COMPARISONS AND EXTENSIONS OF STRUCTURAL AND REDUCED FORM APPROACHES TO THE PRICING OF COMMERCIAL REAL ESTATE SECURITIES AND LOANS IN THE FINANCIAL CRISIS (2007-2010) AND THE RECOVERY (2013-2014)

by

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A Dissertation submitted to the

Graduate School-Newark

Rutgers, The State University of New Jersey

in partial fulfillment of the requirements

for the degree of

Doctor of Philosophy

Graduate Program in Management

Written under the direction of

Professor Robert A. Jarrow and Professor Douglas Jones

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Newark, New Jersey

October 2014

ABSTRACT OF THE DISSERTATION

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To date, the ~$1Trillion CMBS sector in the US does not actively utilize widely accepted advanced derivatives valuation methods. In the absence of risk neutral values for CMBS it is proposed here that risks of default were neither correctly anticipated nor priced in the Crisis (11/2007-12/2010) nor in the Recovery (1/2013-3/2014), thus far. If schisms between market and model prices enable one to secure excess returns then one may reasonably question the weak form efficiency of the CMBS sector. To investigate, I apply four model approaches (structural form, reduced form, generalization of calibrated simulation, and a special case of the generalization) in both the Crisis and the Recovery using two representative loan and bond samples on a daily basis.

The key findings are: *First,* statistical analysis demonstrates the need for the generalized approach. The special case is misspecified and inadequate to the task of modeling CMBS default risk. *Second*, although the structural form yields results in keeping with the generalization, it too is insensitive to risks associated with loan characteristics, borrower behavior, and bond

pricing. *Third*, the reduced form represents a comprehensive and better approach than all others. Building off details that characterize the generalized approach, the Cox Process of the reduced form has embedded within its design the capability to accurately evaluate complex economic relationships that govern the timing and amount of loan defaults. As the reduced form economy is robust, accurate pricing at the bond level is an immediate consequence, given accurate implementation. *Finally*, evidence indicates a sizable disconnect between fair value and market pricing with differentiation amongst the models. Trading tests and statistical analyses suggest an inefficient CMBS market evidenced by the earning of excess returns in backtesting. This dissertation provides valuable insights pertaining to CMBS risk estimation, the pricing of those risks, and CMBS market efficiency.

Acknowledgements

This dissertation would not have been possible without the guidance, friendship and generosity of many people to whom I would like to express my gratitude. I would first like to acknowledge, and thank, my committee of Robert A. Jarrow, Douglas Jones, S. Abraham Ravid and Andrzej P. Ruszczyński. Their generosity, expertise, support and patience with my development in the PhD program is forever appreciated and my work and thinking are much improved due to their guidance and interest over many years. I would also like to thank Jerome Williams and Gonçalo Filipe who provided invaluable experience in helping me design and execute my program efficiently across departments and universities. Additionally, I want to thank Joshua Barratt, Ivan Brick, Abram Connelly, Matthew Garner, Clifford Harris, Barry Schachter and Mark Zifchock, and each of their respective families, for their support of my pursuit into this line of inquiry. Finally, I want to thank my family who continue to see me through thick and thin and provide personal insights, generosity of mind, and more, that have enabled, enhanced and improved this pursuit. I want to thank my parents George and Georgia Christopoulos, my brother Alexander G. Christopoulos, Jim and Sondra Farganis and the Farganis Family, and Frieda Halivopoulos and the Halivopoulos Family.

This path evolves thanks to *all* of you.

Preface

The work contemplated in this dissertation finds it locus in the mid-1990s. At that time, quantitative approaches to the evaluation of mortgage backed securities (MBS) were just beginning. Prepayment models and related option adjusted spread valuation (OAS) techniques were just beginning to be implemented. In that period in the market, however, credit risk evaluation was not actively being contemplated in MBS. In part this was due to the fact that, in the securitized field, the residential market was dominated by conforming government guaranteed loans which necessarily dispensed with prospective losses as a risk to the bondholders. Credit driven 'prepayments' were an anomaly, noise in the context of a well-constructed rate x coupon dominated methodology. As the market evolved post-RTC, loans not guaranteed by the government that faced substantial credit risk began to represent a larger portion of the market overall. The threats of default and loss were argued to be relatively minor inconveniences resulting in prepayment 'speeds' and OAS pricing to be 'off a little bit'. Arguably a more difficult risk to model than, say, prepayments, default risk was not actively pursued.

Modeling credit risk eventually did move to center stage and its developments contributed to the genesis of the credit derivatives market that evolved in the wake of the dot-com bubble and LTCM crisis towards the end of the 20th century. Despite these developments in non-securitized and securitized markets, however, credit risk evaluation and risk neutral pricing within the commercial mortgage backed securities (CMBS) market remained unmoved.

It is my view, based upon direct experience and from several years of formal study, that there continues to be a bias against the use robust derivative pricing technology in the securitized market for commercial real estate loans (CRELs). This in part is due to the complexity of the exercise involved in the modeling of default risk at the loan level and the subsequent pricing exercise required at the bond level. There may also be other reasons for this perceived bias including, but not limited to: small sample sizes relative to loans in RMBS, greater heterogeneity amongst the CMBS loan collateral, more varied idiosyncratic borrower behavior, the historical evolution of the commercial real estate market in the US, the persistent use of cap-rate methods to estimate property values, and a resistance to acknowledging the influence of capital markets on property valuation indirectly through CMBS pricing, among others.

Admittedly, the relationships are complex to model and often difficult to explain. However, technology and theory in the academy are fusing well now to provide us with the ability to approach previously intractable problems rigorously. This dissertation thus seeks in earnest to demystify and formally explain the relationship between risks facing holders of commercial real estate debt and the associated risk adjusted loan level valuation embedded within

CMBS bond pricing. It continues to be my hope that this effort, and others like it, will bring greater insight to academics and practitioners interested in this important area of capital markets and will contribute to the thinking on lending/borrowing, trading/investments and regulation. At its core, this dissertation is about evaluating risk and reward and I hope you find my investigation into the risk estimation, pricing, and efficiency of the CMBS market convincing.

- *Andreas D. Christopoulos*
  July 25, 2014
  Ithaca, NY and Newark, NJ

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Introduction

Commercial mortgage backed securities (hereafter, "CMBS") represent a nearly $1 trillion component of the US economy. CMBS[1] are derivatives collateralized by commercial real estate mortgage loans (hereafter, "CRELs") which are typically 1st lien debt instruments secured by commercial real estate property. Securitized CRELs underlying CMBS debt represent 35-40% of all CRELs outstanding in the US. Despite the important presence of CMBS within capital markets, to date widely accepted derivatives pricing and valuation methods are not actively utilized by CMBS practitioners. By not utilizing advanced derivatives pricing methods, it is my view that CMBS market practitioners are not correctly evaluating the risks of default, loss and concomitant adjustments to the timing of cashflows in the event of default for underlying CREL objects and CMBS bond objects. If this is in fact the case then, when considering market prices of CMBS and related securities, I should see schisms between market prices and *alternative* pricing of the same securities generated under risk neutral conditions (the *model prices*).

This by itself is not necessarily surprising as all financial models are approximations to more complex realities. Nevertheless, even if we consider the weak form of market efficiency (see Fama, 1970), then in an efficient market excess returns cannot be earned in the long run by using

[1] A summary primer of CMBS is provided in Appendix A.

investment strategies based on historical prices. Rather, future price movements should be determined entirely by information *not* contained in the price series itself and market participants should *not* be able to systematically profit from market 'inefficiencies'. In the case of pricing derivatives with default exposure, it should then follow that if we observe (with the benefit of model pricing), a set of signals that enable us to profit from schisms between market price and model price, then the market we are investigating, (in this case the CMBS sector), might possibly be inefficient; for if the CMBS sector were efficient, then no systematic profits should be able to be secured, consistent with Fama's theory. This investigation, identification, and reconciliation are among the main focuses of this dissertation.

To do this, I consider three primary model approaches to the evaluation of risks associated with CRELs that impact the pricing of CMBS and the index swap collateralized by CMBS tranches, CMBX (described in detail in Section 1). In total there are four implemented models discussed with important differences between them. The evolution of the model technology spans 40 years of financial theory and this dissertation provides a comprehensive empirical testing of this theory. Throughout the paper, the analysis of loans and bond pricing and risk values is daily based upon evaluation of loan and bond objects which have monthly payment frequencies.

Two important periods in the history of finance are investigated. The initial period covered is November 2007 thru December 2010 (the *Crisis*). The subsequent period studied is December 2012 thru March 2014 (the *Recovery*). In the Crisis I consider 1 transaction with 175 loans underlying CMBX Series 1 totaling ~$4B. In the Recovery I consider 688 loans totaling ~$13B across 11 transactions underlying CMBX Series 6. The inquiry into the Crisis and the Recovery use the same four model approaches in an effort to determine which one provides the most reliable signals of risk and opportunity and, further, to question assumptions about the efficiency of the CMBS sector overall.

The four model approaches considered rigorously are adaptations of Merton's "On the Pricing of Corporate Debt" (Merton, 1974; the *structural* model; Model 1); a calibrated structural model where the calibrated parameters are used in a simulation technology as introduced by Driessen/Van-Hemert in "Pricing of commercial real estate securities during the 2007-2009 financial crisis" (DVH, 2012; Model 2); a thorough generalization of which DVH, 2012 is a special case (Model 3); and an adaptation of the approach introduced by Jarrow (with Christopoulos and Yildirim) in "Commercial Mortgage Backed Securities (CMBS) and Market Efficiency with Respect to Costly Information" (Jarrow, etal 2008; the *reduced form* model; Model 4). Additionally, in

Appendix B I review other quantitative approaches taken in the literature related to CMBS valuation in Eom, Helwege and Huang, 2004; and Kau, Keenan and Yildirim, 2009 among others.

Model 1, the adaptation of Merton, 1974 to CMBS valuation is critical as it represents the classic foundation for risk neutral valuation of debt instruments. The adaptation I propose uses several new innovations necessary to accurately accommodate complexities related to loan level collateral and bond level priced objects. Model 3, implements a generalized approach to calibrated simulation that builds off Merton, 1974 and which specifically considers the heterogeneity of loan characteristics by accurately incorporating correct cashflows and ruthless default. The generalization is important because it incorporates many realistic features of the loan building blocks of CMBS in a well specified simulated economy. In my study, I secure new insights into CMBS market efficiency and new results that contrast with Model 2, proposed by DVH, 2012 which is incorporated in this dissertation as a special case of the generalized approach. We see, in so doing, that the generalized approach (Model 3) provides a more precise perspective on CMBS efficiency than Model 2. Finally, I turn to Model 4, the adaptation of the reduced form approach introduced by Jarrow, etal 2008. The first three models indicate that quantitative methods applied to CMBS valuation vary in precision due in part to limiting assumptions and restrictions in implementation at both the loan and bond levels. With a robust simulated economy and historically validated default 'triggers' the reduced form further eliminates many unnecessary simplifications. The insights garnered from the different model approaches indicate that assumptions regarding the loan objects given their heterogeneity with respect to timing and amount, clearly need to be eradicated as shown in this study. In the case of the reduced form, broader issues pertaining to the realism of the simulated economy and its interaction with loan objects and the default decision-making behavior of loan borrowers matter further still. The results demonstrated by Model 4 demonstrate increased precision with respect to risk navigation.

The key findings of this dissertation are as follows. *First*, the implementation of the model approaches indicate that the accurate capture of the amounts of principal and interest cashflow payments on both a promised and default adjusted basis is *essential* to any rigorous analysis of risk and pricing of CMBS/CMBX. In their absence erroneous signals as to the risk profile of the securities can occur. Eradication of simplifying assumptions, while difficult, does yield improvements in identifying risks, enriching the simulated values, and capturing key loan characteristics of the objects underlying the derivatives. I consider all the cashflows of all the loans in great detail, and are thus able to make fair value estimates across simulation with default adjusted cashflows. As a result, this study across four models makes more meaningful statements regarding the efficiency of the CMBS market overall than many other studies in the literature.

Second, the market seems to contemplate 'ruthless default' in the expectations process through which prices are arrived at, where ruthless default is defined as the occurrence of default immediately when the borrower has an economic incentive to do so. In the context of Merton, 1974 and DVH, 2012 default is contemplated to only occur at the maturity date of the debt when the equity position in the company drops to zero such that the firm value under such circumstances is entirely debt. In the context of commercial real estate properties if the value of the property declines for a variety of reasons such that a sale of the property at the implied value of the property would be insufficient to pay off the debt obligation/mortgage secured by the property, the borrower then has an incentive to default *immediately* on the mortgage. This may or may not occur in reality and I present arguments for and against implementing this behavior in the modeling.

Third, calibrated simulation approaches benefit greatly from ex-post statistical analysis using publicly available market information. Ex-post statistical analyses improve R-sq to the 0.80-0.90 level across all CMBX classes over 792 trading days in the Crisis with significance across all explanatory variables during the Crisis versus low raw ex-ante ranges of 0.17 to 0.70. An example demonstrating, in the most credit-risk sensitive portion of the capital structure, the increase in precision to be secured from ex-post analyses including variables exogenous to Model 2 can be found in the estimates for the BBB- tranche shown Figure 1 which shows a very good fit[2].

Fourth, there is considerable evidence in both the Crisis and the Recovery of the ability to earn *extraordinary profits* in the CMBS sector through the use of various model approaches that I implement. To test for CMBS efficiency, I backtested models against market pricing using the metric Theta as a barometer of the relative riskiness of observed prices in the market compared with the theoretical risk neutral pricing adjusted for default and loss risks, and implemented a series of long/short and long only trading strategies (described in detail in Section 3). For all historical trading dates, τ , for each time t, on each simulation path l, for a given bond tranche k, Theta, $\theta_k(t,l)$, is defined as the difference between the Fair Value or risk neutral estimate of the bond price and the market price of the bond, $\theta_k\left(t,l\right) \equiv b_k\left(t,l\right) - m_k(t)$. The composite history for fair value, $b(t,l)$, is weighted by the tranche balance for the i-th tranche at time t across k tranches and l simulations. The composite history of the market price, $m(t)$, is also tranche balance

[2] One of the critical questions we have to ask then is 'If the CMBS sector is truly inefficient, what is the purpose of seeking to map model to market price?' In one sense, the use may only be found in disclosing an *absence* of explanatory power. This suggests inquiries into tracking the effectiveness of model driven signals through backtesting trading strategies hold greater promise in arguments related to efficiency than fit.

weighted[3]. The composite Theta across all k bond tranches is depicted in Figure 2a over three years of daily pricing through the crisis[4]. The results show clear differentiation amongst the models perception of default risk at the composite levels. In backtesting (Figure 2b), the 'good' models categorically outperform the long-only CMBS sector benchmark and, during the Crisis, also directly outperform the market portfolio. The model approaches that incorporate accurate cashflows in the simulation and valuation exercise (Models 1, 3 and 4) are grouped generally above the x-axis indicating that on a composite basis, the bond pricing in the market place was relatively inexpensive vs. the risks as contemplated in such model approaches. In contrast, the approach of Model 2 that does not incorporate either a.) accurate cashflows or b.) ruthless default, shows a markedly different profile indicating, consistent with the claims of DVH, 2012 and others that CMBS bonds during the financial crisis were relatively expensive to fairly valued vs. their underlying collateral risks and were *not* sold at fire sale prices.

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[3] The composite history for fair value, $b(t,l)$, is weighted by the tranche balance for the i-th tranche at time = t, $w_i(t)$, or $b(t,l) = \sum_{i=1}^{N} b_i(t,l) w_i(t)$ across N tranches and j simulations. The composite history of the market price, $m(t)$, is also tranche balance weighted, $m(t) = \sum_{i=1}^{N} m_i(t) w_i(t)$.

[4] All models shown in Figure 2a contemplate a.) six types of commercial properties (Multifamily (MF), Retail (RT), Office (OF), Industrial (IN), Hotel/Lodging (LO), and Other (OT), and b.) the accurate maturity date for each of the 172 loans collateralizing the Greenwich Capital Commercial Funding Corp. Commercial Mortgage Trust, Series 2005-GG5 ("GCCFC 2005-GG5", or just "GG5").

Based upon the evidence provided in this study, the three alternative approaches make the strong case that the *opposite* conclusion is true. Namely, relative to the risks of the underlying loan objects, that there were many instances of bonds in the CMBS market that *were* sold at fire sale prices during the financial crisis. As noted in Jarrow, etal 2008 the CMBS market is characterized by the absence of the use of advanced derivatives pricing. Therefore, in the absence of such technology practitioners were exposed to ad-hoc pricing of complex risks in a truly difficult time in the financial market. It is thus not surprising to see, as shown in three of the models, persistent periods in the financial crisis when CMBS were priced 'cheaper' relative to their underlying risks.

"How much cheaper?" In part the answer to that question depends on the model chosen. However, making such choices should not be arbitrary. Thus, *finally*, as initially suggested by the need for ex-post statistical adjustment with exogenous variables in the calibrated approaches, the Model 4 reduced form approach yields better results than any of the other approaches considered. The reduced form actively considers in its structure both static and dynamic information sets within the Cox Process that interact with many of the characteristics of loan and marketplace dynamics more precisely and realistically than any of the Model 1, 2, or 3 approaches. As the reduced form Model 4 is, through careful construction, inherently more sensitive to the risks of the loan objects, the valuation of such loans and the corollary bond capital structure is necessarily more precise. This is evident in the historical analysis. Figure 2b shows the cumulative portfolio returns for the main model approaches investigated on a daily basis through the Crisis using long/short strategies informed by Theta as compared to the long-only, buy and hold, and purely random strategies. Of the model approaches I investigate, and for the reasons I discuss below, the reduced form approach (Model 4) is the most accurate and reliable indicator of risk and reward in CMBS.

In the literature, this dissertation represents a thorough assessment of 40 years of financial theory applied for the first time to CMBS and its derivatives in one paper. It demonstrates an important alternative to real estate economics approaches that focus on cap rate deltas and real estate cashflow analysis (see Conner 2003, Corcoran 2004, Peyton 2009 and others, see Appendix B). These considerations are not necessary[5]. Nevertheless, elements can be helpful at the data level. In fact, in several of the approaches in this dissertation, certain real estate information related to the property value and loan level characteristics are incorporated to estimate values of securities based on loan objects. The approaches in this dissertation utilize important real estate loan object characteristics and place them within the correct derivatives pricing context. The technology

[5] In Appendix C, I provide a working paper study covering 22 years and 2 distinct real estate cycles, I estimated, completely independently from property net operating income and estimated caprates the property value $\hat{V}(t) = b + \sum_k a_k x_k (t - t_k)$ with $\hat{V}(t)$ as the synthetic value for NCREIF at the national level in the OLS. The regression demonstrates.94 R-SQ at the national level simulating NCREIF from i.) Unemployment, ii.) Case-Shiller Housing Index, iii.) Credit Risk Slope, iv.) Mortgage Rates, v.) RiskFree Slope, vi.) CRE Charge-Off Rate, and vii.) Percent of Private CRE Construction. Thereby demonstrating a macro driven property value estimator distinct from traditional real estate economics property specific methodology.

developed allows for evaluation of any CMBS transaction, given data. In light of the financial crisis, it is evident that much can be achieved with respect to risk transparency and cost savings by actively considering alternative approaches such as those proposed herein. The analysis in this dissertation makes evident that not all models are the same, and thus, it is hoped that this and other work stand as testimony in favor of securing rigorous insights into a complex product type that results in more accurate signals of risk and reward for CMBS and better decision-making by practitioners.

The contribution of this dissertation to the literature then is simple: a.) Demonstrate increased realism in modeling risks facing holders of commercial real estate securities by being thorough and attentive to critical real estate loan characteristics and capturing the heterogeneity of the collateral within a more complex economy. Implementing various models that estimate and price default behavior; and b.) Disclose, through high performance simulation and statistical analysis, the efficiency (or lack thereof) of CMBS and the concomitant CREL risks with rigorous theory, tested empirically both within the worst financial Crisis since the Great Depression and the aftermath economic Recovery period. In so doing, this dissertation makes a contribution in support of increased modeling precision using quantitative approaches to evaluate CMBS risks and market efficiency.

The remainder of this dissertation is organized as follows. Section 1 focuses on the Crisis (2007-2010). An overview of the data utilized is provided on a section by section basis. Next, I introduce formally each of the four models implemented for CMBS. To clarify they are, again:

- Model 1: Structural Form (Merton, 1974)
- Model 2: Special Case of Generalized Calibrated Approach (DVH, 2012)
- Model 3: The Generalized Calibrated Approach
- Model 4: Reduced Form (Jarrow, etal 2008)

An in depth statistical evaluation of Model 2 is provided which discloses model misspecification. This analysis prompts the investigation into other models in more depth. I then conclude with comparison of all four models on a daily basis using Theta and query the purpose of 'fitting' within the context of market efficiency inquiry.

Section 2 focuses on the Recovery (2013-2014) following the Crisis. In this section I apply the same model techniques to a new set of loans and bond objects, again on a daily basis.

The comparisons of Theta then invite rigorous analysis in Section 3 which is focused on the statistical analysis of market efficiency of the CMBS sector in both the Crisis and the Recovery. In this section I introduce the trading tests and consider a few statistical evaluations including the intertemporal CAPM to test for the efficiency of the CMBS sector which is called into question.

Finally, in the Conclusion, I provide some further observation on the results, offer suggestions for further extensions to the historical periods studied, and provide supplemental information in supporting Appendices that include proofs and important summaries.

Section 1: The Financial Crisis (2007 thru 2010)

1.1 Data - Crisis: The CMBX pricing data (11/8/2007 – 12/31/2010, daily) was provided through Markit. CMBX is the name of a family of indexed swap derivatives for which the underlying collateral are commercial mortgage backed securities (CMBS). The CMBX data for this study was secured from Markit. There are currently 6 series of CMBX issues outstanding. Each series (1,2…6) is associated with a unique set of 25 CMBS transactions referred to as reference transactions. The CMBX series are partitioned into tranches (AAA, AJ, AM, AA, A, BBB, BBB- and BB) that are secured by the corresponding CMBS tranches, which are the reference assets for the CMBX. The reference assets are in turn backed by loan cashflows according to the tranche cashflow allocation structure from the reference deal. So, for example CMBX.AAA.5 is secured by 25 AAA tranches, 1 from each of the reference transactions and each comprising a weighting of 4% to the CMBX derivative price. Each of the reference transactions is secured by cashflows from hundreds of mortgages in the reference transaction trust that are secured by commercial real estate properties. Across all reference transactions for a given series, the risks of thousands of loans and (indirectly pricing/valuation risks) are represented in the CMBX pricing. The loans underlying the reference transactions are characterized by diversification across all major property types and property submarkets in the US. The reference assets are priced daily by dealers and these prices are then submitted to Markit which, in turn, aggregates the prices from the multiple firms into a single price published at the end of the trading day (4:15PM EST). The 125 CMBS reference transactions and corresponding reference assets amount to hundreds of billions of dollars currently outstanding.

The purpose of the creation the CMBX family of indices was to provide dealers and non-dealer investors with the ability to readily hedge the credit risk exposure associated with CMBS held in portfolios. Investors can hedge the risk free interest rate risk component of CMBS risk premia through Treasuries and futures. With the introduction of CMBX they also were able to hedge the volatility of CMBS specific credit risk premia with varying measures of effectiveness depending on the correlation between the volatility of the risk premia of CMBX compared with that of the actual CMBS underlying the CMBX as securing collateral. As with other credit default swaps, CMBX are essentially a derivative contract between two counterparties. Figure 3 shows the buyer of protection hedge diagram where the investor, X, with exposure to Y cashflows and mark to market risk on the swap pays a fixed coupon to a counterparty Z, and 'receives' or is marked to market on floating spread basis. Quotes and bid/ask spreads change daily and intraday.

In this first study concentrating on the Crisis, I use pricing for CMBX Series 1 and the collateral characteristics from many loans underlying Series 1. The testing is conducted on the 172 loans totaling $4.405 billion that serve as collateral for the Greenwich Capital Commercial Funding Corp Series 2005-GG5 CMBS transaction issued in November 2005 (GG5) within CMBX Series 1. For GG5, a single month of updated loan data was made available for April 2010 from the Trepp Loan file. Additionally, the prospectus supplement and Moody's Pre-Sale Report were used to adjust the data for proforma cashflow origination profiles and the presence of junior (second liens). The economic data used throughout is data provided from ACLI, NCREIF, the Federal Reserve, and CohenSteers/Bloomberg. The maximum likelihood estimates (hereafter, "MLEs") are from Jarrow, etal 2008 are used for the Reduced Form adaptation in this dissertation for the delinquency/current intensity process and the default intensity process. The discussion of the method of the delinquency and default intensity process which considered more than 2.2mm loan life observations can be found in Appendix D. Figure 4 provides a snapshot of the parameter estimates used for multifamily properties located in the Northeast. Ideally, these MLE's would be updated with data from Trepp but that is not currently possible. Nevertheless, because the estimates cover substantial historical relationships between the economy x loan characteristics x event history they seem to perform well.

Independent prices of the loans are not observable after origination except in the case of auction (FDIC). In contrast bonds in the secondary market (and auction) are observable. Since the value of interest here is are bonds the aggregation of the simulated loan level cashflows gets distributed to the bond capital structure in the cashflow algorithms discussed for each Model.

For CMBS loan level information and delinquency status I used information provided to members of the Commercial Real Estate Finance Council. Interest rates were provided through the Federal Reserve Board. REIT prices were provided from Yahoo! Finance. REIT debt levels and 90 day volatilities for REITs and the S&P500 were provided by WRDS. As the data are used differently in each model considered, I will provide summary information in the sub-sections where appropriate.

1.2. Model 1: Structural Form - Merton, 1974: Merton cannot be used to directly evaluate CRELs and CMBS – extensions and adaptation in the form of changes to the assumptions and calibration are required. However, these changes are in fact minor. If carefully constructed, an adaptation of Merton provides a powerful set of insights to the task of CMBS valuation.

The value of a corporation can be characterized by the equation $V = D + E$ (see Brealey, Myers, Allen 2011), where V = the total value of the corporation, D = the value of the debt of the corporation and E = the value of the equity of the corporation. Merton extended this fundamental understanding into the well-known option framework[6].

Since the debt secured by commercial properties typically is non-recourse to the borrowing entity, and typically with a balloon amortization structures (with little, if any principal payments, prior to balloon, the structure of the commercial property is directly comparable to a small corporation (see Jarrow, etal 2008) with a bullet debt obligation. As such, one approach to the valuation of commercial real estate would be to apply Merton.

A defining characteristic of commercial real estate is that it is income producing property (1993 Brueggeman, Fisher). A commercial property is both a physical plant and a business that generates income. The non-recourse provisions and balloon profile of the debt make such a simplified framework reasonable by substituting *the company* contemplated under Merton with *the CRE property*. If the debt secured by the property exceeds the property value at maturity date of the debt, the borrower will default at maturity. Otherwise, the borrower will pay off the debt. Thus, in this basic sense, the Merton framework is consistent, though a simplification of, the profile of a

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[6] The discussion and review of Merton and a proof of Black Scholes Merton is found in Appendix E.

typical CREL and the decision-making process related to debt valuation and default. There are a few differences from the typical restrictions facing initial Merton that have to be addressed.

I must adjust for the fact that CRELs are coupon bearing, possibly amortizing and possibly ballooning[7]. Retaining the maturity default restriction[8], a very simple extension to Merton would change the coupon bearing fact such that the strike at maturity reflects fixed coupons, so

$$D = D_0 e^{coupon*T} \tag{1}$$

However, this change would not adequately address the differences in interest payments associated with ballooning cashflow structures in which interest payments on debt may be made according to say a 30 year regular amortization for 119 months, and then in the 120th month the entire principal payment is due with no interest (or principal) payments after balloon maturity T. The equation would overstate the amount of interest typically paid under a balloon amortizing loan structure, and thus influence the size of D which would influence the closed form solution of Merton applied to

[7] Balloon and amortizing cashflow examples generated from the code are provided in Appendix F.

[8] As supported by several studies including, Jarrow, etal 2008, KKY, 2009, and others, loans frequently default prior to maturity (term defaults) due to failures to meet growth rental targets, failed rent roll re-leasing, declines or flat rents and other reasons. While ruthless default is not part of Model 1 by construction or Model 2 by assumption, but it has been implemented in Model 3 and Model 4.

CRELs. Fortunately, I calculate the correct promised interest and principal cashflows for each loan at each time t, and I thus implement the more precise calculation of:

$$D_i(t) = \sum_{t=\tau}^{T} (c_t + p_t) \tag{2}$$

so $D_i(t)$ represents the remaining interest $\sum_{t=\tau}^{T} c_t$ and principal $\sum_{t=\tau}^{T} p_t$ payments as promised in the loan note from any time t to maturity T for the $i-th$ CREL, where τ is the historical sim date.

The second important extension is to the equity position. In Merton, the equity position of the corporation is observable through the stock market. Additionally, the volatility is also observable. For CRELs there are two distinct periods in which information about CRE and CRELs takes on different characteristics: i.) the date of origination and ii.) every date thereafter until the maturity date of the CREL. Thus, at origination, the value of the CRE, V_0, is actually known as are the value of the debt, D_0, and the value of the equity, E_0. Specifically, I observe in the data the Loan to Value ratio (LTV) for each loan in a collateral pool. Since $\mathrm{LTV}_0 = \frac{D_0}{V_0}$ and $V_0 = D_0 + E_0$, I say,

$$V_0 = \mathrm{LTV}_0^{-1} * D_0 \Rightarrow E_0 = (\mathrm{LTV}_0^{-1} * D_0) - D_0 \Rightarrow E_0 = V_0 - D_0 \tag{3}$$

and thus *know* the equity value at origination. However, for every day after origination, we cannot observe the equity position of the CRE, $E(t)$, we cannot directly observe $D(t)$, and we do not have a reliable volatility for the equity, $\sigma_E(t)$. Since I need to solve for $V_0(t)$ and $\sigma_V(t)$ for all times after origination of the loan, the implied value of the company and the implied company volatility, respectively, I need a proxy for $E_0(t)$ and $\sigma_E(t)$ the equity value and the volatility of the equity, respectively. As is common practice in the literature, (see Appendix B, and others) I have to make assumptions to proceed.

The National Council of Real Estate Investment Fiduciaries (NCREIF) provide a quarterly total return index for commercial real estate properties going as far back as 1978. The properties are held by banks and other institutional investors and are self-reported from sales and mark to market procedures. The property indices are recorded across both property and regional subsets for the entire US. Underlying NCREIF are properties at the submarket level across the 5 major income producing property types. The total return of all property elements underlying NCREIF is comprised of mark to market valuation of CRE assets by the NCREIF member banks and institutions combined with actual sales of any such CRE underlying properties. Information is gathered from major financial institutions (banks, pension funds, and insurance companies) with substantial hard real estate assets in actual (not proxy) submarket locations. The outstanding value of the NCREIF property values exceeds $1 trillion and over time the rolling annualized national NCREIF return index (calculated from quarterly reported total returns) accurately captures and reflects the real estate cycle (Figure 5). As such, NCREIF represents a good proxy for US commercial real estate values.

Since I seek to find an estimator for property values in valuation, I proxy in the simple case for the property value securing a specific CREL with the corresponding NCREIF return volatility. As NCREIF is only reported quarterly, but our project is to estimate CRE value daily, I computed a cubic spline from the actual NCREIF lognormal quarterly index returns $\sigma_{E_{i,y}}(t)$ to form a daily spline estimate of the annualized volatility observed daily, as required under Merton (Model 1). Specifically, I observe NCREIF index values quarterly, $N_y(u)$ for the $y-th$ property type with, u, the quarterly time of observation of the index values for $i = 1-31$; with $1{=}7/2006, 2 = 10/2006, \ldots 31 = 1/2014$, for each of the $y = 1 \ldots 6$ property types (1=Multifamily or MF, 2, =Office or OF, 3=Retail or RT, 4=Industrial or IN, 5=Lodging or LO, and 6=Other or OT). I calculate the lognormal return for each property index over the quarter,

$$R_{y,i} = \ln\left(\frac{N_y(u_i)}{N_y(u_i - 1)}\right) \tag{4}$$

From the set of returns $R_{y,i}$ I calculate the quarterly volatility as standard deviation, so

$$\sigma_{R_{y,i}} = \sqrt{\frac{\sum\left(R_{y,i} - \bar{R}_{y,i}\right)}{(n-1)}} \tag{5}$$

In the context of natural cubic spline interpolation, the set of i quarterly volatility of lognormal returns can be expressed as 31 $(u_i, \sigma_{R_{y,i}})$ 'knots' per $y = 6$ property types. I then interpolate pairwise between $(u_{i-1}, \sigma_{R_{y,i-1}})$ and $\left(u_i, \sigma_{R_{y,i}}\right)$ $\forall\, i, y$ with polynomials of degree 3 setting $\sigma_{R_{y,i}} = q_i(u_i)$, such that under the constraint of passing through all knots $\sigma'_{R_{y,i}}$ and $\sigma''_{R_{y,i}}$ will be continuous everywhere with curvature $k = \frac{\sigma''_{R_{y,i}}}{\left(1+\sigma'^2_{R_{y,i}}\right)^{3/2}}$, $q'_i(u_i) = q'_{i+1}(u_i)$ and $q''_i(u_i) = q''_{i+1}(u_i)$ for all $i, 1 \leq i \leq n-1$. Implementing the natural cubic spline numerical interpolation, from the 31 observed knots $(u_i, \sigma_{R_{y,i}})$ I then calculate $t = 1,2, \ldots 1878$ interpolated daily points (t, σ_{E_y}) required for Merton (Figure 6)[9]. Finally, since I have the daily return index for NCREIF for all 6 property types and since I have the correct inverse LTV at origination, I can estimate the implied equity value for the i-th property of the y-th property type (y= 1…6), $E_{i,y}(t)$, as:

$$E_{i,y}(t) = \mathrm{LTV}_0^{-1} \times \mathrm{NCREIF}_y(\mathrm{t}) \tag{6}$$

[9] We conduct the same procedure in the reduced form where we also need daily volatility values for NCREIF expanding the indexing to $y = 1.,.6$ property types and $k = 1, \ldots 4$ geographic regions.

where NCREIF is the indexed value of NCREIF for each of the 6 property types, with an index start date of 7/1/2006 NCREIF=100. I am not calibrating Merton to any other values observed in the marketplace such as S&P returns or volatilities. However, since I have made changes to the original Merton equations, I have to consider that there may be errors in estimation. As such I incorporate a beta coefficient $\beta_{E_{i,y}}$ for the $i-th$ property of the $y-th$ property type $(y = 1 \ldots 6)$ to modify our NCREIF driven volatility $\sigma_{E_{i,y}}(t)$. The changes in notation are shown below

$$V_i(t)N(d_1) - D_i(t)e^{-r(t)T}N(d_2) - \frac{N(d_1)V_i(t)\sigma_{V_i}(t)}{\beta_{E_{i,y}}\sigma_{E_{i,y}}(t)} = 0 \tag{7}$$

I solve the non-linear system of the final set of equations for parameters V_i, σ_{V_i}, and β_{E_i} simultaneously using the numerical solution,

$$\underbrace{\min \| f(x) \|_2^2}_{x} = \underbrace{\min\left(f_1(x)^2 + f_2(x)^2 + f_3(x)^2 \right)}_{x} \tag{8}$$

for each of the i=172 loans on each of the historical dates.

1.2.1. Outputs and results: The outputs are $V_i(t)$, $\sigma_{V_i}(t)$, and β_i with $V_i(t)$ representing the daily risk neutral estimate for the loan's value under Merton. Additionally, I capture the probability of default $\pi_i(t)$ for each loan over the historical study and aggregate according to the loan's property type, $y = 1...6$, and balance weight $w_i(t)$, giving property type level probability of default based upon this implemented Model 1 as:

$$\Pi_y(t) = \sum_{i=1}^{N} w_{i,y}(t)\pi_{i,y}(t) \qquad (9)$$

With $\pi_i(t) = N(-d_2)$ The computed historical property type probabilities of default Π_y are provided in Figure 7. I notice that they are quite large. At the same time, however, it is useful to consider immediately the actual historical experience of the actual loans considered in my adaptation of Merton for CMBS over this historical period. These loans exhibited nearly 20% actual default rate through the Crisis (Figure 8) with multiple downgrades by the rating agencies up until this past year. As such, while the estimation of risk for Model 1 may be high, given the realization of losses, shown in Figure 8, the estimates for $\pi_i(t)$ are not unreasonable. The loans utilized in this dissertation represent approximately 8% of all loans underlying CMBX Series 1 and are reasonably diverse. The sample is not a large as some studies[10] but it is a larger sample than others[11]. As in DVH, 2012 I assume that loans underlying GG5 proxy for all loans underlying CMBX Series 1.

[10] Jarrow, etal 2008 and, Kau, 2009

[11] DVH, 2012

To test the fair value under Model 1, I have to consider the mock securitization or risk guideline argument. From those perspectives the collateral pool at a bank originator represents an ongoing risk until the loans are distributed through to the capital markets. During normal conditions, the loans are held in the 'held-for-sale' portion of the balance sheet. Under these conditions, the aggregate profile of the loans is considered in bond form. Meaning, estimates for the subordination levels associated with the TBA securitization for which these loans are contemplated to serve as collateral in the near future are determined from observations in the market and discussions with leading rating agencies. When one hedges interest rate and credit risks of the portfolio of loans, one considers the loan no longer as an individual loan, but rather as a component of the TBA securitization. As such, it is not only reasonable, but actually necessary to consider the loans in the aggregate as I do.

The aggregate value of the entire securitization can be arrived at in two ways. Let $b_k(t)$ represent the implied fair value price of the TBA security made up of K bonds. Let $B(t)$ represent the value of the entire securitization and $V_i(t)$ the fair value of the $i-th$ loan in the securitization. Then,

$$\sum_{k=1}^{K} w_k(t) b_k(t) = B(t) \tag{10}$$

but, $B(t)=\sum_{i=1}^{N} w_i(t)V_i(t)$, so:

$$\sum_{k=1}^{K} w_k(t)b_k(t)=\sum_{i=1}^{N} w_i(t)V_i(t) \qquad (11)$$

Therefore, the fair value of the bonds can be determined by simply allocating realized proceeds from the 'sale' of the entire securitization at any time t according to the priority payment rule of the securitization. In this study I (as well as DVH 2012) assume a simple senior subordinate structure based upon the attachment points observed in CMBX Series 1. The weights are found in Figure 9; no Interest Only strip (IO) is contemplated.

The allocation algorithm for Model 1 is straightforward. I have the face amount of the bonds based upon the remaining promised principal cashflows for the trust and the class percent η_k or $h_k(t)=\sum_{t=\tau}^{T} p_t\eta_k$. On each historical evaluation date a sale of all the loan assets at the computed fair value is contemplated as $B(t)=\sum_{i=1}^{N} w_i(t)V_i(t)$. The sale proceeds are allocated from the top down. The AAA class receives all proceeds from the sale up to the maximum of $h_{AAA}(t)$, so, $pmt_{AAA}=\min(h_{AAA}(t),b(t))$. Then, AJ receives all remaining proceeds $pmt_{AJ}=b(t)-pmt_{AAA}(t)$ up to a maximum of $h_{AJ}(t)$; then AM receives all remaining proceeds

$pmt_{AM} = \min\left\{\sum p_{AM}(t),\ b(t) - pmt_{AAA}(t) - pmt_{AJ}(t)\right\}$ to a maximum of $h_{AM}(t)$ and so forth until $pmt_{Other} = \min\left\{\sum p_{Other}(t),\ b(t) - \sum_{K} pmt_k(t)\right\}$. If the proceeds from the dilution of the securitization at fair value are insufficient to cover the promised payment to the bondholder in the payment allocation (aka waterfall/cascade), then the bondholder experiences a loss on that date, t.

Allocation of raw proceeds to the tranches have an upper bound of par, consistent with the value of fixed-income securities at maturity. But the proceeds are generated assuming an immediate distribution artificially compressing the investment period to zero. Therefore, I generate the future value of proceed dollars today at the prevailing risk free rate at time t, such that for each bond I have fair value under Merton of $\dddot{b}_k(t) = pmt_k e^{r(t)T}$ where T is remaining time to maturity of the bond. Figure 10 shows a numerical example.

The top panel of Figure 11 shows the fair value $\dddot{b}_k(t)$ for the bonds[12] and the bottom panel shows the composite fair value $\dddot{B}(t) = \sum_{k=1}^{K} \dddot{b}_k(t)$ compared with the composite market price from the CMBX Series 1 $M(t) = \sum_{k=1}^{K} m_k(t)\eta_k$ with m_k and η_k representing the observed market price for

[12] Market price history is not available for the entire history for AJ and AM and they are deliberately excluded from Theta analyses and depiction.

the CMBX Series 1 tranche and the balance weight of the tranche. What we observe in the top panel is despite the very high probability of default, the realization in the tranche allocation leaves the AAA securities untouched. The pricing of the AAA declines converging as it should to par in an orderly manner. The other securities are exposed in varying degrees to losses as contemplated under the allocation algorithm discussed above. In the bottom panel, however, a very interesting picture emerges. Despite the evidence of losses under the distribution at a fair value mark to market, the overall profile of the entire composite securitization, $\dddot{B}(t)$ (in blue) is categorically above the composite market price equivalent $M(t)$ (in black). The implication is that despite the risk contemplated in the Merton model, the overall risk of the securitization during the financial crisis was lower than anticipated by the market.

I examine the profile further by introducing Theta, $\theta_k(t) \equiv \dddot{b}_k(t) - m_k(t)$ which is a reliable benchmark for the richness or cheapness of individual securities or the entire securitization overall. In Figure 12, I show Theta first for the composite price histories in the upper left in black, and then provide time series comparisons of Theta for each of the bonds in the capital structure compared with the composite Theta (in black). Generally,

$$\theta_k(t) > 0 \rightarrow \dddot{b}_k(t) > m_k(t) \rightarrow \text{"mkt px cheap vs. risks"}$$
$$\theta_k(t) < 0 \rightarrow \dddot{b}_k(t) < m_k(t) \rightarrow \text{"mkt px rich vs. risks"}$$
$$\theta_k(t) = 0 \rightarrow \dddot{b}_k(t) = m_k(t) \rightarrow \text{"mkt px appropriately reflects risks"}$$

What we see is within the capital structure, the AAA classes (blue) are less expensive than the capital structure overall (black), and empirically cheap versus market pricing of the risks with high Theta. At the other end of the capital structure the BBB- class (red) are almost categorically rich relative to the capital structure overall (black) and relative to the market pricing of the risk it faced in the Crisis. Only at the peak of the Crisis in this time series when BBB- prices reached their nadir does the measurement Theta indicate fair valuation of the risks. The classes in between these extremes (AA, A and BBB) move in keeping with the realization of risks in the distribution algorithm at fair value. If one accepts the validity of the Merton model as applied to CMBS valuation as I have cast it, then one would have insights into the relative risk and reward profile of the entire securitization as well as the relative risk/reward profile of the individual bonds.

This analysis is performed by investigating the comparative merits of the different models at this point. As suggested previously in Figure 2a and 2b, the evidence support the view that the Reduced Form approach (Model 4) gives the greatest insight into risks and is thus most reliable. As such, doing statistical analysis on the 'fit' of the Merton Model 1 vs. the market pricing observed is not of particular consequence at this time. Suffice it to say at this point, that this Model 1 is reasonable and its conclusions (Theta's are quite large) and underpinnings (probabilities of default are quite large) seem to point in the intuitively correct direction. Some bonds were priced very cheaply during the Crisis, however, overall, the risk of the securities when considering the entire capital structure was more than offset by chaotic, if not panicked.

1.3. Model 2: Calibrated Merton Hybrid - DVH, 2012: I now turn to an alternative approach to Merton. The authors build upon some work in Jarrow, etal 2008 by considering multiple property types linked to loans, but the approach overall is less rigorous[13]. Importantly, as we shall see, Model 2 (immediately below) is simply a special case of the generalized approach I propose in Model 3 later in this Section.

In DVH, 2012 the valuation method of CMBS implemented is essentially a *two-step* hybrid approach which correctly implements a 'bottom-up' approach to the modeling of the risks of default and loss of underlying loan collateral and then, in turn, transforming the meaning of those loan level risks into meta bond level pricing for which the loan sample serves as collateral. In the *first step*, the authors implement a calibrated parameterization of Merton, 1974 where such parameters

[13] For example, the location of the property is not considered, nor is the historical relationship between defaults and the simulated economy, just to name a few.

are the outputs of a numerically solved non-linear system of six equations which calibrate to daily S&P option volatility, REIT pricing covariances and other observable market metrics relevant to simulation. In the *second step*, the calibrated parameter outputs (determined in the *first step*) are then combined with other values as inputs for use in simulation of REIT prices using a correlated multivariate Wiener process. In the simulation, the simulated REIT prices are linked to loan parameters, where the loan sample of 30 loans (*unreported*) proxy for the set ~1500 loans underlying the tranches that serve as the collateral for the CMBX Series 1 credit derivative swap contract. The trigger for default which is simulated is the inverse LTV metric, $\frac{1}{LTV_t}$, that interacts with the simulated property values captured for each of the loans in the sample. If the metric falls below 1 *at maturity only* then at maturity a default state for the loan is captured. From the set of simulated loan states simulated loan cashflows are generated allowing construction of synthetic tranche level CMBX swap prices under risk neutral conditions independent of actual CMBX tranche prices.

1.3.1. Step1 - Calibration and estimation: An extension to Merton is incorporated that requires solving a system of six non-linear equations for each time step, *t*, in the study. The equations for the non-linear system are defined below:

$$E_j = BS\left(\beta_j^2\sigma_s^2 + \gamma_j^2, \bar{V}_j, \bar{D}_j, \bar{T}_j\right), \quad j = 1, \ldots .3 \tag{12}$$

$$Var\left(\frac{dE_j}{E_j}\right)=\left(\frac{\bar{V}_j}{E_j}\frac{\partial E_j}{\partial \bar{V}_j}\right)Var\left(\frac{d\bar{V}_j}{\bar{V}_j}\right)=\left(\frac{\bar{V}_j}{E_j}\frac{\partial E_j}{\partial \bar{V}_j}\right)^2\left(\beta_j^2\sigma_s^2+\gamma_j^2\right)dt,\quad j=1,\ldots 3 \tag{13}$$

$$Cov\left(\frac{dE_j}{E_j},\frac{dM}{M}\right)=\left(\frac{\bar{V}_j}{E_j}\frac{\partial E_j}{\partial \bar{V}_j}\right)\beta_j\sigma_s^2dt,\quad j=1,\ldots,3 \tag{14}$$

$$\begin{aligned}Cov\left(\frac{dE_j}{E_j},\frac{dE_k}{E_k}\right)&=\left(\frac{\bar{V}_j}{E_j}\frac{\partial E_j}{\partial \bar{V}_j}\right)\left(\frac{\bar{V}_k}{E_k}\frac{\partial E_k}{\partial \bar{V}_k}\right)\beta_j\beta_k\sigma_s^2dt+\\ &\left(\frac{\bar{V}_j}{E_j}\frac{\partial E_j}{\partial \bar{V}_j}\right)\left(\frac{\bar{V}_k}{E_k}\frac{\partial E_k}{\partial \bar{V}_k}\right)\rho_{j,k}\gamma_j\gamma_k dt,\quad j,k=1,\ldots,3\end{aligned} \tag{15}$$

$$Var\left(\frac{dS}{S}\right)=\sigma_s^2dt \tag{16}$$

The LHS of the equations represent the values observed in the historical data which are then "matched empirically on each calibration day." They use 3 property types with 1=Multifamily (MF), 2=Office (OF), and 3=Retail (RT) and they are indexed $j,k=1,\ldots,3$. The subscript s represents the S&P 500. These parameters are for REITs and not commercial real estate loans, underlying CMBS. The equity volatility term instead of being observed as in Merton, is now a composite term of the known volatility on the S&P 500 index σ_s, and unknown REIT property-type parameters β_j and γ_j giving:

$$d_{1,j} = \frac{\log\left(\frac{V_j}{D_j}\right) + \left(r + \frac{\gamma_j^2}{2}\right)T}{\gamma_j \sqrt{T}} \tag{17}$$

and

$$d_{2,j} = d_1 - \gamma_j \sqrt{T} \tag{18}$$

The Black Scholes condition is given by $E_j = V_j \mathbb{N}(d_{1,j}) - D_j e^{-rT} \mathbb{N}(d_{2,j})$, and the Ito's Lemma condition is given by $(\beta_j \sigma_s + \gamma_j) E_j = V_j \sigma_j \mathbb{N}(d_{1,j})$. Having specified, $d_{1,j}$ and $d_{2,j}$ which are used throughout the system, I set the Black-Scholes and Itó conditions equal to zero to obtain the first two of six equations in the system. I thus recast Merton representing the first *two* equations for each date of calibration, in addition to the remaining equations, making the substitution for the cumulative normal distribution function, $\frac{\partial E}{\partial V} \approx \mathbb{N}(d1)$, as appropriate. I numerically solve the non-linear system of equations simultaneously using

$$\underbrace{\min \| f(x) \|_2^2}_{x} = \underbrace{\min \left(f_1(x)^2 + f_2(x)^2 + \ldots f_{16}(x)^2 \right)}_{x} \tag{19}$$

which yields outputs calibrated daily to option prices and other data.

Briefly, $E_j$and E_k are the daily average equity market capitalizations within REIT property-type sector $j, k = 1, \dots, 3$ across the 15 property-type sector specific REITs selected. Specifically there are 4 REITs for apartments/multifamily ($j = 1$), 6 REITs for office ($j = 2$), and 5 REITs for retail ($j = 3$). σ_s^2 is calculated directly from the pricing of three-month (*90-day*) ATM S&P 500 options. D_j is the indexed property specific REIT debt principal value outstanding calculated by taking the daily average of the sum of (long term debt + current liabilities) within property-type sector $j, k = 1, \dots, 3$ across the 15 property-type sector specific REITs selected by DVH as above in with E_j and E_j. The maturity date for REIT debt is assumed to be 5-years[14]. The risk-free rate r is determined from five-year and ten-year swap rates (for each calibration date) as the corresponding linearly interpolated rate for $\tau = T - t$ time to maturity. The dividend rate q is assumed $= 0\%$. The charts (Figure 13) show the calibrated outputs for the system that correspond well with the results of DVH, 2012.

[14] I age the debt over the year and then 'roll' the debt every December 31 such that on January 1 the maturity of the debt is 5 years whereas by December 31 of the year the maturity of the debt is 4 years.

1.3.2. Step 2 - Simulation: Once the calibration is complete, I then simulate REIT values and link the REIT value evolution to individual loans. The risk event for loans is *default only*, initially assumed to occur only at maturity. The multivariate Brownian motion process that generates simulated returns on REITs in implementation is:

$$\frac{d\bar{V}_j}{\bar{V}_j} = (r - q)dt + \beta_j \sigma_s dW_0 + \gamma_j dW_j \tag{20}$$

The Brownian motion, dW_0 , is associated only with the S&P 500 and is 'shared' and constant across all j REITs (modified with the interaction with $\beta_j \sigma_S$) and dW_j is the property-type sector specific Brownian motion (three total) that interacts with their corresponding volatility term γ_j. In the implementation I simulate four Brownian motions dW_0 and dW_j, $j = 1, \dots, 3$ and interact them with the estimated and observed parameters. They anchor the observed simtime values (where each simtime$= 0 = \tau$ corresponds to 1 of 795 historical trade dates) and construct the simulation paths on a daily basis, where each simulation path has 120 monthly time steps (10 years). The 120 month projection is used because at the point of origination of the loan, there are 120 months (per their assumption) until balloon maturity when the entire loan balance is due. The parameters r, q, β_j, σ_S and γ_j are all as of the simulation date. In my rendering there were 795 simulation dates run from 11/1/2007 thru 12/31/2010 reflecting 795 separate sets of calibrated parameters. The observed parameters are r, the constant risk-free rate; q, the dividend rate; and σ_S, the volatility on the S&P 500. The estimated parameters β_j and γ_j are both indexed to j=1,...,3 indexed property types which characterize the REIT returns being simulated. So, for example, VNO (Vornado) is an office REIT (index $j = 3$) whereas EQR (Equity Residential) is an apartment REIT (index $j = 1$). Figure 14 lists the REITs used.

1.3.3. Pricing: The simulations consider changes in the observed and estimated parameters as well as changed anchor points for the value being simulated. In Figure 15, for example, I observe two simulation paths for the Apartment REIT Composite Index ($j = 1$) on two different simulation dates 9/1/2006 (blue) and 12/30/2008 (red). Notice that the blue simulation initiated in 2006 has a higher initial anchor value reflecting the composite value of REITs on 9/1/2006 of about 7.05 whereas the red simulation has a lower anchor value of 3.31 reflecting the accurate decline in REIT value over the 2.25 years. Additionally, the blue simulation path generated in 9/2006 exhibits much lower volatility than the red simulation path generated in 12/2008, as it should. The simulations accurately reflect the differences in the volatility, uncertainty and value in the market at those periods of time[15]. So, what I would expect is that at simulations initiated in 'bad' times the incidence

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[15] It is true, then, that as the loans in the sample age, that it is unnecessary strictly to simulate beyond maturity, but to preserve the robustness of the code for future study, I keep track of the age of the loan and its maturity date. The REIT evolution always is simulated 10 years into the future, the loan maturity which governs the time at which default may occur in simulation ≤ 120. Figure 16 shows 100 simulations for dW_3 for 1 trade date projected 120 months.

of loss events projects should increase resulting in simulated synthetic price compression. In Figure 17, I show for 10,000 simulation paths initialized on each of 9/1/2006 (blue) and 12/30/2008 (red), the rank ordered distribution of the cumulative portfolio loss across the 32 loan sample, with the x-axis showing simulation paths 1 thru 10,000 and the y-axis showing the corresponding portfolio loss generated on such path. The impact of the simulation process on the loans is clear and they in turn govern price. Empirically, the losses from the simulation correspond to the intuition of the earlier plot which showed increased volatility of paths generated in bad times (12/30/2008, red) vs. good times (9/1/2006, blue). The blue points shows low to no simulated portfolio losses on 9/2006 in the 120th maturity month whereas the red points show non-trivial simulated defaults in the 94th maturity month (*aging*) with levels that, interestingly, are consistent with recent history. The losses generated at the portfolio level in the simulation are the result of linking the loans, underlying the tranches, underlying the CMBX Series 1, to the simulated REIT values. The pricing process begins with the simulated REIT value $\overline{V_{J,t}}$ as well as the initial value of the $\overline{V_{J,0}}$ which represents the observed value of the REIT at initialization (simdate= τ) of the simulation.

$$\tilde{V}^{i}_{t,k} = \frac{1}{LTV^{i}_{0,k}} * \frac{\overline{V}_{j,t}}{\overline{V}_{j,0}} * e^{-0.5\sigma_j^2 t + \sigma_j \sqrt{t}\varepsilon_i} \tag{21}$$

$\overline{V_{J,0}}$ is an initialization value visited on each day in the historical record. They introduce the historical factor $1/LTV_{0,k}^{i}$ which represents the historical (at origination) inverse of the loan to value ratio for the $i-th$ loan in the $k-th$ CMBS transaction[16]. The $i = 32$ Brownian idiosyncratic shocks associated with the individual loan risks is captured in the discrete representation of the Brownian random walk in the remaining terms. The value on the LHS of the equation is thus the set of $i = 32$ simulated property values. The values $V_{t,k}^{i}$ act as a rolling barometer for the health of the loan. In the simulation at maturity the loan defaults "if the value per dollar loan drops below a default trigger value which is set equal to the loan amount to be paid at maturity"[17]. Specifically if $\tilde{V}_t^i < 1$ at time of Maturity $(t = T)$ then the loan defaults as the debt obligation is greater than the company value. This borrows from Merton's condition in both restriction and action but the restriction is arguably unnecessary and unrealistic. Consider Figure 18 which shows two simulations of the MF REIT on two separate days (9/1/2006 and 12/30/2008). Figure 19 shows the corresponding calculated value for a single MF loan with $LTV_0 = 0.75$ with $\frac{1}{LTV_0} = 1.33$ and maturity at 120 months (blue) and 92 months (red). Since the barometer is above 1 at maturity for both simulations, no default ensues. One objection to the assumption is that in the interval between

[16] Only one transaction is implemented.

[17] DVH, 2012

the start of the simulation, $t = 0$, and maturity on the loan $t = T$, the value of the property may drop below the value of the debt ("underwater"). Thus, one could make the argument that 'If the property is underwater, why not trigger a 'ruthless' default in the simulation intertemporally if $\tilde{V}_t^i < 1,\ \forall\, t \in (0, T)$ as has been demonstrated in the literature including Jarrow, etal 2008' I return to this later in the paper when introducing my generalization, Model 3.

The contra-argument is that debt service on an IO loan is the coupon which is typically less than principal and interest amortization. Thus if the income generated by the property at any time in the future is sufficient to pay debt expenses (and non-mortgage operating costs of the property) then the borrower might be willing to continue to make debt service payments even if the property was 'underwater' wagering that at maturity, the property value would have turned to a level $\tilde{V}_T^i > 1$. As the vast majority of CRE loans are paid at maturity through refinancing (*'rolling' the debt*), this perspective, implicitly assumes that lenders at such time in the future would be willing to lend at leverage levels at maturity (T) such that the amount that could be borrowed at time T maturity would at least be sufficient to payoff the original mortgage issued at $t = 0$. The loss at the portfolio level is expressed below and represents the aggregation of all the losses at maturity.

$$L_{t,k}^{ptf} = L_{t-1,k}^{ptf} + \sum_{i=1}^{I} w_k^i * D_{t,k}^i * \max\{1.0 - \tilde{V}_{t,k}^i, 0\} \tag{22}$$

The weight w_k^i represents the contribution of the loan balance to the $k-th$ portfolio. D_k^i is the "indicator function taking the value of one if at time t loan i of CMBS deal k defaults." They assume the i loans are equally weighted and that $k = 1$, and, as stated, maturity default decisions are fixed to $t = T$ maturity. Next, I allocate the Losses to the CMBX tranches by deducting the credit enhancement attachment points (low, L and high, H) as maximums from the portfolio loss normalizing by attachment

$$L_{t,k}^{tranche} = \frac{\max\{L_{t,k}^{ptf} - CE^L, 0\} - \max\{L_{t,k}^{ptf} - CE^H, 0\}}{CE^H - CE^L} \tag{23}$$

The insurance responsibility thresholds for which sellers of protection have to provide cash in the event of actual default are tranche insurance boundaries (*attachment points*, shown also with coupons in the Figure 20). The CMBX swap contract has fixed-rate and floating-rate legs. The fixed coupon ('*leg*') is paid to the seller of protection. In exchange, the floating leg is insured against defaults. A typical swap like CMBX is described Figure 3. One way to price out the risk of these responsibilities in the index swap contract is to use Monte Carlo simulation to articulate the impact of the default event on the underlying loan and corresponding tranche cashflows, as well the resultant tranche pricing. The floating rate cashflow which in this example with default simulated only at maturity must be $= 0$ for all t other than $t = T$ is given by:

$$CF_{t,k}^{floating} = L_{t,k}^{tranche} - L_{t-1,k}^{tranche} \tag{24}$$

followed by the computation for the fixed leg

$$CF_{t,k}^{fixed} = \left(1 - L_{t-1,k}^{tranche}\right) * c \tag{25}$$

The notation is imprecise, because if the default can only occur at maturity T and the cashflow for the fixed tranche is based upon the loss of the prior period, then the cashflow at maturity must also be certain as defaults cannot occur prior to maturity. This is inconsistent. I correct for this in the code ensuring that that default occurs at maturity and that the fixed and floating rate summations and difference accommodate the correct cashflow calculation. Finally, the equation below describes calculation of the present value of cashflows for tranches AAA, AM, AJ, AA, A, BBB, and BBB-:

$$P(CE^{L}, CE^{H}) = 100 + 4 * \sum_{k=1}^{25} \sum_{t=1}^{T} e^{r_f t E^{Q}\left(CF_{t,k}^{fixed} - CF_{t,k}^{floating}\right)} \tag{26}$$

The cashflows are discounted at the risk-free rate which is appropriate as the model price is understood to be taken as 'known' and riskless. The risk free rate in this implementation does not vary with time (static term structure), nor is it path dependent (dynamic term structure). In contrast, under the risk-neutral measure Q the cashflows represented within the expectations operator, E^Q, are path dependent. So, for purposes of clarification $r_f = r(\tau)$ is the observed 5-year on the run risk-free rate at historic date τ, as specified by DVH, 2012.

1.3.4 Testing and ex-post analysis of Model 2: I implement an OLS suggested by DVH who claim R-sq overall of 91%:

$$CMBX_{tranchePx} = \alpha + \beta_{tranche} OptMod_{tranche} + \epsilon \tag{27}$$

My initial results are reasonable visually (Figure 21), and statistically (Figure 22), but they do not map consistently to their claim. Figure 21 shows pricing based on the calibrated simulation. I know that the value of REIT prices (equity values, observed) recovered rapidly in the wake of support from the Fed in the QE programs. During this period, volatility on the S&P began to wane and REIT prices more than recovered their levels prior to 11/1/2007. Similarly, AAA CMBX prices which, relative to REIT prices, were relatively stable, also recovered but BBB- CMBX credits did *not* recover (Figure 23). Investigating further, looking at all the CMBX prices over the same period, I see a pattern of persistent muting in the recovery of lower credit rated instruments (Figure 24). Why? Knowing that the volatility terms in Merton are not designed to identify state variables outside of the closed form equations, I ask a few follow-up questions: *First*, what causes the compression in prices; and *Second*, if my simulation and calibration are correct, how is 'compression' captured solely within parameterized Merton where β_j and γ_j capture non-explicit volatility drivers?

I observe empirically a lagging price recovery in CMBX lower credit rated classes relative to REIT equity price recovery over the same period. As such, this faster recovery is coming through into my simulation in the Merton conditions in the E_j terms σ_S^2 terms which are observed and inputs. In fact other sources of signals that were CMBX specific may have been influencing CMBX pricing. If any of such CMBX specific data demonstrated significance, then I could say that the model proposed by Merton and parameterized by DVH in their implementation is not adequately specified. All information public in semi-strong efficient markets needs to be incorporated into pricing. But the pricing estimator as specified first in the Merton calibration, and second in the parameterized simulation did *not* account for explicit changes in the fundamental *credit health* of the CMBS universe. To verify, I incorporated the most general form of a CRE credit warning with 30 plus days delinquency rates (Figure 25) for the entire CMBS Universe released monthly and available to CMBS practitioners constant intra-month. Although Merton's model does not allow for the use of other information such as delinquency as input to valuation, in the context of CRE Loan and CMBS valuation delinquency is significant (see Jarrow etal, 2008). Loan level delinquency status for underlying collateral as well as sector level delinquency should play a formal role in semi-strong form efficient markets. Thus considering the macro delinquency status as an 'environmental input' can be considered as public information that is informing prices of illiquid securities. The information is public to those practitioners trading the objects, though it may be private to others. Adding the delinquency information and expanding the regression to incorporate 30+ delinquency status for the CMBX Universe shows delinquency status to be significant for all tranches increasing the R-sq especially for lower rated credits.

$$CMBX_{tranche} = \alpha + \beta_{tranche} synthCMBX_{tranche} + \beta_{dlq} 30Dlq + \epsilon \quad (28)$$

The results of the regression specified above are summarized in Figure 26 with the impact shown in the plot of BBB- (Figure 27). Additionally, I show all my results for the regression in time series plots below for all classes (Figure 28). Including delinquency ex-post as exogenous and additive to the Merton calibration, *does* seem to interact more significantly with the lower rated tranches, but also demonstrates significance in the higher rated tranches as well.

Continuing my analysis, I want to see if the risk free rate had any explanatory relevance to pricing. To investigate, I layered into the regression the slope of the US Treasury Curve (10s-2s) which is a standard technique to incorporate the prospect of changes in borrowing costs of the US and prospects for inflation into expectations of ancillary instruments. The slope of the treasury curve contributed to the explanation of CMBX prices and demonstrated improved results (Figure 29 and Figure 30 significant in all classes, AAA shown). Looking at the coefficients, higher AAA prices are associated with higher delinquencies which is sensible as managers migrate to better credits in periods of uncertainty. I repeat the same calculations for (Figure 31) BBB- and see similar intuitive results. Higher BBB- prices are associated with lower delinquency (30days plus) and a flatter yield curve two indications of economic health which allay investor concerns thereby raising prices of credit sensitive BBB- securities. In this period, since delinquencies are increasing and the yield curve is steepening (Figure 32), I would thus expect to see the opposite effect, which I do. Finally, given the central role of residential property value deterioration in the Crisis and the concomitant steepening in credit spreads (Figure 33), I consider *both* factor to improve the explanatory profile of the model ex-post and to test for misspecification of the Option Model. As with both treasuries and delinquency status, the Case-Shiller 20 Housing Index, an established indicator of macroeconomic health, demonstrated significance over the period studied and the credit slope were considered in the regression (Figure 34 and Figure 35). In contrast to Treasuries, and to be expected, the credit slope was insignificant for the AAA classes but significant for the lower rated classes, in particular for BBB-. The Credit Slope was insignificant for AAA but significant for BBB-. Additionally as the Credit Slope would be inappropriate for pricing exercises it should be dropped. With the modest selections of relevant explanatory variables, I am able to achieve adjusted improvements.

I observe some significant correlation amongst the Independent Variables (Figure 36), especially between the Credit Slope and the option model, but, no omitted variable from the RAMSEY Reset as $Fitted\ F = 72.13 > F_{crit(3,786)} = 3.78$, (Figure 37). Additionally, the Variance Inflation Factor Test indicates that since none of the $VIF(\beta_i) > 10$ then on my first pass, there does not appear to be a problem with multicollinearity amongst the explanatory variables. Not surprisingly, the Option Model does exhibit the highest VIF (Figure 38). Also, the condition index at n>30 and p>.50 indicates some competing dependency between the option model, CaseShiller and the CreditSlope (Figure 39). Since Credit Slope is insignificant with AAA and for other reasons related to pricing it should be dropped. Finally, the White and Breusch Pagan Tests (Figure 40) does indicate that non-constant variance heteroskedasticity may be present. However, the Durbin-Watson Test for autocorrelation (Figure 41) between the error terms (the residuals) is inconclusive because at 6 degrees of freedom and 795 observations, the statistic is between the upper and lower bounds. While there were some problems with layering in additional variables, the significance of them in many ways is compelling (Figure 42 and Figure 43). The significance of the sparse yet intuitive added variables is that Model 2 can be rejected as an unbiased estimator of market prices. Theta is not white noise and it is correlated with other variables.

1.3.5 Initial extensions to Model 2: I extend Model 2 to incorporate 6 property types[18] by increasing the number of REITs. This allowed me to expand the number of loans in the sample from 30 to 172 reflecting all the loans in GG5. Necessarily each of the equations are re-indexed to $j, k = 1, \ldots, 6$ and the system is solved with more parameter outputs:

$$\underbrace{\min \| f(x) \|_2^2}_{x} = \underbrace{\min \left(f_1(x)^2 + f_2(x)^2 + \ldots f_{40}(x)^2 \right)}_{x} \tag{29}$$

As CMBX Series 1 has more than 1500 loans associated with it, there was still a possibility of sample selection bias. As my sample of 172 loans is the exhaustive set of loans from one of the deals actually associated with CMBX Series 1 and as the loans are accurately weighted and distributed across all property types, this set should more accurately reflect the distribution of risks of CMBX collateral in the form of securing underlying CMBS tranches. The summary statistics of the GG5 transaction sample are shown in Figure 44. As is evident, the weighting of the transaction is not out of line with my weightings using blind property type distributions and sample weightings

[18]The six property types are: Multifamily (MF), Retail (RT), Office (OF), Industrial (IN), Hotel/Lodging (LO), and Other (OT)

though, admittedly, the representation of MF in this trust is less than the universe average for this vintage cohort. The evolutions below are correlated,

$$\frac{dV_{i,j}}{V_{i,j}} = (r-q)dt + \beta_j \sigma_s dW_0 + \gamma_j dW_j + \sigma_j dZ_{i,j}$$
$$\frac{dS}{S} = rdt + \sigma_s dW_0 \qquad (30)$$

where r denotes the risk-free rate, q the dividend rate, dS/S the return on the S&P500 index driven by Brownian motion dW_0. dW_j a Brownian motion representing sector level shocks for the property sector j and $dZ_{i,j}$ a property specific shock. All factors are orthogonal to each other except that the sector level shocks, dW_j are correlated with each other, so the original equation:

$$Corr(dW_j, dW_k) = \rho_{j,k} dt \quad j,k = 1,...,3 \qquad (31)$$

is now, in my extension for 6 property types, written as:

$$Corr(dW_j, dW_k) = \rho_{j,k} dt \quad j,k = 1,...,6 \qquad (32)$$

For each draw governing the sector level shocks I use the correlations determined from the calibration to populate for each simulation date the calibrated correlation matrix:

$$\begin{bmatrix} \rho_{1,1} & \rho_{1,2} & \rho_{1,3} & \rho_{1,4} & \rho_{1,5} & \rho_{1,6} \\ \rho_{2,1} & \rho_{2,2} & \rho_{2,3} & \rho_{2,4} & \rho_{2,5} & \rho_{2,6} \\ \rho_{3,1} & \rho_{3,2} & \rho_{3,3} & \rho_{3,4} & \rho_{3,5} & \rho_{3,6} \\ \rho_{4,1} & \rho_{4,2} & \rho_{4,3} & \rho_{4,4} & \rho_{4,5} & \rho_{4,6} \\ \rho_{5,1} & \rho_{5,2} & \rho_{5,3} & \rho_{5,4} & \rho_{5,5} & \rho_{5,6} \\ \rho_{6,1} & \rho_{6,2} & \rho_{6,3} & \rho_{6,4} & \rho_{6,5} & \rho_{6,6} \end{bmatrix} \tag{33}$$

Each row is associated with a given property-type (1, 2,...,6). This expands the approach where the calibrated correlation matrix is across (1,...,3) property types. I calculate the calibrated correlations with the output shown in Figure 45a; in Figures 45b thru 45d I show the other calibrated output of V_i, γ_i, β_i for the 6 property-type implementation as previously shown for 3 property-types.

The set of charts in Figure 46 show the average simulated inverse LTV evolutions across 1000 simulations for 1092 trading days for all 6 property types. This is the trigger for default when it goes <1 for any loan in any simulation. As is evident, several evolutions show similarities, but there are important differences that are consistent with intuition. The Industrial (IN4) property type exhibits a more muted evolution associated with its lower volatility, but also exhibits some of the most severe triggering on average. This is consistent with intuition that Industrial property types are safe unless they go bad at which point they default with no ready alternative use. The Other/Diversified property type (OT6) appear at times counter cyclical and the lodging property type (LO5) appears to demonstrate a muted response post the worst of the Crisis. What is a bit surprising is that the levels for the triggers for LO5 and OT6 on average appear better than expected and better than all the other property types. This could be attributed to a sample bias previously discussed where the economic profile of these particular loans are *not* representative of the portion of the CMBS universe occupied by these property types.

Simply incorporating 6 property types, however, does *not* produce markedly improved results vs. those analyzed in the 3 property-type case. There are some differences and influences of more REITs and that may be impacting pricing. The top panel of Figure 47 shows the fair value pricing somewhat choppier for the lower rated tranches than with the earlier study. However, the bottom panel showing the composite fair value vs. the composite market price tells the story. The signal of Theta, as in the 3 property type rendering, indicates that the market prices are expensive relative to the risks[19].

1.4. Model 3: A Generalized Calibration Hybrid: A central conclusion of DVH, 2012 (CMBS were *not* sold at firesale prices during the crisis) invites three important questions: *First*, given their results, if their model was comprehensive then there should be few, if any, exogenous variables that demonstrate statistical significance in ex-post analysis. *Second*, as DVH state, many assumptions they made and the small loan sample they use invite inquiries as to whether or not a

[19] Again, perhaps we are asking the wrong 'questions' by seeking a better fit to market prices?

larger, more representative sample reflecting a greater number of loans, more varied property types, and more accurate loan and bond cashflow and maturity profiles, will further explain differences between market and model prices. *Finally*, if the market is efficient then it should not be possible to earn extraordinary profits in backtesting by using model driven signals[20]. Thus by inquiring into the comprehensiveness of the model and generalizing it to accommodate any and all types of loan collateral, I should more readily be able to assess the efficiency of the CMBS sector.

In order to address the question above, it was necessary to examine in greater detail and eliminate many assumptions to create a richer model (Figure 48). Additionally my numerical procedure applies 1000 simulations to each historical date, τ. This section thus presents a general structural model for pricing CMBS using a calibrated hybrid approach. The method of implementation is a calibrated simulation that specifically considers the heterogeneity of loan characteristics by accurately incorporating correct cashflows and ruthless default. The 172 loan sample is examined over 795 trading days in the Crisis. Backtesting indicates a sizable disconnect between fair value and market pricing and simple trading tests suggest that extraordinary profits can be earned with the generalized model. Statistical analysis provides results suggesting an

[20] See Jarrow, etal 2008.

inefficient CMBS market and the need for the generalized model to precisely evaluate CMBS risks and opportunities.

The generalization presented in this section is important because it incorporates many realistic features of the loan building blocks of CMBS in a well specified simulated economy. In the examination, this dissertation gives new insights into CMBS market efficiency and new results that contrast with DVH, 2012 which is incorporated in this dissertation as a special case of the generalized approach. In so doing, this section provides a more precise perspective on CMBS efficiency.

1.4.1. Cashflows and pricing: With the introduction of principal *and* interest cashflows that are correctly timed, the determination of fair value requires some changes to DVH's approach to calculating fair value. The building block of CMBS and CMBX is the mortgage loan collateralized by the income producing property. Commercial mortgages have a variety of profiles that have evolved over time to provide the borrower with important flexibility in both the purchasing of properties and refinancing of existing debt. One of the staples of commercial mortgage lending is the balloon mortgage. In a balloon mortgage maturity at time T, the monthly payments of principal and interest from month 1 to month $T-1$ are based upon a level payment amortization schedule calculated using some fixed multiple, n, of T, or nT. So, for if $T = 120$, $n = 3$, then the level payment amortization schedule for months $1:120-1=1:119$ would be based on an amortization schedule[21] with $nT=3*120=360$.

A typical balloon loan in a CMBS transaction would be a 10/30 which is a 10-year balloon where the mortgage payments monthly for the 1st 119 months of the life of the loan are based upon fully amortizing level pay mortgage formulas, but in the 120th month (10th year) of the life of the loan, the entire loan outstanding principal balance is due. So, for the first 119 months we see in Figures 49a and 49b the familiar level pay mortgage profile with constant monthly payments for a 10mm 6% mortgage of $59,955.05 with increasing amounts of principal and decreasing amounts of interest. When we consider the 120th month of the balloon mortgage the promised principal repayment dwarfs all the prior payments, but to be sure as shown in the summary of month 1:2 and then 116:120, the payments are being made as scheduled per the balloon note terms in Figures 50a and 50b.

In addition to the balloon maturity profile, there are other variations to mortgage terms that provide borrowers with flexibility. These include shorter or longer balloon dates, periods of interest-only payments where no principal is paid, step-up provisions that follow interest only periods where

[21] See Fabozzi, 1994; Hayre, 2001; Jarrow etal, 2008; and several other sources for standard mortgage payment formulas.

payments are increased based upon increases in building occupancy, and many combinations and variations of these themes. The set of inputs for each loan in the GG5 transaction that dictate the $i-$th mortgage payment schedule are straightforward. The inputs for one mortgage in GG5 are summarized in Figures 51a and 51b. Figure 52 shows total promised payments from origination for all loan underlying the GG5 transaction. As is evident, the heterogeneity in the timing and amounts of cashflows is considerable[22]. For valuation purposes, therefore getting the promised cashflows correctly modeled at the loan level linking the payment schedules to the simulation with the algorithms described below, should yield different results.

1.4.2. Cashflow algorithms: Since the priced objects of our inquiry are bonds and not loans, the simulated trust level cashflows must also be allocated accurately through the bond capital structure. Here there are $k = 8$ classes and their weights, coupons and beginning balances are summarized[23] in Figure 53. Although the cashflows of the loans (and the trust and bonds) are monthly, the simulations are conducted daily. So, intramonth the promised cashflow balances of the loans, trust and bonds do not decline, but intermonth they may decline. The promised cashflow

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[22] The loan origination dates of some loans precede others. GG5 is not full until the 19th month (12/1/2005) which precedes $\tau = 1 = 11/13/2007$. Property location attributes are not included in Model 1, 2, or 3 but are collected for use included in the reduced form Model 4.

[23] There is no Interest-Only strip (IO) contemplated, though it could be incorporated. The structure assumed corresponded to that used by DVH as do the coupons and balance weights. All values are estimated from sources I found to be reliable, but could be adjusted further if necessary.

balances of the loans, trust and bonds only decline intermonth based upon the promised cashflow schedules determined at origination. Nevertheless, as market pricing is available daily, a daily pricing exercise intramonth is conducted based on daily simulations which may exhibit simulated defaults based on the implemented simulated methods which also reflect different initialized risk free rates on each initialized simulation date. Let each historical time, τ, with $1 \leq \tau \leq 795$, be the historical date at which a simulation is initialized with daily frequency. I don't have historically updated cashflows from industry vendors or trustees[24] (only promised) and state this data limitation and account for it in the notation and code. The total actual 'trust' principal cashflows across all i-loans determined from promised principal schedules is:

$$P(t) = \sum_{i=1}^{N} p_i(t) \tag{34}$$

and corresponding trust promised interest cashflows is:

$$C(t) = \sum_{i=1}^{N} c_i(t) \tag{35}$$

[24] For this dissertation the balance at each initialization period is assumed to be based upon the promised historic cashflows as opposed to the 'real' historical cashflows which would reflect aberrations to the promised payment schedule due to actual default or actual prepayments.

At the end of each monthly payment period there is an outstanding principal balance for each of the loans, trust and bonds reflecting monthly payment. The allocation of principal at the beginning of each monthly payment period, t, is made from $P(t)$ and such payments are said to be sequential pay, senior/subordinate with 'top-down' priority payment of principal made first to the AAA class until its balance is reduced to zero, then to the AJ class until its balance is reduced to zero,..., then to the Other class until its balance is reduced to zero. In each monthly payment period, t, the beginning balance of the bond, trust and loan objects are adjusted for the principal payment made in the prior period, $t-1$.

