Description: B39, Revisiting the World Series problem (student view), Grade 11, January 29, 1999, raw footage Date: 1999-01-29 Location: David Brearley High School Researcher: Professor Carolyn Maher	Transcriber(s): Kiczek, Regina Verifier(s): DeLeon, Christina Date Transcribed: Spring 2000 Page: 1 of 61

Michael: Ask her for a copy of the problem.

Ankur: Could we have a copy of the problem?

CAM: I don’t think you need it.

Jeff: I don't think we need one.

Romina: [LAUGHS]

RS: Could you walk me through the problem you were working on?

Romina: Okay. There's seven games in a World Series. What are the chances the World Series will be won in four games, five games, six games and seven games.

RS: You mean, <u>exactly</u> four games, <u>exactly</u> five games, <u>exactly</u> six games, <u>exactly</u> seven games?

Romina: [NODS] Hmm-mm.

RS: What did you find?

Romina: Well, we started by writing out the possibilities … didn't we? Well, that’s what we did, we wrote out the possibilities. I don't remember - we got one sixteenth for the first one? Or two?

Ankur: It was two.

Romina: Two. And we linked it back to the, uh, triangle? Take it, Mike.

RS: Could you walk me through that, very slowly?

Michael: We finished …

Jeff: I wish we had the stuff we had.

RS: How did you do that? Okay, how did you do that and what did you find? Could you show me what it looks like and how you thought about it?

Romina: We just, um …

Jeff: Well the first one was kind of - we knew that someone – you know, there's two, two people –

Ankur: In order to win in four games, then one team had to win either all four games or the other team had to win all four games.

Jeff: So out of all the possibilities -

RS: Okay, that's a good place to start. What exactly - what exactly did you find there?

Ankur: That it was – the chance of that happening was one-eighth.

RS: And could you explain how you got that?

Romina: One half, one half, one half, one half --

Ankur: Because –

Romina: Equals one-sixteenth. And there are two possibilities of –

Jeff: Yeah. We did that.

Ankur: And there's four games.

RS: Okay. What's the half?

Romina: The half is - okay, the probability that if they played one game and you have two teams and they are equally, whatever –

Michael: Equally as good.

Ankur: The probability that one team would win. You could win or lose.

RS: Okay.

Jeff: So you're –

RS: So you're making an assumption that the teams are equally matched –

ALL: Yeah.

RS: That it's just as likely for one team to win as the other.

Romina: Yeah.

RS: So that's the way you are framing your problem.

Jeff: Exactly.

RS: Okay.

Romina: So then, the first game you have one half chance.

Jeff: It's either the home team, say, or the away team – either H or A.

RS: Okay.

Jeff: So that would - you know, you have, either one or the other is going to win. And the same thing for each of them. And then, was it, half times half times half times half?

Romina: And you get one-sixteenth. That's why it's – that's for one team winning.

Jeff: That's for one team winning.

Romina: And then for two teams winning - it's two-sixteenths, or one-eighth.

RS: Could I ask a simple-minded question?

Romina: Hmm-mm.

RS: Why did you multiply?

Romina: [LAUGHS]

Jeff: [GROANS] We did this in math.

Romina: Because ...

RS: I'm sorry, but I really want to know.

Jeff: Because adding - it wouldn't work. Because you'd have a half plus a half - you'd get two. And what would that prove?

RS: So you've eliminated adding.

Jeff: Eliminated adding.

Romina: But –

RS: Why did you multiply?

Jeff: And, if you're not going to add, then you're not going to subtract –

Ankur: Because you have either - for this one you have either a home or an away. And for the next one you can either have another home or an away. So for the home you add … Oh! The home team wins or the away team wins, or the home team wins and the home team wins, or the away team wins and the home team wins, or the away and the away.

Jeff: Exactly. That's what it is. So for the first one, it's either –

Ankur: You follow that?

Jeff: It's either home or away. Yeah, exactly.

Romina: The square. [JEFF DRAWS A SQUARE ON HIS PAPER; ROMINA LABELS SIDES WITH H & A.]

Michael: You know about that thing, don't you?

Jeff: [MUMBLES WHILE HE LISTS THE POSSIBILITIES FOR 2 GAMES.]

Michael: How many games - two?

Ankur: All you have to do is that little tree thing.

RS: Now, in your table you have four - it looks like you have four possibilities in your table.

Romina: Yeah. This is the probability.

RS: So, what does the table represent?

Ankur: The two times the two.

Romina: Yeah. We actually have to multiply.

Jeff: The half times the half.

Romina: Yeah. This is –

RS: So here you're looking - are you looking at it in your - are you imagining a two game series at this point?

Romina: Yeah. This is just a two game series. And then if you had another one, we'd have –

Ankur: For each of them, you'd add a home or away.

Romina: Yeah.

RS: But this explains how from - the two possibilities –

Ankur: That's why we're multiplying –

Romina: That is why we're multiplying.

RS: For each of the two games, you're multiplying because of that.

Romina: And then for every one, we kind of were consistent and then we'd multiply again.

RS: So, when you multiply four two's together, you get sixteen.

Ankur: Hmm-mm.

Romina: Hmm-mm.

RS: And what does that sixteen represent?

Jeff: The total possibilities of -

Romina: Yeah, of –

Jeff: Of anything.

Ankur: Anything in the four game series.

Romina: No, the total possibilities of –

Ankur: Anything.

Jeff: Yeah.

Romina: In a four game series.

RS: In a four game series then, there are sixteen possible ways –

Jeff: That something could happen.

Ankur: Out of those, there are only two ways that –

Jeff: If there are two teams. And then out of those, there are only two ways where someone is going to win in four games.

Romina: To the *n*.

Jeff: And that would be – yeah, exactly.

Romina: Two to the *n*.

RS: So it would be two out of sixteen?

Jeff: Hmm-mm.

Romina: Hmm-mm.

RS: And that's how you got one-eighth?

Ankur: Yes.

RS: Okay. It's interesting that there was so much in this, what looked like a simple, obvious possibility. Okay. So that's - so what do you - so the probability of one team or the other winning after just four games, you found was an eighth.

Jeff: Yes.

RS: Okay. So what came next?

Jeff: So next, you have to add another slot, right?

Ankur: Hmm-mm.

Jeff: And then, it could be all sixteen possibilities, with another A or another H on it, but subtracting the two where someone already won, right? Is that right?

Romina: That would be, that would be in four games.

Jeff: So you have - you know what it's going to be in four games. You want H's only in four games, because it's out of sixteen. And then out of all those sixteen possibilities that you had, then all sixteen then can go to an A – like, the next slot can be an A winning, or the next one can be an H, right?

Ankur: Hmm-mm. So that's thirty-two.

Jeff: So that makes thirty-two.

Ankur: Something over thirty-two.

Jeff: Sixteen plus sixteen.

RS: Could you go through that a little more slowly?

Jeff: All right.

RS: I'm hearing two things that are interesting to me and I'd like to know more about them.

Jeff: We're saying that –

RS: Okay.

Jeff: That –

RS: Let me say what the two things are - I'm trying to get a clear question.

Jeff: Hmm-mm.

RS: Okay. One is that - uh - with an additional game, it looks like there’s an additional multiplication.

Jeff: Hmm-mm.

RS: Is that right?

Jeff: Hmm-mm.

RS: So, I'd like to understand how that happens. And then - that's the first thing. The second thing is that you’re taking into account in some way that, as we go into a fifth game, that the series hasn't been won already.

Ankur: Yes.

RS: I think you said something, several of you said something like that.

Jeff: That' why –

RS: So, those are the two things I'd like to see clearly. One is -

Jeff: All right. All right, well –

Romina: You’re adding another one, like he did, so you’re adding – you’re multiplying one-sixteenth. You’re multiplying the one-sixteenth - by another half?

Ankur: Half.

Romina: So you get one thirty-two. But from that you have to eliminate the possibility that – um - it's won in four games.

Michael: One-eighth.

Romina: What?

Michael: One-eighth, right?

RS: Yeah, okay. So you, so you eliminate the possibilities you had before. But is - does that complete the solution for this case?

Jeff: No. Then you got to - from there you got to count out how many of those someone actually wins it.

Ankur: You have to find the ones –

Jeff: So you got like your - you have sixteen total games, right, that you had before. And then you have - out of those sixteen, like with the four, with the four - excuse me - from the first game. When you add the next thing on, that's going to be the A or the H. So there's going to be sixteen – the original sixteen with an A on the end and then the other original sixteen with an H on the end - where the away team wins one of the games. The home team wins the other sixteen of the games.

RS: There's something I'm wondering about, though. Um - if they didn't win it in four games, and –

Ankur: And they had –

Romina: They had to win it –

Ankur: If they go to the fifth game.

