COGNITIVE RADIO NETWORKS: RESOURCE ALLOCATION AND EFFECT OF END-USER BEHAVIOR

By

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ABSTRACT OF THE DISSERTATION

Cognitive Radio Networks: Resource Allocation and Effect of End-User Behavior

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Recent advances in Cognitive Radio (CR) technology are reshaping wireless communications systems with two important technological contributions, Radio Access Technology (RAT) multiplicity and Dynamic Spectrum Access (DSA). Advances in radio design now enable multi-RAT wireless devices that streamlines the design and implementation of DSA that provides spectrum sharing flexibility. The flexibility enabled by CR technology has even permeated up to the application layer where end users have been empowered to a greater degree of freedom to use wireless devices. In this dissertation, we address two important aspects of CR networks, (i) resource allocation in multi-RAT enabled wireless networks; and (ii) impact of end users' behaviors on wireless networks.

In the first part, we study an example of the coexistence of multiple RATs devices in a network, namely a concept of Cognitive Digital Home (CDH) requiring spectrum coexistence of various devices and networks of networks. We develop a resource allocation framework when there is a multiplicity of RATs including cognitive and legacy radios. We design distributed algorithms for maximizing sum rate and maximizing service capacity. Distributed power control and admission control schemes are proposed to improve energy efficiency and system feasibility.

In the second part, we focus on the impact of end users' behaviors on wireless systems by investigating the role of Prospect Theory (PT) in wireless network design. Prospect theory, developed by Kahneman and Tversky, explains real-life decision making that often deviates from the behavior expected under Expected Utility Theory (EUT). As a first step in this direction, we consider a radio resource management problem where users follow PT and compare it to the case when users follow EUT. Specifically, we consider a random access game and prove under mild conditions that deviations from EUT of any player results in system throughput degradation and increased delay and energy consumption in a 2-player game. We also study N-player symmetric homogeneous games where similar system performance disparities are observed. Finally, the framework introduced in the above random access model is extended to study an exemplary two-level data pricing model and compare service choices when users follow EUT and PT.

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Dedication

To my parents and my wife for their support, encouragement and love

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Chapter 1

Introduction

The recent advances in Cognitive Radio (CR) technology have been reshaping the modern wireless communications systems. Among numerous contributions the CR technology have made, flexibility and multiplicity are of paramount importance. The CR technology has greatly boosted the development of the Radio Access Technology (RAT) multiplicity, i.e, the radio design where multiple radio technologies can coexist on the same platform. As early as the dumb cell phones with multiple RATs (e.g., cellular and bluetooth), the wireless communications researchers and practitioners have been eager to exploit the advantages the RAT multiplicity would bring, e.g., flexible spectrum usage and energy efficiency. The recent trend in research on offloading data streams among multiple RATs on the same platform (e.g., software defined radio) has further advanced this exploration. The CR technology aims to provide the RAT flexibility as much as possible. A notable example is the White-Fi radios proposed and developed recently where flexible radio interfaces provide access to multiple available spectrum bands in TV white space [1] [2]. The CR technology has utilized the RAT multiplicity to realize the Dynamic Spectrum Access (DSA), i.e., one of the fundamental reasons why the CR technology was proposed and developed. Extensive studies, e.g., [3], have shown that the "scarcity" of available spectrum bands is nothing but the inefficient usage of the spectrum resources that is a consequence of the traditional and inflexible wireless system design. The CR technology aims to utilize the spectrum resources in a more efficient and intelligent way by avoiding the time and spatial inefficient usage of spectrum resources. Researchers and practitioners have proposed various solutions to achieve DSA. A well-known model [3] is the two-layer CR network where users are divided as primary users and secondary users. Primary users have the license to use a specific spectrum but also have the rights to share it with the

secondary users. The secondary users are all agile CR devices that can flexibly sense the spectrum environment and access the available spectrum bands. The primary users allow the secondary users to share their resources and the secondary users should return either monetary incentives or communication rewards to the primary users. Besides, one of the first implementable solutions of DSA is the White-Fi technology mentioned above where the CR devices try to harvest the abundant available TV white space spectrum [2]. The flexibility enabled by the CR technology and Software Defined Radio (SDR) has even permeated up to the application layer where the end users have been empowered to use the wireless devices in many novel ways with smart phones and smart applications. For example, end users have more RAT options for their communications needs. Meanwhile, various data communications plans and packages are available to end users. Moreover, more heterogeneous and convenient smart device applications make the RATs, data plan and packages selection much easier for end users. Therefore, end users' behaviors and choices will definitely become paramount in shaping the current and future wireless system design and operation. Therefore, the topic on the impact of end users' behaviors on the wireless system design should be considered as a promising and urgent research direction in the future. In this dissertation, we have addressed both aspects of CR networks, namely resource allocation in a multi-RATs device enabled wireless networks and the impact and influence of end users' behaviors on the underlying network protocol design.

In the first part of this dissertation (chapter 2), we study an example of coexistence of multiple RATs devices in a novel network, namely a concept of Cognitive Digital Home (CDH). With advances in radio access technologies and the increasing demand on heterogeneous data services, integration of wireless networks with multiple RATs is expected to be a prevalent feature of future mobile networks. Meanwhile, the growth of multimedia services in a home environment for communications, entertainment and safety has also resulted in the concept of a digital home where a multiplicity of devices and RATs coexist. Further, with the advent of cognitive and multi-platform radios, one can envision the spectrum occupancy of these devices and RATs to range from the TV white spaces (54MHz $\sim$ 698MHz) [4, 5] to unlicensed bands (2.4GHz and 5GHz) [6] and even all the way to 60GHz radio bands [7, 8] in a digital home.

The new features of wireless networks in a digital home have drawn some attention from the research community though there are still plenty of interesting research topics to be addressed. One of the most important topics in a digital home is the effective operation of multi-RAT networks. There have been some efforts directed at the operation of multi-RAT networks in general. In [9], the authors proposed a framework for optimized dynamic usage of radio resources in wireless networks with multi-RATs and multi-operators. Particularly, cognitive radio functionalities have been shown as a must to implement such a system. The authors in [10] studied the network selection process in multi-RAT networks by decomposing it and comparing the common approaches for network selection proposed in literature. In [11], the authors compared the data transmission schemes over multi-RATs with single RAT. They concluded that the algorithm for distributing upper layer data packets over multi-RATs should be carefully designed in order to fully utilize the resources of wireless networks. Radio resource allocation problems in multi-RAT networks were studied in [12] and [13] from the theoretical standpoints. In [12], the authors formulated a utility maximization problem for multi-channel, multi-RAT and multi-hop wireless networks and a dynamic algorithm was proposed based on the decomposition of this problem. The authors of [13] studied a network throughput maximization problem in a multi-user and multi-RAT network where transmission schemes over multi-RATs and single RAT are both allowed. This problem was shown to be convex and a distributed algorithm based on dual decomposition was developed to solve it. Besides the effective operation of the networks with existing RATs, integration of new RATs into a digital home is also studied. In [1], the authors began to investigate the feasibility of a new RAT, i.e., cognitive radio systems over TV white space, for a digital home. Analytical and simulation methods have been used to compare the performance of the cognitive radio system with that of license-exempt systems over other spectral bands, e.g., 2.4GHz and 5GHz bands.

Although the resource allocation in multi-RAT wireless networks has been studied as discussed above, few efforts [14] [15] have focused on efficient resource allocation in a digital home with multi-RATs. Fair and efficient resource allocation is of paramount importance in supporting various data services. In our earlier work in [14], we have developed a framework for centralized spectrum management in a cognitive digital home (CDH) where

a home genie node (HG) coordinates spectrum coexistence across a multiplicity of RATs. Previous works regarding multi-RAT resource allocation usually assume that a RAT and its accessible spectral band are bundled and different RATs' bands don't overlap with each other. However, in a CDH, legacy RATs are allowed to coexist and Cognitive Radio (CR) RATs can access all the spectral resources when they are available. These assumptions make our CDH model different in describing multi-RAT wireless home networks and also add much more complexity to the resource allocation problems within it. For example, the multi-RAT resource allocation problems in [13] can be formulated as a convex optimization and solved by standard methods. However, resource allocation problems in a CDH are generally $\mathcal{NP}$-complete and often heuristic algorithms should be carefully designed to trade off the performance and complexity. General analytical models for resource sharing among legacy devices in the same spectral band are difficult to build even though experimental results provide some insights (see [16] and reference therein). In this work, we consider two models in a CDH for addressing spectrum coexistence of legacy devices: (i) Pessimistic Controllability (PC) Model, and (ii) Switched RAT (SR) Model. Under the PC model whose preliminary results can be found in [15], we assume that the HG is unable to exercise any control over the devices of legacy RATs and hence cannot influence how the resources are shared by legacy devices that share spectrum. In this case, we assume somewhat pessimistically that each legacy device obtains an equal share of the spectrum resource. Under the SR model, we assume that the HG is able to exercise perfect control over the legacy devices within the parameters of the protocol specifications of each legacy device. Based on the two models, two resource allocation problems (i) Maximizing Sum Rate (MSR), and (ii) Maximizing the Service Capacity (MSC), are formulated. Further, distributed algorithms based on partial dual decomposition are proposed for addressing these problems. Pricing indices play an important role in the design of the distributed algorithms since they convey crucial information regarding a service's achievable and target data rates. A distributed power control scheme based on the channel and RAT allocation result is designed for the services to efficiently use the energy. An admission control scheme based on the pricing indices is also proposed to improve system feasibility when the CDH system cannot meet all the service requests.

The chapter 2 is organized as follows. The system model of CDH is presented in section 2.1. In section 2.2, MSR and MSC are formulated. Distributed algorithms, power control scheme and admission control scheme based on partial dual decomposition are designed for them. Extensive numerical results are shown in section 2.3 to evaluate the performance of the system.

In the second part of this dissertation (chapter 3), we focus on the impact and influence of end users' behaviors on the wireless systems by studying the role of Prospect Theory (PT) in wireless networks. This work is a first step in this direction and a game-theoretical approach is used. Since the early works in [17–21], game theory has emerged as a powerful tool for the analysis and design of radio resource management algorithms for wireless systems and networks. As detailed in recent surveys on game theoretical studies of various aspects of wireless communication networks [22] and [23], a great deal of meaningful insights have been gained into a wide range of problems and engineered system solutions have emerged for a variety of systems such as cellular, ad-hoc mesh, sensor and WiFi networks. All these works, including much of traditional game theory (going back to von Neumann and Morgenstern [24]) rely on the precept that users follow expected utility theory (EUT), where decision-making is guided strictly by accepted notions of utility, always rational and uninfluenced by real-life perceptions. This is a very sound assumption that governs the engineered system design of such systems when the actions of end-users do not interfere with such design. Moreover, there is ample evidence of the success of this approach as seen by the phenomenal growth of wireless system and network deployments along with their overarching applications and societal benefits.

On the other hand, the advent of easy to use, smart and programmable radio devices is resulting in the ability of end-users to control devices with a greater degree of freedom than ever. While current radio technologies and associated communication protocols are still for the most part agnostic to the decision- making of end-users, it is conceivable that in the future, users could make decisions that influence the underlying design of various algorithms and impact the performance of the overall system. These decisions could range from choices of access control to cooperation to selection of dynamic pricing plans. Other common examples of end-user actions include repeated refreshing of a browser under a

delayed video stream or slowly loading web link. More sophisticated examples include modifying drivers of radio cards and associated protocols such as is increasingly becoming possible with the advent of programmable cognitive and smart radio devices. Even from a system perspective, the ever increasing capacity crunch faced by service providers is driving the migration of wireless data services in the future towards dynamic spectrum access and dynamic pricing based options, thereby exposing the overall design of the network to the decisions of end-users based on their monetary perceptions of the value of the service. Further, there is ample evidence (anecdotal and otherwise) [25] that decision making in real- life is often guided by perceptions that deviate from the precepts of EUT.

Motivated by these emerging wireless networking scenarios, we turn to Prospect Theory (PT) [25], a Nobel prize winning theory developed by Kahneman and Tversky that explains real-life decision- making and its deviations from EUT behavior. While the main ideas and models behind early PT were developed based on responses/decisions of players involving monetary transactions (prospects), the behavioral deviations from EUT are general enough that they have widespread application in many areas [26–29]. We believe that understanding the role of PT in wireless systems and networks, that are increasingly becoming user-centric is important. It is often recognized that a measurement of user satisfaction must be included in the assessment of the efficiency of the network as a whole [30]. It has also been revealed that service repurchase intention among mobile Internet users was significantly positively related to "experienced value" and "satisfaction" [31]. While these findings highlight the importance of emphasizing user-experience when defining and assessing quality of service, traditionally, improvement in network service has followed a bottom-up approach, assuming that optimization of performance at the engineering design level will translate directly into an improved user experience.

As a first step in this direction, we consider an exemplary radio resource management problem where users follow PT and compare and contrast it to the case when users follow EUT. Specifically, we consider a random access game where selfish players adjust their transmission probabilities over a collision channel according to rewards received for successful transmission but also incur energy and delay costs. In the initial work [32], we only considered a 2-player homogeneous wireless random access game with 0/1 collision

channel. In this dissertation, we significantly extend the work by studying and comparing both 2-player homogeneous and heterogeneous games where general random access channel model can be applied, i.e., the packet reception probability can range within $[0, 1]$. Furthermore, we also extend the investigation to a N-player homogeneous game. Our extended results prove the correctness of our findings in [32] in a much more general fashion, i.e., the deviation from EUT results in degradation of system throughput, increased delay and energy consumption.

While the random access scenario considered here does not exactly reflect the time-scale or granularity of end-user decision making, it nevertheless serves as a useful illustration to open up this new line of investigation. There is a gap between the random access model used here and practical role of PT in the real world. However, we believe this exemplary model can get rid of the unnecessary complexities of real wireless systems at the very initial stage of this new research and thus help us identify the possible impact of end-user behaviors on wireless systems in a clear fashion. Moreover, this simple wireless random access model actually captures the essence of some real-life wireless communication scenarios. Notable examples of end-user decision making as it relates to transmission probabilities are: the decision to access cellular data services when the network may already be congested; the decision to utilize a Wi-Fi service at an airport depending on the usage fee and relative importance of immediate data communications; and the decision to utilize high speed data services even though the battery levels may be very low. The random access model used in this dissertation may not exactly describe the above scenarios, however, it is actually a reasonable abstraction of the transmit choices made by the end user since it focuses on the probability of a user's data transmission.

The chapter 3 is organized as follows. An extensive related work review is provided in section 3.1. In section 3.2, a brief introduction to Prospect Theory is provided. Wireless random access games are formulated in section 3.3 among selfish players under both the EUT and PT models. In section 3.4, the impact of any player's deviation from EUT on the individual player's and system performance is numerically and analytically studied in a 2-player heterogeneous wireless random access game. The scenario where both players follow either only PT or only EUT is further studied as 2-player homogeneous PT game

and EUT game, respectively in section 3.5. In section 3.6, a N-player homogeneous game where all the users follow either only PT or only EUT is discussed.

The above work in chapter 3 has clearly shown that the deviation of end user from EUT can impact on the individual user utility and system performances. A direct extension and an interesting topic beyond this is that how the impact of end users can change the underlying the system protocol design. In chapter 4, we start to tackle this issue from data pricing ([33] and references therein) that is an essential and key part of any real wireless data communications systems. An efficient and properly designed data pricing scheme can motivate the end users utilizing network resources in an optimal way. As a result, the whole system may be operated at an optimal point. Depending on the designers' and operators' objectives, at the optimal operation point, the system may attract the most end users to utilize the network resources, experience the least traffic congestions or generate the most profits to the service providers. As mentioned in the above, the impact of end users' behaviors on the wireless system has been seldom studied. Therefore, the current data pricing schemes do not consider this impact extensively or systematically, either. Moreover, the deviations of people from EUT have constantly been observed in the monetary activities through numerous experiments that are well studied in PT. Therefore, the need to study the role of PT in data pricing schemes also seems to be necessary and urgent. In this dissertation, we also provide a first step study of the impact of end users' behaviors on data pricing.

In chapter 5, we conclude and provide future directions for further research. Since the latter part of the dissertation is essentially a first step in exploring the role of PT in wireless networks, an extensive discussion on the topics in the second part of the dissertation is provided.

Chapter 2

A Framework for Distributed Resource Allocation and Admission Control in a Cognitive Digital Home

2.1 System Model

2.1.1 Cognitive Digital Home Architecture

In a CDH, motivated by [34], a spectrum manager, Home Genie Node (HG), coordinates the spectrum coexistence across the home networks, with the degrees of freedom ranging from the transmission parameters such as frequency, bandwidth, power, etc. to the RAT itself. We assume in our model that the CDH employs devices that support multi-platform radios (MPR) equipped with both CR and legacy RATs. The legacy RATs include technologies such as Bluetooth, Wi-Fi, Zigbee, etc.. The CRs in the CDH are assumed to be generic spectrally agile radios capable of noncontiguous Orthogonal Frequency Division Multiple Access (OFDMA) over the entire range of unlicensed frequency spectrum available. The realization of the CR RAT in a CDH relies on advances in hardware and antenna design such as anticipated in future cognitive radio networks [35] [36]. Though a cognitive radio system may allow multiple users to possibly share a channel as in [13], the noncontiguous OFDMA enabled CR RAT [36] considered in our model is designed to support the high rate craving services which usually has high transmit power, e.g., high definition video streaming. Since orthogonal use of a channel maximizes a single service's data rate on that channel, we assume that cognitive radios are opportunistically able to find and use orthogonal channels whenever they are available. Further, we also stipulate that the cognitive radios avoid channels used by legacy RATs so that their performance is not degraded. Parallel transmission [13] where a MPR (service) can simultaneously employ multiple RATs for transmissions is also allowed.

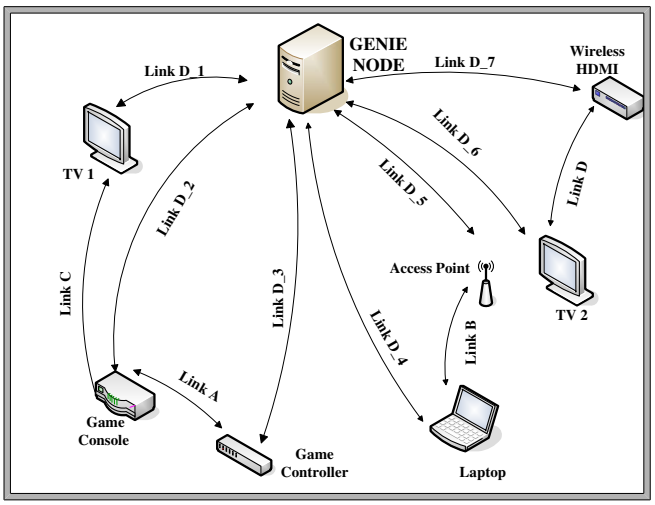


Figure 2.1: Illustration of a Cognitive Digital Home

The classes of devices considered in this CDH model (see Fig. 2.1) include service provision devices (SPD) which directly provide data services to the end users, e.g., TV and Laptop, as well as relay and wireless access network (RWAN) devices, e.g, Wi-Fi access point and wireless High Definition Multimedia Interface (HDMI), which provide access and relay services to SPDs. All these devices are equipped with MPRs. The HG controls the devices via a set of dedicated control channels as shown in Fig. 2.1, e.g., link D_1 for TV 1. The SPDs and RWANs may report their local spectral environment information, data rate requirements or access decisions to HG via control channels.

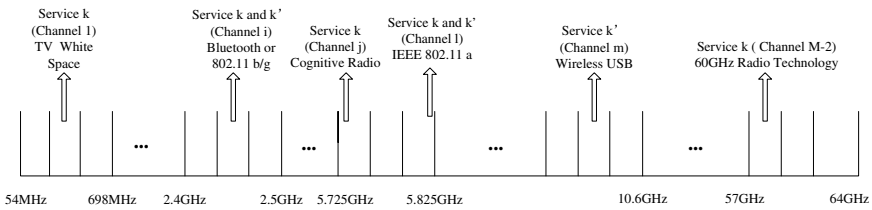


Figure 2.2: Exemplary service's channel allocation. Service k and k' share channel i and l by using legacy RAT, e.g.,Wi-Fi. Service k exclusively uses channel j by using CR RAT.

2.1.2 Preliminaries

Definition 2.1.1. *A channel is the atomic unit of spectrum utilized by legacy RATs in a CDH. It is also the atomic unit of spectrum that can be controlled and allocated and,*

hence, a single orthogonal tone accessed by the noncontiguous OFDMA enabled Cognitive Radio (CR).

The idea of channel allocation is illustrated in Fig. 2.2, where for simplicity only three channels are shown for each portion of spectrum. We assume that the spectrum used by the RATs in the CDH (ISM, TV White Space, Wireless USB and 60GHz radio) is divided into M equal width channels. However, in the less crowded spectrum bands but with more available resources, e.g., 60GHz bands, several contiguous channels may be grouped as a single allocation unit for better efficiency. As mentioned above, a service can obtain resources from various RATs and this service may be allocated with noncontiguous channels. Further, in any channel, there could co-exist multiple legacy RATs and multiple services (e.g., see channel i in Fig. 2.2) for ensuring fairness among services. However, service assigned with CR RAT will exclusively use a channel (e.g., see channel j in Fig. 2.2) for maximizing the network throughput. This idea can be formally described by a set of channel usage constraints which all the services should comply with. In the following, $\mathcal{K} = \{1, 2, ..., K\}$ and $\mathcal{M} = \{1, 2, ..., M\}$ are the set of services and set of channels in a CDH, respectively. $\mathcal{T} = \{1, 2, ..., T\}$ is the set of available RATs on each MPR. $x(k, i, t)$ in constraint (2.1) indicates whether the k-th service uses RAT t in channel i or not. $l(k, i, t)$ in constraint (2.3) shows how the k-th service uses technology t to access channel i.

$$x(k, i, t) = \{0, 1\},\ \forall k \in \mathcal{K},\ \forall i \in \mathcal{M},\ \forall t \in \mathcal{T} \tag{2.1}$$

$$\mathbf{x_k} = [x(k, 1, 1) \dots x(k, i, 1) \dots$$
$$x(k, i, t) \dots x(k, i, T) \dots x(k, M, T)] \tag{2.2}$$

$$\mathbf{l_k} = [l(k, 1, 1) \dots l(k, i, 1) \dots l(k, i, t) \dots l(k, i, T) \dots l(k, M, T)] \tag{2.3}$$

$$0 \leq l(k, i, t) \leq x(k, i, t),\ \forall k \in \mathcal{K}, \forall i \in \mathcal{M}, \forall t \in \mathcal{T} \tag{2.4}$$

$$\sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} l(k, i, t) \leq 1,\ \forall i \in \mathcal{M} \tag{2.5}$$

$$\sum_{t \in \mathcal{T}} x(k, i, t) \leq 1,\ \forall k \in \mathcal{K}, \forall i \in \mathcal{M} \tag{2.6}$$

If the k-th service uses CR as its RAT, it has the flexibility to access all the channels in a CDH and it occupies channel i orthogonally, i.e., $l(k, i, t) = 1$, otherwise $l(k, i, t) = 0$. If the

k-th service uses legacy RAT to access channel i, the sharing of the channel among services (using legacy RATs) is possible. $l(k, i, t)$ indicates the effective sharing portion that the k-th service occupies in channel i and the sharing could occur in the time domain. However, the value of $l(k, i, t)$ is highly dependent on the legacy RATs sharing model employed and we will address this issue below. Different from the CR RAT, legacy RATs can only access a limited number of channels in a specific region, i.e., $\forall i \in \mathcal{M}_t \subset \mathcal{M}$, which is predetermined, e.g., IEEE802.11 b/g in 2.4GHz. The total usage of a channel should be no larger than 1 as shown in constraint (2.5). We assume a service can only employ a single RAT on a channel one time to avoid self-interference as shown in constraint (2.6).

The coexistence of multiple services with possibly different legacy RATs in the same channel has been studied [37] [38] [39]. However, general analytical models for how resources are shared by legacy devices (e.g., IEEE802.11 and Bluetooth) are not easy to construct. The complexity lies in the interference generated from multiple services with multiple legacy RATs, the differing channels seen by the services and heterogeneous access protocols employed by multiple RATs. As mentioned earlier, in the CDH, we address this complexity via two models (i) Pessimistic Controllability (PC) model, and (ii) Switched RAT (SR) model. Under the PC model, we assume that multiple services assigned to the same channel with possibly different legacy RATs obtain an equal share of the resources. For instance, if K services share a channel, service k with RAT t only achieves $\frac{1}{K}$ of the data rates if service k occupies the channel exclusively with the same RAT t. While this is indeed pessimistic, it reflects the reality that the HG is limited in its capability to control the protocol parameters of co-existing legacy RATs. However, in some special cases, the PC model has been shown to be accurate for describing resource sharing among multiple services (as in Carrier Sense Multiple Access (CSMA) networks [40]). Specifically, the PC

model is defined by setting:

$$l(k,i,t)=\begin{cases} x(k,i,t) & \text{if}\, t=\text{CR}, \forall i\in\mathcal{M}, \forall k\in\mathcal{K} \\ \frac{1}{\sum\limits_{k'\in\mathcal{K}}\sum\limits_{t'\neq\text{CR}} x(k',i,t')} & \text{if}\, t\neq\text{CR}\,\text{and}\, x(k',i,t')\neq 0, \\ & x(k,i,t)\neq 0, \forall i\in\mathcal{M}_t, \forall k\in\mathcal{K} \\ 0 & \text{otherwise} \end{cases} \tag{2.7}$$

With the advancement of the radio access technologies deployed in a digital home, a better controllability of the HG over the RATs can be expected [41]. Thus, the SR model assumes that the MPRs employs the switched legacy RATs scheme, i.e., only one MPR with a legacy RAT transmits in a channel at a time. The HG may precisely schedule services with legacy RATs to share the channel. Then, $l(k,i,t)$ should take real number value in interval $[0,1]$. The SR model is defined by setting:

$$l(k,i,t)=\begin{cases} x(k,i,t) & \text{if } t=\text{CR},\ \forall i\in\mathcal{M},\ \forall k\in\mathcal{K} \\ [0,1] & \text{if } t\neq\text{CR},\ \forall i\in\mathcal{M}_t,\ \forall k\in\mathcal{K} \\ 0 & \text{otherwise.} \end{cases} \tag{2.8}$$

The theoretical physical data rate that can be obtained by the k-th service in channel i is given as: $R^P(k,i,t)=w_i\log(1+\frac{h_k^i P(k,i,t)}{N_0 w_i})$, where w_i is the bandwidth of channel i. h_k^i and $P(k,i,t)$ are the k-th service's channel gain in the i-th channel and transmit power in the i-th channel with t-th RAT, respectively. N_0 is the noise level. The effective data rate achieved by the k-th service using technology t in channel i is a RAT dependent function, i.e., $R(k,i,t)=f_t(R^P(k,i,t))$. For simplicity, in this dissertation we assume the relationship is linear and characterized by a factor $\alpha(k,i,t)$, e.g., $R(k,i,t)=\alpha(k,i,t)R^P(k,i,t)$, where $0<\alpha(k,i,t)\leq 1$ and $\alpha(k,i,t)=l(k,i,t)$. The data rate achieved by the k-th service in channel i is $R(k,i)=\sum_{t\in\mathcal{T}}R(k,i,t)=\sum_{t\in\mathcal{T}}l(k,i,t)R^P(k,i,t)$.