Let $\mathbb{P}_k(t)$ represent the principal payment to the $k-$th bond at the beginning of the payment period. Then $\mathbb{O}_k(t)$ represents the end of payment period outstanding principal balance on the $k-$th bond, and $\mathbb{O}_k(0)$ the original bond balance, so:

$$\mathbb{O}_k(t) = \mathbb{O}_k(t-1) - \mathbb{P}_k(t) \tag{36}$$

and for each payment month t, principal payments $\mathbb{P}_k(t)$, for $\forall\, k$ tranches are determined as:

$$\mathbb{P}_k(t) = \begin{cases} for\; k = 1, \left(\max\left(0, \min\left(\mathbb{O}_1(t-1), P(t)\right)\right)\right); \\ for\; \forall\, k > 1, \left(\max\left(0, \min\left(\mathbb{O}_k(t-1), P(t) - \sum_{k=1}^{K} \mathbb{P}_{k-1}(t)\right)\right)\right) \end{cases} \tag{37}$$

Whenever there is excess principal such that at any time t, and for $\forall\ k > 1\ P(t) - \sum_{k=1}^{K} \mathbb{P}_{k-1}(t) > 0$ then such positive principal payment will be captured and allocated to the next k −th tranche in the sequential pay structure. The chart and corresponding table in Figures 54a and 54b build intuition with the exact values in the chart of principal allocation for a fictitious $500mm transaction[25]. The algorithm for allocation demonstrates the payments as expected with the maximum principal amount allocated in any given month equal to the total amount in the trust ("All k") column on the left of Figure 54b. For example in 3/2008 3 bonds $k = 5, 6, and\ 7$ each receive some payment of the $16.789mm principal paid in that month. After the payments of the outstanding principal balance due to the tranches are made in full, the tranche receives no further payments of principal. The totals at the bottom are identical to the initial balances at the top (in grey) as expected.

The interest paid to each of the classes is paid from the trust interest collected from the loans, $C(t)$ as defined above. The algorithm for promised interest payment, $\mathbb{I}_k(t)$ to the bonds is:

[25] A more extensive example showing generated output from the code is provided in Appendix F.

$$\mathbb{I}_k(t) = \begin{cases} for\ k = 1, \left(\max\left(0, \min\left(\mathbb{O}_1(t-1) \times \frac{\iota_1}{12}, C(t) \right) \right) \right); \\ for\ \forall\ k > 1, \left(\max\left(0, \min\left(\mathbb{O}_k(t-1) \times \frac{\iota_k}{12}, C(t) - \sum_{k=1}^{K} \mathbb{I}_{k-1}(t) \right) \right) \right) \end{cases} \tag{38}$$

with ι_k representing the fixed rate coupons for the bonds. Figure 55 shows the corresponding interest for the same \$500mm sample transaction in the previous table above. The total interest collected $C(t)$ in this example is identical to the amount paid in each month (grey), as expected. Finally, the total promised payment for the k-th bond in any month t is then:

$$\mathbb{T}_k(t) = \mathbb{P}_k(t) + \mathbb{I}_k(t) \tag{39}$$

To extend for simulation is largely a matter of notation and capturing the items in the code. Once the loan and bond cashflows are correctly modeled in the promised case as shown above, then the exercise becomes straightforward. For each i loan, on each simulation path, l at each simulated time step τ there is an associated simulated principal cashflow $\tilde{p}_i(t, l)$ and a corresponding simulated interest cashflow $\tilde{c}_i(t, l)$. The total 'trust' simulated principal cashflow on each simulation path, l at each time step t is the aggregated loan level principal cashflows for N loans is:

$$\tilde{P}(t,l) = \sum_{i=1}^{N} \tilde{p}_i(t,l) \tag{40}$$

and the total 'trust' simulated interest cashflow is:

$$\tilde{C}(t,l) = \sum_{i=1}^{N} \tilde{c}_i(t,l) \tag{41}$$

However, since I am substituting (and allocating) the promised loan cashflows with simulated cashflows, I have to adjust in the allocation algorithms. Let $\widetilde{\mathbb{P}}_k(t,l)$ represent the simulated principal payment to the k −th bond, at time t on simulation path l, and $\widetilde{\mathbb{O}}_k(t,l)$ the corresponding pathwise outstanding principal balance on the k −th bond, with $\widetilde{\mathbb{O}}_k(\tau = 0, l)$ the original k −th promised bond balance and $\widetilde{\mathbb{O}}_k(\tau, l)$, the simulated initialized promised outstanding principal balance for the k −th bond initialized as the identical historical value used in all simulations l beginning at historical date τ so that the value corresponds with the outstanding principal balance at time $t = 0$:

$$\tilde{\mathbb{O}}_k(\tau,l) = \mathbb{O}_k(t=0), \forall l, \tau \tag{42}$$

Then, necessarily, for all $t > 0$

$$\tilde{\mathbb{O}}_k(t,l) = \tilde{\mathbb{O}}_k(t-1,l) - \tilde{\mathbb{P}}_k(t,l) \tag{43}$$

where for each simulated payment month $t > 1$, simulated principal $\widetilde{\mathbb{P}}_k(t,l)$, for $\forall$ k tranches are:

$$\tilde{\mathbb{P}}_k(t,l) = \begin{cases} for\, k = 1, \left(\max\left(0, \min\left(\tilde{\mathbb{O}}_1(t-1,l), \tilde{P}(t,l)\right)\right)\right); \\ for\, \forall k > 1, \left(\max\left(0, \min\left(\tilde{\mathbb{O}}_k(t-1,l), \tilde{P}(t,l) - \sum_{k=1}^{K} \tilde{\mathbb{P}}_{k-1}(t,l)\right)\right)\right) \end{cases} \tag{44}$$

The corresponding simulated interest payments, $\mathbb{I}_k(t,l)$ to the bonds is then:

$$\tilde{\mathbb{I}}_k(t,l) = \begin{cases} for\, k = 1, \left(\max\left(0, \min\left(\tilde{\mathbb{O}}_1(t-1,l) \times \frac{\iota_1}{12}, \tilde{C}(t,l)\right)\right)\right); \\ for\, \forall k > 1, \left(\max\left(0, \min\left(\tilde{\mathbb{O}}_k(t-1,l) \times \frac{\iota_k}{12}, \tilde{C}(t,l) - \sum_{k=1}^{K} \tilde{\mathbb{I}}_k(t,l)\right)\right)\right) \end{cases} \tag{45}$$

Finally, the total simulated cashflow payment is then:

$$\tilde{\mathbb{T}}_k(t,l) = \tilde{\mathbb{P}}_k(t,l) + \tilde{\mathbb{I}}_k(t,l) \tag{46}$$

Going back to the promised cashflows for a moment, I need to note that the face amount of the bond based upon the promised principal cashflows is:

$$\mathbb{F}_k(t) = \sum_{t=1}^{T} \mathbb{P}_k(t) \tag{47}$$

and so I can represent the risk-neutral fair value price as a percent of par for the k-th bond as:

$$b_k(t,l) = \frac{\left[\sum_{l=1}^{L} \sum_{t=1}^{T} \tilde{\mathbb{T}}_k(t,l) e^{-r(\tau)t}\right]}{\mathbb{F}_k(t) L} \tag{48}$$

where, as before, τ is the historical simulation date and $r(\tau)$ is the 5 year on the run risk free rate as utilized by DVH, 2012 who do not specify an interest rate process. The observed market price for the k-th bond is $m_k(t)$. So, the risk metric of Theta for the k-th bond is:

$$\theta_k(t,l) \equiv b_k(t,l) - m_k(t) \tag{49}$$

When I want to calculate the composite value for Theta across $\forall k$ I first weight the fair value by the relevant outstanding principal balance and total trust principal such that the weight of the k-th bond is:

$$w_k(t) = \frac{\sum_{t=1}^{T} \mathbb{O}_k(t)}{\sum_{k=1}^{K} \mathbb{F}_k(t)} \tag{50}$$

which of course *is not* the same percentage as the bond weight at origination. As principal pays down according to schedule, the AAA bond, $k = 1$, will decline in relative weight vs. the other bonds. This weight then is used to give a composite fair value price as:

$$b(t,l) = \sum_{k=1}^{K} w_k(t) b_k(t,l) \tag{51}$$

and the composite market price as:

$$m(t) = \sum_{k=1}^{K} w_k(t) m_k(t) \tag{52}$$

which allow us to then express Theta as a composite value across all bonds in the trust as:

$$\theta(t,l) \equiv b(t,l) - m(t) \tag{53}$$

1.4.3. Term vs. ruthless default: Importantly, I pursued further the issue of ruthless default versus default at maturity. In Merton, 1974 the closed form solution restricts default from occurring prior to the debt maturity date. So, in the life span of a loan, the date of primary interest from a creditor perspective to the company is the date at which the loan is meant to be repaid. There are arguments for and against this approach. Jarrow, etal 2008 show that many loans in the sample that exhibited default did so prior to maturity. Similarly, in the small sample of 172 loans used in this analysis, 10 actually defaulted in the historical period following the analysis, and all 10 exhibited default prior to their maturity date. The contra position speaks to ruthless default behavior speaks directly to leverage. If, as is the case with a sizable portion of outstanding loans in the CMBS universe, the borrower has an Interest Only loan, from the borrower perspective, as long as the NOI on the property is sufficient to pay the debt service on the loan, then temporary declines in the price driving it below the amount of debt outstanding may not drive the property holder to default on its obligations to the creditors. A rebuttal of course is that with no principal payments, there is no ‘skin’ in the game and the likelihood of ruthless default increases. The debate is ongoing one and not resolved here. What is resolved is that evidence of ruthless default does exist in the literature and anecdotally in industry business practice. The risk is real and it should be considered.

Shown are two plots (Figure 56 and Figure 57); one for AAA and one for BBB- across 3 years of trading, daily. The *Black* series are the actual prices of CMBX from the marketplace. The *Gold* series are DVH’s model with 30 loans and 3 property types with maturity restricted default and no statistical adjustment ex-post. The *Blue* series is also maturity restricted default, but with all the adjustments made this Fall (172 loans, 6 property types, correct maturity date, etc.). The *Red* is the most recent version of DVH extension incorporating ruthless default. These plots suggest that bond traders anticipate ruthless default as a behavior to be expected of CRE borrowers, thereby *pricing it into* CMBX, resulting in greater convergence to market pricing.

Corresponding to intuition, the lower rated BBB- tranche exhibits greater sensitivity to the prospect of this borrower threat of exercising the default option versus the exercise of default restricted to the maturity date of the loan. As a result, the BBB- Ruthless simulated price remains more compressed in keeping with the market price observations than the maturity default simulation for the same instrument. Finally, particularly for the BBB- series, the incorporation of the improvements in the data and expansion of the property types reflected in both *Red* and *Blue* series show significant improvement over the model from the DVH (*Gold*) with respect to the fit to market prices (*Black*). As such, when I layer in the ex-post statistical analysis, I would expect to see somewhat different, if not better, results those previously calculated. As anticipated the probability of default associated with Ruthless Default behavior is larger for all property types than the pdef restricted to maturity under simulation (Figure 58). Interestingly, Merton's closed form which is restricted to maturity approaches the ruthless behavior under simulation at the REIT driver level with results similar to those discussed at the loan level in the sample in Model 2. Finally, the Expected Losses over the study period generated under the Ruthless Default simulation and non-Ruthless default are as shown in (Figure 59).

1.4.4. Results: In the initial comparison (Figure 60) the profile of fair value across all bonds is much more stable. This reflects the modified incorporation of the cashflows, as well as ruthless default. Most striking is the reversal of the composite profile of fair value in Model 3 vs. Model 2. I see a tighter relationship with market pricing and, consistent with the results of Model 2 a strong indication that the market pricing of the securitization overall (black line) more than compensated buyers for the risks as contemplated in the application of the Model 3 assumptions to the 172 loans underlying GG5. This reflects the importance of the accurate timing of cashflows when making judgments about the relative risk and rewards of securities within the securitized markets. Figure

61 shows again clearly that not all bonds are exposed to default risk equally. Some are more exposed to risks than others; and Theta does a good job of disclosing the relative risk/reward profile of the tranches and when they are more/less sensitive to changes.

1.5. Model 4: Reduced Form - Jarrow, etal 2008: In this section I consider the reduced form approach of Jarrow, etal 2008 as an alternative to the Merton, 1974 approach (Model 1) and the generalized approach to the calibrated simulation (Model 3) and its special case (Model 2). A 27 factor correlated economy is simulated and CREL valuations are conducted under a 9 factor HJM where loan level default barriers are governed by State variables with events modeled using a Cox Process. Despite the difference with other approaches in 'triggering' default the goal of the reduced form approach is the same: specifically, to seek the present value of default/loss adjusted simulated cashflows.

I use the notation and allocation algorithm provided in Model 3 where the simulated cashflow for the k-th bond at time t on simulation path l is given as:

$$\tilde{\mathbb{T}}_k\left(t,l\right)=\tilde{\mathbb{P}}_k\left(t,l\right)+\tilde{\mathbb{I}}_k\left(t,l\right) \tag{54}$$

and the face amount of the bond based upon the promised principal cashflows is:

$$\mathbb{F}_k(t) = \sum_{t=1}^{T} \mathbb{P}_k(t) \tag{55}$$

This enables us to represent the risk-neutral fair value price as a percent of par for the k-th bond as:

$$b_k(t,l) = \frac{\left[\sum_{l=1}^{L}\sum_{t=1}^{T}\tilde{\mathbb{T}}_k(t,l)e^{-r(\tau)t}\right]}{\mathbb{F}_k(t)L} \tag{56}$$

Given the observed market price for the k-th bond is $m_k(t)$ the risk metric of Theta for the k-th bond is then:

$$\theta_k(t,l) \equiv b_k(t,l) - m_k(t) \tag{57}$$

The notable sole distinction between the expression for fair value in Model 4 and Model 3 (above) is that the risk free rates in Model 4 are path dependent (not static and historic as of time τ) and generated within a multifactor Heath-Jarrow-Morton, 1992 ("HJM") term structure framework[26] such that for Model 4:

[26] To ensure that there is continuity with Models 1, 2 and 3 where term structure is *not* contemplated, though I only select $r(t = 60, l)$ the 5 year pathwise simulated forward risk-free rate, $f(60, t)$, so $r(t, l) = f(60, t, l)$ from the vector of rates that make up the pathwise term structure generated under HJM. I thereby eliminate concerns as to whether the different rates that constitute the term structure are disproportionately responsible for differences among Model 4 fair value compared to the other three models. The forward rate are pathwise and determined under HJM, but I only select a single rate for each path and apply it to all simulated cashflows for discounting. In the Cox Process, I use the entire pathwise forward term structure under HJM. This is a useful simplification; it does beg the question, however, as to the impact of the entire

$$b_k\left(t,l\right)=\frac{\left[\sum_{l=1}^{L}\sum_{t=1}^{T}\tilde{\mathbb{T}}_k\left(t,l\right)e^{-r(t,l)t}\right]}{\mathbb{F}_k(t)L} \tag{58}$$

As discussed, I have historic volatilities for NCREIF and each of its property x regional elements, simulation of each of them is as comprehensive for our modeled economy as simulating each of the components of synthetic NCREIF. I seek to generate a composite view of the economy in the future of which actual NCREIF is simulated as one of several cross-correlated random variables. This gives us greater precision in mapping the loan level parameters of interest related to default to elements of our simulated economy.

While NCREIF is quite informative, (see Appendix C) it is surely not the entire US economy. To gain a correlated and distributed perspective on possibilities in the future I broaden the components of NCREIF to include property *x* region sub-indices (traded) and BBREIT indices (also traded) in addition to the entire risk-free term structure. It is worth reiterating a point on the choices of NCREIF and BBREIT/ICF. NCREIF reflects a stable and well regarded source for property values throughout the US. It reflects values of CRE reported by commercial banks, investment banks, pension funds, and life insurance companies. In contrast, BBREIT/ICF reflects

term structure on Theta, as well as the value of interest rate risk, and credit-risk decomposition, all of which are areas of inquiry in current research building on this dissertation, but outside its scope.

property specific REIT prices. By including *both* types of indices (both of which are tradable) I capture a difference between the market's view of property specific risks (BBREIT, much more volatile) versus a property fundamentals perspective on a mostly unlevered long term buy and hold portfolio of CRE assets (NCREIF, less volatile). Figure 62 shows the differences in the historical volatility of the national NCREIF across all property types and the National BBREIT index, which confirms intuition that REITs are much more volatile than the properties owned by them.

To ensure that our inputs were consistent with the historical record, I examined the historical record for the daily returns for the Office (OF) property type for NAREIT (a REIT index) and NCREIF. As expected the NCREIF index historically exhibits more muted volatility than the REIT index as NCREIF is a measure of longer term property value, while REITs are a measure of pricing daily expectations that may be influenced by factors apart from those traditionally associated with CRE valuation. This pattern is consistent across all property types in comparing REITs to NCREIF. The historical daily volatility for REITs exhibits much greater volatility than the spline fitted NCREIF indices for all property types, is consistent with the historical record (Figure 63).

The stochastic processes for all $i = 1, \ldots, 27$ factors of the economy including the property x regional indices, the regional property indices, the REITs and the interest rates are:

$$dV_{i,l} = rdt + \sigma_i dZ_{i,l} \tag{59}$$

Where r denotes the risk-free rate and $dZ_{i,l}$ the $i = th$ Brownian motion representing correlated shocks for the economy. All factors are correlated with one another as described in the procedure above. So, for this model with $i = 27$ correlated components of the economy I write:

$$Corr(dZ_j, dZ_k) = \rho_{j,k} dt \quad j,k = 1,\ldots,27 \tag{60}$$

for each draw.

I adopt the technology in a multifactor approach to simulate the entire forward rate term structure for our economy using Heath-Jarrow-Morton, 1992 allowing the accurate modeling of the evolution of the entire forward rate curve. For our simulation I require the elements of the risk free term structure to be equivalent to a package of zero-coupon bond with unique discount rates that satisfies

$$\sum_{i=t}^{T} c_i e^{-ri} + p_i e^{-ri} = \sum_{i=t}^{T} c_i e^{-z_i i} + p_i e^{-z_i i} \tag{61}$$

where c_i is the coupon of a coupon bearing CMT on the run bond, p_i is its principal payment at maturity, r is the constant CMT yield to maturity, t, is the time of receipt of cashflow, and z_i is the unique vector of discount rates with the interpretation of theoretical zero-coupon bond yields from $i = (0.5, 30)$ and which, together, constitute the spot rate term structure of risk free interest rates.[27] From the spot rates I can construct the entire forward rate curve where each forward rate is noted as[28] $F(t,T)$ and where the set of all forward rates on (t,T) constitutes a forward rate evolution. Since each unique spot rate of interest $r(t) = F(0,0)$ is a forward rate where the maturity date is equal when I say I am simulating forward rates, I use as the core the set of forward rates that are also the spot rates for all (t,T), Unlike the other n parameters, under the multi-factor HJM framework the forward prices each have a drift term that is correlated with their historic volatility where in a n-factor model I will have n corresponding Brownian motions $Z_1(t),...,Z_n(t)$ to work with to generate forward rates from the initialization point of the simulation. So, for our purposes the basic multifactor HJM model is

$$f(t,T) = f(0,T) + \sum_{i=1}^{n} \int_0^t \alpha_i(t,T)dt + \int_0^t \sigma_i(t,T)dZ_n(t) \tag{62}$$

where the forward rate process starts at time $t=0$ with the forward rate $F(0,T)$ and evolves driven by various Brownian motions and a drift and in discrete terms as

[27] For further explanation of the bootstrap method see Fabozzi, 1993.

[28] For example, the 3 month forward rate in 3months, the 10 year forward rate in 10years, etc.).

$$f(t+dt) = f(t) + \sum \alpha_n \sqrt{dt} + \sum \sigma_n Z_n \sqrt{dt} \tag{63}$$

$Z_n = Z \neq Z_k$ where Z_k is defined as the correlated random shock for parameter k across all k parameters,

$$f_k(t+dt) = f_k(t) + \sum \alpha_k \sqrt{dt} + \sum \sigma_k Z \sqrt{dt} \tag{64}$$

and where Z is simply a separate uncorrelated random draw for a given parameter *k*.

Since I want to *correlate* the HJM forward rate evolution of $k = 1, 2, \ldots, 9$ forward rates with the 18 other elements of the economy, I substitute the correlated Z_k for Z_n by implementing the standard Cholesky decomposition applied to the variance-covariance matrix Σ determined from the correlation history of required Z_k values embedded within all historic X_k values within a stochastic process framework. I observe the matrix Σ has three characteristics:

1. It is symmetric such that $\Sigma^T = \Sigma$;
2. The diagonal elements satisfy $\Sigma_{i,l} \geq 0$; and
3. It is positive semi-definite so that $x_k^T \Sigma x_k \geq 0$ for all $x \in \mathbb{R}^n$.

Since these three conditions are met[29], I can use the Cholesky decomposition which satisfies:

[29] See Haugh (2004) for further review of this procedure.

$$\Sigma = C^T C \tag{65}$$

as the matrix Σ is 27x27, and I can find matrix C. Given matrix C, I create a row vector w_i of independent random draws on $\sim N(0,1)$ and take the product of matrix C and vector $w_i(t,l) = rand(N(0,1))$ of 27 random draws to create 27 *correlated* random draws, $Z_k(t,l)$, for each time step

$$Z_k(t,l) = \sum_{i=1}^{27} C_{ki} w_i(t,l) \tag{66}$$

where each Z_k is correlated amongst all 9 forward rates *and* 18 property indices and the discrete form of the HJM evolution is then

$$f_k(t+dt) = \sum f_k(t) + \alpha_k \sqrt{dt} + \sum \sigma_k Z_k \sqrt{dt} \tag{67}$$

which then allows us to simulate $k = 1, 2, \ldots, 27$ State variables, representing the cross correlated US Economy

$$x_k(t+dt,l) = x_k(t,l) + \sigma_k Z_k(t,l)\sqrt{dt} \tag{68}$$

together, $X_k(t,l)$:

$$X_k(t,l) = \begin{bmatrix} x_1(1,l) & x_2(1,l) & \cdots & x_{27}(1,l) \\ x_1(2,l) & x_2(2,l) & \cdots & x_{27}(2,l) \\ \vdots & \vdots & \ddots & \vdots \\ x_1(360,l) & x_2(360,l) & \cdots & x_{27}(360,l) \end{bmatrix} \tag{69}$$

While I simulated the state variables in our economy for each month forward for 30 years, I discount for valuation purposes with an associated path-wise term structure which is transformed from the simulated forward rates. A smoothing procedure of interpolation is used to construct l theoretical monthly term structures that are consistent with our simulated economy and appropriate for loan level simulated cashflows of principal.

In Figures 64a, 64b and 64c I show snapshots of the correlation matrix, the variance covariance matrix and the Cholesky decomposition generated in the simulation code[30]. This procedure is initialized on each simulation date, τ. I show for descriptive purposes a few snapshots of values generated by the correlated simulation in Figure 65. The results are intuitive with respect to the volatility of NCREIF vs. REITs as well as the difference in the periods of the Crisis with REITs exhibiting broader distribution of paths prior to the Crisis versus property values measured by NCREIF and both REITs and NCREIF exhibiting sustained levels of volatility as the worst of

[30] Whenever practical parallelized computation was utilized to optimize speed distributed across internal microprocessors.

the crisis subsides with NCREIF showing relatively more persistent uncertainty than REITS when compared with their pre-crisis levels.

1.5.1. The state of the loan: Now that I have established the simulation of the US economy, I have to discuss the risk of default pathwise under simulation. The state of the loan (current, delinquent, or default) is considered at each step of the simulation. Inputs to the realization of a new state for the loan on the simulation path are the correlated random variables of the economy previously described as well as the state of the loan at the time step. In this sense the state of the loan which is stochastic governs the cashflow and future cashflow of the loans by either defaulting or not as per the method below. What I am going to do is to link the state variables to property characteristics and to employ a choosing process, or modified Cox Process, to visit the risk of defaults. The link between the simulated economy and the loan state is established by using the MLE's as coefficients within the Cox Process. The coefficients that govern our choices at each (t,l) are thus the MLE's where the hazard rate estimation was done separately for fixed-rate and floating-rate loans. Figure 66 shows the loan state transitions over 2.2mm loan life observations from 1998 to 2005; Figure 67 contains a summary of the loans contained in the estimation. For non-CTLs, the focus of this dissertation, there are 94,011 fixed-rate loans. The number of defaults for the fixed-rate loans is 2,153. The parameter estimates for a competing risk current versus delinquent point process and for the default point process are shown in Figure 68. The parameter estimates are based on

$$\text{intensity} = \frac{1}{\left(1 + e^{-\sum_i \text{coefficient}_i \cdot \text{variable}_i}\right)} \tag{70}$$

and I discuss the specific implementation further below.

1.5.2. Property characteristics, $U_i(t)$: To simulate defaults in our economy made up of 27 State variables I need to link the state variable to property specific parameters with significance to the events of default and loss. Formally, I identify 10 property characteristics of any commercial real estate loan that demonstrated significance in Jarrow, etal 2008 with respect to modeling historical defaults, $U(t)$. 2 are time dependent 8 are static determined at origination. The 8 static Property Variables related to a property's potential default in the simulated economy are:

- ACLI foreclosure index at origination
- NOI = (Original NOI/Original Loan Balance)
- Original Loan Balance = Log of Original Loan Balance
- DSCR
- LTV
- Loan Coupon
- Coupon Spread = Coupon – Risk Free Rate at Origination

The 2 time dependent Property Variables related to a property's potential default in the simulated economy are:

- Age of loan (t): = (1 – remaining term/original term)
- Delinquency Status of the Loan (t):
  - 0, current
  - 1, delinquent
  - 2, defaulted loan, real estate owned (REO)

1.5.3. Delinquency Status, $N_i(t)$: The delinquency status of the loan contributes to the likelihood of a loan defaulting at some future point in time. In REE 2008 2.2mm loan life transitions were evaluated. The results of that study showed a tendency for loans at 60-89 days delinquency to transition to a worse state. As such, for continuity with 2008 the study and for reasons supported by it, I compress the characteristic delinquency state of loans at initialization of the simulation into current (0-59 days), delinquent (60-90+), or default (REO/Foreclosure) as of the initialization date of the simulation. This data characteristic for each loan is provided by Trepp, LLC. Once the simulation begins, the delinquency status is no longer historic, but simulated, based on the default process discussed below.

1.5.4. Use of MLE Coefficients: I utilize in this dissertation the MLE's associated with the state variables and the Property Characteristics in the simulation in the choosing process of the simulation to visit whether a loan will default at some time t on some path l in the simulation[31]. I

[31] The detailed account of all MLE's and discussion of the delinquency and default intensity process are provided in Appendices D & H.

assume, necessarily, that the present relationships between the simulated economy and the loan profiles in my sample are substantively similar to those in the historical study which covered 1998-2005 from which the MLEs were determined. Refer to Figure 4, previously mentioned, which shows a sample of MLE for one property x regional pairing of multifamily x northeast. The state variables $X_t = X_k(t,l)$ and are non-deterministic (*random*) as they are simulated and vary through times. Each $X_k(t,l)$ state variable has a corresponding parameter estimate (*coefficient*) ψ_k which are constants and *do not* vary through time, giving:

$$\psi_k X_k(t,l) = \begin{bmatrix} X_1(1,l) & X_2(1,l) & \cdots & X_{14}(1,l) \\ X_1(2,l) & X_2(2,l) & \cdots & X_{14}(2,l) \\ \vdots & \vdots & \ddots & \vdots \\ X_1(360,l) & X_2(360,l) & \cdots & X_{14}(360,l) \end{bmatrix} \begin{bmatrix} \psi_1 & \psi_2 & \cdots & \psi_{14} \end{bmatrix} \tag{71}$$

Notice that only $k = 14$ of 27 state variables are represented. This is of course because the loan level characteristics associated with the MLE are only relevant for the subset of all $k = 27$ state variables that were simulated. So, for example an MLE associated with Office Indices is irrelevant to a Retail property being simulated even though both Office and Retail property indices were simulated under the correlated procedure discussed. Similarly, a property with location in the Northeast does not have a corresponding MLE estimator in the Midwest. Thus, the state variables considered in the code are a subset of the total state variables simulated, $X_{k=14}(t,l) \subset X_{k=27}(t,l)$. Specifically,

- 9 Forward Treasuries (3mo, 6mo, 1yr, 2 yr, 3 yr, 5 yr, 7yr, 10yr and 30 yr)
- 3 NCREIF Property Value indices
    - o 1 All Properties/All Regions
    - o 1 Property Specific Indices (Multifamily, Lodging, Industrial, Office, Retail, Other)
    - o 1Regional Specific Indices (East, West, South, Midwest)
- 2 BBREIT/ICF Stock Price indices
    - o 1 All Properties
    - o 1 Property Specific Indices (Multifamily, Hotel, Industrial, Office, Retail, Other)

Like the state variables, property specific characteristics $U_i(t)$ for all i loans are also modified with parameter estimates ϕ which is a vector of constants (determined again from the MLE study) corresponding to each property specific characteristic, $\phi U_i(t)$. The purpose of $\phi U_i(t)$ is to relate the loan specific characteristics $U_i(t)$ for a given loan i on the simulation path l. Coupled with the state variables, we see the beginning of a joined influence of the simulated events $\psi_k X_k(t,l)$, deterministic loan profiles $\phi U_i(t)$, and the $\theta_d N_i(t)$ updated loan payment status, in $\mathbb{Q}_i(t,l)$ forward through time $(t:\mathrm{T})$ across different outcomes $(l:L)$ in the simulated economy. This leaves us approximately with

$$\mathbb{Q}_i(t) \approx \underbrace{\psi_k X_k(t,l)}_{macro} + \underbrace{\theta_d N_i(t) + \phi U_i(t)}_{micro} \tag{72}$$

where the two influences on the payment status $\psi_k X_k(t,l)$ and $\theta_d N_i(t)+\phi U_i(t)$ can be considered macro- and micro-economic influences, respectively.

1.5.5. Reduced form probability of default: I consider one risk, default, to illustrate the basic technique of comparison between simulated (macro) and loan specific (micro) influences on the loan payment status. This has the characteristic of questioning the valuation of such loans amidst an evolving economy. The basic technique is similar to what I did previously. Earlier I simulated variables that were informed by a calibration. Here I also simulate variables and relate them to coefficients for the purpose of estimating the probability of default, or specifically, default intensity[32] λ_d,

$$\lambda_d[t, N_i(t), U_i(t), X_k(t,l)] = e^{\varphi_d + \theta_d N_i(t) + \phi_d U_i(t) + \psi_d X_k(t,l)} \tag{73}$$

where the interval between sequential time observations is Δ given the loan payment histories and the times series observations for the state variables $X_k(t,l)_{t=1}^{T}$

[32] A primer on the use of the Poisson process in modeling default intensity leading into the use of the Cox Process as introduced by Lando and an advancement from ordinary jump diffusions is found in Appendix G. A detailed account of the 'switching' is found at the end of Appendix H.

$$\lambda_d[t, N_i(t), U_i(t), X_k(t,l)]\Delta = \frac{1}{\left(1 + e^{-[\varphi_d + \theta_d N_i(t) + \phi_d U_i(t) + \psi_d X_k(t,l)]}\right)} \tag{74}$$

has the interpretation[33] of being the probability of default over the interval $[t, t+\Delta]$.

1.5.6. Payment State Transition Process: I consider the simple Cox process which can thought of as a kind of measuring stick with which to gauge a conditional action of default within the simulation. I use the MLE to determine estimates for default payment states on a simulation path informed by their *prior* switching state on the path between current and delinquent. I initialize the simulation in the current state, 0, (it could be initialized as delinquent, 1). At each time step t I calculate an intensity for a current state λ_c,

$$\lambda_c[t, U_i(t), X_k(t,l)] = e^{\varphi_c + \phi_c U_i(t) + \psi_c X_k(t,l)} \tag{75}$$

with associated probability of being current as

$$\lambda_c[t, U_i(t), X_k(t,l)]\Delta = \frac{1}{(1 + e^{-[\varphi_c + \phi_c U_i(t) + \psi_c X_k(t,l)]})} \tag{76}$$

[33] See Jarrow, etal 2008, pg 458.

and an intensity for a delinquency state λ_q where

$$\lambda_q[t, U_i(t), X_k(t,l)] = e^{\varphi_l + \phi_l U_i(t) + \psi_l X_k(t,l)} \tag{77}$$

with associated probability of being delinquent as

$$\lambda_q[t, U_i(t), X_k(t,l)]\Delta = \frac{1}{(1 + e^{-[\varphi_l + \phi_l U_i(t) + \psi_l X_k(t,l)]})} \tag{78}$$

The tree below describes the payment state assignment at each time and each path for each loan.

$$\begin{array}{l}
\mathbb{S}_i(t-,l)=cur(t-,l) \searrow \\
\mathbb{S}_i(t-,l)=dlq(t-,l) \nearrow
\end{array}
\Rightarrow \mathbb{C}_i(t,l)=f(\lambda_q,\lambda_c)
\begin{array}{l}
\nearrow \mathbb{S}_i(t,l)=cur(t,l) \nearrow \left[\mathbb{Q}_i(t,l)=f(\lambda_d)\right]
\begin{array}{l}
\nearrow \mathbb{S}i(t{:}t+1,l)=cur(t{:}t+1,j) \Rightarrow \mathbb{S}i(t+1,l) \\
\searrow \mathbb{S}i(t{:}T,l)=def(t{:}T,l) \Rightarrow \text{absorb/end}
\end{array} \\
\searrow \mathbb{S}_i(t,l)=dlq(t,\mathrm{l}) \searrow \left[\mathbb{Q}_i(t,l)=f(\lambda_d)\right]
\begin{array}{l}
\nearrow \mathbb{S}i(t{:}t+1,l)=def(t{:}t+1,l) \Rightarrow \mathbb{S}i(t+1,l) \\
\searrow \mathbb{S}i(t{:}T,l)=def(t{:}T,l) \Rightarrow \text{absorb/end}
\end{array}
\end{array}$$

(79)

Let $\mathbb{S}_i(t-,l)$ be the payment state of the loan at the beginning of each simulation period. The loan enters the system as either current or delinquent. Default is an absorbing state and thus in the interval from the time of a realized default on a simulation path τ to the stated maturity date T, the loan will remain in default in each period of the simulation. Let $\mathbb{C}_i(t,l)$ in the model structure be the choosing process to determine the state of the loan immediately following entrance into the system. At this stage the loan may transition/switch to another state of delinquent or current based upon the random draw from the Poisson distribution with the stochastic interarrival rates, λ_c and λ_q as described below; I can call $\mathbb{C}_i(t,l)$ the delinquency process. Immediately following this loan payment state assignment, the loan is then exposed to another process $\mathbb{Q}_i(t,l)$ to determine the final payment state for the loan at time t. This also entails a random draw from the Poisson distribution with the stochastic interarrival rates, λ_d. I call $\mathbb{Q}_i(t,l)$ the default process, as described below. When the loan transitions to a worse state the indicator variable is set equal to 1. I make a separate Uniform random draw $Z\,(t,l)$ outside the simulated economy to determine the state prior to the choice with the *threshold* condition here defined as the transitioning value for $\mathbb{C}_i(t,l)$ where:

$$\text{If}\begin{bmatrix} \mathbb{C}_i(t,l) = current\ \&\ Z \le \lambda_q \Rightarrow delinquent = 1; \\ \mathbb{C}_i(t,l) = current\ \&\ Z > \lambda_q \Rightarrow current = 0; \\ \mathbb{C}_i(t,l) = delinquent\ \&\ Z \le \lambda_c \Rightarrow current = 0; and \\ \mathbb{C}_i(t,l) = delinquent\ \&\ Z > \lambda_c \Rightarrow delinquent = 1 \end{bmatrix}$$

with *1=delinquent* and *0=current.* I do not have prepayment penalty criteria information, for the loans in this study and thus I do not consider the intensity process

$$\lambda_p[t, N_i(t), U_i(t), X_k(t,l)] = e^{\varphi_p + \theta_p N_i(t) + \phi_p U_i(t) + \psi_p X_k(t,l)} \tag{80}$$

and associated probability of prepayment[34]. Importantly, by eliminating prepayment, a direct comparison with 'credit-only' models in the future (from rating agencies, for example) is made much easier.

In summary, the delinquency/current process $\mathbb{C}_i(t,l)$ occurs at every timestep for each of the i loans and has the effect of turning the delinquency status coefficient on or off in default intensity, λ_d. Specifically, when $\mathbb{S}_i(t,l)=1 \Rightarrow N_i(t,l)=1 \Rightarrow |\theta_d N_i(t,l)| > 0$ which is then used in the default process $\mathbb{Q}_i(t,l)$. $\mathbb{Q}_i(t,l)$ is the Cox Process for the hazard of default that considers the payment state $\mathbb{S}_i(t,l)$ of the loan determined by the process $\mathbb{C}_i(t,l)$. The delinquent status is not always arrived at, and when it is, it does not guarantee default[35], as default is governed by $\lambda_d[t, N_i(t), U_i(t), X_k(t,l)]\Delta$ which is statistical, *not* deterministic. As in $\mathbb{C}_i(t,l)$, the process $\mathbb{Q}_i(t,l)$ where the absorbing default state[36] may be realized requires:

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[34] Treatment of prepayment is done effectively in Jarrow, etal 2008 and is outside the scope of this study.

[35] Importantly, note that upon arriving to the default process $\mathbb{Q}_i(t,l)$ which utilizes the default intensity λ_d as a lower bound, that if the payment state $\mathbb{S}_i(t,l) = 1$ indicating delinquency, then this has the explicit effect of updating the credit State variable $N(t,l)$ to $N(t,l) = 1$ at time t on path l. The impact on the default intensity λ_d is that $|\theta_d N_i(t,l)| > 0$ at time t on path l. Otherwise, of course, $\theta_d N_i(t,l) = 0$.

[36] In the code we convert the 0,1 non-event/event notation to the familiar industry status *2=default, 1=delinquent* and *0=current.*

$$\text{If}\begin{bmatrix} \mathbb{Q}_i(t,j) = current \ \& \ Z \le \lambda_d \Rightarrow default = 1; \\ \mathbb{Q}_i(t,j) = current \ \& \ Z > \lambda_d \Rightarrow current = 0; \\ \mathbb{Q}_i(t,j) = delinquent \ \& \ Z \le \lambda_d \Rightarrow default = 1; and \\ \mathbb{Q}_i(t,j) = delinquent \ \& \ Z > \lambda_d \Rightarrow delinquent = 0 \end{bmatrix}$$

As we see the model contemplates the realistically unlikely, but nevertheless possible (*as seen in data*), event of a transition from the current state to the default state[37], bypassing delinquency. Default is an absorbing state, such that if default occurs at some time t, on some path j the loan cashflow on that path terminates on (t, l) and the recovery rate process begins for that loan on the path. If default does not occur, the payment state $\mathbb{S}_i(t+1, l)$ of the loan following both choices at each time step becomes the persistent 'new' state of the loan at simulated $t+1$. At T, the loan matures as promised on a path l if no default occurs prior to T. Again, a detailed description of 'switching' is found in Appendix H.

For any simulation path that generates a payment state of default, the loan is immediately captured and stored and indexed with respect to its default time t and path l. In this implementation I assume a simple constant loss rate for a loan that reaches default and a constant time to recover proceeds from sale of the property[38]. Depending on whether one is interested in valuing only the

[37] This can be changed to ignore this possibility resulting in choices from $\mathbb{Q}_i(t, l)$ *current* to delinquent vs. to default.

[38] Recovery and loss rates could be modeled as stochastic processes; see Jarrow, etal 2008.

loan, or the loan in a securitization, or both, the technique is equipped with the HJM interest rate process to immediately discount the default adjusted cashflows reflecting deterministic recovery rates on risk neutral term structure suitable for pricing of all path dependent objects.

1.5.7. Results – Fair Value: I repeat the snapshot for fair value and Theta results presented in each of the prior models. What we see is the 'calmest' of all the models. The risks captured through the Cox Process, the state variables, the incorporation directly of delinquency and default data enable us to secure a perspective on the risk of the loans and valuation for the bonds with the most comprehensive data and technique of the Models presented. The composite pricing shows substantially attractive pricing versus risks in Fair Value versus pricing of the market (Figure 69). Again, as in prior cases, this is categorically the case across all bonds at all times. Figure 70 provides us with a precise view into the risk/reward profile of the bonds under the reduced form technique using Theta. As before there is considerable differences amongst bonds x time both relative to one another and versus the capital structure overall. Finally, we refer back to Figure 2a in which I compare the measurement Theta across all four Model approaches. Based upon the evidence, the reduced form approach is the most compelling. It is unique in its ability to incorporate vital information about the loan profiles in balanced way. As such, the signals appear to counter those proposed by DVH, 2012 regarding efficiency and pricing in the CMBS market and are more consistent with those suggested by Jarrow, etal 2008. As noted the actual default experience of the GG5 transaction with a more than 20% lifetime default rate, bolsters this perspective.

1.5.8. Limitations: As always there are caveats and limitations. *First*, the MLE's are 'stale'; meaning that although they were estimated over more than 2mm loan life observations, the cut-off date for that data was in 2006. As such, while the relationships contemplated in the MLE's is probably still valid, the financial crisis may have impacted them. Nothing can be done about this without access to data which is costly and difficult to obtain. *Second*, the delinquency and default experience is estimated from actual default tables of GG5, but they were not provided for each historical date at the loan level. The mapping of the deal level default experience which was available for all historical dates combine with sporadic loan level defaults for the Crisis mapped well. Nevertheless, more granular and regular data would be beneficial.

Section 2: Recovery Period (January 2013 thru March 2014)

In the previous section, I considered the underlying risk profile of CMBS/CMBX and the risks of the underlying cashflows within the financial crisis. Our bond pricing data for the Crisis was isolated to CMBX Series 1 from the period 11/1/2007 thru 12/31/2010 (the Crisis). In Figure 71 (boxed in purple) we see a spike in probabilities of default for all CRE property types, in early 2012, from the perspective of the Merton model. Since the Merton model is forward looking to the maturity of the debt one implication from Merton is that the *Recovery* began in earnest around the beginning of the 3rd quarter of 2012 (Figure 71, arrow).

2.1. Data – Recovery: While the loan level data is representative of the CMBS Universe it is not comprehensive. Additionally, I only have pricing for CMBX Series 6 tranches, and not the underlying bond tranches (collateralized, by the loans) and may therefore only consider at most 6 priced bond objects on any trading day from which to select a portfolio. This is similar to the limitation of the earlier section during the Crisis where we were limited to discussion of CMBX Series 1 and similar to the limitation of DVH 2012.

To reiterate, however, the prices of CMBX are quite rich and generally must reflect the pricing of the 125 underlying tranche collateral. Where this study and model approach adds insight is that it peers through to the loan collateral securing the tranche collateral that secures the top level CMBX tranche pricing. If there is a disconnect between the loan level risk and bond level risk then we should see differences across model approaches in comparison of model fair value to market prices. A discussion of the cashflows produced by the code can be found in Appendix F.

The CMBX Series 6 capital structure is provided in Figure 72. The subordination levels and coupons are determined from the average subordination levels and from review of the prospectus supplements of the underlying 25 transactions. The CMBX Series 6 structure implemented assumes a simple senior subordinated structure as previously discussed for the CMBX Series 1. The pricing of the CMBX tranches through BB were obtained from Markit. Tranches below BB are not priced but exist and serve as the first loss piece of the capital structure. The data for pricing and the underlying economy is daily and provided thru 3/7/2014.

2.2. Valuation - Recovery: As in Section 1, I compute the fair value price and Theta for each of the tranches in this case for CMBX Series 6. I consider the exact same models, and only alter the loan and capital structure required to reflect the new period and objects under consideration. Otherwise the approach is identical. Since the purpose of the study is loan and bond valuation, I begin the analysis on 1/28/2014 which was the date of issuance of the CMBX Series 6 swap objects.

The overview snapshots for each of the models showing the tranche level Theta as well as Theta for composite price are found for Model 1 (Figures 73 & 74), Model 2 (Figures 75 & 76), Model 3 (Figures 77 & 78), and Model 4 (Figures 79 & 80). As is evident, there are substantial differences in the default adjusted pricing for each of the models. Overall, across all models the difference between fair value and the market price (Theta) is much more muted than what was generated by the models in the sample during the Crisis.

This should not be surprising for a few reasons. Unlike the loan collateral underlying CMBX Series 1 during the Crisis study where the average age of the loans at the outset of the study was 32 months, the loans underlying CMBX Series 6 are new, with the age of the loans as of January 2013 ranged from 2 months to 16 months. It would be highly unusual for new issue loans to exhibit default this early in the loan life cycle and, as stated before, the delinquency profile for all the loans in this transaction is 0 in each month during the study, within a broader market environment shift in credit characterized by substantial declines in the seasoned CMBS universe overall. Generally, loans will exhibit greater delinquency and default manifestation as they get older in the CRE universe and this has been argued to be the result of exposure of CRE property leases to uncompensated termination driven by tenant business failures as well as speculative lease-ups of properties that simply did not occur according to plan, or failure of the management company to secure new tenancy in building when existing tenants decided as their right to not renew. These and many other property specific events may occur in the loan's borrowing period (see Brueggeman/Fisher) and may result in deterioration of cashflow proceeds in the form of rents that cause defaults prior to maturity. However, lockboxes and the integrity of conduit lending programs, at least early on, mitigate such immediate deterioration in loan health early in its life.

Loans do age however, even in the Recovery, and this is surely going to be picked up in the Model 4 reduced form approach that captures the non-static information of the age in the Cox Processes that govern its delinquency default simulation. Additionally, in the case of Model 4 the information related to current delinquency status = 0 also represents a key dynamic factor. Model 4 thus stands apart from the other models considered in having by its structure the unique ability to 'digest' such telling updated loan level information as age and delinquency status of collateral which, using such updated information necessarily informs simulations of future loan cashflows at initialization for each time t. In the recovery period, marked by low volatility marketwise and characterized by current health of the loans, Model 4 will thus generate defaults assuming, accurately, the current state of the loans ($dlq = 0$) but with increasing age. Thus the transition to the default state from the current (non-delinquent) state will be less likely than in the Crisis (marked by observed delinquent profile of loans) and older collateral, but the age (and other characteristics) will also be considered resulting in non-trivial simulated delinquencies and defaults during the Recovery period as is appropriate. Transitions to the default state are taking place because of the interaction between the loan parameters (static and in the case of delinquency status, dynamic) and the simulated economy under a rich simulation which can consider extreme possibilities.

Turning to the tranchewise Theta comparison charts (Figure 80) for the reduced form Model 4, we see the black line (composite Theta) is stable across the study. The tighter AAA Theta vs. the composite Theta, indicates that AAA are more fairly priced than the composite of all tranches in the CMBX Series 6 transaction overall. Moving to the right, the AS class, subordinate to the AAA class, indicates a reasonably stable profile, but shows a shift relative to the composite, not categorically higher than it or lower than it. The AA Class exhibits higher Theta values relative to the composite, indicating greater relative value than the composite transaction overall when considering the market pricing x risk. This perspective is even more pronounced in the case of the A and BBB- tranches that too exhibit 'cheap' pricing in the market vs. the risk contemplated under Model 4. In contrast, the BB tranche is clearly absorbing simulated losses early on. Though it is not the first loss piece, there is only 3.2% subordination to the BB and any loan losses in excess of 3.2% will impact the BB tranche. Early on in the Recovery, there was greater uncertainty about the future with higher anticipated default likelihoods in the CRE sector. During the periods from

1/2013 thru 6/2013, BB pricing appears expensive outright relative to the market pricing with Theta <0. During the summer, however, following discussion by the Fed on continued accommodative policies, the underlying risk considered by the Model 4 clearly indicate a shift in the relative value of BB versus market pricing resulting Theta >0.

These relationships discussed in depth for Model 4, however are *not* consistent across all models. To build the intuition across the four models, we consider Theta for the *composite* CMBX price versus the composite market price for all four models (Figure 81). In the case of Merton, (Model 1) we see there is some differentiation in the risk estimation early on with increased distinction relative to the market price in 5/2013. Following that period, Theta becomes more muted contemporaneous with increases in CMBX Series 6 pricing. If you consider the loan level probabilities of default generated from Merton's structural form model previously discussed we see, overall, a decline in probabilities of default that are occurring contemporaneous with declines in Theta. The implication, from an investment management perspective would be that while risk is declining, and the bonds are 'cheap' overall, the opportunities are more muted than they were at the beginning of 2013. Prices are starting to look 'expensive' vs the underlying risks.

In the second panel on the right of the top row Figure 81, we consider Model 2 which is the basic calibration hybrid approach. Relative to Model 1, Model 2 exhibits a higher risk profile than Model 1. Generally, Model 2 indicates favorable pricing opportunities versus the underlying risks under the calibrated simulation, but there are periods, recently, where Model 2 seems to pick up conditions of increased uncertainty. Anecdotally, we know that there were a few days of precipitous drops in the market responding to events in the Ukraine, and this would be picked up in the volatility parameters calibrated under Model 2. Nevertheless, the serious deficits in the assumptions of Model 2, questions just how accurate estimates of Fair Value (and thus Theta) generated can be[39] and I investigate the statistical veracity of claims from Model 2 in the statistical section below where I consider efficiency.

In the second row of Figure 81 on the left we see composite Theta for in Model 3 (the 'evolved' version of Model 2). In Model 3 all assumptions in Model 2 were addressed, and eliminated[40]. What we see in Model 3 is a very tight relationship between fair value and market pricing that accurately considers the loan level collateral and default adjusted cashflows under calibrated simulated conditions. It is a much richer model than Model 2 and the trend down towards Theta=0 is consistent with Model 1 and its associated loan level probabilities of default. The conjecture that things are getting expensive for the reasons stated are clear and a quick glance

[39] We attempt to answer this question in the Section 3 in the discussion of trading tests.

[40] Specifically, I incorporate the accurate maturity, balance, interest only periods, amortization and balloon dates for the loans in the sample; increase the number of REITs from 10 to 35, informing 6 property type diffusions; and incorporate ruthless default.

at the lower rated tranche Thetas indicate indeed that defaults are being simulated (BB), but *less so* as evidenced by increases in Theta for BBB- tranche.

Finally, on the right panel of row 2 in Figure 81 we see the composite level Theta for the reduced form Model 4. In Model 4 we see a profile similar the other models with a downward compression of Theta indicating decreasing attractiveness in market prices relative to underlying risks. The profile can be said to be the most regularly conservative of the approaches with shifts in risk assessment vs. market pricing appearing to exhibit frequent and precise sensitivity. Anecdotally, when early in the summer 2013, Chairman Bernanke's comments on tapering caused markets to swoon, Theta under Model 4 increased. Similarly, in October 2013 when uncertainty surrounding the nomination of Chairman Yellen (also related to tapering) emerged causing uncertainty in the market, Theta for Model 4 also increased. At the same, time, recent events in the Ukraine did not have the immediate and transitory impact on Model 4 through the volatility parameters as they seem to have impacted Model 2. Nevertheless, prices, from the perspective of Model 4, are judged to be most expensive relative to the underlying risk compared with the three other models.

Section 3: Testing of Efficiency (Crisis and Recovery Periods)

The models discussed provide us with perspectives and insights into the pricing of their underlying risks. In the earlier portion of the study on the Crisis, I investigated with some rigor the pricing capabilities of Model 2. In particular I saw considerable evidence of significance among many explanatory variables *exogenous* to the Model 2 structure. In light of the significance of these explanatory variables I called into question the viability of Model 2 as a source for market price estimation.

As we saw in the Crisis portion of the study as well as the Recovery portion, the estimates for fair value are quite different across the different model technologies at the composite level, and vary broadly across time and rating within and across model approaches. While I had considerable success in determining accurately with high statistical significance and R-sq the missing components of price not contemplated in Model 2, the statistical exercise was motivated by the pursuit of the 'ultimate' model that would map precisely to the market price. This motivation, however, is predicated upon the assumption that market prices are efficient.

But what if market prices are *not* efficient?

In the case of fair value pricing, then, we wouldn't necessarily seek to map the fair value price to the market price and use such mapping as a barometer for success. In fact, if market prices are inefficient, we might rather *want* a low explanatory relationship between market price and the model fair value.

Based on the statistical evidence below, the prior work in this study and in other works[41], there is strong evidence that the CMBS market is inefficient consistent with groupthink behavior of crowds. CMBS has, to date, not adopted derivatives pricing technology to assist in the evaluation of underlying risks at the loan and bond level. As a sector highly exposed to several subtleties in risk exposure and valuation complexity that grapples with possibilities of default, loss, prepayment and dramatic changes of timing at the loan and bond levels, it should not be at all surprising to see evidence of imprecision in market pricing versus the underlying risk exposure. Since the Crisis and the Recovery exhibit dramatically different pricing profiles environments, they provide a rich proving ground to investigate claims of inefficiency. If my claim of CMBS market inefficiency is correct then at least two conditions should be readily disclosed from an analysis of the data across different fair value pricing models:

[41] See Jarrow, etal 2008; Stanton & Wallace, 2012; KKY, 2010; Ashcraft & Scheurmann, 2010, and others

Condition 1a – In High Stress Environments a Bad 'Fit' should indicate a Good Signal: The relationship between fair value prices generated independently from market prices should exhibit low explanatory power for the market price. This should be borne out in both OLS and quantile regressions. In the presence of several fair value estimators, the better the fair value estimator, the lower the explanatory power for market price. The conjecture is that in high stress times, the information related to the risk events cannot be parsed efficiently by market participants in the absence of a robust technology specifically designed to evaluate the likelihood of risks manifesting and quantify, dispassionately, the pricing of such events at the bond level.

Condition 1b – In Low Stress Environments a Good 'Fit' should indicate a Good Signal: In the absence of stress in the marketplace the threat of risk manifestation dissipates. In these market conditions, the market participants are able to determine pricing without having to articulate the underlying risks in a sophisticated manner. As such, the fair value in these conditions should also be sensitive to changes in market conditions concomitant with the market actors and good fair value pricing should begin to map more closely to the market pricing. So, in these environments, the conjecture is: the better the fit, the better the model, though clearly since the pricing methods differ from the market, the relationship should still be relatively weak. Exogenous factors here may contribute to explanation of market pricing in conjunction with fair value and should improve the fit.

Condition 2 - Systematic and Extraordinary returns should be made with the best model earning a.) the greatest relative returns and b.) the best empirical returns: Finally, the 'proof' is in the results of trading strategy returns using Theta. Conditions 1 and 2 in some sense are only valid if Theta generated from the fair value with the worst fit to market price actually generates the best returns. Theta should provide clear, non-random, buy and sell signals such that a portfolio manager following such signals should be able to systematically earn extraordinary returns. Additionally, if the claim of inefficiency is correct, then the better Theta, the better the relative extra ordinary returns for portfolios constructed using Theta where the better Theta is the one generated from the fair value price with the lowest explanatory power for the market price. The implication being that the market price does not reflect well the underlying risks and, as such, the veracity of claims of comprehension and transparency of risks as embedded within the market price may readily be called into question.

Necessarily, as is a common assumption, the fictitious portfolio manager must engage in small enough trades such that such moves do not unduly influence the market price. This is non-trivial, particularly in the CMBX synthetic market. Swaps below investment grade in the market such as BB frequently (as with the underlying collateral) traded on a 'by appointment' basis. The liquidity for such securities is sporadic and bid/offer spreads factor multiples of higher rated classes such as AAA. I keep these institutional subtleties in mind in the design of the trading test below, but they do represent a caveat to the conclusions.

3.1. Condition testing: The testing for Condition 1a is straightforward. I perform the most basic OLS and quantile regressions for each of the four models across the two study periods on a daily basis, the Crisis (11/2007-12/2010) and the Recovery (1/2013-3/2014). For the Crisis, I regress the composite market price CMBX Series 1 against the composite fair value price for each of Models 1 thru 4. The condition 1 testing, provides some initial support for inefficiency in the market with differentiation amongst the four models and consistent ranking across OLS and quantile regressions which are summarized in Figure 82.

All fair value metrics are significant, and it is clear that not all fair values have the same fit to the market price. The model with the tightest relationship to the market price is Model 2, my adaptation of DVH 2012 with an R-sq of .68 using OLS and a 'pseudo-R-sq' of 0.38 in the quantile regression. In contrast, the model with weakest relationship to the market price is Model 4. For that model, the R-sq exhibited is 0.18 using OLS and 0.06 using the quantile regression. If the

Model 4 metric Theta provides greater insight in the Crisis than Theta generated from Model 2, I may say the results stand in support of claims of CMBS inefficiency during the Crisis.

The testing for Condition 1b is identical to 1a. I perform OLS and quantile regressions for each of the four models which are summarized in Figure 83. What we see is a near perfect reversal from the Crisis ranking with Model 2 now showing no explanatory relationship with market pricing and also becoming insignificant. In contrast Model 3 and Model 4 show a relatively higher degree of explanatory power mapping the fair value to the market price. As before, if the reduced form Model 4 provides greater trading insight than Model 2, we may say this ordering also supports the claims of CMBS inefficiency during less stressful times of the Recovery.

For the testing of Condition 2, if we are querying the efficiency of a sector within the market, we do not necessarily seek to predict the market price with our models. Rather we seek to secure reliable signals of risk and reward using the fair value price (established independently from the market price) and then compare it to the market price, with Theta.

3.2. Trading tests: Fundamentally, if one is querying the efficiency of a sector within the market, one does not necessarily seek to match the market price to our model price. Rather one seeks reliable signals of risk and reward using the fair value price (established independently from the market price) and then comparing it to the market price, with Theta, which is a reliable benchmark for the richness or cheapness of individual securities or the entire securitization. The trading backtesting uses Theta as the *sole* means for navigating opportunities amidst risks. Generally,

$$\begin{aligned}
&\theta_k(t,l) > 0 \rightarrow b_k(t,l) > m_k(t) \rightarrow \text{"mkt px cheap vs. risks"}\\
&\theta_k(t,l) < 0 \rightarrow b_k(t,l) < m_k(t) \rightarrow \text{"mkt px rich vs. risks"}\\
&\theta_k(t,l) = 0 \rightarrow b_k(t,l) = m_k(t) \rightarrow \text{"mkt px appropriately reflects risks"}
\end{aligned}$$

While CMBS cannot be shorted outright, CMBX are credit swaps and are readily used, for example, by macro hedge funds and other leveraged investment managers to articulate long/short perspectives on term structure of credit within a sector. Additionally, sell-side firms issuing CMBS will frequently utilize CMBX to hedge the credit risk of the loan portfolio with weightings that correspond to the anticipated weighting of the TBA securitization for which the loans will serve as collateral. Since I have the CMBX price series during the Crisis and since these instruments are used to go long/short the sector, synthetically, I am well positioned to examine the market efficiency of the sector. To test the ability of each of the models to identify and achieve extraordinary returns I implemented the following procedure for a trading test using the historical data.

Step 1 – Calculate Tranche level Thetas: At the beginning of each period (day, month, or quarter) I calculate the value Theta for each of the tranches of the CMBX Series I am investigating.

In the Crisis study this is CMBX Series 1. I exclude from the study the AJ and AM tranches for CMBX Series 1 because they are not priced for the entire history – this leaves me with 5 tranches (AAA, AA, A, BBB and BBB-) from which to select the long/short portfolio[42].

Step 2 – Construct the long/short portfolios: I establish a long/short portfolio at time *t*. From the set of tranches available I purchase (long) the cheapest tranche of the set as indicated by the largest value Theta at that time. Execution is at the observed CMBX price at time *t*.

$$long_t = CMBX\_tranche_{max\theta,\, t} \tag{81}$$

I simultaneously sell (short) the most expensive tranche from the set of tranches available as indicated by the smallest value of Theta at that time. Execution is at the observed CMBX price.

$$short_t = CMBX\_tranche_{min\theta,\, t} \tag{82}$$

An apriori view of the market is not contemplated, only a systematic approach to identify relative value. Additionally, I do not explicitly introduce secondary sensitivity measurements to weight the strategies such as durations in a barbell strategy (see Fabozzi, 1994). For our portfolios, these sensitivity estimators are not relevant as I am testing price x valuation and so I equally weight the long and short positions.

[42] The Other tranche is not priced but is included in the capital structure.

Step 3 – Unwind the Trade: The position is held over the horizon period and is then automatically unwound (sold in the case of the long, and purchased back in the case of the short) at the end of the horizon at the then observed CMBX price. The lognormal raw returns of the positions are calculated over the period based on price with gains/losses for price increase/decreases for the long position and gains/losses for price decrease/increases for the short position. The portfolio return is then the weighted sum of these two returns with each weight =0.50, of the return on the long position and the absolute value of the return on the short position.

Step 4 – Rebalance and Repeat: The process is immediately repeated with the new values for Theta on the trade day in the testing period. The procedure is conducted for all models. It is important to emphasize that in this computation I do not include explicit transaction costs. However, because I am comparing matched portfolios, each exhibiting similar rebalancing across time, the transaction costs would be roughly equivalent for the portfolios. This implies that, as a first approximation, the relative performance differential between the portfolios should be unaffected by exclusion of explicit transaction costs.

3.3. Horizons – daily, monthly, quarterly: The holding period horizon tests are conducted with daily, monthly and quarterly investment horizons. The monthly frequency would represent the perspective of a hedge fund or levered investment managers who may seek to rebalance with greater frequency due to leverage considerations. Such an investor may be seeking to articulate a 'macro' view on the CMBS sector through the use of CMBX instruments. The quarterly horizon test represents the perspective of a sell side bank hedging a large portfolio of loans to be securitized and sold that might take 3 months (or more) to build to a critical mass to sell the market in bond form. I also conduct the test with a daily horizon as the test is ultimately one of efficiency which assumes frictionless markets[43]. It is true that CMBS and CMBX are not as liquid as say RMBS passthroughs, government bonds, or currencies, for example. As such, while trading is not categorically by appointment (except for below investment grade), it is nevertheless slower and OTC, with the understanding that large blocks don't get bought and sold (crossed) readily amongst dealers with great frequency. At the same time, nevertheless, the CMBX market is competitively bid amongst all of the largest dealers and as such, pricing may not be uniform across all of them simultaneously. Since the data observation period is relatively short for the Crisis (~3 years) I want to take advantage of a rich data set with respect to observations. So, while portfolio managers might 'suffer' the bid/ask on a daily basis, this influence is reasonably assumed to dissipate with monthly and quarterly horizons. I investigate the daily horizon also, attentive to the caveat of the bid/ask, and note the potential problems with respect to claims about efficiency. For the moment it is quite

[43] Incorporating bid/ask spreads that might expand and contract across credits (AAA tighter than BBB-) and across time (tighter in low vol periods, wider in high vol periods) would be a nice evolution in the analysis. Without a system like TRACE in place, however, which does not currently incorporate CMBS or CMBX, such analysis is not possible.

simply the best that can be done with the data of mid-market closing prices, and it is reasonable, especially in light of recently improved liquidity in the CMBS sector.

3.4. Return results: For each of the four Theta driven strategies I provide the raw returns. In Figure 84 panel A I show the monthly horizon strategy and in Figure 84 panel B for the quarterly horizon strategy for during the Crisis. In Figure 85 panel A for the monthly horizon strategy and in Figure 85 panel B for the quarterly horizon strategy for the Recovery. Additionally, I calculate a long-only sector portfolio which is the weighted composition of all the tranches based upon their weights in the transaction. As with Theta selected securities, I compute the total return for the long-only portfolio based upon the composite prices.

From the raw returns I assume an initial portfolio size of $100mm and calculate its cumulative value from the raw periodic portfolio returns. The plots reflect the intuition. Figure 86 reflects the monthly strategy during the Crisis (*left*) and the monthly strategy during the Recovery (*right*). The long-only sector portfolio is represented by the grey bars, Model 1 (magenta) is Merton, Model 2 (red) is the basic calibration model, Model 3 (green) is my expansion of Model 2, and Model 4 (blue) the reduced form approach. Consistent with what we anticipated the reduced form model, Model 4, with among the lowest explanatory relationship to market prices categorically outperformed all other models as well as the long-only portfolio. Categorically, Model 2 (red) underperforms having shown the greatest fit with the market price in regression. As such, the efficiency of the CMBS market may be called into question as the necessary conditions for challenging efficiency are met.

We see similar results for the quarterly strategy in Figure 87 for the Crisis (*left*) and the Recovery (*right*). Again, the Model 2 approach significantly underperforms the sector portfolio and the three other models, and the reduced form approach of Model 4 performs well and categorically better than Model 2 and the sector overall. Finally, the results for the daily strategy in Figure 88 for the Crisis (*left*) and the Recovery (*right*) show considerable consistency with both the monthly and quarterly results, again with the reduced form Model 4 approach performing well and categorically better than Model 2 and the sector overall.

To be sure, the compounding of returns matter and it should also be evident that the reduced form techniques lose considerable ground (in these purely automated strategies) during the Crisis. Additionally, it does seem that Model 4 really stands out during the Crisis whereas, during the Recovery (so far at least), it outperforms, but not as dramatically. This represents further demonstration of the value of the reduced form technology and insight into the efficiency of markets and the insights of market actors, or lack thereof.

When times are 'good' as characterized by the observables in the Recovery, the complexity of issues facing securities pricing may not take priority with the prospects of default and loss undervalued by market actors. In such an environment, such as the Recovery, where the observable information set includes no delinquency manifestations at the loan level and no macroeconomic warnings at the economy level as represented in REIT, NCREIF or interest rate pricing, Model 4 should perform better than the long-only portfolio if there are risks, but since the risks are muted at initialization of simulations, governed by data, the simulations will produce low frequency of risk events. Under such conditions, since market pricing is also considering with low likelihood in ad-hoc way the default events, the fair value prices and the market prices should be similar, which is seen in the plots and it should be difficult to differentiate in trading strategies using models. As a result, the performance of Model 4 while better is not dramatically better than the rest. When times are 'bad' however, the information set and the meaning of the information to the default event is systematically considered by all the models, but most rigorously in Model 4, in contrast to market actor ad-hoc approaches. Under these conditions, such as the Crisis, the Model 4 approach should dramatically outperform, and it does.

3.5. The 'perfect' portfolio: All the models provide insights; some provide better advice than others. But none of the models are perfect. Perfection, however, has a role in our analysis of efficiency. If we can consider what the perfect portfolio selection looks like in our trading tests, and the implication of the statistics related to the perfect portfolio, I may be able to shed some further light on the model driven portfolios, the market portfolio and the efficiency of the CMBS sector, overall. I am limited to a set of bonds associated with the CMBX Series 1 in the Crisis and CMBX Series 6 in the Recovery. I define the 'perfect' portfolio as the portfolio which, in hindsight, would have delivered the maximum/(*minimum*) return over the horizon for the long/(*short*) positions. Obviously the 'perfect' portfolio dwarfs all the other models (Figure 89) and I thus report it on a log scale[44] as I consider it in the initial statistical analyses and the intertemporal CAPM efficiency tests in the discussion below.

3.6. Trade strategy results composite: To be sure the compounding of returns, the timing of gains and losses, and their magnitude all interact to create the return of the strategy. So, while it is true that the reduced form Model 4 outperforms in both the Crisis and the Recovery, it is also true that the automated strategy in the Crisis gave up significant gains for Model 4. To get a sense for why this might be so, I reviewed the profile of the returns for the trading strategy portfolios and the long-only portfolio. I computed first the frequency of 'correct' calls, where a correct call is defined as a strategy where the return over the horizon resulted in a positive return (>0).

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[44] However, during the Crisis, the Model 4 Theta's corresponded with the 'perfect' approach for both long and short selections 20% of the time. Model 3 13% of the time. Model 1 and Model 2 corresponded 0% of the time under monthly selections.

Immediately after I wanted to get a sense for the magnitude of the right calls and so I calculated also the average correct call return and the maximum correct call return.

Figure 90 provides a summary of the information discussed and it does provide some interesting information. In panel A I present the frequency of the gain strategies. Not surprisingly, Model 2 underperforms all the other models and the long-only portfolio for both daily and monthly investment horizons in both the Crisis (panel A) and the Recovery (panel B). What is interesting, of course is that the other active trading strategies driven by Theta all outperform the long-only portfolio with gains >50% of the time in the crisis, and generally so in the Recovery. Focusing further, it is also quite interesting that Model 4, the given its cumulative portfolio performance described in the plots, actually did not pick gains as frequently as either Model 1 or Model 3 in the Crisis. Model 4 only picked gains 63.33% of the time while Model 1 picked gains 70% of time and Model 3 66.67% of the time on a monthly basis during the crisis. On a daily basis during the crisis, the good trading strategies perform better than 50% of the time, but still Model 4 lags Model 1.

During the Recovery (panel B) Model 4 outperforms all others strategies on a daily basis with correct calls 57.04% of the time. However, over a monthly horizon, actually Merton's Model 1 significantly outperforms with correct calls 84.62% of the time which is consistent with Model

1's outperformance during the crisis as well (70%). This profile begs the question then "How did Model 4 dramatically outperform during the Crisis?"

The answer is cleared up somewhat in panel C. There we see that during the Crisis the profile of the correct calls made by Model 4 was quite good on for both daily and monthly trading horizons, but not categorically the largest. In the case of the Recovery (panel D) we see that Model 4 lags in terms of the frequency of correct calls and their magnitude. In panel E I show the joint occurrence frequency and correct call. There we see the product of the frequency of correct calls (as a percentage) and the average gain return. With the joint product as a ranking we see that Model 4 performed best across the Crisis and so far in the Recovery on both a daily and monthly basis with respect to gains which is consistent with what is shown in the plots.

One takeaway from this (and the plots) is that there is no perfect model approach. Additionally, if we consider the efficiency of the CMBS sector as questionable, for the reasons stated, during bad times the reduced form technology should significantly outperform the other models. Is it always right? No. Only 20% of the time does it correspond with the perfect portfolio strategy; but this is better than the other technologies, and the implementation in a long/short paradigm, informed by Theta, significantly outperforms the long-only portfolio in the Crisis and the Recovery. At the same time, given the fact that there was a deterioration from the peak performance at the height of the crisis when uncertainty was at its apogee, we must always allow for human judgment to work *with* the model technologies to optimize portfolio performance.

3.7. Initial statistical tests: Since we are considering the efficiency the CMBS market, and we have Theta driven long/short strategies we can compare the portfolio returns across strategies and the perfect portfolio to i.) the long-only portfolio of the CMBX composite index (all ratings) and ii.) the Fama French market portfolio[45]. For the paired t-test the null hypothesis is that the mean returns on the portfolio strategy and benchmark (first the CMBX long portfolio and second the Fama French Market portfolio) are the same such that the difference equals zero. There are no surprises in the results, but they are not as compelling. We are concerned with the direction of the differences and so I consider both the statistical difference for the null hypothesis as well as the alternative hypothesis that the mean of the difference is <0. In Figure 91 I start with the perfect portfolio and we may reject the null in favor of the alternative that any observed mean difference shall be less than the mean difference in our sample with probability 1 across both the t-test and Wilcoxon sign test panels A thru D. Unfortunately none of the other tests are significant at the 95% level, though the signs and direction are intuitive. Models 1, 3 and 4 all point towards positive differences from the mean and median respectively for the t-test and one-sided Wilcoxon sign tests,

[45] The market portfolio consists of all NYSE, AMEX and NASDAQ firms. It is obtained from Ken French's site: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french

in contrast to Model 2 which points to negative differences from the mean and median. Even though these results are strongly supportive of the argument of CMBS inefficiency, they are not conclusive for Theta driven strategies, though they *are supportive* of the perfect strategy. Given the extreme outperformance during the Crisis but absence of conclusive results, I don't consider these pairwise tests for the Recovery period. I turn to the possibility of omitted risk premia are governing outperformance.

3.7.1. Intertemporal CAPM test for omitted risk premia: In order for the Model 1-4 strategies to challenge the efficiency of CMBS the abnormal returns implemented in the long/short trading strategies informed by the given Model's Theta must not be readily explained away by factors exogenous to the Model construction. Given the complexity of the modeling exercises for all four models and in light of the significance of exogenous variables in the initial statistical tests[46]. I test for the possibility of omitted risk premia using a standard intertemporal CAPM (ICAPM). The expected return on each of the portfolios is a multi-beta model:

$$ER_t^p = r_t + \sum_{i=1}^{M} \beta_{pi} \left(ER_t^i - r_t \right) \tag{83}$$

[46] Following Jarrow, etal 2008; and Merton, 1990

where R_t^p is the p portfolio's return over $[t, t+1]$, R_t^i is the return over $[t, t+1]$ on a portfolio perfectly correlated to the $i-th$ systematic risk component, and β_{pi} is the beta of portfolio p to the $i-th$ risk component portfolio. There are M possible risk factors. Using the relation

$$R_t^j = ER_t^j + \epsilon_t^j \tag{84}$$

where ϵ_t^j have zero means and are independent across t and j, I can rewrite the multi-beta model as

$$R_t^p = r_t + \sum_{i=1}^{M} \beta_{pi} \left(R_t^i - r_t \right) + \varepsilon_t \tag{85}$$

where $\varepsilon_t = \sum_{i=1}^{M} \beta_{pi} \epsilon_t^i$ have zero means and are independent across t. To construct the regression model for omitted risk premia it is reasonable to assume that one of the systematic risk factors is a CMBS portfolio of equal credit risk as the model driven portfolio under consideration. Letting the return on the index portfolio be denoted by $i = 1$ I can write this last expression as:

$$R_t^p = r_t + \beta_{p1} \left(R_t^1 - r_t \right) + \sum_{i=2}^{M} \beta_{pi} \left(R_t^i - r_t \right) + \varepsilon_t \tag{86}$$

And, it is reasonable to also assume that the beta of the model portfolio with respect to the index is unity, that is, $\beta_{p1} = 1$, yielding our final regression model to test for omitted risk premia as:

$$R_t^p - R_t^1 = \alpha + \sum_{i=2}^{M} \beta_{pi} \left(R_t^i - r_t \right) + \varepsilon_t \tag{87}$$

where, in this specification, the constant α captures abnormal returns. I estimate four different ICAPM regressions (Regression 1 thru Regression 4) to capture various risk premia across all four Theta driven portfolios (Model 1 thru Model 4) and the 'Perfect' portfolio as described above. Some of the assets considered in Regressions 1 thru 4 were tested in the earlier statistical analysis to map to pricing, but are specifically considered here in the context of their horizon returns compared with the returns of each of the Models 1 thru 4 and the 'perfect' portfolio. I perform the Regressions 1 – 4 for Crisis and for the Recovery evaluating efficiency for Daily investment horizons.

3.7.2. Regressions: In *Regression 1* I use the following assets to capture various risk premia: (i.) the REIT stock price index to capture property value risk premium, (ii.) the return on the CaseShiller housing price index[47] (iii.) the 1-year, 2-year, 5-year, 7-year and 10-year zero-coupon bond prices to capture interest rate risk premium and (iv.) a stock market index, the SMB index (small minus big), and the HML index (high minus low) to capture equity market risk premium. In *Regression 2* I consider only the Fama-French 3 factor model of (i.) the stock market index, (ii.) the SMB index (small minus big), and (iii.) the HML index (high minus low) to capture equity

[47] For the daily analysis, we don't have CaseShiller and so we use the ETF REZ to capture residential exposures.

market risk premium. In *Regression 3* I consider what a sparser 'credit' regression model drawing from both Regression 1 and Regression 3. The factors are credit composite factor returns that were tested earlier in a different form: (i.) the REIT stock price index to capture property value risk premium, (ii.) the return on the housing price index, (iii.) FF model of a stock market index, and (iv.) 10-year zero-coupon bond prices. In *Regression 4* I consider only (i.) the FF model of a stock market index.

3.7.3. Results and discussion: Figure 92 Panel A summarizes the results of Regression 1. While the regressions across all but Model 2 are significant by the F-test, we also see that of the variables considered in the significant regressions (Models 1, 2, and 4) is the Market portfolio. While it is true that returns associated with 10 year zero coupon bond are significant for Model 2, the regression for Model 2 is insignificant overall. Additionally, if all variables were set equal to zero, the perfect portfolio would deliver a 2.9% positive daily return on average and it would be significant. This is perfectly reasonable given that the perfect portfolio has 20/20 hindsight by construction. Models 1, 3, and 4 would also deliver positive daily returns of about 0.01%, whereas Model 2 would deliver negative returns on average. The number of factors in this regression is quite large and I perform a few diagnostic tests to see if I can pare it down. It is possible that some of the terms are interacting with one another and drowning out the significance of each other.

In Figure 93 we observe some significant correlation amongst the Independent Variables. Not surprisingly we see REIT and house price indices highly correlated with the market portfolio. Across the zero coupon bond returns five, seven and ten year zeroes are virtually identical in terms of performance and home prices and REIT performance are also highly correlated. This suggests a possible need to omit some of the variables that are correlated with one another. From the RAMSEY Reset test, it would appear, that some key variables are either omitted or possibly being drowned out by overfitting as the test statistics is less than the critical value $Fitted\ F = 0.47 < F_{crit(3,628)} = 2.60$., (Figure 94). Additionally, the Variance Inflation Factor Test indicates that many of the variables exhibit high degrees of multicollinearity with $VIF(\beta_i) > 10$ not shockingly based on the correlation assessment for *all* of the treasuries as well as the REIT index (Figure 95). To address concerns of dependency among the variables I use the condition index (Figure 96). Instances where the variable exhibits a condition index n>30 *and* p>0.50 indicates some competing dependency between variables. In this I don't see any instances of condition index>30, however I do see many p-values >0.50, specifically amongst the zero coupon bonds. As such, I should be able to express the set of zero coupon bonds in term of the remaining variates. Finally, the White and BreuschPagan Tests (Figure 97) does indicate that non-constant variance heteroskedasticity may be present. However, the Durbin Watson Test for autocorrelation (Figure 98) between the error terms (the residuals) is inconclusive because at 11 degrees of freedom and 642 observations, the statistic of 1.53 is between the lower (1.38) and upper (1.72) bounds.

Based upon the discussion above I take a somewhat extreme view of eliminating all variables specifically related to fixed income and real estate and consider Regression 2, the Fama French three factor regression, referring to Figure 92 Panel B. In this regression, I capture only the model of a stock market index, the SMB index (small minus big), and the HML index (high minus low) to capture equity market risk premia. Interestingly, in this sparser regression, the same general story repeats. The regressions are significant for Models 1, 3, and 4 with only the market portfolio exhibiting significance across these models and the perfect model. The signs are consistent with Regression for the mean returns assuming the independent variable values equaled zero. In *Regression 4* I consider only (i.) the FF model of a stock market index. The results summarized in Figure 92 Panel D mimic the result pattern of statistical significance of the market portfolio for Models 1, 3, and 4 and the perfect portfolio with significance amongst the regression as indicated by the F-test.

