

Description: Working without the ribbons and bows: An excursion into division of non-unit fractions (side view) Date: 1993-12-15 Location: Colts Neck Elementary School Researcher: Professor Carolyn Maher	Transcriber(s): Lee, Dasom Verifier(s): Yedman, Madeline Date Transcribed: Spring 2014 Page: 1 of 20
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Line	Time	Speaker	Transcript
17.2.1	06:20	T/R 1:	Ok, good morning! Um, well, um, Meredith, it's so good to see you back and Meredith is probably curious to know what we did and since you were able to write to us to tell us what you did we have some information, Michael, what did you want to say?
17.2.2		Michael:	Well, we've been working with bows and discussing like um well the length, the, how many bows we could fit into a certain amount of meters if the bow took up a certain amount of ribbon. And uh we were recently doing thirds and uh we we got we got like stuff like we came up with like you times nine times three or something like that to get answers.
17.2.3		T/R 1:	Anybody want to add to that? Thank you, Michael. Any other comments about that? Ok, I guess what I'm curious about is how much you could, um, predict about ribbons and bows, maybe without having the ribbons and bows in front of you, if you try to remember some of the things you did and as you try to explain to me your thinking, um, on some predictions, so I'd like you all to imagine, how many of you can imagine in your heads a meter? A ribbon that is a meter long? How many of you can imagine in your heads? If you can would you raise your hand [all students raise hands] How long that is? Is a meter longer than the width of this room?
17.2.4		Students:	No
17.2.5		T/R 1:	Is it shorter than the width of this room?
17.2.6		Students:	Yes.
17.2.7		T/R 1:	Is a meter about the length of this chalkboard
17.2.8		Students:	No
17.2.9		T/R 1:	Is it bigger.
17.2.10		Students:	No
17.2.11		T/R 1:	Is it smaller?
17.2.12		Students:	Yes
17.2.13		T/R 1:	Yes? Um, ok, that's very interesting, um, most of the hands were up but I guess, Brian yours wasn't up, no?

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- You can't imagine the length of a meter? Can someone um help Brian without getting the meter stick and try to describe it? Caitlin?
- 17.2.14 Caitlin: I think it's about this big.
- 17.2.15 T/R 1: About that big. How many of you think that's reasonable? You want to hold your hand up, Caitlin so the rest of the class can see again? [Caitlin spreads arms] About that big? Is that reasonable? Does that help, Brian? Just keep your hands like that let's test this. It's pretty good, actually [brings over meter stick] Not bad, huh? That's pretty good! Ok, Brian? So what's one kind of way that might help you remember? Let's see how Brian does. Is that going to be too long or too short do you think?
- 17.2.16 Student: A little short.
- 17.2.17 T/R 1: What do you think, Brian?
- 17.2.18 Erik: Yeah, it's a little, it's going to be a little short.
- 17.2.19 T/R 1: Hey, that's pretty good, isn't it? Brian got it exactly. So Brian, you can't imagine but that's a good way of doing it, so you could all imagine the length of a meter stick. And what do you do in your heads to imagine the length of two meters? What do you sort of do to imagine it in your heads to imagine the length? Graham?
- 17.2.20 Graham: You double it?
- 17.2.21 T/R 1: You double it. How many of you do that kind of in your heads? [hands raised] And if you wanted three meter sticks? I know that somebody figured out about the length of nine meters, right Mark, you were working on that. Weren't you also working on that Gregory out in the hall with your, yeah, and can you tell us, can you tell the class something that would give you an idea of the length of three meters- nine meters.
- 17.2.22 Mark: Well, just uh if like eight meters plus one meter.
- 17.2.23 T/R 1: But if you had to tell someone here is like part of the school about nine meters?

