Appendix A: Transcript of Session 1 with Group 1, June 25, 2003.

1	Pantozzi:	Here is the question.
2	Angela:	Thank you. [Angela whispers to Magda].
3	Romina:	That is way to too much college right there.
4	Pantozzi:	Now I want you to know before you begin, this is not a test of what you
5		remember...
6	Angela:	Good I...
7	Pantozzi:	You're not going to be rated upon what you say, whether it's right or
8		wrong, and to help you, as it says there, I have about six or seven
9		different calculus textbooks over there, and some other materials, I have
10		papers you did back in 1999, I have some tests you did in 1999,
11	Romina:	They look like that piece of paper you just had...
12	Pantozzi:	The calculus textbooks over there, I put a bookmark, a fluorescent yellow
13		bookmark on the page where is says what the Fundamental Theorem of
14		Calculus is, so it's not a matter of saying here.
15	Angela:	Search through this.
16	Pantozzi:	So it you want to start talking about it without looking through the books,
17		that's fine, if you want to go straight to the books, [students laughs] et
18		cetera, I'm not really going to be participating, I might come in
19		afterwards, after a half and hour,
20	Romina:	Are you going to ask us questions? No direction?
21	Angela:	We have a little thing here to do, didn't you read the directions?
22	Romina:	We, but I thought we'd get some direction or something, some like
23		whys... all right.
24	Angela:	OK.
25	Romina:	OK.
26	Pantozzi:	OK.
27	Angela:	We should go get a book, because I don't remember what it is.
28	Pantozzi:	If you want to draw any pictures, say anything, demonstrate,
29	Romina:	Should we make a plan.
30	Pantozzi:	And again you can ask...
31	Angela:	You be the leader...
32	Romina:	I'll be the leader because I'm bossy?
33	Angela:	I say I think I actually did well in calculus in high school, but I don't
34		remember anything.
35	Romina:	Angela, so did I
36	Angela:	We should get some of our test papers...
37	Romina:	No...[laughs] OK, yeah, we'll get that, and we'll...I don't even know
38		what it is, so I have to read the actual definition.
39	Magda:	It's the integral divided by b - a or something.
40	Angela:	Oh my god... You remember that?
41	Romina:	Oh this is our calculus book!
42	Angela:	That was?
43	Romina:	Oh, that's bad, Angela.
44	Pantozzi:	Well, we didn't use it that often.

45	Romina:	I just remember the guy on it.
46	Pantozzi:	[inaudible]
47	Angela:	I don't think I looked...We want Mike Aiello's paper, right?
48	Romina:	You guys.. if you're getting your papers, can I have mine? Ha ha.
49	Magda:	This one is...
50	Pantozzi:	Here's a whole bunch of stuff.
51	Romina:	I actually took notes Magda...see now they're helping us..
52	Angela:	Oh Yeah, I got a four on that one!
53	Pantozzi:	[inaudible]
54	Romina:	In class... oh Angela too.
55	Magda:	[inaudible]
56	Romina:	I'm probably the only one of us who took notes...
57	Magda:	We took some notes too.
58	Romina:	Oh, I remember this...the Riemann sum we're going to stop there [laughs]
59		that's the one where he draws the little boxes and you add them up
60	Magda:	Yeah.
61	Romina:	I was I couldn't remember I was like I hope that's what it is... remember
62		he drew the integral and then drew the boxes and then add all the boxes up
63	Angela:	I didn't remember that's what it was called.
64	Romina:	Yeah, that was the first words, so I figured... Guys, I have all this, do you
65		want me to read it... this was our actual textbook.
66	Magda:	I think she's got a different book.
67	Romina:	OK, do you want me to read...guys I have all this, do you want me to read
68		it? This was our actual textbook...
69	Angela:	Do we have to each do this, or can we like come up with one big thing?
70	Romina:	We have to come up with one big thing. ...
71	Angela:	All right, good.
72	Romina:	I think, you're the one who said I should read the question...
73	Angela:	It says we can ask other students for help, but I don't know.
74	Magda:	I don't like how this is written.
75	Angela:	Let's look through papers that might be more helpful, right?
76		(Angela and Magda point at a paper, laughing together)
77	Romina:	Is there not just like a definition of what the Fundamental Theorem is?
78	Magda:	Well I have this, you can read this, but I don't like this book.
79	Angela:	Get another one Magda.
80	Romina:	The Fundamental Theorem of Calculus... If f is an integrable.. I can't
81		even say it.. function... blah blah blah g(b) and g(a) etc. I remember
82		that... all right, we're good to go...
83	Angela:	This one... All right, teach me...
84	Romina:	Angela, I have to look at my stuff... I remember seeing it
85	Angela:	You guys took calculus in college... I can't remember this stuff.
86	Magda:	Basically, isn't it just taking the integral of the thing, of the function and
87		then ...
88	Romina:	[reading out of the Foerster text] OK, g(x) equals the integral of ... OK
89		from point a to b of this, the function... equals g(b) – g(a)
90	Angela:	Do that.

91	Magda:	Come again, OK, then it's.
92	Romina:	So OK…
93	Magda:	Just take the integral between the interval
94	Romina:	OK, isn't that the one where.. the a, b… the integral of all of this minus
95		the integral of all of this equals the area from here to here. [draws figure]
96	Magda:	Yes.
97	Romina:	All right, I got it.
98	Magda:	Well basically…
99	Angela:	I got it now that you drew that.
100	Magda:	The integral is like the area underneath the graph, right?
101	Romina:	I'm not going to be able to understand Mike Aiello's work.
102	Angela:	Then let's get rid of Mike Aiello's work…
103	Romina:	And Brian…
104	Angela:	[laughs]
105	Angela:	There are none of Magda's, by the way…
106	Romina:	Did you do that on purpose?
107	Magda:	Where's yours?
108	Romina:	Robert I can't understand.
109	Angela:	Ah ha, my homework.
110	Romina:	Well.
111	Magda:	What exactly are we looking for?
112	Romina:	I don't know. That's you.
113	Angela:	Something to jog our memory. That's me.
114	Romina:	Michelle.
115	Angela:	That's not me. My handwriting is not that neat.
116	Romina:	That's me, it has hearts on it.
117	Angela:	That's you, too.
118	Romina:	That's me, I don't even know why I…
119	Angela:	That's you, too.
120	Romina:	That's me.
121	Magda:	[inaudible]
122		[laughter]
123	Romina:	Where is he, is this just my pile of work?
124	Angela:	You did a lot of homework.
125	Romina:	Oh Jeff, there we go.
126	Angela:	I saw him this weekend.
127	Romina:	I saw him too… I don't do things in pen. Maybe it is mine. This when we
128		went for the afterschool thing.
129	Angela:	This looks like my handwriting.
130	Romina:	The three of use when we were practicing for stuff… yeah.
131	Magda:	Yeah.
132	Angela:	Something of Magda's.
133	Romina:	Ankur, we can take some of Ankur's stuff because he writes neat.
134	Angela:	Does Angela do anything? That's me writing in red pen.
135	Romina:	Angela, here we're getting into Angela. Robert I won't understand.
136	Angela:	[inaudible]

137	Magda:	He was so smart.
138	Romina:	They think on a different level.
139	Magda:	I took a math class with him... and he only came to class once... and he
140		aced... came to class, took the exam in 40 minutes and left...got A's...
141	Angela:	He didn't do so well on that one though.
142	Romina:	Oh, me neither, 80 percent. Ugh.
143	Angela:	Oh God, Eighty is terrible.
144	Romina:	Now it's good. Take so damn long...
145	Angela:	That's definitely me..., I think it's me.
146	Romina:	[inaudible]
147	Magda:	How do I know what I got on this?
148	Romina:	Angela... oh no, the second one.
149	Angela:	All right, that's it. OK.
150	Magda:	[inaudible]
151	Angela:	Let's look at this stuff.
152	Magda:	Oh my god I wrote like, such a... so much.
153	Angela:	You still write like that Magda.
154	Romina:	What exactly do you want us to look for?
155	Magda:	Yeah, I don't know...
156	Angela:	I just thought this might help jog our memory, kind of thing, I don't know.
157	Romina:	Because they [pointing to the Foerster textbook] go through a big long
158		explanation, and I just, I was like OK.
159	Angela:	OK, we can do that...
160	Romina:	No, I just like what do we...
161	Magda:	What does the theorem mean? Doesn't the theorem mean that it just shows
162		the area underneath like a function?
163	Romina:	Yeah.
164	Magda:	You know, there's like definite integrals and like indefinite integrals, you
165		know what I'm saying?
166	Angela:	What if we...
167	Romina:	I don't remember.
168	Angela:	What if we...
169	Magda:	You know, definite is between a and b,
170	Romina:	Oh, OK, so this gives us the area for a definite integral.
171	Angela:	OK, so we should define it, like start off by saying this is what this is...
172		no?
173	Magda:	Finish it...
174	Angela:	OK.
175	Romina:	You can be the writer, this is the first time ever I'm not the writer – I never
176		worked with you guys
177	Romina:	But after that, OK now what are we looking for?
178		[silence]
179	Romina:	OK, thanks for answering me...
180	Angela:	I'm sorry, I can't talk and write at the same time
181	Magda:	What am I writing down?

182	Romina:	I wrote actual write-ups… so maybe that will… I wrote something about
183		the Fundamental Theorem of Calculus.
184	Magda:	OK, talk.
185	Romina:	This one isn't through… this one is about continuous functions. I don't
186		think I did. (Angela whispers.) Do you want paper?
187	Angela:	Yeah, that could help.
188	Romina:	I don't know what that is…
189		[silence]
190	Magda:	Do you have grades on your stuff?
191	Angela:	4's and stuff like that.
192	Magda:	I have… one paper.
193	Romina:	I don't know what.
194	Angela:	No grade on this test.
195	Magda:	A circle. [inaudible]
196	Angela:	Mmmm.
197	Romina:	Do you think it was the first thing we did?
198	Angela:	What?
199	Romina:	The fundamental theorem of calculus – when we got to class.
200	Magda:	I think we started with
201	Angela:	I don't know.
202	Magda:	Derivatives first.
203	Romina:	And then we…
204	Magda:	And then we did that.
205	Angela:	How the heck do you people remember this?
206	Romina:	I have no idea what half this stuff is
207	Angela:	[inaudible]
208	Magda:	Yeah, well, then…
209	Angela:	Don't write on there.
210	Magda:	You know, then like a, b… f of x dx that equals that [Magda writes
211		$\int_a^b f(x)dx = F(\text{b}) - F(\text{a})$ on the paper]
212	Romina:	Why don't you write it on a clean piece of paper?
213	Magda:	No, I'm just saying…
214	Angela:	Magda… So this is it, this is what this is
215	Magda:	Yeah, it says exactly that on here [pointing at the Foerster textbook]
216	Angela:	Oh look at that, it does, but here it uses g's
217	Magda:	This is basically it, you take the function and then you, you know…
218	Romina:	Read this read after this out loud and see if that…
219	Angela:	Read out loud? You want me to read this, or after this?
220	Romina:	So we can hear… Read, I guess read…I don't know… up to that wasn't
221		that read where it says fundamental theorem of calculus highlighted
222	Angela:	But it doesn't really say anything it just says in this section you are going
223		to use what you discovered to put together the fundamental theorem of
224		calculus
225	Magda:	[inaudible]
226	Romina:	Read out loud so I can hear it.