RS: And they go to a fifth game. Um - let me see how to say this. Sometimes I grope for words. Uh - but sometimes a series is going to go longer than five games and sometimes it could end in five games. How do you deal with that?

Jeff: We looked at it like, we counted up all the possibilities of all the games.

Ankur: And you would have to have at least one team win four of those games.

Jeff: Yeah, right.

Ankur: And make sure it ends in five games.

Jeff: Yeah. And then you count up all the –

RS: Oh, I see.

Jeff: Count up all the chances - that was the next step. Then from here - from the thirty-two that we had - then we counted up all the games where some, where there was four A's and four H's - added them together. The four A's and four H's.

Ankur: Not having like four, four in a row and then losing –

Jeff: Yeah.

Ankur: Because then, that would be a four game series.

Jeff: Subtract two from the first one. So, that's the total number won, over thirty-two, and that's how we got the probability for the fifth game.

RS: Okay. Now could we go into detail? So, you're imagining - uh – you're imagining all the possible five game series. On the one hand, it gets won by the away team.

Jeff: Hmm-mm.

RS: And, also, similarly the other ones where it gets won by the home team. So – so, it looks like - uh - the part we don't see yet, because we haven't explained it yet, is what do the games, what are the possibilities of the first four games when the fifth game is a win for away, or similarly what are the possibilities for the first four games when the -

Michael: Three.

RS: When –

Michael: Three wins.

RS: When the home team wins.

Romina: Three wins.

RS: How do you organize those? How do you see those?

Ankur: It's with the triangle, right?

RS: How do you count it?

Michael: I saw it differently.

RS: So there are different ways of seeing.

Michael: I didn’t, I didn't make slots or something.

Romina: I just wrote down the possibilities.

RS: Are you saying that there are a couple of different ways of seeing it?

Michael: Yeah, there’s a different way of approaching that problem.

RS: Could we walk through all of them?

Michael: I guess so.

RS: What are all the different ideas here? One at a time.

Romina: Very simple –

Ankur: One was writing them, writing them out.

Romina: Yeah, you just write them out – just – yeah.

Michael: I want to explain something.

Romina: There’s the away team –

Michael: You're familiar with Pascal's Triangle, right? Right? You know those.

RS: Yeah.

Michael: I had - there's a way to, um, connect that and binary numbers together.

RS: Yes.

Michael: You are familiar with that?

RS: Hmm-mm.

Michael: Okay. So I used that to find a solution to that problem. Right? Let me just –

RS: How does it go?

Michael: [Writes 5 rows of Pascal's Triangle on his paper, omitting row 1.] All right. Well, the connection would be, like - this is a two number [pointing to 1 2 1], like the binary with two numbers in two places.

RS: Yes.

Michael: The first one is the amount, the total number to only have - no ones in it. The second one would be with one. And the third place would be the amount that had two, right?

RS: So you're looking at the row 1 2 1.

Michael: Well, I'm just going to give you an example – you understand?

RS: Yes.

Michael: This column has just one with whole zeros in it [writes 00].

RS: Hmm-mm.

Michael: This one has - there's two in that, those four combinations that have –

RS: What are the two?

Michael: This one and that one [writes 10 and 01].

RS: Hmm-mm.

Michael: And the last one, all ones [writes 11].

RS: Hmm-mm.

Michael: And this holds true for the next one. [Points to row 1 3 3 1.] This one has, there's only one with all zeros.

RS: Yes.

Michael: There's three with, uh - with one.

Ankur: One.

Michael: And there's –

RS: Why are there three with one?

Michael: ‘Cause there's three places. There's only - there's only three ways to do it. And it’s like that [writes 001 010 100].

RS: Okay.

Michael: The next one would be three, three with two ones.

RS: Hmm-mm.

Michael: And the last one would be one, with all three.

RS: Why are there three with two ones? Just three?

Michael: Well, I was –

Jeff: It’s the same situation as the one, except you’re switching, you’re switching – the zero would be in the ones spot.

Michael: I kind of wrote an explanation about this.

RS: Oh, you mean if it had two ones, it –

Jeff: It's the same as having two zeros in <u>that</u> situation.

Ankur: ‘Cause you're having two of one number and one of the other.

Michael: Um …

RS: Oh, because the ones with two ones in them, if they have three digits, have one zero? I see.

Ankur: Use the board.

RS: It says … Okay. The, the message from on-high is, “Use the board.” [Referring to film crew’s directions to show on board what they are doing. Michael goes to the board.] Yeah, okay. Okay, and this is, and, and here we're just following through one of the ways. There was an eraser down there and chalk by the, the tray. Um...

Michael: I'll explain to you real quick why you add ‘em together, which is kind of saying why is there three. And it's the same probability.

RS: Okay.

Michael: I wrote that up, and I gave it to Dr. Maher - if she's here - and I don't know what she did with it.

Romina: She's probably, probably reading it.

Michael: All right. [Michael has written rows 0, 2, 3, 4 of Pascal’s Triangle on the board.] Um - in the, in the previous, in this category or whatever, you have two numbers. One of them has noth - nothing in it. One of them has nothing in it, like no ones in it.

RS: Um hum.

Michael: The second one has - there's two with just one. The last one, there's all spaces are filled up or whatever.

RS: Okay.

Michael: Then when you add another place to it –

RS: Yes.

Michael: Each combination in there could turn into two different ones, you understand? One will have another - one will have a zero or one at the end of it [uses his hands to illustrate].

RS: Could you show me that? Yeah. I want to be just certain that I understand exactly what you're seeing here.

Michael: Take this first number, which is zero [writes 00]. Uh, the second –

RS: Yeah.

Michael: Is one [01]. This would be two [10]. And this is three [11].

RS: Um hum.

Michael: It's binary numbers. When you add another place to it, now you have three spots to fill up - the, these two, this 00 still stays, could become the same value. And this one could become one. So now, two are formed from every single com - possibility. The same thing will go with this [01] -- to this and this [writes 010 011].

RS: Um hum.

Michael: Right? You understand?

RS: I do.

Jeff: That was like adding the extra slot.

Ankur: That's the extra slot with the two possibilities.

Michael: Now when you, when you add an extra slot, obviously one of them is still going to have one [points to 010] and one of them is going to have two [points to 011]. So the 2 right here [in row 1 2 1], there's going to be two - one for each - that, that still have one –

RS: Um hum.

Michael: That's why you add it to that. And this one will have a - this one will add, get one added - so that's one. So there's your three. They come together like that [indicates with his hands]. This one [the 2 in row 1 2 1], there will be the other two of the -- and now in terms of four combinations, the other two will have, uh, two. And the ones that don't get it will have one. You know what I'm talking about?

RS: Yes, I, I think so.

Michael: Um, so they all - that's why they add together. Um, and then that would explain why there's a three there, why there's a four. Because these three are ones that have an - a 1 added on.

RS: Um hum.

Michael: And they go into this category; the ones that don't get anything added go into that one.

RS: Are you saying, for example, that - that six comes up by adding?

Michael: Yeah. Like you add, there's, there's now there's, with these three there, with these two, these six possibilities, there's twelve of them.

RS: Uh huh.

Michael: Six of them get a one added on to them.

RS: Right.

Michael: There. That way that would give you ... bring it to three. Three ones in the, in the series, whatever, and these three, uh, half of them don't get, don't get a 1added onto them. That would, that would leave them in the same category like with just three.

RS: Uh huh.

Michael: That's why you add, would add them.

RS: Uh hum.

Michael: You understand that?

RS: Okay. And the whole triangle is built that way?

Michael: Yeah, by adding.

RS: Okay.

Michael: Now, when they had their, their possibilities ...

RS: Um hum.

Michael: I saw that, like, the first possibility for the, for one game, or for winning in four games, was two out of eight. And the next one was, uh ...

Jeff: We had two out of sixteen, then one out of eight.

Ankur: It was two out of sixteen, Mike.

Michael: It was two out of sixteen?

Jeff: Yeah, one out of eight.

Michael: Then it was, uh, eight out of thirty-two, the next one?

Ankur: Yeah.

Jeff: Yeah, I think so.

Michael: Then, eight out of thirty-two. And the last one was twenty out of whatever, something. It was out of something.

Ankur: It was out of sixty-four.

RS: How did you find those?

Michael: Well, they, they found it out by actually writing out every single one. But then I, I, I was thinking ... right, there's a 1, a 4 and a 10 here *[circles 4th entry in rows 3, 4, 5]*, and if you, I guess, double them, you come out with these numbers: a 2, an 8 and a 20. Because nothing there had a...You know what I'm talking about? Because one doubled is two.

RS: Yes.

Michael: There's 4, doubled is an 8.

RS: Um hum.

Michael: Ten doubled is a twenty.

RS: Yes.

Jeff: Mike, why can't you start from ... another one?

RS: But why? Why - what do you go by?

Jeff: 'Cause that's from four?

