

Description: B43, Interview with Brandon about the Towers and Pizza problem (student view), Grade 4, April 5, 1993, raw footage Location: Colts Neck Researcher: Professor Carolyn Maher Date:4/5/93	Authors: Elizabeth Snee Verified: Robert Sigley Date: 3/3/14 Page: 1 of 42
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Line	Time	Speaker	Transcript
1		R3	Assuming that I have got it with me. Yes, I do. It got a little chopped off in the photocopier, but you can help me understand what it meant. Like the bottom I think did. Okay, let's see. Now this was on...[Brandon lines up the two sheets of paper]. Yeah we need some tape or something there. Okay. And, you can feel free to add to this. Alls I was I was telling you as we were walking over. The people at Rutgers were very interested with what you were doing here and in order for me to be able to explain to them what you did, I need to understand it.
2		Brandon	Okay.
3		R3	Okay? Do you want to tell me what you were doing here and how this, how these turn out to be pizzas – these zeroes and ones? Let me line that up for you. [Brandon: Okay.] Okay.
4		Brandon	I'll do it over again so then you can see how I did it.
5		R3	Oh, that would be neat. I would love that. [Brendon uncaps and recaps markers.] Are any of the working today?
6		Brandon	Well, these two have no points.
7		R3	Okay. Look at that. They got all mushed down. Okay.
8		Brandon	Well, since there are three toppings, four toppings that is...
9		R3	Do you remember what the problem was?
10		Brandon	Yeah, I remember how many toppings, how many different ways could you make toppings with four toppings. Pepperoni pizza and all that stuff.
11		R3	Yeah, there was pepperoni...

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12		Brandon	Mushroom, sausage, and pepper, pepper. Pepper, mushroom sausage, pepperoni.
13		R3	Yeah, so we selected from those four toppings.
14		Brandon	Okay, first you could have a plain cheese pizza with nothing on it. Then since they're in order, instead of going put checking off and doing one sausage and skip around doing all the pepperoni, pepper, mushroom, it would be easier to go in order. So, first I do pepper with blank. Nothing else. That would be a way too. And you can have a pepper and sausage with nothing else. Then you can have peppers, pepper, sausage, mushroom. Well, this kind of isn't actually the way I did it. I went like one, one had plain of that plain, and that plain, that plain, that and so on. It was the wrong way.
15		R3	Yeah, I remember you did. I think you did that when you started the last time, too, in class.
16		Brandon	First, I started going like one, two, three, four. But...
17		R3	Right. One, one with one, and then a pizza with two.
18		Brandon	It's going to be easier going this way.
19		R3	How are you going to change this now?
20		Brandon	I'm just going to change the way I do it. Okay. A blank. Nothing on the pizza. Then you can have one pepper on the pizza with nothing else. One mushroom on the pizza with nothing else. Then you can have a couple of sausages on the pizza with nothing else. Maybe a couple pepperonis. And then if you don't want any of that, you can start getting fancy and going to twos. So have a pepperoni and mushroom and nothing else. Then, a pepperoni and sausage with nothing else. Pepper and pepperoni and nothing else. And so on. Then since we're all done with pepperoni, you

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			could have a mushroom and sausage with nothing else.
21		R3	How did you know, how did you know to go to mushroom now? I'm, I'm interested in that. Okay.
22		Brandon	Why didn't I use pepper anymore?
23		R3	Yeah, yeah. Why didn't you use pepper anymore?
24		Brandon	Because I all ready used pepper there. It's all ready mushroom and pepper, and if I did mushroom, put one down for mushroom and then pepper, that'd be the same thing.
25		R3	Oh, I see.
26		Brandon	So, that'll be like a zero. So, each time you go 3, 2, 1.
27		R3	What do you mean 3, 2, 1?
28		Brandon	First, you, actually four. First, you could use all the toppings, then you could only, then since you used all ready that, you can't use, it's, that's one less. So, you could only use those three. Then use all that, all those. You get three choices with, with the first one. And with the second one, with mushrooms, you only get two choices because there's only sausage and pepperoni. And then with sausage, you could only do pepperoni.
29	5:06	R3	Okay, but I don't, what I don't understand is why when you move to mushrooms, why you can put it with sausage and you can put it with pepperoni, but you can't put it with peppers?
30		Brandon	Because that'd be the same thing. Because if I do that and put a one there. Right there. I all ready got pepperoni-mushroom, pepperoni-sausage. That'd be the same thing. It's just like saying you have an airplane and a car, saying you got a car and an airplane. It's still the same thing.

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31		R3	I see. Okay. I understand why you did that now. What do these zeroes and ones mean? Like what does the zero represent here?
32		Brandon	You have nothing on that. That's nothing. I don't know why I chose to use zeroes and ones.
33		R3	I was going to ask you about that where you got this idea from.
34		Brandon	I don't know how I got it. It just popped into my head. So then you could have a mushroom and the one pepperoni and since then you got, you already used sausage and pepperoni, you could, so all you have left is, then you can't do anything, then all you could do is a sausage and pepperoni. And now, you can't just do, put one by pepperoni because then that'd be the same as up there.
35		R3	Say that again, that last part.
36		Brandon	Since I, if I do that and go put three zeroes and a one right there because like that. That'd be the same right there.
37		R3	I see. Okay, so that would be the same as the pepperoni pizza with nothing else on it.
38		Brandon	Then...
39		R3	You missed a couple here. How will we know when we're done?
40		Brandon	We'll run out of all ways. I'll show, I could, you could tell when you run out. Now we could go to threes if you really like this stuff and if you're really rich. One pepperoni, a mushroom and sausage, nothing else. Anyway, I noticed also, the numbers, each time you go up into like another group of like if you go from twos to threes, it's like the numbers get higher sort of.

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41		R3	What do you mean by the numbers get higher?
42		Brandon	Well, like you, when you look at two, well, that, I couldn't say that. But, sometimes when you just do those - a pepperoni and mushroom and nothing else. It's like 1,100. Then you do pepperoni, mushroom and sausage and it's 1,110.
43		R3	I see what you mean by the numbers getting higher.
44		Brandon	It's kind of confusing doing it that way. So then 13, you could have, 13 you could have a pepperoni, a mushroom, no sausage, and pepperoni. This is where it gets really tricky. For 14, you could have, since a pepperoni, no mushroom, a sausage, and a pepperoni. For 15, you could have a pepperoni, a mushroom, no sausage. Oh I got that. Okay. That's one I got already. Okay for 15 you may be able to have no pepperoni, a mushroom, and sausage and pepperoni. And 16 would be a pizza with everything.
45		R3	And, what would that look like?
46		Brandon	Oh man, I think I got doubles here somewhere. I have no idea.
47		R3	Oh, let's see.
48		Brandon	Because that you should only have 15 ways.
49		R3	Can we go back and check that? You're looking back at this right?
50		Brandon	Yeah, that. This one's a right easy one. Okay.
51		R3	We check this.
52		Brandon	We have a blank pizza, correct. Correct. Correct. Correct. Correct. All these are correct.