2.2 Distributed Resource Allocation and Admission Control in a Cognitive Digital Home

2.2.1 Joint Channel and RAT Allocation Problems

In the most general setting, the resource allocation in a CDH includes the assignment to each service a set of channels, corresponding RATs along with choice of transmission power, modulation and coding scheme. In this dissertation, we focus on Joint Channel and RAT Allocation (JCRA) problems. The first JCRA problem, i.e., Maximizing Sum Rate (MSR), aims to maximize the sum rates while supporting all the inelastic services (rate constrained). By solving this problem, the network efficiency of the CDH can be maximized while the fairness among services can be guaranteed. We study this problem for both the PC and SR models as follows:

$$\max_{\mathbf{X},\mathbf{L}} \sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{M}}\sum_{t\in\mathcal{T}} R^P(k,i,t)l(k,i,t) \tag{2.9}$$

$$s.t. \sum_{i\in\mathcal{M}}\sum_{t\in\mathcal{T}} R^P(k,i,t)l(k,i,t) \geq R_k^{min},\ \ \forall k\in\mathcal{K} \tag{2.10}$$

$$P(k,i,t) = P_t,\ \ \forall k\in\mathcal{K}, \forall i\in\mathcal{M}, \forall t\in\mathcal{T} \tag{2.11}$$

$$\sum_{i\in\mathcal{M}}\sum_{t\in\mathcal{T}} P(k,i,t)x(k,i,t) \leq P_k^{max},\ \ \forall k\in\mathcal{K} \tag{2.12}$$

and with constraints $(2.1) - (2.6)$ and (2.7) or (2.8) for PC or SR, respectively,

where $\mathbf{X}$ and $\mathbf{L}$ are matrices of control variables as defined in constraints (2.1) - (2.6). The k-th row of $\mathbf{X}$ is $\mathbf{x_k}$ defined in constraint (2.2) which indicates the channel usage of the k-th service over all the channels and RATs. That of $\mathbf{L}$ is $\mathbf{l_k}$ defined in constraint (2.3) and describes the actual physical resource share of the k-th service over all the channels. Note that, under the PC model, $l(k,i,t)$ can be directly calculated from $\mathbf{X}$ as in constraint (2.7). The control variables are the entries in $\mathbf{X}$ though $l(k,i,t)$ appears in problem formulation and derivations below for necessary conciseness. However, under the SR model, beyond assigning services to a channel with a RAT, the portion of the physical resources it gets should be decided. Constraint (2.10) requires each service to be satisfied with its own

minimal data rate requirement. Constraint (2.11) reflects the transmit power assumption that it's pre-determined by the RAT itself where a typical value is associated with a RAT as P_t, $\forall t \in \mathcal{T}$. This assumption reduces the dimensions of the assignment and makes the problems tractable. Each service (a MPR device) is also limited by its maximal transmit power as shown in constraint (2.12). Also, the channel usage constraints (2.1) - (2.6) cannot be violated and either constraint (2.7) or (2.8) is added to it depending on which channel access model is used.

As a special case of the MSR problem (2.9), Maximizing Service Capacity (MSC) problem is formulated in order to support as many rate constrained services as possible.

$$\max_{\mathbf{X},\mathbf{L}} \sum_{k\in\mathcal{K}} u(\sum_{i\in\mathcal{M}}\sum_{t\in\mathcal{T}} R^P(k,i,t)l(k,i,t) - R_k^{min}) \tag{2.13}$$

$$s.t.\ P(k,i,t) = P_t,\ \ \forall k \in \mathcal{K}, \forall i \in \mathcal{M}, \forall t \in \mathcal{T} \tag{2.14}$$

$$\sum_{i\in\mathcal{M}}\sum_{t\in\mathcal{T}} P(k,i,t)x(k,i,t) \leq P_k^{max},\ \forall k \in \mathcal{K} \tag{2.15}$$

$$\text{and with constraints } (2.1) - (2.6) \text{ and } (2.7) \text{ or } (2.8) \text{ for PC or SR, respectively,}$$

where $u(x) = 1\,, \forall x \geq 0$ and $u(x) = 0\,, \forall x < 0$.

The channel usage constraints (2.1) - (2.6) with (2.7) or (2.8) add much complexity to the above problems. Depending on which channel access model is used, i.e., PC model or SR model, the problems are named as PC-MSR, PC-MSC, SR-MSR and SR-MSC. The hardness of the channel usage constraints lies in the assumptions that the CR RAT can access all the channels and multiple services can access the same spectral resources with multiple RATs. These assumptions capture the features of CDH as envisioned in the future along with advances in cognitive radio technology. Thus, these problems are worthwhile to solve regardless of their hardness. Before proceeding to algorithm design, the complexities of the problems are first studied.

Theorem 2.2.1. *The PC-MSR, PC-MSC, SR-MSR and SR-MSC are all $\mathcal{NP}$-complete.*

Proof. In the first place, PC-MSR can be shown to be $\mathcal{NP}$. The decision problem of PC-MSR can be described as "given a real number $P \in \mathbb{R}_+$, can the solution to PC-MSR

problem (9) support sum rates such as $\sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{M}}\sum_{t\in\mathcal{T}} R(k,i,t)l(k,i,t) \geq P$? ". Given a solution, i.e., $\{\mathbf{X}^*, \mathbf{L}^*\}$, the correctness of this solution can be verified efficiently, i.e., in polynomial time. We assume that the calculation time of $R^P(k,i,t)$ is constant, then the total time to calculate the sum rate and verify the individual rate requirements can be bounded by $\mathcal{O}(|\mathcal{K}||\mathcal{M}||\mathcal{T}|)$. Therefore, we can show PC-MSR $\in \mathcal{NP}$.

Next, we examine the PC-MSR problem from a different angle and polynomially reduce a 2-dimensional Multiple Knapsack Problem (MKP) [42] [43] to it. By doing this, we first consider a Simpler variation of PC-MSR (SPC-MSR) where we assume only legacy RAT t' is used and $R(k,i,t')$ is predetermined. Then, we may consider that each service has its own knapsack constrained by its own maximal transmit power P_k^{max} and its minimal data rate R_k^{min}. For each channel i, there are $|\mathcal{K}|$ objects affiliated to it, e.g., the k-th service with RAT t' on channel i. This object has constant transmit power size $P_{t'}$ and transmit rate value $R(k,i,t')$. Then, the SPC-MSR problem aims to maximize the sum transmit rate value of a subset of the $|\mathcal{K}||\mathcal{M}|$ objects while the objects are feasibly packed into the $|\mathcal{K}|$ knapsacks. Thus, the correspondence from a 2-dimensional multiple knapsack problem to SPC-MSR problem is established and we have shown that MKP can be polynomially reduced to SPC-MSR problem, i.e., MKP $\preceq_p$ SPC-MSR. With the facts that MKP is strongly $\mathcal{NP}$-complete, PC-MSR $\in \mathcal{NP}$ and PC-MSR is harder than SPC-MSR, we can prove that PC-MSR is also strongly $\mathcal{NP}$-complete. The strongly $\mathcal{NP}$-completeness of PC-MSC, SR-MSR and SR-MSC problems can be proven in the same manner. PC-MSR (PC-MSC) problem is harder than SR-MSR (SR-MSC) problem since it includes nonlinear constraints. □

From Theorem 2.2.1, none of the four problems is shown to have an efficient algorithm. Thus, sensible heuristic algorithms are preferred to balance the system performance and computational complexity. Further, distributed algorithms are preferred in order to reduce the computation and sensing burden at the HG. Therefore, we will design distributed algorithms for JCRA based on partial dual decomposition as discussed below.

2.2.2 Distributed Algorithms for PC-MSR and PC-MSC Problems

Note that in the PC-MSR problem (2.9), the minimal rate requirements (2.10) and maximal transmit power requirements (2.12) are all service-wise. Further, the RAT power constraint (2.11) does not imply any coupling among the services. Thus, an intuitive idea is to decompose this problem service-wise by relaxing either (2.10) or (2.12) or both into the objective function [44] with the only constraint implying couplings among services being the channel usage constraints (2.1) - (2.6) and (2.7). For the PC-MSR problem, the major concern is to make the system feasible and thus relaxing individual rate requirements into the objective function is preferred. The partial dual function can be obtained by solving the following problem:

$$\max_{\{\mathbf{X},\mathbf{L}\}\in\mathcal{Q}} \mathcal{L}(\mathbf{X},\underline{\lambda}) = \sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{M}}\sum_{t\in\mathcal{T}} R^P(k,i,t)l(k,i,t)$$

$$+\sum_{k\in\mathcal{K}}\lambda_k(\sum_{i\in\mathcal{M}}\sum_{t\in\mathcal{T}} R^P(k,i,t)l(k,i,t) - R_k^{min}) \tag{2.16}$$

$$s.t.\ \ P(k,i,t) = P_t,\ \ \forall k\in\mathcal{K},\forall i\in\mathcal{M},\forall t\in\mathcal{T} \tag{2.17}$$

$$\sum_{i\in\mathcal{M}}\sum_{t\in\mathcal{T}} P(k,i,t)x(k,i,t) \le P_k^{max},\ \forall k\in\mathcal{K} \tag{2.18}$$

$$\text{and with channel usage constraints } (2.1)-(2.6)\text{and}(2.7),$$

where $\mathcal{Q}$ is the feasible set defined by 2.17, 2.18 and constraints (2.1) - (2.6) and (2.7). $\lambda_k \ge 0, \forall k \in \mathcal{K}$ is the Lagrangian multiplier and $\mathcal{L}(\mathbf{X},\underline{\lambda})$ is the Lagrangian function. Assume that $\{\mathbf{X}^*,\mathbf{L}^*\}$ is the optimal solution to problem (2.16), then the dual problem (master problem) can be defined as:

$$\min_{\underline{\lambda}\in\mathbb{R}_+^K} \mathcal{L}(\underline{\lambda}) = \sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{M}}\sum_{t\in\mathcal{T}} R^P(k,i,t)l^*(k,i,t)$$

$$+\sum_{k\in\mathcal{K}}\lambda_k(\sum_{i\in\mathcal{M}}\sum_{t\in\mathcal{T}} R^P(k,i,t)l^*(k,i,t) - R_k^{min}), \tag{2.19}$$

where $\underline{\lambda} = [\lambda_1...\lambda_k...\lambda_K]$ is the Lagrangian multiplier vector and $l^*(k,i,t)$ can be computed directly from $\mathbf{X}^*$. If strong duality holds for the problem (2.9) and its dual problem (2.19), the optimal solution to (2.9) can be identified from the feasible solutions of (2.19)

and additional conditions [45]. Unfortunately, problem (2.16) is still $\mathcal{NP}$-hard and furthermore no strong duality relation can be proven here. Thus, an optimal solution cannot be identified by following the standard dual decomposition methods. However, this partial dual decomposition structure is still helpful for developing heuristic distributed algorithms. Problem (2.16) can be decomposed into K local sub-problems and as for the k-th service:

$$\max_{\{\mathbf{x_k},\mathbf{l_k}\}} \sum_{i\in\mathcal{M}}\sum_{t\in\mathcal{T}} R^P(k,i,t)l(k,i,t)$$

$$+\lambda_k(\sum_{i\in\mathcal{M}}\sum_{t\in\mathcal{T}} R^P(k,i,t)l(k,i,t) - R_k^{min}) \tag{2.20}$$

$$s.t.\ \ P(k,i,t) = P_t,\ \ \forall i\in\mathcal{M}, \forall t\in\mathcal{T} \tag{2.21}$$

$$\sum_{i\in\mathcal{M}}\sum_{t\in\mathcal{T}} P(k,i,t)x(k,i,t) \le P_k^{max} \tag{2.22}$$

and with channel usage constraints $(2.1) - (2.6)$and(2.7),

where the k-th service here can only determine its own channel access strategy, i.e., deciding its channel access indicator vector $\mathbf{x_k}$. Meanwhile, the potential violation of the channel usage constraints (2.1) - (2.6) and (2.7) can be avoided with the help of the HG. Though the HG doesn't make centralized spectrum allocation decisions, it can obtain channel usage information from all the services and maintain them in a Global Spectrum Map (GSM). Then, the HG can pass the channel usage information of the other services from the GSM to the service k. The k-th service can therefore avoid collisions and calculate its own achievable data rates. A modified greedy algorithm is used here by the k-th service. The general idea of the modified greedy algorithm is that the individual service should maximize its data rate when global resources are sufficient. However, when the global resources are scarce, the individual service should aim to achieve its rate requirement while allowing possible sharing of the physical resources for other services. As mentioned above, GSM is only used as a simplified method to meet the channel usage constraints (2.1) - (2.6) and (2.7). It avoids exponentially enumerating all possible channel and RAT allocations of all the services but meanwhile introduces performance degradation to the proposed distributed algorithms. If strong duality holds for the primal and dual problems, i.e., (2.9)

and (2.19), then the optimal solution can be obtained through standard method [45], i.e., iterative improvement of feasible primal and dual solutions. However, the $\mathcal{NP}$-hardness of primal, dual and local problems prevents us from deriving such results. The solution, i.e., $\{\mathbf{X}', \mathbf{L}'\}$, from the K local sub-problems via the local modified greedy algorithm, may not even be feasible. Thus, we need to improve the current solution to make it feasible. We propose a pricing index, i.e., PI_k, $k \in \mathcal{K}$, to indicate the service k's priority to obtain resource assignment to meet its data rate requirement. A larger value of PI_k represents that service k has a larger resource insufficiency. If the current allocation is infeasible, the pricing index is updated as:

$$\begin{aligned} &PI_k(itr+1) = \\ &[PI_k(itr) - \alpha(itr)(\sum_{i\in\mathcal{M}}\sum_{t\in\mathcal{T}} R^P(k,i,t)l^{'}(k,i,t) - R_k^{min})]^+, \end{aligned} \tag{2.23}$$

where itr is the iteration index and $\alpha(itr) > 0$ is the step size in iteration itr. $[\cdot]^+$ denotes the projection onto the nonnegative orthant. The current pricing index is calculated based on both the services' achievable data rates in the current allocation, i.e, $\{\mathbf{X}', \mathbf{L}'\}$, and their pricing index in the last infeasible resource claim iteration. If the k-th service is not satisfied with its data rate requirement in the current resource claim iteration, its pricing index PI_k will be increased according to (2.23) in its next resource claim iteration $itr+1$. Otherwise, if the k-th service has been assigned much redundant resources, its PI_k will be decreased. Then, the services will claim resources in the descending order of their pricing indices. The pricing index update in (2.23) should not be considered the same as the Lagrangian multipliers update in standard dual decomposition method [44]. Here, it only determines the order of the services to claim resources in the next iteration. A threshold for the number of resource claim iterations, i.e., $MaxItr$, must be set in implementation and its impact is discussed in the numerical results section. If a feasible allocation cannot be found within the threshold, the system is claimed to be infeasible. The partial dual decomposition is used as a guideline to decompose the PC-MSR problem (2.9) as in Fig. 2.3 rather than exactly solve it. The HG only needs to update the pricing indices and maintains GSM. The sensing burden and part of the computation are distributed to local

services. Based on this decomposition, a 3-stage Distributed algorithm for the PC-MSR that we refer to as D-PCM can be developed as in Fig. 2.4. Since the first two stages of the distributed algorithm aims to support as many rate constrained services as possible, the PC-MSC problem can also be solved with this algorithm. The D-PCM algorithm can be explained in detail as follows:

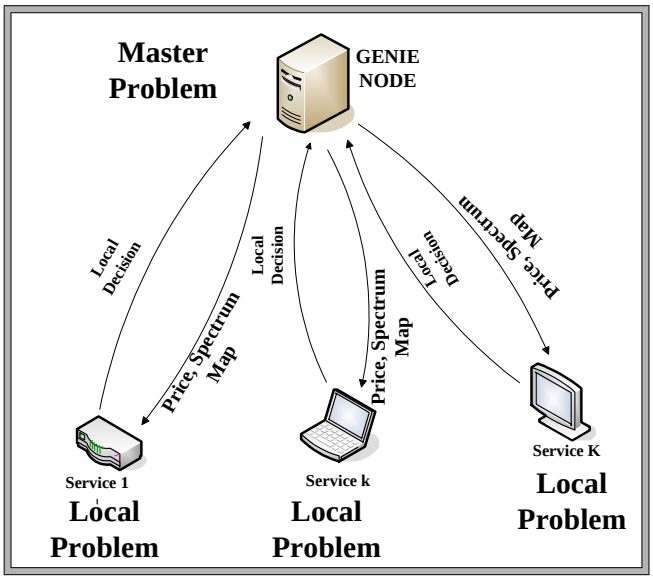


Figure 2.3: Decomposition of Resource Allocation Problems. PC-MSR problem can be decomposed as a master problem and $|\mathcal{K}|$ local problems. The sensing and part of computation burden is distributed to local services. The HG only needs to maintain and update the global spectrum information.

(i) **Priority Initialization** Initial values for pricing indices are computed in this stage. The initial value of pricing index reveals the gap between service's potential achievable data rates and its minimal data rate requirement. The k-th service's potential achievable data rates can be estimated by its average achievable data rates across all the channels, i.e., $R_k^E = \frac{\sum_{i\in\mathcal{M}} w_i \log(1+\frac{h_k^i P_k^{max}}{N_0 w_i})}{M}$. Then, its target rates to potential rates ratio is defined as $TP_k = \frac{R_k^{min}}{R_k^E}$. Pricing indices are initialized as $PI_k(0) = TP_k$, $\forall k \in \mathcal{K}$ and all the services are sorted in the decreasing order of the pricing indices. With a larger pricing index, the service has higher probability of not achieving minimal data rate requirement and thus should be assigned a higher resource claim priority. The HG disseminates the pricing indices to all the services and the services claim the resources in the order mentioned above.

(ii) **Min. Rate Allocation** All the MPR devices in a CDH are assumed to be able to

obtain their own channel gains and the HG node can disseminate the updated channel usage information (GSM) to them through the control channels. The channel usage information records the services which transmit in each channel and the corresponding RATs employed. Based on this information, the services can solve their local problem (2.20). The service with the highest pricing index gets the highest priority to solve the local problem and claims the resources from the HG. The other services solve their own local problems and claim the resources from the HG in the order of decreasing initial pricing indices.

For the k-th service, its local problem aims to maximize its data rate to satisfy the minimal data rate requirement subject to the transmit power and channel usage constraints. Meanwhile, the k-th service should also consider the other services' minimal data rate requirements and not demand too much resources. Hence, depending on the relationship between the number of services $|\mathcal{K}|$ and that of channels $|\mathcal{M}|$, the service can use a different strategy to solve its local problem. The k-th service starts from its best channel (the highest channel gain) and searches over all the available channels in the order of decreasing channel gains. On the channel c, if $|\mathcal{M}| < |\mathcal{K}|$, the k-th service starts from the lowest transmit power RAT and searches over all the RATs in the order of increasing transmit power. On the contrary, if $|\mathcal{M}| \geq |\mathcal{K}|$, the k-th services searches the RATs in the decreasing order of the transmit power. Given that the k-th service can achieve $R_k = \sum_{i=1}^{c-1} \sum_{t \in \mathcal{T}} R^P(k,i,t) l(k,i,t)$, if $R_k + R^P(k,c,t) l(k,c,t) \geq R_k^{min}$ and this possible allocation does not violate the channel usage constraints (2.1) - (2.6) and (2.7), the k-th service will claim this resource usage from the HG. Otherwise, it will check the $(t+1)$-th RAT that has next higher (or lower) transmit power. If $t = T$ and $R_k + R^P(k,c,t) l(k,c,t) < R_k^{min}$ that means no RATs in the channel c can provide the sufficient additional rates to satisfy the service's minimal data rate requirement, the k-th service will claim $R_k = R_k + R^P(k,c,t') l(k,c,t')$ where $t' \in \mathcal{T}$ is the feasible RAT with the largest transmit power for the k-th service in the c-th channel and then go to next channel $c+1$ with next lower channel gain. This search process continues until the k-th service goes through all its accessible channels and sends its channel and RATs usage claim and achievable data rate to the HG.

After the HG gets the k-th service's resource usage claim, it updates the GSM and

sends it to the next service in the priority list to claim the resources. When the HG gets the resource usage claims from all the $|\mathcal{K}|$ services, it can calculate the gaps between each service's achievable and minimal required rates, i.e., $\sum_{i\in\mathcal{M}}\sum_{t\in\mathcal{T}} R^P(k,i,t)l(k,i,t)-R_k^{min}$. If all the services achieve their rate requirements, the HG sends the permissions to all the services for their feasible claims. If some service cannot be satisfied with its rate requirement, the HG updates the pricing indices according to (2.23) and sorts them in the decreasing order that is also the new order for the services to claim the resources. The same procedures of local problem solving, resource usage claim and the pricing indices updates will be executed until the system is feasible or the number of resource claim iterations exceeds a pre-defined threshold $MaxItr$.

(iii) **Marginal Rate Allocation** If the resource allocation in the stage 2 can satisfy all the services' rate requirements, the services will access the residual spectrum resources in this stage. The services access the residual resources in the order that is finalized in stage 2. For each service, it accesses the channel and RAT pair which is available and provides it the largest additional data rate. One important rule of access is that services will not access the channels which are already shared by other services with legacy RATs in stage 2 since the potential sharing will possibly undermine the feasibility of the system.

The stage 1 of D-PCM requires information of $|\mathcal{K}|$ services over $|\mathcal{M}|$ channels to initialize the pricing indices which results in $\mathcal{O}(|\mathcal{K}||\mathcal{M}|)$ complexity. In stage 2, each service $k \in \mathcal{K}$ needs to check each RAT and channel pair in each iteration. Since the number of the resource claim iterations is chosen to be a small constant, the complexity in stage 2 is $\mathcal{O}(|\mathcal{K}||\mathcal{M}||\mathcal{T}|)$. Stage 3 only requires $\mathcal{O}(|\mathcal{K}||\mathcal{M}|)$. In total, the D-PCM is of complexity $\mathcal{O}(|\mathcal{K}||\mathcal{M}||\mathcal{T}|)$ which shows it is a fast algorithm with polynomial time complexity.

2.2.3 Distributed Algorithms for SR-MSR and SR-MSC Problems

As proven in section III, the PC-MSR, PC-MSC, SR-MSR and SR-MSC problems are all strongly $\mathcal{NP}$-complete which entail forbidding complexity to find their optimal solutions. The SR-MSR and SR-MSC problems can be readily solved by the branch and bound algorithm [42] which is of asymptotically exponential complexity and centralized. From (2.7) and (2.8), we can identify that the feasible sets of the PC-MSR and PC-MSC are

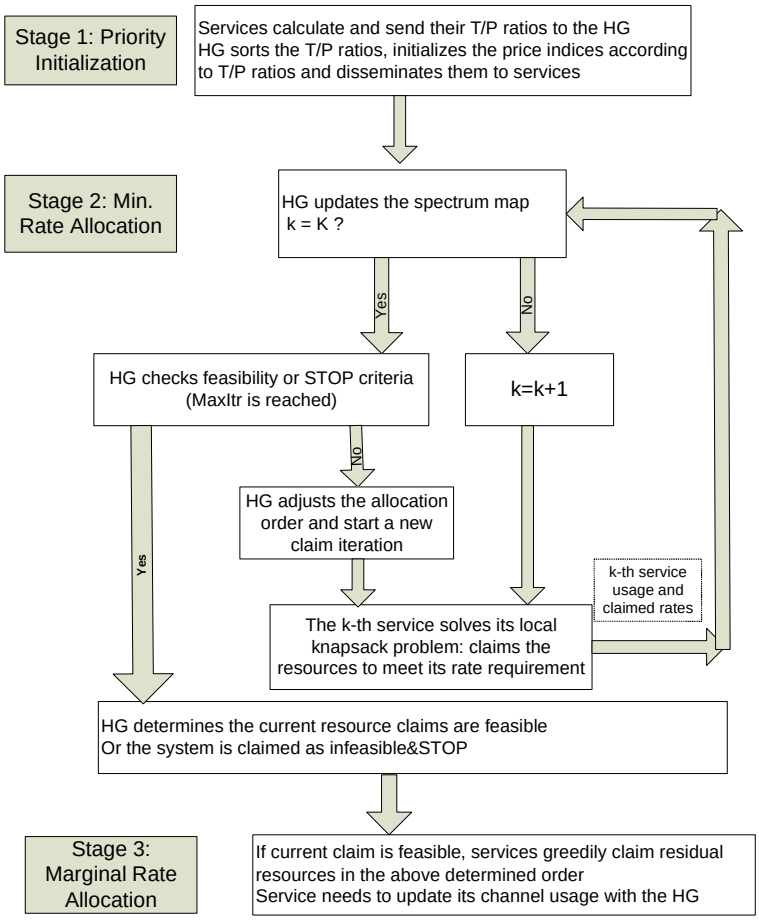


Figure 2.4: **D-PCM**: Distributed Algorithm for PC-MSR/MSC. The algorithm is designed based on partial dual decomposition. Pricing indices are used to indicate the gap between a service's achievable rate and its target rate. It's also an important module of **D-SRM**, the distributed algorithm for SR-MSR/MSC.

proper subsets of that of the SR-MSR and SR-MSC, respectively. Thus, the optimal solution of SR-MSR (SR-MSC) problem gives an upper bound for that of PC-MSR (PC-MSC) problem and can be used as a benchmark to evaluate the performance of D-PCM. As in case of PC-MSR and PC-MSC problems, a two stage Distributed algorithm for the SR-MSR and SR-MSC problems that we refer to as D-SRM is designed as follows.