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- 17.2.24 Mark: Oh, I guess maybe probably as big as the chalkboard or a little bigger. Probably like from the chalkboard, from the end of the chalkboard to Danielle.
- 17.2.25 Erik: No.
- 17.2.26 T/R 1: Gregory? You measured out something in the hallway.
- 17.2.27 Gregory: Oh, I know we measured the hallway.
- 17.2.28 T/R 1: You measured the hallway and what did you find the length of the hallway to be?
- 17.2.29 Gregory: Twenty-seven meters.
- 17.2.30 T/R 1: Was that twenty-seven meters, ok. So you're thinking that that's about twenty-seven. What do you think, Danielle?
- 17.2.31 Danielle: I think it's bigger than.
- 17.2.32 11:16 T/R 1: You think it's bigger than the length of the wall in here. How many of you think it's bigger than the length of of this here? How many of you think that it's smaller?
- 17.2.33 Student: Because this is eight meters.
- 17.2.34 T/R 1: This is eight meters? How do you know that?
- 17.2.35 Students: Because we measured it
- 17.2.36 Danielle: When we were measuring the school.
- 17.2.37 T/R 1: And you remember that?
- 17.2.38 Students: Yes.
- 17.2.39 T/R 1: Ok, so what would nine meters be then, about? What do you think?
- 17.2.40 Brian: Well, it would be about from the chalkboard to there.
- 17.2.41 T/R 1: From the chalkboard to whom?
- 17.2.42 Brian: Graham.
- 17.2.43 T/R 1: From the chalkboard to Graham? That would be about nine meters? Graham, you agree?
- 17.2.44 Graham: No.
- 17.2.45 T/R 1: Why not? You think it's too big, too small?
- 17.2.46 Graham: Too small.
- 17.2.47 T/R 1: You think it's too small.
- 17.2.48 T/R 1: The classroom is only eight meters.
- 17.2.49 T/R 1: The classroom which way? The width of the classroom?

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- 17.2.50 Students: Both
- 17.2.51 Erik: It's eight by eight
- 17.2.52 Students: It's eight by eight.
- 17.2.53 T/R 1: It happens to be an eight by eight classroom?
- 17.2.54 Students: Yeah.
- 17.2.55 T/R 1: So that's interesting, I would have not thought that. That's very interesting. What do you think, Brian?
- 17.2.56 Brian: Twenty-seven meter hall comes to this hall, coming down here, it's about from the door to um to the middle of to when you start going down there.
- 17.2.57 T/R 1: David?
- 17.2.58 12:31 David: I was going to say that um to the door, the outside door down to where you take a right[students talk simultaneously]
- 17.2.59 T/R 1: Oh so all the way where the intersection is where you can make a right, Danielle?
- 17.2.60 Danielle: If you go right to the bathroom then you will go up.
- 17.2.61 T/R 1: If you go into the bathroom, ok that's very interesting, Michael?
- 17.2.62 Michael: Also, I think about the width of it is like a little bit away from the wall and there um and all then to that wall.
- 17.2.63 T/R 1: So if we started at that wall, how much out of the classroom would we have to go to get nine meters?
- 17.2.64 Students: One meter.
- 17.2.65 13:29 T/R 1: One meter. Right, you all agree? That's very, very good, so you can imagine these lengths which I think is wonderful that you can get, um, some idea. So the next thing to, can you imagine in your heads what a third of a meter is? How many of you can imagine in your heads about a third of a meter? I'm not going to ask you exactly but you have some idea what a third of a meter is, you know what we mean by a third of something, don't you? How many of you know that? And you can imagine a third of a meter, right? That's real good, now I want to discuss with all of you, does anyone want to talk about that third of a meter? We did that yesterday in the beginning of class, Meredith wasn't here, but what we did, Meredith, remember

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some of you actually made string of a meter and you made your string into thirds and you had your third of a meter length, I remember that, right? Many of you did this and you did it last week. Um, I guess I wanted to discuss with you a problem that I believe um somebody was working on in the hallway, it might have been Erik, and um the problem had to do with, I remember I asked you how many bows can you make from ribbon that's a third of a meter in length if you have nine meters of ribbon, right? And I think everyone was able to solve that problem, right? You remember that everybody, raise your hands if you think you know the answer to that problem. And what did you get, Laura, what was that? How many, how many bows can you make if you start with nine meters and you have each bow to be a third of a meter in length? How many can you make? Talk to your partner for a minute, just for a minute, see if your partner agrees with you or not, Laura? Twenty-seven Laura says. How many of you think twenty-seven? And you can imagine there are twenty-seven of those one third meter lengths, right, in the nine meters of ribbon. So I'm imagining a ribbon along the floor going out one meter, and I'm imagining one third meter lengths and I'm counting them, right? How many of them will I count out if I count them?

