UNDERSTANDING HOW TEACHERS' ACTIONS IMPACT STUDENT LEARNING

BY ROYA BASU

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ABSTRACT OF THE DISSERTATION

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Lesson study, a collaborative teacher professional development process, affords teachers the opportunity to interact with their peers to analyze, reflect on, and revise their thinking in order to enhance their teaching practices. This case study described what occurred during a lesson study process, including the events that took place in four elementary and middle school mathematics classrooms, with a focus on planning, implementation, and reflection of teachers-both before and after the lesson- and on the ways in which their students solved mathematical problems. The goal of this study was to better understand how the teachers' actions impacted the mathematical thinking that took place in their classrooms.

The following research questions guided this study:

1. What instructional and pedagogical decisions were made by the teachers prior to, during, and after each lesson implementation?

2. What types of interventions (including questions) and interactions with students were enacted by each teacher during his or her lesson implementation?
3. How did teachers' actions and interactions with students impact the mathematical work and reasoning that took place in the classroom?
4. What evidence is there, if any, of teachers' recognition of and reflection on their teaching practices, the lessons, and the students' mathematical work?

The main source of data, the video tapes of the lesson implementations and the debriefing sessions, were used in order to identify, transcribe, and code critical events that occurred during the lessons and debriefing sessions. The coding, coupled with other data sources such as students' work, was used to develop descriptive analyses of the lessons, teachers' reflections, and the whole lesson study process. Collectively, these were used to answer the research questions.

The results of this study highlighted the importance of a) teachers' autonomy-supportive behaviors in the classroom; b) teachers' familiarity with a wide range of solution strategies for problem activities; c) opportunities for teachers to work collaboratively and to reflection on students' mathematical work; and d) understanding the impact of the "in-the-moment" decisions that teachers make on student learning.

By examining the ways in which teachers' actions impacted students' learning during the lesson study process, this study has implications for designing professional development programs that are potentially effective in helping teachers become reflective practitioners.

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"At times our own light goes out and is rekindled by a spark from another person. Each of us has cause to think with deep gratitude of those who have lighted the flame within us."

Albert Schweitzer

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Dedication

To my Mom, Feri Izadi, who encouraged me to start this.

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CHAPTER 1: INTRODUCTION

1.1 Statement of the Problem

According to the National Center of Education Statistics (1999, 2001), teacher preparation is one of the main factors that could potentially impact students' achievement in mathematics. There is a general consensus among researchers and educators that teacher quality matters greatly and that teachers do impact student achievement (ACE, 1999; Carey, 2004; Clotfelter, Ladd, &Vigdor, 2007; Darling-Hammond et al., in press; Mendro, 1998; NCTM, 2000; Stronge, Ward, Tucker, & Hindman, 2007; Wright, Horn, &Sanders, 1997; Sanders & Horn, 1998). However, the sheer complexity of teaching presents a challenge in determining exactly how teachers impact student understanding.

The fact that the instructional skill of the teacher is considered a major factor in student learning highlights the importance of meaningful professional development programs that can improve teacher quality. Schorr et al. (2007) emphasize that "all teachers, no matter the level, should continue to develop, since there can be no final or fixed state of experience or excellence" (p. 433) and Marzano and Walters (2009) believe that one of the goals of every school district should be that every teacher improves every year.

Lesson study as a collaborative teacher professional development process has been commonly used for over a century in Japan (Murata, 2011). During this process the teachers meet in small groups to discuss learning goals for their students, select a focus topic, study it, and write a detailed lesson plan. This lesson is then taught by one of the teachers and is attended by other teachers in the group. During a debriefing session that follows, the teachers reflect on the lesson, discuss their observations, and make necessary modifications. If desired, another teacher implements the refined lesson in a second class while the other teachers observe. This process may be repeated several times and at the end, the teachers write a report reflecting on their own learning experiences during this whole process so that other teachers can benefit from it (Lewis, 2002).

Many Japanese teachers believe that participation in lesson study cycles has positively impacted their teaching practices (Shimizu et al., 2005). Citing this culture of continuous teacher development programs in Japan, a number of scholars (Hill et al., 2008; Ma, 1999; Stigler & Hiebert, 1999) have called for reshaping the nature of teaching and professional development in the United States where teachers' attendance at a few workshops is typically considered an acceptable form of professional development. The learning community model, inherent in the lesson study process, affords teachers the opportunity to interact with their peers and to continually analyze, reflect on, and revise their thinking in order to build upon and enhance their teaching and learning models.

Although there is a large body of research literature that supports and emphasizes the importance of preparing elementary school children for the mathematics they encounter in higher grades (Blanton & Kaput, 2005; Cobb, Whitenack, & McClain, 1997; Driscoll, 1999; Warren, Cooper & Lamb, 2006; Yackel, 1997), more research is needed to shed light on elementary school children's thinking as they tackle mathematical problems and to find out how or whether teachers' actions can foster the development of mathematical reasoning in their students.

1.2 Purpose of Present Study

This case study described what occurred during a lesson study process, including the events that took place in four elementary and middle school mathematics classrooms, with a focus on planning, implementation, and reflection of teachers-both before and after each lesson. In particular, there was a focus on the teachers' interactions with students, their instructional practices, and their reflections on the way the students attempted to informally solve a series of pictorial algebraic problems and justify their solutions.

The goal of this study was to better understand how the teachers' actions impacted the mathematical thinking that took place in their classrooms. The following research questions guided the study:

1. What instructional and pedagogical decisions were made by the teachers prior to, during, and after each lesson implementation?
2. What types of interventions (including questions) and interactions with students were enacted by each teacher during his or her lesson implementation?
3. How did teachers' actions and interactions with students impact the mathematical work and reasoning that took place in the classroom?
4. What evidence is there, if any, of teachers' recognition of and reflection on their teaching practices, the lessons, and the students' mathematical work?

1.3 Validity

In keeping with validity procedures for qualitative research as described by Creswell & Miler (2000), several steps were taken to ensure the credibility of the findings throughout the process of data analysis.

First, the accuracy of transcriptions of the data was verified by a second independent researcher serving as an external audit. To verify the accuracy of the coding, a random sample of 46 instances of coded actions and comments were presented to four independent researchers, together with a list of codes used in this study. After viewing the relevant video clips of the lesson implementations, the researchers were asked to code the 46 instances: 98% of the time they designated the same code from the list of the codes and 6% of the time they selected the same code but suggested the addition of a second code from the list to describe the instance. In case of discrepancies, the disputed coding was discussed until a consensus was reached. Second, the researcher triangulated the data by using the video recordings, transcripts, students' work samples and teachers' written reflections and notes to ensure an accurate rendering of the events that occurred during the course of this study. Third, the researcher performed a member check by soliciting input from the teacher participants on the accuracy of the data and on the subsequent analysis, interpretations, and conclusions of this study.

As a participant in this study, the researcher might have brought in some unintended biases into the analyses and final conclusions. In order to safeguard against such partialities, the member check and peer review procedures helped to keep in check the conclusions and ensure the integrity of this study.

1.4 Limitations

The main source of data for this study, in the form of videotapes, was collected in three days, in the classroom of four teacher participants with a total of 61 students. Each of these four classes was visited only once. With only one roving camera in the classroom, it is reasonable to assume the loss of some potentially valuable and important data for this study. Therefore, when examining the results of this study, one must take into account the limitations imposed by the relatively short timeframe and the small sample size.

Additionally, for most of the participants, teachers and students alike, taking part in a lesson study process was a novel idea and therefore it is impossible to determine if or how the presence of a cameraman and several observers in the classroom impacted the behavior and responses of the sample population in this study.

CHAPTER 2: REVIEW OF THE LITERATURE

2.1 Theoretical Framework

This study is grounded in theories on how people learn and develop knowledge. Davis (1984, 1992) believes that we, as individuals, construct our understanding of the environment we live in by creating our own mental models that we use to make sense of our experiences. Schorr and Koellner-Clark (2003) describe knowledge in terms of internal models, or explanatory systems, for making sense of situations. From this modeling prospective, acquisition of knowledge or knowledge development takes place when new experiences are mapped into existing internal models (Schorr & Koellner-Clark, 2003). These internal models are the building blocks which we use to form mathematical ideas (Davis, 1984, 1992) and constructing strong mathematical ideas goes hand in hand with deep understanding of mathematical concepts.

There are many theorists who call for the structure of a classroom environment which promotes a deeper mathematical understanding in children (Brooks & Brooks, 1993; Clement, 1991; Davis, Maher, & Noddings, 1990; Groves & Doig, 2005; Kamii & Dominick, 1998; Schorr, Warner, Gearhart, & Smuels, 2007; Warner, Schorr, Arias, Sanchez, & Endaya, 2006). The teacher who presents the students with meaningful activities, shows interest and respect for the students' ideas, and encourages them to share and justify their thoughts would, as a result, provide the students with the opportunity to build strong mathematical ideas (Alston & Pedrick, 2011; Maher, Davis, & Alston, 1992).

2.2 Teacher Preparation: What Teachers Need to Know

The goal of most teacher preparation and/or professional development programs is to develop effective teachers and ultimately increase student learning. However, the question regarding the types of teacher knowledge necessary for effective teaching has been the subject of much interest to teacher educators and researchers over the years (Ball, 2011; Cole & Knowles, 2000; Darling-Hammond, 1999; Grossman, 1987; Grossman, Schoenfeld, & Lee, 2005; Hill, Rowen, & Ball, 2005; Michaels et al., 2008; Monk, 1994; Shulman, 1987).

Cole and Knowles (2000) provided an overview of various research perspectives on teacher knowledge, tracing its evolution over the past few decades. They described a shift in focus from measurable and observable teacher skills and behaviors to decision making and cognitive processes involved in teaching practices. However, the current national focus on teacher performance standards and evaluation systems, has created a shift back to more measurable models in which, teacher effectiveness is tied to students' standardized test scores.

There are multiple forms of teacher knowledge that are deemed important in student achievement gains. Shulman (1986) identified three categories of teacher subject matter knowledge: *content knowledge*- including facts, concepts, and the structure of the knowledge; *pedagogical knowledge*- including representations of concepts and an understanding of what aspects of certain topics could present a challenge to the learner; and *curriculum knowledge*- including awareness of how topics are related and concepts are built upon at different stages.

The results of the studies on the impact of content knowledge of elementary and secondary teachers on their students' achievement were somewhat mixed and equivocal. Darling-Hammond (1999) attributed this ambiguity to the fact that "subject matter knowledge is a positive influence up to some level of basic competence in the subject but is less important thereafter" (p.6). A study by Monk (1994) showed similar results. His findings revealed a positive correlation between the number of undergraduate mathematics courses taken by high school teachers and their students' achievement; however, this pattern of growth in students' achievement did not continue beyond five mathematics courses taken by the teachers. Additionally, the different criteria used to measure content knowledge – for example the number of college level courses, often a proxy for mathematical knowledge, as opposed to scores on subject matter exams – contributed to the conflicting results on the effects of teacher content knowledge on student academic gains (Darling-Hammond, 1999).

Ball (2011) believes that for pre-secondary teachers, majoring in mathematics does not necessarily translate into greater gains in their students' learning and Stronge (2007) has contended that "subject matter knowledge positively affects teaching performance; however, it is not sufficient in and of itself" (p.10).

Although Shulman (1986) identified three distinct categories of teacher knowledge, he has also acknowledged that skillful teaching requires a type of emerging knowledge that combines content and pedagogy. He defined pedagogical content knowledge as

> …ways of representing and formulating the subject that make it comprehensible to others……an understanding of what makes the learning of specific topics easy or difficult; the conceptions and preconceptions that

> students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. (pp. 9-10)

The idea of intersecting content with pedagogy has since been suggested by others (Grossman, 1987; Ma, 1999). Michaels et al. (2008) described "pedagogical content knowledge" as a blend of the essential understanding of the discipline with an understanding of how students learn. Grossman, Schoenfeld, and Lee (2005) defined pedagogical content knowledge as the teacher's "ability to anticipate and respond to typical student patterns of understanding and misunderstanding within a content area and the aptitude to create multiple examples and representations of challenging topics that make the content accessible to a wide range of learners" (p. 210).

Hill, Rowen, and Ball (2005) assert that effective teachers need to have Mathematical Knowledge for Teaching (MKT). They describe MKT as "the mathematical knowledge used to carry out the *work of teaching mathematics*" (p. 373). This type of mathematical knowledge uses effective instructional strategies to help the learner develop targeted mathematical concepts. They regard the "work of teaching" as the ability to clarify terms and concepts, use representations, adjust and modify textbook presentations of concepts, and have the diagnostic ability to analyze students' solutions and identify their misconceptions. In essence, MKT is the intertwining of the three categories of teacher knowledge, where a deep understanding of pedagogical content knowledge informs curricular choices.

One study conducted by Hill, Rowen, and Ball (2005) examined the impact of teachers' MKT on first and third graders' mathematical achievement gains over one year. Based on the relationships between subject matter and pedagogy, they assessed teachers' mathematical knowledge for teaching by devising a measure that was task-oriented. They also incorporated, in the design of their study, multiple choice tests developed by Hill, Schilling, and Ball (2004) which measured not just the mathematical content knowledge but also the teaching-specific mathematical skills necessary for the “work of teaching”. They used teachers’ scores on these tests as a predictor of students’ gains in mathematics achievement and found that teachers’ MKT was significantly and positively related to students’ gains during first and third grades.

In a related case study, Hill and colleagues (2008) explored the relationship between the teachers’ Mathematical Knowledge for Teaching and the Mathematical Quality of Instruction (MQI). They also examined specific aspects of teaching that were most impacted by MKT. They used a sample of ten teacher volunteers – teaching various grades from second to eighth grade- who took the MKT assessment and agreed to have nine of their lessons and the follow up interviews videotaped. The researchers also developed a method for scoring the quality of instruction by defining and coding key aspects of MQI, which included such elements as: presence of mathematical errors, inappropriate/appropriate responses to students, connection between mathematics and classroom activities, richness of the mathematics to include representation and justification, and the use of mathematical language.

Hill and colleagues claimed that “the inescapable conclusion of this study is that there is a powerful relationship between what a teacher knows, how she knows it, and what she can do in the context of instruction” (p.496). They found those teachers who had a higher MKT avoided making mathematical errors and implemented a richer and more rigorous mathematical instruction.

By highlighting the positive impacts of teachers' content-oriented instructional skills on students' learning, these findings have significant implications for designing effective professional development programs for practicing teachers.

Mewborn (2003) believes that the design of professional development programs should be based on theories on how people, adults in particular, learn and acquire knowledge. "Just as one cannot expect students to learn something simply by being told that it is so, one cannot expect teachers to change their teaching practice simply because they have been told to do so" (p. 49). Mewborn's belief is consistent with the assertion made by Maher and Alston (1990) that teachers, just as children, "learn in social contexts in which they can interact and make sense of their experiences" (p. 148).

Similarly, Schorr and Lesh (2003) echo Mewborn's (2003) belief on learning. They contend that "telling" teachers about effective teaching practices is just as ineffective – in terms of knowledge acquisition- as "telling" students about mathematical ideas. Schorr and Lesh (2003) offer a view on students' knowledge acquisition which is parallel to teachers' learning. They assert that in the same way that students learn through meaningful problem solving situations, teachers also learn through personally meaningful problem solving experiences.

Research findings (Francisco & Maher, 2010; Schifter, 1998; Schorr, Warner, Gearhart, & Samuels, 2007; Warner, Schoor, Arias, & Sanchez, 2010;) suggest that teachers should have the opportunity to learn about mathematics and mathematics teaching in an environment which is compatible with and simulates a classroom climate that fosters students' learning. In other words, teachers need to learn actively through dialogue, discussions, and interactions and should be afforded the chance to critique each other's work, justify their solutions, and revise their own ideas (Mewborn, 2003).

Schorr et al.'s (2007) view on effective teacher development programs is based on the models and modeling approach (Schorr & Koellner-Clark, 2003). They (2007) contend that when learners (teachers) encounter something for the first time or from a different perspective, the initial models or the explanatory systems they use to make sense of the information may be rudimentary and hazy. Therefore, the learners need the chance to "revise, refine, extend, test, and share their evolving models for teaching and learning mathematics over extended periods of time" (p.432).

Schorr et al. (2007) reported on how a modeling perspective was used in the design of a meaningful professional development program, a key component of an initiative entitled the Newark Public Schools Systemic Initiative in Mathematics (NPSSIM) which began in October 2002. During weekly meetings held at Rutgers University, Newark Public School students, teachers, administrators, undergraduate students, and researchers worked collaboratively on mathematical tasks specifically designed to create models for "constructing, describing, explaining, manipulating, predicting, and controlling complex systems" (p. 435). These mathematical tasks were carried out in an environment that fostered deep conceptual understanding, encouraged the construction and justification of conjectures, and nurtured the development of *powerful mathematical affect*. Goldin et al. (2005) assert that *powerful mathematical affect*, resulting in powerful mathematics, occurs in "safe" classroom environments where students can confront and use their negative feelings toward mathematics- such as anxiety, impatience, and frustration- to develop positive feelings such as curiosity, anticipation, pride, and enjoyment.

To illuminate the effects of the professional development component of the initiative (Newark Public Schools Systemic Initiative in Mathematics), Schorr et al. (2007) reported on the findings of a case study that examined teacher growth and its impact on student learning in the district. The data showed that the NPSSIM Project had a significant positive impact on students' scores on standardized tests. They (2007) proposed that as teachers pay closer attention to their students' thinking and improve their own subject matter knowledge, their students improve as well.

In another case study related to the NPSSIM Project, researchers Warner, Schorr, Arias, and Sanchez (2010) studied two middle school teachers' interactions with their students over a period of time. They documented the teachers' models for helping students to make sense of problem situations and justify their solutions, as well as the impact of these changes on students' knowledge and understanding. The relevance of their study to the present study is twofold, as it examines teacher questioning in the context of an extended professional development program. The researchers (2010) found that both teachers initially asked questions that were based on their own understanding of the topic. While one teacher asked mainly generic questions, the other teacher's questions were directive in nature. Over time, the generic questions were replaced by explicit questions that built on student's understanding and directive questions gave way to guiding questions that were based on students' own ideas and prior knowledge. The researchers report that over the course of this study, as the teachers modified their teaching models for problem solving, the students began to engage in spontaneous classroom discourse about their ideas and solution strategies.

The positive results of these findings is a further testament to the effectiveness of professional development and teacher preparation programs in which teachers are not just passive receivers of information but also active participants in a learning community. This affords them the opportunity to continually analyze, reflect on, and revise their own mathematical thinking in order to build upon and enhance their teaching and learning models.

There are many scholars (Darling-Hammond, 1996; DuFoour & Eaker, 1998; Fullan, & Hargreaves, 1991; Joyce, & Showers, 1995; Louis, Kruse, & Raywid, 1996; Newnann, & Whlage, 1995; Schorr, Warner, Gearhart, & Samuels, 2007; Stigler, & Hiebert, 2009) who advocate the development of a professional learning community as a strategy for continual teacher growth and substantial school improvement. Cochran-Smith and Lytle (1999) outline three types of knowledge relating to teacher development within learning communities: a) knowledge for practice- including subject matter knowledge, pedagogical knowledge, and knowledge about relevant research findings and theories of learning; b) knowledge in practice- referring to knowledge acquisition through experience, reflection, and narratives; and c) knowledge of practice- developed collectively within the learning community and pertaining to the theoretical aspects of both practice and knowledge and the relationship between the two.

Citing a culture of collaborative, continuous teacher development programs in Japan and China, a number of scholars (Hill et al., 2008; Ma, 1999; Stigler & Hiebert, 1999) have called for reshaping the nature of teaching and professional development in the United States where teachers' attendance at a few workshops is typically considered an acceptable form of professional development. Meanwhile, learning community groups such as Collaborative Inquiry and Lesson Study groups are gaining popularity within the United States (Lesson Study Research Group, 2007; Nelson, Deuel, Slavit, & Kennedy 2010).

2.3 Lesson Study

Lewis (2002) describes the lesson study process as a cycle during which, a group of teachers work collaboratively to formulate goals for student learning, select a focus topic, study it, and write a detailed lesson plan, referred to as the "study lesson" or "research lesson". This lesson is then taught in a real classroom by one of the teachers in the group while the other teachers observe and take notes on student learning and development. After this implementation, the group meets again to reflect on the lesson, discuss their observations, and make necessary revisions. If desired, another teacher implements the refined lesson in a second class while the other teachers observe. This process may be repeated until all revisions –deemed necessary by the teachers- are made to the lesson. At the end, the teachers write a report reflecting on their own learning experiences during this whole process with the ultimate goal of improving instruction for their students.

Lesson study as a collaborative teacher professional development process has been commonly used for over a century in Japan (Murata, 2011). Many Japanese teachers believe that participation in lesson study cycles has positively impacted their teaching practices (Shimizu et al., 2005). By providing teachers the opportunity to examine educational approaches such as collaborative learning or problem-based math instruction, curricular content and instructional sequences, lesson study has been used effectively in Japan to connect theory and practice and deepen teachers' understanding of student thinking (Murata, 2011).

In a study conducted by Meyer and Wilkerson (2011), the researchers explored whether participation in lesson study increased teachers' mathematical knowledge and advanced teachers' knowledge for teaching mathematics. A total of twenty-four middle school mathematics teachers from seven schools in a large urban district completed a six-hour summer professional development session to learn about lesson study with the goal to plan and implement a lesson by the end of December of the same year. These teachers were divided into five groups consisting of four to five teachers. One teacher in each group volunteered to implement the research lesson while others observed and took field notes. During the reflection sessions that followed, the groups had the opportunity to modify and re-teach the lessons.

The data from this study indicated that in three out of the five groups, lesson study did afford the participating teachers the opportunity to develop mathematical knowledge for teaching. Interestingly, the analysis of the data revealed three common characteristics that were displayed within those three groups that provided opportunities for improved teacher knowledge. These characteristics, however, were noticeably absent in the other two groups who failed to provide opportunities for the development of mathematical knowledge. In their analysis of the data, Meyer and Wilkerson (2011) described the three potential factors that helped support an increase in teachers' knowledge of mathematics while participating in lesson study: a) lesson plan- paying close attention to creating a lesson plan that addresses students' interests and makes mathematical and real world connections; b) level of anticipated students' questions and responses- putting thought into predicting students' questions and answers supports a better understanding of teachers' own knowledge of both mathematics and students; c) emphasis on conceptual understanding- time spent during the planning sessions on choosing activities that promote conceptual understanding.

The results of this study highlighted the fact that participation in lesson study does not automatically result in teacher growth and was consistent with Fernandez's (2003) belief that for lesson study to work as a feasible form of professional development, the teacher participants must view the lesson study process through three critical lenses: the researcher lens, which encourages the teachers to ask questions about certain aspects of the practice and motivates them to design classroom experiences to address these questions; the student lens, which allows the teachers to examine the lesson from the students' perspective; and the curriculum developer lens, which helps the teachers with the organization and sequencing of the learning experiences.

In a related case study, researchers Honenshield Tepylo and Moss (2011) examined the gains in teacher mathematical knowledge of four grade 5 and 6 teachers in a rural district as they participated in three cycles of lesson study on teaching fractions over a period of two months. The researchers had anticipated that each of the four phases of lesson study- Investigation of Goals; Planning; Research Lesson; Debrief/Reflection- might support increases in teachers' mathematical knowledge for teaching. They considered four components of teacher knowledge: Common Content Knowledge (CCK), Specialized Content Knowledge (SCK), Knowledge of Content and Student (KCS), and

Knowledge of Content and Teaching (KCT) (Ball, Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008). For their analysis, they chose to combine CCK and SCK into one type of knowledge, Teacher Content Knowledge (TCK).

They found that after multiple cycles of lesson study and as the teachers paid more attention to the tasks at each phase of the cycle, the gains in teacher knowledge were increased. For example the anticipated gains in KCS during the planning phase of the cycle did not occur until three cycles of lesson study were completed. Interestingly, they observed gains in TCK not only in the initial phase- Investigation of Goals- but also in the last phase when teachers reflected on the lessons during the debriefing sessions.

Olson, White, and Sparrow (2011) reported on a case study which explored the influence of lesson study on five elementary school teachers' mathematics pedagogy. These teachers were part of three lesson study teams that were formed along grade level bands in a Midwestern school district. Two of the three lesson study teams completed their research lesson, while the third team, after attending the second meeting, abandoned the lesson study research. Two of the five teachers in this study were members of the third team. This provided the researchers (2011) the opportunity to contrast the pedagogy of teachers who completed their research lesson with those who did not.

The analysis of the result of this study showed that the three case study teachers who completed their lesson studies made some positive pedagogical changes. These teachers became more curious about their students' thinking and as a result asked the type of questions that elicited mathematical thinking, initiated student led discussions, and deepened mathematical understanding. Also the lesson study experience had prompted these three teachers to reflect on and discuss their pedagogical practices, which led them to reexamine their beliefs about teaching and learning. As a result they asked questions that encouraged students to reflect, justify, and extend their mathematical thinking. In contrast, the researchers found no changes in the pedagogy of the two teachers who had abandoned the research lesson midstream. “Their beliefs and assumptions did not change and they continued to teach using a comfortable pedagogy of past practice” (p.53).

Olson, White, and Sparrow (2011) also assert that certain factors can limit the success of lesson study as a professional learning process. They found that the success of lesson study appeared to depend on how well the teachers in the study functioned as a team. In successful lesson study groups, teachers formed closely knit communities where leadership was equally divided, and teachers openly shared their experiences and vulnerabilities.

Hart and Carriere (2011) described a case study of eight third-grade teachers from a small urban school district in the southern United States who volunteered to participate in three lesson study cycles spanning over the course of a school year. The mathematics coordinator and university faculty consultant, Carriere and Hart respectively, played a dual role in this project as researchers and facilitators/participants. They wanted to find out whether the teachers participating in the lesson study cycle and supported by expert facilitators would develop the three critical lenses - researcher lens, student lens, and curriculum developer lens (Fernandez at al. 2003) - necessary for becoming a productive lesson study community. The videotapes of the first and last lesson study cycle sessions were transcribed and the teachers’ comments were coded for evidence of the three lenses.

By comparing the substance of the teachers' coded remarks from the first cycle sessions to the final cycle sessions, the researchers found that over the course of the year, the teachers developed a more sophisticated lens in two out of the three lenses, namely student lens and curriculum developer lens. While teacher remarks from the first lesson were primarily on how students behaved and what they did during the lessons, in the final lesson, teachers began to unpack and look at the problems through the student's eye and comment about specific aspects of mathematics that were confusing to the students and what students appeared to understand about the mathematics they were doing. The substance of the comments coded as curriculum developer lens also changed over the course of the study. Initial teacher remarks on how the organization of the lesson or materials kept students on task and avoided confusion, were replaced in the final lesson, by comments which were more focused on how the organization of the lesson impacted the development of conceptual understanding in students.

2.4 Teacher Questioning and Interventions

In line with NCTM‘s (1991) recommendations, good teaching occurs when probing questions are used to engage all students in mathematical discussions. Students involved in mathematical discourse will have the opportunity to meaningfully communicate their thinking, reflect, internalize, justify their reasoning, and as a result, gain a deeper understanding of the mathematical concepts. A timely and appropriate question by the teacher may not only foster the development of an ideal learning environment, but could also be used as a tool to assess students' understanding or misconceptions.

There is a comprehensive body of research on the topic of teacher interventions and classroom discourse (Blanton & Kaput, 2005; Dann, Pantozzi, & Steencken, 1995; Davis & Maher, 1997; Kawanaka & Stigler, 1999; Martino & Maher, 1999; Warner, Schorr, Arias, & Sanchez, 2010; White, 2003). Employing these research supported practices could foster higher student learning. The types of questions that demand justification or generalization encourage mathematical discourse, challenge students to offer alternative solutions, and nurture children's sense of curiosity, which can enhance learning and result in a deeper mathematical understanding (Boaler & Brodie, 2004).

In a study conducted by Blanton and Kaput (2005), the researchers examined the classroom practice of one third-grade teacher to find out how and to what extent the teacher was able to help the development of students' algebraic reasoning skills. This particular teacher was in her second year of a 5-year professional development project, GEAAR (Generalizing to Extend Arithmetic to Algebraic Reasoning). The researchers visited the classroom 38 times during the course of one year, collecting their data in the form of field notes, audio tapes, teacher's reflections, and the students' written works and activities. They found evidence to support the fact that the teacher's instruction was instrumental in promoting algebraic thinking and reasoning among her students. They asserted that the teachers' ability to use a wide range of arithmetic content and integrate it with some forms of algebraic reasoning (generalized arithmetic and functional thinking) throughout instruction, provided evidence of significantly cultivated students' algebraic reasoning skills.

The implication of the results of this study is important in the area of professional development. It indicates that through appropriate training, some elementary school teachers could potentially identify areas in arithmetic content that could be extended and generalized into algebraic ideas. A raised awareness of the importance of mathematical reasoning in young students should encourage the teachers to view the mathematic they teach differently and to listen to students' thinking for opportunities to foster these important skills.

A paper presented by researchers Dann, Pantozzi, and Steencken (1995), described a qualitative study which focused on 13 seventh graders. These students were part of a larger group of students involved since grade 1 in an on-going longitudinal study in a working class district in New Jersey. After two days of working on a combinatorics problem by building concrete representations, a boy named Jeff claimed that he was unconsciously learning something by doing mathematics. This prompted the researchers to investigate the problem solving activity that Jeff and his classmates were engaged in to analyze the details of the learning experience that incited Jeff's response. The researchers kept a focus on how teacher questioning facilitated students as they a) justified their ideas, b) expanded ideas to similar problems, c) made connections to previous tasks, and d) generalized their conjectures in the context of isomorphic problems.

The researchers provided examples of teacher questions designed to elicit explanation and justification from students. They noted that these questions were directed in such a way to encourage participation of other students in the group. Although there was evidence that many students participated in the discussions, it is hard to determine whether it was specifically the questions (questioning technique), a positive classroom climate in general, or a combination of the two that invoked such reaction. There was evidence to support the researchers' claim that rather than acknowledging students correct responses, the teacher used questioning to prompt them into further explanation, justification, and generalization of their ideas. By sharing their mathematical thinking and reasoning, students were able to make a connection to a related problem they had done before and construct a general rule. Dann et.al (1995) believed that opportunity for students to internalize, construct, and share mathematical ideas coupled with timely teacher intervention created a classroom atmosphere in which “unconscious learning” could occur.

In a case study, White (2003) investigated mathematical classroom discourse in two third-grade teachers’ classrooms in an urban school district near Washington, DC. The researcher visited each teacher’s mathematics class eight times to passively observe their teaching. The eight classroom visits to each of the two teachers, resulted in audiotapes whose transcripts and accompanying field notes produced the main source of the data. Some of the findings of her study were of particular relevance to this present study. Through analysis of four selected vignettes, White (2003) found that teacher’s questioning appeared to help students to think about the mathematics they did and to encourage them to share their answers. Teacher’s questioning also enabled the teacher to assess if students had a problem-solving plan and offered the struggling students several strategies. Additionally, the teachers’ questioning patterns encouraged students to connect their problem solving with symbolic representations.

Researchers Martino and Maher (1991), reported on a study that focused on teacher questioning in third- and fourth-grade classrooms from three school districts in New Jersey. The study was associated with a longitudinal study which was then in its eleventh year and involved approximately 150 third, fourth, and fifth grade students. The three districts were comprised of one urban community, a blue collar community, and a suburban community. The data was collected from videotaped lessons, students' work samples, videotaped interviews with students, and researchers' notes. Videotapes from the lessons and the interviews were transcribed and the teacher/facilitator questions were identified. Four episodes of teacher interventions were selected, analyzed and presented. The findings of this qualitative study were of relevance to the present study as they showed how teacher questioning helped students a) build justifications to their solutions, b) explain and generalize their solutions, c) connect two isomorphic problems, and d) understand other peoples' strategies.

In a research study conducted by Doerr and English (2006), the researchers described their findings on how two middle-grade teachers supported their students' mathematical reasoning during a task which involved data analysis. The purpose of their study was to examine how teachers develop subject matter and pedagogical content knowledge and how they use this knowledge to promote their students' reasoning and learning. The researchers chose a sequence of mathematical model-eliciting tasks which were reform-based in the sense that they had the potential for promoting student engagement with a realistic problem that could be solved in multiple ways and had the capacity for students to engage in self-evaluation of the results. Model-eliciting tasks are tasks in which students express their thought processes by explaining and justifying their work while engaged with the task and as they present and report on the result of their findings (Lesh, Hoover, Hole, Kelly, & Post, 2000). The researchers focused and reported on the results of two teachers, Mrs. L and Mrs. R who represented diverse teaching background and experience.

The relevance of this study to the present study was twofold: it highlighted not only the significance of teacher actions and interventions, but also the importance of students' use of representations in the learning of mathematics. Mrs. L, a novice teacher, repeatedly stressed to her students that they should use "numbers, not pictures" and voiced her concerns and frustrations to the researchers that her students were not "thinking mathematically". She did not appear to understand that mathematical models and refined strategies can emerge and develop from early types of mathematical attempts. Mrs. R, on the other hand, seemed to have a solid understanding of the underlying mathematics of the task involved and when some of the students came up with a graphic representation, which neither Mrs. R nor the researchers had considered before, she questioned the students about their reasoning, giving them ample time to explain the relationship between the graphic representation and the structure of their solution and helped to develop and document their strategies.

Kawanaka and Stigler (1999) described their findings from two studies of teachers' use of questions in eighth-grade mathematics classrooms in Germany, Japan, and the United States. Videotapes of eighth-grade mathematics lessons from the three countries were translated, transcribed, coded, and analyzed for this study. The videos, taped in 1994 and 1995, were part of the video component of the Third International Mathematics and Science Study (TIMSS). The collection of videotapes used for the study included 50 lessons from Japan, 100 from Germany, and 81 from the United States.

Some of the interesting findings from the first study, which was a quantitative research study, were: teachers dominated talk in all three countries; in all three countries, the most common type of teacher talk was categorized as “information”; in all three countries, the most common type of student utterance was categorized as “response”; U.S. student responses were significantly shorter than the responses from the other two countries; and that higher order questions were asked more often by Japanese teachers and least often by U.S.

The findings from the second study showed that U.S. mathematics instruction emphasized mastery of principles and production of correct answers, whereas German teachers often asked students to explain their thinking process. The researchers also comment on the fact that Japanese teachers tended to use divergent (open-ended) problem solving methods more than did the other two countries. They noted that in Germany and U.S. more convergent problem solving activities were used in which, students knew what solution method to use to solve a given problem. Perhaps one of the most important conclusions indicated by the findings of this study was that not all questions that demand description or explanation can be categorized as higher level questions unless they elicit higher level thinking and responses.

2.5 Summary

Based on this literature, it is clear that the knowledge necessary for effective teaching is more than the compartmentalized knowledge of content, pedagogy, and curriculum. It is an integration of these three types of knowledge, where the critical understanding of a discipline, coupled with an understanding of how students learn could inform curricular choices; it is the knowledge that helps teachers create a classroom structure in which, artful questioning and interventions could encourage students to reflect, internalize, justify their reasoning, and meaningfully communicate their thinking.

The literature suggests that the design of any professional development program should take into account the theories on how people learn and acquire knowledge. It is believed that teachers, much like the students, learn best through engagement in meaningful activities and by personal experiences. As indicated in this literature review, lesson study, as a collaborative teacher professional development process, affords the teachers the opportunity to continually analyze, reflect on, and revise their own thinking in order to build upon and enhance their teaching and learning models.

CHAPTER 3: METHODOLOGY

3.1 Setting and Sample

A graduate course offered through the Department of Continuing Education of Rutgers University, and structured around a lesson study process, provided the backdrop to this present study. Ten students (including the researcher), all of whom were teachers of mathematics themselves, met every Thursday (3:30 to 6:00 pm), from January to May, at a Middle School located in a small urban city- referred to in this study as Sheffield- in New Jersey. The course was co-taught by a faculty member of the university and a full-time doctoral student, who had worked in Sheffield district as a middle school mathematics teacher in previous years. Five of those teachers taught in Sheffield, while the other five taught in the neighboring districts.

The focus of the course was on the development of algebraic ideas from 5th grade through 9th grade Algebra. In particular, much attention was given to the standard-based curriculum resources such as Everyday Mathematics, Connected Mathematics, and Discovering Algebra, which were used in Sheffield School District for upper elementary, middle school, and Algebra 1 students respectively. During the first half of the course, each class session focused on several units from the algebra strand of these resources in order to examine the development of algebraic ideas horizontally, within a grade level, and vertically, between grade levels.

Each week in class, the class participants investigated, discussed, and solved algebraic problems selected from the curricular resources. The teachers were asked to modify these activities for grade level suitability, try them out with the students in their own classrooms, and report back on how the students solved these particular problems.

As a result of these investigations, much of the class discussions revolved around students' thought process as they do mathematics, the questions they ask, their perceived weaknesses and strengths, the strategies and models they use, and the ways in which they justify their answers.

Additionally, every week the assignment for the next class session included reading of various articles or book excerpts that related to the focused idea of algebraic development in students. The teachers were encouraged to post their reflections on the reading assignments on Sakai- a web-based network at Rutgers University- which provided a forum for interaction about various course activities between the weekly sessions.

The final project in the course was the implementation of a lesson study cycle. This project was embarked upon with clear goals and objectives in mind. The teachers were to attend to students' mathematical reasoning and observe the various strategies used in solving mathematical problems, both across and within grade levels. It was agreed that while supporting students' autonomy, the teachers would encourage the students to work together and communicate their mathematical thinking verbally, through classroom discourse, as well as in writing.

During one class session, the 10 teachers worked in two separate groups to plan two different research lessons, one for Grades 5 and 6 and the other for Grades 7 through 9. All the activities selected for the two lessons involved solving equations or systems of equations represented as pictorial expressions.

Six teachers, including the researcher, agreed to implement these lessons in their classroom, but due to scheduling difficulties, not all the 10 teachers were able to attend all the sessions as observers. Each of the six implementation sessions was followed by a debriefing session, in which the teacher and the observers reflected on the implemented lesson and considered possible modifications prior to the next implementation sessions. For example, the Grade 5 lesson taught on May 14 was revised based on the debriefing and re-taught to a different group of students in Grade 6 later that same day.

Out of the six teachers who agreed to implement these lessons in their classroom, four have been selected as the focus of this present case study. Out of the 10 teachers who had worked collaboratively to create the lessons, six teachers observed lesson #1 and lesson #2, ten teachers observed lesson #3, and only two teachers observed lesson #4.

All of the four teachers in this study taught in Sheffield. Table 1 provides some additional information on these teachers who, together with their students, make up the sample for this case study.

Table 3. 1: Characteristics of the Teachers in the Case Study

Teacher	Gender	Race	Grade level	Teaching Experience (yr)
T1	F	African American	5	3
T2	M	African American	6	16
T3	F	African American	8	10
T4	F	African American	5	8

The racial breakdown of Sheffield school district indicates that there are about 59% African American, 39.9% Hispanic, 0.5% Caucasian, 0.5% Asian, and 0.1% Native American students in the public schools (NCES, 2006-2007)

3.2 Data Collection

The main source of data for this study was the video tapes of the lesson implementation and the debriefing sessions with the teachers. Documents such as teachers' notes, and student work samples supplemented the videotaped data.

3.2.1 Videotaped Observations

The first two lesson implementations took place on the same day. Six teachers, together with the two course instructors observed a fifth and a sixth grade lesson, 91 and 79 minutes long and facilitated by T1 and T2 respectively. The following day, one more lesson was implemented in a grade 8 class facilitated by T3. This lesson was 67 minutes long and was observed by ten teachers, two course instructors, and two school administrators who had been invited to stop by and visit. The last lesson implementation took place in a grade 5 class by T4. This lesson was 76 minutes long and was observed by two teachers and two course instructors. Table 2 provides additional information on data collection pertaining to lesson implementations.

Table 3. 2: Lesson Implementation Sessions

Teacher	Grade level	Date of lesson Implementation (2008)	Duration	Topic
T1	5	May 14	91 minutes	Systems of equations (pictorial)
T2	6	May 14	79 minutes	Systems of equations (pictorial)
T3	8	May 15	67 minutes	Systems of equations (pictorial)
T4	5	May 30	76 minutes	Systems of equations (pictorial)

Each lesson implementation session was recorded using one roving camera, accumulating a total of 313 minutes of recorded classroom data. Each lesson was followed by a videotaped debriefing session, in which the teacher participants reflected on and discussed the lesson that had been taught. There was a total of 122 minutes of recorded data from the debriefing sessions which, coupled with the videotaped lessons, make up the primary source of the data for this study.

3.2.2 Documents

The secondary source of data was comprised of the copies of the activities (see Appendix A), the students' work samples, and the preparatory teachers' notes and comments archived on Sakai. The lesson plans were created through a collaborative effort by the teacher participants who developed two lesson plans, one for grades 5 and 6 and the other for grades 7 through 9. The activities were selected from the curricular resources in Mathematics in Context and Connected Mathematics Program.

3.3 Method of Analysis

The researcher used the analytical model developed by Powell, Francisco, and Maher (2003) for video data analysis. This model was partially used to examine video data and the documents were examined in relation to the videos during the analytic process**.**

First the researcher viewed the videotapes of all the four implementation sessions and the debriefings. The intent of this initial viewing was to become familiar with the data rather than try to analyze the content.

After viewing the videotapes, the researcher wrote a description of what occurred during these sessions. To provide easy reference to segments of the videotapes, data was chunked into approximately 5- minute time intervals and an objective description of observed occurrences was written for each lesson (see Appendices B1-4) and the debriefing session (see Appendices C1-4). The researcher purposely tried not to use any kind of analytical lens in writing these descriptions.

After viewing and describing the lesson implementation videos, *critical events* related to the research questions were identified for each of the four lessons (see Appendices D, E, F, and G). According to Powell et.al (2003), an event is deemed critical when it "demonstrates a significant or contrasting change from previous understanding, a conceptual leap from earlier understanding" (p. 416). For this study, *critical events* identified incidences in which certain occurrences, deemed significant by the researcher, were highlighted and analyzed in order to address the research questions. For instance a *critical event* identified a change in teachers' expectations of how students would go about solving a problem, which in turn resulted in modifications of the lesson plan. The selection of *critical events* involved careful deliberation on the research questions so that these events, taken individually, were snapshots offering glimpses into the lessons and collectively, the string of these snapshots shaped a narrative that provided the researcher an insight into the nature of the interactions that transpired in the classroom, and subsequently helped answer the research questions.

After viewing and describing the debriefing videos, the researcher identified *episodes* of teachers' comments or questions that helped answer the research question regarding the teachers' reflections on the lessons and students' work (see Appendices H1-4). For this study an *episode* refers to a single comment or question posed by one teacher or a cluster of comments/questions about a topic that involved one or more teachers.

After identifying the *critical events* and the *episodes*, the corresponding video segments were transcribed. These transcriptions paved the way for accurate coding of the *critical events* and the *episodes*. The researcher used the transcribed *critical events* from the classroom video data to develop a coding system which identified teachers' and students' actions and responses. These coding schemes are described in sections 3.4.1 and 3.4.2 respectively. The transcribed *episodes* from the videotaped debriefing sessions were used to develop another coding system that identified the nature of the teachers' comments and their reflections on the lessons and the students' work. This coding scheme is described in section 3.4.3.

The next step involved constructing a descriptive analysis of each of the lesson implementations, with references to the *critical events* that occurred during each lesson. For these analyses the coded videotaped data was examined in conjunction with the students' work samples and, when appropriate, the teachers' reflections from the debriefing sessions. During the final step, the descriptions of the debriefing sessions, together with the transcribed and coded *episodes* from these sessions were used to examine the nature of teachers' reflections and comments. The descriptive analyses of the lessons, coupled with the analyses of teachers' reflections, provided a perspective on teacher's actions and students' responses during the whole lesson study process.

3.4 Coding Systems Utilized in Present Study

This section presents a description of the three coding schemes used in this study. These involve coding systems for categorizing: (a) teachers' actions and responses, (b) students' actions and responses, and (c) teachers' reflections on lessons and students' work.

Coding systems identified in previous research (Kawanaka & Stigler, 1999; Fernandez et al., 2003) were utilized in the present study to characterize (a) teachers' actions and responses, (b) students' actions and responses, and (c) teachers' reflections on lessons and students' work. These coding schemes are described in the sections below. The first two coding schemes-teachers' actions and responses and students' actions and responses- aimed to identify those actions and responses that were relevant to the study. Actions and responses were deemed relevant by the researcher if they helped answer the research questions or related to the set of goals that were determined by the teachers for the lesson study process, embedded within this present study. As part of the objectives in doing the lesson study, it was agreed that while supporting students' autonomy, the teachers would encourage the students to work together and communicate their mathematical thinking verbally, through classroom discourse, as well as in writing. These goals influenced the emergence of all codes and specifically the two coding schemes which identify the teacher's and students' actions and responses.

3.4.1 Coding for Teachers' Actions and Responses

The coding system described by Kawanaka and Stigler (1999) was partially employed to develop the coding scheme for teachers' actions and responses in this present study. Kawanaka and Stigler defined "Elicitation (E)" as "A teacher utterance intended to elicit an immediate communicative response from the student(s), including both verbal and nonverbal responses." They further classified elicitation into three categories to highlight the cognitive demands of teachers' questions. Kawanaka and Stigler defined the three categories of elicitation as follows:

1. Yes/no (YN): Any content elicitation that requests a simple yes or no as a response.
2. Name/state (NS): Any content elicitation that (a) requests a relatively short response, such as vocabulary, numbers, formula, single rules, prescribed solution methods, or an answer for computation; (b) requests that a student read the response from a notebook or textbook; and (c) requests that a student choose among alternatives.
3. Describe/ explain (DE): Any content elicitation that requests a description or explanation of a mathematical object, nonprescribed solution methods, or a reason why something is true or not true. (p. 258)

Following Kawanaka and Stigler's coding scheme, the present study includes three codes pertaining to elicitation: Eliciting a response – Yes/No (E-YN); Eliciting a response – Name/State (E-NS); and Eliciting a response – Describe/ Explain (E-DE).

Additionally, Kawanaka and Stigler coded "A teacher utterance intended to cause students to perform immediately some physical or mental activity" as Direction (D). This code has also been adopted by the researcher and appears in the coding scheme as: Directing students to perform some mental or physical activity (DA).

The table below shows 18 codes which were used in this study to identify relevant teachers' actions and responses. These codes are arranged under three broad categories of questions, communication, and academic support.

Table 3. 3: Coding Scheme - Teachers' Actions and Responses

Codes	Questions
E-YN	Eliciting a response –Yes/No
E-NS	Eliciting a response – Name/State
E-DE	Eliciting a response – Describe/Explain
RQ	Rewording a question
	Communication
ECD	Encouraging classroom discourse
EWE	Encouraging student to write down explanations
OCR	Providing opportunity to critique other's reasoning
DA	Directing students to perform some mental or physical activity
VSR	Validating student's reasoning
PSR	Paraphrasing student's response
	Academic Support
AV	Attending to mathematical vocabulary
SPK	Tapping into student's prior knowledge
SDA	Suggesting a different approach
OH	Offering hints
REP	Using representation
SSA	Supporting student's autonomy
BSI	Building on student's ideas
EX	Giving an example

3.4.2 Coding for Students' Actions and Responses

Examination of the classroom video data resulted in the emergence of 16 codes pertaining to student behavior. These codes identify actions and responses that helped answer the research questions and identify instances of student participation and classroom discourse. The table below shows the 16 codes, arranged under two broad categories of communication and others.

Table 3. 4: Coding Scheme - Students' Actions and Responses

Codes	Communication
AQ	Answering a question
SSC	Sharing solution/idea with class
SST	Sharing solution/idea with teacher
JS	Justifying solution
CS	Comparing solution strategies/ideas
MDP	Engaged in mathematical discourse with peers
QRO	Questioning the reasoning of others
ARO	Agreeing with the reasoning of others
MSC	Making a self-correction
CAS	Correcting another student
ACP	Acknowledging the positive contribution of a partner
AH	Asking for help
Others	
DLU	Displaying lack of understanding
AHP	Accepting help from peers
RHP	Reluctant to accept help from peers
VS	Volunteering to share ideas

3.4.3 Coding for Teachers' Reflections

The purpose of coding the teachers' reflections was not to measure the frequency of the use of each lens by the teachers, nor was it aimed at tracing the development of the teachers' ability to adopt a particular lens during the course of the study. Indeed, the time frame was too short in this present study for change to be observed. The coding of the teachers' reflections however, provided an insight into the nature of the conversations that occurred during the debriefing sessions.

The coding scheme for teachers' reflections was developed based on three critical lenses- researcher lens, curriculum developer lens, and student lens- identified by

Fernandez et al. (2003). Fernandez and colleagues used these lenses as a tool to examine the practice of lesson study. They believed that for lesson study to work as a feasible form of professional development, the teacher participants must view the lesson study process through these three critical lenses: the researcher lens, which encourages the teachers to ask questions about certain aspects of the practice and motivates them to design classroom experiences to address these questions; the student lens, which allows the teachers to examine the lesson from the students' perspective; and the curriculum developer lens, which helps the teachers with the organization and sequencing of the learning experiences.

Building on Fernandez's work, the researcher developed a set of operational definitions for three lenses- research, curriculum, and student- which were used in the present study. At times, teachers' reflections indicated a simultaneous use of multiple lenses, which resulted in an over-lapping of the codes. The table below lists the codes for each lens and their corresponding working definitions.

Table 3. 5: Coding Scheme - Teacher's Reflection

Codes	Adopted lens used during teacher reflections
R	**Research lens** [Operational definition: 1. Critiquing pedagogical decisions made with respect to the research goals 2. Restructuring a lesson with the research objectives in mind 3. Verbalizing the research goals 4. Demonstrating fascination/interest in students' mathematical work and behavior 5. Relying on concrete evidence to support the research process 6. Speculating on student outcomes under different circumstances 7. Examining the development of student conceptual understanding]
C	**Curriculum lens** [Operational definition: 1. Relating instruction to future learning 2. Relating instruction to prior knowledge 3. Contemplating the development of content within a lesson 4. Contemplating the development of content within a unit/across grade levels 5. Considering the organization and presentation of instructional material 6. Discussing the lesson plan/task selections]
S	**Student lens** [Operational definition: 1. Discussing student behavior/attitude 2. Discussing student misconceptions and/or understandings 3. Discussing student mathematical work/strategies]

After viewing and describing the debriefing videos, the researcher identified and transcribed *episodes* of teachers' comments or questions that helped answer the fourth research question regarding the teachers' reflections (see Appendices H1-4). While examining the transcribed *episodes* from the videotaped debriefing sessions, the operational definitions were used in order to identify and code the nature of the teachers' comments and their reflections on the lessons and the students' work. This resulted in many examples of coded teachers' reflections (see Appendices H1-4).

CHAPTER 4: RESULTS

4.1 Overview

This chapter is dedicated to the results of the study. The planning that took place by the teachers as part of the lesson study process during the pre-implementation/planning phase will be described in Section 4.2. This will include class discussions and investigations that influenced the task selection for the lesson study project as well as a list of the solution strategies/paths that was generated as the teacher participants attempted to solve the problems in class.

In section 4.3, an overview of the lesson #1 implementation and the following debriefing session will be provided. Also included in this section will be a descriptive analysis of the lesson implementation #1 with reference to the Critical Events that occurred during this lesson. Sections 4.4 through 4.6, pertaining to lessons #2 through #4 respectively, will follow the same format as in section 4.3.

A closer look at the nature of the teacher reflections will be provided in Section 4.7, and in section 4.8 a summary of the findings of this study with respect to each of the research questions will be offered.

4.2 The Planning Phase

Prior to the lesson implementations and in preparation for the lesson study project, the teachers were assigned to read Catherine Lewis's book (2002) on lesson study and throughout the course, various aspects of lesson study process were discussed in class. This project was embarked upon with clear goals and objectives in mind. The teachers were to attend to students' mathematical reasoning and observe the various strategies used in solving mathematical problems, both across and within grade levels. It was agreed that while supporting students' autonomy, the teachers would encourage the students to work together and communicate their mathematical thinking verbally, through classroom discourse, as well as in writing.

During one class session, the 10 teachers worked in two separate groups to plan two different research lessons, one for Grades 5 and 6 and the other for Grades 7 through 9. All the activities selected for the two lessons involved solving equations or systems of equations represented as pictorial expressions. The choice for the selected tasks was influenced by two factors: students' needs; and curricular gaps. The following sections describe how these factors impacted the task selection for the research lessons.

4.2.1 Students' Needs

The teachers had the opportunity to discuss and solve many algebraic problems in class. One of the problems they had collaboratively investigated was called Shorts and Glasses (Meyer and Pligge, 1998) (see Appendix B3). Afterwards several teachers tried this problem in their own classrooms and with their own students. Two of them, a 7th grade Special Education and an 8th grade algebra teacher, both reported that all their students had simply used trial and error to solve the problem and expressed their disappointment with the absence of students' algebraic thinking and proportional reasoning. Another teacher, who taught at 5th grade level reported the following:

> I had a chance to try the Price Combination problems (Shorts and Glasses, Soda and Shirt, see Appendices A3 and A4)….They were all over the place in terms of responses; some pulling other information into the problem like the glasses cost more because they have an eyeglass cleaner with them!...They seem to understand the concept of equality although one student's solution to…"What is the price of one pair of glasses?" was the following: "$20.00 because 20 + 20 = 40 + 10 = 50 and 20 + 10 + 10 + 10 = 50". The student's use of the equal sign as shown above is not a true

> statement. I think this is something we should be "sticklers" about so that the kids understand equality better.

Following these discussions, the class was assigned a reading from *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School* (Carpenter et al., 2003). In this article the authors had reported on a study which showed that a significant number of elementary school students had incorrectly placed the number 12 in the blank, when they were given the equation: 8 + 4 = ___ + 5, and were asked to identify a number that made it a true statement.

Some teachers decided to try this type of simple problems with their students and a few days later, an 8th grade teacher posted the following on Sakai:

> After reading Carpenter's article…I wondered if students in higher grades would make similar mistakes…So I decided to make a worksheet and gave it to 50 of my students of varying abilities. Two of the equations were similar to those mentioned.
> 1. 12 + 6 = ___ + 2
> 2. 18 – 8 = ___ + 1
>
> 18% of the 8th graders got these wrong!

After identifying students' difficulties with problems such as the Shorts and Glasses and Fill in the Blank, the teachers decided it would be a good idea to select activities that involved concepts of equivalency and equations.

4.2.2 Curricular Gaps

The collaborative act of examining the curricular resources used in Sheffield School District led to rich classroom discussions on the development of students' algebraic ideas as they progress from Grade 5 to Grade 9. The 5th grade teachers working in Sheffield, reported that a major component of algebra strand in Everyday Mathematics at Grade 5 was the concept of equivalence. The concept was first introduced using an actual pan balance and then pictures of scales to represent the concept and introduce the idea of balanced expressions. Soon after that the students were exposed to formal algebraic representation, in the form of equations, and manipulation of symbols. The teachers felt that the sudden shift in the curriculum, from concrete to abstract representations, did not afford the students enough time to fully explore the concept and gain a deep understanding of the meaning of equivalent expressions and the use of the equal sign symbol as an indicator for equivalency between expressions.

Additional examination of the middle school curriculum- the Connected Mathematics Program- revealed that, beyond applications of formulas for calculating area and perimeter, the students in 6^{th} and 7^{th} grades had little exposure to algebraic expressions and equations.

After identifying the gaps in the curriculum and recognizing students' struggles with concepts involving equivalence and their lack of understanding of the meaning of the equal sign symbol, the teachers reached a unanimous decision to include in their research lessons, activities/tasks that involved solving equations represented as pictorial expressions.

4.2.3 The Tasks: Solution Paths

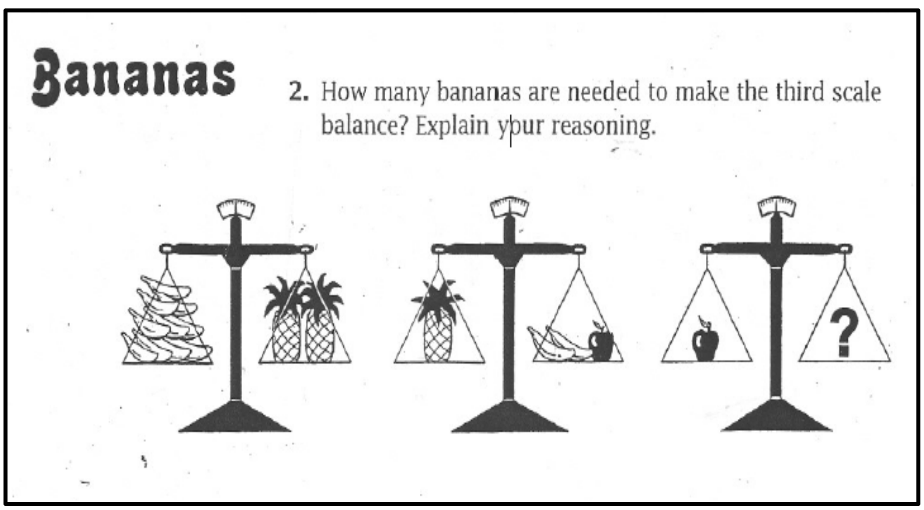
The teachers identified six activities (Tasks 1- 6) from Mathematics in Context (MIC) (Meyer and Pligge, 1998) to be used for the research lessons. Much time was spent in class discussing, solving, and justifying the solutions to these problems and the teachers compiled a list of various solution paths for each task included in the research lessons. These were not exhaustive lists of all the possible solutions, but included all the strategies used by the teachers as they attempted to solve the problems themselves.

Four activities (Tasks 1-4) were chosen for the research lesson in Grades 5 & 6. Two activities (Tasks 5 and 6) were originally written as part of the research lesson intended for Grades 7 through 9, but for reasons explained in section **4.5.2**, Task 6 was never used during the implementation phase. For this reason the possible solution paths to Tasks 1-5 (see Appendix A) only, were described in this study.

4.2.3.1 Task 1 – Bananas

Task 1 (see Figure. 4.1) was essentially a pictorial depiction of a system of two equations with three unknowns and the reader was asked to re-define one of the variables in terms of one of the other two variables. Specifically, the reader had to decide how many bananas weighed as much as an apple.

Figure 4. 1: The Bananas Problem



<u>Solution Path 1A</u>

Some teachers used formal algebra to solve this problem. It was anticipated that some Algebra 1 students might choose a similar path as shown below.

Let *b* represent the weight of one banana, *p* represent the weight of one pineapple, and *a* represent the weight of one apple.

Then the first and second scales can be represented by the equations:

$10b = 2p$

$p = 2b + a$

Simplify the first equation above:

$5b = p$

Substitute $5b$ for p in the second equation above:

$5b = 2b + a$

Subtract $2b$ from both sides of the equation:

$3b = a$

Therefore one apple weighs as much as three bananas.

Solution Path 1B

- 10 bananas weigh as much as 2 pineapples.
- Therefore 5 bananas weigh as much as 1 pineapple (cutting each side in half).
- On the second scale……….2 + ? = 5
- An apple weighs as much as 3 bananas.

Solution Path 1C

- 10 bananas weigh as much as 2 pineapples.
- Therefore 5 bananas weigh as much as 1 pineapple (cutting each side in half).
- On the second scale……….Replace the pineapple with 5 bananas.
- Take away 2 bananas from each side.
- An apple weighs as much as 3 bananas.

Solution Path 1D

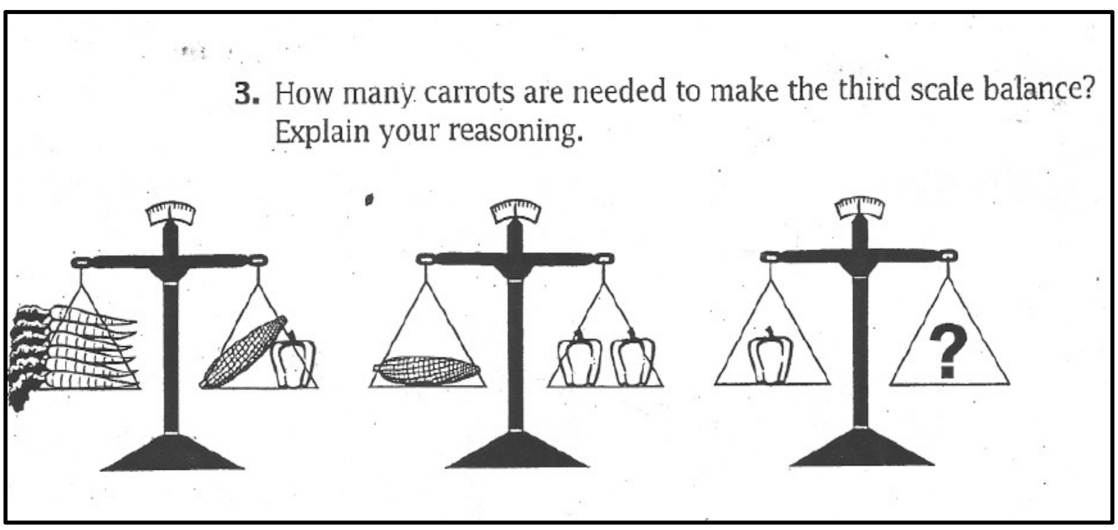
- Consider the relationship shown on the second scale.

- On the first scale, replace each pineapple with 2 bananas and an apple.
- Now you have 10 bananas Vs. 4 bananas and 2 apples.
- Take away 4 bananas from each side.
- Now you have 6 bananas Vs. 2 apples.
- 3 bananas Vs. 1 apple (cutting each side in half).
- An apple weighs as much as 3 bananas.

4.2.3.2 Task 2 – Carrots

This task (see Figure. 4.2), similar to the previous one, was a pictorial depiction of a system of two equations with three unknowns and the reader was asked to re-define one of the variables in terms of one of the other two variables. Specifically, the reader had to decide how many carrots weighed as much as one pepper. This task had the potential of engaging the students in a deeper form of proportional reasoning.

Figure 4. 2: The Carrots Problem



Solution Path 2A

Some teachers used formal algebra to solve this problem. It was anticipated that some Algebra 1 students might choose a similar path as shown below.

Let r represent the weight of one carrot, *c* represent the weight of one corn, and *p* represent the weight of one pepper.

Then the first and second scales can be represented by the equations:

$6r = c + p$

$c = 2p$

Substitute 2*p* for *c* in the first equation above:

$6r = 2p + p$

Add like terms:

$6r = 3p$

Simplify the equation:

$2r = p$

Therefore one pepper weighs as much as 2 carrots.

Solution Path 2B

- Consider the relationship shown on the second scale.
- On the first scale, replace the corn with 2 peppers.
- Now you have 6 carrots Vs. 3 peppers.
- Divide each side by 3 to get: 2 carrots Vs. 1 pepper.
- Therefore a pepper weighs as much as 2 carrots.

Solution Path 2C

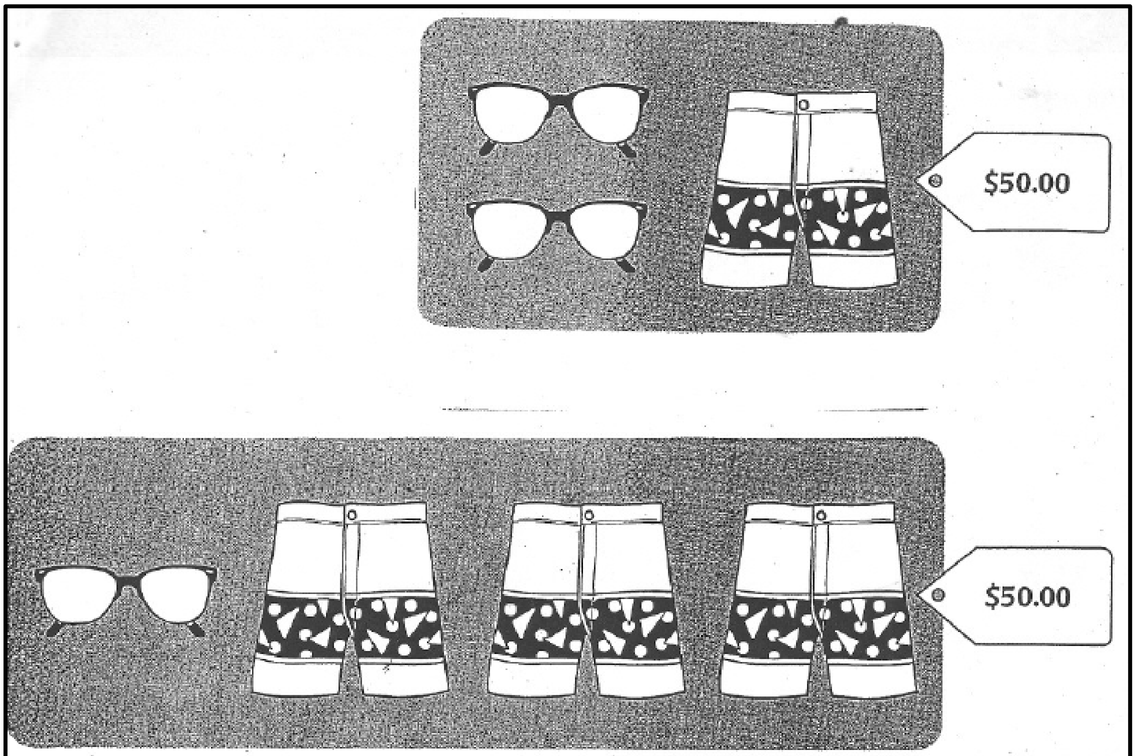
- Identify the 2:1 relationship shown on the second scale.
- Based on this realization, consider the first scale and pick two numbers that have a 2 to 1 relationship, representing a corn and a pepper respectively and add up to 6 carrots (4 + 2 = 6).

- A pepper weighs as much as 2 carrots.

4.2.3.3 Task 3 – Shorts and Glasses

Task 3 (see Figure. 4.3) was a pictorial depiction of a system of two equations with two unknowns and the reader was asked to identify the more expensive item before calculating the actual prices. Specifically, the students were asked to identify the more expensive item before calculating the actual prices. It was speculated that some students, as some teachers had done, would not pay close attention to the first part of the question and determine the more expensive item by first calculating the actual prices. Some teachers used the $50 price tags and formal algebra, while others used the price tags and the guess and check strategy to find the price of each item. Both of these solution paths are included in the list below.

Figure 4. 3: The Shorts and Glasses Problem



1. Without knowing the prices of a pair of glasses or a pair of shorts, you can determine which item is more expensive. Explain how.
2. How many pairs of shorts can you buy for $50?
3. What is the price of one pair of glasses? Explain your reasoning.

<u>*Solution Path 3A*</u>

Let x represent the cost of one pair of shorts and y represent the cost of one pair of glasses.

Then the problem can be represented by the system of equations:

$x + 2y = 50$ …….1st equation

$3x + y = 50$ …….2nd equation

Multiply the 1st equation by 3 to get:

$3x + 6y = 150$

Subtract the 2nd equation from this new equation to get:

$$5y = 100$$

Simplify to get:

$$y = 20$$

Substitute 20 for y in either equation above to get:

$$x = 10$$

Glasses cost $20, and shorts cost $10. Therefore the glasses are more expensive. You can buy five pairs of shorts for $50.

<u>Solution Path 3B</u>

Let x represent the cost of one pair of shorts and y represent the cost of one pair of glasses.

Then the problem can be represented by the system of equations:

$$x + 2y = 50$$

$$3x + y = 50$$

Since both equations are equal to 50, you can write the equation:

$$x + 2y = 3x + y$$

Subtract $x + y$ from both sides to get:

$$y = 2x$$

Therefore the glasses are twice as expensive as the shorts.

In the second picture, the glasses can be traded in for two pairs of shorts. So you can get five pairs of shorts for $50. This implies that the shorts are $10. Therefore the glasses cost $20 a pair.

<u>Solution Path 3C</u>

- Use guess and check to figure out the price of each item.
- Based on the actual prices, name the more expensive item.
- Based on the actual price of one pairs of shorts, calculate how many pairs can be purchased with $50.

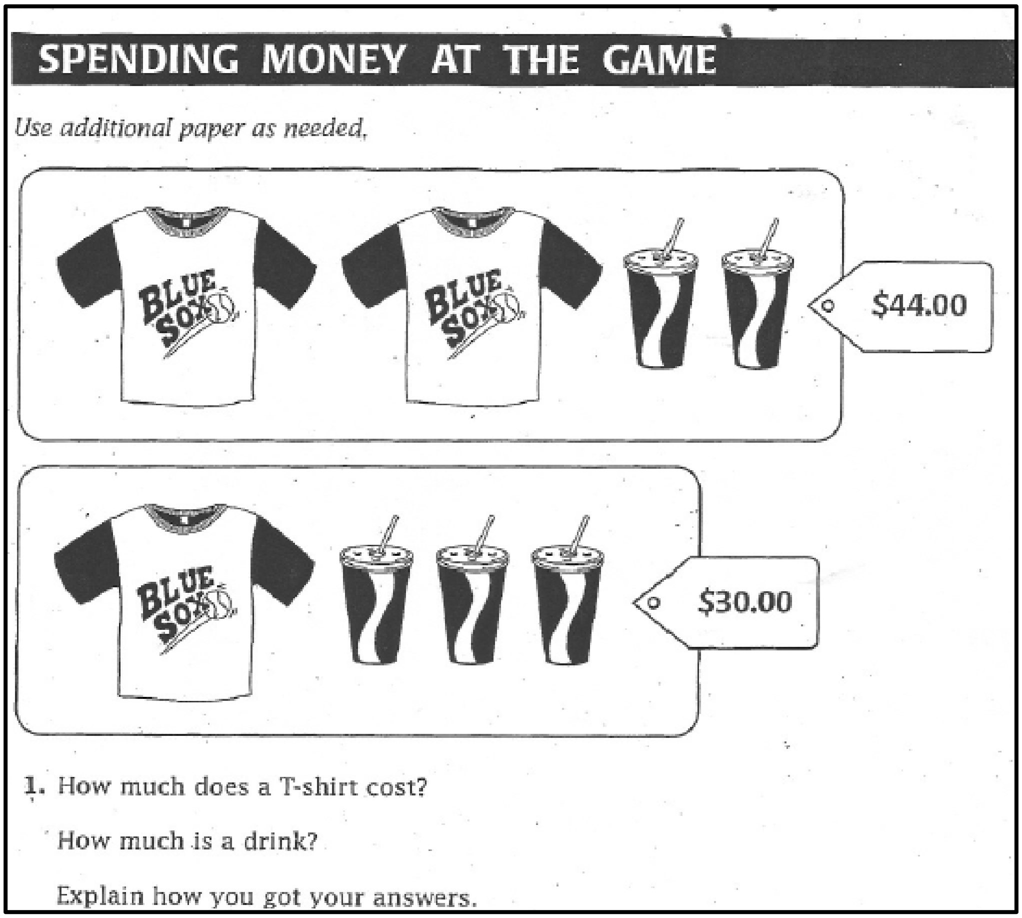
Solution Path 3D

- Cross out one pair of glasses and one pair of shorts in both pictures.
- Note that the $50 price tags will reduce by the same amount in both pictures and therefore will remain equal to one another.
- The new pictures show that one pair of glasses is the same value as two pairs of shorts.
- Therefore glasses are more expensive than the shorts. In fact the glasses are twice as expensive as the shorts.
- In the second picture, the glasses can be traded in for two pairs of shorts.
- Therefore you can get five pairs of shorts for $50.
- This implies that the shorts are $10 a pair.
- Therefore the glasses cost $20 a pair.

4.2.3.4 Task 4 – Soda and Shirts

Task 4 (see Figure 4.4) was a pictorial depiction of a system of two equations with two unknowns and the reader was asked to determine the cost of each item.

Figure 4. 4: The Soda and Shirt Problem



<u>*Solution Path 4A*</u>

Some teachers used formal algebra by writing a system of two equations and solving for the two unknowns. It was anticipated that some Algebra 1 students might choose a similar path as shown below.

Let t represent the cost of a T-shirt and s represent the cost of a soda.

Then, the problem can be represented by the system of equations:

$2t + 2s = 44$ …….1st equation

$t + 3s = 30$ …….2nd equation

Multiply the 1st equation by ½ to get:

t + s = 22 ===> t = 22 - s

Therefore the 2nd equation above, which is the same as (t + s) + 2s = 30, can be re-written as:

22 + 2s = 30

Simplify and solve for s to get:

s = 4 ===> t = 18 (since t + s = 22, which means that t = 22 – s)

Therefore the shirt costs $18 and the soda costs $4.

Solution Path 4B

- Cross out two soda cups and one shirt from each picture.
- The new picture indicates that a shirt is $14 more expensive than the soda (44-30=14).
- In the second picture, trade in the shirt for a soda and reduce the total by $14.
- The new picture indicates that 4 soda cups will cost a total of $16 (30-14=16).
- Divide 16 by 4 to get the price of one soda ($4)
- Add 4 and 14 to calculate the price of one shirt ($18)

[Note: Students may trade in a shirt for a soda or vice versa in either of the pictures]

Solution Path 4C

- Notice that in the top picture, if one shirt is traded in for a soda, the second picture will be formed and the price will drop by $14 (44-30=14).
- This indicates that a shirt is $14 more expensive than the soda.
- In the first picture, trade in the two shirts for two sodas and reduce the total amount by $28 (2x14=28).
- The new picture indicates that 4 soda cups will cost a total of $16 (44-28=16).

- Divide 16 by 4 to get the price of one soda ($4)
- Add 4 and 14 to calculate the price of one shirt ($18)

[Note: Students may trade in a shirt for a soda or vice versa in either of the pictures]

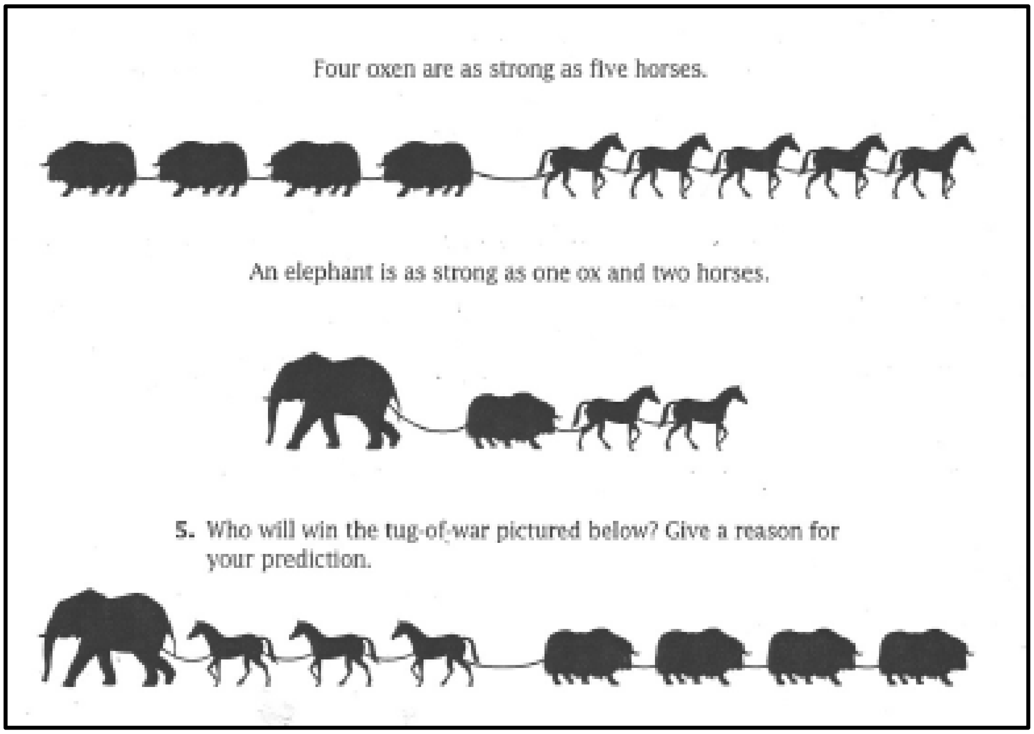
<u>*Solution Path 4D*</u>

- Cross out one shirt and one soda in the top picture and cut the price in half.
- This indicates that one soda plus one shirt will cost $22 (44:2=22).
- In the second picture, bundle one shirt and one soda and note that they cost $22.
- In the second picture, eliminate one shirt and one soda and reduce the total cost by $22.
- This new picture indicates that two sodas cost $8 (30-22=8).
- Divide 8 by 2 to find the cost of one soda ($4).
- Subtract 4 from 22 to find the cost of one shirt ($18).

4.2.3.5 Task 5 – Tug-of-War

Task 5 (see figure. 4.5) was a pictorial depiction of two equations representing equality in the strength of two groups of animals, and an inequality representing the winning team. The reader was supposed to consider the information provided by the equations in order to decide which side of the inequality was the greater side. Specifically, the reader had to identify the winner in the tug-of-war.

Figure 4. 5: The Tug-of-War Problem



Solution Path 5A

- Consider the top picture. Since four oxen are as strong as five horses, replace the four oxen in the bottom picture with five horses.
- The new picture will show one elephant plus three horses Vs. Five horses.
- But we know from the middle picture that an ox and two horses are as strong as one elephant and therefore can replace the elephant in the picture.
- The new picture will show five horses plus an ox Vs. five horses.
- The group on the left side has a greater strength and is clearly the winner.

Solution Path 5B

- Consider the middle picture. Since we know that an ox and two horses are as strong as one elephant, we can replace the elephant in the bottom picture with an ox and two horses.

- The new picture will show five horses plus an ox Vs. four oxen.
- But we know from the top picture that five horses are as strong as four oxen. Therefore the addition of an ox to the five horses on the left will create an imbalance of strength and make the group on the left the winner.

Solution Path 5C

- Consider the top picture. Now, if the two sides were to be of equal strength in the bottom picture, the elephant would have to be as strong as two horses.
- But, the middle picture shows that an elephant is stronger than two horses.
- Therefore the left side in the bottom picture is stronger and will win the tug-of-war.

4.3 Lesson Implementation and Debriefing #1

The following sections provide an overview of the first lesson implementation and the debriefing session. A more comprehensive description of these sessions is also provided which detail the observed occurrences of the video data in approximately 5-minute time intervals (see Appendices B1 and C1). The abbreviations R1 and R2 refer to the two university researchers while T1 through T4 identify the teachers in this study, who facilitated the four implementation sessions respectively. O1 through O4 identify the teachers who, as graduate students, participated in the lesson study and attended all or some of the implementation sessions as observers.

4.3.1 Lesson Implementation #1: Overview

This session was approximately 90 minutes long and was facilitated by T1, a fifth grade teacher. There were 14 students in the classroom. In addition to the classroom teacher, T1, there were eight other adults, R1, R2, O1, O2, O4, T2, T3, and T4, present in the room as observers. An additional person was in charge of taping the session. Students' desks were put together to form four groups of students. There were three or four children in each group. The grouping of the students was decided by the teacher prior to the lesson and was based on the teacher's anticipation of how well they would work together as a group.

The class began with the teacher introducing the university faculty member, R1, who addressed the students and explained, in simple terms, the purpose of the lesson and the presence of all the observers in the classroom. After the introductions were made the teacher handed out the first activity sheet and the mathematical work began.

The lesson consisted of four tasks, which were implemented in the following order: Activity 1- Bananas (approximately 15 minutes); Activity 2- Carrots (approximately 15 minutes); Activity 3 - Short and Glasses (approximately 25 minutes); Activity 4- Soda and Shirts (approximately 30 minutes) (see Appendices A1-4). The teacher asked the students to work with a partner and to explain their solutions in writing. The students worked in pairs or within their group and could be heard discussing the task at hand. Activities 1, 3, and 4 were introduced by handing out a copy of the problem to each student. Activity 2 was first introduced on the overhead and after the students had a chance to discuss it with their partners, a copy of it was distributed to each student. While the students were working on the problems the teacher circulated among the groups, overseeing their progress. The observers, some standing and some seated, situated themselves near specific groups of students. They observed the students' work, took notes and at times were seen to interact with the students. Most of these interactions occurred because the observers needed clarification on the students' thinking and would ask questions such as "How did you get that?" or "Why did you do this?"

Upon completing each task some students volunteered to share their solution strategies with the class. For the first three tasks (Bananas, Carrots, Short and Glasses) each group had the opportunity to report back their findings to the class. On the other hand, due to lack of time, only one group of students was afforded the time to share their solution to the Soda and Shirts problem. During the class presentations, the students were quiet and appeared to be attentive. However, very few comments were made by the students regarding the presentations given by their peers. During the course of the lesson the teacher repeatedly asked the students to put down their solutions in writing.

4.3.2 Debriefing #1: Overview

The debriefing took place in the classroom immediately following the lesson implementation and lasted for about 15 minutes. T1 expressed her frustration with the fact that students complained about the difficulty level of the last activity (Shorts and Glasses) and the time constraints which prevented them from exploring the problem more deeply. T1 also admitted her frustration with the fact that almost all students used guess and check to solve the last two problems, just as she had done herself the first time she had worked on these problems.

R1 suggested that they modify the "Shorts and Glasses" problem in such way that might eliminate the guess and check strategy as an option toward a solution. She proposed that for the next lesson implementation they omit the $50 price tags and just tell the students that the two sets of items cost the same amount. Everyone agreed that it was a good idea and speculated that by eliminating the numbers, the students would perhaps use a different strategy to determine the more expensive item.

T1 solicited the group's opinion on the sequencing of the problems and whether there were too many activities. T3 did not think that the lesson was too long and wondered whether her own lesson, due to take place the following day, might be too short. It was suggested that she might want to pick an additional problem to lengthen her lesson plan.

T4 suggested that in order to make the implemented lesson shorter and have more time for the last two problems, they could split the first two activities and ask half the class to do the Banana problem, while the other half does the Carrot problem. She argued that since the students present their solutions to class, all students would be exposed to both problems. R1 disagreed with her and explained that the progression from the first to the second problem was important in the sense that with the first problem, the students did the problem and reported out on it, but with the second problem, the students began to really listen to each other's justifications and compare their findings. In response, T4 wondered whether the higher level of discourse generated by the second problem was due to the nature of the problem or whether it was due to the fact that the second problem was presented more publicly, on the overhead, and not as a handout which students tend to regard as their own personal paper to work on individually. R1 agreed that might be the case, but reminded the group that the first problem could have served as a warm up for students to get into having more dialog with their peers and part of their goal was to have students communicate their thinking.

They also discussed students’ solution strategies. In the first two problems (Bananas and Carrots), most students explained their answer by assigning weights (in pounds) to the fruits and vegetables. T1 admitted that at first she had thought the students were wrong in introducing pounds in their explanations. R2 noted that Kyla’s answer was different from the rest in that there was no mention of specific amount assigned as weight and she had used substitution and deduction to arrive at her answer. O1 asked whether it was all right to ignore the students’ inability to recognize the unsuitability of equating one banana with one pound. R2 speculated that what the students were doing was to convert things to pounds so it would make sense to them in terms of weight. They chose one pound because “it was easy, one unit, one pound, one anything. Did it have to be a pound? No.” The group agreed that, contextually, they wanted the students to understand that they were comparing weights in these problems and it was the weight of five bananas that equaled the weight of two pineapples.

4.3.3 Descriptive Analysis: Lesson #1

The following is a descriptive analysis of the lesson implementation #1, with references to the *critical events* that occurred during the lesson (for a complete list of all the transcribed critical events, see Appendices D1-10). The selection of *critical events* involved careful deliberation of the research questions so that these events, taken individually, were snapshots offering glimpses into the lesson and collectively, the string of these snapshots contributed in shaping a narrative that provided the researcher an insight into the nature of the interactions that transpired in the classroom, and subsequently helped answer the research questions.

In the following descriptive analysis, the coded videotaped data was examined in conjunction with the students' work samples and, when appropriate, the teachers' reflections from the debriefing session. As a result, in this analysis, references to the teachers' reflections are at times intertwined with discussions on students' work and classroom occurrences. In order to better distinguish the classroom descriptions from the debriefing conversations, any discussion pertaining to the teachers' reflections will be italicized in the following analysis.

This lesson took place in a fifth grade classroom with 14 students. In addition to the classroom teacher, T1, there were eight other adults present in the room as observers: two university researchers, R1 and R2; the other three teachers in this study, T2, T3, and T4, who facilitated the subsequent implementation sessions; and three of the teachers, O1, O2, and O4, who as graduate students, participated in the lesson study and attended this implementation session.

At the start of the session and six additional times throughout the lesson, the classroom teacher, T1, instructed her students to work together within their groups to discuss the problems and the solutions, and repeatedly reminded all of them to put down their mathematical thinking in writing. This effort on the teacher's part was evidence of her commitment to the goals and the objectives that were set by the teachers for the lesson study project. The objective was for the teachers to support students' autonomy, attend to their mathematical reasoning, and observe the various strategies they used in solving mathematical problems. Additionally, the teachers were to encourage the students to work together and communicate their mathematical thinking verbally, through classroom discourse, as well as in writing.

