METHODS FOR ROBUST CALIBRATION OF TRAFFIC SIMULATION MODELS

by

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ABSTRACT OF THE DISSERTATION

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Well-calibrated traffic simulation model predictions can be highly valid if various conditions arising due to time-of-day, work zones, weather, etc. are appropriately accounted for during calibration. Calibration of traffic simulation models for various conditions requires larger datasets to capture the stochasticity. In this study we use datasets spanning large time periods to, especially, incorporate variability in traffic flow and speed. However, large datasets pose computational challenges. With the increase in number of stochastic factors, the numerical methods suffer from curse of dimensionality.

We propose a novel methodology to address the computational complexity in simulation model calibration under highly stochastic traffic conditions. This methodology is based on sparse grid stochastic collocation, which treats each stochastic factor as a different dimension and uses a limited number of points where simulation is performed. A computationally-efficient interpolant is constructed to generate the full distribution of the simulated output. We use real-world examples to calibrate for different times of day and conditions and show that proposed methodology is more efficient than traditional

Monte Carlo-type sampling. We validate the model using a hold-out dataset and also show the drawback of using limited data for macroscopic simulation model calibration.

Modelers could often face situations with limited data in calibrating for a particular condition, often when using traffic sensor data. We augment the current data with other sources when sensor data is missing.

For calibrating microscopic traffic simulation models needing customized models augmenting the default modeling, require detailed site-specific data. In such cases same generic calibration methodology may not be applicable and specialized formulations are required. We propose the use of a simulation-based optimization (SBO) framework for calibration of toll plaza models that economizes on data requirements. The novelty of the SBO framework is that parameters corresponding to unavailable data can be used as calibration parameters. Using case studies the benefits of the SBO framework are demonstrated. Furthermore, we combine the sampling and interpolation using stochastic collocation with the SBO framework. Using this hybrid framework, we perform calibration to obtain distribution of output from the toll plaza model that closely follows the observed measures at the toll plaza.

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Dedication

To my parents and Late Dr. S. Ragahavachari

Preface

The work conducted in this dissertation has been presented and published in several conferences and journals. Below is the list of publication derived from this dissertation with corresponding chapter numbers.

Chapter-4 and 5

- Ozbay, K., S. Mudigonda, E. Morgul, H. Yang, and B. Bartin "Big Data and Calibration and Validation of Traffic Simulation Models", To appear in Transportation Research Circular, Transportation Research Board, Washington, D.C., January 2014.
- Mudigonda, S., K. Ozbay, "Using Big Data and Efficient Methods to Capture Stochasticity for Calibration of Macroscopic Traffic Simulation Models", Presented at the Symposium Celebrating 50 years of Traffic Flow Theory, Portland, Oregon, August 11-13, 2014.
- Mudigonda, S., K. Ozbay, "Robust Calibration of Macroscopic Traffic Simulation Models using Stochastic Collocation", Submitted to 21st International Symposium on Transportation and Traffic Theory, Kobe, Japan, 2015.

Chapter-6

- Mudigonda, S., Bartin, B., and Ozbay, K. "Microscopic Modeling of Lane Selection and Lane-changing at Toll Plazas", Presented at 88th Transportation Research Board Annual Meeting, Washington, D.C., January 2009.
- Mudigonda, S., K. Ozbay, B. Bartin, "Data-driven Heuristic Optimization Methodology for Simulation, Calibration and Design of Toll Plazas",

Submitted to the Transportation Research Board's 93rd Annual Meeting, Washington, D.C., January 2015.

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CHAPTER 1. INTRODUCTION

1.1 Background

Traffic simulation models are computational tools based on certain mathematical principles that govern traffic flow. Over the past century, traffic models came through a significant amount of development. They have increased in complexity ranging from a simple input-output type model to macroscopic models to highly-detailed microscopic simulation model. The microscopic models represent each car separately with a number of complex parameters. In highway transportation, micro simulation tools such as CORSIM (Kim and Rilett (2003)), PARAMICS (Ma and Abduhai (2002), Zhang et al. (2008), Lee and Ozbay (2009), Yang and Ozbay (2011)), and VISSIM (Menneni et al. (2008)), AIMSUN (Hourdakis et al. (2003)) among others, allow traffic engineers and planners assess the performance of existing roadway systems in a detailed manner by constructing a model of the existing facilities, such as toll plazas, signalized and unsignalized intersections and traffic circles, as well as to predict the effects of potential operational or infrastructure changes.

The purpose of constructing a computational model for traffic networks is that the model's outputs are representative of real-world traffic conditions. The traffic conditions are dependent on many factors such as time, weather, road geometry, pavement condition, unpredictable factors such as accidents, etc. The availability of a simulation model that accurately represents different traffic conditions is of paramount importance for any analysis involving traffic modeling. Thus, the value of these tools lies in their ability to stochastically simulate drivers' behavior, such as lane changing, car following, gap acceptance and route choice. The functions or rules that govern drivers' decisions in simulation software tools need to be fine-tuned to reproduce field conditions. Despite the advances in computing power and the ability of available simulation tools to represent complex driver behavior, simulation modeling and analysis is still a long and painstaking procedure, requiring extensive field data for validation/calibration.

Model verification, calibration and validation are important steps in the development of a valid simulation model, and crucial for ensuring reliable information is gathered from these models.

Model verification means building the model correctly. This stage deals with accurately transforming the model concept from a simulation flowchart into a model specification using a computer program (Balci (1997)). Model calibration is the process to obtain a desired confidence level where the model and its results are reasonable for the objective it was developed for. The validation process ascertains that the output data obtained from the simulation model driven by the input data are close to the real system output data. When comparing the system and model output data, if there are substantial differences in the comparison, some correction factors are added in the input data. Then the model and system output data are compared again. This iterative procedure of input modification to meet the target output measures is called "calibration". In this study, for the sake of brevity, we use "calibration" as a generic term to describe the validation and calibration process.

The outputs of traffic simulation models are derived from a particular set of mathematical equations and relationships given a specific input data. The input data consists of two main groups of data: physical input data I_s (e.g., volume counts, origin-destination demands, capacity and physical features of roadway sections) and driver specific parameters C_s (i.e., adjustable components of driver behavior such as free flow speed, reaction time, mean headway, etc.). Output from a simulation model can be expressed as equation (1.1).

$$O_{obs} : f(I_s, C_s) \rightarrow O_{sim} \mid I_s, C_s + \varepsilon \tag{1.1}$$

$f(I_s, C_s)$ = functional specification of the internal models in a simulation system

O_{sim} = simulation output given the input data and calibrated parameters,

ε = margin of error between simulation output and observed field data, and,

O_{obs} = observed field data.

The process of calibration entails adjusting the calibration parameters (C_s) so that the error between the output from simulation and field conditions is minimized as shown in equation (1.2),

$$\min_{C_S} U(O_{obs}, O_{sim}(I_s, C_s)) \tag{1.2}$$

where,

O_{obs}, O_{sim} - observed and simulated outputs

C_S - parameter set

U - error functions for outputs

In transportation engineering an error statistic often used is Geoffrey E. Havers (GEH) statistic, which is very similar, but less rigorous, to the statistic used by statisticians and economists for forecasting called Theil's U index of inequality (Theil, 1961). GEH and Theil's U index are of the form shown in equation (3). Another commonly used error statistic for calibration of simulation models is root mean square percent error (RMSPE) shown in equation (1.3).

$$GEH = \sqrt{\frac{\frac{1}{T}\sum_{i=1}^{T}(O_{sim,i} - O_{obs,i})^2}{\frac{1}{2T}\sum_{i=1}^{T}(O_{sim,i} + O_{obs,i})}}$$

$$U = \frac{\sqrt{\frac{1}{T}\sum_{i=1}^{T}(O_{sim,i} - O_{obs,i})^2}}{\sqrt{\frac{1}{T}\sum_{i=1}^{T}(O_{sim,i})^2} + \sqrt{\frac{1}{T}\sum_{i=1}^{T}(O_{obs,i})^2}} \qquad 1.3)$$

$$RMSPE = \sqrt{\frac{1}{T}\sum_{i=1}^{T}(\frac{O_{sim,i} - O_{obs,i}}{O_{obs,i}})^2}$$

FHWA's Traffic Analysis Toolbox recommends that if GEH < 4 for link volumes for 85% of the links and average travel times are within 15% of observed values, then it is considered as a satisfactorily calibrated model (Dowling et al. (2004)). Theil's U index varies between 0 and 1, with 0 showing a perfect model and 1 implying a completely wrong prediction. In order to achieve this level of calibration for various conditions (peak, off-peak, weekends, workzone conditions, normal and inclement weather, under accident, and other events), detailed level of data is required.

1.2 Motivation

Traditionally traffic simulation models are used to study scenarios for a certain time period of a so called "typical day". However, as shown in Ozbay et al. (2014), the determination of a typical day is not a trivial task or a typical day may not even exist in reality. The above assertion is based on the distribution and spread of traffic demand. However, the actual traffic flows along the section of interest would also vary based on many conditions such as, work zones, driver/vehicular variability and other unobserved phenomena. Thus, the existence of a “typical” day in the supply-side traffic output is also not always likely. Limited data captures only a few specific conditions, or is a dilute sample of different conditions. As depicted in Figure 1-1, using only smaller samples of data will not accurately capture variation in traffic data.

Thus, calibration parameters estimated using limited sample data are not always representative of all possible conditions of the simulated system and might thus result in inaccurate predictions. Hence, it is expected that the model predictions will only be accurate for those specific conditions. Using these models for conditions other than the ones for which calibration data was available for would not yield accurate results. As mentioned earlier, variability can be incorporated within *inputs (demands)* I_s and *calibration parameter set (supply)* C_s during different periods of the day, work zone activity, weather conditions, driver population composition, highway geometry, etc. In order to obtain accurate predictions from a traffic simulation model, it is important to consider not only the demand- and supply-side variations from various conditions, but also the variation of demand- and supply-side variations within each type of condition. Hence, the typical day scenario might not be the best scenario to test the effectiveness of operational strategies. Moreover, there is an increasing trend for using well-calibrated simulation models as predictive tools for real-time traffic control (Vasudevan and Wunderlich (2013), Yelchuru et al. (2013), USDOT ICM, Olyai (2011), Dion et al. (2009)). Clearly, these simulation models have to work under a combination of conditions that will considerably deviate from the typical day scenario.

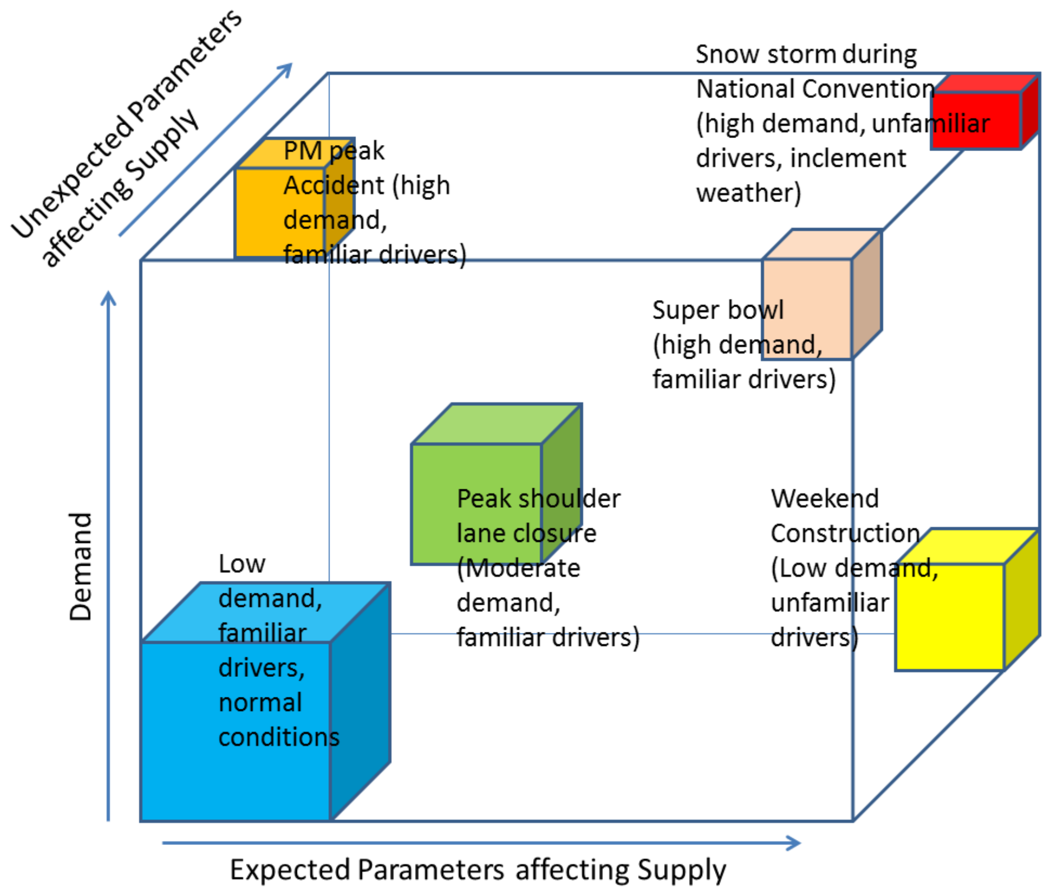


Figure 1-1 Illustration of various traffic conditions for which data is required for calibration (adapted from (Wunderlich (2002))

Traditional sources of traffic data used in the calibration of traffic models are either limited by the availability of the data that only cover typical conditions or may not be reliable enough. However, with the advent of new information technologies, unprecedented wealth of calibration data is on the fingertips of users by means of smart phones, GPS-equipped devices, and RFID readers. This, in turn, has led to massive amount of passively collected location and event data for various time periods. These data provide an opportunity to validate and calibrate traffic simulation models for a variety of spatio-temporal conditions.

There were studies that captured traffic variability: Li et al. (2009), Ngoduy (2011), Zhong and Sumalee (2008), Jabbari, Liu (2012), Lee and Ozbay (2009), to name a few. In cases where large sources of data spanning different conditions are available, to capture the stochasticity in traffic conditions, there is an increase in number of factors of stochasticity. However, the increase in the number of factors affecting stochasticity increases the dimensionality of the calibration process. This in turn results in increased computational effort required in calibrating traffic simulation models for different conditions such as variability within weekday/weekend, and seasonal variability, and special situations including adverse weather, work zones, etc. Thus depending on the size of the network and number of stochastic dimensions, traditional Monte Carlo-type sampling approaches can become prohibitive in terms of computational effort. It may not be possible at all to simulate the output for each and every possible realization of parameter and input. Also, all possible points in the stochastic space of simulation output may not have the corresponding observed data. Thus it is important to obtain an effective sampling and interpolation methodology for predicting output accurately but with lower computational effort.

A number of simulation approaches exist to model roadway sections with varying degree of resolution i.e. microscopic, mesoscopic and macroscopic. Hence, an effective framework needs to be built around the existing simulation (macroscopic or microscopic) models in order to facilitate the robust calibration by taking all types of variation in traffic conditions, albeit, taking into account the computational effort required.

The data to account for the traffic variability can come from many different sources. Sensor station data is one of the most commonly used and available data among these sources. However, sensor station data is a point data i.e. it represents flow or speed of traffic at a single point of the freeway. If a sensor is located in the part of a freeway where there is no entry or exit ramps close by the spatial variation of flow may not be large. But if the sensor is located close to an exit or entry ramp, the variation of flow before and after the ramp is high. If the objective is to capture all forms of variability in traffic, this spatial variation is also of significance.

Additionally, there may be instances where the data from the sensor is missing due to sensor malfunction. To combat such cases, other data sources are needed to supplement the sensor data. In addition to sensor stations that detect vehicles on the freeway, there are other vehicle identification technologies available. The infrared tag used in electronic toll collection is one source of data, used to a widely by TRANSCOM. Another technology which has gained popularity recently is the GPS sensor, attached either to the vehicle or available through the drivers' smart phones. All these sources provide a rich source of traffic data such as flow, speed and travel time. Each of these data can be used, synergistically, in the robust calibration of traffic simulation models. Depending on the quality of the dataset, it is established in literature (Mathison, 1988) that using multiple data sources increases the validity of the model. This type of data fusion for calibration will be another important aspect of this study.

Traffic simulation models can be used to model freeway sections to urban networks to sections with specific geometric features such as toll plazas, traffic circles, etc. As far as the current state-of-practice goes, freeways and urban networks do not, usually, involve much customization on the part of the modeler. The customization, usually, includes adjusting and/or overriding the underlying algorithms for car-following, gap acceptance, lane changing, etc. In most simulation tools and packages these algorithms are based on data collected from freeway sections – hence the relative ease in customization.

On the other hand modeling sections that involve specific geometric features can be quite complex. The complexity can be due to changing driver behavior at that particular geometry or a set of traffic control measures. So the underlying algorithms in the simulation models must be extended or overridden to incorporate this new behavior. This process requires data collection that is site-specific for different conditions. The data collection in itself can be quite cumbersome. Hence extensive calibration of these models will be an extremely time-consuming task. If, for instance, the model involves a large freeway section with some specific geometric features, such as a toll plaza, then executing an iteration may be time-consuming as well. Calibrating and running such large customized simulation models in a robust manner calls for different type of methodology.

In this dissertation, the customized microscopic simulation model chosen is that of multi-lane toll plazas. Simulation modeling of toll plaza operations is a very demanding task due to drivers' complex lane selection. Because most available microscopic simulation software packages do not have credible toll plaza models, some of these studies developed customized toll plaza simulation models (Junga (1990), Correa et al. (2004), Danko and Gulewicz (1991), Burris and Hildebrand (1996) and Astarita et al. (2001)). The drawback of customized models is the fact that they are standalone models and not integrated in the rest of the traffic network. Others studies used commercially available microscopic simulation software such as VISSIM, PARAMICS and used a number of parameters provided by the default simulation engine in the software (Al-Deek et al. (2000), Chien et al. (2005), Ceballos and Curtis (2004), Nezamuddin and Al-Deek (2008)). However, these parameters are generic parameters and may not directly reflect the specific parameters involved in the drivers' decision making at the toll plaza. In addition, all of the developed models represent barrier-type toll plazas only. When toll plazas are located at separate locations away from the mainline - unlike barrier toll plazas - there are additional factors that influence drivers' lane selection decisions.

One of the motivation for this current study stems from need for developing a better toll plaza simulation model that can address the above shortcomings of the existing tools. This requires modeling drivers' lane choices accurately. A driver's lane choice at a toll plaza is based on several factors including queue length, number of lanes changes required, and the direction of entry and exit in and out of the toll plaza. Estimation of a driver lane choice model bears two problems: One is the extent of data required to develop a statistically significant model. Collecting such extensive data is possible using video cameras; yet often this endeavor is not necessarily practical because of the wide area of coverage needed to capture and extract the required variables. Second concern is transferability, in other words whether the model would be valid for other time periods of the day or days of the week or when there is a change in vehicular composition or in the lane configuration of toll plazas.

Hence generic calibration methodology may not be applicable to these customized microscopic traffic simulation models. Specialized optimization framework is required to combat the issue of unavailability of data for calibration for various conditions.

1.3 Outline of the Dissertation

In this dissertation, we propose a novel and practical framework for the calibration of traffic simulation models for various traffic conditions and a robust mechanism to predict simulation output for many different conditions. Following are the sections of the dissertation.

Chapter 2 presents a review of literature on calibration studies for traffic simulation models. Also presented are some of the approaches used to simulate stochasticity in traffic simulation. Literature on modeling complex toll plazas using microscopic simulation models is also presented.

Chapter 3 presents various contributions made in this dissertation.

Chapter 4 illustrates the need for incorporating stochastic inputs and parameters in traffic simulation. It also presents the methodology for incorporating stochasticity in macroscopic traffic simulation models. Initially, it presents exploration of output data to categorize into various distinguishable groups or clusters. Subsequently for each output cluster, the numerical methods using the computationally-efficient sparse grid stochastic collocation for discretizing the stochasticity of input and parameter distributions are illustrated. Finally, the optimization method using simultaneous perturbation stochastic approximation (SPSA) for calibrating the said traffic simulation models is discussed.

Chapter 5 demonstrates the application of the proposed methodology for calibrating a freeway section using real-world data. Also, Chapter 5 demonstrates the proposed framework for larger freeway section when some data is missing. Alternative sources of data are used to supplement the data in the calibration process.

Chapter 6 shows the calibration methodology for microscopic modeling of customized complex toll plazas using a data-driven lane choice heuristic and a simulation-based optimization (SBO) framework. Also in the chapter we demonstrate, via case studies, the usefulness of the SBO framework when data is partially available or not available. The SBO framework is extended further to incorporate the ability to estimate stochastic output using the computationally-efficient sparse grid stochastic collocation.

Finally, in Chapter 7 we present the conclusions of the calibration methodologies proposed in this dissertation. We also discuss the direction of the future work using these methodologies.

CHAPTER 2. LITERATURE REVIEW

Traffic simulation models vary in their degree of complexity and level of detail. Based on the level of detail, the models can be classified as, macroscopic, mesoscopic and microscopic simulation models. The complexity and time consumed to execute the models increase in the same order, namely, macroscopic, mesoscopic and microscopic simulation models. While microscopic simulation models provide an ideal platform for detailed modeling, the number of parameters involved in the modeling and thus the effort in calibration is greater. The data required for simulating different conditions using the microscopic model is difficult to obtain or may not be available.

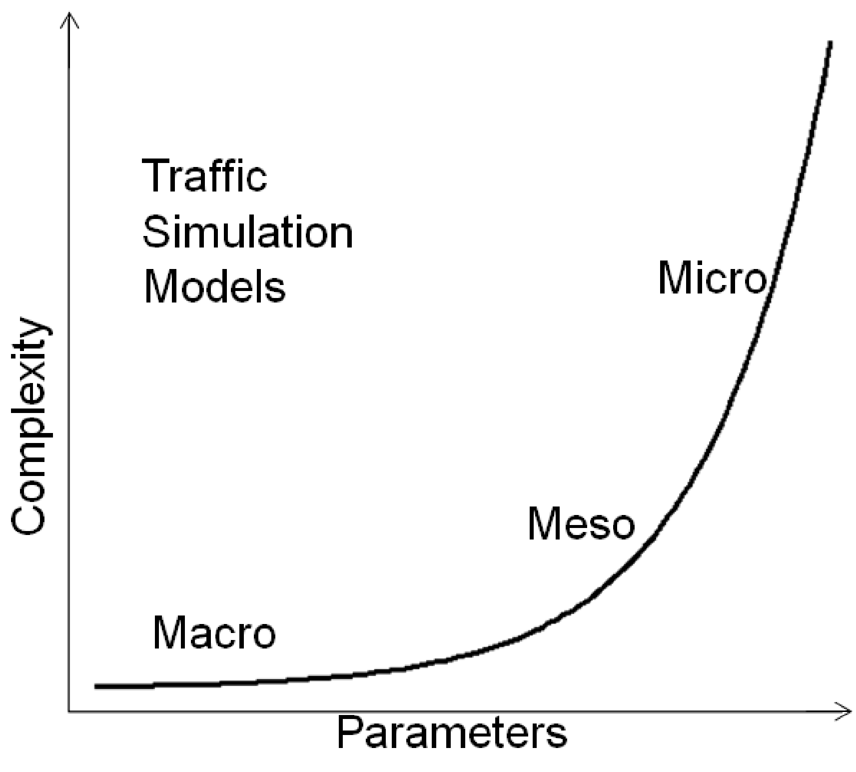


Figure 2-1 Traffic simulation model complexity

A summary of literature in calibration of traffic simulation models is presented in the next subsection.

2.1 Calibration of simulation models of Freeway and Urban networks

There has been extensive amount of work related to the calibration of microscopic traffic simulation models. Ding (2003), Gardes et al. (2002), Ma and Abdulhai (2002), and Lee et al. (2001) used mean target headway and mean reaction time as the parameters to be calibrated. Hourdakis et al. (2003) and Mahut et al. (2004) calibrated global and local parameters, and Toledo et al. (2004) and Jha et al. (2004) used origin-destination (O-D) flows and driver behavior parameters as calibration parameters.

Depending on the scale and complexity of the simulation model, estimating the objective function for the calibration process can get quite expensive, in terms of time and resources. There have been different methodologies used in the optimization problem i.e. the calibration process.

Lee et al. (2001), Schultz and Rilett (2004), Park and Qi (2005), and Ma and Abdulhai (2002) used genetic algorithms (GAs) to calibrate microscopic simulation tools. Kim and Rilett (2003) used the simplex algorithm as the calibration methodology, while Kundé (2002) used the Simultaneous Perturbation Stochastic Approximation (SPSA) algorithm to calibrate a model developed in DynaMIT-P, a mesoscopic simulation tool.

Bayesian optimization is another possible approach in the optimization problem. The advantage of this approach is that is combines the models and a sampling criteria.

Bayyari et al. (2004) proposed a Bayesian approach that takes into account the variability i.e. distribution in demand, turn count and measurement errors therein. Lee and Ozbay (2008) proposed a Bayesian approach that takes into account the variability in demand and uses the SPSA to optimize the parameters.

The selection of parameters to be calibrated and the methodology followed are very important aspects of the overall calibration process. Ding (2003), Gardes et al. (2002), Lee et al. (2001), Toledo et al. (2004), Jha et al. (2004), Kim and Rilett (2003), Schultz and Rilett (2004), Hourdakis et al. (2003), Park and Qi (2005), Ma and Abdulhai (2002), Duong et al (2010), Yang and Ozbay (2011), Korcek et al. (2012), Henclewood et al. (2013), Punzo et al. (2013), and Ge and Menendez (2013) used microscopic simulation tools. In each study, model parameters were selected and various methodologies adopted for calibration, such as the SPSA algorithm (Ding (2003), Kundé (2002), Balakrishna et al. (2007), Ma et al. (2007) , Lee and Ozbay (2009), Yang and Ozbay (2011)), Genetic algorithm (Lee et al. (2001), Schultz and Rilett (2004), Park and Qi (2005), Ma and Abdulhai (2002), Cheu et al. (1998), Kim et al. (2005), Duong et al (2010), Korcek et al. (2012)), and simplex algorithm (Kim and Rilett (2003)). Error statistics used were mean absolute error (MAE) (Ding (2003), Kim and Rilett (2003). Ma and Abdulhai (2002), Schultz and Rilett (2004), Lee and Ozbay (2009), Duong et al (2010), Korcek et al. (2012)), root-mean-square error (Kundé (2002), Hourdakis et al. (2003), Qin and Mahmassani (2004), Balakrishna et al. (2007), Yang and Ozbay (2011), Punzo et al. (2013), GEH (Zhang et al. (2008), Punzo et al. (2013). Kundé (2002) and Qin and Mahmassani (2004) used mesoscopic simulation tools in the validation process. In this chapter, comprehensive reviews of previous calibration studies are summarized in Table 2-1.

Table 2-1 Summary of Literature on Calibration of Microscopic Traffic Simulation Models

Authors	Complexity; Simulation Tool	Calibrated Parameters	Optimization Methodology	Type of Roadway Section	Performance outputs	Validation Measure	Data Used in Calibration
Lee et al. (2001)	Micro; PARAMICS	Mean target headway, mean reaction time	Genetic algorithm	Freeway	Occupancy, Flow	N/A	
Gardes et al. (2002)	Micro; PARAMICS	Mean target headway, mean reaction time	N/A	Freeway	Speed, Volume	N/A	15-min counts during AM peak on 19-mile freeway section
Ma and Abdulhai (2002)	Micro; PARAMICS	Mean headway, mean reaction time, feedback, perturbation, familiarity	Genetic algorithm	Urban	Traffic counts	MAE	Detector data for 1 hr during AM peak
Kundé (2002)	Macro; DynaMIT-P	Speed–density relationship, Capacity	Box complex, SPSA	Network	Free-flow, Minimum	RMSE	
Ding (2003)	Micro; PARAMICS	Mean target headway, mean reaction time	SPSA algorithm	Freeway	Flow, density	MAE	
Kim and Rilett (2003)	CORSIM, TRANSIMS	CORSIM: Car-following factors, driver's aggressiveness factor TRANSIMS: O-D matrix, PT1 parameters	Simplex algorithm	Freeway	Volume	MAER	Data 5 loop detector stations for 13.9-mile section of freeway for 1 hr. during AM, PM and off peak
Hourdakis et al. (2003)	Micro; AIMSUN	Global, local parameters	Trial and error	Freeway	Volume	RMSP	5-min. data from 21 detector stations for a 12-mile freeway section during PM peak for 3 days
Schultz and Rilett (2004)	Micro; CORSIM	Driver behavior parameters, vehicle performance parameters	Automated Genetic algorithm	Freeway	Volume, Travel time	MAE	Data 5 loop detector stations for 13.9-mile section of freeway for 1 hr. during AM and PM peaks
Jha et al. (2004)	Micro; MITSIMLab	Parameters of the driving behavior models and route choice model, O-D flows, habitual	Iterative approach	Urban Network	Travel time	N/A	Detector data for 15 days for AM and PM peaks on a large urban network

Mahut et al. (2004)	Macro; EMME/2	Local parameters, global parameters	Dynamic MSA equilibration algorithm	Urban Network	Travel time, Counts	N/A	10-min counts for 1 hr. during AM peak
Toledo et al. (2004)	Micro; MITSIMLab	O-D flow, behavioral parameters	GLS optimization	Freeway and arterial	Speed, Density	RMSE, RMSP, MAE, MAPE	Data from 68 detector stations on 3 freeways for 5 weekdays
Qin and Mahmassani (2004)	Macro; DYNASMA RT-X	N/A	Transfer function model	Freeway Network	Speed	RMSE	Data from 7 detector stations on 3 freeways during AM peak for 5 weekdays
Park and Qi (2005)	Micro; VISSIM	Eight parameters	Genetic algorithm	Intersectio n	Average travel time	N/A	Detector data for 1 hr. during PM peak for 3 days
Kim et al. (2005)	Microscopic Simulation	Various microscopic simulation parameters in VISSIM	Genetic Algorithm with Non-parametric statistical test	Freeway	Travel Time Distribution	N/A	Travel time data for 1 hr. during AM peak on 1.1 km. freeway section
Balakrishna et al. (2007)	Micro; MITSIMLab	Driver behavior model parameters	SPSA algorithm	Freeway, Parkway	Traffic Counts	RMSN, RMSPE,	15 min. data from 33 detector stations
Ma et al. (2007)	Microscopic simulation	Global parameters (Mean target head, mean reaction time etc) Local parameters (link headway factor, link reaction factor, etc.)	SPSA algorithm	Freeway	Capacity	N/A	Detector data for 1 hr. during PM peak
Zhang et al. (2008)	Micro; PARAMICS	Mean target headway, mean reaction time, driver awareness, aggressiveness	SPSA algorithm	Urban Freeway network	Flow, Occupancy	GEH	5-min detector count during PM peak for 7 days
Lee and Ozbay (2009)	Micro; PARAMICS	Mean target headway, mean reaction time	enhanced-SPSA	Freeway	Speed, Counts	MAE	5-min detector count during AM peak for 16 days

Duong et al (2010)	Micro; VISSIM	Headway, reaction time, standstill distance, max. look ahead distance, safety distance, acceleration parameters	Genetic Algorithm	Freeway	Speed, volume, surrogate safety measures	MAE for speed and volume	NGSIM trajectory data for US-101for 40 min.
Yang and Ozbay (2011)	Micro; PARAMICS	Headway, reaction time, queue gap, speed, link headway, signpost, aggressiveness, awareness, memory	SPSA	Freeway	traffic conflict, lane change, volume and speed	RMSPE for speed, volume, lane change and conflict	NGSIM trajectory data for US-101 for 15 min.
Korcek et al. (2012)	Meso	Free flow speed, speed at capacity, capacity, jam density	Genetic Algorithm	Freeway	speed, density, flow	MAE for speed, density, flow	Loop detector data from two freeways in Czech and Slovak Republic for one year
Henclewood et al. (2013)	Micro	Combinations of parameter sets	N/A (determine the best combination of parameters)	Freeway	Travel time distribution	Kolmogorov-Smirnov , and heuristic form fit tests	NGSIM trajectory data for Peachtree Street in Atlanta, Georgia for 30 min.
Punzo et al. (2013)	Micro	follower's maximum desired speed and acceleration, acceleration rate, desired distance, minimum time headway	OptQuest Multi-start	Freeway	Speed	RMSE and IMSE of speed	NGSIM trajectory data for I-180
Ge and Menendez (2013)	Micro	Average Standstill Distance, Additive Part of Desired Safety Distance, Multiplicative Part of Desired Safety Distance, Accepted Deceleration (Trailing), Lane Change Distance, Emergency Stop Distance	Quasi-OTEE	Urban network	Travel Time	Difference in Travel time	Travel times on 20 road sections in the Zurich inner city network

Ding (2003) performed a calibration study of a microscopic simulation model developed using PARAMICS. She used mean target headway and mean reaction time as the key parameters to be calibrated, since these two parameters affect the car following and lane-changing models. SPSA was used as the optimization algorithm, and the relative error of density and flow was used as the objective function. The selection of these parameters was based on the previous work of Sanwal et al. (1996).

