THE TWO FACES OF IMPULSIVITY

IN THE HYPERBOLIC DISCOUNTING FUNCTION

by

HAEWON YOON

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ABSTRACT OF THE DISSERTATION

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In decision research, hyperbolic discounting has been used for over 25 years to capture two aspects of impulsivity: 1) dynamic inconsistency--the tendency to initially prefer the long term option (e.g., to save money or exercise more) but then to switch to the short term option--and 2) level of discounting--differentiating those who wait for larger later options from those who prefer proximal options. A model simulation and an empirical experiment show that the hyperbolic discounting function does not accurately predict the relationship between dynamic inconsistency and different levels of discounting. Findings were better fit by an alternative model that incorporates subjective time sensitivity and predicts that extreme impulsiveness will lead not to dynamic inconsistency but rather to consistent preference for proximal rewards.

ACKNOWLEDGEMENT

I dedicate this thesis to my advisor Gretchen B. Chapman. Thank you, Gretchen.

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INTRODUCTION

Many people struggle with impulsive desires to spend money on a new device rather than saving for retirement, indulge in a dessert rather than sticking to a diet, or sleep in rather than hitting the gym. How do we define impulsivity? Impulsivity can manifest as a strong preference for smaller sooner options. Alternatively, it can also manifest itself as dynamic inconsistency—a preference whereby an agent initially choses to wait for the larger later option but later switches to the smaller sooner option (1), because she finds that she cannot endure the wait. In the current paper we explore the relationship between these two faces of impulsivity and the implications that relationship has for psychological models of intertemporal choice.

Arguably, the most famous model in intertemporal choice research is the hyperbolic discounting function (2). Researchers have been using this model for over 25 years because it can account for above two phenomena very well. First, the model can represent individual differences in impulsiveness using the discount rate parameter. An impatient individual who prefers smaller sooner rewards is represented with a high discount rate. Research has shown that high discount rates are correlated with smoking, obesity, and low credit scores (3-5). Second, this model can demonstrate dynamic inconsistency using two hyperbolic curves that cross each other (Fig 1B). Dynamic inconsistency has been a key focus in intertemporal choice research because it captures self-control failures in daily life, such as low savings rates or unused gym memberships (6, 7) and because it cannot be explained by the normative (exponential) discounting function (8).

However, research using the hyperbolic discounting function has studied each phenomenon separately, instead of considering these related phenomena together. The current study exams the link between these two sides of impulsivity and tests how the hyperbolic discounting function explains the relationship. We will also compare hyperbolic discounting to a recently formulated alternative model, which proposes a different relationship between these two faces of impulsivity.

Herrnstein (9) conducted landmark theoretical work on the relationship between dynamic inconsistency and the discount rate. He derived the conditions that give rise to a consistent preference for the larger later option (which we call the L-L pattern - Fig. 1A) and dynamic inconsistency (the L-S pattern– Fig. 1B) using the location of the point at which the two discounting curves cross (Dp). According to his analysis, an agent will show the L-L pattern if Dp has a negative value (Fig. 1A) and the L-S pattern if Dp has a positive value (Fig. 1B).

However, there is a third pattern the agent could show: preferring the smaller sooner option at all points in time (the S-S pattern – Fig. 1C). In this case, Dp has a value larger than the delay for the smaller sooner option (Table 1).

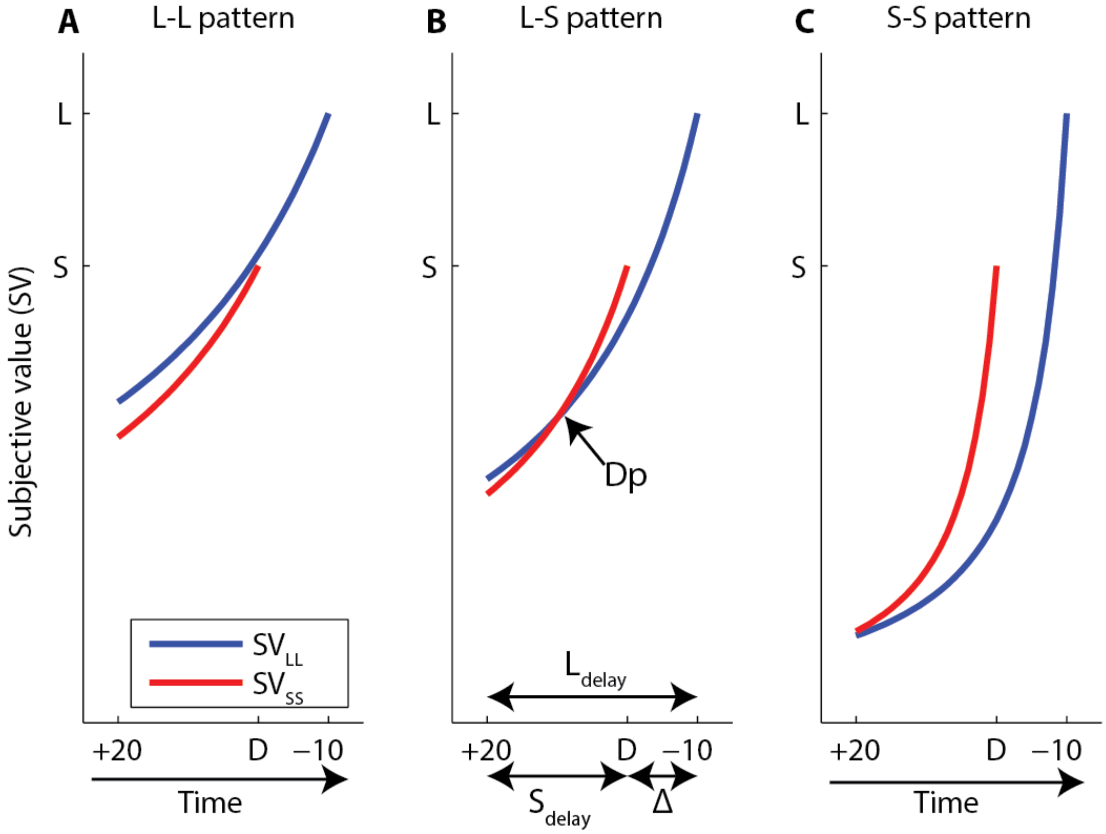

Fig. 1. Different choice patterns as a function of the hyperbolic discount rate (k). As the hyperbolic agents with different discount rates (A: k=.03, B: k=.05, and C: k=.15) move through time (from left to right) while holding constant all other parameters, the agents show the LL (A), LS (B) and SS (C) patterns depending on their discount rates. Dp for the L-L pattern (A) has a negative value and is located to the right of D. Dp for the S-S pattern (C) has a positive value (left of D and >20). SV_{LL}: subjective value of the larger later option, SV_{SS}: subjective value of smaller sooner option, S: the magnitude of the smaller sooner option, L: the magnitude of the larger later option, L_{delay}: delay for the larger later option, S_{delay}: delay for the smaller sooner option, Δ is L_{delay} - S_{delay}, and D is the point in time when S occurs.

