COGNITIVE RESOURCES OF PHYSICS EXPERTS

by

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ABSTRACT OF THE DISSERTATION

Cognitive Resources of Physics Experts

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One important goal of physics education is to help students develop reasoning patterns similar to those of expert physicists. To achieve this goal, physics educators must know what makes physics experts so successful at solving challenging physics problems. However, this dimension of physics expertise has not been fully explored by the physics education research (PER) community. In this dissertation, I describe several studies I have conducted that further the PER community's understanding of physics expertise. In these studies, I investigate how expert physicists reason as they solve unfamiliar, challenging physics problems by using a resource-based model of cognition to analyze videotaped recordings of problem solving sessions. By developing a way to determine when experts are making conceptual breakthroughs I analyze what resources experts use during conceptual breakthroughs. In the first study, I show that physics conceptual breakthroughs are characterized by reasoning which combines resources related to intuitive knowledge, higher level physics based conceptual knowledge, and epistemological knowledge. In the second study, I develop a way to reliably code for epistemological resources and determine what epistemological resources experts rely on most during conceptual breakthroughs. My findings show that experts rely on contrasting cases more often than any other epistemological resource. In the third study, I use variation theory to investigate how experts use contrasting cases. I look for patterns across all instances when experts use contrasting cases to make a conceptual breakthrough and show how scientific epistemology can be used to better understand experts' use of contrasting cases. I discuss how the findings of each study can be used to inform physics education.

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Chapter 1

Introduction

As physics educators, we should aspire to teach our students more than just physics facts and ideas contained in textbooks. While content goals are important, many students who take introductory physics courses, and even many students who take upper level physics courses, will not use a majority of the content oriented material they learn in those courses once they enter the workforce (Czujko, 1997). The modern workface requires individuals to use their knowledge to adapt to new situations, think critically in unfamiliar circumstances, and independently learn new information (Koenig, 2011). As a consequence, we must ask ourselves: what else should our students gain from their coursework? The answer to this question is manifold; however one of the foci of physics education, and science education in general, should be the engagement of students in science practices (Pellegrino & Hilton, 2012). Science practices are activities that scientists (in our case physicists) engage in when they construct and apply knowledge. In other words, we need to immerse our students in the practice of doing physics when they are learning physics (Etkina & Van Heuvelen, 2007). We hope that by doing this we will be able to enhance their capacity to deal with new problems and think critically in any situation.

In order to do this, we must first paint a picture of how people reason in physics in general and then work within this model to determine what exactly physicists do when they encounter a situation in which they must think critically. We must capture the substance of the problem solving strategies which guide the reasoning of physicists in novel situations. This way we can better understand how physics experts solve problems and help our students develop similar proficiencies. Analyzing how physicists critically think in challenging situations and understanding their thought processes in these moments from a fine grained perspective is the primary goal of my thesis.

I. Motivation

I.1 Resource Based Model of Cognition

To better facilitate student learning of physics and specifically of science practices, we need to investigate the learning process in sufficient detail so that we can provide causal explanations of the individual and contextual nature of learning (diSessa, 2006). The resource-based model of cognition, introduced by Hammer (2000) is a model which can support such a detailed investigation into the learning process. This model describes individual cognition as an in the moment, contextually dependent utilization and combination of fine grained bits of information, dubbed "resources," to construct ideas that exist at a larger grain size.

Developing this model into a tool that can be used by educators to help them better understand ways they can improve instruction and encourage student learning of physics requires a focused effort into answering the following questions:

1. What resources do students have available to them?

2. What patterns of resource activation can we identify? This includes identifying the contextual dependency of resource activation as well as ways in which resources may be combined.
3. What resources and activation patterns are productive in specific contexts?
4. What makes certain resources more productive than others?

Previous work has begun to answer some of these questions in the context of novice studies. Researchers have classified resources and identified how different resources are used by students (diSessa, 1993; Hammer, 1994; Hammer, 1996; Hammer, 2000; Hammer & Elby, 2002; Lising & Elby; 2005). While these studies can answer the first two questions by focusing on the reasoning of novices, the third and fourth questions cannot be answered deeply based exclusively on an analysis of novices' reasoning.

Experts have amassed a great deal more content knowledge than novices and notice aspects of a problem that novices do not (Bransford, Brown, & Cocking, 2000). Confining studies to novice reasoning would miss the resources and patterns of resource activation which cause these differences. In order to more completely answer the third and fourth questions, we need to study the reasoning of experts using a resource-based model of cognition.

Furthermore, we might not be able to find the answers to these questions easily by using analysis techniques that have been developed within the resources framework. Developed methods use the resources framework to make extended qualitative arguments for identifying a small number of resources (ex: Harrer, Flood, & Wittman; 2013) or justifying a resource-based interpretation of a single individual or group's reasoning (ex: Louca et al., 2004). Discovering what resources or reasoning patterns are consistently productive and persist over extended sets of qualitative data requires using more quantitative methods, such as the methods described by Chi (1997) for quantifying qualitative data. These methods are less well developed in the resources framework and reliability measurements have achieved mixed results. Scherr and Hammer (2007) were able to achieve 90% inter-rater agreement when using obvious behavioral criteria to code for epistemic frames. However, when Bing and Redish (2009) similarly developed criteria for identifying epistemic frames that were based on slightly less obvious behaviors, inter-rater agreement was only 70%. In order to make these methods accessible, more work needs to be done to develop ways of reliably coding data.

I.2 Personal Epistemology

The modern student preparing to enter the workforce faces a different challenge than a student of past generations. With the incorporation of the internet into everyday life and its power to provide a wealth of information to users in the everyday activities, the emphasis of education is changing. Workers are not simply prized for their capability to possess large amounts of knowledge, but are expected to make decisions with the knowledge they have and acquire new knowledge on their own (Czujko, 1997; Koenig, 2011). Encouraging growth within these areas requires addressing aspects of a student's personal epistemology, i.e. their beliefs concerning the structure of knowledge and the processes that aid in the attainment of knowledge.

Developing our understanding of the importance of an individual student's epistemology in learning physics has been the subject of many studies, all which indicate that an advanced epistemology can encourage students to be more productive learners. Hammer (1994) used a small number of case studies to argue qualitatively that students' epistemological sophistication influences problem solving approaches and the acquisition of new knowledge in physics. May and Etkina (2002) were later able to show that this trend held quantitatively by developing measures to assess students' epistemological sophistication and showing that this measure correlated with student learning gains. Furthermore, Lising and Elby (2005) showed that there was a causal link between a student's epistemological stance and learning. These studies and more (e.g. Hofer & Pintrich, 1997; diSessa, Elby, & Hammer, 2002; Louca et al., 2004; Elby & Hammer, 2010) all support the idea that if we address issues related to beliefs concerning the structure of knowledge and the processes that aid in the attainment of knowledge, our students will be better equipped to tackle the real world problems of efficiently using the knowledge they have and independently constructing new knowledge.

A problem with many of these studies which investigate questions of personal epistemological sophistication is that they define epistemological sophistication either phenomenologically, based on student data, or through the use of theoretical arguments. The former leads to snapshots of students current epistemological state but does not provide data on the evolution of epistemology. For example, Hammer's (1994) framework for describing personal epistemologies was built by analyzing student interview data during the course of one semester to determine different categories of epistemological sophistication the students fell into. The other studies mentioned above are all similarly driven by data collected from students over short periods of time. While this type of analysis is insightful and gives educators an idea of what to look for and expect in their classrooms, there is evidence that an individual's epistemology can develop over time (Perry, 1970). The studies that focus only on students' "static" epistemologies do not address the full extent to which a person's epistemology can be developed. The question still remains as to how we know what should count as a sophisticated epistemology and whether or not there are epistemologies that will serve our students better than the ones they are currently using.

I.3 Expert Epistemology

The correlation of learning gains with students' epistemological preferences, the expert/novice dichotomy, and the confinement of epistemological studies to the student realm point to a need to investigate physics experts' epistemologies to see if we can determine what aspects of their epistemology help them develop new knowledge effectively and be successful in solving complex problems. If we were able to detail an expert's epistemology, it would allow us to determine what a sophisticated epistemology in physics actually looks like and would allow us to find epistemological traits which we might wish to foster in students in order to help them develop reasoning expertise.

However, studies performed to gain insight into physics experts' cognition make it clear that there is not enough evidence available in the literature on physics experts to begin building a model of the epistemologies of physics experts. Aside from a general lack of studies which explicitly look at the personal epistemologies of physics experts, prior research fails to give us a clear model of expert cognition from which we can extrapolate definitive conclusions about epistemology. Previous studies of physics experts fall short in several areas.

First, some studies, examples include Chi et al. (1981) and Lin and Singh (2010), study experts reasoning about problems with which they are fairly familiar. This is problematic because, as originally described by Hatano and Inagaki (1986) and further developed by Schwartz, Bransford, and Sears (2005), expertise is more correctly conceptualized as multidimensional and context dependent, where one dimension is related to routine processes and another to novel experiences. Studies such as those described above only probe the dimension of expertise related to routine processes. If we want to understand what aspects of a physics expert's epistemology enables her/him to be successful in the process of development of new knowledge and in using this knowledge productively in new situations (the scenario which most closely mimics the context which students experience in a learning environment), then we need to begin to look at studies which place experts in challenging, novel situations.

A second shortcoming of prior research is that studies that place experts in contextually relevant conditions, such as Singh (2002) and Kohl and Finkelstein (2008), do not provide the depth of analysis necessary to begin developing a fine-grained model. This is of no fault of the studies themselves because they were not attempting to develop a fine-grained model of expert epistemology and as a result the epistemological aspects were not given a significantly detailed analysis. However, if we want to build a model that is able to explain the complex processes of cognition and conceptual change this level of detail is absolutely necessary (diSessa, 2006).

A third problem is that other analyses (Popper, 1959; Kuhn, 1970; Toulmin, 1972) focus on knowledge construction on a scale which is very broad by looking at how physical theories emerge within a discipline. While these types of studies help us to understand the epistemology of a discipline as a whole, the findings from these studies may or may not be directly applicable to "in the moment" reasoning which occurs on a personal level. Empirical studies of individual physicist's actions and utterances in the moment which they are actively problem solving is necessary to either support or refute the idea that we can use broader scientific epistemologies as a substitute for the personal epistemologies of practicing physicists.

II. Study Overview and Research Questions

In order to more fully understand how expert physicists reason and construct new knowledge, I conducted a study which analyzed expert problem solving from a fine grained perspective. In this study, professional physicists were asked to solve novel, challenging physics problems. The data for this study consisted of videotaped records of the problem solving sessions. By developing methods to reliably analyze significant moments during the problem solving process, I was able to gain insight into the reasoning processes of expert physicists. Specific attention was allotted to understanding how physicists reason within the resource-based model of cognition and the function of specific epistemological resources they were frequently using.

Through the presentation of the study in this dissertation, I will answer the following questions:

1. Are there any patterns in the way that professional physicists activate and combine different resources during critical moments of the problem solving process?
2. How do the patterns of resource activation during professional physicists' problem solving compare to the patterns of resource activation during pre-service physics teachers' problem solving?
3. How can we reliably identify epistemological resources?

4. What epistemological resources do professional physicists use when solving challenging, novel problems? Are there certain resources that experts tend to use more often than others?
5. If there are resources that are used more frequently than others, how do the experts use these resources to solve a challenging novel problem?

III. Dissertation Overview

This dissertation consists of a collection of three studies aimed at answering the questions presented above. Each study analyzes the resources used by physics experts as they solve novel, challenging physics problems through a slightly different lens. In chapter 2, I present a study where I have analyzed how the experts combine their intuitive understanding of the physical world, their higher level conceptual physics knowledge, and their epistemological resources during important moments during the problem solving process. In chapter 3, I analyze these same important moments, establish criteria for reliably identifying epistemological resources, and focus on identifying specific epistemological resources that are used by the experts. By doing this, I determine which resources are used most frequently. In chapter 4, I build off the analysis in chapter 3 and analyze how the experts utilize the most frequently used epistemological resource, contrasting cases, in order to determine the function that this resource plays in the overall problem solving process. Chapter 5 concludes the dissertation by looking at the knowledge gained from all three studies as a whole and discussing how the findings of these studies can be used to inform teaching practices.

Chapter 2

Physicists' Combination of Resources While Solving Novel Physics Problems

I. Introduction

There is a growing trend to examine student learning from a fine-grained perspective. Prior to this trend, student reasoning was modeled as an application of theory-like cognitive structures to problem solving (Clement, 1982; McCloskey, 1983; Hofer & Pintrich, 1997). However, evidence of inconsistent student reasoning patterns (McDermott, 1984; Steinberg & Sabella, 1997, Bao & Redish, 2006) is at odds with a theory-like model of cognition. These inconsistencies led to arguments against these models (Smith, diSessa, & Roschelle, 1993). Alternative, fine-grained explanations of student reasoning were developed (diSessa, 1993; Hammer, 1996). The resource-based model of cognition (Hammer, 2000) was developed to encompass many similar, fine grained models of student reasoning. This model says that student reasoning results from application of fine-grained pieces of knowledge, called resources. Resources are cognitive structures which represent abstract knowledge an individual has gained from their experiences. They can encompass different types of knowledge from intuitive notions of how the physical world works (diSessa, 1993) to somewhat more complex pieces of knowledge about the physical world (Hammer, 2000) to knowledge about how an individual can develop an understanding of the physical world (Hammer & Elby, 2002). More complex ideas are formed by piecing together multiple resources. However, the resources that an individual uses are determined by the context they find different resources useful in. Learning occurs as an individual makes connections between resources and determines which resources are appropriate in which contexts.

Developing the resource-based model of cognition into a useful tool for educators requires answering several questions. First, we must know what resources students have when they enter the classroom. These will be resources that educators can expect to see students use in the classroom. Educators can use these resources as a starting point to build from. Second, we must know if there are patterns in the ways that students use their resources. This will help educators be aware of how students will use their resources and will give educators the ability to anticipate student reasoning processes. Third, we must know if there are any resources or ways in which resources can be used that are more effective at promoting understanding. This knowledge will provide educators with a goal to aim for when designing instruction.

Many studies that investigate student reasoning from a resource-based perspective attempt to answer one or more of these questions in some form. One common trend among resource based studies is that they use students as the subjects of study, with little exception (Kustusch et al., 2014). By studying only the reasoning of students, researchers are limited in the extent to which they can determine what resources are most effective and how resources can be used most effectively. This is because students may not have developed the most effective resources and they may not use their resources in the most effective manner.

In order to completely identify effective resources and ways of using resources, we need to investigate how physics experts reason. To do this, we conducted a study and analyzed several groups of physics experts as they solved novel, challenging physics problems. We focused on analyzing the types of resources that the physics experts used during moments when they make a conceptual breakthrough. Previous research (Richards, 2013) has investigated how pre-service physics teachers construct new knowledge while learning about the physics of solar cells. This research found that pre service physics teachers' conceptual breakthroughs are characterized by the presence of different types of resources, specifically, a p-prim, a higher-level conceptual resource, and an epistemological resource. In this study, we answer the following questions:

1. To what extent can we identify patterns in the types of resources that physics experts use when making a conceptual breakthrough?
2. What similarities exist between the patterns of resources used by pre service physics teachers during conceptual breakthroughs and the physics experts?

II. Theoretical Background – Resources Framework

II.1Phenomenological Primitives (P-prims)

The resources framework is a "knowledge in pieces" model of cognition. One of the first complete descriptions of such a model was proposed by A. diSessa (1993). diSessa developed this model to describe the intuitive knowledge system of individuals related to the physical world and the mechanisms that can change this knowledge system. Contrary to “misconceptions” or “preconceptions” models, which say that the fundamental elements of an individual’s knowledge system are large-scale ideas or conceptions, diSessa proposed that knowledge about the physical world could be broken down into pieces of knowledge that make up these ideas. He argued that an individual’s knowledge could be broken down into pieces of knowledge that are self-explanatory abstractions of real life experiences. These pieces of knowledge are so small that they do not require any justification and require little mental effort to utilize. The name he chose for these pieces of knowledge was *phenomenological primitives*, or *p-prims* for short. This name, *phenomenological primitives*, comes from the fact that they are the most basic, primitive pieces of knowledge that an individual can access at any given instance. They are developed by individuals unconsciously to understand patterns they observe in the physical world.

II.1.a Examples of P-prims

One example of a p-prim from diSessa (1993) is *supporting*. *Supporting* conveys the idea that heavy objects hold smaller objects up just by being stronger. A person would use this p-prim to help them make sense of why books don’t fall through tables, why we don’t make tables out of paper, or how a big, strong person can hold up a small person. Another example from diSessa is *dying away*. *Dying away* is an abstraction about scenarios where some initial condition slowly fades away. The fading of a plucked guitar string or the slowing of a car when the driver takes their foot off the gas pedal are examples where an individual would use this resource. Both examples show how p-prims are pieces of knowledge that are fairly basic and are useful for making sense of everyday situations.

II.2 Physics Based Conceptual Resources

When D. Hammer (2000) developed the resource-based model of cognition, he incorporated p-prims into the model. Hammer, like diSessa argued for a knowledge in pieces perspective. However, while diSessa focused on the role of p-prims, Hammer incorporated different types of knowledge elements into his model. A significant difference between the two models was that Hammer's resource based model of cognition also focused on higher level pieces of knowledge about the physical world, physics based conceptual resources, as well as p-prims. He says that physics based conceptual resources, in some cases, could be basic pieces of cognitive structure, like p-prims. However, in other cases, knowledge elements can have internal structure. Knowledge elements can be distributed over several basic pieces of cognitive structure, a structure which has been referred to by diSessa and Sherin (1998) as a coordination class. Sayre and Wittmann (2008) make the distinction between resources that have internal structure which may be explored by an individual and resources that have external structure that is no longer accessible to the individual. They state that p-prims are the later type of resource. Drawing on an analogy which relates the resource based model of cognition to computing, Hammer says that p-prims are like the most basic pieces of computer code and other, more distributed resources are more like elements of the computer's operating system. These physics-based conceptual resources represent higher level pieces of knowledge that an individual may use to help them understand phenomena in the physical world. Like p-prims, they may be derived as generalizations of common sense or they may be developed within the context of a physics class.

II.2.a Examples of Physics Based Conceptual Resources

Sayre and Wittmann's (2008) analysis of intermediate level physics students' choice of coordinate systems provides two good examples of physics based conceptual resources. In their study, they analyze two students' use of the resources *polar* and *cartesian*, resources for understanding the respective coordinate systems. These resources have a defining feature which we will use to differentiate p-prims from higher level resources. The resource *polar* and *cartesian* are unlikely to be constructed by students without some exposure to cartesian and polar coordinate systems in a classroom. They incorporate some kind of physics based idea that is typically learned only through exposure to physics instruction. Another example of a physics based resource is *force as a push.* When a physics teacher repeatedly talks about exerting a force on an object by pushing it, a student could develop the resource *force as a push.* Conversely, p-prims, such as dying away and supporting are intuitively developed. While it is not a requirement that all higher level conceptual resources be physics based, many are developed and invoked in the context of learning in a physics classroom and therefore will contain some sort of physics based idea.

II.2.b Motivation for Distinguishing Between P-prims and Physics Based Conceptual Resources

While Hammer's original model included p-prims as a subclass of conceptual resources, we believe that the primitive nature of p-prims sets them apart. Our motivation for this comes from a mechanism diSessa proposed, called *distributed encoding*. This mechanism explains how the role and function of p-prims change as expertise develops. diSessa explains that at first, an individual's knowledge system is made up of weak, unstructured sets of p-prims. As an individual becomes more expert diSessa says, "the priority of some p-prims becomes greatly enhanced or reduced, and contexts of activation may migrate, expand, or contract, depending on the elements' new roles in the developing physics knowledge system." (diSessa, 1993, p. 114) Basically, this means that the individual will begin to learn what p-prims are appropriate in different contexts. However, diSessa expands on this. He explains that when expertise is developed, p-prims not only gain or lose priority in different situations, but that p-prims:

> …can no longer be self-explanatory but must defer to much more complex knowledge structures, such as physics laws, for justification. P-prims come to serve weaker roles, as heuristic cues to more formal knowledge structures, or they serve as analyses that do their work only in contexts that are much more particular than the range of application of the general or universal laws of physics. I call this reuse and integration of intuitive knowledge structures into the functional encoding of expertise *distributed encoding*. (diSessa, 1993, p. 115)

Essentially, p-prims are no longer the central element in the knowledge system. More complex, higher level knowledge structures are central. P-prims are now associated with these higher level knowledge structures. In the language of resources, an important aspect of learning is the interaction between p-prims and higher level conceptual resources. Without this interaction, abstract principles would not be grounded in real world experience captured by p-prims. In order for a student to figure out when they should use a certain physics idea, they must be able to associate that idea with a p-prim. Identifying when distributed encoding is being developed requires differentiating between higher level conceptual resources and p-prims. It is for this reason that we refer to p-prims and higher level, physics based conceptual resources separately. From here on, when we say physics based resources, we will mean higher level conceptual resources that relate to physics ideas and will say p-prims when we wish to refer to the special subclass of lower level conceptual resources that have been discussed in detail by diSessa.

