folded on the desk. She pauses for approximately twelve seconds. All five students are writing on their papers. Ms. B looks around the table at their papers.] So did you come up with the best DJ? [Ms. B looks at Ghee]	Ms. B responds to Lenny but she does not validate nor negate what he said. Her tone of voice has no emotion. Ms. B appears ***curious*** about the students' progress toward choosing the best DJ. **[Int]**
24:33	Ghee	Yup, uhhh, Solidus. Solidus.	Solidus Sounds offers the best deal at six hours. Ghee's tone of voice may suggest that he is ***confident*** in his decision.
24:37	Ms. B	And how did you come up with that? [Ms. B abruptly turns around to address the group behind her.]	Ms. B appears ***curious*** about Ghee's reasoning. Her tone of voice is ***serious.*** **[Int, reasoning]** It is possible that Ms. B turns around to address behavior that she perceives to be off-task at the table behind her.
24:41	Ghee	Didn't we explain that to her already? [Ghee poses this question to the members of his group. He cocks his head to the side and furrows his eyebrows.] [Ghee's tone of voice softens and he	Ghee appears ***confused*** and ***annoyed*** by Ms. B's question. His tone of voice is whiny.

		smiles as he speaks to Ms. B.] I thought I explained that to you already.	Yet, when he speaks directly to Ms. B, his tone of voice changes to suggest respect toward Ms. B.
24:48	Ms. B	But then I left, and then I came back. And I was trying to pick up where, where I left off. [Ms. B points to herself and then gestures towards the students. Her tone of voice is apologetic.]	It appears that Ms. B is apologetic as she expresses ***interest*** in Ghee's progress. **[Int]**
24:57	Ghee	Well. [Ghee leans back in his chair and looks down at his paper. He raises his eyebrows. Ms. B is smiling at him.]	
24:58	Shawn	Why you changing it around? [Shawn points towards Ghee's paper with his pen.]	It may be inferred that Shawn is questioning why Ghee changed his equation. Earlier, Lenny had pointed out that "'Cause we put the forty instead of the hundred, times, instead of the hundred first. 'Cause it's a hundred dollars. When, when you first come in, then it's forty added on to it. That is what [Ghee] messed up on." There is no written evidence of the students' work at this point, but it may be inferred that the original equation that they wrote was y = 100x + 40 when the correct answer, that Ghee and Lenny are inferred to have at this point is y=40x + 100.
25:00	Ghee	[Ghee looks at Shawn.] I had to, cause it's a hundred first and then you got uhh…	
25:05	Shawn	[Shawn mutters something inaudible.]	
25:08	Ghee	Oh but…[Ghee looks down at his paper and leans on his desk.]	It may be inferred that Ghee is ***puzzled*** by Shawn's question, "why

			you changing it around?”
25:09	Ms. B	So you, umm, determined what your variables were, so Y is what? [Ms. B looks around the table. Her facial expression is serious.]	It appears that Ms. B ***recognizes that Ghee is puzzled*** and is intervening to help him frame his reasoning when she asks, “y is what?” **[Rsi]** Also, it may be inferred that Ms. B is ***curious*** about the students’ understanding of the contextual meaning of each variable. **[Int, reasoning]**
25:13	Ghee	Y is the…[Ghee looks around]	
25:16	Shawn	The uhh money….X is the…[inaudible]	
25:20	Ghee	Y is the uhh, the amount of money, and X is the number of [Ghee raises his eyebrows] hours. [Ms. B nods her head as Ghee speaks.]	Ghee presents correct reasoning.
25:27	Ms. B	So you determined what your variable is? [Ms. B’s tone of voice is quiet, she has her hand on her forehead and her brow is furrowed.] It gave that to you as well. They said that Y equals, they said let Y be the total, let Y be the, I can’t see. [Ms. B is looking at Lenny’s paper] Let me just see? It gave it to you already, it said. Let Y be the total cost. [Ms. B is pointing at Lenny’s paper.]	Ms. B’s tone of voice suggests that she is ***interested*** in making sure that the students understand the context of the problem. **[Int]** She points out the fact that the students were given the information for the assigned variables.
25:46	Lenny	Yeah.	
25:46	Ms. B	Okay. And X be the number of hours, okay. [Ms. B folds her arms on the desk again.]	Ms. B’s tone of voice conveys that she is ***satisfied*** with the students’ understanding of the variables.
25:48	Ghee	Yeah.	
25:48	Ms. B	Okay, so you picked Solidius Sounds. How did you pick that? [Ms. B shrugs her shoulders.]	Ms. B conveys ***curiosity*** about how the students chose Solidus Sounds when she shrugs her shoulders and asks the students how they made that choice. **[Int]** Ms. B waits ***patiently*** while
25:54	Shawn	Cause we umm…	
25:56	Ghee	We did, ohh! [Ghee drops his pencil and leans down to pick it up.]	
25:58	Shawn	I got mine in …. [Ms. B presses the backs of her hands on	

		her mouth.]	the students appear to struggle to answer Ms. B's question, why they chose Solidius Sounds as the best choice for a DJ.
26:07	Devin	[Devin looks at Ghee and says:] Why couldn't it be like ten hours? Instead of six? [Ms. B straightens in her chair and turns to glance towards Devin. Ms. B then looks around the table.]	
26:09	Shawn	She wanted to, she wanted to do this one, but that's when we found out that.	
26:13	Ms. B	Wait real quick, cause [Devin] just whispered something. Let him whisper it again. [Ms. B puts her hand up to her face as if she is telling a secret as she speaks to Ghee and Lenny. She smiles at Devin and lowers her voice to a near whisper. Devin smirks.]	Ms. B appears ***pleased*** that Devin asked a question. Her gestures and tone of voice [putting her hand next to her face as if she is telling a secret, and lowering her voice to a near whisper] suggest that she is aware that Devin may be shy about participating in mathematical discussions. She conveys that she ***values*** his idea because she offers him an opportunity to say it again by telling the other students to listen rather than speaking directly to Devin. She appears to be ***encouraging student discourse*** when she tells the group, "Let him whisper it again." **[Int, Val]** When Ms. B looks at Devin, her facial expression may be interpreted to show that she is ***proud*** of him. Devin smirks and appears to try to suppress his ***smile, perhaps embarrassed*** that Ms. B noticed his question.
26:18	Ghee	He said, why couldn't it be ten. [Ghee looks at Ms. B and begins pushing buttons on a calculator.]	

26:24	Ms. B	Did you guys hear what [Devin] just said? [Ghee, Dana, Shawn and Lenny are all looking at their papers or their calculators.] [Ms. B smiles as she leans in towards Dana and Shawn. She is smiling.] [Dana], [Shawn]? [Devin is looking down at his paper and glances up as Ms. B says each student's name.]	Ms. B ***encourages student discourse*** when she asks Dana and Shawn if they heard Devin's question. **[Endis]** It appears as though the students are either ignoring Ms. B or they don't hear her. Ms. B responds by calling Dana and Shawn by name. Devin's body language may suggest that he is ***shy.***
26:28	Shawn	Nope.	
26:29	Ms. B	[Dana]?	Ms. B calls on Dana a second time.
26:29	Dana	I heard him. [Dana is writing on her paper, her head is turned away from Ms. B.]	It is possible that Dana ***resents*** the positive attention that Devin is receiving from the teacher. She appears to be ***sulking*** during this entire episode.
26:29	Ms. B	What did he say?	It is inferred that Ms. B recognizes that Dana is upset about something and she is trying to engage Dana with the mathematics. **[Int]**
26:30	Dana	Why can't it be ten hours instead of six? [Dana continues to write on her paper as she answers Ms. B. Her tone of voice has no affect.]	Dana answers Ms. B, but she does not look at her, suggesting that she is ***not interested*** in Devin's suggestion.
26:34	Ms. B	That is a good question! [Ms. B smirks and smiles.] Why *can't* it be ten hours instead of six hours? [Ms. B glances around the table. Dana, Ghee and Shawn are writing on their papers.]	Ms. B exhibits that she is ***pleased*** by Devin's question in her tone of voice, which suggests ***excitement*** and her language, which definitively ***validates*** Devin's question.

			[Val, Int] She repeats Devin's question.
26:39	Lenny	'Cause then it's gonna cost more! [Lenny looks up at Ms. B, then goes back to writing on his paper.]	Lenny appears to disagree with Devin's suggestion of ten hours because "it's gonna cost more!" Ghee supports Lenny's idea with ***enthusiasm.***
26:43	Ghee	It's gonna cost super money!	
26:44	Lenny	It's gonna cost more. [Lenny sits up in his seat and leans across the table to look at Ghee's paper. He furrows his eyebrows.]	
26:45	Ghee	Who gonna party for ten hours, though? [Ghee opens his eyes wide and scans the group.]	It appears that Ghee expresses ***disbelief*** that a DJ would be hired for ten hours. Ghee ***disagrees*** with Devin's suggestion to calculate up to ten hours.
26:48	Shawn	How, how, how you gonna go for that long?	Shawn also agrees with Lenny and Ghee.
26:48	Ms. B	Oh I like to party. [Ms. B opens her eyes wide, smiles and gestures with her hand] I'll party for fifteen hours, I'll be there. Fifteen hours! [Ms. B smiles and shakes her head.]	Ms. B appears to ***excited*** about the possibility of a fifteen-hour party. It may be inferred that Ms. B is expressing ***excitement*** about a fifteen-hour party in order to ***stimulate the students' interest*** in Devin's suggestion. **[Val]**
26:53	Shawn	Daaaag…..it would be super jiving. [Shawn is smiling]	
26:54	Ghee	Yea, I know right. [Ghee sits back in his chair. He is smiling.]	
26:55	Ms. B	That is a good question! [Ms. B shrugs her shoulders and folds her arms, and raises her eyebrows as she looks at Lenny. Her tone of voice expresses excitement.]	Ms. B is ***excited*** about Devin's question. **[Val]**
26:59	Ghee	She be baked. [Shawn is punching numbers in his calculator and is laughing.] [Ms. B smiles as she looks at Ghee and puts her hand up.]	

27:00	Lenny	People are gonna get aggravated, everyone else is taking at one…	
27:04	Ms. B	So can you show me if it costs more money? [Ms. B leans in towards Lenny, her brow is furrowed and she is no longer smiling.]	Ms. B's tone becomes more serious as she leans in towards Lenny and asks him to show her that it costs more money. Ms. B appears to be trying to ***reengage*** Lenny and Ghee's attention to the mathematics. **[Int]**
27:08	Ghee	He'll try. [Lenny grabs calculator from Ghee and holds it up in the air, looking at it.]	
27:09	Lenny	Didn't you just do it?	
27:10	Ghee	Yeah. Lemme just do the last one. Lemme do the last one. Let me do the last one! [Lenny holds the calculator so Ms. B can see it, Ms. B leans toward the calculator. Ghee is looking at Lenny, pointing his hand at Lenny. Lenny hands the calculator to Ghee.] Lemme do the last one! [Ghee looks at the calculator screen.]	Ghee appears ***eager*** to use the calculator that Lenny has. His tone of voice grows increasingly loud as he says, "lemme do the last one", three times.
27:18	Ms. B	That's for what, you are showing what? [Ms. B is looking at the calculator.] [Ms. B turns around to speak to the group behind her.] That was two minutes. Now you need to focus yourself. I gave you two and half minutes. It's over time. [Ms. B turns around and looks at Dana.]	Ms. B is ***curious*** about what calculation Ghee has shown her on the calculator screen. **[Int]** It appears that Ms. B is ***displeased*** with perceived off-task behavior of the students at the group seated behind her. This is the second time that she has addressed this group during this episode. Ms. B appears ***aggravated*** with the off-task behavior that she perceives. **[Dis-b]**
27:29	Ghee	Whooaa, son! …He a beast, son…he a	

		beast, son.	
27:33	Dana	I did, umm ten, I did ten. [Dana looks up and then at her paper.]	
27:35	Ghee	Oh he a beast son, yo we can do the last dude! We were doing Light Plastic, but, but, [Ghee sits back in his seat and throws his pencil down on the paper.] Ten hours is all right. [Ghee taps the table with his hands]	
27:47	Lenny	But how much is it gonna be? [Lenny is looking at Ghee. Ms. B looks back and forth between Lenny and Ghee.]	
27:49	Ghee	Four hundred seventy five dollars. [Ghee is sitting back in his chair and his eyebrows are raised.]	
27:48	Dana	I picked.	
27:49	Lenny	Yea, how much was the last one?	
27:53	Ghee	The one that was picked before?	
27:53	Lenny	No, I am talking about, umm, how much was this? How much was this? [Lenny points at his paper.]	It is not clear what "this" is.
28:00	Ghee	For ten hours? [Ghee asks Lenny.]	
28:00	Dana	For six hours.	
28:00	Lenny	No six.	
28:00	Dana	Tom's Tunes was umm six hundred dollars. [Dana looks at her calculator. Dana's tone of voice is quiet.]	Dana presents the correct answer for Tom's Tunes for ten hours. It is not clear, though, from language whether she understands six hundred dollars to be the total cost for six hours or ten hours.
28:03	Lenny	For six.	
28:03	Dana	Six hundred dollars! [Dana leans in towards the boys and cocks her head to the side. Dana smiles and raises her voice. Dana looks back at her calculator and shakes her head.]	Dana appears to express ***anger*** toward the members of her group when they do not acknowledge her answer.
28:09	Ghee	Hold on, [Ghee sits up in his seat and points to his paper] Tom's Tunes was six hundred, but the dude that we picked was umm…	
28:13	Dana	Would be four hundred.	
28:15	Ghee	Four hundred. [Ghee pauses has he looks	Ghee disagrees with Dana

		at his paper.] No! No! [Ghee raises his voice and raises his eyebrows as he points to his paper] His was three hundred sixty, his was [Ghee pauses] three forty.	and expresses his reasoning with ***enthusiasm.*** Tom's Tunes charges three hundred sixty dollars for six hours. Solidus Sounds charges three hundred forty dollars for six hours.
28:20	Lenny	Yea that's what I'm talking about.	
28:23	Ghee	Three forty? Yeah three forty. But for ten hours, he's five hundred dollars and then and then the other dude is six hundred and then ten hours the last Light Plastic, Plastic is four hundred seventy five dollars… for ten hours.	Ghee explains that Solidus Sounds would charge three hundred forty dollars for six hours and five hundred dollars for ten hours. Ghee synthesizes the information for the cost of each of the three DJs for ten hours. Solidus Sounds would cost five hundred dollars, Tom's Tunes would cost six hundred dollars and Light Plastic would cost four hundred seventy five dollars.
28:39	Lenny	But we won't party for ten hours.	
28:39	Ghee	Nah, nah that's good! Yo cause I'll have some… [Ghee sits back in his seat and smiles.]	Ghee appears ***pleased.***
28:40	Lenny	I'm a leave….I'm a leave at like two. No no no no.	
28:45	Ms. B	What's good? [Ms. B rests her chin on the back of her hand and looks Lenny at Ghee.]	Ms. B appears ***curious*** about what Ghee says is "good". **[Int]** It also may be inferred that she is ***confused*** about the boys' thinking because she furrows her brows. **[Con]**
28:47	Ghee	'Cause, you you party longer for cheap. You party longer for twenty-five dollars less. [Ms. B is looking at Ghee]	Ghee appears to have reasoned that Light Plastic's rate is less expensive for a "longer" party.

			Solidus Sounds charges five hundred dollars and Light Plastic charges four hundred seventy five dollars, a difference of twenty-five dollars.
28:54	Ms. B	Really? [Ms. B turns to look at Shawn, and her chin is resting on the back of her hand. Ms. B has her eyebrows furrowed.]	Ms. B appears ***surprised and possibly impressed*** by Ghee's claim that you party for "twenty-five dollars less". **[Int]**
28:57	Shawn	What, what kind you doing to get this? What kind of DJ you dealing with? [Shawn is smiling as he looks at Ghee]	
29:03	Ms. B	What kind of DJ would you be getting for that amount? [Ms. B raises her voice and points at Ghee.]	Ms. B rephrases Shawn's question. She appears ***curious*** about "what kind of DJ" the students would be getting for twenty-five dollars less. **[Int]**
29:03	Ghee	So ten hours, four hundred seventy five dollars is nothing! [Ghee has his hands out, he is leaning back in his chair.]	It may be inferred that Ghee is ***pleased*** with the prospect of paying four hundred seventy five dollars for ten hours.
29:11	Ms. B	Well the DJ…what DJ would that be? [Ms. B points toward Ghee]	Ms. B appear ***curious*** about which DJ would cost four hundred seventy five dollars for four hours. **[Int]**
29:12	Ghee	Light Plastic!	Ghee is correct. [30 * 10 + 175 = 475] So, Ghee has changed his answer for the best DJ from Solidus Sounds, his original answer during this episode, to Light Plastic.
29:13	Ms. B	That would be Light, Light Plastic. They're, they're showing that Light Plastic will be that much. [Ms. B leans towards Shawn, gesturing toward Ghee] So what do you guys say about that?	Ms. B appears ***excited*** about Ghee's answer and appears to be trying to encourage the other students to engage in a discussion with Ghee. **[Endis]**

29:20	Dana	Yeah!	Dana appears to agree with Ghee.
29:21	Ms. B	Yeah what? [Ms. B furrows her eyebrows when she looks at Dana.]	Ms. B appears ***interested*** in Dana's affirmative comment. **[Int]**
29:24	Dana	The hour…(inaudible)	
29:25	Ms. B	So now you switched over? [Ms. B's eyebrows are still furrowed as she looks at Dana.]	It may be inferred that Ms. B is ***questioning*** Dana's opinion.
29:27	Dana	No, I wasn't with them!	Dana appears to be telling Ms. B that she did not collaborate with the boys at her table. Her tone of voice may be inferred to show that she is ***angry*** with the boys.
29:28	Ms. B	Oh, so what were you doing [Dana]? [Dana is seated with her head resting on her palm.]	It appears that Dana is ***not happy.***
29:29	Lenny	She doing her own little thing.	
29:31	Ms. B	So what were you doing?	Ms. B persists, showing interest in Dana's reasoning. **[Int, reasoning]**
29:32	Dana	Not with them. [Dana rests her head in her hand, and is mumbling when she answers Ms. B.]	It may be inferred that Dana is ***pouting*** because of her posture, tone of voice and language. Dana has been uncharacteristically quiet during this episode.
29:33	Ms. B	So what were you doing?	Ms. B asks Dana a third time, and appears ***interested*** in Dana's mathematical progress. **[Int]**
29:36	Lenny	She trying to say which one would…[inaudible]	
29:37 29:43	Dana Lenny	I was doing all, I was figuring out….I was going up but I was on seven. [Dana has her cheek resting on her hand as she speaks to Ms. B.] You were changing?	

29:44	Ms. B	You were on seven. So what do you mean you were going up? [Ms. B stretches her neck to look at Dana's paper.]	It may be inferred that Ms. B ***recognizes that Dana is pouting*** and that Ms. B is taking ***interest*** in Dana's mathematical progress. **[Int]**
29:48	Lenny	So for ten hours we going.	
29:49	Dana	[Dana points at her paper as she answers Ms. B's question.] I was going up, like when he said five I went to six, when he said six I went to seven [inaudible].	Dana may be explaining that she was one step ahead of the boys in calculating the costs of the DJs for each successive hour. It may be inferred that Dana's tone of voice indicates that she is ***unhappy.***
29:58	Ms. B	Okay so when you were figuring that out, what did you determine? [Ms. B points at Dana, then she rubs her chin with her hand.]	Ms. B expresses ***interest*** in Dana's reasoning. **[Int, reasoning]**
30:03	Lenny	So you gonna go with us?	
30:03	Dana	That umm, when I was on seven, umm, [The noise level in the classroom has become louder, so it is difficult to hear the students in this group.] [Ms. B leans forward as Dana speaks.] [Devin has turned around in his seat, and is looking toward the group behind him. Ms. B glances briefly towards him then returns her attention to Dana.]	
30:10	Shawn	[inaudible]	
30:10	Dana	Huh? [Dana looks at Shawn, she is pointing to her paper with her pen.]	
30:11	Shawn	I'm not talking to you.	
30:15	Dana	[inaudible] [Dana puts rests her cheek on her hand again. Ms. B is leaning in toward Dana.]	It appears that Ms. B is ***interested*** in what Dana is saying and is willing to be ***patient*** as Dana formulates her answer. **[Int]**
30:19	Ghee	Who just touched me? [Ghee leans in, looks towards Dana, and smirks. Ms. B sits up in her chair and looks around.]	
30:24	Lenny	So we gonna change it?	

30:27	Ghee	Yea, I already changed mine! [Ghee does a little dance move in his seat.]	
30:28	Devin	I changed mine! [Ghee looks at Ms. B]	Ghee's tone of voice conveys confidence in his answer.
30:32	Lenny	What you, what we changing?	It appears that Lenny is ***confused*** about what Ghee is changing.
30:34	Dana	[Ms. B] I don't get that!	Dana also appears ***confused.***
30:36	Ms. B	What did you just say, Lenny? Hold on, let him, Lenny just said something. We didn't hear him. [Ms. B leans in and smiles] We talked right over him. What did you say Lenny?	Ms. B does not acknowledge Dana. Ms. B ***shows interest*** in Lenny's idea. Ms. B also says, "we talked right over him", possibly as a reminder for students to practice peer-to-peer ***respect***. **[Int, Endis, Ptp]**
30:45	Ghee	He said why ain't we just figured out all of them?	
30:47	Ms. B	So does anyone want to answer his question? [Ms. B is shuffling papers on the desk in front of her. She does not look at the students when she speaks.]	Ms. B is ***encouraging student discourse*** when she asks if any of the students are going to answer Lenny's question. **[Endis]**
30:50	Ghee	I just figured out all of them for ten hours.	
30:53	Lenny	No I'm talking about at first before. [camera zooms in on Devin]	
30:56	Ghee	Because we we we went with six hours trying to get out it easier. [Ghee shrugs his shoulders] We didn't want to do work!	Ghee makes an honest confession that his group only wanted to do the minimal amount of work to solve the problem.
31:02	Ms. B	Oh you didn't want to do work!? [Ms. B looks at Ghee, leans toward him and raises her eyebrows. Her tone of voice is calm, but her words are deliberate. She points at herself.] In my class, you didn't want to do work? [Ms. B has lowered her voice to a near	Ms. B appears ***surprised*** by Ghee's admission to not want to do any work. Her language and body language (referring to herself) may suggest that Ms. B feels personally ***insulted*** when she says, "in my class, you didn't want

		whisper and she is smiling as she talks to Ghee.]	to do work?"
31:07	Dana	[Ms. B]? I was…[inaudible]	
31:08	Ms. B	You can't, you can't even do the work today, you usually do the work, just today is not your day? [Ms. B shakes her head and smirks as she looks at Ghee.]	It is clear that Ms. B is ***disappointed*** by Ghee's suggestion that he does not want to work in her class. **[Dis-b]** Ms. B's tone of voice and language may be inferred to convey ***sarcasm*** toward Ghee when she says, "Today is just not your day?".
31:15	Ghee	No I'm talking about we didn't want to do like, like so much…we trying to get out of it easy. [Ghee gestures towards himself.]	Ghee appears to be defending his group for "trying to get out of it easy".
31:20	Devin	Yea we don't want so much work…	
31:23	Ms. B	Let me tell you something, when it comes to money, don't you want to give the [Ms. B's eyebrows are raised and she leans in towards the students, emphasizing her words with hand gestures]…. You want to see exactly what you doing! [Ms. B nods her head.] Ms. B points at the table with her finger.	Ms. B appears to be trying to reengage the students with the mathematics by ***appealing to their interest*** in money. **[Enthus]**
31:24	Ghee	Old Sols for six hours, he was still less…Old….	
31:29	Shawn	[inaudible]	
31:31	Lenny	For six hours, now you want to tell us.	
31:34	Ghee	Nah, not will hear all that! [Ghee is pushing buttons on his calculator, leaning back in his chair. Lenny leans back in his chair and smiles.]	
31:36	Shawn	Now you gonna tell us after I've been saying it. Now we hearing it. That's your problem. [or] Shut your pie hole. [Inaudible what Shawn is saying to Ghee. Listened to this from both the still and Lina's camera.] [Shawn is smiling.] [Ms. B stands up and leans on the desk.]	It appears that Shawn is joking with Ghee because he changed his answer.
31:41	Ghee	Yo son if he was three o nine for six hours, he was still too much money. He	

		was fifteen dollars less. [Ghee leans in towards Lenny and puts is calculator down.]	
31:47	Lenny	So we gonna go with the ten?	
31:49	Ghee	Yea, we gonna go with the ten homie, homie, homie, homie. [Ghee sticks his tongue out and wiggles in his seat, then he holds his paper up for Shawn to see.]	Ghee is acting silly.
31:51	Ms. B	[Ms. B walks around the table and leans down toward Dana. Her tone of voice is quiet and serious.] What two variables….?	It may be inferred that Ms. B intervenes privately with Dana because she recognizes that Dana's emotions are interfering with her mathematical engagement.
31:51	Shawn	So we gonna try to change your um…all of them to which one?	
31:53	Ghee	I already changed mine exactly, look.	
31:56	Ms. B	Give me your thought your decision. [Ms. B is speaking to Dana. She does not acknowledge Ghee.]	It appears that Ms. B is ***interested*** in Dana's mathematical ideas. **[Int, Endis]**
31:58	Dana	Like what?	
32:02	Ghee	I ain't changed this on at all.	
32:03	Ms. B	[Ms. B is pointing to Dana's paper] Okay, let Y be the total cost right….	Ms. B leads Dana in a mathematical discussion. **[Endis]**
32:07	Dana	Yes. [Dana is looking at her paper, chewing on her thumb.]	
32:08	Ms. B	[Ms. B is leaning down and pointing at Dana's paper, she is taking quietly directly to Dana]...and the X be the number of hours worked. So you just…will either one of these…will Y will it be…which one will change something, will it be the total cost of something, or the number of hours worked?	Ms. B appears ***curious*** about Dana's mathematical reasoning about which variable is the independent variable and which variable is the dependent variable. **[Int, Endis]**
32:22	Dana	[Dana looks up at Ms. B] Number of hours worked? [This episode ends when the camera pans to another table.]	Dana presents correct reasoning.
End TSMIE [B.02.a.06]			

Discussion of TSMIE [B.02.a.06]

This episode begins when Ms. B sits down at a desk next to Devin and appears curious about what Ghee was showing Lenny in his notebook. Ms. B appears excited that Ghee was referencing his notebook, perhaps because this is evidence that the Ghee is exhibiting autonomy, a characteristic that Ms. B appears to support.

When Ms. B asks the students if they have figured out which would be the best DJ, the students respond that Solidus Sounds would be the best choice yet they do not back their ideas up with mathematical reasoning. It appears that Ghee, Shawn and Lenny are attempting to develop a response for Ms. B's question, "how did you come up with that?" when Devin asks, "why can't it be ten hours instead of six?" Ms. B appears excited by Devin's question possibly for two reasons. First, it is inferred that Ms. B is excited because for ten hours, the lowest cost for a DJ would be Light Plastic, which is different than the best deal for six hours, Solidus Sounds. Also, Devin typically does not engage in mathematical discussions, so it is likely that Ms. B is proud of Devin for speaking up.

When Lenny and Ghee dismiss Devin's idea, Ms. B gets excited and remarks that she loves to party and would "go for fifteen hours" if possible. Lenny, Shawn and Ghee find Ms. B's suggestion humorous, they are laughing and smiling. Ms. B validates Devin's question by saying, "that's a good question.", and she appears to try and reengage the students asking, "So can you show me if it costs more money?"

This intervention is effective because Ghee calculates the cost for each DJ for ten hours and determines that Light Plastic will be the least expensive DJ. Lenny wonders aloud why the group didn't just figure the rates for all three DJs for "all of them" from

the beginning. Ghee admits to Ms. B that the group only calculated the DJ rates for six hours because they didn't want to work, "…we we we went with six hours trying to get out it easier". Ms. B appears to take personal offense to this when she points to herself and says, "In my class, you didn't want to do work?" She exhibits disappointment in Ghee's confession and Ghee appears to try to defend himself, "…we trying to get out of it easy." It is evident that he recognizes Ms. B's disappointment and that he values her opinion.

Approximately four minutes into the episode, Dana tells Ms. B that she was not working with the group when she says, "No, I wasn't with them!" Dana appears to be angry, frowning and resting her head on her hand. It is inferred that Ms. B attempt to engage Dana in the group discussion by expressing interest in Dana's solution. It is inferred that Dana was calculating the rates for each of the three DJs at each successive hour. Dana's mood does not appear to change after this intervention and during the final minute of this episode, Ms. B walks around the table to stand next to Dana. It is inferred that Ms. B intervenes with Dana because she recognizes Dana's anger may be interfering with her engagement. It is inferred that Ms. B tries to foster Dana's mathematical engagement by leading her through the beginning stages of writing an algebraic representation of the DJ rate plan.

Immediately subsequent to this episode, the students appear to remain engaged with the mathematics after Ms. B walks away because Shawn says to Ghee "What did y'all get for that?...for the first equation." Dana and Lenny and Devin are writing on their papers.