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53		R3	Are they all different from each other?
54	9:38	Brandon	Yeah because I checked them with coloring(?). Oh here are the threes. This is where I get, got mixed up. A pepperoni, mushroom, and sausage. And a pepperoni sausage, a pepper sausage and pepperoni then you could have a pepper mushroom and pepperoni. Still different. For 15 you could have a pepperoni, mushroom, sausage and pepperoni and pepper.
55		R3	And what's this down here?
56		Brandon	Well, that would be 16 because we got doubles somewhere. We got doubles. That one was a double. That one would be exactly a mushroom, sausage, pepperoni, sausage, pepperoni, so we cross that one out.
57		R3	Okay. But what about that one?
58		Brandon	That one. Those three?
59		R3	How can we figure out is this one a duplicate?
60		Brandon	No. That one? I thought we draw, drew a line. A mushroom sausage and pepperoni. Don't have it. [R3: Okay.] We need straight ones. [R3: Okay.] And then you could only get up to 12 that's where you're going to stop because then it's all twos. And I wouldn't have a three and a two in the twos group like...
61		R3	Okay, yeah, these are in groups? [Brandon: Yeah, like first there's like...]Then you're thinking about these in groups?
62		Brandon	Yeah, like first there's a one group you would only have one topping. The twos group you only have two toppings. The threes group you only have three toppings. And the final group you have all toppings.

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63		R3	I see. So, you might if you in fact you were reorganizing this you might move this one. [Brandon: Yeah, that one's...] Where would you move it to?
64		Brandon	Well, if a mushroom, pepperoni, sausage I'd move that right there between 14 and 15.
65		R3	Okay, why would you move it there?
66		Brandon	Because it goes after that you have pepperoni-mushroom-sausage, pepperoni mushroom and pepper, and if I move it up one, that would be incorrect because you have still you would have a you would start off with a mushroom and you're still working with pepperoni, pepper so it'd probably go here because then you go into another different way, different group.
67		R3	I see. So is this one we did okay? Does this one work too?
68		Brandon	No, that one's kind of confusing for me. That one wouldn't work that well. That's like the best one.
69		R3	This is the best one?
70		Brandon	Yeah, that's a good, good working one.
71		R3	Why is that one confusing?
72		Brandon	Because I kind of like got confused during the middle.
73		R3	Can you show me what, can you show, you have them in groups here, can you show me what those groups are on here?
74		Brandon	Okay. You could go. There's one group. And, let me use a different color.
75		R3	Okay. And what group is that?

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76		Brandon	Okay. Here. The ones group.
77		R3	Okay. And what does that mean? The ones group?
78		Brandon	You only have one topping in that group.
79		R3	Okay. What about this one right up here?
80		Brandon	That would be a totally separate group. The zero group. Nothing. Now, you go into the ones group...
81		R3	Do we have, can I, I just want to stop you for a second. Do we have all the ones that we could possibly have in the ones group?
82		Brandon	Yeah.
83		R3	How do you know that?
84		Brandon	Because I went down one pepperoni, no mushroom, no sausage, no pepperoni. One mushroom, no pepper, sausage and pepperoni. One sausage, no pepperoni, mushroom and pepper. Then one pepperoni with nothing else.
85		R3	Interesting. Okay.
86		Brandon	Because if I did, because if I did that again right there that would be the same right there. [R3: Okay.] Because no matter where I put it, it would have a same in that same group. Like a one there then a one there.
87		R3	I see. Okay.
88		Brandon	Then you could have a twos group which would go about. A twos groups is like the most.
89		R3	What do you mean the most?
90		Brandon	You get the most out of two because you get more, you get more choices than one, and, and, you get more choices.

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			There's like you could have more on. You get there's more different choices like pepperoni-mushroom, pepperoni-sausage, pepper-pepperoni, and that so on so like the two group is like the biggest. Then you got the threes group...
91		R3	Can we slow down again? This is really very interesting, but I'm wondering now you have one, two, three, four, five, six in the twos group. Can you convince me that there is, there aren't anymore in the twos group. That there aren't seven or eight.
92		Brandon	You go pepper-mushroom. That's one. Pepper-sausage. That's two. Pepper-pepperoni. Three. Then you can't do anymore because you already used sausage once and mushrooms once in to tell that you are a and to see that you made duplicate look over there in one because if you just look there you'll see another one. But, if you see a zero there, that means it's not a duplicate because you got nothing there. So if you there's a one-one, then that'd be the same as there. Then you get into mushrooms. Mushroom-sausage. Mushroom-pepperoni. No more because then you would see another one down there in that same group with the mushroom and the sausage.
93		R3	How come, how come there are no more thought with the mushroom?
94	15:13	Brandon	Because each time you get less. If you start off with pepperoni, you got three choices because there's mushroom, sausage, pepperoni. Get two mushroom, you only got two choices because pepperoni you already used with mushroom.
95		R3	I see.
96		Brandon	Then the same with sausage. Only you do pepperoni. Then if I put pepperoni by itself that'd be the same as up there.

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97		R3	Okay. I'm following.
98		Brandon	So then you get after three, you go peppers-mushroom-sausage, pepper-mushroom-pepperoni, then pepper, pepper-sausage-and pepperoni.
99		R3	Is that all?
100		Brandon	Then you could do, start with mushroom. Mushroom-sausage-and pepperoni and that's all for the threes group. Because, I know you're going to ask why, because since if you do that that if you put that one there, it would still be the same, it would be the same as that. And since you're doing threes and there's only have three left, you could only have all three. All ones and no zero. And no pepper because you already used that. And no matter where you put another one, it would be the same as any one of those threes up there. So then your only choice left is having an all pizza with everything.
101		R3	Interesting. And what are we calling this group?
102		Brandon	The all. I don't know what I'd call that. The total.
103		R3	Okay. The total? We call these the zeroes, the one toppings, right?
104		Brandon	You had two toppings, three toppings, four toppings.
105		R3	You call it four toppings. Sure. This is very, very fascinating, Brandon. I'm really, really understanding what you did here now. Can I ask you a question?
106		Brandon	Yeah. What?
107		R3	Now, we've done some different problems when I've been in this year with Mrs. Zalee (?).
108		Brandon	Yeah with the blocks and all that.