Immediately after the introduction of the first task, the Bananas problem, the students began discussing the problem with their partners. In Event 1 (see Appendix D1) the camera captured students at each of the four tables discussing the task at hand.

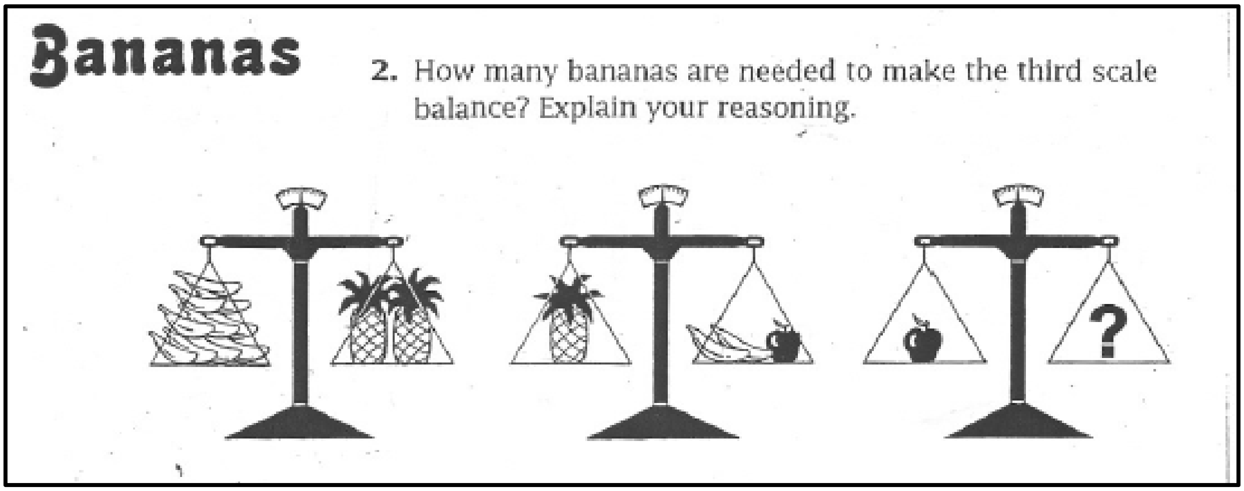
Specific description of some of the students' work will be provided in the following analysis, which details the interactions that took place chronologically, among students situated at different tables.

Table 1

Four girls- Neema, Kyla, Carin, and Titiana- were seated at Table 1. The classroom teacher, T1, had asked the students to work together in pairs within their group of four. So Neema paired up with Kyla, who was seated next to her and Carin partnered with Titiana.

Upon completing the Bananas problem several students volunteered to share their solution strategies with the class. The first student who volunteered to present her findings was Carin, who had been working with Titiana on the Bananas problem. This problem (see Figure 4.6 below, a copy of Figure 4.1) was a pictorial depiction of a system of two equations with three unknowns in which, the reader was asked to re-define one of the variables in terms of one of the other two variables. Specifically, the reader had to figure out how many bananas weighed as much as one apple.

Figure 4. 6: The Bananas Problem



The following is the transcription of Carin's oral explanation of her solution to the Bananas problem (Event 2, Appendix D2).

Line #	**Transcription**
1	**Carin:** Me and Titiana figured this out, but at first we said [holding her
2	paper up and pointing to, presumably, the first scale], if 10 bananas
3	equal two pineapples, equal two pineapples, then each pineapple must
4	be 5. And so, now (in the second picture) if two bananas equal 2, then
5	this apple has to…
6	**T1:** Two bananas equal?
7	**Carin:** Equal, like [pause] 2 pounds. Then one apple has to equal 3
8	pounds. Because the (second) scale is even and this (pineapple) is equal
9	to 5. And 2 plus 3 is 5. So if this apple equals 3, then it has to be, if this
10	apple equals 3 pounds, then it has to be three bananas.

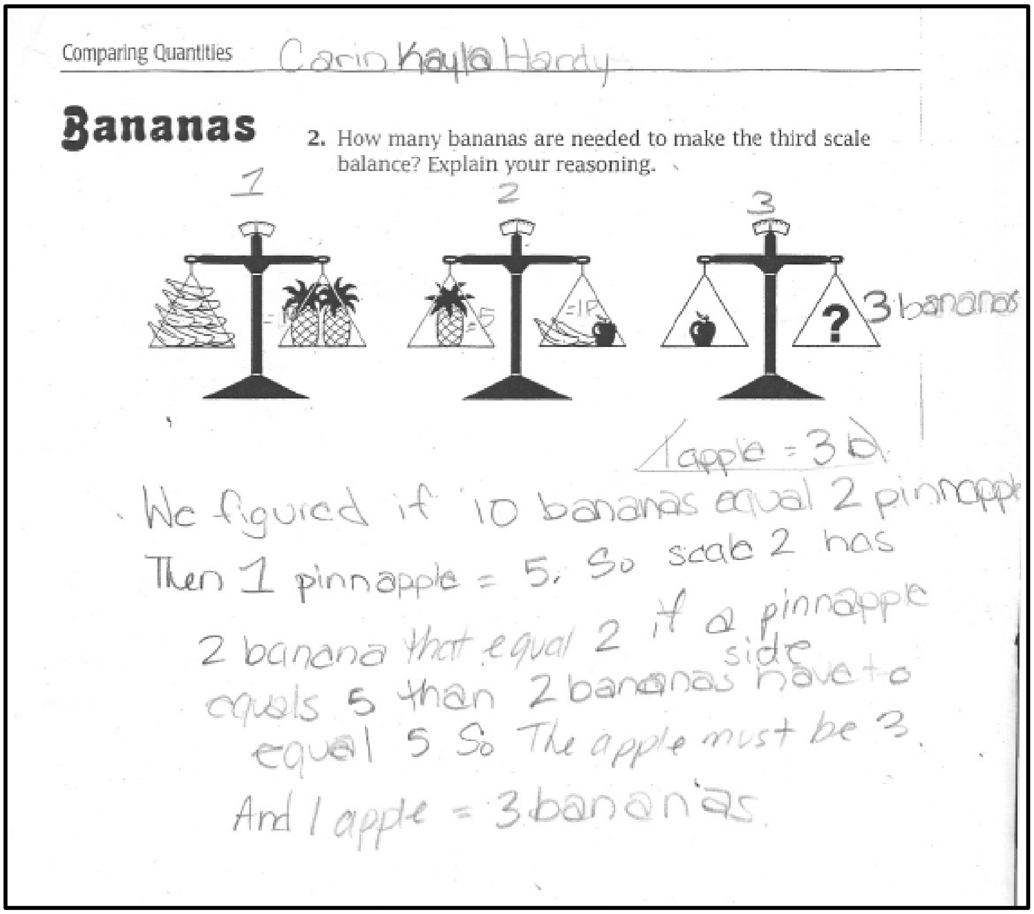
Carin began her explanation by acknowledging the contribution of her partner by simply saying: "Me and Titiana figured this out …" [line 1]. Much like Carin, most of the students who stood up and shared their findings with the class, recognized the efforts of their partner(s) in some form. This could be indicative of a shared recognition by the students of the importance of collaboration and teamwork, which was emphasized by the teacher's repeated reminders for the students to work together.

Before examining Carin's mathematical work, it is important to note that during the planning phase of the lesson study and in preparation for the lesson implementations, One of the strategies used by the teachers, and identified in this study as Solution Path 1B, was as follows:

- 10 bananas weigh as much as 2 pineapples (considering the first scale).
- Therefore 5 bananas weigh as much as 1 pineapple (cutting each side in half).
- On the second scale……….2 + ? = 5
- Therefore an apple weighs as much as 3 bananas.

Carin had essentially used Solution Path 1B to arrive at her answer. Carin's written work is illustrated in Figures 4.7 below.

Figure 4. 7: Carin's Work on the Banana Problem



It is interesting to note that in her written work, Carin had not attached any specific weight to any of the fruits and yet, during the oral presentation that followed, she made several references to pounds as her choice of unit for measuring the weight of apples and bananas. Examination of the transcript of the conversation between the teacher and Carin during her presentation suggests that perhaps the teacher’s question, “Two bananas equal?” (line 6), resulted in Carin’s spontaneous introduction of pounds as a unit of weight and its inclusion in her further explanation (lines 7-10). Following Carin’s presentation, other students in class also adopted pounds as a unit of measure in their presentations, which will be further discussed during the analysis of subsequent events.

During the class presentations, the students were quiet and appeared to be attentive. However, given the fact that very few comments were made by the students regarding the presentations given by their peers, it is impossible to gauge the level of their attentiveness to the solution strategies shared by others. Event 3 highlights, among other points discussed in this section, an instance in which, Carin at Table 1, critically listened to another student’s explanation. In this episode, Jaylen (from Table 3) who had been working on the “Banana” problem with Amir, volunteered to report their findings to the class and was prompted by Carin to clarify his explanation:

Line # **Transcription**

Jaylen: Me and Amir figured out that an apple equals three bananas because on the first scale, there was 10 bananas and two pineapples. So then two pineapples will have to equal about five pounds and then 10 bananas will be equal to five pounds. So on the second scale, there is only one pineapple, which is five pounds. But there is only two bananas, so then three, one apple will have to be three pounds and two bananas will be two pounds.

T1: So at some point we started converting to pounds?
[T1 calls on Carin, who has her hand up.]

Carin: I don’t understand what Jaylen said because he said two pineapples equal five pounds and one pineapple equals five pounds.

T1: That’s right. I thought I heard that too.

Jaylen: Both of the pineapples each equal five pounds.

T1: One pineapple equals five pounds.

Jaylen: And then another one equals five pounds.

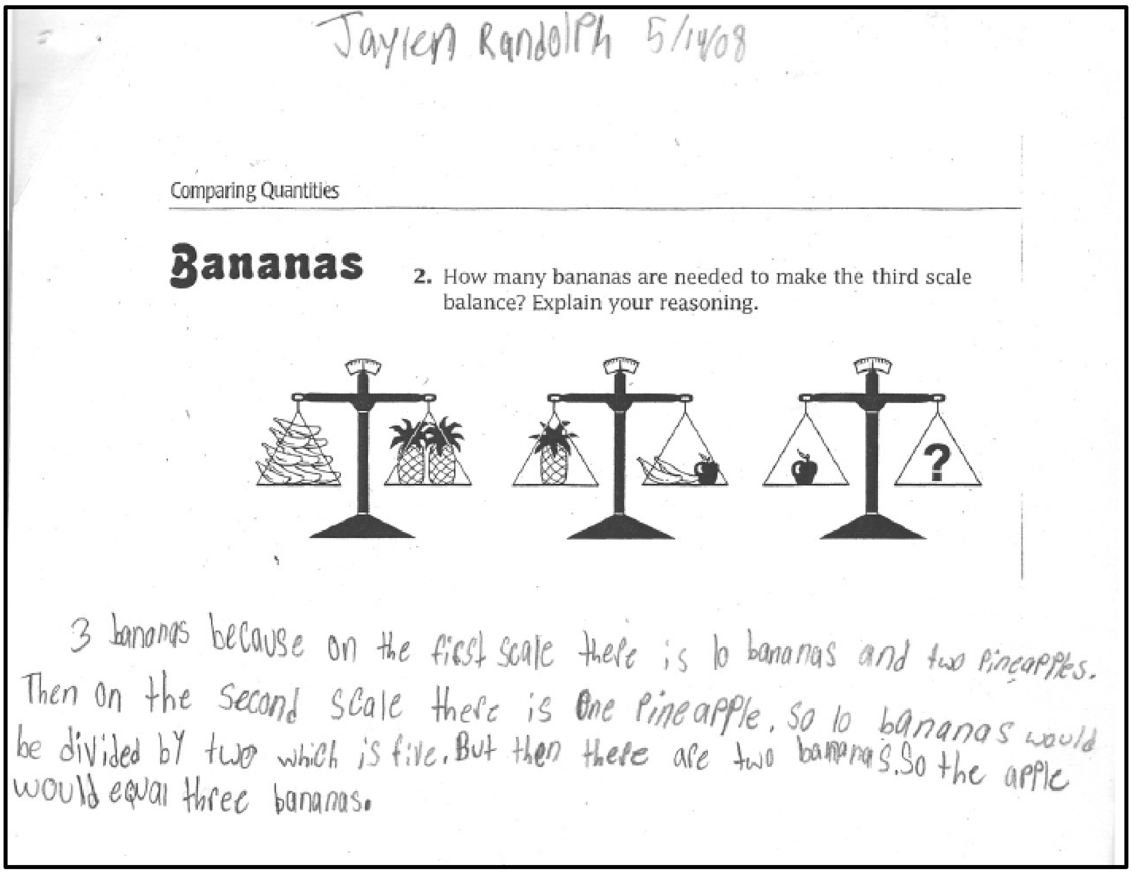
T1: Okay, one pineapple equals five bananas?

17 **Jaylen:** Yes.

18 **T1**: Okay.

Jaylen and Amir had also used Solution Path 1B to arrive at their answer. Jaylen's written work is shown in Figure 4.8 bellow.

Figure 4. 8: Jaylen's Explanation of the Banana Problem



Although his answer was correct, Jaylen's articulation of his thinking resulted in a discrepancy in his explanation [lines 1-7]. After listening to Jaylen's explanation, Carin raised her hand and the teacher gave her the opportunity to critique Jaylen's reasoning. Carin expressed her confusion and pointed out, correctly, that Jaylen, in his explanation had stated that two pineapples equaled five pounds and later on had said that one pineapple equaled five pounds [lines 10 and 11]. Prior to Jaylen's presentation, Carin herself had shared her solution strategy with the class which was similar to Jaylen's. This indicates that Carin was not questioning Jaylen's solution strategy, but the way he presented his reasoning. After Carin's comment on Jaylen's delivery of his explanation, Jaylen re-phrased his earlier statement [line 13] which seemed to satisfy Carin.

In their verbal explanations both Jaylen and Carin had used solution path 1B with a slight modification of attaching weights to the fruits. By assigning one pound to each banana, they were able to use the words, pound and banana, interchangeably in their explanation. For example Jaylen started his explanation by saying that an apple equaled three bananas [line 1] and in conclusion he repeated his assertion by saying that an apple equaled three pounds [line 6]. However, in Jaylen's written explanation (figure 4.8 above), there is no mention of pounds which suggests that after listening to Carin's explanation, he too decided to incorporate pounds as a unit of measuring weight in his oral explanation.

Attaching weights to the various fruits was not a possibility that had been discussed among the teachers during the planning phase of the lesson study and the teacher's question "So at some point we started converting to pounds?" [line 8], was purely rhetorical and reflected her surprise at some of the students' introduction of pounds as a unit of measurement for weight.

Much of the time during the debriefing session that followed this lesson was spent on discussing the fact that some students had decided to attach weights (in pounds) to the items in the Bananas and the Carrots problems. During this debriefing one of the teacher observers, O1, commented: "But isn't it interesting why they (students) introduced pounds (in the scale problems). They were all talking about pounds!" (See Teachers'

Reflections, Appendix H1) *The classroom teacher, T1, responded by admitting that at first she had thought the students were wrong in introducing pounds in their explanations: "I thought that was wrong, and then you (addressing R1) said, no that's what we want to get to"* (see Teachers' Reflections, Appendix H1). *The university researcher, R1, explained that it was important for students to understand the inherent existence of the concept of weight in a picture of a pan balance that depicted the equivalency between 10 bananas and two pineapples. The students' decision to use pounds was reflective of the understanding that the weight of 10 bananas is equal to the weight of two pineapples. O1 asked whether it was all right to disregard the students' inability to recognize the unsuitability of equating one banana with one pound. R2 speculated that what the students were doing was to convert things to pounds so it would make sense to them in terms of the weight. They chose one pound because "it was easy, one unit, one pound, one anything. Did it have to be a pound? No"* (see Teachers' Reflections, Appendix H1). *It was agreed that for the purpose of the lesson on the concept of equivalence, it was acceptable to let the students equate one banana with one pound and to pursue the concept of number sense and the feasibility of assigning one pound to one banana in further discussions and for another lesson.*

About sixteen minutes into the lesson Kyla was asked to share her solution to the "Bananas" problem with the class (see Event 4, Appendix D4):

Line #	**Transcription**
1	**Kyla**: Okay, me and Neema worked together.
2	**T1**: Louder.

Kyla: Me and Neema woked together and we first we figured out we looked at the first scale and said to ourselves that since there is two pineapples and 10 bananas, that must, that means that 10 bananas were equal to two pineapples. And then what we did was we went to the second scale and we saw that there was one pineapple, an apple and two bananas. So we figured if five bananas equaled a pineapple, but there is an apple and only two bananas, that uhm, that the apple would equal to 3. So then we went to the third scale and we figured out that since the apple is there and an apple equals three that the other side to make it equal will have to be an apple and two bananas.

T1: Okay, so in that last one?

Kyla: For the last scale?

T1: Ahum.

Kyla: There will be an apple, An apple on one side and an apple and two bananas on the other side.

T1: For the last scale?

Kyla: Ahum.

T1: So an apple equals an apple and two bananas?

Kyla: Wait, wait, wait.

T1: An apple equals an apple and then two more bananas? [Kyla confers with Neema who points to something on Jasmine's paper]

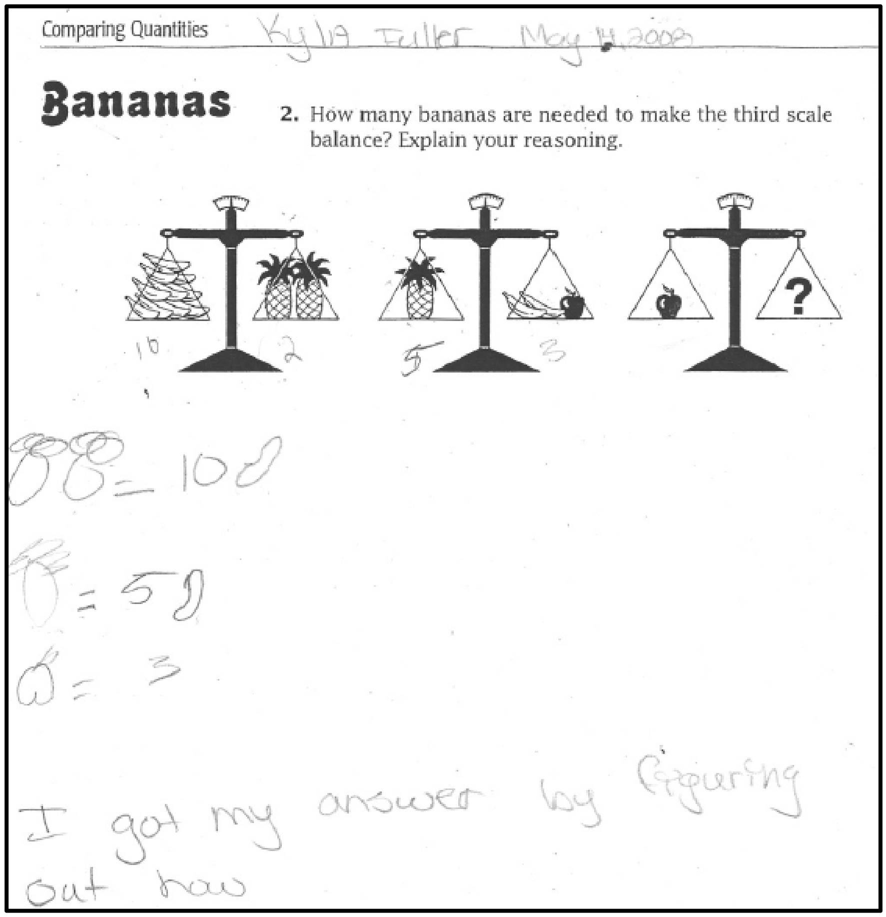
T1: Just go through it again. [The girls start discussing the problem.]

Based on the first two scales, Kyla had correctly identified that one apple equals three bananas [lines 3-10]. However, for her final answer, she incorrectly placed an apple and two bananas in the last scale [lines 10-12]. There are several noteworthy points in this episode which will be discussed below.

Immediately after the Bananas problem was first introduced to the class and following the teacher's instructions, the students had begun discussing the problem and its solution with their partners. Kyla's initial conversation with her partner, Neema, about the Bananas task was captured on tape during this discussion period (see Event 1, lines 26-29, Appendix D1). Kyla had told Neema: "Apple equals three. So now, if an apple equals three pounds, we are trying to figure out what balances that out (pointing to the last scale). Hmm!"

Although Kyla had figured out that an apple would equal three pounds on the second scale, she did not seem to have a robust understanding of the fact that she was equating one pound with one banana and was still pondering over what would balance the apple on the third scale. Kyla's written work is shown in Figure 4.9 below. She had formed three equations by drawing pictures of the fruits and using the equal sign. The first equation simply represented the first scale in which, 10 bananas were equal to two pineapples. The second equation stated that a pineapple was equal to five bananas, a natural deduction from the first equation. The third equation stated that an apple was equal to 3, but did not specify as to 3 what.

Figure 4. 9: Kyla's Explanation of the Banana Problem



Since she had not written anything under the last scale, it is unclear whether she understood that an apple was equal to three bananas. Also in her oral explanation, she never explicitly mentioned that an apple equaled three bananas: “So we figured if five bananas equaled a pineapple, but there is an apple and only two bananas, that uhm, that the apple would equal to 3” [lines 8-10]. In fact it is possible that perhaps Kyla believed that an apple equaled any three items, as evident in her explanation: “So then we went to the third scale and we figured out that since the apple is there and an apple equals three that the other side to make it equal will have to be an apple and two bananas.” [lines 10-12]

After hearing Kyla’s explanation and her incorrect assertion that an apple and two bananas would balance an apple on the last scale, the teacher asked Kyla, twice, to confirm what should be placed on the third scale [lines 13 and 18]. Once it was established that Kyla truly believed her wrong answer to be correct, the teacher paraphrased and then reworded Kyla’s response and posed it as a question in such a way that highlighted the impossibility of Kyla’s answer: “An apple equals an apple and then two more bananas?” [Line 22]

It is impossible to know whether it was the element of surprise in the teacher’s voice as she repeated Kyla’s answer in the form of a question or whether it was Kyla’s own realization of the unfeasibility of her statement that made her doubt her answer and start consulting with her partner, Neema. The teacher provided Kyla and Neema the opportunity to review their answer and report back (see Event 5, Appendix D5). About five minutes later Kyla volunteered to report on her revised answer to the “Bananas” problem:

Line #	**Transcription**
1	**T1:** So you did a self-correction?
2	**Neema:** Yeh.
3	**T1:** Okay talk it through.
4	**Neema:** We….[not audible]
5	**T1:** Nice and loudly.
6	[Neema smiles shyly]
7	**Kyla:** Want me do it? [asking Neema]
8	**Neema:** Yes.

Kyla: Okay. Okay, we had said before that it would be an apple, two bananas. But we were actually wrong. It would be three bananas altogether because we looked at the apple and knew that apple was three so we said that, we said to make it equal, the other side has to equal three also. So what we did was figure out, we took it back to where the pineapple and the five bananas were equal to each other. But then we looked at it and said that is not really equal because each banana is one pound. So then we thought like, okay, since they are equal to one pound then for it to equal three pounds, you can take three bananas, they weigh the same as an apple.

T1: We all agree with what we just heard? [addressing the class]

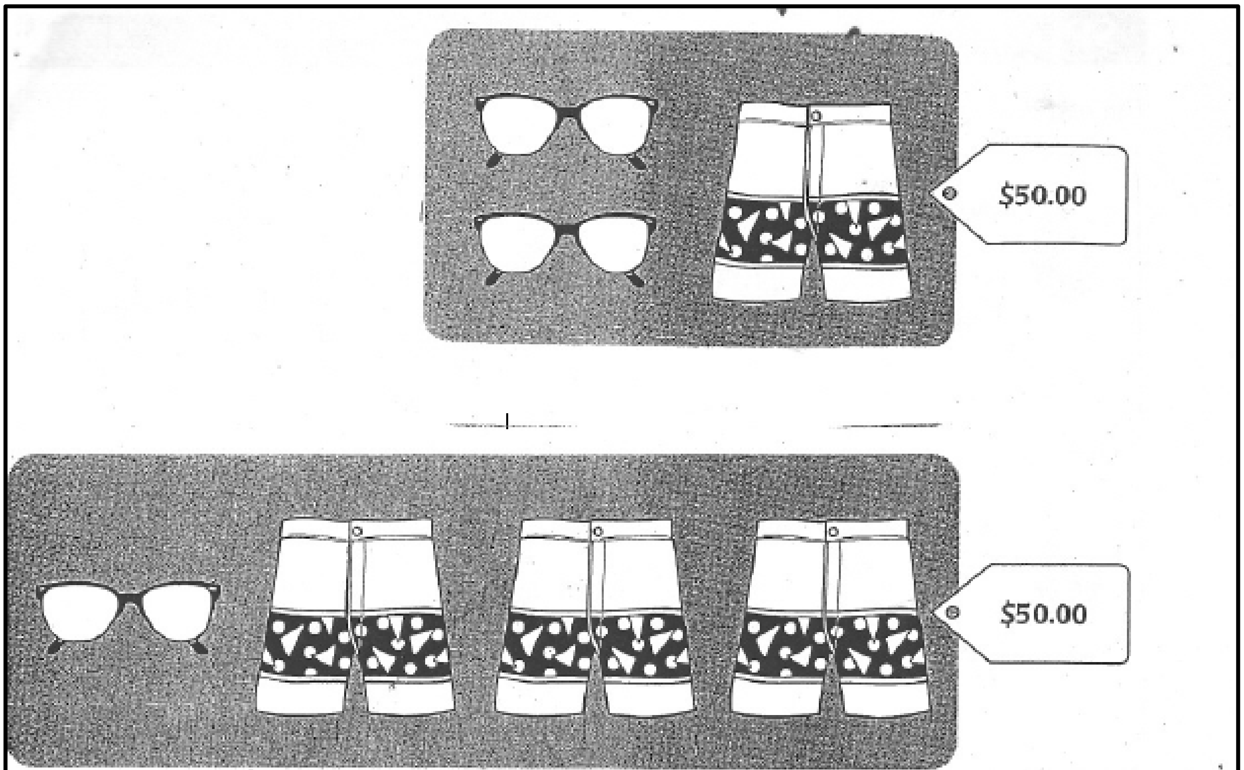
Some students: Yes.

It is evident from Kyla's explanation that once she was able to firmly establish the correlation between a pound and a banana in her mind, she could get the correct answer by placing three bananas on the last scale to balance it with one apple: "…banana is one pound. So then we thought like, okay, since they are equal to one pound then for it to equal three pounds, you can take three bananas, they weigh the same as an apple."[lines 16-18]

By giving Kyla and Neema the opportunity to review their work and by acknowledging that they had made a self-correction, the teacher not only supported student autonomy but also contributed to a classroom environment in which persistence in pursuite of understanding is encouraged.

Another task that was tackled by the students was the Shorts and Glasses problem (see Figure 4.10 below, a copy of Figure 4.3), which was a pictorial depiction of a system of two equations with two unknowns. Specifically, the students were asked to identify the more expensive item before calculating the actual prices.

Figure 4. 10: The Shorts and Glasses Problem



1. Without knowing the prices of a pair of glasses or a pair of shorts, you can determine which item is more expensive. Explain how.
2. How many pairs of shorts can you buy for $50?
3. What is the price of one pair of glasses? Explain your reasoning.

During the planning phase of the lesson study and in preparation for the lesson implementations, the teachers had worked on this problem and created a list of solution paths (see section 4.2.3.3). It was speculated that some students, as some teachers had done, would not pay close attention to the first part of the question and determine the more expensive item by first calculating the actual prices. Some teachers had used the $50 price tags and formal algebra (Solution Paths 3A and 3B), while others had used the price tags and the guess and check strategy (Solution Path C) to find the price of each item. During the lesson implementation, however, it was hoped that at least some students would use Solution Path 3D, or some version of it, which involved proportional reasoning and informal algebraic ideas. The Solution Path 3D for this problem, identified by the teachers was as follows:

- Cross out one pair of glasses and one pair of shorts in both pictures.
- Note that the $50 price tags will reduce by the same amount in both pictures and therefore will remain equal to one another.
- The new pictures show that one pair of glasses is the same value as two pairs of shorts.
- Therefore glasses are more expensive than the shorts. In fact the glasses are twice as expensive as the shorts.
- In the second picture, the glasses can be traded in for two pairs of shorts.
- Therefore you can get five pairs of shorts for $50.
- This implies that the shorts are $10 a pair.
- Therefore the glasses cost $20 a pair.

As predicted, all of the students used the $50 price tags with the guess and check strategy in order to first find the correct prices for each item. They then used the actual prices to determine the more expensive item. The teacher repeatedly reminded the students that they were to identify the more expensive item without knowing the actual prices and urged them to use a strategy other than guess and check. But as discussed later, only one student managed to do so and the rest of the students found the prices first through trial and error. It is important to note that the strategy commonly referred to as the guess and check or trial and error, does not necessarily entail the random selection of a number as the initial guess. In fact all the students in this study, who used this strategy, began this guessing and checking process by selecting “sensible” numbers as their preliminary choice. For example, knowing that the total price of three items was $50, no one started with the assumption that the price of one of the items would be $50 or more. In other words, students’ guesses were not haphazard, but methodical and within the perimeters of the problem and the given facts.

The following is a descriptive analysis of some of the students’ work on the Shorts and Glasses problem and the interactions that took place chronologically, involving students situated at various tables.

<u>Table 3</u>

Four students- Amir, Jaylen, Erica and Sierra- were seated at Table 3. Amir and Jaylen paired up as partners, while Erica and Sierra worked on the problems as a team.

While Jaylen and Amir were working on the Short and Glasses problem, the teacher walked over to their table and had a conversation with the boys about the strategy they had used in solving the problem (see Event 6, Appendix D6):

Line #	**Transcription**
1	**Amir:** Those glasses are really expensive.
2	**T1:** How do you know that?
3 4	**Amir:** Because glasses…[not clear].. usually have to pay a lot of money.
5	**T1:** What if you go to Walmart?
6	**Amir:** Eh, probably still be kind of more expensive than shorts.
7	**T1:** Okay, let’s look at what we have here and you tell me why.
8	**Jaylen:** We thought that this [pointing to one of the shorts in the

second picture], we tried 20, 40 [pointing to a second pair of shorts], that wouldn't work. So then we dropped it down to 10. Ten, 20, 30 [pointing to the three shorts]. And 30 plus, what would this be [pointing to the glasses] to equal 50. And then we kind of thought it was 20. And then [finding the total for items in the second picture] 20, 30, 40, 50. So the sunglasses would be more expensive, which is $50.

T1: Okay, so what strategy was that?

Jaylen: Ahm..

Amir: Addition?

Jaylen: Squeeze method?

T1: Addition is an operation.

Jaylen: Squeeze method?

T1: Squeeze method? What does squeeze mean?

Jaylen: It is like 30 [pointing to a pair of shorts] we thought….[not clear]

T1: So you came up with a high number and a low number and started squeezing? [Jaylen nods]

T1: Okay, where did you get the numbers from?

Jaylen: We randomly picked them.

T1: okay. So when we just pull the numbers out and try them out, what is that strategy called?
[One of the girls at Table 3 is heard saying: Guess and check]

T1: You are guessing. Now, is there any way you could do besides guess and check? That is my question. [T1 walks away]

Amir and Jaylen had used guess and check to find that the shorts and glasses were $10 and $20 respectively. Because the teacher had been emphasizing the fact that they had to determine the pricier item without knowing the actual prices, when she asked

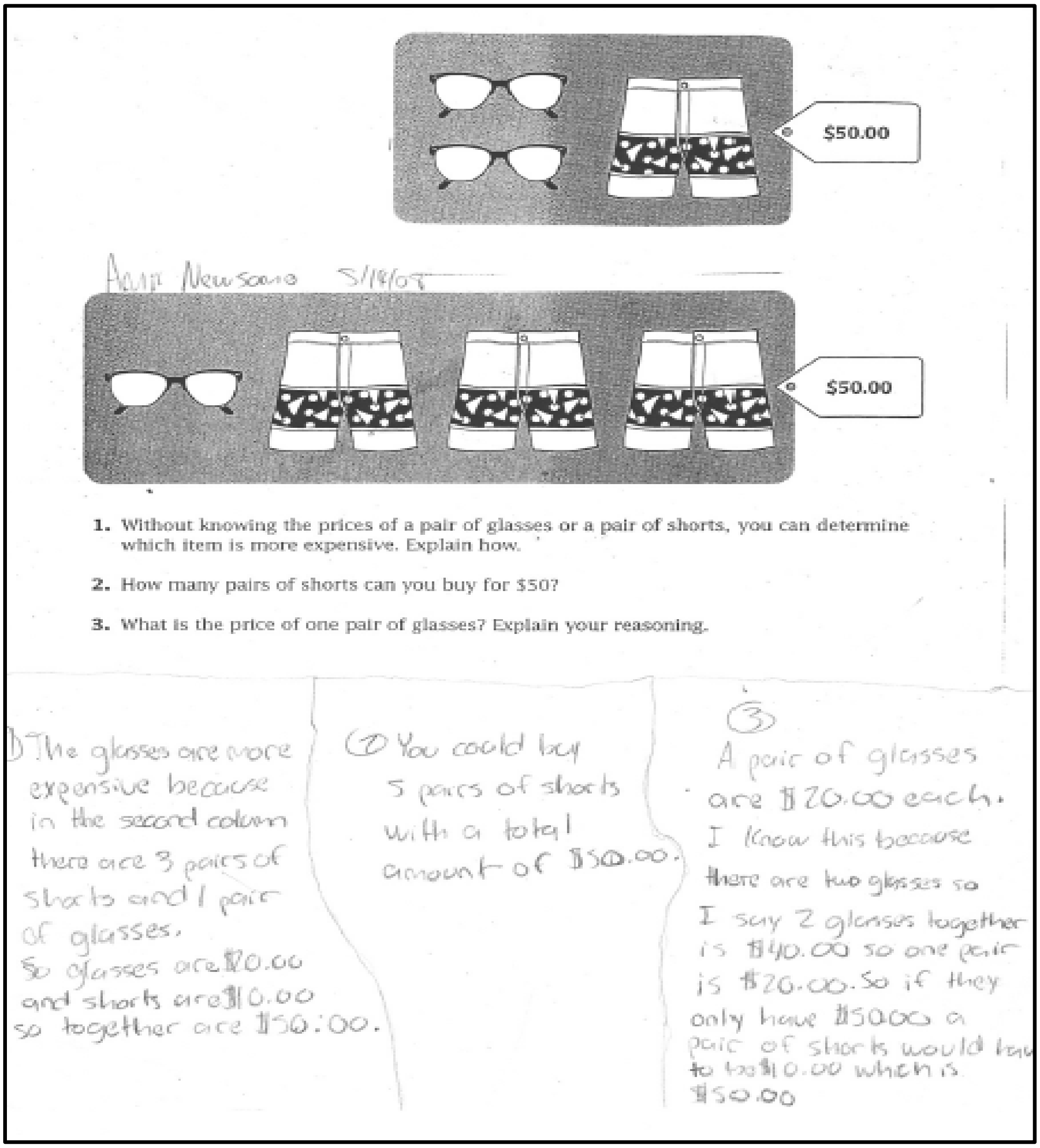
Amir to explain why they thought the glasses were more expensive, Amir tried to justify his claim by appealing to, what was for him, common sense: because glasses are really expensive and you usually have to pay a lot of money for them [lines 1, 3, and 4]. When the teacher asked whether the glasses would still cost a lot in a less expensive store like Walmart [line 5], Amir replied, quite reasonably, that even in Walmart the glasses would still be more expensive than the shorts [line 6]. This claim reflected his conviction that in general glasses cost more than shorts.

Amir's reasoning was solely based on his prior knowledge and personal experiences of the retail market. However, there were other students in this study who used a similar lens in justifying their assertion that the glasses were more expensive than the shorts (more discussions on this topic will follow in later sections).

The teacher then pointed to the pictures and asked Amir and Jaylen to justify their reasoning based on what was shown in the pictures [line 7]. Jaylen described the strategy they had used to determine the prices, which was essentially by trial and error [lines 8-14]. However, when asked by the teacher to name the strategy they had used [line 15], the boys had difficulty identifying guess and check as their strategy of choice. Amir replied that they used the addition strategy and in response to that the teacher, attending to mathematical vocabulary and terminology, simply reminded him that addition was an operation and not a strategy. Jaylen, spontaneously and in a spirit of creativity, came up with a made up word, the "Squeeze" method, to describe the guess and check strategy they had used. The teacher then built on Jaylen's idea by using the word "squeeze" to construct a sentence that captured the essence of the strategy used by Amir and Jaylen: "So you came up with a high number and a low number and started squeezing?” [lines 24 and 25]. After further questioning, Jaylen declared that they had picked the numbers randomly [line 27] and eventually one of the girls at the table identified the strategy used by Amir and Jaylen as guess and check. However, despite Jaylen’s belief that they had randomly picked $30 to be the cost of a pair of glasses, their decision to pick $30 as the starting guess was a conscious choice that was guided by the given facts of the problem ($50 price tag), and therefore not strictly random.

Although the boys had already found the actual prices and knew that the glasses were more expensive than the shorts, the teacher asked if they could find a different strategy from guess and check that would allow them to reach the same conclusion without having to figure out the prices first. After posing the question, the teacher walked away from their table and if Jaylen and Amir made any further attempts to solve the problem differently, there is no record of it, since the camera moved away from that table. Additionally, the examinations of Amir and Jaylen’s written explanation revealed no evidence of any strategy used other than the guess and check. Amir’s written explanation, which was similar to Jaylen’s, is shown in Figure 4.11 below. .

Figure 4. 11: Amir's Explanation of the Shorts and Glasses Problem



<u>Table 2</u>

Three students- Jasmine, Jade, and Tequrra- were seated at Table 3 and worked together as a group. Jade, their spokesperson was the last person to report on the Shorts and Glasses

problem before the class moved on to the next task (see Event 7, Appendix D7):

Line #	**Transcription**
1	**Jade:** The item that we think is more expensive is the sunglasses. We
2	knew they were more expensive because the more glasses you have, the
3	less items you would need in each box to equal the $50.
4	**T1:** Oh, okay. The more glasses you have [pause]. The more glasses
5	you have the less item you would need to kind of supplement to get to
6	$50?
7	[The girls at Table 2 nod.]

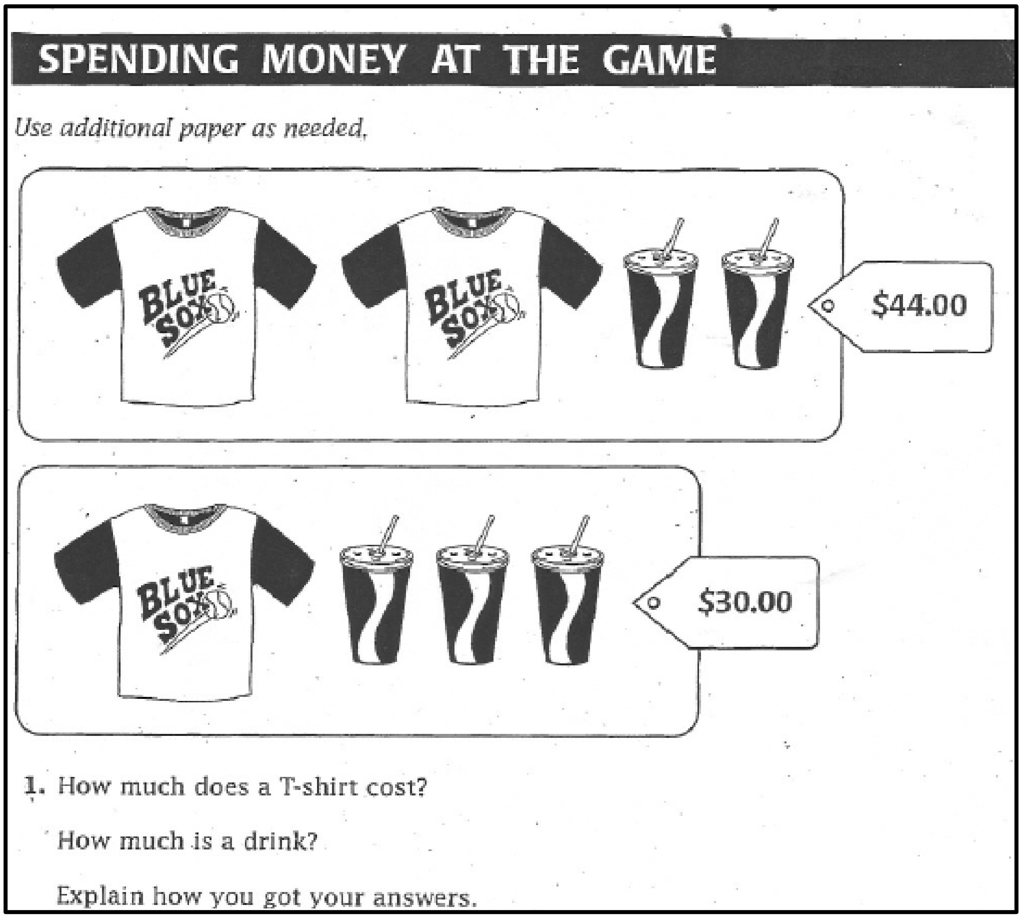
This episode was interesting because all along the teacher had been encouraging the students to identify the more expensive item without making a reference to the actual prices. And yet, when Jade presented her reasoning [lines 1-3], arguably not perfectly articulated but nevertheless entirely logical, it did not receive the attention and enthusiasm that it deserved.

After listening to Jade's explanation the teacher paused to think about what she had said, repeated her statement [lines 4-6], and after saying a quick okay, abruptly moved to introduce the next task. It is important to note that Jade's explanation was not one of the solution paths previously identified by the teachers and therefore it seems reasonable to assume that the teacher did not exactly follow the logic behind it and needed more time to digest and truly understand it. Perhaps under different circumstances- without any time constraints or the pressures of being observed and videotaped- the teacher would have taken the time to not only understand Jade's reasoning herself, but also, through elaboration and examples, help other students follow Jade's thinking.

During the debriefing session that followed, there was no mention of Jade's strategy, but there was a general consensus that the Shorts and Glasses problem needed *certain modifications in order to make it less tempting for students to use the guess and check strategy. It was suggested by R1, and approved by others, that for subsequent lessons the Shorts and Glasses problem would first be introduced to the class on the overhead with the $50 price tags hidden, and the students would be verbally informed that the total prices in the two box were the same. It was speculated that not knowing the actual total prices, eliminated the tendency to find the individual prices through trial and error and encourage the students to use a more algebraic approach in determining the more expensive item.*

The last task attempted by the students was the Soda and Shirt problem (Figure 4.12 below, a copy of Figure 4.4), which was a pictorial depiction of a system of two equations with two unknowns and the reader was asked to determine the cost of each item.

Figure 4. 12: The Soda and Shirt Problem



Unlike the previous problem (Shorts and Glasses), in which the round number of $50 for both price tags made it fairly easy to guess the price of each item, this problem presented a great challenge to the students, who continued to use the guess and check strategy. Jasmine, Jade, and Tequrra worked independently at this point, plugging in various numbers, trying to find prices that worked in both boxes. Jasmine was caught on camera telling Jade that $1.50 for a soda and $25.50 for a shirt worked in the bottom box and that she had to test to see if they would add up to $44 in the top box (which they did not).

The students soon found out that even though they had the option of using a calculator, it was not so easy to find the individual prices through trial and error and despite the teacher's insistence that other strategies existed, none of the students could think of any. Visible signs of frustration could be seen around the class: at Table 1, Carin resting her forehead on her palm and saying: "Oh, darn"; Kyla professing that the problem was too hard; at Table 4, Jarrod punching some numbers on the calculator, tapping his clenched fist on the table several times and making a face, as if in pain; Maurice, after realizing that his chosen numbers didn't work, saying, "Man, I was close".

About 5 minutes into the problem (see Event 8, Appendix D8), the girls over at Table 1 asked the teacher directly to give them a hint: "You are supposed to help us." Mindful of the goals of the lesson study that included the fostering of student autonomy, the teacher turned to the university researcher, R1, and asked if she could provide the students with a hint. R1 suggested that the teacher offer a hint by posing a question to the students. A couple of minutes later the teacher addressed the whole class: "My kids. Can we make this problem with less items? And how can we do it?" This hint generated a new energy among some students and propelled them to consider new avenues, while others continued plowing through various combinations of numbers that would add up to the total prices of $44 and $30 in the two boxes.

During the planning phase of the lesson study and while preparing for the lesson implementations, the teachers had worked on the Soda and Shirt problem and had come up with four possible solution paths for this task (see section 4.2.3.4). Two of the solution paths (4B and 4D) involved elimination of some of the items in the pictures. Quite possibly, it was with these two solution paths in mind that the teacher offered the above mentioned hint to her students. One of the solution paths, Solution Path 4D was as follows:

- Cross out one shirt and one soda in the top picture and cut the price in half.
- This indicates that one soda plus one shirt will cost $22 (44:2=22).
- In the second picture, bundle one shirt and one soda and note that they cost $22.
- In the second picture, eliminate one shirt and one soda and reduce the total cost by $22.
- This new picture indicates that two sodas cost $8 (30-22=8).
- Divide 8 by 2 to find the cost of one soda ($4).
- Subtract 8 from 40 to find the price of two shirts (44- 8= 36).
- Divide by 2 to find the cost of one shirt ($18).

About 10 minutes after the teacher had hinted at the possibility of a simpler problem with fewer items, the girls at Table 2 managed to find the correct prices for each item. In Event 9 (see Appendix D9) Jade and Jasmine shared their findings with the teacher:

Line #	**Transcription**
1	**T1:** What is your answer?
2	**Jasmine:** The juice costs $18 and the soda pop…
3	**Jade:** No, the…
4	**Jasmine:** Oh yeah, the shirt costs $18 and the juice costs $4.
5	[T1 starts adding some numbers up; Audio not clear]
6	**T1:** And how did you get it?

Jade: Because, like we came over here (pointing to her paper) and said to find half of the total price for half of the items which was a shirt and a drink. We then started to think which numbers would…

Jasmine: We forgot about 44, we just worked with 22.

T1: Alright, you worried about 22 then. So you started finding out which numbers….

Jade: which numbers add up to 22 and work with the totals.

T1: Okay.

Jade: And we got 18 and 4.

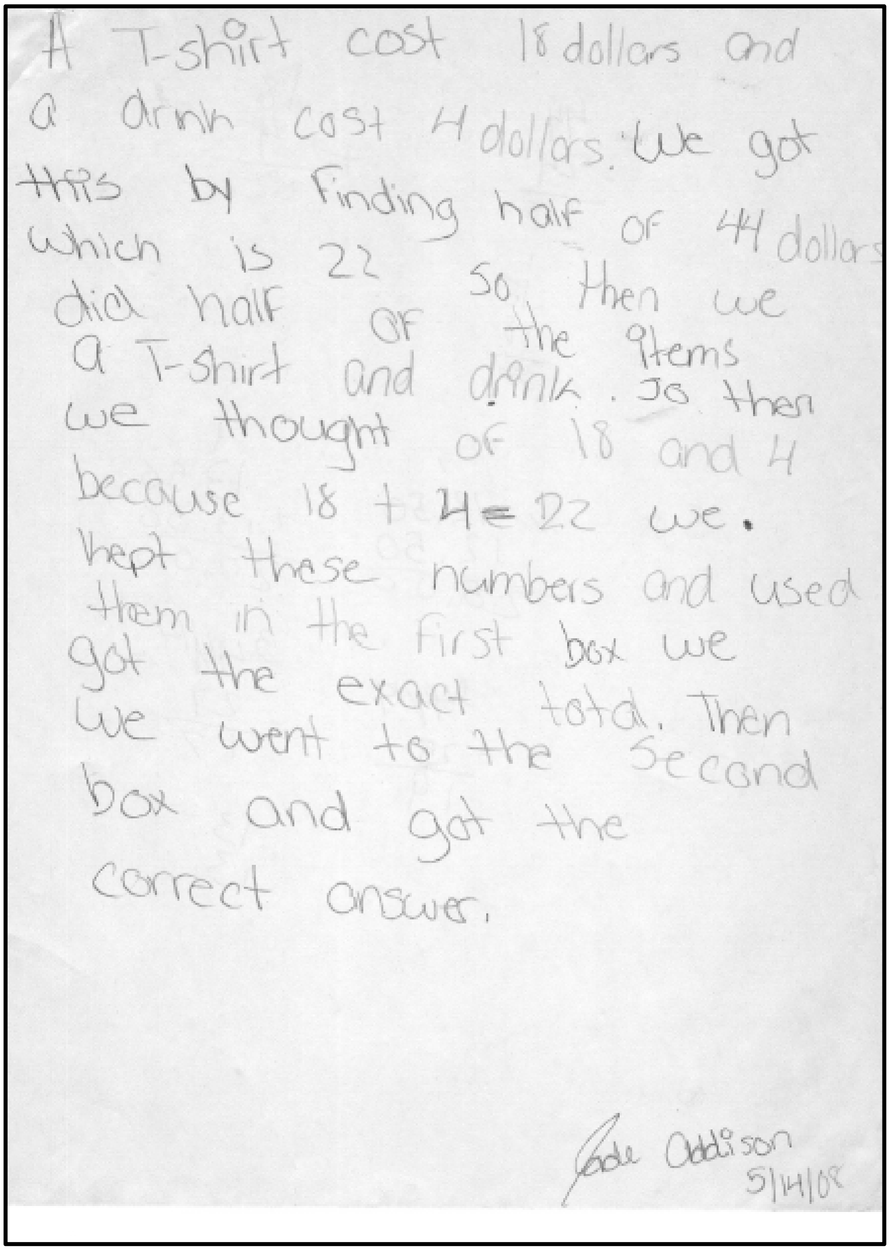
T1: Okay, good, alright. Did you write down your explanations?

Jasmine: And then we did the rest of it. We did 18 plus 4, plus 4…

T1: And it worked for you. Okay, write down your explanation.

After listening to Jade's explanation, the teacher asked the girls to record their reasoning in writing. Jade's written work is shown in Figure 4.13 below.

Figure 4. 13: Jade's Explanation of the Soda and Shirt Problem



A T-shirt cost 18 dollars and a drink cost 4 dollars. We got this by finding half of 44 dollars which is 22 so then we did half of the items a T-shirt and drink. So then we thought of 18 and 4 because 18 + 4 = 22 we. kept these numbers and used them in the first box we got the exact total. Then we went to the second box and got the correct answer.

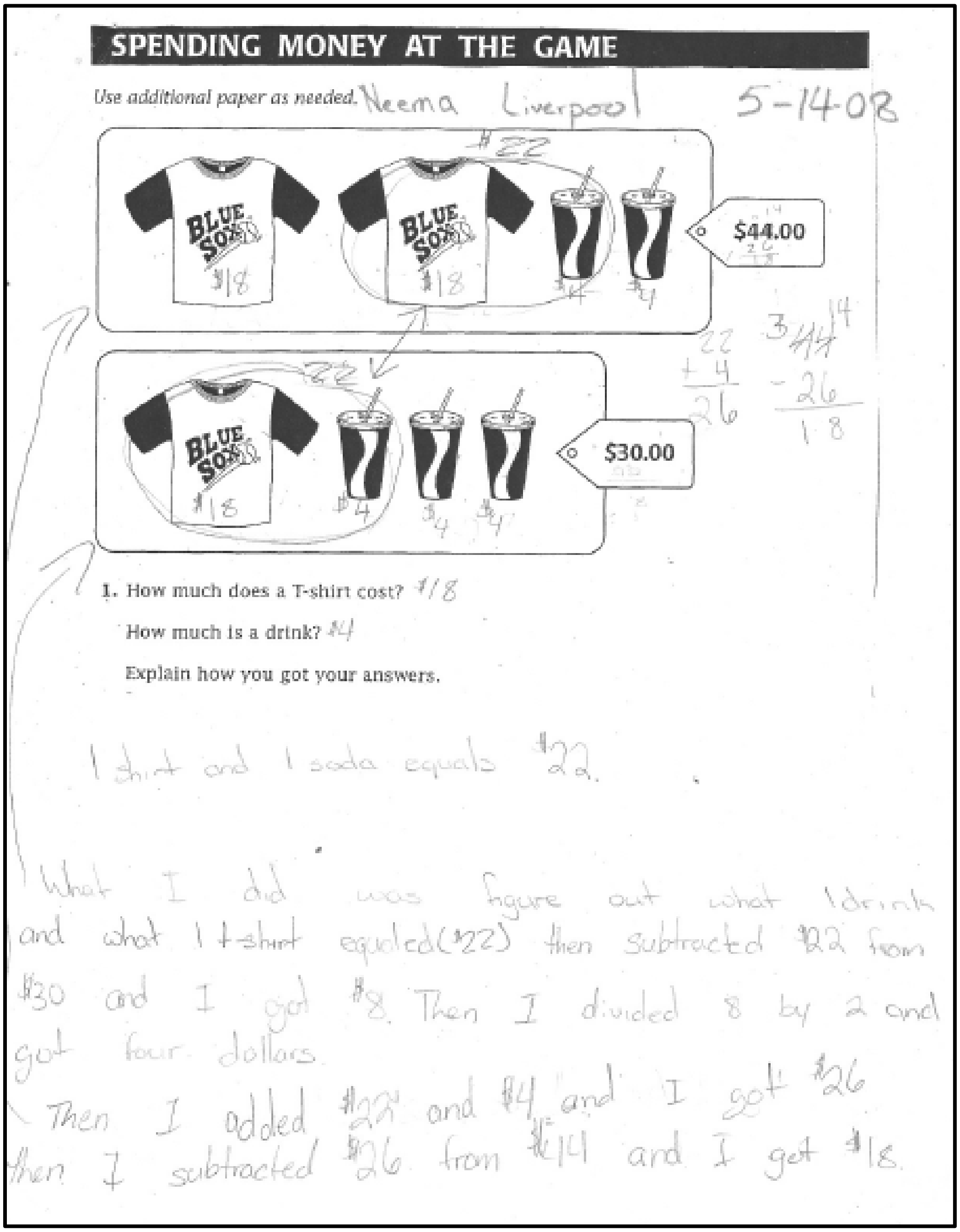
Jade Addison
5/14/08

Ironically enough, although the girls had taken the hint and cut the items and the total price of $44 in the top picture in half, they had reverted back to the guess and check strategy, which was then made simpler, once they had reduced the total price from $44 to a smaller amount of $22. They had then found, through trial and error, that the shirt and soda cost $18 and $4 respectively.

Due to lack of time no one had the opportunity to share their solution strategy for the Soda and Shirt with the class. However, the examination of the students' work revealed that five other students had solved the problem similar to the way that the girls at Table 2 had done (reducing 44 to 22, then guessing and checking). Three students did not pursue the teacher's hint of fewer items, but managed to find the correct prices through trial and error and five students handed in papers that were either blank or showed little evidence of any thoughtful mathematical work. This episode illustrates the difficulty in steering some students toward an alternative strategy if their minds are set on a specific idea. Despite the teacher's repeated suggestion that the students do not use the guess and check strategy and her hint which was meant to nudge them toward a certain path, the majority of students stuck with their initial method of trial and error.

Two students however, Kyla and Neema from Table 1, had picked up on the teacher's hint and had used a strategy which was almost identical to the Solution Path 4D mentioned above. Neema's written work is shown in Figure 4.14 below.

Figure 4. 14: Neema's Explanation of the Soda and Shirt Problem



Neema’s work is indicative of some sophisticated algebraic thinking on her part. One of the teachers, during the planning phase of the lesson study, had solved this problem using formal algebra, which was identified as Solution Path 4A and shown below:

Let t represent the cost of a T-shirt and s represent the cost of a soda. Then, the problem can be represented by the system of equations:

$2t + 2s = 44$ …….1st equation

$t + 3s = 30$ …….2nd equation

Multiply the 1st equation by ½ to get:

$t + s = 22$

Therefore the 2nd equation above, which is the same as $(t + s) + 2s = 30$, can be re-written as:

$22 + 2s = 30$

Simplify and solve for s to get:

$s = 4$ ===> $t = 18$ (since $t + s = 22$, which means that $t = 22 - s$)

Therefore the shirt costs $18 and the soda costs $4.

In the following table (Table 4.1), Neema's explanation is compared and contrasted with the formal algebraic solution above, highlighting the depth and substance of Neema's algebraic thought process. Although she did not use any equations, variables, or formal algebraic terminology, the first four steps in Neema's work are essentially identical to the steps taken by the teacher who solved this problem using formal algebra.

Table 4. 1: Comparison Between Neema's Work and the Formal Algebraic Solution

Neema's Explanation	**Algebraic Solution**
She found the price of one shirt and one soda by bundling a shirt and a soda in the top picture and cutting the price in half ($22).	She found the price of one shirt and one soda by multiply the 1st equation by ½ to get: t + s = 22
She drew an arrow showing the transfer of one bundle of shirt and soda to the second picture, so the second picture can be interpreted as: $22 + 2 soda cups = 30	She substituted 22 for t + s in the second equation and re-wrote it as: 22 + 2s = 30
She subtracted $22 from $30 to get $8.	She subtracted 22 from both sides to get 2s = 8
She divided $8 by 2 to get $4, which is the cost of one soda.	She divide both sides by 2 to get s = 4, which is the cost of one soda.
In the top picture, add the price of the bundle ($22) with the price of a soda ($4) and subtract the sum from the total of $44 to find that a shirt costs $18.	Substitute 4 for *s* in the equation t + s = 22 and solve for *t*. Therefore the shirt costs $18.

It was unfortunate that due to lack of time, Neema never got the chance to share her solution with the class. Since there was no mention of Neema's work during the debriefing session that followed, it is unclear whether the classroom teacher or any of the observers took note of Neema’s explanation while she was working on it in class.

4.4 Lesson Implementation and Debriefing #2

The following sections provide an overview of the second lesson implementation and the debriefing session. A more comprehensive description of these sessions is also provided which details the observed occurrences in approximately 5-minute time intervals of the video data (see Appendices B2 and C2). The abbreviations R1 and R2 refer to the two university researchers while T1 through T4 identify the teachers in this study, who facilitated the four implementation sessions respectively. O1 through O4 identify the teachers who, as graduate students, participated in the lesson study and attended all or some of the implementation sessions as observers.

4.4.1 Lesson Implementation #2: Overview

This session was approximately 80 minutes long and was facilitated by T2, a sixth grade teacher. There were 16 students in the classroom. In addition to the classroom teacher, T2, there were eight other adults, R1, R2, O1, O2, O4, T2, T3, and T4, present in the room as observers. An additional person was in charge of taping the session. Students’ desks were put together to form six groups of students. There were two or three children in each group. The grouping of the students was decided by the teacher prior to the lesson and was mainly based on the teacher’s anticipation of how well they would work together. Also the teacher had tried, to some extent, to have a more academically strong student in each group.

The teacher had originally intended to do four activities with his students, but due to lack of time, was unable to include the last activity, the Soda and Shirt problem. Therefore the lesson consisted of three tasks which were implemented in the following order: Activity 1- Bananas (approximately 20 minutes); Activity 2- Carrots (approximately 22 minutes); Activity 3 - Short and Glasses (approximately 20 minutes) (see Appendices A1-3). Activity 1 was introduced by handing out a copy of the problem to each student. Activity 2 and Activity 3 (which had been modified during the preceding debriefing session) were each first introduced on the overhead and after the students had a chance to discuss the problem with their partners, a copy of it was distributed to each student.

The class began with the teacher introducing the university faculty member, R1, who addressed the students and explained, in simple terms, the purpose of the lesson and the presence of all the observers in the classroom. After the introductions were made the teacher handed out the first activity sheet and the mathematical work began.

The teacher asked the students to work within their groups to discuss the problem, reach a consensus about the solution, and be ready to report back to class their findings. Initially, when the students began working on the first task, they were very quiet. They either worked individually or talked about the problem in hushed voices. The teacher reminded the class to discuss the problem within their groups and one observer was heard telling a group of students that they did not need to whisper and could freely talk to each other. After this initial inhibition, the students became more vocal and as the lesson continued, some students became increasingly more talkative. Part of this classroom noise was generated by student to student or teacher to student mathematical discourse and some was related to off-task conversation among the students.

While the students were working on the problems the teacher circulated among the groups, overseeing their progress. With the exception of R1, who moved around the room during the lesson, the rest of the observers remained seated or stood at the periphery of the classroom for the most part. From time to time some of them walked around and took notes, while others sat next to a group of students and listened in on their conversation. Most of the interactions between the students and the observers occurred because the observers needed clarification on the students' thinking and would ask questions such as "How did you get that?" or "Why did you do this?"

Upon completing each task the teacher asked the students to share their solution strategies with the class. Very few students volunteered to speak up. Some were reluctant and had to be coaxed by the teacher, while others simply said that their strategy was the same as the previous presenter and declined to elaborate further. During some of the class presentations, some students were busy talking to others within their group and several times the teacher had to address this lack of attentiveness.

4.4.2 Debriefing #2: Overview

The debriefing took place in the classroom immediately following the lesson implementation and lasted for about 32 minutes. T2 states that he was happy with the level of interaction between the students in certain groups and that he was satisfied with the overall quality of work that many students had displayed. However, he noted, that in the latter part of the session there had been some behavioral issues. The teacher stated his belief that the students' misbehavior was the result of their frustration with their own level of performance in comparison to the perceived higher level at which, they thought that their peers were handling the same problems.

T4 asked whether the students were normally put together in groups of three. T2 responded that the students usually worked in pairs, but he had decided to deviate from the normal structure and put them in groups of three because he had felt that the third person would help toward a better outcome by bringing additional insight into the conversation. There were some comments on how well some of the students had worked and how some had played around within their assigned groups.

There was much discussion about the students' responses to the modified version of the Shorts and Glasses problem in which, most of the students in the first lesson implementation had used the guess and check strategy to find the actual prices. Subsequently the teachers had decided to hide the price tags in the picture for this lesson implementation, with the hope that the students would perhaps use a different strategy to determine the more expensive item. T1 and T4 argued the merits of some of the responses in which, the students had reasoned that the glasses were more expensive than the shorts because "they were breakable", "made you see", and were "harder to make". T1 declared that she rather liked those answers since they related to the students' life experiences and there was mathematics associated with their thinking. Although T4 granted that the students' thinking was valid, she did not agree that it was mathematical. She reminded T1 that upon hearing such answers, the teacher had asked the students to determine the most expensive item mathematically and based on only what they could see in the pictures, and that the students were unable to do so.

The teachers also discussed the fact that some of the students' written work contradicted the verbal explanation presented by the group's spokesperson and wondered whether a consensus had been reached among the group members. One of the observers noted that after the students had completed the Bananas task and during the class presentation, the students at Table 5 had the wrong answer of two bananas written on their papers, but because four students had previously reported their answer to be three bananas, they falsely claimed that they had the same answer and never presented their solution to the class.

There were further discussions on students' solution strategies to the Shorts and Glasses problem. R1 stated her belief that students were unable to recognize the 1 to 2 ratio between the number of glasses and the number of shorts. T3 reported that although one girl at Table 6 had thought of eliminating a pair of shorts and a pair of glasses from each picture (which left her with one pair of glasses against two pairs of shorts), she could not make the conclusion that the glasses were twice as expensive as the shorts. In fact she maintained her earlier conclusion that the shorts were more expensive. However, T3 speculated that given time and with further probing, the student would have been able to recognize the 1 to 2 relationship between one pair of glasses and one pair of shorts.

T3, who was due to facilitate lesson #3 with her grade 8 students the following day, told the group about her revised lesson plan and her decision to include the Carrots and Bananas problems as warm up activities (more discussions on this decision will follow in later sections), followed by the tug-of-war and the Chicken problems as the main activities. R1 recommended that T3 consider the possible inclusion of the Shorts and Glasses problem in her lesson plan so they could see whether the eighth grade students would be able to identify the 1 to 2 relationship between the glasses and the shorts.

4.5.3 Descriptive Analysis: Lesson #2

The following is a descriptive analysis of the lesson implementation #2, with references to the *critical events* that occurred during the lesson (for a complete list of all the transcribed critical events, see Appendices E1-8). The selection of *critical events* involved careful deliberation of the research questions so that these events, taken individually, were snapshots offering glimpses into the lesson and collectively, the string of these snapshots contributed to shaping a narrative that provided the researcher an insight into the nature of the interactions that transpired in the classroom, and subsequently helped answer the research questions.

In the following descriptive analysis, the coded videotaped data was examined in conjunction with the students' work samples and, when appropriate, the teachers' reflections from the debriefing session. As a result, in this analysis, references to the teachers' reflections are at times intertwined with discussions on students' work and classroom occurrences. In order to better distinguish the classroom descriptions from the debriefing conversations, any discussion pertaining to the teachers' reflections will be italicized in the following analysis.

This lesson took place in a sixth grade classroom with 16 students. In addition to the classroom teacher, T2, there were eight other adults present in the room as observers: two university researchers, R1 and R2; the other three teachers in this study, T1, T3, and T4, who facilitated the subsequent implementation sessions; and three of the teachers, O1, O2, and O4, who as graduate students, participated in the lesson study and attended this implementation session.

The following is a descriptive analysis of some of the students' work and the interactions that took place chronologically at various tables as the students attempted to solve the Bananas problem. This problem was a pictorial depiction of a system of two equations with three unknowns in which, the reader was asked to re-define one of the variables in terms of one of the other two variables. Specifically, the reader had to figure out how many bananas weighed as much as one apple.

Table 1

Three girls- Jasmine, Dina, and Monae- were seated at Table 1. After the introduction of the Bananas task (see Figure 4.15 below, a copy of Figure 4.1), the students began working on the problem. Even though they had been told to work within their groups, the class was very quiet as the students worked independently and there was limited interaction between them. In Event 1 (see Appendix E1), the teacher encouraged students to discuss the problem within their groups and immediately afterwards, the girls at Table 1 were caught on tape having a conversation about the answer to the problem:

Line # **Transcription**

T2: When I said prepare for sharing, what I, what I meant by that was within your group, try to achieve a consensus, meaning see if you can come to some agreement about what the answer is . Alright, so that you are having a little discussion, if there are different responses within the group, discuss it and see how you might come to an agreement.
[The teacher moves to Table 5 to answer a question raised by a student and the camera focuses on Table 1, where Jasmine is explaining her solution to her partners, Dina and Monae]

Jasmine: …..apples….[not clear].

Monae: I know, apple is three bananas. Three plus two is five.

Jasmine: Exactly! This is how I got my answer because three plus two is equal five!

Monae: There is already two bananas there. [pointing to the second scales]

Dina: Yeah. [pointing to the pictures]

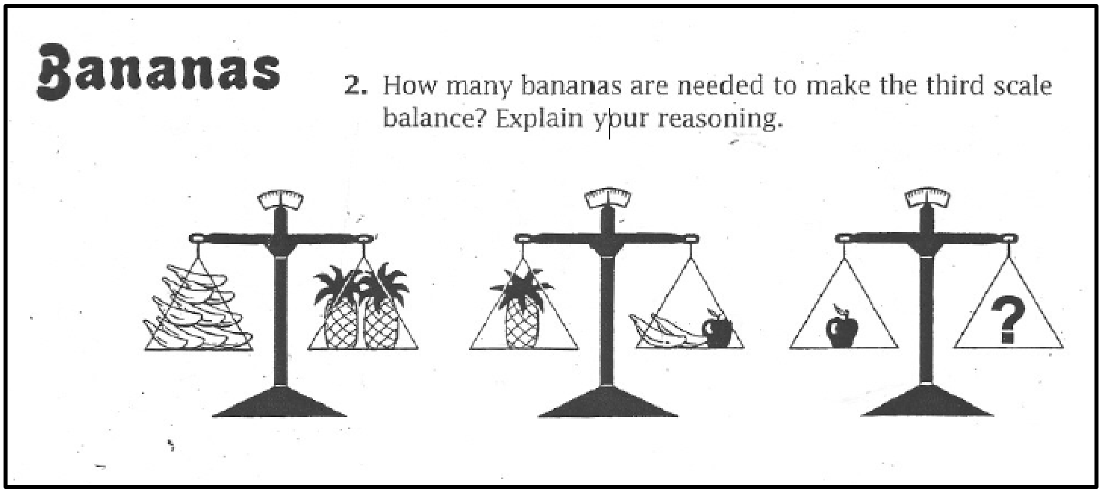
Jasmine: So the apple….[not clear]

Dina: That is three bananas [pointing to the third scale] and that is two [pointing to the second scale.

Jasmine: Oh, I get it now. Oh my god [pressing her palm to her forehead]!

As soon as the teacher clarified his expectations for the students to compare and discuss their solutions [lines 1-5], there was a noticeable increase in the noise level around the room as the students began talking to each other about the problem. Jasmine, who had been working independently up to this point, started discussing her answer with Monae and Dina [lines 9-20]. All three girls had essentially used the reasoning seen in Solution Path 1B (see section 4.2.3.1) in which, based on the first scale (see Figure 4.15 below, a copy of Figure 4.1), one could conclude that a pineapple weighed as much as five bananas and therefore determine that replacing the apple with three bananas in the second picture, would balance the scale. However, Dina and Monae had correctly concluded that three bananas would balance the third scale, whereas Jasmine had thought it would be five bananas. Once Jasmine listened to Dina and Monae argue why the correct answer was three and not five bananas [lines 10, 13-15, 17 and 18], she was quick to acknowledge her error [lines 19 and 20] and her body language (pressing her palm to her forehead) was perhaps indicative of her frustration with her own careless oversight.

Figure 4. 15: The Bananas Problem



Prior to this lesson Jasmine, an outspoken and spirited girl, had always sat by herself in one corner of the classroom and had refused to work in partnership with any other student, arguing that she could work best by herself. As T2 reported during the following debriefing session, Jasmine had been reluctant to work with Monae and Dina during this lesson and had made it clear that her eventual agreement to join the group was a one-shot-deal and for this specific lesson only. However, the three girls worked very well together, not only on the Bananas problem, as showcased during Event 1, but also throughout the lesson and their collaborative attitude and effort was acknowledged by the teachers during the debriefing session afterwards. In fact R2 jokingly suggested that the teacher show Jasmine the video clips of her collaborative work with Monae and Dina, as evidence that she was indeed capable of working well within a group.

There is no evidence that this experience turned out to be a watershed moment for Jasmine to adjust her attitude toward cooperative work with her peers. However, it is clear that the teacher's persuasive insistence on Jasmine joining a group, whether she enjoyed it or not, afforded her the opportunity to experience the dynamics involved in collaborative work.

About 15 minutes into the lesson, the teacher asked the three girls at Table 6 to present their findings to the Bananas problem. The girls, like so many others in this class, were reluctant to speak up and asked whether the teacher would move on to another group. T2 made it clear that rather than skipping them, he would get back to them at a later time. Two groups of boys (at Tables 4 and 2), agreed to share their findings. Both groups had used a strategy similar to the one used by Dina, Monae and Jasmine at Table 1 (Solution Path 1B, section 4.2.3.1). The teacher then moved on to Table 1 and asked Jasmine, the group's spokesperson to report. She stated that they had used the same reasoning to arrive at the same answer that the other two groups had previously reported and despite the teacher's request, she refused to elaborate any further. Two other groups (Tables 3 and 5) used the same excuse and got away with not having to share their findings with the class.

However, as it was noted by one of the observers during the debriefing session that followed, the three students at Table 5 had in fact the wrong answer of two bananas written on their papers, but after hearing four groups of students report their answer to be three bananas, they falsely claimed their answer to be the same as others.

Due to absence of the students' written work samples, the data for this study does not include the documented student responses from this class. Therefore, it is not possible to ascertain whether or not the students at Table 3 had in fact used the Solution Path 1B.

Table 6

The last presentation on the Bananas problem came from Jazmine, Tateanna, and Sakeena at Table 6, who had been reluctant to speak out the first time they were called upon and the teacher, true to his words, went back to them and gave them the opportunity to share their answer with the class. In spite of the fact that up until that point every presenter had claimed that an apple equaled three bananas, these girls disagreed with everyone and their spokesperson, Jazmine, asserted her belief that the correct answer was two bananas (see Event 2, Appendix E2):

Line # **Transcription**

Jazmine: I think that, I think that it wouldn't be three. I think it would be two. Because if one apple and two bananas equal one, equals one pineapple (on the second scale), then it must be equivalent, one apple equals two bananas.
(Kevin from Table 4 is heard saying: I object.)

T2: Okay, go back through it one more time [Jazmine sits down] one more time Jazmine.
Jazmine: I don't think it would be three because if two bananas and one apple equals one pineapple, then one apple equals two bananas. Because that is like, if you have, like if you have a banana and the apple [holding two palms up] that is going to be the same size if you hold them in your hands. That is why, that's why you are holding them in your hands. That's what the scale is like. So, that's why I think it's two.

Jazmine's reasoned that since on the second scale, one pineapple equaled two bananas and one apple, then two bananas must be equal to an apple [lines 1-4]. This was not a logical conclusion and she tried to strengthen her argument, perhaps upon hearing Kevin say: "I object", by saying that the bananas and the apple held in your hands would be the same size (weight) [lines 8-14].

One of the observers commented during the debriefing: "The girls at Table 6 had the courage to disagree with all the answers they had heard so far and claim that the answer was two bananas. I really admired them for that" (see Teachers' Refections, Appendix H2).

Although incorrect, Jazmine's forceful assertion that an apple equaled two bananas had an interesting consequence. After hearing Jazmine, Oscar from Table 5 (who had falsely claimed their answer to be three bananas) was seen on the videotape telling the teacher that he agreed with Jazmine that the correct answer was two bananas. It is conceivable that hearing Jazmine defend her answer, gave Oscar the confidence to think that perhaps he had been right after all! Unfortunately the teacher did not pursue it any further with Oscar and therefore it cannot be established what Oscar's reasoning was and why he agreed with Jazmine.

<u>Table 4</u>

Three boys, Kevin, Caliph, and Cory were seated at Table 6. The boys seemed to work well together and although they completed all the tasks, they often joked and played around loudly. Kevin was the first person to announce his disagreement with Jazmine's assertion that an apple equaled two bananas. In Event 2 (continued), Kevin offered an explanation as to why Jazmine's claim could not be correct:

Line #	**Transcription**
15	**T2:** Okay Kevin, do you have a response?
16 17	**Kevin:** No, no. I thought she said weight. No. I don't agree, but [Kevin shrugging his shoulders].
18	**T2:** Okay and do you, do you have [Kevin talking and laughing at his

table], let's assume that we were debating this issue, what would be your response kevin?

Kevin: Huh?

T2: let's assume, let's assume that we were debating this issue and you're trying to convince Jazmine that your solution is correct. What would you say in response?

Kevin: [sigh, pause, laugh] I would say that I think that it is three, three bananas because, like Caliph said, if you got 10 bananas and two pineapples. Half, half of two pine, half of two is one and half of 10 is five. So if you take [laughs in acknowledgment of something Caliph has just said to him], if you take one pineapple and put two bananas (already there on the second scale) and two bananas (to replace the apple, as claimed by Jazmine), that is four bananas. We don't have eight bananas on the first scale, we have 10. So, you don't have, it will have to be three bananas. One apple equals three bananas because, uhm, how to put this?
Cory and Jasmine put their hands up.

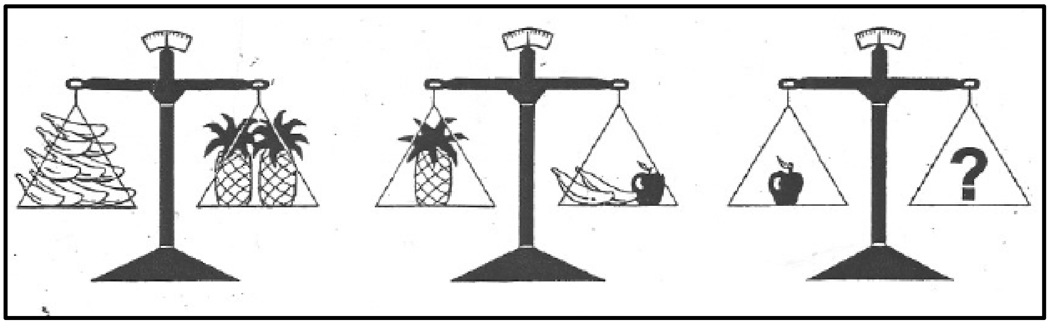
T2: Wait, wait. Hold your thoughts. Let him finish.

Kevin: So, uhm, it had to be three, uhm. Yeah, cause basically five is half of 10 and one is half of two, so if you got, half and half it is.

While Kevin disagreed with Jazmine's argument, he was hesitant to put forth a counter argument [line 16]. By folding his arms and shrugging his shoulders, he displayed his lack of interest in presenting a reason as to why Jazmine was wrong. The teacher, however, persuaded him to offer an explanation [lines 18-20 and 22-24].

Kevin's argument, although not well-articulated, was powerful. He was essentially saying that Jazmine's assertion (one apple weighs as much as two bananas) would mean that on the second scale, four bananas would balance one pineapple (see figure 4.16 below). But since the first scale implied that one pineapple weighed as much as half the number of bananas, for Jazmine's claim to hold true, there would need to be eight bananas on the first scale.

Figure 4. 16: The Scales in the Bananas Problem



He reasoned by contradiction that since there were 10 bananas, not eight, an apple could not possibly be the same as two bananas in weight. Kevin's explanation demonstrated his understanding of the interdependence of the three scales as part of a system that represented the relationship between the weights of the three fruits.

After hearing Kevin's counter argument, Cory, one of Kevin's partners at Table 4, raised his hand to make the following comments (Event 2, continued):

Line #	Transcription
39	**Cory:** But, but bananas could be one and a half.
40	**T2:** Bananas could be one and a half what?
41	**Cory:** Pounds.
42 43	**T2:** Okay, so now we are introducing the notion of pounds. Okay, why is that significant?
44 45 46	**Cory:** Because this could be one and a half, one and a half [pointing to each of the two bananas on the second scale] and that equals three. So the apple could be two pounds.
47	**T2:** So are you in agreement with Kevin's rational or Jazmine L's?
48	**Cory:** kevin's. I am just saying though. [students laughing]
49	**T2:** Mr. Pedrick (R2)?

R2: Mr.[unclear] said that they're one and a half pound each. What's the last name?

T2: That's Cory.

R2: Cory, if they are a pound and a half each, how much would the (ten) bananas weigh?

Cory: one and a half?

T3: No, the total pounds.
[the class goes silent for a few seconds and the students look very thoughtful]

Kevin: I have to think about it, have to think about it.

T2: Think of it perhaps the decimal representation of one and a half.

T3: It's one and a half, so you have 10 (bananas) [addressing Cory].

Although Cory was in agreement with Kevin [lines 47 and 48] that one apple equaled three bananas, he appeared to play the devil's advocate by presenting a scenario in which, Jazmine could be correct in asserting that one apple equaled, not three, but two bananas. Cory simply suggested that each banana (on the second scale) could be one and a half [line 39], and then introduced pound as the unit of measurement for weighting the bananas [line 41]. Up until that point no one in their explanations had attached any specific weight to any of the fruits and much like the situation in lesson implementation #1 (section 4.3.3), it is quite possible that the teacher's clarifying question: "Bananas could be one and a half what?", prompted Cory to spontaneously add the word pound to his vocabulary. Cory further argued that on the second scale, if the bananas were to be 1.5 pounds each, then two bananas would weigh 3 pounds and the apple would have to weigh 2 pounds in order to balance the pineapple on the left pan of the scale.

Cory seemed to have overlooked the interdependency of the scales, focusing only on the second picture. By assigning 1.5 pounds to each banana he was able to balance the scale with 5 pounds in each pan. However he had lost sight of the fact that the pineapple in the second picture weighed as much as five bananas (pounds), if and only if the first picture was true. Hoping to bring Cory's attention to this oversight (as reported by R2 during the debriefing), R2, who was seated at the back of the room, asked the question: "Cory, if they are a pound and a half each, how much would the (ten) bananas weigh?" [lines 53 and 54]. The class went silent for a few seconds while some students tried to figure out the answer to this question. The teacher offered a hint by asking the students to consider the decimal representation of one and a half [line 60]. However, no one could come up with the answer (no calculators at the tables) and about 45 seconds after the question had been asked, the teacher shifted the focus from this question and moved on to another group of students for their presentation.

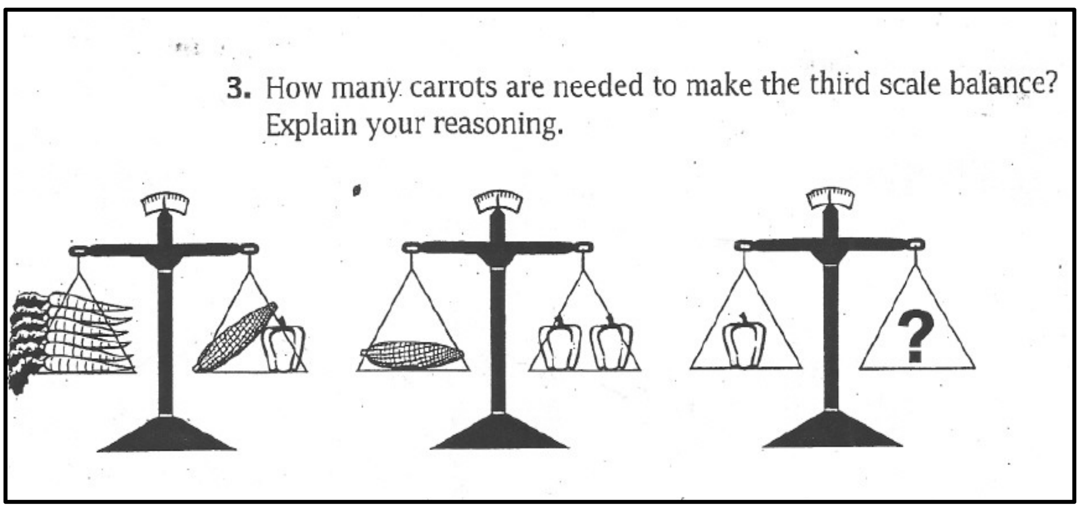
In an effort to move the lesson along, the teacher was not able to afford the students the opportunity to work through this important question, which potentially could have made Cory realize the flaw in his argument and would have been an opportunity to emphasize the interdependency of the pictures as one system of equations. Additionally, it is unclear whether Oscar together with Jazmine and her partners were ever convinced that the correct answer to this problem was three bananas.

The second task introduced to the students was the Carrots problem. Similar to the Bananas problem, this task was a pictorial depiction of a system of two equations with three unknowns and the reader was asked to re-define one of the variables in terms of one of the other two variables. Specifically, the reader was asked to re-define the weight of a pepper in terms of the weight of carrots. The students tackled the Carrots problem in different ways, highlighted in Events 3, 4, and 5 that follow. The students worked on this problem for approximately 10 minutes before the teacher asked the first group to present their solution to the class.

Table 1

The three girls- Jasmine, Dina, and Monae -worked on the Carrots problem (see Figure 4.17 below, a copy of Figure 4.2) together and were the second group to present their solution to the class. Their strategy was similar to the one used and presented by Joel from the preceding group.

Figure 4. 17: The Carrots Problem



Attempting to solve this problem during the planning phase of the lesson study, one of the strategies used by the teachers, and identified in this study as Solution Path 2C, was as follows:

- Identify the 2:1 relationship shown on the second scale.

- Based on this realization, consider the first scale and pick two numbers that have a 2 to 1 relationship, representing a corn and a pepper respectively and add up to 6 (4 + 2 = 6).
- Therefore a pepper weighs as much as 2 carrots.

With the exception of two groups of students, whose strategies will be discussed later, the majority of the students, including the girls at Table 1, had used a strategy similar to Solution Path 2C in order to arrive at their answer. In Event 3 (see Appendix E3), Dina shared her groups answer to this problem with the class:

Line #	**Transcription**
1	**Dina:** I think that a pepper is equal to two carrots, because in scale 1
2	we can see that there are six carrots and a corn and a pepper. We could,
3	there are six carrots, right? We can say that a corn was three carrots and
4	a pepper was equal to three. But, in scale 2 we can see that a corn is
5	equal to two peppers. So a corn should weigh four pounds and a
6	pepper, two pounds. Because two plus two equals four (demonstrating
7	that the numbers work in the second picture). So a pepper is equal to
8	two carrots.
9	**T2:** Okay.

Dina's explanation reflected her understanding that the pictures in this problem should not be viewed and interpreted independent of one another. She pointed out that by examining the first scale only, one could infer that perhaps the pepper and the corn were equal in weight, each weighing as much as three carrots. Then she rejected the validity of this inference by referencing the second picture, which clearly negated the idea that the corn and the pepper were of equal weight [lines 3-5].

