Appendix C: Transcript of Session 2 with Group 1, July 25, 2003

Pantozzi: Whenever I show this calculus thing to somebody, I'm always like, maybe it doesn't really make any sense, maybe it isn't really right, and maybe no one is ever going to pay attention to it... so all that to say, we've been talking about the fundamental theorem of calculus, and the reason I'm doing this research is because I want to write about what it actually means to understand it, and so, certainly all of the things you said last time indicate that you do have an understanding of the fundamental theorem of calculus

Angela: Even me?

Pantozzi: Yes.

Angela: OK.

Pantozzi: Being a researcher and being a teacher is different when I was watching the tape, you know I'd hear you say something, you know, and the other two say something...I'd go, oh oh, oh...as a teacher you'd follow that up with a question but here you have to just, sort of hold back.

Angela: Sit back.

Pantozzi: Yeah, see what happens when you talk to each other. So there was... I watched it on the plane trip to California, ... I just popped it in my laptop and there I was on the plane... nothing else to do...watched it there, watched it a few more times...and suffice it to say I have a lot to write about just from the first session. So what I'm going to try to do in this session is follow it up a little bit, I'm going to try to pose some questions, and I'll leave again sometimes just so, so it's not you talking to me because that changes things, but I will try to be more helpful and at the end hopefully you may feel a little bit better than you did perhaps at the end of the last session.

Angela: (Laughs)

Pantozzi: We're going to do some things with Sketchpad perhaps depending on what you happen to say. I heard there's been comparison... like did my group do something different than the other group? Do they know more than us?

Angela: (waves hand)

Pantozzi: Both groups went in different directions... each group looked at a different aspect of the fundamental theorem, which is why I have a lot to write about. You can pick different things to look at and pick different things to discuss.

Pantozzi: Of course it's always just fun to, you know, watch you go through papers and, you know, make jokes to each other too.

Magda: (inaudible) the simplistic version... like how you went in depth.

Angela: (laughs) No Magda, different, different

Magda: Well there...

Angela: Different, not better.

Pantozzi: OK, so this session will be different you know. from the other one. OK, the aspect of the fundamental theorem I saw you guys focus on last time was…

Angela: We have the evil mac too.

Pantozzi: (Types $\int_a^b f(x)dx = g(b) - g(a)$ on screen.) …and actually the book, I think the book that you were looking at, I think this was *g*.

Angela: Yeah.

Pantozzi: So think what we'll do is start with this and talk about, pose a couple questions based upon what I heard you talk about before and then I'll pose a question and leave.. so OK, so I'll start this session by saying that that is one aspect of the fundamental theorem and that's, there would be more written with it in the book but underneath where it says the fundamental theorem of calculus it would say that , OK? I t would also say that I think it would also say again thinking of the book you were looking at that f is the derivative of g

Angela: Yes. Right?

Magda: Yes.

Pantozzi: So here's, here's how I will start I will pretend to be that student who was in the task last time not for an extensive period… just to ask a question…

Magda: OK.

Pantozzi: So I'm now going to start talking as the student. Since we talked last time, Since I listened to your help that you gave me last time, I took the session, the class session about the fundamental theorem, and I know that was part of it, and what I learned was, had something to do with this: I want to find the area from a to b and of the function f and I know about Riemann sums you guys helped me with that too you talked about that and I want to find the exact area that's under the graph so the graph might be going like this the area between the graph and the x axis if not going down to infinity or something, I was thinking that at the beginning, just, just the area there and what I learned that the fundamental theorem was, well one thing you could do with the fundamental theorem is use it to figure out the area.

Pantozzi: But what you had to do is to figure out the anti… I think they said the anti derivative and I'm not quite sure I get that part but I know that I just said a moment ago I know that if that's the formula then F is the derivative of G so I have to do some sort of formula and then I can get the exact area. I guess what I don't get yet was and I think you guys said this when you're helping me last month like you had an x squared graph you used and an x cubed graph to figure out the area.

Angela: Yeah.

Pantozzi: So I guess what I want to know now that I took my class on the fundamental theorem and had you guys help me a little bit is how they ever came up with this in the first place I know it works, I did my test and I know what to do but I guess I want to help I want to understand where they came up with this idea in the first place…

Angela: The why…

Pantozzi: Like how would they have known that?

Magda: What are you asking like how…

Angela: Why. (laughs)

Pantozzi: I don’t know if I am asking, why,

Magda: X squared, like in simple terms, if your function is x squared…

Pantozzi: I know there is a proof of it, I know that if I’m trying to find the area under x squared…

Magda: Like if you have x squared how do know that you are supposed to go up a power is that what you’re asking?

Pantozzi: Not even specifically that problem, you might help me by using that problem as a specific example but more in general why I have to do this anti derivative thing to figure out area I can’t see why, like it’s my professor said that, and that's the way to do it and I got my problems right but I am just wondering where you know, where that comes from…

Angela: Well…

Pantozzi: And that's a tough question.. I mean the professor didn’t really explain it, but.

Angela: No textbooks today?

Pantozzi: I’ve got a whole bunch of textbooks over there again.

Angela: Good.

Pantozzi: So I know that if I have x squared I'm supposed to do one third x cubed because the derivative of one the third x cubed is that…

Magda: X squared.

Pantozzi: One x squared right, but it just seemed to come out of the blue and so what I'd like you to help me with help me see how this is not just out of the blue, because I hope it’s not, I like to have math make sense. OK, so…

Angela: Well…get a book Magda.

Pantozzi: Yeah, all the books are over there…use graph paper,

Angela: Use black Magda.

Magda: I’m sorry.

Pantozzi: And also what I’m going to do, if during your discussion, you’d like me, meaning the real me, not the student me,

Angela: That was good acting by the way.

Pantozzi: Thanks. You’d like me to do something with the computer, I have the sketch pad and you know I've got a program that might do some things that might visualize it you know with pictures and stuff..

Angela: Umm.

Pantozzi: You don’t know what the program does, you might remember that we used the program in class four years ago but even if you imagine something and you think doing something might help doing something on here might help I might be able to do it so I’ll be a resource for you so I’ll be having multiple personalities.

Magda: (Laughs)

Angela: Can we write down what we have to do just so I can have a better understanding.

Magda: Well I think

Angela: Like if we had to come up with a question.

Magda: How do you know to find the area under the graph with.

Angela: How that works basically.
Magda: No he's asking basically I don't know if that's right to find the area under the graph how do you know that you have to take the anti derivative I think that's what he's asking us.
Magda: Because he told us so… (laughs)
Angela: I'm going to get a book.
[Angela leaves the table.]
Angela: OK how are we doing this.
Magda: I'm going to draw a graph. Hmmm.
(silence)
Magda: Well my thinking of why it has to go to the higher power is because if you're adding up the area underneath OK so you are going here and are going here and going here you are adding up what is under here and you are stacking like this little thing, like, you know more on top of that, so like at this point like here, you have this area and this area, you know?
Angela: That makes sense didn't we say that last time.
Magda: But you know what I'm saying that's why it has to be to a higher power.
Angela: Yeah.
Angela: Does it have to be a higher power or does it just have to be more than… I don't know…
Magda: Because its an integral is to a higher power.
Angela: OK.
Magda: You remember when you're taking a derivative…
Angela: Yeah, OK.
Magda: It's like you have to, it's going to be x… n =1 over n + 1 going to be n.
Angela: I haven't done this in four years.
Magda: Right? That's going to be…
Angela: That's one? What is that?
Magda: That's n.
Angela: OK, It's doesn't look like an n, Magda.
Magda: You know what I'm saying… that you're always going to a higher power.
Angela: OK.
Magda: So if that's what he was asking us then that's why…isn't that what he's asking us, like how do you know to go to a higher power how do you know to take an antiderivative?
Angela: I guess. I don't… I thought we said that last time Mags. I thought you said that last time. You were explaining to Romina like that.
Angela: Is that just the question though, like.
Magda: And if you go back, this point, like this is, this will tell you the slope of that.
Angela: Right.
Magda: How does that tie in? (Magda draws a little tangent segment on her steeper graph, and connects a point in the center of it with a point on her graph of $y = x^2$.
Angela: I don't know.
Magda: This, is one, this is two, this is three…so at four.

Angela: We have calculators.
Magda: At 4 the slope of this, at x equals 4, *m* would be… the slope on this graph is 16 but why?
Angela: (inaudible) oh, OK.
Angela: The slope
Magda: No the slope...
Angela: Here on this graph (pointing to the steeper one) would be 4.
Magda: Yes.
Angela: I mean 16.
Magda: Yes.
Angela: Yeah.
Magda: But why, I don't know, like you know. (Magda has labeled the less steep graph as x^2.)
Angela: Because that's what we were told.
Magda: The thing I remember…
Angela: We need to figure out why this works.
Magda: The thing I remember him doing specifically was when he was like teaching us integrals and the way he explained it to us he was doing it on Sketchpad I don't know if you remember, and he was like dragging it and then adding the area on and then this graph.
Angela: I don't remember I have the worst memory.
Magda: He was like going like point by point like here and this graph like grew and then.
Angela: oh yeah… and here's the next one and it would go up, yeah, OK.
Magda: And then this was filling in underneath.
Angela: Yeah, I remember that.
Magda: But I don't, I know kind of like why it's going to a higher power.
Angela: But why it works out that way?
Magda: And then if you go in reverse why does it.
Angela: Like because like if this (referring to the steeper graph) is this right (filling in the area under the graph of x squared) it makes sense for this to be the slope of that because that is the rate that that is changing.
Magda: Say that again.
Angela: Like if this graph is the area under here it makes sense for like you know for 16 to be the slope at 4 because that is the rate at this is changing like that's rate that the area is increasing you know what I'm saying?
Magda: Mmm humm.
Angela: X^2. I don't know if that is just like pointing out obvious stuff.
Magda: No because that's good because as you go higher
Angela: The slope gets steeper
Magda: But the area,
Angela: Gets larger.
Magda: You're adding more area.
Angela: It's changing at a steeper pace.
Magda: So is that our kind of equation?
Angela: I don't know… I guess.