II.3 Epistemological Resources

When Hammer (2000) introduced the idea of conceptual resources, he also introduced epistemological resources. While conceptual resources are pieces of knowledge related to an individual's conceptual knowledge about the physical world, epistemological resources are pieces of knowledge related to an individual's understanding of the nature of physics knowledge and their understanding of how knowledge about the physical world can be attained.

II.3.a Examples of Epistemological Resources

Hammer and Elby (2002) gave several examples of what epistemological resources are. One example is the resource *knowledge as fabricated stuff*. This resource helps a person understand that new knowledge can be built from other pieces of knowledge. A student might use this resource to understand how they can use their prior physics knowledge to develop their own model of a phenomenon. Another example of an epistemological resource is *accumulation. Accumulation* refers to an individual's knowledge related to gathering information. This resource may be activated when a student decides that they "just don't know enough" to answer a question and need to read their textbook to find an answer. A third example of an epistemological resource is *supporting evidence*. This resource may be activated to understand why a lawyer needs to provide evidence if they want to prove that a criminal is guilty. These three examples represent the broad nature of epistemological resources, covering different epistemological categories. The first resource represents an individual's understanding of the nature of knowledge. The second represents knowledge about an activity to help gain new knowledge. The third represents knowledge related to how we know something is true. These examples do not cover the entire scope of epistemological resources, but represent how epistemological resources cover a range epistemological knowledge.

II.3.b Link Between Epistemological and Conceptual Resources

While epistemological resources are different from physics-based resources and p-prims, they are not isolated. Epistemological resources play an important role influencing how an individual uses p-prims and conceptual resources when they try to learn something new (Hammer et al., 2004). For example, Louca et al. (2004) analyzed third grade students who were developing an explanation of why leaves change color. When the teacher encouraged the students to use their resources for understanding *mechanistic* explanations rather than *teleological* explanations the content of the students' explanations changed. Furthermore, Lising and Elby (2005) showed how an individual's epistemology can cause them to create a mental barrier between formal and intuitive reasoning. This prevents the student from using formal physics based conceptual resources when they are reasoning intuitively and p-prims when they are reasoning formally. Other studies (Hammer, 1994; May & Etkina, 2002) have also shown a link between an individual's epistemology and learning outcomes.

II.4 Example Interpretation of Student Reasoning Using Resources

To see how these different types of resources can be used by students as they reason about physics problems, considering the following example question and a common student response:

> **Q:** In order for an object to move at constant velocity you must apply a constant force. Do you agree or disagree? Explain.
>
> **A:** I agree with the above statement. You can show this is true by thinking about what happens when you apply a force to a box by pushing it across the floor. The box only moves as long as the force is applied. Once you stop pushing the box, the box slows down and eventually comes to a rest.

Rather than interpreting this answer as a whole unit indicative of a coherent, stable idea that a student has, we can break their reasoning down into the resources that they used, in the moment, in order to construct this answer. First, they began their answer by saying that they wanted to "show this is true." This statement indicates that they are reasoning using the epistemological resource *supporting evidence*. Second, they talk about exerting a force on a box "by pushing it across the floor." Here they are using the physics based resource *force as a push* to reason about how a force can be exerted on an object. Third, they say that "the box slows down and eventually comes to a rest." This is evidence that they are using the resource *dying away* to think about how the motion of the object will change once a force is no longer exerted on the object. While these are only some of the resources that are being used by the student in this statement, this shows how a student's reasoning can be broken down into pieces by using the resources framework.

II.5 Resource Activation Patterns

The links between p-prims, physics-based conceptual resources, and epistemological resources suggest that they are interconnected in the learning process. Previous research conducted by AJ Richards (2013) as part of his dissertation work found that analyzing learning episodes in terms of all three of these types of resources leads to interesting results. In the study by Richards, a group of pre-service physics teachers were videotaped while they were learning about solar cells. Richards identified moments when students reasoned to a conceptual breakthrough and referred to these as critical moments or critical events (Powell, Francisco, & Maher, 2003). Richards found that critical events were much more likely than non-critical events to have at least one resource from each of the three categories previously mentioned, i.e. p-prims, physics based conceptual resources, and epistemological resources. Specifically, he found that 88% of critical events showed evidence of all three types of resources while only 24% of non-critical events showed the same pattern.

III. Study Description and Data Collection

In this study, we determine the robustness of the pattern found by Richards. By investigating its applicability across contexts, we will determine whether the pattern Richards observed is evidence of a more general pattern of reasoning or an artifact of the study itself. To do this, we videotape and analyze the reasoning of individuals constructing knowledge through reasoning but we alter the context. We investigate how physics experts construct knowledge while solving complex novel, physics problems. By maintaining a focus on knowledge construction through reasoning we are analyzing the same cognitive process while changing contextual variables.

III.1 Participants

The data gathered to answer these questions were obtained by videotaping expert problem solving sessions in full in order to accurately capture the complexity of their reasoning during the sessions (Jordan & Henderson, 1995). During the problem solving sessions, physics experts solved difficult, novel problems which are described below. Before the experts were given either problem, they were asked to think aloud so we could determine their thought process by analyzing the verbal data (Ericsson & Simon, 1980). After an introduction to the problem, the interviewers interacted with the experts minimally to preserve the spontaneity of the experts' reasoning. Physics experts in this study were physics graduate students who have advanced past their qualifier and physics Ph.Ds. The participating physicists were selected based on their response to an email request asking for volunteers. They were not offered monetary compensation for their participation in the study. Six pairs of physicists were videotaped in total.

III.2 The Physics Problems

The pairs solved one of two problems. Three pairs solved a problem which featured topics from optics while the other three pairs solved a problem about the physics of solar cells. The three pairs who solved the solar cell problem were specifically assigned to that problem because they had background knowledge of solar cells. By having different groups solving different problems, we hoped to further investigate the contextual applicability of the pattern found by Richards (2013).

III.2.a Puzzle Problems vs. Ill-Structured Problems

In choosing the problems used, we wanted to make sure the experts experience was similar to the experience of pre-service teacher who participated in the study by Richards (2013). We decided what aspects of the pre-service teachers' learning experience were contextually relevant based on a definition of problem solving expertise given by Schwartz, Bransford, and Sears (2005) and the distinction between puzzle type problems and ill-structured problems discussed by Kitchener (1983) and others (Churchman, 1971; Mitroff & Sagasti, 1973). Expertise is a two dimensional construct. One dimension is related to an expert's ability to solve routine procedural processes extremely efficiently. The other dimension relates to their ability to learn and solve problems by adapting to circumstances which they are not accustomed to. Kitchener describes a complimentary view of problems. Puzzle type problems are problems that have a distinct procedure and can be solved algorithmically. Ill-structured problems are more open, require gathering data, developing and analyzing assumptions, and often do not have a definitive answer. Since the pre-service teachers in the study by Richards were attempting to build new knowledge about solar cells, their success was dependent on their ability to learn in new circumstances, not their ability to carry out procedural processes. This situation favors ill-structured problems over puzzle type problems. This meant we needed to have our experts solve problems sufficiently difficult and complex so they could not procedurally apply physics ideas and would be forced to use their ill-structured problem solving capabilities.

III.2.b The Light Cone Problem

To achieve this goal we utilized the following two problems. The first was an optics problem described by Etkina, Planinsic, & Vollmer (2013). In this problem, a clear plastic container (shaped like a rectangular fish tank) of water mixed with a few drops of milk is set on a piece of white paper. When you shine a laser beam into the water from the top, a cone of light, originating at the bottom of the container, is clearly visible in the water. We asked the experts to explain why this light cone occurs. Solving this problem requires recognizing that the paper beneath the container scatters light in all directions and that the change in index of refraction upon entering the container causes the light which has been scattered by the paper to bend towards the normal. The pairs that were given this problem to solve spent approximately one hour working on this problem and all were able to develop a satisfactory explanation for the phenomenon they observed.

III.2.c The Solar Cell Problem

The solar cell problem was also described elsewhere by Jones et al. (2013). In the first part of the problem, experts are given two graphs showing the output current of a solar cell as portions of the cell are covered. One graph shows the current as the cell is covered horizontally, the other vertically. We ask the experts to explain the shape of the graphs. Once the experts develop an explanation, they are given the second part of the problem. In this part, the experts have to predict the current in two circuits, one with two solar cells connected in series so they have the same polarity and the other with the solar cells connected in opposition. They must predict the current as the coverage of each solar cell is varied. Once the experts develop a prediction, we show them the actual measurements. We ask the experts to resolve discrepancies between their predictions and the measurements. Solving this problem correctly depends on the experts' ability to recognize that the solar cells have a unique geometry. They also must understand that a solar cell does not act as a simple voltage source. The pairs given this problem spent about two hours working. While all groups could solve the first part of the problem, none developed an answer which explained all aspects of the second part.

IV. Data Analysis

Similar to the study by Richards (2013), we look for p-prims, physics resources, and epistemological resources during critical moments. Our primary goals for the analysis are to: 1. Identify critical moments during the problem solving session. 2. Identify resources that the physicists were using in these moments. 3. Look for patterns in the types of resources that were used in each critical moment. We will focus on how each of these goals is achieved individually. By performing this analysis we hope to answer three major questions. Are there similar resource activation patterns in pre-service physics teachers and physics experts during moments of conceptual breakthroughs? Are the patterns of resource activation found restricted to specific contexts? How can we understand the significance of any patterns that we do find?

IV.1 Critical Events

IV.1.a Establishing a Definition for Critical Events

To identify critical events, we first defined what it meant to make progress through the problem. We started from a definition in line with Powell, Fransisco, &

Maher's (2003) idea of critical events. A critical event is a moment where a conceptual breakthrough occurs, leading to reasoning which is different from reasoning prior to this moment. A further stipulation we imposed is that breakthroughs should occur through reasoning and not by chance or observation. At an early stage, before the definition of a critical event was refined further, a small number of events were classified by a single member of the research team. After this, three members of the team discussed the classifications of these events. In this way, the following criteria for identifying critical events were collaboratively developed (Jordan & Henderson, 1995). To determine when critical moments occurred we first viewed the videos several times to identify the lines of inquiry important for each group to solve their problem. This helped us refine our understanding of the problem solving sessions (Lesh & Lehrer, 2000). We decided that for a moment to be critical, it must initiate, progress, or conclude a line of inquiry by answering an important question about the problem. In this way, we identified parts of the problem solving sessions to analyze further (Derry et al., 2010). Events were chosen so they embodied a cohesive line of dialogue and captured a full exchange between the problem solvers. On average, they were 110 words long. We analyzed these events to determine what questions the problems solvers were answering in each event. Once we identified these questions we analyzed both the question and transcript to determine if the problem solvers were using reasoning to answer these questions, if the question was important to an overall line of inquiry, and if the question was being proposed or answered, rather than remaining unresolved. If all these criteria were satisfied, then the event was classified as critical. Otherwise, it was classified as non-critical.

After these criteria were established, we brought in an outside researcher to determine whether these criteria were reliable.

IV.1.b Establishing Reliability of Critical Events

Using the guidelines above, a single researcher coded 146 critical events and 51 non-critical events. Afterwards, an outside researcher familiar with the light cone problem but not involved in any other aspect of the study helped establish reliability for the classification of critical events. To do this, we provided the outside coder with the list of questions which embodied the substance of each critical event. The outside researcher was asked determine whether or not answering each question would meet the above criteria for a critical event. The outside researcher also had the text from the critical events available to consult. After training the researcher on a few example critical events, the outside researcher independently classified 25% of the events. The primary coder and the outside coder achieved an 82% agreement (k = 0.6) before discussion. After discussion, the primary and secondary coder agreed on the classification of 96% of the sample events.

IV.2 Coding Resources

We coded critical and non-critical events for resources in order to determine which resources the experts were using in each event. To establish reliability of coding we developed the following approach:

1. Describe the event without explicitly mentioning any resources.
2. Identify key elements of the dialogue in each event.

3. Determine if these elements are evidence of knowledge active in the reasoning process.
4. Break down the elements that are part of the reasoning process into the smallest pieces of knowledge the text shows evidence of.
5. Indicate what part of the text shows the clearest evidence that this piece of knowledge is being used in the reasoning process.
    - If uncertain whether there is strong enough evidence that this piece of knowledge is being used in the reasoning process, consult the text around the event.
6. Indicate whether piece of knowledge is a p-prim, physics based conceptual resource, or epistemological resource.
7. Classify these pieces of knowledge based on previously identified resources in the literature if possible.
8. If strong enough evidence exists that a resource is being used, but it doesn't have a commonly used name in the literature or has not been previously identified, give it a tentative name.
9. Compile a running list of resources that have been identified.
10. Use this list as a reference when classifying resources to maintain consistency.

Once an initial list of resources was compiled using the inductive method described above, we created descriptions for each resource. Then we reanalyzed a portion of the critical events to deductively identify resources using only the list of resources that had been generated inductively and the descriptions for each resource. Once this was completed, the results of the deductive approach were compared to the initial inductive approach. If there was a mismatch between the two, the event was further analyzed to resolve the mismatch by refining the description of the resource(s) in question.

IV.2.a Example of a Critical Event Coded for Resources

An example of how we coded a single critical event is presented below. Evidence of resource activation is indicated by different color text or highlighted text. Highlighted text is used when evidence for one resource overlaps with evidence for another resource. Each piece of evidence is given a number which corresponds to the list of resources on the right. Different colors are used to indicate different types of resources. Blue text – P-prim. Red text – Physics Based Conceptual Resource. Green text, yellow highlighting – Epistemological Resource.

Table 2.1 Example of critical event coded for resources

C: Yeah What this (1) diagram is ignoring though is what happens at the paper. Right? .(2,3,4) R: Yeah. C: What happens when the laser hits the paper? (5)We don't – do we get – we don't get the exact same beam coming back, right? (6) Cause that's what you were trying to look at before, I think. (7) Just sort of looking at what (8,9,10) – R: Yeah. Somehow it spreads out (11) in a very well defined way. (12) Uh…in the paper. C: Ok, so what is this paper doing (13)? Beam hits it, the paper is – the paper is what? Optically rough (14)? Is that a term we can use? C and R: Haha. C: Why not? R: It's not a mirror. (15, 16) C: That's true. I'll point it away from you. R: Heheh. C: So the laser hits the paper and has to be diffused (17) somewhere, right? It has to…it scatters (18). I mean if we – The paper is definitely rough (19).	1. Multiple Representations 2. Contrasting Cases 3. Hypothetico-deductive Reasoning 4. Limitations of Models 5. Causal Reasoning 6. Something's Changing 7. Peer Cognitive Awareness 8. Experimentation 9. Inductive Reasoning 10. Knowledge from direct observation 11. Spreading 12. Concentration and/or localization 13. Mechanistic Reasoning 14. Roughness 15. Analogical Reasoning 16. Mirrors 17. Spreading 18. Scattering 19. Roughness

IV.2.b Distinguishing Between P-prims and Higher Level, Physics Based Conceptual Resources

While the distinction between epistemological resources and other types of resources is more transparent, the distinction between p-prims and physics specific conceptual resources is not as transparent. However, it is the interplay between p-prims and more complex resources that is at the heart of developing expertise through distributed encoding. For this reason, we believe it is important to distinguish between p-prims and higher level conceptual physics resources. While we don't argue that we can make the distinction between what a primitive resource is and what a higher level resource is based on the content of a single critical event, we do make the distinction between resources which appear to have a more primitive substance and could be seen as generally applicable in common everyday situations and resources which are more closely linked to scenarios you would only encounter in a physics classroom or when involved in some sort of physics problem solving.

In the example above we've identified the conceptual resources *something's changing, spreading, concentration/localization, roughness, mirrors, and scattering*. Out of these resources we classified *something's changing, spreading, concentration/localization,* and *roughness* as p-prims. These ideas represent resources that have some kind of everyday applicability. *Mirrors* and *scattering* were classified as physics specific resources. While mirrors are present in everyday life, they take on a special significance in the physics classroom. Specifically, their surface obeys the law of reflection. From the dialogue before the statement "It's not a mirror," it's likely that the two experts had the physics specific nature of mirrors in mind when they said this. This is especially evident when they said "We don't – do we get – we don't get the exact same beam coming back, right?" You could also argue that the statement "…it scatters," is just an extension of the idea that the light is spreading out after it hits the paper. However, this statement is used to further clarify the previous statement, "So the laser hits the paper and has to be diffused somewhere, right?" This is a generic statement about what the light is doing. The statement "…it scatters," is more than just a restatement of the previous sentence. It adds significance to this previous statement by re-expressing it in more precise, physics specific terminology.

To provide more insight into the types of resources that we classify as p-prims and physics specific conceptual resources, we've included a table of all the conceptual resources classified during coding of the light cone problem. In this table we've indicated which resources were classified as p-prims and which were classified as physics specific resources. The distinction between physics specific resources and primitive resources is pronounced in these tables.

Table 2.2 Resources coded as P-prims and Physics Based Conceptual Resources

P-Prims		**Physics Based Resources**	
Bending	Losing	"Halo Effect"	Mirrors
Bouncing	Matching	Absorption	Multiple Interfaces
Cancelling	Maximum	Angle - Cone	Optical Interface
Cloudy	More cause/more effect	Angle of incidence	Optical Medium
Concentration/localization	Normal (perpendicular)	Angle of refraction	Orthogonal
Cutoff	Ohm's P-prim	Brewster's Angle	Particles
Effect Size - Greater Effects	Parallel	Color - white	Powerful Light
Effect Size - Less Effect	Reflections	Critical Angle	Ray diagram
Effect Size - No Effect	Roughness	Dependent Variables	Refraction
Effect size - small	Selection	Dispersion	Rotation
Entirety	Sharpness	Emission	Scattering
Figural Primitive - Bottom	Something's Changing	Independent Variables	Snell's Law
Figural Primitive - External	Something's Constant	Index of Refraction	Specular Reflection
Figural Primitive - Internal	Spherical	Interference	Spherical waves
Figural Primitive - Sideways	Spreading	Interference pattern	Total Internal Reflection
Figural Primitive - Top	Thickness	Law of Reflection	Transmission
Limit	Trapping	Light as a wave	
Linear			

V. Findings

V.1 Resource Activation Patterns

After we coded each event for resources, we looked at the critical events and non-critical events to see if there was a p-prim, physics based resource, and epistemological resource in each event. First, we focused on the light cone problem. We found that 46/52, or 88%, of critical events had evidence of all three types of resources. However, when we analyzed the non-critical events, we found that only 27/51, or 53%, of all non-critical events showed evidence of all three types of resources. 100% of all critical and non-critical events had evidence of epistemological resources.

We then focused on the solar cell problem. We found that 70/94, or 74%, of all critical events showed evidence of all three types of resources. This was surprisingly lower than previous results and prompted us to further investigate these critical events. We found that the questions the experts were answering in these critical events were not strictly related to the physics of the problem, but rather engineering or mathematical issues. As a result, we grouped the critical events into two categories: physics critical events and non-physics critical events.

V.1.a Physics and Non-Physics Critical Events

An example of each type of critical event is shown in Table 2.3. A critical event was included in either group based on the questions the experts were dealing with. If the question was related to the physics of the problem, it was a physics critical event. If it was related to the mathematics, engineering, or some other aspect of the problem, it was a non-physics critical event. When we grouped the critical events like this we found that 62/74, or 84%, of physics critical events had all three types of resources and 6/20, or 30%, of non-physics critical events had all three types of resources.