<u>Table 5</u>

Immediately following Dina's presentation, another group of students, Oscar, Maledy, and Jevon at Table 5 reported their findings. They all agreed that based on the first scale, a pepper equaled three carrots and supported their answer by saying that 6 divided by 2 was equal to 3 (number of carrots divided by the number of items on the right- hand-side was equal to three). They never disputed Dina and Joel's reasoning and made no reference to the fact that their answer differed from what had already been presented. During the class presentations, the class was often noisy and some students appeared to be inattentive. Several times the teacher had to address this issue and insist that the students remain quiet and pay attention to the presenters. Given this fact, it is possible that perhaps Oscar, Maledy, and Jevon never paid much attention to the prior presentations. But if they did, there was no indication that they had critically listened to the previous arguments and whether they were or were not convinced by them.

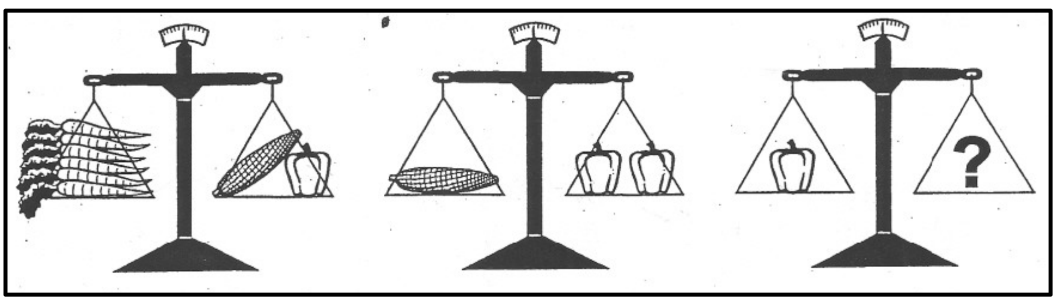
After listening to the students at Table 5, declaring that a pepper was equal to three carrots, the teacher asked the class if anyone had a comment to make. Joel raised his hand and asked: If the pepper and the corn are equal to three carrots each, how come the second scale shows that two peppers are equal to one corn? The teacher paraphrased Joel's question by stating that the second scale conflicted with the assertion made by the group at Table 5 and asked Oscar, Maledy, and Jevon to reconsider their reasoning and answer. The teacher then asked another group to present and since there were no more discussions with the students at Table 5 about this particular problem, it cannot be determined if they ever understood why two carrots, not three, would balance the pepper on the last scale.

Table 4

Kevin, Caliph, and Cory at Table 6 were the only group of students who used a different strategy to arrive at their answer to the Carrots problem (see Figure 4.18 below). One of the strategies used by the teachers during the planning phase of the lesson study and identified in this study as Solution Path 2B (see section 4.2.3.2) was as follows:

- Consider the relationship shown on the second scale.
- On the first scale, replace the corn with 2 peppers.
- Now you have <u>6 carrots</u> Vs. <u>3 peppers</u>.
- Divide each side by 3 to get: <u>2 carrots</u> Vs. <u>1 pepper</u>.
- Therefore a pepper weighs as much as 2 carrots.

Figure 4. 18: The Scales in the Carrots Problem



In Event 4 (see Appendix E4) Kevin shared his solution to the Carrots problem with the class:

Line #	**Transcription**
1	**Kevin:** Alright. There are six carrots, there are six carrots and..[not
2	clear].. and a corn and a pepper equal six carrots. If a corn on the cub
3	equals two peppers then you add the first pepper that you got with the
4	corn on the cub, you get two peppers [Kevin and his partners laughing].
5	So, so, yeah, one pepper had to equal two carrots and three peppers
6	equal six carrots.
7	**T2:** Okay, now I want you to go back to that middle piece there where
8	it sounded like you were referring to the second scale
9	**Kevin:** Oh, I said…

T2: You said something about three peppers.

Kevin: Oh, the, oh…

T2: Cause up there I don’t see anywhere where there is three peppers. Just didn’t know where that notion came from.

Kevin: If you add the first pepper that you got [students laughing]

T2: Hush. Pay attention.

Kevin: If you add the first pepper that you got with the corn on the cub, corn on the cub, with the two peppers, you get three peppers equal six carrots.

T2: Aha, so since three peppers equal six carrots you are drawing what conclusion?

Kevin: That one pepper equals two carrots.

T2: Okay, or it’s the same in terms of weight.

Kevin: Yeah.

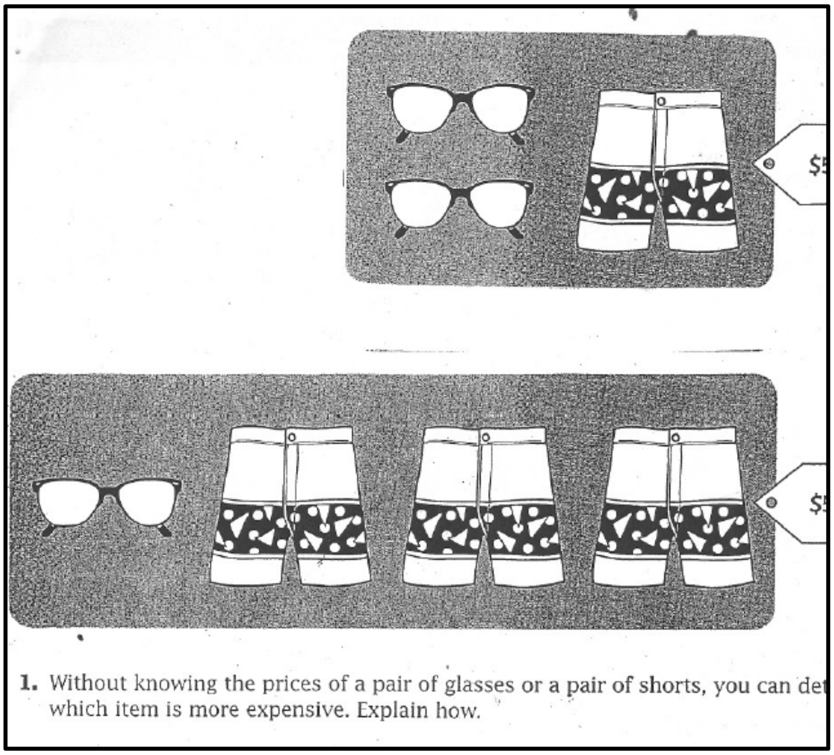
T2: Let’s make that distinction.

Although Kevin had the correct answer, his explanation was unclear. He seemed distracted by a private joke at their table, which made him laugh and giggle several times during his presentation. His first attempt at explaining his reasoning was ambiguous [lines 1-6] and the teacher had to ask him several clarifying questions and push for further explanations [lines 7- 8, 10, 12, 19, and 22]. Kevin’s ill-articulated explanation was basically similar to Solution Path 2B described above. He noticed the 2 to 1 ratio between the number of corn and peppers on the second scale and used that information to substitute two peppers for the corn on the first scale. Although he did not use the word "substitute" or "replace" in his explanation, he essentially used the concept to balance six carrots with three peppers and concluded that a pepper weighed as much as two carrots.

About 50 minutes into the lesson the teacher introduced the third task, the Shorts and Glasses problem, on the overhead projector. During the previous lesson implementation the majority of students had used the guess and check method to figure out the actual prices before determining the more expensive item. During the debriefing session that had followed, the teachers had decided to modify the Shorts and Glasses problem with the hope that the students would perhaps use a different strategy to determine the more expensive item. They had agreed that during the next lesson implementation and to introduce the first part of the problem, they would first put the question up on the overhead projector, cover the $50 price tags, and just tell the students that the two tags showed the same amount.

The modified version of the Short and Glasses problem, as viewed by the students when the teacher presented this problem on the overhead, is shown in Figure 4.19 below.

Figure 4. 19: The Modified Version of the Shorts and Glasses Problem



The students began working on this problem as the camera moved around the room, capturing the conversations within various groups. The teachers had speculated that this modified version of the Shorts and Glasses problem with the hidden price tags would eliminate the guess and check strategy as an option toward a solution. Nevertheless, a number of students used this strategy to determine the most expensive item. For example, the girls at Table 1 (Dina, Jasmine, and Monae) found out, by trial and error, that $2 for a pair of shorts and $4 for a pair of glasses complied with the premise that the total price tags in the two pictures were of equal amount and used this knowledge to declare the glasses as the more expensive item. Others like the girls at Table 6 (Sakeena, Tateanna, and Jazmine) decided to pick their own prices of $12.99 and $2.99 for the shorts and glasses respectively. Although it was brought to their attention by one of the observers, these girls chose to ignore the fact that their assigned prices did not add up to the same totals in the two pictures. Based on their personal experiences, these prices made sense to them and they concluded that the shorts were the more expensive item.

Table 4

About four minutes after the picture was put up, the camera captured Kevin talking to his partners, Cory and Caliph, about this problem (see Event 5, Appendix E5):

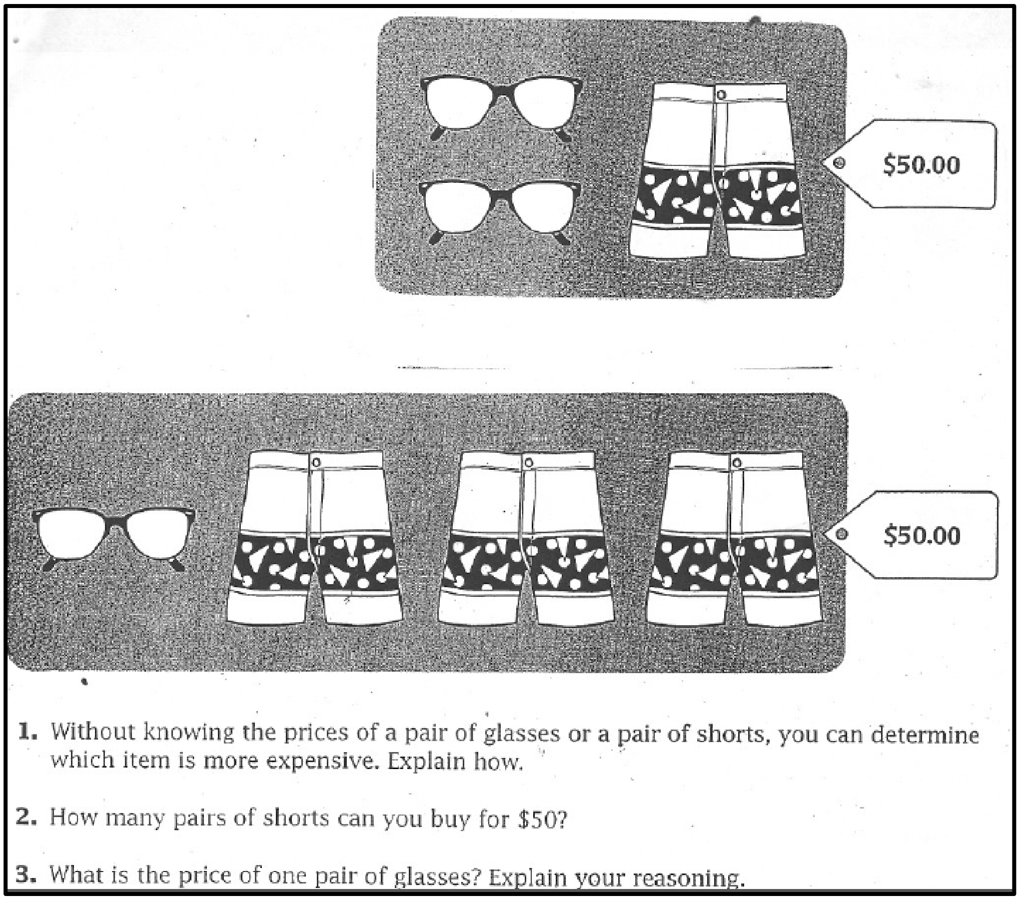
Line #	**Transcription**
1	**Kevin:** Why waste your money on two pairs of glasses and one pair of
2	shorts when the glasses are going to take up most of your money. So,
3	just get two pairs of shorts and one pair of glasses. It is cheaper.

Although Kevin did not explicitly state that the glasses were more expensive than the shorts, his statement above implied that he believed so. He did not refer to any specific amount of money but recognized that in the top picture, if one pair of glasses was replaced by a pair of shorts, the total amount would decrease [line 3]. This would happen only if the glasses were more expensive than the shorts.

Unfortunately, since there is no record of kevin’s written response and due to lack of time, his group never got the chance to share their findings with the class, it cannot be determined if Kevin’s belief that the glasses were more expensive was centered on personal experience, or whether he used the two pictures in the problem to arrive at his conclusion.

After the students had the chance to determine the more expensive item, without knowing the amount of total price tags in each picture, a copy of the original version of the Short and Glasses problem (see Figure 4.20, a copy of Figure 4.3), revealing the $50 price tags, was distributed among the students and the students were asked to record their answers to all the three question.

Figure 4. 20: The Shorts and Glasses Problem



<u>Table 1</u>

Prior to knowing the actual amount written on the price tags in the pictures, the girls at this table, Dina, Jasmine, and Monae, had determined that $2 for a pair of shorts and $4 for a pair of glasses satisfied the premise that the total prices were the same in the two pictures. Once the dollar value of the price tags was revealed, the girls had to figure out new prices for the shorts and glasses that added up to $50 in each picture.

During the debriefing that followed this lesson, the teachers talked about the students' general inability to recognize that the glasses were twice as expensive as the shorts and R1 stated: "I don't believe that they (students) were able to see two shorts for one pair of glasses relationship under any circumstances. And so if you are thinking algebraically, that is really what you are trying to get at" (see Teachers' Reflections, Appendix H2).

However, Event 6 (see Appendix E6) could be used as evidence that perhaps the girls at Table 1 had noticed the 1 to 2 relationship between the prices of shorts and glasses. In this episode the researcher R1 was engaged in a conversation with Jasmine at their table:

Line #	Transcription
1 2	**R1:** How much did you get for the total amount here? [pointing to the top picture and referring to the time when the prices were $2 and $4]
3	**Jasmine:** Ten.
4 5	**R1:** Okay. But now, instead of 10 the total is 50. Is it going to make the items cost more or less?
6	**Jasmine:** More.
7 8 9	**R1:** It's going to cost more. So now you have different prices. So the 2 and a 4 is not going to work. You have to figure out two new prices. But I think, from what Dina said, you still think the glasses are more.
10	**Jasmine:** Eight and 16?
11 12 13	**R1:** Does that work? [The camera moves to the next table, but Jasmine's response can be heard.]
14 15	**Jasmine:** Oh, 20 and 10? Twenty, 20, 40,..[not clear]..Yeah. So yes! [The camera moves back to Table 1.]

R1: Does that work for the bottom picture too, Jasmine?

Jasmine: So, this is 10 [pointing to the shorts in the top picture and then pointing to each item in the bottom picture and silently adding up the prices.] Yes it does. Hold on, hold on. Yeah, it does. Ten, 20, 30, and 30 plus 20 is 50. I am right, glasses cost more.

While trying to come up with a set of prices for shorts and glasses that made a total of \$50 in each picture, Jasmine guessed and checked two sets of prices: {\$8-\$16} and {\$10-\$20} [lines 10 and 14]. Although Jasmine never articulated the fact that the glasses were twice as expensive as the shorts, her choice of these two sets of numbers - {\$8-\$16} and {\$10-\$20}- reflected that perhaps, barring a coincidence, she was aware of the 1 to 2 relationship that existed between the two prices.

One of the strategies used by the teachers during the planning phase and identified in this study as Solution Path 3D (see section 4.2.3.3) was as follows:

- Cross out one pair of glasses and one pair of shorts in both pictures.
- Note that the \$50 price tags will reduce by the same amount in both pictures and therefore will remain equal to one another.
- The new pictures show that one pair of glasses is the same value as two pairs of shorts.
- Therefore glasses are more expensive than the shorts. In fact the glasses are twice as expensive as the shorts.
- In the second picture, the glasses can be traded in for two pairs of shorts.
- Therefore you can get five pairs of shorts for \$50.
- This implies that the shorts are \$10 a pair.
- Therefore the glasses cost \$20 a pair.

The teachers had hoped that after realizing the 1 to 2 relationship between the prices of shorts and glasses, the students would (as done in Solution Path 3D above) use a more sophisticated strategy, such as substitution, to figure out the prices. The teachers had not anticipated that some students might figure out that the glasses were twice as expensive as the shorts, but revert back to the guess and check strategy and use the knowledge of 1 to 2 relationship to make more educated guesses (picking two numbers, where one is twice as large as the other, as in 8 and 16).

Table 5

The students at this table, Maledy, Oscar, and Jevon, were the first group of students to share their solution to the Short and Glasses with the class. In Event 7 (see Appendix E7), Jevon and Oscar explained why they believed the glasses were more expensive than the shorts:

Line # **Transcription**

Jevon: I think the glasses are more expensive, I think the glasses are more expensive because they are breakable and they are more harder to make.

T2: Blacktop voice! I am all the way here by this tree and you are over there at the basketball court.

Oscar: Want me say it? Want me say it [asking Jevon]

Oscar: he said that glasses are more expensive one because it, it can break and it’s harder to make and they help people see.

T2: So, we are determining that which item is more expensive?

Oscar: Glasses.

T2: Now, we are giving that based on what we know about glasses and what we see in the picture?

Oscar: Yeah.

T2: Then I missed that part. Tell me one more time.
[Oscar pauses]

T2: Which is more expensive and why?

Oscar: The glasses.

T2: Because?

Oscar: Because it is more, it's harder to make and it helps people see and they can break.

The students at this table claimed that the glasses were the more expensive item and justified their assertion by stating the functional and physical characteristics of glasses: they help you see; they are fragile; and they are harder to make [lines 1-3, 7 and 8]. The students' reasoning was solely based on their life experiences and did not reflect any understanding of the intended mathematical ideas associated with the problem. This kind of reasoning resonated with the type of responses that the Shorts and Glasses problem had elicited from some of the fifth grade students during the previous lesson implementation.

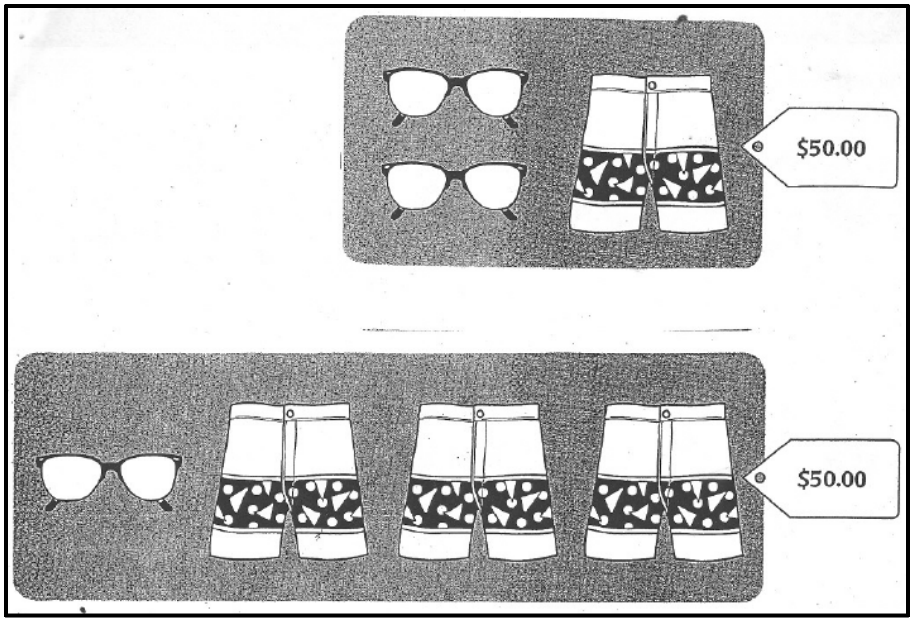
During the debriefing session that followed there were discussions on how some students, particularly those from an urban setting, tended to get lost in the context and were less able to pull the mathematics out of a problem. Opinions were divided on the merits of this type of student responses. T1 expressed an affinity for Jevon and Oscar's response, pointing out its authenticity and validity. She felt that any kind of assessment should take into account students' rich contextual insight into the given problems. T4 argued that Oscar's response, although valid, was not mathematically grounded: "There *wasn't math in that (response) because when he (teacher) asked them for it, they didn't give it to him. They were unable to give that." (See Teachers' Reflections*, *Appendix H2)*

Although no one contested T4's assertion that there was no mathematics associated with Oscar's answer, the video data did not support her viewpoint. In Event 7 (continued), the teacher asked Oscar to base his answer only on what he could see in the picture:

Line #	**Transcription**
21 22	**T2:** Now, see the question they asked you was: Can you determine that based on what is in the picture?
23	**Oscar:** Oh, yeah.
24 25	**T2:** Mathematically, based on what is in the picture how do we know that glasses…
26 27	**Oscar:** Because in part A (first picture) there are only three things, that is two glasses and one pair of shorts that cost $50.
28	**T2:** Aha.
29 30	**Oscar:** And in part B (second picture), there are three pairs of shorts and one pair of glasses and it costs the same.
31	**T2:** So that implies that glasses are …..
32	**Oscar:** More expensive.

Oscar's explanation [lines 26 and 27, 29 and 30], although fundamentally sound, was inadequately expressed and needed further elaboration in order to make it a convincing argument that the glasses were more expensive than the shorts (see figure 4.21).

Figure 4. 21: The Pictures in Shorts and Glasses Problem



Oscar’s reasoning was essentially similar to what Jade, the fifth grader from the previous lesson implementation, had presented: “The item that we think is more expensive is the sunglasses. We knew they were more expensive because the more glasses you have, the less items you would need in each box to equal the $50.” Much like what happened after Jade’s presentation, the teacher did not ask Oscar to elaborate any further and quickly moved on to another group of students.

As noted before, during the planning phase of the lesson study the teachers had not identified Oscar’s argument as a possible way of determining the most expensive item in this problem and therefore it is plausible to assume that perhaps the teacher was caught off guard with Oscar’s reasoning and decided to move on.

4.5 Lesson Implementation and Debriefing #3

The following sections provide an overview of the third lesson implementation and the debriefing session. A more comprehensive description of these sessions is also provided which detail the observed occurrences of the video data in approximately 5-minute time intervals (see Appendices B3 and C3). The abbreviations R1 and R2 refer to the two university researchers while T1 through T4 identify the teachers in this study, who facilitated the four implementation sessions respectively. O1 through O7 identify the teachers who, as graduate students, participated in the lesson study and attended all or some of the implementation sessions as observers.

4.5.1 Lesson implementation #3: Overview

This session was approximately 67 minutes long and was facilitated by T3, an eighth grade teacher. There were 12 students in the classroom. In addition to the classroom teacher, T3, there were 12 other adults, R1, R2, O1, O2, O3, O4, O5, O6, O7, T1, T2, and T4, present in the room as observers. An additional person was in charge of taping the session. The school principal and the district's mathematics coach also attended this session. The grouping of the students was not planned specifically for this lesson and was based on where they normally sat in this class. Students' desks were put together to form five groups of paired students. Two boys sat separately and by themselves. However, following the teacher's direction a few minutes into the lesson, one of the two boys joined the boys at Table 3 while the other opted to work by himself throughout the session.

The teacher had originally intended to do five activities with her students, but due to lack of time, she was not able to include the last activity, the Chickens problem.

Therefore the lesson consisted of four tasks (see Appendices A1-4) which were implemented in the following order: Activity 1- Carrots (approximately 15 minutes); Activity 2- Bananas (approximately 4 minutes); Activity 3 - Short and Glasses (approximately 8 minutes); and Activity 4 – The Tug-of-War (approximately 31 Minutes).

The class began with the teacher introducing the university faculty member, R1, who addressed the students and explained the purpose of the lesson and the presence of all the observers in the classroom. After the introductions were made the teacher handed out the first activity sheet and the mathematical work began.

From the beginning the teacher had asked the students to discuss and solve the problems with their partner, write down their explanation, and be prepared to convince her that their answer was correct. While the students were working on the problems the teacher circulated among the groups, interacting with the students and overseeing their progress. The observers remained seated or stood at the periphery of the classroom for the most part. From time to time some of them walked around and took notes and interacted with the students. Most of the interactions between the students and the observers occurred because the observers needed clarification on the students' thinking and would ask questions such as "How did you get that?" or "Why did you do this?"

Upon completing each task some of the students had the opportunity to share their solution strategies with the class. Many students volunteered to speak up and most of the presentations took place at the overhead projector in front of the room. A very short time was spent on Activity 2 as the students arrived at the correct answer very quickly and after two groups of students reported on their work, the teacher moved on to the next task. The students seemed very attentive and eager to work and remained on task throughout the lesson.

4.5.2 Debriefing #3: Overview

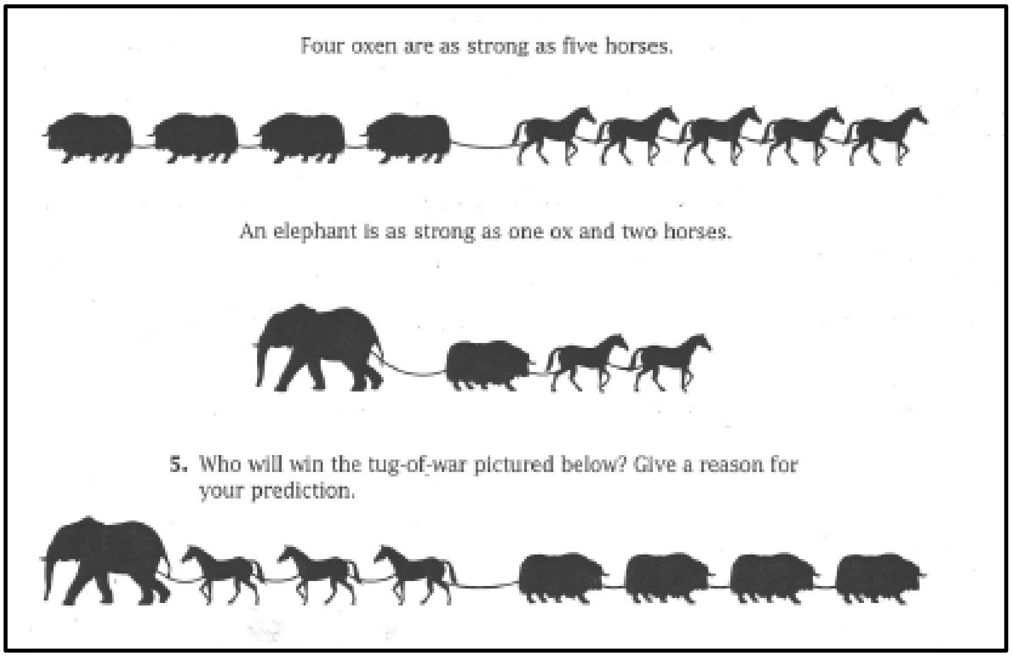
The debriefing took place in the classroom immediately following the lesson implementation and was approximately 30 minutes long. The school principal and the district's mathematics coach were also in attendance.

T3 told the group that she thoroughly enjoyed working with this group of eighth graders, who always worked hard and did their very best when faced with challenging mathematical problems. There were some discussions on the importance of addressing students' misconceptions. R1 told the group that when kids presented contradicting ideas, teachers in general, had an obligation to uncover their mistakes and address their misunderstandings. R1 also noted that during the previous two lesson implementations there were a few occasions in which a group of students had arrived at an incorrect answer and although their reasoning was questioned by either the teacher or their peers, not sufficient amount of time had been spent to determine whether these students had truly realized their error and were convinced by others.

One of the observers asked T3 why she deviated from the original lesson plan, which had the Chickens problem as the main activity. T3 explained that originally, the Tug-of-War problem was the warm up activity and she was planning on doing the Chickens problem as the main activity for the lesson. But after seeing the struggle with the balance problems in the 6th grade class, she had decided to include those problems in her lesson, which left no time for the Chicken problem. Also she had wanted to pave the way for the Chicken problem, which was rather difficult, by first exposing her students to the concept of equality through the balance problems, and then introducing inequality through the Tug-of-War problem.

The teachers mentioned some of the students' responses to the Shorts and Glasses problem. They acknowledged that even though some students had figured out, very quickly, that the glasses were twice as expensive as the shorts, they did not use that piece of information to solve the other parts of the problem. One of the observers referred to the Tug-of-War problem and the difficulty Dieshe had in convincing Jynita that an elephant and three horses would win the tug of war. He said he had noticed four teachers going to Jynita at various points trying to help her. He demonstrated the tactic he had used in order to change Jynita's mind about her answer. He had cut out the picture of the two horses and an ox, which equated to an elephant in the middle picture (see Figure 4.22 below, a copy of Figure 4.5). By placing this cutout on top of the elephant in the bottom picture, he had managed to convince Jynita that her initial answer was wrong. Everyone agreed that this was a fantastic idea and T1 and O2 believed this method could serve as a great modification for special education students.

Figure 4. 22: The Tug-of-War Problem



The teachers also discussed how Keith and Tyreal had independently worked on this problem, had used a similar strategy but had solved it differently. Tyreal had assigned the numbers 1 and 1 ¼ to each horse and ox respectively, whereas Keith had decided on the numbers 4 and 5 to represent the horse and the ox. There was much discussion on how each of the two boys had struggled, and yet persevered, before ultimately getting the correct answer.

4.5.3 Descriptive analysis: Lesson #3

The following is a descriptive analysis of the lesson implementation #3, with references to the *critical events* that occurred during the lesson (for a complete list of all the transcribed critical events, see Appendices F1-16). The selection of *critical events* involved careful deliberation of the research questions so that these events, taken individually, were snapshots offering glimpses into the lesson and collectively, the string of these snapshots contributed in shaping a narrative that provided the researcher an insight into the nature of the interactions that transpired in the classroom, and subsequently helped answer the research questions.

In the following descriptive analysis, the coded videotaped data was examined in conjunction with the students' work samples and, when appropriate, the teachers' reflections from the debriefing session. As a result, in this analysis, references to the teachers' reflections are at times intertwined with discussions on students' work and classroom occurrences. In order to better distinguish the classroom descriptions from the debriefing conversations, any discussion pertaining to the teachers' reflections will be italicized in the following analysis.

This lesson took place in an eighth grade classroom with 12 students. In addition to the classroom teacher, T3, there were 12 other adults, R1, R2, O1, O2, O3, O4, O5, O6, O7, T1, T2, and T4, present in the room as observers.

Approximately six minutes into the lesson the teacher handed out the first activity sheet, the Carrots problem (see Figure 4.23 below, a copy of Figure 4.2). This task was a pictorial depiction of a system of two equations with three unknowns and the reader was asked to re-define one of the variables in terms of one of the other two variables. Specifically, the reader was asked to re-define the weight of a pepper in terms of the weight of carrots by determining the number carrots that would balance a pepper on the scale. The following is a descriptive analysis of some of the students' work and the interactions that took place at various tables and around the classroom as the students attempted to solve this problem.

Within one minute after the Carrots problem was presented to the class, Kevin put his hand up and asked if he could share his answer with the class. T3 acknowledged the fact that, in general, the students were reluctant to explain their reasoning in writing but urged Kevin to write down his explanation first before sharing it verbally. While the students were working on the problem the teacher circulated among the groups, overseeing their progress. The following is a descriptive analysis of some of the occurrences at Table 4.

<u>Table 4</u>

Two girls, Jyanita and Dieshe, were seated together at this table. While most of the students were in the process of writing down their explanation to the Carrot problem (see Figure 4.23 below, a copy of Figure 4.2), these girls had not begun writing and seemed to be unsure as how to proceed. About six minutes after the introduction of the problem, the teacher approached their table and tried to help them get started (see Event 1, Appendix F1):

Line # **Transcription**

T3: Let me ask you a question. What do you believe they are asking?
What are you expected to do? What are the expectations?

Dieshe: [not clear]

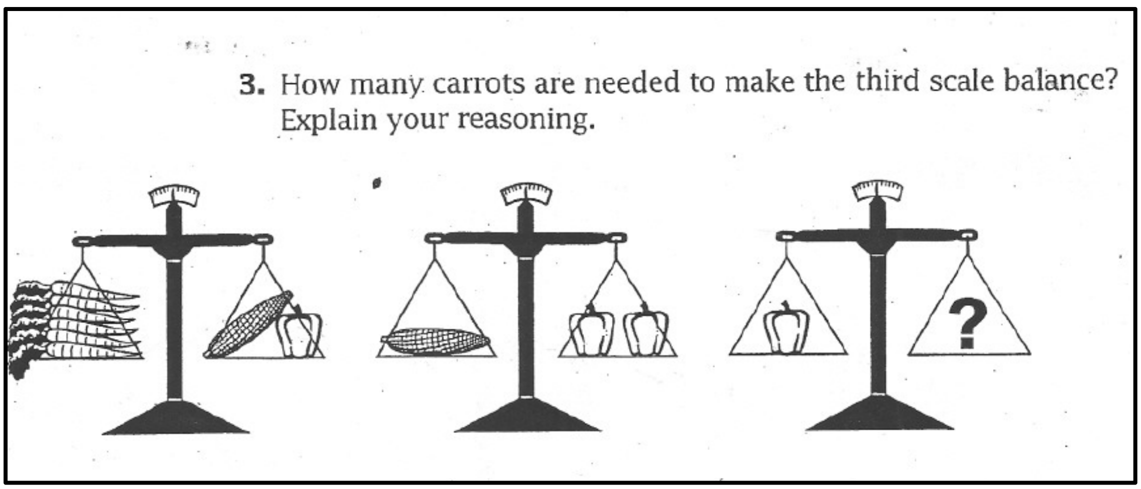
T3: Okay, what do you know about the carrots? Explain to me what
you know about the carrots. What is it that you know about the carrots?

Jynita: They are orange? [Dieshe giggles]

T3: They are orange, okay. But let’s talk about the scale.

The teacher tried to gauge the girls’ understanding of the problem [lines 1and 2] and once she was satisfied that they understood what the problem was asking them to do, she proceeded to question them about the first scale in particular: “What is it that you know about the carrots?” [lines 4 and 5]. Jynita replied that they were orange [line 6] and although is not clear if she meant to be facetious or not, her valid, yet irrelevant response prompted the girls to laugh and giggle. The teacher had hoped to elicit an answer regarding the relationship between the weight of six carrots and a corn and a pepper as represented by the first scale. However, her question (What is it that you know about the carrots?) was vague and lent itself to such a response as Jynita’s.

Figure 4. 23: The Carrots Problem



Whether it was a jokey response or Jynita truly could not make a comment about the carrots in the context of the balance scale, the teacher chose to assume the later and delved into a whole class discussion, reviewing students’ prior knowledge on pan balance and the concept of equality (Event 1 continued):

Line #	**Transcription**
8 9	**T3:** Boys and girls, let me just, remember back in second grade, when you were working with the pan balance?
10	**A boy:** Yeah.

T3: What happened with the pan balance?

Kevin: You had to make each side balance out.

T3: You had to make each side balance out. So explain that a little more in depth to me. You had to make each side balance out.

Tyreal: You had to reduce.

T3: You have to reduce. Okay I am hearing words like reduce.

Kevin: You have to reduce. You have to add to it.

T3: You have to add to it. So Mrs [T3] is going to write some of these words down.
[T3 erases the board. R2 offers to give T3 her a pen, but she says she has one and gets one out of a box on her desk]

T3: Okay, so we are working with the pan balance [writes Pan balance on the board]. We had to keep, and this is the word balance [underlines the word, balance]. We had to make it balance. What were some of the things we had to do? And you just have to go a little bit more in depth. You had to reduce, but what was it that I had to reduce?
[Hands go up]

T3: You said you had to …[pointing to Kevin]

Tyreal: From the side that is the heavier.

T3: Excuse me?

Tyreal: You had to reduce from the side that was heavier.

T3: Okay, so reduce from the heavier side. [writes it on the board]. Okay, what else? You had to add. Excuse me, what did you say? [addressing Keith]

Tyreal: You had to add to the side that was lighter.
[T3 writes it down on the board]

The teacher spent a short time (less than two minutes) reviewing some of the concepts associated with scales by asking questions and writing the elicited students' responses on the board [lines 8-36]. They discussed the idea of balance on a scale and how in order to achieve it, one had to either reduce from the heavier side or add to the lighter side. The teacher then asked the class: “What is it that you know about the first picture? What does that first picture tell you?” Jynita immediately raised her hand and replied: “You had to have a corn and a pepper in order to equal to all the carrots.”

Perhaps it was the re-phrasing of the question from “What is it that you know about the carrots?” to “What does that first picture tell you?” that led to the reasonable and satisfactory response from Jynita. It is also possible that the preceding class discussion on pan balance helped create a framework that guided Jynita’s answer.

The teacher then asked if anyone wanted to present their solution at the overhead. Several students volunteered and the teacher called on Tyreal.

Table 6

Two boys, Tyreal and Al, were seated next to each other. Although they worked together on the problems throughout the lesson, Tyreal seemed to take the lead and was the driving force behind any ideas they came up with. In Event 2 (see Appendix F2), Tyreal explains his solution to the Carrots problem on the overhead (see Figure 4.24 below):

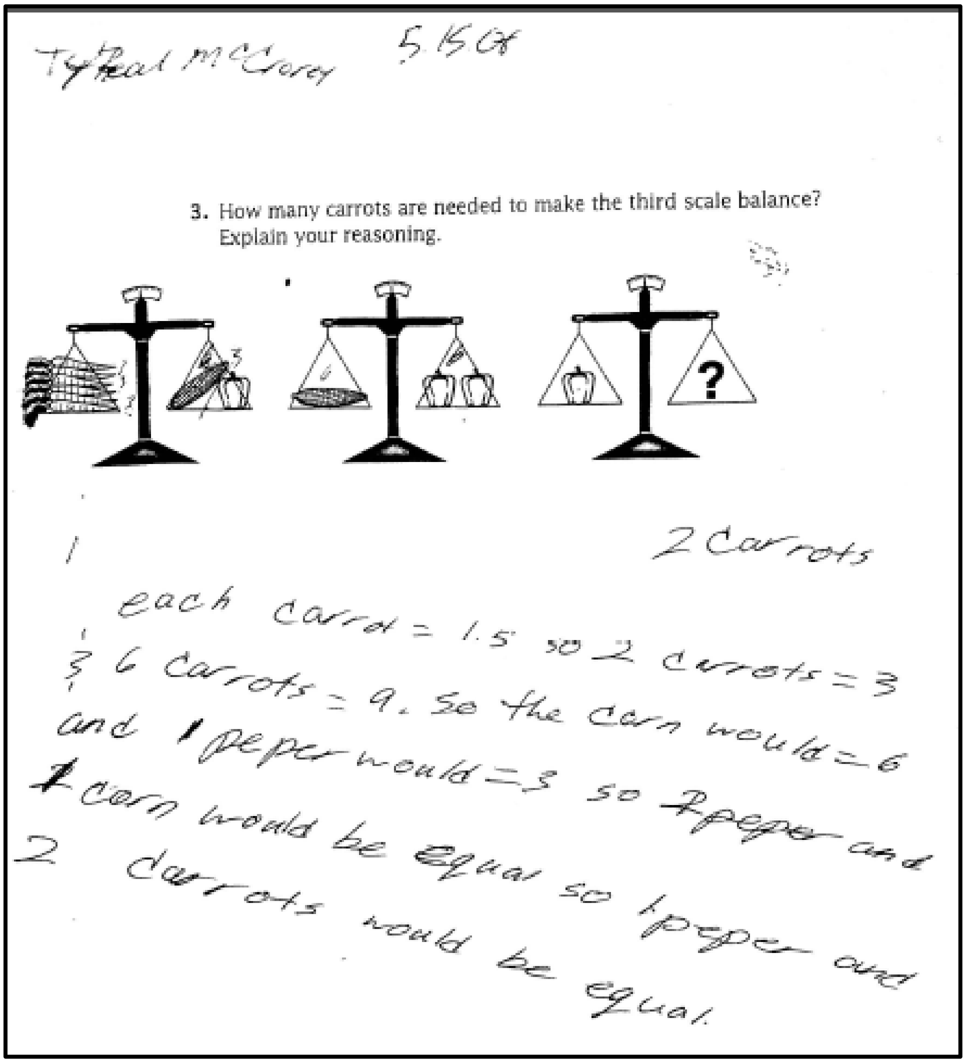
Line #	**Transcription**
1	**Tyreal:** Since there are six carrots, I tried numbers like one point five
2	or one and a half [writes 1.5 next to the carrots on the first scale] for
3	each carrot. And two carrots is three, like three.
4	**Several people:** Ahem.
5	**Tyreal:** So all of them would equal 9. So that means the corn will have
6	to be 6 and the pepper would equal 3 [writes 6 and 3 next to the corn

and the pepper respectively]. So two peppers would equal to 6 and the corn is 6 [writes 6 on both sides of the scale in the second picture].

Tyreal: And then… the pepper, there is one [pointing to the third picture]. Since there is only one pepper, this is equal to 3 [writes 3 next to pepper on the third scale] and two carrots [writes 2 carrots next to the question mark]

The teachers had not anticipated that students might attach a number to each carrot, let alone a decimal number. Since the teacher never questioned Tyreal’s choice in assigning 1.5 to each carrot, it is not clear why he chose to use a decimal number as opposed to a simpler number like 1. However, Tyreal managed to make it work. He demonstrated functional thinking when he mapped one (carrot) to the number 1.5, two (carrots) to the number 3, and extended this pattern to correlate six (carrots) to the number 9 [lines 1-5]. Tyreal’s written explanation is illustrated in Figure 4.24 below.

Figure 4. 24: Tyreal's Written Explanation of the Carrots Problem



One of the strategies used by the teachers and identified in this study as Solution Path 2C was as follows:

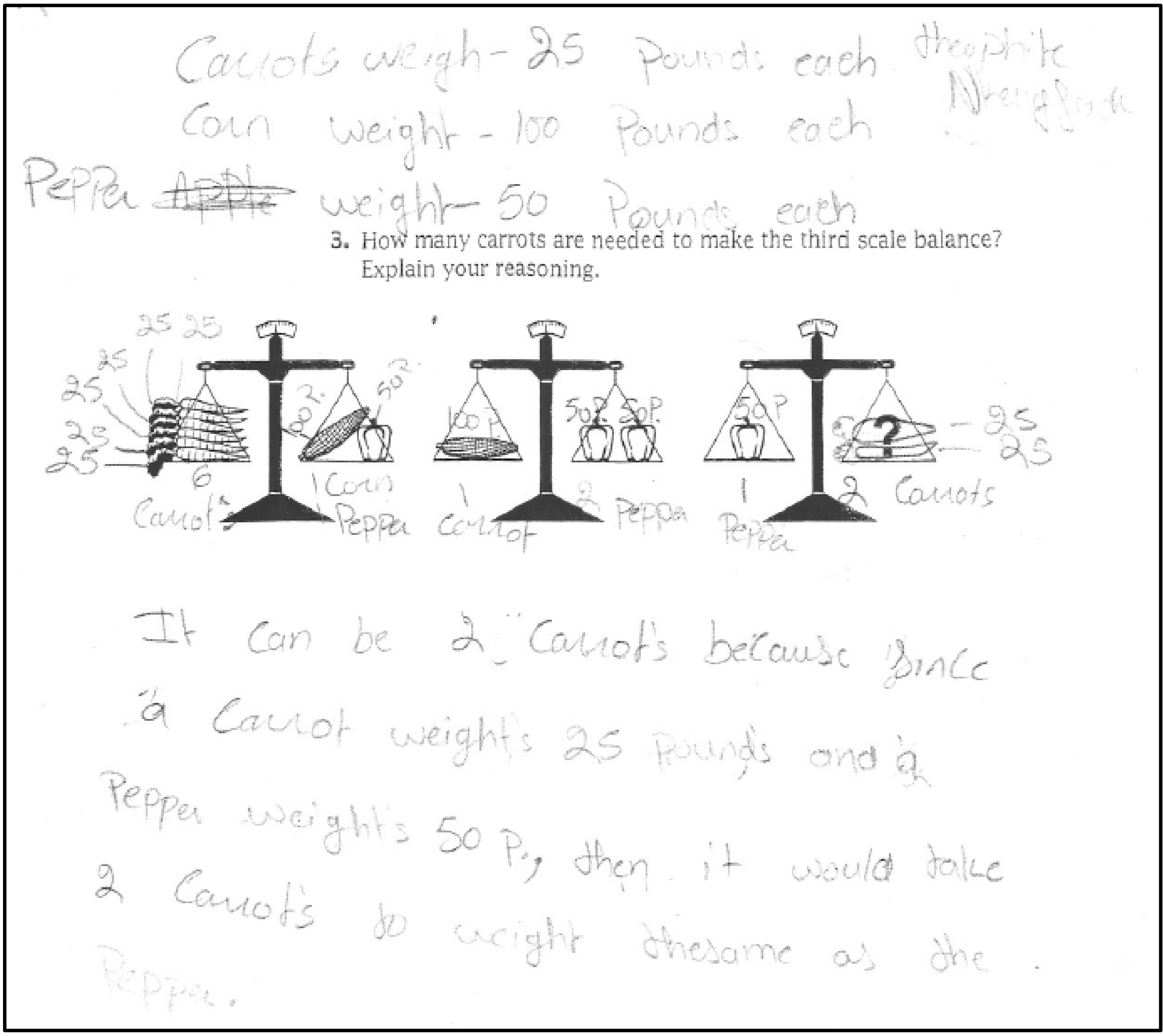
- Identify the 2:1 relationship shown on the second scale.
- Based on this realization, consider the first scale and pick two numbers that have a 2 to 1 relationship, representing a corn and a pepper respectively and add up to 6 carrots (4 + 2 = 6).
- A pepper weighs as much as 2 carrots.

Tyreal’s strategy was a modified version of this Solution Path. Although he did not explicitly mention the 2 to 1 relationship between the number of corns and the number of peppers, he did emphasize that assigning the number 6 to a corn and the number 3 to a pepper worked in the second picture, where a corn was shown to weigh twice as much as a pepper [lines 7 and 8]. This indicated that perhaps with this 2 to 1 relationship in mind, Tyreal had decided on two numbers, 6 and 3, to represent a corn and a pepper respectively, which added up to a total of 9 and represented the six carrots on the first scale.

After Tyreal’s presentation, Kevin declared that he had used a different strategy to arrive at the same answer of two carrots. He presented his work, which was essentially the same as the Solution Path 2C mentioned above. After these two presentations the teacher introduced the next activity, so it was not clear at that point if the rest of the students had the correct answer to the Carrot problem. However, examination of the data regarding the students’ work samples revealed that four students, that is 1/3 of the class, had arrived at the incorrect answer of three carrots. Unfortunately, it cannot be determined whether these students were ever convinced by either Tyreal or Kevin’s reasoning.

Another student work sample that captured the researcher’s attention was that of Theo’s. Theo never presented his work on the Carrots problem in class and there was no mention of it during the debriefing session. Theo’s work is shown in Figure 4.25 below.

Figure 4. 25: Theo's Solution to the Carrots Problem

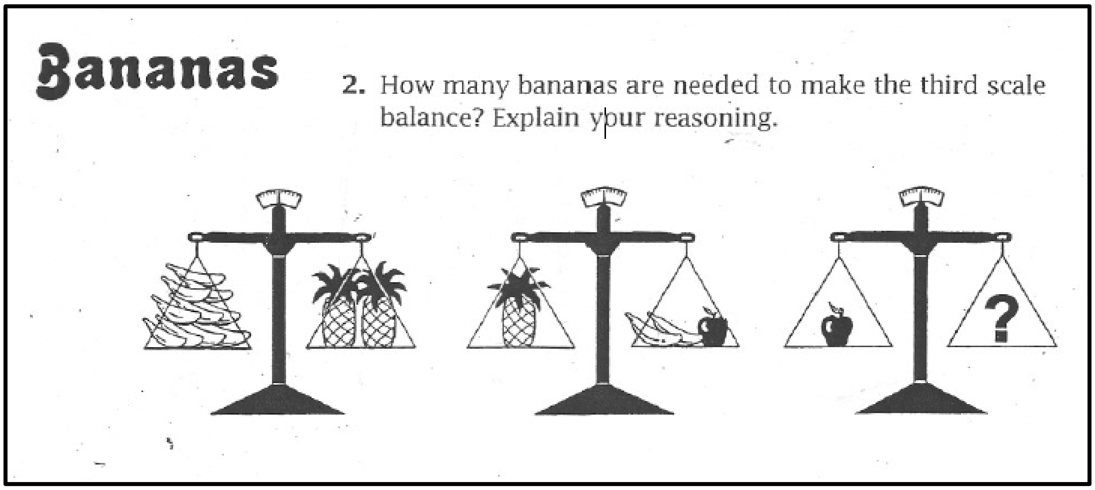


Unlike Tyreal who had assigned the number 1.5 to each carrot, Theo had attached the number 25 to each carrot and had used the word pounds to indicate that each carrot weighed 25 pounds. He had then figured out that a corn weighed 100 pounds and a pepper weighed 50 pounds. In his work he had clearly indicated the weight of each vegetable on the three scales, as well as in three sentences seen at the top of his paper. He had explained in writing that “Since a carrot weight’s 25 pounds and a pepper weight’s 50 p., then it would take 2 carrots to weight the same as the pepper”.

Putting aside the unrealistic nature of his assertion that these vegetables could weigh as much as 100 pounds each, his numbers worked well and satisfied the conditions indicated on each scale. However, it is not clear why or how Theo decided on the numbers, 25, 100, and 50. Perhaps he started with the middle scale, chose a “nice” round number of 100 for the weight of a corn, figured out each pepper to be 50 pounds, and then worked backwards to find the weight of each carrot on the first scale by dividing 150 by six. Regrettably, one would never know for sure the details of Theo’s exact thought process as he solved this problem!

About 20 minutes into the lesson, the teacher introduced the next activity, the Bananas problem. This problem (see Figure 4.26 below, a copy of Figure 4.1) was a pictorial depiction of a system of two equations with three unknowns in which, the reader was asked to re-define one of the variables in terms of one of the other two variables. Specifically, the reader had to figure out how many bananas weighed as much as one apple.

Figure 4. 26: The Bananas Problem



The class spent less than four minutes on the Bananas problem and the examination of the students' work samples showed that all students had arrived at the correct answer. The following is a descriptive analysis of Jynita and Dieshe's work on this problem.

Table 4

Jynita and Dieshe worked on this problem together and within seconds, Jynita raised her hand and told the teacher that she really wanted to explain her answer to the class (Event 3, Appendix F3):

Line # **Transcription**

Jynita: I used Tyreal's technique.

T3: Okay, share.

Jynita: it was 10 bananas, so then, and two pineapples, So each pineapple had to equal 5, so then 10 and 10 (referring to the first scale). And then for the next one the pineapple is 5 again. So two bananas will have to equal 1(each), so then an apple will have to equal 3. Then we have 5 and 5 for the next one (referring to the second scale). And if the apple is equal to 3 then you have to have three bananas.

Jynita and Dieshe used a slightly modified version of the Solution Path 1B, which was as follows:

- 10 bananas weigh as much as 2 pineapples (considering the first scale).
- Therefore 5 bananas weigh as much as 1 pineapple (cutting each side in half).
- On the second scale……….2 + ? = 5
- Therefore an apple weighs as much as 3 bananas.

Their written work, as well as Jynita's oral explanation, showed that they did not use a unit of weight or describe the weight of a fruit in terms of the weight of another fruit. For example rather than saying that a pineapple was the same as five bananas, they simply attached the number 5 to a banana [line 4]. What is noteworthy is the fact that Jynita publicly acknowledged employing Tyreal's strategy (of attaching a number (1.5) to each carrot) [line 1]. This was evidence that the students, at least Jynita and Dieshe, paid attention to Tyreal's reasoning, understood it, learned from it, and were able to apply it in solving a different problem.

Approximately 24 minutes into the lesson, the teacher introduced the modified version of the Shorts and Glasses problem on the overhead, in which the $50 price tags were hidden (see figure 4.27 below, a copy of Figure 4.19). She told the students that the total prices were the same in both pictures and asked them to determine the more expensive item. The following is a descriptive analysis of some of the students' work and the interactions that took place at various tables as the students attempted to solve the Shorts and Glasses problem.

<u>Table 2</u>

Before the teacher had a chance to finish her question: “Which is more expensive, the shorts or the glasses?” several hands went up. In Event 4 (see appendix F4), Kevin explained why the glasses were more expensive than the shorts:

Line # **Transcription**

Kevin: Ahh, the glasses are more expensive, because one glass, one pair of glasses equals two pairs of shorts

T3: How did you get that? [sounds surprised]

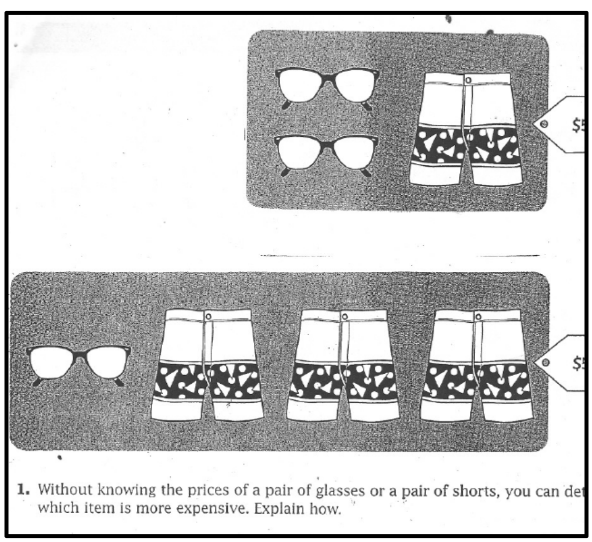
Kevin: Because there are three pairs of shorts down there [second picture] and one glasses and two pairs of glasses up there [first picture] and one pair of shorts.

T3: Wow! Do you agree? Were you about to raise your hand? [pointing to a student at the back]

A boy: I …[not audible]

T3: You had the same thing? What about you? [Pointing to another student who says something, not audible.] You were about to say the same thing?

Figure 4. 27: The Modified Version of the Shorts and Glasses Problem



Kevin was quick to not only identify the glasses as the more expensive item, but also declare that they were in fact twice as expensive as a pair of shorts [lines 1 and 2]. Although his explanation was inadequate and ambiguous [lines 4-6], Kevin was the only student in this study who managed to immediately identify the 2:1 ratio between the number of shorts and the number of glasses. The teacher appeared to be impressed by Kevin answer [line 7] and although she called on several other students who had originally raised their hands, no one offered any additional insight into the problem. In fact they all declared to have had the same answer as Kevin's [lines 7-12]. It seemed at the time that the majority of students, if not all, had arrived at the conclusion that the glasses were twice as expensive as the shorts. However, as evidenced by the following Events, there were students who, even after Kevin's explanation, still either considered the possibility of the shorts and the glasses costing the same, or completely overlooked the 2 to 1 relationship that existed between the glasses and the shorts.

Immediately following Kevin's explanation, the teacher handed out a copy of the Shorts and Glasses, which revealed the $50 price tags, and asked the students to figure out the actual prices of the items. The following is a descriptive analysis of some of the occurrences at Table 3.

<u>Table 3</u>

While the students were working on this problem, Theo was seen punching numbers on his calculator, trying to find the prices for the glasses and the shorts. In Event 5 (see Appendix F5) Theo called the teacher over to his desk and explained how he figured out the prices for each item:

Line # **Transcription**

T3: Yes? [Theo’s hand is raised. T3 walks over to him. Theo is holding a calculator.]

Theo: So these [pointing to the items in the bottom picture] cost twelve thirty, twelve dollars thirty cents, I mean twelve dollars fifty cents. Because four items, divide fifty by four.

T3: But if you just already said that the glasses much cost more than shorts, so is it possible that they all cost the same? Just based upon this here. [She points to the pictures and walks away. Theo starts punching numbers on the calculator.]

It is important to note that about two minutes prior to this episode, Kevin had stated that the glasses were twice as expensive as the shorts and many students, including Theo, had declared their agreement with him. And yet, Theo had divided the total price of $50 among the four items in the bottom picture to arrive at the conclusion that a pair of shorts was the same price as a pair of glasses, each costing $12.50. This indicated that Theo was not convinced that the glasses were more expensive, let alone that, specifically, they were twice as expensive as the shorts. Additionally, Theo’s answer reflected his apparent lack of understanding that the prices had to add up to $50 in both pictures, which given his proposed price of $12.50 per item, they did not.

About five minutes after the copies of the Shorts and Glasses problem were first distributed the teacher asked for volunteers to present their solution at the overhead.

<u>Table 1</u>

Two boys, Chad and Devon who were partners, worked together on the Shorts and Glasses problem. In Event 6 (see Appendix F6) Devon presented their solution at the overhead:

Line # **Transcription**

Devon: first, when we first started, we both noticed, so we started with 17, 17 [pointing to the glasses in the top picture] and 15 [pointing to the shorts; he actually means to say 16]

T3: Okay, wait, boys and girls let's listen.

Devon: And it moved up to 15, I mean 50 [added up to 50]. So we thought, alright, that is right. Then we got down here [pointing to the bottom picture], 17, 15, I mean 17, 16, 16, 16, it equaled to 62. So it is like, you can't do that. So then we started going through the numbers again. That is when I found out that if we do 20 and 20 [pointing to glasses in the top picture], that is 40. You do 20, 20 that is 40, and then the pants need to be 10, so that will be 50. Now if you go ahead at the bottom [bottom picture], it will be 20 plus 10, plus 10, plus 10 [writing the prices on top of each item in the bottom picture]. That equaled 50 again.
[A couple of students say something and there is laughter]

Devon: That is where I got 20, 20, 10 from [writing 20, 20, and 10 next to the items in the top picture to indicate prices]

T3: Okay.

The strategy used by Devon and Chad was similar to the one identified in this study as Solution Path 3C, which was as follows:

- Use guess and check to figure out the price of each item.
- Based on the actual prices, name the more expensive item.
- Based on the actual price of one pairs of shorts, calculate how many pairs can be purchased with $50.

In order to find the actual prices, Devon and Chad had initially tested the set of numbers, 17 and 16, to represent the prices for the glasses and the shorts respectively [lines 1-3]. Although these prices satisfied the total of $50 in the top picture [lines 5 and 6], they resulted in a total of $62, and not $50, in the bottom picture [lines 6-8]. This indicated to the boys that they needed to adjust their prices, so that they would work in both pictures and subsequently led them to the correct prices of $20 and $10 [lines 8-17].

Although in their initial guess, $17 and $16, the glasses were more expensive than the shorts by $1, their choice of these numbers did not reflect the boys' understanding that the glasses were twice as expensive as the shorts. However, less than five minutes earlier, Devan had agreed with Kevin's statement: “The glasses are more expensive, because one pair of glasses equals two pairs of shorts”.

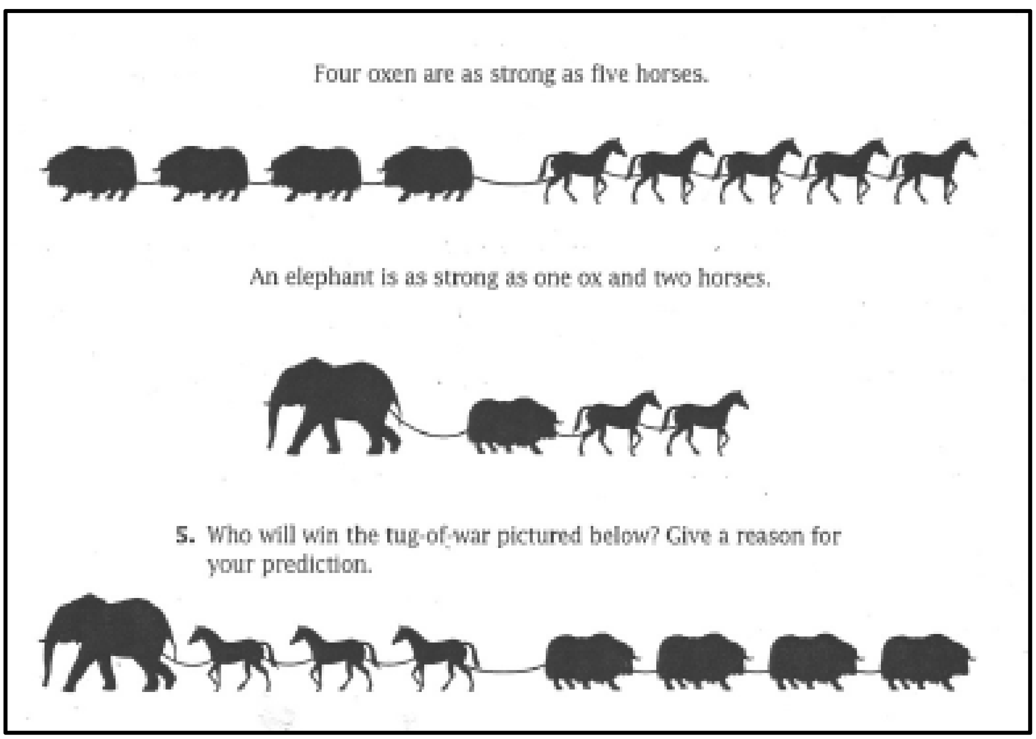
It is possible that the boys partially agreed with Kevin that the glasses were more expensive but did not pay close attention to the second part of his statement, claiming that the glasses were twice as expensive. Also note that in the second part of Kevin's statement: “one pair of glasses equals two pairs of shorts”, there is no reference to the prices of the items. Therefore it is conceivable that Devon and Chad heard Kevin's statement and yet, in their minds, this information did not translate into a relationship between the prices of these items. (Discussions on teachers' assumptions about students' understanding of the mathematics they cover in class will follow in later sections.)

About 30 minutes into the class the teacher held up a photograph, which had been taken on that year's field day and showed the students playing the tug-of-war (Event 7, Appendix F7). She asked the students to describe what they knew about the game and Kevin made a connection between the tug-of-war and the pan balance. He said that prior to pulling on the rope the tug-of-war resembled the pan balance: “It's like the scale, they are even on equal sides” and once the tugging started, the stronger side would win. Devon added: “It goes unbalanced”.

The showing of the photograph was supposed to spike the students' interest and serve as an introduction to the next activity, the Tug-of-War problem. However, the discussion that it incited also served as a bridge between the previous balance scale problems, exploring the concept of equality, and the tug-of-war problem with the inherent notions of unbalance and inequality.

About 34 minutes in to the lesson, the teacher handed out the Tug-of-War problem (see Figure 4.28 below, a copy of Figure 4.5). This problem was a pictorial depiction of two equations representing equality in the strength of two groups of animals, and an inequality representing the winning team. The reader was supposed to consider the information provided by the equations in order to decide which side of the inequality was the greater side. Specifically, the reader had to predict which group of animals would win the tug-of-war.

Figure 4. 28: The Tug-of-War Problem



After explaining the problem, the teacher asked the students to predict the winners in this game of tug-of-war, emphasizing that they had to convince her that their answer was correct. While the students were busy working on the problem the teacher circulated around the room, overseeing their progress.

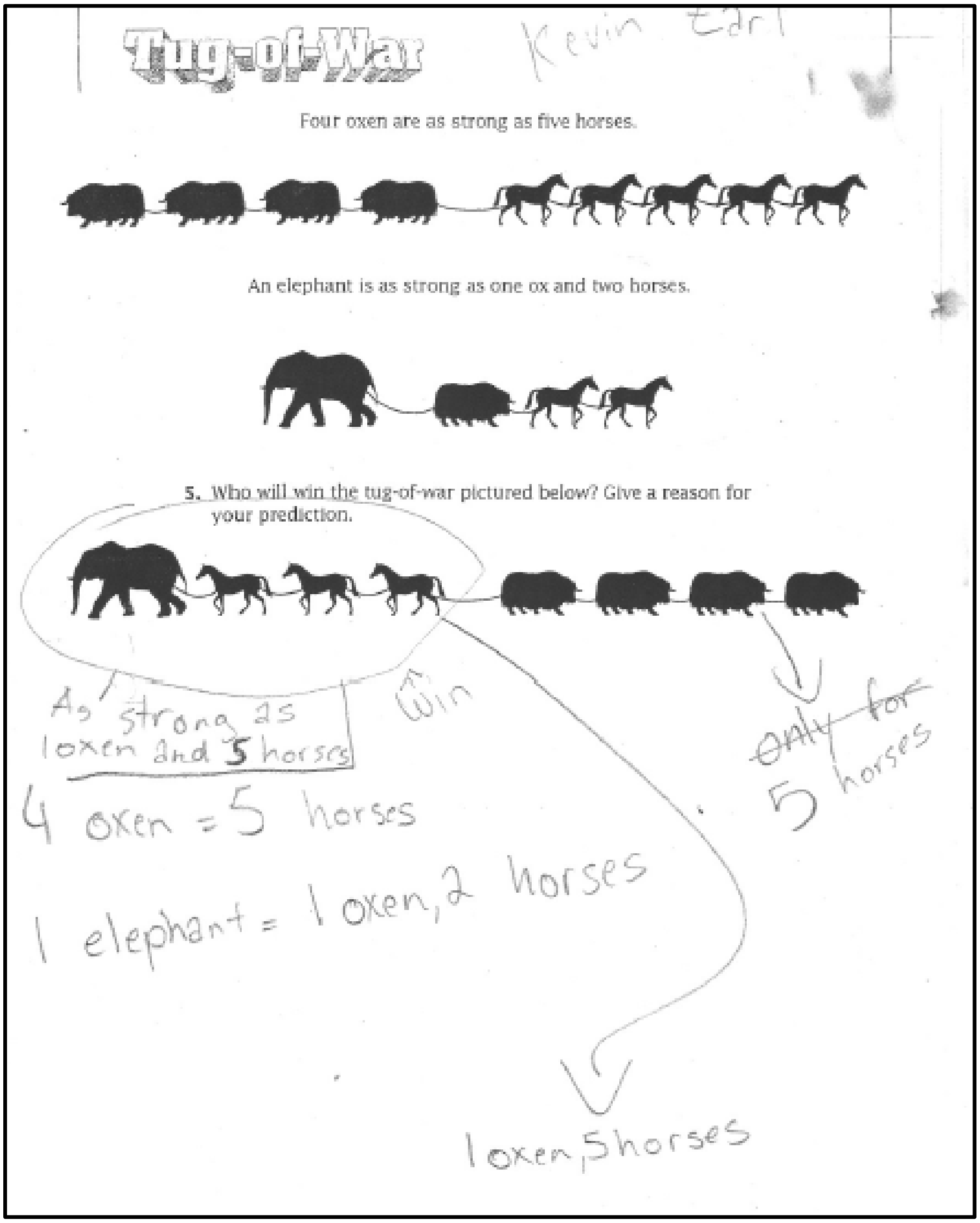
Two minutes later Keith, who was seated by himself at table 5, proceeded to explain to the teacher why he thought that the elephant and the horses would win the tug-of-war (Event 8, Appendix F8). Halfway through his explanation, the teacher stopped him and said, “Why don’t you write all this out? Write it all down. I never said you cannot write on this. You write on it. You can write notes. You can scratch”. Keith replied, “I hate writing because it, it’s mad hard!” The teacher encouraged Keith to put down in writing exactly what he had been telling her earlier on. Keith’s work on this problem will be discussed and analyzed later.

A few seconds later, Kevin raised his hand and asked if he could share his answer with the class (see Event 9, Appendix F9). The teacher asked if he had written down his explanation and upon hearing that he had not, instructed him to do so. Kevin appeared to be unhappy about this and said, “I can’t write my mind”. The teacher replied that she preferred for Kevin to explain his thinking in a paragraph and use his writing to justify his answer. However, she gave Kevin the option of putting down his thinking in a bullet format. She was also heard saying, “Do what you need to do so you have all this on paper.”

The teacher’s insistence on students’ writing of their explanations was an evidence of the teacher’s commitment to the goals and the objectives that were set by the teachers for the lesson study project. The objective was for the teachers to support students’ autonomy, attend to their mathematical reasoning, and observe the various strategies they used in solving mathematical problems. Additionally, the teachers were to encourage the students to work together and communicate their mathematical thinking verbally, through classroom discourse, as well as in writing.

As seen in Figure 4.29 below, Kevin did not write a paragraph explaining his solution strategy, nor did he use bullets to structure his writing. However he did make some attempts at conveying his mathematical thinking in writing. He used arrows and a few words in the bottom picture, indicating that an elephant and three horses were the same as an ox and five horses, and on the other side, four oxen could be replaced by five horses. He identified the winning group by writing the word, “win” at the end of an arrow that pointed to the circled group of animals on the left-hand-side.

Figure 4. 29: Kevin's Explanation of the Tug-of-War Problem



Kevin's strategy in solving this problem was similar in nature to one of the strategies used by the teachers during the planning phase of the lesson study and identified in this present study as Solution Path 5A shown below:

- Consider the top picture. Since four oxen are as strong as five horses, replace the four oxen in the bottom picture with five horses.
- The new picture will show <u>one elephant plus three horses</u> Vs. <u>Five horses.</u>
- But we know from the middle picture that an ox and two horses are as strong as one elephant and therefore can replace the elephant in the picture.
- The new picture will show <u>five horses plus an ox</u> Vs. <u>five horses.</u>
- The group on the left side has a greater strength and is clearly the winner.

Events 10 and 11 took place at Table 4. The following is a descriptive analysis of the interactions that occurred between Dieshe, Jynita, and the teacher while the students were trying to solve the Tug-of-War problem.

<u>Table 4</u>

About five minutes after the students first started working on the Tug-of-War problem, the teacher was seen at Jynita and Dieshe's table, having a conversation with them about the problem (see Event 10, Appendix F10). Jynita and Dieshe had arrived at the wrong conclusion that the four oxen would win the tug-of-war. Jynita seemed convinced that her answer was correct and when questioned by the teacher, tried vehemently to defend her answer. Dieshe, on the other hand, appeared not to be completely committed to her initial assertion that the four oxen would win the tug-of-war and seemed very thoughtful throughout this conversation.

In this episode, Jynita explained their answer to the teacher, who started questioning their reasoning:

Line #	Transcription
1	**T3:** Go ahead, You said the oxen, they can pull five horses, right?

Jynita: Uhum.

T3: Here is the four [pointing to the four oxen in the third picture], here is the four [pointing to the four oxen in the first picture], where is the five horses?

Jynita: But they, they can still pull three horses if they can pull five, right?

T3: What does this say? [pointing to the first picture]

Jynita: Four oxen are as strong as five horses.

Jynita had arrived at her conclusion through faulty reasoning. She argued that if four oxen could pull five horses (top picture), then they should be able to pull three horses (bottom picture) as well [lines 6 and 7]. Although her conditional statement (if…then) was logical, it was irrelevant to the problem in which, as shown in the bottom picture, four oxen competed against not just three horses, but an elephant as well.

None of the three pictures supported the first part of her statement (if four oxen could pull five horses). In fact, the first picture showed that four oxen were as strong as five horses, implying that four oxen could not pull five horses. Interestingly enough, when asked by the teacher to interpret the top picture, Jynita’s answer, “Four oxen are as strong as five horses”, indicated her understanding that the five horses and four oxen were of equal strength [lines 8 and 9]. Or perhaps, she had simply read the line from the problem (which was the same as her reply) and did not quite understand it. As for the second part of her statement (then they should be able to pull three horses), there was no picture showing four oxen against three horses. Jynita, it seemed, had decided to overlook the elephant in the bottom picture, in which case the oxen would win the tug-of-war.

As this episode continued, the teacher tried to focus the girls' attention to the middle picture:

Line #	**Transcription**
10 11	**T3:** But look what you see here [pointing to the middle picture], an elephant is as strong as one oxen and two horses. What does that mean?
12	**Jynita:** That means that it is not as strong as two oxen.
13 14	**T3:** What, What is a horse, elephant as strong as..What does that mean, an elephant is as strong as one ox and two horses?
15	**Dieshe:** they [unclear] to an elephant.
16	**T3:** Excuse me?
17	**Dieshe:** they both, all three equal one elephant.
18 19	**T3:** Okay, so what else, what else could, what else could we assume based upon that message there, that statement? What can we imply?
20	**Jynita:** The oxen are going to win the tug-of-war.
21 22	**T3:** The oxen are going to win the tug-of-war? [Jynita nods]

The teacher kept encouraging the girls to interpret the middle picture and to talk about its meaning and implications [lines10 and 11, 18 and 19]. As reported later, the teacher had hoped that bringing the girls' attention to the middle picture, would prompt them to replace the elephant with two horses and an ox in the bottom picture. However, Jynita's interpretations of the middle picture - "That means that it (elephant) is not as strong as two oxen." and "The oxen are going to win the tug-of-war." – did not appear to lead Jynita in the direction the teacher had hoped for. [lines 12 and 20]. But Dieshe appeared to be pensive as she listened to and answered the teacher's question:

Line # **Transcription**

Dieshe: So that means they will win. [pointing to the elephant and the three horses]

T3: Why?

Dieshe: Because if you know that..

Jynita: No, they are going to win Dieshe [pointing to the four oxen]
[Dieshe looks at the pictures thoughtfully]

T3: Why?

Jynita: The oxen are going to win.

T3: What were you about to say? [addressing Dieshe]

Dieshe: The elephant, the, okay, this is what I am saying. You said that, okay four oxen is equal to five horses, right? But is only three horses over there [pointing to the last picture]. But an elephant is equal to an ox and two horses. So we have three [pointing to horses in the last picture], four, five [adding the horses in the middle picture] that is five horses and another ox.

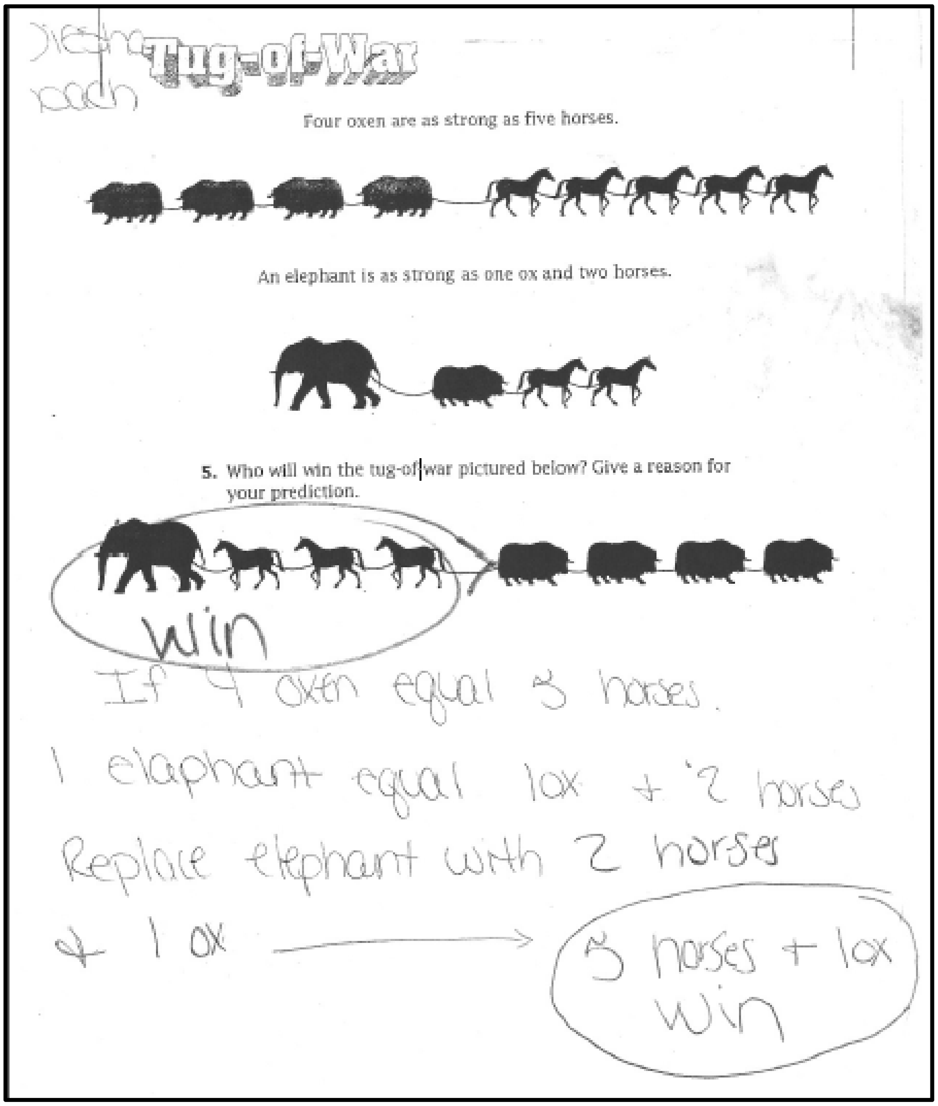
T3: See, why don't you write that down? Just what you said so you won't forget it. Why don't you write it down?

After much pondering about the problem and the implications of the message conveyed by the middle picture, Dieshe came to the realization that the elephant side would win the tug-of-war [lines 33-38]. It was interesting to see that while Dieshe was trying to formulate her reasoning, Jynita tried, twice, to convince her that their original answer was correct [lines 27 and 31]. Eventually Dieshe was able to articulate her reasoning, which was similar to one of the strategies used by the teachers during the planning phase of the lesson study and identified in this present study as Solution Path 5B shown below:

- Consider the middle picture. Since we know that an ox and two horses are as strong as one elephant, we can replace the elephant in the bottom picture with an ox and two horses.
- The new picture will show five horses plus an ox Vs. four oxen.
- But we know from the top picture that five horses are as strong as four oxen. Therefore the addition of an ox to the five horses on the left will create an imbalance of strength and make the group on the left the winner.

Satisfied with Dieshe's explanation, the teacher urged her to write down her reasoning. Dieshe's written work is shown in Figure 4.30 below. Jynita, on the other hand, never handed in her work on this problem.

Figure 4. 30: Dieshe's Explanation of the Tug-of-War Problem



After instructing Dieshe to write down her explanation, the teacher left their table for a few minutes to attend to another group of students. Meanwhile, Dieshe tried to convince Jynita that the elephant and the three horses would win the tug-of-war (see Event 11, Appendix F11):

Line #	Transcription
1	**Dieshe:** Okay look, there is four oxes and five horses. Right? Right?
2	**Jynita:** Yeah, but where are you getting five horses and an ox?
3	**Dieshe:** An elephant is equal to …..
4	**Jynita:** But is that in the picture?
5	**Dieshe:** This here says an elephant is as strong as an ox and two horses
6	[pointing to the middle picture]. So if you take away the elephant and
7	replace it with…
8	**Jynita:** But, does it say anything about taking an elephant away?
9	**Dieshe:** But it is a replacement cause it is equal to that.
10	**Jynita:** How are you replacing the elephant? It says the picture below.
11	It doesn't say anything about taking away.
12	**Dieshe:** It is like replacement, it is the same exact, yeah, it is.
13	**Jynita:** I don't get you Dieshe [Jynita shakes her head dismissively].

Dieshe's explanation indicated an understanding of the algebraic concept of substitution. She tried to convince Jynita that one could take away the elephant in the last picture and replace it with one ox and two horses [lines 1, 3, 5-7, 9, 12]. In her explanation, Dieshe used words and phrases such as "replace", "equal to", and "same exact" to describe the process of re-naming the elephant. Nonetheless, Jynita did not seem to understand Dieshe's reasoning and questioned her rational [lines 2, 4, 8, 10 and 11]. Eventually, while shaking her head, Jynita turned away from Dieshe and said that she did not understand her [line 13].

Not having reached a consensus, the girls stopped talking and began working independently, writing down their explanations. The teacher, who had left their table only a couple of minutes earlier, returned and continued having a conversation with Jynita about the problem (Event 11 continued):

Line #	Transcription
14	**T3:** Okay, and what do you say? [addressing Jynita]
15	**Jynita:** I think the oxes are going to win.
16	**T3:** Why?
17	**Jynita:** Because they look like they are going to win.

Jynita's tone of voice appeared to indicate that she was no longer interested in pursuing the conversation. However, the teacher was persistent and continued with her questions (Event 11 continued):

Line #	Transcription
18	**T3:** They look like they are going to win? [Pause]
19	**T3:** What is it that you know about the oxen?
20	**Jynita:** That they can pull five horses.
21 22	**T3:** Okay, so here you only have three. Are you convinced because of that?
23	**Jynita:** Uhum
24	**T3:** What is it that you know about an elephant?
25	**Jynita:** An elephant is equal to one ox and two horses, but…
26	**T3:** Okay, so let's do that….
27 28	**Dieshe:** See look, see look, this is what I am saying. You see how on this side there is an elephant….

Jynita: How are you going to stop me when I am in the middle of explaining?

T3: Okay, keep going.

Jynita: I think it is the oxen (winning) because if elephant can pull an oxen and two horses, they can't pull four oxen, wait,..[Pause]

Dieshe: Want me to explain one more time?

Jynita: No, because I am talking.

As the teacher continued to question Jynita, Dieshe interjected a few times trying to explain what was very obvious to her, but difficult for Jynita to see [lines 27 and 28, 34]. Jynita seemed somewhat annoyed and that may account for her response to Dieshe, indicating that she did not want to be interrupted [lines 29 and 30, 35]. However, her demeanor had changed and she no longer appeared totally convinced by her own reasoning [lines 32 and 33]. The teacher continued (Event 11 continued):

Line # **Transcription**

T3: Let's look at here. What can you tell me about an elephant, an ox and a horse?

Jynita: Well, oxen and two horses equal one elephant.

T3: Okay, so what can you say about this here? [pointing to the elephant in the bottom picture]. So if that is the case what is this over here?

Diesha: it is one elephant, the same as two horses and an ox.

Jynita: What? [looking hard at the paper]

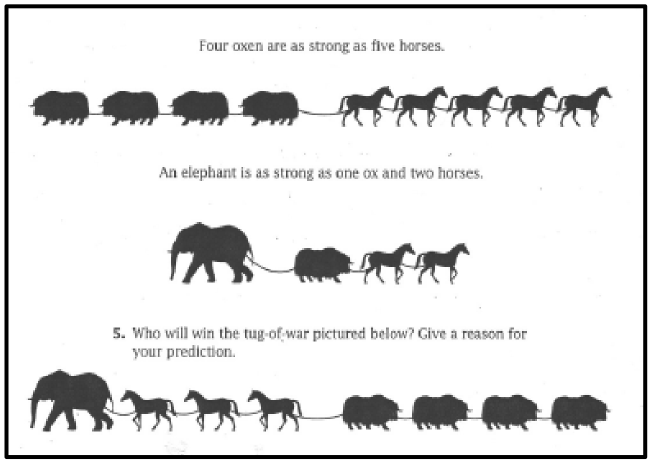
Dieshe: You take the elephant right here [pointing to the middle picture] and the elephant right here [pointing to the bottom picture]. So the elephant is just the same as two horses and an ox. What is this elephant equal to? Two horses and an ox. And then, four oxes [pointing to the bottom picture], four oxes [pointing to the top picture], but there were five horses [pointing to the top picture], and there is one, two, three [pointing to the three horses in the bottom picture], four, five [pointing to the two horses in the middle picture] and an ox.

Jynita: But how do you get five horses if it is not in the picture?

Dieshe: This is equal to this, right? [pointing to the two elephants] And this is equal to this [pointing to the two sides in the middle picture]. Right? So if you add this and this…

Once again, the teacher asked Jynita to interpret the middle picture [lines 36 and 37] (see Figure 4.31 below, a copy of Figure 4.5). Jynita replied that an ox and two horses equaled one elephant [line 38]. The teacher then asked Jynita what she knew about the elephant in the bottom picture, bearing in mind how she had interpreted the middle picture [lines 39-41]. When Jynita paused, Dieshe answered the question by stating that the elephant in the bottom picture was equal to an ox and two horses [line 42]. Jynita looked genuinely puzzled [line 43], so Dieshe delved into an explanation of how an ox and two horses could substitute the elephant in the bottom picture to result in five horses and an ox on the left-hand-side [lines 44-51]. Jynita wondered how there could be five horses on the left-hand-side if one could not see them there [line 52] and Dieshe tried, one more time, to explain the process [lines 53-51].

Figure 4. 31: The Tug-of-War Problem



At this time, the teacher who had kept quiet during the girl's conversation, asked if either one of them had been swayed by the other one's argument (Event 11 continued):

Line #	**Transcription**
56 57	**T3:** Are you convinced by her or is she convinced by you? [addressing Jynita]
58 59	**Jynita:** I, [pauses] I don, [pauses, smiles, keeps looking at and playing with a piece of string in her hands] I don't want her answer!
60 61	**T3:** Okay, then you come up with you own answer. I'm going to give you another chance to answer [T3 walks away]

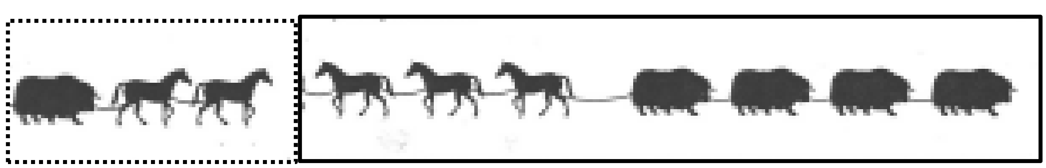
Jynita did not answer the teacher straight away. She kept her eyes on a piece of string that she was playing with. Eventually she smiled and without looking at the teacher, said that she did not want Dieshe's answer [lins 58 and 59]. The teacher acknowledged Jynita's autonomy by affording her the opportunity to formulate her own response [lines 60 and 61].