- 17.2.66 Student: Twenty-seven.
- 17.2.67 T/R 1: Twenty-seven. But some of you didn't count them, some of you found an interesting way of getting your answer without counting, right, Alan? What was that?
- 17.2.68 Alan: It was multiplying nine times three and you get twenty-seven
- 17.2.69 T/R 1: Ok, but we wrote that problem as if we had nine meters divided by one third of a meter, you said that's the same as nine times three or twenty-seven. How many discovered that? How many of you discovered that yesterday? You you found the answer to be twenty-seven. I remember people did it several ways.

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Ok? Let's look at the different ways people thought about this. Some of you counted up how many one third meter lengths there were. Some of you did it by counting. And you, you took the one third meter and you counted them, how many of them when you counted them up?

- 17.2.70 Michael: You had, well you had three.
- 17.2.71 T/R 1: Three for each meter, so for nine meters you had.
- 17.2.72 Michael: Um, you had nine
- 17.2.73 T/R 1: You had the three nine times
- 17.2.74 Michael: Yeah.
- 17.2.75 T/R 1: Is that right? Some of you did it that way. Does that make sense? So some of you had the three, like one third one third one third and you had it nine times. Well that's what we wrote here, didn't we? Nine times three, right? The three nine times? Ok? So that was a way to do it. Some of you did it, said well, what was another way? We have counting, we have this way, nine times three equals twenty-seven, what was the third way that someone did this problem? Audra.
- 17.2.76 Audra: [inaudible]
- 17.2.77 T/R 1: Yeah, how did you do that.
- 17.2.78 Audra: You put the number down three times and you counted
- 17.2.79 T/R 1: Yeah, but, but what you did is you said there are three times and there are three times again and there are three times again, right? How many times did you have three?
- 17.2.80 Audra: Um,
- 17.2.81 T/R 1: One two three four five six seven eight nine, is that right?
- 17.2.82 Audra: Three.
- 17.2.83 T/R 1: That added up to
- 17.2.84 Audra: We did the three three times and we added it up to nine.
- 17.2.85 18:20 T/R 1: Ok so you added this up to nine, you added this up to nine, you added this up to nine, and all those nines gave you twenty-seven? That was another way, is that

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right? That some people did that problem? [Figure O-20-28] Ok? Did anyone do it any other way? Now, I'm wonder- I don't really, um, all these are wonderful ways, right? They're all great ways of doing it. Whatever way you're thinking about it, the important thing is, can you do a different problem and work it out with these ways? So I want you to imagine a different problem, this is the one I think Erik gave us. I still have my nine meters of ribbon, can you imagine that? My nine meters? But now I'm making ribbons that are three meters long. Remember that, Erik?

17.2.86

Erik: Yes.

17.2.87

T/R 1: Are my ribbons going to be bigger ribbons or smaller ribbons? Raise your hands if you think you know? I have nine meters of ribbon but not my ribbons are three meters long. Will my ribbons end up being bigger ribbons or smaller ribbons? That's what I want you to think about. Talk to your partner for a minute. Ok, let's see how many of you think it's going to be smaller? How many of you think the ribbons are going to be bigger? How many of you aren't sure? There are some people who didn't raise their hands either way. We'll go one more time. How many of you think it's going to be smaller? How many of you think the ribbons are going to be bigger? How many of you aren't sure? Ok. So, we now have to help the people who aren't sure - no one thinks smaller

17.2.88

Erik: Come to think of it, you could have, you could have three bows, well, I mean you could have each bow that's three meters long, and you can have one bow that you have to twist and everything using three meters of ribbon making it small and just like twisting it every which way.

17.2.89

T/R 1: That's an interesting idea, ok, maybe that's I hadn't thought about it that way, Erik, but let's suppose we're making identical kinds of bows, we're making, can we talk about a standard bow in here? All the bows are