Magda: Let’s see what the book says
Angela: Nothing… I don’t know I can’t read math its too… dry for me.
Magda: Go write a novel… (inaudible) I’ll read that.
Angela: This is the stuff that we were just talking about right no, that’s with a negative… I think it would be past this...
(They look through the pages in the book.)
Magda: Remember this problem when we were doing it in class.
Angela: No.
Magda: It's if you have two functions and you need to find the area in between you just.
Angela: Oh yeah OK OK OK.
Magda: Take the integral...
Angela: Yeah...
Magda: A way to define integrals.. oh my god here’s the velocity stuff
Angela: Where's Romina...
Magda: (inaudible)
(Silence as they look at the book)
Angela: Is this going to help us?
Magda: No, this is more like how you divide it into intervals blah blah blah (inaudible)
Angela: Simpson’s rule. Next chapter already? The reciprocal function… the population growth problem… this is the people problem. Do you remember this? We definitely did this. Can you sit on the same side as me Magda? Sorry we’re switching Madga around.
Magda: Does this mess you up?
Angela: It probably does.
(Sergei talks)
Magda: We want to switch places so we can read. So she doesn’t have to read upside down.
Angela: Make my head hurt. (laughs)What the heck is this, Magda?
Magda: It's the integral,
Angela: What’s the C.
Magda: Its + C it's where you start your graph.
Angela: Oh yeah that's what I thought.
Magda: The "Initial condition" (she makes quote signs with her hands.)
Angela: I haven’t done math in four years, OK?
Magda: Yeah but like I’m just saying…
Angela: Four years.
Magda: Your initial condition. Remember how we always solved for C to see where the graph would start you know.
Angela: OK, and then go from there.
Angela: Is this...limits and stuff?
Magda: No, this is just ln.
Angela: I don’t remember that…ln.
Magda: It’s just a function, it’s one of those like e to the x, you know like you have, remember when he was explaining to us like compound rates or

something, I don’t know, I just know how it looks.
Angela: When he taught us about like insurance and stuff… and then he told us about how to be a millionaire. and I still have to do that, invest some money. That’s log.
Magda: Log is ln, ln is the natural power…
Angela: OK, this made so much more sense to me in 12th grade.
Magda: Functions and derivatives… derivatives
Angela: We have to be in the anti derivatives.
Magda: What?
Angela: We need antiderivatives, right?
Magda: That’s where is…
Angela: Yeah, it’s just going backwards correct.
Magda: What do you mean?
Angela: Like.
Magda: Yet just like going a power.
Angela: Just going backwards. This is exactly what we did the other day though.
Magda: I know but he is asking us why.
Angela: Why.
Magda: We have to go up to the next power that's basically what he wants to know right.
Angela: I think he means why it works like how it works,
Magda: What you mean like why it works when you take it to the…
Angela: I don’t know, like if you answered one part of it you answer the whole thing no? Why does that equation work? why is that the fundamental theorem of calculus? if we can answer that we can answer everything. I guess.
Magda: Well why it works to me basically going back to this thing, is if you have you know from a to b of F of x our f of x will be x squared right.
Angela: OK.
Magda: So g of b minus g of a
Angela: Umm.
Magda: So say in our case it would be four to one.
Angela: One to four.
Magda: Four to one of x^2 and then that would be one-third 4 to the third minus 1/3
Angela: Minus 1/3 one to the one third
Magda: Right so basically what we’re doing what we’re saying is.
Angela: That is 1/3 x cubed, right?
Magda: This is x^2 this is one third x cubed right
Angela: Um huh.
Magda: So what’s happening, this is one, and this is four (points to graph)
Angela: OK. So that’s up at 16.
Magda: So basically what you are doing, first you're taking this (she circles 1/3 4^3 with her pen in the air) which would be all the area here (she fills in the area under the graph from x = 4 backwards to x = 0)
Angela: And subtracting.

Magda: And subtracting this piece.
Angela: Yeah.
Magda: Here and basically what we said before you know you're adding on the area.
Angela: Um hum.
Magda: So basically you are taking this point right here at four which is this (she draws a point at 4, 16)
Angela: You're subtracting the area from the point at one.
Magda: You know.
Angela: One.
Angela: And you're subtracting the area you piled on up until one.
Magda: Which is the area your piled on when you're starting ...
Angela: Yeah, OK, that makes sense.
Magda: That's why, that's why, that's how I would say why that works (she circles 1/3 4^3 – 1/2 1^3 with her pen in the air) I would say how we got this graph by you know piling the area on on on
Angela: Right.
Magda: And that's how we ended up with this one to the third x cubed graph.
Angela: Okay but why does it just happen to be.
Magda: 1/3 x cubed.
Angela: …that though. Like why…
Magda: Why does it happen to be…
Angela: The integral?
Angela: Is that the right word?
Magda: Yes.
Angela: OK. Why does it happen to be that though? Like I think that's what he's asking us. I think that he gets that it works…
Magda: Um hum.
Angela: …but why does it work, why is it that? Because like we said that all last time.
Magda: So why… why… why does it happen to be going up?
Angela: Like let's just change these, lets change what f is for, for the sake of…cause we keep like.
Magda: Let's use…
Angela: I have to make copies of these…
Magda: F of x could be what do you want it to be?
Angela: I don't know… we have graphing calculators.
Magda: 1/x.
Angela: Sure. So, OK, g would be.
Magda: So g would be, in our case, ln x.
Angela: Let's pick something else… (laughs) stick with x squared, stick with x squared forget it, I was trying to be creative here. OK, why is this the integral?
Magda: Why is this an integral?
Angela: Why does it work out that the integral is... where is that other thing… this point here minus this point here is this area here I understand like you are

adding it but why is it the integral I'm playing teacher… like do you get what I'm saying I know how it works but why?
Magda: Because this function happens to be.
Angela: Why does it happen to be there's got to be a reason right
Magda: Well.
Angela: A mathematical reason... think… why is this true maybe…do you know what I'm saying?
Magda: Why do you…Say like if I didn't know this…
Angela: Right.
Magda: …and I was asked to, you know, draw a graph of the area I would just plot points
Angela: Right but you wouldn't have something really accurate because you have to go with super super tiny.
Magda: Yeah, so if I didn't know that, if I didn't know that this was one third , if I didn't know that…
Angela: Right.
Magda: Then basically were going back to the thing of Riemann sums.
Angela: Right but what I'm saying is we do know this. All right we do know this, even if we didn't, why is… forget that we don't we know this… why does this just happen to be the graph of the area underneath it's antiderivative... is that the right word? I hate math terms.
Magda: Why…
Angela: Why is this graph the area under here besides adding up like that why is it the integral because actually now I'm curious (laughs) we should find out.
Magda: I'm pretty sure we have the answer to that.
Angela: Why wouldn't it.
Magda: You're saying if you know this function…
Angela: Even if you didn't this is still like conceptually this you know what I'm saying if you didn't know that this was the formula for that.
Magda: OK, I'm just saying, if you had this function how would you find the area underneath this?
Angela: You have to do a Riemann sum do it in trapezoids, and keep going, or you can use that other way. If the graph was like this, you could do it like that the other way like this, (draws something) I'm wrong I'm just not going to draw pictures any more… (she scratches out what she drew). Do know what I'm saying though
Magda: Yeah, I it totally know what you're saying I just don't know
Angela: The answer.
Magda: Right.
Angela: OK neither do I Mags,
Angela: Do you know what that I like? Indexes. (Angela looks in the index of the textbook.) What am I looking for, Mags?
Magda: So basically our question is why do we do this…
Angela: Why use the integral/ Why is it the integral. Why does this equation work?
Magda: Well we know why it works but now we need to know why you take the integral

Angela: That's part of why it works I think, no.
Magda: Yeah, well, no,
Angela: In depth. ... What am looking for Mags...
Magda: I don't know what you're looking for.
Angela: Application of... base e... derivative of that, no, maybe? Exponential function...
Magda: Exponential function is where you have x in the exponent.
Angela: Oh, OK, I'm thinking of it the wrong way, heh, heh... compare lower number to a function...
Magda: Why do you go up a power why why...
Angela: (Laughs)
Magda: I like my explanation because it definitely has to be higher it definitely has to be steeper.
Angela: Yeah that definitely makes like its definitely right that works
Magda: The graph has to be steeper, and like how you were saying how you're adding on more area as you go on because this graph is growing.
Angela: Um hum.
Magda: But then also because we have a growing graph , but say we had something like this (she draws a graph that is increasing and decreasing)
Angela: Then it would change to like it would go down and then it would go back up right.
Magda: (Starts to draw a new graph.)
Magda: Cause then it be adding more area,
Angela: No it would keep going up because you're still adding on area It would only go down if it went below the x.
Magda: But it's like slows down here.. goes up...
Angela: The rate is different.
Magda: It goes up... it's like.
Angela: You're starting at 0.
Magda: You can start wherever because that's the whole point of C, what C is.
Angela: But if your area is starting at 0 you have to start at 0 don't you?
Magda: Yes, OK.
Angela: OK, sorry.
Magda: So it's like growing growing growing higher then still growing but at a slower pace here. It never like goes down, it's just going at a slower pace, and then it starts picking up again..
Angela: Yeah.
Magda: So it's one of those, then its concave up, concave down...
Angela: Yeah.
Magda: ...concave up.
Angela: Right. And that's (pointing to the graph below) the slope of that (pointing to the graph above) right?
Magda: Yes. my whole thing is you've got to go to a higher power just because this point it has to be some like faster growing graph then what you originally had you know
Angela: Right I get that a hundred percent... what I'm saying is why does that

graph happen to be the integral?