Since the choices of variables in Regression 2 and 4 specifically ignore issues that we know to be significant from the fair value x price matching in the Crisis analysis, I try to find a middle ground between Regression 2 and the earlier analysis of missing independent variables related to real estate credit and fixed income to test for exogenous significance to the model. In Regression 3 I include (i.) the REIT stock price index to capture property value risk premium, (ii.) the return on the housing price index, (iii.) Fama French model of a stock market index, and (iv.) 10-year zero-coupon bond prices. The issues of borderline codependency seen in Regression 1 amongst the variables with the condition indices all much lower than 10 (Figure 99). I check to see that in the sparser model I have no omitted variables and the Ramsey RESET test confirms this (Figure 100). I also do a check for the presence heteroskedasticity with the White and Breusch Pagan Tests (Figure 101) which has been eliminated with chi-square of 14df test stat of 17.86 < the critical value 23.68 indicating non-constant variance is not present in the sparser Regression 3 model. Finally, the Durbin Watson Test for autocorrelation (Figure 102) between the error terms (the residuals) does seem to exhibit autocorrelation with 5 degrees of freedom and 642 observations. The statistic of 1.53 is below the lower boundary of 1.72 and thus the null of no autocorrelation is rejected in favor of positive autocorrelation and the BreuschGodfrey LM test statistics (Figure 103) reject no autocorrelation in favor of AR(1) and AR(2) processes which is noted.

As in Regressions 1 and 2, the pattern of significance of the market portfolio repeats with correct signs for the constant coefficients and overall significance for the Model 1, 3 and 4 and the perfect portfolio regressions with no significance for the Model 2 regression. Based upon this analysis I do not see evidence of omitted risk premia in any of the models considered as none of the risk premia considered are significant other than the market overall. The statistics show low explanatory power between the returns on the market portfolio (significant) and excess returns over the CMBX benchmark long index as articulated in the long/short strategies governed by bond level Thetas. Therefore, based on the analyses, we can be comfortable that the models are well specified and do not omit risk premia. Since there are significant problems with Regression 1 in the Crisis and since Regression 2 ignores considerations fundamental to fixed income and credit, I do not compute them. In Figure 104 Panel C I repeat sparse credit Regression 3 and the even sparser Regression 4 in Figure 104 Panel D for the Recovery period. None of Model 1 (Merton), Model 4 (reduced form) or the perfect portfolio are significant in either approach. In the case of the Regression 3 (Panel C) there Model 2 and Model 3 regressions are significant and suggest omitted risk premia of REITs. Interestingly, the signs of the significant REIT return variable are opposite for Model 2 and Model 3.

Conclusion

It was my intention to consider the reduced form technology versus other competing approaches robustly with application to CMBS and to contemplate the efficiency profile of CMBS in both the Crisis and the Recovery. This dissertation analyzes the four approaches rigorously and provides an important generalization to the calibrated approach. The power of Merton, 1974 was a bit surprising given the limitation of information considered in the structural form and particularly given the historical periods considered. There is considerable evidence presented in both the Crisis and the Recovery of the ability to earn *extraordinary profits* with model driven trading strategies thereby supporting claims of CMBS market inefficiency. Clearly, seriously implementation for any model must consider the heterogeneity of loan characteristics and accurately incorporating correct cashflows and ruthless default behavior of borrowers. This care builds the realism of the loan collateral and priced bond objects being investigated. By extension as we have seen, efforts to be thorough in the implementation of the loans and bonds *and* their interaction with the simulated economy, increases precision still further. The reduced form approach yields better results than any of the other approaches considered because of the more realistic and thorough approach through the modified Cox Process. As such, from this study it appears to provide the best approach to accurately anticipate and price default risk for CMBS supported by the ability to earn extraordinary profits in two important periods in finance history and compelling statistical results. While I cannot unambiguously reject the market efficiency of the CMBS sector, there is strong support for inefficiency in the statistical analysis of excess returns over two important periods in market history. The insights provided by the reduced form technology, in particular in both the Crisis and the Recovery support the increased use of such technology in areas of investment management and risk evaluation.

There are additional areas worth exploring related to this investigation. One thing to do would be to expand the technology to perform the many risk calculations used in the industry and evaluated in the academic literature. Capturing Basel values for all models as well as option adjusted spread, weighted average lives, duration and convexity all hold interest for analyses of spread decomposition and portfolio management risk optimization. In the area of spread decomposition, some preliminary work was begun and given the analyses that query efficiency of the sector, spread decomposition analyses might be able to provide further evidence supporting the integrity of the model specification. Additionally, to delve deeper into the area of price formation in the area of market microstructure, some interesting testing of computation of fair value with Model 4 might be able to be performed with frequency of minutes (*seconds, milliseconds…*) vs. days. Efforts to compare such computations to market pricing of the bonds would however not be possible given it is still largely a negotiated OTC market without electronic trading. Nevertheless, markets change and the adoption of advanced derivative pricing technology in CMBS may be prompted, ultimately, by competitive needs. Regardless of the state of real time frequency of market prices, real time computation of fair value using Model 4 reduced form technology may also hold interest for managers and academic studies. Given the development of my technology for this work, analyzing the efficiency of the CMBS market is limited only by time and data.

Appendix A – A Short Review of CMBS

CREL's are quite heterogeneous and when they are utilized as collateral for securities they can make valuation of such securities challenging. CRELs are typically quite large with an average loan size >$8mm and when found within securitizations typically number about 200[48]. As such, unlike residential mortgages which are more homogenous, smaller in size, and greater in number within securitizations, CREL valuations do not benefit from statistical techniques sometimes used in the residential mortgage sector associated with the law of large numbers[49]. As such, to make any reasonable statements about the risks associated with CRELs or securities collateralized by such CRELs, and valuations thereto, one must be prepared to implement methods of evaluation that adequately identify and treat the many idiosyncratic risks of CRELs briefly discussed here.

Income Producing Property: CRE is defined as income producing property, where such income is generated from rents charged by property owners to tenants. This key feature of income distinguishes CRE from single family residential property which is underwritten assuming no income is produced on the residential property from rents and where the property is typically the

[48] CRELs >= $20mm represent 85% of the market (ref. Trepp Database 12/2011).

[49] See Hayre, 2000

borrower's primary residence[50]. In this sense while both CRELs and residential mortgages are amortizing debt the fundamental use of the collateral securing such debt are quite different. CREL's are backed by a business enterprise, residential mortgages are backed by someone's home.

Property Types: There are 5 primary property types that make up the CRE sector: Office, Retail, Multifamily, Industrial and Lodging. Within each of these property types there are subdivisions. For example within Lodging there are divisions associated with extended stay hotels, motels, and deluxe hotels. Similarly within retail, there are divisions associated with anchored retail malls and strip malls. Sometimes there are hybrid structures, or so-called mixed use properties in which a portion of the revenue on the property is generated from office tenants and a portion is generated from retail tenants. Despite these sub-divisions, from a macroeconomic perspective, the incentives facing the leasing occupants are consistent within the primary groups. Distinctions between properties within the same category will be evidenced in the observed leverage and coupon compensating the lender for different risks.

Location: Each of the CRE properties can be found in a specific location. In the context of conducting a study, depending on the data each of these locations for types of properties been

[50] This may change in the case of distressed bulk purchases of foreclosed or distressed homes that may be sold with a rental strategy. See L. Goodman and M. Meyers Bloomberg News 11/30/2011.

categorized at from as micro as zip code, and then ascending in order to central business district, county, city, state, region, and finally nation.

Leverage: As with residential real estate, the leverage on a CRE is captured by the loan to value ratio (LTV) which reflects the ratio of the mortgage amount, M, at origination over the value of the property at origination V so,

$$LTV = \frac{M}{V} \tag{88}$$

Borrowers purchasing CRE properties typically invest a portion of their own capital and borrow the remainder. In this sense, the CRE property can be considered to be single purpose corporate entity with a single plant (the CRE property site) and a single product (the space for lease to tenants). As with any corporation (see Brealey, Meyers), the value of the CRE property, V, is:

$$V = D + E \tag{89}$$

At origination, the value of the equity position, E, of the borrower is reflected by the amount the borrower has invested in the purchase of the property, with the remaining value of the property associated with the value of the debt, D. As the value of the property may change through time, so too will the implied value of the debt and the implied value of the equity or,

$$V_t = D_t + E_t \tag{90}$$

NOI: When CRE is underwritten, the income of the property less expenses to operate the property (or net operating income, NOI, O) is of central importance to lenders because mortgage payments to the lender are made from NOI. The debt service coverage ratio (DSCR) is the ratio of the NOI (annual) to the mortgage debt service (annual), S, so,

$$DSCR = \frac{O}{S} \tag{91}$$

DSCR represents a quick measure of the ability of the borrower to pay the debt obligations associated with the mortgage. DSCR > 1.0 reflects adequate revenue produced by the property to service the debt. A DSCR < 1.0 reflects inadequate revenue generated by the property to service the debt.

Standard Industry Value Metric: The industry standard method (see Brueggeman/Fisher) for estimation of value of a CRE property is provided as the ratio of the net operating income, O, to the capitalization rate K:

$$V = \frac{O}{K} \tag{92}$$

This becomes interesting as now we have two equations for the same property allowing us to say:

$$D + E = \frac{O}{K} \tag{93}$$

As CRE may change through time, I will visit these conditions again, indexed to time, so

$$D_t + E_t = \frac{O_t}{K_t} \tag{94}$$

ProForma: While it might seem implausible that mortgages secured by CRE property could be underwritten based on NOI in place at DSCR<1.0x, in point of fact in the realm of CRE, especially in the case of mortgages secured by larger properties, mortgages are frequently underwritten with NOI in place at DSCR<1.0x. These loans, are underwritten on a so-called *pro-forma* or *stabilized* basis. In such instances, a loan is underwritten based upon expectations of the future growth trajectory of rents on the securing property. While NOI in place at the point of origination may be insufficient to service the debt, the anticipated future income generated by the property is expected by lenders to be more than adequate to meet debt obligations.

To offset the inadequacy at origination of the income generated by the property, the lender and borrower will agree to establish a cash reserve account. In the interim period between the loan origination date and the anticipated date of stabilization of the property, the reserve account ensures that the debt service due to the lender over this period is paid in full should income generated by the property income be insufficient in any given month. If the cash reserve becomes depleted following the date of origination and the property does not 'stabilize', the income generated by the property will be insufficient to service the debt and the borrower will have an incentive to default[51].

Loan Structures, Coupon, and Default/Refi Risk: Typically the CREL will have balloon amortization structure. In a balloon structure, the monthly debt service for a period of time (*eg 119 months*) will reflect mortgage payments of principal and interest[52] for a longer term (*eg 360 months*). At the maturity date of the loan, say month 120, the borrower must pay off the outstanding principal balance of the mortgage. Typically, the borrower will pay off the outstanding principal with a new loan. This assumes, of course, that the lending criteria will be favorable to the borrower at such time and that availability of capital will be in place by lenders. As I have discussed previously seen in this financial crisis since 2008, and expected going forward into 2012, not only have lending criteria tightened (*lower LTV*), but they have declined in parallel with declining

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[51] It is worth mentioning, that at times, the cash reserve fund is established by the lender directly, (lender-financed) or a consortium of co-investors in the property in exchange for preferred returns. As such, the reader should not assume that the borrower has a 100% 'hard-equity' position in the property as evidenced by the presence of a cash-reserve. The borrower may have invested a portion of the proceeds in the reserve fund, but often times <100%.

[52] Sometimes only interest payments (so-called IO loans) or sometimes a blend (interest only for an initial period of say 20 months, stepping up 360 amortization principal and interest for 99 months.). These features vary but overall are mechanisms that were implemented by industry professionals to encourage CRE activity.

property values. This is important as it represents two linked criteria for credit availability both of which make it more difficult for borrowers at the point of balloon to secure a new mortgage, thereby increasing the likelihood of default in the current economic crisis[53].

Cross-Collateralization and Subordinate-Leverage: Not all CREL's are standard first lien debt on the property underwritten by lenders secured by income generated from a single property. Sometimes, CRELs are secured by portfolios of properties. Such CREL's are referred to as cross-collateralized, where the loan debt service obligations are secured by all properties in the portfolio. This provides additional flexibility and comfort to lenders using portfolio diversification arguments where if one property falls on difficult times, excess NOI from the other properties can make up such shortfalls.

Additional leverage, subordinate in terms of payment priority to the senior first lien, are also underwritten with interest in single properties or portfolios of properties. In the most basic example, second liens, junior/subordinate loans can represent additional leverage (*junior, or B-Notes*) on the CRE much in the way that a home equity loan may represent additional leverage on a residential property. More complex encumbrances on the property can be accompanied with

[53] This is borne out per the S&P comments and several other data sources.

sophisticated voting rights in the case of Mezzanine Financing. Mezzanine capital can take the form of debt or preferred equity. Because of the highly situational profile of Mezzanine capital, and its relatively small amount (<1% of all loans) only the encumbrance aspect as a junior lien are considered in this study.

Prepayment Lockout and Defeasance Features: Unlike residential mortgages which may be prepaid without restriction, CREL's are typically restricted from prepayment for several years prior to the balloon maturity. Typically CRE debt has one or more type of prepayment restriction according to some schedule. The simplest of these restrictions is the hard lockout in which the loan covenant states that no prepayment may undertaken by the borrower for a certain period of time.

Alternatives to the hard lockout restriction include, yield maintenance in which a 'make-whole' provision is written in which the borrower repays the lender the present value of the future interest payments under the loan covenant discounted at the current risk free rate associated with the remaining maturity on the loan. Sometimes an additional percentage fee or sequence of fees (3% for prepay in first 12 months, 2% for month 12 – 60, 1% thereafter) may accompany yield maintenance or stand alone (in older loans). Finally, a more recent feature was the inclusion of a defeasance option in which the borrower would swap out risk debt payment obligations to the lender and replace them with treasury strips that mimicked identically the cashflow originally intended to be paid to the lender[54].

All lock outs are intended to compensate the lender from the foregone interest payments and to increase the certainty of a return. Such features have changed over time. Thus, from a prepayment incentive modeling perspective one must pit i.) the cash-out refinancing incentive as well as ii.) the pure interest rate savings incentive, against the 'cost' of the prepayment restriction.

Mock Securitization Profile of Origination: Having considered the certain loan characteristics it is worth noting a few key aspects to the business of origination under mock securitization. More than 50% CRELs are originated in securitization warehouses or conduit pipelines. What this means is that investment banks utilize their capital to lend to borrowers interested in purchasing CRE or refinancing existing CRELs. Regardless of the purpose, the CRELs are held on balance sheet until an aggregate amount of loans is built up to satisfy market demand for the CRELs as collateral within a securitized transaction. At the point of securitization, the investment banks sell the loans off their balance sheet into a special purpose vehicle, or trust. The trust's sole operational function is to issue bonds. At closing of the transaction, optimally, all the

[54] This particular type of lockout has the effect of credit enhancement to the lender as the cashflows, once defeased, are no longer generated by the commercial real estate borrower and CRE property but are now cashflows generated by the US Treasury.

bonds issued by the trust are sold to institutional investors and the originating banks collect the present value of the cashflows associated with the bonds collateralized by the underlying CRELs at the clearing price on the closing date. If the investment bank was accurate in their estimate of demand and in their estimate of the rating agency treatment of the originated loans, and if they were hedging both interest rate and credit risk correctly, then at the closing date the investment bank should realize a gain on sale of the loan collateral issued in bond form. From a capital allocation perspective, their balance sheet is freed up from originated loans and the bank can now engage in lending more capital for new loans in keeping with regulatory leverage restrictions. After closing date of securitization, typically originating trading desks of investment banks that originated the loans will make markets for institutional investors in over the counter transactions trading '*their deals*' for the customers as well as others.

There is no rule for aggregation set in stone, but typical CMBS transactions, for example, range from just under $1B to as much as $6B. What this means is that the aggregate principal amount of the CMBS trust at any time t, $C(t)$, is equal to the sum of the principal balance outstanding on each of i loans $L(t)$, so:

$$C(t) = \sum_{i=1}^{n} L(t)_i \tag{95}$$

From this trust, which can be made up of collateral from multiple originators, a corresponding set of bonds is determined through an iterative process such that the principal paying bonds *B(t)* correspond in value to *C(t)*, so:

$$\sum_{i=1}^{n} B\left(t\right)_i = C(t) = \sum_{i=1}^{n} L(t)_i \tag{96}$$

In a typical senior-subordinated structure, originators estimate the rating agencies future evaluation of the credit risk of the collateral which is distilled into a vector of subordination levels for the pool that correspond to different ratings. The interpretation is that the *subordination level* required by the rating agencies is sufficient to adequately protect investors from loss of principal. Recall from Jarrow, etal 2008 that in a typical senior-subordinate structure, payments of principal received are paid in order of seniority until the class is paid off in full (*so AAA, then AA,…then UR*). If losses are incurred through default on any of the i loans, those losses of principal reduce the amount of the classes in reverse order (*so, UR, then B, then…then AAA*)

Ratings agencies have a variety methods for determining the probability of default (PD), loss given default (LGD), for any loan i[55]. Once the expected loss (E[Loss]) is determined at the loan level

$$E\left[Loss\right]_i = PD(t)_i * LGD(t)_i * EAD(t)_i \tag{97}$$

then for any trust *C(t)*:

$$E\left[Loss\right]_{C(t=0)} = \sum_{i=1}^{n} E\left[Loss\right]_i \tag{98}$$

The E[Loss] for *C(t=0)* determines the sizes of the Unrated thru single-B tranches of the transaction. Once the sizes of the Unrated and single-B tranches are determined, further linear multiples are calculated for the remaining tranches in reverse seniority. So,

$$\sum_{i=1}^{n} B_i(t) = (AAA, AA, \ldots\ B, UR) \tag{99}$$

It is important to keep in mind that many loan level parameters used by the rating agencies for loans that exist are considered in the origination process *prior to* the loan being made. In this sense, originators are aware of the approximate judgment ratings agencies will make on a loan in a

[55] Exposure at Default, $EAD_i(t)$, is simply the outstanding principal balance at the time of the default, $B_i(t)$.

trust before the loan is actually originated. By providing guidance on the model parameters of interest (so called drivers of their models) originators frequently reverse engineer the sizing of the loans into their tranches using origination grids. A typical E[Loss] grid will be a *n x m* matrix with *LTV x DSCR* for a given property type. As DSCR increases the E[Loss] declines and as LTV declines E[Loss] decreases.

Tranche/class sizes can simply be expressed as a percentage of the total outstanding principal balance of the transaction. For Subordination levels, the percentage expressed is cumulative and descending with respect to the outstanding principal balance of h classes over the entire trust so,

$$trustprinbal(t) = \sum_{h=1}^{H} totprinbal_h(t) \tag{100}$$

$$SubLevel_h(t) = 1 - \left[\frac{\sum_{h=1}^{H} totprinbal_h(t)}{trustprinbal(t)} \right] \tag{101}$$

Appendix B – Other Alternatives/Literature

B.1. Eom, Helwege & Huang, 2004: Eom, Helwage & Huang (EHH) published a survey of 5 corporate structural form corporate bond pricing models. One is an extension to Merton in which they provide simulation. In, Model 2, I adopt EHH's extension of Merton which adjusts for coupon, term structure and an American style Default Barrier and further their work by making adjustment for CRE. Let V_t, K_t, and r_t represent the time t values of the firm's assets, total liabilities, and the risk-free interest rate, respectively. Assume that

$$dV_t = (r_t - \delta)V_t dt + \sigma_V V_t dZ_{1t}^{\mathbb{Q}} \tag{102}$$

where for CRE properties, the payout ratio , $\delta = 0$, σ_V is the volatility of the CRE property, and $dZ_{1t}^{\mathbb{Q}}$ is a one dimensional standard Brownian motion under the risk neutral measure, $\mathbb{Q}$. For simplicity, consider now a simplified CREL with maturity T and unit face value that with fixed debt service (coupons) at an annual rate of c. Let T_n, $n = 1,2...,T$ be the *n*-th coupon date. In EHH, the extended Merton model, $K_t = K \forall t \in [0,T]$ and default is triggered if the asset value is below the default barrier, K , on coupon dates. In the EHH extension of Merton, the price of a loan is equal to a portfolio of zero coupon bonds and can be written as:

$$P(0,T)=\sum_{i=1}^{T-1}D(0,T_i)E^{\mathbb{Q}}[cI_{\{V_{T_i}\geq K\}}+\min(wc,V_{T_i})I_{\{V_{T_i}<K\}}]$$
$$+D(0,T)E^{\mathbb{Q}}[(1+c)I_{\{V_T\geq K\}}+\min(w(1+c),V_T)I_{\{V_T<K\}}], \quad (103)$$

where $D(0,T_i)$ denotes the time 0 value of a default free zero coupon bond maturing at T_i, $I_{\{\cdot\}}$ is the indicator function, $E^{\mathbb{Q}}[\cdot]$ is the expectation at time 0under the $\mathbb{Q}$ measure, and w is the recovery rate following a default.

From EHH, it is known that:

$$E^{\mathbb{Q}}[I_{\{V_t\geq K\}}]=N(d_2(K,t)) \quad (104)$$

and

$$E^{\mathbb{Q}}[I_{\{V_t<K\}}\min(\psi,V_t)]=V_0D(0,t)^{-1}e^{\delta t}N(-d_1(\psi,t))+\psi[N(d_2(\psi,t))-N(d_2(K,t))] \quad (105)$$

where $\psi\in[0,K]$, $N(\cdot)$ represents the cumulative standard function, and

$$d_1(x,t)=\frac{\ln\left(\frac{V_0}{xD(0,t)}\right)+\left(-\delta+\frac{\sigma_V^2}{2}\right)t}{\sigma_V\sqrt{t}};\ d_2(x,t)=d_1(x,t)-\sigma_V\sqrt{t} \quad (106)$$

Given any term structure $D(0,\cdot)$, with the equations above, I can calculate the price of a defaultable CREL with fixed coupons under Merton's assumptions. The assumption that do remain are that that the loan is a balloon, such that all principal is repaid at time T. If one wanted to convert this to monthly cashflows one could do so.

B.2. Kau, Keenan and Yildirim (2008): In 2008, Kau, Keenan and Yildirim (KKY) approached the implementation of Merton specifically to CRELs. In that paper, the key insight comes from construction of first passage time model with an implied 'current LTV'. The purpose of the study is to determine implicit default probabilities in commercial real estate loans.

Specifically they cast the first time to default, τ, as the first time the underlying process, LTV, crosses the default barrier b; that is

$$\tau = \inf\{t \geq 0 : LTV_t \geq b\}, \tag{107}$$

where $b > 0$ and where LTV follows a geometric Brownian motion of the form,

$$\frac{dLTV_t}{LTV_t} = \mu_l dt + \sigma_l dW_t \tag{108}$$

where μ_l and σ_l are the drift and volatility parameters for implied current LTV and W is a standard Brownian motion process defined on the probability space (Ω, F, P) and initialized at $LTV_0 \leq b$.

KKY recognize the complicated decision making process of default in their suggested approach and thus make the simplified assumptions of the loan as perpetual and non-amortizing (so, IO) with continual constant payments, and where such loan is in an environment of constant interest rates. The borrower retains the right to default and the loan is non-recourse to the lender. They then resolve to simulate the value of the property V_t where,

$$V_t = V_0 e^{(\mu - 0.5\sigma^2)t + \sigma W_t} \tag{109}$$

subject to the default barrier b, such that

$$\frac{dV_t}{V_t} = (\mu - s)dt + \sigma dW_t \tag{110}$$

for service flow, s, such that the current implied LTV ratio, and the value of the mortgage

$$LTV_t \equiv \frac{L}{V_t} \tag{111}$$

And follows the geometric Brownian motion as they claim.

They continue and discuss optimal strike prices as extensions of Black and Cox (1976), and then further construct the implied LTV, $ILTV_t$ as distinct from LTV_t [56], the reason presumably that the volatility of the property value is insufficiently linked to REIT values. In KKY they calculate the implied LTV as:

$$ILTV_t = \frac{current\_balance_t}{implied\_value_t} \quad (112)$$

where,

$$implied\_value_t = (1 + \Delta reit_t) * implied\_value_{t-1} \quad (113)$$

and where, $reit$ is the property type REIT index and

$$implied\_value_0 = \frac{L}{LTV_0} \quad (114)$$

They acknowledge that in the literature (and in industry) that there are several ways to infer current LTV and this is simply one approach.

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[56] They claim that the current LTV is not observable, and of course that is true.

Finally, given the default barrier, b, the express the default probability on the loan as

$$\begin{aligned} P(\tau \le t) &= P\left(\max_{0\le s\le t} LTV_s \ge b\right) \\ &= \Phi\left(\frac{-\ln\left(\frac{b}{LTV_0}\right)+\left(\mu_l-\frac{\sigma_l^2}{2}\right)t}{\sigma_l\sqrt{t}}\right) \\ &+ e^{\frac{2\left(\mu_l-\frac{\sigma_l^2}{2}\right)\ln\left(\frac{b}{LTV_0}\right)}{\sigma_l^2}}\Phi\left(\frac{-\ln\left(\frac{b}{LTV_0}\right)+\left(\mu_l-\frac{\sigma_l^2}{2}\right)t}{\sigma_l\sqrt{t}}\right) \end{aligned} \tag{115}$$

To find this default probability, the drift and volatility parameters, μ_l and σ_l need to be calculated which is straightforward because, the log return of the process is an independently drawn random variable from the Normal, or

$$\ln\left(\frac{LTV_t}{LTV_{t-1}}\right) \sim N\left[\left(\mu_l-\frac{\sigma_l}{2}\right),\sigma_l^2\right] \tag{116}$$

where sample moments of $\ln\left(\frac{LTV_t}{LTV_{t-1}}\right)$ were calculated from the property x region REIT indices to find μ_l and σ_l. KKY go on to test their results and make the claim that no feasible reduced form model based on actual data is likely to assign much probability to a particular loan in a particular period. I find such claim to be *erroneous* from the study above.

Appendix C – NCREIF as Proxy for Property Values (ex-NOI)[57]

In practice and the literature there is a school of thought that posits that property values may only reasonably be estimated directly from property net operating income cashflows using the familiar cap-rate calculation discussed previously. In this section I formulate the regression model on the data described using a multivariate ordinary least squares (OLS) with a modification to accommodate for the optimized lead time of some of the explanatory variables to estimate

$$\hat{y}(t) = b + \sum_{k} a_k x_k (t - t_k) \tag{117}$$

where $\hat{y}(t)$ is the synthetic estimate of the NCREIF Total Return value determined from the OLS Regression on the x_k parameters (*Unemployment, Case-Shiller, etc.*) at time t, and where b is a constant, a_k is the correlation coefficient of the $k-th$ parameter, and $x_k(t-t_k)$ is the value of each such $k-th$ parameter at time t, minus the optimized delay for the $k-th$ parameter, t_k, where such delay was determined by maximizing the correlation between each x_k and NCREIF as previously discussed. Several of the k parameters that contribute to a best prediction of $\hat{y}(t)$ can be considered *leading* indicators for NCREIF Total Returns. For example, the optimal value for the Case-Shiller total return index as a contributing parameter to $\hat{y}(t)$ is determined to be 4 periods, so for any time t, where k= CaseShiller, $t_k = t - 4$. None of the parameters used to estimate NCREIF

[57] From Jarrow, etal 2008.

Total Returns are *lagging* indicators and thus the maximum value for $(t - t_k)$ for all k parameters used is 0 and the minimum value is -4. The initial results for the rolling return for NCREIF are somewhat promising with an R-sq of 0.89 (Figure 105). However as we can see from the Durbin-Watson d-statistic, this OLS has within it some autocorrelation issues.

Before examining closely the regression above in its final form, it is worth taking a moment to consider each of the explanatory variables on a solo basis, taking the remaining x variables to =0. The purpose of this check is to get a further statistical profile on solo value each of the explanatory variables I have chosen (cum lags) over the 22 year history. Figure 106 summarizes the key findings for the individual x variables and 4 autocorrelation tests (discussed below) with Y NCREIF Actual as the dependent variable and each of X1 thru X7 as the sole explanatory independent variable.

With the exception of X4, the FHLMC Residential Mortgage Rate, each of the x-variables have t-stats of magnitude such that the null hypothesis H0: x is insignificantly different from zero must be rejected. As expected then, the validity of the regression overall provided by the F-Test, also show validation, with the exception of X4, evidenced by reasonable to high RSQs. Finally, I look at the signs of the coefficients for each of the explanatory variables to see if intuition in the simple model bears out. We see that X1, Unemployment and property values in synthetic NCREIF have a negative relationship as expected (high/low unemployment correspond with low/high CRE property values). Continuing, we see a positive sign for X2, CaseShiller implying that (high/low residential property values correspond with high/low CRE property values). For X3 we see as expected, steepening credit (more perceived risk)/narrowing credit (less perceived risk) corresponding with lower/higher CRE property values. In X5, Risk Free Slope we see steepening risk free credit (high long term borrowing)/narrowing risk free credit (lower long term borrowing) corresponding with lower/higher CRE property values which also follows intuition. For X6, the CREChgOffRate (or the percentage of CRE holdings that banks writeoff to a value of $0), also corresponds in sign with higher/lower charge offs corresponding with lower/higher CRE property values. Finally, the sign of X7, Private CRE Construction follows the industry knowledge of countercyclicality with higher/lower levels of construction occurring in an environment of lower/higher property values. Finally, though I cannot reject the null that X4, the FHLMC Residential Mortgage is statistically indistinguishable from zero with a p-value of 0.231, I still look at the sign of the coefficient and note that, as with the other six 6 explanatory variables that are significant on a solo basis, the sign of the coefficient of X4 corresponds with intuition indicating higher/lower prevailing mortgage rates correspond with lower/higher CRE property values.

Regardless of the fact that I have already lagged the data, checking for autocorrelation in the residual at this stage is appropriate. We do the check here for each of these sub values for autocorrelation. The four autocorrelation tests are i.) Durbin-Watson[58] which provides the d-statistic to test for first order serial correlation in the disturbance when all the regressors are strictly exogenous; ii.) Durbin's Alternative Test for serial correlation in the disturbance. This test does not require that all the regressors be strictly exogenous. Here we are provided with the chi-sq test value for the df (in this case df=1) with H_0: *no serial correlation*. If the chi-sq test value > chi-sq critical value for df=1, then we reject the null and accept that there is serial correlation; iii.) Breusch-Godfrey LM test for autocorrelation used to determine higher-order serial correlation in the disturbance. This test does not require that all the regressors be strictly exogenous. Again, as with *durbinalt*. we are provided with the chi-sq test value for the df (in this case df=1) with H_0: *no serial correlation*. If the chi-sq test value > chi-sq critical value for df=1, then we reject the null and accept that there is serial correlation; and Engle's ARCH LM test for autoregressive conditional heteroskedacity which tests for time-dependent volatility. In particular, performs Engle's Lagrange multiplier test for the presence of autoregressive conditional heteroskedasticity in the residuals Here we are provided with the chi-sq test value for the df (in this case df=1) with H_0: *no ARCH effects*. If the chi-sq test value > chi-sq critical value for df=1, then we reject the null and accept that there is serial correlation.

[58] i.) estat dwatson; ii.) estat durbinalt; iii.) estat bgodfrey; and iv.) estat archlm, respectively

A set of analyses is summarized in Figure 107 for X7 PrivateCREConstruction. For the standard Durbin-Watson d-statistic we see serial correlation for all x-variables. Additionally for each of *durbinalt*, *bdgodfrey*, and *archlm* the chi-sq value exceeds the chi-square critical value for each x-variable and so in all tests we reject the null that there is no serial correlation and accept that there is some serial correlation present in the residual in all variables.

Serial autocorrelation has the impact of understating the variance in the OLS and in so doing mutes the standard error for positive autocorrelation (generally the case). As such, reliance upon coefficients from OLS with associated standard error may increase t-stats and, as such, may increase the number type I errors where the null of the OLS $(H_0\text{:}\, B_i\, coefficient\, is\, zero)$ is rejected erroneously. The standard adjustment technique for serial autocorrelation is the Prais-Winsten Regression in which we estimate a linear regression for the dependent variable from a set of independent variables that is corrected for serially correlated residuals using the Prais–Winsten, 1954 estimator. This estimator improves on the Cochrane–Orcutt, 1949 method in that the first observation is preserved in the estimation routine. In particular we seek the rho that minimizes SSE (ref, Cameron, Trivedi). In Figure 108, I provide a summary for all seven explanatory variables run solo Prais-Winsten Regressions vs. OLS. I show the updated coefficient, R-sq, t-stat, p-value, and importantly the original Durbin Watson statistic and the adjusted Durbin-Watson Statistic determined post-adjustment. So, as anticipated there is improvement in the Durbin-Watson statistic in all cases, however, only in the case of X1 Unemployment is the improvement sufficient (>1) to indicate no serial autocorrelation in the error term. However, when we consider all variables together we see a different effect in the Prais-Winsten regression (Figure 109), consistent with, yet *improved* over, the initial OLS.

As hoped, the autocorrelation has been removed in the multivariate Prais-Winston with the d-stat increasing from .776107 to 1.350785. All of the signs of the coefficients now map to intuition in concert and each of X1 Unemployment and X2 CaseShiller (Residential Housing), X3 Credit Slope and X6 CRE ChargeOff Rates have t-stats supporting rejecting the null. With respect to the X4 Mortgage Rate and the X5 Risk Free Slope I would expect to see contemporaneous correlation between these two regressors in a Hausman Test. With respect to X7 the CRE Private Construction, some additional testing might also further light on this. Additional testing can always be done. For example, while Prais-Winsten is a GLS estimator[59], the decline in the RSQ to .6459 from .8857 suggests a further GLS analysis might also provide additional insight into estimating the unknown parameters in a linear regression model. Nevertheless, based on this analysis of 22 years, the notion

[59] The GLS is applied when the variances of the observations are unequal (heteroscedasticity), or when there is a certain degree of correlation between the observations. In these cases ordinary least squares can be statistically inefficient, or even give misleading inferences.

that macroeconomic variables *cannot* be used to estimate property value returns, separately and independently from property specific Net Operating Income and cap rates is reasonably challenged.

Appendix D – The Delinquency and Default Intensity Process[60]

In this dissertation I am only concerned with delinquency and default for fixed-rate CRELs. There are no floating-rate loans and no CTLs in our sample. The estimates used in the computation in this dissertation are taken directly from Jarrow, etal 2008. Care to calculate the default and prepayment intensities separately was conducted in that study in which we had access to the loan history database—including defaults, prepayments and loan characteristics—was provided by Trepp[61]. This database contains information on over 100,000 commercial loans. The data provides monthly observations of the relevant variables over the time period June 1998 to May 2005. In this database, the loans are classified as current, 30-59 days delinquent, 60-89 days delinquent, 90 plus days delinquent and defaulted. Loans exhibiting REO or Foreclosure status are considered to be in default. Defaults are distinct from delinquencies. Because our model has only three classifications (current, delinquent or default), not five, I needed to determine a coarser partitioning of the classification. A statistical analysis was done to see if 30-59 days delinquent should be classified as delinquent or current and if 90 plus days delinquent should be classified as delinquent or default.

[60] From Jarrow, etal 2008.

[61] See Reilly and Golub, 2000; and Trepp and Savitsky, 2000.

In Jarrow, etal 2008, we conducted a 6-year study of delinquency transitions of more than 2.3million loan life observations. Recall Figure 66 which shows the transitions over all loans from healthy to worse or conversely over the period June 1998 to June 2004. A healthy state is defined as current (0 days delinquent). A worse state is defined as the next higher delinquency status. So, for example, a loan that is current in month 1 is characterized as having transitioned to a worse state in month 2 if its delinquency status in month 2 is 30-59 days delinquent. Similarly, if a loan in month 1 is 90 plus days delinquent, it is said to have transitioned to a healthy state if it becomes 0 days delinquent in month 2. Loans that persist in non-transition for multiple months either due to aberrations in the data (found in loans exhibiting 30-59 or 60-89 loan delinquency status for multiple months in a row) or due to categorization (90 plus days delinquent is, by definition, a multiple month state) are not transitioned until they migrate to either healthy (0 days delinquent) or a worse delinquency or defaulted (REO, Foreclosure) state.

Historically, more loans that were 30-59 days delinquent went to current then on to a further delinquent status, hence they were so classified as current. In contrast, more loans that were observed in 60-89 days delinquent migrated to a worse state and were therefore classified as delinquent. Finally, the majority of 90 days plus delinquent loans did not default. Hence, they too were classified as delinquent. In summary, in our model current loans are defined as actually current and 30-59 days delinquent, while delinquencies are classified as 60-89 days delinquent and 90 days plus delinquent. Defaulted loans are those loans that are classified as either REO or in foreclosure.

For the intensity process estimation, the loan-specific characteristics included are: (1) age of the loan (as a percent of the life of the loan), (2) the delinquency status of the loan (dlqstatus), (3) an American Council of Life Insurers (ACLI) foreclosure survivor bias variable (fore index)[62], (4) the NOI at origination divided by the loan balance at origination (noi), (5) the prepayment restriction (normalized, monthly) (pen), (6) the logarithm of the original loan balance (origloanbal), (7) the debt service coverage ratio at origination (dscr), (8) the loan-to-value ratio at origination (ltv), (9) the weighted average coupon at origination (wac), (10) the loan spread at securitization (only for fixed-rate loans) (coupon spread), (11) a dummy variable for property type (IN, LO, NF, OF, OT) and (12) a dummy variable for geographical location (R1-R8). The choice of many of the variables were dictated by data availability. Our database contained reliable data on loan characteristics at origination, but not afterwards.[63]

[62] This is the average foreclosure rate over the past 14 years for each property× region, constructed from the ACLI foreclosure database (see Figure 110).

[63] For example, some but not all of the loans had data on NOI after origination. The sparsity of the updated NOI observations made this variable inappropriate to use. In addition, the updated NOI information is self-reported and not reliable. Whereas, at the origination date, the information is audited by the originator and a third party appraiser.

The hazard rate estimation was done separately for fixed-rate and floating-rate loans. Recall that Figure 67 contains a summary of the loans contained in the estimation. For non-CTLs, the focus of this dissertation, there are 94,011 fixed-rate loans. The number of defaults for the fixed-rate loans is 2,153. Recall that the parameter estimates for a competing risk current versus delinquent point process and for the default point process are shown in Figure 68. The parameter estimates are based on the equation:

$$\text{intensity} = \frac{1}{\left(1 + e^{\sum_i \text{coefficient}_i \cdot \text{variable}_i}\right)} \tag{118}$$

The first column contains the variables: $h_i(t) = i\,\text{th}$ property × region stock price index at time t; *P1* = industrial property dummy variable; *P2* = lodging property dummy variable; *P3* = multifamily property dummy variable; *P4* = office property dummy variable; *P5* = other property dummy variable; *P6* = retail property dummy variable (*omitted*); *R1* = East–North–Central region dummy variable; *R-SQ* = Mideast region dummy variable; *R3* = Mountain region dummy variable; *R4* = Northeast region dummy variable; *R5* = Pacific region dummy variable; *R6* = Southeast region dummy variable; *R7* = Southwest region dummy variable; *R8* = West–North–Central dummy variable; *R9* = other region dummy variable (*omitted*); age = (1 – remaining term/original term); dlqStatus = delinquency status; fore index = ACLI foreclosure index; noi = net operating income at origination divided by the original loan balance; pen = penalties divided by outstanding balance at time t; origloanbal = logarithm of the original loan balance; $H_i(t) = i\text{th}$ property stock price index at time t; dscr = debt service coverage ratio at origination; ltv = loan to value ratio at origination; $r(t) =$ spot rate of interest at time t; $\bar{H}(t) =$ Reit stock price index at time t; $f(t,10) - r(t) = 10$ year forward rate minus the spot rate at time t; wac = weighted average coupon at origination; coupon_spread = coupon minus treasury rate spread at loan origination; The remaining columns give the coefficients and standard errors.

For the current and delinquent intensity process note that the coefficients are equal and opposite in sign for current and delinquency. I concentrate on explaining the intensity of going from current to delinquent. For fixed-rate loans: (i) as the age of the loan increases, the likelihood of delinquency increases, (ii) as historical foreclosures increase (fore index), the likelihood of delinquency declines, (iii) the NOI (noi) is insignificantly different from zero[64], (iv) the higher the prepayment penalties (pen), the higher the likelihood of delinquency, (v) the larger the original loan balance, the higher the likelihood of delinquency, (vi) the higher the debt service coverage ratio (dscr), the higher the likelihood of delinquency, (vii) the higher the loan-to-value ratio at

[64] This is probably due to the endogeneity of the origination process. The terms of the loan contract are set to reflect the NOI of the given property, making its explanatory power zero.

securitization (ltv), the lower the likelihood of delinquency[65], and (viii) the higher the weighted average coupon (wac) and coupon spread, the lower the likelihood of delinquency. As the property indices increase ($h_i(t)$, $H_i(t)$, and $\bar{H}(t)$), the likelihood of delinquency declines. As either the spot rate ($r(t)$) or the slope of the forward rate curve ($f(t,10)-r(t)=10$) increases, the likelihood of delinquency increases. All of these comparative statics are as expected.

For the default intensity, the signs of these coefficients are mostly as expected. For fixed-rate loans: (i) the larger the age of the loan, the more likely to default, (ii) if the loan is delinquent, then probability of default increases, (iii) as historical foreclosures increase (fore index), the likelihood of default decreases, (iv) net operating income (noi) appears to have no impact on likelihood of default, (v) the higher the prepayment penalties (pen), the higher the likelihood of default (vi) the larger the original loan balance, the more likely it is to default, (vii) the higher the debt service coverage ratio, the less likely to default, (viii) the higher the loan-to-value ratio at securitization (ltv), the higher the likelihood of default, (ix) the higher the weighted average coupon (wac), the higher the probability of default and the higher the coupon spread at origination, the lower the probability of default. As the spot rate ($r(t)$) increases or the term structure ($f(t,10)-r(t)=10$) becomes more steep default is more likely. Lastly, as the property × region

[65] Again, this is probably due to the origination process. Those loans that have high initial loan-to-value ratios are probably viewed as having less default risk at origination. Otherwise, the originators would have reduced the loan-to-value ratio of the borrowing entity.

index ($H_i(t)$) increases, default is unchanged. As the property index ($h_i(t)$) increases, the likelihood of default declines. Finally, as the REIT index increases ($\bar{H}(t)$) default declines.

A standard method for measuring out-of-sample performance is the area under the receiver operating characteristic (ROC) curve. For comparison across models, a value of 0.5 for the ROC measure indicates a random model with no predictive ability, while a value of 1.0 indicates perfect forecasting. The ROCaccuracy ratios for the different intensity processes estimated for Fixed Rate Loans are all quite high with ROC ratios for Default=0.830, Current=0.886, and Delinquent = 0.886. These numbers are comparable to those obtained in the estimation of corporate bankruptcies (see Chava and Jarrow, 2004).

Appendix E –Merton, BSM Proofs, and Brownian Motion

E.1. Merton, 1974: Suppose for simplicity that a firm has one zero coupon bond outstanding and that the bond matures at time *T*. Merton defines for us the following:

V_0: Value of the company's assets today
V_T: Value of the company's assets at time T
E_0: Value of the company's equity today
E_T: Value of the company' equity at time T
D_T: Amount of debt interest & principal due at time T
σ_V: Volatility of assets
σ_E: Volatility of equity
r: Constant spot risk free rate of interest

In theory, if $V_T < D$ it is rational for the company to default on its debt, *D*, at time *T*. If $V_T > D$ then the company should make the payment at time *T* and the value of the equity, *E*, at this time is $V_T - D$. In this sense, the equity holder has both a junior and contingent claim on the residual value of assets in the future (*time T*), which by Merton can be constructed as the maximum of assets minus debt or zero (0),

$$E_T = \max\left[\left(V_T - D\right),\ 0\right] \tag{119}$$

This shows that the equity holder is effectively a call option holder with the strike price on the call option equal to the value of the repayment of the debt (*D*) at time *T*. Thus the equity holder is exposed to changes in value with respect to time. If at time *T*, the value of *D* is greater than or equal to the total value of the company (V_T), the equity (*modeled thus as a call option*) will expire as worthless (0). Such expiration will evidenced by a default on the debt at time *T*. If the value of the debt is less than the total value of the company (V_T) at time *T*, then the option will expire and the equity value E_T will be positive resulting in a repayment of the debt at time *T*. Thus, the risk of default and reward of equity value are transferred to the equity holder at the time of the expiration of the debt. For our purposes in determining the probability of default under Merton, such expiration will be evidenced by a default on the debt (D) at time T as the company is 'underwater'. Equity (E_T) only begins to exhibit values greater than zero when the value of the company V_T exceeds the amount of the debt (D). Thus, I can consider the value of the debt at expiry as a barrier. If the company value falls below it, it will have an incentive to default. Since the value of the Equity can be said to be contemplated as a European Call option on a non-dividend paying stock, then I simply build upon the familiar notation from BS and make a few minor adjustments to the notation for Merton. Specifically, $c = S_0 N(d_1) - Ke^{-rT} N(d_2)$ can be altered for the Merton notation by noting that $c = E_0$ so I can say, under Merton

$$c = E_0 = V_0 N(d_1) - De^{-rT} N(d_2) \tag{120}$$

where, V_0, the value of the company today is substituted for S_0 the price of the Stock today; and where D, the amount of Debt due at maturity is substituted for K, the Strike Price on the call at expiration. Finally, I substitute σ_V, the volatility of the company for σ, the volatility of the stock price and I then adjust for the notation for each of $N(d1)$ and $N(d2)$ to get,

$$d_1 = \frac{\ln(V_0/D) + (r + \sigma_V{}^2/2)T}{\sigma_V\sqrt{T}} \tag{121}$$

and

$$d_2 = d_1 - \sigma_V\sqrt{T} \tag{122}$$

Given Merton's framework and assumptions, I now have the implied value of the debt based on the parameters and framework, as

$$D_0 = V_0 - E_0 \tag{123}$$

What's missing of course is that that observing the true value of a company, V_0, or the volatility of a company, σ_V, is not typically observable. However, if the company is publicly traded, we can observe E_0 (the LHS) directly and can estimate σ_E, the equity volatility from historical data. Since we know, E_0, to solve for V_0 and σ_V, we simply need one more equation that is also constrained by the same two unknowns. Importantly, by Ito's Lemma , $\partial E/\partial V$ represents a draw on the Normal distribution for the term in Merton, $d1$, so $\partial E/\partial V \approx \mathrm{N}(d1)$. And so I write:

$$\sigma_E E_0 = N\left(d_1\right)\sigma_V V_0 \tag{124}$$

Now I have two simultaneous equations in two unknowns that can be solved for the implied company value and implied company volatility, V_0 and σ_V respectively, where,

$$\text{Merton/BSM condition: } F(x,y) = F(V_0,\sigma_V) = E_0 = V_0 N(d_1) - De^{-rT} N(d_2) \tag{125}$$

and

$$\text{Ito Condition: } G(x,y) = G(V_0,\sigma_V) = E_0\sigma_E = N(d_1)V_0\sigma_V \rightarrow E_0 = \frac{N(d_1)V_0\sigma_V}{\sigma_E} \tag{126}$$

so, $G(x,y) = F(x,y)$ iff

$$V_0 N(d_1) - De^{-rT} N(d_2) = \frac{N(d_1)V_0\sigma_V}{\sigma_E} \tag{127}$$

The perspective brought by solving $F(x,y) = 0$ and $G(x,y) = 0$ by finding the x and y values that minimize $\left[F(x,y)\right]^2 + \left[G(x,y)\right]^2$ allows us to contemplate the payoff or default of the company with respect to its debt obligation in terms of the Normal probability distribution.

E.2. Put/call parity proof for Black Scholes Merton: The Black-Scholes price at time t of a European put option with strike K and maturity T is $F(t, S_t)$ where

$$F(t,x) = Ke^{-r\theta}\Phi(-d_2) - x\Phi(-d_1) \tag{128}$$

The value at time t of a European option whose payoff at maturity is $C_T = f(S_T)$ is $V_t = F(t, S_t)$ where

$$F(t,x) = e^{-r(T-t)}\int_{-\infty}^{\infty} f\left(S_t e^{\left(\left(r-\frac{\sigma^2}{2}\right)(T-t)+\sigma y\sqrt{T-t}\right)}\right)\times\frac{1}{\sqrt{2\pi}}e^{\left(\frac{-y^2}{2}\right)}dy \tag{129}$$

Let $\theta = T - t$ and let $S_t = x$. So I get

$$F(t,x) = e^{-r\theta}\int_{-\infty}^{\infty} f\left(xe^{\left(\left(r-\frac{\sigma^2}{2}\right)\theta+\sigma y\sqrt{\theta}\right)}\right)\times\frac{1}{\sqrt{2\pi}}e^{\left(\frac{-y^2}{2}\right)}dy \tag{130}$$

$$F(t,x) = e^{-r\theta}\int_{-\infty}^{\infty} f\left(\underbrace{xe^{\left(r\theta+\sigma y\sqrt{\theta}-\frac{\sigma^2}{2}\theta\right)}}_{curr\ px\ of\ stock\ at\ t}\right)\times\frac{1}{\sqrt{2\pi}}e^{\left(\frac{-y^2}{2}\right)}dy \tag{131}$$

Now I can write the European put with a strike price K which has value greater than zero so $Put = P_t = max\{K - x, 0\}$ which I could derive by integrating:

$$P_t = e^{-r\theta} \int_{-\infty}^{\infty} f\left(K - \underbrace{xe^{\left(r\theta + \sigma y\sqrt{\theta} - \frac{\sigma^2}{2}\theta\right)}}_{curr\ px\ of\ stock\ at\ t} \right)_+ \times \frac{1}{\sqrt{2\pi}} e^{\left(\frac{-y^2}{2}\right)} dy \tag{132}$$

Instead of deriving the price of the put directly, however, I can use the put-call parity theorem. Suppose portfolios exist, one consisting of a European call option and a riskless discount bond, and the other consisting of a European put option and a share of stock against which both options are written. The call and put both have exercise price K and t periods to expiration. And the riskless bond pays off K dollars at time t. Then these portfolio payoffs are identical since the payoff on the first portfolio is

$$Max\{x - K, 0\} + K = Max\{x, K\} \tag{133}$$

and the payoff on the second portfolio is

$$Max\{K - x, 0\} + x = Max\{x, K\} \tag{134}$$

Consequently, the current value of these portfolios must also be the same; otherwise there would be a riskless arbitrage opportunity. Therefore, the price of a European put option P_t can be constructed from the call C_t, the strike K and the stock Price x ."[66]

$$\begin{aligned} P_t &= C_t + Ke^{-r\theta} - x \\ &= \underbrace{x\Phi\left(d_1\right) - Ke^{-r\theta}\Phi\left(d_2\right)}_{Call\ V_t} + Ke^{-r\theta} - x \\ &= Ke^{-r\theta} - Ke^{-r\theta}\Phi\left(d_2\right) + x\Phi\left(d1\right) - x \\ &= Ke^{-r\theta}\left(1 - \Phi\left(d_2\right)\right) - x\left(1 - \Phi\left(d1\right)\right) \\ &= Ke^{-r\theta}\Phi\left(-d_2\right) - x\Phi\left(-d1\right) \end{aligned} \tag{135}$$

As required. Now, to prove C_t, we derive, and solve for the boundary condition of y in function $f(\cdot)$ so the price condition is:

$$xe^{r\theta}e^{\sigma y\sqrt{\theta}}e^{-\frac{1}{2}\sigma^2\theta} \geq K \tag{136}$$

therefore,

[66] See Garven, James R. 2/26/2012

$$\frac{K}{x} \le e^{r\theta} e^{\sigma y\sqrt{\theta}} e^{-\frac{1}{2}\sigma^2\theta}$$

$$\frac{x}{K} \le \frac{1}{e^{r\theta} e^{\sigma y\sqrt{\theta}} e^{-\frac{1}{2}\sigma^2\theta}}$$

$$\frac{x}{K} \le e^{-r\theta} e^{-\sigma y\sqrt{\theta}} e^{\frac{1}{2}\sigma^2\theta}$$

$$ln\left[\frac{x}{K}\right] \le ln\left[e^{-r\theta} e^{-\sigma y\sqrt{\theta}} e^{\frac{1}{2}\sigma^2\theta}\right]$$

$$ln\left[\frac{x}{K}\right] \le ln\left[e^{-r\theta-\sigma y\sqrt{\theta}+\frac{1}{2}\sigma^2\theta}\right]$$

$$ln\left[\frac{x}{K}\right] \le -r\theta - \sigma y\sqrt{\theta} + \frac{1}{2}\sigma^2\theta$$

$$\sigma y\sqrt{\theta} \le -r\theta + \frac{1}{2}\sigma^2\theta - ln\left[\frac{x}{K}\right]$$

$$y \le \frac{-r\theta + \frac{1}{2}\sigma^2\theta - ln\left[\frac{x}{K}\right]}{\sigma\sqrt{\theta}} = -d_2 \tag{137}$$

which is the lower bound of integration. Changing signs:

$$d_2 = \frac{ln\left[\frac{x}{K}\right] + r\theta - \frac{1}{2}\sigma^2\theta}{\sigma\sqrt{\theta}} \tag{138}$$

Therefore, if $y \le -d_2 \rightarrow y \ge d_2$, and the integrand will be positive on the bounds of integration

$$F(t,x) = e^{-r\theta}\int_{-d2}^{\infty}\left(xe^{r\theta}e^{\sigma y\sqrt{\theta}}e^{-\frac{1}{2}\sigma^2\theta} - K\right)\times\frac{1}{\sqrt{2\pi}}e^{\frac{-y^2}{2}}dy \tag{139}$$

And I multiply through and make the observation

$$C_t = e^{-r\theta} \int_{-d2}^{\infty} x e^{r\theta} e^{\sigma y \sqrt{\theta}} e^{-\frac{1}{2}\sigma^2\theta} \frac{1}{\sqrt{2\pi}} e^{\frac{-y^2}{2}} dy - e^{-r\theta} K \underbrace{\int_{-d2}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-y^2}{2}} dy}_{\substack{std\ Normal \\ integral = \Phi(d2)}} \tag{140}$$

$$C_t = e^{-r\theta} \int_{-d2}^{\infty} x e^{r\theta} e^{\sigma y \sqrt{\theta}} e^{-\frac{1}{2}\sigma^2\theta} \frac{1}{\sqrt{2\pi}} e^{\frac{-y^2}{2}} dy - e^{-r\theta} K\Phi(d2) \tag{141}$$

Re-arranging

$$C_t = e^{-r\theta} \int_{-d2}^{\infty} x e^{r\theta} e^{\sigma y \sqrt{\theta}} e^{-\frac{1}{2}\sigma^2\theta} e^{\frac{-y^2}{2}} \frac{1}{\sqrt{2\pi}} dy - e^{-r\theta} K\Phi(d2) \tag{142}$$

But I can take out $e^{r\theta} in\ the\ first\ integral$

$$\begin{aligned}
C_t &= e^{-r\theta} e^{r\theta} \int_{-d2}^{\infty} x e^{\sigma y \sqrt{\theta}} e^{-\frac{1}{2}\sigma^2\theta} e^{\frac{-y^2}{2}} \frac{1}{\sqrt{2\pi}} dy - e^{-r\theta} K\Phi(d2) \\
C_t &= e^{r\theta - r\theta} \int_{-d2}^{\infty} x e^{\sigma y \sqrt{\theta}} e^{-\frac{1}{2}\sigma^2\theta} e^{\frac{-y^2}{2}} \frac{1}{\sqrt{2\pi}} dy - e^{-r\theta} K\Phi(d2) \\
C_t &= e^{0} \int_{-d2}^{\infty} x e^{\sigma y \sqrt{\theta}} e^{-\frac{1}{2}\sigma^2\theta} e^{\frac{-y^2}{2}} \frac{1}{\sqrt{2\pi}} dy - e^{-r\theta} K\Phi(d2) \\
C_t &= \int_{-d2}^{\infty} x e^{\sigma y \sqrt{\theta}} e^{-\frac{1}{2}\sigma^2\theta} e^{\frac{-y^2}{2}} \frac{1}{\sqrt{2\pi}} dy - e^{-r\theta} K\Phi(d2) \\
C_t &= \int_{-d2}^{\infty} x e^{\sigma y \sqrt{\theta} - \frac{1}{2}\sigma^2\theta - \frac{y^2}{2}} \frac{1}{\sqrt{2\pi}} dy - e^{-r\theta} K\Phi(d2)
\end{aligned} \tag{143}$$

Re-arranging

$$C_t = \int_{-d2}^{\infty} xe^{\frac{-y^2}{2}+\sigma\sqrt{\theta}y-\frac{1}{2}\sigma^2\theta}\frac{1}{\sqrt{2\pi}}dy - e^{-r\theta}K\Phi(d2) \tag{144}$$

Now I complete the square for the exponent

$$\frac{-y^2}{2}+\sigma\sqrt{\theta}y-\frac{1}{2}\sigma^2\theta \tag{145}$$

For any quadratic when I complete the square I use $ax^2 + bx + c = 0$ then solve for the roots $(x-h)^2 + k = 0$, where $h = -\frac{b}{2a}$ and $k = c - \frac{b^2}{2a}$. In the case where a dne 1 I normalize to make $= 1$

$$\begin{aligned}
&-2*\left(\frac{-y^2}{2}+\sigma\sqrt{\theta}y-\frac{1}{2}\sigma^2\theta = 0\right)\\
&y^2 - 2\sigma\sqrt{\theta}y+\sigma^2\theta = 0\\
&So,\ a = 1,\ b = -2\sigma\sqrt{\theta},\ c = \sigma^2\theta,\ h = -\frac{\left(-2\sigma\sqrt{\theta}\right)}{2} = \sigma\sqrt{\theta},\\
&and\ k = \sigma^2\theta - \frac{\left(-2\sigma\sqrt{\theta}\right)^2}{4} = \sigma^2\theta - \frac{4\sigma^2\theta}{4} = \sigma^2\theta - \sigma^2\theta = 0
\end{aligned} \tag{146}$$

Therefore, I get:

$$\begin{aligned}
&(x-h)^2 + k = 0\\
&\left(y-\sigma\sqrt{\theta}\right)^2 + 0 = 0
\end{aligned} \tag{147}$$

But I have to multiply through by the reciprocal I initially normalized with so I get:

$$
\begin{aligned}
&-\frac{1}{2}\left(y-\sigma\sqrt{\theta}\right)^2+0=0\\
&-\frac{\left(y-\sigma\sqrt{\theta}\right)^2}{2}=0\\
&\frac{-y^2}{2}+\sigma\sqrt{\theta}y-\frac{1}{2}\sigma^2\theta=-\frac{\left(y-\sigma\sqrt{\theta}\right)^2}{2}
\end{aligned}
\tag{148}
$$

And I substitute in the exponent in the integral

$$
C_t=\int_{-d2}^{\infty}x\underbrace{e^{-\frac{\left(y-\sigma\sqrt{\theta}\right)^2}{2}}\frac{1}{\sqrt{2\pi}}}_{\substack{\sim N\ density\\ unit\ variance}}dy-e^{-r\theta}K\Phi\left(d2\right)
\tag{149}
$$

Finally, I substitute $u=y-\sigma\sqrt{\theta}$. If y runs from $-d2\ \to\infty\ then\ u+\sigma\sqrt{\theta}=y$, then u runs from

$\underbrace{-d2-\sigma\sqrt{\theta}}_{=d1}\le u\le\infty$. So, $d1=d2+\sigma\sqrt{\theta}$.

Therefore:

$$
\begin{aligned}
C_t&=\int_{-d1}^{\infty}xe^{-\frac{u^2}{2}}\frac{1}{\sqrt{2\pi}}du-e^{-r\theta}K\Phi\left(d2\right)\\
C_t&=x\int_{-d1}^{\infty}\underbrace{e^{-\frac{u^2}{2}}\frac{1}{\sqrt{2\pi}}}_{\substack{\sim N\ density\\ unit\ variance}}du-e^{-r\theta}K\Phi\left(d2\right)\\
C_t&=x\Phi\left(d1\right)-e^{-r\theta}K\Phi\left(d2\right)
\end{aligned}
\tag{150}
$$

which is the Black Scholes Pricing Formula for a European Call Option C_t .■

E.3. Black Scholes Merton with Stochastic Calculus: Suppose that the value of a European Call option can be expressed as $V_t = F(t, S_t)$. Then $\tilde{V}_t = e^{-rt}V_t$, and I may define $\tilde{F}$ by $\tilde{V}_t = \tilde{F}(t, \tilde{S}_t)$. Under the risk neutral measure the discounted asset price follows $d\tilde{S}_t = \sigma\tilde{S}_t dX_t$ where, (under probability measure) $\{X_t\}_{t\geq 0}$ is a standard Brownian motion[67]. To find the stochastic differential equation satisfied by $\tilde{F}(t, \tilde{S}_t)$, I note that I have an SDE for discounted asset price that follows (*satisfies*) $d\tilde{S}_t = \sigma\tilde{S}_t dX_t$. So, the discounted European Call that satisfies (*follows*): $\tilde{F}(t, \tilde{S}_t) = (\cdot)dt + (\cdot)dX_t$. I proceed by Itò,

$$\begin{aligned}
d\tilde{F}\left(t,\tilde{S}_t\right) &= \frac{\partial \tilde{F}}{\partial t}dt + \frac{\partial \tilde{F}}{\partial \tilde{S}_t}d\tilde{S}_t + \frac{1}{2}\frac{\partial^2 \tilde{F}}{\partial \tilde{S}_t^2}\underbrace{\left(d\tilde{S}_t\right)^2}_{dt} \\
d\tilde{F}\left(t,\tilde{S}_t\right) &= \frac{\partial \tilde{F}}{\partial t}dt + \frac{\partial \tilde{F}}{\partial \tilde{S}_t}d\tilde{S}_t + \frac{1}{2}\frac{\partial^2 \tilde{F}}{\partial \tilde{S}_t^2}dt \\
d\tilde{F}\left(t,\tilde{S}_t\right) &= \frac{\partial \tilde{F}}{\partial \tilde{S}_t}\underbrace{d\tilde{S}_t}_{=\sigma\tilde{S}_t dX_t} + \left(\frac{\partial \tilde{F}}{\partial t} + \frac{1}{2}\frac{\partial^2 \tilde{F}}{\partial \tilde{S}_t^2}\right)dt
\end{aligned} \tag{151}$$

So,

$$d\tilde{F}\left(t,\tilde{S}_t\right) = \frac{\partial \tilde{F}}{\partial \tilde{S}_t}\left(\sigma\tilde{S}_t dX_t\right) + \left(\frac{\partial \tilde{F}}{\partial t} + \frac{1}{2}\frac{\partial^2 \tilde{F}}{\partial \tilde{S}_t^2}\right)dt \tag{152}$$

Also, since,

$$\tilde{F}\left(t,\tilde{S}_t\right) = \tilde{V}_t = e^{-rt}V_t \tag{153}$$

67 Discussed at the conclusion of this Appendix.

We can determine that

$$\begin{aligned}
\frac{\partial \tilde{F}}{\partial t} &= e^{-rt} dV_t + V_t\left(-re^{-rt}\right)dt \\
\frac{\partial \tilde{F}}{\partial t} &= e^{-rt} dV_t - r\underbrace{V_t e^{-rt}}_{=\tilde{V}} dt \\
\frac{\partial \tilde{F}}{\partial t} &= e^{-rt} dV_t - r\tilde{V}_t dt
\end{aligned} \tag{154}$$

So I can substitute for $\frac{\partial \tilde{F}}{\partial t}$ in

$$d\tilde{F}\left(t, \tilde{S}_t\right) = \frac{\partial \tilde{F}}{\partial \tilde{S}_t}\left(\sigma \tilde{S}_t dX_t\right) + \left(e^{-rt} dV_t - r\tilde{V}_t dt + \frac{1}{2}\frac{\partial^2 \tilde{F}}{\partial \tilde{S}_t^2}\right)dt \tag{155}$$

The first two terms in parentheses vanish as they are differential products with dt and negligible.

So,

$$d\tilde{F}\left(t, \tilde{S}_t\right) = \frac{\partial \tilde{F}}{\partial \tilde{S}_t}\left(\sigma \tilde{S}_t dX_t\right) + \frac{1}{2}\frac{\partial^2 \tilde{F}}{\partial \tilde{S}_t^2} dt \tag{156}$$

Rearranging, then I get

$$d\tilde{F}\left(t, \tilde{S}_t\right) = \frac{1}{2}\frac{\partial^2 \tilde{F}}{\partial \tilde{S}_t^2} dt + \sigma \frac{\partial \tilde{F}}{\partial \tilde{S}_t} \tilde{S}_t dX_t \tag{157}$$

which is the SDE satisfied by $\tilde{F}(t, \tilde{S}_t)$ in the form as required, and I note $\tilde{S}_t = \tilde{S}_0 + \sigma \int_0^t \tilde{S}_u dX_u$

then, using the fact that V_t is a martingale under the risk-neutral measure, I find the partial differential equation satisfied by $\tilde{F}(t, x)$, and show that:

$$\frac{\partial F}{\partial t} + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2 F}{\partial x^2} + rx\frac{\partial F}{\partial x} - rF = 0 \tag{158}$$

is the Black-Scholes equation. Since V_t is a martingale under the risk neutral measure I write

$$f\left(S_t\right)=V_t=e^{-r(T-t)}E^Q\left[V_T \mid \mathcal{F}_t\right],0<t<T \tag{159}$$

where V_T is the payoff at expiration T, $\mathcal{F}_t$ is the filtration generated by process $\{W_t\}$ under Q.

And, we know from Feynman-Kac that

$$F\left(t,x\right)=E^Q\left[e^{-r(T-t)}f\left(S_T\right) \mid S_t=x\right] \tag{160}$$

which can be written

$$\begin{aligned} F\left(t,x\right)&=E^Q\left[e^{-r(T-t)}V_T \mid V_t=x\right] \\ F\left(t,x\right)&=e^{-r(T-t)}E^Q\left[V_T \mid V_t=x\right] \end{aligned} \tag{161}$$

So, the key claim to make is that conditioning on the filtration $\mathcal{F}_t$ and conditioning on the value V_T are the same things provided that $V_T = \phi\big(x(T)\big)$, which it is.■

E.4. The Wiener Process and Brownian Motion: One example of random processes with independent and stationary increments is a Wiener Process. Schaum and Ross tell us a random process $\{X(t), t \geq 0\}$ is called a Wiener Process if:

- $X(t)$ has stationary independent increments
- The increment $X(t) - X(s)(t > s)$ is Normally distributed
- $E[X(t)] = 0$
- $X(0) = 0$

In the discrete time setting a widely adopted model for stock price dynamics is

$$\frac{S\left(t+\Delta t\right)-S\left(t\right)}{S\left(t\right)}=\mu\Delta t+\sigma\Delta \mathrm{B}\left(t\right) \tag{162}$$

With Δt a time interval, $S(t)$ and $S(t+\Delta t)$ the stock prices at current time t and future time $(t+\Delta t)$, and $\Delta B(t)$ the Brownian motion increment over Δt with μ and σ constants. By Wiersema

> The change in the stock price, relative to its current value at time t grows at a non-random rate of μ per unit of time and that there is also a random change which is proportional to the increment of the Brownian motion over Δt, with proportionality parameter σ. The standard Brownian motion importantly models the rate of return on a stock (and thus can take on negative values). The analogue in the continuous time world, the analogue is the arithmetic[68] Brownian motion stochastic differential equation:
>
> $$dS(t) = \mu\, dt + \sigma\, dB(t) \tag{163}$$
>
> With μ and σ known constants, and $\sigma > 0$. The growth (*drift*) coefficient $\mu[t, S(t)] = \mu$ and the diffusion (*volatility*) coefficient $\sigma[t, S(t)] = \sigma$ are both constant. Expressing this in integral form I get:
>
> $$\int_{t=0}^{T} dS(t) = \int_{t=0}^{T} \mu\, dt + \int_{t=0}^{T} \sigma dB(t) \tag{164}$$

which can be written as

$$S(T) - S(0) = \mu[T-0] + \sigma\left[B(T) - B(0)\right] \tag{165}$$

$$S(T) = S(0) + \underbrace{\mu T}_{non-random} + \sigma \underbrace{B(T)}_{\substack{\sim N(0,t-s) \\ s<t}} \tag{166}$$

And so the solution $S(T)$ can take on negative values, which is required for the modeling of random variables as returns (which will do *extensively*).

Finally, the distribution parameters for the arithmetic Brownian Motion SDE are given by:

---

[68] Contrasting with Geometric Brownian motion $dS(t) = \mu\, S(t)dt + \sigma\, S(t)dB(t)$ which *cannot* take on negative values.

$$\begin{aligned} E\left[S(T)\right] &= E\left[X(0)+\mu T+\sigma B(T)\right] \\ &= X(0)+\mu T+\sigma \underbrace{E\left[B(T)\right]}_{\substack{but,\ \sim N(0,1) \\ so \\ E[\cdot]=mean=0}} \\ &= X(0)+\mu T \\ &= \mu T,\ \text{for } X(0)=0 \end{aligned} \tag{167}$$

and

$$Var\left[S(T)\right] = Var\left[X(0)+\mu T+\sigma B(T)\right] = Var\,\sigma\left[B(T)\right] = \sigma^2 T \tag{168}$$

A discretized representation of generated arithmetic Brownian Motion $W(t)$ is provided in Figure 111 as equivalently $W(t,l)$ with each l representing a realization, or *sample path*, of the Brownian Motion $W(t)$.

Appendix F: Cashflow Verification

Using principal *and* interest cashflows require careful calculation of fair value at the bond level. I implement the exact allocation algorithm for Model 3 and simply adjust for pathwise risk free discount rates for Model 4. Below I provide detailed examples from the R-code implementation. In order to allocate the cashflows to the bond structure, I needed to sum the default adjusted cashflows for each period for each of the loans. This gives us an aggregate, or trust level default adjusted set of periodic cashflows, for each of the simulations. I show a small example of 4 loans across 5 simulations. Figure 111b shows the 1st 36 (of 120) periods for each of the 4 loans across each of the 5 simulations. The organization of the cashflows is #loans*simulations , so, again, in this example, the first 5 columns represent the cashflows for loan 1 across the 5 simulations, the second 5 columns represent the cashflows for loan 2 across the 5 simulations, and so forth. I now must capture the loan cashflows for each of the simulations. Figure 112 shows the aggregation across the 4 loans for each of the 5 simulations for the first 36 periods.

To check for accuracy, consider Loan payment period 23. The cashflows are highlighted in Figure 111b and 112. The period cashflows are then broken out in Figure 113 summing to what is reported in the Trust Cashflow table. The 0.52 cents on more than $24mm in anticipated payments is due to formatting and so the calculations appear to be correct. I now turn to the bond allocation.

I have ensured that the trust cashflows are captured and accurate with the above and many other tests. Additionally, only for those loans that default strictly 'prior' to origination data (only) I retain the default pushing it to the 2nd promised payment date. This preserves the default in highly volatile periods but uses the historical rate as a proxy. Truly this is just a computational convenience for very rare exceptions, but it does what is intended and ensures that 100% loss does not ensue by utilizing the historical loss rate for commercial real estate property types. Additionally, instead of rigidly (and unrealistically) assuming that recoveries are instantaneous, I use recovery periods specific to property types - and this is flexible.

In this summary, I am just looking at the principal cashflows including recoveries (with their timing) for 8 multifamily loans across 10 simulations (for display). The bond capital structure used is identical to the CMBX Series 1 (of which these loans are members). The percentage of the original balance of the loans in this example reflect the subordination as given (Figure 114). For each simulation date each of the 120 months of simulated cashflows must be allocated in full to the classes. At the end of each month that is simulated on a simulation date the Trust Cash Balance must equal 0 reflecting total allocation of all cashflows for that period. I consider 10 simulations and as before I show the trust cashflows which now reflect the sum of the simulated principal cashflow (with recoveries) for the 8 multifamily loans in the trust (Figure 115). I see, as before that the amounts vary in the periods reflecting different default, promised payments and timing of recovery.

In the senior Sub Structure, the principal is allocated first to the AAA, then to the AJ, then to the AM, and so forth thru the BBB-. I show Trust next to AAA (Figure 116-a) in the aggregate followed by the remaining tranche and then Figure 116-b which breaks out the Trust and AAA across 10sims. It is clear that only a portion gets allocated from the Trust to the AAA. But how much? If things are working, only the amount up to the cumulative remaining balance of the AAA in any given period. Consider simulation 10. The Trust (left) generates cashflows which are completely allocated to the AAA in periods 32 and 33, however in period 34 only 673.22 is allocated to the AAA. Why? Because that was the amount of the remaining balance of the AAA in that period under simulation. With the balance paid down in month 34, the AAA is no longer entitled to any other principal and so in months 35:120 (only up to month 50 shown), I see that the AAA receives $0 while the trust is still generating cashflows (Figure 117 and 118). I stay with simulation 10 for convenience and show the cascade of the remaining balance for each of the securities and the trust. It is worth highlighting that in addition to accurate cashflow allocation under simulation, there is of course, the possibility under such simulation that there is insufficient cash for the classes due to default and loss. In this particular simulation 10 the BBB- class *never* receives any payments (there are no principal payments in this simulation after month 50). Additionally, the BBB only gets paid off slightly. All the other classes AAA thru A all get paid off in full. This is reasonable given the loss experience in this Crisis[69].

For the Recovery portion of the study the relevant cashflow schedule information related to amortization and interest is from the EDGAR/prospectus supplement data in the Annex A tables. From this information, I was able to generate the promised cashflows from origination. At each simulation date (daily) I initialize the simulation reflecting the paid down balance of all the loans within the trust at that time and then allocate the cashflows under default adjustment in the simulation across the structure.

I know the allocation of principal[70] to each of the AAA, AS, AA, A, BBB-, BB and Unrated tranches is correct. Figure 119 sums to the maximum amounts of principal at the point of initialization of the simulation with 0.20% associated with rounding and timing mismatches only for the Unrated class on a $13.3B pool which is reasonable. Figure 120 shows the output for the promised cashflows allocation (over three pages) to the bond structure and Figure 121 shows the timing over the 145 month simulation horizon (as some loans expire >120 months in this pool). I

[69] See US CMBS Market Trends – December 2013 Natixis & Moody's, among others.

[70] Similarly, we know that the interest cashflows are being paid correctly and that the pricing reflects this correctly

also know that I am capturing the default adjusted cashflows correctly. Consider Figure 122 in which one loan that defaults on path 8. The recovery time is 2 months and the recovery rate is 74.23% of remaining balance at the time of default. This particular example is from the reduced form technology. The third column is the default adjusted cashflow that correctly captures the promised cashflows to period 29. A default occurs in period 30 and the recovery is realized in period 32. I use a short recovery period simply to be economical with simulation processing time as I must simulate as many months as the longest scheduled maturity of any loan in the sample *plus t*he recovery at maturity of the longest dated loan contemplating a balloon default. The recovery time and rate can always be adjusted. I next show in Figure 123 the allocation of principal cashflows for the promised and an average across paths contemplating default. The sum of the principal balance for each of the classes (on average across all paths is provided). It accurately reflects the default occurrence (cashflows sooner than expected from recoveries from defaults) and also accurately reflects the seasoning. The average across paths for the default plot was computed as of simdate 3/7/2014 and there is also amortization that pays down the AAA over that period. In Figure 124 in which the default adjusted principal is as of simdate 3/7/2014, there is no loss of principal to the AAA, (or for that matter all the classes through BBBmin). Rather, the AAA was paid off partly based on loans that originated as early as 8/2011. On average the BB as of this simdate across all simulations was showing a small loss. This is consistent with the pricing intuition.

Because of shortening in duration of the cashflows, however, there will be less interest paid to them as they will exist over a shorter period of time.

Finally, it is important to note that while I do not have as a resource updated 'real' cashflow data for the loans, what I do have is updated delinquency and default information on each of the transactions for which each of the 688 loans in this sample serve as the exhaustive collateral set. From origination of the loan through March 1, 2014 there were 0 delinquencies and 0 defaults in each of the transactions represented. As such *it must be the case* that the historical updated real cashflows for the simulation initialization period in each of the four models, must follow exactly the promised cashflow schedule. Therefore, I have no information deficit in the underlying sample data used for the Recovery period.

Appendix G– Primer on Poisson and Cox Processes

The Appendix serves as a primer on the use of Poisson and Cox Processes in this dissertation. It attempts to succinctly use references that take us from the basic counting process to the Cox Process.

G.1. Poisson and the Poisson Process as a counting process: Schaum and Ross tell us a Poisson Process $X(t)$ is an important counting process that fulfills the following criteria:

1. $X(0) = 0$
2. $X(t)$ has independent and stationary increments
3. The number of events in any interval of length t is Poisson distributed with mean λt such that for all $s, t > 0$:

$$P\left[X(t+s) - X(s) = n\right] = e^{-\lambda t}\frac{(\lambda t)^n}{n!}, n = 0,1,2\ldots \tag{169}$$

It follows from (3.) that a Poisson Process has stationary increments and that using the moment generating function $\phi(t)$ I can determine the mean $\mu_X = E[X(t)] = \lambda t$ and the variance $Var[X(t)] = \lambda t$ which are the same as is required of a Poisson random variable[71]. The implication is that for any unit length interval (t=1), such as $(0,1), (4,5), (100, 101) \ldots$ etc, the expected number of events that take place in that interval is then just λ, the *intensity*.

