# THE FUNDAMENTAL STRUCTURE OF THE WORLD: PHYSICAL MAGNITUDES, SPACE AND TIME, AND THE LAWS OF NATURE


By

MARCO KORSTIAAN DEES


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ABSTRACT OF THE DISSERTATION

The Fundamental Structure of the World: Physical Magnitudes, Space and Time, and the Laws of Nature

By MARCO KORSTIAAN DEES

Dissertation Director:

Dean Zimmerman

What is the world fundamentally like? In my dissertation I explore and defend the idea that we should look for accounts of reality that avoid redundant structure. This idea plays a central role in science, and I believe its has the potential to be extremely powerful and fruitful in metaphysics as well.

I identify three forms of redundancy in metaphysics: *empirical*, *metaphysical* and *axiomatic* redundancy. Avoiding these forms of redundancy imposes powerful constraints on acceptable accounts in metaphysics; we should look for views that (i) do not posit unnecessary structure, (ii) characterize the world without redundancy, and (iii) avoid unexplained patterns at the fundamental level. I argue against widely accepted accounts of physical magnitudes and space and time on the basis that they suffer from these forms of explanatory redundancy, and in their place I develop novel accounts that are not explanatorily defective in this way.

Chapter one argues that the structure of quantitative properties is reducible to facts about the *dynamical* roles different magnitudes play in the laws of nature, so that *2kg mass* is greater than *1kg mass* in virtue of the fact that these magnitudes give rise to different consequences for how things accelerate.

Chapter two argues that the spatial and temporal arrangement of the world reduces to facts about its causal structure, so that you are closer to your pint of beer than to the moon in virtue of the

fact that you causally interact more strongly with the beer than the moon.

In the final chapter I argue that although physics describes the world in the language of mathematics, there are compelling reasons to think that this description is not fundamental, for it is extrinsic and involves conventional choices of scale. If this is right then corresponding to every mathematical description of the world there is an intrinsic description that characterizes the physical structure of reality directly. I conclude that the fundamental physical laws are not the laws of physics.

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# 1. Introduction

This dissertation is an attempt to tackle the question: *What is the world fundamentally like?* It is natural to think that in order to learn about the fundamental nature of the world we should look to our most fundamental science, physics. But it is far from clear what, exactly, we learn about the world from physics. In this dissertation I develop and defend the idea that we should look for accounts of reality that avoid *explanatorily redundant* structure. This idea plays a central role in science, and I believe its has the potential to be extremely powerful and fruitful in metaphysics as well. Very roughly, the idea is that we should look for accounts of reality that *minimize what is left unexplained.*

I identify three kinds of redundancy in metaphysics that I call *empirical*, *metaphysical*, and *axiomatic* redundancy. Avoiding these forms of redundancy imposes powerful constraints on acceptable accounts in metaphysics; we should look for views that (i) do not posit unnecessary structure, (ii) characterize the world without redundancy, and (iii) avoid unexplained patterns at the fundamental level. These constraints are, I hope, extremely natural and independently attractive. Nonetheless, no mainstream package of views in metaphysics satisfies all three principles. I argue against widely accepted accounts of physical magnitudes and space and time on the basis that they suffer from these forms of explanatory redundancy, and in their place I develop novel accounts that are not explanatorily defective in this way.

A theory contains *empirically* redundant structure if it posits things we dont need in order to make sense of the world. For example, just as we dont need to posit facts about witches to make sense of the world, I argue that we dont need to posit primitive facts about which things are more massive than others.

A theory contains *metaphysically* redundant structure if it fails to satisfy a plausible minimality constraint on the fundamental facts: we should attribute just enough structure to the world to characterize it completely, but no more. For example, once God had decreed that there be elephants, giraffes, and kangaroos, He didn't then need to decree that mammals exist  this decree would be redundant. Many mainstream views in metaphysics fail to satisfy this natural requirement; for example, I argue against primitivism about spacetime on the grounds that there is no way for the primitivist to satisfy this constraint.

A theory contains *axiomatically* redundant structure if makes unnecessary stipulations about how the basic building blocks of the world behave. We should prefer theories on which striking patterns in reality can be explained rather than simply taken as primitive. For example, I argue for a broadly structuralist account of physical quantities on the basis that this allows us to explain why nothing can have both *1kg mass* and *2kg mass*. On the standard approaches to physical quantities, constraints like this must simply be stipulated to hold. But brute patterns like these are mysterious, because it is natural to think that the basic building blocks of the world do not impose constraints on how they may be rearranged.

The properties of physics are quantitative properties like mass and charge that come in determinate magnitudes like *1kg mass* or *2kg mass*. These magnitudes stand in a structure that allows us to make comparisons among them, such as the fact that *2 kg mass* is greater than *1 kg mass*. According to primitivism about quantity these comparisons are simply brute features of the world. The first chapter of the dissertation argues instead that quantitative relations between magnitudes reduce to facts about the dynamical effects of instantiating these magnitudes. So, for example, *2kg mass* is greater than *1kg mass* in virtue of what the laws say about the consequences of instantiating *2kg mass* and *1kg mass*.

Both reductionists and non-reductionists about laws of nature typically take the structure of spacetime itself as primitive. The second chapter of my dissertation develops a view on which the spatial and temporal arrangement of the world is not primitive but instead reduces to facts about causal dependence. On this view you are closer to your coffee mug than to the moon in virtue of the fact that you can causally interact much more easily with your coffee mug than with the moon.

A primary motivation for rejecting primitive facts about physical magnitudes and about space and time is that we can make sense of the world without them: there are simple, elegant and explanatory accounts of the world that do not posit primitive facts about physical magnitudes or the structure of spacetime. The extra structure of the primitivist views is redundant structure.

One worry about this line of argument is that it threatens to opens the door to skepticism, and so the reasoning to which it appeals cannot be sound. After all, one might think that we can fully account for patterns in our phenomenal states without positing any external world objects. But the argument from redundant structure differs from skeptical reasoning in three important ways. First, it is a claim about rational theory choice, about what we ought to believe, rather than about what we know or which of our beliefs are justified. Second, in the case of the external world there is no simple, elegant and explanatory competing theory capable of accounting for patterns in our phenomenal states. Third, it simply incredible that it is *never* rational to abandon one theory in favour of another on the basis that it attributes surplus structure to the world. This means there is a deep and general question about how to draw a distinction between skeptical reasoning and a sound avoidance of explanatorily redundant structure. I do not know the answer to this question; but every plausible candidate answer favours the application to physical magnitudes and the structure of spacetime.

In the fourth chapter of my dissertation I argue that although physics describes the world in the language of mathematics, there are compelling reasons to think that this description is not fundamental, for it is extrinsic and involves conventional choices of scale. If this is right then corresponding to every mathematical description of the world there is an intrinsic description that characterizes the physical structure of reality directly. I conclude that the fundamental physical laws are not the laws of physics.

# 2. Physical Magnitudes

Many familiar properties come in degrees. Hippos are *heavier* than hedgehogs, toads are *taller* than tadpoles, and flamingos fly *faster* than fleas. The properties of physics are also like this; electrons and protons have different magnitudes of *charge*, up quarks and down quarks have different *spin*, and x-rays and radio waves have different *frequencies*. Properties like these are *quantities*, and they have two characteristic features. A quantity like mass is associated with a family of determinate mass properties, like 1kg mass and 3.7kg mass. I'll say these are *magnitudes* of mass. Secondly, the magnitudes of a quantity stand in a structure that allows us to make comparisons among them, as when we say that the average African elephant is *three times* as massive as the average Rhinoceros.

What is the metaphysical basis for comparisons like these? Answering this question is crucial to understanding the role quantities play in science. It is because quantities come in degrees that they are aptly represented by numbers, so that we can say that the elephant has a mass of 5,500kg. And this numerical representation is essential to laws of nature that state quantitative relationships between properties, like $f = ma$.

Things behave the way they do because of the properties they have. So too for quantities; the more mass something has, the greater the gravitational attraction it experiences and the more it resists acceleration. What is the status of this claim? According to the mainstream view of physical quantities, *quantity primitivism*, this is just something that happens to be true given the form the laws take.[1] The fact that an elephant is more massive than an egg is not constituted or explained by the fact that I can lift only one of the two. In this paper I argue that this is a mistake. Instead, I will argue that facts about which things are more massive than others reduce to facts about which things give rise to greater gravitational

[1] A version of quantity primitivism has been defended by almost everyone who has written about quantities, including Armstrong (1978), Bigelow and Pargetter (1988), Field (1980), and Mundy (1987).

attraction and resist acceleration more. On this view, elephants are more massive than eggs just in virtue of the fact that, say, elephants will crush things that eggs will not. I'll call this view *nomic reductionism about quantity.*

I present two principal arguments for favouring nomic reductionism over quantity primitivism. The first is that quantity primitivism is committed to *explanatorily redundant structure*, and so we should prefer nomic reductionism on parsimony grounds. The second argument concerns the fact that magnitudes are not freely recombinable. Magnitudes in the same family are incompatible — it is impossible for something to have both 1kg mass and 2kg mass. Necessary connections between properties call out for explanation, and whereas the nomic reductionism can provide an elegant explanation for this failure of free recombination, the quantity primitivist cannot.

Here's the plan for the paper. Section 1 introduces nomic reductionism and quantity primitivism, and sections 2 and 3 present the argument from redundant structure and the argument from incompatibilities. The final section anticipates some objections.

## 1 Quantity Primitivism & Nomic Reductionism

I will focus on the case of mass, although the issue I will raise arises for all quantities. The structure of mass allows us to make various kinds of mass comparisons.[2] For example, consider the following claims:

(1) An elephant is more massive than an egg.

(2) 1000kg is greater than 0.01kg

Claims like (1) compare objects; they are *first-order* mass comparisons. Claims like (2) instead compare properties, so are *second-order* mass comparisons. One question that arises about quantities is whether first-order or second-order comparisons are prior.[3]

[2]I will focus on ordering relations among massive objects. But many physical quantities, including mass and charge, have more structure than this; they also have *distance* structure, so that it makes sense to say that 2kg mass is much closer in mass to 1kg mass than 1,000kg mass. I'll focus on ordering relations for simplicity.

[3]Field (1980) offers a first-order account of mass. See Mundy (1989) for arguments in favour of second-order over first-order accounts of quantity. The first-order/second-order distinction is related to the distinction between *absolutist* and *comparativist* accounts of quantity. A comparativist about quantity holds that

The issue I wish to raise in this paper is independent of the dispute between first-order and second-order accounts of quantity. Instead, I want to ask whether some mass comparisons are fundamental, or whether they can all be explained in other terms.

Quantity primitivism is the claim that there are fundamental comparisons among physical magnitudes, whether these are first-order or second-order comparisons. For example, Mundy (1987) invokes the two second-order relations, $\leq$ and $*$. Intuitively, $\leq(p_1, p_2)$ encodes the fact that $p_1$ is less than or equal to $p_2$, and $*(p_1, p_2, p_3)$ encodes the fact that $p_3$ is the sum of $p_1$ and $p_2$. Field (1980) posits two first-order relations, *mass-betweenness* and *mass-congruence*. Both Mundy and Field count as quantity primitivists because they recognize fundamental mass comparison facts. According to nomic reductionism, on the other hand, there are no fundamental quantity comparisons. Instead, the only physically significant relations among magnitudes concern the roles those magnitudes play in the laws, and in particular facts about the different consequences the magnitudes have for how things move around.

According to nomic reductionism we can make sense of the structure of mass solely in terms of the fact that 1000kg mass and 0.01kg mass are associated with different consequences for how things accelerate. For example, objects with a mass of 1,000kg *accelerate more slowly* in response to a given force, and dispose other things to *accelerate faster* due to their gravitational attraction. According to nomic reductionism, then, spatiotemporal quantities like acceleration, spatial distance, and temporal duration are special. The world's temporal and spatial arrangement is fundamental, and it gives rise to the structure of physical quantities like mass and charge. I will refer to non-spatiotemporal quantities as *physical magnitudes*.[4]

claims about comparisons among objects, like (1), are prior to claims that attribute intrinsic properties to objects, like 'the Elephant has a mass of 1000kg'. The absolutist must arguably invoke second-order comparative facts like (2), as in Armstrong (1988), Bigelow and Pargetter (1988) and Mundy (1987). So both the absolutist and the comparativist are quantity primitivists since they invoke some kind of fundamental quantity comparisons.

[4]Mass was once thought of not as a quantity associated with inertia or gravitation but as measure of how 'filled in' a region of space is. Conceiving of mass in this way would make it a quasi-spatiotemporal quantity. But I take it that the contemporary conception of mass is not like this.

According to the quantity primitivist, which things are more massive than others is independent of what the laws are like.[5] The nomic reductionist, on the other hand, claims that the elephant has more mass partially in virtue of the fact that it's harder to throw an elephant than an egg.[6]

This difference has two important implications for how to think about quantities. First, the quantity primitivist and the nomic reductionist disagree about which features of our numerical representations of quantities are merely conventional and which are physically significant. And the two views have different accounts of the content of the laws.

We use real numbers to represent magnitudes of mass. Why is this? According to the quantity primitivist this is because the the mass magnitudes stand in various fundamental ordering and distance relations, and the real numbers also stand in ordering and distance relations, and so we can use this numerical structure to represent the physical structure of the mass magnitudes. Not all features of a numerical scale are physically significant. For example, which mass magnitude gets assigned to the number 1 is purely conventional. But, for example, the ordering in a scale for mass is physically significant because it reflects the ordering relations the mass magnitudes stand in.[7]

According to quantity primitivism, quantities like mass are structured independently of the laws. So the primitivist regards laws like $f = ma$ as linking families of properties that

[5] By saying that the structure of a quantity is 'independent' of the laws I mean something stronger than 'modally free', namely that facts about a quantity's structure do not *metaphysically depend on*, or are *metaphysically explained by* facts about laws. I take the phrase 'f obtains in virtue of g' to articulate a distinctively metaphysical type of explanation that can also be expressed with a variety of locutions like *f because g*, *f depends on g*, for f to obtain *just is* for g to obtain, or *g grounds f*. I take the notion to be familiar from various debates in philosophy; for example, Socrates' challenge to Euthyphro was to say whether the pious acts are pious in virtue of the love of the gods or *vice versa*. Similarly, physicalism is the claim that everything obtains in virtue of physical facts. Recently a number of philosophers have explicitly defended the importance of the notion of fundamentality and metaphysical explanation to metaphysics; see, for example, Fine (2001), Schaffer (2009), Rosen (2010), or Sider (2012). But I invite the reader who prefers to understand these debates in terms of supervenience to read my claim as a supervenience thesis as well.

[6] One might think that quantity primitivists do not merely deny this 'in virtue of' claim; they endorse an opposing claim, namely that elephants are harder to throw than eggs in virtue of having more mass. But this second 'in virtue of' claim is plausibly a merely physical explanation. The quantity primitivist need not think that facts about throwing elephants are *grounded* in facts about their masses. Rather, the distinctive claim of quantity primitivism is the denial that mass comparisons are grounded.

[7] The details of what it takes for a scale to be faithful depends on the version of quantity primitivism. For instance, on Mundy's (1989) account, a faithful scale assigns real numbers $r_1$, $r_2$ and $r_3$ to mass magnitudes $m_1$, $m_2$ and $m_3$ if and only if (a) $r_1{+}r_2{=}r_3$ if and only if $*(m_1, m_2, m_3)$ and (b) $r_1 > r_2$ if and only if $\leq (m_1, m_2)$. On Field's (1980) account, a faithful scale assigns real numbers $r_1$, $r_2$, $r_3$ and $r_4$ to objects $o_1$, $o_2$, $o_3$ and $o_4$ if and only if (a)*mass-between*($o_1$,$o_2$,$o_3$) if and only if either $r_1 \leq r_2 \leq r_3$ or $r_1 \geq r_2 \geq r_3$ and (b) *mass-congruent*($o_1$,$o_2$,$o_3$,$o_4$) if and only if $|r_1 - r_2| = |r_3 - r_4|$.

have a prior structure. The fact that one mass property $m_1$ is twice as large as another $m_2$ is not constituted by or grounded in the fact that anything with $m_1$ will accelerate half as fast under a given force than something with $m_2$ is another.

Nomic reductionism is the claim that the only physically significant comparisons among physical magnitudes concern the role those magnitudes play in the laws. But there is an apparent problem with this claim, since the laws appear to take quantity structure for granted. $f = ma$, for example, seems to say that if one mass $m_1$ is twice as large as another $m_2$, then a body with $m_2$ will accelerate twice as fast as one with $m_1$ under the same force. If the law has this content then it cannot, on pain of circularity, be used to account for what makes $m_1$ twice as large $m_2$.

In fact this is not a problem for the nomic reductionist, for she claims that the content of the laws is exhausted by specifying what consequences different magnitudes have for how things accelerate. Consider the case of $f = ma$. Everyone will agree that some features of the statement of this law are purely conventional. For example, the choice to measure mass in kilograms instead of grams does not reflect the fact that there is anything special about objects with a mass of 1 kg. So it is natural to think that stating the law as $f = 1000ma$ with mass measured in grams is physically equivalent to $f = ma$ with mass measured in kilograms. The nomic reductionist claims that the physically significant content of this laws is exhausted by determining which magnitudes of mass and force are associated with which accelerations.

Given an appropriate assignment of masses, and forces and accelerations to numbers, this content can be expressed mathematically as $f = ma$. This encodes the fact that anything with a mass of 3 kg that experiences a resultant force of 6 Newtons accelerates at 2 $ms^{-2}$. Since the nomic reductionist regards the kilogram scale as a conventional choice, it is helpful to be able to refer to the magnitudes independently of a numerical scale. Let $m_1$ be the property of having a mass of 3kg and $f_1$ be the property of experiencing a resultant force of 6 Newtons. Then the statement of $f = ma$ encodes the fact that anything with $m_1$ and $f_1$ has an acceleration of 2 $ms^{-2}$.

But this fact can be encoded by different assignments of numbers to mass and force properties if the statement of the law is adjusted, so writing the law as $f = ma$ involves a conventional choice on our part. There are other combinations of mathematical scales for mass and mathematical statements of the laws that express this same content. For example, consider the 'schmilogram' scale, the assignment of numbers to mass properties with the feature that if a mass property gets assigned to the number $r$ by the kilogram scale, it gets assigned to $\frac{1}{r}$ by the schmilogram scale. Pair this with the mathematical statement $f = a/m$. This combination also entails that anything with $m_1$ and $f_1$ has an acceleration of 2 $ms^{-2}$.[8] So the nomic reductionist will say that $f = a/m$ with mass is measured in schmilograms is equivalent to $f = ma$ with mass measured in kilograms.

Nomic reductionism is the claim that there are no fundamental mass comparisons. All God had to do when he created the world was to settle the laws (that is, determine the ties between physical magnitudes and acceleration.) He did not then also have to settle the facts about the structure the mass properties stand in. This contrasts with quantity primitivism, which holds that some mass comparisons are fundamental.

To make vivid the difference between quantity primitivism and quantity reductionism, consider a scenario in which (by the quantity primitivist's lights) the mass comparisons are quite different but in which things behave just the way they actually do. Suppose the laws of nature were different so that wherever some quantity actually depends on a thing's mass it instead depends on the *inverse* of its mass.[9] And further suppose the distribution of masses differs from that in the actual world so that wherever $x$ kg mass is actually instantiated, $\frac{1}{x}$ kg mass is instantiated instead.

In this scenario everything behaves in exactly the same way that it actually does! Everything that actually has a small mass has a large mass in this scenario; but since the laws treat things with large masses the way that our actual laws treat things with small masses, things behave in the same way. This scenario and agrees with the actual world on the trajectory taken by every massive object.[10]

[8]A mass of 3 kg is equivalent to a mass of 1/3 schmilograms; since in the schmilogram scale $a = fm$, this means that $f = 6.1/3 = 2ms^{-2}$.

[9]That is, $f = a/m$ instead of $f = ma$ and the gravitational attraction between massive bodies were proportional to $\frac{1}{m_x.m_y.d^2}$ instead of to $\frac{m_x.m_y}{d^2}$.

[10]Consider an object $o$, with mass $m_1$ kg, $d$ m from the centre of the Earth, which has mass $m_2$ kg. In $w$ $o$ has a mass of $\frac{1}{m_1}$ kg and the Earth has a mass of $\frac{1}{m_2}$ kg. The force due to gravity on o is actually $\frac{m_1 m_2}{d^2}$; the force due to gravity on o in $w$ is $\frac{1}{\frac{1}{m_1}\frac{1}{m_2}.d^2} = \frac{m_1 m_2}{d^2}$. Assume that the only force on o is that due to

The quantity primitivist must claim that this is a scenario in which things have different masses than in the actual world. But because everything in scenario has a property that plays the same role in laws as in the actual world, the nomic reductionist claims things have the same mass in the two world.

Some might take this to be a count against nomic reductionism, since intuitively the inverse-mass has a different distribution of mass and nomic reductionism fails to vindicate this intuition. I am happy to admit that this is a cost of the view, but one that is outweighed by the arguments in favour of it. Moreover, there is little reason to expect our intuitions about the nature of physical magnitudes to be reliable, and so this is an issue on which our beliefs ought be guided by the arguments rather than by which view is closest to common sense.

If nomic reductionism is correct then the laws of nature play a role in accounting for the structure of physical quantities. But it is neutral on which philosophical account of laws of nature is correct. It is therefore compatible with the view that the laws are fundamental *sui generis* facts, or the view that the laws are explained by the essential dispositions of the fundamental properties, or even, as I argue in section 4, the view that facts about laws are themselves reducible to facts about the distribution of properties over spacetime.