227	Angela:	The top graph in figure 5-8 is a function g, an indefinite integral of F.
228		That is, g(x) = (integral sign) how do I say that?
229	Magda:	Integral of f of x dx.
230	Angela:	By the definition of indefinite integral, g prime of x = f of x. because g is
231		differentiable, the mean value theorem applies to it on a, b, in brackets,
232		or on any subinterval of a, b.. do you want me to continue?
233	Magda:	So…
234	Romina:	OK.
235	Magda:	What exactly are looking for, really? I don't…
236	Romina:	So basically this is like… we learned about the Riemann sum first, which
237		is like our very primitive way of getting the area underneath the
238	Magda:	Integral.
239	Romina:	I remember he taught us this, you make it smaller and smaller.
240	Magda:	You do the… and then we had the midpoint, is what he starting off with, I
241		think.
242	Angela:	[reading a paper] inaudible.
243	Romina:	So then…Isn't it impossible to get the area under like an indefinite…
244		don't you need like two points to … do you know what I'm saying? How
245		would we get the area underneath this (pointing to the book)
246	Magda:	You take the integral between a and b
247	Romina:	So that's how we make an indefinite definite? I don't know what that is, I
248		just remember is that what it kinda is? Do you have any idea what I'm
249		saying?
250	Magda:	No.
251	Romina:	If they gave us a line, we couldn't figure out
252	Angela:	Why don't you just draw things
253	Romina:	How to figure out the area I'm just saying, if they just give as a line, we
254		couldn't… if they… if we…we had like the bell curve for example,
255		[Romina draws graph] we really couldn't… we couldn't figure out the
256		area, because this would always gets smaller… don't we have to kind of
257		make it an approximation so we take a really far corner here and a really
258		far corner here…and take the area between like that.
259	Magda:	Yes.
260	Romina:	So this is basically what that is.
261	Magda:	But it doesn't have to go to infinity, it could be like any curve.
262	Romina:	But if it's any curve, wouldn't I be able to figure it out? Like, [draws on
263		paper] can't I figure it out?
264	Angela:	But not if it was like this and this and that [draws a wiggle that crosses the
265		x axis and then increases]
266	Romina:	But if it goes like that then.
267	Angela:	I don't know…
268	Romina:	How do you do… break it up…
269	Angela:	I guess…
270	Romina:	You're the math one…
271	Angela:	I'm not.
272	Romina:	She is. Do we really have to break it up, is there a way, I don't remember.

273	Magda:	Well…
274	Romina:	There's not a way you could just figure it out, is there? you'd have to get
275		the equation of the line,
276	Magda:	No well, if you take the integral, knowing that, if you take any integral
277		between a set of points, you don't even have to know how the graph looks
278	Angela:	Yeah.
279	Magda:	To figure the area underneath it cause you could be taking like sine of blah
280		blah blah of like some ridiculous equations, and some of the equations,
281		you wouldn't even know what they look like… you wouldn't even have
282		to know what the graph looks like, you can take the integral of it, you can
283		just plug the numbers and get the answer
284	Romina:	I don't remember how to take an integral I don't think.
285	Magda:	You know, it's like to a higher power.
286	Romina:	Well that, OK, but in practice, I don't know if I'd be able to do it … none
287		of this is going to be useful to us…
288	Angela:	(inaudible) There are three parts to the question… and it might help if we
289		like attack each individual question…
290	Magda:	What does the theorem mean?
291	Angela:	What does the theorem mean? Magda?
292	Magda:	I don't know what the theorem means… it means; it lets you know how to
293		find the area underneath the graph that's what I always thought of.. I
294		would define them maybe…
295	Angela:	[inaudible]
296	Romina:	I'm just looking because I wrote about everything… all right, this might…
297	Magda:	I'm just trying to like draw simple graphs and kind of..
298	Romina:	I think I have a write-up from him about… [silence] all right, read
299		this…[silence as Romina reads a handout from her high school calculus
300		class (appendix E)] …remember this equation you said at the beginning
301		F(b) – F(a) / (b-a)
302	Magda:	That's the slope, isn't it?
303	Romina:	Yeah.
304	Magda:	It gives you the slope
305	Romina:	Why did you say that?
306	Magda:	Well, OK, no, this is a simple x squared graph, [pointing to the graph she
307		has drawn] and this is an integral of that, an x to the third graph and then
308		basically when you take the integral you want to find the area underneath
309		the thing so it's basically you find the value of the point here minus the
310		value of the point here what the value here is minus the value right here
311		that gives you the area, but when you connect them, it's like a line, which
312		is…the slope… and I don't know what I'm trying to say but…
313	Angela:	[laughs] that was the first thing I started to understand… I don't know.
314	Romina:	Is it, when you do, so this is the integral?
315	Magda:	This is the integral of x squared.
316	Romina:	OK, so when we're doing that, you want to find, what we're doing is
317		finding all the area under this
318	Magda:	Yes.

Romina: Minus all the area under this, but how does that relate to the actual... do you understand what I'm asking?

Magda: Well, the actual graph, the point right here, [pointing to the points labeled F(b) and F(a) minus this point right here, is the area underneath the graph

Romina: I don't understand that; how can... say this point is two, and this point is one, you can't, what do you mean?

Magda: OK well,

Romina: Two two.

Magda: Say this point is two and this point is one, like this, so its 2 –1 = 1 so the area underneath that is one.

Romina: I'm not getting that

Magda: OK, let's use...

Angela: Yeah.

Magda: This is x squared, and the integral of that is 1/3 x^3, alright, so and say you want to take the integral between 2 and 0, and say this is like 2 and this is 0.

Angela: [inaudible]

Romina: Good call, Mags.

Magda: So that 2, and that's zero, so plug in 2 here, two to the third is 8, 8/3, right.

Romina: Uh hummm.

Magda: Minus ... minus that is zero, so the slope...no, not the slope, the area under here is 8/3.

Romina: OK, now I understand what you're saying.

Magda: So that's and this... I kind of like drew it in the wrong direction, this graph is supposed to be on top here

Romina: Where's my...

Magda: And then if you go back...

Romina: Do we have down what the theorem means.

Angela: I don't have it written down.

Romina: Like I'm saying, is that all that it...

Angela: What the theorem means, what the theorem is for, and why the theorem is true.

Romina: Someone read this, because I mean... I think that's like right after we probably did it.

Magda and Angela read what the paper Romina gives them (see appendix)

Romina: I got all excited and then I got to limits and then I stopped...

Angela: [laughs]

Romina: [inaudible]

Angela: This is this, right? [pointing to paper (see appendix)]

Magda: F(t) will be this, this line...

Angela: Right.

Magda: The integral will be this line.

Angela: OK, sorry, I'm like all I'm Englished out I can't read anything that's math anymore.

Romina: Can you guys read my handwriting?

Angela: Absolutely not!

Romina: We're getting closer – this is what I learned right before I learned the Fundamental Theorem…

[laughs]

Romina: I swear, it really is.

Magda: Wow, you have… Did we have to write journals like this?

Romina: Apparently not!

Magda: I could have sworn I did work in this class.

Romina: You did it with me.

Magda: So why isn't my name on here?

Romina: Ask him. No. I wrote my own Magda. What's the date on that?

Magda: October 8.

Romina: We're

Angela: This is October 14.

Romina: Almost there.

Angela: Kind a.

Magda: …[inaudible] intervals…

Angela: A calculator…

Magda: A calculator wouldn't….

Angela: I don't know Magda, You know more than me…

Magda: I don't know anything… this is like Analysis I wanted to plug and chug the numbers.

Magda: You don't understand, I learned so many different ways of taking integrals, it's… I don't even… now my sister is taking calc 2 so I'm like refreshing my memory

Romina: This is too old. Or we could have looked at a test that said the FTC… All right, here it is, let's see…

Angela: [inaudible]

Romina: You look at this, I'll look at the other ones.

Angela: What am I going to look at?

Magda: Oh my god, that's… [inaudible]

Angela: You just handed me something.

Romina: No, I was keeping that, I was going to show that to you in a second.

Angela: I'm not going to understand this.

[students are looking at papers]

Romina: I formed the name of a country using all of our initials,

Angela: [laughs]

Romina: And I wonder why I don't know what the fundamental theorem of calculus is.

Angela: I used to know…

Romina: Did you read my statement?

Angela: No, it had something to do with limits and derivatives…

Romina: OK, what the FTC is

Magda: [inaudible]

Angela: I remember this paper.

Romina: Would it have stuff to do with, like uh, tying in the whole idea like, like you know, how a derivative and an integral is kind of like tied together,

410		and the whole limit, finding the specific slope, and using the integral to
411		find a specific slope of a point.
412	Magda:	Yeah, because if you take an integral,
413	Romina:	Isn't that what the question...
414	Magda:	And you take it back to it that's kind of like the integral.
415	Romina:	The derivative of the integral
416	Magda:	The derivative of the integral is the actual function.
417	Romina:	And when we take the integral to find like the slope of certain points on.
418	Magda:	On the integral?
419	Romina:	Say there's a line.
420	Magda:	OK.
421	Romina:	And you like, you want to know the slope at a certain.
422	Magda:	So you take the derivative.
423	Romina:	OK.
424	Magda:	Slope is derivative, area is the integral
425	Romina:	Area is the integral... OK.. I don't..
426	Angela:	[inaudible]
427	Romina:	I don't... did you get anything off that test on the thing...
428	Magda:	Well, I don't know what I'm looking at – I can like do, figure out the
429		problems, but that doesn't tell me what...[laughs]
430	Angela:	Everyone's got this test but it doesn't quite help.
431	Magda:	Which...what is the graph
432	Romina:	I don't know... do you guys... Is that the graph? I don't even know what
433		the graph is.
434	Angela:	I have no idea.
435	Magda:	Hold on...
436	Romina:	Which one's the graph?
437	Magda:	When the graph of f is shown in the graph... I'm guessing this is the graph
438		right here.
439	Romina:	No I drew that in. Is that the graph?
440	Angela:	This... it probably is, you found that out, that out (pointing to the area)
441		that and that, you shaded that in so this is probably the graph.
442	Magda:	That's the graph, OK. Then what is it asking?
443	Angela:	No, that's the graph...
444	Romina:	That's the graph Magda it goes boom boom boom boom... that's, I don't
445		know what that is yet.
446	Magda:	OK.
447	Angela:	You should be more thorough in your explanations.
448	Magda:	Oh, this one's the integral, probably, because you're adding this area, this
449		area up, and then this area.
450	Angela:	And it keeps going up... and here it's negative, so it goes down.
451	Romina:	Didn't we take these tests in uh...blue books... didn't we, because we
452		don't have the answers to the tests?
453	Angela:	Didn't we do it with loose leaf? We didn't take these in blue books...
454		maybe the final or the midterm...

Romina: Well I'm just saying maybe... I have something written in there that I don't have now?
Angela: Probably. Yeah, probably.
Romina: All right, here's something... the integral from a to b minus the integral from a to c equals the integral... a to b minus a to c .
Magda: Can I see?
Romina: [inaudible]
Magda: [inaudible]
Romina: [inaudible]
Angela: What are we trying to figure out?
Romina: I just want to see it has anything if this is leading us anywhere I'm not sure what he wants.
Magda: So the integral of this is this, so say this is "a",
Angela: We basically have to teach a class on the fundamental theorem of calculus. That's...
Romina: It's from b to c, right?
Magda: Area right here, that's the area... ... from a to b minus the area from a to c is the area between here and here... so it's like.
Romina: Zero.
Magda: No, You're subtracting more, so it's the area left over...so it's the area between b to c. [Magda draws on the graph representing x^2 she drew earlier]
Romina: That's what I said Magda...[laughs]
Magda: Anyway, I like drawing stuff...uh yeah.
Romina: I need to talk to him because I don't know exactly what he wants.
Angela: 5 points...
Romina: Is he out there? I'm like, I don't have a direction here. ..
Angela: We have like 400 books... like I think the point of this is to be teaching ourselves... reteaching ourselves how to do this. That's what the point of this is.
Romina: Angela...so stand in a corner and don't listen...
Angela: I'm just thinking... the point of this...
Romina: We have a question for you.
Pantozzi: Sounds like I'm being called down. to the principal's office.
Romina: Please, sit... no. We're like just like going through our old stuff.
Magda: What exactly are we looking for?
Romina: We knew what it was right off the bat, you'd be pretty impressed.
Pantozzi: OK.
Romina: We knew.
Pantozzi: Well, the collective we.
Romina: We know what it is, what exactly are you looking for?
Pantozzi: Well, lets go back to the task...I was doing this for Anna you know, a couple of months ago, she asked me, something I learned, something that.
Romina: So if I told someone...
Pantozzi: If someone came up to you and said that they're in calculus now, and they've taken the first three chapters, four chapters of the book, and

they've gotten to this fundamental theorem section... I think I might have told you the story of what happened at my final exam...in calculus I, I took it early because I needed to help my dad with catering...and so they just gave it to me in a room in the math department office...so I was in there, I got to question 10, it was something about the FTC but I didn't really know what it meant, but I knew it was a big F and a little f and a g, and an integral sign, so I tried to string some of that together.