Following Shreve, I let the sequence $\tau_1, \tau_n, \ldots \tau_n$be independent exponential random variables, all with the same mean $\frac{1}{\lambda}$. I note the attribute on the memorylessness property of the

[71] The mathematics associated with this are provided at the end of this Appendix.

exponential random variable and in particular emphasize "After waiting S time units, the probability that I will have to wait an additional t time units is the same as the probability of having to wait t time units starting at time $t = 0$. The fact I have already waited s time units does not change the distribution of the remaining time". The τ_k random variables are, the interarrival times. The arrival times (or jump times) are then:

$$S_n = \sum_{k=1}^{n} \tau_k \tag{170}$$

where S_n is the time of the $n - th$ jump and the *Poisson Counting Process*, $N(t)$, which counts the number of jumps that occur at or before time t is:

$$N(t) = \begin{cases} 0 \quad if\ 0 \le t < S_1 \\ 1 \ \ if\ S_1 \le t < S_2 \\ 2 \ \ if\ S_2 \le t < S_3 \\ \quad\quad \vdots \\ n \ \ if\ S_n \le t < S_{n+1} \\ \quad\quad \vdots \end{cases} \tag{171}$$

with jump times S_n right continuous. Because the expected *time* between jumps is $\frac{1}{\lambda}$ the jumps are arriving at an average *rate* of λ per unit time and the Poisson Counting Process has intensity λ. In the first panel of Figure 124 I see that for constant intensity $\lambda = 0.25$ over the course of 15 years in one simulation l, the first jump occurs in the 97$^{\text{th}}$ time step, with each step $k = \frac{1}{12}$ so $s_n =$

$\sum_{i=1}^{97} \frac{1}{12} = 8.083$ and $N(t) = 1$. In the second panel of Figure 124 the first jump occurs at the 1st time step, the second occurs at the 78th time step, and the third jump occurs at the 96th time step so $s_n = \sum_{i=1}^{96} \frac{1}{12} = 8.0$ and $N(t) = 3$. I describe some of mathematics required to describe the distribution of the Poisson Process $N(t)$ at the end of this section.

G.2. The Compound Poisson Process and Jump Diffusion: As before I note that $N(t+s) - N(s)$ is independent of the filtration $\mathcal{F}(s)$. For the stationarity claim Shreve shares with us "When a process has the property that the distribution of the increment depend only on the difference between the two time points, the increments are said to be stationary. Both Poisson and Brownian Motion have stationary independent increments.

One direction we could pursue would be to simulate asset prices or the components of our simulated economy using Merton using Jumps where the combination of the Brownian Motion and the jumps is a Jump Diffusion. To get there using Shreve's notation I restate Corollary 11.3.4, Let $y_1 \dots, y_M$be a finite set of nonzero numbers and let $p(y_1) \dots p(y_M)$ be positive numbers that sum to 1. Let $Y_1, Y_2, \dots$be a sequence of iid random variables with $P\{Y_i = y_m\} = p(y_m), m - 1, \dots, M$. Let $N(t)$ be a Poisson process with parameter λ and $\{Y_i\}_{i \geq 1}$be a sequence of independent identically distributed random variables and define $Q(t) = \sum_{i=1}^{N(t)} Y_i$ as a *Compound Poisson Process.*

For $m = 1, \dots, M$ let $N_m(t)$denote the number of jumps in Q of size $\mathrm{y_m}$up to and including time t. Then, $N(t) = \sum_{m=1}^{M} N_m(t)$ and $Q(t) = \sum_{m=1}^{M} y_m N_m(t)$. The processes $N_1, \dots, N_M$ defined this way are independent Poisson Processes and each N_m has intensity $\lambda p(y_m)$. Shreve provides us with a graph in Figure 125 of one path of a Compound Poisson Process. The Compound Poisson Process is a generalization where the waiting times between jumps are exponential but the jump sizes can have an arbitrary distribution. So, the key point to take away from this is that while a process is incrementing through time the Compound Poisson Process times between jumps are following an exponential distribution. Both Shreve and Tankov/Voltchkova tell us that the amplitude of the jump when it occurs is of random size/arbitrary. The amplitude/size could be Normally Distributed/Gaussian, or could follow some other distribution or process.

Focusing more on the question of "when" the jump occurs, I revisit the exponential distribution for Poisson. Recall a continuous random variable that follows an exponential distribution with parameter $\lambda(> 0)$has probability density function (pdf) of

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & x < 0 \end{cases} \tag{172}$$

Let's assume that parameter $\lambda = 1$. This reduces to the form

$$f_X(x) = \begin{cases} e^{\zeta x}, & x > 0 \\ 0, & x < 0 \end{cases} \tag{173}$$

This means that if a process is following an exponential distribution through time, I would adjust the notation for time index t, or a Unit Exponential. Using Lando's notation:

$$F_E\left(E_1(t)\right)=\begin{cases} e^{E_1(t)}, & x(t)>0 \\ 0, & x(t)<0 \end{cases} \tag{174}$$

If $E_1(t)$ is allowed to be random I can simulate the value of $E_1(t)$ using the Uniform distribution similar to what we would do to simulate a normally distributed random variable, where in both instances, the input is the Uniformly distributed random value between 0 and 1 and the output is the distributed value. This is consistent with the explanation provided in the algorithm of Tankov/Voltchkova in which they simulate the Compound Poisson Process as described below with a Gaussian amplitude (see Figure 126)

> "Conditionally on $N_T = n$, the jump times $T_1, \ldots, T_n$ of a Poisson process on the interval $[0, T\,]$ are distributed as n independent ordered uniforms on $[0, T\,]$. This leads to the following algorithm:
>
> - Simulate N_T from the Poisson distribution with parameter T .
> - Simulate N_T uniform random variables $\{U_i\}_{i=1}^{N_T}$ on $[0, T]$.
> - Simulate N_T independent variables variables $\{Y_i\}_{i=1}^{N_T}$ with law f (meaning with distribution/density of Y_i).
>
> The process is then given by $X_t = \sum_{i=1}^{N_T} Y_i 1_{U_i \le t}$"

Combining a Brownian motion with drift and a compound Poisson process, I obtain the simplest case of a *Jump Diffusion* — a process which sometimes jumps and has a continuous, but random evolution, between the jump times. Following Tankov/Voltchkova they present us with the simplest form of Jump Diffusion proposed which is a Levy process

$$X_t = \mu t + \sigma B_t + \sum_{i=1}^{N_T} Y_i \tag{175}$$

The best known model of this type is proposed by Merton, 1976 model where stock price (modeled as an exponential to ensure positivity) is $S_t = S_0 e^{Xt}$ with X_t as expressed above and the jumps $\{Y_i\}$ following a Gaussian distribution. Tankov/Voltchkova provide us with a simulation and in the figure below show us one sample path of the jump diffusion process X_t (Brownian motion + compound Poisson). As we see in Figure 127 "in between jumps, the process evolves like a geometric Brownian Motion, and after each jump, the value of S_t is multiplied by e^{Y_i}." As such, the model proposed by Merton can be considered a generalization of Black Scholes:

$$\frac{dS_t}{S_{t-}} = \bar{\mu} dt + \sigma dB_t + dJ_t \tag{176}$$

where "J_t is the compound Poisson Process such that the i-th jump of J is equal to $e^{Y_i} - 1$. Whenever there is a jump the value of the process before the jump is used on the left hand side of the formula (S_{t-})." Shreve also provides us with similar expression in his Definition 11.4.3.

G.3. Reduced Form Default Intensity Models: The discussion of Jump Diffusion above has direct application to the modeling of asset price evolutions (stocks and bonds). In the reduced form approach of Model 4, however, I will be taking a different route. Instead of modeling the asset price evolution directly with a jump process as per Merton, I will be using an adaptation of the Poisson Process found in the default modeling literature.

Following Trueck/Rachev, reduced form models (in general) allow for surprise defaults. "At the heart of the reduced form models lies the instantaneous rate of default, the default intensity λ. Let $\mathcal{F}_t$ be the information up to time t, and τ the default time, Δt a marginally short time interval, and $\lambda(t)$ the default intensity as a function of time only. Assuming no defaults up to time t the basic default intensity is expressed as:

$$P\left(\tau \in \left(t+\Delta t\right) | \mathfrak{F}_t\right) \approx \lambda\left(t\right)\Delta t \tag{177}$$

which is approximately the proportionality factor between the default probability within a given time interval Δt and the length of this time interval." In other words, λ is the intensity of the process that specifies the default time τ. In the literature, Poisson processes are used to model the default times of rare and countable events. In this context the time of default is interpreted as the first jump of the Poisson process. So, revisiting the Poisson Process in Figure 124, as a default process, the default time in the first example would be $\tau = 97$ and the default time in the second example would be $\tau = 1$.

The reduced form (Model 4) default process I implement is more complex than this. Further, it is distinct amongst the other models studied in my dissertation. In Model 1, the default event is relegated to maturity and based upon the option value of the equity determined under Merton. In Models 2 and 3, the default event is triggered by the single factor (the inverse LTV) dependent on the REIT evolution corresponding to the property type. In Model 2, the event is restricted to the implied property value vs the debt value at Maturity (like Model 1, though it is a simulated event and property type specific); in Model 3 the default event may occur anywhere on the interval $[t, T]$ on any simulation path l.

In contrast, Model 4 considers the event of default to be a function of the relationship between loan level characteristics and the entire simulated economy. What I am going to do is consider use the intensities of states of the loans (current, delinquent or default) as time and path dependent random variables interacting with loan and economy characteristics. The method will be the Cox Process as introduced by Lando and Duffie in the literature and expanded on in Jarrow, etal 2008. The end result will be the simulation of the correlated economy (as discussed in the main text) and a default process that will perturb the cashflows with respect to the historical experience, statistically, using Maximum Likelihood Estimates. Then, using the cashflow algorithms of loan to bond allocation, the default adjusted bond level cashflows will be produced pathwise, and ultimately valued using the HJM term structure. The linkage between the loan and the simulated economy is the maximum likelihood estimates.

G.4. Use of Maximum Likelihood Estimates: Briefly, in the context of my dissertation, the MLE's were estimated using three logisitic regressions to capture the binary response of an event (the delinquent state, q, or the default state, d,) or non-event (the current state, c, aka the paying as promised state) from a set of 100,000 loans over a seven year period 1998-2005. The logistic function always takes on values between 0 and 1 with

$$F(y) = \frac{e^y}{1+e^y} = \frac{1}{\frac{1+e^y}{e^y}} = \frac{1}{\frac{1}{e^y}+1} = \frac{1}{1+e^{-y}} \tag{178}$$

If we view y as a linear function of an explanatory variable x and its associated coefficients β and intercept I may write:

$$E\left(y \mid x\right) = F(\beta_0 + \beta_1 x) = \frac{1}{1+e^{-(\beta_0+\beta_1 x)}} \tag{179}$$

which is the logistic CDF with the interpretation as the probability of success, or presence of the event. The intensity associated with the events is written as the multiple regression:

$$\text{intensity} = \frac{1}{\left(1+e^{-\sum_i \text{coefficient}_i \cdot \text{variable}_i}\right)} \tag{180}$$

G.5. Duffie on State variables & Parameter Estimates in Reduced Form: To interact the intensities described above with the loans, Duffie provides a succinct overview of the procedure for establishing a dependence of the default intensity upon State variables vis a vis parameter estimates (in this case MLEs). I learn that the maximum likelihood estimation of term structures of conditional default probabilities require both the estimation of default intensities at each point in time and the estimation of the probabilistic behavior of default intensities *over* time. Thus the default estimation problem is partitioned into a two part procedure: i.) estimate the parameter vector β determining the dependence of each default intensity $\lambda_d(X_t, \beta)$ on the underlying state variable vector X_t; and ii.) estimate the probabilistic time series behavior of X_t. Specifically, Duffie tells us:

> We fix some probability space $(\Omega, \mathcal{F}, P)$ and information filtration $\{\mathcal{F}_t : t \geq 0\}$. For a given stopping time τ, say a default time, we wish to estimate the term structure $\{P(\tau > t): t > 0\}$of survival probabilities. We suppose that τ is doubly stochastic driven by a *d*-dimensional Markov process X with intensity $\lambda_d(X_t, \beta)$, where $\beta \in \mathbb{R}^{\ell}$ is a vector of parameters. We suppose for simplicity that X is constant between integer observation times, $t = 1, 2, \ldots$

Therefore in the intensity expression above, the $coefficient_i$ term corresponds to Duffie's parameter coefficient vector, β, and the $variable_i$ term corresponds with Duffie's underlying state variable X_t, such that each default intensity $\lambda_d(X_t, \beta)$ is dependent on the simulated parameters which are state variables X_t and the state variable coefficients β (which in this dissertation are the MLEs). In this dissertation, the time dependent state variable X_t include the simulation of REITs, NCREIF property indices, and forward risk free rates under HJM. There are 27 such state variables all of which have an explanatory relationship with the historical events of default and delinquency as captured in the logisitic regression[72] such that the intensity using Duffie's notation is restated as:

$$\text{intensity} = \frac{1}{\left(1 + e^{-\sum_i \beta_i \cdot \mathrm{X}_i}\right)} \tag{181}$$

The intensity then is said to be Stochastic allowing the default intensity to change over time. As the method for implementation is simulation, one could express the intensity then as $\lambda_d(X_{t,j}, \beta)$. According to Duffie, I say that a stopping time τ with intensity λ_d is doubly stochastic, driven by X, if, conditional on the covariate path $\{X = X_t : t \geq 0\}$, τ is the first event time of some Poisson process with time-varying intensity $\{\lambda_t : t \geq 0\}$. This Poisson property implies that

$$P\left(\tau > t \,|\, (X, \beta)\right) = e^{-\int_0^t \lambda(s)ds} \tag{182}$$

Applying the law of iterated expectations (over the simulations), then

$$P(\tau > t) = E\left[P\left(\tau > t \,|\, (X, \beta)\right)\right] = E\left(e^{-\int_0^t \lambda(s)ds}\right) \tag{183}$$

Additionally, stopping times, $\tau_1, \ldots, \tau_n$ that are doubly stochastic driven by state variable $X$with respective intensities $\lambda_1 \ldots \lambda_n$ are said to be *jointly* doubly stochastic if these times are X-

[72] To estimate we use a numerical procedure, such as Newton Raphson, for MLE to maximize the log likelihood of the event of interest.

conditionally independent. An implication is that $\tau_1, \ldots, \tau_n$ are correlated only through the joint dependence of their intensities on the covariate state variable process X. For example, for any time

$$P\left(\tau_i > t, \tau_j > t\right) = E\left[P\left(\tau_i > t, \tau_j > t \mid \left(X, \beta\right)\right)\right]$$
$$= E\left(e^{-\int_0^t \lambda_i(s)ds} e^{-\int_0^t \lambda_j(s)ds}\right) \tag{184}$$
$$= E\left(e^{-\int_0^t \left[\lambda_i(s)+\lambda_j(s)\right]ds}\right)$$

Therefore for some probability space $(\Omega, \mathcal{F}, P)$ where Ω contains the possible state of the world, the set $\mathcal{F}$ consists of the subsets of Ω, called "events" to which a probability can be assigned, the probability measure $P: \mathcal{F} \longrightarrow \mathbb{R}$ assigns a probability $P(A)$ to each event A. I also fix an information filtration $\{\mathcal{F}_t : t \geq 0\}$ satisfying the conditions that specify for each time t the set $\mathcal{F}_t$ of events that are observable at that time. Then, given a stopping time τ for say default, I say that a progressively measurable process λ is the intensity of τ if a martingale $M$is defined by

$$M_t = 1_{\{\tau < t\}} - \int_0^t \lambda_s 1_{\{\tau > s\}} ds \tag{185}$$

where for any event A, the indicator 1 has an outcome of 1 on the event A and 0 otherwise. This means that at any time t before τ, conditional on the current information $\mathcal{F}_t$, the mean rate of arrival of default is λ_t, the conditional default intensity conditioned on all information up to t. For example, with time measure in years, a default intensity of $\lambda_t = 0.1$ means that default arrives at a conditional mean rate of once every 10 years, given all information available at time t.

G.6. Lando on Cox Processes, State variables & Default Intensity: While Duffie provides support for the use of Lando extends the discussion further. Specifically, a primary focus of Lando is to allow for dependence between default intensities and state variables. The timing of the jump event (default, delinquency) is also considered a conditional Poisson Process, where the distribution of the "when" of the jump is conditioned on the state variable $X(t)$. Where I extend the discussion is in the linking of the default time with the intensity process λ which is a function of state variables so, where $\lambda(X_s)$ is the Cox Process. Lando tells us:

> A Cox Process is a generalization of the Poisson Process in which the intensity is allowed to be random but in such a way that if we condition on a particular realization $l(\cdot, \omega)$ of the intensity, the jump process becomes an inhomogeneous Poisson process with intensity $l(s, \omega)$…where the random intensity
>
> $$l(s,\omega) = \lambda(X_s) \tag{186}$$
>
> is an R^d valued stochastic process and $\lambda: R^d \rightarrow [0, \infty)$is a non-negative continuous function. The assumption that the intensity is a function of the current level of the state variables, and not the whole history, is convenient in applications, but it is not necessary…The state variables will include interest rates on riskless debt and may include time, stock prices, credit ratings, and other variable deemed relevant for predicting the likelihood of default. Intuitively, given that a firm has survived up to time t, and given the history of X up to time t, the probability of defaulting within

the next small time interval Δt is equal to $\lambda(X_t)\Delta t + o(\Delta t)$[73]. Let E_1be a unit exponential variable which is independent of *X,* given also is $\lambda: R^d \to R$ which we assume non-negative and continuous. From these two ingredients we define the default time, τ as follows:

$$\tau = \inf\left\{t : \int_0^t \lambda\left(X_s\right) ds \geq E_1\right\} \tag{187}$$

Thus, if the intensity is greater than or equal to the unit exponential variable (which is independent of *X*), a default event occurs. X_s is very rich on two levels. First, because the conditional intensity is independent of the Exponential variable, the mathematics of the Poisson Process apply. Second, the intensity itself is informed by X_s which incorporates a lot of information. In Jarrow, etal (2008) some of X_s is current information as Lando suggests such as parameters for time, current payment status of the loan, outstanding loan balance, and many other factors. Many of these factors, are in turn, driven by correlated Brownian Motion $dZ_k(t,l)$ which serve as dynamic inputs to a regression in which the amplitude for the jump is determined by a regression of the form:

$$Y_t = \alpha + \beta_k Z_k\left(t, j\right) \tag{188}$$

where β_k represents the k-th corresponding coefficient for a time dependent variable determined using MLEs and Y_t represents the dependent variable of default.

To make some of the mathematics more explicit we just consider a little theory. I assert that what Lando is saying is that the compound Poisson Process, call it C, is a conditional Poisson

---

[73] Where $o(\Delta t)$ represents any function of Δt such that in the limit $f(\Delta t)/\Delta t \to 0$ faster than Δt.

Process, where $\mathbb{P}(C|X_t)$ is Poisson, conditional on the information set in X_t state variables. The assumption is that we 'know' everything in the State variable and have complete path-wise information. This is consistent with the idea of generating the correlated Brownian Motions and linking such continuous processes which represent the economy, to the actual historical default experience based upon loan level and economy wide characteristics experienced by the loan under simulation on each path l at each time t, found together in the state variables X_t, or actually $X_{t,l}$. With this perspective, then, over the interval $[0,t]$, the integral can be divided into

$$\begin{aligned}
&\int_0^t \lambda(X_s)\,ds \ge E_1 = \left\{ \int_0^{t-\Delta} \lambda(X_s)\,ds + \underbrace{\lambda(X_t)\Delta t}_{\approx \int_{t-\Delta}^{t} \lambda(X_s)ds} \right\} \ge E_1 \\
&\int_0^t \lambda(X_s)\,ds \ge E_1 = \left\{ \int_0^{t-\Delta} \lambda(X_s)\,ds + \int_{t-\Delta}^{t} \lambda(X_s)\,ds \right\} \ge E_1 \\
&\int_0^t \lambda(X_s)\,ds \ge E_1 = \left\{ \int_{t-\Delta}^{t} \lambda(X_s)\,ds + \int_0^{t-\Delta} \lambda(X_s)\,ds \right\} \ge E_1
\end{aligned} \tag{189}$$

So the takeaway is that the conditional Poisson Process is fully independent from the economy under simulation, as it should be, but it is embedded in historical sense as the realization of the intensity at any time t on any path l. $\lambda(X_s)$ informs the threshold of the default time τ. This completes the main portion of the supplement to the text which discusses the implementation of Model 4 in detail. The remainder of this Appendix provides some mathematics supplements referred to in this Appendix.

G.7. Basic Poisson mathematics: A discrete random variable X with parameter $\lambda > 0$ is said to be a Poisson random variable if its probability mass function (pmf) given by $p_X(k) = P(X = k) = e^{-\lambda}\frac{\lambda^k}{k!}$ for $k = 0,1,2,\ldots$ and corresponding cumulative distribution function (cdf) given by $F_X(x) = e^{-\lambda}\sum_{k=0}^{n}\frac{\lambda^k}{k!}$ for $n \leq x < n+1$ with $\mu_X = E(X) = \lambda$ and $\sigma_X^2 = Var(X) = \lambda$. Proof of the expectation of a Poisson Random Variable is given by:

$$E[X] = \sum_{n=0}^{\infty}\frac{ne^{-\lambda}\lambda^n}{n!} = \sum_{n=1}^{\infty}\frac{e^{-\lambda}\lambda^n}{(n-1)!}$$
$$= \lambda e^{-\lambda}\sum_{n=1}^{\infty}\frac{\lambda^{n-1}}{(n-1)!} = \lambda e^{-\lambda}\underbrace{\sum_{k=0}^{\infty}\frac{\lambda^k}{k!}}_{but\ =e^{\lambda}} \tag{190}$$
$$\therefore \lambda e^{-\lambda}e^{\lambda} = \lambda$$

To get the variance, I use the mgf of a Poisson random variable with mean :

$$\phi(t) = E\left[e^{tX}\right] = \sum_{n=0}^{\infty}\frac{e^{tn}e^{-\lambda}\lambda^n}{n!}$$
$$= e^{-\lambda}\sum_{n=0}^{\infty}\frac{\left(\lambda e^t\right)^n}{n!} = e^{-\lambda}\sum_{n=0}^{\infty}\frac{\left(\lambda e^t\right)^n}{n!} \tag{191}$$
$$= e^{-\lambda}\sum_{n=0}^{\infty}\frac{\left(\lambda e^t\right)^n}{n!} = e^{-\lambda}e^{\lambda e^t}$$

So,

$$\phi^{'}(t) = \frac{d}{dt}e^{\lambda\left(e^{t}-1\right)} = e^{\lambda\left(e^{t}-1\right)}\lambda e^{t} \tag{192}$$

and

$$\begin{aligned}\phi^{''}(t) &= \frac{d}{dt}e^{\lambda\left(e^{t}-1\right)}\lambda e^{t} \\ &= e^{\lambda\left(e^{t}-1\right)}\lambda e^{t} + \lambda e^{t}e^{\lambda\left(e^{t}-1\right)}\lambda e^{t} \\ &= \lambda e^{t}e^{\lambda\left(e^{t}-1\right)}\left[1+\lambda e^{t}\right]\end{aligned} \tag{193}$$

So,

$$\begin{aligned}\phi^{'}\left(t=0\right) &= E\left[X\right] \\ &= e^{\lambda\left(e^{0}-1\right)}\lambda e^{0} \\ &= 1\times\lambda\times 1 = \lambda\end{aligned} \tag{194}$$

$$\begin{aligned}\phi^{''}\left(t=0\right) &= E\left[X^{2}\right] \\ &= \lambda e^{0}e^{\lambda\left(e^{0}-1\right)}\left[1+\lambda e^{0}\right] \\ &= \lambda\left[1+\lambda\right] = \lambda+\lambda^{2}\end{aligned} \tag{195}$$

$$\begin{aligned}Var\left(X\right) &= E\left[X^{2}\right]-\left\{E\left[X\right]\right\}^{2} \\ &= \lambda+\lambda^{2} - \left\{\lambda\right\}^{2} = \lambda\end{aligned} \tag{196}$$

So I conclude for a Poisson Random Variable the mean and the variance are the same. ■

G.8. Mathematics for the distribution of the Poisson Process: Beginning with the convolution for the gamma density:

$$\int_0^s g_n(\nu)\, f(s-\nu)\, d\nu = \int_0^s \frac{(\lambda\nu)^{n-1}}{(n-1)!}\lambda e^{-\lambda\nu}\lambda e^{-\lambda(s-\nu)} d\nu$$

$$= \frac{\lambda^{n-1}\lambda\lambda}{(n-1)!}\int_0^s \nu^{n-1}e^{-\lambda\nu}e^{-\lambda s}e^{\lambda\nu}d\nu$$

$$= \frac{\lambda^{n+1}}{(n-1)!}\int_0^s \nu^{n-1}e^{-\lambda\nu-\lambda s+\lambda\nu}d\nu$$

$$= \frac{\lambda^{n+1}}{(n-1)!}\int_0^s \nu^{n-1}e^{-\lambda s}d\nu$$

$$= \frac{\lambda^{n+1}e^{-\lambda s}}{(n-1)!}\int_0^s \nu^{n-1}d\nu$$

$$= \frac{\lambda^{n+1}e^{-\lambda s}}{(n-1)!}\nu^n \Big|_{\nu=0}^{\nu=s}$$

$$= \frac{\lambda^{n+1}e^{-\lambda s}}{(n-1)!}s^n$$

$$= \frac{\lambda^n\lambda^1 s^n e^{-\lambda s}}{(n-1)!}$$

$$= \frac{(\lambda s)^n}{(n-1)!}\lambda e^{-\lambda s} \tag{197}$$

But I note from the determination of S_n as having the gamma density that

$$g_n(s) = \frac{(\lambda s)^{n-1}}{(n-1)!}\lambda e^{-\lambda s} \Rightarrow \frac{(\lambda s)^n}{(n-1)!}\lambda e^{-\lambda s} = g_{n+1}(s) \tag{198}$$

which matches the solution in Shreve, and for the Lemma for the distribution of the Poisson Process $N(t)$ with intensity λ. (Shreve, pg 465),

$$P\left\{N\left(t\right)\geq k+1\right\}=P\left\{S_{k+1}\leq t\right\}=\int_0^t\frac{(\lambda s)^k}{k!}\lambda e^{-\lambda s}ds \tag{199}$$

By Parts, let $u=\lambda\frac{(\lambda s)^k}{k!}=; du=\lambda\frac{k\lambda(\lambda s)^{k-1}}{k!}=\frac{\lambda^2(\lambda s)^{k-1}}{(k-1)!}ds; dv=e^{-\lambda s}ds; v=\frac{e^{-\lambda s}}{-\lambda}$, so

$$\begin{aligned}
&=\left(\lambda\frac{(\lambda s)^k}{k!}\right)\left(\frac{e^{-\lambda s}}{-\lambda}\right)\Bigg|\begin{matrix}s=t\\ s=0\end{matrix}-\int_0^t\left(\frac{e^{-\lambda s}}{-\lambda}\right)\left(\frac{\lambda^2(\lambda s)^{k-1}}{\left(k-1\right)!}\right)ds\\
&=-\frac{(\lambda s)^k}{k!}e^{-\lambda s}\Bigg|\begin{matrix}s=t\\ s=0\end{matrix}+\int_0^t\frac{(\lambda s)^{k-1}}{\left(k-1\right)!}\lambda e^{-\lambda s}ds\\
&=-\frac{(\lambda t)^k}{k!}e^{-\lambda t}+\underbrace{\int_0^t\frac{(\lambda s)^{k-1}}{\left(k-1\right)!}\lambda e^{-\lambda s}ds}_{\substack{\textit{but this is the}\\ \textit{gamma density from } 11.25}}\\
&=-\frac{(\lambda t)^k}{k!}e^{-\lambda t}+P\{N(t)\geq k\}
\end{aligned} \tag{200}$$

Therefore, from the initial expression

$$P\left\{N\left(t\right)\geq k+1\right\}=P\{S_{k+1}\leq t\}=-\frac{(\lambda t)^k}{k!}e^{-\lambda t}+P\{N(t)\geq k\} \tag{201}$$

Then, with this result from Shreve for $k\geq 1$,

$$P\left\{N\left(t\right)=k\right\}=P\left\{N\left(t\right)\geq k\right\}-P\left\{N\left(t\right)\geq k+1\right\}=\frac{(\lambda t)^k}{k!}e^{-\lambda t} \tag{202}$$

and for $k=0$,

$$P\left\{N\left(t\right)=k\right\}=P\left\{S_1>t\right\}=P\left\{\tau_1>t\right\}=e^{-\lambda t} \tag{203}$$

which is as Shreve tells us with $k = 0$. ■

Appendix H – Implementation of Switching

In the implementation, the payment state transition process (switching) is the result of the delinquency/current process $\mathbb{C}_i(t,l)$ and the default process, $\mathbb{Q}_i(t,l)$. Beginning with the process $\mathbb{C}_i(t,l)$, I have stochastic intensities for lambda delinquent, λ_q and lambda current, λ_c. The payment state $\mathbb{S}_i(t-,l)$ is the payment state of the loan at the initialization of any simulation. In all cases, the real payment state of the loan based on historical data will be used at initialization time $t = 0$. For all subsequent dates, $[t + 1{:}T]$, the loan state $\mathbb{S}_i(t+1{:}T,l)$ will be determined by the payment state transition process as described in the tree. Each period thereafter and updated data (monthly) behaves as a unit exponential Poisson process, with the payment state the result of a two stage transitioning to either the delinquent or default state indicated by 1 over the period so

$$P(=k=1)=\frac{\left(\lambda_q(t,l)\right)^k}{k!}e^{-\lambda_q(t,l)} \tag{204}$$

becomes

$$P(=1)=\lambda_q(t,l)e^{-\lambda_q(t,l)} \tag{205}$$

which has the interpretation of the lower bound for the current state such that if a Uniform random draw $Z \leq P(\mathbb{S}_i(t,l) = 1)$, then $\mathbb{S}_i(t,l)=1$ otherwise $\mathbb{S}_i(t,l)=0$. I simulate and capture the intensities, λ_q and λ_c and λ_d and payment state $\mathbb{S}_i(t,l)$ resulting from the delinquency/current process $\mathbb{C}_i(t,l)$ and the default process, $\mathbb{Q}_i(t,l)$ for all times t and all simulations j.

I see from the cascading screen shots (Figure 125) the first 13 months of 20 of 250 simulations for one individual loan on one simulation date. The loan delinquency status entering the simulation was current (delinquency status=1.) The loan takes on a simulated delinquency status for each of the simulated times t=1:120 of either delinquent or current. The instance of delinquency of 250 possible simulated instances for each of the 120 simtimes are shown. 11,012 instances of delinquency status show up out of 30,000 simulated states resulting in average realized delinquency status =1 frequency of 0.37.

Next I have the default process, $\mathbb{Q}_i(t,l)$ which uses the default intensity, λ_d. I concentrate on the case where the payment state $\mathbb{S}_i(t,l)$ is delinquent such that the delinquency state variable, $N_i(t,l)$, is turned on to a value of 1 such that $|\theta_d N_i(t,l)| > 0$ on a particular path j. In one example this occurs in a case in the Crisis where this loan was also conveniently in a delinquent state entering the system (it doesn't have to be, it could have transitioned to delinquent from current at $\mathbb{C}_i(t,l)$), so in this case $\mathbb{S}_i((t=0)-,l) = delq((t=0)-,l), \forall\ l$.

In Figure 126 we observe the simulated default draw from an initial delinquent state for the first 10months of simulation across 10 of 250 simulations. Next I show the default boundary for the loan as determined by λ_d. Next I show the result of the delinquency process which "precedes' the default process. As we see in the initial state, the loan is categorically in the delinquent state. I then show the result of the default process $\mathbb{Q}_i\left(t,l\right)$ which is determined by

$$P\left(=k\right)=\frac{\left(\lambda_d\left(t,l\right)\right)^k}{k!}e^{-\lambda_d\left(t,l\right)} \tag{206}$$

which becomes,

$$P\left(=1\right)=\lambda_d\left(t,l\right)e^{-\lambda_d\left(t,l\right)} \tag{207}$$

and which has the interpretation of the lower bound for the default state such that if a Uniform random draw $Z \le P\left(\mathbb{S}_i(t,l)=1\right)$ then $\mathbb{S}_i(t,l)=1=\text{default}$. Though default may occur from the current state, it is quite rare and in this rendering it does not occur. Additionally, simply by being in the delinquent state does not guarantee going into default. For default to occur in the process, the draw must be less than or equal to the boundary. I show the delinquent/current status of the loan for the first 10 months, the result of $\mathbb{C}_i(t,l)$. The first month in this example is always delinquent. On path 9, the payment state persists through the tree as delinquent for the first 3 months. In the 4th month the loan transitions to default which is an absorbing state. In the code I distinguish this payment status with a 2 corresponding to the industry data convention, but it is only a code/visual designation. The loan has terminated at period $t=4$ on path $j=9$. Why? Because in that month the Uniform draw $Z=0.218 \le 0.2840666$, the default boundary[74].

[74] Default is an absorbing state captured in the code, but for technical reasons, all default instances are generated pathwise even though only the 'first' default realization matters in the sense of perturbing the cashflow reflective of such absorbing event on the simulation path.

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Figures

Figure 1a: Ex-post analysis of Model 2 Pricing vs. Market Price (BBB-)

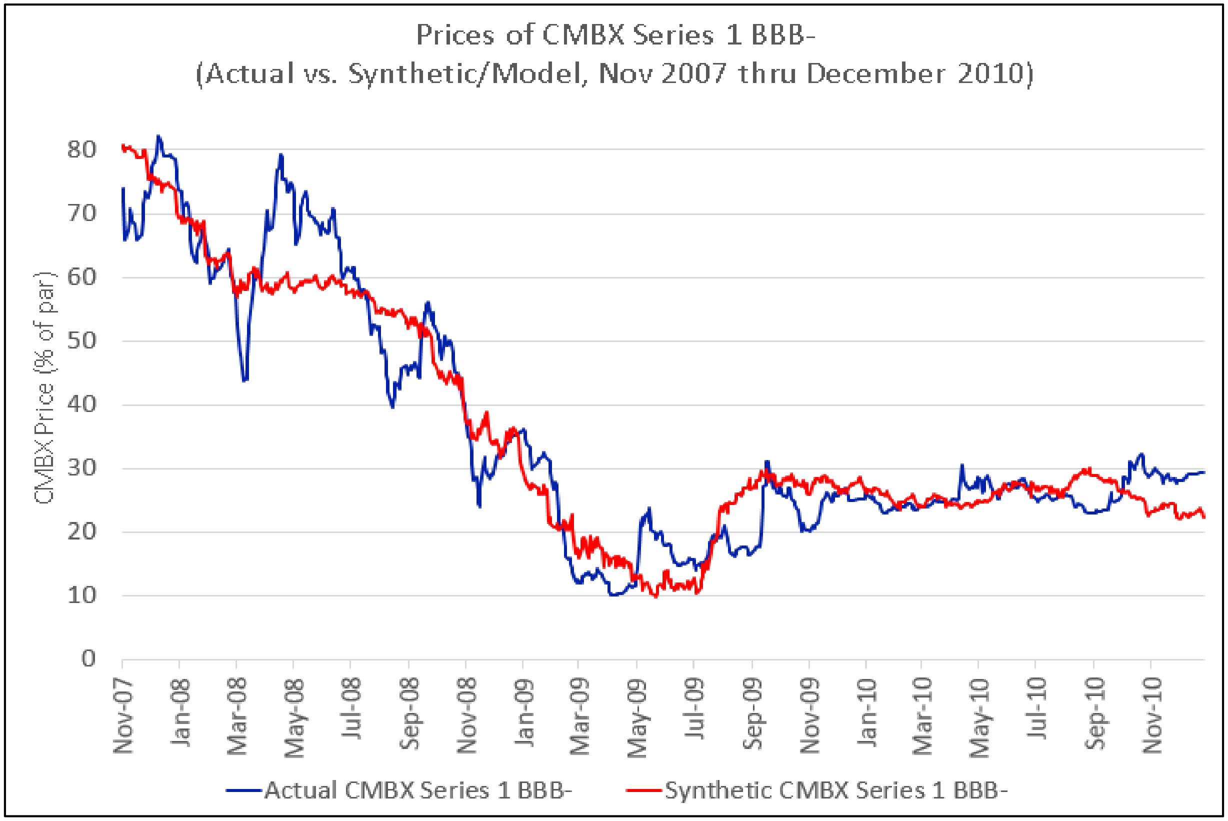


Figure 2a: Composite Theta – Crisis (daily)

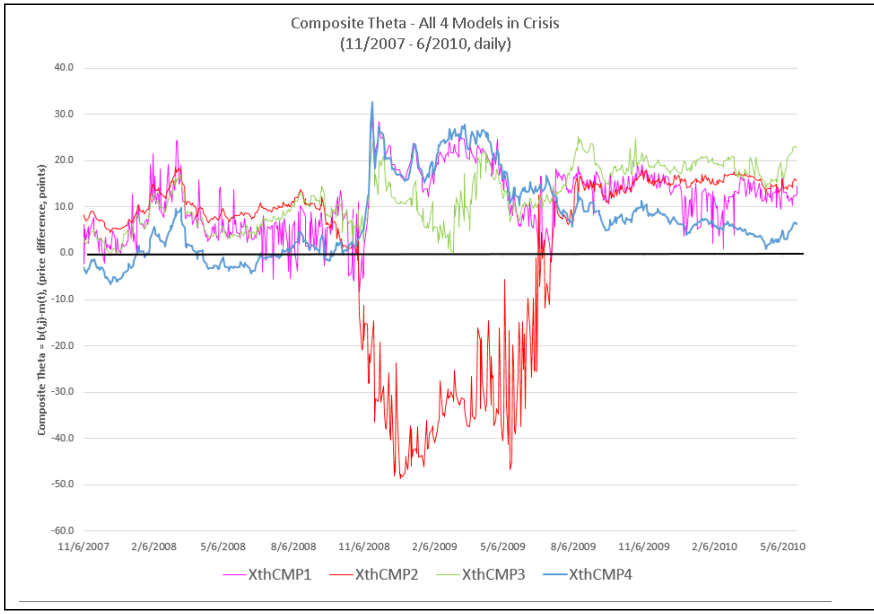


Figure 2b: Theta Driven Returns – Crisis (daily)

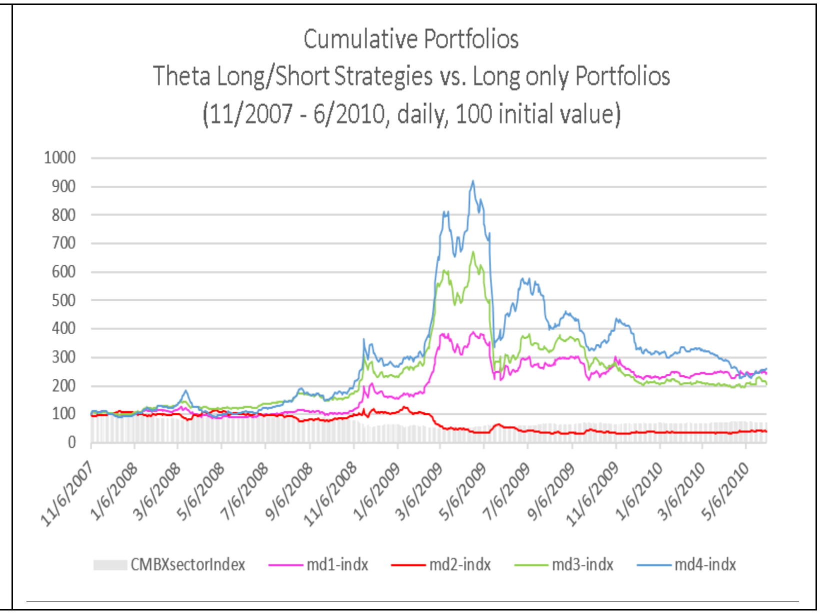


Figure 3: Basic Credit Default Swap

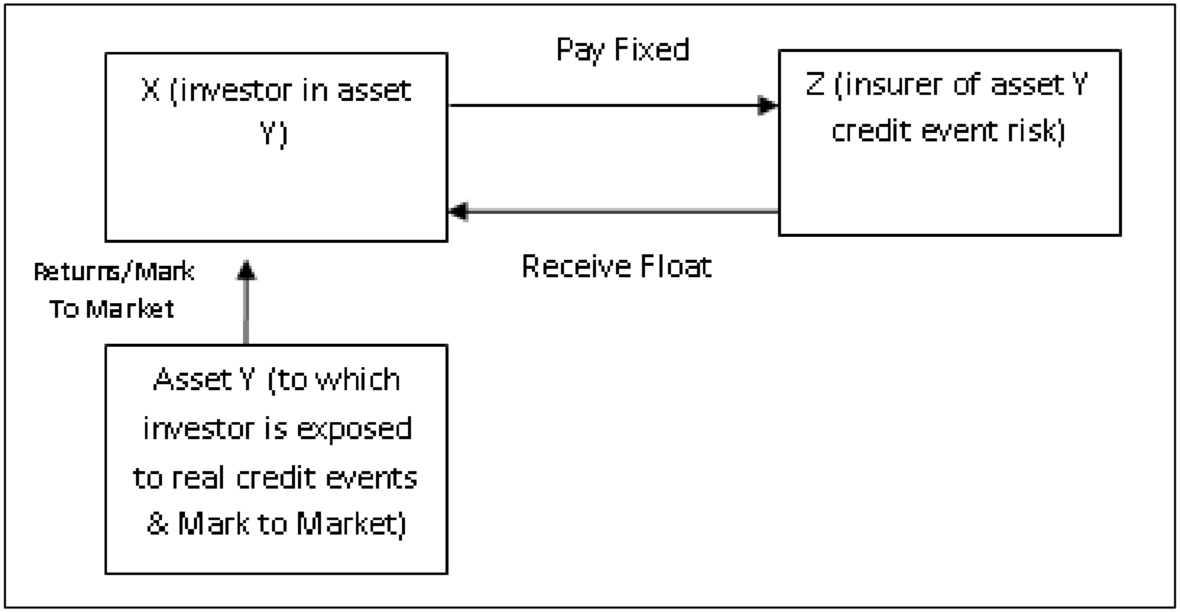


Figure 4: MLE Parameter Estimates for Default

Parameter Estimates/Coefficients for Probability of Default, λ_d: ϕ_d	
Variable	**Def**
Intercept	-10.9695

Parameter Estimates/Coefficients λ_d State Variables, X(t): ψ_d	
Variable	**Def**
h_i(t): NCREIF - National (t)	0.0012
P3: NCREIF - Multifamily (t)	-0.0547
R4: - NCREIF - Northeast (t)	0.1062
H_bar(t): BBREIT - National (t)	-0.0092
H_i(t): BBREIT - Multifamily (t)	-0.0095
spot: Spot rate (t)	0.2195
f(t,10)-r(t): 10yr fwd rate (t) - spot (t)	0.3287

Parameter Estimates/Coefficients λ_d Property Variables, U(t): Φ_d	
Variable	**Def**
age = (1-remterm/origterm)	3.2446
dlqstatus = (0,1, or 2)	5.3898
fore_index = .0025	-6.5445
noi = (orig_noi/orig_loanbal)	0.0000
origloanbal = log of orig_loanbal	0.0552
dscr = orig_dscr	-0.4287
ltv = orig_ltv	0.0113
wac = cpn_t	0.1130
coupon_spread = cpn_0-rf_0	-0.0521

Figure 5: NCREIF Rolling Returns

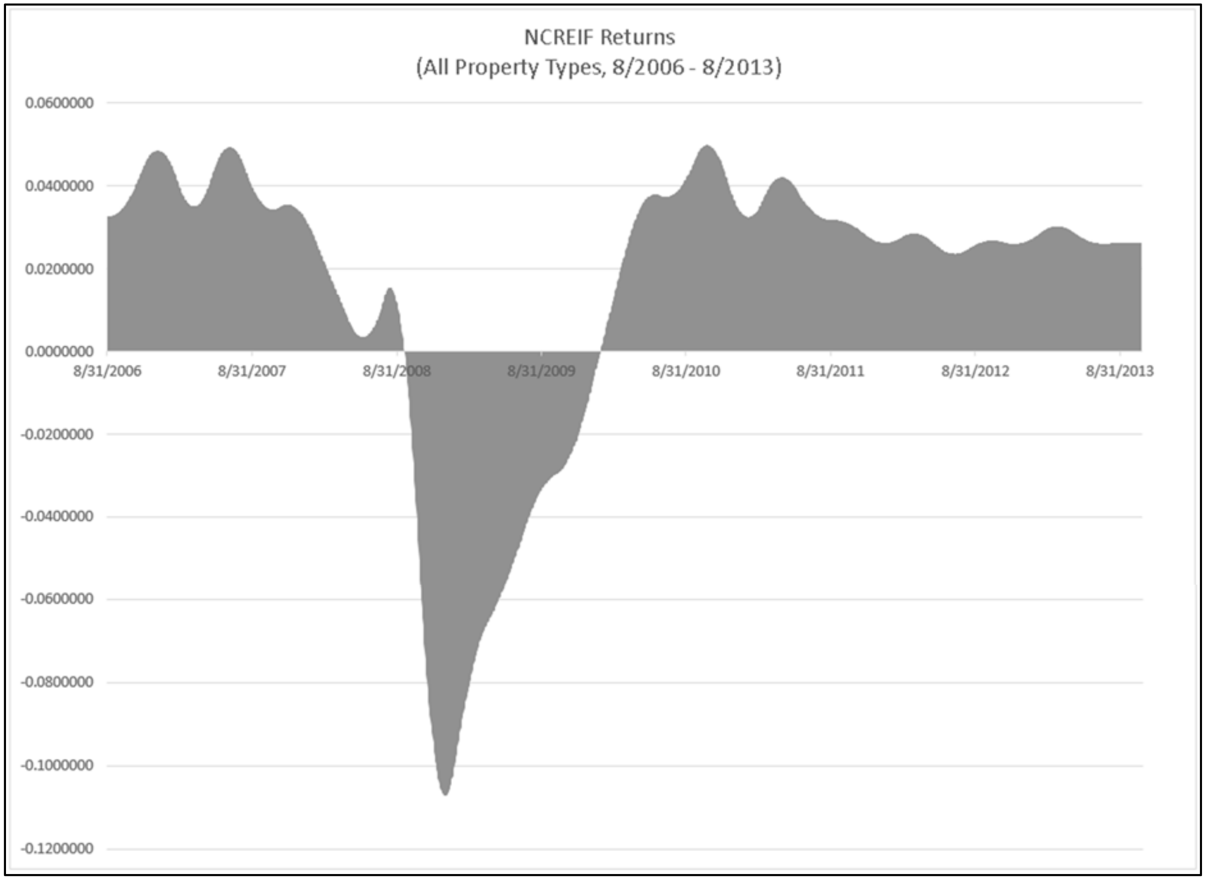


Figure 6: Actual vs. Spline fit NCREIF

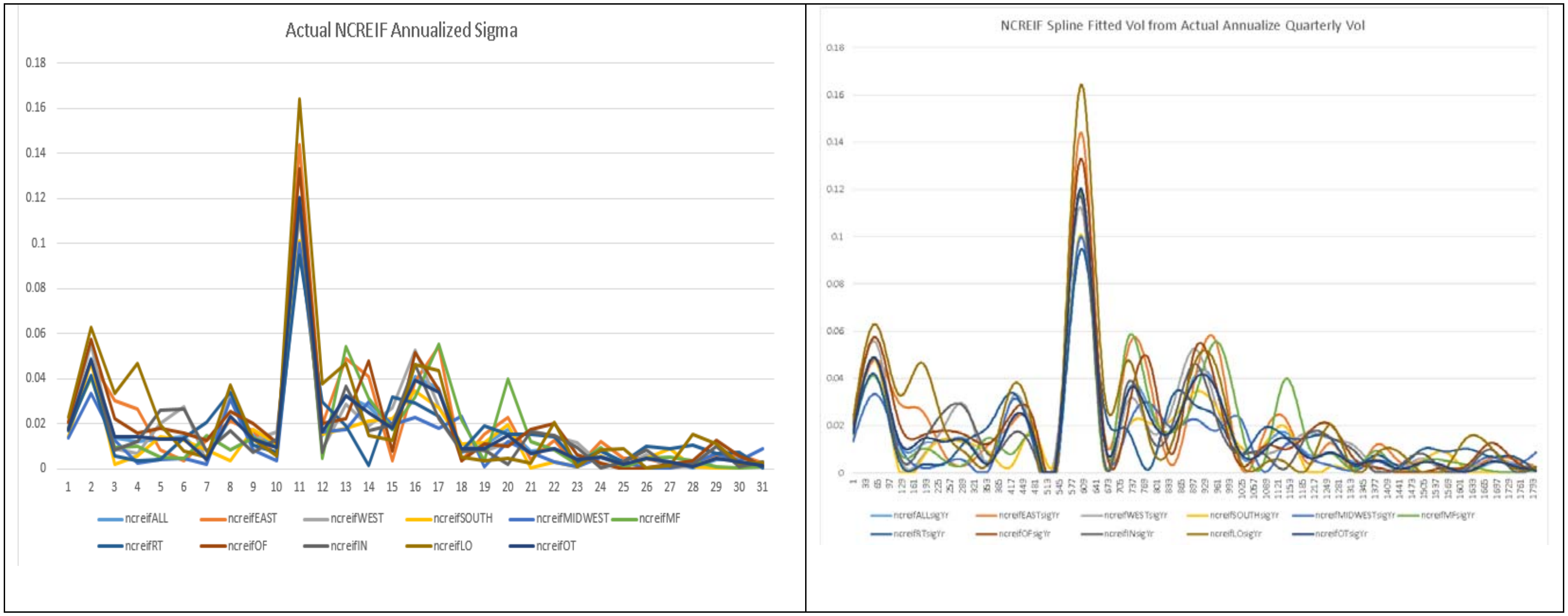


Figure 7 – Loan Level Probability of Defaults aggregated by Property Type

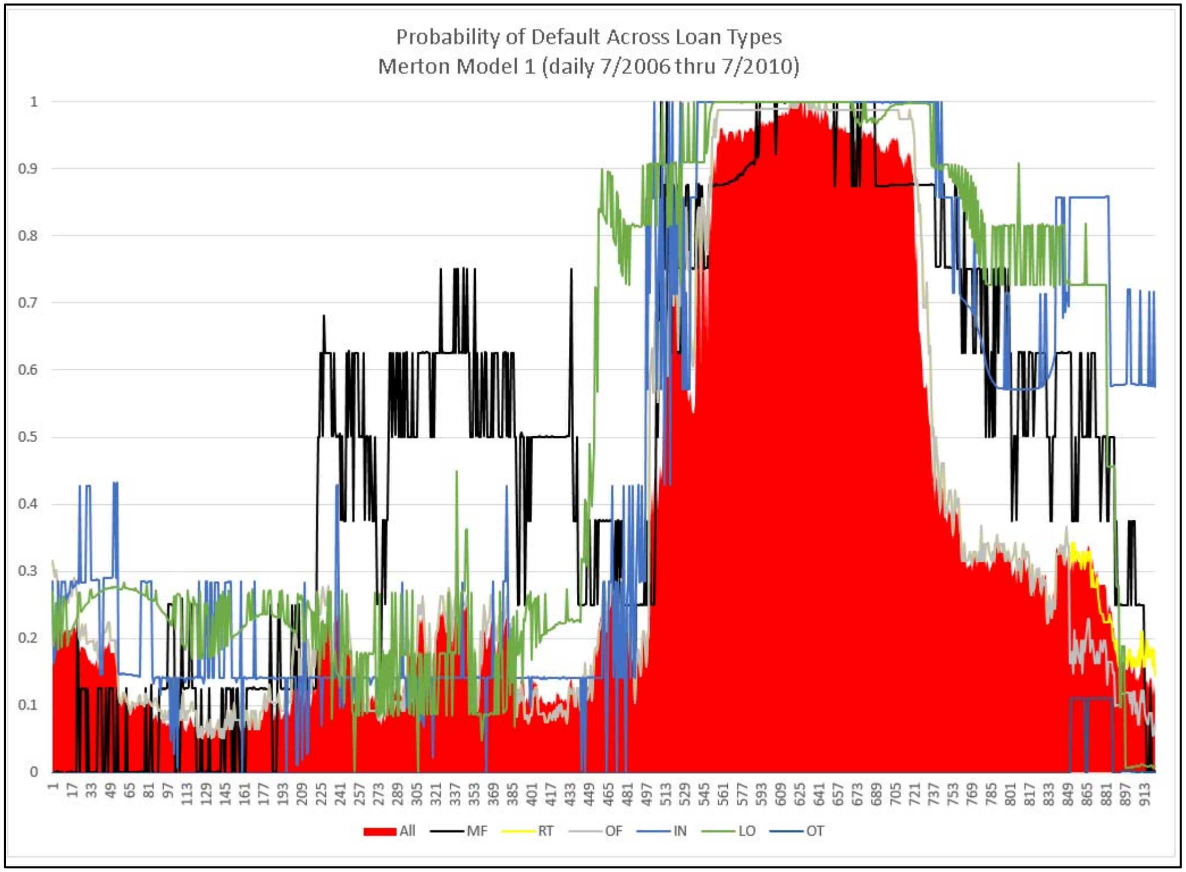


Figure 8: GG5 Historical Default Experience

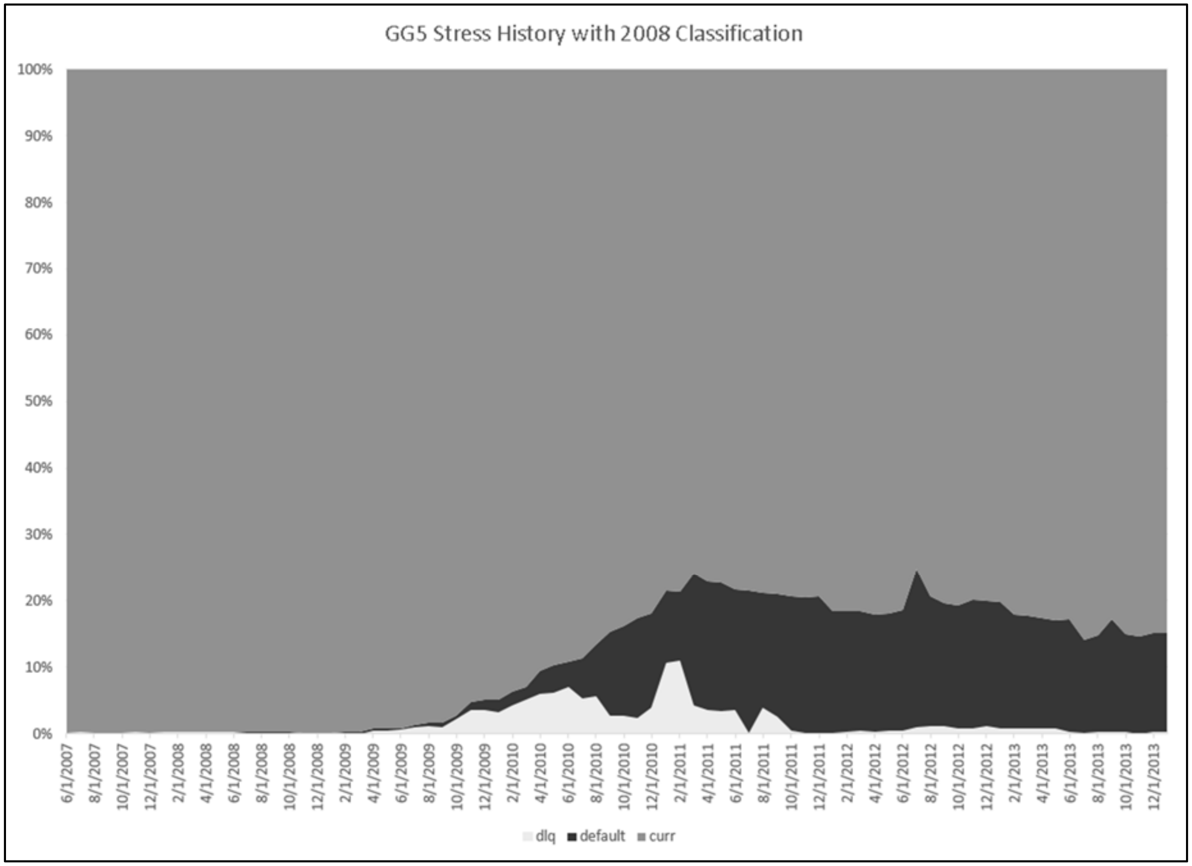


Figure 9: Tranche Structure for CMBX in Crisis

Class/Tranche	Class Percent
AAA:	0.6440
AJ:	0.0584
AM:	0.0984
AA:	0.0742
A:	0.0205
BBB:	0.0274
BBB-:	0.0451
Other:	0.0320

Figure 10: Numerical Example of Merton Fair Value Allocation

B(t) - FairValue Trust:	780
H(t) - Rembal of the Trust:	936

k=Tranche	origbal	Rembal h_k	Disposition at Fair Value pmt_k	RawRecovery	T	r(t) (monthly)	FairValue bdot_k
k=AAA	644.00	580.00	580.00	100.00%	75	0.000416667	103.1743
k=AJ	58.40	58.40	58.40	100.00%	75	0.000416667	103.1743
k=AM	98.40	98.40	98.40	100.00%	75	0.000416667	103.1743
k=AA	74.20	74.20	43.20	58.22%	85	0.000416667	60.31997
k=A	20.50	20.50	-	0.00%	85	0.000416667	0
k=BBB	27.40	27.40	-	0.00%	98	0.000416667	0
k=BBBminus	45.10	45.10	-	0.00%	98	0.000416667	0
k=Other	32.00	32.00	-	0.00%	98	0.000416667	0
Total	1,000.00	936.00	780.00				

Figure 11: Fair Value Merton (All classes except AJ and AM)

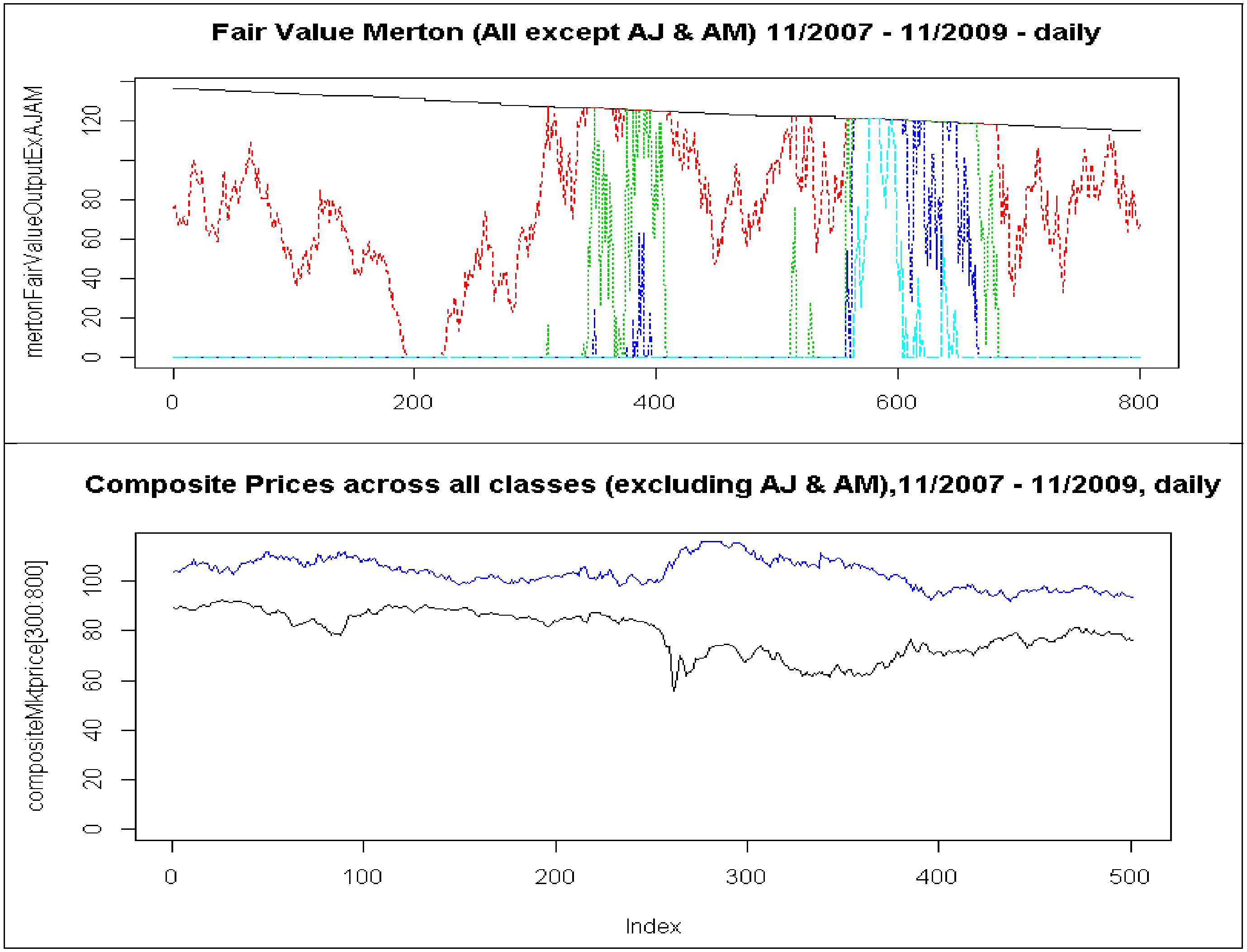


Figure 12: Relative value comparisons using Theta for Merton

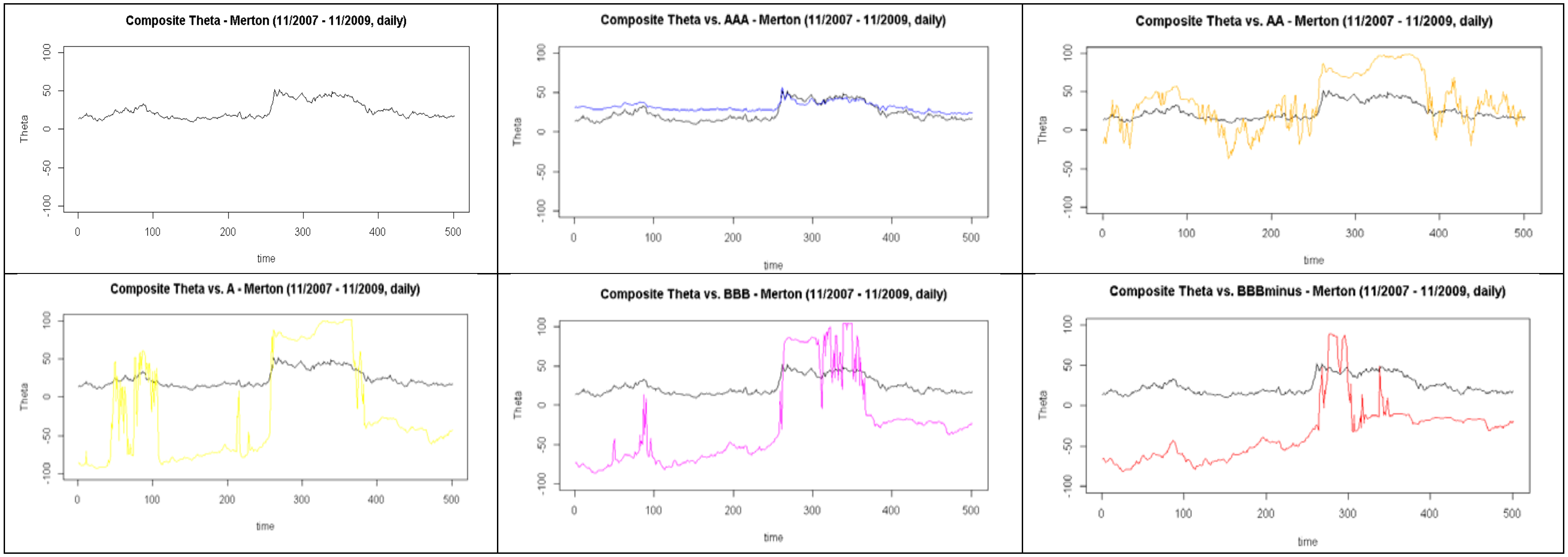


Figure 13: Calibrated Values 3 Property Case (9/2006-12/2010, Daily)

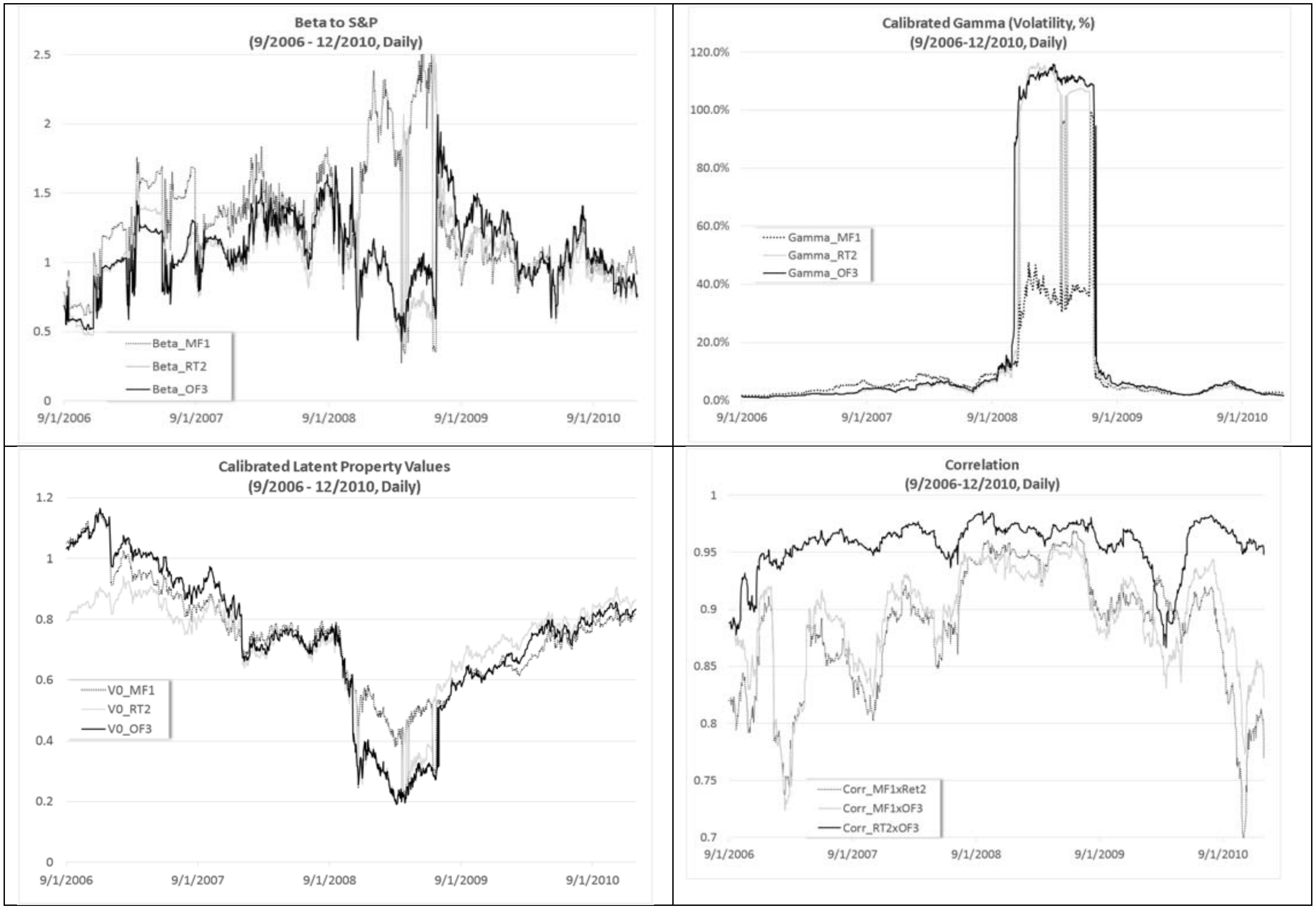


Figure 14: REITs used in Models

symbol	REIT	Prop	M2	M3
AVB	AvalonBay Communities	MF	*	*
BKD	Brookdale Senior Living	OT		*
BMR	Biomed Realty Trust	OF		*
BRE	BRE Select Hotels	LO	*	*
BXP	Boston Properties	OF	*	*
CLI	Mack-Cali Realty Corp	OF	*	*
DDR	DDR Corp	RT	*	*
DRE	Duke Realty	IN	*	*
ELS	Equity Lifestyle	MF		*
EQR	Equity Residential	MF	*	*
ESC	Emeritus Corp	OT		*
FCH	Felcor	LO		*
FR	First Industrial Realty	IN		*
GRT	Glimcher Realty Trust	RT		*
HCN	HealthCare REIT	OT		*
HIW	Highwoods REIT	RT	*	*
HOT	Starwoods Hotels	LO		*
HST	Host Hotels & Resorts	LO		*
KIM	KIMCO Realty	RT	*	*
LHO	Lasalle Hotel Properties	LO		*
LRY	Liberty Property Trust	OF	*	*
NNN	National Retail Properties	RT		*
PLD	Prologis Inc.	OT		*
PSB	PS Business Parks	OF		*
REG	Regency Centers Corp	RT	*	*
SPG	Simon Property Group	RT	*	*
SSS	Sovran Self Storage	OT		*
TCO	Taubman Centers	RT	*	*
UDR	UDR, Inc.	MF	*	*
VNO	Vornado	OF	*	*
WPC	WP Carey	IN		*

Figure 15: Two simulations for Apt REIT (1)

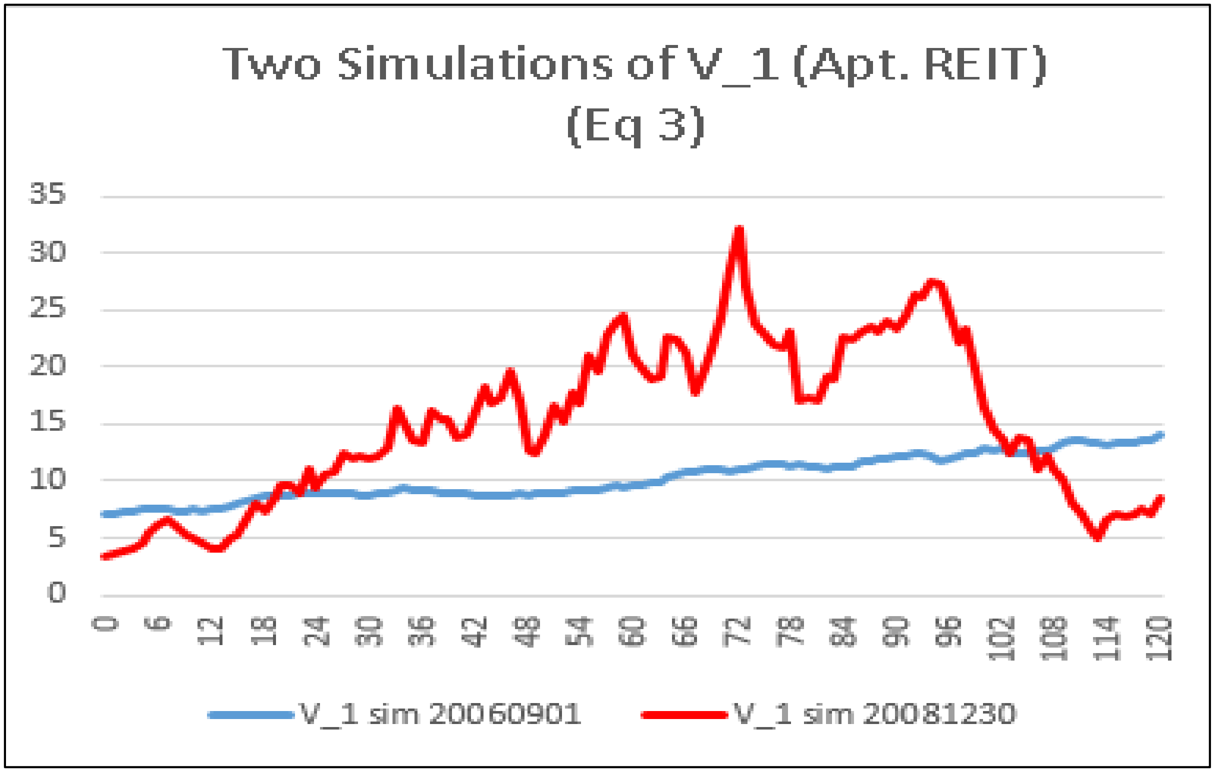


Figure 16: Implementation of Simulations for Apartment REITs

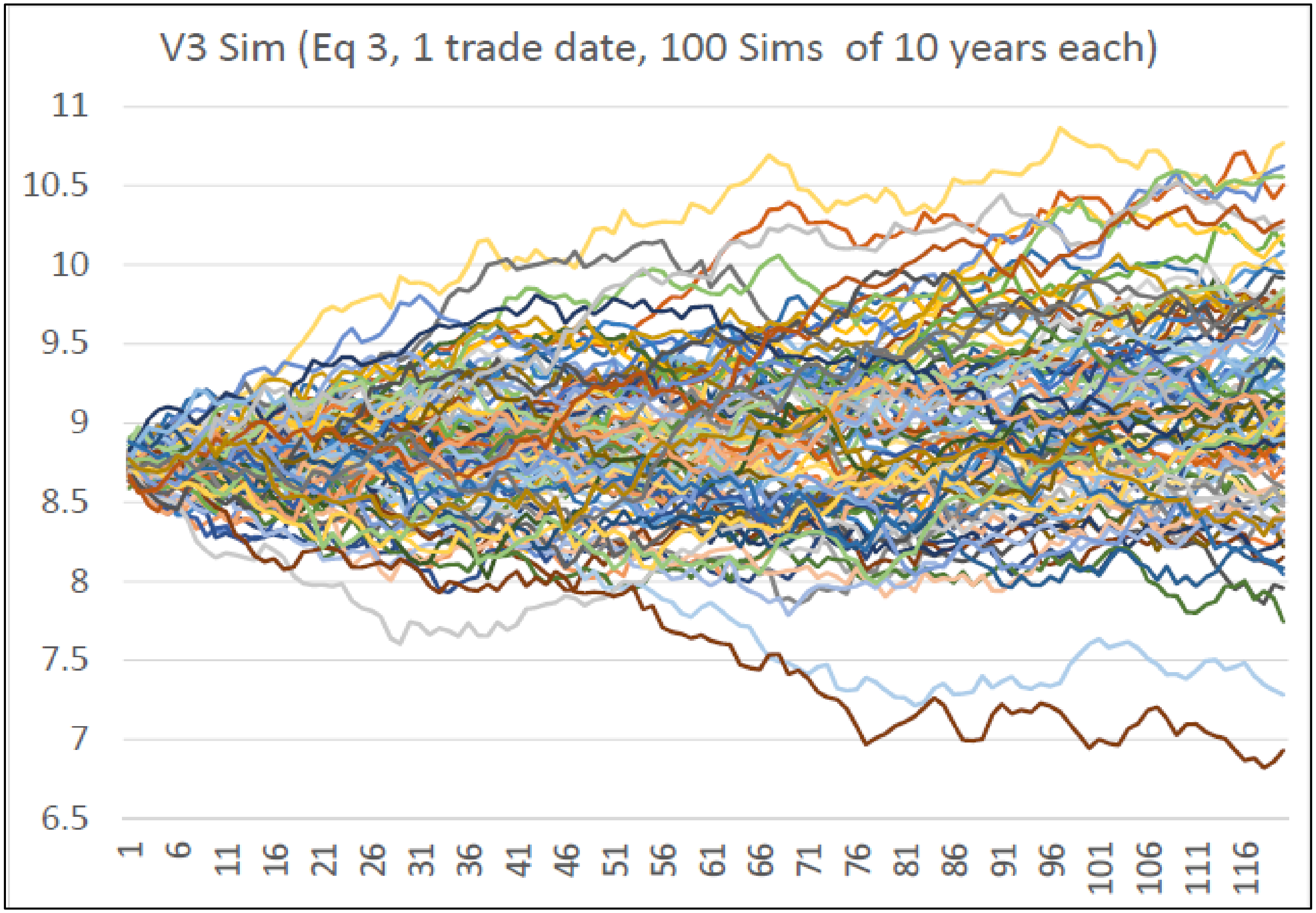


Figure 17: Simulated Portfolio, Cumulative across 10,000 simulations

Figure 18: Two simulations of Apartment REITs

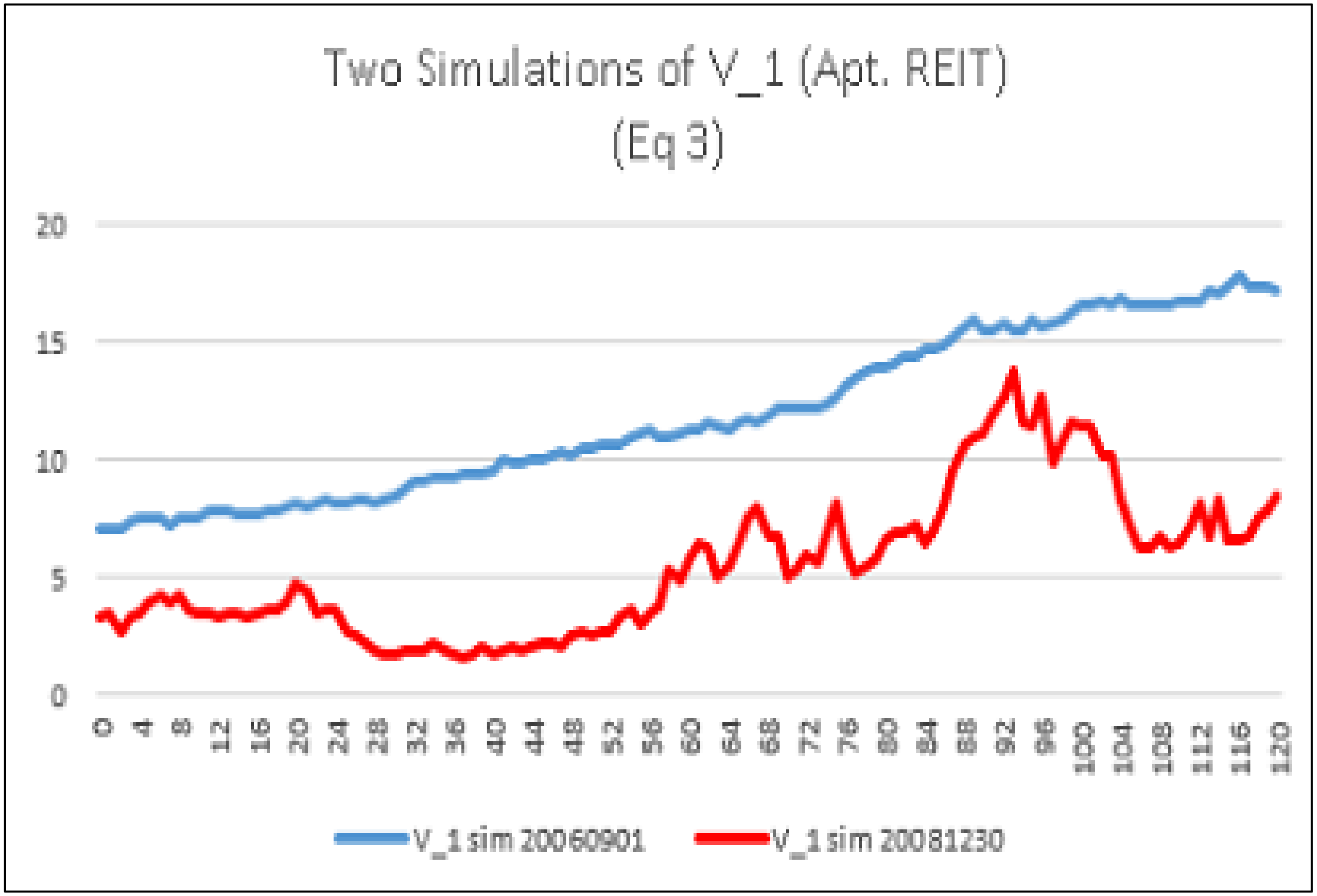


Figure 19: Simulated Inverse LTV for Apartment Loans

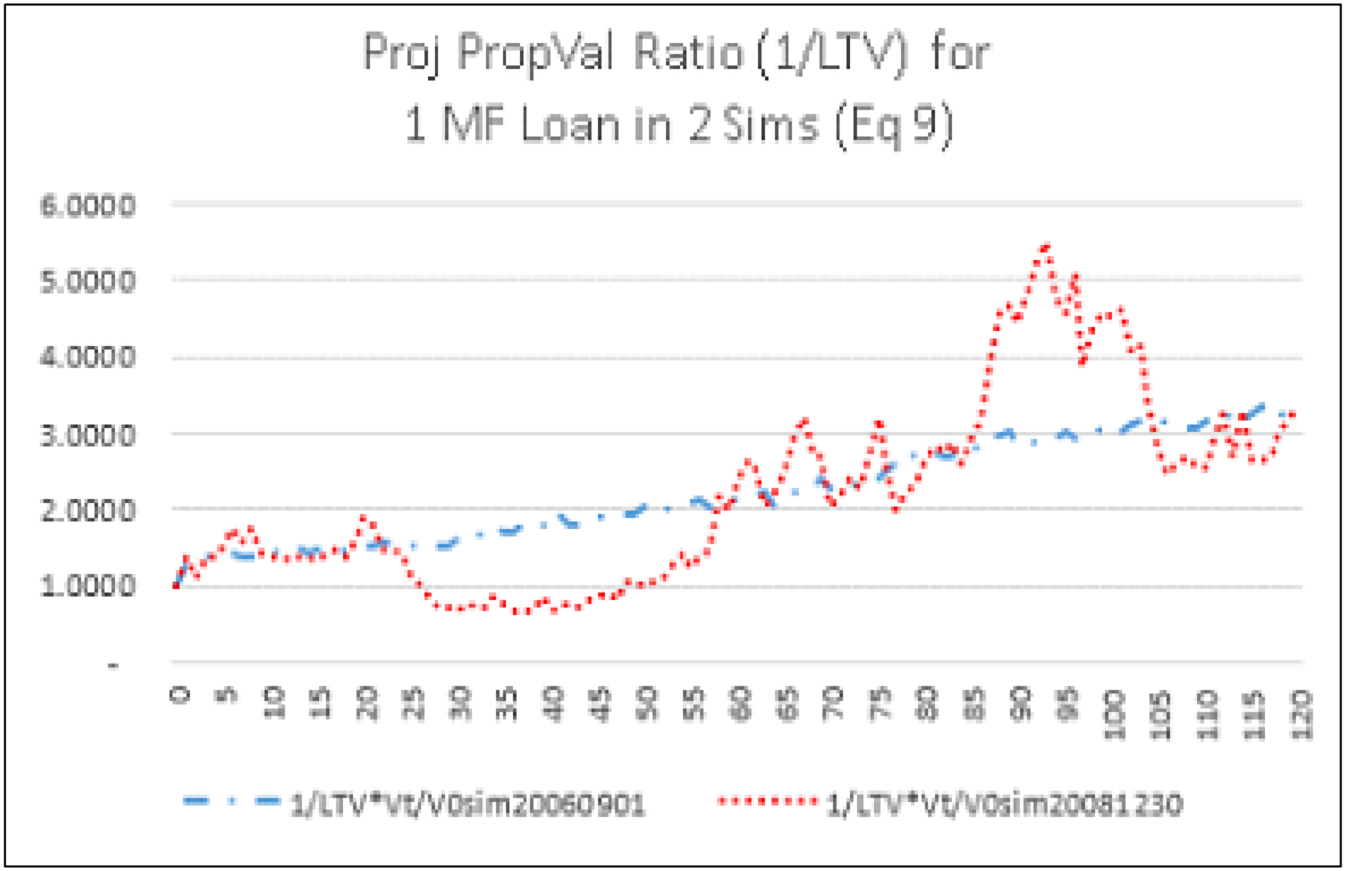


Figure 20: CMBX Attachment Points and Coupons

	CMBX Series 1						
Class	AAA	AM	AJ	AA	A	BBB	BBB-
Fixed Coupon, c	10	50	84	25	35	76	134
CE^H	46.00%	29.76%	19.92%	12.50%	10.45%	7.71%	3.20%
CE^L	29.76%	19.92%	12.50%	10.45%	7.71%	3.20%	0.00%

