

Description: Working with Larger Ribbons and Bows (side view) Date: 1993-12-14 Location: Colts Neck Elementary School Researcher: Professor Carolyn Maher	Transcriber(s): Yankelewitz, Dina Verifier(s): Yedman, Madeline Date Transcribed: Spring 2009 Page: 1 of 27
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Line	Time	Speaker	Transcript
16.1.1	00:07	T/R 1:	Now, I know Beth wasn't here, she's, she's, I... I understand that umm she knows about the activities some people have shared, uhhh but uh, let's see what can we tell Beth about what we did last time? Any, any discoveries that we made in our project? Anything we remembered about making these ribbons that would be an important kind of thing to have noticed? Jessica?
16.1.2		Jessica:	Well, I noticed that after a while like it started making a pattern.
16.1.3		T/R 1:	Ok. You want to say a little bit more about that?
16.1.4		Jessica:	Well, um, I forget what pattern but I think it was going like it started going in three, six, nine, like... like when it said when you had like different size ribbons and every time it got like ...like three times bigger and it kept doing it in all different kinds of patterns, I thought.
16.1.5		Michael:	Yeah, because at first it went two, three, four, five
16.1.6		Jessica:	And then it went...
16.1.7		Michael:	and the second one went, uh, the second one went four, eight, something like four, six, yeah
16.1.8		T/R 1:	I don't remember any two, four, six or four, eight.
16.1.9		Michael:	No, it's four, it's four, six, eight, ten... and then there was that odd, and then there was that two thirds one.
16.1.10		T/R 1:	Ok, let's, let's, let's hold out... Brian what were you just saying?
16.1.11		Brian:	Well, if we, remember we had the three meters, you would always like times the number by three. Like you go three, six, nine?
16.1.12		T/R 1:	Yeah, yeah Michael's asking the question I had which number. Let's use that as an example. I have ribbons three meters long and I'm making bows how long? For example. Michael?
16.1.13		Michael:	Uh, one half
16.1.14		T/R 1:	One half a meter long, so if I have, I could sort of imagine ribbon three meters long, three of these sticks long, that's how long, and I'm making bows a third of a meter long, how can I imagine a third of a meter? How could I imagine one third of a meter? You could imagine a meter, right?

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You can see a meter? How can you imagine a third? Can you all in your heads imagine a third? How many of you can, imagine a third? So what are you imagining when you imagine a third? Not everyone is imagining it. Beth, what do you imagine?

16.1.15 Beth: [hems and haws]

16.1.16 T/R 1: Is it longer than this? No? Is it shorter than this? [students yeah] Is it shorter than this length?

16.1.17 Beth: Uhh huh.

16.1.18 T/R 1: Ok, so it's shorter than this length. About how short, much shorter is it than this length? What are you imagining? You're the only ones who can imagine how much shorter it is? I think more of you can imagine. Can you imagine a third of a meter? I have some half hands up. Jessica, what do you imagine?

16.1.19 Jessica: Well, I imagine if you like pull the ruler into like three pieces and then it would be like, like, up to the um I think wait, um thirty-three mark, I think.

16.1.20 T/R 1: Well how, how did you decide on the thirty-three mark?

16.1.21 Jessica: Well that's what I think because um, um, thirty-three plus thirty-three plus thirty-three is ninety nine and that's,

16.1.22 Michael No, but there's a hundred...

16.1.23 Jessica Yeah, and then a hundred, around like thirty three and like a half almost.

16.1.24 T/R 1: What do you think? Jackie, your hand up partially?

16.1.25 Jackie: Something around.

16.1.26 T/R 1: Something around that.

16.1.27 Alan: I think there, it's thirty-three and one third because if you take two more thirds you can get it to a hundred.

16.1.28 T/R 1: What do you think, Jessica? Thirty-three and a third?

16.1.29 Jessica: Yeah.

16.1.30 T/R 1: That what you're imagining, so this...

16.1.31 Michael: I'm, I'm imagining it just being cut into three equal halves

16.1.32 4:53 T/R 1: Equal parts. Three equal parts. How many of you imagined it cut into three equal parts? [many hands raised] Ok, and Jessica and Alan were a little more explicit they were trying to actually tell me the... how long those parts are, right? And uh, and so you're telling me in this meter stick, because there...you're telling me there are a hundred

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- 16.1.53 Beth: How many [inaudible]
- 16.1.54 Laura: I think it's nine
- 16.1.55 Jessica: I forget what I wrote on my paper.
- 16.1.56 Laura: Three meters, so...
- 16.1.57 Jessica: Yeah you can make three bows.
- 16.1.58 T/R 1: We have three meters of ribbon, and we're making bows, we have three meters of ribbon to start with and our bows are to be one third of a meter in length. How many bows can I make from three meters of ribbon?
- 16.1.59 Andrew: One third of a meter. So if this [using pens] is a meter and then.. [\[Figure S-7-13\]](#)
- 16.1.60 James: and then this is a meter, then this is a meter
- 16.1.61 Andrew: so this would be divided into thirds
- 16.1.62 James: Three six nine.
- 16.1.63 Andrew: Yeah.
- 16.1.64 CT: You've got, you've got ribbon how long? Three meters. Alright, but a bow is going to be, what do you think one third of what?
- 16.1.65 Jessica: I think I thought in order to make bows, I thought you can make three of them
- 16.1.66 T/R 1: From one meter.
- 16.1.67 Laura: Yeah.
- 16.1.68 Jessica: From, from here to here.
- 16.1.69 T/R 1: Mmm hmm.
- 16.1.70 Jessica: For one meter, and like, like, what I was saying, like thirty three and it would be somewhere around there
- 16.1.71 T/R 1: Right, but now I have three meters of ribbon.
- 16.1.72 Jessica: Three meters? Oh, three, nine
- 16.1.73 T/R 1: So why don't you talk - nine? You agree? Laura?
- 16.1.74 Beth: Yeah [Laura nods] because three times three is nine
- 16.1.75 Jessica: Because three times three is nine.
- 16.1.76 T/R 1: Ok, James?
- 16.1.77 James: Um, Andrew [inaudible] um we think um it's nine and there's nine in three meter sticks.
- 16.1.78 T/R 1: Ok.
- 16.1.79 James: That's what we think.
- 16.1.80 T/R 1: Ok, and you could persuade us, everybody, that that's the case?
- 16.1.81 James: Yeah.