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109		R3	Does this problem with the pizzas remind you of any other problems we've done this year?
110		Brandon	Like what do you mean? Like the way I did it?
111		R3	In any way, does it remind you of any of the problems that we've done? [Brandon: Does it?] It could be in the way you've done them.
112		Brandon	Oh yeah, it kind of a little reminds me of the blocks 'cause you would because my partner and I, whoever that was, I think it was Colin, did it in order. Like one, like you would do yellow, red, yellow, red then switch it around. Do the opposite red, yellow, red, yellow. It's kind of what we do here. Just do it in groups. Like that's what we did with the blocks with my partner.
113		R3	Now we built towers that were...
114		Brandon	Yeah. Opposite. How many ways could we make towers? How many ways could we make pizzas? The same problem.
115		R3	The same kind of thing. Do you remember how many towers there were? [Brandon: There was...] That goes way back.
116		Brandon	I think I could remember.
117		R3	Would you like, would you like these? Or would you like these?
118		Brandon	[Writes out tower combinations] Oh wait. I think I could do it. We had an all tower. Two an all tower. Shaded. Non-shaded. We had another tower. We were working with fours, right? [R3: Yes.] Then that then you would have the opposite of that which that would be two. That would be a group of four. And six because you would have, maybe you could have and maybe let's say umm one and one then you could maybe have over there. One and one. Yeah, there was

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			six towers. Six towers I remember.
119		R3	And that was, that was all the towers you could make?
120		Brandon	Yeah, six towers.
121		R3	That's interesting. Could you show me how those looked like with blocks? [Brandon: Okay.] I'll give you some blocks. You could sit down. I just want to get these for you.
122		Brandon	[Puts together blocks] Two yellows two reds. One and put red on the bottom yellow on top. One group. [Combines two reds and two yellows. Then makes red-yellow-red-yellow and yellow-red-yellow-red] Another group. [Combines four yellows and four reds] Another group. Another pair. Okay, 2, 4, 6, and now let's try something, oh wait, there's more than 6. [Pieces red-yellow-yellow-red and yellow-red-red-yellow] Another group. Okay, I think that's all. Two, four, six, eight. Eight. I think that's all.
123	20:33	R3	We talked about this before. Is there a way you could be sure you have them all?
124		Brandon	I don't, I kind of don't not know.
125		R3	Like when you did pizzas, you really seemed sure that you had them all. Don't you?
126		Brandon	Yeah, yeah, because that was on a graph.
127		R3	Is there a way to do that with towers?
128		Brandon	Yeah, you could do a graph with anything. Towers.
129		R3	Now, that would be interesting. Could you, do you think you could do it with towers? How would we make a graph with towers? What would that look like?
130		Brandon	Shaded equals red. Not shaded equals yellow. Okay. You could have a shade. Okay, it would be either like a shaded

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			like I was doing before a shaded, all shaded and then not shaded. Just a plain old tower. A red, one with red yellow like that one would be with that, that would be that. That would be one group. [R3: Okay. What connects?] You could have...then you could maybe have shaded, not shaded and shaded, red-yellow-red-yellow. Then you could have yellow-red-yellow-red.
131		R3	Okay. I'm con...you know what I'm confused about, Brandon? [Brandon: What?] Now, what I'm confused about is how can I, how am I going to keep track of this? Like here you showed me very nicely how I could keep track, but how am I going to know that I've, I've thought them all up if they're...
132		Brandon	I know. Well, it's harder to do it with the graph because you can't do like...
133		R3	Could we do a graph for this?
134		Brandon	You can't do like, you can't do yellows and reds because that would be too confusing because then you would have to draw the blocks. And if you just go like blank that, it would be too confusing, so you have to draw the blocks. Because here with blocks, that's one group. You could do two blocks.
135		R3	Would you, would you have to draw the blocks to do this? Could you...
136		Brandon	Oh wait. Now I remember how we found it out. We kept trying different ways. We tried, and since there was a, you could only use four blocks, we kept trying every single way we could think of. That'd be the same as that. Whatever we tried, we still got the same.
137		R3	What I'm wondering about now. I want to take this a step higher than we talked about it in December.

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138		Brandon	Oh wait, wait, wait. There's some more.
139		R3	Ah. There is more. Okay.
140		Brandon	I knew there was more.
141		R3	You can see I wasn't convinced you had them all. Yeah. You want some more blocks?
142		Brandon	I knew there had to be some more. Okay. What was that? You could do two yellows. Two red, a yellow, and a red. [Created yellow-red-yellow-yellow and red-yellow-red-red] That would be another group. Here we go, here we go.
143		R3	Well, you can think these all up in your head very quickly.
144		Brandon	Well, all you gotta do is make one that you could make the opposite of. All I had to do is make this one tower. Then you, I, it instantly came in my head. I could put red there, red there, and red there. And so then, you could make total opposites like that. [R3: I see. So these are all opposites.] Like where there was a yellow, you could put a red. Where there was a red, you would put a yellow.
145		R3	So these are all pairs of opposites?
146		Brandon	Yeah. You could have a yellow-red, a yellow.
147		R3	I'll line these up so you could see them.
148		Brandon	Yellow-red-red-red.
149		R3	Oh, I'll get you more blocks. You don't have to keep jumping up. Looks like you need some more yellows while you're over there.
150		Brandon	This should be way more than I need. Okay, then you could have those. Then since, once you've seen one group, you could make then you could make just the same with doing opposites. Another group. But actually, it would be easier

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			when I did this. Wait a minute, same. Okay, that group's gone. Okay, so you could start off with a bottom then work your way up. You could work your way up like this.
151	25:25	R3	What do you mean by working your way up?
152		Brandon	Well, once you've, when you get to one of one color like one yellow and all reds or one red all yellow, you can start with the bottom and move like for the red group, you could start with one red three yellows, and go a yellow-red-yellow-yellow, yellow-yellow-red-yellow, and [R3: You're moving the red.] yellow-yellow-yellow-red.
153		R3	You're moving the red. Okay, so you're moving it up one each time. [Brandon: Yeah, then maybe you could have...] That's a good idea.
154		Brandon	[Makes yellow-red-yellow-yellow and yellow-yellow-red-yellow] Then you can do that. That'd be another one. It's kind of like stairs. Then you could have all yellows and one red. Now you just switch it around and do the opposite. That one would go with what was that? That?
155		R3	These guys?
156		Brandon	Yeah, I made, then you could make one of these. I would go...
157		R3	So you already have that first one. I see.
158		Brandon	Yeah, I didn't notice that. [R3: That's interesting.] You could do that one. Two reds-yellow-red. Instead of two yellows-red-yellow, two reds-yellow-red. That'd be another pair. Then you could do maybe all reds and then a yellow on top. And it's kind of like the pizza problem. You start off with maybe group like this one would be the ones group.
159		R3	Let's see what you're talking about.