Table 1. The relationship between three choice patterns, Dp, and S_{delay}.

Initial choice	Later choice	D_p	Choice Patterns
Larger later option	Larger later option	$D_p < 0$	L-L pattern
Larger later option	Smaller sooner option	$S_{delay} > D_p > 0$	L-S pattern*
Smaller sooner option	Smaller sooner option	$D_p > S_{delay}$	S-S pattern
Smaller sooner option	Larger later option		Improbable†

*The condition for the L-S pattern is slightly different from Herrnstein's definition (Dp>0) due to the condition for the S-S pattern, which was not examined in his paper (9).

†A preference switch from the smaller sooner option to the larger later option, also known as reverse time inconsistency (17), is improbable for convex discounting functions.

For an agent with hyperbolic discounting, Dp can be denoted as follows.

$$SV_{SS} = \frac{S}{1 + k \cdot D_p}$$

$$SV_{LL} = \frac{L}{1 + k(D_p + \Delta)}$$

Dp occurs when the subjective values (SV) of the smaller sooner option and the larger later option are equal (SV_{SS}=SV_{LL}); thus,

$$D_p = \frac{\Delta}{L/S - 1} - \frac{1}{k}$$

A combination of four parameters, three parameters from Dp (k, Δ, and L/S) and the delay for the smaller sooner option (S_{delay}), predicts whether an agent using the hyperbolic discounting function shows the L-L, L-S, or S-S pattern (Table 1). Using our newly formulated iPRP (intertemporal preference reversal prediction model) framework, we ran a simulation by testing all possible combinations of those parameters. The results characterize the relationship of the discount rate to each of the choice patterns (Appendix. A).

Figure 2 shows the simulation results using the hyperbolic discounting function. As the agent's discount rate increases, the proportion of the parameter space that yields the L-L pattern decreases and the proportion of L-S and S-S patterns increase. This makes sense because the L-L pattern will be observed when the agent can patiently wait for the larger later option, while the L-S and S-S patterns will be observed when the agent fails to do so. Oddly, however, the proportion of the S-S pattern reaches an asymptote at 50% in this analysis. This means that even if the hyperbolic agent has an extremely high discount rate, which implies a very strong preference for smaller sooner options, the agent is predicted to show the S-S pattern in only 50% of the parameter space.

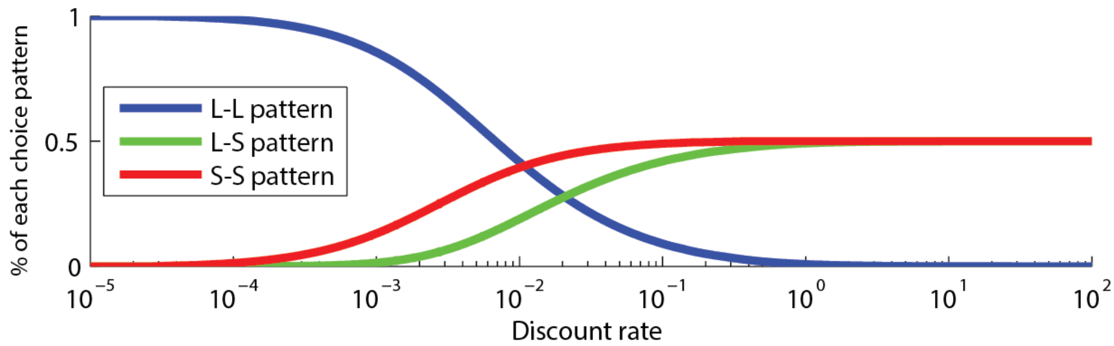

Fig. 2. Model predictions using the hyperbolic function, showing the percentage of the parameter space occupied by each choice pattern as a function of the agent's discount rate.

Herrnstein stated that the mechanism underlying dynamic inconsistency may be "a systematic psychological distortion of time perception" (10). That is, perceived or projected duration can differ from actual, objective time duration, and this discrepancy can affect choices between delayed outcomes. Although he suggested hyperbolic discounting as the underlying mechanism for subjective time perception, we consider an alternative conceptualization of subjective time preference, given that the hyperbolic model yields a prediction that does not fit the empirical data.

Recently, several alternative discounting models that explicitly represent subjective time perception have been proposed (11-13). Among them, we used the constant sensitivity (CS) model (11), which uses a minimal modification from the exponential discounting function by simply adding a time-sensitivity parameter (s) onto the delay parameter:

$$f(D) = e^{-k \cdot D^s}$$

where k represents the discount rate and *s* represents the time sensitivity parameter. When s=1, the CS model becomes the exponential discounting function.

A comparison between the hyperbolic and CS model is not straightforward because the hyperbolic discounting function has a single free parameter (k), while the CS model has two free parameters (k & s). We modified the CS model to create a single free parameter model by freezing the time sensitivity parameter at 0.5, simply using a square root function for time sensitivity. We refer to this version of the model as the simple CS model.

$$f_{simple}(D) = e^{-k \cdot \sqrt{D}}$$

In this case, Dp becomes,

$$D_p = \frac{(k^2\Delta - \ln^2(S/L))^2}{4k^2 \ln^2(S/L)}$$

Figure 3 shows the results from the iPRP simulation using the simple CS function. Unlike the simulation using the hyperbolic discounting function, the proportion of the parameter space showing the S-S pattern reaches 100% as we increase the discount rate.

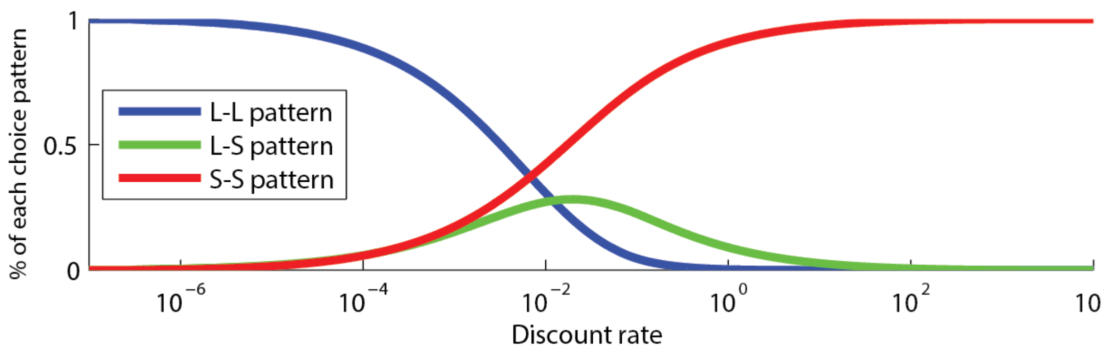

Fig. 3. Model predictions using the simple CS function, showing the percentage of the parameter space occupied by each choice pattern as a function of the agent's discount rate.

