SYSTEM RELIABILITY AND MAINTENANCE MODELING WITH CHANGING AND UNCERTAIN FUTURE STESS PROFILES

By

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ABSTRACT OF THE THESIS

System Reliability and Maintenance Modeling with Changing and Uncertain Future Stress Profiles

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The US Navy has increased interest in the reliability of aircraft launching and recovery equipment. Data is readily available and failure time distributions can be estimated; however, the aircraft equipment will not be operated with the same stress profile in the future as the data provided. In fact, the US Navy will increase the stress profile on the equipment by incorporating heavier aircraft into the fleet, while downsizing the lighter aircrafts. This creates an uncertain stress profile the aircraft carrier systems will be subjected to. Since the composition of the fleet is uncertain, determining reliability and component redundancy and/or replacement is difficult. Thus, new models and optimization algorithms are proposed involving data analysis at the component-level based on Weibull shape parameters modeled after using a general log-linear model based on the mean and variance of critical stress measures in a changing environment, and Weibull shape parameters modeled using a general log-linear model based on the distributional form of critical stress measures in a changing environment.

Traditional system reliability considers a set of failure data which is analyzed to estimate a failure time distribution. This failure time distribution can be utilized to estimate reliability at some point in time. This thesis pertains to design problems with a probabilistic future stress profile, but using models based upon the current failure data. Since a future

stress profile can be probabilistic and distinctly different, the traditional system reliability model will be unable to estimate future reliability from the existing failure data. Instead an estimate of the future failure time distribution must be made utilizing accelerated life concepts, and the optimal component reliability becomes difficult to determine. Depending on the level of usage, the optimal component redundancy might change. This research tries to develop a heuristic for system reliability optimization considering a probabilistic future stress profile in which the stresses can increase to different levels.

A failure time distribution is determined for each system component as a function of usage stress distribution. The component models are then assembled into a system model. This system model tests different composition of fleet data based upon different probabilities. Although these probabilities are ambiguous it is certain that the stress profiles will increase. This system model was evaluated to determine what preventative maintenance or component replacement can be done in the present so that the unknown future stress profile will not cause high costs in labor and replacement parts.

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Dedications

To my parents, Kyoung and Kishimitsu Hada, for raising me.

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1.0 Introduction

This thesis evaluates empirical data obtained from a discrete loading system with predictable, quantifiable and changing loading patterns. Every usage cycle can be different, due to a different load, pressure or force being applied to the system. Taking this cyclic data and forming a stress distribution only describes past occurrences. However, the future loads on the system are anticipated to be increasing due to changing user preferences or system requirements. This creates a shifting stress distribution with time. An example of this shift is depicted in Figure 1.

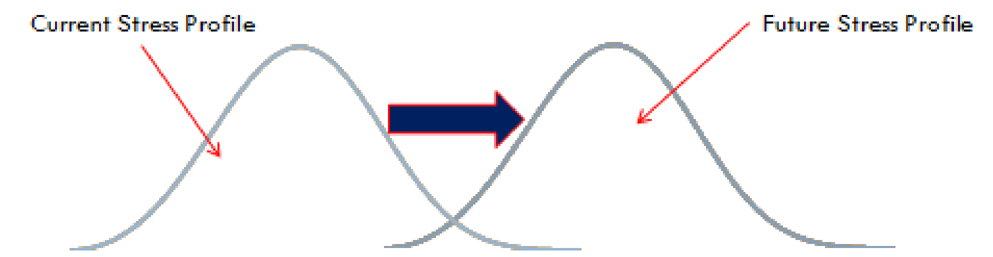


Figure 1. Movement of the current stress profile to the future stress profile with time

This future stress profile represents a single possible future with a certain probability, and there are several possible future scenarios. In practice these probabilities would only be estimates, but consideration of this probabilistic future scenarios lend to more robust designs and maintenance plans. After different future stress profiles with different probabilities are determined, a simulation model was constructed and run to determine the optimal component replacement as well as preventive maintenance schedule for the system.

1.1 Background

NAVAIR (Naval Air Systems Command) provides systems and material support for the US Navy. NAVAIR Headquarters is a tenant of the Naval Air Station in Patuxent River (Pax River) in Maryland. Ten other locations exist, eight within the United States and two international locations. The Lakehurst NJ, branch specializes in support equipment for both the Aircraft Launch and Recovery Systems.

The recovery gear or arresting system is designed to rapidly decelerate an aircraft when it lands on a naval vessel. The major systems used within a typical arresting system are the hook cable or pendant(s), purchase cables, sheaves, and arresting engine. The arresting engine absorbs and dispels the energies of a landing plane. The sheaves redirect the purchase cable and the hook cable or pendant attaches itself to a landing aircraft and is connected to the purchase cable.

The launcher or catapult system is an aircraft catapult device used to deploy aircrafts from the Navy aircraft carriers. It consists of a track, a large piston, and shuttle. To launch an aircraft, steam pressure is built up in the cylinders and then released. This causes the piston to release which in turn pulls nose gear assembly which is attached to the aircraft. The aircraft is dragged along a track and the velocity due to this release will be sufficient to allow the aircraft to take flight.

1.2 Problem Statement

Improvements in aircraft technology coupled with heavier equipment and the discontinuation of the lightweight T-45 aircraft will cause an increase in the average weight of an aircraft in the Navy air fleet. This expected increase in weight pushed Navy officials to consider that the extra weight may cause accelerated wear in both the arresting and the catapult systems. Both systems will still be utilized for the next 20 years and the future reliability of each must be calculated. In essence, the heavier air fleet and the responding heavier loads will cause reliability to decrease and in order to assure that the equipment can withstand the new stresses the reliability must be calculated. After the future reliability is obtained a redundancy design as well as an optimal corrective and preventative maintenance policies are simulated. Figure 2 depicts the average cable tension per arrestment (landing) from 1976 to 2007. As shown in the graph, the gradual increase in tension is what Navy officials have concern over. The last three entries are projected future values of tension.

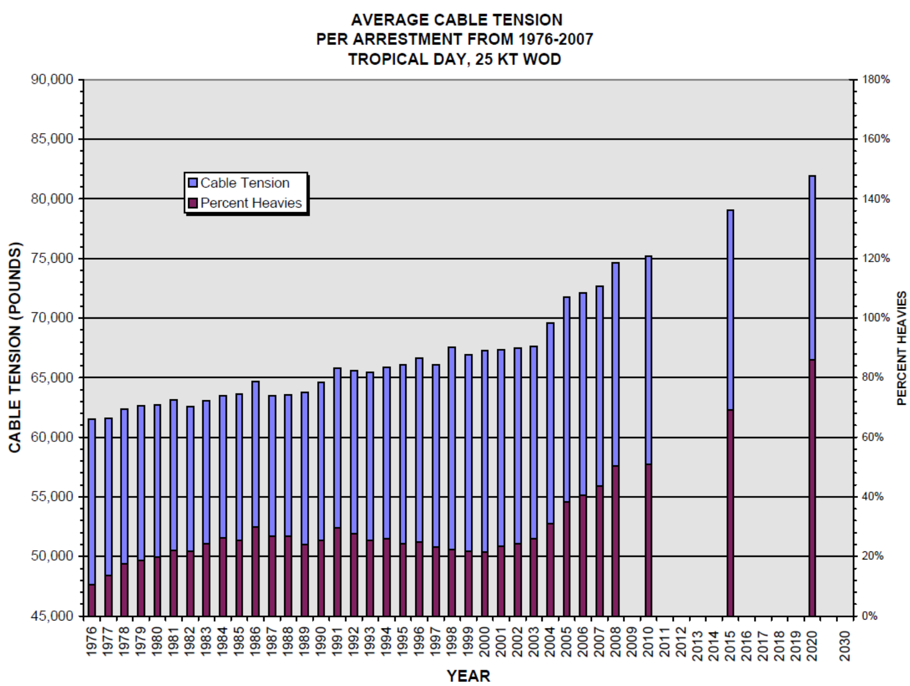


Figure 2. Projected Stress Increase on Pendant

2.0 Literature Review and Background Concepts

There have been many different papers and articles written about component replacement, each with unique applications and methods.

2.1 Component Replacement

A replacement of a component is optimal usually when the maximum useful life of a component has been consumed. Although preventive maintenance may prolong the useful life, the optimal replacement time is the time right before a component fails, as you have gotten the most usage out of the component without having to experience a failure or any unplanned downtime. Age replacement policy (ARP) is one method to try and optimize the replacement of a component. However, implementation of ARP requires continuous tracking of a component's service life. Many industries with large systems, each having a number of components, find this difficult to achieve in practice. Another option for maximizing the useful life of components is to continuously monitor the condition using sophisticated on-line instruments and to replace them just before failure. However, such a proposition is expensive and the time indicated for immediate replacement may not be suitable for a plant shutdown, because of production conflicts or any other kind of scheduling (Das and Acharya, 2004).

Two effective policies to be considered and compared are Age Replacement during Delay Time (ARDT) and Opportunistic Age Replacement during Delay Time (OARDT). Age replacement, as previously mentioned, is the policy where the component is replaced on failure or after a fixed period of service after the fault. Opportunistic failure utilizes the first random occurring opportunity for preventive replacement of a faulty component if it has given service for a fixed period after the fault.

Das and Acharya (2004) use the long run cost per unit time according to the renewal reward or,

$$G_d(t_d) = \frac{\text{expected cost during life cycle}}{\text{expected length of life cycle}} = \frac{C_d(t_d)}{L_d(t_d)} \tag{1.1}$$

$G_d(t_d)$ = long run cost per unit time for ARDT policy
$C_d(t_d)$ = expected cost during the life cycle
$L_d(t_d)$ = expected length of the life cycle
t_d = time

The expected cost in a renewal cycle is sum of the expected preventive replacement cost, the expected failure replacement cost, and the expected cumulative degradation cost per renewal cycle. This is expressed as :

$$C_d(t_d) = C_p\left[1 - F_H(t_d)\right] + C_f\ F_H(t_d) + \left\{C_r\left(\min\left(H, t_d\right)\right)\right\} \tag{1.2}$$

C_p = cost per preventive maintenance
$F_H(t_d)$ = probability of failure due to degradation at time t_d
C_f = cost per failure
H = degradation
t_d = time

$C_r(\min(H,t_d))$ = expected cumulative degradation cost over a renewal cycle.

These equations are used for the ARDT policy.

For the OARDT policy (Das and Acharya 2004) the long run cost per unit time is

$$\frac{\text{expected cost during the life cycle with opportunistic replacement}}{\text{expected length of the life cycle with opportunistic replacement}} = \frac{C_{od}(t_d)}{L_{od}(t_d)} \tag{1.3}$$

$G_{od}(t_{od})$ = long run cost per unit time for OARDT policy
$C_{od}(t_{od})$ = expected cost during the life cycle with opportunistic replacement
$L_{od}(t_{od})$ = expected length of the life cycle with opportunistic replacement
t_d = time

The renewal cycle cost is the sum of the expected cumulative degradation costs, and the expected replacement cost. By plotting the ARDT and OARDT long run cost per unit time and calculating both breakeven points, the authors suggest that one must prefer opportunistic age replacement during delay time policy to age replacement during delay time policy.

When creating a preventive maintenance schedule or component replacement for anticipated future stress levels, Das and Archaya (2004) have noted that instead of replacing any age related component it might be more beneficial to follow an OARDT policy.

Yamada and Osaki (1981) wrote a paper on optimal replacement policies for nonessential and essential units. Many papers comparing age and block replacement policies have been written and published, one such paper is Barlow and Proschan (1965). In Barlow and Proscha's model two costs are evaluated, one associated with a corrective action (replacing the unit) and one cost associated for a preventive maintenance cost (non-failed unit being replaced). This paper concentrates on nonessential units and develops a method to estimate the appropriate numbers of spares that should be provided given both the preventive maintenance cost and corrective maintenance cost for a componen

2.1.1 Architecture for Component Replacement

While technology advances, software systems must evolve due to improved technology and changing requirements. Postma, America, and Aijnstra (2004) use a 3RDBA (three cycles consisting of steps Requirements, Design, Build, and Analyze) approach that facilitates replacing a key component in a long-living architecture. The approach consists of an exploration, consolidation and migration cycle. Each cycle contains four steps: Requirements, Design, Build and Analyze (Postma et al., 2004).

The example Postma et al (2004) used to illustrate the 3RDBA approach was a medical imaging system, a system which would be in use for 15 years. A decision making tool to decide whether a component should be replaced by a functionally similar component, one with extended functionality, or the same component. 3RDBA represents a different approach that could be utilized for aging systems with increased usage requirements. However, the approach is nonmathematical and only aids in a decision, and no actual quantitative methodology is provided.

2.1.2 Markov Chains

Albin and Chao (1992) utilize Markov chains to model a multi-component series system to determine the optimal preventive replacement in which the component deteriorates with time. The time causes the operational characteristics of the component to change and consequently increasing the failure rate of the component near the deteriorated one. The replacement policies involve inspections, and if the deterioration exceeds a critical level, replace the component, or continuously monitor the deteriorating component until failure. The replacement policies are evaluated by mean cost rate and by the ratio of the reduction in the number of failures to the number of preventive replacements.

There are other sources for extensive bibliographies on maintenance models for deteriorating systems, including Barlow and Proschan (1965), McCall (1965), and Pierskalla and Voelker (1976). Much of the work focuses on one-component systems and is based on the original Markov chain model for describing the deterioration process.