Angela: A's and b's

Pantozzi: So I tried to write something and make some sense out of it but I wasn't really sure what to say about it... so as I said, imagine this person has come to you and has just done this section and I really didn't get it so and you took calculus before, so.

Romina: Isn't it just the area underneath the curve...

Pantozzi: Well, I can't answer that question, right now.

[laughter]

Pantozzi: However what I want you to do is put together, you know, I can be that person when I come back in again but you want to put together something to say to this person.

Magda: Oh, so we're going to present to you...

Angela: A presentation...

Magda: Present to you...and then you're going to be asking us questions...

Pantozzi: I might ask you some questions, I don't know what I'll say.

Angela: That's WRONG... sorry...

Pantozzi: No, I definitely won't do that, because that's not my role in what I'm doing now...no but that's the way I want you to think about it... you looked, looked at some textbooks, you knew something right off the bat...but imagine you're telling, you're trying to help this person do those three things that you underlined before. So you are going to put together, you are going to meet with them tomorrow morning, and I want to sound like I know what I'm talking about because I took calculus and this person's coming to me for help, you know, so plan together what you would say to them. You know, start here, you understand this...what exactly would you say to them... plan that out, you can put diagrams on the board...reenact it, try it out first with yourselves. And then try it out on me, that sort of thing.

Angela: When we try it out I'll be the student who doesn't know anything Kidding... not really.

Magda: I think what it means is, I'd say it's the area under a graph [Pantozzi leaves]

Angela: Of any graph?

Magda: Under a function, under some kind of function

Romina: I guess we would have to would we have to go into Riemann sums ?

Magda: The example of this and this... it's really not a function... I don't know...

Angela: Shouldn't we define under – under could be like all the way under.

Magda: The x axis cutting off at the x axis.

Angela: Yeah, but How do we word that?

547	Magda:	I don't know... like copy it right from of the book.
548	Romina:	What if the graph goes underneath the...
549	Angela:	That's plagiarism...
550	Romina:	Magda...
551	Angela:	Copy my papers... I'm not just going to steal something out of the book
552		we're not going to learn anything it if we do it like that.
553	Romina:	[inaudible]
554	Magda:	Page 49.
555	Romina:	I have a question for you: What's the integral of that? Is it all of this
556		[pointing at an area that extends off the page to the left]
557	Angela:	This and this, right?
558	Romina:	Just this.
559	Angela:	This stuff, or is it that?
560	Magda:	It's all that. It depends on...
561	Romina:	What's under this, nothing?
562	Angela:	It would be here too?
563	Magda:	Yes. That's what it would be.
564	Angela:	So between the graph and the x axis.
565	Magda:	Um hum.
566	Romina:	How would we find.. like is it all this?
567	Magda:	Well that could go into infinity.
568	Romina:	Does it go to infinity?
569	Magda:	There could be a cut off point.
570	Romina:	Like here? (She draws a point)
571	Magda:	You can take limits like as x approaches infinity or something like that
572		and then.
573	Angela:	Can you do this without a graph? Is it formalized?
574	Magda:	You can have definite or indefinite integrals and then take limits I
575		remember I did something with indefinite integrals where you take limits
576	Romina:	He saved that for 4 years and you're writing on it!
577	Magda:	Like integral from infinity to infinity...infinity to infinity it's something
578		like the limit as x approaches or is it t approaches infinity from in or is it
579	Romina:	You're getting into letters here... what's h?
580	Magda:	F of t ... I don't remember you change it to t .. it's something with t's and
581		you start taking limits.
582	Angela:	What?
583	Magda:	I don't know, I was actually helping my sister do this yesterday.
584	Angela:	This is why you know things.. right?
585	Romina:	We're not getting very far here.
586	Angela:	OK. can we talk about defining this I know you said it's the area, but what
587		about if you're like not doing a graph.
588	Romina:	Should we like...
589	Angela:	Do we have to do a graph to do this
590	Romina:	Should be start really basic?
591	Angela:	Yeah, we should.

592 Magda: Like a real life problem – what is it like acceleration, velocity, and like
593 something [she moves her hands in a downwards motion]
594 Angela: Oh, god,
595 Romina: Speed, velocity, acceleration,
596 Magda: How does it go?
597 Angela: Is it like when Mr. Pantozzi taped himself in the car.
598 Romina: [inaudible]
599 Magda: That's the real life.
600 Romina: If we have like a graph, that's the.. I don't remember.
601 Magda: It's like something...velocity, acceleration, it goes up
602 Romina: The derivative is acceleration... no... if you have speed, speed, the
603 integral is...
604 Romina: Do you remember this?
605 Angela: I don't remember this.
606 Romina: It's probably in those books.
607 Angela: The last time I did any of this was in high school.
608 Romina: Oh, and my major is physics...
609 Magda: But you know everything is a function, you know speed, it's some kind of
610 a function.
611 Angela: You took math classes... my math classes were like what's 2 + 4.
612 Magda: Hold on. If you have a function of speed, OK first you are driving, say OK
613 it's zero, lets say you were going 5, and then 10, right, then it goes like
614 this, then you can like level out, so your speed no your.
615 Angela: Your acceleration.
616 Magda: Your acceleration would be
617 Angela: Like would be the difference between these points, from here to here, and
618 then from this to here, like that...
619 Romina: Cat! ... Or not...
620 Angela: Please Romina, I was understanding things,
621 Magda: The speed.
622 Angela: Don't bring the cat back.
623 Magda: Hold on, what is it, the speed, the velocity, If you have the speed, you're
624 looking for the acceleration, and what else?
625 Romina: Distance.
626 Magda: OK, distance.
627 Romina: I can't remember...
628 Magda: The distance would be...
629 Romina: Do you know?
630 [laughter]
631 Romina: We're not allowed to ask.
632 Magda: So...
633 Romina: This is such a sad display,
634 Angela: They're sitting back there laughing... these students...
635 Magda: [inaudible]
636 Romina: Wouldn't distance be the integral, how much area you went.
637 Magda: Yeah, distance.

638 Romina: Oh, so it was distance.
639 Magda: Distance is the integral, then it was speed, acceleration. Speed is the
640 function…
641 Angela: Speed is the function… So let's write that down, so Angela can
642 understand things. Speed is the function, this would be like f(x),
643 Romina: Distance…
644 Angela: And distance
645 Magda: Do distance is the integral,
646 Angela: I did something right.
647 Romina: I don't know if that's right… and acceleration is the derivative,
648 Angela: Distance, acceleration.
649 Magda: [inaudible]
650 Romina: D/dx or the little thing.
651 Angela: What did you just say?
652 Romina: D/dx.
653 Magda: Intervals…
654 Angela: [inaudible]
655 Romina: All I'm saying is this Angela.
656 Angela: Oh, OK.
657 Romina: Doesn't sound… Are we sure that's right.
658 Angela: No.
659 Magda: I'm pretty sure that's right. That makes sense. If you have speed you
660 travel, you accumulate distance.
661 Romina: Cause then, we could explain that, break that down, with rectangles, and
662 trapezoids, and then whatever
663 Magda: No, no no, I agree agree agree
664 Romina: This theorem lets you evaluate definite integrals exactly by algebra using
665 indefinite integrals – so that's what it does – I guess we missed that line
666 before. (reading from the book.)
667 Angela: We can't copy that out of the book.
668 Romina: We saw that before we were like wa ha? That's what we were saying
669 before.
670 Magda: [inaudible]
671 Romina: You get a definite with an indefinite.
672 Magda: Well an indefinite integral just means you don't have bounds on it, isn't
673 that it.
674 Angela: [inaudible]
675 Romina: Yeah…you're the one who taught me what…
676 Angela: What it does.
677 Romina: It makes an indefinite…no, you don't need to write that down, it's just.
678 Angela: Don't we have to answer…Isn't that part of the question? No, what it's
679 for.
680 Romina: What it means.
681 Angela: What it means, what it is for.
682 Romina: Well, we have the equation…
683 Angela: Why is it true? That's the third part of the question.

684	Magda:	Because we read it in every single book.
685	Romina:	No, didn't we just explain that?
686	Angela:	The textbook tells us so. Mr. Pantozzi...
687	Romina:	You just sat there before and explained it,
688	Magda:	Oh with my little...sign.
689	Romina:	With the.. yeah...put in the numbers.
690	Angela:	Can we...
691	Romina:	Do we assume they know what an integral is?
692	Angela:	Let's assume that they don't...
693	Romina:	We need to explain what an integral is?
694	Angela:	[inaudible]
695		[Magda excuses herself to use the restroom.]
696	Angela:	All work ceases until Magda returns. I think we should start with the
697		basics, because it makes it easier to explain other things... because if we
698		like start in the middle he'll ask us questions and we might not be prepared
699		to answer them... get away from it...
700	Romina:	Do we have to get into derivatives, explain that?
701	Angela:	I don't know.
702	Romina:	I don't think I can... I don't...
703	Angela:	Excuse me...I'm going to steal Magda's book. Is that the textbook we
704		used?
705	Romina:	Yeah.
706		[Romina is reading.]
707	Angela:	Remember this guy Euler?
708	Romina:	What's the mean value theorem?
709	Angela:	A what?
710	Romina:	The mean value theorem.
711	Angela:	I have no idea. I'm telling you I really remember nothing, it's terrible. It
712		makes me sad. I should take calculus next year.
713	Romina:	Do you know what the mean value theorem is?
714	Angela:	Of course she does, she's Magda.
715	Magda:	Mean mean... isn't that over b – a I don't know. I think that's what it is.
716	Magda:	I came up with, how about we start, OK, say you have a function [she
717		draws a parabola) maybe this.
718	Angela:	X squared.
719	Magda:	And then the, OK well, the integral of it would be, well, it's the area
720		underneath the graph, say, all right, so basically, lets start plotting it little
721		by little kind of deal.
722	Angela:	Can we use graph paper?
723	Romina:	Here, wouldn't.
724	Magda:	[inaudible]
725	Romina:	Here wouldn't, Magda, look at this, isn't this like the mathematical
726		explanation of it [pointing out the explanation in Foerster page 216) Do
727		you understand it?
728	Angela:	I'll plot x squared.
729	Romina:	I don't really OK, so g is the, is the integral of the function?