EXPERIMENT

We ran an experiment to see whether people actually show response patterns that fit this unique model prediction. We presented participants with a series of intertemporal choice questions selected based on iPRP and identified the proportion of each choice pattern for each participant.

Participants

Participants (N=100) were recruited from Amazon Mechanical Turk. 40 participants were female and the mean age was 31.7 years. 69 participants were college graduates, 11 participants had education levels above a college degree, and 20 participants had education levels below a college degree. The median income range was $35,000-$49,999. Participation was restricted to US residents aged 18 years or older.

Attention check questions

Because in an online experiment it is difficult to supervise participants' attention, we included 16 attention check questions randomly placed among the main set of questions. The check questions were choice pairs with an obvious right answer (i.e., would you prefer $7.69 in 196 days vs. $10.00 today?), and they used the same range of magnitude and delay as did the main questions so that participants would need to pay careful attention to avoid making mistakes. Prior to the experiment, we established the rule that participants who made more than two mistakes on the check questions would be removed from analysis. However, no participants made a mistake more than twice (Appendix. B).

Payment

We advertised our experiment as paying \$1.50 for a 30-minute task. Only once participants entered the experiment were they informed about the complete payment structure. In addition to the base pay of \$1.50 for entering the experiment, participants were also paid a \$1.00 bonus if they correctly answered at least 14 of the 16 attention check questions. Finally, all participants were given the outcome of one of their actual choices, randomly selected. These payments ranged from \$0.77 to \$10 with delays from 0 to 364 days. The monetary amounts were paid in the form of Amazon Digital Gift cards that were activated on the appropriate date. The average participant received \$8.36 with 162 days of delay from the random selected choice paid out for real.

Experiment

The experimental stimuli were 304 intertemporal choice questions presented in random order. These included 288 main questions and 16 attention check questions. The 288 questions formed the bottom and top “layers” of the parameter space (Figure A1 top and bottom cells). The first layer (144 questions) consisted of choices between an immediate option and a larger delayed option. The second layer (144 questions) consisted of questions where both options were delayed and were created by adding a common delay to the first layer questions. The 144 questions in each layer were formed by crossing 12 levels of Δ with 12 levels of S (L was fixed at \$10). By comparing the responses to questions from one layer with the corresponding answers from the other layer, one of the three choice patterns could be identified. Participants received payment based on their response to one randomly question.

Analysis

We used maximum likelihood estimation (MLE) to approximate each participant's discount rate. First, we created arrays that contain the proportion of each choice pattern (pSS_k, pPR_k, and pLL_k) implied by each discount rate level (k) using the hyperbolic discounting function and the simple CS function. Figures 2 and 3 show changes of those arrays as a function of discount rate. Next, each participant's (n) observed frequency of each choice pattern was counted (cSS_n, cLS_n, and cLL_n) from their experiment responses. Finally, we calculated the likelihood for different values of the discount rate using a multinomial distribution.

$$L(k \mid n) = \frac{cSS_n + cLS_n + cLL_n}{cSS_n!\, cLS_n!\, cLL_n!}\, pSS_k\, pLS_k\, pLL_k$$

For each participant, we chose a discount rate (k) that shows the maximum likelihood within the discount rate range. With the estimated discount rate, each participant's predicted and observed proportions of each choice pattern were compared.

We calculated the sum of the maximum log-likelihood (LgLk) for the model, which is the same as the formula for Bayesian Information Criteria (BIC), except for the parameter adjustment. With the negative sign, lower score implies better model fitting.

$$LgLk = -\sum \log_{10} L(k \mid n)_{\max}$$

RESULTS AND DISCUSSION

Figure 4A-C shows the predicted and observed proportions of each choice pattern for all participants. Most notably, some participants showed the S-S pattern in more than 50% of the parameter space, which hyperbolic discounting cannot predict (Fig. 4C). This incongruence between the model prediction and empirical data is not just a simple quantitative fitting issue alongside the 45 degree line. Rather, it raises qualitative questions about whether this class of model can serve as a valid descriptive theory. The hyperbolic discounting model predicts that very high discount rates do not lead to high levels of the S-S pattern.

This counterintuitive model prediction is not limited to the hyperbolic discounting function. The predecessors of the hyperbolic model, the response strength model (9) and the simple reciprocal model (14), also predict that the S-S pattern can occupy only 50% of the parameter space. This is because the formula for Dp in these two discounting functions is identical to the Dp formula for the hyperbolic discounting function if the agent's discount rate is high (Table 2). All of these models originate from the Matching Law principle introduced by Herrnstein (15). Consequently, it is not surprising that this entire family of models shares the same characteristic.

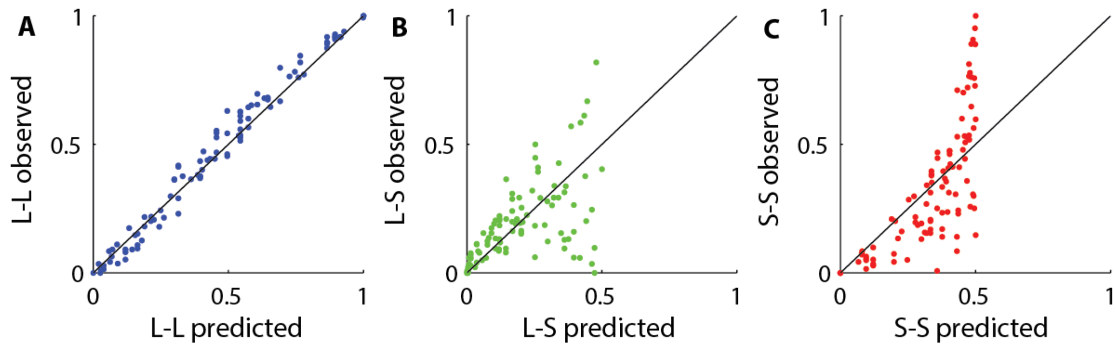

Fig. 4. Predicted-observed plot for three choice patterns using the hyperbolic discounting function (A-C).

Table 2. Similarity of the formulae for Dp in the Response strength hypothesis (9), Simple Reciprocal Model (14), and Hyperbolic discounting function (2). The three Dp equations become indistinguishable from one another when the agent's discount rate is high.