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- going to look like this blue one. Like the blue one, so they're going to be all like the red one. So the question is, if I make my bows to be like this, would my one third meter bows that I'm making for my nine meters, be smaller or bigger than my three meter bows that I'm making from my nine meter ribbon?
- 17.2.90 Michael: The one third meter bow would probably be smaller, unless you had two different kinds of ribbon.
- 17.2.91 T/R 1: But we're not, though, we're keeping them the same.
- 17.2.92 21:22 Michael: But if you did, then you could make this the three meters really complicated bows so it is going to be very little and that one basically
- 17.2.93 T/R 1: Which is what Erik and David are saying, now some people aren't convinced of that, so what can we do to convince them that we are going to have bows that are bigger? Brian, you're not convinced, right? Ok. I'm imagining my nine meters on the floor. Are you imagining that? Ok. Now I'm going to cut that ribbon to make bows, right? But I'm going to cut it how long? How long will I cut it? Starting with nine meters, now I'm making three meter bows, right? So what's happening? How can we explain what's happening? What do you think, Brian?
- 17.2.94 Brian: It would be in thirds.
- 17.2.95 T/R 1: Ok, so can you, can you sort of tell me where one of the first cuts would be if I rolled this ribbon on the floor and it went out the room.
- 17.2.96 Brian: Um, on three meters.
- 17.2.97 T/R 1: On three meters, how many of you agree that the first cut would be on three meters? Do you imagine that? Is there anyone who can't imagine that? So my first cut's going to be on three meters. Where's my second cut going to be? Audra?
- 17.2.98 Audra: Um, on six meters.
- 17.2.99 T/R 1: On six meters. Is there a third cut?
- 17.2.100 Students: No
- 17.2.101 T/R 1: It's all done, right? Ok, so how many cuts will there be? How many cuts? Kimberly?

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- 17.2.102 Kimberly: Two.
- 17.2.103 T/R 1: There are going to be two cuts, right? How many pieces of ribbon will there be? Erin?
- 17.2.104 Erin: Three.
- 17.2.105 T/R 1: Three. Ok, so how many bows will I make?
- 17.2.106 Students: Three
- 17.2.107 T/R 1: I'll make three bows. You know that. Nine divided by three is three, that's one way, the other way is if I do it by cutting, right? One cut, my second cut, my second cut gives me three cuts each three meters long, right? To give me my nine meters, is that right? [Figure O-26-08] Ok, that's very good. How many of you understand that? Ok, let's give you some more. Could you imagine twelve meters of ribbon? How many of you can imagine? Sort of? Is it going to be more than nine meters?
- 17.2.108 Students: Yes.
- 17.2.109 T/R 1: Right? It's going to go past that hallway, you think?
- 17.2.110 Students: Mmm hmm
- 17.2.111 T/R 1: Yeah, who's class is that on the corner?
- 17.2.112 CT: Mrs. Warwick
- 17.2.113 T/R 1: It's going into Mrs. Warwick's room, isn't it? If we're rolling out that ribbon, ok? So you could imagine twelve meters. Now I'm making my bows a half a meter in length.
- 17.2.114 Student: A half meter!
- 17.2.115 T/R 1: A half a meter in length. So now I'm starting with twelve meters of ribbon, one half meter bow, right? How many bows am I going to make? Talk to your partners. Ok, ok, if you've solved that one, you can also solve the one if you're making them two meters long, while you're waiting for the other people. I want to know how many you can make that are half meter bows, and how many you can make that are two meters. Ok, if you think you have a way of getting those answers, if you've found a sort of secret about

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how to do it, I'd like to know what your secret is.

[Figure O-28-33]

1	19:12	Teacher	Now I am making my bows half a meter in length. Now I am starting with 12 meters of ribbon. One half meter goes? How many bows am I going to make? Two parts (questions).
2	19:51	Brian	(To Danielle) Because half of 12 is 6. You are not paying attention.
3	20:05	Beth	We have it.
4	20:06	Teacher	How?
5	20:07	Beth	Because we talked about it. *Jessica whispers something* Oh, yeah, because you double it.
6	20:14	Teacher	*To the entire class* If you solved that one, also solve the *inaudible*.
7	20:24	Beth	I just made *inaudible*
8	20:52	Teacher	*To the entire class* If you think you have a way of getting those answers if you found a secret on how to do it, I would like to know what the secret is.
9	21:08	James	2 meters is six.
10	21:18	Andrew	6 is 3 times 2. It's 3 times 2. Yeah. *Both Andrew and James write down $3 \times 2 = 6$ * And now 6 *inaudible* is just 2. So 6 meters equals to *inaudible* $\frac{1}{3}$ equals and $\frac{1}{2}$ equals, no, 2 equals, and half equals 2 *inaudible*
11	22:45	Jessica	We got 24.
12	22:48	Teacher	Okay, the next one?
13	22:49	Jessica	We got 6.
14	22:51	Teacher	For the next one? *Walks over to Beth* What did you guys get for the next one?
15	22:55	Beth	We are trying to, we are thinking. We got 24 and 6.
16	22:59	Teacher	What about *inaudible*?
17	23:02	Beth	*inaudible* six.
18	23:04	Teacher	So we got red $\frac{1}{3}$ meter bows and 6 *inaudible*
19			Brian is drawing models to show his reasoning.
20	24:00	Jessica	12 meter got *inaudible* for one half meter bows. We got. What did we get? *Laura answers* Oh yeah, 24 for half meter bows, right? So we have 24 for half

