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ABSTRACT OF THE THESIS<br>COGNITIVE INFLUENCES IN THE PERCEPTUAL BRAIN<br>\title{ EXPERIMENTS AND COMPUTATIONAL MODELS FOR THE AMES WINDOW ILLUSION }<br>By:<br>MARIA ELENI KARAKATSANI<br>Thesis Director:<br>Thomas V Papathomas, PhD<br>Steven M. Silverstein, PhD

The current study is part of a more comprehensive project that aims to explore potential differences between schizophrenia (SZ) patients and healthy controls in perceiving depthinversion illusions (DII). Previous work with two types of DII, namely the hollow mask and the reverse perspective illusions, has indicated that SZ patients tend to rely less on experience and stored knowledge, in this case the experience with faces and linear perspective, than healthy controls. The present study explores how healthy controls perform on variants of the "Ames window illusion" that uses humans' experience of viewing rectangles. The Ames window is based on a rotating trapezoid, typically rotating about a vertical axis that is located in the middle between the vertically oriented long and short bases. Because the trapezoid is perceived as a slanted rectangle, viewers perceive the Ames window illusion, which is a type of DII: the window appears to oscillate back and forth even though it rotates continuously in the same direction. The most plausible explanation is that viewers perceive the inverse depth when the short base is closer than
the large base, because of prior experience in viewing slanted rectangles. We investigated the strength of the illusion by using nine computer-generated windows that were displayed on a screen. The nine windows were designed to vary systematically three key parameters: (1) The long-to-short base ratio; (2) the height-to-short base ratio, and (3) the presence or absence of shadows. These stimuli were used in two experiments to assess illusion strength using two measures: (A) Asking observers to report which base was in front at selected instances, signaled by auditory beeps; (B) Asking observers to indicate reversals in rotation direction. The two measures produced results that had a high degree of correlation, thus confirming the validity of the methods. The data were fed to an optimization algorithm for a model that was based on a linear combination of the weights of the three parameters. The model produced results that were significantly correlated with the experimental data. The next phase will involve experiments with SZ patients, based on the results of the present study.

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## Introduction

A controversial issue in cognitive science has been whether people perceive the world in the same way across individuals. It has been a challenging task to determine whether the spatial and temporal arrangements, the structures, the colors, the textures and every feature that leads the brain to a conclusion of the objects’ identity are universal or whether there is a commitment to universal rules in order to communicate. Scientists suggest (Ungerleider 1982) that it is the interaction between previous experiences and sensory input that leads to the perception of the world as it is. However, physiological differences such as mental disorders (Silverstein 2006, Dima 2009) as well as personal experiences affect the degree of interaction between prior knowledge and visual cues as well as how much each adds to the final percept.

The motivation for the current study is the need to explore the differences in perception between patients with schizophrenia (SZ) and healthy people by utilizing visual illusions as an interesting and insightful tool to investigate and explain the processes that take place in the brain and the visual system. It is important to examine if the same sensory input is perceived differently by the two populations. It is these differences that could indicate the causal risk factors leading to the existence and expression of the disorder. In order to accomplish this task a class of visual illusions called "Depth-Inversion Illusions" - or DII for short - is utilized because of their particular characteristic of giving rise to illusory percepts that are obtained from the healthy population but not from patients with schizophrenia (Silverstein 2006, Dima 2009). The most representative and well studied
examples of DII phenomena are the "Hollow Mask", the "Termespheres" and the "Ames Window" illusion.

## "Reverspectives"

From Ptolemy's years to Patrick Hughes's time, artists, writers and ordinary people showed a great interest in structures that produced the depth inversion effect. One class of stimuli that exhibit DII is the so called "Reverse Perspectives" or "Reverspectives" (Papathomas 2013). Namely, reverspectives are cleverly painted 3D structures comprised of protruding surfaces that are smaller than the base. So, features that are physically closer to the observer are perceived further away than the large base following the rules of linear perspective. According to their presentation mode, linear-perspective structures can be viewed in three different forms: (1) as a reverse-perspective that produces the DII illusory percept; (2) as a forced or proper perspective that doesn't elicit the DII effect; (3) a planar (conventional) perspective that is painted on a 2-D planar surface that everyone is familiar with, exhibiting weaker illusory effects.

Two major corollaries of this phenomenon are: (A) a perceived illusory motion of the stationary object when the viewer moves in front of it and (B) the illusion of a rotating object to be perceived as rotating in the direction opposite to that of of the physical direction of rotation.

Sceneries are commonly used in order to introduce and explain the depth inversion phenomenon. Figure 1 shows a structure of two convex truncated pyramids introducing the scenery from Kastoria, a city in northern Greece. The top view of the orthographic projection shows in proper perspective two small rectangles protruding in the viewer's direction, suggesting that they are


Figure 1: Schematic representation of the truncated pyramids in both physical orientation and illusory perception. The figure is utilized in order to explain illusory motion and was taken from Papathomas (2007) by permission from the author.


Figure 2: The upper Figure is a reverse perspective of Kastoria. It suggests how the observer perceives the scenery. The lower part is an orthographic representation of the structure from different points of view. The Figure is taken from http://ruccs.rutgers.edu/~papathom/biography.htm by permission from the author.
closer to the observer than the rest of the structure, while the building at the center of the structure recedes to the distance. However, if the structure is seen from the front view the illusory percept dominates, thus the buildings jut out and the lake lies at the back of the scene.

Figure 2 shows the two truncated pyramids of the physical stimulus in thick black solid lines, as well as the outline of the perceived object at position 1 in dashed lines and is employed in order to explain the illusory percept. Points on the physical stimulus are connected to the eye with the lines of sight. As the observer moves to the right, a typical point on the actual scene, denoted by point "F",


Figure 3: Schematic explanation of why the reverspective is perceived to rotate in the opposite to the physical direction when viewed from a stationary observer. The figure was taken from Papathomas (2007) by permission from the author. remains stationary. On the contrary, point "F1", the upper right edge of the "illusory percept", moves to the left towards point "F2" giving the impression that the object moved in he direction opposite to that by the observer's direction of motion. Points "F1" and "F2" are attached to the lines of sight and therefore change position during motion, as it can be seen in "Perceived Scene 2".

The same explanation holds for the case of a stationary observer in front of a rotating object. The perceived object follows the lines of sight that connect the physical object to the observer's eye and is therefore perceived to be rotating in the direction opposite to the
actual direction of motion, as seen in Figure 3. In what follows, the perceived scene rotates clockwise while the actual counterclockwise.

Patrick Hughes, an ingenious and famous British artist, invented and created a huge series of reverse-perspective paintings that can be classified as depth-inverting (DII) objects, the structure and logic of which is the same as the one illustrated in "Kastoria". The "Sticking Out Room", "Vanishing Venice", and "Day Dreaming" are some of his masterpieces. Hughes shows apparent talent in combining the three different types of perspective in one work in "Day Dreaming".

Sceneries explain in the simplest way how the brain can be misled by the depth inversion effect produced by challenging perspective rules. The reasoning behind the motion of stationary objects, when the observer remains in the illusory percept, is helpful in understanding other DII stimuli as well.

## "Hollow Mask"

The "Hollow Mask" illusion is the foremost representative example of this class of objects because of its properties and its extensive employment by researchers (Papathomas 2007,2013). The principle of this illusion lies in the tendency to perceive objects, especially faces, as convex even if they are concave. This phenomenon could hold locally, meaning that it occurs only for some parts of the object or globally when referring to the whole object. In particular, the hollow mask illusion refers to a concave face, like a mold, that is perceived as a normal convex one when viewed from sufficiently
large distances. This bias is due to the familiarity with convex objects that surround us, especially faces while it is less likely to find objects that are concave in our daily life.

The explanation as to why the mask appears to move when the viewer moves in front of it lies in the geometrical analysis of the lines of sight. The actual features of the mask are anchored to the lines of sight. By focusing on the left eyeball of the mask, which is marked with the letter " $F$ ", feature " $F$ " retains its place - since the mask is stationary - while the viewer moves from position 1 to 2 when the veridical concept is obtained. However, when the viewer obtains the illusory percept, the hollow mask is viewed as convex and thus the physical left eye is viewed as a right eye. At position 1 the illusory eye lies along the dashed physical line of sight

Figure 4: Schematic representation and explanation of the Hollow Mask Illusion. The figure was taken from Papathomas (2007) by permission from the author. and is marked with the letter
"F1". At position 2, the
illusory eye lies along the solid line of sight and is marked with the letter "F2". "F1" and
"F2" do not overlap, showing that, when the viewer moves from position 1 to position 2, the illusory eye shifts from F1 to F2, demonstrating an apparent motion as shown in Figure 4.

The same explanation holds for the case of the "Hollow Mask Rotation". In detail, when the concave mask rotates clockwise the motion that is perceived is in the opposite direction, thus counterclockwise, because the illusory characteristics lie along the changing lines of sight, yielding an illusory percept.

Cues that affect this phenomenon that can be manipulated either to strengthen or to weaken the illusory percept are the shadowing, binocular viewing, motion parallax, viewing distance, familiarity with the object, and painted random or facial features (Papathomas 2004, Sherman 2011). All these parameters question and test the significance of prior experience and visual cues in terms of brain processing as well as variations among different groups of people that share common features. Studies show that people with schizophrenia, drug and alcohol abusers, and people with sleeping disorders are less susceptible to the "Hollow Mask Illusion" because of the inhibition of the proper transaction of the top-down influences with the visual input. The Hollow Mask Illusion has been proved to be a very helpful tool in investigating the effect of the convexity bias in perception. This is insightful especially when experimenting with SZ patients, who are expected to be less biased than controls. Moreover, this kind of DII stimuli furnishes the rationale for our experiments as it proposes the lack of processes related to previous experiences in the brain of SZ patients. This attenuation is supposed to occur in SZ patients when experimenting with other classes of DII stimuli as well, and this is going to be tested.

## "Termespheres"

Another artist, named Dick Termes, impressed by this interesting field of science introduced a new kind of DII stimuli, the "Termespheres", picturing the scenery that an eye would capture if placed at the center of a transparent sphere (Papathomas 2011). Termespheres are painted like the artist was looking through a very thin lens while light rays pierced the surface of the sphere. Although the sphere is painted on the outer surface, the viewer might get the impression that it is a concave object after prolonged observation and especially when the scenery suggests this percept. Such objects that exhibit both the veridical and the illusory percept are called bistable because the observer might switch percepts from veridical to illusory and vice versa arbitrarily and frequently. As is usually the case, when the moving viewer is in the veridical percept, the sphere seems to move in the actual direction of the motion. On the other hand, if the sphere is rotating and the observer is in the illusory percept, the sphere seems to rotate in the opposite to the physical direction. The explanation follows the general rules of perceiving DII stimuli.


Figure 5: Three views of a Dick Terms' famous piece of art utilizing the "six point" technique. The picture depicts "St. Mark's Square". The figure was taken from Papathomas (2007) by permission from the author.

## Theories on the interaction of top-down and bottom-up mechanisms.

All DII stimuli mentioned above share one important characteristic despite their profound differences: they all produce the depth inversion effect which is related to familiarity cues and consequently to familiarity and past experiences. The interaction of this prior knowledge and the effect of visual cues varies among the population and this is going to be tested by another class of DII stimuli in order to confirm previous findings and strengthen the argument. However, it has to be clarified why the brain chooses to invert depth and what this interaction between processes is like. In other words, how the brain functions in terms of perceptual strategies and rules followed.

Scientific interest has been always engaged in figures and statements that questioned common sense and the basic principles of understanding (Papathomas 2013). DII stimuli are multistable stimuli that lead to semistable solutions that retract each other through questioning the perceptual processes like multistable puzzles give insight to logical systems through competing results. The reasoning for the absence of a solid percept lays in the understanding of the pathway that the sensory input follows from the visual system until the formulation of the percept. It is crucial to be aware of the way the information is being processed in the brain, depending on the object being observed, in order to relate the bottom-up visual cues and the top-down visual "knowledge" to arrive at the final percept.

Visual input is essential in the brain's sensory and perceptual function, although it is based on partial information due to occlusions or self occlusions. However, even under optimal circumstances, the visual input would be only a subset of the available
information. As Adelson and Bergen (1991) argued, factors that are included in the visual input are the object's orientation, the variation in eye angle, the corresponding geometry, the sampling frequency and other relevant limitations.

During the 1920s and 1930s Gestalt psychologists argued that perceptual mechanisms are based on grouping principles thus; a structure's percept is related to the interaction and combination of the partial percepts of the pieces that it consists of (Frisby 2010). "Gestalt" is a German word that means shape or configuration. In other words, the brain looks for features that are familiar and their configuration is expected. This information is integrated into the perception, which results in the configuration of the object viewed. For instance, when someone looks at the object of interest, the brain receives external input at first such as, wheels, window shields, doors, panels, bumpers, wipers and mirrors and then steering wheel, seats, gears and board. All these separate pieces of this unknown structure are processed in order to be appropriately combined to something similar to previous objects seen, such as a car. A generalized description of this brain process that interprets environmental cues is described by the flowchart in Figure 6. As already mentioned, information from environmental cues enters the sensory system, and is compared to knowledge obtained from previous experiences, leading to the construction of several hypotheses on visual percepts. Whenever the bottom-up data and top-down hypotheses come to an agreement, the observer arrives at a stable percept, but if they are competing, there may be multiple competing semistable percepts.


Figure 6: The flowchart shows the pathway the human brain follows in order to interpret sensory input taking into account the contribution of previous experiences to the final percept. Hypotheses are formed and tested until arriving at a stable or semistable result.

It is emphasized that the formation of percepts is guided by a combination of "top-down" and "bottom-up" influences. The contribution of each process to the final percept is the point of interest in this study as it differs in healthy people from people suffering from mental disorders (Frisby 2010)..

Top-down processes refer to concept driven information that the visual system utilizes in order to come to a safe conclusion of the object or scene observed.

Bottom-up processes refer to data driven information that the visual system receives from the environment in order to shape a perception of the world.

The terms "bottom-up" and "top-down" are associated with the direction of the pathway that the brain utilizes in order to process the information that would lead to the formation of a percept. As seen in Figures 7, the direction starting from the eyes and arriving at the frontal lobe has heuristically an upwards orientation and therefore data driven processes are called bottom-up. On the other hand, the feedback from the extrastriate cortex toward the primary visual cortex, as well as other areas of the brain, has a downwards orientation that is known as top-down.

Physiologically, the eyes capture an "image" and the data arrive at the Lateral Geniculate Nucleus (LGN) through the optic tract. Optic radiation conveys the information to the primary visual cortex V1 at the occipital lobe that transmits them further to the extrastriate visual cortical areas $\mathrm{V} 2, \mathrm{~V} 3, \mathrm{~V} 4$ and V 5 . The processing of information from V1 through V2 all the way to V5 is called "The Dorsal Stream", described usually as the "Where Pathway" or the "How Pathway" and is associated with the object orientation in space. The ventral stream or the "What Pathway" refers to the transmission of
information from V1 through V2 all the way to area IT and is associated with shape recognition. All cortical areas interact with each other as well as with other brain areas exchanging feedback before processing the information to the frontal lobe (Ungerleider 1982).


Figure 7: A schematic representation of the areas in the human brain. The numbering of brain areas is explained in more detail in Figure 8.


Grossberg (1987) argued that the human brain needs stability in order to arrive at safe conclusions over time when exposed to familiar input but also agility in order to adjust to unexpected circumstances. The degree of participation of each of the two characteristics varies among situations. Moreover, Johnson and Hawley (1994) showed that despite the favorable baseline of familiarity and stability, the human brain is more attuned to novelty. This controversial theory can be explained by the interaction of top-down and bottom-up processes. As a matter of fact, all theories point out that schema-driven (top-down) processes prevail in case of expected input while data-driven (bottom-up) mechanisms take over control when the input is novel. It is this interaction that leads the brain to the final percept (Papathomas 2013).