I have contrasted two accounts of the structure of physical quantities. Quantity primitivism invokes fundamental quantitative structure to make sense of physical quantities. Nomic reductionism simply locates this structure in the nomic connections among magnitudes. The following section argues that the extra structure of quantity primitivist is explanatorily redundant. Section 3 argues that nomic reductionism is preferable because it affords an explanation for the fact that magnitudes fail to be freely recombinable.

## 2 The Argument From Redundant Structure

This section argues that quantity primitivism is committed to redundant structure. My case against quantity primitivism is analogous to the case against endorsing facts about

gravitation attraction of the Earth. The acceleration of $o$ is actually given by $a = \frac{f}{m} = \frac{\frac{m_1 m_2}{d^2}}{m_1} = \frac{m_2}{d^2}$. The acceleration of o in $w$ is given by $a = f.m = \frac{m_1 m_2}{d^2}.\frac{1}{m_1} = \frac{m_2}{d^2}$, which is precisely what it actually is.

absolute velocity in the context of Newtonian gravitational mechanics.

You are moving at different speeds relative to different things. You are stationary with respect to your armchair, moving at about 66,500 mph around the sun, and at about 515,000 mph around the centre of the Milky Way. But how fast are you *really* going? Do you also have an *absolute* velocity in addition to all these relative velocities?

The answer depends on how much structure spacetime has.[11] If spacetime has 'Newtonian' structure then, since there is a fact about the spatial distance between any two points, there is a fact about your absolute velocity (just find the distance between the region you are located in now and the region you were located in a moment ago). But if spacetime merely has 'Galilean' structure[12] then there are no facts about the distance between non-simultaneous points, and so there is no such thing as absolute velocity.[13]

The consensus among scientists and philosophers of science is that, assuming the laws are those of Newtonian gravitational mechanics, we should think spacetime has only Galilean structure. This conclusion is typically supported by one of two arguments. Both arguments ultimately depend on the observation that facts about absolute velocity are *undetectable*. As Newton himself was aware, what the laws say about how things in a system interact is completely independent of how fast the system is moving. But this means that even if you have an absolute velocity it's impossible to detect it.

What does it take for some quantity **q** to be detectable?[14] On a natural way of thinking about detectability, there must at least be some measuring procedure for **q** such that (a) its outputs are reliably correlated with the value of **q** and (b) its outputs are accessible to us, so that the procedure allows us to form reliable beliefs about the value of **q**.[15] It's natural to think that if there is such a measurement procedure then the results of a measurement can be recorded with the position of a pointer, or by being written down on a piece of paper, or by being described verbally, or by being displayed on a computer screen. After all, if a

[11] As Maudlin (1993) points out the issue of whether there are absolute velicities is independent of whether substantivalism is correct, since the relationist could recognize absolute velocities by, for example, positing distance relations between the temporal parts of material bodies.

[12] Sometimes called 'neo-Newtonian spacetime'.

[13] As Newton's Bucket argument showed, there had better be facts about emphabsolute accelerations. In Galilean spacetime absolute acceleration cannot be defined in terms of absolute velocity. Instead, a basic distinction between accelerating and non-accelerating trajectories is baked in.

[14] Thanks to an anonymous referee for urging me to clarify the for of the argument in what follows.

[15] This way of thinking about detectability comes from Albert (1996) and Roberts (2008).

measurement procedure allows me to form reliable beliefs about **q** then surely I can decide to record my belief by writing it down. Let's say that measurement procedures like this are *empirical measurement procedures*.[16]

That is, if there is an empirical measurement procedure for **q**, then there must be some condition C (i.e., whatever the set-up conditions for the procedure are) such that if C, for any value of **q**, $x$, the laws guarantee that the procedure results in a recording of '$x$' only if the value of **q** is $x$.[17]

But given Newtonian gravitational mechanics it is impossible for there to be an empirical measurement procedure for absolute velocity! Suppose there were such a procedure and that it is carried out by Sally the scientist. Sally writes down the result on a piece of paper: *My absolute velocity is 5 mph.* Now imagine a world that is just like ours, except that everything is moving 1000 miles an hour faster in a certain direction. The two worlds agree on the relative motions and positions of every object. Therefore Sally writes down *My absolute velocity is 5 mph.* in this world too. But Sally's absolute velocity is different in the two worlds, and so the measurement procedure must have produced a false result in at least one of them. Therefore the procedure can't have been reliable after all.

Thus we have an argument that absolute velocities are undetectable that appeals to the following principle:

> (P1) A quantity $q$ is empirically detectable in a world $w$ only if there is an empirical measurement procedure for $q$ in $w$.

Since there is no empirical measurement procedure for absolute velocities in NGM, absolute velocities are empirically undetectable.

However, this principle is arguably too limited in scope. Consider the hypothesis ('STATIONARY') that spacetime has Newtonian structure, and the laws are those of Newtonian gravitational mechanics together with the stipulation that it is a law that the center

[16] Even if there is no empirical measurement procedure for some quantity it doesn't quite follow that it is undetectable. Perhaps there could be beings that have the ability to directly sense their absolute velocity, even though they would be in the bizarre position of being unable to communicate their sensations in the form of letters or in spoken conversation or by sign language. (This question is taken up in Roberts (2008).) But I take it to be eminently plausible that we are not like these beings, and so the only quantities detectable to us are those for which there exist empirical measurement procedures.

[17] This condition should obviously be relaxed to accommodate statistical errors.

of mass of the universe is stationary. There *is* an empirical measurement procedure for absolute velocities given STATIONARY: to find the absolute velocity of some body, simply find its motion relative to the center of mass of the universe.

But there is an important sense in which absolute velocities would still be undetectable given STATIONARY. For the measurement procedure described above is only a reliable measurement procedure for absolute velocities if the laws are those of STATIONARY. So our having evidence concerning the absolute velocities of things depends on our having evidence that the laws are those of STATIONARY. But we don't have any such evidence, since the world according to STATIONARY is indiscernible from a world in which spacetime only has Galilean structure and the laws are simply those of NGM.[18]

The general point is that in order for something to be detectable, not only must there be laws that allow us to construct a certain measuring device, we must also know what the laws are that govern our measuring devices.[19] This suggests that we adopt a more general principle concerning detectability:

> (P2) If there is a measurement procedure for some quantity q if the laws are L, but not if the laws are L*, and we have no evidence that the laws are L rather than L*, then q is undetectable.

This principle correctly predicts that even if STATIONARY is true, absolute velocities are undetectable.

We have looked at two reasons for thinking that absolute velocities are undetectable in NGM. One argument against Newtonian spacetime appeals directly to the fact that positing undetectable structure is a theoretical vice.

> (D1) Newtonian spacetime requires empirically undetectable structure that is not endorsed by Galilean spacetime.
>
> (D2) All else equal, if one theory $T_1$ posits less undetectable structure than another theory $T_2$, this a reason to prefer $T_1$ over $T_2$.
>
> (D3) So, all else equal, we should prefer positing Galilean to Newtonian spacetime.

[18] Dasgupta (2013) appeals to this reasoning to argue that absolute mass facts, as opposed to merely mass ratios, are undetectable.

[19] Many thanks to an anonymous reviewer for drawing my attention to the importance of this issue.

Note that this argument is not committed to verificationism; it is compatible with (D2) that we are justified in endorsing theories that posit plenty of undetectable structure. (D2) merely says that when two theories are otherwise equally worthy of belief, we should prefer the one with less undetectable stuff.[20] While I think this argument has some merit, there is a closely related but importantly different argument that appeals instead to *redundant structure*.

One worry with the argument against undetectable structure is that is relies on a false premise, (D2), since there is nothing intrinsically suspect about positing undetectable facts. We should believe the hypothesis that provides the best explanation of our evidence, and this hypothesis may well appeal to undetectable features of the world. Rather, the correct diagnosis for why we shouldn't posit facts about absolute velocities is simply because we should attribute as little structure to the world as we can get away with.

And the fact that absolute velocities are empirically undetectable shows that we *can* get away with attributing less structure to the world than is required by Newtonian spacetime. Absolute velocities aren't needed to make sense of world, and so they are *explanatorily redundant*.[21]

Thus I think that the following argument captures the best case against positing the full structure of Newtonian spacetime in the context of Newtonian gravitational mechanics (NGM):

(N1) Galilean spacetime has less structure than Newtonian spacetime.

(N2) All else equal, if two theories are both empirically adequate we should prefer the theory that attributes the least structure to the world.

(N3) NGM with Newtonian spacetime and NGM with Galilean spacetime are both empirically adequate.

(N4) So, all else equal, we should prefer positing Galilean spacetime to Newtonian spacetime.

While this argument does not rely on the claim that undetectable structure is itself

[20] The case against Newtonian spacetime is framed in these terms by Maudlin (1993) and Dasgupta (2012).

[21] The case against Newtonian spacetime is put this way, for example, by Earman (1989), Brading and Castellani (2005), Roberts (2008), North (2009), Baker (2010), and Belot (2011).

problematic, undetectability considerations play a crucial role in providing a justification for (N3). A world with Galilean spacetime differs from a world with Newtonian spacetime only with respect to features that are undetectable. So if NGM combined with Newtonian spacetime is empirically adequate, so too is NGM combined with Galilean spacetime. If absolute velocities *were* detectable, then a theory that dispensed with them would be no good.

I take something like (N2) to be ubiquitous in both scientific and common sense reasoning, and enshrined in inference to the best explanation. Again, (N2) is not the claim that simpler hypotheses are always better; just that, faced with two hypotheses that are otherwise equally worthy of belief, we should prefer the one that attributes less structure to the world.

The claim that Galilean spacetime attributes less structure may be justified in a number of ways. One method is to appeal to a modal test for having more structure: there are a great many distinctions made by Newtonian spacetime that Galilean spacetime ignores, since for every way of arranging things over Galilean spacetime, there are many different arrangements in Newtonian spacetime that agree on the relative motion but not the absolute motions of things. But this modal test is only a rough heuristic. For as Dasgupta (2013) observes, someone might believe that spacetime has Newtonian structure, but also think that the actual world is the only possible world. (Perhaps because he thinks Spinoza was right about modality). In this case there are no more ways of arranging things in Newtonian than Galilean spacetime — there is exactly one. But surely this Newtonian's eccentric beliefs about modality does nothing to alter the fact that Newtonian spacetime has more structure than Galilean spacetime. Exactly how to spell out what it takes for one theory to have more structure than another is a vexed question that I won't try to settle here.[22] One thought is that in Galilean spacetime there are no matters of fact about the spatial distance between non-simultaneous points, whereas there are in Newtonian spacetime. Another is that Galilean spacetime has more symmetries than Newtonian spacetime. But I hope it is clear enough that however the notion is spelled out, Newtonian spacetime has more structure than Galilean spacetime.

[22] North (2009) contains a detailed discussion of what it means to minimize structure in a physical theory.

I've endorsed a particular analysis of why we should reject Newtonian spacetime in the context of Newtonian gravitational mechanics, and I will argue that quantity primitivism should be rejected for similar reasons. But my case against quantity primitivism won't depend on this analysis, for whatever theoretical vice Newtonian spacetime exemplifies it is one that is shared by quantity primitivism. If Newtonian spacetime should be rejected because it contains undetectable structure then, since analogous considerations support the claim that quantity primitivism also contains undetectable structure, quantity primitivism should be rejected on these grounds as well.

The quantity primitivist holds that there are primitive facts about which things are more massive than others. I will now argue that facts like these are just like absolute velocities. Worlds that differ only about which things are more massive than others are indiscernible, and so we don't need facts like that to make sense of the world. Quantity primitivism is committed to redundant structure, for the additional fundamental facts it requires perform no explanatory work, and so we should prefer nomic reductionism. This argument mirrors the one given above against Newtonian spacetime.

(Q1) Nomic reductionism attributes less structure to the world than quantity primitivism.

(Q2) *Ceteris paribus*, if two theories are both empirically adequate we should prefer the theory that attributes the least structure to the world.

(Q3) Nomic reductionism and quantity primitivism are both empirically adequate.

(Q4) So, *ceteris paribus*, we should prefer nomic reductionism to quantity primitivism.

This argument is valid, and so it remains only to defend the premises.

The premise (Q2) just *is* the premise (N3) in the argument against Newtonian spacetime, and so I won't say more about it here.

As for (Q1), nomic reductionism attributes less structure to the world in the same way that Galilean spacetime posits less structure than Newtonian spacetime. And again, we may appeal to the heuristic that the nomic reductionist *ignores distinctions* made by the quantity primitivist. But this test suffers from the same limitation as when applied to Newtonian spacetime. A quantity primitivist could deny that the inverse mass world is possible, and so deny that that quantity primitivism recognizes any more distinctions than the nomic reductions. (She might think this, for example, because she thought Spinoza was right about modality.) But in this case too, whatever quirky views the quantity primitivist might have about what's possible they surely aren't relevant to the question of how much structure quantity primitivism attributes to the world. Since the quantity primitivist posits extra facts that the nomic reductionist takes to be reducible, I take it to be clear enough that quantity primitivism requires more structure than nomic reductionism.

On to (Q3). The case for thinking that nomic reductionism is empirically adequate is also analogous to the case for thinking that Newtonian gravitational mechanics in a Galilean spacetime is empirically adequate. In the case of spacetime, we argued that since NGM with Newtonian spacetime is empirically adequate and NGM with Galilean spacetime agrees in all detectable respects, it too must be empirically adequate. Similarly, I'll argue that since there are no detectable differences between worlds that differ *only* about which things are more massive than others, nomic reductionism agrees with quantity primitivism in all detectable respects, and so it too is empirically adequate.

Why think fundamental quantity comparisons are undetectable? Well, consider whether there is a measurement procedure for mass ordering facts, for example. There must be a procedure that takes two objects, x and y, and results in a recording of 'x is more massive than y' only if x is in fact more massive than y. If the laws are those of Newtonian gravitational mechanics then placing the two objects on a balance and writing down which way the balance tips is a reliable measurement procedure for mass orderings.

But now consider the 'inverse-mass' world described in the previous section, in which wherever there is actually something with x kg mass there is something with 1/x kg mass instead; and the laws there are the result of replacing every appearance of 'm' in statements of the actual laws with '1/m'. In this world the balance procedure is not a reliable way of measuring mass-orderings. So in order to obtain evidence about mass-orderings we need first to know that the laws are those of the NGM, not those of the inverse-mass world.

But we don't have any such evidence! The two sets of laws are equally simple and elegant. And the two worlds agree perfectly on the trajectories things take. But *this* means that they agree perfectly on where dials in detecting devices point, and on what anyone ever wrote down, said or, for that matter, thought.[23] Since we don't have any evidence that the laws are those of NGM and not inverse-NGM, we have no reason to think that balances provide us with evidence about mass orderings.

The situation is analogous to the case of Stationary, the hypothesis that spacetime has Newtonian structure but it is a law that the center of mass of the universe is stationary. If Stationary is correct, then there is a measurement procedure for absolute velocities. But absolute velocities are still undetectable, since we could have no evidence that the laws are those of Stationary rather than simply those of NGM.

So for every world recognized by the quantity primitivist there is a an empirically equivalent world recognized by the nomic reductionist. Thus quantity primitivism is empirically adequate only if both are. This completes the defense of premise (Q3) in the argument. Even though nomic reductionism attributes less structure to the world than quantity primitivism is still able to account for the data. The extra structure of quantity primitivism is redundant structure.

Is it possible the quantity primitivist to respond that there are still primitive mass comparisons that are preserved across the inverse mass world? Perhaps a quantity primitivist could adopt a radical version of comparativism, and claim that the only fundamental mass comparisons are betweenness facts, such as the fact that 2kg mass is between 1kg mass and 3kg mass. Such betweenness facts are preserved under the operating of taking inverses, and so my chosen example does not yet show that this version of quantity primitivist has less structure than nomic reductionism[24]

In response, note first just how impoverished this version of quantity primitivism is: it is obviously impossible to capture mass orderings or ratios with just the resources of mass betweenness facts. So this radical comparativist agrees with the nomic reductionist that, for example, writing Newton's second law as $f = ma$ was a conventional choice on our part because it is physically equivalent to $f = m/a$ with mass measured schmilograms.

Because this comparativist regards mass betweenness facts as physically real, however, she – unlike the nomic reductionist – thinks that worlds with a different distribution of

[23]Assuming that fixing the configuration of your brain and environment settles the thoughts you have.

[24]I thank an anonymous referee for pressing this objection to me.

mass betweenness facts are genuinely different ways for the world to be.[25] But now the nomic reductionist can argue along precisely the same lines as before that such differences are undetectable, and so we have no need to posit irreducible mass betweenness facts to make sense of the world.[26]

This section argued that quantity primitivism attributes unnecessary structure to the world and that we should prefer nomic reductionism instead. But perhaps all else is not equal, and we are justified in positing the extra structure of quantity primitivism because we thereby obtain a theory with greater explanatory power. The next section argues that this is not the case and that nomic reductionism is preferable precisely because of its greater explanation power.

## 3 The Argument Against Brute Necessary Connections

If science is a guide to which properties are fundamental, then the fundamental properties are physical magnitudes, like 1kg mass or 3 Coulombs charge. It is determinate properties like these, rather than determinables like mass or charge, that are fundamental, since settling the distribution of mass and charge underdetermines the distribution of magnitudes of mass and charge.

The fact that the fundamental properties are physical magnitudes is puzzling, since physical magnitudes are not freely recombinable: the fact that something has 1kg mass entails that it has no other magnitude of mass. The distribution of one magnitude, say 1kg mass, imposes constraints on allowable distributions of other magnitudes of mass. But we expect the basic building blocks of the world to be, in David Hume's words, "entirely loose and separate."[27] Although property incompatibilities do not concern 'distinct existences' since they constrain the properties of a single *particular*, positing necessary connections

[25] For example, consider a world ($w_{cut}$) in which mass is distributed in the following way: for all objects, $o$, if $o$ actually has a mass of $x$ kg then (i) if $x$ is greater than 1000 then $o$ has a mass of $x$ kg in $w_{cut}$; (ii) if $x$ is less than or equal to 500 $o$ has a mass of $x + 500$ kg in $w_{cut}$; (iii) if $x$ is greater than 500 but less than or equal to 500, $o$ has a mass of $x - 500$ kg in $w_{cut}$.

[26] Could the quantity primitivism admit that there are no mass betweenness facts, but adopt some even sparser conception of which mass comparisons are physically real? It is unclear how she could, for *whatever* comparisons she regards as real will serve to distinguish worlds the nomic reductionist regards as identical. The nomic reductionist, recall, denies that there are *any* physically significant mass comparisons, except those that concern how magnitudes are linked with acceleration.

[27] David Hume (1975) [1748] p. 61.

between fundamental properties is a serious theoretical vice.

First, necessary connections are *evidence of dependence.* Entailments between distinct properties – for example, between natural and normative properties – are usually taken as a strong indication that the properties in question are not both fundamental but instead that one obtains in virtue of the other, or that each obtains in virtue of some further fact. So if a theory posits fundamental properties or relations that stand in necessary connections this is evidence against that theory.

Secondly, necessary connections *call out for explanation.* This is plausibly what drives many to infer from the supervenience of normative on natural properties that both are not fundamental. For if natural and normative properties *were* both fundamental, it would be mysterious why they were so nicely choreographed. We might imagine that God creates the world, one fundamental property at a time. Once he has settled the distribution of the natural properties, he goes on to specify the distribution of normative properties — but does so in precisely such a way that one class of properties supervenes on the other. Why would God's creative powers follow this pattern?

Thirdly, in general necessary connections between fundamental properties *mean redundancy at the fundamental level.* Suppose that property P necessitates property Q, but that P and Q are both fundamental. Now suppose that when God creates the world, He settles all the fundamental facts, one by one. First He decrees that some object o has P, and everything that is required for o to have P appears. Then God decrees that o is also Q – and now nothing new happens, because o's having Q was already settled by God's first decree. The fact that o is Q provides no new information about the world since it is entailed by o's having P. The natural conclusion is that Q is not fundamental after all, since this results in a sparser set of fundamental facts that still characterizes reality completely. A preference for sparser accounts of the world militates against theories with redundancy, and therefore against theories according to which there are necessary connections between fundamental facts.

Finally, necessary connections between fundamental properties appear to *rule out Humean reductionism about laws of nature.* Although I take Humean reductionism to be supported by powerful arguments, it is a controversial position, and so this last consideration won't be persuasive to everyone. Humean reductionists regard facts about laws — and related concepts like *dispositions*, *powers*, or *causation* to reduce ultimately to non-nomic facts. The most promising form of Humean reductionism is the 'best system analysis' defended most notably by David Lewis, according to which the laws are the theorems of the axiomatization of the distribution of properties that achieves an optimal balance of informativeness and simplicity.

Does the best system analysis count as a version of Humean reductionism? It does, as long as facts about the distribution of properties are themselves non-nomic. Lewis claimed that the fundamental properties are non-nomic, insofar as they obey a *principle of recombination*. Lewis offered various formulations of this principle of different strengths. But on the most natural reading, the fundamental properties are freely recombinable as long as the fact that one property is instantiated somewhere has no entailments for where any other property is instantiated. But if the fundamental properties are physical magnitudes as conceived by the quantity primitivist, then this is surely false! Magnitudes in a quantity are incompatible, and so the fact that some object *o* has 1kg mass *does* entail something about where other magnitudes of mass are instantiated: that o does not also have 2kg mass. So it would appear that necessary connections between fundamental properties rule out one of the most plausible accounts of laws.[28]

I conclude that there are compelling reasons to avoid positing necessary connections between fundamental properties and relations. As I'll argue, however, quantity primitivism, unlike nomic reductionism, requires endorsing problematic brute necessities.