730 Magda: G, I don't know what g is, how do they define it. G is... Well, g(x) is the
731 integral of f(x) [on page 216 it says integral (no limits of integration) of
732 f(x)dx = g(x)
733 Romina: So the derivative of the integral is the actual function.
734 Magda: So the derivative is... do you have a pencil so I can like...
735 Romina: Rewrite it on...
736 Angela: Pencil doesn't show up on camera.
737 Romina: Here, rewrite it on the thing. Give me the paper...
738 Magda: OK, basically this says that, OK...hold on. G(x) equals the integral of f(x),
739 right.
740 Romina: Yeah.
741 Magda: So that means...
742 Romina: What does c1 mean – I just read this I really don't remember.
743 Magda: Let c1 be the points, the first and second... that's the different cut-off
744 points.
745 Romina: OK.
746 Magda: So $g(c_1)$ is [she writes $g'(c_1)$ on her paper]
747 Romina: Is the function.
748 Magda: Is the function It's f(x)
749 Romina: So here, write like an arrow underneath it, so we know it's f(x).
750 Magda: Which is f(x), [she draws an arrow between $g'(c_1)$ and $f(c_1)$] no, $f(c_1)$
751 Romina: Is equal to
752 Magda: $G(x_1) - g(a)$ over the change in x. [She writes $\frac{g(x_1)-g(a)}{\Delta x}$ equal to
753 $g'(c_1)$.]
754 Angela: What's this then?
755 Magda: It's f of c_1. The actual function.
756 Angela: OK.
757 Romina: Go g of...can you draw.
758 Magda: The graph of..
759 Angela: Do that.
760 Magda: So f(x) would be, say this, right?
761 Romina: Um hum.
762 Magda: Use like simple functions, [she draws a parabola]
763 Romina: Then the integral would be...
764 Magda: Then the integral of it would be the x^3 graph.
765 Romina: OK.
766 Magda: So then here this says that the actual function at some point c_1.
767 Angela: Just draw a point on there.
768 Romina: Is our "a" constant?
769 Angela: What?
770 Magda: Hold on. "a" is the interval that it's on. So say your interval would me like
771 from zero, OK, this is your a and this is your b...
772 Angela: [inaudible]
773 Romina: [inaudible]

774 Magda: X sub 1, into intervals of equal width...g of one so OK, so basically,
775 you're dividing this into smaller intervals.
776 Angela: Umm.
777 Magda: So.
778 Romina: So it's kind of like...You're dividing them until, you get like one... is that
779 what that's saying?
780 Magda: Well yeah this would be like x1, x2, x3 right.
781 Angela: Um hum.
782 Magda: So this is x1, so that would be $g(x_1)$ [pointing to the x axis and the cubic
783 graph] So that would be like this is the x1 here, so this, minus g(a) which
784 is the original point which is here, over the change which is the distance
785 here.
786 Romina: OK, so that gets us the area? Does it?
787 Magda: Did I... gets us c_1... (as she points to the $g'(c_1)$ she wrote on the paper.]
788 Angela: That gets us this...right?
789 Magda: Which is on the original curve is... yes, that gets you the area.
790 Romina: The slope of...
791 Magda: No, that gets you the area.
792 Romina: Isn't the slope on our f the area on the g?
793 Magda: Isn't the slope on our f this is our f... no...
794 Romina: Isn't that what an integral is...
795 Magda: This is our f...
796 Romina: By finding the area you find the actual slope...
797 Magda: This is our f(x) so what are you saying?
798 Romina: Like the slope from..
799 Magda: Of a...
800 Romina: Of a point is the area under it.
801 Magda: The slope of a point%
802 Angela: Slope between these two points is the area here is what she's saying. Like
803 from a to x1 would the area under here.
804 Romina: No, isn't that why we take the integral?
805 Magda: No no no no. The slope would be this... like the slope would be this...
806 like the slope at x one. [Magda draws a linear graph underneath the graph
807 of the parabola.]
808 Romina: The derivative is the slope.
809 Magda: Yeah.
810 Romina: So what is the purpose of the integral? Why do I need to know the area?
811 Magda: The whole thing like with the distance, with traveling the distance, and
812 stuff like that
813 Angela: Ok, what is that whole thing?
814 Magda: Ok, if you're traveling some speed over like whatever, and you want to
815 find the
816 Romina: Wouldn't that be the slope?
817 Magda: Well, the slope...
818 Romina: What's velocity?
819 Angela: Distance over...no...[laughs]

820	Romina:	I was just asking...I thought you knew I don't know...
821	Angela:	I feel so...
822	Romina:	Go on.
823	Magda:	I don't know if I'm explaining this right.
824	Angela:	I don't know Magda. I have no idea what you're talking about.
825	Magda:	How about if we go ahead and draw...the graph.
826	Romina:	I'm saying because you can figure...
827	Magda:	This is what it's saying.
828	Romina:	I understand by looking at it, because I know what the final product is
829		supposed to be but I don't know how anyone looking at that [the three
830		graphs now on the paper] would understand it, because we can't explain
831		it.
832		[Silence]
833	Angela:	What if we do it formula wise...like would that...would that help?
834	Romina:	Hold on, is each x, are the x's the intervals...?
835	Magda:	Say it again.
836	Romina:	Are the x's the intervals?
837	Magda:	Well, if you divide them into intervals , x1, x2, x3... I'm still not getting
838		what the c1, c2 is.
839	Romina:	Bring out another book and see if they have it the same...maybe an easier
840		proof.
841	Angela:	Is there a third grade version of this...that I can handle? [Angela is
842		drawing a parabola on the graph paper]
843	Romina:	Are these people kidding me with these books? This one's not any better I
844		don't think.
845	Angela:	Here's your graph Magda.
846		[Magda continues to look at the Foerster page 216]
847	Magda:	The points the first second and third...
848	Angela:	Should I do...
849	Magda:	The conclusion of the mean...
850	Romina:	This one's not working for me... do you get that? [She hands over the
851		Contemporary Calculus textbook] It's a completely different proof.
852	Magda:	[inaudible]
853		[Romina goes leaves the table to pick up another book]
854	Romina:	How about a teacher's guide?
855	Angela:	[laughs]
856	Magda:	So this is going to be 1, 2, 3, maybe we should like...
857	Angela:	Double.
858	Romina:	Photocopy this?
859	Magda:	Here's more graph paper. Here. Use this.
860	Angela:	I feel like we're in the middle of finals or something.
861	Romina:	We could just draw on the board... maybe that would be easier.
862	Angela:	Are we taking the middle or what
863	Magda:	Yeah just take the middle... I'm still wondering what the c1 means.
864	Romina:	Yeah, I don't... did you see the time?
865	Magda:	C one.

Romina: There's another one with like time and height...it might.. that might...
Magda: It looks like it's the points on the first, second, and third subintervals... so it basically like c3 is any point on this interval? Is that what it's saying?
Romina: I don't know, that's why I didn't learn like that. Isn't it something about like our intervals getting smaller and smaller and smaller... is that what it is for?
Magda: Yeah, well, if the intervals are smaller and smaller, it's more accurate...
Romina: Yeah, so is that what that is trying to say? Then we add together like the, like if our intervals are getting smaller, so we have more intervals, and we add them together, and it's getting smaller, it's more accurate,
Magda: Because if this... doesn't make sense, OK because if this is actually that, and you are multiplying it by the change of x so this like say this is two...this is two...[pointing a point on the graph of the parabola]
Romina: Is the C in the point or the area.
Magda: No, it has to be the point, it's a point.
Romina: So if we're doing height times width,
Magda: Yes.
Romina: OK. SO we're doing height, that's
Magda: Well, no, it's not height, it's whatever,
Romina: [inaudible]
Magda: Say this is 2, you're multiplying whatever the function, so whatever the function is, if it's 2, then it's 4, times whatever the change is so basically what you are doing is like 2... if it's 4 times .5 say that's like change of x, so that little thing equals 2.
[Romina draws a graph, and labels points a and b on the x axis.]
Romina: What are the x's ?
Magda: X is this.
Romina: Well, but.
Magda: This is your x.
Romina: So what they're saying is...like at the bottom [pointing to the bottom of page 216]
Magda: You divide by the change in x.
Romina: No like I don't, so... so like x1.
Magda: X2.
Romina: They're taking x2, x3, x4.
Magda: Yeah.
Romina: So they're taking this minus this which will get me this [draws area under her graph between x1 and x2.) Am I wrong?
Magda: G is.. yes, that's right.
Romina: And they're adding this plus this to get this...so they're doing all this, so eventually, you get $b - a$.
Romina: OK, I got the bottom half figured out.
Magda: Where are you looking? [Romina points to the bottom of page 216 in the Foerster text] OK that makes sense.
Romina: OK, here, they're doing... this isn't our g function, this is our g prime (referring to the graph she has drawn.)

912 Magda: Yes,
913 Romina: So here they're doing g of x1.
914 Magda: No, no, this is your g function, because f of… no.
915 Romina: Then, how is that.
916 Magda: No it is g prime.
917 Romina: So here… x one minus g(a)
918 Magda: Hold on… g prime.. Which is also f of x.
919 Romina: I don't get why you're dividing the change over x, I don't get why you're
920 doing that, if it's the derivative.
921 Magda: Well draw the integral of that… no, the integral? [Romina begins to draw
922 a new graph.]
923 Romina: I don't know the integral, I was just guessing x to the third. Is that right?
924 Angela: Is that it? [inaudible] Forget it, never mind.
925 Magda: Yeah…
926 Romina: So I did it backwards. I'm not going crazy here. [Referring to what the
927 integral of the graph she drew would look like.] This is a negative x
928 squared, yeah, so it's that.
929 Magda: Yeah.
930 Romina: You guys looked at me like I had five heads. It's the same…
931 Angela: Wait a second… did I just do this wrong?
932 Romina: OK, so what is this?
933 Magda: So now…
934 Romina: G(x1) – g(a) over.
935 Magda: You're doing this?
936 Romina: Yeah, I'm just writing it down cause I can't…
937 Magda: G of x.
938 Romina: Then what's the next one just so I can see what the pattern.. is that g.. oh,
939 that's g prime.
940 Magda: No it's g.
941 Romina: Equals g prime (c2) I don't understand what this is.
942 Magda: That's what I'm trying to figure out. So let's try this.
943 Magda: G(x1) so this is your x1…so g(x1) [she places a point on the *x* axis of the
944 "integral" graph Romina has drawn] is right here.
945 Romina: Um hum.
946 Magda: So g(x1) is right here.
947 Romina: OK. Minus…
948 Magda: Minus g(a) which is this point right here,
949 Romina: All right…divided by…
950 Magda: The change.
951 Romina: When they say, OK, the change, they mean this part [she indicates the
952 change in *x* between the two points she has drawn on the *x* axis.
953 Magda: Yes. Which is just x1 – a .
954 Romina: All right. Why did I have to make it so damn complicated.
955 Magda: So…
956 Angela: I think we have to figure that out.
957 Magda: Explain this.

Romina: OK, see that's where I was going wrong, I wasn't looking at this as this is the integral function. That's why I was not getting it.

Angela: But this is the integral function, right?

Romina: No see how she... this all...All this (referring to the symbols in the text) was happening on this graph. I was not understanding that... that all is happening on that.. am I not...

Angela: Isn't this... this?

Magda: Isn't this the slope at the point there (pointing to Romina's g' graph.)

Romina: Yeah. Isn't that what you just did? And the slope that's what I was saying – isn't the slope the area?

Angela: Yes! Yes it is.

Romina: No, no I don't know, that's why I'm.. I don't know...

Magda: If you're dividing by the change, it is...

Romina: That's the slope. That's why I was thinking this was the slope from that.

Angela: This is also the change, is it not?

Magda: Yes, yes this is the whole... we did it, somewhere... here, isn't that it?

Angela: Yeah.

Romina: Yeah.

Angela: All right. Now, how does that pertain to what we're doing? Now that we've figured out what the book is trying to say...

Romina: No I am just... I still am lost. This is the slope. [pointing to the calculation of slope on her paper.]

Angela: Right.

Magda: Yes.

Romina: Oh, so this is saying the derivative is the slope. Isn't that.. that's all it's saying.

Angela: Yeah.

Romina: I'm sorry we wasted all that time trying to figure that out.