Begin TSMIE [B.02.a.07] 32:20 – 33:50 (L)			
Participants: Rita, Ms. B, Emmy Rita initiates this episode when she asks Ms. B for help.			
32:24		[The camera pans to Shay, he is leaning on his table, looking at his calculator and writing on his paper. Ms. B is present at the table, but the camera is focused at Shay.]	
32:26	Rita	[inaudible] [Ms. B] we need your help.	
32:37	Ms. B	You need my help? [Ms. B's tone of voice is inquisitive.]	Ms. B appears ***surprised*** by Rita's appeal for her help.
32:38	Rita	I I don't know…	
32:39	Ms. B	I don't know I'm much help, but okay? Let me see.	Ms. B appears to express ***doubt*** in her ability to help the students, possibly trying to promote student ***autonomy.***
32:46	Rita	[Ms. B is standing between Frank and Emerald, but is only visible from the waist down. She is holding a piece of chart paper.] Umm about the graph, he did the window, but I don't understand how to do it.	
32:51	Ms. B	And he did it already?	
32:51	Rita	I don't know whether he did it, but I don't understand.	
32:56	Ms. B	So did you, what are you doing [Shay]? [Shay is leaning on his desk, looking at his calculator and writing on his paper.]	Ms. B expresses ***interest*** in Shay's calculations. **[Int]**
32:58	Rita	[Shay], he still, he he did his table, [Rita points toward Shay's paper] but I'm talking about like how this says to graph the equation and plot all three in the same window.	
33:10	Ms. B	So what does it mean to graph all three equations in the same window? [Ms. B raises her voice.]	Ms. B appears to understand Rita's impasse and ***expresses interest*** in Rita's reasoning about graphing three equations in one window on the graphing calculator. **[Int, reasoning]**

33:15	Rita	To graph all of them in the same, umm, graph paper.	It is likely that Rita means that all three equations need to be graphed on the same graph.
33:17	Ms. B	Okay.	
33:20	Rita	But, I, I did the hours and then…..it cost more money.	Rita's tone of voice may be interpreted to convey ***uncertainty***. It may be inferred that Rita was able to easily plot (the x-values) which range from one to twelve for the tables that she and Shay generated. But, she is experiencing difficulty plotting the dollar amounts (the y-values) because they increase into the hundreds, at a much faster rate than the hours.
33:27	Ms. B	So what do you guys think?	Ms. B poses the question back to the students in the group, conveying ***interest*** in their ideas. Ms. B also ***encourages student discourse*** by asking the other students to comment. **[Int, Endis]**
33:28	Emmy	You have to go really really high.	
33:30	Ms. B	You have to go really really high, so then when you go really really high, then what might you want to do? [The camera shows Ms. B leaning on the empty desk between Emmy and Frank. She is looking at Rita.]	Ms. B repeats Emmy's suggestion then expresses ***curiosity*** about the students' next step. **[Int]**
33:37	Emmy	Increase the multiple that it was talking about, but I tried to do it and like… [Ms. B begins to nod her head, then the camera pans back to Shay's paper.]	
33:42	Ms. B	Then what happened?	Ms. B expresses curiosity about Emmy and Rita's procedure of setting up the graphing calculator. **[Int]**
33:45	Rita	I started with twenty but ….	
33:47	Ms. B	So then you just increased the number so… [The camera pans to Ghee's table.]	

The camera is focused on Ghee's table for approximately one minute before returning to Rita's table.			
34:36	Ms. B	[Ms. B is leaning on the desk in between Emmy and Frank, she is looking at Shay and smiling. Ms. B points toward Rita.] So can you explain to Rita what to do?	It is inferred that Ms. B recognizes that Shay has completed the graph in his calculator. Ms. B ***recognizes Rita's impasse*** and she intervenes by ***encouraging student discourse*** when she asks Shay to help Rita. **[Rsi, Int]**
34:40	Shay	[Shay reaches to take the calculator from Frank' hands.] Look. The y-minimum gonna be one, the y-maximum gonna be twelve. The x scale gonna go by twos then the y- I mean that was the x's. [Shay looks at Rita, then he leans over and smiles as he looks at Frank' paper.]	Shay's tone of voice may be inferred to convey confidence in his mathematical understanding. He has assumed the "let me teach you" engagement structure.
34:57	Rita	So the y-minimum gonna be what? [Rita looks at Shay.]	
35:00	Shay	Y-minimum is sixty! [Shay raises his eyebrows as he reads from his calculator screen.] Then the y-maximum be seven twenty. Then the scale gonna go by sixties. [Ms. B walks behind Shay and away from the table.] And then the x-minimum gonna be one.	
35:13	Rita	Nah, I got that. I mean with the like x umm..	
35:16		Let me see it. [Rita hands her calculator to Shay.]	
35:18	Rita	I did everything you said, but…the last part.	
35:23	Shay	That's gonna be one. [Shay hands his calculator back to Rita.]	
35:25	Shay	And then graph…view your graph. [Ms. B returns to stand next to Shay.]	
35:30	Rita	Uhhhhh….[Rita shows her calculator to Shay.]	
35:35	Shay	Uhhhh….[Shay looks at Rita's calculator, he raises his eyebrows and smiles.]…your junk must be broken.	
35:38	Ms. B	[Ms. B rests her hand on the corner of	Ms. B expresses ***curiosity***

		Shay's desk. Her facial expressions cannot be determined because the camera does not show her face.] So why didn't it graph?	about why Rita's calculator did not show the graph. **[Int]**
35:42	Shay	Lemme see. [Shay reaches for Rita's calculator.] His graphed! [Shay shrugs his shoulders.]	Shay and Rita are engaged in trying to figure out why Rita's calculator is not correctly displaying the graph.
35:49	Rita	Redo it!	
35:54	Shay	[Ms. B leans down to look at the calculator screen.] How you clear it? Clear it. [Shay holds his calculator and Rita's calculator side by side and looks at the screens.]	
36:00	Shay	We gotta graph it? [Shay sits up in his chair and looks over towards Ghee's table then looks at Ms. B. Shay asks Ms. B something that is inaudible. Ms. B points at the table.]	
36:04	Frank	Graph it on here?	
36:05	Ms. B	Right. [Ms. B leans towards Frank] 'Cause that's what it said you had to do. Graph it on paper. [Ms. B gestures to the large piece of graph paper on the table.] You might want to graph it on here, [Ms. B points to Rita's paper] on paper first, then on here.	Ms. B suggests that the students make a draft of a graph on the smaller graph paper before putting the graph on the large paper to present.
36:12	Rita	Yeah, that's what I was tryin' to do.	
36:14	Ms. B	Okay. [Camera pans to Ghee's table.]	
End TSMIE [B.02.a.07]			

Discussion of TSMIE [B.02.a.07]

In this episode, Ms. B expresses surprises when Rita asks for her help to set up a graph of three DJ rates on her graphing calculator. It appears that Rita and Emmy are experiencing impasse as they try to determine the range for the y-axis on the graph. Emmy says, "You have to go really really high", likely referring to the dollar amounts. It is possible that the difficulty stems from the range of the hours (x values) is from one to twelve while the dollar amounts range from sixty into the thousands (this group has made some mathematical errors, so the costs for their DJs go into the thousands).

Ms. B does not address the arithmetical error. It is inferred that Ms. B realizes that Shay has already completed his graph on the calculator she asks him to help Rita, "So can you explain to Rita what to do?"

This interaction shows Ms. B encouraging students to work together and encouraging student autonomy as she silently observes the students' interaction. Shay explains the process for setting up the graph on the calculator.

Immediately subsequent to this episode, the students appear to be engaged with the mathematics as they make a rough draft of the graphs on grid paper to be shared later with the whole class.

Begin TSMIE [B.02.a.08] 36:36 – 44:00 (L)			
Participants: Ghee, Ms. B, Lenny, Shawn, Devin, Dana Immediately prior to the start of this episode, Ghee leans across the table towards Shawn and yells. "Forty times ten plus a hundred". Emphasizing each word with a tap of his pen on the table. Shawn responds by saying, "Son, you told me a hundred times forty!"			
36:36	Ghee	Not again.	It appears that Ms. B is talking to Ghee in response to his complaint, "not again". Ms. B appears ***curious*** about Ghee's comment. **[Int]**
36:36	Ms. B	[Ms. B approaches the table and stands next to Devin] What's going on? What? What?	
36:39	Ghee	Nothing! [Ghee looks up at Ms. B.]	
36:38	Ms. B	What happened? [Ms. B's facial expression is out of camera range.]	
36:38	Ghee	Nothing! [Ghee looks down at Devin's desk.]	
36:40	Ms. B	Is it cause I brought the paper? [Ms. B smiles at Ghee.] Okay, so did we get to the point where we're graphing it? Did we come up with, umm, who we're gonna pick to DJ this? [Ms. B looks from Ghee to Lenny and leans on the table.]	
36:50	Lenny	Yea.	
36:54	Ghee	Light Plastic.	
36:54	Ms. B	Who? [Ms. B looks at Devin.]	
36:54	Devin	Light Plastic	
36:54	Ghee	Light Plastic	
36:58	Ms. B	Light plastic. [Ms. B nods her head.] Okay.	
	Shawn	[Shawn looks at Ghee and asks:] Forty times ten, forty times ten plus a hundred? [Ghee nods]	
36:58	Lenny	So we could start at an earlier time and a lot, a lot of people gonna come.	
37:06	Ghee	[Ghee bangs hands on desk] Son! [Ghee he looks at Shawn's paper.]	Ghee appears to express ***anger*** at Shawn for writing the equation with the "hundred" last.

		It's supposed to be a hundred first off son.	
37:11	Shawn	Yea, that's how I *was* doing it right. Yea!	
37:13	Ghee	Son, you supposed to do it like this, look. [Ghee's tone of voice softens as he uncaps his marker and reaches for Shawn's paper.]	
37:15	Shawn	Why you confusing me?	
37:16	Lenny	Like this right here.	
37:20	Shawn	But how, but how am I supposed to be adding a hundred though?	
37:20	Ghee	Look, look! I got this look. [Ghee writes on Shawn's paper.] A hundred plus forty times *ten!*	
37:31	Dana	[Ms. B] how we supposed to…?	
37:31	Ms. B	Can you explain to be exactly what you wrote down, the that equation? [The camera pans in to the equation that Ghee just wrote on Shawn's paper. Y=100 + 40 x 10 (x)]	Ms. B expresses ***interest*** in Ghee's reasoning about the equation that he wrote. **[Int, rs]**
37:37	Dana	Why would you…never mind?	It appears that Dana has a question, but is frustrated and disengages from the conversation, "never mind".
37:37	Lenny	A hundred, a hundred when umm….	
	Ms. B	What? [Dana is looking at Ms. B.]	Ms. B appears to recognize Dana's negative emotion when she hesitated to ask her question and Ms. B ***encourages*** her to share her thoughts. **[Int, Endis]**
37:40	Dana	Why don't you add, why don't you put forty, umm, times ten and then put the variable and then you add a hundred?	It may be inferred that Dana is feeling ***confused*** about why Ghee did not write the equation as y = 40(10) + 100.
37:48	Ghee	[Ghee imitates mustache with his marker, the marker falls] Cause you remember when we had to do last time? She said, put fifty cent plus, plus five dollars. [Ghee does not look at Dana when he answers her. He is scowling.]	It is inferred that Ghee is recalling a strategy from a different lesson, "remember …last time".
37:55	Dana	No it's not. It's five dollars plus fifty	Dana appears to be ***angry***

		cent. [Dana does not look at Ghee. She is scouling.]	with Ghee.
37:57	Ghee	And what that, what is that? [Ghee is shouting as he points to the paper where he wrote his equation. He is looking around wildly.] Power rangers always trying to kill somebody. [Ghee is flailing his arms. He pushes back in his seat.]	Ghee's tone of voice and gestures convey that he may be ***angry*** at Dana's challenge to his mathematical thinking.
38:00	Lenny	That's a hundred plus forty.	
38:04	Ms. B	So can you explain what that equation means? [Ms. B is out of camera range. Her tone of voice is calm.] So what does the equation mean? [Ghee] you drew it, what does that equation mean?	Ms. B intervenes, and it is inferred that she recognizes the students veering off task. She poses a mathematical question, expressing interest in Ghee's reasoning. **[Rsa, Endis, Int, rs]**
38:12	Shawn	I'm about to [inaudible]	
38:16	Ms. B	So you just picked any numbers? [Ghee has his forehead down on the desk, then he shakes his head "no" and looks at Ms. B.]	It may be inferred that Ms. B has gotten Ghee's attention when she asks, "so you just picked any numbers?" because he looks up at her. **[Int, Endis]**
38:18	Lenny	That was four g's.	
38:20	Ms. B	So how did you get "a hundred plus forty times ten X"? [Ms. B is pointing at the paper. Ghee is rocking in his chair.]	Ghee may be ***trying to calm himself*** down by rocking in his chair, he may still be ***agitated*** by Dana's challenge. Ms. B appears ***interested*** in understanding Ghee's reasoning for writing the equation. **[Int, rs, rp]**
38:27	Ghee	Cause X is the, uhh, [Ghee is leaning forward in his chair, he looks up at Ms. B] is the number of hours.	Ghee is reengaged with the mathematics and appears to have ***calmed*** down.
38:30	Ms. B	Ok.	
38:33	Ghee	Y is…[Ghee looks at the paper, his eyebrows are raised.]	

38:33	Ghee	Uhh uhhh, forty [Ghee glances at his paper, then looks back to Ms. B. He is rocking side to side in his chair.] is the amount of uhhhh…money.	
38:37	Devin	What's it gonna be?	
38:40	Ms. B	What? Forty is what?	
38:42	Ghee and Devin	Amount of money.	
38:44	Devin	For each hours.	
38:44	Ghee	For the hours.	
38:45	Ms. B	Okay.	Ms. B is showing ***interest*** in Ghee's explanation as she says, "ok". **[Int]**
38:48	Ghee	Then a hundred is added. Well is what you pay first. [Ghee looks up at Ms. B.]	
38:51	Ms. B	And what is the ten? [Ms. B leans forward and points to the equation.]	Ms. B expresses ***curiosity*** about Ghee's understanding of the meaning of "ten". **[Int, rs]**
38:53	Ghee	The hours! [Ghee looks at Ms. B]	Ghee's tone of voice may be inferred to show that he is confident in his idea.
38:53	Devin	That's the hours. [Devin looks at Ms. B]	
38:56	Ms. B	And what is the X? [Ms. B points to the paper.]	Ms. B expresses ***curiosity*** about Ghee's understanding of the meaning of "X". **[Int, rs]**
38:58	Devin	That's the….	
38:58	Ghee	That's the variable! [Ghee looks up at Ms. B] We just put X so you wouldn't be like, what you just said. [Ghee looks at Ms. B]	Ghee's tone of voice may be interpreted to convey that Ms. B asked an "obvious question".
39:03	Ms. B	Oh, I see what you saying, okay. [Ms. B raises her voice and scratches her head. Ms. B nods her head.] But you know what, in, for the future. Bring that over here. [Ms. B gestures towards Shawn's paper.] I see I understand what you saying, so you can say it to me, but…let me see your marker. [Ms. B takes a marker from Devin] Just write the equation right at	Ms. B's tone of voice and language may convey that she now understands Ghee's equation. She ***validates*** his reasoning, but corrects the syntax of the equation. **[Val]**

		the top of that cause you are confusing me, I thought you were saying times Ten X [Ms. B points to the equation on Shawn's paper, the equation reads y=100 + 40 * 10 x]. But you are just saying the X, you are using 10 for that. [Ms. B leans down and draws an arrow from the x to the ten. So don't do that at all, don't put that at all, cause I thought you were saying. [Ms. B crosses out the x in the equation.] So what equation would you use for any number, [Ms. B writes "y=" on Shawn's paper, then holds the pen out in front of her] can you write that up there?	Ms. B asks the students to write a general equation for Light Plastic's rate. Ms. B expresses ***interest*** in the students' reasoning. **[Int, rs]**
39:31	Ghee	Me? [Lenny takes the marker she is holding.]	
39:31	Lenny	I can write it.	
39:31	Ms. B	I don't care who. [Ms. B leans on the desk, watching Lenny as he begins to write.]	
39:35	Lenny	So a hundred plus forty right, I'm stuck now.	
39:45	Ms. B	He is stuck he said.	Ms. B recognizes Lenny's impasse and intervenes by ***encouraging student discourse*** when she tells the other students, "he's stuck he said." **[Rsi, Endis]**
39:48	Ghee	[Ghee is resting his chin on his desk.] Times. [Ghee looks at Lenny] Times! Times! [Ghee raises his voice as he speaks.] Ten!	It may be inferred that Ghee is growing ***impatient*** with Lenny as Lenny hesitates to write anything on the paper.
39:52	Dana	[Ms. B] how they answering, how they doing the equation first and they didn't even answer the second half of the question? Cause they, they just put any answer, they didn't even think of like….	Dana appears ***unhappy*** as she complains about the boys doing the equation first.
39:53	Lenny	You said that right?	

39:55	Ghee	Times ten. [Lenny writes y = 100 + 40*10]	
39:58	Lenny	That is the same thing as this!	It appears that Lenny disagrees with the equation that he wrote, pointing out that it is the same equation as the one that they had written before. Lenny is correct because Ms. B had asked the students to write a general equation and
39:59	Ghee	No she said after that.	
40:00	Shawn	Son, for the last one you got four seventy-five.	
40:04	Lenny	There it is right there. [Ms. B leans in to look at what Lenny wrote.]	
40:07	Ms. B	So it's Y equals a hundred plus forty times ten?	
40:10	Ghee	Yea	
40:11	Ms. B	[Ms. B lowers her voice.] What do you think [Dana]? We're not in a good mood today? [Dana has her chin resting in her hand, she is looking away from Ms. B.]	Ms. B ***recognizes that Dana's affect*** is interfering with her mathematical progress. Ms. B acknowledges Dana's bad mood, her tone of voice may be interpreted to sound ***sympathetic*** towards Dana. **[Rsa]**
40:14	Dana	[Dana shakes her head "no"]	
40:16	Shawn	She don't want to be in this.	Shawn refers to Dana not wanting to participate in the group.
40:16	Ghee	[Ghee looks at Dana.] See! Show me what you got little momma.	
40:18	Ms. B	Do you want to move to another group?	Ms. B's tone of voice has ***no emotion,*** yet she ***acknowledges the possible source of Dana's mood*** by suggesting that she move to a different group.

40:20	Dana	[Dana has her mouth resting on her hand, so she is barely audible. She is not looking at Ms. B.] No.	
40:20	Ms. B	Just not a good day?	Ms. B ***recognizes Dana's negative affect***. **[Rsa]**
40:24	Ghee	(inaudible response to Lenny waving his hand)	
40:25	Ms. B	[Ms. B raises her tone of voice again as she points to the equation that Lenny wrote.] Okay so, when we are talking about…an equation, how many variables do we need?	Ms. B changes the focus back to the mathematics when she asks the students a specific question, "how many variables" do we need?
40:31	Lenny	[Lenny holds up two fingers] [The camera is at Lenny's back, showing Ghee with his chin resting on the desk.]	Lenny is correct.
40:31	Ms. B	Two, where are your two variables?	Ms. B ***validates*** Lenny's idea by following it up with a question. **[Val]**
40:35	Lenny	Y and X	
40:36	Ms. B	Where is it in this equation? [Ms. B points her finger to the equation.] For any number, I just want to know for any number of hours. [Ms. B's tone of voice is raised.]	It may be inferred that Ms. B is becoming ***eager*** for the students to write a two-variable equation.
40:37	Ghee	There and there, there and there. There there there there.	
40:40	Ms. B	So what is that? [Ms. B leans forward on the desk.] [Devin stands up and leans forward on his desk.]	Ms. B's body language and tone of voice suggest that she is eager for the students to solve the problem. Devin's body language suggests that he is engaged, as he mimicks Ms. B's posture, leaning in to the table.
40:42	Ghee	Forty and ten.	
40:44	Dana	So how ya'll gonna, so what does that ten for, if that is a variable, where the variable at? [Dana leans forward and points to the equation.]	Dana appears ***curious*** about where the variable is and her tone of voice conveys that she is ***angry.***
40:51	Ghee	Forty and ten.	

40:51	Lenny	You said it's forty and ten.	
40:54	Dana	Won't you, [Dana writes on the equation] won't you have to do that?	
40:55	Devin	She said don't do that!	Devin appears to disagree with what Dana is writing. It may be inferred that he is referring to Ms. B.
40:56	Ms. B	I didn't say…	Ms. B's tone of voice is ***defensive.***
40:58	Lenny	She never say that.	
40:58	Ms. B	Okay! So what do you mean just do this, can you write it down here? Because they were just saying that that ten is for the X. [Ms. B points at the equation that Lenny wrote.] So I was, I was just getting confused. What, what, what, what equation are you working with?	Ms. B is ***confused*** about how the students are interpreting the algebraic equation. **[Con, reasoning]**
41:08	Lenny	Jump in, jump in.	
41:11	Ms. B	For any number, for any number of hours.	Ms. B continues to express ***interest*** in the students writing an equation. **[Int]**
41:15	Shawn	Ya'll don't even get along today, ya'll used to be snuggle up. Now ya'll, I don't even know. [Lenny gestures back and forth between Dana and Ghee. Both Dana and Ghee are looking down, frowning.]	It may be inferred that Shawn is referring to a possible social conflict between Dana and Ghee, "ya'll used to be snuggle up". This may be the source of Dana's anger and disengagement.
41:30	Dana	[Camera zooms to show Dana writing an equation. Her chin is resting in her hand. She mumbles as she speaks.] I didn't do that what they did. I didn't do what they did.	
41:33	Ms. B	Okay. [Ms. B is off camera, her tone of voice is upbeat.]	Ms. B's tone of voice may be interpreted to provide ***encouragement*** to Dana. **[Raa]**
41:35	Dana	I didn't do it. I did this. [Dana is looking at her paper, her cheek is resting on her hand. She copies an equation from her calculator to the paper and looks down as she talks. Ghee has his head resting on the desk, turned away from Dana.]	

41:42	Dana	And they said use any variables, so I used J. [Dana looks at her paper as she speaks. Ghee is resting his head on the desk, he is looking away from Dana.]	
41:46	Lenny	Now it's confusing.	
41:49	Ghee	Whoa, how do you party for ten hours? [Ghee is resting his chin on the desk. He is smiling at Lenny.]	Ghee appears to be ignoring Dana.
41:53	Ms. B	Okay, so can you explain the equation to the group?	Nobody responds to Ghee, and Ms. B ***encourages*** Dana to share her different approach with her group. **[Int, Endis, Raa]**
41:55	Lenny	Come on, do you want to do that all day?	
41:58	Ms. B	Did you have a different way? [Ms. B's facial expression is off camera.] Let's listen to what [Dana] is going to say. [Ms. B puts the palm of her hand on Ghee's desk, her voice is raised over the students talking.] Cause [Dana] has a thought, two different thoughts.	Ms. B appears to acknowledge that there may be ***more than one solution to the problem*** when she says, "did you have a different way?" and "Two different thoughts". Yet, she also ***encourages peer-to-peer respect*** when she says, "Let's listen to what [Dana] is going to say." **[Raa, Endis, Int, Ptp]**
42:03	Dana	Yea I don't know if I'm right. [Ghee, Shawn and Lenny are all talking at once.]	Dana appears uncertain of her answer.
42:06	Ms. B	Are we listening? [Ms. B leans in towards Lenny and Shawn and furrows her eyebrows.] Go ahead Dana, so there is two different thoughts. [Ms. B gestures towards Ghee and Lenny.]	Ms. B appears ***angered*** by Shawn and Lenny's behavior as she chastises them and tells them to listen. Ms. B expresses interest in Dana's alternate approach to writing the equation. **[Dis-b, Ptp, Int, Raa]**
42:13	Dana	I did Y equals thirty times ten j plus one hundred seventy five. [Dana looks down at her paper as she reads the equation then looks at Ms. B. Her voice trails off at the end.]	
42:21	Lenny	What happened?	

42:23	Shawn	She said thirty times ten [Lenny reaches across the table and takes Dana's paper to read the equation.] plus seventy five.	
42:23	Ghee	…said ten J plus one seventy five.	
42:32	Shawn	What is that, what is that?	
42:34	Dana	Any variable, she said we can use any variable.	
42:38	Shawn	Stop talking to me like I'm stupid.	It may be inferred that Shawn was ***offended*** by Dana's tone.
42:41	Lenny	The umm, one seventy-five is supposed to go first! [Lenny leans across the table towards Dana.]	Lenny appears ***angry*** that Dana placed the one hundred seventy five at the end of the equation instead of at the beginning.
42:43	Dana	Well I didn't do it like that, I multiplied. Cause if I was gonna do it, I was gonna do it [inaudible].	Dana's tone of voice may be inferred to show that she is ***angry*** with Lenny as he tried to correct her.
42:50	Lenny	All right, all right, we'll do it the way you do it. I don't have no problem with that.	It is inferred that Lenny is trying to stay out of trouble with Dana. He agrees to "do it" her way.
42:55	Ms. B	Okay so.. [Ms. B stands up in her chair and leans down on the desk with her elbows and looks at the students.]	Ms. B appears to recognize that the social conflict between Lenny and Dana is interfering with the students' engagement with the mathematics. Her body language suggests that she is ***eager*** for the students to make progress.
42:55	Dana	Cause I…	
42:57	Lenny	I don't got NO problem with it.	Lenny expresses anger through sarcasm perhaps?
43:01	Ms. B	So now…	
43:01	Ghee	I quit. [Ghee has his chin resting on his desk.]	Ghee appears ***frustrated*** when he exclaims that he wants to "quit".
43:02	Ms. B	You quit, are we giving up in here, no way. [Ms. B raises her voice and slaps her hand on the desk.]	Ms. B demonstrates ***disappointment*** in Ghee's defeat and remarks that giving up is not an option, "no way". **[Dis-b]**
43:04	Ghee	Didn't you hear what she just said.	

43:05	Ms. B	What she just say? What she say? [Ms. B looks at Ghee and furrows her eyebrows.] [Ms. B raises her voice.] I missed it! [Ghee leans forward, with his face resting on his fists, he is smiling.] Okay so now you guys made a decision, you guys said who are you going to pick? [Devin points at Lenny's paper.]	Ms. B intervenes, asking the students to name the DJ they would choose. **[Int, Endis]**
43:15	Ghee	Show me what you got?	
43:18	Lenny	Umm….Light Plastic!	
43:20	Ms. B	And why are you picking Light Plastic. [Lenny rocks back and forth in his seat.]	Ms. B expresses ***interest*** in the reasoning behind the students' choice. **[Int, rs; Endis]**
43:21	Devin	Cause that is more cheap, cause…	Devin and Lenny engage in a discussion about the idea that the DJ may play past the time that most people leave the party.
43:22	Lenny	For ten hours we can party for how much?	
43:25	Devin	You get more hours and you don't even get.	
43:28	Lenny	I would rather be dancing.	
43:32	Devin	[Devin looks down at his notebook and he plays with his pen while he speaks to Lenny.] No would wouldn't be dancing for the whole ten hours, so you would start leaving. By that time people gone and you wouldn't have to pay for all the hours because you would say he didn't play for ten hours. You would say he played for six hours.	
43:49	Lenny	The problem is gonna be wacked if we bought for ten hours he gonna be playing the same songs.	
43:50	Ms. B	So, okay, can you graph all three of these equations in the same window? Can you do that now? Can you graph this for me? I'll, I'll be back, can you graph this for me? I need to see all three of these graphs. [Ms. B points to a paper on the desk] [camera turns to Shay who has his hand raised, the following dialogue was captured from the still camera.]	Ms. B may recognize that the boys are getting distracted by the context of the problem and are not focused on graphing the linear equations. She intervenes, redirecting the students to graph the equations using the

			graphing calculator. Before she leaves, Ms. B alerts the students that she will return to check on their progress. **[Int, rpEndis]**
43:34	Lenny	Graph three equations…Where? [Lenny points to his paper and looks at Ms. B as she is walking away.] On here?	
43:36 (Still)	Ms. B	[Ms. B furrows her brow.] Graph it on the graphing calculator it says. [Ms. B walks away toward Shay's table.]	
End TSMIE [B.02.a.08]			

Discussion of TSMIE [B.02.a.08]

This episode begins when Ms. B approaches the group and stands next to Devin. Ms. B is distributing large graph paper to each group likely for each group to present their graphs to the class. It appears that Ghee is disappointed by Ms. B's presence when he says, "Not again". Ms. B briefly acknowledges his complaint then she asks the students if they have graphed the equation. She also asks them if they have decided on the best DJ. This is the second time that Ms. B has posed this question to this group.

Lenny, Ghee and Dana disagree on how to write a linear equation and Ms. B engages in a discussion with them. Ghee, Lenny and Dana each express anger in varying degrees and at various times during this discussion. Ms. B recognizes that Dana has a different approach to writing the equation and encourages the boys to listen to her ideas, "Let's listen to what [Dana] is going to say. [Ms. B puts the palm of her hand on Ghee's desk, her voice is raised over the students talking.] Cause [Dana] has a thought, two different thoughts." The students are teetering on the edge of becoming disengaged with the discussion about how to write the equation.

Dana has been very quiet during this class and is withdrawn today as compared to

her typical level of engagement. It appears that Ms. B recognizes this and is sympathetic towards Dana having a bad day. Although Dana does not offer an explanation for her bad mood, Shawn says "She don't want to be in this [group]. Ms. B then offers for Dana to sit with a different group, but Dana declines Ms. B's offer.

At the end of the episode, Ms. B appears to recognize that Devin and Lenny are becoming drawn into the debate about the sensible amount of time that a DJ should be hired for. This may distract the students from the mathematics, so Ms. B attempts to redirect their attention, telling them to graph the equations in the calculator. She mentions that she will be back to check their work on the calculator. "I'll, I'll be back, can you graph this for me? I need to see all three of these graphs."

Immediately subsequent to Ms. B's departure, it appears that Ghee, Dana and Shawn are not engaged with the mathematics. While Lenny appears to do computations on his calculator, Ghee and Shawn try to console Dana who is now crying. Shawn appears concerned about Dana and tells Ghee that Dana is in a bad mood because Ghee and Dana are not agreeing on anything today. Ghee puts his arm around Dana. These three students do not reengage with the mathematics for the remainder of the period

Begin TSMIE [B.02.a.09] 44:10 – 46:50 (L)			
Participants: Ms. B, Shay, Rita Ms. B initiates this interaction when she approaches Shay's table and says, "I'm back, yes?" It may be inferred that Ms. B is curious about how Shay and Rita have progressed since the last interaction, which ended approximately eight minutes earlier. In the previous interaction, Ms. B had instructed the students in Shay's group to graph their equations on paper.			
44:10	Ms. B	I'm back, yes. What you turning up?	Ms. B addresses Shay and Rita, her tone of voice suggests that she is ***eager*** to see their progress.
44:15	Shay	Look I got it wrong, those supposed to be…[The camera pans back to Ghee's table for a few seconds then returns]	
44:22	Ms. B	How you know that?	Ms. B's question conveys ***curiosity*** about Shay's reasoning. **[Int]**
44:23	Shay	Because you can't go across, like you can't go to eight and then go across. [Shay is holding his paper up, he looks at Ms. B. Ms. B is looking at Shay's paper, she is standing between Shay and Ruth, her hand is on her hip.]	Ms. B's body language suggests that she is ***interested*** in understanding Shay's question. **[Int]**
44:28	Ms. B	Well did you look on your, umm, paper.	
44:30	Shay	What do you mean?	
44:31	Ms. B	[Ms. B's facial expressions are out of camera range.] What is the independent variable? Where is your umm…	
44:34	Rita	The Y is the independent variable	
44:36	Ms. B	Y is independent and Y represents what?	Ms. B's tone of voice and language may suggest that she ***validates*** Rita's statement, "The Y is the independent variable". **[Val]**
44:41	Rita	Y represents the the…[Rita taps on her paper with her pen.]	
44:41	Shay	The number of hours.	
44:42	Rita	The the the number of hours. [Rita bites her pen and turns around, looking towards the door.]	
44:53	Ms. B	So which on is the Y, can you label X and	It may be inferred that Ms.