Table 2.3 Physics vs. non-physics critical events

Non-physics	Physics
Question: How does the construction of the solar cell make the two situations described in the problem unique?	**Question:** How do the individual p-n junctions function differently in the two situations and how does this explain the difference in the observed current vs. coverage graphs?
D: And so, so, that would mean that as you bring the paper in from this side, you're only partially covering them. A: So that, yeah. D: And then as you bring in the paper from this side… A: The long way. D: You're, you're actually covering entire – A: Individual cells. D: Yeah. Yeah. A: Yeah…Yeah. So, so the difference between the two approaches is in one situation we're covering up like one cell at a time and on the other side we're covering up parts of all the cells.	A: Yeah. When you cover up, when you cover up one of the cells, that cell is now just like a chunk of silicon that's not excited. D: Right, it becomes like a…uh – A: Big resistor. D: Resistor, yeah. A: Yeah. So that's why it's going to cut the current down a lot more. If we, when we come in from the long side and just cover up parts of each cell, then we're going to, um, none of the cells really become a resistor, it's just like, um like, each of them is still contributing something. D: Ok. A: So the effective resistance is less than if we cover up one whole cell. Does that make sense?

VI. Discussion

VI.1 Resource Activation Patterns

The goal of this study was to determine if there were patterns in the way physics experts used resources during conceptual breakthroughs and compare these patterns to those identified by Richards (2013). We will now discuss how we achieved these goals. First, we identified that when solving the light cone problem, 88% of all critical events showed evidence of a p-prim, a conceptual resource, and an epistemological resource. Only 53% of non-critical events showed the same pattern. This means that a conceptual breakthrough is more likely to occur when the experts' reasoning incorporates a p-prim, physics based conceptual resource, and epistemological resource. The percentage of critical events which had evidence of all three types of resources was comparable to the percentage of critical events that had the same pattern in the study by Richards (2013). Moreover, 100% of all critical and non-critical events had evidence of an epistemological resource. Earlier, we chose to distinguish between resources which appeared to be more primitive and resources which appeared to be higher level, physics based conceptual resources. Since epistemological resources appear in every event, there would be no difference between critical and non-critical events without distinguishing between p-prims and physics based conceptual resources. Whether or not the distinction we made between p-prims and higher level, physics based conceptual resources was artificial are not, it allowed us to differentiate between critical and non-critical events. This suggests that the distinction we have made is significant.

When we first analyzed the solar cell problem, we found that the three-resource type pattern was not as strong. Only 74% of all critical events had evidence of all three types of resources. We were able to regain the strength of this pattern by differentiating between physics and non-physics critical events. When we made this distinction 84% of all physics critical events had evidence of the pattern, while only 30% of non-physics critical events did. These percentages are similar to the percentages present in critical events and non-critical events in the light cone problem. We needed to make the distinction between physics and non-physics critical events in order to regain the strength of the pattern in this problem. However, no non-physics critical events appeared in the light cone problem or in the previous study performed by Richards (2013). This is due to the different nature of the solar cell problem. In the solar cell problem, the experts devoted much time to understanding how the solar cell was constructed. Experts do not need to call on their physics knowledge to do this, but it is a very important part of the problem. The light cone problem did not require an understanding of the experimental setup that was removed from the physics of the problem. Additionally, the study performed by Richards was carried out during a concept construction lesson and not independent problem solving sessions. It is likely that the pre-service teachers taking part in this study did not deal with as many non-physics critical events because the instruction helped draw their attention from non-physics details.

Since there is evidence of all three types of resources in physics critical events but not non-physics critical events, this shows that there are different types of reasoning during these moments. One interesting pattern to help explain this is that out of the non-physics critical events that did not have evidence of all three types of resources, 10/12 had a p-prim, but not a physics based resource. The higher level conceptual resources we used to analyze these critical events were physics based conceptual resources. If we investigated other types of conceptual resources, like engineering or math, it is likely that we would find evidence of a p-prim, higher level conceptual resource, and epistemological resource in these events as well. This evidence of non-physics reasoning patterns while solving a physics problem is indicative of the multi-faceted nature of physics problem solving.

VI.2 Significance of the Pattern

So far we have shown that all three types of resources are used by physics experts during a large number of critical events, a pattern which is also evident in pre-service teachers. We have also found that the strength of this pattern is determined by whether we differentiate between p-prims and physics specific resource. What might be the significance of this pattern and the differentiation we are making between p-prims and higher level, physics based conceptual resources? Why is it important that experts use p-prims and higher level conceptual physics resources during critical moments? To understand the purpose of this pattern and this differentiation, we'll take a deeper look at the critical event in Table 2.1.

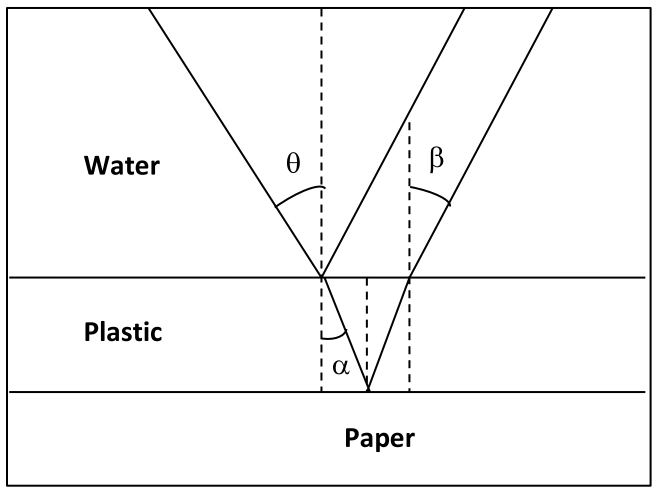


Figure 2.1 Initial diagram constructed by experts to represent the light cone problem.

Prior to this critical event, experts C and R were making a Snell's Law type diagram to help explain the light cone. The diagram they were drawing is shown in Figure 2.1. This diagram has several optical interfaces. At the interfaces there is either refraction or reflection of a single ray of light which obeys Snell's Law when refracting and the Law of Reflection when reflecting. At the start of the critical event, expert C draws attention to the predictions of this diagram. He compares the predictions of this diagram with R's previous observation, that when the laser beam hits the paper, it does not reflect perfectly and "we don't get the exact same beam coming back…" R recounts what they know about the light, that it spreads out but that it does so in a well-defined way to make the cone. C then thinks about what about the paper might make this happen.

He mentions that the paper is rough. R comments that paper is not like a mirror. C focuses on the diffuse, spread out nature of the light after it hits the paper and concludes that the light must be scattering, rather than reflecting, off the paper.

In this passage, the experts make a decision about what physics related resources to rely on as they continue with the problem. From the reflections in the diagram which obey the law of reflection, it appears that C and R are using physics resources related to mirrors. This is confirmed when R brings up mirrors later in the passage. At the beginning, C has activated several epistemological resources. By using these resources he identifies that their model does not account for the observation that the beam does in fact change. This causes the p-prim *something's changing* to become activated. R then specifies how the laser beam changes when he says, "Somehow it spreads out…" This indicates that he is using the p-prim *spreading*. Later R comments that the paper is "not a mirror." By using epistemological resources, C activates a p-prim, *something's changing,* that is at odds with his physics resources related to understanding mirrors. This causes the experts to question the validity of this resource and ultimately abandon it. Once they question the applicability of *mirrors*, C activates the epistemological resource *mechanistic reasoning* to try to understand why the light spreads out. This causes him to activate the p-prim *roughness*. Through the combination of the p-prims *roughness* and *spreading*, C activates the higher level physics resource *scattering*. As the two continue to solve the problem, they no longer model the paper as a mirror, but include light scattered in all directions from the paper. This indicates that from this point forward, they no longer use the resource *mirrors*, but do use the resource *scattering*.

This is evidence of distributed encoding being utilized in the moment to help the experts make an active decision about which higher level physics resources they should use to explain the problem at hand. As diSessa suggested, p-prims are acting as "heuristic cues to more formal knowledge structures." The p-prims are not sufficient justification for an explanation, but help to identify which more formal knowledge structures are appropriate in a given situation. In this situation, the p-prims *something changing, spreading, and roughness* all indicate that formal knowledge for understanding *mirrors* is not appropriate, but that formal knowledge related to *scattering* should be used instead. The experts appear to be using blended processing, combining their intuitive knowledge with their more formal conceptual physics knowledge, to make a decision about what formal physics knowledge to use (Kuo et al., 2013). The epistemological resources act as control mechanisms for focusing one's attention so that appropriate p-prims may be activated. In this case, the epistemological resources that were activated by consulting the diagram encouraged activation of the p-prim *something's changing* and activation of the epistemological resource *mechanistic reasoning* encouraged activation of the p-prim *roughness.*

VI. Implications for Instruction

The link between p-prims, physics based conceptual resources, and epistemological resources suggests that physics educators should design instruction so that students have an opportunity to have all three types of resources activated at once. This finding suggests that unless we give students the chance to reconcile how their understanding of the world can be used to help them decide what more sophisticated physics ideas they should be using, they will be less likely to make a conceptual breakthrough on their own. In order to do this, students must also be aware of the need to reconcile their intuitive understanding of the world with more formal physics ideas. Helping students develop such an awareness is not possible without focusing on epistemological issues as a part of instruction (Elby, 2001). Therefore, effective instruction must incorporate epistemological goals. Furthermore, the need to distinguish between physics critical events and non-physics critical events and the fact that there may be important non-physics resources implies that if we only focus on students developing their physics resources, they might be unable to become expert problem solvers because they will lack the resources to help them solve problems that require the use of math or engineering resources. Therefore future work lies in the area of extending resource work to mathematics and engineering.

Chapter 3

What Epistemological Resources do Physics Experts Use when Solving Novel Problems?

I. Introduction

The question of what constitutes physics expertise is one of the unresolved PER problems (Maloney, 2011). While there have been studies of expert problem solving in physics (Chi, 1981), in many of them experts solve introductory problems. Such tasks are usually easy for experts and although they can be used to study routine expertise, they cannot capture the true nature of a physicist's expertise. A better model of expertise is a two dimensional model in which one dimension is occupied by an expert's capacity to deal with familiar situations, and the second dimension accounts for their ability to consistently and effectively adapt to unfamiliar situations. This dimension is known as adaptive expertise (Hatano & Inigaki, 1986; Schwartz, Bransford, & Sears, 2005). This model helps us understand both why physicists are so good at solving traditional physics problems and how they are able to carry out research and construct new knowledge.

To understand what aspects of a physics expert's adaptive expertise enables him or her to gather more knowledge and reach understanding from this knowledge, we need to study experts in challenging, novel situations. While the most obvious way to study the cognitive processes of experts solving authentic problems would be to observe them in their own laboratories, we can also place experts in an educational laboratory and ask them to tackle science problems purposefully designed to challenge them (Dunbar, 2000). There have been several studies which do place physics experts in contextually relevant, challenging situations, such as Singh (2002), Kohl and Finkelstein (2008), and Kustusch et al. (2014).

In the study by Kohl and Finkelstein, physics graduate students were asked to answer several freshman level physics questions, one of which had a twist that made it especially challenging and proved such, as only three out of five graduate students were able to answer it correctly. However, the challenging problem for experts was not the main focus of the study and the authors did not provide in depth detail of the problem solving process but only presented a final analysis of the types of behavior the experts were engaged in.

In Singh's study, physics faculty members were asked to think about a non-intuitive introductory level physics problem. The task proved to be challenging for experts and none of them indicated that they knew what the answer should be offhand or had solved a similar problem before. Many of them stated a first impression of the problem that would lead them to an incorrect answer. However, the professors were only asked to think about the problem they were given and were not required to provide a full answer. While the professors' initial reactions may show how adaptive expertise manifests itself in the early stages of problem solving, this does not cover the entire problem solving process and does not give us insight adaptive expertise.

More recently, Kustusch et al. (2014) studied the reasoning of 10 faculty members with backgrounds primarily in physics as they solved a challenging problem in thermodynamics. Only half of the experts were successful. Kustusch et al. focused on the use of the mathematics of partial derivatives in thermodynamics. While such focused analysis is important for understanding the contextual details of how physicists reason, studies have shown that some aspects of expertise are domain general (Schunn & Anderson, 1999). We believe that it is important to look for these cross-cutting, domain general features of expert reasoning as well as domain specific features. Doing so requires a wider lensed analysis.

Previously, we have explored physics experts' adaptive expertise by analyzing the different types of resources they used while solving challenging, novel experimental physics problems. We found that when the physics experts made a conceptual breakthrough, 88% of the time their reasoning contained evidence of three different types of resources: p-prims (diSessa, 1993), higher level physics based conceptual resources (Hammer, 2000), and epistemological resources (Hammer & Elby, 2002). In this paper, we continue to explore physics experts' adaptive expertise by focusing on analyzing the different epistemological resources that experts use as they solve challenging physics problems. To do this, we code transcripts of the problem solving sessions for epistemological resources using a fine-grained, resources-based framework (Hammer, 2000; Hammer & Elby, 2002). The integrity of our results will depend on reliable coding schemes for epistemological resources, an issue that has received little attention and is relatively unexplored within the resources framework.

Typically, data analysis using the resources framework involves extended qualitative argumentation to justify the identification of a small number of resources or to analyze how a small number of resources are influenced by contextual details. For example, Harrer, Flood, and Wittmann (2013) dedicated an entire short paper to arguing for three resources and a paper by Louca et al., (2004) shows how a teachers' epistemologically based intervention helps students transition from using teleological and anthropomorphic resources to mechanistic resources. While these types of analyses are important, they will not help us to reliably draw out patterns from large sets of qualitative data. In order to identify reliable patterns, we must stray from extended qualitative argumentation and develop a different method of data analysis relying on a more quantitative approach.

This requires using methods similar to those described by Chi (1997) for reliably codifying and quantifying qualitative data. These methods have not been used often in the resources framework and have been met with mixed success. Scherr and Hammer (2007) developed codes for identifying epistemic frames and were able to achieve 90% inter-rater agreement. However, the epistemic frames that they identified were strongly linked to very obvious behavioral clusters such as focusing on a worksheet, discussing the worksheet with a group, talking to the TA, and joking. Another study performed by Bing and Redish (2009) attempted to reliably identify students' epistemic frames. These epistemic frames were linked to less obvious behavioral cues related to making a calculation, discussing similarity between mathematical and physical arguments, invoking authoritative rules, or searching for mathematical consistency. When attempting to code these less obvious cues, Bing and Redish were only able to achieve 70%

reliability and discuss how the “fuzziness” of human cognition makes achieving high reliability difficult.

By codifying and examining specific epistemological resources that physics experts use during key moments in the problem solving sessions, we answer the following research questions:

1. How can we reliably identify different epistemological resources over large sets of qualitative data?
2. To what extent do physics experts rely on different epistemological resources while solving novel, challenging physics problems?
3. What are the most commonly used epistemological resources?

II. Theoretical Background

In this section we overview the resource-based model of cognition and situate this study within the framework.

II.1 The Resources Framework

The resources framework (Hammer, 2000) models cognition at a fine grained level and is aligned with a broader “knowledge in pieces” philosophy (Redish, 2004). In this model, individuals form complex ideas by assembling small-scale pieces of knowledge, called resources, into larger scale ideas or “conceptions.” By using this framework, researchers can better understand the contextual details of student reasoning and identify pieces of knowledge that students use across contexts.

For example, when students reason about pushing a box along a floor at constant velocity, they may think that they should exert a constant force to maintain constant velocity. Rather than treating this constant force/constant velocity idea as a fundamental knowledge element, we can consider the resources that the student might have used to build this idea (Hammer, 1996). When a student thinks about an object moving across a surface, they will remember many times when objects slide to a stop, activating the resource "dying away." Students could then reason that to counteract this "dying away" they need to continually push the object, activating the resource "maintaining agency." Finally, they might consider that if you push the box too hard or too soft the box will not move at a constant velocity, activating a number of resources, such as "equilibrium" and "something's constant." The student would then piece together "dying away," "maintaining agency," "equilibrium," and "something's constant" to form the larger scale idea that constant velocity motion requires a constant force.

The "resource" has several important, defining features. First, a resource captures knowledge which an individual gains by abstracting common features from previous experiences. An individual uses this knowledge when they try to understand new experiences. Because resources capture knowledge that has been gained by generalizing prior experiences, a resource is not right or wrong, but is applicable or inapplicable in different situations (Hammer, 2000).

A second feature of resources is that they can form larger cognitive structures (diSessa, 1993; diSessa & Sherin, 1998; Wittmann, 2006). Resources that are used together become linked, or coordinated. The strengths of these connections can grow and fade. As connections between resources grow stronger, resources become very strongly linked so that activation of one resource guarantees activation of strongly linked resources. A cluster of strongly linked resources acts as a single entity, called a coordination class, and is treated as a larger grained resource. For example, Wittmann (2001) describes students using a variety of resources to reason about the physics of waves, such as "bouncing" and "cancelling" to describe interacting waves and "smaller is faster" or "working harder" to describe the motion of waves. These resources coordinate to form the coordination class "waves as objects." This highlights a third feature of resources. Resources are not restricted to a particular cognitive grain size (Conlin, Gupta, & Hammer, 2010). In some cases it is more productive to focus on specific resources that make up a coordination class while in others it may be more productive to think about the coordination class as a whole.

A fourth feature of resources is that they are context dependent. While an individual has resources at all times, the resources are not actively used to reason until the individual is in a context which activates those resources. This means that an individual's reasoning is also determined by the context they find resources relevant in, rather than just the resources they have. A consequence is that even if an individual has all the resources they need to understand a physics problem, they may not be successful at solving the problem if they don't think the appropriate resources are contextually applicable. One example, provided by Lising and Elby (2004), is the case study of a student Jan. When Jan is learning about the formation of shadows, she disregards common sense explanations in favor of more technical sounding explanations. She justifies this to her classmates, saying that she is trying to make her explanations more "physics-oriented." In this case, Jan thinks the physics classroom is a place where technical sounding resources are preferred over common sense resources.

II.2 Epistemological Resources

The resources model also helps researchers better understand the nature and content of students' epistemological knowledge, the knowledge students use to understand the nature of knowledge in physics and how to construct new knowledge in physics. Epistemological knowledge answers questions like: how is physics knowledge learned, where does physics knowledge come from, what is the structure of physics knowledge, and what is the content of physics knowledge, among many others (Hammer, 1994). Early models of epistemological knowledge identified developmental stages of epistemological sophistication (Perry, 1970). Later models, such as Hofer and Pintrich's model (1997), dissected an individual's epistemological knowledge into dimensions that an individual could progress or regress along, such as the simplicity of knowledge and justifications for knowing. These models represent an individual's epistemological knowledge as fully formed theories, indicative of monolithic, coherent cognitive structures at odds with a "knowledge in pieces" framework.

A "knowledge in pieces" epistemological framework was developed by Hammer and Elby (2002), building on the fundamental idea of Hammer's (2000) resource based model of cognition that knowledge systems are made up of fine-grained cognitive structures. In their model, the smallest unit of epistemological knowledge is called an epistemological resource, best described as an element of common sense with epistemological implications. An example of an epistemological resource that Hammer and Elby give is "knowledge as propagated stuff." This is a resource abstracted from situations where an individual receives knowledge from a source. Such situations occur when a teacher tells their students a definition, an individual gathers facts from a book, or a parent tells a child what time their grandparents will be visiting.

Each epistemological resource is neither right nor wrong, but appropriate or inappropriate depending on the context. Consider “knowledge as propagated stuff.” It would be inappropriate for a student to believe they could fully understand Newton’s Laws by listening to their teacher talking. In this case “knowledge as propagated stuff” would be inappropriate. However, a student who forgot what Newton’s Laws are might consult their teacher or their textbook for a definition. In this case “knowledge as propagated stuff” is appropriate. Epistemological growth happens by learning when specific resources should be used and what resources should be used together.

The epistemological resource framework has successfully predicted and explained many aspects of student reasoning in the physics classroom and beyond. Hammer, Elby, Scherr, and Redish (2004) used the epistemological resources framework to explain why we seldom see successful knowledge transfer. Lising and Elby (2005) used the model to show a causal link between a student’s epistemological stance and their learning outcomes. Louca, Elby, Hammer, and Kagey (2004) demonstrated how teachers can use the model to positively influence students’ reasoning. Furthermore, Bing and Redish (2007) examined the type of resources students use when they “get stuck,” while Tuminaro and Redish (2007) showed how epistemological resources can produce student behavioral patterns in the classroom.