Magda: It has to be a faster growing graph, an exponential

Angela: Well can't it be… I know it's not... but I'm saying like… why is it the integral I don't even know how to phrase that.

Magda: OK say you had.

Angela: I get that it has to be a greater growing graph.

Magda: A function of one then this would go one to two, to three.

Angela: Its just going to keep going, it's just going to be a straight line right?

Magda: Yeah.

Angela: OK. Yeah.

Magda: So what I'm saying is I don't know why exactly it goes up by "one" power but like the reason it has to be it can't be going down it has to be going up a higher power, to a higher power it's because when you think of it you have to keep adding on.

Angela: Yeah, I know, that makes sense, that makes perfect sense I'm just curious as to why it's that in particular. I don't expect you to answer me I expect us to find this answer together.

Magda: I don't know why technically you do it, you know.

(Romina enters the room.)

Angela: What's up Ro, Banana Republic shirt.

Romina: Nope.

Angela: (States another shirt company).

Romina: Yep.

Angela: We have some thing with the Banana Republic.

Romina: Yep. Different color but same shirt as the last time I was on the train.

Angela: Your hair is lighter, it looks good.

Magda: Basically, I don't really know what we're looking for.

Romina: Got really far in a half hour, huh.

Angela: Why is that, Mags? I don't know.

Magda: Basically I think what he's asking us …

Angela: Yes.

Magda: Basically what he's asking us is… he understands that, (points to the board) he knows that the integral is area underneath the graph but why is it that you're taking the integral.

Angela: Why is it that?

Magda: Is that what you're asking?

Angela: Is that what you're asking? That's what we understood it to be.

Pantozzi: I'll repeat it for Romina since she just started she just got here. It's a month since your guys last helped me, I'm acting as the student.

Angela: It's really good acting.

Pantozzi: And been a month since you helped me and I've taken that course, taken that section of the book and had it taught in class I know that the thing up there (on the board) is the integral from a to b of f of x and that means it's the area between a and b and I know that in order to figure out the area, you're supposed to… g is the function that's, well, f is the derivative of g. Romina's like… I know you just got off the train.

Romina: Give me a second, f is the derivative of g

Magda: Since g is the integral of f

Romina: No, I'm just trying to think how we did it last time. OK.

Pantozzi: OK, so that means, I don't know if I can't remember if I said it last time so g is the anti derivative of f did I say that last time?

Romina: [inaudible]

Magda: Yes.

Pantozzi: Because I think that's what that means if the derivative of g is f then the anti derivative of f is g…

Angela: The opposite.

Pantozzi: Did I say that right.

Romina: It's the opposite.

Angela: Yes.

Pantozzi: So what I asked them, I was trying to understand… I took the section, I know how to solve problems if you tell me to figure out the integral from 1 to 5 of x^2 I know I'm supposed to do 1/3 x cubed and then substitute in one and five and subtract and get the area but what I missed in the lesson that I took in school in my class was where did it come from the anti derivative… like why is it the anti derivative.

Magda: Yeah like why are we taking the integral

Angela: So I was right… yay I got it right

Romina: Why are we taking the integral or why is g the integral of f

Angela: Why does it work out that in this equation that.

Magda: You have to take.

Angela: That's the integral like why… do you know what I'm saying? We'll explain.

Romina: Because if you take the integral of a function it's the functions integral.

Angela: (laughing) yeah but like why.

Magda: I think, are you asking us what an integral is like why.

Angela: Basically.

Magda: Like you would be go up one higher is that

Pantozzi: Sure.

Angela: Do want us to explain what we've got to you so far.

Pantozzi: Yeah that would be good, that would be good, bring Romina up to.

Angela: We came up this funky little thing going on here

Magda: Basically what we were saying and I totally remember this from Sketchpad when you were teaching it to us.

Pantozzi: Um.

Magda: Is that what you were doing, you were taking the function and going at and at one of this is how much area you filled in and like this would grow

Angela: The graph.

Pantozzi: Show Romina too.

Romina: I see it. Oh that I can't.

Angela: Special powers.

Magda: And as you go on this area would be added on and kind of like this graph would show up so what did we say… so basically that's the we're saying and the reason it's a higher power is because you are adding on more area and this graph is always going to be growing faster

Pantozzi: OK. What if it's not something with a higher power what if it is some other sort of graph that doesn't have you know, exponents like that in it

Angela: Like this.

Magda: No that's also.

Angela: Kind of does.

Magda: Kind of like that too it does because you're going up one thing….

Romina: You mean like a whole equation with a whole bunch of different exponents?

Pantozzi: Yeah like, if I had something like e to the x, you something without an exponent if I remember correctly from class, all the anti derivative rules don't work that way like they don't always up one power like with everything

Magda: Like sine and cosine you don't…

Romina: Oh, please.

Angela: Wonderful.

Magda: Let's explore sine and cosine maybe.

Angela: Use a different piece of paper.

Pantozzi: I have, you mentioned sketchpad and I have it here so if there's anything you'd like me to make.

Romina: This might be, I still don't get the question; is the question why we take the integral?

Angela: Yeah.

Pantozzi: Well, like, I want you to I have a split personality here I'm me and I'm this student you're talking to also when I watched your video last time you focused on one aspect of the fundamental theorem and clearly you knew what it's about and as I said to your fellow students here.

Romina: My colleagues.

Pantozzi: Your colleagues. Good answer.

Angela: We are colleagues.

Romina: (Inaudible)

Angela: (Inaudible)

Pantozzi: I'm not sure what exactly what it means to understand the fundamental theorem that's why we're talking. We're talking about what different kinds of understanding you could have about it and this is the aspect you seemed to focus on in your talk the last time so going back to being the student again I just want to, I just missed something in that lesson that I took so I'm hoping you guys can fill it in since you've done lots of discussion about this… all right now back to being me again I've got, I have Sketchpad I can fill in the area, like you were talking about, I can show it you and show it happen if that would help any.

Magda: Yeah why don't you do that for us.

Pantozzi: Is me as the student is my question clear enough, or no?

Angela: You want to get how it works.

Romina: Can you say something since you're taking the area, something it goes to the next power because it's like the cumulative function, you're talking the area at every single point and that's why it has to be bigger.

Angela: Yeah, that's what Magda said…

Magda: Yeah but with sine and cosine that you're not exactly taking it to the next power.

Romina: I know that you're not taking it to the next power but isn't it like steeper in some way?

Romina: Is this even e? I haven't done this in a while. Yeah, right (pointing to the graph that she drew.)

Angela: Because log is the other way.

Romina: Do you remember what the integral of e looks like.

Magda: E to the x is its e to the x.

Romina: Whispers.

Angela: Laughs.

Pantozzi: You guys remember quite a lot.

Angela: Magda remembers quite a lot. She refreshes our memory.

Romina: She's taken all these math courses (inaudible)

Magda: But I don't know why though.

Angela: The last math class I took was in high school.. well real math class.

Romina: Infinitely big… infinitely small.

Angela: I don't know. I don't remember. I don't want to remember.

Magda: No, because…if you take between one…

Romina: So is the derivative of log log?

Angela: Du du du du.

Magda: Integral…

Romina: Or is the integral of log log if it has the same integral does it have to have the same derivative?

Magda: I learned how to take integrals of logs and stuff

Angela: I don't remember.

Magda: Integral of ln.

Romina: Log, ln…

Magda: No, like that's too hard… the integral of one over x is like ln of x.

Pantozzi: See you guys talked a lot about this a lot in that last session and again me acting as a student I also heard a lot that I heard about anti derivatives and about how you're supposed to do that...

Angela: Um hum.

Pantozzi: But what me as the student is missing, like I was, like my mind wandered during that part of the lesson maybe where they said, here's why you take an anti derivative to figure out the area.

Magda: What are you doing?

Romina: Are you going to show us.

Pantozzi: Well I can show you this, I don't know if it will have any use to you

Romina: We'll show that here is (inaudible)

Angela: You can read that?

Romina: Yep. (inaudible)

Pantozzi: What this thing does is it fills in the area and that red graph is.

Angela: The integral?
Pantozzi: I guess, the integral, it tells you how much area you've got so no this is just an estimate because there's actually really trapezoids there but it will tell me how much you've got.
Romina: This is really advanced since we saw the first version of this.
Angela: You had to type something in then slowly craft…
Angela: This is exciting.
Magda: I totally remember doing this.
Romina: Yeah, me too.
Angela: Well I didn't until Magda reminded me.
Romina: Doesn't the integral, does the integral measure the slope?
Angela: Umm.
Magda: No the original graph is the slope of the integral.
Angela: Yeah.
Romina: So then see how it goes, the integral increases and then decreases.
Magda: Because you have negative area That's why.
Angela: Down it takes them away.
Pantozzi: What's that?
Angela: When goes below the x axis it goes down who because you're taking area away
Magda: Away, yeah.
Angela: And when it hits the x axis again going up like to the positive section.
Romina: Point of inflection…
Angela: I don't remember what the word is , I don't know.
Magda: Concave up concave down.
Angela: I don't remember math terms I just kind of concept things… why is it the integral.
Romina: Why do you take the integral for the area…
Angela: Feels like I'm at a laser light show.
Romina: I'm (inaudible) So basically we're measuring our area Maybe I'm simplifying this…
Angela: You're not, we're thinking the same way.
Romina: Yeah if we, let's just say that an integral didn't exist.
Angela: Exactly.
Romina: Let's say there's no integral if we went and graphed the area underneath the graph, there's another, another.
Angela: Even if the integral didn't exist... the concept, like if we didn't know it was the integral, the concept is still the same.
Romina: Exactly.
Angela: So we have to think of it that way though we have to think of it as being the integral.
Romina: Isn't it if I just graphed the area underneath…
Angela: Yeah.
Romina: …this function I get this other line.
Angela: Yeah.
Romina: …which essentially would be the integral, it that not like what.