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- 16.1.82 T/R 1: Ok.
- 16.1.83 8:40 Danielle: [standing with CT, Brian, Jackie, and Amy] It would be like [points with fingers]
- 16.1.84 Jackie: Can I ask Mrs. Palmer ...
- 16.1.85 Amy: Can we ask Mrs. Palmer if we could borrow her meter sticks or no?
- 16.1.86 CT: Well, well you're bothering her, ok, well, you've got a meter stick right here, help us with this. We're trying to figure out what one third of a meter is so we can figure out how many one thirds go into three meters. So you think right here is one,
- 16.1.87 Brian: Yeah, cuz...
- 16.1.88 CT: And then where would the other one be? [Brian points] Right here?
- 16.1.89 Brian: Yeah.
- 16.1.90 CT: Alright,
- 16.1.91 Brian: And then the other one right there [pointing]
- 16.1.92 CT: Alright so then this is the third of a meter, this is a third, between these two? And it's a third between Amy and myself, well how many to a meter then?
- 16.1.93 Students: Three.
- 16.1.94 CT: Three, well how many to three meters?
- 16.1.95 Erin: Nine.
- 16.1.96 CT: How did you get that?
- 16.1.97 Amy: Three times three, three times
- 16.1.98 CT: Well, is that right?
- 16.1.99 Brian: I think so
- 16.1.100 Danielle: Yeah.
- 16.1.101 CT: Well, here's here's one meter stick and I have how many, how many bows from here?
- 16.1.102 Brian: You have three meters in one and then three meters
- 16.1.103 Danielle: But doesn't it
- 16.1.104 CT: Go ahead, go ahead, we don't know if we're right, so go ahead. Doesn't it what, hon?
- 16.1.105 Danielle: I don't know.
- 16.1.106 CT: Alright, well how many bows do you have here?
- 16.1.107 Danielle: Three.
- 16.1.108 CT: Three, and how many meters is it?
- 16.1.109 Danielle: One.

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- 16.1.110 CT: Now let's just take a flight of imagination and keep this one here in your mind let's move this one, here's a second one. How many, how many uhh bows do you have in this one?
- 16.1.111 Danielle: Three.
- 16.1.112 CT: So how many do you have to make two?
- 16.1.113 Danielle and Amy: Six.
- 16.1.114 CT: And here, keep that in your mind. Here's one, here's one, you've got that in your mind. Here comes the third one, how many do you have here, in this third one?
- 16.1.115 Danielle: Three.
- 16.1.116 CT: How many is that all together?
- 16.1.117 Danielle: Nine.
- 16.1.118 CT: You're sure.
- 16.1.119 Danielle: Yeah.
- 16.1.120 CT: Are you really sure?
- 16.1.121 Danielle: [nods head]
- 16.1.122 CT: What do you think [to others]? What do you think? Well we'll see.
- 16.1.123 Amy: [speaks, but inaudible]
- 16.1.124 CT: Do whatever you need, do, use whatever you need to use. If you think you know your means then use it.
- 16.1.125 Jessica: Can we take our papers back and start, um
- 16.1.126 T/R 1: Well, you won't need it yet.
- 16.1.127 Jessica: Ok.
- 16.1.128 Danielle: [this group is measuring meters of ribbon] Here's two meters, I mean one meter.
- 16.1.129 T/R 1: You mean twenty-seven three times is eighty one, now tell me what you did here.
- 16.1.130 Alan: I did twenty, I did seven times three and that equals this, never mind.
- 16.1.131 T/R 1: Tell me what you did, I want to know what you did Alan.
- 16.1.132 Alan: Well, it's wrong, anyway.
- 16.1.133 T/R 1: Well, what did you do, though?
- 16.1.134 Alan: I did, I multiplied seven times three and got twenty one
- 16.1.135 T/R 1: [interjecting] Twenty-one.
- 16.1.136 Alan: And then I put the two up there, added that, times it and got twelve, now it's wrong.
- 16.1.137 T/R 1: Ok, so that particular rule didn't work, now what did you do, Kimberly?

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- 16.1.138 Kimberly: I did twenty-seven times three.
- 16.1.139 T/R 1: And how did you do it?
- 16.1.140 Kimberly: I times twenty, I times three times seven, I got twenty-one, so I carried the two, then I did three times two and added the two to my answer. [\[Figure S-12-53\]](#)
- 16.1.141 T/R 1: Why does that work?
- 16.1.142 Kimberly: Umm
- 16.1.143 T/R 1: Or does it work? I mean, Alan showed me here three times, if you have three times twenty-seven that means you have a twenty-seven three times, and he proved to me that it's eighty-one, how does, why does that work?
- 16.1.144 Kimberly: because you add that. [\[Figure S-13-13\]](#)
- 16.1.145 T/R 1: I don't understand why it works.
- 16.1.146 Alan: Because you're basically doing is writing, you have twenty-seven three times
- 16.1.147 T/R 1: Mmm hmm
- 16.1.148 Alan: And then you get your answer eighty-one.
- 16.1.149 Kimberly: It's just, you're just adding it faster.
- 16.1.150 T/R 1: I don't know why that works, that adding faster. See, Alan added faster and it didn't work. Does it always work?
- 16.1.151 Kimberly: No.
- 16.1.152 T/R 1: Doesn't always work.
- 16.1.153 Kimberly: But it does sometimes.
- 16.1.154 T/R 1: But Alan's was different.
- 16.1.155 Kimberly: Yeah.
- 16.1.156 T/R 1: I'm kind of curious about that. See if you can come up with a rule that works all the time. You know what it means, right?
- 16.1.157 Kimberly: Yeah.
- 16.1.158 T/R 1: Something to think about, right?
- 16.1.159 Alan: Yeah.
- 16.1.160 T/R 1: You said you have twenty seven three times. Would it work if you had twenty three times and seven three times?
- 16.1.161 Alan: Yeah
- 16.1.162 Kimberly: Uh, maybe, uh I don't think so.
- 16.1.163 T/R 1: Kimberly isn't sure. But you think so
- 16.1.164 Kimberly: I don't think so
- 16.1.165 T/R 1: She doesn't think so, you do.
- 16.1.166 Alan: Yeah, I do, cuz you have twenty [writes]