Ma and Abdulhai (2002) performed a calibration study of microscopic simulation models based on combinatorial parametric optimization using a GA. They used GENOSIM (a GA-based simulation-optimization system) to solve a combinatorial parametric optimization problem; it minimized the relative error between field data and simulation output by searching for an optimal value of microsimulation parameters. Ma and Abdulhai (2002) employed four types of GAs—namely, simple GA, steady-state GA, crowding GA, and an incremental GA. In the case of the simple GA, the population was displaced with new individuals; on the other hand, in the steady state GA, some parts of the population overlapped with new individuals. The crowding-based GA was the same as the simple GA type in selection and reproduction, but replacement was discriminated. As the objective function, the MAE of the difference between real-world and simulated traffic was used.

Gardes et al. (2002) performed a calibration and application study of a PARAMICS model of Interstate 680, located in the San Francisco Bay Area. In order to calibrate input parameter values, significant checks and changes were performed for four major categories: network (network geometry, signposting, link speeds), demand (vehicle proportions, vehicle mean top speed), overall simulation configuration (time steps per seconds, speed memory), and driver behavior factors (mean target headway, mean reaction time). Gardes et al. (2002) then included a new ramp-metering strategy, in addition to auxiliary lanes and an HOV lane, to evaluate a range of operational strategies applicable to the modeled network.

Lee et al. (2001) calibrated the parameters of a PARAMICS model of a Southern California network using a GA. Mean target headway and mean reaction time were employed as key parameters. Mean target headway affects the acceleration and deceleration times of each vehicle and mean reaction time affects the acceptable gap of the lane-changing model. Using a GA, a number of input parameters were repeatedly generated until the parameters were optimized. As a fit test, the differences between occupancy and volume, as obtained from the PARAMICS model and field data, were used.

Kundé (2002) conducted a calibration study of a supply simulator that is a part of DynaMIT-P, a mesoscopic traffic simulator. He studied various methodologies classified into three categories—namely, path search methods, pattern search methods, and random methods. Path search methods estimate a direction to move, from an initial vector to an improved point. Response Surface Methodology (RMS) and SA are major path search methods, and pattern search methods—such as the Hooke and Jeeves method, the Nelder and Mead (simplex search) method, and the Box Complex method—search for a characteristic or pattern from the observations. Random search methods look for an improved point, without the aid of previous information. The stochastic ruler algorithm, stochastic comparison method, and simulated annealing are random search methods.

The calibration methodology used by Kundé (2002) involves three stages, at the disaggregate level, the sub-network level, and the entire network level. The first stage of the methodology is used to calibrate a study of speed-density relationships and the capacities of each segment. The second stage is performed when an accurate O-D can be collected from the sensors. In the last step, the stochastic optimization problem is carried out, which is the calibration at the entire network level. At this stage, the Box Complex method and the SPSA algorithm are employed.

Kim and Rilett (2003) performed a calibration study of micro-simulation modeling using the simplex algorithm. Two micro-simulation models—namely CORSIM and TRANSIMS—were tested using the simplex algorithm. The CORSIM O-D matrix—which uses the CORSIM O-D estimation model and the automatic vehicle identification (AVI) O-D matrix—is the information maximization estimation model they used. In this calibration study, the parameters used in TRANSIMS were the O-D matrix and the PT parameters, described as the deceleration probability (PT_1), the lane-change probability (PT_2), and the plan look-ahead distance (PT_3). CORSIM calibration parameters were car-following factors, acceleration/deceleration factors, and lane-changing factors. Both CORSIM and TRANSIMS were calibrated when parameter sets were default values. Preliminary calibrations for the AM, PM, or off-peak time periods were conducted using the simplex algorithm mean absolute error ratio (MAER) was used to compare observed CORSIM and TRANSIMS volumes. Models calibrated via the simplex approach were found to have a lower MAER than the models that used default values.

Schultz and Rilett (2004) conducted the calibration of a microscopic simulation model using the FHWA's corridor simulation model (CORSIM), and also used the distribution of car-following sensitivity factors as the main calibration parameter. Car-following sensitivity factors included driver behavior characteristics that depended on the car ahead, with specified sensitivity. The author classified different car-following sensitivity factors and identified them to explain variability among driver types. A GA was used for the calibration methodology and the new distribution was examined to fit the data. For the car-following sensitivity analysis, the author outlined two alternatives that were lognormal and normal car-following sensitivity analyses, and compared those with initial distributions.

Jha et al. (2004) performed a calibration of a large-scale network using MITSIMLab, a micro traffic simulation model. They calibrated driving behavior parameters with a single freeway section without considering route choice. After driving behavior parameters were calibrated, the values were fixed; the calibration work of route-choice parameters, an estimation of O-D flows, and habitual travel times was then jointly performed for a large-scale network, using an iterative solution approach. As the function of the calibration process, the travel times of field data were compared with the output of the simulation model.

Toledo et al. (2004) performed a calibration study of microscopic traffic simulation models. They focused on the interactions of O-D flow estimations and calibrations of behavior parameters. He proposed an iterative solution approach that starts with habitual travel times, because O-D flow estimation needs to generate an assignment matrix based on route-choice behavior and experienced travel times. Habitual travel times are important variables in solving a driver's route-choice problems. The assignment matrix was generated based on these travel times, and O-D flow estimation was performed using a generalized least squares (GLS) formulation. The new O-D matrix is used to recalibrate route choice and driving behavior parameters, and this iterative procedure was repeated to minimize the least square error. To demonstrate the efficiency of this approach, it was applied to two different case studies developed with the use of MITSIMLab, a microscopic traffic simulation model (Toledo et al. (2004), Jha et al. (2004)). In the first case study, O-D flows were known and route choice was not present in the network. In this case, driving behavior was the only parameter to calibrate. As a function of the fit test, the RMSE, the root-mean-square percent error (RMSP), the MAE, and the mean absolute percent error (MAPE) of the difference between observed and simulated speed measurements were used. For the other case study, O-D flow estimations and calibration work with regard to the travel time coefficient of the route choice were performed. As a function of the fit test, Toledo et al. (2004) used RMSE and MAE to compare the observed and simulated counts. Both of the case studies were good fits.

Hourdakis et al. (2003) proposed a calibration procedure for microscopic traffic simulation models for a 20-km freeway section in Minneapolis. For this calibration study, they divided the simulator parameters into global parameters such as length, width, desired speed, maximum acceleration/deceleration, and minimum headway, as well as local parameters like the speed limits along individual sections of the freeway model.

Global parameters were closely related to the performance of the entire model, and local parameters affected specific parts of the network. Hourdakis et al. (2003) divided the calibration process into volume-based calibrations and speed-based calibrations. The objective of volume-based calibrations was to obtain the volumes from simulation that were as close to the real-world counts as possible; the objective of the speed-based calibrations was to obtain the speeds from simulation that were as close to the real-world speeds as possible. The sum of the squared errors was used as the optimization technique to calibrate the simulator parameters, and traffic volumes were used as an objective function to be minimized.

Park and Qi (2005) performed a calibration of a microscopic and stochastic simulation model developed in VISSIM, based on a parameter optimization using a GA. The traffic simulator included car-following and lane-changing logic, as well as the signal state generator that can decide signal control logic. The location was the intersection of U.S. Route 15 and U.S. Route 250 in Virginia, and the average travel time was used as the measure of effectiveness. In order to acquire an accurate simulation result, the acceptable ranges of eight parameters were determined and multiple simulation runs performed to reduce the stochastic variance with default parameter values. A GA approach was applied to calibrate parameter values. In order to verify whether the calibrated parameters were statistically significant, a t-test and visualization check were also performed.

Cheu et al. (1998) performed a calibration of a FRESIM-based model of a Singapore expressway, using GAs. The existing representative optimization algorithms used a gradient approach that lacks robustness. This algorithm has only one solution, and this may lead only to a local solution; therefore, a GA was applied to reduce this problem and find a global solution. The calibration work was performed for a 5.8-km section of the Ayer Rajar Expressway in Singapore. The parameters calibrated for FRESIM are free-flow speed and driver behavior parameters. As a fit test, Average Absolute Error (AAE) was used between FRESIM simulation output and field data from the loop detector.

Milam (2005) recommends guidelines for the calibration and validation of traffic simulation models. The calibration requires modifying traffic control operations, traffic flow characteristics, and driver behavior. He summarizes the default values of the parameters to be calibrated in CORSIM, as well as their effective range. The parameters presented in the validation guidelines were traffic volume, average travel time, average travel speed, freeway density, and average and maximum vehicle queue lengths. The author recommends each parameter's acceptable range of error between CORSIM simulation results and field data. Tables 2.2 and 2.3 show the calibrations and their effective range and validation guidelines.

Dowling et al. (2004) recommend the calibration/validation of microsimulation models in three steps. The simulation model is first calibrated for capacity and then traffic flow, both at a bottleneck section. Finally, the model is calibrated for system performance at the entire network level. According to the author, for the capacity calibration procedure, the capacity of the given model was estimated by counting the maximum possible flow rate of the target section, and parameters that directly affect the capacity were selected. The mean squared error (MSE) was used for the objective function, and the optimal parameter values were obtained at the point where the MSE was minimized. In the case of matching observed traffic flow, the route choice algorithm parameters were adjusted until predicted volumes fit field counts. Finally, overall traffic performance was compared with various field data, including travel time, queue lengths, and duration of queuing. As an example of satisfying the three-step application, Dowling et al. (2004) used the sample problem offered by Bloomberg et al. (2003), which compared six simulation models with the *Highway Capacity Manual* (*HCM*).

Mahut et al. (2004) performed calibration work based on the dynamic traffic assignment (DTA) model. DTA is a procedure where network users choose the best route, to minimize overall travel cost. The DTA model used in this study is based on an iterative approach, where flows are updated with successive iterations that are based on travel times from the simulation model. The EMME/2 software package was used to calibrate a DTA trip table that was modified through a matrix adjustment. As part of the verification process, three consecutive 15-minute counts were compared with calibrated model results.

Qin and Mahmassani (2004) performed a calibration study of dynamic speed-density relationships by using data collected from Interstate Freeways I-5 and I-405 in California. They estimated the parameters to find the minimum discrepancy between observed and simulated speed, using transfer function, one-regime modified Greenshields, and two-regime modified Greenshields models. The RMSE of speed was used as a goodness-of-fit test. As a result of the comparison, the transfer function approach was found to be more accurate than static modified Greenshields models in estimating dynamic traffic speed.

Lee and Ozbay (2009) proposed a Bayesian sampling approach in conjunction with the application of the SPSA stochastic optimization method. The enhancement in the calibration procedure is by considering statistical data distribution. A microscopic simulation model based on PARAMICS, was used in conjunction with the proposed methodology. The calibration was performed with the data obtained from a complete input distribution for a section of I-880 freeway in California.

Duong et al (2010) proposed a calibration and validation prodecure for the performance of safety measures in microscopic traffic simulation models. The authors used a multi-criteria optimization methodology using Genetic algorithm. The objective function in the optimization procedure includes the mean squared difference of speed and volumes as two competing objectives. The authors implemented their procedure for a freeway section on US-101 with an on- and off-ramps, which is a part of Next Generation Simulation (NGSIM) initiative. The authors compared a surrogate safety measure of crash prediction index predicted from the model that is calibrated using single criteria optimization and multi-criteria optimization. The authors found that the model calibrated using MOP that the set of parameters for which the model has minimum error in speed is not the same for minimum error in volume or minimum error in crash prediction index.

Yang and Ozbay (2011) proposed an optimization approach to calibrate a traffic simulation model for rear-end traffic conflict risk analysis on highways. The proposed calibration approach is developed based on the stochastic gradient approximation algorithm to find optimal parameters. The calibration methodology accounts for multiple calibration criteria using traffic conflict, lane change, traffic count and speed. Simulated

operational measurements and traffic conflict risk in terms of surrogate safety measures are quantified and compared with observations derived from real-world vehicle trajectory data from the NGSIM program. The calibrated traffic model has been validated by using independent vehicle trajectory data saved as a hold-out sample. The results show that the fine-tuning of parameters using the proposed calibration approach can significantly improve the performance of the simulation model to describe actual traffic conflict risk and operational performances.

Korcek et al. (2012) proposed an effective calibration method for a simple microscopic traffic simulation model. The proposed model is based on the cellular automaton, which can easily be accelerated. Genetic algorithm was used to find suitable parameters of the CA model for a given field data. For those test road segments, we increased the precision of simulator by 20.09% in average in comparison with a manually updated and tuned model. With the proposed procedure, the authors claim that it is possible to readjust the model to given field data.

Henclewood et al. (2013) employed a Monte Carlo approach to generate candidate parameter sets for calibration. This procedure was applied to calibrate the VISSIM model of the NGSIM study area. One thousand potential parameter sets were generated and these parameter set simulations were evaluated against a robust set of calibration criteria to determine which were calibrated. Two calibration criteria were applied: 1) evaluation of startup and saturation flow and 2) statistical evaluation of travel time distributions. The parameter sets that satisfy both these criteria are considered as adequately calibrated The results suggest that parameters determining distance between cars under various conditions are dominant meeting the evaluation criteria. The results suggest that this approach offers a robust and effective method of calibrating simulation models where disaggregate level vehicle data are available.

Punzo et al. (2013) propose a time-series based approach to evaluate the overall performance of the simulation model in the objective function. Observed traffic measurements are indeed autocorrelated, and thus methods applicable to independent observations cannot be adopted. Spectral analysis, by means of estimated correlograms, is applied here instead, to study time series data generated by simulated stochastic models. As a result, the objective function of the optimization problem reproduces the distance between the spectra of the real and the simulated traffic measure. Minimizing this distance allows having a simulated trajectory that reproduces as better as possible the actual dynamics involved in the car-following process and which is not concerned with local "compensation" effects typical of the GOF usually applied in car-following calibrations

Ge and Menendez (2013) propose a Sensitivity Analysis (SA) as a preliminary step for the model calibration. Through SA the modeler can obtain a better knowledge about the relationship between the model inputs and outputs, and hence focus on the most important parameters for further calibration. An improved SA method, quasi-Optimized Trajectories Elementary Effects method, was proposed that was applied in a case study to screen the most important parameters of VISSIM. The results show that the use of SA as a screening approach can be an effective way to deal with large and complex model calibrations. In addition, they show that the proposed SA method is accurate and efficient for other networks and simulation models.

2.2 Stochastic macroscopic traffic simulation models

While microscopic simulation models provide an ideal platform for detailed modeling, the data required for different conditions is difficult to obtain or may not be available. Additionally, the model building, calibrating and executing may be time-consuming and computationally expensive. When studying the effects of various stochasticities, we are going to focus on macroscopic models because they are mainly mathematical models that do not have the extended level of heuristics incorporated like the microscopic models that attempt to capture driver level decisions such as lane change, familiarity, etc. Hence, most of the previous studies focusing on capturing and modeling traffic variability have used macroscopic simulation models.

2.2.1 Macroscopic First-order Traffic Flow Models

Macroscopic models involving traffic flow are represented by a set of partial differential equations (PDE) (such as the conservation of traffic flow). The first-order macroscopic traffic flow model can be formulated as shown in equation (2.1).

$$\begin{array}{ll} \partial_t \rho + \partial_x \left(\rho v\right) = 0 & \text{Conservation of vehicles} \\ q = \rho v \sim f(\rho) & \text{Flow-density relationship} \\ \rho(x,0) = \rho_I(x) & \text{Initial Conditions} \\ \rho(x_i,t) = \rho_i^B(t) & \text{Boundary Conditions} \end{array} \tag{2.1}$$

The first two equations represent the conservation of flow and the flow-density relationship. The third shows the spatial distribution of density/vehicles at $t = 0$. The fourth is the boundary conditions representing the inflow and outflow at various points i of the roadway section to be modeled.

The flow-density relationship shown in equation (2.1) is also called as the traffic flow fundamental diagram. The fundamental diagram between flow and density follows a concave shape. It can be used to determined quantities such as the maximum flow rate and maximum density that can be reached on a roadway section. These quantities characterize the roadway section and are important in the solving the first order traffic flow model.

Solutions to equation (2.1) are dependent of the differentiability of the initial conditions $\rho_I(x)$. Thus only weak solutions, constrained by the "correct" compression and rarefaction wave speeds, are admissible (Leveque, 1992). Solutions to macroscopic traffic flow models follow the numerical methods for solving hyperbolic PDEs. More popular among these methods is the Godunov scheme also known, in the traffic simulation parlance, as the cell transmission model (CTM) due to Daganzo (1994). This involves discretization of the roadway into cells as shown in Figure 2-2. Each cell has a capacity and can "sending" and "receiving" functions governed by the fundamental traffic flow diagram.

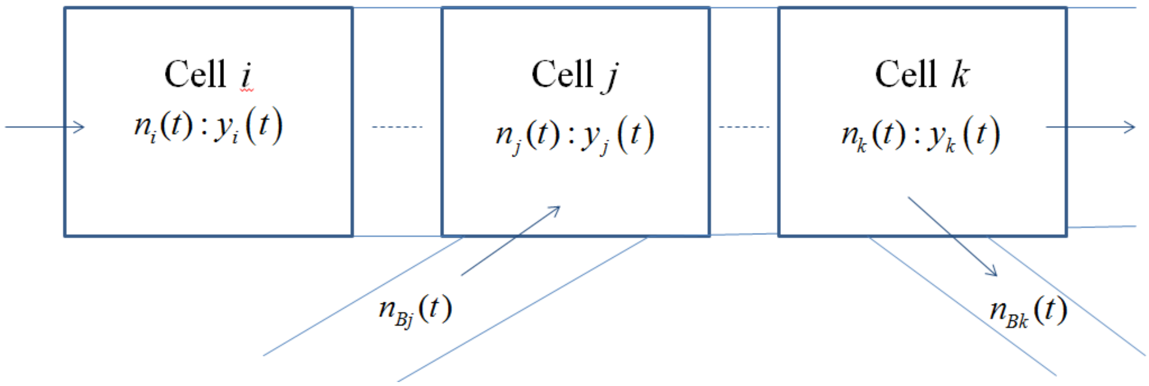


Figure 2-2 Discretization of Highway

The solution to CTM can be represented by estimating the discretized form of density using flow as shown in equation (2.2) (Daganzo (1994)).

$$
\begin{aligned}
&n_i(t+1) = n_i(t) + y_i(t) - y_{i+1}(t) \\
&y_i(t) = \min\{n_{i-1}(t), Q_i(t), (w/v)[N_i(t) - n_i(t)]\} \\
&\text{where,} \\
&n_{i-1}(t)\text{: no.of vehicles in cell } i\text{-1 at time } t \\
&Q_i(t)\text{: capacity flow into cell } i \text{ at time } t \\
&N_i(t) - n_i(t)\text{: amount of empty space in cell } i \text{ at time } t \\
&w\text{: wave speed for the fundamental diagram} \\
&v\text{: free flow speed for the fundamental diagram}
\end{aligned}
\tag{2.2}
$$

This method is equivalent to the numerical method developed for gas dynamics calculations by Godunov (1959). Godunov's method involves solving a series of Riemann problems forward in time at each discretized space section or cell. For a conservation law represented by,

$$\partial_t u + f'(u)\partial_x u = 0$$

the Godunov's solution is given by equation (2.3), (Leveque (1992))

$$U_i^{n+1} = U_i^n - \frac{k}{h}[F(U_i^n, U_{i+1}^n) - F(U_{i-1}^n, U_i^n)]$$

where,

$$F(\text{numerical flow function}) = \frac{1}{k}\int_{t_n}^{t_{n+1}} f(u(x_{i+1/2}, t))dt \tag{2.3}$$

k: size of time step

h: size of cell

U_i^n: is the discretized value of $u(x,t)$ in cell i at time interval t_n

In the conservation law, if u is represented by density ρ from the traffic flow equation then in equation (2.3) F is the flow q. The quantities U_i^n in equation (2.3) are number of vehicles in cell i at time t_n and U_i^{n+1} represents number of vehicles in cell i at time t_{n+1} respectively. The quantities $F(U_{i-1}^n, U_i^n)$ in equation (2.3) represent flow entering and $F(U_i^n, U_{i+1}^n)$ represents flow leaving cell i at time t_n respectively. Hence, $-[F(U_i^n, U_{i+1}^n) - F(U_{i-1}^n, U_i^n)]$ is net flow into cell i at time t_n which is nothing but $y_i(t)$ from equation (2.2).

2.2.2 Summary of Studies on Stochastic First-order Models

Variability in traffic is a very important aspect to be captured in the modeling approach. The variability could be a result of the change in:

1. demand,
2. vehicle composition,
3. driver behavior,
4. other unobservable factors.

The variability in demand can easily be captured by varying the boundary conditions of the corresponding models. In order to capture variability in vehicle composition, different vehicle classes can be used in either macroscopic, mesoscopic (such as cellular automata) or microscopic modeling approaches. Each vehicle class with a bounded speed, acceleration and deceleration parameters is supposed to model the stochasticity due to vehicle composition. The influence of driver behavior on traffic variability is an area that is still being explored and models thus created have only been ad-hoc. The stochasticity due to unobservable factors is usually modeled as a noise or error in the modeling parameters.

The macroscopic modeling methodology, as compared with others, is easier to implement and faster to execute. Hence capturing the stochasticity in traffic, due to all or most of the above mentioned factors is more tractable using macroscopic modeling approach. A brief summary of literature on stochastic macroscopic models is presented below.

Boel and Mihaylova (2006) proposed a stochastic compositional model by extending the cell-transmission model (CTM) by means of adding white noise to the speed that is estimated using the speed-flow fundamental diagram (FD). Boel and Mihaylova (2006) introduced stochasticity into the CTM by introducing noise terms into the sending and receiving functions. For very light traffic flow, they used binomial distribution for the noise term. For very heavy traffic flow, they used Gaussian distribution. For traffic conditions between very light and very dense, the authors' model is not well-defined.

A similar approach was performed by Sumalee et al. (2009) with a framework based on CTM with stochastic demand and flow-density relationship. Sumalee et al. (2009) introduced a stochastic CTM by using random variables in the sending and receiving functions via random parameters for the free-flow speed, jam density and backward wave speed of the fundamental diagram. In addition, the authors utilized the switching-mode model proposed by Munoz et al. (2003) to deal with the non-linear nature of the fundamental diagram.

Bladin et al. (2010) similarly formulated an approach using a noise term to the speed in the speed-density fundamental diagram. The fundamental diagram was divided into free-flow and congested regime. This noise term was introduced in the speed term of the congested regime.

Kim and Zhang (2008) constructed a stochastic flow-density curve that can be applied to macroscopic traffic flow models of first or second order. The basis of the stochastic flow-density curve is the random fluctuations of gap time and reaction times and, transitions between traffic states on the fundamental diagram.

Li et al. (2009) also constructed a stochastic speed-density relationship and applied it to a first order traffic flow model. The stochasticity is in the form of a noise term to the free-flow speed. The first order model based on the stochastic speed-density function was evaluated using Monte Carlo simulations of a finite difference numerical scheme.

Wang et al. (2009) developed a stochastic speed-density relationship following a spectral decomposition of covariance of speed as a function of density.

Khoshyaran and Lebaque (2009) formulated a framework for capturing variability using second order traffic flow model that is in the same mould as Aw-Rascle-Zhang (ARZ) model.

Ngoduy (2011) has also developed a stochastic first order model using different classes of vehicles whose behavior influences the variability in traffic. The model is an extension of first-order model by including a free flow speed term derived from a stochastic capacity derived from a Weibull distribution. Monte Carlo simulations of a numerical scheme were performed in the implementation.

Jabari and Liu (2012) incorporated stochasticity into CTM by representing number of vehicles in a cell as a counting process which is expressed as a random function of time headway.

A comparison of various aspects of these models can be seen in Table 2-2.

All of these models capture stochasticity by means of obtaining the simulation output for each condition separately, by means of Monte Carlo-type exhaustive simulation runs. Depending on the size of the network this approach can become computationally expensive and time-consuming.

Table 2-2 Summary of Literature on Stochastic Macroscopic Traffic Simulation Models

Authors	Type of Model	Solution Methodology	Stochastic Parameter	Data Used in Calibration
Boel and Mihaylova (2006)	First-order	Godunov scheme or CTM	Noise in speed-density FD	Loop detector data on a freeway section

Kim and Zhang (2008)	Flow-density curve	Only validation of fundamental diagram	Free flow speed, jam density	NGSIM data
Khoshyaran and Lebaque (2008)	Second order (ARZ-type)	Godunov scheme		A model is proposed but not implemented
Li et al. (2009)	First order	Essentially non-oscillatory (ENO)	Free flow speed	Loop detector data
Sumalee et al. (2009)	First-order mode switching	Godunov scheme	Free flow speed, jam density, wave speed	Loop detector data for 7 hours on three days in two years
Bayen et al. (2010)	First-order	Godunov scheme	Jam density	A model is proposed but not implemented
Ngoduy et al. (2010)	Multi-class first order	Godunov scheme	Free flow speed, jam density	
Jabari and Liu (2012)	First-order	Godunov scheme	Jam density	Compared with CTM

2.3 Calibration of Traffic Simulation Models with Specialized Model Components

Macroscopic models provide a simpler platform on an aggregate level of modeling for building simulation models on a bigger scale. On the other hand, microscopic simulation models present other challenges due to their very detailed nature. In addition to considering stochasticities observed in basic traffic flow parameters, there is a need to consider the effect of special geometric characteristics of the individual components of the modeled network such as merge locations on freeways and traffic circles, lane configurations for toll plazas, etc. Moreover, user behavior in relation to these specific geometric characteristics need to be captured by calibrating model parameters such as, mean reaction time, mean headway, route choice parameters, aggressiveness and familiarity of drivers with the system, etc.

Microscopic simulation models of freeways have been studied extensively as discussed in the literature review in the previous section. The calibration of these models involves calibrating parameters for car-following such as mean reaction time and target headway, parameters for lane change and gap acceptance and even parameters for route choice.

Microscopic simulation models involving specialized modeling of geometric features also involve many parameters. For example, Ozbay et al. (2006) modeled and calibrated a microscopic simulation model of New Jersey Turnpike (NJTPK) including its 26 toll plazas. The modeling process involved customization of driver behavior at the toll plazas based on entry and exit ramps. Along with the global mean reaction time and headway the other parameters used in the calibration process were link-level reaction time and headway at the toll plazas.

Mudigonda et al. (2009) developed a generic approach for modeling toll plazas and calibrated the models for different toll plazas. Their methodology entails modeling the drivers' lane choice decision process using a linear utility model. The utility model is expressed as a function of the entry ramp of the driver, the queue at each toll booth of the toll plaza and the exit ramp that the driver intends to take after exiting the toll plaza (as shown in equation (2.4)).

$$U_i = \alpha^e p^e{}_i + \alpha^x p^x{}_i + \alpha^q p^q{}_i \tag{2.4}$$

Where, $p^e{}_i$, $p^x{}_i$, $p^q{}_i$ are the probabilities of choosing lane i depending on the approach ramp (e), exit direction (x) and the queue conditions (q) respectively, and α^e, α^x, α^q are the weights for each variable where $\alpha^e + \alpha^x + \alpha^q = 1$. They evaluated their algorithm for three different types of toll plazas, namely, one with two entry and exit ramps, two entry and one exit ramp and one entry and one exit (barrier) toll plaza. The authors implemented the model in PARAMICS and compared the lane usage at the toll plazas (example shown in Figure 2-3). These lane usages are during the peak hour on specific days. In order to predict the outputs such as, lane usage and lane delays during other time periods and lane configurations, the data may not be available or too tedious to collect.

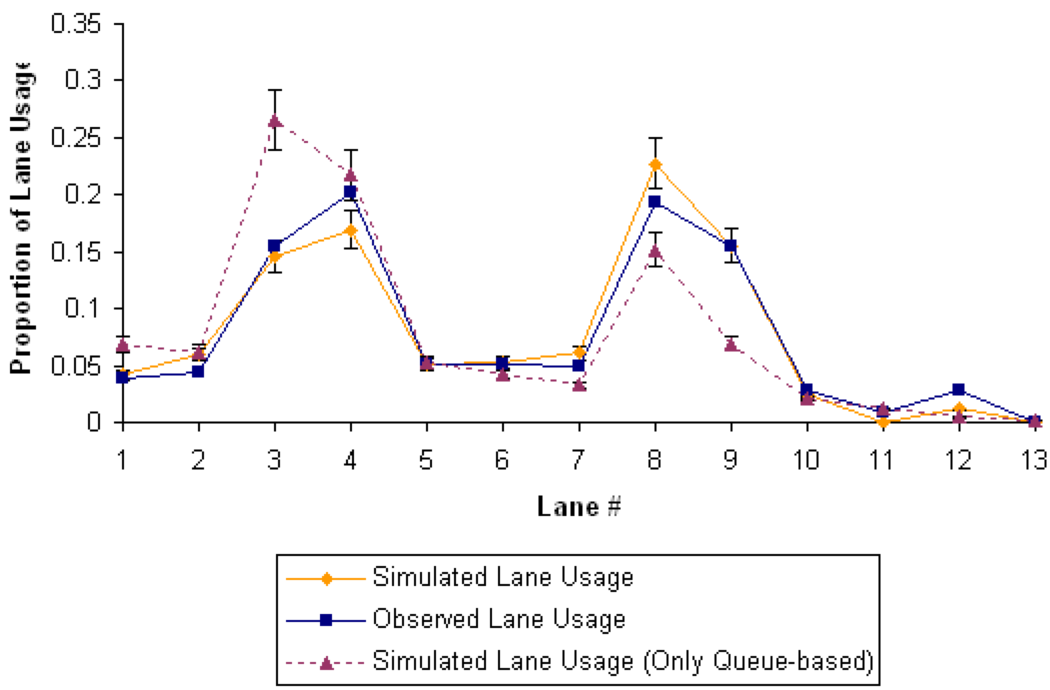


Figure 2-3 Lane Usage Comparison using customized Toll Plaza Driver Behavior Methodology (Mudigonda et al. (2009))

Ozbay et al. (2010) modeled the driver behavior at the toll plazas on the New Jersey Turnpike using a discrete choice model. They use approach ramp, exit ramp and queue lengths at the toll booths as the model parameters as shown in equation (2.5).

$$P_i = \frac{1}{1 + \exp-(b_i + a_i^q p^q{}_i + c_i R)} \qquad 2.5)$$

The variable $p^q{}_i$, in effect, represents how much less queue the lane i has. The variable R is a binary variable with $R = 0$ if vehicle is approaching the toll plaza from the toll plaza through the middle, $R = 1$ otherwise. The authors implemented the model in PARAMICS.