BS: Yeah.

Michael: 'Cause this is the, the fourth place over. Now, um, these are the, how many games are won with three. All right? How many ... I'm not saying A's and H's. I'm using ones and zeros.

RS: Yeah, that I ... Okay, so your ones and zeros are representing home and away ...

Michael: A's and H's. Now, these have ... how many came out with all wins - all ones, right? Out of three games.

RS: This may seem like a really dumb question, but I hope it isn't pointless, but, um, suppose the home team won the first two games, the away team won the third game, and the home team won the fourth game. How does that fit into your picture?

Jeff: You know, like ...

RS: The home team wins two games in a row. Then the away team wins one.

Michael: Out of five games?

Romina: No, erase the zero.

RS: Let's make it a four game series.

Michael: Okay.

RS: Yes. So you'd, so the home team wins the first two games. The away team wins the third game and the home team wins the fourth game. So you write that as one, one, zero, one. And how does that connect to the Pascal triangle that you drew?

Michael: Um ...

RS: Okay. Let's get a World Series where the home team wins. So let's put another 1, let's put another 1on the end.

Michael: All right.

RS: Yeah. Okay. So they win the last game, yeah.

Michael: *[Writes 11011]* This combination would be, to this ... There should be out of part of these, out of these ten?

RS: It would be one of those ten?

Michael: Yeah. Um, you've got, uh, am I right?

Romina: Yeah, I think you are. *[INAUDIBLE]*

RS: Could you show me how you would check that?

Michael: *[INAUDIBLE]*

RS: Yeah.

Michael: I had it. Um ... I don't know.

Ankur: It would be one of the four.

Michael: One of the four?

Ankur: 'Cause the zero can go in ...

Michael: Yeah. All right.

Ankur: Actually, no.

RS: Wait.

Romina: No.

Ankur: It would be one of the five. 'Cause it ...

Michael: All right. Well, I think it would be one of the four because … Let's just take this one *[indicates 1101]*. All right - so - this is one of the, one of the four ...

Ankur: And the next one would be one of the five.

Michael: For some reason, I don't know why, but like the amount of possibilities that, uh, a three, uh, that three ones will come up ...

RS: Um hum.

Michael: In with, uh, four places or something ...

RS: Um hum.

Michael: But, uh, that, that's the probability of winning on the fifth game. I don't know why, but if you, if you win three games, for four, that's the probability of winning in the fourth.

RS: Are you saying that if the, if the home team won three out of the, <u>exactly</u> three out of the four, then it's in a position to win - if it wins the fifth game, or ... ?

Michael: It's not in a position to win, but if that, the probability of winning three out of four…

RS: Uh huh.

Michael: Is the probability of winning in, in the, in the fifth game.

Jeff: Of winning four out of five.

Michael: As, not, not, yeah, of winning, having four, of winning in the fifth game ... I, I wouldn't say four out of five.

Jeff: Yeah, but if you won in the fifth game, you'd be winning four games of five.

Ankur: That's just a way to express how, that's one of four ways to express how the home team won three out of the four games.

RS: Well, suppose we're the home team and that we've won three out of the first four games, what do we think about the fifth game?

Michael: The fifth game?

RS: Yeah.

Michael: Um ...

Jeff: I just think we should refer to it as the home team because it switches back and forth, so the home team could win four games in a row and, you know, it could actually be split two and two or something like that, you know? It's been bothering me. 'Cause that, it brings a whole other problem in.

Michael: Well, just pretend all the games were played on one field. That way, there's only one home.

Jeff: Well, I'm just saying that ...

Michael: Well, I ...

Jeff: That's a whole other problem.

Michael: Team one, team two. All right? Instead of H and A?

RS: Okay.

Michael: Now, what was the question?

RS: Is … Suppose we're the home team. And, and it was one, one, zero, one. So we won three games out of four ...

Michael: Right.

RS: And we're going into the fifth game. Uh, what do we think about that situation? What does it mean?

Michael: What we think? Um -

RS: We want to win the World Series, right? So what might we be thinking?

Michael: Well, the probability of winning, tomorrow or the next game, is a half -

RS: Ah, hah!

Michael: But the probability of, the probability of you actually coming up, you know, winning on the fifth game is eight out of thirty-two.

RS: Okay, why eight?

Michael: Eight, why eight? Well, there's, there's a probability of four out of *[appears to be* *mentally adding the entries 1 4 6 4 1]* … sixteen, right?

Romina: Yeah.

Michael: Four out of sixteen of winning first three, three games out of four.

RS: Okay. Because there are four of those, of those numbers with three ones and one zero. Okay. I see.

Michael: So the probability of winning those three games are four out of sixteen.

Romina: You're doing both of them?

Michael: Yeah - um - we were just talking about the away team.

Ankur: *[inaudible]*

Michael: All right. I got it now. The probability of you winning would be four out of sixteen. Then to win the next game would be, that's the probability of winning the next game. Four out of thirty-two. Obviously, there's two teams. So you double it here. You add another four out of thirty-two.

RS: I see. Did any of the others around the table think of it in a different way? It <u>does</u> make sense, I think.

Jeff: 'Cause I mean, I think I was ...

RS: Does that make sense to you?

Jeff: Well, that was - seriously, that was kind of, yeah, that was kind of the same thing that we were doing. We were just using A's and H's. And then we'd multiply them in the next part or like the, by putting in the next slot ...

Ankur: That's how we got the bottom number.

Jeff: And that's how we'd get the bottom number.

Romina: Yeah, those are all our ideas together.

Jeff: And A and H's would be, like that's a nice representation of all of our ideas.

RS: Okay. So you looked at, so it sounds to me like you looked at two probabilities. The probability of getting where we are ...

Michael: And the probability of winning in the next game.

RS: And then the probability of what happens in the next game. Okay, I think, I think I understand it. Are you all, how did you look at it?

Michael: I looked at it as *[waves hand to indicate Pascal's triangle written on the board]*. I, I really wasn't too sure. It's all those numbers in the triangle. It had some kind of relationship and after you said that, now it makes sense. Does it make sense to you?

RS: Excuse me - my microphone fell off. Okay, now - okay, so Mike's approach was to make this binary coding, to code into these two digit binary numbers, and to connect to Pascal's triangle, uh, what were the other approaches that you did for this, to get the eight over thirty-two?

Romina: Well, Michael does the binary and we do it, but just with the H and the A.

RS: Uh huh.

Romina: Like in "home" and "away." And he just makes it look a lot nicer than we do.

Jeff: We didn't tie it in with -

Ankur: We found the thirty-two by multiplying - we didn't write all those thirty-two out.

Jeff: Yeah, like Brian did.

RS: Okay.

Ankur: And then we wrote the eight out, which, which would satisfy that.

RS: Could you show me how you wrote them?

Jeff: That's how we did it, like this.

RS: Did you just pick them at random or did you have uh, did you have a way of …

Romina: For four? Well, we have like an organized way, like you bring the A through and-

RS: Could you show me?

Jeff: I didn't write them like that.

RS: Yeah.

Romina: I did. Um, are we doing it for four, four games?

Ankur: Five games or four games? What do you want, the two or the eight?

RS: Could somebody go to the board and do that?

Ankur: *[To Romina]* Knock yourself out.

Romina: How many games, four or five?

Jeff: Well, you start with four and then you could add another thing on the end, to explain five better.

Romina: *[At the board]* All the possibilities or just for one team winning?

Ankur: Just for, just the eight, right?

RS: Show me what you did. In other words …

Romina: That's for five, though - I don't know, for the first one, the only way in four games they'd win is home, home, home, home or away, away, away, away *[writes on board: HHHH AAAA]*.

RS: Mmh-hmm.

Romina: And that's where we got our two.

RS: And what about five games?

Romina: And five games -

Ankur: Five games - four would have to be of one, like, letter.

Romina: We picked something to go at the end, like away *[writes on board: AAAHA*

AAHAA AHAAA HAAAA].

Ankur: And the other four would be the opposite.

Jeff: Yeah, and that just *[inaudible]*

RS: I see. Yeah, okay. So, you didn't just pull them out of the air?

Romina: No we didn't.

RS: You had a way of thinking about them. Yeah. So now, okay. So that's - the probability, so let's ... see where we are, do we have all the alternative approaches out in the open so far? Okay, good. So now .. that's how we looked at the probability, that's how you looked at the probability of winning the series in exactly five games, was that right, and you got eight over thirty-two?

Romina: Yeah.

Jeff: Mmh-hmm.

RS: Okay - so - let's see, two-sixteenths, what's that? One-eighth?

Students: Mmh-hmm.

RS: And eight thirty-seconds, what's that?

Students: One-fourth.

Ankur: You just did it over sixteen, we did two sixteens and that was four sixteens.