		Y for me? [Ms. B speaks quickly and points towards Rita's paper. Rita writes a Y above the left hand column, representing hours and X above the right hand column, representing cost.] Raheem what do you think about this, [Emmy] what do you think about this?	B recognizes that Shay and Rita may be incorrect. Ms. B intervenes by expressing curiosity about Raheem and Emmy's thoughts. **[Rsi, Int, Endis]**
45:07	Rita	That looks like it's supposed to be the other way around though, cause it's normally X and Y. [Rita taps on Shay's paper with her pen. Shay is biting his nail and looks up at Ms. B. Ms. B's face is out of camera range.]	Emmy and Raheem do not reply to Ms. B's question but Rita corrects her mistake and Shay agrees.
45:14	Shay	YUP YUP YUP YUP YUP!	
45:18	Ms. B	Why you say Yup? You just agreeing with her or something…?	Ms. B expresses ***curiosity*** about why Shay agrees with Rita. **[Int, reasoning]**
45:23	Shay	Uh….She right! [Shay is writing on his paper, looking down as he answers Ms. B.]	
45:27	Ms. B	So how do you know that…[inaudible]	Ms. B does not accept Shay's simply agreeing with Rita, and she encourages Shay to explain his reasoning. **[Int, Endis]**
45:29	Shay	Well look, X and Y. [Shay points at his paper and raises his eyebrows. He does not look at Ms. B.]	Both Shay and Rita exhibit ***confidence*** in their reasoning because they each continue to write on their papers.
45:31	Rita	Cause Y depends on the X. The Y depends on the X. Yeah. [Ruth nods her head.]	
45:37	Ms. B	Okay, so now what is your question?	
45:43	Shay	How you supposed to do it like that? [Shay holds a paper up towards Ms. B and looks up at her.]	
45:44	Ms. B	What do you mean? I am not following your question.	Ms. B expresses confusion about Shay's reasoning. **[Con]**
45:51	Shay	Now how go to eight. [Shay pushes buttons on his calculator.] So now how I start from eight, to four twenty? If I don't know where eight at.	

46:04	Ms. B	So you going up first? [Ms. B points at Shay's graph. He has graphed the dollar amounts on the y-axis and the hours on the x-axis.]	Ms. B expresses interest in Shay's process for graphing points on the graph. When he asks Ms. B how to graph, she responds with a question, ***promoting Shay's autonomy***.
46:06	Shay	Don't you gotta go up and then across?	
46:09	Ms. B	How do you graph?	
46:09	Rita	Oh yeah he supposed to go, no you go umm, [Rita is pointing at Shay's paper.] Count like start of at one, no you start of at zero, zero.	
46:13	Shay	So I start over here. Zero.	
46:20	Rita	And then at one you know that it would be sixty dollars. So you're doing it right. You're doing it right because one hour, one hour is sixty dollars. Zero hours, you get no money.	
46:33	Shay	No that is the first one, I'm on the second one now. [Raheem is sitting to Shay's left. He looks on silently.]	
46:36	Rita	Yeah, one twenty.	
46:42	Shay	Oh yea, I see it, I see it.	
46:42	Rita	I'm seeing what you did. You did one, two, four, six, eight	
46:44	Shay	And then it's for the second one, it would be seven hundred.	
46:51	Ms. B	[Ms. B raises her voice and addresses the whole class.] At this time, I want us to end in um, not rush, get everything organized. Oh wait, [Ms. B turns off the lights.] You know what that means? You know what that means?	
End TSMIE [B.02.a.09]			

Discussion of TSMIE [B.02.a.09]

Ms. B initiates this interaction when she approaches Shay and Rita and says, "I'm back, yes." Rita and Shay appear to be experiencing impasse with graphing the equations. Ms. B intervenes, posing questions about which variable would be independent and which would be dependent. Initially, the students make a mistake by

assigning y to the hours and x to the cost. Ms. B appears to recognize this mistake and she poses the question to Raheem and Emmy and she asks Rita to label the columns of her table with the variables. This intervention appears effective because Ruth quickly realizes that the x and y should be reversed.

This episode highlights an example of the teacher recognizing a student error, yet intervening in a method so that the student is able to see and fix her own error. This type of interaction may encourage student autonomy and boost the students' mathematical self-efficacy because it empowers the student to remedy his or her own errors without direct input from the teacher.

This episode ends when Ms. B makes an announcement to the whole class that it is time to conclude the class period.

Ms. B: Classroom observation analysis, "The DJ Problem"

Day Two

Eight *teacher/student(s) mathematical interaction episodes* were identified in the video data collected in Ms. B's classroom on December 7, 2006, Day Two of Cycle Two.

Ms. B stands near the middle of the classroom in front of the coat closet. She tells the students that they will continue to work on the first part of the DJ problem, "Parts a, b and c". She holds up a large piece of centimeter grid paper and explains to the students that they will graph their findings on the paper. She tells the students that she will also give them a large chart paper where they will write their answers for parts a and b. She announces that the students will have twenty-five minutes to prepare their graph and their answers on the large paper.

Time	Speaker	Transcript	Descriptions / Comments
Begin TSMIE [B.02.b.01] 3:13 – 6:18 (L)			
Participants: Ms. B, Ghee, Dana, Shawn, Devin, Lenny During the two minutes leading up to this interaction, Ghee, Shawn and Devin appear disengaged from the mathematics because they are looking around, talking and laughing. Dana appears unhappy because she is frowning, it is inferred that Dana is unhappy because she is experiencing impasse with using a calculator to graph the equations. She says, "We don't know how to do it" and hands her calculator to Lenny. Lenny is pushing buttons on the calculator and at 2:20, Dana asks Lenny, "You did it? Lemme see." Lenny continues to push buttons on the calculator and does not show the calculator to Dana. She hastily grabs a calculator, says, "Just like this" and tosses the cover off the calculator and begins pushing buttons. Ghee appears disengaged because he has his head resting on his arm and is mumbling expletives to Shawn. At 3:08 he looks up, and abruptly sits up as Ms. B approaches the table. It appears that Ghee engages as a result of Ms. B's presence near his table. He picks up a calculator, and begins talking to Ms. B. "We are about to graph [inaudible]".			
3:13	Ms. B	So, I wanna see what you guys come up with. [Ms. B approaches the table and sits down next to Devin. She is not smiling.]	It may be inferred that Ms. B is ***interested*** in the students' progress at this table because she says, "I wanna see what you guys come up with." **[Int]**
3:19	Ghee	On the calculator!	

		[Ghee is smiling and laughing.]	
3:20	Ms. B	What are you graphing? [Ms. B is looking down at the paper and her facial expression is serious.]	Ms. B appears ***curious*** as she asks Ghee to clarify what they are graphing on the calculator. **[Int, rp]**
3:22	Ms. B	Each what? [Ms. B looks at Ghee.]	Ms. B may be ***curious about*** Dana's answer, "each one" because she asks for her to elaborate. "each what?" **[Int]**
3:23	Dana	Wait! What you doing? [Dana raises her voice as she takes the calculator back from Shawn. Shawn sits back in his seat, he is holding a different calculator in his lap.]	Dana appears ***angry*** with Shawn as she protests and takes the calculator back.
3:26	Ghee	Each variables.	
3:28	Ms. B	You're graphing each variable? [Ms. B looks at Ghee.] So you're graphing an X and a Y? [Ms. B stretches her neck, looks at Ghee and glances from Devin to Dana. Ghee yawns, he is resting his face on his hand.]	Ms. B reiterates Ghee's answer, neither validating nor negating his idea. It may be inferred that Ms. B is not satisfied with Ghee's answer because she lends specificity to his answer by identifying the two variables. It appears that Ms. B recognizes that Ghee is disengaged (or pseudo-engaged) from the mathematics.
3:32	Ghee	That, that that that. [Ghee points three times to his paper with his pen.]	
3:33	Ms. B	So what are those three things called? [Ms. B is leaning in towards Ghee, her hands are folded in her lap and her facial expression is serious.]	Ms. B's intervention is inferred to be an attempt to engage Ghee in a discussion that may reveal his reasoning. Ms. B appears ***curious*** about what Ghee means by "that, that, that, that". It may be inferred that she is ***dissatisfied*** with Ghee's level of engagement because she continues to ask him to be more specific in his language. **[Int, reasoning, Dis-e]**
3:36	Dana	Look I graphed it [Ms. B]. Look. [Dana holds up her calculator, the screen facing	Dana appears ***eager*** to show Ms. B her graph.

		Ms. B. Dana raises her eyebrows and is not smiling.]	
3:40	Ms. B	What did you graph? [Ms. B has her hands in her lap as she looks at Dana]	Ms. B's tone of voice conveys ***no emotion*** in response to Dana's graph, yet her language conveys ***curiosity*** about what Dana graphed on the calculator. **[Int, rp, rs]**

3:41	Ghee	[Ghee looks at Dana's calculator screen then points towards Shawn.] That what you got, ha!	
3:42	Shawn	I told you you wrong man.	
3:43	Dana	Each umm, umm… …yeah each equation. [Dana looks at her paper, then cocks her head to the side and looks at Ms. B]	It may be inferred that Dana is ***seeking feedback about her graphs on the calculator from Ms. B*** when she cocks her head to the side.
3:48	Ms. B	[Ms. B's tone of voice is soft.] Oh. Okay, so you graphing each equation, [Shawn reaches across the table and takes the calculator from Dana's hand, Dana does not react] and so did you, and it said what? [Ms. B is leaning towards Ghee and Dana, her hands are in her lap and her facial expression is serious.]	It may be inferred from Ms. B's soft tone of voice, posture and facial expression that she is ***patiently*** waiting for Dana to interpret the graph on her calculator. **[Int, rp, rs]**
3:55	Dana	It didn't say nothing?	
3:56	Ghee	It just split up that little, thingy.	
3:59	Ms. B	So what happens if it didn't come up with anything? [Ms. B is leaning in looking at Ghee and Dana, she furrows her brow and shakes her head.]	It may be inferred that Ms. B expresses ***puzzlement*** in her facial expression and ***curiosity*** in her language when she asks the students "what happens?" Ms. B does not impose her ideas about what might have gone wrong in the calculator. **[Int, rs]**
4:02	Dana	We did it wrong.	
4:02	Ghee	Nope, your scale and stuff might be wrong. [Ghee looks at Ms. B and gestures back and forth with his hand. Ms. B looks at Ghee, she is not smiling.]	Ghee and Dana offer suggestions about what might have gone wrong in the calculator.

4:06	Dana	Or your window setting.	
4:06	Ghee	…your window setting, settings…	
4:08	Ms. B	Okay…so [Ms. B nods her head, she is still not smiling, Ms. B looks at Lenny.]	It is inferred that Ms. B is ***pleased*** with Dana's and Ghee's idea because she nods her head. **[Val]**
4:11	Ms. B	So can you take that part out of your mouth? [Ms. B furrows her brows as she leans toward Lenny.]	
4:13	Dana	Somebody [inaudible] bit my wrong my calculator [Dana reaches across Ghee's desk and takes a calculator off of Devin's desk].	
4:14	Ms. B	So the window settings, you might need to change the window settings? [Ms. B looks at Ghee, then leans back in her chair to look at Devin's calculator.]	Ms. B's tone of voice conveys ***no emotion***, but she builds on Dana and Ghee's ideas that the window settings on the calculator might be wrong. **[Blds]**
4:18	Dana	Yeah.	
4:19	Ms. B	Okay. [Ms. B looks at Ghee.]	
4:19	Ghee	That ain't got no battery. [Ghee takes the battery door off of the calculator and shows it to Ms. B.]	
4:21	Dana	Somebody took my calculator, it was sitting right here, it ain't got a top on it, its mine.	Dana expresses ***anger*** accusing "somebody" of taking her calculator.
4:26	Ghee	It was all taken… [Ms. B leans back and looks inside Devin's desk.]	
4:32	Dana	[Ms. B stands up and walks away.] Where my cal…	The students are disengaged from the mathematics while they sort out problems with the calculators. Ms. B intervenes when she briefly leaves the group to retrieve a calculator for Dana.
4:35	Ghee	I'm about to break that.	
4:36	Shawn	[inaudible] man I'm sick of doing this…[inaudible] you took it didn't you	
4:38	Ghee	I told you was like [Dana	

		and Ghee both jump to grab the calculator Ms. B put on the desk.]	
4:39	Dana	[Ms. B reaches over Shawn and hands a calculator to Dana.] Thank you.	
4:41	Shawn	Dang, ya got both of them	
4:42	Ghee	I told ya'll she's about to get the gray, great one. [Ghee stands up and walks away from the table.]	
4:45	Ms. B	Okay, [Ms. B sits down in the seat next to Devin again] so what were you talking about with your window settings? [Ms. B is leaning in directing her question to Dana.]	Ms. B returns to the group and tries to reengage the students with the mathematics by appearing ***curious*** about how Dana plans to set up her window settings to show the graphs of the three equations. **[Int, Blds, Auto]**
4:47	Dana	That we gotta change it. [Dana is pushing buttons on the calculator that Ms. B delivered.]	Dana does not articulate how she plans to adjust the window settings.
4:49	Ms. B	Okay so how do you change it? [Ms. B is seated next to Devin with her hands in her lap. She is not smiling.]	Ms. B expresses ***curiosity*** about the specific process required to change the window settings on the calculator. **[Int]**
4:52	Dana	Go to the window, for the x-minimum put…[Dana looks at her paper]…he put, we, I put positive ten. [Dana shrugs her shoulders and looks at Ms. B.]	
5:04	Ms. B	Okay so why did you put positive ten?	Ms. B's tone of voice conveys ***no emotion***, but her language conveys ***curiosity*** about Dana's reasoning for using positive ten for the x-minimum. **[Int, rs]**
5:08	Dana	[Dana drops her hand on the table, looks back at her paper, she is has her elbows resting on the desk and is	It may be inferred that Dana does not understand why she used positive ten because she falters in response to Ms. B's question. It appears that Dana is

		holding her calculator.] Cause of sixty, cause of I we I'm about to graph this one…wait no, I still don't know how you get…[Dana looks at the paper in the middle of the table, then looks up toward Ghee who is returning to the table] Which one had the highest? [Dana scratches her head and smiles.]	experiencing ***impasse*** as she tries to determine the window settings in order to show the graphs of the three equations. It is possible that Dana understands that she needs to use the maximum value for the cost of the DJs when she asks, "which one is the highest?" It appears that Dana is the only student who is engaged in the problem in this point. Ghee is slowly meandering back to his seat, while Shawn, Devin and Lenny sit silently. It may be inferred that Dana is smiling in response to something unrelated to the math because she appears to try to hide her smile.
5:27	Ms. B	What do you mean which one had the highest? [Ms. B is off camera, her tone of voice is low and serious.]	It appears that Ms. B is ***curious*** about Dana's comment, "which one had the highest." Ms. B ***encourages*** Dana to be more specific in her language. **[Int,rs; Endis]**
5:30	Dana	Cause I was gonna umm, if it was positive, I mean negative ten I would move it up some.	
5:39	Ms. B	Whatever they are doing back there, let them go. [Ms. B's eyebrows are are raised, she is looking at Lenny and Shawn as she speaks. She looks at the boys for approximately three seconds before looking away.]	It may be inferred that Ms. B is ***annoyed*** with Shawn for behavior that may be perceived to be off-task when she advises him to ignore the students "back there". Ms. B emphasizes her ***displeasure*** when she pauses and stares at the boys for approximately three seconds. **[Dis-b]**
5:43	Ghee	They just talking about me. [Ghee looks down at his paper and begins to write.]	

5:44	Dana	Oh no, [inaudible]…I wanna x-umm minimum I put a positive ten! [Dana looks up at Ms. B.] A positive ten.	Dana's tone of voice may be inferred to convey ***confidence*** in her idea to use positive ten for the x-minimum.
5:48	Ms. B	Do you guys agree with [Dana], or disagree? Or did you not even hear what she said? [Pause.] You didn't hear what she said. So [Shawn]…because he said he wasn't paying attention, [Ms. B is resting her cheek on her fist, she looks at Dana] so can you repeat what you trying to do?	Ms. B expresses her ***displeasure*** in Shawn and Larry's engagement when she tells Dana that the boys were not paying attention to what she was saying. **[Dis-e, Endis]** Ms. B's posture appears to reflect that she is ***disappointed*** by the students' disengagement. At this point, it appears that Dana is the only student who is engaged with the math.
6:04	Dana	Was I trying to move it down? [Dana is looking at the calculator screen.] Oh! Go to the one, we trying to fix the window setting. [Ms. B taps on Devin's desk and points towards Dana.] For x-minimum I put negative ten… [Ms. B stands up and walks away from the table] Wait we ain't have no negative dollars [Dana shakes her head and looks toward Ghee's desk, Ghee stands up and walks toward the back of the room] …Where is he keep moving to? [Dana looks toward Ghee and points her finger at him. Shawn holds his calculator display up to face Dana then sits back in his seat.]	It may be inferred that Dana's tone of voice and language conveys ***uncertainty*** but then she has a ***revelation*** and begins to "teach" Shawn and Lenny about the window settings. It is inferred that Ms. B is signaling to Devin to pay attention to Dana as she speaks. **[Endis, Ptp]**
End TSMIE [B.02.b.01]			

Discussion of TSMIE [B.02.b.01]

Ms. B appears to be checking in with the students' mathematical progress when she approaches the table and sits down next to Devin. Ms. B's presence appears to influence Ghee's engagement because he straightens his posture and begins to tell Ms. B that they are going to graph the equations on the calculator. The students are experiencing impasse with generating graphs of the three equations on the graphing calculator because they have not determined the correct window settings, specifically the x and y minimums and maximums as well as the range for each axis.

Ms. B's tone of voice is devoid of emotion as she poses questions, which are inferred to encourage Dana and Ghee to be more specific in their mathematical language. "So what are those three things called?" "What did you graph?" Ms. B also asks the students to provide reasons for their actions, "Okay so why did you put positive ten?" "What do you mean which one had the highest?" Ms. B does not impose her own mathematical ideas and it appears that she provides validation for mathematical ideas only once during this episode when she nods in response to Ghee and Dana saying that they need to change their window settings.

Dana appears to make some mathematical progress during this episode as she experiments with entering values in the window settings. However, her numbers are not appropriate for the data and this may lead to future impasse. It is not evidenced whether or not Ghee, Lenny, Shawn and Devin make mathematical progress. Devin, Lenny and Shawn do not engage in the mathematical discussion during this episode and Ms. B addresses off-task behavior once when she speaks to Lenny and Shawn telling them to ignore the behavior of the students at a nearby table.

Immediately subsequent to this episode, Ghee gets up from the table and walks across the room, gets a calculator and then walks back to his table slowly, talking to students at other groups along the way.

Ghee's behavior at the beginning and the end of this episode may be inferred to show that in Ms. B's presence, Ghee is likely to engage or at least appear engaged, but in her absence, he may tend to disengage from the mathematics.

Begin TSMIE [B.02.b.02] 6:20 – 10:00 (C)			
Participants: Ms. B, Natasha, Kenny, Jay, OJ, Al, Frank Immediately prior to this interaction, it appears that the students are not engaged in the mathematics because they are laughing and shuffling papers around the desk. Ms. B initiates this interaction when she approaches the group and looks around at the students, pursing her lips.			
6:24	Ms. B	[Ms. B approaches the table and stands next to boy seated across from Kenny] So…[Ms. B looks at Kenny's paper.] This is nice, this is what the DJ is going to look like over here? [Ms. B nods her head as she looks at Kenny. Kenny is smiling and looking down at his paper.]	It may be inferred that Ms. B is being ***sarcastic*** as she looks at Kenny's depiction of a dinosaur on his paper.
6:30	Natasha	He got a basketball on the team! [The camera pans in to show a student's drawing of a dinosaur with a basketball in between his legs. Ms. B looks at Natasha and she is slightly smiling.]	
6:32	Ms. B	Okay. So what price does he charge? [Ms. B is resting her hand on the boy's desk and glances at OJ, she is not smiling.]	
6:32	Natasha	Oh okay that why D…(inaudible)	
6:38	Kenny	He charges one seventy-five…Which one? [Kenny is sitting back in his seat, he is not smiling and looking down]	
6:41	Ms. B	I don't know which one this goes to. [Ms. B gestures toward a student paper and smiles.]	Ms. B expresses that she is uncertain. **[Int]**
6:43	Jay	The second one.	

6:43	Kenny	I had…I did that yesterday. [Ms. B smiles and nods as she is looking at Kenny]	It appears that Kenny is telling Ms. B that he drew the dinosaur yesterday. It may be inferred that Kenny is ***embarrassed*** that Ms. B discovered his drawing. He explains that the drawing was from yesterday.
6:46	Natasha	Okay, no you did that yesterday.	
6:46	Ms. B	I know, but I'm, okay so yesterday you were trying to make pictures of what DJs look like …so what is that? [Ms. B is smiling slightly]	Ms. B uses subtle ***humor*** as she incorporates Kenny's drawing into the discussion in an apparent attempt ***to reengage the students with the mathematics.*** **[Int, Endis, humor]**
6:52	OJ	Light plastic.	
6:52	Ms. B	Light plastic, okay so how much does he charge? [Ms. B purses her lips as she looks at Kenny.]	
6:55	Kenny	[Ms. B is standing, facing Kenny with her hand resting on her hip] He charge uhh… he chargest one seventy-five every, every hour we pay him thirty dollars. [Kenny fidgets with his pen as he answers Ms. B, he looks down at his paper. Kenny looks at Al.]	
7:01	Al	What is you doing it's the second one…(inaudible)	
7:03	Natasha	We're not doing the third, see you not with us, you not with us. 'Cause the second one is the cheapest [Ms. B looks at Natasha's paper and raises her eyebrows.]	It appears that Natasha is speaking to Jay who is seated next to her and may be ***annoyed*** because Jay is not engaged in the discussion with the rest of the group, "you not with us".
7:11	Ms. B	So how do you know the second one is the cheapest? [Ms. B glances toward a different group, then looks back at Natasha.]	Ms. B's tone of voice is flat as she speaks to Natasha, but her language may be inferred to convey ***interest*** in Natasha's reasoning for choosing "the second one". **[Int]**
7:13	Natasha	Because where is my paper,	Natasha appears ***confident*** in her

		bust it out.	answer as she looks for evidence to support her claim, "the second one is the cheapest".
7:16	Jay	The third one is the cheapest.	
7:16	Al	I got it right here [Al flattens a crumpled piece of paper and hands it to Jay.]	
7:19	Jay	Let me see. [Jay looks at the paper.]	
7:22	Ms. B	So if it's right, how come it's all balled up? [Ms. B furrows her brow and looks at Al, she is smiling.]	Ms. B expresses ***surprise*** and possible ***confusion*** when she asks Al why his paper is all "balled up". Her smile suggests that she finds this ***humorous.*** **[Int]**
7:23	Natasha	[Natasha laughs]	
7:28	Ms. B	Okay. [Ms. B is smiling, she shrugs her shoulders]	
7:30	Frank	[Ms. B], may I please… [Frank is seated at a different table, he is leaning back in his seat with his hands raised. Ms. B glances at him briefly, but does not speak to him.]	
7:36	Ms. B	So why did you pick the second one? [Ms. B scratches her head and speaks softly as she addresses Jay]	Ms. B expresses ***interest*** in Jay's reasoning for choosing the second DJ. **[Int]**
7:37	Jay	[Jay reads from his paper] Cause the…[inaudible]… I mean the third…down payment is a a hundred seventy-five dollars…he only receives thirty dollars per hour.	Ms. B looks at Ja's paper as he speaks, she appears to be expressing ***interest*** in his reasoning. **[Int]**
7:51	Ms. B	So where did you? [Ms. B points to Jay's paper, then looks at Natasha]	
7:51	Natasha	Yeah so [Ms. B], we said,	

		look [OJ] we said eight hours or six hours? [Natasha looks across the table at OJ as Ms. B glances back and forth between OJ and Natasha]	
7:57	Al	Six	
7:57	Natasha	No seven…seven right, seven. Seven or six	
7:57	Al	You said six…we said six	
8:00	OJ	Six, six, six	
8:02	Natasha	All right so we did…times six [Natasha pushes buttons on the calculator] so that is two forty [Natasha looks at Ja and points to her calculator] plus a hundred...plus a hundred…[Ja takes the calculator off of Natasha's desk] then is three forty and then if you do thirty times six. [Natasha leans on the desk.]	It appears that Natasha is ***confident*** as she is engaged in "Let me teach you" as she explains the process for calculating the cost of Solidus Sounds.
8:19	Jay	You said thirty times six? [Jay looks at Natasha]	Jay appears ***uncertain*** as he looks to Natasha for help.
8:22	Natasha	Thirty times six	
8:26	Jay	For which one? [Jay looks at Natasha]	
8:27	Natasha	What you doing [Natasha grabs the calculator from Jay and pushes buttons, Jay looks on as Natasha performs calculations] okay thirty times six that's one eighty and then you add one seventy-five and that's three fifty five. But the other one was, ummm, ow much was the other one?	It may be inferred that Natasha is ***impatient*** with Jay because she hastily grabs the calculator from him.
8:43	OJ	Dude, dude. Thirty and six…that's one eighty.	

8:47	Natasha	That was two forty plus [Natasha points at the calculator] yeah that's three forty. But that one was…[inaudible] umm so you save fifteen dollars. [Natasha sits up and glances toward Ms. B]	
8:52	OJ	The third one is cheaper, the third one is cheaper	OJ and Natasha disagree about which DJ is the cheapest.
8:53	Natasha	No it's not.	
8:53	Al	Let me see.	
8:55	OJ	I did, I did thirty, thirty times six…ooo, I was looking, I'm supposed to do one seventy five…	
8:57	Natasha	It's the second one!…It's the second one cause see this one is three umm forty [Natasha points at the calculator], and the third one is…yeah	Natasha appears to defend her answer that the "second one" (Solidus Sounds) is the cheapest DJ by stating the costs of the other DJs.
9:08	OJ	[OJ drops his calculator on his desk and sits back in his chair] Yea this one is three fifty five. I did, I just did plus a hundred. [OJ points to his paper and looks up at Ms. B]	
9:17	Natasha	See and this one, the first one, this is three hundred and sixty, we ain't doing that one [Ms. B walks away]	
9:23	Jay	Okay, so if we picked the second one, it would be number A	
9:28	Natasha	Hmmm, all right which DJ? [Natasha uncaps her pen, and shuffles her papers around on her desk.]	It may be inferred that Natasha is satisfied with her groups' solution and is ready to write the answer to part a of the problem which reads: "Which DJ would you choose? What variables might effect your decision?"

9:35	Ms. B	[Ms. B returns to the table] Okay let me give you a shee---uhh [Ms. B walks away to get a sheet of paper]	
9:37	OJ	All right, we gonna, we gonna DJ	
9:41	Kennedy	Solidus sounds, right?	
9:43	Natasha	It fines…What you giving me this for…o it's nice.	
9:49	Ms. B	[Ms. B brings grid paper back to the group] So, can you share your umm your answers on this paper in the marker [Ms. B looks at Natasha as she gives directions] and then make sure you put how you got your answer…okay? [Ms. B walks away from the group]	It may be inferred that Ms. B is ***satisfied*** with this group's mathematical progress because she gives them a large piece of chart paper to record their answers. This episode ends when Ms. B walks away. **[Sat]**
End TSMIE [B.02.b.02]			

Discussion of TSMIE [B.02.b.02]

This interaction begins when Ms. B playfully acknowledges Kenny's drawing of a dinosaur and comments that he may have drawn a sketch of what one of the DJs would look like. She incorporates the drawing into her questioning when she says, "So what price does he [the dinosaur] charge?" Kenny tries to explain that he did the drawing yesterday, Ms. B asks, "I know, but I'm, okay so yesterday you were trying to make pictures of what DJs look like …so what is that?" Ms. B briefly listens to Jay as he attempts to explain his mathematical reasoning. When he falters, Natasha engages in "Let me teach you" as she explains how to calculate the cost of Solidus Sounds for a six-hour party. Ms. B stands as a silent observer as Natasha explains the calculations to Jay.

It appears that the students at this table made some mathematical progress toward identifying the "best" DJ during this episode as they calculated the cost for all three DJs for a six-hour party. Ms. B appears satisfied with their progress as she hands them a large piece of chart paper to record their answer.

Immediately subsequent to this episode, Natasha appears engaged with the mathematics because she reads a question from the paper, posing the question to her group. When she gets no response from her group members, she appears angered because she yells at Kenny and calls out to Ms. B. Ms. B is at Dana's table and she does not respond to Natasha's calls for help. For the next several minutes, Natasha and Kenny argue about who was "doing the work" yesterday. Natasha expresses anger as she defends herself against Kenny's claims that she did not do the work yesterday.

