Description: Building large models to show equivalence: A generalization (classroom view) Date: 1993-10-11 Location: Colts Neck Elementary School Researcher: Professor Carolyn Maher	Transcriber(s): Yankelewitz, Dina Verifier(s): Yedman, Madeline Date Transcribed: Spring 2009 Page: 1 of 22

Line	Time	Speaker	Transcript
11.0.1	3:24	T/R 1:	Well, good morning! I surprised you, I came back! Yeah! I just couldn't stay away. I heard such really wonderful things happened on Friday and I watched a tape on Thursday, so I had to rearrange my schedule so I could be here, and I'm just so happy to be here. Um, I was watching the tapes, you know I do that, and I was reading your papers, and I did that, and I was also talking to some of the people who were here when I wasn't here, and trying to figure out some of the things you were doing. And I understand that, uh, let's see, I think it was on Thursday, that you were working on all sorts of problems, everybody was working on something, I know this group here with Amy James and Jackie, they discovered a secret, they told me, do you remember? Yes, and then I know that there was a group here, I think that was Alan, Erik, David, and Meredith, who were testing a theory? Is that right, you were testing a theory? So we have a secret that we want to hear about, we have a theory that was being tested, and then I know that, um, Andrew's not here?
11.0.2		CT:	Yes he is but [some excuse]
11.0.3	3:45 (S)	T/R 1:	Ok, but he and Jessica had also built a model, right, that was rather interesting for one of the problems, that they were sharing with Brian and Michael. Remember that, and also with Erik was over there, I noticed talking about their model and some other people came over. And um, there were all kinds of interesting things happening, uh, and I read some of the papers of the different kinds of thinking you were doing, and um, Sarah had built a new model that I hadn't seen for one of her problems and Kimberly and Audra and Erin and Jackie, and oh, just such exciting things, I mean, how can we get to all of this? And, um, back there, Graham and Kelly were building their models as well. So I thought that, you know, maybe what we'll do is we'll start with some of the things that confused me and maybe you can help straighten me out. Um, I know that one of the things that happened was when the group built a model on the floor, I looked at that tape, um, on Thursday, and the group built a model to test David's theory which I'm sure David and his friends will share with you, and then the next day, when I think it was David, Erik, and Meredith

			trying to build it again they had some trouble. Uh, and I wondered if they figured it out, I didn't finish watching that tape, but I watched the one the day before. Some of you are already working on different problems by now, now we have Andrew back he can tell us his theory, so I thought you know, maybe we would start with that and then, you know, try to have you all contribute and share and, wouldn't it be nice to know what other people are doing now? Aren't you interested in what other groups are doing and the way they're thinking about some of these problems? I get kind of curious, you know, once I've worked on something after a while, I wonder I say, I wonder how other people are thinking about this, I wonder what they're doing, what do you think, Alan? Do you get sick of it after a while or do you want to be curious about it. You know, it depends, I suppose. Um, do you remember the problem that, uh, I think everybody in this class has now spent a bit of time working on the problem, which is larger, three quarters or two thirds, and, if you decide which one is larger you were asked, by how much. Do you remember, how- Everybody here has worked on it, isn't that true? Raise your hand if you've worked on that problem? I believe everybody, yeah. Some of you started a new one, but everybody has worked on that. Ok, the question, how many of you have built two models for that problem? For that problem, which is larger, three quarters or two thirds, how many of you built two models? [Some students raise hands] Ok, how many of you have built more than two models? [Three raised hands are seen]. So we have some, you know, really differences here and some time to share, and Gregory you built more than two also and Danielle, you've got two models. Um, how many you think there are? How many models do you think you can build? Michael.
11.0.4	6:52	Michael:	Um, if you know what you're doing and you know what strategy, you could probably build, you could probably build one for every single rod.
11.0.5		T/R 1:	What do you mean, one for every single rod, tell me what you mean by that.
11.0.6	7:14	Michael:	You could, you could build a thing, you could build fractions of every single rod if you know what you're doing

			and you have a strategy or a secret that, that you know will work.
11.0.7		T/R 1:	What can be such a secret?
11.0.8		Michael:	That's what I was trying to figure out.
11.0.9		T/R 1:	That's what you were trying to figure out? Does anybody else have any ideas about that? Those of you who built three models, do you think that's all? Can you build more? This table here, Jackie and James, Amy, you think there can be more? [mmm hmm] How many do you think?
11.0.10		Amy:	Well, we got six
11.0.11		T/R 1:	You think that's it?
11.0.12		Jessica, Amy, James:	[Shake heads] No.
11.0.13		T/R 1:	No? Anybody else? Any thoughts? [Meredith raises her hand] Meredith? What do you think?
11.0.14	8:00	Meredith:	Well, maybe if you, say you had a white rod, and you divided the white rod, maybe you could make more models that way, if you divided the white rods.
11.0.15		T/R 1:	So you're thinking if you had more, different size rods?
11.0.16		Meredith:	Yeah, yeah.
11.0.17		T/R 1:	Like if you took the white rod and designed one smaller than the white rod? [mmm hmm] Ok. Anybody else, what do you think? There was a theory, I know David's looking at what we built here, David you had a theory that you were testing about uh building some models. There were some models that everyone here built, maybe we should put two of them on the overhead, if you can help me do that, to compare which is bigger three quarters or two thirds. Um, to decide with the rods you have, what was the smallest model you can build? With the rods you had available, not being able to cut them or make them different. Did you find- is there a smallest model, when you compared three quarters or two thirds? [pause] Can you remember? What, what do you think, Mark?
11.0.18		Mark:	um, I think, (Pause) I'm not that sure.
11.0.19		T/R 1:	You're not sure, you want to think a little bit more? Sarah, what do you think? I see Sarah and, uh Beth? What do you think?
11.0.20	9:42	Sarah:	Um, there’s a smaller one.
11.0.21		T/R 1:	What do you think it is? You want to come up and build it really quickly? [Sarah and Beth get up] Maybe while they're, they're building the small one, you can think of the