RS: Okay. So the next one down is twenty over sixty-four. How'd you get that? That's winning in six games?

Romina: You add the - you add the other half ...

Jeff: Add it?

Romina: Multiply the other half, to get sixty-four.

Ankur: Multiply the other half, and then -

RS: Uh-huh.

Romina: And in this one, we wrote out the twenty -

Ankur: Ten.

Romina: Ten and then we -

RS: You wrote out the ten.

Romina: Yeah.

RS: I see it. Okay, but Michael's approach was he could calculate the ten from Pascal's Triangle.

Michael: I think so. I tried to find something that - when you're writing out things, there's so much room for error.

Jeff: Oh, there really is, and you can't prove it.

Michael: I know. How do you know that's it? I don't know. Just, that's it. I checked it five times already.

RS: All right. So that's uh, twenty over sixty-four is the probability of winning the series, of somebody winning the series in exactly ... six games. Is that right? What about the probability of winning the series in exactly seven games?

Jeff: That's what's left over.

Ankur: It's the same -

Jeff: It's the same thing.

RS: That's what's left over? What do you mean?

Romina: You just add those *[pointing to the numbers on board]*.

Ankur: 'Cause the series has to end in either four, five, six or seven games.

RS: Okay.

Ankur: So, if you add up four, five and six -

Jeff: Five and six, the only thing left is -

Ankur: And subtract that, from the whole -

Jeff: From the whole, then -

Ankur: Then that's what you get.

RS: I see, okay. What if you had to calculate it directly?

Ankur: We can still do what he did.

Jeff: You still - you just continue on.

Ankur: We could always do what he did.

Michael: I don't know …

Romina: Divide by half for the other line and go down *[referring to Pascal's Triangle]*.

RS: Wanna take a shot at it?

Michael: *[Looking at Pascal's Triangle on board]* Uh, that's four games, five games, six games …

Jeff: Four or five, six, one more, it would be … wouldn't it be …

Romina: Four or five.

Michael: *[Writes on paper]* One, six, fifteen, ... twenty, right? I would say, forty out of ... one-twenty.

Ankur: One twenty-eight, which is -

Michael: I don't, I don't know if that equals a whole.

Ankur: It does.

Michael: Does it?

Ankur: 'Cause it's twenty out of sixty-four, which was the same -

Jeff: As ten out of -

Ankur: Which is five out of sixteen.

Romina: Mmh-hmm.

Ankur: Yeah.

Romina: That's what we got last time.

Ankur: It's two out of sixteen, four out of sixteen, five and five -

Jeff: Yeah.

Ankur: And that equals sixteen.

RS: Are you checking that this agreed with the number you found earlier?

Ankur: Yeah.

Romina: Yeah.

RS: How do you ... could you show me that check?

Jeff/Ankur: Well, we added up the two-sixteenths, four-sixteenths, five-sixteenths -

Ankur: That's what they reduce to.

RS: Mmh-hmm.

Jeff: And we got -

Ankur: And that came to -

Jeff: Six -

Ankur: Eleven sixteens.

RS: Uh-huh.

Jeff: And then we needed five more, so you get sixteen sixteenths, to get a whole.

Ankur: Which is forty out of one twenty-eight.

RS: Okay.

Ankur: Five-sixteenths.

RS: Okay. So all these different approaches are arriving at the same place. I like the way you did that. So you .. in the cases where you actually sort of got in and had to do the calculations, had to sort of slog through the calculations in one way or another, you're treating the last game as special and then you're looking at the possibilities that could lead up to it, is that fair?

Ankur: Yeah.

Romina: Mmh-hmm.

RS: Okay. Um, now you're assuming that the two teams are equally matched.

Jeff: Yes.

RS: And that was your explanation for the ... the halves that came into it.

Jeff: Yeah, but that, that was -

Romina: It was just one out of two, right?

Jeff: Yeah, that was, that was stated in the in the problem, too, that they were equally matched.

RS: Uh-huh. Have you, was it stated in the problem? Okay.

Jeff: Yes. We didn't … I don't think we did, but in the actual paper that we had written on …

RS: Yeah. Did you wonder what you would do if, uh, they weren't equally matched?

Ankur: That would be a problem.

Romina: No. You'd have ...

Jeff: Well, how do you judge how much more … yeah, how would you judge who's better and how much? Yeah, how would you say that?

Michael: You'd need a percent.

Ankur: Yes. How much better this team is than … better and by how much.

Jeff: How much.

Romina: You would need a fraction or a percentage.

Jeff: Yeah. I mean ...

Michael: Well, if he just gives us a percentage - you know what I'm saying?

Jeff: No, but if he just says that, all right, suppose team two's better.

Michael: By how much?

Jeff: Well, that's what I'm saying.

RS: How might you … okay, this is an interesting question. Uh, you're looking at maybe a whole season of games, right?

Jeff: Mmh-hmm.

RS: How might you get a handle on it -

Ankur: You just - you just change the bottom number, 'cause -

RS: The idea that one team might be better than the other? What would you look at?

Jeff: 'Cause we, we look at their records and say -

Ankur: Just look, yeah.

Jeff: And you could find out how - it, like, shows you how much better Team A would be than Team B.

Michael:

STUDENT

[OVERLAPS] That's kind of like a opinion. You think it's better because the records --

STUDENT

[OVERLAPS] Yeah but I mean where they played and --

STUDENT

[OVERLAPS] I would be better because I ____.

STUDENT

[OVERLAPS] Yeah, you really don't know. Anything can happen ____.

STUDENT

Then that way each team doesn't have a fifty percent ___.

STUDENT

[OVERLAPS] It don't matter after thirty one to one, that one champion, by one.

STUDENT

Exactly. [PAUSE] Yeah it's just ___.

STUDENT

I mean if you make purpose, like --

STUDENT

A concrete decision.

STUDENT

[OVERLAPS] It's kind of hard to put a value ____. Uh, that teams better than that team because they have a better record. Well I think the other team is better because they got a better pitcher.

STUDENT

Uh-huh.

STUDENT

It's hard to -- but if you were to say, all right ...forget that, the ones -- one little win, forty, forty times -- forty percent more than the other, then ____. You understand what I'm saying? Then, regardless of what the team -- like you could get us a percentage. Sure you could do it but [MUMBLES].

ROBERT SPEISER

Okay, let's think about real life experience. You've been following baseball for a long time right? Um, how often do you, do you think that the teams are even, evenly matched?

STUDENT

Rarely.

STUDENT

Probably not.

STUDENT

Yeah but that's left up to opinion. I mean you could say that, uh --

ROMINA

[OVERLAPS] Some teams just don't do good -- don't do well against the other teams because they take it for granted and then they play --

ROBERT SPEISER

[OVERLAPS] But would it be fair to ___ money most of the time?

STUDENT

That's why they have lines ____.

STUDENT

Yeah there's never ____ anyway.

ROBERT SPEISER

Cause you're saying, you're saying that in general people have views that one team is likely to be better than the other.

STUDENT

Well that's where the whole live situation comes in.

STUDENT

[OVERLAPS] That's why they try to make it even.

STUDENT

[OVERLAPS] You figure that - this team is gonna touchdown better. And that's, that's _____. And they're usually -- I don't know what they do but they're usually right on dude let me tell ya.

STUDENT

[OVERLAPS] Yeah.

STUDENT

It's pretty good. Cause it's always like -- they just ___.

STUDENT

[OVERLAPS] They have to ___.

STUDENT

Like say like if my teams playing ____'s team, my team will be ____ by say seven points.

STUDENT

[INAUDIBLE]

STUDENT

[OVERLAPS] If I don't, if I don't beat em by more than seven then people at the ___ even know I won.

STUDENT

It's like giving me a seven nothing --

STUDENT

[OVERLAPS] Yeah. Exactly.

ROBERT SPEISER

Suppose the home team, suppose people -- some, you know enough people thought the home team was twice as likely to win as the away team?

STUDENT

[COUGHS]

ROBERT SPEISER

how would that effect it?

ROMINA

You have more of a chance to win that ____ in the third games.

STUDENT

Well if they're twice as likely then ....instead of multiplying it by a half it would be like two thirds.

STUDENT

Two thirds?

STUDENT

A,A -- I mean H,H,A. [OVERLAPPING VOICES]

STUDENT

[OVERLAPS] That's another thing like .....you would have like your -- however many combinations. All right, how many -- like sixteen games, you know like the first one?

ROBERT SPEISER

Mmh-hmm.

STUDENT

There was sixteen for one team and sixteen for another. And she would multiply that one, sixteen by two because it will have -- it will happen twice as many times as the other one.

ROBERT SPEISER

[COUGHS] I'm not sure I follow you yet. Suppose, for example, you were working on -- oh I don't know - why don't we start -- follow the same program you guys were explaining to me before. UH, how would you look at the probability? Suppose you, Suppose that you felt that one team was -- any given game -- say the home team was twice as likely to win as the away team. How would you calculate the probability of one of those teams? One or the other in other words? Winning the series in exactly four games.