Begin TSMIE [B.02.b.03] **10:14 – 14:35 (L)**			
Participants: Dana, Ms. B, Ghee, Shawn, Lenny This episode begins approximately four minutes after the end of the previous interaction between Ms. B and these students. When Ms. B walked away from the students in the previous episode, Dana was entering negative ten as the x-minimum in the window settings but then later realized that negative ten would be inappropriate because "you can't have negative dollars". In Ms. B's absence, Shawn and Ghee exhibit behavior that is perceived to be off-task, discussing video games and talking about students from other tables. Lenny is sitting quietly, pressing buttons on his calculator and Devin sits quietly, writing on his paper. Dana appears to remain engaged with the mathematics as she appears to enter data from her paper into the calculator. In the minute leading up to this interaction, Dana angrily exclaims, ""I don't know how to change the window settings!" Shawn responds, "Don't get angry, calm down." Ghee shows his calculator display to Lenny and says, "the lines go straight up".			
10:14	Dana	I don't know how to fix the window settings! [Ms. B approaches the group and stands next to Dana's desk and Dana looks at Ms. B and scratches her head.]	Dana's tone of voice appears ***whiny*** and conveys her apparent ***frustration*** that she has been unsuccessful in changing the window settings on the calculator.

10:16	Ms. B	Ok so what can you tell me about the window settings?	Ms. B's tone of voice may be conveyed to show ***interest*** in Dana's understanding of the window settings. **[Int]**
10:21	Dana	It tells how long the uhhh y-ax, axis, and the uhhh the x-axis.	
10:27	Ms. B	Okay, [Ms. B nods] so with that said, what do you think you have to do? [Ms. B is standing next to Dana, looking at her paper. Only the side of Ms. B's face is visible from the camera angle, so her facial expressions cannot always be described.]	Ms. B appears to ***agree*** with Dana's answer. Ms. B's tone of voice is low and it is inferred that she ***recognizes Dana's frustration and her impasse*** with determining window settings. Ms. B appears ***calm*** as she ***encourages*** Dana to think of the next steps. **[Rsa, Rsi, Enc]**
10:35	Dana	I don't know…I don't, the umm, like when the Y axis, since it's gonna be the ummm number tot, the total cost, [Natasha is calling from a different table, Ms. B looks up, then turns her attention back to Dana.]	Dana appears ***annoyed*** by Ms. B's question when she says, "I don't know", her tone of voice is whiny.
10:45	Ghee	[Ghee is leaning on his desk, pushing buttons on his calculator. He looks up at Ms. B.] It shouldn't be any negative though…It shouldn't be any negative.	
10:50	Dana	[Dana looks at Ghee] This is gonna be total cost umm put like the high, the higher number, the higher number you have, you could like add the highest number up there? [Dana looks up at Ms. B, her voice trails off at the end of her question. Ms. B points toward the other students in the group and Dana rests her cheek in her hand.]	It appears that Dana is ***uncertain about the maximum value*** because she poses her answer as a question, studying Ms. B's face as she answers. **[Endis]** It may be inferred that Ms. B is suggesting that Dana consult with her group members and that Dana appears ***frustrated*** by this.

10:50	Shawn	Don't start arguing, please, please please, what you acting like a third grader, somebody gotta take you out.	
11:07	Ms. B	Talk to the group. I'm listening! [Ms. B raises her voice, then lowers her voice.] I'm listening!	It may be inferred that Ms. B is trying to ***encourage student discourse*** about the mathematics. Dana appears ***frustrated*** with Ms. B's response. It may be inferred that Dana is ***angry*** at Ms. B because Ms. B is not giving Dana specific direction for determining the window settings. **[Endis, Int]**
11:08	Dana	No you're not! [Dana is looking toward Ghee, her tone of voice is soft.]	It may be inferred from Dana's facial expression and tone of voice that Dana is ***frustrated*** with the fact that Ms. B is telling Dana how to enter the calculator settings.

11:10	Ms. B	But maybe if you say something to him, something may you know click, from everything that we did. [Ms. B's tone of voice is direct and calm.]	Ms. B appears ***patient*** as she is trying to ***encourage student discourse*** and group autonomy by telling Dana to talk to Ghee. **[Endis]**
11:13	Ghee	How do you not finding him yet?	
11:15	Ms. B	How do you know if he has or not? So okay, so what's the question? What's the question?	Ms. B appears to be trying to ***encourage student discourse*** by asking Dana to pose her question to the group. **[Endis]**
11:25	Dana	I don't know how to do the window setting! [Dana is looking down at her calculator, tapping her pen on her desk.]	It may be inferred that Dana's tone of voice conveys that she is ***angered*** by Ms. B's question. It may also be inferred that Dana is ***frustrated*** that she still does not know how to fix the window settings.
11:28	Ghee	Jack and the bean stalk. [Ghee's tone of voice is mocking, he puts his head down on his desk.]	Ghee's outburst is unrelated to the mathematics and Ms. B does not acknowledge the outburst.
11:30	Ms. B	So with the window settings, what does the window settings tell us? [Ms. B's facial expression is serious and she is speaking more rapidly.]	Ms. B asks the students to think about the conceptual meaning of the window settings. Ms. B appears ***eager*** to engage the students with the math because her speech becomes more rapid. **[Int, anxious]**
11:39	Ghee	[Ghee stands up, pulls up his pants and sits down.] Oh brother! The window settings tell us uhhh, [Ghee leans down and presses buttons on his calculator] the x-maximum	
11:45	Ms. B	What does x-maximum mean?	Ms. B expresses ***interest*** in the students' understanding of the x-maximum.
11:48	Ghee	The amount of the scale! [Ghee dramatically drops his calculator on the desk and gestures his arms wide as he looks at Ms. B.	

		Ghee's voice is raised.]	
11:50	Dana	The highest number on the x-axis [Dana is looking at Ms. B, her tone of voice is soft.]	Dana is correct.
11:52	Ms. B	Okay the highest number on the X axis. [Ms. B nods her head and then points to Ghee, her tone of voice is soft] You threw out scale, what is that?	Ms. B ***validates*** Dana's correct answer by nodding her head and repeating Dana's words, then she asks Ghee to describe "scale". **[Val, Int]**
11:57	Ghee	The scale…one…[Ghee has his head on his desk, he covers his face with his arm as he laughs.] I don't know!	It may be inferred that Ghee is ***confused*** and that he responds with ***humor.***
12:04	Ms. B	Okay, so is that what you want?	
12:07	Ghee	I meant the amount on the graph, damn my bad, I'm sorry. I…I messed up. [Ghee still has his head on the desk. Al approaches the table and puts a piece of paper on the table.]	
12:10	Ms. B	So do you want it to go by ones?	Ms. B appears ***curious*** about Ghee's idea for the scale as Ghee suggests different possible scales. **[Int]**
12:13	Ghee	What.	
12:14	Ms. B	Your scale on the X axis.	
12:16	Ghee	I could go y ones, two, it could go by ten… [Ghee looks at Ms. B and shrugs his shoulders, his head is still resting on the desk.]	
12:21	Ms. B	Ten…so which..	
12:22	Larry	Oh never mind	
12:22	Ghee	[Ghee lifts his head off of the table.] Cause the most amount is	Ghee recognizes that the maximum value must be considered when determining the scale.

		three hundred, three hundred seventy five dollars.	
12:26	Ms. B	[Ghee sticks his tongue out] So for which axis?	
12:30	Ghee	For the umm	
12:30	Dana	For, umm, the x-axis, x	Dana and Ghee are incorrect because the hours should be represented on the x-axis, since the way the students wrote the equations on the previous day have x as the independent variable and y as the dependent variable.
12:32	Ghee	For the x-axis, the amount of money	
12:35	Ms. B	Ok, so did you put that in there?	Ms. B does not correct the error but expresses ***curiosity*** whether they have entered values into the calculator. **[Int]**
12:38	Ghee	Lemme fizzale	
12:39	Ms. B	You did?	
12:40	Dana	Yeah!	
12:41	Ms. B	There is nothing there. [Ms. B's tone of voice is soft as she speaks directly to Dana.]	
12:41	Ghee	Wait.	
12:42	Dana	Where?	
12:44	Ms. B	On the x-axis	
12:46	Shawn	Oh I thought	
12:49	Ms. B	What did you say the highest was again? [Ms. B leans across Dana's desk as she speaks to Ghee]	
12:51	Ghee	Three seventy-five.	
12:56	Ms. B	Ok. And what do you guys think about the x-minimum?	It is not clear whether or not Ms. B validates nor negates Ghee's answer, "three seventy-five".
12:58	Ghee	Could be one!	
13:01	Ms. B	Okay.	
13:02	Ghee	Cause you can't get negatives, you can't get negatives	Ghee is correct.
13:05	Ms. B	What do you mean you	Ms. B appears ***curious*** about Ghee's

		can't get negative?	reasoning why "you can't get negative". **[Int, reasoning]**
13:08	Dana	You can't have negative dollars.	
13:11	Ms. B	Ok, well you *could* have negative.	Ms. B disagrees with Ghee and Dana that "you can't have negative dollars."
13:14	Ghee	You could have negative dollars, but you got to have a credit card.	Ms. B's challenge, "you *could* have negative" provokes a mathematical discussion that reveals Dana's understanding of why the graph minimums will be positive. When Ms. B expresses ***curiosity*** about Dana's reasoning to support, "you can't have negative dollars", Dana articulates that the DJ is going to be paid a positive number of dollars. **[Int]**
13:15	Dana	But you gotta	
13:18	Ms. B	But in this case…	
13:19	Dana	You can't have negative dollars	
13:22	Ms. B	Okay why, why not?	
13:24	Dana	Because if we umm, because we gotta pay… we gotta pay Solidius sounds, umm a ***hundred dollars*** before he is gonna do something. [Dana is looking at Ms. B, scratching her head.]	
13:34	Ms. B	Okay. [Ms. B nods her head.]	Ms. B ***validates*** Dana's reasoning and appears ***satisfied*** with the students' understanding as she nods her had in agreement. **[Val, Sat]**
13:34	Ghee	It's thirty…It's thirty! [Ghee looks up at Ms. B]	Ghee appears confident in his answer, "it's thirty", but his language is not specific. It is possible that he is referring to the x-minimum.
13:38	Shawn	Stop yelling.	
13:39	Ms. B	For what?	
13:40	Shawn	Thirty!	

13:42	Ghee	For the x-minimum. [Ghee looks at Ms. B and bobs his head in a dancing motion.]	Ghee's body language suggests that he is ***confident*** about his answer.
13:44	Ms. B	How you get thirty?	Ms. B appears ***curious*** about Ghee's reasoning. **[Int, reasoning]**
13:44	Ghee	Because that's how much you gotta pay for each, umm hour	
13:50	Ms. B	Okay. [Ms. B looks at Shawn and Lenny] so did you guys agree about the X minimum being thirty? You just saying yes, just to say "Yes [Ms. B] leave me alone" [Ms. B shakes her head] or "yes" because you really agree with him? [Ms. B is slightly smiling.]	Ms. B's language suggests that she is ***displeased*** with the level of Shawn and Larry's engagement with the mathematics. Although her language may appear stern, Ms. B offers a hint of a smile, which may convey that she ***is friendly*** about the boys at the same time that she is serious about them becoming engaged. **[Dis-e]**
13:59	Shawn	Yes cause I really agree with him.	
14:01	Ms. B	Ok, so where is your calculator?	Both Lenny and Shawn look at Ms. B, but do not take action to get started doing anything. So, Ms. B suggests specific action the boys should take to become engaged with the math. Ms. B ***encourages student discourse*** when she asks Ghee to repeat the x-maximum for Shawn. **[Endis]**
14:03	Ms. B	So if you agree, maybe you should enter it in your calculator? [Lenny and Shawn look at Ms. B, Shawn is smiling.] So you said x-maximum should be, cause he doesn't have it in there.	
14:15	Ghee	Oh the X maximum is three hundred seventy five dollars, three hundred seventy five. [Ghee is sitting back in his	

		seat, reading from his calculator. Dana has her chin resting on her fist, she turns to Ghee and yawns.]	
14:17	Shawn	Should be thirty…oh three hundred, three hundred seventy five??	
14:21	Ghee	Yea, then the X minimum is thirty	
14:26	Shawn	thirty	
14:28	Ms. B	So can you guys continue the rest of it on your own…without me standing here?	Here is a clear indication supporting the inference that Ms. B's presence in the group positively influences student engagement.
14:33	Ghee	Yes, Yes we can. [Ms. B walks away from the group.]	
14:34	Dana	What's the scale by? [Dana looks at Ghee]	
End TSMIE [B.02.b.03]			

Discussion of TSMIE [B.02.b.03]

This episode begins when Ms. B approaches the group and Dana complains that she is having trouble setting the window on her graphing calculator. Dana appears angered because Ms. B does not directly help her, but instead encourages Dana to discuss her trouble with the other group members. It is likely that this is not good news for Dana as the other members of her group do not appear engaged with the problem. Ms. B intervenes by suggesting how Dana might engage the students in her group, "But maybe if you say something to him, something may you know click, from everything that we did." Ms. B is encouraging student discourse as a possible strategy to foster student engagement with the mathematics. It is inferred that Ms. B recognizes the students' impasse and she intervenes asking the group, "So with the window settings, what does the window settings tell us?", "what does the x-maximum mean?" This intervention is

effective in fostering the students' engagement with the mathematics.

Ms. B addresses Larry and Shawn's behavior, asking the boys whether they agree or disagree with Ghee's idea that the x-minimum be thirty. She implies that the boys may be agreeing with Ghee so that she would leave them alone, this appears to get the boys' attention because they are both looking at her with thoughtful expressions. Before leaving this group, Ms. B implies that her presence influences student engagement. "So can you guys continue the rest of it on your own without me standing here?"

Immediately subsequent to Ms. B's departure, the students appear to be engaged in entering the window settings in their calculators because they are pushing buttons on their calculators.

<table>
<tr><th colspan="4">Begin TSMIE [B.02.b.04]
22:25 – 25:05 (L)</th></tr>
<tr><td colspan="4">Participants: Ms. B, Dana, Ghee
This episode begins approximately six minutes after the last interaction between Ms. B and the students at this table.
After Ms. B's previous interaction with this group, the students determined the minimum and maximum for the x and y-axes as well as the scale for each of the axes. Ghee stated the window settings as follows: "The x-maximum is six hundred…The x-minimum is thirty...X scale is 10, the …y-minimum is one, the y-maximum is 10, the y-scale is one," [16:25- 16:38, L]. Although the numbers appear appropriate, their placement on the axes is reversed. The students have placed the dependent variable (cost) on the x-axis instead of on the y-axis.</td></tr>
<tr><td>22:25</td><td>Ms. B</td><td>[Ms. B approaches the table and stands next to Lenny. Her tone of voice is soft, her facial expressions cannot be determined because her face is off camera.]
What I need you guys to do is to start putting your thoughts on this paper here.</td><td></td></tr>
<tr><td>22:26</td><td>Dana</td><td>We, we trying to figure out our window settings cause where it's supposed to be!
[Dana is pressing buttons on</td><td>Dana's tone of voice may be inferred to convey frustration that they have yet to figure out the window settings for the graph.</td></tr>
</table>

		her calculator, her voice is raised when she speaks to Ms. B.]	
22:32	Ms. B	You still working on the window settings? [Ms. B's tone of voice is soft and flat.] So what did you come up with? [Ms. B is standing next to Lenny, her hands are on her hips and she is leaning over to look at Lenny's calculator. Lenny looks up at Ms. B and they smile at one another.]	Ms. B appears ***surprised*** that the students are still working on setting the window on their calculator and ***curious*** about their progress. **[Int]**
22:41	Dana	Look we we know everything, we don't know…see this is the only our graph. Our thing did! [Dana turns her calculator so Ms. B can see the screen.]	
22:48	Ms. B	Oh so I see a start.	Ms. B appears to be ***encouraged*** that the students have "a start".
22:50	Ghee	Yea I see it too! [Ghee looks up at Ms. B]	
22:51	Ms. B	And where is it.	
22:52	Ghee	At the top! [Ghee looks at his calculator screen, then leans over to look at Dana's calculator]… where it's supposed to be.	Ghee conveys ***confidence*** in his answer when he says, "where it's supposed to be!"
22:54	Ms. B	It's further up. It's further up or is it lower on the Y axis? [Ms. B leans towards Ghee, her brow is furrowed.]	Ms. B appears ***curious*** about the students' graphs as leans in and furrows her brow. It is inferred that she seeks clarification about where the line is on the y-axis. **[Int]**
22:59	Ghee	Up.	
23:00	Ms. B	So what do you think you might need to do? [Ms. B rubs her chin as she looks at Ghee.]	Ms. B appears ***curious*** about Ghee's next steps for fixing his graph. **[Int]**
23:05	Ghee	Change our scale. On the y-axis. [Ghee raises his eyebrows and looks at Ms. B, then he draws back.]	
23:06	Ms. B	Why? Oh! Maybe I don't	At first, Ms. B appears to express

		know, change it to what? [Ms. B is looking at Ghee, her hand is on her hip.]	***surprise***, then Ms. B appears ***indifferent*** about Ghee's idea to change the scale on the y-axis. **[Sur]**
23:12	Ghee	One.	
23:13	Dana	What is our scale?	
23:18	Ghee	[Ghee punches buttons on his calculator]…Flips…and it still went back to the same thing! [Ghee straightens in his seat and raises his voice.]	It may be inferred that Ghee's tone of voice expresses ***surprise*** at what appears on his calculator screen.
23:18	Dana	Ain't even doing that yet.	
23:27	Ghee	[Ghee is looking at his calculator, pressing buttons] What she spent? Why is our y-scale, I mean our x-scale…and not even it…[Ghee shakes his head and looks at Dana] yeah its still doing the same thing.	
23:40	Dana	It's still doing the same thing!	Dana's tone of voice appears to convey ***surprise*** and ***frustration*** that the calculator stills shows the same thing despite the fact that they changed the scale.
23:41	Ms. B	So it's one, two, [Ms. B is out of camera range, both Ghee and Dana look up at Ms. B]	Ms. B appears to recognize student impasse. **[Rsi]**
23:44	Dana	Two	
23:48	Ms. B	So what do you think you might wanna try now?	
23:53	Dana	Something higher…I ain… I put one, I mean two. [Ghee rests his head on his arm.]	
23:56	Ms. B	So you said something higher, how about you put something higher? [Ms. B is off camera. Her tone of voice is soft.]	Ms. B appears to ***encourage*** Dana to use her own idea of inputting a higher range in the window settings. **[Rsi, Enc]**
24:00	Dana	I punched nine. It still didn't, it went the same thing.	
24:03	Ghee	[Ghee looks at Dana] What on the x-axis? Or the y-axis?	

24:06	Dana	[Ghee sits up and pushes buttons on his calculator.] My y-axis, it's still the same thing. [Dana's tone of voice is soft.]	
24:12	Ms. B	Same thing, okay. [Ms. B's tone of voice is soft, she is leaning over, looking at Dana's calculator.]	It appears that Ms. B conveys ***sympathy*** for Dana's struggle as Dana appears not to be making any progress towards successfully graphing the equation.
24:21	Ghee	Ohh! I got…I pressed twenty! [Ghee looks up at Larry and Shawn]	
24:21	Ms. B	Can you guys do me a favor and can you…you pressed twenty and what happened? [Ms. B looks at Ghee]	Ms. B appears ***curious*** about the result on Ghee's calculator. **[Int]**
24:25	Ghee	It went right back to the same thing, except it just made a dotted line. [Ghee looks up at Ms. B]	Ghee is referring to the graphical display on his calculator.
24:28	Ms. B	Just made a dotted line...Can you guys do me a favor and just put what you have on the grid paper?	
24:33	Dana	We don't got nothing?	Dana expresses anger that her group "don't got nothing".
24:34	Ms. B	You have something! [Ms. B's tone of voice loud, then soft.] You don't have anything?	It may be inferred that Ms. B is trying to ***encourage*** Dana to record their solution thus far on the grid paper. Ms. B appears to recognize Dana's ***dismay*** that they don't have anything to report, Ms. B's tone becomes ***sympathetic.*** **[Rsi, Val]**
24:36	Dana	No.	
24:36	Ms. B	Can you put your thought at least here, [Ms. B puts the chart paper in the middle of the table] what you came up with, her your equations for A and B	Ms. B appears to recognize that the students have reached a point of impasse with the window settings on the calculator so that they will be unable to complete the large graph for presentation. So, Ms. B assigns the students to write equations for parts A and B. **[Rsi]**
24:46	Dana	Yea 'cause I don't know how	It may be inferred that Dana is

		to change the window setting.	***dismayed*** because she was unable to produce a graph on the calculator.
24:49	Ms. B	You did change the window setting when it was one equation, you were able to change it. So can you get, just get something down on that. What you have here, I want you to get it together, okay. Use the other side of the marker so it's thicker.	It may be inferred that Ms. B ***recognizes Dana's impasse*** and she tries to ***encourage*** Dana. Ms. B reminds Dana that she was able to change window settings when solving a problem with one equation. It is inferred that Ms. B recognizes that Dana's group may not complete the graph during this class period but that she wants them to be able to contribute to the whole class discussion, which is forthcoming. **[Rsi, Val, Enc]**
End TSMIE [B.02.b.04]			

Discussion of TSMIE B.02.b.04

Ms. B initiates episode approximately twenty-two minutes into the class period. At the beginning of the class period, Ms. B announced that the students would have twenty-five minutes to complete the problem. It is inferred that Ms. B is eager for the students to record their mathematical solutions on the chart paper to share in the whole class discussion.

Dana and Ghee are struggling with determining the appropriate settings on the graphing calculator to represent the three equations for the DJ's rate schedules. It is not clear that Ms. B recognizes why the students are having difficulty. Ms. B does, however acknowledge Dana's negative affect when Dana complains, "We don't got nothin'". It is inferred that Ms. B tries to provide encouragement by recalling that Dana was able to graph one equation in the calculator.

At the end of this episode, it is inferred that Ms. B wants the students to focus their efforts on preparing their work to share with others. She redirects the students to present

what they have completed up to this point. “So can you get, just get something down on that. What you have here, I want you to get it together, okay”.

Immediately subsequent to this episode, Dana begins to write on the chart paper while Ghee, Larry and Shawn appear to disengage from the mathematics because they are not discussing the mathematics.

Begin TSMIE [B.02.b.05] 2:02 – 8:14, (C, Title 2)			
Participants: Frank, John, Ms. B Prior to this interaction, it appears that Frank, Jen and John have a question for Ms. B because they have their hands raised at 1:10. At 1:25, it be inferred that Jen is becoming angry that Ms. B has not recognized that she has a question when she slams her hand on the table and exclaims, “My arm hurt!”, then she raises her hand again.			
02:02	Frank	[Ms. B approaches the table and stands next to Frank’ desk] How we supposed to graph that? [inaudible] [Ms. B tucks her hair behind her ear and leans toward Frank, looking at his paper as he talks.]	Ms. B appears ***interested*** in Frank’ graph as she leans down and looks at his calculator display. **[Int]**
02:08	Ms. B	[Ms. B leans on Frank’ desk and laughs, girl is also laughing] What did they say, say that again?	It appears that Ms. B is ***amused*** by something that Frank said.
02:12	Frank	We supposed to graph that, uhh babies.	
02:15	Ms. B	So did you graph it? [Ms. B is not laughing, she is still leaning on Frank’ desk, looking at his paper.]	Ms. B is still ***smiling,*** and it appears that she is ***curious*** about whether or not Frank wrote the graph on paper. **[Int]**
02:17	Frank	No!	
02:18	Ms. B	Why not? You graphed it and that’s what’s happened? [Boy sitting next to Frank shows Ms. B the display on his calculator screen, Ms. B looks at the calculator. She is not smiling.]	Ms. B appears ***interested*** in the students’ graph as she leans in toward the students. **[Int]**
02:22	Frank	Yea, yea, it’s not a regular graph. [The camera briefly zooms to show boy’s calculator screen displaying a four quadrant graph.]	

02:24	Ms. B	What you mean it's not a regular graph?	Ms. B's tone of voice has no emotion, she appears ***confused*** about Frank' comment, "it's not a regular graph". **[Con, Int]**
02:25	Frank	Not a graph that they doin'.[Frank gestures toward Mark] We doing…[inaudible]	
02:31	Ms. B	Ok, [Ms. B looks at Frank' calculator display and pauses] So when you….what kinds of numbers are you looking at? Positive or negative numbers?	It appears that Ms. B ***recognizes that the students are experiencing impasse*** due to the window settings in their calculator and it may be inferred that she asks, "what kinds of numbers…" as an intervention to help the students to recognize that their graph will show strictly positive numbers. **[Rsi, Int]**
02:42	M and F	Positive	The students are correct.
02:45	Ms. B	Positive not negative numbers?	It is inferred that Ms. B's language is intended to help the students recognize that their window settings should not include negative numbers. **[Rsi, Int, Blds]**
02:47	Frank	Eight numbers [inaudible]	
02:48	Ms. B	So are you dealing with negative numbers? [Ms. B straightens and looks around the table at the students]	
02:49	Ms. B	No	
02:50	Ms. B	You said yes, [Ms. B points at M, then points at Frank.] you said no.	Frank and M are not in agreement.
02:53	Mark	Both!	
02:54	Ms. B	Both and you said no.	
02:59	Student	Plus thirty	
03:01	Mark	Uhh… [Mark smiles and looks up at Ms. B, John looks at camera and smiles]	It may be inferred that Mark is ***confused*** as he smiles and looks up at Ms. B.
03:05	Mark	The eight positive per hour and the rest of the numbers is negative. [Mark looks up at Ms. B whose facial expression cannot be determined because she is off	Ms. B does not respond verbally to either boys' comments.

		camera]	
03:16	Frank	I don't get it! [Frank is smiling and looking at his calculator]	
03:18	Mark	No no no no, ain't no negative numbers. [M shakes his head no]	It may be inferred that Mark is ***confident*** about his reasoning as his tone of voice is a bit stronger when he realizes that there are no negative numbers in the solution.
03:26	Frank	No [inaudible]	
03:29	Ms. B	So how come now you saying there is no negative? [Ms. B is looking at M, her head is cocked slightly to the side and she is not smiling.]	Ms. B appears ***curious*** about M's reasoning why there are no negative numbers in this problem. **[Int, reasoning]**
03:33	Mark	Cause ain't no money going down. [M looks up at Ms. B]	
03:36	Ms. B	There is no money going down?	Ms. B reiterates M's reason, she ***does not validate nor negate M's*** reasoning.
03:37	Mark	Not not losing….[Mark glances at Frank]	
03:37	Frank	[inaudible] [Ms. B puts her hands behind her back, purses her lips and looks at Frank, then looks at Jen and looks at Mark.]	
03:45	Mark	I don't know. [Mark shakes his head and puts his calculator on his desk]	It appears that M may be ***frustrated*** because he is still not clear about whether or not negative numbers will come into play.
03:46	Ms. B	What are our two variables? [Ms. B looks around the table at the students, and shifts from side to side]	It is possible that ***Ms. B recognizes her students' impasse*** and she intervenes by asking the students to think about a different aspect of the problem. Ms. B appears ***curious*** about the students' understanding of what the variables represent and she asks them a for a concrete example (using numbers) for "hours worked". **[Rsi, Int, Blds, Endis]**
03:48	Frank	X and Y	
03:49	Ms. B	And what does X represent? [Ms. B raises her eyebrows]	
03:50	Frank	The hours worked	
03:51	Ms. B	The hours worked, so can you give me and example of the hours worked? [Ms. B looks at Frank]	
03:56	Frank	So the DJ worked for one hour and gets paid a hundred seventy five dollars plus the …[inaudible, Ms.	