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- 16.1.167 T/R 1: Right.
- 16.1.168 Alan: if you have seven three times
- 16.1.169 Kimberly: Add them together, you'd have to add them together.
- 16.1.170 T/R 1: Ok, right.
- 16.1.171 Kimberly: You would have to add them together.
- 16.1.172 T/R 1: And you add them together and what do you get. [Alan's paper has the addition of three twenties and the addition of three sevens, and then the sum of those two sums] Ok, so that worked, didn't it? Does that help you figure out a way to make it work every time? I bet you can invent a rule that works, Alan. If you think about what you did. Ok? You have to add them together. What do you mean you have to add them together, Kimberly? [\[Figure S-14-22\]](#)
- 16.1.173 Kimberly: You would have to add those two answers together to get
- 16.1.174 T/R 1: Why?
- 16.1.175 Kimberly: Because, if you wanted to do it faster.
- 16.1.176 T/R 1: Here.
- 16.1.177 Kimberly: Because you wouldn't be able to get the answer for this if you were using this, and you would try to get the answer eighty-one, you wouldn't be able to get the answer unless you added the two answers together.
- 16.1.178 T/R 1: But why?
- 16.1.179 Kimberly: [shakes head] I don't know.
- 16.1.180 T/R 1: You don't know. Well, that's what I was asking you to think about. [to class] Ok, just for a time out for a minute while you're working on this, for those of you who are finished with that problem, I asked you, how many ribbons one third meter in length can you make from three meters of ribbon, right? And then I said suppose you had nine meters of ribbon, how many ribbons can you make one third meter in length and then I said suppose you had twenty-seven meters of ribbon, how many ribbons can you make one third meter in length? So those are the problems you're working on, I just want to be sure you know all know the problems you're working on now.
- 16.1.181 15:51 Jackie: Ok, now we have eighty-one, that's just extra.
- 16.1.182 Jessica: Nine, you got nine, right?
- 16.1.183 Jackie: We got nine.
- 16.1.184 Jessica: So did I.

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- 16.1.185 Amy: Let's get some paper to write this down on.
- 16.1.186 Danielle: I like your sweatshirt.
- 16.1.187 Brian: [Jackie and Jessica are talking] Guys, this is math, you're on camera. You're on camera and you're like oh nice sweatshirt.
- 16.1.188 Amy: You can do that out at the playground
- 16.1.189 Sarah: Guys are you on the first one?
- 16.1.190 Alan: ... Seven times three, and you get the twenty-one. You add the sixty and the twenty-one and you get the eighty-one. Now you get it? [\[Figure S-17-00\]](#)
- 16.1.191 Student: Neither do I.
- 16.1.192 Kimberly: Got it.
- 16.1.193 Alan: What I'm doing is, you have your twenty-seven, so you take off the seven, and you get and you only have twenty. So then you do twenty times three and you get sixty, which brings me to step two. You don't have two, so you have the seven. So you do seven times three and that equals twenty-one. So you add the sixty and the twenty-one and you get eight one.
- 16.1.194 Kimberly: Ok. I think I got it. Alright, I don't get it.
- 16.1.195 Alan: You still don't get it. Ok, I'll put it in a lot more words.
- 16.1.196 Kimberly: Cuz I'm not sure about something! [takes Alan's paper] I think we're supposed to try make a rule for that.
- 16.1.197 Alan: Not that one, it was wrong.
- 16.1.198 Kimberly: Ok. I'm confused.
- 16.1.199 Alan: Ok, you added twenty-seven, before you multiply you take off the seven and then you get twenty. And then you have twenty times three and that equals sixty. So then you go to step two. You don't have the two there anymore so you have the seven. You do seven times three and that equals twenty-one so you add your two answers and you get eighty one.
- 16.1.200 Kimberly: Got it.
- 16.1.201 Alan: Good. So,
- 16.1.202 Kimberly: Ok, what do we have to do now? Ok, what do we have to do now?
- 16.1.203 Alan: What do you want to do now?
- 16.1.204 Kimberly: I don't know.
- 16.1.205 Alan: We finished the problem, so
- 16.1.206 T/R 1: How much ribbon do you have?

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- 16.1.207 Michael: We have six thousand five hundred and sixty one yards, um meters of ribbon.
- 16.1.208 T/R 1: Did you have a calculator check your computation?
- 16.1.209 Erik: And right now, we're tying ourselves down to get nine meters of ribbon! We're tying ourselves down.
- 16.1.210 T/R 1: Literally, Erik, you're literally tying yourself down?
- 16.1.211 Erik: Yay, we got-
- 16.1.212 T/R 1: Well, did you figure it out, Alan?
- 16.1.213 Alan: For the strategy, for the strategy.
- 16.1.214 T/R 1: Tell me
- 16.1.215 Alan: Twenty-seven meters.
- 16.1.216 T/R 1: Ok. Did you discuss it with Kimberly?
- 16.1.217 Kimberly: Yeah.
- 16.1.218 T/R 1: Did you both agree on this?
- 16.1.219 Kimberly: Yeah.
- 16.1.220 Alan: Finally, she got it.
- 16.1.221 T/R 1: Oh, I can't wait.
- 16.1.222 Kimberly: Yeah, I got confused.
- 16.1.223 Alan: Alright, so you have the twenty-seven before you multiply it and you take off the seven and you have twenty so then you multiply twenty three times and you get sixty.
- 16.1.224 T/R 1: Mmm hmm.
- 16.1.225 Alan: And so you go to step two. Then you don't have the two anymore and you only have the seven. And you multiply seven times three and you get twenty-one. So then step three you add sixty and twenty-one and get eighty-one.
- 16.1.226 T/R 1: Ok. Ok. Now, I'm curious, I was very intrigued by what Kimberly used some kind of procedure here that I don't quite understand but is there any way on the basis of what you did you could make sense of what she did?
- 16.1.227 Alan: Well
- 16.1.228 T/R 1: Can you, suppose someone...cause I don't understand why this procedure works, I understand what you explained to me, but, I want to know why this works cuz this seems to work too
- 16.1.229 Alan: It does work.
- 16.1.230 T/R 1: It does work. But why does it work is my question to you.
- 16.1.231 Alan: First can you explain the problem.

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- 16.1.232 Kimberly: Ok, well, all it is is you have the twenty-seven but on his you took the seven away. And all I did was multiply the twenty-seven and
- 16.1.233 Alan: You did
- 16.1.234 Kimberly: I multiplied it together instead of having to multiply it separately and add them together.
- 16.1.235 T/R 1: Ok, but tell me how you multiplied it together.
- 16.1.236 Kimberly: Ok.
- 16.1.237 T/R 1: In other words tell me what you did what did you do when you multiplied.
- 16.1.238 Kimberly: Well, I put twenty seven times three equals [writes] so I, I times the seven and the three and I got twenty one
- 16.1.239 T/R 1: Ok, so
- 16.1.240 Kimberly: I carried the two, but...
- 16.1.241 T/R 1: Is that a two?
- 16.1.242 Alan: In multiplication you don't carry.
- 16.1.243 T/R 1: Is that a two?
- 16.1.244 Kimberly: Well, I learned to do that.
- 16.1.245 T/R 1: Ok, Kimberly, is that a two. This is a one, but my question is, is that a two?
- 16.1.246 Alan: Yeah, but why do you have the two up there? Because two, because two added to two times three equals six, you had, hold it, two times two, two plus two...
- 16.1.247 Kimberly: No, what I learned to do was
- 16.1.248 Alan: times three equals twelve.
- 16.1.249 Kimberly: What I learned to do was do was multiplication, then you add that to your multiplication answer.
- 16.1.250 T/R 1: But what are you adding? I don't. When you say three times seven is twenty-one, write that down, three times seven is twenty-one, [Kimberly writes] now I always, I always learned that that twenty-one, that isn't a two, this is the one, but this isn't a two. This is two tens, I learned.
- 16.1.251 Kimberly: But what I learned is you put the one there, and then you carry the two like you do in adding but you times the number so I times three times two and then whatever you got as your multiplication answer you added that number to that and you put, and then once you got there you got your answer.
- 16.1.252 T/R 1: I understand that, but I want to know why it works.

