## Transcript of Problem Solving Session 1/22/99

Note: Explanatory notes and brief descriptions of sections of transcript not included here are enclosed in brackets.

```
T/R1: We've been looking at the tapes, you know, and there's
        some interesting ideas that different people have presented
        and some questions that came out of looking at them that
        we wanted to address again. We'ii start with something-
        with a new problem. Is that okay?
    Jeff: That's fine.
    Michael: That's fine.
    T/R1: And- and you can use anything else, you could use each
        other. The rule again for this ... [materials are distributed]...
        you can use any tools you want, you know. We, we have
        things around if you want to use them and you're welcome to
        use them. But the idea of course, as you know, is not just to
        tell us about what the probability is that these games will be
        won, but you have to be able to convince us. I should, I
        should introduce this by saying l've given this problem to a
        bunch of folks lately and I have found people think about it in
        very different ways. Is that good enough? And so, there
        isn't a particular one way to think about it. It's just what
        makes it interesting. So you may, you know, you may all
        think about this initially differently and then you share your
        thinking. But the important thing is, there is only one right
        answer and what you ought to try to think about is, if
        someone else is thinking about it differently than you are,
        how do you connect your thinking? Do you see what l'm
        saying? Does it come together? Does it make sense in any
        way? That's all I have to say. Do you understand? Do you
        all know about World Series?
Jeff: Mm hm.
Brian: Yeah.
T/R1: You all know that the World Series is played in seven
        games? Did you know that Romina because I didn't know
        that.
    Romina: No, yeah, I know that.
T/R1: And, you know, you can win in four games or five games or
        six games or seven games. Right? There's certain
        probabilities. Any questions? If you don't like the way
        you're seated you can change it some, you know, if you
        want to get into groups. But, you know, we want you to be
        able to talk to each other, in any way you want.
Romina: Couldn't we do like a - you know how we do, like -
```

T/R1: $\quad$ Okay, I'm going to leave you alone.
Romina: They can go all seven or they could go all four. So, it would be A, A, A, A and B, B, B, B -Team A and Team B?
Jeff: Wait, what's the - wait - wait -
Romina: So those are the only possibilities for four?
Jeff: Mm hm .
Romina: So, in four games, would it be, like, one-half of a chance? Or would we have to write it out with -- using all seven?
Jeff:
Brian: See, $I$ think that it's the hardest to win it in four games.
Four games.
Jeff: Definitely the hardest.
Romina: Yeah, exactly.
Jeff: So, it wouldn't be one-half.
Brian: Isn't it the odds - the odds of winning one game, times the odds of winning one game, times the odds of winning one game?
Jeff: That's what I'm thinking.
Ankur: Look, it's a fifty percent chance of winning the first game.
Brian: All right.
Romina: One-half.
Brian:
So, it's like, half times a half - no, wait - remember the odds get harder to win two in a row, like a coin flip?
Ankur: Yeah, so that's -
Romina: Yeah, that's how you do it: a half times a half times a half times a half.
Brian: All right.
Jeff: If that's the case, what is it?
Romina: Four - hold on - four times -
Brian: That's sixteen.
Romina: Is it one thirty-two?
Brian: $\quad$ Two times two is four, times two is eight. Three times three -
Romina: Oh, never mind, I get it. Now, would you have, for five games, like, would it be like that [writes $1 / 2 \cdot 1 / 2 \cdot 1 / 2 \cdot 1 / 2 \cdot 1 / 2]$ ?
Ankur: $\quad$ Hopefully, the odds of winning are -
Jeff:
Brian:
We're never going to get - it's never gonna equal up to one, though.
Does it have to?
Romina: No, but, I was thinking, you know how we do, like, uh, like A -
Jeff: Yeah.
Romina: For four - you know what l'm saying?
Jeff: Mm hm.

| Jeff: | Then that would be that number and that number. Yeah, that's what I was thinking, but - |
| :---: | :---: |
| Ankur: | So then we got to do it like that. |
| Jeff: | But, how - all right, wait. |
| Romina: | How many possibilities? |
| Ankur: | There can only be two ways. |
| Jeff: | Well, wait. Just - so, wait. Before we do that, let's look at, um, how do you get to that point in the first place? To finding out? 'Cause there's like a lot of different combinations - two to the seventh. Is that two to the seventh? |
| Romina: | Isn't it - yeah, two n? |
| Jeff: | Yeah. All right, so say it's two to the seventh. |
| Romina: | How much is that? I don't know. |
| Ankur: | For this, you've gotta find all possibilities with - |
| Romina: | Do you want to write down the possibilities so we just see it? |
| Jeff: | Wait. Do you know how many two to the seventh is? |
| Brian: | Yeah, it's the order you win, though, too. |
| Jeff: | Yeah, and we also have to look at the fact that [inaudible] |
| Romina: | Yeah, I know. |
| Jeff: | For like five to be - five - five - |
| Romina: | First we look at it like this and then we move over. |
| Jeff: | Five B's is not - |
| Ankur: | It's like, three A's, it could be a B and then - |
| Jeff: | Yeah. Like five B's - or five A's and then - |
| Romina: | Yeah, I know. I'm just saying, like, each time we'd look over like, well five, we'll see how many - you know? [they are writing strings] |
| Ankur: | To have - |
| Romina: | I can do the ones just having A winning, like having - |
| Ankur: | It would be eight, for the five. |
| Romina: | Do you understand, just like, you know, having [inaudible]. |
| Jeff: | Yeah, but you're not going to know if you have them all, though. How are you gonna know? |
| Ankur: | To have - this will be, five games will be eight. |
| Jeff: | Think so? |
| Ankur: | 'Cause like it'd be these four, 'cause look, you can only lose one game, right? |
| Jeff: | Mm. Oh, so it's only - |
| Ankur: | You can't lose the last one, or, 'cause, or they already won four, you know what I mean? |
| Jeff: | Yeah, yeah, yeah, yeah. |
| Ankur: | So it'd be these, and then, like, A there and then B's in the other place. You know what I mean? |
| Jeff: | All right. So after the four, for winning in four games - |
| Romina: | Should it be over seven, though? |


| 219 | Ankur: | Eight of these. It'd be over, like, total possibilities of - |
| :---: | :---: | :---: |
| 220 | Jeff: | Yeah, the total possibilities is eight, right? |
| 221 | Ankur: | They have eight ways of winning but it'd be over - |
| 222 | Jeff: | Oh. Eight over one - no - well, how do we find out? |
| 223 | Ankur: | l'd be over - the total possibilities of two, like two - two |
| 224 |  | colors and five things. |
| 225 | Jeff: | Yeah. |
| 226 | Romina: | It wouldn't - they wouldn't be over the whole thing, like a |
| 227 |  | seven? |
| 228 | Jeff: | It wouldn't be. |
| 229 | Michael: | It should be over - over seven, 'cause it's four out of seven |
| 230 |  | games. |
| 231 | Ankur: | But this one wouldn't be over seven. |
| 232 | Jeff: | It wouldn't be. |
| 233 | Ankur: | It wouldn't. None of this would be over seven. |
| 234 | Romina: | So why would it be eight? I missed that. See, I didn't hear |
| 235 |  | you. |
| 236 | Jeff: | Because he's going like this [writes out strings for B winning |
| 237 |  | in 5 games] - you're just moving it over and then - |
| 238 | Romina: | Eight? |
| 239 | Jeff: | Yeah, well, the last one would be B, B, B, B, A. |
| 240 | Romina: | Mm hm . |
| 241 | Jeff: | In that case, they would have won at this game right here. |
| 242 |  | But that would count, though. |
| 243 | Romina: | Wouldn't it only be four? How is that eight, though? |
| 244 | Ankur: | It'd count in the total possibilities, but it couldn't count in the |
| 245 |  | wins. I counted that one, the B and then the four A's, but |
| 246 |  | you can't do four B's and then an A. |
| 247 | Jeff: | No, it'd just be four B's. |
| 248 | Ankur: | You know what I mean? |
| 249 | Jeff: | Then B, A, A, A, A. |
| 250 | Romina: | Okay. So should we stick with one team winning or either |
| 251 |  | team winning? |
| 252 | Ankur: | It has to be either. |
| 253 | Jeff: | Well it's gotta be - what's the probability in the World Series |
| 254 |  | that they would win in five, four games? It's - |
| 255 | Ankur: | So it'd be either. Now you just gotta find the total |
| 256 |  | possibilities of - you know what I mean? |
| 257 | Jeff: | First of all - |
| 258 | Romina: | Wouldn't it be two to the fifth? Or would it be two to the |
| 259 |  | seventh? |
| 260 | Jeff: | Two to the seventh is a hundred twenty-eight. And, but, like |
| 261 |  | B, B, B, B, B - like B seven times wouldn't count. But on the |
| 262 |  | other hand, the sum - |