		next one. I don't think all will fit on the overhead, but at least we'll get some of these models up, we might have something to talk about. [Sarah and Beth build a model of a light green and white train, two red rods, and four white rods] Ok, what do you think?
11.0.22	Erik:	I think it's, they're right, but one green and uh, and a one I think equals one purple, because if you would put that to that it would just equal one purple and put the
11.0.23	T/R 1:	Ok, tell me what you're showing up here, uh, Sarah and Beth. You can put a purple up there, too. Erik is suggesting above it.
11.0.24	Sarah:	Yeah [She places a purple rod above the model]
11.0.25	T/R 1:	Ok, but tell us what you're showing. We're trying to show three quarters and two thirds, is that right? [mmm hmm] Uh, can you show us how that shows three quarters and two thirds? [Sarah and Beth whisper to each other]
11.0.26	Beth:	Oh, we don't have, [to T/R 1] We don't have thirds in there.
11.0.27	T/R 1:	oh, ok.
11.0.28	Beth:	We only have the half and the whole.
11.0.29	T/R 1:	Ok, so you have three quarters and a half. Ok. Can you build one for three quarters and two thirds? Does someone have any other suggestions? Alan? Did your hand go up or did you just wave it? Anybody else remember how you did that? Kelly, you think you have something, you want to come and help? Graham, you can come and help.
11.0.30	Kelly:	I don't have fourths in my one that I made
11.0.31	T/R 1:	You don't have fourths? How about here? Amy? James? Jackie? [Amy James and Jackie come to OHP] Oh we're starting to get think- we're starting thinking now. [Amy, James, and Jackie build the model using orange and red train as one, purple as one third, and light green as one quarter.] Ok, you want to tell the class about what you have there?
11.0.32	James:	Ok, um, well um, we had this model and we uh this is two thirds and three fourths and we think that three fourths is bigger by one twelfth. [He separates two purple rods and three light green rods from the original model to show the comparison].
11.0.33	T/R 1:	Questions, anybody? What's one twelfth?
11.0.34	James:	Because, um, twelve whites equal up to this. Let me show, here. [He puts two white rods on his model]

11.0.35	T/R 1:	Are you all convinced?
11.0.36	Jessica:	But didn't you say to make a smaller one?
11.0.37	T/R 1:	Well I said you make the smallest one you can make?
11.0.38	Jessica:	But I don't think that's the smallest one.
11.0.39	T/R 1:	Can you make a smaller one? It's an interesting question. If you don't think there's a smaller one you should be able to show it, or at least-. [James and his partners continue placing the twelve white rods on the OHP]. What do you think?
11.0.40	James:	Well, we just put twelve whites on there and it takes one white to equal the two pinks, to the three, oh yeah, purple to the three greens. So that's why we think it's one twelfth.
11.0.41	T/R 1:	How many of you agree with that? How many of you agree with that model? How many of you found that same model when you worked it out? Raise your hand if you found the same model. [All students visible raise hands] Ok, it looks as if everyone did. I have a question that Jessica is raising, I'm listening to Jessica next to me. Jessica says she thinks there's a model that you can build that's smaller, now, when I use the word smaller, you use the word smaller, what do you think we mean? What do you think, Erik?
11.0.42	Erik:	Smaller in size-wise? Like size for the thirds, the fourths, and, and the whole. Smaller by size.
11.0.43	T/R 1:	Ok, so what what we called one in this problem was what, what did we call one? Brian?
11.0.44	Brian:	Well, um, the orange and the red.
11.0.45	T/R 1:	Yeah that train we called one, right? And I guess the question is, that train has a particular length, right? You can see the length of that train? Is it possible to build a model to show the comparison of three quarters and two thirds with a train that has lengths smaller than that, now if you think it isn't, you've gotta convince me with some argument.(Oh I know) Oh, Erik thinks he has an argument to convince me there's not a smaller one because Jessica doesn't believe it. Right, Jessica? Jessica seems to think there's one that can be made with a train that has length shorter than the one up there. So if you think you have an argument, Erik thinks he does, raise your hand. I want you to think about this. You might want to talk to your partner about your argument and see if your partner buys it. You don’t have to talk to your