It is impossible to know for sure whether or not Jynita was convinced by Dieshe's reasoning. However, her demeanor (as observed on the videotape) and her apparent refusal to "adopt" Dieshe's answer suggested that perhaps Jynita no longer believed her own answer to be correct, but had a hard time admitting it.

During the debriefing session that followed this lesson, one of the observers referred to the Tug-of-War problem and the difficulty Dieshe had in convincing Jynita that an elephant and three horses would win the tug of war. He believed that Jynita was genuinely unable to envision five horses and an ox in the bottom picture once the substitution was made. He reported having gone to Jynita after the teacher had left their table. As he approached her desk, Jynita told him, "I already have an answer, don't come to me" (see Teachers' Reflections, Appendix H3).

Nonetheless, he stayed and tried to help her. He demonstrated the tactic he had used in order to change Jynita's mind about her answer. He had cut out the picture of the two horses and an ox in the middle picture. He believed that by placing this cutout on top of the elephant in the bottom picture (see Figure 4.32 below), he had made it possible for Jynita to visualize the result of the substitution and the formation of five horses and an ox on the left-hand-side. The observer told the group that after this demonstration, Jynita was happy and began erasing her incorrect answer.

Figure 4. 32: Visualizing the Substitution in the Tug-of-War Problem



He claimed that this demonstration had convinced Jynita that her initial answer had been wrong. But, his claim cannot be substantiated as Jynita never handed in her written response to this problem. However, everyone agreed that the visual enforcement *strategy he had employed was a fantastic idea and T1 and O2 speculated that this technique could serve well as a modification to help some Special Education students.*

About 15 minutes after the Tug-of-War was initially handed out and while the students were still working on the problem, R1 called the teacher over to Table 5 and asked her to listen to Keith's explanation of his solution strategy.

<u>Table 5</u>

Keith preferred to sit alone and worked by himself throughout the lesson. He was the ideal student to observe since he talked to himself aloud while he worked through the problems, revealing his thought process. R1 had listened to Keith's reasoning and was keen for the teacher to hear what Keith had to say. In Event 12 (see Appendix F12), Keith explained his initial strategy and the subsequent steps that led to his final answer:

Line # **Transcription**

Keith: before your professor helped me out I had one, two, three, four (oxen) against these four (horses, in the top picture) and I took the other horse out because I thought that if two horses were equal to one ox, it would work, but I was stuck and I took the extra horse out because I was trying to equal it out, it would not work for half a horse.

T3: Okay

Keith: So then I progressed with that strategy. Then I came to every, for every one ox it will equal two horses and I will get three, this will be 3 [pointing to the elephant in the middle picture] I guess, and then, or 4. So then I tried 4 plus 1, plus 1, plus 1 [pointing to the animals on the left hand side in the bottom picture] and I got 7, the first time and then I tries 2 and 2 and I got 4 [oxen on the right] and I said it would be the left side (to win). But first I started equal, cause it looked the same, but then I had to think about it.

Previously, Keith had shared (off camera) his strategy with R1. Considering the top picture, where four oxen balanced five horses, he had concluded that an ox was equal to one horse plus a fraction of a horse. Being adamant about not killing a horse, since a fraction of a horse implied a dead horse, he had decided to “even out” the first picture by eliminating one of the horses all together. He had then decided to assign the number 1 to each horse and the number 2 to each ox. Based on these number assignments, he had calculated that the elephant, as seen in the middle picture would be a 4 (one ox and two horses; 2+1+1=4). In the bottom picture, he had added his assigned numbers to arrive at the number 7 for the animals on the left-hand-side and the number 4 for the animals on the right-hand-side, declaring the elephant and the three horses as the winners in the tug-of-war.

There were several flaws in Keith’s reasoning: a) by crossing out a horse he had changed the given facts in the problem and had essentially created a whole new problem; b) he had not used the assigned numbers consistently as he had used 2 for an ox in the middle picture and 1 for an ox in the bottom picture; and c) his assigned numbers (1 for horse, 2 for ox) did not create a balance in the first picture, with or without the crossed out horse.

Despite his faulty reasoning, Keith had arrived at the correct answer that the animals on the left hand side would win the tug of war. R1 had brought it to his attention that his assigned numbers of 2 and 1 for each ox and horse respectively, did not work in all the pictures. With a focus on maintaining the balance in the top picture, R1 had encouraged Keith to think of different numbers to represent the ox and the horse. In Event 12 (continued), Keith explained to the teacher how he finally figured out the answer:

Line # **Transcription**

Keith: Then I noticed that if four go into five I'll have 20. So I said this would be 5, 5, 5, 5 [pointing to oxen] and this would be 4, all these (five horses) would be 4. So I will have one ox [pointing to the middle picture], it will be 5 plus 4 and another 4 (the two horses) and I will have 13 plus. Now, moving on to the bottom picture, since I have the elephant that is 13, then I have three horses, that would be 16 and I would have 25. Wait, yeah I'll have 25. Twelve and 13 would be 25. Then I have four 5's is 20 (oxen), so still the left side will be more than the four oxen put together.

R1: Why is it that these four, that you gave the 5 to the oxen and the 4 to the horse?

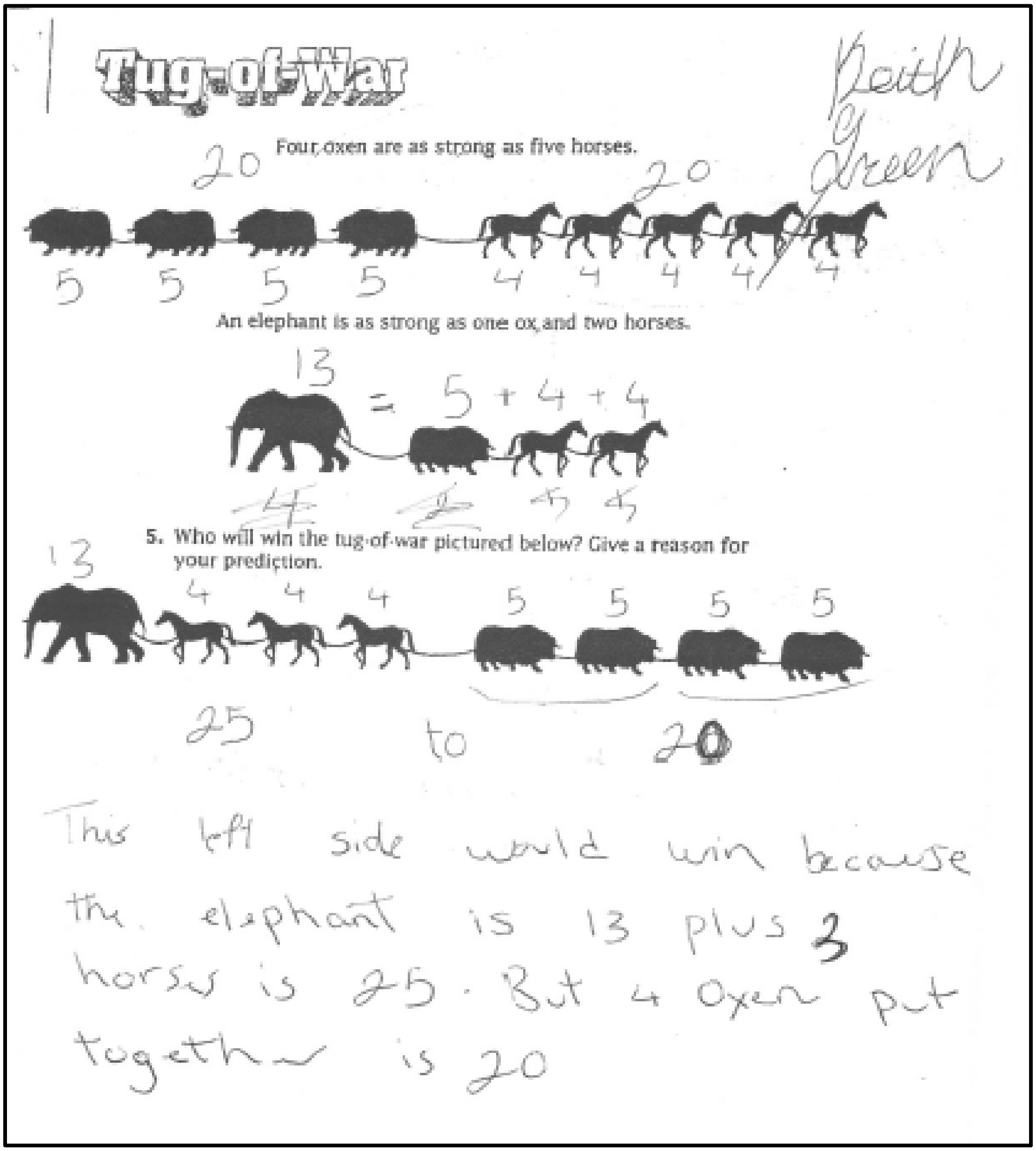
Keith: Because 5 times four equals 20 and 4 times five equal 20

T3: So you made sure they were balanced.

Keith had decided to attach the number 5 to each ox and the number 4 to each horse and when questioned as to why he had picked these numbers, his answer reflected his understanding that the two sides had to be of equal value in the first picture [lines 24-26]. However, as it became apparent later on (Event 14), in Keith's mind, these numbers did not signify the strength of the animals.

Based on these numbers, and as illustrated by his written explanation in Figure 4.33 below, Keith had correctly evaluated that in the bottom picture, the animals on the left and the right side could be represented by the numbers 25 and 20 respectively, and therefore had identified the elephant and the three horses as the winners of the tug-of-war.

Figure 4. 33: Keith's Explanation of the Tug-of-War Problem



The students worked for approximately 20 minutes on the Tug-of-War problem before the teacher asked for volunteers to share their solution with the class. Five people including Keith, Dieshe, and Kevin, whose solution strategies were discussed earlier, presented their findings. The following is a descriptive analysis of Theo and Tyreal's mathematical work from Tables 3 and 6 respectively.

<u>Table 3</u>

Theo, unlike the others who had shared their solution before him, believed that the four oxen on the right-hand-side were the winners in the tug-of-war. In Event 13 (see Appendix F13) Theo presented his solution to the Tug-of-War problem at the overhead. . First he placed his lengthy written answer on the overhead and read it. He was asked to read it a second time, but it was difficult to follow the logic of his explanation, which concluded (incorrectly) that the four oxen were stronger than one elephant and three horses. T3 then displayed the transparency of the original problem on the overhead and asks Theo to use the pictures to explain his thinking more clearly:

Line # **Transcription**

Theo: Right here [underlines the horses and ox in the middle picture] it
says that this ox and these two horses are as strong as this elephant. So
I took out this ox and these two horses [crossing out the animals from
the opposite sides in the bottom picture] because they make up one
elephant. Now there is three oxes and one elephant and one horse. So I
did, since two horses can make up almost an ox, I said two oxes to take
one elephant and the other ox is stronger than the horse. So the oxes
[on the right] are stronger than this group [on the left].

T3: Do you have any question?
[Some hands go up and some students start talking.]

T3: Wait, wait, wait. [unclear] Jynita?

Dieshe: I wonder how did you assume that two horses is equal to an
ox?

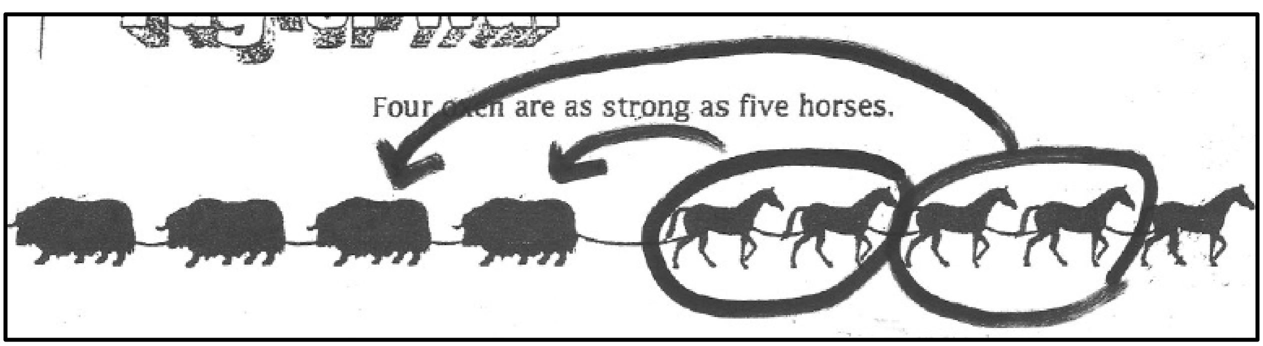
Theo: I said that because up here [pointing to the top picture] four ox,
four oxes is equal to five horses.

Theo's reasoning included many incorrect assumptions and subsequent steps. He referred to the second picture and based on the fact that one elephant was as strong as one ox and two horses, he crossed out one ox and two horses from the opposite sides in the third picture [lines 1-5]. He also assumed, incorrectly, that two horses were as strong as one ox and used that notion to conclude that two oxen were stronger than an elephant [lines 6 and 7].

After listening to Theo's explanation, the teacher gave the class the opportunity to critique Theo's reasoning [line 9]. Several hands went up and Dieshe asked why Theo had assumed that two horses were as strong as an ox [line 20]. Theo justified his assumption by referring to the first picture in which, four oxen were shown to be as strong as five horses. There were some students, including Kevin, who were clearly not convinced by Theo's justification: "But you can't split the horses because it is uneven." The teacher asked Kevin to clarify his point, using the picture on the overhead (Event 14, Appendix F14).

Theo returned to his seat and taking his place at the overhead (see Figure 4.34 below), Kevin explained why Theo's claim that two horses were as strong as one ox, was not correct: "Because he is trying to put two horses equal to one ox. But he couldn't because if there was two split, those two split, like this (in the top picture, circles two horses, connects to one ox) and this (circles and connects another two horses to a second ox), then you can't (pointing to the single horse remaining). There are only five horses."

Figure 4. 34: Kevin's Counter Argument



Kevin had difficulty articulating his counter argument, but as seen before, he managed to convey the gist of his mathematical thinking through drawing. After connecting two pairs of horses to two oxen, Kevin indicated that there were not enough horses to continue assigning a pair of them to an ox, implying that an ox could not be as strong as two horses.

Since the focus of the class shifted to the next presentation, Theo did not have the opportunity to comment on Kevin's counter argument and did not try to defend his answer any further. Therefore, it cannot be determined if Theo was convinced by Kevin's statement, or whether he realized why his final answer was incorrect.

However, during the debriefing that followed, R1 began the session by talking about Theo (see Teachers' Reflections Appendix H3):

> *The real issue that I like us to think about is Theo, who came up with what he thought was a very logical solution and it didn't work. And so what I was trying to do, and I know what (T3) was trying to do, is to get into his head to know what question to ask. Because ultimately, the only thing that happened was, we just sort of over-rode him. Because the other kids had two or three ways of coming up with something. But Theo still believed that his solution, which was the opposite, was correct.*

One of the observers, O5, who had been observing Theo at his desk while working on the Tug-of-War problem, told the group (see Teachers' Reflections Appendix H3):

> *I was watching him and what he did was, when he substituted for that elephant, he crossed off the oxen on the other side, but he crossed off the horses from the elephant side. So I talked to him at the end. I said, do you realize that when you did that you didn't take off horses against thetook off horses that would have been helping the elephant. He got that part. He saw that. That is where it ended.*

The teachers agreed that they really had not fully understood Theo's mathematical thinking and R1, alluding to the complexity of teaching, told the group (see

Teachers' Reflections Appendix H3): "But just look how complicated that is and how, you know, all of us together are struggling with it. And so a piece of what we need to help each with over the time is when you get into this kind of situation, when the kids are sharing and, and you have a responsibility to help uncover these mistakes and have kids come together. Because there is a right answers. You can't leave them thinking, that's okay, or this is okay. And that is hard."

Tyreal, from Table 6, was the last person to share his solution to the Tug-of-War problem with the class.

Table 6

Tyreal started his presentation by acknowledging the fact that his strategy, in terms of structure, was similar to Keith's, except that he had assigned fractional numbers to the oxen as oppose to the whole numbers used by Keith. In Event 15 (see Appendix F15), Tyreal explained his solution strategy:

Line #	**Transcription**
1	**Tyreal:** Since there are five horses and four oxen, that means each ox
2	is equal up to at least one of the horses. And since each, if you take off
3	the last horse that means they would, they would be equal (four on each
4	side).
5	Kevin: Oh, so you are splitting a horse?
6	**Tyreal:** Yeah, so it would be one point, would be one and one fourth
7	(each ox)
8	So then [pointing to the middle picture], if you write 1 ¼ (pronounced
9	as one fourth) for one ox and one horse is 1, so the elephant will have
10	to equal to 3 ¼. But then [pointing to the bottom picture] all you have
11	to do is add them all up. You get 6 ¼ (pronounced as six and a quarter)
12	[pointing to the left side] and 5 [pointing to the right side].
13	**Tryshon:** Ooooh I get you. [snapping her fingers and swaying]

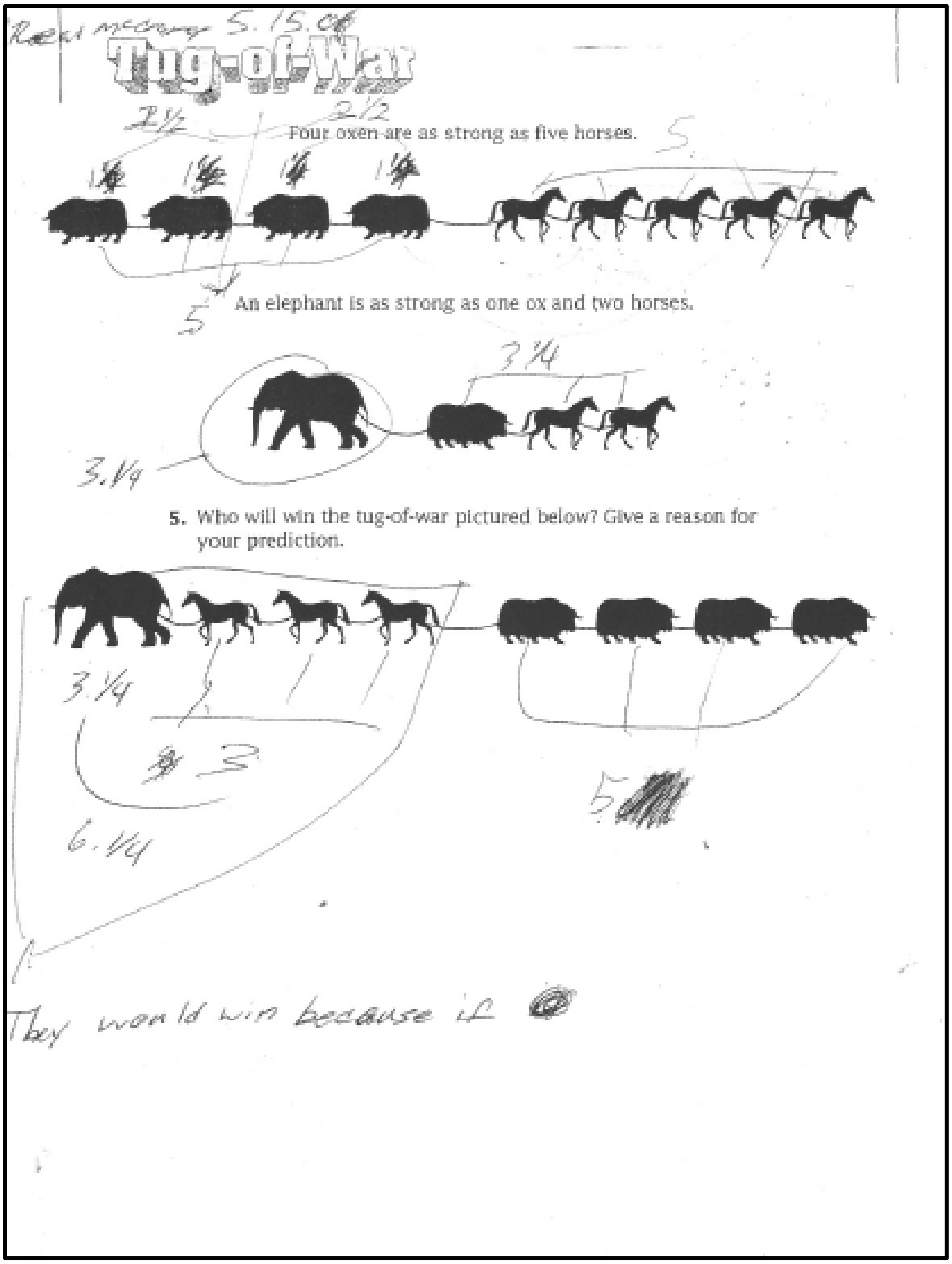
14 **Kevin:** Which side won? Which side won?

15 **Tyreal:** That one [pointing to the left side]

16 **Kevin:** Yeah, but yours is mad difficult.

Tyreal had assigned the number 1 to each horse and the number 1 ¼ to each ox. He had understood that each ox was as strong as at least one horse and that since the number of horses exceeded the number of oxen by one, he had to split the strength of that extra horse between the four oxen [lines 1-9]. As illustrated by his written work in Figure 4.35 below, Tyreal had then figured out that in the middle picture, the elephant's strength could be represented by the number 3 ¼ [lines 9 and 10]. He had then proceeded to add up all the numbers corresponding to each of the animals in the two opposing groups shown in the bottom picture [lines 10-12]. With the sum of 6 ¼ on the left and a total of 5 on the right, Tyreal had concluded that the elephant and the three horses would win the tug-of-war [lines 14 and 15].

Figure 4. 35: Tyreal's Explanation of the Tug-of-War Problem



During Tyreal's presentation, the students were very quiet and appeared thoughtful and attentive. Tryshon, Kevin's partner at Table 2, seemed particularly pleased to have comprehended Tyreal's reasoning [line 13]. Kevin, who had asked some clarifying questions during Tyreal's presentation [lines 5 and 14], appeared to be satisfied with Tyreal’s solution method, but declared it difficult to follow [line 16]. Keith, on the other hand, questioned the validity of Tyreal’s reasoning (Event 15 continued):

Line #	**Transcription**
17 18	**T3:** Is there anyone who wants to share but didn’t get the opportunity to share?
19	**Keith:** Oh, wait. I have a question. [hand up]
20	**T3:** Yes, Mr…, Keith.
21 22	**Keith:** How can you get a decimal of an animal? It won’t work, because it will be dead!
23 24	**Tyreal:** Because I am not splitting the animal, I am splitting how…strength [not audible; kids laughing]
25 26 27	**T3:** What did you say? What did you say? What did you say exactly? What did you say? What are we measuring? Are we measuring the animal or what are we measuring Tyreal?
28	**Tyreal:** Strength.
29	**T3:** Strength. We are measuring the strength.

Keith had used a similar strategy of assigning numbers to the animals. But, he had insisted on avoiding fractional numbers and had settled on whole numbers (4 and 5) to represent a horse and an ox. Questioning Tyreal’s choice of fractional numbers [lines 21 and 22] was a further indication of Keith’s lack of understanding that these assigned numbers were merely a symbolic representation of the animals’ strength, and not the discrete number of the animals themselves. On the other hand, Tyreal seemed to understand this [lines 23 and 24, 28] and was therefore comfortable with splitting 5 (number of horses) into 4 (number of oxen) and using the fractional number, 1 ¼, to represent the strength of each ox.

The teachers noted that arriving at number 1 ¼ to represent each ox had not come easy to Tyreal. One of the observers, O1, had been watching Tyreal during the class as he worked on this problem. During the debriefing session that followed, O1 reported the following (see Teachers' Reflections Appendix H3):

> *But you know, this is really interesting because he (Tyreal) really struggled, when he had five (horses) on one side, he really had a tough time getting 1 ¼ (for each ox) because he was putting 1.4 to begin with… I could see he was adding it up and it wasn't becoming 5. So he would change it to something else… He really did a lot of trying to, and he couldn't understand that it was 5 divided by 4 that is going to give him 1 ¼. He never used the calculator… And then once he had the ¼, he went and checked all the answers at the end. So towards the end, when he was actually writing his answer, I said, well what mathematical operation could you have used to get from 5 to this answer that you have? He said "I don't know". And I said, well what did you do to 5 in order to get to 1 ¼? He said "I don't know, I just, I just was trying to split it into 4". I said, well what mathematical, what is the mathematics in it? He still didn't know. And I couldn't resist the urge because I told him, well what would you get if you do, on the calculator, 5 divided by 4? And he said "I don't know". He picked up his calculator and you should have seen his face when he did that and he got 1.25 and he said "Oooh!" Then he realized that it was 1 ¼, that it was 1.25.*

O1's account of Tyreal's struggle with figuring out that 5 divided by 4 would result in 1 ¼ was as a shocking revelation to the classroom teacher, who, a few minutes earlier, had proudly alluded to Tyreal's fluency in working with fractions.

While the students had been working on the Tug-of War problem, the teacher had spent a lot of time at Table 4, trying to understand Jynita's thinking and quite understandably, could not have been present at Table 6 to witness Tyreal's struggle with fractions. This is a further testament to the complexity of teaching and the difficulty, if not impossibility, of attending to each and every student's mathematical thinking.

4.6 Lesson Implementation and Debriefing #4

The following sections provide an overview of the fourth lesson implementation and the debriefing session. A more comprehensive description of these sessions is also provided which detail the observed occurrences of the video data in approximately 5-minute time intervals (see Appendices B4 and C4). The abbreviations R1 and R2 refer to the two university researchers while T3 identifies one of the teachers in this study, who facilitated the third implementation session. O3 identifies the teacher who, as a graduate student, participated in the lesson study and attended some of the implementation sessions as an observer.

4.6.1 Lesson Implementation #2: Overview

This session was approximately 75 minutes long and was facilitated by T4, a fifth grade teacher. In addition to the classroom teacher there were four other adults, R1, R2, T3 and O3, present in the room as observers. R2 was in charge of videotaping the session. There were 19 students in the classroom seven of whom, had not returned the signed consent forms and although they completed the activities and participated in the class discussions, they did not appear on the videotape.

The lesson included three tasks which were implemented in the following order: Activity 1- Bananas (approximately 13 minutes); Activity 2- Carrots (approximately 21 minutes); Activity 3 – Tug-of-War (approximately 19 minutes) (see Appendices A1, A2, A5).

The class began with the teacher introducing the university faculty member, R1, who addressed the students and explained, in simple terms, the purpose of the lesson and the presence of the observers in the classroom. After the introductions were made some time was spent on rearranging the room, so that the 12 students who had brought back the signed consent forms could be grouped together. Six long tables, arranged to form a large U shape, were set up in one section of the room to accommodate these 12 students, who could be videotaped. The students worked together in pairs on the problems during the lesson.

While the students were working on the problems the teacher and the observers circulated among the groups, overseeing their progress. The students appeared to remain on task throughout the lesson. Upon completing each task the teacher asked the students to share their solution strategies with the class. There was not an overhead projector used during this lesson and all the presenters who shared their findings with the class, remained seated at their table during their presentations.

In an effort to promote student understanding, the teacher used various techniques during this lesson. For example, while a student described his solution strategy for the Carrots problem, the teacher drew pictures of scales that represented the steps the student had taken in his reasoning. This visual re-enforcement proved useful as it appeared to help the student keep track of the steps and eventually led him to the correct answer. Also in order to demonstrate the concept of substitution on a balance scale, the teacher solicited the participation of four students and created two imaginary scales, where a quantity of weight in one scale, replaced an equivalent weight on the second scale.

Not all students were able to complete the Tug-of-War problem. At the end of the lesson the teacher told the class that they would continue working on this problem at a later date.

4.6.2 Debriefing #4: Overview

The debriefing took place in the classroom immediately following the lesson implementation and lasted for about 45 minutes. The classroom teacher, T4, expressed her satisfaction with the way the lesson had turned out: “I think it went beautifully”. She talked specifically about one of her students, Kieshe. She shared with the group that Kieshe, who was considered well below grade level, was the type who always worked very hard, but often came up with incorrect answers. The teacher had been supportive of Kieshe’s efforts all throughout the year, encouraging her to keep on doing her best and was very pleased that she had been such an active participant during this class.

There were some discussions on how several students were unable to think proportionally as they tackled the Carrots problem. R1 reminded the group that proportional reasoning did not come naturally to a lot of children at that age. They discussed Nazeer’s solution to the Carrots problem, which involved the concept of substitution. The teacher explained why she had started drawing scales on the board as Nazeer was explaining his thinking. It had seemed to her that Nazeer was moving things around on the scales and had a difficult time keeping track of the changes and explaining them at the same time. T4 told the group that being a visual person herself, she had begun losing track of what Nazeer was doing and drawing the scales on the board had helped her, as well as her students, to better follow Nazeer’s explanation.

The teacher also felt that her drawings of the scales had helped Nazeer realize his mistake and led him to make a self-correction. She wondered if she had offered Nazeer too much help: “Was that too much help? You know how teachers lead things sometime. I didn’t want to do that.” R2 replied: “I think if you really listen to the child and understand truly what they are saying, then represent it for them to help them get through it, you are not doing anything but making that tiny little bridge between A and B. But if you listen to them and hear what you want to hear and then make your representation, then it's whole other story. But I think you truly listened to him"

There were some discussions on specific students' behavior: there was a girl who was sulking because she had not brought the permission slip and could not be videotaped; there were some students who were feeling sleepy; there was a student who was talkative, but very bright and the teacher did not understand why he had been retained the previous year.

T4 showed the group an Algebra book she was planning to use as a resource for supplementary materials. R1 and T3 started looking through the book and R1 noted that, as a class, they had been working on all the big ideas mentioned the book, such as representation, balance, functions, proportional reasoning, variables, and inductive reasoning.

Some people thought that the third problem, the Tug-of-War, was too difficult for the students in this class. R2 thought that the students had been really engaged with the first two problems, but the third one had stumped them.

4.6.3 Descriptive Analysis: Lesson #4

The following is a descriptive analysis of the lesson implementation #4, with references to the *critical events* that occurred during the lesson (for a complete list of all the transcribed critical events, see Appendix G1-7). The selection of *critical events* involved careful deliberation of the research questions so that these events, taken individually, were snapshots offering glimpses into the lesson and collectively, the string of these snapshots contributed in shaping a narrative that provided the researcher an insight into the nature of the interactions that transpired in the classroom, and subsequently helped answer the research questions.

In the following descriptive analysis, the coded videotaped data was examined in conjunction with the students' work samples and, when appropriate, the teachers' reflections from the debriefing session. As a result, in this analysis, references to the teachers' reflections are at times intertwined with discussions on students' work and classroom occurrences. In order to better distinguish the classroom descriptions from the debriefing conversations, any discussion pertaining to the teachers' reflections will be italicized in the following analysis.

This session was facilitated by T4, a fifth grade teacher. There were 19 students in the class. In addition to the classroom teacher, T4, there were four other adults, R1, R2, T3 and O3, present in the room as observers. R2 was in charge of videotaping the session.

The class began with the teacher introducing the university faculty member, R1, who addressed the students and explained, in simple terms, the purpose of the lesson and the presence of all the observers in the classroom. After the introductions were made and about 12 minutes into the lesson, the teacher handed out the first activity sheet and the mathematical work began. T4 asked the students to work together and the camera showed all six pairs of students engaged in a conversation about the Bananas problem (see Figure 4.36 below, a copy of Figure 4.1). This problem was a pictorial depiction of a system of two equations with three unknowns in which, the reader was asked to re-define one of the variables in terms of one of the other two variables. Specifically, the reader had to figure out how many bananas weighed as much as one apple.

The students worked for approximately seven minutes on the Bananas problem before the first presentation began. The following is a descriptive analysis of some of the students' work and the interactions that took place chronologically at various tables as the students attempted to solve this problem.

<u>Table 3</u>

Two girls, D'nea and Destiny, were seated at Table 3 and worked on the Bananas problem together. In Event 1 (see Appendix G1), the teacher asked D'nea to present their findings to the Bananas problem:

Line # **Transcription**

T4: Alright, let's see. Who likes to start, who likes to share first?
[pause] D'nea, can you tell me about something you worked on?

D'nea: Ten, ten, ten bananas. Take away five.

T4: Take away five what? Chickens?

D'nea: No, five bananas [smiles and looks over at her partner, Destiny].

T4: Oh [pause], keep going [pause]. You took away five bananas.

D'nea: Yeah.

T4: You want to tell me why you took away five bananas?

D'nea: Because [pause].

T4: Your partner is right next to you.
[D'nea looks at Destiny and giggles. Destiny is busy writing something
down and does not look up.]

T4: You remember why you took five bananas?

[After a long pause D'nea smiles and shakes her head. Destiny looks up and smiles.]

T4: So why did you guys take away five bananas? You are trying to remember or process it?
[Destiny nods.]

T4: You want to finish and write it down and I come back?
[Destiny nods.]

As soon as D'nea uttered the first sentence of her explanation [line 3], the teacher asked her a clarifying question, which was meant to be slightly humorous [line 4]. Although D'nea responded to the teacher's question, she and her partner appeared reluctant to continue with their explanation or perhaps they were simply unable to explain their solution. The teacher encouraged the girls to justify their reason for taking away five bananas [lines 8, 10, 13, 16 and 17] and gave them ample time (about 80 seconds) to speak up, but when they remained silent, she asked them to put their thoughts in writing and report later [line 1].

Due to absence of students' written work samples, the data for this study did not include the documented student responses from this class. Therefore, it is not possible to know how D'nea and Destiny solved this problem or whether or not they arrived at the correct answer.

The focus of the class shifted to another student, Oscar, who was eager to answer the question the teacher had asked D'nea: Why did you take away five bananas?

<u>Table 1</u>

Oscar and Maybelea were partners and had worked on the Bananas problem together. In this episode Oscar explained why they had taken five bananas away (Event 1, continued):

Line # **Transcription**

Oscar: We did that because we divided 10 into 2, which gave us 5. And then we knew that take away 5 will be how much a pineapple weighs, as much as five bananas.

T4: where did you get 10 from and where did you get divided by 2?

Oscar: Because we counted how much bananas were on one side and we counted how much pineapple were on the other side.

T4: What do you mean by how much bananas?

Oscar: The amount.

T4: Okay, so how many bananas do you have?

Oscar: ten.

T4: Okay, and then [pause].

Oscar: Then we divided it by 2.

T4: And why did you divide it by 2?

Oscar: To see how much one pineapple weighs.

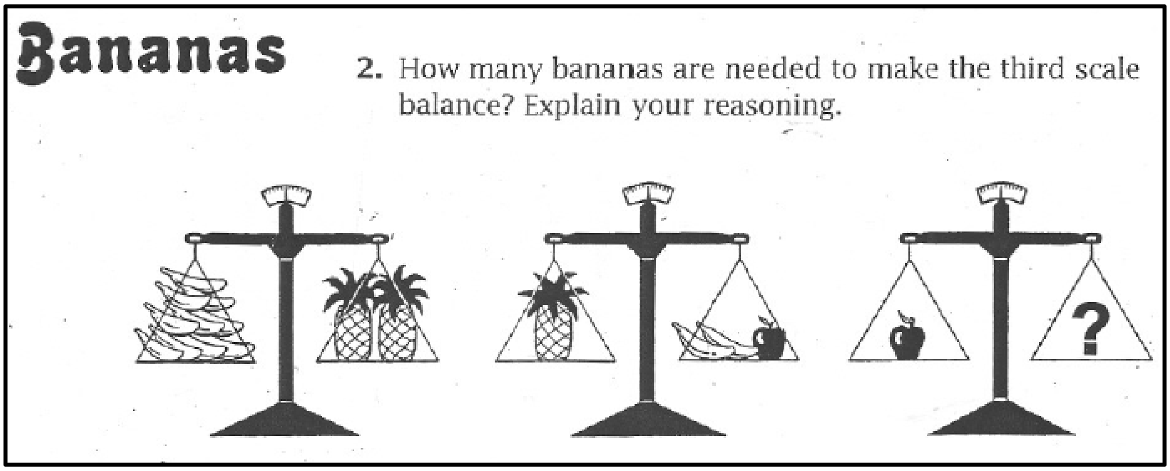
T4: Okay.

Oscar: And which gave us 5. And the next scale says one pineapple equal as much as two bananas and an apple. And since we know that a pineapple weighs as much as five bananas, we said, then we tried to figure out what number adds, goes equally with 2. And then we got 2 plus 3 which equals to 5 and then we knew an apple equals three pounds, three bananas.

T4: Do you have all that down on your paper?
[Oscar nods.]

T4: Okay, good.

Figure 4. 36: The Bananas Problem



Oscar explained how they took away five bananas from the first scale in order to determine the weight of one pineapple in terms of the bananas [lines 21-23]. The teacher however, kept on questioning Oscar, asking him to justify his every step [lines 24, 27, 29, 31, 33]. Oscar concluded his explanation by declaring that an apple weighed as much as three bananas [lines 36-41]. The strategy Oscar and Maybelea had used to solve this problem was similar to one of the strategies used by the teachers during the planning phase and identified in this study as Solution Path 1B, shown below:

- 10 bananas weigh as much as 2 pineapples (considering the first scale).
- Therefore 5 bananas weigh as much as 1 pineapple (cutting each side in half).
- On the second scale……….2 + ? = 5
- Therefore an apple weighs as much as 3 bananas.

After listening to Oscar's explanation, the teacher asked if he had written everything down on his paper. She then asked Keisha at Table 4 to present.

<u>Table 4</u>

Kieshe and Arlin were partners at Table 4 and worked well together throughout the lesson. Kieshe was seen on the videotape as often taking the lead in any mathematical ideas the girls came up with.

During the debriefing session that followed this lesson, the classroom teacher, T3, spoke about Kieshe: "The one over here that was talking a lot, Kieshe? She would be considered extremely below level, you know. And you know, she is the one who would work, work, work and her response would still be way over here."(see Teachers' Reflections, Appendix H4)

In Event 2 (see Appendix G2), Kiesha explained her reasoning as to why one apple was equal to two bananas in weight:

Line # **Transcription**

Kieshe: It's 10 bananas in all because we counted it on the first scale. So we took away two and put it, put it in the third scale. Because one apple weighs 3 pounds and 2 ounces because, [pause] oh, and 2 pounds, I mean 2 ounces and a banana weighs 1 pound [pause], 2 pounds and 3 ounces [pause]. Two bananas because an apple weighs 3 pounds, 2 ounces because, because I counted all the bananas on the first one [pause]. A banana, the banana has 10 and two pineapples. So if you take away two bananas of the first one, it'll be eight in all and the third scale will have one apple and two bananas because two pineapples weigh more than one apple. So therefore [pause], so therefore the first scale I left eight and the last scale which is the third, the third scale there is two bananas.

T4: Okay. Because you said a lot and I am trying to process what you were saying. Are you saying that you took two bananas off the first scale?

Kieshe: Yes.

T4: Okay. Did you do anything to the pineapples?

18	**Kieshe:** No.
19 20	**T4:** Okay. So if you take two bananas off and you leave eight bananas there and the two pineapples, does it, does the scale stay the same?
21	**Kieshe:** Yes [Arlin, ever so slightly, shakes her head, no].

Kieshe explained her reasoning as to why one apple is equal to two bananas in weight. Her explanation [lines 1-12] was difficult to follow and did not make much sense (except perhaps to herself and possibly to her partner, Arlin). The teacher questioned Kieshe about her decision to transfer two bananas from the first scale to the third scale. She did not ask why Kieshe had done this, but wanted to know whether this action had any impact on the first scale [lines 19 and 20]. Kieshe replied that the scale would stay the same, but her partner, Arlin, shook her head no. The teacher however, asked another student, Nazeer, if he agreed with Kiesea (Event 2, continued):

Line #	**Transcription**
22	**T4:** Do you agree? Nazeer, do you agree?
23 24 25	**T4:** She said she took two bananas off the first scale. She had eight bananas on one side of the scale and two pineapples on the other side. She said the scale won’t change. Do you agree?
26	**Nazeer:** No.
27	**T4:** Why not? Tell why not.
28 28 30	**Nazeer:** Because 10 bananas weigh as much as two pineapples and you take off two of the bananas, then the pineapples are going to, pineapples are going to weigh more.
31	**Kieshe:** I get it.
32	**T4:** Do you get it? Alright.

Nazeer disagreed with Kieshe that the scale would remain the same after the removal of two bananas. Although he did not say how the position of the two pans on the scale would change (uneven pans or pineapples lower and the bananas), he reasoned that once the two bananas were removed, the pineapples would weigh more than the remaining eight bananas [lines 28-30]. Immediately, Kieshe indicated that she understood Nazeer's explanation.

Further examination of the wording of the teacher's questions (does the scale stay the same? Does the scale change?), coupled with the fact that English was not Kieshe's first language, raised the possibility that perhaps Kieshe had not truly understood the question she had been asked. In the teacher's questions there were no references to weight or balance, only if the scale had stayed the same or had changed. It is possible that perhaps Kieshe was thinking to herself: Of course the scale has not changed; it is still the same scale! And if Keisha had indeed understood the question but truly did not know how the removal of weight from one pan would impact the balance on a scale, then a prior class discussion on pan balance related concepts, as done by T3 during the previous lesson implementation, might have been beneficial to the students in this class, and in particular, to Kieshe.

No other student was asked to report on the Bananas problem and the teacher started handing out the next activity sheet. In the absence of students' work samples from this class it is impossible to know how many students, apart from Oscar and his partner, arrived at the correct answer to the Bananas problem.

However, for approximately 8 minutes, while the students were working on the second task (the Carrots problem), R1 was still engaged in a conversation with Nazeer and his partner, Jarrod, about the Bananas problem. Parts of their conversation was captured on tape, which showed R1 questioning the boys' reasoning and trying to make them aware of the fact that although their answer was correct, their reasoning was flawed. At some point in his explanation, Nazeer had told R1 that a pineapple weighed as much as five bananas and had also asserted that a pineapple weighed as much as three apples. R1 tried to make the boys realize the infeasibility of their claim that a pineapple weighed as much as three apples:

R1: You claim that one apple is the same weight as three bananas (pointing to the last scale). So how many bananas would be equal to two apples?

Nazeer: six bananas.

R1: And how many bananas would equal to three apples?

Nazeer: Nine bananas.

R1: So you are saying three apples equal nine bananas. You also claim that three apples equal one pineapple, which means that one pineapple is equal to nine bananas. But you had convinced me before, using the first scale, that one pineapple is equal to five bananas. Which one is correct, five or nine?

Nazeer: Five bananas.

Nazeer was able to correctly conclude that if an apple weighed as much as three bananas, then three apples would weigh as much as nine bananas. Using the Transitive Property of Equality (if a = b and b = c, then a = c), R1 argued that Nazeer's claim that three apples weighed the same as nine bananas and also the same as one pineapple, implied that a pineapple weighed as much as nine bananas, which contradicted his earlier assertion that a pineapple weighed as much as five bananas. Nazeer however, seemed unable to follow the logic of R1's reasoning. Mindful of the fact that Nazeer had not even begun working on the Carrots problem, which some students had already an answer for, R1 discontinued her conversation with Nazeer and allowed him to start working on the next task.

Trying to catch up with the rest of the class, Nazeer began working on the Carrots problem. Similar to the Bananas problem, this task was a pictorial depiction of a system of two equations with three unknowns and the reader was asked to re-define one of the variables in terms of one of the other two variables. Specifically, the reader was asked to re-define the weight of a pepper in terms of the weight of carrots.

Approximately 10 minutes after the Carrots problem was first introduced, the teacher asked the boys at Table 5, Rene and Dajuan, to share their findings with the class. The boys claimed that a pepper weighed as much as one carrot, but could offer no justification for their assertion. The teacher asked them to think some more about the problem, write down their thinking, and report later. The teacher asked Nazeer and Jarrod to report on their findings. The following is a descriptive analysis of Nazeer and Jarrod's work on the Carrots problem (see Figure 4.37 below, a copy of Figure 4.2) and the interactions that took place between them and the teacher as they presented their solution.

<u>Table 2</u>

Jarrod and Nazeer had about six minutes to work on the Carrots problem before they were asked to report on it (see Event 3, Appendix G3):

Line # **Transcription**

Jarrod: Well. Well, me and Nazeer said that, uhm, uhm the first scale which has six carrots and a corn, uhm and a pepper equals uhm,… [Nazeer whispers something to Jarrod, who then pushes the paper toward Nazeer].

Nazeer: Three peppers equal six carrots, because on the second, on the second, uhm scale we have one corn and two peppers and on the first scale we have one corn and one pepper. So we, uhm replaced the one corn with two peppers in [pause].

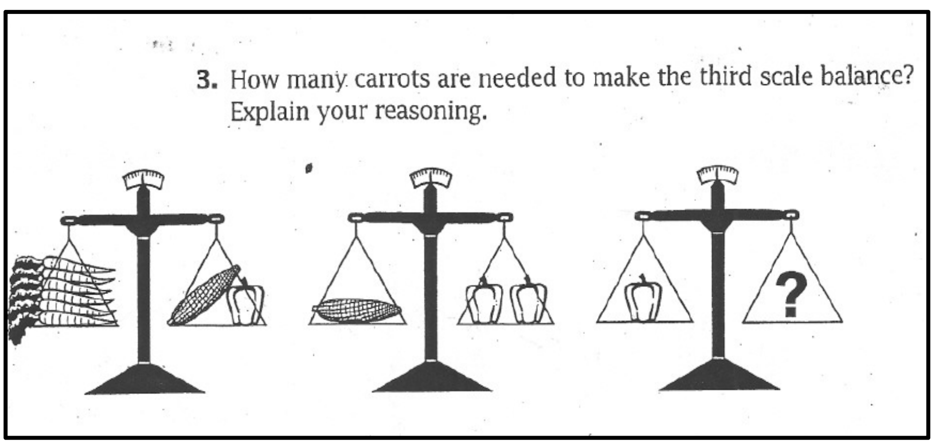
T4: Where?

Nazeer: We replaced the two peppers from the second scale and put it on, replaced it with the corn on the first scale. And so, three peppers equal the same amount as six carrots.

T4: Does everyone hear what he saying? He is saying he took the ear of corn from here (1st scale), okay? And put it over here (2nd scale). And took these two peppers and put them here (1st scale). So now he has a corn equals a corn (2nd scale) and he has three pepper equals six carrots. Is that what you just said Nazeer?

Nazeer: Yes.

Figure 4. 37: The Carrots Problem



Jarrod started the presentation but since he was struggling to formulate his explanation, Nazeer volunteered to take over [lines 1-4]. Nazeer explained that since two peppers were equal (in weight) to one corn, they swapped the positions between the corn on the first scale and the two peppers on the second scale and concluded that six carrots were the same (weight) as three peppers [lines 5-12]. The teacher then repeated Nazeer's explanation and asked him to verify its accuracy [lines 13-17]. T4 encouraged Nazeer to continue with his explanation (Event 3 continued):

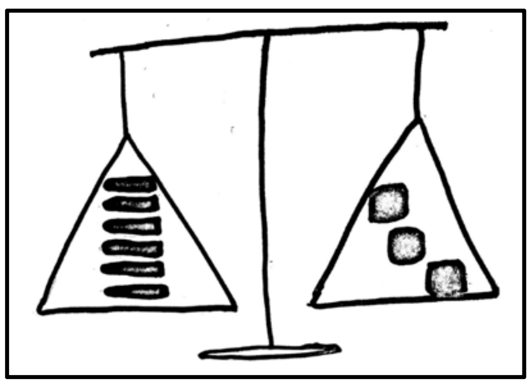
Line #	**Transcription**
19	**T4:** Okay. Keep going [addressing Nazeer].
20	**Nazeer:** One ear of corn equals three carrots. Because if we took off
21	half the carrots and one pepper, it would be the same, it would be the
22	same weight because..Help me out Jarrod.
23	Jarrod: Uhm, uhm. Because if you uhm, take half of the uhm, six
24	carrots and you took away three carrots and you took away a chilli
25	pepper, they both equal to, uhm, uhm, same amount as, as, as three
26	carrots [pause].
27	**Nazeer:** Oh, I get it now. Three carrots is the same amount as two
28	corns, I mean two peppers. Because one corn equals two peppers and if
28	we take away both the corn and one pepper, if we take away a corn and
30	add one pepper, we would have the same amount as three carrots. And,
31	and then, ugh… [pause], no that's because uhm, one corn is two
32	peppers and two peppers equals three carrots because [long pause].

It appeared as if Jarrod and Nazeer had not had the chance to finish working on this problem as Nazeer was clearly struggling to decide on what the next step should be [lines 27-32]. It was interesting to note that although the boys had initially substituted two peppers for a corn on the first scale, they did not follow up on that idea. They even considered the possibility of a pepper weighing the same as three carrots [lines 20-26], which contradicted the 2 to 1 relationship that existed between the number of peppers and the number of corns.

During the debriefing session that followed, R1 commented on the boys' work: "They weren't thinking at all proportionally. We frequently don't realize, is not natural for their thinking right now. So of course it is hard for them to see that the corn is double the pepper. You know what I am saying?" (See Teachers' Reflections, Appendix H4)

As Nazeer and Jarrod tried to come up with the next step in solving the problem, the teacher drew a picture of a scale on the whiteboard. This picture reflected the changes on the first scale after the boys' initial step of substituting two peppers for a corn and showed six carrots balancing three peppers. A reconstructed version of the teacher's initial drawing as observed on the video is shown in Figure 4.38 below.

Figure 4. 38: Teacher's Drawing on the Whiteboard - Stage 1



The teacher continued her conversation with Nazeer (Event 3 continued):

Line # **Transcription**

T4: Okay. Does this help you out a little bit?
[Camera shows a diagram that T4 has drawn on the whiteboard of a scale with six carrots on one pan and three peppers on the other pan.]

T4: Because it seems to me like you are trying to visualize something but you didn't change it on your paper. So you are trying to remember and you are trying to say what you are trying to do. Here is what you said. You said you took, you took the two peppers and you put them here, and you took the ear of corn and put it here [pointing to the pictures on the paper]. Okay? So we are looking at the one scale, we are looking at this one scale. So the new scale looks like this [pointing to her drawing on the board], six carrots and the three peppers. Okay? So, now tell me, because what are we looking for? What are we trying to find?

Nazeer: How many carrots equal one pepper?

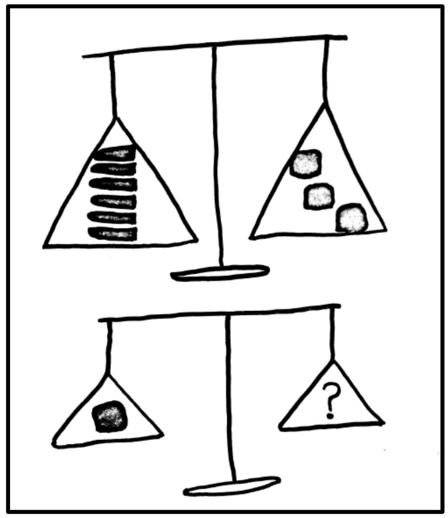
T4: Okay. So, then what we are trying to find is, the scale, what does this scale look like.[T4 draws a second scale on the board with one pepper on one pan and a question mark in the other pan.] And I am such an artist! So we trying to figure out what goes in here [pointing to the question mark], right? Does that help you any?

Nazeer: Yes.

After drawing a picture of the “new” first scale, the teacher reviewed the steps Nazeer had previously taken in the substation of two peppers for a corn [lines 39-43]. She then questioned Nazeer on what the problem was asking him to do [lines 44 and 45]. When Nazeer replied that the objective was to find the number of carrots that balanced one pepper [line 46], the teacher drew a picture of a second scale on the whiteboard. The new drawing represented the objective of the problem, showing a pepper balancing a “?” on a scale.

A reconstructed version of the teacher’s second drawing as observed on the video is shown in Figure 4.39 below.

Figure 4. 39: Teacher's Drawing on the Whiteboard - Stage 2



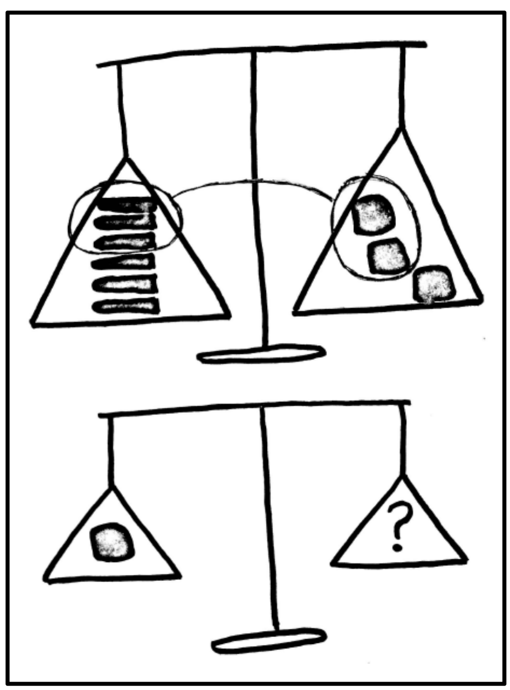
During the debriefing session that followed, the teacher spoke about her decision to draw the scales on the whiteboard. She reported that she often drew pictures and diagrams during the lessons because she herself was a visual person and she had decided to draw the scales because she had been losing track of what Nazeer was saying. She had hoped that the students themselves would draw pictures of the scales to express their thinking, but since they had not, she began to draw scales on the board as Nazeer was explaining his thinking. It had appeared to her that Nazeer was moving things around on the scales and had a difficult time keeping track of the changes while explaining it at the same time. The teacher also felt that seeing the new scales on the board might have helped several other students follow Nazeer's reasoning. The group agreed that it had been a good idea to represent Nazeer's thinking through a diagram.

Having shifted Nazeer's focus from the pictures on his paper to the drawings on the whiteboard, the teacher asked him to continue with his search for an answer to the problem (Event 3 continued):

Line #	Transcription
53	**T4:** Okay, so now explain to us what’s, what we do next.
54 55	**Nazeer:** We have to take away the same amount of peppers, the same weight of peppers as we did, uhm, the weight of carrots.
56	**T4:** Okay.
57	**Nazeer:** And two peppers equals three carrots.
58 59	T4: So you are saying these two peppers is the same, is the equivalent of three carrots?
60	**Nazeer:** yes.
61	**T4:** So then three carrots equals, is equivalent to this last pepper?

When Nazeer claimed that two peppers weighed as much as three carrots [line 57], the teacher drew two connecting circles, one around three carrots, the other around two peppers, as shown in Figure 4.40 below.

Figure 4. 40: Teacher's Drawing on the Whiteboard - Stage 3



Using Nazeer"s claim, the teacher inferred that three carrots were equivalent to the one remaining pepper and she put that idea to him in the form of a question [lines 58 and 59]. Although the teacher's tone of voice, when posing the question, did not indicate an element of surprise, which might have hinted at the inaccuracy of the assertion that three carrots were equivalent to the one pepper, Nazeer appeared to be confused and unsure as how to answer the question (Event 3 continued):

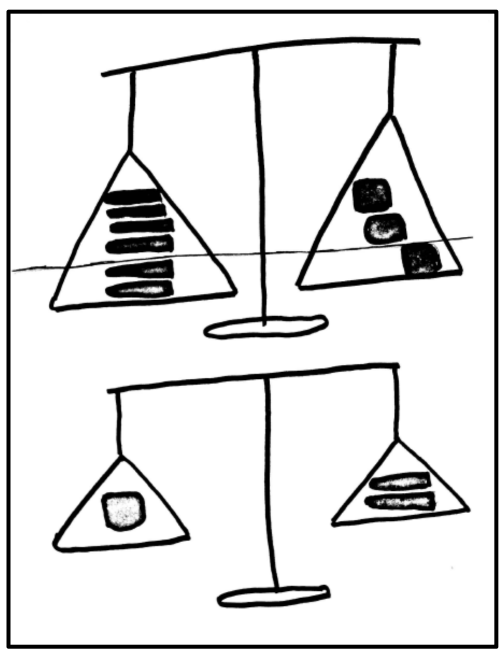
Line #	**Transcription**
62 63	**Nazeer:** wait [looks confused and doubtful, then smiles]. No, I meant two peppers, two peppers is the same as four carrots.
64	**T4:** Okay. So then here it is. So now two carrots is equal to this pepper.
65	**Nazeer:** Yes. [Jarrod nods.]
66	**T4:** And these four carrots are equal to the two peppers.
67	**Nazeer:** Yes.
68	**T4:** Okay. So now, are we done?
69	**Nazeer:** One pepper equals two carrots.

After a few seconds of looking pensive, Nazeer broke into a smile and declared that two peppers were the same weight as four carrots [lines 62 and 63] and concluded that a pepper weighed as much as two carrots [line 69]. As shown in Figure 4.41 below, the teacher made a final adjustment to her drawings on the whiteboard to represent Nazeer's correct answer.

One of the strategies used by the teachers during the planning phase, and identified in this study as Solution Path 2B, was as follows:

- Consider the relationship shown on the second scale.
- On the first scale, replace the corn with 2 peppers.
- Now you have <u>6 carrots</u> Vs. <u>3 peppers</u>.
- Divide each side by 3 to get: <u>2 carrots</u> Vs. <u>1 pepper</u>.
- Therefore a pepper weighs as much as 2 carrots.

Figure 4. 41: Teacher's Drawing on the Whiteboard - Stage 4



Nazeer's strategy in solving the Carrots problem was similar to Solution Path 2B in the way that the peppers substituted the corn on the first scale. However, there was no evidence that Nazeer was dividing six (carrots) by three (peppers), as was done in Solution Path 2B, in order to get to his answer of two carrots. The fact that he first figured out that two peppers were the same as four carrots and used that information to conclude that one pepper was the same as two carrots, indicated that perhaps he was using the drawings to visually distribute the six carrots evenly among the three peppers.

During the debriefing session that followed, there were further discussions on this episode. T4 spoke about her decision to correlate three carrots to two peppers after Nazeer's incorrect assertion: "I was really trying to do it so I could help him. But I wanted to make sure that was what he said." (see Teachers' Reflections, Appendix H4) R1 praised the teacher's technique, saying that Nazeer's "self -correction once he saw the picture was very good".

The teacher however, wondered if she had offered too much help: "I don't know if, okay, was that too much help? You know how teachers lead things sometimes? I didn't want to do that" (see Teachers' Reflections, Appendix H4) R1 reminded the teacher that she had not corrected Nazeer and it was Nazeer who had made the correction himself. R2 added: "I think if you really listen to the child and understand truly what they are saying, then represent it for them to help them get through it, you are not doing anything but making that tiny little bridge between A and B. But if you listen to them and hear what you want to hear and then make your representation, then it's whole other story. But I think you truly listened to him" (see Teachers' Reflections, Appendix H4)

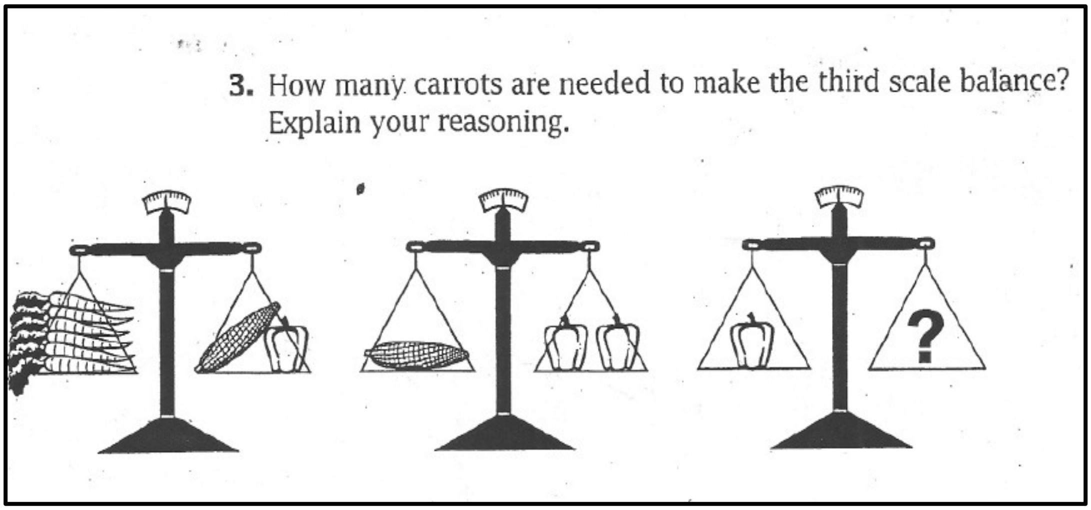
Following Nazeer's presentation, the teacher asked if anyone in class had used a different strategy to solve the carrots problem. Oscar raised his hand and the teacher called on him to explain his solution. The following is a descriptive analysis of Oscar and Maybelea's work on the Carrots problem (see Figure 4.42 below, a copy of Figure 4.2).

Table 1

Oscar and Maybelea had solved the Carrots problem using a different strategy than the one previously described by Nazeer. In Event 4 (see Appendix #), Oscar volunteered to share their solution with the class:

Line #	**Transcription**
1	**Oscar:** We did it differently.
2	**T4:** Okay.
3	**Oscar:** On the first scale, six carrots, a corn and a pepper.
4	**T4:** Okay.
5	**Oscar:** So we assumed that a corn and pepper was as much as three
6	carrots until we got to the second scale. A corn was three carrots and
7	two peppers for three, wait. Each pepper weighs as much as three
8	carrots. So then we added 3 plus 3 (on the second scale), because there
9	were two peppers and they were 6 and 3. We noticed that they were not
10	balanced. So then we tried with other ones, 2 and 4. The pepper, the
11	corn weighs as much as four carrots and the pepper weighs as much as
12	two. On the second one (scale) is corn, four carrots, and two peppers,
13	each two carrots. So then we add 2 plus 2, which gave 4 and they were
14	balanced. So we, and then the third one (scale) is one pepper equals as
15	much as a question mark and the question mark is two carrots.
16	**T4:** Okay. Any questions for any of the groups?

Figure 4. 42: The Carrot Problem



Oscar's explanation indicated a fundamental understanding of the interdependence of the three scales which could not be viewed and interpreted in isolation from one another. He described how the examination of the second scale had helped them recognize their initial incorrect assumption that a corn and a pepper each weighed as much as three carrots [lines 5-10] and how they had used the second picture to test the validity of their final answer [lines 10-14].

At first glance it appeared as if Oscar and Maybelea had used the strategy identified in this study as Solution Path 2C in which, a focus on the 2 to 1 relationship between the number of peppers and the number of corns, as represented on the second scale, guided the discovery of the correct answer. Solution Path 2C was as follows:

- Identify the 2:1 relationship shown on the second scale.
- Based on this realization, consider the first scale and pick two numbers that have a 2 to 1 relationship, representing a corn and a pepper respectively and add up to 6 carrots (4 + 2 = 6).
- A pepper weighs as much as 2 carrots.

However, in his explanation Oscar never alluded to the fact that a corn weighed twice as much as a pepper. Although Oscar and Maybelea had tested the numbers 4 (for corn) and 2 (for pepper) to verify that they balanced the second scale, there is no evidence that they had picked these two numbers (4 and 2) because one was twice as large as the other. It is possible that perhaps Oscar and Mayblea had also tried the numbers 1 and 5 and rejected them since they did not balance the second scale. Since there was no reference to the 2 to 1 ratio between the number of peppers and corns in Oscar's explanation, it cannot be determined whether Oscar and Maybelea had recognized the existence of such relationship.

After Oscar's presentation, the teacher asked if anyone had any questions to ask or if anyone has any difficulties with what they had discussed in class thus far. Kieshe raised her hand and said she did not understand how items were moved from one scale to another. In order to help Kieshe, the teacher solicited the participation of four students and created two imaginary scales, where a quantity of weight in one scale, replaced an equivalent weight on the second scale (see Event 5, Appendix G5). The following is a descriptive analysis of what occurred as the teacher attempted to construct the imaginary scales.

The teacher asked Rene to join her in front of the room so he could help her with a demonstration:

Line # **Transcription**

T4: Okay. Now. Rene, could you stand up for me please. If we were to get on a scale, stand up like this. And I were on one side of the scale and Rene was on the other side of the scale, do you think it would be balanced?

Kieshe: No.

T4: Okay, you don’t think it would be balanced. Why not?

Kieshe: Because you are heavier.

T4: Ohhh, you say I am heavier? Okay, yes, I am heavier. So what we need to do, because at this rate what would the scale look like? Would it look, as you said, at a line, would it be in a line [holding two palms up]?

Kieshe: No.

T4: It would look like this [palms up at different levels]. And Rene would be up here, right? And I would be here.

Dajuan: Because you are heavier, you will bring it down.

T4: I am here [lower hand] and Rene is here [higher hand]. So what do we have to do with this [higher hand]? Tell me what we have to do with this. With this part of the scale where Rene is.

Kieshe: [unclear]…two people go to Rene.

Rene and the teacher stood a few feet apart, pretending to be on the opposing sides of a scale. The teacher asked Kieshe whether the scale would be balanced [lines 3 and 4]. Kieshe said that the scale would not be balanced because the teacher was heavier than Rene [lines 5, 7]. Holding her two palms up, the teacher then asked Kieshe whether the two pans would be aligned line 10]. Kieshe said they would not be on the same line and Dajuan noted that because the teacher’s side was heavier, it would be lower than Rene’s side [line 15]. The teacher asked Kieshe for suggestions on how to balance the scale and Kieshe proposed that they add two more students to Rene’s side [lines 16-19].

Following Kieshe’s suggestion, the teacher asked Arlin and Dajuan to join Rene on the imaginary scale and noted that according to the scale, which was supposedly balanced, she weighed the same as Arlin, Dajuan, and Rene combined and therefore the two sides of the scale stood in alignment. The teacher then asked a fourth student, Cristian to join the scale on her opposing side and Dajuan immediately remarked that their side would then be heavier and therefore would stand at a lower level than the teacher's side. Once again the teacher asked Kieshe to balance the scale (Event 5 continued):

Line #	**Transcription**
20	**T4:** your side goes down and my side goes up. So kiesha, tell me what
21	I have to do to make it balance.
22	**Kieshe:** You could take out two people?
23	**T4:** Take two people off? Which two people?
24	**Kieshe:** Arlin and Dajuan.
25	**T4:** Arlin and Dajuan. So move, move Arlin and Dajuan. Thank you.
26	**T4:** Before we knew that those three and I balanced out, right? What
27	makes you think Cristian and Rene balance it out?
28	[Some kids try to call out.]
28	**T4:** Hold on. Hold on one second.
30	**Kieshe:** Because they are the same size.
31	**T4:** But, what does that, in relationship to me and my weight. We have
32	to figure out what Cristian weighs. We have to figure that out, because
33	we don't know if this would work, unless we know how much Cristian
34	weighs. If there is a scale that says Cristian weighs as much as these
35	two (Arlin and Dajuan stand opposite Cristian on a second imaginary
36	scale)) okay? So now, when I am on the scale over here and then
37	Cristian comes over here [Cristian moves to the first scale and stands
38	next to Rene], right? When Cristian comes over here, now we can
39	validate, or we can say, yes, since he (Cristian) weighs the same as
40	those two (Arlin and Dajuan), right? And when they (Arlin and Dajuan)
41	were on the scale with Rene, it was balanced, right? We took those two

off and we replaced it with Cristian, which weighs as much as those
two. So now the scale is balanced again. Because we took off
something, but we put the same weight back to keep it balanced.
Because if we took off something off one side, they would be lopsided,
right? We want to be balanced. Right? That’s what the scales are about.

T4: Does that help you?

Kieshe: Yes.

T4: It helped you too [addressing Yaasmyn and her partner]?

After adding Christian to the scale, where T4 had previously balanced the three students, Kieshe claimed that taking Arlin and Dajuan off would adjust the scale back to a balanced position [line 22, 24]. She justified her claim by saying that Arlin and Dajuan appeared to be the same size [line 30]. Perhaps Kieshe was misapplying the notion of removing equal weights from the opposing sides of a balanced scale when she removed Arlin and Dajuan (same size) from the same side of an already unbalanced scale. Since the teacher did not ask her for clarification, it is not clear what Kieshe exactly meant by the statement that Arlin and Dajuan were the same size.

At the debriefing session that followed this lesson, T4 spoke about Kieshe’s suggestion to remove Arlin and Dajuan from the scale: “I try to let them lead it, you know, the kids. I always try to take whatever the child says and make it work somehow. So when she said, when at first she said take them out, instinctually I was about to say, no, just take Cristian out. But you know I am glad that made me think of the other scale to show her that way. So I think that might have helped her” (see Teachers’ Reflections, Appendix H4.)

The teacher decided to build on Kieshe’s idea of removing Arlin and Dajuan from the scale by setting up a condition that accommodated her suggestion. She set up a second imaginary scale on which, Arlin and Dajuan balanced Cristian on the opposing side [lines 34-36]. She then demonstrated that replacing Arlin and Dajuan with Cristian on the first scale would restore the equilibrium only because the combined weight for Arlin and Dajuan was the same as Cristian's weight [lines 40-46].

By constructing the imaginary scales and recruiting the students to model as weights, the teacher was able to demonstrate the concepts of equality, inequality, and substitution. However, in spite of some students' claim that they found this demonstration helpful [lines 47-49], the degree of its effectiveness cannot be determined in this study.

Approximately 45 minutes into the lesson the teacher began preparing for the next activity, the Tug-of-War. The teacher told her students that one of the activities they did on field day every year reminded her of scales (see Event 6, Appendix G5):

Line # **Transcription**

T4: Oh, there is an event. There is an event that kind of reminds me the scales. At field day, every year.

A girl: Tug-of-war.

T4: Tug-of-war? Why does that remind you of the scales?
[Some raise their hands and some attempt to call out.]

T4: Hold on one second, Maybelea?

Maybelea: Because, if there is more people, like there is more people on this side, this side will win. Because there is more force…

Cristian: How about the other side is stronger than the side that have more people?

T4: Okay, let's say there are 25 kindergarteners and there is 19 of you. Who do you think, do you think the 25 kindergarteners are definitely going to win?

14 **Some students:** No!

15 **T4:** Why not?

16 **Some students:** We are stronger.

17 **T4:** Is that what you were saying [asking Cristian]? He is saying, what
18 about the strength? So, is it about the number on each side?

19 **Some students:** No.

20 **T4:** Or is it about, what is it about? Strength or the weight?

21 **Some students:** Strength.

22 **Some students:** weight.

23 **T4:** Okay, so I am going to let you tell me.

As soon as the teacher said that one of the activities on field day reminded her of scales, one girl identified the tug-of-war as the activity the teacher had in mind [lines 1-3]. It was interesting to note that during the previous lesson implementation, Kevin had made the same connection, but in reverse. Unlike in this lesson when the teacher mentioned the scales and a students related it to the tug-of-war, in the previous lesson T3 had held up a picture of the students playing the tug-of-war and Kevin had immediately said that it was like the scale.

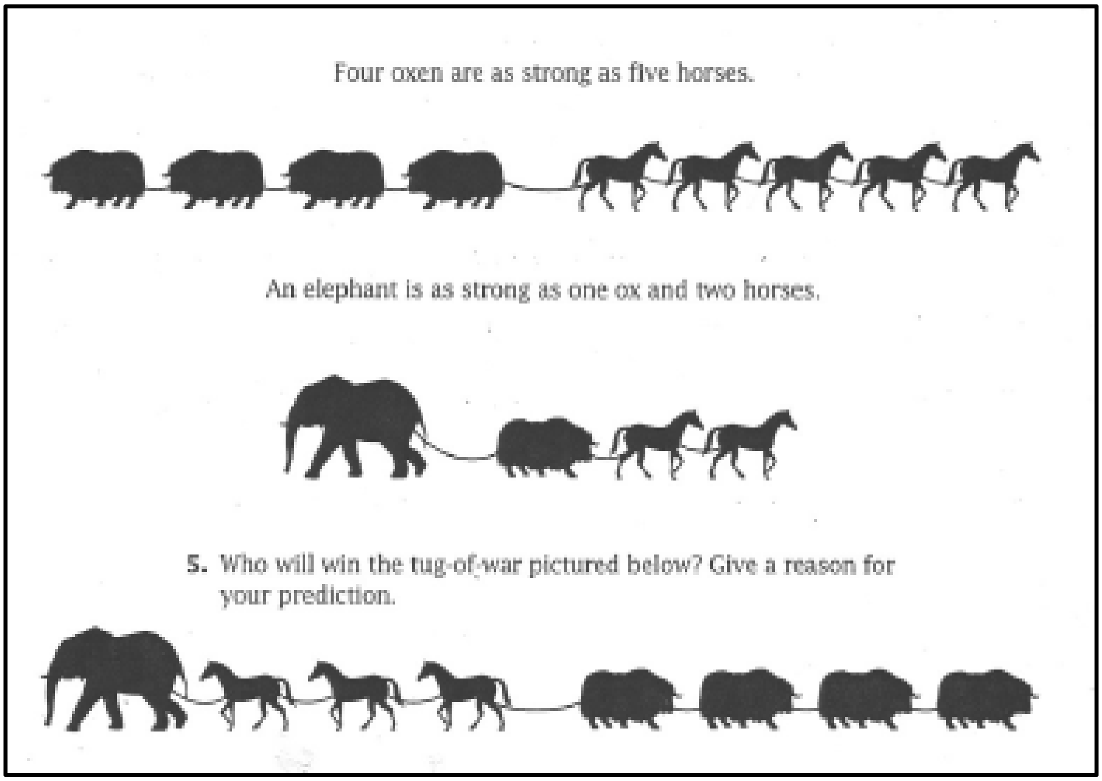
When the teacher asked how the scale resembled the tug-of-war, several students raised their hands and Maybelea replied that in a tug-of-war, the side with more people, and therefore more force, would win the game [lines 4-8]. Cristian questioned the validity of Maybelea’s statement, alluding to the fact that it was not the sheer number of the people on one side, but their combined strength that determined the winner of the game [lines 9 and 10].

Rather than agree or disagree with Cristian, T4 provided the opportunity for students to draw their own conclusion. She presented a scenario in which, 25 Kindergarteners played against 19 (the number of students in this class) fifth graders and asked the students if they thought that the Kindergarteners would definitely win the game [lines 11-13]. Some students excitedly shouted, no, that they were stronger than the kindergarteners [lines 14, 16]. The teacher emphasized the conclusion by asking whether it was the number of people that determined the winning side and some students replied no [lines 17-19]. The teacher then asked the class to decide whether it was the combined strength or the weight of the people that determined the winner in the game [line 20]. Some students thought it was the weight, while others said it was the strength [lines 21 and 22]. The teacher decided to leave it at that and let the students come to a conclusion at a later time [line 23].

It is possible that those students, who regarded the weight to be the deciding factor in determining the winner in the tug-of-war, were perhaps remembering its connection to balance scales, which inherently represent the concept of weight.

Approximately 56 minutes into the lesson the teacher handed out the Tug-of-War problem (see Figure 4.43 below, a copy of Figure 4.5). This problem was a pictorial depiction of two equations representing equality in the strength of two groups of animals, and an inequality representing the winning team. The reader was supposed to consider the information provided by the equations in order to decide which side of the inequality was the greater side. Specifically, the reader had to predict which group of animals would win the tug-of-war.

Figure 4. 43: The Tug-of-War Problem



The students worked for about 10 minutes on this problem before the teacher asked if anyone wanted to share their thinking about the problem with the class. She told the class that since some people had not finished working on the problem, they were not to give out their answer, but rather talk about things they had noticed about the problem.

Arlin from Table 4 said that the second picture helped her decide the winner in the tug-of-war and that the elephant is stronger than the oxen. Daniel, who was not being videotaped, said that according to the first picture, an ox was as strong as two horses, which implied that in the second picture, an elephant was as strong as four horses. Oscar from Table 1 believed that an ox was stronger than two horses and based on that assumption he eliminated the ox from the second picture and concluded that an elephant was as strong as two horses.

While reminding Oscar that the tug-of-war was about strength and not weight, the teacher gathered the three students who had previously helped her in constructing the imaginary scale. She reminded Oscar that the three students weighed the same as her and asked Oscar whether the scale would remain balanced if she took one of the students off the scale. Oscar acknowledged that it would not. She then brought Oscar's attention to the second picture in the Tug-of-War problem and asked if one could take out the Ox and claim that the strength of an elephant still balanced the strength of two horses. Oscar agreed that one elephant could not be as strong as two horses.

The class ended with the teacher collecting the students' work, saying that they would revisit and complete the work on the Tug-of-War problem at a later date.

During the debriefing session that followed the teachers spoke about the Tug-of-War problem, which was originally selected by the teachers to be used in Grades 7 through 9, but T4 had decided to include it in her lesson plan and try it out with her students. R2 remarked: "The first two (Scale problems), they (students) were really engaged in, but I think the third one (tug-of-war) really stumped them" (see Teachers' Reflections, Appendix H4). The classroom teacher disagreed with R1's observation and said: "I don't think so. I don't think that they were, I don't think they recognize. I don't think that they think that they are stumped, so to speak. I think it is taking them a little longer to come up with some stuff but I don't know yet" (see Teachers' Reflections, Appendix H4).

One of the observers, O3, thought that the Tug-of-War problem was too difficult for Dajuan and Rene. He told the group: "They were stumped. They were just sort of sitting there, you know. You could tell they were overwhelmed. I asked them if they *needed help getting started and I think Rene said, no. So I said, well if you change your mind I'll be happy to help you get started. And then they asked, they said, okay" (see Teachers' Reflections, Appendix H4).*

R1 asked O3 to describe the kind of help he had offered them. O3 explained: "I just asked him to look at the elephant and the oxen and the horses and to substitute. I just said look down. What equals an elephant down in the last one? And as soon as I said that, he got it [snapping his fingers] and he replaced the elephant with the horses and the oxen" (see Teachers' Reflections, Appendix H).

R2 reminded the group of the strategy that one of the observers had used during the previous lesson implementation, in which a cutout of the ox and horses from the second picture was placed on top of the elephant in the bottom picture in order to help the student visualize the concept of substitution. R1 commented: "What I keep wanting to do is, what kind of question can you ask that doesn't do it for them? I mean for me cutting it and putting it down here is giving them the answer, and I don't want to do that" (see Teachers' Reflections, Appendix H4).

4.7 The Nature of Teacher Reflections

Many examples of teachers' reflections and comments from the debriefing sessions were embedded within the descriptive analysis of the lesson implementations in sections 4.3.3, 4.4.3, 4.5.3, and 4.6.3 above. In this section, additional examples of teachers' reflections during the debriefing sessions will be presented and the nature of the comments will be examined.

Fernandez (2003) believed that for lesson study to work as a feasible form of professional development, the teacher participants must view the lesson study process through three critical lenses: the research lens, which encourages the teachers to ask questions about certain aspects of the practice and motivates them to design classroom experiences to address these questions; the student lens, which allows the teachers to examine the lesson from the students' perspective; and the curriculum developer lens, which helps the teachers with the organization and sequencing of the learning experiences.

Building on Fernandez's work, the researcher developed a set of operational definitions for three lenses, research, curriculum, and student (see Chapter 3, section 3.4.3), which were used in the coding of teacher reflections. The researcher identified *episodes* of teacher reflections from the four debriefing sessions (see Appendices H1-4). In this study an *episodes* referred to a single comment or question posed by one teacher or a cluster of comments or questions about a topic that involved one or more teachers. While examining the *episodes* from the videotaped debriefing sessions, the operational definitions for the three lenses were developed and used to identify and code the nature of the teachers' comments and reflections. This resulted in a number of examples of coded teachers' comments/reflections. Some of the teacher comments were assigned more than one code when appropriate. Examples in which an overlapping of the lenses occurred will be provided later.

The coding of the teachers' reflections provided an insight into the nature of the conversations that occurred during the debriefing sessions. Teachers' reflections from the four debriefing sessions will be collectively examined in three sections, each relating to the use of one of the three lenses: student, curriculum, and research.

4.7.1 Adopting the Student Lens

After viewing the debriefing videos, the researcher identified and transcribed *episodes* of teachers' comments or questions that helped answer the research question #4 regarding the teachers' reflections on the lessons and students' work. Using the operational definitions, the transcribed *episodes* (see Teachers' Reflections, Appendix H1-4) from the videotaped debriefing sessions were examined to identify the teachers' comments and reflections that pertained to students and their work.

The following working definitions were used in this study to describe the situations when student lens was used by the teachers:

1. Discussing student behavior/attitude
2. Discussing student misconceptions and/or understandings
3. Discussing student mathematical work/strategies

The teachers appeared to adopt the student lens more often than any other lens during the debriefing session. This was perhaps due to the fact that these sessions always began by R1 prompting the classroom teacher to talk about how he/she felt about the preceding lesson, with a focus on the students' work. As a result, much time was spent discussing students' mathematical work, their understandings of and misconceptions about the mathematics they encountered. To a lesser degree, some of the comments related to student behavior and attitude. The students in this study, in general, behaved well and seemed to be on task during the lesson implementations. However, T2 reported during the second debriefing session, the chronic problems he faced with students' misbehavior in his class and reflected more on behavior and attitude than any other aspect regarding his students: “I guess towards the latter portion there were some behavioral issues”; “Now you see Jasmine G, the Jasmine who was over here with glasses, she doesn’t work with anybody in any class, she always sits by herself.”; and “Now these gentlemen over here worked pretty well, Kevin, Caliph, and Cory. But then occasionally there was this off task…yeah” (see Appendix H2).

The teachers also discussed students’ misconceptions and/or understanding of the mathematics they encountered in class. During the fourth debriefing session one of the observers, O3, addressed an important issue on how sometimes teachers made incorrect assumptions about their students’ understanding of the mathematics they did. He told the group: I had made the assumption that the students would know that a corn would be the same as any other corn in the picture. However, I saw a couple of kids in this class who were not starting with the premise that all the corns or bananas were the same. Some students thought that the goal of the problem was to make each of the three scales balance (see Appendix C4). During the same session T3, who had had a conversation with Cristian about the Carrots problem, told the group: I sat with Cristian, who was working with Yaasmyn, and he was not thinking proportionally. As he was looking at the scales, each scale had its own identity and he wasn’t seeing the connection between the first scale and the second and third scale (see Appendix C4).

In the descriptive analysis of lesson #3, one episode described Keith’s reservations in assigning a fractional number to each ox in the Tug-of-War problem. He was trying to re-name each of the four oxen in terms of a horse and was convinced that assigning a fractional number to an ox implied cutting up and killing the horse (see section 4.5.3). During the debriefing session R1 spoke about a conversation she had had with Keith regarding this problem: “The real reason that Keith bugged down and went to 20, 20 was because he was adamant , he had this notion of equating, of doing oxen in terms of horses. Even though it was a context issue, it was a mathematical idea too. And so when I said, are there some numbers you could use that won’t cut up the horse, he said, “Oh, 20, 20” and went into 4’s and 5’s” (see Teachers’ Reflections, Appendix H3).

The teachers also discussed students’ mathematical work and made comparisons between different strategies. During the first debriefing session the group talked about the different approaches the students had considered in solving the Bananas problem. R2 told the group: “Well, Kyla had a much different strategy than the rest. Like I think three groups were doing the five pounds, three pounds, one pound thing. But she (Kyla) did it more substitution and kind of like deduction” (see Teachers’ Reflections, Appendix H1).

During the debriefing that followed the grade 6 lesson (implementation #2), one of the observers, O2 told the group: “I saw a progression from this morning to this afternoon and I think that was partly the age because we went from 5th grade to 6th grade; Just the whole mentality, the whole thinking. Your class (5th grade teacher), it wasn’t, I don’t think putting even like a variable or anything to anything was an option in your class. Whereas, who was it? Joe? Joel? He was up there already putting 1p equals the bananas and the apples and he had everything in letters and numbers” (see Teachers’ Reflections, Appendix H2).

4.7.2 Adopting the Curriculum Lens

Deliberations on students’ mathematical work led to the adoption of curriculum lens by the teachers, when they engaged in discussions on how the organization of the lesson impacted student understanding of the concepts at hand, how the tasks were sequenced, and how the instruction was related to student prior and future knowledge.

The following working definitions were used in this study to describe the situations when curriculum lens was used by the teachers:

1. Relating instruction to future learning
2. Relating instruction to prior knowledge
3. Contemplating the development of content within a lesson
4. Contemplating the development of content within a unit/across grade levels
5. Considering the organization and presentation of instructional material
6. Discussing the lesson plan/task selections

Using the operational definitions, the transcribed *episodes* (see Teachers' Reflections, Appendix H1-4) from the videotaped debriefing sessions were examined in order to identify the teachers' comments that related to the choice of the instructional material and the sequencing of the tasks used during the lessons.