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- 16.1.253 Kimberly: I don't know why.
- 16.1.254 T/R 1: Alan?
- 16.1.255 Alan: Well, what she is doing is she multiplied seven times three and got twenty-one. She carried the two and added those and multiplied it twice.
- 16.1.256 T/R 1: Ok, well you gotta think about that one.
- 16.1.257 Alan: But wait. But.
- 16.1.258 22:42 T/R 1: I understand what you did here, it makes sense to me, I'm not so sure I understand that. I'm not saying I don't agree that it works, but I don't know why it works. [[Figure S-22-43](#)]
- 16.1.259 Alan: Kim you might have to rephrase your number problem here. Because what you're doing is your doing seven times three is twenty-one, you're carrying that, and you only multiply every number by two.
- 16.1.260 Kimberly: Maybe I should divide it into steps or something.
- 16.1.261 Alan: If you can explain it that way.
- 16.1.262 Kimberly: I think I can.
- 16.1.263 Alan: Alright, put it in steps.
- 16.1.264 Kimberly: Let me try.
- 16.1.265 Alan: And then I'll read your.
- 16.1.266 Kimberly: Alright, I have an idea, I gonna put it a little bit like that, ok? [writes] No, I keep making messing up. I'm gonna do it my own way. I keep making mistakes on this.
- 16.1.267 Alan: You want me to get a calculator?
- 16.1.268 Kimberly: No. Alright go use a calculator. No, I got it, I got it.
- 16.1.269 Alan: Where is x? Where is the times symbol?
- 16.1.270 Kimberly: Ok, ok, ok, I only have one little step, hold on, hold on.
- 16.1.271 Alan: Alright, step one, twenty-seven times three equals sixty one. What?
- 16.1.272 Kimberly: No, come here. Look, see this right here? If you, I brought that over. And then you do that, you do those and then you do that if you didn't, if that number didn't exist you'd have sixty-one, but then you take that and you add that two, but that two becomes a twenty and then you add it. So I, I can't explain this problem. I can't explain how I did it, I just know how to do it that way. [[Figure S-26-41](#)]
- 16.1.273 Alan: Wait, let's see. I know that twenty-seven times three equals eighty one [uses calculator]

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- 16.1.274 Kimberly: Right.
- 16.1.275 Alan: It says right here.
- 16.1.276 Kimberly: Right.
- 16.1.277 Alan: And if you do twenty-seven times four it only equals, it equals one oh nine. Right there, I typed that in. Anyway, um, twenty-seven equals, so there's your eighty-one. Now the way you're doing it can't be done on the calculator.
- 16.1.278 Kimberly: I know. You're ignoring the two. Forget that two.
- 16.1.279 T/R 1: What did you times three? Twenty-seven?
- 16.1.280 Beth: Twenty-seven.
- 16.1.281 T/R 1: Ok show me how you did three times twenty-seven and got seventy-eight.
- 16.1.282 Laura: We just kept on adding.
- 16.1.283 T/R 1: You added? Well, rather than adding three is there another way you can do it?
- 16.1.284 Laura: Times
- 16.1.285 T/R 1: Yeah that's one way is there another way? What does three times twenty-seven mean? You said you could have twenty-you said you could add three twenty-seven times.
- 16.1.286 Jessica: Yeah.
- 16.1.287 T/R 1: That's twenty-seven times three. What does three times twenty-seven mean?
- 16.1.288 Jessica: Three times twenty-seven that's seventy-eight.
- 16.1.289 T/R 1: Show me.
- 16.1.290 Jessica: I did-
- 16.1.291 T/R 1: What does it mean to have three times twenty-seven?
- 16.1.292 Beth: Twenty-seven three times.
- 16.1.293 T/R 1: Ok, so why don't you have put twenty-seven three times, you could add twenty-seven three times. Ok, that's true.
- 16.1.294 Beth: Eighty one
- 16.1.295 28:25 T/R 1: Eighty one. Ok, now what do you get when you get three twenty-seven times? Seventy-eight, is that possible?
- 16.1.296 Jessica: Twenty-seven three times?
- 16.1.297 T/R 1: You told me you got seventy-eight when you added three twenty-seven times. You kept adding threes.
- 16.1.298 Jessica: [Beth laughs] No, I guess we counted wrong.
- 16.1.299 T/R 1: Maybe you added twenty-six times?
- 16.1.300 Jessica: Yeah, that would be eighty one, and that number would be eighty-one.

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- 16.1.301 T/R 1: Ok, now you said there was another way you could do it, three times twenty-seven, you said you could multiply it rather than add it three times? How do you do that? How do you multiply three times twenty-seven?
- 16.1.302 Jessica: Uh,
- 16.1.303 T/R 1: Can you show me how to do that? [Laura uses the standard multiplication algorithm.] [[Figure S-29-19](#)]
- 16.1.304 Beth: What do we have to do?
- 16.1.305 T/R 1: You said you could multiply three times twenty-seven. You know how to do that? [Beth begins to write] That says twenty-seven times three. Beth wrote three times twenty-seven you wrote, well, depends on how you how you read it, I guess. How do you do that? Do you know how to do that? Did you learn that? Three times twenty-seven. [Jessica writes the same as Laura][[Figure S-30-29](#)]
- 16.1.306 Jessica: Yeah, and then you get eighty-one.
- 16.1.307 T/R 1: You got the same answer? Laura?
- 16.1.308 Laura: Yeah.
- 16.1.309 T/R 1: How, how does, why does that work? How does that work? Three times twenty-seven, what did you do there?
- 16.1.310 Beth: First, I did three times seven is twenty-one, put down a two
- 16.1.311 Jessica: Carry the two.
- 16.1.312 Beth: and then three times two is six, plus two is seven, I mean eight. And you get eighty-one.
- 16.1.313 T/R 1: You said you carried- three times seven is twenty-one why don't you write that down, three times seven is twenty-one? [Beth does so] Now, when you say twenty-one, what does that mean, twenty-one? Does that mean two plus one? Or three? What does that mean, the twenty-one?
- 16.1.314 Jessica: The twenty-one means that you're seven, fourteen, twenty-one, that you're taking the seven.
- 16.1.315 Beth: You're taking the seven three times
- 16.1.316 Jessica: Three times.
- 16.1.317 T/R 1: Yeah, but what does twenty-one mean? What does twenty-one mean?
- 16.1.318 Laura: Twenty-one means that two times
- 16.1.319 T/R 1: That means two tens and one one, Laura? That means two tens and one one. Ok