			partner. Ok, we'll let you guys go first and Erik is listening to see if he agrees. Ok.
11.0.46	17:41	Amy:	We say that there was no more, that you can't get a smaller one because every one you use equals up to an orange and a red, and the secret is that every one has three purples and four greens. And so you can't possibly make one smaller because you won't be able to fit, it won't work because every one you make equals up, equals up to the orange and red.
11.0.47		T/R 1:	So you're telling me the six models that you made
11.0.48		Amy:	Were the same length.
11.0.49		T/R 1:	Were all the same length. In all of your cases, the, the, what you called one had the same length as the orange and red. That's very interesting.
11.0.50		James:	Well, well, we could make another model to show that.
11.0.51		T/R 1:	We believe you. Does anybody need to have that shown, what they would've done? How many of you did that too? You found different ways to show one, that had the same length as orange and red,
11.0.52		Beth:	Yeah, Sarah just built another model that uses the exact same length as the other one.
11.0.53		T/R 1:	And what did, what was, how did she make her train?
11.0.54		Jessica:	Yeah, that's what I was going to do.
11.0.55		Beth:	Blue, light green, and then the half is dark green, and the third is purple and the fourths were the dark greens- were the light greens.
11.0.56		Jessica:	See, that's what I did, I was gonna make one that was the same exact size.
11.0.57		T/R 1:	Ok, so many of the models you made were really models where your, what you called one, that train, really had the same length as the orange and red. Is that all you can make? How do you know that, that, that there's not one smaller? Erik? I'm still not convinced that there's not one smaller. They didn't convince me. How would you convince me?
11.0.58	19:22	Erik:	Well, see, I agree that, that, I agree with them just at the part that there's no, there's no other smaller. I think, because at their model, they use the twelfth as the white ones, and there's no rod smaller than the white rod. So, therefore, if you make it a rod smaller than it, they can't, you can't divide it into twelfths.
11.0.59		T/R 1:	Ok. Did you hear what he said? Yeah!

11.0.60		Erik:	Because the twelfths right here are the smallest rod possible.
11.0.61		T/R 1:	Ok, so
11.0.62		Erik:	Unless you made a new rod.
11.0.63		T/R 1:	So unless we use Meredith's idea of creating new rods that had, that were smaller than the white rods, then you could make a smaller model, Erik?
11.0.64		Erik:	Yeah.
11.0.65		T/R 1:	The rest of you agree with that? So, so then, ok, I'll buy that, how many of you buy that argument? That seems reasonable. So you've made the smallest one already. Jessica, is that reasonable to you?
11.0.66		Jessica:	Yeah.
11.0.67		T/R 1:	Ok, my next question is, can you make one that shows the comparison of three quarters and two thirds, that's bigger than this? Are there others?
11.0.68		Students:	Yeah.
11.0.69		Michael:	I know, I just did one.
11.0.70		T/R 1:	Ok, um, you think there are others, ok. You have another one?
11.0.71		Michael:	Yeah.
11.0.72		T/R 1:	I would like one, that uh, is the next smallest.
11.0.73		Michael:	Next sma-
11.0.74		T/R 1:	Can you predict something about the one that would be next smallest? I mean next largest, I'm sorry. The one that's the next largest. Brian, what's your prediction?
11.0.75		Brian:	I think it would be twenty-four.
11.0.76		T/R 1:	You think what would be twenty-fourths? What rods?
11.0.77		Brian:	Well, the next, the next, the next larger one will be, I think the whole will be twenty-four.
11.0.78		T/R 1:	But we, we call the whole one.
11.0.79		Brian:	Yeah, I know, but what I mean
11.0.80	21:01	Michael:	No, no, it would take twenty four ones to equal a whole.
11.0.81		T/R 1:	What would be, what would be twenty-fourths?
11.0.82		Brian:	Like the, there would be, there would take twenty-four white cubes to equal up to a whole
11.0.83		Student:	I, I also have a strategy
11.0.84		T/R 1:	Wait, wait a second, you're saying twenty-four white ones would equal your train.
11.0.85		Brian:	Yeah, yeah.

11.0.86	T/R 1:	That you're going to call one. So then what would one white one be called in that next model do you think?
11.0.87	Michael:	One twelfth
11.0.88	Brian:	I think, um
11.0.89	T/R 1:	What would that white one be called?
11.0.90	Michael:	Well it's not gonna, we're not gonna. Let me see this [Michael begins to build the model]
11.0.91	Brian:	Um, one twenty-fourth I think.
11.0.92	T/R 1:	Brian thinks then white ones in the next train would be one twenty-fourth. You think that too?
11.0.93	Erik:	Yeah, Alan and I made that same model. We made the same model that was, I think it was two oranges and like one purple, yeah it was two oranges and one purple and then it had the thirds
11.0.94	T/R 1:	Ok, why don't the rest of you sit down and let's have Erik and Alan make that model. Did you make it too, Michael? Is that what you had?
11.0.95	David:	I made that also
11.0.96	T/R 1:	Ok, you watch what they're doing, and seeing if -leave that other one up there ohhh! Ok, can you leave the other one up there maybe while you're making that?
11.0.97	Erik:	Sure we can.
11.0.98	T/R 1:	Keep the other one up there. Erik: Just move this over.
11.0.99	T/R 1:	James, why don't you make the other one too, so it doesn't go away on the bottom.
11.0.100 22:14	Brian:	Kaitlin, can I borrow some oranges. [whispering, inaudible] 2 more, no [starts to count something in his model] 6 more
11.0.101 23:20 (F)		T/R 1: The rest of you could be making these if you haven't already, in your seats just so you have them in front of you. I would suggest you to make both models in your seats and keep them in front of you.
11.0.149 25:50(F)	Amy:	Wait, no, twelve plus two, wait twelve plus twelve, twenty-four, twenty-four here. You've got twenty-four plus twenty-four is forty-eight.
11.0.150	James:	I need those. I need another red. I need thirty-six whites.
11.0.151	Amy:	Thirty-six? Why?
11.0.152	Jacquelyn:	Why? Why are you saying thirty-six?
11.0.153	Amy:	I need the reds, I'm trying to prove a point here.
11.0.154	Jacquelyn:	You can have all the reds you want. I'm going to sit here and watch you guys.