Angela: Well, maybe it will help.

Romina: I still don't know where... did the other people go about this a lot faster with this?

Magda: This is one, this is...

Angela: Where's Mike Aiello when you need...this is one two, three, oh man. Nice job Magda.

Magda: Say our g(x) was this, we're doing this area manually. This is point five.

Angela: OK.

Magda: So the area...

Angela: Can we get different colors. Can we get different color pens, is that possible?

Romina: This is why I don't like working with girls.

Elena: Do you need pens?

Angela: We can use those, that will work. No, just because everything like would look the same...

Magda: [inaudible]

Romina: [reading from the Teacher's guide to AP Calculus] Use the Fundamental Theorem to evaluate definite integrals, That's what we're doing.

1004	Magda:	[inaudible]
1005	Romina::	Wait wait wait… [reading from the Teacher's guide to AP Calculus] Use
1006		the FTC to represent a particular antiderivative and the analytical and
1007		graphical analysis of functions so defined.
1008	Angela:	I have no idea what that said. So the area…
1009	Romina:	The antiderivative. Isn't that the integral… of the derivative?
1010	Angela:	You're asking me? Magda?
1011	Romina:	I'll just it here and talk to myself.
1012	Angela:	I don't know what that is. I have no idea. I'm like re-learning this all right
1013		now.
1014	Romina:	This is a very poorly made teachers manual.
1015	Angela:	He gave us a teacher's manual?
1016	Romina:	Yeah, that's what I'm saying…
1017	Angela:	… isn't it?
1018	Romina:	It's like a how to teach.
1019	Angela:	Oh, that should help a lot, right, Because that's what we have to do?
1020	Romina:	Thanks…
1021	Magda:	What are we going to use – a midpoint kind of deal.
1022	Romina:	You can use all of them…
1023	Angela:	Can we do that thing…
1024	Magda:	Can you elaborate?
1025	Angela:	Are you going to do this? Is that going to help? [She draws the graph of a
1026		function and rectangles under it.]
1027	Magda:	Yes, that's what I'm trying going to do.
1028	Angela:	Midpoints…
1029	Magda:	That's what I'm… so we're going to use the midpoints.
1030	Angela:	Yeah, that's what I meant.
1031	Magda:	OK, so this is one bar.
1032	Angela:	So we have to find the midpoint there. I'm sorry Magda, I should have
1033		made it better.
1034	Magda:	This is the second point.
1035	Angela:	Is there an exact…
1036	Magda:	Hold on, hold on. Which one is which?
1037	Angela:	Ignore that, it's this one. It's x squared…x squared.
1038	Magda:	Right here.
1039	Angela:	No, Magda, I think halfway would be higher up I think.
1040	Magda:	No, about right here.
1041	Angela:	Oh, from here, to here, I was like, what are you talking about Magda?
1042	Magda:	Then here, it would probably be like…right?
1043	Angela:	Do you want me to start getting numbers for you?
1044	Magda:	Yeah.
1045	Angela:	Ooh, a graphing calculator, I haven't used one of these in a long time.
1046		[Whispers to Magda.]
1047	Magda:	Well, just…
1048	Angela:	Find the area of the rectangles.
1049	Magda:	Um hum.

1050 Angela: I'll wait until you're done.
1051 Magda: Yeah. So like you're going to be doing .25 squared.
1052 Angela: Why .25?
1053 Magda: Because we're using our...
1054 Angela: Oh, because it's the midpoint.
1055 Magda: Yeah.
1056 Angela: [inaudible]
1057 Romina: You know what I think we should do? We should first, explain, explain
1058 the calculus and the area,
1059 [laughter]
1060 Romina: Then we should explain definite integrals, and then we're going to do
1061 calculus and area by the Riemann sums,
1062 Angela: Isn't that cheating, using the teacher's manual?
1063 Romina: It's not telling me how to do it, because apparently the teacher's supposed
1064 to know, because they've taken these math classes over and over. And
1065 then we're going to go into definite integrals and antiderivatives, and then
1066 comes the FTC. We have this stuff, just don't have integrals and
1067 antiderivatives, that's the whole thing that I made you look at that you
1068 apparently didn't ...
1069 Magda: Well, integrals and antiderivatives, aren't they the same thing?
1070 Romina: Well that's what I thought, but why did they write it out like that?
1071 Angela: Shouldn't we assume that the student knows that?
1072 Magda: Well, antiderivative, it's like one has like how the graph moves up and
1073 down because you can kind of start taking the integral at any point. It's
1074 something...
1075 Romina: I'll keep that open just in case we decide to uh...
1076 Magda: Isn't that the whole issue with plus C. that's the difference between and
1077 antiderivative and an integral.
1078 Romina: That sounds really familiar, Magda, but I don't know.
1079 Angela: it's not really helping much.
1080 Romina: I just, there was, there was a thing... antiderivative
1081 Magda: So the height right here is what?
1082 Angela: Oh, I just have to do that thing... squared...point 0625.
1083 Magda: Times the change in x is point 5.
1084 Angela: I knew that Magda.
1085 Magda: So the area of this little... the first triangle.
1086 Angela: So it's like, hold on. Times the change in x, that's what we're
1087 doing...equals area of..
1088 Magda: Area of...
1089 [laughter]
1090 Romina: What are you guys doing?
1091 Angela: I'm trying to write this down so I know what I'm doing.
1092 Magda: It's the area, OK. It's the area, that's it.
1093 Romina: I can't find it in here.
1094 Angela: I'm going to make a little chart. Yeah!
1095 Romina: What are you...?

1096	Angela:	I was good at this stuff back then.
1097	Romina:	… oh doing a Riemann sum.
1098	Magda:	Using the midpoint.
1099	Angela:	This is .5 though, right? X is .5? No?
1100	Magda:	Yes. No, change of x is point 5.
1101	Angela:	And.. wait that doesn't help, that messed up my chart. So why are we
1102		using point 5… oh
1103	Magda:	Because it's the mid.
1104	Angela:	Because it's the midpoint? I'm sorry. And that's the area. [Angela is
1105		making a chart of values of x and A. Her first entry is .25 and .03125.]
1106	Magda:	Yes, that's the area.
1107	Angela:	So we're doing… if the change is .5 then we're doing .75, right?
1108	Magda:	No, no no, the change is always point 5.
1109	Angela:	Right. But from .25 to .75 it's .5.
1110	Romina:	I don't have any clue what you guys are doing and I'm sitting right here.
1111	Angela:	We're just finding the areas of rectangles
1112	Magda:	Rectangles.
1113	Romina:	The whole rectangle?
1114	Magda:	Finding the area and like …our change in x is .5. So that's our change in x
1115		right here.
1116	Romina:	Why is it .5 if it goes over… oh.
1117	Magda:	And we're using the midpoint which is the midpoint between .5 which is
1118		.25…to figure it out. So .25 into that function.
1119	Romina:	But you're not doing… It looks like you're using the rectangles on the left.
1120	Magda:	Well no, we're using the midpoints, we're using the whole thing.
1121	Romina:	It's not… it just doesn't look like that OK.
1122	Magda:	But what are you saying?
1123	Romina:	When you're saying midpoints aren't you supposed to be taking …
1124	Magda:	[inaudible]
1125	Romina:	Which ones are… oh that graph. Midpoint. OK, now I… it just looked
1126		funny. It looked like it was.. I don't know. And you're going to add all
1127		those together to get the area.
1128	Magda:	Um hum.
1129	Angela:	How far are we going up?
1130	Magda:	Like 3.
1131	Angela:	OK.
1132	Angela:	So this is 2.75.
1133	Magda:	Uh hum.
1134	Angela:	Oh, that was wrong. [laughter]
1135	Romina:	All right now what are we doing after this?
1136	Magda:	We kind of want to prove…
1137	Romina:	That's our intro to area and calculus right there.
1138	Angela:	Yeah, we did something.
1139	Magda:	1, 2, 3.
1140	Angela:	You really should label these things, Magda.

1141 Magda: Alright so, the area, like of the little things, the little things, when you add
1142 that all up, that's our estimate of the area between the interval of 0 to 3.
1143 Angela: Um hum.
1144 Romina: Hold on, what are we doing , right now, are you finding the area
1145 underneath
1146 Magda: X squared.
1147 Romina: OK, and then...
1148 Magda: OK, can you add that up so, that's our estimate of the area.
1149 Romina: How does that connect to the integral?
1150 Magda: Well now, this is our integral.
1151 Romina: Yes.
1152 Magda: of an x^2 function, so this is f(x) = 1/3 x^3, right, and basically this is...
1153 we're finding the area on the interval of 0 to 3. So using our like theorem
1154 of calculus, or... just substitute that in. 3 cubed is 27 divided by 3.
1155 Romina: But why , why does that work?
1156 Magda: Just because it does.
1157 Romina: I think that's what he's asking, we have to know that.
1158 Magda: Hold on, well first we're explaining what it means,
1159 Romina: OK.
1160 Magda: and then why it works. Isn't that the last question.
1161 Romina: What is the theorem for,
1162 Angela: That's the area.
1163 Romina: And why the theorem is true.
1164 Magda: So this is the estimate. So hold on 3^3 is what, 27, divided by three is 9,
1165 and it's nine,
1166 Angela: So this is very close, yeah, we did something.
1167 Magda: OK, so that's like the estimate.
1168 Angela: Ok, now explain what you just explained to her to me, because I was
1169 typing in numbers and I missed it.
1170 Romina: No, we didn't get anywhere, that's what we have to do like, why does that
1171 like, why does that work?
1172 Angela: This is this, right.
1173 Magda: Yes.
1174 Angela: OK.
1175 Romina: Yeah, that's it. When we plug in, 3, we get the area,
1176 Angela: We get 9, which is close to.
1177 Romina: Why does that work, that's the question.
1178 Angela: [writes F(x) = 1/3 x^3 on the paper.] Well it's there already.
1179 Romina: Rewrite it. Forget the teacher's manual. This is the part where we're
1180 [Magda draws a new figure]
1181 Magda: So basically what we're doing here is we're doing the distance, isn't that
1182 the distance right here.
1183 Romina: I don't know if that's right though, I think but...
1184 Magda: I don't know...
1185 Romina: I think it's speed acceleration, distance.

1186	Magda:	Hold on, I'm not saying that, I'm not going into that...I'm just drawing
1187		this line,
1188	Romina:	What are you asking me then?
1189	Magda:	I'm trying to see, I mean, what f(x) I mean F(3) – F(0)
1190	Angela:	Whatever, that's close.
1191	Magda:	Is this number minus this number. [Magda points to two points on the
1192		graph she has just drawn.] So what does that give us? I know that gives us
1193		the area, because we know that.
1194	Angela:	[whispering] but why...
1195	Romina:	Here, let me look...[Romina picks up the Foerster book.]
1196	Magda:	Well basically what we're doing here is%OK, basically what we're doing
1197		here is taking the area... OK, give me the numbers...
1198	Angela:	Point 3.
1199	Magda:	Ok, it's 1, 2, 3, 4, 5, right, it's like point 3, right? so it's here. I mean, it's
1200		at .25 OK, so it's here,
1201	Angela:	No, it's .28.
1202	Magda:	So it's here...
1203	Angela:	It doesn't help, it's tiny.
1204	Magda:	This is one, so at .25, it's point 3 so it's somewhere here.
1205	Angela:	OK.
1206	Magda:	Then at .75 let's see this is one. it's like a third way up.
1207	Angela:	Um hum.
1208	Magda:	then at 1.5.
1209	Angela:	1.25.
1210	Magda:	1.25.
1211	Angela:	It's .78, it's like here, right?
1212	Magda:	Then at...
1213	Angela:	1.75.
1214	Magda:	1.75 it's 1.5.
1215	Angela:	1.5.
1216	Magda:	So this is two. [making a mark on the *y* axis.] So this is like here.
1217	Angela:	OK, and 2.25 it's 2.5.
1218	Magda:	2.5?
1219	Angela:	Um hum.
1220	Magda:	OK.
1221	Angela:	And then it's 3.7 at 2.75. ... maximum, I'm sorry.
1222	Magda:	So basically, if you plot those points you get this.
1223	Angela:	Um hum...
1224	Magda:	That's what it looks like, right?
1225	Angela:	That's that? Is that that? [Referring to the previous graph that Magda drew
1226		f(3) and f(0) on.]
1227	Magda:	Well, yes, that's what it looks like.
1228	Romina:	[whispering] Speed velocity, distance.
1229		[laughter]
1230	Angela:	You can explain that one.
1231	Romina:	No, I.