Scientists proposed various theories that explain physical visual processes when a moving observer stares at stationary objects (Papathomas 2013). The "theory of direct perception" suggests that perception under self motion is based only on retinal information, while "inferential theories" argue that a combination of retinal and extraretinal signals is utilized in order to lead to an unambiguous decision. According to the latter, the observer creates a representative model of the object in the brain. Retinal flow coupled with the model as well as the eye and head movements determine the expected retinal flow which finally is compared to the actual. According to the resulting differences, the brain concludes as to whether the object is stationary or not. Although it is still debated, this explanation holds for understanding the depth inversion effect.

Familiarity, environmental information as well as basic grouping principles are coupled in order to interpret the input that the visual system receives. The resulting percept is the outcome of testing and evaluating possible interpretations. However, several studies
demonstrate results showing that people suffering from mental disorders such as schizophrenia are less susceptible to DII-type of visual illusions. Interestingly, the information processing in their brain is based primarily on the received input in order to represent the environment, while further interaction with prior experiences is suppressed.
"Schizophrenia" was firstly introduced by Eugen Bleuler, the famous Swiss psychiatrist of the early $20^{\text {th }}$ century, as a more thorough and elaborate description of a disorder described previously by Emil Kraepelin as "dementia praecox" or "early dementia". "Schizophrenia" is a Greek word translated as "split mind" proposing an abnormal integration of feeling and thinking, thus a pathological loss of connectivity in mental functioning. Bleuler observed that it is about a family of related deficits and not about a single symptom or disorder and pointed out that its importance lies in the expression of the disease through a variety of symptoms as well as its progression (Silverstein 2006).

Research on patients with schizophrenia exposed to visual illusions shows that they manage to perceive the depth as physically presented when exposed to DII stimuli and are not affected by perspective cues, prior experiences or familiarity (Dima 2009). Experiments employing the "Hollow Mask" illusion revealed that SZ patients perceive the hollow mask as a hollow mold while controls observed a normal convex face under the same conditions of lighting, distance and viewing mode.

A narrow interpretation of these results suggests a profound deficit of SZ patients' bias in favor of convexity during perception. It is of great interest to investigate whether a dysfunction in neuronal connectivity related to convexity is the underlying cause or a more specific impairment for face processing or for linear perspective processing.

As a matter of fact, a broader plausible explanation implies that the disequilibrium in the function of the brain results in a dysfunction of the interaction of top-down and bottomup processes leading to the prevalence of data-driven information. This imbalance coupled with a deficiency in the correcting systems suggests that the brain of SZ patients groups several pieces of the stimulus together without comparing them to similar previously seen objects that could be expected. The efficacy of the broader explanation should be examined under extensively experimenting on patients with other classes of DII stimuli.

## Studying the "Ames Window Illusion"

Having identified the populations whose perception is about to be examined and having clarified the data processing network in the human brain, it is significant to carefully choose the stimuli that will be utilized. Among the DII stimuli mentioned, there is a class that hasn't been studied as extensively as the "Hollow Mask" and the "Reverspectives" and is called the "Ames Window Illusion". It is about a trapezoidal structure that produces an illusory motion effect due to the depth inversion effect, due to its geometrical characteristics. It is suggested in the literature that, when viewing several variants of the "Ames Window", the strength of the illusion depends on several parameters.

The hypothesis explores if varying the appearance of a window - from looking like a perfect rectangle to a trapezoid with large aspect ratio - could be the key component for the strength of the illusion. The illusory percept is based on the bias to perceive
trapezoids such as the "Ames Window" as slanted rectangles because of the daily exposure to rectangular forms such as doors, windows, tables and floors (Papathomas 2013). Furthermore, other parameters are examined for the degree of their contribution to the illusory percept as well. To accomplish these tasks, computer-based models of the "Ames Window" were constructed by altering long to short side ratio, width to short side ratio and shadowing. The study consisted of two experiments in which the same stimuli were set on rotation but the participant's task was altered. The comparison of the two experiments is anticipated to confirm the hypothesis that observers' prior experiences gives rise to the illusory percept in controls while the suppression of prior influences and the prevalence of stimulus-driven information gives rise to the veridical percept in patients with schizophrenia.

Furthermore, it is argued (Silverstein 2006, Dima 2009) that patients do not only lack in convexity bias as suggested by previous experiments on SZ patients with the "Hollow Mask" (Keane 2013) but also in perspective and rectangularity bias. It is hypothesized that SZ patients should remain at the veridical percept when exposed to any illusion that depends on top-down cognitive information. Finally, we should investigate whether patients do not obtain the illusory percept at all or there is a threshold in strength beyond which the illusory percept is obtained.

The "Ames Window" is selected because of its ability to produce stable illusory percepts, and it is a powerful proof of the importance of schema-driven processes in perception. The importance of this study is to investigate the contribution of top-down influences in the process of perception.

## Details on "The Ames Window"

The lawyer and artist Adelbert Ames developed another class of objects exhibiting the depth inversion effect, some of which are considered important in psychology courses as well as in science museums. The "Ames Chair", the "Ames Room" and the "Ames Window" are only some examples of his work (Papathomas 2013).

The "Ames Window" as presented in Figure 9 is a planar trapezoid that is painted as a window. The theory behind this craftwork is that people have the tendency to perceive trapezoids as slanted rectangles, especially when the former are set in motion and viewed from adequate distance. Even a planar representation of the object under linear perspective is ambiguous about whether it is a rectangle that is rotated over the vertical axis or a frontoparallel trapezoid. During 3D rotation, there are periods during which the short side comes closer to the observer than the long side. However, the rules of perspective suggest that objects that are closer to us cast a lager


Figure 9: Schematical view of the Ames Window. The short base of the trapezoid is indicated by the letter " $A$ " and the long base by the letter "B". The rotation axis is set in the middle with a clockwise direction. The Figure was taken from Papathomas (2013) by permission from the author. retinal image than objects of the same size that lie further away. Based on this fact, it is obvious that when the physical short side of the object is closer to the viewer than the
long side, the depth is inverted leading to an illusory oscillation instead of the physical 360-degree rotation. The same illusory percept rises if the viewer moves in front of a stationary trapezoid. The result is the perception of an illusory motion.

Figure 10 illustrates a possible explanation considering a stationary observer, who faces a trapezoid. The window AB (solid black line) rotates clockwise to position CD (dash-dotted black line). The letter A denotes the short side while the letter B denotes the long side. The trapezoid is rotating clockwise and therefore the solidblack arrow AB will rotate to position CD ,
 indicated by the dash-dotted black arrow.

Figure 10: Schematic explanation of the Ames However, if depth is inverted $A$ is perceived at the back as A' and B closer as B'. The perceived trapezoid is now $A^{\prime} B^{\prime}$ and after the rotation it moves to Window illusion. The plain letters indicate the actual shape of the trapezoid and its physical rotation while the primed letters denote the illusory percept and the rotation in the opposite to the physical direction. The figure was taken from Papathomas (2013) by permission from the author.

C'D' which suggests a counterclockwise rotation, thus opposite to the physical direction.
The same explanation holds for a moving observer in front of a stationary trapezoid.

Ames's original apparatus consisted of rectangular and trapezoidal physical windows that shared the same vertical axis and rotated at the same direction and speed (Ames 1951). Each window had a rod in the middle at an angle of $45^{\circ}$ and an attached cube at the upper
left edge of the short side. Light was shed from above and the angle between the windows was adjustable but held during rotation while the apparatus was hanging from the ceiling and rotated under the force of a motor. This design aimed at the observation of different percepts during rotation with respect to the geometrical characteristics of the windows. According to Ames's original experiment, when people with normal vision looked at the rectangle they saw a rotating rectangle of constant size, speed and direction with the cube and the rod coming along. On the other hand, when the trapezoid was observed, people saw a rectangle of changing form that oscillated at varying speed while the rod penetrated it at some time points and the cube sailed around.


Figure 11: Photograph of Ames original apparatus. The surfaces of the rectangular as well as the trapezoidal window are in the same plane, which is perpendicular to the camera. The picture is taken from Ames (1951, page 2).

It should also be mentioned that these percepts change by varying parameters such as the size of the trapezoid, the distance between the observer and the apparatus, the direction as well as monocular or binocular viewing.

Chart 1 demonstrates schematically the top view of the physical (Row I) and the perceived (Row II) motion of the rotating trapezoid as well as the motion of the cube (Row III) and the rod (Row IV). The first two rows will be elaborately analyzed in order to understand the differences between physical rotation and of the physical (Row I) and the perceived (Row II) motion of the rotating trapezoid as well as the motion of the cube (Row III) and the rod (Row IV). The first two rows will be elaborately analyzed in order to understand the differences between physical rotation and perceived oscillations. Although the next two rows give a great insight into the effect of cognitive influences, they are not studied in the current project. Letters A and B are assigned to the short and long side respectively, so that line $A B$ shows the apparent position of the trapezoid at various instances. A' and B' stand for the apparent localization of the corresponding sides and therefore $\mathrm{A}^{\prime} \mathrm{B}$ ' shows the perceived position of the trapezoid. Each row counts for one full rotation and the same pattern is repeated.

The rotation started at instances I1 and II1 but the trapezoid was perceived to rotate at a lower than the actual speed at I2 and II2. After $90^{\circ}$ of physical and $50^{\circ}$ of perceived rotation the trapezoid seemed to slow down until it reached a dead stop and rotated to the reverse direction as shown from instances I3 and II3. It oscillated until instance II9 after which it returned to its original position. The oscillation angle was measured to be $100^{\circ}$. During each revolution the trapezoid seemed to be of the same size, smaller or larger than the rectangle according to the angle seen from the observer as well as its configuration in
space while sometimes left the impression of being nearer to the observer. It is interesting to note that 10 feet distance was sufficient for the illusory percept to take place when the observer used monocular vision while the double distance was needed in order to catch the same effects for an observer who saw binocularly.
 Figure 12. Char I presents the rotating trapezoidal window as viewed from above. The circle is a full rotation while the solid the trapezoid in its physical and illusory orientation. The first row refers to the physical orientation while the second to the illusory percept. A and $A^{\prime}$ denote the short base of the trapezoid while $B$ and $B^{\prime}$ the large base. The Chart is obtained from Ames (1951, page 6).

Ames concluded on the most important attributes that were observed by the participants and by coupling their responses with global rules that define perception, he argued on the explanation of the illusory effect of the trapezoid carrying his name. Moreover it is important to acknowledge that the analysis of these observations revealed the parameters that contribute the most towards this effect. To begin with, innumerous occasions have been observed, at which the human brain interprets trapezoidal forms as rectangular. Unconsciously, objects such as doors, desks and windows are seen as rectangles even when the line of sight is such that trapezoids should be seen. Having this fact in mind, one could consider that when the brains view a physical trapezoid, it is biased to perceive a slanted rectangle. Moreover, as a physical window rotates the apparent lengths of the sides change due to perspective. The side that is further away seems smaller than the side that is closer to the observer, but they appear equal when the window is at its frontoparallel orientation. These changes give feedback to the brain suggesting the translation of shape distortions as changes of position and therefore the trapezoid is perceived as a slanted rectangle. When the long side of the trapezoid is closer to the observer the side differences are even bigger and it is seen larger and nearer compared to the case where the closer side is closer.

These considerations furnish a rather reasonable explanation for the changing shape and size of the trapezoids although at a different degree (Ames 1951). Moreover, the perceived oscillation can be explained by the direction reversals that occur during each revolution, since as the trapezoid rotates from instance I3 to I6 the horizontal angle between the trapezoid and the eye decreases. However, the perceived position of a window placed physically at a frontoparallel orientation towards the observer (instance

I3) is that of a slanted rectangle that recedes away as it can be seen from instance II3. Based on prior experience, the trapezoid has to move even further away in order for the horizontal angle to decrease; thus to the opposite than the natural direction. Such reversals occur every time the trapezoid is in frontoparallel position to the eye. All of the above suggest that the sides of a rectangle change according to their relevant distance from the observer when it rotates due to perspective, thus it is self evident that a trapezoid would be perceived as a rotated rectangle. However, because of the side difference that a trapezoid naturally encounters, it never reaches the frontoparallel view with equally long vertical sides as a rectangle would. Therefore the observer is left with the impression of a shortened revolution that lasts longer.

An important question that rises is why the phenomena described change when viewing the trapezoids under different conditions such as various angles, orientation in space and viewing time while the perception of a rectangular window remains constantly the same. The answer is summarized in Table 1, which demonstrates the similarities and dissimilarities that rectangles and trapezoids encounter under different conditions. The rectangle is seen as it is anytime from any point of view and only the angle from the observer changes according to the corresponding position as well as the direction of rotation. On the other hand, the unchanged characteristics for a trapezoid are the "Windowness" and "Rectangularity", while all other features depend on the position of the observer and the time point. If the trapezoid is seen above or below the middle of the structure, then the only characteristic that is definitely the same is its "Windowness". Apart from that, everything else changes according to the observer.

Table 1: Similarities in perception experienced when the same participant observes the trapezoid from different points of view retaining the same distance. The Table is obtained from Ames (1951).

|  | Rectangular Window |  | Trapezoidal Window |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Similar | Dissimilar | Similar | Dissimilar |
| "Windowness" | $\checkmark$ |  | $\sqrt{ }$ |  |
| Rectangularity | $\checkmark$ |  | $\checkmark$ |  |
| Inclination |  | $\checkmark$ |  | $\checkmark$ |
| Size | $\checkmark$ |  |  | $\sqrt{ }$ |
| Shape | J |  |  | $\sqrt{ }$ |
| Motion | $\checkmark$ |  |  | $\checkmark$ |
| Direction |  | $\checkmark$ |  | $\checkmark$ |
| Distance | $\checkmark$ |  |  | $\checkmark$ |

The next question that comes up is whether the observations explained are common among different population or they change across individuals, depending on pathologies and the state of the observer (intoxicated, sleep-deprived, etc.). Table 2 illustrates that in one revolution the observations vary amongst observers according to the space and time characteristics for the case of trapezoid while remain basically unchanged for rectangles.

Table 2: Similarities and differences in perception experienced between two participants observing the same stimulus under common circumstances and especially at the same moment. The Table is obtained from Ames (1951).

|  | Rectangular Window |  | Trapezoidal Window |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Common Aspects | Unique Aspects | Common Aspects | Unique Aspects |
| "Windowness" | $\checkmark$ |  | $\checkmark$ |  |
| Rectangularity | $\checkmark$ |  | $\checkmark$ |  |
| Inclination |  | $\checkmark$ |  | $\checkmark$ |
| Size | $\checkmark$ |  |  | $\checkmark$ |
| Shape | , |  |  | $\sqrt{ }$ |
| Motion | $\checkmark$ |  |  | $\checkmark$ |
| Direction |  | $\checkmark$ |  | $\sqrt{ }$ |
| Distance | $\checkmark$ |  |  | $\checkmark$ |

Although this holds for one revolution, during the entire experiment the observer exhibit similar phenomena for multiple revolutions, however these perceptual transitions are experienced at different time points. In other words, the observers see the same distortions at different times as seen in Table 3.

Table 3: Similarities and differences in perception experienced between two participants observing the same stimulus under common circumstances during the entire trial. The Table is obtained from Ames (1951).

|  | Rectangular Window |  | Trapezoidal Window |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Common <br> Aspects | Unique <br> Aspects | Common <br> Aspects | Unique <br> Aspects |
| "Windowness" |  |  |  |  |
| Rectangularity |  |  |  |  |
| Inclination |  |  |  | $\sqrt{*}$ |
| Size |  |  |  | $\sqrt{*}$ |
| Shape |  |  |  | $\sqrt{*}$ |
| Motion |  |  |  |  |
| Direction |  |  |  |  |
| Distance |  |  |  | $\mathbf{V}$ |

Variations of the "Ames Window Illusion" include tilted windows, windows with convex and/or concave parts, windows rotating around horizontal axis, combinations of more than two "Ames Windows" to form a new object or combinations of an "Ames Window" with 3D objects.