11.0.155	James:	We're in battle
11.0.157	T/R 1:	Ok. Now I know, I know you're building these models and I know you don't have enough rods so I know that you have to uh, share some of your uh rods and sometimes you can only build one model on a desk, and I know some of you are able to imagine the models now too. How many of you could imagine what it looks like, even though you haven't quite built it? Raise your hand if you could imagine what it looks like. [A few children raise their hands] I'm kind of curious, what do you imagine that you don't have there, Jessica? I see that you built a model that has two oranges and a purple that you're calling one.
11.0.158	Jessica:	Well, I imagine the white ones.
11.0.159	T/R 1:	And you're imagining the whites, and how many do you imagine are there?
11.0.160	Jessica:	Twenty-four.
11.0.161	T/R 1:	You're imagining twenty-four. And Andrew, I see, built it. And how many do you have there, Andrew?
11.0.162	Andrew:	Um, twenty-four whites.
11.0.163	T/R 1:	Andrew has twenty-four. And can you see on the overhead how many whites, those of you who don't have enough? Can you see? I know it's hard, I have trouble counting when it's not nearby when there's so many little pieces. But you built it too, Amy, how many do you have, Amy?
11.0.164 27:08	Amy:	Twenty-four.
11.0.165	T/R 1:	Twenty-four? Yes?
11.0.166	Student:	Twenty-four.
11.0.167	T/R 1:	Twenty-four also. How many of you are convinced that with the model, that Alan and Erik have up on the overhead, how many of you are convinced of the number of white cubes there? [Two hands visible are raised]. How many of you are convinced how many are up there? Raise your hand if you're convinced how many. Because if you're not, you may want to go up and count them to be sure or go to someone else who built the model and count them.
11.0.168	Michael:	I, I built the model.
11.0.169	T/R 1:	How many of you have a model that shows that there are twenty-four? Raise your hand if you have a model nearby. Ok, so Sarah and Beth, you built one, you don't have enough?
11.0.170	Beth:	No.

11.0.171	T/R 1:	I'm sure Andrew will lend you a few if you want to complete your model. I guess you need some whites and some reds. Ok, they don't need quite that many. Ok, do you believe there are - Sarah and Beth?
11.0.172	Beth:	Yeah [Sarah nods]
11.0.173	T/R 1:	Are you convinced, Kelly and Graham? You're convinced? Let's see, Michael and Brian I know you're convinced, I saw you had that built. Kimberly and Audra, are you convinced? [Audra nods] What about Gregory and Danielle - are you convinced [mmm hmmm] You're convinced also? Erin and Jackie? Ok, I know David and Meredith are convinced, and ok, so it sounds as if everyone is convinced that that's the case. So now, let's talk about, um, Erik's theory, Erik says now that the white one on this model, the larger model, where we called one the train that was made up of two orange and one purple, right? That particular train that he built? That he's now going to give the white one the number name, what, class?
11.0.174	Students:	One twenty-fourth.
11.0.175	T/R 1:	One twenty-fourth. So you agree with Brian's conjecture. Right? Brian says one twenty-fourth. How many of you agree with Brian's conjecture? [All students visible raise hands] The white one in that model has the number name one twenty-fourth. Now how does that help you solve the problem what is the difference between two thirds and three quarters, gentlemen who are up there? We know the white one has number name one twenty-fourth in that model.
11.0.176 29:13	Erik:	Uh, see what we did here was we have the fourths and the thirds.
11.0.177	Alan:	Yes, mmm hmmm.
11.0.178	Erik:	And then the twelfths and they, they said that the twelfths would do it.
11.0.179	Alan:	mmm hmmm
11.0.180	Erik:	So the twelfths would be the reds which is one, which is two whites, and then people think the twelfths would be the answer, but if you take two of the twenty-fourths
11.0.181	Alan:	It would equal up to a red rod.
11.0.182	Erik:	It would equal up to a red rod.
11.0.183	Alan:	Which would be equal to twelfths.
11.0.184	Erik:	Which would be one twelfth. So, see, we think, I think that the answer is either two twenty-fourths or one twelfth.

11.0.185	Alan:	Mmm hmmm.
11.0.186	T/R 1:	How many of you agree with what they said?
11.0.187 29:46	Alan:	So there are two answers. Both the same
11.0.188	T/R 1:	You agree the answer is either two twenty-fourths or one twelfth. Does anyone think the answer is one twenty fourth? Ok, that's very interesting, that's very nice, gentlemen, thank you that's lovely. That was very helpful. And then I remembered - let's leave that up there, you can sit down. I have another question. I remember then some people had written - do you have a question, Amy?
11.0.189	Amy:	No, I just want to say something.
11.0.190	T/R 1:	Ok.
11.0.191	Amy:	Um, that, but two reds equal up to a purple.
11.0.192	T/R 1:	So?
11.0.193	Amy:	So you could put six purples down to make
11.0.194	Erik:	What do you mean six purples?
11.0.195	Amy:	Becau- six purples, because look. [holds up a purple and two reds]
11.0.196	Andrew:	Ok, but why would you need the purples?
11.0.197	Erik:	Why would we need the purple? It's only one twelfth.
11.0.198	Amy:	I know, but you could also do it that way. You could also put six.
11.0.199	Alan:	Why would we need that?
11.0.200	Erik:	We only need one twelfth. It's either two twenty-fourths or one twelfth.
11.0.201	Alan:	They're both the same answer.
11.0.202	Erik:	They're both the same answer.
11.0.203	Amy:	I know.
11.0.204	Alan:	[inaudible drowned out by Erik]
11.0.205	Erik:	But why would we need the purple. The purple would be too big.
11.0.206	Amy:	Six purples equal up to the whole train you made.
11.0.207	Erik:	But why do we need - we don't need sixths. We only need thirds and fourths.
11.0.208	Alan:	Yeah why would we need that?
11.0.209	Erik:	And twelfths and twenty-fourths. That's all we need.
11.0.210	Amy:	Ok [sighs]
11.0.211	Erik:	I don't think we need sixths.
11.0.212	T/R 1:	So it sounds like Amy is answering a different question. I think I hear the que- I hear what Amy is saying because I heard her say it earlier. Amy is saying there are other ways

you can uh make trains, right? That have length [Amy nods] of two orange and one purple. And I know that this table spent a lot of time doing that, of course what Erik and Alan and James are saying that's true, but it really isn't necessary or related to solving this problem. Do you agree with that, Amy? [Amy nods and says mmm hmmm] But that's an interesting thought that there is another way and when you think about all those purples there, what number name would you give to a purple then?