1232	Magda:	Well no, we're wrong, because area is change of x multiplied some by
1233		some height, so it's not like a point on the axis, so scratch that [she
1234		crossed out the plotted points]
1235	Angela:	Why did we scratch that?
1236	Magda:	That's not the y, the y is the height. You know what I'm saying.
1237	Angela:	Right…
1238	Magda:	So basically I'm not graphing this, basically I'm supposed to graph this,
1239		against the h?
1240	Angela:	But doesn't that work, though? If this is x and this is y (pointing to the
1241		column of area values) doesn't that work?
1242	Romina:	That would be the integral, right? This book is not very…
1243	Magda:	No, no, because if you actually think about what we know already, then if
1244		this is.. the integral is supposed to the x^3 function, 1/3, hold on, is it? 1/3,
1245		if you put 2.75, what is that?
1246		[Angela uses the calculator.]
1247	Magda:	Divided by three?
1248	Angela:	6.9.
1249	Magda:	So it doesn't work .
1250	Angela:	It doesn't work.
1251	Magda:	So that's not what… that's not the y.
1252	Romina:	This… you… you can't have the x and the y, this x and the A on the same
1253		graph. You can have these two on the same graph, can't you, and this is
1254		your… the integral.
1255	Angela:	But if you plotted this, wouldn't it be, like, but thinking in terms of like
1256		speed velocity, acceleration,
1257	Romina:	We obviously don't know that, so we should stop using that!
1258	Magda:	Hold on, hold on…but that's the area. So this is this…
1259	Romina:	I don't…didn't you do like…If you do the x… then you did the… what
1260		did you multiply by.
1261	Angela:	That.
1262	Magda:	We're saying here that on the interval…
1263	Angela:	And the function was x squared.
1264	Magda:	…from 0 to .5,
1265	Romina:	What do you mean… [she writes $x^2 \cdot \Delta x = A$.]
1266	Magda:	The area…equals .03125, right?
1267	Romina:	Isn't x^2 our height here?
1268	Angela:	Yeah.
1269	Romina:	So wouldn't we have to graph x and x^2 on our g' and then we have…
1270	Angela:	Yeah, that's this graph, that's what that is…
1271	Romina:	So what are we trying to graph on the same graph? And then we're… I
1272		don't know what you're trying to do then.
1273	Magda:	Hold on, oh my god, this is the area.
1274	Angela:	Yes, we've established that.
1275	Magda:	And then using the integral, is supposed to give you the area on that.
1276	Angela:	Isn't that what we just did?
1277	Magda:	Hold on.

1278	Angela:	And you said it was wrong.
1279	Romina:	Magda, try to explain it to us instead of being like just sh sh. We don't
1280		follow you if you're four steps ahead.
1281	Angela:	[inaudible] Come on, help us out Mags. You're the accounting major. You
1282		take math.
1283	Romina:	.03125 is the area of...? The first?
1284	Angela:	That first little section there.
1285	Romina:	See, we need...
1286	Angela:	That whole rectangle thingie.
1287	Romina:	OK.
1288	Angela:	Yes, rectangle, not rectangle thingie.
1289	Magda:	So now the point at ... isn't it at point .5, is supposed to equal the area, so
1290		this is .5, so this is supposed to equal....03125... I mean, around.
1291	Romina:	Yes.
1292	Angela:	That's what I'm doing.
1293	Magda:	Point 5 [cubed] divided by 3...
1294	Angela:	.41. .041.
1295	Magda:	Well, it's because it's an estimate.
1296	Romina:	But OK, I understand what you're doing but, then, don't we, to take the
1297		integral of this, blah blah blah blah it's this, right.
1298	Magda:	[inaudible]
1299	Romina:	My question was not important.
1300	Magda:	Hold on, hold on, hold on. I think I got it now.
1301	Angela:	We have clean paper.
1302	Romina:	Yeah.
1303		[laughter]
1304	Angela:	...to keep crossing things out.
1305	Magda:	At .5, the area is..
1306	Angela:	The area is .03.
1307	Romina:	Is this the integral? that you're.
1308	Magda:	The same thing...
1309	Angela:	Are we drawing the same thing we drew 2 minutes ago?
1310	Romina:	Yeah, and we can't go on...Magda, my only question, is going to be really
1311		basic, just listen to me.
1312	Magda:	OK.
1313	Romina:	You know what I... what function is this? [she draws a graph with area
1314		filled in underneath.]
1315	Magda:	Of what though?
1316	Romina:	What is this.
1317	Angela:	F of x?
1318	Romina:	This is my g prime?
1319	Angela:	Integral?
1320	Magda:	What function?
1321	Romina:	That's what I don't get...
1322	Magda:	OK, f(x) is x^2 that is our example.
1323	Romina:	OK. [Pointing to the graph she just drew] Is this our f(x)?

1324	Magda:	No.
1325	Angela:	No. x squared is our f of x.
1326	Romina:	This is our integral.
1327	Angela:	Yeah.
1328	Romina:	But see You know how when we draw the integral, and then we do the
1329		area underneath…
1330	Magda:	Yes.
1331	Romina	Isn't that what we do with our g prime?
1332	Magda:	No, it's not the area underneath the integral, it's the area underneath the
1333		function.
1334	Romina:	But when we.. I understand that but I'm saying that's how we've been
1335		drawing it so every time we say this is our g of x…this really is, this is…
1336		if you figure out the area of this [she traces the area under the graph] this
1337		[pointing to the graph that she has drawn the area under] would be the
1338		integral of our g of x [she writes integral symbol g(x)]
1339	Magda:	Correct.
1340	Romina:	So why do we keep drawing this [She traces the graph] and trying to
1341		figure that out [she traces over the area]
1342	Angela:	Were not doing that this is the F of x.
1343	Romina:	So my question is how does this [She traces over the area she has filled in]
1344		change into that other graph that's the part I'm not getting.
1345	Magda:	What do you mean.
1346	Angela:	I think that's what we're trying to get too.
1347	Magda:	This…
1348	Romina:	So we're all lost at the same spot
1349	Angela:	I think so
1350	Romina:	Do you understand what I'm saying - how does this OK, this is our f of x,
1351		or our G prime this is my G prime [she traces over the graph again] and I
1352		want to figure out the area under my g prime to get to my G yes.
1353	Angela:	When did we switch to G's.
1354	Romina:	That's what the problem was in the book.
1355	Angela:	OK. I'm like…
1356	Romina:	G is the integral and G prime is the derivative.
1357	Angela:	OK.
1358	Romina:	OK no it's the function and not the derivative of the function
1359	Angela:	I know what you meant.
1360	Romina:	Yeah OK. This is our g prime, this is our g, when I get all this [she runs
1361		her pen up and down across the area under the graph] how do I graph this
1362		to get my I don't know I don't even know whatever.
1363	Magda:	Ok well this is your f of x.
1364	Romina:	OK, so we went from…[she draws a new graph, the graph of a parabola]
1365		so I want to go from here to here [She marks off points on the x axis
1366		labeling them a and b] and when I graph my integral [she draws the graph
1367		of a cubic function]

Romina: I don't get how this [she fills in the area between the graph of the parabola and the x axis] turns into all this [she fills in the area above the cubic graph)

Magda: No no no, no area.

Angela: The line.

Magda: The point.

Romina: That do you mean equals this point [she draws a point on the cubic graph above the location of b on the x axis.] you mean this point is the area of all these together.

Magda: Yes. Well this point...

Romina: Say this area here, this area is 2 [she traces over the area between the x axis and the parabola between the points at $x = a$ and $x = b$)

Magda: OK. And this areas is 2...

Romina: And this starts at –1... is this 1? [Pointing to the point above the point at $x = b$.]

Magda: Well.

Angela: Is it?

Magda: Whatever b is.. oh this point is 1?

Romina: This point is equal with my b and this point is equal with my a.

Magda: OK.

Romina: And the area of this is 2.. so this is negative one, say [she points to the lower point] does this have to be 1.

Magda: Well, 1minus1 minus...is, yeah, um hum... Correct 1 minus minus... is a plus, correct.

Romina: So each point on this [she makes points on the graph of the cubic function] is like a really skinny rectangle kind of [she makes little rectangles between the x axis and the parabola]

Magda: Yes and that's what I was saying here.

Romina: But I wasn't getting you and that's why I wanted to ask this.

Magda: Yes.

Romina: All right now go.

Magda: Well basically what we're graphing here is the areas at point 5 the area is that and at 1 the area if that [she points to points she has just drawn above the x axis at x = .5 and x = 1] and it kind of builds up so when you the... get to the final point you get that 9 [she draws a third point to the right of the first two at $x = 3$, with a greater y value than the first two points.]

Angela: And this graph [pointing to what Magda has just drawn] is that? [pointing to 1/3 x cubed on the graph paper drawn earlier]

Magda: Is that, yeah...[pointing to the symbols F(x) = 1/3x^3] so you're kind of like stacking it up.

Magda: Yeah.

Romina: So you're just putting it on top of each other%

Magda: Um hum.

Romina: Yeah, OK...so I think we know what the integral is.

[laughter]

Romina: I think we've beat that to the ground... why does that.. like why does that math work.

Angela: And on to the fundamental theorem of calculus.

Romina: No I mean well then.

Magda: [inaudible]

Romina: ... the a and the b that's the fundamental theorem of calculus.

Magda: Well then of you take like the area between - not of this graph of course - like the point between here and here that's going to give you this little area right here [pointing to the graph on the graph paper]

Angela: Approximately.

Magda: Approximately.

Romina: The fundamental theorem of calculus is just an easier way...

Magda: No, look.

Romina: ...to do the integral it's like the definite integral, right?

Magda: Well no because...

Romina: It's a way to figure it out...

Magda: ...you've got to take the integral to figure out the actual area

Romina: Yeah.

Magda: It's not an easy way of taking the integral because you have to take the integral anyway you know what I'm saying.

Romina: Yeah, I... so you're saying that that we didn't discuss what the fundamental theorem of calculus is.

Angela: No.

Romina: We didn't.

Magda: Will technically we did.

Romina: All I thought we did because I thought were moving on after that.

Magda: Yeah because if you think about it if OK then you have this point.

Romina: We don't even know we discussed the fundamental theorem.

Magda: We did, OK.

Angela: We did?

Magda: Yeah.

Romina: Because I thought we're done with the theorem part like what it is... are you not done?

Angela: I am done.

Romina: Really... Are you not done? Angela, for all I know we could have just breezed right over that part.

Angela: what we just did that's what that is right... that this area is this graph [She points to the graph of the cubic function on the graph paper.]

Magda: Yes.

Angela: OK... I'm hearing voices... and that's what the fundamental theorem of calculus is.

Magda: And this would be .03125... hold on a second...this will be .03125 and this will be .03125 plus .28125.

Romina: Yes.