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			meter ones. So we got 2 meter bows for... how many meters do we have?
21	25:21	Teacher	Can you tell me more about this Andrew? What you talked about? *teacher walks away and comes back*When you work that out, I would like for you to tell me 12 meters bows *inaudible* two thirds *inaudible*
22			Really loud chair squeaking noise...James is writing down his explanation
23	26:51	Beth	See, the meters, bows are going to be half meters. So they can't be 2.
24	27:03	Meredith	Say this was *inaudible*. It's not, I know that. You divide it in to half. You have two pieces. Do you know what I mean? You are not going to have 24.
25	27:13	Beth	But you have TWELVE meters. So you have to divide each meter in half. <u>*Beth knows that she has to divide not one, but all 12 meters of ribbon in order to get 24 ribbons.*</u>
26	27:25	Teacher	We are talking about 12 meters of ribbons. You guys are on half a meter (problem)? So what is the question? How many bows can we make if each one is half a meter?
27	27:41	Meredith	For each one?
28	27:43	Teacher	Right.
29	27:45	Meredith	Oh, I thought you meant, one...
30	27:49	Teacher	I didn't say *inaudible* 12 meter in half. I want to know how many half meter bows I can make. Do you change your mind?
31	27:57	Meredith	Yeah because I thought you meant divide the whole thing (12 meter ribbon) into two.
32	27:58	Teacher	So do you agree?
33	27:59	Meredith	Yes.
34	28:00	Teacher	So all of these questions I am asking you here are how many two meter bows you can make, how many 1/3 bows you can make. Is that clear now?
35	28:06	Meredith and Beth	Yes.
36	28:07	Teacher	Okay, now tell me how many 2/3 meter bows you can

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			make from 12 meters of ribbon.
37	28:20	Meredith	The fourth one would be 12.
38	28:24	Beth	Wait, we didn't get the third one.
39	28:34	Jessica	Third is you have to get $\frac{1}{3}$ meters *inaudible*
40	28:47	Meredith	Twelve times six, right? I got it. You have to try. 12 times 6 *inaudible*
41	29:01	Jessica's partner	This is the rule. *inaudible* half. So there will be two.
42	29:10	Meredith	No. <u>*Maybe Meredith is having difficulty because now she has to divide by a number that is not a whole number*</u>
43	29:11	Jessica's Partner	Yes.
44	29:50	Someone?	So you can only cut it in half so *inaudible*
45	30:02	Someone? 2	36!
46	30:04	Andrew	No, wait. I could do these. 8 times 2 is 16. ... 76 and twelve... So $76+12$ is 88.
47	30:58	James	What are you doing?
48	31:00	Andrew	We are making 12 meters...
49	31:13	Teacher	*To Brian* Let's go back. You said that if they are half meters, you can make 24 of them, right? But if they are $\frac{1}{3}$ meter you can only make 3 of them? Does that make sense? You've got 12 meter, right? So with half a meter (ribbon), you can make 24 of them. Now for $\frac{1}{3}$ of a meter... Do you understand my problem? You two need to talk to each other. *Brian and Danielle* I am not convinced that with your two meters, how many did you make? *Brian shows his paper* You made six. How did you do that?
50	32:26	Brian	I put each meter together. Well, twelve is an even number. You can make two's out of it. If I put two's together, I count two, four, six, eight, ten and twelve. If you put them together, you only get six.
51	32:45	Teacher	*To Danielle* Do you know *inaudible* the same thing?
52	32:50	Danielle	I made these.