2.2 Stress Models with Covariates

A model to alter the life parameter in the Weibull distribution depending on the mean and standard deviation of the stress loads of the system was developed to access the availability of a system due to future loads. Covariate models are used to represent the effect of different treatments or usage conditions in a lifetime model. A covariate is defined as a treatment or explanatory variable that influences the failure time of the component. Typical covariates include those that represent mechanical forces, material properties, and environmental factors. There are two rather popular approaches for linking these covariates to the failure time probability function. The first method, known as Accelerated Life Testing (ALT), is based on modifying the time axis of the survivor function. The original application of ALT was to reduce the time to test production components by increasing, or accelerating, the primary explanatory factors and using the resulting model to predict component lifetimes under standard in-service conditions. The premise of the second approach, called Proportional Hazard Model (PHM), is to modify the hazard rate function to include the covariates (Wallace, 2004).

Wallace (2004) simulated and demonstrated a multi-response component failure distribution as a function of operational parameters. Although Wallace generated his data from a sophisticated system model, the data gathered in this thesis was simulated following a certain distribution. Furthermore for real applications, physical data is used to calculate the loads on the system. A significant difference is the use of joint probability models that Wallace utilized to represent the joint randomness. Assuming that *z*, the joint randomness term using a function of standard normal variants, is standard normal then the joint probability density function is given by

$$f(x) = \prod_{i=1}^{n} f_{x_i}(x_i) \frac{\varphi(z, C')}{\varphi(z_1)\varphi(z_2)...\varphi(z_n)} \tag{2.1}$$

Where $\varphi(z, C')$ is the *n*-dimensional standard normal probability density function of the standard normal variables, *z*, and *C*' is the correlation coefficient matrix of the transformed space with elements ρ_{ij} . Wallace then further considers a joint covariate model and selects a turbine blade engine to test the reliability of the simulated data. First 10,000 simulations are used to determine appropriate parametric distribution for the overstress and fatigue life failure modes. The Anderson-Darling Test statistic was used to compare the fit for the data. An Accelerated Life Test is used with the log-quadratic link function to account for covariate models in the ALT model. A quadratic polynomial function is assumed for the exponential component of the link function is

$$\Psi(z) = e^{g(\beta z)} = e^{(\beta_1 z_1 + \beta_2 z_2 + \beta_3 z_1 z_2 + \beta_4 z_1^2 + \beta_4 z_1^2)} \tag{2.2}$$

Wallace demonstrated the use of simulated data as well as ALT testing for a design of experiment of a turbine blade engine. The use of a joint probability distribution and a quadratic polynomial are not used in this thesis. However, ALT and a covariate models are developed.

2.3 Heuristics

There are many heuristics to consider when trying optimizing parameters for design problems. Some of the most common are explained in this section. A metaheuristic is a method that optimizes a problem by iteratively trying to improve a feasible solution. Metaheuristics make few assumptions and search large areas of feasible solutions.

The most recent metaheuristic is the cuckoo search by Yang and Deb (2009). However, the most common methods are simulated annealing (Kirkpatrick et al., 1983), Tabu search (Glover, 1983), genetic algorithms (Holland, 1975) and ant colony optimization (Dorigo et al., 1991).

2.3.1 Genetic Algorithms

Genetic algorithms (Deb and Goel, 2001) are a population based search that can evaluate multiple solutions in each generation (i.e., run). The use of genetic algorithms stems from the versatility of the heuristic as well as the simplicity.

The steps for a simple genetic algorithm are:

1. Start with a randomly generated population or candidate solutions.
2. Calculate the fitness or quality of the solutions (called chromosomes) in the populations.
3. Repeat the steps until *n* off-springs have been generated
    a. Select a pair of parent chromosomes from the current population, with the probability of selection being an increasing function of fitness or quality. The same chromosome can be used more than once to become a parent.
    b. With another probability called the crossover rate, crossover a randomly chosen solution chromosome to form two off-spring solutions. If there is no crossover, then form two off-spring that are exact copies of the parents.
    c. Mutate the offspring at each locus with a mutation probability and place the new chromosome in the new population. If *n* is odd, one new population member can be discarded at random.
4. Replace the current population with a new population.
5. Go to Step 2

These steps are from Mitchel (2005). For more on genetic algorithms Srinivas and Deb (1994) and Deb, Agrawal, Pratap and Meyrivan (2000) are a few relevant summaries among many papers written on genetic algorithms. There are many different variants and genetic algorithms can only produce a good solution and not the optimal.

2.3.2 Simulated Annealing

Simulated annealing is a local search for locating a good approximation of an optimum given a large space; it was introduced by Kirkpatrick (1983) and is based upon the annealing of metal. When metal is heated and then slowly cooled at a controlled pace, the number of defects in the crystal the metal forms can be reduced. The simulated annealing algorithm replaces the current solution for a problem with a solution closely related and then begins searching for a better solution in the neighborhood of the closely related solution.

In some simple steps the simulated annealing process can be described:

Step 1: Decide on the number of iterations for the program to run.

Step 2: Start at an initial solution S_0.

Step 3: Calculate the objective function and store it.

Step 4: Generate a neighborhood solution and calculate the objective function and store this new value.

Step 5: Based upon the acceptance probability, accept or reject the new objective value.

a.) If accepted, set the new solution or objective value as the best and update and store the value.

b.) If not accepted, than disregard the new objective value.

Step 6: Repeat steps 5 and 6 till the number of iterations is reached.

These steps are a summary of Muralikrishnan (2008) which provides optimization of a portfolio. Muralikrishnan also compares simulated annealing to generic greedy algorithms and states that if the probability of acceptance is zero, then simulated annealing operates as a greedy algorithm and moves to all solutions with the highest objective function value. When this greedy algorithm and the simulated annealing were compared, the simulated annealing outperformed the greedy algorithm in almost every case.

Many other variations of simulated annealing have been created such as MOSA (Multiple Objective Simulated Annealing) developed by Ulungu, Ost and Teghem (1998) or Enhanced Simulated Annealing Algorithm (Loganantharaj, 1997). There are also a myriad of works that have compared and contrasted different types of simulated annealing as well as genetic algorithms.

2.3.3 Tabu Search

Tabu search is attributed to Glover (1986). It is a mathematical optimization procedure which is similar to simulated annealing involving a local search method. This method utilizes a memory in which solutions are put on a "taboo" list, a set of solutions that the algorithm does not revisit. There are three main strategies to Tabu search. The first is the forbidding strategy in which the algorithms control what enters the Tabu list. The second is a freeing strategy, which controls what exits the Tabu list. The final method is a short-term strategy that manages both the forbidden and free strategies to select a trial solution. The basic components of a Tabu search consist of a memory or list to classify moves or searches that are Tabu. A neighborhood is calculated and identified for closely related solutions that can be reached from the current solution. The Tabu list can be overridden, this is called an aspiration criteria where the solution in a Tabu list is better than any visited.

The Tabu search steps are explained below:

Step 1: Start with an initial solution in a set.

Step 2: Generate a subset of solutions such that either one of the Tabu conditions is violated or at least one aspiration condition holds.

Step 3: Choose the best solution in the subset.

Step 4: If the best solution is better than the global best solution, then set the solution as a global solution.

Step 5: Update the Tabu list and the aspiration conditions.

Step 6: Count the iterations or go back to step 2.

The Tabu search may be terminated in many conditions such as, if there is no feasible solution, when the noted number of iterations has been reached. The number of iterations since the last improvement has been met. Figure 3 shows a simple flow chart (Lei, Liu, and Roberto, 2010)

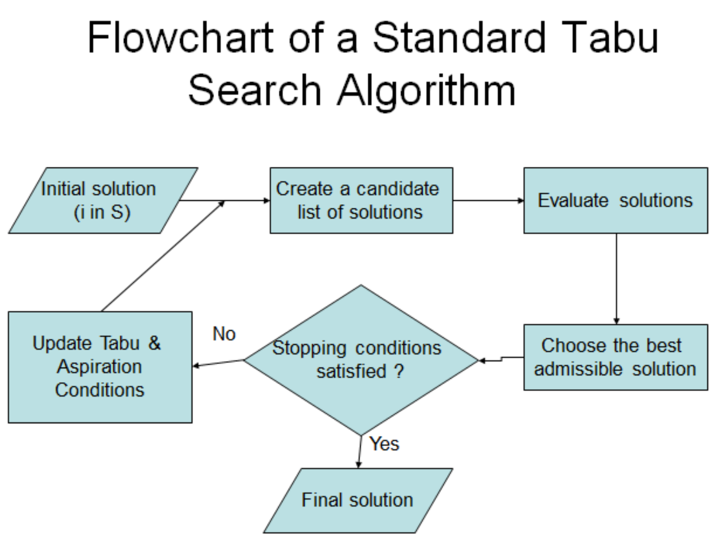


Figure 3. Flow Chart of Tabu Search

2.3.4 The Cuckoo Search

The cuckoo search is based upon the cuckoo species which lay their eggs in the nests of other host birds (Yang and Deb, 2009). Each egg in a nest represents a solution, and a cuckoo egg represents a new solution. The aim is to use the new and potentially better solutions (cuckoos) to replace a mediocre solution in the nests. In a nest there are multiple solutions, and a new solution or cuckoo egg is added. The objective is for a new and potentially better solution to replace a mediocre solution in the set. There may be multiple sets or only one solution in each set (Yang and Deb, 2010).

The cuckoo search has three rules:

1. Only one new solution can be inserted per cuckoo, and each new solution is placed in a randomly chosen set of solutions or nests;
2. The best nests or set of solution with high quality of eggs will carry over to the next generation.

The number of available host's nests is fixed, and the egg laid by a cuckoo is discovered by the host bird with a probability from 0 to 1. Discovering the new solution depends on some set of inferior solutions.

2.4 Availability

The main focus of this research is to maximize availability given some constraints. A brief review of availability is presented here. Availability contains both reliability and maintainability, which makes it a valuable metric to industry. Availability can be classified either into the time interval consideration or the type of downtime (Elsayed, 2009). This review summarizes average up-time availability, steady-state availability and the inherent availability.

In many systems it is vital to know the availability in certain time intervals. This is referred to as the average up-time availability and can be expressed as

$$A(T) = \frac{1}{T}\int_0^T A(t)dt \tag{2.3}$$

$A(T)$ = average availability

T = time (2.4)

$A(T)$ can be estimated by obtaining an expression for $A(t)$ as a function of time or by numerically solving the probabilistic state transition states and summing the probabilities. The average uptime availability may be the most useful for systems whose usage is defined by a duty cycle. Steady-state availability is the system availability when the time interval is very large. Steady-state availability is a metric utilized for systems that operate indefinitely such as communication cables.

The final availability described in this thesis is inherent availability, which only includes the corrective maintenance of the system and excludes ready time, preventive maintenance downtime, and logistics down time. This is expressed as

$$A_i = \frac{MTBF}{MTBF + MTTR} \tag{2.5}$$

MTBF = Mean Time Between Failure
MTTR = Mean Time To Repair

The steady state and inherent availabilities are the same when all of other logistics times are ignored and only the corrective maintenance time is considered.

There are many other types of availability, such as achieved availability, operational availability (includes the logistics time), mission availability, etc. The inherent availability is more widely used as logistics time and ready time can be difficult to determine. Choosing the proper availability and proper metric is necessary and determined by what key performance indicators are necessary.

2.5 Accelerated Life Testing

The purpose of accelerated life testing is to induce failures at a faster rate in a harsher environment. The underlying assumption in relating the failure data to the accelerated life is that the components operating under the normal conditions experience the same failure mechanisms under normal conditions in the accelerated environments. In other words, the harsh environment should not impose entirely different failure modes.

An accelerated life model usually consists of a life distribution and a life-stress model. A life distribution is a distribution models failure time data. Some common life distributions are the normal or Gaussian distribution, the exponential distribution, a popular distribution where the failure rate is constant; and the Weibull distribution, in which parameters can be altered to mimic other distributions. Choosing a life distribution can be based upon previous data or physics models. Electrical systems are commonly modeled as exponential and mechanical fatigue as log-normal.

The life stress model relates the incremental increase in stress of the harsh environment in the environment to the failures; for example if an experiment is set up with three levels of humidity; normal, high, and intense. The failure for each will be recorded and observed and the life stress model relates the levels of humidity (stress) to the failure times (actual time). The most common accelerated conditions are temperature, humidity, and voltage.

2.5.1 Arrhenius

The Arrhenius relationship is commonly used for analyzing data for which temperature is the accelerated stress. The relationship is as follow:

$$R(T) = Ae^{-\frac{E_a}{KT}} \tag{2.6}$$

where R is the speed of the reaction, A is a non-thermal constant, E_a is the activation energy and K is the Boltzmann constant and T is the absolute temperature.

2.5.2 Eyring

The Eyring model is commonly used for analyzing data for which temperature or humidity is the accelerated stress. The model was formulated from quantum mechanics principles. The expression is as follows:

$$L(V) = \frac{1}{V} e^{-\left(A - \frac{B}{V}\right)} \tag{2.7}$$

L represents a quantifiable life measure, such mean life, characteristic life, median life, $B(x)$ life, etc. V represents the stress level (temperature values in absolute units, i.e. degrees Kelvin or degrees Rankine). A is one of the model parameters to be determined. B is another model parameter to be determined.

2.5.3 General Log-Linear Relationship

The general log-linear relationship describes a life characteristic as a function of vector of n stress.The mathematical relationship is given as:

$$L(\mathbf{x}) = e^{\alpha_o + \sum_{j=1}^{n} \alpha_j x_j} \tag{2.8}$$

α_j are model parameters. $\mathbf{x}$ is a vector of n stresses. This relationship can be further modified through the use of transformations and can be reduced to the relationships discussed previously, if so desired. As an example, consider a single stress application of this relationship and an inverse transformation on x, such that $v = 1/x$ or:

$$L(v) = e^{\alpha_o + \frac{\alpha_1}{v}} \tag{2.9}$$

$$= e^{\alpha_o} e^{\frac{\alpha_1}{v}} \tag{2.10}$$

2.5.4 Life Stress Models Using Stress Strength Interference

A unique portion of this research plan comes from deriving a future stress profile. While in this plan the method utilized is based upon the standard deviation and mean of the accumulated stress in previous data, there have been other approaches. A similar utilization of three types of usage data and their treatment for performing reliability predictions is explained by Mettas and Vassihiou (2002). He explains that the stress conditions depend on the way the product is used and not every customer uses the product in the same way. Certain customers operate the product at higher stress levels than others. For example, every user does not accumulate 12,000 miles a year on a vehicle and every user does not print the same number of pages per week on a printer (Mettas, 2005).