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- 16.1.320 Jessica: Yeah, like if you have, like last year we were doing about these things, and they were like ten blocks in there and then we had two of them,
- 16.1.321 T/R 1: Oh ok, yeah, two tens.
- 16.1.322 Jessica: And then we had the one.
- 16.1.323 T/R 1: Ok, so I'm confused when you say carry the two you're not carrying two of these?
- 16.1.324 Jessica: No, we're carrying two tens.
- 16.1.325 T/R 1: You're carrying two tens, you're carrying two tens, so how does this work? Three times seven is twenty-one ones, or two tens and one one. Right? So, how does that work? Why does that work? What do you think, Laura? What does this carrying the two mean?
- 16.1.326 32:25 Beth: Because-
- 16.1.327 T/R 1: Beth? What are you thinking?
- 16.1.328 Beth: I was thinking it would be alright because this, this two is in the tens column
- 16.1.329 T/R 1: That's a ten also, ok, so here you have three times two tens that's six tens and this is two more tens? That gives you eight tens. So your answer is eight tens and one one? Does that make sense? [[Figure S-32-48](#)]
- 16.1.330 Beth: Yeah.
- 16.1.331 T/R 1: Ok, I'm wondering if you can share that with Jessica who didn't hear what you just said because I, I might ask you later why does that work. Do you think you can explain it?
- 16.1.332 Beth: Yeah.
- 16.1.333 T/R 1: Because some people don't know why that works, so I want you to think about that.
- 16.1.334 Jessica: And that-
- 16.1.335 T/R 1: Ok, well, Beth, let's see if Beth can explain it to you and be sure you all agree and come up and write up why you think that works. Ok?
- 16.1.336 Beth: Because two is in the tens column, and so is that so.
- 16.1.337 Jessica: Yeah, I know. So it would be, so it's like you're carrying two tens.
- 16.1.338 Beth: Yeah, and plus two tens. Now, let's keep doing this.
- 16.1.339 Jessica: How many were we up to?
- 16.1.340 Beth: Well, we have to change that to an eighty-one.
- 16.1.341 Laura: How much, wait?

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- 16.1.342 Jessica: You have to change this.
- 16.1.343 Beth: Change it to an eighty-one.
- 16.1.344 Jessica: Now, let's go see how much eighty-one meters is outside.
- 16.1.345 Beth: Why?
- 16.1.346 Laura: Now I now I have to change this to an eighty-one.
- 16.1.347 Beth: What?
- 16.1.348 Laura: This.
- 16.1.349 Beth: Yeah
- 16.1.350 Laura: This.
- 16.1.351 Beth: Now we gotta do this eighty-one times. Ok, keep working.
- 16.1.352 Laura: What were we up to?
- 16.1.353 Jessica: Ok, that's thirty-three down there. And that's thirty-three.
- 16.1.354 Beth: It's thirty-three too! It's sixty six. Because it's thirty-three here and thirty-three here, sixty-six!
- 16.1.355 Jessica: Fifty-seven, fifty-eight, fifty-nine, sixty!
- 16.1.356 Laura: [Says something inaudible] Sixty, now I'm going to go to the next page. I guess, right? Ok.
- 16.1.357 T/R 2: [inaudible]
- 16.1.358 Alan: Two for every meter.
- 16.1.359 T/R 2: Ok, and
- 16.1.360 Alan: That means if you had [inaudible] divided by how many other meters you have, but I think you should do two times, wait, if you have eighty-one meters and you want to find out how many ribbons should be in that, you know that two ribbons can be made out of each meter
- 16.1.361 T/R 2: Ok,
- 16.1.362 Alan: So that means two times eighty-one and your answer is one hundred and sixty-two, which is obviously the answer you'd have to give.
- 16.1.363 T/R 2: Mmm hmmm, Ok, what did Kim- what if instead
- 16.1.364 Alan: If you had a thousand
- 16.1.365 T/R 2: What if instead of a half a meter, what if they were um uh a fourth of a meter? Then what would you do?
- 16.1.366 Kimberly: That would be times four.
- 16.1.367 T/R 2: Ok, why does that work? Why does multiplying by two or three or four work?
- 16.1.368 Kimberly: Uh, because uh that's the num- that's the, it's like a four, and if you're using um a fourth, and you use four, it's sort of like, you're just using regular numbers.

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- 16.1.369 T/R 2: Ok, so if I have one meter of ribbon, and they were a fourth of a meter, how many bows could I make?
- 16.1.370 Kimberly: Four,
- 16.1.371 T/R 2: Do you agree with that?
- 16.1.372 Kimberly: eight, twelve...
- 16.1.373 Alan: So that would be if you had eight one and then you'd have to multiply that by four you get three hundred and twenty-four bows
- 16.1.374 T/R 2: Three hundred and twenty-four bows if I had eighty-one meters of ribbon?
- 16.1.375 Kimberly: Ok,
- 16.1.376 Alan: Three hundred and twenty-four meters would be the entire perimeter of this school.
- 16.1.377 T/R 2: Wow, are you up to measuring that out?
- 16.1.378 Alan: No actually this long hall is eighty-two so it would be only one meter less than that.
- 16.1.379 Kimberly: So if you would do it again you would have one thousand, two hundred thirty-six.
- 16.1.380 T/R 2: Ok
- 16.1.381 Alan: Times four. If you use the answer of eighty-one times four, you'd get I don't know. You'd just keep going and then divide it by the number of
- 16.1.382 Kimberly: In this [inaudible]
- 16.1.383 T/R 2: What happened? The calculator's not going past a million?
- 16.1.384 Kimberly: The calculator quit.
- 16.1.385 T/R 2: You need a bigger calculator.
- 16.1.386 Kimberly: No, the calculator quit, it said error.
- 16.1.387 T/R 2: Error. This is all very interesting. Do you feel better about being able to explain this?
- 16.1.388 Kimberly: I think so.
- 16.1.389 T/R 2: Can you try it, can you practice on me before she asks you to explain this?
- 16.1.390 Kimberly: I don't think I can.
- 16.1.391 T/R 2: Yes you can
- 16.1.392 Alan: Oh, I could tell you how I could explain mine.
- 16.1.393 T/R 2: Ok, I'm not concerned about that, but Kim's nervous because what if she gets called on now? You listen too, ok, listen to her argument. Ok, Kim, why does this work?
- 16.1.394 Kimberly: I don't know. I'm confused, that's why I can't do it.