263 Romina: But, I'm saying for this one, it's eight over - would it be eight

264
265
266 Jeff:
267
268
269
270
271
272
273
274
275 Ankur:
276 Jeff:
277 Ankur:
278
279 Jeff:
280 Ankur:
281
282 Jeff:
283 Ankur:
284 Jeff:
285 Ankur:
286 Jeff:
287 Ankur:
288 Jeff:
289 Ankur:
290 Jeff:
291 Ankur:
292 Jeff:
293 Ankur:
294 Jeff:
295
296 Michael:
297 Jeff:
298 Ankur
299 Brian:
300 Jeff:
301 Romina: Hold on. What do you - you got eight?
302 Brian:
303 Jeff:
304 Brian:
305 Jeff:
306 Ankur:
307 Romina over two to the seventh or two to the fifth? Maybe to the fifth? Well, because It wouldn't be two to the fifth because some games would have to go more than five games.
Well - well, what's the first one?
Than five slots. This only goes five slots, but then what happens when it goes to six games?
Romina: Yeah. Are you doing that for seven?
Brian: For six.
Jeff:
Only - there's a total of two ways they could win in four games, right?
Yeah, it's two over two to the fourth.
Why is it two to the fourth?
Because that's the total, like - two to the fourth will give you
the total possibilities of four things -
All right.
You know what I mean? It has to be over - do you know what l'm talking about or not?
Yeah, it's got to be over two to the fourth - four spaces.
The total possibilities of A, B - yeah, four spaces.
Yeah, all right. It makes sense.
That's what -
And then that'd be eight over two to the fifth, you think?
That's four, eight, that's sixteen.
And then eight over two to the fifth?
I guess.
Which would be -
Thirty-two.
What does eight over thity- two reduce to?
There's more than thirty-two, though.
Four, four over sixteen? One over eight? That's one eight, right?
But is there thirty-two possibilities in five games?
Yeah.
That's what l'm-- I think there's more.
For how many games?
Five.
How many possibilities?
Five spaces.
Total possibilities?
Thirty-two for five.
Well, it could be -
$\mathrm{He}-$

308
Ankur: Yeah, it is thirty-two.
309 Romina:
Brian -
Ankur:
Jeff:
312 Ankur
313 Jeff:
314 Anku
315 Jeff:
316
317 Anku
318
319 Brian
320 Jeff:
321 Ankur:
322 Jeff:
323 Anku
324 Jeff:
325 Ankur:
326 Romina:
327 Brian:
328
329

337 Ankur:
338 Jeff:
Yeah. Where you just go two -

Yeah. And for six it'd be mean?
You can skim them out.
Yeah, how do you -
Five -
How do you skim them out?
Yeah. two for five.
Ankur: No, we got eight for five.
Jeff: No, no.
Brian: Eighteen, for six.
Jeff: Total?
Brian: For six.
Ankur: What about for five?
Brian: Eight. over one twenty-eight?
Jeff: Excuse me?

Like one hundred percent? game. seven games.
‘Cause remember you do the blanks, like the five blanks?
Two, two - you can put either A or B, A or B, A or B.
Yeah, well then, it's just two to the fifth.
And that'd be sixty-four. And then the last one would be a
hundred - over a hundred and twenty--eight?
Yeah but we gotta find the other things. You know what I

Like, you see how I skimmed these out?
You've got to have two B's in this - just make sure that Brian's doing them for six. How many did you get?
It's not going to be what they got, 'cause I didn't get thirty-

Romina: They got, they got eight for five. You got eight.

Weah, okay. Weil do the same thing for six.
Two for four. Eight for live. Uh, eighteen for six.
Romina: And seven - what's - wouldn't this be like one twenty-eight

Romina: Would that be one twenty-eight out of one twenty-eight?
Jeff: One twenty-eight out of one twenty-eight?
Romina: 'Cause there's, there's - a team has to win by seven games.
Jeff: I know, but it's not saying, what's the chances, what's the what's the probability that a World Series will be won in seven games, not that someone will win in the seventh

Romina: No. Yeah. That - the World Series has to be won in the

352
353
354
355
356
357
358
359
360
Jeff:

Jeff:
Brian:
Jeff:

Jeff:

Romina: The sixth.

Ankur:

Romina: Do you want me to write out possibilities for seven? This will take -
Michael: No, it's saying, what's the possibility that's it's won -
Romina: This is going to take forever.
Michael: Let's say you just take the seventh game. Obviously, that would be the hundred percent.
Jeff: Yeah, that was--
Romina: I'm going to write that.
Michael It's not saying there's a probability that World Series is won - is won in - you know, what's the probability that it's in one of, you know, one of these four.
Romina: All possibilities -
Jeff: You see, well I had thought of -
Ankur: Are we doing it right, then, or not?
Romina: I'm going to write that.
Jeff: Oh, well I thought it was.
Michael: They're asking, will be won in seven games. And, obviously, that's a hundred.
Jeff: Yeah, but I-
Ankur: How about this?
Michael: The probability has to be a hundred, though.
Jeff: I know, what's the chance that it, that out of -1 thought that all the games -
Ankur: It - it will, it will.
Michael: No, I'm not saying it has to add up to a - seven has to be a hundred.

Michael: They're, they're not asking, like, what's the probability that you win the World Series. What's the probability that it's in, that it was won in the seventh game or the sixth game.
Ankur: See I don't think that's what -
Jeff: $\quad$ No, it's what's the probability it would of won in -
Michael: It's easy if you really just read it how it is.
Ankur: It can't be a hundred. Because then it can be won in four games.
Well, yeah, what [inaudible].
No, no. I'm saying what [inaudible].

$$
400 \text { T/R1: }
$$

401 Jeff:
402 T/R1:
Yes? What's the confusion? saying -

Then, then it's a hundred percent.
No, but asking, out of all the chances -

Yeah, that's what - that's what I figured.
All right, so it's -thirty-two. thirty-two's -
I'm not going to get that far.

Ankur: Yeah.
Jeff:
Ankur: all out? what I mean? looking at it there?
Yeah, I was trying to do something like that.
Yeah, but you got two. I mean -
It's all messed up. they're all the same. It's like two [inaudible]. can't have - you know what I mean?
Yeah. Exactly. It's, I think it's, uh -

You said if it reaches the seventh game, it's going to be won. Yeah. That's obvious, but they're not asking that.
Can we factor this out [looks to T/R across the room]?
We're not, uh, exactly sure what's being asked here.
I mean, are you saying that, like, in seven games, are you saying, what's the chances that someone will win it all in seven games? Like, for the, the just $D$ part. Or are you

Ankur: Are you saying, if it gets to the seventh game, then it's an obvious answer. You won in the seventh game.

But you're asking, you're asking it the other way. You're Out, out of all the games, what's the probability it's won in the seventh. Right. You interpreted it the way I did.

That way it all - you - it all adds up to, uh, it all adds up to

So, wait, wait. If eighteen out of sixty-four is right, then nine thirty-two's, eight thirty-two's and two thirty-two's, is nineteen

Then, that means that that game will have to be won in ten in thirteen. That'll be - in seven, it's thirteen thirty--twos. If we did everything else right. You know what l'm saying?

I don't know if that's right, but - are you going to write them
Just not all of them, but just like the ones that - you know
Yeah, I was looking at the two like that, you know how you're

You got here - it could be two, if these are different or one if
And then you've got to minus something out because you

Jeff:

Jeff:

Jeff:

Romina: How many do you have so far?
Ankur: But you really don't have two right here because you have something like -
Romina: Oh.
Ankur: It's like you can have A or B , right?
Jeff: $\quad \mathrm{Mm} \mathrm{hm}$.
Ankur: If you choose B -
Romina: I messed up so bad. Like, I'm so lost.
Ankur: You can only have another B here and you don't have a choice of two any more.
Romina: But, see, because it's so hard to organize them -
Jeff: Yeah, I hear you. It's - but you could get either one. For the next one it could be A or B , too.
Ankur: Yeah.
Jeff: $\quad$ And then for the next one, even if it is B , it could be A or B again.
Ankur: Yeah.
Jeff: $\quad$ So right now, even if it could be $B, B, B$ or $A, A, A-$ Ankur: That's right.