The most important characteristic about "Ames Window" is that observers could be trained under repetitive exposure to perceive the illusory oscillation of the object or obtain the physical rotation under volition. This could be a powerful tool for studies on people with mental disorders such as schizophrenia, who may not experience the illusory percept as strongly as controls

Thorough studying of the parameters listed in the Tables follows in order to analytically observe how and why each cue affects the strength of the illusion caused by the "Ames Window". This process will inform which attributes should be adopted in order to
construct stimuli that strengthen or weaken the illusion in the current experimental process. In this way we are on track with the hypothesis of the parameters being the key component of the strength of the illusory percept.

## Parametric Analysis

Parametric analysis refers to the elaborate investigation of the effect that attributes and sensory signals have on perception as well as to the examination of the variables' interactions (Braden 1978, Sherman 2011). The importance of knowing the way and the extent to which specific parameters affect the strength of the illusion is that it will enable us to construct various trapezoids of different illusory levels in the main experimental component of the current project. Consequently the most important aspects are the perspective, movement parallax, viewing conditions, distance, dimensions, speed of rotation and shadowing.

## Perspective

Graham (1963) was the first who suggested the importance of linear perspective when viewing trapezoidal objects as the main factor influencing the strength of the illusory percept. Linear perspective refers to the angle between the observer and two constant points of an object, causing the visual angle to decrease with increasing distance. Moreover, in the case of trapezoids, the perspective is also produced by the length difference of the two vertical sides. Therefore, it is empirically expected to observe the shorter side always to be further away when the trapezoid is not parallel to the observer.

Apart from linear perspective, there is another category that is favored in computer-based models, namely orthographic projection, which is a type of parallel projection that
contributes to the understanding of the configuration and orientation of several parts of an object. The drawback in this case is that the representation is in two dimensions and there is no way to distinguish longer from shorter sides due to lack of perspective. As it is described, orthographic projection could be a tool to increase the strength of illusory percepts but it could also lead to increased ambiguity which would eventually disrupt the results.

## Motion Parallax (MP) and the Kinetic Depth Effect (KDE)

Motion parallax (MP) refers to the differential angular velocity between a moving observer and two static points on the static object being viewed. A related cue is the kinetic depth effect (KDE). KDE is the differential amplitude of motion trajectories of different points of a rotating object for a stationary viewer: generally, distant points generate smaller motion amplitudes than near points (Braden 1978). Both cues are considered to be rich cues to depth, because they enable viewers to recover the 3D shape of objects. Furthermore, both cues provide accurate sensory input for the orientation, sometimes contradicting linear perspective information, especially in the case of trapezoids.

In more detail, if MP or KDE cues are more robust than linear perspective, then the physical rotation is going to be observed. On the contrary, if the former are ambiguous, then linear perspective leads to the illusory percept of an oscillating object. The conflict arises when both types of cues are similarly strong or weak. Moreover, when the visual angle decreases, motion cues give less accurate input due to ambiguity and prevalence of
perspective cues. As proposed by Zegers (1965) smaller visual angles increase the frequency of oscillations by cutting down parallax's saliency.

## Viewing Conditions

Monocular and binocular viewing conditions of the rotating trapezoids are experimentally employed in order to decide on their degree of participation in the illusory effect. Several studies suggest that monocular vision not only strengthens the illusion but also prolongs the duration the observer spends on the illusory percept (Braden 1978, Hill, Cahill 1975). Theoretically, binocular disparity is expected to lead mainly to the recovery of the veridical percept. As a matter of fact, experiments proved that stereopsis is a quite powerful cue for the recovery of veridical depth. It is experimentally proven that motion parallax coupled with monocular viewing leads to the recovery of the physical object depth, but still weaker than binocular disparity (Papathomas 2012).

## Viewing Distance

The distance between the observer and the stimulus is crucial and interacts with most of the other cues in both positive and negative ways (Braden 1978, Cahill 1975). As a matter of fact, the amount of apparent reversals increases with distance. Movement cues are depend on the distance as it decreases when the observer diverges from the stimulus leading to the strengthening of illusory percepts.

## Dimensions

The term "dimensions" refers to the geometrical characteristics of the trapezoids used in the experiments and influences how linear perspective will affect the illusion. In other words, the degree of perspective under which the object is viewed is specified by the aspect ratio between the lengths of the vertical and horizontal sides of the trapezoid. Previous work has identified the effect of the ratio of long to short vertical side as well as horizontal to short side as important components for the strength of the illusion. Cross (1969) argued that the ratio of the two vertical sides is directly proportional to the amount of perceived oscillations. Many other studies (Braden 1978) have examined the strength of the illusion as a function of the aspect ratio of the long to the short side in trapezoids. On the other hand (Graham and Gillam 1970), there is relatively little work on the effect of the horizontal to short side ratio on the illusory percept, which is investigated in the current study.

## Speed

According to Borjesson (1971), increasing the speed of rotation in trapezoids from 3 rpm to 30 rpm leads to a decrease in oscillations' frequency. However, Braden suggested that increasing the speed from 5 rpm to 20 rpm is sufficient to lead to a decrease in oscillations indicating a lower boundary for the speed threshold. Finally Borjesson decided that the optimum velocity for maximum illusory percept is between 3 and 6 rpm .

Braden (1978) proved experimentally that the frequency of oscillations, hence the strength of the illusory percept, increases at lower speeds of rotation close to 5 rpm .

Shadowing

Shadowing is extremely important, especially in computer-based models in order to yield the perception of a 3D object (Borjesson 1971). There is evidence showing that shadowinterposition cues contribute to the strength of the illusion. Generally, trapezoids with shadows produce stronger illusory percepts compared to windows lacking shadows.

## Experimental Methods

## Cohort

The experimental process was set up in two stages. The first stage involved the recruitment of healthy people usually referred to as the control group. Twenty people from Rutgers University were randomly selected to participate. Despite the absence of specific guidelines they all share some common characteristics such as the age that ranges from 19 to 30 years as well as the educational level which varies from undergraduate students to post doctoral fellows.

The second stage involves the recruitment of the patients with schizophrenia. However, it was decided that in order to examine this more vulnerable group of people, some results should be obtained from the control group indicating whether the stimuli constructed verify the hypothesis. After this validation process the experiments with patients would start.

Before proceeding with the main experiments, a pilot study was conducted from five members working on this project. These people had the same ranges both in age and in educational level as in the official cohort but were familiar with the experiments due to their involvement in the design. The pilot study indicated the range of parameters to be used in the main experiments.

## Stimuli

It is of first priority to construct stimuli producing different degrees of illusory strength by altering the attributes described in Table 4 . The variation as well as the effect of each
parameter on the resulting strength of the illusion should be clarified. Some of the parameters are kept unchanged in the current study while others vary in order to produce the desired effect.

Table 4: Presentation of the parameters changed or maintained constant in the current study .

|  | Constant | Variable |
| :--- | :--- | :--- |
| Dimensions |  |  |
| Perspective |  |  |
| Kinetic depth effect |  |  |
| Distance |  |  |
| Speed of rotation |  |  |
| Shadowing |  |  |
| Viewing conditions |  |  |

## Dimensions

Since the original "Ames Window" was a comparison between a rectangular and a trapezoidal window, these two instances should count as the extremes in the current study (Ames 1951). The actual dimensions of these windows are exactly the same as Ames proposed in his experiment, while the rest of the instances are intermediate variations based on previous studies. The sides that are varied are the long vertical side and the horizontal side, usually referred to as the width, while the short side is kept constant as reference. Table 5 provides information about the actual lengths of the sides as well as the corresponding ratios (Braden 1978). The Long to $\underline{\text { Short vertical side ratio is }}$ abbreviated as LS and the Width to Short vertical side ratio is abbreviated as WS.

Table 5: Analysis on the structural parameters that were changed, their corresponding values and ratios.

| L | W | S |
| :---: | :---: | :---: |
| 19 | 19 | 19 |
| 20.5 | 28.5 |  |
| 21.82 | 42.75 |  |
| 28.5 |  |  |
| LS |  | WS |
| 1 |  | 1 |
| 1.07 |  | 1.5 |
| 1.15 |  | 2.25 |
| 1.5 |  |  |

According to Ames's original experiment as well as the literature reviewed, it is expected that the strength of the illusion will increase with increasing LS ratio. The amount of the produced strength is going to be measured and validated. Although there is strong evidence in that the LS ratio increases illusory strength, there are only some indications on the effect of WS variation and its effect on the illusion. It is speculated that the strength of the illusion increases when the WS ratio drops from 2.25 to 1 . This hypothesis is based on the alteration of the perceived lengths of the vertical sides for different widths when the trapezoid is viewed under perspective. In order to obtain a clearer view of the hypothesis, we consider two trapezoids varying only in width. When both are set in rotation under the same conditions, the short side comes closer to the observer while the longer side moves away. At this position, the sizes of the vertical sides are perceived slightly differently due to linear perspective. As described in the introduction, the short side will appear larger and the long side will appear shorter than the physical dimensions. If the width increases, then the effect of perspective is even stronger. In other words,
when the short side is parallel and close to the observer's line of sight, the vertical sides' difference (VsD) is minimized as seen in Figure 13 due to linear perspective.


Figure 13: The dark and the light blue trapezoids share the same rotation axis as well as the same bases' length. Their difference is in width with $W_{L}>W_{s}$. When the trapezoids are viewed at $90^{\circ}$ from the presented orientation the short side is parallel to the line of sight. Due to perspective cues the short side from both trapezoids is going to be observed longer than the physical side and the longer side shorter than the physical side. The vertical side difference is going to be larger for smaller widths than from larger $\mathrm{VsD}_{\mathrm{S}}>\mathrm{Vs}_{\mathrm{L}}$.

The corollary of this phenomenon is that the strength of the illusory effect will be reduced by increasing WS ratio due to linear perspective's contribution. This relation between WS and illusory effect will be validated in the current study.

Having described the differences in dimensions that have to be tested, twelve stimuli were constructed accounting for all possible LS and WS combinations. The pilot study revealed some important stimuli producing variations of illusory percepts as well as some overlapping occurrences. After eliminating the latter, the stimuli that were selected for the experiments are presented in Table 6

Table 6: a. Analytical representation of the nine stimuli utilized according to their geometrical characteristics. The short side (S), the long side (L) and the horizontal side (or width W) length as well as their corresponding ratios are shown in centimeters.

| Stimulus <br> Number | Condition | S | L | W | LS | WS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 12 | 19 | 19 | 19 | 1 | 1 |
| 2 | 8 | 19 | 19 | 28.5 | 1 | 1.5 |
| 3 | 3 | 19 | 20.5 | 42.75 | 1.07 | 2.25 |
| 4 | 16 | 19 | 20.5 | 28.5 | 1.07 | 1.5 |
| 5 | 19 | 19 | 20.5 | 19 | 1.07 | 1 |
| 6 | 13 | 19 | 21.82 | 42.75 | 1.15 | 2.25 |
| 7 | 18 | 19 | 21.82 | 19 | 1.15 | 1 |
| 8 | 14 | 19 | 28.5 | 28.5 | 1.5 | 1.5 |
| 9 | 17 | 19 | 28.5 | 19 | 1.5 | 1 |

## Perspective

Computer-based animations could be presented under orthographic projection as well as under perspective projection. At first orthographic projection seemed promising in terms of increasing the illusory effect but this idea was abandoned due to ambiguity in the perception of orthographic projection animations. The pilot study revealed that the absence of perspective caused uncertain observations. Therefore only linear perspective is used for all stimuli.

## Kinetic Depth Effect (KDE)

The participants remained seated during the entire experimental process. However, because of the Kinetic Depth Effect (KDE), i.e., the differential angular velocity between
the observer and two points on the rotating object viewed, it is obvious that the KDE magnitude changes, depending on the parameters of the rotating window (Braden 1978). The differences in angular velocity are caused by the variation of the stimuli' dimensions.

## Distance

Since the experiment is computer based, the virtual distance that the stimulus is placed from the observer is set at 12 feet (Braden 1978). The observer is considered to be seated 50 cm in front of a computer screen at a natural distance.

## Speed of rotation

For the pilot study, two extreme speeds were tested at 5 rpm as the most adequate speed for observing the illusory percept, and at 20 rpm as the speed at which the illusion breaks. The results obtained were consistent with previous studies (Braden 1978). Since the fast rotation didn't contribute significantly to the process, this speed value was abandoned for the experiments, and 5 rpm was chosen as the preferred speed for all stimuli.

## Shadowing

Shadowing is a significant factor contributing to the illusory percept (Borjesson 1971). The stimuli that were expected to produce a weak effect did not have shadow in order to cluster them to the weakest stimuli. On the other hand, moderate and strong illusory effects were produced by stimuli that were assigned with shadow.

Table 7 summarizes the characteristics of each stimulus constructed for the experimental process.

Table 7: Analytical representation of the nine stimuli employed according to the parameters utilized.

|  | Condition | LS | WS | Shadow | Speed | Projection |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 12 | 1 | 1 | 0 | 5 | Perspective |
| $\mathbf{2}$ | 8 | 1 | 1.5 | 0 | 5 | Perspective |
| $\mathbf{3}$ | 3 | 1.07 | 2.25 | 0 | 5 | Perspective |
| $\mathbf{4}$ | 16 | 1.07 | 1.5 | 1 | 5 | Perspective |
| $\mathbf{5}$ | 19 | 1.07 | 1 | 1 | 5 | Perspective |
| $\mathbf{6}$ | 13 | 1.15 | 2.25 | 1 | 5 | Perspective |
| $\mathbf{7}$ | 18 | 1.15 | 1 | 1 | 5 | Perspective |
| $\mathbf{8}$ | 14 | 1.5 | 1.5 | 1 | 5 | Perspective |
| $\mathbf{9}$ | 17 | 1.5 | 1 | 1 | 5 | Perspective |

## Viewing Conditions

Although observers viewed the stimuli binocularly (both eyes open), the very nature of the stimuli is inherently monocular. The order the stimuli are presented is based on the informal results obtained from the pilot study coupled with the knowledge gained from the thorough review of the literature. The illusory effect is expected to increase from stimulus 1 to 9. In detail, the nine stimuli are divided in three subgroups; stimuli 1 to 3 are expected to produce weak illusory effects, stimuli 4-6 represent the moderate class, and instances 7 to 9 are stimuli causing strong or very strong illusions. The hypothesis suggests that reordering of the stimuli inside the subgroups is not only expected but will be insightful in terms of factors' contribution to the effect. However, the classes have to hold in order to maintain consistency with the literature and the hypotheses.

## Experimental Setup

The stimuli described were constructed in Photoshop and then manipulated in MATLAB R2012b utilizing Psychtoolbox (add references). The experiments run on a laptop that was could be carried anywhere, making it convenient for people who do not like to or cannot be transported easily. However, no computer knowledge was required of the participants. Observers had to observe the rotating windows and report their percept according to the task assigned.

Before each experiment a live demonstration with a physical sample stimulus took place, as seen in Figure 14. The administrator held the sample and explained to the participant how the stimulus would look, and what the experiment was about. The sample stimulus was a red rectangle that had 3 white dots on one vertical side and 3 black dots on the other vertical side. The rectangle had a fixed spill (axis) in its center, so that it could rotate clockwise or counter-clockwise. This structure exhibited all the characteristics of the stimuli used in the main experiments. The rotation axis was placed at the center of the windows and the dots were assigned on each vertical side respectively in order to address clearly the task to the observer.