		B looks at Frank as he talks]	
04:05	Ms. B	Okay, and what is your Y variable? [Ms. B looks at Frank]	Ms. B appears ***satisfied*** with Frank' reasoning as she poses the next question. **[Sat]**
04:07	Frank	Total cost. [Ms. B briefly turns to look toward the door.]	
04:11	Ms. B	Total cost and the total cost is what total cost? [Ms. B turns back toward Frank and purses her lips]	**[Blds, Int, reasoning]**
04:14	Frank	The amount all together for the eight hours of [inaudible]	
04:17	Ms. B	Okay so.	
04:20	Frank	Like for one hour that would be sixty dollars for two hours it would be a hundred and twenty dollars.	Frank and M offer correct reasoning.
04:22	Mark	It would be a hundred and twenty dollars	
04:25	Frank	And then, and then at the end it would be a hundred and eighty, [Frank is looking at his calculator screen] I got [inaudible]	
04:35	Ms. B	So you have it in here? [Ms. B leans over and looks at Frank' calculator display] So you are….	Ms. B appears ***curious*** about Frank' calculations. **[Int]**
04:38	Frank	That's for all, I put all three…	It may be inferred that Frank entered all three equations in his calculator.
04:39	Ms. B	So you put all three in there, okay so now did you graph it? [Ms. B stands up straight] And what do you say about that graph?	Ms. B appears ***curious*** about whether or not Frank graphed the equations. **[Int, Endis]**
04:48	Frank	It looks weird.	
04:49	Ms. B	It looks weird. [Ms. B shifts her weight back and forth and purses her lips. She is not smiling.]	Ms. B repeats Frank' judgment of his graph but she does not offer her own judgment.
04:50	Frank	Well like its got the one that's negative, like half of it goes bigger [Ms. B leans in and looks at Frank' caclculator display]	Based on Frank' description of the graph, it may be inferred that he has graphed all three graphs on a four quadrant coordinate grid.

		then the rest and half of it goes down….so this side is the negative side.	
05:06	Ms. B	So what do you guys have to say about this graph?	It can be inferred that Ms. B recognizes that Frank is experiencing impasse because he has represented the three equations on a four-quadrant graph instead of a single quadrant graph. Ms. B ***does not offer her own opinion,*** but rather appears ***interested*** and ***curious*** about the opinions of the other students. **[Int, Endis]**
05:08	Frank	It's negative.	
05:11	Mark	Cause you loosing money. You paying for the DJ.	It appears that Mark may think that the negative numbers on the graph represent the debits from the executive board's budget.
05:15	Ms. B	[Ms. B's tone of voice lightens and she points to the desk] Okay so can you guys do me a favor and what you umm graphed in here [Ms. B touches Frank' calculator] and what you wrote down, I'm gonna give you a little piece of chart paper, and then can you also graph it here…	It is inferred that Ms. B is offering a different mode for representation by giving the students chart paper to record their graph. Ms. B does not respond to M's comment, she begins to give directions for the students to record their work for the class discussion. **[Endis, Int]**
05:27	Frank	You want us to graph this one? [Frank holds up his calculator display]	
05:29	Ms. B	Is that what you want to graph? I'm not telling you exactly what to graph, but you have to graph...they want you to graph…it says "graph all three equations in the same window of your calculator, make a sketch of the graph you see." [Ms. B's face is off camera]	Ms. B does not specify which graph the students should display. Rather, she redirects the students to the statement of the problem, reading it verbatim.
05:47	Frank	We got to fix the window…[Frank gestures toward his paper and looks up at Ms. B]	Frank recognizes that his graph must be "fixed" before they write their graph on the chart paper.
05:52	Ms. B	What do you mean fix the window?	Ms. B appears ***curious*** about Frank' suggestion to fix the

		[Ms. B is facing Frank, she is not smiling]	window settings.
05:54	Frank	Change the setting so it would be, [inaudible] this graph, it would be like umm, a coordinate graph. [Frank points to his paper]	
06:00	Ms. B	So what quadrants are you looking at? What quadrants do you want to look at? [Ms. B is turned toward Frank, she has one hand on her hip]	It may be inferred that Ms. B's question is intended to help the students to fix their window settings.
06:07	Mark	Quadrant one.	The students are not in agreement and appear uncertain about which quadrant they should use for their graph.
06:12	Frank	Quadrant, I think quadrant one	
06:14	Quanee	I think it is quadrant number two.	
06:15	Ms. B	How many quadrants are there?	
06:17	Frank, Q	Four	
06:17	Frank	Three…yea four.	
06:19	Ms. B	Three? [inaudible, Frank looks up at Ms. B and smiles] you see three equations… Okay so there are four quadrants, so when you said change the setting so you want the settings to look like, how do you want your graph to look, [Ms. B points to Frank' paper] can you just draw me…what you want it look like… [Ms. B steps away from Frank' table and raises her voice to address students who are standing up approaching the door] I only need one person to answer the door! [Jay and Lenny walk back to their seats while Ghee answers the door] [Ms. B looks at Frank' paper as he writes] Okay so you're gonna change your settings, you said, to look like this, [Ms. B taps Frank' paper] so my question is if you just want to	Ms. B builds on Frank's idea when she asks him to sketch a graph for her, as he wants it to look. **[Blds, Int, Endis]** Ms. B briefly addresses student behavior that may be perceived to be off-task when she raises her voice and says, "I only need one person to answer the door". **[Dis-b]** **[Blds, Int]** Frank' paper cannot be seen on video but it may be inferred that he drew a sketch of Quadrant I where all coordinates are positive.

		make this graph what quadrant is that? [Ms. B purses her lips and looks at Frank.]	
06:58	Mark	Four!	
06:58	Frank	[Frank looks down at the floor] One!	
07:03	Mark	Four	
07:05	Ms. B	You said four you said one…[Ms. B points at Mark, then points at Frank] [Ghee approaches with papers in his hand, Ms. B turns to look at the papers, raises her eyebrows then points toward her desk.]	
07:08	Frank	Three	
07:10	Mark	(shakes head no) Four!	
07:12	Ms. B	Ooh, one, four, two! [Ms. B leans on girl's desk and smiles]	Ms. B appears ***humored*** by the many different answers that the students are suggesting.
07:13	Mark	Four	
07:15	Ms. B	Anyone think three? [Ms. B smiles as she looks around at the students.]	Ms. B appears ***playful.***
07:18	Frank	I think three, two, one… [Frank smiles]	
07:18	Quanee	I, I think five.	
07:20	Ms. B	You looked on, umm, the floor just now, why did you look on the floor?	Ms. B is ***curious*** about why Frank looked at the floor. The coordinate plane is represented on the floor of Ms. B's classroom. The students' groups are each positioned in one of the four quadrants.
07:24	Frank	[Frank leans down and looks at the floor, he is inaudible]	**[Int, Endis, Blds]**
07:28	Ms. B	Okay so, when you drew this, [Ms. B points to Frank' paper] what quadrant was that?	
07:32	Frank	One. [Frank looks at Ms. B]	Frank appears confident in his answer as he looks directly at Ms. B.
07:34	Ms. B	Do you guys agree or disagree? If you need to stand up and see….[John, Frank and girl look at	Ms. B does not validate nor negate Frank' answer, but poses the question to the whole group.

		the floor]	
07:47	Mark	Three…	
07:49	Frank	Whatever.	
07:49	Ms. B	What quadrant are you sitting in here?	Ms. B expreses interest as she builds on Frank's idea to use the floor grid as a guide to determine which quadrant is appropriate to represent the DJ problem. **[Blds, Int]**
07:51	Frank	One	
07:52	Ms. B	One. So can you come over here so that Frank can show you what he is talking about? [Mark stands up] 'Cause you are saying that you are sitting in quadrant one and he thinks that this is quadrant one right here. [Ms. B raises her eyebrows] So you need to see this. [Mark walks around the table toward Ms. B and Frank stands up.]	Ms. B invites the students to note their position on the classroom floor grid. **[Int, Blds, Endis]**
08:10	Mark	[M picks up a paper from the table and looks back and forth from the paper to the floor. Ms. B looks at him.] Oh yeah.	Mark appears to experience a breakthrough in his understanding.
08:10	Frank	Told you	
08:13	Mark	You're quadrant one. [Mark and Frank return to their seats]	
08:14	Ms. B	So if you want to put your settings, in Quadrant One that where you'll be, I'll be back. [Ms. B walks away from this group.]	Ms. B appears ***satisfied*** with the students' conclusion that they are seated in Quadrant I and that they need to set their calculator settings to show Quadrant I. **[Sat]**
End TSMIE [B.02.b.05]			

Discussion of TSMIE [B.02.b.05]

This episode begins when Ms. B responds to John, Frank and Jen who have their hands raised. The students are experiencing impasse because their graph on their

calculator shows four quadrants and it appears that the three lines representing the costs three different DJs are very close together. Ms. B appears interested and curious about the students' graph, as she looks at Frank' calculator screen and questions him when he states: "it's not a regular graph". It appears that Ms. B recognizes that the students need to readjust the window settings on their calculator, yet she does not explicitly tell them what to do. Instead, she poses a series of questions asking the students what types of numbers should be represented on their grid. It is inferred that Ms. B intends for her questions to help the students recognize that they only need to use positive numbers in the DJ graph.

Ms. B then asks the students to record their solution, including the graph, on chart paper, yet the students still seem to struggle with the graphical representation. Frank states, "we got to fix the window?" and his question appears to capture Ms. B's interest. Ms. B asks the students to identify the quadrant in which their graph will appear and she appears humored when the students offer three different answers. The coordinate grid is represented on the floor of the classroom and when Ms. B notices Frank looking at the floor, she encourages the students to use the floor to help answer the question.

Ms. B appears satisfied with the students' mathematical progress when she departs from the table.

This episode illustrates Ms. B's skill in fostering student engagement in the face of student impasse. When Frank and Mark disagree about the appropriate quadrant for graphing the data, Ms. B recognizes that Frank is using a resource (the floor grid). She invites Mark to also use this resource and the intervention is effective. Mark is enlightened by this intervention. Ms. B does not impose her own mathematical ideas

despite the fact that the students appear frustrated. Also, when Frank asks her, “You want us to graph this one?”, Ms. B responds, “I’m not telling you exactly what to graph”.

Ms. B’s sense of humor is evidenced in this episode, as she appears genuine in her humor when she smiles and laughs along with her students, while attending to the mathematics.

Immediately subsequent to Ms. B’s departure, Frank, John and Mark appear to remain engaged as they press buttons on their calculator screen. At 9:40, Frank exhibits frustration when he says, “Oh man, nothing works!”

Begin TSMIE [B.02.b.06] **14:26 – 14:55 (C, Title 2)**			
Participants: Ms. B, Frank, Mark Ms. B approaches this group in response to Frank who holds up his calculator for toward Ms. B. Previously, this group had been struggling with setting the graphing calculator window to show only Quadrant One.			
14:26	Ms. B	[Ms. B approaches the table as she looks at Frank' calculator display. Ms. B smiles broadly at Frank] Okay.	Ms. B shows ***interest*** in Frank' calculator display and appears ***very pleased*** by what she sees. **[Int]**
14:35	Mark	[Other students are speaking inaudibly. Ms. B is standing next to Frank' desk leaning toward the students with her hands behind her back.] You didn't even tell us.	
14:38	Frank	Yep.	
14:39	Ms. B	And is it right?	Ms. B expresses ***interest*** in Frank's reasoning and supports Frank's ***autonomy*** when she asks, "is it right?"
14:40	Frank	Yes	
14:42	Ms. B	How do you know?	
14:43	Frank	Cause it look like that. [Frank gestures toward his calculator screen.]	
14:44	Ms. B	Okay.	Ms. B ***validates*** Frank's reasoning.
14:45	Frank	It look like Quadrant One.	It may be inferred that Frank has entered correct window settings to show Quadrant One.
14:47	Ms. B	So now are you gonna graph what's on there?	Ms. B expresses ***curiosity*** about what Frank plans to graph. **[Int]**
14:53	Frank	I need, I need that paper.	
14:55	Mark	This one, this one [Off camera, a student from a different table is calling, "Ms. B, Ms. B." Ms. B walks away from the table]	
End TSMIE [B.02.b.06]			

Discussion of TSMIE [B.02.b.06]

This episode is brief but significant because Frank, who had been experiencing impasse during an earlier episode, has made mathematical progress in Ms. B's absence. He expresses pride in his work as he shares his progress with Ms. B: he has Quadrant One displayed on is calculator screen. Ms. B appears very pleased when she looks at Frank' calculator display and smiles broadly.

Begin TSMIE [B.02.b.07] 15:42 – 16:26 (C, Title 2)			
Participants: Frank, John, Quanee, Ms. B This episode begins when Ms. B responds to Frank who appears to have a question because he has his hand raised.			
15:42	Frank	When I did [inaudible] [Ms. B approaches Frank' desk, leans down and smiles as she looks at his calculator screen. Frank is holding his calculator in his hand]	Ms. B appears ***friendly*** and ***interested*** in Frank' work as she looks at Frank' calculator. **[Int]**
15:46	Ms. B	Okay… [Ms. B glances at Frank] So what do you have to say about that? [Ms. B's tone of voice is soft, she is leaning on Frank' desk, the camera zooms in to show Frank' calculator screen – insert screen shot if possible]	Ms. B does not offer a mathematical answer. Instead, she appears ***curious*** about Frank' thoughts about the graph. **[Int]**
15:48	Frank	…..It's weird [The camera is positioned behind Frank, so his facial expressions cannot be determined.]	Frank seems to recognize that the graph is incorrect.
15:54	Ms. B	It's weird? [Ms. B is looking at the calculator screen.]	Ms. B reiterates Frank's comment, and her tone of voice conveys s***ympathy*** towards Frank.
15:58	Jack	But this, I think, that's why	

		you gotta graph it	
16:04	Frank	Then I was…[inaudible]	
16:09	John	That's why you gotta graph it all together in the same window [The camera is positioned behind John, so his facial expressions cannot be determined. He turns his head when he speaks to Frank]	It may be inferred that Jack is referring to all three equations for all three DJs.
16:17	Frank	Okay	
16:24	Quanee	(inaudible)	
16:26	Ms. B	[Ms. B walks away from the group]	
End TSMIE [Ms.B.02.b.07]			

Discussion of TSMIE [B.02.b.07]

It appears Frank initiates this episode when he summons Ms. B by raising his hand. He appears frustrated about the representation on the calculator, which appears to show no lines. It may be inferred that source of the problem is that Frank' has set the range incorrectly in the window settings. Ms. B appears sympathetic in her tone of voice and interested in his question based on her body language, but she offers no mathematical input that may help Frank to proceed. It appears that no mathematical progress was made during this episode, which ends when Ms. B walks away from the group. It is not clear what prompts her departure. Immediately subsequent to Ms. B's departure, the students are disengaged from the mathematics. Frank exhibits frustration and slams his calculator on the desk as he says, "I quit man!". His group members respond with emotional support, La: "Don't get pissed"; Q: "Don't get mad, get glad!" but no mathematical suggestions that would lead to the solution of this problem.

Begin TSMIE [B.02.b.08] [17:35-27:03, C, Title 2]			
Participants: Ms. B and the whole class Immediately prior to this episode, Dana turns off the lights in the classroom and the noise level in the classroom becomes quiet. It may be inferred that turning out the lights is one method that Ms. B uses to get the students' attention. Ms. B waits until all students have stopped talking before she speaks. This episode begins as Ms. B stands at the front of the classroom and makes an announcement to the whole class.			
17:35	Ms. B	[Ms. B speaks to the whole class from the front of the room. The noise level in the classroom is very loud] At this time, what we are going to do, we are going to sh—share our thoughts. [Ms. B turns around to glance at the clock.] We are probably not going to have enough time to get through everything, but we gonna probably…we are gonna to get into two groups, right now. We are gonna go from group to group, like we've been doing. So at this time we are gonna stand up and push your chairs in. [Ms. B gestures movements of standing up and pushing in a chair with her hand. Students begin to talk] Just stand up, push your chairs in, you move now. Stand up push your chairs in…..[As the noise level in the classroom gets louder from students talking, Ms. B's tone of voice remains flat, but she increases the volume of her voice] Stand up push your chairs in. [some students begin to move around the classroom while others remain seated]	Ms. B announces to the whole class the students are going to share their solutions at this time and explains that the students will "go from group to group" as they did in previous lessons. Ms. B announces, "stand up, push your chairs in" five times, yet some students are still seated.

		Stand up, push your chairs in.	
18:48	Al	Can I use the bathroom?	
18:49	Ms. B	Like last period, I'm not writing passes….Stand up push your chairs in. [Ms. B turns toward Dana's table and raises her voice] I said don't talk! [Nastasia] we are waiting for you… [Ms. B stands in the middle of the classroom, her finger is on her chin, she scans the classroom] At this time we are going to start in Quadrant Two. So I need everyone to meet me over in Quadrant Two now! Move yourselves over in Quadrant Two. [The students begin to assemble around Natasha's table as Ms. B remains standing in the middle of the classroom with her arms folded.] Wherever that is, go to Quadrant Two. You should know at this point and I will meet you there. I'm gonna wait for the class to get there first, [Ms. B folds her arms as she turns to look toward Dana's table] and I'll meet you there. [The students are assembled, standing around Quadrant Two's table, Ms. B is standing near the front of the table with her arms crossed.] Okay so now, Quadrant Two, they are going to, because we need to go back to the executive board, and umm discuss what they think	Ms. B denies Al's request to use the bathroom since it is the last period of the school day. When the whole class is assembled around Quadrant Two's table, Ms. B invites someone or everyone from the group to explain their solution to the problem.

		the best deal is and *why*. So someone from Quadrant Two [Ms. B waves her hand in the air and furrows her brow] or the whole group, anyone.	
19:57	Al	Aight, Aight….We, we picking two because… [The camera is positioned behind Al, so his facial expressions cannot be determined]	Al and Natasha begin to explain their answer, but do not give any specific details, or mathematical information to support their decision for choosing Solidus Sounds. It may be inferred that Natasha and Al may be feeling ***shy*** in the context of explaining their answer to the whole class because they both stumble over their words.
20:04	Natasha	We think that Solidius Sounds has got the best umm… [Natasha looks down at her paper]	
20:09	Al	It's the cheapest, that's what we got	
20:09	Natasha	It's got the best offer because, it is the cheapest	
20:14	Al	They don't want to spend all that money… [Ms. B is seated in a student chair and is looking up at Natasha]	
20:23	Ms. B	Does anyone have anything to say? [The class is silent for a few seconds before Ms. B quietly speaks]	It is likely that Ms. B is not satisfied with Natasha and Al's explanation up to this point because they did not offer mathematical reasoning to support their decision. However, Ms. B appears to support the students' autonomy by asking the class, "does anyone have anything to say?"
20:24	Al	Anybody got any questions raise their hand?	
20:28	Rita	I got questions. [Ruth raises her hand]	

20:30	Al	Yes Ruth! [Al points his ruler at Ruth]	
20:30	Rita	Can ya'll repeat it	
20:32	Al	Ah ah	
20:33	Natasha	He said that we choose "Solidus Sounds" cause she got the best offer.	
20:38	Al	It's a he. It's a he she. [Al smiles and looks at the camera]	
20:42	Natasha	Don't come over here acting like this. [Natasha looks to her left]	Natasha is looking at another student who is off camera. Natasha appears ***annoyed*** by the way the student is acting.
20:44	Rita	I didn't, I can't just sit there…the question	
20:48	Ms. B	Devin just said something. [Ms. B is off camera, so her facial expression cannot be determined]	Ms. B appears ***interested*** in Devin's question.
20:49	Al	What you…Yes Devin?	
20:50	Natasha	What you gotta ask? [Natasha speaks to Devin]	Natasha's tone is ***direct*** and it may be inferred that her tone could ***intimidate*** a students like Devin because he is typically a quiet student.
20:52	Devin	…how you know that's the cheapest…	
20:53	Natasha	How we know cheapest one, that's a good question, thank you for asking…[Natasha looks down at her paper and smiles] because what we did was..see I say…okay down payment was a hundred dollars so we said our party was gonna last for six hours	Natasha appears ***pleased*** and ***engaged*** as a result of Devin's question as she looks at her paper and starts talking quickly.

		so it's forty dollars a hour, so what we did was forty times six and that's two hundred forty and then we added the down payment which is a hundred dollars so that equals three hundred forty dollars. [The camera zooms in to show Natasha's paper. 1. 60 x 6 =360 + 60 = 420 2. 40 x 6 = 240 + 100 = 340 3. 30 x 6 = 180 + 10] But when we did Tom's Tunes we did sixty times six and we got three hundred sixty… [Natasha straightens and raises her voice] Ya'll being real rude and ignorant! [she stares toward the door, then looks back to her paper and lowers her voice] Sorry…[Natasha smiles] That's sixty times six is three hundred sixty and then we added sixty and that equals four hundred twenty. So then for …. Where the paper at? Okay well wait a minute, then we did Light Plastic and we did thirty times six and that's one hundred and eighty…we didn't even finish that one.	
21:54	Rita	Can I ask about this one.	
21:56	Natasha	Okay go ahead	
21:56	Rita	Ummm, I disagree, I disagree with the one that ya'll picked because each--	

22:00	Dana	How, how you how you know that you picked the same hours that they picked…	
22:05	Rita	Huh?	
22:05	Dana	You picked the same hours as they picked?	
22:07	Rita	No, I'm saying, I'm saying I'm starting with one hour…this is this is why I disagree. Cause if you would go one hour for all of them, then the first one would be	
22:16	Al	You would still get the same answer	
22:17	Rita	Yea this would be one forty and that would be two o five. But if you increase by, if you go another hour you would have to add sixty more dollars, one hundred forty more dollars and two twenty, …like for instance if you go to two hours at Tom's Tunes they'd be one twenty, and then if you go to Solidus what ever the name is Sounds, it would be two eighty and but if you go to Light Plastic it'd be four ten, so each hour its increasing by sixty, one forty, and two o five, and that's how come I disagree with that. [Rita points at Natasha's paper]	Ruth has made math errors. It appears that for each DJ, she has added both the down payment and the hourly rate, then multiplied this sum by the number of hours. For example, Tom's Tunes charges a sixty dollars an hour with no down payment, so two hours would cost 60 * 2 = 120 dollars. Solidus Sounds charges forty dollars an hour plus one hundred dollars as a flat fee, so Ruth added 40 + 100 = 140, then multiplied 140 * 2 hours = 280 dollars Light Plastic charges thirty dollars an hour plus a one hundred seventy five dollar flat fee, so Ruth added 30 + 175 = 205, then multiplied 205 * 2 hours = 410 dollars.
22:51	Natasha	Okay but the reason why *we* agree with this is because it says… [Natasha points to the	It appears that Natasha does not recognize the mathematical errors in Ruth's argument. Natasha appears to be defending the

		problem paper].	reputation of her group when she emphasizes, "why *we* agree".
22:57	Al	We wanted to pick that one is cheaper. [Al is seated in his chair, Natasha looks at him briefly]	
22:57	Natasha	Yea and it says, sixty per hour, we said we was gonna party for six hours so if you do sixty times six that is three hundred sixty. But when you look at this one right here, that's three hundred forty.	
23:09	Ruth	But.. [Ruth looks down a paper]	It appears that Ruth disagrees but Ruth cuts her off.
23:10	Natasha	Plus	
23:11	Al	But it's cheaper!	
23:12	Natasha	Yea it's cheaper, so why not choose that, why not pay one forty, yea why pay four twenty between.	
23:18	Al	…..[inaudible]….it's old people in that house. [many students are talking at once]	
23:21	Ghee	Yea that's cause you have ten hours you could have picked, Slash the Sounds!	
23:21	Dana	Shut up wait till we to ours! [Dana furrows her eyebrows and raises her voice in response to Ghee's comment]	
23:23	Rita	I don't get this because I don't get this because [Ms. B is seated, looking up at Ruth, her arms are folded.]	
23:23	Natasha	Solidia Sounds!	
23:30	Ghee	Plastic, Plastic…no Light Plastic	
23:30	Natasha	Oh.	

23:32	Ghee	You would have did ten hours, you would have got a better, a better, deal a what you name a better jam	
23:35	Dana	Why don't you just be quiet and wait till we get over there to our group?	It may be inferred that Dana is ***annoyed*** with Ghee because he is talking about their solution when it is not their turn to present.
23:42	Ms. B	So what do you think for yourselves? [Ms. B looks at Natasha and rubs her chin, she glances rapidly back and forth between Natasha, Ruth and Dana]	
23:42	Natasha	Well we don't wanna party for no ten hours, we said six. [Natasha leans her upper body toward Ghee as she speaks]	It appears that Natasha is ***angry*** *that her group's answer is being questioned.*
23:45	Rita	I about to use yours, you said ya'll partying for, ya'll wanna party for six hours. If you partying for six hours at Time Tunes [Ruth points at her paper as she speaks. Ms. B looks on silently rubbing her chin] it would be three hundred sixty, but if you go to Solidius Sounds eight forty.	Ruth's erroneous mathematical reasoning is confirmed. She has calculated Solidus Sounds using the calculations: (100+40) * 6 = 840.
23:55	Natasha	No it would not. [Natasha looks at her own paper and shakes her head, no] …..no it would not!	Natasha disagrees and raises her voice in anger as she says, "no it would not" a second time.
23:57	Rita	Yes it would because it is increasing by one forty. It said, it said right here [Ruth points at the problem paper] Solidius Sounds charges one hundred dollars plus forty dollars per hour.	Further confirmation of Ruth's incorrect reasoning. Ms. B does not intervene, she is seated, looking on silently as Natasha and Ruth argue. Ruth appears confident in her own reasoning as she disputes with Ruth.
24:04	Natasha	That's exactly what we did! [Natasha points at her own	Natasha correctly describes the process for calculating the cost of

		paper, her voice is raised] The down payment is a hundred dollars, okay we did that. We wanna party for six, umm hours, so we did forty times six, that's two hundred forty, yea then we added a hundred, so that's three forty, so how did it equal all that.	Solidius Sounds for six hours. Natasha appears agitated that she and Ruth have different answers for the same question. She appears confident in her groups' process as she reviews the calculations.
24:16	Dana	And then she added a hundred	
24:19	Natasha	So how that equal all that?	
24:20	Dana	[Dana leans across the table to look at the problem paper] Cause she wasn't doing this first she was gonna do last after they….like after the whole part finished then that's what she was gonna pay cause she did forty times six, which equals two hundred forty then she added a hundred..cause she wanted it for six hours.	It appears that Dana is defending Natasha's reasoning to Rita but Rita does not seem convinced.
24:32	Rita	Well..one forty, we'll I did one forty	
24:36	Al	But we did all of them to see what was gonna come out with the bettest deal.	
24:38	Students	The bettest deal, what is a bettest deal?	It appears that the students are mocking Al's pronunciation of "bettest".
24:40	Al	The better, better deal…better…better [Al smiles as he waves his ruler around in the air. He is the only student who is seated.]	
24:46	Ghee	Bet on, zoom in shine on…	
24:48	Al	Any more questions? [Ruth looks down at her paper, her lips are pursed]	
24:48	Ms. B	Do you want to bring yours over Dana? You can bring	Ms. B expresses interest in Dana's group solution.

		yours over. [Ms. B's tone of voice is gentle.]	
24:50	Dana	Nah I don't wanna bring yours over, I'll sit here and then do it. [Ms. B's eyebrows are raised as she looks at Dana]	Dana appears agitated, and Ms. B remains firm in her suggestion for Dana to bring her group's solution to share with the whole class.
24:54	Ms. B	Bring it over since we are all over here. Bring it over. [Ms. B remains seated, she leans forward in her seat, holding her knee with both hands]	
24:58	Dana	I ain't bringin' that big thing over there!	
25:00	Ms. B	Larry and [inaudible] [Ms. B is leaning back in her chair, she is looking toward the door, pointing her finger, the camera pans to show Larry and Ghee standing near the door of the classroom] he and ….okay so go ahead. I'm listening cause I wanna see. [Ms. B looks around at the students, then pushes her chair backwards]	Ms. B appears agitated by the perceived of-task behavior exhibited by Lenny and Ghee. Ms. B expresses interest in Dana's solution.
25:25	Dana	Yes it is it's right here. [Dana raises her voice] Oh well we picked ten hours because we wanna party all night long so…[Dana smirks] why ya'll didn't put the….	
25:27	Student	During the first five hours we gonna make it that romantic date, like that diner, then for the other five hours it's gonna be all club	
25:41	Ms. B	Go ahead Dana	
25:25	Dana	We picked ten hours because we wanna party all night	It may be inferred from Ms. B's facial expression that she is ***concerned***

		long so we did…[Ms. B is looking at Dana, she is not smiling] We did…well we picked umm Light Plastic because we umm multiplied three hundred I mean thirty times ten…[Ms. B glances around at the students]..it's three hundred	about the progress of the mathematical discussion as Dana stumbles over her answer. It appears that student engagement is diminishing as students begin to shuffle and the noise level increases.
26:05	Al	Why did ya'll pick Light Plastic? [Al taps his shoulder with his ruler]	It may be inferred that Al is truly ***curious*** about Dana's choice.
26:07	Dana	Because we wanted the party, since we wanted to pick, since we wanted to party for ten hours straight, we multiplied each one and umm light plastic had the umm best deal. [Dana looks at Al]	
26:17	Ghee	Yes for ten hours cause you know we wanted to party all night long…all night long..siscuo (dancing)	It may be inferred that Ghee is clowning around, as he dances while he speaks.
26:45	Natasha	For ten hours?	
26:47	Al	Exactly…Exactly	
26:48	Ghee	Yea cause the last two hours is gonna be a wu tang party, ya heard. [Students laugh]	
26:50	Dana	The last four hours is gonna be a wu tang party	
26:57	Ghee	Oh aight, you the party planner.	
26:59	Lenny	The first two hours is gonna be romantic [everyone in the class laughs, including Ms. B]	
27:03	Ms. B	At this time…because we are already over by I think five minutes, my homeroom is outside, what I need for my responsible students in three o nine to do…my	Ms. B ends the class without offering any of her mathematical ideas regarding the students' answers. She refers to her students as "responsible" as she suggest that she

		responsible students to gather up your work very very neatly along with the main (inaudible) and put it together….(chatter insues as students pack up, lesson is over)	has ***confidence*** in them to clean up their work and return her supplies.
End TSMIE [B.02.b.08]			

Discussion of TSMIE [B.02.b.08]

This episode begins when Ms. B announces to the whole class that they are going to share the groups' work. She acknowledges that they will not have enough time to review all students' work, but only the work of two groups. Ms. B is calm and stern when she instructs the students to assemble in Quadrant Two. She conveys trust and high expectations in her students when she tells the students that she is not going to direct them to Quadrant Two, stating that they should know how to get there by now.

Ms. B invites Natasha's group to share their solution with the class. Natasha and Al get off to a slow start, stumbling over their answers at first. When Ms. B draws attention to a question posed by the quiet student Devin, Natasha's level of engagement markedly increases. Rita disagrees with Natasha's solution and describes her own solution, which illustrates that she has made errors in her procedure. To calculate the DJ costs, Rita added both the hourly rate and the down payment, before multiplying by the number of hours.

Ms. B does not address these errors, but invites Dana to present her solution to the class. The class period ends before Dana can provide a complete explanation of her group's solution.

This episode illustrates an example of how Ms. B encourages the student autonomy and student discourse.