In the first stage of the D-SRM, an initial channel and RAT allocation should be obtained in a distributed manner. The D-PCM designed in Fig. 2.4 has two attractive features: (i)D-PCM will output an initial channel and RAT allocation efficiently, and (ii)D-PCM is distributed where the majority of the computation and sensing burden is assigned to the services. Thus, the D-PCM becomes a proper subroutine candidate here. In the second stage, with the initial channel and RAT allocation, i.e., $\widetilde{\mathbf{X}}$, the HG solves a simple linear program (LP) to refine the resource allocation on the channels with legacy RATs to determine the exact portion of the physical resources each service will obtain. The legacy RATs refinement LP can be formulated as:

$$\max_{\mathbf{L}} \sum_{k\in\mathcal{K}} \sum_{i\in\mathcal{M}} \sum_{t\neq \mathrm{CR}} R^P(k,i,t) l(k,i,t) \tag{2.24}$$

$$s.t. \sum_{i\in\mathcal{M}} \sum_{t\neq \mathrm{CR}} R^P(k,i,t) l(k,i,t) \geq R_k^{res}, \ \ \forall k \in \mathcal{K} \tag{2.25}$$

$$\sum_{k\in\mathcal{K}} \sum_{t\neq \mathrm{CR}} l(k,i,t) = 1, \ \ \forall i \in \mathcal{M} \tag{2.26}$$

$$0 \leq l(k,i,t) \leq \widetilde{x}(k,i,t), \ \ \forall k \in \mathcal{K} \ \forall i \in \mathcal{M}, \tag{2.27}$$

where $R_k^{res} = R_k^{min} - \sum_{i\in\mathcal{M}} \sum_{t=\mathrm{CR}} R^P(k,i,t) l(k,i,t)$ is the residual part of the rate constraint which should be satisfied with legacy RATs and part of the rate constraint may have been satisfied by CR RAT in stage 1. The initial allocation $\widetilde{\mathbf{X}}$ results in an feasible allocation. Thus, no power constraint is required here. Meanwhile, only the services assigned with legacy RAT on a channel in $\widetilde{\mathbf{X}}$ can be further considered for allocation on that channel here as in (2.27). The above LP can be efficiently solved by the HG since it readily obtains the necessary information to solve the LP in the first stage and the problem size is usually small in a digital home.

2.2.4 Power Control in a Cognitive Digital Home

Besides channel and RAT allocation discussed above, power control is another important issue for managing resources in a CDH. However, as we have mentioned in Theorem 1, the joint channel and RAT allocation itself is already $\mathcal{NP}$-complete. Adding power control as an additional degree of control, i.e., joint channel, RAT and power allocation, will make the problem even more intractable. To overcome the intractability, we propose power control in a CDH as an additional functionality which can improve the system performance based on the allocation results from D-PCM or D-SRM algorithms. Given the channel and RAT allocation result obtained from D-PCM or D-SRM, i.e, $\mathbf{X}^*$ and $\mathbf{L}^*$, the power control can be implemented locally by each service as follows:

$$\max_{\mathbf{p_k}} \sum_{i\in\mathcal{M}}\sum_{t\in\mathcal{T}} R^P(k,i,t)l^*(k,i,t) \tag{2.28}$$

$$s.t. \quad \sum_{i\in\mathcal{M}}\sum_{t\in\mathcal{T}} P(k,i,t)x^*(k,i,t) \leq P_k^{max}, \ \ \forall k \in \mathcal{K} \tag{2.29}$$

$$0 \leq P(k,i,t) \leq P_t^{max}, \ \ \forall i \in \mathcal{M}, \forall t \in \mathcal{T}, \tag{2.30}$$

where each service tries to locally maximize its achieved data rates given the channel and RAT assignment. As in equation (2.11), the joint channel and RAT assignment is based on some fixed nominal transmit power value P_t. Subsequently, each service adjusts its transmission power $P(k,i,t)$ for each channel and RAT here, corresponding to the constraints (2.29) and (2.30), respectively. For fixed constraints P_k^{max} (service) and P_t^{max} (RAT), the above problem (2.28) is a convex program and thus can be solved efficiently. Moreover, in a typical CDH environment, given the relatively short range of communications, the dynamic range of power control will typically be less than in a macrocell setting.

2.2.5 Admission Control in a Cognitive Digital Home

As emphasized in previous sections, the JCRA problems, e.g., PC-MSR and SR-MSR, are all $\mathcal{NP}$-hard and thus their feasibility is hard to achieve. This is the reason why the D-PCM and D-SRM algorithms weigh more importance on the feasibility of the problems besides the total sum rates. The infeasibility of the systems when the physical resources

are limited requires the CDH system to employ an admission control scheme. From the system aspect, the admission control scheme should help the HG efficiently determine which services should be accepted for data services. From the optimization aspect, the admission control scheme should serve as a feasibility seeking algorithm to reduce the constraint set size so that the reduced optimization problem becomes feasible.

In general, seeking feasibility of an optimization problem can be formulated as the minimum-cardinality IIS (Irreducible Infeasible Set) set-covering problem that is known to be $\mathcal{NP}$-hard [46]. Heuristic methods such as in [47] [48] have been developed for it, but the current state of the art mainly targets linear systems [46]. The JCRA problems in a CDH are in general nonlinear. Therefore, specific admission control methods should be developed to address the infeasibility issue. The pricing indices PI_k in (2.23) used in resource allocation provide a nice indication for admission control since they represent the gap between each service's target data rate and its achievable data rate in the resource claim iteration. One example of a service rejection scheme based on the pricing indices first we consider is the Average Pricing Index (API) scheme. The API scheme rejects the service with the largest average pricing index, i.e., $k = \arg\max_{i\in\mathcal{K}} \overline{PI_i} = \arg\max_{i\in\mathcal{K}} \frac{\sum_{itr=1}^{MaxItr} PI_i(itr)}{MaxItr}$, where $MaxItr$ is the maximum number of resource claim iterations before the system is determined infeasible. The API scheme will reject the services until the remaining system is determined feasible by either D-PCM or D-SRM or all the services are rejected. The API scheme can be appended to the end of the D-PCM or D-SRM algorithms as shown in Fig. 1.5 where the power control block is also shown. The performance of the API scheme will be discussed next.

2.3 Numerical Results

We first illustrate our results in a simple setting that exemplifies a CDH environment with Wi-Fi, bluetooth and cognitive RATs which use fixed transmit power on each channel for each service. The spectral environment parameters used in the MATLAB-based simulation are shown in Table 1.1. Due to the directional nature of the wireless links in 60GHz bands, the link gains of each service in 60GHz bands can be described by a static path loss channel

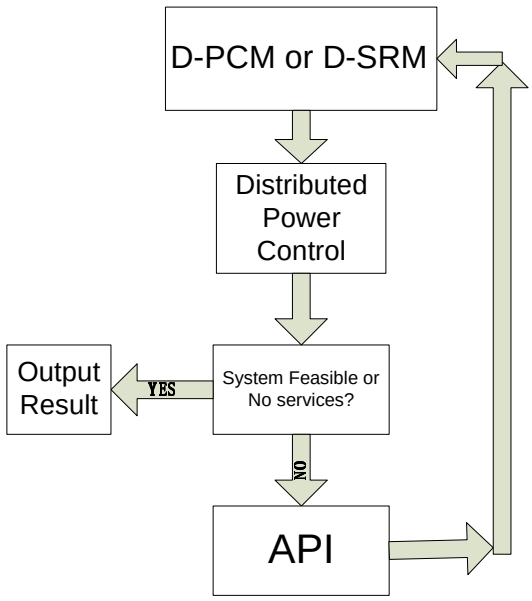


Figure 2.5: Distributed Power Control Scheme and Admission Control Scheme(API). **D-PCM** and **D-SRM** are the distributed algorithms for PC-MSR(MSC) and SR-MSR(MSC), respectively. Power control is completed locally by each service based on a feasible joint channel and RAT assignment. The pricing index reveals the gap between a service's achievable rates and target rates and thus is a proper indication on the service rejection candidate.

model as

$$\frac{P_r}{P_t} = (\frac{c}{4\pi d_0 f})^2 \times (\frac{d_0}{d})^\gamma, \tag{2.31}$$

where P_r and P_t are received and transmit powers. d is the link distance between the transmitter and receiver. $d_0 = 1$m is the indoor reference distance. f and c are the channel center frequency and speed of light, respectively. γ is the indoor path loss coefficient that is chosen to be $\gamma = 1.8$ for line of sight (LOS) links in 60GHz bands [49] [50]. For the 2.4GHz and 5GHz bands, the Rayleigh fading model is used to calculate the link gains of each service as

$$\frac{P_r}{P_t} = (\frac{c}{4\pi d_0 f})^2 \times (\frac{d_0}{d})^\gamma \times X_g, \tag{2.32}$$

where the path loss coefficient is chosen to be $\gamma = 3$ [51] and X_g is a random variable with Rayleigh distribution with zero mean and unit variance.

Table 2.1: Exemplary Resource Allocation in a CDH

Available Channels in the Allocation							
Channel Index	1	2	3	4	5	6	7
Center Frequency(GHz)	2.412	2.437	2.462	2.484	57.1	57.2	57.3
Bandwidth(MHz)	20	20	20	20	100	100	100
Service Requirements							
	Link A	Link B	Link C	Link D			
Required Data Rate(Mbps)	1	10	25	60			
Power Constraint(mW)	10	60	100	300			
Channel Allocation Result							
	Link A	Link B	Link C	Link D			
Wi-Fi(30mW)		4	3, 4	1, 2, 3			
CR(60mW)				5, 6, 7			
Bluetooth(10mW)	1, 2, 3, 4						
Achieved Data Rate(Mbps)	1	13	27	76			

The first numerical simulation is for an exemplary scenario where a CDH has service provision devices such as a game console, game controller, laptop and High Definition TV (HDTV) as shown in Fig. 2.1. The link A (game controller) is assumed to be a legacy bluetooth link at 1Mbps and we consider channel and RAT allocation for the link B (web surfing), C (game display) and D (wireless HDTV). The available channels and their bandwidths are shown in Table 2.3. We assume Wi-Fi and bluetooth can operate only on the first 4 channels in the 2.4GHz band while the CRs can operate on all available channels. The degradation of Wi-Fi due to bluetooth is modeled via a degradation factor $\alpha_t = 0.9$ [37]. The allocation results also shown in Table 2.3 verify that our D-PCM algorithm efficiently utilizes the integrated spectrum and RATs resources in the CDH. The services with high data rate requirements which usually also have high transmit power margin, e.g., the wireless HDTV, are pushed to higher frequency bands with CR RAT. The services with low transmit power margin and moderate rate demands, e.g., web surfing, are allocated to the ISM bands with Wi-Fi legacy RATs. The sharing nature of some legacy RATs such as Wi-Fi can accommodate more such services to meet their data rate requirements than the orthogonalized CR RAT while CR RAT can provide more flexibility to support high rate craving services.

In the following, we will investigate the performance of D-PCM and D-SRM algorithms. The performance of D-PCM algorithm will be compared with the upper bound of its optimal solution generated by the SR-MSR problem. It will also be compared with that of

the D-SRM algorithm to study the gain due to the relaxation of the legacy RATs access. The channels of 20MHz wide each which are centered at 5.18, 5.2, 5.22, 5.24, 5.26, 5.28, 5.3, 5.32, 5.745, 5.765, 5.785 and 5.805GHz are used here. In the first place, we will investigate the performance of D-PCM and D-SRM as the function of the number of services $|\mathcal{K}|$ and the number of channels $|\mathcal{M}|$. Two metrics, i.e., the sum rates of all the services and the system feasibility rate, are compared in the following figures. If all the services' rate requirements are satisfied, the system is claimed to be feasible. The sum rate is defined as the sum of all the services' rates if the system is feasible. Here, all the services are of the same rate requirements, i.e., $R_k^{min} = 10$ Mbps for each $k \in \mathcal{K}$. Their link distances are uniformly generated over [5, 10]m. The services also have the same individual transmit power constraint, i.e., $P_k^{max} = 200$ mW for $\forall k \in \mathcal{K}$. Two RATs, i.e., Wi-Fi with typical transmit power 30mW and CR with typical transmit power 60mW, are employed in the CDH where Wi-Fi RAT is allowed in the first 3 channels and the CR RAT can possibly be used in all the channels. For each choice of number of services and number of channels, the results (the actual allocation for each of 100 trials is not shown) shown are averaged over 100 trials.

In Fig. 2.6, the performance of D-PCM in terms of the system feasibility rate is compared with that of D-SRM. The system feasibility rate is defined as the percentage of the feasible trials out of the total 100 trials. The D-PCM algorithm almost obtains the optimal solution in Fig. 2.6. As the number of services increases, its performance will degrade as expected. The D-SRM algorithm which is based on D-PCM algorithm obtains the optimal solution in this case. The system feasibility degradation of D-PCM shown in the figure actually reflects the limited controllability of legacy RATs in the PC-MSR problem.

The performance of D-PCM and D-SRM in terms of the sum rate are compared in Fig. 2.7. Both D-PCM and D-SRM have less than 25% performance degradation relative to the optimal solution of the SR-MSR problem. This also indicates that the gap between the performance of D-PCM and the optimal solution of PC-MSR is lower than 25%. As shown in the figure, when $|\mathcal{K}| > |\mathcal{M}|$, there is a sum rate drop for both D-PCM and D-SRM. This is due to the RAT access strategy used in the two algorithms. As mentioned above,

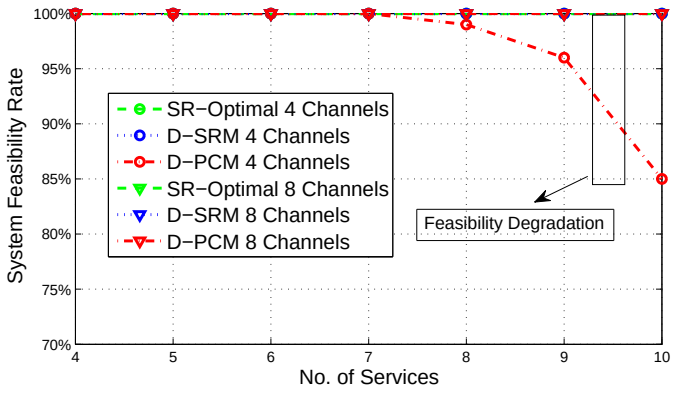


Figure 2.6: System Feasibility Rate. The services have same individual rate requirement of 10Mbps.

when $|\mathcal{K}| \leq |\mathcal{M}|$, both the algorithms push the services to aggressively access the RATs to maximize their rates. However, when the physical resource is scarce, the algorithms encourage the services to conservatively share spectrum with legacy RATs which is important to ensure the system feasibility. As a result, the sum rate achieved decreases and as a trade off, the system feasibility rate increases. The performance improvement of D-SRM over D-PCM is greater when the system approaches infeasibility.

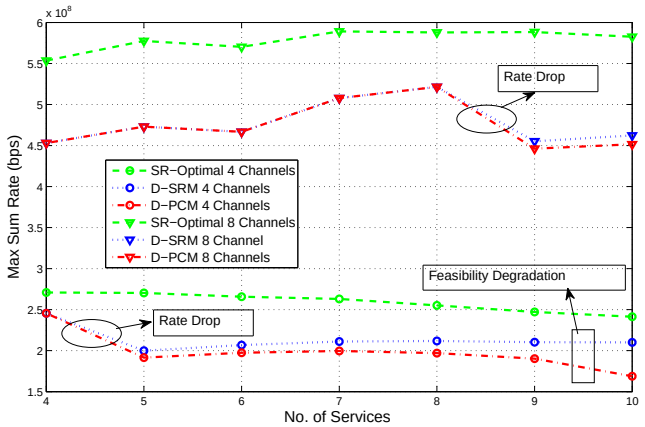


Figure 2.7: Max Sum Rate. The services have same individual rate requirement of 10Mbps.

Pricing indices are used by the HG to determine the services' priority for claiming resources. They are updated according to (2.23) to find a possible feasible allocation when the current allocation is infeasible. However, the number of resource claim iterations is limited by the iteration threshold $MaxItr$. In Fig. 2.8, the impact of $MaxItr$ is shown in

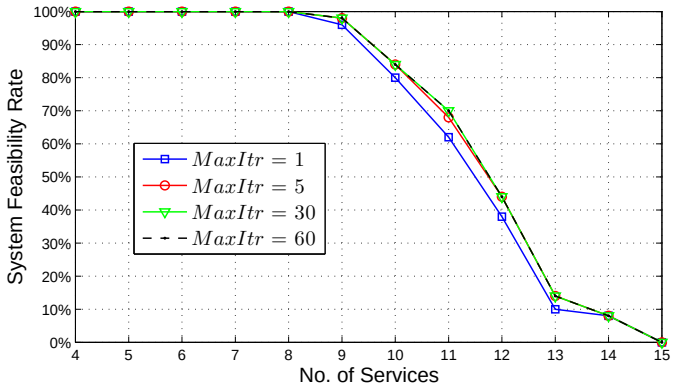


Figure 2.8: Convergence of resource claim iterations (D-PCM). The services have same individual rate requirement of 10Mbps. The effect of the iteration threshold $MaxItr$ is shown for the D-PCM algorithm implementation.

terms of system feasibility for D-PCM. 4 channels are considered. The results show that when $MaxItr$ increases from 1 to 5, the performance gain is largest. The reason for this observation is due to our sensible design of heuristic algorithms for exploring the feasibility of system and even within a small number of $MaxItr$, the proposed algorithm D-PCM could well detect the nature of the system feasibility. Our extensive experiments indicate that $MaxItr = 5$ is a reasonable choice for balancing the computation and accuracy.

In the following, the performance of both the D-PCM and D-SRM are evaluated in a more general setting. Rather than identical rate requirement for each service, three classes of services with differing rate requirements exist, i.e. low-rate services, medium-rate services and high-rate services, are considered. Also, the number of services approaches to 20 so that the limiting behaviors of algorithms can be shown. 6 channels are considered while the fraction of each class of service can vary.

In Fig. 2.3 and Fig. 2.3, the performance of D-SRM with power control is shown. To illustrate the performance improvement introduced by power control, the individual maximal transmission power is set to $60mW$ for each service while other settings stay the same. The services can use the proposed power control scheme to efficiently assign its transmission power so that channel diversity can be explored. As a result, D-SRM with power control has obvious improvement over that without power control in terms of system feasibility and sum rate both.

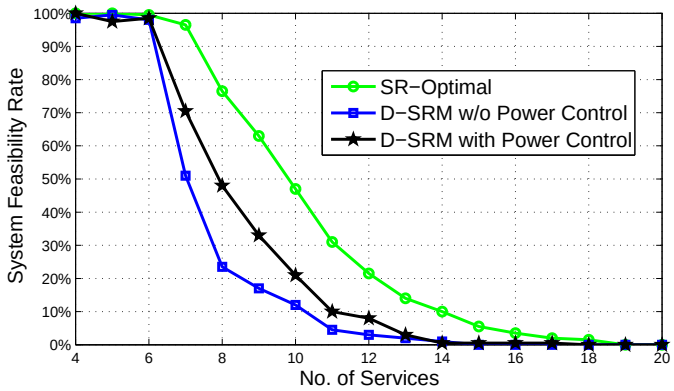


Figure 2.9: Power Control (System Feasibility Rate). Results are shown for different service distribution. 6 channels are used. Each service's transmission power is within 60mW. $P_{wifi}^{max} = 35mW$ and $P_{CR}^{max} = 65mW$. For each problem instance, the composition of the three classes of services is generated uniformly. Low rate: [5,15]Mbps. Medium rate: [15,25]Mbps. High rate: [30,40]Mbps

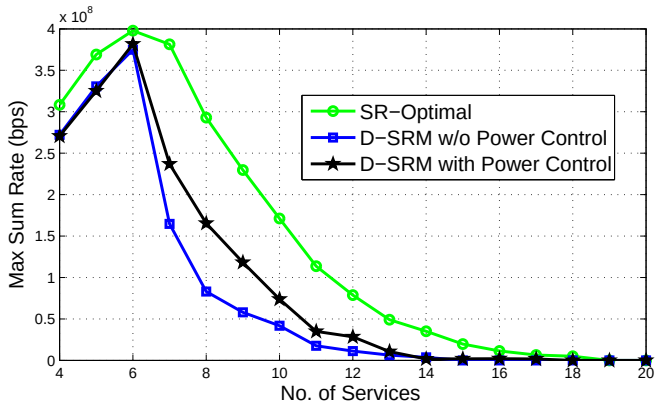


Figure 2.10: Power Control (Sum Rate). Results are shown for different service distribution. 6 channels are used. Each service's transmission power is within 60mW. $P_{wifi}^{max} = 35mW$ and $P_{CR}^{max} = 65mW$. For each problem instance, the composition of the three classes of services is generated uniformly. Low rate: [5,15]Mbps. Medium rate: [15,25]Mbps. High rate: [30,40]Mbps

When the system is determined infeasible, an admission control scheme is necessary as stated in section 2.2.5. In Fig. 2.3, 6 channels are used for simulation. The number of services varies from 4 to 10. The same three classes of services described earlier are considered. The SR channel access model is used. The modified D-SRM algorithm with the API admission control scheme is compared with D-SRM and the optimal solution obtained from the branch and bound method [42]. The simulation result clearly shows that the admission control scheme will help the CDH support more services than otherwise.

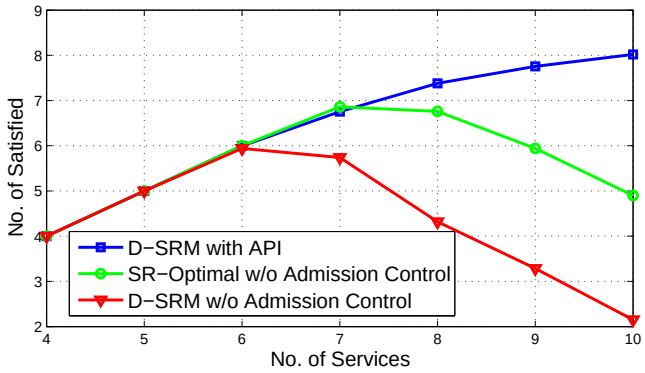


Figure 2.11: Admission Control Scheme. Results are shown for different service distribution. 6 channels are used. For each problem instance, the composition of the three classes of services is generated uniformly. Low rate: [5,15]Mbps. Medium rate: [15,25]Mbps. High rate: [30,40]Mbps. Each service's transmission power is within 200mW.

Chapter 3

Effect of End-User Behaviors on Wireless Random Access

3.1 Related Work

As mentioned in chapter 1, the advances in smart and programmable radio devices have provided the end users with a greater degree of control [52–54]. As a result, the understanding of end users' real-life decisions are becoming of paramount importance in designing and managing current and future wireless networks. Prospect Theory (PT) is well known to be able to explain real-life decision making and thus becomes a candidate tool to address this need. Though historically, PT has been developed based on monetary transactions, it has been successfully applied to various fields as mentioned in the above section, such as [26–29]. Game theory has also been a powerful tool to describe the interactions among end users in wireless networks as some exemplary works can be found in [17–21,55]. The game theoretical approach heavily relies on the precept that users follow Expected Utility Theory (EUT). In [56], the authors have provided basic characterizations of non-cooperative games where players may follow PT for decision making.

As indicated in chapter 1, research is needed to identify how resource allocation mechanisms impact the value of that resource to users and conversely, how end-user actions impact resource allocation. The end users' subjective perception over the received service and their behaviors have more and more significant influence on the design, operation and performance of communication networks and systems [57]. The authors in [58] explore the possible usage of psychology in the field of autonomous networks and systems. They claim that the end users' behaviors and preferences can have direct impact over the operation and design of the such systems. They used PT as a possible candidate to model end users' preferences. In [59], the authors identify the importance of the end users' behaviors and preferences and proposed a "local community networking system with ad hoc networking

technology". Moreover, they used Prospect Theory to model the end users' behaviors in such a relaying network environment.

Though the above works have conceptually mentioned the need for Prospect Theory in communication networks and systems, a more in-depth technical and quantitative study on this topic is needed. Therefore, we select an exemplary wireless random access model as a starting point to identify the impact of end users' decisions on the wireless networks with the help of PT. The work in [18] is one of first works that applies game theory to wireless random access and power control. Specifically, the authors applied game theory to a simplified Aloha system. In [60], a more detailed game-theoretical study was provided on an Aloha system. The authors in [61] have investigated the probabilities of retransmission in a distributed manner under both cooperative model and non-cooperative game model in a slotted Aloha system. In [62], the authors have performed an extensive study on the end users' transmission probabilities at the equilibrium of a random access network considering end users' throughput rewards, energy costs and delay penalties. Non-cooperative game theory has been applied there and the case of both selfish and malicious end users were studied.

3.2 Background: Prospect Theory

Expected Utility Theory (EUT) [24] has been a fundamental part of modern economics. It provides an approach to valuate a prospect L [25], i.e., a contract that will yield M different outcomes o_i, $i = 1, ..., M$ and each outcome occurs with probability p_i, $\forall i = 1, .., M$ where $\sum_{i=1,...,M} p_i = 1$. EUT determines that the prospect is valuated as

$$u^{EUT}(L) = \sum_{i=1,...,M} p_i v^{EUT}(o_i), \tag{3.1}$$

i.e., the expected value of all possible outcomes. $v^{EUT}(\cdot)$ is a value function of the outcomes and it is often assumed to be concave in EUT. However, in Prospect Theory (PT) [25] that was postulated by Kahneman and Tversky, extensive experiments and measurements

suggest that in decision-making in real life, the prospect L is valuated as

$$u^{PT}(L) = \sum_{i=1,\ldots,M} w(p_i) v^{PT}(o_i). \tag{3.2}$$

This valuation is significantly different from EUT in the following two ways.