The participant sat in front of the laptop at a viewing distance of about 50 cm during the entire process. After clarifying that the task was understood, the experimenter started the computer program and entered the code that was used anonymously and confidentially for each occupied approximately $1 / 3$ of the screen.


Figure 14: The photograph of the physical 3D sample stimulus utilized in the live demonstration.

Figure 15 demonstrates a typical stimulus that the participant viewed during both experiments.


Figure 15: The photograph of one of the nine actual stimuli employed in both experiments.

The participants viewed the nine versions of the Ames Window, each one presented in four different ways, in a 2 (position of white dots) x 2 (direction of rotation) design: white dots on the short and black dots on the long side, or vice versa, as well as clockwise and counter- clockwise rotation. Overall, the observer was exposed to 36 stimuli in a random sequence for 40 seconds each. Every trial started with a window on the screen, rotating at a rate of 5 rpm . Trials followed each other in quick succession that was comfortable for the participant. Breaks took place at any time the participant felt like needing them and lasted as long as the participant wished before proceeding to the next trial.

With all parameters taken into consideration, each experiment lasted approximately 30 minutes, as determined in pilot experiments. This time span would be longer if the participant took longer breaks. The experiment guaranteed safety without any risk for the participant. In order to obtain meaningful and understandable data, the criteria that define the strength of the illusion should be clearly identified.

Previous studies provided insight into how the illusion is measured in the case of the "Ames Window". Scientists prefer to measure the illusory effect of the current stimulus by the amount of perceived oscillations (Ames 1950). As a matter of fact, observers are likely to see two direction reversals per full revolution of the window. This statement could easily be explained by Chart I, and especially rows I and II that give insight to the maximum amount of oscillations that could be experienced, as well as the phase at which these seem to occur. Row I stands for the actual revolution of the trapezoid with "A" denoting the short and " B " the long side. Row II refers to the perceived motion of the window. By comparing the two rows it is obvious that two changes in direction occur per
cycle. When the rotation starts, the stimulus is observed to rotate counterclockwise until instance II3, where it seems to slow down and reverses direction. From II4 to II9 the window rotates with a clockwise direction. The second time the window changes direction is at II9 when it starts rotating counterclockwise again until II11. Thus, for each physical rotation there are two perceived oscillations.


Figure 16: The first two rows of Chart I.

In the current study the total amount of the illusory percept is determined by the criteria described that vary the illusion from weak to strong.

## Experiment 1

The goal of this experiment was to measure the illusion perceived for each stimulus by analyzing participants' reports. By comparing the strength of illusory percept, the stimuli would be sorted in increasing strength order. The participant's task in this session was to report whether the side with the white dots was closer or further away from him/her than the side with the black dots, at certain strategically selected instances. Before the actual experiment, the experimenter performed the live demonstration described, by specifically asking about whether the side with the white dots were closer or further away than the one with the black dots.

Experimentally, a sound (beep) was heard at a time that corresponded to 10 degrees after each frontoparallel position was attained (Chart 1 instances I3, II3); this beep prompted the participants to report whether the side with the white dots was closer than the side with black dots, or was further away. The first beep occurred after a certain accommodation period (intended to familiarize observers with the spatial arrangement of the stimulus), the length of which was randomized across trials. One full revolution lasted 12 seconds and attained the frontoparallel position twice, resulting in two prompts per revolution. The participants had to report their percept by pressing and releasing on of two keys on the keyboard every time they heard the beep, depending on whether the white dots appeared closer or further away than the black dots. The response "closer" was assigned to the down arrow, while the answer "further" to the up arrow. We randomized the order that the stimuli were presented to avoid ordering effects. Moreover, the angle at which the window started its rotation (Chart 1 any instance) varied randomly across trials. With all parameters taken into consideration, the entire experiment lasted approximately 30 minutes, as determined in pilot experiments. Ultimately, it is evident that a maximum of one illusion was to be perceived per full revolution. Since each rotation took 12 seconds to occur and the trial lasted for 40 seconds there were three full revolutions per trial and consequently a maximum of three illusions to be perceived for stimuli that produce strong illusions.

The experiment was coded so that reports and corresponding physical orientations would be automatically recorded and compared, leading to four possible interpretations, in a 2 (physical orientations) x 2 (reported orientations), i.e., white dots being closer or further than black dots. Table 8 demonstrates how the interpretation of the comparison between
perception and reality is formed according to the orientation of the side with the white dots. The notation for the symbols used (I, NI, T, F) is explained in what follows.

Tables $8 \mathbf{a}$ and $\mathbf{b}$ : Explanation of the interpretation method followed in Experiment 1 according to the comparison of the physical orientation (Reality) and the perceived (Response) configuration of the stimulus.

White dots on the short side

| Reality/Res <br> ponse | Closer | Further |
| :---: | :---: | :---: |
| Closer | NI | I |
| Further | F | T |

a.

White dots on the long side

b.

An example of one possible configuration is chosen in order to explain the interpretation possibilities: the white dots are on the short side, the window is placed vertically to the observer with the tracking side on the back rotating counterclockwise as seen in Rows I and II of Chart I.

Illusion (I): Although the short side is physically closer to the observer, it was reported that it is perceived to be further away. Instances I6 and II6 verify the perception of the illusion when the short side is physically located closer to the observer than the long side.

No illusion (NI): In this case, even though an illusion is expected to be perceived the observer reports no illusion. This would be the case at which the observer perceives instance I6 instead of II6.

True ( $T$ ): Perspective rules are followed; there is no illusion to be perceived and the observer reports no illusion. This is the case in both I1~II1 as well as I11~II11.

False $(F)$ : Although perspective rules are followed, the participant perceives a depth reversal unexpectedly. The report disagrees with the physical locations of the two sides, and no illusion is expected to be perceived. At this case the reverse of instances II1 and II11 would be observed.

After collecting the data, the experimenter counted the number of times each answer appeared according to the stimulus viewed and argued on the illusory strength by assigning different weights to each answer. How these calculations were obtained will be discussed in the results section.

## Experiment 2

The second experiment was designed to validate the results obtained from the first experiment. The same process was followed as in the first experiment, but the task assigned to the observer was different. Participants observed the rotating windows and decided on the direction of rotation or oscillation perceived.

A live demonstration occurred with the same physical sample stimulus in order to clarify the difference from the first task. The administrator held the rectangular sample and
rotated it in both clockwise and counter-clockwise direction asking the participant to report the direction of the motion. The window could be perceived rotating in one direction during the entire trial or several reversals in direction within the period of a trial could be experienced.

Because the task encountered some ambiguity as well as confusion of the terms clockwise and counterclockwise, the administrator asked the participants to state verbally their perception during the experiment and recorded the percept accordingly, instead of having them do it. Whenever the report was clockwise, the administrator pressed and released the right arrow and in the case of the counter clockwise perception the left arrow was pressed and released on the keyboard. Observers decided freely on when to repotrt the direction of motion although the administrator reminded them to respond when judged necessary. It was clear that they had to choose a direction as soon as the stimulus was in the screen and maintain or reverse their report according to their percept. It was speculated that stimuli producing strong illusory percepts would be seen to oscillate frequently while less strong stimuli would be seen to rotate as they naturally were. No prompt sound was employed.

The experiment was coded so that reports and corresponding physical orientations would be recorded and compared leading mainly to four possible interpretations, in a 2 (actual direction of rotation: CW or CCW ) $\times 2$ (reported direction of rotation: CW or CCW ) combination [CW: Clockwise; CCW: Counterclockwise]. The interpretation depended on the orientation of the window, i.e., whether the long side of the window was on the left or right.

Table 9demonstrates how the interpretation of the comparison between perception and reality is formed according to windows conformation.

Tables 9 a,b: Explanation of the interpretation method followed in Experiment 2 according to the comparison of the physical direction of rotation (counter- clockwise) and the perceived (Response) direction of rotation of the stimulus.

| CW / <br> Response | CW | CCW |
| :---: | :---: | :---: |
| LS on the <br> Left | NI | I |
| LS on the <br> Right | C | F |

a.

| CCW / Response | CW | CCW |
| :---: | :---: | :---: |
| LS on the Left | W/S | C |
| LS on the Right | I | NI |

b.

An example of one possible configuration is chosen in order to explain the interpretation possibilities as seen in Rows I and II in Chart I. By the time the window appears at a random angle, its motion is ambiguous. As soon as it arrives at the orientation described by instance I3 it is obvious that the long side is on the right and the window is rotating counterclockwise. However, every time the long side is moving further away (from I3 to I9), the participant is supposed to perceive the illusion (from II3 to II9) according to the stimulus' characteristics. The first row of the chart represents the physical rotation that is in counterclockwise direction as seen. The second row demonstrates vividly reversals in direction. The window rotates counterclockwise from II1 to II3, clockwise from II4 to II9 and then counterclockwise again.

Illusion (I): Since the physical rotation is counterclockwise and the long side is on the right, each time the participant reports clockwise direction, then the illusion is perceived (II3).

No illusion (NI): This is the case at which the window is perceived to rotate counterclockwise during the entire trial, even when the illusory percept should be perceived. (Row I instead of Row II)

True ( $T$ ): There is no illusion to perceive and the participant does not report an illusion. (II1 to II3 and II9 to II11)

False $(F)$ : Although perspective rules are followed, the participant perceives oscillations unexpectedly. This report disagrees with the physical rotation.

Since the illusory percept rises when the window is moving from the parallel (I3~II3 or I9~II9) to the vertical plane (I1~II1 or I6~II6 I11~II11) it was expected that observers would report their percept during that period, although that was not always the case. During the pilot study, it was observed that some responses of rotational direction reversal were recorded at unexpected in stances; these responses are known to occur spontaneously and are not relevant to the strength of the illusion. Since reversals were expected to occur right after frontoparallel orientation (instance I3, I9), and since we know that participants respond with a certain delay (reaction time RT), a response angle of $60^{\circ}$ was assigned to the program as a threshold to account for the reaction time. This scheme was designed to distinguish the expected from the spontaneous responses. The degree level was chosen at 60 degrees which corresponds to the $\frac{1}{6}$ of the revolution, thus 2
seconds. This time interval was chosen as an upper bound for the reaction time in similar situations. RT is defined as the time elapsed between the appearance of a sensory stimulus and the corresponding action or perception. It usually lasts 0.2 to 1 second, but in order to include any unexpected disturbances or difficulties, it is set to 2 seconds. Figure 17 demonstrates the case.


Figure 17: A top view of the rotation. The threshold is shown in degrees $\left(60^{\circ}\right)$ for both cases of counter- and clock-wise rotation.

The reports obtained, starting from the frontoparallel-plane position until the threshold of 2 seconds later were interpreted as proposed above. However, if the participant's response occurred at a time slot after the threshold of 2 seconds, then the prefix " S " (" S " standing for "Spontaneous") was assigned to each interpretation, SI, SNI, ST, SF accordingly, to indicate a spontaneous reversal in the direction of rotation.

The assessment of the strength of the illusory percept follows the logic of the first experiment, by which a maximum of one illusion is expected to be observed per full revolution. Since each rotation took 12 seconds to occur and the trial lasted for 40
seconds, there were three full revolutions per trial and consequently a maximum of three illusions to be perceived.

## Results

## Experiment 1

The experimental process was thoroughly designed so that the results would automatically be stored and displayed facilitating the interpretation and validation process. A data sheet was exported from MATLAB R2012b for each observer. Table 10 demonstrates the information provided in this sheet as the columns suggest. The physical characteristics of the stimulus, the special features of each trial, the actual orientation and observer's perception as well as their comparison are reported. Each row stands for the observer's response to the prompt sound heard. The information sheet is analytically explained.
A. Column Trial: The sequence of the trial occurring is presented. The trial number is repeated six times accounting for the six answers to the prompt sound. Fewer responses denote that the participant missed a prompt.
B. Column ID: The ID refers to the special identification number the administrator assigned to the stimuli in order to distinguish them.
C. Column Short Side: This column contains the physical length of the short side of the stimulus presented in this trial.
D. Column Long Side: This column contains the physical length of the long side of the stimulus presented in this trial.
E. Column Width: This column contains the physical length of the horizontal side of the stimulus presented in this trial.
F. Column LS Position: The LS position shows the orientation of the stimulus' long side. $L$ stands for left and $R$ for right from the rotation axis.
G. Column Rotation Speed in RPM: Although the rotation speed is constant it is displayed in order to discriminate the clockwise from the counterclockwise rotation. A negative sign of the speed value signifies the former while the latter is described by a positive sign.
H. Column Initial Angle: This column demonstrates the angular orientation of the window when the trial first started. The value of 0 degrees is assigned to the frontoparallel plane.
I. Column Shadow: This column informs the administrator whether the window had shadow or not. This is conveyed by assigning the value of " 1 " for the first case and the value of " 0 " for the second case.
J. Column White Dots Side: This column demonstrates the side of the stimulus to which the white dots are assigned to during the trial.
K. Column Response Time: The reaction time to the prompt is measured in order to ensure that the threshold time is sufficient for every observer. The timer is reset at each prompt.
L. Column Response Angle: The angle of the window is measured when the observer responds to the prompt in order to ensure that the threshold angle is sufficient for every observer. The angle is set to zero at each prompt.
M. Column Rotation Angle: The angle of the window at the time the observer responds. The angle is set to zero only when the trial starts.
N. Column Reality: This column demonstrates the physical orientation of the side with the white dots by the time the observer responds.
O. Column Response: This column demonstrates the observer's response according to the perception.
P. Column True or False: If the reality cell matches with the corresponding response cell the answer is reported as true while in the case of disagreement it is false.
Q. Column Illusion: The comparison between the reality and the corresponding response cell coupled with the white dots side cell is utilized in this column in order to interpret the responses by making use of Tables 8 a and b .
Table 10: A shortened version of the data sheet as obtained from MATLAB for Experiment 1.