But the fourth one, it could be either one, unless one of these two were all three.
Ankur: Yeah. That's what I was trying to [inaudible].
That's the thing. When you get to the next one, I think there's so many different things that are pulling on this that it doesn't -
Ankur: I think these are it.
Jeff: How many do you have there?
Ankur: Ten, and then there's the others, so it's twenty. You know what I mean?
Well then we messed up somewhere if that's the one.
How come? It'd be twenty out of sixty--four.
Oh.
You know what I mean?
So it'd be ten thirty--two's?
Guessing so far.
Then we're - then we still messed up somewhere. It's got to be thirteen, if we did everything eise right.
Ankur: Thirteen out of what?
Jeff: Thitty-two. Six, twenty-six.
Ankur: How come?
If we did all the other ones right, they're gonna equal up to thirty-two thirty-two's. So if [part] C is nine thirty-two's, like they said - I don't know if that's right, though - then, that's gotta be thirteen.

| $\begin{aligned} & 486 \\ & 487 \end{aligned}$ | Ankur: | Oh. Well. I started by putting the two games together that they would win. |
| :---: | :---: | :---: |
| 488 | Jeff: | Mm hm. |
| 489 | Ankur: | One, two, three, four. |
| 490 | Jeff: | Then you got four more. |
| 491 | Ankur: | Is that - and then - but I don't know. You can't put B on the |
| 492 |  | last game, obviously. |
| 493 | Jeff: | Yes. |
| 494 | Ankur: | Got two - one, two, three. I already got it together, then - |
| 495 |  | that's it. [He and Jeff examine his list of strings.] Unless we |
| 496 |  | don't do the opposite - you know what I mean? The other |
| 497 |  | team winning? |
| 498 | Jeff: | What do you mean? |
| 499 | Ankur: | Like, you know, how we're doing - |
| 500 | Jeff: | Oh, yeah. |
| 501 | Ankur: | Like, there's two out of sixteen; maybe just one out of |
| 502 |  | sixteen? |
| 503 | Jeff: | I hear you. Then that'd be what? Four out of thirty-two? |
| 504 | Ankur: | Right now this is coming to twenty out of sixty-four. So, ten - |
| 505 | Jeff: | It's still too short. |
| 506 | Romina: | I am so lost. |
| 507 | Jeff: | We're down by three. We only have twenty-nine. |
| 508 | Romina: | Are you doing the opposites? |
| 509 | Ankur: | Mm hm . |
| 510 | Jeff: | Is that for - how many games? |
| 511 | Romina: | See, I got eighteen, but I don't know if I have them all, I don't |
| 512 |  | know if I repeated some. I'm just so, like - |
| 513 | Jeff: | For seven? |
| 514 | Romina: | Because B can't and A can't be in the first four. |
| 515 | Brian: | I had thirty for seven. |
| 516 | Romina: | Oh. |
| 517 | Jeff: | Can I see that for one second? Because, you know, if you |
| 518 |  | look at something for long, you just - |
| 519 | Romina: | That's what l've been thinking about. |
| 520 | Brian: | Actually, I got thirty-two for seven. |
| 521 | Romina: | How many possibilities are there for that? Because all I did |
| 522 |  | was that. Sixteen. |
| 523 | Jeff: | So then, you're saying there's double this total because you |
| 524 |  | could do the opposites of each? |
| 525 | Ankur: | Should've got thir - should've got thirty, Bri. |
| 526 | Romina: | See, I don't know if we should do the opposite. |
| 527 | Ankur: | Brian - |
| 528 | Brian: | What? |
| 529 | Ankur: | You should have got thirty, I think. |

530 Jeff:
531
532
533
534
535
536
537 Jeff:
538
539
540
541
542
543
544
545
546
547
548
549
550
551
552
553
554
555
556
557

Jeff:

Jeff: No. It's, it's what's the probability that the World Series will
Romina: See, I would have, if I double mine -
Ankur: $\quad$ There's like - when you add these up, what's six out of - six and five, that's eleven. That's sixteen. If this comes to be five out of sixteen, then you know it's right. You know what I mean?
Jeff: Oh yeah, you went - oh, I didn't even realize. See - wait. But how come I had eighteen sixty-fours, too?
Ankur: Out of where?
Jeff: $\quad$ On the eight thirty-two's, uh -
Romina: What did you guys do while I was doing this?
Ankur: Was it five-sixteenths?
Jeff: Well, that's weird, because I had - eighteen -
Romina: What were you guys doing while I was doing this?
Jeff: Um, nothing much.
Romina: Uh, did you see doubles?
Jeff: I really didn't look yet.
Romina: I didn't - I have [inaudible] from there. I didn't do it for, like, B's winning. I only did it for A's winning.
Jeff: It's hard to read in this kind of set up, you know what I'm saying? Do you see doubles in here?
Ankur: [Looking at Romina's paper] Is that a seven?
Jeff: Yeah.
Romina: With A winning.
Ankur: Did you just randomly write them or did you do them in some order?
Romina: I started in some order, then I-it's hard, though, because you're just like - I don't know.
Ankur: $\quad$ The only way $I$ can think of, is do it again and see if you get the same amount. [Hands paper back to Romina.] You know what I mean?
Romina: Thanks, Ank.
Jeff: How sweet.
Romina: Do it again.
Ankur: Or, like, compare with Brian. See if you got the same amount.
Romina: No, we didn't.
Brian: That was [inaudible].
Did he have more or less?
Romina: He has less than I do. But see, like, I didn't switch them to do the other.
Ankur: Then l'll do it now.
Romina: But I think his is, like, an organized - but I didn't have, like, two B's to begin with.
575 Brian: What?
576 Romina: Two B's don't start over here.
577 Brian: What do you mean two B's?
578 Romina They could not have like $B, B, A, B, A, A$.
579 Brian: Like the B-B, it'd be two B -
580 Jeff: So wait, what'd you get for six? For [part] C?
581 Brian: I got them somewhere. I got them over here.
582 Ankur: Oh. Give me a second.583 Brian:The reason l've [inaudible]. I didn't want to -584 Ankur:I only have sixty -585 Jeff
Where'd you get that from?
586 Romina:B wins -
Ankur: $\quad$ I got ten and then the other half is -
589 Jeff:Romina: Because A could still win it.
Brian: No, this is what I got. Start off with three - three to one team592
593594Ankur: Romina, let me see yours real fast.595
596Brian
Three to one team, three of another team winning - and out
of - for the first team winning -1 just moved the first $B$ over.
Romina: That's what I did.
597 Rrian:
Not only do I do that. I keep it one spot over.
Ankur:
Sixteen.
599 Romina Okay, but this is like A winning, right?
600 Brian: Okay.
601 Romina: But couldn't A win if you did -
602 Ankur: Ten, fifteen, twenty - oh.
603 Romina: See like, if you went -
604 Jeff: How many should have the -
605 Brian: $B, B$, like $A, A, A-$
606 Romina: $B, A$.
607 Brian: B, A, A, or something like that. Something, something's still-Eighteen? If this is included. I don't know.
609 Romina: Yeah, that is. Eighteen.
610 Ankur: Eighteen.
611 Romina: I don't know why I did that.
612 Michael Are you sure that's it?
613 Romina:
614
615 Jeff:
Hold on. l'll try.616 RominaBecause A has to win the last game, or seven.
617 Jeff:618 Ankur: I know what you're saying.

619 Romina: Yeah, like do it just from one team winning. You can always

| Romina. | Yean, like do it just from one team winning. You can always double it after that. |
| :---: | :---: |
| Ankur: | Like four. |
| Jeff: | Do you have more now? |
| Ankur: | Hm ? |
| Jeff: | Do you have more? |
| Ankur: | I only got seven so far. |
| Brian: | I only got five extras when I did it. Nine, ten, eleven, twelve, thirteen, fourteen, and then - |
| Romina: | Five extras and how many do you have? Eighteen? |
| Brian: | I got forty-two. |
| Romina: | I mean, how many do you have just on the side of that? |
| Brian: | Twenty-one. What do you got? |
| Romina: | I only got eighteen, but l I know l'm doing it wrong. |
| Brian: | This is like a second-grade technique. |
| Jeff: | 1 couldn't do it, dude. |
| Romina: | Hm. |
| Brian: | Did you do that whole, uh, seven games, exponent thing for seven games? Did you come up with any number for seven games, like total number of possibilities? |
| Romina: | One twenty-eight. |
| Jeff: | That's the total number of, of - |
| Romina: | Then I'm - |
| Jeff: | Of seven there could be. But that includes $A$ seven times which would never happen. Well, actually, it's one twentyeight minus four, minus five, minus six. |
| Romina: | If they - the bottom number [inaudible]. |
| Jeff: | Exactly. |
| Romina: | They do two $n$. |
| Jeff: | So, yeah - you know, we could do this. Ankur - |
| Ankur: | Hm [still writing strings and checking lists]? |
| Jeff: | There's a hundred twenty-eight possibilities. You subtract the possibilities from four games. You subtract, then, the possibilities from five games, then the possibilities from six games, and that leaves you with the only ones that are left. |
| Ankur: | Yeah, I know, but - |
| Jeff: | So what is - |
| Ankur: | The only way that we could prove that this, this holds true, you know what I mean? I could of told - remember we didn't we say it's going to be five out of - |
| Jeff: | Yeah, but what I'm - but, all right, how many games do, how many do, can you win in four games? How many possibilities did you get? Two? |
| Ankur: | I got eighteen, too. |