Angela: But why is it the integral.
Romina: Because someone named it Angela.
Angela: No but why did it end up being that I don't remember why and I really want to know now.
Romina: Isn't that what the integral is by definition that's what I'm not understanding, is that not what it is, it just happens coincidence that the area under the graph is an integral.
Angela: It can't just be coincidence.
Romina: I thought an integral is the area under a function...
Angela: Come on, this is math it doesn't just happen.
Romina: See what I'm seeing... we have to take this next step.
Angela: We have to figure out, I don't know where we're taking it, I just know we have to.
Pantozzi: I as the student know that the integral is the area I just don't understand why where anti derivatives come in like why you're doing this whole... whole formula thing.
Romina: We're saying anti derivatives and integrals are the same thing right?
Pantozzi: Are we?
Angela: Laughs.
Magda: Well the difference between an anti derivative and an integral.. no, no...
Pantozzi: Well I think.
Romina: I use them interchangeably but I keep noticing that they keep saying anti derivative and I keep saying integral so I'm starting to wonder (?)
Magda: How I understand it is the anti derivative can start anywhere like you know you can like start drawing it anywhere and you have that like + C thing.
Angela: I remember that.
Magda: But an integral is just a function OK maybe I'm not understanding this... integral may.
Romina: So I could move this whole red line up.
Magda: Yes.
Romina: But I can't do it with an integral?
Magda: What is.
Angela: The red line is what.
Romina: It's the anti derivative.
Angela: OK.
Romina: Is this red line the anti derivative?
Magda: Yes. I've...
Angela: (inaudible)
Pantozzi: Both as the student and as myself, all I can say about this is is that what this sketchpad thing does is count how much area there is there and plots how much there is.
Magda: See there even though you have a positive area between zero and two.
Angela: Collectively right.
Romina: This is... (gets up to point at the screen)
Magda: No, over, zero and two

Romina: This.
Magda: But the line is below but it's still growing it's still going up because the area is positive but the reason it's in the negative because the whole thing is like minus 6 that...that's your initial, your initial condition, which is the C.
Angela: Right there...
Magda: Which is negative
Angela: That makes sense.
Romina: Yeah, that makes sense to me. The initial condition is that its 6 where do you see - 6 here I see where that crosses, but I'm saying OK when we start it
Pantozzi: You can tell me to do anything with this that you want me to
Romina: When you started here, why did you like.
Magda: So, OK, can you start drawing the anti derivative from zero.
Romina: So OK.
Magda: See stop... Can you stop it at 2 see it's six.. and.
Romina: OK.
Angela: OK.
Magda: ...and its kind of like he moved that Ok and can you see that when he started at 2s that whole line got moved down to, got moved down to, like down six. six.
Romina: That's not the same graph I saw 2 seconds ago.
Magda: Yeah it is.
Angela: Yes it is. It's just higher up.
Romina: Yeah, OK I say so where did you start from last time?
Magda: So hold on (Magda goes to the board.) Before he started drawing it from here (Magda points to $x = 2$ on the x axis) from right here. 2 so basically this point right here (points to the graph of the integral, and then to the the x axis at $x = 2$) was down at 0 so this point was moved down 6.
Romina: The antiderivative, OK the integral is the anti derivative but the anti derivative doesn't always have to be its integral but can the integral be the same as anti derivative can we write the integral as a function + C.
Magda: It definitely has to do with C and where you start drawing the graph that that like.
Angela: Wait...
Romina: But isn't C just like our error and what not?
Angela: Why is it different there?
Magda: What, what do you mean, where is it different?
Angela: Oh no...
Romina: At the, at that end?
Magda: It's the same.
Angela: It's the same.
Magda: Just move it up,
Angela: OK, OK.
Magda: Because it is growing.
Angela: I'm thinking...

Romina: Are you trying to find the definition?
Magda: No I'm trying to see what the real difference between integral and anti derivative is (She looks through the Foerster book)
Angela: They have it it should be because…
Magda: They do…
Angela: (Angela reading from the text.) g(x) is an anti derivative of f(x) if and only if g'(x) = f(x) an anti derivative is the same as an indefinite integral an indefinite integral...
Magda: Is that…an indefinite integral is where you are not defining a and b.
Angela: Yeah.
Romina: So it is the same
Magda: What.
Romina: So it is the same. Did we not just read that it is the same it is.
Angela: Yeah.
Magda: It's the same as an indefinite but its not the same as a definite.
Angela: OK.
Romina: Last time we were doing indefinite.
Magda: No, indefinite just means it's not just defined on that like..
Romina: Yeah.
Magda: Specific
Angela: A to b.
Magda: Interval. Definite is a to b
Angela: (Angela reading from the text.) Indefinite integral: g(x) is the integral of f(x)dx if and only if g'(x) = f(x)
Romina: So if we have a…
Angela: So an integral is the same as an anti derivative
Romina: If we're finding…
Angela: So we can use the integral as a loose term here (laughs)
Romina: So if we're doing… take the integral from *a* to *b* then there has to have a set integral because you start at that point *a*.
Angela: Yeah, you're going between A and B
Romina: So if they asked you can't add a C to that, right?
Magda: To where.
Romina: If you're taking the integral from a to b there's no place to add a C
Angela: There's no spot for it on this thing.
Magda: You can't because.
Romina: It has to be the same (inaudible)
Angela: why is it the integral…is there a formula kind of way we can figure it out.. indefinite integrals…
Romina: Someone has to like throw me in a direction here… take the area under something…
Angela: I'm very un directional
Magda: (inaudible)
Romina: Rectangle…
Angela: We did this already this is what we spent the whole time doing last time
Romina: The area of this, the area of this, and then you plot them

Angela: Right. But why, why is it the integral? Why does it end up being the integral.

Romina: See I think you're being philosophical and I think I'm sticking to math.

Angela: Yeah but I think if we figure that out we figure out how that equation that was up there is true right isn't that what we're trying to explain?

Romina: Like that is why do we call water water? That's what you're saying to me right now.

Angela: But it's not the same thing.. not the same thing.

Romina: That's why we're having such a hard time. Because that is why do you add when there's… when you add.

Angela: Because you're.

Romina: See I.

Angela: Conceptually taking three of something and four of something.

Romina: Here I'm conceptually taking the area (laughs) take the integral.

Angela: I don't know but why does it… I don't know.

Romina: That's where I'm a little… are you seeing it like that?

Magda: Yeah no.

Angela: Once you get to sine and cosine it's not like ... that's different.

Romina: Are you still not taking in the areas?

Magda: It's negative positive negative positive that's why it's a flipped thing.

Romina: Where do you get that we're not…

Angela: I'm thinking more in language ways because that is the way I think and I need to stop doing that in math

Romina: You're obviously getting something that I'm not here.

Angela: I don't think I am, I don't know.

Magda: This is sine.

Angela: Ok, sine.

Magda: The derivative of the sine would be.

Romina: Derivative?

Magda: I mean integral.

Angela: The indefinite integral.

Magda: You start at (Magda is drawing)

Romina: (Romina is drawing) I was going to trace if over I think that's wrong So is that the integral of sine? Is the integral of sine sine? Its decreasing.

Magda: (Magda is drawing). It's going like this…

Angela: Let's see if I can remember how to use this thing… y = sin (using the graphing calculator) Where's x? (inaudible) on this thing.

Romina: What are you trying to do?

Angela: A lot of things…

Romina: …plot it?

Angela: I'm done. that would be sine. So how do you do…

Romina: So what's the integral of sine? (Laughter) Do you remember asking me that?

Magda: Cosine…something cosine… negative cosine…

Romina: Negative cosine, or.

Magda: Positive.

Romina: You don’t add any numbers to it I thought maybe it was negative two cosine.

Angela: No, I don’t know…

Romina: [inaudible]

Magda: No I think it's sine is negative cosine and cosine is positive sine or it's the other way around I don't know that's why I'm drawing it

Angela: I had one of those little devices…

Romina: I think sine is cosine.

Angela: little things to help you remember it.. I don’t remember anymore. I had one of those little things to help you remember

Romina: That’s the triangle, isn’t it?

Magda: (Magda is drawing) OK it’s actually growing slowly.

Angela: And here is where it starts decreasing, and her, increasing.

Magda: And this is growing more on this interval.

Angela: Right there.

Romina: Decline.

Angela: That’s where it starts going down.

Magda: It’s still going up.

Angela: Yes, but this is where it goes down…here is where it stops increasing so much

Romina: It's negative cosine isn't it.

Magda: So this is where it crosses the zero so this is pi

Angela: [inaudible]

Romina: Oh and she's getting all detailed.

Angela: Draw the circle. Draw the circle.

Romina: Magda, oh jeez.

Angela: I have this on one of those papers last time. I don’t remember.

Romina: Do we not get notes this time.