	Response strength hypothesis (9)*	Simple Reciprocal Model (14)	Hyperbolic discounting function (2)
Equation	$B_1 = \frac{eR_1}{R_1 + R_e + kD}$ $B_2 = \frac{eR_2}{R_2 + R_e + k(D+\Delta)}$	$SV_{SS} = \frac{S}{D}$ $SV_{LL} = \frac{L}{D+\Delta}$	$SV_{SS} = \frac{S}{1+k \cdot D}$ $SV_{LL} = \frac{L}{1+k \cdot (D+\Delta)}$
Dp, when $SV_{SS}=SV_{LL}$ or $B_1=B_2$	$D_p = \frac{\Delta}{R_2 / R_1 - 1} - \frac{R_e}{k}$	$D_p = \frac{\Delta}{L / S - 1}$	$D_p = \frac{\Delta}{L / S - 1} - \frac{1}{k}$

*Herrnstein's "Reinforcement 1 (R_1) or 2 (R_2)" correspond to the magnitude of the smaller sooner reward (S) and the magnitude of the larger later reward (L) and "Behavior 1 (B_1) and 2 (B_2)" correspond to the subjective values of the smaller sooner reward (SV_{SS}) and the larger later reward (SV_{LL}). Herrnstein's *e* and R_e are curve-fitting parameters (9).

Figure 5A-C shows the agreement between predicted and observed proportions of each pattern in our experiment using the simple CS model. This model showed a better fit to our experimental data compared to the hyperbolic model, especially for the S-S pattern (Fig. 5C). In addition, the sum of the maximum log-likelihood for the simple CS model (=2150) was better than for the hyperbolic model (=3217). However, the most important finding is not the better quantitative model fit, but rather the more plausible qualitative predictions from the simple CS model. This model predicts that very impatient agents who have very high discount rates will show the SS choice pattern almost exclusively, a finding that the hyperbolic model cannot account for. This is true not just for the simple CS model but also for the general CS model at all time-sensitivity levels (Fig. 6). Moreover, this model explicitly suggests the non-linearity of time perception, rather than hyperbolic discounting or the matching principle, as the cause for dynamic inconsistency. The proportion of the parameter space occupied by the L-S pattern (dynamic inconsistency) approaches zero as the time sensitivity parameter (s) approaches 1, where s=1 implies perfect agreement between subjective time perception and objective time (Fig. 6).

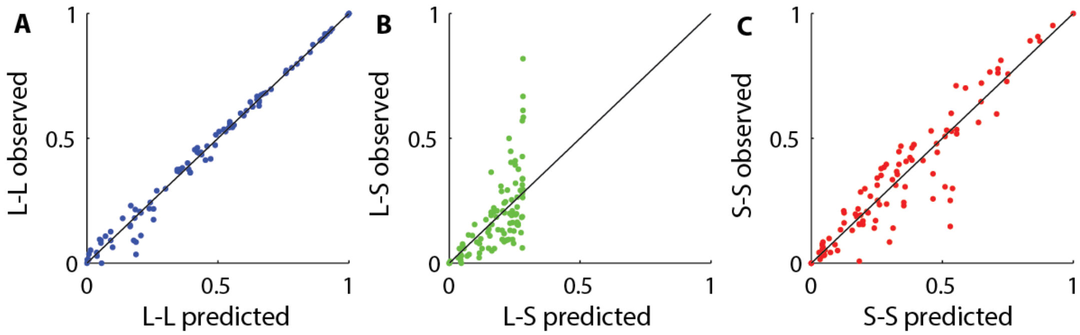

Fig 5. Predicted-observed plot for three choice patterns using the simple CS function (A-C).

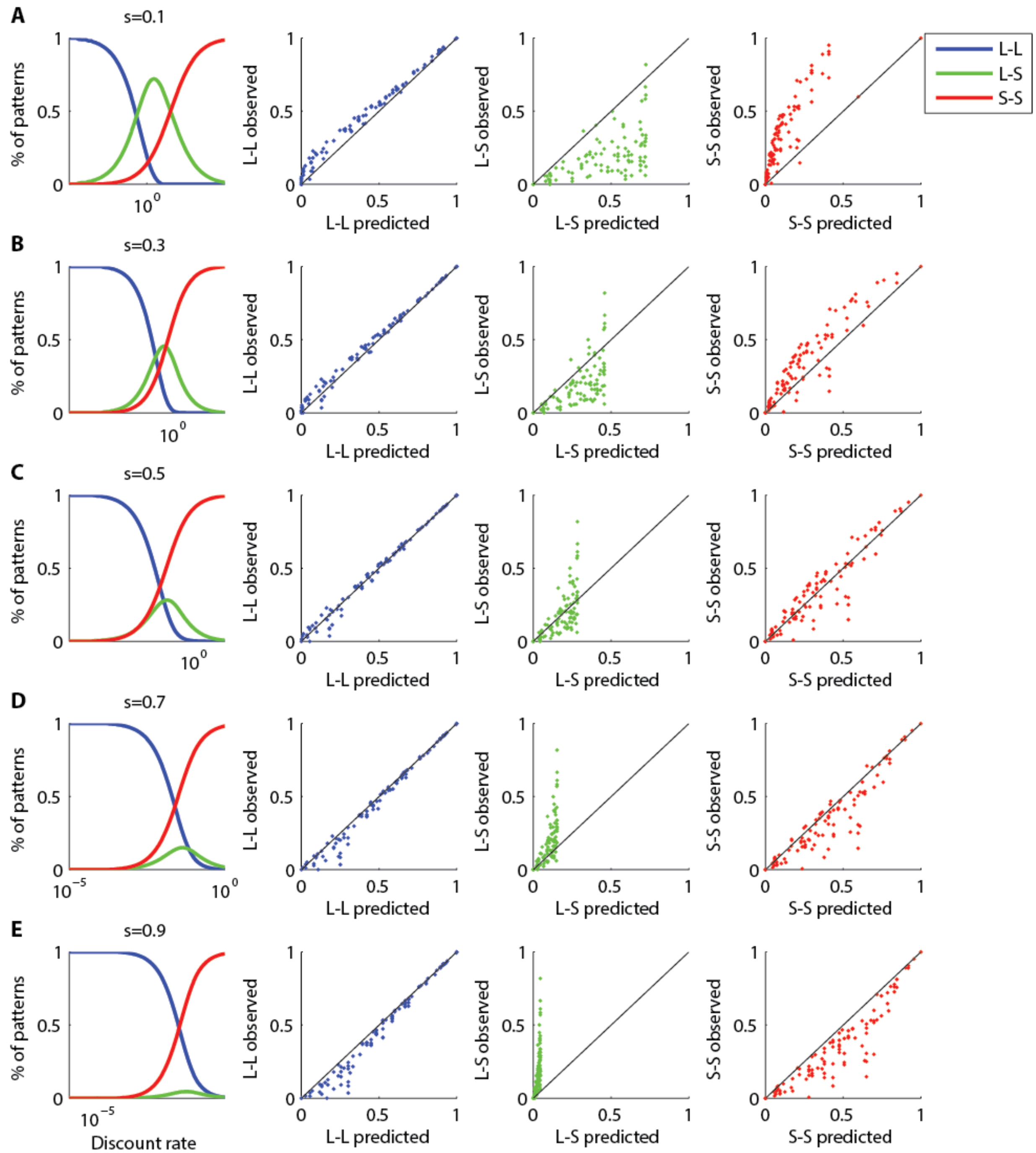

Fig 6. Predictions from the CS model with various time sensitivity levels (A: s=0.1, B: s=0.3, C: s=0.5, D: s=0.7, and E: s=0.9). The panels in the first column show model predictions, and the panels in the 2nd to 4th columns show predicted-observed plots for the three choice patterns.