Nezamuddin and Al-Deek (2007) modeled the Holland East Plaza on SR408 in Orlando Florida using PARAMICS micro-simulation package. The authors used many different default model parameters mean reaction time and headway, next lanes, lane choice rules, etc and calibrated the model using ramp and link volumes.

Bartin et al. (2005) developed customized models for simulating traffic circles with different traffic controls and driver behavior. Apart from the normally used parameters of global reaction time and headway, the authors developed gap acceptance models for the gap acceptance behavior of drivers at the merge locations of traffic circles. The probability of accepting a gap is modeled as a probit model,

$$\Pr ob(V_{Acc} + \varepsilon_{Acc} > V_{\mathrm{Re}\,j} + \varepsilon_{\mathrm{Re}\,j}) = \Phi\left[\frac{V_{Acc} - V_{\mathrm{Re}\,j}}{\Omega}\right] \qquad (2.6)$$

$$V_{Acc} = \alpha + \beta g$$

V_i is the observed part of the gap utility function, ε_i is the observed part of the gap utility function for acceptance (i=*Acc*) and rejected (i=*Rej*) gaps. Φ is the standard cumulative normal function. g is gap, α, β probit model coefficients.

Vaiana et al. (2007) used VISSIM to build a microscopic simulation models for roundabouts in Italy. In the modeling procedure, the authors used parameters such as approach speed, circulating flow, circulating radius of roundabout, and width of the splitter island.

Previous studies that aimed at developing toll plaza simulation models is summarized in Table 2-3.

Table 2-3 Summary of Toll Plaza Simulation Literature

Study	Simulation Tool	Features	Validation
Junga (1990)	GPSS (Standalone, Macroscopic)	Lane assignment based on queue and payment type	Validated for barrier tolls only.
Correa et al. (2004)	TOLLSIM (Standalone, Macroscopic)	Lane assignment based on shortest queue	Validated for barrier tolls only.
Danko and Gulewicz (1991)	Spreadsheet (Standalone, Macroscopic)	Lane assignment based on shortest queue	Validated for barrier tolls only.
Burris and Hildebrand (1996)	Standalone, Microscopic	Lane assignment based on Queue length, and Proximity of the preferred payment-type lane	Validated for barrier tolls only.
Al Deek et al. (2000)	Standalone, Microscopic	Queue length, Faster vehicle always changes lane. Reaction times, service times and arrivals from field data	Validated for barrier tolls only.
Astarita et al. (2001)	Standalone, Microscopic	Utility model based on: Aggressiveness of drivers, Queue length and Number of lanes to cross to reach at a particular lane	Validated for barrier tolls only.
Chien et al. (2005)	PARAMICS (Microscopic)	Default PARAMICS driver behavior, lane changing models.	Validated for barrier tolls only.
Ceballos and Curtis (2004)	VISSIM (Microscopic)	Default VISSIM parameters. Lane selection based on queue, payment type.	Validated for parking toll plaza.
Ozbay et al. (2006) and Bartin et al. (2007)	PARAMICS (Microscopic)	Customized PARAMICS driver behavior, path-based lane changing models.	Validated for non-barrier and barrier tolls on NJTPK.
Nezamuddin and Al-Deek (2007)	PARAMICS (Microscopic)	Default PARAMICS driver behavior, lane changing models.	Validated for barrier tolls only.
Mudigonda et al. (2009)	PARAMICS (Microscopic)	Utility-based lane choice model as a function of approach ramp, exit ramp and queue.	Validated for different barrier and non-barrier tolls.

Ozbay et al. (2006) and Bartin et al. (2007) developed a toll plaza model in

Paramics that is integrated with the freeway model of NJTPK. It was shown in Ozbay et al. (2006) that the default Paramics lane selection at toll plazas was not sufficient to simulate real-world toll plaza operations. Therefore, the default lane selection at toll plazas was improved using Paramics Application Programming Interface (API). The authors developed a path-based lane choice model that takes into account the ramp drivers select after crossing toll plaza.

As an extension of Ozbay et al. (2006) and Bartin et al. (2007), Mudigonda et al. (2009) proposed a model that enhances the modeling of the drivers' decision making at a toll plaza. By the use of Application Programming Interface (API) feature of PARAMICS, positives from both standalone models (natural decision making process) and the microscopic simulators (detailed car-following and gap acceptance models) are combined.

From the above summary of literature on the simulation of complex traffic flow features, it can be inferred that modeling of all these additional features of driver behavior requires many parameters. In turn, the number of parameters that are needed to be adjusted in order to calibrate the simulation model multiply. This increases the number of scenarios to be evaluated to obtain a robust calibrated model. This in turn increases the complexity of the experimental design of traffic simulation models. Hence, there is a need for a procedure that can reduce the number of scenarios in evaluating the complex simulation models albeit without significant loss in predictive accuracy of the model.

CHAPTER 3. ISSUES IN CALIBRATION OF TRAFFIC SIMULATION MODELS AND CONTRIBUTIONS OF THIS DISSERTATION

Traffic simulation models are mathematical abstractions of the transportation system in which output is derived from a particular set of mathematical equations and relationships given a specific input data. The input data consists of two main groups of data: physical input data I_s (e.g., volume counts, capacity and physical features of roadway sections) and driver specific parameters C_s (i.e., adjustable components of driver behavior such as free flow speed, reaction time, mean headway, etc.). Output from a simulation model can be expressed as equation (1.1). The process of calibration entails adjusting the calibration parameters (C_s) so that the error between the output from simulation and field conditions is minimized as shown in equation (1.2).

In order to capture variability in modeling and simulating traffic simulation models, stochasticity in both input (I_s) and calibration parameters (C_s) need to be considered. In order to model the stochasticity in inputs, i.e., demand-side variability, demands from various time periods have to be obtained. Additionally, the supply-side variability, similarly, involving observed outputs, such as speeds and flows, during various time periods are also needed. Traditionally, observed output data comes from traffic roadway sensors. However, the accuracy of traffic sensor data may not be good all the time (Rajagopal and Varaiya (2007), Li and Li (2009)). In such cases, alternative sources of data may be needed to supplement the sensor data.

For calibrating microscopic traffic simulation models, specifically, those that require several customized models augmenting the default modeling, requires data in much greater detail. Customization of microscopic models is required when the default modeling capability is inadequate. Such customization is performed often for modeling traffic circles (Bartin et al. (2005), Vaiana et al. (2007)), toll plazas (Astarita et al. (2001), Ozbay et al. (2006) and Mudigonda et al. (2009)), freeway merging sections (Yang et al. (2006), Gardes et al. (2002), Yang and Ozbay (2011)), etc. Thus the same generic calibration methodology may not be applicable to traffic simulation models of much greater detail. The detailed data for calibrating such models may need to be collected via video data captured at specific locations of the section to be modeled. Such data may not always be available. Thus better means and methods to calibrate such models for various conditions are required.

From the discussion above and the review of literature in the chapter 2 the following are the important issues identified in the calibrating traffic simulation models for various conditions:

1. Characterizing demand- and supply-side variation
2. Stochastic modeling – computational complexity
3. Calibration of microscopic simulation models requiring customization
4. Performing calibration to address limited data availability

Below, we present a short description of each identified issue and in the end present the contributions of this dissertation. Figure 3-1 illustrates the flowchart of the proposed calibration methodology and various contributions of this dissertation.

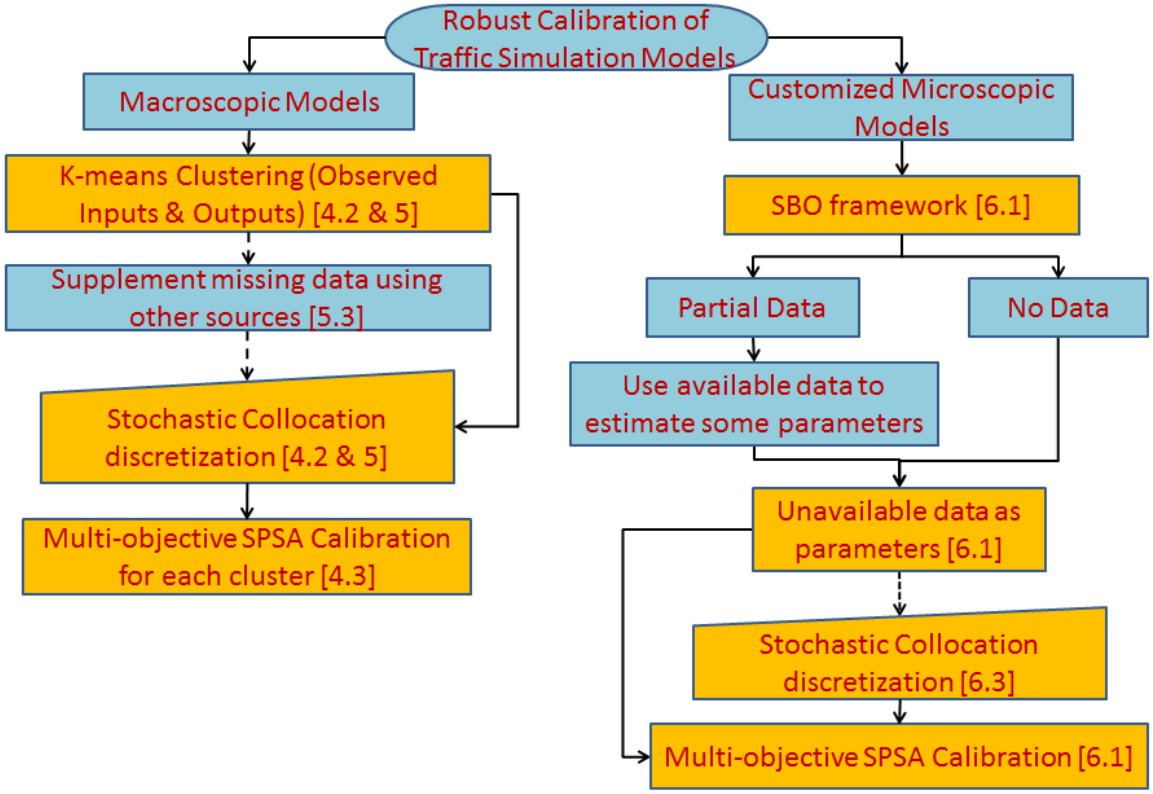


Figure 3-1 Flowchart of the Research Methodology and Contributions Presented in the Dissertation

In chapter 4, the methodology employed in the calibration of macroscopic models using stochastic collocation is presented. In chapter 5, case studies for the implementation of the proposed calibration methodology are presented. Chapter 6 presents the calibration of customized microscopic simulation models and the simulation-based optimization (SBO) framework also illustrating the methodology using case studies. The SBO framework is extended further to incorporate the ability to estimate stochastic output using the computationally efficient sparse grid stochastic collocation.

3.1 Capturing Demand-side and Supply-side Variation

Based on the discussions above, making accurate predictions using traffic simulation models requires calibration data over much greater span of time as compared to the studies described in the previous chapter. Most of the past traffic calibration is based on the assumption of a typical weekday or weekend day at best (Kim and Rilett (2003),Ma and Abdulai (2002),Hourdakis et al. (2003),Toledo et al. (2004),Qin and Mahmassani (2004),Balakrishna et al. (2007)). Variations in traffic data are twofold. One is the demand-side variation, also characterized as variation of input to the simulation model. Second is the supply-side variation such as speed, flow, etc., characterized as variation in parameters of the simulation model. It is important to capture the variations in both these

Ozbay et al. (2014) and Mudigonda and Ozbay (2015) analyzed demand and sensor data to investigate whether representative days do exist in traffic. Depending on how close or distant the demand or speed values are to each other, attempt is made to classify the demand or speed data for each time period into clusters. Chapter 4.1 presents the clustering methods used to group and classify the observed inputs and outputs. Using clustering approach the demand-side and supply-side variation is studied. In chapter 5, case studies using freeway section from NJTPK are used to obtain the demand- and supply-side variation for the section. Each cluster represents a group of demands that are similar to each other and can be represented by the centroid of the cluster. The basic hypothesis is that the greater the number of clusters, the lower is the likelihood of existence of a "typical" day. The analysis illustrates that the existence of a "typical" day in traffic demand is not always likely. Hence, to obtain accurate predictions from a traffic simulation model, it is important to consider not only the demand from various clusters, but also the variation of demand within each cluster. The above discussion is based on the distribution and spread of traffic demand. However, the actual traffic flows along the section of interest would also vary based on many conditions such as, work zones, driver/vehicular variability and other unobserved phenomena.

For each of the cluster, the proposed calibration methodology is applied to obtain the optimal set of parameters that represents the observed output distribution as closely.

Thus, when building a credible traffic simulation model, it is important to capture variations on demand-side and supply-side. This variation includes not only different groups into which the demand and supply fall into, but also the variation within each group.

3.2 Computational Complexity

After acquiring the demand- and supply-side variability, the next step would be to incorporate it into the simulation modeling framework. Since the demand- and supply-side variability could follow any distribution, an analytical solution may not readily exist. Hence, numerical methods have to be adopted to solve the traffic simulation problem with variability.

There were studies that captured traffic variability (Li et al. (2009), Ngoduy (2011), Zhong and Sumalee (2008), Jabbari, Liu (2012), Lee and Ozbay (2009)) to name a few. The computational models used to solve for this stochastic traffic simulation problem use a Monte Carlo (MC)-type independent sampling for various traffic conditions. However, the increase in the number of factors affecting stochasticity increases the dimensionality of the calibration process. This in turn results in increased computational effort required in calibrating traffic simulation models for different conditions such as variability within weekday/weekend, and seasonal variability, and special situations including adverse weather, work zones, etc.

It may not be possible at all to simulate the output for each and every possible realization of parameter and input. Also, all possible points in the stochastic space of simulation output may not have the corresponding observed data. Solution methods that use repeated intensive sampling for various dimensions in the stochastic space are not desirable in the construction of robust methods for calibration of traffic simulation models. Hence, it is important to obtain an effective sampling and interpolation methodology for predicting output accurately but with lower computational effort. Alternative methods to mitigate the issues of computational complexity need to be explored.

To address the above issues of computational burden, in chapter 4, a sampling and interpolation methodology using stochastic collocation is proposed. This approach uses the output from deterministic model runs and interpolates to obtain furthermore outputs without actually running the model. The traditional Monte Carlo-type sampling has a convergence rate of the order of $O(1/\sqrt{M})$ (Loh (1996)), where M simulation runs for various traffic conditions. Convergence rate of the interpolant is of the order, $O(M^{-2}|log_2 M|^{3(N-1)})$ (for piecewise linear basis), $O(M^{-k}|log_2 M|^{(k+2)(N-1)})$ (for k-polynomial basis) where M is the total number of collocation points, Q-th order of interpolation, N-

dimensions and *k*-polynomial basis. This rate can be controlled by the interpolation level and polynomial order k (Ganapathysubramaniam and Zabaras (2007), Klimke (2006)).

In chapter 5, case studies that use freeway section at interchange 7 of the NJTPK as the modeling section, are presented. The proposed calibration methodology is applied to this model for various traffic conditions. These traffic conditions are determined by the clustering approach mentioned in the previous subsection. The observed flow distribution is closely matched using the calibrated set of parameters using the proposed methodology. The calibrated parameters are also validated using a hold out dataset. Additionally, we show, that computational efficiency of the proposed methodology as compared to more commonly used Monte Carlo method is shown empirically.

3.3 Calibration of microscopic simulation models with customization

In the previous subsection, the computational complexity that the numerical methods capturing stochasticity suffer from is illustrated. This problem is further exacerbated when microscopic traffic simulation models require additional customization. Customization of microscopic models is required when the default modeling capability is inadequate. Such customization is performed often for modeling traffic circles (Bartin et al. (2005), Vaiana et al. (2007)), toll plazas (Astarita et al. (2001), Ozbay et al. (2006) and Mudigonda et al. (2009)), freeway merging sections (Yang et al. (2006), Gardes et al. (2002), Yang and Ozbay (2011)), etc. Thus the same generic calibration methodology may not be applicable to traffic simulation models of much greater detail such as the toll plaza models.

Calibration of such customized microscopic models for various conditions has two primary difficulties. First is the number of parameters, not only default model parameters but also customization parameters, which need adjustment when calibrating. Second, is the availability of detailed data required for such detailed modeling for various conditions. An alternative calibration approach that can economize on the data requirement will be very useful.

In chapter 6, we propose the use of a simulation-based optimization framework for calibration of toll plaza models. The toll plaza modeling approach proposed in Mudigonda et al. (2009) is adopted. The flexibility of this SBO framework is that it can be used with the toll plaza modeling approach:

- in cases where site-specific video data is available to estimate all the required parameters of the toll plaza model
- when only partial data to estimate these parameters are available
- or even when no data is available.

The novelty of the SBO framework is that whichever data is not available, the corresponding parameters can be used as calibration parameters.

Using case studies the benefits, mentioned above, of the SBO framework are demonstrated. Furthermore, we combine the sampling and interpolation using stochastic collocation (proposed in chapter 4) with the SBO framework. Using this hybrid framework, we calibrate the parameters to obtain distribution of output from the toll plaza model that closely follows the observed measures at the toll plaza.

3.4 Calibration with limited data

Modelers could often face situations when calibrating for a particular condition that there may be some data missing and only limited data is available. Missing data often arises when using traffic sensor data. Traffic sensors may be malfunctioning or not accurately recording the speed or flow data. In chapter 5, the use of alternative sources of data when sensor data is missing is demonstrated. This aspect is elaborated using a larger freeway section of eight miles encompassing interchanges 7 and 7A of NJTPK. On this section few sensors have 25% of missing data and one with 70% missing. To mitigate this issue, ETC data is used to estimate the section travel times. In the proposed calibration methodology, travel times are used as an additional output measure.

Limited data availability could be a more ubiquitous issue when calibrating microscopic models that require detailed data (such as video data) at specific sites of the section of interest. An example could be the need for video data downstream of a toll plaza to calibrate the user behavior downstream of the toll plaza. To deal with such issues, as mentioned in the previous subsection, a SBO framework is proposed to calibrate toll plaza models. The novelty of this framework is that when some of the data is unavailable, the parameters associated with the unavailable data can be used as parameters in the calibration process.

CHAPTER 4. METHODOLOGY FOR CALIBRATION OF MACROSCOPIC TRAFFIC SIMULATION MODELS

A simulation model, like any mathematical model, can be represented as a prediction model using a set of parameters along with an additive error term. This error term can further be split into a sum of modeling and estimation errors as shown in equation (4.1). The modeling error represents how accurately the model can capture various aspects of traffic flow whereas the estimation error represents how accurately the created model can reproduce various traffic parameters, such as flow, density and speed.

$$\begin{aligned}
&\boldsymbol{\Xi}(\mathbf{x}, I_S) = \boldsymbol{\mu}(C_S(\mathbf{x}), \mathbf{x}, I_S) + \boldsymbol{\varepsilon}_M(\mathbf{x}) + \boldsymbol{\varepsilon}_E(\mathbf{x}) \\
&\text{where,} \\
&\boldsymbol{\Xi} \text{ - vector of outputs observed} \\
&\boldsymbol{\mu} \text{ - vector of outputs predicted by the model} \\
&C_S(\mathbf{x}) \text{ - parameter vector} \\
&\boldsymbol{\varepsilon}_M \text{ - modeling error} \\
&\boldsymbol{\varepsilon}_E \text{ - estimation error} \\
&\mathbf{x} \text{ - dependent variable (space and time)} \\
&I_S \text{ - vector of inputs}
\end{aligned} \tag{4.1}$$

The estimation performed by simulation can be depicted using equation (4.1). It can be seen that the vector of observed outputs $\boldsymbol{\Xi}$ is approximated by simulation output $\boldsymbol{\mu}$. $\boldsymbol{\mu}$ is a function of input vector I_S and parameter set $C_S(\mathbf{x})$. Translating to traffic simulation models, the outputs are flows, densities, travel times etc., the inputs, I_S, are demand entering the freeway or arterial that is being modeled, the parameter set, $C_S(\mathbf{x})$, can be free flow speed, jam density, or reaction time, headway, etc. It can be noted that the inputs represent the demands and parameter set depicts the supply on the freeway or arterial of interest.

The variability in observed traffic metrics, $\mathbf{\Xi}$, and simulation output, $\boldsymbol{\mu}$, is a result of variability in inputs I_S (demands) and parameter set $C_S(\mathbf{x})$ (supply) during different time periods of the day, different weather conditions, different driver population composition, different geometry, etc. Other stochastic factors that could influence traffic dynamics include, the unpredictability of driver behavior and vehicular performance, random and unquantifiable events affecting drivers, random and unquantifiable of the driving environment and factors such as choice of destination and path.

When accounting for variability in traffic simulation, the stochasticity can be captured by making the inputs ($I_S(\omega)$) and parameter set ($C_S(\mathbf{x},\omega)$) stochastic. The stochasticity resulting from other unquantifiable and random factors can be represented by the modeling error $\boldsymbol{\varepsilon}_M$.

In this chapter we give a detailed explanation of the methodology that we propose for modeling inputs ($I_S(\omega)$) and parameter set ($C_S(\mathbf{x},\omega)$) stochastically in the calibration of macroscopic simulation models. As mentioned in chapter 3 of this dissertation, the problems that are identified for a robust calibration of traffic macroscopic simulation models are the following:

1. Characterizing demand- and supply-side variation
2. Modeling stochasticity

3. Parameter Optimization for calibration
4. Calibration using limited data via fusion of multiple sources of data

Each of these issues and possible solution methodologies are discussed in the subsequent sub-sections.

4.1 Supply- and Demand-side Variation

As mentioned in the previous chapter, analysis of demand data distribution can provide a useful insight into whether representative days do exist in traffic. For this purpose 15-minute demand data extracted from E-ZPass data from the New Jersey Turnpike (NJTPK) for a year is analyzed as an example. Depending on how close or distant the demand values for each 15-minute time interval are to each other, attempt is made to classify the demand data for each time period into clusters. Each cluster represents a group of demands that are similar to each other and can be represented by the centroid of the cluster. The basic hypothesis is that the greater the number of clusters, the lower is the likelihood of existence of a "typical" day.

In order to separate or classify the section outputs, we use clustering techniques. Clustering techniques are usually applied in initial investigation of data. However, it is an effective method to separate data into groups by minimizing variance within the group and maximizing variance between groups. The section output that falls into a group can be considered to be subjected to similar conditions. Hence, the simulation inputs and parameters that can be used to generate these conditions are similar. For clustering time series data, some of the common algorithms used are K-means clustering, hierarchical clustering and fuzzy c-means clustering (Liao (2005)). For the electronic toll collection data, we use K-means clustering. In order to determine the optimum number of demand clusters, silhouette statistics are generated for each of the links. Silhouette statistics show how dissimilar a particular demand value is from its demand cluster centroid.

There are 28 interchanges on NJTPK spread over different spacing. Considering the roadway between each interchange as a link, there are 65 links in northbound and southbound directions on the NJTPK system. For the purpose of clustering demand, the 15-minute demand data between September 2011 and August 2012 for 5 AM – 9 PM is analyzed.

It can be seen from Table 4-1 that there are links for which the demand falls into multiple clusters. 24 links have demand falling into two optimal clusters, 32 links have three clusters and so on. More than 63% of the link demands have three or more clusters. Among these clusters there are different weekend or weekday demand distributions. This means that considering a single distribution of demand for a weekday or weekend is not sufficient to accurately calibrate a simulation model that can be used throughout the year.

Table 4-1: Distribution of number of links on NJTPK and optimum number of clusters

Optimum Number of Clusters	Number of Links
2	24
3	32
4	7
5	1
6	1

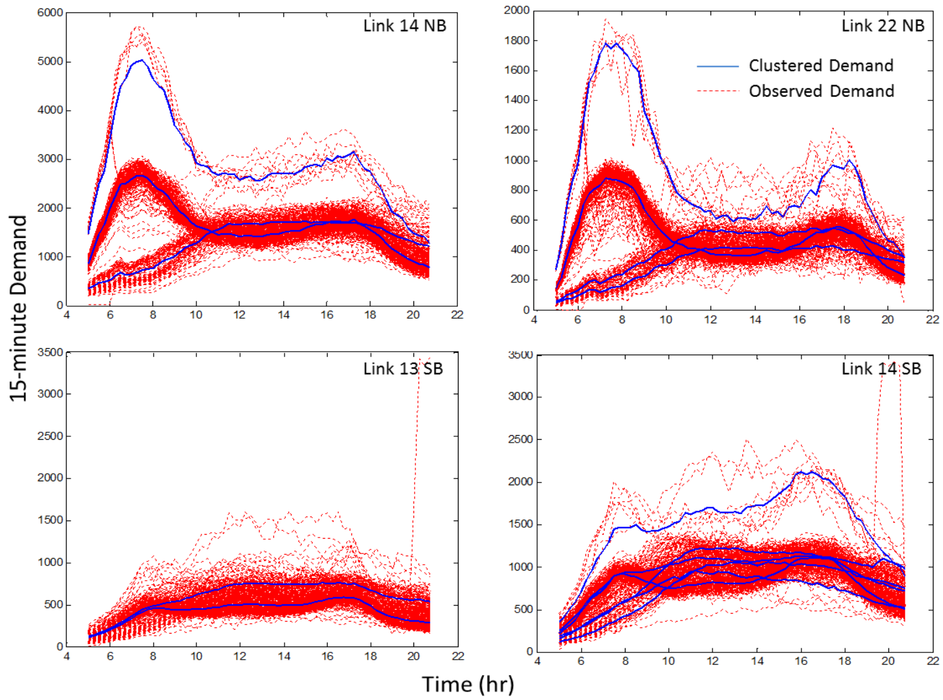


Figure 4-1. Illustration of clustered demand for four different links on NJTPK

In order to show the representativeness of the clusters, we show the frequency of observations vs. their cluster number. Figure 4-2 depicts the likelihood of an observation (i.e., the demand on a link for a day in the whole year) to fall into a particular cluster. It shows that 35% of observations fall into clusters one or two and 20% of demands fall into four other clusters. Although 35% of observations do fall into one or two clusters, the distribution of the observations within the cluster is fairly large, as can be seen from the spread of observed demands around the clustered demand in Figure 4-2.

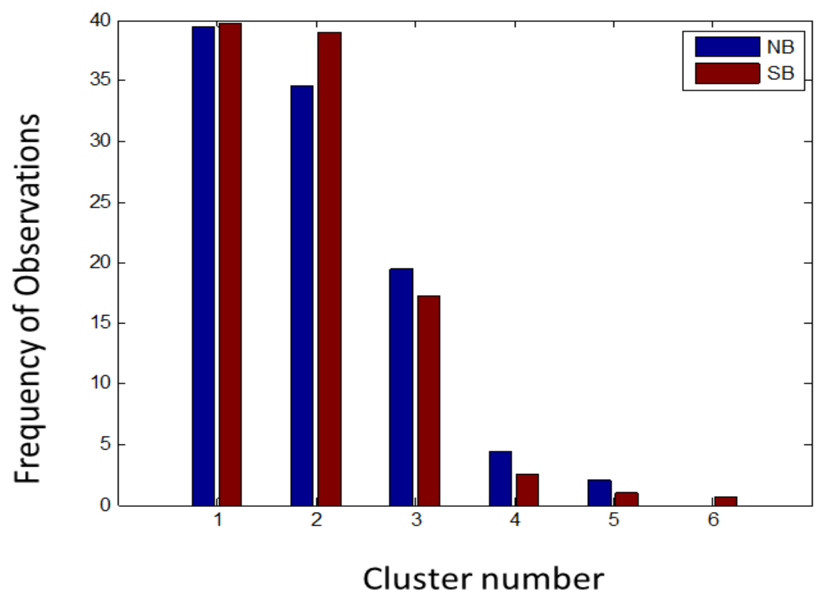


Figure 4-2. Frequency of number of observations for all links in each cluster

In order to capture various supply-side traffic conditions, we use the traffic sensor data. As mentioned in the earlier, traditionally, an arbitrarily chosen day or few days from the sample has been used for calibration. However, as depicted in Figure 1-1, this may not be representative of the speed and flow variation of the section. In order to capture the true variation in speed and flow of the section, we would require using the output from a much larger sample of the population. The variation in the output of the freeway section could be due to reasons such as time of day, weather, construction, incidents, geometry, or even due to differences in acceleration or deceleration of drivers. One or more of these conditions could result in variation of the section output. It is prudent to calibrate the simulation model separately for each of such condition resulting in the observed output.

Simulation inputs form the demand-side of the freeway section and simulation parameters form the supply-side of the freeway section. The variation in the observed output of a freeway section could be a result of variation in either of the supply- or demand-side of the freeway section. In general, changes in speed (at least free flow speed) is a result of changing pavement condition, geometry or surrounding conditions, and thus is a property of the supply-side of the freeway section. Unlike speed, flow output is a function of both supply-side as well as demand-side of the freeway section. Thus, in this study, we use the speed output for separate the conditions that govern the supply-side of the freeway section. Subsequently, to capture the demand-side variation, we consider the variation in demand to capture the variation in output within each supply-side condition.

In this study, we use a hybrid of electronic toll collection (ETC) data for demand and traffic sensor data for speed and flow. The ETC data is collected for all toll ways in the U.S. and in New Jersey. Taking toll facilities in New Jersey as an example, New Jersey Turnpike (NJTPK) is spread over 150 miles with 28 interchanges and 366 toll lanes. Garden State Parkway (GSP) is about 170 miles long with 50 toll plazas and 236 toll lanes. Each freeway carries up to 400,000 vehicles per day (NJTA (2013)). The ETC data is collected at toll plazas on these freeways. (NJTA (2013)) The ETC dataset consists of the individual vehicle-by-vehicle entry and exit time data. It also consists of the information regarding the lane through which each vehicle was processed (both E-ZPass and Cash users), vehicle types, number of axles, etc.

Speed output from the traffic sensors is used to categorize various traffic conditions. The demand from the ETC data is obtained for various clusters of traffic conditions so as to use a distribution of demand for each condition. The simulation is performed using the clustered demand data distribution and simulation output of flow and density is compared to the observed distribution from sensor data.

Thus the traditional approach of calibrating for a typical day is not sufficient, especially, if the calibrated models are used as predictive models. Hence, to obtain accurate predictions from a traffic simulation model, it is important to consider not only the demand-side variation but also and supply-side variation. Additionally, the demand- and supply-side variation within each cluster is also essential.

4.2 Modeling Stochasticity

When accounting for variability in traffic simulation, the stochasticity can be captured by making the inputs ($I_S(\omega)$) and parameter set ($C_S(\mathbf{x},\omega)$) stochastic. Below we describe the stochastic modeling of inputs and parameter set.