II.2.a Development in the Epistemological Resource Framework

Many of the accomplishments of the epistemological resource framework have resulted from development of the details of the model itself. When Hammer et al. (2004) developed an alternative explanation to classical transfer, they did so by developing the idea of epistemological framing. As explained by Hammer et al., epistemological framing happens when a person is in a situation, like sitting in a classroom, and asks themself, "What epistemological knowledge is useful in this situation?" This is not necessarily done explicitly, and often happens subconsciously. When a person asks themself this question, epistemological resources are activated. What resources are activated depends on what resources that person has found useful in similar situations. These resources, activated together, form connections. Over time, if someone encounters the same situation repeatedly and activates the same resources consistently, the resources will form strong connections and become a locally coherent epistemological frame. In this way, Hammer et al. applied a general rule about resources, they activate in sets that can become a resource itself when the connections are reinforced, to the epistemological resources framework to develop epistemological frames. The epistemological frame which students activate determines what activities they engage in during learning opportunities.

Examples of epistemological frames were found by Scherr and Hammer (2007). When they investigated the behavior of students in introductory physics labs, they found that students primarily switched between a "worksheet frame," a "discussion frame," and a "TA frame" during the lab. These frames showed consistent evidence of epistemologically significant behaviors. In the worksheet frame the students focused on their worksheets, interacted minimally with their peers, and did not show much evidence of original thought. In the discussion frame students interacted with their peers, showed evidence of original thought, and engaged in mechanistic thinking. In the TA frame, the students focused on the TA and listened to the TA or reiterated what they had discussed as a group.

An individual develops expertise by constructing sophisticated networks of epistemological resources, such as epistemic frames, and learning the contextual appropriateness of specific resources. diSessa (1993) describes that expertise is attained in a knowledge in pieces framework by reorganizing and prioritizing existing pieces of intuitive knowledge and the connections between them. As these pieces of knowledge form a more systematic, coherent knowledge system, more complex knowledge structures emerge. He terms this integration of intuitive knowledge pieces into a more complex system "distributed encoding." When experts form epistemic frames and other sophisticated networks of epistemological resources they achieve distributed encoding. While much has been said about what expertise should look like, very few studies have actually focused on analyzing experts from a resource based framework and no studies have been done that focus on the epistemological resources used by experts as they construct new knowledge. Understanding what expertise is requires that we work to identify these more sophisticated epistemological networks and processes that experts use to help them construct new knowledge.

III. Study Design

The study that we designed to investigate the epistemological resources physics experts use was a laboratory type study in which we analyzed videotaped problem solving sessions. During these sessions, post-qualifying physics graduate students, physics post docs, and physics professors served as experts and worked in pairs as they solved one of two challenging, novel physics problems. In total, six pairs of experts participated in this study and were divided evenly among the two physics problems. We chose a small sample size to allow us to gain an in depth understanding of each problem solving session (Cresswell, 2007). Since we were interested in the thought processes of the physics experts, we instructed each pair of experts to think aloud so their thinking would be more transparent (Ericsson & Simon, 1980). During problem solving sessions, the interviewers took on a passive role, interjecting only to remind the problem solvers to think aloud and answer clarifying questions. This was done to capture more natural thought processes of the physics experts. Problems solvers were not informed if they had come up with a sufficient answer, but instead were asked to come up with an answer which they were satisfied with. After the experts were satisfied with or felt that they could go no further, the interviewers asked clarifying questions. All problem solving sessions were videotaped in full.

III.1. Ill-Structured vs. Puzzle Problems

One aspect of the study that was very important was selecting proper problems for the experts to solve. These problems needed to be designed so that the experts would utilize their adaptive expertise to solve the problem, rather than their routine expertise. It was important that the problem we gave our experts would be significantly challenging, require the experts to develop an understanding of the problem they were solving, and would not be solvable through the application of a well-known procedure. Essentially, we wanted our experts to solve ill-structured problems and not puzzle type problems (Churchman, 1971; Mitroff & Sagasti, 1973; Kithchener, 1983). While the distinction between puzzles and ill-structured problems has been discussed in many places, the description we give most closely follows that as described by Kitchener.

Puzzles are problems for which there is a single solution that can be reached by mechanical application of a deductive algorithm. To solve a puzzle a problem solver does not need to gather additional data or make considerations about the applicability of data, the validity of assumptions, or multiple ways to interpret data. Examples of puzzles are number games, such as Sudoku and traditional physics textbook problems that can be solved by algorithmic application of an equation. Becoming adept at solving puzzle problems means becoming proficient at recognizing when algorithms apply and applying them efficiently. These are the skills captured by routine expertise. While efficiency and routine expertise are important, it was not what we wanted to focus on.

On the other hand, ill-structured problems are problems for which a solution cannot be immediately obtained. In some cases there is no one correct solution. The solution of an ill-structured problem may depend on varying ways to interpret available data and assumptions that can be made. These problems require gathering, synthesizing, and evaluating data while generating multiple different solutions and determining the strengths and weaknesses of those solutions. Ill-structured problems are more aligned with research problems that practicing physicists deal with. Becoming adept at ill-structured problems requires negotiating a myriad of higher level skills simultaneously. The types of skills required to be successful at solving ill-structured problems overlap with the skills captured by the adaptive dimension of expertise.

To make sure we chose ill-structured problems, we picked two problems which were not immediately solvable by algorithmic means, required the experts to gather and analyze data, and allowed the experts to analyze the data through several alternative possible solution paths. During the problem solving sessions, we saw sufficient evidence that the problems we had picked met these criteria.

III.2 The Light Cone Problem

The first problem we chose was a "light cone problem" (Etkina, Planinšič, & Vollmer, 2013). Since this problem is described in full elsewhere, we will only describe it briefly here. In the light cone problem, a clear container constructed of either plastic or glass is placed on a white piece of paper. The container is filled with water mixed with a few drops of mix. A green laser is shone into the water from above so that it strikes the bottom of the container. What you observe when you do this is a cone of laser light which has a vertex at the point where the laser hits the bottom of the container. A picture of the experiment is shown below in Figure 3.1. The task we give our experts is to develop an explanation for their observations.

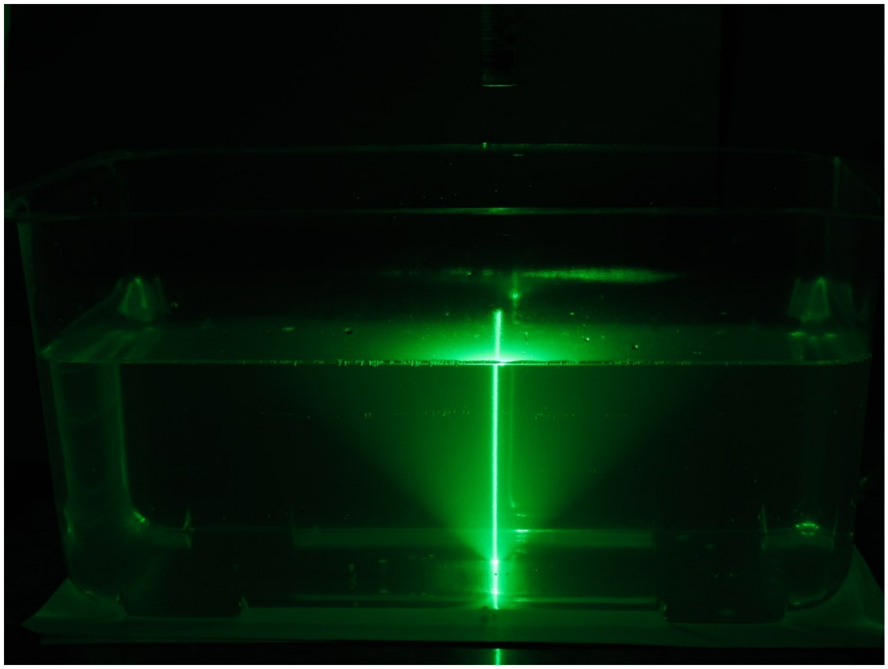


Figure 3.1 A picture of the light cone experiment.

III.3 The Solar Cell Problem

The second problem was about the physics of solar cells (Jones et al., 2013). The problem was about identifying how a solar cell operates in a circuit and how the output of a solar cell is dependent on illumination. We recruited physics experts with experience in solar cell physics to solve this problem. The problem had two parts. At the start of the problem solving session each pair of experts was given a handout for part 1 which explained an experiment and the measurements that were taken, shown to the left in Figure 3.2. In this experiment, a rectangular solar cell was connected in series to an ammeter and a resistor. A white light was turned on and brought near the solar cell so that the solar cell was illuminated. The light was fixed in place and we recorded a current measurement. We took a black piece of paper and began covering the solar cell. We first covered the solar cell from the long side. When the solar cell was 25%, 50%, 75%, and

100% covered we measured the current. We repeated the same steps but covered the solar cell from the short side. When we had current measurements for the long side and the short side, we plotted them together on a graph. We asked the experts to explain why the graphs (shown in Figure 3.2) looked the way that they did. We gave the experts the equipment that we used to perform the experiment and told them that if they wanted any other equipment they should ask us. If we had the equipment available, then we would give it to them.

After the experts had developed an explanation for part 1 that they were satisfied with, they were given another handout describing part 2, as shown to the right in Figure 3.2. In part 2 the physics experts had to make predictions for another experiment and explain the reasoning behind their predictions. In the first part of this experiment, two solar cells were connected to each other in series so that the positive lead of one solar cell was connected to the negative lead of the second solar cell and the positive lead of the second solar cell was connected to an ammeter which was in series with a resistor that was connected to the negative lead of the first solar cell. In this way, all four circuit elements were connected in series. This connection was referred to as the “series” connection. The two solar cells were placed side by side and then illuminated with the white light that was used in part 1. A black piece of paper was used to cover the solar cells and generate the set of scenarios depicted in the table in Figure 3.2. In the second part of this experiment, the polarity of the second solar cell was reversed, such that the negative lead of the first solar cell was now connected to the negative lead of the second solar cell and the positive leads were connected through the series connection which incorporated the resistor and ammeter. This connection was referred to as the “anti-series" connection. The black paper was again used to generate the same covering scenarios that were generated in the first part of this experiment. The experts' goal for part 2 was to develop predictions for the current measured by the ammeter in each covering scenario. After the experts developed predictions and explained the rationale behind each prediction, they were given the results of the experiment and asked to reconcile any discrepancies between the results and their predictions. Once they completed their reconciliations, the interviewers asked questions to clarify anything that was unclear.

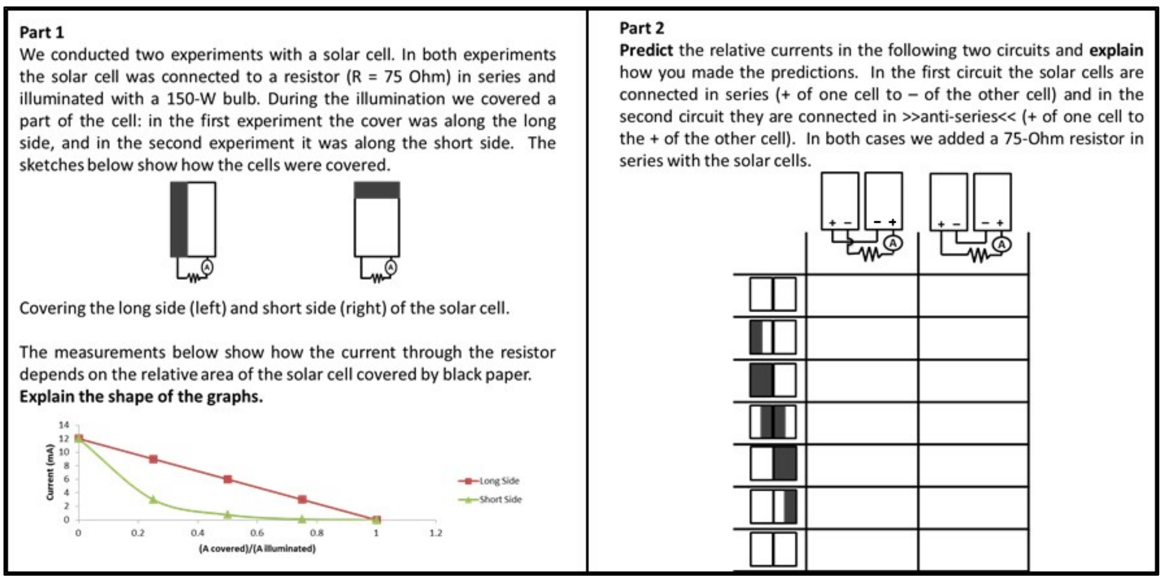


Figure 3.2 Handouts given to experts who solved the solar cell problem.

IV. Data Analysis

The major goal of our study was to answer, "What makes experts so good at solving challenging problems that they have never seen before?" To answer this question with the data we gathered, we adopted a phenomenological approach (Moustakas, 1994; Creswell, 2007). This is appropriate because we are essentially attempting to answer the question, “What does it mean to gain understanding of a problem to a physicist?” from an epistemological resources perspective. To do this, we first engage in *horizontilization* of the data. This process has two parts, described below. First, we look through problem solving sessions and pick out moments when the experts make significant progress towards developing an understanding of the problem. By analyzing the contents of these moments, we can gain insight into the thought processes of the physics experts as they build an understanding of the problem. To determine what is important about an expert’s thought processes during these moments, we broke down each moment and identified clusters of meaning (Moustakas, 1994) within the transcript. We analyzed these clusters using a resource based analysis and identified epistemological resources used by the physics experts. By looking for common epistemological resources across the different groups of physicists we describe the essence of problem solving from an epistemological resources perspective.

IV.1 Critical Events

To begin the analysis process the videotapes were transcribed in full and the problem solving sessions were watched repeatedly. This helped us develop a refined description of what took place during each problem solving session (Lesh & Lehrer, 2000). After we developed descriptions of the problem sessions, we narrowed the focus of our analysis by identifying moments during the problem solving process which we felt deserved deeper investigation (Lesh & Lehrer, 2000; Powell, Francisco, & Maher, 2003; Derry et al., 2010). The moments that we chose most closely resemble the definition given by Powell, Francisco, and Maher of “critical moments” or “critical events.” A critical event is a moment during the problem solving session where the problems solvers reason to a conceptual breakthrough. This leads to reasoning that differs from reasoning prior to this moment. These critical moments exemplify a moment where the problem solvers make significant progress towards developing a full understanding of the problem.

By engaging in collaborative discussions during preliminary analysis of candidate critical events we developed a shared set of criteria for determining if a portion of the text should be classified as a critical event (Jordan & Henderson, 1995). First, a critical event is a moment during the problem solving process when a breakthrough occurs, during which time the problem solvers identify or answer important questions about the problem for the first time. This criterion meant that critical events captured moments when lines of inquiry were either initiated or ended. Second, the breakthrough must happen through reasoning. This means that breakthroughs which resulted from chance observations are not considered critical moments. Finally, critical events are short pieces of the transcript which concisely capture complete exchanges between problem solvers or reflections of an individual. To identify critical events, one of the researchers, Darrick Jones, first selected portions of the transcript that captured complete dialogue during which it seemed important breakthroughs were occurring. Once these candidates were identified the same researcher determined what question the problem solvers were answering during the event. If the question was important to developing a deeper understanding of the problem at hand and would require reasoning to answer, then the event was deemed critical. Using this process, 146 critical events were identified. In a similar manner 51 non-critical events were classified.

IV.1.a Establishing Reliability of Critical Events

After events had been classified as either critical or non-critical events, an outside researcher, familiar with the light cone problem but not involved in any other aspect of the study, was brought in to establish the reliability of the classifications. To do this, the outside coder was provided with the list of questions which embodied the substance of each critical event so that they could determine whether or not answering each question would meet the above criteria for a critical event. They were also given the text from the critical events to consult when the questions were unclear. After training the researcher on a few example critical events, the outside researcher independently classified 25% of the events. The primary coder and the outside coder achieved an 82% agreement (k = 0.6) before discussion. After discussion, the primary and secondary coder agreed on 96% of the events. It was clear from the discussion that much of the disagreement arose from situations where questions remained unresolved in the critical event.

IV.1.b Example of Critical vs. Non-Critical Event

To help show how we classified critical events, we present an example of a critical event and a non-critical event in Table 3.1. In the non-critical event, two of the physics experts, C and R, are solving the light cone problem. Up to this point, C and R had performed the light cone experiment themselves, performed slight variants of the experiment, and observed the effects. At this point, C and R noticed that there was a small piece of cardboard under the white paper beneath the experiment. They decided to see what happened when they performed the light cone experiment with the piece of cardboard directly beneath the container instead of the white paper. Their primary observation during this event was that there was no light cone when the experiment was performed without the white paper, there was only a specular reflection. During this event they make observations and for the first time during the problem solving session provide an answer to the question “What is observed when the laser beam hits the white paper versus the cardboard?”

The critical event that we selected focuses on C and R later during problem solving session. Prior to the excerpt, C and R created a diagram to represent reflections of the laser beam off the white paper beneath the container of water as if the paper were a mirror. Just before the excerpt, R noticed that light doesn’t reflect off paper in this way. In this excerpt C realizes that their diagram fails to incorporate this and tries to explain why paper might cause light to reflect differently than a mirror. This critical event represents the first time where C and R answer the question “What mechanism causes the paper to scatter light in all directions?”

In the non-critical event, the primary breakthrough is attributed to C and R answering “what is observed when the laser beam hits the white paper versus the cardboard?” This is an important moment in the problem solving process because it provides the experts with a scenario during which the light cone doesn’t appear. By drawing comparisons between scenarios when the cone does appear and when it doesn’t appear, the experts begin to develop ideas about the nature of the cone. They answer this question by performing experiments and noting observations that they made. There is no evidence of reasoning. Even though this is a breakthrough, it is not a critical event because the experts do not reason to this breakthrough. Contrarily, when C and R answer the question “What mechanism causes the paper to scatter light in all directions?” they ask themselves "What happens when the laser hits the paper?" and then discuss how the beam spreads out, combine this observation with the idea that the paper is "optically rough," and are able to determine that the laser scatters when it hits the paper. The fact that C and R make a breakthrough in this event through a line of reasoning and not just observations defines this excerpt as a critical event.

Table 3.1 Example Critical Event and Non-Critical Event

Question being answered What is observed when the laser beam hits white paper versus cardboard?	**Question being answered** What mechanism causes the paper to scatter light in all directions?
Non-Critical Event R: So does it, do you the same thing if you don't shine it onto the paper? (C moves laser beam off of the paper) R: No. C: No we do not. R: You still get that kind of uh, specular beam reflection. I don't know if you can see that from where you are. Cause from certain – Yeah, uh, you definitely get a just a straight line reflection. C: Oh yeah I see. I have to go on an angle. R: But you don't get the cone away from the paper. C: Yep. I agree. Ok. So off the paper we have the weak specular reflection, so then if we…(directs laser beam onto the paper) R: But I think you still get that on the paper too if you hold it at an angle. C: Specular? Yeah. R: Yeah.	**Critical Event** C: Yeah. What this diagram is ignoring though is what happens at the paper. Right? R: Yeah. C: What happens when the laser hits the paper? We don't – do we get – we don't get the exact same beam coming back, right? Cause that's what you were trying to look at before, I think. Just sort of looking at what – R: Yeah. Somehow it spreads out in a very well defined way. Uh…in the paper. C: Ok, so what is this paper doing? Beam hits it, the paper is – the paper is what? Optically rough? Is that a term we can use? C and R: Haha. C: Why not? R: It's not a mirror. C: That's true. I'll point it away from you. R: Heheh. C: So the laser hits the paper and has to be diffused somewhere, right? It has to…it scatters. I mean if we – The paper is definitely rough.

IV.2 Coding for Epistemological Resources

Once critical events were extracted from the text, we determined the content of experts' reasoning during critical events by analyzing each event from a resource-based perspective. This involved first inductively identifying potential resource candidates, developing criteria for reliably identifying the most common resource candidates, and finally deductively identifying resources using the criteria developed. To begin we analyzed each critical event to determine the pieces of knowledge that were being used to reason in each critical event, as has been done in other previous studies of a similar nature (Richards, 2013). After discussing sample sets of data, the following method for coding potential resources was established:

1. Describe the critical event without making explicit mention of any resources.
2. Identify key components of the event.
3. Determine if these components have evidence of knowledge that is being used to reason.
4. Break down individual components into the smallest pieces of knowledge for which there is evidence.
5. Indicate where the text most clearly shows evidence of this piece of knowledge.
    - If it is unclear that there is strong enough evidence of this piece of knowledge being actively used, consult the text around the event.
6. Indicate if the piece of knowledge is a conceptual resource or epistemological resource.
7. Classify resources by giving them names according to previously identified resources in the literature if possible.
8. If there is strong evidence that a resource is being used, but there is no common name in the literature or it has not been previously identified, give it a tentative name and a working description.
9. Compile a running list of resources that have been identified in the critical events. Use this list as a reference to maintain consistency.