For example, during the first debriefing session, T1 wondered if she had included too many tasks in her lesson plan. This led to a conversation involving T1, T3, T4, and R1 (see Teachers' Reflections, Appendix H1, Episodes 6 and 7):

Line #	**Transcription**
1 2	**T1:** Now what are you thinking about the sequence? Cause that seemed long to me, and of course rushed at the end.
3	**T3:** You know what; I didn't think it was too long. I think it was fine.
4	**T1:** Really?
5	**T3:** I think mine is going to be too short, or, I just, because I was like,

wow!

R1: I don’t know. You [addressing T3] may want to add something
else.

T4: If you wanted to have time, like more time for the last two
(problems), what you might want to do was half the class gets one of
the warm-ups (Bananas) and the other half get the other half, the other
warm-up. And they report out and they are not doing, they are not
reporting the same. You know what I mean?

R1: I sort of disagree with that. I found the progression from the first to
the second one really interesting. And what I wrote in my notes was
that I realized that in the first one (Bananas), they sort of get it; each
one of them presented their solution. In the second one (Carrots),
because they had messed around with making a corn and a pepper the
same, they began, they began to listen to each other.

During the second briefing session, T3 spoke about her decision to include the Bananas and Carrots problems in her lesson plan for the upcoming lesson implementation the following day: “I am going to use the Bananas (problem) so the kids get an understanding of the balance because the tug-of war deals more with the inequalities. So, just so that we can examine equalities, take a look at both of them (both Bananas and Carrots). You know, just to look at the scales, scales are supposed to be equal, and then go into the tug-of-war” (see Teachers’ Reflections, Appendix H2). During the next debriefing session, which followed T3's implemented lesson, T3 explained to one of the observers the reason she had decided to include the scale problems in her lesson: “As I saw the kids struggling in your class [pointing to T2, the grade 6 teacher] with that balance (problems), I felt it was necessary to do it with the 8^{th} graders” (see Teachers’ Reflections, Appendix H3)

During the last lesson implementation, several of the students appeared to have difficulty understanding the concept of substitution as it applied to the transfer of objects from one pan balance to another in the Bananas and Carrots problem. Realizing the need to address this issue, T4 had contemplated revisiting the basic facts regarding pan balances in a whole-class discussion format, much like T3 had done during her lesson implementation. During the debriefing session T4 told the group: “At one point I was thinking I need to go over some things, you know, so that we can move forward. But I chose not to because after listening to them, you know I was thinking, oh okay, I don’t have to.” Instead, she had decided to gauge the prior knowledge of one student in particular, who appeared to lack the basic understanding of the concepts related to scales. During the debriefing she told the teachers: “When I went over there I wasn’t sure whether she was just, ‘okay, I am done’, or whether or not it was too much for her. I didn’t really know. So I brought out this other algebra book and so I figured let me back up to see exactly what she knows, to get to see where is it that she is stuck” (see Teachers’ Reflections, Appendix H3). Eventually T4 had arranged a class discussion on the concept of substitution by soliciting the participation of four students, who helped to create two imaginary scales, where a quantity of weight in one scale, replaced an equivalent weight on the second scale (Event 5, see Appendix G5).

Since several students in T4’s class had appeared to have had difficulty with substituting items in the pan balances, during the debriefing session that followed, the teachers re-examined the sequence of the tasks used in the implementation sessions. Referring to a book they had used as resource for task selection, they noted that the balance problems (Bananas and Carrots) were preceded by a problem in which, the students explored the concept of trading or bartering. The teachers wondered if T4’s students could have benefited from having done this bartering problem prior to the balance problems. R1 told the group: “It’s how you get into this idea of having to substitute or barter. And so with our discussion about how hard it was for them to get into that idea. Maybe that’s why they did this (in the book).” One of the teachers replied: “It’s interesting though because we thought, as a group, that this (balance problem) being the earliest and beginning ideas of algorithm that they need to get that. But it is interesting that maybe you need to trade before you can balance” (see Teachers’ Reflections, Appendix H4).

4.7.3 Adopting the Research Lens

While examining the transcribed instances (see Teachers’ Reflections, Appendix H1-4) from the videotaped debriefing sessions, the operational definitions were carefully deliberated on in order to identify teachers ‘comments which were made from a researcher’s point of view: relating to the lesson study goals, examining teaching practices that impacted student learning, and seeking concrete evidence to support student conceptual understanding.

The following working definitions were used in this study to describe the situations when research lens was used by the teachers:

1. Critiquing pedagogical decisions made with respect to the research goals
2. Restructuring a lesson with the research objectives in mind
3. Verbalizing the research goals
4. Demonstrating fascination/interest in students’ mathematical work and behavior
5. Relying on concrete evidence to support the research process
6. Speculating on student outcomes under different circumstances

7. Examining the development of student conceptual understanding

The teachers adopted the research lens during the debriefing sessions, when they critiqued their own or each other's pedagogical decisions that impacted student learning. During the first lesson implementation, the students had a difficult time with the Soda and Shirt problem, complaining that it was too hard. Afterward, T1 shared with the teachers her frustrations with the students' struggle and admitted that perhaps there had been some way of guiding the students while supporting their autonomy: "I probably could have done some different questioning to help them get where they needed to be without being so, without being too intrusive" (see Teachers' Reflections, Appendix H1).

In the descriptive analysis of lesson #4 (section 4.6.3), one episode (Event 3, Appendix G3) described how T4's drawings on the whiteboard had helped Nazeer keep track of his mathematical reasoning. During the debriefing that followed this lesson, the teachers talked about T4's decision to document Nazeer's thinking through drawings and its impact on Nazeer's ability to self-correct. R1 told R2: "I hope you got her (T4's) drawing on the video because his (Nazeer's) self-correcting when he saw the picture was great."

T4, however, critiqued her own decision to draw the diagrams and wondered if she had offered too much help: "I don't know if, okay, was that too much help? You know how teachers lead things sometimes? I didn't want to do that" (see Teachers' Reflections, Appendix H4). R1 reminded the teacher that she had not corrected Nazeer and it was Nazeer who had made the correction himself. R2 added: "I think if you really listen to the child and understand truly what they are saying, then represent it for them to help them get through it, you are not doing anything but making that tiny little bridge between A and B. But if you listen to them and hear what you want to hear and then make your representation, then it's whole other story. But I think you truly listened to him" (see Teachers' Reflections, Appendix H4).

The teachers often referred to students' written work when discussing their solution strategies. For example during the second debriefing, while discussing the students' responses to the Bananas problem, T1 told the group: "I am curious though when we look at the documentations, because this group over here, Monae, this lady over here, she had the correct answer but the other two didn't. And she was the spokesperson, right? She was the spokesperson for the group. So I am curious to see if they have, if they still have two bananas equals one apple on the other two papers or if they changed it. I am curious to see. Because when I was over there they had still two (bananas)" (see Teachers' Reflections, Appendix H2).

The teachers also reflected on student conceptual understanding and speculated on student outcome under different circumstances. During the second lesson implementation, students were presented with a modified version of the Shorts and Glasses problem. The teachers had speculated that the modification would encourage the students to think proportionally rather than use trial and error to solve the problem. During the debriefing session that followed this lesson, R1 told the teachers: "I don't believe that they (students) were able to see two shorts for one pair of glasses relationship under any circumstances. And so if you are thinking algebraically, that is really what you are trying to get at" (see Teachers' Reflections, Appendix H2).

In response, T3 described the conversation she had had with one of the students, Sakeena, regarding the Short and Glasses problem. T3 described how Sakeena had thought of crossing out a pair of shorts and a pair of glasses from each of the pictures, which left her with one pair of glasses against two pairs of shorts. T3 told the group: "And she saw there were two shorts and there was one glasses and she still said it was shorts (more expensive). And then I am thinking she [unclear] in terms of quantity, not looking at the value. I think she began to see quantity… So I believe given the time and if we were to probe her more, she (Sakeena) would have immediately said, well this is one (glasses), it's equal to this (two shorts), so this (glasses) must be more expensive" (see Teachers' Reflections, Appendix H2).

4.7.4 Overlapping of the Lenses

At times, teachers' reflections indicated a simultaneous use of multiple lenses. Some of the teachers' comments mentioned in the previous sections, and categorized under a specific lens, were also coded for the adoption of a second lens.

4.7.4.1 Student and Research Lenses

Consider the following remark made by one of the observers during the second debriefing session: "I saw a progression from this morning to this afternoon and I think that was partly the age because we went from 5th grade to 6th grade; Just the whole mentality, the whole thinking. Your class (5th grade teacher), it wasn't, I don't think putting even like a variable or anything to anything was an option in your class. Whereas, who was it? Joe? Joel? He was up there already putting 1p equals the bananas and the apples and he had everything in letters and numbers" (see Teachers' Reflections, Appendix H2). This instance of teacher reflection was presented in section 4.7.1 as an example, for using the student lens. Although the observer was commenting about what the students had done, she was also using the research lens when addressing and comparing the development of conceptual understanding in students.

Another example of the overlapping of student and research lenses was evident during the third debriefing session, when R1 spoke about Theo's work (see Event 13, Appendix F13): "The real issue that I like us to think about is Theo, who came up with what he thought was a very logical solution and it didn't work. And so what I was trying to do, and I know what (T3) was trying to do, is to get into his head to know what question to ask. Because ultimately, the only thing that happened was, we just sort of over-rode him" (see Teachers' Reflections, Appendix H3). In this example, R1 was addressing the issue of teacher's interventions which allow students uncover their mistakes and one of the goals of the lesson study project was to attend to student's mathematical thinking while supporting their autonomy. In this case both student and research lenses were adopted, when R1 reflected on Theo's work and an appropriate teacher intervention.

4.7.4.2 Student and Curriculum Lenses

One instance of teacher reflection, presented as an example of adopting the curriculum lens (see section 4.7.2), described how T4 had contemplated whether to review some basic concepts regarding pan balances with her students: "At one point I was thinking I need to go over some things, you know, so that we can move forward. But I chose not to because after listening to them, you know I was thinking, oh okay, I don't have to… I can let them keep doing it. They are doing fine" (see Teachers' Reflections, Appendix H4). In this example, the teacher had made a decision about what instructional material to cover, based on her students' demonstrated level of conceptual understanding. Therefore, both student and curriculum lenses were used when T4 reflected on this issue.

During the third debriefing session, there were some discussions on the choice and sequencing of the tasks that T3 had used in her lesson. Originally, she had not planned on using the Bananas and Carrots problems in her lesson, but after seeing some students' struggles with these problems during the previous lesson implementation, T3 had decided to use them as warm-up activities in her class: "As I saw the kids struggling in your class [pointing to T2] with that balance, I felt it was necessary to do it with the 8^{th} graders". Another example of the overlapping of the curriculum and students lenses could be seen in R1's comment, when she noted how the inclusion of the Bananas and Carrots problems at the beginning of the lesson had helped some students in solving the Tug-of-War problem: "Then, for the girls it was obvious that the notion of replacing had become a part of their vocabulary, which I don't think would have happened if you hadn't done those (Bananas and Carrots problems) earlier (see Teachers' Reflections, Appendix H3).

4.7.4.3 Research and Curriculum Lenses

One instance of teacher reflection, presented as an example of adopting the curriculum lens (see section 4.7.2), explained why T3 had decided to include the balance problems in her lesson. During the second debriefing session, T3 told the group: "I am going to use the Bananas (problem) so the kids get an understanding of the balance because the tug-of war deals more with the inequalities. So, just so that we can examine equalities, take a look at both of them (both Bananas and Carrots). You know, just to look at the scales, scales are supposed to be equal, and then go into the tug-of-war" (see Teachers' Reflections, Appendix H2). This example shows the adoption of research lens as well as the curriculum lens, when T3 considered how the task selection might help the development of conceptual understanding in her students.

Overlapping of research and curriculum lenses also occurred during the first debriefing session, when R1 spoke about modifying the Shorts and Glasses problem for the subsequent lessons. The teachers had speculated that the modification would encourage the students to consider the 2:1 relationship between the number of shorts and the glasses and help the development of proportional thinking. R1 told the group: “Can I throw in what I just said to (T2) because that is, I think, what he is going to do, and I want to know what you all think because we can change it. But to my mind the better question for the beginning of the second one (Shorts and Glasses problem) would be not to have any dollars there at all, which what I think he is going to do on the overhead” (see Teachers’ Reflections, Appendix H1).

4.7.4.4 Research, Student, and Curriculum Lenses

The following conversations involving R1 and T4 provide two examples of teacher reflections, when all three lenses were simultaneously adopted.

During the first debriefing session T4 suggested that perhaps, in order to shorten the amount of time spent on the scale problems, they should assign the Bananas problem to half of the class, while the other half worked on the Carrots problem. She argued that since the students shared their findings with the class, all students would end up being exposed to both of the problems. R1 replied: “I sort of disagree with that. I found the progression from the first (Bananas problem) to the second one (Carrots problem) really interesting. And what I wrote in my notes was that I realized that in the first one, they (students) sort of get it; each one of them presented their solution. In the second one, because they had messed around with making a corn and a pepper the same, they began, they began to listen to each other" (see Teachers' Reflections, Appendix H1). She also reminded the teachers that a part of their goal for the lesson study was to get the students to begin to talk to and hear each other.

In the preceding lesson, the Bananas problem had been handed out to the students as a worksheet, whereas the Carrots problem had been first introduced to the students on the overhead and each student was given a copy of it a few minutes later. T4 wondered whether it had been the difference in the nature of the two problems or the way they had been presented to the class, which had caused the Carrots problem to generate more mathematical discourse among the students: "Because, you know, like you said they were talking more. So I wonder if they would've still talked more if, if it was on the overhead, because they didn't have anything to personalize. Like, this is mine [clutching her notepad to her chest], have to do my own thing" (see Teachers' Reflections, Appendix H1).

4.8 Summary

This Chapter was dedicated to the results of the study. The previous sections described what occurred during the three stages of the lesson study process: the planning phase, the lesson implementation sessions, and the debriefing meetings. This section will provide a brief response to each of the four research questions.

1. What instructional and pedagogical decisions were made by the teachers prior to, during, and after each lesson implementation?

During the course of this study, the teachers made a range of instructional and pedagogical decisions. Choosing an appropriate mathematical concept and relevant tasks for the lesson implementations was a collaborative effort by the teachers during the planning phase. Various decisions, related to student autonomy, were made primarily by the teacher facilitators during the lessons. During the debriefing sessions, some decisions were made by the teachers collectively, concerning task modifications and lesson plan revisions.

Decisions Made Prior to the Lessons

As described in section 4.2, after considering student needs and curricular gaps during the planning phase, the teachers collectively decided to explore the concept of equality with their students for the lesson study project. They read and discussed various articles on the concept of equality and solved and discussed a variety of related mathematical problems. After examining various curricular resource materials, the teachers worked together to select appropriate tasks and generate a list of possible solutions for each problem. They worked collaboratively to create two lesson plans, one for Grades 5 through 7, another for Grades 8 and 9. The teachers also agreed on a set of goals for the lesson study project.

Decisions Made During the Lessons

During the lesson implementations, several autonomy-supportive decisions were made by the facilitating teachers, which were manifested in the classroom in different ways. For example T3 displayed flexibility regarding the students' seating arrangement and grouping. She decided to let her students work with a partner of their choice and allowed one of the students, Keith, who normally preferred to sit by himself, to work individually. Keith worked very well by himself and remained engaged with the tasks throughout the lesson. The other three teachers decided to consider the group dynamics and ended up grouping their students based on the anticipation of how well they would work together. These teachers encouraged the students not to work in isolation, but rather work collaboratively with other students in their assigned group. One of T2's students, Jasmine, was also used to working by herself in class. However, T2 succeeded in persuading Jasmine to join a group of two other girls for this implementation session.

During the lessons, when the students were stuck and did not know how to proceed or had an incorrect answer to a problem, the teachers made a conscious decision not to give directives as hints. Instead they tried to offer instructional support by asking helpful questions or by providing information. For example, when students had the incorrect answer to the Shorts and Glasses problem, the teachers approached the students, asked whether their answer would work in the second picture (satisfy the situation represented by the second picture), and then walk away from the students.

During the third lesson implementation, several students came up with solution strategies for the Tug-of-War problem, which had not been previously identified by the teachers during the planning phase of the study. Embracing these unexpected responses, T3 felt that her students needed extra time to explore this problem and for her and the observers to fully understand the students' reasoning. She made the decision to allow her students to set the pace and continue working on the problem in their own way. As a result of this autonomy-supportive decision, the last intended task (the Chickens problem) had to be omitted from the lesson.

Decisions Made After the Lessons

During the first debriefing sessions the teachers collectively decided to modify the Shorts and Glasses problem, with the hope that the modified version would encourage proportional reasoning in the students during the subsequent lesson implementations.

In the course of the second debriefing meeting, T3 suggested revising the task selection for her lesson to include the Bananas and Carrots problems in her lesson plan for the upcoming lesson implementation the following day. Having observed several students in T2's class struggle with these balance problems, T3 felt it was necessary to do these problems with her 8th graders as warm-up activities. She also felt that exposing her students to the Bananas and Carrots problems, which involved the concept of equality, was a good stepping stone for tackling the Tug-of-War problem, with its inherent concept of inequality. The teachers agreed with T3's suggestions and the proposed changes to her lesson plan.

2. What types of interventions (including questions) and interactions with students were enacted by each teacher during his or her lesson implementation?

There were many actions and interventions that were enacted by the teachers during the lessons. These included: promoting mathematical communication, through discourse as well as writing; building on students' responses; asking a variety of clarifying and probing questions while supporting student autonomy; attending to unexpected student solution strategies; and providing opportunities for students to self-correct.

The teachers in this study appeared to make a deliberate effort to promote mathematical communication in their classrooms. They encouraged student-to-student mathematical discourse by repeatedly reminding their students to discuss the problems and reach consensus on solution strategies within their groups. For example, T1 instructed her students -at least six times during the course of the lesson- to work together within their groups to discuss the problems and the solutions. The teachers also promoted classroom discourse by asking students to share their findings with the class and by giving students the opportunity to critique each other's reasoning. At the end of a student's presentation, the teachers would often ask the class if they agreed with the presenter's point of view. For example, after Neema and Kyla presented their solution to the Bananas problem, T1 asked the class: "We all agree with what we just heard?" (Event 5, section 4.3.3) We also witnessed how T4 provided Nazeer with the chance to comment on an incorrect assertion Kieshe had made while explaining her solution to the Bananas problem (Event 2, section 4.6.3).

Sometimes the teachers had to create the opportunity for students to comment on each other's solution strategies. As we saw in T2's class, when Kevin disagreed with Jazmine's solution strategy, and yet appeared reluctant to put forth an argument as to why Jazmine's answer was incorrect, the teachers had to coax Kevin into offering a counter argument (Event 2, section 4.5.3). Also in T3's class, the teacher's question: "So how different is Theo's (solution) than yours?" encouraged Kevin to critique Theo's solution strategy (Event 14, section 4.5.3).

The teachers also promoted mathematical communication in their classrooms by encouraging students to record their answers and explanations on paper. For example, T1 instructed her students -at least six times- to work together within their groups to discuss the problems and the solutions, and repeatedly reminded all of them to put down their mathematical thinking in writing. Also, when several students, including Keith (Event 8, section 4.5.3) and Kevin (Event 9, section 4.5.3), in T3's class complained that the task of writing their explanation was too difficult, T3 told them that they could put down their thinking on paper in any way they wanted, as long as it conveyed their reasoning. This autonomy-supportive intervention on the teacher's part resulted in some creative explanations written by several students.

There were a couple of incidents in which, the teachers used student responses and built on their ideas in order to deepen student understanding. For example in lesson #4, T4's construction of a human pan balance was a spontaneous response to Kieshe's request for clarification on the concept of substitution. When Kieshe unexpectedly (and incorrectly) suggested the removal of two students from one side of the scale, the teacher made a conscious decision not to reject her idea. Instead, T4 decided to build on Kiesha's idea by setting up a condition that accommodated her suggestion (Event 5, section 4.6.3). Also in lesson #1, when Jaylen referred to the guess and check strategy as "Squeeze method", T1 decided to build on Jaylen's idea by using the word "squeeze" and constructing a sentence that captured the essence of the strategy used by Amir and Jaylen (Event 6, section 4.3.3).

Throughout the lessons, the teachers asked a variety of questions. Some of these questions were directive in nature, asking students to perform a physical or mental act (i.e. asking a group of students to review their answer and report back later, or asking a student to write down his explanation). Some of the questions were meant as hints, providing instructional support when students needed help. For example when several students asked T1 to give them a hint in solving the Shirt and Soda problem, T1 offered them a hint by asking if the problem could be solved by considering fewer items (Event 8, section 4.3.3). Also with Shorts and Glasses problem, whenever students came up with incorrect prices that satisfied the situation shown in only one of the pictures, the teachers simply asked whether those prices “worked” in both pictures (Event 6, section 4.4.3).

Some questions were designed to recall prior knowledge, as in the case when T3 led a whole-class discussion and asked her students a series of questions on pan balances. T3’s impromptu decision to review the topic was a form of intervention and was in response to Jynita’s apparent lack of focus on the underlying mathematical concept in the Carrots problem (Event 1, section 4.5.3).

Some of the clarifying questions that the teachers asked required a simple yes/no answer but many elicited longer responses that explained or justified the students’ mathematical thinking. However, it was noted that sometimes when students came up with unexpected solution paths, the teachers acknowledged the correct final answer, but did not question the students on their strategy, nor did they ask them to justify their answer. For example, when Jade (lesson #1, 4.3) and Oscar (lesson #2, section 4.4.3) explained why the glasses were more expensive than the shorts, the classroom teachers acknowledged that the answers were correct, but did not examine the reasoning behind the answers. This was possibly due to the fact that the strategy used by Jade and Oscar was not in the list of strategies that had been compiled by the teachers prior to the lesson implementations.

On the other hand, unexpected student responses were at times fully embraced by the teachers. For example, when several students in T3's class solved the Tug-of-War problem by using numbers to represent the strength of the animals (Events 12 and 15, section 4.5.3), the teacher decided to allow extra time for students' ideas to be fully explored, understood, and shared.

There were a couple of episodes in which, the teachers provided opportunities for students to self-correct. We saw in lesson #1 (Event 4, section 4.3.3), how Kyla and Neema were afforded the opportunity to reflect on their incorrect answer and report back to class on their self-correction at a later time during the lesson. We also witnessed the incidence in lesson #4, when T4's unplanned drawing of the scales, depicting a crucial step in Nazeer's explanation to the Carrots problem, resulted in an eventual self-correction and led Nazeer to the correct answer (Event 3, section 4.6.3).

3. How did teachers' actions and interactions with students impact the mathematical work and reasoning that took place in the classroom?

Teachers' actions appeared to impact students' mathematical work and attitudes in different ways: an emphasis on student-to-student dialogue appeared to result in high level of collaborative work and public recognition of peer contributions by the students; modification of the Shorts and Glasses problem appeared to promote proportional reasoning in several students; encouraging students, in spite of their reluctance to write down their explanations, appeared to prompt several students to document their mathematical thinking in creative ways.

There appeared to be a shared recognition among the students of the importance of collaboration and teamwork, which was emphasized in all the classes by the teachers' repeated reminder for the students to work together. When sharing their findings with the class, a vast majority of students began their explanation by simply saying: Me and ---- (partner's name) figured out that … (Events 2, 3, and 4, section 4.3.3; Event 3, section 4.6.3).

Many students also acknowledged the positive contributions of their partners or other classmates. For example, in T3's class, before explaining her solution to the Bananas problem, Jynita acknowledged that she had used the strategy of assigning numbers to the items, the same way as Tyreal had done in the previous task, the Carrots problem (Event 2, section 4.5.3).

In all four classes, the students were engaged in mathematical discourse with their peers and appeared to collaborate and help each other through the problems. We saw in lesson #1 that immediately after the introduction of the first task, the students began discussing the problem with their partners at each of the four tables (Event 1, section 4.3.3).

In Lesson #2, even Jasmine, who was cajoled by T2, and had reluctantly agreed to join a group of two other girls, ended up being an active participant and collaborated well with the other members in her group. With the exception of many students in T2's class, the majority of students in this study were willing and eager to speak up and share their findings with the class, particularly when their solution strategies differed from the ones previously reported (Event 4, section 4.6.3; Event 15, section 4.5.3).

The students in the second and third lesson implementations were presented with the modified version of the Shorts and Glasses problem. This modification appeared to be helpful as it encouraged several students to think proportionally while tackling this problem. For example, Kevin in T3's class, not only figured out that the glasses were more expensive than the shorts, but also acknowledged the 2:1 relationship between the glasses and the shorts by stating that the glasses were twice as expensive as the shorts (Event 4, section 4.5.3).

Teachers in this study put a great deal of emphasis on written explanations by repeatedly reminding students to write down their justifications for the answers to the problems. Examination of the available student work samples revealed that not all students had the correct answers to all the problems, even though there were multiple presentations of the solutions in class prior to collecting students' papers at the end of each class. Some students had written down the correct answers but there was insufficient explanation for the reader to follow the thought process leading to the indicated answers. We saw two episodes in T3's class, where Keith and Kevin had worked on the Tug-of-War problem independently and although their solution strategies were very different from one another, they had both arrived at the correct answer to the problem. However they protested that writing their explanations was too hard and they could not write their "mind". T3 insisted that they put down their thinking, if not in full sentences, in a bullet format or in any other way they pleased (Events 8 and 9, section 4.6.3). Eventually, Keith used a series of numbers and three sentences to satisfactorily describe his solution strategy. Kevin did not write any sentences explaining his solution strategy, nor did he use bullets to structure his writing. However he used arrows and a few words to convey his mathematical thinking in writing.

4. What evidence is there, if any, of teachers' recognition of and reflection on their teaching practices, the lessons, and the students' mathematical work?

During the debriefing sessions, the teachers reflected on a variety of topics relating to the students, the curriculum, and their teaching practices. They discussed students' solution strategies, their understanding of and misconceptions about the mathematics they did, and to a lesser extent, their behaviors and attitudes. They also reflected on the task selection and sequencing as well as the development of students' conceptual understanding within a lesson and across grade levels. The teachers also commented on their own and each other's pedagogical decisions that were made during the preceding lessons.

There were many discussions on the different ways the students tackled the problems. For example, in the second debriefing session one observer reflected on the progression she had noticed from grade 5 to grade 6. She noted that the fifth grader simply described the relationship between the fruits/vegetables in the Bananas and Carrots problem, whereas several students in Grade 6 had used the equal sign and various letters (i.e. b for banana and p for pineapple) to form equations that represented the scales (see Teachers' Reflections, Appendix H2).

The teachers also reflected on students' misconceptions of the mathematics they did. During the third debriefing session, the teachers talked about Keith's reservations in assigning a fractional number to each ox in the Tug-of-War problem. He was trying to re-name each of the four oxen in terms of a horse and was convinced that assigning a fractional number to an ox implied cutting up and killing the horse (see section 4.5.3). Also during the fourth debriefing session, T3, who had had a conversation with Cristian about the Carrots problem, told the group: I sat with Cristian, who was working with Yaasmyn, and he was not thinking proportionally. As he was looking at the scales, each scale had its own identity and he wasn't seeing the connection between the first scale and the second and third scale (see Appendix C4).

There were some discussions on how, in general, teachers have a responsibility to address and clarify students' misconceptions. In particular, they reflected on Theo's mathematical work on the Tug-of-War problem, which he had presented to the class on the overhead. He had arrived at an incorrect answer and his reasoning included many incorrect assumptions and subsequent steps (Event 13, section 4.5.3). Although the validity of Theo's answer was challenged by several students, it could not be determined if Theo was convinced by the counter arguments put forth by other students, or whether he realized why his final answer was incorrect (Teachers' Reflections, Appendix H3).

Some of the discussions were centered on students' behavior and attitude. During the debriefing session, T2 spoke about the chronic problems he was facing with students' misbehavior in class. It was noted that T2's reflections were more focused on behavior than any other aspect regarding his students: "I guess towards the latter portion there were some behavioral issues"; "Now you see Jasmine G, the Jasmine who was over here with glasses, she doesn't work with anybody in any class, she always sits by herself."; and "Now these gentlemen over here worked pretty well, Kevin, Caliph, and Cory. But then occasionally there was this off task…yeah" (see Appendix H2).

The teachers also reflected on the impact of task selection, sequencing, and presentation on students' mathematical work. For example, after the first lesson implementation they decided to modify the Shorts and Glasses problem with the hope that the modified version would encourage the students in the subsequent lessons to solve the problem using proportional reasoning, rather than the guess and check strategy (see Teachers' Reflections, Appendix H1). Also during the second briefing session, T3 spoke about her idea to include the Bananas and Carrots problems in her lesson plan for the upcoming lesson implementation the following day: "I am going to use the Bananas (problem) so the kids get an understanding of the balance because the tug-of war deals more with the inequalities. So, just so that we can examine equalities, take a look at both of them (both Bananas and Carrots). You know, just to look at the scales, scales are supposed to be equal, and then go into the tug-of-war" (see Teachers' Reflections, Appendix H2).

Another example of teachers' reflections on the impact of task selection, sequencing, and presentation on students' mathematical work occurred during the first debriefing session, when T4 suggested that perhaps, in order to shorten the amount of time spent on the scale problems, they should assign the Bananas problem to half of the class, while the other half worked on the Carrots problem. She argued that since the students shared their findings with the class, all students would end up being exposed to both of the problems. R1 replied: "I sort of disagree with that. I found the progression from the first (Bananas problem) to the second one (Carrots problem) really interesting. And what I wrote in my notes was that I realized that in the first one, they (students) sort of get it; each one of them presented their solution. In the second one, because they had messed around with making a corn and a pepper the same, they began, they began to listen to each other" (see Teachers' Reflections, Appendix H1). T4 wondered whether it had been the difference in the nature of the two problems or the way they had been presented to the class (one as a hand-out, the other presented on the overhead), which had caused the Carrots problem to generate more mathematical discourse among the students: "Because, you know, like you said they were talking more. So I wonder if they would've still talked more if, if it was on the overhead, because they didn't have anything to personalize. Like, this is mine [clutching her notepad to her chest], have to do my own thing" (see Teachers' Reflections, Appendix H1).

The teachers also discussed the development of students' conceptual understanding within a lesson as well as across grade levels. There were some discussions on students' prior knowledge, relating it to their current instructional material and their future learning. For example, during the last debriefing session, the teachers reflected on how the tasks used in this study could pave the way for formal algebra later on, but re-examined the sequencing of the problems and wondered whether the students needed to know how to "barter" before they were exposed to the balancing problems (see Teachers' Reflections, Appendix H4).

The teachers also critiqued the impact of their own and each other's pedagogical decisions on student learning. For example, the teachers agreed that T4's unpremeditated decision to draw pictures representing Nazeer's explanation (Event 3, section 4.6.3), as well as her construction of the human scale (Event 5, section 4.6.3) had been beneficial to many students in the class.

CHAPTER 5

5.1 Discussions

Several emerging themes associated with the findings of this study will be discussed in this section. Aspects of teacher decisions will be considered with a focus on the teacher's awareness and in-depth understanding of a wide range of possible solution strategies, both correct and incorrect, as well as the use of student responses in ways that generate rich and productive class discussions. There will also be discussions on teacher reflections, and the social norms that provide opportunities for students to engage in productive mathematical discourse with their peers.

In this study, it was noted that several times the teachers encountered unexpected students' solution strategies. On these occasions, the student's reasoning was not always thoroughly explored by the teachers and at times did not receive the enthusiasm and attention deserved. This intermittent lack of follow up on students' responses was perhaps due to the unexpected nature of the students' proposed solution strategies. Through planning and by predicting an array of possible students' responses, the teachers could have solid expectations for what is likely to happen in response to a problem and avoid being caught off guard by what the students may say and do (Stein, Engle, Smith, & Hughes, 2008). Although the teachers had generated a list of possible solutions for each of the problems during the planning phase of the lesson study, the list was, understandably, not exhaustive. However, a more comprehensive list of both correct and incorrect strategies might have been beneficial to the teachers in identifying, understanding, and embracing various student responses.

For example, in the case of the Shorts and Glasses problem, there were a couple of instances in which, students correctly determined the glasses to be more expensive than the shorts, sighting the fact that for a fixed amount of money you would get fewer number of items if you had more glasses than shorts. The teachers had not anticipated such response from the students and as a result this line of reasoning was not further explored. Had the teachers been prepared for such student response, perhaps they could have further illustrated this type of reasoning by providing an example. For instance, they could have presented the following scenario: John has $5 to spend at the school store. A notebook costs $2 and a pen costs $1. Examining the different combinations of these two items that John could spend his $5 on, could have potentially moved the entire class forward in understanding the above reasoning suggested by their classmate.

It is of course impossible to predict all possible student responses, and therefore encountering unexpected student answers is inevitable. It is also understandable that, from a practical point of view, teachers cannot always spend the time during a lesson to examine, analyze, and understand unexpected student strategies. However, teachers should be encouraged to habitually examine their own reactions to students' actions, and make those unexpected student responses the subject of their reflective moments after a lesson.

There were instances in this study when, incorrect student responses were questioned by the teacher or other students, but insufficient amount of time was spent on diagnosing and discussing the source of the misunderstandings that had led to the incorrect responses. Attending to each and every student's mathematical thinking and addressing their misconceptions is a very difficult, if not impossible, task. However, students' correct or incorrect responses to instructional tasks could be used by the teacher in whole-class discussions in such ways that could potentially advance the mathematical learning of the entire class (Ball, 1993; Lampert, 2001). For example, in justifying his incorrect claim that the oxen would win the tug of war, Theo had made an incorrect assumption and had taken several subsequent incorrect steps to arrive at his answer. Although Kevin had demonstrated on the board why Theo's assumption that an ox was as strong as two horses was incorrect, no other aspect of Theo's reasoning was further examined and discussed. At some point Theo had made the error of eliminating unequal "strengths" from the two sides of a "balanced" tug of war game. Addressing the consequences of such action would have been an opportunity for the teacher to guide the class toward a deeper understanding of the inherent notion of balance in an equation and, taking a longer-term view, pave the way for the concept of subtracting equal values from both sides of an equation or an inequality in formal algebra. We don't know if Theo was convinced that his answer was incorrect or was aware that he had made other mistakes along the way. This lack of follow up on Theo's misguided reasoning was brought up by the university researcher during the debriefing session, who told the group that when kids present contradicting ideas, teachers in general, have an obligation to uncover their mistakes and address their misunderstandings.

The examination of the data regarding the students' work samples in this study, not surprisingly, revealed that not all students had arrived at the correct answers to the given problems. However, in one class, 1/3 of the students' written work showed an incorrect answer to the Carrots problem. Two students had shared their solution to the Carrots problem with the class, each using a different strategy, but both arriving at the correct answer. Nevertheless, it cannot be determined whether the 1/3 of the students, who had handed in their work bearing the incorrect answer, were ever convinced by either of the two presentations. Perhaps some, if not all of them, were convinced but simply did not get the chance to make the necessary corrections before handing in their work.

This brings out the notion that students' reporting on their solution strategies needs to move beyond what Ball (2001) has called a “show and tell” in which, students with correct answers volunteer to share their solution strategies with the class. Stein, Engle, Smith, & Hughes (2008) believe that careful monitoring of students' work as they discuss and solve the problems within their small groups should inform the teacher of the range of answers that the students have come up with. Using that information the teacher could call on select students to present their findings and use their responses as jumping boards for whole-class discussions that not only compare the different strategies presented, but also address the misconceptions that resulted in incorrect answers for some students. Therefore it is essential for the teacher to be familiar with a wide range of likely, both correct and incorrect, student responses to a problem so that while she monitors the student's progress as they work on the problems, she can plan on productive class discussions that are thoughtfully arranged around student responses.

The teachers in this study tried to promote mathematical communication in several ways. One way was by repeatedly reminding the students to discuss the problems and share their mathematical thinking within their small groups or with their partners. Through communication students have the opportunity to discuss, reflect on, and refine their mathematical thinking and by listening to others' explanations, students can develop their own understandings (NCTM, 2000). However, in his analysis of a case study, Cobb (1995) noted that the social context of students' small-group relationship affected the level of mathematical learning that occurred as a result of small group discussions. He found that if one student was regarded as the mathematical authority in the group, his or her reasoning was generally never challenged or questioned by other group members. His analysis indicated that this was not productive for either the student giving the explanations or for those listening to it.

Cobb's findings (1995) and other's (e.g., Webb, 1991) suggest that learning opportunities don't necessarily present themselves to students who explain their mathematical thinking to their peers, unless they know that other students are listening critically and might question their reasoning. These findings highlight the importance of considering the group dynamics when teachers assign students to work together in a small group or pair them up to work as partners. Most of the students in this study appeared to work well together. However, there were instances in which, one student seemed to assume the leadership role while the other became the almost "silent" partner. This was particularly noticeable with Tyreal and Al, where Tyreal dominated most of the conversations and was the source of all the mathematical inputs, while Al simply followed along with Tyreal's ideas.

Teacher reflection is an integral part of the lesson study process. The proponents of lesson study, as a reflective process, agree that the teachers' collaborative involvement in reflections and discussions on student learning and how their teaching affects it, is a major component of lesson study as a productive form of professional development (Lewis 2000; Murata 2011). Collaboration and teamwork, inherent in the lesson study process, provided the teachers in this study with the opportunity to learn together and from each other, in an environment that was non-judgmental, and where authority was shared equally among the group members.

The teachers who attended the lesson implementations as observers, tried to limit their interactions with the students within the confines of lesson study practice and for the most part, approached the students in order to observe their work. In order to better understand the students' thinking, the observers occasionally asked clarifying questions such as "How did you get that?" or "Why did you do this?" It is not clear whether the observers' clarifying questions from the students impacted student learning in this study. It is possible that articulation of their reasoning, helped some students to better understand and perhaps modify or correct their thinking. There were documented instances, such as R1's interaction with Keith (Event 12, section 4.5.3), that led to Keith's discovery of a correct solution to the Tug-of-War problem. But we cannot determine for sure if it was R1's intervention that resulted in Keith's self-correction or whether, given time, Keith would have come up with the correct answer by himself.

We do know however, that the observers' interactions with the students benefited the facilitating classroom teachers. During the debriefing sessions, by reporting on students' mathematical thinking, the observers were able to provide the classroom teachers with an expanded knowledge of the students' understanding and/or misunderstanding of the mathematics they had done. T1 reported enjoying having had the extra pairs of eyes and ears in the classroom to observe and take note of students' mathematical thinking.

During the debriefing sessions the teachers, collectively and publicly, reflected on students' mathematical work, the content and the sequence of the material they covered in class, as well as their teaching practices that impacted student learning. In this study, the teacher reflections were examined in terms of three lenses - Student, Curriculum, and Research (as described in this study). However, it was noted that many of the discussions on the development of student conceptual understanding, which involved the adoption of the research lens, were often initiated or prompted by the primary university researcher, R1. This suggests that perhaps lesson study communities could benefit from the support of expert facilitators when practical or possible. As Hart and Carriere (2011) put it: "There is room for the active support of external facilitators who are knowledgeable about the lesson study process and who embrace the values of lesson study: A culture of self-criticism, openness to the ideas of others, and a willingness to embrace mistakes" (p. 37).

5.2 Conclusion

Although the teachers in this study drew from a shared pool of tasks for their individual lesson implementation, we saw, as one might expect, that no two lessons were exactly alike. Indeed, certain amount of teacher adaptation is inevitable, but lesson modifications and improvisations that occur organically and in response to students' needs were embraced and encouraged in this study. For example, in the third lesson, several students had solved the Tug-of-War problem using an unexpected strategy that had not been previously identified by the teachers during the planning phase of the study. Consequently, the classroom teacher and the observers spent a fair amount of time listening to and understanding the students' reasoning. The teacher's decision to afford her students extra time on this problem, resulted in the omission of one of the intended tasks for the lesson, but allowed the students to fully explore the problem and resulted in five class presentations, showcasing different solution strategies. Also in the fourth lesson we witnessed how one student's ambivalence about an important concept, prompted the teacher to depart from her original plan and spend a good amount of the class time on constructing a physical representation of the concept in question, which appeared to deepen understanding in several students.

Of course the teachers in this study, as researchers participating in a lesson study project, did not feel hard-pressed to get through all the intended tasks. But, in the "real" world and with the time constraints put on teachers for covering an array of topics while keeping up with a closely watched pacing guide, it is understandable that some might deem it necessary to adhere to a more rigid timetable. It should also be noted that a departure from the original plan may not always be productive. Teachers' impromptu adaptations or modifications during a lesson may result in an unintended deletion of important aspects of the mathematical ideas inherent in the original problem activity.

There were a few instances in this study, where the students' lack of fluency in arithmetic got in the way of doing the mathematics. For example in the second lesson implementation R2 asked the class, addressing Cory in particular, to find the total weight of 10 bananas given that each one weighed 1 ½ pounds. When no one could immediately come up with the answer, the classroom teacher offered a hint by asking the students to consider the decimal representation of one and a half. In the absence of calculators, the students were unable to perform this multiplication within a short time period and about

45 seconds after the question had been asked, the teacher, in an effort to move the lesson along, shifted the focus from this potentially important question and moved on to another group of students for their presentation. Also in Lesson #3, as reported by one of the observers during the debriefing session, Tyreal had a difficult time distributing five horses equally between four oxen. Even though he had a calculator on his desk, he was not using it because, as it became apparent later, he did not recognize that what he was trying to do conceptually was the same as dividing five by four. It took him several minutes to realize that the correct answer was 1 ¼ and not 1.4.

While providing instructional support in the form of a hint, it is natural for teachers, as it occurred in this study, to nudge students toward a certain solution path that they themselves are familiar with and feel comfortable about. Nevertheless, it is imperative for teachers to know and be conversant about a wide array of, both correct and incorrect, possible students' responses to a problem. It is also crucial for teachers to fully understand the mathematics involved in various solution strategies. This level of preparedness on the teacher's part would hopefully safeguard against what happened at times in this study, when some teachers skipped over or minimized the importance of perfectly logical, yet unexpected students' responses. This was perhaps due to the teachers' lack of familiarity with the solution strategies proposed by the students.

The teachers in this study tried to promote mathematical communication by encouraging mathematical discourse. They also emphasized that the students had to put down their explanations on paper before sharing their reasoning with the class. As important as writing seems to be across curriculum, it remains to be the Achilles' heel for many students. With the exception of a few, the majority of students in this study did not write well-structured, coherent paragraphs to explanation their mathematical thinking. Despite some students' protests - "I can't write my mind" and "I hate writing because it, it's mad hard!"- one of the teachers encouraged her students to do whatever they can to convey their reasoning to the reader. This encouragement produced some interesting work by several students, such as Kevin, who used arrows and circles together with a few choice words to explain his answer to the tug-of war problem. Without using a single complete sentence, Kevin displayed his ability to communicate his mathematical thinking and reasoning through words and drawings.

Sometimes it is extremely difficult, if not impossible, to steer students toward a certain path when their minds are set on a different approach. We saw an example of this in the first lesson implementation, when despite the teacher's repeated suggestions that the students do not use the guess and check strategy and her dispense of several hints, meant to nudge the class toward a certain solution strategy, the majority of students remained with their initial method of trial and error. Upon reflecting on students' handling of this problem during the debriefing session that followed this lesson, the teachers decided to modify this question in such a way to limit the guess and check strategy as an option toward the solution. During the subsequent lesson implementations, when students were presented with this modified version of the Shorts and Glasses problem, it was noted that the modification had encouraged the intended proportional thinking in some students.

In this study we also witnessed the episode involving Jynita and her partner Dieshe, tackling the tug-of-war problem. Despite Dieshe's earnest efforts to convince Jynita of the correct answer, Jynita appeared reluctant to concede that her initial answer had been incorrect. Even when she seemed convinced by Dieshe's reasoning, Jynita remained adamant about "sticking" with her own results, telling the teacher, "I don't want her (Dieshe's) answer"! Several teachers/observers had separately approached Jynita during the lesson, trying to understand her thinking and hoping to help her get the correct answer to this particular problem. One of the observers reported during the debriefing that when he had approached Jynita, she had immediately said that she already had the answer and did not need any further help. Nevertheless, the observer had decided to go beyond the typical lesson study code of behavior for observers and demonstrate how an elephant can be substituted, by placing a cutout of the two horses and an ox on top the elephant. The observer believed that by doing so, he had made it possible for Jynita to visualize the result of the substitution and accept that her initial answer had been incorrect. Whether Jynita was truly convinced or not, we cannot tell for sure, as she never handed in her written response to this problem.

The debriefings that followed each lesson implementation session, afforded the teachers a chance to discuss a variety of topics. Examination of the nature of teacher reflections in this study indicated that the teachers adopted the Student lens (as defined in this study) and spent much time discussing the students' solution strategies, their misconceptions, their understandings, and to a lesser degree their behaviors and attitudes. The students, in general, behaved well and seemed on task during the lesson implementations in this study. However, one of the teachers had reported, during the debriefing session, the chronic problems he faced with students' misbehavior and disturbances in his class and, not surprisingly, this teacher reflected more on behavior than any other aspect regarding his students.

At times the examination of student solution strategies, led the teachers to reflect through the Research lens (as defined in this study), inspecting the nature of the tasks and the way they were presented to the students. For example, since the majority of students during the first lesson used the guess and check strategy to solve the Shorts and Glasses problem, the teachers decided to modify the problem, with the hope that in the subsequent lessons the students would use a strategy other than the guess and check. This modification appeared to be helpful as it encouraged several students to think proportionally while tackling this problem. The research lens was also used by the teachers when they examined the development of student conceptual understanding or critiqued their pedagogical decisions with respect to the lesson study goals. For example, during the debriefing session, one of the teachers told the group: “I probably could have done some different questioning to help them get where they needed to be without being too intrusive.”

Adopting the Curriculum lens (as defined in this study), the teachers related instruction to prior or future learning. For example in one debriefing session, they examined the sequence of the tasks used in the implementation sessions. One of the teachers said: “It’s interesting though because we thought, as a group, that this [balance problem] being the earliest and beginning ideas of algorithm that they need to get that. But it is interesting that maybe you need to trade before you can balance.” Using the curriculum lens, the teachers also contemplated the development of content within a lesson, a unit, or across grade levels.

Close examination of the substance of conversations, revealed the simultaneous use of more than one lens in several instances of teacher reflections. For example during the conversations around the Shorts and Glasses problem, which led to a modification for use in subsequent lessons, the teachers were viewing the topic through not only the Research lens, as suggested above, but also the Curriculum lens as they critically examined the effectiveness of the instructional material.

On a personal note, participation in the lesson study project described in this study, had a profound impact on me as a teacher. In a field, where professional peer interaction is not always the norm, it was refreshing to be part of a learning community and have the opportunity to make joined decisions about the lessons and reflect on them afterward. Our reflections on student's mathematical work supported the adoption of other lenses through which, we began to discuss the curriculum and reflect on our teaching practices. Engagement in such rich conversations left an impression on me that was carried over into my every day classrooms. I found myself more attentive to my students' needs and began to question the questions I asked of them. I can honestly say that my involvement in the lesson study project was the key determining factor in choosing my dissertation topic.

5.3 Implications for Practice

This study highlighted the significance of teacher's careful consideration of students' needs and the positive outcomes of thoughtful teacher reflections on lessons and students' mathematical thinking. Therefore the findings of this study have implications for practice, both for teachers in examining their practices before, during, and after each and every lesson, as well as for policy makers and administers in providing meaningful professional development programs that have lasting positive effects on teachers and their practices.

When teachers are familiar with a wide array of solution strategies for problem activities, they are less likely to get caught off guard by what students may say or do. Therefore, predicting a comprehensive list of both correct and incorrect strategies might be beneficial to the teachers in identifying, understanding, and embracing various student responses.

The teachers in this study benefited from opportunities to work together in order to prepare for the lessons, and to reflect on students' mathematical work, as well as their own teaching practices. The teachers who attended the lesson implementations as observers, tried to limit their interactions with the students according to the general guidelines of lesson study protocol. But, there were a couple of instances in which, the observer's interaction with a student was more of an intervention rather than a mere attempt to observe and understand the student's thinking. However, it was found that this did not impede the reflective process and in fact enhanced student learning. Therefore, this study has implications for designing professional development programs that promote teacher collaboration and are effective in helping teachers become reflective practitioners.

5.4 Implications for Future Studies

This lesson study could potentially have implications for future studies. A follow up study on the four teachers in this present study could be designed, perhaps through classroom visitations and interviews, to determine whether their participation in the lesson study have in anyway impacted their teaching practices. Additionally, a follow up study on teacher reflections could utilize the recorded data from the debriefing sessions in this study. A quantitative analysis of the teachers’ reflections could perhaps examine the number of times each lens (Student, Curriculum, and Research) was used by the teachers and study the circumstances that lead to the adoption of a particular lens. Also, there were six lesson implementations during this lesson study cycle, four of which were selected for the purpose of this present case study. A future study can perhaps be dedicated to analyzing the remaining two lessons.

Appendix A: The Tasks

Appendix A1: The Bananas problem

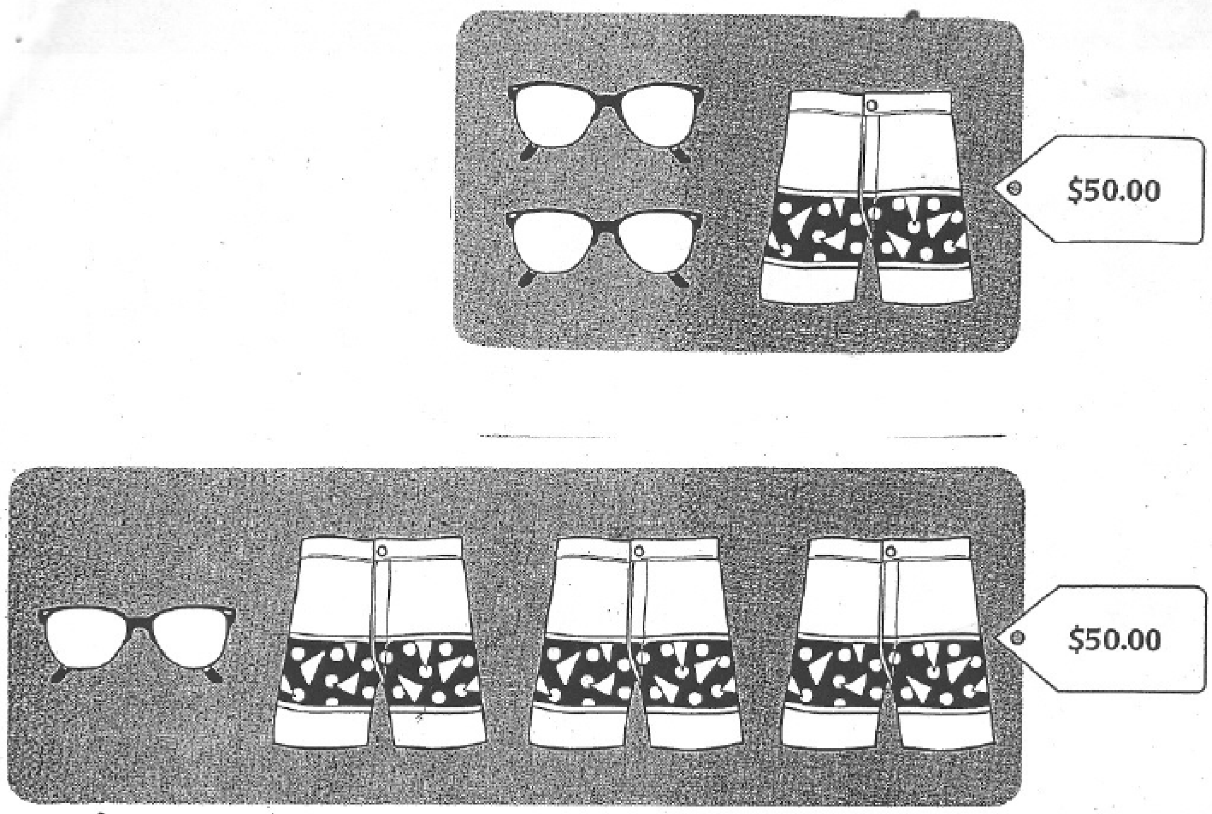
Bananas

2. How many bananas are needed to make the third scale balance? Explain your reasoning.

Appendix A2: The Carrots problem

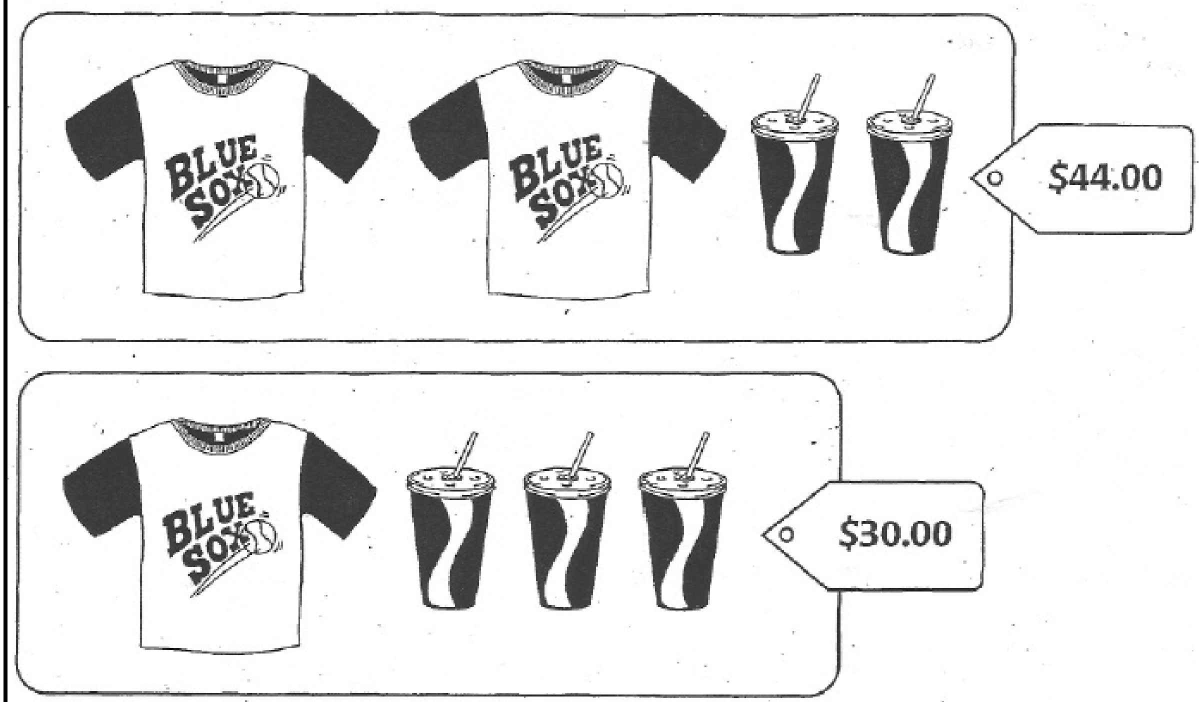
3. How many carrots are needed to make the third scale balance? Explain your reasoning.

Appendix A3: The Shorts and Glasses problem



1. Without knowing the prices of a pair of glasses or a pair of shorts, you can determine which item is more expensive. Explain how.
2. How many pairs of shorts can you buy for $50?
3. What is the price of one pair of glasses? Explain your reasoning.

Appendix A4: The Soda and Shirt problem



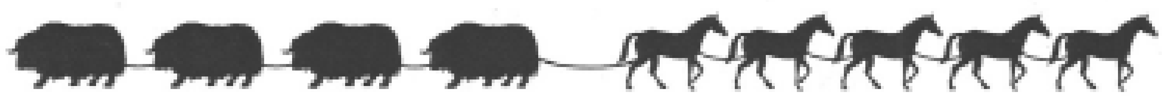
1. How much does a T-shirt cost?

   How much is a drink?

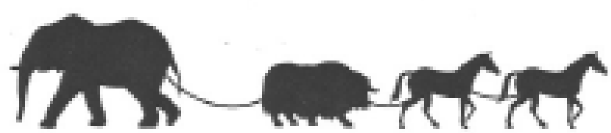
   Explain how you got your answers.

Appendix A5: The Tug-of-War problem

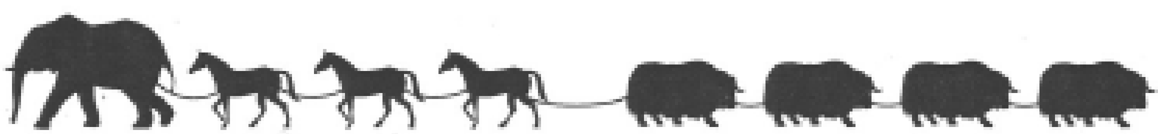
Four oxen are as strong as five horses.



An elephant is as strong as one ox and two horses.



5. Who will win the tug-of-war pictured below? Give a reason for your prediction.



Appendix B: Descriptions of the Lesson Implementations

Appendix B1: Description of lesson #1

Roles T1-1 & T1-2	Place data recorded: Sheffield, NJ Date data recorded: 5/14/08 Classroom teacher: T1 Grade 5
Time Interval	**Description**
00:00-5:00	There are 14 students in the classroom. In addition to the classroom teacher, T1, there are eight other adults, R1, R2, O1, O2, O4, T2, T3, and T4, present in the room as observers. An additional person is in charge of taping the session. The class begins with the teacher (T1) and the observers helping students tape their nametags on their desks. Students' desks are put together to form four groups of students. There are three or four children in each group. (The grouping of the students was decided by the teacher prior to the lesson and was based on students' personalities and how well they would work together.) T1 introduces the university faculty member, R1, who addresses the students. She explains that the adults, present in the classroom, are interested in becoming better teachers and a good way to do that would be for them to observe a lesson. She also asks the students if it would be okay for them to be videotaped and for the observers to walk around and interact with them once they start doing the math problems. T1 asks the observers to introduce themselves to the class.

5:00-10:00	The observers introduce themselves to the students by stating their name, the grade they teach and the school they work at. T1 hands out the first worksheet (Bananas). She asks the students to work in pairs to solve the problem. She also tells them that they should explain their solution. Students begin working. Some work in pairs, conversing about the task at hand, while others quietly read through and work on the problem by themselves.
10:00-15:00	Students are busy writing out their answers. T1 reminds the class that in a few minutes they would share their answers and that they would have to convince others that their answer is correct. After a couple of minutes T1 asks, “Okay, who wants to share?” Carin raises her hand and is asked to stand up to explain her solution. Carin says that she and Titiana worked together and they first figured out that if 10 bananas are equal to two pineapples, then one pineapple would be equal to five bananas. Therefore in the second picture where one pineapple equals two bananas and an apple, two bananas are two pounds which means that the apple would be three pounds. She then concludes that in the last scale, the one apple would equal three bananas. T1 says, “Okay, three bananas”, and asks another group to share. Jaylen, who has been working with Amir, raises his hand. The two boys stand up to present their solution. Jaylen says that one apple equals three bananas because: In the first picture, 10 bananas equal two pineapples, so the two pineapples are about five pounds. On the second scale, there is only one pineapple which is about five pounds, but there are only two bananas, so the apple must be three pounds and the two bananas are two pounds. T1 asks, “so, at some point we started converting to pounds?” Carin raises her hand to point out that Jaylen, in his explanation, stated that two pineapples equal five pounds and later on said that one pineapple equals five pounds. Jaylen clarifies by saying that two pineapples equal five pounds each and therefore one pineapple equals five pounds. T1 rephrases Jaylen’s statement by saying that one pineapple equals five bananas. She moves on and asks another group to share their findings.
15:00-20:00	Jasmine stands up and says that she worked with Jade and Tequrra. She explains that since 10 bananas equal two pineapples, they divided 10 by two and concluded that each pineapple is equal to

	five bananas. Then they looked at the second scale and remembered that one pineapple equaled five bananas, so they knew that an apple would be the same as three bananas because "five bananas will make it leveled". She ends by repeating that one apple equals three bananas. The teacher says, "good, okay" and asks for another group to share. Kyla, who has been working with Neema, stands up. She explains that since 10 bananas equal two pineapples, then on the second scale, one apple must equal three bananas. Then she finishes by saying that in the last picture there would be an apple and two bananas. T1 repeats their final answer in the form of a question: "So an apple equals an apple and two bananas?" Kyla bends down to discuss the answer with Neema. T1 asks them to work on it again. T1 asks Maurice, Elyce, and Jarrod to share their solution with class. They say that they are still working on it. The teacher asks Sierra and Erica to present. Erica stands up and reads from her paper which explains that since 10 bananas equal two pineapples, each pineapple is equal to five bananas and therefore in the last picture, the apple will be the same amount as three bananas. T1 asks the class whether 10 bananas are really equal to two pineapples. Some students say yes and some say no. T1 asks: In what sense are they equal? Someone says: In pounds. T1 states that they are equal in weight. T1 asks Maurice, Elyce, and Jarrod if they have a different solution strategy to the ones already discussed. They say no, and the teacher starts introducing the next problem. R1 speaks up and says she is really interested in hearing what Neema and Kyla have to say about the solutions discussed because their answer was not the same as others.
20:00-25:00	R1 asks,"Did you rethink it? What do you think?" Neema and Kyla have done a self-correction and T1 asks them to talk through it. Neema starts talking, but then asks Kyla to take over. Kyla says that they were wrong before in saying that the final answer is an apple and two bananas. She explains that in the second picture where one pineapple equals two bananas and an apple, an apple is one pound, but they need three pounds which means that the apple would be three pounds or three bananas. T1 introduces the next problem. She puts a transparency of the problem on the overhead projector and asks a student to read the problem. A boy reads: "How many carrots are needed to make the third scale balance? Explain your reasoning." Kyla and Neema approach the board to get a closer look at the

	pictures. T1 makes sure that all students can clearly see that there are six carrots on the first scale. A couple of the students say they already have the answer. T1 encourages them to write their answer on the scrap paper so they can report on it later. Students start working on the problem.
25:00-30:00	Students are busy working, writing, and discussing the problem and its solution with their partner. Carin is trying to convince Titiana that the answer is two carrots. T1 tells the class that they have one minute left. T1 asks Maurice to share his answer with the class. He stands up and says that there will be two carrots in the last scale. He explains that according to the second scale, a corn is double a pepper. So a corn is four pounds. T1 repeats: A corn is four pounds? A girl clarifies by saying: Because the pepper is two pounds. A boy says: And a carrot is one pound. T1 says okay and asks another group to present. Carin stands up and says since there are six carrots that equal one pepper and one corn, at first they thought it would be three and three but Tatiana had said: “You can’t do that”.
30:00-35:00	T1 asks: “Why can’t you do it?” Carin explains that if the corn and the pepper were each equal to three carrots, then the second scale would not be balanced, because you would have three carrots on one side and six on the other side. Carin continues her explanation: They then decided that a corn was equal to four carrots and a pepper equal to two carrots. The second scale was consistent with that guess because as Carin notes, 2x2=4. T1 then asks Carin about the last scale. She explains: A corn is equal to four carrots, but if we put a corn, it won’t equal one pepper. So we have to put two carrots to equal a pepper. T1 agrees and points out that the question posed on the last scale was not about corn anyway, but carrots! T1 asks Kyla to present. Kyla stands up and says: We started off thinking like them (Carin and partner), that it is three and three. But then Neema told me that it does not work for the second scale. So I went back to the first scale to see what other multiples will equal to six, and I thought of four and two. T1 asks: Two and four are multiples of six? Kyla immediately replies, no, that she looked for two other numbers that equaled six. Kyla continues: So four carrots equal a corn and two carrots equal a pepper. This also works in the second scale. So on the last scale, it will be two carrots, and each carrot is

	one pound. T1 asks someone from another group to present. Tequrra stands up and starts reading her written answer: We counted the carrots on the first scale. Looking at the second scale we knew that the corn and the pepper were not equal and that a corn equals four carrots and a pepper equals two carrots. So we knew that we needed two carrots in the last scale. Sierra raises her hand and volunteers to present next. She stands up and reads her answer, which states the items in the first scale and that according to the second scale, four carrots equal a corn and two carrots equal a pepper. A pepper is two pounds, but if you put two carrots which are one pound each, it will equal one pepper. T1 asks who wants to present next. Jaylen, who has been working with Amir, raises his hand and says: From the first picture, we found out that each carrot is one pound, making a total of six pounds. Then I tried three and three (three carrots for each pepper and corn), but it didn’t work. Then I went up and I went down and noticed that the corn could be four carrots and the pepper could be two carrots.
35:00-40:00	Jaylen continues to say: On the second scale, pepper is two pounds and 2x2=4, and the corn is four pounds. So on the last scale, one pepper is equal to two pounds, which is the same amount of two carrots. T1 asks Jaylen to explain how he knew that the corn and the pepper were not three pounds each. Taylen says that with three and three, the second scale would not be balanced, the two peppers would be three plus three which is six pounds and the corn on the other side would be just three pounds. Jaylen is asked to repeat this explanation, which he does. T1 moves on to the next problem. She hands out copies of the problem and tells the class that this may look familiar to some of them. The problem is about sunglasses and shorts. Some kids acknowledge having done this problem before. T1 reminds the students to show their work and the strategy they use. Students start working in pairs.
40:00-45:00	Students are busy discussing the problem with each other. T1 approaches Kyla and Neema. She asks them to explain how they answered the first part of the question. Neema starts by saying that the shorts are $20 each, but the teacher interrupts her to point out that the question is asking which one is more expensive without knowing the prices. Kyla takes over and explains: We said if the shorts were $20 a pair,

	then three pairs of shorts would already be over \$60. Then we thought that sunglasses are more expensive than the shorts, since there are three pairs of these (pointing to the shorts) and only one of these (glasses). Neema says that she first priced the shorts at \$15 each and that worked for the second picture but not for the first picture. T1 wants to know how it worked for the second picture. She starts adding up, pointing to the shorts: 15, 15, and 15 is 45. Kyla takes over and says that the glasses are \$5, so it would be 50. But if you… Neema addresses Kyla and says: “can I speak?” She then continues to explain that since \$15 for shorts and \$5 for glasses did not work in the first picture, she went down by five and tried \$10 for shorts and \$20 for glasses, which satisfied both pictures. T1 asks the girls to name the strategy they had used to find the prices. They do not know. T1 tells them that it is called guess and check. T1 asks if they could have done it any other way. Titiana says they did it differently and explains their method. She explains that they started with the first picture assigning \$40 to the shorts and \$5 for a pair of sunglasses. Since that didn’t work for the second picture, they lowered the price of the shorts to \$30 and tried again. Eventually and after several trials, they found the correct prices. T1 points out that they too used trial and error and asks if there is another way of doing it. Carin says no and T1 says she thinks there is. Tequrra, Jasmine, and Jade are working together. T1 is interacting with Elyce, Maurice, and Jarrod. Maurice is explaining something to the teacher and she asks where they got the \$20 from. Maurice responds that they tried to come up with numbers that added to \$50 in both pictures. T1 reminds the whole class that the first question is very important and that is the one they are going to report out on. The question is asking for a comparison of the prices without knowing the price of each item. T1 asks Elyce, Maurice, and Jarrod if they could have done it a different way. Elyce says that she was thinking of dividing 50 by 3. T1 asks: “What is three? These three?” (pointing to the first picture) Elyce nods yes and Jarrod picks up a calculator.
45:00-50:00	T1 walks around the room, looking at the students’ work and at

	times she points to something on their paper and engages in short conversations with students. T1 stops to have a conversation with Jaylen and Amir. She asks Amir to explain why he thinks glasses are more expensive than shorts. Amir says that usually you have to pay a lot of money for them at the store. T1 asks: What if I go to Wal-Mart? Amir replies that it would still be probably more expensive than shorts. T1 points to the paper and encourages the boys to identify the more expensive item based on the pictures in the problem. Jaylen explains how they looked at the pictures and tried different prices for the items until they found that $20 and $10 worked. T1 asks the boys if they know the name of this strategy. Amir answers: “Addition?” T1 tells him that addition is not a strategy, but an operation. Jaylen says: Squeeze method? T1 inquires whether that means starting with a low number and a high number and then squeezing. Jaylen nods yes. T1 asks how they came up with the initial numbers and Jaylen says that they randomly picked them. T1 presses the boys to name the strategy where you pick a number and try to see if it works. One of the girls at the table answers that it is guess and check. T1 acknowledges that it is the correct answer and asks the group if they could identify the more expensive item by using a strategy other than guess and check. She then walks away.
50:00-55:00	Students continue to work on the problem. T1 reminds students that the focus is going to be on the first question and asks them to write the solution down and be prepared to report out on it. Elyce, who has been working with Maurice and Jarrod, stands up to report on her group’s finding. She reads the question first and answers it by saying that the glasses are more expensive than the shorts. However, when T1 asks her to justify her answer, Elyce admits that they found the actual prices first and based on that they determined which item is more expensive. On teacher’s request, Elyce explains how they found the actual prices and when T1 asks her to name the strategy they used, Maurice chimes in to identify it as guess and check.
55:00-01:01:41	Jaylen stands up to share his result with the class. He claims that the glasses are more expensive and justifies it by explaining how they figured out the actual prices. T1 emphasizes that the question says “without knowing the prices”,

	and asks another group to present. Erica also stands up and presents her solution to the class but she, same as Jaylen, has used trial and error to determine the prices and has come to the conclusion that sunglasses are more expensive than shorts. T1 says: Okay, so you have attached money too. She then asks R1 if it is okay to move on to the next problem, but R1 indicates that she likes to hear how the students at Table 2 tackled the problem. Jade stands up and says that they know sunglasses are more expensive than shorts because the more glasses you have, the fewer items you need to make the total reach $50. T1 repeats Jade's answer and asks someone at Table 1 to report. Kyla says that they too used numbers, but she also adds that since glasses are breakable, they are more expensive. T1 comments that they are being practical and asks Titiana to stand up and share what she and her partner, Carin, have found out. Titiana says that they divided $50 by 3 and got $16.20. Then they multiplied 16.20 by 3, and got $48.60, which they subtracted from 50 to get $1.40. They concluded that the shorts cost $16.20 and the sunglasses were $1.40. R1 asks Titiana a clarifying question: You divided 50 by 3 to get $16 and something. Does that mean that the shorts and sunglasses are the same price? Titiana says yes, but immediately changes her answer to a no. Carin restates the prices they had found for each item. T1 asks whether the prices work in both pictures, if they need to satisfy both situations shown in the pictures, and if so, why. Carin immediately says yes and tries to explain. Meanwhile Titiana is calculating something and says that it does not work in both pictures. T1 says that they need to assess their work and "go back to the drawing board". T1 is about to introduce the next problem. **[End of Role T1-1;Lesson continues on Role T1-2]**
Time Interval	**Description**
00:00-5:00	The class starts working on a new problem involving shirts and cups of soda. T1 explains that a shirt and a soda are the same price in both pictures, but as shown, the total costs are different in the two boxes. The students at Table 3, Jaylen, Sierra, Erica, and Amir, are trying to apply the guess and check method to come up with the correct prices. They try out a few numbers, but they don't work. At Table 2, Jasmine tells Jade that she has found prices of each item that work in one box and that she has to try and see if they

	work in the second box as well. At Table 1, Neema is telling the teacher that $2 per soda and $24 per shirt work in the bottom picture, but not in the top one. T1 says that is why she is trying to push them to use another strategy besides guess and check. Kyla responds by saying that they tried but could not think of any other way. T1 assures them that there are other ways and Neema and Kyla ask for hints. T1 asks R1 if she can give the girls a hint and is advised that she should provide hints through asking questions. T1 Asks the girls if they can "make that problem smaller in any way….make that problem with less item". Kyle says: "Oh, I think I know what you mean". She starts writing. T1 also shares that hint with the whole class by asking: Can you make that problem with less item and if so, how? Carin, who is busy working with Titiana, asks T1 if the soda cups are of different sizes. T1 replies that they are all the same. Carina and Titiana are using guess and check method. Carina points to the second box and adds $5 three times (three soda cups) and a $15 shirt to get the sum of $30. She then tries these assigned prices in the first box. When she realizes that they don't work, she rests her forehead on her palm and says: "Oh, darn". At Table 4, Elyce, Jarrod and Maurice are also using trial and error. They work independently and guess several numbers, using calculators to check whether the assigned prices add up to the indicated sum.
5:00-10:00	Elyce, Jarrod and Maurice are visibly struggling to find the correct prices. Jarrod, after punching some numbers on the calculator, taps his clenched fist on the table several times and makes a face (as if in pain). Maurice, after realizing that his chosen numbers don't work, says: "Man, I was close" and Elyce say: "Me too". The camera moves to Table 3. T3 is seated close by. Amir suggests that a shirt and a soda are $10 each. Jaylen disagrees by pointing out that if they were both $10, the sum would be $4 short in the top picture. Sierra and Erica try $12 per shirt and $8 per drink and after adding the prices in the bottom picture and getting a sum greater than $30, realize that the assigned prices don't work. T1 comes over and asks Sierra and Erica: "What if I had only one shirt and one soda?" Erica says that it would be $22 and half of it would be $11. Sierra says that each shirt would be $11. T1 asks whether that would work. The girls start adding the prices in the top picture, assigning $11 for both the shirt and the soda. T1 asks if it works. Erica says yes and Sierra says no because the total for the bottom picture is

	$30. T1 tries to give them a hint by pointing to two shirts, one in the top picture and the other in the bottom one. She then asks: What does that mean? What can you do about that? The girls have no answer to that and after a few seconds, Erica resumes the guess and check strategy with two new prices of $6 and $12. T3 questions the girls about their choice of $6 and $12. She finds that their original guess for a soda was $7 and for a shirt was $15. She asks why they made those changes. Sierra answers something (not clear) and T3 smiles, shakes her head, says okay, and turns away slightly.
10:00-15:00	T3 asks Sierra: “What can you interpret, from the picture, about the shirt and how it relates to the soda?” The camera moves to Table 4. Jarrod claims that the price of each item is $11. T1 asks if it works in both pictures. Maurice says that it does not work for the bottom picture. T1 moves away and tells the class that during the remainder of class, they should make sure that they record their thinking on the paper. R1 is looking over Carin’s paper and having a conversation with her and her partner, Titiana. As she is walking away she says that their idea is very good and that they should keep on working. Titiana asks T1 if the prices have to be “even”. T1 answers that whatever the prices, they should add up to the indicated sum in each picture. Kyla stretches and tells T1 that the problem is too hard. Carin and Titiana are working together. T1 approaches, looks over Carin’s work and pointing to the paper she asks whether their answer would work. As Carin finds out that it doesn’t, Titiana, rests her head on her folded arms on the table. Camera moves to Table 2. Jasmine, Tequrra, and Jade have figured out the prices. Jasmine says to Jade and Tequrra that she knew the shirt would be a lot of money and “that was a good way of looking at it though, to cut it in half”. R1 and T1 approach the table and the girls tell them that the shirt is $18 and the soda is $4.
15:00-20:00	T1 wants to know how they got the answer. Jade explains that they started with the top picture and since there were two of each item, they divided the total sum by 2 to get the price of a shirt and a soda combined. Jasmine adds that this way they could forget about the $44 and just work with $22.

	Jade further explains that they considered numbers that add up to 22 and work with the given totals in the pictures. T1 says: Okay, good. Write down your explanation. T1 moves away. The camera shows Table 3, where T2 is discussing the problem with Sierra and R1 is having a conversation with Erica and Amir. She is trying to help Erica and Amir see that the shirt and soda are not the same price. She uses her hand to cover two sodas in each picture, leaving exposed two shirts in the top picture and a shirt and a soda in the bottom picture. The camera goes back to Sierra and focuses on her paper, where she is subtracting 30 from 44. The camera shows Table 4, where Maurice, Elyce, and Jarrod are writing. Maurice says: “Oh, I am a genius”. T1 asks the class to write the solution down and justify their thinking.
20:00-25:00	T1 reminds them to put their names on each piece of paper, including the scrap paper, and staple them all together. Students are following directions. At Table 4, students are having a conversation with R2, who wants to know what their final answer is. Jarrod tells him that the answer is $18 for a shirt and $4 for a soda. R2 asks if there could be another answer. Jarrod’s response is no, but Elyce says there could be. She also adds that the problem they just did was too frustrating and nerve wrecking. R2 inquires whether there could be an easier way to get the answer. Elyce says, maybe. Students who have finished organizing their work, walk to T1 and hand in their stack of papers. T1 looks through the papers to make sure there are written explanations for each problem. Some students are still writing. At Table 3, Sierra (referring to the bottom picture) tells R1 that three sodas are $12 and a shirt is $16. R1 points out that 12 and 16 do not add up to 30. Sierra offers a new price for the shirt, mentioning 17, then changing it to 20. R1 wants to know what plus 12 will make 30. Sierra responds, $4. T2, who is seated next to Sierra says: “You know that Sierra, we talked about it before”. Erica suggests 18, and sierra confirms that 12 and 18 add up to 30. R1 asks her to check these prices in the top picture to see if they add up to $44. She then asks Sierra to name and explain the strategy she used. Sierra says she used guess and check by going up

	and down with the numbers.
25:00-29:00	R1 asks her to explain how and why she went up and down with the numbers. Sierra says that she started with $12 for a shirt and $6 for a soda. Then she decided to go up with the shirt price and lower the price of the soda. R1 encourages her to write down her thinking. The students who have finished and handed in their work have already left the room. At Table 4, O4 places a piece of paper in front of Jaylen and asks what the total price would be of the shirts and sodas he has drawn in the box. It is not very clear how many shirts and soda cups he has drawn on the paper. Also at that table, T3 starts a conversation with Amir. She is trying to get Amir to compare the two total prices, but there is no time and the class ends.

Appendix B2: Description of lesson #2

Roles T2-1 & T2-2	Place data recorded: Sheffield, NJ Date data recorded: 5/14/08 Classroom teacher: T2 Grade 6
Time Interval	**Description**
00:00-5:00	There are 16 students in the classroom. In addition to the classroom teacher, T2, there are eight other adults, R1, R2, O1, O2, O4, T1, T3, and T4, present in the room as observers. An additional person is in charge of taping the session. [Two school vice principals pop in for a visit sometime during the lesson.] Students' desks are put together to form six groups of students. There are two or three children in each group. [The grouping of the students was decided by the teacher prior to the lesson and was mainly based on students' personalities and how well they would work together. The teacher had tried, to some extent, to have a more academically strong student in each group] The class begins with the teacher (T2) introducing the university faculty member, R1, who addresses the students. She explains that the adults, present in the classroom, are interested in becoming better teachers and a good way to do that would be for them to observe a lesson. She also asks the students if it would be okay for them to be videotaped and for the observers to walk around and interact with them once they start doing the math problems. The observers introduce themselves to the students by stating their name, the grade they teach and the school they work at. Several kids show excitement as they recognize one of the observers as their 5th grade teacher from the previous year. T2 tells the class that they will be working on some problems similar to some of the problems they had done in the past, and in particular, the one they had done the previous day. He also asks them to work within their groups, discuss the problems, and be ready to report back to class their solutions.
5:00-10:00	R2 helps T2 hand out the first worksheet (Bananas problem). T2 reminds students to put their names at the top of the page, read the question carefully and discuss the problem with each other. Most of the time students quietly read through and work on the problem by themselves. However at Table 4, Kevin, Caliph, and Cory are engaged in a three-way discussion about the problem. Also a few words are exchanged between the two students seated at Table 3.