| A | B | C | D | E | F | G | H | 1 | 」 | K | L | M | N | 0 | P | Q |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Trial | ID | Short Side | Long Side | Width | LS Position | Rotation Speed | Initial Angle | Shadow | White Dots Side | Response Time | Response Angle | Rotation Angle | Reality | Response | T/F | Illusion |
| 1 | 19 | 19 | 20.5 | 19 | R | 5 | 90 | 1 | Is | 0.75036 | 32.5486 | 392.5486 | F | C | F | I |
| 1 | 19 | 19 | 20.5 | 19 | R | 5 | 90 | 1 | Is | 1.2174 | 31.0773 | 571.0773 | C | C | T | R |
| 1 | 19 | 19 | 20.5 | 19 | R | 5 | 90 | 1 | Is | 1.1509 | 29.606 | 749.606 | F | F | T | NI |
| 1 | 19 | 19 | 20.5 | 19 | R | 5 | 90 | 1 | Is | 0.81722 | 19.6333 | 919.6333 | C | C | T | R |
| 1 | 19 | 19 | 20.5 | 19 | R | 5 | 90 | 1 | Is | 0.76716 | 18.162 | 1098.162 | F | C | F | I |
| 2 | 16 | 19 | 20.5 | 28.5 | R | -5 | 0 | 1 | Is | 0.38378 | 21.5324 | -201.5324 | F | C | F | I |
| 2 | 16 | 19 | 20.5 | 28.5 | R | -5 | 0 | 1 | Is | 0.75031 | 17.5607 | -377.5607 | C | C | T | R |
| 2 | 16 | 19 | 20.5 | 28.5 | R | -5 | 0 | 1 | Is | 0.81722 | 19.5899 | -559.5899 | F | C | F | I |
| 2 | 16 | 19 | 20.5 | 28.5 | R | -5 | 0 | 1 | Is | 0.83382 | 20.1189 | -740.1189 | C | C | T | R |
| 2 | 16 | 19 | 20.5 | 28.5 | R | -5 | 0 | 1 | Is | 0.86721 | 21.148 | -921.148 | F | C | F | I |
| 2 | 16 | 19 | 20.5 | 28.5 | R | -5 | 0 | 1 | Is | 1.0506 | 26.6778 | -1106.6778 | C | C | T | R |
| 3 | 12 | 19 | 19.01 | 19 | L | -5 | 0 | 0 | 55 | 0.4333 | 23.0326 | -203.0326 | F | C | F | W |
| 3 | 12 | 19 | 19.01 | 19 | L | -5 | 0 | 0 | 55 | 0.78368 | 18.5608 | -378.5608 | C | C | T | NI |
| 3 | 12 | 19 | 19.01 | 19 | L | -5 | 0 | 0 | 55 | 0.76707 | 18.0897 | -558.0897 | F | F | T | R |
| 3 | 12 | 19 | 19.01 | 19 | L | -5 | 0 | 0 | 55 | 0.95065 | 23.6195 | -743.6195 | C | C | T | NI |
| ... |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 34 | 8 | 19 | 19.01 | 28.5 | R | 5 | 90 | 0 | 55 | 0.91728 | 22.6627 | 1102.6627 | C | C | T | NI |
| 35 | 19 | 19 | 20.5 | 19 | R | 5 | 0 | 1 | 55 | 0.4505 | 23.5327 | 203.5327 | F | F | T | R |
| 35 | 19 | 19 | 20.5 | 19 | R | 5 | 0 | 1 | 55 | 1.2009 | 30.5628 | 390.5628 | C | F | F | I |
| 35 | 19 | 19 | 20.5 | 19 | R | 5 | 0 | 1 | 55 | 0.76716 | 18.0897 | 558.0897 | F | F | T | R |
| 35 | 19 | 19 | 20.5 | 19 | R | 5 | 0 | 1 | 55 | 0.73376 | 17.1185 | 737.1185 | C | F | F | I |
| 35 | 19 | 19 | 20.5 | 19 | R | 5 | 0 | 1 | 55 | 0.90054 | 22.6483 | 922.6483 | F | F | T | R |
| 35 | 19 | 19 | 20.5 | 19 | R | 5 | 0 | 1 | 55 | 0.76742 | 18.1765 | 1098.1765 | C | F | F | I |
| 36 | 16 | 19 | 20.5 | 28.5 | R | 5 | 0 | 1 | 55 | 0.60038 | 28.0334 | 208.0334 | F | F | T | R |
| 36 | 16 | 19 | 20.5 | 28.5 | R | 5 | 0 | 1 | 55 | 0.85076 | 19.561 | 379.561 | C | F | F | I |
| 36 | 16 | 19 | 20.5 | 28.5 | R | 5 | 0 | 1 | 55 | 0.78389 | 19.0898 | 559.0898 | F | F | T | R |
| 36 | 16 | 19 | 20.5 | 28.5 | R | 5 | 0 | 1 | 55 | 0.76728 | 18.1186 | 738.1186 | C | F | F | I |
| 36 | 16 | 19 | 20.5 | 28.5 | R | 5 | 0 | 1 | 55 | 1.1173 | 28.6492 | 928.6492 | F | F | T | R |
| 36 | 16 | 19 | 20.5 | 28.5 | R | 5 | 0 | 1 | 55 | 1.1005 | 28.1781 | 1108.1781 | C | F | F | I |

Columns A to M are necessary to the administrator for reconstructing the stimulus viewed by the observer as well as the special characteristics of the rotation. This information is also valuable for debugging the program and making sure of the efficacy and efficiency of the experiment. Columns N and O give insight to what was physically presented and what was perceived from the observer. Their comparison is shown in column P and is interpreted in column Q utilizing the information provided from column J. The evaluation of the responses and the measurement of illusory strength follow by utilizing the information provided by column J .

The next step is to count how many times each answer appeared within a trial for each observer, weight the possible responses in different ways and turn the absolute numbers into percentages. It has already been mentioned that the maximum amount of illusions to be perceived in this experiment under the defined parameters is 3 .

## Weighting Process I (WPI)

For this process the value of " 1 " was assigned to the illusory percept while " 0 " to every other response. Table 11 indicates the percentages of the results obtained from one observer.

The rows denote the stimulus ID and columns 2-5 represent the observer's responses for each different presentation of the stimuli. The sixth column is the average of the four preceding columns and the prediction denotes the values obtained from the pilot study.

The way the stimuli were sorted is indicated by the results obtained from the pilot study. Some interclass order variations are not only anticipated but also insightful.

Table 11: The percentage of perceived illusion as indicated by a single observer for the WPI of Experiment 1. The four columns from I to IV refer to the four different presentations of the stimuli. The average column indicates the averaged value of the four preceding cells while the prediction is constructed based on the results obtained from the pilot study.

| Stimulus | I | II | III | IV | Average | Prediction |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\mathbf{1 2}$ | $0 \%$ | $100 \%$ | $33 \%$ | $0 \%$ | $33.25 \%$ | $1.60 \%$ |
| $\mathbf{3}$ | $33.00 \%$ | $33 \%$ | $100 \%$ | $0 \%$ | $41.50 \%$ | $4.98 \%$ |
| $\mathbf{8}$ | $0 \%$ | $100 \%$ | $33 \%$ | $0 \%$ | $33.25 \%$ | $13.32 \%$ |
| $\mathbf{1 6}$ | $100 \%$ | $0 \%$ | $33 \%$ | $100 \%$ | $58.25 \%$ | $33.32 \%$ |
| $\mathbf{1 9}$ | $66 \%$ | $100 \%$ | $100 \%$ | $100 \%$ | $91.50 \%$ | $60 \%$ |
| $\mathbf{1 3}$ | $33 \%$ | $66 \%$ | $100 \%$ | $100 \%$ | $74.75 \%$ | $75 \%$ |
| $\mathbf{1 8}$ | $100 \%$ | $100 \%$ | $66 \%$ | $100 \%$ | $91.50 \%$ | $87 \%$ |
| $\mathbf{1 4}$ | $100 \%$ | $100 \%$ | $100 \%$ | $66 \%$ | $91.50 \%$ | $95 \%$ |
| $\mathbf{1 7}$ | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ | $100.00 \%$ | $100 \%$ |

The strength of the illusion produced from each stimulus as well as the correlation between the averaged responses and the predicted reports are seen in Figures 18-19.

The averaged illusory percent for each stimulus was measured from the 20 participants following the same process as proposed for the previous individual and the results are summarized in Table 12.

Participant's Illusory Percept


Figure 18: The histogram reflects the differences in illusory strength between the predicted results as obtained from the pilot study (red columns) and the averaged results obtained from the experimental process for a typical observer (blue columns).

## Correlation of Participant's Experimental to Predicted Illusion



Figure 19: The correlation between the experimental and the predicted results for the typical participant.
Table 12: The summary of the participants' responses for the WPI of Experiment 1. The averaged results from each participant are shown as well as the average of these averages (blue column). The prediction is the one obtained from the pilot study. The green column refers to the standard error of the mean (SEM) followed by their average (green column).

| AS | BK | KD | DP | FB | 1 | JA | IR | JT | 12 | LH | MQ | MT | NB | SC | SS | WY | YW | EA | PM | Average | reelicion | 0 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $12.41 .25 \%$ | 8.25\% | 33.25\% | 0.00\% | 0.0\%\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 16.50\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.0\% | 0.00\% | 4.96\% |  | 0.11821 | 0.026432184 |
| $325.0 \%$ | 16.50\% | 41.5\% | 8.25\% | 33.0\% | 24.75\% | 0.0\% | 0.0\% | 91.50\% | 0.00\% | 91.5\% | 0.00\% | 0.00\% | 0.00\% | 16.5\% | 0.00\% | 497.75\% | 0.00\% | 49,75\% | 16.5\% | 23.238 |  | 0.28874 | 0.044563794 |
| 8 33.0\% | 0.00\% | 33.25\% | 0.00\% | 0.0\%\% | 16.5\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 16.5\% | 0.00\% | 8.25\% | 8.25\% | 5.79\% |  | 0.10773 | 0.022090205 |
| 16 25.0\% | 8.25\% | 58.25\% | 16.5\% | 16.5\% | 83.25\% | 8.25\% | 0.00\% | 58.0\% | 0.00\% | 66.25\% | 49.50\% | 8.25\% | 0.00\% | 16.5\% | 0.00\% | 91.5\% | 8.25\% | 91.50\% | 0.00\% | 30.29\% |  | 0.32958 | 0.077697076 |
| $19.49 .50 \%$ | 497.75 | 91.5\% | 25.0\% | 16.5\% | 74.75\% | 33.25\% | 16.50\% | 75.0\% | 8.25\% | 91.50\% | 74.75\% | 25.0\% | 75.00\% | 4200\% | 16.5\% | 58.0\% | 100.0\% | 49.75\% | 16.50\% | 49.45\% |  | 0.29294 | 0.065503264 |
| 13 49.75\% | 0.00\% | 74.75\% | 8.25\% | 33.00\% | 33.0\% | 8.25\% | 0.0\% | 100.00\% | 8.25\% | 74.75\% | 33.00\% | 0.00\% | 0.00\% | 25.0\% | 0.00\% | 91.5\% | 0.00\% | 991.50\% | 0.00\% | 31.55\% |  | 0.359 | 0.080031116 |
| 18 91.50\% | 66.5\% | 91.50\% | 58.0\% | 49.50\% | 83.0\% | 33.25\% | 41.50\% | 100.00\% | 16.50\% | 91.50\% | 66.25\% | 33.0\% | 83.25\% | 24.75\% | 83.00\% | 100.0\% | 66.5\% | 75.0\% | 91.50\% | 67.30\% |  | . 262 | 0.058683361 |
| $14.83 .0 \%$ | 74.75\% | 91.5\% | 74.75\% | 16.5\% | 91.0\% | 0.0\% | 41.50\% | 91.50\% | 74.75\% | 83.00\% | 100.0\% | 74.5\% | 66.50\% | 33.25\% | 91.5\% | 91.5\% | 25.0\% | 74.71\% | 83.00\% | 11\% |  | 0.8874 | 0.066273326 |
| 17 10000\% | 83.00\% | 100.0\% | 83.0\% | 66.50\% | 83.0\% | 1000\% | 66.5\% | 100.00\% | 91.50\% | 100.0\% | 74.75\% | 91.5\% | 100.0\% | 50.0\% | 91.50\% | 91.5\% | 74.75\% | 91.50\% | 100.0\% | 86.95\% |  | 0.1422 | 0.03179633 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 0.054385629 |

All personal responses were averaged indicating the final illusory strength obtained from the corresponding stimulus and are plotted against the predicted responses as seen in Figure 20. The Standard Error of the Mean (SEM) is computed per stimulus and the average is plotted in the graph as well. SEM is defined as the standard deviation of the means over the total number of samples and is described by the following equation:

$$
S E M=\frac{\sigma}{\sqrt{n}}
$$

where, $\boldsymbol{\sigma}$ : the standard deviation of the means in each stimulus,
$\mathbf{n}$ : the total number of participants.

Illusory Percept


Figure 20: The histogram reflects the differences in illusory strength between the predicted results as obtained from the pilot study (red columns) and the summarized averaged results obtained from the experimental process for the entire cohort (blue columns). The error bars are estimated based on the averaged SEM.

## Correlation of Experimental to Predicted Illusion



Figure 21: The correlation between the experimental and the predicted results for the entire cohort.

## Weighting Process II (WPII)

For this process the value of " 1 " was assigned to the illusory percept while " -0.5 " to the absence of illusion "NI". Every other response was zero. Table 13 indicates the percentages of the results obtained from the same observer.

The configuration is exactly the same as in the previous process.

Table 13: The percentage of perceived illusion as indicated by a single observer for the WPII of Experiment 1. The four columns from I to IV refer to the four different presentations of the stimuli. The average column indicates the averaged value of the four preceding cells while the prediction is constructed based on the results obtained from the pilot study.

|  | I | II | III | IV | Average | Prediction |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 12 | $0 \%$ | $-33 \%$ | $-33 \%$ | $-50 \%$ | $-29.00 \%$ | $1.60 \%$ |
| 3 | $17.00 \%$ | $-50 \%$ | $-33 \%$ | $-33 \%$ | $-24.75 \%$ | $4.98 \%$ |
| 8 | $-50 \%$ | $-50 \%$ | $-33 \%$ | $-50 \%$ | $-45.75 \%$ | $13.32 \%$ |
| 16 | $66 \%$ | $-50 \%$ | $-33 \%$ | $-50 \%$ | $-16.75 \%$ | $33.32 \%$ |
| 19 | $100 \%$ | $0 \%$ | $-33 \%$ | $-17 \%$ | $12.50 \%$ | $60 \%$ |
| 13 | $0 \%$ | $-33 \%$ | $-33 \%$ | $-33 \%$ | $-24.75 \%$ | $75 \%$ |
| 18 | $100 \%$ | $33 \%$ | $33 \%$ | $66 \%$ | $58.00 \%$ | $87 \%$ |
| 14 | $100 \%$ | $33 \%$ | $100 \%$ | $66 \%$ | $74.75 \%$ | $95 \%$ |
| 17 | $100 \%$ | $66 \%$ | $100 \%$ | $66 \%$ | $83.00 \%$ | $100 \%$ |

The strength of the illusion produced from each stimulus as well as the correlation between the averaged responses and the predicted reports are compiled in Figures 22-23.

The averaged illusory percent for each stimulus was measured from the 20 participants following the same process as proposed for the previous individual and the results are summarized in Table 14.

All personal responses are averaged again indicating the final illusory strength assigned to the corresponding stimulus and are plotted against the predicted responses as seen in Figures 24-25.

## Participant's Illusory Percept



Figure 22: The histogram reflects the differences in illusory strength between the predicted results as obtained from the pilot study (red columns) and the averaged results obtained from the experimental process for the same typical observer (blue columns).