APPENDIX C

Thesis Data Record

Ms.E, "Adding and Subtracting Fractions", Day One & Day Two

TSMIE	Time codes	Camera angle	Duration	Themes Discussed in Results
E.01.a.01	**11:10 – 12:30**	Lina	1:20	
E.01.a.02	**13:30 – 14:53**	Cecilia	1:23	Interest
E.01.a.03	**15:08-17:11**	Lina	2:03	
E.01.a.04	**18:43 – 22:14**	Cecilia	3:31	Validation
E.01.a.05	**22:16 - 23:39**	Lina	1:23	Validation
E.01.a.06	**23:40 - 30:26**	Lina	6:46	Interest
E.01.a.07	**33:43 – 35:04**	Lina	1:21	
E.01.a.08	**38:10 – 40:59**	Lina	2:49	
E.01.b.01	**1:37 – 4:23**	Cecilia	2:46	No interest
E.01.b.02	**1:34 – 6:29**	Lina	5:55	Interest
E.01.b.03	**10:15 – 12:40**	Cecilia	2:30	
E.01.b.04	**9:30 – 15:03**	Lina	5:33	
E.01.b.05	**16:44 – 19:28**	Lina	2:44	
E.01.b.06	**25:56 – 34:10**	Cecilia	8:14	

Ms. E, "Finding Areas and Other Products", Day One & Day Two

TSMIE	**Time codes**	**Camera angle**	**Duration**	
E.02.a.01	**9:45 – 18:33**	Cecilia	8:48	Autonomy / not autonomy
E.02.a.02	**23:00 – 25:42**	Cecilia	2:42	
E.02.a.03	**30:28- 32:35**	Cecilia	2:12	
			Total: 13:42	
TSMIE	**Time codes**	Camera angle	Duration	
E.02.b.01	**3:11 – 13:19**	Cecilia	10:08	
E.02.b.02	**12:39- 38:51**	Lina and Cecilia	26:12	
E.02.b.03	**45:07 – 46:38**	Lina	1:31	
E.02.b.04	**47:07 -50:44**	Lina	3:37	
			Total: 41:28	

Approximate total for Ms. E: 99:28 (21 episodes)

Student engagement immediately subsequent to episode (within 2 minutes)
Engaged = 13 episodes

Not engaged= 3 episodes

Not determined = 2 episodes

Ms. B, "Fences for Grazing", Day One

TSMIE	Time codes	Camera angle	Duration	Themes Discussed in Findings
B.01.a.01	**3:35 – 5:51**	Still	2:20	
B.01.a.02	**8:35 – 13:48**	Still	5:10	
B.01.a.03	**12:40 – 19:14**	Cecilia	6:30	
B.01.a.04	**19:43 – 22:51**	Cecilia	3:10	
B.01.a.05	**21:41 – 27:40**	Lina	6:00	
B.01.a.06	**29:20 – 34:06**	Cecilia	4:40	
B.01.a.07	**35:01 – 37:59**	Cecilia	3:00	
B.01.a.08	**36:40 – 39:58**	Lina	3:20	Validation
			Total: 34:10	

Ms. B, "Fences for Grazing", Day Two

TSMIE	Time Codes	Camera angle	Duration	
B.01.b.01	**1:38 – 3:34**	Still	2:00	
B.01.b.02]	**3:09-4:49**	Cecilia	1:40	
B.01.b.03	**6:40 – 11:21**	Cecilia	4:20	
B.01.b.04	**15:22 – 16:39**	Cecilia	1:10	
B.01.b.05	**24:00 – 25:45**	Lina	1:45	
B.01.b.06	**26:37 – 28:05**	Cecilia	1:30	
B.01.b.07	**28:34 – 30:11**	Lina	1:30	
B.01.b.08	**31:04 – 31:31**	Lina	0:30	
B.01.b.09	**32:06 – 33:13**	Cecilia	1:00	
B.01.B.10	**33:40 – 35:54**	?	2:00	
B.01.B.11	**38:59 – 40:46**	Still	1:45	
			Total: 18:45	

Ms. B, "The DJ Problem", Day One

TSMIE	Time codes	Camera angle	Duration	Themes Discussed in Results
B.02.a.01	00:00 – 7:15	Lina	7:15	
B.02.a.02	10:25 – 12:53	Lina	2:30	
B.02.a.03	12:32 – 17:35	Cecilia	4:30	Validation
B.02.a.04	17:49 - 18:57	Lina	1:10	
B.02.a.05	20:04 - 23:21	Cecilia	3:20	Validation
B.02.a.06	23:51 - 32:22	Lina	8:30	Validation
B.02.a.07	32:20– 36:14	Lina	1:30	
B.02.a.08	36:36 – 42:36	Lina	7:30	
B.02.a.09	44:10 – 46:51	Lina	2:40	Validation
			Total: 39:00	

Ms. B, "The DJ Problem", Day Two

TSMIE	Time codes	Camera angle	Duration
B.02.b.01	3:13 – 6:04	Lina	3:00
B.02.b.02	6:20 – 9:49	Cecilia	9:40
B.02.b.03	10:14 – 14:34	Lina	4:30
B.02.b.04	22:25 – 25:05	Lina	2:40
B.02.b.05	2:02-8:14	Cecilia, Title 2	
B.02.b.06	14:26-14:55	Cecilia, Title 2	
B.02.b.07	**15:42 – 16:26**	Cecilia, Title 2	
B.02.b.08	**17:35-27:03**	Cecilia, Title 2	
			Total: 36:00

Approximate Total for Ms. B: 127 minutes (36 episodes)

Student engagement immediately subsequent to episode (within 2 minutes)
Engaged= 19 episodes

Not engaged = 7 episodes

Not determined = 8 episodes

APPENDIX D

Retrospective Stimulated Recall Interview Protocol

Part III: Follow-up/Post-taping Interview (45 minutes)

Purpose: This interview will be conducted immediately after each cycle of videotaping a teacher's classroom. The purpose is to see how the teacher feels the lesson went and how the students felt during the lesson. In particular, we will show the teacher videotape of the class we observed and ask the teacher to react to what he or she sees.

Instructions: The questions are meant to be broad and open-ended and will hopefully encourage the teacher to speak at length. The questions are organized into sections, with a leading statement (sometimes a leading question) followed by subquestions. The teacher may end up answering some of these questions in their response to the other questions, in which case the interviewer should take care to not ask something which has already been answered. Some of the questions will usually be redundant. However, these questions are included for the interviewer's sake, should the teacher need some prodding or should the interviewer need to fill up more time.

We would also like the interviewer to pick up and respond to particular *cues* the teacher may give. By cues, we mean general statements the teacher might make which could lead to interesting examples or discussions or statements about which the interviewer would like some clarification. For example, a teacher might state he or she thinks the students found the lesson hard, to which the interviewer might respond, “Why do you think they found it hard?” or, “Could you describe an aspect of the lesson which the students found hard?” or even, “Did any of the students find the lesson easy?” This is

largely left to the discretion of the interviewer, but it should be apparent when we would like the interviewer to pick up on a cue. In general, responses to cues should be of the form:

- Could you talk more about that?
- Could you give me an example?
- Could you explain why you think that?
- Could you go into more detail about that?
- Could you describe that to me?

In certain places, we may explicitly suggest that the interviewer ask similar questions. However, as the preceding discussion suggests, this in no way means that these are the only instances when the interviewer is encouraged to interject such questions.

Finally, some specific instructions are interspersed throughout the interview in italics, including pieces of information which the interviewer must specify him- or herself (e.g., his or her name) and suggestions for follow-up questions which depend heavily on the responses the teacher has given during the interview. *Note: some of these italicized instructions require planning prior to the interview.*

Questions:

Hello, *(teacher's name)*, my name is *(interviewer's name)*. We're meeting today to talk about the lessons we observed *(appropriate quantifier, for example "two" or "a few")* days ago. I'm interested in how you thought the lessons went and how your students reacted to the lessons. *(Include a brief recap of the lesson so as to refresh the teacher's mind.)*

1. First, could you share some of your thoughts about the lessons we observed?
   a) What about these lessons stands out in your mind?
   b) How well do you think your students understood the lessons? What convinces you of that? Could you give me an example?
   c) How did the group we observed compare to the other groups you have?
   d) How do you feel about the way you handled the lessons? Would you change anything if you were to do it again?
   e) Did you accomplish everything you wanted to?
   f) How did this lesson compare to the lessons you teach when we're not videotaping?

2. I would like to show you a few video clips we found particularly interesting. I'd like you to focus on your students' feelings and their mathematical thinking when you watch these episodes. *(Show a clip. Follow up with questions a-e below. Then show the next clip and repeat.)*
   a) Could you describe what is happening at this point in the class?

b) In your mind, what happened in the clip you just saw?

c) *(turn the focus to a particular student; repeat this for each student in the clip)* What emotions do you think this student is feeling?

d) *(if appropriate)* What sort of mathematical thinking and behavior do you see here?

e) How do you feel about this part of the class? Is there anything in this clip that surprises you?

3. Now I'd like to ask you about your students.

a) How did your students react to the lessons?

b) Do you think your students were engaged by the lessons?

- *(if so)* What convinces you of this? Do you think your students enjoyed the lesson? Were there any parts of the lesson that did not engage your students? *(if so)* Why do you think this happened?
- *(if not)* What convinces you of this? Why do you think the lesson failed to keep their attention?

c) Did any particular students like the lessons? Did any particular students dislike the lessons? *(in either case)* Why do you think this student felt this way?

d) Did any particular students do well with the lessons? Did any particular students fail to grasp the lessons? *(in either case)* Why do you think this student performed this way?

e) Did your students work in groups? *(if so)* How were the groups chosen? How

well did they work together?

f) Did your students share their mathematical ideas?

- *(if so)* Could you give me an example?
- *(if not)* Why do you think this is?

g) *(if students did share ideas)* Did your students listen to each other? Did they respond to each other's ideas?

- *(if so)* Could you give me an example?
- *(if not)* Why do you think this is?

h) Did your students get into arguments?

- *(if so)* Could you talk about this in more detail? How did you handle this situation? How did the students respond to the situation?

i) What was the most difficult part of the activities for your students? Why did they have trouble with this part?

j) Did any students reach an impasse during the activities?

- *(if so)* Could you give me an example? Why do you think this occurred? Were the students frustrated? How did you deal with this situation? How did the students respond to this situation?
- *(if not)* Were any of the students frustrated with any parts of the activities? *(if so)* Why do you think this occurred? How did you deal with this situation? How did the students respond to this situation?

4. I would like to talk about the strategies you used to present these lessons.

a) *Recall some of the strategies the teacher discussed in the previous interview.*

Ask the teacher if they used these strategies and, if so, how and when they were used. Ask the teacher how well these strategies worked.

b) Will you teach these lessons again in the future? How will you teach them? Will this be any different from the way you taught them this time? How so?

5. I would like to conclude this interview by asking you to reflect for a moment on the lessons we observed. Could you talk about what sticks out most in your mind about the lessons? Is there anything that occurred in the lessons that we didn't talk about that you think is important? *(if so)* Why is this important to you?

Thank you very much for your time, *(teacher's name)*, and I have enjoyed working with you during this project!

APPENDIX E.1 MS. E RETROSPECTIVE STIMULATED RECALL INTERVIEW

"Adding and Subtracting Fractions"

Speaker	Transcript
I	My name is [C], we have met before, we are meeting here today to talk about the lessons we observed and you know we were here Wednesday and Thursday and when I refer to or when I ask you a lot of these questions I want you to think back about how you were feeling or what you were thinking on Wednesday and Thursday and not so much what you are thinking or feeling right now, kind of go back.
Ms. E	Ok, right now, I am going to try, try. backtrack
I	So we are really interested in how you thought the lessons went and what your students reacted to the lessons, so. Of course we are working on adding and subtracting fractions and we did the first investigation working with just section 18 on the drawing. So first could you share some of your thoughts about the lessons we observed
Ms. E	Umm…this particular class…umm…I was ***stuck*** in the beginning Because I was looking for a certain answer which is 64, ***I was hoping someone would say 64 and that didn't come up and so I decided when I didn't come up and I had to make a decision rather that to say 64 or to just let it be and at that point*** **I decided I will just let it be** ***and see how it goes so*** that's how I started the lesson that day but I felt very very uneasy ***because I wanted them to start with 64 because I thought it would have been easy but also didn't want to suggest my own thinking at that point I wanted them to just come up with something and see how it went***, so that class on Wednesday became a class of more more like ok proportions rather than what I intended it to be and its quite quite, **I was very *uneasy* because it was the fourth class that I was doing that day and the three classes went the way I expected them to go and this class didn't** so the 64 came up with the 3 classes without me giving them a hint or anything and that was that one line that I wanted to hear and I didn't hear from them so I didn't know which way to go first
I	Umm okay, anything else about these lessons that stands out in your mind
Ms. E	Umm it seemed to me that the ***students are still not umm familiar with naming fractional parts*** so it became also that the last two days that we were doing was about naming the fractional parts, the original intention for this lesson was to add and subtract fractions ***but the students were not ready to do that*** I was thinking unless they can name the fractional parts they cant start adding so ***it took longer than I thought it would***
I	Umm how well do you think your students understood the lesson, I know you started to answer this a little bit umm so how well do you think your students understood the lesson and kind of how things convinced you of them

Ms. E	Wednesday and Thursday both days ***they struggled*** and they they, if I am going to talk only about Wednesday and Thursday, there ***was very little umm progress done.*** Umm the first day was just about the drawing, was purely the drawing and the second day they were developing their concepts of how to name the fractions. ***So there was very little of what I intended to do which was adding and subtracting,*** if there was even any. Umm and there were ***students who were umm I guess getting confused with the model in terms of naming the fractions***, I can name [Felipe], [Juan] and there was Richard I think was naming the fractions with dimensions of a (laughs) particular portion within the different sections so umm. But I was going to let that go which I did eventually I did I thought myself this is a good piece for, for presentation so that's what we did today. But at that point this is a good conversation piece and it would show that some kids are able to name them correctly and ***others are still struggling and that could start umm what could be a very good presentation.*** So I thought umm even on Wednesday and Thursday of starting off with students who thought about naming the fractions with a dimensions of a side and see how it goes because I have seen other kids name them umm with 64 as the denominator and counting individual boxes. Obviously that is one way and they have some of them are able to do equivalent fractions. So there is the concept umm of naming the fractions correctly some of them are able to do it, I would say if I am not mistaken ***about 3/4ths of the class were at least able to name in that manner umm several parts of section 18*** and attempted to name it in section 19 as well.
I	Okay…umm how did the group we observed compare to other groups that you have.
Ms. E	Yes I don't know if it's me if I was giving it, if I was giving them more time but the three other classes are now on a different lesson. Three classes are now on 4.2 umm my focus with the three other classes was to name equivalent fractions and then today we started 4.2 which would lead to adding fractions and later on the algorithm but this class is still at the point where they are naming the fractions so umm it's a three other classes are in 4.2 and this class is still in 4.1 and ***I am thinking am I taking my time so much on this class because I am looking for affect in particular and in the other classes I wasn't looking for affect in particular at all and I was just looking to find the correct answers and find the equivalent fractions*** so I am also trying to reflect on how if that's what ***I am trying to do with this particular class since I know I am looking for affect am I tailoring this class so much on the affect and*** allowing the other classes to go on with the lesson which like I said is my focus in those three and keep my focus in this class on affect so and so it's the same teacher and the same lesson and the same group of kids, or comparatively the same so I don't know ***I have to reflect on that and make sure I am not focusing too much on the affect and also I am doing the same thing I am doing with the other classes.***
I	Umm...Is there anything else as you started talking about this a little bit?

	Would you change anything if you were to do this again, if you were to teach this lesson again is there anything in particular that you would want to change?
Ms. E	If, I am very much concerned with my timing with the pacing. Umm I will do the same thing that I was doing with the other two classes umm I am able to take my time with this class and because of another factor. ***This class is small. And I can individually talk to them and find out what they are thinking and then again it's making this lesson***, it's extending this lesson, ***more then I want it to be extended. But there is a different focus too and so I am accomplishing something different in this class, which is individualizing instruction*** I don't know, for a lack of a better term. Umm yeah.
I	Okay…umm did you accomplish everything you wanted to? (turns the page)
Ms. E	On Wednesday and Thursday no, even up to today ***I didn't accomplish the things I wanted to accomplish*** with that group, their pacing as I said earlier they're behind compared to the other three classes that I have
I	Okay…umm so now we are going to get into the video clips
Ms. E	Okay
I	So you know you are going to have the laptop in front of you and you can watch the clips, so let me know, I guess the sound is ok ("yeah" in the background by a female) so it shouldn't be that bad so now this one here I just want to kind of introduce it a little bit, umm basically this is going to be [Manuel D] and as you can see it's when they are day 1, this is Wednesday and they are working in the group umm and let's see what happens. **[Ms. E views KAE 1 on laptop]**
Ms. E	Okay…uh huh
I	You want me to pause it (looking at someone else) we will just kind of go with what you saw so far…okay…so could you describe what's happening at this point in the class? (beep)
Ms. E	Okay…umm Manuel is trying to find out if N'yette, N'yette's model is workable? And umm..umm its becoming as I said earlier a lesson of proportion (beep) and ***<u>I was trying to point out to them the problem is asking for square so some of the models are not going to fit what the problem is asking, which is a square, umm and I didnt know how to go about it at that point so I was just asking questions</u>*** and then I couldn't even follow how Manuel was computing and how he was deciding the model could be used or the model could not be used, he wasn't focusing on the square but he was focusing on umm if I am not mistaken the divisibility by 640, is is the number of boxes divisible by (someone takes the laptop and fixes the screen) 640 evenly and if it was then he would say its okay and you can do it. I thought that was what is going on
I	Okay..umm so now you already started to focus on particular students but anything else so for example umm Manuel D, also have Jose umm N'yette, anything in particular, ***what emotion did you think these students were feeling***?

Ms. E	Umm Jose is just observing and umm I don't know if he understood it but he wasn't asking any questions umm N'yette is trying to defend her model, she has created a model with umm Tiffany ***and she's trying to defend that.*** Umm (points to the laptop screen) Manuel and umm Daniel were trying to check on N'yette's model as far as their computations are concerned, if you notice there was also calling in a couple of different students to watch and ***I do that when I think that they, whatever they are talking about here can influence or inspire the other students, at least one person from each group so when they go back to their group they have something to bring back to their groups so I was calling Noah and I was calling*** Manuel (interviewer says something in the background) to stay and watch what was going on because I think there wasn't much that was going on in their group and something was going on in this group and I was hoping they could see what's going on here and get some ideas before they go back to their own group
I	Umm okay…how do you feel about this part of the class?
Ms. E	Umm right now or when this was happening?
I	When that was happening.
S	When this was ***happening I was trying to understand and follow everyone's thinking,*** I was trying to N'yette's reasoning even after I said 1 square mile I was pointing at the book and there was one point where she wanted to say something I said lets focus on this one first because I really wanted her to see that it specified that it's one square mile so anything of the model that doesn't fit the square shouldn't be used at that point, that is what I was thinking and then umm [Manuel] was insisting on his computations and even agreeing to N'yette, N'yette's umm model is okay because its divisible by sixty-four or I think six-hundred forty was same.
I	Okay…umm is there anything in this clip that surprises you?
Ms. E	Umm no this is how it goes usually, umm students go with their thinking ***no matter how much I try to sway them on something that I want to think of when they are not ready to listen to it or when they are not ready to focus on what I want them to focus on they will just keep on going*** on with what they want to say and ***it doesn't matter what the teacher says and it's not surprising to me.***
I	Umm okay…umm so now we are going to do this with the next clip
Ms. E	okay
I	So let me just make sure it's on the correct title (works with the laptop) are we good with the battery? (to another person), we are good **KAE 2**
Ms. E	Should be...i changed it (watches video)
I	So what do you think about that? Describe what was happening at that point in class?
Ms. E	Yes, they were caught up in a drawing and it doesn't seem like its progressing at all so I wanted to at least set some parameters so lets just say

	its 64 and I didn't know how it was going to go because they were just going with their thinking, there was another group of students umm making 640 smaller boxes so I was trying to umm I was trying to set some parameters saying that okay…do a square with 64 inside which was my original plan anyway just didn't come up but I thought in the beginning I was allowing them, the previous clip if I may go back to that I was just allowing them because there is some mathematical value to what they are doing, it wasn't necessarily what I wanted to do that day but the thinking that was going on was very powerful, if, if you know that the lesson was on proportion they were definitely on target but that wasn't the lesson at that time so I think I did this I don't know what time I did this umm during class but towards the middle or towards a few minutes toward the end of the class when umm ***I see that people are just struggling with the drawing and its becoming the drawing class and not the math class so I said okay let me set some parameters and see where it goes***.
I	Okay…so umm this particular clip I want you to focus on and we can play it again if you would like, but I would like you to focus on what N'yette and Tiffany were doing, now you said that you tried to focus parameters so you announced to the class what those parameters were so umm ***what emotions do you think that [N'yette] was feeling at that point.***
Ms. E	Can I see it again? I need to watch it again
I	You want to watch it again? Yeah no problem
Ms. E	Play that again…its paused (as she watches the video on the laptop)
I	Yeah…how do I do this? … Nope (trying to get the video on the laptop to work)
Ms. E	It's still on pause ….its not doing anything.
I	Yeah I know maybe I should just go find another copy
Ms. E	Press down…down um hmm
I	Wow, too fast, yea there you go
Ms. E	[KAE 2 – SECOND VIEWING]
I	Can I pause it…yeah aright (stops the video)
Ms. E	Yeah, I don't think they were listening at all...umm yeah, Tiffany looked like she was umm, her, her, she was staring in the air. And [***N'yette] even with her body language she was looking the other way and wasn't listening to what I was saying so they just wanted to do what they started.*** Umm and umm ***I am not going to insist on any of the students if they don't do 64***, I wasn't about to insist on anyone to do what I wanted them to do. But that is what they were doing; they were going with what they already had.
I	Umm…okay. Let's see. (Pauses for a couple seconds) aright we are going to show you episode 3 now. Again the same thing umm with here we are going to focus on N'yette (mumbles under her breath and plays the video on the laptop)

Ms. E	Ms. E views KAE 3
I	You can pause it now (pauses the video on the laptop)…okay
Ms. E	Was this Thursday or Wednesday?
I	This was Thursday…yeah
Ms. E	This was Thursday?
I	Yeah this was Thursday umm also it didn't appear here in the clip, not sure where it did but umm she, you called her over to this group umm she was actually at her own group and then umm you showed her a drawing and you said do you agree this is 3/4ths and umm and and you had her look at the drawing and she's like I don't agree with this and that's when she went over and started talking and was wait a minute and was like I don't agree with you that its 3/4ths
Ms. E	And what did she say it was 1/4th and the other one was 1/2? Umm I think what happened was that she is calling a certain portion ½, one that was smaller then 1/4th and if my memory serves me right, oh no I am trying to go to Friday…if I..i am not ***sure but she as far as emotions are concerned she would be sure about something and I am not even sure if it's correct at this point and she wanted to prove that and she's saying others don't make sense***, other things don't make sense, mine must be right…something to that effect like I make a lot of sense so I am right because everything else I heard didn't make any sense so I must be right…you know. Yeah I think that was what she was saying.
I	Umm okay …anything else regarding their emotions that you she was feeling at that moment because right now, now she's working with a this group of boys
Ms. E	She seems, ***she seems empowered because I called her to explain something to another group which umm she obviously liked very much and umm she was prepared to do it.*** Whatever she believed in she was ready to approve that she was right and she didn't get intimidated with the questions with what the others were saying and umm the body language, the children were, looked like they were engaged in this one…whatever it is they were talking about so if anything, **if we didn't even accomplish what we wanted to accomplish that day there was another layer to the lesson that I think we accomplished which is you know sharing their ideas** and defending their answers and if anything these two days did that for some of the kids, I think
I	Okay…umm let's see, how do you feel about this particular part of the class, this was kind of to give you a time frame I believe this was in the beginning of the class, early early in the beginning of the class.
Ms. E	Just this particular part of class…I think I set parameters in this one and I had said 64 meters umm 64 boxes and umm they tried that but I still limited

	them to section 18 and **umm they accomplished very little as far as my goal was concerned** which was to at least name the different parts of that section. ***And at the end of the day, okay if I am going to focus on this particular part, umm <u>what I am building on at this point is just the environment.</u>*** Okay I can call in somebody to join a group and ***<u>that it is okay that your answers are different and you can talk about it umm you can defend your answer and the teacher doesn't have to be there all the time.</u>*** Umm I saw that at that point I don't think I am in this group at all. Am I in this group?
I	Umm if I keep playing it I believe you, you walk over to them eventually
Ms. E	Right but I had left that group and I saw them still interacting and umm that was powerful for me, ***that the teachers not around and still they are engaging in what they are doing.*** And the conversation was limited to math and, and they look like they are engaged even without adult supervision. So…umm yeah I like to see things like that in class
I	We are going to show you now…umm…it's going to be N'yette and Elijah. Again same group umm still second in. (starts the video on the laptop) [KAE 4]
Ms. E	Okay…okay (watches the video)
Ms. E	They are getting up. It seemed like umm, N'yette had to remove her sweater and Noah had to fix his jacket…
Ms. E	That's when I joined them
Ms. E	Half of 4 is 2, oh okay, there were 2 different ideas here that I didn't follow, one was the idea that they were creating a certain algorithm for multiplication I think, ½ times something then its 1/6 so they are thinking maybe we should add the top, then Elijah thought he was right so he rest his case, it looked like you know he was satisfied. N'yette looks like she was convinced so she had a name for it. Now she went back to explain it to Julia and that's the only time I followed her thinking, why she was calling that smaller portion 1/2. She said if this is ¼ then half of 4 is 2 so this little piece right here must be ½ and she wasn't thinking in terms of how big ½ is and ¼ is and I believe that I questioned her on that umm after this episode. That she had realized that of course ½ should be bigger then ¼ so this cant be ½ if it's smaller then 1/4, I think I remember that. But the way she reasoned that that smaller portion is ½ because 4 divided by 2, half of 4 is 2. So she diving the denominator. Umm she had, she had a certain algorithm, its not out of the blue I want to decide this is 1/2, it's not like she was guessing, she had an explanation for what she was doing. The body language showed a lot umm I think umm when N'yette had to remove her sweater and Noah was fixing his umm umm jacket, it seemed like the ***conversation is getting heated*** and umm umm Elijah even had, umm he was ***very proud of himself and umm he had*** explained something and N'yette agreed with him. ***He looked satisfied when he stood up and he looked like he was proud of himself so***. And Julia looked very very

	confused with the quick talking. I don't know if Manuel in the back, Manuel C is even engaged. I don't know if he is but that day I know he was thinking. Many days he's not thinking but that day I think he was. Definitely Noah was engaged, N'yette was engaged, Elijah was engaged, Julia wanted to, Julia was engaged she just can't follow the thinking. I think that's what's going on.
I	Okay…umm so umm now I would like to ask you about your students overall. So not only the ones we have been seeing in clips but your class all together. So how did your students react to the lessons?
Ms. E	Okay…to the lessons…umm the way that I was doing the class ***they are happy about umm they are happy to defend their answers.*** They said in the end I want to do this again tomorrow and you would hear them umm after this class, we were very late that day, umm and then even going down to the cafeteria, we took a long time going, we saw the principal and she was saying how late they are already. Not, she wasn't blaming me but she was pointing at the kids, kids at that point were talking about the math class, at the stairwell, ***they were talking about how fun the math class was, umm that day.*** Umm and I saw, I also saw that in the other kids in the other classes ***that they feel empowered if I ask them to come and talk to the other kids about their thinking, they like that***. Umm some are still not willing to do so, there is a couple of different kids here that I can name, such as Philix and umm umm Isle I think who didn't want to explain their ideas, didn't want to go up to explain, same thing happened today but I think they are getting their idea that you are going to have to explain your thinking in class and that you are going to have to defend your answers, you are going to have to convince me. Today I found myself saying, if you want to, if you don't agree with it, you can say something like convince me. ***So I am actually giving them the words that I think they need so that they can converse in an environment like this.*** (bell rings) I actually need to go but if you have another 5 minutes maybe
I	Okay…umm let's see. Did any particular students do well with the lessons?
Ms. E	In this class or in the other classes?
I	Umm…I think this class
Ms. E	This class…umm as far as you know in general not these days right (pointing to the laptop) umm Gabriella and umm Tiffany got the correct answers for fractional parts and they were able to name some equivalent fractions as well so and umm surprisingly there were other 2 girls that I think I was asking naming questions that were able to name fractional parts with a denominator of 64, Petrice and Janice's umm and today when they had their presentations, Julia was still confused with the same idea that N'yette was explaining she named $1/4^{th}$ a $1/64^{th}$ that's in the clip today. ***But she was definitely out there defending her answer and umm*** Tiffany and Gabriella ended up convincing her and she was convinced in the end. Jose stayed with his idea that naming a simple fraction is by naming the dimensions of umm the shape and he was still confused, by the end of the

	lesson he was still confused. Some of them were convinced that he was confused.
I	Just a last question. You answered a lot of them already. Is umm did your students work in groups and umm so how were the groups chosen?
Ms. E	The students ***choose their own groups and umm there are days where I would tell them work in groups of no more then 3*** but usually ends up like they would usually go with their friends, or ***whoever they are comfortable with that day.*** There is no set group but I would ask them first to work on their own for a couple of minutes and then I would encourage them to be in a group. Students who don't want to be in a group are not discouraged to do so; if they want to work alone they are allowed to.
I	Umm okay…and lastly umm just moment for you to reflect on the lessons and is there anything else that occurred that maybe we didn't talk about now that you would like to add that you think is important?
Ms. E	Umm just my pacing, I am a little bit concerned with what I am doing, ***I think I am focusing more in this class on affect and then with a lesson proper*** with the other two classes so I am thinking what is making the difference and if so, if I can continue with this, if I allow it to continue are they going to get the same lessons that they need to get or am I going to be able to cover the same lessons with them. Umm so there is just a couple of different concerns that I have, umm but as far as setting the environment is concerned I think we are on the right track, we are able to set the environment umm in both umm different classes with this and the other classes
I	Umm okay, aright so thank you very much for your time I know that this is crazy
Ms. E	Sure .