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160		Brandon	Oh, now. I see this now. This is like the ones group. You only have one of the opposite color in there. This isn't how I did it, but I, but I just noticed it.
161		R3	This is fascinating to me.
162		Brandon	I just noticed it. Then you would have, that would be the ones group. You only have one in there.
163		R3	I guess you got this pair. [Brandon: And that pair. And that pair.] And this pair.
164		Brandon	And that pair. That pair would be the ones group.
165		R3	Okay. These are the ones then.
166		Brandon	[Has yellow-yellow-yellow-red/red-red-red-yellow, yellow-yellow-red-yellow/red-red-yellow-red, yellow-red-red-red/red-yellow-yellow-yellow, yellow-red-yellow-yellow/red-yellow-red-red lined up.] Then you, then in pizza, this would be like the whole group. [Holds red-red-red-red/yellow-yellow-yellow] All groups. Save that for last. One group. Now you have the two groups. [Holds red-yellow-yellow-red/yellow-red-red-yellow] The twos. You have a red, you have two of the opposite colors in there. Same with here. That's one group. Two of these.
167		R3	These are the ones.
168		Brandon	Here are the twos group. [Has yellow-yellow-red-red/red-red-yellow-yellow and red-yellow-yellow-red/yellow-red-red-yellow lined up in front of him] You have two of the opposite color in there. These would be like plain pizzas. Plain pizzas or all pizzas. Oh wait. These would be in the twos groups. [Adds red-yellow-red-yellow/yellow-red-yellow-red] They've got two of the opposite color.
169		R3	How many, how many towers do you have in the twos group

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			here?
170		Brandon	Pairs or each separate?
171		R3	Separate towers.
172		Brandon	Two, four, six. Six.
173		R3	Six of them. Now, that's interesting. Isn't it?
174		Brandon	<p>Eight. Eight and oh yeah. Eight and ones. Each time you go to a higher level of towers like you have most of ones because you could do one there, one there, one there and one there, and that would have a ton of those. But with the twos you could only have, you must have two in each so that takes away from the other, the opposite color so that you can't pile them as high. What I'm trying to say is like with one, with the ones you could do like that [makes yellow-red-red-red], but then since you have the others when you try to do it, you can't pile them as high with the opposite color. And if you tried to do that, you couldn't do that. You couldn't pile it as high as the opposite color.</p>
175		R3	I'm not following.
176		Brandon	I know. I'm not following myself either. Okay.
177		R3	I'm very interested in what you did here. The way you said it's like groups?
178		Brandon	<p>I know. Yeah, what you do is then you could have one, you have one of the opposite color in there that means there would be more ways I'm going to say opposite because there are more yellows of the opposite. I'm going to say opposite because if I was using green and yellow, I won't say red and yellow. So then you would have an opposite which would make it more but here you go, you use...</p>

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179		R3	Aren't there opposites over here?
180		Brandon	Yeah, but here, over here, you must use two of each. Two reds and two yellows. If you use two reds and one yellow, it'd be three. Two reds and three yellows it'd be five. The same for yellow, the same would go for yellow. So you'd have to have less, because you would have to use two of the opposite. So you could have these. Like that right there. And since, if we're working with fives, there'd have many more. But since we aren't not working with fives, you would only be able to do it like this. There must be at least two of the opposite color for it to go into that group or go into the ones group.
181		R3	Now, can I ask you a question? This is very interesting.
182		Brandon	Because if you go like this, [makes yellow-yellow-red-yellow] [R3: Maybe you can help me.] ones groups. For the ones groups, you'd have to have at least two, two colors. Like...
183	31:02	R3	I think I'm starting to understand. Like there's. Is this what you're saying to me: that there's more ways to rearrange something like this [holds yellow-red-yellow-yellow] than there is to rearrange something like this [holds red-yellow-red-yellow].
184		Brandon	Yeah, yeah, because this, you could start with the bottom and put one up each step. That would be a lot. But this, if you do it, you would have to go like. This would be a good example [holding yellow-red-yellow-red]. Put these two there [pointing to red], then you could move them one step higher. You moved them another step higher, they would look like this. How would I do that? You would start from this, the low part, move it up to one higher part. Now if you moved to a higher part, it would look like this [creates red-yellow-red-yellow-yellow]. [R3: Yeah.] But that's five so

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			you can't. [R3: I see.] But if you take off one, then it'd still be the same, no matter which you take off. The top same. The bottom same. [R3: Okay now you had...] The bottom, take that off, it's the same as that. Take off the top, it's the same as that.
185		R3	Oh, I see. Now you had, I just wanna, I just wanna finish what you had here. You had this guy and you had this guy also. [Creates 4 yellows and 4 reds] Okay, and they were in a group by themselves. Okay, so you said that these all had two. Two yellows or two reds?
186		Brandon	Two. They had, they had two of each color. They must to be in the twos groups. If they had three of each color and one of the opposite, they would be in ones. And you won't have any threes group because that would be the same because...
187		R3	Why not?
188		Brandon	Because for threes group, a three group, you have three yellows one red. That would be the same. A three group is like that group because three opposite color so that would be a three group would be the same as one group.
189		R3	Can I ask you now, if, could we call, if I, if I wanted to, could I call this all a threes group?
190		Brandon	Yeah. You could call it a one or three group.
191		R3	Okay. Why?
192		Brandon	Because you could call it a three group because it has three opposite colors. Three colors of one and one opposite. Or you could call it a ones group because it has one opposite color. You could call it three or ones. It doesn't matter which.
193		R3	What if I asked you to focus now on the color in these

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			towers?
194		Brandon	What do you mean? Oh just use color?
195		R3	Now if I asked you to focus on the colors, and to take another look at your groups. Say we focused on, which color do you want to focus on? Red or yellow?
196		Brandon	I don't care.
197		R3	Your favorite?
198		Brandon	Let's do yellow.
199		R3	Okay, if I asked you to focus on a particular color like yellow, okay, and then I asked you to tell me what the ones towers were, could you do that?
200		Brandon	What the ones towers with yellow would be? Only if you used yellow?
201		R3	We're looking at yellow.
202		Brandon	Oh, so there's no red? If you only used yellow?
203		R3	No, what I'm saying is if we're putting our focus on the yellow because there is red and yellow in all of these, right?
204		Brandon	Yeah, because if you used just yellow, you would use only that tower [holds 4 yellow tower].
205		R3	Sure would, wouldn't you? If were looking at these 8 towers here, and we're looking at yellow [Brandon: Oh like almost yellow], and we're looking for ones, what would, what would be a ones tower in?
206		Brandon	A ones yellow tower?
207		R3	Yeah.

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208		Brandon	[Holding red-yellow-red-red] That would be a ones yellow tower. And that would be a threes red tower.
209		R3	Okay, so it's a one yellow and three red.
210		Brandon	Yeah.
211		R3	Okay, what else would be a, a one yellow here?
212		Brandon	[Selects red-red-red-yellow] One yellow.
213		R3	You, can you, that's interesting. Can you pull the rest of them out that would be?
214		Brandon	One yellow. And that's all the one yellows. No, there's one more one yellow. There must be one more.
215	34:56	R3	Why? Why did you say there must be one more?
216		Brandon	Because, because before I only saw three and there's 5 up there. So three, five take away three is two. So there must be, so you could put another one in that group which would even it out.
217		R3	Oh okay. So you're evening the groups out?
218		Brandon	Anyway, I just looked at the tops because I saw no yellows bottom pieces, right there. [R3: I see.] And also, it's like the pizza problem. You work your way down. [Points to ones yellow towers] Like pepperoni, mushroom, sausage, and pepper.
219		R3	Wait, do that again for me. [Brandon: Yeah.] This is, this one here.
220		Brandon	Yeah, because we're kind of like see how this sheet says, how I...
221		R3	Let's look at the ones. Where was that?