## Correlation of Participant's Experimental to Predicted Illusion



Figure 23: The correlation between the experimental and the predicted results for the typical participant.
Table 14: The summary of the participants' responses for the WPII of Experiment 1. The averaged results from each participant are shown as well as the average of these averages (blue column). The prediction is the one obtained from the pilot study. The green column refers to the standard error of the mean (SEM) followed by their average (green column).

| AS | BK | KD | DP | FB | 1 | IA | JR | J | 12 | LH | MO | MT | NB | S | SS | WY | YW | EA | PM | Averages | Peliction | 0 | $S=0 /\left(1{ }^{\wedge} 0.5\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1224.75 \%$ | .2900\% | 4.25\% | . $29.00 \%$ | -37.25\% | . $4.575 \%$ | . $4.50 \%$ | .33.5\% | . $45.75 \%$ | .21.00\% | . $45.75 \%$ | . $45.75 \%$ | . $45.75 \%$ | . 4.7 .75 | . $45.75 \%$ | . $45.75 \%$ | . $45.75 \%$ | .50.0\% | . $45.75 \%$ | . 4.7 .75 | 578\% |  | 19074 | 0.042650288 |
| $3 \cdot 12.50$ | -20.75\% | 16.75\% | . $24.75 \%$ | 1250\% | .12.25\% | . $37.50 \%$ | . $33.25 \%$ | 91.50\% | .41.5\% | 91.50\% | . $41.5 \%$ | . $45.75 \%$ | . $45.75 \%$ | -16.75\% | . $41.75 \%$ | 29.0\% | . $45.75 \%$ | . $41.75 \%$ | .41.75\% | -13.10\% |  | 0.42106 | 0.094151054 |
| 8 4.25\% | . $45.75 \%$ | 4.25\% | . $45.75 \%$ | .33.25\% | -20.75\% | . $45.75 \%$ | . $37.25 \%$ | . 4.7 .75 | .29.25\% | .8.25\% | . $41.5 \%$ | . $41.75 \%$ | . $45.75 \%$ | . $45.75 \%$ | . $45.75 \%$ | . $45.75 \%$ | . $45.75 \%$ | 4.25\% | 0.00\% | .30.55\% |  | (19.12 | 0.04539535 |
| $16.40 \%$ | . $29.00 \%$ | 46.00\% | . $16.75 \%$ | . $1250 \%$ | .2900\% | . $29.00 \%$ | -20.75\% | 45.50\% | .37.5\% | 58.25\% | 4.25\% | . $33.25 \%$ | . $4.5 .5 \%$ | . $45.75 \%$ | . $45.75 \%$ | 87.50\% | 8. 8.55 | 87.50\% | .41.75\% | 3.29 |  | 0.43728 | 0.09777984 |
| 19 4.0\% | 33.25\% | 87.50\% | 12.5\% | .1250\% | 66.75\% | 4.25\% | .8.25\% | 62.50\% | .24.75\% | 87.50\% | 70.75\% | .8.25\% | 62.5\% | 54.25\% | -20.75\% | 41.5\% | 70.75\% | 54.25\% | 4.0\% | 3209\% |  | 0.768 | 0.08460078 |
| 13 29.25\% | . $37.5 \%$ | 66.50\% | . $24.75 \%$ | 37.0\% | 4.0\% | -16.75\% | .8.25\% | 91.50\% | .12.5\% | 66.50\% | 4.25\% | . $45.75 \%$ | .45.75\% | .8.25\% | . $45.75 \%$ | 83.0\% | .41.50\% | 83.00\% | 8.25\% | 9.33\% |  | 0.471 | 0.105350333 |
| 18 91.50\% | 54.0\% | 91.50\% | 58.0\% | 41.25\% | 79.0\% | 16.50\% | 41.5\% | 91.50\% | 4.25\% | 83.00\% | 54.0\% | 4.0\%\% | 79.25\% | 4.00\% | 87.50\% | 91.50\% | 46.0\% | 91.5\% | 87.50\% | 59.65\% |  | 0.33187 | 0.07420269 |
| 14 79.00\% | 74.75\% | 91.50\% | 74.75\% | 33.0\% | 91.5\% | . $33.25 \%$ | 37.25\% | 91.50\% | 66.50\% | 91.50\% | 100.0\% | 66.5\% | 54.00\% | 4.25\% | 91.5\% | 91.50\% | 16.75\% | 100.00\% | 91.50\% | 65.70\% |  | 0.36515 | 0.081649645 |
| $1710000 \%$ | 79.0\% | 100.0\% | 8300\% | 58.25\% | 8300\% | 10000\% | 54.25\% | 10000\% | 8750\% | 100.0\% | 83.0\% | 9150\% | 100.00\% | 25.0\% | 100.0\% | 9150\% | 66.50\% | 100.0\% | 100.0\% | 85.13\% |  | 0.20218 | 0.0452049 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 0.0744003 |

## Illusory Percept



Figure 24: The histogram reflects the differences in illusory strength between the predicted results as obtained from the pilot study (red columns) and the summarized averaged results obtained from the experimental process for the entire cohort (blue columns). The error bars are estimated based on the averaged SEM.

## Correlation of Experimental to Predicted Illusion



Averaged Illusion
Figure 25: The correlation between the experimental and the predicted results for the entire cohort.

## Experiment 2

For the second experiment a similar data sheet was exported from MATLAB R2012b for each observer. Table 15 demonstrates the information provided in this sheet as the columns suggest. The physical characteristics of the stimulus, the special features of each trial, the actual orientation and observer's perception as well as their comparison are reported. Each row stands for the observer's response. The information sheet is analytically explained.
A. Column Trial: The sequence of the trial and the viewer's responses are presented. The trial number might appear once indicating that the observer maintained the original percept or could be repeated many times implying that the viewer perceived oscillatory motion.
B. Column ID: The ID refers to the special identification number the administrator assigned the stimuli in order to distinguish them.
C. Column Short Side: This column contains the physical length of the short side of the stimulus presented in this trial.
D. Column Long Side: This column contains the physical length of the long side of the stimulus presented in this trial.
E. Column Width: This column contains the physical length of the horizontal side of the stimulus presented in this trial.
F. Column LS Position: The LS position shows the orientation of the stimulus' long side. $L$ stands for left and $R$ for right of the rotation axis.
G. Column LS Position: This column informs the administrator of the long side orientation at the time the participant reports a perception.
H. Column Rotation Speed in RPM: Although the rotation speed is constant it is displayed in order to discriminate the clockwise from the counterclockwise rotation. A negative sign of the speed value signifies the former while the latter is described by the positive sign.
I. Column Initial Angle: This column demonstrates the angular orientation of the window when the trial first started. The values of 0 degrees is assigned to the frontoparallel plane.
J. Column Shadow: This column informs the administrator whether the window had shadow or not. This is conveyed by assigning the value of " 1 " for the first case and the value of " 0 " for the latter.
K. Column White Dots Side: This column demonstrates the side of the stimulus to which the white dots are assigned to during the trial.
L. Column Response Time: The time elapsed since the start of the trial.
M. Column Response Angle: The angle of the window when the observer reports a percept. The angle is set to zero at the beginning of each trial.
N. Column Response: This column demonstrates the observer's response according to the perception.
O. Column True or False: If the sign of the speed value matches with the corresponding response cell, the answer is reported as true while in the case of disagreement it is false. Thus, is the speed value has a negative sign indicating clockwise rotation and the response is "Right" then the answer is counted as true.
P. Column Illusion: The comparison between the sign of the speed value, the response angle and the corresponding response cell coupled with LS position cell is utilized in this column in order to interpret the responses by using Tables 9 a and $b$.
Table 15: A shortened version of the data sheet as obtained from MATLAB for Experiment 2.

| A | B | C | D | E | F | G | H | I | 」 | K | L | M | N | 0 | P |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Trial | ID | Short Side | Long Side | Width | LS Original Position | LS Position | Rotation Speed | Initial Angle | Shadow | White Dots Side | Response Time | Response Angle | Response | T/F | Illusion |
| 1 | 14 | 19 | 28.5 | 28.5 | R | L | 5 | 90 | 1 | Is | 4.0259 | 210.9596 | L | T | C |
| 2 | 16 | 19 | 20.5 | 28.5 | L | L | 5 | 0 | 1 | Is | 2.2049 | 66.9776 | L | T | C |
| 3 | 19 | 19 | 20.5 | 19 | L | L | -5 | 0 | 1 | Is | 2.7219 | -82.4724 | R | T | S-NI |
| 4 | 3 | 19 | 20.5 | 42.75 | L | R | -5 | -90 | 0 | Is | 3.8066 | -204.9616 | R | T | C |
| 5 | 3 | 19 | 20.5 | 42.75 | L | L | -5 | 0 | 0 | Is | 2.5677 | -77.9739 | R | T | S-NI |
| 6 | 13 | 19 | 21.82 | 42.75 | R | R | -5 | 0 | 1 | 55 | 2.8181 | -85.4714 | R | T | S-C |
| 6 | 13 | 19 | 21.82 | 42.75 | R | R | -5 | 0 | 1 | 55 | 11.288 | -335.3879 | L | F | S-F |
| 6 | 13 | 19 | 21.82 | 42.75 | R | R | -5 | 0 | 1 | 55 | 14.2225 | -418.3602 | R | T | C |
| 6 | 13 | 19 | 21.82 | 42.75 | R | L | -5 | 0 | 1 | 55 | 21.1248 | -620.7926 | L | F | S-1 |
| 6 | 13 | 19 | 21.82 | 42.75 | R | R | -5 | 0 | 1 | 55 | 26.7102 | -784.7378 | R | T | C |
| 6 | 13 | 19 | 21.82 | 42.75 | R | L | -5 | 0 | 1 | 55 | 32.3957 | -950.1825 | L | F | I |
| 6 | 13 | 19 | 21.82 | 42.75 | R | R | -5 | 0 | 1 | 55 | 37.9479 | -1111.6285 | R | T | C |
| 7 | 12 | 19 | 19.01 | 19 | L | R | -5 | 0 | 0 | 55 | 4.4115 | -132.9556 | R | T | S-C |
| 7 | 12 | 19 | 19.01 | 19 | L | L | -5 | 0 | 0 | 55 | 10.7638 | -319.3933 | R | T | S-NI |
| 7 | 12 | 19 | 19.01 | 19 | L | R | -5 | 0 | 0 | 55 | 17.433 | -514.828 | L | F | S-F |

$\dot{\dot{n}} \bar{\Sigma}-\bar{z}-\cup-\cup \overline{\dot{n}} \cup \dot{\dot{n}} 0$
 $\propto \rightharpoonup \propto \propto \propto \rightharpoonup \propto \rightharpoonup \propto \rightharpoonup \propto \propto \rightharpoonup$



.."








Columns A to M are necessary to the administrator for reconstructing the stimulus viewed by the observer as well as the special characteristics of the rotation. This information is also valuable for debugging the program and making sure of the efficacy and efficiency of the experiment. Moreover, the response angle indicates whether the response was before the threshold value or exceeded it leading to different interpretations. Columns N and O give insight in what was perceived from the observer and whether this answer corresponds to the physical direction or not. The comparison of columns $\mathrm{M}, \mathrm{N}$ and O leads to the interpretation of the responses shown in column P .

The next step is to count how many times each answer appeared within a trial for each observer weighting the possible responses in different ways and turn the absolute numbers into percentages. It has already been mentioned that the maximum amount of illusions to be perceived in this experiment under the defined parameters is 3 as well.

## Weighting Process I (WPI)

For this process the value of " 1 " was assigned to the illusory percept while " 0 " to every other response. Table1 16 indicates the percentages of the results obtained from one observer.

The rows denote the stimulus ID and columns 2 to 5 represent the observer's responses for each different presentation of the stimuli. The sixth column is the average of the four preceding columns and the prediction denotes the values obtained from the pilot study.

The way the stimuli were sorted is indicated by the results obtained from the pilot study. Some interclass order variations are not only anticipated but also insightful.

Table 16: The percentage of perceived illusion as indicated by a single observer for the WPI of Experiment 2. The four columns from I to IV refer to the four different presentations of the stimuli. The average column indicates the averaged value of the four preceding cells while the prediction is constructed based on the results obtained from the pilot study.

|  | I | II | III | IV | Average | Prediction |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 12 | $66 \%$ | $33 \%$ | $33 \%$ | $33 \%$ | $41.25 \%$ | $1.60 \%$ |
| $\mathbf{3}$ | $0.00 \%$ | $0 \%$ | $0 \%$ | $0 \%$ | $0.00 \%$ | $4.98 \%$ |
| $\mathbf{8}$ | $0 \%$ | $0 \%$ | $0 \%$ | $0 \%$ | $0.00 \%$ | $13.32 \%$ |
| 16 | $0 \%$ | $0 \%$ | $0 \%$ | $0 \%$ | $0.00 \%$ | $33.32 \%$ |
| 19 | $0 \%$ | $100 \%$ | $33 \%$ | $0 \%$ | $33.25 \%$ | $60 \%$ |
| 13 | $66 \%$ | $0 \%$ | $0 \%$ | $0 \%$ | $16.50 \%$ | $75 \%$ |
| 18 | $100 \%$ | $100 \%$ | $100 \%$ | $33 \%$ | $83.25 \%$ | $87 \%$ |
| 14 | $0 \%$ | $100 \%$ | $100 \%$ | $100 \%$ | $75.00 \%$ | $95 \%$ |
| 17 | $66 \%$ | $33 \%$ | $100 \%$ | $100 \%$ | $74.75 \%$ | $100 \%$ |

The strength of the illusion produced from each stimulus as well as the correlation between the averaged responses and the predicted reports are seen in Figures 26-27.

The same process is followed for the 20 participants leading to the summarizing Table 17 which demonstrates all averaged responses.

All personal responses were averaged indicating the final illusory strength obtained from the corresponding stimulus and are plotted against the predicted responses as seen in Figures 28-29. The Standard Error of the Mean (SEM) is computed per stimulus and the average is plotted in the graph as well.


Figure 26: The histogram reflects the differences in illusory strength between the predicted results as obtained from the pilot study (red columns) and the averaged results obtained from the experimental process for the same typical observer (blue columns).

## Correlation of Participant's Experimental to Predicted Illusion



Averaged Illusion
Figure 27: The correlation between the experimental and the predicted results for the typical participant.
Table 17: The summary of the participants' responses for the WPI of Experiment 2. The averaged results from each participant are shown as



Stimuli ID
Figure 28: The histogram reflects the differences in illusory strength between the predicted results as obtained from the pilot study (red columns) and the summarized averaged results obtained from the experimental process for the entire cohort (green columns). The error bars are estimated based on the averaged SEM.
Correlation of Experimental to Predicted Illusion


Figure 29: The correlation between the experimental and the predicted results for the entire cohort.

## Weighting Process II (WPII)

For this process the value of " 1 " was assigned to the illusory percept (I), " 0.5 " to the spontaneous illusion (SI) while "- 0.5 " to the absence of illusion (NI) and "- 0.25 " to the spontaneous absence of illusion (SNI). Every other response was zero. Table 18 indicates the percentages of the results obtained from the same observer. The configuration is exactly the same as in the previous process.

Table 18: The percentage of perceived illusion as indicated by a single observer for the WPII for Experiment 2. The four columns from I to IV refer to the four different presentations of the stimuli. The average column indicates the averaged value of the four preceding cells while the prediction is constructed based on the results obtained from the pilot study.

|  | I | II | III | IV | Average | Prediction |
| :---: | :---: | :---: | :---: | :---: | :---: | ---: |
| 12 | $58 \%$ | $0 \%$ | $25 \%$ | $8 \%$ | $22.75 \%$ | $1.60 \%$ |
| 3 | $0.00 \%$ | $-8 \%$ | $-8 \%$ | $-8 \%$ | $-6.00 \%$ | $4.98 \%$ |
| 8 | $0 \%$ | $-8 \%$ | $-17 \%$ | $0 \%$ | $-6.25 \%$ | $13.32 \%$ |
| 16 | $0 \%$ | $-8 \%$ | $-8 \%$ | $0 \%$ | $-4.00 \%$ | $33.32 \%$ |
| 19 | $-8 \%$ | $100 \%$ | $8 \%$ | $-8 \%$ | $23.00 \%$ | $60 \%$ |
| 13 | $50 \%$ | $0 \%$ | $-8 \%$ | $-8 \%$ | $8.50 \%$ | $75 \%$ |
| 18 | $100 \%$ | $100 \%$ | $75 \%$ | $-8 \%$ | $66.75 \%$ | $87 \%$ |
| 14 | $0 \%$ | $100 \%$ | $100 \%$ | $58 \%$ | $64.50 \%$ | $95 \%$ |
| 17 | $50 \%$ | $33 \%$ | $83 \%$ | $66 \%$ | $58.00 \%$ | $100 \%$ |

The strength of the illusion produced from each stimulus as well as the correlation between the averaged responses and the predicted reports are seen in Figures 30-31.

The same process is followed for the 20 participants leading to the summarizing Table 19 which demonstrates all averaged responses.

## Participant's Illusory Percept



Stimuli ID
Figure 30: The histogram reflects the differences in illusory strength between the predicted results as obtained from the pilot study (red columns) and the averaged results obtained from the experimental process for the same typical observer (green columns).