Pantozzi: I didn’t bring notes this time probably because I don’t know where they are anymore…

Magda: It’s negative cosine.

Romina: I told you that like how long ago Magda.

Magda: I’m sorry.

Romina: Cosine sine.

Magda: So what are we trying to figure out?

Romina: Angela, what are you thinking?

Angela: I don’t know I ‘m done thinking… what makes sense.

Romina: Enlighten me here.

Angela: That's right?

Magda: Yes, It’s a flipped cosine. But this doesn't help us.

Angela: I thought maybe it would trigger something in Magda’s wonderful memory. I’m just saying like…

Magda: You want to know why does it happen to be the integral. Or antiderivative.

Pantozzi: The integral is area.

Magda: OK.

Pantozzi: If I’m trying to figure out area I know its an integral, and someone came up with that word, called it an integral I have a book that tells me where that is... but what I'm missing in the lesson is why do I use an anti derivative to figure out the area, because that's what the fundamental theorem that you guys, as I understood what you talked about last time , you know, you were talking about this *g* and its... the derivative of *g* was *f*, so *g* is the anti derivative of *f*.

Romina: [inaudible]

Pantozzi: I think in terms of your discussion before... I don't know if I can answer...the integral is area and I accept that but it just seems, do anti derivatives come out of the blue to equal area or something like that formula says

Romina: When we write to the + C is that (pointing to the board) on the *f* function side?

Magda: No when you take you wouldn't do it here.

Angela: It would be someplace else.

Romina: It would be after you take it.

Magda: It would be f of x = g(x) + C and that C, what I always understood it it's where you kind of start the graph like this C is like something on the y axis like so C could equal like so negative 2 and this is kind of like where the graph you know meets.

Romina: Did we do something like... were... you remember something where we went 2 down from every single point something like that do you remember this?

Magda: That something like that?

Angela:

Magda: As the area between two curves.

Angela: That was between.

Romina: But this is where like I pushed it down all 2 it's still the same function

Angela: Yeah, it just drops.

Magda: It's starting lower.

Angela: If its 2.. every point...

Romina: T would it have the same area like to

Magda: Yes, you would have to...

Angela: You mean the same area between the graph and the x axis? No it would be different.

Romina: Is this area, this is all positive, this goes positive dips into the negatives...

Angela: Here, and then here,

Magda: Oh, I thought you were talking about the area in between

Romina: Even though the graph is the same amount that's like getting into the anti derivative of the anti derivative.

Angela: What.

Magda: No the whole thing is here that you'd have negative 2 in your function and every time this little area here like here would be 2 less than the area up here you know what I'm saying? you would have this little square like left over because it's moved down you know what I'm saying.

Romina: Yeah. (Inaudible)

Magda: No, that's like a good point… if this was some function minus 2, say,
Romina: When you take the um, you take anything thing if it just falls out doesn't it, right
Angela: No that's with the derivative
Magda: This will get an x
Romina: So let's take the derivative of.
Angela: *X*.
Romina: What's like the integral of f, is g plus c
Angela: Right.
Romina: So then this integral of that is g plus c.
Magda: Um hum.
Romina: then when you take it again it's g + c squared, no cx.
Magda: C is a constant.
Romina: No, but when you take the integral you have C X.
Angela: Doesn't that.
Romina: This is a function how does that work.
Angela: You need to keep adding some primes or something it's not the same G right.
Romina: And the derivative is…
Magda: It becomes f of x.
Romina: This becomes f of x?
Magda: Um hum.
Angela: It does?
Romina: So the integral of this is that, so the integral of that is that so then you take the derivative of that…
Magda: Hold on, you're going up?
Angela: Wait, this becomes this again?
Romina: No, now, OK, so hold on. I'm going to take the integral of this,
Magda: I don't understand, hold on.
Romina: If I'm going to take the integral of this, now I'm going to take the integral of this..
Magda: Becomes a big g or something.
Angela: Like g prime or something,
Romina: That was derivative.
Magda: No, big G. Plus c of x.
Romina: So we just went, when we step down to this and we step back up it's not the same thing anymore.
Angela: Yeah but you're not stepping back up you're stepping down from here.
Romina: I want to step up I just don't have the right terminology… The integral of this function equals g.
Angela: Show me.
Romina: How come we don't write it like this?
Magda: Not x, its
Angela: Plus C.
Romina: Give me another piece of paper. (inaudible) Equals g + C because it's indefinite, right?

Magda: Right.
Romina: And when we take the integral of this function to move back to that we get big G + C x does that make any difference at all.
Magda: You not going back to that function.
Angela: You don't go back to that,
Magda: Wait you're not going backwards.
Angela: this is like the opposite of what you're doing
Romina: I always think of as if I'd take a.
Angela: Like if you have this graph OK, and then you do
Romina: No if I What I was trying to do?
Angela: If you do the integral of tha.
Romina: That's where I was confused, that's what I was asking. Do the integral.
Angela: Well even if you started someplace else if you're taking the integral of this it's not going to go back to F of x you're taking the integral it's going to be completely different graph then.
Romina: I don't think it can be a completely different graph?
Magda: What do you mean a different graph?
Angela: If you do this and then you do that, it's going to be x to the.
Romina: Integral derivative if you take the derivative and then go to its integral shouldn't the integral be what you started with.
Angela: No it would be x to the 4.
Romina: I mean if you take the integral and then you if you take the derivative of the integral wouldn't it be the function
Magda: Yes.
Romina: that's what I'm trying to do here and I just couldn't write it.
Angela: I don't know what you just said could you.
Romina: Ok, I have the integral of a function is the integral so then the derivative of this function has to be the original function.
Angela: Right.
Romina: That's all I was trying to do I couldn't write it.
Angela: OK.
Romina: See that… didn't he make us take double integrals… do the integral, and then the integral of the integral… and then we tried… am I completely out of it?
Magda: No no,
Romina: Am I completely out of it?
Magda: If you take an integral of an x function, but still taking the integral, you know what I'm saying…
Romina: Talk about C now?
Magda: C kind of just moves the function.
Angela: Yeah, I don't think that's
Magda: Yeah because like if you go the other way like from the red graph to the purple graph you're finding…
Romina: Shift left or right.
Magda: What?
Angela: Learned all that stuff. I had one of those things to remember, mnemonic

devices is that the word? I had one of those too. Move left or right,

Magda: What do you mean, left or right.

Romina: C moves it down… Shifted to the right or shifted to the left?

Angela: Like if you wanted to move the graph…like there.

Magda: Oh, yeah.

Romina: This doesn't really matter,

Angela: It's something.

Romina: I was just curious.

Magda: Inside like the x squared minus 2, or something, that moves to the right.

Romina: How do you remember this stuff?

Angela: She's good, that's why. She's math girl.

Magda: Minus 2.

Angela: She's going to be an accountant.

Magda: Whatever. Moves it to the other side. But what I was saying…

Romina: Could we…

Magda: The red line, if you look at the red line and you take the derivative, so basically it doesn't matter where the graph is because the slope is going to be always the same you know what I'm saying.

Angela: They're parallel to each other,

Romina: So I don't…

Angela: Right.

Magda: The C it just moves that up or down.

Angela: I think it just that moves it up or down. (Moves papers). Sorry, I'm big on organizing today. … the whole office downstairs.

Romina: I'm not getting the question.

Magda: I'm not getting it either.

Romina: We're not getting the question. Did the other group get the question?

Pantozzi: They had different questions.

Angela: They had different questions.

Pantozzi: Because they talked about different things. The way I was thinking about it outside was… Did somebody just, maybe it happened this way, who knows. Someone just sat down one day and say the way to figure out area is to find the anti derivative.

Romina: So are you talking about the process the steps you take to just write the equation, you know how you take it up an exponent, or differently if its sine or cosine or are you taking about that, or theoretically why you take the anti derivative.

Pantozzi: Me as that student that you're helping feels that antiderivative… doesn't know where, it just seems the anti derivative just seems to be plucked out of thin air we were talking about area, we were talking about figuring out area, and then we said oh, to find the area, do the antiderivative, and then you'll have the area. and I missed the part about where.. why is that… why is that what you have do… why not take the derivative… why doesn't that give you the area?

Angela: Why not add two?

Pantozzi: Does that help any?

Romina: Well if you take the area and we're plotting the area on top of each other like the amounts, I understand when someone first did this a graph, did they know what the derivative was at that time I don't know when we graph that it wasn't the derivative so I just figure they named it something.. like this is where... what do you... this is where I...
Magda: I'm totally... I totally don't know where to go from here.
Angela: See I was thinking about it the other way.
Romina: I thought it was area first and then integral or anti derivative.
Magda: You mean integral and then derivative?
Angela: Anti derivative like it which came first kind of deal.
Magda: Like integral and then derivative is that what you're saying.
Romina: No I thought it was function, slope.
Angela: I don't think it can just be that only because you don't only think about math like in graphing terms or in visual terms you have to be like...
Romina: I'm sure that when someone drew that other graph they somehow correlated it to the original function and if I plot that area then I get to this function I'm going to call it the integral you're saying someone said the word integral and then they defined it as something
Pantozzi: Well they... I'm sorry.
Angela: I'm sorry... no I'm thinking I don't even know how to describe what I'm thinking, go talk.
Pantozzi: My question as the student is, I agree with what both of you are saying, they drew it someone was trying to figure out area in the past and I don't know, I don't know whether they drew a graph but somewhere along the line they were looking at something with area and then they said oh anti derivative.
Romina: So that's what... so we're on the same page about that.
Pantozzi: I think... so where did that anti derivative stuff come in, not historically, but where, how would you, I'm the student again,
Romina: The opposite of taking the derivative
Pantozzi: What does that have to do with area though
Romina: See if you have a derivative you can graph the area of the derivative...
Pantozzi: Um hum.
Romina: ...you get its integral.
Angela: But why is it like that graph like that specific why is it like this.
Romina: Oh, now I understand the question,
Angela: Wait.
Romina: I thought you were asking why is it the anti derivative.
Magda: This is your original function and then at some point your slope is 2...
Pantozzi: Well I was thinking this is one question I was thinking of asking but go ahead with what you're going to say.
Magda: If you have a function
Romina: If you have your function.
Magda: Yeah, if you have your function.
Romina: Really, can we recap here... function... the derivative is the slope function.
Magda: Can you go back to the graph?