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222		Brandon	Here's that sheet. How I...
223		R3	Is this it? This was your last one.
224		Brandon	Yeah, that's page one. It's kind of like that. You start with zero. You have, you could...
225		R3	What would the zero one look like if we're looking at yellow?
226		Brandon	Zero one?
227		R3	Zero yellow.
228		Brandon	[Takes away blocks] Blank.
229		R3	Now, I don't get what you just did.
230		Brandon	It would be nothing. A zero one's tower would be...
231		R3	A zero, a zero tower, if we're looking at yellow, would be nothing?
232		Brandon	Yeah, it would be nothing.
233		R3	Well, what was a ones yellow?
234		Brandon	A ones yellow. If I could find it.
235		R3	You showed me one yellow.
236		Brandon	Yeah. A ones yellow tower. Here's a twos. One's red. One yellow. Now just look at the top of the groups. Ah here we go. And all these are different groups. All the rest are threes. All the rest are fours.
237		R3	You go so fast for me, Brandon. Tell me again how this is like the pizzas.
238		Brandon	Well, you have one pepperoni. That'd be like, one pepperoni is like, since we're looking at yellow, the yellow would be

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			one and the reds would be zeroes. You could have one pepper, like it shows here, and right there, I got then it's like stairs, you were, if I draw a line down...
239		R3	You need a pen? Let's get these out of the way.
240		Brandon	If I draw a line down here like this, it'd go like sort of look like stairs.
241		R3	I see.
242		Brandon	Then you'd go across, draw a line down here. Go across, draw a line down here. Go across, draw a line down there. Go across so you would have like one, one, one, one. Sort of like here you have one pepperoni, one mushroom, one sausage, and one pepper.
243		R3	Is what you're saying to me then like a yellow cube here is like a number one when in your chart?
244		Brandon	If we're focusing on red, then red would be a number one.
245		R3	Well, let's continue with yellow. This is interesting. I think this is really neat. Now, what would come next with what we have here if we want to reorganize? You said these would be like the one, yellows. [Takes y-r-r-r/r-y-r-r/r-r-y-r/r-r-y]
246		Brandon	These would be the ones group. [R3: Now, what about...] Now you would start with the two yellow group. [R3: Okay.] But since, but since we're working with opposites, the two yellows group, you could only use one 'cause that would be just the same. Two pepper, no it wouldn't. Then you would have maybe a pepperoni, mushroom...
247		R3	Where were all our twos towers? There's, is this one? How many were there? [Brandon: Here, I'm gonna lie these down.] How many were there? I forgot.

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248		Brandon	There was...here's a two. If you lie 'em down and face them sideways, it would, okay like you're focusing on yellow, you put yellow right there [puts blocks down on pizza number chart], that'd, if these would stretch them out far enough like put down for that you would have yellow-yellow-yellow, nothing. Oh, oh, that's page two.
249		R3	Let's go back to page one.
250		Brandon	Page one is always going everywhere.
251		R3	Page one keeps flying south.
252		Brandon	Okay. First, you would have like one yellow-yellow-red-red. Same here. Because if you lie 'em down, stand them up, it'd be harder to have to stand up the paper. So, yellow-yellow, one red. [R3: Now, I understand.] That would be a two. Then you could have...
253		R3	Yeah, where would the tower be that would look like this pizza?
254		Brandon	Right here. Right here you would have a yellow stand for one. So it would have a yellow one, red zero, yellow one, red zero. [R3: I see.] That would be another one.
255		R3	So this would come next.
256		Brandon	Yeah. Now, you could have...
257		R3	We have other ones.
258		Brandon	Yeah, right here. Now right here, you could have a yellow, yellow-red-red-yellow. Yellow-red-red-yellow.
259		R3	[Holding yellow-red-red-yellow] So what would this pizza look like? This one?
260		Brandon	That would be pepperoni and pepper. Pepper and pepperoni. But now if we're doing it in opposite groups, this would be.

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			<p>Wait. Let's do this one. [Holding yellow-red-yellow-red] Now, we'd do, that'd be yellow-yellow-yellow. Wait a minute, There's, I need that one. [Holds red-yellow-yellow-red] Then you'd do like red is zero. So you have nothing there. That's correct. Yellow, yellow, yellow, yellow correct. Red zero. Correct. So that'd be another one. Now, you could do, you use this for that on page two. Now, page two you would have this one. [Holding red-yellow-red-yellow] Red zero, yellow one, red zero, yellow one. That go, that's another goodie. [R3: Okay.] Now, all you would have left was two reds two yellows. Two reds two yellows.</p>
261		R3	<p>Fascinating. Okay. And then are we out of them? And are we out of towers with two?</p>
262	40:46	Brandon	<p>Yeah, because there would be no more. Look at the chart. Now, now, where is the threes? [R3: Okay, now we...]But now, if we do threes...</p>
263		R3	<p>Can we, are we still sticking with yellow as the color we're focusing on?</p>
264		Brandon	<p>If we're focusing on playing yellow, then you would swipe out, then these would be the ones for yellow.</p>
265		R3	<p>These are the twos.</p>
266		Brandon	<p>Yeah. [Holding y-r-y-y and y-y-y-r] This would be the threes for yellow because then you would have. 1-2-3. 1-2-3-blank. 1-2-3-blank. That's in for threes. Then you would have a two, a two zero one. One one zero one. [Pointing to y-r-y-y] One one zero one. And that's in again. Then you would have one zero one one. [Pointing to y-r-y-y] One zero one one. That's in. Now you could have a zero one one one. [Holding r-y-y-y] Zero one one one. That's in. Now, if we're just focusing on yellow, this then would be the pizza with everything.</p>

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267		R3	Oh. I see. Okay. And are we missing any?
268		Brandon	No.
269		R3	You know what I'm wondering? We have this guy left. [Picks up 4 reds] Right? [Brandon: Yeah, because we're not focusing on him.] Because he's the opposite of this guy.
270		Brandon	Yeah, we're not focusing on red.
271		R3	If we had to call him a name...
272		Brandon	This would be the zero. Oh yeah, since the red would stand for zero, this would be the zero guy.
273		R3	This is neat. This is really neat, Brandon.
274		Brandon	Finally found out what the red would be. Red zero guy.
275		R3	Okay. Could we have done it the, I just wanna ask you. You don't have to do it, but could we have done it the other way around? Could we have just focused on red and, and gotten it to work the same way?
276		Brandon	The same way. Just, just look like this. [Transposes ones standing towers with threes standing towers] Here is the ones group. Twos group. [R3: One red. Okay.] But the twos group would be the same. And then all you do...
277		R3	And, and what would these be? What would these be?
278		Brandon	That would be the threes group. Just switch, then just switch those around. Same thing.
279		R3	Neat. Now, would, would we be changing the number names for red and yellow? In other words, when we did this...
280		Brandon	Now the reds would be one and the yellow would be zero.
281		R3	This is really nice. Are you convinced that you've found all