Now if thought of differently, a future stress profile can be thought of as a different customer usage profile, one with higher stress and higher loads. Mettas (2005) explains the traditional theory of accelerated life models with a life-stress relationship. Represented in Table 1 are some of the common life-stress relationships

Table 1. Common Life-Stress Relationships

Relationship	Model
Arrhenius	$t_p = Ae^{\frac{B}{S}}$
Eyring	$t_p = \frac{1}{S} e^{-\left(A - \frac{B}{S}\right)}$
Inverse Power	$t_p = \frac{1}{KS^n}$

The life characteristic, t_p, can represent any percentile of the distribution. The percentile is selected according to the assumed underline distribution. Some typical life characteristics are presented in Table 2 and by using the maximum likelihood estimator (MLE) the parameters for the distribution as well as the life characteristic can be obtained.

Table 2. Typical Life Characteristics

Distribution	Parameters	Life Characteristic
Weibull	β, η	Scale Parameter (η)
Exponential	λ	Mean Life ($1/\lambda$)
Lognormal	μ, σ	Median (T)

In the following example is taken from a technical paper (Mettas and Zhao, 2005). An electric motor with a warranty of 1,000 cycles has three loads that are being tested; 6 lbs., 8 lbs., and 12 lbs. loads. The Weibull-inverse power model was fitted to the data set and the life characteristic and Weibull parameters were calculated. It is stated that an average customer is assumed to use the 7 lbs. stress level. However, this is done purely through test data. Through surveys, Mettas (2005) collected additional data that showed the actual customer usage profile.

With this new data a new problem developed, namely how to relate the new load distribution based upon the customer usage profile to the life of the different motors. In other words, if a percentage of the motors are utilized at stress level *S* what would be the calculated reliability? This needs to be repeated for all stress levels that are experienced in the field in order to estimate the overall percentage of units failing by time *t*. For this, a stress-strength interference analysis will be used to obtain the percentage failing during warranty from the whole range of load sizes applied in the field. The equation of stress-strength interference is given by:

$$P(x_2 \geq x_1) = \int_0^\infty f_2(x) R_1(x) dx \tag{2.11}$$

where x_2 = stress

x_1 = strength

$f_2(x)$ = stress function

$R_1(x)$ = reliability of the strength of material

Given a distribution that describes the different stress level (custom surveys), and a distribution that describes the strength of this unit, the probability of failure can be calculated as the probability of stress exceeding the strength. The test data on the motor of the different stress levels provides a strength distribution. Using these distributions, one distribution can gives the percentage of units operating at each load size, and another can give the percentage of units that fail at each load size during the warranty period of 1,000 cycles. Using the stress-strength interference model, the probability of failure can be calculated for different customer profiles.

Considering stress profiles and calculating reliability, this method provides solid quantifiable results. Stress-strength interference is the alternative method utilized together with an accelerated life method from empirical data. Mettas' paper points out the alternative in developing a stress profile as well as calculating reliability through stress-strength interference.

3.0 Probabilistic Futures

This thesis consists of several related tasks. The first is to identify a correct stress usage profile from current data. This data is then utilized to create distinct future stress profiles. Each future stress profile has a probability associated with it. Figure 4 describes the current data and the possible different stress distributions that can manifest in the future. The different composition of aircraft creates different loads which affect reliability.

The future air wing composition is unknown and thus, a future stress profile can only be estimated.

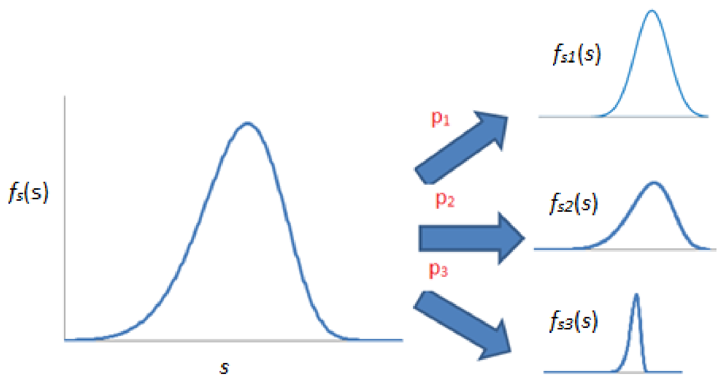


Figure 4. Stress Profile and Possible Outcomes

Once the stress profiles are estimated, reliability block diagrams are used to model the system. This block diagram represents an example arresting or launching gear system. With this model, the optimum preventive maintenance schedule as well as optimum component replacement strategy and layout can be determined. In short, with failure data currently available, a decision on how redundancy or system designs should be changed to meet the cost and manual restraints in anticipation of the future stress profiles.

The basic steps to this research were

1. Gather simulated data.
2. Assign a probability to each stress profiles to accurately represent each possible future stress profiles.
3. Estimate the overall stress profile.
4. Use this profile to develop a preventive maintenance strategy as well as component replacement strategy.
5. Utilize Tabu search or simulated annealing to develop the component replacement strategy as well preventive maintenance schedule.

3.1 Stress Profiles

A review of research concerning distribution parameters with regard to loading and life conditions have revealed that concentrating on failure time might not always be the correct direction. The conventional method is to create a life distribution from failure or test data.

Every cycle can be different, due to a different load, pressure, or force being applied to the system. Taking this cyclic data and forming a stress distribution only mathematically describes past occurrences. In fact, the future loads on the system may be increasing due to user preferences or changing system requirements, which create a shifting stress distribution. The system would still be used in the foreseeable future, and the simulation model must predict system performance and the most unreliable components given the changing stresses associated with the new user requirements.

The basic method for component reliability models considering stress cycle distribution counts all stress cycles regardless of any metric and acts as a baseline explanation. The basic method also would provide the same results as the conventional method of calculating a failure time distribution.

3.1.1 Without Load Adjustment

Assuming that a particular system or component or failure mechanisms are not impacted by stress and that stress would not account for any differences, a simple counting of the cycles would suffice as the rate of failure or where

$$t = \sum_i t_i, \tag{3.1}$$

where t = number of load with only one category

This is the conventional way to record failure data where there is no metric or measurement of force. The advantage of this method is that it provides a definite answer and provides a failure time distribution. However, there are many disadvantages. This method cannot account for a future stress profile as stress is not even considered. This method also does not account for different levels of stress. In summary, the lack of stress cycles and profiles in this traditional method proves to be disadvantageous.

3.1.2 Mean and Standard Deviation of Cycle Stress

The mean and standard deviation method requires more data as well as a useful stress metric unit. By taking the standard deviation and average of each type of load, a Weibull parameter can be calculated to represent the failure distribution. When η is a function of the cycles or load and is altered by the mean and standard deviation, covariates η is shifted into a more appropriate value to represent the future stress profile.

$$f(\tau) = \frac{\beta}{\eta}\left(\frac{\tau}{\eta}\right)^{\beta-1} e^{-\left(\frac{\tau}{\eta}\right)^{\beta}},\ \tau > 0, \beta > 0, \eta > 0 \tag{3.2}$$

$$\lambda(\tau) = \frac{\beta}{\eta}\left(\frac{\tau}{\eta}\right)^{\beta-1} \tag{3.3}$$

η $= \eta_0 \exp(-b_1\mu_{stress} - b_2\sigma_{stress})$
$f(t)$ = Weibull distribution probability density function
β = Shape parameter
η = Adjusted Life Parameter
η_0 = Initial Life Parameter
t = Failure Cycle
μ_{stress} = average stress
σ_{stress} = standard deviation of stresses
b_1 = coefficient for mean stress
b_2 = coefficient for standard deviation

Mathematically, an increased stress profile can be considered by adjusting the mean and standard deviation of every cycle to a higher level. As the mean and standard deviation increase, the exponential function decreases, causing η to decrease.

The advantages of this method is that impact of a force can be directly monitored and can distinguish the load on a component due to increased force. The disadvantage would be that a large and comprehensive data set is needed, large variability will also significantly affect results, and the mean and standard deviation might not be sufficient in describing different load profiles.

3.1.3 Relative Frequency of Cycle Stress

This last failure model provides the most robust parameters by organizing the failure data, cyclic loads, or any other kind of metric unit into subcategories. Once these subcategories are determined, a percentage number is given to each subcategory or in this case x_i. Any number of subcategories can exist and the load may increase or decrease reliability by the simple calculation of changing x_i into either a positive or negative value. This method provides the most organized conditions as well as directly relates the increased force, or stress on the subcomponent. However, the data must be representative and excess variability impacts results. This method also requires more failure data points than other methods.

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^{\beta}},\ t > 0, \beta > 0, \eta > 0 \tag{3.3}$$

$$\lambda(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \tag{3.4}$$

$$\eta = \eta_0 \exp(b_0 x_0 + b_1 x_1 + b_2 x_2 + \ \dots.)$$

$$\sum_i x_i = 1$$

x_0 = 1st relative frequency
x_1 = 2nd relative frequency
x_2 = 3rd relative frequency

3.2 Examples

Since each cycle causes a distinct stress, each failure is associated with a different stress profile. For example if an old Hornet airplane is launched, for that launch the stress on the system will be less than that of the new Hornet which is 7,000 lbs. heavier. Now if only the old Hornets were launched until failure, the system should last longer than that of the system which launched only the new Hornet aircraft. This example illustrates the simplest case of how a failure cycle must be adjusted for stress as the stress of each failure cycle contributes to accelerated failure times.

3.2.1 Stress and Standard Deviation of Stress

With each failure cycle, a stress and standard deviation is calculated. Table 3 is a truncated example of the mean and standard deviation of the simulated data

Table 3. Truncated Data for Mean and Standard Deviation

Cycles to Failure	Mean Stress (lbs) *	Standard Deviation
184	635.98	200.22
320	723.16	202.31
377	599.88	226.54
170	730.699	174.35
540	478.50	200.03
141	685.75	228.69
369	508.51	227.00

This is only a subset from an original example data set. From Table 3 each system use produced a unique stress and these stresses are averaged until a failure occurs. Once a failure occurs, the cycles between failure or failure count as well as the mean and standard

* This is simulated data and does not represent actual aircraft data.

deviation are recorded. The example data includes types of planes each with different stresses. A failure cycle was determined according to a Weibull distribution and the stress was assigned for one of the three planes at each cycle.

With this data, the next step is to define the stress-life relationship. The future stress profile is calculated based upon this current data set according to the mean and standard deviation. Using accelerated life testing and the general log-linear model and a Weibull life distribution, the general log linear equation parameters b_o and b_1 are -0.0038 and -0.0014, and an η_0 of 4,712 cycles. This new Weibull distribution represents the future stress profile as a function of the mean and standard deviation of the future aircraft fleet stress. Figure 5 is the Weibull plot of failure data as of the parameters mentioned.

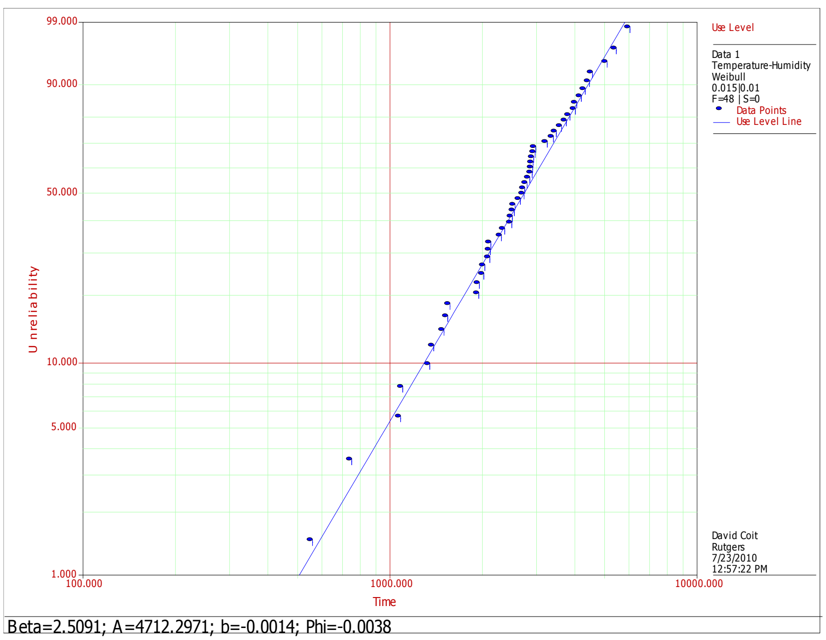


Figure 5. Mean and Standard Deviation Weibull Life Distribution Plot for future Stress Profile

3.2.2 Relative Frequency

The relative frequency method operates under the same principal as the mean and standard deviation but changes the input variables into a relative frequency; the relative frequencies of the stresses are used to categorize stress into three different ranges. The stresses are divided into proportions and are represented by the variables x_1, x_2 and x_3. These relative frequencies must add up to one and can be divided even further, but for this example only three categories were used.

The accelerated life model uses the relative frequencies to calculate an associated coefficient b_i. In the case of the mean and standard deviation each failure cycle had an associated mean stress and deviation. Now each failure cycle has a proportion of categories assigned to a failure cycle. This proportion is used by the accelerated life model to calculate the necessary coefficients using the Maximum Likelihood Estimate (MLE) method. Table 4 is a small example of data analyzed.