Quantity primitivism is the claim that there are fundamental mass comparisons. Recall that a quantity primitivist could hold that either first-order (as in 'the elephant is more massive than the egg') or second-order comparisons (as in '2kg mass is greater than 1kg

[28]Ned Hall (unpublished manuscript) has recently argued, in effect, that there is nothing problematic about entailments like this. It is part of Humean reductionism about laws that laws reduce to facts that are themselves non-modal. This is supposed to rule out, *inter alia*, essentialism about physical quantities, the view that properties play their nomic roles essentially. But the Humean cannot say that a property is non-modal as long as it respects a strong form of recombination because physical magnitudes fail such a condition. Hall recommends that the Humean endorse the following version of the claim the laws reduce to non-modal facts: '[t]he fundamental ontological structure of the world is given by the distribution of perfectly natural magnitudes in it, where these magnitudes respect an inter-magnitude principle of recombination. All other facts, including facts about the laws, reduce to these facts.' This Humean has in some robust sense fewer fundamental modal facts. On the face of it, however, the claim that satisfying this limited recombination principle suffices for being entirely non-modal just looks *ad hoc*.

mass') are fundamental.

The first-order quantity primitivist posits fundamental mass comparison relations that hold between objects. But while these relations are supposed to be fundamental they are not freely recombinable. For example, Field (1980) invokes the two relations *mass-betweenness* and *mass-congruence*. These relations are stipulated to obey certain constraints; for example, *mass-betweenness(x,y,z)* and *mass-between(w,x,z)* entail that *mass-betweenness(w,y,z)*. Similarly, *mass-congruence(x,y,w,z)* and *mass-congruence(x,y,u,v)* entail *mass-congruence(u,v,w,z)*. But this is just to say that these fundamental relations violate our ban on brute necessary connections.

The second-order quantity primitivist posits fundamental mass comparisons among properties. For instance, Mundy (1979) posits two second-order relations, $\leq$ and $*$, where intuitively $\leq(p_1, p_2)$ means $p_1$ is less than or equal to $p_2$, and $*(p_1, p_2, p_3)$ means that $p_3$ is the sum of $p_1$ and $p_2$. The second order quantity primitivist must also recognize brute necessary connections.

The following question arises for the second-order quantity primitivist: is it essential to a magnitude of mass that it stand in the mass comparisons it actually does? Take two mass magnitudes, $m_1$ and $m_2$, and suppose that $\leq (m_1, m_2)$. Does it follow that in every world in which they are instantiated, $\leq (m_1, m_2)$? Suppose that it does, so that our quantity primitivist has an *essentialist* second-order account. Then her theory entails that $m_1$ and $m_2$ are necessarily incompatible. But as I argued, since $m_1$ and $m_2$ are fundamental properties we expect them to be recombinable, and so this is a reason to reject essentialist second-order quantity primitivism.

But adopting a non-essentialist version of second-order quantity primitivism instead is little help since it requires brute necessary connections of a different form. Suppose that second-order mass comparisons are not essential to the magnitude they relate. If so then the second-order quantity primitivist is free to regard first-order properties as being freely recombinable after all. Suppose that the predicate 'has 1kg mass' refers to the property $m_1$ and 'has 2kg mass' refers to the property $m_2$. Since the fundamental properties are freely recombinable, there is a world something has both $m_1$ and $m_2$. But this does *not* mean that the quantity primitivist must claim that it is possible for an object to have both 1kg mass and 2kg mass! That is, this quantity primitivist can distinguish the following claims:

(3) Nothing can have both 1kg mass and 2kg mass.

(4) Nothing can have both $m_1$ and $m_2$.

Our primitivist denies (4). But she can still endorse (3), on the grounds that 'has 1kg mass' and 'has 2kg mass' do not rigidly designate $m_1$ and $m_2$. The primitivist could instead offer a semantics for these predicates so that they only refer to a pair of properties if they stand in the appropriate mass comparisons. The quantity primitivist would then have an explanation of (3) on the basis that it is analytically true, and yet deny (4), and so avoid positing necessary connections among fundamental properties.

But while some brute necessary connections are avoided using this strategy, others are not. For suppose the primitivist regiments the structure of mass properties in terms of Mundy's relations $\leq$ and $*$. While the non-essentialist allows that $m_1$ and $m_2$ are recombinable because they are only contingently related by $\leq$, the primitivist must still endorse the following brute necessities:

(5) necessarily, for any properties $p_1$, $p_2$, $p_3$ if $\leq(p_1, p_2)$ and $\leq(p_2, p_3)$ then $\leq(p_1, p_3)$.

(6) necessarily, for any properties $p_1$, $p_2$ if $\leq(p_1, p_2)$ then nothing has both $p_1$ and $p_2$.

So even if the non-essentialist quantity primitivist can explain magnitude incompatibilities, she still posits necessary constraints associated with second-order mass comparisons like (5) and (6).[29]

So all versions of quantity primitivism involve positing necessary constraints that govern the fundamental properties or relations. This is a reason to avoid quantity primitivism if we can.

[29] Could the quantity primitivist claim that it is merely a contingent fact that $\leq$ behaves like an ordering relation, and so deny (5) and (6)? This would make it mysterious what the content of the quantity primitivist's view is; it amounts to the claim that there are *some* second-order relations that happen to behave in a certain way in the actual world. But it is unclear what these relations are and what they have to do with the structure of mass: merely labeling a relation '$\leq$' or calling it 'less-than-or-equal-to' on its own does not explanatory work. Perhaps a version of quantity primitivism could be developed that escapes these problems, but the ball is certain in the quantity primitivist's court.

The nomic reductionist does not require positing any special necessary connections. For one, the nomic reductionist denies that there are any fundamental mass comparisons and so avoids the necessary connections associated with them like (5) and (6).

The nomic reductionist also has a natural way to account for magnitude incompatibilities. The nomic reductionist holds that the only physically significant quantity comparisons concern which properties are associated with which acclerations. Again, the nomic reductionist could develop an essentialist or a non-essentialist version of her view.

The non-essentialist nomic reductionist can, like the non-essentialist quantity primitivist, hold that the fundamental properties are freely recombinable because they play their role in the laws contingently. On this view, it is a contingent fact that the properties $m_1$ and $m_2$ play the role of 1kg mass and 2kg mass in the laws, and so it is possible that something instantiate both $m_1$ and $m_2$, as long as they play different roles in the laws. And again, the nomic reductionist can offer a semantics for 'has 1kg mass' so that it is analytic that nothing has both 1kg mass and 2kg mass.

According to the essentialist nomic reductionist, *what it is* to be the 1kg mass property is to play a certain role in the laws; that is, to have certain consequences for how things move around. This means that in order for a thing to instantiate multiple magnitudes from the same quantity it would have to be disposed to follow incompatible trajectories. It is part of playing the 1000 kg mass role that if one object has 1000 kg mass, then a second massive object separated from it by 1 mm and under no other influences accelerates at 66.7 mm $s^{-2}$. And it is part of playing the 2000 kg mass role that if one object has 2000 kg mass, then a second massive object separated from it by 1 mm and under no other influences will accelerate at twice that rate, 133 mm $s^{-2}$. So if an object has both 1000kg mass and 2000kg mass, then nearby massive objects under no other influences accelerate at 66.7 mm $s^{-2}$, and they accelerate at 133 mm $s^{-2}$. Since this is impossible, we have an explanation for why nothing can have more than one magnitude of mass.

Of course, it might seem that all that has been achieved is that the bulge has been moved in the spatiotemporal carpet, for surely it is just as mysterious that nothing can have two different magnitudes of acceleration as it is that nothing can have two magnitudes of mass!

The nomic reductionist has two responses to this worry. The nomic reductionist *could* accept that in order to explain magnitude incompatibilities she must simply take for granted that nothing can have more than one acceleration. Even if this were so, the nomic reductionist would have the advantage over the quantity primitivist of having reduced the number of unexplained necessary connections: instead of being burdened with physical magnitude incompatibilities as well as spatiotemporal incompatibilities, she need appeal only to the former. And of course, this response leaves open the possibility that spatiotemporal incompatibilities may themselves be explained.

But the nomic reductionist needn't be this concessive. Even without assuming that nothing can accelerate at multiple rates, there are independent reasons to think that there could not be *laws* that require things have more than one rate of acceleration. First, laws of temporal evolution describe how the states evolve over time as a function of how objects and properties are distributed. If things have multiple accelerations then as a result they must end up in multiple locations. But the causal influences on some object depend on its location. For example, if two objects are both 1m and 2m apart, do they experience the forces they would experience if they were 1m apart or the force they would experience if 2m apart? If both, then the object that already has two locations must accelerate in two different ways, and so it must have even more locations in the future; and every object that interacts with the multiply located object must itself have multiple accelerations and therefore multiple locations. So it is somewhat implausible that there could be laws like this.

Secondly, even if laws with multiple outputs were *coherent*, if laws of temporal evolution are to be fully general they must generate a unique output. For example, consider the law $f = ma$. On a natural way of regimenting this law, it says that for any object, the unique resultant force on it is equal in magnitude to the product of its unique mass and its unique acceleration. Suppose the law were weaker and simply required that for any object, it has a mass $m_i$, a resultant force $f_i$ and an acceleration $a_i$ such that $f_i$=$m_i a_i$. This law leaves it open that there are other triples of mass, force and acceleration that do not satisfy $f = ma$. So if laws are to be fully general they must give unique outcomes for how states evolve over time.

If this is right then laws of temporal evolution arguably do not merely require that states evolve in a certain way; they also entail that any *other* way the states evolve violates the laws. Therefore it is a consequence of the laws that if one object has 1000kg mass then any massive object at a distance of 1 mm has exactly one acceleration, namely, 66.7 mm $s^{-2}$. The same goes for 2000kg mass. And it is a straightforward logical impossibility for the *unique* acceleration of some object o to be 66.7 mm $s^{-2}$ and for the *unique* acceleration of o also to be 133 mm $s^{-2}$. But there is nothing mysterious about necessary connections that are logical connections, such as the fact that it is impossible for something to exist and not exist at the same time, or that it is impossible to be both square and not square at the same time.

The nomic reductionist, therefore, can explain magnitude incompatibilities without having to invoke brute necessary connections. This a compelling reason to prefer nomic reductionism.

## 4 Objections Anticipated

In this section I will respond to what I take to be the most pressing objections to nomic reductionism.

The first objection is that nomic reductionism is not compatible with Humean reductionism about laws of nature. According to Humeanism reductionism, facts about laws are grounded in the global distribution of properties. The most promising story about how facts about laws are so grounded is the best system account associated with John Stuart Mill and Frank Ramsey and developed by David Lewis, according to which the laws are the axioms of the systematization of the facts that achieves the best possible balance of informativeness and simplicity.[30] Humean reductionism is motivated by a desire to reduce all nomic facts to non-nomic facts, like facts about how properties are distributed over spacetime. For this reason Humeans typically claim that the fundamental properties are themselves freely recombinable, in the sense that the fact that one property is instantiated at one location places no constraints on where any other property is instantiated. This provides a clear

[30] The *loci classici* are Mill (1973), Ramsey (1970), and Lewis (1973).

sense in which the facts about the distribution of properties are non-modal.

Now, discussion about the merits of Humean reductionism has largely ignored the fact that the fundamental properties are clearly *not* freely recombinable in this sense, for they are physical magnitudes, and physical magnitudes in the same family are incompatible! So not only is nomic reductionism compatible with Humean reductionism, it provides the best way for the Humean reductionism to uphold the free recombination claim, thereby ensuring that the reduction is to wholly non-nomic facts.

The Humean nomic reductionist takes the distribution of determinate properties over spacetime as fundamental, but does not recognize any fundamental facts about how these determinate properties are structured, or even about which properties are members of the same family. According to nomic reductionism comparisons among properties – including facts about which properties are members of the same family – are grounded in the role of those magnitudes in the sparse dynamical laws. And the nomic reductionist is free to claim that what makes the sparse dynamical laws *laws* is simply that they are part of the systematization that achieves a best balance of informativeness and simplicity. The sparse dynamical laws contain less structure than the richer laws recognized by the quantity primitivist. But the Humean mosaic recognized by the nomic reductionist has less structure too, since she does not regard any facts about comparisons among properties as fundamental like the quantity primitivist does. So I see no reason to suspect that the best system analysis should be adequate only on the assumption that quantity primitivism is correct.[31]

The second objection concerns the explanation of magnitude incompatibilities. I have so far ignored what might seem to be the most natural account of quantities, which holds that quantities are functions from objects to numbers. On this view, if I have a mass of 75kg, this is because the *mass-in-kilograms* function maps me to the number 75. Call this the *Pythagorean view*. The Pythagorean view captures the structure of quantities in a simple and direct way. And since part of what it is to be a function that it has a unique output,

[31]This position resembles in some respects the view proposed by Ned Hall (unpublished ms.), according to which the Humean mosaic only includes facts about the trajectories things take. Facts about physical magnitudes are not fundamental. Instead, they are made true by the fact that the best system of these trajectories attributes physical magnitudes to material objects together with laws about how things move around given their properties. The proposal in the text differs in that the Humean mosaic, the facts being summarized, also includes which properties things have. But it is similar in emphnot including facts about the structure those properties stand in, such as facts about which things are more massive than others.

it would seem that there is no mystery as to why nothing can have two distinct masses at the same time.

For all its appeal, the Pythagorean view faces formidable problems and should be rejected. The Pythagorean view either requires implausibly privileging a choice of scale or is committed to massive redundancy in the fundamental facts. Take the fact that I have a mass of 75kg. According to the Pythagorean view this is so in virtue of the fact that the *mass-in-kilograms* function maps me to the number 75. But the fact that we chose to measure masses with the kilogram scale, as opposed to the grams scale or the solar masses scale was an arbitrary decision on our part. I have a mass of 75,000 grams, so the *mass-in-grams* function maps me to the number 75,000. Are both mass functions fundamental, or is one privileged? It is hugely implausible that we have just happened to hit upon the one that is fundamental, since there is nothing physically special about objects that have 1kg mass. But if instead each mass function that corresponds to a choice of scale is fundamental, then the view requires a vast proliferation of fundamental facts. On the amended version of the view it is a fundamental fact that the *mass-in-kilograms* function maps me to one number, and it is a fundamental fact the mass-in-grams function maps me to another number, and so on.

Another family of problems with the Pythagorean View is that it is not an intrinsic feature of an object that a certain function maps it to one number rather than another. If the fundamental properties are quantities, and quantities are just functions from objects numbers, we are left with a picture of the world on which nothing has any interesting intrinsic nature. But surely the world is a certain way intrinsically. Moreover, it is one of our most fundamental convictions about properties and explanation that things behave the way they do at least partially because of the way they are intrinsically. Balls roll because they're spherical, for example. But if the fundamental facts about the nature of things only concern functions from objects to numbers, then we lack this form of explanation for how things behave.

I conclude that the Pythagorean view is not an attractive way to explain magnitude incompatibilities.

## 5 Conclusion

I conclude that we should adopt nomic reductionism over quantity primitivism. We should prefer a picture of the world that is not unduly mysterious nor unnecessarily complex; quantity primitivism fails on both counts.

The arguments presented in this paper generalize to other disputes in metaphysics. Most obviously, many of the same considerations given here arise for spacetime as well, and I believe this means we should conclude that facts about the spatial or temporal separation of two points is reducible to facts about potential causal interactions between those points. Another promising application for the argument against redundant structure is in the debate over quidditism, the issue of whether the identity of a property is fixed by its nomic role, or haecceitism, the thesis that an object's identity is not fixed by its qualitative features. It has been a mistake to see the principal obstacle to quidditism or haecceitism as revolving around the problem of whether we can come to know quiddistic or haecceitistic facts; a more compelling objection is that these views are committed to explanatorily redundant structure.

# 3. The Causal Theory of Spacetime.

We naturally think that the way things are arranged in space and time is a fundamental feature of the world. This paper argues that this is a mistake, and instead defends the *causal theory of spacetime*, the view that facts about the spatial and temporal distance between material bodies reduce to facts about how they interact.

This is a radical claim, for the vast majority of philosophers at implicitly accept *spacetime primitivism*, the claim that the spatial and temporal arrangement of the world is irreducible – whether this arrangement is to be understood in terms of relations between material objects or in terms of the structure of substantival spacetime. The causal theorist, on the other hand, holds that the spatiotemporal arrangement of the world reduces to facts about lawful dependence. On this view, all God had to do when he created the world was determine how the inhabitants of the world interact and the spatiotemporal arrangement of the world emerged from this basis.

I present three arguments in favor of the causal theory. First, if the spatiotemporal arrangement of the world were independent of its causal structure then it would be empirically inaccessible. I'll argue that this means that we don't need irreducible facts about space and time to make sense of the world – they are *explanatorily redundant.* Second, if spacetime primitivism is correct then we must give up a plausible minimality constraint on the fundamental – primitive spacetime facts are *metaphysically redundant.* Third, adopting the causal theory of spacetime allows us to explain why spatial and temporal relations fail to be freely recombinable. Spacetime primitivists must instead posit unexplained necessary connections between the basic spatiotemporal relation. As I'll explain this is a form of *axiomatic redundancy.*

Here's the plan for the paper. Section 1 develops and clarifies my thesis. Section 2 presents the argument from explanatory redundancy, section 3 presents the argument from metaphysical redundancy, and section 4 presents the argument from axiomatic redundancy. Finally, sections 5 and 6 describe how the causal theory may be combined with both non-Humean accounts of laws of nature and Humean reductionism.

## 1 Causal Structure

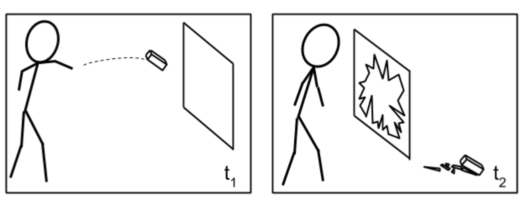
How strongly things interact depends on how far apart they are. An erupting volcano in the Galapagos Islands might hurt some turtles there but will probably do little harm to turtles in Japan.

Given spacetime primitivism this is something that merely happens to be true given the form the laws take. But according to the causal theory, facts about causal influence are constitutive of distance in space and time; it is partly in virtue of the fact that the Japanese turtles would be less harmed by the volcano that they are farther away.

For the spacetime primitivist it is one thing to say that two things are far apart, and quite another to say how they causally interact. It is therefore coherent to suppose, if spacetime primitivism is correct, that the facts about how far apart things are might vary independently of the facts about causal influence.

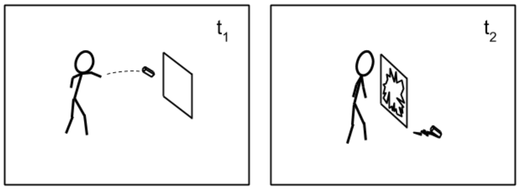
Suppose that in the actual world Billy smashes a window by throwing a brick at it:

*Actual World:*



Billy throwing the brick caused the window to smash. If spacetime primitivism is correct then we can keep this causal fact fixed while varying the spatiotemporal arrangement of the world. For example, consider a world in which everything is half its actual size.

*Halved World* ($w_{half}$):

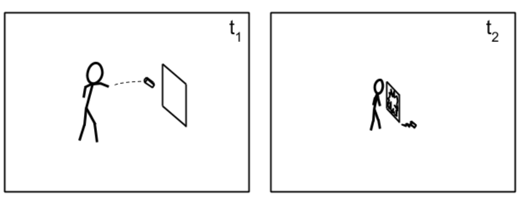



$w_{half}$ is just like the actual world except that every material object is half as large as it actually is, but the laws are also scaled down so that things interact just as they actually do.[1] A smaller Billy throws a smaller brick and smashes a smaller window.

The *ratios* among distances are unchanged in $w_{half}$. Although Billy is half his actual size, his size *relative to the window* is unchanged. The spacetime primitivist could hold that only distance ratios are physically real, and conclude that since $w_{half}$ agrees on all the distance ratios it is merely a rediscription of the actual world.[2]

But if facts about distance are independent of facts about causal interaction then we can consider worlds in which things causally interact just as they actually do but in which distance ratios are not preserved. Of course, since these worlds have different spatiotemporal arrangements there is a clear sense in which things *don't* causally interact the way they actually do. But I take it that there is an equally intuitive sense in which the following worlds describe worlds with the same causal relations.

*Shrinking World* ($w_{shink}$):

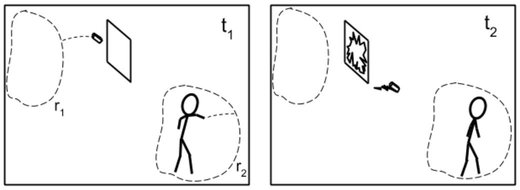



In $w_{shrink}$ Billy is half his actual size at $t_1$. But the next second, at $t_2$, he is half as large as he was at $t_1$. The next second his size has halved again. Everything is constantly shrinking in size, but the strength with which things interact weakens to compensate perfectly.

[1] That is, replace every occurrence of $d$, $x$, $v$, and $a$ in the laws by $d/2$, $x/2$, $v/2$ and $a/2$.