1. Probability Weighting Effect: It is revealed in PT that people use their subjective probabilities $w(p_i)$ rather than objective probabilities p_i to weigh the values of possible outcomes. Moreover, people tend to over-weigh low probability outcomes and under-weigh moderate and high probability outcomes. Based on the experimental results, an original form for the probability weighting function was proposed in [63]. While there have been several efforts to identify appropriate probability weighting functions, in this work, we will use the one identified by Prelec [64] that captures the over-weighting and under-weighting of probabilistic outcomes as follows (see Fig. 1):

$$w(p) = \exp(-(-\ln p)^{\alpha}),\ 0 < \alpha \leq 1, \tag{3.3}$$

   where α is the parameter which reveals how a person's subjective evaluation distorts the objective probability and a smaller α describes a more curved probability weighting function. From Fig. 1, we can see that the probability weighting function has several features [64]: (1) It is asymmetrically reflected at a point, i.e., $1/e$, where $w(1/e) = 1/e$; (2) It is concave if $0 \leq p < 1/e$ and convex if $1/e \leq p \leq 1$; (3) $w(p) > p$ if $0 \leq p < 1/e$ and $w(p) \leq p$ if $1/e \leq p \leq 1$.

2. Framing Effect: PT [25] states that in decision-making in real life, the value of an outcome is determined by considering the relative gains or losses regarding a reference point. PT also proposes that the value function should be a concave function of gains and a convex function of losses with the convex part usually having a steeper slope. In other words [25], "losses looms larger than gains." The framing effects can be demonstrated in Fig. 3.2.

It has been consistently observed that EUT fails to interpret people's real-life decisions that could be well explained by PT, e.g., the famous Allais' paradox. In [25], the authors

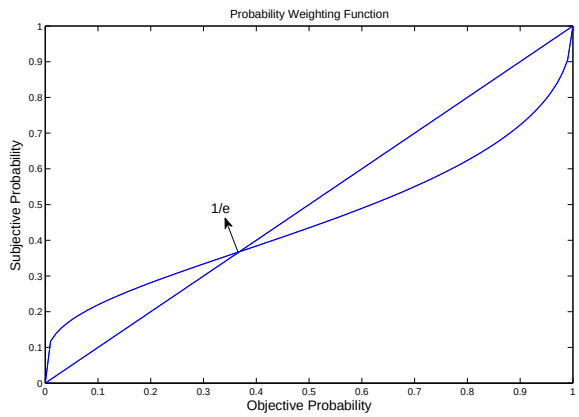


Figure 3.1: Probability Weighting Function. The curve shows the probability weighting effects when $\alpha = 0.5$. The straight line represents the objective probability.

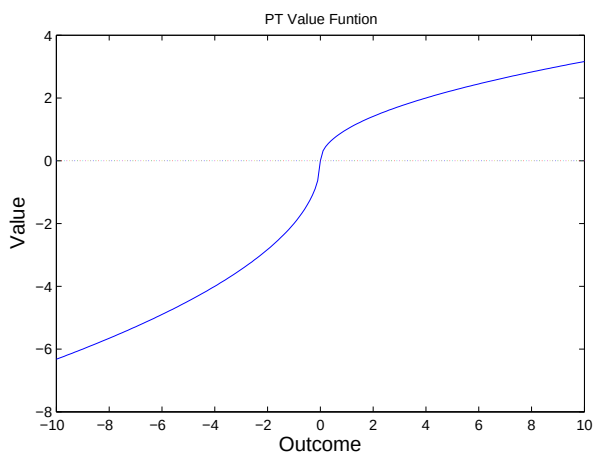


Figure 3.2: Framing Effects in Prospect Theory. A generic value function is shown about reference point 0.

Table 3.1: An Example of EUT Violation

Problem \ Prospect	A	B
1	\$2500 *with probability* 0.33 \$2400 *with probability* 0.66 \$0 *with probability* 0.01	\$2400 *with certainty*
2	\$2500 *with probability* 0.33 \$0 *with probability* 0.67	\$2400 *with probability* 0.34 \$0 *with probability* 0.66

provided a variation of the Allais' paradox as in Table 3.1. There were two problems in the experiment and for each problem, the respondents were asked to choose between two prospects (A or B). For example, in Table 3.1, the respondent had two prospects in problem 1. If she chose A, she would win 2500 dollars with probability 0.33 or 2400 dollars with probability 0.66 or nothing with probability 0.01. If she chose B, she would win 2400 dollars for sure. In [25], it was found that a majority of the respondents (61 *per cent*) chose B for problem 1 and A for problem 2. According to EUT, a respondent would valuate a prospect, e.g., problem 1A, as the expectation of all the prospect's outcomes, e.g., $0.33v^{EUT}(2500) + 0.66v^{EUT}(2400) + 0.01v^{EUT}(0)$. Thus, a preference of 1B over 1A implies $0.33v^{EUT}(2500) + 0.66v^{EUT}(2400) + 0.01v^{EUT}(0) < v^{EUT}(2400)$ that is equivalent to $0.34v^{EUT}(2400) > 0.33v^{EUT}(2500)$. Meanwhile, the choice of 2A over 2B implies $0.34v^{EUT}(2400) < 0.33v^{EUT}(2500)$. These two results produce a paradox. This observation as well as numerous other ones are used in [29] to illustrate the situations where EUT fails to accurately describe people's real-life decisions. However, PT can successfully explain the decisions the respondents made in the above experiment. Given zero is the reference point, $\alpha = 0.5$ and $v^{PT}(\cdot)$ is linear, it can be easily verified that $w(0.33)v^{PT}(2500) + w(0.66)v^{PT}(2400) + w(0.01)v^{PT}(0) < w(1)v^{PT}(2400)$ (1B over 1A) and $w(0.34)v^{PT}(2400) < w(0.33)v^{PT}(2500)$ (2A over 2B) are established simultaneously. As a special case of the framing effect shown in Fig. 3.2, in this work we apply PT by assuming a linear value function towards the outcomes of communication activities, i.e., $v^{PT}(x) = ax$ where $a \geq 0$ is a constant. Note that, x is the gain or loss relative to fixed reference point zero, i.e., values are framed about the reference point zero.

3.3 A Wireless Random Access Game

In this work, we consider a time slotted wireless random access network with $N = |\mathcal{N}|$ selfish players accessing a single access point. Each player is assumed to have a saturated packet queue and able to transmit one packet in a time slot [62]. Given any time slot, there is a subset $\mathcal{J} \subseteq \mathcal{N}$ of players that transmit packets to the base station simultaneously while players in $\mathcal{J}^c \subseteq \mathcal{N}$ decide to wait. For player $i \in \mathcal{J}$, let $p_{i|\mathcal{J}}$ be the probability that the base station captures her packet. Note that, player i's transmission probability is denoted as p_i. For a successful packet delivery, a player obtains a unit reward $c_i \geq 0$ while incurring a unit energy cost $e_i \geq 0$. However, if a player $i \in \mathcal{J}$ fails to deliver her packet to the base station, she incurs a unit energy cost $e_i \geq 0$ and a unit delay penalty $d_i \geq 0$. If she decides to not transmit (wait), i.e., $i \in \mathcal{J}^c$, she also incurs a unit delay penalty $d_i \geq 0$. Note that, we implicitly assume that c_i, e_i and d_i are chosen such that they are in uniformly the same units.

All the players have the same pure strategy set, i.e., $A_i = \{t, nt\}$, $i \in \mathcal{N}$, where the strategy t denotes transmission and nt denotes wait (no transmission). The pure strategy profile of all N players in a time slot can be represented as a vector $\mathbf{a} = [a_i]_{\forall i \in \mathcal{N}} \in \mathbf{A}$ where $\mathbf{A} = A_1 \times A_2 \times ... \times A_N$ is the set of all possible pure strategy profiles. For any given $\mathbf{a} \in \mathbf{A}$, let $\mathcal{J}(\mathbf{a}) \subseteq \mathcal{N}$ denote the set of players who transmit. Each transmission generates two possible outcomes with probabilities for each player $i \in \mathcal{J}(\mathbf{a})$, i.e., a successful packet delivery with probability $p_{i|\mathcal{J}(\mathbf{a})} \in [0, 1]$ or a packet delivery failure with probability $1 - p_{i|\mathcal{J}(\mathbf{a})}$. Note that, the successful packet delivery probability $p_{i|\mathcal{J}(\mathbf{a})}$ is actually determined by the underlying channel model, e.g., path loss, AWGN and rayleigh. The exact channel model adopted is not crucial in this model and thus $p_{i|\mathcal{J}(\mathbf{a})}$ is assumed to be available for each player. If the player i does not transmit, i.e., $i \in \mathcal{J}(\mathbf{a})^c$, she gets a delay penalty d_i. Thus, if we assume a linear value function, then a player valuates the possible outcomes associated with an arbitrary pure strategy profile as follows:

$$v_{i|\mathbf{a}} = \begin{cases} p_{i|\mathcal{J}(\mathbf{a})}(c_i - e_i) + (1 - p_{i|\mathcal{J}(\mathbf{a})})(-e_i - d_i) & \text{if } a_i = t \\ -d_i & \text{if } a_i = nt. \end{cases} \tag{3.4}$$

Under this setting, we will consider random access games with 2 types of players: (a) players who follow the precepts of EUT and (b) players who follow the precepts of PT. We will refer to these players as EUT players and PT players, respectively. Further, depending on the player composition, two types of wireless random access games are possible: (1) homogeneous game and (2) heterogeneous game. A homogeneous game consists of either only EUT players or only PT players. The heterogeneous games must simultaneously have both PT and EUT players, i.e., some players are PT players and others are EUT players. Furthermore, under linear value function and framing about 0, the PT and EUT players have identical value functions over a pure strategy profile, i.e., $v_{i|\mathbf{a}}^{EUT} = v_{i|\mathbf{a}}^{PT} = v_{i|\mathbf{a}}$. We assume each player may adopt a mixed strategy, i.e., player i will transmit a packet in a time slot with probability $p_i \in [0, 1]$ and wait with probability $1 - p_i$. A mixed strategy vector of all players can be represented as $\mathbf{p} = [p_1, p_2, ..., p_N]$ where p_i is the i-th player's transmission probability. An EUT player's utility can be represented as,

$$u_i^{EUT}(\mathbf{p}) = \sum_{\mathbf{a}\in\mathbf{A}} \left(\prod_{j\in\mathcal{J}(\mathbf{a})} p_j \prod_{k\notin\mathcal{J}(\mathbf{a})} (1 - p_k) v_{i|\mathbf{a}} \right), \tag{3.5}$$

where $\mathcal{J}(\mathbf{a})$ is the set of players who transmit in the time slot, i.e., $a_j = t$, $\forall j \in \mathcal{J}(\mathbf{a})$. Note that, the probability that a specific pure strategy profile $\mathbf{a}$ is chosen is $\prod_{j\in\mathcal{J}(\mathbf{a})} p_j \prod_{k\notin\mathcal{J}(\mathbf{a})}(1-p_k)$ and $v_{i|\mathbf{a}}$ is the corresponding value for player i as defined in (3.4).

A PT player knows that $\mathbf{a}$ will occur with probability $\prod_{j\in\mathcal{J}(\mathbf{a})} p_j \prod_{k\notin\mathcal{J}(\mathbf{a})}(1-p_k)$. However, she cannot objectively evaluate this probability and this makes her utility function fundamentally different from that of an EUT player as discussed in section 3.2. A PT player's subjective utility can be formally defined as:

$$u_i^{PT}(\mathbf{p}) = \sum_{\mathbf{a}_1\in\mathbf{A}, a_{1i}=t} SP(\mathbf{a}_1) v_{i|\mathbf{a}_1} + \sum_{\mathbf{a}_2\in\mathbf{A}, a_{2i}=nt} SP(\mathbf{a}_2) v_{i|\mathbf{a}_2}. \tag{3.6}$$

Note in the above, the player i subjectively determines that $\mathbf{a}_1$ will be chosen with probability $SP(\mathbf{a}_1) = p_i w_i \left(\prod_{j\in\mathcal{J}(\mathbf{a}_1)\setminus\{i\}} p_j \prod_{k\in\mathcal{J}^c(\mathbf{a}_1)}(1 - p_k) \right)$, where $\mathbf{a}_1 \in \mathbf{A}$ denotes the pure strategy profiles where the i-th player transmits, i.e., $a_{1i} = t$, and $w_i(\cdot)$ is the i-th player's probability weighting function. Similarly, she weighs outcomes of $\mathbf{a}_2$ with

$SP(\mathbf{a}_2) = (1-p_i)w_i\left(\prod_{j\in\mathcal{J}(\mathbf{a}_2)} p_j \prod_{k\in\mathcal{J}^c(\mathbf{a}_2)\setminus\{i\}}(1-p_k)\right)$ where $\mathbf{a}_2 \in \mathbf{A}$ denotes the pure strategy profiles where she chooses to wait, i.e., $a_{2i} = nt$. Note that, we assume a PT player only nonlinearly transforms other players' strategy probabilities rather than her own since a person is able to choose her own strategy probability objectively.

For the i-th player, she plays the random access game by solving an optimization problem as follows:

$$\max_{p_i} u_i(p_i, p_{-i}), \tag{3.7}$$

where $u_i(\cdot)$ can be either EUT utility (3.5) or PT utility (3.6) and p_{-i} represents the collection of all other players' transmission probabilities. A list of important notations is list in Table 3.2.

3.4 A 2-Player Heterogeneous Wireless Random Access Game

In our earlier work [32], a 2-player homogeneous wireless random access game with linear value function was studied using a simple 0-1 random collision channel model for random access. Here, we investigate a 2-player heterogeneous wireless random access game with linear value function for a general random access channel. We extensively study the differences between a 2-player homogeneous EUT game and a 2-player heterogeneous game . In the following, we use i to denote the player we refer to and j to denote her opponent. According to (3.5) and (3.6), the utility functions of an EUT and a PT player can be specified as follows, for $i = 1, 2$:

$$u_i^{EUT}(\mathbf{p}) = p_i p_j v_{i|\{t,t\}} + p_i(1-p_j)v_{i|\{t,nt\}} + (1-p_i)(-d_i), \tag{3.8}$$

$$u_i^{PT}(\mathbf{p}) = p_i w_i(p_j) v_{i|\{t,t\}} + p_i w_i(1-p_j)v_{i|\{t,nt\}} + (1-p_i)(-d_i), \tag{3.9}$$

where $v_{i|\{t,t\}} = p_{i|\{i,j\}}(c_i - e_i) + (1 - p_{i|\{i,j\}})(-e_i - d_i),\ i = 1, 2$ is the value the i-th player assigns for the pure strategy $\{t, t\}$, i.e., both players transmit. $v_{i|\{t,nt\}} = p_{i|\{i\}}(c_i - e_i) + (1 - p_{i|\{i\}})(-e_i - d_i),\ i = 1, 2$ denotes the value the i-th player assigns for the pure strategy profile $\{t, nt\}$, i.e., only the i-th player transmits. Note that, the first element in a pure strategy profile is always the referred player's strategy. $p_{i|\{i,j\}}$ and $p_{i|\{i\}}$ denote the

Table 3.2: Notations Reference Table

Notations	Interpretations
$\mathcal{N}$	Set of players
$\mathbf{a}$	A pure strategy profile vector of all players
$\mathbf{p}$	A mixed strategy profile vector of all players, i.e. $\{p_i\}$
$\mathcal{J}\{\mathbf{a}\}$	Set of players choosing transmission given $\mathbf{a}$
$v_{i\|\mathbf{a}}$	The value i-th player obtains given $\mathbf{a}$
p_i	The i-th player's transmission probability
$p_{i\|\mathcal{J}(\mathbf{a})}$	The i-th player's successful package delivery probability given $\mathcal{J}(\mathbf{a})$
$w_i(\cdot)$	The i-th player's probability weighting function
$u_i^{EUT}(\cdot)$	The i-th EUT player's utility
$u_i^{PT}(\cdot)$	The i-th PT player's utility
$v_{i\|\{t,nt\}}$	The i-th player's value when she transmits and her opponent stays quite
$p_{i\|\{i,j\}}$	The i-th player's package reception probability when both players transmit

i-th player's successful packet delivery probabilities given that her opponent player j also transmits or not, respectively. p_i and p_j represent the transmission probabilities of the i-th player and her opponent, respectively. Under the game setting, players' communication performance are compared for three metrics, namely average throughput, average energy and average delay. They can be calculated as follows, for $i = 1, 2$:

$$T_i(\mathbf{p}) = p_i p_j p_{i|\{i,j\}} + p_i(1 - p_j)p_{i|\{i\}} \tag{3.10}$$

$$E_i(\mathbf{p}) = p_i \tag{3.11}$$

$$D_i(\mathbf{p}) = p_i p_j(1 - p_{i|\{i,j\}}) + p_i(1 - p_j)(1 - p_{i|\{i\}}) + (1 - p_i) \tag{3.12}$$

3.4.1 The Existence and Uniqueness of Mixed NEs of a Heterogeneous Game and a Homogeneous EUT game

The performance of players and system in (3.10) - (3.12) can only be meaningfully evaluated at Nash Equilibrium (NE) [65]. In the following, the mixed NE characteristics of a heterogeneous game and a homogeneous EUT game are studied. The players' utilities associated with pure strategy profiles can be compactly described in table 3.3. Note that, if the strategy profile $\{t, t\}$ where both players transmit is more preferred by player i than the strategy where player i does not transmit, i.e., $v_{i|\{t,t\}} \geq -d_i$, $i = 1, 2$, it is also true that $v_{i|\{t,nt\}} > -d_i$ since the profile $\{t, nt\}$ generates less interference to player i's transmission and thus more preferred than the strategy profile $\{t, t\}$. As a result, player i will

Table 3.3: Players' Values towards pure strategy profiles

Player 1 \ Player 2	t	nt
t	$(v_{1\|\{t,t\}}, v_{2\|\{t,t\}})$	$(v_{1\|\{t,nt\}}, -d_2)$
nt	$(-d_1, v_{2\|\{t,nt\}})$	$(-d_1, -d_2)$

always transmit, i.e., $p_i = 1$. Similarly, if $-d_i \geq v_{i|\{t,nt\}} > v_{i|\{t,t\}}$, $i = 1, 2$, player i will always not transmit by adopting $p_i = 0$. In the following, we exclude these two trivial scenarios by assuming for $i = 1, 2$, $v_{i|\{t,t\}} < -d_i$ and $v_{i|\{t,nt\}} > -d_i$. Furthermore, in the following, no further discussion on the resulting two trivial pure NEs, i.e., $\{t, nt\}$, $\{nt, t\}$, will be made either.

Theorem 3.4.1. *There exists a unique mixed NE $[p_1^H, p_2^H]$ for a heterogeneous game given $v_{i|\{t,t\}} < -d_i$ and $v_{i|\{t,nt\}} > 0$ for $i = 1, 2$.*

Proof. Without loss of generality, we assume player 1 is a PT player with utility function (3.9) and player 2 is an EUT player with utility function (3.8). In this game, each player, e.g., player i, $i = 1, 2$, will choose a probability for transmission, e.g., $p_i \in \Delta(A_i) = [0, 1]$ for t and thus $1-p_i$ for nt. It can be easily seen that the interval $\Delta(A_i)$ is a nonempty compact convex subset of a Euclidian space for $i = 1, 2$. Further, the PT player's utility function in (3.9) is linear in $p_1 \in \Delta(A_1)$. Given a fixed p_2 (i.e., the EUT player's mixed strategy), we pick any feasible $b_1, f_1, g_1 \in \Delta(A_1)$ and $u_1^{PT}(b_1, p_2) \geq \max\{u_1^{PT}(f_1, p_2), u_1^{PT}(g_1, p_2)\}$. Then, $u_1^{PT}(\lambda f_1 + (1-\lambda) g_1, p_2) = \lambda u_1^{PT}(f_1, p_2) + (1-\lambda) u_1^{PT}(g_1, p_2) < u_1^{PT}(b_1, p_2)$, $\forall \lambda \in [0, 1]$. Hence, the subset of the PT player's mixed strategies which yield less utility than an arbitrary feasible mixed strategy $b_1 \in \Delta(A_1)$ is convex and we have shown the PT player's utility function is quasi-concave in $\Delta(A_1)$. Similarly, it can be easily shown that the EUT player's utility function $u_2^{EUT}(p_1, p_2)$ is also quasi-concave in $\Delta(A_2)$. According to the Proposition 20.3 in [65], at least one mixed NE exists in the heterogeneous game.

At the mixed NE $\mathbf{p}^* = [p_1^*, p_2^*]$, by definition, the PT player should always obtain the identical utility by playing either of the pure strategies, i.e., $u_1^{PT}(1, p_2^*) = u_1^{PT}(0, p_2^*)$. Note that, $u_1^{PT}(1, p_2) = w_1(p_2) v_{1|\{t,t\}} + w_1(1 - p_2) v_{1|\{t,nt\}}$ and $u_1^{PT}(0, p_2) = -d_1$. If $v_{1|\{t,t\}} < 0$

and $v_{1|\{t,nt\}} > 0$, the PT player assigns a negative value to a collision and a positive value to an interference-free transmission. Then, it follows that

$$\frac{\partial u_1^{PT}(1,p_2)}{\partial p_2} = \frac{\alpha w_1(p_2)(-\ln(p_2))^{\alpha-1}}{p_2} v_{1|\{t,t\}} + \frac{\alpha w_1(1-p_2)(-\ln(1-p_2))^{\alpha-1}}{p_2-1} v_{1|\{t,nt\}} < 0. \tag{3.13}$$

Therefore, $u_1^{PT}(1,p_2)$ is a strictly decreasing function of $p_2 \in [0,1]$. The points where the function $u_1^{PT}(1,p_2)$ intersects with the horizontal line $u_1^{PT}(0,p_2) = -d_1$ are the solutions to the equation $u_1^{PT}(1,p_2) = u_1^{PT}(0,p_2)$ and thus the EUT player's mixed strategies at the mixed NE. Since $u_1^{PT}(1,p_2)$ has been shown as a strictly decreasing function of $p_2 \in [0,1]$, the intersection point is unique and therefore the EUT player's mixed strategy (transmission probability) p_2^{H} at the mixed NE is also unique. Meanwhile, the EUT player's utility when she always transmits, i.e., $u_2^{EUT}(p_1,1)$, can be similarly shown as a strictly decreasing function of $p_1 \in [0,1]$. Hence, the PT player's mixed strategy p_1^{H} at the mixed NE is also unique. Furthermore, as mentioned earlier, $v_{i|\{t,t\}} < -d_i$, $i = 1,2$ and $v_{i|\{t,nt\}} > 0 > -d_i$, $i = 1,2$ ensure that neither of the players would choose a pure strategy. Thus, a unique mixed NE $[p_1^{\mathsf{H}}, p_2^{\mathsf{H}}]$ exists for the 2-player heterogeneous game. □

Note that, the conditions in Theorem 1 are mild and intuitive in that they reflect a communications scenario where a "positive" value ($v_{i|\{t,nt\}} > 0$, $i = 1,2$) results when only player i transmits and a "negative" value ($v_{i|\{t,t\}} < -d_i$, $i = 1,2$) results when the 2 players collide. Further, this negative value is worse than when the player i does not transmit.

Corollary 3.4.2. *There exists a unique mixed NE $[p_1^{EUT}, p_2^{EUT}]$ for the homogeneous EUT game given $v_{i|\{t,t\}} < -d_i$ and $v_{i|\{t,nt\}} > 0$ for $i = 1,2$.*

Proof. The existence and uniqueness of the mixed NE for the homogeneous EUT game can be established in the same fashion as in the heterogeneous game. □

Due to the analytical intractability of the probability weighting function in equation (3.3), the mixed NE involving PT players cannot be derived in closed form even for a 2-player scenario. For example, in a 2-player homogeneous game, the players' transmission probabilities $[p_1, p_2]$ at the mixed NE can be derived by solving the following nonlinear equations:

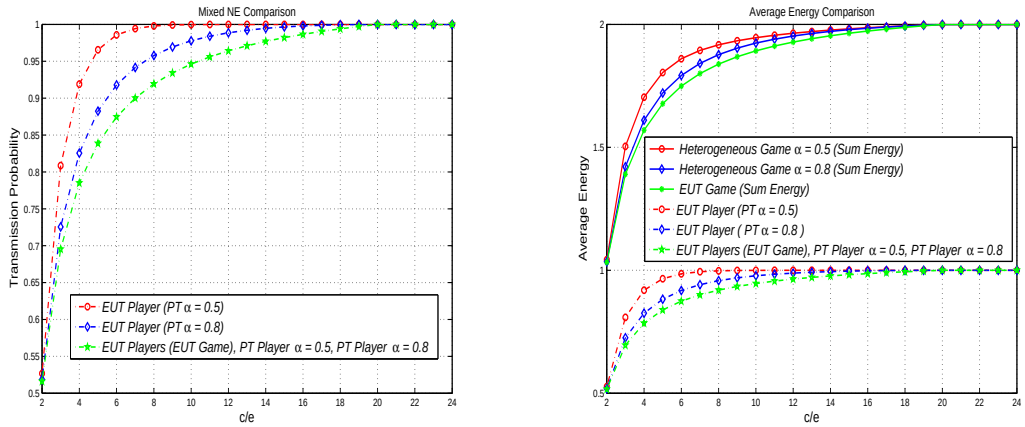
$$\exp(-(-\ln p_1)^\alpha)v_{2|\{t,t\}} + \exp(-(-\ln(1-p_1))^\alpha)v_{2|\{t,nt\}} = -d_2 \tag{3.14}$$

$$\exp(-(-\ln p_2)^\alpha)v_{1|\{t,t\}} + \exp(-(-\ln(1-p_2))^\alpha)v_{1|\{t,nt\}} = -d_1. \tag{3.15}$$

Even though the above 2 equations are independent, owing to the analytical intractability in arriving at closed form solution, we use the numerical solvers in MATLAB to find the transmission probabilities at the mixed NE in the rest of this work. In spite of the analytical intractability, we will prove many intuitive properties of the mixed NE as will be seen in the remainder of this chapter.