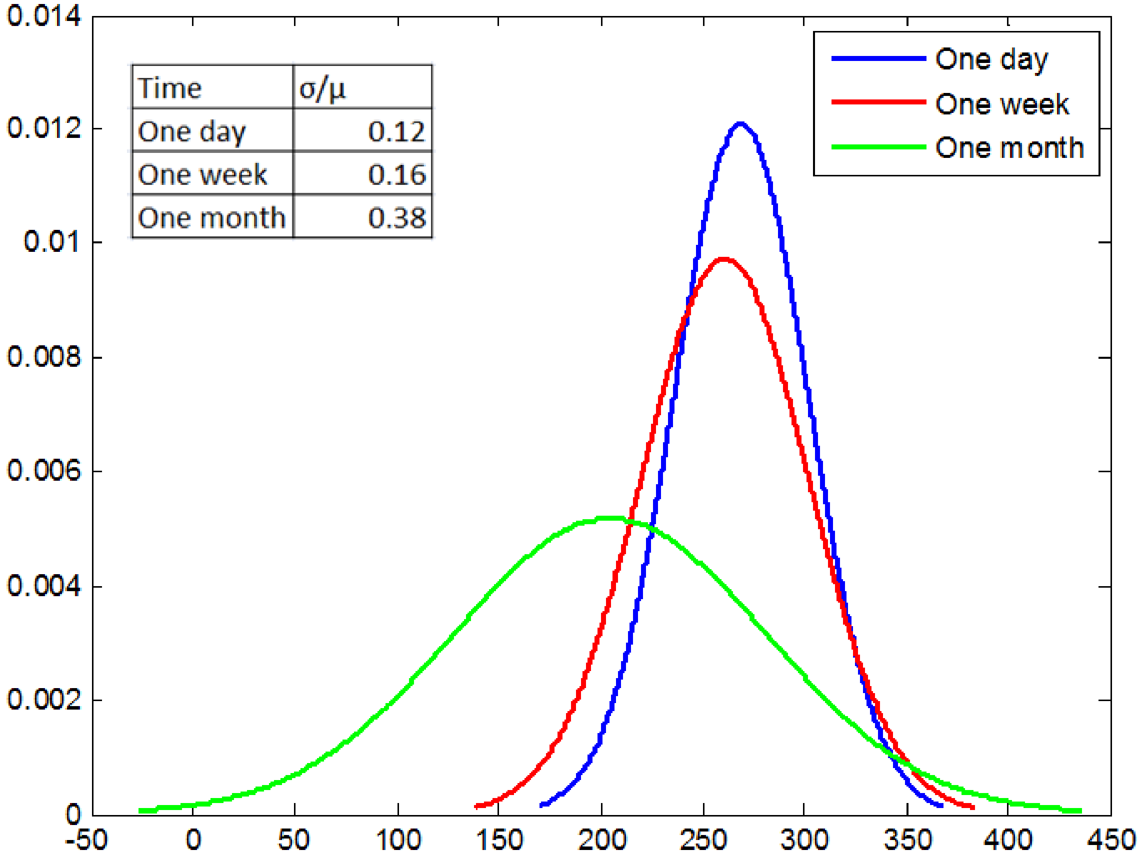
4.2.1 Quantification of Stochastic Inputs

The data used in some of the recent studies in the calibration of traffic simulation models are shown in Table 4-2. This is a summary of the data used in the literature presented in Table 2-1. It spans about three to 16 days during AM and/or PM peaks. Thus, these data may be limited to specific conditions or may be a diluted sample of few conditions. This approach of using limited demand in the calibration process is a deterministic approach. Hence the models calibrated using such data and the predictions from such models are not robust enough.

Table 4-2 Data used in Calibration of Traffic Simulation Models

Study	Data
Hourdakis et al. (2003)	5-min. data; 21 detector stations; 12-mile freeway section; PM peak; 3 days
Jha et al. (2004)	Detector data;15 days; AM and PM peaks; large urban network
Toledo et al. (2004)	68 detector stations; 3 freeways; 5 weekdays
Qin and Mahmassani (2004)	7 detector stations; 3 freeways; AM peak; 5 weekdays
Kim et al. (2005)	Travel time data for 1 hr.; AM peak;1.1 km. freeway section
Balakrishna et al. (2007)	15 min. data; 33 detector stations
Zhang et al. (2008)	5-min detector count; PM peak; 7 days
Lee and Ozbay (2009)	5-min detector count; AM & PM peaks ; 16 days
Duong et al (2010)	NGSIM trajectory data for US-101 for 40 min.
Korcek et al. (2012)	Loop detector data from two freeways in Czech and Slovak Republic for one year
Ge and Menendez (2013)	Travel times on 20 road sections in the Zurich inner city network

In order to illustrate the variability in traffic demand over time, the distribution of demand over different sampling time periods can be shown. The distribution of flow for a loop detector at milepost 60.3 on the NJTPK for the month of April 2011 during the weekday AM peak period is used for this purpose. Figure 4-3 shows the fitted normal distribution probability density curve for one day, one week and one month. The coefficient of variation for these time periods is also shown in Figure 4-3. It should be noted that normal distribution is not the best fit for the data but it is used to illustrate the increasing spread (variance) as the sampling time period increases, which is illustrated by the increasing coefficient of variation.



Time	σ/μ
One day	0.12
One week	0.16
One month	0.38

Figure 4-3 Fitted Normal Distribution for one day, one week and one month of loop detector data at milepost 61.2 northbound direction on NJTPK

As mentioned in the section on characterizing demand-side variation, the demand from the ETC data is obtained for various clusters of traffic conditions. Since this demand is from many different days, the demand-side variation is captured as a distribution for each condition. This input (demand) drawn from a distribution is a representative sample for the traffic supply conditions The simulation is performed using the clustered demand data distribution and stochastic simulation output of flow and density are generated.

4.2.2 Quantification of Stochastic Parameter Set

It is assumed that the traffic processes are represented by the first-order model with just the conservation of flow (equation (2.1)). Note, however, that the framework presented herewith can be applied to second-order models or simulation model of any detail. Given this assumption, the uncertainty in traffic can be incorporated by introducing stochasticity into the second, third and fourth condition of equation (2.1). This stochasticity could span in space and time as well.

The second aspect of incorporating stochasticity, after demand, is stochastic parameters. The parameter uncertainty in traffic can be incorporated by introducing stochasticity into the second and third conditions of equation (2.1). This stochasticity could span in space and time as well.

In order to illustrate the stochastic variation in parameters, consider the discretization of the highway that is being modeled as shown in Figure 4-4. The highway is divided into *n* sections. Stochasticity in the boundary condition would mean that the demand could be drawn from a distribution of demands on the highway from different days for the same modeling time period. Stochasticity in the fundamental flow-density (q-k) relationship, $f(\rho)$, means, that the maximum flow, free flow speed and jam density could be drawn from a distribution rather than using fixed values. Stochastic fundamental flow-density relationship is evidenced from various loop detector data and also proposed by studies such as Kim and Zhang (2009), Muralidharan et al. (2011). Stochasticity in time would mean that the parameters that go into the model may vary with time of day. Lee (2008) has considered the stochasticity in boundary conditions and time in his methodology. Additionally, it can be observed from studies such as Lee and Ozbay (2008), Balakrishna et al. (2008), Zhong (2010) calibration traffic simulation models for different time periods/conditions could result in different sets of parameters. Stochasticity in space would mean that, instead of using the same set of parameters for the whole highway, each section could have a different set of parameters. Hence it would be more appropriate to derive/obtain a distribution of the parameters based on the type of condition. Note that this distribution can span both in the temporal (time of day, season, weather, etc.) and spatial (changing geometry or pavement condition in different parts of the network) dimensions. However, the variability in traffic conditions has been modeled using Monte Carlo-type exhaustive simulations representing each and every condition. But this approach, depending on the size of the network, could be computationally time-consuming.

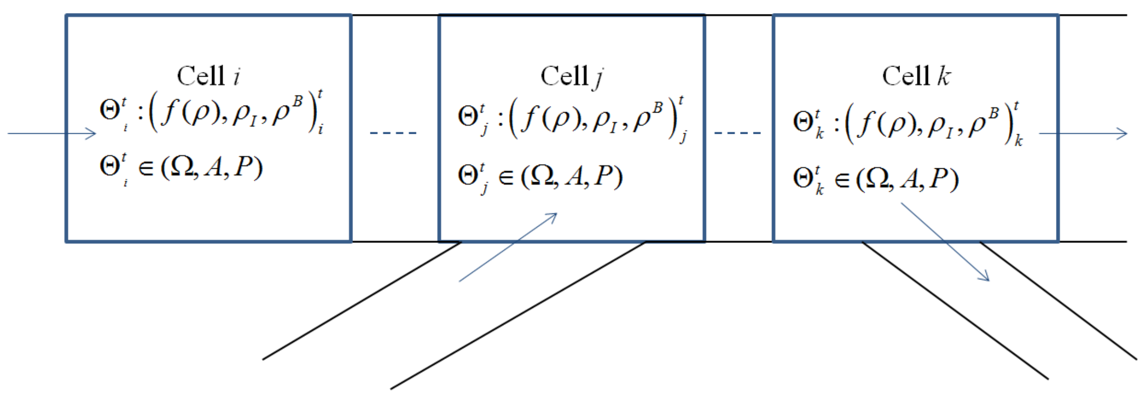


Figure 4-4 Discretization of a Highway Section

The stochastic version of the macroscopic traffic flow model in equation (4.2) can be shown as a stochastic partial differential equation (SPDE),

$$\begin{aligned}
&\partial_t \boldsymbol{\rho}(x,t,\omega) + \partial_x \left(\boldsymbol{\rho}\mathbf{v}(x,t,\omega)\right) = 0 \\
&\boldsymbol{\rho}(x,t,\omega) \in D \times \Omega \\
&\text{D: deterministic } x\text{-}t \text{ domain} \\
&\Omega\text{: stochastic space} \\
&q = \rho v \sim f(\rho,\omega)_i\text{: fundamental relationship for } i\text{-th output cluster} \\
&\rho_i(x_j,t) = \rho_j^B(t,\omega)_i\text{ : stochastic demand in cell } j \text{ for } i\text{-th output cluster}
\end{aligned} \tag{4.2}$$

written in short as,

$$B(\boldsymbol{\rho} : x,t,\omega) = 0 \tag{4.3}$$

Thus the parameter set is defined as a stochastic function of time and location and having a probability density function $p(\mu,\sigma)_i^t$. Thus, the set of parameters of interest with stochasticity can be expressed as $(f(\rho(\omega)), \rho_I(\omega), \rho_i^B(\omega))$. Note that there can be additional parameters in $f(\rho)$ based on which regime of traffic flow the stochasticity is modeled. In the language of measure theory, $(f(\rho(\omega)), \rho_I(\omega), \rho_i^B(\omega))$ belongs to a probability space (Ω, A, P) whose event space is Ω and is equipped with σ-algebra A and probability measure P. Thus for $\omega \in \Omega$, we intend to solve for $\rho(x,t,\omega) : D \times \Omega \to \mathbb{R}$ such that equation (4.2) is satisfied. Using this framework, the next three sections describe a generic solution methodology for solving the first-order traffic flow model with stochastic parameters.

4.2.3 *Solving the Stochastic Traffic Flow Model*

In the case of models of physical processes represented by a set of partial differential equations (such as the conservation of traffic flow), uncertainty can be incorporated by modeling the process as an SPDE. The uncertainty in various parameters is modeled using various numerical methods. Among these numerical methods, quantification of uncertainty in model parameters and, initial and boundary conditions is an actively researched area.

The solution methods to solve the SPDEs can broadly be classifies into Monte Carlo-type statistical methods and non-statistical methods. Monte Carlo-type methods do not approximate the solution space. They use the deterministic solution method to repetitively solve the problem at each sample point of the stochastic space. Non-statistical methods approximate the solution space and model the stochasticity in the approximated space. Stochastic spectral methods are examples of non-statistical methods and are among the more effective and recently explored approaches.

While choosing the method of solution for the SPDEs, it is important to consider the computational complexity involved in the problem.

4.2.3.1 Complexity in Design of the Traffic Simulation Experiment

Calibration of traffic simulation model entails repeated execution of the simulation by varying the supply-side parameters and demand-side inputs until the error in outputs is minimized according to certain criteria. With so many variables, it is necessary to have a systematic approach to determine which parameters are important and how many replications of the simulation are necessary. Also, it is important to find the effect of each of these parameters and/or any interaction between them, before running the simulations for the combinations of parameters. This is the process generally called the experimental design.

Suppose there are *k* parameters. One approach is to vary the level of one parameter and keep all other *k -1* parameters fixed. However, this is not an efficient approach and it may not be effective in determining the interaction between the parameters. A more efficient method is to have two levels for each parameter and execute the simulation at each of the 2^k number of combinations. This approach is called the 2^k factorial design. (Law and Kelton (2003))

When designing for an experiment with 2 distinct values in the discretized form for the n stochastic inputs (demands) would involve a full-factorial design i.e. with 2^n replications. Additionally, the model has, say, 2 parameters with l and m distinct values (traffic conditions), the number of replications would be $m*l*2^n$.

Furthermore, due to the stochastic nature of the inputs, just two levels of inputs may not be enough. The objective of the simulation is to approximate the mean μ with $\bar{\Xi}$. The precision, γ, can be expressed as $\gamma = |\mu - \bar{\Xi}| / \mu$. For statistically significant results, the number of replications needed to be at a level of precision γ, estimated standard deviation *S*, and t-statistic for *M-1* degrees of freedom, significance level α is given in equation (4.4) (Law and Kelton (2003))

$$n = \left(\frac{t_{M-1,1-\alpha/2} * S}{\gamma} \right)^2 \tag{4.4}$$

Most studies capturing stochasticity in computational traffic models use a Monte

Carlo (MC)-type independent sampling of *M* simulation runs for various traffic conditions. However, the convergence rate for MC-type method or Latin hypercube sampling is slow, $O(1/\sqrt{M})$ (Loh (1996)).

To illustrate the computational burden for MC-type sampling, suppose we intend to simulate a freeway section with an on- and off-ramp. There are three independent demand inputs, namely, mainline, on- and off-ramp demands. Let's suppose there are two stochastic parameters sampled at 10 points each and let's also suppose that the standard deviation of demand is 140. To achieve a precision of 100 veh/hr in flow at 90% level of significance, the number of replications for variance reduction is 8. Thus the total number of runs required is $10*10*8^3 = 51{,}200$. If each run takes 5 s (for instance), then the computational time taken = 71 hrs. If we want to increase our precision to further reduce the variance, the number of runs and computational time increases exponentially, as illustrated in Figure 4-5.

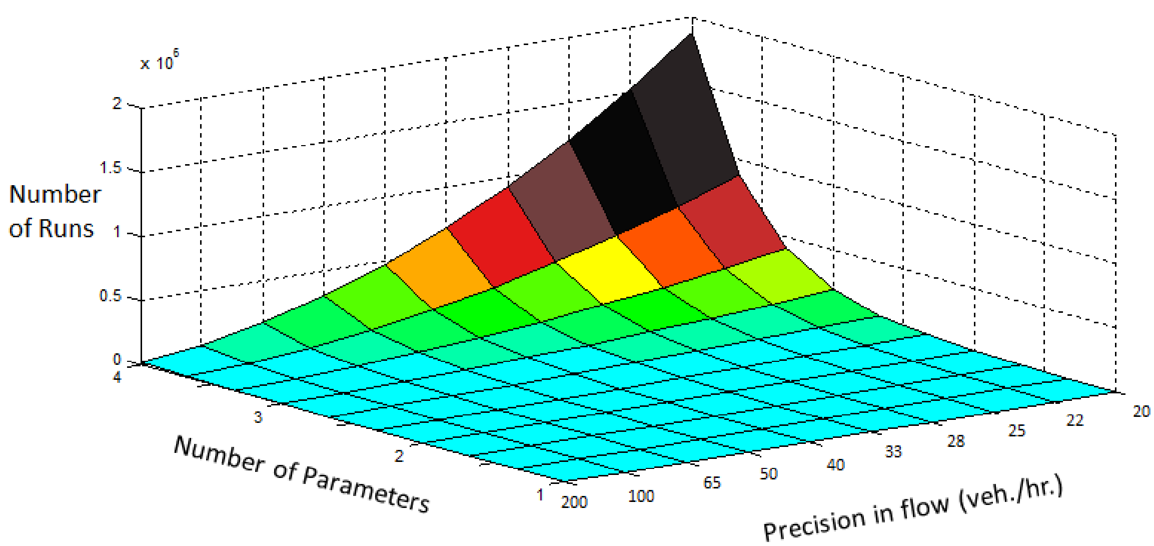


Figure 4-5 Illustration of number of runs required for statistically significant

results using MC-type sampling with number of parameters and precision of estimation

In cases where large sources of data spanning different conditions are available, to capture the stochasticity in traffic conditions, there is an increase in number of factors of stochasticity. This in turn increases the dimensionality of the calibration process. Thus depending on the size of the network and number of stochastic dimensions, MC-type sampling approaches can become prohibitive in terms of computational effort. It may not be possible at all to simulate the output for each and every possible realization of parameter and input. Also, all possible points in the stochastic space of simulation output may not have the corresponding observed data. Thus it is important to obtain an effective sampling and interpolation methodology for predicting output accurately but with lower computational effort. Thus solution methods that use repeated intensive sampling for various dimensions in the stochastic space are not desirable in the construction of robust methods for calibration of traffic simulation models. Alternative methods to mitigate the issues of computational complexity need to be explored.

4.2.3.2 Stochastic Spectral Methods

Stochastic spectral methods provide an effective alternative to computationally-intensive Monte Carlo methods. In this method each stochastic factor is treated as another dimension. The spectral methods involve decomposing the infinite dimensional solution domain Ω is approximated by an N-dimensional space Γ. This N-dimensional space is spanned by orthogonal polynomials Φ_i such that $\Gamma = span\{\Phi_i(\xi_i)\}_{i=0}^{N}$ for $i = 1,...,N$. The choice of the type of polynomial is made depending on the PDF $p_i(\xi_i(\omega))$. This process is called as generalized polynomial chaos (g-PC) expansion and is used as a basis for spectral methods. Polynomial chaos methods were first investigated by Ghanem and Spanos (1991) in finite element modeling of solids. g-PC methods have been successfully applied to stochastic modeling of elastic materials, conduction of heat (Wan and Karniadakis (2005)), incompressible flows (Xiu and Karniadakis (2003), Mathelin et al. (2005)), etc. Implementation of these methods to flows involving discontinuities (such as vehicular traffic flow) is an actively researched area.

Once these polynomials are obtained, the g-PC approximation of the solution $\rho(x,\xi)$ is obtained as a projection onto the space Γ using a linear combination of the orthogonal polynomials that span the space Γ as in equation (4.5),

$$\rho_N^P(x,\xi) = \sum_{m=1}^{M} \rho_m(x,\xi)\Phi_m(\xi_m(\omega)),\ M = \begin{pmatrix} N+P \\ N \end{pmatrix} \tag{4.5}$$

where the coefficient $\rho_m(x,\xi)$ is given by $\rho_m(x,\xi) = \int \rho(x,\xi)\Phi_m(\xi)p(\xi)dy$. g-PC solution methods essentially entail estimating the coefficient $\rho_m(x,\xi)$. Thus, using the g-PC expansion equation (4.5) can be written as,

$$B(\sum_{m=1}^{M} \rho_m(x,\xi_m)\Phi_m(\xi_m) : x,t,\xi) = 0 \tag{4.6}$$

By projecting it onto the polynomial basis Φ_m ,i.e., transferring the stochasticity from the dependent variables to the basis polynomials, it can be written as equation (4.7),

$$\left\langle B(\sum_{m=1}^{M} \rho_m(x,\xi)\Phi_m : x,t,\xi), \Phi_m \right\rangle = 0, M = \begin{pmatrix} N+P \\ N \end{pmatrix} \tag{4.7}$$

where $\langle a,b \rangle$ denotes the inner product of functions a and b. This method of projection is called as the stochastic Galerkin approach (Ghanem and Spanos (1991)). Thus, in the stochastic Galerkin method, the spatial and time domain are approximated using a finite element discretization and the stochastic space is also approximated using a g-PC approximation as shown in equation (4.5). The equation (4.7) is equivalent to a set of M coupled deterministic set of PDEs. The coupled nature of equation (4.7) makes the solution method non-trivial since the M coupled deterministic PDEs are dependent on each other. Hence regular solution methods for deterministic PDEs cannot be applied for solving equation (4.7). A methodology using the existing solution methods for deterministic PDEs is more generic and useful. Stochastic collocation is one such method and is explained in the next subsection.

4.2.3.3 Stochastic Collocation Method

An alternative approach to using stochastic spectral methods is to have a finite element approximation for the spatial domain and approximate the multi-dimensional stochastic solution space using interpolating functions along with deterministic solutions in each independent dimension. These interpolating functions are so chosen to be mutually orthogonal so that the resulting equations are decoupled. This approach is called the collocation approach, using which we can compute the deterministic solution at various points in the stochastic space and then build an interpolated function that best approximates the required solution over the stochastic solution space. Stochastic collocation can be used to greatly reduce the computational burden without foregoing the modeling accuracy that much. Instead of executing the simulation for each and every condition i.e., using a Monte Carlo-type of exhaustive simulation runs, the interpolating functions can be used to approximate the output at the intermediate conditions. In addition, the time consumed by the collocation approach can be further reduced by parallelizing the simulation under each condition, since each of them may be independent of the other.

The central tenet of the collocation method is to construct an interpolation function for the dependent variables using their values at particular points in the stochastic space. The difference with the stochastic spectral methods is that they approximate the stochastic solution space using g-PC expansion of orthogonal polynomials of random variables.

Stochastic collocation involves decomposing or parameterizing the model outputs using N independent random variables, $\boldsymbol{\xi} = \{\xi^i(\omega)\}, i = 1,...,N$ at prescribed set of collocation points. In other words the infinite dimensional solution domain Ω is approximated by an N-dimensional space Γ. The probability density function of each random variable $\xi^i(\omega)$ can be defined as $p_i(\xi^i) : \Gamma_i \to \mathbb{R}$ $\mathbb{R}$. The joint density for $\boldsymbol{\xi}$ can be written as $p(\boldsymbol{\xi}) = \prod_{i=1}^{N} p_i(\xi^i)$ and the corresponding support as $\Gamma = \prod_{i=1}^{N} \Gamma_i \subset \mathbb{R}$. This representation can be extended to any N-dimensional space based on the set of parameters. Thus for $\boldsymbol{\xi} \in \Gamma$, we solve for $p(x,t,\boldsymbol{\xi}) : D \times \Gamma \to \mathbb{R}$ such that equation (4.5) is satisfied for $p(x,t,\boldsymbol{\xi}) \in D$, the space-time domain. So, the output can be expressed as $\boldsymbol{\rho}(x,t,\boldsymbol{\xi}(\omega)) = \boldsymbol{\rho}(x,t,\xi^1(\omega),...,\xi^N(\omega))$,

$$\boldsymbol{\rho}(x,t,\boldsymbol{\xi}) = \sum_{j=1}^{N} \tilde{\rho}_j \ \ {}_j$$

where,

$$\Psi_j = j\text{-th orthogonal functional} \tag{4.8}$$

$$\tilde{\rho}_j \quad {}_j \rangle = \int_{\Gamma} \boldsymbol{\rho}(x,t,\boldsymbol{\xi})\Psi_j p(\boldsymbol{\xi})d\boldsymbol{\xi}$$

$$= \int_{\Gamma} \boldsymbol{\rho}(x,t,\xi^1(\omega),...,\xi^N(\omega))\Psi_j p_1(\xi^1)...p_N(\xi^N)d\xi^1...d\xi^N$$

Thus estimating the output at a generic point entails estimating the coefficients $\tilde{\rho}_j$, i.e., the integrals shown in equation (4.8). This is accomplished by some deterministic integration techniques by approximating the stochastic N-dimensional space $\left\{\boldsymbol{\xi}^{(k)}\right\}_{k=1}^{Q} \in \Gamma$, by means of an interpolation function built using deterministic solutions evaluated at each of a set of Q collocation points. One such cubature is using the Lagrangian interpolation of $\rho(x,\boldsymbol{\xi})$ can be shown as equation (4.9), (Xiu and Hesthaven, 2005; Babuska, Nobile, Tempone, 2007)

$$\hat{\rho} = LI\rho(x,\boldsymbol{\xi}) = \sum_{j=1}^{Q} \tilde{\rho}_j \quad L_j(\boldsymbol{\xi}), \forall x \in D, \boldsymbol{\xi} \in \Gamma \subset \Re^N \tag{4.9}$$

where $L_k(\boldsymbol{\xi}^{(j)}) = \delta_{kj}, 1 \le j,k \le Q$ are Lagrangian polynomials and $\tilde{\rho}_k$ is the deterministic solution at a given point $\boldsymbol{\xi}^{(k)}$. In other words, at any point $\boldsymbol{\xi} \in \Gamma$ the Lagrangian interpolation approximates ρ by $\hat{\rho}$. Substituting this to SPDE shown in equation (4.5), gives,

Since $L_j(\boldsymbol{\xi})$ are orthogonal interpolating functions, the above equation, in the interpolation form, becomes into Q deterministic decoupled equations,

$$B(\tilde{\rho}_j \quad : x,t,\xi^{(j)}) = 0, j = 1,...,Q \tag{4.10}$$

Thus stochastic collocation is classified as a non-intrusive method since the deterministic equations are not coupled and the numerical methods applicable in solving deterministic form of PDEs can be applied directly.

Although the deterministic integration approaches are fairly straightforward, the implementation in multi-dimensional space is not trivial. Computationally efficient schemes to approximate the multi-dimensional stochastic space, such as the Smolyak algorithm (Ganapathysubramanian and Zabaras, 2007), are available. Smolyak algorithm reduces the number of collocation points Q in multiple dimensions by using tensor products of each one-dimensional interpolants in a particular way shown below.

Consider the one-dimensional interpolant for function f using the set of points used as $\Theta^{(k)}$.

$$\begin{aligned}
&U^i(f) = \sum_{j=1}^{m_i} f(\xi_j) L_j, \\
&f(\xi_j) = \text{deterministic soluation at } \xi_j, \\
&L_j = \text{interpolation basis polynomial}, \\
&m_i = \text{no. of nodes at level of interpolation } i.
\end{aligned} \tag{4.11}$$

However, when extending to multiple dimensions, since each stochastic dimension is independent, the interpolant in N-dimensions involves tensor products of one dimensional interpolants $U^{i_1},\ldots,U^{i_N}$. The multi-dimensional tensor product can be written as

$$(U^{i_1}\otimes\ldots\otimes U^{i_N})(f)=\sum_{j_1=1}^{m_1}\cdots\sum_{j_N=1}^{m_N}\ \cdots\ \cdots$$

where

$U^{i_1},\ldots,U^{i_N}$: one dimensional interpolants in the dimension $i_1,\ldots,i_N$ respectively (4.12)

$L_{i_1},\cdots$ re basis polynomials for each dimension

$\xi_{j_1}^{i_1},\cdots$ re respectively the $j_1,\ldots,j_N$ th points in $1,\ldots,N$ dimension

$m_1,\ldots,m_N$: total no. of points in $1,\ldots,N$ dimension

It can be seen that the tensor product in equation (4.12) requires $m_1 * \ldots * m_N$ evaluations.

The Smolyak algorithm constructs a sparse interpolant $A_{q,N}$ (N is the stochastic dimensions and $q-N$ is the order of interpolation) using a product of one-dimensional functions (Klimke (2006)) is in equation (4.13),

$$A_{q,N}=\sum_{q-N+1\le|\mathbf{i}|\le q}(-1)^{q-|\mathbf{i}|}\cdot\binom{N-1}{q-|\mathbf{i}|}\cdot(U^{i_1}\otimes\ldots\otimes U^{i_N})$$

where

$A_{N-1,N}=0,$ (4.13)

$U^{i_1},\ldots,U^{i_N}$: one dimensional interpolants

$\mathbf{i}=(i_1,\ldots,i_N)$ with $|\mathbf{i}|=i_1+\ldots+i_N,$

Here, $i_k, k=1,\ldots,N$ is the level of interpolation in dimension *k*. Smolyak algorithm builds the multi-dimensional interpolant using one-dimensional interpolants of order $i_k, k=1,\ldots,N$ with the constraint that across all dimensions, the sum $|\mathbf{i}|=i_1+\ldots+i_N$ follows $q-N+1\le|\mathbf{i}|\le q$. This same sparse interpolation can be shown using an incremental interpolant $\Delta^i=U^i-U^{i-1};U^0=0$ as,

$$A_{q,N}(f) = \sum_{|\mathbf{i}|\le q} (\Delta^{i_1} \otimes \ldots \otimes \Delta^{i_N})(f) = A_{q-1,N}(f) + \sum_{|\mathbf{i}|=q} (\Delta^{i_1} \otimes \ldots \otimes \Delta^{i_N})(f) \tag{4.14}$$

Thus to construct the interpolant $A_{q,N}(f)$ (in equation (4.13)) from scratch, we need to compute the function at the nodes covered by the sparse grid $H_{q,N} = \bigcup_{|\mathbf{i}|=q} \quad {}_1^{(i_N)})$. Thus this interpolation process allows us to utilize all the previous interpolants generated. Using appropriate points such that $\Theta_1^{(i)}$ is nested, $\Theta_1^{(i)} \subset \Theta_1^{(i+1)}$, an extension from the i-th interpolant to i+1 only needs the evaluation at points that are unique to $\Theta_1^{(i+1)}$. Thus the Smolyak algorithm provides great savings in computational resources.

It can be seen that determining the grid $H_{q,N}$ is an important part of the Smolyak algorithm for interpolation. The distribution of points on the grid is usually performed using piecewise linear basis functions (Clemshaw-Curtis grid) or polynomial basis functions (Chebyshev-Gauss-Lobatto grid). Examples of these grids are shown in Figure 4-6. In cases where the original function is relatively smooth and higher accuracies are required, the polynomial bases are recommended (Klimke (2006)). Since the smoothness of the simulation output is not known, we use the piecewise linear bases. More specifically, we use the Clemshaw-Curtis grid for basis for constructing the interpolant.

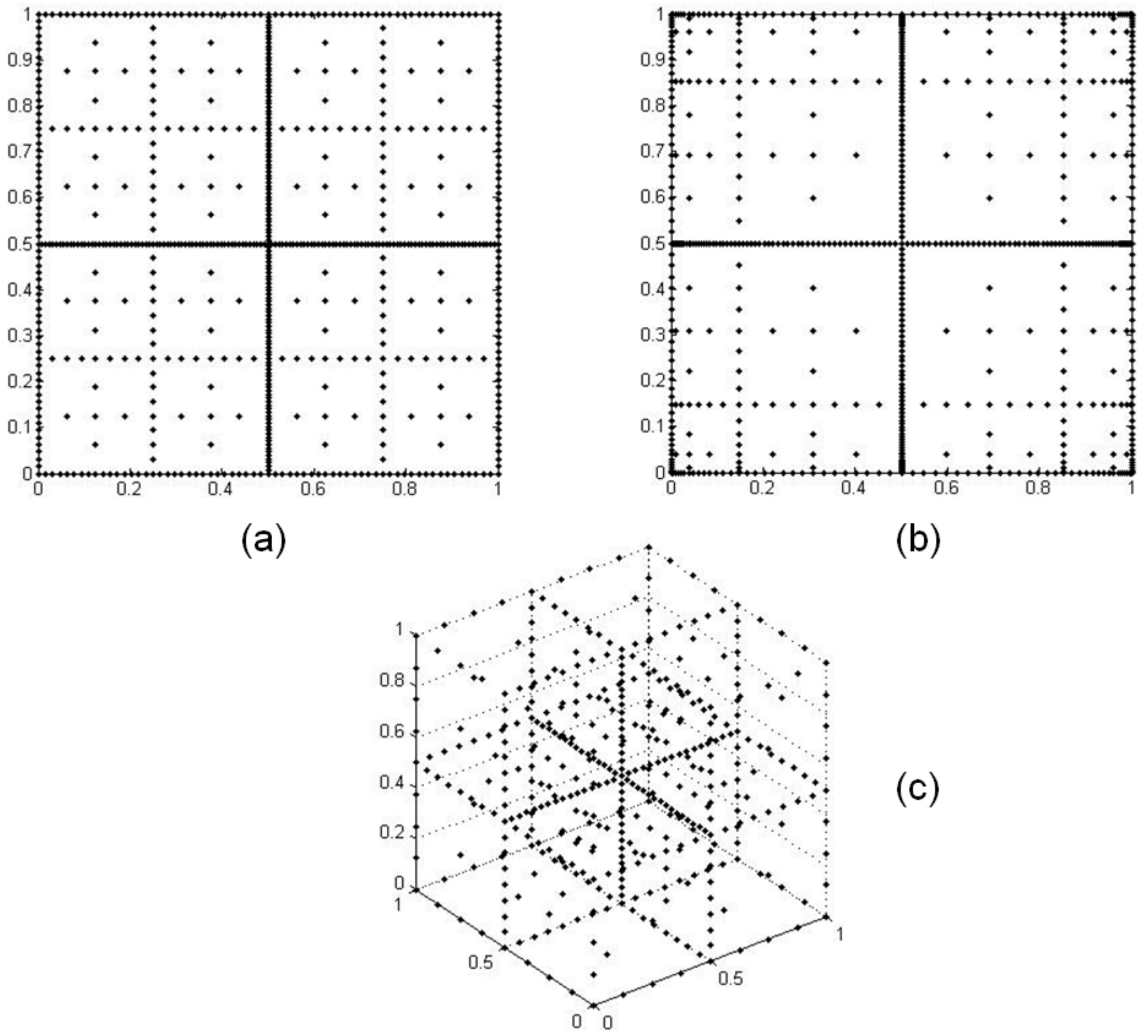


Figure 4-6 (a) Clemshaw-Curtis grid in two-dimensions, (b) Polynomial basis grid in two-dimensions, (c) Polynomial basis grid in three-dimensions

Thus the stochastic collocation points at various points in the grid are used to discretize the stochastic dimension of the dependent variables. Also as shown in equation (4.12) and equation (4.13), the interpolation schemes can be used to predict the outputs at intermediate points where traffic data may not be available. An illustration of the discretization of the stochastic space using a Clemshaw-Curtis grid in two dimensions is shown in Figure 4-7.