As we coded critical events, our understanding of various resources evolved. To maintain consistency, we included working definitions of resources we had identified. These definitions were refined as we coded critical events.

IV.2.a Example of a Coded Critical Event

An example of a critical event coded for resources is shown in Table 3.2. In this excerpt, green text and yellow highlighting indicates evidence of an epistemological resource, while red indicates a conceptual resource. Highlighting is used to show evidence for multiple resources when the evidence overlaps. We present a more detailed explanation for why we coded this example for certain resources in Appendix A.

Table 3.2 Example of critical event coded for resources

C: Yeah. What this diagram (1) is ignoring though is what happens at the paper. Right? (2,3,4) R: Yeah. C: What happens when the laser hits the paper? (5) We don't – do we get – we don't get the exact same beam coming back, right? (6) Cause that's what you were trying to look at before, I think. (7) Just sort of looking at what (8,9,10) – R: Yeah. Somehow it spreads out (11) in a very well defined way. (12) Uh…in the paper. C: Ok, so what is this paper doing (13)? Beam hits it, the paper is – the paper is what? Optically rough (14)? Is that a term we can use? C and R: Haha. C: Why not? R: It's not a mirror. (15,16) C: That's true. I'll point it away from you. R: Heheh. C: So the laser hits the paper and has to be diffused (17) somewhere, right? It has to…it scatters (18). I mean if we – The paper is definitely rough (19).	1. Multiple Representations 2. Contrasting Cases 3. Hypothetico-deductive Reasoning 4. Limitations of Models 5. Causal Reasoning 6. Something's Changing 7. Peer Cognitive Awareness 8. Experimentation 9. Inductive Reasoning 10. Knowledge from direct perception 11. Spreading 12. Concentration and/or localization 13. Mechanistic Reasoning 14. Roughness 15. Analogical Reasoning 16. Mirrors 17. Spreading 18. Scattering 19. Roughness

Once all critical events were coded the way described above, we adopted a more quantitative approach. We gave each epistemological resource a code (Chi, 1997). In total, 46 different epistemological resources were identified. To begin to determine whether or not there were any patterns in the epistemological resources used by the physics experts and focus our analysis, we counted the number of critical events in which each epistemological resource was found. We found that while experts used a significant number of epistemological resources, only 19 were found in at least 5% of all critical events. While these were only preliminary results, we elected to focus our analysis on these 19 most common resources and analyze them in more detail.

IV.2.b Establishing Reliability and Recoding Most Common Epistemological Resources

We developed criteria for reliably coding these 19 most common epistemological resources. Critical events that showed evidence of a code were analyzed together in order to determine what about a critical event classified it as a specific code. By doing this, we developed criteria that served as guidelines to help coders reliably classify a critical event as showing evidence of a particular code. Once the criteria were developed, the primary coder, Darrick Jones, who developed the criteria, and a secondary coder coded critical events for reliability. This served not only to help determine whether or not the criteria were reliable, but also helped determine whether or not the criteria adequately captured what the primary coder wanted. As a preliminary test of the criteria and to help train the secondary coder, the primary and secondary coders coded 10% of all critical events for a single code at a time. After, the coders compared answers and discussed their selections.

Based on these discussions and the agreement between the two coders, the codes were either refined or left as they were. After the criteria were refined, the coders individually coded another 20% of all critical events for each resource. Once the 20% had been coded, the coders discussed disagreements and made final alterations to the criteria only if there were systematic disagreements. Reliability established in this way is shown in Table 3.3.

Table 3.3 Epistemological Resources Used by Physics Experts and Accompanying Inter-Rater Reliability Statistics

Epistemological Resource	Percent Agreement	Cohen's Kappa
Analogical Reasoning	0.80	0.69
Attention to Novelty	0.77	0.59
Causal Reasoning	0.83	0.67
Consistency	0.77	0.57
Contrasting Cases	0.80	0.62
Deductive Reasoning	0.90	0.80
Experimentation	0.97	0.96
Hypothetico-Deductive Reasoning	0.83	0.77
Inductive Reasoning	0.77	0.65
Knowledge From Direct Observation	0.80	0.61
Limitations of Models	0.77	0.58
Mathematical Reasoning	0.83	0.67
Mechanistic Reasoning	0.87	0.73
Multiple Representations	0.87	0.76
Peer Cognitive Awareness	0.87	0.74
Personal Cognitive Awareness	0.80	0.60
Plausibility	0.93	0.92
Relative Value of Knowledge	0.90	0.87
Sense Making	0.83	0.68

V. Findings

After reliability was established, the primary coder re-coded all critical events based on the criteria developed as described in the previous section. Once all critical events were coded, the resources were again counted so that we could determine the percentage of critical events each resource was found in. The results of the tally are shown in the graph in Figure 3.3. Five epistemological resources were found in less than 20% of all critical events. Two epistemological resources were found in 20-30% of all critical events, 5 in 30-40%, 4 in 30-45%, one in 50-60%, and two in the 60-70% range. Overall, the most commonly used epistemological resource, contrasting cases, was found in 67% of all critical events, with causal reasoning coming in a close second at 64%. We found that the epistemological resource that was used most commonly by physics experts during these challenging, novel problem solving processes was the "contrasting cases" resource. In a companion paper we will further analyze the details surrounding physics experts' use of contrasting cases.

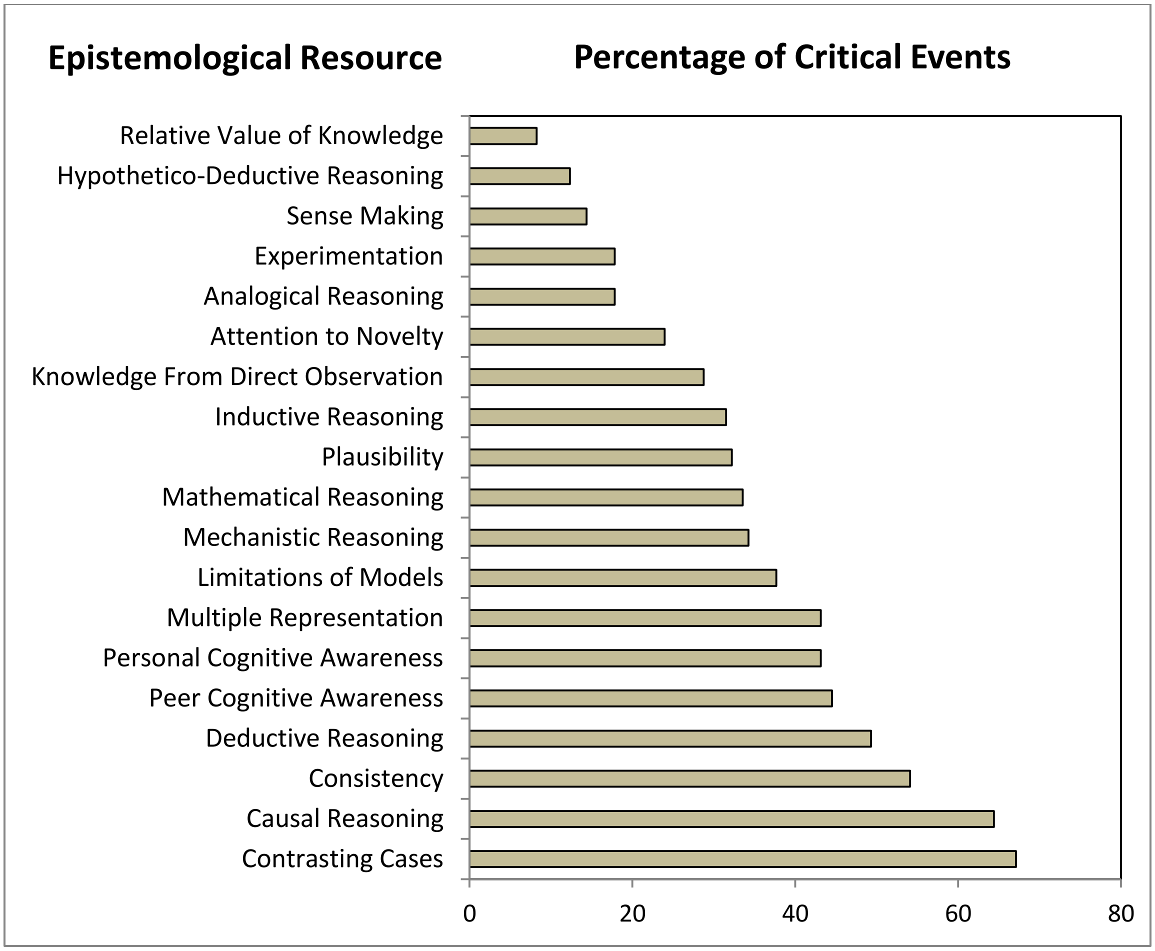


Figure 3.3 Bar graph showing the percentage of critical events in which each of the 19 most common epistemological resources were found

VI. Common Epistemological Resources

In this section, we give detailed descriptions of the 6 most common codes (after recoding) and the criteria we established for identifying them. These resources are (starting with the most common): contrasting cases, causal reasoning, consistency, deductive reasoning, peer cognitive awareness, and personal cognitive awareness. Descriptions and criteria for the other 13 codes can be found in Appendix B.

VI.1 Contrasting Cases

We coded a critical event for evidence of contrasting cases if we found that experts were drawing on the similarities and differences between two or more situations and/or ideas. This resource borrows its name and core idea from the successful interventions developed by Bransford and Schwartz (1999). In order for a critical event to be coded for evidence of contrasting cases it needed to satisfy all the following criteria:

1. Either:
    a. A primary case and secondary case can be identified.
    b. A continuous set of cases can be identified.
2. Evidence that there is a comparison between the primary and secondary cases.

Below is a critical event that was coded for contrasting cases. In this critical event, experts C and R are trying to solve the light cone problem. Prior to this event, C and R have performed the light cone experiment on white paper, cardboard, a brown wooden table, and a piece of plastic. They noticed that there is a difference between these situations and the light cone is only visibly present when white paper is used. In this critical event, C and R reflect on these experiments.

> C: Ok. Ok, so reflected laser beam. So when it [the laser] hits the white paper there should be more reflection, right?
> R: Yeah.
> C: Than if it was just hitting the plastic or if it was just hitting the cardboard or the table.
> R: Yeah. Which makes sense. I guess it's just cause it's a white piece of paper. That's reflecting more.
> C: So then is this cone, the cone is due primarily to the reflected beam…hmm.

Evidence of C and R using contrasting cases is most clearly identifiable when C says "So when it [the laser] hits the white paper there should be more reflection…than if it was just hitting the plastic or if it was just hitting the cardboard or the table." In this statement C first references a situation where the light cone experiment is performed on a white piece of paper when he says, "…it hits the white paper…" This serves as the primary case. He later references the variations of the experiment when he says "…it was hitting the plastic or if it was just hitting the cardboard or the table." These act as the secondary case. Between these two statements he says, "…there should be more reflection than…" indicating that he is drawing a comparison between the two situations. Overall, C is drawing a comparison, that there should be more reflection, between two groups of situations, light shining off a white piece of paper and light shining off cardboard, plastic, or the table. This comparison is the final, most important criterion that causes us to classify this critical event as having evidence of contrasting cases.

VI.2 Causal Reasoning

Russ et al. (2008) describe in detail a framework for identifying mechanistic reasoning in student inquiry. In their framework, they differentiate mechanistic reasoning from strictly causal reasoning. They describe causal reasoning as reasoning that identifies the causal agents which lead to later events. Mechanistic reasoning is described as a slightly more complex form of reasoning which not only looks at what is happening, but how it is being brought about. Mechanistic reasoning describes processes that link cause and effect. Causal reasoning is only concerned with identifying causes and effects. Several criteria were developed to aid in the identification of critical events where causal reasoning was present. These criteria were:

1. Identification of an agent(s) (object or phenomenon) responsible for the cause.
2. A phenomenon that is the effect.

3. A consequential link between the two OR that the two are not causally related.
4. Utterance containing the cause should have some kind of action word.

All four criteria need to be satisfied for an event to be coded for causal reasoning. Below is an excerpt from a critical event where we identified causal reasoning. Prior to this excerpt, P has been experimenting with a solar cell to determine if the voltage of the solar cell is linearly related to the area of the solar cell exposed. After P has taken some measurement he says:

P: So, covering half of one cell is not equivalent to reducing its voltage by a half.

In this utterance, "covering half of one cell" acts as the causal agent. "…reducing its voltage by half" is the phenomenon that is the effect. In this critical event, P identifies that these two are not causally related when he says that the two are "not equivalent." Finally, the cause contains the word "covering," which is an action word. An example of a critical event which makes use of mechanistic reasoning is shown in the appendix.

VI.3 Consistency

Bing and Redish (2012) have identified two epistemic frames that are important in the development of expertise in physics. These frames are "physical mapping," which allows people to make and support arguments by emphasizing the coherence between mathematical and physical reasoning, and "mathematical consistency," which allows learners to judge the arguments they make based on their mathematical consistency. The consistency code is motivated by this work, but is a bit more general than the aforementioned frames and focuses on the broader theme of consistency, which underlies these frames. The criteria for this category are as follows.

1. Identification of a model.
2. Identification of a prediction based on that model.
3. Either:
    a. An expression that the prediction is, isn't, or should be met by observations or internal knowledge.
    b. A second prediction based on that model and a check for consistency between the two.

OR

1. An observation.
2. A bid to align model with observation.

This code has two sets of criteria because it arose from the merging of two separate codes from an earlier stage in the analysis. Either set needs to be satisfied in full for a critical event to be coded for consistency. Below is an example of an excerpt from a critical event that we coded for consistency. In this critical event, experts C and R are working on the light cone problem. C discusses an argument they made trying to explain the light cone. R objects to this argument.

> C: You have some point and it's going to emit, scattering and emitting, in all directions to make this halo, right?
> R: Mhmm.
> C: But beyond this crit – whatever, uh, if we say this is the normal – there's some critical angle. So anything beyond that you won't see, these will just get reflected back.
> R: Right, but the angle's very close to that you should get something coming off almost parallel to the –
> C: Oh! I see what you're saying.
> R: To the surface of the plastic. Which we're not seeing.
> C: So you're saying we need to – Yeah. Very very close – if this is the critical angle then you should see something like that –

R: Yeah.
C: That, that, and that. So we should be getting a, uh…
R: We should still get –
C: Hemisphere instead of a cone. That's a good point.

In this excerpt, C starts off by explaining the model they are using. This starts with "You have some point…" and ends with "…these will just get reflected back." R makes a prediction based off that model saying that "angle's very close to that you should get something coming off almost parallel." This leads C to predict that they "should be getting a…hemisphere…" C continues and acknowledges that this is inconsistent with their observations when he completes his statement saying "…instead of a cone."

VI.4 Deductive Reasoning

Deductive reasoning is a form of logic where one begins a line of reasoning from an idea that acts as a premise and reasons logically from this premise to some sort of conclusion. In order for a critical event to be coded for evidence of deductive reasoning, we needed to be able to identify all of the following criteria in the critical event:

1. A statement based on prior knowledge that that acts as a premise.
2. A conclusion derived from that premise.
3. A linguistic link showing evidence that the conclusion is based on the premise.

An example of an excerpt of a critical event that was coded for deductive reasoning is presented below. In this critical event, A and D are working towards solving the first part of the solar cell problem. Previously, they have identified that the solar cells are constructed of several p-n junctions and that covering from one side of the solar cell covers whole p-n junctions one at a time, while covering from the other side partially covers each junction simultaneously. They remembered that a covered p-n junction acts as a resistor and are trying to use this information to help them understand the shape of the graphs that they have been given.

> A: Once we cover up that first one [p-n junction] we've introduced a huge series resistance.
> D: Yeah.
> A: So like a lot of the current's gonna die right away. So like covering up a few more isn't gonna make as big a difference, which is why we see the curve drop very sharply at the beginning but then flatten out a bit.

In this excerpt, A begins by stating his premise, "Once we cover up that first one [p-n junction] we've introduced a huge series resistance." From this he draws a few conclusions: "a lot of current's gonna die right away," "covering up a few more isn't gonna make as big a difference," and finally concludes that this "is why we see the curve drop very sharply at the beginning but flatten out a bit." These are linked to the premise with the phrases "so like" and "which is why."

V.5 Personal Cognitive Awareness

An important aspect of developing expertise is learning to be metacognitive. One aspect of metacognition is an individual's capacity to consciously be aware of their own thought process. This code represents times where we found evidence of this awareness during critical events. In order for a critical event to be classified as having evidence of this code, it needed to satisfy the following criteria.

1. A statement made by an individual where they make reference to what they are thinking (or not understanding) and explicitly reference the personal nature of what they are thinking.

In the critical event excerpt below, experts A and D have just begun working on the first part of the solar cell problem. At this point, they have begun to look at the construction of the solar cell. In this critical event, expert D explains how he typically pictures a solar cell and questions whether or not that is appropriate for this problem.

> D: (Grabs a solar cell) I mean, it looks like...hmm. I'm trying to look at this solar cell to see if I can tell how exactly it's constructed cause it's – when I normally, when I try to think about a solar cell, I usually think about a p-n junction.
> A: Right.
> D: But an entire, like an uh, like an actual unit is more than just one p-n junction, right?

In this excerpt, D says "…when I try to think about a solar cell, I usually think about a p-n junction." This is a reference made by D about what he is thinking. The fact that he uses the first person pronoun "I" tells us that he is explicitly addressing his own personal thought process. As a result, we would code this passage as having evidence of personal cognitive awareness.

V.6 Peer Cognitive Awareness

An important aspect of working in a team is learning to work with others. To capture this, we developed the "peer cognitive awareness" code. In order for a critical event to be coded as having evidence of peer cognitive awareness, it needed to satisfy at least one of the four following criteria.

1. Asking another person for clarification.
2. Asking for another person's input.
3. Making reference to something someone else said and explicitly that they were the one who said it.

4. Trying to clarify or explain something for the other person when they indicate that they are unsure.

Following is a critical event we coded for peer cognitive awareness. In this critical event, experts C and R are nearing the end of the light cone problem. They have already developed the correct solution and they are trying to perform any tests that would make them confident of their answer. Previously, R had come up with a test. Rather than shining the laser beam into the container from above, he tried shining it into the bottom at parallel incidence to see if the laser beam was refracted into the water at the same angle as the outside of the cone. Initially, C was unsure what R was doing, even though R had tried explaining it. In this critical event, C comes to understand what R was trying to explain earlier.

> C: I see what you're saying.
> R: Yeah, this is still the same order of uh –
> C: Yeah. You're just not having the light scatter the same way. Is that pretty much the only difference, right? Air, plastic, water, but you don't have the reflection off the paper.
> R: You're just, you're just not dispersing the light like you do off the paper.
> C: Yeah. Yeah. Yeah I see what you mean. It doesn't go past that particular angle which looks the exact same as the cone angle.

The event begins with C saying "I see what you're saying." He then goes on to restate R's idea when he says "You're just not having light scatter the same way." And then ends with "I see what you mean. It doesn't go past that particular angle which looks the exact same as the cone angle." This evidence is all sufficient to classify this critical event as having evidence of peer cognitive awareness under the third criteria.

VI. Discussion

At the beginning of this paper, we set out to answer three research questions. In this section, we will discuss how we answered each of these questions and the significance of these answers.

VI.1 Reliability of Resource Coding

The first research question we proposed was: How can we reliably identify different epistemological resources over large sets of qualitative data? In this study we showed that we were able to reliably code large amounts of qualitative data for epistemological resources. By using the steps below, we were able to develop and refine criteria that enabled us to achieve at least 77% inter-rater agreement, and often much higher, with an average inter-rater agreement of 84%, for each of the 19 most common epistemological resources. The procedure we used was:

1. We narrowed the range of our data set by identifying moments that we were interested in analyzing.
2. We defined these critical moments, developed criteria for identifying them, and tested these criteria on 25% of all events to determine that they were reliable.
3. We performed a preliminary analysis of these moments by inductively searching for resources using the steps described in section 3.IV.2 and determining which resources were most common at this preliminary stage.
4. In order to perform a more rigorous and reliable analysis, we narrowed the focus of our primary analysis to the 19 most common resources.