CONCLUSION

In summary, we examined the relationship between the three choice patterns and discount rates, whereas previous studies have examined these two phenomena separately. The hyperbolic discounting function and related models that use the Matching Law principle yielded a counter-intuitive prediction that did not fit the results of our experiment. In contrast, theoretical predictions from the alternative model using subjective time perception was aligned both with our intuition and our empirical results. Our iPRP analysis enabled the comparison between the predictions of these two models by considering multiple parameters together and thus went beyond the insights that traditional discounting curves can provide (Fig. 7).

Our result is closely related to recent findings that the phenomenon of decreasing discount rates over a time period, which is the hallmark of hyperbolic discounting, disappears, reflecting exponential discounting, when the delay term is adjusted to account for participants' subjective time perception (13, 16). The CS model, with the exponential discounting function as a special case, can explicitly account for these empirical results.

Finally, the current study addresses the relationship between the two faces of impulsivity: dynamic inconsistency and high discount rate. According to the hyperbolic model, an agent with a high discount rate is characterized as showing frequent dynamic inconsistency. In contrast, according to the CS model, an agent with a high discount rate is characterized by showing frequent consistent preference for the smaller, sooner option. For the CS model, dynamic inconsistency is an intermediary phenomenon that appears when the discount rate neither high nor low. Our empirical data support the CS model, implying that dynamic inconsistency is not a marker of impulsivity or high discount rate. Instead, dynamic inconsistency reflects the level of time sensitivity, not discount rate (Fig 6).

Who is more impulsive: the person who plans to forgo sweets, save money, and give up cigarettes but then later falls prey to temptation, or the person who plans from the beginning to indulge? According to our analysis, the latter is the true face of impulsivity.

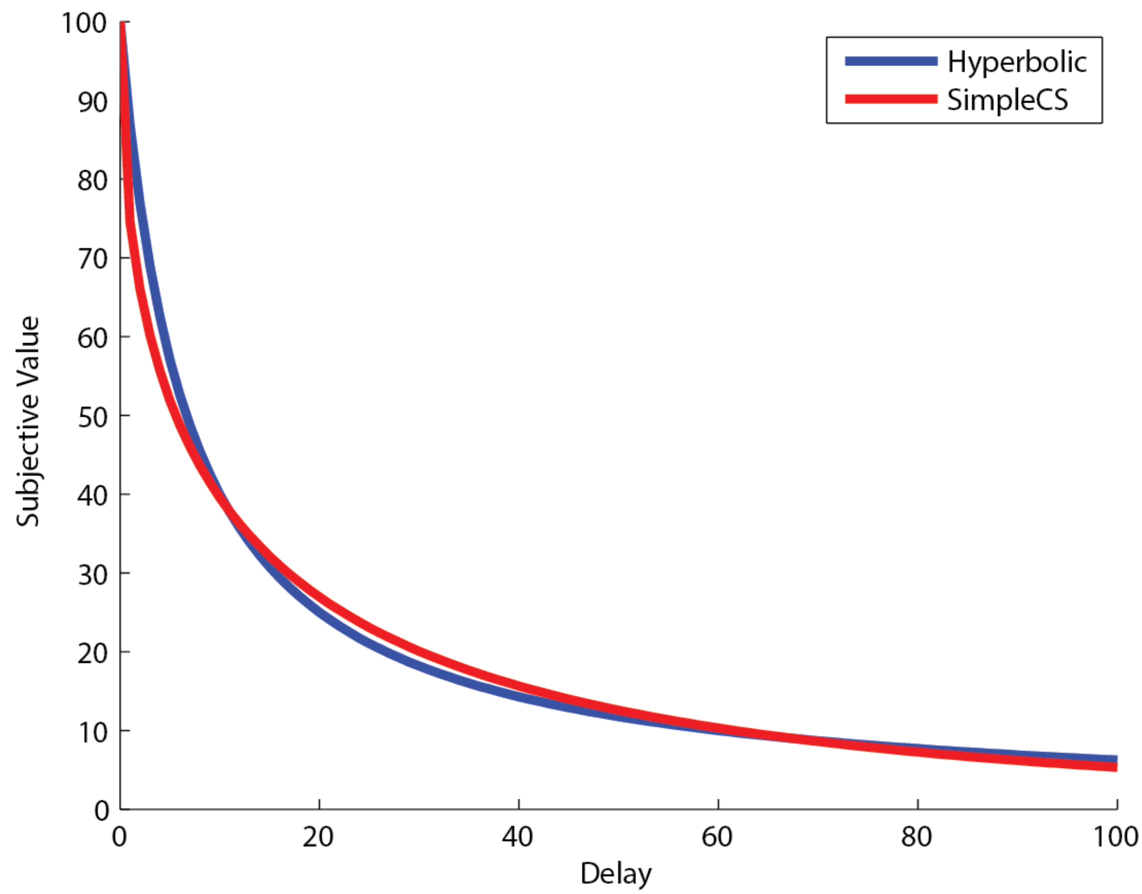

Fig 7. Discounting curves using the hyperbolic discounting function (k=1.8) and the simple CS function (k=1). These two functions make different predictions about percentage of each choice pattern (Fig 2 and Fig 3), yet they produce discounting curves that are indistinguishable from each other.

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APPENDICES

Appendix A. iPRP simulation

We defined a parameter space using four parameters (Δ, L/S, S_{delay}, and k) and examined where within that parameter space the three choice patterns (L-L, L-S, and S-S) would occur. We will review how the combination of Δ, L/S, and S_{delay} predicts a specific choice pattern under a fixed discount rate (k), and later expand to various k levels.

Let's begin defining the parameter space with L/S, the ratio between the magnitudes of the larger and smaller options. For simplicity, we will hold the L magnitude constant and consider various S levels that are smaller than L (L>S). A model simulation with varying L levels will be considered later. Also, we will hold the maximum delay for the larger later option (MLD) constant to simplify our simulation. Δ and S_{delay} together determine L_{delay} ($\Delta + S_{delay} = L_{delay}$). Thus, various combinations of Δ and S_{delay} can result in various levels of L_{delay}. Simulations with different levels of MLD will be considered later.