11.0.213 Amy and Erik: One sixth.

11.0.214 T/R 1: So I think that Amy is saying look! On this same train I could show one sixth! Which almost suggests I could ask you another problem. [Erik laughs]

11.0.215 Erik: Which is if you asked another, Which is Which is if you asked a question with a sixth, then you

11.0.216 Alan: Then you do it

11.0.217 Erik: Then you could do it.

11.0.218 T/R 1: Yeah, but I could ask you a question which is bigger a sixth or three quarters. Don't do that now!

11.0.219 Erik: Then, then you do a sixth!

11.0.220 T/R 1: Don't do that now, but I could ask you that question, couldn't I, which is bigger a sixth or three quarter?

11.0.221 Erik: Yeah.

11.0.222 T/R 1: And you should be able to answer it with this model. [Erik says yeah] Let's hold that question for a minute. I don't want to lose the question but I don't want to lose what we're talking about so we'll put that aside. I remember last Thursday when I walked around the room then I said could you make another model and a lot of you said "oh you know I don't have enough. I don't have enough of these blocks so I said can you imagine it and I remember talking and I know Andrew actually made the model when David had a theory that he shared with um Erik and Alan and Meredith, right, David? And so he shared a theory and I remember Erik said hey wait a minute that's what Andrew built! And then Jessica said that they already built what the theory was, that's what I heard, so I'd like to hear um, David's theory again, if you don't mind, David, if you think you can remember your theory and Andrew I want you to listen very carefully and Jessica and the rest of you I want you to listen carefully to David's theory because it really has

		to do with if I were to make another model, is it possible do you think to make another model if we had more blocks, it is a possible thing to do? [Student says yes]. How many of you think we can [Most/all students visible raise hands]. Ok. How many of you think we can make another model? Some of you aren't sure, how many of you aren't sure? Meredith's not sure? Erik's not sure? Danielle's not sure? Audra's not sure. Ok. How many of you are sure we can make another model? [All other students raise their hands.] Ok, that looks like that's James and Alan and Andrew and Jessica and Beth and Sarah, Kelly, Graham, Brian, Michael, Caitlin, did I leave anybody out? David is sure. Ok. Let's listen to David's theory and see if we could convince those or else they have to show us our theory doesn't work.
11.0.223	David:	Well, first, um, Meredith made um, a model which had one orange, one blue, and one black.
11.0.224	T/R 1:	Ok, she made a model with an orange a blue and a black. That's what you told me?
11.0.225	David:	Yeah. And then she had, um, the whites, I think they were something like
11.0.226	Erik:	Twenty-fourths.
11.0.227	David:	Yeah, one twenty-fourth and the reds were one twelfth and, um,
11.0.228 34:28 35:26(F)	Erik:	Just like the one up there.
11.0.229	David:	Yeah.
11.0.230	T/R 1:	So you're saying that if I had an orange, a blue and a black, that the model should look like the one up here.
11.0.231	Erik:	Just about.
11.0.232	T/R 1:	But it doesn't.
11.0.233	Erik:	well...
11.0.234	T/R 1:	Right? See what happens?
11.0.235	Erik:	But then, then the one, then the, the uh, um, I don't know
11.0.236	Alan:	Then the reds couldn't be twelfths.
11.0.237	Erik:	Yeah, then the reds couldn't be twelfths and the whites couldn't be twenty-fourths.
11.0.238	Alan:	Right, it would either take one [inaudible]
11.0.239	T/R 1:	Andrew, what do you think? Andrew and Jessica, what do you think?
11.0.240	Andrew:	[Refers to twenty-four cm model on desk] Well, I made a model that had the white was one forty-eighth and the

		purples were twelfths and the white was, I mean the red was twenty-fourths and I took two browns as the thirds and two dark greens as the fourths and they I called them the fourths and then the whole was four oranges and two purples.
11.0.241	T/R 1:	Now, you're telling me that you used browns, two browns to be
11.0.242	Jessica:	One, like one, one third.
11.0.243	Andrew:	Yeah.
11.0.244	T/R 1:	One brown was one third, two browns was two thirds?
11.0.245	Andrew:	No
11.0.246	T/R 1:	Is that what you're telling me?
11.0.247	Erik:	No
11.0.248	Andrew:	Two browns was one third
11.0.249	Erik:	Two browns was one third.
11.0.250	Andrew:	I took two browns and put them together
11.0.251	T/R 1:	Two browns to be one third!
11.0.252	Andrew:	Yeah.
11.0.253	T/R 1:	Oh, ok, that's not going to fit. But maybe, um, you want to come up here and do that? [Andrew and Jessica come to front of class.] Ok, here you go. Why don't you build that right here. Do it up front here, uh, why don't you come all the way around, Jessica. Ok, let's see what they're doing here because, um, it looks to me as if you need a bunch of rods to do this. [They work for about two minutes to build the model of a train of four oranges and two purples, six brown rods and eight dark green rods, and twelve purple rods, twenty-four red rods, and white rods]
11.0.254	Andrew:	It might not be enough.
11.0.255	T/R 1:	Now, I want all of you to see what Jessica and Andrew are building, and, now you all can't come up at one time, so I'm gonna, if it's ok with Mrs. Phillips, I'm gonna ask you in little groups to go up there and take a look at their model and so um we can be able to talk about it and then some of you maybe can look at it from where you're sitting. I know that Gregory and Danielle are very fortunate - they have front row seats. I think, can you see Alan and Erik?
11.0.256	Erik:	Not really.
11.0.257	T/R 1:	Not really. So some of you are going to have to go up in a minute to see what they're doing. So why don't we start at least with Erin and Jackie, why don't you sort of kind of come up and see what they're building. Be careful there's a