[2] Shamik Dasgupta (2013) argues that since halving every distance would not make any detectable difference to the world, facts about absolute distances are 'empirically redundant' and this is a reason only to regard facts about distance ratios as physically real.

*Cut and Paste World* ($w_{paste}$):




In $w_{paste}$ Billy is not located in front of the window. However, the region he occupies, $r_2$, stands in all the relations of causal dependence that $r_1$, the region that is *actually* occupied by Billy, stands in. So when Billy throws the brick, it emerges from $r_1$ as if Billy were located there.[3]

It's easy to multiply examples. $w_{grow}$ is just like the actual world except that everything doubles in size every second but the strength with which things interact weakens to compensate perfectly. The shards of glass produced are much larger than in actuality, although they are no more dangerous. $w_{faster}$ is just like the actual world except that the time between events decreases, so that everything happens faster and faster. Billy is able to escape the crime scene much sooner, although he's no more likely to evade capture. And so on.

These scenarios differ radically about the spatial and temporal arrangement of the world. But it is clear that they have something in common; in a quite intuitive sense, things causally interact just as they actually do. And they agree not only about actual causation but also about causal dependence. For example, in each world if Billy were to throw a feather at the window the window would fail to break, and if he were to throw a grenade it would smash the window more thoroughly.

I'll say that these worlds have the same *causal structure*. The causal theory of spacetime is the claim that the spatiotemporal arrangement of the world reduces to its causal structure. As I'll make clear, I understand this an explanatory claim: the causal theorist claims that spacetime structure obtains *in virtue of* causal structure.[4] For example, it is partly *because*

[3]David Albert (1996) considers a similar scenario to argue that the geometrical appearances are accounted for by the dynamical laws.

[4]The claim that spacetime is not fundamental (or 'emergent') comes up in discussions of two ares of physics. David Albert (1996) defends an interpretation of Bohmian mechanics on which the fundamental space is an extremely high-dimensional *configuration space*. Similarly, the idea that four-dimensional spacetime is not fundamental is a feature of some versions of string theory. But both of these claims involve taking some kind of spatial or geometrical structure as fundamental, and so I will treat these claims as versions of spacetime primitivism.

a nuclear explosion in the vicinity of my coffee mug would cause me to die, while a nuclear explosion on the moon would cause me no harm, that I am closer to my coffee mug than to the moon. Like many philosophers, however, I do not take causation to be fundamental, so I will characterize causal structure in terms of the laws of nature.

If the causal theory of spacetime is correct then these scenarios do not, after all, correspond to different possibilities. Is this a count in its favor, because intuitively these scenarios involve making distinctions without a difference? Or does this count against the causal theory because the causal theory fails to recognize intuitively distinct possibilities? I'm not sure; I find my intuitions to pull in both directions. So one response is to say with David Armstrong, *spoils to the victor!* and conclude that we should let our intuitions be guided by theory, not *vice versa*. But I am happy to grant that there is some weak intuitive pressure against the causal theory. Intuitions about the nature of fundamental reality must sometimes be revised in the face of countervailing evidence, as with the appearance that the sun revolves around the Earth, or that some events are objectively simultaneous. Similarly I will argue that the evidence from intuitions against the causal theory is outweighed by the arguments in favor of the view.

The issue at stake between the causal theory of spacetime and spacetime primitivism is independent of the dispute between substantivalism and relationism about spacetime. Substantivalism is the claim that spacetime regions or points do not depend for their existence on material objects.[5] According to relationism, spatiotemporal relations only hold between material objects and claims about spacetime itself are derivative relative to claims about spatiotemporal relations between material objects.[6] This is a dispute about which kinds of entities instantiate fundamental spatiotemporal properties and relations: material objects or spacetime points and regions? The question at issue in this paper cross-cuts the substantivalism-relationalism debate since it concerns the spatiotemporal *relations* themselves, not their relata. If substantivalism is correct, we may ask whether it is a fundamental

[5] I don't think the dependence at issue is merely modal dependence, since in principle a substantivalist could deny that facts about spacetime are modally independent of material bodies. Rather, substantivalism properly construed is the claim that spacetime points and regions do not exist in virtue of material objects and their properties. See Dasgupta (2011).

[6] I'll frame the discussion in terms of spatial and temporal *relations*, but I intend this to be neutral about whether nominalism or realism about properties and relations is correct.

fact that two points are one meter apart or whether this obtains in virtue of other facts. And if relationism is correct, we may ask whether it is a fundamental fact that two material objects are one meter apart, or whether this obtains in virtue of other facts. I will assume substantivalism for the sake of presentation but the extension to relationism should be straightforward.

The causal theory of spacetime is the claim that spacetime structure reduces to causal structure. In what sense do the scenarios above describe worlds with the same causal structure? In each world, Billy's throwing a brick causes a window to break. But we cannot explain what the worlds have in common by saying that a *duplicate* of Billy causes a *duplicate* of the window to break, since duplication is standardly defined so that two objects are duplicates only if their parts stand in the same spatial relations.[7]

Let us use a slightly different notion instead. Say that two objects are *purely qualitative duplicates* if and only if their parts have the same perfectly natural properties.[8] Objects can be purely qualitative duplicates even though they are not duplicates; one window may be a purely qualitative duplicate of another even though it's twice as large. I'll say that purely qualitative duplicates have the same *purely qualitative profile.* We can extend this notion to spacetime regions: the purely qualitative profile of a spacetime region $r$ is given by saying which perfectly natural properties are instantiated at each subregion of $r$.[9]

In the actual world, Billy's throwing the brick caused the window to break. In $w_{half}$ smaller Billy caused a smaller window to break. Billy and smaller Billy are purely qualitative

[7]For example, see Lewis (1986) p. 60. Lewis requires that if objects are duplicates then their parts must stand in the same perfectly natural relations. I should note that I am happy to take mereological relations among regions as primitive.

[8]That is, there is a one-one mapping between their parts such that every part of one object is mapped to a part of the other object with the same perfectly natural properties. (Note that everything is a part of itself). The restriction to *perfectly natural* properties is required so that objects can be purely qualitative duplicates even though they fail to share properties like *being exactly ten feet from President Obama*.

[9]Three remarks. First, while I am assuming substantivalism I shall try to remain neutral about *supersubstantivalism*, the thesis that material objects are identical to spacetime regions. So I will remain neutral on whether the perfectly natural properties are instantiated merely *at* spacetime regions (by being instantiated by a material object located there) or *by* the regions themselves. I assume that spacetime is continuous. Second, spacetime is not made up of enduring points, but instantaneous points-at-a-time, *events*. Spacetime regions perdure: they exist at multiple times by having parts at those times. So the notion of location appealed to in a claim like 'the ball started at $l$, bounced off the wall and returned to $l$' encodes information about the relation between two regions that make up $l$: the spacetime region initially occupied by the ball, $l_i$, and the region occupied later $l_f$. Third, if spacetime is discrete then some worlds that differ by a scale factor will not count as having the same causal structure. For example, if spacetime is a lattice made of points one 'unit' apart, then objects can differ in size by being different finite numbers of units across. Therefore no discrete spacetime has the same causal structure as any continuous spacetime.

duplicates, and so too are the broken window and the smaller broken window in $w_{half}$. So the regions occupied by Billy in the two worlds ($r_{Billy}$ and $r_{SmallBilly}$) are purely qualitative duplicates, as are the regions occupied by the broken windows ($r_{Window}$ and $r_{SmallWindow}$).

But these two pairs of regions have something else in common: they stand in the same relations of causal dependence. For example, if $r_{Billy}$ contained an exploding bomb, this would cause $r_{Window}$ to contain a more thoroughly smashed window. And the same is true of $r_{SmallBilly}$ and $r_{SmallWindow}$. I'll say these regions are *causally similar.*

We can characterize causal similarity in terms of the what laws say about how the regions interact. I write $L(p)$ to express the fact that it is a consequence of the laws that $p$. Suppose that the laws entail that if there is an explosion of a certain magnitude sufficiently close to a glass window then that window breaks. In particular suppose that if the laws entail that $r_{Billy}$ contains an exploding bomb then $r_{Window}$ contains a thoroughly smashed window. Then we can write this as $L(BOMB(r_{Billy}) \rightarrow SMASH(r_{Window}))$. Since we are interested in causally similar regions in worlds with different spatiotemporal arrangements we will focus on lawful conditionals that do not presuppose facts about how things are arranged in space and time. We can do this by selecting just those conditionals that describe the interaction between regions in non-spatiotemporal terms. For example, if $p_B$ and $p_S$ are the purely qualitative profiles that the regions $r_{Billy}$ and $r_{Window}$ would have if they contained an exploding bomb and a smashed window, then $L(p_B(r_{Billy}) \rightarrow p_S(r_{Window}))$ characterizes the dependence between the regions without presupposing spacetime structure. I will say that these lawful conditionals that ignore spacetime structure express *direct dependence* relations among regions.[10]

Causal similarity can now be defined in terms of relations of direct dependence. Two regions are causally similar when they have the same *forward-looking causal profile* and the same *backward-looking causal profile.*

Two regions have the same forward-looking causal profile (CAUSAL PROFILE$_F$) if and

[10] It is worth noting that this lets us distinguish two different senses in which a window can be said to be broken. In one sense (the spatial sense) a window is broken if and only if its proper parts are no longer in spatial contact. But in the other sense (the causal sense) a window is broken if and only if its parts no longer compose a causally cohesive object, so that, e.g. pushing one part causes the whole window to move. These senses can come apart – imagine a version of the cut and paste world, but swapping out a section of an unbroken window instead. In this new cut and paste world the window is spatially but not causally broken.

only if what goes on in the regions has the same affects on other regions:

> Causal Profile$_F$: two regions, $r_1$ in $w_1$ and $r_2$ in $w_2$, have the same forward-looking causal profiles if and only if for any purely qualitative profiles $p_1$ and $p_2$, if there is a region $r_3$ such that $L(p_1(r_1) \rightarrow p_2(r_3))$ then there is a region $r_4$ such that $L(p_1(r_2) \rightarrow p_2(r_4))$.[11]

Similarly, two regions have the same backward-looking causal profile (Causal Profile$_B$) if and only if they are affected in the same way by what is going on in other regions:

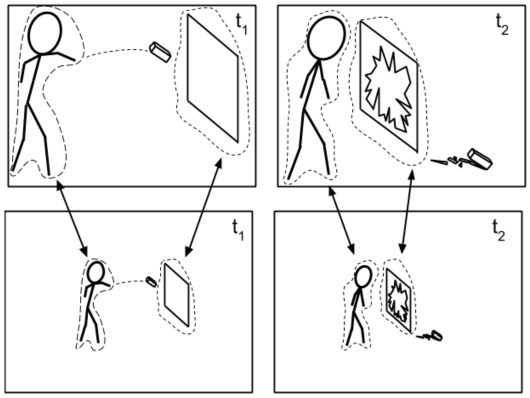
> Causal Profile$_B$: two regions, $r_1$ in $w_1$ and $r_2$ in $w_2$, have the same backward-looking causal profile if and only if for any purely qualitative profiles $p_1$ and $p_2$, if and only if there is a region $r_3$ such that $L(p_1(r_3) \rightarrow p_2(r_1))$ then there is a region $r_4$ such that $L(p_1(r_4) \rightarrow p_2(r_2))$.

If two regions are both causally similar and purely qualitative duplicates, I will say that they are *causal duplicates*. Now we can characterize causal structure in the following way:

> Causal Structure: two worlds $w_1$,$w_2$ have the same causal structure if and only if there is a one-one mapping between regions in $w_1$ and regions in $w_2$ that maps every region to a causal duplicate region.

This captures what is shared between the worlds we looked at previously. In each world, we can identify regions that are causal duplicates of actual regions. For example, the region that actually contains Billy is a causal duplicate of the smaller region in $w_{half}$ that contains the smaller Billy.

[11] To incorporate chancy causation we need to require that if $r_1$ having $p_1$ lawfully entails that the chance that $r_3$ has $p_2$ is $c$, then $r_2$ having $p_1$ lawfully entails that the chance that $r_4$ has $p_2$ is $c$.



Causal duplicate regions in $w_{actual}$ and $w_{half}$.

We are now in a position to state the causal theory of spacetime:

> THE CAUSAL THEORY: facts about the world's spatiotemporal structure obtain in virtue of its causal structure.

Since the causal structure of the world is determined by the laws of nature, there is an obvious problem with this claim. The laws as we know them make claims about spatiotemporal structure: they say, for example, how things *accelerate* under given forces, and how gravitational attraction varies as a function of the *distance* between two massive bodies. So we might worry that since the laws themselves appeal to spatiotemporal notions, we cannot use the laws to analyze spacetime itself.

Precisely what fundamental reality is like according to the causal theory – and therefore how the causal theorist responds to this problem – depends on which account of the laws of nature is correct.

The most promising version of reductionism about laws is the best system analysis defended notably by David Lewis.[12] On this view, a statement counts as a law when it encodes a lot of information in a particularly efficient way. Compare all the ways of summarizing the contingent facts about the world. Some are very informative, but extremely complicated: we could just list every property instantiated by each spacetime point. Others are extremely simple, but not very interesting: we could simply state the number of material particles that exist. According to the best system analysis, the laws of nature are those statements that together achieve the best balance of informativeness and simplicity.

[12]See Lewis (1983). Lewis was building on the regularity accounts of John Stuart Mill and Frank Ramsey.

Proponents of the best system analysis have typically held that the contingent facts being summarized are those that make up the 'Humean mosaic:' the facts about the distribution of fundamental intrinsic properties over spacetime. But a Humean causal theorist may adopt a *best system analysis of spacetime* where the facts being summarized are the *non-spatiotemporal mosaic*: just those facts that describe the physical magnitudes instantiated at different points. On this view, when God created the world all he had to do was to create an infinity of simple unstructured objects and then sprinkle physical magnitudes over them. In some worlds, the best way to provide a lot of information about these facts in a simple way is to attribute distance structure to the points and add laws that describe how things interact as a function of the distance between them. According to the best system analysis of spacetime, a claim about the spatiotemporal structure of the the world is true if and only if it is entailed by the best summary of the non-spatiotemporal mosaic.[13] The claim that two particles are 1m apart is made true, if it is true, by the fact that the best system says so, even though fundamentally speaking nothing stands in distance relations.

It may seem wildly implausible that such a sparse basis could give rise to the rich spatiotemporal structure we take our world to have. I respond to this worry in section 6.

To be a non-Humean about laws is to hold that there are irreducible facts about laws, dispositions, nomological possible, or some related notion. If the causal theory is to give a non-circular account of spacetime the non-Humean causal theorist must identify nomic features of the world that do not presuppose facts about its spatiotemporal arrangement.

Causal structure is characterized in terms of the direct dependence relations among regions. For example, it is part of the causal structure of the world that if a nuclear warhead is detonated in the region actually occupied by my coffee mug then the region I actually occupy will contain an explosion.

The spacetime primitivist will naturally regard direct dependence relations as determined by the laws of nature together with facts about the spatiotemporal relations between

[13]The best system analysis of spacetime resembles the Humean reductionism about physical magnitudes suggested by Ned Hall (forthcoming). Hall proposes a version of the BSA on which the Humean basis consists of facts about particle trajectories and nothing else. In worlds with sufficiently rich particle trajectories, the mosaic may be summarized very simply by attributing physical magnitudes to particles and then writing down laws that describe how the trajectory of a particle depends on its physical magnitudes and environment. On this view the claim that some particle has 1g mass is made true by the best system saying so even though, fundamentally speaking, nothing has mass. The causal theorist can make a similar move.

regions. But the non-Humean causal theorist can instead reverse the order of explanation. On this view these relations of direct dependence linking particular regions are fundamental, and they give rise to the spatiotemporal arrangement of the world. The familiar laws of nature that are stated in terms of spatiotemporal structure are just simple and elegant ways of encoding facts about these direct dependence relations. Since metaphysical explanation comes apart from scientific explanation, the non-Humean causal theorist can claim that the law $f = ma$ is scientifically fundamental while direct dependences are metaphysically fundamental. I develop this position in more detail in section 6.

As I understand it, the causal theory of spacetime is an explanatory claim. To say that facts about how far apart things are reduce to facts about how things interact is to say they obtain *in virtue of* them.[14] I take the locution *f in virtue of g* to articulate a non-causal flavor of explanation that is familiar from various issues in philosophy. Socrates' challenge to Euthyphro was to say whether the pious acts are pious in virtue of the love of the gods or *vice versa*. A promising way of understanding physicalism is as the claim that everything obtains in virtue of physical facts.[15] Many hold that dispositional properties are instantiated in virtue of categorical properties: the glass is fragile in virtue of having a certain crystalline structure.[16] Normative properties are had in virtue of natural properties: an act is wrong in virtue of the fact, say, that it causes gratuitous suffering. I take this notion to be familiar to common sense judgments too, as when one says that Mary has a headache because her brain is in a certain state $b$.[17]

The causal theory of spacetime contrasts with spacetime primitivism, the claim that space and time are fundamental. The following section argues that we should reject space-

[14]We can make a distinction between two kinds of 'in virtue of' claim. One kind consists of cases in which A-facts obtain in virtue of B-facts, although it is possible for the B-facts to obtain in virtue of other facts instead, or for there to be nothing in virtue of which the A-facts obtain. Another consists of cases in which A-facts obtain in virtue of B-facts in any world in which they obtain. One might think, for instance, that even if consciousness facts obtain in virtue of physical facts, there could have been worlds in which consciousness facts obtain in virtue of ectoplasm facts. But it is less plausible to think that while knowledge facts actually obtain in virtue of facts about reliable processes (say), they might have failed to obtain in virtue of anything. I won't take on whether the causal theory of spacetime belongs with the former or the latter cases.

[15]See, e.g. Barry Loewer (1996).

[16]See Prior, Parghetter and Jackson (1982)

[17]For more explicit defenses of this notion see Fine (2001), Schaffer (2009), Rosen (2010), or Sider (2012). I avoid using the term 'ground' because it is used in different ways in the contemporary literature. Some, such as Fine, use it to refer to the type of explanation I have in mind, but by others, like Schaffer, use it to name a *relation* that is supposed to back the explanations in question.

time primitivism because primitive spatiotemporal relations are explanatorily redundant.

## 2 The Argument from Parsimony

The argument from parsimony is simple. The causal theory of spacetime attributes less structure to the world than spacetime primitivism. All else equal, we should prefer theories that attribute less structure to the world. So, all else equal, we should prefer the causal theory over spacetime primitivism.

My case against spacetime primitivism is analogous to the case against endorsing facts about absolute velocity in the context of Newtonian gravitational mechanics (NGM).

You are moving at different speeds relative to different things. You are stationary with respect to your armchair, moving at about 66,500 mph around the sun, and at about 515,000 mph around the center of the Milky Way. But how fast are you *really* going? Do you also have an *absolute* velocity in addition to all these relative velocities?

The consensus among philosophers of science is that we should think not. As Newton himself was aware, what the laws of NGM say about how things in a system interact is completely independent of how fast the system is moving. But this means that even if you have an absolute velocity, it is impossible to detect it. The fact that absolute velocities are undetectable shows that we don't need them to make sense of the world. Since we should prefer theories that attribute less structure to the world, we should prefer an account of the world that does not recognize absolute velocities.[18]

Why aren't absolute velocities detectable? For a physical quantity $q$ to be detectable requires that there is a *measuring procedure* for $q$, a nomologically possible process whose outputs (a) are reliably correlated with the value of $q$ and (b) are accessible to us, so that the procedure allows us to form reliable beliefs about the value of $q$.[19] For example, a measurement procedure might correlate the value of $q$ with the position of a dial in some measuring device, or what is displayed on a computer screen, or the arrangement of ink

[18] For discussion of this case see Earman (1989), Brading and Castellani (2005), Roberts (2008), North (2009), Baker (2010), and Belot (2011). Some philosophers (for example Dasgupta (2013) and Maudlin (2007)) present the case against absolute velocities as revolving around the vice of positing undetectable structure rather than the vice of positing redundant structure.

[19] This way of thinking about detectability comes from Albert (1996) and Roberts (2008).

particles on a piece of paper, so that by observing the dial, computer screen or paper, we can form reliable beliefs about $q$.

If a quantity $q$ is detectable by any means then we can argue that in particular there must be a measurement procedure that correlates the value of $q$ with the positions of ink particles on a piece of paper. After all, if there is any measurement procedure for $q$ that allows me to form reliable beliefs about $q$, then I could decide to write down the content of my beliefs on a piece of paper, and so the procedure that includes my recording the result on paper will itself be a reliable measurement procedure.[20]

But given NGM there is no measurement procedure like this for absolute velocity! Suppose there were such a procedure and that it is carried out by Sally the scientist. Sally writes down the result on a piece of paper: *My absolute velocity is 5 mph.* Now imagine a world that is just like ours, except that everything is moving 1000 miles an hour faster in a certain direction. The two worlds agree on the relative motions and positions of every object. Therefore Sally writes down *My absolute velocity is 5 mph.* in this world too. But Sally's absolute velocity is different in the two worlds, and so the measurement procedure must have produced a false result in at least one of them. So the procedure can't have been reliable after all.

This suggests the following necessary condition for some quantity to be detectable:

(P1) A quantity $q$ is detectable in $w$ only if there is a measurement procedure for $q$ in $w$.

Since there is no measurement procedure for absolute velocities in NGM, absolute velocities are undetectable.