So far, we have held k, L, and MLD constant for simplicity. Let's start with a case where k=.1, L=$500, and MLD =100 days. Given these fixed parameters, we can create an L/S array by changing S levels (i.e., S=$1 to $499). Also, we can make S_{delay} and Δ arrays that do not exceed the MLD (i.e., S_{delay} from today to 99 days and Δ from 1 day to 100 days). With these three different arrays, we can create a 3D array that includes all those parameters (fig. A1).

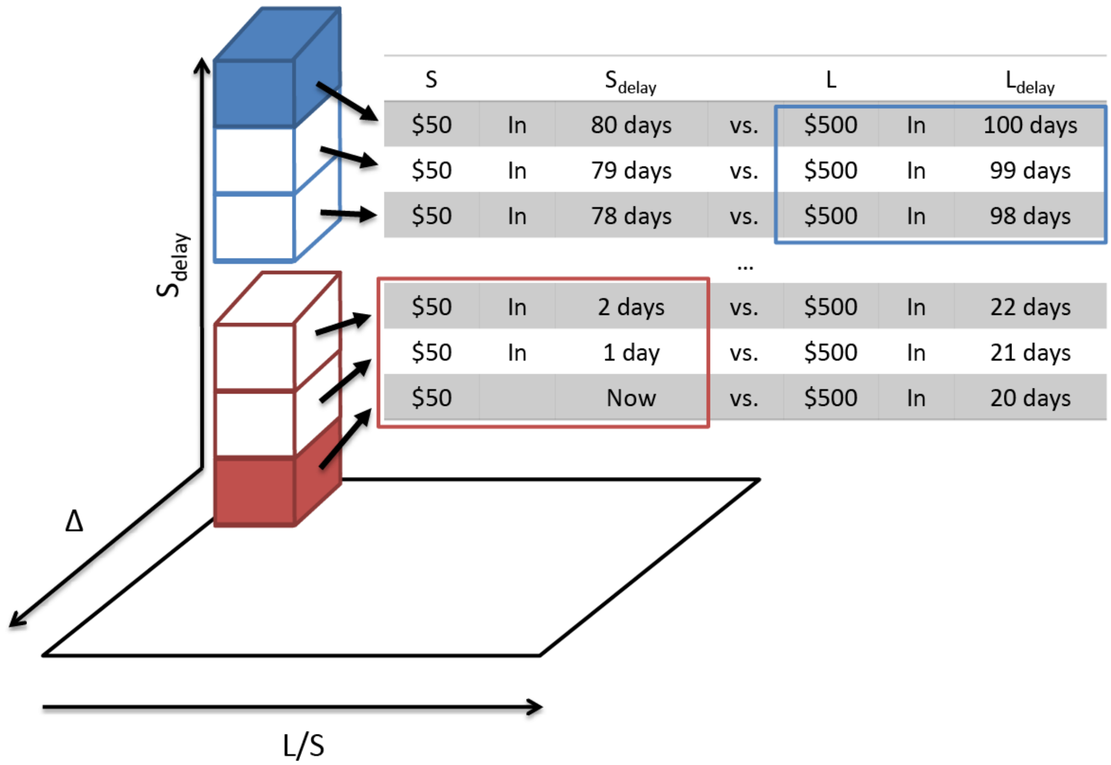

Fig A1. Parameter space in iPRP. Boxes represent series of intertemporal choice questions with S=\$50, L=\$500, Δ=20 days. S_{delay} ranges from 0 to 80 days. In this figure, the agent shows the L-S pattern by preferring larger later options for larger S_{delay} questions (blue boxes) and by preferring smaller sooner options for smaller S_{delay} questions (red boxes). The agent's choice pattern in this specific parameter set can be inferred either by comparing the value of Dp and S_{delay} (Table 1) or simply by comparing the top (the solid blue box) and bottom (the solid red box) responses.

Now, consider the specific combinations of these arrays (S=\$50, L=\$500, Δ=20 days, and S_{delay} ranging from 0 to 80 days) displayed in Figure A1. Each box (specific combination of L/S, Δ, and S_{delay}) comprises a specific choice pair. For example, a combination of S=\$50, L=\$500, Δ=20 days, S_{delay}=today implies a choice between \$50 now vs. \$500 in 20 days.

Now, consider an agent faced with a choice between \$50, which will be delivered in 80 days. and \$500, which will be delivered in 100 days. This agent will be asked the same question repeatedly over 80 days (for example, tomorrow this agent will be given a choice between \$50 in 79 days and \$500 in 99 days) until the S_{delay} becomes 0. This is the same situation as that shown in figure 1. Depending on the agent's location in time (Fig. 1), S_{delay} gradually decreases, as represented by the S_{delay} dimension in figure A1.

The agent's pattern of responses across the series of choice questions created along the S_{delay} dimension can follow one of three choice patterns. If the agent prefers the smaller sooner option in every choice in the series, we categorize that as the S-S pattern in this specific parameter set (S=\$50, L=\$500, Δ=20 days). If the agent prefers the larger later option in every choice in the series, we categorize that as the L-L pattern. If the agent changes her preference such that she prefers the larger later option in the first few choices of the series, but then switches to the smaller sooner options later in the series, that comprises the L-S pattern.

Which of these three patterns occurs is determined by the value of Dp and S_{delay} (Table 1). For example, if Dp has a negative value (-7.78) when S=\$50, L=\$500, Δ=20 days, k=0.1, and S_{delay}=5 days, it means that the agent will show the L-L pattern in this parameter set. If Dp is positive (3.33) when S=\$200, L=\$500, Δ=20 days, k=0.1, but less than S_{delay} (=5 days), it means the agent will show the L-S pattern. And finally, if Dp (=170) is greater than S_{delay} (=5 days) when S=$450, L=$500, Δ=20 days, k=0.1 then the agent will show the S-S pattern.

These three patterns can be also identified by simply comparing the responses from the top layer and the bottom layer of the parameter space. As in Figure A1, if the agent prefers the larger later option on the top layer (solid blue box) and prefers the smaller sooner option on the bottom layer (solid red box), this series of questions can be identified as the L-S pattern. By doing so, each choice pattern can be identified without using the Dp equation. This is useful when Dp is insolvable (i.e., when using the exponential discounting function).