		cord here too. Ok, if you've seen what they're building and you think you understand how they built their model, then if you can sit down and then I'd like another group to come up and see.
11.0.258	Jessica:	[Explaining to Erin and Jackie] And what we did is, this was our whole and this was like, these, like, um, two, [takes two brown rods and holds them together] two browns equal one third, like we counted two as one. And those were our thirds, and two greens was one, and one two three four, so those were our fourths, and
11.0.259	Andrew:	Purples were the
11.0.260	Jessica:	Purples were the twelfths and the reds were the
11.0.261	Andrew:	Twenty-fourths
11.0.262	Jessica:	And the white ones were forty-eighths.
11.0.263	T/R 1:	Ok, it looks like it's going to be a little harder than I think, I think that you're gonna need an explanation when you go up. And so I think maybe we should have, rather than have you do this a lot of times, maybe we should have a few explanations. Maybe we should have more people up here. Um, some of you can come around while we can hear Andrew and Jessica - would you mind doing this a couple of times?
11.0.264	Jessica:	Yeah.
11.0.265	T/R 1:	So why don't you come around, a few of you can come around the table and listen to their explanation. I think that's the best way. Some of you can come behind the table, I think.
11.0.266	Andrew:	So what we did was, we, um.
11.0.267	T/R 1:	Ok, just a second, let's wait till as many people, uh, can
11.0.268	Andrew:	We had, the, um, whole, was four oranges and two purples, and then we, our strategy was we took two browns and we put em together and they were the third. And then we took two dark greens and put them together and they were the fourths. And then the purple was the twelfth
11.0.269	Jessica:	Twenty-twelfth.
11.0.270	Andrew:	And the red was the twenty-fourth and the white was the forty-eighth.
11.0.271	Jessica:	Forty-eighths.
11.0.272	Andrew:	And,
11.0.273	Jessica:	Cuz, what we did really to figure it out.
11.0.274	T/R 1:	Well, what's the difference?

11.0.275	Andrew:	The difference is one twelfth.
11.0.276	T/R 1:	Can you show us?
11.0.277	Jessica:	Yeah.
11.0.278	Andrew:	Ok, we don't have enough, we'll just take them over here. Two thirds, that's all we need, and then
11.0.279	Jessica:	So, um, do we need anything?
11.0.280	Andrew:	Here, we need brown.
11.0.281	Jessica:	We need some of those.
11.0.282	Andrew:	Ok, well, we came up with the conclusion of that
11.0.283	Jessica:	That two thirds.
11.0.284 43:06	Andrew:	That two thirds - that four, three fourths [As Andrew speaks, Jessica points to relevant rods on model] were bigger than two thirds by one twelfth
11.0.285	Jessica:	By one twelfth [holds up purple rod].
11.0.286	Andrew:	Or two twenty-fourths [Andrew and Jessica place two red rods and four white rods, to complete the model of a purple, two reds, and four whites]
11.0.287	Jessica:	Or
11.0.288	Andrew:	Or four forty-eighths.
11.0.289	T/R 1:	Ok, now, I'm wondering if some of you can pull aside and maybe the rest of you can come up and I'd like Jessica and Andrew to say it nice and loud so in back of the room, the people who are sitting could hear. Now those of you who think you understood their argument um, you can go back to your seat. The rest of you can come and sit on the floor, but if they can turn around and try to share it so the people in the back of the room could understand, would you mind going through it one more time particularly the different names for the way you've represented the difference? Ok, hold on, I'd like you all, now if you're back in your seats, if you want to try to stand up and look while they're explaining it, that's ok. Ok, are we all ready? Hold on a minute, Jessica, I think we're having a little discussion about uh solutions there, Ok, can you nice and loud for the people back there show them what you're doing?
11.0.290	Jessica:	Well, what we did was we made a model and we counted um
11.0.291	Andrew:	Four oranges
11.0.292	Jessica:	four oranges and two purples as our whole and for our thirds we counted, we counted two oranges as one, I mean