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- 16.1.395 T/R 2: Well, you just told me some beautiful things about all the patterns and relationships here.
- 16.1.396 Kimberly: Well, I'm confused. [to students approaching instructor] It quit!
- 16.1.397 T/R 1: [to other students] Ok, How far did you go?
- 16.1.398 Danielle: We went up to three point eight seven four two six four eight.
- 16.1.399 T/R 2: Ok, that's a lot of bows, ok you two get to work on making those! [to Kimberly] Ok, tell me about this. You have twenty-seven times three is eighty-one
- 16.1.400 Kimberly: Twenty-seven times three is eighty-one but if and if you have one meter and it was times four by fourths you get four bows, and if it was by thirds you get three bows so the third or the fourth would be three or four.
- 16.1.401 T/R 2: Ok.
- 16.1.402 Kimberly: [inaudible]
- 16.1.403 T/R 2: Ok. Alright, if you had to say where the twenty-seven came from, do you remember?
- 16.1.404 T/R 1: [to Alan] Beth, Laura and Jessica may have figured out, Alan, why that, why Kimberly's algorithm works.
- 16.1.405 Kimberly: Well
- 16.1.406 T/R 2: I understand you got it by multiplying by 3.
- 16.1.407 Kimberly: We got an answer. She asked us what, what would you get how many bows would you get if you had three times nine and we got twenty-seven and she said how many bows would you get if it was three twenty-seven?
- 16.1.408 T/R 2: Ok, so this is how much ribbon you have.
- 16.1.409 Kimberly: Yeah.
- 16.1.410 T/R 2: You have twenty-seven meters of ribbon.
- 16.1.411 Kimberly: Right.
- 16.1.412 T/R 2: Ok, and tell me again why you're multiplying by three.
- 16.1.413 Kimberly: Because she said, how many ribbons can you make out of twenty-seven meters, out of, um, if you're making three thirds.
- 16.1.414 T/R 2: If you're making a third, ok, I understand that perfectly well, I think you're fine.
- 16.1.415 Kimberly: But I'm confused.
- 16.1.416 39:37 T/R 2: Are you still feeling confused about it? It's kind of a hard idea

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- 16.1.417 Kimberly: Yeah
- 16.1.418 T/R 2: Yeah, isn't it? Yeah, I think that's what it is.
- 16.1.419 Kimberly: It's easy to learn it but it's hard to explain it.
- 16.1.420 T/R 2: It's hard, it really is hard but you know we always ask you to explain. ok.
- 16.1.421 Alan: [to beth] Carrying the two. Now what is your way of doing that? You multiplied the three times, what Kim did is she multiplied the three times the seven and then carried the two up there. Right, but
- 16.1.422 Beth: We carried because that's
- 16.1.423 Alan: Just show me how you did yours.
- 16.1.424 Beth: That's, I did the same thing as Kimberly.
- 16.1.425 Alan: I know, but
- 16.1.426 Beth: And it works because this two is ten and that two is ten and when you add them.
- 16.1.427 Alan: It's forty.
- 16.1.428 Beth: No, because you do three times two
- 16.1.429 Alan: Right, and that would be six
- 16.1.430 Beth: And then plus two more
- 16.1.431 Alan: Is eight
- 16.1.432 Beth: Right,
- 16.1.433 Alan: But wait, what you're doing is you're only multiplying that two and adding that twenty onto that, you're not multiplying that two. What you're doing is you're just adding that onto there. You're not multiplying that two.
- 16.1.434 Jessica: You're not supposed to
- 16.1.435 Beth: [shaking head] You're not supposed to, you're not.
- 16.1.436 Kimberly: ...you have, you learned that last year right? And you had Ms. Firestone right? Then I know why you're getting a different answer than him. He had [inaudible] and I had Warwick and you had Firestone. So maybe Warwick and Firestone taught the same thing, but [inaudible] didn't. So that's why Alan's confused and we know what we're doing.
- 16.1.437 Alan: Well, I made up one of mine, and this is what I did. First you have your um twenty-seven, then you take off the seven and you only have twenty. So you multiply twenty by three and you get sixty. So then in the step two you only have your seven left so you multiply seven times three and you

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- get twenty-one. You add sixty and twenty-one and you get eighty-one.
- 16.1.438 T/R 2: It's a different way isn't it?
- 16.1.439 Jessica: Very different.
- 16.1.440 T/R 2: It seems to work. Have you tried it for any other numbers to see if it works?
- 16.1.441 Jessica: We did, um, we did um, we can make out of nine meters we can make twenty-seven bows, out of twenty-seven meters you can make eighty-one bows, and then out of eighty-one meters we got two hundred and twenty-fourty, but now's it's even so now we think it's two hundred and forty-three.
- 16.1.442 Kimberly: Yeah, it is,
- 16.1.443 Jessica: I got the same thing with the calculator. And we were doing it like this
- 16.1.444 Kimberly: Yikes, yikes, yikes.
- 16.1.445 T/R 2: Oh, ok.
- 16.1.446 Jessica: So we must have made an error.
- 16.1.447 T/R 2: What, can I ask you, I mean you probably have said this to Dr. Maher but I wasn't over here, why are you multiplying by three?
- 16.1.448 Jessica: Well because she asked us
- 16.1.449 Alan: The first problem that we had to do was if we had three meters,
- 16.1.450 T/R 2: Does it have to do with that three meters of ribbon?
- 16.1.451 Alan: Cause, you had to multiply it by the number of three, by the number of meters you had.
- 16.1.452 Jessica: Because it's a pattern or something.
- 16.1.453 T/R 2: Ok, it's a pattern, I'm real
- 16.1.454 Jessica: It just seems to be working.
- 16.1.455 43:06 T/R 2: I'm real confused though about why the three why, why multiply by three, why not multiply by two?
- 16.1.456 Alan: Because the problem was to only have three meters.
- 16.1.457 Jessica: Right.
- 16.1.458 Alan: That wasn't the problem.
- 16.1.459 T/R 2: Ok, what if I had
- 16.1.460 Alan: Had it been two meters, this would have only been [inaudible]
- 16.1.461 T/R 2: What if I had started with um six meters?