Magda: Yes so we accomplished that

Romina: I am with you there, but does that explain the fundamental theorem of calculus.
Magda: Well technically yes I guess.
Romina: Well the student should be able to jump from this point to that conclusion now why does it work.
Magda: Well if you take that and say you want to the integral between .5 and 1
Romina: You just…
Magda: Of the F of x function right so you take this number and you take that so you'll get…
Romina: Scary…Ok, so why does it work.
Angela: That's f of a, right….I'm wrong…I don't know…forget what I just said…
Romina: What do you mean? No, ask.
Angela: It doesn't make sense to me. I'm confused.
Romina: What's not making sense?
Angela: Nothing I'm OK.
Magda: Well the slope of that.
Romina: Angela, you know how we always say I get it,. and then they ask us questions and we don't know… Just ask us a question then.
Angela: I'm ok I was just going off on a tangent because I have…
Magda: Well the slope between this would will be f(1) minus f(.5) over. 5
Angela: Over the change in x.
Magda: That will be the slope of this.
Angela: Yes.
Magda: Of our "integral" (signals with quotes with her hands)
Angela: Right.
Romina: Which will be the derivative of our integral.
Angela: Which would be the…
Magda: Which would be the…
Romina: Which would be our g prime.
Angela: Wouldn't that be the area…isn't that the area like Romina just drew here… cause it's like.
Romina: You just told me slope was derivative.
Magda: Yes slope is derivative.
Angela: Isn't that the same thing? [inaudible]
Romina: So if that's our integral, we just figured out of the slope of the integral that would be the derivative…. Yeah, We're not doing well.. Am I completely off?
Magda: I don't know what you're asking.
Romina: This what you did right here didn't you just figure out the slope from here to here.
Angela: Yeah.
Romina: And isn't this our integral.
Magda: Correct.
Romina: So the slope of the integral would be the derivative of the integral would be the function we started with% I just didn't understand what you guys for doing that's why I said that.

1504	Angela:	It's like what you just did here (pointing to Romina's graph) that's what
1505		that is...this is 2 and that's going up 2 and that's why it ended up at 1
1506		right... I'm thinking in very elementary terms here
1507	Romina:	Oh boy.
1508	Angela:	Yeah? No?
1509	Romina:	This is drawn...
1510	Magda:	Do you have that book with the g's in it.
1511		[Pantozzi enters the room]
1512	Romina:	Can I just ask did the first group go in a completely different way.
1513	Pantozzi:	I didn't watch most of the first group just as I didn't watch most of this.
1514	Romina:	You guys did.
1515		[laughter]
1516	Pantozzi:	Their lips are sealed.
1517	Romina:	I feel our group can't really work apart we're not the same in parts.
1518	Angela:	What?
1519	Romina:	If everyone else was here we wouldn't be doing this.
1520	Angela:	Aren't the 3 of us working here together
1521	Romina:	The whole group I mean whole group.
1522	Angela:	If I had to work on these by myself I'd be in a lot of trouble.
1523	Magda:	OK so what are we saying.
1524	Romina:	I just asked you a question% I had no idea what you guys were doing.
1525	Angela:	how about we use this to explain that... that should be our first part we
1526		need to organize ourselves a bit better it's driving me crazy.
1527	Romina:	I think we have the integral part down what the integral is all that stuff.
1528	Angela:	OK this and this can go together.
1529	Romina:	You can throw this in.
1530	Angela:	OK next step.
1531	Romina:	What the theorem means.
1532	Angela:	Isn't that we just did.
1533	Romina:	Yeah that we just did% Ok, and what the theorem is for.
1534	Angela:	No I think we just did what it is for.
1535	Romina:	To find the area%
1536	Angela:	Right, that is what it's for.
1537	Magda:	Hold on
1538	Angela:	That it means is that [pointing to Foerster textbook]we can't copy that
1539		because I don't plagiarize.
1540	Romina:	Angela, I'm going to hit you, we're not plagiarizing it... obviously if we
1541		don't understand I can't plagiarize it. OK, what the theorem means can we
1542		do that like whole thing about the theorem is as our intervals reach zero...
1543	Angela:	What?
1544	Romina:	The Riemann sum, as our intervals reach zero
1545	Angela:	Getting smaller?
1546	Romina:	OK can we get that far.
1547		[laughing]
1548	Angela:	Words like get smaller that I understand.
1549	Angela:	Is that what it means Magda? (Magda looks)

1550	Magda:	[inaudible]
1551	Romina:	That's what an integral means but the fundamental theorem of calculus is
1552		an integral from a to b.
1553	Magda:	Yes. it's like on a defined area.
1554	Romina:	OK.
1555	Magda:	OK.
1556	Romina:	So what it means is if we take the Riemann sum from a to b as the
1557		intervals get smaller
1558	Magda:	It becomes more accurate.
1559	Romina:	It becomes the integral and as it reaches zero and That's like the whole
1560		limit thing if something is going to reach zero you can switch it and it's b
1561		– a.
1562	Magda:	Um hum.
1563	Romina:	OK.
1564	Angela:	I missed that.
1565	Romina:	Remember the limit as h approaches zero.
1566	Angela:	I'm telling you, I don't remember
1567	Romina:	The limit...as h approaches infinity,
1568	Angela:	That looks familiar to me but I don't know what it means though.
1569	Romina:	No that's wrong as h approaches zero.
1570	Magda:	The intervals get smaller and smaller.
1571	Romina:	Does that% am I on crack here.
1572	Magda:	How about you use slope.
1573	Angela:	OK you mean they're just getting smaller the change in x%
1574	Romina:	Yet to the point that they're not actually squares they're just [she motions
1575		up and down with her hands]
1576	Angela:	And what about.. blah blah blah this making it more accurate like making
1577		it not being a rectangle but a trapezoid.
1578	Romina:	No that's with a Riemann sum remember they did first they would have
1579		these all even with the left and the then they centered it and then we got
1580		really high powered and use the trapezoid did trapezoid come first.
1581	Angela:	OK. that's just what we did, OK.
1582	Romina:	[inaudible]
1583	Magda:	Basically that's why it works because it isn't that cause like [looking at the
1584		book and pointing]
1585	Angela:	You need to complete a sentence first for us to understand.
1586	Magda:	This will cancel that and this will cancel that and x will cancel that you
1587		know what I'm saying, then you'd be left with.
1588	Romina:	B minus a.
1589	Magda:	Yeah.
1590	Angela:	All those steps in the middle don't count .
1591	Romina:	So we do have what it is for what it means and why it's true.
1592	Angela:	Why it's true?
1593	Pantozzi:	I sat down because you have about 15 minutes left on the tape.
1594	Romina:	We haven't gotten very far for you.

Pantozzi: That's not it like I said with the other group I don't know what you talked about, I gave this to you as a task to get you talking not for you specifically to answer this isn't a test of what you remembered or anything like that... so if you talked.. like I heard you say did we even talk about the fundamental theorem of calculus for the last hour... I mean even if you didn't if you talked around it that would be great stuff for me in terms of this research also so there is no problem there...um...reading my question now and listening to you for the last 10 minutes or so, perhaps I should give asked what does the theorem say.

Romina: Isn't that kind of like we did.

Pantozzi: Or maybe that's what it means, I don't know.

Angela: That area is that.

Romina: I don't get like "what it means" is that just stating the theorem.

Pantozzi: See I'm not sure what I meant by what I said either.

[laughter]

Romina: If you don't know...

Angela: You want us to directly answer each question

Pantozzi: Well sometimes people will say a theorem like you can read a sentence to me.

Romina: That's what I thought it means...

Pantozzi: And not understand what it means...so when you're going to meet with the student tomorrow who is taking calculus now, and wants to know about this what might you say to them first to help them.

Angela: We like started with the graph.

Romina: But probably tell them what the book says.

Magda: We'd start with like saying that, like a simple graph.

Angela: ...actually counts for something.

Pantozzi: OK , this is something you've talked about for a while,

Romina: Yeah...

Pantozzi: So for the last 10 minutes or so you can pretend I'm the student or pretend one of you are the student and just go through whatever you want to present to them just to summarize.

Angela: I can be the student.

Romina: OK, so you know what the fundamental theorem is, I mean you know what the equation states.

Pantozzi: I've seen the equation.

Romina: SO we have that.

Magda: [inaudible]

Pantozzi: Like I've seen that I've read that. [I point to the theorem in the Foerster textbook.]

Romina: OK. we went a lot of places with this.

[laughter]

Pantozzi: I know what a derivative is.

Angela: Well we basically did that but we made it with a graph.

Romina: The first thing we did was we took the function.

Angela: We used f of x, x squared as our function.

Romina: And essentially we talked about what the integral was, how we want to find the area underneath our x squared from point we designated points from like a …

Magda: From like one to three, that's what it was.

Romina: So we did that and do you know what the Riemann sum is?

Pantozzi: Yes.

Romina: OK.

Angela: We did that.

Romina: So we took a Riemann sum underneath that area and then basically we… what the integral is is stacking on the each area% under.. yeah I don't know where that went…

Angela: Points.. there you go…

Romina: So do want to explain that? you wrote it.

Magda: Well basically what we did is that we figured out that at .5 the area would be .03125 and basically that is doing the change of x which in our case was .5 times the height which would be the.

Angela: [inaudible]

Magda: …y if you plug it into here [points to the equation f(x) = x^2] and that's our area, so at .5 that would be that [she points to the point on her graph at .5, .03125] so at .5… and at one you would just add this and this together and then you just keep going.

Angela: Keep adding.

Magda: Keep adding it up and then you get to the integral.

Pantozzi: You get a graph?

Magda: Yes which is the integral of the f of x.

Pantozzi: OK.

Romina: So then we went on to.. so if we add up all those areas right there, we get our area from… did we start at zero? Zero to 3.

Magda: To 3 which equals 9 like which is if you actually take the function you get that.

Angela: It's right down there.

Magda: If you take that.

Magda: But with our estimates how we're showing it we're doing with the area of the rectangles it came out to 8.937 which is like the estimate and so basically and then as you make your change of x smaller and smaller will become.

Angela: More accurate.

Magda: More accurate… which is actually what the integral is.

Romina: So then if you take, that's kind of sloppy that was our first but if we have a graph and we want to know the area from a to b what you basically do is% and this like after we know what a Riemann sum all that so we have an integral, you know what an integral is… you take the integral of all of this [from the left up to b] of all of b and then you take subtract the integral of a which is all of this, then you know exactly the integral from a to b and that's the fundamental theorem of calculus.

Angela: And it took that long for us to figure it out.

Pantozzi: This is the same question I asked the other group near the end given that you've just been talking about this for a long time there's going to be a second session where after I've watched the tape I'm going to see what ideas you guys brought out I'm going to have, going to bring some more things to that here I just gave you some books and said go ahead talk about it in the second session I'll bring some things that specifically you might be interested in knowing... so after you've talked about the fundamental theorem of calculus for this amount of time what questions do you still have about it if any.. like what might you want me to bring to the next section... next session... to either help you explain it more or to help you understand it more.

Romina: I'm not getting... go ahead.

Angela: May be a specific problem with numbers or like what you were saying before speed velocity acceleration kind of thing.

Romina: Could you just answer that really quick?

[laughter]

Romina: Speed velocity distance acceleration you know how one's a function one's the integral of the function...one's the derivative of the function... could you just tell us which one's the function and which one's the derivative and which one's the integral.

Pantozzi: Velocity is the derivative of position or distance

Romina: Position...

Pantozzi: Or distance.

Angela: Distance.

Romina: OK, and acceleration is the derivative of velocity.

Magda: [inaudible]

Angela: See I said that.

Magda: What is it again?

Pantozzi: It's position velocity acceleration (Pantozzi: Moves his hands in a downward vertical motion) position is the first thing, where you are, and the derivative of that is the velocity and the derivative of that is acceleration.

Magda: So we were right, we were saying we had velocity.