3.4.2 A Heterogeneous Game: Consequence of Deviation from EUT

A heterogeneous game consisting of one PT player and one EUT player is considered and we will highlight the consequences on the performance compared to a 2-player homogeneous EUT game. The heterogeneous game can be considered as a deviation from the homogeneous game as one of the EUT players is replaced by a PT player who differs from the EUT players in the homogeneous game only in the probability weighting index α. We specifically study three issues: (a) the impact on the performance of the EUT player in the heterogeneous game, (b) the impact on the performance of the whole system, and (c) the performance difference between the PT and EUT players in the heterogeneous game. We consider the following 3 scenarios for comparison: (1) Both players are EUT (EUT game), (2) One EUT and one PT player with $\alpha = 0.8$, and (3) One EUT and one PT player with $\alpha = 0.5$. For both the heterogeneous game and the homogeneous EUT game, two players randomly access the channel with $p_{i|\{i\}} = p_{j|\{j\}} = 0.98$, $p_{i|\{i,j\}} = p_{j|\{i,j\}} =$



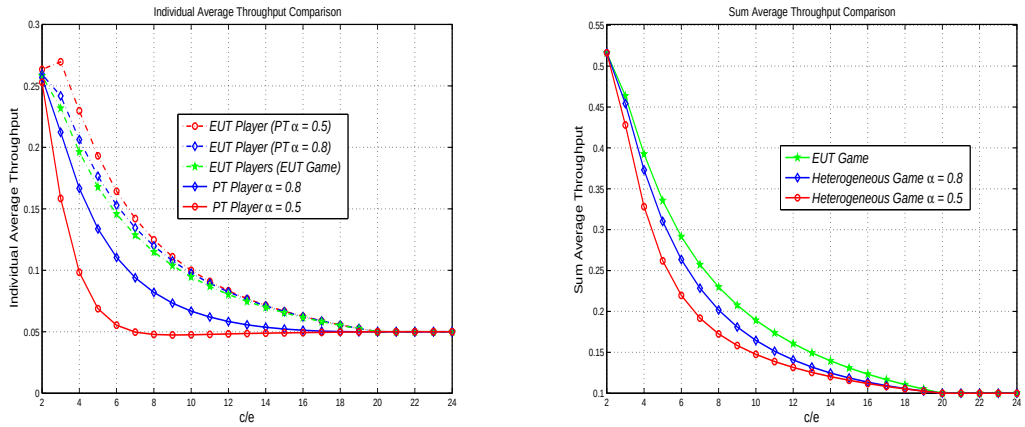
(a) Transmission probability of each player at mixed NE

(b) Average energy comparison

Figure 3.3: (a) Transmission probability of each player at mixed NE for each game. (b) Average energy comparison at individual player level and system level. Three games (heterogeneous game ($\alpha = 0.8$), heterogeneous game ($\alpha = 0.5$), EUT game) are studied and compared where no delay penalty is considered ($d = 0$).

0.05, $i = 1, 2$. We begin with a special case where there is no delay penalty for each player as $c_1 = c_2 = c$, $e_1 = e_2 = e = 2$, $d_1 = d_2 = d = 0$. The system performances are studied and compared as a function of c/e. Due to the symmetry here, the EUT players in the homogeneous EUT game have the identical strategy and performance at the mixed NE. We observed in Fig. 3.3(a) that with the introduction of the PT player , the EUT player transmits more aggressively in the heterogeneous game than the EUT players in the homogeneous EUT game. Note that, the PT player in the heterogeneous game under two different scenarios chooses the same transmission probability at the mixed NE since her opponent's (the EUT player) utility function stays unchanged. Within the heterogenous game, the EUT player is more aggressive in transmission than the PT player. Both the trends are more obvious as the PT player deviates more from EUT (smaller α). Further, both the players' transmissions under the 3 scenarios become more probable as the unit throughput reward increases and finally converge to transmission with probability 1 when $v_{i|\{t,t\}} = -d = 0$, $i = 1, 2$ ($c/e = 20$). Note that, all the curves do not increase to 1 until $c/e = 20$ though in the figure, they are seen very close to it before $c/e = 20$.

In Fig. 3.3(b), it can be seen that the introduction of the PT player actually causes the EUT player in the heterogeneous game consume more average energy than the EUT



(a) Individual average throughput at mixed NE

(b) Sum average throughput of both player at mixed NE

Figure 3.4: Average throughput comparison at individual player level and system level. Three games (heterogeneous game ($\alpha = 0.8$), heterogeneous game ($\alpha = 0.5$), EUT game) are studied and compared where no delay penalty is considered ($d = 0$).

players in the homogeneous EUT game. Meanwhile, the PT player's consumed energy stays unchanged as explained in Fig. 3.3(a). As a result, the players in the heterogeneous game consume more sum energy in average than those in the homogeneous EUT game. Moreover, a more deviated PT player causes an increased sum average energy consumption.

Figure 3.4(a) shows that the introduction of the PT player actually increases the EUT player's average throughput compared to the EUT players in the homogeneous EUT game. However, the PT player herself suffers an obvious average throughput degradation. Moreover, within the heterogenous game, the PT player achieves an obviously less average throughput than the EUT player. Further, as the PT player deviates more from EUT, the above trends are also more exaggerated. Note that, each player's achieved average throughput eventually decreases as the unit throughput reward increases since players also transmit more aggressively and thus generate more collisions. The deviation of one player from EUT to PT can also significantly impact the system level performance as in Fig. 3.4(b). It can be observed that the players in a heterogeneous game achieve less sum average throughput than those in the homogeneous EUT game. As the PT player deviates more from EUT, the heterogeneous game suffers an even larger sum average throughput loss.

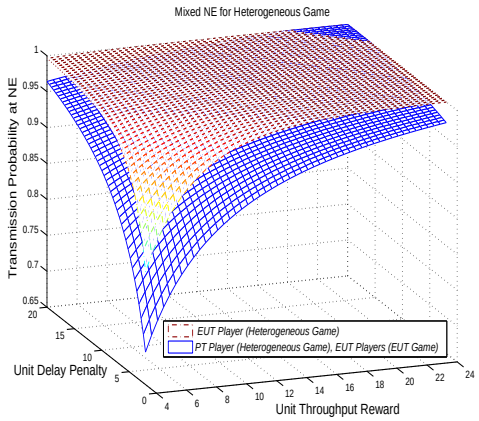
In Fig. 3.5, we study a heterogeneous game in a more general setting where both unit throughput reward and unit delay penalty are non-zero and varying. For comparison purpose, we choose $c_1 = c_2 = c$, $e_1 = e_2 = e = 2$, $d_1 = d_2 = d$ and $p_{i|\{i\}} = p_{j|\{j\}} = 0.98$, $p_{i|\{i,j\}} = p_{j|\{i,j\}} = 0.05$ for both players in both the heterogeneous game and the homogeneous EUT game. The PT player's probability weighting index is $\alpha = 0.6$. In Fig. 3.5(a), it can be seen that the introduction of the PT player causes the EUT player of the heterogeneous game transmit more aggressively than the EUT players in the homogeneous EUT game. Within the heterogeneous game, the EUT player is more aggressive than the PT player. Both players' transmission probabilities converge to 1 when $v_{i|\{t,t\}} \geq -d_i$, $i = 1, 2$. In Fig. 3.5(b), the impact of the PT player's deviation on the average delay is shown. The EUT player in the heterogeneous game suffers slightly less average delay than the EUT players in the homogeneous EUT game. However, within the heterogeneous game, the PT player suffers more average delay than the EUT player. Further, the deviation of the PT player also causes the heterogeneous game suffer a larger sum average delay than the homogeneous EUT game. In Fig. 3.5(c), it has been shown that more sum average energy is consumed in the heterogeneous game than that in the homogeneous EUT game. Fig. 3.5(d) has shown that the heterogeneous game suffers a sum average throughput degradation compared to the homogeneous EUT game. Moreover, the PT player is shown to achieve less average throughput than the EUT player within the heterogeneous game.

Therefore, together with the first set of numerical result this subsection has demonstrated 3 main consequences of the PT player's deviation from EUT as listed below:

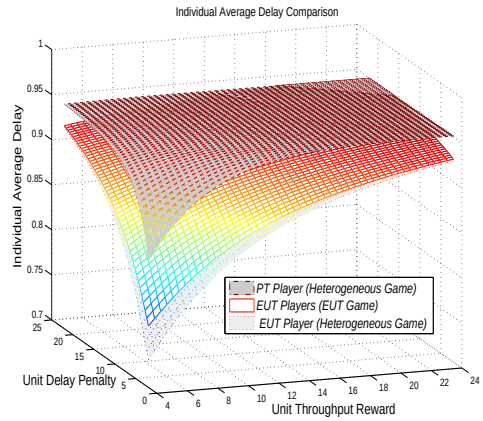
- (**C1**) The deviation of PT player causes the EUT player in the heterogeneous game achieve more average throughput, suffer less average delay but consume more average energy than the EUT players in the homogeneous EUT game.

- (**C2**) Within the heterogeneous game, the PT player achieves less average throughput but suffers more average delay. The EUT player consumes more average energy.

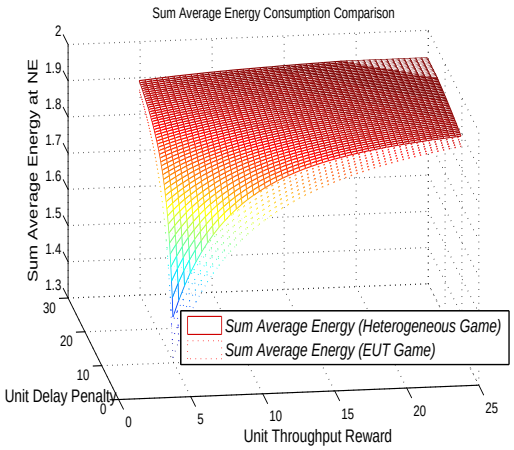
- (**C3**) The deviation of the PT player undermines the overall system level performance in every aspect as the heterogeneous game achieves less sum average throughput, suffers more sum average delay and consumes more sum average energy than the



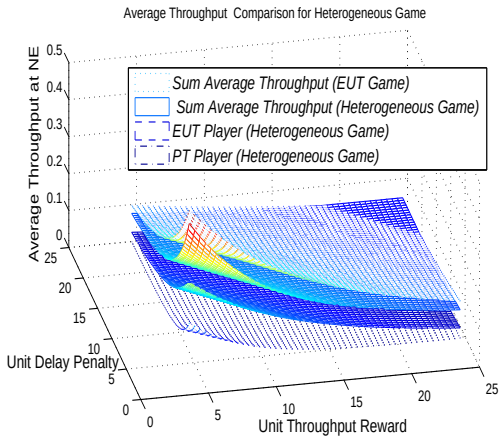
(a) Transmission probability at mixed NE



(b) Individual average delay at mixed NE



(c) Average Energy at mixed NE



(d) Average Throughput at mixed NE

Figure 3.5: A heterogeneous game ($\alpha = 0.6$) and a homogeneous EUT game are compared where unit throughput reward and unit delay penalty are non-zero.

homogeneous EUT game.

3.4.3 Analytical Insights into Consequence C1 - C3

We will now prove the three main consequences (1), (2) and (3) in subsection 3.4.2 under mild conditions. We begin with the following Lemmas.

Lemma 3.4.3. *Each player's average throughput function* (3.10) *is an increasing function of her own transmission probability and a decreasing function of her opponent's transmission probability.*

Proof. It can be easily verified that $\frac{\partial T_i(\mathbf{p})}{\partial p_i} = p_j p_{i|\{i,j\}} + (1-p_j)p_{i|\{i\}} > 0$ and $\frac{\partial T_i(\mathbf{p})}{\partial p_j} = p_i p_{i|\{i,j\}} - p_i p_{i|\{i\}} < 0$ since $p_{i|\{i\}} > p_{i|\{i,j\}}$ that simply reflects the fact that player i gets a higher successful packet delivery probability for a collision-free transmission than a collision. □

Lemma 3.4.4. *Each player's average delay function* (3.12) *is a decreasing function of her own transmission probability and an increasing function of her opponent's transmission probability.*

Proof. This also follows the fact that $p_{i|\{i\}} > p_{i|\{i,j\}}$. □

Lemma 3.4.5. *If $v_{i|\{t,t\}} < 0$, $v_{i|\{t,nt\}} > 0$, there exists a unique number $p^{\text{int}} \in (1/\rho, 1-1/\rho)$ (ρ denotes the Euler's number here) such that $u_i^{PT}(1, p^{\text{int}}) = u_i^{EUT}(1, p^{\text{int}})$. Further, $u_i^{PT}(1, p_j) \leq u_i^{EUT}(1, p_j)$ and $u_i^{PT}(1, p_j)$ is convex when $p_j \in [0, p^{\text{int}})$. Meanwhile, $u_i^{PT}(1, p_j) \geq u_i^{EUT}(1, p_j)$ and $u_i^{PT}(1, p_j)$ is concave when $p_j \in [p^{\text{int}}, 1]$.*

Proof. Note that, $u_i^{PT}(1, 1/\rho) = w_i(1/\rho)v_{i|\{t,t\}} + w_i(1-1/\rho)v_{i|\{t,nt\}} < 1/\rho \times v_{i|\{t,t\}} + (1-1/\rho) \times v_{i|\{t,nt\}} = u_i^{EUT}(1, 1/\rho)$. This holds because $v_{i|\{t,t\}} < 0$, $v_{i|\{t,nt\}} > 0$ and the probability weighting function has the property that $w_i(1/\rho) = 1/\rho$ and $w_i(1-1/\rho) < 1-1/\rho$. Similarly, we can show that $u_i^{PT}(1, 1-1/\rho) = w_i(1-1/\rho)v_{i|\{t,t\}} + w_i(1/\rho)v_{i|\{t,nt\}} > (1-1/\rho) \times v_{i|\{t,t\}} + 1/\rho \times v_{i|\{t,nt\}} = u_i^{EUT}(1, 1-1/\rho)$. Thus, there must exist a point $p^{\text{int}} \in (1/\rho, 1-1/\rho)$ such that $u_i^{PT}(1, p^{\text{int}}) = u_i^{EUT}(1, p^{\text{int}})$ due to the continuities of both $u_i^{PT}(1, p_j)$ and $u_i^{EUT}(1, p_j)$. We define the difference function of the PT utility and the EUT utility when the player always transmits as $\Delta(p_j) = u_i^{PT}(1, p_j) - u_i^{EUT}(1, p_j) = f_1(p_j)v_{i|\{t,t\}} + f_2(p_j)v_{i|\{t,nt\}}$ where $f_1(p_j) = w_i(p_j) - p_j$ and $f_2(p_j) = w_i(1-p_j) - (1-p_j)$. According to the definition of probability weighting function (3.3) and from Fig. 3.1, it can be easily seen that the function $f_1(p_j)$ is a monotonically increasing function in $[0, \epsilon_1(\alpha)]$, decreasing function in $(\epsilon_1(\alpha), 1-\epsilon_2(\alpha))$ and increasing function in $[1-\epsilon_2(\alpha), 1]$. Similarly, $f_2(p_j)$ can be shown to be a monotonically decreasing function in $[0, \epsilon_2(\alpha)]$, increasing function in $(\epsilon_2(\alpha), 1-\epsilon_1(\alpha))$ and decreasing function in $[1-\epsilon_1(\alpha), 1]$. Note that, $\epsilon_1(\alpha)$, $\epsilon_2(\alpha) \in (0, 1)$ depend on the probability weighting index α and $\epsilon_1(\alpha) < \epsilon_2(\alpha)$. Also, it can be shown that $\partial w_i(p_j)/\partial p_j = 1$ when $p_j = \epsilon_1(\alpha), 1-\epsilon_2(\alpha)$. The above

properties can be shown in Fig. 3.6.

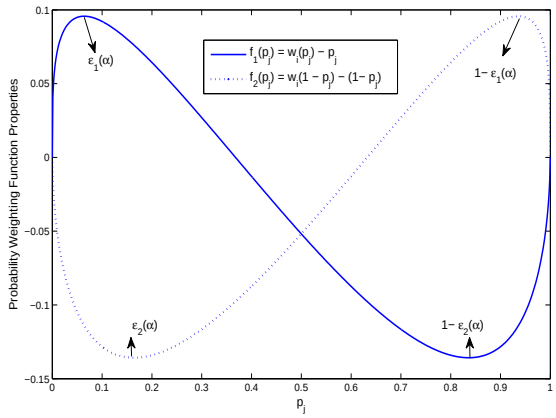


Figure 3.6: $w_i(p_j) - p_j$ and $w_i(1-p_j) - (1-p_j)$ as functions of p_j for $\alpha = 0.6$.

Moreover, for an arbitrary $p_j \in [0, \epsilon_1(\alpha))$, $\Delta(p_j) < 0$ since $f_1(p_j) > 0$, $f_2(p_j) < 0$, $v_{i|\{t,t\}} < 0$ and $v_{i|\{t,nt\}} > 0$. $\Delta(p_j)$ decreases further as p_j approaches $\epsilon_1(\alpha)$ and then there must be a point $\zeta_1 \in [\epsilon_1(\alpha), \epsilon_2(\alpha)]$ where $\Delta(p_j)$ begins to increase. When $p_j \in [\epsilon_2(\alpha), 1-\epsilon_2(\alpha)]$, $\Delta(p_j)$ keeps increasing further since $df_1(p_j)/dp_j < 0$, $df_2(p_j)/dp_j > 0$ and thus $d\Delta(p_j)/dp_j > 0$. Note that, $\Delta(p^{\text{int}}) = 0$ and this zero is unique due to the above construction. Hence, we can deduce that $u_i^{PT}(1, p_j)$ must lie below $u_i^{EUT}(1, p_j)$ and be convex when $p_j \in [0, p^{\text{int}})$ if we draw $u_i^{PT}(1, p_j)$ and $u_i^{EUT}(1, p_j)$ together. At the other end, for an arbitrary number $p_j \in [1-\epsilon_2(\alpha), 1]$, we can show that $\Delta(p_j) > 0$. By following a similar approach as in the previous scenario, we can show that $u_i^{PT}(1, p_j) > u_i^{EUT}(1, p_j)$ and $u_i^{PT}(1, p_j)$ is concave when $p_i \in [p^{\text{int}}, 1]$. The above characteristics can be seen in Fig. 3.7. □

Theorem 3.4.6. ***(Consequence (C1))*** *In a 2-player heterogeneous game, at the mixed NE, the EUT player achieves more average throughput, suffers less average delay and uses increased average energy, compared to that in a 2-player homogeneous EUT game if the PT player's (player i) value function is such that* $v_{i|\{t,nt\}} > -\rho d_i + (1-\rho) v_{i|\{t,t\}}$.

Proof. In the homogeneous EUT game, the EUT player's (player j) transmission probability p_j^{EUT} at the mixed NE can be solved by setting $u_i^{EUT}(1, p_j) = u_i^{EUT}(0, p_j) = -d_i$. From lemma 3.4.5, when $p_j \in [p^{\text{int}}, 1]$, $u_i^{PT}(1, p_j)$ lies above $u_i^{EUT}(1, p_j)$ and is concave. If

$v_{i|\{t,nt\}} > -\rho d_i + (1-\rho)v_{i|\{t,t\}}$, it can be easily shown that $p_j^{\mathsf{EUT}} = \frac{-d_i - v_{i|\{t,nt\}}}{v_{i|\{t,t\}} - v_{i|\{t,nt\}}} > 1 - 1/\rho$. Correspondingly, the horizontal line $u_i^{EUT}(0, p_j) = -d_i$ must intersect $u_i^{PT}(1, p_j)$ in the heterogeneous game at p_j^{H} where $p_j^{\mathsf{H}} > p_j^{\mathsf{EUT}}$ (A graphical example can be found in Fig. 3.7). Meanwhile, since the EUT player (player j) in the heterogeneous game is identical to the EUT player j in the homogeneous EUT game, it can be devised that $p_i^{\mathsf{H}} = p_i^{\mathsf{EUT}}$. From lemma 3.4.3, the EUT player's (player j) average throughput function T_j in (3.10) is an increasing function of her own transmission probability and a decreasing function of her opponent's transmission probability. Thus, the EUT player (player j) in the heterogeneous game achieves a larger average throughput than in the homogeneous EUT game due to her own transmission probability increase while her opponent's stays unchanged. Similarly, the decline in the EUT player's average delay in the heterogeneous can be explained accordingly. However, due to the EUT player's increasing aggressiveness, she consumes more average energy in the heterogeneous game. □

Theorem 3.4.7. ***(Consequence (C2))*** *In a heterogeneous game, if the PT player (player i) and the EUT player (player j) only differ in the probability weighting index, i.e., $\alpha_i < 1$ and $\alpha_j = 1$ and $v_{i|\{t,nt\}} > -\rho d_i + (1-\rho)v_{i|\{t,t\}}$, it can be shown that $p_i^{\mathsf{H}} < p_j^{\mathsf{H}}$. In other words, the PT player transmits less aggressively than the EUT player at the mixed NE. Further, the EUT player achieves more average throughput and suffers less average delay but consumes more average energy than the PT player.*

Proof. From lemma 3.4.5, $u_i^{PT}(1, p)$ lies above $u_j^{EUT}(1, p)$ and is concave when $p \in [p^{\mathsf{int}}, 1]$. Thus, by following a similar approach in theorem 3.4.6, it can be shown that the PT player transmits less aggressively than the EUT player. Correspondingly, the EUT player incurs higher average energy consumption than the PT player in the heterogeneous game. Together with lemma 3.4.3 and 3.4.4, the claims regarding the players' average throughput and delay are also established. □

Theorem 3.4.8. ***(Consequence (C3))*** *In a heterogeneous game, at the mixed NE, if $p_i^{\mathsf{H}} > \frac{p_{j|\{j\}}}{\sum_{i=1,2}(p_{i|\{i\}} - p_{i|\{i,j\}})}$ for the PT player, the two players achieve less sum average throughput, suffer more sum average delay and consume more sum average energy than*

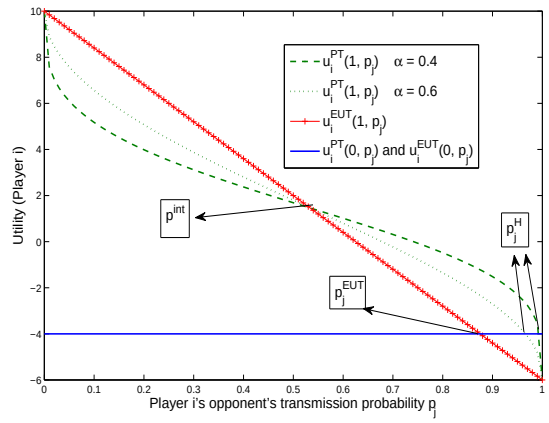


Figure 3.7: Players'transmission probabilities at the mixed NE. $v_{i|\{t,t\}} = -6$, $v_{i|\{t,nt\}} = 10$ and $d = -4$.

those of the corresponding homogeneous EUT game. In other words, the deviation of the PT player harms the system performance in every aspect.

Proof. From theorem 3.4.6, it is already known that in the heterogeneous game, the PT player chooses an identical transmission probability as the EUT players in the homogeneous EUT game as $p_i^{\mathsf{H}} = p_i^{\mathsf{EUT}}$. However, the EUT player in the heterogeneous game becomes more aggressive than that in the homogeneous EUT game as $p_j^{\mathsf{H}} > p_j^{\mathsf{EUT}}$. The two players' sum average throughput, sum average delay and sum average energy can be written as:

$$T_s = p_i p_j p_{i|\{i,j\}} + p_i(1-p_j)p_{i|\{i\}} + p_i p_j p_{j|\{i,j\}} + (1-p_i)p_j p_{j|\{j\}} \tag{3.16}$$

$$\begin{aligned} D_s = {} & p_i p_j (1 - p_{i|\{i,j\}}) + p_i(1-p_j)(1-p_{i|\{i\}}) + 1 - p_i \\ & + p_i p_j (1 - p_{j|\{i,j\}}) + (1-p_i)p_j(1-p_{j|\{j\}}) + 1 - p_j \end{aligned} \tag{3.17}$$

$$E_s = p_i + p_j. \tag{3.18}$$

If $p_i > \frac{p_{j|\{j\}}}{\sum_{i=1,2}(p_{i|\{i\}} - p_{i|\{i,j\}})}$, then it can be easily verified that $\partial T_s/\partial p_j < 0$ and $\partial D_s/\partial p_j > 0$. Therefore, the sum average throughput function and sum average delay have been shown as a decreasing and an increasing function of p_j, respectively. Therefore, if $p_i^{\mathsf{H}} > \frac{p_{j|\{j\}}}{\sum_{i=1,2}(p_{i|\{i\}} - p_{i|\{i,j\}})}$, then the two players in the heterogeneous game achieve less sum average throughput, suffers more sum average delay and consume more sum energy than that of the homogeneous EUT game due to the increased aggressiveness of the EUT player. □

3.5 A 2-Player Homogeneous PT Wireless Random Access Game

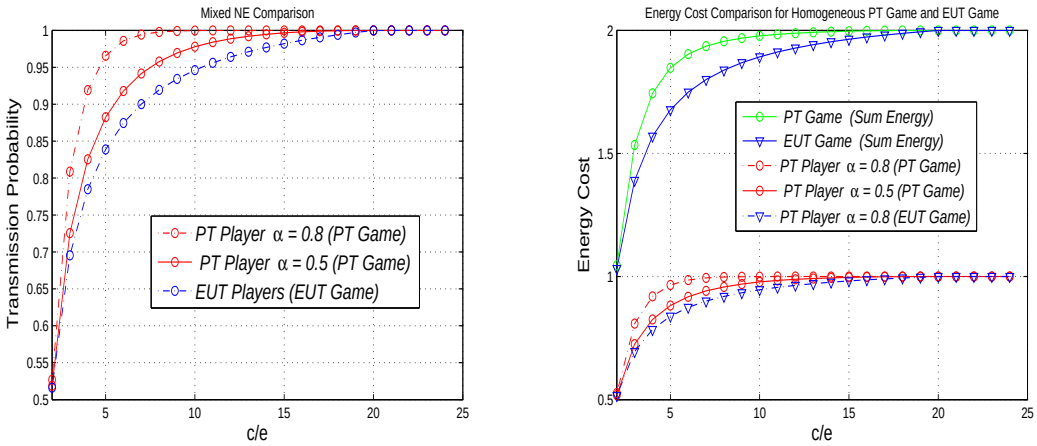
In the above section 3.4.2, we studied the scenario when one player deviates from EUT in a 2-player wireless random access game, i.e., a 2-player heterogeneous game. In this section, a more general scenario where both players follow the precept of PT is further analyzed. As in our earlier work [32], a 2-player homogeneous PT wireless random access game has been studied under a 0/1 collision channel model. We now extend this work by considering the homogeneous game under a general collision channel model. Further, without loss of generality, we assume that the two players, i.e., player $1, 2$, both follow the precepts of PT with probability weighting indices $1 > \alpha_2 > \alpha_1 > 0$. Thus, both players' utility functions should be described as in (3.9) and the two utility functions differ only in the probability weighting index. Similar to section 3.4, we will compare the 2-player homogeneous PT game to a benchmark, i.e., the corresponding 2-player homogeneous EUT game. Note that, for player $i = 1, 2$, her value function regarding a pure strategy profile stays the same with all value functions being linear and about reference point 0 in both the homogeneous PT and EUT games, i.e., $v^{PT}_{i|\{t,t\}} = v^{EUT}_{i|\{t,t\}} = v_{i|\{t,t\}}$ and $v^{PT}_{i|\{t,nt\}} = v^{EUT}_{i|\{t,nt\}} = v_{i|\{t,nt\}}$.