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			the towers and all the pizzas?
282		Brandon	Yeah. Yeah. All the towers. All the pizzas.
283		R3	They both come out to how many?
284		Brandon	16. 2-4-6-8-10-12-14-16.
285		R3	You convinced of this now?
286		Brandon	Yeah.
287		R3	Yeah? This is really nice.
288		Brandon	Oh, and you could also do the opposites on the pepperonis and sausage thing.
289		R3	Oh. How would that work?
290		Brandon	It, it would be easy. But it would be a little more. It would be a little harder.
291		R3	Yeah, but you could just show me how to start. You don't have to do the whole thing now.
292		Brandon	I won't. I'm not going to do the whole thing.
293		R3	Because I don't want to make you nuts here.
294		Brandon	Okay. So, you could have a pepperoni, a mushroom, no sausage, no pepperoni. No pepper, no mushroom, sausage, pepperoni. Two opposites. [R3: Interesting.] And that would be just like these two. Nope. If we're focusing on yellow, it would be sort of like [R3: Yeah, which ones would it look like?] 1-1-0-0. These two. [Selects r-r-y-y/y-y-r-r] [R3: If we're focusing on...] Yeah, if we're focusing on yellow, it would be 1-1-0-0. [Holds y-y-r-r] 1-1-0-0. 0-0-1-1. [Holds r-r-y-y] 0-0-1-1.
295		R3	Oh. Can I ask you what you think now? Which do you think,

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			<p>which argument do you think is more convincing: grouping them by zero, one, two, three, and four, those four categories, which is five categories, [Brandon: Or by opposites.] or by opposites? Which convinces you that you have them all? Which way?</p>
296		Brandon	Tough.
297		R3	Because they're are both ways of going about them, right?
298	45:09	Brandon	It depends on what you like. If you're better with opposites, do opposites. If you're better with grouping or playing by categories, do categories.
299		R3	Interesting. I guess my one concern with opposites is that, how do I not know there's another one out there in space somewhere that I haven't thought of with an opposite? You know what I'm saying?
300		Brandon	<p>We're only using four blocks, so you could only start out with four, you could, well, to put, let's put it, let's make it easier. You could only have two, you could only use four blocks, so you, there, there, there wouldn't be that many opposites. The, an opposite would be like take that and that. [Selects r-y-r-r/y-r-y-y] One opposite. Another opposite would be [Selects y-y-y-r/r-r-r-y] that and that. Another opposite would be [Selects y-y-r-y/r-r-y-r] this. This. And this. This. [Selects y-r-r-r/r-y-y-y] Those were two opposites and to prove that those are all the opposites to ones, you could, you would make stairs, different colors, multi-colored stairs. Y-r-y-r. Then you could have those. All yellows, all the yellows would go up, in stairs, like stairs. The three, the one group for yellows or three groups for red would look like that. And the same would go for here. And now, since we're only using four, any more would be in the twos or threes, twos or threes groups.</p>

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301		R3	<p>Okay. Now the twos, I can see that for these making a staircase, [Brandon: Yeah, those were easy.] but the twos, [Brandon: Are hard.] thinking about that in opposites, and knowing that I in fact know them all, I find that difficult to do.</p>
302		Brandon	<p>Okay. You have two of those. [Selects y-r-r-y/r-y-y-r] And since you're only using four, you've got two of those. Two of these. [Selects r-r-y-y/y-y-r-r] Two of these. [Selects r-y-r-y/y-r-y-r] And that's all for the twos group. 2-4-6.</p>
303		R3	<p>But, how do you know that though? I mean, I know you showed it to me with your pizza drawings, [Brandon: I know.] but if I'm doing it in terms of opposites now, which is what you're saying, how do I know, how do I know in terms of opposites that that's all of them?</p>
304		Brandon	<p>Find them. You can't do anymore if you try, whatever you try, it comes out one of those. Because since they're opposites, if, if we never had that one, and we looked hard enough, you could see that you have y-r-r-y, but you don't have r-y-y-r. No matter what you try to do, you'll always end up with one of those. Like for that, whatever you do, you try to do one, you will, you will either end up with like y-y-r-r, r-r-y-y. Anyway, it's just like, it's just like turning them upside down. You start a y-y-r-r like that would turn upside down would become that. That's sort of, and no matter what you try, you would always end up with one of those.</p>
305		R3	<p>I'm fascinated by what you showed me with how you can lay this like this. In fact, look, it fits. And, and how this, let's see this one, [Selects r-y-y-r] number 10 here, would be this one, right, because we call the ones yellow. That's really neat the way you thought of that. You know I can, I can see that. And you've convinced me with this picture that you</p>

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			found all the ones and the zeroes. And it's very nice.
306		Brandon	And all the others.
307		R3	You're a very good thinker, you know that?
308		Brandon	It was Colin's idea for the graph. I was, I was thinking about what to do because I didn't know how to divide. Once Colin got...
309		R3	Colin thought of the graph?
310		Brandon	Yeah, so then we, so then I, we started doing, working separately. We split up then we went and came back together and compared answers.
311		R3	That's really a good strategy. Can I ask you a question? I'm just curious. What do your mom and dad do for a living?
312		Brandon	My dad makes pool liners. My mom just works at home.
313		R3	That's hard work, Brandon. Take my word for it. That's really hard work. That's what my mom does, too. Okay. I want to ask you one more question before I let you go. [Brandon: What?] Have you thought at all about the towers of different sizes anymore? Remember, we started, we started to talk about that. We talked about towers that were three high and five high. We were talking about different heights.
314	49:58	Brandon	Oh three high. Those would be, there would be about 6, less than 6 ways.
315		R3	How do you know that?
316		Brandon	Maybe. 6. There would be 6 ways.
317		R3	Can I ask you a question?

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318		Brandon	I'm going to make a theory there is 6.
319		R3	Okay. Let's test, let's test that theory. Can we test it? [Brandon: No.] Well, you could do whatever way you like. I almost wish I could see you test it with your graph. Can you make a graph to show that for threes?
320		Brandon	I don't think so. It'd be hard. When you're using only, when you're using only colors and cubes, and, and, it's hard to make a graph. More likely it'd be easier to make the cubes than the graph, and make like, but you can't say like yellow red. It'd be easier if you called them something like maybe pepper, peppers, tomato, carrot, and tomato.
321		R3	You know what I'm thinking?
322		Brandon	No, zucchini and tomato.
323		R3	Which, kind of like what you showed me here where this kind of went with this? [yeah] Okay. Could you do that for threes if you had a key that kind of told [oh yeah] you that zero and one were whatever color they are?
324		Brandon	Yeah, but even if I do threes, I could, a three would never fit on that chart no matter what I do.
325		R3	Why not?
326		Brandon	Because since we're using four different toppings, and since we're only stacking them in three stories high...
327		R3	Well, could we make our chart look a little different?
328		Brandon	Yeah, if we take off one. It would be wrong then because then you would have to make it less. There would be many opposites. There would be many sames. So you could have a yellow-yellow, a yellow-red-yellow, or a red-yellow-red. That's one pair. And since you're only working with three, if