## Correlation of Participant's Experimental to Predicted Illusion



Figure 31: The correlation between the experimental and the predicted results for a single participant.
Table 19: The summary of the participants' responses for the WPII of Experiment 2. The averaged results from each participant are shown as well as the average of these averages (blue column). The prediction is the one obtained from the pilot study. The green column refers to the standard error of the mean (SEM) followed by their average (green column).

|  | As | BK | KD | DP | FB | 1 | IA | IR | J | 12 | LH | MQ | MT | NB | SC | Ss | WY | YW | EA | PM | Averges | Predicition | 0 | $\mathrm{S}=0 /(\mathrm{n} 0.50$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 12 | 12.50\% | 0.00\% | 31.25\% | 4.25\% | 0.00\% | 0.00\% | 0.00\% | 2.00\% | 0.00\% | 0.00\% | 12.50\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 14.75\% | 0.00\% | 0.00\% | 3.86\% |  | 0.0804 | 0.017993488 |
| 3 | 3 4.17\% | 0.00\% | 0.00\% | 0.0\% | 0.00\% | 0.00\% | 0.00\% | -2.00\% | 4.25\% | 0.00\% | 0.00\% | 0.0\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | -2.00\% | 0.00\% | -200\% | -200\% | . $0.40 \%$ | 4.5 | 0.01593 | 0.003561013 |
| 8 | 4.25\% | .6.25\% | . $2.00 \%$ | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 4.25\% | 0.0\% | 0.00\% | 0.00\% | 0.0\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | . $2.00 \%$ | 0.00\% | 0.00\% | . $0.44 \%$ |  | 0.01858 | 0.004155193 |
| 16 | 8.25\% | 4.25\% | . $4.25 \%$ | 18.75\% | 16.75\% | 12.5\% | 10.25\% | 16.5\% | 0.00\% | 0.00\% | 6.25\% | 4.25\% | 0.00\% | 4.42\% | -4.25\% | 0.00\% | 0.00\% | . $6.25 \%$ | -4.25\% | 16.5\% | 4.55\% |  | 0.0823 | 0.018342243 |
| 19 | 14.42\% | 18.50\% | 25.0\% | 16.75\% | 24.75\% | -4.25\% | 21.0\% | . $14.75 \%$ | 27.0\% | 4.25\% | 35.25\% | 6.25\% | 0.00\% | 18.75\% | 0.0\% | -2.00\% | 0.00\% | 18.75\% | 0.00\% | 4.25\% | 10.70\% |  | 0.12999 | 0.02906763 |
| 13 | 4.25\% | 0.00\% | 10.5\% | 6.25\% | 25.0\% | 0.00\% | 14.50\% | 18.75\% | 0.00\% | 0.00\% | -2.00\% | 0.00\% | 0.00\% | 4.25\% | 4.25\% | 0.00\% | 0.0\% | 8.25\% | 0.00\% | 8.25\% | 5.11\% |  | 0.07283 | 0.016284351 |
| 18 | 20.92\% | 64.50\% | 66.75\% | 79.0\% | 68.50\% | 29.25\% | 87.5\% | 23.0\% | 95.75\% | 33.25\% | 50.0\% | 22.75\% | 85.25\% | 41.5\% | 47.75\% | 91.75\% | 27.0\% | 60.5\% | 18.75\% | 91.75\% | 55.27\% |  | 0.27211 | 0.060844609 |
| 14 | 5208\% | 83.25\% | 62.50\% | 87.25\% | 45.75\% | 8.25\% | 66.5\% | 41.5\% | 83.25\% | 31.25\% | 64.75\% | 54.0\% | 45.75\% | 91.0\% | 85.5\% | 72.50\% | 35.5\% | 33.25\% | 35.5\% | 83.25\% | 58.13\% |  | 0.23397 | 0.052316317 |
| 17 | 52.0\% | 93.75\% | 54.0\% | 91.75\% | 54.25\% | 14.5\% | 98.0\% | 46.0\% | 98.0\% | 89.50\% | 99375\% | 91.5\% | 85.25\% | 85.25\% | 27.0\% | 8995\% | 56.0\% | 77.0\% | 56.0\% | 91.75\% | 72.24\% |  | 0.25136 | 0.056252388 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 0.02875222 |

All personal responses were averaged indicating the final illusory strength obtained from the corresponding stimulus and are plotted against the predicted responses as seen in Figures 32-33. The Standard Error of the Mean (SEM) is computed per stimulus and the average is plotted in the graph as well.


Figure 32: The histogram reflects the differences in illusory strength between the predicted results as obtained from the pilot study (red columns) and the summarized averaged results obtained from the experimental process for the entire cohort (green columns). The error bars are estimated based on the averaged SEM.

## Correlation of Experimental to Predicted Illusion


-20.00\%-10.00\% 0.00\% 10.00\% 20.00\% 30.00\% 40.00\% 50.00\% 60.00\% 70.00\% 80.00\%
Averaged Illusion
Figure 33: The correlation between the experimental and the predicted results for the entire cohort

Weighting Process III (WPIII)

For this process the value of " 1 " was assigned to the illusory percept (I), " 0.5 " to the spontaneous illusion (SI) while "-0.5" to the wrong answers (W) and " -0.25 " to the spontaneous wrong reports (SW). Every other response was zero. Table 20 indicates the percentages of the results obtained from the same observer.

The configuration is exactly the same as in the previous process

Table 20: The percentage of perceived illusion as indicated by a single observer for the WPIII for Experiment 2. The four columns from I to IV refer to the four different presentations of the stimuli. The average column indicates the averaged value of the four preceding cells while the prediction is constructed based on the results obtained from the pilot study.

|  |  | II | III | IV | Average | Prediction |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 12 | $58 \%$ | $17 \%$ | $50 \%$ | $0 \%$ | $31.25 \%$ | $1.60 \%$ |
| 3 | $0.00 \%$ | $0 \%$ | $0 \%$ | $0 \%$ | $0.00 \%$ | $4.98 \%$ |
| 8 | $-8 \%$ | $0 \%$ | $0 \%$ | $0 \%$ | $-2.00 \%$ | $13.32 \%$ |
| 16 | $0 \%$ | $-17 \%$ | $0 \%$ | $0 \%$ | $-4.25 \%$ | $33.32 \%$ |
| 19 | $0 \%$ | $92 \%$ | $8 \%$ | $0 \%$ | $25.00 \%$ | $60 \%$ |
| 13 | $42 \%$ | $0 \%$ | $0 \%$ | $0 \%$ | $10.50 \%$ | $75 \%$ |
| 18 | $92 \%$ | $100 \%$ | $83 \%$ | $-8 \%$ | $66.75 \%$ | $87 \%$ |
| 14 | $0 \%$ | $100 \%$ | $100 \%$ | $50 \%$ | $62.50 \%$ | $95 \%$ |
| 17 | $25 \%$ | $25 \%$ | $83 \%$ | $83 \%$ | $54.00 \%$ | $100 \%$ |

The strength of the illusion produced from each stimulus as well as the correlation between the averaged responses and the predicted reports are seen in Figures 34-35.

The same process was followed for the 20 participants leading to the summarizing Table 21 which demonstrates all averaged responses.

Participant's Illusory Percept


Stimuli ID
Figure 34: The histogram reflects the differences in illusory strength between the predicted results as obtained from the pilot study (red columns) and the averaged results obtained from the experimental process for the same typical observer (green columns).

> Correlation of Participant's Experimental to Predicted Illusion


Figure 35: The correlation between the experimental and the predicted results for the typical participant.
Table 21: The summary of the participants' responses for the WPIII of Experiment 2. The averaged results from each participant are shown as well as the average of these averages (blue column). The prediction is the one obtained from the pilot study. The green column refers to the standard error of the mean (SEM) followed by their average (green column).

| AS | BK | KD | DP | FB | 1 | JA | IR | J | 12 | LH | MQ | MT | NB | SC | SS | WY | YW | EA | PM | Averoges | ediction | 0 | $S=0 /\left(\wedge^{\wedge} 0.5\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 12 12.5\% | .16.75\% | 22.75\% | 2.25\% | .6.0\% | .190\% | . $8.50 \%$ | -10.5\% | . $4.25 \%$ | .6.25\% | 10.25\% | 0.00\% | . $2.0 \%$ | 0.0\% | .6.25\% | .8.25\% | -2.00\% | .2.25\% | -20\% | 0.00\% | -2.31\% |  | 0.0951 | 0.021264991 |
| $30.00 \%$ | 4.00\% | .6.0\% | 6.25\% | 4.00\% | . $6.25 \%$ | .4.25\% | .8.25\% | . $4.00 \%$ | .8.25\% | .8.25\% | 0.00\% | 0.0\% | . $4.00 \%$ | .6.25\% | 0.00\% | . $6.25 \%$ | . $12.50 \%$ | . $6.25 \%$ | -2.0\% | . $4.84 \%$ |  | 0.03323 | 0.007480102 |
| $8.6 .25 \%$ | .17.0\% | . $6.25 \%$ | . $6.25 \%$ | .10.50\% | -10.25\% | . $4.25 \%$ | .600\% | .4.25\% | -10.5\% | .6.25\% | 0.00\% | . $6.00 \%$ | 6.25\% | . $4.00 \%$ | .12.50\% | . $2.00 \%$ | . $6.25 \%$ | 0.0\%\% | .12.50\% | .6.86\% |  | 0.04278 | 0.009565348 |
| 16 6.00\% | -200\% | 4.00\% | 12.50\% | 10.50\% | 10.25\% | 12.25\% | 8.50\% | 0.00\% | . $4.00 \%$ | 6.25\% | 4.25\% | .12.5\% | -200\% | .6.25\% | 4.00\% | .4.00\% | -10.25\% | 4.0\%\% | .4.00\% | 0.68\% |  | 0.07547 | 0.016876194 |
| 19 21.0\% | 12.50\% | 23.0\% | 22.75\% | 35.25\% | -200\% | 14.50\% | . $4.00 \%$ | 29.00\% | . $6.25 \%$ | 33.0\% | 12.25\% | . $6.25 \%$ | 12.50\% | .6.25\% | .8.25\% | -10.50\% | 8.25\% | .8.25\% | .6.25\% | 8.3 |  | 0.15297 | 0.03420459 |
| $13.4 .25 \%$ | . $8.00 \%$ | 8.5\% | 2.25\% | 23.0\% | . $6.25 \%$ | 4.25\% | 8.25\% | -2.00\% | . $6.25 \%$ | 4.00\% | 0.00\% | 2.25\% | 2.25\% | . $4.00 \%$ | .8.25\% | 4.00\% | 6.25\% | . $4.0 \%$ | . $6.25 \%$ | -0.01\% |  | 7529 | 0.016834601 |
| 18 29.25\% | 64.50\% | 66.75\% | 85.25\% | 68.75\% | 27.0\% | 91.50\% | 27.25\% | 98.0\% | 18.75\% | 48.0\% | 22.75\% | 77.0\% | 52.00\% | 50.00\% | 95.75\% | 33.25\% | 56.25\% | 33.25\% | 95.75\% | 57.05\% |  | 0.26886 | 0.060322662 |
| 14 399.75 | 83.25\% | 64.50\% | 91.75\% | 43.50\% | 6.25\% | 64.50\% | 47.5\% | 85.25\% | 35.25\% | 66.75\% | 52.0\% | 41.75\% | 899.5\% | 79.25\% | 72.5\% | 39.50\% | 33.25\% | 41.75\% | 72.50\% | 57.51\% |  | 0.22836 | 0.051062228 |
| 17 51.75\% | 91.75\% | 58.00\% | 93.75\% | 66.75\% | 31.25\% | 91.75\% | 56.25\% | 98.0\% | 89.50\% | 91.75\% | 91.50\% | 83.25\% | 83.25\% | 31.00\% | 91.50\% | 52.0\% | 73.0\% | 52.0\% | 98.00\% | 73.80\% |  | 0.22094 | 0.049403215 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 0.029664881 |

All personal responses were averaged indicating the final illusory strength obtained from the corresponding stimulus and are plotted against the predicted responses as seen in Figures 36-37. The Standard Error of the Mean (SEM) is computed per stimulus and the average is plugged in the graph as well.

Illusory Percept


Figure 36: The histogram reflects the differences in illusory strength between the predicted results as obtained from the pilot study (red columns) and the summarized averaged results obtained from the experimental process for the entire cohort (green columns). The error bars are estimated based on the averaged SEM.

## Correlation of Experimental to Predicted Illusion


$-20.00 \%-10.00 \%$ 0.00\% 10.00\% 20.00\% 30.00\% 40.00\% 50.00\% 60.00\% 70.00\% 80.00\%
Averaged Illusion
Figure 37: The correlation between the experimental and the predicted results for the entire cohort.

## Model

The rationale behind the construction of a model is that the appropriate combination of the main parameters would lead to the accurate prediction of the illusory strength. In more detail, a thorough study of the results described in the discussion section reveals the contribution of each parameter to the final percept. Intuition suggests that the LS ratio is the most determinant variable followed by the shadowing and the WS ratio. Based on these attributes, it is explored whether a linear combination of weights could produce satisfactory fits with the data. It is plausible that the parameters can be linearly related since their gradual change leads to the gradual change of the illusory percept with only minor errors. Of course, it is quite possible that better fits can be achieved with non-linear combinations. For a linear analysis, the following normalized equation is constructed:

$$
I=\frac{W_{L S} * L S+W_{W S} * \frac{1}{W S}+W_{S H} * S H}{W_{L S}+W_{W S}+W_{S H}}
$$

Where:
$\mathrm{W}_{\mathrm{LS}}=$ the unknown weight assigned to the LS ratio,
$\mathrm{W}_{\mathrm{WS}}=$ the unknown weight assigned to the WS ratio,
$\mathrm{W}_{\mathrm{SH}}=$ the unknown weight assigned to the shadow.

The inverse of the WS ratio is used as the last parameter because the hypothesis argues on the reverse relation between the width and the illusory strength as explained in the introduction.

The first step in the track of finding the values of the weights is to align them in order of increasing contribution to the illusory percept. According to results of previous studies, as well as our hypotheses, the weights are expected to be aligned in the order presented, thus

$$
W_{L S}>W_{S H}>W_{W S}
$$

It is essential to carefully choose one weighting process from each experiment, utilizing the results as the lead for the illusory percept produced respectively. For that purpose, the higher correlation between each process and the predicted results is employed as an indicator. According to Figures 21 and 25 the weighting process II is preferred for the first experiment and as for the second experiment the weighting process I is chosen based on Figures 29, 33 and 37.

Tables 22 and 23 summarize the results for each experiment separately.

Table 22: The nine stimuli with their specific geometrical characteristics are presented and related to the produced percent of illusory effect as obtained from the WPII of Experiment 1. WPI showed weaker correlation with the prediction than WPII.

| Exp I | LS | WS | SH | Results |
| :---: | :---: | :---: | :---: | :---: |
| 12 | 1 | 1 | 0 | -0.3525 |
| 8 | 1 | 1.5 | 0 | -0.3479 |
| 3 | 1.07 | 2.25 | 0 | -0.0787 |
| 16 | 1.07 | 1.5 | 1 | -0.0126 |
| 13 | 1.15 | 2.25 | 0 | 0.0938 |
| 19 | 1.07 | 1 | 1 | 0.3289 |
| 18 | 1.15 | 1 | 1 | 0.5799 |
| 14 | 1.5 | 1.5 | 1 | 0.6389 |
| 17 | 1.5 | 1 | 1 | 0.8389 |

Table 23: The nine stimuli with their specific geometrical characteristics are presented and related to the produced percent of illusory effect as obtained from WPI of Experiment 2. The rest of the weighting processes showed lower correlation values with the prediction than WPI.

| Exp II | LS | WS | SH | Results |
| :---: | :---: | :---: | :---: | :---: |
| 8 | 1 | 1.5 | 0 | 0 |
| 3 | 1.07 | 2.25 | 0 | 0.0043 |
| 12 | 1 | 1 | 0 | 0.0739 |
| 16 | 1.07 | 1.5 | 1 | 0.1043 |
| 13 | 1.15 | 2.25 | 0 | 0.1046 |
| 19 | 1.07 | 1 | 1 | 0.2351 |
| 18 | 1.15 | 1 | 1 | 0.6997 |
| 14 | 1.5 | 1.5 | 1 | 0.7395 |
| 17 | 1.5 | 1 | 1 | 0.8499 |

In order to decide on the sequence of the weights, the values of the main parameters are applied to the equation and the value of " 1 " is assigned to one weight while the other two weights are set to be zero. Finally the computed values of the illusory percept are compared with the experimental data computing their correlation. The rationale of this process is to find out how much do the computed results resemble to the experimental when only one parameter is contributing to the illusory percept. The process is applied in both experiments.