Table 4. Truncated Data for Relative Frequency Method

	200-500 lbs	500-700 lbs	700-1000 lbs
Cycle to Failure	x_1	x_2	x_3
184	0.26	0.34	0.40
320	0.20	0.10	0.70
377	0.41	0.18	0.41
170	0.12	0.24	0.64
540	0.68	0.12	0.20
141	0.16	0.34	0.50
369	0.67	0.16	0.30

For this example the stress levels were separated into three groups. One of high stress (700-1000 lbs.), moderate stress (500-700 lbs.) and low stress (200-500 lbs.). Each failure cycle has a composition of each category. In the first row, the component failed at 184 cycles with 26% of cycles having low stress, 34% of cycles having moderate stress, and

40% having high stress.

In this example, b_2 is equal to 0, which means that stresses in the most central category do not increase or decrease η. The variables b_1 and b_3 are calculated using the MLE using software, and a log-linear model. The Weibull plot is shown in Figure 6.

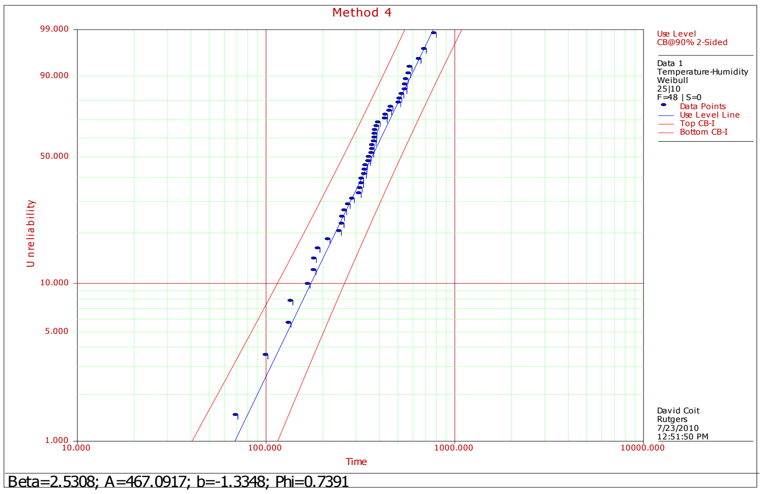


Figure 6. Relative Frequency Weibull Plot

In Table 4 the relative frequencies as well as the failure cycles are presented and the data indicates that b_1 and b_3 are equal to 0.7 and -1.3 respectively. η_0 is 467 cycles and β is equal to 2.53. The future Weibull stress plot is shown in Figure 6.

Choosing a composition of only middle ranked stress cycles ($x_1 = 0$, $x_2 = 1$, $x_3 = 0$) produces the same mean stress as a mixed set ($x_1 = 0.2$, $x_2 = 0.6$, $x_3 = 0.2$); however this method may likely produce a lower η for the more diverse frequency set. The relative frequency method penalizes the heavier stresses and causes the η to decrease due to a negative exponential function. The lighter stress cycles cause η to increase; thus increasing life.

Both methods can create an accelerated life Weibull model based upon current data. For the mean and standard deviation model, a mean and standard deviation must be selected based on future operating profiles and an appropriate Weibull distribution represents the failure cycles of that particular future stress profile with those chosen set of values for the Weibull parameters. The relative frequencies also alter the current Weibull distribution data into a future distribution in which the stresses change into a selected frequency. Both methods are viable options when considering the overall stress because the mean shifts as heavier and heavier aircraft are introduced into the fleet. With the relative frequency method, the high stress category is increased, and thus, causing a shift in the distribution and changing the stress profile.

4.0 Optimal Replacement Times

For every component in the Reliability Block Diagram, there is a preventive maintenance schedule that must be calculated. However, when calculating the optimal time the decision maker must be careful to balance the risks of the failure versus the risks of performing very conservative maintenance. As with everything in the world today, excess maintenance costs money and time. This maintenance may not even be necessary. For example, if a car had to replace the brakes every time it was driven, it would become extremely expensive to maintain. This would be because of the constant purchasing of brake pads. However, the labor involved in installing the pads also takes time. While this will surely decrease the chance of the break pad being overused, it is not necessary for all the excess maintenance. Although the car brake example is an extreme case the underlying message was that for a preventive maintenance to be effective a balance must be found between the cost of unexpected repair and the cost of the preventive maintenance. “Maintenance experts agree that replacing a component before it fails (preventively) may, under certain circumstances, make better economic sense than replacing the component when it fails (correctively). The key is to determine whether the preventive replacement of a specific component is appropriate and, if so, to identify the best time to replace the component. This article presents an examination of the simple concept of determining an optimum replacement time for a single component" (Reliasoft 2009).

Although the preventive maintenance schedule is quickly calculated using a button in BlockSim, the calculation done by the program is balancing the cost factor of the unwanted repair and the preventive maintenance. Two conditions must be met for an optimal preventive maintenance time to be scheduled

Once the failure distribution is known or assumed then a preventive maintenance schedule can be determined. However, the failure rates less than or equal to one. A Weibull shape parameter (β) of 1 or exponential failure distribution would mean that the doing preventive maintenance would serve no purpose as the failure rate would remain the same. A Weibull shape parameter (β) less than 1 means that the component seems to be more reliable as time passes and thus replacing the component is unwise. Of course when encountering problems in a practical setting data issues must be observed and accounted for. Once it is known that the Weibull shape parameter (β) is greater than 1 another condition must be satisfied before continuing. "The second requirement to justify preventive replacement depends on the component and can be satisfied if the cost of a planned or preventive replacement (C_P) is less than the cost of an unplanned or corrective replacement (C_U)" (Reliasoft 2009). If the cost of preventive maintenance is more than the cost of the failure, the wise and least expensive choice would be to just replace the components.

When the two conditions are met the optimum preventive maintenance time can be calculated. The corrective cost increases as time increases due to the failure rate increasing, thus indicating that as time passes the component is more likely to fail. “The preventive replacement costs will decrease as the time interval increases because the more time passes, the fewer preventive replacement actions will need to be performed. The total cost will be the sum of these two costs. At one point (time *t*), a minimum cost point exists that determines the optimum preventive replacement time for the component” (Reliasoft). Figure 7 is the graph that depicts the optimal replacement point. It is the point at which the corrective cost and the preventative cost meet.

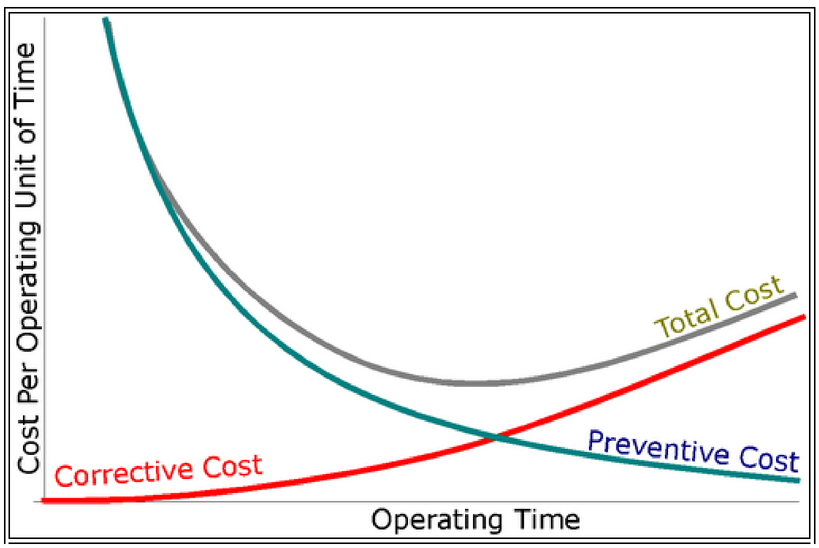


Figure 7. Operating Time Vs Cost Per Unit of Time

Figure 7 is a representation of the following formula:

$$f(t) = \frac{TotalExpected\ Replacement\ Cost\ Per\ Cycle}{Expected\ Cycle\ length} \tag{4.1}$$

$$= \frac{C_p \cdot R(t) + C_u[1 - R(t)]}{\int_0^t R(s)ds} \tag{4.2}$$

Where C_p is the cost for a preventative action, C_u is the cost for each unplanned or unwanted repair, and $R(t)$ is the reliability function of the particular component. The optimum replacement time can be obtained by solving for t when:

$$\frac{\partial[f(t)]}{\partial t} = 0 \tag{4.3}$$

For every component in this thesis an optimal preventive maintenance schedule time has been calculated. Each preventive maintenance cost is independent of the cost for the system. Including the preventive maintenance with the unknown system configuration would be troublesome as the input criteria will expand thus exponentially increasing the search area for the objective function. No longer is there a single variable for cost. Now the redundancy cost and the preventive maintenance cost would have a relationship and make this problem a two variable neighborhood search, which may be too arduous for manual inputs. For this case, preventive maintenance costs are calculated separately and not included in the cost for system redundancy.

5.0 Estimating and Optimizing System Availability

If future stress distribution and the associated probability are known or can be estimated, the expected availability can be calculated. It is the metric used to calculate an optimal or acceptable solution, for a particular preventive maintenance time and redundancy level.

An optimization problem is as follows:

$$\begin{aligned} &\max E[A_S(\mathbf{x}, \boldsymbol{\tau})] \\ &\text{st} \sum_i c_i x_i \le C \\ &x_i \in \{1, 2, 3, \ldots\}\ ,\ \tau_i \ge 0 \end{aligned} \tag{5.1}$$

$\mathbf{x} = (x_1, x_2, x_3, \ldots..)$
$\boldsymbol{\tau} = (\tau_1, \tau_2, \tau_3, \ldots..)$
x_i = the redundancy level for component i
τ_i = preventative maintenance for component i in the environment

Adding a stress level the expected availability is a function of a stress vector.

$$\max_{\boldsymbol{x},\boldsymbol{\tau}} E[A_s(\boldsymbol{S}, \mathbf{x}, \boldsymbol{\tau})] \tag{5.2}$$

The stress has an element which coincides with different probabilistic future with probability p_i. Each S represents a different future stress profile. Each stress profile has a defined distribution with a mean or variance or is represented by a relative frequency of the composition of aircraft.

$$S \in \{S_1, S_2, S_3, S_{4,} \ldots \ldots .., S_r\}$$

$$S_1 \sim F_{s_1}(S_1), \quad \mu_1 = E[S_1], \quad \sigma_1 = Var[S_1]$$
$$S_2 \sim F_{s_2}(S_2), \quad \mu_2 = E[S_2], \quad \sigma_2 = Var[S_2]$$
.
.
.
$$S_r \sim F_{s_r}(S_r), \quad \mu_r = E[S_r], \quad \sigma_r = Var[S_r] \tag{5.3}$$

where S_r = stress profile in future r
μ_r = mean stress of future r
σ_r = standard deviation of stress of future r

The constraints are to minimize cost as each redundancy has an associated cost with parts and maintenance. To calculate the expected availability, the sum of the probable availabilities for each stress, redundancy, and preventive maintenance must be added and weighted to the probable outcome.

$$E[A_s(\boldsymbol{S}, \mathbf{x}, \boldsymbol{\tau})] = \sum_{i=1}^{p} p_i\, A(S_i, \mathbf{x}, \boldsymbol{\tau}) \tag{5.4}$$

6.0 Redundancy Allocation Problem

Once all the reliability coefficients are calculated using accelerated life testing, a design decision must be made. Each component has a Weibull distributed time to failure, a preventive maintenance schedule, a corrective maintenance schedule, the cost for the component, and the cost for the repair. It is assumed that the cost for preventive maintenance is independent of the cost of redundancy. This is to assure that once a redundancy is chosen the preventive maintenance costs for a component is the same, although this might not be the case as less preventive maintenance may be necessary for the redundant system. The data is all collected into a simulation program, but all the redundancy levels are calculated manually.

The Redundancy Allocation Problem (RAP) has been solved many times and in different ways. Mathematical programming techniques such as integer programming and dynamic programming have been used to solve a redundancy problem. This thesis utilizes a Tabu search method to solve the RAP for this paper. Two different types of RAP are presented in this paper. One is where only one vender is available for a component, the other RAP is where a choice can be made to purchase from a different vendor with a higher reliability.

The most common RAP is the series system of *s* independent *k*-out-of-*n*:*G* systems. The subsystem is working if *k* out of the *n* components is operational. According to (Kulturel-Konak et al., 2003) this problem has been studied many times over and different approaches can be presented in (Tillman et al. 1997). This paper only deals with a Tabu Search Redundancy Allocation Problem (TSRAP) and thus the other approaches are not fully disclosed.

In the Tabu search approach the moves are deterministic which reduces variability in the search parameters. The approach for this paper is based upon the paper written by Kulturel-Konak et al. (2003). Some changes to Kulturel-Konak (2003) TSRAP and the problem presented in this paper are:

1.) the initial solution always starts from a simple 5 component series system

2.) there is a cost associated with each component and a budget. A solution that exceeds the budget is no longer a feasible solution. Taken from (Kulturel-Konak, 2003). The following terms are used to describe the problem: BEST MOVE (best solution that would be a result from taking any of the current available moves), BEST SO FAR (best solution so far in the search, it may be feasible or infeasible), BEST FEASIBLE SO FAR (best feasible solution found so far in the search.) These steps are altered to fit the needs of the paper and will thus reflect changes. An example will be provided in the later called simulation. This is a mathematical as well as theoretical explanation of TSRAP regarding this paper.

Step 0: Start with an accelerated life altered Weibull distribution of the 5

components in the system.

Step 1: Search the neighborhood for all possible defined moves for each subsystem.