Figure 21: Initial Model 2 (top) compared with DVH, 2012 (bottom)

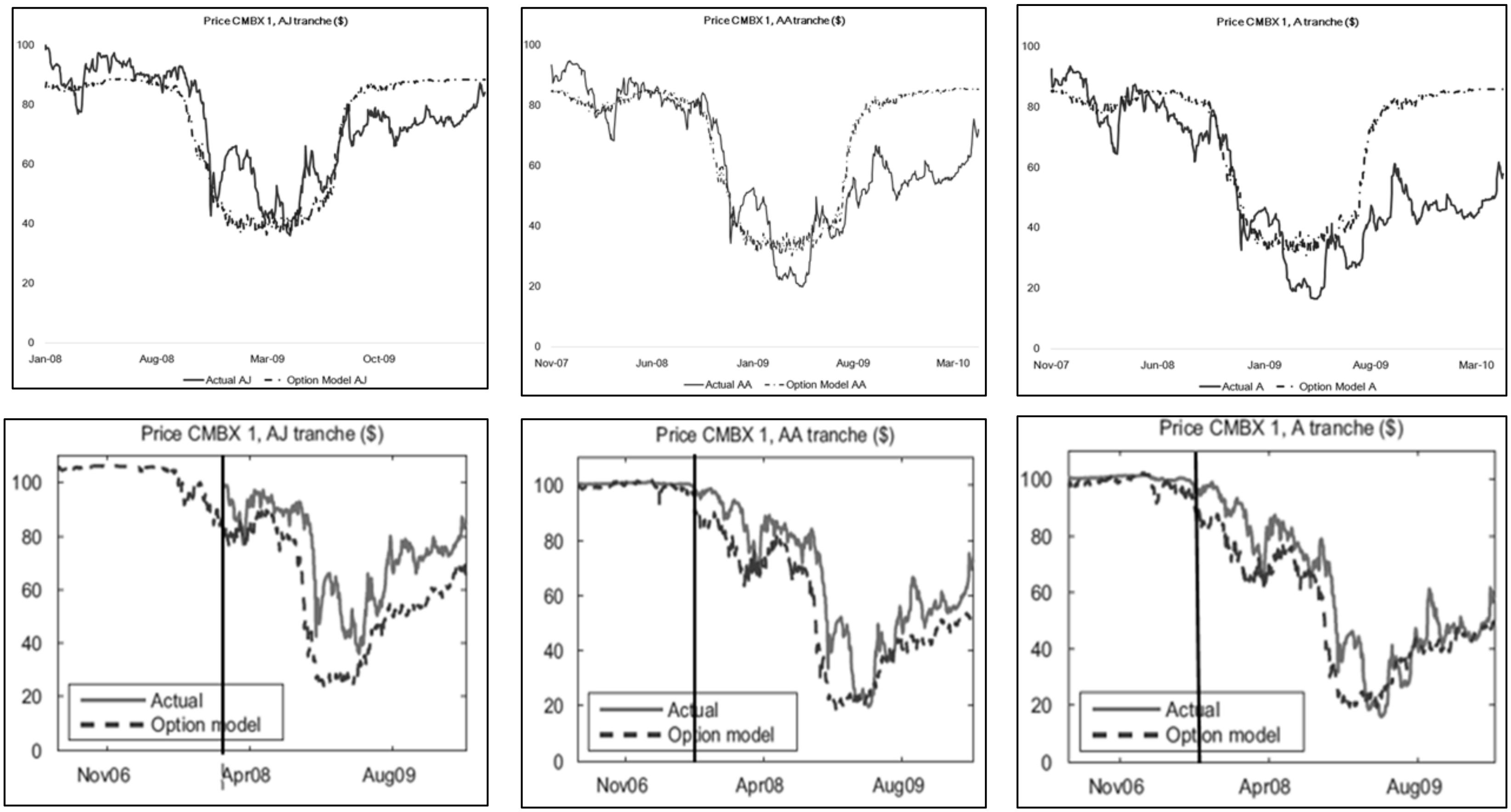


Figure 22: Initial Statistical Results of OLS

CMBX 1	R-sq	t-stat	p-value	obs (days)
AAA	0.73010	46.31000	0	795
AJ	0.70637	18.27894	0	752
AA	0.59290	33.98000	0	795
A	0.46110	26.05000	0	795
BBB	0.21150	14.58000	0	795
BBBminus	0.14000	11.36000	0	795

Figure 23: Volatility vs. Indexed Price Series (AAA and BBB-)

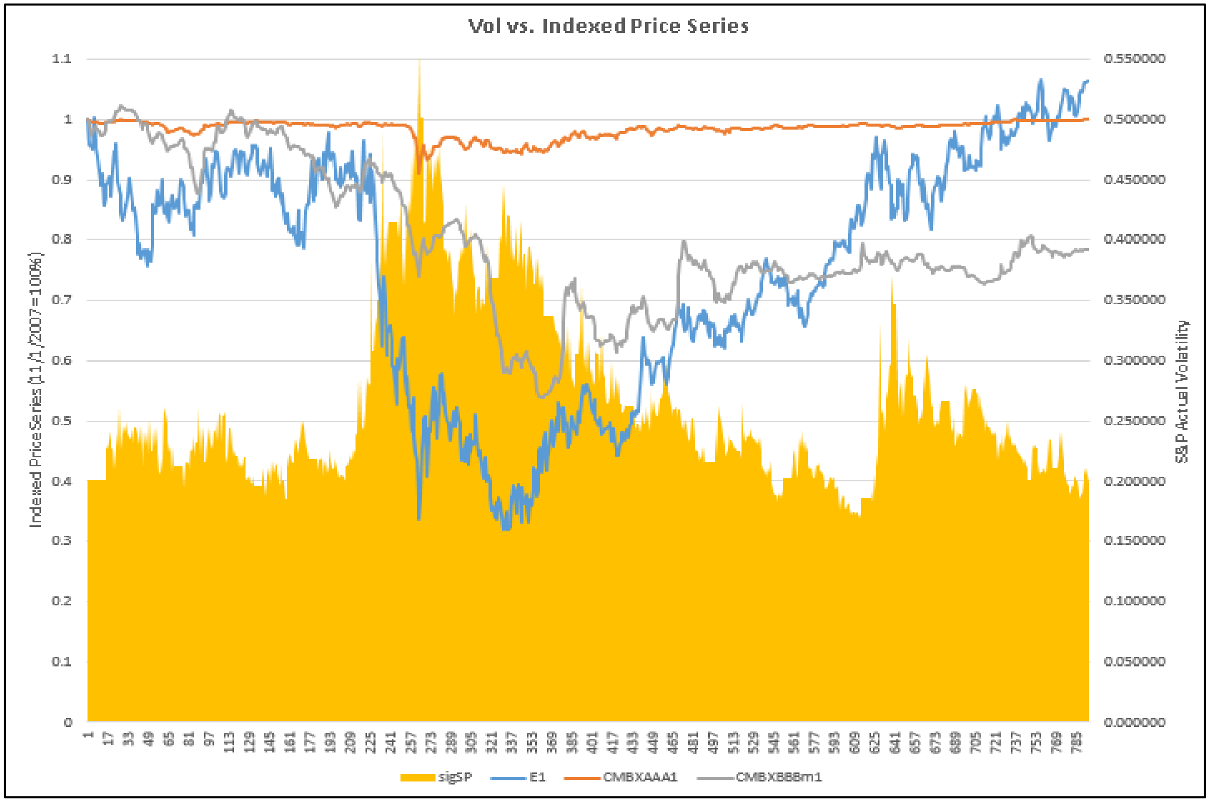


Figure 24: CMBX Price Series for Crisis

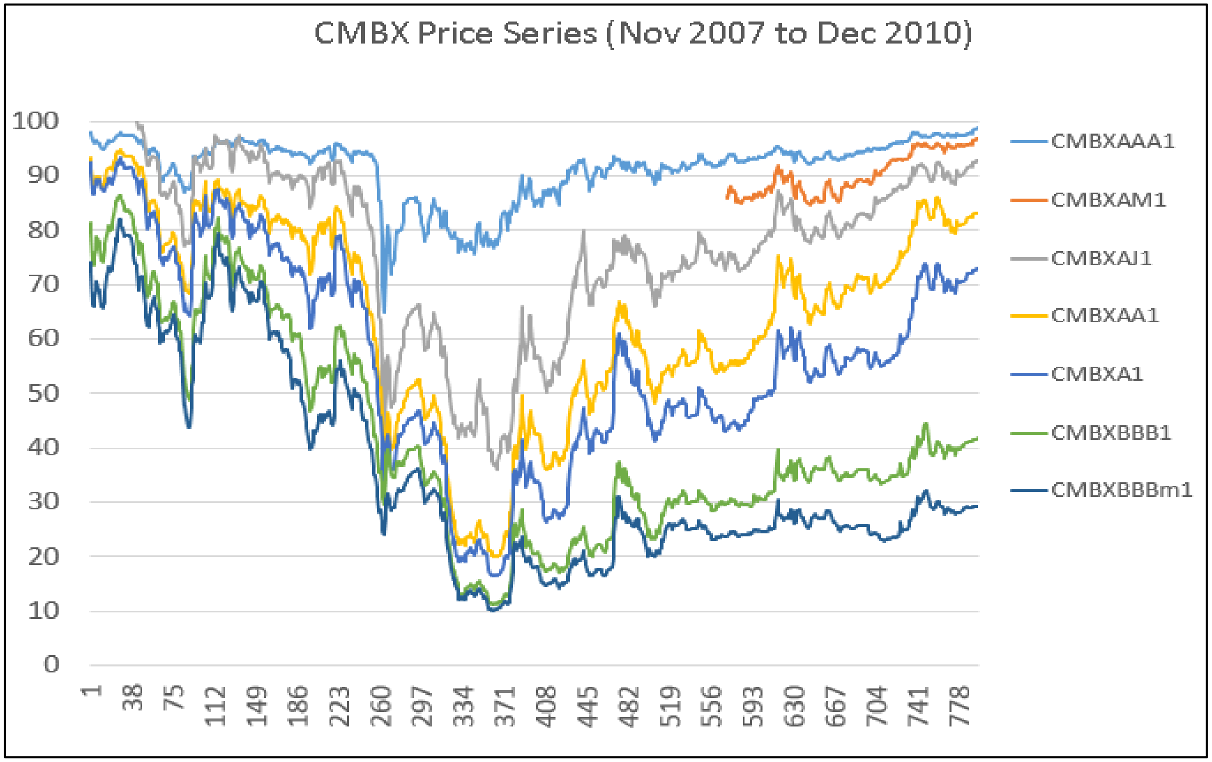


Figure 25: 30 Plus Days Delinquency History in Crisis

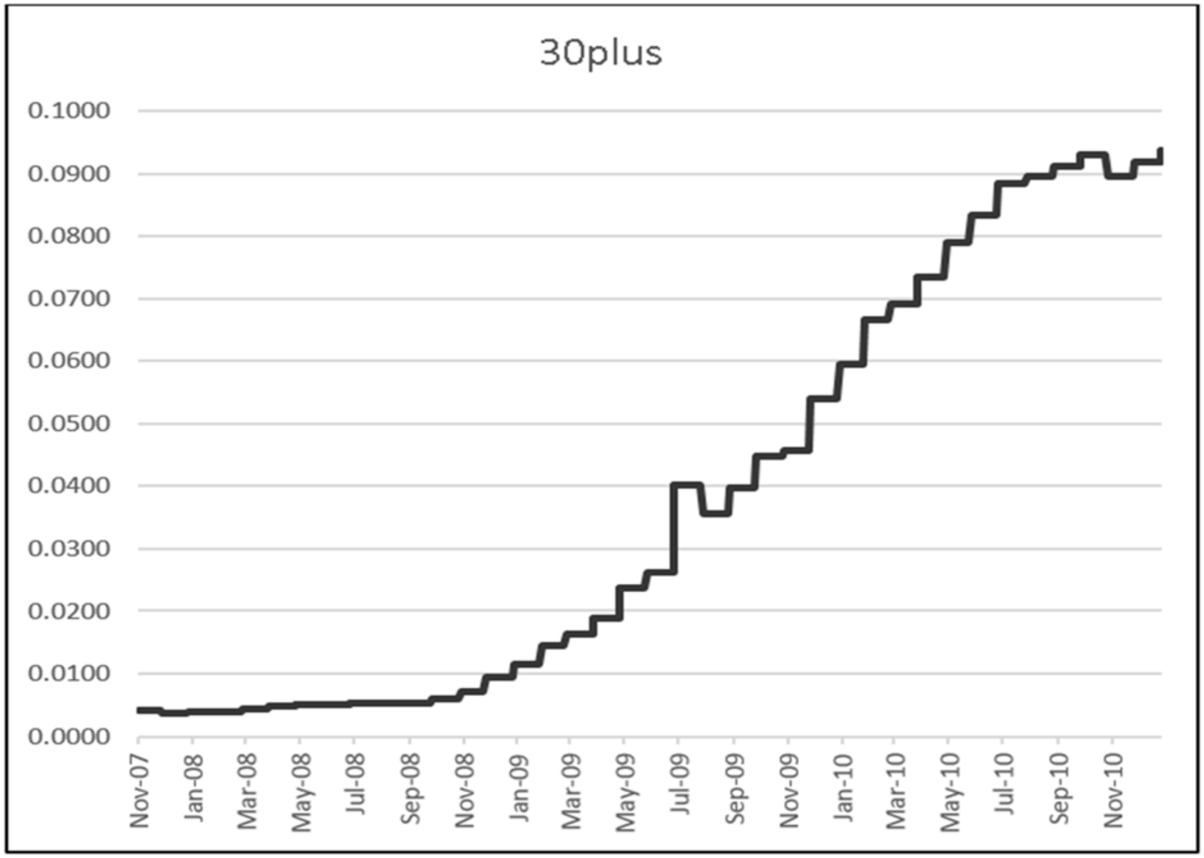


Figure 26: Ex-Post OLS Results with 30 Plus Dlq Exogenous Variable included

Comparitive Regression Results Summarized				
CMBX 1	R-sq	R-sq w DLQ	p-value	obs (days)
AAA	0.73010	0.73030	0	795
AJ	0.70637	0.70598	0	752
AA	0.59290	0.69330	0	795
A	0.46110	0.68760	0	795
BBB	0.21150	0.70890	0	795
BBBminus	0.14000	0.77200	0	795

Figure 27: BBB- ex-post with 30Plus Dlq included

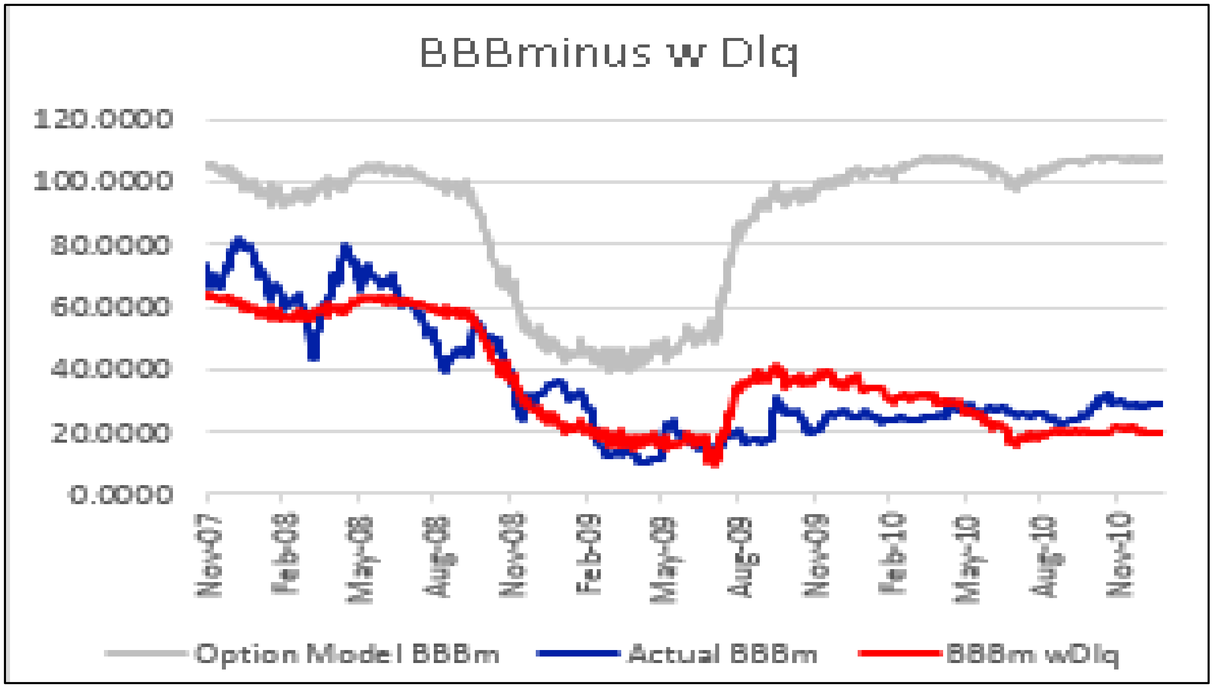


Figure 28: All other CMBX Classes with 30Plus Dlq included ex-post

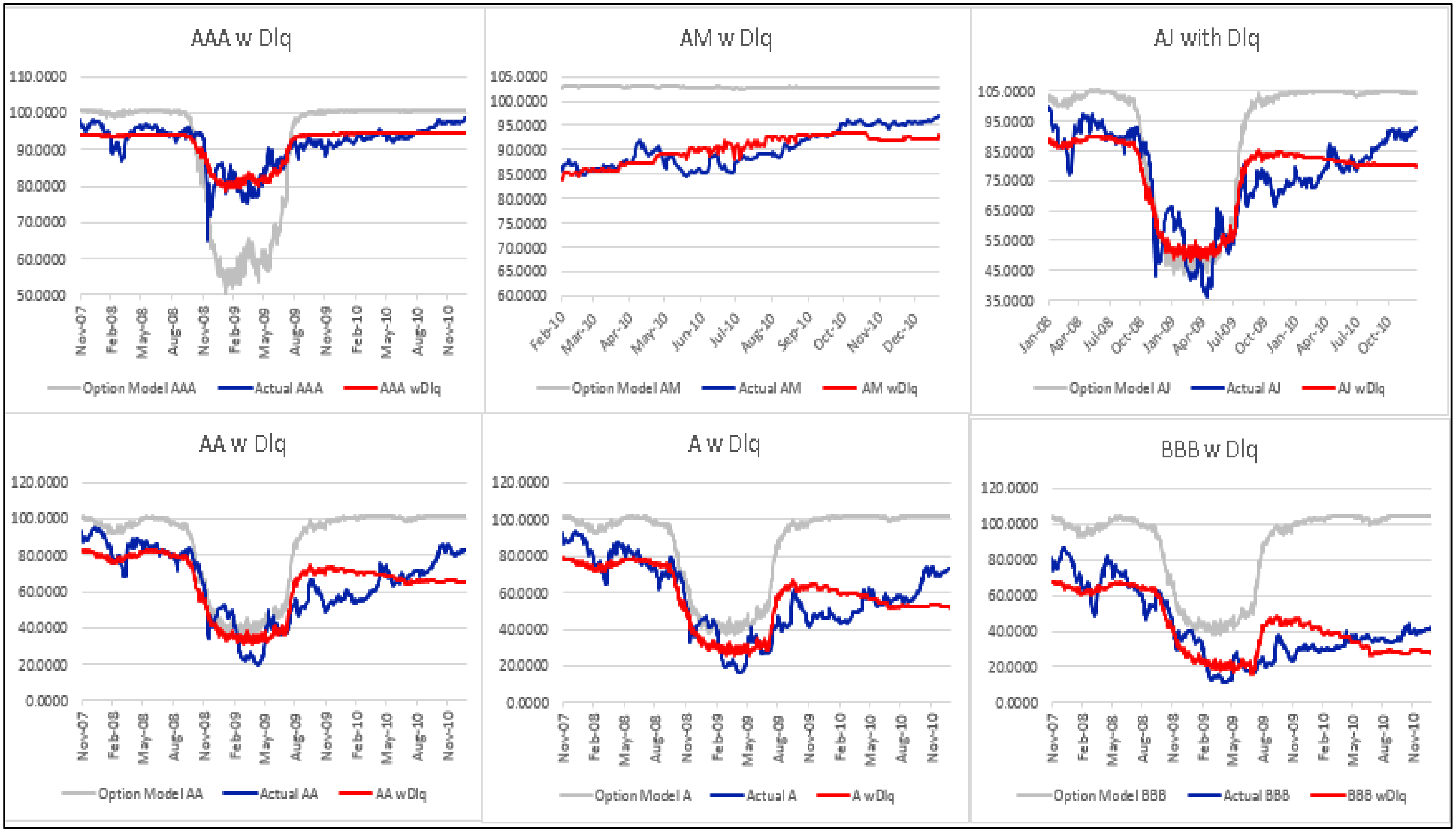


Figure 29: Regression results for 30Plus and the Treasury Slope (AAA)

. **regress ActualAAA OptionModelAAA plus TsySlope**

Source	SS	df	MS
Model	19621.9867	3	6540.66225
Residual	6358.04166	791	8.03797934
Total	25980.0284	794	32.7204388

Number of obs	=	795
F(3, 791)	=	813.72
Prob > F	=	0.0000
R-squared	=	0.7553
Adj R-squared	=	0.7543
Root MSE	=	2.8351

ActualAAA	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
OptionModelAAA	.2773369	.0073061	37.96	0.000	.2629953	.2916784
plus	47.35722	5.719217	8.28	0.000	36.13059	58.58386
TsySlope	-.0663141	.007483	-8.86	0.000	-.0810029	-.0516253
_cons	71.19482	1.050414	67.78	0.000	69.1329	73.25675

Figure 30: AAA CMBX with 30Plus Dlq and Treasury Slope

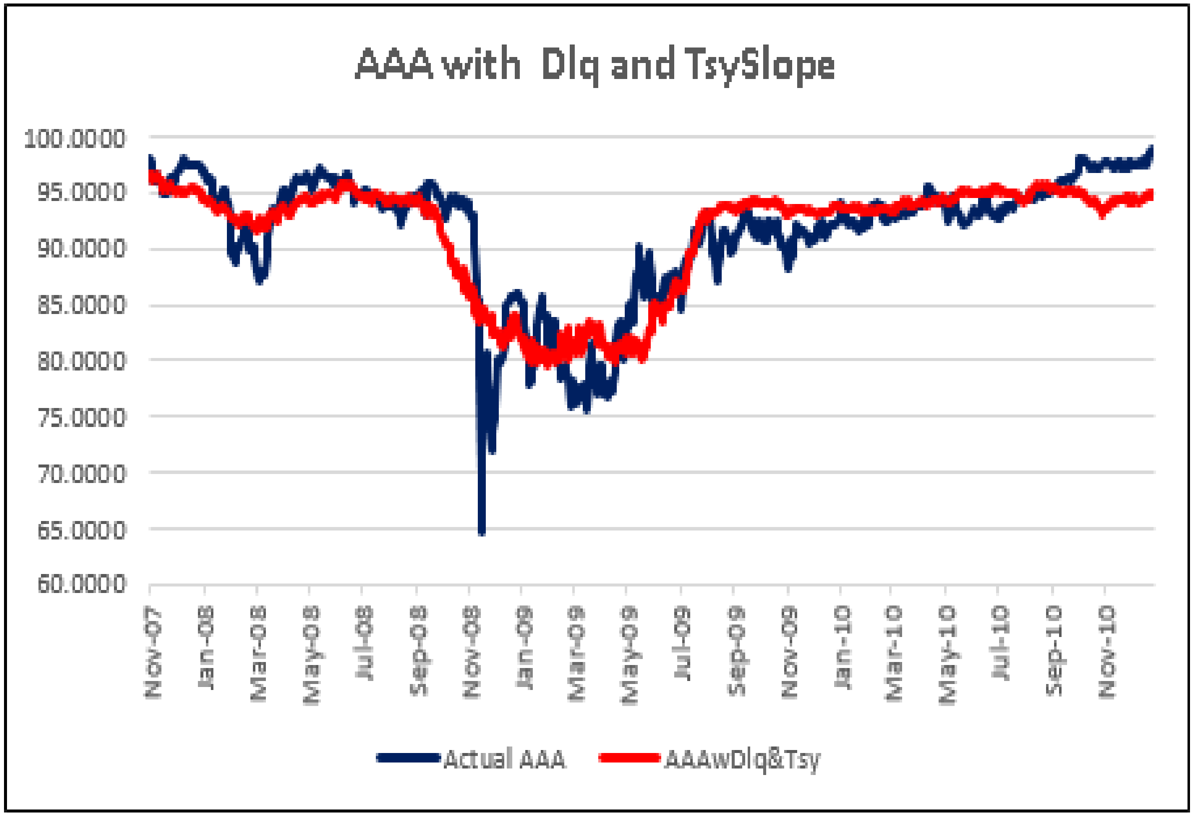


Figure 31: BBB- CMBX with 30Plus and Treasury Slope Ex-post

```
. regress ActualBBBm OptionModelBBBm plus TsySlope
```

Source	SS	df	MS
Model	229815.516	3	76605.1719
Residual	60382.4173	791	76.3368108
Total	290197.933	794	365.488581

Number of obs = 795
F(3, 791) = 1003.52
Prob > F = 0.0000
R-squared = 0.7919
Adj R-squared = 0.7911
Root MSE = 8.7371

ActualBBBm	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
OptionModelBBBm	.5640927	.0170428	33.10	0.000	.5306382	.5975471
plus	-369.8638	19.04782	-19.42	0.000	-407.2541	-332.4736
TsySlope	-.2057768	.0240209	-8.57	0.000	-.252929	-.1586245
_cons	21.34939	2.90185	7.36	0.000	15.65315	27.04563

Figure 32: Treasury Slope vs. 30 Day Plus Delinquency

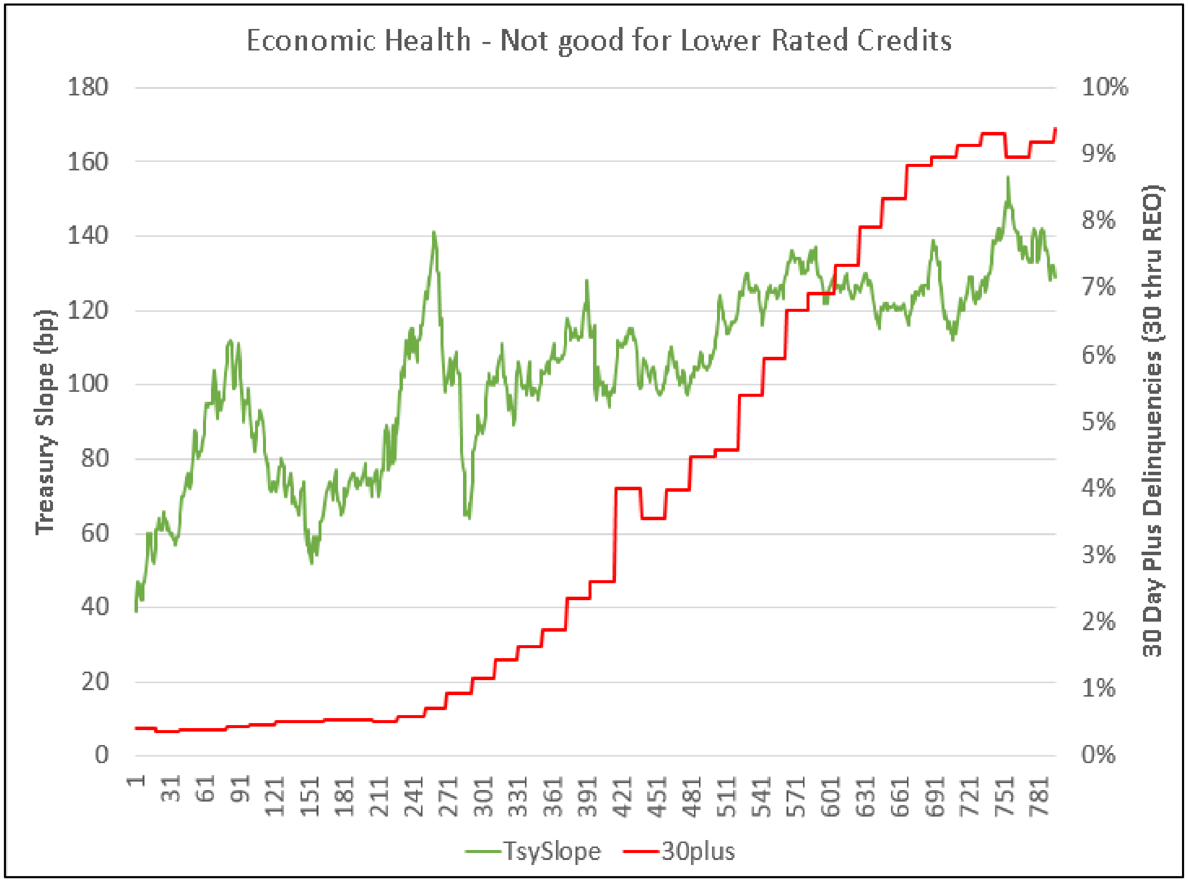


Figure 33: Case-Shiller Housing vs. Corporate Credit Slope

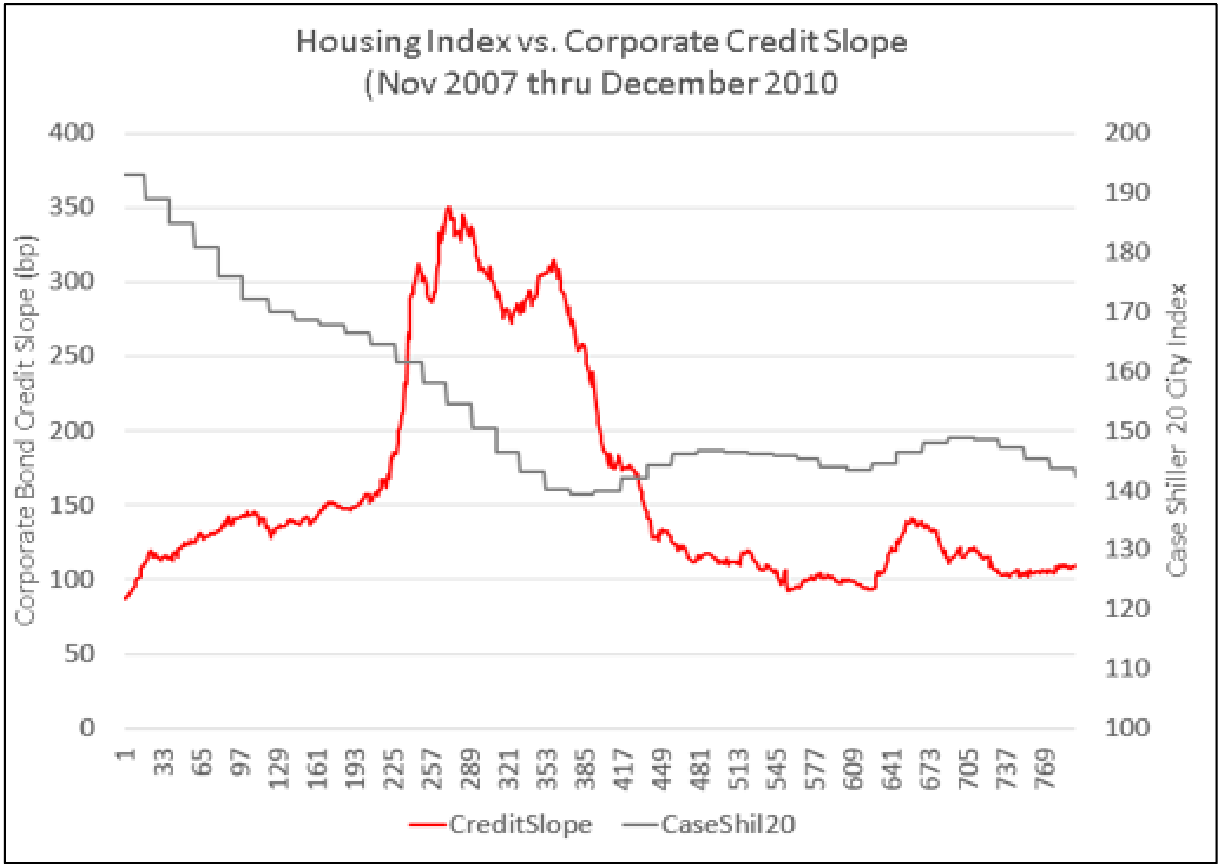


Figure 34: OLS for AAA w/ 30Plus, Treasury Slope, Case-Shiller and Credit Slope

```
. regress ActualAAA OptionModelAAA plus TsySlope CaseShil20 CreditSlope

      Source |       SS       df       MS              Number of obs =     795
-------------+------------------------------           F(  5,   789) =  524.16
       Model |  19968.4987     5  3993.69973           Prob > F      =  0.0000
    Residual |  6011.52975   789  7.61917586           R-squared     =  0.7686
-------------+------------------------------           Adj R-squared =  0.7671
       Total |  25980.0284   794  32.7204388           Root MSE      =  2.7603

------------------------------------------------------------------------------
   ActualAAA |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
OptionModelAAA |  .2177384   .0161335    13.50   0.000     .1860688     .249408
        plus |   64.30839   6.930139     9.28   0.000     50.70469    77.91208
    TsySlope |  -.0442892   .0082061    -5.40   0.000    -.0603976   -.0281808
  CaseShil20 |   .0904925   .0135009     6.70   0.000     .0639906    .1169943
 CreditSlope |  -.0047408   .0035251    -1.34   0.179    -.0116605    .0021789
       _cons |   60.46176   2.583161    23.41   0.000     55.39108    65.53244
------------------------------------------------------------------------------
```

Figure 35: OLS for BBB- w/30Plus, Treasury Slope, Case-Shiller and Credit Slope

```
. regress ActualBBBm OptionModelBBBm plus TsySlope CaseShil20 CreditSlope

      Source |       SS       df       MS              Number of obs =     795
-------------+------------------------------           F(  5,   789) = 1508.69
       Model |  262719.009     5  52543.8019           Prob > F      =  0.0000
    Residual |  27478.9237   789  34.8275332           R-squared     =  0.9053
-------------+------------------------------           Adj R-squared =  0.9047
       Total |  290197.933   794  365.488581           Root MSE      =  5.9015

------------------------------------------------------------------------------
  ActualBBBm |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
OptionModelBBBm |   .3530238   .0260952    13.53   0.000     .3017995    .4042481
        plus |  -76.87351   16.01391    -4.80   0.000    -108.3084   -45.43861
    TsySlope |  -.0942483   .0171094    -5.51   0.000    -.1278335   -.060663
  CaseShil20 |   .8835934    .031105    28.41   0.000      .822535    .9446518
 CreditSlope |   .0498789   .0070512     7.07   0.000     .0360375    .0637203
       _cons |  -127.4544   5.285171   -24.12   0.000    -137.8291   -117.0798
------------------------------------------------------------------------------
```

Figure 36: Correlation Table

```
. correlate OptionModelBBBm plus TsySlope CaseShil20 CreditSlope
(obs=795)

             | Option~m     plus TsySlope CaseS~20 Credit~e
-------------+---------------------------------------------
OptionMode~m |   1.0000
        plus |   0.4314   1.0000
    TsySlope |   0.0833   0.7910   1.0000
  CaseShil20 |   0.2850  -0.6222  -0.7373   1.0000
 CreditSlope |  -0.9018  -0.4889  -0.1354  -0.1425   1.0000
```

Figure 37: Ramsey Reset Test

```
. ovtest

Ramsey RESET test using powers of the fitted values of ActualBBBm
       Ho:  model has no omitted variables
                  F(3, 786) =     72.13
                   Prob > F =      0.0000
```

Figure 38: Variance Inflation Factor

. **vif**

Variable	VIF	1/VIF
OptionMode~m	**8.08**	**0.123768**
plus	**6.46**	**0.154715**
CreditSlope	**6.00**	**0.166637**
CaseShil20	**4.67**	**0.214312**
TsySlope	**3.80**	**0.263012**
Mean VIF	**5.80**	

Figure 39: Variance decomposition test

. **colldiag()**

Proportion of variance associated with the decomposition

Cond Number	OptionModelBBBm	plus	TsySlope	CaseShil20	CreditSlope	_cons
1	**0.0003**	**0.0016**	**0.0004**	**0.0001**	**0.0008**	**0.0001**
3.28533	**0.0001**	**0.0860**	**0.0002**	**0.0001**	**0.0171**	**0.0001**
5.98041	**0.0180**	**0.0612**	**0.0027**	**0.0012**	**0.0704**	**0.0002**
20.5402	**0.0162**	**0.5458**	**0.7737**	**0.0038**	**0.2050**	**0.0000**
37.5846	**0.8787**	**0.1147**	**0.0378**	**0.1064**	**0.6969**	**0.1017**
75.7698	**0.0868**	**0.1906**	**0.1852**	**0.8884**	**0.0097**	**0.8979**

Figure 40: White test and IM-test

```
White's general test statistic :  298.0449  Chi-sq(20)  P-value =  2.0e-51

. imtest

Cameron & Trivedi's decomposition of IM-test
```

Source	chi2	df	p
Heteroskedasticity	298.05	20	0.0000
Skewness	31.95	5	0.0000
Kurtosis	2.37	1	0.1235
Total	332.37	26	0.0000

Figure 41: Durbin Watson Test

```
. dwstat

Durbin-Watson d-statistic(   6,    795) =  .0483615
```

Figure 42: AAA final comparison in Crisis

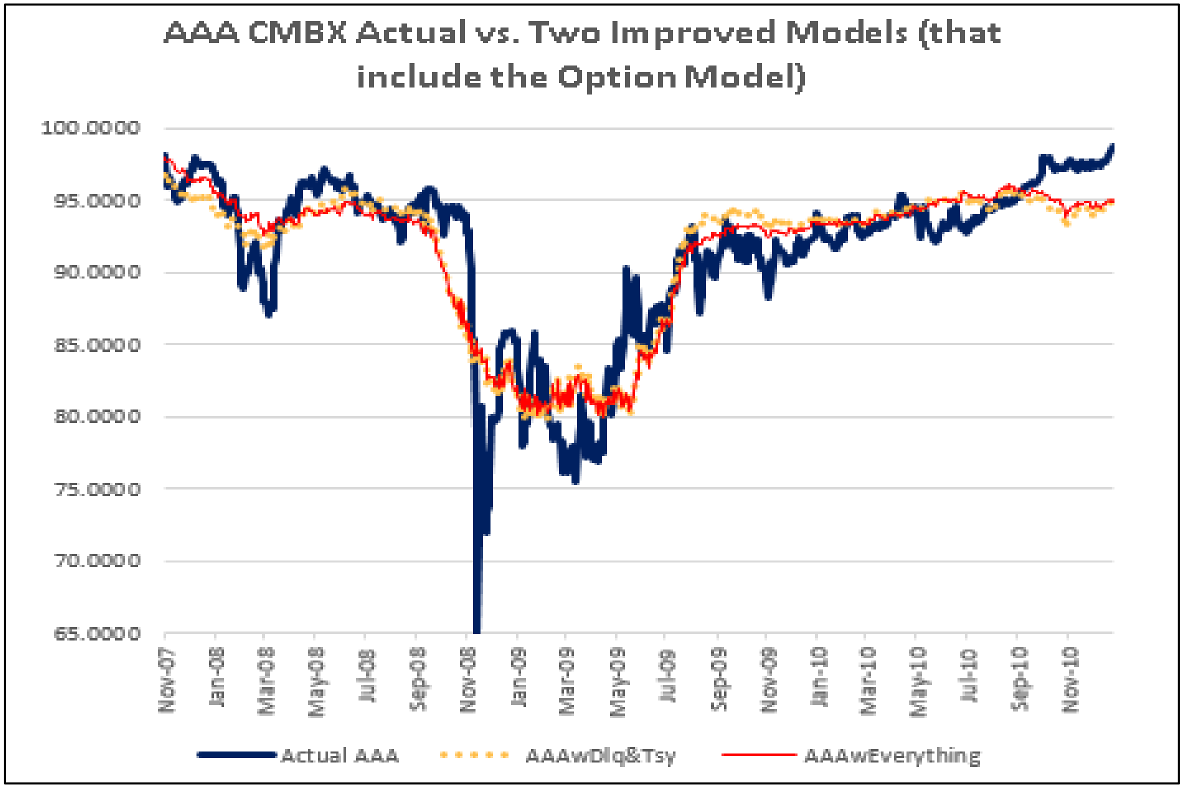


Figure 43: BBB- final comparison in Crisis

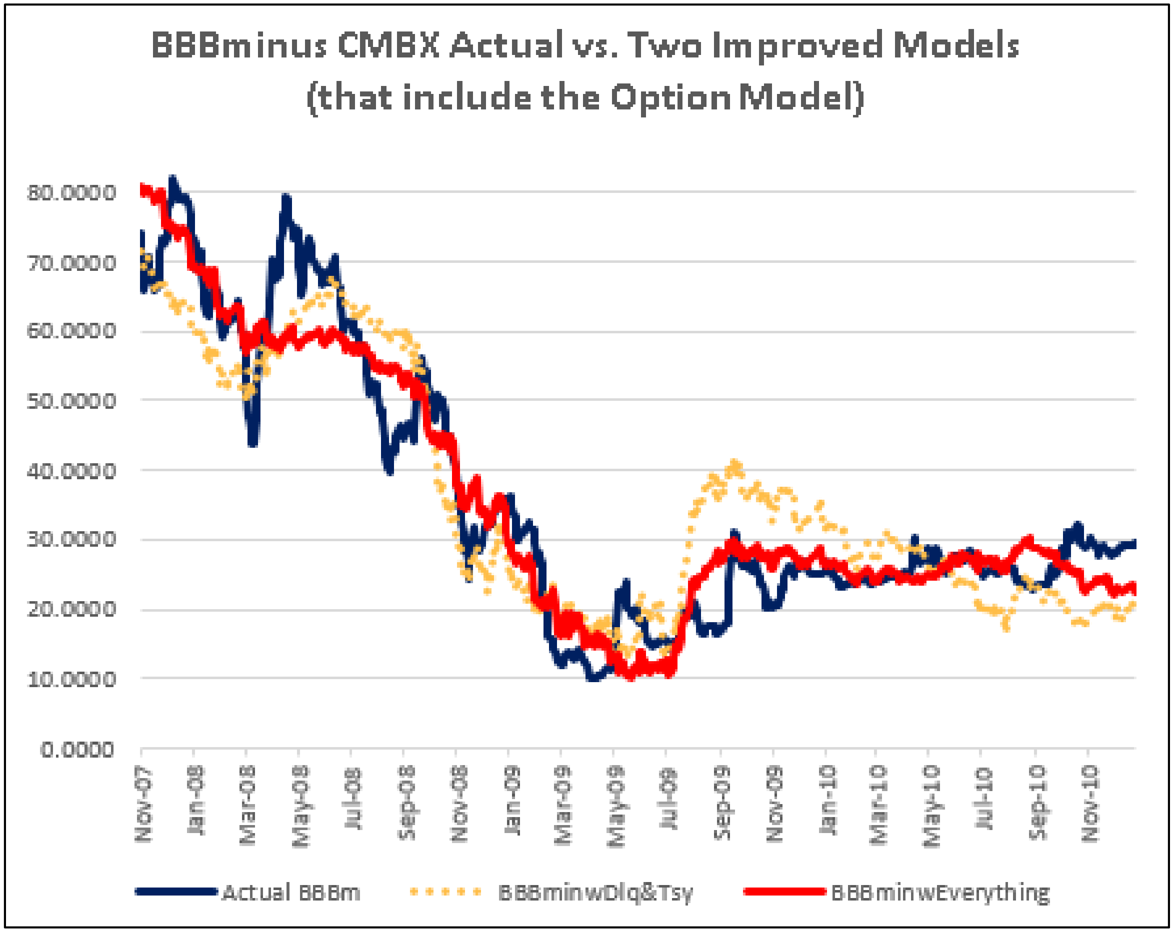


Figure 44: Property Type Composition in CMBX deal GG5

Row Labels	Count of prop_type_o	Sum of balwgt	Average of invLTV	Average of oterm
IN	7	8.82%	1.31	120.00
LO	11	15.32%	1.61	104.73
MF	8	6.33%	1.45	88.50
OF	61	33.13%	1.49	108.79
OT	9	2.11%	1.45	106.67
RT	76	34.29%	1.47	116.41
Grand Total	**172**	**100.00%**	**1.48**	**111.30**

Figure 45a: Rhos 6 Property Type Calibration
(MF:RT=Red, MF:OF=Blue, MF:IN=Black, MF:LO=Yellow, MF:OT=Orange).

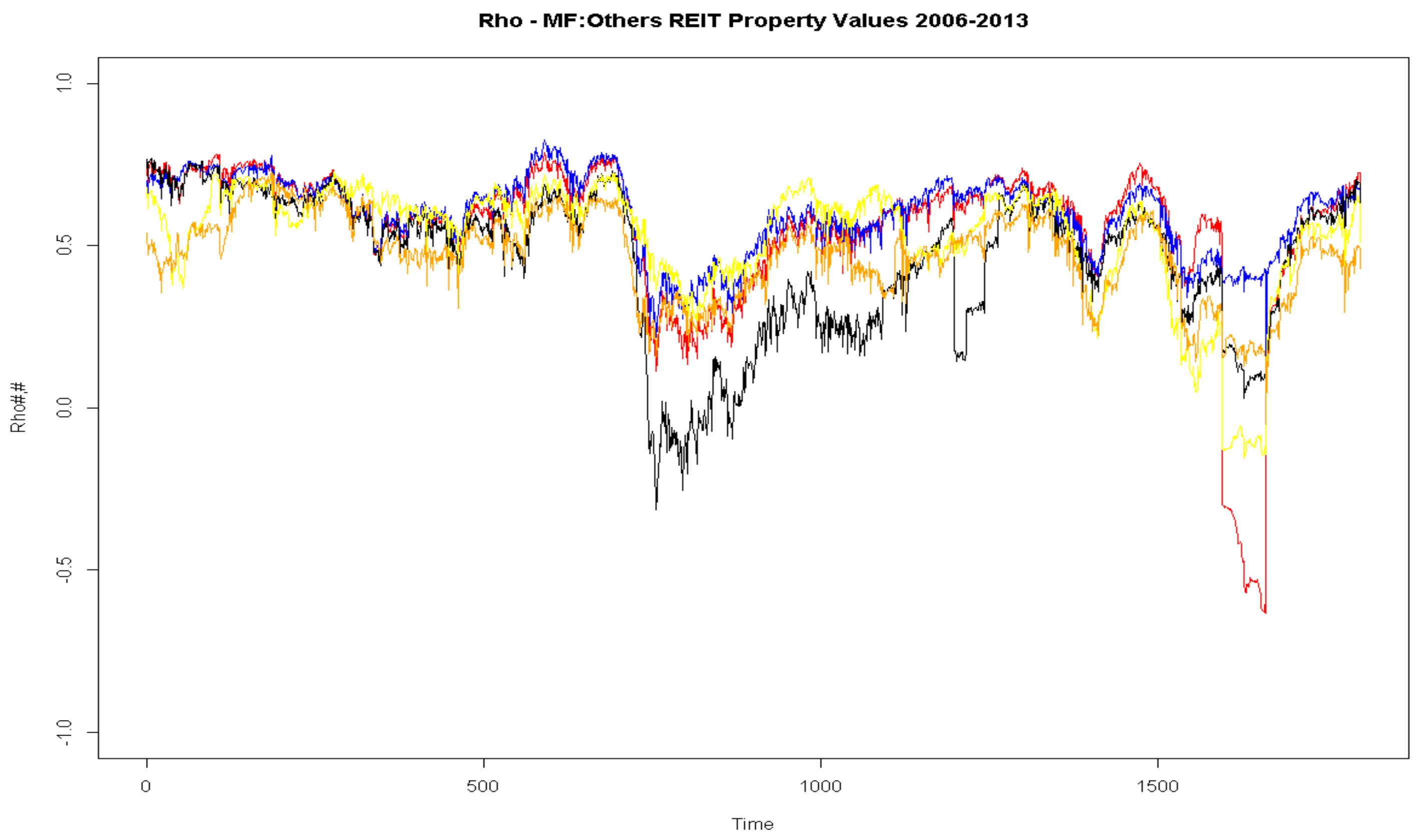


Figure 45b: Latent Property Type Values – 6 Property Type Calibration
(MF=Magenta, RT=Red, OF=Blue, IN=Black, LO=Yellow, OT=Orange)

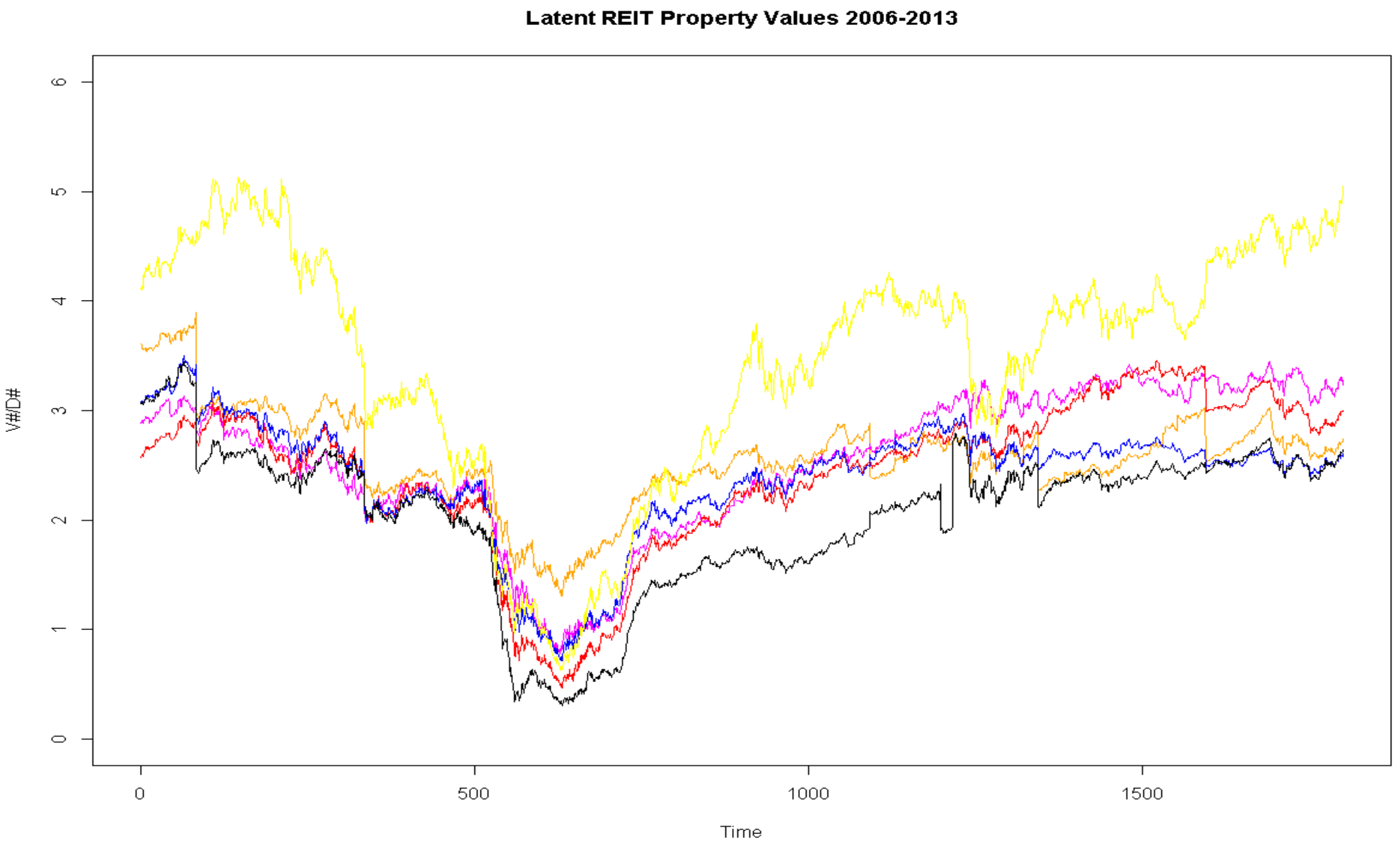


Figure 45c: Gamma 6 Property Type Calibration

(MF:RT=Red, MF:OF=Blue, MF:IN=Black, MF:LO=Yellow, MF:OT=Orange).

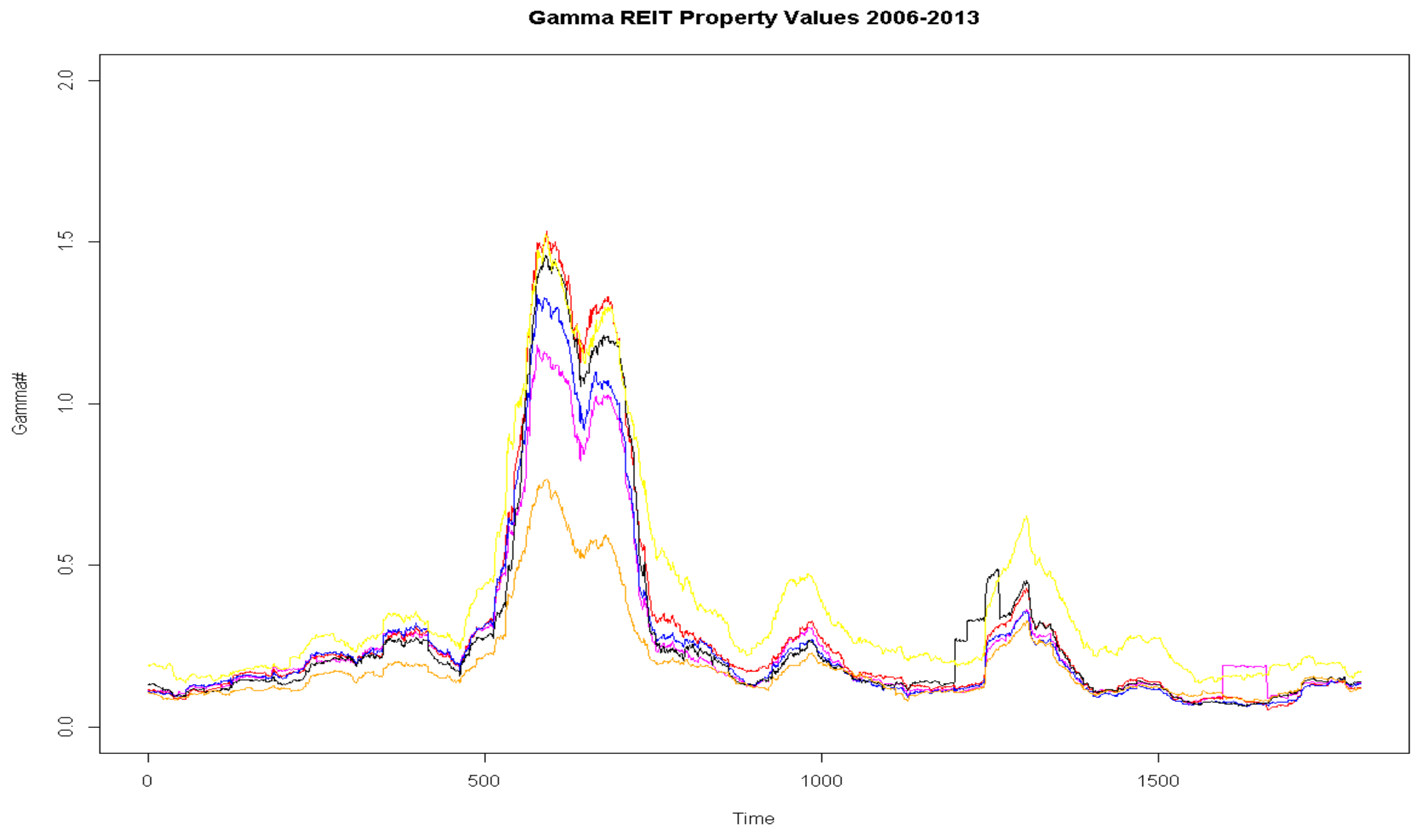


Figure 45d: Beta 6 Property Type Calibration

(MF=Magenta, RT=Red, OF=Blue, IN=Black, LO=Yellow, OT=Orange)

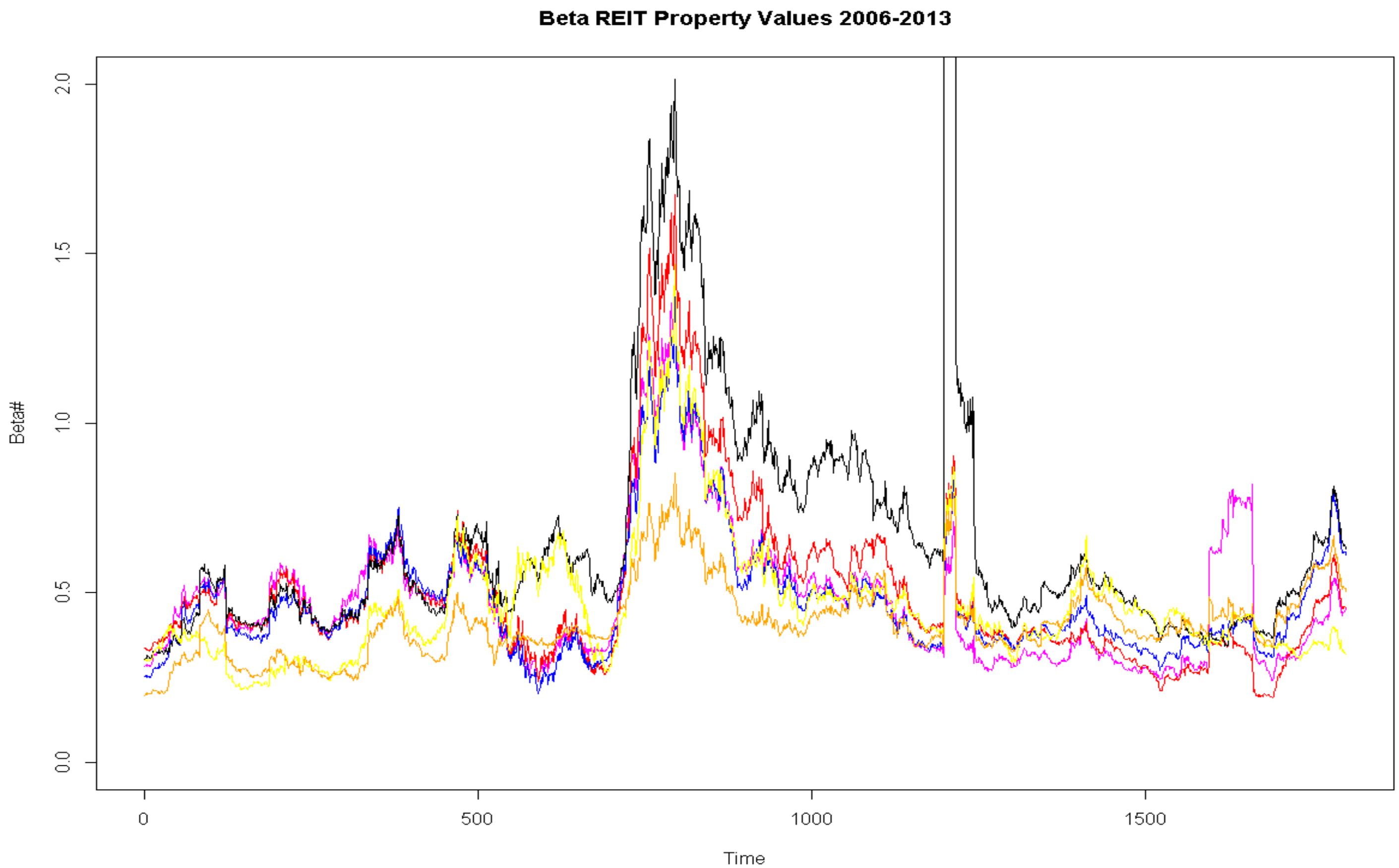


Figure 46: Inverse LTV by property type (simulated)

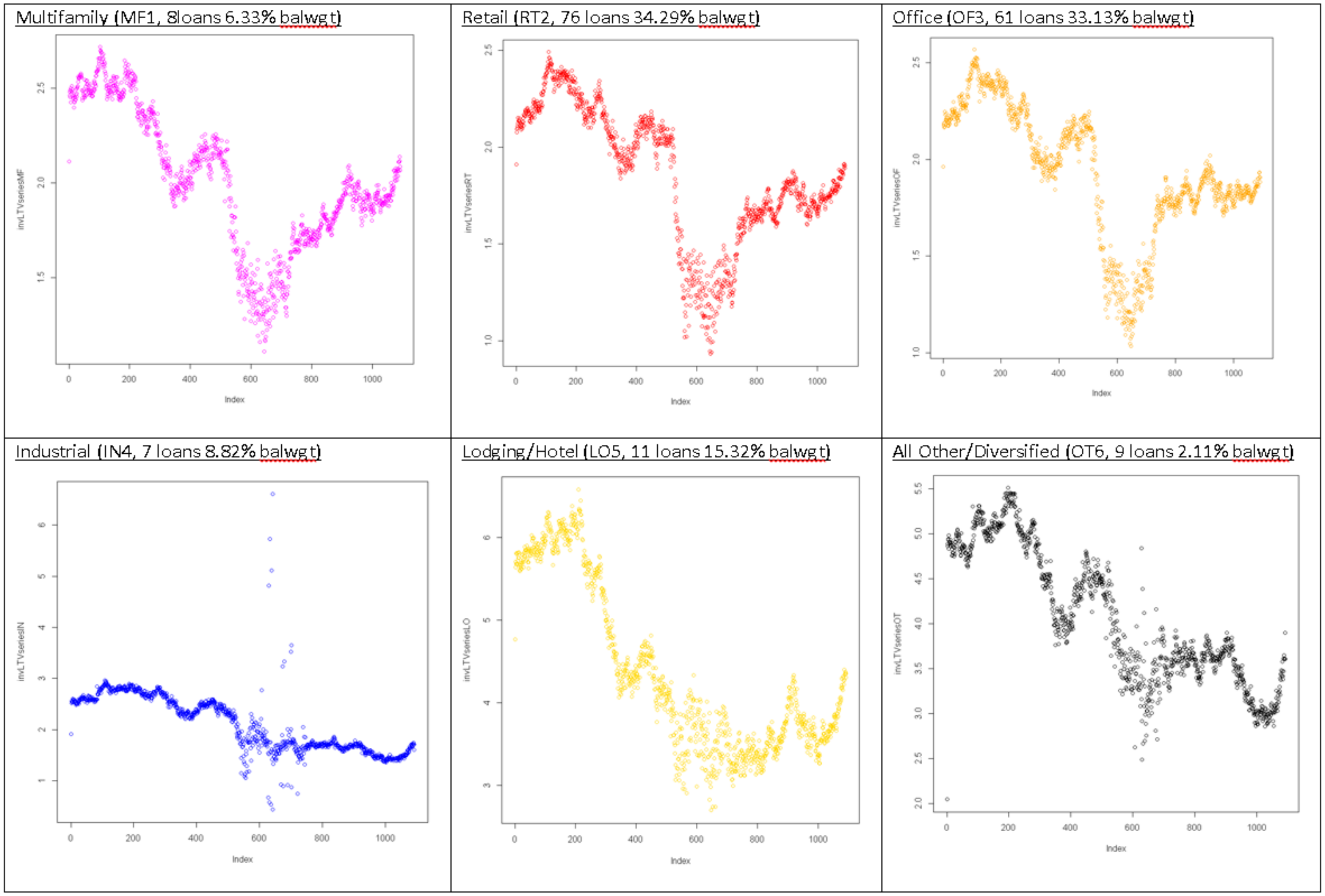


Figure 47: Fair Value Pricing No Cashflows, 6 Property Types.