Figures $38 \mathrm{a}, \mathrm{b}$ and c demonstrate the results of the process described for the first experiment.
$\mathrm{W}_{\mathrm{LS}}=1, \mathrm{~W}_{\mathrm{ws}}=0, \mathrm{~W}_{\mathrm{sH}}=0$

a.
$\mathrm{W}_{\mathrm{Ls}}=0, \mathrm{~W}_{\mathrm{ws}}=1, \mathrm{~W}_{\mathrm{sH}}=\mathbf{0}$


Computed Illusion
b.

$$
\mathrm{w}_{\mathrm{Ls}}=0, \mathrm{w}_{\mathrm{ws}}=0, \mathrm{w}_{\mathrm{sH}}=1
$$



Figures 38 a, b and $\mathbf{c}$ : The correlation between the experimental results and the computed for Experiment 1 by employment of the equation proposed and the assignment of the weighting values indicated by the titles.

Figures $39 \mathrm{a}, \mathrm{b}$ and c demonstrate the results of the process described for the second experiment.


c.

Figures $39 \mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ : The correlation between the experimental results and the computed for Experiment 2 by employment of the equation proposed and the assignment of the weighting values indicated by the titles.

According to the correlation coefficient the weights are consistent in both experiments and are arranged as expected,

$$
W_{L S}>W_{S H}>W_{W S}
$$

Since the relative order of the weights is defined, the remaining component of the process is to define their values. Several variations could be used verifying the equation as well as taking into account the constraint of the order. However, not every choice would lead to high correlations between the computed and experimental data. Therefore, an optimization process is employed. Specifically the Solver in Excel is utilized as the analysis tool that finds the optimal value of a target cell by changing values in cells used to calculate the target. The target cell is the correlation coefficient between the computed and the experimental results that is set to approach its maximum possible value of one. The changing cells correspond to the three weights $\mathrm{W}_{\mathrm{LS}}, \mathrm{W}_{\mathrm{WS}}$ and $\mathrm{W}_{\mathrm{SH}}$ that could change during the iteration procedure in order to reach the goal. Three initial arbitrary values have to be assigned to them for the process to begin. However, the optimization is subjected to three constraints:

$$
\begin{aligned}
& W_{L S}>W_{W S} \\
& W_{L S}>W_{S H} \\
& W_{S H}>W_{W S}
\end{aligned}
$$

When the iteration process is over, the optimum value of the target cell is displayed as well as the values of the three cells that led to the final result. This method was employed in the current study and applied to both experiments yielding the following results.

## Optimization process of the first experiment.



## Optimized



Figure 40: A schema of the optimization process followed and the results obtained for Experiment 1. The correlation of the experimental and optimized data at the lower left side of the diagram indicates the accuracy and efficacy of the method.

## Optimization process of the second experiment.



Optimized



Figure 41: A schema of the optimization process followed and the results obtained for Experiment 2. The correlation of the experimental and optimized results at the lower left side of the diagram indicates the accuracy and efficacy of the method.

## Discussion

## Experimental Evaluation

The rationale behind the utilization of several weighting processes in order to measure the strength of the illusion that each stimulus produces is based on the investigation of the best approach to measure the participants' responses. Figures 21 and 25 demonstrate that the difference between the correlations of each process and the predicted results is of the order of $5 \%$. This is an appropriate percentage to account as a discriminating factor between the two processes, indicating that the more elaborate method leads to results that are highly correlated with the predicted estimations. On the other hand, the percentage remains relatively small compared to the huge differences in the weights used, indicating the accuracy of the data obtained in experiment 1. This remark is demonstrated by the correlation between the two weighting processes in Figure 42.


Figure 42: The correlation between the two processes in the first experiment.

Accordingly, the variance of the correlations between the experimental and predicted results in Experiment 2 lies between $0.56 \%$ and $4.25 \%$ as can be measured from Figures 29, 33 and 37. This is verified from Figures 43 a, b and c that demonstrate the high correlation between the processes.


c.

Figures 43 a, b, c: The correlation between the three processes interchangeably for the second experiment.

It has to be emphasized that the weighting processes vary significantly from each other but their effect on altering the results is minor. In other words, the correlations between the experimental data from the processes in each experiment do not differ, despite the extremely different values used in order to weight the three parameters. This consistency in the results is attributed to the robust experimental design that yields accurate results.

It the sense that Experiment 2 functions as a validation process for the reliability and replicability of Experiment 1 it is important to investigate whether there is a consistency in the strength of the illusory percept between the two experiments. Therefore, all possible cross correlations between the two experiments are constructed as seen in Figures 44a-f. Although the task in Experiment 2 was originally thought of increased ambiguity, the resulting correlation coefficients are so high that the results obtained are robust.

## Correlation of WI of Exp2 to WI of Exp1



Exp2
a.

Correlation of WI of Exp2 to WII of Exp1

b.

## Correlation of WII of Exp2 to WI of Exp1


c.

Correlation of WII of Exp2
to WII of Exp1

d.

Correlation of WIII of Exp2 to WI of Exp1

e.

Correlation of WIII of Exp2 to WII of Exp1

f.

Figures 44a-f: The cross correlation between the several processes among the two experiments as indicated by their legends.

## Parametric Evaluation

The parametric evaluation is essential in terms of confirming the accuracy of the hypotheses constructed as well as validating the reliability of the rationale on which the model was based. The first step in this process is to have a closer look at the results obtained from both experiments and interpret them in terms of answering the hypotheses constructed. By focusing on the first experiment it is essential to let the participants decide on the order of the stimuli. Sorting the stimuli according to the illusory strength reported and not to the pilot study is presented in Tables 24 a and b .

Tables 24 a and b: The nine stimuli and their illusory effect as obtained from the WPI (a) and the WPII (b) of Experiment 1. The stimuli are ordered according to the observers' perception in
ascending order.

|  | ascending order. | Average |
| :---: | ---: | ---: |
| 12 | $33.25 \%$ | $1.60 \%$ |
| 8 | $33.25 \%$ | $13.32 \%$ |
| 3 | $41.50 \%$ | $4.98 \%$ |
| 16 | $58.25 \%$ | $33.32 \%$ |
| 13 | $74.75 \%$ | $75 \%$ |
| 19 | $91.50 \%$ | $60 \%$ |
| 18 | $91.50 \%$ | $87 \%$ |
| 14 | $91.50 \%$ | $95 \%$ |
| 17 | $100.00 \%$ | $100 \%$ |

a.

|  | Average | Prediction |
| :---: | :---: | ---: |
| 12 | $-35.25 \%$ | $1.60 \%$ |
| 8 | $-34.79 \%$ | $13.32 \%$ |
| 3 | $-7.87 \%$ | $4.98 \%$ |
| 16 | $-1.26 \%$ | $33.32 \%$ |
| 13 | $9.38 \%$ | $75 \%$ |
| 19 | $32.89 \%$ | $60 \%$ |
| 18 | $57.99 \%$ | $87 \%$ |
| 14 | $63.89 \%$ | $95 \%$ |
| 17 | $83.89 \%$ | $100 \%$ |

b.

It is significant to notice that the reordering matches in the two different weighting processes, but analyzing the special characteristics of the rearranged stimuli would give greater insight in the parametric contribution to the illusion.

Table 25: The nine stimuli and their parametric analysis of construction.

| Condition | LS | WS | Shadow |
| :---: | :---: | :---: | :---: |
| $\mathbf{1 2}$ | 1 | 1 | 0 |
| 8 | 1 | 1.5 | 0 |
| 3 | 1.07 | 2.25 | 0 |
| $\mathbf{1 6}$ | 1.07 | 1.5 | 1 |
| $\mathbf{1 3}$ | 1.15 | 2.25 | 1 |
| $\mathbf{1 9}$ | 1.07 | 1 | 1 |
| $\mathbf{1 8}$ | 1.15 | 1 | 1 |
| $\mathbf{1 4}$ | 1.5 | 1.5 | 1 |
| $\mathbf{1 7}$ | 1.5 | 1 | 1 |

According to Table 25 some important observations are extracted:

- The LS ratio is gradually increasing with the illusory percept as expected. There is only a switch with stimuli 13 and 19 but the explanation could lay in the interaction of the LS and WS effect.
- The first two stimuli differ only in their WS ratio and, although it is expected that increasing WS ratio should weaken the illusory percept, it strengthens it. However this stimuli show small difference in their illusory strength.
- Stimuli 3, 16 and 19 that have the same LS ratio follow the intuitive expectation of strong illusory percept with smaller WS ratio. The same observation holds for all remaining groups of stimuli sharing the same LS ratio, thus $13 \sim 18$ and 14~17.
- It is observed though that stimuli 13 and 19 have switched order although the first encounters greater LS ratio. This could be explained though by their strong difference in WS ratio that possibly exceeds the effect of the LS ratio.

Finally, as it is anticipated, the stimuli without shadow are the weakest while all remaining stimuli have shadow.

Accordingly, the same process is followed for the second experiment.

Tables 26 a, $\mathbf{b}$ and $\mathbf{c}$ : The nine stimuli and their illusory effect as obtained from the WPI WPII and WPIII of Experiment 2. The stimuli are ordered according to the observers perception

|  | Average | Prediction |
| :---: | :---: | :---: |
| 8 | $0.00 \%$ | $13.32 \%$ |
| 12 | $7.39 \%$ | $1.60 \%$ |
| 16 | $10.43 \%$ | $33.32 \%$ |
| 13 | $10.46 \%$ | $75 \%$ |
| 19 | $23.51 \%$ | $60 \%$ |
| 3 | $43.00 \%$ | $4.98 \%$ |
| 18 | $69.97 \%$ | $87 \%$ |
| 14 | $73.95 \%$ | $95 \%$ |
| 17 | $84.99 \%$ | $100 \%$ |

a.

|  | Average | Prediction |
| :---: | :---: | :---: |
| 16 | $-0.99 \%$ | $33.32 \%$ |
| 12 | $-0.32 \%$ | $1.60 \%$ |
| 8 | $4.07 \%$ | $13.32 \%$ |
| 13 | $4.14 \%$ | $75 \%$ |
| 3 | $4.95 \%$ | $4.98 \%$ |
| 19 | $11.04 \%$ | $60 \%$ |
| 18 | $53.79 \%$ | $87 \%$ |
| 14 | $56.81 \%$ | $95 \%$ |
| 17 | $71.21 \%$ | $100 \%$ |

b.

|  | Average | Prediction |
| ---: | ---: | ---: |
| 8 | $-6.67 \%$ | $13.32 \%$ |
| 12 | $-4.99 \%$ | $1.60 \%$ |
| 16 | $-2.43 \%$ | $33.32 \%$ |
| 19 | $0.32 \%$ | $60 \%$ |
| 13 | $0.92 \%$ | $75 \%$ |
| 3 | $8.95 \%$ | $4.98 \%$ |
| 18 | $55.01 \%$ | $87 \%$ |
| 14 | $56.61 \%$ | $95 \%$ |
| 17 | $72.53 \%$ | $100 \%$ |

C.

Tables $26 \mathrm{a}, \mathrm{b}$ and c demonstrate the reordering of the stimuli. By observation it is noticed that:

- Despite the differences in rearrangement there is a trend that follows the first experiment.
- Stimuli 8,12 and 16 occupy the first positions meaning that they produce the weakest effects. According to the corresponding ratios this attribute is consistent with the observations from the first experiment.
- Stimuli 13 and 19 follow in success as expected because of the increase in LS ratio.
- Stimulus 3 seems to produce stronger than the expected effect and this could be explained by the ambiguity that increases due to the relatively rectangular window with very large WS ratio.
- The last three stimuli remain consistently the strongest in the same order over all processes in both experiments.

The experimental and parametric evaluations are consistent with literature in terms of the definition of the relationship between the LS ratio and the strength of the illusion. As originally suggested, it is proved that the bigger the ratio, the stronger the illusory effect. This is obvious in both experiments as well as the observation that the variance of the LS ratio is the primary parameter of the illusory strength. Furthermore, LS is the key component in ordering the stimuli with the corresponding percept.

Finally, the parametric analysis proved the hypothesis concerning the relationship between the WS ratio and the illusory strength to be valid. Apparently, the longer the horizontal length of the window is, the weaker the illusory percept. This is explained by the increased effect of linear perspective in case of large WS ratios.

## Model Evaluation

The parametric analysis used a linear relation between the variables examined and the illusory strength that was employed as the basis for the model rationale. The algorithm was based on the equation that relates the stimulus to the illusory strength. It actually describes the relation between the parameters that were weighted in different ways with the prediction of the illusory percept yielded. The correlation coefficients obtained from the comparison of experimental and computed data showed consistency of both experiments with literature. The order of the variables followed as speculated:

$$
\mathrm{W}_{\mathrm{LS}}>\mathrm{W}_{\mathrm{SH}}>\mathrm{W}_{\mathrm{WS}}
$$

The optimization process that followed utilized this constraint in order to find the best combination of values that should be assigned to the three weights yielding the maximum possible correlation between the experimental and computed results. The weighting values do not differ much in the two experiments, indicating again the high correlation amongst experiments revealing the robustness of their design. Significantly, the implementation of the computational model could be employed in predicting the illusory strength of the stimuli described in future studies. As a matter of fact it could be the basis for a more complicated algorithm that would take into account several other variables.

## Future Work

The results obtained from the healthy controls do not only verify previous knowledge but contribute to the expansion of the understanding of the Ames Window Illusion. Since it is proved that the experiments are designed in a very thorough and detailed way, it is essential to consider testing patients with schizophrenia with the stimuli described. The robustness of the experiments guarantees the accuracy of the data to be obtained and the efficacy of the model secures the validity of the predicted results. It is expected that very weak illusions are not going to be observed by neither the controls nor the patients and very strong illusions are going to be observed by both groups. In the latter case the illusory percent might be lower for patients but it will remain above threshold of illusory percept. However the intermediate cases are questioned in terms of the percept that they will cause. According to the psychometric curve presented in Figure 45 the most important region is the area around the middle of the curve, near the point where the illusory strength is about $50 \%$, because this is the most sensitive part of the curve In other words, stimuli that produce moderate illusory percepts could be the subject for further research because they produce effects that half of the samples take as illusory while the rest as veridical.


Figure 45: Psychometric curve with the area of interest circled.

These stimuli should be tested on patients with schizophrenia in order to examine how much of the illusory percept they perceive. The hypothesis states that patients should remain less at the illusory percent than the controls. In other words they should show lower percentages even with the moderate stimuli. The hypothesis is based on the fact that the integration of data-driven and schema-driven processes is deficient. Therefore, patients should be able to break the illusion when presented by moderate strength stimuli. However, because of their sensitive nature, an inability to maintain their concentration focused for prolonged periods of time the experimental process should be limited. Although both experiments showed robust results, the task in the first one is less ambiguous in terms of understanding. Experimenting with controls revealed that participants understood easier the task of reporting whether the white dots are closer to or further away from the black dots than indicating the direction of rotation. Training them led to robust results but this would be a trivial process in case of patients. Finally, the number of stimuli should be limited to four in order to minimize the duration of the
experimental sessions. The four stimuli would include two catch trials, thus a very weak and a very strong stimulus and two stimuli near the most sensitive part of the psychometric curve, thus producing more clear differences between patients and controls.

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