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			you have my favorite, then it's 6.
329		R3	You said 6.
330		Brandon	So then since you can't do it anymore, you could only do. Then you could do a red. Yup. It's 6 alright. The answer is 6 already. You could tell. You could tell it's 6.
331		R3	How? I couldn't tell.
332		Brandon	You could tell it's 6 already just by making that one more. Actually, you could tell it's 6 by just looking at that. By just in your head. Since there's only 3, you could draw it actually too.
333		R3	Let's get another piece of paper.
334		Brandon	No, there's enough room on the back of this. Don't waste paper. So you could do just like shaded would be red. 1-2-3. Shaded-shaded-shaded. Oh wait. No, it would be more than 6, I think. Then you could have blank-blank-blank. Same as always. Maybe shaded-blank-shaded. Or this, this one would go with that one.
335		R3	Okay. These are opposites then?
336		Brandon	Yeah, then that then you could have blank-shaded-blank. [R3: Okay.] Another one would be, this would be group of two. Four. That would be 4. Here's 6. You could have one blank, blank. Blank-blank-blank one-one. Then you could have blank-blank-shaded. Shaded-shaded-blank. Six. Oh, the answer would be 8.
337		R3	Okay, so wait, now these stand for shadeds? [Brandon: Yeah.] These things that look like little squiggles? [Brandon: Those are reds.] And these are blanks so they're zeroes? [Brandon: Yeah.] Can you fix these so I can see that too because I'm looking at two different systems here? This is good. Interesting. Okay, now how do you know you have

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			them all?
338		Brandon	Well, I'll make the ones into cubes. These two would go with that would be that. [Combines y-r-y/r-y-r] This would be this.
339		R3	Okay, that's a pair of opposites. I see that.
340		Brandon	Now, where is that other one? Then you could have those two, which would be together. [Combines y-y-y/r-r-r] Then maybe, where's that other pair of threes I had? Okay. This, and you could have a red and two yellows. Then a yellow and two reds. Okay, now after that, oh, I confused myself kind of. Now, now it's right. Then you could have those two. You already have 6 groups. It should be 8 because I forgot about, without counting these [holding r-r-r/y-y-y]. If you didn't count it, it would be 6, but those are colors, so they count. Then you could work like from bottom up just like over there. You start with a bottom red and yellow, you work your way up to those two. And you could only do these, it depends on what color you're looking at. That would be one then you could have two reds and a yellow. And that would be all.
341	55:57	R3	What do you mean by work your way up? I think I see it, but I want to be sure.
342		Brandon	You start with two yellows and a red. A red, a yellow, and a red on the bottom. [Okay] You can hook those on and make that one group. Just for one, pretend those are not there. You start off with those wherever you see the main opposites, then you would have like another one right there. Those would be the two main opposites. Then you could work, then you would go to the highest, and those would be the two main opposites. [R3: I see. Okay] Take out all the yellow, yellow ones and they'd look like that.

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343		R3	You're going to make the...
344		Brandon	Staircase.
345		R3	Staircase.
346		Brandon	And when you do it, the staircase should only have three stairs for it to be right.
347		R3	Why?
348		Brandon	Because if, to tell if you have one of the same, it would look like this. Then you say, "Oh I got another one." Put it somewhere. The stair goes down. You have, you walk up a stair. Walk across two stairs, and go up. Fine, switch around. You go up three stairs, down one. But there you would have to crawl, jump to a stair, walk up and go down, [R3: I see.] so it would make no sense. So that, then, you would take that away, and the same, is true for the red.
349		R3	So you got these, these, and these. [Points to r-r-y/r-y-r/y-r-r; y-y-r/y-r-y/r-y-y; r-r-r/y-y-y] And, you sort of showed me with this picture here, sort of like what you did on this chart here, right? [R3: Yeah] Is there a way, I guess what I'm wondering is now, could we apply this chart you made, which I think is a very interesting nice strategy to towers of different heights or pizzas with different amounts of toppings?
350		Brandon	Yeah. Easy.
351		R3	How, how would that work? Like I'm wondering...
352		Brandon	Okay. Now, let's focus on red now. I'm going to focus on red. Right there. 1-1-0-0. 1-1-0-0. Okay. But if you use the yellow, this one [r-y-r], it wouldn't be right. The first three would be right. You would have red 1-1-0-0. 1-1, what happened to the other? Take out, you would have to...

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353		R3	Could we take off a column maybe and then look at it?
354		Brandon	You would have, it would be, you would have a lot of opposites.
355		R3	I see what you're saying.
356		Brandon	Because they have opposites. [R3: Like what happened to the...] Like now you have a two, now you have groups of, twos become, threes become twos, groups of twos become ones. Sort of. Some of the groups. Wait. Okay. You would have opposites like here's one group. Let me just see, let me just find that twos group. There's the ones group. The twos group. Take that off, you would have in this group, where's the other sheet with that? Alright. Here's how we have lots of opposites. I mean lots of same. Okay. Now, take that off. You would have. Now, here's a threes group. Fine, you take off one. Now, you only have a pepperoni, pepper and sausage. Pepper-sausage. Take one, just take off one column. That screws the whole thing up. Pepper-sausage. Pepper-sausage.
357		R3	I see. I see duplicates.
358		Brandon	Yeah, for all. Oh, take off one. 1-2-3. 1-2-3.
359		R3	How many duplicates do you think there are going to be, now that we took off a topping here? [Brandon: A lot.] Is every tower here, is everything here now, this number going to have a duplicate, do you think?
360	1:00:01	Brandon	Yeah. About. Yeah. Every one probably. In the twos group 1-2. Threes group, that'd be here. 1-2. 1-2. Every one has a duplicate. We'll call them. take off two columns, then everyone, then you would, you would have way less. You would have 1. You would have lots of duplicates
361		R3	That's a good question. How many, how many possibilities

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			would there be if we just had the two columns with our zeroes and ones? You're right, we're going to have a lot of duplicates.
362		Brandon	Only two.
363		R3	What would they be?
364		Brandon	Pepperoni and. No. Only one. Only pepperoni with mushroom.
365		R3	You sure?
366		Brandon	Wait. There'd be 3. There'd be 3. Pepperoni no mushroom. That'd, that would be one. Pepperoni-mushroom. That would be two. No pepperoni-no mushroom that would be three. And that's all you could do. [R3: I see]. Right here you have a number 10. Like 1-0, 1-0. And there'd be more 1-0s. One, another 1-0. And now for, and now you've got 11s. 1-1. 1-1. Wait, wait. You'd have 1. 1-0. 0-1. 0-1. Oh now, you would have 3, you would have 3 duplicates. Three of each one would have 2 more. Two others the same.
367		R3	Two others the same?
368		Brandon	Yeah, if you, wait if you use 3, each one would have only one other the same.
369		R3	So, if I'm looking at 0-0, [Brandon: Yeah.] how many times do I, will 0-0 appear?
370		Brandon	One. One 0-0. Two 0-0. And, there's one more somewhere. Three 0-0.
371		R3	So there's one...
372		Brandon	Two, three.
373		R3	Two. What about this one?