663 Jeff:
664
665 Ankur:
666 Jeff:
667 Ankur:
668 Jeff:
669
670
671
672 Ankur:
673 Romina:
674 Brian:
675 Romina:
676 Brian:
677 Michael:
678 Ankur:
679 Brian:
680 Jeff:
681 Ankur:
682 Jeff:
683 Ankur:
684 Jeff:
685 Ankur:
686 Jeff
687 Ankur:
688 Romina:
689
690 Jeff
691 Michael:
692
693
694
695
696
697
698
699 Ankur:
700 Michael:
701 Romina:
702 Brian:
703

706 Brian:

All right. We'll see if that checks out that way. And then how many total games did we have in five? Sixteen?
There it is. Four -
Ankur, how many different games are there in five? To win Yes?
In five?
Yeah, because I really don't think that we're going to get it [inaudible].
It's All right. Hold on, just give me a second.
Thirty-six and one twenty-eight's, uh - it's sixty-four.
[Inaudible] know better.
Even though if you do this, that's [inaudilbe].
Maybe.
This - uh -
It's kind of like the [inaudible].
What'd you say, now?
Would you do, like, a half by a total number [inaudible]?
How many games do you have total, of five that equal one?
Eight.
Or - so - total eight.
Yeah.
And then in, uh, six?
Six, I got twenty.
Twenty?
Uh huh.
You feel like each time you move up, like, every time they go to five games they eliminate like most of the possibilities.
And then -
This is kind of like the, um, the craps problem because I was looking at the first game, you know, he has a one in sixteen chance of winning, one in sixteen chance of losing -
Romina: $\quad \mathrm{Mm} \mathrm{hm}$.
Michael: And the rest is going on to the next one.
Ankur: Is going on to the next one, yeah.
Michael: Remember doing that? And he just started making branches out and branches out.
Mm hm .
Do you remember how he ended that one?
Does anyone have their notebook in here?
Ankur: I didn't take notes, so -
Romina: I'm just so lost. Like, I completely messed this up [her listing].
Those are problem - possibilities, though.

| $\begin{aligned} & 707 \\ & 708 \end{aligned}$ | Romina: | I - do you want me to go through some of mine to see if we have them? You did the same. You went in the same pattern |
| :---: | :---: | :---: |
| 709 |  | I did. |
| 710 | Jeff: | Ankur, do you see what I'm saying, though, with that? |
| 711 | Romina: | You did. And then you moved the B over and then you |
| 712 |  | moved the other B over. |
| 713 | Jeff: | That these numbers are really odd. |
| 714 | Ankur: | Brian, did you get eighteen? |
| 715 | Brian: | For six. |
| 716 | Ankur: | For six? |
| 717 | Brian: | For the six games. |
| 718 | Romina: | He has twenty-one for seven. |
| 719 | Brian: | Actually - |
| 720 | Ankur: | Total like, like - |
| 721 | Brian: | Forty-two. |
| 722 | Romina: | Yeah. |
| 723 | Ankur: | Forty-two? |
| 724 | Brian: | Forty-two for game seven. |
| 725 | Romina: | For, for $A$ and $B$. Like $A$ winning and $B$ winning. |
| 726 | Jeff: | See, because I'm just saying that - |
| 727 | Ankur: | Are you sure none of them are like - |
| 728 | Jeff: | I think it's a high number [inaudible]. |
| 729 | Ankur: | Throw them down [his list of strings]. |
| 730 | Brian: | What? |
| 731 | Ankur: | Throw them down. |
| 732 | Jeff: | Gee. |
| 733 | Romina: | I just completely lost myself here. |
| 734 | Ankur: | This is the, this is the sixteen. This is the four, oh, no, I mean |
| 735 |  | the sixth. |
| 736 | Brian: | The long one is seven, the one above that is six. The shorter |
| 737 |  | one's five and the one above that's four. The whole thing on |
| 738 |  | the bottom, even though the two were separated - |
| 739 | Ankur: | They're part of - oh, you wrote them both ways. |
| 740 | Brian: | It's getting blurry after a while. |
| 741 | Ankur: | Well, you got eight - |
| 742 | Romina: | I had - would you think that for four games it'd be like this? |
| 743 | Brian: | That's what I would think. |
| 744 | Romina: | Okay. |
| 745 | Ankur: | You got eighteen for six, Bri. |
| 746 | Brian: | That's what I said. |
| 747 | Ankur: | Oh. And I got twenty. |
| 748 | Brian: | Maybe there's one l forgot. |
| 749 | Romina: | Where did he take this - |
| 750 | Brian: | All I got to do is find one. |
| 751 | Ankur: | Yeah, I know what you mean. |

794 Ankur

Jeff:

Ankur
Jeff:
Ankur:
Jeff:
Ankur
Jeff:
Ankur:
Jeff:
Ankur:

Brian: And then the opposite for it, so -
Romina: Mike, you know how you were saying for that craps problem, we can't - but how do you branch off the first one?
Michael: I, I don't know. I left that idea a long time ago.
Romina: [Inaudible] a half an hour. I don't know. Maybe you ought to throw us in some direction here.
Brian: I think this is the direction.
Romina: Yeah. \{To T/R] You want to throw us in some direction here?
T/R1: [Inaudible].
Brian: We're all coming up with relatively close numbers. Like,
Ankur's getting twenty for six.
Ankur: [To Brian] I got the two that you missed.
Brian: What?
Ankur: I got the two that he missed.
Brian: Oh, so you got twenty for that now, so if it's all right -
Jeff: [Inaudible] do anything without that.
Romina: Wait, Ankur what are you doing? I, I don't -
Brian: However you got twenty, do that for seven games.
Ankur: What? All right.
Romina: Ank, what are you doing down there?
Ankur: Just finding the two that Brian missed and I found them.
Romina: I know, but, like, how did you get twenty? Like, did you write them all out?
Ankur: I wrote them out.
Romina: Oh, you did?
Jeff: This is, uh -
Ankur: I wrote out ten, and then the other ten would be the other half, like he said.
It'd be the only thing [inaudible], Ankur, except there's a hundred and twenty-eight, twenty-eight total things for - you know. You figure that if you subtract all these, then you'd get the answer.

Yeah.
You'd get the last one. But you get like, um, ninety It should be forty out of one twenty-eight, I think.
No. But I'm not even saying like that. I'm saying that we subtract all the -
And - all the - other possibilities.
You have twenty-eight here, so you'd subtract two because of the four games. There's two that would cancel out, like What do you mean?
And then -
You'd subtract eight.
Eight. Ten - no, then eight, twenty Twenty.
797 Brian: Those aren't the best [inaudible].

798 Jeff:
799 Brian:
800 Ankur:
801 Jeff:
802 Ankur:
803 Jeff:
804 Brian:
805 Ankur:
806 Jeff:
807 Brian:
808
809
810 Jeff:
811
812
813
814
815
816
817
818
819 Jeff:
820
821
822
823 Jeff
824 Ankur
825 Jeff:
826 Ankur
827
828

Thirteen.
They can't be factors in the seventh game.
That's what I'm thinking.
Well that's why they're not - that's why you subtract them.
That's why you subtract them. Yeah.
But then you get like a number like ninety-six.
Out of what? One twenty-eight?
Out of what?
I'm not sure.
Because - that sounds right, though, because you've got higher odds of winning in seven games. So find the total and then subtract -
All right, wait, wait - it's - the first one's two over thirty-two and eight out of thirty-two and -
Brian: Yeah, but we - wouldn't you just take half of them, too?
Because one team's going to win, because we got combinations in both teams. So we could just take that team. You know, like, I could be -
Romina: The only - that's what I was saying, like, only one team.
Brian: And to the [inaudible] - that's the total number, but it's one team that's going to be winning. So it's one--half.
Jeff: See, the other thing is that if this is the case, the first three games with, uh, [part] C being twenty out of thirty-two - is that what [part] C is, Ankur?
Ankur: Um -
Jeff: $\quad$ Or is it ten?

Ankur: You know what I mean. Yeah.
Brian: $\quad$ One - twenty out of sixty-four for six?
Romina: How did we do that? What was the probability of sixty-four coming before seven?
Brian: How'd you get sixty-four?
Ankur: Sixty-four total, two to the sixth.
Romina: Jeff?
Jeff: What?
Romina: How did we do that probability with six?
Brian: $\quad$ For four - for four, five and six?
Ankur: Just six.
Jeff: Oh, I don't know.
Romina: How did we do that thing - you know, like, how it's the probability of a six coming up before a seven.
Because it's fifty-fifty.
Yeah, that's how you do it.