Angela: [inaudible]

Magda: We had that...

Romina: What we did...for a long time.

Pantozzi: So you still have some questions about that issue velocity acceleration.

Magda: But we didn't really know what acceleration was.

Romina: No we didn't know what velocity was.. I get...

Angela: What was the formula for that?

Romina: All that stuff we explained to you that I understand about it but.

Magda: That more is there to it?

Romina: Yeah, like what?

Magda: This is definitely like the big the most important thing but like what else do you like is there to it?

Angela: Like specific problems would be the only%

1733 Romina: Why is it true like?
1734 Pantozzi: So that's something you didn't get to...
1735 Angela: We were supposed to figure that out.
1736 Romina: Yeah like we just hit a rut like we couldn't really I understand integral I
1737 understand all the stuff under... but I just don't see why the uh...
1738 Magda: But isn't this kind of why? (pointing to the book)
1739 Romina: Yeah, I understand that but%
1740 Angela: Isn't what why we were doing like why isn't doing it out like that and
1741 plotting it out and figuring it out ... isn't that why.
1742 Romina: I don't know...
1743 Pantozzi: I don't know what to say...
1744 Romina: What more do you want for us to answer this?
1745 Pantozzi: I can't answer that question because I didn't watch the whole thing that you
1746 did.
1747 Romina: You're going to beat yourself over the head when you watch this
1748 [laughter]
1749 Romina: [inaudible]
1750 Angela: [inaudible]
1751 Pantozzi: Well as I said at the beginning the reason I'm researching this is that this
1752 is the fundamental theorem of calculus and they name it that for some
1753 reason and there's some interest in if you've if you've learned all the
1754 separate things what happens when you try to you know you've mentioned
1755 integrals you mentioned limits.. I don't know if you mentioned derivatives
1756 at all in your conversation.
1757 Angela: Sort of.
1758 Romina: That's where we sort of got into problems I understand integrals and the
1759 limits
1760 Pantozzi: Um hum.
1761 Romina: And I understand that the derivative of the integral would be the function
1762 and the integral of a function is that just didn't make sense but but I don't
1763 see how they're all tied together too much [looks to the other two students]
1764 do you understand what I'm trying to say?
1765 Pantozzi: You said the integral of a.
1766 Romina: No that was bad.. like the derivative of an integral would be the function
1767 [see motions down with her hands] and like the integral of the derivative
1768 would be the function.
1769 Magda: Um.
1770 Romina: I don't know why I was trying to say that but I see how that process works
1771 see that
1772 Angela: [inaudible]
1773 Romina: But I don't understand it all.
1774 Pantozzi: I'll end this way then and then we can chat. suppose you had to put a
1775 bumper sticker on the back of my car about the FTC, perhaps you
1776 wouldn't put it on your car... is it possible to put it on a bumper sticker?
1777 Or um...
1778 Angela: How big is the font?

Pantozzi: Good question.
Romina: See, I understand that when I look at it. (Referring to the book.)
Pantozzi: Yeah?
Romina: [inaudible]
Angela: Well I didn't when I looked at that I had to relearn this because it's been so long since I've done it I'm not going to say I understood it
Romina: Isn't this just what it is?
Angela: Now looking at it I get it...
Pantozzi: Well I can't answer that right now.
Angela: [inaudible]
Pantozzi: I could say yes, I could say no.
Romina: I mean like
Pantozzi: I'm interested in what you thought...
Romina: We probably have a shallow understanding of it.
Pantozzi: Why do you think that?
Romina: Just wait until you watch the tape.
Pantozzi: Well why do you think that right now. I didn't watch the tape.
Romina: Because it can't be that simple, it can't just be the graph from a to b... I think it is.
Angela: I think we learned why at one point didn't we? I remember knowing things.
Romina: We figured out why we didn't use the textbook in class.
Pantozzi: [inaudible]
Romina: We did... did we not go through this...We tried to understand this for a good like half hour.
Angela: Who needs a textbook when you got Pantozzi?
Romina: Yeah, I was really badgering.
Pantozzi: So let me end this way this time and I'll really end this time... the student hasn't taken this section yet – the way I posed it in the task is that they already took it and want some help with it so hey, you took calculus, and I'm supposed to learn the fundamental theorem of calculus, what's that going to be about? Now I'm going to leave and I don't know how much time there is on the tape, but answer that question in a short, bumper sticker sort of thing and the font can be about this big.
Magda: Basically what I would say...
Angela: Yeah, but we could say this and this... but generally speaking what is it.
Magda: The fundamental theorem is I would say its taking the integral on a defined interval.
Romina: Function.
Magda: Taking the integral of a function on a defined interval.
Romina: I like it.
Angela: Taking what?
Romina: Having never taken this class, would they know what an integral is, is that?
Angela: Well I'm sure they would if they're going to be learning the fundamental theorem next.

1825 Romina: Well I think that's good for me.
1826 Angela: Should we write that down for him.
1827 Magda: [inaudible]
1828 Angela: OK.
1829 Magda: I don't know, right.
1830 Romina: That's the fundamental theorem...it's kind of simple for... isn't it?
1831 Angela: What?
1832 Magda: Well basically what you're doing when%
1833 Romina: No I agree with you, that's why, I agree, but I just don't always, uh... I
1834 don't know how to go about...
1835 Magda: I would probably draw a graph and be like.
1836 [laughter]
1837 Magda: Do what we did here draw a graph.
1838 Angela: Yeah but we're just putting something on a bumper sticker you can't just
1839 draw a graph you have to say something right.
1840 Magda: I'd be like the area in green is this minus the area in blue.
1841 Angela: Obviously he's trying to get us to articulate everything we just did in, like
1842 a few short words.
1843 Romina: This is what I would write. [She writes integral sign b – integral sign a]
1844 Angela: That's what you're going to say to somebody.
1845 Romina: I don't know what else to say that makes sense – and I'd give them the
1846 picture.
1847 Magda: You need an f there [she adds a f to Romina's equation]
1848 Romina: OK, if we got all like.
1849 Angela: So this is our bumper sticker.
1850 Romina: No, but it's not, it's a point on the f. Oh yeah, you're right, you're right.
1851 And I would draw them that picture.
1852 Magda: Hold on, hold on, then it wouldn't be the integral?
1853 Romina: I know Magda, it just made sense to me. I'm just kidding.
1854 Magda: Well then...
1855 Romina: You guys took me all serious.
1856 Angela: OK, this is our bumper sticker, what does it say?
1857 Romina: No, but the area, the integral at point b is that, minus the integral at point a
1858 is that, equals integral from point b to a...how else would you write that?
1859 It does kind of that does make sense to me, that's why I'm not a calculus
1860 teacher.
1861 Angela: The book...
1862 Magda: That would be *a* and *b* [writes integral of f(x) from a to b]
1863 Romina: Oh, yeah yeah yeah.
1864 Magda: That's how I would write it.
1865 Romina: I have no idea.
1866 Magda: And that's what it equals.
1867 Romina: And then draw.
1868 Angela: OK.
1869 Romina: I just don't think its...
1870 Magda: And then that would equal F(b) – F(a).

Angela: But how do we say this not using like math language and graphs though?
Magda: It's the area under...
Romina: The integral from negative infinity to b of f of x minus the integral of negative infinity...
Angela: No, no, I mean like, I don't know...
Magda: What's the... using position, velocity acceleration.
Romina: Shut up with that, we don't know what it is...can't use that, we don't understand it.
Angela: You're not supposed to be telling the person exactly what it is, he just wants the general idea.
Romina: If someone didn't understand it, I'd draw a graph, and be like, you have a function...
Angela: But that's what we're not supposed to do.
Magda: If they don't know about integrals, I don't think they would be asking us about the fundamental theorem of calculus. What he's saying is...
Romina: In my sophomore year of high school, I was driving with Mr. Pantozzi, I believe it was here, and his car was dusty... and we were talking about... or maybe it was precalculus, I don't know we were talking about calculus and whatever, and I told him, I really don't want to take calculus, and he was like, that was like blasphemy, but whatever, and he goes, well, you'll be fine, and he drew a graph, and he asked how much distance did they cover from here to here, and we like shaded it in, like with the dust, and all that distance, and he's like oh, the area, I'm like yeah, the area that is how someone explained it to me...
Angela: OK.
Romina: And he's a teacher, so I think that's fine that I explain it to someone like that.
Romina: And then we did...someone...and he was like, explain acceleration. He's like if someone starts here, if this is like speed, and this is distance, you go like this, what does that mean
Magda: You're accelerating.
Romina: That's how we learned it, in dust. If you can handle that, you'll be fine in calculus.
Magda: Well that's why you put in real life kind of like.
Angela: Words.
Magda: Terms.
Angela: I'm just a person who likes words.
Magda: And basically like the f(x) would be like the different like accelerations you could be accelerating at like 5, you could be accelerating at 10, and that's going to be like your function, you know.
Romina: What more could we say? ... Call him back in. Tozzi! OK, good because that was just going from bad to worse.
Sergei: One minute left.
Romina: Just wrap it up, we're done.
Sergei: [inaudible]

Romina: That's a wrap, people. Do you remember how you explained calculus to me?
Pantozzi: Do I remember?
Romina: Yes. In your car on the dust?
Pantozzi: [silence]
Romina: He doesn't remember...
Pantozzi: Which one was this?
Romina: We were in your car... we had to have been coming here, where else would we go together?
Pantozzi: [inaudible]
Romina: And you were driving and you did it with the dust on your dashboard. [laughter]
Pantozzi: And what did I draw.
Romina: You drew...
Angela: A graph.
Romina: You drew this first, because I was like, I don't want to take calculus.
Pantozzi: I do remember this day, so don't worry about it.
Romina: I don't want to take calculus you said, well, it's going to be easy, you did this, and you're like, if this is speed, and no... no this is distance or time... time.. I think it's time... whatever, or something like that, and you're like what's happening here, and I said you're accelerating and then you did how much distance did you cover and I said all this... Do you remember this? ... why isn't it good enough for me to explain to you?
Pantozzi: What, just now, you mean, or...
Romina: That's exactly how we would explain it if you asked us what the fundamental theorem of calculus was.
Pantozzi: Did we get that on tape before.
Romina: What?
Angela: Yeah.
Romina: Yeah, oh yeah.
Pantozzi: What you just said before...
Romina: Yeah.
Pantozzi: OK.
Romina: Why is that bad?
Angela: OK.
Pantozzi: Has anyone said it was bad.
Angela: ... language.... I was like how would you say it with words?
Romina: I don't like learning math with language.
Angela: See I can't... I don't think I could do it any other way. It's the way I think.
Pantozzi: That's why I love talking to students, especially you guys, because there's always a difference of opinion.
Romina: Hey, you used to make us write remember?
Pantozzi: Um hum.
Angela: See how that's how I remember things... writing...
Romina: These books are really bad.

1961	Angela:	I can't read math language... I mean graphs help me more than other
1962		things but like just articulating it in regular words is the best way for me.
1963	Romina:	That's regular words for me.
1964	Angela:	Yeah, but without a graph. And without a formula...
1965	Romina:	[inaudible]
1966	Pantozzi:	You guys must have read my dissertation proposal, because that's one of
1967		the things I'm interested in, what representations of the idea do you like to
1968		use... so you said words are good...
1969	Magda:	Symbols and graphs...
1970	Romina:	Yeah, and I'll do words... but words are kind of just the filler, because
1971		you're explaining the graph.. if you sat there with hands folded...and the
1972		fundamental theorem of calculus, say you have a function... you have a
1973		function point a to point b its so much easier to just draw the graph if you
1974		just sat there hands folded if you have a function, a function point a to
1975		point b, from negative infinity... it's so much easier to just do this
1976	Angela:	I don't even mean like that. I mean like using plain simple.