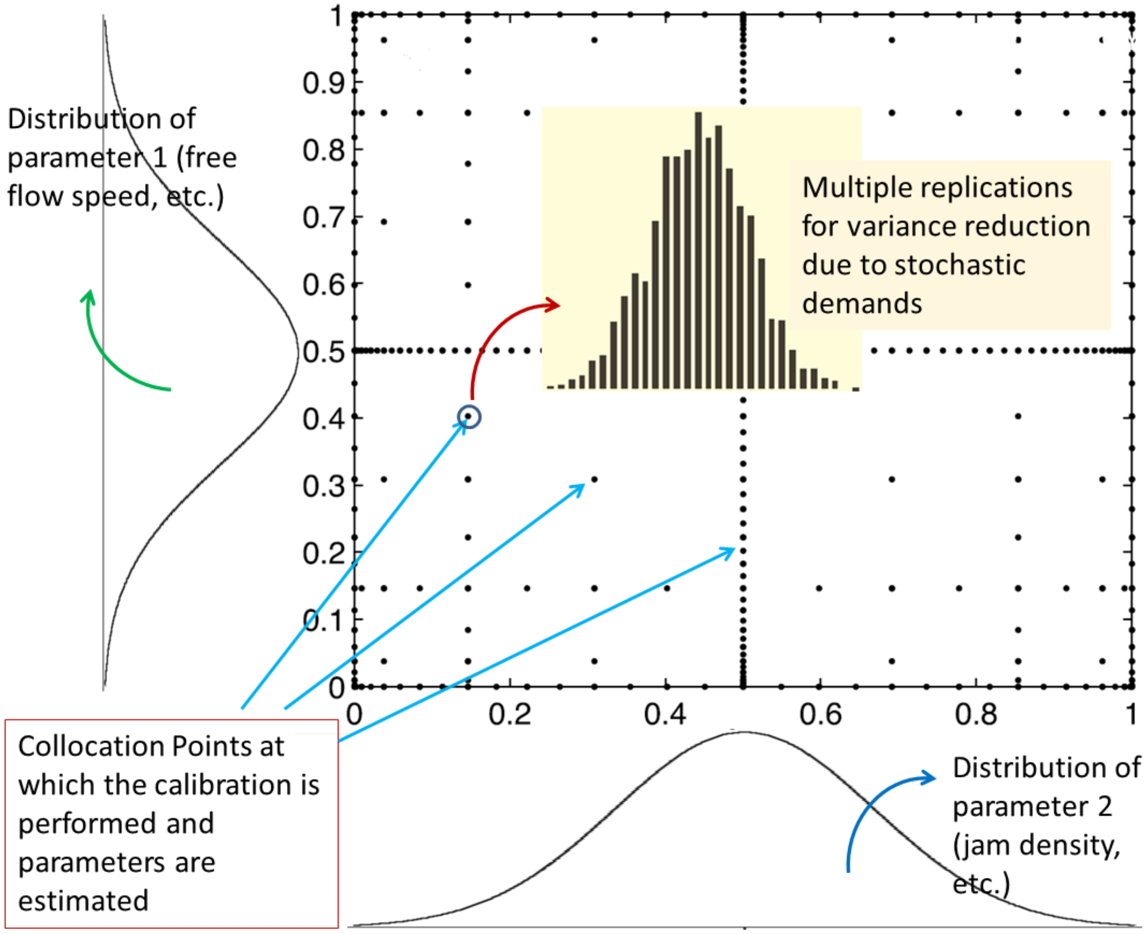


Figure 4-7: Example of approximation of stochastic space by collocation points

The advantage of this recursive/nested structure is that to increase the order of interpolation (accuracy) we can use all the deterministic solutions from the previous steps: $A_{q-1,N}$, by adding a few more deterministic solutions. When new data is available, additional deterministic solutions can be evaluated and accuracy of interpolant is improved.

Convergence rate of the interpolant is of the order, $O(M^{-2}|log_2M|^{3(N-1)})$ (for piecewise linear basis), $O(M^{-k}|log_2M|^{(k+2)(N-1)})$ (for k-polynomial basis) where M is the total number of collocation points. This rate can be controlled by the interpolation level q-N and polynomial order k (Ganapathysubramaniam and Zabaras (2007), Klimke (2006)). Thus, we show, empirically, that convergence of this interpolant is better than the more commonly used Monte Carlo method.

The methodology described in this subsection illustrates the solution approach to stochastic macroscopic traffic PDE. However, the parameters in the PDE need to be optimized to calculate the flows and densities close to the observed values. This optimization procedure is described in the next subsection.

4.3 Parameter Optimization

The output generated from a macroscopic traffic simulation model is the density, speed and flow over time at different sections of the roadway section or network. When constructing a robust simulation model, the objective is to obtain accurate outputs for varying conditions. In other words, the parameters for the simulation model are to be estimated by minimizing the error in the simulated and observed outputs. As a part of this exercise, the set of parameters that best represent each condition have to be estimated. This presents a stochastic inverse problem.

The stochastic collocation points in the grid (illustrated in the previous subsection) are used to discretize the stochastic dimension of stochastic inputs as well as stochastic parameters. The process involved in the estimation of calibrated parameters is described below.

From each realization of the parameter set, using the demand distribution as an input, the simulation output distribution (e.g., flow or density distribution) is generated. This distribution is compared with the observed output distribution and using a test statistic (such as the test statistic from the Kolmogorov-Smirnov (KS) test), the error is estimated. This error is used as an objective function and is minimized as part of the multi-objective parameter optimization forming the stochastic inverse problem is shown in equation (4.16), using the simultaneous perturbation stochastic approximation (SPSA) algorithm (Spall (1992)).

$$\begin{aligned}
&\min_{\Theta_S} U(O_{obs}, O_{sim}(I_s, \Theta_s)) \\
&\text{where,} \\
&O_{obs}, O_{sim} \text{ - observed and simulated outputs} \\
&\Theta_S \text{ - parameter set} \\
&U \text{ - error functions for outputs}
\end{aligned} \tag{4.15}$$

The methodology employed to solve the inverse problem in equation (4.15) depends on the ease and feasibility in calculating the gradient of the function to be minimized in equation (4.15). For instances where the gradient is tractable the optimization can be performed using methods such as steepest decent algorithm. In cases of problems with lower dimension of parameters, such as freeway sections with lower uncertainty and/or sections with lesser number of on- and off-ramps set the gradient is tractable. However, in order to calculate the gradient the solution search space needs to be discretized and sampled. Although there are fairly efficient sampling schemes such as Latin hypercube sampling, the size of the sample space could get very large with increase in number of variables (on-ramp, off-ramp demands, etc.). Therefore, stochastic approximation methods can be used to approximate the gradient using with minimal sampling required. The gradient can be approximated using finite difference methods (Kiefer and Wolfowitz (1952)) or the more efficient simultaneous perturbation stochastic approximation (SPSA) algorithm (Spall (2003)). The sampling for parameter updating is performed using successive averaging developed by Robbins and Monro (1951).

In this study the inverse problem of finding the parameters is solved using the SPSA algorithm. The SPSA algorithm has been used extensively in dynamic O-D matrix estimation (Cipriani et al. (2011)), traffic control (Spall and Chin (1997), Ma et al (2007a)), solving the stochastic inverse problem in the calibration process (Lee and Ozbay (2008), Balakrishna et al. (2007), Ma et al. (2007b)) and network analysis (Ozguven and Ozbay (2008)). The SA algorithm normally focuses on finding the vector value $\Theta \in \Gamma$, which either minimizes the loss function $L(\Theta)$ or makes the gradient equation $g(\Theta)$ equal to zero. The SPSA is an applicable stochastic optimization method for multivariable equations, and the standard SPSA algorithm has the following form (Spall (2003)):

$$\hat{\Theta}_{k+1} = \hat{\Theta}_k - a_k \hat{g}_k(\hat{\Theta}_k) \tag{4.16}$$

Here, $\hat{g}_k(\hat{\Theta}_k)$ is the SP of the gradient $g(\Theta) = \dfrac{\partial L}{\partial \Theta}$ estimated, based on the loss function measurements, at $\Theta = \hat{\Theta}_k$ at the k^{th} iteration. a_k indicates the step size and is a nonnegative scalar coefficient. The basic solution to an optimization problem is to minimize the loss function $L(\hat{\Theta}_k - a_k \hat{g}_k(\hat{\Theta}_k))$ at the k^{th} iteration. The new value of Θ, obtained for every iteration, is calculated by subtracting the product of step size and the gradient at the present value from the previous value of Θ .

The gradient approximation $\hat{g}_k(\hat{\Theta}_k)$ is the most important part of the SPSA algorithm. With Stochastic Perturbation (SP), loss measurements are obtained by randomly perturbing the elements of $\hat{\Theta}_k$. Assuming that Θ is p -dimensional, the Stochastic Perturbation (SP) gradient approximation can be shown in the following form:

$$\hat{g}_k(\hat{\Theta}_k) = \begin{bmatrix} \dfrac{L(\hat{\Theta}_k + a_k\hat{g}_k(\hat{\Theta}_k)) - L(\hat{\Theta}_k - a_k\hat{g}_k(\hat{\Theta}_k))}{2c_k\Delta_{k1}} \\ \bullet \\ \bullet \\ \bullet \\ \dfrac{L(\hat{\Theta}_k + a_k\hat{g}_k(\hat{\Theta}_k)) - L(\hat{\Theta}_k - a_k\hat{g}_k(\hat{\Theta}_k))}{2c_k\Delta_{kp}} \end{bmatrix} = \frac{L(\hat{\Theta}_k + a_k\hat{g}_k(\hat{\Theta}_k)) - L(\hat{\Theta}_k - a_k\hat{g}_k(\hat{\Theta}_k))}{2c_k} \begin{bmatrix} \Delta_{k1}^{-1} \\ \Delta_{k2}^{-1} \\ \bullet \\ \bullet \\ \Delta_{kp}^{-1} \end{bmatrix} \tag{4.17}$$

Here, the, p -dimensional random perturbation vector, $\mathbf{\Delta}_k = \left[\Delta_{k1}^{-1} \quad \Delta_{k2}^{-1} \quad \bullet \quad \bullet \quad \Delta_{kp}^{-1}\right]^T$ is a user-specified vector for which the components of $\mathbf{\Delta}_k$ are normally distributed ± 1 Bernoulli variables. Here, c_k is a positive scalar.

The problem of minimizing $L(\Theta)$ for a differentiable loss function is equivalent to finding a solution of the gradient approximation $g(\Theta) = \dfrac{\partial L}{\partial \Theta} = 0$. The loss function for this study is a standard quadratic measure, $L(\Theta) = E\left[x^T x \mid \Theta\right]$. $E\left[x^T x \mid \Theta\right]$ denotes an expected value that is conditional on the set of controls with weights Θ (Spall (1997)).

The SPSA algorithm is not a greedy algorithm i.e., it does not result in the optimal solution, but converges quickly towards the optimal solution. Also it requires only two iterations to compute the gradient at each point.

The objective function in equation (4.15) can have two components, error in flow and error in density at each section. It is likely that during the optimization procedure each of the errors could conflict with each other. In other words, one set of parameters could reduce the error in flow which the other set could increase the error. This may result in oscillation of the optimal solution. Thus there are multiple objective functions that need to be optimized.

$$\min_{\Theta_t} F(\Theta_t) = \min_{\Theta_t} \begin{bmatrix} U_1(q_i^{Ob}, q_i^S(\Theta_t)) \\ U_2(\rho_i^{Ob}, \rho_i^S(\Theta_t)) \end{bmatrix}$$

This problem constitutes the methodology of multi-criteria optimization problems (MOP). Generally, the solution methods for MOP are classified as scalarization approaches and Pareto-approaches (Steuer (1986)). Scalarization approaches attempt to convert the vector form of the objective function to a scalar form. This conversion is based on using preferences for each objective in the form of weighting or other additional constraints (Ehrgott (2005)). The Pareto-approaches search the solution space based on the preferences a posteriori. In this study, we adopt a weighting mechanism for each objective i.e. error. Thus the new formulation of the inverse problem can be expressed as shown in equation (4.18). In equation (4.18) the weight parameter w signifies the importance in the calibration process that can be assigned to each error measure. The weights can be assigned values of the variance in flows and densities observed.

$$\min_{\Theta_t} \sum_{i=1}^{N} \left\{ w_1 U_1(q_i^{Ob}, q_i^S(\Theta_t^k)) + w_2 U_2(\rho_i^{Ob}, \rho_i^S(\Theta_t^k)) \right\}$$

where,

q_i^{Ob}, q_i^S - observed and simulated flows at location i

ρ_i^{Ob}, ρ_i^S - observed and simulated densities for location i (4.18)

Θ_t^k - parameter set for time period t and iteration k

w_1, w_2 - weights for the error measures

U_1, U_2 - functions representing the error in flow and density

To summarize the proposed approach, first the output data exploration is performed to categorize output into statistically separable clusters. For each output cluster, a corresponding input/demand-side distribution and parameter/supply-side distribution are generated. These distributions are discretized using the stochastic collocation method. For each realization of the parameter set, the simulation is performed and the output distribution is generated using the demand-side distribution. This is compared to the observed distribution and error statistic is generated. If the error statistic is not satisfactory, the parameter set is updated using the SPSA algorithm (Spall (1992)) and the output distribution is re-generated. A schematic representation of the sequence of steps in our proposed methodology is shown as a flowchart in Figure 4-8. The main advantages of using this proposed calibration methodology are the following:

1. Flexibility in applying to any type of traffic simulation (1^{st} order, 2^{nd} order, meso/microscopic, etc.),
2. Computationally more efficient than MC-type exhaustive sampling methods with effective interpolant to generate full distribution of simulation output.
3. Time consumed by the collocation approach can be further reduced by

parallelizing the simulation under each condition,

4. Nested form of the algorithm is useful in refining the interpolant as and when there is new data available.

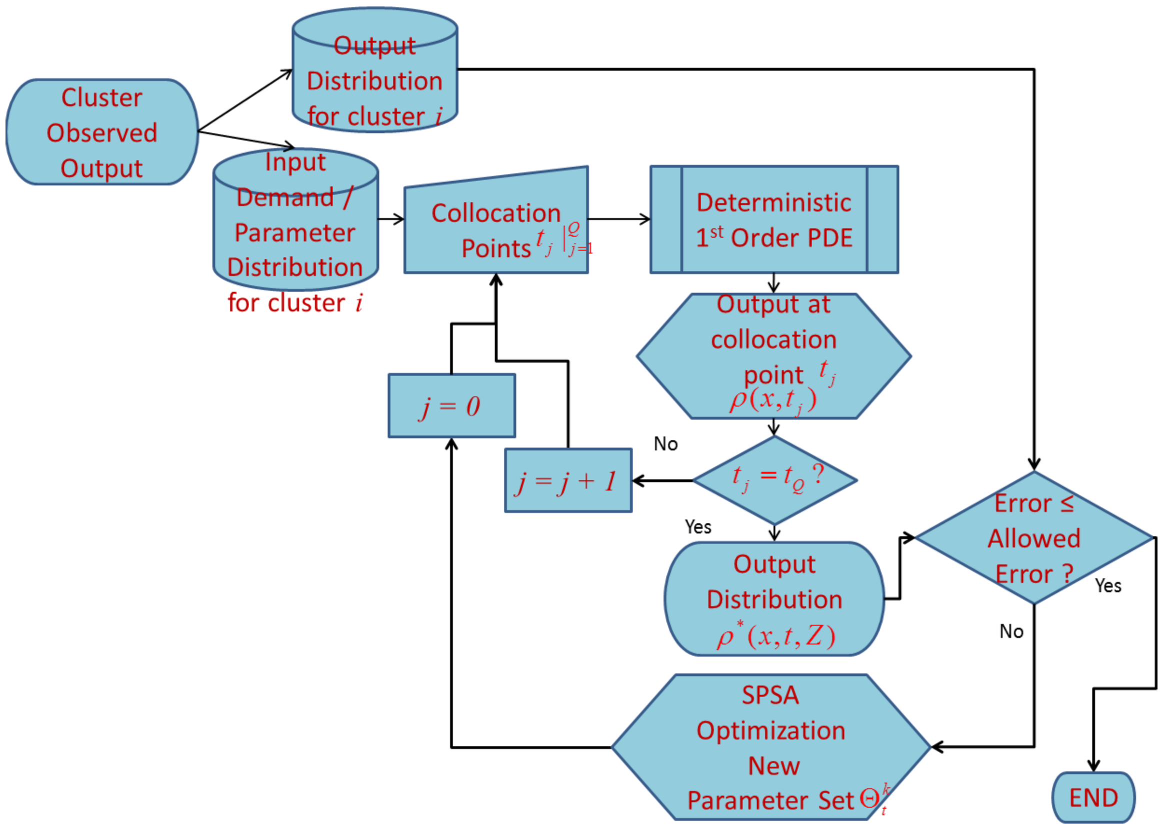


Figure 4-8 The logic of the proposed calibration methodology

Efficient collocation methods improve efficiency of solving PDE-based simulation model compared to Monte-Carlo methods. This resulted in the recent development of stochastic gradient-based tools for performing optimization (Sankaran (2009); Zabaras and Ganapathysubramanian (2008)) that significantly improve efficiency compared to Monte-Carlo methods. Methods for solving stochastic inverse problems (Faverjon and Ghanem (2006)) have also been recently developed. The proposed methodology is designed along similar lines.

The simulation output $\rho(x,t_j)$ is obtained at each of these points, t_j, is a result of discretization of stochastic space using stochastic collocation and SPSA optimization for the parameters. Thus evaluating $\rho(x,t_j)$ requires multiple simulation runs for considering the stochastic inputs. Thus the output at each of the collocation points is a distribution rather than a single point value. The output at any of the intermediate points is obtained using the interpolation scheme shown in equations equation (4.12) and equation (4.13).

In the next chapter we present several case studies illustrating the application of the proposed calibration methodology.

CHAPTER 5. CASE STUDIES USING PROPOSED CALIBRATION METHODOLOGY

In order to illustrate the proposed calibration methodology to capture stochastic variation in traffic conditions, a three-lane section of the NJTPK turnpike at interchange 7 is chosen. Although, microscopic traffic simulation tools, such as PARAMICS or VISSIM, provide a detailed and relatively accurate platform for modeling, the model building, calibration and execution can be very time consuming. When studying the effects of various stochasticities, we are going to focus on a first order macroscopic traffic simulation model to model the traffic flow in the section. The stochastic version of the first order macroscopic traffic flow model can be represented as follows.

$$\begin{aligned}
&\partial_t \boldsymbol{\rho}(x,t,\xi) + \partial_x \left(\boldsymbol{\rho}\mathbf{v}(x,t,\xi)\right) = 0 \\
&\boldsymbol{\rho}(x,t,\xi) \in D \times \Omega \\
&\text{D: deterministic } x\text{-}t \text{ domain} \\
&\Omega\text{: stochastic space} \\
&q = \rho v \sim f(\rho,\xi)_i\text{: fundamental relationship for } i\text{-th output cluster} \\
&\rho_i(x_j,t) = \rho_j^B(t,\xi)_i : \text{ stochastic demand in cell } j \text{ for } i\text{-th output cluster}
\end{aligned} \tag{5.1}$$

We discretize the time and space for the model using the cell transmission model. A schematic representation of the discretized simulated section, the stochastic input and model parameters is shown in Figure 5-1.

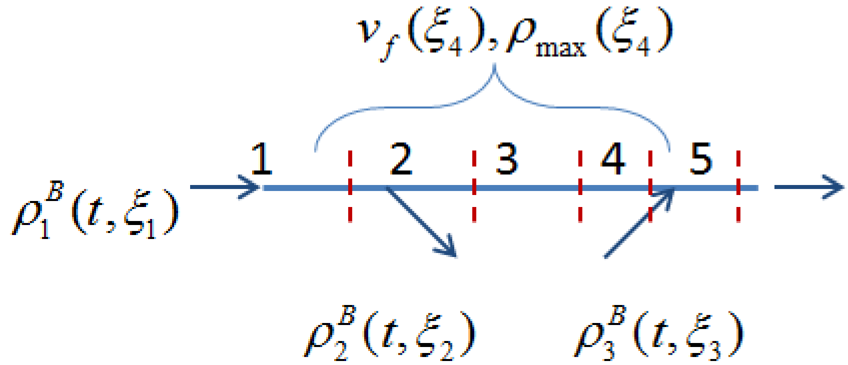


Figure 5-1: Schematic representation of the study section

In order to capture various traffic conditions, we use the traffic sensor data between for every 5 minutes between January 1, 2011 and August 31, 2011.This is, however, a very large dataset and can be considered as the population. We use a smaller sample of peak period during the months of April and May 2011 for calibration and use other parts of the larger dataset for validation. Even this sample of peak period during the months of April and May 2011 has a wide range of variation in speed and flow, as can be seen in Figure 5-2. Traditionally, an arbitrarily chosen day or days from this sample is used for calibration. However, as can be seen in Figure 5-2, this may not be representative of the speed and flow variation of the section. In order to capture the true variation in speed and flow of the section, we would require using the output from the whole sample.

As mentioned in the methodology section, in order to separate or classify the section outputs, we use k-means clustering. We followed the steps mentioned below in the clustering process:

1. Set up a desired number of clusters.

2. Group speed data from various days into clusters so as to minimize the sum of the differences between the day values and the mean for each cluster. (the procedure involved in k-means clustering)
3. As mentioned in the methodology section, the objective is to minimize the differences between data within clusters and maximize the difference between clusters. Hence, if the coefficient of variation (CoV) is more than 0.25 for any cluster, increase the number of clusters.

5.1 AM Peak Calibration Results

From Figure 5-2 (a), it can be seen that the speed output fell into two distinct groups with CoV less than 0.25 for within each group. Hence, we consider two distinct supply-side conditions in calibrating the macroscopic model for the AM weekday peak. There are 24 and 19 days, respectively, falling under clusters 1 and 2. This shows that the possible reason for significant number of days with lower speed in cluster 1 is some activity that consistently takes place, such as long-term construction or maintenance activity. We did not observe significant variation in speeds due to weather. It is likely that the demand-side could have been impacted due to weather. Thus considering the distribution of demand during all the days encompasses the variation due to weather conditions as well.

We calibrate the first order simulation model for each of these conditions separately and estimate the corresponding optimal parameters. Due to the variation in speed (shown in Figure 5-2 (a)), in this case study we propose to have a stochastic fundamental diagram that has a stochastic free flow speed. The distribution of the free flow speed is assumed to be Gaussian (Zhong and Sumalee (2008), Mihaylova and Boel (2006)). Thus the mean (μ_{vf}) and standard deviation (σ_{vf}) of free flow speed form a part of the parameter set to be estimated along with critical density (ρ_{max}) and jam density (ρ_{jam}).

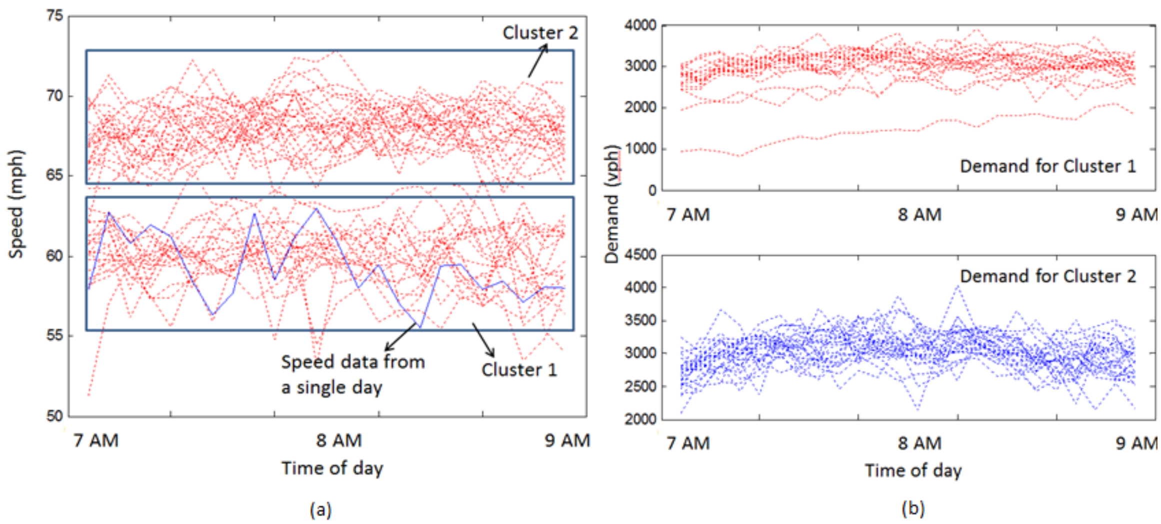


Figure 5-2 (a) Illustration of Speed Data Variation for AM weekday Peak period during April and May 2011 (b) Distribution of demand for each condition (cluster) for the mainline section at interchange 7 of NJTPK during AM weekday peak

To capture the demand-side variation, we obtain the demand distribution from days falling into each cluster. The variation in demand at this section is captured using the ETC data for every 5 minutes between January 1, 2011 and August 31, 2011. The demand variation corresponding to each condition/cluster during the AM weekday peak period is shown in Figure 5-2 (b). Additionally, the on- and off-ramp demand distributions are also generated using the ETC data.

Thus for each cluster, the distribution of demand during each 15-minute time period is generated as illustrated in Figure 5-3. The calibration of the macroscopic simulation model is performed for AM peak (7-9AM). As mentioned in the methodology, with the demand distribution as an input, for each realization of the parameter set, the simulation output flow distribution is generated for cells 2 and 5 in Figure 5-1. This distribution is compared with the observed flow distribution at locations corresponding to cells 2 and 5, and using a test statistic (such as the test statistic from the KS test), the error is estimated. This error is used as an objective function and is minimized using the SPSA algorithm. The result of calibration is demonstrated using the comparison of simulated and observed flow.

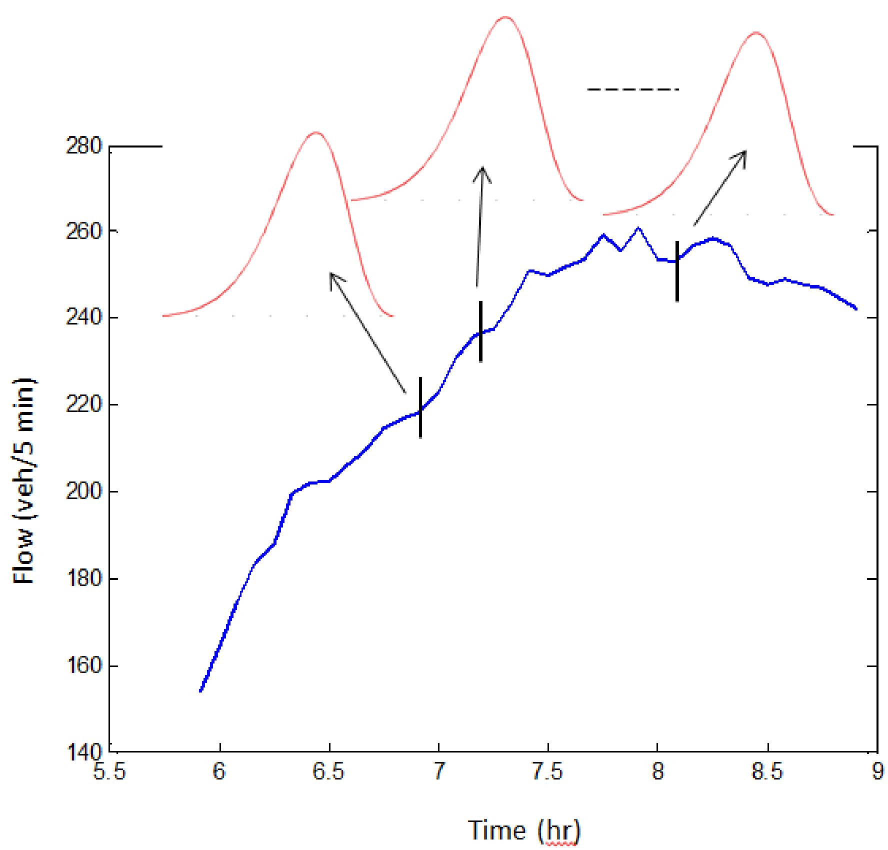


Figure 5-3 Schematic depiction of demand distribution at various sampled

time intervals

For this study, the Clemshaw-Curtis grid (two-dimensional version of which can be seen in Figure 4-7) is selected as the appropriate sparse grid to discretize the stochastic demand. The simulation is calibrated using the demand values at each of these grid points. The objective function for calibration is the test statistic used in the Kolmogorov-Smirnov test at 90% significance, maximum separation between two distributions. As mentioned in equation (4), a sparse grid interpolation is performed for the output of the simulation and a Smolyak interpolant is constructed. Distribution of simulated flows is obtained by repeated evaluation of the Smolyak interpolation function. The simulated flow distribution is compared to the observed distribution from the sensor data.

The comparison of observed and simulated flow distributions in cells 2 and 5 (Figure 5-1) from the calibrated model for AM weekday peak period for condition 1 is shown in Figure 5-4 (a). We noticed during the process of calibration that some cells have different calibrated parameter set from the others. In other words, the stochasticity in simulation parameter set is not only temporal but also spatial. The calibrated parameters ([μ_{vf}, σ_{vf} ρ_{max}, ρ_{jam}]) for AM weekday peak period for condition 1 are [53 2.11 85 150, 62 1.93 82 150] in appropriate units. The objective function (KS test statistic) after calibration is calculated as 0.09.