842 Ankur: $\quad \mathrm{Mm} \mathrm{hm}$.
843 Romina: I, I don't know.
844 Jeff: You know what l'm saying?
845 Brian: That was, like, yesterday.
846 Romina: [Laughs] I don't remember.
847 Brian: You checking those answers good?
848 Ankur: That's what l'm doing.
849 Romina: Mine is not - I don't mind. Which one are you doing, Ank,

850
851
Brian: $\quad$ I know it's important to get more -
Jeff: Yeah - like during the physics test, my hand was, like, blue. Like, I was like all nervous and I was just, like, killing myself.
Brian: Do you know what to do?
Romina: Hm? Oh. Okay. I'm so lost.
Brian: Oh, there's got to be a way to use the whole fraction thing without even having to get involved in the stuff we're doing. So how many total are there? Both teams winning.
Romina: Let's look - can you do that, like, two at the end? You had to take things out because, like, because you know we had combinations where the first two letters would be A and that wouldn't be a seven, that'd be a fifth.
Brian: I thought we had [inaudible].
Romina: What?
Brian: I thought we had [inaudible].
Jeff: Yeah, hold on, hold on. It's crazy.
Romina: I think we're missing something really big.
Brian: Are we missing something?
Romina: Are we missing something?
Brian: We always think we're missing something.
Romina: What - oh [inaudible] miss something we're not getting here.
Brian: We might have it. We're just checking it. You never know.
Romina: Did you find more on his?
Michael: No matter how many you find, they're going to ask you why.
Jeff: Yeah, we can't prove just one combination.
Michael: I know. No matter how many you find.
Romina: Is that - is that like the - do we know out of how many? Did you guys figure that part out?
Jeff: I don't know. [Inaudible] It's - it's only ten for six games?
Ten thirty-two?
Romina: That would [inaudible]. The twelve out of thirty-two.
Jeff: Huh?
Romina: Twelve out of thity-two?
Jeff: I don't know.
Romina: Oh.

887 Jeff:
888
889
890
891
892
893
894
895
896
897
898
899

Jeff:

Jeff:

See, if it was, uh - this sixty-four, I reduced to - what's that? Just [inaudible] them up and reduced them.
Romina: How'd you get ten-thirteenths? I'm just asking. I don't know what you guys were doing over here. I [inaudible].
Jeff: I got ten because it was five-sixteenths that Ankur had, so it was ten and thirty-two and if that's ten, that's eight and that's two and that's -
Romina: That would have been nice, two, eight, ten.
Jeff: Yeah, well, say that is the case with twenty. That makes that twelve out of thirty-two. It's got to be the only ones left.
Romina: [To Ankur] Did you find more?
Ankur: I don't know. I'm not done yet. Stop it.
Romina: Do you want me to check 'em for you?
Ankur: No, l'll do it.
Romina: [Inaudible] winner or loser is, your chance of winning in three games. We did this stuff.

Romina: Huh?
Jeff: And does it change it from [inaudible].
Romina: What?
You know, who wins the first game, does that change everything?
Brian: That doesn't sound good [reference to noise in hallway]. How many do you got left?
Ankur: Like four.
Jeff: Did you check it? Are you taking this home?
Ankur: $\quad$ No, because he's got four and l've got three. I'm trying to find which one.
God bless Ankur. I just don't have the patience to do it now. Look, got it doubled now.
Oh, you found one?
Yeah, it's near the end.
Did you find one?
It's right there.
Yeah, it is one. How many more does he have than you?
No, we both have, three and three.
I have no clue.
It's something [inaudible], Romina. Four, five, six, seven, eight, nine, ten.
What do you recommend?
Twenty. It is right. Forty out of one twenty-eight. Then the whole thing adds up to one.
Do they match?
They match.
Wait, forty out of one twenty-eight?

| 932 | Ankur: | Yeah, it works. |
| :---: | :---: | :---: |
| 933 | Jeff: | Wait, twelve is - six - twenty-four - |
| 934 | Ankur: | Just add those - if you add up all of those, it'll equal one. |
| 935 |  | Forty - |
| 936 | Romina: | Hold on. What did you - |
| 937 | Ankur: | Forty out of one twenty-eight will reduce to twenty out of |
| 938 |  | sixty-four, reduces to five-sixteenths. |
| 939 | Romina: | What'd you do? I, I can't see. |
| 940 | Jeff: | See - wait, wait, wait. That doesn't equal up, then. |
| 941 | Ankur: | Yeah, it does. |
| 942 | Jeff: | You got two, and four - |
| 943 | Ankur: | Mm hm . |
| 944 | Jeff: | That's six, and ten, sixteen. |
| 945 | Ankur: | Sixteen out of sixteen. |
| 946 | Jeff: | Yeah, you're right. But what - that's the same [referring to |
| 947 |  | the probabilities of a series ending in six and seven games]? |
| 948 | Ankur: | I guess. It turned out to be the same. 'Cause this was out, |
| 949 |  | 'cause it was out of sixty-four. |
| 950 | Jeff: | Because you had - that one was two-sixteenths? |
| 951 | Ankur: | Yeah. All A's or all B's. |
| 952 | Jeff: | All right, that was two-sixteenths. You see, I had it as two |
| 953 |  | thirty-two's. Yeah, you're right. Two-sixteenths, in that case, |
| 954 |  | it's four thirty-two's. |
| 955 | Ankur: | You had two thirty-two's the whole time? That's why you |
| 956 |  | were coming up with, like, ninety-six or whatever, probably. |
| 957 | Jeff: | Yeah. And then I had ten - well, I said - in the beginning, I |
| 958 |  | was saying that it was ten thirty-two's, just as - well, I was |
| 959 |  | saying it was like twelve. |
| 960 | Ankur: | [To Michael] You're doing binary? |
| 961 | Michael: | I'm doing the same thing as you. It's just that I'm not using |
| 962 |  | A's and B's. |
| 963 | Romina: | Down here, what'd you do? |
| 964 | Ankur: | Binary numbers. |
| 965 | Michael: | How many did you win in five games? Eight? |
| 966 | Jeff: | He checked that and then he just said, all right, well, that's |
| 967 |  | ten because that's - |
| 968 | Romina: | Out of thirty-two? Is that it? |
| 969 | Jeff: | Yeah. |
| 970 | Romina: | Ten out of thirty-two. This one's ten. What'd you get for the |
| 971 |  | one above? |
| 972 | Ankur: | He got twenty-one, but he had a double, so it's twenty, and |
| 973 |  | then when you do the other side, it's another twenty, so it's |
| 974 |  | forty. |
| 975 | Romina: | And now, what - what is it for six games? Is it the same |
| 976 |  | thing? |


| 977 | Jeff: | Yeah. |
| :---: | :---: | :---: |
| 978 | Ankur: | It's twenty out of sixty-four. |
| 979 | Romina: | And for five games? |
| 980 | Jeff: | It's - four-sixteenths. |
| 981 | Ankur: | It's eight out of thirty-two, or four-sixteenths. |
| 982 | Brian: | So, reduce that - |
| 983 | Jeff: | Yeah, you could even reduce them again and do all eighths, |
| 984 |  | couldn't we? |
| 985 | Ankur: | Mm hm . |
| 986 | Jeff: | We could do, uh - no, you couldn't. |
| 987 | Romina: | Why are [parts] C and D the same? |
| 988 | Ankur: | Oh, no, you couldn't. |
| 989 | Romina: | How did you guys get those numbers [for parts C and D ]? |
| 990 | Ankur: | Five-sixteenths? |
| 991 | Jeff: | Yeah, it's five-sixteenths. |
| 992 | Romina: | How'd you guys get - did you guys just guess 'em or what? |
| 993 | Ankur: | Well, five-sixteenths - |
| 994 | Jeff: | That equals - fifteen - |
| 995 | Ankur: | This one was supposed to be two. |
| 996 | Jeff: | Yeah, [inaudible]. |
| 997 | Brian: | [Inaudible] five-sixteenths. What about four sixteenths or |
| 998 |  | something? |
| 999 | Ankur: | Both. Six [game series] and seven [game series]. |
| 1000 | Jeff: | Six and seven. |
| 1001 | Brian: | They're both the same thing. |
| 1002 | Romina: | How did you guys get that? I'm just curious. |
| 1003 | Jeff: | Once you got a possibility of six - |
| 1004 | Ankur: | [Inaudible] the possibilities - |
| 1005 | Jeff: | Once you finish getting six, then we just went, five - ten, |
| 1006 |  | eight, two - |
| 1007 | Romina: | Oh, okay, so - |
| 1008 | Ankur: | Like, the top number is, like, the games that you can win - |
| 1009 |  | like, the ways that you can win - and the bottom number is |
| 1010 |  | the total possibilities for that number. |
| 1011 | Jeff: | Two and [inaudible]. |
| 1012 | Romina: | I put two sixteenths. |
| 1013 | Ankur: | Exactly. |
| 1014 | Jeff: | Wait, Ankur. Of course not - |
| 1015 | Brian: | That's five-sixteenths, like - |
| 1016 | Jeff: | The first one's not two out of sixteen. |
| 1017 | Ankur: | Why not? |
| 1018 | Jeff: | So - now, why is it two-sixteenths? |
| 1019 | Ankur: | 'Cause, like, there's - |
| 1020 | Romina: | Isn't it supposed to be, like, one-sixteenth? |
| 1021 | Ankur: | [Inaudible] the one sixteenth. Why wouldn't it be? |

1022 Jeff: Why - why is it two-sixteenths?
1023 Ankur: Because it's four numbers, right?
1024 Jeff:
1025 Ankur:
1026 Jeff:
1027 Ankur
1028
1029 Jeff:
1030 Ankur:
Mm hm.
There's only two ways that you can win, all A's or all B's.
Mm hm .
And then, the total way - total possibilities of four numbers for either - this could be A or B, A or B, or A or B.
All right.
1031
1032 Jeff:
1033 Ankur:
1034 Jeff:
Two times two times two times two. Two to the fourth is -two-sixteenths.