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- 16.1.462 Alan: Twenty-seven times six.
- 16.1.463 T/R 2: Ok, and I wanted to make bows that were a third a meter
- 16.1.464 Alan: And twenty-six.
- 16.1.465 T/R 2: Six meters of ribbon
- 16.1.466 Alan: Hold it
- 16.1.467 T/R 2: Bows that were a third a meter in length each.
- 16.1.468 Alan: Times three, nine, no. Ok, I got that too. I think it's
- 16.1.469 T/R 2: Where did you get twenty-seven from?
- 16.1.470 Kimberly: We just did, [inaudible] only we pulled out the three and put the six in, we just doubled the three. Right, all we did was we kept the twenty-seven but we just doubled the three.
- 16.1.471 T/R 2: Ok, listen to this now, I want you to , I want you to start fresh, ok? I don't want you to think about any of the past stuff we've been working on today.
- 16.1.472 Alan: Ok.
- 16.1.473 T/R 2: Ok. New problem, the problem is I have seven meters of ribbon.
- 16.1.474 Alan: Seven.
- 16.1.475 T/R 2: Ok? and I want to make bows that are a third of a meter each. How many bows would I get?
- 16.1.476 Jessica: You'd get twenty-one. Because seven times three is twenty-one.
- 16.1.477 Kimberly: Right
- 16.1.478 T/R 2: Ok, but you're multiplying by three again and we didn't start with three meters, so I don't understand. We started with seven meters.
- 16.1.479 Alan: Right, so that would be seven times seven.
- 16.1.480 T/R 2: So is that where the three is coming from? That's what I don't understand.
- 16.1.481 Alan: And you'd get forty-nine.
- 16.1.482 Jessica: No
- 16.1.483 T/R 2: Ok, now you're saying something different here, ok, why?
- 16.1.484 Alan: So you multiply the number of meters you got by the number, by the fraction you're making.
- 16.1.485 Kimberly: The third is just like the three, it's like a regular number.
- 16.1.486 T/R 2: Is it?
- 16.1.487 Kimberly: It isn't, it's sort of used as a regular number but it's really a third.

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- 16.1.488 Alan: Yeah, what you do is you take the number of ribbon you have, and then make the fraction, the fraction like one third, the three, multiply the number of meters you have and then you get your answer of how many bows can be made out of them.
- 16.1.489 T/R 2: Oh, you're using some sort of a rule here.
- 16.1.490 Alan: Yeah cause say I had fifty, fifty meters, and I wanted a third of each of those meters. That would mean each meter gets three parts, so you multiply this by three, and I get a hundred and fifty, so that's how many bows you can get.
- 16.1.491 T/R 2: So you're starting, the light is starting to go on for me, ok? I'm starting to see what you're doing.
- 16.1.492 Alan: [interjects, inaudible]
- 16.1.493 T/R 2: You'll have to say that again.
- 16.1.494 Alan: Actually, the fraction that you have, the second digit in fraction is the number you multiply the number of meters that you have. That means if I had seven and I wanted to divide it into fourths, you go seven times four equals twenty-eight.
- 16.1.495 T/R 2: So when you say the second number of the fraction, you mean the number on the bottom in the fraction?
- 16.1.496 Alan: So the second number of the fraction, like it, one fourth, [uses calculator]
- 16.1.497 T/R 2: Ok, I see, you have a slash line it's the second number.
- 16.1.498 Alan: The second number on the right side of the slash. And then you multiply by the meters that you've got and then you get your answer of how many bows can be made out of em.
- 16.1.499 T/R 2: Ok, you, are you all in agreement with that? That seems to work?
- 16.1.500 Others: Yeah.
- 16.1.501 T/R 2: Ok, I want you to think about something else then, ok? Let's go back to, [T/R 1 starts speaking], I guess we'll think about it later.
- 16.1.502 T/R 1: Ok, I wonder if I could ask you to give me your attention for a moment. We have only a few minutes left I know you've been working very very hard, I know there have been some wonderful thinking and wonderful mathematics going on, I have some questions that may be. Ok, let's start with some things that I know we all know the answer to,

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- you can answer it together if you all stop what you're doing for a moment we'll have more time to finish. First question, three meters of ribbon, how many bows one third of a meter in length can we make? Class.
- 16.1.503 Students: Nine.
- 16.1.504 T/R 1: Does anybody disagree? You're all absolutely convinced? How many of you are convinced? How many of you can prove it? How many of you know how to prove it? Ok, that looks like everybody, I think, Danielle, is your hand up? Your hand is not up. So Danielle, you don't know how to prove it?
- 16.1.505 Danielle: Kind of.
- 16.1.506 T/R 1: Kind of over here? Kind of. Sarah, how would you prove it?
- 16.1.507 Sarah: Um, you go three-
- 16.1.508 T/R 1: Nice and loud so they can hear you. We're listening to the proof, gentlemen.
- 16.1.509 Sarah: You go three plus three plus three and that would equal nine. And
- 16.1.510 Jackie: Or three times
- 16.1.511 Michael: That's why because you have three meters and take... and you have three one thirds in each meter so three, three threes, and that equals nine.
- 16.1.512 49:05 T/R 1: Jackie, Danielle, does that make any sense?
- 16.1.513 Jackie: I think it's three meters times three meters equals nine meters.
- 16.1.514 T/R 1: Danielle, do you agree or disagree?
- 16.1.515 Danielle: Yeah, that's what I did.
- 16.1.516 T/R 1: You think that's a good idea.
- 16.1.517 Michael: Well, you can times it, but you can add it too.
- 16.1.518 T/R 1: What confuses me is that you don't have three meters, you have a third of a meter, so you're telling me that you multiply by three. So how did you do this? What are some ways of doing this?
- 16.1.519 Michael: Three times three.
- 16.1.520 T/R 1: So you-, I'm asking you three meters of ribbon, and I'm making bows, I'm dividing it into one third meter length bows, and you're telling me that I can do that answer by multiplying it three times three and getting nine. How many

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of you did it that way? You said three divided by a third gave me three times three or nine? [some students raise hands] Some of you did it differently, some of you said three divided by a third is equal to three plus three plus three or nine? How many of you did it that way? A couple of you did it that way. How many of you did it the first way? Some of you raised your hands for one way, and only a couple- how many of you did it a different way then? How many people measured it out? How many of you took nine meters of ribbon and measured it out? [other hands raised] And how did you do it, to convince yourself, uh, yes? Erin?