Angela: Here you should use this paper, because that doesn’t have our names on it.

Romina: The derivative is the slopes of the function, the slope of the function.

Magda: The derivative, yeah.

Romina: Derivative is slope function.

Magda: OK say the function is the red thing (referring to the red graph on the board)

Romina: OK.

Angela: Which red thing?

Romina: [inaudible]

Angela: I know, but which

Magda: And so you take derivative of that is going to be the purple thing.

Romina: Yeah, right.

Magda: So basically that is going to tell you the slope.

Romina: So the red thing is the anti derivative.

Pantozzi: OK wait, I am the student that's the part I don't get I don’t get that. She just said that.. didn’t you just say that if I take the derivative of the red graph I get the purple graph.

Magda: Correct.

Pantozzi: I don't get that. I thought that red graph was a graph of the area.

Romina: But they're obviously correlated because you had one to draw the other.

Pantozzi: Had what to draw the other.

Pantozzi: I started with the purple graph, that was my function.

Romina: And when we drew, we graphed the area

Magda: You got the red graph

Pantozzi: You got the red graph, OK. I’m fine with that

Romina: So they have to be tied together somehow.

Pantozzi: OK.

Angela: We're figuring out why they're tied together.

Magda: So when you have that red graph,

Pantozzi: Yeah…

Magda: …and you take the derivative of the red graph.

Pantozzi: Um hum…

Magda: …you're going to get the purple graph.

Pantozzi: O.K. I don't know why that's true.

Romina: Because when you plot the slope of the red graph like we the way that.

Angela: That’s the rate that it's changing.

Romina: When we did okay because when we were graphing the red one we were taking amount of… amount of area underneath.

Pantozzi: That was me…I’m sorry.

Romina: Because when we did that… the area… because the slope is changing at a rate.

Magda: Yes.

Romina: When we take the derivative of the red one we're actually graphing the initial slope that we had to figure out the area.. you see there (pointing to the projected graph) it's going faster and then it peaks and then it’s slowing down and your graph is going like the slope hits zero and then the slope goes up again.

Pantozzi: OK say what you said again you are saying again and I need you to say the red graph the purple graph or something to help me out.

Romina: When we took the area of the purple graph.

Pantozzi: OK.

Romina: We accumulated the area underneath the purple graph.

Pantozzi: OK.

Romina: The way the area is formed like you have a lot of area and then the area gets smaller is because of the rate, the way the purple slope is changing which is the way in the purple line is changing which is the slope the area is caused by the changes in the slope like more area less area so the inflection points on your red one is caused by when your slope goes down because you're gaining area at less speed and when your going from negatives you're gaining more negative faster and then you're losing negatives… yeah.. and then when you take some so then we take the slope of the red graph we're essentially getting the purple graph again because that's the slope… do you want to add? I know that sounds really confusing but.

Angela: It's like the rate that you're accumulating area or losing area as you go along the x axis that's what slope is.

Romina: We should have known that was the question a long time ago.

Magda: We did we saw that at the very beginning.

Angela: We said that, but I thought it was more complicated than that, I don't know.

Romina: Articulate that. Do you see what I'm saying?

Angela: Give that girl a point.

Romina: Do you see what I'm saying.

Pantozzi: Well I do of course.

Romina: Well I know you do but as the student.

Pantozzi: As the student? Um…Well let me go back to myself again, when I say I do when I say I do, I mean I know what I'm thinking in my head I can't be sure what you're thinking in your head other than what you said, you know what I'm saying.

Romina: And obviously it sounds like mumbling to you.

Pantozzi: Well, when I listen to you, whenever you listen to somebody else you read into what they're saying from what you yourself believe so you may say something and I'll go, oh yeah that's what she means but you might not actually mean that I might be thinking what I mean… so I guess as the student again here is what I understand… here is what I think you said… I build up the area… and stop me at any point if what I'm saying doesn't sound like what you said. You said I build up the area you build up the area and the area sometimes increases

Romina: Well at each point because of the way this slope of the purple graph is going say you had an area I don't know if you could assign at like 5 units of some sort, the area of at point a would be 5 units at some point.

Pantozzi: 5 total units?

Romina: Yeah just at that point I don't think that's like.
Pantozzi: Let me make something that does that.
Romina: Is that permissible can you take a point.
Angela: Have like a perfect.
Romina: Yeah, say like a.
Pantozzi: That's 2 1/2.
Romina: Yeah that's what I mean
Pantozzi: And this would be…
Romina: That's going to obviously be like more because your slope is increasing.
Pantozzi: What's going to be more?
Romina: Your area under from point… I could count… your point at 2 is. is less than your point at… OK your point at 4 has more area underneath than your point at 2 because your slope is increasing you are allowing more area.
Pantozzi: The slope of what is increasing
Romina: The slope of the line is increasing.
Angela: The slope of the purple function is increasing
Romina: So if you take a look at the red line.
Magda: No no no the slope of your function is not increasing.
Angela: It's staying the same but it's going higher up.
Romina: The slope, yeah.
Magda: The whole thing is you're adding on more area.
Pantozzi: I'm listening, go ahead.
Magda: As you go, you know,
Romina: See your area is not going to be as much wait, is that the same.
Pantozzi: I changed it from x to one half x I can change it to anything you want.
Romina: Can you keep this and do 1 x
Pantozzi: Good question.
Romina: Can you keep this whole thing on and then do a function as 1 x.
Romina: If you can't I understand…
Pantozzi: Oh, I can do this. I'm here to grant your wishes today.
Romina: You act like you designed this or something.
Pantozzi: I give a lot of credit to the people who actually did design it… all right so I need to…we were on page 9 of the other one.
Romina: (inaudible)
Angela: Transparence,
Romina: I'm impressed, Tozzi.
Pantozzi: OK, so this one is .5 x, and you want me to change this one to like x like it was before.
Romina: Yeah: see that one has a steeper slope, it doesn't really have a.
Angela: it's twice as big
Romina: You're right, it doesn't…that slope thing is messing me up.
Angela: The slope is not changing its actually increasing at the same rate right, no that's the slope… well no its not.
Romina: Like if you're going for at the same speed, you're always going to be covering, you're covering the same amount of distance per minute but

you're always going to be increasing your distance unless you're going forwards and backwards in which case your slope would change from negative to positive across zero so it's not more slope… because you're increasing your area…

Angela: More accumulation.

Romina: …and then when you take the derivative of the red one.

Magda: No but, if you look at the red graph and at x = 6 of the red graph.

Romina: Which.

Magda: The one to the right. The slope of the red graph…

Angela: That's 8?

Magda: It's 6. At x = 6 look at the red graph

Angela: No I meant…

Magda: The slope of that graph at 6 would be 3 and then but

Angela: Oh, OK.

Magda: If you move up if you are accumulating more area the slope is going to be bigger because.

Romina: The slope is…

Angela: It's getting bigger there is more area like per interval.

Romina: If you take the derivative of the red line.

Magda: It's getting greater because you're adding more area it's getting more…

Romina: The rates going to stay the same but.

Magda: No it's not it's going to get greater its going higher up.

Romina: The rate of the rate is getting greater.

Angela: the distance between the original function and the x axis is increasing.

Magda: If you think about it.

Romina: OK let's say this is going to go up… this rate is increasing at a particular rate a constant rate

Magda: [inaudible]

Angela: Forget about the line and look at the area, you kno what I'm saying, like

Magda: Here it's like 2 1/2, the slope would be 2 and 1/2 and but here, you move one over, over here, you move only one over and hold on I can't find… the slope already is I can't see

Angela: Magda's blocking the projector our chance to see.

Pantozzi: Three is this is three.

Magda: Oh this is three.

Romina: (pointing to the red parabola) the rate of this one is increasing at a constant rate of 0.5

Romina: What I'm saying is the rate of this one (pointing again to the red parabola) is increasing at a constant rate of 0.5 I don't know, maybe I'm wrong

Angela: the slope of that line yes, but not the area underneath, right?

Romina: I'm not talking about the area underneath the red one.

Angela: I'm not even talking about the red line.

Romina: I'm talking about the rate of the red line because if when if you take the derivative of the red line to have to get the purple line and this is the slope is increasing for the red line so it can't be that so obviously we know it's the red line not the purple line because the purple has a constant slope but the way we get from the red line to the purple line is taking the rate of the rate is that making any sense.

Magda: I don't.

Magda: I don't understand what you are saying, the rate of the rate.

Romina: Push me there Magda, I'm almost there

Magda: Well what are you saying the rate of the rate I don't understand the wording.

Romina: The red one is not constant it has, it's growing.

Angela: It's a curve.

Romina: But at the rate at which the curve is growing is a constant so it doesn't like it's not like growing at 25% at one point, then it increases, not even grows, then it increases at 25%, then it increases at 50% it's not like that it increases at a steady pace

Angela: Does it do that for every kind of graph though?

Romina: Just this one.