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53	32:51	Teacher	Those are lines? So you got six. So what about $\frac{1}{3}$ meters? *teacher walks away*
54	33:11	Andrew	I did that right.
55	33:12	James	Oh.
56	33:28	Meredith	So each meter...
57	33:30	Beth	No, I didn't say... I said you count. Six, twelve, counting two times.
58	33:47	Meredith	One second. You said you *inaudible* each *inaudible* six.
59	33:54	Beth	Yes. No. Wait, say that again.
60	33:55	Meredith	That's what you said about the half.
61	34:00	Beth	No. I said *inaudible*
62	35:18	Danielle	*counting* 36.
63			Camera focuses to the problem. 12 meters cut into $\frac{1}{2}$, 2, $\frac{1}{3}$, 6, $\frac{2}{3}$ meter bows.
64			Two partners are using transparent paper to make a strip of 12 'meter' ribbon.
65	37:06	Boy 1	This will be 6 and we will make another one down here.
66			Jessica, Laura, Meredith, and Beth are using a meter stick
67	37:19	Jessica	That is one third. So there are two in each meter. No... So 33 (centimeter) is $\frac{1}{3}$. So I'm going to put a pen there. And 66 (centimeter) is $\frac{2}{3}$. And then 96...
68	38:19	Teacher	I can do one better. *Inaudible* You can make a picture. I want you to imagine what you know already and try to draw a picture and sketch it.
69	38:32	Jessica	No, we can't. All we know is that this is one third.
70	38:43	Teacher	If you want to use that thick of a *inaudible*, here.
71	38:49	Meredith	Guys, we need three $\frac{1}{3}$'s. We only took two $\frac{1}{3}$'s. We need three.
72			The teacher hands them a large paper with a meter taped fixed to the paper.
73	39:44	Meredith	Guys, what we need is, we need three thirds because we don't need two halves. <u>*This shows me that Meredith knows that there are 3 $\frac{1}{3}$'s in a meter instead of 2 $\frac{1}{3}$'s in a meter.*</u>
74	39:52	Jessica	No, we need two thirds.

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75	39:54	Meredith	*inaudible* two thirds but we have to divide it to three thirds.
76			Meredith and Jessica divide the meter stick at 33 centimeter, and 66 centimeter.
77	Laura	40:34	That and that. And we only have that much left (66-100 centimeter).
78			Students are labeling where they divided the meter stick. Jessica says that the last piece is too big to be $\frac{1}{3}$. <i>So students negotiate where to put the second tally.</i> *This shows that students are not completely aware of how to equally divide the meter stick in thirds.*
79	43:10	Jessica	So try it now. (She takes the last $\frac{1}{3}$ and compares the length of it to the second $\frac{1}{3}$.)
80	43:22	Beth	I don't think so.
81	43:23	Meredith	It can't be there. It has to be farther up. (Further down the ruler to make the length of the second $\frac{1}{3}$ equal the length of the first $\frac{1}{3}$.) I know where. I can estimate. If we put it here, no...
82	44:17	Teacher2	Do you want to explain this again in front of the camera? (A boy with his explanation written on a piece of transparency paper)
83	44:22	Boy	What I did was I made 12 meters. The long line is the divider for each meter. In the meters, there are three thirds. The bracket shows these two thirds and there are two thirds and so on. So when you go down, the final answer is 18. And that's the number of bows you can make with $\frac{2}{3}$ meter bows.

17.2.210 50:11 T/R 1: Ok, I'm going to ask you to, we're going to need to stop for a moment, I'd like you all to just stop I know you're all in the middle of this and maybe Mrs. Phillips will let you um continue a little bit of this, it's up to her, but I would like us to do some sharing, because there are some ways people have been thinking about it. Ok, I'm interested in your sharing the way you're thinking about it, and maybe we'll have

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another couple of minutes to finish up. Ok, um, Alan and Kimberly wanted to share with you the way they were thinking about the twelve meters divided by bows of two thirds of a meter, right? We had twelve meters of ribbon and we're making the bows two thirds - can you kind of look to see because I'm going to ask you to write about what they did, I want to ask you to write about what they did and I want to make sure that what they did makes sense to you or doesn't make sense to you, because if it doesn't make sense their job is to uh either convince you or you convince them, so can you all give us your attention here for a minute? Ok, Kimberly and Alan, tell us what you did.

17.2.211 51:24 Alan: [Figure O-53-09] This entire thing is twelve meters. The long line is the divider of each meter [inaudible] The brackets are dividing the thirds up so there are two thirds, there are two thirds, there are two thirds, there are two thirds, and if you count up how many two thirds there are, you'll eventually get down to eighteen, and that's how many bows you can make of two thirds out of twelve meters.