1035 Romina: I hope we can discuss it.
1036 Ankur: I don't know how to explain it.
1037 Romina: We just, we just -
1038 Jeff: Well, we can explain it - we explain up to six.
1039 Romina: We went through the method with it.
1040 Jeff:
1041 Ankur:
1042
1043 Jeff:
1044 Ankur:
Yeah.
But then we'll be - then we can't prove that - we have all the possibilities, you know what I mean?

1045 Jeff:
1046 T/R1:
1047 Ankur:
Right.
Um -
We should - uh, [to T/R] you want to talk?
[Inaudible].
1048 T/R1: Um, do all of you agree? Some people are [inaudible] each
1049
1050 Jeff other too.

1051 Romina: Yeah, l'm - I asked. They told me. I understand.
1052 Brian:
Now they're going to ask you.
1053 Ankur: No, that [inaudible] How we going to prove that?
1054 Brian: Jeff.
1055 Ankur: Why - how do you have all the possibilities?
1056 T/R1: Say Brian.
1057 Ankur: And I'm starting to ask Brian and Mike.
1058 Jeff
1059 Romina:
Who do you think they'll ask first? Brian and Mike.
1060 Ankur:
How did you - how'd you get the bottom denominator?
1061
1062 Brian:
Oh. Romina, they're not going to ask me anything - I
guarantee you that.
1063 Romina:
Everybody get in here. Group discussion.
How'd you get the bottom number? How'd you get the denominator, because I didn't understand it.
It's like this: one, two, three, four. You got a chance of two for this, two for this, two for this and two for this.

1067 Romina: No, I did that too, but I just thought -

1068 Jeff:
1069
1070 Romina
1071 Jeff:
1072
1073 Romina
1074 Jeff:
1075 Romina
1076
1077 Jeff:
1078 Ankur:
1079 T/R1:
1080 Jeff:
1081 T/R1:
1082 Brian:
1083 T/R1:
1084 Brian:
1085 Jeff:
1086
1087
1088
1089
1090
1091
1092
1093
1094
1095
1096
1097
1098
1099 T/R1:
1100 Ankur:
1101 T/R1:
1102 Brian:
1103 Michael:
1104 T/R1:
1105 Jeff:
1106 Alice
1107
1108 Ankur:
1109 Romina:
1110 Alice:
1111 Ankur:

Two for this, two for this, two for this and then you multiply them together, then the two to the n thing -
Mm hm.
So then, only two out of those entire ones can win. So that's two- sixteenths.
Can I say something? That's the first one.
Yeah, I understood, I understood that, but how'd you get the same one [inaudible].
Then we just reduced terms.
Okay.
Jeff, do you want to go to the blackboard and show me?
All right.
And we're all going to ask questions, aren't we, Brian?
Yes, we are.
Romina, we're going to ask -
I'm ready to tear him apart.
Yeah, that'd be cool, huh? All right, what we did is we took, uh - [draws four horizontal lines on board]. All right, so For the first one, out of four games, you could have - this could be, um, an A or a B. That could be an A or a B and, you know, so on. So, basically what we did was this: that could be two possibilities, that could be two possibilities, that could be two and that could be two. And that was like when - we went back to the old days and it was like, two to the $n$. And then, four. So two times two, times two, times two, and that's how we got to sixteen. And that would be the bottom number. And then in order to win in four games, these have to either be all A's or all B's, so we got two out of sixteen for winning in four games, which is the probability of winning in four games. Does that make sense?
I think so.
Well, we all agree.
Brian, does that make sense?
Yeah.
Yeah.
Questions? Alice?
You all okay with this?
That makes sense to me. I wonder if - did, did all the others of you come up with the same thing?
[inaudible]
Yeah that's -
In the same way?
Same way -

1112 Jeff:
1113 Brian:
1114
1115 T/R3:
1116 T/R1:
1117 Brian:
1118 T/R1:
1119 Brian:
1120 Romina:
1121
1122 Brian:
1123 T/R3:
1124 Michael:
1125 T/R1:
1126 Michael:
1127
1128
1129 Jeff:
1130 T/R1:
1131 Jeff:
1132
1133
1134
1135
1136
1137
1138
1139
1140
1141
1142
1143
1144
1145
1146 T/R1:
1147
1148 Brian
1149
1150 Ankur
1151 T/R1:
1152 Anku
1153 T/R1:
1154 Ankur
1155 Romina:

I don't think we went in the same way. Just little bits of information from everybody helped create the answer.
Yeah.
But some of you didn't write numbers like you did.
The numbers are right here.
I see strings of A's and B's.
Well, these just show, like, like, what can happen.
The first one, they had to win them straight, so we couldn't really -
Couldn't put them into numbers.
Okay. And Michael, did you get [inaudible]?
Um -
What did you do, Michael?
Something else. I don't know, I'm still trying to - you see, they have something that works for that first one, but does it work for every -
[inaudible] Do you want to go on that?
Do you buy that one, Michael?
That was the first one, so for the first one, it's two-sixteenths. All right, for the next one, we're going to do the same situation, but this will be two to the fifth, so it's gonna be out of thirty-two, and thirty-two is the bottom number. And then for - I think, for these we were just kind of - we went through 'em. We were - that's why there's strings of A's and B's on everyone's paper. And in order to get these, we went through all the possibilities where there was only - there was five, five different, five places, and A or B was in four of them. And we went through all of them and that's how we got that. And then we ended up with, um, eight of thirty-two for that. Now, that's not too convincing 'cause we just went through 'em, but we went through all the ones that were out of five with four A's and so we got that. I don't think we have a really concrete mathematical backing for that.
Is that right? You don't have a very concrete mathematical backing?
I don't think we ever have a concrete mathematical backing for anything.
Like, we can - we can convince ourselves that we did it.
How is - how is -
But I don't think we can -
How did you convince yourselves?
Just by how we -
We looked at all these.

| $\begin{aligned} & 1156 \\ & 1157 \end{aligned}$ | Ankur: | Like, how we wrote out the pos - like, just the possibilities of - not all the possible - |
| :---: | :---: | :---: |
| 1158 | T/R1: | Yeah. |
| 1159 | Romina: | And they all add up right. |
| 1160 | Ankur: | Just the ones that would allow you to win. |
| 1161 | T/R1: | So - so, you tried to write the strings of the wins? |
| 1162 | Ankur: | Mm hm. |
| 1163 | T/R1: | Okay. In five? |
| 1164 | Jeff: | All right. So once we did that, we did the same thing for six, |
| 1165 |  | too. Six times - it, it would be out of sixty-four. |
| 1166 | Ankur: | We got twenty out of sixty-four. |
| 1167 | Jeff: | And, and then we got twenty out of that. |
| 1168 | Ankur: | And then - |
| 1169 | Jeff: | And like - |
| 1170 | Ankur: | Before we did seven, we - |
| 1171 | Jeff: | We counted all of them up and - |
| 1172 | Ankur: | We added them up, the two [for four games], five [game |
| 1173 |  | result] and six [game result], and then seven should have |
| 1174 |  | been the remainder. |
| 1175 | Jeff: | Of them. |
| 1176 | Ankur: | And so - and - |
| 1177 | Jeff: | If they're all right. |
| 1178 | T/R1: | What if they're not? |
| 1179 | Jeff: | Then it's wrong. |
| 1180 | Ankur: | But then - |
| 1181 | T/R1: | So, you didn't actually compute seven? |
| 1182 | Ankur: | No, but then we did compute seven and it matched. |
| 1183 | T/R1: | Oh, you did. |
| 1184 | Ankur: | Yeah. |
| 1185 | T/R1: | So, that was a double-check. |
| 1186 | Ankur: | Mm hm . |
| 1187 | T/R1: | How do you know you're not double-counting? |
| 1188 | Jeff: | [inaudible] |
| 1189 | Ankur: | [inaudible] |
| 1190 | Romina: | We individually worked through them. |
| 1191 | Brian: | How do you know you're not missing any? |
| 1192 | Jeff: | That's the big question. |
| 1193 | T/R1: | Mm hm . |
| 1194 | Jeff: | That's why we're kind of - |
| 1195 | Brian: | From the information we were given, it seems like we have |
| 1196 |  | them all. |
| 1197 | T/R1: | Okay. Is there a way of coming up with a representation or a |
| 1198 |  | way of writing it, so that you could persuade somebody else |
| 1199 |  | that you really do have them all, and they don't just believe |