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374		Brandon	That's....oh yeah, that would be four. 0-0. 0-0. 0-0. 0-0.
375		R3	Do you agree with that? [Yeah] Did I count right? Interesting. Okay, so let's take this back to towers now. Okay, we did pizzas with three toppings, two, three, four toppings. Now, you're telling me about pizzas with two toppings. Okay? If we go back to towers, selecting from two colors, red and yellow again, how many towers do you think there would be that would be two cubes tall?
376		Brandon	Two cubes? There would only be three?
377		R3	What does it look like?
378		Brandon	No, make that 4, make it 4. [Creates r-r/y-y/y-r/r-y] That's all.
379		R3	Okay. Now, let's go back to pizzas when we have two toppings. Let me take another sheet here.
380		Brandon	What color do you want to focus on?
381		R3	I'm going to call, I'm going to say we're gonna have mushroom and [Brandon: Pepperoni.] pepperoni as our choice. Okay? Okay, I'll just write "m" for mushroom. Okay. Now those are the two we're selecting from, right? [Brandon: Yeah.] How does this match up to towers now? We made. You said that there were three pizzas with two toppings, right? What would those be again? Could you show me with?
382		Brandon	With just these two, you would have 1-0. 0-0-1.
383		R3	And what would, what would, where would the?
384		Brandon	Oh, no, wait. We gotta to start over again. First, I want to, first I have to know, are we going to do it by how, you could have like just one pepperoni, how many pieces of that could you have on like you could have on just one pepperoni on a

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			slice, two pepperonis on a slice, three pepperonis on a slice.
385		R3	Does that matter?
386		Brandon	Yes, it does a lot. Okay. Let's do mushroom and anchovies. You could have one mushroom, zero anchovies. Two mushrooms, zero anchovies. Three mushrooms, zero, zero anchovies. Four mushrooms, zero anchovies. [R3: I see what you're saying.] You could go as, you could go up to a billion.
387		R3	Is that what we were doing though before when we were looking at these?
388		Brandon	No, no before when we were doing the pizza problem, you would just do one.
389		R3	What did the zero and one mean? I...
390		Brandon	Like one could mean, yeah, you have it. Zero, zero would mean no. That's nothing.
391		R3	I see. If we go back to that system, of it, of what you called one and zero. You said one meant it had it. Zero meant it didn't have it. Then how many pizzas could we make selecting from anchovies and mushrooms?
392		Brandon	Oh, without depending on how many... [R3: Yeah.] Just with that, it would have one mushroom-zero anchovies. Zero mushrooms-one anchovy. And I think...
393	1:05:26	R3	So there were how many?
394		Brandon	Four.
395		R3	Four. Does that match up to what we did over here with these?
396		Brandon	Yeah. Yeah. Let's focus on it now. Red, red zero-zero.

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			Yellow one-one. 0-0. 0-1. 1-0. 1-0. And there would be 0-0. 0-0. Yeah.
397		R3	That is really fascinating. Do you think that's always going to work? Like does it matter how many toppings we have? Could we do something like this?
398		Brandon	Yeah, but if we're going to do it, like how I said earlier, how much, how many toppings you're going to have on one pizza, like, we're going to have four pieces, four mushrooms on a slice, you would have to tell us what the top number of [R3: Sure.] pieces you could have on a pizza or else you would go onto a billion.
399		R3	Oh sure. See, I guess, I guess, when I was thinking about the pizzas in my head, [Brandon: Yeah.] I was just thinking that we we're kind of throwing the stuff on the pie. Have you ever seen those machines like at Domino's Pizza where it comes out of, like it throws it on the pie, and I wasn't worrying because some pieces may get four slices of pepper, but some may only get one or two you know because it's kind of scattered by the machine. So, I was just thinking about whether it has some on the pie or it doesn't have some on the pie. Not worrying about the individual pieces.
400		Brandon	Well, if it was like, if it was being handmade, pulling a certain amount each, you would. Someone would have to tell such top number to stop at. And if it was 10, and you're only using mushrooms-and anchovies, there would be 20. You could have 20 in total.
401		R3	You think so?
402		Brandon	Yeah. Ten. You could only go up to 10 slices, 10 pieces on each pizza. 10 x 2 because there is only two of those.
403		R3	Two toppings. And what would the 20 mean here?

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404		Brandon	That in total, in total you could have 20, 20 on each, if, card if like on each card you could have like 20. It would be 20 if you wanted the total amount. The biggest. You would have 10 mushrooms. 10 anchovies.
405		R3	On the pie?
406		Brandon	But. Mainly, if it were medium there's 8. You would have to do 10x8. And 8 on each slice, there would be 10. 10x8. 80. 80 pieces in total.
407		R3	Oh. Okay, so you've told me number of pieces and this is the amount of toppings.
408		Brandon	Yeah, if you only ordered one pizza with everything, the top amount would be 20.
409		R3	The top amount of what?
410		Brandon	How many pieces of each you could get in total, since you could get 10 mushrooms in total and 10 anchovies together. [R3: Ten mushrooms and 10 anchovies around the pie.] And since [R3: Or only 10 on a slice?] you're only using two, that would be 2. And times it by two.
411		R3	Is this on a slice or a pie? Ten of each? Is that what you're...
412		Brandon	Oh that would, with that. [R3: I'm imagining this pizza.] This would, this would be just one piece. Because, because mainly a medium pizza would be 8 pieces, so you would have to do 10x8.
413		R3	I see. To tell me, to tell me how much mushroom is on the pie or mushroom and anchovy?
414		Brandon	If, if you want a pizza with everything, it would be, you would have 20 thingamabobs on each.
415		R3	I see. Okay. I understand what you're saying. Did you think

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			this was kind of interesting doing this? [Brandon: Yeah.] Connecting this up? This was really fun for me. Would you come back and talk to me some other time? [Brandon: Okay.] I'd love that.
416		Brandon	What time is it? I'm a little worried.
417		R3	It's late. It's getting to be late. It's quarter to eleven.
418		Brandon	Quarter to eleven? Oh. That's okay. [R3: Is that okay?] Yeah. It doesn't.
419		R3	!here are you supposed to be now?
420		Brandon	We're just coming out of reading.
421		R3	Okay. Can I take you back then? If you're concerned about the time.
422		Brandon	No, I may, I may have missed my music lesson, but that's okay.
423		R3	Is that okay? Are you going to get in trouble?
424		Brandon	It's okay. No, it's okay.
425		R3	Do I need to talk to anybody for you? Or...
426		Brandon	I could tell Mr. Franco during band practice.
427		R3	Okay. I'm sorry you missed your lesson. You know, I didn't want you to do that, but this was just so fascinating to me. You could leave those, Brandon. I'll fix those later. I think I probably should get you back to your class. I, I'm, I'm glad that you said that we could come back and talk with you though if we, you know, if we would like because I bet there, after people see this tape from Rutgers, that they're going to have more questions for me. I'm going to need to talk to you again, I think.

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428		R2	There you go.
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