However, for some quantity $q$ to be detectable it is not sufficient for there to be a measurement procedure for $q$. Consider the hypothesis ('STATIONARY') that there are facts about absolute velocities and the laws are those of NGM together with the stipulation that it is a law that the center of mass of the universe is stationary. There *is* an measurement procedure for absolute velocities given STATIONARY: to find the absolute velocity of some

[20] This is at least this case for what is detectable *for us*. Perhaps there could be beings that have the ability to sense their absolute velocity directly, even though they would be in the bizarre position of being unable to communicate their sensations in the form of letters or in spoken conversation or in sign language. (Roberts (2008) discusses the possibility of such beings, and the implications this has for the claim that absolute velocities are undetectable.) But I take it that we are not like these beings.

body, simply find its motion relative to the center of mass of the universe.

But there is an important sense in which absolute velocities would still be undetectable given STATIONARY. For the measurement procedure described above is only a reliable measurement procedure for absolute velocities if the laws are those of STATIONARY. So our having evidence concerning the absolute velocities of things depends on our having evidence that the laws are those of STATIONARY. But we don't have any such evidence, since the world according to STATIONARY is indiscernible from a world in which there are no absolute velocities and the laws are simply those of NGM.[21]

The general point is that in order for something to be detectable, not only must there be laws that allow us to implement a measuring procedure, we must also know *what the laws are* that govern our measuring procedure. This suggests that we adopt a more general principle concerning detectability:

(P2) If there is a measurement procedure for some quantity $q$ if the laws are $L$, but not if the laws are $L*$, and we have no evidence that the laws are $L$ rather than $L*$, then $q$ is undetectable.

This principle correctly predicts that even if STATIONARY is true, absolute velocities are undetectable.

The fact that absolute velocities are empirically undetectable shows that we don't need facts about absolute velocities to make sense of the world; the extra spacetime structure required to make sense of them is superfluous structure.

The spacetime primitivist holds that there are primitive facts about the spatiotemporal arrangement of the world. I will now argue that facts like these are just like absolute velocities. Worlds that differ only in how things are arranged in spacetime are indiscernible, and so we don't need primitive spacetime facts to make sense of the world. Spacetime primitivism is committed to redundant structure, for the additional fundamental facts it requires perform no explanatory work.

The argument from redundancy against spacetime primitivism is analogous to the case against positing facts about absolute velocities:

[21] Dasgupta (2013) appeals to similar reasoning to argue that absolute mass facts, as opposed to merely mass ratios, are undetectable.

(S1) The causal theory attributes less structure to the world than spacetime primitivism.

(S2) *Ceteris paribus*, if two theories are both empirically adequate we should prefer the theory that attributes the least structure to the world.

(S3) The causal theory and spacetime primitivism are both empirically adequate.

(S4) So, *ceteris paribus*, we should prefer the causal theory to spacetime primitivism.

This argument is valid, and so it remains only to defend the premises.

As for (S1), we could appeal to a modal test for when one theory attributes more structure than another. The claim that endorsing absolute velocities requires extra structure is typically motivated in this way: if there are absolute velocities then there are possibilities that differ only in that everything is moving at a different constant velocity. Similarly, if spacetime primitivism is correct then the actual world and the shrinking world are distinct possibilities. But this is not so according to the causal theorist. Since the causal theory ignores distinctions recognized by spacetime primitivism, spacetime primitivism contains extra structure.

But this modal test is at best a useful heuristic. For consider someone who believes that there are absolute velocities but denies the relevant claims about possibility. For example, as Dasgupta (2013) points out, she might believe that Spinoza was right and there is only one possibility, the actual one. Or she may just endorse STATIONARY. A spacetime primitivist could deny that the shrinking world and the actual world are distinct possibilities on similar grounds. But surely these quirky modal beliefs are simply irrelevant to how much structure a theory attributes to the world. A better test is simply to look at the fundamental facts the theories posit. After all, attributing excess structure is a matter of what the world is actually like, not what it could have been like. The spacetime primitivist recognizes all the fundamental facts the causal theory does, and more besides: primitive facts about how things are arranged in spacetime. So spacetime primitivism attributes more structure to the world than the causal theory of spacetime.[22]

I take the principle expressed in (S2) to be ubiquitous in both scientific and common

[22] The causal theory of spacetime is the claim that facts about spacetime are not fundamental, not that they are false. So the principle appealed to is: attribute as little structure to *fundamental* reality as possible.

sense reasoning, and enshrined in inference to the best explanation. (S2) is not the claim that simpler hypotheses are always better; just that, faced with two hypotheses that are otherwise equally worthy of belief, we should prefer the one that attributes less structure to the world.

On to (S3). The case for thinking that the causal theory is empirically adequate is analogous to the case of absolute velocity. Since absolute velocities are undetectable, an account of the world that doesn't recognize absolute velocities is alike in all detectable respects with an account that does, and so both theories are empirically adequate as long as one is.

I will argue that primitive spacetime facts are undetectable, and since the causal theory agrees with spacetime primtivism on all the detectable facts, the causal theory is empirically adequate if spacetime primitivism is.

Why think primitive spacetime facts are undetectable? Well, consider whether there is a measurement procedure for distance facts, for example. There must be a procedure that, given two points $p_1$, $p_2$, results in a recording of '$p_1$ is $x$ meters from $p_2$' only if $p_1$ is in fact $x$ meters from $p_2$. Suppose that placing the end of some measuring tape next to one point, holding the tape taut so that it lies on the second point, and recording the number on the tape adjacent to the second point is such a measurement procedure.

Suppose we try to measure my height by this method. Now consider the halved or shrinking worlds described in the previous section, in which my height is different from what it actually is. Since these worlds have the same causal structure as the actual world, they agree about the output of the measurement procedure. So in order for the tape measure to give me evidence about my height I need to have evidence that the laws are those of the actual world and not those of the halved world or the shrinking world. But we don't have any such evidence. These worlds have the same causal structure, and so they are perfectly indiscernible. The same things happen, and for the same reasons. Suppose you actually form the belief that there is beer in the fridge on the basis of your perceptual evidence. Then in any world with the same causal structure as the actual world, a purely qualitative duplicate of you forms the same belief on the basis of seeing a purely qualitative duplicate of the beer (and drinks it for the same reason!)

So for every world recognized by the spacetime primitivist there is a an empirically equivalent world recognized by the causal theorist. Thus spacetime primitivism is empirically adequate only if both theories are. This completes the defence of premise (S3) in the argument. Even though the causal theory attributes less structure to the world than spacetime primitivism it is still able to account for the data. The extra structure of spacetime primitivism is redundant structure.

The argument from redundant structure has applications in other, more mundane cases. Suppose you are choosing between two theories that are alike except one ('Goblin') adds that there is an undetectable goblin collocated with each massive object. It's extremely natural to think that we should give Goblin lower credence because it posits things, goblins, that aren't needed to explain the data.

Note that the dispute at issue between the causal theory and the spacetime primitivist is about *what we ought to believe*, not about what we know, given the beliefs we have. A believer in Goblin, if Goblin were true, would plausibly know where the goblins are. But this does not address the question of whether he should be believe Goblin in the first place. Similarly for spacetime primitivism. I grant that if spacetime primitivism *were* true then we would know how things are spatiotemporally arranged — as long as we were not in the shrinking world. But this does not yet answer the question of whether or not to believe that spacetime primitivism *is* true.

One might worry that the argument from redundant structure is just skeptical reasoning. Worlds with different spatiotemporal arrangements but the same causal structure are simply skeptical hypotheses, the worry goes. And just as we don't need to be able to rule out skeptical hypotheses like brain-in-vat worlds to obtain evidence for claims like *there is a table*, we don't need to rule out these spatiotemporally deviant worlds to have evidence for claims like *I am six feet tall.*

Saying exactly what distinguishes cases of bad, skeptical, reasoning from good cases of inference to the simpler explanation is a deep and difficult problem. But surely we are *sometimes* justified in rejecting theories with surplus structure, as in the case of absolute velocities or Goblin. And whatever the correct principles of theory choice are, it's plausible that the case against spacetime primitivism belongs with the case against Newtonian absolute velocities, not the skeptic's case against the external world hypothesis.

One might be tempted to argue that any sensible epistemological principles that allow us to avoid skepticism also undermine the argument from redundant structure against space-time primitivism. But it appears that any such principle will also, incorrectly, undermine the case against absolute velocities or against GOBLIN.

We have considered one such principle. One might claim that something is undetectable only *relative* to some laws, and conclude that any argument for undetectability that appeals to possibilities with different laws is unsound. For example, perhaps some fact $f$ is undetectable only if there is no reliable measurement procedure for $f$, where measurement procedures are defined in terms of laws in the way discussed above. Or perhaps a fact it undetectable if it fails to be invariant under the symmetries of the laws. But while these features might pick out sufficient conditions for something's being undetectable they do not identify necessary conditions, since they fail to predict that if the laws were those of STATIONARY then facts about absolute velocity would be undetectable.

Suppose instead that we should believe whatever theory best explains our evidence, where our evidence includes whatever we know.[23] If I can come to know that I have hands on the basis of perception, my evidence includes the fact that I have hands, and so I can dismiss the hypothesis that I'm a brain in a vat. However, this principle predicts that the believer in GOBLIN would be unjustified in coming to accept the goblin-free theory instead, because as long as she believes the goblin theory, she takes herself to know the whereabouts of the goblins. Such facts are therefore part of what she takes to be her evidence, and so she will reject the Goblin-free theory on the ground that it can't explain what she believes that she knows.

But this is the wrong result. Surely we want to say that the believer of GOBLIN, by reflecting on the available theories, should rationally be able to adopt the goblin-free theory instead. Similarly, someone who believes STATIONARY, assuming he or she possesses the relevant astronomical information, could come to know facts about absolute velocities. If she is rational in rejecting theories that fail to account for what she takes to be her evidence,

[23]Timothy Williamson (2000) defends this account of evidence. Note that *norms for belief* may come apart from *principles of theory choice*. For we might think the brain in a vat is not justified in believing she has hands while someone with a body is, even though it is rational for them to have the same beliefs.

she is rational in maintaining her belief in Newtonian absolute space. But intuitively this isn't right.[24]

A quite different objection to the argument from redundancy is that while, all else equal, we should prefer theories that attribute less structure to the world, all else is not equal between spacetime primitivism and the causal theory. Perhaps by allowing the extra structure required by spacetime primitivism we obtain a theory that is much more explanatory, or otherwise superior, and so the extra structure earns its keep. The causal theory may attribute less structure to the world, but it is so unwieldy, or disjunctive, or ugly that it is overall unworthy of belief. The next section, which presents the argument from explanatory power, will argue that the causal theory is not just theoretically on a par with spacetime primitivism, it is to be preferred precisely because it is explanatorily superior.

## 3 The Argument from Metaphysical Redundancy

David Lewis said of the perfectly natural properties and relations that "there are only just enough of them to characterize things completely and without redundancy."[25] There is something very intuitive about this thought. When God created the world, we might imagine, he didn't do unnecessary work. The fundamental facts should plausibly form a *minimal supervenience base*, so that everything supervenes on the fundamental facts but not on any proper subset of them. If the fundamental facts failed to form a minimal supervenience base, then some of them wouldn't be needed to characterize the world. I'll say facts like this are *metaphysically redundant.*

According to the spacetime primitivist, spatiotemporal relations are fundamental. But

[24] Suppose instead that our evidence consists of what we know *non-inferentially*, where a belief is non-inferential if we don't believe it on the basis of inference. (See Alexander Bird (2004).) Against the skeptic we can claim that we know non-inferentially that we have hands. But, we might think, we have plenty of non-inferential knowledge about the spatiotemporal arrangement of the world. Surely I can know where things are just by looking at them! But if STATIONARY were true, we arguably could also have non-inferential knowledge about absolute velocities. (As long as our true beliefs about the absolute velocities were caused by and sensitive to the absolute velocities, say.) So again, this principle has the incorrect result that the believer in absolute velocities would be justified in rejecting theories that failed to account for absolute velocity facts.

[25] Lewis (1986) p. 60. Lewis makes a similar remark in his (1983) p. 12: "The world's universals should comprise a minimal basis for characterizing the world completely." Lewis clearly means something modal by 'characterizing reality': a collection of facts characterize a world $w$ completely if and only if they are true only at $w$.

as I will argue, they do not form a mimimal supervenience base. So there must be metaphysical redundancy in the spatiotemoral primivist's account of the world. This is a reason to reject spacetime primitivism.

I'll first explain why this is the case for the most naïve version of spacetime primitivism, and then explain why any more sophisticated version fails to deliver a minimal supervenience base as well.

Consider a spacetime primitivist who regiments the structure of spacetime by positing a family of external relations, the distance relations.[26] That is, the relations *one meter apart*, *two meters apart*, *seventeen meters apart* and so on are all fundamental.

On this view, in order for God to determine how things are arranged in space at a given time he must decide separately, for every material object, which distance relations it stands in. Suppose he starts with my fridge; it is two feet from my coffee maker, 200 miles from Obama, 4000 miles from Putin, and so on.[27] Next, he determines all the distances the Eiffel Tower stands in: it is 95,000 miles from the South Pole, 239,000 miles from the moon, and so on. And thirdly he determines all the fundamental distance relations the Sun stands in. If this way of thinking about distance is correct, God has done only a tiny fraction of the work he needs to do to settle the distance facts once he has settled which distances these three objects, my fridge, the Eiffel Tower, and the Sun, stand in. But any additional work he does is unnecessary, for the distance relations these three objects stand in is enough to determine the distance between any arbitrary objects in the universe.

Say we want to know how far apart Obama and Putin are. According to the spacetime primitivist this is to ask which fundamental distance relation holds between them. But how far apart they are is already determined by the relations we have specified! For if we know how far Obama and Putin are from my fridge, the Eiffel Tower, and the Sun, then

[26] Note that in a relativistic setting it is neither spatial nor temporal distance relations that will be primitive but rather spatiotemporal interval relations. But nothing hinges on this and for familiarity I will use spatial distance relations as my example.

[27] Composite objects plausibly inherit their locations, and therefore the distances they stand in, from their parts: my toaster is two meters from my coffee mug in virtue of the fact that the atoms making up my toaster are two meters from the atoms making up my coffee mug. And if substantivalism is true then it is plausible that the distances between material objects are inherited from the distances between the regions at which they are located: my toaster is two meters from my coffee cup in virtue of the fact that my toaster is located at $r_1$, my coffee cup is located at $r_2$, and $r_1$ is two meters from $r_2$. But I will ignore all of these complications to keep the discussion simple.

by triangulating we know how far apart they are. So this fact about the distance between Obama and Putin is metaphysically redundant. The spacetime primitivist could claim that only some facts about distances are fundamental. But any choice of some distance relations over others will be implausibly arbitrary.

Rather than taking distance relations as fundamental, the spacetime primitivist could instead encode facts about distance in other terms. But however she regiments the structure of spacetime her account will entail that there are metaphysically redundandant fundamental facts.

For example, she could posit two fundamental relations, *betweenness* and *congruence*.[28] *Congruence* holds between four points $p_1, p_2, p_3, p_4$ just in case the distance between $p_1$ and $p_2$ is the same as the distance between $p_3$ and $p_4$. But this account suffers from the same problem: if my coffee mug and my toaster bear *congruence* to your coffee mug and toaster and also to Fred's coffee mug and toaster, then this entails that your and Fred's mugs and toasters stand in *congruence* too. So this last fact is redundant.

The primitivist could instead take facts about *path length* to be basic. The distance between two points can then be defined as the length of the shortest path between them. This avoids the redundancy that arose with distance relations, since the length of the shortest path between Obama and Putin is not determined by the lengths of the paths between my fridge, the Eiffel tower and the Sun and everything else. But taking path lengths to be fundamental results in redundancy of another form. Let a path be a fusion of points, and suppose we assign each path a positive real number that represents its length in meters. Since we are assuming that space is dense, every path $p$ is composed of two subpaths $p_1$ and $p_2$. The length of a path is determined by the length of all the subpaths that compose it. So if the length of $p_1$ and $p_2$ is determined, there is no need to then go on to determine the length of $p$. So any fundamental fact about the length of $p$ would be metaphysically redundant. But there was nothing special about $p$; and therefore every path length fact is metaphysically redundant.[29]

[28] David Hilbert's (1899) axiomatized Euclidean geometry in these terms. Field (1980)

[29] The primitivist could take facts about the *metric tensor* $g$ associated with each point to be basic. The metric tensor of a point $p$ encodes information about distances within an infinitesimal neighborhood of $p$.[30] But since the metric tensor at $p$ provides information about the distance structure *nearby* $p$, redundancy re-arises. We can illustrate this point with another example of a neighborhood-dependent property, velocity.

If spacetime primitivism is correct then there is no non-arbitrary, non-redundant supervenience base for the world. This is a reason to prefer the causal theory, for which these problems do not arise since the world's minimal supervenience base doesn't include facts about spatial or temporal distance.

## 4 The Argument from Brute Necessary Connections

The spatiotemporal primitivist holds that spatiotemporal relations are fundamental. This is puzzling, because spatiotemporal relations are not freely recombinable. The fact that $a$ is two meters from $b$ and $b$ is two meters from $c$ imposes constraints on possible distances between $a$ and $c$. But we expect the basic building blocks of the world to be, in David Hume's words, "entirely loose and separate."[31] When possible, we should avoid theories that posit necessary connections between fundamental properties and relations.

Necessary connections call out for explanation. Normative properties supervene on natural properties. Many philosophers conclude that normative properties are grounded in natural properties, precisely because this supervenience ought to be explained. For if natural and normative properties were both fundamental, it would be mysterious why they were so nicely choreographed. We might imagine that God creates the world, one fundamental property at a time. Once he has settled the distribution of the natural properties, he goes on to specify the distribution of normative properties, but does so in precisely such a way

The velocity of some object at time $t$ is defined in terms of what the object does *nearby* $t$: the velocity of $o$ at $t$ is the limit of the average velocity of $o$ in smaller and smaller intervals of time containing $t$. This means that specifying the instantaneous velocity of an object at every time involves some redundancy. Suppose $o$ traveled on some smooth trajectory between $t_1$ and $t_2$, and that the velocity of $o$ at every time between $t_1$ and $t_2$ is given except for some some instant $t_i$. Because velocity is defined in terms of nearby instants, the velocity of $o$ at $t_i$ is already settled by velocities at other times. So specifying the velocity at $t_i$ would be redundant. The stipulation that $o$ traveled smoothly is doing some work here since the claim about redundancy only follows given that $o$ *has* a velocity at $t_i$. The situation with metric tensor facts is precisely analogous. Suppose the metric tensor at every point in some space except for $p$ is given. Then it is determined exactly what the metric tensor at $p$ is. So specifying the metric tensor at $p$ in addition would be redundant. Phillip Bricker (1992) argues on this basis that we should invoke novel fundamental properties that behave like metric tensors but are intrinsic to points, and therefore which aren't defined in terms of their neighborhoods. These properties are analogous to the intrinsic velocities invoked by Michael Tooley (1988). These properties would seem encode a lot of information, since they have the structure that metric tensors have. But in fact Bricker's metric tensors only provide this information given that the laws happen to tie them to the neighborhoods of points that instantiate them. But then encoding this structure in the properties themselves is doing no work, which is all by the laws. While this view may escape the argument from metaphysical redundancy it makes the argument from explanatory redundancy more pressing.

[31] David Hume (1975 [1748]) p. 61.

that one class of properties supervenes on the other. Why should God's creative powers follow this pattern?

Consider David Lewis' complaint about David Armstrong's account of laws of nature. On Armstrong's account it is a law that anything with F has G if and only if F bears the second-order relation *nomic necessitation* (or $N$) to G. Lewis objects that no explanation has been given for why the fact that N(F,G) should entail that anything with F also has G:

> Whatever N may be, I cannot see how it could be absolutely impossible to have N(F,G) and Fa without Ga ... The mystery is somewhat hidden by Armstrong's terminology ... who would be surprised to hear that if F 'necessitates' G and a has F, then a must have G? But I say that N deserves the name of 'necessitation' only if, somehow, it really can enter into the requisite necessary connections. It can't enter into them just by bearing a name, any more than one can have mighty biceps just by being called 'Armstrong'.[32]

Lewis seems to think that there is something especially problematic about Armstrong's theory. I don't think that's right. Armstrong posits a special second-order relation to make sense of laws. But it is a virtue, not a vice, of Armstrong's account that he does not merely posit and stop there. He says something about how his chosen machinery is supposed to behave. The phenomenon Lewis is objecting to is utterly mundane: any theory must have some entities or primitives that aren't explained in other terms, and any interesting theory will say something about how these primitive features behave.

Lewis is a spacetime primitivist. He recognizes a family of perfectly natural external relations, the distances. But for them to play the role of distances they must obey certain constraints, like the triangle inequality: it had better be the case that for any three points $a$, $b$ and $c$, the distance between $a$ and $b$ added to the distance between $b$ and $c$ is not more than the distance between $a$ and $c$. And it had also better be the case that a given pair of points only ever stand in one of these fundamental external relations: two points cannot stand in multiple distance relations. How does Lewis explain these constraints? He doesn't. That $a$ is $1m$ from $b$, $b$ is $1m$ from $c$, and $a$ is $1m$ from $c$ are all distinct, basic states of

[32] Lewis (1983) p. 366.

affairs. We might imagine Armstrong offering a parody of Lewis' complaint:

> Whatever these distance relations may be, I cannot see how it could be absolutely impossible to have *one-meter(a,b)*, *one-meter(b,c)* and *twenty-meters(a,c)*. I say that these relations deserve the name 'distances' only if, somehow, they can really obey the necessary constraints. They cannot obey them just by bearing a name, any more than [etc.]