In the current study we are interested in the three choice patterns, not the particular value of Dp. Consequently, the S_{delay} dimension can be collapsed, changing the three-dimensional array (Δ, L/S, and S_{delay}) into a two-dimensional array (Δ and L/S). We note where each of the three choice patterns will occur within that two-dimensional array (coded with blue for consistent L-L patterns, red for consistent S-S patterns, and green for L-S patterns), as shown in a single slice in figure A2. Each slice depicts how much of each choice pattern would be observed from an agent with a specific discount rate.

Now, we will vary k, L, and MLD levels one by one. Let's start with various k levels. By changing the k value used in the single slice in figure A1, many different slices can be made as we vary the k values (fig. A2).

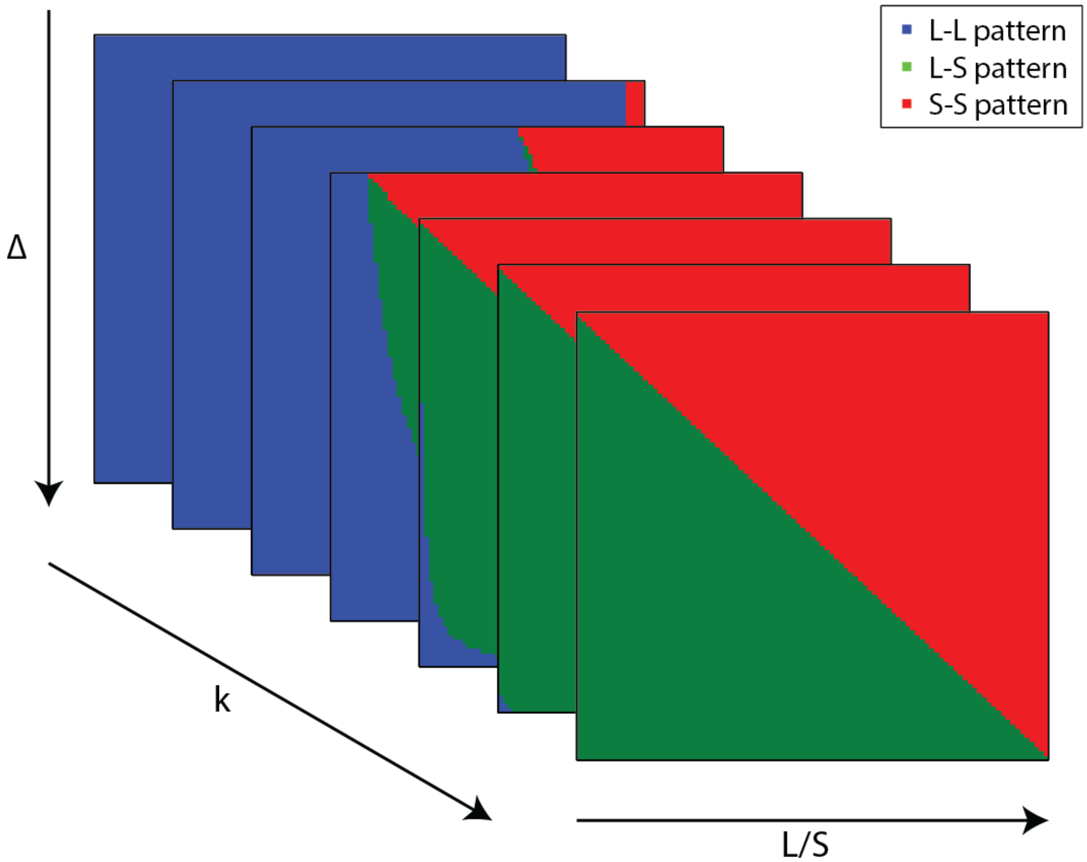

Fig A2. iPRP simulation results as a function of an individual discount rate (k). The parameter space in Figure A1 can be collapsed across the S_{delay} dimension, creating the 2D slices (defined by the Δ and L/S dimensions) shown in this figure. Blue, green, and red spaces imply the parameter combinations that yield the L-L, L-S, and S-S patterns.

This illustrates the relationship among the L/S, Δ, and k parameters and the choice patterns (L-L, L-S, and S-S) in a three-dimension array. We can take one-step further to focus just on the relationship between k and the choice patterns by collapsing the L/S and Δ dimensions, and displaying the proportion of each choice pattern in each slide (fig. 2 and fig. 3). This display provides a simple view of how the proportion of choice patterns changes as a function of discount rate, which is impossible to see from the typical discounting curves (fig. 7).

Now we will vary L levels. However, different levels of L do not have an impact on the model prediction. Figure A3 shows the model prediction with varying levels of L ($100, $1000, and $10,000), but the predictions are identical. The reason is that changing the value of L only changes the scale of the graph.

Finally, we will vary MLD levels. Different levels of MLD scale the parameter space up or down (and hence influences the model prediction), but do not yield qualitatively different predictions. Figure A4 shows the model prediction with varying levels of MLD (100 days, 1000 days, and 10000 days). Note that different levels of MLD only change the x-axis of the model predictions. Changes in the value of MLD equally influences both S_{delay} and Δ (L_{delay}=S_{delay}+ Δ, MLD is maximum value of L_{delay}). The change in Δ influences the value of Dp. For example, a larger MLD increases the magnitudes of both S_{delay} and Δ equally, but yields a smaller Dp because of the denominator. A smaller Dp yields more impatient choice patterns (more S-S or L-S patterns). This is exactly what happened in Figure A4 as we increased the MLD. Increasing MLD moved the model prediction to the left, implying more impatient choice patterns at the lower discount rate levels.