10:00-15:00	T2: “Let’s try and ready ourselves for sharing in three minutes”. Some students seem to have finished working on the problem and some are busy writing out their answers. The class is very quiet. O1 is heard telling the students that it is okay to talk to each other and that they don’t have to whisper. T2 turns on and adjusts the overhead projector. He then tells the students that they should discuss their answers within the group to make sure an agreement is reached before they present to class. This generates some discourse. At Table 1, Jasmine shares her answer with Monae and Dina. Monae and Dina don’t agree with her answer and try to explain to her why she is wrong. Jasmine finally says that she gets it. T2 is heard asking different groups of students if they are in agreement with each other or whether they have reached consensus. T2 says to class: Raise your hand if you have different answers within your group.
15:00-20:00	No hand is raised and the teacher concludes that the students have reached consensus within the groups. T2 tells the students that each group is going to present their solution to the class and they should decide on who is going to be the spokesperson within each group. T2 asks a group of three girls to present, but they ask if they can be skipped. T2 tells the girls that he would come back to them later and asks for another group to volunteer. The boys at Table 3 raise their hands and Caliph stands up and is told by T2 that, if needed, he can use the overhead projector for his presentation. Caliph remains at this desk. He explains that it takes 10 bananas to balance two pineapples. If you take out one pineapple, you take out half of the bananas and that is five bananas. Since one pineapple equals two bananas and an apple, it means that one apple is equal to three bananas. T2 asks Jasmine at Table 1 to report out next. Jasmine puts her head down on the table indicating her apparent unwillingness to share. T2 tells her that if she does not have anything to add to Caliph’s explanation, she should at least repeat, loudly, what she was just saying. Jasmine says that they had the same answer. T2 asks if they used the same reasoning, to which Jasmine responds with a long sigh and a yes. T2 asks Nekaybaw and Jayananoh to report, but they decline and T2 asks Jose and Joel to share their thoughts with class. Joel says: “Ten bananas and two pineapples. On the next scale there

	is only one, so we put five bananas. But since there is only two, one apple must be equal to three bananas". T2 asks in what way one apple is equal to three bananas. Some students are heard to reply, "weight". T2 asks Jevon, Meledy, and Oscar to present next, but they say that they have the same answer and nothing new to report.
20:00-25:00	T2 asks Jazmine, Tateanna, and Sakeena to share their findings. Jazmine stands up and says that they believe the answer to be two and not three bananas. She explains: "Since one pineapple is equal to two bananas and one apple, then one apple must be equal to two bananas, because, the same as scales, it is like having an apple in one hand and two bananas in the other hand." Kevin, at Table 4, says that he disagrees with Jazmine and T2 encourages him to tell Jazmine why he disagrees with her answer. Kevin explains why one apple cannot be equal to an apple by pointing out that on the second scale, if the apple is replaced by two bananas, then we are saying one pineapple is equal to four bananas. This is wrong because we know from the first scale that one pineapple is equal to five bananas. Jasmine, at Table 1, says that in the first picture half of 10 is five and half of two is one. So one pineapple is equal to five bananas. But on the second scale, there are only two bananas. So we need three bananas because 2+3=5. Cory has had his hand up and the teacher calls on him. Cory says that the bananas could be 1.5 pounds each and an apple could be 2 pounds. So on the second scales, two bananas and an apple will be a total of 5 pounds. T2 is about to move on to another group when R2, seated at the back of the classroom, indicates that he has a question for Cory. He asks Cory what the weight of 10 bananas would be. Cory says to himself: "One and a half times 10" as he tries to calculate the answer. No answer is given and the class moves on.
25:00-30:00	There is a lot of off task chatting going on in the classroom. Before moving on to the next task, T2 repeats how he expects the students to work together and discuss the problem within their group. T2 puts a transparency of the next problem- with carrots, corns, and peppers- on the overhead and also hands in a copy of it to each student. Students start working on the problem Joel and Jose, at Table 2, are immediately engaged in a discussion about the problem. Joel notes that two "pumpkins" equal a corn

	(They take the peppers to be pumpkins). Jose adds, pointing to the first scales, that it would be three (he is saying, correctly, that three pumpkins could replace a corn and a pumpkin). Joel disagrees and says that it should be four (he is saying, correctly, that four carrots equal one corn). Jose says: "what do you mean?" Joel points to the six carrots and says: This is six, so this (corn) is four and this (pumpkin) is two. The camera moves to another table.
30:00-35:00	At Table 5, Maledy asks Jevon if he gets it and when he nods yes, Maledy wants to know what he has for an answer. Jevon, smiling shyly starts reading from his paper (not audible). Oscar asks him to speak up, but Jevon covers his face with his paper. Maledy snatches the paper away from Jevon's face, places it on the table, and pointing to the pictures, gives him her answers. She then makes a gesture that indicates her apparent impatience, looks at the camera, and smiles. Oscar concludes that Jevon does not want to share and turns to Maledy and asks what she thinks a corn will be. Maledy reads out loud as she writes down her answer: It will be three carrots…(not clear)… one pepper. Oscar mentions that pepper is not that heavy. Camera moves to Table 3, where T3 is discussing the problem with Jayananaoh. Jayananaoh claims that if three carrots are moved from the first scales and put on the last scales, then the right pan will be heavier than the left pan in the first scales. T3 asks Jayananaoh to interpret the second scales. He says that the right pan is heavier than the left pan because there are two items on the right and only one item on the left. The camera moves to Table 6, where T2 asks Tateanna how many carrots will be in the last scale. Tateanna says two, but immediately changes her answer to three carrots. T2 says okay and clarifies that he is not saying yes, or no to her answer and that she will have to justify her answer later on. T2 asks Sakeena and Jazmine if they agree with Tateanna, and when they say, no, the answer is two carrots, T2 tells them to discuss the answer with Tateanna so they can reach consensus. Jazmine is thinking about formulating her explanation, when the camera moves away. At Table 5, T2 finds out that the students do not have the same answer.
35:00-40:00	Oscar thinks the answer is ½ a corn. T2 points out that the question

	is asking how many carrots and not corns. Maledy thinks the answer is three carrots and explains that since there are six carrots in the first scales, then six divided by two is three, which means that pepper and corn each equal three carrots. Jevon agrees with this explanation. T2 asks them to discuss their answer with Oscar and moves away. At Table 2, Joel explains his answer to T2. Jose is in agreement with Joel. T2 asks for volunteers to present to class. Two groups raise their hands. It takes a few minutes to quiet down the class. T2 asks Table 6 to present, but changes his mind and asks Table 2 to go first. Joel, at Table 2, says a pepper is equivalent to two carrots in weight. T2 asks him to justify his answer. Joel says that according to the second picture, one corn equals two peppers. So in the first scale, the corn is four and the pepper is two.
40:00-45:00	T2 asks Dina from Table 1 to present. Dina says that from the first scale, where six carrots equal a pepper and a corn, you can say that the pepper and the corn each equal three carrots. But this is not true because according to the second picture, a corn and a pepper are not equal and it takes two peppers to equal one corn. Therefore a pepper is two carrots. T2 asks Table 5 to present. The three students in this group appear reluctant to be the spokesperson, but finally Maledy speaks up. She claims that six carrots divided by two will give you three carrots and that means that the pepper and the corn are three carrots each. Jevon and Oscar confirm their agreement with Maledy. T2 asks the class if anyone has a question about Maledy's statement. Joel raises his hand and says that Maledy's claim that a pepper and a corn each equal three carrots is in conflict with the second picture, where a corn equals two peppers. T2 asks Table 5 to think about Joel's statement and turns the attention of the class to another group of students. T2 asks Tateanna at Table 6 to present. Tateanna is hesitant. Eventually, Sakeena volunteers to read off Tateanna's paper. From what Sakeena says, it is not clear what the final answer is and Tateanna keeps saying no.
45:00-50:00	T2 asks Sakeena what would replace the question mark in the last picture and she replies that Tateanna thought it would be two carrots. She is not asked to justify her answer. There are two groups of students who have not yet shared their

	solution with the class. T2 asks the class to pay close attention to the last two presentations since there has not been a consensus amongst the groups regarding the final answer. T2 asks Table 4 to present. Kevin says that since, according to the second picture, one corn equals two peppers, if you replace the corn in the first picture with two carrots you will have six carrots equal three peppers or one pepper equals two carrots. T2 makes the distinction that one pepper equal two carrots in terms of weight. T2 asks Table 3 to present. Jayananoh says that their answer is two carrots and tries to explain by saying that since one corn is equal to two peppers, then one pepper is probably equal to two carrots. Nekaybaw adds that since a corn is half of six and the pepper is half of six, then they are equivalent. Then she stops and says, “ never mind” and in spite of encouragement from the teacher, she offers no further explanation. T2 introduces the next problem, “Shorts and Glasses”.
50:00-55:00	T2 places a transparency of the problem on the overhead projector and, with some delay, covers the price tags showing the two totals. He explains the problem to the students and asks them to determine which item is more expensive. R1 gives the class the very important piece of information that the two totals are of equal value and that their task is to find out which is more expensive, a pair of shorts or a pair of glasses. One boy immediately says glasses and R1 asks how he knows that. Students start discussing the problem in their groups. Joel, who had seen the $50 tags before they were covered, has already figured out the prices and tells his partner, Jose, that the glasses are $20 and the shorts are $10 and therefore the glasses are more expensive. At Table 5, Maledy tells Oscar that one pair of shorts might cost $10 and one pair of glasses might be $20. At Table 4, Kevin wonders why anyone would waste money by buying two pairs of glasses and one pair of shorts “when glasses are going to take up most of your money” and adds that it is cheaper to buy two pairs of shorts and one pair of glasses. R1 poses a question to the boys at Table 4.
55:00-00:59:51	She asks how many pairs of shorts they will get if they traded in a pair of sunglasses. The camera moves to Table 6 where T3 is having a conversation with Sakeena. The camera zooms on Sakeena’s written work which shows she has added $2 (for two pairs of glasses at $1 each) and

	$12.50 (for one pair of shorts) and has written $14.50 as the total. T3 reminds her that the total is the same for both pictures and asks her to calculate the total price in the bottom picture. The camera zooms on Titeanna's work and shows that she has added 12.99 three times to get $38.97. One of the observers asks her how much a pair of glasses would be. Titeanna says that the three shirts are $38.97 and a pair of glasses is $2.99. T3 tells someone off at the next table and asks him to mind his own business. She then turns to Sakeena and encourages her to finish calculating 12.50 x 3. The camera moves to Table 4, where R1 is listening in on the conversation. Cory says one pair of glasses is worth two pairs of shorts. T2 tells the class that they should write down which item is more expensive and why. The students at Table 1 tell R1that the glasses are $2 and the shorts are $4. R1adds the prices in the first picture to get $8 and asks them to calculate the total in the second picture. Dina says that in the second picture, each pair of shorts is $2. R1says that each item is the same price in both pictures. Jasmine says she gets it now and starts adding some prices up. **[End of Role T2-1; Lesson continues on Role T2-2]**
Time Interval	**Description**
00:00-5:00	At Table 6, Tateanna tells T2 that the shades are $2.99 and the shorts are 12.99. T2 asks her what makes her think that the shorts are more expensive than glasses, but she does not have an answer to this question. T2 walks away. Students are given a copy of the problem they have been working on. There are three questions about the shorts and glasses on this worksheet and the students have already been working on the first one which was presented on the overhead. T2 asks them now to write down the answers to all three questions on their paper. Students get to work. At Table 1, Dina, Jasmine, and Monae are trying guess and check to find the prices. Jasmine wants to stick with the original prices of $2 and $4 for shorts and glasses respectively. R1 points out to her that since they now know that the total is $50, $2 and $4 would no longer work, as the total for that purchase in the first picture comes out to be $10 and not $50. Jasmine suggests the new prices of $8 and $16. Dr. Alston asks whether it works. The camera moves to Table 4 briefly, but returns to Table 1. The girls must have guessed that $10 for shorts and $20 for glasses

	work in the top picture because Dr. Alston is heard asking Jasmine if their guess works in the bottom picture. Jasmine adds up the prices and shows excitement over the fact that it does. Jasmine tells T2 that they have all the answers. T2 comes over to listen to their explanation.
5:00-10:00	Jasmine starts talking, but she repeatedly makes mistakes and has to repeat sentences to correct herself. She says that at first they thought that the prices were $2 and $4. But then, since they did not add up to $50, they found out that the prices were $30 for glasses and $20 for shorts. T2 points out that with those prices, the total would be more than $50. Monae corrects Jasmine and says that the shorts are $10 and the glasses are $20. T2 asks if these prices work in the second picture. The girls say yes, and start adding the prices up to demonstrate. T2 says, good job and asks them to write everything down. T2 walks around and checks students' work to make sure everyone has written responses to all three questions. T2 asks R2 to see him outside the classroom as he has a favor to ask. The two leave the classroom and T2 returns after a few seconds. T2 encourages some students to add more to their explanation.
10:00-15:00	T2 asks if every group is ready to share and they all say yes. R2 returns to the room. Jevon at Table 5 presents first. He says that glasses are more expensive than shorts because they are breakable and harder to make. T2 asks Jevon to repeat it louder and Oscar asks Jevon if he could step in as the spokesperson. Jevon agrees and Oscar says that glasses are more expensive because they can break, are harder to make, and they help people see. T2 notes that their conclusion that glasses are more expensive than shorts is based on what they know about these items and not based on what they see in the pictures and encourages Oscar to offer another explanation. Oscar says that in the first picture there are more glasses and a total of three items, but in the second picture, for the same amount of money, there are four items and only one pair of glasses. T2 asks the class if they all agree that glasses are more expensive than the shorts. The three girls at Table 6 shout their disagreement.

	It appears as if Nekaybaw at Table 3 also disagrees. T2 asks her to read her written answer out loud. Nekaybaw says that she no longer agrees with what she has written and offers her justification as to why shorts are more expensive than glasses. The boys at Table 4 are chatting and it is hard to hear her explanation. But T2 replies that the explanation she presented is based on her prior knowledge about shorts and glasses and not on the pictures provided. Tayananaoh, Nekaybaw's partner at Table 3, offers his explanation.
15:00-18:40	Tayananaoh begins speaking but before he has a chance to explain, Nekaybaw takes over and says that glasses are $20 and shorts are $10, so the glasses are more expensive. Tayananaoh complains that he did not get a chance to speak. T2 asks the girls at Table 1 to present. Jasmine reports that glasses are more expensive because they are $20, whereas the shorts are $10. T2 wants to move on to the next question, but they have to stop the lesson as the afternoon announcements start through the PA system. The class ends.

Appendix B3: Description of lesson #3

Roles T3-1 & T3-2	Date data recorded: 5/15/08 Place data recorded: Sheffield, NJ Classroom teacher: T3 Grade 8
Time Interval	**Description**
00:00-5:00	There are 12 students in the classroom. In addition to the classroom teacher, T3, there are 12 other adults, R1, R2, O1, O2, O3, O4, O5, O6, O7, T1, T2, and T4, present in the room as observers. An additional person is in charge of taping the session. The math coach and the school principal are also attending the session. All students' desks are put together in pairs. There are five groups of two students. Two students sit separately and by themselves. [The students are seated based on where they normally sit in this class.] T3 addresses the class: Today we are going to be doing some algebra problems similar to what we have been doing in the past. We want to expose you to more algebra1 material and also we want to find out how "advanced" 8th grade students tackle algebraic problems. T3 introduces the university faculty member, R1, who addresses the students. She explains that the grownups, present in the classroom, are all graduate students at Rutgers and as a group, they have been working on some algebra problems. R1 continues: We are interested to know how students would deal with these types of algebra problems and your teacher offered to do this lesson in your class today. She also asks the students if it would be okay for them to be videotaped and for the observers to walk around and interact with them once they start doing the math problems. T3 asks the observers to introduce themselves. Observers introduce themselves, stating their name, the grade they teach and the school they work at. The principal is among the observers, standing at the back of the room.

5:00-10:00	After the introductions are over, T3 puts a transparency of the "Carrots" problem on the overhead and asks the students to read it quietly. While the students are busy looking at the problem, T3 hands out a copy of the problem to each student and asks them to explain how many carrots would balance the last scales. Kevin puts his hand out to give the answer, but T3 tells him to write down his answer first. Some students start discussing the problem with their partner. T3 tells them to write down their explanation. She repeats several times that the students need to convince her that their answer is correct. T3 says to class: Someone just asked me if the answer can be a half. Write down how you got it and convince me. The students are busy writing. The camera zooms on Tyreal's paper while he explains his answer to T3.
10:00-15:00	Tyreal tells T3: If one carrot is 1.5, then two of them are 3 and six of them are 9 So the pepper is 3 and the corn is 6. Then, in here [pointing to the second picture], this [corn] is 6 and on this [two peppers] is 3 and 3, is 6. T3 asks: What will this be [pointing to the pepper in the last picture]? Tyreal says, 3. T3 says, wow! Write that down. She moves away. As the teacher walks away, Tyreal quietly says, two carrots. His partner, Al, says: You never told her [the teacher] that it would be two carrots. While the students work on the problem, T3 and some of the observers walk around, looking at students' work. T3 approaches Table 4, where Jynita and Dieshe are working together. T3 asks: What do you think the problem is asking you to do? Jynita says something which is not very clear. T3: What do you know about carrots? Dieshe says that they are orange and the two girls start giggling. T3 acknowledges that carrots are orange, and adds, let's talk about the scales. T3 suddenly stops and decides to address the whole class. T3: Boys and girls, do you remember in second grade when you worked with pan balance? What happened with pan balance? Kevin: We had to make the two sides balance. T3: How did we make the sides balance? Students say add to or reduce from a side.

	T3 starts writing the answers on the board and asks the students to be more specific. Keith says: You have to reduce from the heavier side. Another student says that you have to add to the lighter side. T3 writes these comments on the board and says that in general, you are making both sides equal to each other. She then writes "equal to" on the board and underlines the word, equal. T3: Now, in the first picture we have six carrots on the left side and a corn and a pepper on the right side. What is it that we know about the first picture? Dieshe: We know that one pepper and one corn is the same weight as six carrots.
15:00-20:00	T3 asks Dieshe if she would like to go to the overhead to do the problem. Dieshe hesitates and a couple of students volunteer. Tyreal goes up front to demonstrate his solution to the class. He explains that he assigned the number 1.5 to each carrot, so two carrots would be 3. Then he says that a corn would be 6 and a pepper would be 3 and he points out that the second picture verifies his assertion. Then he points to the last picture and says the pepper which is equal to 3 is the same as two carrots. One girl is heard saying: I knew the answer was two carrots. Kevin claims that he has a different way of doing it and is asked to go to the overhead and share his answer with the class. Kevin writes on the board: 6 carrots = 1 corn + 1P 1 corn = 2P 1 P = ??? He is not able to articulate his reasoning very well, but what he says amounts to the following: Two peppers equal one corn, so in the first picture we know that a corn is the same as four carrots and a pepper is equal to two carrots. Therefore the answer is two carrots.
20:00-25:00	T3 hands out copies of the "Bananas" problem and asks the students to work on this problem in pairs. She also puts a transparency of the problem on the overhead projector. After about two minutes, students volunteer to share their answers. Jynita admits that she used the same technique Tyreal had used in the previous problem. Jynita explains that if 10 bananas are equal to two pineapples, then one pineapple would be equal to five bananas. Therefore in the second picture where one pineapple equals two bananas and an apple, the apple would be three bananas. She then concludes that in the last scale, the one apple would equal three bananas.

	Both Davon and Kevin offer a similar explanation. K3 introduces the next problem, “Shorts and Glasses”, on the overhead relating it to a story about one of her own shopping trip experiences. The price tags showing the $50 value are covered in both pictures, but K3 mentions that the total prices are the same. K3: Without knowing the prices of glasses and shorts, can you tell me which one is more expensive, and why? A few hands go up immediately and K3 calls on Theo to answer. Theo says that glasses are more expensive. T3 asks: In general? Theo nods yes.
25:00-30:00	T3 asks the class to decide which one is more expensive based on the pictures. Kevin says: Glasses are more expensive because one pair of glasses is the same price as two pairs of shorts. T3: How did you get that? Kevin: Because in the bottom picture, there are two extra shorts and one pair of glasses, and in the top picture, there are two pairs of glasses and one pair of shorts. Keith and Chad say that they had the same answer as Kevin. K3 then hands out a copy of the problem, which indicates that the total prices in the two pictures are $50. She asks the students to figure out the price of each item. Students start working on the problem. Theo who is using a calculator raises his hand and T3 approaches him. Theo says (pointing to the bottom picture), that the glasses and shorts each cost $12.50, because they add up to $50. T3 reminds him that glasses are more expensive than the shorts and she walks away. Students are working on the problem. Most of them are using a calculator. T3 asks the students to write down their explanations as they would say it verbally. Dieshe and Jynita are talking to each other about the problem They have figured out that the glasses are $20 and the shorts are $10. K3 looks over their paper, confirms that the prices are correct and instructs them to write down their explanation. K3 moves over to Tyreal, looks over his paper, and says: Explain how you got your answer.
30:00-35:00	T3 reminds the class that the items shown in the top picture are identical to the ones shown in the bottom picture.

	Several students have their hands up and are keen to share their findings. Davon goes to the overhead projector. He says: First we said the glasses are $17 and the shorts are $16, which worked in the first picture but added up to more than $50 in the second picture. Then we found out that $20 for glasses and $10 for shorts worked in both pictures (he actually wrote the prices down and calculated the sum to show the total of $50). T3 goes to the overhead and asks the class: If I wanted to write an equation for these pictures, what would it be? A few responses from the students (not clear). T3: What can I say about the total prices in the top and the bottom pictures? Kevin: They are the same. T3: What symbol did you learn in the kindergarten that means the same? Someone says: Equal sign. T3 draws an equal sign between the two pictures and says: Here we are talking about things that are equal. Now let's change gear. I have something for you here. Do you remember this? (T3 holds up a picture of students playing Tug-of-Wars.) Students acknowledge playing the game as one of the activities offered on field day. T3 asks the students to describe the game. Jynita: You pull on the rope and you cannot go over the line. Kevin: It is like the scales; both sides are equal. T3: Yes, first both sides are equal. Then you start pulling on the rope and it goes off balance and one side wins. T3 hands out copies of the "Tug-of-Wars" problems to the students. She reads the problem aloud and tells the students they need to prove that their answer is correct.
35:00-40:00	Students are busy working on the problem. They discuss the problem and listen to each other's ideas. T3 visits several groups, listens to their explanations and asks them to write their thinking down. Some students are reluctant to write anything down, but they listen to their teacher and begin to write. The observers are walking around, listening in, and taking notes.
40:00-45:00	T3 is working with Dieshe and Jynita. K3 asks (pointing to the second picture): What does it means when we say an elephant is as strong as one ox and two horses? Dieshe: It means they are equal on both sides. T3 asks how that information can be used to determine who wins

	the tug of war in the last picture. Jynita says the four oxen will win, but she has no explanation for her answer. After some thinking Dieshe figures out that she can replace the elephant in the last picture with an ox and two horses and end up with four oxen on one side and five horses and an ox on the other side. She refers to the equality in the first picture and concludes that the elephant and three horses will win. T3 asks Dieshe to write down her explanation before she forgets. R1 is heard asking Dieshe for clarification and she repeats her explanation to R1. Dieshe tries to convince Jynita that her answer is correct. She explicitly says that you can take away the elephant in the last picture and substitute one ox and two horses in its place. Jynita rejects Dieshe's idea, reasoning that nowhere in the picture it tells you that you can remove the elephant. Jynita shakes her head and says: I don't get you Dieshe. The girls stop talking and start writing their solution down. T3 come over to the girls again. Dieshe tells her again why one elephant and three horses are going to win. T3 asks Jynita about her prediction. Jynita says that the four oxen are going to win and when T3 asks her why she thinks that, Jynita says: "Because they look like they are going to win". T3 tries to make Jynita articulate what she knows about oxen. Jynita describes what she sees in the pictures, but also adds her own interpretation by saying: If four oxen can pull five horses, then I am sure they can also pull three horses and an elephant. Jynita is adamant that her answer is correct and when Dieshe tries again to explain the solution, Jynita cuts her out and reminds her that she is the one who is doing the talking.
45:00-50:00	Jynita talks some more about the picture and Dieshe tries again to make her see how she can use substitution to end up with five horses and an ox on one side in the last picture. This time Jynita has kept quiet and after listening to Dieshe's explanation she says: How can you end up with five horses? Do you see five horses in this picture? T3 asks Jynita: "Are you convinced by her or is she (Dieshe) convinced by you?" Jynita: "I don't know. I don't want her answer." T3: "Okay you come up with your answer. I will give you another chance to write your answer." T3 moves away, asking the class if anyone is ready to share. Several people are heard saying that they are ready.

	Camera moves to the other side of the room, where Keith is explaining his solution to R1. R1 asks T3 to listen to Keith's explanation. Keith has arrived at his answer by assigning appropriate numbers to each animal.
50:00-55:00	R1 points to the top picture and asks Keith: Why did you assign each ox a 5 and each horse a 4? Keith: "Because there are four oxen and Five horses, and both side will be 20." T3 asks Keith to write his solution down. Dieshe is the first one to share her solution with class. She puts her answer which is written on transparency on the overhead projector and explains her solution to the class. Theo is the next person to present. He places his written answer on the overhead and starts reading it.
55:00- 01:00:24	He is asked to read it a second time, but it is difficult to follow the logic of his explanation, with the conclusion that the four oxen are stronger than one elephant and three horses. T3 displays the transparency of the original problem on the overhead and asks Theo to use that to explain his thinking more clearly. Theo refers to the second picture and based on the fact that one elephant is as strong as one ox and two horses, he crosses out one ox and two horses in the third picture. He also goes on to say that one elephant is almost as strong as two oxen and an ox is as strong as two horses. Some students are not convinced and raise their hands to ask questions. Jynita: "How did you assume that two horses are as strong as an ox?" Theo: Because in the first picture it says that four oxen are equal to five horses. Kevin: It could be six horses for the four oxen. Jynita: Then it will be half a horse. Kevin: But you can't split the five horses here, because it is not even. Theo has no answer to give. Kevin goes to the overhead to explain his solution. He has used the substitution method and his explanation is identical to Jynita's. K3 asks Kevin to explain why he thinks Theo's explanation is incorrect. Kevin reject Theo's assertion that one ox is equal to two horses and justifies it by circling horses in groups of two and connecting each

Time Interval	Description
	circle to an ox in the first picture. This demonstration shows that there are not enough horses in the picture to support Theo'a claim. Kevin puts up his written explanation on the overhead and reads it to the class. Some students clap. **[End of Role T3-1; Lesson continues on Role T3-2]**
Time Interval	**Description**
00:00-6:27	Keith is now in front of the class, presenting his solution. [The first few minutes of his presentation has not been captured on video] His paper on the overhead shows that Keith has assigned the number 5 to each ox and the number 4 to each horse. He has written the number 20 above the oxen as well as the horses, indicating the equality of the two sides. In the second picture, Keith has written the number 4 above each horse and the number 5 above the ox. He has added these numbers to get the number 13, which he has written above the elephant. He has used the numbers 13, 5, and 4, representing an elephant, an ox, and a horse respectively, to calculate a numerical value for the combined strength of the animals in the last picture. He has written 25 on the left side and 20 on the right side, concluding that the elephant and the three horses will win the race. Several students are heard saying that they understand his reasoning. R1 addresses Jynita and asks: Do you think his method was similar to yours or different? Jynita: It was basically the same, but he used numbers. A boy is heard saying that Keith's method was faster and Jynita agrees. Chad: His (Keith's) was easier, because when he showed it on the overhead and explained it, I understood it. Next, Tyreal goes to the board to present. He starts by saying that he too assigned numbers to an ox and a horse, just as Keith had done, but his numbers are different and include decimals. He has assigned each horse to be a 1, making a total of 5 on the right side in the first picture. He then explains that each of the four oxen will be at least a 1, correlating with the four horses, with one horse left to be equally divided between the four oxen. He has figured out that each ox would be represented by the number 1¼. He then uses these numbers to show that the number associated with the elephant in the second picture would be 3 ¼. He uses these assigned numbers to calculate a total for each side in the third picture. He ends up with 6 ¼ for the left side and 5 for the right side, concluding that the elephant and three horses would win. Students appear to be paying close attention to his explanation. Tryshon seems excited that she has been able to follow Tyreal's

	reasoning. She says: “Ooh, I get you”. Kevin offers his opinion about Tyreal’s method: “Yeah, but yours is mad difficult”. Students laugh. T3 asks Tyreal what sign can be placed between the two numbers in the last picture to show the relationship between the two sides. Tyreal says that it would be the “greater” sign, but he is unsure as how to draw it. Some students help by telling him which way “the mouth” opens. T3 asks: Is there anyone who likes to share, but didn’t get the opportunity? Keith raises his hand and says he has a question for Tyreal. Keith: “How can you get a decimal of an animal? It will be dead.” Tyreal: we are not measuring the animals. We are measuring the strength. T3 asks Tyreal to repeat his statement for emphasis. T3 tells the students that she would like to collect all their work and reminds them to write their name on their papers. R1 addresses the class by thanking them for their good work and asking their feelings about the type of problems they just did in class. Students say that it was new to them, but they enjoyed doing this kind of problems. R1 asks whether they thought the problems were in anyway related to Algebra. The answers were mixed: Yes, kind of. Davon says something about adding negative numbers. Jynita says it was related to algebra because you could replace the animals with numbers. One boy wants to know if T3 is doing this lesson in any other of her classes. T3 says that she is doing it for this class only, as this is her favorite class. The students thank her for that. Class ends.

Appendix B4: Description of lesson #4

Roles T4-1 & T4-2	Date data recorded: 5/30/08 Place data recorded: Sheffield, NJ Classroom teacher: T4 Grade 5
Time Interval	**Description**
00:00-5:00	There are 19 students in the classroom. In addition to the classroom teacher, T4, there are four other adults, R1, R2, T3 and O3, present in the room as observers. R2 is in charge of videotaping the session. The desks are arranged around the room forming a large U shape. T4 explains that not all students brought back their permission slip, consenting to be videotaped. Some students had expressed concerns about the videos appearing on "America's Funniest Home Videos" at some point in the future. Even though T4 had explained to them the purpose of taping the lesson, there were still some students who had not handed in their signed permission slip. Those students were all grouped together on one side of the room and although they were not videotaped, they participated in all the classwork and had interactions with the observers. T4 introduces the university faculty member, R1, as her college professor and her math teacher. R1 addresses the students: We have been studying different math problems that are suitable for your age group. The only way we can be sure that these problems are good is by studying the ways you would tackle these problems. Would it be okay for us to walk around and look at your work and at times interact with you? The children say, yes. R1 also explains that the lesson is videotaped so it can be viewed later on for detailed analysis, not to look at the students, but to look at their mathematical work.

5:00-10:00	About five minutes is spent rearranging the desks in order to separate the kids who would appear on the video from those who would not. Two students change their minds in the last minute and decide to hand in their permission slips. Another table is moved over to accommodate the new arrivals. A total of 12 students will be videotaped and they are seated at long tables arranged in a U shape. Each student has a large nametag in front of them. T4 pairs up the students and asks them to work as partners.
10:00-15:00	T4 addresses the students: As you remember, a couple of weeks ago I gave you a problem to do in class. Some of you did the “Carrots” problem and some did the “Bananas” problem. We did not get the chance to discuss the answers then, but now I am going to give you a copy of the “Bananas” problems and I would like you and your partner to work together to solve it and prepare to share your responses with the class. Please have a conversation about it with your partner. T4 reminds the students who are not being videotaped that they are included in this activity and they should feel free to participate in the discussions. T4 distributes the papers and students begin to discuss the problem with their partner. The camera focuses on Dajuan and Rene. Dajuan: Two divided into ten (pointing to the last picture). Rene: Of course, we have to divide. Why don’t we take two bananas out to make it even to the two pineapples? Dajuan: It is already even. Two divided into ten is five. Rene: That’s right. Dajuan: So one pineapple is equal to five bananas. Rene: Five bananas, of course (rolling his eyes). Dajuan refers to the second picture and points out that one pineapple equals two bananas. Rene cuts in to say: And one apple. Dajuan: How many more bananas are needed to make it five bananas? Rene: Three. Camera’s focus shifts to Kieshe and Arlin who have already solved the problem. R2 (cameraman) asks Kieshe to explain her solution. Kieshe: There are ten bananas in all and two pineapples. There is one apple in the third scale and it looks like it only weighs 3 pounds. And if it weighs 3 pounds and 2 ounces, then you can pick two bananas from the first scale and the remaining eight will weigh the same as the two pineapples.

	R2 asks Arlin if she agrees with Kieshe. Arlin nods yes, and adds that her answer was also two bananas. Kieshe starts writing her explanation. The camera moves to Maybelea and Oscar, who have already figured out that three bananas would be needed in the last scale. Maybelea is talking through the explanation, step by step, trying to really understand the reasoning. Oscar seems very confident about the answer. Maybelea: "So one pineapple is…" Oscar: "Five bananas." Maybelea: "Exactly, and one apple is equal to (pointing to the second picture and glancing over at Oscar)." Oscar: "Three bananas." Maybelea nods and pointing to the last picture says: "So, an apple (she pauses)." Oscar: "Weighs as much as three bananas." Maybelea smiles and says: "They will be equal."
15:00-20:00	T4 reminds students to put their thoughts down on paper. Maybelea and Oscar look around and say they have to find a pencil. At another table Destiny and D'nea are discussing the problem. They figure out that one pineapple is equal to five bananas, but they end up concluding that an apple is equal to two bananas as opposed to three. The camera moves over to Nazeer and Jarrod. (While the camera was still on Destiny and D'nea, Nazeer and Jarrod could be heard saying that five bananas equal one pineapple.) Nazeer points to the 10 bananas and say: "We have to add another set to these, five more. That means 10 bananas and 15 apples, no 10 apples, no." Jarrot: "So how is 10 apples going to ..." Nazeer: "No, 10 is so wrong. It is 15. What is half of 15?" Jarrot tries to figure out half of 15. Nazeer: "There is no half of 15. So we have to choose a number." The boys are still trying to figure it out, when T4 asks the class if anyone is ready to share. T4 calls on D'nea to present her solution. D'nea: 10 bananas, take away five. T4 cuts in: Take away five what, chickens? D'nea smiles, says no, five bananas, then shakes her head, shrugs her shoulders, and glances over at her partner, Destinay. T4 asks D'nea why she took away five bananas, to which she has no answer. T4 urges D'nea to continue, but she smiles shyly and refuses to add anything further.

20:00-25:00	T4 turns to Destiny: "Do you want to tell me why you took five bananas away?" When Destiny shakes her head, T4 tells them to finish writing up and that she would come back to them later. T4 asks Oscar to present. Oscar: We took away five bananas in order to measure how much a pineapple weighs. We divided 10 by two and got five. T4 asks Oscar where they got the 10 from and why they divided it by two. Oscar explains that 10 is for the 10 bananas in the first picture and they divided it by two in order to find the weight of one pineapple. Oscar continues: Then we looked at the second picture which showed that one pineapple was equal to two bananas and an apple and since we know that one pineapple is equal to five bananas and 2+3=5, we knew that one apple must weigh the same as three bananas. T4 asks Oscar if he has all his explanation down on paper and when Oscar nods yes, she asks Kieshe to share her answer with class. Kieshe: "There are 10 bananas in the first scale. We took two bananas from the first scale and put them in the third scale, because one apple weighs three pounds and two ounces and a banana weighs two pounds and three ounces and two pineapples weigh more than one apple." T4: Okay, you said a lot and I am trying to understand what you said. You said that you took two bananas away from the first scale. Did you do anything to the pineapples on that scale? Kieshe: No. T4: With the eight bananas on one side and two pineapples on the other side, do you think the scale will remain the same? Kieshe says yes, while her partner, Arlin, is shaking her head no. T4 asks Nazeer if he agrees with Kieshe that the scale would remain the same. Nazeer says no and T4 asks him to tell Kieshe why he disagrees with her. Nazeer: Because two pineapples weigh the same as 10 bananas and when you take two bananas away, the pineapples are going to be heavier that eight bananas. Kieshe says that she gets it.
25:00-30:00	T4 starts handing out the next task which is the "Carrots" problem. The camera shows R1 talking with Nazeer and Jarrod, trying to understand their solution to the "Bananas" problem. Nazeer tries to explain to R1 that one pineapple is equal to five bananas, which is also equal to three apples, concluding that one apple is equal to

	three bananas. R1 says that she does not understand his reasoning as to why one pineapple is equal to three apples. Nazeer asks his partner, Jarrod, if he wants to help with the explanation. R1 asks Jarrod if he agrees with Nazeer. When Jarrod does not offer any explanation, R1 tries to make the boys see that one pineapple cannot be equal to three apples. R1: You claim that one apple is the same weight as three bananas (pointing to the last scale). So how many bananas would be equal to two apples? Nazeer: six bananas. R1: And how many bananas would equal to three apples? Nazeer: Nine bananas. R1: So you are saying three apples equal nine bananas. You also claim that three apples equal one pineapple, which means that one pineapple is equal to nine bananas. But you had convinced me before, using the first scale, that one pineapple is equal to five bananas. Which one is correct, five or nine? Nazeer: Five bananas.
30:00-35:00	The camera moves to Maybelea and Oscar who are working on the "Carrots" problem, but the discussion between R1 and Nazzer can be overhead. Nazeer explains to R1 that according to the second picture and using the fact that one pineapple is equal to five bananas, it can be concluded that one apple is the same as three bananas. However, he still maintains that one pineapple is equal to three apples. Mayblea and Oscar have finished solving the "Carrots" problem and Mayblea is reviewing the steps they took. Oscar helps by finishing a few of her sentences. Mayblea: Six carrots weigh as much as a corn and a pepper. But since two peppers are the same weight as a corn, a corn must be four carrots and a pepper must be two carrots. Oscar: And in the second picture, the two peppers are 2+2 which is equal to 4, which is a corn. One pepper is equal to two carrots. Oscar and Mayblea try to match their verbal explanation to a combination of numbers and mathematical operations. Mayblea: Divide? Oscar: "Yes, divide 2 into 6, because there are two items (first picture)." Mayblea: "Six into 2, it will be 3." Oscar: "It is?" Mayblea: "Yes, because 2x3 is 6." Oscar: "No, what we are doing is incorrect. It can't be odd, otherwise it is not balanced. If this was odd (pointing to the six

	carrots) then it would not be balanced with the other side." T4 asks if anyone needs more time. Both Oscar and Mayblea raise their hands. Camera moves away and shows T4 working with Kieshe. Kieshe: You can move three carrots from here (first picture) and put them here (last picture). T4: How did you go from here to here? How do you know that a pepper is equal to three carrots? Kieshe: You can test it out. If they are equal, you can keep the carrots there. K4: How do you test it?
35:00-40:00	Keisha has no answer to that question. Arlin and Kieshe start a discussion about the problem as T4 moves away and tells the class that Rene and Dajuan will be presenting their solution first. Rene: I think two peppers will weigh as much as two carrots, because one ear of corn probably is the same as four carrots. So one pepper weighs the same as one carrot. T4 asks Dajuan, Rene's partner, if he agrees and he nods yes. T4: How did you come up with that? Rene: From the second picture we know that one corn weighs the same as two peppers. T4 asks Kieshe: How do we know that from the second picture? Kieshe: Because there is one corn in one triangle and two peppers in the other triangle and they are on the same line. T4: They are balanced. T4 asks Rene to continue. Rene whispers to Dajuan that they need help. Dajuan says that they are done. T4: You are done? Rene: "Dajuan says we are done, but that is not what I am saying." Rene continues: Then we looked at the picture where six carrots are equal to a corn and a pepper and we said that a pepper was equal to one carrot. Dajuan agrees with this conclusion. However, despite repeated effort by T4 to elicit some kind of justification, neither Rene nor Dajuan offer further explanation. T4 asks the boys to think about some explanation for their solution and that she would get back to them later. Jarrod is asked to share his findings with the class.
40:00-45:00	Jarrod starts by saying that in the first scale six carrots equal a corn and a pepper, but then he hesitates and when his partner, Nazeer,

	whispers something to him, he passes the paper towards Nazeer and lets him take over. Nazeer: Three peppers equal six carrots because according to the second picture, two peppers equal one corn. So in the first picture, we replaced the corn with two peppers and got six carrots equal to three peppers. T4 repeats the steps Nazeer had taken thus far and asks Dajuan if it has helped him to understand the problem a little better. Dajuan replies, yes and T4 asks Nazeer to continue. Nazeer: One ear of corn equals three carrots because if we take away half the carrots and the pepper (in the first picture), it will be the same weight, because… Nazeer asks Jarrod to take over, who repeats what Nazeer had just said, but hesitates and cannot justify why three carrots weigh the same as one pepper. Nazeer: "Oh, I get it now. Three carrots are the same weight as two peppers because one corn is equal to two peppers and if take away one corn and add one pepper, we will have the same amount as three carrots. No, one corn is the same as two peppers and two peppers are the same as three carrots because…" Nazeer and Jarrod are visibly frustrated. T4: Does this help you? Camera shows a diagram that T4 has drawn on the whiteboard of a scale with six carrots on one pan and three peppers on the other pan. T4 says to Jarrod and Nazeer: I think it is hard to visualize what you say you are doing on the scale. You told us earlier that you replaced the corn in the first scale with two peppers. So this is the scale you will end up with. Now what is it we are trying to find? Nezeer: How many carrots equal one pepper? T4 draws a second scale on the board with one pepper on one pan and a question mark in the other pan. T4: Now we have to decide what goes on the second scale. Explain to us what we have to do next. Nazeer: "We have to remove peppers and carrots in equal weights and two peppers are the same as three carrots." T4: "So are you saying that in the last picture three carrots are equal to one pepper?" Nazeer frowns and says, no.
45:00-50:00	Nazeer: I meant to say two peppers are the same as four carrots. T4: So two carrots are the same as one pepper. Both Jarrod and Nazeer say, yes. T4: And these four carrots are equivalent to two peppers. Jarrod and Nazeer agree.

	T4: So, are we done? Nazeer: yes. T4: Are we? What takes the place of the question mark then? Nazeer: Two carrots, which are the same as one pepper. T4: Okay, good. Did anyone do it differently? Dajuan says he knows how to do it a different way and T4 asks him to share his method with the rest of the class. Dajuan looks at his paper for a little while and says he cannot remember. T4 tells him to think about it and write it down. Oscar claims that he has a different way of doing the problem. Oscar: We looked at the first scale with six carrots and a corn and a pepper and thought that a corn and a pepper are the same as three carrots each. But when we got to the second scale, we noticed that three and six don't balance. So we tried different numbers like two carrots for a pepper and four carrots for a corn. That balanced the second scale with two plus two on one side and four on the other side. So in the third scale you put two carrots in place of the question mark to balance the one pepper. T4: Okay. Any questions for any of the groups? What are some of the things you were stumbling on? Please tell me if you have any difficulties with anything we have done so far before we move on to another problem. Kieshe says she finds it difficult to understand how the carrots were moved from the first scale. T4 asks Rene to volunteer for a demonstration. T4 and Rene stand a few feet apart. T4: Suppose Rene and I are standing on the two pans of a big scale. Do you think the scale would be balanced, or as you said it earlier, would we be on the same line? Kieshe: No T4: Why not? Kieshe: Because you are heavier. T4: What would the scale look like then? Several students model the situation by holding up their open palms at different levels. T4: Which one would be me? Most students recognize that the lower palm would represent the teacher. However, Dajuan says that the palm which is higher is the teacher and T4 explains to him that the heavier person would be at a lower level. T4: What do we have to do in order to make the scale balance? Students suggest adding two more students to Rene's side. T4 asks Arlin and Dajuan to stand next to Rene. T4: Suppose the scale is now balanced, which means that the three of them combined are the same weight as me.

50:00-58:30	T4 then asks Cristian to join the three students on the scale and asks the class to describe what the scale would look like. Students respond that the scale is no longer balanced and side with the kids will be heavier. T4 asks Kieshe how they can make the scale balance again. Kieshe: Take out two people. T4: Which two people? Kieshe: Arlin and Dajuan. T4: But we know that Rene together with Arlin and Dajuan balance me. What makes you think that Rene and Cristian would balance the scale now? Kieshe: Because they look the same size. T4 creates another imaginary scale by asking Arlin and Dajuan to stand together and placing Cristian a few feet apart from them. T4 tells Kieshe: Suppose we had a scale that confirmed that Arlin and Dajuan's combined weight was the same as Cristian's weight. Then, going back to the first scale, we could take Arlin and Dajuan off the scale and replace them with Cristian, because we know for sure that they weigh the same. This is what scales are about; you remove some weight from one side and it will be lopsided. We have to replace it with something the weighs the same so it would be balanced. T4 continues: It is the same way in the "Carrots" problem. We are trying to figure out how to manipulate the carrots and the second scale is helping us figure out how to get to the third scale. Do you understand? Was this helpful? Kieshe and other students say yes to both questions. T4 makes sure that the students have no more questions to ask about the scales before moving on to the next task. T4: There is an activity that we do at field day every year that kind of reminds me of scales. Some students say: Tug of war. T4: How does it remind you of scales? Several people have their hands up and T4 calls on Mayblea to answer. Mayblea: Because it you have more people on one side, they will win the game because there will be more force. Cristian asks: What if the people on the side with fewer people are stronger? T4: Imagine if on one side there are 25 kindergarteners and on the other side are 19 fifth graders. Do you think that the 25 kindergarteners are going to definitely win? Students say, no. T4: why not? Students: Because we are stronger.

	T4: Do you think it is about weight or strength? Some students respond, strength and some say weight. The teacher says she would let them decide later and introduces the next task, “Tug-of-War”. T3 helps distribute the papers. T4 asks the students to put their name and date on the paper and read the instructions. After a little while three students are asked to take turns and read the instructions aloud. T4 tells the class that the same way they had predicted the winner between the kindergarteners and 5th graders, they have to predict the winner in this problem. **[End of Role T4-1; Lesson continues on Role T4-2]**
Time Interval	**Description**
00:00-05:00	Students are busy working on the problem. T4 asks Nazeer and Jarrod why and whether the first two pictures are important in making the prediction in the last picture. T4 asks Destiny to join Kieshe’s group because D’nea has put her head down on the table and is no longer working. Kieshe tells Destiny: I think the elephant and horses will win the game because in the second picture it says an elephant is as strong as an ox and two horses. And this is three (pointing to two horses and an ox in the second picture) and if you add one more (pointing to the four oxen in the third picture) it is still going to be stronger. Destiny starts writing down the explanation on her paper. The camera moves to Yaasmyn who is explaining her reasoning to her partner, Cristian. Yaasmyn: The elephant and the three horses are stronger than four oxen because an elephant weighs 10 pounds… Cristian: Ten pounds? Yassmyn nods, yes. Cristian: More, I think. Yes, 10 tons (Rene has suggested 10 tons). Yaasmyn: the elephant weighs 10 tons and the four oxen weigh… Cristian: No, four oxen have the same strength as five horses, and… Cristian and Yaasmyn are studying the pictures trying the make sense of them. The camera moves to Kieshe, who is trying to explain her thinking to Arlin and Destiny. Arlin is not convinced and argues about something while pointing to her paper. However, Kieshe is adamant about the point she is making and says: If there was no elephant in the picture (third picture), the four oxen will win. But when you add the elephant to the horses, then they will win. The camera moves to Jarrod and Nazeer.

05:00-10:00	Nazeer: One elephant is equal to 1.5 oxen. And 1.5 oxen are equal to two horses. Therfore, the odds are…. Nazeer is busy writing down what he has been saying (It is not clear what his final answer is). Oscar and Mayblea are going through the steps they took to solve the problem. Mayblea is writing everything down. T4 asks the class: How many of you have finished with the "Tug-of-War" problem. Some hands go up and T4 says: I know some of you are not finished, but let's hear about what you think of it so far. T4: Who likes to say something about what they noticed in this problem. Some hands go up and T4 calls on Arlin. Arlin hesitates and says: Something I noticed? T4: I like to hear whatever you want to say about the problem. We are wrapping it up and we not going to give the answer yet. We are going to talk about something that we noticed that helped us with the problem. We don't want to ruin it for people who are not finished. Arlin: The second picture that tells us an elephant is as strong as an ox and two horses helped me predict the winner, because I knew an elephant was stronger than an ox. T4 say, okay and calls on one of the students who are not videotaped. Student: I found out that two horses are as strong as one ox, and two oxen are the same as an elephant. So one elephant is the same as four horses.
10:00-17:15	T4 calls on Oscar. Oscar: I noticed that an elephant weighs as much as two horses. From the first picture we decided that one ox is stronger than two horses. Then we took the two horses from the second picture and put them in place of the elephant in the last picture. T4: How did you come up with the idea that two horses are as strong as an elephant? Oscar: In the second picture, we just eliminated the ox, because we know that the ox is stronger than the two horses. T4: Can we just take the ox out in the second picture? Will the two horses, without the ox, be still as strong as an elephant? T4 creates an imaginary scale by grouping three students together and equating their combined weight to her own weight. She then removes one of the kids from one side of the scale and asks if the scale is still balanced. After seeing this demonstration, Oscar agrees that the elephant

	won't be the same strength as two horses. R1 addresses the class: I know it is not true in real life, but can we pretend here that each horse is as strong as every other horse in this problem. Also I want you to pretend that every ox is as strong as every other ox. So, looking at the first picture, which is stronger, an ox or a horse? Some students are heard saying, ox. T4: I heard some people say an ox. This is something for us all to think about. You will have to convince us that an ox is stronger than a horse or vice versa. But before the announcements start on the PA system, I like to hear Kieshe's thoughts on the "Tug-of-War" problem. Kieshe starts talking, but is interrupted by the loud school announcements over the PA system. Kieshe continues afterwards and says that one elephant is stronger than the oxen. T4 collects the papers and tells the class that they would continue working on this problem later on. The class ends.

Appendix C: Descriptions of the Debriefing Sessions

Appendix C1: Description of the debriefing #1

Role T1-2	Date data recorded: 5/14/08 Place data recorded: Sheffield, NJ Classroom teacher: T1 Grade 5
Time Interval	**Description**
29:00-35:00	The debriefing takes place in the classroom immediately after the lesson implementation. T1 together with the university facilitators and the observers are seated around a table. O2 is standing up, taking notes. R1 asks T1 to talk about what she felt good about and what she found frustrating about the lesson. T1 states that for the most part, it was okay, but she felt frustrated about the fact that they ran out of time and that the students kept complaining that the last problem was too hard. She also says that she should have posed better questions to the students to guide them without being too intrusive. T1 also admits her frustration with the fact that almost all students used guess and check to solve the last two problems, as she had done herself the first time she had worked on these problems. R1 replies that it is okay for these fifth graders to use the guess and check method. However, she suggests that they modify the “Shorts and Glasses” problem in such way that eliminates the guess and check strategy as an option toward a solution. She proposes that for the next lesson implementation they cover the $50 price tags and just tell the students that the two tags show the same amount. The hope was that by eliminating the numbers, the students will have to use a different strategy to determine the more expensive item. Everyone around the table agrees that it is a good idea. T1 asks the group’s opinion on the sequencing of the problems and whether there were too many problems. T3 says that the lesson was not too long and wonders if her own might be too short (T3’s lesson implementation was due to take place the next day). It is suggested that she may want to pick an additional problem to lengthen her lesson plan, perhaps one of the four problems from today’s lesson. T4 suggests that in order to make today’s lesson shorter and have more time for the last two problems, they could take the first two problems (warm up problems) and ask half the class to do the “fruits” problem, while the other half does the “vegetable” problem. Since the students present their solutions to class, all students will be exposed to both problems.

	R1 disagrees with her and explains that the progression from the first to the second problem was important in the sense that with the first problem, the students did the problem and reported out on it, but with the second problem, the students began to really listen to each other's justifications and compare their findings. She also reminds T1 that part of the goal was to have students communicate their thinking.
35:00-42:57	R2 asserts that sometime asking the students whether they agree or disagree can be problematic because some students may just say, yes they agree and know that they don't have to present. R1 says: You could ask how was yours different? Because they really went about it differently. T1 claims that there were no different ways of doing the first problem and R2 points out that Kayla's answer was different to the rest. Most people explained the problem in terms of five pounds and three pounds, whereas she did it more like substitution and deduction. O1 points out that Kayla's work on paper was correct, but did not match her verbal explanation. This is confirmed by another observer and they agree that something threw Kayla off at the end of her verbal explanation, even though she had even drawn the picture of the correct number of bananas for her final answer. R1 says that she cannot wait to look at the students' work the next day. O1 comments that it was interesting that all the students introduced pounds to explain their solution. T1 says to R1: I thought it was wrong, but you said this is what we want them to get to. R1 says: "It was fascinating to see one boy, who was the only one in his group insisting on changing it to pounds because it had to make sense to him and bananas equal to pineapples did not make sense to him." R2 says that it was not a requirement for students to use pounds and it was nice that T1 let them come up with it and make sense out of it. The group agrees that contextually, they want the students to understand that they are comparing weights in these problems and it is the weight of five bananas that equals the weight of two pineapples. R1 explains that in the "Shorts and Glasses" problem, a price tag was attached so it was explicit that the equality referred to prices, whereas in the problems with the scales, the children had to recognize that various items were equal to each other in terms of their weights.

	R1 further illustrates that it is the same thing with Cuisenaire rods: An orange is not equal to two yellows, they are equal in length. O2 comments that she did some similar problems once with her students. The problems involved different types of candy on scales and the students had to do several problems before they truly understood the concept. R1 says that when you deal with the algebraic concept of equality, it is very important to be clear with the language and tell the kids that two Tootsie Rolls are not equal to some other type of candy. R2 notes that with manufactured items, there is consistency with the weight and we would have been in trouble if a student argued that not all carrots weigh the same. R1 agrees and adds that in the “Shorts and Glasses” problem, since the shorts are identical, it is understood that they are the same price. T3 shares with the group that in preparation for her lesson implementation, she did a problem with her students about downloading gigabytes and the number of megabytes involved. She says that they discussed the number of songs and the number of minutes that could be put on a CD. They were then considering music and were told that “one song is 2.5 music”. One of her students kept on working in terms of minutes and seconds. T3 adds, “But now I am like, that’s kind of okay”. O1 asks whether it is okay to ignore the students’ lack of precision in equating one banana with one pound. R1 says that it is a good question and if you want to pursue that, you can ask them why they chose one (pound). She goes on to say that they chose one because “it was easy, one unit, one pound, one anything. Did it have to be a pound? No. ….Suppose a banana had weighted a quarter of a pound, then what would a pineapple weigh? There is tons you can do there”. She further explains that what the students were doing that day was to convert things to pounds so it would make sense to them in terms of weight. They end the debriefing session. They break up for lunch and arrange to meet15 minutes before the start of the next lesson implementation in the afternoon.

Appendix C2: Description of the debriefing #2

Role T2-2	Date data recorded: 5/14/08 Place data recorded: Sheffield, NJ Classroom teacher: T2 Grade 6
Time Interval	**Description**
18:40-20:00	The debriefing occurs in the classroom immediately after the lesson implementation. T2 together with the university facilitators and the observers are seated in a circle. R1 asks T2 to talk about things that went the way he wanted them to go and what he found frustrating about the lesson. T2 states that he was happy with the level of interaction between the students in certain groups and that he was happy with the overall work that many students did. However, he notes, that in the latter part of the session there were some behavioral issues. He believes that the students' misbehavior was the result of their frustration with their level of performance in comparison to their perceived higher level at which, their peers handled the same problems.
20:00-25:00	In response to T4's question (not clear), T2 says that the students had not done these problems before. T2 is asked whether the students were in their usual grouping for this lesson. He replies that normally the students work in pairs and not in groups. T4 asks why he deviated from the normal structure and put them in groups as opposed to leaving them in pairs. T2 says that he felt the third person would help toward a better outcome by bringing additional insight into the conversation. R1comments on how well the three girls, Dina, Jasmine, and Monae worked together as a group. T2 informs the group that Jasmine never likes to work with anybody and always sits by herself and works on her own, but on that day, she was great. R1agrees and acknowledges that grouping the three girls together was a good decision on the teacher's part. R2 jokingly tells T2 to let Jasmine watch the videotape of the day's lesson, so she can see that it is possible for her to work with others. T2 shakes his head and some people laugh. T4 says she overheard Jasmine telling T2 at the beginning of class that she did not want to work within a group and T2 told her that it would be for this session only. T3 who had worked with Monae on some project in the past, tells

	the group that she never thought Monae had the confidence and the capability to work as she did within the group. T3 thinks that having Jasmine in the group really helped Monae. T2 agrees and says that was one of the groupings that worked because there were no behavioral distractions. T2 says, and everyone agrees, that Cory, Caliph, and Kevin also worked well together, even though they were playing around at times. T1 adds that they only played around once they were done with the problem. Another observer notes that toward the end of the lesson, the three boys were shutting down and had to be coaxed into writing their answers down. O1 asks T2: Did you purposely try to group them in such a way that there would be an academically strong person in each group? T2 replies that he aimed for that, but it was not always possible to do it because of some behavioral concerns. He adds that he decided on the groups just before the lesson by considering, mainly, the dynamics of the groups and how well the students would work together. R1: Were there anything that surprised you? T2 says that Tateanna's level of engagement was very nice to see, since she had been struggling all throughout the year and only recently had started to show confidence in herself. He also adds that although her test scores have not increased, her class participation has improved remarkably.
25:00-30:00	T1 points to Table 5 and says: How about those answers that glasses cost more because they are breakable? "I like those answers though!" People comment that students are being practical and thinking contextually. R1adds that according to an article she read, a characteristic of students from a more urban setting is that they are less able to pull the mathematics out of a problem and they get more lost in the context of it. T1 questions the validity and reliability of tests, where the focus is only on math and does not take into account students' rich contextual insight into the given problems. T4 notes that in the same way that words could mean different things at different times, the students have to understand that this is a math class and their answers (Table 5) did not have anything to do with math. T1: There was math associated with their answer. T4 disagrees and says: There was no math involved in the answer, because when the teacher asked for it, the students did not have any

	answer. She goes on to give an example of a situation that occurred in her 5th grade class, where the students were given the prices for a large, a medium, and a small carton of milk and had to decide which size they would buy to get the best value for their money. Some students said the smallest one because there were only two of them in their household. T4 asserts that although the students' reasoning for wanting to buy the small carton was perfectly valid, she, as the teacher, had to make them realize that there was only one correct answer regardless of the number of people in the household. T1 thinks that in time and through more questioning, the students in T2's class would have eventually seen the mathematics in the problem. R1 reminds the group that these things take time to develop. O1 comments: I thought T1 handled it very well. When the students gave reasons as to why glasses are more expensive, T1 validated their points, saying that your answer is based on what you know and not based on what you see in the picture. R1 talks about how sometimes students see what they want to see about a picture. She explains: Jasmine was adamant that in the last picture there should be two bananas (instead of three) to balance an apple. When I asked her how that would affect the second picture, she immediately said that the second picture would not balance then, because the pineapple would be heavier.
30:00-35:00	R1 continues: I told Jasmine that the pans do balance in the second picture and she said that in that case another banana would be needed to balance the scales in the third picture. O1 asks R1 whether Jasmine eventually agreed that there would be three bananas in the last picture. R1 says: I don't know if she did or not, but she certainly convinced me that she understood that for the scales in the second picture to be balanced, you would need five bananas on the right hand side. O1 addresses T2: You know how you went around and asked different groups to present and some groups were a little shy about speaking and just basically said that they had the same answer? At Table 5, Oscar, Maledy, and Jevon had the wrong answer of two bananas written on their papers, but because most students had reported their answer to be three bananas, they falsely reported that they had the same answer. Then you (T2) moved to the last group to present and these girls at Table 6 had the courage to disagree with all the answers they had heard so far and claim that the answer is two bananas. I (O1) really admired them for that. I also noticed that Oscar at Table 5 said that he has changed his answer and it is two bananas. Obviously hearing Jazmine at Table 6 argue about it

	gave Oscar the confidence to think that maybe he was right after all!! T1: I am interested to look at the documentation later on because at Table 1, Monae was the spokesperson for the group and she had the correct answer on her paper. But the other two girls had the wrong answer of two bananas written on their papers. I wonder if they changed their written answer. R1: I am not sure, but I think she convinced them of the right answer. R1 reminds the group that there was so much going on and urges them to write down and make notes on what occurred in class as much as they remember. R1 asks T3 if she has revised her lesson plan for her lesson implementation tomorrow afternoon. T3 say that she is planning on doing the "Bananas" and "Carrots" problems first, which represent equalities, followed by the "Tug of War", which represents inequalities. R1: "So you are going to make that kind of progression. It should be very interesting, and if you have time, are you going to do the Chicken problem?" T3 says, yes and since she has an 80-minute block, she would hopefully have enough time. "I think I will spend about 10 minutes on the first two problems and about 15 to 20 minutes on the Tug of War problem." R1: Once you have done these two problems first, the Tug of War may go faster. R2 notes that the sequencing of the problems, as arranged by T3, matches the order these problems appear in the book. T3 informs the group that her class starts at 10:30 the next morning and gives them the room number. R1 asks the group about a young man who came into the classroom during the lesson and stayed as an observer for a few minutes. Someone says that he is the new vice principal and the group agrees that it was nice to have the vice principal visit the classroom during the lesson implementation. T3 says that the principal was absent; otherwise she would have come in too.
35:00-40:00	R1 gets back to analyzing students' mathematical work and says: In the "Shorts and Glasses" problem, Jasmine had initially reversed the prices of $2 and $4 and when I asked her if the prices worked in both pictures, she realized that they did not and eventually got them right. And when the total price was increased to $50, she knew that the price of each item had to be increased. They all got that and used the same technique to find the new prices of $10 and $20. But when I was working with Monae and gave her a new total of $60, I

	could tell that she really had not caught on to the idea that the prices had a 2:1 relationship. R1 goes on to say: I am interested to know if any of you saw any evidence that the children really understood that two pairs of shorts cost the same as one pair of glasses. When we talk about algebraic thinking, that is what we want to get at and I don't believe they could see the 2:1 relationship under any circumstances. R1 turns to T3 and says: I wish you could do this problem with your students tomorrow. T3 says that maybe she would do it; maybe she would do it instead of Tug of War. They debate whether they should take out Tug of War or just do one of the scales problems in order to make room for the Shorts and Glasses problem. T3 is asked to have all the problems at hand for tomorrow's lesson and decide on which one she wants to do. T3 shares with the group what she observed at Table 6, when she was working with Sakeena. T3 says: Sakeena crossed out one pair of glasses and one pair of shorts from each picture and was left with one pair of glasses in one picture and two pairs of shorts in the other picture. But she still kept on saying that shorts are more expensive.
40:00-45:00	T3 continues: Then I thought she is just thinking about the quantity and not the value. But I think that given more time and if we had probed her more, she would have been able to see that if two shorts are the same price as one pair of glasses, then the glasses are more expensive. R1: If you can have twice as many shorts as glasses, the obvious question is which one is cheaper. O2 says: It is almost the same mentality when you start with fractions and have to decide which one is bigger, 1/10 or 1/8. R2 says: Since you mentioned fractions, I was blown away when Cory said that a banana is 1 ½ pounds and I asked how much would 10 bananas weigh, but he didn't have an answer. Did anyone see if anyone in their group figured the answer? Teachers say, no, the students just moved on. R1 says it was a good question and "if we were not under pressure to move on, you (addressing T2) could have been a teachable moment to tell the kids that we really have to try and figure this out." R2 says: That would have made the problem really challenging to say if one banana weighs 1.5 pounds, then what goes on the last scale. If that is the given information, it makes the problem a good one!

	R1 acknowledges the fact that some kids had difficulty with the arithmetic part of it. She says that when working with Jayananoh on Shorts and Glasses, she realized how much he was struggling with the Middle math. R1 adds: He could figure out that half of 4 was 2, half of 20 was 10, but could not figure out half of 30. With a stretch, he probably could get from 100 to 50, but when I asked him to find half of 5, he said he didn't know and guessed the answer to be 2. R1 asserts: The arithmetic really stands in the way. R2 says: I know with tracking we always say the kids in lower groups feel bad about it. But, I wonder how these kids feel in a mixed ability groups when they hit a brick wall and are afraid to say something wrong in front others who get the answers more quickly. I wonder if tracking is appropriate for that reason only. R1 says: you are going to see the disparity in every classroom. The key is to group the kids in such a way that they feel comfortable working with each other and also create a classroom community where kids are encouraged to listen to each other. T2 mentions that sometimes when a group of kids have managed to solve a problem, he asks them to move around, as scouts, to help and explain the problem to other students. T1 says: At Table 3, neither Nekaybaw nor Jayananoh wanted to be the spokesperson, but when I told Nekaybaw that her answer of $10 and $20 was correct, she was very keen to present. Unfortunately her write up did not match her correct verbal answer at the end. R1 tells O2 that she has been very quiet and O2 responds that she is new to this and is just absorbing everything. T2 says: "Welcome to lesson study". R1 asks O2 if she has any reflections she likes to share. O2 says: I saw a progression from this morning to this afternoon and I think it was partly the age, because we went from 5th grade to 6th grade.
45:00-50:40	O2 continues: And since are going to do 8th grade tomorrow, it will be curious to see what happens. R1: What kind of progression do you mean? O2 explains: The mentality was quite different. None of the 5th graders even tried to use variables, whereas in this class, Joel was using letters and numbers to describe situations. For example he had written 1P = 2B+1A to describe the relationship between a pineapple, two bananas, and an apple. O2 points to a problem on the board and asksT2: I am curious to know if your students can do that "Taxicab" problem. Will they be able to write an equation for it? T2 says that some could.

	R2 addresses T1: "I was curious to know what if next week you gave your students a problem in which they had to substitute numbers, or more of a standard pre-algebra substitution problem." T1 replies: "Where we shift from the pan balance to the algebra, it will be perfect and I have not done that yet. It will be a transition." T4 says that her kids did not have any trouble with the concept of a variable, but were completely baffled by the idea of pan balance. T4 continues to say: "I had to explain to them that space between the two trays was like an equal sign, which means that what is in one tray is equal (in weight) to what is in the other tray. And it progressed from there." T1: The problem I have with the pan balance is the calibration; you don't always get the visual for the equality since the trays don't end up being balanced. Some of the kids get really hung up on that and try balancing it by throwing in additional paper clips. T4 asks T1 to clarify what she said. R1says that those pan balances are not perfect and therefore hard to calibrate. For example if you put 2 grams on each tray, they should balance and sometimes they don't. The group then talks about the problem with using standard objects like paper clips, where there is a slight variation in terms of weight. R1 brings the session to a close by asking everyone to arrive at the school around 10:00 am the next morning for the next lesson implementation that starts at 10:30.

Appendix C3: Description of the debriefing #3

Role T3-2	Date data recorded: 5/15/08 Place data recorded: Sheffield, NJ Classroom teacher: T3 Grade 8
Time Interval	**Description**
06:27-10:00	The debriefing occurs in the classroom immediately after the lesson implementation. T3 together with the university facilitators and the observers are seated in a circle. The school principal and the math coach are also attending this debriefing. R1 starts the conversation: I would like us all to think about the changes we observed, developmentally and otherwise, as we progressed from 5th to 6th and on to 8th grade. Also there are some socioeconomic issues we have to address and how important it is to keep these students engaged and interested in the work they do. The conversation is suddenly disrupted by the news that they have to vacate the room and move to the school library at some point during this debriefing. R1 resumes the debriefing by asking T3: What are the most important things that you think they (students) did. T3: I am really proud of my students. When I first started working with them, I found out that their regular teacher never challenged them and only gave them boring worksheets to do. Then I started giving them more advanced and interesting work and all of them rose to the occasion.
10:00-15:00	The other day I came in and the kids were doing a grade 6 worksheet. They were done in 15 minutes, which means they had over an hour to do nothing. T1asks T3: So you don’t see these kids every day? T3: I am the support teacher for 6th and 8th grade students, but from January to April I worked exclusively with 6th grader because their teacher had left. Now, I am back to teaching both 6th and 8th grade. When I work with these 8th graders, we do challenging problems in cooperative learning groups. Their regular teacher has just given up on these kids and does not regard these kids capable of doing these types of problems. The mathematics coach asks T3 if the regular teacher follows the curriculum and uses the CMP (Connected Math Program). T3 refrains from answering the question. R1: Let us not forget how difficult it is for a teacher to do this kind of problems. Here we are all supporting each other and pointing out to each other when we get excited with the work a student is doing.

	It is a lot of hard work to do this kind of instruction. One of the things I would like us to work together on is "what kind of questions do you ask to get the best, or to get good deep thinking from these kids". R1 continues: The real issue I want us all to think about is Theo, who came up with what he thought to be a very logical solution and it didn't work. What I was trying to do and I know what T3 was also trying to do was to get into his head to figure out what questions to ask him. Ultimately the only thing that happened was that we just overrode him, because the kids had several other ways of getting the answer which was the opposite of what he had and I am not sure if he was convinced by others. They all agree that Jynita's question from Thoe (How did you assume that two horses are as strong as an ox?) was a great question. R1: The great thing about this class was that once they got into it, they started questioning each other, which is in a way better than the teacher questioning them.
15:00- 20:00	R1 continues to say: We did not hear from Theo enough to know how he was substituting for the elephant and why he was crossing out the oxen and horses the way he did. O5: I was watching what he was doing. He was trying to substitute the elephant in the third picture. He crossed an ox out from the right side and two horses from the left side. So I questioned him about it at the end of class. R1: What did he say? O5 had asked Theo: Do you realize that when you crossed these two horses off in the last picture, you got rid of the horses that were helping the elephant and not working against the elephant? O5 continues: He (Theo) got what I was telling him, he saw that. R1: It goes to show how complicated this is. Here we are all working together and helping each other and still, we are struggling around it. We need to help each other develop, over time, so that when kids present contradicting ideas, we can uncover their mistakes and address their misunderstandings. We have a responsibility to do that because there is a correct answer and we cannot leave them thinking it is okay to get this or the other answer. T3: I think what happened today was because of the time factor. R1: I am not talking about you or today's lesson. I am talking about what happens in the classrooms every day. In fact a few instances occurred in T2's classroom yesterday. T2: Yes, but some of those difficulties were contextual, when students were considering the price of glasses in general as opposed to what they could see in the picture.

	R1: Sometimes contextual difficulties have mathematical components. For example Keith wanted to use a number (4) to equate the four oxen with five horses. But he kept on saying that he couldn't do it because that would mean breaking up a horse and killing it. R1 continues: So I asked him if he could use another number that would work. And he came up with the number 20. O2: Yes, he (Keith) kept on asking how a horse can be broken into parts. That is why I was encouraging him to pose his question to Tyreal who had used fractional numbers to solve the problem. R1 addresses T2: When I mentioned the instances about yesterday's lesson, I was not referring to contextual problems with glasses. I was specifically thinking about the "Carrots" problem, where some students thought the answer was 2 carrots and some thought it was 3 carrots. They start talking about where to go for lunch.
20:00- 25:00	O6 has a question for T3. He wants to know why she deviated from the original lesson plan, which had the "Chickens" problem as the main activity. T3 explains that originally, the "Tug-of-War" problem was the warm up activity and she was planning on doing the "Chickens" problem as the main activity for the lesson. But after seeing the struggle with the balance problems in the 6th grade class, she decided to include those problems in her lesson, which left no time for the "Chickens" problem. She adds that at some point she will definitely do the "Chickens" problem with her students later on. R2: I will come back to your class for that and if anybody else wants to join me, it will be great. O2 tells T3: I did the "Chickens" problem with my 7th graders. I have an emotionally disturbed kid in that class who likes to do things his way. He is very "math bright" but struggles with language arts and has difficulty with explaining his reasoning through writing. I guess I should have used a tape recorder to capture his response because he was very good and immediately, within 10 seconds, he knew that he could compare two of the boxes and find the difference between a small chicken and a medium chicken. So I think your kids will do okay with the "Chickens" problem. O6 addresses T3: I agree with you and think you were right about doing the balance problems, however, based on the work your students displayed today, I think they would have been just fine if you had stayed with the original lesson plan. R1 tells T3: Yes, the students might have done just fine with the original lesson, but your decision to include the balance problems

	helped your students understand what it means to replace. One powerful thing that happened today was with the "Shorts and Glasses" problem. The kids could see that one pair of glasses could take the place of two pairs of shorts and this is a concept that the 6th grade students were only beginning to see in yesterday's lesson. On the other hand, today, even though the students figured out that "the glasses are double the shorts", they did not use this piece of information to solve the other parts of the problem. R1 addresses T3: If I were you, after the kids had found the price of each item that added up to $50, I would have changed the total from $50 to, say $60, to see whether they use the 2:1 relationship or they start over trying to guess and check.
25:00-30:00	R1 continues: I think the progression from the first problem to the last one was very important because by the time they got to the "Tug-of-War" problem the notion of substitution was already in place and had become part of the students' vocabulary. You could see it clearly in the conversation when Diesha was trying to explain to Jynita how the elephant can be replaced by an ox and two horses. T3: I also think it was better that I did the "Carrots" problem first, because after that the "Bananas" problem went really fast. R2: I think there is a delicate balance between giving students challenging work and making sure that they don't become frustrated. R1: You want some frustration, but not to make it an over reach. R2 says that at some point during the lesson, he thought about the original lesson plan and was tempted to whisper "Chickens" in T3's ear. He also adds that it is a hard decision whether to move on or to just give students more time to work on a given problem. T4: I think the lesson went very well. It flowed very nicely. T3: I knew the "Chicken" problem was hard, so I wanted to expose my students to the concept of equality first through the "balance" problems and then introduce inequality, which was the "Tug-of-War". The "Chicken" problem is also a kind of inequality. O2: The "Shirts and Soda" problem is also inequality because the two total prices are different. So you may want to do that before you give them the "Chickens" problem. R1 tells T3: Yes, the "Shirts and Soda" problem followed by the "Chicken" make a good sequence, since the former involves two variables and the later has three variables. R2 addresses T3: Let us know when you decide to do this follow up lesson. I will bring my video camera.
30:00- 37:16	O4 refers back to the struggle Dieshe had convincing Jynita that the

	elephant and three horses would win the tug of war. He says: I saw four teachers going to Jynita trying unsuccessfully to make her see how the substitution is done. When I approached her, she immediately said that she had her answer and that the Oxen would win. O4 demonstrates the tactic he used to change Jynita's mind about her answer. He had cut out the picture of the two horses and an ox, which equated to an elephant in the middle picture. By placing this cutout on top of the elephant in the bottom picture, he had managed to convince Jynita that her initial answer was incorrect. Everyone agrees that this is a fantastic idea. T1 and O2 say this method could serve as a great modification for special education students. R1 says it was important for Dieshe to have the opportunity to explain the problem to Jynita which must have given her a sense of power. R2: Dieshe used to be one of my students in 6th grade. So after she was done with the "Tug-of-War" problem, she gestured for me to go to her. She wanted to know what I thought of her solution. I read it and said it was good. Then I asked her if she could assign numbers to the animals and solve it that way. She seemed hesitant. After Keith's presentation, I asked her what she thought of his method. She agreed that the problem could be solved by using numbers. R1: It was interesting, when after the presentations I asked the class to comment about the different methods, Davon and Chad said that the numbers made it very easy for them to understand. T1: I thought it was brilliant the way Keith used the numbers 4 and 5. Did he come up with those numbers himself? R1: He wanted to use numbers, but he was hesitant as he thought it would kill the horse. Then I prompted him by asking if he could come up with a number that would not "kill" the horse. He then came up with the numbers 4 and 5 all by himself. O2: Keith was talking to himself all through this problem. T4: Keith and Tyreal had a long conversation about this problem. O6: Do we know if Keith and Tyreal came up with the numbers independently. Several people nod and say, yes. T3: Keith was the one who assigned 1.5 to each carrot. And I am thinking 1.5? Then he said 6 and 3, and I thought, yes, that is right! O1: I know Tyreal really struggled to come up with 1 ¼ for each of the four oxen after he had assigned 1 to each horse. At the beginning he had 1.4 for each ox and he knew it was wrong because when he added 1.4 four times, they did not add up to 5. He tried several numbers, always checking by calculating the sum, before he finally came up with 1 ¼ that added to 5 for the four

	oxen. He never used a calculator and he was not aware that 5 divided by 4 would have given him the answer he worked so hard to get. He had crossed out one horse and knew that that one horse had to be split between four oxen and he had a really tough time with it. O1 continues: Once he was done and was writing everything down I asked him what mathematical operation he could have used to get from 5 to his answer (1 ¼). He didn't know. I asked him what he had done to 5 that got him 1 ¼? He said that he didn't know, he was just trying to split 5 into 4 groups. He did not know that what he was doing was the same as performing the mathematical operation, division. I could not resist the urge and asked what he would get if he divided 5 by 4. He started punching up numbers on his calculator and you should have seen the look on his face when he saw 1.25 or 1 ¼ as the answer. O2: Keith was a great one to watch because everything he thought came out of his mouth. He talked to himself the entire time. You don't know how many times Keith said to himself, "No, I can't do this. I will have a dead horse". The debriefing ends.

Appendix C4: Description of the debriefing #4

Role T4-2	Date data recorded: 5/30/08 Place data recorded: Sheffield, NJ Classroom teacher: T4 Grade 5
Time Interval	**Description**
17:16-20:00	The debriefing takes place in the classroom immediately after the lesson implementation. T4 together with the university facilitators and the two observers, T3 and O3 are present in the room. The beginning of the tape is cut, but R1 must have asked T4 how she feels about the way the lesson went. T4: I think it went beautifully. Kieshe, who was talking a lot, is considered well below grade level, but I have been pushing her a lot all throughout the year. She is the type who works very hard, but her answers all always incorrect. Today when I asked the class if anyone was still having difficulty with the scale problems, she was the one who raised her hand and spoke out and I was so excited. R1: I did not quite get what she said when she raised her hand. What did she have difficulty with? T4: She couldn’t understand how items were moved from one scale to another. That is why I thought of building a scale to try and help her understand. T3: That was good. T4: I guess I got stuck at some point, but I tell them when I get stuck and I try to process it through. R1: But you pulled it off and you were able to replace the one student with two. That was good. T4: And I was trying to let them lead it. I try to take whatever the child says and make it work somehow. For example when Kieshe suggested that we take Dajuan and Arlin out, my first instinct was to say no, just take Cristian out. I am glad I did not say that, because that gave me the opportunity to build the second scale and I think that might have helped her understand better. Overall, I am always pleased with what they do.
20:00-25:00	R1: After the first three lesson implementations, O2 had made a comment on the progression with students’ thinking as we went from 5th grade to 6th and then to 8th grade. Now we see it in reverse as we went from 6th to 5th grade. The gist of it is to see how these ideas are developing over time and identify some of the stumbling blocks. R1 continues: For example Nazeer and Jarrod were not thinking

	proportionally at all when they were doing the Carrots problem. They only thought of removing items and subtracting. It is something very important that the corn is double the pepper, but we frequently fail to realize that it is not natural for them to think this way right now. T3: Did they come up with that idea themselves? It was amazing! T4: Nazeer probably came up with the idea. R1: Yes it was their idea. R2: They used substitution. R1: It blew my mind because what Nazeer said was way beyond what they (Nazeer and Jarrod) had been talking about. All of a sudden he could see the replacement. T3: I was amazed by that. T4: here is a problem I have with my kids. Most teachers struggle with classroom management issues, but my problem is, and I have had all throughout the year, how to get the kids to want to do well. How do I motivate them to want to learn? R1: Do you feel that they were engaged today? T4: Yes, absolutely. R2: I think they were really engaged with the first two problems, but the third one stumped them. T4: They were stumped, but I don’t think they felt like they were stumped. R1: That is a good perspective. T4: They just thought it is what I think it is. R1 tells T4: I think you are right; they approached the third question much more intuitively, but like I said, they were not thinking proportionally. O3: I think Rene and Dajuan were stumped. They were overwhelmed. T4: It was a long day. O3: I asked them if they needed help getting started. Rene said, no and that Dajuan was thinking about it. But Dajuan was not thinking about it! I told them I would be willing to help them get started, should they change their mind. Eventually they said okay and all I had to do was just one little thing. R1: What did you do? O3: I just ask them to look at the second picture which showed one elephant was equal to an ox and two horses.
25:00-30:00	O3 continues: And I asked them what the elephant in the third picture could be replaced with. They immediately knew to substitute and could then see that the left side would win. R2: I am thinking of what O4 did the other day in T3’s class, when he cut out the picture of one ox and two horses and replaced the

	elephant with that cutout. It was very powerful and the key question with this problem is how to get the kids to see how to proceed and figure out which side is stronger. R1: What I am interested about is finding out what questions to ask the kids, which doesn't do the problem for them. For me, cutting the picture and placing it on top of the elephant, is giving them the answer and I don't want to do that. T4: I agree with you. I am so time-challenged; I normally just let them have more time, if they need it, to work through a problem. R2: You did well today. T4: Well, I structured it this way for today. Maybe I will teach like this next year. O3: Don't you go by the bell? T4: I use it as a guideline. I wanted the students today to breeze through the first two problems so they would have more time for the last problem. R1: But they really needed the time to work through those two problems. Were you surprised that it took so long? T4: Not really, I didn't have any expectations. I did not expect anything because we did not have any conversations about those problems at all and at some point I thought I had to go over some things so that we could move forward. But I chose not to, because after listening to them I felt they were doing fine and that I didn't need to take charge of that. R1: they really needed that. T4: That is why I was asking if anyone was struggling with anything and whether they wanted any help. I don't think they wanted any help. They wanted to show what they knew and I was excited about that. R2: Prior to the lesson, you were debating on whether to start with the "Tug-of-War" problem. T4: I am glad I didn't. T3: I sat with Cristian, who was working with Yaasmyn, and as you (R1) mentioned he was not thinking proportionally. As he was looking at the scales, each scale had its own identity and he wasn't seeing the connection between the first scale and the second and third scale. So I had a conversation with him about that. Yaasmyn wanted to take the carrots off the first scale and put them on the third scale. Cristian kept looking at it, saying that they were the same. Then he said (T3 refers to and reads from her notes), "I believe the pepper is going to be nine carrots".
30:00-35:00	T3 continues: Because he (Cristian) had identified the corn and the pepper as being three carrots each. After our conversation, he looked at the second scale and said it won't work in the second

	picture because with two peppers on one side and one corn on the other side, we end up having six and three which don't balance. So I saw that level of reasoning. R1: "It will be interesting to study what he did. What so many of them did was to consider these two things and each one is three. Then it turns into three things, so it becomes 3, 6, 9." T3: That is what Cristian said. He said (reading from her notes), "Three carrots because it will balance out one green pepper; nine carrots because one pepper equals three, therefore everything is equal to three". He kept on saying, "It is the same thing". So I think he sort of saw the proportional reasoning, but he couldn't quite apply it. O3: I had made the assumption that the students would know that a corn would be the same as any other corn in the picture. However, when I taught this lesson, not all my students were thinking that way and I saw a couple of kids in this class who were not starting with the premise that all the corns or bananas were the same. Some students thought that the goal of the problem was to make each of the three scales balance. R1: They saw it as three unique problems. O3: yes. R1: It is very interesting for all of us to think about all the things we take for granted. I think they were okay with the notion of scales and how equal weights will balance. But in the "Tug-of-War" problem it was more difficult for them to understand the concept of balance and a couple of the kids in your class (addressing O3) were looking at the first two pictures and were not at all convinced that they were balanced, so they were trying to figure out the winners. O3: Yes, we assume that they know. T4: But I wanted to see what they knew. First I thought that maybe I should talk about scales and the concept of balance, but I was very proud of Kieshe when she talked about the pans being on the same line. For her, it is a big achievement. R1: And Oscar and his partner? O3: Boy, is he bright? R1: And he is very articulate. T4 receives a call from her daughter and has a short conversation on her cell phone.
35:00-40:00	R1 leads a group conversation about the sequence of the problems presented in the research lessons. R1 notes that the problems used for the lessons were presented in the same order as they appear in the Connected Math Program (CMP). However, the series of these problems start with a bartering problem, which was not used in the

	research lessons. R1 wonders if they had done this type of bartering problem prior to doing the scale problems, it would have been beneficial to those kids who had a hard time with the concept of substitution. R2: It is interesting. As a group, when we were planning the lessons, we thought it is essential for the kids to know the concept of balancing before we introduce them to equality and formal algorithms. But I guess the kids need to know how to trade before they learn how to balance. T4: I think they did okay. They all agree that most of the kids were fine. T4: I have a diverse group. Initially when I gave them these problems, I offered no help. I put them in pairs to work on the “Carrots” and “Bananas” problems. After they were done, I put two pairs together, making a group of four. Each group put their findings on chart papers. They were so in to it that they didn’t want to go to their next class. I have not looked at the charts yet to see how they did on their own and without any help. R1: Maybe this will be your assignment as you write your observation. Look them over and see what they did and also bring them with you to Monday’s class, so we can all see.
40:00-45:00	T4: I looked at their work briefly and it seems like they drew the same scales as shown in the problems. I was hoping that they would draw new scales to show their thinking. That is why I started drawing scales on the board as Nazeer was explaining his thinking. It seemed to me that he was moving things around on the scales and had a difficult time keeping track of the change and explaining it at the same time. R1 addresses O3: I had the same experience in your class when the kids were working on the “Bananas” problem. I wanted them to draw a new scale and at one point I even pushed a kid to do it. But he refused saying, “No, that is not what I am thinking of. I am not thinking five bananas and (verses) two bananas and an apple and then take two (bananas) off. That is not what I am thinking”. He said, “It is the pineapple over here that I know equals five bananas, so it is this scale that I want to take the apple off and add the two bananas to”. R1 adds: And I realized he did not want to be told to draw something that he was not thinking of. T4: I drew the new scale because I am a visual person myself and I was losing track of what he (Nazeer) was doing. I think it also helped several other students to see the new scales on the board. R1: Oh, I think what you did on the board was very good! T4: I was really trying to help him, but at the same time I wanted to

	make sure I understood what he was saying. R1 tells R2: I hope you captured the drawings on the board, because his self-correction once he saw the picture was very good. T4: Now, do you think it was too much help? R1: No, he did the correction by himself. T4: But you know how teachers sometime lead? I didn't want to do that. R2: I think if you truly listen to the child and understand what they are saying and then make a visual representation of what they are saying in order to help them get through it, you are not doing the problem for them. But, if you listen to them and hear what you want to hear and then make a representation of it, then it is whole different story. But you truly listened to him. R1: you really did, R2: You understood that he wanted to put three peppers on one side. T4: And he said put the corn back here. It was the first time I heard that and I thought, okay. R1: That was good. But in his head something had shifted. He was splitting the carrots into three and three and he had put the three peppers there, but then it was two and one. R2: He must have thought originally, like many other kids had done, to split the six carrots evenly between the corn and the pepper. But when they got to the next scale that logic fell apart. T3: I was amazed by Nazeer. T4: Yes, I don't know why he was ever retained. He does well for me. He is a talker though. R2: Some kids just manage to "butt head" with certain teachers. R1: In some way, he is also relatively assertive, in a good way. When I would say something to him, he would correct me and say that was not what he meant. Some teachers may not like that.
45:00-50:00	T4: The girl who sat over there and did not want to be videotaped, she was pouting today because she really did want to be videotaped and maybe it was because she was not getting enough attention today. She is very bright and normally works very hard. But today she just was not working well with her partner. Maybe it was just a long day for them. But, I still think they did well. After lunch is usually hard for them to get into the work. R1: I think she was falling sleep. T4: At the beginning of class she was doing the work. And Daniel, he is resource too but he is strong in math, but with her, you never know what you are going to get. R1: "She was fine, but what I saw in her the inability to think conditionally. The only things she could come up with what she

	saw in the pictures. So it is not only the proportional thinking, it is also the conditional thinking - if this, then that – which is very hard for kids.” T4: I am very interested to know what the problem is with her. When I went to her, I was trying to figure out whether she truly felt she was done working on the problem or she had no clue how to do the problem and had just written something down for the sake of it. So I got this other algebra book out to back up and see where she was stuck. I wanted to see if she understood the concept of balancing weights on a scale, but she was just not interested in talking at all. I showed her a picture with different scales in it and asked if they were all balanced. She just shook her head, no. I asked her which one was not balanced and she pointed to it. So I know she at least understands the concept of balance and now I have to think of other questions to ask her later on. R1: These are big mathematical ideas that kids need to understand.
50:00-55:00	T4 takes a book out of her bag and says she is going to use it as a resource for supplementary materials. R1 and T3 start looking through the book. T4 addresses R2 and O3: When I was building a scale using the kids, I thought I was going to embarrass some of the students. I wasn’t worried about my own weight, but I thought it might be awkward for some of the students. T4 laughs and adds: When I was looking for volunteers for the next scale, I could see some kids were looking at me with their eyes wide open and thinking, “not me!” R1: This is interesting. In this book they mention all the big ideas and we have been working with all of them. Ideas like representation, the notion of balance, functions, proportional reasoning, variables, and inductive reasoning. R2: At some point Oscar was in a situation that he didn’t know that what he was saying didn’t make sense. T4: Oscar is very bright, but he rushes too much and is always the first to finish his work. I always have to make him check his work and he often finds mistakes after checking. R1: You want to make sure that you don’t take the spark away from the kid. If he is truly finished, it is really boring to just check the work. What you may want to do, say in the “Tug-of-War” problem, if a kid is finished with it way before everyone else in class, you challenge him by changing the problem slightly and asking him to make a second prediction. For example, you can change the second picture to show an elephant has the same strength as an ox and one horse instead of two horses. T4: But Oscar is not often the one who needs a challenge. It is

	usually Nazeer who could do with a follow up question. With Oscar, it is a question of fixing the mistakes that he made. R1 gives back the book to T4 and asks her to bring it to class on Monday.
55:00-01:02:05	There is a discussion about the revamping of the mathematics curriculum in the district. The debriefing session ends.