For larger problems a list of candidate solutions may have used (Glover, 1997) Two kinds of moves are possible for this problem. For the TSRAP for only one component choice only includes the first move; however, the multiple component choice has two possible moves. The first type of move is to change the number of components by adding one ($x_{ij} \rightarrow x_{ij} + 1$). The second type of move is to change the type of component is to change the component choice ($x_{ij} \rightarrow x_{ik}$ for $j \neq k$), for each subsystem. Any addition or change in component is considered a singular move. Subsystems are changed out one at a time; thus the reliability in theory can be recalculated and updated accordingly. The moves are performed independently and compared to the best move so far. If this solution, the best move, is infeasible; due to over budget, or is on the Tabu list, then the move is disallowed and must be restarted. If the solution is not Tabu and is under the cost constraint, then the best so far solution is accepted.

Step 2: Update the Tabu List

The move is accepted and added into the Tabu list. If the Tabu list is full, the older Tabu list entry is deleted. To know if an entry on the Tabu list is feasible or infeasible, the system cost and weight are noted.

Step 3: Check the stopping criterion

> Finally the stopping criterion is checked. It is defined as the maximum number of iterations without finding an improvement in the BEST FEASIBLE SO FAR. If it is reached the search is concluded and the BEST FEASIBLE SO FAR solution is the TSRAP recommended solution.

Again these steps are taken from (Kulturel-Konak et al, 2003) with alterations. In this TSRAP subtracting a component is not a feasible move because the initial solution always needs 5 components and subtracting a component will not add reliability. However, without the subtracting of a component the Tabu search cannot move as quickly and thus it may be useful to incorporate, but this leads to another dilemma. Simulation is done manually and thus takes an extraordinary amount of time. This factor leads to shorting the number of models that most calculated and accounted for.

For the stopping criterion, the assumed number of iterations was first set to 8 solutions and then later changed to 5. It is noted in the simulation examples which examples and used the 8 iterations without improvement and which used 5 iterations.

The basic rule of any simulation is to get the best and most accurate data possible. Since the simulation only runs on what inputs are given if the inputs have uncertainties the simulation will not give a result that will accurately depict future metrics. Thus the adage, “garbage in garbage out” comes from bad data. Bad data can be issues of data collection, clear on uncertain metric, data formatting, and even simple typos. Although it may be expensive, the best data might come from computer automated systems, as the human error component may be significantly reduced.

Once the data is gathered and collected an Accelerated Life Test (ALT) will be run to model future stresses. The general log linear will be chosen to represent the life stress relationship because of the availability of multiple covariates to represent an increasing or decreasing life as a function of, in this case mean and standard deviation or frequency of the stresses of the present data.

Once the stress profiles’ variables are determined the first part of the simulation begins. Since each component’s optimum component replacement time is determined by the cost of the part and the failure distribution, this is calculated first. Each component has an optimal preventive maintenance schedule in which preventive maintenance is performed to prolong the life of a component. This preventive maintenance schedule and the time for preventive and corrective maintenance to complete are strong factors in calculating the availability.

When all of the stress profiles and preventive and corrective maintenance times are inputted, the redundancy level must be set. To determine an acceptable redundancy level, a certain cost threshold is chosen. In most cases the more reliable component will cost more and thus this simulation model will also follow that same concept. Once all the redundancy is set in place, the optimal system configuration is complete but for only one possible future. The accelerated life test, preventive maintenance, corrective maintenance, and Tabu search must be repeated for any amount of possible futures desired.

After creating a model for each possible future, the expected availability will be calculated using the sum of expected probabilities. Figure 8 is a flow chart which represents the steps in finding the availability of the system with an uncertain future stress profile from computing the times to failure to the final calculation of the expected future system availability.

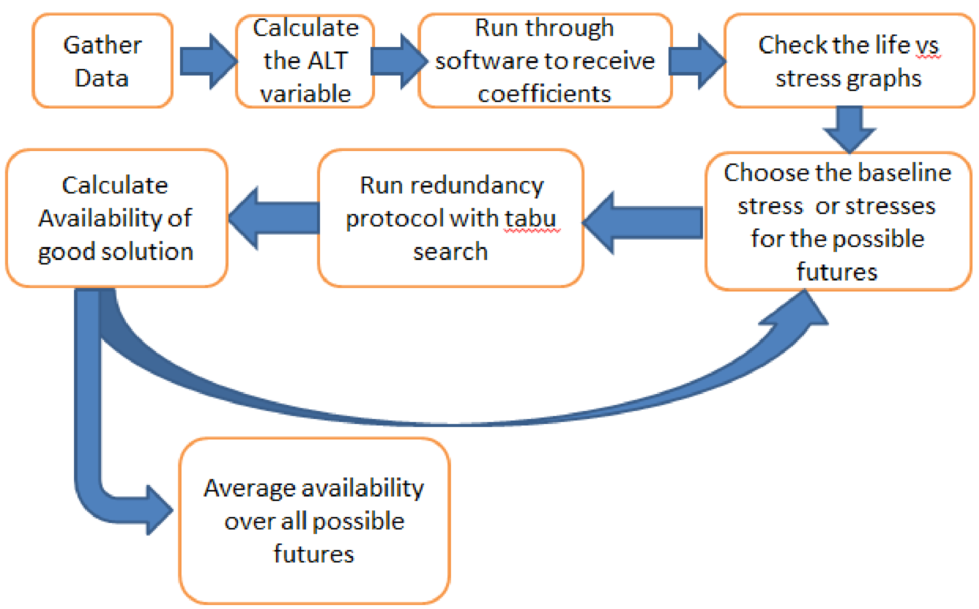
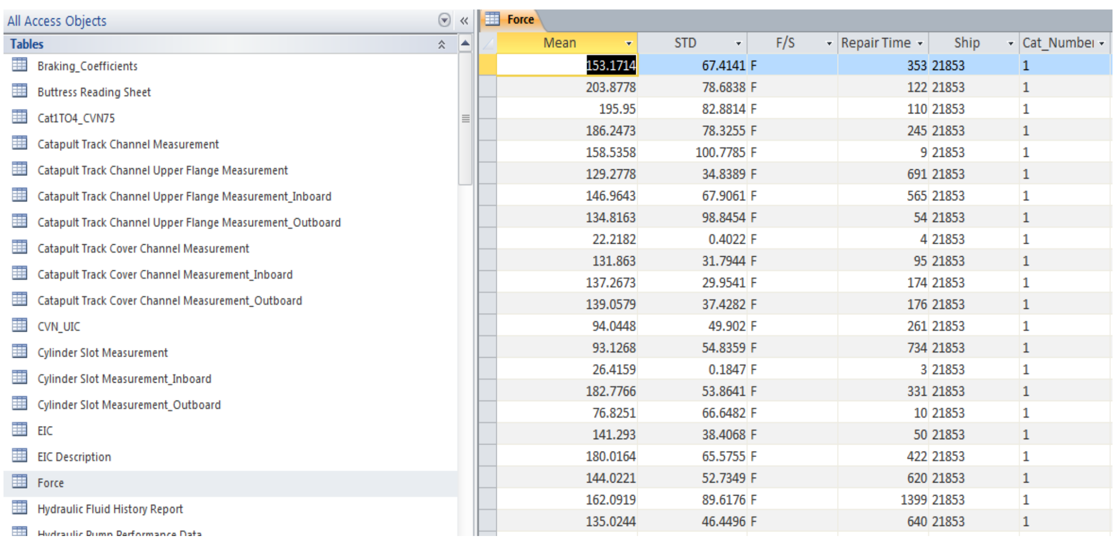


Figure 8. Flow Chart of Data to Simulation Model

7.0 Examples

The general log linear life stress relationship is utilized to create a futuristic stress profile with data acquired in the present. This example uses the mean and standard deviation method as an example model. The inspiration of this comes from the Navy air fleet. As aircrafts land on a carrier, each landing impacts the arresting gear system and causes stress. Since the number of arrestments between each corrective action as well as the tension recorded both the mean and standard deviation of each time to failure can be recorded with relative ease. Shown in Figure 8 is a database that calculates the mean time to failure as well as a corresponding mean tension to failure and standard deviation to failure.



Mean	STD	F/S	Repair Time	Ship	Cat_Number
153.1714	67.4141	F	353	21853	1
203.8778	78.6838	F	122	21853	1
195.95	82.8814	F	110	21853	1
186.2473	78.3255	F	245	21853	1
158.5358	100.7785	F	9	21853	1
129.2778	34.8389	F	691	21853	1
146.9643	67.9061	F	565	21853	1
134.8163	98.8454	F	54	21853	1
22.2182	0.4022	F	4	21853	1
131.863	31.7944	F	95	21853	1
137.2673	29.9541	F	174	21853	1
139.0579	37.4282	F	176	21853	1
94.0448	49.902	F	261	21853	1
93.1268	54.8359	F	734	21853	1
26.4159	0.1847	F	3	21853	1
182.7766	53.8641	F	331	21853	1
76.8251	66.6482	F	10	21853	1
141.293	38.4068	F	50	21853	1
180.0164	65.5755	F	422	21853	1
144.0221	52.7349	F	620	21853	1
162.0919	89.6176	F	1399	21853	1
135.0244	46.4496	F	640	21853	1

Figure 9. Database Calculates Mean and Standard Deviation of Tension along with Frequency

Once the Repair Time (Mean Time Between Failure), Mean (Average Force Unit to Failure), and Standard Deviation (Standard Deviation of Force Units) are calculated a futuristic baseline must be determined. As a general rule with any ALT the baseline should not cause any failure not caused by the original dataset. For this example set, the raw data is shown in Table 5. The Repair Time, Mean, and Standard Deviation are calculated for the raw data to be inputted into ALT software. The MTBF (Mean Time Between Failures) is the time between each repair not including logistics downtime and the actual repair time. This MTBF is counted in cycles or loads. In the Navy example one cycle would represent a single launch or single recovery. These launches/recoveries are counted until a failure occurs, the time between failures is the mean time to failure or the MTBF. However in each cycle there is a tension recorded. Once a failure occurs all of these tensions are summed and averaged over the course of the failure cycle. This is the mean force unit recorded in the column.

$$\frac{Tension}{Mean\ Time\ To\ Repair} = \frac{\sum_{x=1}^{MTTR} Tension}{MTTR} \tag{6.1}$$

The Std column is simply the standard deviation over the course of the cycles to failure (CTF).

Table 5 Raw Data Table

CTF	Mean	Std
495	463*	443
759	490	253
333	536	643
554	678	245
547	905	543
561	986	546
682	1070	242
611	1122	760
345	1543	234
250	1789	683

After calculating the desired variable needed for the ALT, the a_1 and a_2 parameters must be calculated to shift the failure distribution according to an increasing stress load.

* Simulated data is all in pound and does not represent actual aircraft data.

This is done simply through software; however some check still must be made. The first check is to see the fit of the Weibull distribution with the parameters and the future stress chosen. Figure 9 is a probability plot of a the component raw data which is exposed to the future stresses of 2400 force units and a standard deviation of 600 units. The distribution parameters are β = 4.4889, α_0 = 6.7852, α_1= -0.0004 α_2= -0.0002. By looking at Figure 9, one can deduce that the distribution fits the data accurately.

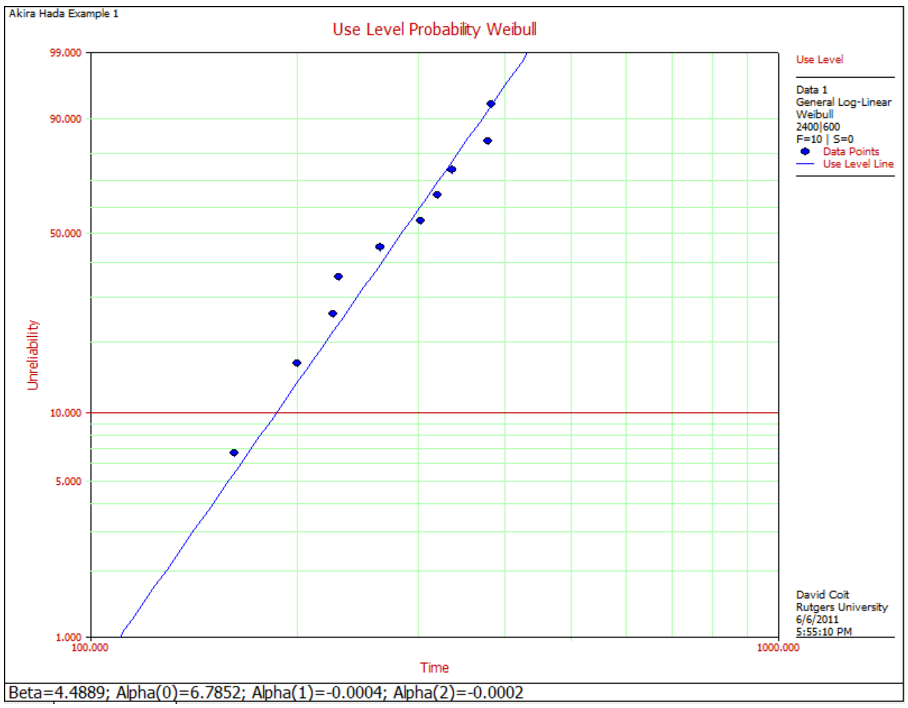


Figure 10. Probability Plot of Table 5

While the data in this paper shows simulated times, a thorough quality assessment should be performed on real data. A simple life versus stress plot should show that as the load increases the life parameter of the component should decrease. Depending on the type of product, an increase in the variance may or may not increase the life parameter. What must be determined beforehand is if the component is more durable for heavy loads. In most cases a heavier load will correspond to more wear than a lighter load. An increase in variance means that the component accrues a diverse set of stresses both heavier and lighter. However, increasing stresses tend to impact a typical component more severely than a linearly related lighter load, although this might not be case all the time. For the arresting and launching systems of the Navy, a higher variance and higher mean of stress should decrease the life parameter. In Figure 10 the stress (Average Force Unit) is compared to the life and in Figure 11 another stress (Std of the Force unit) is shown. Figure 10 depicts a cleaner and more comprehensible 3-dimensional graph of the pdf vs. force vs. life.