In order to compare the efficiency of the stochastic collocation approach, the distribution of simulated flow after model calibration is also generated using Monte Carlo sampling method. In order to achieve the flow distribution, the SC approach required 4034 evaluations for various stochastic demand combinations. However, using a MC-type sampling required 180,000 runs of the simulation model. The reason, as mentioned earlier, is due to the ability to construct efficient Smolyak interpolant that uses the simulation output from much fewer runs.

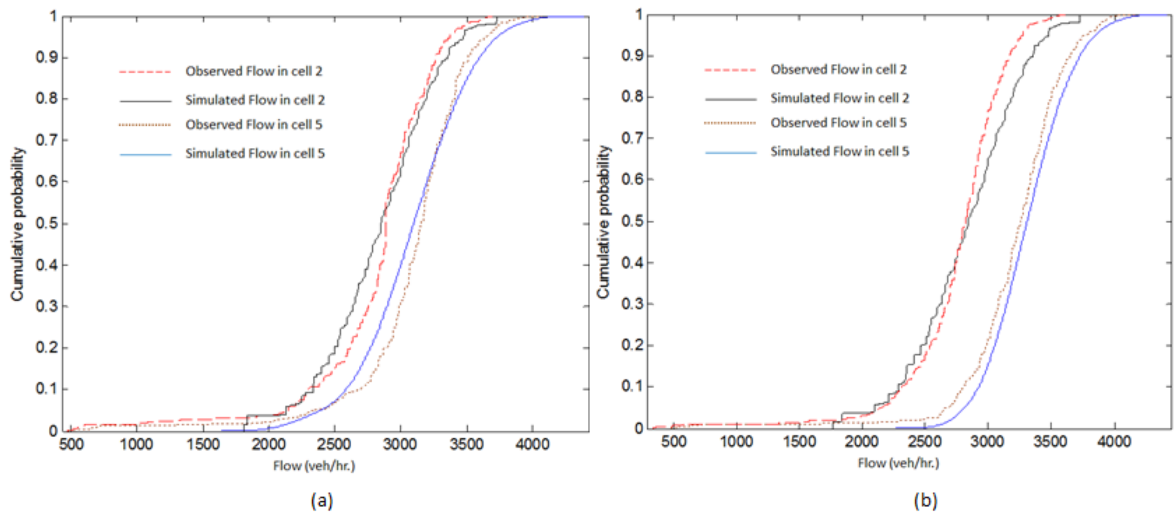


Figure 5-4 (a) Comparison of observed and simulated link flow distributions during AM peak period for condition 1 (b) Comparison of observed and simulated link flow distributions during AM weekday peak period for condition 2

The comparison of observed and simulated flow distributions in cells 2 and 5 (Figure 5-1) from the calibrated model for AM weekday peak period for condition 2 is shown in Figure 5-4 (b). Similar to condition 1, we noticed during the process of calibration that some cells have different calibrated parameter set from the others. In other words, the stochasticity in simulation parameter set is not only temporal but also spatial. The calibrated parameters ([μ_{vf}, σ_{vf} ρ_{max}, ρ_{jam}]) for AM peak period for condition 2 are [50 1.64 83 150, 70 1.58 100 150] in appropriate units. The objective function after calibration is 0.08.

As in the first condition, the distribution of simulated flow after model calibration is also generated using Monte Carlo sampling method. In order to achieve the flow distribution, the SC approach required 3330 evaluations for various stochastic demand combinations. However, when using a MC-type simulation 180,000 samples were required.

The motivation behind using data from a variety of conditions is to capture the stochasticity in traffic conditions. To illustrate the drawback of using limited data, we compare the distribution of flow for AM period by using only one day's speed and flow to calibrate the AM peak model. The simulated flow distributions (shown in Figure 5-5) from limited data model does not match, not only the AM peak flow data under cluster one but also the AM peak flow data under cluster two. In addition, the objective function after calibration is 0.25. The objective function (test statistic of KS test) for calibration using the data from 43 days for condition 1 and 2, respectively, are 0.09 and 0.08. This illustrates the drawback in using limited data for model calibration and the importance of considering stochasticity in traffic conditions when calibrating traffic simulation models.

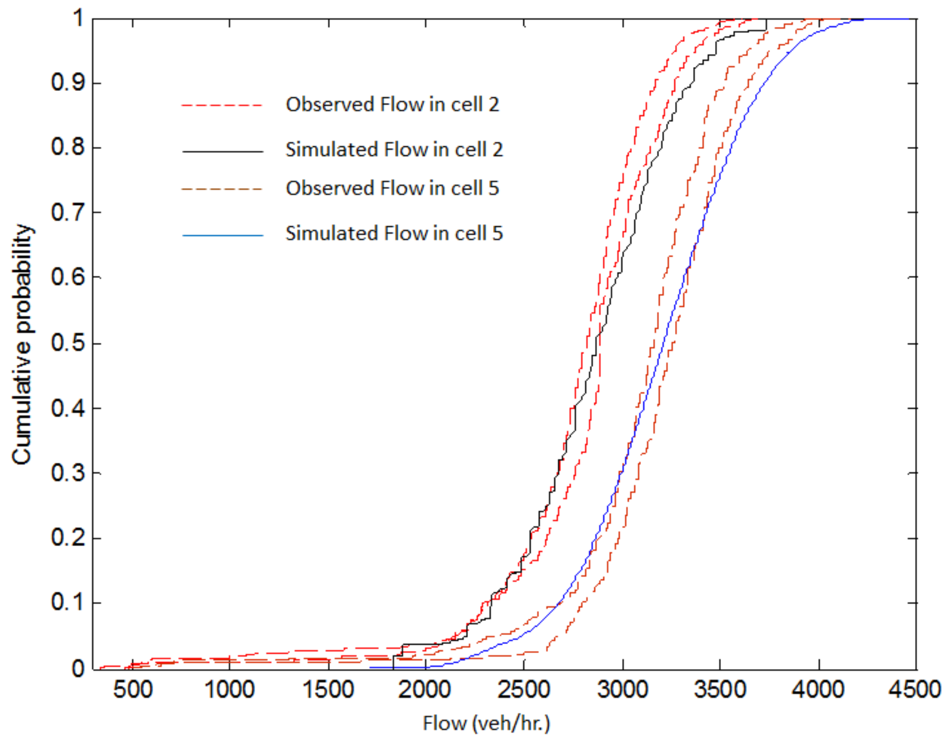


Figure 5-5 Comparison of observed and simulated link flow distributions using limited data in calibrating AM peak period

5.2 PM Peak Calibration Results

Similar to the AM weekday peak, clustering of speed data for PM (4-6PM) peak periods weekday is performed. Unlike the AM peak, the PM peak period speed data fell into six clusters. However, speed data from 33 out of 43 days (78% of data) fell into two clusters with CoV less than 0.25 for within each cluster. The CoV for the other clusters was in the range of 0.3-0.7. Also, the frequency of number of days within each cluster is not more than 4, indicating the speed data corresponding to these days as outliers due to work zone conditions or incidents. Hence, we use the two major clusters are representative clusters. Figure 5-6 shows the speed variation among the two major clusters during PM weekday peak period. Hence, we consider two distinct supply-side conditions in calibrating the macroscopic model for the PM peak.

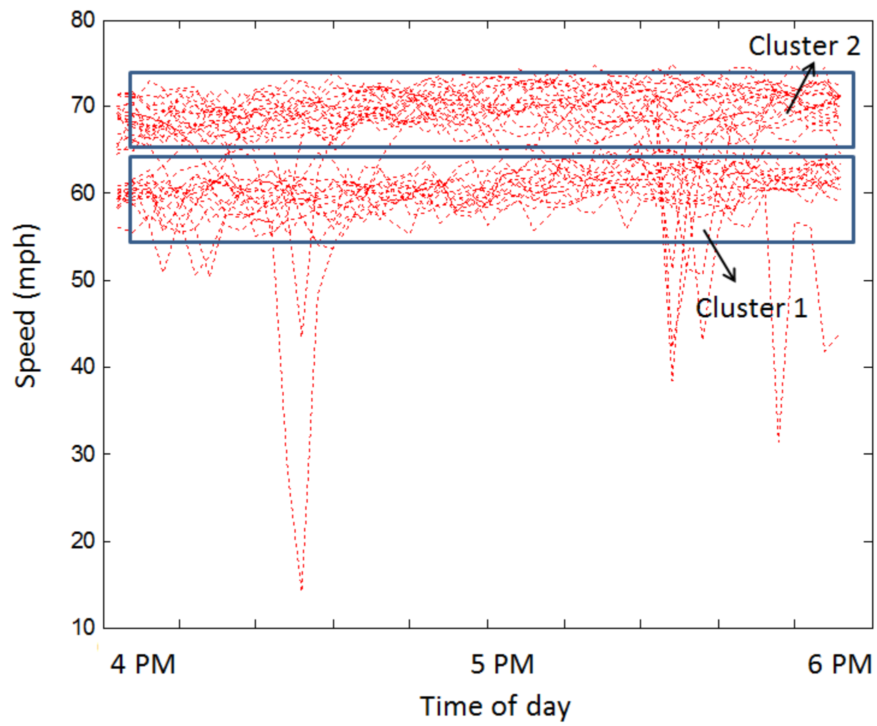


Figure 5-6 Illustration of Speed Data Variation for PM weekday Peak period during April and May 2011 at interchange 7 of NJTPK

The comparison of observed and simulated flow distributions in cells 2 and 5 (Figure 5-1) from the calibrated model for PM weekday peak period for condition 1 is shown in Figure 5-7(a). Similar to AM peak, we noticed during the process of calibration that some cells have different calibrated parameter set from the others. In other words, the stochasticity in simulation parameter set is not only temporal but also spatial. The calibrated parameters ([μ_{vf}, σ_{vf} ρ_{max}, ρ_{jam}]) for PM weekday peak period for condition 1 are [52 4.16 83 150, 62.2 2.85 90 150] in appropriate units. The objective function after calibration is 0.08. In order to achieve the flow distribution, the SC approach required 1506 evaluations for various stochastic demand combinations. However, a MC-type sampling method to achieve the same accuracy required 180,000 runs of the simulation model.

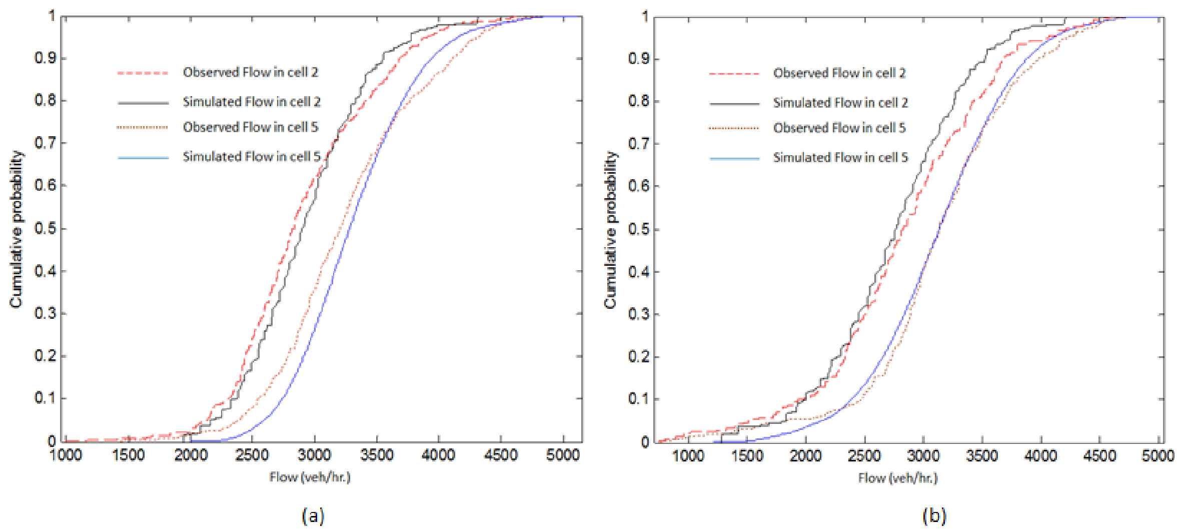


Figure 5-7 (a) Comparison of observed and simulated link flow distributions during PM weekday peak period for condition 1 (b) Comparison of observed and simulated link flow distributions during PM weekday peak period for condition 2

The comparison of observed and simulated flow distributions in cells 2 and 5 (Figure 5-1) from the calibrated model for PM weekday peak period for condition 2 is shown in Figure 5-7(b). Similar to AM peak, we noticed during the process of calibration that some cells have different calibrated parameter set from the others. In other words, the stochasticity in simulation parameter set is not only temporal but also spatial. The calibrated parameters ([μ_{vf}, σ_{vf} ρ_{max}, ρ_{jam}]) for PM weekday peak period for condition 2 are [48 2.26 80 150, 65 3.32 87 150] in appropriate units. The objective function after calibration is 0.05. In order to achieve the flow distribution, the SC approach required 4034 evaluations for various stochastic demand combinations. However, using a MC-type sampling required 180,000 runs of the simulation model.

In order to validate the estimated parameters using the proposed calibration methodology, we chose the month of July. Using the clusters generated for the speed observations for the weekday PM peak speed data April and May, we classify the speed data in July at interchange 7. This process resulted in 80% of the data falling into cluster two among the clusters generated for April and May. We generate the demand distributions for the mainline, on- and off-ramps using the ETC data for the days falling into the aforementioned cluster. Then we run the simulation separately for each clusters using the corresponding parameter set and demand distributions. The comparison of flow distributions is shown in Figure 5-8. The values of the objective function (KS test statistic) are 0.084.

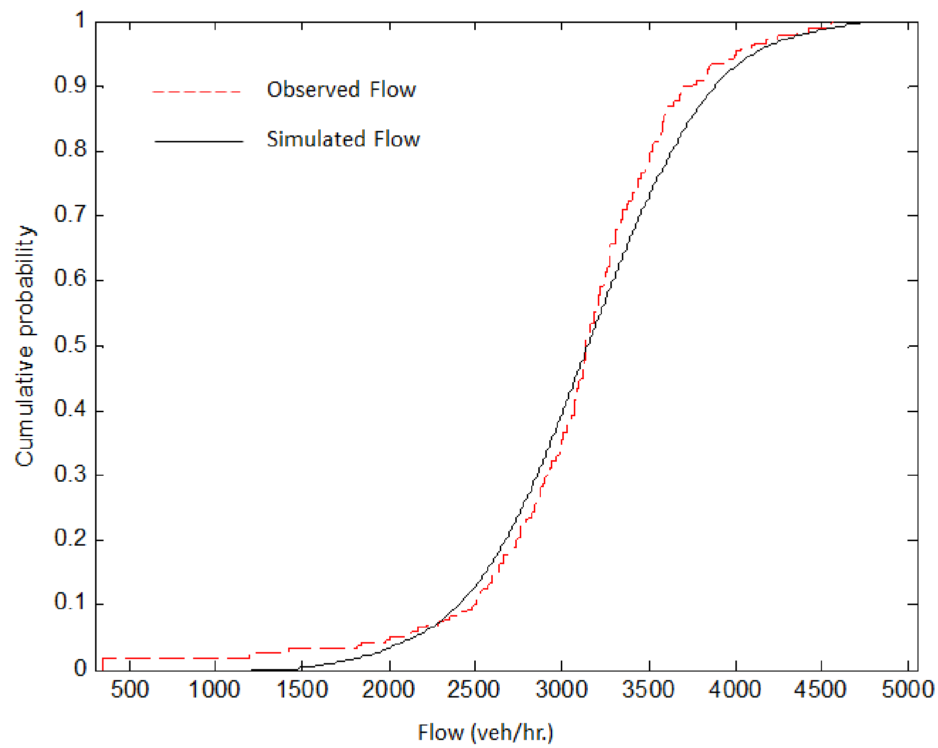


Figure 5-8 Validation of estimated parameters by comparison of flow distributions for major weekday PM peak days in July

5.3 Illustration of Proposed Calibration Methodology with Limited Data

5.3.1 Introduction

As noted in the first section, the lack of data near an entry or exit ramp, toll plaza, lane drop (merge), traffic circles, work zones, or in general road sections that are challenging to model (due to difference in grade, low visibility) may lead to the model not capturing the complexity of traffic flow for these sections. This drawback is more pronounced in macroscopic models which, unlike the microscopic models, do not take into account the movements or decisions of each individual vehicle. However, macroscopic models are much simpler to build and consume much less time and computational resources than microscopic models. Thus it can be said that the modeling error equation (4.1) for macroscopic models is higher than for microscopic models.

Also, the speed and flow data collected using traffic detectors is not very reliable. In cases where the availability of sensor data is limited or unreliable, alternative sources of data have to be combined and supplemented with sensor data. This fused dataset is more accurate.

Aside from temporal variability, the traffic conditions can show spatial variability. As an illustration the speed-flow scatter plots for two traffic sensors separated by one mile for a single day on a three-lane section of the NJTPK is shown in Figure 5-9. The free flow speed (v_f) and maximum flow ($q_{\max}$) show significant differences for the two locations.

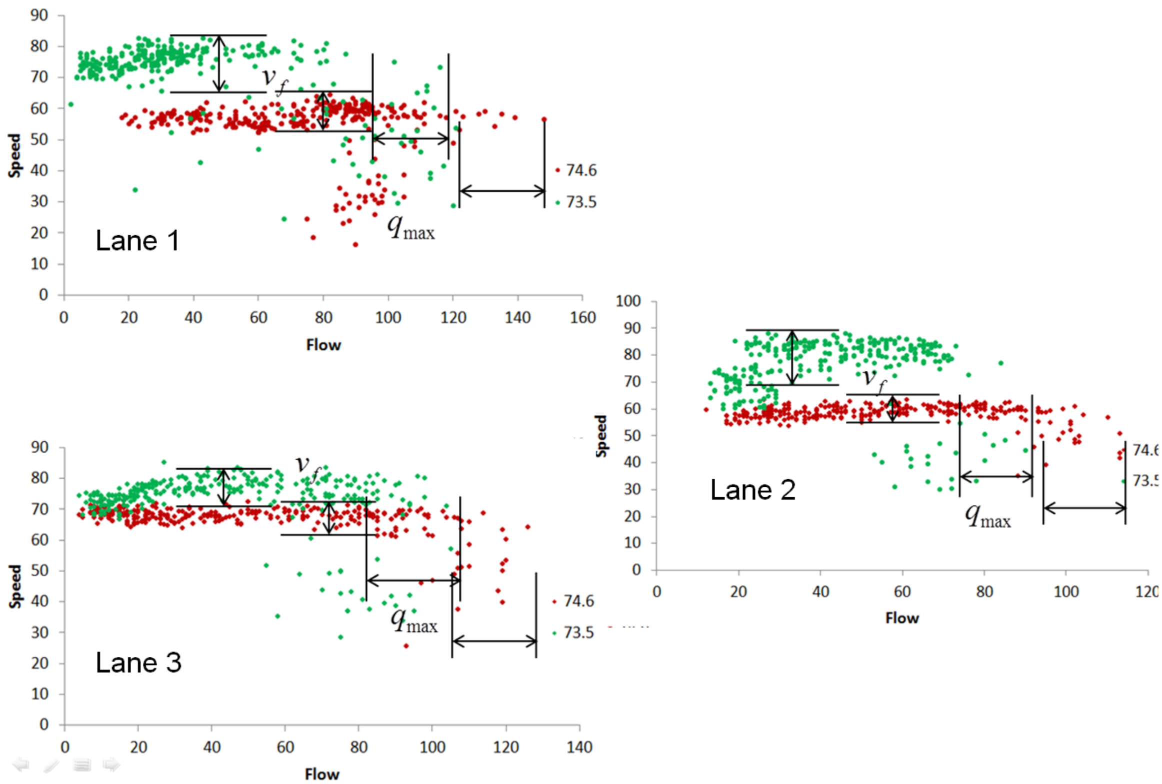


Figure 5-9 Spatial Variation of Traffic Flow

This brings up the question of using sensor station data in the calibration process. Sensor station data is one of the most commonly used and available data. However, sensor station data is a point data i.e. it represents flow or speed of traffic at a single point of the freeway. If a sensor is located in the part of a freeway where there is no entry or exit ramps close by the spatial variation of flow may not be large. But if the sensor is located close to an exit or entry ramp, the variation of flow before and after the ramp is high. If the objective is to capture all forms of variability in traffic, this spatial variation is also of significance. One of the approaches by which this aspect of spatial variation will be addressed is by choosing the appropriate discretization scheme depending on the location of the sensor.

In addition to sensor stations that detect vehicles on the freeway, there are other vehicle identification technologies available. The infrared tag used in electronic toll collection is one source of data, used to a widely by TRANSCOM. Each of these data can be used, synergistically, in the robust calibration of traffic simulation models. Depending on the quality of the dataset, it is established in literature (Mathison, 1988) that using multiple data sources increases the validity of the model. This type of data fusion for calibration will be another important aspect of this study.

5.3.2 Illustrative Example and Discussion

The stochasticity in space for the parameter vector is evidenced by a calibration study performed as part of this study. The study section is a three lane, 8-mile section of the New Jersey Turnpike covering interchanges, 7 and 7A in the northbound direction of traffic flow. A schematic representation of the section can be seen in Figure 5-10. Also shown is the cell discretization of the section into 11 cells. Sensor data for flow and speed is available for each mile along the section. A macroscopic first order simulation model is built for this section for the same time period as the data used to calibrate the freeway section in section 4.2. For this time period, sensor data for four locations is available. However, since this data is from traffic sensors, the speed and flow data is missing for 20% of the time for one sensor and around 5% of time. Additionally, the data from sensors may not be as accurate (Rajagopal and Varaiya (2007), Li and Li (2009)).

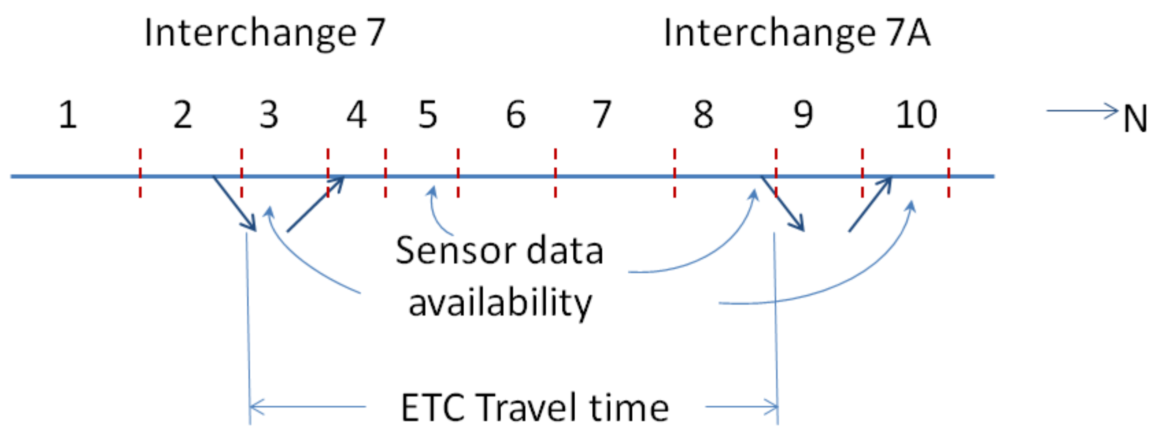


Figure 5-10 Schematic representation and data availability of the model of NJTPK between interchanges 7 and 7A

In order to mitigate the issue of missing and partly inaccurate data, we use the travel time estimated from the ETC data. The travel time between interchange 7 and 7A is estimated for the month of April for every five minutes during the PM peak period (4-6 PM). Using ETC travel time as another calibration measure would result in a more robust model due to two reasons. Firstly, the ETC data is continuously available and the exact time of exit and entry at every toll plaza of the interchange of each and every vehicle is available. Secondly, travel time data is a measure which has a much greater spatial extent than speed data collected as a point measure using sensor data. Also, travel time is an important measure when using simulation models as predictive tools.

We use the same two representative clusters during the PM peak from the previous chapter for the purpose of calibration. The demand and travel time distribution data is also separately collected for the two clusters. The same methodology described in section 3 is used to calibrate the freeway section depicted in Figure 5-10. However, we use flow and travel time as output measures in the calibration process.

The comparison of observed and simulated flow distributions from the calibrated model for PM weekday peak period for condition 1 is shown in Figure 5-11. Similar to AM and PM peaks, the stochasticity in simulation parameter set is not only temporal but also spatial. The calibrated parameters ([μ_{vf}, σ_{vf} ρ_{max}, ρ_{jam}]) for PM weekday peak period for condition 1 for cells [{1,2}; {3,4,5}; {6} and; {7,8}] are [52 4.16 83 150; 65 7 90 150; 70 17 90 150; 66 15 90 150] in appropriate units. The objective function after calibration for flow is 0.08 and 0.15 for travel time. In order to achieve the flow distribution, the SC approach required 15,121 evaluations for various stochastic demand combinations. However, a MC-type sampling method to achieve the same accuracy required 200,000 runs of the simulation model.

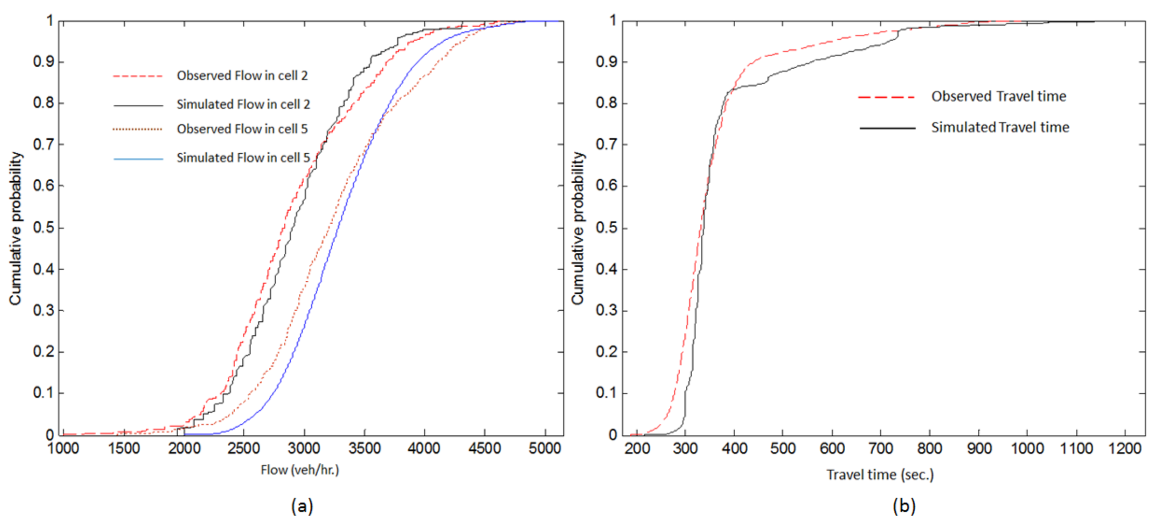


Figure 5-11 Comparison of observed and simulated (a) flow distribution and (b) travel time distribution during PM peak for interchange 7-7A freeway section of NJTPK

As compared to the calibrated parameters using only flow data from sensor data, the variance in free flow speed is higher when flow and travel time data are used for calibration. The likely reason for this finding is the differences in speeds between different lanes and also differences in speeds among different vehicle types. The variance in speed over smaller sections among different lanes and/or vehicle types may not be as pronounced as in the smaller section. However, variation among different lanes and vehicle types would be more pronounced over larger section. Hence, since we use travel time, which is a measure with greater spatial extent than point flow data, the variance in speed is higher.

CHAPTER 6. CALIBRATION OF MICROSCOPIC TRAFFIC SIMULATION MODEL OF TOLL PLAZAS

As mentioned in the previous chapters, for building robust traffic simulation models the calibration process has to be performed for many different conditions. This applies to both macroscopic and microscopic models. In this chapter, we adopt simulation-optimization approach to the calibration of a microscopic simulation model of a toll plaza similar to the one presented in chapter 4 and 5. This approach is a variant of the calibration approach presented the previous chapters in this dissertation in that it formulates a special simulation-based optimization problem targeted to the calibration of a multi-lane toll plaza model. The prime motivation of this dissertation is that it is important to consider variability in both the inputs, $I_{s,}$ demand-side variability, as well as calibration parameters, C_s, supply-side variability. Hence, is essential to consider demand from various days than using average demands or demands from a smaller sample. Thus, in this chapter we consider demands from many days in the calibration process.

In chapter 4, the computational complexity the numerical methods capturing stochasticity suffer from is illustrated. This problem is further exacerbated for microscopic traffic simulation models, specifically, those that require several customized models augmenting the default modeling, requires data in much greater detail. Customization of microscopic models is required when the default modeling capability is inadequate. Customization of microscopic models is required when the default modeling capability is inadequate. Such customization is performed often for modeling traffic circles (Bartin et al. (2005), Vaiana et al. (2007)), toll plazas (Astarita et al. (2001), Ozbay et al. (2006) and Mudigonda et al. (2009)), freeway merging sections (Yang et al. (2006), Gardes et al. (2002), Yang and Ozbay (2011)), etc. Thus the same generic calibration methodology may not be applicable to traffic simulation models of much greater detail such as the toll plaza models. The detailed data for calibrating such models may need to be collected via video data captured at specific locations of the section to be modeled. Such data may not always be available. Thus better means and methods to calibrate such models for various conditions are required.

One important contribution of this chapter is its emphasis on the importance of the development of specialized optimization formulations for different kind of simulation calibration problems. This is aimed at demonstrating the difficulties in using a generic calibration approach for all kinds of simulation models. Moreover, we also discuss the idea of calibration with missing data in the same way we discussed in Chapter 5. Furthermore, we extend the proposed SBO framework to be combined with the computationally-efficient sparse grid stochastic collocation method to generate distribution of outputs rather than average values.

Accurate modeling of toll plazas can suffer from the lack of adequate models in off-the-shelf traffic simulation packages. Despite having a representative modeling methodology for toll plaza lane choice, it is imperative that these models are calibrated appropriately for different conditions. Calibration entails adjusting the model parameters so that the toll plaza simulation output (such as lane usage, throughput, etc.) matches the observed output within a certain amount of error. However, when there is no observed output available, for instance for evaluating a proposed toll plaza design, we need a framework to evaluate the performance of the design. In this study, we present a simulation-based optimization (SBO) framework that

- applies a data-driven lane choice decision heuristic model proposed in an earlier study (Mudigonda et al. (2009)) by the authors for modeling lane selection behavior at toll plazas,
- provides the flexibility of,
    - using existing individual lane choice data, if available, as input to the SBO framework,
    - using lane choice measures in the heuristic model as calibration parameters in the SBO framework when lane choice data is partly or completely not available.

We use the electronic toll collection (ETC) data from NJTPK to illustrate the usefulness of the proposed SBO framework. Since these data used in the lane choice heuristic are easily and continuously available, the coefficients of the lane choice heuristic can be obtained and the model can be calibrated for different times of the day and days of the week. Thus this SBO framework allows the modeler to generate simulated lane choice behavior similar to the observed behavior using easily and abundantly available ETC data.

We use the toll plaza at Interchange 14A of the NJTPK for case studies with data partially available or not available. We show the usefulness of the SBO framework for toll plaza model calibration. Additionally, we mine the existing data on drivers' lane decision making to establish a system-wide measure to characterize toll plaza performance. This system-wide measure is also useful as an objective in modeling proposed toll plaza designs where there are no lane choice data available.

6.1 Modeling and Calibration Methodology

The variables that influence the drivers' lane choice decision making process at toll plazas are:

(1) **Approach direction of vehicles to toll plaza**: In general, drivers tend to use the toll lanes closer to their current lanes. Depending on which direction they approach a toll plaza from, the possibility of reaching a toll lane that is far from their current lane is lower. For example, in Figure 6-1, using approach 1 are likely to choose lanes that are on the right side of the toll plaza where conditions permit and vice versa. This stems from the fact that drivers try to avoid excessive weaving at the toll plaza entrance where vehicles access the plaza from different directions.