5. We developed criteria for deductively identifying each specific resource in the critical events. These criteria were developed in order to both communicate and refine the principle ideas that were being used to code for these resources in the preliminary analysis.
6. Using these criteria, two researchers, both familiar with the solar cell problem that the experts had solved, coded 10% of all critical events for each resource individually to train the secondary coder and determine that our criteria were reliable.
7. After separately coding the 10%, the two coders discussed their selections. If agreement was poor or there was systematic disagreement between the two coders, the codes were refined. Otherwise, they were left as is.
8. After the codes were refined, the coders individually coded another 20% of all critical events to establish reliability for the new criteria.

VI.2 Variety of Epistemological Resources

The second research question we set out to answer was: To what extent do physics experts rely on different epistemological resources while solving novel, challenging physics problems? First, we found that experts rely on a variety of different epistemological resources. In this study, we identified 46 different epistemological resources used by physics experts. This supports the idea that physics expertise is partly achieved by building large amounts of domain-specific knowledge (Chi, 2006). The physics experts have developed a broad range of epistemological knowledge over time and this knowledge comes to bear when they solve challenging problems. Such a high number represents that a variety of epistemological knowledge is useful when solving novel problems.

Second, we found that the experts used some epistemological resources more than others during critical moments of the problem solving process. Specifically, 19 out of the 46 epistemological resources were found in 5% or more of the 146 critical events during preliminary coding. Further investigation showed that out of these 19 most common resources usage varied greatly. What does this variability represent? First, it represents a measure of what reasoning processes are most likely to help a person make a breakthrough when solving a novel physics problem. Second, this variability represents that certain resources have gained priority in the reasoning processes of physicists during critical moments. Most likely, these resources have gained priority because physics experts have found them useful in the past (Hammer et al., 2005).

VI.3 Contrasting Cases

The third research question was: What are the most commonly used epistemological resources? By looking at the number of critical events that each epistemological resource was identified in, we determined that the most commonly used resource was contrasting cases. This may not be surprising since instruction which makes use of contrasting cases has been successful (Bransford & Schwartz, 1999; Schwartz et al., 2011). Evidence of experts utilizing contrasting cases to construct knowledge in novel situations is not confined to this study either. In a study performed by Wineburg (1998), an expert historian reasoning outside his specific historical domain of expertise was able to develop a coherent understanding of Abraham Lincoln's views on slavery by stringing together an argument which cross referenced and compared several different documents.

In another study by Samarpungavan, Westby, & Bodner (2006) expert chemists were interviewed about how they deal with empirical anomalies that arise in their research. They found that the expert chemists had developed multiple, triangulating tests that they would carry out to better understand the anomaly and determine if it was the source of routine error or a new phenomenon. It would appear that the use of contrasting cases to better understand novel situations might not be specific to physics expertise but rather a domain general element of expertise (Schunn & Anderson, 1999).

Hammer and colleagues (2005) proposed a mechanism to explain why contrasting cases are so useful for constructing new knowledge. They argue that the usefulness of contrasting cases can be explained by examining the role contrasting cases play in building more complex networks of resources. When observing phenomena, contrasting cases help draw an individual's attention to important patterns or features across phenomena. The features that a person focuses on determine what resources they associate with the phenomena. Once this happens, these resources become linked with each other and the important features they observe, creating a network of resources which becomes an individual's understanding of the phenomena. This mechanism helps explain why our experts would rely on contrasting cases so heavily. Contrasting cases appear to be a vital aspect of determining what resources should be associated with particular phenomenon and thus construction of more complex knowledge structures in novel situations.

Similar explanations are also offered by variation theory (Marton & Booth, 1997; Marton & Tsui, 2004; Ling, 2012**).** In this theory of learning, learning happens when an individual is able to recognize aspects of a phenomenon that they were not previously able to. Learning is a process of discerning what about a phenomenon sets it apart from other phenomena. These elements are known as critical aspects. The best way to draw out these critical aspects is to introduce "patterns of variation" across phenomena. In this scenario, contrasting cases function to expose these patterns of variation, which lead to the discernment necessary for learning.

VII. Implications for instruction

While physics experts use some epistemological resources much more often than others, their problem solving process cannot be characterized by only one or two epistemological resources. The physics experts relied on many epistemological resources to solve the problems they were given. These resources were: analogical reasoning, attention to novelty, causal reasoning, consistency, deductive reasoning, experimentation, hypothetico-deductive reasoning, inductive reasoning, knowledge from direct observation, limitations of models, mathematical reasoning, multiple representations, personal cognitive awareness, plausibility, relative value of knowledge, sense making, and peer cognitive awareness. It would do our students a disservice if instruction was not designed to develop all these epistemological resources. Instruction needs to focus on developing a multitude of epistemological resources.

However, it is clear that contrasting cases do play an important role and should be emphasized. To help students become more expert-like, instruction should develop and encourage students' use of contrasting cases. Instructors can do this by using the cognitive apprenticeship model (Barab & Hay, 2001). To start, teachers should model contrasting cases, allowing students to see how contrasting cases are used effectively.

Then, students should be given a chance to develop their use of contrasting cases through scaffolded activities with preselected contrasting cases. Then, the scaffolding can be removed and students will have the opportunity to construct their own contrasting cases. Throughout this process, students will need to reflect on how they use contrasting cases (White & Frederickson, 1998).

Instructors and researchers also need to identify contrasting cases that are useful for exposing important patterns. For example, contrasting motion diagrams of objects at different velocities may help students understand what velocity measures and contrasting diagrams of objects at different accelerations may help students understand what acceleration measures. However, these contrasting cases might not help students distinguish acceleration and velocity. Understanding how velocity and acceleration are different may require showing a motion diagram with constant velocity alongside a diagram with constant acceleration and explicitly discussing the similarities and differences between the two diagrams. The task of instructors would then be to find appropriate contrasting cases to use (see Etkina, Van Heuvelen, & Gentile, 2014 for examples).

Chapter 4

Using Variation Theory and the ISLE Cycle to Better Understand Physics Experts' Use of Contrasting Cases

I. Introduction

Contrasting cases are simultaneous presentations of two or more different ideas, phenomena, or objects, which are presented together to highlight their similarities or differences. Many studies have demonstrated that integrating contrasting cases into instructional materials leads to success. Schwartz and Bransford (1998) have shown that showing contrasting cases before a lecture helped to better prepare students to learn from lecture. Schwartz and colleagues (2011) found that students who learned about ratios using contrasting cases outperformed students who learned about ratios from direct forms of instruction. In addition to the success of teaching strategies which make use of contrasting cases, experts also use contrasting cases when solving novel problems. In a study performed by Wineburg (1998), a history expert was asked to determine Abraham Lincoln's views on slavery from a collection of documents. While the history expert was not an expert on Abraham Lincoln's views of slavery, he was able to determine Abraham Lincoln's views by analyzing the documents collectively and comparing different statements made across documents. Samarpungavan, Westby, and Bodner (2006) documented that chemist experts have several different mechanisms for dealing with data anomalies in their labs that they use to better understand an anomaly.

Previously, we've shown that physics experts make use of contrasting cases when solving novel physics problems. However, simply knowing that physicists use contrasting cases isn't enough to inform the development of instructional materials. We need to know how physics experts use contrasting cases so we can teach students to use them in similar ways. To develop this knowledge, we further investigate physics experts' use of contrasting cases by building an understanding of the function of contrasting cases, starting with a fine grained look at moments when experts use contrasting cases during problem solving sessions. In our study, we focus on the following questions:

1. When do physics experts use contrasting cases while solving novel problems?
2. What are the purposes of the contrasting cases that expert physicists use?
3. What are the patterns in physics experts' use of contrasting cases?
4. How can we link experts' use of contrasting cases to more complex epistemological processes?

II. Contrasting Cases and Variation Theory

II.1 Preparation for Future Learning

Many teaching methods do not meet traditional measures of success (Detterman & Sternberg, 1993). Students appear to fail to transfer the knowledge they learn in class to other similar tasks. Students have difficulty solving problems they have never seen before, even if the problems are only slightly different from ones they can solve. It appears the best efforts of teachers and researchers are failing. However, traditional transfer goals and the traditional view of transfer have come under question (Bransford & Schwartz, 1999). Bransford and Schwartz argue that traditional transfer focuses on expert-like goals where perfected finished products are the measure of success. They argue that it is important to measure other types of learning, such as the development of learning skills that help an individual adapt and learn in new situations. This more modern model of transfer is called "preparation for future learning." In this model, the main goal of an education is to help students develop skills, enabling them to learn independently. In this model, successful transfer happens when an individual uses the skills effectively to learn something new. In this model the educator must identify the knowledge and skills that will prepare a student for future learning. To start identifying these skills, we need to decide what constitutes learning.

II.2 Variation Theory

Variation Theory (Marton & Booth, 1997; Marton & Tsui, 2004; Ling, 2012) is built from the assertion that learning happens when someone is "capable of being simultaneously and focally aware of other aspects or more aspects of a phenomenon than was previously the case" (Marton & Booth, 1997, p 142). For an individual to understand a phenomenon, or object of learning, they must identify important aspects of that phenomenon. This happens through a process called discernment. In this process, important parts of a phenomenon are differentiated from the phenomenon as a whole. Discernment cannot happen without comparing the object of learning to other phenomena. Comparisons draw out differences between the object of learning and other phenomena. This helps students decide what parts of the object of learning they should focus on. Aspects of a phenomenon which are systematically different from other phenomena are called “critical aspects.” The precise value that a critical aspect takes for a phenomenon is called a “critical feature” of that phenomenon. In variation theory, learning which aspects of a phenomenon are critical and which can be ignored is a key process called developing the relevance structure of a phenomenon. Once an individual has determined what parts of the object of learning are critical features, she/he pieces these critical features together to build an understanding of the phenomenon as a whole.

III.2.a An Example of Variation Theory Applied to One Dimensional Kinematics

These abstract ideas of variation theory require an example. Take a look at Figure 4.1. This figure contains a dot diagram of an object moving in one dimension. The dots represent the position of an object at 1-second intervals as it moves along a straight line from left to right. This diagram by itself does not help students discern where they should focus their attention.

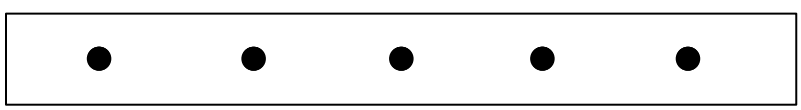


Figure 4.1 A dot diagram for an object moving at constant velocity.

Now consider Figure 4.2. In this figure there are three dot diagrams. Each diagram is slightly different. In, the first two diagrams, the space between the dots differs. This dimension of variation would draw a student’s attention to the distance between dots. This represents how fast the object is moving and would help a student begin to develop the concept of speed. In the language of variation theory, speed is identified as a critical aspect. The exact spacing of the dots in each diagram would be the critical feature for that diagram. The third diagram is different from the first two diagrams because the space between the dots increases as time increases. This represents how an object's motion can be characterized by constant speed or the speed can change. The visual representation of the difference helps students construct the concept of acceleration.

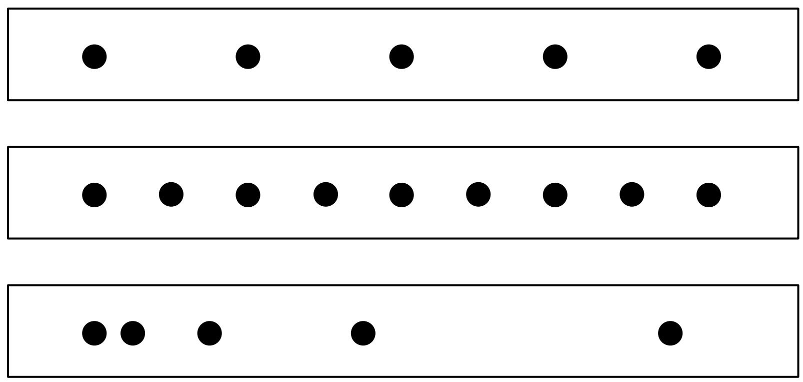


Figure 4.2 Three different dot diagrams. The top depicts an object moving slowly at constant velocity. The middle shows an object also moving at constant velocity, but slower. The bottom shows an object moving with changing velocity.

III.3 Contrasting Cases

An individual's ability to identify and make use of patterns of variation is exactly what Bransford and Schwartz (1999) argue is at the center of a person's successful learning. However, Bransford and Schwartz discuss them under a different name, contrasting cases. Contrasting cases are nothing more than different manifestations of a similar object or phenomenon shown together with some aspect varied across the cases. Contrasting cases use patterns of variation to draw out important aspects of a phenomenon. These ideas appear to complement those of variation theory.

If we want to prepare our physics students for future learning, then we need to focus on developing students' ability to use contrasting cases. An understanding of how contrasting cases are used best precedes our capability to achieve this goal. Therefore, we must first develop an understanding of how contrasting cases are used in physics. In this study, we do this by using variation theory to analyze how physics experts use contrasting cases as they solve novel physics problems. Then we search for patterns across a multitude of contrasting cases to identify features which can be used as focal points for instruction.

III. Description of the Study

We asked 6 pairs of physics experts to take part in advanced problem solving sessions. Physics experts ranged in experience from post-qualifying Ph.D. candidates to physics professors. The 6 pairs were split evenly across two different problems, a light cone problem (Etkina, Planinšič, & Vollmer, 2013) related to optics and another problem related to the physics of solar cells (Jones et al., 2013). The experts were split so that the three pairs of experts who had experience with solid state physics and solar cell physics solved the solar cell problem. Each pair was videotaped independently. Problem solving sessions lasted from one to two hours. Afterwards, the videotapes they were transcribed in full.

III.1 The Physics Problems

An important feature of this study was the selection of problems that were given to our experts. We did not want the experts to be able to recognize the problem and remember a solution. We wanted them to construct a solution in the moment. To ensure that we succeeded, we chose problems that were not puzzle type problems, but were ill-structured (Churchman, 1971; Mitroff & Sagasti, 1973; Kithchener, 1983), problems that did not have a clear cut, immediately accessible solution that could be obtained algorithmically, required the experts to gather and analyze data, and required the experts to make, develop, and evaluate multiple solutions. While we will not go into detail about it here, we have shown elsewhere that the light cone problem and the solar cell problem meet the criteria for ill-structured problems.

III.1.a The Light Cone Problem

The light cone is briefly described as follows. A clear plastic container, which is open on top, is filled with water and placed on top of a white piece of paper. A few drops of milk are mixed into the water. A green laser pointer is shone from above the container into the water and directed so that it strikes the white piece of paper beneath the container. What then happens is that a green light cone forms in the water with the point of the cone at the bottom of the container, as shown in Figure 4.3. For a more complete description of the light cone problem, please see Etkina, Planinšič, & Vollmer (2013). Experts who solved this problem were simply shown this experiment and then asked to explain the cone.

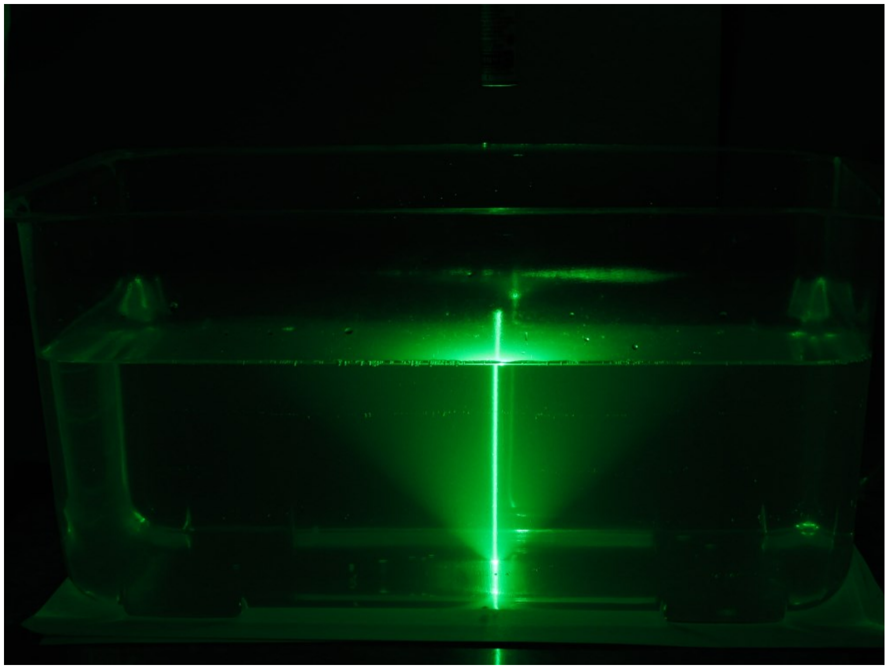


Figure 4.3 A picture of the light cone experiment.

III.1.b The Solar Cell Problem

The second problem we gave our experts was a solar cell problem. The handout which accompanied this problem is shown in Figure 4.4. The problem had two parts. In the first part, the experts had to explain data from the following experiment. A solar cell was illuminated with a 150-W bulb and connected in series to an ammeter and a resistor. The experts were asked to explain current measurements that were taken as the solar cell was covered with a black piece of paper from the side and from the top, as shown in the handout. In the second part of the experiment, the experts were asked to predict current output when two solar cells were connected in series and covered with a black piece of paper as shown in the second handout in Figure 4.4. The experts were then asked to predict the current when the same experiment was performed, but with the polarity of one of the solar cells reversed. The experts were shown actual measurements for this experiment and asked to resolve discrepancies between their predictions and the results.

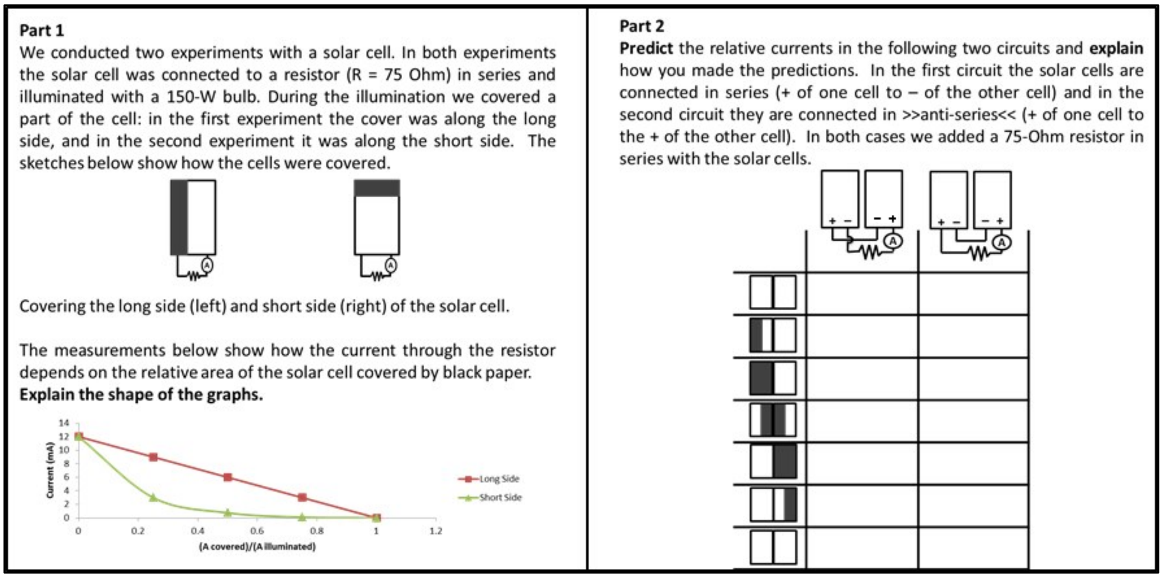


Figure 4.4 Handouts given to experts that solved the solar cell problem.

IV. Physics Experts' Use of Contrasting Cases to Explain the Light Cone

To help provide insight into how physics experts use contrasting cases in novel situations, we will present an account of how experts C and R developed a solution to the light cone problem. While there were six groups of experts that took part in this study, we chose to present the problem solving session of experts C and R because their problem solving session provided clear and concise examples of patterns that we identified across all the groups of experts. The general process that C and R followed was nearly identical to the process followed by all the groups who solved the light come problem. It started with C and R making observations related to the light cone and continued with them making sense of these observations and developing a preliminary model. Then they found that the predictions of their model did not match their observations. They modified their model, developed a satisfactory explanation, and performed experiments to gain confidence in their model. The analysis of this process will be broken down into segments of this process. During each segment of the problem solving process we will highlight the significant features of C and R's behavior. Where contrasting cases are present we will explain how variation theory can help us understand the purpose of these contrasting cases.