STUDENT

Just like you said [OVERLAPPING VOICES]

ROBERT SPEISER

Why don't we just --

STUDENT

[OVERLAPS] What is it? Home, home, away, away, home, away home?

STUDENT

[OVERLAPS] OH, oh you have to --

ROBERT SPEISER

[OVERLAPS] Just four games.

STUDENT

The first -- when they're equally matched you have, um, the home team has a one sixteen chance of winning. A one eighth chance, and the other one has a one eighth chance. You would add them together. But this one eighth with um, -- [OVERLAPPING VOICES] no it's one sixteenth. One sixteenth. One sixteenth. Now let's say this is the better team.

ROBERT SPEISER

Yeah.

STUDENT

They have twice -- they come out twice as many times as this. So you would, um, --

STUDENT

Multiply by two?

STUDENT

Just double it.

STUDENT

Just double it so it would be a two sixteenth chance of --

STUDENT

Just add another one sixteenth ____.

STUDENT

[OVERLAPS] Add another one sixteenth and the probability of, uh, --

STUDENT

It would be over -- we'd have forty eight then?

STUDENT

What was the question? Probably that --

ROBERT SPEISER

Suppose we believe that -- say the home team is twice as likely to win any given game as the away team, what's the probability of the World Series, that the World Series would take exactly four games?

STUDENT

Three out of sixteen.

ROBERT SPEISER

How do you --

STUDENT

[OVERLAPS] You can't do three out of sixteen cause -- All right.

ROBERT SPEISER

How do you see that?

STUDENT

I got three out of sixteen and it makes a little more sense because three out of sixteen is bigger than two out of sixteen. Now if you -- [OVERLAPPING VOICES]

STUDENT

Yeah but wouldn't it be the same because the home team plays twice and the away team plays twice at home? So you figure they have the same advantage?

STUDENT

[OVERLAPS] But if -- if one team is better than the other-

STUDENT

[OVERLAPS] We're not saying that, we're saying that the home team is twice as likely to win.

STUDENT

[OVERLAPS] You said the home team - [OVERLAPPING VOICES]

STUDENT

No the ___ at their home stadium, all right?

ROMINA

Or a specific team.

STUDENT

Just say Team A and B, don't worry about home.

STUDENT

[OVERLAPS] Then you're saying -- yeah, then you're saying -- well that's like every ___ question because like if you ask ___ differently, it's all different.

STUDENT

All right, you have Team A what was more likely to win --

STUDENT

[OVERLAPS] All right, all right well --

STUDENT

[OVERLAPS] So therefore it's gonna be, so Team A is gonna -- they have a better shot at winning it for -- than the other one.

ROBERT SPEISER

Yeah.

STUDENT

So, obviously it's gonna be a little bigger.

ROBERT SPEISER

All right. All right.

STUDENT

So, like I -- so the better team has a two out of sixteen chance of winning.

STUDENT

And the ___ also has a sixteen --

STUDENT

Yeah.

ROBERT SPEISER

Are you sure?

STUDENT

It would be -- am I sure? No. I just said it.

ROBERT SPEISER

Okay.

STUDENT

Cause that one event of having Team A winning four times in a row is twice as likely -- twice as more likely to happen to us. The other team ____.

ROBERT SPEISER

Let's back up a little bit. Um, if the team's are equally matched, what's the probability of Team A winning one game?

STUDENT

Fifty Fifty.

ROBERT SPEISER

How would you express that as a probability?

STUDENT

[INAUDIBLE]

ROBERT SPEISER

How would you express -- well the probabilities you wrote on the board were numbers right? What would be the numbers?

ROMINA

One out of two?

STUDENT

One out of two?

ROBERT SPEISER

Are you reading the fraction 2/16 as symbolizing two out of sixteen?

STUDENT

Yeah.

ROMINA

Yeah.

STUDENT

Yes.

ROBERT SPEISER

Okay, so now suppose Team A is twice as good as Team B, what's the probability of Team A winning one game?

STUDENT

Two out of three.

STUDENT

Mmh-hmm. They have twice the chance so it leaves three chances to win.

STUDENT

So would that be three out of seventeen then? Is that what you're saying?

ROMINA

[INAUDIBLE]

STUDENT

Like you say how you go from one to two, to two to three?

STUDENT

Yeah but if you're going from .....

STUDENT

Cause you're doing -- look, if there's a chance of Team A winning one game is two out of three, winning game two is another two out of three. So it would be what? Four out of nine, of winning the first two games?

STUDENT
Yeah. And then --

STUDENT
Two out of three again.

STUDENT
Would be eight out of twenty-seven.

STUDENT
[OVERLAPS] Eight out of twenty-seven, and then two out of three again for winning four in the row.

STUDENT
Which would be --

STUDENT
Sixteen out of eighty-one?

STUDENT
Or seven times three. Yeah, so it was a --

STUDENT
Sixteen out of eighty-one?

STUDENT
Sixteen, eighty-one, that's not gonna reduce.

STUDENT
That's with just Team A winning.

STUDENT
Team A winning, Team A winning.

STUDENT
Four in a row.

STUDENT
And then --

STUDENT
[OVERLAPS] First four and then you do the other thing.

STUDENT
Then it would be one, one, one --

STUDENT
[OVERLAPS] One out of three --

STUDENT
One times -- one out of three times -- it would be one eighty-one wouldn't it?

ROMINA
Mmh-hmm.

STUDENT
Because one ____ not changing.

STUDENT

It would be seventeen out of eighty-one.

STUDENT

Yeah seventeen out of -- seventeen out of eighty-one, it's gonna be one and four --

STUDENT

[OVERLAPS] It's gonna be over in four games.

STUDENT

[OVERLAPS] A sixteen out of eighty-one shot that team -- the better team that's --

STUDENT

[OVERLAPS] Yeah.

STUDENT

...twice as better's gonna win ....in those games. And one out of eighty one chance that it ____ twice as bad.

ROBERT SPEISER

Could you walk me through that slowly? Um, and maybe using the board wouldn't be bad if you have a lot of writing to do. I want, I want to understand how you see the eighty-one. [OVERLAPPING VOICES] And then finally the seventeen and the sixteen.

STUDENT

Cause the probability is -- you said Team A is twice as better as Team One. So the chance of Team A winning, and winning one game at random is two -- one -- two out of three. Right?

STUDENT

Cause they have twice as -

STUDENT

Cause they have twice, twice as much shot as winning a game as Team B does. So if for every two games Team A wins, Team B will win one game.

ROBERT SPEISER

Okay.

STUDENT

So it's two out of three.

ROBERT SPEISER

Okay so it's like -- it looks like you're taking the games in groups of three then when you're thinking of that.

STUDENT

[OVERLAPS] Yes.

ROBERT SPEISER

Okay.

STUDENT

And then another game is two out of three again. [OVERLAPPING VOICES] -- multiplying together.

ROMINA

Just like we did like the one half ____.

ROBERT SPEISER
[OVERLAPS] Okay.
STUDENT
[OVERLAPS] ___ in the same situation --
STUDENT
[OVERLAPS] That's how we got -- and then we do that four times to get the probability that Team A would win.
STUDENT
And then since the other one is only one third, if you multiply one third times one third, times one third, one third before the top number is just one times one, times one, times one.
ROBERT SPEISER
[OVERLAPS] Uh-huh.
STUDENT
[OVERLAPS] Saying three to the fourth equals eighty-one. And so that would be one out of eighty-one and then we add that one to the sixteen out of eighty-one and you ____ eighty-one.
ROBERT SPEISER
Okay. So I see your calculation as multiplying two thirds by itself four times. And then multiplying one third by itself four times, and then adding the results. Is that right?
STUDENT
Yeah. Mmh-hmm.
ROBERT SPEISER
So it's not like the double that you did before.
STUDENT
Mmh-hmm.
ROBERT SPEISER
Okay. Um, I guess I'm still wondering about how you think it through for four games.
STUDENT
[INAUDIBLE]
ROBERT SPEISER
Because you're talking about two games out of three as your way of organizing the possibilities. What happens when --
STUDENT
[OVERLAPS] [COUGHS]
ROBERT SPEISER
[INAUDIBLE] four games? Right?
STUDENT
That's why we did two out of three --
STUDENT
[OVERLAPS] That's why we did that yeah, yeah.
ROBERT SPEISER

[OVERLAPS] I see, okay. Okay. All right. Now what, what would you do to figure out the probability for one of the, one of the teams? One or the other of the two teams winning the World Series in five games, in exactly five games?

STUDENT

Same situation where the other team is twice as better?

STUDENT

Is Team A twice as better as Team One?