3.5.1 The Existence and Uniqueness of Mixed NEs of a Homogeneous PT Game

Before investigating the performance deviation of a 2-player homogeneous PT game from the corresponding 2-player homogeneous EUT game, we first address the existence and uniqueness of the NE.

Theorem 3.5.1. *There exists a unique mixed NE $[p^P_1, p^P_2]$ for a homogeneous PT game given $v_{i|\{t,t\}} < -d_i$ and $v_{i|\{t,nt\}} > 0$ for $i = 1, 2$.*

Proof. The existence of the NE in a homogeneous PT game can be proven by following the same argument in theorem 3.4.1. Furthermore, given the identical condition as before, i.e., $v_{i|\{t,t\}} < -d_i$ and $v_{i|\{t,nt\}} > 0$ for $i = 1, 2$, it can be shown that $u_i(1, p_j)$ is a decreasing function of p_j for $i = 1, 2$ as shown in (3.13). Since player j's mixed strategy at the NE is solved by equating $u_i(1, p_j) = u_i(0, p_j) = -d_i$ and $u_i(1, p_j)$ is a strictly decreasing function, p^H_j must be uniquely determined. Thus, the mixed NE $[p^P_1, p^P_2]$ is also unique. □



(a) Transmission probability of each player at mixed NE

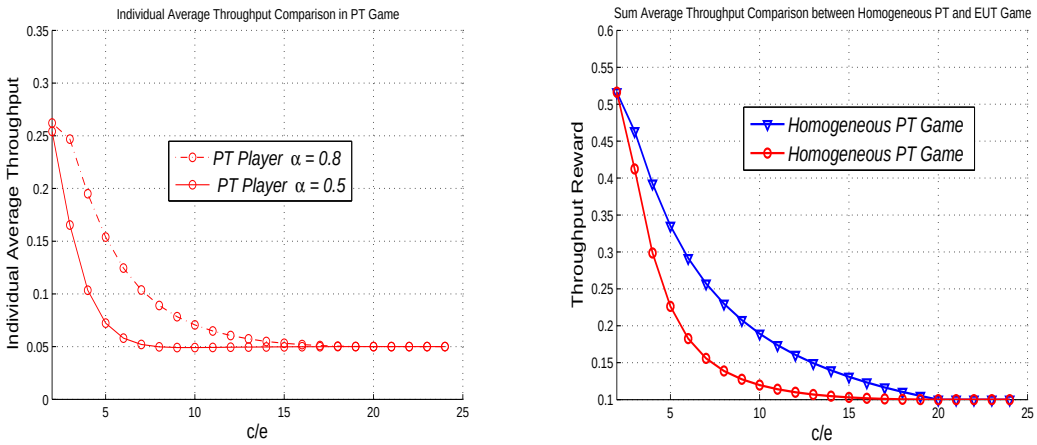
(b) Average energy comparison

Figure 3.8: (a) Transmission probability of each player at mixed NE for each game. (b) Average energy comparison at individual player level and system level. Homogeneous PT game ($\alpha_1 = 0.5$, $\alpha_2 = 0.8$) and homogeneous EUT games are studied and compared where no delay penalty is considered ($d = 0$).

3.5.2 A Homogeneous PT Game: More Consequences of Deviation from EUT

In the following, we will compare the performance of the 2-player homogeneous game to that of the 2-player homogeneous EUT game. Specifically, we will study the following issues: (1) Performance differences between two PT players within the 2-player homogeneous PT game. (2) The difference on the performance of the whole system between two games. Without loss of generality, in the 2-player homogeneous PT game, we assume $\alpha_1 = 0.5$ and $\alpha_2 = 0.8$. For both games, two players randomly access the channel with $p_{i|\{i\}} = p_{j|\{j\}} = 0.98$, $p_{i|\{i,j\}} = p_{j|\{i,j\}} = 0.05$, $i = 1, 2$. We begin with a special case where there is no delay penalty for each player as $c_1 = c_2 = c$, $e_1 = e_2 = e = 2$, $d_1 = d_2 = d = 0$. The system performances are studied and compared as a function of c/e.

In Fig. 3.8(a), the PT players in the homogeneous PT game are shown to transmit more aggressively than the EUT players in the homogeneous EUT game. Note that, both the EUT players have the same performance in the homogeneous EUT game due to the same parameter setting. Further, it can be observed that the PT player that deviates less from the EUT (i.e., with choice of $\alpha = 0.8$) transmits more aggressively in the homogeneous PT



(a) Individual average throughput at mixed NE for homogeneous PT game

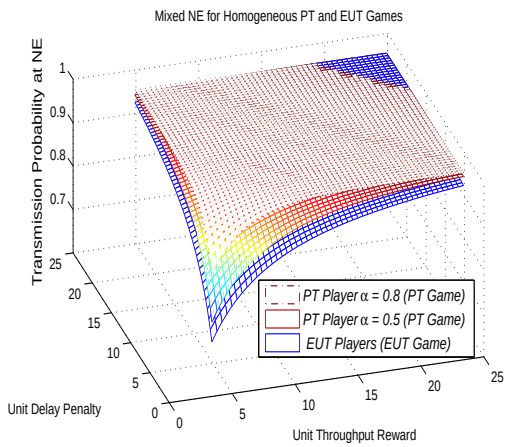
(b) Sum average throughput of both player at mixed NE

Figure 3.9: Average throughput comparison at individual player level and system level. Homogeneous PT game ($\alpha_1 = 0.5$, $\alpha_2 = 0.8$) and homogeneous EUT games are studied and compared where no delay penalty is considered ($d = 0$).

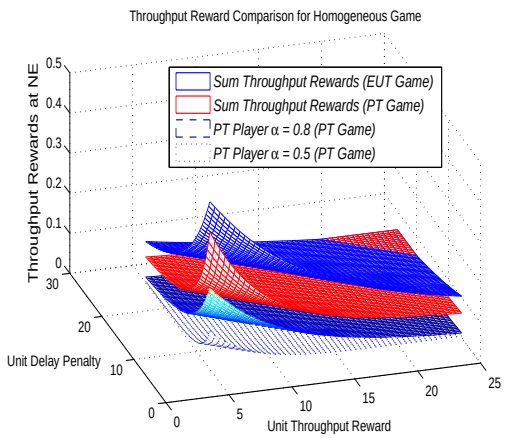
game. As a result of the transmission aggressiveness of the PT players at the NE shown in Fig. 3.8(a), the PT players consume more average energy than the EUT players as shown in Fig. 3.8(b). Between the PT player, the one with a larger probability weighting index ($\alpha = 0.8$) consumes more average energy than the other PT player. More importantly, the deviation of the PT players from EUT costs the system more energy at the mixed NE compared to the homogeneous EUT game.

In Fig.3.9(a), the throughput performance difference between two PT players in the homogeneous game is shown. The PT player that deviates less from EUT ($\alpha = 0.8$) achieves more average throughput than the one with a smaller probability weighting index. On the system level, PT players in the homogeneous PT game also incur sum throughput degradation compared to the homogeneous EUT game as shown in Fig.3.9(b).

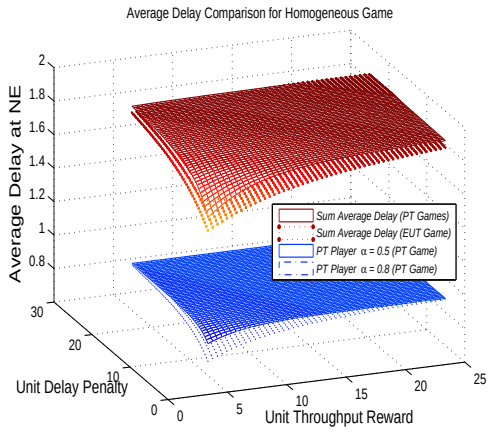
While the above results are for the case of no delay penalty, we now study the performance deviation of the PT game in a more general setting. Specifically, we consider a case where unit throughput reward and unit delay penalty both vary while the unit energy cost is fixed, i.e., $c_1 = c_2 = c$, $e_1 = e_2 = e = 2$, $d_1 = d_2 = d = d$. For comparison, we choose $p_{i|\{i\}} = p_{j|\{j\}} = 0.98$, $p_{i|\{i,j\}} = p_{j|\{i,j\}} = 0.05$, $i = 1, 2$ for PT players and EUT players in both games. In the homogeneous game, we choose $\alpha_1 = 0.5$ and $\alpha_2 = 0.8$ so that PT player



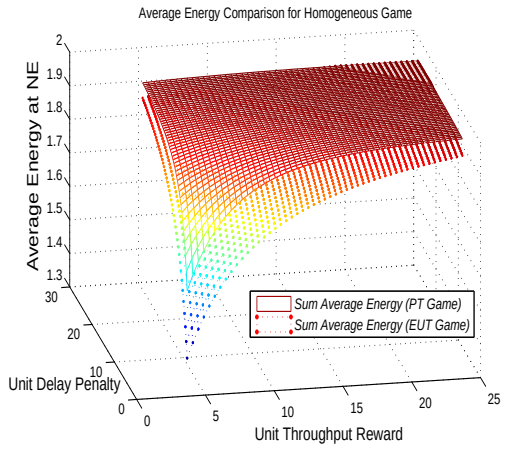
(a) Transmission probability at mixed NE



(b) Throughput Comparison at mixed NE



(c) Delay Comparison at mixed NE



(d) Energy Comparison at mixed NE

Figure 3.10: A homogeneous PT game ($\alpha_1 = 0.5$, $\alpha_2 = 0.8$) and a homogeneous EUT game are compared where unit throughput reward and unit delay penalty are non-zero. In each figure, the order of plots are according to the order of the legends from top to bottom.

1 deviates more from EUT than PT player 2. In Fig. 3.10(a), a similar trend in to that in Fig. 3.8(a) can be observed. The PT players in the homogeneous PT game transmit more aggressively than the EUT players in the homogeneous EUT game. Further, the PT player deviating less from EUT (with a larger α) transmits more aggressively than the other one. The difference in the transmission aggressiveness decreases as the unit throughput reward and unit delay penalty increase. As a result of the transmission aggressiveness of the PT players at the mixed NE, Fig. 3.10(b) shows that the homogeneous PT game suffers a sum average throughput degradation compared to the homogeneous EUT game on the system

level. This degradation becomes less severe as the unit throughput reward and unit delay penalty increase. Within the homogeneous PT game, the PT player that deviates more from EUT (with a smaller α) achieves less average throughput than the other one. In Fig. 3.10(c), homogeneous PT game suffers a greater sum average delay than the EUT game. Within the homogeneous PT game, the PT player who deviates more from EUT (with a smaller α) suffers a larger average delay than her opponent. Fig. 3.10(d) shows that the deviation of the PT players of the homogeneous PT game also consumes the system more average energy than that of the EUT game. Since average energy is proportional to a player's transmission aggressiveness, from 3.10(a), we can see that PT players in homogeneous PT game consumes more average energy than EUT players in the EUT game. Within the homogeneous PT game, the PT player who deviates less from EUT consumes more average energy.

The above results shown in Fig. 3.8(a)- 3.10(b) captures the behavior and performance deviation of the PT players in the homogeneous PT game from the homogeneous EUT game. Two distinct features on individual player and system levels can be summarized as below:

- (**C4**) In the homogeneous PT game, the PT player that deviates less from EUT (with a larger α) achieves more average throughput and suffers less average delay than the PT player who deviates more from EUT. However, the PT player that deviates less from EUT consumes more average energy.

- (**C5**) Similar to the observation from the heterogeneous game in section 3.4, the deviation of both PT players in the homogeneous PT game still hurts the overall system performance in every aspect. The homogeneous PT game achieves less sum average throughput but suffers more average delay and incurs more average energy than the homogeneous EUT game.

3.5.3 Analytical Insights into Consequence C4 - C5

In the following, we will analyze the above consequences of the deviation of the homogeneous PT game from EUT. The general idea and techniques used here are based on the

results in subsection 3.4.3 section 3.4. The following lemma describes the features of the utility functions of the PT players when they transmit with probability one.

Lemma 3.5.2. *If $v_{i|\{t,t\}} = v_{j|\{t,t\}} < 0$, $v_{i|\{t,nt\}} = v_{j|\{t,nt\}} > 0$, there exists a unique number $p^{int} \in (1/\rho, 1 - 1/\rho)$ (ρ denotes the Euler's number here) such that $u_i^{PT}(1, p^{int}) = u_j^{PT}(1, p^{int})$. Further, $u_i^{PT}(1, p_j) \leq u_j^{PT}(1, p_i)$ and $u_i^{PT}(1, p_j)$ and $u_j^{PT}(1, p_i)$ are convex when $p_j, p_i \in [0, p^{int})$, respectively. Meanwhile, $u_i^{PT}(1, p_j) \geq u_j^{PT}(1, p_i)$ and $u_i^{PT}(1, p_j)$ and $u_j^{PT}(1, p_i)$ are concave when $p_j, p_i \in [p^{int}, 1]$.*

Proof. This lemma shows that when both PT players transmit with probability one, the two PT players' utility functions have similar shape, i.e., first convex and then concave. Further, the PT player with a smaller probability weighting index α has a utility function of larger curvature than the other PT player that is a direct result from Fig. 3.1. Note that, $u_i^{PT}(1, 1/\rho) = w_i(1/\rho)v_{i|\{t,t\}} + w_i(1-1/\rho)v_{i|\{t,nt\}} < w_j(1/\rho) \times v_{i|\{t,t\}} + w_j(1-1/\rho) \times v_{i|\{t,nt\}} = u_j^{PT}(1, 1/\rho)$. This holds because $v_{i|\{t,t\}} < 0$, $v_{i|\{t,nt\}} > 0$ and the probability weighting function has the property that $w_i(1/\rho) = 1/\rho$ and $w_i(1 - 1/\rho) < w_j(1 - 1/\rho)$. Similarly, we can show that $u_i^{PT}(1, 1 - 1/\rho) = w_i(1 - 1/\rho)v_{i|\{t,t\}} + w_i(1/\rho)v_{i|\{t,nt\}} > w_j(1-1/\rho) \times v_{i|\{t,t\}} + w_j(1/\rho) \times v_{i|\{t,nt\}} = u_j^{PT}(1, 1-1/\rho)$. Thus, there must exist a point $p^{\text{int}} \in (1/\rho, 1 - 1/\rho)$ such that $u_i^{PT}(1, p^{\text{int}}) = u_j^{PT}(1, p^{\text{int}})$ due to the continuities of both $u_i^{PT}(1, p_j)$ and $u_j^{PT}(1, p_j)$. Similar to the proof of Lemma 5, the rest of the lemma can be proven based on the definition of probability weighting function (3.3) and from Fig. 3.1 and the fact that both utility functions above are strictly decreasing functions. The above properties can be seen in Fig. 3.6. □

Theorem 3.5.3. ***(Consequence (C4))*** *In a 2-player homogeneous PT game, the PT player i who deviates more from EUT (with a smaller probability weighting index α) achieves less average throughput, suffers more average delay but consumes less average energy than the other PT player j. Further, both PT players transmit more aggressively than the EUT players of the corresponding homogeneous EUT game if $v_{i|\{t,t\}} = v_{j|\{t,t\}}$, $v_{i|\{t,nt\}} = v_{j|\{t,nt\}}$ and $v_{i|\{t,nt\}} > -\rho d_i + (1-\rho)v_{i|\{t,t\}}$.*

Proof. From Theorem 3.4.6, it can be shown that given $v_{i|\{t,t\}} = v_{j|\{t,t\}}$, $v_{i|\{t,nt\}} = v_{j|\{t,nt\}}$ and $v_{i|\{t,nt\}} > -\rho d_i + (1-\rho)v_{i|\{t,t\}}$, at the mixed NE $p_i^{\text{P}}, p_j^{\text{P}} \in [p^{\text{int}}, 1]$ and therefore $p_i^{\text{P}} < p_j^{\text{P}}$

and $p_i^{\mathsf{P}}, p_j^{\mathsf{P}} > p_i^{\mathsf{EUT}} = p_j^{\mathsf{EUT}}$. Moreover, from Lemma 3.4.3 and 3.4.4, it can be concluded that player i achieves less average throughput and suffers more average delay. However, she employs less energy than player j since she transmits less aggressively. □

Theorem 3.5.4. *(**Consequence (C5)**) Suppose the mixed NE of the homogeneous EUT game $[p_i^{EUT}, p_j^{EUT}]$ is such that $p_i^{EUT} > \frac{p_{j|\{j\}}}{\sum_{i=1,2}(p_{i|\{i\}} - p_{i|\{i,j\}})}$. Then in a 2-player homogeneous PT game, the two PT players achieve less sum average throughput, suffer more sum average delay and consume more sum average energy than those of the corresponding homogeneous EUT game. In other words, the deviation of the PT players in the homogeneous PT game harms the system performance in every aspect.*

Proof. From theorem 3.4.8, it is shown that if $p_j > \frac{p_{i|\{i\}}}{\sum_{i=1,2}(p_{i|\{i\}} - p_{i|\{i,j\}})}$, then $\partial T_s/\partial p_i < 0$ and $\partial D_s/\partial p_i > 0$. Consider a heterogeneous game derived from the homogeneous PT game where player j becomes an EUT player. Then, $p_j^{\mathsf{P}} = p_j^{\mathsf{H}} > p_j^{\mathsf{EUT}}$ and $p_i^{\mathsf{P}} > p_i^{\mathsf{H}}$ since player j faces the same opponent in the two games but player i faces different opponents. Therefore, if $p_j^{\mathsf{EUT}} > \frac{p_{i|\{i\}}}{\sum_{i=1,2}(p_{i|\{i\}} - p_{i|\{i,j\}})}$, it is true that $p_j^{\mathsf{P}} > \frac{p_{i|\{i\}}}{\sum_{i=1,2}(p_{i|\{i\}} - p_{i|\{i,j\}})}$. Then, from theorem 3.4.8, sum average throughput and sum average delay will be a decreasing and an increasing function of p_i at the mixed NEs, i.e., $\partial T_s/\partial p_i < 0$ and $\partial D_s/\partial p_i > 0$, respectively. Thus, it can be concluded the homogeneous PT game achieves less sum average throughput but suffers more average delay than the heterogeneous game as $p_i^{\mathsf{P}} > p_i^{\mathsf{H}}$. Together with theorem 3.4.8, the statement in the theorem can be shown. □

3.6 N-Player Homogeneous Games

We further consider a N-player symmetric homogeneous setting where all the players have identical utility functions and experience the same channel conditions. Note that, the N-player symmetric game reflects a realistic scenario where every player has a collective (aggregated) view of the set of the players in the game. Further, analyzing and evaluating the perceptions of each of the other $N-1$ players is beyond the feasibility of a single end-user's action. We now state the following theorem.

Theorem 3.6.1. *If each player's utility function* (3.6) *is a strictly decreasing function of her opponent's transmission probability and neither $\{t\}$ nor $\{nt\}$ is the pure dominant*

strategy for each player, there exists a unique mixed NE for an N-player homogeneous wireless random access game with symmetric utility functions.

Proof. The existence of the mixed NEs can be established by generalizing the results in theorem 3.4.1. Since the game is symmetric, all the players share the identical utility function, i.e., $u_i^{PT}(\cdot) = u_j^{PT}(\cdot),\ \forall i \neq j$. Further, each player will choose the identical transmission probability at the mixed NE. Otherwise, assuming there exists a mixed NE such as $\mathbf{p}^* = [p^*, ..., p_i^*, ..., p_j^*, ..., p^*]$ and without loss of generality, two players have different transmission probabilities, i.e., $p_i^* \neq p_j^*$, at the mixed NE, we have $u_i^{PT}(1, \mathbf{p}_{-i}^*) \neq u_j^{PT}(1, \mathbf{p}_{-j}^*)$. However, this contradicts the symmetry condition where at the mixed NE all the players' utilities should be the same, i.e., $-d_i = -d, \forall i \in \mathcal{N}$. By following the same rationale, it can be shown that at the mixed NE, each player's transmission probability must be equal. Next, we will show the uniqueness of this symmetric mixed NE. Assuming $\mathbf{p}^* = [p^*, ..., p^*]_{N\times 1}$ and $\mathbf{p}' = [p', ..., p']_{N\times 1}$ are two different mixed NEs for the game, we can establish that:

$$u_i^{PT}(1, \mathbf{p}_{-i}^*) = u_i^{PT}(1, \mathbf{p}_{-i}') = -d_i,\ \forall i \in \mathcal{N}, \tag{3.19}$$

where $\mathbf{p}_{-i}^*$ and $\mathbf{p}_{-i}'$ denote the i-th player's opponent transmission probability vector, i.e., $\mathbf{p}_{-i}^* = [p^*]_{(N-1)\times 1}$ and $\mathbf{p}_{-i}' = [p']_{(N-1)\times 1}$. If $p^* < p'$, it can be shown that $u_i^{PT}(1, \mathbf{p}_{-i}^*) > u_i^{PT}(1, \mathbf{p}_{-i}')$ since $u_i^{PT}(\cdot)$ is a strictly decreasing function of each i-th player's opponent's transmission probability. If $p^* > p'$, $u_i^{PT}(1, \mathbf{p}_{-i}^*) < u_i^{PT}(1, \mathbf{p}_{-i}')$. Thus, if $p^* \neq p'$, (3.19) will not be established and a contradiction exists. Then, the mixed NE of the N-player symmetric homogeneous wireless random access game is shown to be unique. Note that, an N-player symmetric homogeneous EUT game is just a special case where every player's probability weighting index is 1. □

In the following example, we compare the characteristics of mixed NEs of 3-player homogeneous PT and homogeneous EUT games where $\alpha_i = 0.6$, $p_{i|\{i\}} = 1$ and $p_{i|\mathcal{J}(\mathbf{a})} = 0, \forall \mathcal{J}(\mathbf{a}) \setminus \{i\} \neq \emptyset$, $\forall i \in \mathcal{N}$. Further, we assume $e = d = 2$ and c varies as multiples of e. Note that, due to symmetry, the three players in either PT or EUT games have identical performance.

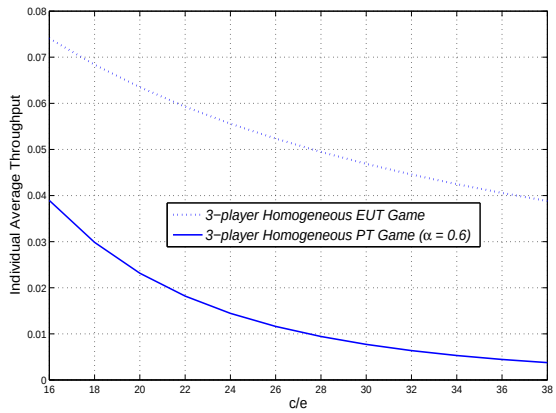


Figure 3.11: Average throughput comparison between PT and EUT Games. All the players in the each game have the same average throughput due to the symmetry.

From Fig. 3.11, it can be observed that the players in the homogeneous EUT game achieve higher individual and thus sum average throughput than that in the homogeneous PT game. Moreover, the homogeneous PT game is also observed to have larger individual and sum average delay and average energy consumption than the homogeneous EUT game.

However, the uniqueness of the mixed NE can still be established for the scenario where $N = 3$. Assuming $\mathbf{p}_1 = [p_1^*, p_2^*, p_3^*]$ and $\mathbf{p}_2 = [p_1', p_2', p_3']$ are two different mixed NEs, it can be established that:

$$u_1^{PT}(1, p_2^*, p_3^*) = u_1^{PT}(1, p_2', p_3') = -d_1 \tag{3.20}$$

$$u_2^{PT}(p_1^*, 1, p_3^*) = u_2^{PT}(p_1', 1, p_3') = -d_2 \tag{3.21}$$

$$u_3^{PT}(p_1^*, p_2^*, 1) = u_3^{PT}(p_1', p_2', 1) = -d_3. \tag{3.22}$$

Without loss of generality, we can assume that $p_2^* < p_2'$ and $p_3^* > p_3'$. To establish (3.21), it is necessary that $p_1^* < p_1'$. Then, (3.22) cannot be established since $u_3^{PT}(p_1, p_2, 1)$ is a strictly decreasing function of p_1 and p_2. Thus, (3.20) - (3.22) can be simultaneously established only when $\mathbf{p}_1 = \mathbf{p}_2$.

Chapter 4

Impact and Influence of Prospect Theory on Data Pricing

We start from a basic two-tier data pricing scheme as shown in Fig. 4.1. The service provider charges the end users by providing them with data services. After the service provider announces the pricing scheme to the end users, the end users make the decision on how to utilize this service. Depending on the detailed design, the response could be whether using the service or not, the frequency to use the service, the probability to use the service and etc.. Then, the end users will utilize the resources according to their decisions and the wireless network is operated at a corresponding point. The service provider will collect the fees and information on the status of the whole network. Then, depending on the designer's or operator's objective, the service provider can adjust the pricing scheme to push the whole network working at an optimal point, e.g., the service provider can collect the most revenues.

In this work, we extend the above exemplary wireless random access model to study the data pricing scheme. Similarly, the practical pricing schemes in the real systems resort much more complex designs and operations. However, given the complexities of the end users' behaviors, these unnecessary details will not help us better understand the role of PT in the data pricing scheme at the initial stage. Therefore, we further utilize this random access example to study the impact of end users' behaviors on the data pricing schemes.