17.2.212 T/R 1: Questions?

17.2.213 Andrew: Well, me and James did uh the same thing that did the twelve and we got eighteen too.

17.2.214 T/R 1: You did it the same way. Any other questions, comments?

17.2.215 52:32 Erik: How are we going to be able to write what they did? I mean, if we write that cause we're going to have to diagram it, there's no way we're going to be able to write it.

17.2.216 Alan: Should I explain it again.

17.2.217 Erik: No, I know what you mean, but we'd have to diagram it to write it, we couldn't write it in words, we'd have to diagram it.

17.2.218 T/R 1: Ok, now first of all, I heard somebody say they'd like to hear a second explanation. How many of you would like another explanation? Ok, now in your explanation, my suggestion is, go through each part,

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- be sure people understand each part, Kimberly, and don't move onto the next part until each little part they understand. Fair enough? Ok, so one more time please?
- 17.2.219 Alan: Ok, this, this is all twelve meters, the line, this line is what divides each meter up, in each meter there are three thirds. The bracket has two thirds under it, which means those are the two thirds to make your bow and if here are two thirds, here are two thirds, here are two thirds, here are two thirds, and you keep going on to the end until you get up to eighteen, and that's how many bows you can make out of two thirds each meter, um, of twelve meters of ribbon.
- 17.2.220 T/R 1: Question?
- 17.2.221 Beth: I agree with that, because in my book, we had the [inaudible] book, for two thirds, uh, I did the same thing like that, and that's how I got my answer.
- 17.2.222 T/R 1: Ok, other comments? How many of you, um, understood this explanation? Raise your hand if you understood the explanation? How many of you would like the explanation broken down again? Raise your hand if you'd like it again. What I'd like you to do Alan is each part say how many of you know where I got the twelve? How many of you know where I got the one? Ok, break it down. Why don't you give it a try, Kimberly? Ok, go very slowly, Kimberly.
- 17.2.223 54:56 Kimberly: This one here, all together is twelve meters. And these here, the long lines separate between 'em. And there are three meters in each meter and the brackets separate two meters and two thirds in each meter.
- 17.2.224 T/R 1: I think I heard you say there are three meters in each meter. I don't think you meant to say that.
- 17.2.225 Kimberly: I know, I didn't
- 17.2.226 T/R 1: What did you mean then?
- 17.2.227 Kimberly: I meant three thirds
- 17.2.228 T/R 1: Three thirds of a meter in each meter

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- 17.2.229 Kimberly: And then the brackets separate two thirds in each meter
- 17.2.230 T/R 1: How many big lines are there? How many big lines are there? Many of you said there are twelve meters, and the big meter marks off each meter. How many big lines would there be to mark off each meter? What do you think, Laura?
- 17.2.231 Laura: Twelve.
- 17.2.232 T/R 1: Laura thinks twelve. Someone think something else? Andrew? What do you think?
- 17.2.233 Andrew: um, Eleven?
- 17.2.234 T/R 1: Andrew thinks eleven.
- 17.2.235 Andrew: Wait no ten.
- 17.2.236 T/R 1: Andrew thinks ten. Brian.
- 17.2.237 Brian: Thirteen.
- 17.2.238 T/R 1: Brian thinks thirteen. James.
- 17.2.239 James: I think eleven.
- 17.2.240 T/R 1: James thinks eleven. David.
- 17.2.241 David: Well, I think, I think ten.
- 17.2.242 T/R 1: You think ten, Erik?
- 17.2.243 Erik: I think eleven.
- 17.2.244 56:02 T/R 1: Well, we just really aren't agreeing. Well, how can we find out? Let's actually count them. Can we point it out, let's count them we'll check it with Alan as we're doing it here. So here's the first one, let's count together, [students join] one two three four five six seven eight nine ten eleven. Would you have to cut the last one or is it cut for you already?
- 17.2.245 Student: No
- 17.2.246 T/R 1: Is the last cut, is the last piece of ribbon or is it cut for you already.
- 17.2.247 Andrew: There is one more.
- 17.2.248 Kimberly: Alan made a mistake.
- 17.2.249 T/R 1: Ok, but if you're cutting this ribbon, how many cuts do you make?
- 17.2.250 Students: Eleven.
- 17.2.251 T/R 1: You make eleven cuts and how many pieces do you get when you cut it?