Angela: Because it's a straight line

Romina: Increases as .5.

Magda: It doesn't increase by point 5.

Romina: Then how do we get the slope here to be .5 Magda?

Angela: This line is.

Magda: That lines' slope is .5.

Angela: The purple line is .5.

Romina: This line's slope, this line somehow the derivative of this has to equal that slope, no?

Magda: Yes, I understand that.

Romina: I'm obviously not saying this right…

Romina: If this grew, if this was one slope is… here one slope, here's another slope, this slope is…

Magda: This slope is 0.5 times at whatever point you are.

Romina: The rate at which the rates… the rates at which the slope of the red line is growing increasing in this case is.

Magda: Point 5.

Romina: 0.5 which is the derivative that's what I mean.

Magda: OK that makes sense but like say you have cosine.

Romina: You know what it is I really don't think about cosine very often.

Angela: Who needs cosine!

Romina: Cosine has to be the same thing.

Angela: Circles stink.

Romina: Cosine is like a lot of little parabolas.. the word parabola… so.. if there were parabolas… it depends like though I don't know how the slope here goes.

Angela: There you go, have a graph. (hands Romina a graphing calculator)

Romina: Is this the integral for this?

Angela: The integral for what? Cosine? Oh yeah, yes, I didn't know if you were trying…

Romina: This.

Angela: 1/3 x^3 for x^2 I graphed that wrong last time I only graphed x^3, not 1/3 x cubed.

Magda: That's right.

Romina: So this, the area. Cause this is all, like I'm trying to think how the rate of the rate would work, yeah it would because this is like…

Angela: Yeah like on this one the slope is changing all the time

Romina: And this time it's decreasing well but is this is a perfect x squared the then wouldn't this be decreasing at the same rate.

Magda: Well of this is decreasing and the slope is negative obviously (referring to the x squared graph that Romina drew)

Romina: Yeah so this is increasing and the so its positive but then if you like so if you have a cosine.

Magda: This function is always increasing. (referring back to the cubic graph that Romina drew)

Romina: Yes but what I'm saying is that when I look at these I think of a bunch of like these So this would have to be going du du... like this, you know, but this goes negative.

Magda: Well but it doesn't because this goes into the negatives.

Angela: So it goes down

Romina: Yeah, um.

Angela: So it doesn't keep increasing it goes down like this

Romina: This is how I think about it… as separate entities.. that's just how I think about it what about e to the x?

Angela: What are you doing?

Romina: She was the one who asked me… I was happy with my explanation before. Have we reached the level of

Pantozzi: Me as student I listen to you guys talk, I say wow. They're talking about derivatives, they're talking about integrals, When you ay, that purple graph, what did you say? the purple graph, how is that related to the red graph?

Magda: It's the area… no, it's the…

Angela: [inaudible]

Romina: [inaudible]

Magda: The purple graph is the derivative of the red graph

Pantozzi: I don't see how you know that.

Romina: Because if i…

Pantozzi: I'm not saying that its wrong, I'm just saying I don't know how you know that. You guys just seem to say that, here I am as the student… I was talking about area, and all of a sudden, bang you're talking about derivatives. Whoa I learned that two months ago, that was chapter one.

Magda: That's old news, right?

Romina: The derivative is the slope, right.

Pantozzi: OK.

Romina: You know that.

Pantozzi: I know that.

Romina: When we look at that we take the slope of the red line which is the purple line.

Pantozzi: How do you know it's that purple line and not some other purple line?

Romina: Because…

Pantozzi: Do you understand what I'm saying?

Romina: Yeah.

Angela:

Pantozzi: How about some other purple line with some other slope.

Angela: The rate on which the slope of that is changing.

Romina: See I don't necessarily…I don't know this is kind of hard to say.. the derivative of x squared is 2 x is that easier for you to visualize are we not ready for that? This is harder for me to think about conceptually because it's not…

Magda: The way I think about it if you take the rate at seven, right, at x = 7 on the red line that would equal around 11 right?

Angela: (laughs) Yeah, 11.

Pantozzi: 7, yeah.

Magda: That would equal around 11. and then you want to find the slope between six and seven of the red line. right.

Romina: When you do that Magda isn't like the slope of that there that (Romina traces her hand from the x axis at (7,0) to the graph of 1/2 x over to the y axis at 3.5.

Magda: Yeah but, you're trying to find the slope of that (the red one) line at well basically what you were doing you're going11 – 9, no 8.

Pantozzi: 8.

Magda: 11- 8 is 3, divided by the change which is one.

Angela: So the slope is 3.

Magda: Yes.

Angela: Approximately

Pantozzi: So that's 3, right, I understand that, the slope is 3, the slope from here to here is 3.

Magda: Yeah so when you go down and look at 6 on the other graph you get 3.

Romina: That's exactly my reasoning.

Pantozzi: Is that what you were saying?

Romina: And obviously I didn't articulate it well… and then when you… and then that has a slope of 0.5 because it, I mean it shows you how the slope increases on the red line but it's increasing at a steady rate making a steady rate of

Magda: A rate of 0.5.

Romina: Good job Magda.

Romina: Really good explanation.

Pantozzi: I need to think about what you said.

Angela: That's the rate that that line's slope is changing.

Romina: The purple line just plots the slopes at certain points on the red line; plots if the slope is 3, it plots three, if the slope is 3.1, it plots at 3.1

Pantozzi: Say that again.

Romina: It just plots the actual numerical value, the slope's numerical value then when you take the slopes of the graphing... the points of the slope of the red line, why do I even bother to try, its...

Magda: Basically.

Romina: Point 5.

Magda: It's growing at 0.5 the slope is increasing at .5 because if you look at that, you're every time OK between five and six you added up like 2.75 of area and between six and seven you added 3.25 of area which is 0.5 more so your slope is increasing by 0.5.

Pantozzi: So here I added on 2.75 I see that and here added on 3.25 of area I understand as the student that means that this graph went up 3.25 and I guess that means from also from here to here.

Magda: Um hum.

Pantozzi: It went up how much? (pointing with the mouse to the red parabola)

Magda: 2.75.

Pantozzi: 2.75 so by going up by 2.75 that makes the slope of this 2.75 is that what you're saying.

Magda: No.

Angela: Yeah doesn't it because you're going over 1, and up 2.75.

Magda: Yeah that what I'm saying.

Pantozzi: Hmmm... I'm doing my double identity again... this particular thing, the fundamental theorem is something I've thought a lot about and I keep thinking about... which is why we're having this conversation today about it... so tell me if what I'm saying, if what I'm saying matches what you think you're saying.

Pantozzi: So I do the area, the red graph plots how much area I have got

Magda: Uh huh.

Pantozzi: And it goes up by certain amounts it goes up by amounts equal to the area under here.

Magda: Uh huh.

Pantozzi: And then because of that? what when I take the slope of this I get the slope like this slope here is 3.25.

Magda: Um hum.

Pantozzi: And this area right here is 3.25 so the slope of this equals that whole area?

Magda: Yes.

Angela: It doesn't equal the area it equals the point on the line right? Am I wrong? Wait? I don't think it equals the area.

Romina: The area equals

Angela: The area equals where it is on the line, not the slope, the slope... the slope doesn't equal the area...

Romina: The way I see it this part 3.5 whatever, (pointing to the area under the graph) equals the slope there.

Angela: Oh, OK OK.

Romina: these got stacked on top of each other you may be thinking of the area under there...

Angela: No, I was just, I wasn't, I was straightening things out in my head.

Magda: Yes that's what we're saying.

Pantozzi: OK so
Romina: A unanimous yes.
Pantozzi: So does that help me make sense of what this says then can you relate what you said to that? (Referring to the statement $\int_a^b f(x)dx = g(b) - g(a)$.) Maybe it doesn't, I don't know. I understand as the student and I think as myself what you said.
Romina: Integral of the purple function (pointing to the statement) isn't that exactly, isn't that exactly what we just said.
Pantozzi: Take it apart for me bit by bit.
Romina: the integral from here to here a to b equals.
Angela: The slope.
Romina: This kind of like got stacked up there so it equals the slope the slope tells you the area no yes…that's why I keep.
Pantozzi: What does this…
Angela: That means like it.
Romina: One.
Angela: The red line is the integral no.
Romina: Yeah, We're taking the integral of the purple line isn't that what the first part is saying, integral of the purple line.
Angela: Isn't that just talking about the area I'm confused
Romina: Coming from me that's right. You know the purple line.
Angela: Hum.
Romina: Integral of which
Angela: the integral of the purple line is the red line, right.
Romina: Yeah but I mean that part of it would be the integral of the purple line from a to b would be 3.5
Angela: G.
Romina: Equals the area under just that point, so it's telling you, kind of like you are sectioning off that particular area on the red line
Angela: From a to b
Romina: on the red line that particular area, that particular spot... to take the slope of.
(silence for about a minute)
Magda: Yeah because.
Romina: This is hard because…
Magda: Like if you take g of b right,
Angela: Um hum.
Magda: Say b is in our case seven whatever we say in our example b is 7, and the a would be the six, right.
Angela: Yeah.
Magda: So if you think about it that gives you the change,
Romina: Slope is change.
Magda: That gives you the slope of that.
Romina: Magda, you're being very articulate person today… happens every day.
Magda: But then if say if like if it's not one, don't, wouldn't you have to divide it

by interval.

Romina: Are you saying you want to see x squared? This is messing me up because they are so similar. I work better with something

Magda: Can you give us a different… like it harder graph.

Pantozzi: A harder graph?

Magda: like an x squared or something.

Pantozzi: Which graph you want me to make x squared?

Magda: F(x)

Pantozzi: So make this x squared?