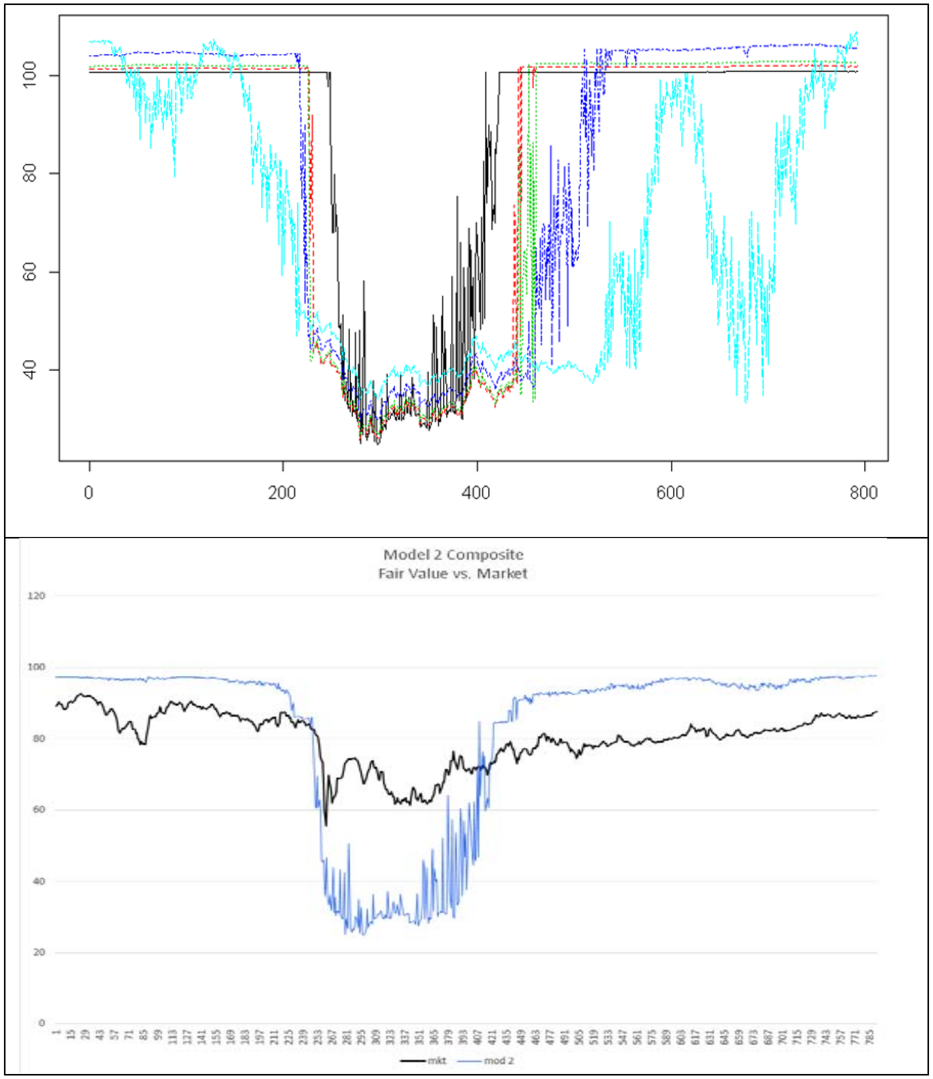
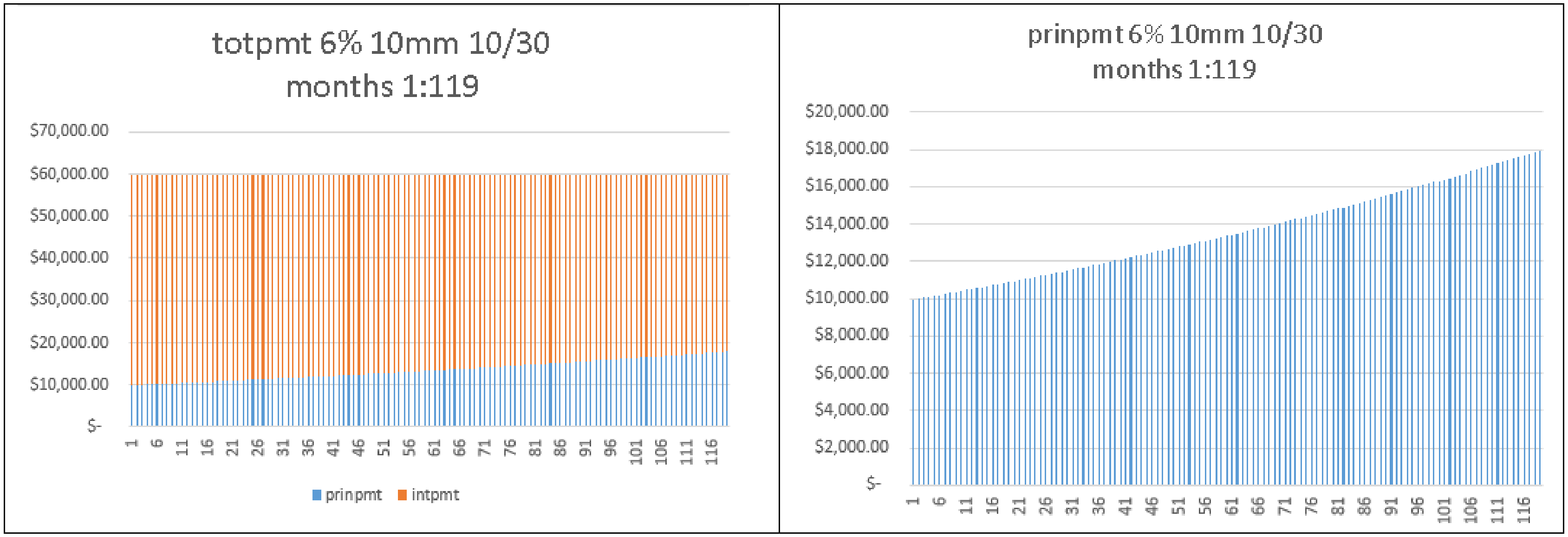


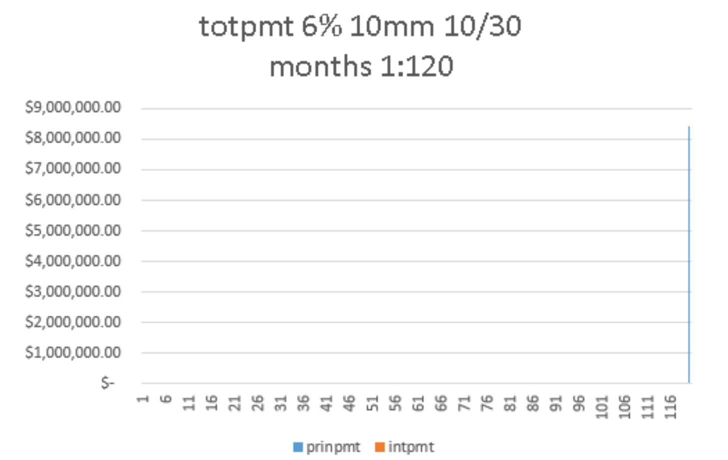
Figure 48: Comparison of Special Case and Generalized Form of Calibration Hybrid Attributes

Special Case of Generalized Form (DVH, 2012; Model 2)	Generalized Form of Calibration Hybrid (Model 3)
33 loans with undisclosed relationship to CMBX Series 1	172 loans from GG5 in CMBX Series 1
3 of 6 property types (MF, RT, & OF)	6 of 6 property types (MF, RT, OF, IN, LO, OT)
Uniform Maturity Dates for all loans	Accurate Maturity/Balloon Dates for all loans
Uniform $Balance across all loans	Accurate $Balance across all loans
10 REITs combined for 3 diffusions	35 REITs combined for 6 diffusions
Assumed Maturity Default	Accurate Maturity and Term/Ruthless Default
Assumed Interest only Balloon Loans	Accurate Amortization of Interest & Principal

Figures 49a and 49b: Mortgage Cashflow Composition



Figures 50a and 50b: Mortgage Cashflows with Balloons



6% coupon
120 balloon month

	begbal	prinpmt	intpmt	totpmt	endbal
1	$ 10,000,000.00	$ 9,955.05	$ 50,000.00	$59,955.05	$ 9,990,044.95
2	$ 9,990,044.95	$ 10,004.83	$ 49,950.22	$59,955.05	$ 9,980,040.12
116	$ 8,457,790.72	$ 17,666.10	$ 42,288.95	$59,955.05	$ 8,440,124.62
117	$ 8,440,124.62	$ 17,754.43	$ 42,200.62	$59,955.05	$ 8,422,370.20
118	$ 8,422,370.20	$ 17,843.20	$ 42,111.85	$59,955.05	$ 8,404,526.99
119	$ 8,404,526.99	$ 17,932.42	$ 42,022.63	$59,955.05	$ 8,386,594.58
120	$ 8,386,594.58	$ 8,386,594.58	$ 41,932.97	$8,428,527.55	$ -

Figures 51a and 51b: Loan 1 in GG5 promised Profile

Loan #		1
OrigBal		17985284
Loan BalanceCurre		16979920
Coupon		6.052
OrigTerm		132
AmTerm		360
Seasoning		71
RemTerm		61
AsOfDate		4/1/2010
Orig Date		4/12/2004
Orig Period		1
PropType	RT	
Location, State	MD	
OrigLTV		55.34
CurLoanLifePeriod		72
OrigIO Per		17
CurAmLifePeriod		55
MaxHist1stPmtPer		1
Balloon		1
RegAmPmt		$108,432.88

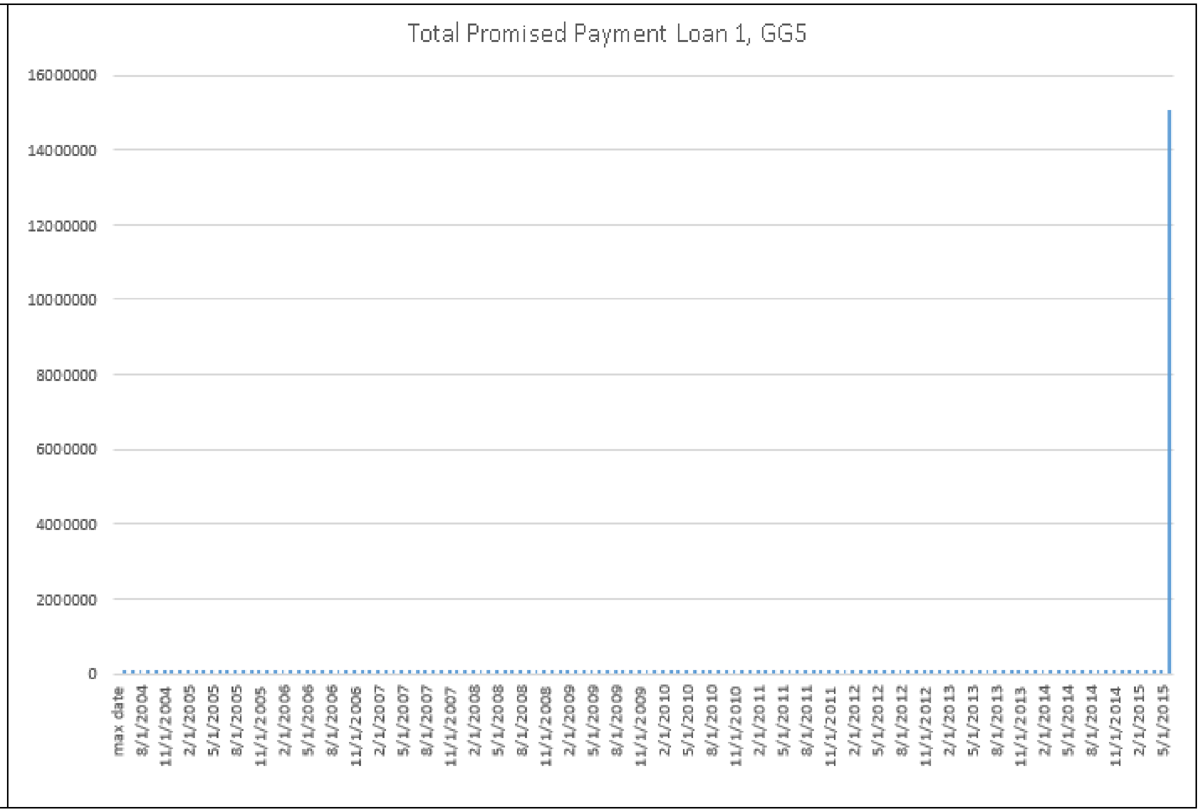


Figure 52: All Promised Cashflows for GG5 from origination to maturity

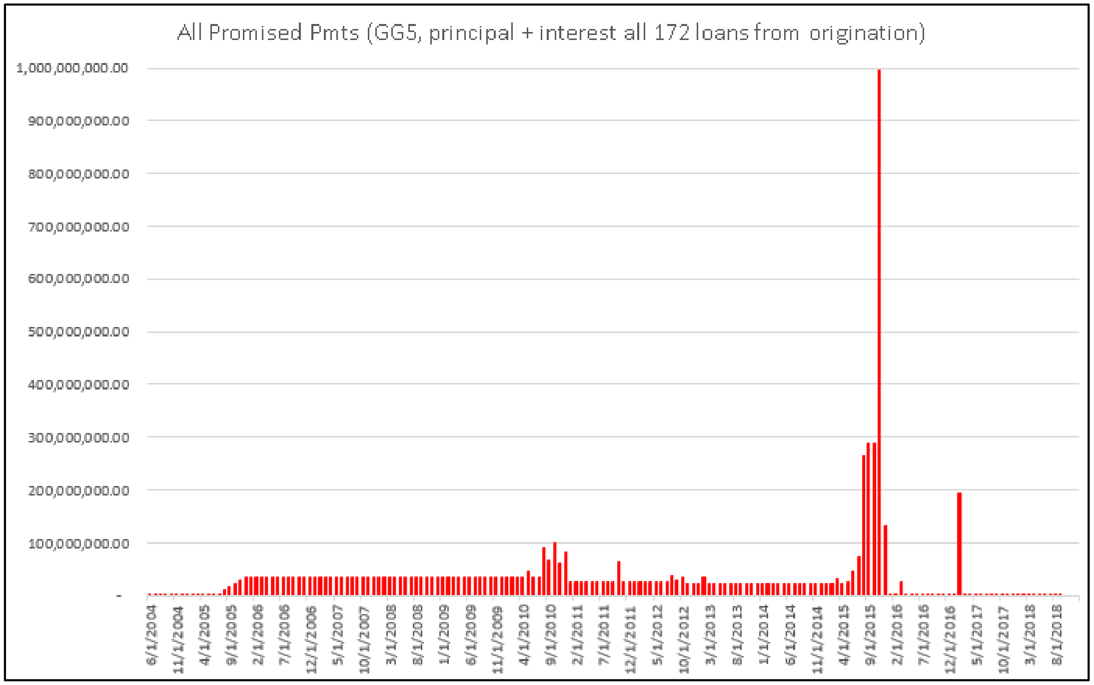


Figure 53: Allocation of CMBX Transaction for Model 3

Class/Tranche	Class %	Annual Coupon	Original Balance
AAA	64.4%	5.68%	$2,667,278,356
AJ	5.8%	6.44%	$241,877,416
AM	9.8%	6.09%	$407,546,879
AA	7.4%	5.88%	$307,316,854
A	2.1%	5.98%	$84,905,600
BBB	2.7%	6.45%	$113,483,582
BBBminus	4.5%	7.04%	$186,792,320
Other	3.2%	8.00%	$132,535,570
Total/avg	100.0%	5.94%	$4,141,736,578

Figure 54a: Example Tranche Allocation of Principal

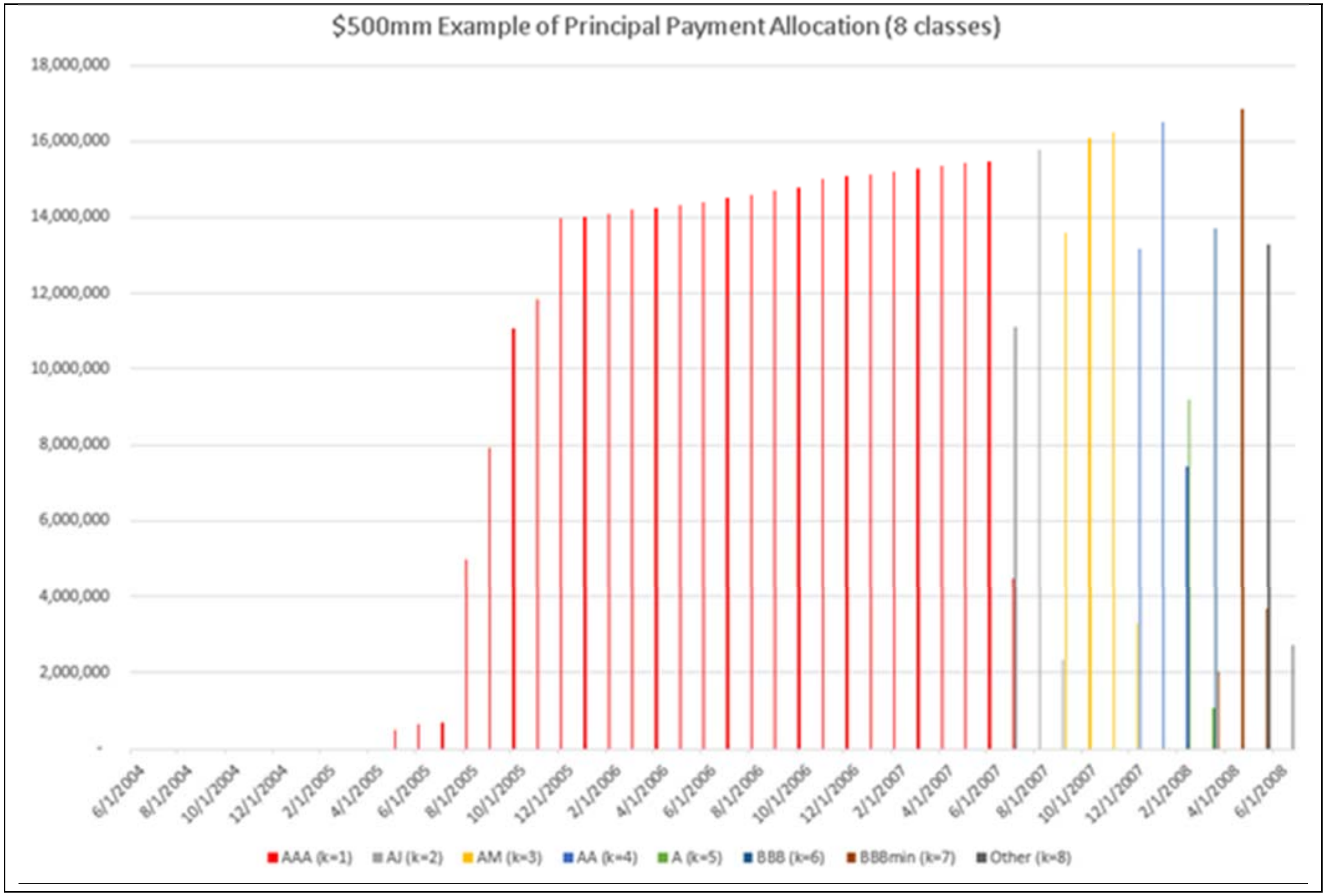


Figure 54b: Principal Allocation from Trust to Bonds

init size %	100.00%	64.40%	5.84%	9.84%	7.42%	2.05%	2.74%	4.51%	3.20%
balance	500,000,000	322,000,000	29,200,000	49,200,000	37,100,000	10,250,000	13,700,000	22,550,000	16,000,000
pmt month	All k	AAA (k=1)	AJ (k=2)	AM (k=3)	AA (k=4)	A (k=5)	BBB (k=6)	BBBmin (k=7)	Other (k=8)
6/1/2004	-	-	-	-	-	-	-	-	-
7/1/2004	-	-	-	-	-	-	-	-	-
8/1/2004	-	-	-	-	-	-	-	-	-
9/1/2004	-	-	-	-	-	-	-	-	-
10/1/2004	-	-	-	-	-	-	-	-	-
11/1/2004	-	-	-	-	-	-	-	-	-
12/1/2004	-	-	-	-	-	-	-	-	-
1/1/2005	-	-	-	-	-	-	-	-	-
2/1/2005	-	-	-	-	-	-	-	-	-
3/1/2005	-	-	-	-	-	-	-	-	-
4/1/2005	-	-	-	-	-	-	-	-	-
5/1/2005	496,899	496,899	-	-	-	-	-	-	-
6/1/2005	634,738	634,738	-	-	-	-	-	-	-
7/1/2005	699,102	699,102	-	-	-	-	-	-	-
8/1/2005	4,993,225	4,993,225	-	-	-	-	-	-	-
9/1/2005	7,916,097	7,916,097	-	-	-	-	-	-	-
10/1/2005	11,082,441	11,082,441	-	-	-	-	-	-	-
11/1/2005	11,822,236	11,822,236	-	-	-	-	-	-	-
12/1/2005	13,972,802	13,972,802	-	-	-	-	-	-	-
1/1/2006	14,036,713	14,036,713	-	-	-	-	-	-	-
2/1/2006	14,100,918	14,100,918	-	-	-	-	-	-	-
3/1/2006	14,196,818	14,196,818	-	-	-	-	-	-	-
4/1/2006	14,261,764	14,261,764	-	-	-	-	-	-	-
5/1/2006	14,327,010	14,327,010	-	-	-	-	-	-	-
6/1/2006	14,392,557	14,392,557	-	-	-	-	-	-	-
7/1/2006	14,521,242	14,521,242	-	-	-	-	-	-	-
8/1/2006	14,598,938	14,598,938	-	-	-	-	-	-	-
9/1/2006	14,697,777	14,697,777	-	-	-	-	-	-	-
10/1/2006	14,780,718	14,780,718	-	-	-	-	-	-	-
11/1/2006	15,004,445	15,004,445	-	-	-	-	-	-	-
12/1/2006	15,073,132	15,073,132	-	-	-	-	-	-	-
1/1/2007	15,142,137	15,142,137	-	-	-	-	-	-	-
2/1/2007	15,211,461	15,211,461	-	-	-	-	-	-	-
3/1/2007	15,281,106	15,281,106	-	-	-	-	-	-	-
4/1/2007	15,351,073	15,351,073	-	-	-	-	-	-	-
5/1/2007	15,421,363	15,421,363	-	-	-	-	-	-	-
6/1/2007	15,491,978	15,491,978	-	-	-	-	-	-	-
7/1/2007	15,589,687	4,491,311	11,098,376	-	-	-	-	-	-
8/1/2007	15,763,649	-	15,763,649	-	-	-	-	-	-
9/1/2007	15,922,933	-	2,337,975	13,584,958	-	-	-	-	-
10/1/2007	16,085,230	-	-	16,085,230	-	-	-	-	-
11/1/2007	16,246,311	-	-	16,246,311	-	-	-	-	-
12/1/2007	16,445,715	-	-	3,283,501	13,162,214	-	-	-	-
1/1/2008	16,520,939	-	-	-	16,520,939	-	-	-	-
2/1/2008	16,596,510	-	-	-	7,416,847	9,179,663	-	-	-
3/1/2008	16,789,009	-	-	-	-	1,070,337	13,700,000	2,018,671	-
4/1/2008	16,865,857	-	-	-	-	-	-	16,865,857	-
5/1/2008	16,943,061	-	-	-	-	-	-	3,665,472	13,277,589
6/1/2008	2,722,411	-	-	-	-	-	-	-	2,722,411
TOTAL	500,000,000	322,000,000	29,200,000	49,200,000	37,100,000	10,250,000	13,700,000	22,550,000	16,000,000

Figure 55: Interest Allocation from Trust to Bonds

Coupon	5.94%	5.68%	6.44%	6.09%	5.88%	5.98%	6.45%	7.04%	8%	
init size %	100.00%	64.40%	5.84%	9.84%	7.42%	2.05%	2.74%	4.51%	3.20%	
balance	500,000,000	322,000,000	29,200,000	49,200,000	37,100,000	10,250,000	13,700,000	22,550,000	16,000,000	
pmt month	All k	AAA (k=1)	AJ (k=2)	AM (k=3)	AA (k=4)	A (k=5)	BBB (k=6)	BBBmin (k=7)	Other (k=8)	month match
6/1/2004	-	-	-	-	-	-	-	-	-	
7/1/2004	-	-	-	-	-	-	-	-	-	
8/1/2004	-	-	-	-	-	-	-	-	-	
9/1/2004	-	-	-	-	-	-	-	-	-	
10/1/2004	-	-	-	-	-	-	-	-	-	
11/1/2004	-	-	-	-	-	-	-	-	-	
12/1/2004	-	-	-	-	-	-	-	-	-	
1/1/2005	-	-	-	-	-	-	-	-	-	
2/1/2005	-	-	-	-	-	-	-	-	-	
3/1/2005	-	-	-	-	-	-	-	-	-	
4/1/2005	-	-	-	-	-	-	-	-	-	
5/1/2005	2,475,996.67	1,524,133.33	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	2,475,997
6/1/2005	2,473,644.68	1,521,781.34	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	2,473,645
7/1/2005	2,470,640.25	1,518,776.92	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	2,470,640
8/1/2005	2,467,331.17	1,515,467.83	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	2,467,331
9/1/2005	2,443,696.57	1,491,833.23	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	2,443,697
10/1/2005	2,406,227.04	1,454,363.71	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	2,406,227
11/1/2005	2,353,770.15	1,401,906.82	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	2,353,770
12/1/2005	2,297,811.57	1,345,948.23	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	2,297,812
1/1/2006	2,231,673.64	1,279,810.30	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	2,231,674
2/1/2006	2,165,233.20	1,213,369.86	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	2,165,233
3/1/2006	2,098,488.85	1,146,625.51	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	2,098,489
4/1/2006	2,031,290.58	1,079,427.24	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	2,031,291
5/1/2006	1,963,784.90	1,011,921.56	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	1,963,785
6/1/2006	1,895,970.39	944,107.05	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	1,895,970
7/1/2006	1,827,845.62	875,982.28	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	1,827,846
8/1/2006	1,759,111.74	807,248.40	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	1,759,112
9/1/2006	1,690,010.10	738,146.77	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	1,690,010
10/1/2006	1,620,440.62	668,577.29	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	1,620,441
11/1/2006	1,550,478.56	598,615.23	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	1,550,479
12/1/2006	1,479,457.52	527,594.19	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	1,479,458
1/1/2007	1,408,111.36	456,248.03	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	1,408,111
2/1/2007	1,336,438.58	384,575.25	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	1,336,439
3/1/2007	1,264,437.66	312,574.33	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	1,264,438
4/1/2007	1,192,107.09	240,243.76	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	1,192,107
5/1/2007	1,119,445.35	167,582.02	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	1,119,445
6/1/2007	1,046,450.90	94,587.57	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	1,046,451
7/1/2007	973,122.21	21,258.87	156,706.67	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	973,122
8/1/2007	892,302.05	-	97,145.38	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	892,302
9/1/2007	807,703.80	-	12,547.13	249,690.00	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	807,704
10/1/2007	726,213.00	-	-	180,746.34	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	726,213
11/1/2007	644,580.46	-	-	99,113.79	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	644,580
12/1/2007	562,130.43	-	-	16,663.77	181,790.00	51,079.17	73,637.50	132,293.33	106,666.67	562,130
1/1/2008	480,971.82	-	-	-	117,295.15	51,079.17	73,637.50	132,293.33	106,666.67	480,972
2/1/2008	400,019.22	-	-	-	36,342.55	51,079.17	73,637.50	132,293.33	106,666.67	400,019
3/1/2008	317,931.35	-	-	-	-	5,333.85	73,637.50	132,293.33	106,666.67	317,931
4/1/2008	227,117.13	-	-	-	-	-	-	120,450.46	106,666.67	227,117
5/1/2008	128,170.77	-	-	-	-	-	-	21,504.10	106,666.67	128,171
6/1/2008	18,149.40	-	-	-	-	-	-	-	18,149.40	18,149

Figure 56: Price comparison of AAA fair value across models

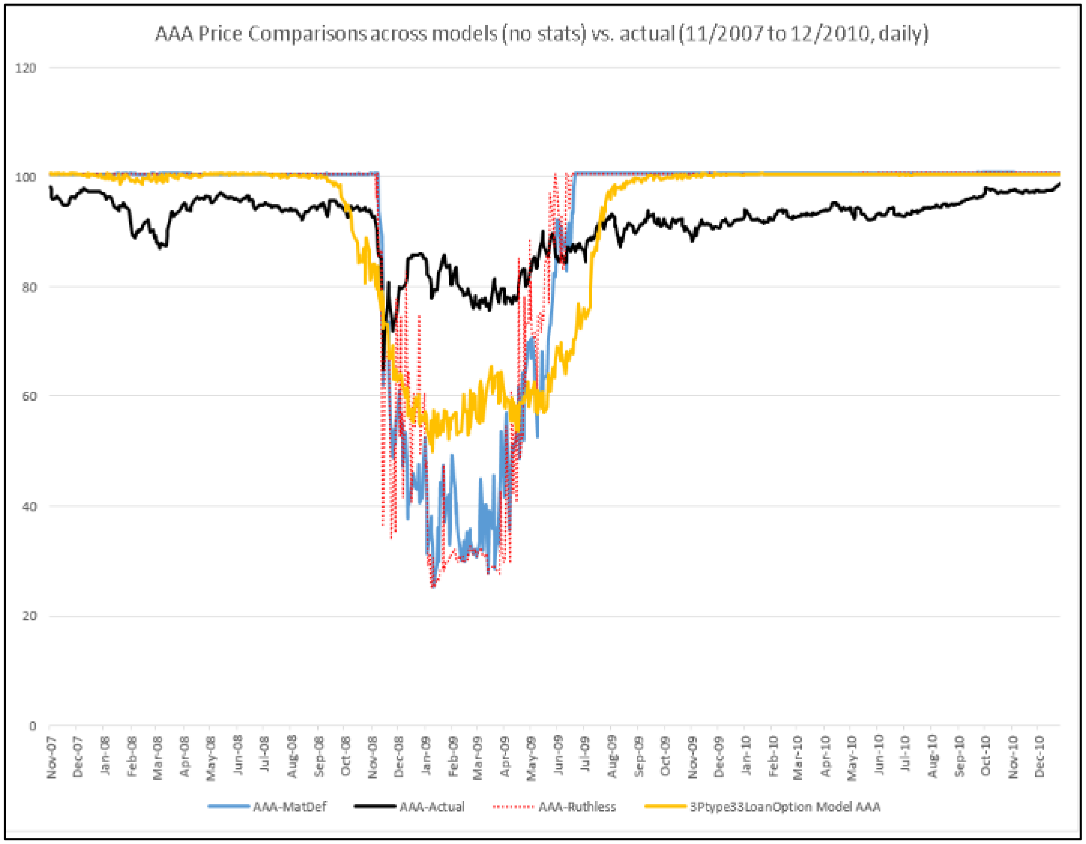


Figure 57: Price comparison of BBB- fair value across models

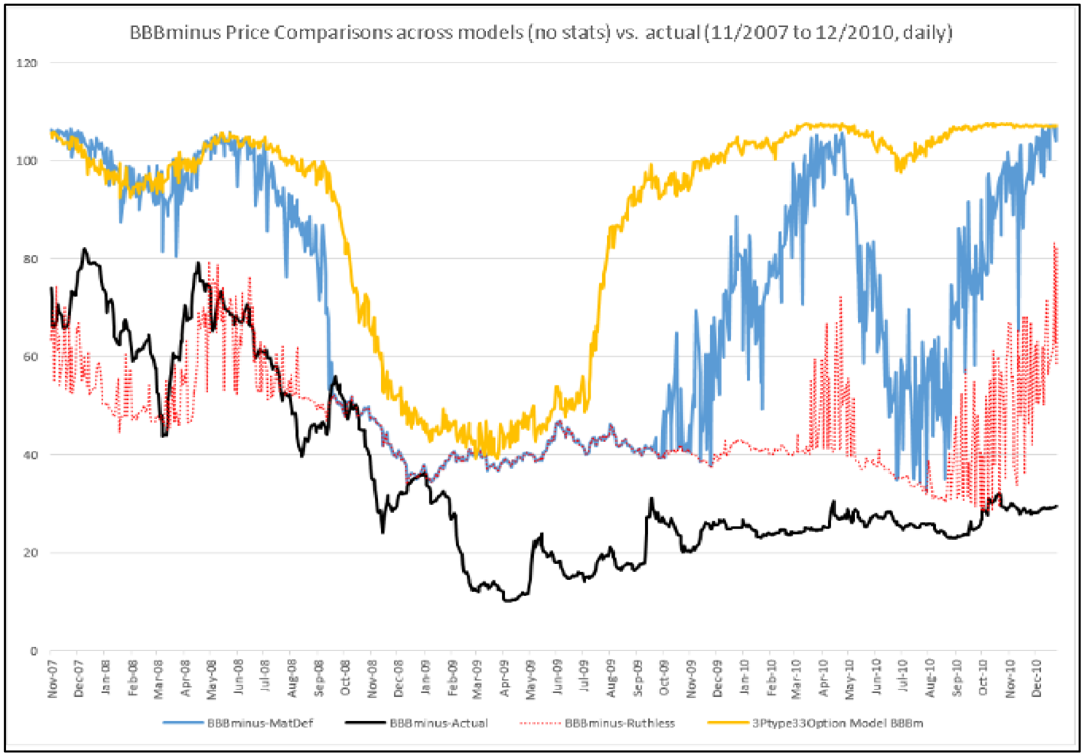


Figure 58: Probability of Default comparisons by property type

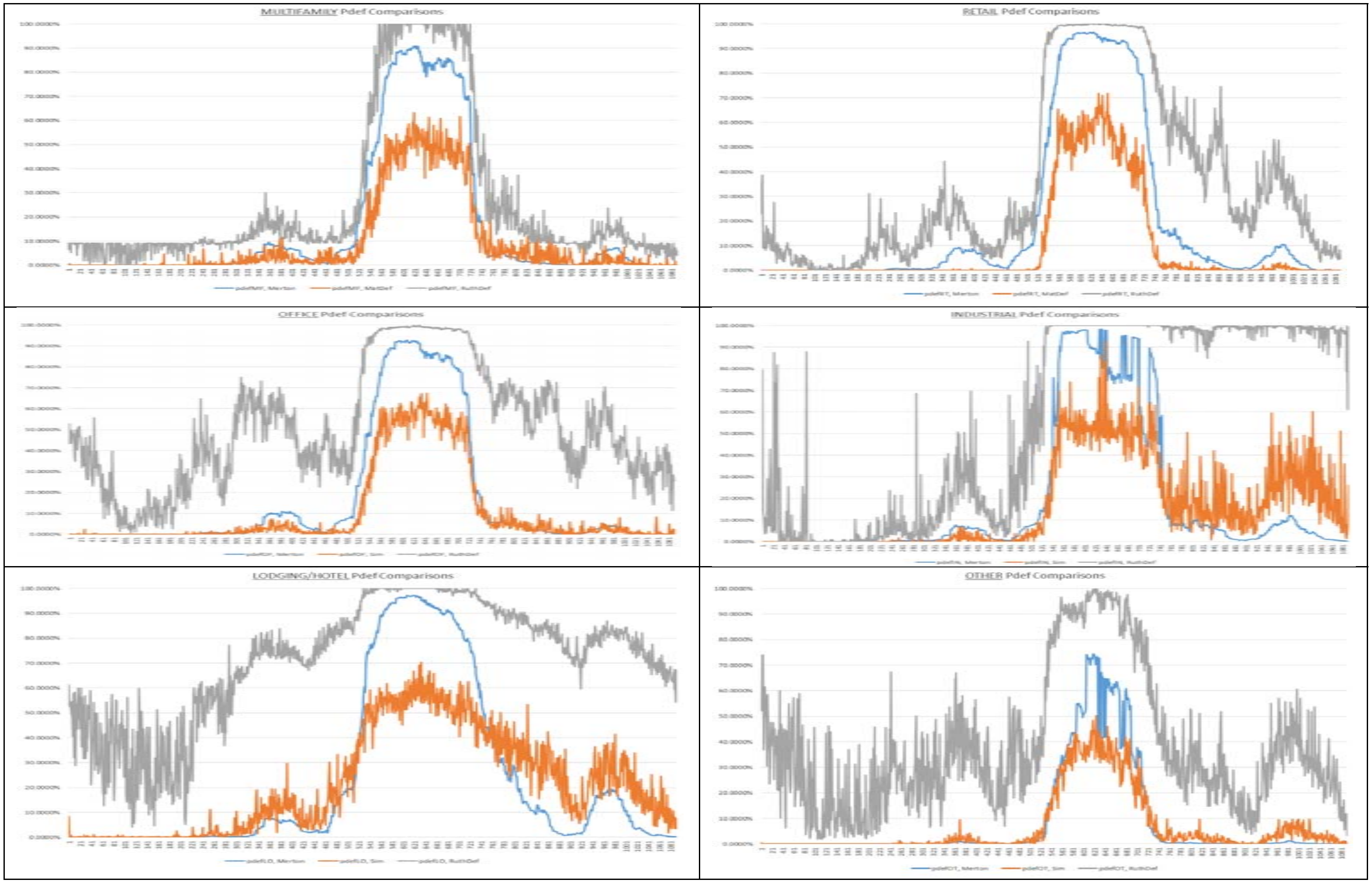


Figure 59: Expected Losses under Ruthless Default Simulation

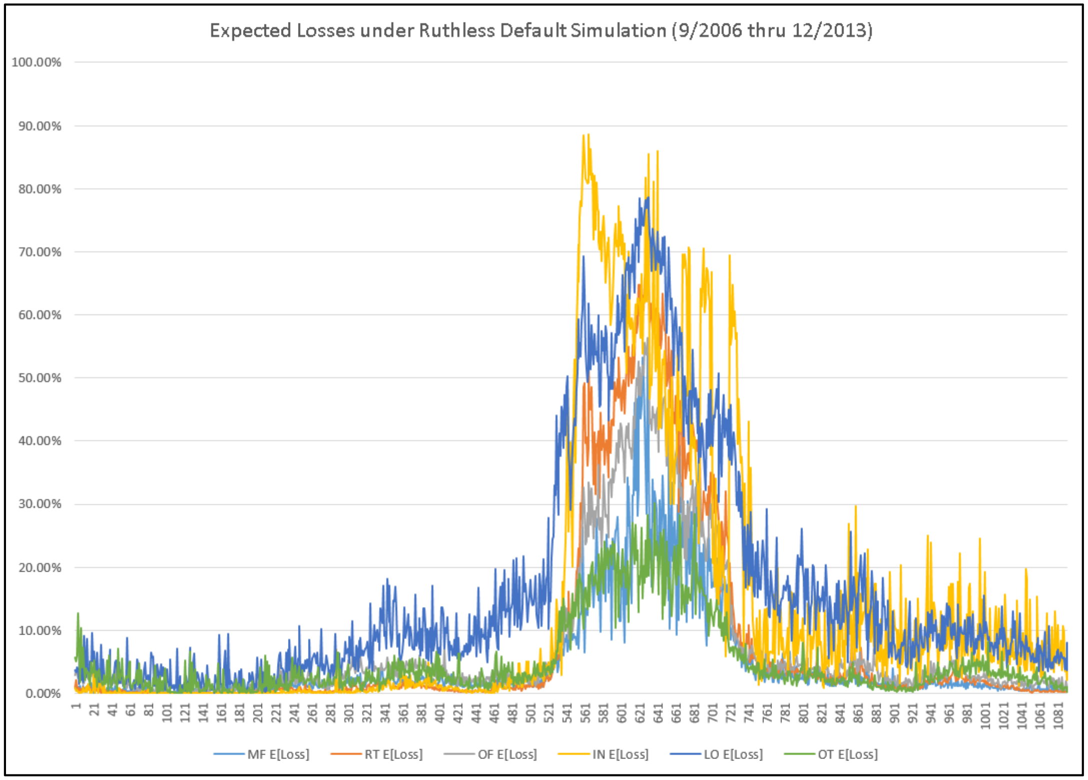


Figure 60: Final Generalized (Model 3) Fair Value

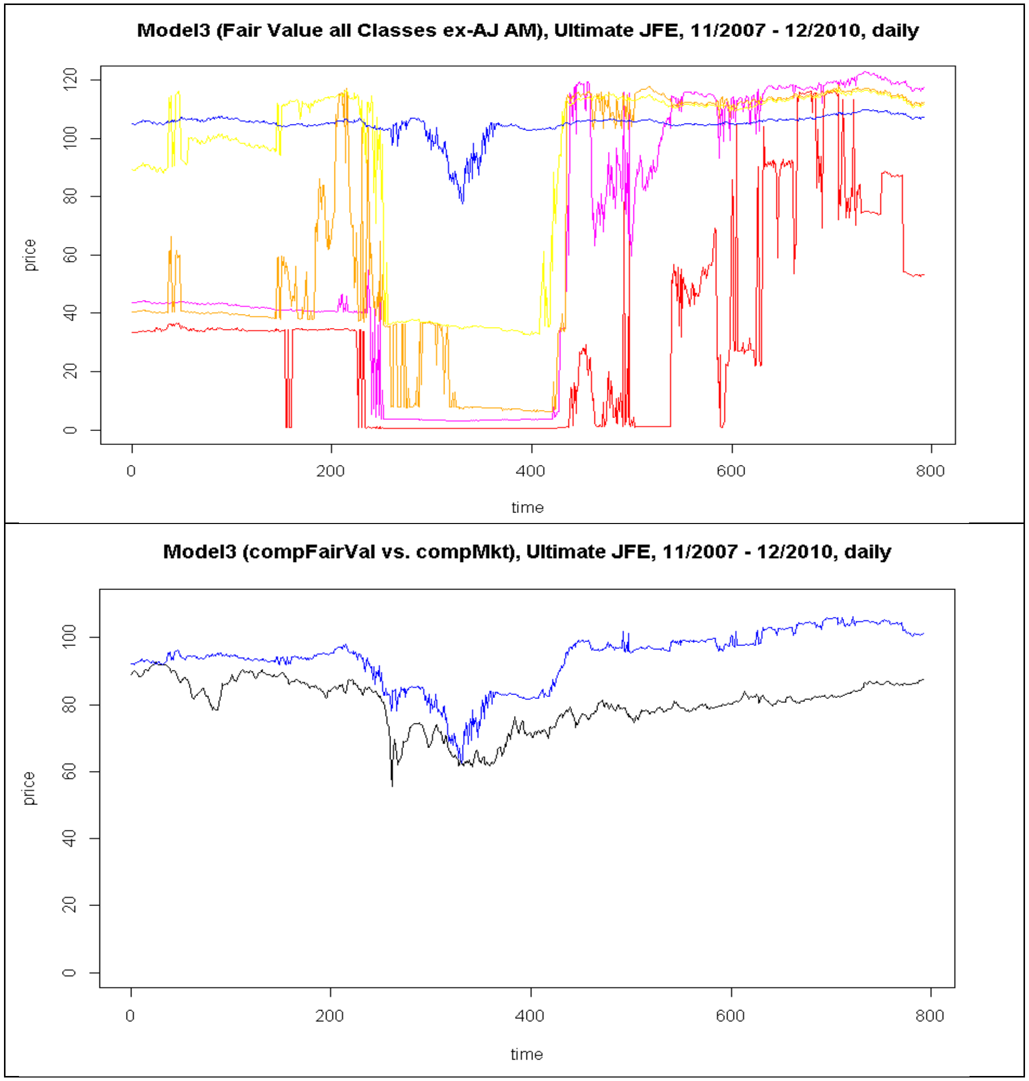


Figure 61: Theta Comparison of Model 3 by Tranches

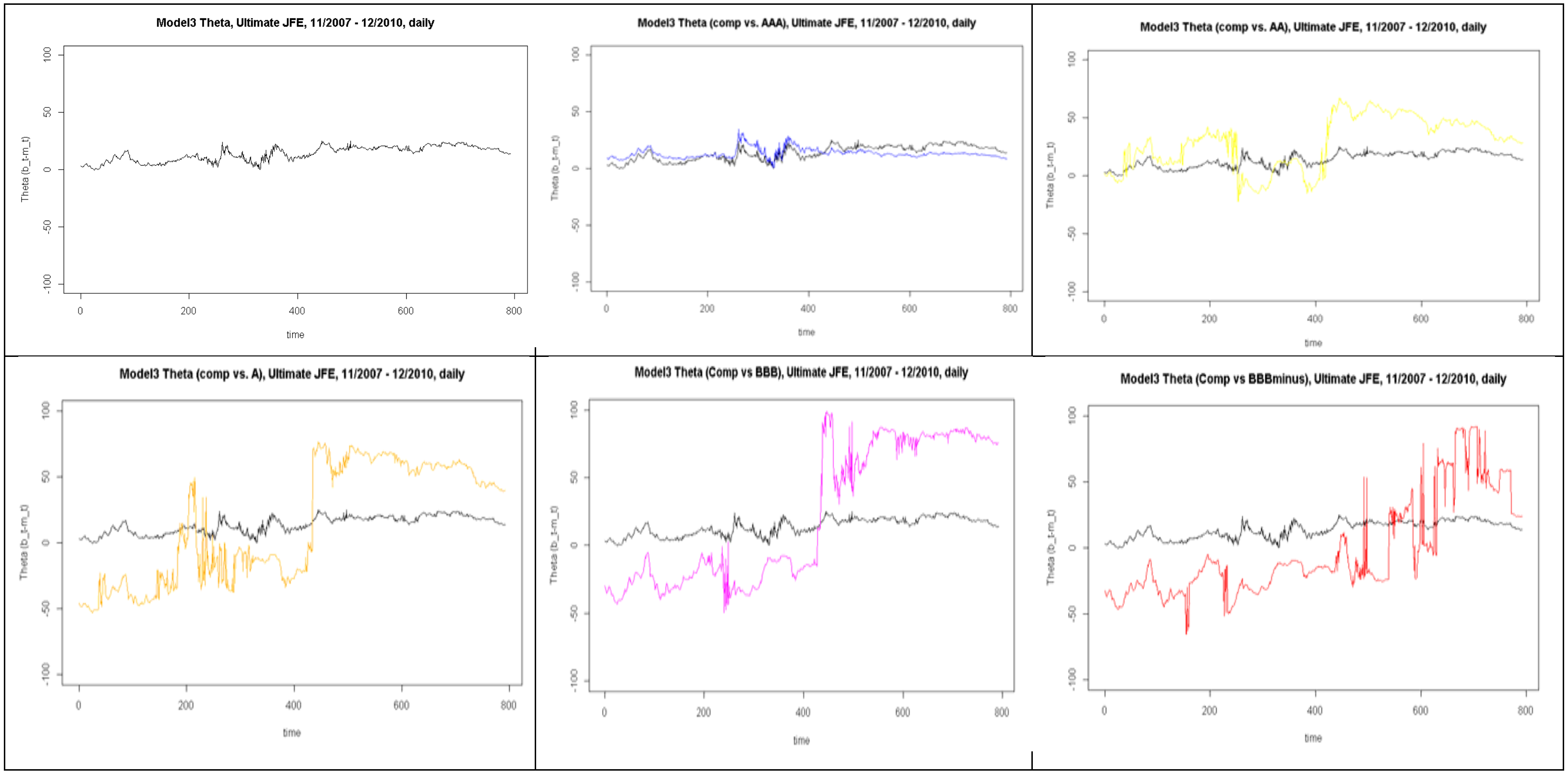


Figure 62: Historical Volatility REITs vs. NCREIF (7/1/2006 to 1/1/2014)

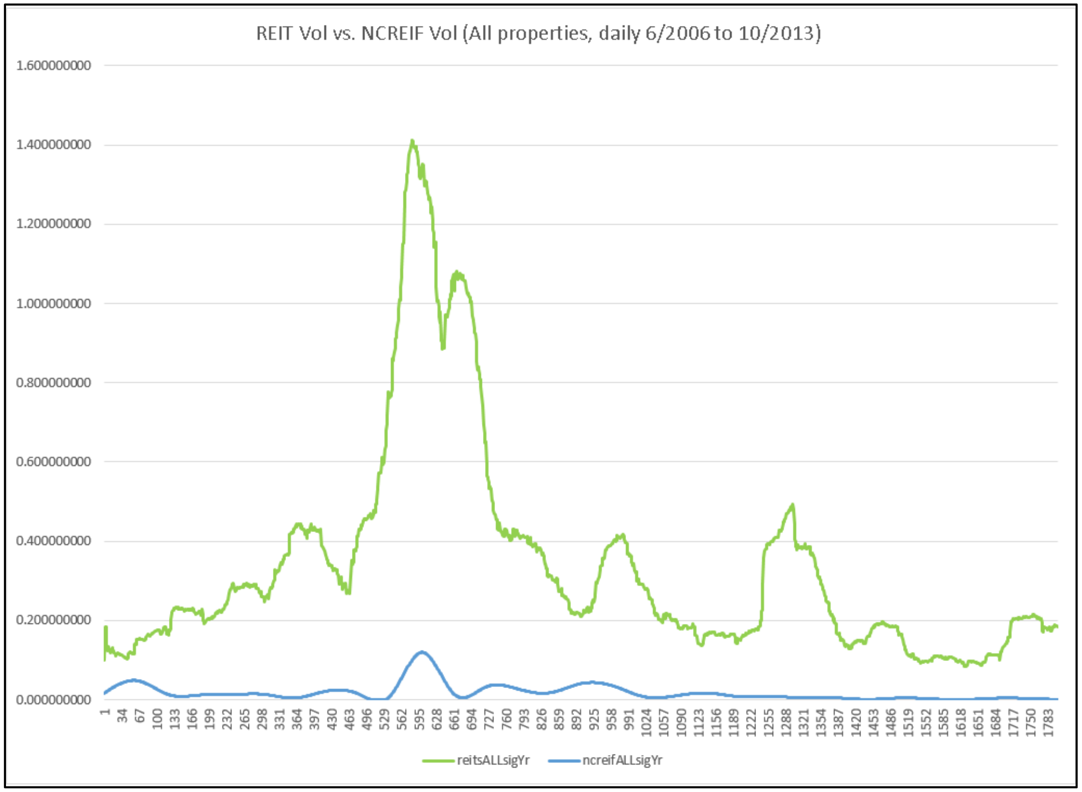


Figure 63: Volatilities for Simulated Values in the Correlated Economy

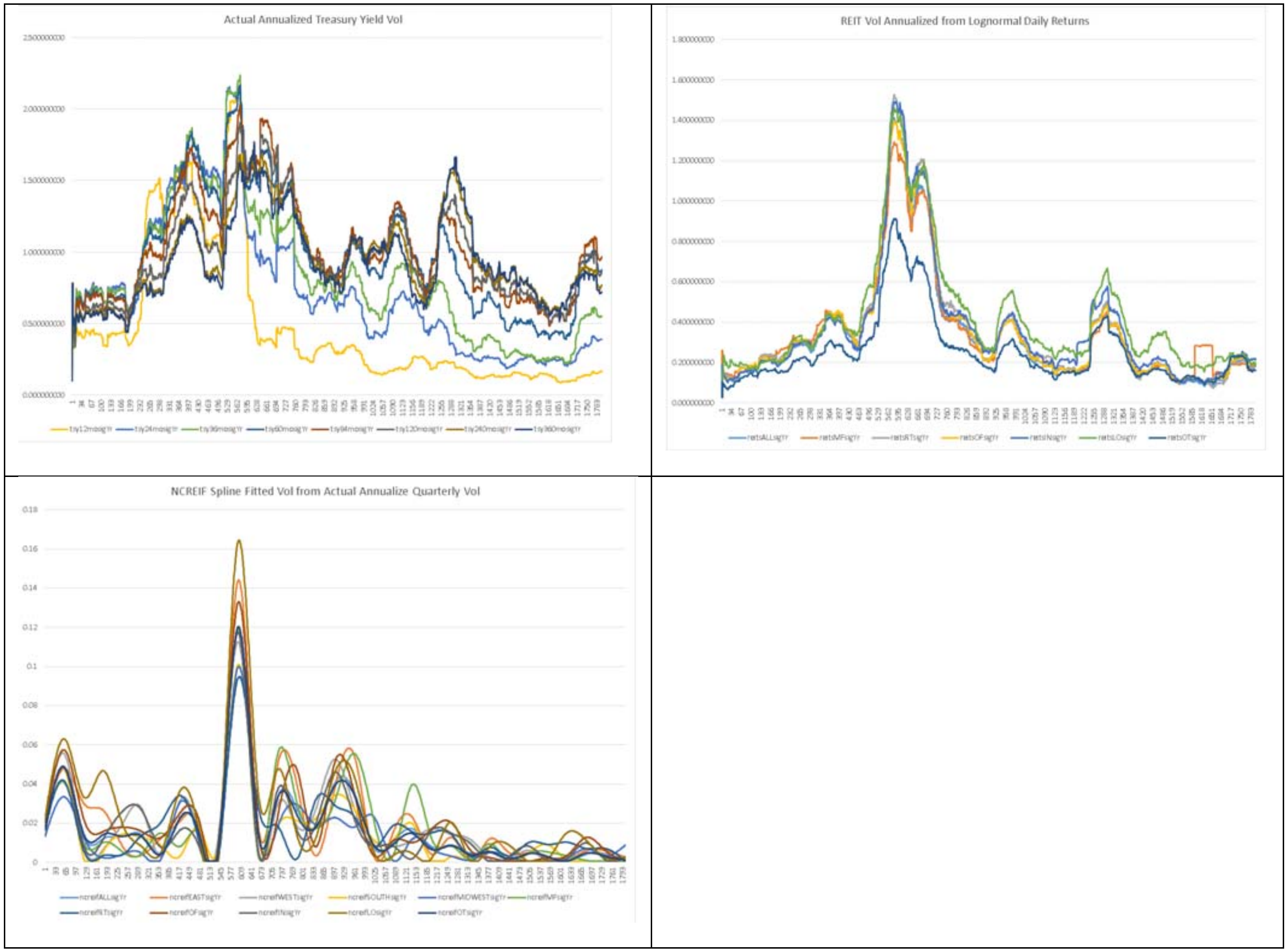


Figure 64a: Correlation to VarCovar (partial, 27 x 10 (of 27), 1 day snapshot)

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> corrmtrxrdcdform
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	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]
[1,]	1.00000000	0.962107883	0.97276721	0.97655936	0.940814217	0.86644582	0.88502858	0.018629671	0.15066525	0.2803891957
[2,]	0.96210788	1.000000000	0.93295372	0.94708199	0.902085870	0.80819080	0.84115394	0.007689710	0.12534588	0.2084322986
[3,]	0.97276721	0.932953718	1.00000000	0.96477198	0.925316242	0.83777508	0.83985338	-0.027926857	0.10714519	0.2590630732
[4,]	0.97655936	0.947081985	0.96477198	1.00000000	0.912226195	0.84013271	0.87985742	-0.042583714	0.09977400	0.2567703528
[5,]	0.94081422	0.902085870	0.92531624	0.91222620	1.000000000	0.75902056	0.78289844	-0.066711215	0.09947134	0.2157904332
[6,]	0.86644582	0.808190804	0.83777508	0.84013271	0.759020562	1.00000000	0.81971950	0.067581787	0.21100983	0.2692332769
[7,]	0.88502858	0.841153939	0.83985338	0.87985742	0.782898436	0.81971950	1.00000000	0.117440695	0.15583608	0.2634485523
[8,]	0.01862967	0.007689710	-0.02792686	-0.04258371	-0.066711215	0.06758179	0.11744069	1.000000000	0.34011769	0.2726594207
[9,]	0.15066525	0.125345883	0.10714519	0.09977400	0.099471338	0.21100983	0.15583608	0.340117692	1.00000000	0.6222864177
[10,]	0.28038920	0.208432299	0.25906307	0.25677035	0.215790433	0.26923328	0.26344855	0.272659421	0.62228642	1.0000000000
[11,]	0.55622559	0.525158799	0.55760382	0.52842747	0.484466075	0.49005171	0.48445967	0.144290077	0.57148432	0.6769843862
[12,]	0.53656822	0.497797394	0.54974956	0.51500624	0.472167058	0.46184538	0.45520197	0.063159284	0.52739702	0.6297475850
[13,]	0.51805766	0.474689161	0.52478443	0.49283135	0.460589577	0.44508380	0.44961080	0.063618524	0.45146732	0.5875960394
[14,]	0.50260962	0.465362695	0.50941290	0.48799487	0.444535606	0.43447319	0.44392934	0.046299365	0.38370616	0.5317571746
[15,]	0.46518239	0.429421251	0.46893260	0.44799635	0.413342158	0.40296380	0.40494180	0.065134388	0.37157866	0.4935327847
[16,]	0.39354558	0.348234222	0.40292487	0.38232756	0.341126150	0.37189481	0.36212336	-0.014342973	0.33239629	0.4237584737
[17,]	-0.02343541	-0.003462480	-0.02570960	-0.02797202	-0.016993009	-0.05388285	-0.03817904	-0.075415255	0.05514446	0.0408356488
[18,]	0.07173349	0.089361037	0.09217091	0.07865500	0.010416732	0.09012336	0.15811991	0.020544486	-0.11433645	0.0045377981
[19,]	-0.04570753	-0.031359667	-0.05186602	-0.04779629	-0.023684776	-0.08104352	-0.08637989	-0.091307962	0.07560382	0.0426631725
[20,]	-0.03757104	-0.025110827	-0.04102776	-0.04297909	-0.021524678	-0.06882118	-0.06415459	-0.071689902	0.07208592	0.0455909638
[21,]	-0.02886908	-0.013856291	-0.02759578	-0.03553153	-0.024115001	-0.05528235	-0.04377715	-0.064054682	0.06590443	0.0530230292
[22,]	-0.04245558	-0.028017497	-0.04151467	-0.04782513	-0.030315708	-0.06862841	-0.06677683	-0.079259419	0.07837645	0.0554253372
[23,]	-0.05801451	-0.050872674	-0.06574182	-0.06282328	-0.027138930	-0.09401023	-0.10894366	-0.085655775	0.08674652	0.0420810108
[24,]	0.05725896	0.090193932	0.07323963	0.06465198	0.000817093	0.07389826	0.13105870	0.002518585	-0.10041669	0.0006838918
[25,]	-0.05998339	-0.045727381	-0.06474285	-0.06063616	-0.032059097	-0.08505248	-0.10460360	-0.098956964	0.07106392	0.0268673535
[26,]	-0.02202567	-0.007849578	-0.02708178	-0.01712321	-0.007979431	-0.03916834	-0.06406674	-0.094778984	0.06238893	0.0265730843
[27,]	-0.01812252	-0.005037435	-0.01747435	-0.02088030	-0.008443465	-0.04208227	-0.04254625	-0.083645552	0.05227577	0.0478762441

Figure 64b: VarCovar Matrix, , based on Correlations and Standard Devs (partial, 27 x 10 (of 27), 1 day snapshot)

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	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]
[1,]	2.497710e-03	2.361225e-03	2.800832e-03	2.600194e-03	3.163755e-03	2.916215e-03	1.672792e-03	2.135775e-04	8.363629e-04	1.097241e-03
[2,]	2.361225e-03	2.411488e-03	2.639428e-03	2.477800e-03	2.980701e-03	2.672782e-03	1.562182e-03	8.662272e-05	6.836964e-04	8.014520e-04
[3,]	2.800832e-03	2.639428e-03	3.319053e-03	2.961201e-03	3.586950e-03	3.250437e-03	1.829887e-03	-3.690698e-04	6.856311e-04	1.168645e-03
[4,]	2.600194e-03	2.477800e-03	2.961201e-03	2.838391e-03	3.270142e-03	3.014332e-03	1.772810e-03	-5.204260e-04	5.904242e-04	1.071152e-03
[5,]	3.163755e-03	2.980701e-03	3.586950e-03	3.270142e-03	4.527474e-03	3.439448e-03	1.992265e-03	-1.029689e-03	7.434241e-04	1.136921e-03
[6,]	2.916215e-03	2.672782e-03	3.250437e-03	3.014332e-03	3.439448e-03	4.535387e-03	2.087787e-03	1.044038e-03	1.578413e-03	1.419730e-03
[7,]	1.672792e-03	1.562182e-03	1.829887e-03	1.772810e-03	1.992265e-03	2.087787e-03	1.430301e-03	1.018853e-03	6.546247e-04	7.801524e-04
[8,]	2.135775e-04	8.662272e-05	-3.690698e-04	-5.204260e-04	-1.029689e-03	1.044038e-03	1.018853e-03	5.262095e-02	8.666011e-03	4.897447e-03
[9,]	8.363629e-04	6.836964e-04	6.856311e-04	5.904242e-04	7.434241e-04	1.578413e-03	6.546247e-04	8.666011e-03	1.233734e-02	5.412164e-03
[10,]	1.097241e-03	8.014520e-04	1.168645e-03	1.071152e-03	1.136921e-03	1.419730e-03	7.801524e-04	4.897447e-03	5.412164e-03	6.131121e-03
[11,]	2.313442e-03	2.146199e-03	2.673435e-03	2.342924e-03	2.712864e-03	2.746539e-03	1.524783e-03	2.754562e-03	5.282649e-03	4.411490e-03
[12,]	1.777525e-03	1.620373e-03	2.099384e-03	1.818731e-03	2.105928e-03	2.061691e-03	1.141137e-03	9.603651e-04	3.883007e-03	3.268559e-03
[13,]	1.343714e-03	1.209789e-03	1.569083e-03	1.362675e-03	1.608422e-03	1.555632e-03	8.824869e-04	7.573920e-04	2.602524e-03	2.387848e-03
[14,]	1.111450e-03	1.011166e-03	1.298570e-03	1.150375e-03	1.323496e-03	1.294668e-03	7.428750e-04	4.699401e-04	1.885809e-03	1.842347e-03
[15,]	8.657479e-04	7.852777e-04	1.006039e-03	8.888076e-04	1.035702e-03	1.010579e-03	5.703002e-04	5.563997e-04	1.536946e-03	1.439074e-03
[16,]	6.044385e-04	5.255332e-04	7.133741e-04	6.259762e-04	7.053898e-04	7.696857e-04	4.208782e-04	-1.011125e-04	1.134627e-03	1.019705e-03
[17,]	-1.159205e-07	-1.682855e-08	-1.465950e-07	-1.474948e-07	-1.131658e-07	-3.591489e-07	-1.429076e-07	-1.712203e-06	6.062186e-07	3.164653e-07
[18,]	2.022567e-07	2.475715e-07	2.995787e-07	2.364136e-07	3.954299e-08	3.424164e-07	3.373730e-07	2.658793e-07	-7.164821e-07	2.004588e-08
[19,]	-3.336446e-07	-2.249258e-07	-4.364308e-07	-3.719252e-07	-2.327677e-07	-7.971697e-07	-4.771465e-07	-3.059238e-06	1.226534e-06	4.879198e-07
[20,]	-2.720681e-07	-1.786722e-07	-3.424824e-07	-3.317773e-07	-2.098544e-07	-6.715567e-07	-3.515563e-07	-2.382817e-06	1.160151e-06	5.172519e-07
[21,]	-1.993377e-07	-9.401020e-08	-2.196521e-07	-2.615384e-07	-2.241820e-07	-5.143741e-07	-2.287424e-07	-2.030092e-06	1.011372e-06	5.736143e-07
[22,]	-2.489133e-07	-1.614040e-07	-2.805764e-07	-2.989060e-07	-2.392975e-07	-5.421925e-07	-2.962661e-07	-2.132911e-06	1.021266e-06	5.091207e-07
[23,]	-6.643416e-07	-5.724149e-07	-8.678261e-07	-7.669032e-07	-4.184124e-07	-1.450662e-06	-9.440598e-07	-4.502149e-06	2.207733e-06	7.549878e-07
[24,]	2.294575e-07	3.551465e-07	3.383304e-07	2.761885e-07	4.408463e-09	3.990516e-07	3.974361e-07	4.632588e-08	-8.943434e-07	4.293832e-09
[25,]	-2.777008e-07	-2.080147e-07	-3.455207e-07	-2.992560e-07	-1.998271e-07	-5.306024e-07	-3.664675e-07	-2.102814e-06	7.311979e-07	1.948812e-07
[26,]	-4.690199e-08	-1.642404e-08	-6.647761e-08	-3.886981e-08	-2.287658e-08	-1.123915e-07	-1.032375e-07	-9.263658e-07	2.952632e-07	8.865496e-08
[27,]	-9.130109e-08	-2.493667e-08	-1.014833e-07	-1.121397e-07	-5.727104e-08	-2.856884e-07	-1.622039e-07	-1.934232e-06	5.853256e-07	3.778996e-07

Figure 64c: Matrix C, the Cholesky Decomposition of Matrix , Σ , (partial, 27 x 10 (of 27), 1 day snapshot)

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> Cmtrxrdcdform
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	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]
[1,]	0.04997709	0.04724614	0.0560423125	0.052027713	0.063304105	0.0583510353	0.033471178	0.004273508	0.0167349260	0.021954887
[2,]	0.00000000	0.01338993	-0.0006239881	0.001470601	-0.000759781	-0.0062792449	-0.001434029	-0.008609753	-0.0079884044	-0.017612610
[3,]	0.00000000	0.00000000	0.0133387900	0.003475979	0.002906364	-0.0017699038	-0.003509307	-0.046026623	-0.0192834256	-0.005453853
[4,]	0.00000000	0.00000000	0.0000000000	0.010828791	-0.002993020	-0.0005681266	0.004219115	-0.052648327	-0.0186058811	-0.002424320
[5,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.022407193	-0.0114132874	-0.004679756	-0.059381324	-0.0143560132	-0.011500807
[6,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.000000000	0.0309418149	0.002213143	-0.001567407	0.0110918366	-0.003692291
[7,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.015843308	0.041004942	0.0001344338	-0.002178881
[8,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000	0.205981207	0.0282452496	0.017836328
[9,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000	0.000000000	0.1007397523	0.040958292
[10,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.056186407
[11,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000
[12,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000
[13,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000
[14,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000
[15,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000
[16,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000
[17,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000
[18,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000
[19,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000
[20,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000
[21,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000
[22,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000
[23,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000
[24,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000
[25,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000
[26,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000
[27,]	0.00000000	0.00000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000	0.000000000	0.0000000000	0.000000000

Figure 65: Time Snapshots of All REITs vs. All NCREIF over 3 historical dates

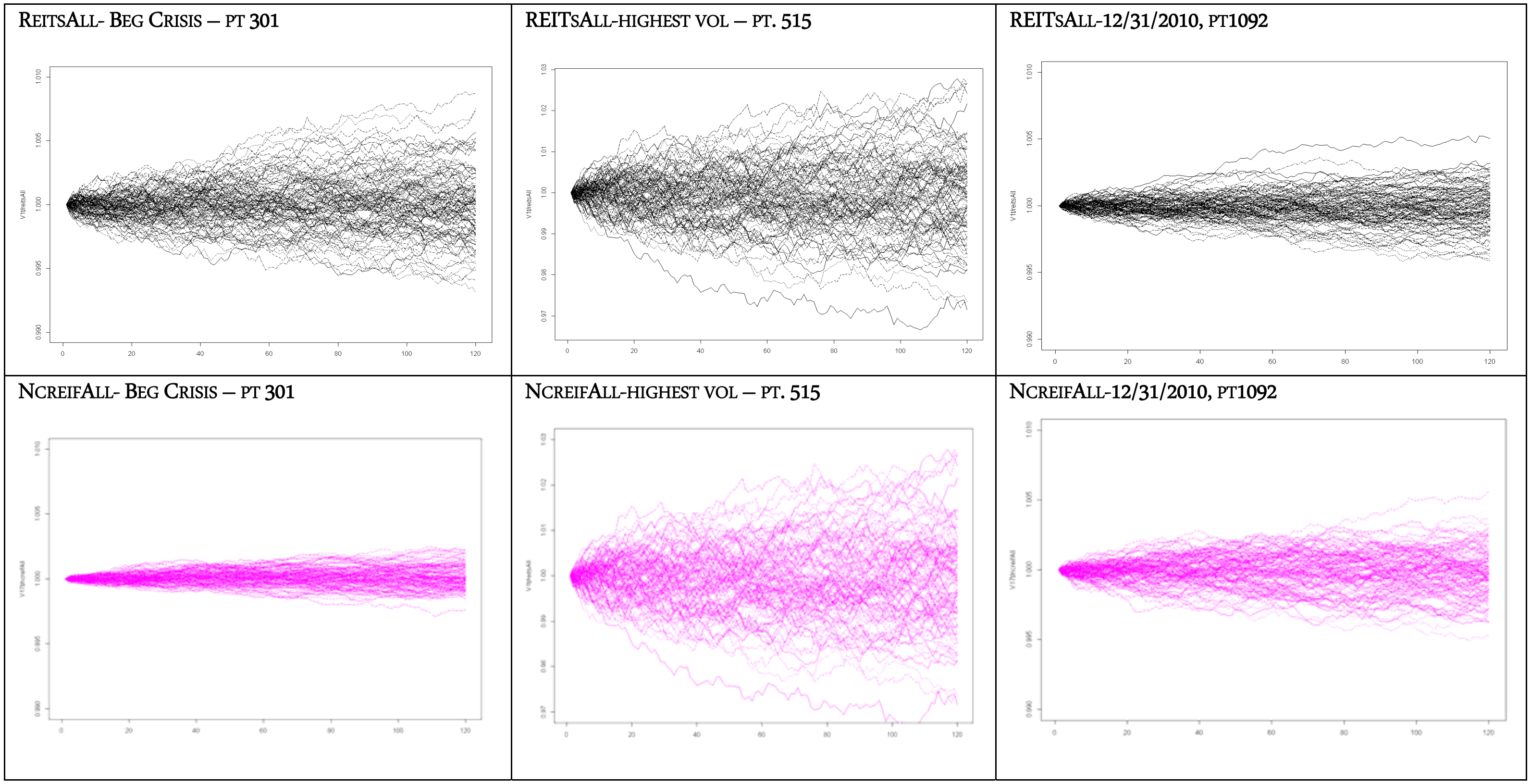


Figure 66: Loan State Transitions

	Transition to the Same or Better State	Transition to a Worse State
Current (0 days delinquent)	99%	1%
Delinquent (30–59 days)	62%	38%
Delinquent (60–89 days)	36%	64%
Delinquent (90+ days)	61%	39%

Notes: This table gives the monthly transition frequencies of moving from the present state (column 1) to the same or better state (column 2) versus a worse state (column 3), for all commercial mortgage loans over the time period June 1988 to June 2004.

Figure 67: Commercial loans in database from June 1998 to June 2004 (from JCY 2008)

	Non-CTLs		CTLs	
	Fixed	Floating	Fixed	Floating
Prepaid	8989	2960	102	4
Default	2153	130	56	1
Total	94011	7198	1358	10

Notes: The loans are partitioned into those that are credit tenant leases (CTLs), fixed- versus floating-rate, prepaid and defaulted. There are a total of 102,577 loans in the sample.

Figure 68: Intensity process parameter estimates based on monthly observations (June 1998 to May 2005, Jarrow, etal 2008)

	FixedRate - Current		FixedRate - Dlq		FixedRate - Def	
Intercept	8.381950	0.158490	-8.381950	0.158490	-10.96945	0.62179
hi(t)	-0.000370	0.001160	0.000370	0.001160	0.00120	0.00361
P1	1.574940	0.039860	-1.574940	0.039860	0.61829	0.12335
P2	-0.948430	0.039080	0.948430	0.039080	-0.31815	0.13029
P3	0.948300	0.024580	-0.948300	0.024580	-0.05465	0.07467
P4	1.318660	0.033980	-1.318660	0.033980	0.99022	0.10278
P5	-1.396210	0.029570	1.396210	0.029570	-1.84097	0.14846
R1	-3.704560	0.079460	3.704560	0.079460	0.38011	0.21373
R2	-3.034860	0.080140	3.034860	0.080140	0.57580	0.21331
R3	-3.308650	0.080930	3.308650	0.080930	0.27483	0.21869
R4	-2.619160	0.080490	2.619160	0.080490	0.10622	0.21279
R5	-2.289500	0.082310	2.289500	0.082310	-0.29227	0.21897
R6	-3.809760	0.078550	3.809760	0.078550	0.41256	0.21108
R7	-3.144090	0.079140	3.144090	0.079140	0.72144	0.21030
R8	-3.767120	0.079520	3.767120	0.079520	0.69119	0.22325
age	0.459910	0.031540	-0.459910	0.031540	3.24458	0.11491
dlqstatus	[redacted]		[redacted]		5.38979	0.05646
foreindex	-7.715300	0.755990	7.715300	0.755990	1.73865	-6.54448
noi	0.000000	0.000000	0.000000	0.000000	0.00000	0.00000
Pen	0.000360	0.000010	-0.000360	0.000010	0.00109	0.00006
origloanbal	0.338350	0.006220	-0.338350	0.006220	0.05524	0.02320
Hi(t)	-0.004550	0.000230	0.004550	0.000230	-0.00954	0.00107
dscr	0.005910	0.003080	-0.005910	0.003080	-0.42866	0.06687
ltv	-0.019760	0.000440	0.019760	0.000440	0.01134	0.00123
spot	0.663370	0.011200	-0.663370	0.011200	0.21948	0.04018
H(t)	-0.009940	0.000510	0.009940	0.000510	-0.00924	0.00194
f(t,10)-r(t)	0.272950	0.014420	-0.272950	0.014420	0.32873	0.05438
wac	-0.384940	0.008970	0.384940	0.008970	0.11301	0.03162
coupon_spread	-0.459250	0.010747	0.459250	0.010747	-0.05213	0.03074

Figure 69: Reduced Form Fair Value across classes and Composite view

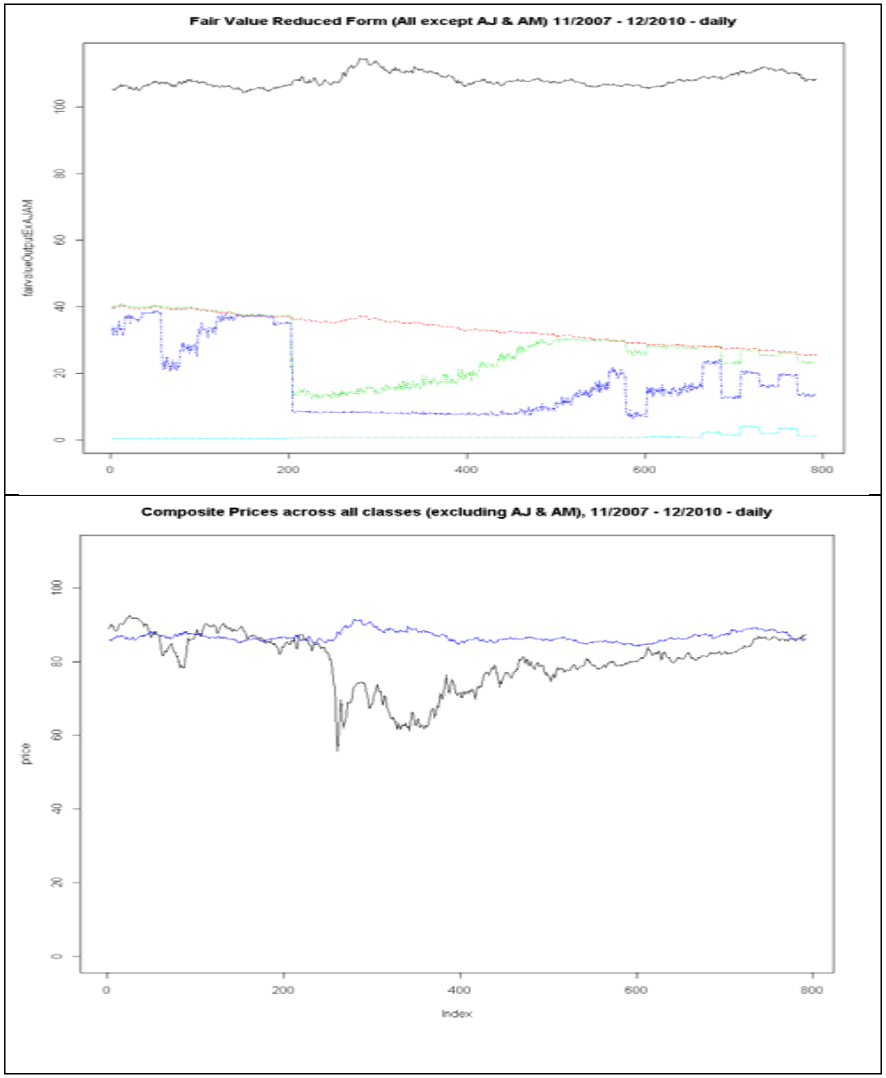


Figure 70: Reduced Form Theta across different classes and Composite view

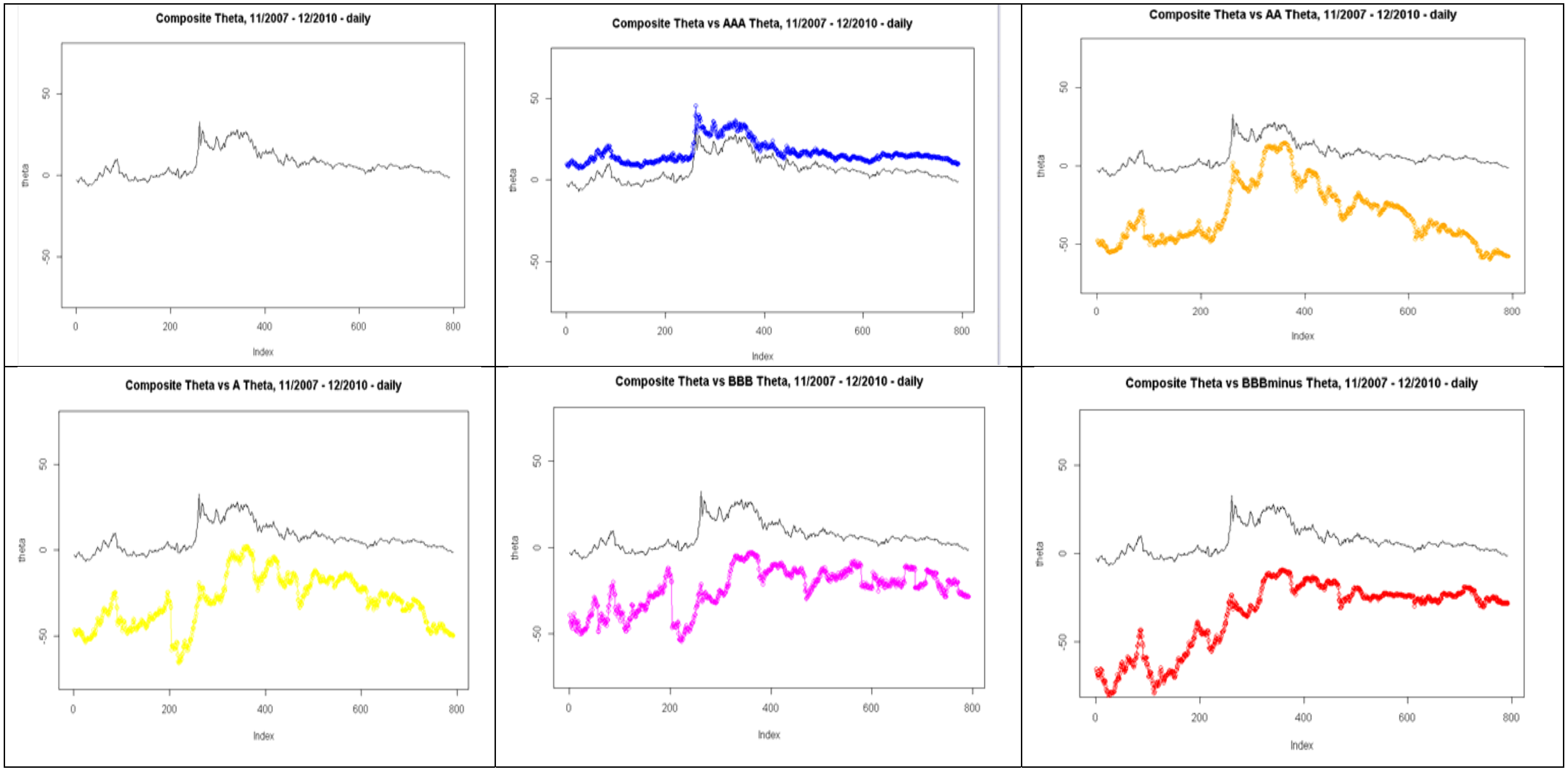


Figure 71: Merton Pdefs for REITs (6/2007 – 3/2014)

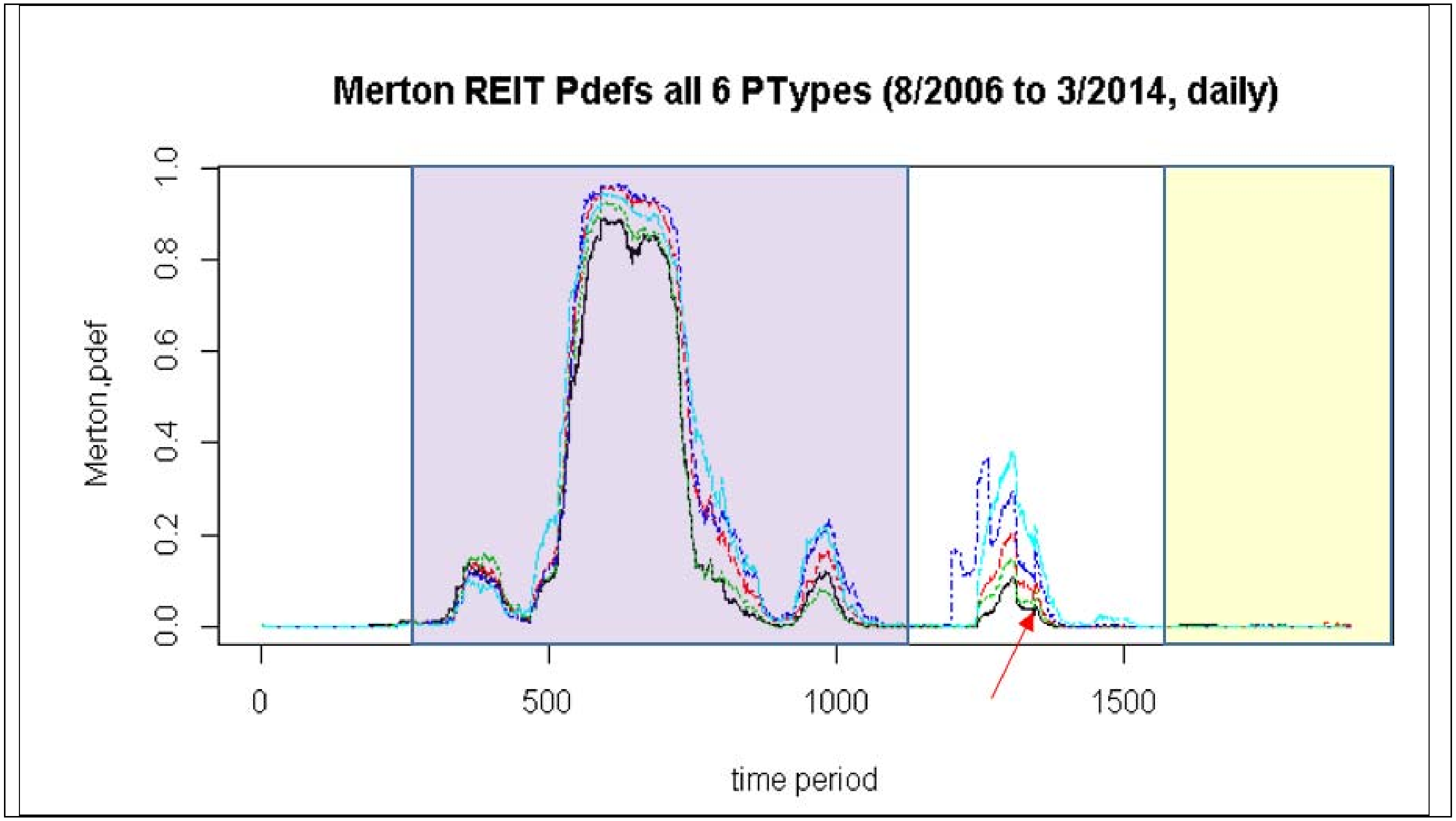


Figure 72: CMBX 6 Capital Structure

Class	Size	% of Deal	Subordination	Coupon
AAA	$ 9,244,254,655.33	69.50%	30.50%	2.50%
AS	$ 1,064,086,866.80	8.00%	22.50%	3.00%
AA	$ 798,065,150.10	6.00%	16.50%	3.50%
A	$ 532,043,433.40	4.00%	12.50%	4.00%
BBBminus	$ 665,054,291.75	5.00%	7.50%	5.00%
BB	$ 266,021,716.70	2.00%	5.50%	7.00%
UR	$ 731,559,720.93	5.50%	0.00%	NA
Total	$13,301,085,835.00	100.00%		

Figure 73: Model 1 Merton Plot Composite

Composite Prices (panel A) and Composite Theta (panel b), Pure Merton JFE Series 6 Sample (1/28/2013 – 3/7/2014, daily)