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- 17.2.252 Students: Twelve.
- 17.2.253 T/R 1: Twelve, twelve meter lengths. Ok, do you all understand how we get twelve of those one-meter length? How many of you understand that? With the eleven cuts? But there are twelve marks, that's right what Laura said, if you count, if you're looking at marks. Now, what did they do after that? After they marked off these one meter lengths, what did they do next?
- 17.2.254 Student: They put the two thirds in.
- 17.2.255 T/R 1: Well, before they put the two thirds in, what did they do before they put the two thirds in, Meredith?
- 17.2.256 Meredith: Brackets.
- 17.2.257 T/R 1: Well, they did something before then, I think. Before they marked two thirds, what did they mark first, Andrew?
- 17.2.258 Andrew: Well, the, um, the thirds.
- 17.2.259 T/R 1: They marked the thirds first. Is that what you did, Alan and Kimberly? [mmm hmm] The marked the thirds first. Why do you suppose they marked the thirds first? Why do you think they did it that way? What would your guess be? What were they after? Meredith?
- 17.2.260 Meredith: Well, so they could know where to put the brackets.
- 17.2.261 T/R 1: So they know where to put the brackets. And what did the brackets show in this problem? Did the brackets show one third?
- 17.2.262 Student: No
- 17.2.263 T/R 1: What did the brackets show? Graham?
- 17.2.264 Graham: Two thirds.
- 17.2.265 T/R 1: Two thirds. Can you see that? They had to mark one third, and each meter they marked a third, you see how they did that? And then they marked two thirds, right? And then they put brackets. Now, what did they do after they marked all those two thirds off with the brackets? What did they do after that? Andrew?

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- 17.2.266 Andrew: They numbered them.
17.2.267 T/R 1: They numbered them. Why do you think they numbered them? Why do you suppose they used the strategy of numbering them? That was kind of clever of them to number them, at first they didn't number them, and later on they came with those numbers. Why did you start numbering them, Kimberly?
- 17.2.268 Kimberly: So that we can find out the answer, because we lost count a few times before we put the numbers.
- 17.2.269 T/R 1: Oh, yeah, you lost count, I remember one time you said seventeen and sixteen and
- 17.2.270 Kimberly: Yeah.
17.2.271 T/R 1: A couple of you also lost count didn't you I noticed as I walked around. I noticed what you were doing the same thing, but some of you lost count. So the numbering was a very good strategy. And what did, how many numbers did they end up having when they counted two of the thirds?
- 17.2.272 Student: Eighteen.
17.2.273 T/R 1: Eighteen. How many of you are convinced that eighteen is the answer? How many of you think that you can write up or try to write what they did?
- 17.2.274 Erik: I think I can diagram it, I don't think I can write about it.
- 17.2.275 Michael: We did basically what they did
17.2.276 T/R 1: Now, before you go I have just one question to ask you, it's Alan's birthday so we're not letting you off so easily. [inaudible] It's twelve divided by two thirds, now before when I asked you how many one third ribbon lengths when there were twelve meters of ribbon, what did you tell me the answer was? Everybody? If there are one third meter lengths, how many?
- 17.2.277 Student: Seven.
17.2.278 T/R 1: Twelve meters of ribbon, one third meter each.
17.2.279 Student: Oh.
17.2.280 Graham: Twenty-four

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- 17.2.281 T/R 1: No one third.
- 17.2.282 Graham: Oh thirty-six.
- 17.2.283 T/R 1: Thirty-six. One half meter, Graham, was twenty-four. You said thirty six, and some of you found a secret for finding it, what was that secret? Andrew?
- 17.2.284 59:56 Andrew: Um, I said, I um multiplied either um one third two third six or two to twelve and I got the answer.
- 17.2.285 T/R 1: Ok, so you said there was a rule like this you found the secret, twelve divided by one third, you found that by multiplying twelve times three and getting thirty-six. My question to you is does that secret work here? Twelve multiplied by three and a two. That's a big question mark. Maybe when we come back we'll think about that secret. I think our time is up.
- 17.2.286 CT: Um, Dr. Maher, would you like them to explain how many two third meters you can make out of twelve meters?
- 17.2.287 T/R 1: You can get out of twelve meters. And you can draw a picture, Erik, if you can try words, I'd like that, you can draw a sketch, any way you want to do it. Thank you.
- 17.2.288 1:00:56 [end of class, focus on Jackie finishing her transparency]