IV.1 Making Initial Observations

The problem solving session began with an interviewer showing C and R the light cone and asking them what they saw. The experts identified the cone and were told to explain why this cone appears. The interviewer then left the experts to work independently. C and R immediately begin by observing how the light cone changes when the material underneath the container is changed. They make observations of the light cone phenomenon with white paper, a wooden table, and a piece of cardboard underneath the container. They conclude that the cone changes when the material underneath the container is changed. They notice that while there is a cone with white paper, there is no cone when the table is directly beneath the container, and that there is a weak cone when cardboard is used. Then they attempt to justify their observations.

> C: Ok. Ok, so reflected laser beam. So when it hits the white paper there should be more reflection, right?
> R: Yeah.
> C: Than if it was just hitting the plastic or if it was just hitting the cardboard or the table.
> R: Yeah. Which makes sense. I guess it's just cause it's a white piece of paper. That's reflecting more.

C: So then is this cone, the cone is due primarily to the reflected beam…hmm.

In this quote there is clear evidence that C and R are using contrasting cases. C and R are varying the material beneath the container and noting differences in the light cone across these cases. To understand the purpose of this contrasting case, we first notice that C and R pay attention to the variant across the cases, the material beneath the container and look for a reason why this might cause the light cone to change. By coupling this focus on the variant with the knowledge white paper should reflect more, C and R conclude that the light cone is caused by the reflected laser beam.

It should be noted at this point that experts C and R were unique in the fact that, other than a brief mention of the possibility that the light cone was due to interference, they did not investigate many possible solutions at the beginning stages of their inquiry. Other groups of experts mentioned many other possibilities. What C and R have in common with other groups is that this particular contrasting case, where the material beneath the container acted as a variant, also drew other groups' attention to the reflective nature of the light cone. In this sense, the purpose of the this contrasting case was not just to identify what might be causing the light cone, but first draw their attention to the variant and then cause them to search for a reason why this variation would cause the light cone to change.

After realizing that the material beneath the container was important, C and R continue experimenting and shine the laser beam into the container at many different angles. This leads them to make another observation.

> R: …The fact that the angle of the cone doesn't change [when you change the incident angle of the laser] makes me think that it has something to do with the, the –
> C: Interfaces?
> R: Yeah the interface since the –

C: … Interfaces aren't changing…Ok so let's say it's based on the interfaces then.

In this quote, the experts are contrasting the light cone as the angle of the laser beam varies. They notice that across the contrasting cases, the light cone is invariant. When they look for another invariant to relate to the invariance of the light cone, they decide that the optical interfaces beneath the container act as that invariant. This contrasting case focuses the experts' attention from the entire apparatus and light cone phenomenon as a whole to the interfaces beneath the container.

By viewing the experts' activities through the lens of variation theory, we can say that different observations of the light cone phenomenon helped the experts identify patterns of variation and separate critical features from the whole object of learning. First, they changed the material beneath the container and were able to identify that the material beneath the container was a critical aspect of the phenomenon. They also realized that the white paper beneath the cone, and the interface the paper creates, was a critical feature of the phenomenon. They experimented with the angle of the incident laser beam and observed no variation in the light cone. From this they concluded that the incident angle of the laser beam is not a critical aspect of the light cone.

IV.2 Constructing an Initial Model

Once C and R agree that the interfaces are important, they begin constructing a model of the light cone. The model is centered on the construction of a Snell's Law type diagram. A recreation of the initial diagram that C and R construct is shown in Figure 4.5. Such diagrams are often used when two optically different materials come into contact with each other. The diagram focuses on the interfaces at the bottom of the container. The fact that these interfaces are the focal point is significant because the experts have just used contrasting cases to identify that these interfaces are a critical feature of the light cone. Contrasting cases helped the experts identify critical features of the phenomenon which they use as the focal point of their model.

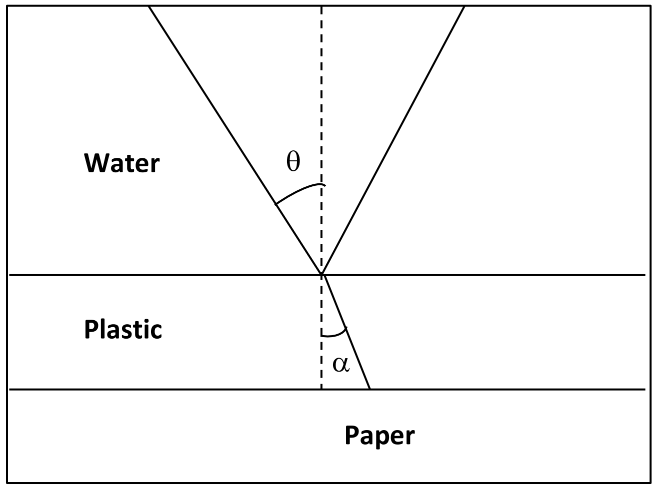


Figure 4.5 The diagram C and R construct as they begin to model the light cone phenomenon. This diagram focuses on the optical interfaces at the bottom of the plastic container, the water-plastic interface and the plastic-paper interface. C and R have drawn the laser beam incident on the water-plastic interface at the angle θ, a specular reflection off of the water-plastic interface, and the refracted beam as it travels through the plastic at the angle α. C and R have yet to address how they should represent the laser beam after it hits the paper.

IV.3 Further Observations and Model Refinement

In their diagram C and R have broken the interface below the container into two interfaces, a top interface and a bottom interface as shown in Figure 4.5. They have not addressed whether the top or bottom interface is important. Once they have constructed the diagram shown in Figure 4.5, this issue arises and the experts decide to perform an experiment to confirm their suspicion that the bottom interface causes the light cone. They first observe the light cone on the white paper and then slowly raise the container off the paper as shown in Figure 4.6. By performing this experiment, the experts produce contrasting cases where the reflection off the top interface remains invariant, while the bottom interface varies. They notice "the further away it (the paper) is, the more diffuse the cone is." They decide that

> C: … reflection off the plastic seems minimal at best right? The first interface?
> R: Yeah. I think it just gives you that specular beam.
> C: Yeah.
> R: And then the, the paper is what gives you that cone.

From this, we see that the model has helped them develop other contrasting cases that they use to clarify the model. Again, variation theory can offer insight into the function of these contrasting cases. When a learner recognizes what parts of the object of learning are more important than others, they are assigning those aspects relevance; they are building the "relevance structure" of the object of learning. In this segment, R has identified two parts of the object of learning, the top interface and the bottom interface. By performing the experiment described above, the experts notice that varying the bottom interface does influence the light cone and decide that the bottom interface is more relevant. This is confirmed by the statement that "…the paper is what gives you that cone." This determination of the relevance structure of the phenomenon helps again to focus their attention and analysis on the bottom interface.

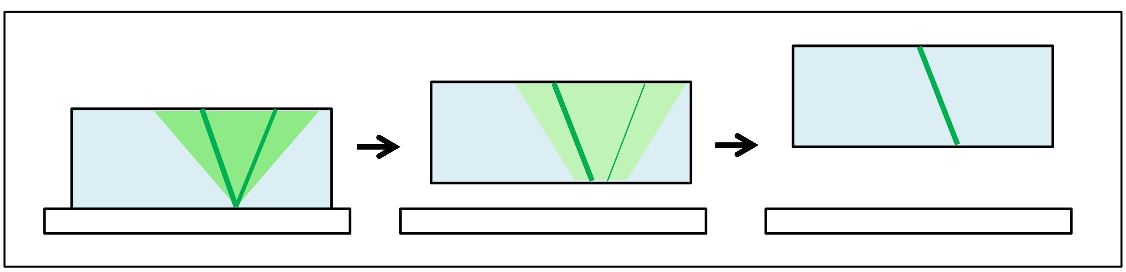


Figure 4.6 A representation of the experiment carried out by R to determine whether the water-plastic interface or paper-plastic interface was causing the cone. In this experiment, R observed the light cone phenomenon with the plastic container flush against the paper and then made further observations as he slowly raised the container off the paper. He concluded that as the container was raised, the cone got more diffuse.

After performing the experiment just discussed, R shines the laser beam at just the white piece of paper and says the following:

> R: Is it, when you shine the light off just the paper…you can see it, um, reflecting off at a lot of different angles. Like the…I dunno you get this diffuse looking spot.

However, this observation is not addressed further by either expert at this time. Instead, the experts focus on completing the diagram's specular reflections at each interface. The progress that they have made on their diagram is shown in Figure 4.7.

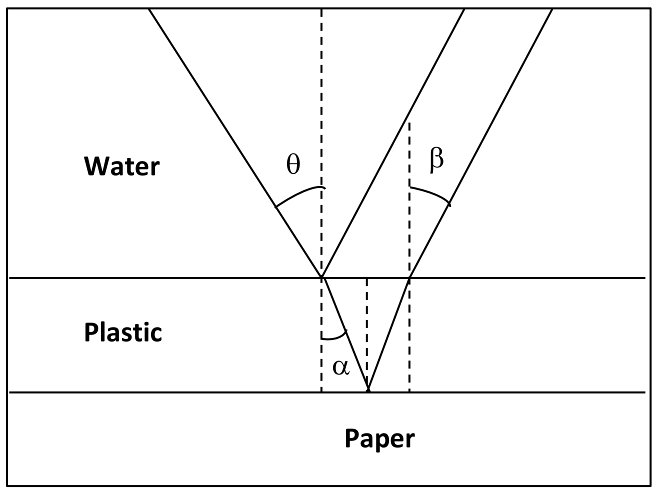


Figure 4.7 The diagram that is on the board when C and R decide that their diagram does not incorporate what happens to the laser when it hits the paper. In this diagram, C and R have added a specular reflection of the laser off the paper-plastic interface and the refracted laser beam that would result when the paper travelled from the plastic to the water.

After staring at the diagram for a moment, C comes back to the previous statement made by R.

> C: Yeah. What this diagram is ignoring though is what happens at the paper. Right?
> R: Yeah.
> C: What happens when the laser hits the paper? We don't – do we get – we don't get the exact same beam coming back, right? Cause that's what you were trying to look at before, I think. Just sort of looking at what –
> R: Yeah. Somehow it spreads out in a very well defined way. Uh…in the paper.

What is interesting about these two passages is that the comment R made in the first passage was initially ignored. However, when taken in context of the model, the comment garners significant attention in the second passage. After this moment, C and R spend a good deal of time trying to better understand why the light appears as a diffuse spot on the paper.

Again contrasting cases aid the physicists' reasoning. When C recounts R's observation, he does so immediately after stating "What this diagram is ignoring though is what happens at the paper." In this statement, C is contrasting the model of the light cone with R's observation. Through this contrast he identifies a mismatch; the model does not incorporate what happens at the paper. This mismatch gives priority to R's previous statement. In the first passage, the group is unsure whether or not this piece of information is relevant. However, when a contrast is identified in the second passage, this observation gains relevance and is brought into the group's focus. Contrasting cases once again identify areas where the model can be improved, this time by drawing attention to differences between the model and the phenomenon. Once a mismatch is identified, priority is given to observations that can help eliminate the mismatch.

Once C and R agree that the way the light comes off the paper is important, they talk about how to incorporate this into their model. R then mentions that the "halo of light" that you observe when the laser beam hits just the paper is different from the overall light cone phenomenon. Instead he says that the sides of it "are so well defined." Immediately after, R proposes a mechanism to describe how the light can go from being dispersed in all directions to a well defined cone and makes two additions to their diagram, shown in Figure 4.8.

> R: So if it's coming off in every direction from the paper…. But then there's, there's an interface here…. So there's a, I forget what they call it. The magic angle or whatever where you have total internal reflection beyond a certain angle…. So, um maybe, I don't know, up to some angle here, um –
> C: Beyond that angle it's reflected and it's not transmitted through.
> R: Beyond that angle the light just, yeah.
> C: Ok. That sounds pretty good actually.
> R: Whereas close to normal incidence you get that cone coming off.

Another important aspect of learning, according to variation theory, is understanding how several different critical features come together to produce a whole phenomenon. In this segment R has identified one critical feature of the phenomenon, the specific way that light scatters off a piece of paper, and is trying to understand how that critical feature fits into the whole light cone phenomenon. By contrasting a critical feature of the phenomenon, the scattering of light in all directions, with the conical nature of the whole phenomenon, R recognizes that their model is incomplete. This causes him to reason that there must be another critical feature they have not identified which produces the well-defined nature of the light cone. He proposes that the new critical feature is a critical angle at the bottom interface. He pieces together these two critical features to develop a model which can explain the whole light cone phenomenon.

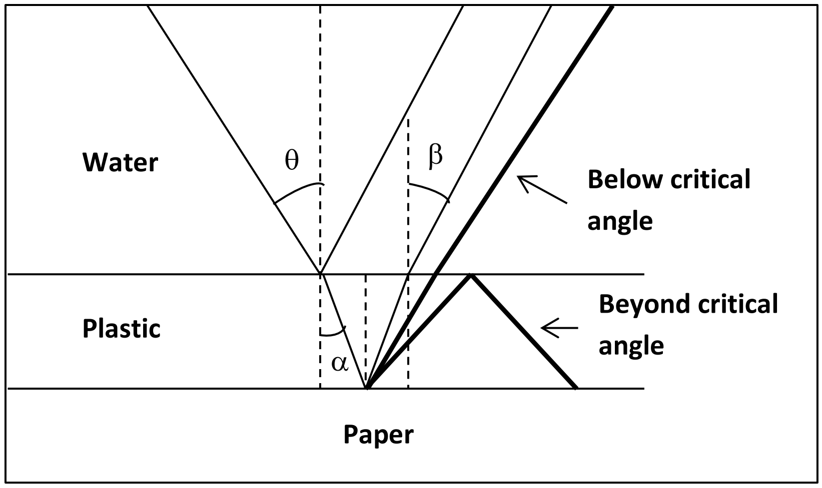


Figure 4.8 This is the diagram that R uses to explain his critical angle mechanism. He draws the two bold light rays to emphasize the point that certain light rays beyond the critical angle will be cutoff at the plastic-water interface, while others will not be cutoff and form the light cone.

IV.4 Identifying Flaws in the Model

C and R continue by discussing how to calculate the critical angle and start constructing a new diagram and mathematical representation to go along with the critical angle model until R notices that something is wrong. In the dialogue that follows C draws a sketch recreated in Figure 4.9.

> R: Right. I'm just trying to figure out why, um, because the way we set this up –
> C: Mhmm.
> R: You should be able to get, um, light coming off in all directions still. Through the water instead of just some specific angle.
> …
> C: But beyond this crit – whatever, uh, if we say this is the normal – there's some critical angle. So anything beyond that you won't see, these will just get reflected back.
> R: Right, but the angle's very close to that you should get something coming off almost parallel to the –
> C: Oh! I see what you're saying.
> R: To the surface of the plastic. Which we're not seeing.
> C: So you're saying we need to – Yeah. Very very close – if this is the critical angle then you should see something like that (at this point C draws the diagram in Figure 4.9 and points to the bolded line segments) –
> R: Yeah.

C: That, that, and that. So we should be getting a, uh…
R: We should still get –
C: Hemisphere instead of a cone.

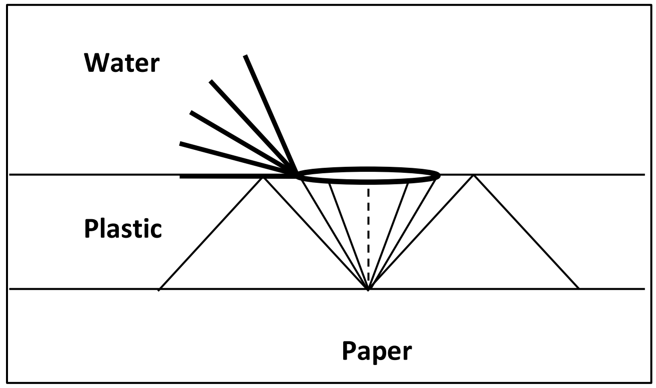


Figure 4.9 This is the diagram that C draws on the board to show that, if R's critical angle model were true, they would expect to see a hemisphere of light instead of a cone. The bolded portion is what he draws as he emphasizes this point.

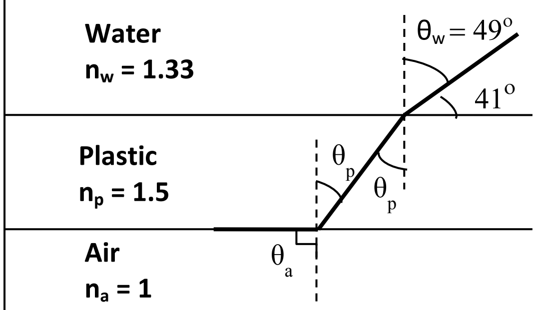
In this segment, the experts are contrasting their model with the actual phenomenon they observe. This is most evident when C says, "Hemisphere instead of a cone." This coupled with his previous statement that "angle's very close to that you should get something coming off almost parallel to the … To the surface of the plastic. Which we're not seeing," provides strong evidence that he is drawing a comparison between what the model predicts, as shown in Figure 4.9, and what is actually observed. He uses this comparison to identify what is different between the two situations. In this case, the contrasting case draws out the differences between predictions of the model and the actual phenomenon. This allows the experts to decide if their model should be refined. After R identifies this contrast, C and R go on to alter their model. In this way, the experts judge whether or not they have properly integrated all the critical features they have identified into a whole understanding of the phenomenon.

IV.5 Building a New Model

After they realize their model is flawed, C and R remain silent for some time. R breaks the silence saying, "Do we have to assume that there's a little bit of air in-between? ... I'm wondering, do you only get certain angles that are allowed going into the plastic to begin with?" R then proposes a new model.

> R: … you should be able to get, um, very glancing angle light that still reflects up in here…if even the light that's coming in almost parallel to the surface gets reflected up at an angle then this is the, the –
> C: The limit?
> R: The biggest angle that you can have going through the plastic and that'll be bent even further still into the water, but not as much as it was bent from air into the plastic. So that still gives you some limiting angle through the water. So, so that what I'm thinking is the air to plastic Snell's law and then the plastic to water.

C and R draw a new diagram and begin working out the details of R's ideas with Snell's Law equations. The finished diagrams and equations they have written on the board are recreated in Figure 4.10. Similar to initial construction of the model, contrasting cases are not directly present in this quote. However, contrasting cases were used to draw attention to the deficiencies in the previous model. It is precisely these deficiencies that are addressed in this new model. The contrasting cases identified which critical aspect of the model (how the light gets "cut-off") needed to be reassessed or refined in order to build a whole understanding of the phenomenon.



$$n_a \sin\theta_a = n_p \sin\theta_p$$

$$\frac{n_a}{n_p} = \sin\theta_p$$

$$\frac{2}{3} = \sin\theta_p$$

$$\theta_p = 42^o$$

$$n_w \sin\theta_w = n_p \sin\theta_p$$

$$\sin\theta_w = \frac{n_p}{n_w}\sin\theta_p$$

$$\theta_w = 49^o$$

Figure 4.10 This figure recreates diagrams and equations grouped together on the board used by C and R to develop a new model explaining the light cone. The diagram on the left shows how light scattered at a ninety-degree angle off the paper would be ”bent up” when it entered the water. The equations on the right show how C and R used Snell’s Law to predict the angle of the cone.

IV.6 Testing the New Model

Once C and R develop their new model, they come up with ways to determine if their model is accurate. First, using Snell’s Law as shown in Figure 4.10, they calculate that the angle between the edge of the cone and the bottom of the container should be 41 degrees. They check to see if this matches what they observe.

> C: So is it about 41, 41? 41 degrees with respect to…
> R: I’m gonna say it’s 45ish.
> C: Haha.
> R: Plus or minus ten degrees.
> C: So our observation is roughly consistent
> …
> C: … I would like to compare directly, but we need some way to measure angles but it looks, it looks really close.

They agree that the angle of the cone looks the same as the angle that they’ve calculated. Then, R has an idea for an experiment. A schematic of the experiment is shown in Figure 4.11. They carry out this experiment and C comments on it.

> R: I was just thinking, can we show that there’s some limit to the angle of uh the light that comes up through the water?
> …

R: Cause even shining this through uh almost, almost parallel to the bottom of this you get about a 45 degree beam through the water….
C: Yeah. You're just not having the light scatter the same way. Is that pretty much the only difference, right? Air, plastic, water, but you don't have the reflection off the paper.
R: You're just, you're just not dispersing the light like you do off the paper.
C: Yeah. Yeah. Yeah I see what you mean. It doesn't go past that particular angle which looks the exact same as the cone angle.