- 16.1.521 Erin: Uh, we took string and went out in the hallway and measured the nine meters out.
- 16.1.522 T/R 1: So, you measured out nine meters, and how did you get umm, how did you measure out nine? You measured nine bows or nine meters?
- 16.1.523 Erin: Nine meters.
- 16.1.524 T/R 1: I'm confused, we started with three meters.
- 16.1.525 Erin: Ok, um, I didn't have to um measure it out.
- 16.1.526 T/R 1: You didn't have to measure that one, so that one you had the three meters, and what did you, what was the question you were asking, you didn't have to measure it, so how did you do it?
- 16.1.527 Erin: Um, I did the first way, umm, three times three.
- 16.1.528 T/R 1: How did you know to multiply it three times three?
- 16.1.529 Erin: [laughs]
- 16.1.530 T/R 1: Do you understand my question, how did you know to multiply three times three? Jackie?
- 16.1.531 Jacqueline: Well, well, see, we had three meters so you put three down, and you're trying to divide it into thirds so you put another three down and then you times it and that would equal up to nine.
- 16.1.532 T/R 1: Ok, so you're telling me that in the one meter, you have three thirds, is that what you're telling me?
- 16.1.533 Jacqueline: Mmm hmm.
- 16.1.534 T/R 1: How many of you did it that way, in one meter you have three thirds so in the nine meters you have a total of nine

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- thirds - you have three one thirds, another three one thirds, and another three one thirds. You didn't do it that way.
- 16.1.535 Jacqueline: No, I'm trying to think.
- 16.1.536 T/R 1: Did anybody do it that way? I'm confused how you got your answer. I'm so confused. Andrew?
- 16.1.537 Andrew: Well, me and James did three times three like that and we got the three and three because, um, you eventually have three meters and so one third, three, so you have three thirds of a meter so that's three thirds of a meter, so that's three times three meters equal nine meters, nine meters. Yeah.
- 16.1.538 T/R 1: Ok, maybe, maybe... James? Do you agree with that?
- 16.1.539 James: Yeah.
- 16.1.540 T/R 1: Anybody else? Maybe we should move on to the next question. Now we have nine meters of ribbon and bows are a third of a meter. Is that when you measured it in the hall, Erin?
- 16.1.541 Erin: Yeah
- 16.1.542 T/R 1: So tell me what you did in the hall? You had nine meters of ribbon.
- 16.1.543 Erin: Umm, and we measured it out, and um,
- 16.1.544 T/R 1: So what did you do out in the hall we couldn't see you [Erin laughs] What were you doing out there?
- 16.1.545 53:27 Erin: Well, um,
- 16.1.546 T/R 1: So what's the question you measured out nine meters out there, and you're making bows, how long were the bows?
- 16.1.547 Erin: One third.
- 16.1.548 T/R 1: One third. Did you have one third meter string?
- 16.1.549 Erin: Yeah.
- 16.1.550 T/R 1: And how many of those one thirds?
- 16.1.551 Erin: Twenty-seven
- 16.1.552 T/R 1: There were twenty-seven of them. You measured it out, that's really neat. Anybody else measured it out like that? I saw some other people out in the hall measuring. In fact, we lost some people. Did you measure it out like that? What did you do, Mark?
- 16.1.553 Mark: Well, we measured out um, yeah we measured twenty-seven meters.
- 16.1.554 T/R 1: You ended up with twenty-seven of them?
- 16.1.555 Mark: Yeah, we...

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- 16.1.556 T/R 1: Twenty-seven of those one thirds? And I know David and Erik you did something like that too.
- 16.1.557 Erik We did, we did it with Erin, we did it with Erin
- 16.1.558 Graham: We did it with twenty-seven meters
- 16.1.559 T/R 1: Ok, so you said to me that nine divided by one third, right, when you measured it out you found out that that was twenty-seven, and some of you did it differently. Who did it differently, without measuring it? Those of you who did it without measuring it, Sarah, what did you do?
- 16.1.560 Sarah: We timesed.
- 16.1.561 T/R 1: You said nine divided by a third is the same as nine times three?
- 16.1.562 Sarah: Yeah, and then,
- 16.1.563 T/R 1: Or twenty-seven
- 16.1.564 Sarah: Yeah and then we kept on timesing by three whatever the answer was.
- 16.1.565 T/R 1: Ok, I know that time is running out but I have this other question I want to ask you. Um, when you have nine meters of ribbon, I think Erik and David did this, and now we're making our, our ribbons three meters in length, not one third of a meter in length. Do you understand my question? How many bows can you make?
- 16.1.566 Erik: We're using nine meters, right?
- 16.1.567 T/R 1: You have nine meters of ribbon and now your bows are three meters in length.
- 16.1.568 Erik: Ok, you have nine meters of ribbon and your bows are three meters in length. If you have wait, yeah, if you have three meters all you have to do is multiply three times three and you get nine meters because you, if you have if each...
- 16.1.569 T/R 1: Ok, so how many can you make?
- 16.1.570 Erik: You can make three, three bows
- 16.1.571 T/R 1: So you're saying if I have nine meters and I'm making them three meters in length we could make three bows.
- 16.1.572 Erik: Yes.
- 16.1.573 T/R 1: What do you think, class? David?
- 16.1.574 David: I think the same thing, because, um, if each one takes up like a meter, um, nine divided by three, that, that would be three ribbons.
- 16.1.575 T/R 1: Each one takes up three meters.

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- 16.1.576 Erik: Yeah, each one takes up three meters.
- 16.1.577 David: Oh, yeah, wait a minute, um, it would be, it's like three times three would equal nine so uh nine divided by three equals three, um, [laughs] it's just because if you have three plus three plus three so you can if each one takes up three meters then you can make three bows out of nine. Because you have three meters and then, um, alright one bow would take up three so there'd be six meters left another bow would take up three so then there would be uh three meters left and then there'd be a third one and there wouldn't be, there wouldn't be any ribbon left.
- 16.1.578 T/R 1: Alright, I don't, I don't know the way the rest of you think about that. Do you agree with that? If you have nine meters bow and the three meters in length, you could make three of them. I think we have to stop now. What I'd like you to do, many of you did different things, right? I would like you to write to us and tell us what you did and why you did it. I also would like, particularly, the table of Beth, Jessica, Laura, Kimberly and Alan to write up your, why your rule works. As best as you can explain why your rule works. Ok? So if you're using a particular rule of multiplying, if you can explain to me why that works, we're going to share that tomorrow, we're coming back tomorrow, and we can start sharing, so whatever you did to get your answers, I want you to write up a story to us to explain it to us. That's your assignment. What you did and why.
- 16.1.579 Erik: So whatever answer you did? Whatever answers you did.
- 16.1.580 T/R 1: And how. How you did it.
- 16.1.581 57:22 [End of Class]