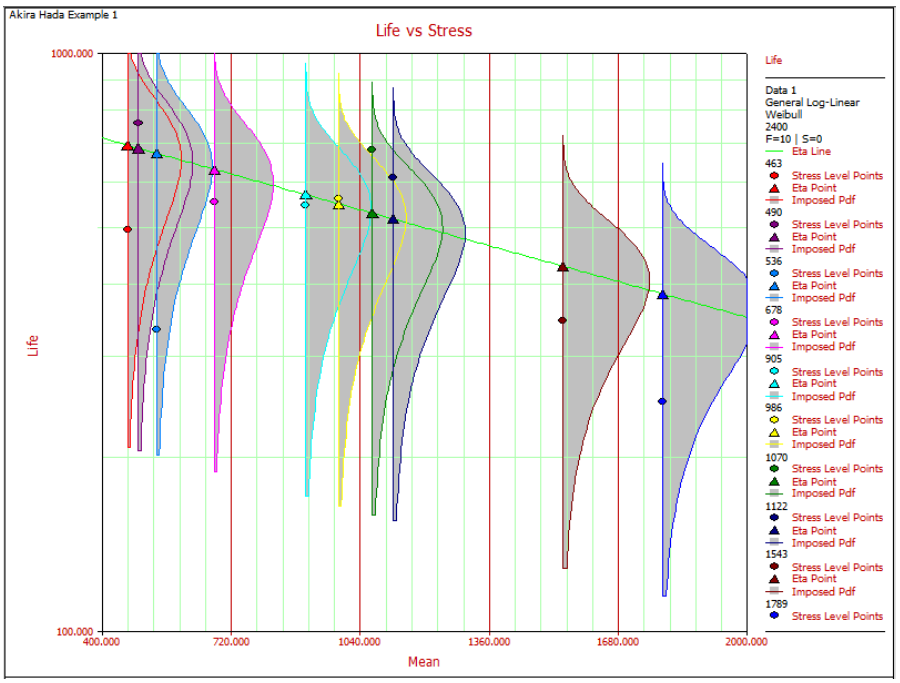


Figure 11. Force Units (2400) vs Life

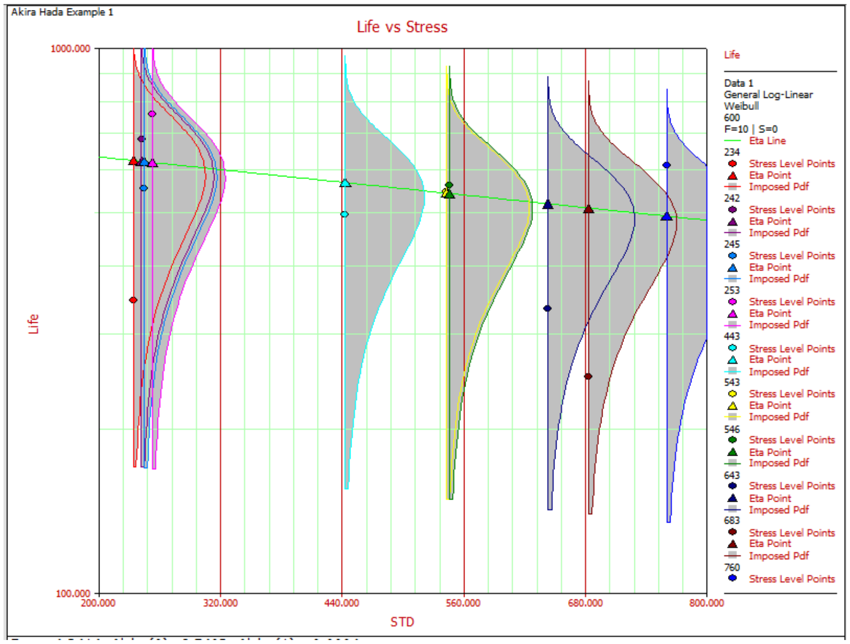


Figure 12. Standard Deviation vs Life

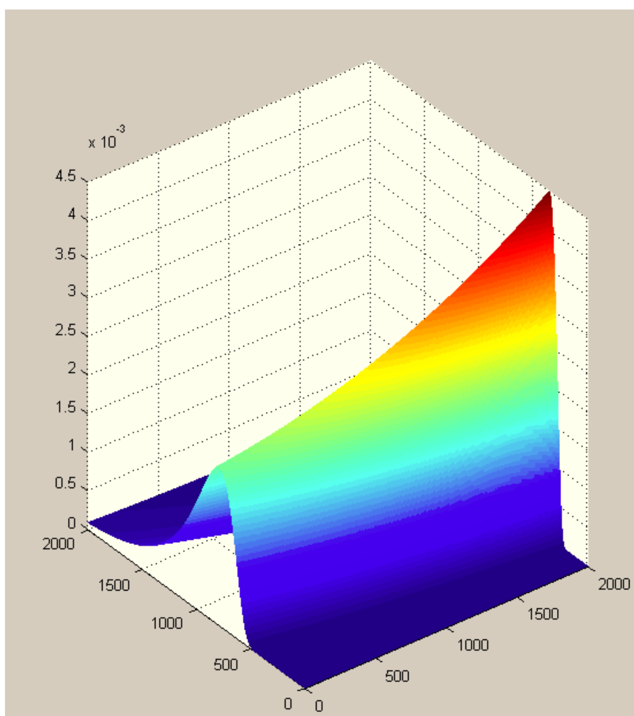


Figure 13. 3D Model Standard Deviation vs Life

After obtaining the accelerated life parameters and results, the optimal component replacement must be calculated. The basis of optimal component replacement takes the cost of the failure and compares that cost to the accumulated cost of preventive maintenance accounting for the failure distribution. This optimal component replacement time is utilized to determine when a component should be replaced in the system. This is all done when creating a policy is created.

After determining all the inputs to the availability metric for one possible future, the redundancy of the level of the system must be determined. In this example no component switching is assumed which means that only one component is available so the only possible action to improve reliability would be to add a redundancy. A cost unit will constrain the system and the objective is to maximize availability within the cost constraints. Multiple component switching has more possible actions due to the ability to choose a different more reliable component albeit at a higher price and also increases the iterations for the Tabu search.

Some assumptions are made to simplify this problem. The preventive maintenance and optimal replacement schedules remain constant and do not change while adding the redundancy. This assumption is made to reduce the problem set, but in a typical scenario adding redundancy to certain systems can reduce the amount of preventive maintenance costs and also depending on the type of stand-by system the optimal replacement maybe different as well. Think for example of a cold stand-by system, no preventive maintenance will be needed on the part that is not active as it does not accumulate any wear, the replacement component will also not be purchased if the first failure occurs but will probably be replaced sometime when the second fails. Although the optimal preventive maintenance times does not change mathematically as continually providing preventive maintenance on the non-redundant component will provide a longer life and a more cost efficient system. The slack provided by the redundancy might enable decision makes to hold off on preventive maintenance or purchasing a replacement due to the availability of the redundancy. This may especially be true if the preventive maintenance is a long and arduous procedure or if the replacement component is expensive. However, the assumption in this paper is that all optimum replacement times and preventive maintenance schedules are independent of the redundancy levels and thus can be calculated separated. In future possible works this issue may be addressed; however currently this is beyond the scope of this project.

The redundancy level is determined by a Tabu search, for this example the Tabu list is made short and the Tabu list is set to 4 items because the solution set is rather small. After calculated all parameters and redundancy levels for one possible future, the process must be repeated for multiple futures. For this example, the preventive maintenance times and repair times have been held constant but in a more realistic example these times will not be a constant value but a distribution of times as well as different components having different repair and preventive maintenance times. In other words, using a car as an example, the time it takes to change a tire will be significantly less than restoring a transmission. Later in this thesis other problems will demonstrate changes in the preventive maintenance times as well as the changes in the costs associated with the optimal replacement time.

For all examples done, only three possible futures will be addressed. Other possible profiles maybe be added to the method and would be done by simply repeating the steps explained above. The estimated availability is calculated as the sum of the weighted probabilities of each future. The probability of each future is chosen on the likelihood of the event to occur; there is no absolute way to predict this occurrence, so one must choose wisely on either experience, intuition, or some other basis. When calculating the future stresses the base or future stresses will be inputted and thus will change the parameters.

7.1 Tabu Search : Single Component

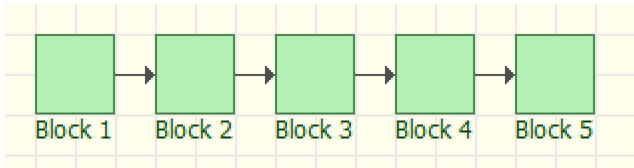


Figure 14. Basic Reliability Block Diagram

Figure 14 is designated as the initial solution because the system does not have any redundancy as this system is still in operation. All improvements are made to this initial system and the goal is to reduce the cost of the reliability while maintaining the maximum reliability allowed. The cost is set at a certain value. In other words a budget is given and the Tabu search will not find a solution in which the budget is exceeded.

Each block has a Weibull failure time distribution determined by data collected at the component level. The failure time distribution is not the only important property inputted into the simulation. The corrective maintenance time as well as the preventive maintenance time is inputted as well. These times represent the length of time it takes for a corrective or preventive maintenance action to occur, which effects system availability for during maintenance components are being repaired. If the system must be taken down for repair, then this will affect availability.

In a previous section the move is mentioned for only one component choice.

> "The move is to change the number of components by adding one ($x_{ij} \rightarrow x_{ij} + 1$). Any addition or change in component is considered a singular move. Subsystems are changed out one at a time; thus the reliability in theory can be recalculated and update accordingly. The moves are performed independently and compared to the best move so far. If this solution, the best move, is infeasible; due to over budget, or is on the Tabu list, then the move is disallowed and must be restarted. If the solution

is not Tabu and is under the cost constraint than the best so far solution is accepted."

The initial system starts with 5 subsystems and adds a redundant component at each move. The first neighborhood is defined in Figure 15. (Dummy blocks are used to allow the software to complete the simulation.)

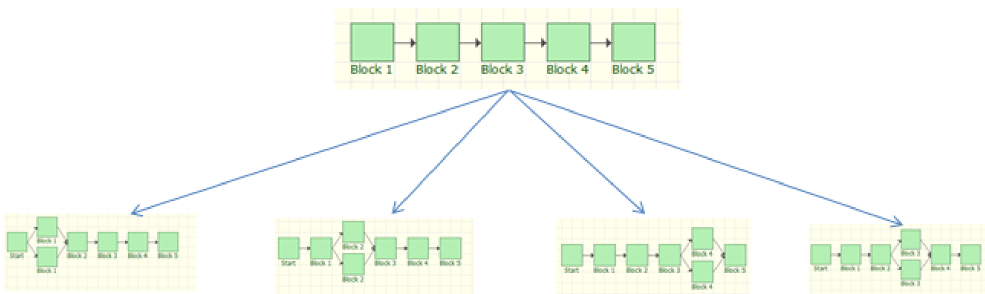


Figure 15. Possible Solutions for One Iteration in One Possible Future

Each component has a redundancy added iteratively and a simulation is run to calculate the reliability for each system. When a redundancy is added the cost of the component is noted. In the first few iterations the cost is not a factor as the budget is far below the limit. However, in as the redundancy levels increase the cost will increase as well and the eye must be kept on the budget to ensure that the cost is not greater than the budget. When running the simulation for this example adding a redundancy to component 2 showed the greatest increase in availability to the system and thus the candidate solution now becomes Figure 16 or the solution which is under budget and has the greatest calculated availability.

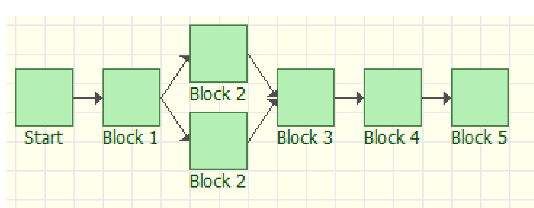


Figure 16. Optimal Solution for the First Iteration

The optimal solution does this is by no means surprising. The component, block 2, has the most unreliable failure distribution and providing redundancy to this component would seem intuitive. This BEST SO FAR system is now used as a basis for the Tabu search. However, adding the same component will now be forbidden and force a search to multiple neighborhoods. Each solution will then be compared and a new solution will be chosen as the BEST SO FAR system. Each BEST SO FAR system will eventually be compared until the stopping condition is met.

The parameters for the Weibull are as follows, with a time to replace set at 100. This is depicted in the table below. The cost constraint used in the single case is 80. In a more robust model this cost would be defined either in monetary value, time, or some other predetermined metric.

After inputting the variables in Table 6 below, the Tabu search was run. Then the most reliable solution of the iterations is chosen and the move added to the Tabu list.

Table 6. Eta Values for Future Events

Component	Beta	η1	η2	η3
Block 1	3.4	305*	346	479
Block 2	2.6	451	634	834
Block 3	3.78	221	567	743
Block 4	1.4	103	293	409
Block 5	2.7	266	854	1002

7.1.1 Single Component: Results

The single component case is fairly simple as there is no competing component. Each subsystem only has one option and the stopping condition and cost constraint become

* Data used for this thesis was provided by the Navy. This data needed another conversion rate to get actual units. At the time of calculating these examples, the Navy did not authorize the release of actual units in this thesis and thus simulated or base line Navy units were used to calculate these results.

the main factors as the algorithm adds a redundant component to increase availability until the stopping condition is met.

In this example, the eta and the beta parameter are already calculated for each future. Each future represents a possible loading condition. In this case, all futures experienced increasing loads. Future 1 is the mildest; however the force experienced on this system is still more than the current force the system is experiencing. Future 2 represents a slightly heavier load and future 3 represents an even heavier load than future 2.