Magda: Yeah.

Romina: Can you stop it from the next point over from, yeah. The area underneath that part is about 1.25 so the slope is about 1.25 of that line

Magda: No it's like 5 6 7.

Romina: Oh, I didn't see..

Magda: Uh huh.

Romina: Can you draw that line on a different point

Pantozzi: How do you mean,

Romina: Starting like at five going to is, the red line.

Pantozzi: You want to see, like this over here.

Romina: I want to see just this portion,

Pantozzi: Just this portion, not that.

Romina: Yeah. Or anywhere, you can start it at three.

Pantozzi: Is that what you meant?

Romina: Yeah, so the area, see the area from 5 to 6 is like 1 2 3 4 5.

Magda: Like 6.

Romina: 6 units.

Pantozzi: These are10… 5 each this way right now.

Romina: So it's about 30.

Magda: Um hum.

Romina: The slope from five to six of the red line is 30 over 1. Thirty. Saying exactly what that was saying.

Pantozzi: So that's what that says?

Romina: Maybe we should take it over a bigger span so we do over all the areas, because the slope could be changing from b to a when you take a

Angela: May be like from 3 to 7.

Magda: Yeah but then you'd have to divide it by the interval that you're taking the thing over.

Angela: But it's less accurate if it's a bigger… right.

Magda: No, it would be very accurate.

Romina: That's how I think about with area when I take a big span,

Magda: Um hum.

Romina: Then you subtract…

Magda: Can you like go…

Romina: Can you start like over there (to the left) increasing, increasing … zero… actually can you move the graph down… our purple graph down to like x squared 3, I mean minus 5.

Romina: When the slope is decreasing when have negative area and it like cancels itself out negative area of like a half no yeah, a half.

Magda: So say you go from 1 2, 3, 4, at 4.

Romina: Negative 4.

Magda: Negative 4. From like negative four and negative 2, you want find that slope.

Romina: The slope changes… you can't find it, it doesn't have one slope

Angela: Lots of.

Romina: Because the slope increases and then decreases because it's a positive area and a negative area.

Angela: That just shows how the rate changes.

Magda: Hold on…

Romina: Even if you take from two things to the left of b... no, you don't have to move it …it's just like, if you start two points behind that, and we go to b, that area, that slope right there went from, you know, well it's pretty similar at the beginning, then slope is always changing… flattens out, then negative, … negative… so you can't think of it like that…and the slope of the red one is changing at a rate of the purple one.

Angela: The rate of the rate.

Pantozzi: As the student I have a question and I'd like to use just a very simple example (Pantozzi changes the graph to f(x) = 3) Can you recap what you said with this example right now.

Magda: [inaudible]

Romina: Our purple graph at each unit that it moves requires three units of area and our red graph is growing at no it's not growing it has a rate of 3 because that's how much our purple one is growing by like the accumulated area.

Angela: The area under the purple.

Pantozzi: The area under the purple is doing what?

Romina: It's increasing at a rate of three

Angela: Constant.

Romina: Per point per unit.

Pantozzi: So how about if I only went like halfway there like this.

Romina: You increase half of 3

Magda: Which is one half which gives you the s...

Romina: That's the slope of the red line is 1.5.

Pantozzi: The slope of the red line is…

Angela: No… that's where the red line goes up to.

Romina: Oh hold on the slope of the red line.

Magda: The slope is still three… the slope is still three.

Romina: Oh, right, sorry.

Magda: Because you took only half of the block before you were increasing by threes but now you only increase by before you're taking going over on the x's by ones but now you are going on the x's by no you went over by 0.5 only 0.5 so you got to multiply so you've got to one times three it was growing by three so now it's 0.5 times three is 1.5

Pantozzi: So this area right here right now in purple is 1.5

Magda: 1.5.

Angela: 1.5.

Pantozzi: But this doesn't make the slope of that to 1.5.

Magda: No.

Angela: No.

Romina: If you move it it still makes it three. I don't ... because your slope from, from point 4 to 5 the slope of the red line is still three because it's a constant it's constant.

Magda: It's less, it made less of a jump between the things.

Angela: The interval that you're going from is smaller; so I mean like, the slope's is going to be the same... where the lines ends up where the line stops...

Romina: Can you take the slope from 4 to 4.5 is 1.5 and you cut everything down like that then the slope of the line would be 1.5 when.

Angela: ?

Magda: So you're dividing by 0.5 so basically you're kind of multiplying by that you know what I'm saying so here you're going like

Romina: [inaudible]

Magda: 4.5 minus the four and then dividing it over 0.5.

Romina: Point 5.

Magda: Which is you know like multiplying.

Romina: Ohhh. I'll be quite honest with you...

Angela: ...not even one little bit.

Pantozzi: Don't worry... ignore that... that's just a graph that's up there now. Do you have any comments about the slope of the red graph or what you've been talking about, the slope again.

Angela: At the cusps...is that what you call it?

Magda: The derivative is undefined at the cusps

Angela: That I remember.

Pantozzi: Like there's, would you, there's the red graph and it got some... I don't even know how much area it is exactly here, but say I get to here,

Magda: OK.

Pantozzi: Can you tell me what the slope of the red graph is just by looking at it like this.

Romina: A little bit over one.

Magda: Well take that's whatever... negative 0.5 minus.

Angela: What points are you using it takes two points to make a slope It doesn't have one slope.

Magda: No it is...

Angela: Here where... I'm just saying the slope from where.

Magda: (inaudible) graph...like.. negative .5 over...

Romina: Can't you look directly at the graph? from seven to eight it's a little bit over one, the slope because the area is a little bit over one unit and then over there from 4 to 5 its negative a little but over one because the area is a little bit below one just by looking at it...

Angela: It's not, there.. Where that one point is it's like the slope of that on the purple.

Magda: At what points are you taking?

Romina: At any point, at any one point

Angela: You're looking at the end, right…

Romina: What, oh oh, looking at the end, at any one point the slope equals the area whether it is positive or negative area that's the way I always think about under its negative area over positive area so like from 4 to 5 the slope is negative 1.15

Magda: OK.

Romina: ?

Angela: But you can't look at the whole thing at that little pokey spot it changes.

Romina: It's still negative just less negative or its still positive just less positive so your slope decreases because it is less positive you're still accumulating positive area just accumulating less amounts of it.

Angela: Right and so… and that those spots its undefined.

Pantozzi: Let me ask you a question as the student maybe as me too… let's suppose the area under here is exactly two if that area is 2 I don't know how I know that let's just suppose it is… I made it with water and poured it out or something. If this area is 2, what does that tell me about the red graph… anything?

Magda: That.

Angela: In that section that's how much it's going to increase right, from like that point to that point, it will go up to.

Magda: It will go up two.

Angela: No no wrong.

Magda: But you've got to divide it by the interval that you are going over the slope, the slope is gonna… no the line is going to grow, it's going to increase so that it has a slope of 2 over that interval like so that interval looks like three.

Pantozzi: From here to here you're talking about so that interval is about three.

Magda: Say three. So it's two divided by three… your slope would be like two-thirds.

Pantozzi: The slope of what is 2/3.

Magda: The slope of the red line.

Pantozzi: Where?

Romina: You lost me at two thirds.

Pantozzi: This area is 2, and Magda said from here to here let's say it's three.

Romina: OK, So the slope you're saying is … but you can't say that…

Angela: But that's not the slope because it is a curve like if that were straight line that would be the slope but it's not because that's not just

Romina: The way I think about it as if you're driving a car from point A to point B like you could drive really fast and then really slow and then really fast or really fast and then really slow to get 3 mi. or.. 3 miles… so the area that you'd cover would be kind of does that mean the average rate that you are going… I'm so wrong…

Pantozzi: Well continue what you were going to say.

Romina: I don't want to be more wrong….

Pantozzi: Remember the more you talk to the better this is for other people… for humanity…

Romina: Like, this is the cat.

Angela: No pressure.

Pantozzi: Now, not that kind of pressure.

Romina: You can't you're constantly well not constantly you're changing your slope on the red line yeah if you're changing, well almost constantly.

Angela: It is isn't it like you could keep going smaller and smaller and smaller with your intervals.

Romina: You're changing your slope constantly but from point A to point B you're covering x amount of area

Angela: Two.

Pantozzi: Two.

Romina: So the average rate at which your red line is the slopes of your red line you know what I'm

Magda: Increasing or decreasing…

Romina: Are an average of your slopes… the average is the slope of.. no…

Angela: If it is just that chunk then it is two thirds

Magda: That's what I'm saying it's the average…

Romina: I was just thinking, when you said 2/3.

Angela: That's just like so… That's just like so taking the graph and being like…

Magda: But that's not the average because you're no going by one interval

Romina: … numbers…

Angela: That's why we're good as a team…

Pantozzi: Were about to run out of tape,

Magda: That's like taking the average between because you're going like one interval…

Pantozzi: We're close to running out of tape so can you look back at that… you were talking about something being 2 and were talking about something being 3 and let say this is 6.2 and this is 9.2. Can you put those numbers in here?

Angela: So *b* is 9.2.

Pantozzi: So what what's what?

Angela: So *B* is 9.2 and *a* is 6.2.

Pantozzi: F is the purple function.

Romina: F of 3 equals 2.

Pantozzi: So What equals 2?

Romina: No, that doesn't equal 2, wait.

Magda: That $g(b) - g(a) = 2$.

Romina: *A*.

Pantozzi: That whole thing equals 2.

Romina: Yeah.

Pantozzi: Thank you.

Romina: Do you not agree?

Pantozzi: I'm saying thank you.

Angela: You're welcome.