1200
1201
1202
1203
1204
1205
1206
1207
1208
1209
1210
1211 Ankur:
1212 Michael:
1213 Jeff:
1214 Ankur:
1215 Michael:
1216 Ankur:
1217 Michael:
1218
1219 Ankur:
1220 Romina:
1221 T/R1:
1222
1223 Michael:
1224 T/R1:
1225 Ankur
1226 Jeff:
1227 T/R1:
1228
1229
1230
1231
1232
1233 T/R1:
1234 Michael:
1235 Ankur:
1236 T/R1:
1237 Jeff:
1238 Ankur:
1239 Michael:
1240 T/R1:
1241 Michael:
1242 Jeff:
1243 Romina:
you that, "I sort of went through them and it looks like I have them all"?
Ankur: Do an equation or -
Yeah, I mean, I think that's what we didn't know.
Listen to this [showing his paper to Ankur] You have - well, let's go to the fourth one from each one [fourth coefficient in rows 3-6]. One team has one and the other team also has one, so that's two. The second one, you get double that and get an eight. A twenty -
Mm hm.
And -
Ten and ten, and then the What'd you get for the seven games?
Same thing as -
Forty.
Twenty.
Yeah, it's twenty.
So, obviously, you have some kind of connection with the [makes a triangle with his hands].
Yeah, I know.
They - they did the triangle?
You want to show us up there what you're doing, Michael, please?
I don't know what type of connection this displays, but-
Well, put it up and show us -
The ones that are crossed out?
Yeah.
Michael you always say you don't know how to explain it, but when you're done, I do understand what you're saying. All right. You're all familiar with this [writes the first rows of Pascal's Triangle on board].
Ankur: Yeah, this is how you - this is how you prove that we didn't miss any.
Oh, okay.
I don't know how to prove it. I'm actually just saying that-Actually, it does because you did it - watch.
All right. Ankur's going to prove what you have.
Oh, gee.
Because you have, you actually taught us this one day.
One, right [writing on board]?
You give me more credit than I deserve.
I'm going to run out of room
Five.
Five.

1244 Michael:

All right, um, I just found, like, if you take the fourth number in each one [circles these entries]-- that way, if you double each number, 'cause you have two teams, you can get the possibilities of four games. Four games, um, equals two, right? You got eight, twenty and forty, like they said.
Ankur: It makes sense. It makes sense from what you said that one time.
I don't know - I don't know if you, she, she, she - we found a connection between binary numbers and this. [To T/R] You remember that, right?
Help me remember.
Remember we found a connection between the binary numbers, you know, and then this triangle?
Mm hm.
I don't know how l'm going to explain it.
If you show me -
It's just like binary numbers 'cause you're just writing A's and B's, ones and zeros, so, like - all right.
Exactly.
So - I'm just going to just tell you - like, I don't want to, to explain it because it will take too long. This is - if you have, um, bin - like four places that, um, yeah, four places, it would be one out of those sixteen -
Yeah.
One out of those sixteen, that has all zeros - nothing, all right? This is the three. There will be three of them that have two, three of them that have -
Two.
Three of them have one.
Three. There's three.
All right.
Three of them that have two and one of them will have three. Um, now when you go to the next step, those, uh, that last - those last - those three games that they won - The first three games, if they win that, that'll be like, those three possibilities without - would be - if they win the next game or those three - if they win - Uh, I don't know how to explain it. Uh, on the third game - I don't know. I - I have trouble explaining things. I don't even know what I'm trying to do now.
You're - you're doing fine.
But, um - Do you guys see anything?
Well, obviously, there's something going on with the one, four, ten and twenty.
Well, I missed you.

| 1289 | Michael: | Yeah, there's obviously something going on. |
| :---: | :---: | :---: |
| 1290 | Jeff: | I mean, that's not - they wouldn't be like that. And I guess |
| 1291 |  | you were gonna say if it was out of eight games, it would be |
| 1292 |  | thirty-five? The probability would be thirty-five out of - |
| 1293 | Michael: | I think [inaudible]. |
| 1294 | Jeff: | Do you know what I'm saying? |
| 1295 | Ankur: | Yeah. |
| 1296 | Michael: | It would be one, seven [writing next row of triangle] - |
| 1297 | Ankur: | You just add the fifteen and the twenty, and you get thirty- |
| 1298 |  | five. |
| 1299 | Jeff: | So, I mean, there's gotta be something there, because it |
| 1300 |  | wouldn't all - |
| 1301 | Michael: | No, it wouldn't be - it would be thirty-five, doubled. |
| 1302 | Ankur: | Yeah, seventy. |
| 1303 | Jeff: | That's what it would be. Yeah. |
| 1304 | Michael: | Right. |
| 1305 | Jeff: | Thirty-five from one team. |
| 1306 | Michael: | Eighty, seventy. But it only goes - but the limits of the |
| 1307 |  | problem are to win four out of seven, not four out of eight. |
| 1308 | Jeff: | Oh yeah, I know. What I'm saying all along. |
| 1309 | T/R1: | Okay. Um, don't go away Michael. |
| 1310 | Michael: | I won't. |
| 1311 | T/R1: | Because one of the other questions I wanted to ask is that |
| 1312 |  | Michael showed me something last time, when we talked |
| 1313 |  | ourselves, that I guess you all didn't hear and I wasn't sure |
| 1314 |  | how much of it he was going to share with this. And I guess |
| 1315 |  | a little bit of it I connect to from what you talked about the |
| 1316 |  | last time. But - but what you explained to me, Michael, I |
| 1317 |  | remember the last time, is - you see that addition of ten, you |
| 1318 |  | know, the six and four? |
| 1319 | Michael: | Yeah. No, no. Why do you add them together? Then why is- |
| 1320 | T/R1: | Or the twenty. Why you add them together and you had an |
| 1321 |  | explanation - |
| 1322 | Michael: | Yeah. |
| 1323 | T/R1: | And you were using pizzas to explain it to me. Any of you |
| 1324 |  | ever heard this before? |
| 1325 | Ankur: | Yes. |
| 1326 | T/R1: | You were talking about toppings on pizzas. |
| 1327 | Romina: | Oh yeah, I remember that. |
| 1328 | T/R1: | Is that right? |
| 1329 | Michael: | Why are they adding on - |
| 1330 | T/R1: | And so, um, you took a road there - |
| 1331 | Michael: | Yeah, uh - |
| 1332 | T/R1: | [To Jeff, who has to leave] l'll see you next week. |
| 1333 | Jeff: | Yes. I'm very sorry. |


| 1334 | T/R1: | At tweive. |
| :---: | :---: | :---: |
| 1335 | Jeff: | Twelve o'clock. |
| 1336 | Brian: | We're on at twelve o'clock. |
| 1337 | T/R1: | Uh, twelve. We'll have one. |
| 1338 | Jeff: | All right. |
| 1339 | Alice: | Good luck. |
| 1340 | T/R1: | Good luck in your game. |
| 1341 | Jeff: | Thank you very much. |
| 1342 | Michael: | All right. Um - |
| 1343 | T/R1: | Do you remember? |
| 1344 | Michael: | Yeah, I remember. Right. You have, what? Three toppings |
| 1345 |  | and this one has four? |
| 1346 | T/R1: | Okay, which one is this? |
| 1347 | Michael: | Three toppings. |
| 1348 | T/R1: | If you're thinking - |
| 1349 | Michael: | [Pointing to the row with entries 133 1] This is like a three- |
| 1350 |  | topping pizza. There will be one with, uh - |
| 1351 | Ankur: | Plain? |
| 1352 | Romina: | Plain. |
| 1353 | Michael: | Plain, right? Three with just two toppings, three with, uh, just |
| 1354 |  | one topping, three with just two and one with all toppings. |
| 1355 |  | And when you have that one pizza, what - if you don't add |
| 1356 |  | on a - a topping, it'll still stay in that zero place. But then you |
| 1357 |  | add, you add, a mush - if you do add a topping, that - those |
| 1358 |  | ones will become into four different pizza pies. |
| 1359 | T/R1: | Show me the one and the three giving you the four, in terms |
| 1360 |  | of pizzas. Can you tell me that? |
| 1361 | Michael: | All right. Hold on. l'm trying. |
| 1362 | T/R1: | Sure. Just tell me that in pizza toppings. |
| 1363 | Michael: | In pizza toppings - |
| 1364 | T/R1: | How one plus three equals four. |
| 1365 | Michael: | I'm trying to think - I - I had it. Last time I talked to you, I |
| 1366 |  | had, had it so good. |
| 1367 | Ankur: | Why don't you just roll the tape? |
| 1368 | Michael: | Yeah, you got it on tape. |
| 1369 | T/R1: | Well, why don't you help him figure it out? Yeah. Let's - |
| 1370 |  | let's go back and - and think of what that means. Can you |
| 1371 |  | show me - can you show the one and the three being a four, |
| 1372 |  | so everyone knows what we're focusing on, Michael? |
| 1373 | Michael: | What - what are you talking about? |
| 1374 | Ankur: | How the - |
| 1375 | T/R1: | Draw the lines. Okay. Now, why - l'm asking you why that |
| 1376 |  | works, with pizzas. |
| 1377 | Michael: | All right. You're going to add a topping to every single pizza |
| 1378 |  | on there, right? There's going to be twice as many pizzas. |