Noticing that in the single component case where the cost of a component is set equally a pattern begins to emerge when running future scenarios. Although this section's intention was to go into detail of the single component case, there seemed to be no particularly interesting finding. The final structure of the component is shown in Figure 17.

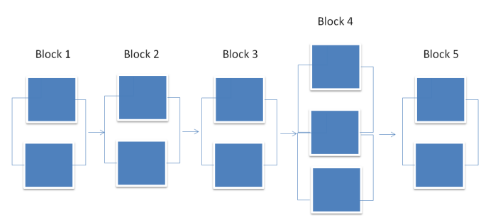


Figure 17. Single Component Final Structure

All three futures converged to the same solution, a 2-2-2-3-2 system. For each future the configuration may be the same. Combining the availabilities and the probabilities, the probabilistic availability is calculated as 0.8841, which means that the system will be available 88.41% of the time.

Table 7. Single Component Availability

Future	Availability	Probability
1	0.8904	0.8
2	0.8607	0.1
3	0.8569	0.1

7.2 Tabu Search: Multiple Components

In this example a multiple component system represents a system in which there are two choices available for each component. One choice is the standard component in which the cost is the same; however, the designer may choose an alternative component with increased reliability but at a higher cost. Each component is different so there is no mixing of choices within the system; however, it is possible to mix within the subsystem.

To identify the difference between the original component and the more expensive and reliable component, the original component is labeled with the name “Block” and the more reliable component is labeled with the name “Part”.

Table 8. Original Component vs Alternative Component Costs

	Component				
Type	1	2	3	4	5
Block	5	6	8	9	6
Part	7	9	10	12	9

Table 9 represents the difference between the costs from a block component (original) to a part component (more reliable). The cost represents a cost unit not associated with a dollar or currency value. The first multiple component examples choose arbitrary cost with the only rule being that the more reliable component is more expensive than the original. In the second example, the cost of the more reliable component’s cost is reflected in the reliability increase of the component.

All components are assumed to have a Weibull failure time distribution; thus all components have a Beta and Eta parameter. A larger numerical eta usually corresponds to a longer average life when holding the beta parameter constant. In Table 10, all the equivalent components (i.e., Part 1 and Block 1) have equivalent betas. However, the more expensive component (Part) has a higher life parameter; thus making the component more reliable. The other columns in the table are the corrective times or the time it takes for a corrective action to be completed and the preventive time, the time it takes for a preventive action to be completed. The column values are not calculated nor were they pulled from a data source. However, in this paper these times were made to be consistent and followed the logic that a corrective maintenance action would take longer than a preventative maintenance action. The final column is the optimal replacement time of each component given the distribution parameters of the Weibull distribution and a 4 to 1 ratio of unexpected replacement cost to a planned replacement cost or the cost of a failure versus a replacement.

Table 9. Information Used for Components

Type	Beta	Eta	Corrective Time	Preventive Time	Optimal Replacement
Block 1	3.3	890	100	20	498.2027
Block 2	2.6	1455	100	20	804.8099
Block 3	3.9	247	100	20	142.2642
Block 4	1.25	609	100	20	1014.3987
Block 5	2.6	450	100	20	248.9103
Part 1	3.3	1000	100	20	559.7783
Part 2	2.6	1700	100	20	940.3277
Part 3	3.9	800	100	20	460.7748
Part 4	1.25	750	100	20	1249.2595
Part 5	2.6	700	100	20	387.1937

Once all the variables are entered into the reliability block diagram, the search for the optimal solution must be conducted. As stated in a previous section, every system starts with a 5 subcomponent system which contains the “Block” components. Similar to the single component choice, one by one a component has a redundant component added, switched, or changed. However, a key difference in the multiple component example is more choices are available for each subsystem since the “Part” component can now be implemented. This widens the possible solutions that can be accessible as there are more possible combinations due to the availability of another choice. Figure 18 depicts the additional choices available for the multiple component case.

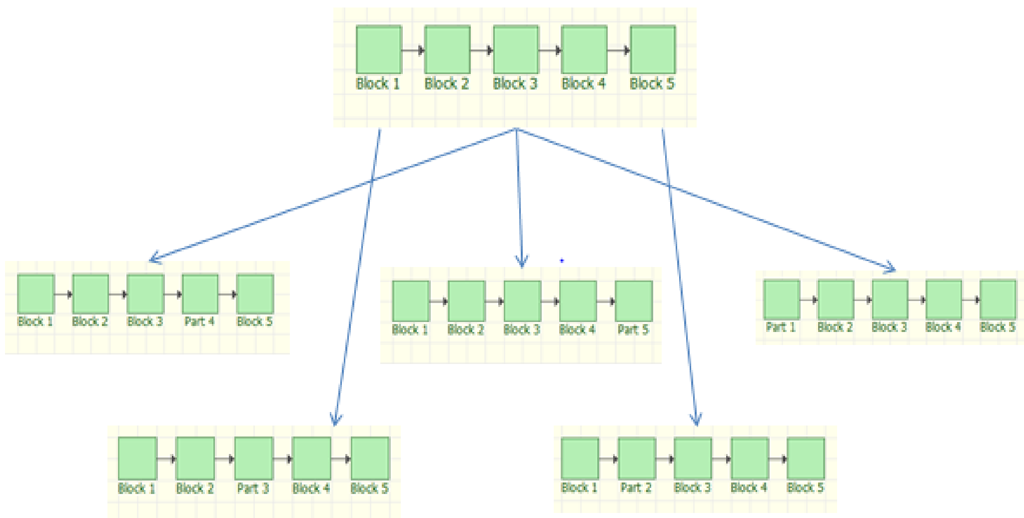


Figure 18. "Block" Component is Replaced with a Single "Part" component

In the multiple component case each neighborhood search has more solutions; however, the more reliable component is initially being compared with a redundant configuration of the original. Although not a dire problem for a computer, the manual Tabu search is slow moving and to get a fair comparison the Tabu search must have a high number of iterations; this also requires a proper stopping condition.

7.2.1 Greater Cost for Components with Higher Availability

As stated in the previous section, the initial solution starts with a complete system with no redundancy. Keeping everything consistent, the standard component is labeled “Block”, while the more reliable component is labeled “Part”. Table 9 is the information table that contains all the basic information obtained by test data or empirical data. The corrective distribution is the distribution of the repair time for an unexpected failure event. This corrective distribution is set for a constant of 20 units. The preventive distribution is the distribution of the repair time of an expected failure event. This preventive distribution is also set as a constant of 5 units. The assumption is that the system does not occur downtime while the redundant components are being worked upon, however when a subcomponent is down there is a downtime. This downtime is equivalent to the corrective distribution constant of 20 time units. The last column depicts the replacement policy. This can be altered to the user’s discretion but in this thesis the replacement policy is calculated by the individual component and replaced at an optimal condition explained in the optimal component replacement section.

Table 10. Variables for Alta Model

	Block Name	Number of Identical Blocks	Identical Blocks Series (Y) Parallel (N)	Block Failure Distr	Block Failure Distr P1	Block Failure Distr P2	Corrective Distr P1	Preventive Distr P1	Preventive: Policy	Preventive: Misc. Cost Per Action
1	Block 1	N/A	N/A	WBL	2.5	1100	20	5	Preventive Policy6	0
2	Block 2	N/A	N/A	WBL	1.6	637	20	5	Preventive Policy7	0
3	Block 3	N/A	N/A	WBL	4.3	583	20	5	Preventive Policy8	0
4	Block 4	N/A	N/A	WBL	3.7	820	20	5	Preventive Policy9	0
5	Block 5	N/A	N/A	WBL	1.8	358	20	5	Preventive Policy10	0
6	Part 1	N/A	N/A	WBL	2.5	1200	20	5	Preventive Policy1	0
7	Part 2	N/A	N/A	WBL	1.6	700	20	5	Preventive Policy2	0
8	Part 3	N/A	N/A	WBL	4.3	650	20	5	Preventive Policy3	0
9	Part 4	N/A	N/A	WBL	3.7	890	20	5	Preventive Policy4	0
10	Part 5	N/A	N/A	WBL	1.8	450	20	5	Preventive Policy5	0

The original data is summarized in the Table 10. However, with the introduction of new stresses the ALTA software will calculate new parameters to reflect the predicted stress profiles. In this example, there are three possible futures, a future in which no changes occur and the stress on the component remains the same; a future in which the mean and standard deviation of the component slightly increase; and finally a future in which the mean significantly rises and the variation also rises. The respected covariates for each future stress profiles are as follows in Table 11.

Table 11. Mean and Standard Deviations of Loads

Future Profile 1		Future Profile 2		Future Profile 3	
Mean	Std	Mean	Std	Mean	Std
58000	28000	65000	30000	80000	32000

Each component's Weibull variables are recalculated to represent the three distinct futures. All components were checked to make sure that each mean vs. life and mean vs. std graph had a negative correlation. The results and the cost are depicted in Table 9.

Cost is calculated in two methods. The first method is to arbitrarily assign an increased cost to the more reliable component. For example, block1 can be replaced by part1, which has a higher availability. However, part1 is more expensive by some cost with no real basis aside from the higher availability. The second method used to calculate cost is by taking a relationship of the cost unit multiplied by the ratio of the reliability, or the Block reliability over the Part reliability. These costs were only determined using the multiple component examples, as the component has two options in which the price can be changed. In the previous example, (single component) there was only one cost.

7.2.2 Multiple Component Basic Model: Future 1

The first profile to go through the redundancy allocation Tabu search is the profile with a mean of 58,000 and a standard deviation of 28,000. This profile represents a future in which no additional force has been added and thus this profile mimics the present stress profile. The system model has 5 subsystems and each subsystem has a choice of 2 components. The nomenclature used to describe this redundant system is simple, a numerical value represents the number of components from the less reliable component or the Block, an alphabetical value represents the more reliable component or the Part. The actual value of the number or letter determines how many of that particular component is in the subsystem. Each subsystem is also separated by b – or _ . For example a 1_b_2_bb system is a system in which the first subsystem has one block component, the second system one part component, the third subsystem having two part components. Table 12 is the first iteration of the first future profile example and the associated move added to the Tabu list. The stopping condition is 6 consecutive iterations without an improvement and a cost constraint of 80 units.

The first future is a future with the lightest aircrafts. Although an increase is experienced, it is now one which is not too different from the current loads the system is facing now. The initial iteration and availabilities are given in the Table 12 below.

Table 12. Initial Iteration

Structure	Availability
2-1-1-1-1	74.2481
1-2-1-1-1	73.8604
1-1-2-1-1	75.7653
1-1-1-2-1	82.65232
1-1-1-1-2	74.7439
b-1-1-1-1	74.01
1-b-1-1-1	73.5959
1-1-b-1-1	75.7653
1-1-1-b-1	79.0843
1-1-1-1-b	74.2716

The common factor among all systems in all futures is that intially redundancy is added to components 3 and 4. This is logical as these are the most unreliability components and are thus availability will increase more dramatically as redundancy is built for these components. Althought these are different possible futures with different calculated failure functions (different parameters) the change is not drastic enough to see an initial difference of the systems redundancy in any future. It is safe to say that these are components are the low hanging fruit of the system.

Table 13. Iteration 2

Structure	Cost	Availability
1-1-1-1b-1	45	83.1672
2-1-1-2-1	48	83.4027
1-2-1-2-1	49	82.8337
1-1-2-2-1	51	85.5401
1-1-2-2-1	49	84.0462
b-1-1-2-1	45	83.05973
1-b-1-2-1	46	82.5971
1-1-b-2-1	47	83.8725
1-1-2-1-b	52	82.6232

A unique instance occurs during iteration 4, at this point the chosen feasible solution is 1-1-2-2-b with an availability of 87.2991. Table 14 shows iteration 3 and 4 with the structure and the availabilities of each system. Highlighted in red are the most reliable (highest availability) in this iteration. As seen the move that takes 1-1-2-2-b to 1-1-b-2-2 or from iteration 3 to iteration 4 is in actuality a switching of components. While this move is not significant in itself, in this project finding a better solution usually does not occur by switching. This is the first and only case in which switching the structure of the components gives a better solution. Switching the structure provides the Tabu search to change the searchable area so the algorithm is limited to the local area. In this case, the searchable region for the solution switched areas moving the local of the best solution for the problem.

Table 14 Iteration 3 and 4

it3		it4	
switch		Switch to b	
2-1-1-2-1	83.4027	b-1-2-2-b	87.2991
1-2-1-2-1	82.8337	1-b-2-2-b	87.3609
1-1-2-2-1	85.5401	1-1-b-2-b	87.7835
1-1-1-2-2	84.0462	1-1-2-1b-1b	88.1295
switch to b comp		Switch	
b-1-2-2-1	86.2643		
1-b-2-2-1	86.2812	b-1-2-2-1	86.3031
1-1-1b-2-1	86.4706	1-b-2-2-1	86.3396
1-1-2-1b-1	87.1935	1-1-b-2-2	88.5637
1-1-2-2-b	87.2991	1-1-2-b-2	84.5036
add regular		add regular	
2-1-2-2-1	86.6421	2-1-2-2-1-b	87.9385
1-2-2-2-1	85.798	1-2-2-2-b	87.6462
1-1-2-2-2-	87.2823	1-1-2-2-b	88.326

Each iteration added a component to the least reliable subsystem until the 9th iteration, shown in Table 15. In the 9th iteration, any additionally component added to the feasible solution causes a violation of the cost parameter and thus no longer becomes a feasible solution. Switching provides no new benefit as those solutions have been explored or are over cost as well. Changing the component to an alternate component is the only method which can possibly allow the availability to increase and at the same time meet the constraint of 80 cost units, but this does not occur during the stopping condition.

Table 15. Over Cost

Iteration 9	2_2_1b_1b_1b	
add	all over cost	
change		
1b_2_1b_1b_1b	78	93.59
2_1b_1b_1b_1b	79	93.87
rest over cost		
switching will have no effect		
adding will have no effect		
changing will go over		

The Tabu search finds a good solution which has the structure, 2 – 1b-1b-1b-1b and an availability of 93.87 percent.