		two browns as one. [holds up two brown rods end to end] And we had
11.0.293	Andrew:	that was our third.
11.0.294	Jessica:	That was our thirds, and for our fourths we counted two greens as one [holds up a train of two dark green rods], two dark greens as one.
11.0.295	Andrew:	Purples were our twelfths, the reds were the twenty-fourths [Jessica says twenty fourths] and the whites were forty-eights.
11.0.296	Jessica:	Forty-eighths. And we think that, we think that, three um, fourths are bigger than two thirds by either, um, one forty- I mean four forty-eighths um, two twelfths, or, um
11.0.297	Andrew:	No, two twenty-fourths.
11.0.298	Jessica:	Two twenty-fourth or what's that? One twelfth.
11.0.299	T/R 1:	What do you think about that, Michael?
11.0.300	Michael:	I guess I agree with it, it's what I came up with.
11.0.301	T/R 1:	You came up with the same model, didn't you?
11.0.302	Michael:	Yeah
11.0.303	T/R 1:	Did anyone else come up with that same model? That's very lovely. Thank you so much, Andrew, and does anybody have a question to ask Andrew and Jessica before they're finished? Does anybody have a question? Does anybody have a comment? You sure you don't want to ask them any of that? Sarah what do you think?[Sarah says no.] Is that interesting [Sarah says mmm hmmm]? It's very interesting Ok, um, I'm going to ask you to sit down and I want to thank you very much for making that model for us. But I guess I'm asking the question, uh, to Meredith and James and to Erik and Alan right now, uh, does this have anything to do with your theory and the theory you tested? Meredith and David and Erik and Alan - does this model have anything to do with the theory you tested, David?
11.0.304	David:	Uh, yes because we thought that the ones would be one forty-eighth
11.0.305	Erik:	And and the
11.0.306	David:	And then the reds would be, um, one
11.0.307	Erik:	Twenty-fourth and the purple, well originally, we thought that the light greens would be, well David thought that the light greens would be twelfths, but then we tried it and they would become the sixteenths, so then we tried the purple,

		yeah the sixteenths and we tried the purple and then that was the twelfths.
11.0.308	Alan:	Since whites are doubles, they're forty-eighths
11.0.309	Erik:	So, in other words we doubled everything.
11.0.310	Alan:	Yeah. You basically just added, like, there originally were just two oranges, now there are four oranges and an extra purple. Now there are six, there are six browns.
11.0.311	T/R 1:	So let's see, on this model here we had an orange and a red, and then on that model there we have two orange and a purple and in this orange here we have four orange and two purple. All of these represent one, is that a surprise?
11.0.312	Alan:	It could have been two purples changing into a brown.
11.0.313	T/R 1:	It could have been two purples changing into a brown
11.0.314	Alan:	Yeah.
11.0.315	T/R 1:	That's true.
11.0.316	Alan:	And
11.0.317	T/R 1:	I guess my question is what you called one in each of these models? Are they related in any way? The lengths? All of these you called one, are the lengths related to each other, if you study each of the models you built. You see this one here you called the orange and red one, isn't that right, and here you called one two orange and purple, right?
11.0.318	Alan:	So basically it's just doubled. That's
11.0.319	T/R 1:	What do you mean by that "basically it's doubled", Alan? That's an interesting idea. In what way is it doubled?
11.0.320	Alan:	Um, ok, it's doubled because it now it has four oranges and two purples or a brown, so
11.0.321	T/R 1:	But the first one doesn't have any purples.
11.0.322	Alan:	Well, that's because this had nothing to do with the first problem because of the first question, but
11.0.323	T/R 1:	I'm not sure I understand what you're saying.
11.0.324	Alan:	Had there have been sixths.
11.0.325	Erik:	I know.
11.0.326	T/R 1:	We didn't have sixths, we had twelfths here.
11.0.327	Alan:	Mmm hmm.
11.0.328	Erik:	I think I know what he's saying.
11.0.329	Alan:	Right, there you have twenty-fourths and the whites are forty-eighths this time. Now, up there, there are no purples, because they weren't put on. But had they have been, on the bottom, which they are, they are twelfths, because
11.0.330	Erik:	Purples? In that

11.0.331	Alan:	Purples are twelfths.
11.0.332	Erik:	In that model they became twelfths, but over there they would be the sixths. Like Amy said, if
11.0.333	Alan:	Right, because if you double each of them, it would come out to twice the number.
11.0.334	Erik:	Exactly!
11.0.335	T/R 1:	James?
11.0.336	James:	Uh, I think um, that um, because there are two oranges and two purples I agree with Alan that it's double but why the red's there, it's two reds make a purple and that, that means the two oranges and the red make two oranges and a purple.
11.0.337	Alan:	Yeah, cuz if you took the two oranges out of that model and a purple, and then two more oranges and a purple, and you put them on top of each other, they'd be equal. But if you put em side to side you'd have four oranges and two purples, or the two purples could be a brown. So it's basically doubled, each of the length is doubled.
11.0.338	T/R 1:	I wonder if the rest of you see this, I'm saying, this is an orange and it's not a purple, it's an orange and a red, right? Now, how does this get doubled to be this? I see there are two oranges, instead of one orange, I see the one orange length got doubled, instead of one orange there's two, right? Isn't that true? But how did the red get doubled?
11.0.339	Alan:	The red-
11.0.340	T/R 1:	I'm confused, how did the red get doubled here?
11.0.341	Alan:	The red
11.0.342	T/R 1:	I see the orange got doubled here because there are two oranges, right? From one orange to two oranges, I don't know how did the red get doubled? I don't see that. Jessica? Kimberly.
11.0.343	Kimberly:	Well, they used a purple and the red, two reds make a purple, so now if they have a purple, they doubled the red.
11.0.344	T/R 1:	Is that what you were going to say?
11.0.345	Jessica:	Yeah.
11.0.346	Alan:	I was going to say something different
11.0.347	T/R 1:	So you're tellling me that instead of the one orange and one red, we have two oranges and two reds in this model. But they just called it a purple rather than two reds. Do the rest of you see that? [mmm hmm] Ok, so this model is doubled of this, now you have to convince me that this model is double of this, so instead of two oranges and a purple, what