Still, Lewis does have a legitimate complaint against Armstrong. It is that Armstrong posits necessary connections where he doesn't need to. Armstrong must simply stipulate that *nomic necessitation* behaves in the way he claims it does. The Humean reductionist about laws need not make any such stipulation.

We can profitably think of Lewis's complaint as an appeal to a certain kind of parsimony.

Theories that make fewer assumptions are, all else equal, better theories. This principle takes on a few different guises in metaphysics. It's familiar to distinguish between the *ontology* of a theory (which things it says exist) and its *ideology* (those expressions of the theory which are unexplained, the primitives of the theory.) We can distinguish between varieties of simplicity correspondingly. Ontologically simpler theories posit fewer (types or tokens of) entities. Ideologically simpler theories use fewer primitive expressions.

But there is a further notion of simplicity that does not take either of these forms. Say *axiomatically* simpler theories are those that contain fewer stipulations about how the primitives of the theory behave.

Suppose the spacetime primitivist accounts for the structure of space and time by positing a family of perfectly natural external relations, the distance relations. These relations must be stipulated to behave in certain ways if they are apt to play the role of *distance* relations. First, they exclude one another. It had better not be possible for two things to stand in hundreds of different distance relations. And second, they must obey broader constraints in their distribution, like the triangle inequality: it had better be true that for any three objects, $o_1$,$o_2$,$o_3$, the distance between $o_1$ and $o_3$ is at most the sum of the distances between $o_1$ and $o_2$ and $o_2$ and $o_3$.[33] This a cost that the causal theorist avoids.

[33] As Maudlin (2007) points out, there are many more constraints once we consider the distribution of distances for more than three objects.

As we saw in the previous chapter a more sophisticated spacetime primitivist need not regard distance relations as fundamental. But however the spacetime primitivist accounts for the structure of space the same problems will reemerge for similar reasons.

For example, suppose that path lengths are fundamental and facts about the distance between two points obtain in virtue of facts about the length of the shortest path between them. Maudlin (2007) claims that the benefit of taking path length to be prior is that constraints like the triangle inequality emerge by definition instead of having to be stipulated. But as we saw in the previous section there are still plenty of constraints on path lengths that must be postulated. For example, we must stipulate that the length of a path is always equal to the sum of the lengths of the subpaths composing it.[34]

These unexplained stipulations are theoretical costs. The distance relations are supposed to be fundamental and we expect fundamental relations to be freely recombinable. But distance relations, as we saw, are not freely recombinable.

We should avoid positing necessary constraints whenever we can; all else equal they make a theory worse. Spacetime primitivists must simply postulate that their favored primitives obey certain constraints such as the triangle inequality. The causal theorist, on the other hand, has no need to, since these constraints naturally emerge from the causal facts to which spacetime reduces.

The constraints emerge for slightly different reasons depending on whether the causal theory is combined with a Humean or non-Humean view of laws of nature.

Given the best system analysis of spacetime, constraints like the triangle inequality hold simply because the best system, if it attributes spatiotemporal structure to the world, will do so in a way that respects the triangle inequality. The Humean causal theorist is free to accept that the fundamental properties are truly freely recombinable. But any summary of the world that attributes distances to things in a way that does not obey the triangle inequality, or which attributes multiple distances to pairs of objects, will be a worse summary, since it will be impossible to write down empirically adequate laws for how things

[34] Suppose instead that distances are encoded with the relations *congruence* and *betweenness*. It must be stipulated that *congruence* is transitive and *betweenness* is transitive. Finally, consider the view that facts about the metric tensor are basic. It must be stipulated that if the metric tensor at some point $p$ represents a locally positively curved space then p is not surrounded by a locally negatively curved space, for example.

interact depending on the distances between them.

The non-Humean causal theorist takes as primitive direct dependence relations of the form $L(p_1(r_1) \rightarrow p_2(r_2))$. We saw that Armstrong must stipulate that *nomic-necessitation* behaves in a certain way. But there is nothing unique about Armstrong's account in this respect; *any* non-Humean must make an analogous claim.[35] So the non-Humean causal theorist should stipulate that $L(p)$ entails $p$. But once this constraint is in place the non-Humean causal theorist has a ready explanation for the constraints that the primitivist must take for granted. Take the fact that no two points can stand in more than one distance relation. This would require that one point is disposed to be affected in two different ways by goings on at the other points. This requires that, say, a massive object at $p_1$ disposes the gravitation field at $p_2$ to be $g_1$ and disposes it to be $g_2$. But this would require the same point to have two gravitational field values, and this is impossible. More generally, physical magnitudes within a determinable family, like 1kg mass and 2kg mass, exclude one another. Given this fact and the causal theory of spacetime, it follows that it is impossible for two points to stand in more than one distance relation.

Of course, the non-Humean causal theorist is left with the unexplained incompatibility of physical magnitudes. But so too is the spacetime primitivist. The causal theorist has no need, unlike the primitivist, to *additionally* stipulate that constraints like the triangle inequality hold.[36]

Spacetime primitivists must posit brute necessary connections between the basic building blocks of the world, whereas these connections emerge naturally on the causal theory of spacetime. This is another reason to prefer the causal theory.

[35] For example, the dispositionalist will leave principles like the following unexplained:

> Dispositional Principle: if something is disposed to $x$ given $y$, and $y$ occurs, then (absent finks and masks) it $y$'s.

[36] I argue for an account of physical magnitudes that allows us to explain the incompatibility of magnitudes in chapter two of my dissertation; this project crosscuts the question of whether the causal theory of spacetime is correct.

## 5 Objections I: The non-Humean Causal Theory of Spacetime

The non-Humean causal theorist takes relations of direct dependence between regions to be fundamental. Recall that direct dependence relations encode lawful conditionals that do not presuppose any spacetime structure, since they only relate regions' purely qualitative profiles.

The standard approach to laws and spacetime regards direct dependences as metaphysically derivative. On this picture God settled the arrangement of things in spacetime, and then settled the laws, and it is in virtue of these facts that direct dependences hold. The non-Humean causal theorist reverses this order of explanation. In her view, when God created the world he settled the direct dependences immediately, and facts about the spatial and temporal arrangement of the world emerged in virtue of them.

According to the causal theorist this was also enough to fix the laws. Scientific laws like Newton's law of universal gravitation, together with an assignment of distance structure to spacetime, serve to encode facts about direct dependences among regions in an elegant and efficient way.

I anticipate the following objections to this claim.

One objection concerns the fact that we expect the fundamental nomic facts to be *qualitative*, in the sense that they do not discriminate between objects that are qualitatively alike. For example, if laws say that if a brick $b$ with mass $m$ traveling at velocity $v$ hits a glass window $w$ then $w$ smashes, then surely the laws say that *any* brick with mass $m$ hitting a window like $w$ breaks that window. Laws are not sensitive, we might have thought, to the identities of things – all that matters is *how* things are, not *which* things they are. But direct dependences do not satisfy this condition, for they express dependence relations between particular regions.

It is unclear that there is anything intrinsically problematic about this. After all, according to spacetime primitivists, it is a brute fact that some points but not others are close together; the causal theorist merely replaces these individualistic facts with facts about direct dependence.

A more precise version of this worry is that since direct dependences relate particular regions they must be horribly complicated and list-like; but nothing law-like is complicated in this way. In response, recall that the causal theorist recognizes that the laws are the familiar regularities that presuppose spacetime structure. The causal theorist merely claims that these laws are not metaphysically fundamental.

We can distinguish between *scientific* explanation and *metaphysical* explanation. For example, while the fact that a certain atom is ionized at $t_2$ might be scientifically explained by the fact that it absorbed some radiation at $t_1$. But the fact that it absorbed some radiation is not what *makes it true* that it is ionized. It is ionized in virtue of having a different number of protons and electrons.

Humeans about laws appeal to this distinction in response to the charge that their account of laws is circular. Humeans claim that the fact that some regularity $R$ is a law obtains at least partially in virtue of the fact that $P$ is a regularity. But laws are supposed to explain their instances. So $R$ is a law is explained by $R$ (from Humeanism) and $R$ is explained by the fact that $R$ is a law (from the explanatoriness of laws.) David Armstrong (1983) claims that this makes the account circular. Humeans (like Loewer (2012)) may respond by pointing out that the senses of explanation at issue are different. Laws are metaphysically explained by their instances, but instances are scientifically explained by the laws.[37]

The non-Humean causal theorist can make a similar move; she can claim that the familiar laws that are framed in terms of spacetime are scientifically fundamental, while direct dependences are nonetheless metaphysically fundamental.

There are independent reasons to think that the scientifically fundamental laws are not metaphysically fundamental.

The fundamental scientific laws are differential equations; they say how the rate of change of one quantity relates to other quantities. But facts about rates of change, like acceleration, are not metaphysically fundamental. The acceleration of some body at time $t$ is defined as the limit of the rate of change of velocity in successively smaller time periods

[37]Lange (2013) argues that while the two notions of explanation are distinct, scientific explanations are transmitted over metaphysical explanations, and that this means the Humean account of laws is circular after all. Hicks and van Elswick (2014) respond by arguing against brdige principles linking scientific and metaphysical explanation.

containing $t$. Velocity is similarly defined in terms of position. But scientists feel no need to state laws about rates of change as very complicated claims about limits, and if they did the laws would become dramatically less simple.[38]

Another reason for thinking that the scientifically fundamental laws are not metaphysically fundamental is that scientific laws are *mathematical* claims. A very plausible explanation of this fact is that even though the world does not have fundamental mathematical structure itself (a two meter rod doesn't stand in the same relation to a three meter rod as the number 2 stands in to the number 3), we may usefully use mathematical structures to represent physical structures. But there are obvious reasons for physicists to describe the world mathematically even if the world has no fundamental mathematical structure: precisely because it is simpler and more elegant and easier to reason about the mathematical description

The lesson, it seems, is that scientists deliberately state the laws in non-fundamental terms for the sake of the simplicity gained.

Of course this is perfectly compatible with there being a close connection between the scientifically fundamental and metaphysically fundamental laws, so that the fact that some property appears in the scientifically fundamental laws is defeasible evidence that it is metaphysically fundamental. But there are already independent compelling reasons to think that the metaphysically fundamental laws are quite complicated. So it is no objection to the non-Humean nomic theorist that her view has this feature as well.

## 6 Objections II: Against the Best System Analysis of Spacetime

According to the best system analysis of spacetime, facts about the spatiotemporal arrangement of the world are fixed by the facts about which points have which properties. On this view, all God had to do was to say, of each spacetime point, which properties it has, and that was enough to give rise to the rich spatiotemporal structure we take our world to have.

It may seem incredible that such a sparse basis could put enough constraints on the structure for world. And admittedly, the most naive version of the best system analysis of

[38] Moreover, if these claims about limits were themselves defined in the standard way, in epsilon-delta terms, the full statement of the laws would be even more complicated.

spacetime is hopeless. For it to succeed, it must be the case that a wide range of worlds that lack fundamental spatial and temporal structure are best systematized by attributing a spatial and temporal arrangement to them.

But many such worlds that we take to be possible can be systematized simply without appeal to the spatiotemporal arrangement of things. Consider a world governed by Newtonian gravitational mechanics that contains some point masses interacting according to Newtons law of universal gravitation, where the only fundamental properties are the mass magnitudes. We have some reason to think that such a world is possible. But in this world, non-spatiotemporal reality can be characterized exhaustively in an extremely simple way, since all there is to tell is the cardinality of the points that instantiate each mass magnitude. So if this world is possible the causal theory cannot be true.

But there is a way of developing a more promising version of the best system analysis of spacetime. For independent reasons we should recognize more structure in the Humean basis.

One way to add structure is to recognize more physical magnitudes. Supposing that we can sensibly make a distinction between fundamental and non-fundamental properties, the question arises of how can we obtain *evidence* about which properties are fundamental. Perhaps the most popular answer, following David Lewis, is that our best physical theories aim to describe the world in fundamental terms, and so our best source of evidence about which properties are fundamental is to look at which properties appear in our best scientific theories. If this is right, then it is artificially restrictive to recognize only mass magnitudes as fundamental even in the context of Newtonian gravitational mechanics, since the theory concerns facts about forces, or gravitational potential energy, or gravitational field values, or gravitational potential field values, depending on the formulation of the theory. Call these latter magnitudes *gravitational magnitudes.* If we take the physics at face value, then the Humean mosaic includes not just facts about mass magnitudes but also about gravitational magnitudes. And in complicated worlds like ours there is no simple way to describe the relationship between the points that instantiate a given mass magnitude and those that instantiate a given gravitational magnitude that does not go via spacetime structure.[39]

[39]This might suggest that the best system analysis of spacetime is much less ambitious than it might at

If this is right then the Humean basis consists of uncountably many points, and facts, for each of those points, about the mass density and the gravitational magnitude at that point. The amount of information to be systematized is vast. What are the competing systematizations? No list, no matter how long, can specify even a small part of this information, and even if it could it would fail drastically to be simple. But in worlds like ours there is an available systematization that provides a huge amount of useful information in a simple way: *there is some way of assigning distances to pairs of points so that the mass density and gravitational potential are related by the laws of Newtonian gravitational mechanics.* Given the vastness of the Humean basis it is implausible that any other systematization comes close to matching this balance of informativeness and simplicity.

But one might still worry that this approach does not place enough constraints on which assignment of distances are part of the best system. And it does seem that in simple worlds the best system analysis of spacetime gives the wrong predictions.[40] But in this respect the amended best system analysis is no worse off than the canonical best system analysis, which is well known to conflict with our intuitive judgments about simple worlds. But in more complex worlds like our own, the task of finding alternative ways of assigning spacetime structure to points that facilitates dynamical laws that are anywhere near as strong and simple as those of Newtonian gravitation mechanics looks to be hopeless, for in general, variations in the distances assigned to points will result in false predictions about the relationship between mass and gravitational potential.

I conclude that the best system analysis of spacetime is is a promising way for the Humean to adopt the causal theory. The Humean need posit only points and their properties

first appear, because various of these candidate fundamental properties have spatial notions baked in, since they are vectorial magnitudes. If things with directions are fundamental, then something geometrical is in the Humean basis after all. It might seem that the Humean should welcome this fact, since it becomes much easier to see how to recover the geometry of spacetime if she is starting with something geometrical. But I suspect that this undermines the motivations for the nomic theory of spacetime in the first place, since precisely the same arguments against spacetime primitivism look like they will undermine fundamental vectorial properties. For instance, we might imagine worlds in which the direction of each force is inversed, but in which things accelerate in the opposite direction to the resultant force they experience. This world will again look just like the actual world. So we can run the parallel argument that fundamental force directions are explanatorily idle. Similarly, forces look like they obey constraints just like spatiotemporal relations, for forces are necessarily additive. For these reasons I think the Humean should only recognize scalar fields, like the gravitational potential, in her basis.

[40] Consider a two dimensional world where how fast something moves vertically depends on how far it is horizontally from the center of mass. We could re-describe this world as one in which the vertical spatial dimension plays the role of time, and there is a law that says things speed up over time.

at the fundamental level, and hold that claims about how those points are arranged in space and time serve to summarize this basis.

## 7 Conclusion

The vast majority of philosophers hold that space and time are fundamental. I hope to have shown in this paper that the causal theory of spacetime deserves serious consideration. This project clearly interacts with various other debates in metaphysics. For example, the kinds of parsimony considerations that militate against spacetime primitivism also have traction elsewhere (facts about laws, the identities of material bodies, the identities of properties, the structure of physical magnitudes, to name a few). But it is far from clear that one can always opt for the reductionist option without pushing the redundant structure bulge under the carpet. So a difficult and interesting question arises of how to choose the overall package of views. Nonetheless I conclude that there is a formidable case against the spacetime primitivist.

# 4. Maudlin on the Triangle Inequality

In "Considerations from Physics for Deep Metaphysics," Tim Maudlin argues that we should take facts about distance to be analyzed in terms of facts about path lengths. His reason is that if we take distances to be fundamental we must stipulate that constraints like the triangle inequality hold, but we get these constraints for free if we take path lengths to be prior.

Maudlin claims that this is a reason to favor substantivalism over relationism about space: the substantivalist, who believes in paths even when they are not occupied, can define distance in terms of path lengths and thereby explain the triangle inequality; but the relationist must take distances to be fundamental, and so must take the triangle inequality to be a brute fact.

I will argue that Maudlin is mistaken. Even if we take path lengths as primitive, the triangle inequality follows only if we stipulate brute constraints among the fundamental properties and relations that are at least as puzzling as the triangle inequality. There may be other reasons to define distances in terms path lengths, and so other reasons to favor substantivalism, but being able to explain the triangle inequality is not one of them.

## 1 Maudlin's Argument

Consider the following two ways of accounting for the world's spatial structure.

On one view the the structure of space is explained in terms of a family of fundamental external *relations*, the distance relations.[1] The geometry of space is ultimately explained by the distribution of distance relations between material bodies or spacetime points. Call

[1] A relation is *external* if whether or not it holds between some things is not determined by their intrinsic features. A relation is *fundamental* if facts about its instances do not metaphysically depend on other facts.

this the *distances first* view.

Another way of accounting for the structure of space is to take as primitive facts about *path lengths*. I'll assume that a path is just a mereological fusion of points of space.[2] We can represent facts about path lengths with a function, $L$, from paths to real numbers. Call this the *path-length first* view.

As Maudlin points out, we can define distances in terms of path lengths, and *vice versa*. We can analyze the distance between two points as the length of the *shortest path* between them. And we can analyze the length of a path in terms of the sum of the distances along its subpaths.[3]

Distances obey the following constraint, the *triangle inequality*:[4]

(1) TRIANGLE INEQUALITY: for any three points, $x$, $y$, $z$, the distance between $x$ and $y$ is no greater than the distance between $x$ and $z$ plus the distance between $z$ and $y$.

Maudlin argues that we should prefer the path-length first view over the distance first view, because on the path-length first view we can explain the triangle inequality, whereas on the distance-first view it must be stipulated to hold.

Before we examine Maudlin's argument that the triangle inequality falls out automatically on the path-length first view, it is worth asking why it would count against the distance-first view that the triangle inequality must be stipulated.

One way of motivating this thought is that we expect the fundamental properties and relations to be freely recombinable, and if distance relations were fundamental then the triangle inequality would state a way in which they fail to be recombinable. But even those who doubt that fundamental properties and relations are freely recombinable in general might still reasonably think that necessary connections between properties are *evidence* that they are not fundamental. For example, many infer from the fact that moral properties

[2] I'll assume that for any collection of points there exists a path which is the fusion of those points, but I don't think anything hangs on this assumption.

[3] Suppose path $p$ has endpoints $p_i$ and $p_f$. Take a number of points on $p$: $p_1$, $p_2$, ... $p_n$. We can approximate the length of $p$ as the sum of the distance between $p_i$ and $p_1$ and between $p_1$ and $p_2$, and so on. The sum becomes a better approximation as more points in $p$ are included and the farthest distance between points decreases. The *length* of path $p$ can then be defined as the limit of this sum as the distance between points approaches zero.

[4] The triangle inequality is central to our conception of distance; the standard definition of a distance function requires that it satisfy the constraint.

supervene on natural properties that moral properties are instantiated *in virtue of* natural properties. A natural justification for this inference is the thought that if both natural and moral properties were fundamental then the supervenience of one on the other would amount to a mysterious conspiracy by the fundamental properties to line up nicely. So a plausible reason for avoiding necessary connections at the fundamental level is simply that unexplained patterns at the fundamental level are theoretical costs; all else equal a theory is better if it leaves fewer patterns unexplained. This plausible general principle indeed gives us a reason to avoid the distance first view: if distance relations were fundamental then the triangle inequality would amounts to an unexplained pattern at the fundamental level.

Let us now consider Maudlin's argument in favor of the path-length first view. Here is the crucial piece of reasoning:

> If distance is defined in terms of paths ... then the triangle inequality falls out automatically. Since a path from $x$ to $z$ connected to a path from $z$ to $y$ *is* a path from $x$ to $y$, the minimal length of a path from $a$ to $y$ cannot be greater than the sum of the length of the minimal path from $x$ to $z$ and the length of the minimal path from $z$ to $y$.[5]

To see what's at issue, let us first state explicitly what the triangle inequality amounts to if distances are defined in terms of path lengths:

(2) Path Length Triangle Inequality: for any three points, $x$, $y$, $z$, the length of the shortest path between $x$ and $y$ is no greater than the length of the shortest path between $y$ and $z$ added to the length of the shortest path between $x$ and $z$.

Consider three points in particular, $a$, $b$, and $c$. Let the shortest path from $a$ to $b$ be $\mathbf{p}_1$, and the shortest path from $b$ to $c$ be $\mathbf{p}_2$. (I will use boldface names like $\mathbf{p}_1$ to refer to particular paths and italicised names like $p_1$ as a variable for paths). Now, Maudlin points out that $\mathbf{p}_1$+$\mathbf{p}_2$, the fusion of $\mathbf{p}_1$ and $\mathbf{p}_2$, *is* a path from $a$ to $c$, so the shortest path from $a$ to $c$ cannot be longer than $\mathbf{p}_1$+$\mathbf{p}_2$. Maudlin concludes from this that the shortest path from $a$ to $c$ is no longer than the length of $\mathbf{p}_1$ added to the length of $\mathbf{p}_2$.

[5]Maudlin (2007) p. 88. For stylistic consistency I have used $x$,$y$, and $z$ instead of Maudlin's A, B and C.