ROBERT SPEISER

Yeah where the first team is twice as good.

STUDENT

[INAUDIBLE]

ROBERT SPEISER

[OVERLAPS] Is twice as likely to win at giving ___ the second team.

STUDENT

So for the next shot you figure that ....have a -- at a -- for the fourth game two out of the, the -- they have two out of three chance of winning that game.

STUDENT

Why don't we just do what he did I guess somehow.

STUDENT

With that team? The triangle?

STUDENT

No like--

STUDENT

[OVERLAPS] The triangle's not gonna work with this. It's not --

STUDENT

[OVERLAPS] I know. Would you multiply just by two thirds?

STUDENT

Is that what we did?

STUDENT

[OVERLAPS] Seventeen out of -- seventeen out of eighty-one times two thirds.

STUDENT

Yeah.

STUDENT

Seventeen, eighty-one times one-third. And then add that --

STUDENT

[OVERLAPS] ____ would be -- yeah well seventeen, eighty-one times two thirds will be what? Seventeen times -- seventeen times two will be, uh, --

STUDENT

Thirty-four?

STUDENT

That's the probability of winning. You have to take a --

STUDENT
[OVERLAPS] And then we have to add --
STUDENT
[OVERLAPS] You have to take the probability of now -- that's a -- out of four games ___ need the probability of three out of four. And then adding the last part in.
STUDENT
[OVERLAPS] [INAUDIBLE]
STUDENT
Because you have the -- that, that -- sixteen, eighty-one, or that seventeen out of --
STUDENT
[OVERLAPS] That's winning four.
STUDENT
That's winning four games. And you can't play another game because someone's already ___.
STUDENT
Then you have to subtract those too.
STUDENT
You need, you need, you need to find out the probability of three out of four games instead of, you know, out of four. And it's not seventeen eighty-one or anything.
STUDENT
What is it?
STUDENT
I don't know.
ROBERT SPEISER
How would you go after this?
STUDENT
Like what are you --
ROBERT SPEISER
[OVERLAPS] Looks like there's a question on the table right? How would you go after it?
STUDENT
[LONG PAUSE] I got the probability of winning three out of four games. With Team A having twice a better chance of winning as Team B.
ROBERT SPEISER
[COUGHS]
ROMINA
____ like three games.
ROBERT SPEISER
Want to talk for a while and I'll join you in a couple of minutes? [GETS UP]

B ROLL [FIGURING OUT PROBABILITY INAUDIBLE]

STUDENT
So if it's fifty/fifty ___ three out of four ....what's that?
ROMINA
[OVERLAPS] Half, one half, one half. [BACKGROUND NOISE] [INAUDIBLE VOICES] You couldn't write that in possibilities could you?
STUDENT
See I don't know if it's two thirds like that. I mean it makes sense but in a game [OVERLAPPING VOICES] --
ROMINA
There's like-
STUDENT
[OVERLAPS] That made sense though.
ROMINA
[OVERLAPS]. There's one -- there's two --
STUDENT
[OVERLAPS] Four in a row made sense.
STUDENT
Yeah, I don't know. I'm still on this whole two thirds business. Cause the other game, I mean, all right, out of --
STUDENT
[OVERLAPS] What are you saying like Team A --
STUDENT
[OVERLAPS] This is probably three out of four if it's fifty/fifty.
STUDENT
Do you ____?
STUDENT
What, three games out of four?
STUDENT
That's fifty/fifty.
STUDENT
You wrote Team A and Team H,. Let's say they win one game, Team A won two games.
STUDENT
[OVERLAPS] It's not -- it's eight right?
STUDENT
Eight out of thirty-two?
ROMINA
The A Team?
STUDENT
Yeah.
STUDENT

It is two, it is two out of three. [OVERLAPPING VOICES] -- they win, they win two.

STUDENT

They win two.

ROMINA

It's like having this, H or J. Or the other way around.

STUDENT

Half.

ROMINA

Show me what this is _____.

STUDENT

All right if they had -- if three times is ____. [OVERLAPPING VOICES INAUDIBLE] But that's -- two out of three -- I think you should only use that for the last one like we used ___. We can just multiply all this together.

STUDENT

That's what I was thinking.

STUDENT

And there's a two thirds chance of that happening, or one third, one third, one third --

STUDENT

[OVERLAPS] One third, one third, one third and then switch it up -- we're at two third, two third, two third, one third?

STUDENT

[OVERLAPS] Yeah, so it would be -- and we need -- four of those right?

STUDENT

Eighty-one -- two over eighty-one.

STUDENT

Then four of them right?

STUDENT

[INAUDIBLE]

STUDENT

Because then we put the A over here.

STUDENT

Yeah, and then you figure that out of all those two eighty ones ....

STUDENT

And then the other way is ....

STUDENT

And that'll be two times --

STUDENT

[OVERLAPS] That's eighty or eighty-one. And then there's four of those.

STUDENT

So that'll be three, yeah.

STUDENT

[OVERLAPS] So that'll be three out of eighty-one?
STUDENT
So it--
STUDENT
[OVERLAPS] Add that so it's forty out of eighty-one?
STUDENT
All right, so say forty out of eighty-one chance, that's almost half a chance it's gonna run a fifth game. Well that makes sense though. Because the one ____ manage.
STUDENT
[OVERLAPS] Yeah.
STUDENT
So it's gonna be -
STUDENT
[OVERLAPS] ____ to seven.
STUDENT
[OVERLAPS] Chances are, chances are the high amounts are gonna be in the early games.
STUDENT
Yeah. Well that's half. And that one was eight thirty-two.
STUDENT
Which ones are thirty --
STUDENT
[OVERLAPPING VOICES] Yeah but the other side has an advantage it's almost twice --
STUDENT
[OVERLAPS] Not that much of an advantage.
STUDENT
Yeah they twice as good.
STUDENT
[OVERLAPS] Twice as good.
ROMINA
[OVERLAPS] Twice as good.
STUDENT
Three out of thirty-two, I mean .....
STUDENT
I mean if you even look at --
STUDENT
[OVERLAPS] Cause look -- if you look at it, it's not supposed to go to seven games cause then you need four and three instead of four and two.
STUDENT
[OVERLAPS] I mean but --
STUDENT

[OVERLAPS] It could go up to seven.
STUDENT
[OVERLAPS] Yeah it could.
STUDENT
[OVERLAPS] I know it's not, it's highly unlikely -- [OVERLAPPING VOICES]
STUDENT
It's less likely. It's still likely.
STUDENT
[OVERLAPS] Less likely, and that one is only eight out of --
STUDENT
[OVERLAPS] That's not gonna be zero.
STUDENT
And if you look at it in terms of eighty-one, I mean eight out of thirty-two, that's like almost -- it's more -- if you put it in terms of eighty-one. Because eighty-one --
STUDENT
[OVERLAPS] Want to do this out of five games then?
STUDENT
Hm. Let's see what it looks like. See if we're ___ on the same ....
STUDENT
This will be different.
STUDENT
Yeah. So the one team has to win three games. ___ one three.
STUDENT
We screwed this one up.
STUDENT
There's got to be less than, there's got to be less than eighty-one.
STUDENT
[OVERLAPS] ____ up isn't there? There's another H isn't there?
STUDENT
Well you look at it --
ROMINA
Are we still winning those?
STUDENT
That's what I was gonna say. No one's winning this. You know what I mean. Have to get another H. So we just did eighty-four games and it should have been -- it's over eighty-one times three. Forty-three.
STUDENT
Two forty-three. Um....does this make more sense then?
STUDENT
Yeah. [COUGHS]
STUDENT

Sixteen ....[PAUSE] sixteen out of ..... did you get a half or something?

STUDENT

Not it's not. That's cause we did, cause we did ____ this and that's only three games out of ____. Actually it's, almost a half chance of winning three out of four games. Then multiply that by ____.

STUDENT

I did like ....for the -- before we had .....Team A -- when it was a fifty/fifty chance they had a four out of sixteen chance of winning three games out of four. Right? You know that? The four and that. And then ....um, but this is the better team. It has a -- for every one game this team wins, this team wins too. You know what I'm saying? Like ....the twice ___ to win.

STUDENT

[OVERLAPS] Sixty-four.

STUDENT

So you -- they're probably now eight sixty-four. It's double. And then to win the last game, it's still two-thirds cause ....one is two thirds more than the other. This one comes out to eight.

ROMINA

Why is that one two thirds? I thought this one was better?

STUDENT

Um, yeah. That would be four to ___. This is sixteen. Uh, this would be twenty out of forty-eight. Yeah twenty out of forty-eight. [OVERLAPPING VOICES] It'll be over in five days.

STUDENT

What's ___ games?

STUDENT

Four games what?

ROMINA

Which one is ____? [OVERLAPPING VOICES]

STUDENT

Yes.

STUDENT

All right. Fifty-one out of ____.