7.2.3 Multiple Component Basic Model: Second Future

The second possible future's Tabu list is provided by Table 16

Table 16 Tabu list

Tabu	
add to 4th	release
add to 3rd	release
change 3 to b	release
add 5th	release
add to first	release
change 1st to b	release
add to 4th	
second component to b	
change 3 to b	
switch 3rd with 1st	
switch 2nd with third	

Iterations 1 to 3 follow a generic pattern, where the most unreliable component has redundancy added. It is not until Iteration 4 where a new component is introduced and is chosen as the best solution for that iteration. (Availabilities are listed on the right, all solutions are under cost).

Table 17. Iterations of the Second Future

Iteration 4	
add	
2-1-1b-2-1	82.308
1-2-1b-2-1	79.8618
1-1-1b-2-2	82.795
change to b	
b-1-1b-2-1	80.9623
1-b-1b-2-1	79.6777
1-1-1b-1b-1	79.8256
1-1-1b-2-b	0.7808
switch	
1b-1-1-2-1	78.0805
1-1b-1-2-1	71.2589
1-1-2-1b-1	0.7983
1-1-1-2-1b	71.605

iteration 5	
2-1-1b-2-2	0.8638
1-2-1b-2-2	0.8342
change to b	
b-1-1b-2-2	0.8499
1-b-1b-2-2	0.731
1-1-1b-1b-2	0.7326
1-1-1b-2-1b	0.8326
switch	
2-1-1b-2-1	0.8234
1-2-1b-2-1	0.7986
1-1-2-2-1b	0.79
1-1-2-1b-2	0.7946

Iteration 6		I
2-2-1b-2-2	0.8707	
		1
change to b		1
1b-1-1b-2-2	0.8721	
2-1-1b-2-2	0.8638	c
2- 1-bb-2-2	0.8643	b
2-1-1b-1b-2	0.8291	1
2-1-1b-2-1b	0.8736	1
		1
switch	2-1-1b-2-2	1
1-2-1b-2-2	0.8342	
1b-1-2-2-2	0.8133	s
		3

In iteration 5, a component is added to block 1 and the reliability is increased and in iteration 6 the block 1 component is changed into a part as availability increases. In iteration 8, 9 and 10 a few solutions have gone over the cost threshold and cannot be considered a feasible solution. These iterations are depicted in both Table 17 and 18. However, beyond iteration 12 the system is now too costly to improve and switching will not allow the system to go under cost. Thus out of the final solutions, bb – 1b – b – 3 – 2 has the best availability at 0.8873 and is chosen as the best available solution. It may not be the most optimal but it represents a good reliability structure given the cost and the reliability parameters. This solution is met as the Tabu search conditions were exhausted.

Table 18. More Iterations of 2nd Future

J	K	L	M	N	O
Iteration 8		Iteration 9		It11	
	1b-1-1b-3-2	adding	over cost	adding	overcost
1b-2-1b-3-2	over cost			change b	overcost
1b-1-2b-3-2	over cost	change to b			
		bb-b-1b-3-2	0.8903	switch	
change to b		1b-b-bb-3-2	0.8946	bb-1b-b-3-2	0.8588
bb-1-1b-3-2	0.8667	1b-b-1b-2b-2	over cost	bb-b-3-1b-2	0.8359
1b-b-1b-3-2	0.8944	1b-b-1b-3-1b	over cost	bb-b-2-3-1b	overcost
1b-1-bb-3-2	0.8877			b-bb-1b-3-2	overcost
1b-1-1b-2b-2	0.8886	switch		3-b-1b-bb-2	overcost
1b-1-1b-3-1b	0.8892	b-1b-1b-3-2	0.8703	2-b-1b-3-bb	overcost
		1b-1b-b-3-2	0.8583		
switch		1b-3-1b-b-2	0.7838		
3-1-1b-1b-2	0.8714	1b-2-1b-3-b	0.8232	It12	
1b-3-1b-1-2	0.7927	it10		adding	overcost
1b-1-3-1b-2	0.8305	adding	overcost	change to b	overcost
1b-1-1b-2-3	0.8688	changing to b			
		bb-b-bb-3-2	overcost	switch	
		1b-b-bb-2b-2	overcost		
		1b-b-bb-3-1b	overcost		
		switching			
		bb-b-1b-3-2	0.8873		
		1b-bb-b-3-2	0.8595		

The 2nd future represents a heavier load of aircraft. The solution initially starts off in the same pattern as the first future by adding a block to the 3rd and the 4th components. This is not a surprise as the 3rd and 4th components are the least reliable components of the system. The Tabu search initially improves the availability of the system efficiently and cost effectively but once these moves are forbidden or Tabu, the efficient move is now unavailable. This may hinder the search for the best solution but it also forces the system to calculate different reliability structures that would otherwise not be developed till later in the algorithm.

7.2.4 Multiple Component Basic Model: Third Future

In the 3rd future, the system structure diverges quickly from the first future's structure relatively quickly. In the first future and the second future, the 3rd move was to change the 3rd component into a different more expensive (yet, more reliable) part. However, in the 3rd future, the 3rd move is to add redundancy to the 5th component. This shows that a component will deteriorate at a faster rate relative to different levels of stress, thus a system with 200 units of extra load maybe need to be built differently than a system which will experience only 100 units of extra load, as different components will have different failure functions and different expected lives and different rates of failures.

The system structure diverges from the first future's structure from the 3rd move. The Tabu list is shown in Table 19.

Table 19. Tabu List

Tabu
add 4th
add 3rd
add 5th
change 5th to b
add to 1st
add 4th
change 3rd to b
change 2nd to b
overcost
overcost

In iteration 7, the system becomes very limited as movement is not restricted due to the high cost. Only a few solutions are now feasible without going over cost. The availabilities of the solutions are provided in Table 20. The best for now solution has reached a tipping point at a cost of 76 where only a few moves can be considered. Some of these moves are infeasible as they are located on the Tabu list. This provides no issue as these feasible solutions have an availability lower than the best for now solution.

Table 20. Iteration 7

it7	
2-2-2-3-1b	
switch	
3-1-2-2-1b	89.31
2-3-2-1-1b	89.5
2-1-3-2-1b	89.4963
2-1-2-1b-3	89.44
change	
1b-1-2-3-1b	90.16
2-b-2-3-1b	90
2-1-1b-3-1b	92.21
2-1-2-2b-1b	89.25

The feasible solutions are listed below. The only available move that can be made is replacing the 2nd component with a more reliable alternative. Some of these moves are unfeasible as they are on the Tabu list. All other solutions are over cost.

Table 21. Final System

1b-1-1b-3-1b	92.257	78
2-b-1b-3-1b	92.4369	79
2-1-bb-3-1b	89.89	78
2-1-1-2b-1b	88.99	79
2-1-1b-3-bb	90.93	79

Iteration 8 only has two moves. After choosing 2-b-1b-3-1b as the best for now solution, the search continues, however no better solution is found as all solutions are over cost or the availability never rises above this current solution. Once the stopping condition has been met the best for now solution is the solution of choice.

7.2.5 Multiple Component: Basic Model

With all three future and availabilities calculated, the likely availability of the system across all futures is calculated (probability of future 1)(availability of future 1) +(probability of future 2)(availability of future 2) + (probability of future 3)(availability of future 3). These futures probabilities are to be chosen with an expert or decision maker to approximate the possibilities of each future. Table 22 shows the corresponding probabilities and availabilities with respect to the future it represents.

Table 22. Final Step

Future	Probability	Availability
1	0.2	93.87%
2	0.5	92.44%
3	0.3	88.73%

The final estimated availability across all futures is then calculated as 91.61%. This is the estimated availability for the multiple component case with cost units of 80. Although the more reliable components are more expensive, there is no actual formula which decided the cost or increased cost. The next example demonstrates a slightly more complex model, as a cost ratio was used to balance the increased availability of the more reliable components with cost.

7.3 Multicomponent: Cost Ratio

The previous examples showcased a situation in which the cost of a more reliable part was set as a higher amount with no mathematical basis. To make the problem a little more complex, the more reliable component is now equivalently priced. In other words, a 5% increase in reliability of the component will cost 5% more. This differs from the previous situation in which the more reliable component was just set at a higher cost. The table below depicts the cost of each block and component. The budget was minimized to 75 due to simulations taking an excessive amount of time.

Table 23. Original vs Alternative Costs

	Component				
Type	1	2	3	4	5
Block	5	5	5	5	5
Part	6.5	6.1	5.2	5.8	6

Table 24. Raw Inputs of Weibull Model

	A0	a1	a2	Future1_η	Future2_η	Future3_η	Cost
Block 1	14	-2.0705E-06	-0.0002	3943.84	2605.60	1693.17	5
Block 2	11.3	-0.000013117	-0.0001	2296.67	1715.39	1153.60	5
Block 3	15.3	-0.0000512	-0.0002	837.48	392.29	122.00	5
Block 4	10.205	-0.0000175	-0.00009589	668.53	488.24	309.98	5
Block 5	17.7	-0.000082766	-0.0002	1479.67	555.69	107.63	5
Part 1	14.2682	-2.0705E-06	-0.0002	5157.00	3407.10	2214.01	6.5
Part 2	11.5	-0.000013117	-0.0001	2805.15	2095.18	1409.01	6.1
Part 3	15.34	-0.0000512	-0.0002	871.66	408.30	126.98	5.2
Part 4	10.3603	-0.0000175	-0.00009589	780.85	570.26	362.06	5.8
Part 5	17.89	-0.000082766	-0.0002	1789.29	671.97	130.15	6

7.3.1 First future:

The first possible future had some interesting occurrences during the simulations. In most instances having a more reliable component in the system actually decreased availability, this may be misleading due to the allowable moves defined in the solution neighborhood. In this neighborhood, a more reliability component was always compared to a component with redundancy. This makes a move to the more reliable component difficult as it is continually being evaluated against two redundant components. However, eventually the solution incorporates the more reliable parts and the end result of the Tabu search resulted in a system which had the configuration of 1B-2B-2B-2B-2B with the cost units of 74.6.

7.3.2 Second Future:

The second possible future followed the same pattern as the first future, but strictly diverged towards the end of the Tabu search. The final system configuration was set to 1B-3-1BB-1BB-2B with a cost of 74.5. The drastic difference comes from the usages of the more reliable components. Switching never provided a more optimal solution and thus never became a viable option.

7.3.3 Third Future:

The third future allow had a slightly different path, followed the same system configuration as the first future, with the same cost. From this one can infer that the Accelerated Life Testing may have the same relationship and thus future one and future three maybe linear related. The components will then wear at the same rate. The availability of the system drops to 88.98% but this is expected with the heavier load.

7.3.4 Results

The availability of the systems drastically differed in all three scenarios. As the load of the futures increased, the reliability of the system decreased. Table 24 shows the different futures and different probabilities associated with each future.

Table 25. Cost Ratio Results

Future	Probability	Availability
1	0.2	99.83%
2	0.5	99.65%
3	0.3	89.39%

Multiplying the probability by the availability and summing all possible futures together should net the average availability of the system, considering all three scenarios. In this case, (0.2 *99.83) + (0.5*99.69) + (0.3*89.39) will produce a system with 96.608% availability across three possible futures.

8.0 Future Research

This model can provide a good solution to a system which will incur an uncertainty of the future. For a move in depth analysis, different applicable future profiles can be developed, more complex reliability block diagrams can be introduces, or taking into account more variables such as the cost of increasing preventive maintenance or the cost of time. However, when running a study of a large magnitude, data management and data processing will become an issue. The researcher will have to take into account the logistics of the data as well as the processing of the simulation computer to ensure a timely result. This thesis shows the surface of what could be done modeling uncertain futures there is room to expand upon this model. Some of the areas that can be explored are:

1. Timing of maintenance
2. Preventive maintenance costs
3. Corrective maintenance costs
4. Different crew availability
5. Tools and equipment
6. Different reliability structures (i.e cold standby, 2 out of 3 sub-systems)
7. Assigning costs to labor

This model uses the optimal replacement theory but tolerances could be added and expanded on. Preventive maintenance time was set as a constant in this model, however it is possible to gather data and calculate an accurate distribution to model the preventive maintenance. The availability of the crew must also be accounted for, as well as the different tools used to fix these components. The structure of the system itself may be altered to provide a more complex system. This model also assigns a fixed cost allotted to the cost of component; however, in reality there are more cots such as the cost of preventive maintenance, the cost of installing the new component. These costs can be added to make the model more complex. While it may seems like a good idea to add more variables, there are other logistical issues that will occur. Some of these variables are not easily determined and others are subjective. These are just a few additional components to the simulation model that can be added. However, the additional variables will add significant simulation time and may take hours to run a single model, but these are factors to be considered when considering a complex system.

Tabu search is a good search method used for many different applications. The one caveat that could be changed for this Tabu search method would be to allow a way for a component to be able to duplicate itself, so that a system such as 1-1-1-7-1 can exist. Extending the Tabu list to allow an optimal move twice may be a way to modify the search to allow duplicate components. However, this may trap the search in a localized area and it may make the solution unable to move to a drastically different structure.

9.0 Conclusion

The examples in this thesis demonstrate how Accelerated Life Testing can be used to show the impact of loads on different components in different possible futures. Each future will change the component's ALTA variables which will in turn change the reliability of the component for that particular future scenario. Since each component's reliability is affected differently by the load, the system may develop drastically different reliability models depending on the future scenario. A safe availability metric can be obtained by combining all the availabilities across the different future scenarios. The problem can be made more complex by adding many different conditions. Thus the model in this thesis is a baseline model to illustrate how Tabu search in conjunction with ALTA can determine the availability of a system with uncertain stress profiles.

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