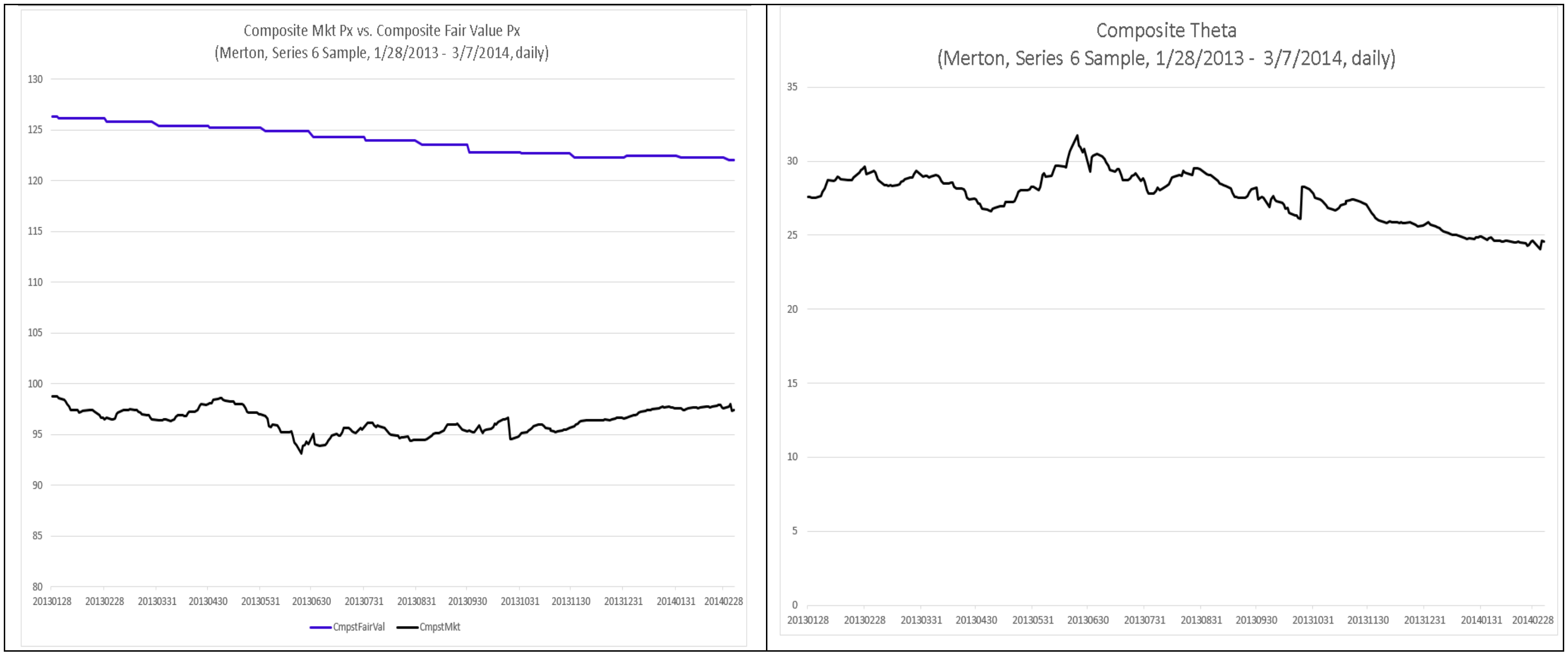


Figure 74: Model 1 – Merton Plots Individual Thetas v. Composite

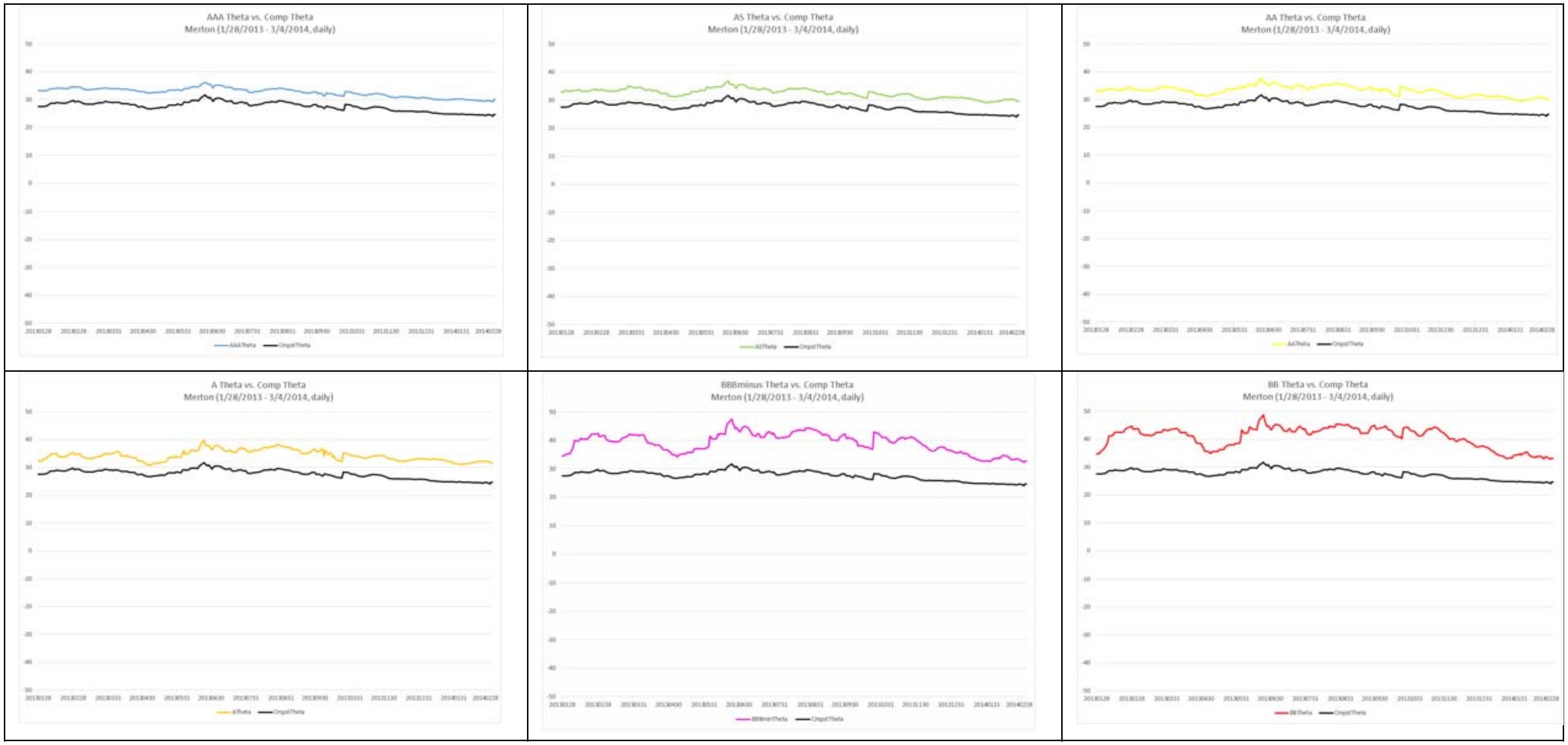


Figure 75 – Model 2 Composite Theta

Composite Prices (panel A) and Composite Theta (panel b), NO CFlow JFE Series 6 Sample (1/28/2013 – 3/7/2014, daily)

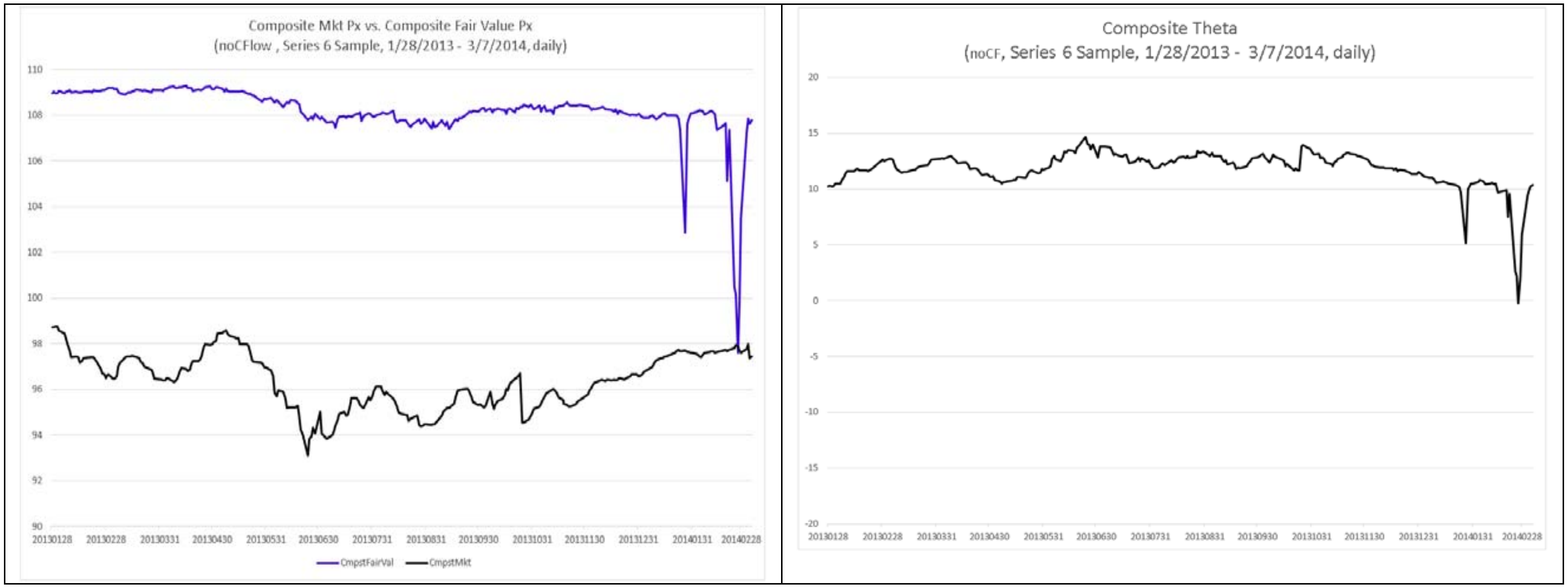


Figure 76 – Model 2 Tranchewise Theta

Composite Prices (panel A) and Composite Theta (panel b), NO CFlow JFE Series 6 Sample (1/28/2013 – 3/7/2014, daily)

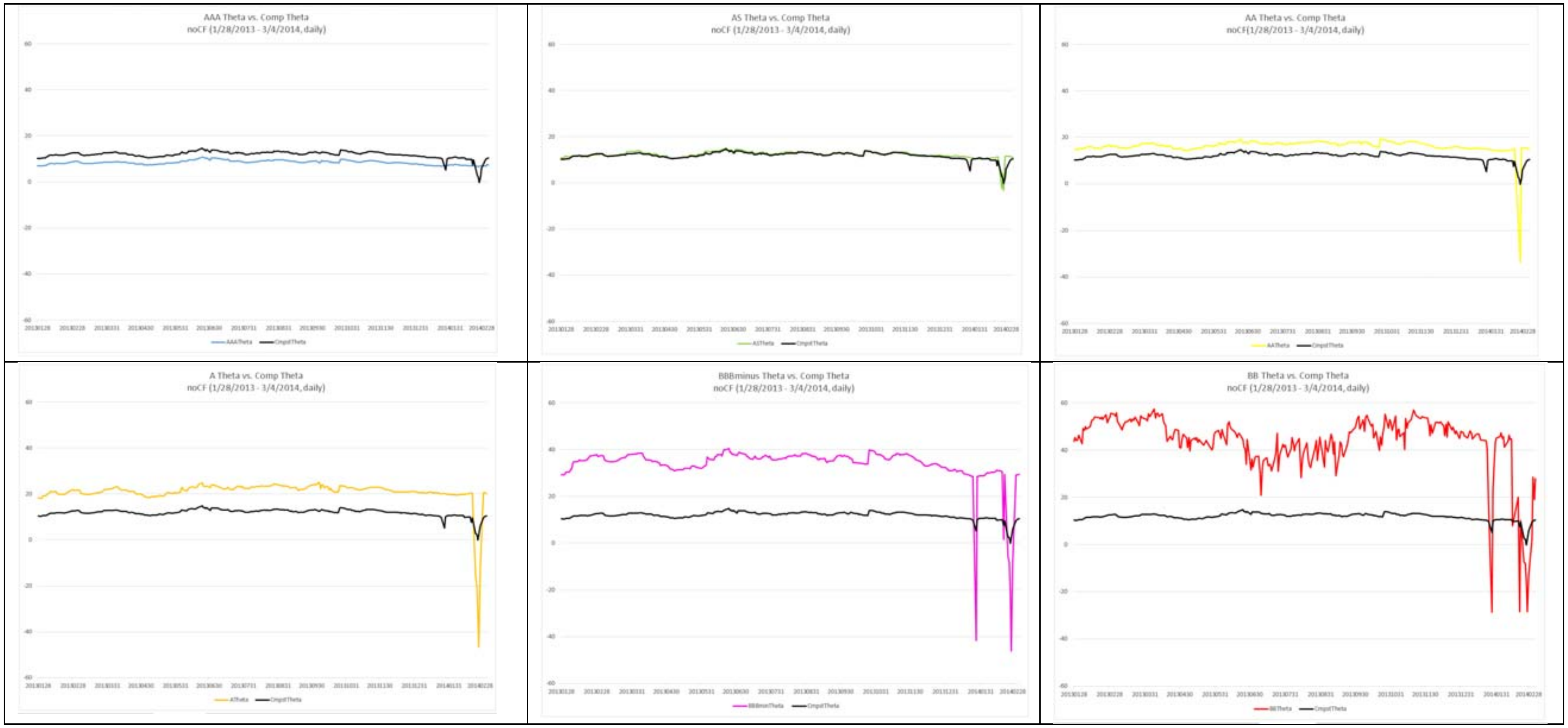
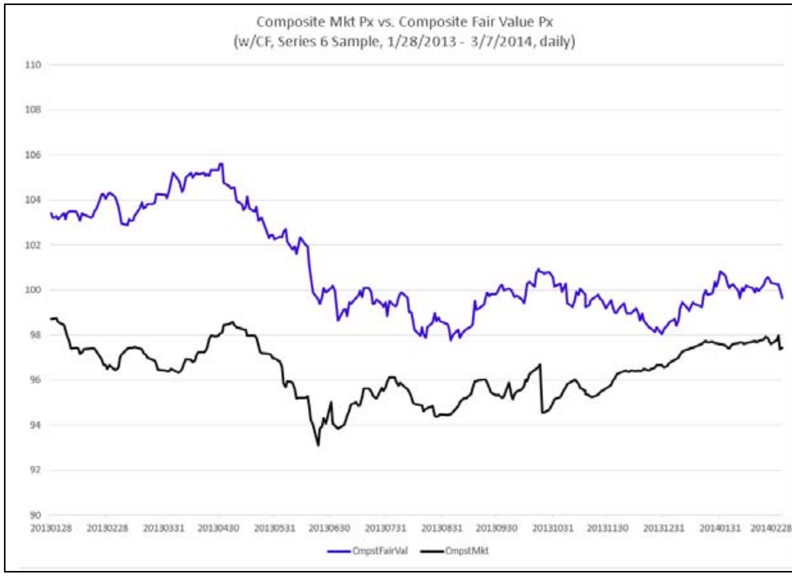


Figure 77 – Model 3 Composite Theta

Composite Prices (panel A) and Composite Theta (panel b), W/CFlow JFE Series 6 Sample (1/28/2013 – 3/7/2014, daily)



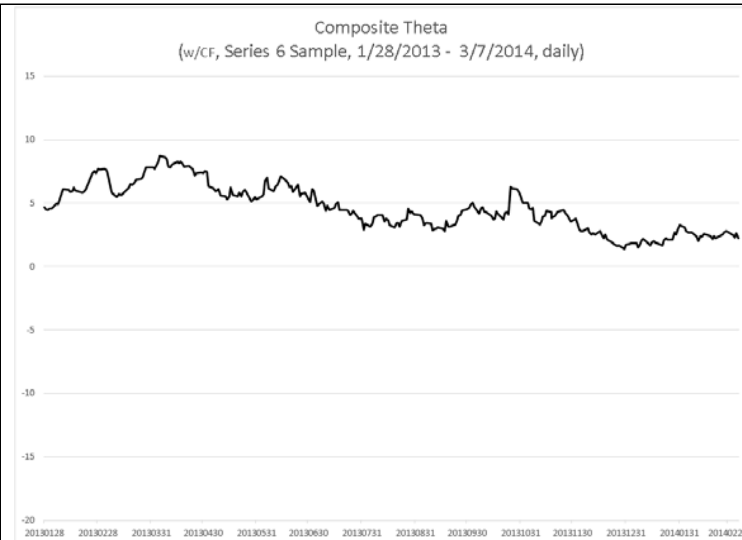


Figure 78 – Model 3 Tranchewise Theta

Composite Prices (panel A) and Composite Theta (panel b), W/CFlow JFE Series 6 Sample (1/28/2013 – 3/7/2014, daily)

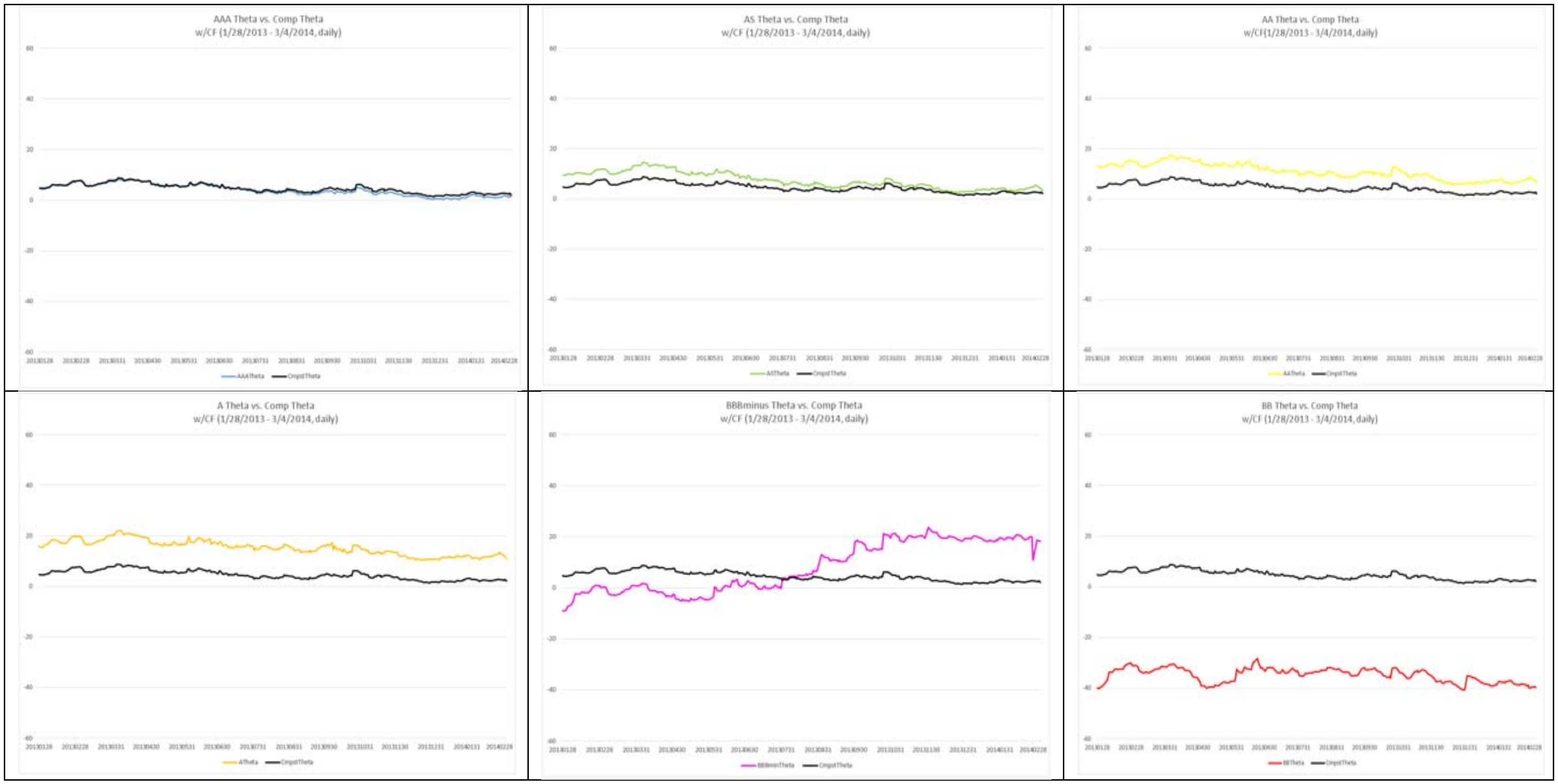


Figure 79 – Model 4 Composite Theta

Composite Prices (panel A) and Composite Theta (panel b), Reduced Form Series 6 Sample (1/28/2013 – 3/7/2014, daily)

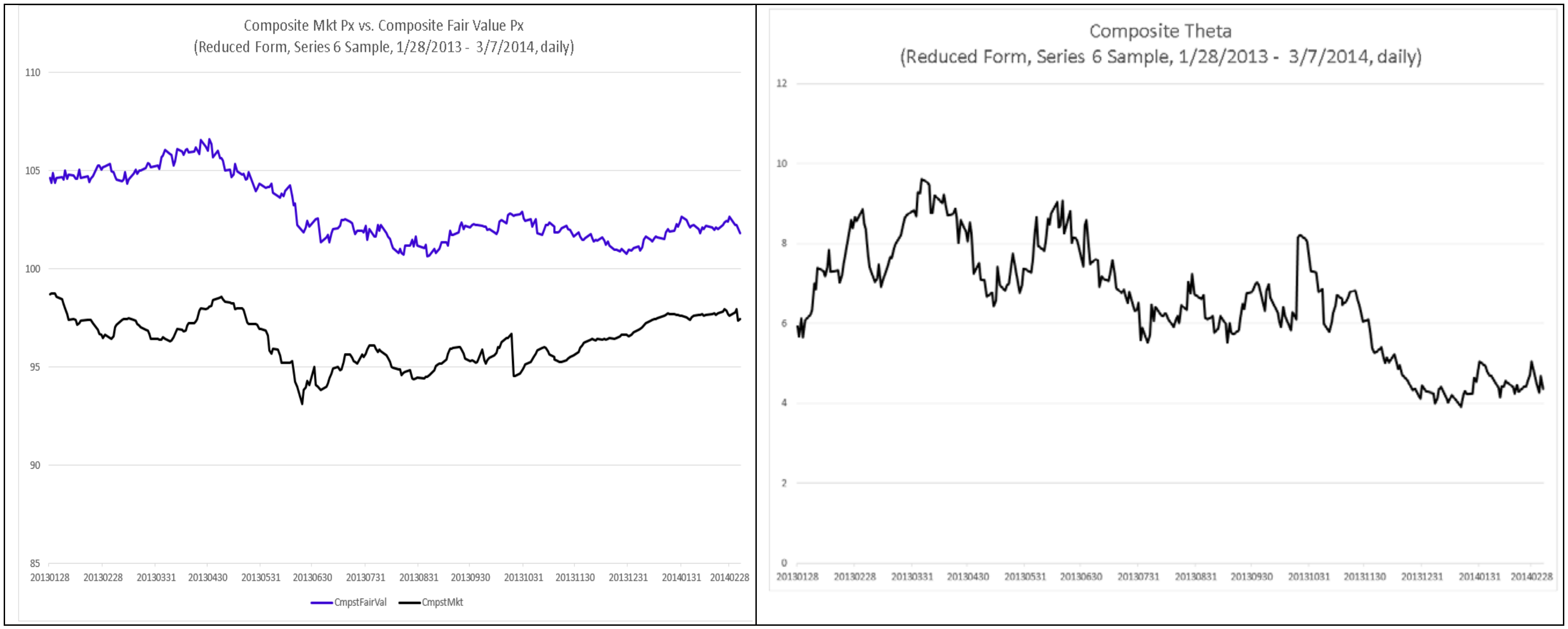


Figure 80 Model 4 Tranchewise Theta

Composite Prices (panel A) and Composite Theta (panel b), Reduced Form Series 6 Sample (1/28/2013 – 3/7/2014, daily)

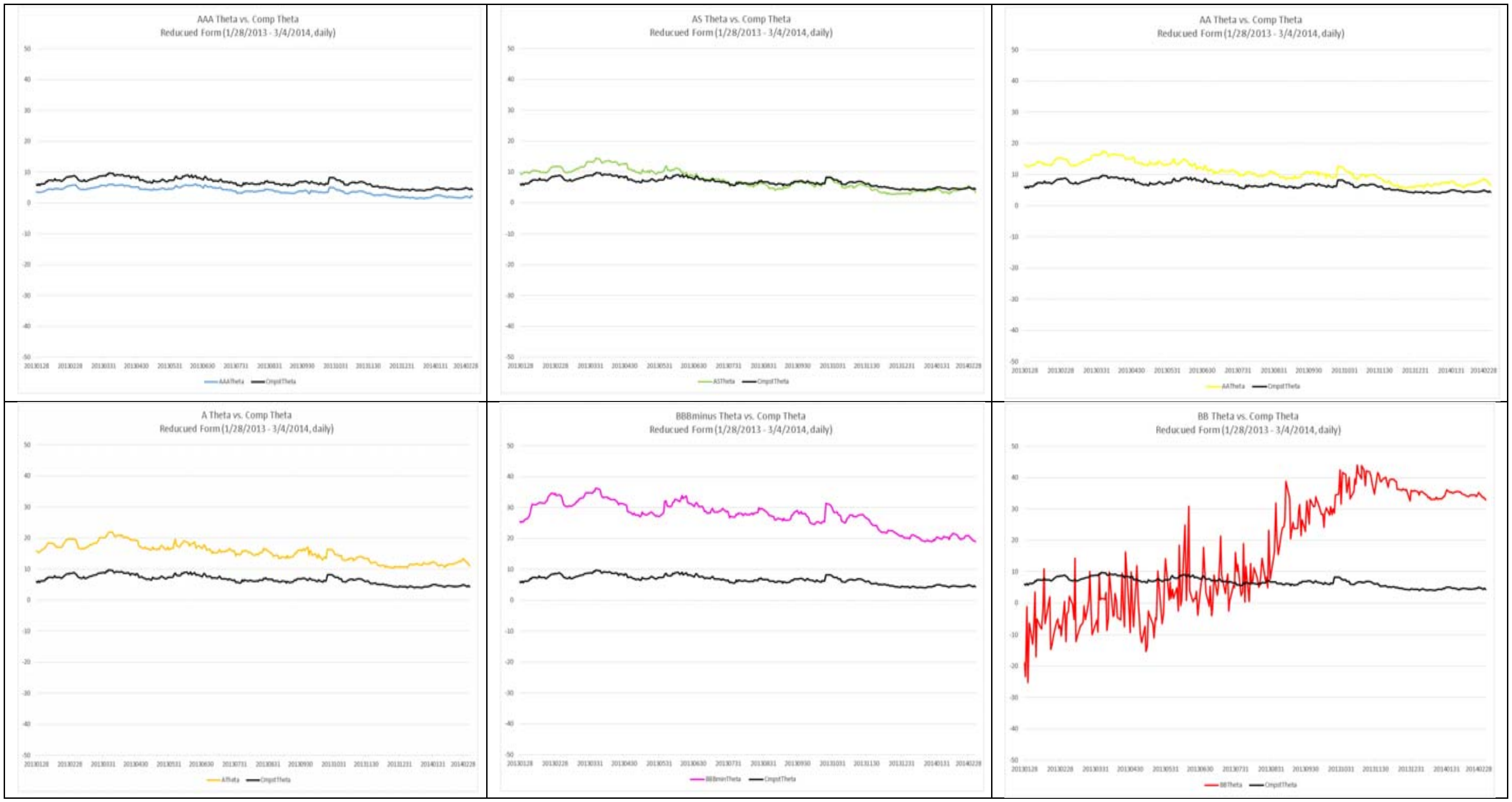


Figure 81: Composite Thetas, All Models

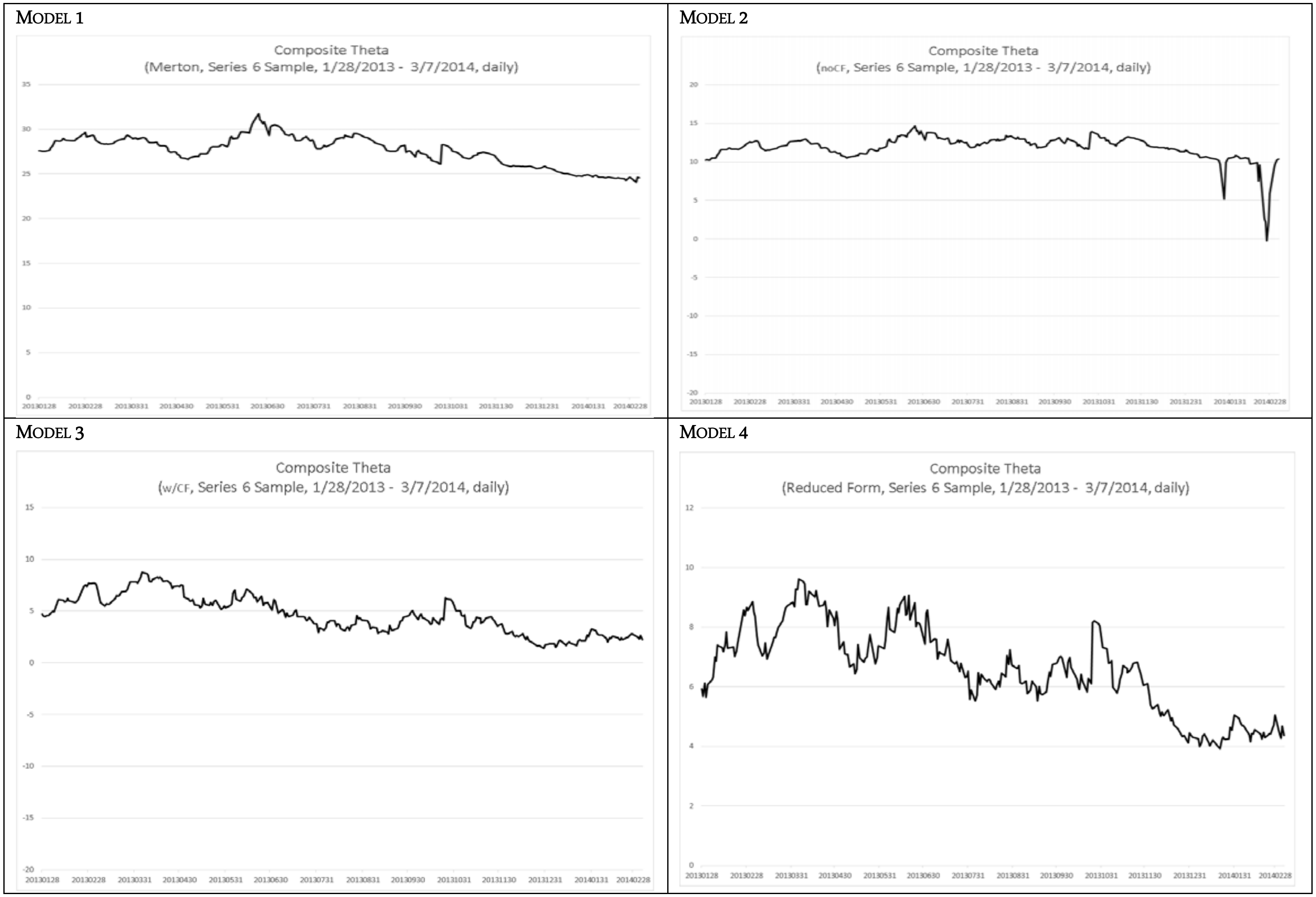


Figure 82: Efficiency OLS and Quantile Regression Results - Crisis

	Crisis			
	OLS		Quantile	
Model	R-SQ	p-val	R-SQ	p-val
Model 2	.68	0.000	0.38	0.000
Model 3	.47	0.000	0.24	0.000
Model 1	.26	0.000	0.12	0.000
Model 4	.18	0.000	0.06	0.000

Figure 83: Efficiency OLS and Quantile Regression Results -Recovery

	Recovery			
	OLS		Quantile	
Model	R-SQ	p-value	R-SQ	p-value
Model 3	0.310	0.000	0.195	0.000
Model 4	0.271	0.000	0.168	0.000
Model 1	0.027	0.006	0.024	0.003
Model 2	0.003	0.344	0.017	0.000

Figure 84: Monthly and Quarterly lognormal horizon return across models (Crisis)

Monthly Returns (11/2007 - 6/2010)

regperiod	mthlnrtMod1	mthlnrtMod2	mthlnrtMod3	mthlnrtMod4	mnthlnrtSector
1	0	0	0	0	0
22	(0.05)	0.03	0.01	(0.03)	0.03
43	0.05	(0.04)	0.01	0.04	(0.02)
64	0.04	(0.13)	0.09	0.13	(0.09)
85	0.02	(0.17)	0.04	0.19	(0.05)
106	(0.12)	0.24	(0.02)	(0.27)	0.10
127	0.00	(0.03)	(0.01)	0.02	0.02
148	0.01	(0.00)	0.02	0.00	(0.00)
169	0.06	(0.09)	0.01	0.09	(0.02)
190	0.08	(0.12)	0.02	0.21	(0.03)
211	0.04	(0.08)	0.10	0.07	0.01
232	(0.12)	0.05	(0.11)	0.02	(0.03)
253	0.08	0.16	0.00	0.16	(0.03)
274	0.23	0.15	0.09	0.25	(0.16)
295	(0.13)	0.08	(0.03)	(0.08)	0.02
316	0.20	(0.24)	0.25	0.24	(0.00)
337	0.51	(0.68)	0.70	0.68	(0.12)
358	0.10	(0.23)	0.05	0.23	(0.01)
379	(0.34)	0.42	(0.44)	(0.42)	0.12
400	0.15	0.04	(0.12)	(0.04)	0.03
421	0.14	(0.23)	0.22	0.23	0.02
442	(0.32)	(0.04)	(0.14)	(0.28)	0.06
463	0.09	(0.05)	0.05	0.05	0.00
484	(0.18)	0.26	(0.18)	(0.17)	0.01
505	0.12	(0.24)	0.04	0.20	(0.02)
526	(0.12)	0.10	(0.11)	(0.22)	0.02
547	0.06	0.01	0.06	0.03	0.02
568	(0.07)	(0.05)	0.03	0.11	(0.01)
589	0.00	(0.00)	(0.00)	(0.04)	0.00
610	0.05	0.06	0.02	(0.09)	0.03
631	0.05	0.09	0.03	(0.10)	0.01

Quarterly Returns (11/2007 - 6/2010)

regperiod	qtrlnrtMod1	qtrlnrtMod2	qtrlnrtMod3	qtrlnrtMod4	qtrlnrtSector
1	-	-	-	-	-
64	(0.05)	0.03	0.01	(0.03)	(0.08)
127	0.02	(0.17)	0.04	0.19	0.07
190	0.01	(0.00)	0.02	0.00	(0.05)
253	0.04	(0.08)	0.10	0.07	(0.05)
316	0.23	0.15	0.09	0.25	(0.14)
379	0.51	(0.68)	0.70	0.68	(0.01)
442	0.15	0.04	(0.12)	(0.04)	0.10
505	0.09	(0.05)	0.05	0.05	(0.00)
568	(0.12)	0.10	(0.11)	(0.22)	0.03
631	0.00	(0.00)	(0.00)	(0.04)	0.04

Figure 85: Monthly and Quarterly lognormal horizon return across models (Recovery)

Monthly Trading Returns - Recovery (1/2013 - 3/2014)

regperiod	mlnrtMod1	mlnrtMod2	mlnrtMod3	mlnrtMod4	mlnrtLongSector
1	-	-	-	-	-
22	0.01	(0.09)	0.07	0.02	(0.02)
43	0.01	0.01	(0.01)	(0.01)	(0.00)
64	0.03	0.04	(0.03)	0.01	0.02
85	0.02	0.01	(0.02)	(0.01)	(0.01)
106	0.02	(0.05)	0.03	(0.00)	(0.03)
127	0.01	(0.00)	(0.00)	0.01	0.01
148	0.01	(0.01)	0.01	(0.02)	(0.01)
169	0.01	0.00	0.01	0.02	0.01
190	0.01	(0.00)	0.01	(0.02)	(0.01)
211	0.01	(0.01)	0.02	0.01	0.01
232	(0.03)	0.05	(0.02)	0.05	0.01
253	(0.04)	0.04	(0.00)	0.04	0.01
274	0.01	(0.01)	(0.01)	(0.01)	0.00

Quarterly Returns - copied - all Models (1/2013 - 3/2014)

regperiod	qlnrtMod1	qlnrtMod2	qlnrtMod3	qlnrtMod4	qlnrtLongSector
1	-	-	-	-	-
64	0.01	(0.09)	0.07	0.02	(0.01)
127	0.02	0.01	(0.02)	(0.01)	(0.02)
190	0.01	(0.01)	0.01	(0.02)	(0.01)
253	0.01	(0.01)	0.02	0.01	0.03

Figure 86: Cumulative monthly returns from Theta strategies vs. long only

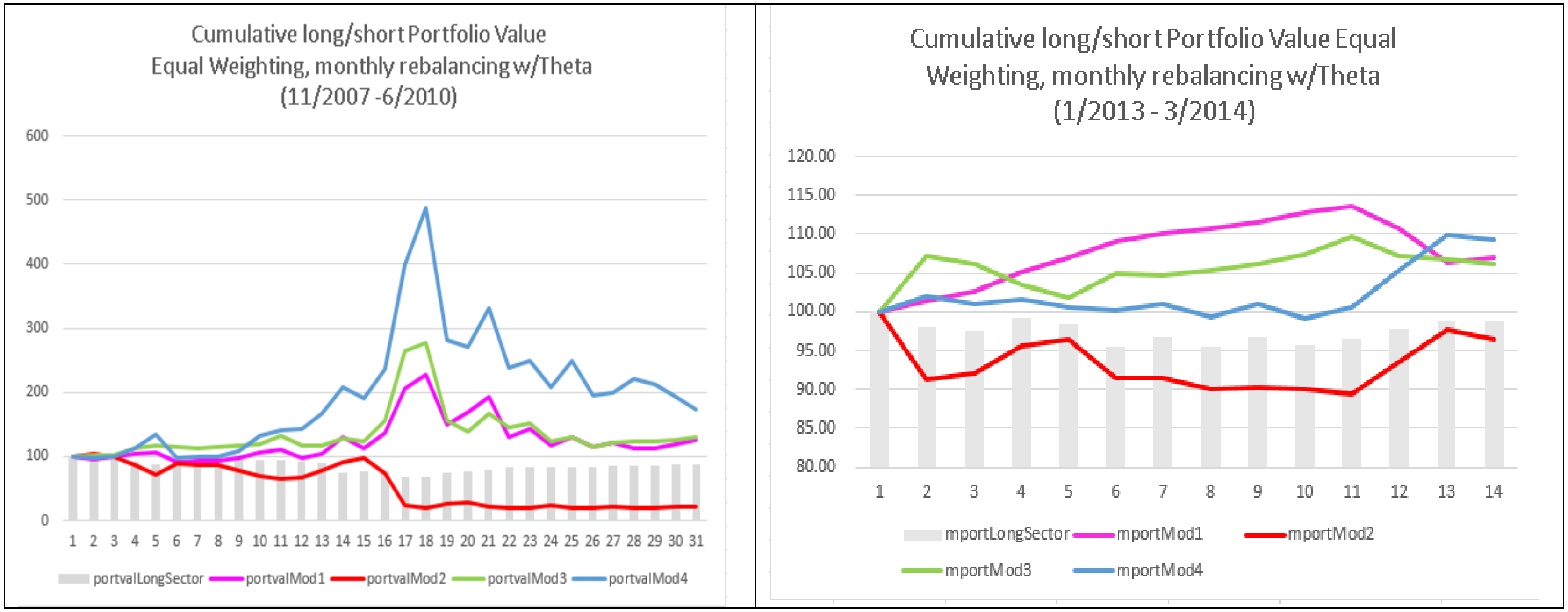


Figure 87: Cumulative quarterly returns from Theta strategies vs. long only

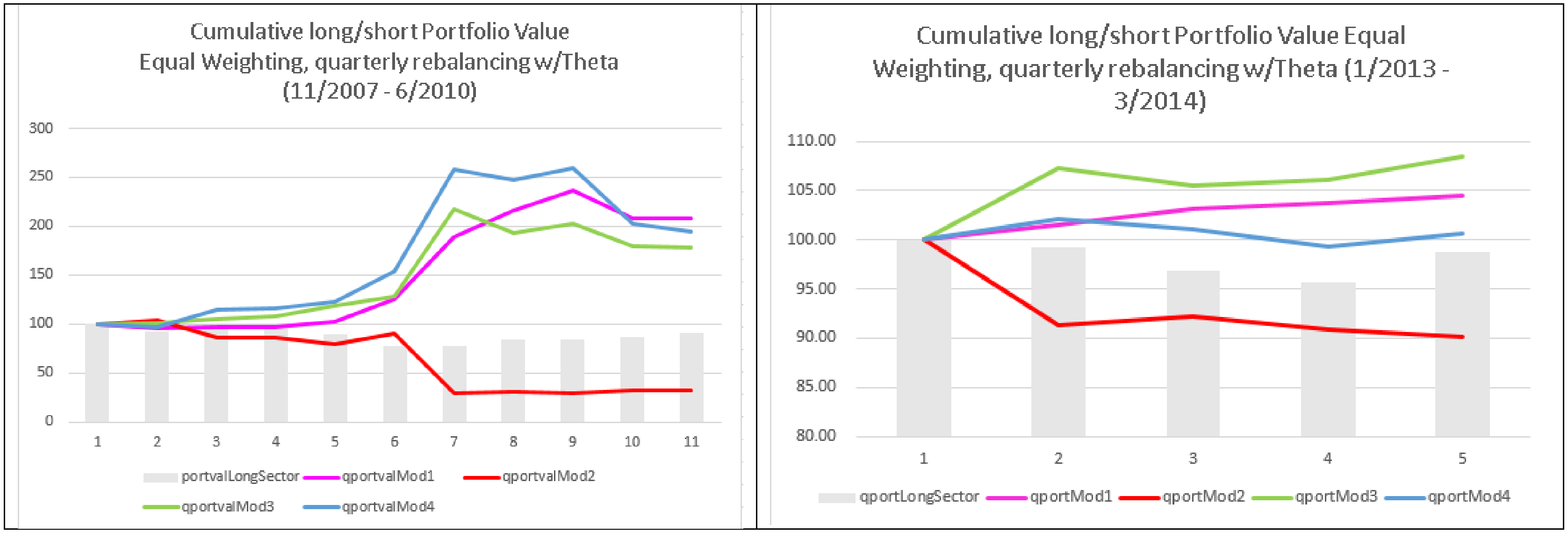


Figure 88: Cumulative daily returns from Theta strategies vs. long only

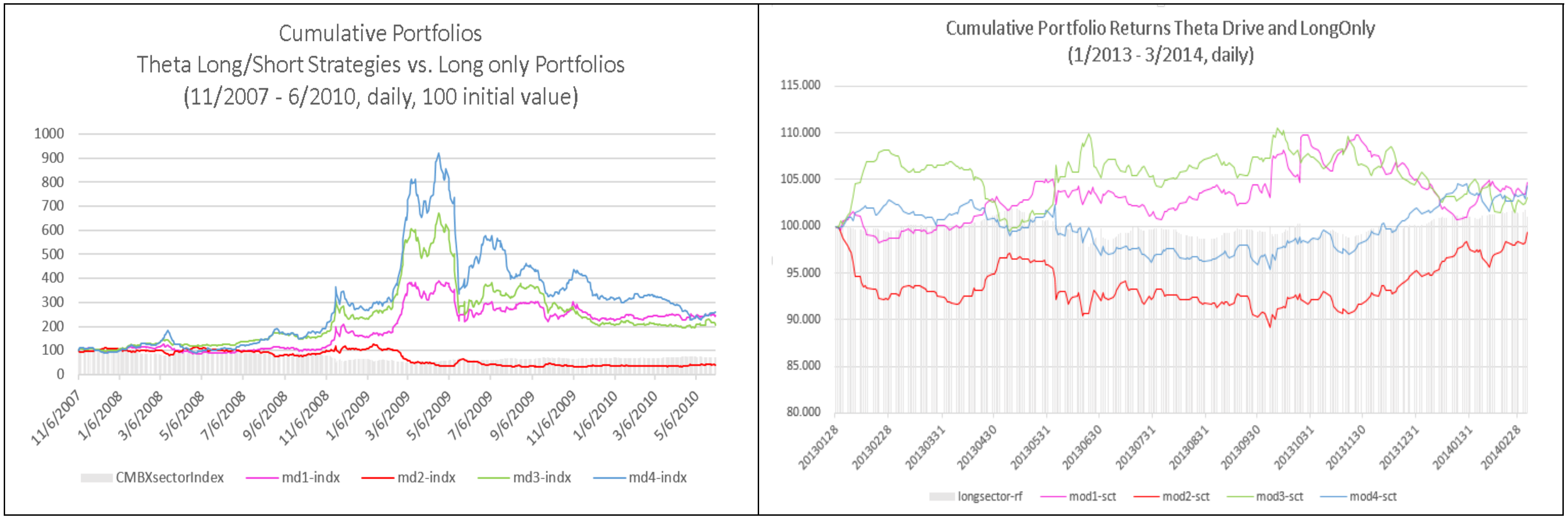


Figure 89: Log daily returns from Theta strategies and long only (w/'Perfect' port)

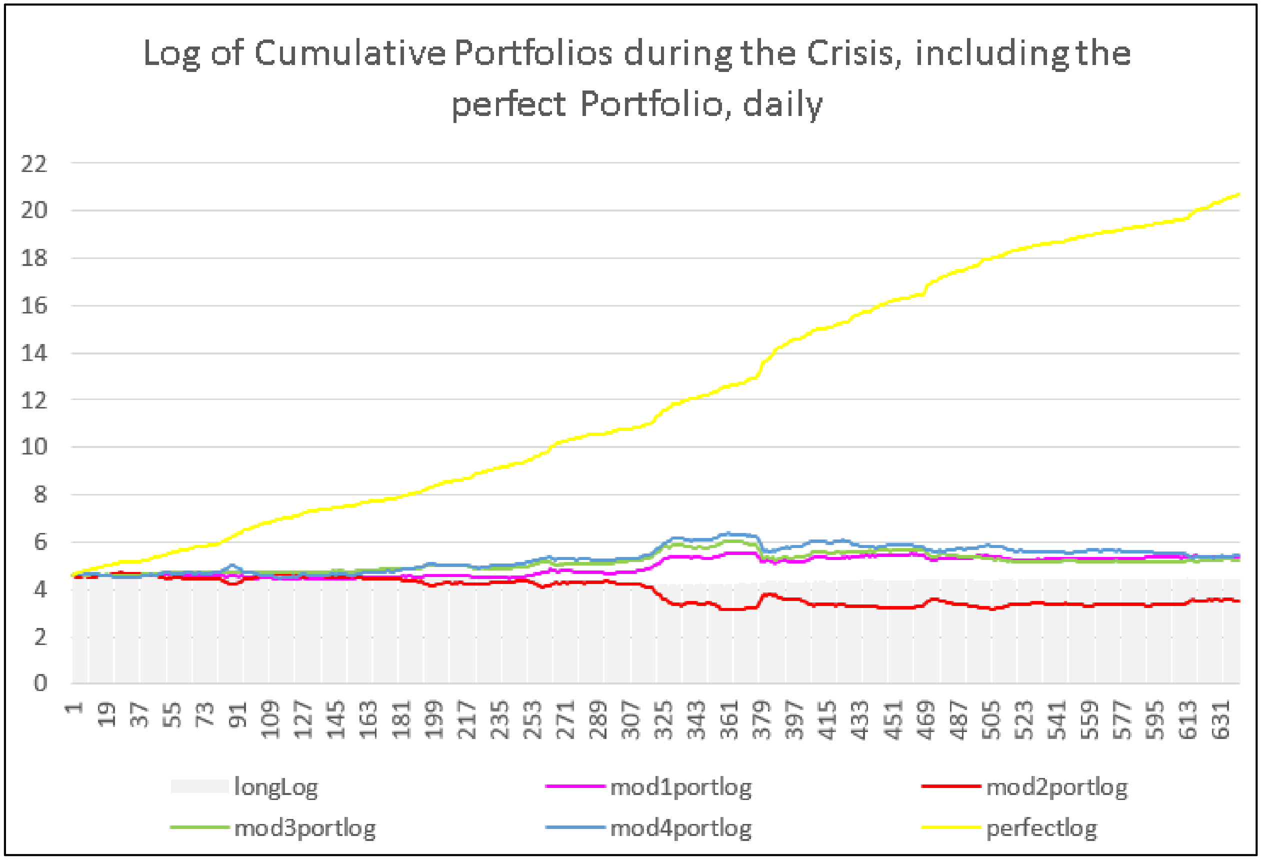


Figure 90: Frequency and Marginal Returns

Panel A: Crisis (11/2007 – 6/2010) – Frequency of Gain Strategies (Return >0)						
Crisis	Model 1	Model 2	Model 3	Model 4	LongOnlyCMP	N
Daily	351 (54.8%)	285 (44.5%)	340 (53.1%)	350 (54.7%)	322 (50.3%)	640
Monthly	21 (70.0%)	13 (43.3%)	20 (66.7%)	19 (63.3%)	16 (53.3%)	30
Panel B: Recovery (1/2013 – 3/2014) – Frequency of Gain Strategies (Return >0)						
Recovery	Model 1	Model 2	Model 3	Model 4	LongOnlyCMP	N
Daily	147 (53.1%)	140 (50.4%)	143 (51.6%)	158 (57.0%)	151 (54.5%)	277
Monthly	11 (84.6%)	6 (46.2%)	6 (46.2%)	7 (53.9%)	7 (53.9%)	13
Panel C: Crisis (11/2007 – 6/2010) – Averages and Maximums for Gain Strategies (Return >0)						
Daily	Model 1	Model 2	Model 3	Model 4	LongOnlyCMP	N
Average	1.2%	1.7%	1.3%	1.7%	0.8%	640
Maximum	10.5%	19.3%	12.2%	11.2%	9.8%	640
Monthly	Model 1	Model 2	Model 3	Model 4	LongOnlyCMP	N
Average	13.2%	8.1%	14.4%	20.5%	6.1%	30
Maximum	51.4%	14.7%	69.7%	67.7%	9.9%	30
Panel D: Recovery (1/2013 – 3/2014) - Averages and Maximums for Gain Strategies (Return > 0)						
Daily	Model 1	Model 2	Model 3	Model 4	LongOnlyCMP	N
Average	0.3%	0.4%	0.3%	0.3%	0.1%	277
Maximum	3.1%	1.6%	1.9%	1.2%	1.0%	277
Monthly	Model 1	Model 2	Model 3	Model 4	LongOnlyCMP	N
Average	1.2%	2.5%	2.5%	2.3%	1.1%	13
Maximum	2.5%	4.8%	7.3%	4.8%	1.5%	13
Panel E: Product Ranking Frequency (Frequency * Average Return)						
Crisis	Model 1	Model 2	Model 3	Model 4	LongOnlyCMP	N
Daily	0.68%	0.77%	0.68%	0.92%	0.39%	640
Monthly	9.20%	3.50%	9.62%	13.00%	3.26%	30
Recovery	Model 1	Model 2	Model 3	Model 4	LongOnlyCMP	N
Daily	0.65%	1.27%	1.29%	1.30%	0.59%	277
Monthly	1.03%	1.16%	1.15%	1.22%	0.58%	13

Figure 91: Paired t-test and Paired Wilcoxon Sign tests

PANEL A: Paired t-test : H0: mean (port-CMBXlong)=0

port	Ha: mean(diff) < 0	Ha: mean(diff) != 0	Ha: mean(diff) > 0
perfect	1.0000	0.0000	0.0000
mod1	0.9576	0.0847	0.0424
mod2	0.1547	0.3093	0.8453
mod3	0.9092	0.1817	0.0908
mod4	0.9436	0.1128	0.0564
CMBX long			
FFMkt	0.4730	0.9460	0.5270

PANEL B: Paired t-test : H0: mean (port-FFMkt)=0

port	Ha: mean(diff) < 0	Ha: mean(diff) != 0	Ha: mean(diff) > 0
perfect	1.0000	0.0000	0.0000
mod1	0.9121	0.1757	0.0879
mod2	0.1848	0.3996	0.8152
mod3	0.8784	0.2431	0.1216
mod4	0.9102	0.1796	0.0898
CMBX long	0.5270	0.9460	0.4730
FFMkt			

PANEL C: Wilcoxon Sign Paired test : H0: med (port-CMBXlong)=0

port	Ha: med(diff) > 0	Ha: med(diff) < 0
perfect	0.0000	1.0000
mod1	0.2637	0.7614
mod2	0.9978	0.0028
mod3	0.2902	0.7363
mod4	0.1522	0.8656
CMBX long		
FFMkt	0.1924	0.8284

PANEL D: Wilcoxon Sign Paired test : H0: med (port-FFMkt)=0

port	Ha: med(diff) > 0	Ha: med(diff) < 0
perfect	0.0000	1.0000
mod1	0.0667	0.9430
mod2	0.9964	0.0045
mod3	0.2386	0.7852
mod4	0.1924	0.8284
CMBX long	0.8284	0.1924
FFMkt		

Figure 92: ICAPM Regressions (Crisis, daily)

PANEL A: REE 2008 Efficiency Regression for Omitted Risk Premia, (11/2007 - 6/2010, daily)

MODEL 1	*Intercept*	*REZ-spot*	*RT-spot*	*1yr-spot*	*2yr-spot*	*5yr-spot*	*7yr-spot*	*10yr-spot*	*Mkt-Rf*	*SMB*	*HML*	*N*	*F-test*	*adjR2*
coef	0.002	-0.063	0.086	-0.782	1.458	-1.405	1.838	-0.844	-0.294	-0.019	-0.226	642	3.605	0.039
serr	0.001	0.075	0.084	1.653	2.294	1.569	1.128	0.658	0.107	0.157	0.148		**0.000**	
t-stat	1.661	-0.840	1.015	-0.473	0.636	-0.896	1.629	-1.282	-2.747	-0.119	-1.533			
p-val	0.097	0.401	0.310	0.636	0.525	0.371	0.104	0.200	**0.006**	0.905	0.126			
MODEL 2														
coef	-0.001	0.084	-0.067	-1.398	3.728	-2.236	-1.397	1.457	0.042	0.027	-0.023	642	1.283	0.004
serr	0.001	0.080	0.090	1.765	2.450	1.675	1.205	0.703	0.114	0.168	0.158		0.236	
t-stat	-0.487	1.050	-0.741	-0.792	1.522	-1.335	-1.159	2.072	0.370	0.163	-0.148			
p-val	0.626	0.294	0.459	0.428	0.129	0.182	0.247	**0.039**	0.712	0.870	0.883			
MODEL 3														
coef	0.001	-0.024	0.066	0.142	-0.700	-0.419	1.623	-0.899	-0.263	0.083	-0.135	642	1.877	0.013
serr	0.001	0.079	0.089	1.734	2.407	1.646	1.184	0.691	0.112	0.165	0.155		**0.045**	
t-stat	0.840	-0.300	0.746	0.082	-0.291	-0.255	1.370	-1.302	-2.339	0.505	-0.869			
p-val	0.401	0.764	0.456	0.935	0.771	0.799	0.171	0.194	**0.020**	0.614	0.385			
MODEL 4														
coef	0.001	-0.128	0.214	-0.675	-0.129	0.115	1.668	-1.350	-0.547	-0.041	-0.070	642	4.339	0.050
serr	0.002	0.086	0.097	1.894	2.628	1.797	1.293	0.754	0.123	0.180	0.169		**0.000**	
t-stat	0.963	-1.493	2.217	-0.356	-0.049	0.064	1.290	-1.790	-4.455	-0.225	-0.412			
p-val	0.336	0.136	**0.027**	0.722	0.961	0.949	0.198	0.074	**0.000**	0.822	0.681			
PERFECT														
coef	0.029	0.087	-0.047	-1.716	4.772	-1.764	1.021	-0.558	-0.186	-0.114	-0.048	642	2.515	0.023
serr	0.001	0.082	0.093	1.815	2.520	1.723	1.239	0.723	0.118	0.173	0.162		**0.006**	
t-stat	19.428	1.056	-0.511	-0.945	1.894	-1.024	0.823	-0.771	-1.579	-0.659	-0.296			
p-val	**0.000**	0.291	0.609	0.345	0.059	0.306	0.411	0.441	0.115	0.510	0.768			

PANEL B: FamaFrench 3Factor Regression , (11/2007 - 6/2010, daily)

MODEL 1	*Intercept*	*Mkt-Rf*	*SMB*	*HML*	*N*	*F-test*	*adjR2*
coef	0.002	-0.239	-0.010	-0.201	642	10.262	0.042
serr	0.001	0.066	0.145	0.138		**0.000**	
t-stat	1.623	-3.609	-0.068	-1.453			
p-val	0.105	**0.000**	0.946	0.147			
MODEL 2							
coef	-0.001	0.047	0.039	-0.029	642	0.176	-0.004
serr	0.001	0.071	0.156	0.148		0.912	
t-stat	-0.847	0.664	0.247	-0.198			
p-val	0.397	0.507	0.805	0.843			
MODEL 3							
coef	0.002	-0.191	0.117	-0.095	642	5.189	0.019
serr	0.001	0.069	0.152	0.145		**0.002**	
t-stat	1.315	-2.754	0.765	-0.659			
p-val	0.189	**0.006**	0.444	0.510			
MODEL 4							
coef	0.002	-0.374	0.008	0.032	642	11.257	0.050
serr	0.001	0.076	0.167	0.159		**0.000**	
t-stat	1.528	-4.908	0.045	0.203			
p-val	0.127	**0.000**	0.964	0.840			
PERFECT							
coef	0.026	-0.151	-0.045	-0.051	642	2.494	0.007
serr	0.001	0.073	0.161	0.153		*0.059*	
t-stat	21.475	-2.054	-0.279	-0.332			
p-val	**0.000**	**0.040**	0.781	0.740			

Figure 92: ICAPM Regression (Crisis, daily - continued)

PANEL C: My Sparse Regression for Omitted Risk Premia, (11/2007 - 6/2010, daily)								
MODEL 1	*Intercept*	*REZ-spot*	*RT-spot*	*10yr-spot*	*Mkt-Rf*	*N*	*F-test*	*adjR2*
coef	0.002	-0.071	0.071	0.022	-0.320	642	7.407	0.038
serr	0.001	0.072	0.080	0.154	0.102		**0.000**	
t-stat	1.576	-0.994	0.885	0.145	-3.126			
p-val	0.115	0.321	0.377	0.885	**0.002**			
MODEL 2								
coef	-0.001	0.068	-0.051	-0.286	0.035	642	1.121	0.001
serr	0.001	0.077	0.086	0.165	0.110		0.345	
t-stat	-1.197	0.881	-0.597	-1.735	0.316			
p-val	0.232	0.379	0.551	0.083	0.752			
MODEL 3								
coef	0.002	-0.010	0.046	0.000	-0.285	642	3.773	0.017
serr	0.001	0.075	0.084	0.161	0.107		**0.005**	
t-stat	1.342	-0.138	0.544	-0.002	-2.662			
p-val	0.180	0.891	0.587	0.998	**0.008**			
MODEL 4								
coef	0.002	-0.134	0.209	-0.072	-0.553	642	9.839	0.052
serr	0.001	0.082	0.091	0.176	0.117		**0.000**	
t-stat	1.538	-1.624	2.284	-0.406	-4.722			
p-val	0.125	0.105	0.023	0.685	**0.000**			
PERFECT								
coef	0.026	0.042	0.000	0.112	-0.220	642	2.113	0.007
serr	0.001	0.080	0.089	0.170	0.113		0.078	
t-stat	21.078	0.527	-0.002	0.656	-1.943			
p-val	**0.000**	0.599	0.998	0.512	**0.052**			

PANEL D: Theta vs. the FF 1Factor Market Portfolio, (11/2007 - 6/2010, daily)					
MODEL 1	*Intercept*	*Mkt-Rf*	*N*	*F-test*	*adjR2*
coef	0.002	-0.293	642	28.655	0.041
serr	0.001	0.055		**0.000**	
t-stat	1.577	-5.353			
p-val	0.115	**0.000**			
MODEL 2					
coef	-0.001	0.039	642	0.434	-0.001
serr	0.001	0.059		0.510	
t-stat	-0.847	0.659			
p-val	0.397	0.510			
MODEL 3					
coef	0.002	-0.219	642	14.606	0.021
serr	0.001	0.057		**0.000**	
t-stat	1.325	-3.822			
p-val	0.186	**0.000**			
MODEL 4					
coef	0.002	-0.365	642	33.830	0.049
serr	0.001	0.063		**0.000**	
t-stat	1.539	-5.816			
p-val	0.124	**0.000**			
PERFECT					
coef	0.026	-0.164	642	7.309	0.010
serr	0.001	0.061		**0.007**	
t-stat	21.509	-2.704			
p-val	**0.000**	**0.007**			

Figure 93 - Correlation

```
. correlate housertrn reitrtrn oneyr twoyr fiveyr sevyr tenyr FFmktrf FFsmb FFhml
(obs=642)
```

	houser~n	reitrtrn	oneyr	twoyr	fiveyr	sevyr	tenyr	FFmktrf	FFsmb	FFhml
housertrn	1.0000									
reitrtrn	0.9110	1.0000								
oneyr	0.0435	0.0592	1.0000							
twoyr	0.0085	0.0464	0.8996	1.0000						
fiveyr	-0.0163	0.0125	0.5781	0.8319	1.0000					
sevyr	-0.0315	-0.0026	0.4721	0.7324	0.9683	1.0000				
tenyr	-0.0337	-0.0061	0.4048	0.6555	0.9277	0.9673	1.0000			
FFmktrf	0.7341	0.8412	0.0134	-0.0072	-0.0206	-0.0240	-0.0169	1.0000		
FFsmb	0.2031	0.1261	0.0003	0.0458	0.0809	0.0815	0.0824	-0.0471	1.0000	
FFhml	0.5409	0.6185	-0.0249	-0.0124	0.0075	0.0075	0.0211	0.5617	0.0035	1.0000

Figure 94: Ramsey RESET

```
. ovtest

Ramsey RESET test using powers of the fitted values of mdlrtrn
       Ho:  model has no omitted variables
                 F(3, 628) =      0.47
                  Prob > F =      0.7058
```

Figure 95: VIF test of Multicollinearity among variables

. vif

Variable	VIF	1/VIF
fiveyr	43.99	0.022735
sevyr	37.01	0.027021
twoyr	30.47	0.032822
tenyr	18.40	0.054353
oneyr	11.75	0.085128
reitrtrn	10.60	0.094361
housertrn	6.58	0.152071
FFmktrf	3.82	0.261575
FFhml	1.67	0.599424
FFsmb	1.17	0.852670
Mean VIF	16.54	

Figure 96: Condition Index among variables

. colldiag()

Proportion of variance associated with the decomposition

Cond Number	housertrn	reitrtrn	oneyr	twoyr	fiveyr	sevyr	tenyr
1	0.0000	0.0000	0.0020	0.0011	0.0010	0.0011	0.0021
1.17172	0.0130	0.0089	0.0000	0.0000	0.0000	0.0000	0.0000
1.88189	0.0000	0.0000	0.0075	0.0007	0.0007	0.0019	0.0054
2.07126	0.0036	0.0001	0.0014	0.0001	0.0002	0.0006	0.0016
2.86141	0.0166	0.0064	0.0015	0.0004	0.0000	0.0000	0.0002
3.1133	0.0018	0.0004	0.0308	0.0074	0.0000	0.0017	0.0076
4.34017	0.2367	0.0098	0.0006	0.0002	0.0001	0.0001	0.0001
7.98225	0.4886	0.6692	0.0685	0.0182	0.0128	0.0001	0.0562
8.88612	0.1836	0.2683	0.1569	0.0399	0.0818	0.0015	0.2762
20.3212	0.0454	0.0305	0.5255	0.7584	0.8846	0.2609	0.0240
15.1286	0.0106	0.0063	0.2053	0.1736	0.0187	0.7321	0.6266

Cond Number	FFmktrf	FFsmb	FFhml	_cons
1	0.0000	0.0002	0.0000	0.0094
1.17172	0.0210	0.0012	0.0337	0.0000
1.88189	0.0005	0.1258	0.0019	0.1890
2.07126	0.0077	0.6285	0.0263	0.0443
2.86141	0.0345	0.0304	0.7818	0.1359
3.1133	0.0122	0.0036	0.1166	0.5980
4.34017	0.6392	0.2041	0.0000	0.0003
7.98225	0.1864	0.0007	0.0278	0.0082
8.88612	0.0956	0.0004	0.0098	0.0134
20.3212	0.0024	0.0029	0.0001	0.0000
15.1286	0.0005	0.0024	0.0020	0.0015

Figure 97: White test for heteroskedasticity in the error term

```
. estat imtest, white

White's test for Ho: homoskedasticity
         against Ha: unrestricted heteroskedasticity

         chi2(65)     =    137.66
         Prob > chi2  =    0.0000

Cameron & Trivedi's decomposition of IM-test

---------------------------------------------------
              Source |       chi2     df      p
---------------------+-----------------------------
  Heteroskedasticity |     137.66     65    0.0000
            Skewness |       9.74     10    0.4634
            Kurtosis |       7.88      1    0.0050
---------------------+-----------------------------
               Total |     155.28     76    0.0000
---------------------------------------------------
```

Figure 98: Durbin Watson

```
. dwstat

Durbin-Watson d-statistic( 11,   642) =  1.535462
```

Figure 99: Condition Index for the Credit Regression

```
. colldiag

Proportion of variance associated with the decomposition
  Cond    |
 Number   |housertrn  reitrtrn     tenyr   FFmktrf     _cons
---------+------------------------------------------------
       1 |  0.0205    0.0141    0.0000    0.0333    0.0006
 6.32882 |  0.7588    0.9771    0.0058    0.3245    0.0005
 3.12459 |  0.2206    0.0087    0.0008    0.6410    0.0072
 1.47564 |  0.0000    0.0000    0.3880    0.0003    0.3839
 1.85141 |  0.0000    0.0001    0.6053    0.0009    0.6079
```

Figure 100: Ramsey RESET for Credit Regression

```
. ovtest

Ramsey RESET test using powers of the fitted values of mdlrtrn
       Ho:  model has no omitted variables
                 F(3, 634) =      0.17
                  Prob > F =      0.9194
```

Figure 101: White Test for Credit Regression

```
. estat imtest, white

White's test for Ho: homoskedasticity
         against Ha: unrestricted heteroskedasticity

         chi2(14)      =     17.86
         Prob > chi2   =    0.2133

Cameron & Trivedi's decomposition of IM-test

---------------------------------------------------
              Source |       chi2     df      p
---------------------+-----------------------------
  Heteroskedasticity |      17.86     14    0.2133
            Skewness |       5.26      4    0.2617
            Kurtosis |       7.61      1    0.0058
---------------------+-----------------------------
               Total |      30.72     19    0.0433
---------------------------------------------------
```

Figure 102: Durbin Watson test for autocorrelation

```
. dwstat

Durbin-Watson d-statistic(  5,   642) =  1.535353
```

Figure 103: BreuschGodfrey test of AR(1) and AR(2)

```
. estat bgodfrey, lags(1,2)

Breusch-Godfrey LM test for autocorrelation
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       1     |         34.793               1                   0.0000
       2     |         35.592               2                   0.0000
---------------------------------------------------------------------------
                        H0: no serial correlation
```

Figure 104: Panel C and D: Recovery

PANEL C: My Sparse Regression for Omitted Risk Premia, (1/2013 - 3/2014, daily)

MODEL 1	*Intercept*	*Mkt-RF*	*10yr-spt*	*REITs-rf*	*REZ-rf*	*N*	*F-test*	*adjR2*
coef	0.000	-0.033	-0.186	0.027	-0.029	277	1.674	0.010
serr	0.000	0.046	0.085	0.034	0.033		0.156	
t-stat	0.462	-0.718	-2.195	0.804	-0.878			
p-val	0.645	0.474	**0.029**	0.422	0.381			
MODEL 2								
coef	0.000	0.053	0.007	0.090	-0.014	277	3.150	0.030
serr	0.000	0.038	0.070	0.028	0.028		**0.015**	
t-stat	-0.281	1.407	0.100	3.248	-0.506			
p-val	0.779	0.160	0.920	**0.001**	0.614			
MODEL 3								
coef	0.000	-0.054	0.038	-0.132	-0.030	277	5.009	0.055
serr	0.000	0.043	0.079	0.031	0.031		**0.001**	
t-stat	0.638	-1.260	0.476	-4.192	-0.970			
p-val	0.524	0.209	0.634	**0.000**	0.333			
MODEL 4								
coef	0.000	0.010	0.024	0.029	-0.029	277	0.644	-0.005
serr	0.000	0.037	0.068	0.027	0.027		0.632	
t-stat	0.549	0.271	0.348	1.075	-1.074			
p-val	0.584	0.786	0.728	0.283	0.284			
PERFECT								
coef	0.006	-0.017	-0.077	0.004	0.041	277	0.764	-0.003
serr	0.000	0.043	0.079	0.031	0.031		0.549	
t-stat	19.318	-0.410	-0.972	0.137	1.327			
p-val	**0.000**	0.682	0.332	0.891	0.186			

PANEL D: Theta vs. the FF 1Factor Market Portfolio, (1/2013 - 3/2014, daily)

MODEL 1	*Intercept*	*Mkt-Rf*	*N*	*F-test*	*adjR2*
coef	0.000	-0.042	277	0.846	-0.00056
serr	0.000	0.046		0.359	
t-stat	0.647	-0.920			
p-val	0.518	0.359			
MODEL 2					
coef	0.000	0.049	277	1.659	0.002375
serr	0.000	0.038		0.199	
t-stat	-0.215	1.288			
p-val	0.830	0.199			
MODEL 3					
coef	0.000	-0.051	277	1.333	0.0012
serr	0.000	0.044		0.249	
t-stat	0.526	-1.154			
p-val	0.599	0.249			
MODEL 4					
coef	0.000	0.007	277	0.041	0.000149
serr	0.000	0.037		0.840	
t-stat	0.564	0.203			
p-val	0.573	0.840			
PERFECT					
coef	0.006	-0.017	277	0.156	-0.00306
serr	0.000	0.042		0.693	
t-stat	19.432	-0.395			
p-val	**0.000**	0.693			

Figure 105: Initial NCREIF OLS Regression

```
. regress ActualNCREIFRollingReturnsY X1Unemployment X2CaseShiller X3CreditSlope X4MortgageRate X5RiskFreeSlope X6CREChargeoffRate X7PrivateC
> REConstruction

      Source |       SS       df       MS              Number of obs =      90
-------------+------------------------------           F(  7,    82) =   90.78
       Model |  .616142563     7  .088020366           Prob > F      =  0.0000
    Residual |  .079509099    82  .000969623           R-squared     =  0.8857
-------------+------------------------------           Adj R-squared =  0.8759
       Total |  .695651662    89  .007816311           Root MSE      =  .03114

------------------------------------------------------------------------------
ActualNCREIFRollingRet~Y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------------------+----------------------------------------------------------------
          X1Unemployment |   .1210283    .702552     0.17   0.864    -1.276571    1.518628
           X2CaseShiller |   .5872378   .0836527     7.02   0.000     .4208259    .7536498
           X3CreditSlope |  -.0683469   .0099054    -6.90   0.000    -.0880519    -.048642
          X4MortgageRate |  -.0199576     .00262    -7.62   0.000    -.0251696   -.0147456
         X5RiskFreeSlope |  -.0229351   .0035865    -6.39   0.000    -.0300697   -.0158004
      X6CREChargeoffRate |  -.0105854   .0109289    -0.97   0.336    -.0323265    .0111557
X7PrivateCREConstruction |   .1223219   .0530472     2.31   0.024     .0167941    .2278497
                   _cons |    .248462   .0480129     5.17   0.000      .152949     .343975
------------------------------------------------------------------------------------------

. estat dwatson

Durbin-Watson d-statistic(  8,    90) =  .7761072
```

Figure 106: Summary Statistics for solo regressions each of 7 x-variables & associated autocorrelation tests for synthetic NCREIF

x-variable	Coeff	StdError	t-stat	pval	F	Prb>F	RSQ	Dwtsn	daltX2	bgX2	arch
X1 Unemployment	-4.630307	0.4048245	-11.44	0.000	130.82	0.000	0.5978	.139699	740.454	80.537	71.063
X2 CaseShiller	0.9076193	0. 0822733	11.03	0.000	121.70	0.000	0.5804	.1400557	561.166	77.920	57.820
X3 CreditSlope	-0.1293126	0.0176354	-7.33	0.000	53.77	0.000	0.3793	.2563417	268.912	68.000	51.575
X4 MortgageRate	-0.0072094	0.0059769	-1.21	0.231	1.45	0.231	0.0163	.0983784	820.009	81.367	69.177
X5 RiskFreeSlope	-0.0389995	0.0050204	-7.77	0.000	60.34	0.000	0.4068	.2090902	375.331	73.064	54.970
X6 CREChgOffRate	-0.0850968	0.0066281	-12.84	0.000	164.83	0.000	0.6519	.2365583	301.704	69.856	60.757
X7 PrivateCREConstr	-0.3386575	0.1023531	-3.31	0.001	10.95	0.014	0.1106	.1112442	758.625	80.741	59.401

Figure 107: Sample of solo regression for X7 Private CRE Construction & associated autocorrelation tests

```
. regress ActualNCREIFRollingReturnsY X7PrivateCREConstruction

      Source |       SS       df       MS              Number of obs =      90
-------------+------------------------------           F(  1,    88) =   10.95
       Model |  .076967279     1  .076967279           Prob > F      =  0.0014
    Residual |  .618684384    88  .007030504           R-squared     =  0.1106
-------------+------------------------------           Adj R-squared =  0.1005
       Total |  .695651662    89  .007816311           Root MSE      =  .08385

------------------------------------------------------------------------------------------
ActualNCREIFRollingRet~Y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------------------+----------------------------------------------------------------
X7PrivateCREConstruction |  -.3386575   .1023531    -3.31   0.001    -.5420627   -.1352523
                   _cons |   .2101974   .0424434     4.95   0.000     .1258501    .2945447
------------------------------------------------------------------------------------------

. estat dwatson

Durbin-Watson d-statistic(  2,    90) =  .1112442

. estat durbinalt

Durbin's alternative test for autocorrelation
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       1     |        758.625               1                   0.0000
---------------------------------------------------------------------------
                        H0: no serial correlation

. estat bgodfrey

Breusch-Godfrey LM test for autocorrelation
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       1     |         80.741               1                   0.0000
---------------------------------------------------------------------------
                        H0: no serial correlation

. estat archlm
LM test for autoregressive conditional heteroskedasticity (ARCH)
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       1     |         59.401               1                   0.0000
---------------------------------------------------------------------------
         H0: no ARCH effects      vs.  H1: ARCH(p) disturbance
```

Figure 108: Solo Prais-Winston Regressions for all 7 XVariables and associated DurbinWatson Stats

x-variable	Coeff	StdError	t-stat	pval	F	Prb>F	RSQ	dwtsn (orig)	dwtsn (adj)
X1 Unemployment	-6.740923	.6688781	-10.08	0.000	101.57	0.0000	0.5386	.139699	1.077457
X2 CaseShiller	.9514761	.1280446	7.43	0.000	55.22	0.0000	0.3883	.1400557	0.772544
X3 CreditSlope	-.0374255	.0104165	-3.59	0.001	12.91	0.0005	0.1292	.2563417	0.752473
X4 MortgageRate	-.0002898	.0071868	-0.04	0.968	0.00	0.9679	0.0000	.0983784	0.412529
X5 RiskFreeSlope	-.0121825	.0045507	-2.68	0.009	7.17	0.0089	0.0761	.2090902	0.573268
X6 CREChgOffRate	-.053743	.0113599	-4.73	0.000	22.38	0.0000	0.2046	.2365583	0.756099
X7 PrivateCREConstr	-.1273278	.0896643	-1.42	0.159	2.02	0.1592	0.0227	.1112442	0.463369

Figure 109: Prais-Winsten Multivariate for all 7 X variables

```
. prais ActualNCREIFRollingReturnsY X1Unemployment X2CaseShiller X3CreditSlope X4MortgageRate X5RiskFreeSlope X6CREChargeoffRate X7PrivateCRE
> Construction, rhotype(dw) corc ssesearch

Iteration 1:  rho = 0.8944 , criterion =  -.02849443
Iteration 2:  rho = 0.8944 , criterion =  -.02849443
Iteration 3:  rho = 0.8944 , criterion =  -.02849443
Iteration 4:  rho = 0.9480 , criterion =  -.02693768
Iteration 5:  rho = 1.0236 , criterion =  -.02537404
Iteration 6:  rho = 1.0236 , criterion =  -.02537404
Iteration 7:  rho = 1.0524 , criterion =  -.02522207
Iteration 8:  rho = 1.0506 , criterion =  -.02522186
Iteration 9:  rho = 1.0506 , criterion =  -.02522186
Iteration 10:  rho = 1.0513 , criterion =  -.02522178
Iteration 11:  rho = 1.0512 , criterion =  -.02522178
Iteration 12:  rho = 1.0512 , criterion =  -.02522178
Iteration 13:  rho = 1.0512 , criterion =  -.02522178
Iteration 14:  rho = 1.0512 , criterion =  -.02522178
Iteration 15:  rho = 1.0512 , criterion =  -.02522178
Iteration 16:  rho = 1.0512 , criterion =  -.02522178

Cochrane-Orcutt AR(1) regression -- SSE search estimates

      Source |       SS       df       MS              Number of obs =      89
-------------+------------------------------           F(  7,    81) =   21.11
       Model |  .046003888     7  .006571984           Prob > F      =  0.0000
    Residual |  .025221777    81   .00031138           R-squared     =  0.6459
-------------+------------------------------           Adj R-squared =  0.6153
       Total |  .071225665    88  .000809383           Root MSE      =  .01765

------------------------------------------------------------------------------
ActualNCREIFRollingRet~Y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------------------+----------------------------------------------------------------
          X1Unemployment |  -3.422226   .8613997    -3.97   0.000    -5.136141    -1.70831
           X2CaseShiller |   .5624644    .125466     4.48   0.000     .3128264    .8121024
           X3CreditSlope |  -.0202848   .0076906    -2.64   0.010    -.0355867   -.0049829
          X4MortgageRate |   .0003061   .0045049     0.07   0.946    -.0086573    .0092695
         X5RiskFreeSlope |  -.0001548    .003175    -0.05   0.961     -.006472    .0061624
       X6CREChargeoffRate |  -.0181193   .0090195    -2.01   0.048    -.0360653   -.0001733
X7PrivateCREConstruction |  -.0305621   .0615191    -0.50   0.621    -.1529658    .0918416
                   _cons |   .2376305   .0721733     3.29   0.001     .0940284    .3812327
-------------------------+----------------------------------------------------------------
                     rho |    1.05124
------------------------------------------------------------------------------------------
Durbin-Watson statistic (original)    0.776107
Durbin-Watson statistic (transformed) 1.350785
```