T/R1:

But these three pizzas - three of them got a topping, went there, and three of them didn't, went there. One of them had a topping, right there, and one of them didn't, went there. 'Cause these three pizzas are going to turn into six pizzas. Now I got it, right? And three of them, which had three toppings and gained another one, are in the next category. They moved a step up. These guys stayed in the same place 'cause they didn't get one.
Ankur: [inaudible]
Michael: That's why they had -
Ankur: $\quad \mathrm{Mm}$ hm.
Michael: Did I get that, like the last time?
I don't know. Did they - Brian doesn't understand. I grasped it. Yeah. This pizza -
Can you tell it to me, then, with some toppings? Make up some toppings and see what it says.
Michael: All right, you got, um - cheese pizza, no, plain pizza and you got a pizza with mushrooms. You're going to add a topping is that right? You have three toppings in this one [row of entries 133 1]. You have, uh, you can have one plain, one with peppers and mushrooms and sausage. And then you're going to add another topping - I don't know, more cheese? Uh, you're gonna - you're either gonna add more cheese to, to one of them, either - you're going to - when you add another topping - you're gonna, like - the possibilities are going to double because some of them you're going to add, you're going to add cheese to them, and some of them you're not. Like, you're going to have this one pizza, it's plain. The one that's not going to get another topping is gonna go here. The one that's going to get the cheese is going to go here. And three of these which only had the mushrooms, three are going to get cheese. They're going to go here.
Wait a minute. Is cheese the plain?
Cheese - well, I think -
Extra cheese.
Extra cheese. Anchovies. Whatever you want.
That helps a lot.
These with the mushrooms, if they get anchovies, they're gonna be three different, three different pizzas with two toppings. They're gonna go here. If they don't, they're still going to be three different pizzas -
Mm hm .

1423
1424
1425
1426
1427
1428 Ankur:
1429 Michael:
1430
1431
1432 T/R1:
1433 Romina:
1434 Brian:
1435 Michael:
1436 Brian:
1437 Michael:
1438 T/R1:
1439
1440
1441
1442
1443 Ankur:
1444 Romina:
1445 Michael:
1446 Ankur:
1447 Michael:
1448 Romina:
1449 Brian:
1450 Michael:
1451
1452
1453
1454
1455 T/R1:
1456 Michael:
1457 Ankur:
1458 Michael:
1459
1460 Ankur:
1461 Michael:
1462
1463 Ankur:
1464 Michael:
1465
1466
1467

That have one topping. So, do you understand? Like, um, when you add - . When you add a topping or whatever, uh, this many is gonna - is like - if you think of it as steps, like one, two - this one's going to, like, move up a step - 'cause it's -
[inaudible]
Either gonna get one or not, all right? So there's twice as many possibilities now. And three of them will have, and three then will not have, that extra one.
Does that make sense?
Yeah, we understood this.
You got something, you move up one.
I - I don't know if that makes sense.
If you don't, you join with the one of those over there.
I can't explain it any other way.
Could you say that again, Brian? One more time. If you've got something -
If you move up -
If you get something, you move up, and if you don't, then you just join the other one.
If you got something you go on.
You join in.
You stay - you stay -
Is that's what you're trying to say?
Yeah, I - but then -
You could have helped him out there, Brian.
That wouldn't have done anything.
Now with the - with the one, three, three, one
[entries 1331 1], that circled one is, um, I guess you win those three games in a row. There's only one possibility. Now, your next time you either win a game or you don't. And that's how it goes with that, you know?
So, that's the two possibilities there?
That - the one would be -
One -
You know what l'm saying, like to - in order to - how many is up there? One plus three -
Eight.
Eight up there? What was our probability? Two out of sixteen. All right.
And that's one out of eight.
Yeah, it's one out of eight. You have a - you have a one out of eight possibility of, uh, winning three in a row. And then the next game, there's a fifty-fifty - All right, you would either win, or you can go to that four category. I'm saying, like, the

1468
1469
1470
1471 Brian:
1472 Michael:
1473 T/R1:
1474
1475
1476
1477 Ankur:
1478 T/R1:
1479 Ankur:
1480 Brian:
1481 Romina:
1482 T/R1:
1483 T/R3:
1484 Michael:
1485 Ankur:
1486 Michael:
1487
1488
1489
1490
1491
1492
1493
1494
1495
1496
1497
1498
1499
1500
1501
1502 Ankur:
1503 Michae
1504 Ankur:
1505 Michael:
probability of, uh, I guess getting - of winning, would, would have to - you - you have to count the number before it. Like, winning, uh, three - let's see. It's confusing. Uh, huh. Right.
I - I know what it means in my - I can see it in my head.
I - I can - someone want to help Michael out? Do you know what he's saying here? He's trying to summarize this. He's sort of suggesting, gosh, if you follow sort of that path you can get those probabilities pretty fast, right?
Mm hm .
Kind of handy on tests, the SAT or Do they ask this on the SAT?
That's a good question, is that going to -
Forget it.
The advanced math one -
Mike, that circled one is?
Is the - all right, is the, uh -
Is the - in the four games, that's winning all four.
Let's say in - in the games - no, no that's just three games.
All right. Your probability of winning three times in three games. The first one you have a one out of eight chance losing all three.
Romina: So that's the one, three, three, one there?
Michael: And the second one, you have three possibilities of winning one: you could win it the first time, the second time or the third time. Uh, the third one would be winning twice.
T/R1: Okay.
Michael: The first one, you - you can understand. And there's only one other, one way to win three times.
Okay, there's one way to win -
Now, if you win those three games - uh -
If you go to the, if you went to the right, what would that indicate?
If you went to the right Then you won.
No, you see that - that wouldn't -
No, if you -
Don't think of going over there like going on to the next game, 'cause, 'cause, um, the - you know, the one and the four, that'd be like you lost, then, 'cause you stayed in the same place, but it's not. It's like you won those three games and now you have a - like, you have a fifty-fifty chance of winning the next one - which might be why we double it. I don't know.

| 1512 | Ankur: | Actually, I was going to say, like, that one represents |
| :---: | :---: | :---: |
| 1513 |  | like, winning three games in a row, or like three A's. |
| 1514 | Michael: | That's the probability. |
| 1515 | T/R1: | Okay. |
| 1516 | Ankur: | And then, if you go to the right, that's like getting another A, |
| 1517 |  | and there's only one way to get four A's. If you go to the left |
| 1518 |  | that's like getting a B, and that's like three A's and a B, and |
| 1519 |  | there's four different ways you can write that. |
| 1520 | T/R1: | Now, you said, Michael, that you have a fifty-fifty chance - |
| 1521 | Michael: | I don't think that has anything to do with it, though. I don't |
| 1522 |  | know if it does. |
| 1523 | T/R1: | Well, that's an interesting question. Is that an assumption in |
| 1524 |  | this problem? |
| 1525 | Michael: | Yeah. |
| 1526 | T/R1: | Does all this work if this one team has a higher chance of |
| 1527 |  | winning than the other? |
| 1528 | Michael: | If one team is better than the other. |
| 1529 | Romina: | Yeah, then they have a - |
| 1530 | T/R1: | You said it twice and you sort of - you said it almost as if, |
| 1531 |  | well, if you have a fifty--fifty chance, either the other team |
| 1532 |  | could win or the other. |
| 1533 | Michael: | Of winning. I mean if, if - |
| 1534 | T/R1: | Are you retracting that? Because you did say that. |
| 1535 | Michael: | If one team is better than the other, then probably the thing |
| 1536 |  | [Pascal's Triangle] wouldn't even matter. |
| 1537 | T/R1: | So this may not work if one team is better than the other. |
| 1538 | Michael: | Probably not. |