Appendix D: Transcribed and Coded Critical Events for Lesson #1

Appendix D1: Lesson #1- Transcribed and coded Critical Event 1

<table>
<tr><td colspan="2"><u>Event 1</u>
Role T1-1
Teacher: T1
Students: Elyce, Maurice, Jarrod, Amir, Jaylen, Erica, Sierra, Jasmine, Jade, Kyla, and Neema</td></tr>
<tr><td>Transcript of Event

Time (as shown on RoleT1-1): (00:07:45- (00:09:52)</td><td>[It is about 8 minutes into the lesson. The students are working on the Banana problem. The camera captures students at each of the four tables discussing the task at hand. The clip starts with the students at Table 4.]
Elyce: Okay, so it has to be three.
Maurice: Look, it’s 10 bananas and it won’t equal. So each pineapple could be five, five bananas.
Elyce: five…
Maurice: Hold on. So five bananas equals… Then how much equal the bananas?
Elyce: One.
Maurice: One, two. No.
Elyce: It has to equal one.
Maurice: Two bananas and an apple equal to that.
Elyce: It has to equal to five pounds.
Jarrod: Yeah.
[The camera moves to Table 3.]
Amir: So… One two three four five six seven eight nine ten.
Jaylen [addressing Amir and pointing to his paper]: Right here it’s 10 and this is two pineapples. And then one pineapple, two bananas and one apple. One apple would be three bananas.
Erica [addressing Sierra]: …for two pineapples [pointing to the first picture], but one pineapple [pointing to the second picture] is five bananas.
[The camera moves to Table 2.]
Jasmine: We have to explain how we came that apple equals three.
[Jasmine, Tequrra and Jade start writing and the camera moves to Table 1.]
Kyla:…apple equals three.
Neema: Oh, I get it.
Kyla: Apple equals three. So now, if an apple equals three pounds, we are trying to figure out what balances that out (pointing to the last scale). Hmm!</td></tr>
<tr><td>Description of the event</td><td>In this episode, the students at each table are seen discussing the Banana problem with their partners.</td></tr>
<tr><td>Teacher’s Actions(s)/ Response(s)</td><td></td></tr>
</table>

Students' Action(s)/ Response(s)	1) Students discuss the Banana problem with their partner [lines 4-29]. Code: Engaged in mathematical discourse with peers (MDP) Code: Justifying solution (JS) 2) Maurice questions Elyce about her reasoning [lines 4-14]. Code: Questioning the reasoning of others (QRO)

Appendix D2: Lesson #1- Transcribed and coded Critical Event 2

Role T1-1 **Teacher: T1** **Students: Carin**	**<u>Event 2</u>**
Transcript of Event **Time (as shown on RoleT1-1): (00:12:57-(00:13:38)**	[It is about 13 minutes into the lesson. Carin who has been working on the “Bananas” problem with Tatiana volunteers to share their findings with the class] Carin: Me and Tatiana figured this out, but at first we said if 10 bananas equal two pineapples [holding her paper up and pointing to, presumably, the first scale], equal two pineapples, then each pineapple must be 5. And so, now if two bananas equal 2, then this apple has to… T1: Two bananas equal to? Carin: Equals, like [slight hesitation] 2 pounds. Then one apple has to equal 3 pounds. Because the (second) scale is even and this (pineapple) is equal to 5. And 2 plus 3 is 5. So if this apple equals 3, then it has to be, if this apple equals 3 pounds, then it has to be three bananas.
Description of the event	In this episode, Carin who has been working on the “Banana” problem with Tatiana, volunteers to share their findings with the class. After the teacher asks a clarifying question, Carin spontaneously adds the word, pounds to her explanation (even though her written work does not indicate that she had initially attached any specific weight to the any of the fruits).
Teacher’s Actions(s)/ Response(s)	1) In her explanation, when Carin says that two bananas equal to 2, the teacher asks Carin to clarify her comment [line 8]. Code: Eliciting a response – Name/State (E-NS)
Students’ Action(s)/ Response(s)	1) Carin volunteers to share her thinking with the class [lines 4-7; lines 9-12]. Code: Sharing solution/idea with class (SSC) Code: Volunteering to share ideas (VS) Code: Acknowledging the positive contribution of a partner (ACP)

Appendix D3: Lesson #1- Transcribed and coded Critical Event 3

Event 3 **Role T1-1** **Teacher: T1** **Students: Jaylen and Carin**		
	Line	
Transcript of Event **Time (as shown on RoleT1-1): (00:13:50- (00:14:58)**	1	[It is about 14 minutes into the lesson. Jaylen and Amir have been working on
	2	the "Bananas" problem together. Jaylen volunteers to share their findings with
	3	the class]
	4	Jaylen: Me and Amir figured out that an apple equals three bananas because
	5	on the first scale, there was 10 bananas and two pineapples. So then two
	6	pineapples will have to equal about five pounds and then 10 bananas will be
	7	equal to five pounds. So on the second scale, there is only one pineapple,
	8	which is five pounds. But there is only two bananas, so then three, one apple
	9	will have to be three pounds and two bananas will be two pounds.
	10	T1: So at some point we started converting to pounds?
	11	[T1 calls on Carin, who has her hand up.]
	12	Carin: I don't understand what Jaylen said because he said two pineapples
	13	equal five pounds and one pineapple equals five pounds.
	14	T1: That's right. I thought I heard that too.
	15	Jaylen: Both of the pineapples each equal five pounds.
	16	T1: One pineapple equals five pounds.
	17	Jaylen: And then another one equals five pounds.
	18	T1: Okay, one pineapple equals five bananas?
	19	Jaylen: Yes.
	29	T1: Okay.
Description of the event		In this episode, Jaylen who has been working on the "Banana" problem with Amir, volunteers to share their findings with the class. Carin raises her hand to point out that Jaylen, in his explanation, stated that two pineapples equal five pounds and later on said that one pineapple equals five pounds. Jaylen clarifies by saying that the two pineapples equal five pounds each and therefore one pineapple equals five pounds.
Teacher's Actions(s)/ Response(s)		1) The teacher provides the opportunity for Carin to comment on Jaylen's Explanation [line 11]. Code: Providing opportunity to critique other's reasoning (OCR) Code: Encouraging classroom discourse (ECD) 2) The teacher paraphrases Jaylen's response [line 16]. Code: Paraphrasing student's response (PSR)

Students' Action(s)/ Response(s)	1) Jaylen reports his and his partner's findings to class [lines 4-9]. Code: Sharing solution/idea with class (SSC) Code: Volunteering to share ideas (VS) Code: Acknowledging the positive contribution of a partner (ACP) 2) Carin questions Jaylen's explanation [line 12 and 13]. Code: Questioning the reasoning of others (QRO) 3) Jaylen corrects his earlier assertion [line 15; line 17]. Code: Making a self-correction (MSC)

Appendix D4: Lesson #1- Transcribed and coded Critical Event 4

Event 4 **Role T1-1** **Teacher: T1** **Students: Kyla and Neema**		
Transcript of Event	**Line**	
	1	[It is about 16 minutes into the lesson. Kyla and Neema have been working on
	2	the “Bananas” problem together. Kyla shares their findings with the class]
Time (as shown on RoleT1-1): (00:15:58- (00:17:21)	3	Kyla: Okay, me and Neema worked together.
	4	T1: Louder.
	5	Kyla: Me and Neema woked together and we first we figured out we looked
	6	at the first scale and said to ourselves that since there is two pineapples and 10
	7	bananas, that must, that means that 10 bananas were equal to two pineapples.
	8	And then what we did was we went to the second scale and we saw that there
	9	was one pineapple, an apple and two bananas. So we figured if five bananas
	10	equaled a pineapple, but there is an apple and only two bananas, that ohm,
	11	that the apple would equal to three. So then we went to the third scale and we
	12	figured out that since the apple is there and an apple equals three that the
	13	other side to make it equal will have to be an apple and two bananas.
	14	T1: Okay, so in that last one?
	15	Kyla: For the last scale?
	16	T1: Aham.
	17	Kyla: There will be an apple, An apple on one side and an apple and two
	18	bananas on the other side.
	19	T1: For the last scale?
	20	Kyla: Aham.
	21	T1: So an apple equals an apple and two bananas?
	22	Kyla: Wait, wait, wait.
	23	T1: An apple equals an apple and then two more bananas?
	24	[Kyla confers with Neema who points to something on Jasmine’s paper]
	25	T1: Just go through it again.
	26	[The girls start discussing the problem.]
Description of the event		In this episode, Kyla who has been working on the “Banana” problem with Neema, reports their findings to the class. Based on the first two scales, they have correctly identified that one apple equals three bananas in weight. However they seem unsure about how many bananas would balance the last scale. The teacher asks them to review their work. As the girls begin talking to each other about the problem, the focus of the class moves on to another group.
Teacher’s Actions(s)/ Response(s)		1) After hearing Kyla’s explanation with the placement of the wrong number of bananas in the last scale, the teacher asks Kyla, twice, to repeat what goes on the third scale. [line 14; line 19]

	Code: : Eliciting a response- Name/State (E-NS) 2) Once it is established that Kyla believes her wrong answer to be correct, the teacher paraphrases Kyka's response and poses it as a question in such a way that makes Kyla doubtful about her answer. [line 21] Code: Paraphrasing student's response (PSR) Code: Offering hints (OH) Code: Eliciting a response-Yes/No (E-YN) 3) The teacher rewords her previous question to further highlight the impossibility of Kyla's answer. [line 23] Code: Rewording a question (RQ) Code: Offering hints (OH) 4) The teacher asks Kyla and Neema to re-examine their work and report later. [line 25] (see Event 5). Code: Directing students to perform some mental or physical activity (DA) Code: Supporting student's autonomy (SSA)
Students' Action(s)/ Response(s)	1) Kyla reports her and her partner's findings to class. [lines 5-13; lines 17 and 18] Code: Sharing solution/idea with class (SSC) Code: Acknowledging the positive contribution of a partner (ACP) 2) When Kyla seems unsure about her answer, she is seen consulting with Neema. Following the teacher's direction, the two of them discuss and review their solution. Code: Engaged in mathematical discourse with peers (MDP)

Appendix D5: Lesson #1- Transcribed and coded Critical Event 5

<u>Event 5</u> **Role T1-1** **Teacher: T1** **Students: Kyla and Neema**	
Transcript of Event **Time (as shown on RoleT1-1): (00:20:14-(00:21:28)**	[It is about 20 minutes into the lesson. Kyla and partner, Neema, were given the opportunity to revise their answer to the “Bananas” problem and report back.] T1: So you did a self-correction? Neema: Yeh. T1: Okay talk it through. Neema: We….[not auidable] T1: Nice and loudly. [Neema smiles shyly] Kyla: Want me do it? [asking Neema] Neema: Yes. Kyla: Okay. Okay, we had said before that it would be an apple, two bananas. But we were actually wrong. It would be three bananas altogether because we looked at the apple and knew that apple was three so we said that, we said to make it equal, the other side has to equal three also. So what we did was figure out, we took it back to where the pineapple and the five bananas were equal to each other. But then we looked at it and said that is not really equal because each banana is one pound. So then we thought like, okay, since they are equal to one pound then for it to equal three pounds, you can take three bananas, they weigh the same as an apple. T1: We all agree with what we just heard? [addressing the class] Some students: Yes.
Description of the event	Kyla and Neema who had presented before and had come up with the wrong answer to the “Bananas” problem, were given the opportunity to review their answer and report back. In this episode Kyla explains why the answer should be three bananas.
Teacher’s Actions(s)/ Response(s)	1) The teacher acknowledges that Kyla and Neema have made a self-correction. [line 4] Code: Supporting student’s autonomy (SSA) 2) After Kyla’s explanation, the teacher provides the students the opportunity to agree/disagree with Kyla. [line 21] Code: Providing opportunity to critique other’s reasoning (OCR) Code: Encouraging classroom discourse (ECD)

Students' Action(s)/ Response(s)	1) Kyla reports on her revised answer to the "Bananas" problem. [lines 12-20] Code: Sharing solution/idea with class (SSC) Code: Making a self-correction (MSC)

Appendix D6: Lesson #1- Transcribed and coded Critical Event 6

<u>Event 6</u> **Role T1-1** **Teacher: T1** **Students: Amir and Jaylen**	
Transcript of Event **Time (as shown on RoleT1-1): (00:47:50-00:49:37)**	[It is about 48 minutes into the lesson. At Table 3 Amir and Jaylen have been working on the “Shorts and Glasses” problem. Amir is explaining to T1 why he thinks the glasses are more expensive than the shorts.] Amir: Those glasses are really expensive. T1: How do you know that? Amir: Because glasses…[not clear].. usually have to pay a lot of money. T1: What if you go to Walmart? Amir: Eh, probably still be kind of more expensive than shorts. T1: Okay, let’s look at what we have here and you tell me why. Jaylen: We thought that this [pointing to one of the shorts in the second picture], we tried 20, 40 [pointing to a second pair of shorts], that wouldn’t work. So then we dropped it down to 10. Ten, 20, 30 [pointing to the three shorts]. And 30 plus, what would this be [pointing to the glasses] to equal 50. And then we kind of thought it was 20. And then [finding the total for items in the second picture] 20, 30, 40, 50. So the sunglasses would be more expensive, which is $50. T1: Okay, so what strategy was that? Jaylen: Ahm.. Amir: Addition? Jaylen: Squeeze method? T1: Addition is an operation. Jaylen: Squeeze method? T1: Squeeze method? What does squeeze mean? Jaylen: It is like 30 [pointing to a pair of shorts] we thought….[not clear] T1: So you came up with a high number and a low number and started squeezing? [Jaylen nods] T1: Okay, where did you get the numbers from? Jaylen: We randomly picked them. T1: okay. So when we just pull the numbers out and try them out, what is that strategy called? [One of the girls at Table 3 is heard saying: guess and check] T1: You are guessing. Now, is there any way you could do besides guess and check? That is my question. [T1 walks away]
Description of the event	In this episode Jaylen and Amir explain to T1 why they think sunglasses are more expensive than shorts. However, they have difficulty identifying guess and check as their strategy of choice and make up a name for it, the “Squeeze” method. The teacher uses the word “squeeze” to construct a

	sentence that captures the essence of the strategy used by Amir and Jaylen.
Teacher's Actions(s)/ Response(s)	1) When Amir claims that sunglasses are more expensive than shorts, the teacher asks for justification. [line 6] Code: Eliciting a response – Describe/Explain (E-DE) 2) When Amir tells the teacher that glasses usually cost a lot of money, the teacher asks him to consider a specific low budget store. [line 8] Code: Eliciting a response – Name/State (E-NS) Code: Tapping into student's prior knowledge (SPK) 3) The teacher points to the pictures and asks Amir and Jaylen to justify their reasoning based on what is shown in the pictures. [line 10] Code: Eliciting a response – Describe/Explain (E-DE) Code: Directing students to perform some mental or physical activity (DA) Code: Suggesting a different approach (SDA) 4) The teacher asks Amir and Jaylen to name the strategy they used in solving the problem. [line 18] Code: Eliciting a response – Name/State (E-NS) 5) When Amir names Addition as a strategy, the teacher simply reminds him that addition is an operation. [line 22] Code: Attending to mathematical vocabulary (AV) 6) When Jaylen mentions the "Squeeze" method, the teacher encourages him to elaborate on that. [line 24] Code: Eliciting a response – Describe/Explain (E-DE) 7) Using the word "squeeze", the teacher constructs a sentence that captures the essence of the strategy used by Amir and Jaylen, which was guess and check. [lines 26 and 27] Code: Eliciting a response –Yes/No (E-YN) Code: Building on student's ideas (BSI) Code: Paraphrasing student's response (PSR) 8) The teacher asks Jaylen and Amir how they came up with the initial numbers for guessing and checking. [line 28]

	Code: Eliciting a response – Name/State (E-DE) 9) The teacher further questions Amir and Jaylen about their strategy, hoping they would identify it as the guess and check method. [lines 30 and 31] Code: Eliciting a response – Name/State (E-NS) 10) Once it is established that Amir and Jaylen have used the guess and check method, the teacher asks if they could have done it a different way and walks away from their table. [lines 33 and 34] Code: Eliciting a response – Describe/Explain (E-DE) Code: Suggesting a different approach (SDA) Code: Supporting student's autonomy (SSA)
Students' Action(s)/ Response(s)	1) Amir tells the teacher that glasses are more expensive than shorts because you normally have to pay a lot of money for glasses. (line 5; line 7; line 9] Code: Sharing solution/idea with teacher (SST) 2) Jaylen describes the guess and check method they used in order to determine the prices for shorts and sunglasses. [lines 11-17] Code: Justifying solution (JS) Code: Sharing solution/idea with teacher (SST) Code: Acknowledging the positive contribution of a partner (ACP) 3) Amir replies to the question: What problem solving strategy did you use? [line 20] Code: Answering a question (AQ) 4) Jaylen replies to the question: What problem solving strategy did you use? [lines 21 and 23] Code: Answering a question (AQ) 5) Jaylen tries to tell the teacher how the squeeze method works and that they had randomly picked numbers to try out and see if they worked [lines 25 and 29]. Code: Sharing solution/idea with teacher (SST)

Appendix D7: Lesson #1- Transcribed and coded Critical Event 7

Role T1-1 **Teacher: T1** **Student: Jade**	**<u>Event 7</u>**
Transcript of Event **Time (as shown on RoleT1-1): (00:58:17-00:58:44)** **Line**	[It is about 58 minutes into the lesson. At Table 2, Jade, Jasmine, and Tequrra have been working on the "Shorts and Glasses" problem. Jade presents their finding to the class] Jade: The item that we think is more expensive is the sunglasses. We knew they were more expensive because the more glasses you have, the less items you would need in each box to equal the $50. T1: Oh, okay. The more glasses you have [pause]. The more glasses you have the less item you would need to kind of supplement to get to $50? [The girls at Table 2 nod.]
Description of the event	In this episode Jade, Jasmine, and Tequrra have been working on the "Shorts and Glasses" problem at Table 2. Jade stands up and reads her explanation as to why they think the glasses are more expensive than the shorts. The justification she offers is solely based on the two pictures given in the problem.
Teacher's Actions(s)/ Response(s)	1) After hearing Jade's reasoning, the teacher repeats her response in the form of a question. [lines 7 and 8] Code: Paraphrasing student's response (PSR) Code: Validating student's reasoning (VSR)
Students' Action(s)/ Response(s)	1) Jade shares her reasoning with the class as to why the glasses are more expensive than the shorts. [lines 4-6] Code: Sharing solution/idea with class (SSC) Code: Justifying solution (JS)

Appendix D8: Lesson #1- Transcribed and coded Critical Event 8

<u>Event 8</u> **Role T1-2** **Teacher: T1** **Students: Neema and Kyla**		
Transcript of Event **Time (as shown on RoleT1-2): (00:01:51-00:03:24)**	**Line**	[It is about one hour and two minutes into the lesson. Neema and Kyla, at Table 1, have been working on the “Soda and Shirt” problem. They are using the guess and check method to determine the price of each item. Neema is explaining to the teacher what they have come up with so far.] Neema: I was thinking that maybe these are $2 each (Pointing to the three soda cups in the bottom picture), and now that would be ($) 6. And then 30 minus $6 would be $24. And 24 and 24 (total price for two shirts in the top picture), like if you would do this, it would be too much. T1: Okay, so the guess and check did not work. That is why I was pushing you before that for any other strategies you could use besides guess and check. Kyla shakes her head, no. T1: No? Kyla: We couldn’t think of anything. T1: Ha? Kyla: We couldn’t think of anything. T1: There are other things you could do. Kyla: Like what? Neema turns around smiling and looking at T1 expectantly. Titiana who is also seated at Table 1 is heard saying: You are supposed to help us. T1: Like a hint (she laughs). T1 (addressing R1): Dr. Alston, am I able to give any hint? R1: Ask a question. T1: Ha? R1: Ask a question. T1: Ask a question; in the form of a question (she laughs). T1: Can we make that problem smaller, in any way? (addressing the girls at Table 1) T1: Can we make that problem with less items? Kyla: Oh. I think I know what you mean. Kyla and Neema start talking about the problem and T1 addresses the whole class. T1: My kids. Can we make this problem with less items? And how can we do it?
Description of the event		At Table 1, Neema tells the teacher that $2 per soda and $24 per shirt will work in the bottom picture, but not the top one.

	T1 says that is why she was trying to push them to use another strategy besides guess and check. Kyla responds by saying that they tried but could not think of any other way. T1 assures them that there are other ways and Neema and Kyla ask for hints. T1 asks R1 if she could give the girls a hint and is advised that she should provide hints through asking questions. T1 asks the girls if they can "make that problem smaller in any way….make that problem with less item". Kyla indicates that the hint has been useful. T1 also shares that hint with the whole class.
Teacher's Actions(s)/ Response(s)	1) After hearing that Neema' strategy of guess and check has not resulted in correct answers, T1 reminds the girls to use a different strategy other than guess and check [lines 10-12]. Code: Suggesting a different approach to solving the problem (SDA) 2) T1 offers a hint to the students at Table 1 by posing a question [lines 29-31]. Code: Offering hints (OH) 3) T1decides to extend the hint to the whole class [lines 34-35]. Code: Offering hints (OH)
Students' Action(s)/ Response(s)	1) Students at Table1 ask T1 for a clue to help them with the solution method [lines 19-22] Code: Asking for help (AH)

Appendix D9: Lesson #1- Transcribed and coded Critical Event 9

<u>Event 9</u> **Role T1-2** **Teacher: T1** **Students: Jasmine and Jade**		
Transcript of Event	**Line**	
	1	[It is about one hour and 15 minutes into the lesson.
	2	At Table 2, Tequrra, Jasmine and Jade have been working on the “Soda and
Time (as shown on RoleT1-2): (00:14:53- (00:16:03)	3	Shirt” problem. T1 approaches their table.]
	4	T1: What is your answer?
	5	Jasmine: The juice costs $18 and the soda pop…
	6	Jade: No, the…
	7	Jasmine: Oh yeah, the shirt costs $18 and the juice costs $4.
	8	T1 starts adding some numbers up [Audio not clear]
	9	T1: And how did you get it?
	10	Jade: Because, like we came over here (pointing to her paper) and said to find
	11	half of the total price for half of the items which was a shirt and a drink. We
	12	then started to think which numbers would…
	13	Jasmine: We forgot about 44, we just worked with 22.
	14	T1: Alright, you worried about 22 then. So you started finding out which
	15	numbers….
	16	Jade: which numbers add up to 22 and work with the totals.
	17	T1: Okay.
	18	Jade: And we got 18 and 4.
	19	T1: Okay, good, alright. Did you write down your explanations?
	20	Jasmine: And then we did the rest of it. We did 18 plus 4, plus 4…
	21	T1: And it worked for you. Okay, write down your explanation.
Description of the event		In this episode, Jasmine and Jade who have been working on the “Soda and Shirt” problem, explain to the teacher how they cut the total price of $44 in half to find the price of half of the items. They used this smaller number ($22) in the guess and check strategy and found the correct prices. [About 12 minutes earlier, the teacher had given a hint to the whole class: “Can we make this problem with less items? And how can we do it?” Prior to the teacher’s hint, Jasmine was captured on camera using the given totals ($44 and $30) and struggling to find the correct answers]
Teacher’s Actions(s)/ Response(s)		1) The teacher asks the girls to explain how they came up with their answer. [line 9] Code: Eliciting a response – Describe/ Explain (E-DE) 3) The teacher indirectly acknowledges the merit of the girls’ decision to cut the total price in half. [lines 14 and 15]

	Code: Validating student's reasoning (VSR) Code: Repeating student's response (RSR) 4) The teacher acknowledges the correct answer and asks the girls to explain their thinking in writing. [line 19; line 21] Code: Validating student's reasoning (VSR) Code: Encouraging student to write down explanations (EWE)
Students' Action(s)/ Response(s)	1) Jasmine tells the teacher what prices for each item they came up with. [line 5; line 7] Code: Sharing solution/idea with teacher (SST) 2) As Jasmine begins to tell the teacher the prices they have found for each item, Jade alerts her to the fact that she is making a mistake by switching the prices for the two items. [line 6] Code: Correcting another student (CAS) 3) Jade and Jasmine explain and justify their thinking to the teacher. [line 10-12; line 13; line 16; line 18; line 20] Code: Justifying solution/idea (JS) Code: Acknowledging the positive contribution of a partner (ACP)

Appendix D10: Lesson #1- Summary table of Critical Events

Teacher: T1	
Event / Time	**Description**
Event 1 Time (as shown on RoleT1-1): 00:07:45 - 00:09:52	In this episode, the students at each table are seen discussing the Banana problem with their partners.
Event 2 Time (as shown on RoleT1-1): 00:12:57 - 00:13:38	In this episode, Carin who has been working on the "Banana" problem with Titiana, volunteers to share their findings with the class. After the teacher asks a clarifying question, Carin spontaneously adds the word, pounds, to her explanation (even though her written work does not indicate that she had initially attached any specific weight to any of the fruits).
Event 3 Time (as shown on RoleT1-1): 00:13:50 - 00:14:58	In this episode, Jaylen who has been working on the "Banana" problem with Amir, volunteers to report their findings to the class. Carin raises her hand to point out that Jaylen, in his explanation, stated that two pineapples equal five pounds and later on said that one pineapple equals five pounds. Jaylen clarifies by saying that the two pineapples equal five pounds each and therefore one pineapple equals five pounds. Solution Path: 1B
Event 4 Time (as shown on RoleT1-1): 00:15:58 - 00:17:21	In this episode, Kyla who has been working on the "Banana" problem with Neema, reports their findings to the class. Based on the first two scales, they have correctly identified that one apple equals three bananas in weight. However, for their final answer, they incorrectly place an apple and two bananas in the last scale. Once it is established that Kyla believes her wrong answer to be correct, the teacher paraphrases and then rewords Kyka's response and poses it as a question in such a way that makes Kyla doubtful about her answer. The teacher encourages Kyla and Neema to reexamine their work and report back later on.
Event 5 Time (as shown on RoleT1-1):	Kyla and Neema who had presented before and had come up with the wrong answer to the "Bananas" problem, were given the opportunity to review their answer and report back. In this episode Kyla explains how they made

00:20:14 - 00:21:28	a self-correction and why the answer should be three bananas. Solution Path: 1B
Event 6 Time (as shown on RoleT1-1): 00:47:50 - 00:49:37	In this episode Jaylen and Amir explain to T1 why they think sunglasses are more expensive than shorts. However, they have difficulty identifying guess and check as their strategy of choice and make up a name for it, the “Squeeze” method. The teacher builds on student’s ideas and uses the word “squeeze” to construct a sentence that captures the essence of the strategy used by Amir and Jaylen. Solution Path: 3C
Event 7 Time (as shown on RoleT1-1): 00:58:17 - 00:58:44	In this episode Jade, Jasmine, and Tequrra have been working on the “Shorts and Glasses” problem at Table 2. Jade stands up and reads her explanation as to why they think the glasses are more expensive than the shorts. The justification she offers is solely based on the two pictures given in the problem. Solution Path: Not identified previously
Event 8 Time (as shown on RoleT1-2): 00:01:51 - 00:03:24	In this episode Neema and Kyla, who have been working on the “Soda and Shirt” problem, ask T1 to give them a hint. T1 asks R1 if she can give the girls a hint and is advised that she should provide hints through asking questions. T1 offers the girls a hint by asking them a question. Kyla indicates that the hint has been useful. T1 also shares that hint with the whole class.
Event 9 Time (as shown on RoleT1-2): 00:14:53 - 00:16:03	In this episode, Jasmine and Jade who have been working on the “Soda and Shirt” problem, explain to the teacher how they cut the total price of $44 in half to find the price of half of the items. They used this smaller number ($22) in the guess and check strategy and found the correct prices. [About 12 minutes earlier, the teacher had given a hint to the whole class: “Can we make this problem with less items? And how can we do it?” Prior to the teacher’s hint, Jasmine was captured on camera using the given totals ($44 and $30) and struggling to find the correct answers.] Solution Path: 4D –Modified (Partial 4D + Guess and Check)

Appendix E: Transcribed and Coded Critical Events for Lesson #2

Appendix E1: Lesson #2- Transcribed and coded Critical Event 1

Event 1 **Role T2-1** **Teacher: T2** **Students: Dina, Monae, and Jasmine**	
Transcript of Event **Time (as shown on RoleT2-1): (00:13:25-00:14:37)**	[It is about 13 minutes into the lesson. Classroom is very quiet and there is very little interaction between the students. Most students are working independently until the teacher makes a push for discussion within the groups] T2: When I said prepare for sharing, what I, what I meant by that was within your group, try to achieve a consensus, meaning see if you can come to some agreement about what the answer is . Alright, so that you are having a little discussion, if there are different responses within the group, discuss it and see how you might come to an agreement. [There is a noticeable increase in the noise level around the room as the students start talking to each other. The teacher moves to Table 5 to answer a question raised by a student and the camera focuses on Table 1, where Jasmine is explaining her solution to her partners, Dina and Monae] Jasmine: …..apples…. Monae: I know, apple is three bananas. Three plus two is five. Jasmine: Exactly! This is how I got my answer because three plus two is equal five! Monae: There is already two bananas there. [pointing to the second scales] Dina: Yeah. [pointing to the pictures] Jasmine: So the apple….[not clear] Dina: That is three bananas [pointing to the third scale] and that is two [pointing to the second scale. Jasmine: Oh, I get it now. Oh my god!
Description of the event	In this episode the students are working on the "Bananas" problem. Even though they had been told to work within their groups, the class is very quiet and there is little interaction between the students until the teacher encourages the students to compare and discuss their solutions. This generates classroom discourse and at Table 1 Dina and Monae help Jasmine realize the mistake in her solution.
Teacher's Actions(s)/ Response(s)	1) The teacher asks the students to discuss their solutions within the group and reach consensus [lines 5-9]. Code: Directing students to perform some mental or physical activity (DA) Code: Encouraging classroom discourse (ECD)

Students' Action(s)/ Response(s)	1) Dina and Monae help Jasmine realize the mistake in her solution. Code: Engaged in Mathematical discourse with peers (MDP) Code: Correcting another student (CAS) Code: Justifying solution (JS)

Appendix E2: Lesson #2- Transcribed and coded Critical Event 2

<u>Event 2</u> **Role T2-1** **Teacher: T2** **Students: Jazmine, Kevin, Jasmine, and Cory**	
Transcript of Event **Time (as shown on RoleT2-1): (00:20:12-00:24:50)**	[It is about 20 minutes into the lesson. Jazmine, at Table 6, shares her group's findings with the class. She disagrees with the previous two presenters and claims the answer to be two bananas as opposed to three.] T2: Okay ladies. [After some debating at the table, Jazmine reluctantly stands up to present.] T2: Thank you Jazmine L. Jazmine: I think that, I think that it wouldn't be three. I think it would be two. Because if one apple and two bananas equal one, equals one pineapple, then it must be equivalent, one apple equals two bananas. (Kevin is heard saying: I object.) T2: Okay, go back through it one more time [Jazmine sits down] one more time Jazmine. Jazmine: I don't think it would be three because if two bananas and one apple equals one pineapple, then one apple equals two bananas. Because that is like, if you have, like if you have a banana and the apple [holding two palms up] that is going to be the same size if you hold them in your hands. That is why, that's why you are holding them in your hands. That's what the scale is like. So, that's why I think it's two. T2: Okay, now when you say one apple is equal to two bananas, is that what you are saying? Are you referring to weight? T2: Okay. She is saying they are equivalent in weight [addressing kevin]. Okay Kevin, do you have a response? Kevin: No, no. I thought she said weight. No. I don't agree, but. T2: Okay and do you, do you have [Kevin talking and laughing at his table], let's assume that we were debating this issue, what would be your response kevin? Kevin: Huh? T2: let's assume, let's assume that we were debating this issue and you're trying to convince Jazmine that your solution is correct. What would you say in response? Kevin: [sigh, pause, laugh] I would say that I think that it is three, three bananas because, like Caliph said, if you got 10 bananas and two pineapples. Half, half of two pine, half of two is one and half of 10 is five. So if you take [laughs in acknowledgment of something Caliph has just said to him], if you take one pineapple and put two bananas (already there on the second scale) and two bananas (to replace the apple, as claimed by Jazmine), that is four bananas. We don't have eight bananas on the first scale, we have 10. So, you don't have, it will have to be three bananas. One apple equals three bananas

	because, uhm, how to put this? Cory and Jasmine put their hands up. T2: Wait, wait.Hold your thoughts. Let him finish. Kevin: So, uhm, it had to be three, uhm. Yeah, cause basically five is half of 10 and one is half of two, so if you got, half and half it is T2: Jasmine G. Jasmine: Huh? Oh, oh, right. I think it is three bananas because on the first scale we have 10 bananas, two pineapples. Half of that is five and one. But on the scale we only have one pineapple, two bananas and an apple. So it has to five apples (she means bananas!). So then, two plus, two plus three equals five. T2: Cory. Cory: But, but bananas could be one and a half. T2: Bananas could be one and a half what? Cory: Pounds. T2: Okay, so now we are introducing the notion of pounds. Okay, why is that significant? Cory: Because this could be one and a half, one and a half [pointing to each of the two bananas] and that equals three. So the apple could be two pounds. T2: So are you in agreement with Kevin's rational or Jazmine L's? Cory: kevin's. I am just saying though. [students laughing] T2: Mr. Pedrick (R2)? R2: Mr.[unclear] said that they're one and a half pound each. What's the last name? T2: That's Cory. R2: Cory, if they are a pound and a half each, how much would the (ten) bananas weigh? Cory: one and a half? T3: No, the total pounds. [the class goes silent for a few seconds and the students look very thoughtful] Kevin: I have to think about it, have to think about it. T2: Think of it perhaps the decimal representation of one and a half. T3: It's one and a half, so you have 10 (bananas) [addressing Cory]. [After about 45 seconds, when no one has offered an answer to the question posed by R2, the teacher moves on to the next group.]
Description of the event	Five minutes earlier, the teacher had asked the girls at Table 6 to be the first group to report their findings on the "Banana" problem. The girls had asked if they could be skipped. T2 had replied that he would not skip over them, but will let them present later on. In this episode, T2 has brought everyone's attention back to Table 6 and has asked them to present. Jazmine stands up and disagrees with the previous two presenters by claiming the answer to be two bananas as opposed to three. Kevin, encouraged by the teacher, offers an explanation as to why Jazmine L. cannot be correct. Jasmine G. explains her reasoning for her answer and Cory, although in agreement with Kevin, plays the devil's advocate and offers a possible scenario, where Jazmine L. could be

	right.
Teacher's Actions(s)/ Response(s)	1) The teacher asks for explanation [lines 11 and12; lines 53 and 54]. Code: Eliciting a response – Describe/Explain (E-DE) Code: Directing students to perform some mental or physical activity (DA) Code: Encouraging classroom discourse (ECD) 2) The teacher asks for clarification of the explanation [lines 19 and 20; line 52; line 57] Code: Eliciting a response – Name/State (E-NS) Code: Encouraging classroom discourse (ECD) 3) The teacher encourages students to respond to Jazmine's comment [lines 21 and 22; lines 24-16; line 44; line 50]. Code: Providing opportunity to critique other's reasoning (OCR) Code: Encouraging classroom discourse (ECD) 4) The researcher, R2, asks a question, which potentially could have highlighted the flaw in Cory's assertion. Code: Eliciting a response – Name/State (E-NS) 5) When the students don't come up with a quick solution to the product of 10 and 1 1/5, the teacher suggests they work with the decimal representation of 1 ½ in order to calculate the product. Code: Suggesting a different approach (SDA)
Students' Action(s)/ Response(s)	1) Jazmine shares her group's finding with the class, which is not in agreement with any other solution presented thus far [lines 8-10; lines 13-18]. Code: Sharing solution/idea with class (SSC) Code: Justifying solution (JS) Code: Questioning the reasoning of others (QRO) 2) Kevin disagrees with Jazmine and tries to explain why [line 23; lines 31-39; lines 42 an d 43]. Code: Questioning the reasoning of others (QRO) Code: Justifying solution (JS)

	3) After listening to Jazmine claiming the answer to be 2 bananas, Jasmine offers her reasoning as to why the answer is 3 bananas. Code: Sharing solution/idea with class (SSC) Code: Justifying solution (JS) 4) Cory, although in agreement with Kevin, plays the devil's advocate and offers a possible scenario, where Jazmine L. could be right. Code: Sharing solution/idea with class (SSC)

Appendix E3: Lesson #2- Transcribed and coded Critical Event 3

<table>
<tr><th colspan="3"><u>Event 3</u>
Role T2-1
Teacher: T2
Students: Dina</th></tr>
<tr><td>Transcript of Event

Time (as shown on RoleT2-1): (00:41:28-00:42:13)</td><td>Line
1
2
3
4
5
6
7
8
9
10</td><td>
[It is about 42 minutes into the lesson.
Dina justifies her solution to the “Carrots” problem.]
Dina: I think that a pepper is equal to two carrots, because in scale 1 we can
see that there are six carrots and a corn and a pepper. We could, there are six
carrots, right? We can say that a corn was three carrots and a pepper was
equal to three. But, in scale 2 we can see that a corn is equal to two peppers.
So a corn should weigh four pounds and a pepper, two pounds. Because two
plus two equals four (demonstrating that the numbers work in the second
picture). So a pepper is equal to two carrots.
T2: Okay.</td></tr>
<tr><td colspan="2">Description of the event</td><td>Dina who has been working with her partners, Jasmine and Monae, shares their answer to the “Carrots” problem with the class.</td></tr>
<tr><td colspan="2">Teacher’s Actions(s)/ Response(s)</td><td></td></tr>
<tr><td colspan="2">Students’ Action(s)/ Response(s)</td><td>1) Dina shares her solution with the class [lines 3-8].

Code: Justifying solution (JS)
Code: Sharing solution/idea with class (SSC)</td></tr>
</table>

Appendix E4: Lesson #2- Transcribed and coded Critical Event 4

Role T2-1 **Teacher: T2** **Students: Kevin**	**<u>Event 4</u>**
Transcript of Event **Time (as shown on RoleT2-1): (00:45:56- 00:47:26)**	[It is about 46 minutes into the lesson. Kevin who has been working with Caliph and Cory at Table 4, presents their solution to the “Carrots” problem to the class.] Kevin: Alright. There are six carrots, there are six carrots and..[not clear].. and a corn and a pepper equal six carrots. If a corn on the cub equals two peppers then you add the first pepper that you got with the corn on the cub, you get two peppers [Kevin and his partners laughing]. So, so, yeah, one pepper had to equal two carrots and three peppers equal six carrots. T2: Okay, now I want you to go back to that middle piece there where it sounded like you were referring to the second scale Kevin: Oh, I said… T2: You said something about three peppers. Kevin: Oh, the, oh… T2: Cause up there I don’t see anywhere where there is three peppers. Just didn’t know where that notion came from. Kevin: If you add the first pepper that you got [students laughing] T2: Hush. Pay attention. Kevin: If you add the first pepper that you got with the corn on the cub, corn on the cub, with the two peppers, you get three peppers equal six carrots. T2: Aha, so since three peppers equal six carrots you are drawing what conclusion? Kevin: That one pepper equals two carrots. T2: Okay, or it’s the same in terms of weight. Kevin: Yeah. T2: Let’s make that distinction.
Description of the event	In this episode Kevin tries to explain his solution to the “Carrots” problem. The teacher has to ask him some questions in order to clarify his answer.
Teacher’s Actions(s)/ Response(s)	1) The teacher asks Kevin to explain his reasoning [line 9 and 10; line 12; lines 14 and 15]. Code: Eliciting a response – Describe/Explain (E-DE) 2) The teacher asks a question to clarify Kevin’s conclusion [line20 and 21]. Code: Eliciting a response – Name/State (E-NS)

	3) The teacher paraphrases Kevin's response in order to underscore an important point [line 23]. Code: Paraphrasing student's response (PSR)
Students' Action(s)/ Response(s)	1) Kevin presents his solution to the Carrot problem [lines 4-8; line 16; lines 18 and 19; line 22]. Code: Sharing solution/idea with class (SSC) Code: Justifying solution (JS)

Appendix E5: Lesson #2- Transcribed and coded Critical Event 5

<table>
<tr><th colspan="3"><u>Event 5</u>
Role T2-1
Teacher: T2
Students: Kevin</th></tr>
<tr><td rowspan="7">Transcript of Event

Time (as shown on RoleT2-1): (00:54:21-00:54:36)</td><td>Line</td><td></td></tr>
<tr><td>1</td><td>[It is about 54 minutes into the lesson.</td></tr>
<tr><td>2</td><td>Kevin makes a comment to R1, which indicates his understanding of the fact</td></tr>
<tr><td>3</td><td>that sunglasses are more expensive than the shorts]</td></tr>
<tr><td>4</td><td>Kevin: Why waste your money on two pairs of glasses and one pair of shorts</td></tr>
<tr><td>5</td><td>when the glasses are going to take up most of your money. So, just get two</td></tr>
<tr><td>6</td><td>pairs of shorts and one pair of glasses. It is cheaper.</td></tr>
<tr><td colspan="2">Description of the event</td><td>In this episode Kevin makes a comment to R1, which indicates his understanding of the fact that sunglasses are more expensive than the shorts. However, he does not offer an explanation.</td></tr>
<tr><td colspan="2">Teacher's Actions(s)/ Response(s)</td><td></td></tr>
<tr><td colspan="2">Students' Action(s)/ Response(s)</td><td>1) Kevin's comment indicates his understanding of the fact that sunglasses are more expensive than the shorts [lines 4-6].

Code: Sharing solution/idea with teacher (SST)</td></tr>
</table>

Appendix E6: Lesson #2- Transcribed and coded Critical Event 6

Event 6 **Role T2-1** **Teacher: T2** **Students: Jasmine**	
Transcript of Event **Time (as shown on RoleT2-2): (00:03:07-00:4:31)**	[It is about 64 minutes into the lesson. The girls at Table 1 had used guess and check and found out that $2 per shorts and $4 per glasses worked in both pictures. Now that it has been revealed that the price tag for each group of items is $50, Jasmine speculates that the prices could be $8 and $16 for shorts and glasses respectively.] R1: How much did you get for the total amount here? [addressing Jasmine and pointing to the group of items] Jasmine: Ten. R1: Okay. But now, instead of 10 the total is 50. Is it going to make the items cost more or less? Jasmine: More. R1: It's going to cost more. So now you have different prices. So the 2 and a 4 is not going to work. You have to figure out 2 new prices. But I think, from what Dina said, you still think the glasses are more. Jasmine: Eight and 16? R1: Does that work? [The camera moves to the next table, but Jasmine's response can be heard.] Jasmine: Oh, 20 and 10? Twenty, 20, 40,..[not clear]..Yeah. So yes! [The camera moves back to Table 1.] R1: Does that work for the bottom picture too, Jasmine? Jasmine: So, this is 10 [pointing to the shorts in the top picture and then pointing to each item in the bottom picture and silently adding up the prices.] Yes it does. Hold on, hold on. Yeah, it does. Ten, 20, 30, and 30 plus 20 is 50. I am right, glasses cost more.
Description of the event	The girls at Table 1 have been working on the "Shorts and Glasses" problem. Initially, with the $50 price tags hidden, the girls had concluded that the glasses were more expensive because they had used the guess and check strategy and found out that $2 per shorts and $4 per glasses worked in both pictures. Now that it has been revealed that the price tag for each group of items is $50, Jasmine speculates that the prices could be $8 and $16 for shorts and glasses respectively. However, after R1 asks whether 8 and 16 would work, Jasmine comes up with the new prices of $10 and $20 that satisfy both situations shown in the pictures.
Teacher's Actions(s)/ Response(s)	1) The researcher, R1, asks Jasmine a series of questions to help her figure out the correct answer [lines 6 and 7; lines 9 and 10; line 16; line 20]. Code: Eliciting a response – Name/State (E-NS)

Students' Action(s)/ Response(s)	1) Jasmine responds to R1's questions and finally figures out the correct answer [line 8; line 11; line 15; line 18; lines 21-23]. Code: Sharing solution/idea with teacher (SST)

Appendix E7: Lesson #2- Transcribed and coded Critical Event 7

Event 7 **Role T2-1** **Teacher: T2** **Students: Jevon and Oscar**	
Transcript of Event **Time (as shown on RoleT2-2): (00:11:05-00:12:41)**	[It is about 71 minutes into the lesson. Jevon, Meledy, and Oscar at Table 5 have been working on the “Shorts and Glasses” problem. Jevon shares their findings with the class.] Jevon: I think the glasses are more expensive, I think the glasses are more expensive because they are breakable and they are more harder to make. T2: Blacktop voice! I am all the way here by this tree and you are over there at the basketball court. Oscar: Want me say it? Want me say it [asking Jevon] Oscar: he said that glasses are more expensive one because it, it can break and it’s harder to make and they help people see. T2: So, we are determining that which item is more expensive? Oscar: Glasses. T2: Now, we are giving that based on what we know about glasses and what we see in the picture? Oscar: Yeah. T2: Then I missed that part. Tell me one more time. [Oscar pauses] T2: Which is more expensive and why? Oscar: The glasses. T2: Because? Oscar: Because it is more, it’s harder to make and it helps people see and they can break. T2: Now, see the question they asked you was can you determine that based on what is in the picture? Oscar: Oh, yeah. T2: Mathematically, based on what is in the picture how do we know that glasses… Oscar: Because in part A (first picture) there are only three things, that is two glasses and one pair of shorts that cost $50. T2: Aha. Oscar: And in part B (second picture), there are three pairs of shorts and one pair of glasses and it costs the same. T2: So that implies that glasses are ….. Oscar: More expensive. [The bell rings and the teacher asks another group to present]
Description of the event	Jevon, Meledy, and Oscar at Table 5 have been working on the “Shorts and Glasses” problem. In this episode, Jevon and Oscar reason as to why the glasses are more expensive than the shorts.

Teacher's Actions(s)/ Response(s)	1) The teacher asks Oscar a question in order to clarify his answer [line 11; line 33]. Code: Eliciting a response – Name/State (E-NS) 2) The teacher asks Oscar a question in order to clarify his reasoning [lines 13 and 14]. Code: Eliciting a response –Yes/No (E-YN) 3) The teacher asks Oscar to repeat his answer and his reasoning [line 16; line 18; line 20; lines 23 and 24; lines 26 and 27]. Code: Eliciting a response – Describe/Explain (E-DE)
Students' Action(s)/ Response(s)	1) Jevon and Oscar explain why the glasses are more expensive than the shorts [lines 4 and 5; lines 9 and 10; lines 21 and 22]. Code: Sharing solution/idea with class (SSC) Code: Justifying solution (JS) 2) Oscar gives a different reason to justify his answer [lines 28 and 29; lines 31 and 32]. Code: Sharing solution/idea with class (SSC) Code: Justifying solution (JS)

Appendix E8: Lesson #2- Summary table of Critical Events

Teacher: T2	
Event / Time	**Description**
Event 1 shown on RoleT2-1): 00:13:25 - 00:14:37	In this episode the students are working on the “Bananas” problem. Even though they had been told to work within their groups, the class is very quiet and there is limited interaction between the students until the teacher encourages the students to compare and discuss their solutions. This generates classroom discourse and at Table 1 Dina and Monae help Jasmine realize the mistake in her solution. Solution Path: 1B
Event 2 Time (as shown on RoleT2-1): 00:20:12 - 00:24:50	Five minutes earlier, the teacher had asked the girls at Table 6 to be the first group to report out to class their findings on the “Banana” problem. The girls had asked if they could be skipped. T2 had replied that he would not skip over them, but will let them present later on. In this episode, T2 has brought everyone’s attention back to Table 6 and has asked them to present. Jazmine stands up and disagrees with the previous two presenters by claiming the answer to be two bananas as opposed to three. Kevin, encouraged by the teacher, offers an explanation for why Jazmine L. cannot be correct. Jasmine G. explains her reasoning for her answer and Cory, although in agreement with Kevin, plays the devil’s advocate and offers a possible scenario, where Jazmine L. could be right. Jasmine G’s Solution Path: 1B
Event 3 Time (as shown on RoleT2-1): 00:41:28 - 00:42:13	Dina justifies her solution to the “Carrots” problem. Solution Path: 2C
Event 4 Time (as shown on RoleT2-1): 00:45:56 - 00:47:26	Kevin tries to explain his solution to the “Carrots” problem. The teacher has to ask him some questions in order to clarify his answer. Solution Path: 2B
Event 5 Time (as shown on RoleT2-1):	In this episode Kevin makes a comment to R1, which indicates his understanding of the fact that sunglasses are more expensive than the shorts. However, he does not offer an explanation.

00:54:21 - 00:54:36	
Event 6 Time (as shown on RoleT2-2): 00:03:07 - 00:4:31	This episode can be used as evidence that the girls at Table 1 had noticed the 1:2 relationship between the prices in the “Shorts and Glasses” problem. Initially, with the $50 price tags hidden, the girls had concluded that the glasses are more expensive because they had used the guess and check strategy and found out that $2 per shorts and $4 per glasses had worked in both pictures. Now that it has been revealed that the price tag for each group of items is $50, Jasmine speculates that the prices could be $8 and $16 for shorts and glasses respectively. However, after R1 asks whether 8 and 16 would work, Jasmine comes up with the new prices of $10 and $20 that satisfy both situations shown in the pictures. Solution Path: Not previously identified (The girls had used guess and check even in the absence of the price tags. This had not been anticipated by the teachers.)
Event 7 Time (as shown on RoleT2-2): 00:11:05 - 00:12:41	In this episode Oscar reasons as to why the glasses are more expensive than the shorts. The teacher encourages Oscar to reason based on the observable facts provided by the pictures. Oscar is then able to justify his answer based on the pictures and his reasoning, although not very well articulated, is correct. Solution Path: Not previously identified

Appendix F: Transcribed and Coded Critical Events for Lesson #3

Appendix F1: Lesson #3- Transcribed and coded Critical Event 1

<u>Event 1</u>

Role T3-1
Teacher: T3
Students: Jynita, Dieshe, Kevin, and Tyreal

Transcript	**Line**	
of Event	1	[It is about 12 minutes into the lesson.
	2	The students have been working on the "Carrots" problem. When the teacher
Time (as	3	asks a question about the problem, the answer is far from what she expected.]
shown on	4	T3: Let me ask you a question. What do you believe they are asking? What
RoleT3-1):	5	are you expected to do? What are the expectations?
(00:12:10-	6	Dieshe: [not clear]
00:15:02)	7	T3: Okay, what do you know about the carrots? Explain to me what you know
	8	about the carrots. What is it that you know about the carrots.
	9	Jynita: They are orange? [Dieshe giggles]
	10	T3: They are orange, okay. But let's talk about the scale.
	11	[T3 addresses the whole class]
	12	T3: Boys and girls, let me just, remember back in second grade, when you
	13	were working with the pan balance?
	14	A boy: Yeah.
	15	T3: What happened with the pan balance?
	16	Kevin: You had to make each side balance out.
	17	T3: You had to make each side balance out. So explain that a little more in
	18	depth to me. You had to make each side balance out.
	19	Tyreal: You had to reduce.
	20	T3: You have to reduce. Okay I am hearing words like reduce.
	21	Kevin: You have to reduce. You have to add to it.
	22	T3: You have to add to it. So Mrs [T3] is going to write some of these words
	23	down.
	24	[T3 erases the board. R2 offers to give T3 her a pen, but she says she has one
	25	and gets one out of a box on her desk]
	26	T3: Okay, so we are working with the pan balance [writes Pan balance on the
	27	board]. We had to keep, and this is the word balance [underlines the word,
	28	balance]. We had to make it balance. What were some of the things we had to
	29	do? And you just have to go a little bit more in depth. You had to reduce, but
	30	what was it that I had to reduce?
	31	[Hands go up]
	32	T3: You said you had to …[pointing to Kevin]
	33	Tyreal: From the side that is the heavier.
	34	T3: Excuse me?
	35	Tyreal: You had to reduce from the side that was heavier.
	36	T3: Okay, so reduce from the heavier side. [writes it on the board]. Okay,
	37	what else? You had to add. Excuse me, what did you say? [addressing Keith]
	38	Tyreal: You had to add to the side that was lighter.
	39	[T3 writes it down on the board]

	T3: Okay, so we are working with the pan balance. [Devon raises his hand] T3: Yes, Devon. Devon: We reduce from the …[not clear] and if it is too light, we have to add. T3: Okay. So that is basically what he said. But in general what were you doing to the pan balance? Dieshe: You were trying to figure out what would equal to another. T3: That is it. You had to make it equal to. Now here, equal to [write the words, equal to and underlines the word, equal]. Now here we have a pan balance, okay. Now I want you to think a little deeper about this here. Left side we have some carrots and on the right side there is some corn and green pepper. Hmmm, let's think about this here. T3: What is it that you know about the first picture? What does that first picture tell you? [Jynita's hand goes up] T3: Jynita. Jynita: That, ahm, would've equal weight. You had to have a corn and a pepper in order to equal to all the carrots. T3: okay, now I know that I need a corn and a pepper to equal the six carrots. Right? Jynita: Uhum.
Description of the event	In this episode T3 asks Jynita and Dieshe a question about the problem and the answer she gets is true, but irrelevant to the problem and far from what she expected. She decides to review some prior knowledge with the whole class after which, she reposes the same re-worded question and gets a reasonable answer from Jynita.
Teacher's Actions(s)/ Response(s)	1) Trying to help Jynita and Dieshe get started on the Carrot problem, the teacher asks some questions about the problem [lines 4 and 5; lines 7 and 8]. Code: Eliciting a response – Name/State (E-NS) 2) Addressing the whole class, the teacher reviews previously learned concepts by asking a series of questions [lines12 and 13; line 15; lines 17 and 18; lines 26-30; line 32; lines 36 and 37; lines 44 and 45]. Code: Eliciting a response – Name/State (E-NS) Code: Tapping into student's prior knowledge (SPK) 3) The teacher reposes a question (re-worded) to Jynita [lines 52 and 53]. Code: Eliciting a response – Name/State (E-NS) Code: Rewording a question (RQ)

Students' Action(s)/ Response(s)	1) Jynita responds to the teacher's question [line 9]. Code: Answering a question (AQ) Code: Struggling with the task (ST) Code: Displaying lack of understanding (DLU) 2) Several students volunteer to answer the teacher's questions [line 14; line 16; line 19; line 21; line 33; line 35; line 38; line 43; line 46; lines 55 and 56]. Code: Answering a question (AQ) Code: Volunteering to share ideas (VS)

Appendix F2: Lesson #3- Transcribed and coded Critical Event 2

<u>Event 2</u> **Role T3-1** **Teacher: T3** **Students: Tyreal**	
Transcript of Event **Time (as shown on RoleT3-1): (00:15:18-00:16:35)**	[It is about 15 minutes into the lesson. The students have been working on the "Carrots" problem. Tyreal presents his solution to the "Carrots" problem at the overhead.] T3: Who would like to add, who would like to demonstrate based on what Jynita said? Jynita, would you like to come up here and share? [Jynita does not respond, but several students are heard saying that they would] Tyreal: I can do the whole problem. T3: To do the whole problem? Okay. Tyreal. [Tyreal moves to the front of the room, where the problem is displayed on the overhead projector.] T3: This is just a warm-up for us to start, to begin to get our minds open. Tyreal: …..there? [not clear; pointing to the overhead] T3: yes. Do it on the overhead, hon. [students chatting] Tyreal: Since there are six carrots… T3: Hushsh! Now we are sharing. [students quiet down] Tyreal: Since there are six carrots, I tried numbers like one point five or one and a half [writes 1.5 next to the carrots on the first scale] for each carrot. And two carrots is three, like three. Several people: Ahem. Tyreal: So all of them would equal 9. So that means the corn will have to be 6 and the pepper would equal 3 [writes 6 and 3 next to the corn and the pepper respectively]. So two peppers would equal to 6 and the corn is 6 [writes 6 on both sides of the scale in the second picture]. Tyreal: And then… the pepper, there is one [pointing to the third picture]. Since there is only one pepper, this is equal to 3 [writes 3 next to pepper on the third scale] and two carrots [writes 2 carrots next to the question mark] Kevin: I did it a different way. T3: You did a different way? Okay.
Description of the event	In this episode Tyreal presents his solution to the "Carrots" problem at the overhead. He has assigned the number 1.5 to each carrot and has correctly decided that two carrots should be placed on the last scale. (His solution strategy was not listed amongst the solution paths identified by the teachers during the planning phase)
Teacher's Actions(s)/ Response(s)	

Students' Action(s)/ Response(s)	1) Tyreal volunteers to present his solution to the "Carrots" problem at the overhead [line 8; lines 18-28]. Code: Volunteering to share ideas (VS) Code: Justifying solution (JS) Code: Sharing solution/idea with class (SSC)

Appendix F3: Lesson #3- Transcribed and coded Critical Event 3

Role T3-1 **Teacher: T3** **Students: Jynita**	**<u>Event 3</u>**
Transcript of Event **Time (as shown on RoleT3-1): (00:21:44-00:22:15)**	[It is about 22 minutes into the lesson. The students have been working on the “Bananas” problem. Jynita shares her answer with the class.] T3: Here you have: How many bananas are needed to make the third scale balance? Jynita: I like to answer. [Jynita and several other students have raised their hand] T3: Okay Jynita. [Dieshe and Jynita giggle about something Tyreal has said to them] Jynita: I used your technique, that’s why [addressing Tyreal]. T3: You said what? You did what? Jynita: I used Tyreal’s technique. T3: Okay, share. Jynita: it was 10 bananas, so then, and two pineapples, So each pineapple had to equal 5, so then 10 and 10 (referring to the first scale). And then for the next one the pineapple is 5 again. So two bananas will have to equal 1(each), so then an apple will have to equal 3. Then we have 5 and 5 for the next one (referring to the second scale). And if the apple is equal to 3 then you have to have three bananas.
Description of the event	In this episode Jynita explains her solution to the “Bananas” problem. She claims to have used Tyreal strategy of assigning numbers to items.
Teacher’s Actions(s)/ Response(s)	
Students’ Action(s)/ Response(s)	1) Jynita volunteers to present her solution [line 6]. Code: Volunteering to share ideas (VS) 2) Jynita and partner use the strategy described by Tyreal in a previous problem [line 10; line 12]. Code: Comparing solution strategies/ideas (CS) 3) Jynita explains her reasoning [lines 14-18]. Code: Sharing solution/idea with class (SSC) Code: Justifying solution (JS)

Appendix F4: Lesson #3- Transcribed and coded Critical Event 4

<u>Event 4</u> **Role T3-1** **Teacher: T3** **Students: Theo and Kevin**		
Transcript of Event **Time (as shown on RoleT3-1): (00:23:45-00:25:35)**	**Line**	
	1	[It is about 24 minutes into the lesson.
	2	The teacher introduces the modified "Shorts and Glasses" problem on the
	3	overhead and gets two responses.]
	4	T3: The other day, Miss..[T3] went to the store. Miss..[T3] is always here to
	5	tell you about her..
	6	A girl: shopping.
	7	T3: Her shopping, exactly [students laugh] . And this is what I saw [puts up
	8	on the overhead a copy of the "Shorts and Glasses", with the price tags
	9	hidden]
	10	This is what I saw over there. I saw some sunglasses, I saw some shorts.
	11	A boy: On a rack?
	12	T3: Not on a rack J.C. on a, on a shelf. Now, ironically enough, the two
	13	sunglasses and the shorts were the same, was the same price as the one
	14	sunglasses and the three shorts.
	15	A Boy: Wow, wow. What did you say?
	16	T3: Ironically enough, there were two, there were two shelves. One of the
	17	shelves had two, one of the shelves had two glasses and shorts and in the
	18	other, on the other side of the shelf was one glasses and three shorts. And I
	19	said to myself, hmm, they are the same price, then without knowing, which is
	20	more expensive, the shorts or the glasses?
	21	A couple of boys are heard saying: The glasses [several hands go up]
	22	T3: And how do you know? I haven't heard from you [pointing to Thoe who
	23	has his hand up]. How do you know?
	24	Theo: The glasses are more expensive.
	25	T3: In general?
	26	[Theo nods, yes]
	27	T3: Okay, but let's just get away from that here. Just away, just get away
	28	from because in general, no. Let's just look at those are the same price, which
	29	one is more expensive and based upon what you see in front of you. We will
	30	come back to you [Theo]. Yes? [addressing Kevin]
	31	Kevin: Ahh, the glasses are more expensive, because one glass, one pair of
	32	glasses equals two pairs of shorts
	33	T3: How did you get that? [sounds surprised]
	34	Kevin: Because there are three pairs of shorts down there [second picture] and
	35	one glasses and two pairs of glasses up there [first picture] and one pair of
	36	shorts.
	37	T3: Wow! Do you agree? Were you about to raise your hand? [pointing to a
	38	student at the back]
	39	A boy: I …[not audible]

	T3: You had the same thing? What about you? [pointing to another student who says something, not audible]. You were about to say the same thing? Okay.
Description of the event	In this episode the teacher introduces the modified “Shorts and Glasses” problem on the overhead and is surprised when Kevin immediately identifies the 2:1 relationship between the glasses and the shorts.
Teacher’s Actions(s)/ Response(s)	1) The teacher introduces the modified version of the Shorts and Glasses problem on the overhead [lines 16-21; lines 22 and 23; lines 27-30; line 33] Code: Eliciting a response – Name/State (E-NS) Code: Eliciting a response – Describe/Explain (E-DE) Code: Rewording a question (RQ) 2) The teacher questions Theo’s reasoning [line 25]. Code: Eliciting a response –Yes/No (E-YN) 3) The teacher calls on other students to see if they agree with kevin’s solution [line 37 and 38; lines 40 and 41]. Code: Encouraging classroom discourse (ECD) Code: Eliciting a response –Yes/No (E-YN) Code: Validating student’s reasoning (VSR)
Students’ Action(s)/ Response(s)	1) Theo responds to the teacher’s question [line 24]. Code: Answering a question (AQ) Code: Volunteering to share ideas (VS) 2) Kevin explains why the glasses are more expensive than the shorts [lines 31 and 32; lines 34-36]. Code: Answering a question (AQ) Code: Volunteering to share ideas (VS) Code: Justifying solution (JS) Code: Sharing solution/idea with teacher (SST)

Appendix F5: Lesson #3- Transcribed and coded Critical Event 5

<table>
<tr><td colspan="3"><u>Event 5</u>
Role T3-1
Teacher: T3
Students:Theo</td></tr>
<tr><td rowspan="13">Transcript of Event

Time (as shown on RoleT3-1): (00:26:18-00:26:51)</td><td>Line</td><td></td></tr>
<tr><td>1</td><td>[It is about 26 minutes into the lesson.</td></tr>
<tr><td>2</td><td>Theo explains to the teacher how he found the cost of the sunglasses and the</td></tr>
<tr><td>3</td><td>shorts to be $12.50 each.]</td></tr>
<tr><td>4</td><td>T3: Yes? [Theo’s hand is raised. T3 walks over to him. Theo is holding a</td></tr>
<tr><td>5</td><td>calculator.]</td></tr>
<tr><td>6</td><td>Theo: So these [pointing to the items in the bottom picture] cost twelve thirty,</td></tr>
<tr><td>7</td><td>twelve dollars thirty cents, I mean twelve dollars fifty cents. Because four</td></tr>
<tr><td>8</td><td>items, divide fifty by four.</td></tr>
<tr><td>9</td><td>T3: But if you just already said that the glasses much cost more than shorts,</td></tr>
<tr><td>10</td><td>so is it possible that they all cost the same? Just based upon this here. [She</td></tr>
<tr><td>11</td><td>points to the pictures and walks away. Theo starts punching numbers on the</td></tr>
<tr><td>12</td><td>calculator.]</td></tr>
<tr><td colspan="2">Description of the event</td><td>The students have been working on the “Shorts and Glasses” problem. A few minutes earlier it had been established, through a class discussion, that the glasses are more expensive than the shorts and one student had said that the glasses are twice as expensive as the shorts. The students are now trying to find the prices for each item.
In this episode, Theo calls the teacher over to his desk and explains how he found out the prices by dividing 50 (total price) by 4 (number of items), concluding that the shorts and glasses cost the same ($12.50).
The teacher asks Theo if it is possible for the prices to be the same when we know that the glasses are more expensive. The teacher walks away.</td></tr>
<tr><td colspan="2">Teacher’s Actions(s)/ Response(s)</td><td>1) After listening to Theo’s explanation, the teacher asks him a question and walks away [lines 9-12].

Code: Eliciting a response –Yes/No (E-YN)
Code: Supporting student’s autonomy (SSA)</td></tr>
<tr><td colspan="2">Students’ Action(s)/ Response(s)</td><td>1) Theo shares his findings with the teacher [lines 6-8].

Code: Sharing solution/idea with teacher (SST)
Code: Volunteering to share ideas (VS)
Code: Displaying lack of understanding (DLU)</td></tr>
</table>

Appendix F6: Lesson #3- Transcribed and coded Critical Event 6

Event 6	
Role T3-1 **Teacher: T3** **Students: Davon**	
Transcript of Event **Time (as shown on RoleT3-1): (00:30:55-00:33:03)**	[It is about 31 minutes into the lesson. Devon at the overhead explains how he and his partner, Chad, used trial and error to figure out the prices for shorts and glasses] Devon: first, when we first started, we both noticed, so we started with 17, 17 [pointing to the glasses in the top picture] and 15 [pointing to the shorts] T3: Okay, wait, boys and girls let's listen. Devon: And it moved up to 15, I mean 50. So we thought, alright, that is right. Then we got down here [pointing to the bottom picture], 17, 15, I mean 17, 16, 16, 16, it equaled to 62. So it is like, you can't do that. So then we started going through the numbers again. That is when I found out that if we do 20 and 20 [pointing to glasses in the top picture], that is 40. You do 20, 20 that is 40, and then the pants need to be 10, so that will be 50. Now if you go ahead at the bottom [bottom picture], it will be 20 plus 10, plus 10, plus 10 [writing the prices on top of each item in the bottom picture]. That equaled 50 again. [A couple of students say something and there is laughter] Devon: That is where I got 20, 20, 10 from [writing 20, 20, and 10 next to the items in the top picture to indicate prices] T3: Okay. [Sounds of clapping and laughter as Devon returns to his seat] T3: Okay. Everyone….[not clear]. Did anyone do it differently? What can you tell me, just looking at it here, what can you tell me about the values of, okay so, do me a favor [addressing Davon] write this down here with this pen [handing a pen to Davon who has come back to the overhead]. Just write what you did here. [Davon writes the prices next to each item in the picture.] T3: So in looking at these two [pointing to the top and bottom pictures] pictures and the 50 (price tags), what can you tell me about…If I had to write an equation for this, what would that equation look like? [A few hands are up] Davon: Twenty to the third power? T3: No. What can you tell me about 20, 20, and 10 (first picture) and 10, 20, 20, and 20? What are they? A girl: …..? [unclear] T3: Yeah, he is going [unclear] after math. [Kevin has his hand up and T3 points to him] T3: Yes. If I had to give a sign for this, for this value here [pointing to the top picture] and this value here [pointing to the bottom picture], what would I, what could I say about the price in the top and the price in the bottom

	[circling the two pictures], they are what? Kevin: They are supposed to be the same. T3: Okay, so what, what symbol that you learned about in kindergarten means the same? Kevin: Equal. A boy: Equal to. T3: Equal. So I can just put an equal sign here. [writes the symbol = between the two pictures]
Description of the event	In this episode, Devon is explaining at the overhead how he and his partner, Chad, used the guess and check method to find the prices for shorts and glasses. Even though, it was previously established in class that the two prices had a 2:1 relationship, the boys' choice of initial numbers ($17 and $16) did not reflect that understanding. Also T3 illustrates the meaning of the equal sign by placing one between the two pictures, indicating the two equal price tags.
Teacher's Actions(s)/ Response(s)	1) After Devon's presentation of his solution, the teacher asks if anyone solved the problem using a different strategy [line 21]. Code: Eliciting a response –Yes/No (E-YN) Code: Encouraging classroom discourse (ECD) 2) The teacher asks the class a series of leading questions in order to get the answer she is looking for, which is " the equal sign" [lines 27-29; lines 32 and 33; 37-40; lines 42 and 43; lines 46 and 47]. Code: Eliciting a response – Name/State (E-NS)
Students' Action(s)/ Response(s)	1) Devon explains how he used the guess and check method to arrive at his answer for the Shorts and Glasses problem [lines 7-18]. Code: Volunteering to share ideas (VS) Code: Justifying solution (JS) 2) Several students respond to the series of questions asked by the teacher and eventually Kevin comes up with the answer the teacher was looking for [line 31; line 34; line 41; line 44; line 45]. Code: Answering a question (AQ)

Appendix F7: Lesson #3- Transcribed and coded Critical Event 7

Event 7 **Role T3-1** **Teacher: T3** **Students: Kevin and Devon**		
Transcript of Event	**Line**	
	1	[It is about 33 minutes into the lesson.
	2	The teacher is about to introduce the “Tug-of-War” problem to her students]
Time (as shown on RoleT3-1): (00:33:05-00:34:12)	3	T3: Now, I have something for you now. We’ve been talking about things that
	4	are equal, but let’s change gears. Does anyone remember this here? [holding
	5	out a picture of the students playing tug-of-war on field day]
	6	[Several students respond at the same time. Some say no and others say yes]
	7	Kevin: Oh yeah, that is [unclear] on field day.
	8	[T3 puts the picture up on the overhead]
	9	T3: At field day. What, what activity, what were you doing?
	10	Several students: Tug-of-war.
	11	T3: Who can tell me about tug-of-war? What is tug-of-war? What is it? What
	12	is tug-of-war?
	13	Jynita: When you pull a rope and you have to make sure the person doesn’t go
	14	over the line. If the person goes over the line he loses.
	15	T3: It’s like what? [addressing Kevin who has his hand up]
	16	Kevin: It’s like, like the scale. They are even on equal sides.
	17	T3: Okay, they are equal sides, but then..
	18	Kevin: Whoever is stronger
	19	T3: When one goes…[pretending to pull a rope]
	20	Devon: It goes unbalanced.
	21	T3: It goes unbalanced, right?
	22	Devon: And one side has a fixed side…[unclear].
	23	T3: And one side has a fixed side, okay. Take a look at this tug-of-war. First it
	24	starts off equal. We are standing there. You are making up teams. Everyone is
	25	equal, right? But then it goes like this here [rocking back and forth and
	26	pulling on an imaginary rope] and then the side, one side, okay.
	27	A boy: People start tugging.
	28	T3: And people start tugging.[teacher starts handing out the “Tug-of-War”
	28	problem]
Description of the event		In this episode, T3 shows the class a picture which was taken during the tug-of-war activity on field day. This was supposed to spike the students’ interest and serve as an introduction to the “Tug-of-War” problem. Some students were able to make a connection between tug-of-war and the pan balance.
Teacher’s Actions(s)/ Response(s)		1) The teacher introduces the “Tug-of-War” problem by showing the class a picture which was taken during the tug-of-war activity on field day [lines 3-5; line 9; lines 11 and 12]

	Code: Tapping into student's prior knowledge (SPK) Code: Eliciting a response – Name/State (E-NS) Code: Encouraging classroom discourse (ECD) 2) When one student makes the connection between the tug-of-war and the scale, the teacher further questions the class about the concept [line 15; line 17; line 21; lines 23-26]. Code: Eliciting a response – Name/State (E-NS) Code: Encouraging classroom discourse (ECD)
Students' Action(s)/ Response(s)	1) Kevin makes the connection between the tug-of-war and the scale [line 16; line 18]. Code: Volunteering to share ideas (VS) Code: Comparing solution strategies/ideas (CS)

Appendix F8: Lesson #3- Transcribed and coded Critical Event 8

Event 8 **Role T3-1** **Teacher: T3** **Students: Keith**	
Transcript of Event **Time (as shown on RoleT3-1): (00:36:45-00:37:27)**	[It is about 37 minutes into the lesson. Keith begins to explain to the teacher why he thinks the elephant and the horses will win the tug-of war. Despite his reluctance, the teacher persuades him to write his thinking down on paper.] T3: Go ahead. Keith: I think the elephant and three horses will beat the four oxen because it takes one elephant against one ox and two horses and it takes five horses to match up four oxen. Then… T3: Why don't you write all this out? Write it all down. I never said you cannot write on this. You write on it. You can write notes. You can scratch. Keith: I hate writing because it, it's mad hard! T3: It is not that. You just, and I did not say explain. Write what you wrote, just scratch out. Do what you need to do so you have all this on paper. [Keith begins to write and T3 walks away.]
Description of the event	In this episode, Keith begins to explain to the teacher why he thinks the elephant and the horses will win the tug-of war. The teacher asks Keith to put his thinking down on paper. Keith follows the teacher's direction after protesting that it is too difficult.
Teacher's Actions(s)/ Response(s)	1) The teacher asks Keith to write his explanation down even though he claims that he "hates writing because it is mad hard" [lines 9 and 10; lines 13 and 13]. Code: Encouraging student to write down explanations (EWE) Code: Directing students to perform some physical or mental activity (DA)
Students' Action(s)/ Response(s)	

Appendix F9: Lesson #3- Transcribed and coded Critical Event 9

<u>Event 9</u> **Role T3-1** **Teacher: T3** **Students: Kevin**		
Transcript of Event **Time (as shown on RoleT3-1): (00:37:46-00:38:50)**	**Line**	[It is about 38 minutes into the lesson. Kevin has finished working on the "Tug-of-War" problem and wants to share his answer with T3. The teacher insists that he writes down his thinking] Kevin: Miss..[T3]. Miss ..[T3]. T3: Yes Kevin. Kevin: I think I got it. T3: Okay. Did you explain and give reasons [unclear]? Kevin: Yes. Yes I have. T3: Write it. No, I can't listen to you [unclear]. You want Miss..[T3] to get everything. Every time [unclear], I forget.[T3 walks away] Kevin: I can't write my mind. [addressing no one in particular] [Kevin starts writing. After a few seconds (17), T3 addresses Kevin from across the room] T3: kevin, you can bullet it. You don't have to write it out. You know, you should put this is what dede dede dede. I mean I want you to write a paragraph and prefer that you put your thoughts on paper and use that to justify...[inaudible]
Description of the event		In this episode, Kevin wants to share his answer to the "Tug-of-War" problem with the teacher. However the teacher insists that he writes down his thinking and use his writing in order to justify his answer.
Teacher's Actions(s)/ Response(s)		1) Despite Kevin's protests: "I can't write my mind", the teacher insists that he puts down his explanations on paper [line 7; lines 9 and 10; lines 14-17]. Code: Encouraging student to write down explanations (EWE) Code: Directing students to perform some physical or mental activity (DA)
Students' Action(s)/ Response(s)		

Appendix F10: Lesson #3- Transcribed and coded Critical Event 10

Event 10 **Role T3-1** **Teacher: T3** **Students: Jynita and Dieshe**		
Transcript of Event	**Line**	
	1	[It is about 39 minutes into the lesson.
Time (as shown on RoleT3-1): (00:39:27-00:42:18)	2	Jynita and her partner Dieshe have arrived at the wrong conclusion that the
	3	four oxen will win the tug-of-war. Jynita is explaining their answer to the
	4	teacher. The teacher questions their reasoning and eventually, Dieshe realizes
	5	what the correct answer is.]
	6	T3: Go ahead, You said the oxen, they can pull five horses, right?
	7	Jynita: Uhum.
	8	T3: Here is the four [pointing to the four oxen in the third picture], here is the
	9	four [pointing to the four oxen in the first picture], where is the five horses?
	10	Jynita: But they, they can still pull three horses if they can pull five, right?
	11	T3: What does this say? [pointing to the first picture]
	12	Jynita: Four oxen are as strong as five horses.
	13	T3: Okay, here are four oxen, I don't see the five horses though [pointing to
	14	the third picture]
	15	Dieshe: [pointing to various pictures] It is one elephant here, the same as
	16	there [unclear] one ox and two of those, so that's why I think [unclear] one
	17	elephant and three horses.
	18	T3: What is it that you know about..So it is the..I still don't see five. Do you,
	19	do you see five horses here? [pointing to the third picture]
	20	Jynita: No [shakes her head]
	21	Dieshe: No [unclear] was trying to get the first one, how for, like use the
	22	number, know what I'm saying? Four and then five, but I was trying to figure
	23	out. Couldn't get.
	24	T3: But look what you see here [pointing to the middle picture], an elephant is
	25	as strong as one oxen and two horses. What does that mean?
	26	Jynita: That means that it is not as strong as two oxen.
	27	T3: What, What is a horse, elephant as strong as..What does that mean, an
	28	elephant is as strong as one ox and two horses?
	28	Dieshe: they [unclear] to an elephant.
	30	T3: Excuse me?
	31	Dieshe: they both, all three equal one elephant.
	32	T3: Okay, so what else, what else could, what else could we assume based
	33	upon that message there, that statement? What can we imply?
	34	Jynita: The oxen are going to win the tug-of-war.
	35	T3: The oxen are going to win the tug-of-war?
	36	[Jynita nods]
	37	T3: An oxen is as strong, agh. An elephant is as strong as an oxen and two
	38	horses.
	39	Dieshe: If you add, okay, this is one ox. No hold on. [pause] It will be easier

	if could figure out how much each horse weighs. Not weigh, but like. T3: But we know that [pause] Jynita: Five horses are as strong as four oxen, so three horses aren't as strong as four oxen. Dieshe: So that means they will win. [pointing to the elephant and the three horses] T3: Why? Dieshe: Because if you know that.. Jynita: No, they are going to win Dieshe [pointing to the four oxen] [Dieshe looks at the pictures thoughtfully] T3: Why? Jynita: The oxen are going to win. T3: What were you about to say? [addressing Dieshe] Dieshe: The elephant, the, okay, this is what I am saying. You said that, okay four oxen is equal to five horses, right? But is only three horses over there [pointing to the last picture]. But an elephant is equal to an ox and two horses. So we have three [pointing to horses in the last picture], four, five [adding the horses in the middle picture] that is five horses and another ox. T3: See, why don't you write that down? Just what you said so you won't forget it. Why don't you write it down?
Description of the event	The students have been working on the "Tug-of-War" problem. Jynita and her partner Dieshe have arrived at the wrong conclusion that the four oxen will win the tug-of-war. In this episode, Jynita explains their answer to the teacher who starts questioning their reasoning. Eventually Dieshe figures out the correct answer.
Teacher's Actions(s)/ Response(s)	1) The teacher questions Jynita and Deisha about their reasoning that the four oxen will win the tug-of-war [line 6; lines 8 and 9; line 11; lines 13 and 14; lines 18 and 19; lines 24 and 25; line 27 and 28; line 32 and 33; line 35; line37 and 38; line 41; line 46; line 50; line 52]. Code: Eliciting a response – Name/State (E-NS) Code: Eliciting a response – Describe/Explain (E-DE) 2) The teacher encourages Keisha to write her explanation down. Code: Encouraging student to write down explanations (EWE)
Students' Action(s)/ Response(s)	1) Dieshe eventually realizes her mistake and come up with the correct answer [lines 44 and 45; lines 53-57]. Code: Sharing solution/idea with teacher (SST) Code: Making a self-correction (MSC)

Appendix F11: Lesson #3- Transcribed and coded Critical Event 11

Event 11 **Role T3-1** **Teacher: T3** **Students: Jynita and Dieshe**		
Transcript of Event	**Line**	
	1	[It is about 43 minutes into the lesson.
	2	Dieshe is trying very hard to convince Jynita that the elephant and three
Time (as	3	horses will win the tug-of-war. Jynita is adamant about not accepting
shown on	4	Dieshe's reasoning.]
RoleT3-1):	5	Dieshe: Okay look, there is four oxes and five horses. Right? Right?
(00:42:40-	6	Jynita: Yeah, but where are you getting five horses and an ox?
00:46:32)	7	Dieshe: An elephant is equal to …..
	8	Jynita: But is that in the picture?
	9	Dieshe: This here says an elephant is as strong as an ox and two horses
	10	[pointing to the middle picture]. So if you take away the elephant and replace
	11	it with…
	12	Jynita: But, does it say anything about taking an elephant away?
	13	Dieshe: But it is a replacement cause it is equal to that.
	14	Jynita: How are you replacing the elephant? It says the picture below. It
	15	doesn't say anything about taking away.
	16	Dieshe: It is like replacement, it is the same exact, yeah, it is. [Jynita is
	17	shaking her head dismissively]
	18	Jynita: I don't get you Dieshe.
	19	[They stop talking and each girl starts writing on her paper. The teacher
	20	approaches the girls and sits at their table]
	21	T3: Okay, okay. What did you say?
	22	Dieshe: I said there are four oxen and five horses and the elephant is equal to
	23	an ox and two horses, right? If you replace the elephant and just put the two
	24	horses and ox down here that is five horses and an ox. And the oxes can only
	25	pull five horses so if you add another ox it means they will win.
	26	T3: Okay, and what do you say? [addressing Jynita]
	27	Jynita: I think the oxes are going to win.
	28	T3: Why?
	29	Jynita: Because they look like they are going to win.
	30	T3: They look like they are going to win?
	31	T3: What is it that you know about the oxen?
	32	Jynita: That they can pull five horses.
	33	T3: Okay, so here you only have three. Are you convinced because of that?
	34	Jynita: Uhum
	35	T3: What is it that you know about an elephant?
	36	Jynita: An elephant is equal to one ox and two horses, but…
	37	T3: Okay, so let's do that….
	38	Dieshe: See look, see look, this is what I am saying. You see how on this side
	39	there is an elephant….

	Jynita: How are you going to stop me when I am in the middle of explaining? [T3 is asked a question by one of the observers and she has a brief conversation with him] T3: Okay, keep going. Jynita: I think it is the oxen (winning) because if elephant can pull an oxen and two horses, they can't pull four oxen, wait,.. Dieshe; Want me to explain one more time? Jynita: No, because I am talking Dieshe: Jynita: If oxes can pull five horses, I am sure they can pull three horses. And if one elephant can pull an ox and two horses, then it is basically not going to be able to pull all four oxes. Because oxen weighs more than a horse. T3: Did you say, well I don't know, I don't know. Jynita: But if… T3: Let's look at here. What can you tell me about an elephant, an ox and a horse? Jynita: Well, oxen and two horses equal one elephant. T3: Okay, so what can you say about this here? [pointing to the elephant in the bottom picture]. So if that is the case what is this over here? Diesha: it is one elephant, the same as two horses and an ox. Jynita: What? [looking hard at the paper] Dieshe: You take the elephant right here [pointing to the middle picture] and the elephant right here [pointing to the bottom picture]. So the elephant is just the same as two horses and an ox. What is this elephant equal to? Two horses and an ox. And then, four oxes [pointing to the bottom picture], four oxes [pointing to the top picture], but there were five horses [pointing to the top picture], and there is one, two, three [pointing to the three horses in the bottom picture], four, five [pointing to the two horses in the middle picture] and an ox. Jynita: But how do you get five horses if it is not in the picture? Dieshe: This is equal to this, right? [pointing to the two elephants] And this is equal to this [pointing to the two sides in the middle picture]. Right? So if you add this and this… T3: Are you convinced by her or is she convinced by you? Jynita: I, [pauses] I don, [pauses, smiles, keeps looking at and playing with a piece of string in her hands] I don't want her answer! T3: Okay, then you come up with you own answer. I'm going to give you another chance to answer [T3 walks away]
Description of the event	The students have been working on the "Tug-of-War" problem. In this episode, Dieshe tries very hard to convince Jynita that the elephant and three horses will win the tug-of-war. Jynita however, is adamant about not accepting Dieshe's reasoning and is sticking with her initial belief that the oxen will win. Finally, when it becomes apparent that Jynita no longer believes her solution to be correct, she still refuses to "adopt" Dieshe's answer.