Figure 110: ACLI Historical Foreclosures

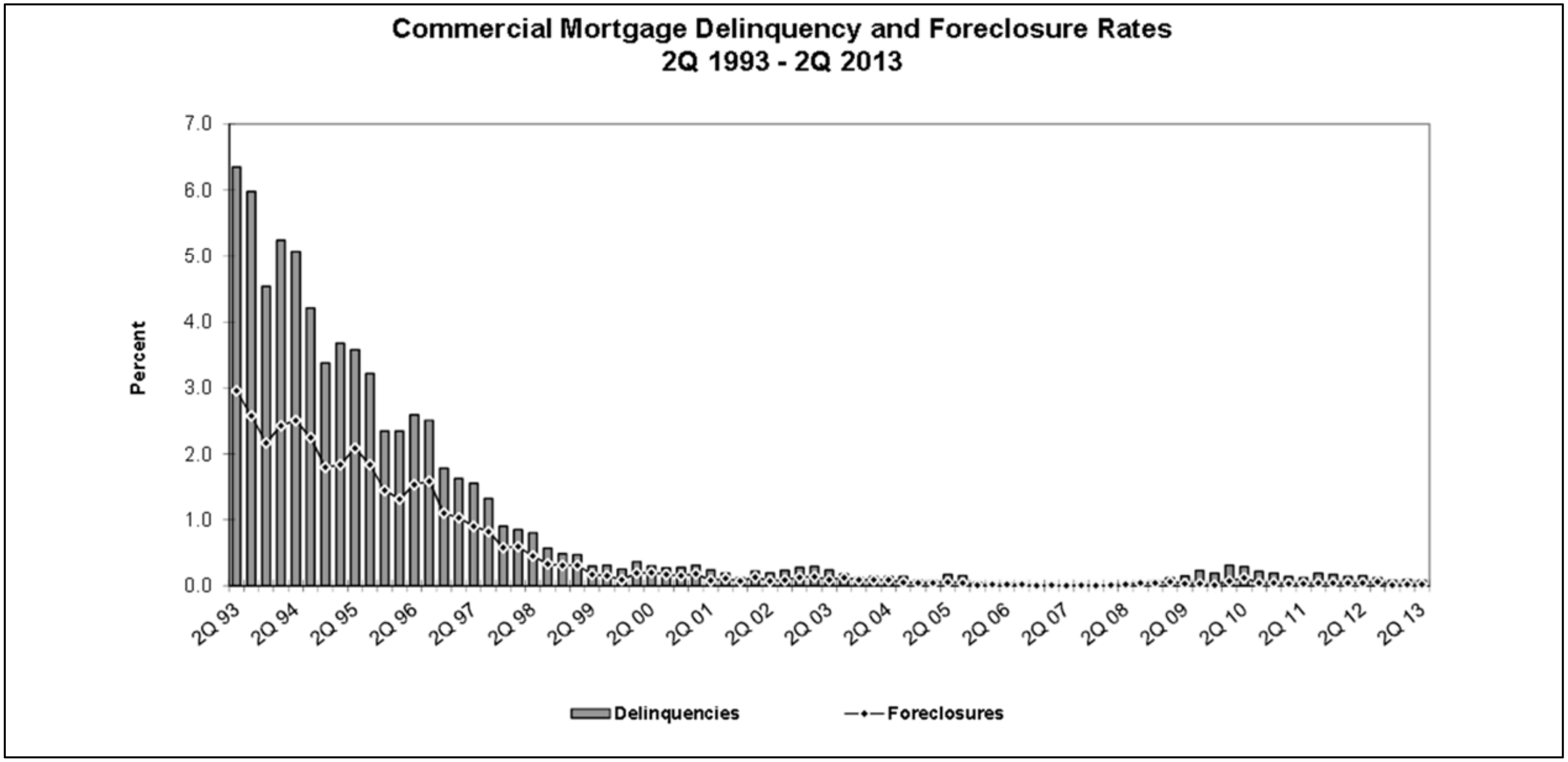


Figure 111a: W(t,l) Discretized Simulation (from Wiersema)

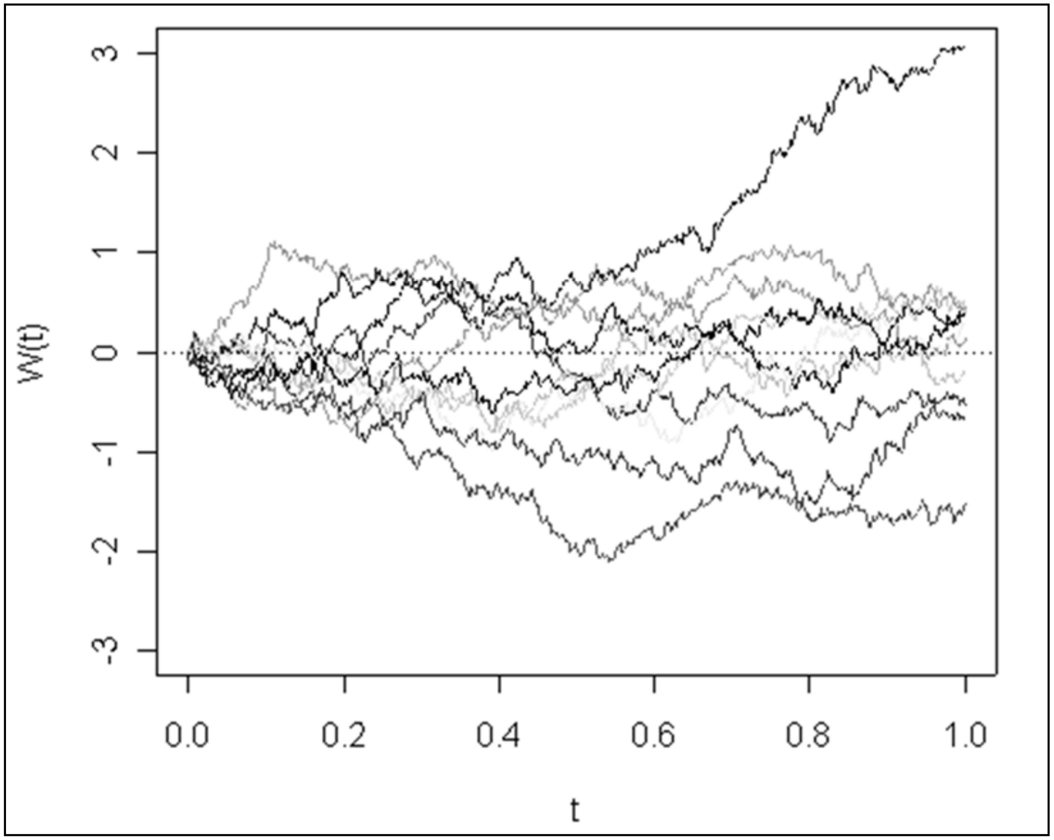


Figure 111b: 5 simulations for 4 loans

```
> MFdefadjprinCF[1:50,]
```

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]	[,11]	[,12]	[,13]	[,14]	[,15]	[,16]	[,17]	[,18]	[,19]	[,20]
[1,]	4182.07	4182.07	4182.07	4182.07	4182.07	2332117	2332117	2332117	2332117	2332117	0.00	0.00	0.00	0.00	0.00	7748.96	7748.96	7748.96	7748.96	7748.96
[2,]	4201.69	4201.69	4201.69	4201.69	4201.69	2342262	2342262	2342262	2342262	2342262	0.00	0.00	0.00	0.00	0.00	7783.67	7783.67	7783.67	7783.67	7783.67
[3,]	4221.40	4221.40	4221.40	4221.40	4221.40	2352450	2352450	2352450	2352450	2352450	0.00	0.00	0.00	0.00	0.00	7818.54	7818.54	7818.54	7818.54	7818.54
[4,]	4241.20	4241.20	4241.20	4241.20	4241.20	2362684	2362684	2362684	2362684	2362684	0.00	0.00	0.00	0.00	0.00	7853.56	0.00	0.00	0.00	0.00
[5,]	4261.10	4261.10	4261.10	4261.10	4261.10	2372961	2372961	2372961	2372961	2372961	0.00	0.00	0.00	0.00	0.00	7888.73	0.00	0.00	0.00	0.00
[6,]	4281.09	4281.09	4281.09	4281.09	4281.09	2383284	2383284	2383284	2383284	2383284	0.00	0.00	0.00	0.00	0.00	7924.07	0.00	0.00	0.00	0.00
[7,]	4301.18	0.00	0.00	0.00	0.00	2393651	2393651	2393651	2393651	2393651	0.00	0.00	0.00	0.00	0.00	7959.56	0.00	0.00	0.00	0.00
[8,]	4321.36	0.00	0.00	0.00	0.00	2404063	2404063	2404063	2404063	2404063	0.00	0.00	0.00	0.00	0.00	7995.21	0.00	0.00	0.00	0.00
[9,]	4341.63	0.00	0.00	0.00	0.00	2414521	2414521	2414521	2414521	2414521	0.00	0.00	0.00	0.00	0.00	8031.03	0.00	0.00	0.00	0.00
[10,]	4362.00	0.00	0.00	0.00	0.00	2425024	2425024	2425024	2425024	2425024	0.00	0.00	0.00	0.00	0.00	8067.00	0.00	0.00	0.00	0.00
[11,]	4382.47	0.00	0.00	0.00	0.00	2435573	2435573	2435573	2435573	2435573	0.00	0.00	0.00	0.00	0.00	8103.13	0.00	0.00	0.00	0.00
[12,]	4403.03	0.00	0.00	0.00	0.00	2446168	2446168	2446168	2446168	2446168	0.00	0.00	0.00	0.00	0.00	8139.43	0.00	0.00	0.00	0.00
[13,]	4423.69	0.00	0.00	0.00	0.00	2456809	2456809	2456809	2456809	2456809	0.00	0.00	0.00	0.00	0.00	8175.88	0.00	0.00	0.00	0.00
[14,]	4444.44	0.00	0.00	0.00	0.00	2467496	2467496	2467496	2467496	2467496	0.00	0.00	0.00	0.00	0.00	8212.51	0.00	0.00	0.00	0.00
[15,]	4465.29	0.00	0.00	0.00	0.00	2478229	2478229	2478229	2478229	2478229	0.00	0.00	0.00	0.00	0.00	8249.29	4503762.32	4501889.33	4842954.73	4421722.99
[16,]	4486.24	0.00	0.00	0.00	0.00	2489010	2489010	2489010	2489010	2489010	0.00	0.00	0.00	0.00	0.00	8286.24	0.00	0.00	0.00	0.00
[17,]	4507.29	0.00	0.00	0.00	0.00	2499837	2499837	2499837	2499837	2499837	0.00	0.00	0.00	0.00	0.00	8323.36	0.00	0.00	0.00	0.00
[18,]	4528.44	21259669.03	21685331.51	18853162.38	20289742.19	2510711	2510711	2510711	2510711	2510711	0.00	0.00	0.00	0.00	0.00	8360.64	0.00	0.00	0.00	0.00
[19,]	4549.68	0.00	0.00	0.00	0.00	2521633	2521633	2521633	2521633	2521633	0.00	0.00	0.00	0.00	0.00	8398.09	0.00	0.00	0.00	0.00
[20,]	4571.03	0.00	0.00	0.00	0.00	2532602	2532602	2532602	2532602	2532602	0.00	0.00	0.00	0.00	0.00	8435.70	0.00	0.00	0.00	0.00
[21,]	4592.48	0.00	0.00	0.00	0.00	2543619	2543619	2543619	2543619	2543619	0.00	0.00	0.00	0.00	0.00	8473.49	0.00	0.00	0.00	0.00
[22,]	4614.02	0.00	0.00	0.00	0.00	2554683	2554683	2554683	2554683	2554683	0.00	0.00	0.00	0.00	0.00	8511.44	0.00	0.00	0.00	0.00
[23,]	21753910.91	0.00	0.00	0.00	0.00	2565796	2565796	2565796	2565796	2565796	0.00	0.00	0.00	0.00	0.00	8549.57	0.00	0.00	0.00	0.00
[24,]	0.00	0.00	0.00	0.00	0.00	2576957	2576957	2576957	2576957	2576957	4898.49	4898.49	4898.49	4898.49	4898.49	8587.86	0.00	0.00	0.00	0.00
[25,]	0.00	0.00	0.00	0.00	0.00	2588167	2588167	2588167	2588167	2588167	0.00	0.00	4920.04	0.00	4920.04	8626.33	0.00	0.00	0.00	0.00
[26,]	0.00	0.00	0.00	0.00	0.00	2599426	2599426	2599426	2599426	2599426	0.00	0.00	4941.69	0.00	4941.69	8664.97	0.00	0.00	0.00	0.00
[27,]	0.00	0.00	0.00	0.00	0.00	2610733	2610733	2610733	2610733	2610733	0.00	0.00	4963.43	0.00	4963.43	8703.78	0.00	0.00	0.00	0.00
[28,]	0.00	0.00	0.00	0.00	0.00	2622090	2622090	2622090	2622090	2622090	0.00	0.00	4985.27	0.00	4985.27	8742.76	0.00	0.00	0.00	0.00
[29,]	0.00	0.00	0.00	0.00	0.00	2633496	2633496	2633496	2633496	2633496	0.00	0.00	5007.21	0.00	5007.21	8781.92	0.00	0.00	0.00	0.00
[30,]	0.00	0.00	0.00	0.00	0.00	2644952	2644952	2644952	2644952	2644952	0.00	0.00	5029.24	0.00	5029.24	8821.26	0.00	0.00	0.00	0.00
[31,]	0.00	0.00	0.00	0.00	0.00	2656457	2656457	2656457	2656457	2656457	0.00	0.00	5051.37	0.00	0.00	8860.77	0.00	0.00	0.00	0.00
[32,]	0.00	0.00	0.00	0.00	0.00	2668013	2668013	2668013	2668013	2668013	0.00	0.00	5073.59	0.00	0.00	8900.46	0.00	0.00	0.00	0.00
[33,]	0.00	0.00	0.00	0.00	0.00	2679619	2679619	2679619	2679619	2679619	0.00	0.00	5095.92	0.00	0.00	8940.33	0.00	0.00	0.00	0.00
[34,]	0.00	0.00	0.00	0.00	0.00	2691275	2691275	2691275	2691275	2691275	0.00	0.00	5118.34	0.00	0.00	8980.37	0.00	0.00	0.00	0.00
[35,]	0.00	0.00	0.00	0.00	0.00	2702982	2702982	2702982	2702982	2702982	0.00	0.00	5140.86	0.00	0.00	9020.60	0.00	0.00	0.00	0.00
[36,]	0.00	0.00	0.00	0.00	0.00	2714740	2714740	2714740	2714740	2714740	2223361.56	2223361.56	5163.48	2223361.56	0.00	9061.00	0.00	0.00	0.00	0.00

Figure 112: Trust Composite across 5 simulations

> MFtrust[1:36,]

	[,1]	[,2]	[,3]	[,4]	[,5]
[1,]	2344048	2344048	2344048	2344048	2344048
[2,]	2354247	2354247	2354247	2354247	2354247
[3,]	2364490	2364490	2364490	2364490	2364490
[4,]	2374778	2366925	2366925	2366925	2366925
[5,]	2385111	2377222	2377222	2377222	2377222
[6,]	2395489	2387565	2387565	2387565	2387565
[7,]	2405912	2393651	2393651	2393651	2393651
[8,]	2416380	2404063	2404063	2404063	2404063
[9,]	2426894	2414521	2414521	2414521	2414521
[10,]	2437453	2425024	2425024	2425024	2425024
[11,]	2448059	2435573	2435573	2435573	2435573
[12,]	2458710	2446168	2446168	2446168	2446168
[13,]	2469408	2456809	2456809	2456809	2456809
[14,]	2480153	2467496	2467496	2467496	2467496
[15,]	2490944	6981992	6980119	7321184	6899952
[16,]	2501782	2489010	2489010	2489010	2489010
[17,]	2512667	2499837	2499837	2499837	2499837
[18,]	2523600	23770380	24196043	21363873	22800453
[19,]	2534580	2521633	2521633	2521633	2521633
[20,]	2545608	2532602	2532602	2532602	2532602
[21,]	2556685	2543619	2543619	2543619	2543619
[22,]	2567809	2554683	2554683	2554683	2554683
[23,]	24328257	2565796	2565796	2565796	2565796
[24,]	2590444	2581856	2581856	2581856	2581856
[25,]	2596794	2588167	2593087	2588167	2593087
[26,]	2608091	2599426	2604367	2599426	2604367
[27,]	2619437	2610733	2615697	2610733	2615697
[28,]	2630833	2622090	2627075	2622090	2627075
[29,]	2642278	2633496	2638503	2633496	2638503
[30,]	2653773	2644952	2649981	2644952	2649981
[31,]	2665318	2656457	2661509	2656457	2656457
[32,]	2676913	2668013	2673086	2668013	2668013
[33,]	2688559	2679619	2684715	2679619	2679619
[34,]	2700255	2691275	2696393	2691275	2691275
[35,]	2712003	2702982	2708123	2702982	2702982
[36,]	4947163	4938102	2719904	4938102	2714740

Figure 113: Aggregation

	Per 23 CF Sim1
Loan1	21,753,910.91
Loan2	2,565,796.00
Loan3	-
Loan4	8,549.57
TotalCalc	24,328,256.48
TrustCalc	24,328,257.00
Diff	0.52

Figure 114: Class Percentages

Class/Tranche	Class Percent
AAA:	0.6440
AJ:	0.0584
AM:	0.0984
AA:	0.0742
A:	0.0205
BBB:	0.0274
BBBminus:	0.0451
Other:	0.0320

Figure 115: Trust CFs across 10sims

> MFtrust[1:50,]

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]
[1,]	2561462.48	2561462	2561462	2561462	2561462	2561462.48	2561462	2561462	2561462	2561462
[2,]	2558098.23	2558098	2558098	2558098	2558098	2558098.23	2558098	2558098	2558098	2558098
[3,]	2569261.48	2569261	2569261	2569261	2569261	2569261.48	2569261	2569261	2569261	2569261
[4,]	2580473.46	2518791	2518791	2518791	2518791	2572619.90	2518791	2572620	2518791	2572620
[5,]	2591734.35	2529774	2529774	2529774	2529774	2583845.62	2529774	2583846	2529774	2583846
[6,]	2603044.41	2392967	2392967	2392967	2392967	2447281.75	2392967	2447282	2392967	2447282
[7,]	2614403.83	2403382	2403382	2403382	2403382	2457941.63	2403382	2457942	2403382	2457942
[8,]	2625812.83	2413842	2413842	2413842	2413842	2468647.96	2413842	2468648	2413842	2468648
[9,]	2637271.62	2424348	2424348	2424348	2424348	2479400.91	2424348	2479401	2424348	2479401
[10,]	2648780.43	2434899	2434899	2434899	2434899	2490200.72	2434899	2490201	2434899	2490201
[11,]	2660339.45	2445497	2445497	2445497	2435573	2501047.56	2445497	2501048	2445497	2501048
[12,]	2671948.95	2456141	2456141	2456141	2446168	2511941.67	2456141	2511942	2456141	2511942
[13,]	28131467.32	27914689	27914689	27914689	27904667	27970741.45	27914689	27970741	27914689	27970741
[14,]	2695320.15	2477567	2477567	2477567	2467496	2533872.44	2477567	2533872	2477567	2523801
[15,]	2707082.30	7113200	7081932	6867917	7161629	7193780.49	6923434	7372884	7002665	7219449
[16,]	2718895.80	2499180	2499180	2499180	2489010	2555994.71	2499180	2555995	2499180	2489010
[17,]	2730760.86	2650304	2664764	2648092	2631377	2702707.64	2637535	2719694	2644203	2637116
[18,]	2742677.70	2520982	2520982	2520982	2510711	2578310.13	2520982	2578310	2520982	2510711
[19,]	2754646.55	2531955	2531955	2531955	2521633	2589540.80	2531955	2589541	2531955	2521633
[20,]	2766667.64	2542975	2542975	2542975	2532602	2600820.40	2542975	2600820	2542975	2532602
[21,]	2778034.17	2554043	2554043	2554043	2543619	2612149.14	2554043	2612149	2554043	2543619
[22,]	2632038.66	2565159	2565159	2565159	9618040	2623527.22	2565159	2623527	2565159	2554683
[23,]	2643504.45	2576323	2576323	2576323	2565796	2634954.88	2576323	2634955	2576323	2565796
[24,]	2659918.67	2592435	2592435	2592435	2581856	2651330.81	2592435	2651331	2592435	2581856
[25,]	2671506.11	2598798	2598798	2603718	2588167	2657959.74	2603718	2662880	2598798	10108971
[26,]	2683144.06	2610109	2610109	2615051	2599426	2669537.40	2615051	2674479	2610109	2655378
[27,]	10230740.97	10157378	10157378	10162341	2610733	10217073.76	10162341	10222037	10157378	2610733
[28,]	2695783.17	2622090	2622090	2627075	2622090	2682055.14	2627075	2687040	2622090	2622090
[29,]	2707520.71	2633496	2633496	2638503	2633496	2693731.58	2638503	2698739	2633496	2633496
[30,]	2719309.35	2644952	2644952	2649981	2644952	2705458.85	2649981	2710488	2644952	2644952
[31,]	2731149.31	2656457	2656457	2661509	2656457	2717237.17	2661509	2722289	2656457	2656457
[32,]	2743040.83	2668013	2668013	2673086	2668013	2729066.78	2673086	2734140	2668013	2668013
[33,]	2754984.13	2679619	2679619	2684715	2679619	2740947.88	2684715	2746044	2679619	2679619
[34,]	2766979.44	2691275	2691275	2696393	0	2752880.73	2696393	2757999	2691275	2691275
[35,]	2779026.96	2702982	2702982	2708123	0	2764865.50	2708123	2770006	2702982	2702982
[36,]	2791126.95	4938102	4938102	2719904	2223362	5000264.03	2719904	2782066	4938102	4938102
[37,]	2803279.62	2726549	2726549	2731735	0	2788991.83	2731735	2794178	2726549	2726549
[38,]	2815485.22	2738410	2738410	2743619	0	2801133.84	2743619	2806343	2738410	2738410
[39,]	2827743.95	2750322	2750322	2755554	0	2813328.70	2755554	2818561	2750322	2750322
[40,]	2840056.05	2762286	2762286	2767541	0	2825576.65	2767541	2830832	2762286	2762286
[41,]	2847143.70	2774302	2774302	2774302	0	2837877.94	2774302	2774302	2774302	2774302
[42,]	2859540.04	2786370	2786370	2786370	0	2850232.78	2786370	2786370	2786370	2786370
[43,]	2871990.35	2798490	2798490	2798490	0	2862641.40	2798490	2798490	2798490	2798490
[44,]	2884494.86	2810664	2810664	2810664	0	2875104.04	2810664	2810664	2810664	2810664
[45,]	2897053.84	2822890	2822890	2822890	2743801	2887620.95	2822890	2822890	2822890	2822890
[46,]	2909667.49	2835170	2835170	2835170	0	2900192.35	2835170	2835170	2835170	2835170
[47,]	2910003.07	2835170	2835170	2835170	0	2900485.49	2835170	2835170	2835170	2835170
[48,]	75170.29	0	0	0	0	65610.08	0	0	0	0
[49,]	75213.11	0	0	0	0	65610.08	0	0	0	0
[50,]	9646.05	0	0	0	0	0.00	0	0	0	0

Figure 116: Trust and Tranche Allocations

	> as.matrix(MFtrust[,10])	> AAArembal	> AJrembal	> AMrembal	> AArembal	> Arembal	> BBBrembal	> BBBminusrembal
	[,1]	[,1]	[,1]	[,1]	[,1]	[,1]	[,1]	[,1]
[1,]	2561462	1.195717e+08	17109769	12901879	3564535	4764305.6	7841977	5251169
[2,]	2558098	1.170136e+08	17109769	12901879	3564535	4764305.6	7841977	5251169
[3,]	2569261	1.144443e+08	17109769	12901879	3564535	4764305.6	7841977	5251169
[4,]	2572620	1.118717e+08	17109769	12901879	3564535	4764305.6	7841977	5251169
[5,]	2583846	1.092879e+08	17109769	12901879	3564535	4764305.6	7841977	5251169
[6,]	2447282	1.068406e+08	17109769	12901879	3564535	4764305.6	7841977	5251169
[7,]	2457942	1.043826e+08	17109769	12901879	3564535	4764305.6	7841977	5251169
[8,]	2468648	1.019140e+08	17109769	12901879	3564535	4764305.6	7841977	5251169
[9,]	2479401	9.943459e+07	17109769	12901879	3564535	4764305.6	7841977	5251169
[10,]	2490201	9.694439e+07	17109769	12901879	3564535	4764305.6	7841977	5251169
[11,]	2501048	9.444334e+07	17109769	12901879	3564535	4764305.6	7841977	5251169
[12,]	2511942	9.193140e+07	17109769	12901879	3564535	4764305.6	7841977	5251169
[13,]	27970741	6.396066e+07	17109769	12901879	3564535	4764305.6	7841977	5251169
[14,]	2523801	6.143685e+07	17109769	12901879	3564535	4764305.6	7841977	5251169
[15,]	7219449	5.421741e+07	17109769	12901879	3564535	4764305.6	7841977	5251169
[16,]	2489010	5.172840e+07	17109769	12901879	3564535	4764305.6	7841977	5251169
[17,]	2637116	4.909128e+07	17109769	12901879	3564535	4764305.6	7841977	5251169
[18,]	2510711	4.658057e+07	17109769	12901879	3564535	4764305.6	7841977	5251169
[19,]	2521633	4.405894e+07	17109769	12901879	3564535	4764305.6	7841977	5251169
[20,]	2532602	4.152634e+07	17109769	12901879	3564535	4764305.6	7841977	5251169
[21,]	2543619	3.898272e+07	17109769	12901879	3564535	4764305.6	7841977	5251169
[22,]	2554683	3.642803e+07	17109769	12901879	3564535	4764305.6	7841977	5251169
[23,]	2565796	3.386224e+07	17109769	12901879	3564535	4764305.6	7841977	5251169
[24,]	2581856	3.128038e+07	17109769	12901879	3564535	4764305.6	7841977	5251169
[25,]	10108971	2.117141e+07	17109769	12901879	3564535	4764305.6	7841977	5251169
[26,]	2655378	1.851603e+07	17109769	12901879	3564535	4764305.6	7841977	5251169
[27,]	2610733	1.590530e+07	17109769	12901879	3564535	4764305.6	7841977	5251169
[28,]	2622090	1.328321e+07	17109769	12901879	3564535	4764305.6	7841977	5251169
[29,]	2633496	1.064971e+07	17109769	12901879	3564535	4764305.6	7841977	5251169
[30,]	2644952	8.004762e+06	17109769	12901879	3564535	4764305.6	7841977	5251169
[31,]	2656457	5.348305e+06	17109769	12901879	3564535	4764305.6	7841977	5251169
[32,]	2668013	2.680292e+06	17109769	12901879	3564535	4764305.6	7841977	5251169
[33,]	2679619	6.732211e+02	17109769	12901879	3564535	4764305.6	7841977	5251169
[34,]	2691275	0.000000e+00	14419167	12901879	3564535	4764305.6	7841977	5251169
[35,]	2702982	0.000000e+00	11716185	12901879	3564535	4764305.6	7841977	5251169
[36,]	4938102	0.000000e+00	6778083	12901879	3564535	4764305.6	7841977	5251169
[37,]	2726549	0.000000e+00	4051534	12901879	3564535	4764305.6	7841977	5251169
[38,]	2738410	0.000000e+00	1313125	12901879	3564535	4764305.6	7841977	5251169
[39,]	2750322	0.000000e+00	0	11464681	3564535	4764305.6	7841977	5251169
[40,]	2762286	0.000000e+00	0	8702396	3564535	4764305.6	7841977	5251169
[41,]	2774302	0.000000e+00	0	5928094	3564535	4764305.6	7841977	5251169
[42,]	2786370	0.000000e+00	0	3141725	3564535	4764305.6	7841977	5251169
[43,]	2798490	0.000000e+00	0	343234	3564535	4764305.6	7841977	5251169
[44,]	2810664	0.000000e+00	0	0	1097105	4764305.6	7841977	5251169
[45,]	2822890	0.000000e+00	0	0	0	3038520.5	7841977	5251169
[46,]	2835170	0.000000e+00	0	0	0	203350.7	7841977	5251169
[47,]	2835170	0.000000e+00	0	0	0	0.0	5210158	5251169
[48,]	0	0.000000e+00	0	0	0	0.0	5210158	5251169
[49,]	0	0.000000e+00	0	0	0	0.0	5210158	5251169
[50,]	0	0.000000e+00	0	0	0	0.0	5210158	5251169

Figure 117: Trust and AAA allocation 10 sims

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> MFtrust[1:50,]
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	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]
[1,]	2561462.48	2561462	2561462	2561462	2561462	2561462.48	2561462	2561462	2561462	2561462
[2,]	2558098.23	2558098	2558098	2558098	2558098	2558098.23	2558098	2558098	2558098	2558098
[3,]	2569261.48	2569261	2569261	2569261	2569261	2569261.48	2569261	2569261	2569261	2569261
[4,]	2580473.46	2518791	2518791	2518791	2518791	2572619.90	2518791	2572620	2518791	2572620
[5,]	2591734.35	2529774	2529774	2529774	2529774	2583845.62	2529774	2583846	2529774	2583846
[6,]	2603044.41	2392967	2392967	2392967	2392967	2447281.75	2392967	2447282	2392967	2447282
[7,]	2614403.83	2403382	2403382	2403382	2403382	2457941.63	2403382	2457942	2403382	2457942
[8,]	2625812.83	2413842	2413842	2413842	2413842	2468647.96	2413842	2468648	2413842	2468648
[9,]	2637271.62	2424348	2424348	2424348	2424348	2479400.91	2424348	2479401	2424348	2479401
[10,]	2648780.43	2434899	2434899	2434899	2434899	2490200.72	2434899	2490201	2434899	2490201
[11,]	2660339.45	2445497	2445497	2445497	2435573	2501047.56	2445497	2501048	2445497	2501048
[12,]	2671948.95	2456141	2456141	2456141	2446168	2511941.67	2456141	2511942	2456141	2511942
[13,]	28131467.32	27914689	27914689	27914689	27904667	27970741.45	27914689	27970741	27914689	27970741
[14,]	2695320.15	2477567	2477567	2477567	2467496	2533872.44	2477567	2533872	2477567	2523801
[15,]	2707082.30	7113200	7081932	6867917	7161629	7193780.49	6923434	7372884	7002665	7219449
[16,]	2718895.80	2499180	2499180	2499180	2489010	2555994.71	2499180	2555995	2499180	2489010
[17,]	2730760.86	2650304	2664764	2648092	2631377	2702707.64	2637535	2719694	2644203	2637116
[18,]	2742677.70	2520982	2520982	2520982	2510711	2578310.13	2520982	2578310	2520982	2510711
[19,]	2754646.55	2531955	2531955	2531955	2521633	2589540.80	2531955	2589541	2531955	2521633
[20,]	2766667.64	2542975	2542975	2542975	2532602	2600820.40	2542975	2600820	2542975	2532602
[21,]	2778034.17	2554043	2554043	2554043	2543619	2612149.14	2554043	2612149	2554043	2543619
[22,]	2632038.66	2565159	2565159	2565159	9618040	2623527.22	2565159	2623527	2565159	2554683
[23,]	2643504.45	2576323	2576323	2576323	2565796	2634954.88	2576323	2634955	2576323	2565796
[24,]	2659918.67	2592435	2592435	2592435	2581856	2651330.81	2592435	2651331	2592435	2581856
[25,]	2671506.11	2598798	2598798	2603718	2588167	2657959.74	2603718	2662880	2598798	10108971
[26,]	2683144.06	2610109	2610109	2615051	2599426	2669537.40	2615051	2674479	2610109	2655378
[27,]	10230740.97	10157378	10157378	10162341	2610733	10217073.76	10162341	10222037	10157378	2610733
[28,]	2695783.17	2622090	2622090	2627075	2622090	2682055.14	2627075	2687040	2622090	2622090
[29,]	2707520.71	2633496	2633496	2638503	2633496	2693731.58	2638503	2698739	2633496	2633496
[30,]	2719309.35	2644952	2644952	2649981	2644952	2705458.85	2649981	2710488	2644952	2644952
[31,]	2731149.31	2656457	2656457	2661509	2656457	2717237.17	2661509	2722289	2656457	2656457
[32,]	2743040.83	2668013	2668013	2673086	2668013	2729066.78	2673086	2734140	2668013	2668013
[33,]	2754984.13	2679619	2679619	2684715	2679619	2740947.88	2684715	2746044	2679619	2679619
[34,]	2766979.44	2691275	2691275	2696393	0	2752880.73	2696393	2757999	2691275	2691275
[35,]	2779026.96	2702982	2702982	2708123	0	2764865.50	2708123	2770006	2702982	2702982
[36,]	2791126.95	4938102	4938102	2719904	2223362	5000264.03	2719904	2782066	4938102	4938102
[37,]	2803279.62	2726549	2726549	2731735	0	2788991.83	2731735	2794178	2726549	2726549
[38,]	2815485.22	2738410	2738410	2743619	0	2801133.84	2743619	2806343	2738410	2738410
[39,]	2827743.95	2750322	2750322	2755554	0	2813328.70	2755554	2818561	2750322	2750322
[40,]	2840056.05	2762286	2762286	2767541	0	2825576.65	2767541	2830832	2762286	2762286
[41,]	2847143.70	2774302	2774302	2774302	0	2837877.94	2774302	2774302	2774302	2774302
[42,]	2859540.04	2786370	2786370	2786370	0	2850232.78	2786370	2786370	2786370	2786370
[43,]	2871990.35	2798490	2798490	2798490	0	2862641.40	2798490	2798490	2798490	2798490
[44,]	2884494.86	2810664	2810664	2810664	0	2875104.04	2810664	2810664	2810664	2810664
[45,]	2897053.84	2822890	2822890	2822890	2743801	2887620.95	2822890	2822890	2822890	2822890
[46,]	2909667.49	2835170	2835170	2835170	0	2900192.35	2835170	2835170	2835170	2835170
[47,]	2910003.07	2835170	2835170	2835170	0	2900485.49	2835170	2835170	2835170	2835170
[48,]	75170.29	0	0	0	0	65610.08	0	0	0	0
[49,]	75213.11	0	0	0	0	65610.08	0	0	0	0
[50,]	9646.05	0	0	0	0	0.00	0	0	0	0

```
> AAAcapture[1:50,]
```

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]
[1,]	2561462.5	2561462.5	2561462.5	2561462.5	2561462	2561462	2561462.5	2561462	2561462.5	2.561462e+06
[2,]	2558098.2	2558098.2	2558098.2	2558098.2	2558098	2558098	2558098.2	2558098	2558098.2	2.558098e+06
[3,]	2569261.5	2569261.5	2569261.5	2569261.5	2569261	2569261	2569261.5	2569261	2569261.5	2.569261e+06
[4,]	2580473.5	2518791.1	2518791.1	2518791.1	2518791	2572620	2518791.1	2572620	2518791.1	2.572620e+06
[5,]	2591734.4	2529774.1	2529774.1	2529774.1	2529774	2583846	2529774.1	2583846	2529774.1	2.583846e+06
[6,]	2603044.4	2392966.5	2392966.5	2392966.5	2392967	2447282	2392966.5	2447282	2392966.5	2.447282e+06
[7,]	2614403.8	2403381.5	2403381.5	2403381.5	2403382	2457942	2403381.5	2457942	2403381.5	2.457942e+06
[8,]	2625812.8	2413841.9	2413841.9	2413841.9	2413842	2468648	2413841.9	2468648	2413841.9	2.468648e+06
[9,]	2637271.6	2424347.7	2424347.7	2424347.7	2424348	2479401	2424347.7	2479401	2424347.7	2.479401e+06
[10,]	2648780.4	2434899.4	2434899.4	2434899.4	2434899	2490201	2434899.4	2490201	2434899.4	2.490201e+06
[11,]	2660339.4	2445496.9	2445496.9	2445496.9	2435573	2501048	2445496.9	2501048	2445496.9	2.501048e+06
[12,]	2671949.0	2456140.5	2456140.5	2456140.5	2446168	2511942	2456140.5	2511942	2456140.5	2.511942e+06
[13,]	28131467.3	27914688.8	27914688.8	27914688.8	27904667	27970741	27914688.8	27970741	27914688.8	2.797074e+07
[14,]	2695320.1	2477567.0	2477567.0	2477567.0	2467496	2533872	2477567.0	2533872	2477567.0	2.523801e+06
[15,]	2707082.3	7113200.4	7081932.3	6867917.1	7161629	7193780	6923433.9	7372884	7002664.8	7.219449e+06
[16,]	2718895.8	2499180.5	2499180.5	2499180.5	2489010	2555995	2499180.5	2555995	2499180.5	2.489010e+06
[17,]	2730760.9	2650304.2	2664764.4	2648091.9	2631377	2702708	2637535.2	2719694	2644203.2	2.637116e+06
[18,]	2742677.7	2520982.5	2520982.5	2520982.5	2510711	2578310	2520982.5	2578310	2520982.5	2.510711e+06
[19,]	2754646.5	2531954.7	2531954.7	2531954.7	2521633	2589541	2531954.7	2589541	2531954.7	2.521633e+06
[20,]	2766667.6	2542974.7	2542974.7	2542974.7	2532602	2600820	2542974.7	2600820	2542974.7	2.532602e+06
[21,]	2778034.2	2554042.6	2554042.6	2554042.6	2543619	2612149	2554042.6	2612149	2554042.6	2.543619e+06
[22,]	2632038.7	2565158.7	2565158.7	2565158.7	9618040	2623527	2565158.7	2623527	2565158.7	2.554683e+06
[23,]	2643504.4	2576323.2	2576323.2	2576323.2	2565796	2634955	2576323.2	2634955	2576323.2	2.565796e+06
[24,]	2659918.7	2592434.9	2592434.9	2592434.9	2581856	2651331	2592434.9	2651331	2592434.9	2.581856e+06
[25,]	2671506.1	2598798.3	2598798.3	2603718.3	2588167	2657960	2603718.3	2662880	2598798.3	1.010897e+07
[26,]	2683144.1	2610109.2	2610109.2	2615050.9	2599426	2669537	2615050.9	2674479	2610109.2	2.655378e+06
[27,]	10230741.0	10157377.6	10157377.6	10162341.1	2610733	10217074	10162341.1	10222037	10157377.6	2.610733e+06
[28,]	2695783.2	2622089.9	2622089.9	2627075.2	2622090	2682055	2627075.2	2687040	2622089.9	2.622090e+06
[29,]	2707520.7	2633496.0	2633496.0	2638503.2	2633496	2693732	2638503.2	2698739	2633496.0	2.633496e+06
[30,]	2719309.4	2644951.7	2644951.7	2649980.9	2644952	2705459	2649980.9	2710488	2644951.7	2.644952e+06
[31,]	2731149.3	2656457.2	2656457.2	2661508.6	2656457	2717237	2661508.6	2722289	2656457.2	2.656457e+06
[32,]	2743040.8	2668012.8	2668012.8	2673086.4	2668013	2729067	2673086.4	2734140	2668012.8	2.668013e+06
[33,]	2754984.1	2679618.7	2679618.7	2684714.6	2679619	1611546	2684714.6	1375484	2679618.7	2.679619e+06
[34,]	912322.5	614961.6	631769.4	817389.4	0	0	772429.3	0	731598.2	6.732211e+02
[35,]	0.0	0.0	0.0	0.0	0	0	0.0	0	0.0	0.000000e+00
[36,]	0.0	0.0	0.0	0.0	1213196	0	0.0	0	0.0	0.000000e+00
[37,]	0.0	0.0	0.0	0.0	0	0	0.0	0	0.0	0.000000e+00
[38,]	0.0	0.0	0.0	0.0	0	0	0.0	0	0.0	0.000000e+00
[39,]	0.0	0.0	0.0	0.0	0	0	0.0	0	0.0	0.000000e+00
[40,]	0.0	0.0	0.0	0.0	0	0	0.0	0	0.0	0.000000e+00
[41,]	0.0	0.0	0.0	0.0	0	0	0.0	0	0.0	0.000000e+00
[42,]	0.0	0.0	0.0	0.0	0	0	0.0	0	0.0	0.000000e+00
[43,]	0.0	0.0	0.0	0.0	0	0	0.0	0	0.0	0.000000e+00
[44,]	0.0	0.0	0.0	0.0	0	0	0.0	0	0.0	0.000000e+00
[45,]	0.0	0.0	0.0	0.0	0	0	0.0	0	0.0	0.000000e+00
[46,]	0.0	0.0	0.0	0.0	0	0	0.0	0	0.0	0.000000e+00
[47,]	0.0	0.0	0.0	0.0	0	0	0.0	0	0.0	0.000000e+00
[48,]	0.0	0.0	0.0	0.0	0	0	0.0	0	0.0	0.000000e+00
[49,]	0.0	0.0	0.0	0.0	0	0	0.0	0	0.0	0.000000e+00
[50,]	0.0	0.0	0.0	0.0	0	0	0.0	0	0.0	0.000000e+00

Figure 118 – AAA Rembal

```
> AAArembal
              [,1]
 [1,] 1.195717e+08
 [2,] 1.170136e+08
 [3,] 1.144443e+08
 [4,] 1.118717e+08
 [5,] 1.092879e+08
 [6,] 1.068406e+08
 [7,] 1.043826e+08
 [8,] 1.019140e+08
 [9,] 9.943459e+07
[10,] 9.694439e+07
[11,] 9.444334e+07
[12,] 9.193140e+07
[13,] 6.396066e+07
[14,] 6.143685e+07
[15,] 5.421741e+07
[16,] 5.172840e+07
[17,] 4.909128e+07
[18,] 4.658057e+07
[19,] 4.405894e+07
[20,] 4.152634e+07
[21,] 3.898272e+07
[22,] 3.642803e+07
[23,] 3.386224e+07
[24,] 3.128038e+07
[25,] 2.117141e+07
[26,] 1.851603e+07
[27,] 1.590530e+07
[28,] 1.328321e+07
[29,] 1.064971e+07
[30,] 8.004762e+06
[31,] 5.348305e+06
[32,] 2.680292e+06
[33,] 6.732211e+02
[34,] 0.000000e+00
```

Figure 119: Cashflows promised by Tranche

	AAA_prin	AS_prin	AA_prin	A_prin	BBBmin_prin	BB_prin	Unrated
max	9,171,411,251.00	1,064,086,867.00	798,065,150.00	532,043,433.00	665,054,292.00	266,021,717.00	727,320,157.72
actual	9,171,411,246.00	1,064,086,867.00	798,065,150.00	532,043,433.00	665,054,292.00	266,021,717.00	764,949,012.36

Figure 120 – Promised Cashflow allocation to the Bond Capital Structure

```
cbind(apply(AAAcapture,1,mean),apply(AScapture,1,mean),apply(AAcapture,1,mean),a
```

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]
[1,]	10953123	0	0	0	0	0	0.00
[2,]	11067599	0	0	0	0	0	0.00
[3,]	11112896	0	0	0	0	0	0.00
[4,]	11158381	0	0	0	0	0	0.00
[5,]	11253483	0	0	0	0	0	0.00
[6,]	11306745	0	0	0	0	0	0.00
[7,]	11425100	0	0	0	0	0	0.00
[8,]	11591366	0	0	0	0	0	0.00
[9,]	11840766	0	0	0	0	0	0.00
10,]	12118292	0	0	0	0	0	0.00
11,]	12317406	0	0	0	0	0	0.00
12,]	12528421	0	0	0	0	0	0.00
13,]	12712917	0	0	0	0	0	0.00
14,]	12764761	0	0	0	0	0	0.00
15,]	12816819	0	0	0	0	0	0.00
16,]	12890975	0	0	0	0	0	0.00
17,]	13252341	0	0	0	0	0	0.00
18,]	13484154	0	0	0	0	0	0.00
19,]	14127295	0	0	0	0	0	0.00
20,]	14415323	0	0	0	0	0	0.00
21,]	14563556	0	0	0	0	0	0.00
22,]	14664532	0	0	0	0	0	0.00
23,]	14823114	0	0	0	0	0	0.00
24,]	14967310	0	0	0	0	0	0.00
25,]	15028359	0	0	0	0	0	0.00
26,]	15089660	0	0	0	0	0	0.00
27,]	15151213	0	0	0	0	0	0.00
28,]	15213020	0	0	0	0	0	0.00
29,]	15318210	0	0	0	0	0	0.00
30,]	15380694	0	0	0	0	0	0.00
31,]	15490974	0	0	0	0	0	0.00
32,]	15724480	0	0	0	0	0	0.00
33,]	15876092	0	0	0	0	0	0.00
34,]	16081918	0	0	0	0	0	0.00
35,]	16147480	0	0	0	0	0	0.00
36,]	16574274	0	0	0	0	0	0.00
37,]	16641475	0	0	0	0	0	0.00
38,]	16708952	0	0	0	0	0	0.00
39,]	16776705	0	0	0	0	0	0.00
40,]	16844736	0	0	0	0	0	0.00
41,]	16913046	0	0	0	0	0	0.00
42,]	16981636	0	0	0	0	0	0.00
43,]	17050507	0	0	0	0	0	0.00
44,]	17119661	0	0	0	0	0	0.00
45,]	35216391	0	0	0	0	0	0.00
46,]	17215728	0	0	0	0	0	0.00
47,]	75817648	0	0	0	0	0	0.00
48,]	35284010	0	0	0	0	0	0.00

Figure 120, cont'd. – Promised Cashflow allocation to the Bond Capital Structure

[49,]	17310285	0	0	0	0	0	0.00
[50,]	54169953	0	0	0	0	0	0.00
[51,]	36164688	0	0	0	0	0	0.00
[52,]	70731153	0	0	0	0	0	0.00
[53,]	35495546	0	0	0	0	0	0.00
[54,]	78396954	0	0	0	0	0	0.00
[55,]	306518042	0	0	0	0	0	0.00
[56,]	115206547	0	0	0	0	0	0.00
[57,]	267580714	0	0	0	0	0	0.00
[58,]	162292895	0	0	0	0	0	0.00
[59,]	98632358	0	0	0	0	0	0.00
[60,]	196931745	0	0	0	0	0	0.00
[61,]	166750390	0	0	0	0	0	0.00
[62,]	16924223	0	0	0	0	0	0.00
[63,]	16992548	0	0	0	0	0	0.00
[64,]	17061152	0	0	0	0	0	0.00
[65,]	17130037	0	0	0	0	0	0.00
[66,]	17199202	0	0	0	0	0	0.00
[67,]	17268651	0	0	0	0	0	0.00
[68,]	17338383	0	0	0	0	0	0.00
[69,]	17408399	0	0	0	0	0	0.00
[70,]	17478702	0	0	0	0	0	0.00
[71,]	17549292	0	0	0	0	0	0.00
[72,]	17620170	0	0	0	0	0	0.00
[73,]	17691338	0	0	0	0	0	0.00
[74,]	17762797	0	0	0	0	0	0.00
[75,]	17834548	0	0	0	0	0	0.00
[76,]	17906591	0	0	0	0	0	0.00
[77,]	17978930	0	0	0	0	0	0.00
[78,]	18051564	0	0	0	0	0	0.00
[79,]	18124494	0	0	0	0	0	0.00
[80,]	51542256	0	0	0	0	0	0.00
[81,]	18273781	0	0	0	0	0	0.00
[82,]	38948311	0	0	0	0	0	0.00
[83,]	56866599	0	0	0	0	0	0.00
[84,]	18440810	0	0	0	0	0	0.00
[85,]	18515324	0	0	0	0	0	0.00
[86,]	18590142	0	0	0	0	0	0.00
[87,]	18781260	0	0	0	0	0	0.00
[88,]	150624705	0	0	0	0	0	0.00
[89,]	18816438	0	0	0	0	0	0.00
[90,]	18892487	0	0	0	0	0	0.00
[91,]	18968847	0	0	0	0	0	0.00
[92,]	19045520	0	0	0	0	0	0.00
[93,]	19122506	0	0	0	0	0	0.00
[94,]	19199807	0	0	0	0	0	0.00
[95,]	19277424	0	0	0	0	0	0.00
[96,]	19355358	0	0	0	0	0	0.00
[97,]	19433611	0	0	0	0	0	0.00
[98,]	19512185	0	0	0	0	0	0.00
[99,]	19591079	0	0	0	0	0	0.00
100,]	19670296	0	0	0	0	0	0.00
101,]	19749837	0	0	0	0	0	0.00
102,]	19829704	0	0	0	0	0	0.00

Figure 120, cont'd – Promised Cashflow allocation to the Bond Capital Structure

[102,]	19829704	0	0	0	0	0	0.00
[103,]	19909897	0	0	0	0	0	0.00
[104,]	19990419	0	0	0	0	0	0.00
[105,]	96886219	0	0	0	0	0	0.00
[106,]	23286472	0	0	0	0	0	0.00
[107,]	93719509	0	0	0	0	0	0.00
[108,]	111931662	0	0	0	0	0	0.00
[109,]	57423186	0	0	0	0	0	0.00
[110,]	284117167	0	0	0	0	0	0.00
[111,]	288768770	0	0	0	0	0	0.00
[112,]	435308641	0	0	0	0	0	0.00
[113,]	859780337	0	0	0	0	0	0.00
[114,]	659710591	0	0	0	0	0	0.00
[115,]	1238538724	0	0	0	0	0	0.00
[116,]	970766442	0	0	0	0	0	0.00
[117,]	638919088	545386402	0	0	0	0	0.00
[118,]	0	518700465	798065150	532043433	1715071	0	0.00
[119,]	0	0	0	0	663339221	266021717	12094092.99
[120,]	0	0	0	0	0	0	704243118.05
[121,]	0	0	0	0	0	0	40732737.78
[122,]	0	0	0	0	0	0	21026.58
[123,]	0	0	0	0	0	0	21109.81
[124,]	0	0	0	0	0	0	7836927.15
[125,]	0	0	0	0	0	0	0.00
[126,]	0	0	0	0	0	0	0.00
[127,]	0	0	0	0	0	0	0.00
[128,]	0	0	0	0	0	0	0.00
[129,]	0	0	0	0	0	0	0.00
[130,]	0	0	0	0	0	0	0.00
[131,]	0	0	0	0	0	0	0.00
[132,]	0	0	0	0	0	0	0.00
[133,]	0	0	0	0	0	0	0.00
[134,]	0	0	0	0	0	0	0.00
[135,]	0	0	0	0	0	0	0.00
[136,]	0	0	0	0	0	0	0.00
[137,]	0	0	0	0	0	0	0.00
[138,]	0	0	0	0	0	0	0.00
[139,]	0	0	0	0	0	0	0.00
[140,]	0	0	0	0	0	0	0.00
[141,]	0	0	0	0	0	0	0.00
[142,]	0	0	0	0	0	0	0.00
[143,]	0	0	0	0	0	0	0.00
[144,]	0	0	0	0	0	0	0.00
[145,]	0	0	0	0	0	0	0.00

Figure 121: Cashflows promised by tranche

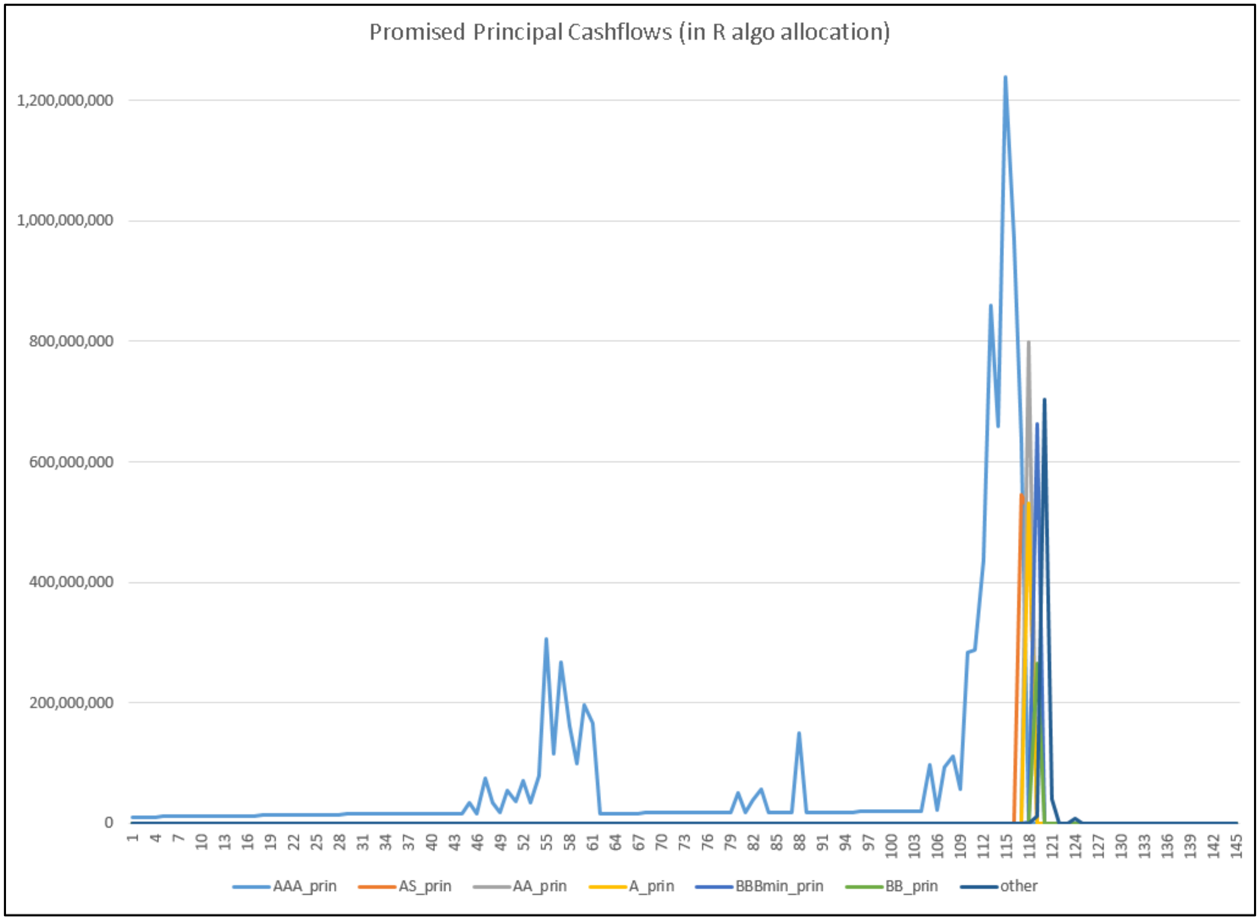


Figure 122: Default adjusted cashflows

```
> cbind(LEMFdefprinCF[,8],LEMFrecoveryCF3[,8],LEMFdefadjprinCF[,8])
          [,1]    [,2]        [,3]
 [1,] 7539.282       0    7539.282
 [2,] 7570.680       0    7570.680
 [3,] 7602.209       0    7602.209
 [4,] 7633.869       0    7633.869
 [5,] 7665.661       0    7665.661
 [6,] 7697.585       0    7697.585
 [7,] 7729.642       0    7729.642
 [8,] 7761.833       0    7761.833
 [9,] 7794.158       0    7794.158
[10,] 7826.617       0    7826.617
[11,] 7859.212       0    7859.212
[12,] 7891.942       0    7891.942
[13,] 7924.809       0    7924.809
[14,] 7957.812       0    7957.812
[15,] 7990.953       0    7990.953
[16,] 8024.232       0    8024.232
[17,] 8057.650       0    8057.650
[18,] 8091.207       0    8091.207
[19,] 8124.903       0    8124.903
[20,] 8158.740       0    8158.740
[21,] 8192.718       0    8192.718
[22,] 8226.837       0    8226.837
[23,] 8261.098       0    8261.098
[24,] 8295.502       0    8295.502
[25,] 8330.050       0    8330.050
[26,] 8364.741       0    8364.741
[27,] 8399.577       0    8399.577
[28,] 8434.557       0    8434.557
[29,] 8469.684       0    8469.684
[30,]    0.000       0       0.000
[31,]    0.000       0       0.000
[32,]    0.000 5597249 5597248.747
```

Figure 123: Promised cashflows versus default adjusted (3/7/2014)

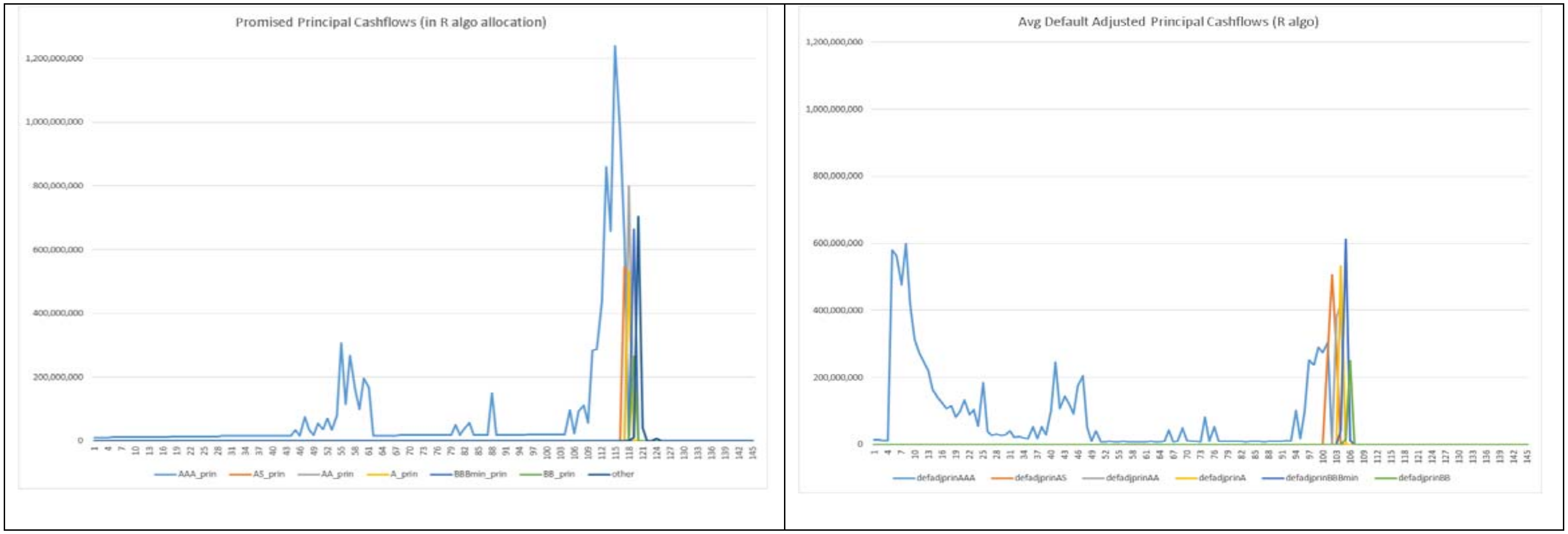
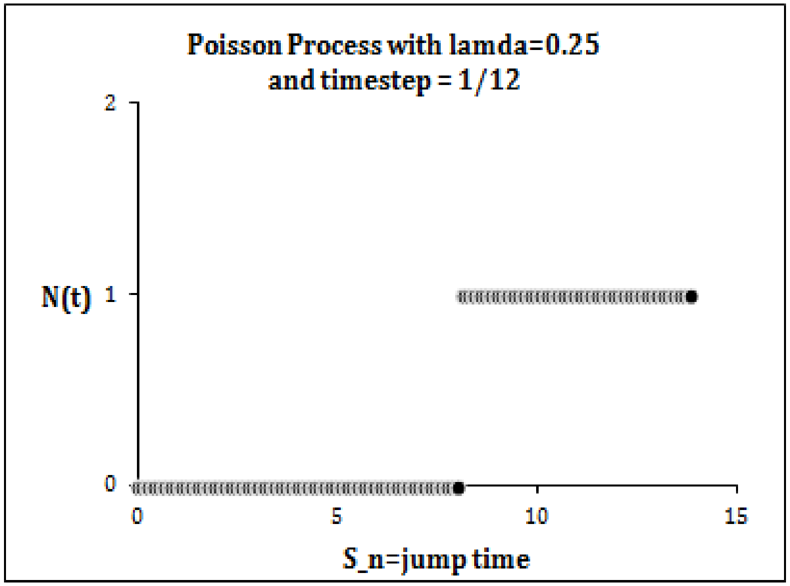


Figure 124: Promised vs. default adjusted (aggregated)

AAA_prinorig	AS_prinorig	AA_prinorig	A_prinorig	BBBmin_prinorig	BB_prinorig
9,171,411,251.00	1,064,086,867.00	798,065,150.00	532,043,433.00	665,054,292.00	266,021,717.00
AAA_defadjprin	AS_defadjprin	AA_defadjprin	A_defadjprin	BBBmin_defadjprin	BB_defadjprin
9,003,572,922.00	1,064,086,867.00	798,065,150.10	532,043,433.40	665,054,291.80	265,666,217.07

Figure 125: Poisson Process as a counting process



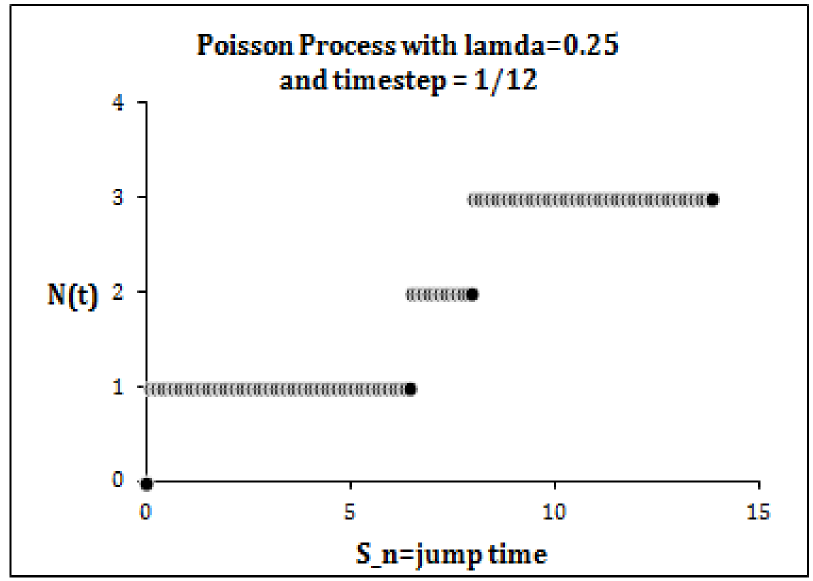


Figure 126: One path of a Compound Poisson Process (from Shreve)

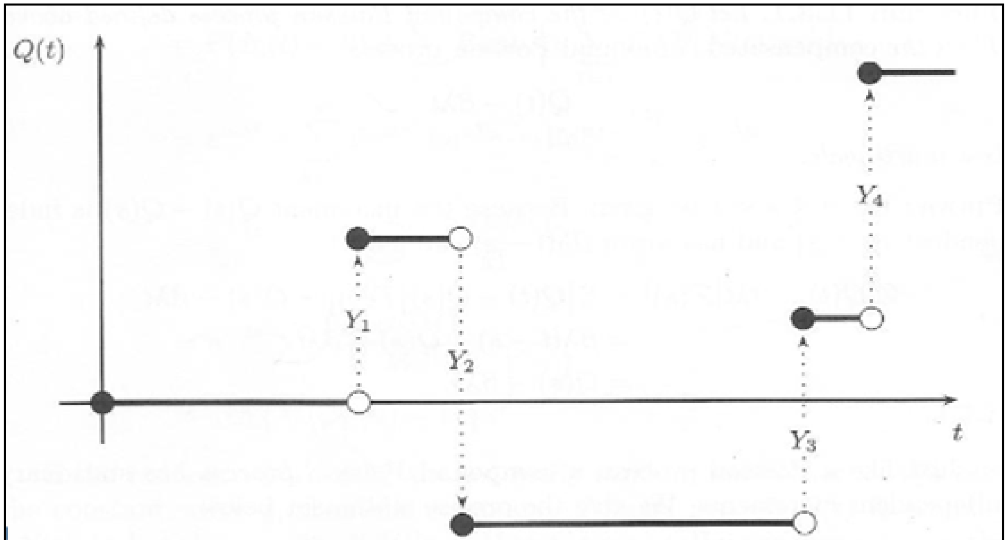


Fig. 11.3.1. One path of a compound Poisson process.

Figure 127: Simulated one path of the Compound Poisson Process (from Tankov/Voltchkova)

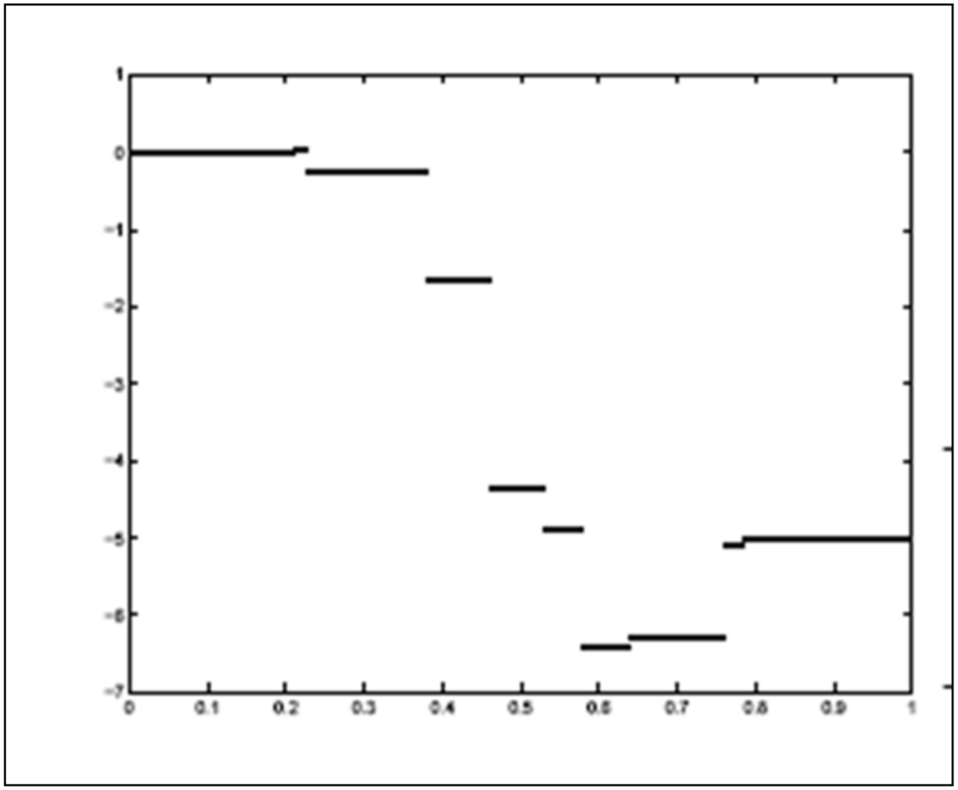


Figure 128: Merton's Jump Diffusion Process (from Tankov/Voltchkova)

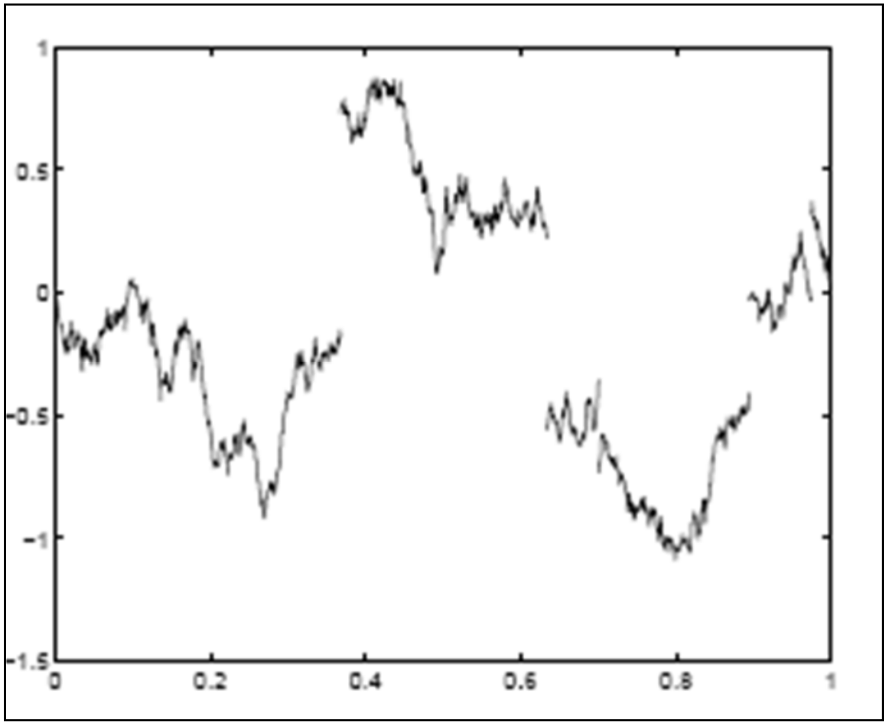


Figure 129: Lambda Current and Delinquent Switching Realizations in Simulation

```
  dlqtry
      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13] [,14] [,15] [,16] [,17] [,18] [,19] [,20]
 [1,]    0    0    1    0    1    0    0    0    1     0     0     1     0     1     0     0     1     0     0     1
 [2,]    0    1    1    0    0    1    0    0    1     0     1     1     0     0     0     0     0     0     1     0
 [3,]    1    0    0    1    1    1    0    0    1     0     0     1     0     1     1     0     0     0     0     1
 [4,]    0    1    0    0    0    1    0    1    0     0     1     0     1     1     0     1     0     0     0     0
 [5,]    0    1    0    0    0    0    0    0    1     0     0     1     1     1     0     0     1     0     0     1
 [6,]    0    0    0    0    0    0    1    0    0     0     0     1     1     1     0     0     0     1     0     0
 [7,]    1    1    0    1    0    1    0    0    0     0     0     0     1     0     0     1     1     0     0     0
 [8,]    1    1    1    0    1    0    0    0    0     0     1     1     0     0     0     0     0     0     0     0
 [9,]    0    1    0    1    0    1    0    0    1     0     1     0     1     1     1     0     0     1     0     1
[10,]    1    0    1    0    1    1    0    0    0     0     1     1     1     1     0     0     0     1     1     0
[11,]    0    1    1    0    1    1    0    1    0     0     0     1     0     0     1     1     1     0     0     1
[12,]    1    1    0    1    1    1    0    0    0     1     0     1     0     0     1     1     1     0     1     0
[13,]    0    0    1    1    0    0    1    0    0     0     1     0     0     1     1     0     0     1     0     0

> apply(dlqtry,1,sum)
  [1]  92  89  88  94  96  83  87  92  88  87 104  95 100  96  90  94  99  96  75  89  94  94  91  91  78  95  90  86 102
 [30]  92  93  85  98  97  89  97  82  86  85  95  91  86  97  87 108  87  90  92 101  88  93  79 101  85  92  94  90  86
 [59]  95  92  85  86  88  95  87  87 102  98  92 102  93  80  91  87  90  92  74  90  84  87  93  92  95  99  98 100 101
 [88]  94  94  89  90  86  85  98 100  93  85  94 100 111  84  91  82  90  86 102  80  96  96  90  97  77  85 109  93  96
[117]  94 102 101  86
> sum(apply(dlqtry,1,sum))
[1] 11012
> sum(apply(dlqtry,1,sum))/(250*120)
[1] 0.3670667
> apply(dlqtry,1,mean)
  [1] 0.368 0.356 0.352 0.376 0.384 0.332 0.348 0.368 0.352 0.348 0.416 0.380 0.400 0.384 0.360 0.376 0.396 0.384 0.300
 [20] 0.356 0.376 0.376 0.364 0.364 0.312 0.380 0.360 0.344 0.408 0.368 0.372 0.340 0.392 0.388 0.356 0.388 0.328 0.344
 [39] 0.340 0.380 0.364 0.344 0.388 0.348 0.432 0.348 0.360 0.368 0.404 0.352 0.372 0.316 0.404 0.340 0.368 0.376 0.360
 [58] 0.344 0.380 0.368 0.340 0.344 0.352 0.380 0.348 0.348 0.408 0.392 0.368 0.408 0.372 0.320 0.364 0.348 0.360 0.368
 [77] 0.296 0.360 0.336 0.348 0.372 0.368 0.380 0.396 0.392 0.400 0.404 0.376 0.376 0.356 0.360 0.344 0.340 0.392 0.400
 [96] 0.372 0.340 0.376 0.400 0.444 0.336 0.364 0.328 0.360 0.344 0.408 0.320 0.384 0.384 0.360 0.388 0.308 0.340 0.436
[115] 0.372 0.384 0.376 0.408 0.404 0.344
> apply(dlqtry,1,sum)
  [1]  92  89  88  94  96  83  87  92  88  87 104  95 100  96  90  94  99  96  75  89  94  94  91  91  78  95  90  86 102
 [30]  92  93  85  98  97  89  97  82  86  85  95  91  86  97  87 108  87  90  92 101  88  93  79 101  85  92  94  90  86
 [59]  95  92  85  86  88  95  87  87 102  98  92 102  93  80  91  87  90  92  74  90  84  87  93  92  95  99  98 100 101
 [88]  94  94  89  90  86  85  98 100  93  85  94 100 111  84  91  82  90  86 102  80  96  96  90  97  77  85 109  93  96
[117]  94 102 101  86
> sum(apply(dlqtry,1,sum))
[1] 11012
> sum(apply(dlqtry,1,sum))/(sims*simMos)
[1] 0.3670667
```

Figure 130: Lambda Current and Default Switching Realizations in Simulation

```
> defZdraw[1:10,1:10]
             [,1]      [,2]       [,3]      [,4]       [,5]       [,6]       [,7]       [,8]      [,9]     [,10]
 [1,] 0.161801678 0.4131677 0.93128738 0.7473191 0.33278869 0.56428315 0.89742654 0.28223441 0.5802670 0.1987496
 [2,] 0.904220750 0.1502735 0.72257637 0.7351538 0.75583481 0.44093657 0.40401873 0.01220087 0.8185097 0.8752844
 [3,] 0.461951720 0.2687959 0.96948804 0.7227733 0.31674224 0.38038878 0.50409254 0.76894884 0.5202700 0.6371548
 [4,] 0.429416354 0.7706819 0.18199287 0.2867008 0.62945073 0.13400977 0.27760433 0.33775054 0.2181774 0.5644142
 [5,] 0.462774547 0.4345343 0.31213407 0.3945369 0.74666053 0.49875220 0.41623113 0.81826810 0.9290802 0.3480856
 [6,] 0.387759448 0.6331299 0.16650903 0.8261276 0.08402253 0.35697583 0.45866295 0.66943660 0.5845819 0.6252287
 [7,] 0.347381163 0.2232542 0.48983977 0.2070705 0.72234787 0.17797691 0.01969225 0.37344731 0.8437117 0.4040148
 [8,] 0.002383945 0.6850360 0.60077952 0.6971256 0.12055056 0.91956518 0.94637620 0.56709772 0.2422204 0.9260755
 [9,] 0.790134650 0.6371770 0.04017813 0.9439457 0.32202212 0.36695626 0.84059824 0.39709427 0.8289708 0.1031227
[10,] 0.625990448 0.5497723 0.80578174 0.2299211 0.09686590 0.02053038 0.40134113 0.57366470 0.2475501 0.9141139
> defaultbound[1:10,1:10]
           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]      [,8]      [,9]     [,10]
 [1,] 0.2862096 0.2862096 0.2862096 0.2862096 0.2862096 0.2862096 0.2862096 0.2862096 0.2862096 0.2862096
 [2,] 0.2861989 0.2849438 0.2860790 0.2866399 0.2853873 0.2863828 0.2848082 0.2863265 0.2864765 0.2858101
 [3,] 0.2846526 0.2866620 0.2867481 0.2852731 0.2854829 0.2851759 0.2837226 0.2870972 0.2842593 0.2844479
 [4,] 0.2850404 0.2864559 0.2862597 0.2830951 0.2835646 0.2873556 0.2821227 0.2887980 0.2840666 0.2840966
 [5,] 0.2858223 0.2852983 0.2846154 0.2820854 0.2838015 0.2863431 0.2819903 0.2876154 0.2839227 0.2828652
 [6,] 0.2859909 0.2843546 0.2836718 0.2830619 0.2850092 0.2884589 0.2818667 0.2879561 0.2835012 0.2838259
 [7,] 0.2868498 0.2833660 0.2828278 0.2834334 0.2846018 0.2899793 0.2822945 0.2887248 0.2842595 0.2844421
 [8,] 0.2875629 0.2852492 0.2860244 0.2833589 0.2855208 0.2903511 0.2819712 0.2890400 0.2856339 0.2848445
 [9,] 0.2890805 0.2820211 0.2876673 0.2850430 0.2864725 0.2893289 0.2837932 0.2906035 0.2849618 0.2836693
[10,] 0.2901103 0.2820707 0.2882899 0.2863156 0.2858422 0.2893772 0.2833059 0.2910559 0.2848305 0.2839493
> dlqcurrstate[1:10,1:10]
      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
 [1,]    1    1    1    1    1    1    1    1    1     1
 [2,]    1    1    0    0    1    0    0    1    1     0
 [3,]    1    1    0    1    0    1    1    1    1     0
 [4,]    0    1    1    1    1    1    1    1    1     1
 [5,]    1    0    0    0    0    0    0    0    1     1
 [6,]    0    1    1    0    1    0    1    0    0     0
 [7,]    0    1    1    1    1    1    1    0    0     1
 [8,]    1    0    1    0    1    0    1    0    1     0
 [9,]    1    0    1    0    0    1    1    1    0     1
[10,]    1    0    1    1    1    1    1    0    1     0
> defstate[1:10,1:10]
      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
 [1,]    2    0    0    0    0    0    0    2    0     2
 [2,]    0    2    0    0    0    0    0    2    0     0
 [3,]    0    2    0    0    0    0    0    0    0     0
 [4,]    0    0    2    0    0    2    2    0    2     0
 [5,]    0    0    0    0    0    0    0    0    0     0
 [6,]    0    0    2    0    2    0    0    0    0     0
 [7,]    0    2    0    2    0    2    2    0    0     0
 [8,]    2    0    0    0    2    0    0    0    2     0
 [9,]    0    0    2    0    0    0    0    0    0     2
[10,]    0    0    0    2    2    2    0    0    2     0
```

Curriculum Vitae

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Born:	September 14, 1966; Brooklyn, NY; USA
Secondary:	Metuchen High School; 9/1980-6/1984; High School Diploma
College:	Vassar College; 8/1984-12/1988; Artium Baccalaureus (AB)
Graduate:	Cornell University; 8/1995-5/1987; Master of Business Administration (MBA)
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Professional:	Lehman Brothers; 8/1992-8/1995; Securitization Analyst
	JPMorgan Chase; 8/1997-6/2001; Head of CMBS Research
	WOTN, LLC; 7/2001-9/2006; CEO
	Nomura Holdings America; 6/2007-1/2008; Head of CMBS Risk Management
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Publications:

1.) Jarrow, R.A. (w/ Christopoulos, A.D. & Yildirim, Y.), "Commercial Mortgage Backed Securities (CMBS) & Market Efficiency with Respect to Costly Information", Real Estate Economics, Vol. 36, Issue 3, pp 441-498, Fall 2008.

2.) Christopoulos, etal "Structured finance securities option pricing architecture" U. S. Patent & Trademark Office, Patent # 8788404, 2014.