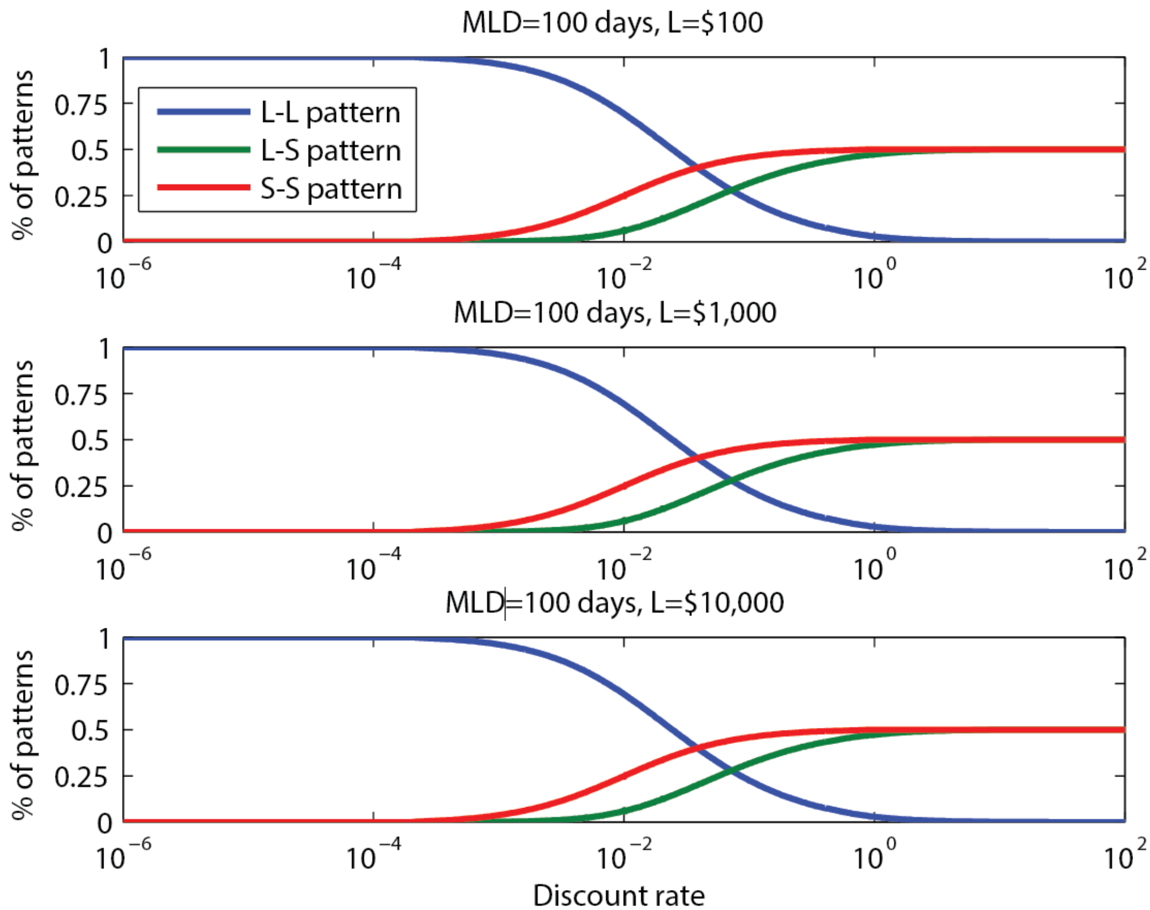

Fig A3. iPRP simulation results with varying L magnitude using the hyperbolic discounting function. L magnitude change does not affect the model prediction.

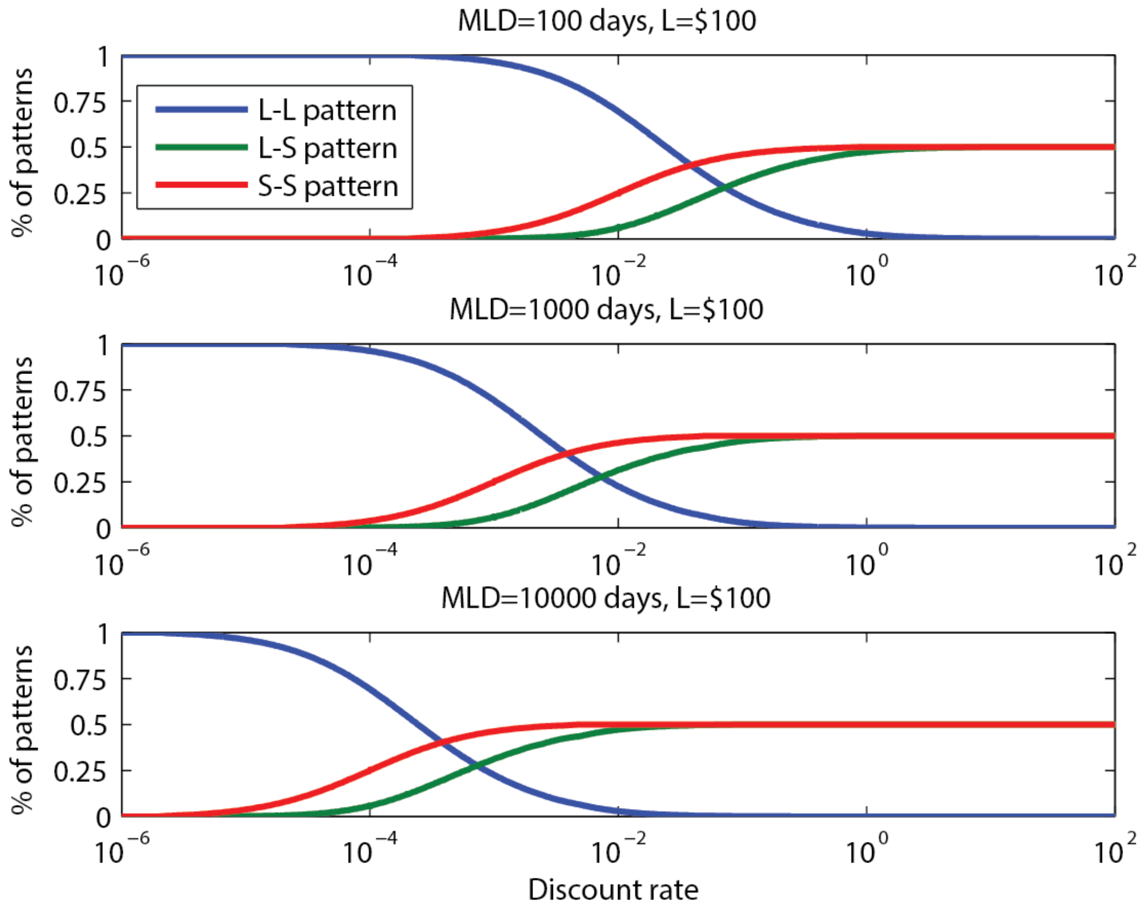

Fig A4. iPRP simulation results with varying MLD. Increasing MLD pushes the overall percentage of choice patterns to the left.

Appendix B. Attention check questions

SS options	LL options
$ 3.08 in 252 days	$ 10.00 in 252 days
$ 10.00 today	$ 10.00 in 140 days
$ 2.31 in 364 days	$ 10.00 in 224 days
$ 3.08 in 364 days	$ 10.00 in 364 days
$ 10.00 in 168 days	$ 7.69 in 364 days
$ 8.46 in 364 days	$ 10.00 in 364 days
$ 10.00 in 308 days	$ 3.85 in 364 days
$ 10.00 in 56 days	$ 1.54 in 364 days
$ 9.23 in 364 days	$ 10.00 in 336 days
$ 10.00 in 28 days	$ 10.00 in 364 days
$ 10.00 today	$ 6.15 in 56 days
$ 10.00 in 224 days	$ 10.00 in 364 days
$ 6.92 in 364 days	$ 10.00 in 364 days
$ 5.39 in 224 days	$ 10.00 today
$ 10.00 today	$ 10.00 in 196 days
$ 7.69 in 196 days	$ 10.00 today