Teacher's Actions(s)/ Response(s)	1) The teacher asks Jynita to explain her reasoning [line21; line 26; line 28; line 43]. Code: Eliciting a response – Describe/Explain (E-DE) 2) The teacher requests a relatively short response from Jynita [line 31; lines 54 and 55; lines 57 and 58; line73]. Code: Eliciting a response – Name/State (E-NS) 3) The teacher requests a simple yes or no as a response from Jynita [line 33]. Code: Eliciting a response –Yes/No (E-YN) 4) When Jynita rejects Dieshe's reasoning, the teacher leaves her to formulate and come up with her own explanation [lines 76 and 77]. Code: Supporting student's autonomy (SSA)
Students' Action(s)/ Response(s)	1) Dieshe tries to convince Jynita that the elephant and three horses will win the tug-of-war. Jynita however, believes that the oxen will win [lines 5-18; lines 38 and 39; lines 59-72]. Code: Engaged in mathematical discourse with peers (MDP) Code: Justifying solution (JS) Code: Questioning the reasoning of others (QRO) Code: Displaying lack of understanding (DLU) 2) Dieshe and Jynita respond to the teacher's questions and explain their reasoning [lines 22-25; line 27; line 29; line 32; line 34; line 36; lines 44 and 45; lines 49-51; line 56]. Code: Answering a question (AQ) Code: Sharing solution/idea with teacher(SST) Code: Justifying solution (JS) 3) Jynita refuses to "adopt" Keishe's answer [lines 74 and 75]. Code: Answering a question (AQ) Code: Reluctant to accept help from peers (RHP)

Appendix F12: Lesson #3- Transcribed and coded Critical Event 12

Event 12 **Role T3-1** **Teacher: T3** **Students: Keith**		
Transcript of Event	**Line**	
	1	[It is 48 minutes into the lesson.
	2	Keith explains to the teacher his wrong strategy in solving the “Tug-of-War”
Time (as shown on RoleT3-1): (00:48:00-00:50:31)	3	problem and how with guidance from R1 he was able to modify his work.]
	4	Keith: before your professor helped me out I had one, two, three, four (oxen)
	5	against these four (horses) and I took the other horse out because I thought
	6	that if two horses were equal to one ox, it would work, but I was stuck and I
	7	took the extra horse out because I was trying to equal it out, it would not work
	8	for half a horse.
	9	T3: Okay.
	10	Keith: So then I progressed with that strategy. Then I came to every, for every
	11	one ox it will equal two horses and I will get three, this will be 3 [pointing to
	12	the elephant in the middle picture] I guess, and then, or 4. So then I tried 4
	13	plus 1, plus 1, plus 1 [pointing to the animals on the left hand side in the
	14	bottom picture] and I got 7, the first time and then I tries 2 and 2 and I got 4
	15	[oxen on the right] and I said it would be the left side (to win). But first I
	16	started equal, cause it looked the same, but then I had to think about it.
	17	T3: Okay, because of the essence of time and I really want you to share…
	18	R1: And then what did you do? [addressing Keith]
	19	Keith: Then the teacher helped me.
	20	R1: You are the one who did it.
	21	Keith: Then I noticed that if four go into five I’ll have 20. So I said this would
	22	be 5, 5, 5, 5 [pointing to oxen] and this would be 4, all these (five horses)
	23	would be 4. So I will have one ox [pointing to the middle picture], it will be 5
	24	plus 4 and another 4 (the two horses) and I will have 13 plus. Now, moving
	25	on to the bottom picture, since I have the elephant that is 13, then I have three
	26	horses, that would be 16 and I would have 25. Wait, yeah I’ll have 25. Twelve
	27	and 13 would be 25. Then I have four 5’s is 20 (oxen), so still the left side
	28	will be more than the four oxen put together.
	28	R1: Why is it that these four, that you gave the 5 to the oxen and the 4 to the
	30	horse?
	31	Keith : Because 5 times four equals 20 and 4 times five equal 20
	32	T3: So you made sure they were balanced.
Description of the event		Keith had been working on the “Tug-of-War” problem by himself. Previously, he had shared (off camera) his strategy with R1 in which he had decided to assign the number 1 to each horse and the number 2 to each ox. Because he did not think it possible to have a fraction of a horse, Kevin had eliminated one of the horses by crossing it out in the first picture. Coincidentally and through faulty reasoning, he had arrived at the correct

	answer that the animals on the left hand side would win the tug of war. R1 had brought several points to his attention: a) by crossing out a horse he had changed the question and created a whole new one; b) he had not used the assigned numbers consistently (Ox = 2 in the middle picture, Ox = 1 in the bottom picture); c) his assigned numbers (1 for horse, 2 for ox) did not create a balance in the first picture, with or without the crossed out horse. With a focus on maintaining the balance in the top picture, R1 encourages Keith to think of assigning different numbers to the ox and the horse. In this episode Keith explains to T3 his initial strategy and the subsequent steps that led to his final answer.
Teacher's Actions(s)/ Response(s)	1) The researcher, R1, asks Keith to explain his reasoning [line 18]. Code: Eliciting a response – Describe/Explain (E-DE) 2) The researcher, R1, questions Keith about his choice of numbers assigned to each animal [lines 30 and 31]. Code: Eliciting a response – Name/State (E-NS).
Students' Action(s)/ Response(s)	1) Keith explains his reasoning to the teacher and the researcher, R1. Code: Sharing solution/idea with teacher (SST) Code: Justifying solution (JS) Code: Answering a question (AQ)

Appendix F13: Lesson #3- Transcribed and coded Critical Event 13

<u>Event 13</u> **Role T3-1** **Teacher: T3** **Students: Theo, Dieshe, and Kevin**	
Transcript of Event **Time (as shown on RoleT3-1): (00:56:14-00:57:31)**	[It is 56 minutes into the lesson. Theo, at the overhead, explains how he came to the (wrong) conclusion that the oxen would win in the “Tug-of-War” problem.] T3: Say that, say that again. This time demonstrate it [gives Theo some markers]. Keith: Excuse me Miss! T3: Yes? You are ready? Okay, you can go right after Theo. Go ahead hon [addressing Theo]. Theo: Right here [underlines the horses and ox in the middle picture] it says that this ox and these two horses are as strong as this elephant. So I took out this ox and these two horses [crossing out the animals from the opposite sides in the bottom picture] because they make up one elephant. Now there is three oxes and one elephant and one horse. So I did, since two horses can make up almost an ox, I said two oxes to take one elephant and the other ox is stronger than the horse. So the oxes [on the right] are stronger than this group [on the left]. T3: Do you have any question? [Some hands go up and some students start talking.] T3: Wait, wait, wait. [unclear] Jynita? Dieshe: I wonder how did you assume that two horses is equal to an ox? Theo: I said that because up here [pointing to the top picture] four ox, four oxes is equal to five horses. Kevin: But it should be six horses, two each of the oxes. Dieshe: yeah, and it would be half. Kevin: But you can’t split the horses because it is uneven.
Description of the event	Theo has been presenting his solution to the “Tug-of-War” problem at the overhead. First he placed his written answer on the overhead and read it. He was asked to read it a second time, but it was difficult to follow the logic of his explanation, which concluded (wrongly) that the four oxen are stronger than one elephant and three horses. T3 then displayed the transparency of the original problem on the overhead and asks Theo to use that to explain his thinking more clearly. In this episode, Theo refers to the second picture and based on the fact that one elephant is as strong as one ox and two horses, he crosses out one ox and two horses in the third picture. He also goes on to say that one elephant is almost as strong as two oxen and an ox is as strong as two horses. Some students are not convinced and raise their hands to ask questions. Dieshe and Kevin question Theo’s reasoning.

Teacher's Actions(s)/ Response(s)	1) The teacher asks Theo to use a pen at the overhead in order to clearly demonstrate his explanation [line 3 and 4]. Code: Directing students to perform some mental or physical activity (DA) 2) Following Theo's presentation to the class, the teacher asks if anyone has any questions for Theo [line 17]. Code: Providing opportunity to critique other's reasoning (OCR) Code: Encouraging classroom discourse (ECD)
Students' Action(s)/ Response(s)	1) Theo explains his reasoning to the class [lines 9-16]. Code: Sharing solution/idea with class (SSC) Code: Displaying lack of understanding (DLU) 2) Dieshe and Kevin question Theo's reasoning [line 20; line 23-25]. Code: Questioning the reasoning of others 3) Theo justifies his reasoning [line 21 and 22]. Code: Justifying solution (JS) Code: Answering a question (AQ)

Appendix F14: Lesson #3- Transcribed and coded Critical Event 14

Role T3-1 **Teacher: T3** **Students: Kevin**		**<u>Event 14</u>**
Transcript of Event **Time (as shown on RoleT3-1): (00:58:52-00:59:15)**	**Line** 1 2 3 4 5 6 7 8 9	[It is 59 minutes into the lesson. Kevin explains why Theo's claim that two horses are as strong as one ox, is not correct.] T3: Okay, So how different is Theo's than yours? Kevin: Because he is trying to put two horses equal to one ox. But he couldn't because if there was two split, those two split, like this [in the top picture, circles two horses, connects to one ox] and this [circles and connects another two horses to a second ox], then you can't [pointing to the single horse remaining]. There are only five horses.
Description of the event		Following Theo's presentation, Kevin has presented his solution to the "Tug-of-War" problem at the overhead. He has used the substitution method and his explanation is identical to Jynita's. In this episode, K3 asks Kevin to explain why he thinks Theo's explanation was incorrect. Kevin reject Theo's assertion that one ox is equal to two horses and justifies it by circling horses in groups of two and connecting each circle to an ox in the first picture. This demonstration shows that there are not enough horses in the picture to support Theo'a claim.
Teacher's Actions(s)/ Response(s)		1) The teacher encourages Kevin to compare his solution to Theo's [line 4]. Code: Providing opportunity to critique other's reasoning (OCP) Code: Encouraging classroom discourse (ECD) Code: Eliciting a response – Describe/Explain (E-DE)
Students' Action(s)/ Response(s)		1) Kevin explains why Theo's reasoning is flawed [lines 5-9]. Code: Answering a question (AQ) Code: Questioning the reasoning of others (QRO) Code: Justifying solution (JS)

Appendix F15: Lesson #3- Transcribed and coded Critical Event 15

<u>Event 15</u> **Role T3-2** **Teacher: T3** **Students: Tyreal, Tryshon, Kevin**		
Transcript of Event	**Line**	
	1	[It is 62 minutes into the lesson.
	2	Tyreal presents his solution to the "Tug-of-War" problem at the overhead.]
Time (as shown on RoleT3-2): (00:02:08-00:04:25)	3	T3: Go ahead.
	4	Tyreal: What I did was similar to how Keith did, but I did different numbers
	5	and I used decimals.
	6	[A couple of girls ask him to move the transparency up]
	7	T3: Go ahead hon.
	8	Tyreal: Since there are five horses and four oxen, that means each ox is equal
	9	up to at least one of the horses. And since each, if you take off the last horse
	10	that means they would, they would be equal (in number; four on each side).
	11	Kevin: Oh, so you are splitting a horse?
	12	Tyreal: Yeah, so it would be one point, would be one and one fourth (each ox)
	13	So then [pointing to the middle picture], if you write 1 ¼ for one ox and one
	14	horse is 1, so the elephant will have to equal to 3 ¼. But then [pointing to the
	15	bottom picture] all you have to do is add them all up. You get 6 ¼ (here
	16	pronounced as six and a quarter) [pointing to the left side] and 5 [pointing to
	17	the right side].
	18	Tryshon: Ooooh I get you. [snapping her fingers and swaying]
	19	Kevin: Which side won? Which side won?
	20	Tyreal: That one [pointing to the left side]
	21	Kevin: Yeah, but yours is mad difficult.
	22	[students laughing as Tyreal starts to go back to his desk]
	23	T3: Wait, wait wait Tyreal. Basically what could you tell me about, what sign
	24	could you put here to show the relationship between the two [pointing to the
	25	two sides in the bottom picture]
	26	Some students: Greater.
	27	T3: Write it.
	28	Tyreal: Which way is it going? [asking the class]
	29	Some Students: It is going to the right; It's going that way [signaling]; Then
	30	go for the alligator; Open mouth that way.
	31	[Tyreal places the sign correctly to indicate the winner.]
	32	T3: Is there anyone who wants to share but didn't get the opportunity to
	33	share?
	34	Keith: Oh, wait. I have a question. [hand up]
	35	T3: Yes, Mr…, Keith.
	36	Keith: How can you get a decimal of an animal? It won't work, because it
	37	will be dead!
	38	Tyreal: Because I am not splitting the animal, I am splitting how…strength
	39	[not audible; kids laughing]

	40 41 42 43 44	T3: What did you say? What did you say? What did you say exactly? What did you say? What are we measuring? Are we measuring the animal or what are we measuring Tyreal? Tyreal: Strength. T3: Strength. We are measuring the strength.
Description of the event		In this episode, Tyreal presents his solution to the “Tug-of-War” problem at the overhead. He acknowledges that his solution strategy is similar to Keith’s in terms of structure except that he has assigned fractional numbers to the oxen as oppose to the whole numbers used by Keith. Kevin thinks Tyreal’s solution method is hard to understand. Keith questions the choice of assigning fractional numbers to animals, which in his mind, implies splitting the animals and therefore killing them. Tyreal replies that he is splitting the strength of an animal and not the animal itself.
Teacher’s Actions(s)/ Response(s)		1) The teacher asks Tyreal to present his solution to the class [line 3; line 7]. Code: Eliciting a response – Describe/Explain (E-DE) Code: Encouraging classroom discourse (ECD) 2) The teacher asks Tyreal to name the mathematical symbol that describes the relationship between the two pictures [line 23-25]. Code: Eliciting a response – Name/State (E-NS) 3) The teacher instructs Tyreal to place the equal sign between the two pictures [line 27]. Code: Directing students to perform some mental or physical activity (DA) 4) The teacher asks Tyreal to repeat his response to Keith’s question [lines 40-42]. Code: Eliciting a response – Name/State (E-NS) Code: Rewording a question (RQ) 5) The teacher repeats Tyreal’s one-worded response and presents it in a complete sentence [line 44]. Code: Paraphrasing student’s response (PSR)
Students’ Action(s)/ Response(s)		1) Tyreal compares his solution strategy to Keith’s solution method [line 4 and 5].

	Code: Comparing solution strategies/ideas (CS) 2) Tyreal presents his solution to the class [lines 8-10; lines 12-17]. Code: Sharing solution/idea with class (SSC) 3) Kevin comments on Tyreal's solution strategy [line 21]. Code: Comparing solution strategies/ideas (CS) 4) When the teacher asks Tyreal to identify and place the mathematical sign which describes the relationship between the two pictures, he turns to his classmates for help [lines 28-31]. Code: Asking for help (AH) Code: Accepting help from peers (AHP) 5) Kevin questions Tyreal's reasoning [lines 36 and 37]. Code: Questioning the reasoning of others (QRO) 6) Tyreal justifies his reasoning [line 38; line 43]. Code: Answering a question (AQ) Code: Justifying solution (JS)

Appendix F16: Lesson #3- Summary table of Critical Events

Teacher: T3	
Event / Time	**Description**
Event 1 Time (as shown on RoleT3-1): 00:12:10 - 00:15:02	In this episode T3 asks Jynita and Dieshe a question about the problem and the answer she gets is true, but irrelevant to the problem and far from what she expected. She decides to review some prior knowledge with the whole class after which, she reposes the same re-worded question and gets a reasonable answer from Jynita.
Event 2 Time (as shown on RoleT3-1): 00:15:18 - 00:16:35	In this episode Tyreal presents his solution to the "Carrots" problem at the overhead. He has assigned the number 1.5 to each carrot and has correctly decided that two carrots should be placed on the last scale. Solution Path: Not identified previously
Event 3 Time (as shown on RoleT3-1): 00:21:44 - 00:22:15	In this episode Jynita explains her solution to the "Bananas" problem. She claims to have used Tyreal strategy of assigning numbers to items. Solution Path: 1B, - Modified (rather than defining a pineapple in terms of five bananas, she has assigned the number 1 to each banana and subsequently has assigned the number 5 to each pineapple.)
Event 4 Time (as shown on RoleT3-1): 00:23:45 - 00:25:35	In this episode the teacher introduces the modified "Shorts and Glasses" problem on the overhead and is surprised when Kevin immediately identifies the 2:1 relationship between the glasses and the shorts and asserts that the glasses are more expensive than the shorts. Solution path: Not clear – Possibly not identified previously
Event 5 Time (as shown on RoleT3-1): 00:26:18 - 00:26:51	A few minutes earlier it had been established, through a class discussion, that the glasses are more expensive than the shorts and one student had said that the glasses are twice as expensive as the shorts (see event 4). The students are now trying to find the prices for each item. In this episode, Theo calls the teacher over to his desk and explains how he found out the prices by dividing 50 (total price) by 4 (number of items), concluding that the shorts and glasses cost the same ($12.50). The teacher asks Theo if it is possible for the prices to be the same when we know that the glasses are more expensive. The teacher walks away.

Event 6 Time (as shown on RoleT3-1): 00:30:55 - 00:33:03	In this episode, Devon is explaining at the overhead how he and his partner, Chad, used the guess and check method to find the prices for shorts and glasses. Even though, it was previously established in class that the two prices had a 2:1 relationship, the boys' choice of initial numbers ($17 and $16) does not reflect that understanding. Also T3 illustrates the meaning of the equal sign by placing one between the two pictures, indicating the two equal price tags. Solution Path: 3C
Event 7 Time (as shown on RoleT3-1): 00:33:05 - 00:34:12	In this episode, T3 shows the class a picture which was taken during the tug-of-war activity on field day. This was supposed to spike the students' interest and serve as an introduction to the "Tug-of-War" problem. Some students were able to make a connection between tug-of-war and the pan balance.
Event 8 Time (as shown on RoleT3-1): 00:36:45 - 00:37:27	In this episode, Keith begins to explain to the teacher why he thinks the elephant and the horses will win the tug-of war. The teacher asks Keith to put his thinking down on paper. Keith follows the teacher's direction after protesting that it is too difficult.
Event 9 Time (as shown on RoleT3-1): 00:37:46 - 00:38:50	In this episode, Kevin wants to share his answer to the "Tug-of-War" problem with the teacher. However the teacher insists that he writes down his thinking and use his writing in order to justify his answer.
Event 10 Time (as shown on RoleT3-1): 00:39:27 - 00:42:18	The students have been working on the "Tug-of-War" problem. Jynita and her partner Dieshe have arrived at the wrong conclusion that the four oxen will win the tug-of-war. In this episode, Jynita explains their answer to the teacher who starts questioning their reasoning. Eventually Dieshe figures out the correct answer. Solution Path: 5B
Event 11 Time (as shown on RoleT3-1): 00:42:40 - 00:46:32	The students have been working on the "Tug-of-War" problem. In this episode, Dieshe tries very hard to convince Jynita that the elephant and three horses will win the tug-of-war. Jynita, however, is adamant about not accepting Dieshe's reasoning and is sticking with her initial belief that the oxen will win. Finally, when it becomes apparent that Jynita no longer believes her solution to be correct, she still refuses to "adopt" Dieshe's answer.

	Solution path: 5B
Event 12 Time (as shown on RoleT3-1): 00:48:00 - 00:50:31	Keith had been working on the “Tug-of-War” problem by himself. Previously, he had shared (off camera) his strategy with R1 in which he had decided to assign the number 1 to each horse and the number 2 to each ox. Because he did not think it possible to have a fraction of a horse, Kevin had eliminated one of the horses by crossing it out in the first picture. Coincidentally and through wrong reasoning, he had arrived at the correct answer that the animals on the left hand side would win the tug of war. R1 had brought several points to his attention: a) by crossing out a horse he had changed the question and created a whole new one; b) he had not used the assigned numbers consistently (Ox = 2 in the middle picture, Ox = 1 in the bottom picture); c) his assigned numbers (1 for horse, 2 for ox) did not create a balance in the first picture, with or without the crossed out horse. With a focus on maintaining the balance in the top picture, R1 encourages Keith to think of assigning different numbers to the ox and the horse. In this episode Keith explains to T3 his initial strategy and the subsequent steps that led to his final answer. Solution Path: not previously identified
Event 13 Time (as shown on RoleT3-1): 00:56:14 - 00:57:31	Theo has been presenting his solution to the “Tug-of-War” problem at the overhead. First he placed his written answer on the overhead and read it. He was asked to read it a second time, but it was difficult to follow the logic of his explanation, which concluded (wrongly) that the four oxen are stronger than one elephant and three horses. T3 then displayed the transparency of the original problem on the overhead and asks Theo to use that to explain his thinking more clearly. In this episode, Theo refers to the second picture and based on the fact that one elephant is as strong as one ox and two horses, he crosses out one ox and two horses in the third picture. He also goes on to say that one elephant is almost as strong as two oxen and an ox is as strong as two horses. Some students are not convinced and raise their hands to ask questions. Jynita and Kevin question Theo’s reasoning.
Event 14	Following Theo’s presentation, Kevin presented his solution to the “Tug-of-War” problem at the overhead. He

Time (as shown on RoleT3-1): 00:58:52 - 00:59:15	had used the substitution method and his explanation was identical to Jynita's. In this episode, K3 asks Kevin to explain why he thinks Theo's explanation was incorrect. Kevin reject Theo's assertion that one ox is equal to two horses and justifies it by circling horses in groups of two and connecting each circle to an ox in the first picture. This demonstration shows that there are not enough horses in the picture to support Theo'a claim.
Event 15 Time (as shown on RoleT3-2): 00:02:08 - 00:04:25	In this episode, Tyreal presents his solution to the "Tug-of-War" problem at the overhead. He acknowledges that his solution strategy is similar to Keith's in terms of structure except that he has assigned fractional numbers to the oxen as oppose to the whole numbers used by Keith. Kevin thinks Tyreal's solution method is hard to understand. Keith questions the choice of assigning fractional numbers to animals, which in his mind, implies splitting the animals and therefore killing them. Tyreal replies that he is splitting the strength of an animal and not the animal itself. Solution Path: not previously identified

Appendix G: Transcribed and Coded Critical Events

for Lesson #4

Appendix G1: Lesson #4- Transcribed and coded Critical Event 1

Event 1 **Role T4-1** **Teacher: T4** **Students: D’nea and Oscar**		
Transcript	**Line**	
of Event	1	[It is about 19 minutes into the lesson.
	2	The students have been working on the “Bananas” problem. The teacher asks
Time (as	3	D’nea and partner to talk about their solution. D’nea begins the explanation,
shown on	4	but when she is asked to be more specific, she smiles shyly and refuses to
RoleT4-1):	5	talk. Oscar steps in and gives his explanation]
(00:19:00-	6	T4: Alright, let’s see. Who likes to start, who likes to share first? [pause]
00:22:12)	7	D’nea, can you tell me about something you worked on?
	8	D’nea: Ten, ten, ten bananas. Take away five.
	9	T4: Take away five what? Chickens?
	10	D’nea: No, five bananas [smiles and looks over at her partner, Destiny].
	11	T4: Oh [pause], keep going [pause]. You took away five bananas.
	12	D’nea: Yeah.
	13	T4: You want to tell me why you took away five bananas?
	14	D’nea: Because [pause].
	15	T4: Your partner is right next to you.
	16	[D’nea looks at Destiny and giggles. Destiny is busy writing something down
	17	and does not look up.]
	18	T4: You remember why you took five bananas?
	19	[After a long pause D’nea smiles and shakes her head. Destiny looks up and
	20	smiles.]
	21	T4: So why did you guys take away five bananas? You are trying to
	22	remember or process it?
	23	[Destiny nods.]
	24	T4: You want to finish and write it down and I come back?
	25	[Destiny nods.]
	26	T4: Okay. Alright, Oscar?
	27	Oscar: We [unclear] with them. We took away five bananas because to
	28	measure how much a pineapple weighs.
	28	[T4 looks over at some students and signals them to be quiet.]
	30	T4: Because how can we say whether we agree with them or not, or anything
	31	if we don’t hear them? Thank you. Go ahead Oscar. I am sorry.
	32	Oscar: We did that because we divided 10 into 2, which gave us 5. And then
	33	we knew that take away 5 will be how much a pineapple weighs as much as
	34	five bananas.
	35	T4: where did you get 10 from and where did you get divided by 2?
	36	Oscar: Because we counted how much bananas were on one side and we
	37	counted how much pineapple were on the other side.
	38	T4: What do you mean by how much bananas?
	39	Oscar: The amount.

	T4: Okay, so how many bananas do you have? Oscar: ten. T4: Okay, and then [pause]. Oscar: Then we divided it by 2. T4: And why did you divide it by 2? Oscar: To see how much one pineapple weighs. T4: Okay. Oscar: And which gave us 5. And the next scale says one pineapple equal as much as two bananas and an apple. And since we know that a pineapple weighs as much as five bananas, we said, then we tried to figure out what number adds, goes equally with 2. And then we got 2 plus 3 which equals to 5 and then we knew an apple equals three pounds, three bananas. T4: Do you have all that down on your paper? [Oscar nods.] T4: Okay, good.
Description of the event	In this episode the teacher asks D’nea and her partner to talk about their solution to the “Bananas” problem. D’nea begins the explanation, but when she is asked to be more specific, and to justify her solution, she smiles shyly and refuses to talk. The teacher gives D’nea and her partner, Destiny, ample time to speak up, and when they remain silent, she asks them to put their thoughts in writing and report later. Oscar steps in and gives his explanation.
Teacher’s Actions(s)/ Response(s)	1) The teacher asks D’nea to share her findings with class [lines 6 and 7]. Code: Encouraging classroom discourse (ECD) Code: Eliciting a response – Describe/Explain (E-DE) 2) The teacher wants D’nea to be more specific with her explanation and encourages her to continue with her explanation [line 9; line 11; line 13; line 18; lines 21 and 22]. Code: Eliciting a response – Name/State (E-NS) 3) The teacher Gives D’nea time to formulate her answer and write it down [line 24]. Code: Encouraging student to write down explanations (EWE) Code: Supporting student’s autonomy (SSA) 4) The teacher asks Oscar to explain his reasoning [line 26; line 35; line 42]. Code: Eliciting a response – Describe/Explain (E-DE) Code: Encouraging classroom discourse (ECD)

	5) The teacher asks Oscar specific questions about his solution strategy [line 35; line 38; line 40; line 44]. Code: Eliciting a response – Name/State (E-NS) 6) The teacher ensures that Oscar has written his explanation down [lines 52-54]. Code: Encouraging student to write down explanations (EWE)
Students' Action(s)/ Response(s)	1) D'nea attempts to explain her reasoning [line 8; line 10]. Code: Sharing solution/idea with class (SSC) Code: Answering a question (AQ) 2) Oscar explains his reasoning [lines 27 and 28; lines 32-24 lines 36 and 37; line 39; line 41; line 43; line 45; lines 47-51]. Code Sharing solution/idea with class (SSC) Code: Answering a question (AQ) Code: Justifying solution (JS)

Appendix G2: Lesson #4- Transcribed and coded Critical Event 2

<u>Event 2</u> **Role T4-1** **Teacher: T4** **Students: Kieshe and Nazeer**	
Transcript of Event **Time (as shown on RoleT4-1): (00:22:17-00:24:56)**	**Line** [It is about 22 minutes into the lesson. Kieshe and her partner, Arlin, have been working on the "Banana" problem and believe the correct answer to be two bananas. Kieshe explains her reasoning, which is confusing and does not make much sense.] T4: Kieshe looks like she is ready to share. Go ahead Kieshe. Kieshe: It's 10 bananas in all because we counted it on the first scale. So we took away two and put it, put it in the third scale. Because one apple weighs 3 pounds and 2 ounces because, [pause] oh, and 2 pounds, I mean 2 ounces and a banana weighs 1 pound [pause], 2 pounds and 3 ounces [pause]. Two bananas because an apple weighs 3 pounds, 2 ounces because, because I counted all the bananas on the first one [pause]. A banana, the banana has 10 and two pineapples. So if you take away two bananas of the first one, it'll be eight in all and the third scale will have one apple and two bananas because two pineapples weigh more than one apple. So therefore [pause], so therefore the first scale I left eight and the last scale which is the third, the third scale there is two bananas. T4: Okay. Because you said a lot and I am trying to process what you were saying. Are you saying that you took two bananas off the first scale? Kieshe: Yes. T4: Okay. Did you do anything to the pineapples? Kieshe: No. T4: Okay. So if you take two bananas off and you leave eight bananas there and the two pineapples, does it, does the scale stay the same? Kieshe: Yes [Arlin shakes her head, no]. T4: Do you agree? Nazeer, do you agree? T4: She said she took two bananas off the first scale. She had eight bananas on one side of the scale and two pineapples on the other side. She said the scale won't change. Do you agree? Nazeer: No. T4: Why not? Tell why not. Nazeer: Because 10 bananas weigh as much as two pineapples and you take off two of the bananas, then the pineapples are going to, pineapples are going to weigh more. Kieshe: I get it. T4: Do you get it? Alright.
Description of the event	In this episode Kieshe explains her reasoning as to why one apple is equal to two bananas in weight. Her explanation is confusing and does not make much

	sense. The teacher asks her some questions that reveal Kieshe's lack of understanding of the notion of equivalence on a balanced scale. Nazeer is asked to explain why her method is incorrect.
Teacher's Actions(s)/ Response(s)	1) The teacher asks Kieshe to share her findings with class [line 5]. Code: Eliciting a response – Describe/Explain (E-DE) Code: Encouraging classroom discourse (ECD) 2) The teacher asks Kieshe questions in order to better understand her reasoning [lines 17 and 18; line 22; lines 22 and 23]. Code: Eliciting a response –Yes/No (E-YN) Code: Eliciting a response – Name/State (E-NS) 3) The teacher paraphrases Kieshe's response and asks Nazeer if he agrees with Kieshe's response [lines 25-28]. Code: Providing opportunity to critique other's reasoning (OCR) Code: Encouraging classroom discourse (ECD) Code: Paraphrasing student's response (PSR) 5) The teacher asks Nazeer to explain why he disagrees with Kieshe [line 30]. Code: Providing opportunity to critique other's reasoning (OCR) Code: Eliciting a response – Describe/Explain (E-DE)
Students' Action(s)/ Response(s)	1) Kieshe explains how she and her partner arrived at their answer to the Banana problem [lines 6-16]. Code: Sharing solution/idea with class (SSC) Code: Displaying lack of understanding (DLU) 2) Nazeer explains why he disagrees with Kieshe's answer [line 28, lines 31-33]. Code: Questioning the reasoning of others (QRO) Code: Correcting another student (CAS) Code: Sharing solution/idea with class (SSC) 3) After hearing Nazeer's reasoning, Kieshe claims to have understood the folly of her earlier assertion [line 34]. Code: Agreeing with the reasoning of others (ARO)

Appendix G3: Lesson #4- Transcribed and coded Critical Event 3

Event 3 **Role T4-1** **Teacher: T4** **Students: Nazeer and Jarrod**	
Transcript of Event **Time (as shown on RoleT4-1): (00:39:40-00:45:50)**	[It is about 40 minutes into the lesson. Nazeer and Jarrod struggle to explain their solution to the “Carrots” problem. With some help from the teacher, Nazeer finally comes up with the correct answer.] Jarrod: Well. Well, me and Nazeer said that, uhm, uhm the first scale which has six carrots and a corn, uhm and a pepper equals uhm,…[pause] [Nazeer whispers something to Jarrod, who then pushes the paper toward Nazeer]. Nazeer: Three peppers equal six carrots, because on the second, on the second, uhm scale we have one corn and two peppers and on the first scale we have one corn and one pepper. So we, uhm replaced the one corn with two peppers in [pause]. T4: Where? Nazeer: We replaced the two peppers from the second scale and put it on, replaced it with the corn on the first scale. And so, three peppers equal the same amount as six carrots. T4: Does everyone hear what he saying? He is saying he took the ear of corn from here (1st scale), okay? And put it over here (2nd scale). And took these two peppers and put them here (1st scale). So now he has a corn equals a corn (2nd scale) and he has three pepper equals six carrots. Is that what you just said Nazeer? Nazeer: Yes. T4: Okay. Everybody see that? Does that help you Dajuan? Does that help you? Some students: Yes. Dajuan: Yes that helps. T4: That helps you a little bit? T4: Okay. Keep going [addressing Nazeer]. Nazeer: One ear of corn equals three carrots. Because if we took off half the carrots and one pepper, it would be the same, it would be the same weight because..Help me out Jarrod. Jarrod: Uhm, uhm. Because if you uhm, take half of the uhm, six carrots and you took away three carrots and you took away a chilli pepper, they both equal to, uhm, uhm, same amount as, as, as three carrots [pause]. Nazeer: Oh, I get it now. Three carrots is the same amount as two corns, I mean two peppers. Because one corn equals two peppers and if we take away both the corn and one pepper, if we take away a corn and add one pepper, we would have the same amount as three carrots. And, and then, ugh… [pause], no that’s because uhm, one corn is two peppers and two peppers equals three

	carrots because [long pause]. T4: Okay. Does this help you out a little bit? [Camera shows a diagram that T4 has drawn on the whiteboard of a scale with six carrots on one pan and three peppers on the other pan.] T4: Because it seems to me like you are trying to visualize something but you didn’t change it on your paper. So you are trying to remember and you are trying to say what you are trying to do. Here is what you said. You said you took, you took the two peppers and you put them here, and you took the ear of corn and put it here [pointing to the pictures on the paper]. Okay? So we are looking at the one scale, we are looking at this one scale. So the new scale looks like this [pointing to her drawing on the board], six carrots and the three peppers. Okay? So, now tell me, because what are we looking for? What are we trying to find? Nazeer: How many carrots equal one pepper? T4: Okay. So, then what we are trying to find is, the scale, what does this scale look like.[T4 draws a second scale on the board with one pepper on one pan and a question mark in the other pan.] And I am such an artist! So we trying to figure out what goes in here [pointing to the question mark], right? Does that help you any? Nazeer: Yes. T4: Okay, so now explain to us what’s, what we do next. Nazeer: We have to take away the same amount of peppers, the same weight of peppers as we did, uhm, the weight of carrots. T4: Okay. Nazeer: And two peppers equals three carrots. T4: So you are saying these two peppers is the same, is the equivalent of three carrots? Nazeer: yes. T4: So then three carrots equals, is equivalent to this last pepper? Nazeer: wait [looks confused and doubtful, then smiles]. No, I meant two peppers, two peppers is the same as four carrots. T4: Okay. So then here it is. So now two carrots is equal to this pepper. Nazeer: Yes. [Jarrod nods.] T4: And these four carrots are equal to the two peppers. Nazeer: Yes. T4: Okay. So now, are we done? Nazeer : yes. T4: Are we? But I still have a question mark here. Nazeer: One pepper equals two carrots. T4: Okay, and you know what I like? I like how you said the weight of. I like how you used that. You said the weight of two carrots. Aren’t those pretty carrots? So you are saying this is your response, this is your answer. [Nazeer nods.]
Description of the event	In this episode Nazeer and Jarrod are struggling to explain their solution to the “Carrots” problem. Their solution method begins with the logical

	substitution of two peppers for a corn on the first scale. However the boys look confused and keep forgetting about what the first scale looks like after the substitution is made. The teacher draws a picture of a new scale based on the boys' initial explanation, which eventually helps Nazeer come up with the correct answer.
Teacher's Actions(s)/ Response(s)	1) The teacher asks a question to help Nazeer clarify his answer [line 13]. Code: Eliciting a response – Name/State (E-NS) 2) The teacher paraphrases Nazeer's response and asks Nazeer to verify the accuracy of her understanding of it [lines 17-21]. Code: Paraphrasing student's response (PSR) Code: Eliciting a response –Yes/No (E-YN) 3) The teacher encourages Nazeer to continue with his explanation [line 28; line 60]. Code: Eliciting a response – Describe/Explain (E-DE) 4) In order to help Nazeer visualize the steps he has taken toward solving the problem, the teacher draws picturse of the scales on the whiteboard which reflects all the changes made during the process described by Nazeer. The teacher then questions Nazeer and Jarrod as she refers to the new drawings [lines41-52; lines 54-5; lines 65 and 66; line 68]. Code: Using representation (REP) Code: Building on student's ideas (BSI) Code: Paraphrasing student's response (PSR) Code: Eliciting a response – Name/State (E- NS) Code: Eliciting a response –Yes/No (E-YN)
Students' Action(s)/ Response(s)	1) Jarrod and Nazeer struggle to explain their solution to the Carrots problem [lines 5 and 6; lines 9-12; lines 14-15; lines 28-40]. Code: Sharing solution/idea with class (SSC) Code: Justifying solution (JS) 2) Nazeer responds to the teacher's questions [line53; line 59; line 62; line 64; line 67; line 72; line 74; line 76; line 78]. Code: Answering a question (AQ) 3) Nazeer recognizes his mistake and changes his answer [lines 69 and 70].

	Code: Making a self-correction (MSC) Code: Answering a question (AQ)

Appendix G4: Lesson #4- Transcribed and coded Critical Event 4

Role T4-1 **Teacher: T4** **Students: Oscar**	**<u>Event 4</u>**
Transcript of Event **Time (as shown on RoleT4-1): (00:46:19-00:47:23)**	[It is about 46 minutes into the lesson. Oscar explains how he and his partner solved the “Carrots” problem.] Oscar: We did it differently. T4: Okay. Oscar: On the first scale, six carrots, a corn and a pepper. T4: Okay. Oscar: So we assumed that a corn a pepper was as much as three carrots until we got to the second scale. A corn was three carrots and two peppers for three, wait. Each pepper weighs as much as three carrots. So then we added 3 plus 3, because there were two peppers and they were 6 and 3. We noticed that they were not balanced. So then we tried with other ones, 2 and 4. The pepper, the corn weighs as much as four carrots and the pepper weighs as much as two. On the second one (scale) is corn, four carrots, and two peppers, each two carrots. So then we add 2 plus 2, which gave 4 and they were balanced. So we, and then the third one (scale) is one pepper equals as much as a question mark and the question mark is two carrots. T4: Okay. Any questions for any of the groups?
Description of the event	In this episode Oscar explains how he and his partner, Maybelea, solved the “Carrots” problem different to the way Nazeer and partner had done.
Teacher’s Actions(s)/ Response(s)	1) The teacher provides the opportunity for students to question each other [line 17]. Code: Encouraging classroom discourse (ECD) Code: Providing opportunity to critique other’s reasoning (OCR)
Students’ Action(s)/ Response(s)	1) Oscar compares his solution method to Nazeer’s [line 3]. Code: Comparing solution strategies/ideas (CS) 2) Oscar describes how he and his partner solved the Carrots problem [line 5; lines 7-16]. Code: Volunteering to share ideas (VS) Code: Sharing solution/idea with class (SSC) Code: Making a self-correction (MSC) Code: Justifying solution (JS)

Appendix G5: Lesson #4- Transcribed and coded Critical Event 5

<u>Event 5</u> **Role T4-1** **Teacher: T4** **Students: Kieshe**	
Transcript of Event **Time (as shown on RoleT4-1): (00:47:31-00:53:50)**	[It is about 48 minutes into the lesson. In order to help Kieshe understand the concept of substitution, the teacher constructs two imaginary balance scales.] T4: What are some things you are stumbling on? Please tell me some of the things you're stumbling on before we move forward [addressing the whole class]. Kieshe: The part I am stumbling on is the part you have to take out how many carrots from the first one and put them into, into groups. T4: Okay. Now. Rene, could you stand up for me please. If we were to get on a scale, stand up like this. And I were on one side of the scale and Rene was on the other side of the scale, do you think it would be balanced? Kieshe: No. T4: Okay, you don't think it would be balanced. Why not? Kieshe: Because you are heavier. T4: Ohhh, you say I am heavier? Okay, yes, I am heavier. I am taller. I am probably, we are probably about the same width or whatever, right? If, if we are the same width, I don't know. But I am probably bigger, okay? Usually, you know, the taller people are heavier. So I am probably bigger. So what we need to do, because at this rate what would the scale look like? Would it look, as you said, at a line, would it be in a line [holding two palms up]? Kieshe: No. T4: What would it look like? Tell me what it would look like. [Students hold their palms up at different levels.] T4: It would look like this [palms up at different levels]. And Rene would be up here, right? And I would be here. Kieshe: No. T4: I am here and he is…? Dajuan: Because you are heavier…. T4: I am heavier, so I am up? Dajuan: Because you are heavier, you will bring it down. T4: I am here [lower hand] and Rene is here [higher hand]. So what do we have to do with this [higher hand]? Tell me what we have to do with this. With this part of the scale where Rene is. Kieshe: [unclear]…two people go to Rene. T4: [laughs] She adds two more people to my weight. Okay, so three of them equal my weight? Okay, fine, fine, fine, fine. If we put, two more people, so Dajuan, stand up. Uhm, Sorry, Arlin, stand up. Okay, now, let's say, let's say, let's say these three and then me. They are on that side and I am on this side and we are in a line. We are balanced, okay? We weigh the same. I weigh as

much as the three of them. The three of them weigh as much as me, okay? So, if now, let’s see we do, we put [T4 and R2 laugh]. Okay, wait, wait a minute. If we take Cristian, and now Cristian you have to go stand over here because Cristian is another scale, Okay? Cristian is another scale alright? We know that Cristian, let’s see. I think I am getting stuck here. Let me see. Trying to make it. Okay, how about this. Cristian, you get on this side of the scale [Cristian joins the other three students].

T4: I am feeling lighter already. So now, what happens?

Dajuan: Our side goes down and…

T4: your side goes down and my side goes up. So kiesha, tell me what I have to do to make it balance.

T4: Tell Oscar [Oscar’s hand is up]. Oscar, see if you agree with her.

Kieshe: You could take out two people?

T4: Take two people off? Which two people?

Kieshe: Arlin and Dajuan.

T4: Arlin and Dajuan. So move, move Arlin and Dajuan. Thank you.

T4: Now how do you know that Cristian and. Because before we knew that those three and I balanced out, right? What makes you think Cristian balances, Cristian and Rene balance it out?

[Some kids try to call out.]

T4: Hold on. Hold on one second.

Kieshe: Because they are the same size.

T4: Because they are the same size. But, what does that, in relationship to me and my weight. We have to figure out what Cristian weighs. We have to figure that out, because we don’t know if this would work, unless we know how much Cristian weighs. If there is a scale that says Cristian and there is, there is a scale that says Cristian weighs as much as, come on you two [asking Arlin and Dajuan to come to the front again]. There is a scale, and the scale says that [arranging the kids on the imaginary scale] Cristian weighs as much as these two, okay? So now, when I am on the scale over here and then Cristian comes over here [Cristian moves to the first scale and stands next to Rene], right? When Cristian comes over here, now we can, we can validate, or we can, we can say, we can actually say, yes, since he (Cristian) weighs the same as those two (Arlin and Dajuan), right? And when they (Arlin and Dajuan) were on the scale with Rene, it was balanced, right? We took those two off and we left, we put, we replaced it with Cristian, which weighs as much as those two. So now the scale is balanced again. Because we took off something, but we put the same weight back to keep it balanced. Because if we took off something off one side, they would be lopsided, right? We want to be balanced. Right? That’s what the scales are about. We have to look at the carrots. We are looking at the carrots and saying, well, how are we going to manipulate the scale, okay, change things from the scale and put things back so that we can figure out the third scale. That is what the second scale is about, okay? So you are using the scales, you are looking at how they relate to each other to get to the third scale.

T4: Does that help you?

	86 87 88	Kieshe: Yes. T4: It helped you too [addressing Yaasmyn and her partner]? [Yaasmyn nods and T4 gives her a high five.]
Description of the event		In this episode the teacher asks the class if anyone has any difficulties with anything they have done so far. Kieshe says she doesn't quite understand how items are replaced on a balance scale. In order to demonstrate the concept, the teacher solicits the participation of four students and creates two imaginary scales, where a quantity of weight in one scale, replaces an equivalent weight on the second scale.
Teacher's Actions(s)/ Response(s)		1) The teacher constructs an imaginary pan balance scale using herself and another student to represent the weights on the scale. She asks Kieshe to describe the position of the two pans [lines 9-30]. Code: Using representation (REP) Code: Directing students to perform some mental or physical activity (DA) Code: Eliciting a response – Name/State (E-NS) Code: Eliciting a response –Yes/No (E-YN) 2) In order to balance the scale, the teacher follows Kieshe's suggestion and asks two more students to join the side opposite to her on the imaginary scale [lines 31-40]. Code: Using representation (REP) Code: Directing students to perform some mental or physical activity (DA) Code: Eliciting a response – Name/State (E-NS) 3) The teacher adds a fourth student to the already balanced scale and asks Kieshe how the scale can be balanced again [lines 45-55]. Code: Using representation (REP) Code: Directing students to perform some mental or physical activity (DA) Code: Eliciting a response – Name/State (E-NS) 4) When Kieshe suggests, unexpectedly, that removing Arlin and Dajuan will balance the scales, the teacher builds on Kieshe's idea and uses the opportunity to demonstrate the concept of substitution [line 56-84]. Code: Building on student's ideas (BSI) Code: Using representation (REP) Code: Directing students to perform some mental or physical activity (DA)
Students' Action(s)/		1) Some of Kieshe's responses indicate her apparent lack of understanding of the concept of equal weights and balance [line 26; line 52; line 55; line 61].

Response(s)	Code: Answering a question (AQ) Code: Displaying lack of understanding (DLU)

Appendix G6: Lesson #4- Transcribed and coded Critical Event 6

Event 6 **Role T4-1** **Teacher: T4** **Students: Maybelea and Cristian**		
Transcript of Event **Time (as shown on RoleT4-1): (00:54:20-00:55:48)**	**Line**	
	1	[It is about 54 minutes into the lesson.
	2	When T4 mentions that one of the activities they do on field day reminds her
	3	of the scales, some students immediately recognize the activity to be the tug-
	4	of-war.]
	5	T4: Oh, there is an event. There is an event that kind of reminds me the
	6	scales. At field day, every year.
	7	A girl: Tug-of-war.
	8	T4: Tug-of-war? Why does that remind you of the scales?
	9	[Some raise their hands and some attempt to call out.]
	10	T4: Hold on one second, Maybelea?
	11	Maybelea: Because, uhm, because..
	12	T4: Hold on one second [addressing some other students].
	13	Maybelea: Because, if there is more people, like there is more people on this
	14	side, this side will win. Because there is more force…
	15	Cristian: [Unclear].
	16	T4: Say that again. Say that again. Ask her.
	17	Cristian: How about the other side is stronger than the side that have more
	18	people?
	19	T4: In other words, for example, visualize. Twenty kindergarteners and 18
	20	fifth graders, wait, there is 19 of you? Okay, let's say there are 25
	21	kindergarteners and there is 19 of you. Who do you think, do you think the 25
	22	kindergarteners are definitely going to win?
	23	Some students: No!
	24	T4: Why not?
	25	Some students: We are stronger.
	26	T4: Is that what you were saying [asking Cristian]? He is saying, what about
	27	the strength? So, is it about the number on each side?
	28	Some students: No.
	28	T4: Or is it about, what is it about? Strength or the weight?
	30	Some students: Strength.
	31	Some students: weight.
	32	T4: Okay, so I am going to let you tell me.
Description of the event		In this episode the teacher is trying to introduce the next task, the "Tug-of-War" problem. She mentions that one of the activities they do on field day reminds her of the scales. Some students immediately recognize the activity to be the tug-of-war. In describing the similarities between scales and the tug-of-war, issues such as equal weight, equal strength, and equal number of students at each end of the rope are discussed. The teacher gives an example

	to clarify.
Teacher's Actions(s)/ Response(s)	1) As a way of introducing the next task, the tug-of-war, the teacher mentions that an activity during the field day reminds her of the scales. Code: Tapping into student's prior knowledge (SPK) 2) After a girl identifies the activity as the tug-of-war, the teacher questions the students about the connection between the scales and the tug-of-war [line 8]. Code: Eliciting a response – Describe/Explain (E-DE) 3) The teacher encourages Cristian to challenge Maybelea's reasoning [line 16]. Code: Providing opportunity to critique other's reasoning (OCP) Code: Encouraging classroom discourse (ECD) 4) The teacher gives an example to highlight Cristian's point [lines 19-22]. Code: Giving an example (EX) Code: Eliciting a response –Yes/No (E-YN) 5) The teacher questions the class about whether it is the strength, weight, or the number of people on either side that determines the winner in a tug-of-war [line4; lines 26 and 27, line 28]. Code: Eliciting a response – Name/State (E-NS) Code: Eliciting a response –Yes/No (E-YN) Code: Paraphrasing student's response (PSR)
Students' Action(s)/ Response(s)	1) When Maybelea mentions that the greater number of people on one side in a tug-of-war determine the winner, Cristian is quick to bring up strength as the main factor [line17]. Code: Questioning the reasoning of others (QRO)

Appendix G7: Lesson #4- Summary table of Critical Events

Teacher: T4	
Event / Time	**Description**
Event 1 Time (as shown on RoleT4-1): 00:19:10 - 00:22:12	In this episode the teacher asks D’nea and her partner to talk about their solution to the “Bananas” problem. D’nea begins the explanation, but when she is asked to be more specific, and justify her solution, she smiles shyly and refuses to talk. The teacher gives D’nea and her partner, Destiny, ample time to speak up, and when they remain silent, she asks them to put their thoughts in writing and report later. Oscar steps in and gives his explanation. Solution Path: 1B
Event 2 Time (as shown on RoleT4-1): 00:22:16 - 00:24:56	In this episode Kieshe explains her reasoning as to why one apple is equal to two bananas in weight. Her explanation is confusing and does not make much sense. The teacher asks her some questions that reveal Kieshe’s lack of understanding of the notion of equivalence on a balanced scale. Nazeer is asked to explain why her method is incorrect.
Event 3 Time (as shown on RoleT4-1): 00:39:40 - 00:45:50	In this episode Nazeer and Jarrod are struggling to explain their solution to the “Carrots” problem. Their solution method begins with the logical substitution of two peppers for a corn on the first scale. However the boys look confused and keep forgetting about what the first scale looks like after the substitution is made. The teacher draws a picture of a new scale based on the boys’ initial explanation, which eventually helps Nazeer come up with the correct answer. Solution Path: 2B
Event 4 Time (as shown on RoleT4-1): 00:46:19 - 00:47:23	In this episode Oscar explains how he and his partner solved the “Carrots” problem differently. Solution Path: Not previously identified
Event 5 Time (as shown on RoleT4-1):	In this episode the teacher asks the class if anyone has any difficulties with anything they have done so far. Kieshe says she doesn’t quite understand how items are replaced

00:47:31 - 00:53:50	on a balance scale. In order to demonstrate the concept, the teacher solicits the participation of four students and creates two imaginary scales, where a quantity of weight in one scale, replaces an equivalent weight on the second scale.
Event 6 Time (as shown on RoleT4-1): 00:54:20 – 55:48	In this episode the teacher is trying to introduce the next task, the “Tug-of-War” problem. She mentions that one of the activities they do on field day reminds her of the scales. Some students immediately recognize the activity to be the tug-of-war. In describing the similarities between scales and the tug-of-war, issues such as equal weight, equal strength, and equal number of students at each end of the rope are discussed. The teacher gives an example to clarify.

Appendix H: Transcribed and Coded Teachers' Reflections

Appendix H1: Debriefing #1- Teacher reflections

Teachers' Reflections - Debriefing 1 – Role T1-2 (00:29:05 – 00:42:58) **Coding Scheme : Student Lens (S), Curriculum Lens (C), Research Lens (R)**			
Time (on RoleT1-2)	**Instance**	**Transcript**	**Code**
00:29:20 – 00:29:27	1	T1: I just felt that they, they were able to zoom through the first few, then they went "this is too hard", on the last one.	S
00:29:35 - 00:29:44	2	T1: I probably could have done some different questioning to help them get where they needed to be without being so, without being too intrusive.	RC
00:29:50 – 00:30:00	3	T1: They really struggled with the last one and they really wanted to end with guess and check, which is what I did the first time too.	S
00:30:12 – 00:31:17	4	R1: Can I throw in what I just said to (T2) because that is, I think, what he is going to do, and I want to know what you all think because we can change it. But to my mind the better question for the beginning of the second one (Shorts and Glasses) would be not to have any dollars there at all, which what I think he is going to do on the overhead. R2: That is a good Idea. T1: Yeah [nodding]. R1: And say, you know just introduce it by saying, I am at the store and discovered that I could get a shorts and two glasses for the same amount as this [pointing to three pairs of shorts]and this[pointing to one pair of glasses]. What are your thoughts? And let them work on just that part of it on paper for a while and after they have come to that kind of a conclusion, then give them the dollars. What do you all think? [They all agree that this is a good idea.]	RC C
00:31:45 -00:31:58	5	T1: Actually I think the fact that they had a quick success with this one (Shorts and Glasses) was fine and they really saw the challenge with this next one (Soda and Shirt), you know, so that first one was a quick success.	S

00:32:07 -00:32:19	6	T1: Now what are you thinking about the sequence? Cause that seemed long to me, and of course rushed at the end. T3: You know what; I didn't think it was too long. I think it was fine. T1: Really? T3: I think mine is going to be too short, or, I just, because I was like, wow! R1: I don't know. You may want to add something else.	C C C C
00:32:23 -00:32:12	7	T4: If you wanted to have time, like more time for the last two (problems), what you might want to do was half the class gets one of the warm-ups (Bananas) and the other half get the other half, the other warm-up. And they report out and they are not doing, they are not reporting the same. You know what I mean? R1: I sort of disagree with that. I found the progression from the first to the second one really interesting. And what I wrote in my notes was that I realized that in the first one (Bananas), they sort of get it; each one of them presented their solution. In the second one (Carrots), because they had messed around with making a corn and a pepper the same, they began, they began to listen to each other.	C RC S
00:33:27 -00:34:27	8	R1: A part of your goal, I thought, was to have them to begin to talk to each other and to hear each other. R2: Which was, just to cut in, when you introduced it (Carrots) with the overhead, without the paper, like if there was a way to measure the decibels in the room, all of a sudden, bam [holding his hand up to show a high level]. For the first one (Bananas), it was like quiet, reading on my own and you were "remember you are in partnership, talk", you know what I mean? You reminded them. The second time (Carrots) you just did it, they had to talk to each other to like, confer, yeah exactly, just to confer the information about the problem. But then I think it jumped into more… T4: So that is why I wasn't sure whether it was what (R1) was saying or whether it was the presentation was different. R1: interesting! T4: Because, you know, like you said they were	R CR S RC RCS

		talking more. So I wonder if they would've still talked more if, if it was on the overhead. Because they didn't have anything to personalize. Like this is mine [clutching her notepad to her chest], have to do my own thing. R1: That is really true. A part of it also is that they were just getting into it.	 S
00:35:08 -00:36:05	9	R1: They really did go about it differently (in solving the problems). T1: Not the first one (problem) though, I don't think. The first one (Bananas). R2: Well, Kyla had a much different strategy than the rest. Like I think three groups were doing the five pounds, three pounds, one pound thing. Or at least five, three, one I think it was. But she (Kyla) did it more substitution and kind of like deduction, like where… O1: Actually Kyla's work on paper was correct but her explanation did not match her work. She did not explain what she had written down. R2: But she only lost it on the last one (scale). T4: Her explanation is going and then it seemed like she was either distracted or something but it throw her off. R2: But she got the three bananas part right. O1: No, she said, two bananas and one apple, right? R2: Oh, did she? I thought she said three. T4: At the end. O1: At the end. But on the paper, it was interesting because she had drawn pictures, you know.	S S S R RS S S S S S
00:36:12 -00:36:34	10	O1: But isn't it interesting why they (students) introduced pounds (in the scale problems). They were all talking about pounds. T1: I thought that was wrong, and then you [addressing R1] said, no that's what we want to get to. R1: Well, what is fascinating is that the little guy who talked about the pounds was almost the only one who was saying, "you got to change it to something that makes sense, that bananas are not equal to pineapples". T1: Okay.	RS R RS
00:37:29 -00:38:27	11	T3: [unclear] we would have gotten something	R

		different (student response) if we just put two pictures with an equal sign. I think the students would have been more direct if there was not a scale there at all. R1: Oh, I just think- and this is the last because you have to go- that that was one of the most important things that happened is that they were able to begin to give you, to give (T1) the opportunity to understand what we mean that you have to say what you mean by equivalence. Because in this one [pointing to the “Shorts and Glasses” problem], you see, we do it for them; we say they were the same amount of money. Here (scale problems), what do you mean when you are saying, and how could we say that? What is equal? It is like the Cuisenaire rods you know. An orange is not equal to two yellows. It is the same length, and you know how careful we had to be in IML.	C RS C
00:40:51 -00:42:02	12	O1: Do you think they are using the idea of weight here correctly though? Because if I am not mistaken, I couldn’t hear that well from this side of the room, but were they not equating one item with one pound? Were they not assigning a pound? R1: I think that is a good point. If you are willing to pursue that, say “well, why did you choose one?” They chose one because it was easy. One unit, one pound, one anything. Does it have to be pounds? No. Suppose the banana, suppose the banana had weighed ¼ pound. Then what about, what would a pineapple be? There is tons you could do there. But they (students) had a [unclear] of it. What they were saying is that we’ve got to convert what we are doing here to pounds, or to weight, to make it, to make it make sense in their head. And to me that was really great, I mean they did it to the simplest unit.	R S RS C S R

Appendix H2: Debriefing #2- Teacher reflections

Teachers' Reflections - Debriefing 2 – Role T2-2 (00:18:43 – 00:50:41) Coding Scheme : Student Lens (S), Curriculum Lens (C), Research Lens (R)			
Time (on RoleT2-2)	**Instance**	**Transcript**	**Code**
00:18:59 – 00:20:00	1	T2: Well, within certain groups I was pleased with the level of interaction and how they were sharing their ideas, working towards consensus. And I was pleased with the overall work that, I would say many of the students did. I guess towards the latter portion there were some behavioral issues and I guess some students were frustrated because they were not catching on I guess as quickly as they perceived their peers to be. So I feel there were a little uneasy about that, almost like shutting down at this point. So that was a concern.	S
00:20:17 - 00:21:37	2	T2: Now you see Jasmine G, the Jasmine who was over here with glasses, she doesn't work with anybody in any class. She normally sits there right in the front, where that computer is. R1: Which Jasmine, the one, which? T2: Jasmine, the short one who sits over there. R1: That little girl. T2: She doesn't work with anyone in any class, science, ELA, nothing, she always sits by herself. But she was, today [nodding and thumb up]. R1: She was great and then it was, whatever you did to put the three, they were three complementary… T2: Right!	S S
00:22:00 – 00:22:22	3	T3: I was surprised at Monae because Monae, I never thought that she, I don't want to say bright, but I just think she was very, she lacked confidence and working with Jasmine G, it built her confidence. T2: Yeah, that was good. That was one of the groups that worked nicely because there weren't any behavioral distractions.	R S S
00:22:23 – 00:23:00	4	T2: Now these gentlemen over here worked pretty well, Kevin, Caliph, and Cory. But then occasionally there was this off task… Yeah. T4: But, usually that is when they are done with	S S

		their work. It looks like they were…	
		T4: Kevin and Caliph. Cory is generally, I would say 60% of the time, the slacker.	S
		T1: But they were not documenting for the last one, so kind of was starting to shut down. They worked well the whole time, but the last one, I would say, get something, get it documented, get something on paper. "I thought you were doing it, I thought you were doing it", but they finally got something down.	S
00:22:03 – 00:23:24	5	O1: Did you try to group them so that there is a strong student in each group or not?	S
		T2: Uhm, this, that would have been my goal, but it is kind of hard to do because of the behavioral things as well. So in some, whereas possible, yes, but in some groups, no.	S
00:25:02 -00:25:44	6	T1: What about these ones (Oscar and Jevon) over here. Because I mean I like those old answers when they say, "well it cost more to make the glasses and you know, they can break" [laugh]. I like those answers though.	RS
		R1: But you know that's going to come.	
		R2: Yeah, over contextualization.	RS
		T1: It's the practical thought.	RS
		R1: But it is also, that article we read, that was talking about the differences between kids from a more urban situation is that they get, that they are less able to pull the math out of it and get more lost in the context of it.	
00:25:51-00:26:58	7	T1: But isn't that the whole issue around, when we talk about the validity and reliability of test that. Because we have our children who are, that is the concern they hear and talk about, you know "well, the glasses cost more, you know, Walmart had a sale", things like that they are saying verses going right into the math. That is the same thing with testing.	R S
		T4: But also that is just like we teach them that words mean different things at different time. You do different things with this information at different times. This is a math class. So, you know	
		T1: There was math associated with their thinking.	S
		T4: there wasn't math in that because when he asked them for it, they didn't give it to him. He said, yes,	S

		and now based on the picture, based on, you know math, where is the math in it? They were unable to give that. I am not saying that it is not realistic that we look at, you know… T1: In their thinking there was math associated with that. T4: But, but what I am saying is… R2: It was off task.	S
00:27:32 - 00:28:25	8	T4: When there is an open-ended question, when there is, you know, things that, you know, they are clearly looking for some math in it, that is what we expect you to do. Yes, there was a problem I had. I forget what it was, but I know there was milk involved. And we were trying to figure out, I think there were three different size milk and they had to figure out which one is better for your money. You know they were like" well there are only two people in my family, so we will get this one". That is very valid. However, I had to as a teacher teach them, but do you think that is the answer they are looking for? Because what about my household? They are looking for an answer, there is a correct answer. And do you think, like which, which answer do you think? You know, like how do we go about getting that answer? T1: And I think they would have given us, with time and with questioning, they would have finally gotten to (referring to students who could not give a mathematical reason as to why the glasses are more expensive than the shorts). T4: I do too.	S S RS
00:28:37- 00:28:58	9	O1: I was just going to make the comment that I really like the way (T2) actually handled it. Because he validated their point. He said, yes, but this is based on what you know about glasses, but I want you to give me an answer based on the picture that you see. So he was trying to bring out the mathematics. And I thought that was great the way he basically handled that.	R
00:29:01 - 00:30:41	10	R1: I found the same, not the same idea, but the same sort of making the picture what you want it with the little girl over here. What's her name? T2: Jazmine L?	R

		R1: Jasmine, when she was convinced it was two bananas in the first one. But then she came back to it and she said “it has got to be two bananas”. And I said, well if it is two bananas, then, if it is two over here [pointing to the last scale]. And she said “it is going to be two over there, it really is”. I said, then what about here [pointing to the middle scale]? She said ”oh, then that one doesn’t balance, that is not the right picture”. And so she said “if it is two over here (last scale), which is what I want it to be, then when you put it over here (middle scale)” and I said, what do you mean? She said “oh, this (pineapple in the middle scale) would be heavier”. And I said, oh [laugh], because that was logically fine. Then I said, but this (middle scale) does balance. She said “oh, then it will have to be three”. Or what I said was, what I did say was, what do you have to add to this (middle picture) to make them balance? And she said “another banana”. And so it was just very interesting. Almost the same sort of remaking of the picture to go with what she was thinking.	S R
		O1: Did they (Jasmine and partners) eventually agree that it is three bananas? Did they see it? Because I am not sure if they actually..	S
		R1: I am not sure if they ever did or not, but she certainly convinced me that she understood that if this (middle scale) was going to balance, they had to put five bananas over there.	RS
00:31:50 -00:32:23	11	T1: I am curious though when we look at the documentations, because this group over here, Monae, this lady over here, she had the correct answer but the other two didn’t. And she was the spokesperson, right, she was the spokesperson for the group. So I am curious to see if they have, if they still have two bananas equals one apple on the other two papers or if they changed it.	R S
		R1: It will be interesting to see. We will have to study this afternoon, but I think she convinced them.	R S
		T1: She convinced them.	S
		R1: I think she did.	S
		T1: I am curious to see. Because when I was over there, they had still two (bananas).	RS
00:32:36 -00:33:43	12	R1: Now, you have revised what you are going to do	C

		[addressing T3]?	
		T3: Yes, what I am going to revise, I am looking at what I see from (T2)'s students, I am going to use the Bananas so the kids get an understanding of the balance. Because the tug-of war deals more with the inequalities. So, just so that we can examine equalities, take a look at both of them (both Bananas and Carrots). You know, just to look at the scales, scales are supposed to be equal, and then go into the tug-of-war.	C
		R1: So you are going to make that kind of progression?	C
		T3: Yes, that kind of progression.	
		R1: I think it is going to be very interesting. And then if you have time?	C
		T3: We are going to do the Chickens. I think we have a 80-minute block. And I am just thinking knowing the kids I'll be working with, yeah this (scale problems) should not take that long. I may just be taking most of the, I think, [unclear] time. I am thinking 10 minute for this here (scale problems) and then maybe 15 to 20 minutes for the tug-of-war and the remainder for the Chicken.	C
		R1: If you do this, the tug-of-war may go faster.	R
		T3: Yes.	
		R1: So it will be interesting to see.	R
00:37:40 -00:38:07	13	R1: I don't believe that they (students) were able to see two shorts for one pair of glasses relationship under any circumstances. And so if you are thinking algebraically, that is really what you are trying to get at, which almost makes me wish you (T3) could do that one (Shorts and Glasses) along the way too, at some point.	RS C
00:39:30 – 00:40:20	14	T3: She (Sakeena) kept on telling, saying it is shorts because shorts are more expensive. Then (T2) heard me say [unclear] shorts and glasses, but she is the one who identified that, she, I said share with Tateanna what you told me. So she said "you could take one from here (top picture), one from here (bottom picture)". She did that! So I said why don't you cross that out? And she saw there were two shorts and there was one glasses and she still said it was shorts (more expensive). And then I am thinking she [unclear] in terms of quantity, not	S

		looking at the value. I think she began to see quantity, but she was the one to relate to that and say "well this one is here, this one is here, one short here, one short here". So I believe given the time and if we were to probe her more, she would have immediately said "well this is one (glasses), it's equal to this (two shorts), so this (glasses) must be more expensive".	R R
00:40:35 – 00:41:12	15	R2: I was so blown away that, when he, Cory said a banana is a pound and a half. Well, how much is all ten? T1: And we never got an answer. R2: I didn't. Did anybody see? But did anybody see anybody in their groups come up with an answer? T2: No. T4: I think they moved on. T1: that (question from R2) was good. R2: That was a nice pause, but R1: Well it was good and if, if we were not under pressure of moving, it would have been a teachable time for you to say, hay we got, you know, we got to stop and figure this out now. T2: Oh, yeah.	RS S S R R
00:44:51 – 45:33	16	O2: I saw a progression from this morning to this afternoon and I think that was partly the age because we went from 5th grade to 6th grade. And if we do these tomorrow, I am curious, that is how I am trying to think. R1: What kind of progression? O2: Just the whole mentality, the whole thinking. Your class [addressing T1], it wasn't, I don't think putting even like a variable or anything to anything was an option in your class. Whereas, who was it [pointing to a seat]? Joe? Joel? He was up there already putting, 1p equals the bananas and the apples and he had everything in letters and numbers.	R R S

Appendix H3: Debriefing #3- Teacher reflections

Teachers' Reflections - Debriefing 3– Role T3-2 (00:06:29 – 00:37:17) **Coding Scheme : Student Lens (S), Curriculum Lens (C), Research Lens (R)**			
Time (on RoleT3-2)	**Instance**	**Transcript**	**Code**
00:06:28 – 00:06:53	1	R1: What I am wanting us to think about is the differences, you know, that occur, both developmentally and for other reasons, as you go from 5th grade to 6th grade to 7th grade to 8th grade. And that was fascinating, that was really fascinating. And it was really interesting that we did it in the order we did with the 5th grade first, 6th grade, and then this one [people nodding in agreement]. Don't you think?	R
		T1: Yeah, it was really clear.	R
00:13:30 – 00:16:37	2	R1: The real issue that I like us to think about is Theo, who came up with what he thought was a very logical solution and it didn't work. And so what I was trying to do, and I know what (T3) was trying to do, is to get into his head to know what question to ask. Because ultimately, the only thing that happened was, we just sort of over-rode him.	S R
		T3: Yeah, [unclear].	
		R1: Because the other kids had two or three ways of coming up with something. But Theo still believed that his solution, which was the opposite, was correct.	S R
		O3: Is he the one, where the girl (Dieshe) asked the question "how did you make the assumption that…"? That was a great question [people nodding].	RS
		R1: That was a great question. And so one of the really strong things about this class was, once they got into it, they began to question each other. And that's so, generally, so much better than you questioning [T3 nods in agreement]. But, but even so, we didn't hear from Theo. I am not sure that he was assuming that an ox was (equal to) two horses. That is what they (other students) said, "an ox is two horses, you can't do that".	R S RS
		T3: oh yeah [she looks through her folder and presumably at Theo's work].	
		R1: I have a notion that he was, was getting an	S

		elephant from two sides, from two sides of the equation. Because he said an elephant is worth two horses and whatever… O2: (O5) had a conversation. O5: I was watching him and what he did was, when he substituted for that elephant, he crossed off the oxen on the other side, but he crossed off the horses from the elephant side. So I talked to him at the end. R1: What did he say? O5: When I talked to him he said, well I helped him, I didn't really ask him [unclear]. Do you realize that when you did that you didn't take off horses against the elephant, you took off horses that would have been helping the elephant? He got that part. He saw that. That is where it ended. It was 15 seconds. R1: Because to me that's what was going on, it is that he wasn't really equating an ox and two horses. But just look how complicated that is and how, you know, all of us together are struggling with it. And so a piece of what we need to help each with over the time-and I think it is true with Connected Math as well-is when you get into this kind of situation, when the kids are sharing and, and you have a responsibility to help uncover these mistakes and have kids come together. Because there is a right answer. You can't leave them thinking, that's okay, or this is okay. And that is hard.	S S S R
00:17:17 – 00:18:07	3	R1: The real reason that Keith bugged down and went to 20, 20 was because he was adamant, he had this notion of equating, of doing oxen in terms of horses and he said, "but I can't do that because I'll have to break up a horse and it would kill it. [Laughter] O2: Yeah, that's what he said. That is when I asked him, I said…The boy who did the fractions? T3: Tyreal. O2: That is when I said asked your question from Tyreal. R1: Even though it was a context issue, it was a mathematical idea too. And so when I said, are there some numbers you could use-he knew he had to make them equal- are there some numbers you could use that won't cut up the horse, he said,"oh, 20, 20 and went into 4's and 5's.	S S R S

00:20:10 – 00:20:43	4	O6: Because I wasn't here yesterday, the changing from the Chickens, what was the reasoning? T3: Based upon, you know what, now that you [turning to R1] said it, I am happy. Based on what I saw [pointing to T2], because originally this was supposed to, this [holding up a transparency of the tug-of-war] was going to be a warm-up and then we were going to do the Chickens. But what happened, as I saw the kids struggling in your class [pointing to T2] with that balance, I felt it was necessary to do it with the 8th graders. So basically it was just replacing. We had planned, I had planned to do the Chickens, it's just we didn't have time because we spent so long on this.	C C S
00:21:47 – 00:25:18	5	O6: I am just trying to use my imagination and figure if we did it, we followed that original plan, and I think it would have gone well. Just, I mean that is my impression. R2: Based on how they performed. O6: Yeah, yeah, based on how they performed I think if we had…But, but I agree with, I agree with the switch. I mean, I think the switch was a good idea. But I am trying to use my imagination to see how… R1: I think, you know, maybe the other (original lesson plan) would have done well. I think what you did uncovered some algebraic issues that were really important. The two girls-that is when I pulled you over…. ….. ….. …..	RC C R
00:22:25 00:25:08		R1: Then, for the girls it was obvious that the notion of replacing had become a part of their vocabulary, which I don't think would have happened if you hadn't done those (problems) earlier.	RSC
00:25:55-00:26:50	6	T3: Another thing that I am really kind of happy that, and it kind of help work itself out, that I did this one first [holding up the Carrots problem]. I really think it was powerful that I chose to do this one, and it was just like, okay… R2: And then the Bananas went like that (quickly). T3: Bacause it went like that (snapping fingers) and had I not done this first, it would have been [unclear]. R2: But there is a, there is a, just through this process, there is a delicate balance sometimes in	C C

		over doing it with the task and causing too much frustration with early successes. You know what I mean? Like, I am not saying to dumb stuff down by any means, but I've been seeing this balance is really, really important because you want to challenge them without frustrating them. Because if you over-challenge them they shut down.	C
		R1: [unclear] frustration too much [laughter].	
		R2: Yeah. No, no, I agree.	
		R1: You want some frustration.	C
		R2: Some, it's just that, to the point where…	
		R1: It can't be an over reach.	
		R2: Yeah.	
00:29:52 - 00:30:51	7	O4: I think the professor (R1) made a very important point about replacing the value and I saw Jynita had a problem with replacing the elephant. Okay, so I went to her and she said "I already have an answer, don't come to me" [laughter]. I know it's frustrating, like (R2) said. I saw at least four teachers helped her, but she still has the answer that the oxen, the right side will win. So, so what I do is, cut this [holds up the cut-out picture of the horses and ox from the middle picture].	S
		T3: That is a good idea.	C
		O4: I cut this and I paste it here [placing it on top of the elephant in the bottom picture]. I said, okay, look at here. You have five horses and an ox.	C
		[Many teachers are heard saying that it is a great idea.]	
		O4: And then she was happy and erased the answer.	R
		T1: That is a modification.	
00:33:53 – 00:36:19	8	O1: But (T3) you know, this is really interesting because he (Tyreal) really struggled, when he had five (horses) on one side, he really had a tough time getting 1 ¼ (for each ox) because he was putting 1.4 to begin with.	RS
		T3: Wow! Okay.	R
		O1: And I was, was trying to figure out what he was doing and I, I just said, couldn't even read, I said, is it 1.4? He said, yes. And I could see he was adding it up and it wasn't becoming 5. So he would change it to something else. And he had a calculator…	S
		R1: Who is that?	
		O1: Tyreal. He really did a lot of trying to, and he	S

		couldn’t understand that it was 5 divided by 4 that is going to give him 1 ¼. He never used the calculator. He never actually did 5 divided by 4. What he did, the five horses, he crossed one of them out and said, now if I want to, if I want to split the 5 into 4 sections. But he couldn’t understand that mathematically it was that operation, the division, 5 divided by 4. So initially he had 1.4, then he, but he would check his answer; add 1.4 four times and he was struggling why isn’t it becoming 5. He went back and forth a few times before he got it. And then once he had the ¼, he went and checked all the answers at the end. So towards the end, when he was actually writing his answer, I said, well what	S
		mathematical operation could you have used to get from 5 to this answer that you have? He said “I don’t know”. And I said, well what did you do to 5	R
		in order to get to 1 ¼? He said “I don’t know, I just, I just was trying to split it into 4”. I said, well what mathematical, what is the mathematics in it? He still didn’t know. T3: Wow!! O1: And I couldn’t resist the urge because I told him, well what would you get if you do, on the calculator, 5 divided by 4? And he said “I don’t	S
		know”. He picked up his calculator and you should have seen his face when he did that and he got 1.25	R
		and he said “Oooh!” Then he realized that it was 1 1/4; that it was 1.25. But it, he really could not, he was just thinking so abstractly in terms of the animals and he actually crossed out one horse and said, now instead of five, if I have four, how much would…. T3: Yeah, it’s right here [holding Tyreal’s paper up].	R
		One forth, one forth, one forth, one forth. O1: Yeah, that’s what he did. O7: he sliced up one of the animals. [Laughter]	S
		O1: He sliced it into a quarter, right?	S

Appendix H4: Debriefing #4- Teacher reflections

Teachers' Reflections - Debriefing 4– Role T4-2 (00:17:13 – 01:02:06)

Coding Scheme : Student Lens (S), Curriculum Lens (C), Research Lens (R)

Time (on RoleT4-2)	Instance	Transcript	Code
00:17:30 - 00:19:45	1	T4: The one over here that was talking a lot, Kieshe?	
		R1: Oh she is so beautiful!	
		T4: Yes, she is, and, but she would be considered extremely below level, you know. Like she is, you know. But I do push her and I've been pushing her all year to work. And you know, she is the one who would work, work, work and her response would still be way over here.	S
		R2: She got it. She got it, she understood it.	S
		T4: But today, but today when I said is there anyone who still needs something and she raised her hand and I was really excited about that.	S
		R1: The question when you said, stumbling or struggling, something like that.	
		T4: yeah, and so she spoke. Normally she doesn't.	S
		R1: I can't remember, I want to be more specific. I can't remember exactly what she said then.	
		T4: About what she stumbled on?	
		R1: Yeah, what was her issue?	
		T4: She was talking about well why do, how do you take the carrots out? And how do you, you know, how do you move…?	S
		R1: Not understanding replacement?	S
		T4: Yes, not really understanding that. And so, then I thought of, well let me try to build the scale and show her how the scale is built. I guess that's what was in my head, to try and help her.	C R
		R1: I thought that was kind of good. Don't you think [addressing T3]?	
		T3: Yeah	
		T4: I got stuck, kind of.	
		T3: No, but that was good.	
		T4: And I try to tell them when I get stuck and I try to process it through.	
		R1: Well you pulled it off, when you were able to replace the one with the two. That was good.	C
		T4: And I try to let them lead it, you know, the kids. Remember, it was funny because I was trying, I	C

		always try to take whatever the child says and make it work somehow. So when she said, when at first she said take them out, instinctually I was about to say, no, just take Cristian out. But you know I was like, why and what and you know. So I was trying to I guess figure out what was she thinking or whatever. And I am glad that, when that, made me think of the other scale to show her that way. So I think that might have helped her.	S R
00:19:55 – 00:21:52	2	R1: I was reading (O2)'s paper that she got in and she was saying what was so fascinating from last week, and today it was sort of in reverse, but it was to see the 5^{th} grade and 6^{th} grade and then 8^{th} grade, sort of in that sequence. And today we went from 6^{th} to the 5^{th} and so coming together as a whole, I do hope you as well [loud announcement on PA system], for the algebra study group. Because what to me is the real gist of it is to look at these ideas that are developing over time and what it is that is a stumbling block or that isn't there. I mean with, for instance I noticed with a couple of them, but it was especially with the boys [pointing behind her], the boys that I was watching? T4: Nazeer and Jarrod. R1: yeah, was that they weren't thinking at all proportionally. What they would do is "and we took out that, and we subtracted out", and this kind of thing, which is something that is so important in that we frequently don't realize is not natural for their thinking right now. So of course it is hard for them to see that the corn is double the pepper. You know what I am saying? T3: Did he come up with that or [unclear]? I thought it was amazing. T4: Nazzer probably came up with it. T3: With "I subtracted, I put.." R2: He used substitution. R1: It really blew my head, because it was way beyond what they had been talking about. Just all of a sudden he saw the replacement. T3: I was amazed by that.	R S R S R S S R S
00:22:39 – 00:26:27	3	R2: The first two (Scale problems), they (students) were really engaged in, but I think the third one (tug-of-war) really stumped them.	S C

		T4: I think it stumped them but…	
		R1: I don’t think so.	
		T4: I don’t think that they were, I don’t think they recognize. I don’t think that they think that they are stumped, so to speak. Like I think that they…	S
		R1: That is a good way of...	
		T4: I think that they believe that, oh, it’s, it’s going to be this or whatever. I think it is taking them a little longer to come up with some stuff but. You know, I think that, I don’t know yet.	S
		R1: I think, I think you are right. I think they approached that last problem much more sort of intuitively [O3 and R2 agree]. And it is almost like I was just saying, they don’t think proportionally, as yours were [addressing O3].	CS S R
		O3: These two, Dajuan I think his name is, and the other.	
		T4: Rene.	
		O3: And Rene, they were stumped (with the tug-of-war problem). They were just sort of sitting there. You know, you could tell they were overwhelmed.	S
		T4: And that I think it was a long day.	
		O3: I asked them if they needed help getting started and I think Rene said “no, Dajuan was just thinking” and Dajuan was [unclear, laugh]. Daujan was not thinking. So I said, so I said, well if you change your mind I’ll be happy to help you get started. And then they asked, they said “okay”. And all I had to do was just one little…	S
		R1: What did you do?	
		O3: I just asked them to look at the second line	
		… … …	
-00:24:26 … … 00:24:54-		O3: And I just said to him, I just asked him to look at the elephant and the oxen and the horses and to substitute. I just said look down. What equals an elephant down in the last one? And as soon as I said that, he got it [snapping his fingers] and he replaced the elephant with the horses and the oxen. And then he was able to jump up to the top line and say he knew that the left side would win because five horses were stronger than four oxen.	RS
		R2: I think (O4) [unclear] cut it.	
		T3: Oh, yeah.	
		R2: But that I think is a really good question. The key question in this problem is to get them to start thinking about how to proceed and how to figure out	R

		which side is stronger and think about if I literally can pick this piece up, replace it in the other problem to even it out. R1: I think that is really smart. What I keep wanting to do is, is just what just (O3) was saying, what kind of question can you ask that doesn't do it for them? R2: That does it for them. R1: Because I want them to see that. I mean for me cutting it and putting it down here is giving them the answer, and I don't want to do that.	R R
00:27:18 – 00:28:10	4	T4: I wanted to kind of breeze through the first two (scale problems) to get them to have more time with the others. That is what was in my mind. I don't know if I really got to do that. R1: But they really needed time with those questions. T4: Right, right. And so… R1: Were you surprised at that? Did you think it would be…? T4: No, I wasn't surprised. I wasn't, I didn't have true expectations. I was, I wasn't, I wasn't expecting anything because of the fact that we didn't have a conversation about this at all. So, right, so when, when I came, like at one point I was thinking I need to go over some things, you know, so that we can move forward. But I chose not to because after listening to them, you know I was thinking, oh okay, I don't have to. No, I was like, I don't have to take charge of that. I can, I can let them keep doing it. They are doing fine. R1: They really needed that (time).	C R C R
00:28:46 – 00:29:23	5	R1: What did you see [addressing T3]? T3: What did I see? I sat with Cristian and... What's that little girl's name? Yaasmyn (while doing the Carrots problem). And he kept saying, as you spoke about proportion, he kept saying the same every time he did certain things. Ironically enough, he didn't look at it, though he saw them the same, I didn't see that level of proportional reasoning in his work. The kids, he kept saying "these are the same, these are the same" (referring to the two pans on each scale). However, as they look at the scale, each scale basically has its own identity. And so that's why… T4: And they don't know how to make that the	S RS RS S

		relationship. T3: They did not make the connection between the first scale, the second scale, and the third scale.	RS
00:34:12 – 00:37:36 - 00:35:33 … … … 00:36:48– - 00:37:03 … … … 00:37:17-	6	R1: Did you do the Bartering (problem)? [Addressing T; Pause as T4 responds to a telephone call]. You know the first one (in the unit, as it appears in the book), where you trade off goats and sheep and corn and stuff. It is the first in the sequence and (R2) and I were talking about that what we've done today, even more than last week, is we really followed the development that the unit has. But the Barter. [R2 hands over the book to R1]. T4: I think what'll do before I finish the tug-of-war [unclear]. R1: [unclear] before you had done that, that, that I would… O3: So the Barter, is this first? R1: Yeah. O3: Because I actually think it's, well to me it was hard. R1: Well, it's sort of [unclear].It's how you get into this idea of having to substitute or barter. And so when our discussion about how hard it was for them to get into that idea. Maybe that's why they did this. … … … R1: I am just wondering how it, I don't know what it'll do if you did it after the other, but if doing it at the beginning, begins to give you some sense of [unclear]. … … … R2: It's interesting though because we thought, as a group, that this [unclear] being the earliest and beginning ideas of algorithm that they need to get that. But [laugh] it is interesting that maybe you need to trade before you can balance.	C C RCS C RC RC
00:41:52 – 00:43:36	7	T4: I usually, I try to (draw a picture), I think it's because I am visual that I try, and I know there are several kids in here, after you look at it, they are like " oh, okay' But I knew that I was losing track of what he (Nazeer) was saying. R1: Oh, no, I thought what you did was just fine. T4: So I had to, like I was like, let me draw this. And I was trying, I was really trying to do it so I could help him. But I wanted to make sure that was	RC C C R

		what he said. So I was like, now, did you…? And he was like "yeah", because he would tell me, like, he would say. R1: Whatever, I hope you [addressing R2] got her drawing on the video because his self-correcting from the three and three, or whatever it was, when he saw the picture was great. T4: I don't know if that, okay now that, was that too much help? Like did I..? R1: Oh, no no. He did it himself. He self-corrected. T4: but you know how teachers lead things sometimes, like I didn't want to do that, and I didn't know if… R2:, you understood truly that he wanted to put three peppers on one side… T4: And he said "put the corn back". And I was like, ooh, it's a first time I've heard of that, putting a corn here and taking that, like, I was like, okay.	R R S S
00:47:10 – 00:49:18	8	R1: What you saw with hers (D'nea's reasoning) was almost, just classic, was an inability to think conditionally. The only thing she could come up with were the things she saw, which was this is equal to this and this is equal to this. And so not only is it the proportional thinking, but it is also the if-then, the conditional stuff, which is really hard for the kids. T4: I think I want to, like I am very interested in, because when I went over there I wasn't sure whether she was just, "okay, I am done", or whether or not it was too much for her. I didn't really know. So I brought out this other algebra book and so I figured let me back up to see exactly what she knows, to get to see where is it that she is stuck. So, you know the first part when we were talking about the balance, you know, and I was trying to make sure that she knew that they are balanced. So what does that mean? What would it look like if it is not balanced? And she wouldn't, she wasn't ready, she didn't want to talk any more. So I showed her a picture. R1: She had actually gone to sleep. T4: yes. R1: So she was like that [indicating drowsiness]. R2: Who?	RS R RS C S S S

		T4: D'nea. So she, she, oh, I showed her a picture. And in the picture it had a scale that wasn't balanced, so you could see it. So I said, well I was asking her, are all of these balanced? She was like [shaking her head]. Then I said, well which one is not balanced? So she pointed [R2 pointing]. Yeah, exactly. And then, so I was thinking, okay, let me come back to you another day. But I, so I know she knows balance. So now my next step would be to say, okay, so why are these balanced, or something like that trying to…	S R
00:49:51 – 00:51:40 - 00:50:10 … … … 00:51:00 -	9	T4: I think I am going to use this book [takes a book out of her bag], this book that I was showing you, you know, I was just showing her the picture. R1: What is this, 4th grade? [T4 shrugs]. Let me see. [R1 and T3 start looking through the book.] … … … R1: You know what is really interesting is [unclear] this big ideas [shows T4 something in the book] and think about how we've been working with every one of them in this set of problems. Here is the representation which, especially with Jameel, who said "well, I have done it in two ways". But, and then there is this notion of balance, the notion of function, proportional reasoning, variables, and inductive reasoning, which is the what if-then, you know [unclear]. But then we are doing every one of these things.	C C S C
00:52:24 – 00:52:58	10	R1: What (O3) and I were talking about was when somebody, when somebody is done or somebody is ahead, I mean it's boring to say check your work if you have already done it. But the challenge, you give Oscar this challenge, if you (T4) re-visit this at all on Monday and you too (T3) for the tug-of-war, suppose the elephant was only as strong as an ox and one horse, not two horses, who would win then?	C

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