(2) **Exit direction of vehicles after leaving toll plaza**: Drivers tend to select lanes that are close to their exit locations to avoid excessive weaving at the downstream of the toll plaza. However, this is not as significant a variable as the approach ramp measure, because drivers have a better view of the relative position of other vehicles when they leave the plaza as opposed to approaching the plaza from different ramps.

(3) **Queue Lengths at toll plaza**: It could be claimed that drivers choose shorter queues to reduce their wait times at the toll plaza. They could possibly change their decisions based on the perceived wait times.

(4) **Vehicle Type:** Depending on the whether the vehicles is car or a truck or bus,

the maneuverability varies. This in turn influences the lane choice of vehicles at the toll plaza.

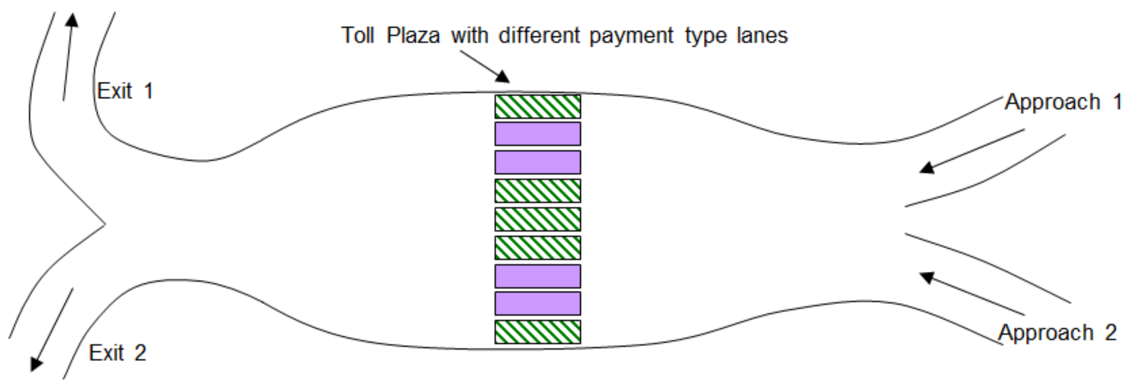


Figure 6-1 Schematic representation of approach, exit directions and lane types at a toll plaza

A heuristic model to evaluate the lane choice measure (LCM) of each lane is formulated based on the variables mentioned earlier in this section, which were also proposed in an earlier study by Mudigonda et al. (2009). The LCM of a given lane i can be modeled as a linear function shown in equation (6.1):

$$
\begin{aligned}
&LCM_i = \alpha^e p^{ej}{}_i + \alpha^x p^{xj}{}_i + \alpha^q p^q{}_i \\
&p^{ej}{}_i = \text{proportion of vehicles choosing lane } i \\
&\text{based on the approach } (\text{e}) \text{ direction } j \text{ (LCM approach)}, \\
&\sum_i p_i^{ej} = 1, \ \forall \text{lanes } i, \text{ for each approach } j \\
&p^{xj}{}_i = \text{proportion of vehicles choosing lane } i \\
&\text{based on the exit } (\text{x}) \text{ direction } j \text{ (LCM exit)} \\
&\sum_i p_i^{xj} = 1, \ \forall \text{lanes } i, \text{ for each exit } j \\
&p^q{}_i = \text{proportion of vehicles choosing lane } i \\
&\text{based on the queue (q) conditions (LCM queue)} \\
&\alpha^e, \alpha^x, \alpha^q = \text{weights for each measure, } e,\ x,\ q
\end{aligned}
\tag{6.1}
$$

When a driver approaches the toll plaza, the driver makes a decision about which lane to choose based on their LCM and selects the lane with maximum LCM. The LCM for approach direction and based on exit i.e., *proportion* of vehicles choosing lane *i* based on which approach or exit they choose, can be calculated from toll transaction data or revealed through video data of the toll plaza. Thus, these LCM's can be either estimated from data if available, or used as a calibration parameter in case the video data of toll plaza is unavailable. The measure for queue is estimated as the proportion of number of vehicles in lane *i* to the total number of vehicles in the lane of the same transaction type as the given vehicle. The default values for α^e, α^x, α^q, are issued an initial value of 0.4, 0.1 and 0.5, respectively. These values are based on the relative importance of each variable in the lane selection and visual verification of the simulation. For a more detailed description of the heuristic please see (Mudigonda et al. (2009)).

6.1.1 Simulation-based Optimization Framework for Calibration of Lane Choice

Models at Toll Plazas

The LCM heuristic described above is implemented into a SBO framework. The SBO framework will be applied to calibrate the toll plaza model when partial data are available. When no data are available, the SBO framework is used to establish a system-wide measure to characterize toll plaza performance. This system-wide measure is also useful as an objective in modeling proposed toll plaza designs where there is no lane choice data available.

6.1.1.1 SBO Framework for Partial Data

The simulation output using the parameters for the LCM methodology can be expressed as follows:

$$\begin{aligned}
&f(I_{Obs}, \theta, C) \rightarrow \text{(simulation model)} \rightarrow \hat{S}(I_{Obs}, \theta) \\
&I_{Obs} = \text{observed input data (origin-destination demand,} \\
&\text{geometric design, operational rules, service time),} \\
&\theta = \text{estimated set of parameters } (p^{x}_{j,i}\ \forall \text{exits } j \text{ and lanes } i) \text{ for} \\
&\text{cumulative lane selection decision measure,} \\
&C = \text{other calibration parameters,} \\
&\hat{S} = \text{simulation output estimated,} \\
&O_{Obs} = \text{observed output}
\end{aligned} \tag{6.2}$$

As mentioned earlier in the proposed methodology, calculating LCM based on approach or exit direction requires appropriate data. It is not possible to have this data for all cases. So, for instance, consider the case where the data is available to calculate LCM based on entry direction but not for exit direction. Such cases can be considered as those where partial data is available for the LCM model. In such cases the LCM based on exit direction can be included as parameters to be estimated from the calibration process. The optimization problem involved in calibrating the toll plaza model i.e., estimating the parameters (θ) when partial data is available can be expressed as shown in equation (6.3). The outputs estimated using the SBO framework for the toll plaza are lane usage percentages and, throughputs.

$$
\begin{aligned}
&\text{Objective function} : \min_{\theta}\Big(\sum_{\text{lane } i} (e_i^{LU} + e_i^{T})\Big) \\
&\text{subject to:} \\
&0 < p_i^{xj} < 1, \sum_i p_i^{xj} = 1, \ \forall \text{lanes } i, \text{ for each exit } j \\
&\text{where,} \\
&e_i^{LU} = \left| \frac{\hat{S}_{LU} - O_{LU}}{O_{LU}} \right|, \text{ percent error in lane usage for lane } i, \\
&e_i^{T} = \left| \frac{\hat{S}_{T} - O_{T}}{O_{T}} \right|, \text{ percent error in throughput for lane } i, \\
&\hat{S}_{LU} = \text{ simulated lane usage estimated for lane } i, \\
&O_{LU} = \text{ observed lane usage estimated for lane } i, \\
&\hat{S}_{T} = \text{ simulated throughput estimated for lane } i, \\
&O_{T} = \text{ observed throughput estimated for lane } i, \\
&\theta = \text{estimated set of parameters } (p_i^{xj} \forall \text{exits } j \text{ and lanes } i)
\end{aligned}
\tag{6.3}
$$

We assume that there is equal importance given to the percent error in each output, lane usage and throughput. Since the lane measures based on exit is *proportion* of vehicles choosing lane i based on the exit, there is a constraint on parameters p^x for all lanes to add up to 1 for each exit.

6.1.1.2 <u>Hybrid SBO Framework using Stochastic Collocation</u>

The SBO framework for calibrating the toll plaza model when partial data is available (shown in section 6.1.1.1), uses average lane usage at the toll plaza as the output measure in the calibration process. However, the calibration can be performed for many different days to obtain calibrated parameters for a much more generic data rather than average output measures. In order to achieve this objective, we use distributions of inputs i.e. consider the demand-side variability. The demand-side variability is discretized using the computationally efficient sparse grid stochastic collocation methodology, elaborated in section 4.2. The same methodology is used to interpolate and generate the distribution of output i.e. lane usage distribution given the demand distribution. The lane usage distribution is compared to the observed lane usage distribution for each lane and the error between them is minimized in the objective function for this hybrid SBO framework shown in equation (6.4)

$$
\begin{aligned}
&\text{Objective function}: \min_{\theta}\Big(\sum_{\text{lane } i} (U_i^{LU})\Big) \\
&\text{subject to:} \\
&0 < p_i^{xj} < 1, \sum_i p_i^{xj} = 1, \ \forall \text{lanes } i, \text{ for each exit } j \\
&\text{where,} \\
&U_i^{LU} = KStest(\tilde{C}_{LU}, C_{LU}), \text{ KS-test statistic in lane usage for lane } i, \\
&\tilde{C}_{LU} \quad \text{distribution of simulated lane usage estimated for lane } i, \\
&C_{LU} \quad \text{distribution of observed lane usage estimated for lane } i, \\
&\theta = \text{estimated set of parameters } (p_i^{xj} \, \forall \text{exits } j \text{ and lanes } i)
\end{aligned}
\qquad (6.4)
$$

6.1.1.3 SBO Framework for Unavailable Data

Proposed design of toll plazas are always hypothetical cases. For such cases there is no available data for LCM based either on entry or exit. Thus, in addition to the existing parameters, the lane probabilities for approach and exit are also included in the SBO framework. Since the LCM based on exit is *proportion* of vehicles choosing lane *i* based on the entry (exit), there is a constraint on parameters $p^{ej}{}_{i}$ ($p^{xj}{}_{i}$) for all lanes to add up to 1 for each entry (exit).

During the design of the toll plaza a system-wide measure is required to characterize the toll plaza performance. Also, in proposed toll plaza designs there is no data available on drivers' LCM. And establishing a system-wide measure would help is estimating the ideal LCM's behind the drivers' decision making. The ideal way to establish this measure is to mine the observed data and construct a likely measure that is minimized in the observed LCM data. This measure can be constructed using a reasonable assumption that the drivers' objective is to minimize a combination of travel times and number of lane changes. The number of lane changes is a surrogate to the drivers' consideration for safety. Thus the system measure is a weighted sum of travel time and number of lane changes as shown in equation (6.4).

$$
\begin{aligned}
&\text{Objective function}: \min_{\theta}\Big(\frac{1}{n}\sum_{n}(\omega_1 TT_n + \omega_2 LC_n)\Big) \\
&\text{subject to:} \\
&\sum_{i} p_i^{ej} = 1, \ \forall \text{lanes } i, \text{ for each approach } j \\
&\sum_{i} p_i^{xj} = 1, \ \forall \text{lanes } i, \text{ for each exit } j \\
&\alpha^e + \alpha^x + \alpha^q = 1 \\
&\text{Written in short as,} \\
&\mathbf{A}^{\mathrm{T}}\boldsymbol{\theta} = \mathbf{1} \\
&\text{where,} \\
&TT_n = \text{ travel time of each user } n, \\
&LC_n = \text{ number of lane changes of each user } n, \\
&\omega_1 = \text{ weight assigned to travel time,} \\
&\omega_2 = \text{ weight assigned to number of lane changes,} \\
&\theta = \text{estimated set of parameters} \\
&\{\alpha^e, \alpha^x, \alpha^q, p_i^{ej} \forall \text{approaches } j \text{ and lanes } i, p_i^{xj} \forall \text{exits } j \text{ and lanes } i\}
\end{aligned}
\tag{6.5}
$$

In equation (6.4) it is assumed that the driver population is homogenous with respect to their choice of the linear combination of travel time and number of lane changes. The weight for number of lane changes, firstly, serves to have both travel time and lane changes to a similar scale of magnitude. The second use is to signify the relative importance of travel time and number of lane changes likely to be in the driver behavior. If we are able to validate a particular weighted average of travel time and number of lane changes as the one that represents the system-wide measure behind drivers' lane decision making, then that weighted average can be used as the objective function to be minimized for proposed toll plaza designs.

For this purpose, we estimate the objective function of the SBO framework as a weighted average of mean travel time and mean number of lane changes with the weights being variable. For each set of weights, we minimize the multi-objective function and compare the LCM measures estimated using the SBO framework the observed measures from video data. The hypothesis is that the particular weighted objective function for which the optimized LCM measures closely mimic the observed measures, then it can be considered as the likely objective behind the drivers' decision making.

6.1.1.4 Optimization Methodology in Calibration

The problems described in equations (6.3) and (6.4) are minimization problems. One of the main complications of this problem is that simulation is used as the "function" to quantify values in the objective function so this is a simulation-optimization problem without a closed form objective function. The solution for the SBO is thus evaluated using simultaneous perturbation stochastic approximation (SPSA) algorithm (Spall (1992)), described in Chapter 3, that is shown to work well for simulation-optimization problems (Zhang et al. (2008), Yang and Ozbay (2011), Balakrishna et al. (2007)). The optimization parameters, entry and exit lane decision measures, and weights, are used for calibrating the heuristic methodology. The fact that the calibration parameters are continuous and the objective function is differentiable with respect to the parameters, aids in the evaluation of the gradient for the SPSA. We recognize that the calibration is a constrained optimization problem. SPSA has been used in other constrained optimization problem before. (Zhang et al. (2008), Yang and Ozbay (2011), Balakrishna et al. (2007))

The SPSA algorithm described above is useful for unconstrained optimization problems. However, the optimization problem for the calibration of the toll plaza model shown in equation (6.2) is a constrained optimization problem with constraints

$0 < p^x_{j,i} < 1, \sum_i p^x_{j,i} = 1,$ on parameters. Use of SPSA for constrained optimization problems usually involves a projection of any infeasible point during the search of parameters, onto the feasible parameter space. In this study we use the constrained SPSA algorithm proposed by Sadegh (1997). The updated set of parameters at each iteration in estimated using a projection *P(θ)* of an infeasible point *θ* onto the parameter space *G*,

$$\hat{\theta}_{k+1} = P(\hat{\theta}_k - a_k \hat{g}'_k(\hat{\theta}_k)),$$
$$\hat{g}'_k = \hat{g}_k - \mathbf{A}(\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}(\mathbf{A}^{\mathrm{T}}\hat{\boldsymbol{\theta}}_{\mathbf{k}} \mathbf{-1}) \tag{6.6}$$
A = Coefficient matrix in the constraints shown in eq. (3)

The projection *P* is defined as point *P(θ)* in *G* closest to point *θ*. In order to account for cases where $\hat{\theta}_k \in G$ but $\hat{\theta}_k \pm c_k \Delta_k \notin G$, at each iteration a closed set G_k contained within *G* is constructed only to estimate the gradient. The gradient is adjusted so that the parameter set θ_k does not violate the constraints. This adjustment is based on projection and restoration moves prescribed in Rosen (1960). For further details please refer to Sadegh (1997) and Rosen (1960).

6.2 Case Studies using the SBO Framework

In this study, our focus is to model the driver behavior at the toll plazas which are not located on the mainline. Examples of such facilities are toll plazas on NJTPK, Garden State Parkway (GSP) in New Jersey and NY Thruway, and Pennsylvania Turnpike. These are located away from the mainline. The toll plazas are connected to several ramps from different directions of the mainline, thus increasing the complexity of the lane choice behavior. In this study we use the toll plaza at interchange 14A of the NJTPK (shown in

Figure 6-2) to illustrate the usefulness of the proposed SBO framework.

We implement the lane choice measure heuristic models developed in Mudigonda et al. (2009) to realistically model the driver behavior at the toll plazas as part of a customized toll plaza model developed using PARAMICS micro-simulation package. PARAMICS' API is used to implement the model to simulate the driver behavior at a toll plaza using the following modeling inputs:

1. Toll plaza geometry was obtained using satellite images as overlays.
2. Toll plaza lane configuration, namely E-ZPass and manual (cash) toll payment was obtained from the NJTPK.
3. Origin-Destination demand Matrix was created using the Electronic Toll Collection (ETC) dataset with individual vehicle-by-vehicle entry, exit time and transaction lane.
4. Service time distribution was obtained from data from earlier studies (Bartin et al. (2007)) where it was shown that it follows a lognormal probability distribution.

The ETC dataset consists of the individual vehicle-by-vehicle entry and exit time data. It also consists of the information regarding the lane through which each vehicle was processed (both E-ZPass and Cash users). It should be noted that NJTPK is a closed system tolled highway. Vehicles enter the mainline through an entry toll plaza at an interchange located separately from the mainline. Similarly, they exit the highway through an exit toll plaza, each interchange therefore has entry and exit toll lanes. Hence, in NJTPK, LCM based on the approach ramp can be deduced from the ETC dataset. LCM based on the exit direction, on the other hand, are not readily available from the

ETC dataset. This is because vehicles are out of the NJTPK system after crossing the toll plaza and the information as to which exit direction they choose are not revealed in the transaction data. Therefore, exit choices can be obtained only with video data. We collected the detailed movement of vehicles involving different combinations of vehicle type, approach ramp, lane choice, exit direction, for the exit toll plaza at interchange 14A of NJTPK for the PM peak period.

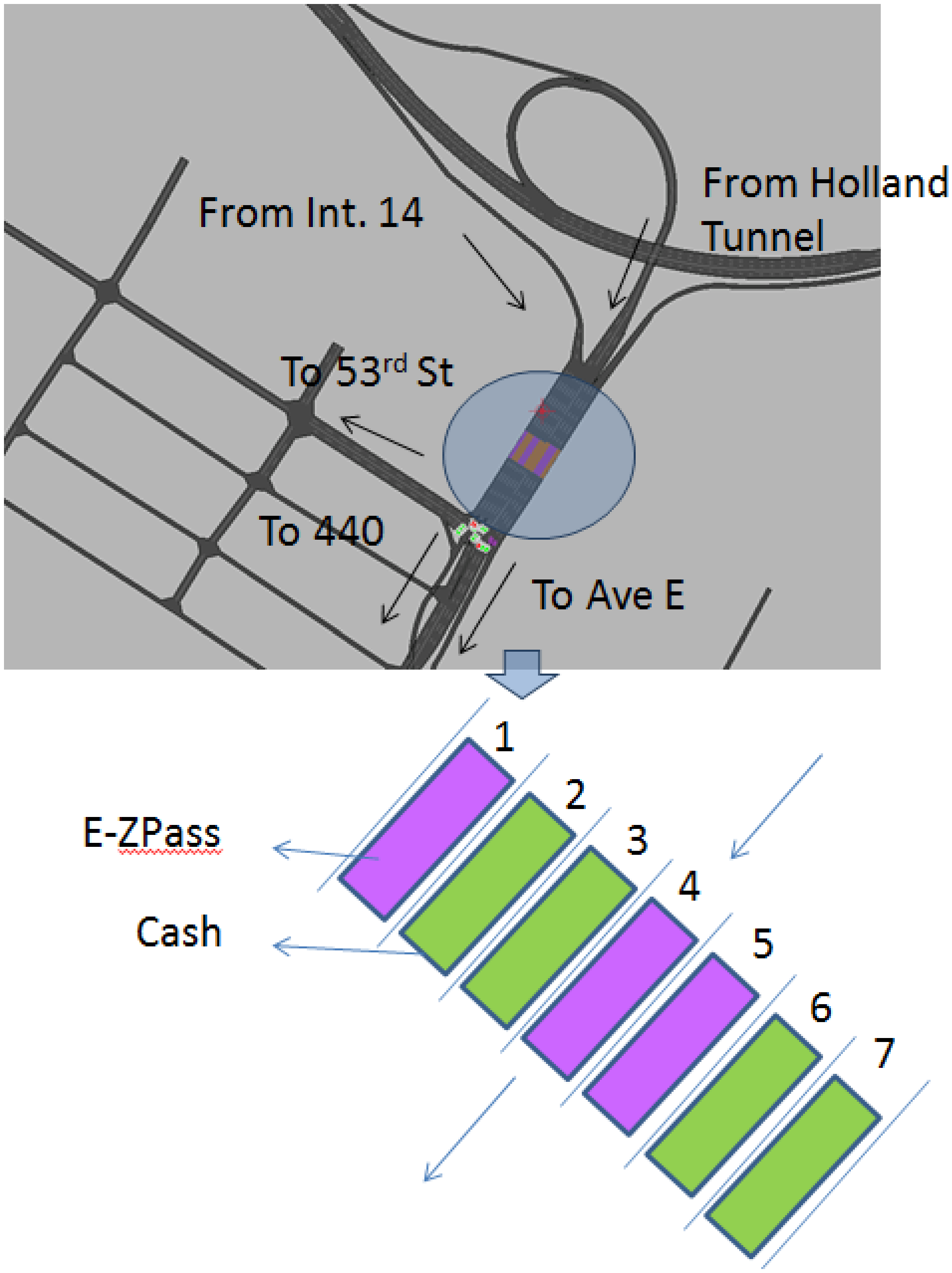


Figure 6-2 Interchange 14A Exit Toll Plaza Simulation Model Developed in PARAMICS and Lane Schematics

The calibration and optimization framework presented in the Methodology section is implemented in MATLAB. The simulation model, constructed in PARAMICS, is run in a batch mode and incorporated into the calibration framework in MATLAB, summarized in Figure 6-3. First, we present a case study of the application of the SBO calibration framework for calibrating the toll plaza at interchange 14A in NJTPK. The second case study is validating the likely objective of the drivers when making their decision at the toll plaza using the LCM framework.

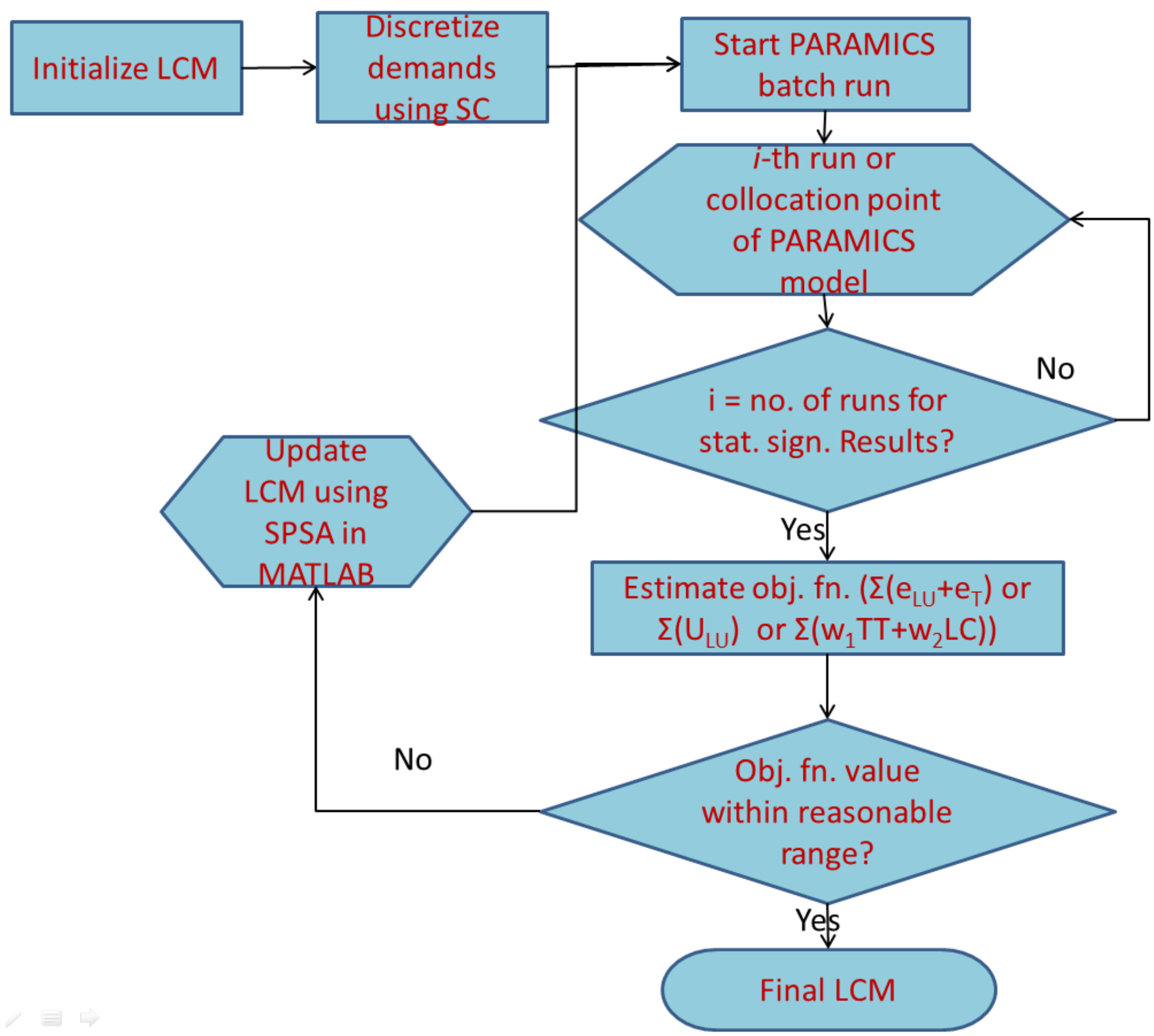


Figure 6-3 Flowchart illustrating the SBO framework

6.2.1 Case Study 1: Calibration of 14A Toll Plaza with Partial Data

From Figure 6-2 it can be seen that the exit toll plaza at interchange 14 has seven lanes with two entries and three exits. The LCM for each of the seven lanes, *i*, at the toll plaza involves two entry measures $p^{ej}{}_i$ (j=1,2) , three exit measures $p^{xj}{}_i$ (j=1,2,3) and one queue measure $p^q{}_i$.

The first application of the calibration framework is performed during the AM peak period (6-9 AM) for the interchange 14A toll plaza depicted in Figure 6-2. During this period, the measure for choosing a lane based on the entry direction $p^{ej}{}_i$ (j=1,2, i=1,…,7) is known from the ETC data. The measure for queue $p^q{}_i$ is measured during the simulation. But the measure for choosing a lane based on exit direction $p^{xj}{}_i$ (j=1,2,3, i=1,…,7) is not known, since the video data is available from the PM period. So all the measures based on exit direction, 21 in number, are chosen as parameters for calibration. Each run of the SBO framework is performed for multiple replications to obtain 90% significance.

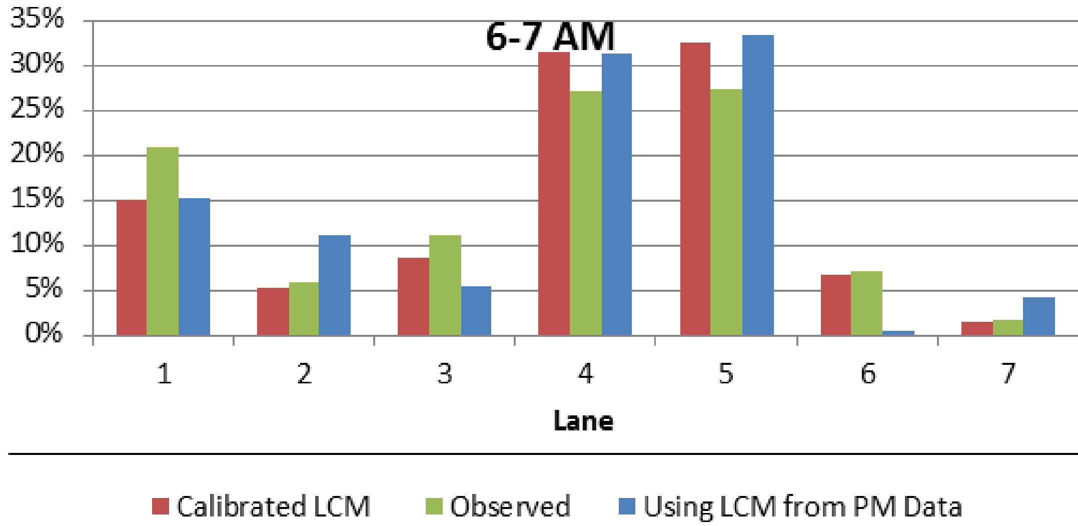
As mentioned in Spall (1992), the number of iterations required for reaching a near-optimal set of parameters is dependent on the starting values. To this end, the duration of calibration is divided into each hour, 6-7, 7-8, 8-9 AM. The starting value for 8-9AM is the calibrated parameter set of 7-8AM and so on. The starting value for 6-7 AM is chosen as all $\theta_{k|k=1}$ = 0.25.

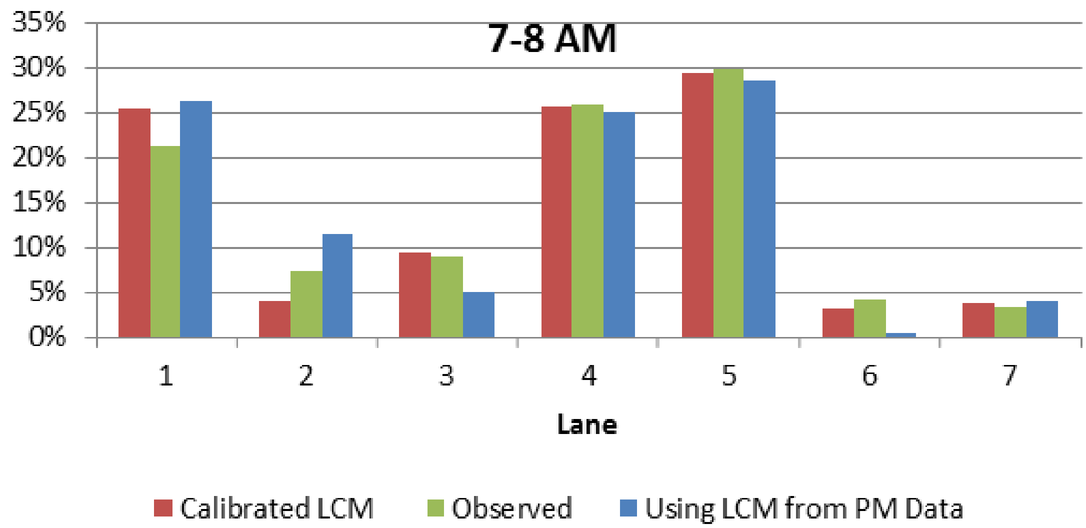
The value of objective function, namely, percent error in lane usage, after the iterations and number of iterations used is shown in Table 6-1. Using the calibrated set of parameters from the previous time period, significantly improves the objective function of the calibration process for the subsequent time period. This can be seen from the number of iterations required to reach the objective function value.

Table 6-1 Details of the calibration output

Time period	**6-7 AM**	**7-8 AM**	**8-9 AM**
Number of iterations	50	50	50
Objective function (% error in lane usage)	0.210	0.108	0.13

The lane usage for each time period is shown in Figure 6-4. The correlation between observed and simulated lane usage for 6-7AM is 96% and for 7-8 and 8-9AM is 98.3%.