Key: I = Interviewer
Ms. E = Teacher

APPENDIX E.2 MS. E RETROSPECTIVE STIMULATED RECALL INTERVIEW

"Finding Areas and Other Products"

Speaker	Transcript
I	Just so you know my name is Cathleen.
Ms. E	Cathleen. Okay.
I	So what have you… what actually happened over the past two days?
Ms. E	Over the past two days… yeah …the first day we actually planned to do 5.2 because we did 5.1 the day before.. **So I was hoping I would go over it for about a couple minutes then move on to 5.**2. As I was trying that I found out that we can't, it was not like they understood what they were doing in 5.1. And then when I asked who did who was comfortable with number 2 I saw like three hands raised. I asked questions and they obviously uncomfortable with that. So we stayed with that problem. Oh I cant remember exactly what happened on the day I remember not being able to do what I wanted to do. Which then translated into not being able to do what I wanted to do in the next day**. So I was getting frustrated** Which led to what happened today which was just escalating from Wednesday to Thursday to Friday. But basically there was a good idea that came out especially the 3 eights and the three sixteenths. I was only listening to what Maria was saying. You can call it the three16s you can also call it the three eights o no you can call it the three eights you can also call it the three sixteenths… I need to get my numbers straight now but she was also, she was saying that it was three eights when its half but I didn't ask those line of questions . I only heard what she said three 16s and three eights it could be three 16 three six… it could be both and I was getting very frustrated with that like how can it be both. So we had that conversation there conversations there and I think we have a whole clip of that is that one of the clips to day ayy so confusing
I	Uhh what thing I noticed was that you had students set up in rows in groups. Is that normal?
Ms. E	Uhh these desks are not easy to put in groups I don't even think they are even designed to be situated so you that can actually have groups. And I Ms. E ind that in the beginning its very helpful that students understand that they are accountable for their own work. So I would put them in groups with the purpose of not having too much noise in the classroom because I noticed that in the beginning if I let them go in groups right away, I noticed that they tend to either just chat or wait for somebody to answer something. So I'm trying that this year to have them in rows . Look at their work first before they allow, before I allow them to go into groups so I would say as soon as everybody has something then we can go into groups otherwise well stay in rows.
I	Umm I'm trying to think of something to say. So even though they are in rows

	are they supposed to kind of stay in their seats or do they talk to each other. What is your what is your take on that?
Ms. E	I don't' prevent them if they start showing their work to the person next to them I don't start I don't' stop saying anything unless it its something **I really want them to do on their own first but most of the time they would start on their own or show or would show a classmate I don't' stop them from doing that.** Umm And before long actually they would gravitate to different groups I don't' like assigning groups… I want them to work in a group where they are comfortable. I would call some students to a group sometimes to look at somebody's work for different reasons sometimes I **find that one person can't start and an idea from another group can probably help him or her to start something or if they have opposing ideas I want them to hear someone else's ideas** so I would call them sometimes they are not doing anything and I want them to see something and one thing that also umm going back to the other question I wasn't paying attention things that are happening in the middle I wasn't following so **I'm not that is even a good model for the kids so I am trying to address the kids that are not involved in the presentation so that's probably half the class so I am trying to address half the class**. Do you have any question trying to make them focus on what the group is discussing and I would tend to allow the other students that I think are actually doing something to just do just do their own thing without my intervention.
I	Umm I think you said in your interview before the cycle started something I obviously wasn't there something about how when you were teaching addition and subtraction of um fractions you were tried to get them to work it out on their own but they had difficulty so you had to tell them and you were trying to not just tell them how to do it this time. How did that…
Ms. E	Oh boy I forgot about that umm hmm I think I was showing them an algorithm. And I realized after showing them that after giving them a test that they still didn't' understand so I don't know if its because they are a new group we can't quite get that communication going or I really need to give them more time but we've being doing it for a while now since September actually and its already November this week is the only time that no no wait no we haven't been doing it for a few months the first one was discounts seems like a long time ago can't remember the first time was percents then fractions but still probably a month or a month and a half doing addition and subtraction fractions dealing with equivalent fractions but still it doesn't seem like we are getting anywhere I started multiplication of fractions today um… no this week. Hoping they'd encounter additional fractions and this time it would click it seem like it is. It seems like I didn't make a mistake I was thinking whether I should give them more exercises in addition or if I should just more on to multiplication and I went ahead and tried that so it seems like it is working seems like it is working we'll see.
I	And um I think u started u started to mention how on one day u didn't get to start where you wanted to. So how did that kind of go into the whole class, how did you adjust to that?

Ms. E	I just have to follow what they are doing and and uh what happened today **I was trying to finish one too many things and that didn't work *out I was frustrated and the kids were frustrated*** also and the usual noise here and there but I was not prepared for that today **I was just focused on trying to finish something the real focus was to finish something today finish, finish, finish. I didn't even give it much thought as to you know. what are we accomplishing. I was just trying to finish something today**
I	So what do you think about math what do u think about the kind of math you are teaching now
Ms. E	What do I think of the kind of math that I am teaching now at first I thought it was the concepts are the same as when I was learning them obviously the approach is different and um I think it is more real than just saying this is how u get equivalent fractions you multiply the numerator you multiply the denominator um or instead of just saying this is how this is the algorithm for multiplying. You can say it in different ways. Such as there is two and one fourth pans of brownies I want one half of that you know half of that and how much would it cost if a pan of brownies is 24 dollars so 24 dollars times 1 and 1 eighth. They don't' have to remember a lot of the things I had to memorize instead they have to think about how to solve the problem. What are the different ways I can do to solve the problem? Instead of what do I do when I do multiplication or what do I do when I do addition what do I do when I do subrogation. And you hear students saying I hate fractions I hate fractions I don't' like fractions. And um I don't' know what happened before this that they have that love hate relationship with fractions umm I hear that a lot I don't' like fractions I don't like fractions when I see how much they can do .with fractions I am not surprised why they don't like it they can't do much actually at this point. Looks at clock. I have about five minutes.
I	Ok umm Is that enough time to go through a clip no? Ok.
	Boy runs in. Where is Mrs. T?
Ms. E	I think she's next door close the door Felix.
I	We are just going to start looking at the clips since we talked about all the other stuff before.
	Teacher is watching screen. She begins laughing and then begins watching quietly. Clip is over she smiles.
I	So can you tell me what was going on in that clip?
Ms. E	[Tora] was trying to explain something but before she can even finish a sentence um the two boys in front, I believe that's Daniel and um Miguel, had a lot to say their answers were different. And I think um [Tora] had since its twenty four, she had written twelve six two and two is that right? two and two, wait I can't remember what the numbers are but she's missing, she's missing some to add up to twenty four. Yes so its twelve, eighteen, nineteen, twenty, twenty-one, twenty-two she thought about the lower part being two and two instead of three and three. **So they were actually questioning the math but they were interrupting her a lot so I think it was very important for me to point out that they have to be respectful those are things that I find I**

	always find that I have to say things like that in the beginning .since its not really a bad thing they are doing they are listening to probably not following [Tora] they were interrupting her also they were talking about the math but I think to create an environment where questioning and justification is the norm everyone should be respectful it is very important that people are respectful o each other.
I	Um so in this particular instance what prompted you to stop and adjust this?
Ms. E	What prompted me to what?
I	To just adjust it right then and there
Ms. E	Yes um as soon as I see an action that needs to be addressed I address it right away I try to make my sentences as minimal as possible because it is also an interruption but um I think it is important that **I point it out because they they really when they are talking to each other they are talking like that they don't even allow another person to talk let alone finish a sentence so that's just how they are and um its very hard to create an environment where people** **Would be encouraged to talk if there is not respect it's very important.**
I	You said that you have to do this a lot in the beginning in the beginning of the school year or umm…
Ms. E	***In the beginning of the school year when the environment is when I am trying to set a certain environment in eh classroom at this point thought they are starting to question each other in the beginning they are not even saying anything when a presentation is going on they would just listen even when they are questioning an idea you will just see it in their faces or sometimes*** they wont' even have a reaction but the ones that I would see making faces or not making faces or whatever what I am trying to say if they are reacting to what the other person is saying they wouldn't' even say something to the person they feel like they have to talk to me and direct a question to me and I would say you can ask the presenter so its something that I think is new to them that they can actually talk tot each other while a presentation is going on.
I	Do you think that it was effective that you stopped and talked to them about being respectful?
Ms. E	It was a what? It was a what?
I	Do you think it was effective?
Ms. E	Uhh I think at that point umm I think at that point it was addressed and I know I will have to address it a couple more times and even throughout the year its an issue theses kids are just so used to talking in this manner that they don't' see it they don't' see anything wrong about it its just how they talk to each other
I	How do you think the students uh feel when they are the ones being interrupted?

Ms. E	Uh they notice it they notice it when they're in front that's the time they realize that that's the time when they're really sensitive to the noise when they are not in front you'll see like [Tora], ***[Tora] has great ideas she's talking the whole period talking just talking talking talking at the time when she's there she gets she was getting annoyed at people who was um was saying something and the thing that annoys me too is when um when somebody would say but what about this what about that and they'll answer back shut up shut up . it annoys me a lot and sometimes*** I can't I don't' know in the beginning there's a lot of that and its hard when you are trying to create an environment where you want them to talk to each other and they're so used to this kind of exchange its very difficult to establish a certain uh and ideal environment .
I	Do you think this has improved since the beginning of the year since September?
Ms. E	Umm improved in such a way that now they're questioning each other in terms of them still saying shut up shut up and all of that comments, they still can't let go. I tell them to just ignore if somebody is saying something to them to focus on what they are saying to ignore the other person that's very hard for them to do that's very hard for them.
I	So what are your biggest concerns about the types of behavior you… that you're seeing here ?
Ms. E	Um mainly that mainly that they can't ignore when somebody makes a comment which is not mathematical they can't ignore that they have to address it they are more willing to obviously that stuff more concerned to them that they're um that their stature in the classroom is at stake their personal … its almost like they are being challenged and there more inclined to answer to that or they are more inclined to address that instead of um they have more interest in dealing with that than getting the math across it seems like more a priority to them and understandably so, these are teenagers.
I	Um how do you think the students feel at the point when you're addressing you know being respectful
Ms. E	***I think little by little they are understanding that it is very important for them to be respectful um there's less and less of the um the back and forth of um throwing of verbal abuse*** there's not that much anymore in the class room even when I called them they used to answer back and defend themselves I would tell them when I call your name don't' answer back we can talk about it after class or you can call me to decide if I say something bad to you you don't' have to answer at that point because we have something to do this is very important we can't keep stopping the lesson for every little thing so we can talk on the side or you can tell me after school I tell them I make mistakes ***I will make a lot of mistakes and when I do I apologize ahead of time for making mistakes and calling them and I also tell them before I call somebody*** I've already seen something so um they're responding well they're responding well they're responding to that so the environment is

	getting more and more respectful I think .
I	Umm I think we're ready to take it to the second clip. Selection yep.
Ms. E	That voice is too high (laughing as she is watching the clip)
Ms. E	***I'm getting frustrated***
I	So what do you see, can you tell me about it?
Ms. E	***I'm getting very frustrated with the idea that they think that three eights and three sixteenths could be the same*** but the one thing that I did follow there is that they actually know its three eighths of a half three eights of a half and um I was just looking at the idea that they said three eights and three sixteenths are the same and I was hung up on that and I obviously wanted to address it so I did that today I addressed it today when I started the class I had them draw two squares two same size squares and I told them to show me three eights in one square and three sixteenths in another square and then I had them write a sentence that tells me whether they're the same or not and if they are the same can they how can they be how are they the same if they're different can they ever be the same. Is there any situation where they are the same because in this case they are saying it's the same three eights and then its also saying three sixteenths and we went as far as I went a far so they were talking about that idea. That three sixteenths is smaller than three eighths three eighth and then I posed the question how about three twelfth where doe three twelfth go at that point they were talking about the denominator sixteen when you have one whole you have to cut the pieces smaller so even though the number is bigger the pieces are actually smaller and somebody had come and said that come come up to me to say three sixteenths is actually uhh two three sixteenths no uh wait wait wait two sixteenths is actually equal to one eight so there were a lot of those idea and these are the same ideas that we were working on with equivalent fractions .a couple of weeks ago but when they came up to talk to me about their findings today it seemed like they'd never saw those things when I thought that they did.
I	So in this particular clip at least how do you think your students were feeling?
Ms. E	Umm if I just look around I see a lot of students that are not paying attention to what's going on in the front mostly what I am saying actually but its interesting to know sometimes ***I've seen that in other clips when you would see a student who looks like he or she's not paying attention and all of a sudden would come up with an answer to the question I am posing*** so you really can't tell whether the students are actually listening or not by just looking at them some of them could be listening attentively and doing something on their seats I've seen that happen a lot when a student is actually is almost sleeping and then all of a sudden would raise his or her hand and say something that solves the problem for the class. I've seen that happen so If I'm just looking at them ***I can see people who are not paying attention um that***

	aggravates me a lot but then I have to think also they may also be listening without me knowing that they are so um I was basically talking to at this in this clip I was basically talking to Daniel and Richard, ***Monyette wasn't paying attention and she's the one that I wanted to address mostly because she's the one that was who had strong feelings about this three eights and three sixteenths being the same.*** So I want to actually let her into this conversation actually I was trying to pull her into this conversation ***but she wasn't latching on so um I was my voice was so high I must have been very frustrated at that point.***
I	Ok um how do you think your students were reacting to the lesson?
Ms. E	Um they were trying to think um and provide answers its funny how Richard in the beginning when he answered he answered correctly and after his classmates said some things about the three eights covering the half and all of that he changed his answers and he was on their side. ***I'm like I lost the one person that I thought was on my side.*** And the other frustration I had while doing this part this was this wasn't the lesson that I wanted to have at that point this wasn't the discussion that I wanted to have I wanted to finish umm five point I forgot what it was five point one five point two. Maybe I wanted to finish that lesson and I couldn't' even get there ***I was getting very frustrated that I couldn't so that was another reason why I was uh my voice was so high eek didn't' sound good***
I	Umm in this clip and maybe in general do you think the students are sharing their ideas?
Ms. E	I think they are I think they are I think umm when uhh when I had the post lesson interview **I didn't' trust them as much I think I said in the pre-interview lesson that umm I didn't' I don't expect them to question each others ideas but a lot of that came out Didn't' expect any arguments with this lesson I was wrong about that too**. It seems to me that some of them are able to follow each others ideas umm and it seems to me that um **I think Daniel and Miguel and Richard they're a couple of kids that are showing interest almost 100 percent of the ti**me.
I	Do you looking back on this clip now um would you have handled it differently if you could go back a little bit?
Ms. E	Hmm that's uh yea that uhh hard to tell umm sometimes I push it and I see that they're getting tired of the topic without actually resolving anything that's why I switched gears even if there were I think we had about ten minutes fifteen minutes left into the period, at the point
I	I think your going to see that.
Ms. E	Okay yea I switched gears because umm I wanted something else to happen nothing was happening well something was happening but not what I thought would happen that day so it was a lot of frustration on my part.
I	Ok well why don't for… take a minute to look at the clip.

Ms. E	Ok is that the next clip? (laughing)
Ms. E	One fifth. (Laughing while watching laptop)
Ms. E	Somebody was still giving an answer (Observation made while watching laptop)
I	So is there anything that you see going on there that we haven't touched on yet?
Ms. E	**We can't resolve yea we can't resolve that story with the three eighths we just got stuck with that um story and I was hoping that we could resolve it somehow and at that point when I saw that I can at least introduce the lesson that I wanted to do that day I took advantage of that we were not getting anywhere so I said to myself ok there not paying attention I'm not getting the answers at least the participation that I want they were just giving me any fraction that they could come up with** . one fifth one fourth and um there's not a lot of participation there's a lot of me in that clip in the whole class actually ummm so I just dropped the whole idea and this morning we addressed it for ten minutes and we were done we were done ten minutes they were quiet they were doing their drawings and they were just all unanimous three eights and three sixteenths are different. it was so easy I should have just given up the idea you know now that I am thinking about this and I'm talking about it I could have given up the idea long before I did um cut it there and said well go back to it I was thinking about that a lot actually when all this was going on lets give up on this today and go back to it tomorrow but there are times when I say that and I forget about it and its a very good concept to address its its um something very important to address it's a foundation for a lot of things in fractions so I was actually pushing it so that I could get answer but that didn't happen and when I when I did address it this morning we didn't do anything before that I **just wrote three eights three sixteenths two squares on the transparency I had them do um shade a parts three eights and three sixteenths write a sentence or two about it. They were just talking about it they were talking about how three eights three sixteenths were different three sixteenths are smaller and when I asked about the three twelfths they knew where the three twelfths belong and I asked how about three thirty-seconds they knew what they were talking about I even went as far as saying why is it that the denominator is bigger and the drawing gets smaller they're saying your cutting the whole into more pieces so there's even some, there's even some there's even a generalization** there ahh we did it for ten minutes.
I	So um when you finally decided to change the topic how were you feeling at that point?
Ms. E	**Tired. I was very tired I was very tired and they all looked tired we were all tired actually it wasn't just me they all looked very tired** to me um and um as I said I just wanted to start something so that we can use that for the next hoping that they would do something at home maybe if not all of them because homework is um its very hard to expect one hundred percent homework there's usually more homework but today there is a better turnout, yesterday the homework turnout was bad but there was um something we

	could talk about today .
I	Umm what do you think would especially in light of how today went what do you think might have been causing the confusion between the three eights and the three sixteenths? For some of them
Ms. E	Umm I'm not sure as I said earlier the idea of calling the three sixteenths a three eighth Monyette said its three eighths of a half but they didn't' specify three eighths of a half I don't' know if they knew it was three eights of a half and couldn't' give it a name or that they thought it was ok to just call it three eighths of a half and they called it three eighths of a half and now that I'm thinking its ok but if I want a one fraction name for it umm I was aiming for I was really aiming for the answer three sixteenths and three eights was the because they are different.
I	Do you think on this day that any of the students understood what was the difference?
Ms. E	Miguel I thought knew that they were different but he was the only one who knew Richard I thought in the beginning had an idea and got very confused once everyone started taking at the end he changed sides I actually asked him again he said they were the same.
I	How would you compare the first day and the second day. Well These clips are from the second day.
Ms. E	What happened on the first day umm I can't remember what happened on the first day what day is the three eighths? is that the was that the that was yesterday's the three eighths was yesterday so what did we do on the first day I forgot I reviewed five point one I don't know, can't, can't remember. What was different? The question is what's different from the first day?
I	How would you compare them if they were different or if they were the same?
Ms. E	I can't even, I can't even remember how they were what they were talking about in the first its almost like its just lumped into one session it feels like it was one session to me now I know what we talked about but as far as what happened I'm not sure maybe if I look at the transparencies maybe I can have an idea of um what went on but at the top of my head I can't I can't
I	I mean some things that must have happened the first day Jose was there so he was up at the front a little bit Miguel was at the front a little bit
Ms. E	Ok I think I remember I had asked a question and um the first day is **when I was seated in the back trying to encourage some students to participate the students that were not writing I was telling them do you agree what about this can you write something I was doing more of that for the students who were not participating or didn't have anything on their notebooks I am really concerned about that particular group and there's a lot of them probably half of the class um that are not able to accomplish a lot maybe not able to accomplish anything** some days because were were talking about different ideas so I sat with them and I was asking them what do

	you think what do you think I was really not really paying attention to what's going on in the front thinking that they can handle themselves and for the most part I think I was right I got the feeling that those who were trying to explain were able to explain to those who wanted to listen that's the idea that I got umm **Jose was there to explain something and I wasn't listening and I asked him to explain it again and he was like ughh and I asked him today or I don't' know when I asked him I said oh I had to ask you again I'm sorry about that I apologize I wasn't listening I will pay more attention um next time he said I wasn't frustrated with you I was frustrated at the kids who weren't listening he said but when he reacted to me when I said explain it again and he reacted he said ughh I felt bad that I had to have him repeat because I was the one who's not listening I was the one who wasn't paying attention so he was actually done he was able to explain I think he was able to convey his message and the people who wanted to listen were able to hear it and I asked him to repeat and for a person like Jose who really doesn't' want to be up there to be up there explaining and I wanted him to repeat it was a little bit too much for me to ask I** didn't' realize that but I am torn because I was trying to accomplish something with the students who were not paying attention I think at this point there are some students that I can let go some of them that would do I was **looking over some of them when I was trying to this is still on the first day I was trying to gather some of the students in one part of the class and I was looking over my shoulder to see what was going in the front an**d they looked they were engaged I think there was a camera were they did I get the feeling did I get the right feeling that they were engaged that they were actually discussing I think at the end of that they actually told me we came up with something so yesterday when they were explaining it oh it was me and so on and so on they knew what was going on they had an idea I didn't need to be there umm they were able to accomplish something so um what I wanted to accomplish in this session was to bring some of the other kids who have not been participating to at least participate if not to participate to have something on the notebook because sometimes they leave without anything in the notebook and I feel bad that I am not able to address everyone's concerns I am not able to not address peoples concerns that's not what I meant to say .but I feel like I'm I'm I'm it's a disservice if some kids go home without anything on their notebook anything to remember that day by or anything important that they learn just anything if not anything that's on the board and some of them leave the class with nothing and I feel strongly that I should be able to reach out to every single student in my classroom if not on the daily basis at least when I **make my rounds I** am able to talk to some kids one day and to another group another day because my attention was very focused on the kids that were participating that I'm ignoring not on purpose but I tend because of the environment I am trying to establish I have come to ignore the other students who are to participating so that was my goal for that day I am actually reaching out to the other students
I	Ok um those are all the questions I have unless you have any other thoughts

	you want to …
Ms. E	**I felt very tired, I was very tired.** No, umm how can I do that lesson differently? Maybe just post a question, if there are no answers just let it go and address it another day and move onto the lesson at that point when I switched gears they weren't even they didn't' even have the energy to switch gears. **Um it was umm real exhausting experience**
I	Did you know how they were able to come up with it today was it because of the task that they did or do you think maybe just thinking about it?
Ms. E	Umm I'm not sure and I'm not sure we accomplished a lot besides the three eights and the three sixteenths, the most that we did today was distinguishing the two and umm generalizing that when the denominator is bigger the pieces are actually smaller. So umm yea in that respect we were able to do something not necessarily what I intended to do and what I wanted to do still today, you know, not what I wanted to do but its something that we can check off at least for now.
I	Thank you very much .Appreciate it.
Ms. E	You're very welcome.
I	We'll see you again.
Ms. E	Ok. Well see you. Alright I'm off and off I need to go.
I	Have a good weekend.
Ms. E	You too.

Key: I = Interviewer
Ms. E = teacher

APPENDIX F.1 MS. B RETROSPECTIVE STIMULATED RECALL INTERVIEW

"Fences for Grazing"

Speaker	Transcript
----	Classroom scene with teacher and students.
I	Hi [C]. My name is [Cn]. We're meeting today to talk about the lessons that we observed last week, and I'm interested in how you thought the lessons went, and how your students reacted…(*couldn't hear last few words. I don't think they'd be significant to the question as a whole) So what are your thoughts on the lessons that we observed?
Ms. B	I think it went well. It started off kind of quiet, but then as the second day came along, they were a little bit more into it. There was still confusion, what area is, what perimeter is, and **that was one of the questions that I thought they were gonna have a little bit of a problem with.** The discussion went **well** because of two students who actually started getting into it, one, their answer being right, and they just wanted to, they weren't listening to each other, and they both thought they were right. So I thought the discussion went well. They're still not finished with the problem, it's still an ongoing problem. They're still not done, they haven't convinced each other of what the 'maximum area' is yet.
I	Okay. The lesson you selected was not the connecting math one, right?
Ms. B	No
I	So is there any particular reason you chose to do that instead of the Connectedmath?
Ms. B	Well, what we're also doing here is we're working on different skills in eighth grade, and one of the skills that we worked on, **began** working on is area and perimeter. Seeing um, it's just base line. Basically seeing if they know the area, know the perimeter. This work, um, this problem was put together, to determine, exactly, 'fencing', they had to understand that meant perimeter. So without using exactly the word 'perimeter', they had to go over with more in depth, with their understanding of vocabulary. So we just chose this because… (at 2:00, a voice from the intercom talks to Ms. Barnes, and interview resumes at 2:04)
Ms. B	Um, yes, we just basically chose that problem just because it's just something, we just wanted to see if they had an understanding of the area and the perimeter.
I	Was there anything in particular that stood out?
Ms. B	Uh, yes. What stood out was, they still are in **confusion of what area is and what perimeter is on their own, but then that really kind of put them at a disadvantage on answering the problem.** Because now they have to waste time on figuring out how to find the area, how to find the perimeter. So that took basically the whole first period. Because they couldn't really really know what was going.

I	Now, this past week you said there was testing (*I believe this is what Isaid), were any of the other classes working on this problem at the same time?
Ms. B	No, this class actually worked on it first. The next classes will actually be working on it **today**… and tomorrow.
I	So, let's go to some video clips. (T says "Okay" at 3:01) I'm going to show you the video clips, and I'd like you to focus on the student's feelings and their mathematical thinking as you're watching them, and we'll kind of talk about them.
Ms. B	Okay
I	And we're going to do one at a time. (*interview resumes after video clip showing, at 3:17)
Ms. B	There wasn't much really going on. They weren't trying to help each other, they just really wanted guidance at this point on what to do.
I	Was that the one where you said something (*background sounds coupled with question make it hard to comprehend—may not be vital in interview) being you?
Ms. B	Right, yes.
I	So what do you think (child's name) was thinking?
Ms. B	I just think (female student's name) wanted an answer at this point. She didn't want to try to think about how to get it, she just wanted to know, 'okay, what's the formula for it so I can plug in numbers and get the answer.'
I	What do you think (student's name) might be thinking?
Ms. B	(student's name) as she kept on saying, she kept on saying, 'What do we have to do? What do we have to do?' I just saw him kind of looking at the problem and looking down, so he had a thought. I don't know what his thought was, but he was going to say something, he was going to answer. Some kind of answer.
I	And how did you think about that at that point?
Ms. B	Um, well I was, I'm thinking that **D(sn) was very dependent on me at this point, and not on her group.** She was just, she kept on saying, 'Ms. Barnes, Ms. Barnes, what do we have to do? What do we have to do?' instead of maybe asking like (student's name) or anyone else in her group. She just kept on, 'Ms. Barnes, Ms. Barnes.'
I	(*C says something not comprehendible, but by her gestures towards the laptop between them, it seems like C is motioning for Ms. Barnes to play and view a video clip on the laptop. Interview resumes at 5:00)
I	What was going on in that clip?
	D(sn) and N(sn). D was in quadrant 2 with her group, and N(sn) was in quadrant 3, but then I saw D(sn) didn't seem like she was paying attention at first. She was playing in L's(sn) hair. She heard what was going on in the other group, and then she went over to the other group it seems like. And she wanted to know what they were saying about what they put. They weren't listening to each other.
I	They were looking at **her** work?