		should we have now if it's doubled? Don't look. What would you expect we would have then if it's doubled? Danielle.
11.0.348	Danielle:	Um, four oranges and two purples.
11.0.349	T/R 1:	Let's see. Do we have four oranges and two purples?
11.0.350	Erik:	One, two, three, four, yup, or four oranges and one brown.
11.0.351	T/R 1:	Or four oranges and one brown.
11.0.352	Alan:	Yep
11.0.353	T/R 1:	Ok, this is the question I ask you. If I were to make another model, Andrew's hand is up, Andrew knows my question, what do you think my question is, Andrew?
11.0.354	Andrew:	If you were gonna make another model, what, um, the doubles be?
11.0.355	T/R 1:	Ok, what would my one look like in terms of rods? Brian!
11.0.356	Brian:	Um, forty-eight.
11.0.357	T/R 1:	What would I call one? Imagine in your head what I would call one?
11.0.358	Brian:	Forty-eight? Cuz there would be, well, cuz there would be forty-eight whites equal up to one and then.
11.0.359	T/R 1:	Well, we have forty-eight whites going up to one here, don't we?
11.0.360	Brian:	Oh!
11.0.361	T/R 1:	In this model.
11.0.362	Erik:	So we have to double that?
11.0.363	Alan:	But, no!
11.0.364	T/R 1:	I don't know, I'm asking you, that's my question, Andrew what do you think?
11.0.365	Erik:	Well you're saying what-
11.0.366	Alan:	No, it can't
11.0.367	Andrew:	Well, the whole would be eight orange rods and
11.0.368	Alan:	It can't be done
11.0.369	T/R 1:	Eight orange rods, I'm listening.
11.0.370	Erik:	Eight orange rods and two browns
11.0.371	Andrew:	And two browns.
11.0.372	T/R 1:	And two brown rods.
11.0.373	Alan:	You can't double that. You can't double that model because if you did, then you wouldn't be able to third it.
11.0.374	Erik:	You wanna make a bet - all you had to do is train it - you just train it!
11.0.375	Alan:	Right because if you doubled that it would be eight oranges and two browns, now is there any rod that could third that?

11.0.376	Erik:	Well if you use a train
11.0.377	Andrew:	Yeah
11.0.378 53:00	Erik:	If you use a train, just like in Andrew's theory.
11.0.379	Alan:	Well, if you train the rod, but that would make it not equal.
11.0.380	Andrew:	It would probably be-
11.0.381	Alan:	Up there, it's just plain, except for the whole.
11.0.382	Andrew:	It would probably be three browns would be the thirds and three dark greens would be the fourths.
11.0.383	Alan:	Right, but that would be using more than one rod to make another rod to fit, fit the same thing.
11.0.384	Erik:	Yeah, so you can do that! Just like, you, Andrew said, you can use a train to make a third and a fourth. Cuz he, like, I, I overheard, they said that if you can use a train to make a whole why can't you use it to make a third and a fourth?
11.0.385	Andrew:	Yeah.
11.0.386	T/R 1:	David?
11.0.387	Andrew:	And a half
11.0.388	Alan:	But then it wouldn't be equal.
11.0.389	Erik:	Yeah they would! Cuz the third could be, like in that model, Andrew used the two browns, that's equal!
11.0.390	Alan:	But in that model, the three browns don't have anything attached on so it's totally equal
11.0.391	Erik:	So? They just doubled it!
11.0.392	Alan:	But if you added something on
11.0.393	Erik:	We just doubled, we doubled that model to equal that model.
11.0.394	Andrew:	Yeah, and I doubled the brown - two browns,
11.0.395	Erik:	Yeah, exactly.
11.0.396	Andrew:	So in the next model
11.0.397	T/R 1:	David, what do you think? Did you want to say something?
11.0.398	David:	Um, I agree with Erik
11.0.399	T/R 1:	What part of what Erik said?
11.0.400	David:	Well, Alan didn't think that you could uh third it, but like Erik said that you can train it and put the other blocks onto the other one
11.0.401	Alan:	What I meant, what I meant is, you can't third it just using one rod.
11.0.402	T/R 1:	Ok, Alan.
11.0.403	Erik:	Exactly. You can't third it using one rod, but you can third it using trains.
11.0.404	T/R 1:	Ok, so

11.0.405	Alan:	You could double that, but you would have to use two rods to make it
11.0.406	T/R 1:	Ok, so you think you can double it and you think you can imagine - can you make one bigger than that?
11.0.407	Erik:	If you doubled that, it would be sixteen oranges [laughter] and, sixteen oranges and four browns!
11.0.408	T/R 1:	Ok, the question I want to leave you all to think about, I'd like you to uh, first I'd like to thank you for the wonderful models you built, but the question I'd like you to think about is, uh, is there, is there a biggest model?
11.0.409	Erik:	Thirty-two oranges! [laughs]
11.0.410	T/R 1:	Is there a biggest model? And if you don't have enough, uh, rods, you could imagine, we could write to Cuisenaire and we can have them ship us buckets and buckets and buckets and buckets
11.0.411	Erik:	Or we could combine all our stuff.
11.0.412		We could start by that but my question to all of you is there a biggest model? Why or why not? And I'd like you to write to me about, about that. Would you do that? Would you write to me? Maybe Mrs. Phillips can let you combine and build together, that might take a little while and a camera. Ok, I think we have to stop, I'll see you in two weeks, and if you could
11.0.413 57:00		[end of class]
11.0.414 58:22	Erik:	I wonder how many oranges
11.0.415 58:17(F)	Alan:	No, I wonder how many oranges it would take to get from here to California