6-7 AM
35%
30%
25%
20%
15%
10%
5%
0%
1
2
3
4
5
6
7
Lane
Calibrated LCM
Observed
Using LCM from PM Data


7-8 AM
35%
30%
25%
20%
15%
10%
5%
0%
1
2
3
4
5
6
7
Lane
Calibrated LCM
Observed
Using LCM from PM Data

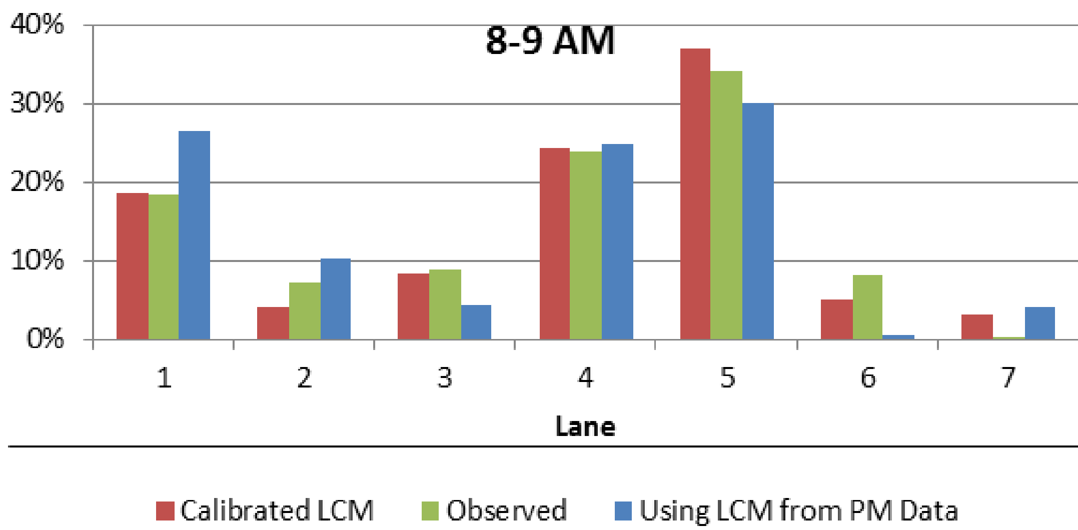

8-9 AM
40%
30%
20%
10%
0%
1
2
3
4
5
6
7
Lane
Calibrated LCM
Observed
Using LCM from PM Data

Figure 6-4 Comparison of AM lane usage observed, estimated using calibrated LCM model and PM period data

As mentioned earlier, the LCM model can be adaptable whenever video data for the lane measures is available or using the LCM measures as parameter in the calibration when video data is absent. To show the benefit of calibration, we compare the objective function, namely, percent error in mean lane usage for the AM peak period using (a) calibrated LCM parameters as shown in Figure 6-4, and, (b) LCM calculated from video data from PM peak period. This comparison showed that the objective function for AM period estimated using PM LCM data as 0.242 and 0.15 when calibration is used. Additionally, the correlation between observed and simulated lane usage for AM period estimated using PM LCM data is, for 6-7AM is 90%, 7-8AM 95% and 8-9AM 89% as opposed to 96%, 98.3% and 98% respectively when LCM are estimated using calibration. These lane usages are compared in Figure 4.

This result clearly illustrates the need for calibrating the simulation model for different time periods. It also shows that the calibration parameters estimated for one time period may not be able to replicate the output for another time period.

6.2.2 Case Study 2: Calibration of 14A Toll Plaza with Partial Data using the hybrid SBO framework considering demand-side variability

To illustrate the use of the hybrid SBO framework we use the same scenario as case study 1 i.e. calibration during the AM peak period (6-9 AM) for the interchange 14A toll plaza depicted in Figure 6-2. However, instead of using average lane usage as the calibration measure, we generate the distribution of lane usage for each lane. The lane usage distribution is generated using the computationally-efficient sparse grid stochastic collocation approach. In addition, the demand-side variability is also considered.

For modeling the demand-side variability, we generate the demand distributions for AM weekday peak period for the months of August and September 2011. There is demand entering via two ramps (as shown in Figure 6-5), one from NJTPK and another from Holland Tunnel (HT). Each of this demand includes E-ZPass vehicles and cash vehicles. Thus there are four components of demand, as shown in Figure 6-5. The 5-minute demand distribution for the two NJTPK demands and two HT demands are shown in Figure 6-5.

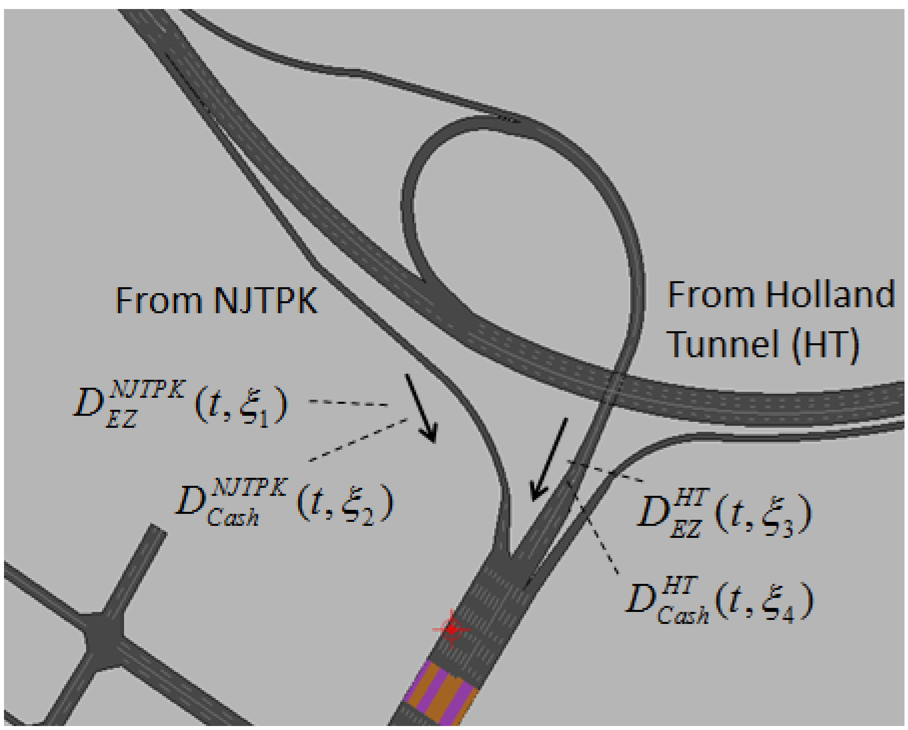


Figure 6-5 Demand-side variability at interchange 14A toll plaza

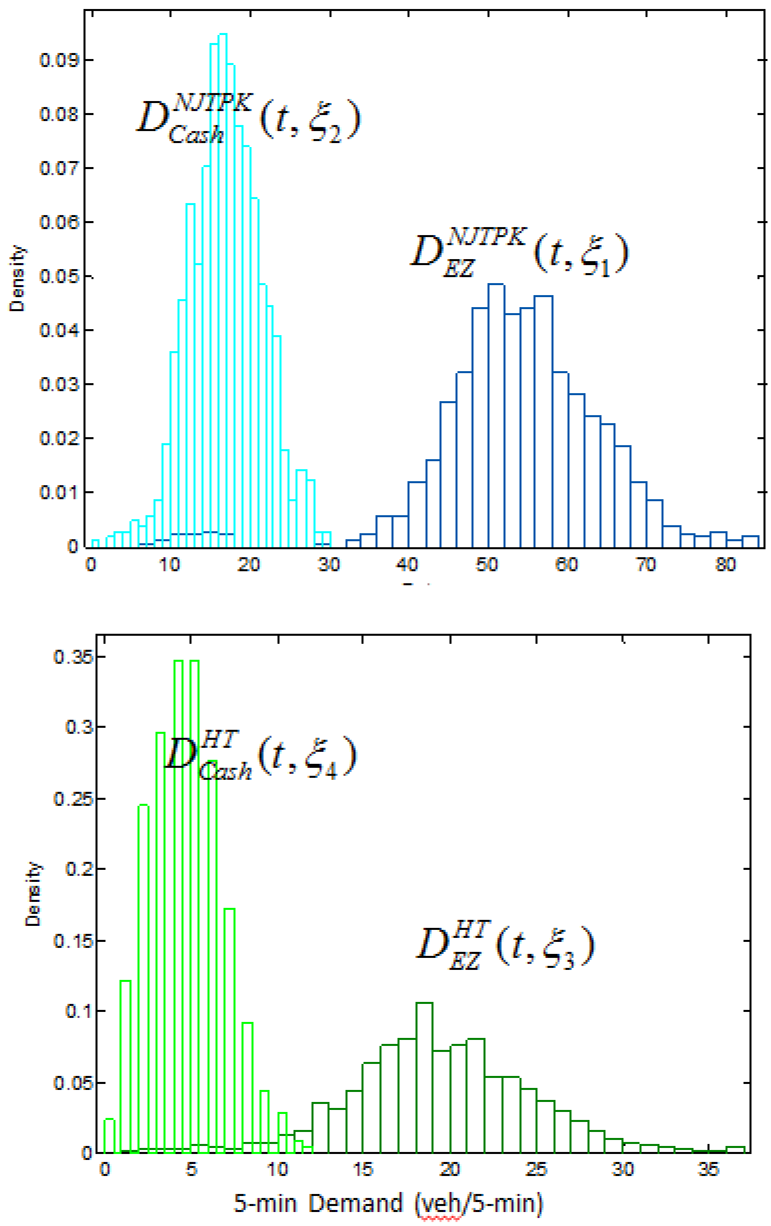


Figure 6-6 Demand distributions at 14A toll plaza during AM weekday peak period

As mentioned in case study 1, during this period, the measure for choosing a lane based on the entry direction p^{ej}_i (j=1,2, i=1,…,7) is known from the ETC data. The measure for queue $p^q{}_i$ is measured during the simulation. But the measure for choosing a lane based on exit direction $p^{xj}{}_i$ (j=1,2,3, i=1,…,7) is not known, since the video data is not available. So all the measures based on exit direction, 21 in number, are chosen as parameters for calibration.

The comparison of observed and simulated lane usage distribution after calibration is shown in Figure 6-7. The value of the objective function, sum over all lanes of KS test statistics comparing observed and simulated lane usage distribution is 0.52.

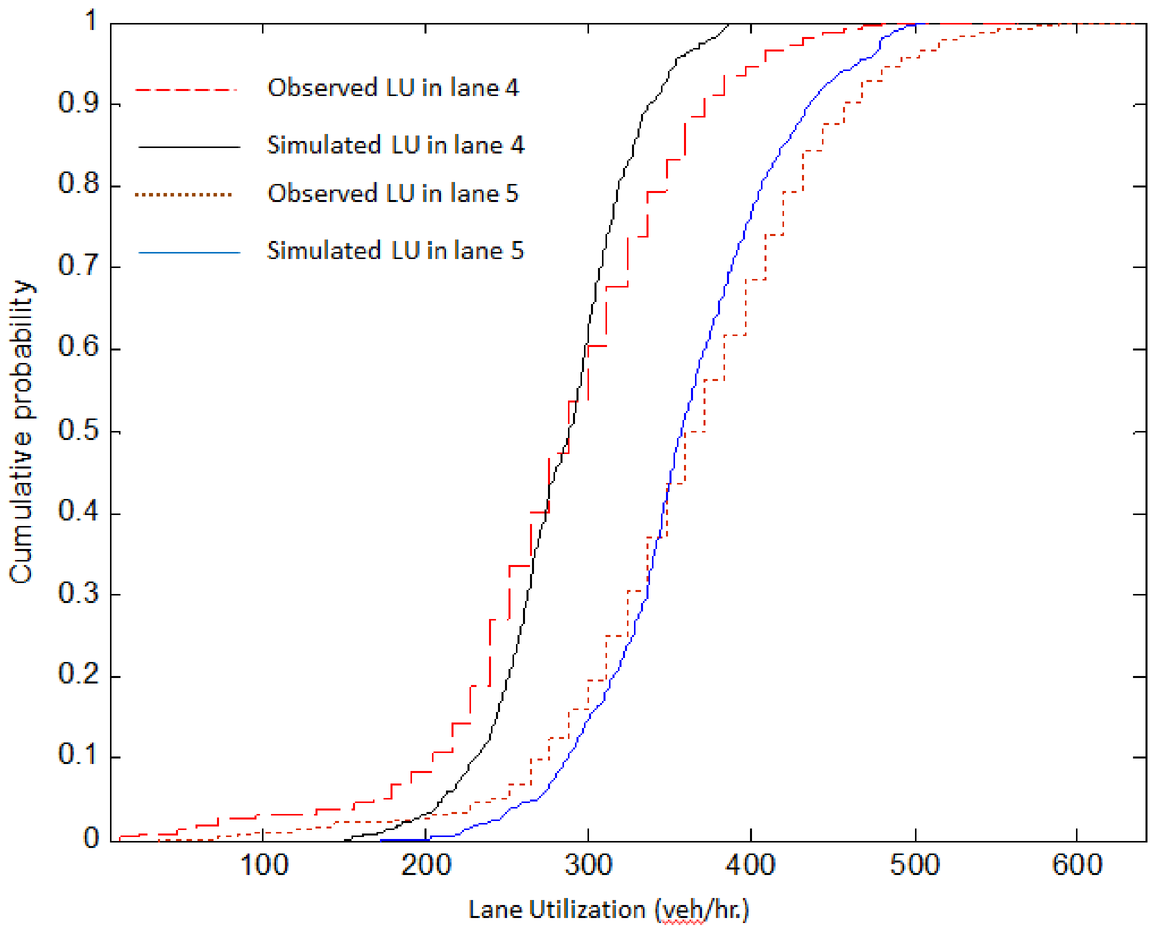


Figure 6-7 Observed and simulated lane utilization distributions for 14A toll plaza model calibrated using hybrid SBO for AM weekday peak

The generation of the simulated distributions required only 65 runs of the microscopic simulation model. However, the number of runs required for an MC-type sampling are 9,000. In terms of computational time, since each run takes about 111 s. The total time taken to generate the simulated LU distributions is 2 hours. The same time when using an MC-type sampling is 162.5 hours.

6.2.3 Case Study 3: Using SBO framework for Case with No Data Available for 14A Toll Plaza

As mentioned in the Methodology section, when no data is available, it is necessary to understand the measure that the drivers tend to optimize during their lane choice process. The purpose of the second case study is to validate and establish a system-wide measure behind the drivers' lane decision making at the toll plaza. For this purpose, we estimate the objective function of the SBO framework as a weighted average of mean travel time and mean number of lane changes with the weights being variable. For each set of weights, we minimize the multi-objective function and compare the LCM estimated using the SBO framework the observed LCM from ETC and video data. In order to validate the system-wide measure, we run the simulation for PM period but without using any of the available data for entry or exit LCM. The hypothesis is that the particular weighted objective function for which the optimized LCM measures closely mimic the observed measures, then it can be considered as the likely objective behind the drivers' decision making.

From Figure 6-1 it can be seen that the exit toll plaza at interchange 14 has seven lanes with two entries and three exits. The LCM (please refer to eq. 1) for each of the seven lanes, i, at the toll plaza involves two entry measures $p^{ej}{}_i$ (j=1,2) , three exit measures $p^{xj}{}_i$ (j=1,2,3) and one queue measure $p^q{}_i$. As an application we implement the SBO framework during the PM peak period (5-6 PM) for the interchange 14A toll plaza for which ETC and video data are available. During this period, the LCM for choosing a lane based on the entry direction $p^{ej}{}_i$ (j=1,2, i=1,…,7) is known from the ETC data. The LCM for choosing a lane based on exit direction $p^{xj}{}_i$ (j=1,2,3, i=1,…,7) is available from video data collected. The LCM for queue $p^q{}_i$ is measured during the simulation based on the queue at each lane. This measure could not be estimated from video data due to the lack of the appropriate camera angle. Hence the measures based on entry direction (14 in number) and exit direction (21 in number) and the weights assigned to the entry, exit and queue measures (α^e, α^x, α^q), 38 in total are chosen set of parameters. These parameters are used with the SBO framework and compared with the observed values of these LCM's. Each run of the SBO framework is performed for multiple replications to obtain 90% significance. The SPSA algorithm for optimization is performed for 50 iterations. The comparison of the observed and optimized LCM is shown in Table 6-2. The LCM in each column for the purple rows (E-ZPass lanes) and white rows (Cash lanes) add up to 1 separately.

Table 6-2 (a) Observed and Simulated LCM Based on Approach Ramp for Interchange 14A

Lane No.	From 14		From Holland Tunnel	
	Observed	**Optimized**	**Observed**	**Optimized**
1	0.50	1.000	0.05	0.000
2	0.45	0.465	0.10	0.294

3	0.35	0.326	0.20	0.184
4	0.35	0.000	0.45	0.395
5	0.25	0.000	0.50	0.605
6	0.15	0.156	0.30	0.294
7	0.00	0.053	0.40	0.228

(b) Observed and Simulated LCM Based on Exit Direction for Interchange 14A

Lane No.	To 53rd St		To NJ 440		To Ave E	
	Observed	**Optimized**	**Observed**	**Optimized**	**Observed**	**Optimized**
1	0.60	0.699	0.15	0.384	0.00	0.116
2	0.55	0.409	0.35	0.187	0.05	0.399
3	0.40	0.200	0.30	0.170	0.15	0.209
4	0.25	0.150	0.42	0.293	0.35	0.218
5	0.15	0.151	0.43	0.324	0.65	0.666
6	0.05	0.409	0.20	0.353	0.30	0.222
7	0.00	0.181	0.15	0.300	0.50	0.170

(c) Optimized entry, queue and exit weights

Entry Weight	Queue Weight	Exit Weight
0.42	0.42	0.15

The LCM's do not exactly match the observed data but from Table 1, the general trend is that drivers closer to each entry ramp tend to chosen lanes closer to that ramp. Similarly, for the exit ramp users tend to choose lanes closer to the exit direction, albeit to a lesser extent than entry. This is the same trend observed in the LCM estimated from the video data collected for the PM peak, as can be seen from Table 6-2. The correlation between the observed and simulation optimized LCM's is 0.6. The default values of weights used in the LCM model (in equation [1]), α^e, α^x, α^q are 0.4, 0.1, and 0.5. The values estimated from the SBO framework for the PM period (shown in Table 6-2(c)) are fairly close to the default values. Thus this validates the default values of the weights assumed based on the authors' judgment.

The average system travel time observed from the video data for PM period is around 25 seconds. The simulated mean system travel time is 24.9 seconds. Hence, the LCM estimated using the SBO framework closely replicated the observed mean travel time. The linear combination of travel time and number of lane changes that yields the above results is *0.5 * travel time + 8 * number of lane changes*. This measure can be used as an objective that is minimized for the toll plaza system as a whole when evaluating proposed toll plaza designs.

Clearly, there could be multiple system measures (combination of travel time and number of lane changes) that can be implemented to match the simulated and observed LCM measures from the observed data. Thus a bi-level optimization framework to establish the best system measure can be developed to address this issue.

CHAPTER 7. CONCLUSIONS AND FUTURE WORK

The predictions of a well-calibrated traffic simulation model will be robust and reliable if the predictions made for various most likely real-world conditions are accurate. Variations in traffic conditions can arise due to many factors such as time of day, weather, existence of work zones, etc. Calibration of simulation models for a realistic range of traffic conditions requires larger than traditionally used datasets capturing the stochasticity in traffic conditions. Although larger datasets provides greater variation in data, this approach poses a challenge in terms of computational effort. With the increase in number of stochastic factors, numerical methods employed for calibration of simulation models suffer from the curse of dimensionality. If, for example, traditional MC-type sampling is used, the computational effort required to simulate and calibrate traffic simulation models for various conditions could become intractable (as illustrated in Figure 4-5).

In this study, we use electronic toll collection data and sensor data for which period from January to August, 2011 to capture various traffic conditions. Also, we propose a novel calibration methodology to encapsulate stochasticity into macroscopic traffic simulation models and their calibration with much lower computational effort. We use stochastic collocation, a type of stochastic spectral method, to capture stochasticity in traffic. This method treats each stochastic factor as a separate dimension. Each dimension is discretized using a set of collocation points and an interpolant for the output is constructed using the simulation output at these points. In particular, we use the Smolyak sparse grid interpolation method due to the high number of stochastic dimensions.

The main advantages of using this methodology are the following:

1. Flexibility in applying to any type of traffic simulation (1st order, 2nd order, meso/microscopic, etc.),
2. Computationally more efficient than MC-type exhaustive sampling methods with effective interpolant,
3. Time consumed by the collocation approach can be further reduced by parallelizing the simulation under each condition,
4. Nested form of the algorithm is useful in refining the interpolant as and when there is new data available.

To demonstrate the usefulness of our methodology, we test it for an on-ramp-off-ramp section of NJTPK in the vicinity of interchange 7. The variation in supply- and demand-side parameters and inputs at this section is captured using the ETC and sensor data for every 5 minutes between January 1, 2011 and August 31, 2011. In order to calibrate the model we use the AM peak period during April and May 2011. The supply-side variation is observed to be clustered into groups. The speed data is divided into clusters using k-means algorithm into two conditions during the AM and PM peak. Due to a significant number of days falling into each cluster, (24 and 19 for AM and 20 and 13 for PM), it is likely that the variation we observed is due to long-term construction or maintenance activity. We did not observe significant variation in speeds due to weather. It is likely that the demand-side could have been impacted due to weather. Thus considering the distribution of demand during all the days encompasses the variation due to weather conditions as well.

The proposed methodology is applied to calibrate a macroscopic first order traffic simulation model for AM peak (7-9AM) and PM peak (4-6PM) for each condition/cluster. For calibrating the simulation model, we use the test statistic from the KS test for flow distributions on the link as the objective function. This objective function is minimized using the SPSA optimization algorithm (Spall (1992)). Due to the variation in speed (shown in Figure 5-2), in this case study we propose to have a stochastic fundamental diagram that has a Gaussian free flow speed distribution. Thus the mean (μ_{vf}) and standard deviation (σ_{vf}) of free flow speed form a part of the parameter set to be estimated along with critical density (ρ_{max}) and jam density (ρ_{jam}). We show that the comparison of simulated and observed flow distributions for the weekday AM and PM peak period for both conditions match well. We obtain completely different parameter sets not only for each condition but also two different parameters for different sections of the freeway section. For AM, PM peak and conditions 1 and 2 the parameter sets are, respectively, as [53 2.11 85 150, 62 1.93 82 150], [50 1.64 83 150, 70 1.58 100 150], [52 4.16 83 150, 62.2 2.85 90 150], and [48 2.26 80 150, 65 3.32 87 150]. Additionally, we notice that the stochasticity in parameters is not only limited to time but also space. We show that the proposed methodology requires much fewer replications – about 98% less – than MC-type sampling approach. Also we illustrate the advantage the proposed calibration approach by comparing simulated flow distributions generated from a model calibrated with a large set of demand and flow data and a model calibrated using limited days' data. We validate the parameters estimated using the proposed methodology by running the model for the weekday PM peak days in July. The KS test statistic obtained for the flow distributions in July are 0.084.

Speed and flow data from traffic sensors may not be the most reliable source of data. Poorly functioning sensors could lead to erroneous data or missing data. In order to mitigate this issue of limited availability of sensor data, we supplement the flow data from sensors by extracting the travel time data from ETC data. To illustrate the proposed calibration methodology using multiple data sources, we use an eight-mile section between interchange 7 and 7A for calibration using macroscopic model. We show that similar to the smaller section considered earlier we observed spatial and temporal stochasticity of parameters. However, the calibrated variance in free flow speed is higher when considering travel time as a calibration measure. The likely reason for this finding is the differences in speeds between different lanes and also differences in speeds among different vehicle types. The variance in speed over smaller sections among different lanes and/or vehicle types may not be as pronounced as in the smaller section. However, variation among different lanes and vehicle types would be more pronounced over larger section. Hence, since we use travel time, which is a measure with greater spatial extent than point flow data, the variance in speed is higher.

Calibration of microscopic simulation models involves much higher number of stochastic factors than macroscopic models. Modeling sections that involve specific geometric features can be quite complex. The complexity can be due to changing driver behavior at that particular geometry or a set of traffic control measures. So the underlying algorithms in the simulation models must be extended or overridden to incorporate this new behavior. This process requires data collection that is site-specific for different conditions. The data collection in itself can be quite cumbersome Hence extensive calibration of these models will be an extremely time-consuming task. If, for instance, the model involves a large freeway section with some specific geometric features, such as a toll plaza, then executing an iteration may be time-consuming as well. Calibrating and running such large customized simulation models in a robust manner calls for a different type of methodology.

In this dissertation, we present a simulation-based optimization framework for the calibration and design of toll plazas. In order to model the drivers' decision making at the toll plaza realistically, we use an intuitive toll plaza lane choice model. This model is validated in a previous study by the authors. The simulation-optimization approach for the calibration of a microscopic simulation model of a toll plaza is a variant of the calibration approach presented in the previous chapters of this dissertation in that it formulates a special SBO problem targeted to the calibration of a multi-lane toll plaza model. The same generic calibration methodology may not be applicable to traffic simulation models of much greater detail such as the toll plaza models. The detailed data for calibrating such models may need to be collected via video data captured at specific locations of the section to be modeled. However, such data may not always be available. Thus better means and methods to calibrate these specialized models for various conditions are required. We emphasize the importance of the development of specialized optimization formulations for different kind of simulation calibration problems. This is aimed at demonstrating the difficulties in using a generic calibration approach for all kinds of simulation models. Thus, the SBO framework provides the flexibility for

- using existing data, if available, as input to the SBO framework,
- employing unavailable or partially available data the modeler can use the unavailable inputs as calibration parameters.

Using the proposed SBO framework, we also discuss the idea of calibration with missing data in the same way we discussed in Chapter 5. Furthermore, we extend the proposed SBO framework to be combined with the computationally-efficient sparse grid stochastic collocation method to generate distribution of outputs rather than average values.

We implement this framework in MATLAB by running PARAMICS in a batch mode and modifying the lane choice measure (LCM) model parameters using the framework. We implement the SBO framework for three cases,

(a) when partial data is available and average lane usage is the output measure,

(b) when partial data is available and lane usage distribution is the output measure, and,

(c) when no data is available.

When partial data is available we use the SBO framework to calibrate the model parameters for the AM peak period for that toll plaza at interchange 14A. We use the error defined as the difference between the simulated and observed mean lane usage summed over all lanes as the objective in the minimization problem of calibration. The simulated lane usage at the toll plaza closely matches the observed lane usage. In order to illustrate the importance of calibrating the simulation model for different time period, we compare the simulation output for the AM period for the two cases (a) using the LCM estimated from PM period when video data is available, and (b) estimating the LCM using calibration. This comparison showed that the objective function for AM period when PM data is used as 0.242 and 0.15 when calibration is used. Additionally, the correlation between observed and simulated lane usage for AM period estimated using PM LCM data is, for 6-7AM is 90%, 7-8AM 95% and 8-9AM 89% as opposed to 96%, 98.3% and 98% respectively when LCM are estimated using calibration.

We extend the SBO framework by combining the sampling and interpolation approach based on the stochastic collocation (proposed in chapter 4) with the SBO framework. Using this hybrid framework, we calibrate simulation parameters to obtain distribution of output from the toll plaza model that closely follows the observed measures at the toll plaza. Regular SBO framework uses average lane usage as a measure in the calibration process. However, the calibration can be performed for many different days to obtain calibrated parameters for a much more generic data rather than average output measures. In order to achieve this objective, we use distributions of inputs for AM weekday peak period for the months of August and September 2011 and discretize the demand-side variability using the computationally-efficient sparse grid stochastic collocation methodology. The same methodology is used to interpolate and generate the distribution of output i.e. lane usage distribution given the demand distribution. The lane usage distribution is compared to the observed lane usage distribution for each lane and the KS test statistic between them is minimized in the objective function for this hybrid SBO framework. The value of the objective function, namely, sum over all lanes of KS test statistics comparing observed and simulated lane usage distribution is 0.52.

The generation of the simulated distributions required only 65 runs of the microscopic simulation model. However, the number of runs required for an MC-type sampling approach is 9,000. In terms of computational time, since each run takes about 111 s., the total time taken to generate the simulated lane usage distributions is 2 hours. The same time when using an MC-type sampling is estimated to be 162.5 hours. This reduction in computational time is a significant benefit of the proposed calibration methodology. This finding is especially important when it is applied to a microscopic simulation model that takes longer time to complete a run than macroscopic models.

When designing a toll plaza, i.e. the case when no data is available, a system-wide measure is required to characterize the toll plaza performance. Establishing a system measure would help to understand the measure that the drivers tend to optimize during their lane choice process. The ideal way to establish this measure is to mine the observed data and construct a likely measure that is minimized in the observed LCM data. This measure can be constructed using a reasonable assumption that the drivers' objective is to minimize a combination of travel times and number of lane changes. The number of lane changes is a surrogate to the drivers' consideration for safety. Thus the objective function is a weighted sum of travel time and number of lane changes. The hypothesis is that the particular weighted objective function for which the optimized LCM measures closely mimic the observed measures then it can be considered as the likely objective behind the drivers' decision making. We also use the SBO framework to validate the likely system measure behind the drivers' decision making at a toll plaza.

The LCM estimated using the SBO follows the observed trend of drivers closer to the entry ramp choosing the lanes closer to the ramp and vice versa. Additionally, the default weights assigned to entry, exit and queue, 0.4, 0.1, 0.5 are close to the optimized weights of 0.42, 0.15, 0.42 respectively. The observed mean system travel time of 25s is closely replicated by the simulated value of 24.9s. The result of the validation is that, the objective behind the drivers' decision making is *0.5 * travel time + 8 * number of lane changes*. This measure can be used as an objective that is minimized for the toll plaza system as a whole when evaluating proposed toll plaza designs.

The proposed calibration methodology could have a larger impact in microscopic simulation models. Some PARAMICS microscopic simulation models described in Ozbay et al. (2013) such as the NJTPK and Jersey City models have about 2000 links with 100,000 vehicles traveling at any given time. Running one hour of such complex models takes about 30 minutes. Calibrating these models using an MC-type sampling method is computationally impossible for obvious reasons. Thus the proposed methodology can be a very useful approach for calibrating microscopic traffic simulation models with varying complexity. This will be one of the major future directions.

The following is a summary of the future work planned based on the findings and work conducted in this dissertation:

1. The stochastic collocation framework described in the methodology section will be extended using Bayesian methodology. The sparse grid interpolation will be used in an adaptive setting using methods described in Gerstner and Griebel (2003) that can be flexible when greater number of points are required in certain

dimensions. This updated methodology will be applied to a larger scale network.

2. The stochasticity in the q-k relationship, $f(\rho)$, can also vary across lanes. This could be due to reasons such as, presence of on and off-ramps, presence of concrete barriers due to work zone conditions, etc. However, in order use different $f(\rho)$ for different lanes, the commonly-used Eulerian formulation for solving the first order macroscopic model, i.e. the cell transmission model will not be sufficient. Instead, a lagrangian formulation will be used to incorporate multi-lane variations. Lagrangian formulation has been shown to be able to model the multi-lane variations. (Laval and Leclercq (2008), van Wageningen-Kessels et al. (2011)). Applying a Lagrangian formulation can be useful in not only modeling lane changes in a better fashion but, it can also be useful in estimating larger spatial measures such as travel time more accurately.
3. Models with Limited Data - Hierarchical Models and Data Fusion using Bayesian framework: The problem of limited data availability will be simulated by not using data for some sections of the real-world data. The data requirement for the macroscopic model will be supplemented using the section flows and speeds from the microscopic model and/or real world data using a hierarchical model structure.

For extending the SBO framework for the toll plazas, there can be multiple system measures (combination of travel time and number of lane changes) that can be implemented to match the simulated and observed LCM measures from the observed data. Hence, as part of future work, we intend to frame the validation of possible objective behind the drivers' lane decision process as a bi-level optimization problem. The first level is to find the optimal combination of weights for travel time and number of lane changes representing the drivers' decision making. This combination of weights is used to frame an objective function. The second level involves minimizing this objective function using the LCM model parameters as variables. Another modification would be to relax the assumption of homogenous driver population with the same linear combination for travel time and number of lane changes, by considering multiple driver classes.

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