At the service provider level, the service provider's utility function is composed of two parts, i.e., the revenue $R(\mathbf{p})$ and the cost $C(\mathbf{p})$ where $\mathbf{p}$ is the vector of probabilities of the transmission for all the end users as in the above wireless random access game. $\mathbf{p}$ actually captures the willingness of end users to utilize the data services. The revenue $R(\mathbf{p})$ and the cost $C(\mathbf{p})$ are both the results of end users' decisions on how to utilize the

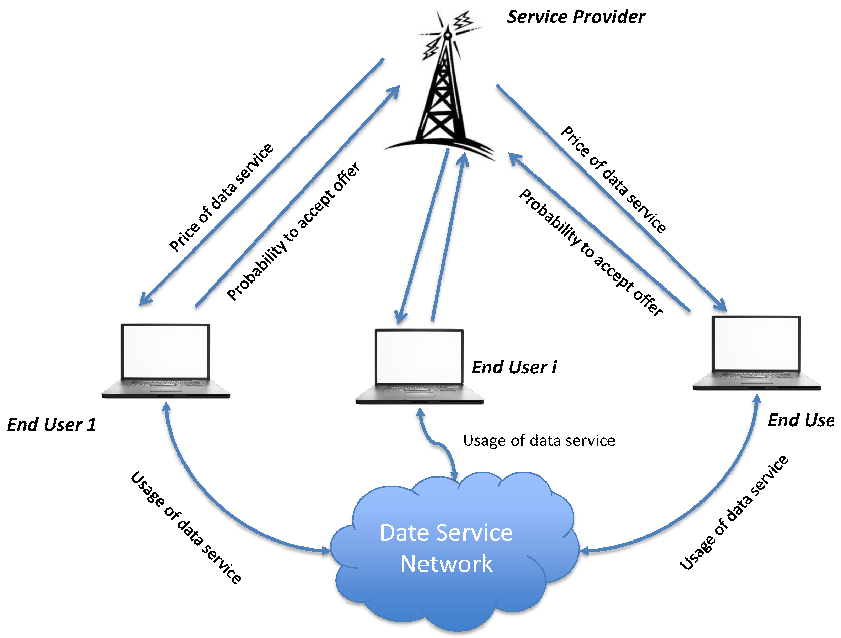


Figure 4.1: A Two-tier Wireless Data Service Pricing Model

data services. In general, $C(\mathbf{p})$ should be an increasing function of $\mathbf{p}$. However, the exact forms of $R(\mathbf{p})$ and $C(\mathbf{p})$ depend on the specific data pricing schemes and will be discussed later. Generally, the service provider aims to maximize her profits as follows:

$$\max_{\mathbf{r}} u_{sp}(\mathbf{p}) = R(\mathbf{p}) - C(\mathbf{p}), \tag{4.1}$$

where $\mathbf{r} = [r_1, ..., r_i, ...r_N]$ is the price vector imposed by the service provider and r_i represents the unit fee the i-th end user needs to pay to use the data service. Note that, revenue and cost are both functions of transmission probability vector $\mathbf{p}$ that is a function of the price vector $\mathbf{r}$. Thus, the service provider's decision on the price vector $\mathbf{r}$ can determine end users' usage on the network resources. The service provider can tune the price vector $\mathbf{r}$ to obtain the optimal utility. In the current work, we consider a constant operational cost for the service provider, i.e., $C(\mathbf{p})$ is constant. Thus, the service provider's

revenue optimization problem can be further derived as:

$$\max_{\mathbf{r}} u_{sp}(\mathbf{p}) = R(\mathbf{p}), \tag{4.2}$$

In this work, we consider two pricing schemes, namely usage based pricing and throughput based pricing schemes. The usage based pricing scheme allows the service provider to charge the end users based on their actual usage of the resources. Per time slot, if the i-th end user chooses to transmit, the service provider will charge r_i for the data service. Therefore, in each time slot, the service provider will collect a revenue as follows:

$$R^u(\mathbf{p}) = \sum_{i \in \mathcal{N}} r_i p_i, \tag{4.3}$$

where $r_i p_i$ represents the expected fee the service provider can collect from the i-th end user.

The throughput based pricing scheme only allows the service provider to charge the end users on the successful transmissions. Per time slot, if the i-th end user chooses to use the channel with probability p_i, then the fee will only be charged proportional to her final throughput. The exact form of the service provider's revenue is as follows:

$$R^t(\mathbf{p}) = \sum_{i \in \mathcal{N}} r_i p_i T_i, \tag{4.4}$$

where $T_i = \sum_{\mathbf{a} \in \mathbf{A}} \left(\prod_{j \in \mathcal{J}(\mathbf{a})} p_j \prod_{j \in \mathcal{J}^c(\mathbf{a})} (1 - p_j) p_{i|\mathcal{J}(\mathbf{a})} \right)$ is the expected throughput of the i-th end user given she chooses to transmit in this time slot.

At the end user level, given the price announced by the service provider, the end users decide to access the data services via a wireless random access game. Their final decisions on how to use the services, i.e., the transmission probabilities, are reflected by the mixed NE of the corresponding game. The resulting mixed NE, i.e., the transmission probability vector $\mathbf{p}$, and the price vector $\mathbf{r}$ compose of the operation point of the network. Therefore, the mathematical model of the end user level is almost the same as the wireless random access games discussed above in this chapter. However, the end users' value functions for a given strategy vector as shown in Eq. (3.4) should be modified to reflect the impact

of pricing schemes on end users' utilities. Note that, the end users in the data pricing model are also players in their underlying wireless random access game. Hence, in the following, "end user" is consistently used to avoid confusion of terms. For the usage based pricing scheme, whenever the i-th end user chooses to transmit in the time slot, she will be charged by the service provider by r_i regardless of the delivery result. Therefore, her value function given a strategy vector should be modified as follows:

$$v^u_{i|\mathbf{a}} = \begin{cases} p_{i|\mathcal{J}(\mathbf{a})}(c_i - e_i - r_i) + (1 - p_{i|\mathcal{J}(\mathbf{a})})(-e_i - d_i - r_i) & \text{if } a_i = t \\ -d_i & \text{if } a_i = nt, \end{cases} \tag{4.5}$$

where the notations carry the same physical meanings as in Eq. (3.4). For the throughput based pricing scheme, the end user is charged only for her throughput. No fee will be paid by the end user if the delivery of her packet is failed. Therefore, the i-th end user's value function is:

$$v^t_{i|\mathbf{a}} = \begin{cases} p_{i|\mathcal{J}(\mathbf{a})}(c_i - e_i - r_i) + (1 - p_{i|\mathcal{J}(\mathbf{a})})(-e_i - d_i) & \text{if } a_i = t \\ -d_i & \text{if } a_i = nt. \end{cases} \tag{4.6}$$

As discussed in earlier part of this chapter, both PT and EUT players retain the same value function. In other words, players in a usage based pricing scheme have the same value function as shown in Eq. (4.5) regardless of their preferences over prospects, either PT or EUT. The same holds true for the throughput based pricing scheme.

From the discussion in the wireless random access game regarding the impact of end users' behaviors, we have seen that the involved and intractable form of the probability weighting function and the resulting utility functions usually make any analysis a difficult task. Therefore, we start the study on the data pricing schemes from the a basic scenario, e.g., a 2-user model. Moreover, we can spell out the mathematical models under different

data pricing schemes. The service provider's utility functions can be shown as:

$$u_{sp}^{u}(\mathbf{p}) = \sum_{i=1,2} r_i p_i \tag{4.7}$$

$$u_{sp}^{t}(\mathbf{p}) = \sum_{i=1,2} r_i p_i (p_j p_{i|\{i,j\}} + (1-p_j) p_{i|\{i\}}), \tag{4.8}$$

where the j-th user refers to the i-th user's opponent.

Under the usage based pricing scheme, end user's value function when the end user transmits is as follows:

$$v_{i|\{t,nt\}}^{u} = p_{i|\{i\}}(c_i - e_i - r_i) + (1 - p_{i|\{i\}})(-e_i - d_i - r_i) \tag{4.9}$$

$$v_{i|\{t,t\}}^{u} = p_{i|\{i,j\}}(c_i - e_i - r_i) + (1 - p_{i|\{i,j\}})(-e_i - d_i - r_i). \tag{4.10}$$

Similarly, the value functions for the throughput data pricing scheme is as follows:

$$v_{i|\{t,nt\}}^{t} = p_{i|\{i\}}(c_i - e_i - r_i) + (1 - p_{i|\{i\}})(-e_i - d_i) \tag{4.11}$$

$$v_{i|\{t,t\}}^{t} = p_{i|\{i,j\}}(c_i - e_i - r_i) + (1 - p_{i|\{i,j\}})(-e_i - d_i). \tag{4.12}$$

Finally, the end user's utility functions can be shown as follows:

$$u_{i,u}^{PT}(\mathbf{p}) = p_i w_i(p_j) v_{i|\{t,t\}}^{u} + p_i w_i(1-p_j) v_{i|\{t,nt\}}^{u} + (1-p_i)(-d_i) \tag{4.13}$$

$$u_{i,t}^{PT}(\mathbf{p}) = p_i w_i(p_j) v_{i|\{t,t\}}^{t} + p_i w_i(1-p_j) v_{i|\{t,nt\}}^{t} + (1-p_i)(-d_i) \tag{4.14}$$

$$u_{i,u}^{EUT}(\mathbf{p}) = p_i p_j v_{i|\{t,t\}}^{u} + p_i(1-p_j) v_{i|\{t,nt\}}^{u} + (1-p_i)(-d_i) \tag{4.15}$$

$$u_{i,t}^{EUT}(\mathbf{p}) = p_i p_j v_{i|\{t,t\}}^{t} + p_i(1-p_j) v_{i|\{t,nt\}}^{t} + (1-p_i)(-d_i) \tag{4.16}$$

where $u_{i,u}^{PT}(\mathbf{p})$ and $u_{i,t}^{PT}(\mathbf{p})$ denote a PT user's utility function under usage based and throughput based pricing schemes, respectively. Correspondingly, $u_{i,u}^{EUT}(\mathbf{p})$ and $u_{i,t}^{EUT}(\mathbf{p})$ refer to an EUT player's utility functions under both the data pricing schemes, respectively. Mathematically, an EUT user's utility function can be considered as a special case of a PT user's, i.e., the probability weighting index $\alpha = 1$. In the following, we denote the Usage-Based Pricing model as UBP where service provider maximizes Eq. (4.7) and end

users maximize Eq. (4.15) or Eq. (4.13). Similarly, the Throughput-Based Pricing model (TBP) can be defined. For each data pricing scheme, we will consider 2 scenarios: (1) both end users are EUT users (denoted as UBP-EUT or TBP-EUT) and (2) both end users are PT users (denoted as UBP-PT or TBP-PT). .

4.1 The Existence and Uniqueness of the Operating Point of a Two-Level Data Pricing Model

According to the structure of the data pricing model, the existence and uniqueness of the operating point is determined by the relationship among service provider's price and end users' utility functions. Furthermore, given a service provider's price vector $\mathbf{r}$, the existence and uniqueness of the operation point is equivalent to that of the mixed NE of the end-user level wireless random access game. Therefore, the conclusions on the existence and uniqueness of the operation point can be established as follows.

Theorem 4.1.1. *There exists a unique operation point* $[\mathbf{r}^*, \mathbf{p}^*]$ *for the UBP with either PT end users or EUT end users if* $v^u_{i|\{t,nt\}} > 0$ *and* $v^u_{i|\{t,t\}} < -d_i$ *for* $i = 1, 2$.

Proof. The scenarios where both users are EUT, i.e., UBP-EUT, can be mathematically considered as special cases of the scenario where both end users are PT users. Thus, we will establish the existence and uniqueness of operation point for UBP-PT and that of the other scenario can also be established. From theorem 3.4.1, it can be shown that, for a fixed a pricing vector $\mathbf{r} = [r_1, r_2]$, if $v^u_{i|\{t,nt\}} > 0$ and $v^u_{i|\{t,t\}} < -d_i$ for $i = 1, 2$, there exists a unique mixed NE $[p_1^*, p_2^*]$ in the corresponding end user level game, i.e, a homogeneous PT game. By applying Eq. (4.9) and Eq. (4.10) in the conditions, we can conclude that if $v^u_{i|\{t,nt\}} = p_{i|\{i\}}(c_i - e_i) + (1 - p_{i|\{i\}})(-e_i - d_i) > r_i$ and $p_{i|\{i,j\}}(c_i - e_i) + (1 - p_{i|\{i,j\}})(-e_i - d_i) > r_i - d_i$, there exists a unique operation point in UBP-PT. The same argument holds true for UBP-EUT. □

Corollary 4.1.2. *There exists a unique operation point* $[\mathbf{r}^*, \mathbf{p}^*]$ *for the TBP with either PT end users or EUT end users if* $v^t_{i|\{t,nt\}} > 0$ *and* $v^t_{i|\{t,t\}} < -d_i$ *for* $i = 1, 2$.

4.2 Impact and Influence of End-User Behavior on Usage Based Pricing Scheme

In the above discussion over wireless random access game, it has been clearly shown that end users' behavior which usually deviates from EUT can obviously impact and change the system and individual performance regarding throughput, energy and delay. As a result, service provider's utility will also be altered by this deviation. In the following, we will compare the end users' actions and the service provider's utility between two scenarios, i.e., UBP-EUT and UBP-PT. We will study the following issues: (1) Variations of operation point of the data pricing scheme under the two scenarios and (2) Differences of service provider's utility at the operation point under the two scenarios. In the following, due to the mathematical intractability mentioned in the chapter of wireless random access game, we use numerical methods to demonstrate the characteristics of the pricing models and comparison results. However, we also analytically prove and validate these findings under mild conditions. Without loss of generality, in UBP-PT, we assume $\alpha_1 = 0.5$ and $\alpha_2 = 0.8$. For both UBP-EUT and UBP-PT, two users randomly access the channel with $p_{i|\{i\}} = p_{j|\{j\}} = 0.98$, $p_{i|\{i,j\}} = p_{j|\{i,j\}} = 0.1$, $i = 1, 2$. We begin with a special case where there is no delay penalty for each user as $c_1 = c_2 = c = 20$, $e_1 = e_2 = e = 2$, $d_1 = d_2 = d = 0$ and the price charged varies, i.e., $r_1 = r_2 = r$. As a result, $v^u_{i|\{t,nt\}} = v^u_{j|\{t,nt\}}$ and $v^u_{i|\{t,t\}} = v^u_{j|\{t,t\}}$. The system performances are studied and compared as a function of r/e.

Lemma 4.2.1. *An end user's utility when she chooses to transmit, i.e., $u^{PT}_{i,u}(1, p_j)$ or $u^{EUT}_{i,u}(1, p_j)$, is a decreasing function of the charged price r_i given a fixed p_j.*

Proof. Under the usage based pricing scheme, whenever the end user chooses to transmit, she will be charged a unit price r_i. A larger value of r_i will result a less value of the user's utility. Moreover, given a fixed p_j, it is true that $\partial u^{PT}_{i,u}(1, p_j)/\partial r_i = -(w_i(p_j)+w_i(1-p_j)) < 0$. In particular, $\partial u^{EUT}_{i,u}(1, p_j)/\partial r_i = -1 < 0$. Thus, the statement is established. □

The above lemma 4.2.1 can provide us with some hint about the influence of the price $\mathbf{r}$ to the transmission probabilities. The transmission probabilities at the mixed NE is determined by solving $u^{PT}_{i,u}(1, p_j) = -d_i$ or $u^{EUT}_{i,u}(1, p_j) = -d_i$. Thus, a decrease in the

value of $u_{i,u}^{PT}(1, p_j)$ or $u_{i,u}^{EUT}(1, p_j)$ must result smaller transmission probabilities at the mixed NE. This qualitative argument can be shown as in the following figure. In Fig. 4.2,

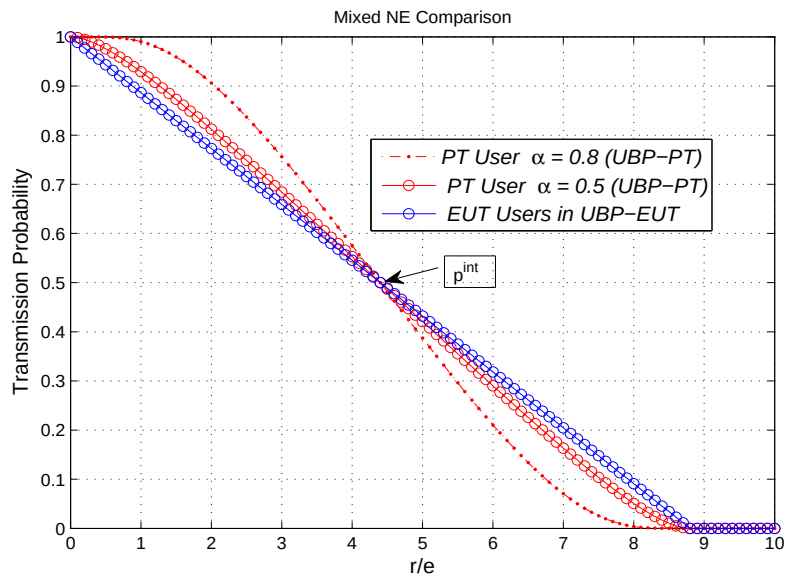


Figure 4.2: Transmission probabilities at the operation point for UBP

it can be seen that, in both UBP-EUT and UBP-PT, end users' transmission probabilities at the operation point decreases as price r increases. Further, as proven in theorem 3.5.3, PT users in UBP-PT transmit more aggressively given $v^u_{i|\{t,nt\}} > -\rho d_i + (1-\rho)v^u_{i|\{t,t\}}$, $v^u_{i|\{t,nt\}} = v^u_{j|\{t,nt\}}$ and $v^u_{i|\{t,t\}} = v^u_{j|\{t,t\}}$. This indicates that the service provider will expect a lower service utilization level if she charges a larger price if $v^u_{i|\{t,nt\}} > 0$ and $v^u_{i|\{t,t\}} < -d_i$ for $i = 1, 2$. This argument holds true for both EUT user and PT user scenarios and can be analytically proven in theorem 4.2.2. Another interesting observation in Fig. 4.2 is that when the price charged becomes relatively large, the PT users in UBP-PT will choose a more conservative utilization of the data services. In particular, there exists a transmission probability where $p_i^{EUT} = p_j^{EUT} = p_i^{PT} = p_j^{PT} = p^{int}$ as shown lemma 3.4.5. This phenomenon can be analytically proven in lemma 4.2.3 in a similar approach used in theorem 3.5.3.

Theorem 4.2.2. *End users' transmission probabilities p_1, p_2 will be a decreasing function of the price r, i.e., end users will choose a lower service utilization level if service provider charges a larger price for both UBP-PT and UBP-EUT if $v^u_{i|\{t,nt\}} > 0$ and $v^u_{i|\{t,t\}} < -d_i$*

for $i = 1, 2$.

Proof. End user i's transmission probability p_i at the operation point can be determined by solving $\mathcal{F}(r, p_i(r)) = v^u_{j|\{t,nt\}} w_j(1-p_i) + v^u_{j|\{t,t\}} w_j(p_i) + d_j = 0$. By differentiating both sides with regard to r, it can be shown that $\partial\mathcal{F}/\partial r + \partial\mathcal{F}/\partial p_i \cdot dp_i/dr = 0$. Finally, it is true that

$$dp_i/dr = -(\partial\mathcal{F}/\partial r)/(\partial\mathcal{F}/\partial p_i) = \frac{w_j(p_i) + w_j(1-p_i)}{v^u_{j|\{t,nt\}}\partial w_j(1-p_i)/\partial p_i + v^u_{j|\{t,t\}}\partial w_j(p_i)/\partial p_i} < 0, \tag{4.17}$$

where $w_j(p_i) + w_j(1-p_i) > 0$ and $v^u_{j|\{t,nt\}}\partial w_j(1-p_i)/\partial p_i + v^u_{j|\{t,t\}}\partial w_j(p_i)/\partial p_i < 0$ as shown in theorem 4.1.1. □

Lemma 4.2.3. *PT users in UBP-PT will use the data service in a less aggressive manner than the EUT users in UBP-EUT if* $v^u_{i|\{t,nt\}} > (1+\rho)v^u_{i|\{t,t\}} + \rho d_i,\ i = 1, 2$.

Proof. The EUT users' transmission probabilities in UBP-EUT can be easily determined as $p_i^{EUT} = p_j^{EUT} = p^{EUT} = \frac{-d - v^u_{i|\{t,nt\}}}{v^u_{i|\{t,t\}} - v^u_{i|\{t,nt\}}}$. From lemma 3.4.5, it can be shown that $u^{PT}_{i,u}(1, p_j) \leq u^{EUT}_{i,u}(1, p_j)$ and $u^{PT}_{i,u}(1, p_j)$ is convex when $p^{EUT} < 1/\rho$. As a result, the horizontal line $u^{PT}_{i,u}(0, p_j) = u^{EUT}_{i,u}(0, p_j) = -d_i$ must intersect with $u^{PT}_{i,u}(1, p_j)$ at a point $p_j^{PT} < p_j^{EUT} = p^{EUT}$. Thus, the PT users will transmit less aggressively than the EUT users in this scenario. Further, $p^{EUT} < 1/\rho$ is equivalent to $v^u_{i|\{t,nt\}} > (1+\rho)v^u_{i|\{t,t\}} + \rho d_i$. □

In figure 4.3, the sum average throughput of both users are compared between UBP-EUT and UBP-PT. The result shows that with usage based data pricing scheme, the deviation of PT users from EUT still hurts the system performance in terms of throughput. This is actually a direct result from theorem 3.5.4. In Fig. 4.4, service provider's revenues are shown for both UBP-EUT and UBP-PT as she charges varying prices to the end users. It can be seen that service provider will gain more revenues when facing PT users when she charges a relative small fee. However, if the service provider keeps increasing the price, the revenue collected from PT users will drop more than that in the UBP-EUT. This can be intuitively explained from Fig. 4.2 and service provider's utility function $u^u_{sp}(\mathbf{p}) = \sum_{i=1,2} r_i p_i$. This can be theoretically analysed in the following corollary.

Corollary 4.2.4. *If $v^u_{i|\{t,nt\}} = v^u_{j|\{t,nt\}}$ and $v^u_{i|\{t,t\}} = v^u_{j|\{t,t\}}$, the service provider will obtain larger revenue when facing PT users (UBP-PT) than that when facing EUT users (UBP-EUT) given $v^u_{i|\{t,nt\}} > -\rho d_i + (1-\rho)v^u_{i|\{t,t\}}$. If $v_{i|\{t,nt\}} > (1+\rho)v_{i|\{t,t\}} + \rho d_i$, the trend is reversed and the service provider will obtain more revenue from EUT users in UBP-EUT.*

Proof. The service provider's revenue is defined as $u^u_{sp}(\mathbf{p}) = \sum_{i=1,2} r_i p_i$. Thus, given the same r, the revenue is totally determined by the transmission probabilities of end users. Thus, with the results in theorem 3.5.3 and lemma 4.2.3, the statement can be established. □

A more important result can be derived from Fig. 4.4 is the difference of optimal operation points between UBP-PT and UBP-EUT. It can be seen that both UBP-PT and UBP-EUT have unique optimal operation point. Meanwhile, the optimal revenue collected from UBP-PT is slightly larger than that from UBP-EUT. However, the optimal price charged in UBP-PT is slightly smaller than that in UBP-EUT. We can theoretically analyze the above results as follows.

Theorem 4.2.5. *The service provider's revenue collected from UBP-EUT is a strictly concave function of price r. As a result, there exists a unique optimal operation point in UBP-EUT where the service provider obtains the largest revenue.*

Proof. The second order derivative of service provider's revenue from the i-th EUT user to the price can be derived as $\partial^2(rp_i)/\partial r^2 = 2\partial p_i/\partial r + d^2 p_i/dr^2$. Moreover, $p_i = \frac{-d-v^u_{j|\{t,nt\}}}{v^u_{j|\{t,t\}}-v^u_{j|\{t,nt\}}}$. Thus, it can be easily derived as $\partial^2(rp_i)/\partial r^2 = 2/(v^u_{j|\{t,t\}} - v^u_{j|\{t,nt\}}) < 0$. In other words, the service provider's revenue from the i-th user is a strictly concave function of the price r and a unique optimal price can be found by equating $\partial(rp_i)/\partial r = p_i + r/(v^u_{j|\{t,t\}} - v^u_{j|\{t,nt\}}) = 0$, i.e., $r^* = v^u_{j|\{t,nt\}} + d_j$. The similar conclusion can be drawn for the revenue collected from the j-th user. Hence, given $v^u_{i|\{t,nt\}} = v^u_{j|\{t,nt\}}$ and $v^u_{i|\{t,t\}} = v^u_{j|\{t,t\}}$, a unique optimal price can be obtained as $r^* = v^u_{j|\{t,nt\}} + d_j = v^u_{i|\{t,nt\}} + d_i$. □

The similar observation regrading the uniqueness of the optimal price is made through numerous numerical simulation for the UBP-PT. However, due to the mathematical intractability of the utility function of PT users in UBP-PT, it is difficult to analytically

prove this argument. We provide this result as the following proposition.

Proposition 4.2.6. *There exists a unique optimal operation point in UBP-PT where the service provider obtains the largest revenue.*

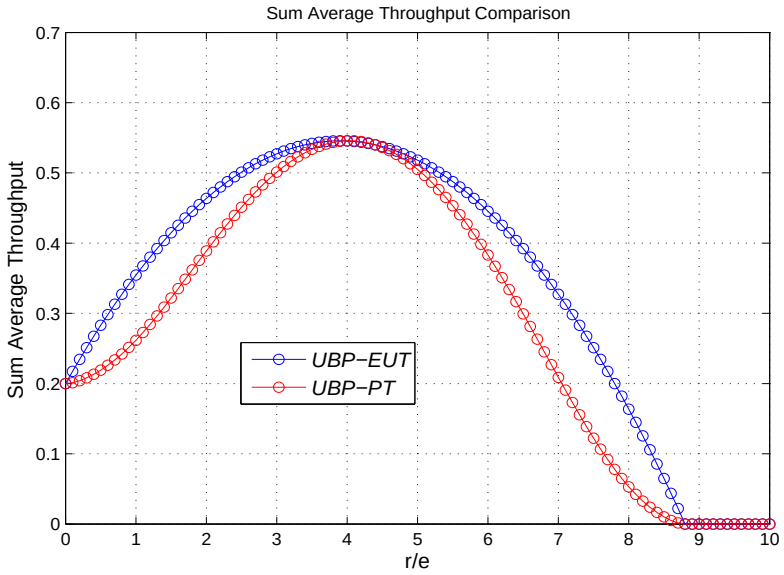


Figure 4.3: Sum Average Throughput at the operation point for UBP

In conclusion, for usage based pricing scheme, the type of the underlying users will influence system performance and service provider's revenue. First, a larger price will discourage the users from using the data service for both UBP-EUT and UBP-PT. Second, the deviation of PT users from EUT still degrades the system sum average throughput under the data pricing scheme. Third, under mild conditions as in corollary 4.2.4, the service provider will gain more revenue in UBP-PT than that in UBP-EUT for a small price. However, as the price increases, the trend will be reversed. Finally, service provider will find a unique optimal price to collect the most revenue in both UBP-PT and UBP-EUT. The optimal revenue collected from UBP-PT is slightly larger than that in UBP-EUT and the the optimal price in UBP-PT is slightly smaller than that in UBP-EUT.