Ms. B	Right. (T responds with an, 'Okay' at 5:31) N(sn) was looking at **her** work at that point, so D(sn) I assume wanted to hear what they had to say about what they wrote and how they wrote it**. They didn't listen to each other, they heard each other because a couple of the things they had the same idea, but they were saying it differently,** it seems so wrong to each other. N(sn), he was trying to explain how to get the area, he was clearly saying, 'length times width', and she said, 'Alright. I know, I know. That's what we did, that's what we did.' But then he was saying, 'no, **this** is what you did.' And she was like, 'Well, that's what we did. Oh well, that's how we did it.' So, I think, I don't know if she knew she was wrong, but because she put something, she was going to stand strong with it and she didn't want to be proven wrong, I'm not sure at this point. But it kind of seems like she knew what to do, but she's saying, 'yeah, I know what we did. Oh well, that's what we did.' So, instead of saying, 'yes, I was wrong, I know what you're saying,' **She just stuck with her story.**
I	What do you think that N(sn) was feeling then?
Ms. B	N(sn) was trying to prove that he was right. **Like, he just, he had a thought and he was sticking with his thought, and he wasn't backing down from it. And he was saying, 'No, this is what you have to do**. And he's like, 'Tell them. I don't know who you're speaking to, but tell them. That's how you have to find…' and he's trying to confirm his thought with someone else. So, he was sticking with his thought.
I	How did you feel about that part of the class?
Ms. B	Um, I was actually with another group. I was in quadrant one, I remember. And, I didn't hear what was going on. You hear noise at first, and you hear them talking about it, so it's okay. But I turned around at one point, because they did get loud, and they were talking about the problem so it was okay. One thing I don't mind is the talking in class. It can get loud, people might walk in sometimes and wonder, 'what's going on in class?' But sometimes you need these conversations to get your thoughts out. I thought it was good. What I'm trying to get them to do is listen to each other, and not just hear each other, because they both have valid points, but if they really would just listen to each other, and prove each other, then it would be a little differently. I know at one point, they were throwing out numbers, two of the boys R(sn) and F(sn), they were laughing and kind of tossed the calculator over to N(sn) so he can show on the calculator, N(sn) grabbed the calculator and showed to D(sn), 'Well what did you get?' and he plugged it in. So that's a point of them listening kind of. **Because now he's trying to show, and she said,** 'Yes, that's what I got. That's what I got.' They were in agreement **right there**. But still, she was stuck on her thought, so she's said, 'Well, that's what we did.' So the conversation I **thought** was fine, I thought was fine**. It did get a little loud, but it's okay.** We learned how to listen to each other, but

	just the fact we were talking about it, and I wasn't even over there. I wasn't talking at all. Um, usually that's what happens in class. **Like, I won't intervene,** as long as they're not fighting, it's okay that we get loud. We all talk differently, you know. So, I thought it was fine.
I	Was there anything that surprised you in this clip that you were watching?
Ms. B	Um, well, I'm not going to say 'surprised' because I do know my students work without me **directing** them all the time, but I'd like to see that even though I wasn't in a group that stuck out to me, I wasn't in a group, I was working in another group. And those two groups kind of joined together to discuss what was going on. **Even though they were suppose to be in their own quadrants, they did move over so they were talking about the problem, so it's okay**. It's not that they were off task. **So, I'm not going to say surprised, but that stood out, and I like that. That my students really are working without me.** They don't need that guidance 100%. They **still are dependent** on me, but you can see here that they are working on their own.
I	Did you have any comments about the way they kind of left it?
Ms. B	Um, the problem? With the problem?
I	Well, it looks like they just kind of walked away.
Ms. B	Right
I	I mean, I'm having trouble seeing it, so…
Ms. B	Right, um. D(sn), I remember she did walk away because she was mad. Because, they started talking about, 'We ripped you, we ripped you.' Like you know, they're saying they **got** them. You know, 'I'm telling you I **proved** you wrong.**' So, I think she just got mad. Um, it's okay that they're like that.** Class was about to be over, and we'll pick back up on it and that'll be another conversation starter in the next lesson when we do revisit this problem again. **Will I answer their question right now? No. No. And they know that. It's okay.** It's an ongoing thing. One thing that **did** stand out, now that I'm thinking is, because this was over the two days, the second day a couple students did go home to find out what area and perimeter were, so when they came back, they were able to have a little bit of clarity because they **knew I wasn't going to answer them. So, either they were still going to be stuck where they left off, but a couple of them did go back and** find out, so I think that's how some of them were kind of able to move on and know what's going on.
I	Let's move to the….(*becomes hard to hear, but it can be assumed they're going to view more videos, because C's head turns towards the laptop. C and T view video clip on laptop until 12:00) Okay, umm, so why do you pull the students to the middle like that? (referring to the video clip)
Ms. B	Um, that **actually** was my first time doing that. I noticed, and I actually **like** pulling them in like that because when they went back to their quadrants, they wanted to see what the comments, what comments were

	written for their group. And that's understandable, but now that just brings a whole 'nother conversation because now they want to see why you wrote that, now they want to explain **why** they did what they did, but there was no time for. They weren't listening to me at that point, because they were still wrapped up in their problems, so I pulled them **away** from their work, and thinking fast, in the middle, there was nothing there. We had to meet in the middle of the grid, and so I can give them instructions so we can end the class.
I	Um, did you think any of your students were feeling anything at that point? Any emotions that might have been…
Ms. B	**Yeah, some of them still were a little mad,** I'm sure, I remember D(sn) after we left back, I gave her a job to collect something, but as she was collecting papers, I saw her reading the problem over again. So she still, and that was the one who was arguing, she still had questions in her head that I guess, were not answered or not clear, I'm not sure. But I just saw her as she was collecting it, she still was reading that problem. **A few of them, I think, could probably care less what was going on, because class was over**. It's okay, you know, they weren't really **drawn** into it the way others were, there are a couple who are just very quiet, who take everything in. They absorb things, and then maybe the next day, they might think about what was said, and then show it, and then tell about it, but they're not one to argue. And those two are very outspoken students, so to come battle against them, and argue against them, you have to have **heart** to do that, you know. **Because they really, really were getting into it,** where as if they were quiet, they may not do that. So I think a couple of them were kind of just taken aback a little bit, heard what was going on, but didn't really get into it.
I	I think we're going to move on. (T reaches for laptop. I assume they're going to view another video clip. Interview resumes at 14:31) So, what was going on at that point?
Ms. B	I just wanted to make the comments on because someone did say, 'That was N's(sn) group.' Because N(sn) doesn't always work. You know, N(sn) would walk around in the hall way, and just drag in, not doing anything. **But when N(sn) works, he works. He knows what he's doing, and you can tell he was actually enjoying himself because he was actually on the floor getting work done, and you know, when you start seeing their behaviors and noticing the things that they do, I was proud of him.** We do give a round of applause often in class, or even one or two people or even if they come up and present, we do the thumbs up, thumbs down, because they do need to feel some type of success. **Yes they were arguing, no they didn't get an answer, but did they work? Yes they did.**
I	Is there anything else in the clip that you kind of gotten so out? (*not sure what she asked)

Ms. B	Just N(sn), as I was seeing that he kind of just listened, and he shook his head, like this, yes (T nods her head, imitating her student N) and he actually did the round of applause as well. So I just noticed this about him as well.
I	**So, what do you think he might have been feeling at that point?**
Ms. B	**He probably felt good. He sees, because we battle, him and I go at it, and it's okay. Because I see what he can do. He's capable of, so I won't back down on him, so I think he felt good, and he even said after day one, "I did a good job, didn't I?" And I said, "Yes, you did a good job." And he said, "Well, I'm going to have another good day." So, for him to say that, and he said, "And, I'm going to come to class on time." So, he felt success, and he felt happy.**
I	I had a little trouble seeing the clip. Is there any mathematical thinking or behavior going on here, or was it more just about acknowledging
Ms. B	Just more acknowledging at this point, just more acknowledging.
I	And how did you feel about that part of the class?
Ms. B	I think that it went very well. I think that part was, **is** needed. Because we all want to know when we do a good job. Adults, children, anyone. **When you do a good job, you want to be recognized at times, so I think it's needed so that they know they are doing a good job. They didn't get the answer, it's okay. You're working towards an answer, and they need to know that they're doing a good job. They are, I'm proud of them.**
I	I think we also have a few more questions. Do you have any time? Maybe five more minutes? (T replies, 'Yes' at 16:57) So how do your students, as a class, react to these lessons, reflecting back on the past two days?
Ms. B	Well, the next day they wanted to go **back** into it again because they wanted to keep working on it, so like that is a good feeling for them, not to just dismiss it, and it's like, 'Okay. There's something to do.' They're trying to figure it out. I think when they first got the problem, I saw a little confusion, because it didn't quite say, 'Perimeter', it said 'fencing'. And I knew that that was going to throw them off, and I know that that's something that **wording** can just turn us all away. So, I think they were just withdrawn at first, they know I'm not going to answer the question, but if they keep reading it over, talk to each other, just start jotting things down. It can be the wrong thing, but as long as your thought is coming out, it's okay. Then I saw them getting into it a little bit more, so I think **withdrawn**, and then I think a little bit more **into** it, and **excited**, and they still want to go on.
I	Were there any parts of a lesson that you think **did not** engage your students?
Ms. B	Yes, they were engaged but I think once they get an answer, and this is a problem across the board, once they get an answer, they think they're done. So a couple, then I said, "Well how do you know that that's the

	maximum area?" "Well", they said, "Well, just because we did this." "But **how** do you now? What are you comparing it with?" So, I think once they got an answer, they kind of just, sat back and were done. so right there, they weren't trying to prove themselves right or wrong at that point, they got an answer, they're done.
I	Did any students in particular like the lesson?
Ms. B	Well, you can see the ones that were really, really into it. Speaking out about the lesson. **F(sn), he's very quiet, but he worked through the lesson as well. So you can see it in a different way, and I know him, so like, he was sitting there, he listens and he will write things down, and that's okay too, and then he might pass it along, and have someone else tell his thought.** But he won't ever say, "That was my thought." He'll let the other person take credit for it, so I see that he was working. S(sn), S(sn) also was working. He hears a lot that's going on, and he might comment on it, and that's okay. Whether he agrees or disagrees. He may not have that thought himself, but just the fact that he can agree or disagree on something, he's thinking about it. 'Why do you disagree?' Something didn't sound right. Or 'why do you agree?' Something sounded right. So I think they were pretty engaged.
I	Did any particular students **dislike** this lesson?
Ms. B	Um…(*long pause, T ponders question, and begins answering the question at 19:49) I don't see where they just disliked it, they might not have been **as interested**, but I don't see where, yeah, I don't see where they disliked it, you know.
I	Are there any other students you haven't mentioned yet who you felt did well with this lesson?
Ms. B	Ne(sn), K(sn) and Al(sn). That was a small group of three. They all had different thoughts, no, actually **two** had the same thoughts, but because they were saying it differently, it made it three different thoughts, because they weren't hearing each other. A(sn), actually, he um, he said he went home and did some research. And he came back, and he had papers, his notebook paper drawn with the rectangles, and he was trying to find the **area**, he still didn't have a clear understanding, but he did have the boxes drawn inside, the squares drawn insides, but he had it numbered wrong, so he has a sense of, 'it's **inside**', but the numbers weren't correct. So, he doesn't have a clear understanding, but he's trying, you know. For him to go home and say he did research**, I'm impressed. Because he did research, you know.**
I	Are there any students that you haven't mentioned yet that you think failed to grasp the lesson?
Ms. B	Well, Al(sn), um, he didn't grasp it. And, it's still in question, because they're still working on it, so right now there's still just a lot of questions going on. They have a lot of thoughts they were able to see **two** other groups, so now there's just more questions on whose right and whose wrong. So right now, they're still just a question.
I	(C whispers something faintly) Um, so, what kind of strategies did you

	feel you used to present this lesson?
Ms. B	**I try not to give away too much information because I just want to see what they know,** but just with the questioning, questioning each other, then they're beginning to question each other a little bit more. They're drawing pictures. Just the prior knowledge, also. Some have it and some don't remember it at all. But I didn't want to give away too much, I didn't open it up in any particular lesson. We've had, over the last couple of weeks, we've worked on open-ended problems, just with perimeter, and just with area. So without me even going back to tell them, 'What did we do in this problem?' I wanted them to just try and come up with it themselves. So I really am not telling them too much at this point.
I	I think we have to conclude this interview. Is there anything that sticks out the most in your mind about the class?
Ms. B	A couple things. Just um, they still don't understand area and perimeter, and even when I left here, I was thinking, 'How come they still don't get it?' Because I taught this to them last year, we were going over it again this year. **They're not retaining that information. Why not?** You know, why aren't they? Um, that's a question because this is something they should've been able to kind of just work through. Maybe not have the maximum area, but they should have been able to have different examples, and just different ways. And they were stuck on just one answer. So that stuck out to me, because now I'm just trying to think, 'How can I get them to get it?' You know.
I	I think we're good. Thank you very much.
Ms. B	No problem.

Key: I = Interviewer
Ms. B = Teacher

APPENDIX F.2 MS. B RETROSPECTIVE STIMULATED RECALL INTERVIEW

"The DJ Problem"

I	How are you?
Ms. B	Fine
I	How was your weekend?
Ms. B	It was good
I	Good? Good. Never long enough though
Ms. B	No, not at all
I	Not long enough for me either. Um, we're meeting today to talk about the lesson we observed last week. Um, so I'm interested to, to know how… and then I'd like to show you a few video clips of your students and um..
Ms. B	Okay
I	If you could talk some more about that
Ms. B	Okay
I	Um, first of all could you just share us your thoughts about Wednesday? I guess we need to sort of, refresh your memory. I do have a copy of the actual lesson that we're talking about.
Ms. B	Okay
I	So you can refer to it. How do you think it worked out?
Ms. B	Um, I think it went okay. They didn't um, get to the level that I wanted them to get to because we've been talking about variables and any number, like how X can represent any number of hours, or any number of um, … whatever the um, independent variable is. And in this case, the independent variable, um, was the number of hours. We wanted to um, we wanted to, I wanted them to just understand that it should be for any number of hours, but they were just putting, like maybe 10 hours, 6 hours or something like that. They didn't get exactly where I wanted them to, um, a couple students did, and um, when they went to go share, one of the students was absent, so she was, that was kind of like a backbone to work together when they were in a group, they didn't agree with what everyone was saying. And um, she kind of was giving, was maybe confused, I'm not sure, because she had a different view point but when the whole group was agreeing and it was just here, quietly she was just like "I still don't agree, I still don't agree"
I	And which student was that?
Ms. B	That was Ruth
I	Ruth?
Ms. B	Right. So she had a different view, she was thinking for any number, and she was thinking for hours between 1 and 3 it would be better for 1 dj group, for this hour, this group is better. So she wasn't expressing that thoroughly, so she didn't convince the whole group that her view point was correct.

I	Well when we talked before the lesson, um, you expressed some concern about where students might get stuck or come to an …, so this idea of variable, was one of those times? One of those things?
Ms. B	Yes um, they, I don't think they did too bad with graphing it after a long while, it took them a really long time to, they should have been able to find the equation and put it right into the graphing calculator and then graph it. But it took them a long time to get to that. It didn't take, it was on Day 3 I believe, most of the groups got to it.
I	And um, you think they should be able to do that because of their prior experience?
Ms. B	Yes
I	They've had experiencing doing that?
Ms. B	Yes
I	So, can you think of any reason why this might have been more difficult for them?
Ms. B	Um, I think maybe when they see a whole bunch of things they have to do on one piece of paper, that gets confusing, it's not just one skill. When your working on one skill for a whole class period and then the next day your doing the same thing, it becomes a little bit repetitive. So you can understand, you might have to really really understand what you're doing, you just know you did this yesterday. But now when you have about five different things that you have to do, it gets confusing, like 'okay, what do I do first?' or 'did we do this?' and now everything is becoming, like all in one big bowl. So it's like, it's just mixed up and they have to kind of like, pull things out so they know what to do.
I	Okay, tell me a little bit more about this lesson. Is it connected math lesson?
Ms. B	Yes it is
I	And um, I noticed that you, that every student received a sheet of their own to work on
Ms. B	Yes, yes
I	Is this the way it was presented in the um, the book, or did you make modifications?
Ms. B	No, this is the same exact problem from the book. It's just one of our, um, base questions, it's not, it's actually an investigation, which is like follow up questions to different skills that we have worked on previously.
I	Uh huh
Ms. B	So I just um, typed it over and put it on one sheet of paper.
I	Okay, based on the student outcome and how many worked through this, um, over the course of several days, do you think you would do anything differently with this particular lesson?
Ms. B	Um, no I don't think so. I think they also just got caught up on, because I did make it um, relate to our real life, so when I made it relate to our real life, when I made it relate to our real life, then they

	got caught up on the DJ on how long they were going to party, would they be getting tired if they partied for 10 hours, so they kind of lost scope of what they were going to do. It's kind of relative I guess, because I was bringing out, okay, if we don't party for 10 hours, lets party for 6 hours, so maybe that might have brought out, "oh well it's cheaper here" only for a couple more hours, even if they are tired, it still is cheaper with this DJ. So you know,
I	Were you pleased with the level of engagement? Because it seemed to connect with..
Ms. B	Yes
I	Because you enjoyed talking about it
Ms. B	Yes. Because the more they talked about it through the whole class, I did notice that, it did keep their attention, um, I think they liked it, because it did relate. We are going to be having an 8th grade dance and we need a DJ, so this is real. We are trying to raise some money.
I	So.. and do you notice when the task has more relevance or interest to them that they stick with it longer?
Ms. B	Yes
I	Do you have any ideas about how that may or may not impact how they learn actual math?
Ms. B	Yes because there's always a question on "when are we going to use this", or "how come we have to learn this" so um, when their lessons where they could relate to every day life or things that are going on in our own personal life, then it makes sense. So this made sense to them because they have to raise this money, so money, when we talk about money, they have to raise this money, they're into it, because it's coming from them, it has something to do with them. So yes, when it has something to do with every day life, they're more into it, they're more in tune to figuring out why.
I	Um, ok we have, um, several video clips that I want you to take a look at and um, I would like for you to take particular interest in what the students are doing, and um, just focus on the students feelings and um, the mathematical thinking that is going on.
Ms. B	Okay
	(tape starts playing)
	(tape ends)
I	Could you describe, um, what's going on? Do you remember?
Ms. B	Yes
I	Do you remember what's happening at this point?
Ms. B	Yes, when I first read the task, when I first finished reading the task, Ny Shay began to first start working by himself, that stood out to me. Everyone else starting talking in their groups, but that group in particular just started working right by themselves. Right away, made a table. And that's exactly what we were doing in class, he right away began comparing all three. Um, Ruth did the same thing,

	they were able to not just pinpoint one DJ and this was the first day and in the first few minutes of class, I remember this was very very much the beginning of the class. On day one
I	On day one?
Ms. B	Yeah, on day one. Yes, mm-hmm. And I said that in the interview that he was going to get that. In the pre-interview. Um, Raheem when I asked him what does he think about that, I realized that I broke his train of thought. He was actually um working through it, he didn't get as far as the other group but he was on the right track but I kind of just broke his train of thought. That's why I just let him go back because he didn't hear what was going on.
I	Um how do you think he might've felt when you did that?
Ms. B	Um, maybe like he wasn't paying attention. Because I asked him what does he think about that so now put him on the spot of what he thinks about that, but he wasn't really paying attention because he had his own thought going on, so he couldn't have done both. So, that just probably not the best thing on my part, to just jump right in and ask him a question, but not realizing he wasn't working either. I didn't realize it until I after I asked him and he looked up and back down like, he was working.
I	So what emotions do you think he was feeling
Ms. B	Um, maybe confused because I'm asking him to answer the question here and his thought on what was happening, but he couldn't answer it so it just made him feel like he didn't know what was going on, but he might've, but just, he wasn't paying attention.
I	Um, as you said, this was towards the beginning of the class, do you, does anything stand out in your mind? I'm gonna show you another clip, but does anything stand out in your mind about this one, which was sort of early on in the class, and how the class progressed after that?
Ms. B	Um, this group kinda worked through the problem very quickly whereas the other 3 groups they didn't start getting a table until day 3. Um, the other groups were just, they were just saying what they thought, they just were picking one, they just put in 10 hours, or 6 hours, but this group never chose a specific hour to party. They were just saying from here to here, this is good or from here to here, but the other groups picked the specific hour. So that group stood out to me, this group right here, stood out to me (pointing to video clip)
I	So how would you describe um the sort of math thinking and behavior that um this clip reminds you of? That we see in this clip and that reminds you of?
Ms. B	Um, he was into it. um, he was focused, um, Ny Shay is very very active but just to see how he settled down as soon as he came in, got right to working on that problem, I was impressed because he really started right away. And he worked through it, the whole

	entire problem. So, he was into it.
I	Um, I have another clip here that I'd like you to take a look at. Well, we think it's the 6th one. Was it that one or the one that was 7? Yeah, it's sort of the longer one. (figuring out clips with technical people)
	(video starts)
	(video ends)
I	Ok, do you remember when you were sitting with that particular group?
Ms. B	Yes
I	Just remind us again, who was in that group?
Ms. B	That was Danaya, Delwin, um La-Gee, uh Savon and Larry. And um,
I	Can you tell us what you remember about this particular group?
Ms. B	There wasn't much conversation going on, there was a little bit, um Danaya was saying that she was working ahead when , when someone said 5 hours, she went ahead an put 6 hours. When someone said 6 hours, she went onto 7 hours. But when she was explaining that, I saw Delwin turn around to the group and he was playing with the other group. But then he came back and he started working on something, um his own thought. Larry, um, I don't remember what he just said but Larry was also asking a question but they were kind of not really, I'm not gonna say into it, but they weren't into it as much as the first group was. They were kind just like, talking about it, they were just saying 6 hours and Lah-Gee said they put 6 hours because that was the easy way out. So, um, I could've asked him maybe, what do you mean was the easy way out? Cause I don't know if he was thinking just pick a number, an hour, and then whichever one is the cheapest, then you go with that one. But if they picked another one, would they have found that the other DJ company was cheaper.
I	So can I ask you to describe their mathematical behavior, um either as a group or individually, would you talk to us a little about what was going on there, in the math activity of those students
Ms. B	Um, well, I guess I would say the math activity, they, I don't think they were all on the same page. They were all kind of in their own world. At their own level because Danaya was doing something different than what Lah-Gee said, she was saying she was going ahead, but she also didn't share, as they were talking about, maybe 6, she didn't say "Oh, look I put 7" so she was kind of just working by herself. Delwin was working by himself and Savon was just kind of sitting there and just agreeing. So there's juts, I think a lot going on
I	If I ask you to describe what you thought those particular students were feeling, would you…
Ms. B	Confusion. I think Danaya was confused a little bit, I think she

	knew she had to do something else, I don't think she was satisfied with her answer, but she didn't know exactly which way to go, at that point. I don't think she was done. But she didn't know where to go.
I	Alright, and again, since this is day 1, how did you feel about what was going on at this particular time, with this group at this point in time?
Ms. B	Being that it was day 1, it was ok. Because um, that's when the questioning comes about, like when they were saying "let's party for 10 hours" but then someone said, um "no, that's too long," so then they tried a different number, the next, when they keep on, when they kept throwing out different numbers, then that's when they'll realize, maybe it is cheaper somewhere else. Just them throwing out numbers to each other was ok. For right now. But I was just surprised that no one was really keeping track, not every group was just keeping track of what they were saying, they were just putting it in the calculator, and kept on going at it
I	And by the end of the task, um, by day 3, how would you describe what was going on?
Ms. B	There were um, there was um, a little bit more understanding going on. Not all groups still came up with a conclusion, depending on the number of hours, but they were starting to get tables, um going on. This was on day 3, we have one more day and I think we should be able to wrap it up and have a discussion. Um, one thing I did notice was with Ruth, when um Ny Shay came through, Ruth...I'm sorry, Ny Shay was absent on the second day. And um, Ruth had to kind of um hold her own without Ny Shay there. But on the third day, Ny Shay and Ruth, Ny Shay came back and Ruth and Ny Shay were in a group and they started talking about what they did on Day 1. She was like "I was right, I was telling that group I was right." So she was kind of starting to get convinced that maybe from the other group, I think wasn't all the way convinced with her answer, cause she didn't have that support from her partner that she worked with. But she realized that she was right, so in the discussion, when we have today, she'll be able to express it. So, when Ny Shay is with her, I think the two working together is good.
I	And on day 2, he was not there?
Ms. B	Right.
I	She was working. How do you think she felt about that?
Ms. B	She still understood what she was doing, but when she came against the whole group, she wasn't, she's not very vocal. She just was, she was just saying to herself, "I still don't understand, I still don't agree, I still don't agree with them." Whereas if she was loud and outspoken, where Ny-Shay would be like "No, you're wrong, it's this way." So she kinda just, she just didn't agree. You know, she didn't argue it down.

I	So you still have another day of working, the students working on this task?
Ms. B	Yes
I	But um, at this point, um, you talked a little bit in the beginning about your students, um seemed to enjoy your activity.
I	Yes
I	And they see um, continue to work till the end. What do you hope, um, the outcome would be for the student. What are the specific…
Ms. B	Um that they can see that when we're talking about, when we're using variables X, it's for any number of hours. So we don't wanna just say 1 hours. We wanna open up our options, and this is what we have to do in life. We don't wanna just say "We wanna party for 4 hours." We might party for 5 because it's only 5 dollars more. So we might use that extra hour, or we just need to be open to seeing the different, um, variety of numbers. Not just stick on one and that's it.
I	So, the expectation is that your students will be able to um, compare and make a determination and support that, their decision?
Ms. B	Right. By looking at a table, they should be able to easily uh, find the 3 equations, because we've been working on the equations, they should be able to find the equations for each DJ. Put it into a table, compare the table, but also with the graph. They should be able to compare it using the graph. So, to see which one is the better deal.
I	And, is there going to be an opportunity for students to share? How do you think students will feel at the end, sharing um their work, and coming up with their decision?
Ms. B	They like to share their work, I think when they share their work, those who didn't get it, they're gonna realize "Oh, that's all we had to do?" Because we've been doing things like this and they're gonna relate it to another problem we've had that's similar to this, but we only used 2 stores. So I think, they're gonna make the relationship between the 2 problems as soon as they get the answer
I	So you think your students have gotten past the most difficult parts of this task?
Ms. B	Yes, for the most part, there are still some haven't gotten it yet and that's ok, um the window setting, some of them, when they're graphing their equations, they're not seeing their graph on the equation, because they're not changing their window settings right, so some could really make that um, that relationship, by just saying "Oh, the x minimum is zero, x maximum is 10." Some were able to put it right in the calculator, but some were just fooling around with trial and error. So I think after they go over it and discuss it and then put it in, they're gonna say "Oh, I got it."
I	Um, would you use this activity again?
Ms. B	Yes, I think it was a good one because um it related to what we're going to be going through. So, it put interest into umm, in trying to

	figure out the answer.
I	Um, ok. Is there anything that stands out in your mind so far? I know you have to finish off the activity with the students, that I haven't asked you about, something particular that stands out in your mind that you wanna talk about?
Ms. B	Um, no its just interesting that they still haven't um, well right away, I was just thinking that they would've um determined X's for any number of hours and they would've just made a chart, a table, quickly from zero hours to like 20 hours and its just interesting that they just arbitrarily just picked a number of hours, so that stood out to me cause we've been working on that, and working on that and they've been making tables for days and they didn't even start with the table. So, that stood out to me. That did.
I	And um, any particular student stand out?
Ms. B	Um, Ny-Shay stood out, Ny-Shay stood out to me just because um, he worked through it very quickly and he is a worker, the class knows him to joke around a lot, and not really do work but lately he has been doing all of his work and he's been engaged, he's been answering questions, so he's really standing out to me at this point because of the way he's working, the way he's working, he seems like he's excited to come to the class and…
I	And that's a change for him?
Ms. B	Yes. Coming on time to class on time, he would come late to class and just, whatever, like if he wants to come in the beginning, or he'll come in ready to joke around but now he's coming in on time, and ready to work.
I	And he seems to be um, this Ny Shay?
Ms. B	Yes
I	At this particular time partnered with Ruth?
Ms. B	Yes
I	And they seem to be working together?
Ms. B	Yes
I	Do you have any ideas on why that might be?
Ms. B	I just think Ny Shay is starting to really like the math class. Ruth also wasn't really working, wasn't really working in math class. Over just the last couple of weeks, she started working a little more. So just seeing her really involved and engaged too. So two of them who aren't really doing anything now working together and they're getting through the tasks. That stood out to me.
I	Um, ok. Do we have time for one? One more clip? Clip 9.
	(video starts)
	(video ends)
I	Ok, that was Danaya, um it appeared that you were highlighting her idea and you'd asked her to talk about her idea, present it to the rest of the group...thinking during that time?
Ms. B	Um, well I saw she was trying to work on getting the equation, I

	didn't correct her, but she was using J as a variable, because um in class, they get confused when they see variable, um equations with different variables. If they don't just see X and Y being used, they think they can't. So I try and stress that you can use any variable as long as you label it. But in this case, they told us to use X and Y. But she still went off and said "Well Ms. Barnes said we could use any variable." And I did but I didn't say it for this particular case. I didn't correct it though. That wasn't, um, what I was focusing on, I was just trying to focus on that she was trying to get her thought down with the equations and um, it, it wasn't right, but I wanted them to try and tell me what each number determined. She I think said 10? She said 175 plus 30J plus 10. Where did that 10 come from? So, and I still don't know where that 10 came from, and they still, their equations are wrong, and they don't know that yet, I want to let it be so when we have a discussion, other students, they're gonna question, like "Where did they get that 10?" So, um, I just wanted, I'm just trying to get her to explain how she chose those numbers.
I	And how do you think she might've felt when, when she was explaining?
Ms. B	Um, well that was Day 1. I-I did notice that um, she wasn't herself. Um, she wasn't as outspoken as she usually was throughout the class, she was a lot quieter. So um, even like there, she said a little bit, usually she goes on and on and on, so I really didn't know, how to…
I	So you really don't know if there was a reason that she…
Ms. B	Yeah, she was trying to work but she wasn't like how she usually was.
I	Oh, ok. Um, so we do wanna, is it…just 1 more clip? Is it? It must be 9, that's the one we just did. This is the one we just showed again. Cecilia's view. That helps. That doesn't help.
	(video starts)
	(video ends)
I	So, can you describe, did you know that that was…
Ms. B	No, I didn't even know that that was going on. Um…
I	…or what you saw
Ms. B	She looked upset. Um, sad, something was going on because I did ask her, um "Is everything ok?" And she said she doesn't wanna talk about it, ask at the end of class. And I said, cause I know you weren't asking your same way. That was um, cute. Lah-Gee put his arm around her cause he was trying to, I guess make her feel better, asking what was wrong. I think he blew in her ear though. So, but, um, you can definitely tell.
I	How does that make you feel when you see students who are comforting each other? I mean I don't know…
Ms. B	I like it and I encourage it because um, it's easy for us to um, down

	someone but it's hard for us to kind of like talk nice and you know pick someone up from feeling down, so I liked what he did, blowing in the ear part (laughs), but he showed, that was his way of just trying to make her smile or whatever.
I	Now again, that was Day 1.
Ms. B	Right
I	Is there anything, having seen that clip now, even though the task wasn't completely over, is there anything that you can reflect on what happened in the second day, or the third day?
Ms. B	She was more outspoken on the second day and the third day. She was talking more and I even said, "So you're back to yourself now?" She was talking a lot more and she was more involved, because she was definitely somewhere else. She was trying to be focused and on task, but I could clearly see that there was something else going on. So you know, when we say we want them to be focused and to do what they're supposed to be doing, we don't know if something is going on. Cause we all have days sometimes, some of us could focus better than others, but I could tell she was trying, but she wasn't there.
I	When you think of that group of students, um, um, Lah-Gee, Larry, Danaya, Savon, and Delwin, um, does anything stand out about them working together? Danaya's the only girl in the group and how do you, um, how do you think they worked with each other, make each other feel?
Ms. B	Um, I never thought about her being the only girl in the group, cause you just said it, because I try not to always do like, all girls, all boys, and we just mix, um. She's very outspoken, so she's, she can hold her own and just be the only girl in the group but um, just the group overall, like Savon and Delwin, they're quieter, so they might have thoughts but they might not say it all the time. And Larry, sometimes he is not always on task, and he loses focus easily so when he is focused he can contribute but a lot of times, if something else is going on, instantly, he turns and he's there. So it's just a lot of not just…workers. They're jut solely geared into it. So,
I	Ok, anything else, um, that you can share with us at this point? About this task, about this project, anything about the clips you've seen?
Ms. B	Um, well the clips…
I	Anything surprise you?
Ms. B	Well I can just say that as a teacher you don't see everything that's going on because I didn't see how she got up and got the tissue and then how she sat back down, then I would come back over and then maybe say, "Well how come you're not working, how come you're not doing this," but not even realizing because you're so in the mix of things that are going on, you don't, you're not, I guess were just not insensitive all the time, as much as I try to be. You have to get

	things done, so you don't have time to…past, why you're not doing what you're supposed to be doing, so clearly, that just kinda made me just step back for a minute and just be a little bit more sensitive, I think.
I	And did you learn about anything that was going on? Or that you didn't know about…coming to class…with Danaya or anyone?
Ms. B	No, she just…no, no, um I just told her I was there, if she wanted to talk. And that was it and she said ok and she smiled. I did see that she wasn't herself. Um, other than that, they they worked but you could tell, like in the other group, the group behind, like when we were on Savon and um Delwin, Danaya's group, that group behind, they weren't focused on the entire time. Or they could've been focused and I'm trying to learn this as well, just because they're not writing and really into it, even though they're laid back and giving their thought, they could still be contributing, and doing what they're supposed to be doing. And I did try and act, try and be a little bit more sensitive to that was well. Because you have some people who work through it, who will just sit back and say their thought. Well, "I think it should be 10 hours," and they're just sitting back.
I	And ultimately, how will you find out what each student will learn through this particular task?
Ms. B	Just by the conversation, um, when we go through each group and share our thoughts um, and then just um, just talking to them, um "Why you think this DJ is better?" Just talking to them, questioning them.
I	Ok, um well I really appreciate you taking this time, especially since we didn't have anyone to, to look after your class, but obviously, you have a great…in the class, so that's…appreciate it, and we'll be speaking to you again about how you feel about the lessons and how your students feel.
Ms. B	Okay
I	Thank you

Key: I = Interviewer
Ms. B = Teacher