But this last step only follows if we grant Maudlin the following constraint:

(3) PATH-LENGTH ADDITIVITY: for any two paths that share an endpoint, $p_1$ and $p_2$, the length of $p_1 + p_2$ is equal to the length of $p_1$ added to the length of $p_2$.

And PATH-LENGTH ADDITIVITY is a substantive claim about path lengths. This is obscured by the fact that there are two different constraints about distance that are easily confused: a *mathematical* constraint on apt numerical representations of distance, and a *physical* constraint on allowable distributions of path-length properties.

We represent the structure of space mathematically. One way of doing so is by means of a *distance function*, a function from pairs of points to real numbers. But not just any such function is an apt representation of the structure of space. Say that a *faithful* distance function is one that accurately represents the physical distances between points, so that mathematical relations among numbers mirror the physical relations among distances. For example, a faithful distance function will assign a smaller number to one pair of points than another pair of points only if the first pair are physically closer.

Another way of representing the structure of space is by means of a *path-length function*, a function from paths to real numbers. Again, a *faithful* path-length function assigns numbers to paths so that the comparisons between assigned numbers mirror the physical comparisons among paths. For example, a faithful path-length function will assign a larger number to $p_1$ than to $p_2$ if and only if $p_1$ is physically longer than $p_2$.

It is plausible that if a path-length function $L$ faithfully represents facts about physical lengths then it will satisfy the following condition:

(4) PL-FUNCTION ADDITIVITY: for any two paths that share an endpoint, $p_1$ and $p_2$, $L(p_1 + p_2) = L(p_1) + L(p_2)$.

But the fact that faithful path-length functions obey PL-FUNCTION ADDITIVITY is just a reflection of the nature of the physical structure being represented. And we can very sensibly ask what this structure must be like for it to be faithfully representable with functions that obey PL-FUNCTION ADDITIVITY.

Compare the case of path-lengths with that of physical magnitudes like mass. We can represent facts about how massive things are with a function from objects to real numbers that gives their mass in kilograms. Now consider: why can an object have only one mass magnitude? To appeal to the fact that masses are represented by a function is to get things backwards: it is *because* every object has only one mass magnitude that we can faithfully represent the facts about mass by using a function, not the other way around.

Path-Length Additivity makes a claim about how the length of a path is related to the lengths of subpaths that compose it. Exactly what this claim amounts to depends on how we account for the structure of path lengths. I will briefly describe what Path-Length Additivity looks like on a *first-order* and on a *second-order* theory of path lengths; but however path lengths are understood, Path-Length Additivity is only true if we stipulate that the fundamental properties and relations follow certain patterns.

On a second-order theory of path lengths, there is a family of determinate *properties*, the path length properties. These properties stand in a structure that allow us to make comparisons among them; for example, they are ordered, so that we can say that some path-length properties are larger than others. We can then understand comparisons among paths in terms of the comparisons among the path-length properties those paths instantiate: one path $p_1$ is larger than another $p_2$ just in case the path-length property instantiated by $p_1$ is larger than the path-length property instantiated by $p_2$.

We might understand the comparisons among path-length properties in a few different ways. One option is to implement Brent Mundy's (1987) theory of quantity, and posit two fundamental second-order relations, *less-than* and *sum-to*, that hold among the path-length properties.[6]

Does Path-Length Additivity come for free in this second-order theory of path lengths? No, since it merely encodes the following constraint:

(5) Second-Order Additivity: for any paths $p_1$ and $p_2$ that share an endpoint, if $p_1$ has path-length $l_1$ and $p_2$ has path-length $l_2$ then there is a path-length $l_3$ such that $p_1 + p_2$ has $l_3$ and $\textit{sum-to}(l_1, l_2, l_3)$.

[6] These are not Mundy's labels, but I use them to make it clear how the relations are supposed to behave. For example *less-than* is stipulated to be transitive, *sum-to* is associative, and so on.

And all Second-Order Additivity does is stipulate how some fundamental properties and relations interact. It's no more innocent than the distance triangle inequality.

So much for the second-order version of the path-length first view. On a first-order theory, path lengths are not understood in terms of path-length properties but instead in terms of relations that hold between the paths themselves. For example, we might invoke two fundamental first-order relations that hold between paths; *shorter-path-than* and *sum-to-path*.[7]

Does Path-Length Additivity come for free on this approach? No, since it merely encodes the following constraint:

(6) First-Order Additivity: for any paths $p_1$ and $p_2$ that share an endpoint, and where $p_3 = p_1 + p_2$, *sum-to-path*$(p_1, p_2, p_3)$.

And again, (6) simply stipulates a constraint on how a fundamental relation, *sum-to-path*, behaves – just like our original triangle inequality. Moreover, for the first-order theory of path lengths the triangle inequality is just one among many constraints that the fundamental relations must be postulated to obey. The distribution of *shorter-than* and *sum-to-path* only give rise to the structure of path lengths if *shorter-than* is transitive, *sum-to-path* is commutative, and *sum-to-path*$(p_1, p_2, p_3)$ entails that *shorter-than*$(p_1, p_3)$, to list just a few constraints.

The advantage of the path-length first view over the distance first view was supposed to be that on the distance first view, the triangle inequality amounts to a brute stipulation that various fundamental relations behave in a certain way. But precisely the the same thing is true of either version of the path-length first view. Maudlin has not given any good reason to take path lengths rather than distances as primitive.

## 2 The Metric Tensor

In other work Maudlin suggests that while he thinks distance ought to be defined in terms of path lengths, he does not, in the final analysis, think the notion of the length of a path is

[7] Again, assume that the relations behave in such a way as to deserve their names.

primitive.[8] Instead, what is fundamental is the *metric tensor*. In effect, the metric tensor of a point $p$ encodes information about distances within an infinitesimal neighborhood of $p$.[9] The length of a path $p$ can then be obtained by integrating along $p$.

So one might think that the claim Maudlin really intends is that we can avoid having to make stipulations like the triangle inequality by taking the metric tensor as basic and defining both path lengths and distances in terms of it.

But this proposal is problematic. The metric tensor at $p$ encodes information about the geometry of space *nearby* $p$; in Phillip Bricker's (1993) terminology, the metric tensor is a *neighborhood-dependent* property. Compare the metric tensor with another neighborhood-dependent property, velocity. The instantaneous velocity of some object at time $t$ is defined in terms of what the object does (temporally) *nearby* $t$: the velocity of $o$ at $t$ is the limit of the average velocity of $o$ in smaller and smaller intervals of time containing $t$. This means that velocities, as standardly conceived, are not fundamental. Similarly for the metric tensor: since it is characterized in terms of the tangent vectors at p, and tangent vectors, like velocity, are derivatives, the metric tensor is also a neighborhood dependent property. So the metric tensor cannot be fundamental after all.

But perhaps there is a way to make sense of fundamental metric tensors, along the following lines.

Call the derivative of an object's position with respect to time its *extrinsic velocity*. Michael Tooley (1988) argues that we should posit novel vectorial properties, *intrinsic velocities*. The intrinsic velocity of an object $o$ at time $t$ would be fundamental, not defined in terms of where $o$ is at other times. The laws of nature ensure that if an object has a certain extrinsic velocity then it has the appropriate intrinsic velocity.[10]

Phillip Bricker (1993) argues that we should take a similar approach to the metric tensor: we should invoke novel fundamental properties that behave like the local metrics of points but are not neighborhood dependent. Call these properties *intrinsic local metrics*.[11] Let

[8] Maudlin (2009) p. 424.

[9] More precisely, the metric tensor at a point is an inner product on the tangent space of that point. Note that this raises the question of the status of topological facts, for the metric tensor can be defined only on differentiable manifold with a baked-in topology.

[10] But perhaps not *vice versa*: Tooley claims e.g. that an object could have an intrinsic velocity even it existed only for an instant.

[11] It is worth nothing that Bricker also thinks we should posit new objects that instantiate intrinsic local

us suppose that this is Maudlin's preferred account of the fundamental structure of space. Now we can ask: does this view escape having to stipulate constraints like the triangle inequality?

This view faces the following dilemma. Either intrinsic local metrics stand in unexplained necessary connections or they do not. Either way, understanding the structure of space in terms of intrinsic local metrics offers no advantage over the distance first view.

Suppose that a given point $p$ has an intrinsic local metric that corresponds to being in a space with positive curvature nearby $p$. Now consider: does this entail that $p$ is in fact surrounded by space with positive curvature? If it does, then we are left with precisely the same stipulated constraints that upsets Maudlin about distance relations. If local intrinsic metric properties are really fundamental, what is to stop God from freely recombining them as He sees fit? What explains their conspiring to match up nicely?

Now suppose instead that intrinsic local metrics are freely recombinable. We might entertain a similar attitude to intrinsic velocities. Perhaps intrinsic velocities are only *nomically* connected to extrinsic velocities. Then we can entertain possibilities in which stationary objects have any conceivable distribution of intrinsic velocities – perhaps an object's intrinsic velocity could vary discontinuously from instant to instant. In most of these worlds, intrinsic velocity would have very little to do with our ordinary concept *velocity*. Similarly for local intrinsic metrics. They are freely recombinable, but only in those worlds in which they happen to be well-behaved do they play the role of *metrics*.

This move neatly avoids having to make any stipulations about how intrinsic local metrics behave. But notice that this move was also available all along to the proponent of the distance-first view. She can claim that distance relations are freely recombinable, and so need not necessarily obey the triangle inequality. In those worlds, they do not play the role of distances, and they do not give rise to spatial structure. The distance-firster could hold that the triangle inequality expresses a merely *nomological* necessity, just like the proponent of intrinsic local metrics.

Accounting for the structure of space in terms of fundamental distance relations, there-

metrics: these new properties are instantiated by what we might call *primitive neighborhoods* – objects that are spatially complex but not defined in terms of sets of points. But this will not make a difference to the point of this paper.

fore – as many relationists do – does not face any special problem with having to stipulate the triangle inequality. Every theory must have some primitives; and every interesting theory will have to say something about how those primitives behave.

# 5. The Fundamental Physical Laws are Not the Laws of Physics

> Even if there is only one possible unified theory, it is just a set of rules and equations. What is it that breathes fire into the equations and makes a universe for them to describe?
>
> — Stephen Hawking, *A Brief History of Time*

One of the goals of scientific inquiry is to discover laws of nature. Physics is the most fundamental science. It's therefore natural to think that the fundamental laws are those investigated by physicists. I will argue that this is a mistake. The laws of physics are mathematical descriptions of the world. But these descriptions are *extrinsic,* concern *causally inert* mathematical objects, and are stated using *arbitrary* choices of scale. We should conclude that these laws are reducible to more fundamental laws that characterize reality directly in non-mathematical terms.

Physics describes the world in the language of mathematics. There has been a recent resurgence of interest in trying to make sense of the role that mathematics plays in our best scientific theories.[1] Most of this attention has focussed on the question of what this tells us about *whether mathematical objects exist*.[2] The question I wish to take up in this paper is different: what does the mathematical nature of physics tells us about *what the world is like*?

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[1] For recent work on the question of how mathematics is applicable to the concrete world see Batterman (2010), Colyvan (2001), Pincock (2007).

[2] According to the indispensability argument, due to Hilary Putnam and W. V. O. Quine, reference mathematica objects is indispensable to science, and we are ontologically committed to any entities that are indispensable to our best scientific theories. See Quine

Consider a few ways in which mathematics is used to describe the world. In 1736 it was impossible to walk across Königsberg crossing each bridge exactly once.[3] Here is an explanation for this fact:

(1) In 1736, Königsberg's graph was non-Eulerian.[4]

Real numbers are used to represent physical magnitudes like mass or charge, as when we say things like:

(2) The satellite has a mass of 100 kg.

More complex mathematical structures are used to describe the structure of space-time, as with claims like:

(3) Space has the topology of $\mathbb{R}^3$.

Each of these claims is a *mixed fact;*[5] a claim partially about a mathematical object and partially about a non-mathematical object. Each can be thought of as asserting a relation between a part of concrete reality and some mathematical object:

(1*) Königsberg bears the *has-as-a-graph* relation to a non-Eulerian graph.

(2*) The satellite bears the *has-a-mass-in-kilograms* relation to the number 100.

(3*) Space-time bears the *has-as-topology* relation to a set *U* of open sets of points.

We may distinguish two opposing explanations of the use of mathematics in physics. According to what I will call Pythagoreanism, there are fundamental mixed facts. The fact that parts of the world are related to mathematical objects cannot be explained in non-mathematical

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(1981), Putnam (1971) and Colyvan (2001) for statements of the indispensability argument, and Burgess and Rosen (1997) for a survey of responses.

[3] This example comes from Pincock (2007). I'll assume that it is 1736 for the purpose of the example.

[4] A graph is Eulerian if it is connected and has zero or two nodes of odd degree.

[5] This is terminology comes from Balaguer (1998)

terms.[6] There is nothing in virtue of which the satellite bears the *has-a-mass-in-kilograms* to the number 100.[7]

According to Anti-Pythagoreanism, on the other hand, every mixed fact obtains in virtue of facts that are either purely non-mathematical or purely mathematical. Facts about the mathematical structure of the world, like (1*)-(3*), obtain in virtue of facts that do not concern any mathematical objects like sets or functions or numbers. We can explain why parts of reality stand in relations to mathematical objects; they are not basic features of the world.

It is perhaps most clear in the case of (1*) what form this explanation will take. Königsberg bears a relation to a certain graph in virtue of the arrangement of particles that make up Königsberg, something that it is possible to describe without mentioning graphs. But I will argue that (2*) and (3*) also obtain in virtue of purely non-mathematical facts.

The argument of the paper is straightforward:

i. Anti-Pythagoreanism is true.

ii. If anti-Pythagoreanism is true then the fundamental laws are not mathematical laws.

iii. So, the fundamental laws are not mathematical laws.

The argument is valid and so to establish the conclusion all that remains is to defend the premises. The plan for the paper is as follows. Section 1 will clarify the claim I mean to defend, that the fundamental laws are not mathematical laws; section 2 argues for anti-Pythagoreanism;

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[6] According to Aristoxenus, "Pythagoras most of all seems to have honored and advanced the study concerned with numbers, having taken it away from the use of merchants and likening all things to numbers" (Fr. 23, Wehrli)

[7] The notions of fundamentality and metaphysical explanation have received considerable attention recently. I take the notion of metaphysical explanation to be familiar in metaphysics. It is the notion at work when Socrates asks Euthyphro if the pious acts are pious because the Gods love them, or in the claim that everything obtains in virtue of the physical. See Fine, (2001), Schaffer (2008), Rosen (2010), or Sider (2013).

and the final section argues that anti-Pythagoreanism entails that mathematical laws obtain in virtue of non-mathematical laws.

*1. The Claim that the Laws of Physics are not Fundamental Laws*

By 'the laws of physics' I mean the propositions expressed by the statements of laws that appear in physics journals and are reproduced in physics textbooks - for example, Schrödinger's equation or the Einstein field equations. To keep the discussion as simple as possible I will use Newton's law of universal gravitation as my example of a law of physics, although the arguments that this law is not fundamental will extend to any law stated in mathematical terms.

Distinguish two related propositions associated with Newton's law of universal gravitation. One proposition, which I'll call NGR, states a regularity; the other, NGL, states *that this regularity is a law*:

> NGR: every two massive objects give rise to an attractive force proportional in magnitude to the product of the masses and inversely proportional to the distance between them.

> NGL: *It is a law that* every two massive objects give rise to an attractive force proportional in magnitude to the product of the masses and inversely proportional to the distance between them.

The debate about the metaphysical status of laws concerns the status of law statements like NGL. According to Humeanism about laws, NGL is true in virtue of non-nomic facts like the pattern of distribution of properties over spacetime, while non-Humeans claim NGL is not so reducible.[8] In order to remain neutral on the metaphysical status of laws I will say that the laws are regularities like NGR.

[8] The most developed Humean account of laws is the best system account defended especially by Lewis (Lewis 1983). The leading non-Humean accounts of laws fall into three types: the necessitarian accounts due to Dretske, (Dretske (977), Tooley, (Tooley 1977), and

When I say that some fact is fundamental, I mean that it is not metaphysically explained by and does not obtain in virtue of other facts. The claim that Newton's law of universal gravitation is a fundamental law is neither the claim that NGR is fundamental nor the claim that NGL is fundamental. NGR is not fundamental since NGR is a mere regularity and regularities are plausibly explained by their instances.[9] The claim that Newton's gravitational law is a fundamental law is not the claim that NGL is fundamental because in the sense at issue it should be possible for a Humean reductionist about laws to claim that Newton's law is a fundamental law. Rather, the claim that Newton's gravitational law is a fundamental law is the claim that Newton's gravitational law does not obtain in virtue of other laws. The laws of chemistry are thus plausibly non-fundamental laws because the fact that a given regularity of chemistry is a law plausibly obtains at least partially in virtue of the laws of physics (and perhaps partially in virtue of the initial conditions).

When I claim that the laws of physics are also non-fundamental laws what I mean is that the mathematical laws typically found in physics textbooks, like NGT, obtain in virtue of laws that make no reference to numbers. My claim, then, is that:

(L) For every law of physics L, there is some distinct law $L_F$ such that L is a law in virtue of the fact that $L_F$ is a law, and such that $L_F$ does not concern any mathematical objects.

*2. Fundamental Physical Structure and Mathematics*

This section argues for anti-Pythagoreanism, the claim that facts about relations between parts of concrete reality and mathematical objects are not fundamental but obtain in virtue of

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Armstrong (Armstrong 1983); dispositional essentialism as defended by, e.g. Bird, (Bird 2010) Maudlin's primitivism about laws (Maudlin 2010).

[9] Once the distinction between NGR and NGL is made, it seems to be a mistake to characterize Humeanism as the claim that laws are statements of regularities - the non-Humean too should accept that NGR states a regularity. What is distinctive about non-Humean accounts of laws is *not* that they deny that laws are regularities, but rather that *what makes something a law* involves something that does not depend on what regularities obtain.

entirely non-mathematical facts. I'll present an argument from arbitrariness, from intrinsicality, and from causal efficacy.

The problem of arbitrariness is that Pythagoreanism fails to account for for the conventions involved in mathematical representations and the fact that different mathematical representations of the same system may be *physically equivalent*. Consider, for instance, the fact that the satellite has a mass of 100kg. According to a naive version of Pythagoreanism the *mass-in-kilograms* relation hold between the satellite and the number 100. But the fact that we chose to measure masses with the kilogram scale, as opposed to the ounces scale or the solar masses scale was an arbitrary decision on our part – there is nothing physically significant about the choice of the kilogram scale over other, say, the ounces scale. The satellite has mass of *3530* ounces, so it also bears the *mass-in-ounces* relation to the number *3530*.

The naive Pythgorean view faces a nasty dilemma: are both relations to numbers fundamental, or is one privileged? It is hugely implausible that we have just happened with the kilogram scale to hit upon the one that is fundamental, since there is nothing physically special about objects that have 1kg mass.[10] But if instead each relation to numbers that corresponds to a choice of scale is fundamental, then the view requires a vast proliferation of fundamental facts. On this view it is a fundamental fact that the satellite bears the *mass-in-kilograms* relation to one number, and it is a fundamental fact that it bears the *mass-in-ounces* relation to another number, and so on. This requires there to be massive redundancy at the fundamental level. Moreover, there would myriad unexplained necessary connections between these fundamental facts. If something bears the *mass-in-kilograms* relation to *x* it necessarily also bears the *mass-in-ounces* relation to

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[10] What if mass turns out to be quantized, so that there is a smallest possible mass magnitude; wouldn't this be a physically privileged unit? It's unclear. An explanatory account satisfying of quantities should be an account of quantity in general, not just mass; so as long as one quantity turns out to be continuous this response fails. Moreover, a theory of *what it is* to be a quantity should apply to even possible cases of quantitative properties, and since continuous quantities are surely possible it won't help if the only actually instantiated quantities are discrete.

*x/35.3;* but if these relations are both fundamental we would expect them to be able to vary independently. Pythagoreanism makes a mystery of the physically equivalent ways of representing mass where anti-Pythagoreanism provides a satisfying explanation: the various numerical representations of mass are apt because they are representations of the same intrinsic non-mathematical structure.[11]

There are other sources of arbitrariness in mathematical representation than choice of scale. For example, the Pythagorean may provide an account of the topology of space-time by positing fundamental relations to certain sets. But *which* such sets show up in her account of the fundamental structure of space-time will depend on which axiomatization of topology she endorses. One way of characterizing the topology of a space considered as a set of points is by saying, for each point p of the space, which sets of points are *neighborhoods* of p. So the Pythagorean could help herself to the fact that the space bears the *has-the-topology-of* relation to a function N from points to sets of points. But another way of characterizing the same topological space is by specifying which sets of points are *open sets.* So the Pythagorean could instead hold that the space bears the *has-the-topology-of* relation to a set of subsets of points of the space. Given the appropriate axioms that govern the behavior of open sets and neighborhoods each choice of primitive can recover the other.