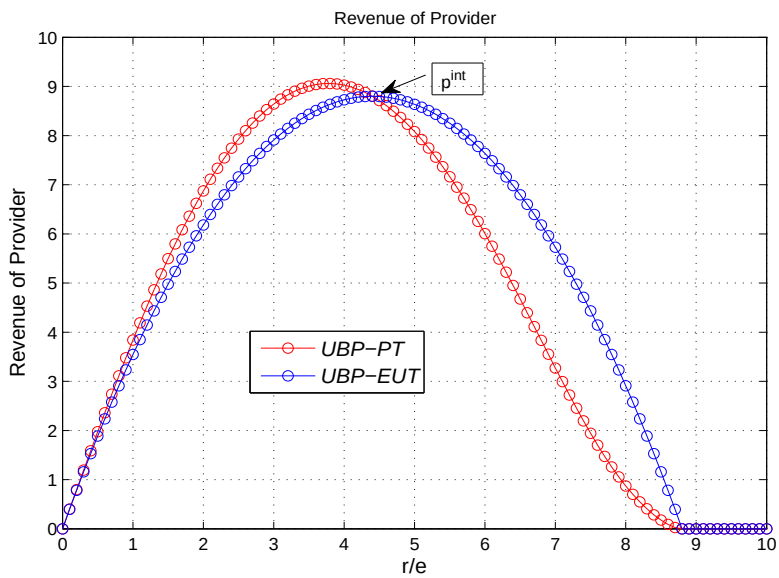


Figure 4.4: Service Provider's Revenue for UBP

4.3 Impact and Influence of End-User Behavior on Throughput Based Pricing Scheme

Unlike the usage based pricing scheme, the throughput based pricing scheme only charges the end users for successful transmissions. In the following, we will compare the service provider's revenue and system performance between two scenarios, i.e., the end users are both PT (TBP-PT) and the end users are both EUT (TBP-EUT). We follow a similar approach as in above section 4.2. In the following, in TBP-PT, we assume $\alpha_1 = 0.5$ and $\alpha_2 = 0.8$. For both TBP-EUT and TBP-PT, two users randomly access the channel with $p_{i|\{i\}} = p_{j|\{j\}} = 0.98$, $p_{i|\{i,j\}} = p_{j|\{i,j\}} = 0.1$, $i = 1, 2$. We begin with a special case where there is no delay penalty for each user as $c_1 = c_2 = c = 20$, $e_1 = e_2 = e = 2$, $d_1 = d_2 = d = 0$ and the price charged varies, i.e., $r_1 = r_2 = r$. As a result, $v^t_{i|\{t,nt\}} = v^t_{j|\{t,nt\}}$ and $v^t_{i|\{t,t\}} = v^t_{j|\{t,t\}}$. The system performances are studied and compared as a function of r/e.

In the usage based pricing scheme, we have shown that a larger price charged by the service provider will discourage the end users from using the data services for both PT and EUT users. The same argument can also be established for the throughput based pricing scheme as in the following lemma 4.3.1 and theorem 4.3.2. A numerical result can be found in Fig. 4.5.

Lemma 4.3.1. *An end user's utility when she chooses to transmit, i.e.,* $u_{i,t}^{PT}(1, p_j)$ *or* $u_{i,t}^{EUT}(1, p_j)$*, is a decreasing function of the charged price* r_i *given a fixed* p_j*.*

Proof. Given a fixed p_j, it is true that $\partial u_{i,t}^{PT}(1, p_j)/\partial r_i = -(p_{i|\{i,j\}} w_i(p_j) + p_{i|\{i\}} w_i(1 - p_j)) < 0$. In particular, $\partial u_{i,t}^{EUT}(1, p_j)/\partial r_i = -p_{i|\{i,j\}} p_j - p_{i|\{i\}}(1 - p_j) < 0$. Thus, the statement is established. □

Theorem 4.3.2. *End users' transmission probabilities* p_1, p_2 *will be a decreasing function of the price* r*, i.e., end users will choose a lower service utilization level if service provider charges a larger price for both TBP-PT and TBP-EUT.*

Proof. Similar to proof of theorem 4.2.2, it is true that

$$dp_i/dr = \frac{w_j(p_i) p_{i|\{i,j\}} + w_j(1 - p_i p_{i|\{i\}})}{v_{j|\{t,nt\}}^t \partial w_j(1 - p_i)/\partial p_i + v_{j|\{t,t\}}^t \partial w_j(p_i)/\partial p_i} < 0, \tag{4.18}$$

where $w_j(p_i) p_{i|\{i,j\}} + w_j(1 - p_i) p_{i|\{i\}} > 0$ and $v_{j|\{t,nt\}}^t \partial w_j(1 - p_i)/\partial p_i + v_{j|\{t,t\}}^t \partial w_j(p_i)/\partial p_i < 0$ as shown in 4.1.2. □

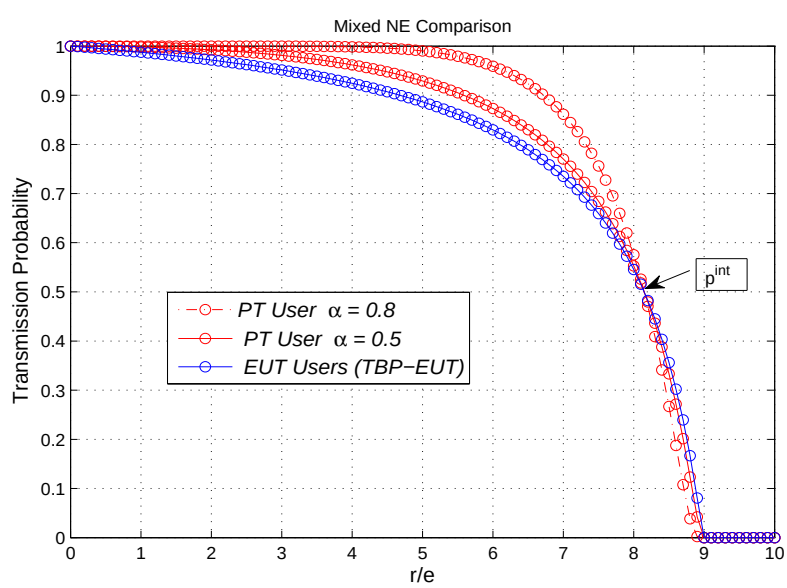


Figure 4.5: Transmission probabilities at the operation point for TBP

Similarly, in throughput based pricing scheme, UBP-PT still suffers a sum average throughput degradation due to PT users' deviation from EUT compared to TBP-EUT as

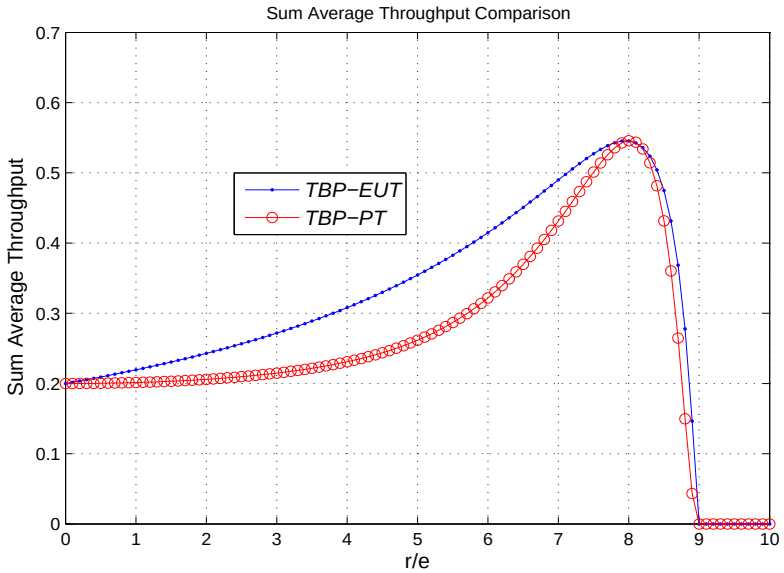


Figure 4.6: Sum Average Throughput at the operation point for TBP

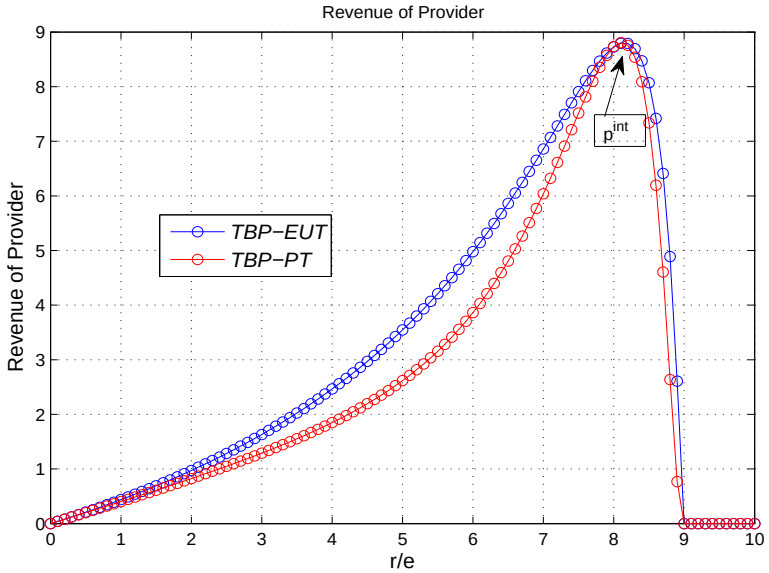


Figure 4.7: Service Provider's Revenue for TBP

shown in Fig. 4.6. Moreover, in throughput based pricing scheme the service provider's revenue is a linear function of the sum average throughput. Thus, it can be expected that the service provider will obtain a larger revenue in TBP-EUT than TBP-PT for the same price r charged under mild conditions shown in Fig. 4.7. This can be analytically proven in the following theorem.

Theorem 4.3.3. *If $v^t_{i|\{t,nt\}} = v^t_{j|\{t,nt\}}$ and $v^t_{i|\{t,t\}} = v^t_{j|\{t,t\}}$, the service provider will obtain a larger revenue when facing EUT users (TBP-EUT) than that when facing PT users (TBP-PT).*

Proof. From theorem 3.5.3 and 3.5.4, it is true that given $v^t_{i|\{t,nt\}} > -\rho d_i + (1-\rho)v^t_{i|\{t,t\}}$, $v^t_{i|\{t,nt\}} = v^t_{j|\{t,nt\}}$ and $v^t_{i|\{t,t\}} = v^t_{j|\{t,t\}}$, TBP-EUT will reserve a larger sum average throughput than that in TBP-PT. Moreover, $u^t_{sp}(\mathbf{p}) = \sum_{i=1,2} r_i p_i T_i = r \sum_{i=1,2} p_i T_i$. Thus, the service provider will obtain a larger revenue in TBP-EUT. Further, as shown in lemma 3.4.5 and Fig. 4.5, as r increases, the revenue collected will also increase until both TBP-PT and TBP-EUT arrive at the same transmission probability, i.e., p^{int}. Beyond this point, the revenue collected will decrease and TBP-EUT still generates more revenue than TBP-PT. □

From Fig. 4.7 and numerous numerical simulation, it can observed that there exists unique optimal price in TBP-EUT and TBP-PT, respectively. Therefore, we have the following proposition.

Proposition 4.3.4. *There exist unique optimal operation points in TBP-PT and TBP-EUT, respectively.*

In conclusion, for the throughput based pricing scheme, we have the following results under mild conditions. First, end users' willingness to use the service will be discouraged by a larger price for both TBP-PT and TBP-EUT. Second, TBP-PT suffers a sum average throughput degradation compared to TBP-EUT. As a result, finally, the service provider obtains a larger revenue in TBP-EUT than that in TBP-PT.

Finally, it is interesting to compare two pricing schemes, namely usage based pricing and throughput based pricing, for two scenarios (end users are both PT or EUT). In

Fig. 4.8, the revenues the service provider obtains in UBP-PT, UBP-EUT, TBP-PT and TBP-EUT are shown and compared. It can be seen that for all the four scenarios, the service provider can find an optimal price to achieve the largest revenue. In general, the service provider requires a smaller price to achieve the optimal operation point in usage based pricing scheme than throughput based pricing scheme. The optimal revenues obtained in the four scenarios are similar. In particular, in the throughput based pricing scheme, the service provider can achieve the same optimal revenue regardless the type of the underlying users. This is a useful feature in pricing scheme design since the service provider can employ the same pricing policy without worrying the valuation preference of end users. However, it should be emphasized that there is obvious disparities between the revenues when the optimal price cannot be applied. The service provider should also consider this feature when designing pricing policy.

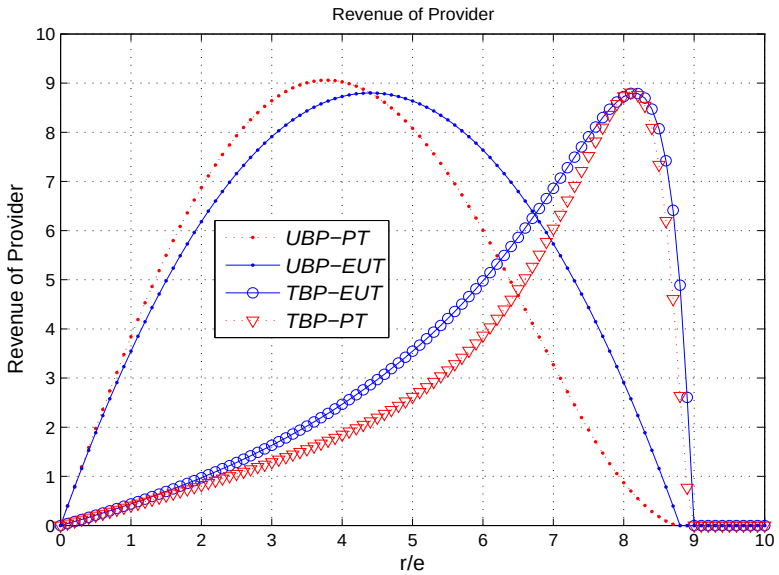


Figure 4.8: Service Provider's Revenue Comparison between UBP and TBP

Chapter 5

Conclusion and Discussion

5.1 Summary and General Discussion on Future Work

In this dissertation, we have studied two important research topics motivated by the recent development of CR technology, (i) resource allocation in multi-RAT enabled wireless networks; and (ii) impact and influence of end users' behaviors on the underlying protocol design. In the first part of this dissertation (Chapter 2), we proposed a framework for distributed resource allocation and admission control in a Cognitive Digital Home (CDH). Two channel access models were considered in the CDH for addressing spectrum coexistence of legacy devices: (i) Pessimistic Controllability (PC) Model where the HG had no influence over legacy devices, and (ii) Switched RAT (SR) Model where the HG had perfect control of legacy devices. Two resource allocation problems (i) Maximizing Sum Rate, and (ii) Maximizing Service Capacity were solved in a distributed manner to reduce the sensing and computation burden. Distributed algorithms were designed using partial dual decomposition techniques. A distributed power control scheme was developed for efficient use of energy. Based on the pricing information obtained from the distributed algorithms, an admission control scheme was designed to improve the system feasibility. The extensive numerical simulation showed that the proposed resource allocation algorithms could efficiently manage the spectral resources allocation in a CDH. The power control and admission control schemes were also shown to greatly improve the system performance.

While this work presents an initial attempt at managing spectrum coexistence in a CDH, there are many challenging open issues that require further attention such as dynamism and scaling in such networks. Moreover, as more novel data and multimedia services, e.g, home automation, wireless HDMI and smart grid, are introduced into the digital home and new radio access technologies in TV white space and 60 GHz radio, are developed for

supporting them, efficient spectrum coexistence of multiple RATs becomes an interesting and urgent task.

In the second part of this dissertation (chapter 3), we have provided a first step in exploiting the role of Prospect Theory (PT) in wireless networks. Motivated by the increasing amount of end-user control afforded in programmable radio devices, we have envisioned a scenario where end-user actions essentially "interfere" with the underlying engineered system design. In this part, as an exemplary scenario, we have considered a random access game where players follow the precepts of Prospect Theory (PT), a theory developed by Kahneman and Tversky to explain real-life decision making that often deviates from the behavior expected under Expected Utility Theory (EUT). A specific game where selfish players adjust their transmission probabilities over a random access channel under throughput rewards, delay penalties and energy costs has been considered. By analyzing the Nash Equilibrium (NE) achieved in a 2-player game, we have proved under mild conditions, that deviations from EUT of any player results in degradation of system throughput and increased delay and energy consumption. We have also studied an N-player homogeneous game with symmetric utility functions and observed similar results at the NE. Moreover, the framework introduced in the above random access model is extended to study an exemplary two-level data pricing model. We compare the service provider's revenues and system performances in the scenarios where end users follow the precepts of PT and EUT, respectively. Two pricing schemes, namely per usage pricing and per successful transmission pricing, are considered and studied.

The results in this work have only characterized the properties of the mixed NE under both EUT and PT settings. An important aspect of wireless transmission in such networking scenarios is the need for distributed algorithms that can be used by the users in making decisions about transmission probabilities. This future work will necessarily have to rely on an algorithmic game theoretic formulation of the problems considered here or consider heuristic distributed algorithms for updating the transmission probabilities using local information available to the end users. The second important aspect that needs further study is the characterization of the probability weighting functions (different from the parametric weighting function used in this work) that are specifically pertinent to wireless

device usage. This requires psychophysics experiments involving real wireless users and devices and is a topic for future study.

In the third part of this dissertation (chapter 4), we provided an initial effort in exploiting the role of PT in data pricing schemes. We extended the wireless random access model used in chapter 3 to a 2-tier wireless data pricing model. Two pricing schemes, one that considers usage based pricing and another that considers throughput based pricing, are studied and compared for two scenarios, i.e., end users both follow PT or EUT. We have shown the differences in data service utilization and service provider's revenues under different scenarios. In general, a larger price will discourage the end users from using the data service for both the EUT and PT users. This result may seem intuitive but it is actually not straight-forward when the end users follow the precepts of PT. More importantly, we have shown that the optimal revenues collected by the service provider are of similar amount for both pricing schemes with either PT or EUT end users. However, for a given price, the type of end users (PT or EUT) will have a direct influence on the service provider's revenue that should be considered by the service provider when designing the pricing policy. Similar to the topic on the role of PT in wireless random access, psychophysics experiments involving real wireless users and devices are necessary in designing and validating any novel pricing scheme that considers end-users' real-life behaviors.

Chapter 3 and chapter 4 have provided very initial approaches to understanding and perhaps exploiting the role of PT in wireless networks. We now provide some pointers to future research directions.

5.2 An Extensive Discussion on the Future Work of Prospect Theory in Wireless Networks

The current wireless random access game model with the Prospect Theory players focused on the non-linear probability weighting effects of the users. A part of the future work will adopt a more comprehensive model in [29] to address more features regrading the value functions of the Prospect Theory players. A second part of the future work may investigate the impact of users' real - life decisions on the demand responsive pricing

schemes of wireless data networks.

5.2.1 Extension on the Study of Effect of User Behaviors on Wireless Random Access

In the current work (chapter 3), we use a linear value function for each player to calculate their value regarding an outcome. Meanwhile, we assume zero as the reference point for all PT players. However, this work can be extended by considering a more general value function that is introduced in section 3.2. For convenience, we rephrase the PT theoretical value function (Framing Effects) as follows: PT [25] states that in decision-making in real life, the value of an outcome is determined by considering the relative gains or losses regarding a reference point. PT also proposes that the value function should be a concave function of gains and a convex function of losses with the convex part usually having a steeper slope. In other words [25], "losses looms larger than gains." An exemplary value function was introduced in [63] is as follows:

$$v(x) = \begin{cases} x^{\beta} & \text{if } x \geq 0 \\ -\lambda(-x)^{\gamma} & \text{if } x < 0, \end{cases} \tag{5.1}$$

where β, γ, λ are all parameters to tune the shape of the value function. Note that, x represents the relative gains or losses here regarding a reference point. Also, other value functions such as in [66] [67] [68] may also be used with caution. In particular, the current work studied the behaviors of best- effort users with linear value functions. With the help of framing effects, we could study the behaviors of the rate constrained user that has a minimum data rate requirement R_i^{min}, e.g., a user that wants to watch a high definition video. Then, R_i^{min} will serve as the reference point for the rate constrained user. For an elastic user (best effort), zero could be set as a reference point.

5.2.2 Demand Responsive Pricing in Data Communication Networks

Demand responsive pricing [67] [68] [66] has been an important part of economics based approaches to studying data communication network. A pricing scheme that incorporates

customers' demand is a key to precisely modelling data communication networks. However, it has been proven that in everyday life, people tend to follow the precepts of PT to valuate a prospect. Thus, it will be an interesting topic if we can incorporate PT into the demand responsive pricing scheme of a data communication market. The basic idea here is that whenever customers face uncertainty in the offered service from the service providers, the differences between PT and EUT in every aspect of the market, e.g., behaviors of service providers and users, should be significant and well studied. The associated results could be utilized by the service provider to realize a larger revenue and improve her system efficiency and by the spectrum policy manager to improve the social welfare. This work should proceed step by step as follows:

1. A single Service Provider (SP) and a single user. The SP is assumed to be EUT for now. A key issue here is to use PT to model the relationship between user's willingness of offer acceptance and a SP's service offer. An example could be a concept of Data Package (DP), i.e., the SP offers a data plan with two rate levels R_{min}, R_{max} and a probability $p_a \in [0, 1]$. This plan $DP(p_a)$ will offer the user data rate R_{min} with probability $1 - p_a$ and R_{max} with probability p_a. The expected data rate the SP provides is $E(DP(p_a)) = p_a R_{max} + (1 - p_a) R_{min}$. The SP incurs a cost as a function of the expected data rate she offers, i.e., $c^{sp}(DP(p_a))$. This function could be a convex function. Meanwhile, the SP charges a price for the DP as $p(DP(p_a))$, e.g., the price can be an increasing linear function of p_a. At the user side, the user will choose the DP that can give her the maximum non-negative utility. The user actually faces a prospect and PT will determine how she valuates every DP. Thus, given a SP's price for a DP, e.g., $p(DP(p_a))$:

$$\max_{p_a \in [0,1]} u_i(DP(p_a)) = w_i(p_a) v_i(R_{max}, p(DP(p_a))) + w_i(1 - p_a) v_i(R_{max}, p(DP(p_a))), \tag{5.2}$$

$$s.t.\ u_i(DP(p_a)) > 0,$$

where $w_i()$ is the PT user's probability weighting function. $v_i()$ is her value function towards a data rate and its associated price. The user will accept a DP (the p_a)

that generates maximum utility to her. Therefore, the SP's offered price $p(DP(p_a))$ actually influences the user's demand p_a. The interesting issue here is to compare the demand difference between an EUT user and a PT user.

2. A Single Service Provider and Multiple Users. The interactions between a SP and users could be studied as a Stackelberg game. The SP is the leader and she knows the users responsive demands regarding the price she sets. There are two scenarios for SP to calculate the users demands regarding a fixed price: (1) the method used in single SP and single user scenario. (2) the users can form a user-level game and they arrive at a demand vector (NE of that game) to the SP. The user locally tries to maximize her utility while subjecting to the constraints. The SP tries to maximize her revenue by setting the proper price with the knowledge about the users' responsive demands. Two novel topics of significant importance can be studied: (1) Differences in terms of SP and users behaviors between two systems, a system with all EUT users and a system with all PT users. The latter one may be a better description for the data communication market than the prior one. (2) Facing a user group of both PT and EUT players, what should the SP do to maximize her revenue?

3. Multiple Service Providers Competition. This work can be used to model the competition among multiple SPs and their interactions with users. The SPs compete with each other by setting prices for their offered data service and the users choose the optimal data service (quantity) according to the offered prices. Similarly, the impact and influence of end-user behaviors on every aspect of the system and individual performance is an interesting and profound research topic.

Appendix A

Notations in Chapter 2

- CDH Cognitive Digital Home
- HG Home Genie Node
- RAT Radio Access Technology
- MPR Multi-Platform Radio
- CR Cognitive Radio
- MSR Maximizing Sum Rate Problem
- MSC Maximizing Service Capacity Problem
- PC Pessimistic Controllability Model
- SR Switched RAT Model
- API Average Pricing Index admission control scheme
- D-PCM Distributed Algorithm for PC-MSR(PC-MSC)
- D-SRM Distributed Algorithm for SR-MSR(SR-MSC)
- GSM Global Spectrum Map
- $\mathcal{K}$ the set of services
- $\mathcal{M}$ the set of channels
- $\mathcal{T}$ the set of RATs
- $x(k,i,t)$ the indicator whether service k occupies channel i with RAT t
- $l(k,i,t)$ the portion of channel for service k on channel i with RAT t
- $P(k,i,t)$ the service k's transmit power on channel i with RAT t
- $R^P(k,i,t)$ the theoretical physical rate of service k on channel i with RAT t
- $R(k,i,t)$ the effective rate service k can achieve on channel i with RAT t
- $\mathbf{X}$ the k-th row is $\mathbf{x_k}$ as in (2)
- $\mathbf{L}$ the k-th row is $\mathbf{l_k}$ as in (3)
- w_i bandwidth of channel i
- h_k^i service k's channel gain on channel i
- N_0 noise level
- R_k^{min} service k's minimum data rate requirement
- P_t nominal transmit power for RAT t
- P_k^{max} service k's maximal transmit power
- $PI_k(itr)$ service k's pricing index in itr-th claim iteration
- P_t^{max} RAT t's maximal transmit power
- $MaxItr$ the threshold for the number of resource claim iterations

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