STUDENT

Probably just when you ____ winning, winning the fifth game, it'll be twenty, forty-eighty. Um, the probability that I was about Team A won, won those twenty games -- sixteen out of twenty.

ROMINA

Okay.

STUDENT

And the other one will be four out of twenty. But the questions not this. It's this one right here.

STUDENT
I don't think so.
STUDENT
___ before the guy told me to stop.
ROMINA
___ told him I couldn't --
STUDENT
[OVERLAPS] I was saying that before and he was like, oh let's think of something else. So it's probably wrong.
ROMINA
I don't know.
STUDENT
I can't concentrate. [WHISPERING INAUDIBLE VOICES]
STUDENT
Just see where that leads. From there we could --
STUDENT
Sometimes it gives it away.
STUDENT
What?
STUDENT
Sometimes it asks you a question that will give you the answer.
STUDENT
That was why I asked the questions. You know the questions [BACKGROUND NOISE]
STUDENT
He'll come back.
STUDENT
Go get him.
STUDENT
[INAUDIBLE]
STUDENT
I have no idea.
STUDENT
He just walked out [OVERLAPPING VOICES].
STUDENT
[INAUDIBLE]
STUDENT
He hasn't knocked yet.
STUDENT
Yeah ___ get up. [LAUGHS]
STUDENT

He's cool.
STUDENT
Lazy.
STUDENT
Go get him. [OVERLAPPING VOICES]
ROMINA
I got to write, I got to serve you guys dinner. No, you go get it. [LAUGHS]
STUDENT
Throw some ____. [LAUGHTER]
STUDENT
Just go please.
STUDENT
What are you gonna say to him when he comes out?
STUDENT
Just ____ we'll all talk.
STUDENT
Yeah, we'll talk about that.
STUDENT
And the --while we're doing this you can ____.
STUDENT
I went up on the board -- I went to the board.
ROMINA
So did I. [OVERLAPPING VOICES]
STUDENT
Just take the ___. [RIPPING PAPER]
STUDENT
Are you serious?
ROMINA
Yeah, you guys are all arguing about the __ and I'm not gonna go there.
STUDENT
Just ____ and let me go.
ROMINA
Pick one, two, three or four.
STUDENT
She knows which one is his. She's not [LAUGHTER]
STUDENT
Never mind.
STUDENT
All right. [DOOR OPENS] [LAUGHTER]
ROMINA
It's not like it's on paper or anything [LAUGHS] I can't watch it.

STUDENT
___ that.
ROMINA
I was number one.
STUDENT
What?
ROMINA
I was number one.
STUDENT
That's the number I was gonna pick too.
STUDENT
Who's number one?
ROMINA
I was.
STUDENT
You're ___ or anything.
STUDENT
I would have picked ____ regardless. I didn't know which one he was in order.
STUDENT
We still don't got enough.
ROMINA
That was huge, I heard ___ be coming back. [LAUGHS]
STUDENT
[INAUDIBLE]
STUDENT
___ that thing yet.
STUDENT
Yeah, four thirty.
ROMINA
[INAUDIBLE] get through. [LAUGHTER]
STUDENT
Explain this one. Me and ____ worked on a little something. And we -- this is what we got.
STUDENT
There's a five game series right? When this is like one way that one team could win.
ROBERT SPEISER
Could you do this at the board.
STUDENT
No.
STUDENT
Yeah. [LAUGHTER]

ROBERT SPEISER
I don't believe you. [APPLAUSE] [LAUGHTER]
STUDENT
Can I erase this?
ROBERT SPEISER
Yes.
STUDENT
You can erase that.
ROBERT SPEISER
Okay.
STUDENT
You got that problem?
ROMINA
[INAUDIBLE]
STUDENT
It's 13.09 miles off the ....
STUDENT
I didn't even do it.
ROMINA
I didn't even attempt that.
STUDENT
I'll bring you there in two and a half hours.
WOMAN
Ankur would you erase the ____ in that whole section.
STUDENT
Yeah. This?
STUDENT
We used five heads _____. [BACKGROUND NOISE]
STUDENT
This was one of the ways of winning, uh, the world series.
ROBERT SPEISER
Uh-huh.
STUDENT
And then since Team A's twice as likely to win as Team B, the chances of this happening is two thirds. And this is one third.
ROBERT SPEISER
Why don't you write, Team A and B? Cause you're gonna talk in A's and B's.
STUDENT
A's the better one.
STUDENT
Why don't you make B better.

STUDENT
___A better.
STUDENT
Go for it man. A better.
STUDENT
And then just multiply that across, right? Cause that's the -- the probability of this happening.
ROBERT SPEISER
I can't help but being curious. Why do you multiply this time?
STUDENT
The same reason the last time.
ROBERT SPEISER
Which is? [COUGHS]
STUDENT
Jump in.
STUDENT
Cause it's the right thing to do. Cause you know it, that's what you're supposed to do. You know you're not supposed to add.
STUDENT
It's an instinct. [LAUGHS] I was born with it.
ROBERT SPEISER
Because the --
STUDENT
[OVERLAPS] I really do, I _____.
STUDENT
[OVERLAPS] Every time you're putting two probabilities because like I said we're always -- well not always, but like we -- whether we know it we have to multiply em.
ROMINA
When they're constructed as --
STUDENT
[OVERLAPS] So why should this be different than any other case?
ROBERT SPEISER
Last time you actually showed me why you multiplied.
STUDENT
I don't even want to show you this time.
ROMINA
Yeah _____.
STUDENT
[OVERLAPS] How did we do that? So just remember what we said. [OVERLAPPING VOICES] It's different.
ROBERT SPEISER

That was a different situation where the two teams were equally matched.

STUDENT

But how is that different? It's just -- we're just doing [OVERLAPPING VOICES]

STUDENT

There was a fifty percent chance of each thing happening and we multiplied it this time which was sixty, sixty.

STUDENT

Thirty-three. And we're still multiplying.

STUDENT

We're changing anything but the chance of the team winning. So it can't be any different.

STUDENT

[INAUDIBLE]

ROBERT SPEISER

Let's talk about that later. Go on, go on and explain the rest of this.

STUDENT

All right.

ROBERT SPEISER

Okay.

STUDENT

If you multiply it across you get ....two out of -- what was it -- two, two eighty-three? [INAUDIBLE] Two forty-three, of that happening. And then you can have the A in each of the four places.

ROBERT SPEISER

Uh-huh.

STUDENT

You can't have it in the last cause then ...one ____ four game series and it's over. So you multiply this four times.

ROBERT SPEISER

Wait you're doing this, you're doing this for a five game series or a four games series?

STUDENT

Five game series.

STUDENT

You just add that.

ROBERT SPEISER

[OVERLAPS] What would you do for a four game series?

STUDENT

You would multiply it three times.

STUDENT

You're adding that.

ROBERT SPEISER

Yeah but what would you do for the four game series?
STUDENT
You --
STUDENT
Four games it's either-- remember the other one we did before?
ROBERT SPEISER
[INAUDIBLE]
STUDENT
Remember he was, he was watching us.
ROBERT SPEISER
Oh yeah you were here. [BACKGROUND NOISE]
STUDENT
You would add that four times. [OVERLAPPING VOICES]
ROBERT SPEISER
Let's go over the five game series.
STUDENT
Yeah.
ROBERT SPEISER
[OVERLAPS] So, you've looked at one possible five game series where B can -- where B comes out.
STUDENT
Mmh-hmm.
ROBERT SPEISER
Okay. Is that the only way?
STUDENT
[OVERLAPS] -- can happen one, two, three or four, it's eight out of two forty-three.
STUDENT
It can't happen in the last one.
ROBERT SPEISER
[OVERLAPS] You mean that the A could be in one out of four places?
STUDENT
Yes.
STUDENT
[OVERLAPS] Yes.
ROBERT SPEISER
So then, so there are four possibilities -
STUDENT
Yeah, so it would be eight out of two hundred and forty-three that ....Team A -- Team B would win.
ROBERT SPEISER
And in each of those cases, were the A's in a different place? Are you taking the same

product in the fractures?

STUDENT

Yes.

ROBERT SPEISER

You are?

STUDENT

Wait, wait what did you say? [LAUGHTER]

ROBERT SPEISER

Okay suppose A won the second game instead of the first, so it would be A --

STUDENT

[OVERLAPS] It would be the same thing. B - A.

STUDENT

[OVERLAPS] It's gonna be the same.

ROBERT SPEISER

Will there be a two thirds under the B at the beginning?

ROMINA

No , same as A.

STUDENT

The two thirds would be here --

STUDENT

[OVERLAPS] It's the other way around.

ROBERT SPEISER

Uh-huh.

STUDENT

So instead of just moving the two thirds --

STUDENT

[OVERLAPS] The ___ matters because the other game is ____.