The anti-Pythagorean sees no mystery here, since there may be any number of similar mathematical objects that are equally apt to represent a single physical structure, just as in the

[11] While there are more sophisticated versions of Pythagoreanism that avoid these problems, but each of them is independently problematic. According to Pythagorean Comparativism, facts about physical quantities like the mass of the satellite are explained in terms of relations between *pairs* of objects to real numbers which represents the *ratio* of their mass. Since mass ratios are scale invariant, there is no problem with arbitrariness. But Pythagorean Comparativism requires brute necessary connections of a different kind, since for any objects x, y, and z, settling the mass ratio of x and y and that of y and z settles the mass ratio of x and z. This means that there is massive redundancy in the relations invoked by the Comparativist Pythagorean. And moreover if the ratio facts are each fundamental then we expect them to be able to vary independently, so it is mysterious that they should be so perfectly choreographed. See Dasgupta (2013) for a recent defense of comparativism about quantity in general.

case of multiple faithful numerical scales for mass. But since the Pythagorean claims that the relation between parts of the world and mathematical structures is a basic feature of reality she must treat the choice of mathematical structure as a basic feature of reality as well.

Whereas the anti-Pythagorean can hold that reality does not have enough structure to decide between treating open sets or neighborhoods as prior, the Pythagorean cannot vindicate this thought. And in general whenever there are multiple ways to represent some physical structure mathematically the anti-Pythagorean she can claim that each of the mathematical claims obtain in virtue of the same non-mathematical facts. But since the Pythagorean claims that there are no such non-mathematical explanation such must either implausibly take one to be privileged, or accept that there is redundancy at the fundamental level.[12]

The second problem with Pythagoreanism stems from the fact that specifying which relations to mathematical objects, like numbers or sets, a thing stands in tells us nothing about what that object is like intrinsically. We naturally think that learning the mass of an object tells us something about what that object is intrinsically like. Perhaps with the case of mass this judgment must be given up. But surely it is something about the nature of space itself that justifies using a certain function from pairs of points to real numbers to represent distances, or it is something about the nature of the physical electromagnetic field that justifies using a function from points to real numbers to represent field values. the world is a certain way intrinsically independently of the relations its parts bear to numbers. If *all* physical features of the world are just relations to

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[12] Perhaps the Pythagorean can escape the problem of arbitrariness in each case by claiming that in cases of physically equivalent representation, we should invoke a relation not to one mathematical representation but to a *class* of representations. In the case of numerical scale for mass, then, the satellite does not bear the *has-mass-in-kilograms* relation to a number. Rather, it bears the *has-mass* relation to a set of ordered pairs of numerical scales and numbers: {<*mass-in-kilograms*,100>,<*mass-in-ounces*, 3530>, … }. But part of the arbitrariness worry was that facts about ounces or kilograms shouldn't show up in fundamental facts at all. And moreover while the Pythagorean leaves it brute the anti-Pythagorean has an elegant explanation for why the kilogram and the ounces scale are both faithful: they are both representations of the same non-mathematical structure.

mathematical objects, then we are left with a picture of the world on which nothing has any substantive intrinsic nature.[13]

The final problem with Pythagoreanism stems from the fact that it renders the fundamental properties of things relations to mathematical objects. But it is broadly agreed that mathematical objects do not causally interact. It is one of our most central convictions about causal explanation, however, that the way things are (at least partially) explains the way they behave. Balls roll because they are round, planets accelerate in gravitational fields because they are massive, and so on. But Pythagoreanism undermines this conviction, for on her view being massive, say, consists in standing in a relation to a number, an abstract object. Abstract objects are paradigmatically non-causal objects, so it is mysterious how bearing a relation to an abstract object could play a role in explaining how a thing behaves. The anti-Pythagorean, on the other hand, holds that planets stand in relations to number in virtue of something purely non-mathematical and so she is free to claim that it is the intrinsic nature of the planet itself that explains why it behaves the way it does.[14]

I conclude that Pythagoreanism is false. Mathematical claims about the world are not fundamental but obtain in virtue of purely non-mathematical claims that characterize reality intrinsically. Recall that Anti-Pythagoreanism is entirely independent of nominalism. It is no part of anti-Pythagoreanism that descriptions of the world that make reference to a mathematical object must be *false*. Anti-Pythagoreanism is a claim about the fundamental physical natural of the world, whereas nominalism is a claim about the existence of mathematical objects.

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[13] Field (1980: 43) argues in a similar vein that mathematics must be dispensable, appealing to the principle that "underlying every extrinsic explanation there is an intrinsic explanation."
[14] Balaguer (1998) argues for a version of anti-Pythagoreanism on similar grounds, saying that "while (A) ["The physical system S is forty degrees Celsius"] does express a mixed fact, it does *not* express a *bottom-level* mixed fact; that is, the mixed fact that (A) expresses supervenes on more basic facts that are *not* mixed. In particular, it supervenes on a purely physical fact about S and a purely platonistic fact about the number 40." (Balaguer (1998) p. 131).

Of course, if anti-Pythagoreanism is true this raises the question of what it is in virtue of which mathematical claims about the world obtain. While the details of the answer to this question will depend on judgments in the metaphysics of quantity and spacetime, there are quite general reasons to believe that an answer does exist. All of the accounts of the applicability of mathematics defended in the recent literature agree that the 'mapping account' provides at least part of the story. See, for example, Pincock (2004), Bueno and Colyvan (2011) and Batterman (2010).[15]

According to the mapping account, when a scientific theory makes reference to a mathematical object, there (typically)[16] is some non-mathematical structure that this mathematical object represents. A mathematical structure may be used to accurately represent some physical structure as long as there is an appropriate mapping from the physical structure to the mathematical structure. A mathematical object accurately represents a part of the world if there is a homomorphism, a structure preserving function, from a part of the world to a mathematical object. A statement that such a homomorphism exists is a representation theorem.

For example, we can adopt a *coordinate system* to reason about facts about spacetime; one way of doing this is to associate each point in spacetime with an ordered quadruple of real numbers - i.e. a point in the four-dimensional mathematical space $\mathbb{R}^4$. If we assign numbers to points in the right way then the structure of $\mathbb{R}^4$ reflects the structure of spacetime, so that points that are *physically* close get assigned coordinates that are *numerically* close. There are two

[15] Batterman (2010) writes that "recent investigators do seem to have an approach to applicability on which, in broad outline, they agree. Bueno and Colyvan, following Pincock, call such accounts 'mapping accounts'. In a nutshell, mapping accounts seek to explain the utility of mathematics in some applied situation by demonstrating the existence of the right kind of map from a mathematical structure to some appropriate physical structure." (Bueno and Colyvan 2011, Pincock 2005).

[16] As Batterman (2010) and Bueno and Colyvan (2011) point out mathematical explanations sometimes involve idealizations, in which cases there will be nothing physical that corresponds to a part of some mathematical structure.

spaces: the physically real space that we move around in and a mathematical space, $\mathbb{R}^4$, that represents it.

It is also convenient to represent physical quantities like mass numerically. Magnitudes of mass have a structure that is aptly represented by the positive real numbers: there are continuum-many mass magnitudes, and they instantiate a distance structure similar to that of the positive real numbers. So it is convenient to assign a positive real number to each magnitude of mass so that we can reason about mass by reasoning about numbers. Again, there are then two structures: the positive real numbers and the collection of mass magnitudes. Let a *mass scale* be a function from objects to positive real numbers. If a mass scale $S_M$ is faithful then for any two massive objects $o_1$ and $o_2$, $S_M(o_1)<S_M(o_2)$ if and only if $o_1$ is less massive than $o_2$.

According to the mapping account, mathematics applies to concrete reality because are exist structure-preserving mappings from parts of the world to the mathematical objects, as in the case of the mass magnitudes and spacetime. In order to assess whether some physical structure is aptly represented, then, there must be a way of characterizing the physical structure intrinsically.[17] There are various theories of quantity that offer intrinsic characterizations of the relevant physical structures; examples include the accounts given in Armstrong (1989), Bigelow and Pargetter (1990), Field (1980), and Mundy (1987).[18]

This section argued for anti-Pythagoreanism, the claim that mathematical descriptions of the world obtain in virtue of non-mathematical facts, which was the first premise in the argument of the paper. The following section argues for the second premise: that if this is right then the

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[17] Reading off physical structure from these mathematical representations requires a *uniqueness* theorem. Not every feature of $\mathbb{R}^4$ represents an intrinsic feature of ST. For example, the universe doesn't have a privileged centre and so it's arbitrary which point gets assigned to (0,0,0,0). And since there is no privileged unit length it's arbitrary which points of physical space are mapped to points in $\mathbb{R}^4$ that are 1 unit apart. The meter scale represents one such arbitrary choice. So if one coordinate system C is faithful, then so is any coordinate system that is a result of uniformly translating or stretching C. Similarly, since it is arbitrary which mass magnitude is assigned to the number 1, if one scale $S_m$ is faithful then so is any scalar multiple of $S_m$.

[18] Armstrong (1978), Bigelow and Pargetter (1988), Field (1980) and Mundy (1987).

laws of physics, which are claims about mathematical representations of the world, are also derivative.

*3. Mathematical Laws are Derivative Laws*

Physicists use mathematical objects to represent the physical structure of spacetime and physical quantities. It is natural that they should then also state the *laws* as relations between these mathematical representations. To simply discussion I'll use Newton's second law of motion, $f=ma$, as my example of a fundamental physical law, and I will ignore the fact that force and acceleration have direction. This law states that the resultant force on a body in Newtons is equal to its mass in kilograms multiplied by its acceleration in meters per second per second.

This law states a relationship between three mathematical objects; a function $f_m$ from objects to real numbers that represents its mass in kilograms, a function $f_f$ from objects to real numbers that represents the magnitude in Newtons of the resultant force acting on it, and a function $f_a$ from objects to real numbers that represents the magnitude of the body's acceleration.

Now we can state Newton's law explicitly as:

> ($L_{\text{EXTRINSIC}}$): *for every massive object o, the value of* $f_f(o)$ *is equal to* $f_m(o)$ *multiplied by* $f_a(o)$*:* $f_f(o)=f_m(o).f_a(o)$

This law states a relationship between three mathematical objects: a numerical representation of force, a numerical representation of mass, and a numerical representation of acceleration. These mathematical objects each represent some physically real non mathematical structure. Given anti-pythagoreanism, if $L_{\text{EXTRINSIC}}$ is true then corresponding to $L_{\text{EXTRINSIC}}$ there is a true claim about how facts about mass, about forces and accelerations relate *intrinsically* that makes no reference to numbers.[19]

[19] In particular, the representation theorems that govern $f_f$, $f_m$, and $f_a$ will determine what form this claim takes.

Let me spell this out. Given anti-Pythagoreanism, corresponding to the mathematical representation of mass given by $f_m$, there is an intrinsic, non-mathematical characterization of the distribution of mass magnitudes. Precisely what this intrinsic characterization looks like depends on which theory of quantity is correct, and we needn't take a stand for our purposes here. For concreteness, I will frame the discussion using Hartry Field's (1980) account of mass, which invokes two fundamental relations, *mass-between*$(o_1,o_2,o_3)$ and *mass-congruent*$(o_1,o_2,o_3,o_4)$. Let's extend this account to cover force and acceleration as well, so that we help ourselves to four more fundamental relations, *force-between*$(o_1,o_2,o_3)$ and *force-congruent*$(o_1,o_2,o_3,o_4)$, and *acceleration-between*$(o_1,o_2,o_3)$ and *acceleration-congruent*$(o_1,o_2,o_3,o_4)$.[20]

On this theory of quantity, a assignment of numbers to represent, say, mass, is faithful when it accurately represents the facts about *mass-congruent*, *mass-between.* For example, an assignment of numbers to to objects $S$ will faithfully represent mass only if for any three objects $o_1$, $o_2$, and $o_3$, $S(o_1) \leqq S(o_2) \leqq S(o_3)$ only if *mass-between*$(o_1,o_2,o_3)$. Claims about the numerical representation of mass, force and acceleration correspond to facts about about the distribution of these relations.[21] For example, the claim that a planet has a mass of 100,000,000 kg accelerating under the influence of a certain force corresponds to a claim purely about the distribution of *mass-congruent*, *mass-between, force-between* and *force-congruent,* and *acceleration-between* and *acceleration-congruent*. The same goes for very general statements about how mass, force and acceleration are related. For every physically significant claim stated in terms of $f_f$, $f_m$, and $f_a$ there is a direct statement in terms of *mass-congruent*, *mass-between, force-between* and *force-*

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[20] I'm assuming that acceleration magnitudes are basic and not defined in terms of rates of change of rates of change of position. Nothing essential except simplicity of the discussion depends on this assumption - we might have started with supposing that the intrinsic structure of spacetime is given Hartry Field's (1980) extension of David Hilbert's axiomatization of Euclidian geometry according to which coordinate systems represent the distribution of two fundamental relations between spacetime points: *spatial-congruent*$(p_1,p_2,p_3,p_4)$ and *spatial-between*$(p_1,p_2,p_3)$.

[21] Or at least, those claims that are physically significant claims so correspond.

*congruent,* and *acceleration-between* and *acceleration-congruent.* Since Newton's law of gravitation is one such general statement there is statement of it in intrinsic, non-mathematical terms.

Recall the distinction from section 1 between the fact that some regularity is *true* and the fact that it is a *law*. $L_{EXTRINSIC}$ is the statement of a regularity. Since I am granting that $L_{EXTRINSIC}$ is a law and only denying that it is a fundamental law, I grant that the following statement is also true: *it is a law that $L_{EXTRINSIC}$ holds.* Call this claim $LL_{EXTRINSIC}$. We can make a similar distinction for the intrinsic statement of the law. Given anti-Pythagoreanism, the truth of $L_{EXTRINSIC}$ guarantees the truth of $L_{INTRINSIC}$, where this is a statement of a regularity involving the predicates *mass-congruent*, *mass-between, force-between* and *force-congruent,* and *acceleration-between* and *acceleration-congruent*. The question now rises: *is it a law that $L_{INTRINSIC}$ holds*? Call the claim that $L_{INTRINSIC}$ is a law $LL_{INTRINSIC}$.

The contention of this paper is that the laws of physics are not fundamental laws, and in particular that $LL_{EXTRINSIC}$ obtains in virtue of $LL_{INTRINSIC}$; that is, $L_{EXTRINSIC}$ is a law in virtue of the fact that $L_{INTRINSIC}$ is a law. I will offer two arguments for this claim: the *argument from arbitrariness* and the *argument from extrinsicness*. The argument from arbitrariness is that since the mathematical form of the law depends on an arbitrary choice of scale, $L_{EXTRINSIC}$ is not a fundamental law. The argument from extrinsicness is that we should think that the fundamental laws relate the *intrinsic* state of the world at one time to the intrinsic state of the world at other times.

*3.1 The Argument from Extrinsicness.*

If anti-Pythagoreanism is true then mathematical descriptions of the world, like those that invoke the kilogram scale to represent facts about mass or a coordinate system to represent spacetime, are *extrinsic*. A mathematical description of a physical entity x is extrinsic (to x) because it describes a relation x bears to something wholly distinct from x, a number. For example, the structure of mass magnitudes is described by the relations they bear to things that are not masses - namely, numbers.

The argument from extrinsicness exploits the simple fact that the intrinsic state of the world at a time is explained by the intrinsic state of the world at previous times. But $L_{EXTRINSIC}$ only relates the extrinsic state of the world at a time to the extrinsic state of the world at later times, since $L_{EXTRINSIC}$ states the relationship between mathematical representations the state of world at each time. If $L_{EXTRINSIC}$ were a fundamental law, then $L_{INTRINSIC}$ would be at best merely derivative, since it is implausible that both laws be fundamental. (I will defend this last claim in more detail while outlining the argument from arbitrariness.) That means that the intrinsic state at one time, $t_1$, explains the intrinsic state at a later time, $t_2$, partly in virtue of the fact that the extrinsic state at $t_1$ explains the extrinsic state at $t_2$.

But this strikes me as patently false. The fact that two negatively charged particles are accelerating away from each other at $t_2$ is surely explained by the *intrinsic* features of the particles, not by facts about what mathematical objects the particles are related to. *Perhaps* we can understand a sense in which the mathematical representation of the state at one time explains the mathematical representation of the state at a later time. But this sense is surely a derivative sense of explanation – what is explanatorily efficacious in the first instance is the worlds' intrinsic nature.

*3.2 The Argument from Arbitrariness*

$L_{EXTRINSIC}$ states a relationship between functions from objects to real numbers. But the fact that we chose to measure mass with the kilogram scale and spatial distance with the meter scale was an arbitrary choice: there is nothing physically privileged about the mathematical field that represents mass density in kilograms per cubic meter. For example, if we measure mass in grams, then the law is $f=1000ma$. There are many physically equivalent ways of stating $L_{EXTRINSIC}$. This means that in order to avoid implausibly privileging one scale, these various relationships between different mathematical objects should be treated on a par.

$L_{EXTRINSIC}$, then, is a statement of only one of a great many relations between relevant mathematical objects. The function from objects to numbers that corresponds to measuring mass in kilograms is related in one way to the function that represent forces in Newtons and acceleration in meters per second squared; the function that corresponds to measuring mass in ounces is related in a different way, and so on. There is thus a multitude of physically equivalent extrinsic laws: $L_{EXTRINSIC}$, $L_{EXTRINSIC2}$, $L_{EXTRINSIC324}$, and so on. These laws somehow conspire to make precisely identical requirements on the intrinsic nonmathematical fields that the mathematical scales represent; namely, the mass density field and the gravitational potential field. This surely calls out for explanation. The natural explanation, of course, is that there is an underlying law that relates the fields themselves and not merely their mathematical representations, namely $L_{INTRINSIC}$, and that this law is gives rise to each of the intrinsic laws. In other words, that $L_{INTRINSIC}$ is the more fundamental law.

What are the prospects for competing explanations? If facts stating laws are themselves fundamental, then it looks especially problematic to claim that extrinsic laws are fundamental, for each of $LL_{EXTRINSIC}$, $LL_{EXTRINSIC2}$, $LL_{EXTRINSIC324}$, and so on would state fundamental facts. Since fundamental facts are typically taken to be independent, it's hard to see what could explain their conspiring together to require the same regularity in the intrinsic features of the world. But the prospects for the Humean reductionist about laws look little better. Recall that the question under discussion is whether laws like $L_{EXTRINSIC}$ are fundamental laws, in the sense that they are not laws in virtue of more fundamental laws. This is consistent with *the fact that they are laws* being itself grounded in some non-nomic facts. On this picture, $LL_{EXTRINSIC}$ isn't fundamental, even though $L_{EXTRINSIC}$ is a fundamental law. So there is a possibility that the account of how each of $LL_{EXTRINSIC}$, $LL_{EXTRINSIC2}$, $LL_{EXTRINSIC324}$ are grounded would explain them in a similar way. If so, then the Humean reductionist about laws could explain why so many fundamental laws are equivalent.

The most promising reductive account of laws is the best system account, according to which the laws are the axioms of the systematization of the distribution of fundamental properties and relations over spacetime that maximises strength and simplicity.[22] But if a systematization had to include each of $L_{EXTRINSIC}$, $L_{EXTRINSIC2}$, $L_{EXTRINSIC324}$, and so on, it would contain a simply vast amount of redundant information. If there were a nonredundant systematization that is not especially complicated then it would seem to be guaranteed to beat a systematization that includes each of the extrinsic laws. And, of course, we have seen that there *is* such a systematization: the direct statement of the intrinsic laws that includes $L_{INTRINSIC}$. So it looks implausible that the Humean reductionist can defensibly take each of the extrinsic laws to be fundamental.

We've seen that whether or not laws are reducible, we should not take extrinsic laws to be fundamental laws. Precisely similar considerations militate against regarding both $L_{INTRINSIC}$ and $L_{EXTRINSIC}$ as fundamental laws. These laws make precisely the same requirements on reality, so taking both to be fundamental would mean the fundamental laws contain a significant amount of redundancy. And the very fact that they are equivalent is something that calls out to be explained.

I conclude that reflection on the conventions involved in mathematical representation of physical states, and therefore in the statement of mathematical laws, create problems with the view that those mathematical laws are fundamental laws. Together, the argument from extrinsicness and the argument from arbitrariness create considerable pressure in favour of the view that mathematical laws are derivative laws and that the fundamental laws are those that concern the intrinsic, non-mathematical, physical structure of the world.

But perhaps there are also formidable reasons to doubt that $L_{INTRINSIC}$ is a law. Let me anticipate one such reservation. Laws of nature are simple, general principles. If the only way to state $L_{INTRINSIC}$ were as an infinite collection of conditionals then this would provide us with excellent reason to doubt that $L_{INTRINSIC}$ is a law. In response, however, there is very little reason

[22] Lewis, (1983).

to think that there is no simple statement of $L_{INTRINSIC}$. Consider the vast amount of thought that has gone into finding elegant *mathematical* statements of the laws. By comparison, almost no effort has been invested in finding simple intrinsic statements of the laws. So *even if* the prospects for finding fairly simple intrinsic laws looked hopeless, this shouldn't be taken to provide much evidence at all for the claim that $L_{INTRINSIC}$ must be complex. But this isn't the situation we find ourselves in. In the few philosophical works devoted to finding intrinsic statements of the laws, there are promising indications that elegant and simple intrinsic statements can be found.[23] Finally, *even if* the mathematical statement of the laws turns out to be considerably simpler and more elegant than their intrinsic statement, this consideration must be weighed against the pressure created by the arguments from extrinsicness and arbitrariness to think that the fundamental laws are the intrinsic laws.

If the arguments of this paper succeed, then none of the paradigm examples of fundamental physical laws are in fact fundamental physical laws. This may appear to be a bold and surprising claim. It is motivated, however, merely by reflection on the implications of anti-Pythagoreanism, an extremely compelling claim about the relation between the physical world and mathematical representations of it.

[23] See H. Field's (1980) and Dorr and Arntzenius, (2012)

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