ROBERT SPEISER

[OVERLAPS] Uh-huh. Are you saying you ....would you get the same product with the two third in those different places?

STUDENT

Yes, why wouldn't you? Cause you still have four, one thirds, and one, two thirds.

ROBERT SPEISER

Why wouldn't you? Why would you?

STUDENT

[OVERLAPS] It's like A times B times C. [OVERLAPPING VOICES]

STUDENT

[OVERLAP]You're multiplying the same numbers except in different order.

ROBERT SPEISER

Okay. That's one of the [OVERLAPPING VOICES] Okay so that's one of your, that's one of your ground rules. Okay.

STUDENT
So that would be eight out of two forty-three.
ROBERT SPEISER
Okay.
STUDENT
It's working now right?
ROBERT SPEISER
I think so. Okay. So --
STUDENT
[OVERLAPS] If Team A won the four games right?
ROBERT SPEISER
Yeah.
STUDENT
It would just be ..... [WRITING ON CHALKBOARD] I love that noise. Like that.
ROBERT SPEISER
Oh.
STUDENT
And then when you multiply that twice you get ...sixteen out of ....two forty-three. And for the same reason as before there's -- it can be in four different places?
ROBERT SPEISER
Mmh-hmm.
STUDENT
Then that would be sixty-four and forty three. [WRITING]
STUDENT
Add them together.
STUDENT
Then you add em together and you get seventy-two out of two forty-three.
ROBERT SPEISER
What?
STUDENT
That reduces right?
ROBERT SPEISER
Ahhh.
STUDENT
Yeah. Yeah. Seventy-two is a multiple of nine right?
STUDENT
Two forty-three out of --
STUDENT
[OVERLAPS] Is a multiple of nine. What's uh, two forty-three's?
STUDENT
[OVERLAPS] Bring it over into five and six.

STUDENT
What's nine times -- what's two ____?
STUDENT
Who cares?
ROBERT SPEISER
I do.
STUDENT
It's always -- it's probably come up in the same ___ order.
ROBERT SPEISER
Wouldn't we want to compare that with the equally matched case ...see if there's a difference? What does it come out to be?
STUDENT
[COUGHS] Twenty-seven?
STUDENT
Twenty-seven. [WRITING] Divide that by nine? Twenty-seven?
STUDENT
Yeah. Twenty seven out of --
STUDENT
Eight.
STUDENT
The other one was eight out of ...
STUDENT
The other one was what?
STUDENT
Eight out of thirty-two.
ROBERT SPEISER
Eight out of thirty-two, it's right below it. Oh.
STUDENT
[WHISTLES] That's something.
ROBERT SPEISER
Why did you whistle?
STUDENT
I'm excited. [LAUGHS]
ROBERT SPEISER
All right. Okay. [PAUSE] So that's a five game series.
STUDENT
[OVERLAPS] ____ for a week. [LAUGHTER]
ROBERT SPEISER
Are there other points of view on this? Other ways of seeing it? Other ____?
STUDENT
I was just guessing. Maybe the top number stays the same and the bottom numbers the

one that changes. [LAUGHTER] I'm not willing to prove that because I don't know why.

STUDENT

Yeah you can't prove it.

ROBERT SPEISER

Tell me why you're guessing then?

STUDENT

Guessing?

ROMINA

It happens.

STUDENT

Because the five games series has eight out of x or whatever. And the other one has eight out of another number. So I'm just thinking maybe --

STUDENT

[OVERLAPS] We get the four game series ___ on there?

STUDENT

Seventeen out of eighty-one.

STUDENT

Maybe that's a coincidence, maybe it's not. I hope it's not.

ROBERT SPEISER

Okay, so what about a six game series?

STUDENT

They didn't tell us. [LAUGHTER] Check it out.

ROBERT SPEISER

You want to check it out?

STUDENT

What is three new things? Twenty-seven right? [LAUGHTER] ___ is twenty-seven.

ROBERT SPEISER

I always thought so.

STUDENT

[OVERLAPS] ___ third is eighty-two.

ROBERT SPEISER

Two to __ , isn't that eight?

STUDENT

It's eight.

STUDENT

All right. I don't know. I'm not good.

ROBERT SPEISER

Okay.

STUDENT

No, what's thirty-two? It's two times ____, fifth.

STUDENT

Two, four --
STUDENT
Eight.
STUDENT
Eight.
STUDENT
Sixteen.
STUDENT
[OVERLAPS] Sixteen. I'm bad with math.
STUDENT
In my head.
ROBERT SPEISER
Now I want to come back to the multiplication.
STUDENT
Mmm.
ROBERT SPEISER
Okay?
STUDENT
It's like that ____ in the first place. Like [OVERLAPPING VOICES] two things right?
ROMINA
Oh this?
STUDENT
Yeah cause --
ROBERT SPEISER
[OVERLAPS] Yeah.
STUDENT
But like you have a fifty percent chance of winning and you have a fifty percent chance of winning, right, and you multiply them together. Right?
ROBERT SPEISER
Yes. Now what I remember here was that -- it was like we -- we chose to look at a simpler situation which is what happens in two games in a row, right?
STUDENT
Mmh-hmm.
ROBERT SPEISER
[OVERLAPS] And that was your strategy and you found there were four possibilities and ...uh, it seemed in that case that the four possibilities might have been equally likely. Right? And ___ right? Now, suppose you had a two game, a, a, two game series but with these teams. In other words they're not evenly matched. What would the probabilities look like that time? That way?
STUDENT
____ anything.

ROMINA
You haven't said anything in a while huh?
ROBERT SPEISER
I want to hear from everybody.
STUDENT
I want to hear from her.
ROMINA
[LAUGHS] I don't know.
STUDENT
The probabilities of what? Of the team winning both those games?
ROBERT SPEISER
Well, let's say you have a two game series. So, you know, Team A could win both games. All right. Then -- or you could have Team A win the -- win one game and Team B win the other. Or Team B could win both games, right? What are the probabilities of those things happening?
STUDENT
In the first game we may have a two thirds chance of winning right?
ROBERT SPEISER
[OVERLAPS] How do you see that? Right.
STUDENT
They always tell us to multiply. They never tell us why. Then you tell us ___ why and we don't know.
ROBERT SPEISER
Am I wrong to wonder?
STUDENT
No.
STUDENT
[OVERLAPS] No that's ____.
ROMINA
[OVERLAPS] You're asking us why we're doing something, that's what you told us to do.
STUDENT
If you say that ____.
ROBERT SPEISER
Are you curious?
STUDENT
No.
ROMINA
No. [LAUGHS]
STUDENT
If it works man.

STUDENT
But only if someone would tell us once.
ROBERT SPEISER
[OVERLAPS] How do you know it works?
ROMINA
Well because it worked for this one.
STUDENT
Cause it's the same reasoning.
ROMINA
Yeah, we're assuming if it works with that one.
STUDENT
It only changes the percentage of the chance to win.
ROBERT SPEISER
Okay. You're assuming it. Um, is it, is it a safe assumption? How would you, how would you --
STUDENT
[OVERLAPS] It's not a safe assumption.
STUDENT
[OVERLAPS] When, when multiplying uh, we need to flip two coins. [INAUDIBLE]
STUDENT
[OVERLAPS] [COUGHS]
STUDENT
[OVERLAPS] ...multiplying those two. Is that the same question you're asking us? Do you understand why you multiply by half?
STUDENT
Yeah he's --
STUDENT
[OVERLAPS] By two halves? [OVERLAPPING VOICES]
ROBERT SPEISER
[OVERLAPS] I'm not worried, I'm not worried when the chances are fifty/fifty.
STUDENT
Personally I would not --
ROBERT SPEISER
[OVERLAPS] But I'm wondering what happens when they're not.
STUDENT
[OVERLAPS] When you, when you -- when the chances are fifty/fifty, you multiply probably the first one coming, and probably the second one.
ROBERT SPEISER
Right.
STUDENT
Now it will make a difference if one ___ is different.

STUDENT

You're just multiplying --

STUDENT

[OVERLAPS] If you've got a fixed ___--

ROMINA

[OVERLAPS] You've still got those.

STUDENT

From the first one and you would still do the ___, just that ones different wouldn't change anything.

STUDENT

He just wants to straighten out the possibilities and ____.

STUDENT

But when you have two events happening you -- for the first one, the first one to happen and the second one to happen [BACKGROUND NOISE] -- by the probabilities. Why? I don't know. Why -- I mean you ____ the first event, the first coin coming up heads, the second event -- the second coin coming up heads. Half and half is one in four chance of them both happening, you know, the same time. So if one coin comes out, has more than the other, why wouldn't you multiply that? I mean, you understand? If you have -- if you're doing one half, you're still multiplying both probabilities.

ROBERT SPEISER

Why wouldn't you add?