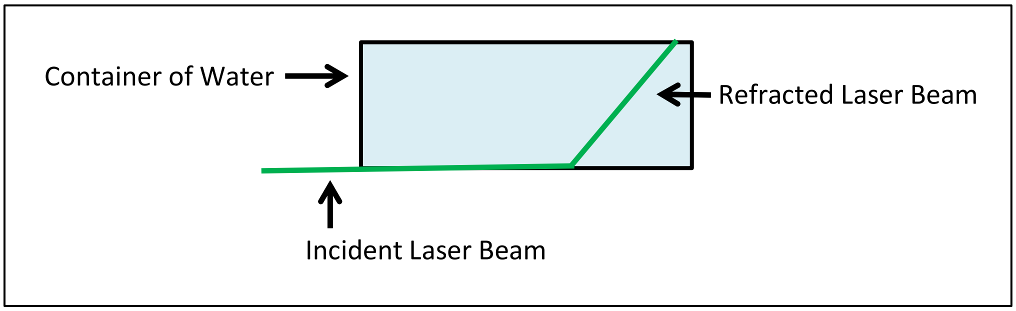


Figure 4.11 This is a schematic representation of the experiment R ran to see if their model was correct.

In both scenarios C and R contrast the predictions of their model with observations that they make. In the first case, they predict that the side of the cone should be elevated 41 degrees from the bottom of the container. Their observations verify this. In the second case, they predict that if they hold the container in the air and shine a laser at nearly parallel incidence towards the bottom of the container, then they should see the laser beam pass through the container at some limited angle. Again their observations verify this. By contrasting the predictions of the model they have built with observations that they make, C and R determine whether or not they have integrated the critical features of the light cone phenomenon into a coherent whole understanding. In this case, they observe a match, gain confidence in their model, and do not attempt to identify any more critical features. Shortly after this, the interview comes to an end.

V. Identifying Patterns Across Groups of Physics Experts

In the analysis above, we showed that experts use contrasting cases in a variety of ways for a variety of reasons. This analysis will help us answer our first and second research questions. However, the above analysis does not help us to answer our third and fourth research questions. Answering these questions are important because these answers can provide us with generalized rules and processes that easily communicate the main ideas of physics experts' use of contrasting cases, generalized rules that would help us to develop instructional materials that make the varied uses of contrasting cases accessible to our students. Therefore, we continued our analysis of physics experts' use of contrasting cases to develop generalizable rules governing physics experts' use of contrasting cases. To do this, we searched for patterns across the six groups of physics experts by analyzing how each group used contrasting cases from the perspective of scientific epistemology.

V.1 Critical Events

The first step in the analysis process was picking out important moments for analysis. To do this, we went through the transcript and identified "critical moments" or "critical events" during the problem solving process (Powell, Francisco, & Maher, 2003). A critical event is a moment during the problem solving process where the experts make a reasoned conceptual breakthrough. We characterized this breakthrough by relating it to an important question about the problem the experts are trying to solve. During a critical event, experts either propose an important question or answer an important question for the first time. These questions are not determined a priori, but by the content of the dialogue during a critical event. After a critical event, there is a notable change in the way the experts discuss the problem. In all, 146 critical events were identified. Critical moments tended to contain about 110 words of dialogue on average. This text was selected to embody a clear coherent piece of dialogue.

To determine the reliability of the selection of critical events we also selected non-critical events from the light cone problem solving sessions. The number of non-critical events we selected was equivalent to about 30% of all critical events. Characteristic questions were developed for these non-critical events. An outside researcher familiar with the light cone problem but not involved in the study in any other way was brought in to determine reliability. The outside researcher was given a list of the questions that characterized the critical and non-critical events and allowed to consult the text that accompanied them to clarify any instances when the questions were unclear. The outside researcher determined which of the questions would: 1) Require reasoning to answer. 2) Result in a deeper understanding of the light cone phenomenon. The outside researcher was trained on a list of 5% of all critical events and an equivalent number of non-critical events and then performed an analysis on a list that contained 25%. The outside researcher's selection of critical and non-critical events was compared with the original selection of critical and non-critical events. We achieved 82% agreement between these selections before discussion and 96% agreement after discussion.

V.2 Analysis of Critical Events with Contrasting Cases

Once critical events were identified they were coded to determine if they contained evidence of contrasting cases. A critical event was coded positively when it met the following criteria:

1. Either:
    a. A primary case and secondary case can be identified.
    b. A continuous set of cases can be identified.
2. Evidence that there is a comparison between the primary and secondary cases.

Using these criteria, we determined inter-rater reliability in a similar manner to the method used to determine reliability of critical events described above. Inter-rater agreement of the coding of contrasting cases was 80% ($k = 0.62$).

We then narrowed our analysis to only critical events with contrasting cases. We looked over all these events and developed categories based on how the experts were using contrasting cases during the critical events. We developed guidelines that described what criteria needed to be satisfied in order to include a contrasting case in a particular category. Because we believed that use of a contrasting case could result from a protracted process extending beyond the critical event, we also analyzed the context around the critical events. Because of this, not all criteria needed to be explicitly identified in the text of the critical event. Criteria could be met by analyzing the surrounding text as well.

As we analyzed more critical events, we refined categories and guidelines. This was done when a contrasting case either didn't fit in any existing categories or could be included in more than one category. After analyzing and reanalyzing the critical events, we developed six different types of contrasting cases that captured the different ways contrasting cases were used by experts. The categories, their selection criteria, and examples are shown in Table 4.1.

Table 4.1 Types of Contrasting Cases with Explanation of How Example Meets Criteria

Criteria for Specific Type of Contrasting Case	Examples of a Contrasting Case of that Type	Explanation of how the Example Meets the Criteria
<u>**Contrasting Observed Experiments**</u> 1. Observations related to either: a. Multiple cases of a single phenomenon b. Multiple different phenomena 2. Pattern observed by comparing phenomena 3. Hypothesis developed based on this pattern	P: (put's the container on the paper) Yeah so there's a cone. T: Ok, but now you take it off the piece of paper… P: Yeah, yeah. (lifts slightly) But there is still something there. (puts it on the table) So, it's about like the reflective, reflective thing. (Shines laser into container) Yes? No? T: It's more like a haze. P: Yeah it's not, but it's [inaudible]. It's something that has to do with this, this paper. I mean with the, this medium. (moves container back onto paper and shines laser into it) Yeah so, ok. So if there is a paper, there is a cone. No paper, no cone.	**1.** The cone on the paper vs. the cone on other materials. **2.** The strength of the cone changes when the material beneath the container changes. **3.** The cone is dependent on the reflective material below the container.
<u>**Contrasting Hypotheses**</u> 1. A single experiment is being discussed. 2. Two different hypotheses. 3. Two different predictions based on those hypotheses.	D: (gets up and moves to the board) But I still think if it [the cone] were, if it were, like if it were a top, happening – something happening at the top surface to cause that, then it would, (begins drawing on the board) it [the light from the laser] would come in like this and then it would start to spread out up here. Instead of coming in here and then spreading out at the bottom. (diagram of each scenario is on the board)	**1.** Laser light being shined into the container, creating a cone. **2.** Hypotheses a. The top interface causes the cone b. The bottom interface causes the cone **3.** Predictions a. The vertex of the cone is at the top interface. b. The vertex of the cone is at the bottom interface.
<u>**Contrasting Assumptions**</u> 1. A single case is being discussed. 2. Two different assumptions. 3. Two different predictions based on these assumptions.	M: But the issue is that we could have this connection. (draws several voltage sources and resistors in series on board) Right? Then if the internal voltage of this guy [the multimeter] is much greater then we'll have the whole sum. P: Mhmm. M: But this is a cheap voltmeter, so maybe the resistance is not that much greater than uh the sum of those guys.	**1.** Voltage of several voltage sources and resistors in series is being measured by a multimeter. **2.** Assumptions a. Internal resistance of multimeter is not much greater than equivalent resistance of the circuit. b. Internal resistance of the multimeter is much greater than equivalent resistance of the circuit **3.** Predictions a. Measurement not close to true value. b. Measurement close to true value.

Table 4.1 (continued) Types of Contrasting Cases with Explanation of How Example Meets Criteria

Criteria for Specific Type of Contrasting Case	**Examples of a Contrasting Case of that Type**	**Explanation of how the Example Meets the Criteria**
<u>Contrasting Testing Experiments</u> 1. A single clearly identifiable model. 2. At least two distinct situations (testing experiments). 3. Predictions made for distinct cases using the model.	P: So for each one it's efficiency for individual cell is proportional to the relative illuminated area. Right? So if you cover it completely with the screen it becomes a huge resistor and it doesn't help at all. But if you cover it like this, (from the long side) it's efficiency drops by 10% and 10% for this guy, 10% for this guy. So overall, a series connection would be a sum of contribution from each one and it would go down linearly for this case (covered from the long side) and somehow very rapidly for this one (covered from the short side).	**1.** Solar cell is constructed of elements with an efficiency that is proportional to the relative illuminated area. **2.** Testing Experiments a. The solar cell is covered from the short side. b. The solar cell is covered from the long side. **3.** Predictions a. A reduction in current output that is very rapid. b. A reduction in current that is linear
<u>Contrasting Predictions vs. Testing Experiments</u> 1. A model can be identified 2. Prediction based on the model 3. An outcome of the experiment compared to the prediction	C: I see what you're saying. R: Yeah, this is still the same order of uh – C: Yeah. You're just not having the light scatter the same way. Is that pretty much the only difference, right? Air, plastic, water, but you don't have the reflection off the paper. R: You're just, you're just not dispersing the light like you do off the paper. C: Yeah. Yeah. Yeah I see what you mean. It doesn't go past that particular angle which looks the exact same as the cone angle.	**1.** Light scattered at all angles by the white paper and then refracted by the water causes the cone. **2.** Any light incident on the bottom of the container at almost parallel incidence should be refracted at the same angle of the cone. **3.** The angle of the laser light refracts at looks to be the same as the cone.
<u>Contrasting Assumptions vs. Testing Experiments</u> 1. A model can be identified. 2. Assumption under which the model can be used to predict the outcome of a testing experiment 3. Fact about the testing experiment is compared to the assumption.	R: Yeah, so there are a couple of interfaces. C: Yeah. R: Well there are two different interfaces at the bottom cause it reflects right off the surface of the plastic. And then it can go through the plastic – C: And reflect back. R: Reflect off the paper. Which makes me think of interference effects, but the thickness would be way too big for real wavelength based interference.	**1.** Thin film interference **2.** For thin film interference to apply, the thickness of the film needs to be on the order of the magnitude of the wavelength of light. **3.** The interfaces involved in this situation are too big for this assumption to apply.

V.2.a Contrasting Observed Experiments

In this type of contrasting case, the experts compare observations to develop a hypothesis. The three criteria that needed to be satisfied for this category were as follows. First, the experts discussed multiple different observations they made. Second, the experts needed to discover a pattern based on these observations. Third, the experts would develop a hypothesis to explain the pattern. In the example in Table 4.1, the experts observed variations of the light cone experiment by changing the material beneath the container and discovered the pattern, "…no paper, no cone…" This leads the experts to develop the hypothesis "…it's about, like, the reflective, reflective thing," meaning that the light cone is dependent on the reflective material beneath the container. While we expect the reasoning to proceed logically in the order that the criteria have been presented, what someone says does not directly mirror their reasoning. Therefore, we do not expect to find these criteria in the same order we present them.

V.2.b Contrasting Hypotheses

Contrasting cases were coded as contrasting hypotheses when the physics experts identified how different hypotheses would manifest themselves in the same situation. We looked for the following criteria. First, the experts discussed a single experiment. Second, the experts reasoned using two separate hypotheses. Third, they made different predictions using the different hypotheses. In the example above, experts B and D talk about what happens in the light cone experiment when you shine the laser into the container. B and D's hypotheses are that the light cone is caused by something that happens at the top of the water ("…something happening at the top surface to cause that…") and, conversely, that the light cone is caused by something at the bottom interface ("…something's happening in here (points to the bottom interface) that's causing it to do that."). The experts make two distinct predictions. First, if the light cone were caused by the top interface, the light cone's vertex would be at the top of the water. Second, if the cone were caused by the bottom interface, the light cone's vertex would be at the bottom interface.

V.2.c Contrasting Assumptions

In this type of contrasting case, experts analyzed how different assumptions would affect their predictions. Each contrasting case in this category met the following criteria. First, the experts discussed only a single situation. Second, they mentioned two different assumptions they could make for that situation. Third, they made separate predictions for each assumption. In the example above, experts M and P are solving the solar cell problem and discussing a circuit consisting of resistors and voltage sources connected in series. As M and P talk about this circuit, they discuss measuring the potential difference across the circuit using a multimeter. They note two assumptions they could make. They can assume that the multimeter has an internal resistance much greater than the circuit. In this case, they predict that the measured potential difference would be close to the actual potential difference. Conversely, they could assume that the multimeter has an internal resistance that is not much greater than the circuit. In that case, they predict they would not measure the true value of the potential difference.

V.2.d Contrasting Testing Experiments

In this category, the experts used contrasting cases to compare different experiments they could use to test a hypothesis. All contrasting cases in this category met the following criteria. First, the experts used a single model or hypothesis to reason. Second, they discussed two different experiments. Third, they used their hypotheses to make predictions for each experiment. In the critical event above, expert P is discussing the solar cell problem and has modeled the solar cell as a series of current generating elements with efficiencies proportional to the illuminated area. P uses this model to make two different predictions for two different experiments. P says that if the solar cell is covered from the short side, whole elements will be covered causing them to act as big resistors. P predicts that this will cause a rapid reduction in current as you cover the solar cell this way. P also talks about covering the solar cell from the long side. In this case, the individual elements are partially covered, causing their efficiency to drop by the percentage of the solar cell that is covered. P predicts that for this experiment the current would be reduced linearly as the solar cell is covered.

V.2.e Contrasting Predictions vs. Testing Experiments

In this category of contrasting cases, experts compare a prediction they have made with the results of a testing experiment. Contrasting cases that fell into this category met the following three criteria. First, we were able to identify a hypothesis that the experts were working with. Because these types of contrasting cases often happen after our experts have developed their hypotheses and because our critical events capture such a short period of time, sometimes clear evidence of the hypothesis must be inferred from the context surrounding the critical event. Second, the experts needed to make a prediction based on this hypothesis. Third, they needed to compare this prediction with the results of an experiment. In the example in the table, C and R have previously developed the hypothesis that the light cone is formed when the scattered laser beam transitions from air to plastic and eventually water. As this happens the light moves from a region with a lower index of refraction to a region with a higher index of refraction causing the light which was originally scattered at 90 degrees to bend towards the normal. C and R don't directly say that this is the hypothesis they are testing, but it is clear from the context. Almost immediately prior to this critical event, R stated, "…can we show that there's some limit to the angle of uh the light that comes up through the water?" R's prediction is that there should be some cutoff to any light that comes into the container. C and R test this prediction by shining a laser at almost parallel incidence into the bottom of the container. They compare their prediction with the outcome of this experiment noting, "It doesn't go past that particular angle which looks the exact same as the cone angle."

V.2.f Contrasting Assumptions vs. Testing Experiments

In this category, experts consider how the assumptions they are making match up with the testing experiments that they are running. For this category, the experts first need to discuss some kind of hypothesis or model. Second, they discuss an assumption under which the hypothesis is valid. Third, they make a comparison between the assumption and the experiment. In the example above, C and R are considering whether or not they can use a thin film interference model to reason about the light cone experiment. For thin film interference, there is the assumption that the optical layers are the same order of magnitude thickness as the wavelength of the light. C and R comment that in this case, the assumption does not match the experimental setup because the thickness of the container is too big.

V.3 Comparing Usage of Different Types of Contrasting Cases

To determine if the experts used different types of contrasting cases more frequently than others, we tallied the fraction of contrasting case critical events that were coded for a particular type of contrasting case. The results of this tally are shown in the graph in Figure 4.12. We found that Contrasting Observational Experiments in 16% of the contrasting case critical events, Contrasting Hypotheses in 7%, Contrasting Assumptions in 4%, Contrasting Testing Experiments in 28%, Contrasting Predictions vs. Testing Experiments in 26%, and Contrasting Assumptions vs. Testing Experiments in 16% of all contrasting case critical events.

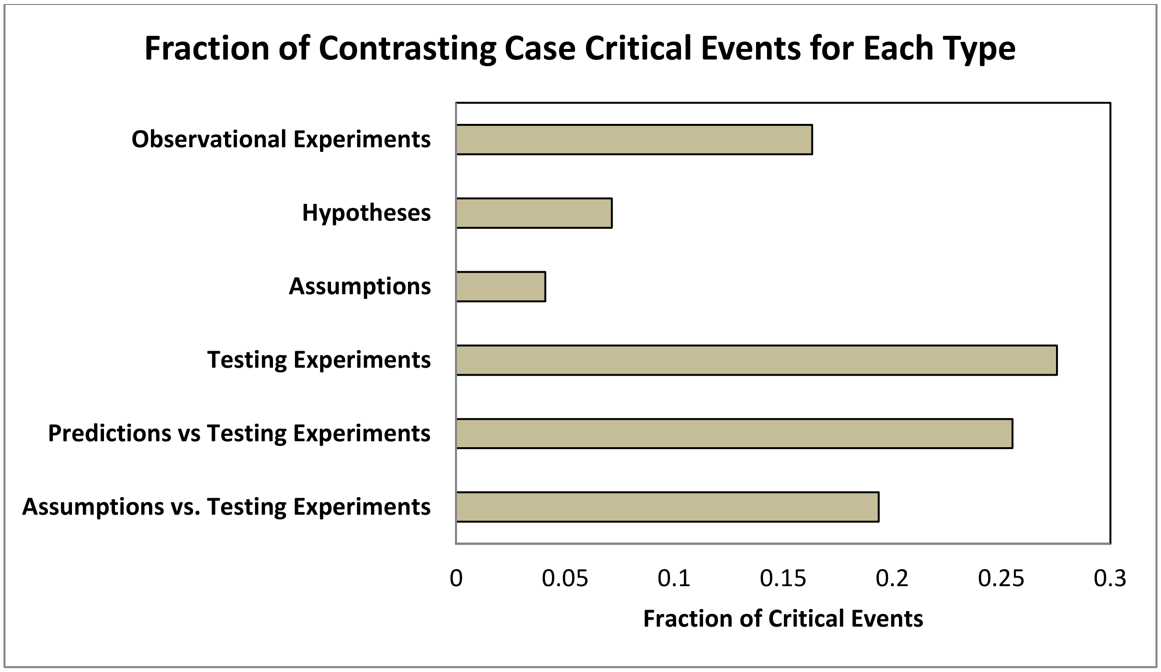


Figure 4.12 A graph showing the fraction of critical events that were coded for contrasting cases that corresponded to a particular type of contrasting case.

VI. Discussion

At the beginning of the paper we set out to answer four research questions. In this section, we will discuss how we answered each of these questions.

VI.1 Use of Contrasting Cases throughout the Problem Solving Process

Our first research question was: When do physics experts use contrasting cases while solving novel problems? Through the above analysis of the problem solving session of C and R, it is clear that contrasting cases are a driving factor behind the initial observations that the experts make, the process of initially constructing a model of a phenomenon, making further observations to refine the model, identifying problems with the model, further model building, and the final testing of a model to ensure that it represents the phenomenon as desired.

VI.2 Purpose of Contrasting Cases

The second research question we set out to answer was: What are the purposes of the contrasting cases that expert physicists use? To answer this question, we used variation theory and analyzed the function of contrasting cases at different points during the problem solving process. We found the following. During initial observations, contrasting cases drew out patterns of variation that the experts used to identify critical features of the light cone. The experts constructed a model of the light cone with the critical features at the focal point. The experts continually contrasted the model that they were building to the phenomenon itself during several stages of the problem solving process. During refinement of the initial model, these contrasts confirmed and clarified previously identified critical features of the phenomenon. The contrasts also drew experts' attention to additional critical features of the phenomenon. In this sense, the experts used contrasting cases to determine which features of the phenomenon were more relevant. When experts found that a feature was more relevant they analyzed it in more detail in the model. Finally, once the experts figured out which parts of the phenomenon were important, they built a coherent explanation of the whole phenomenon by integrating these features. Contrasts between predictions based on this explanation and observations of the phenomenon allowed the experts to determine if they had correctly identified the parts and constructed them into a whole model properly. When the experts identified a contrast between observations and predictions, the experts refined the critical features that were creating this contrast. After the experts constructed a new explanation in this way, they made predictions based on this model once again and contrasted these predictions with observations. Since the experts were unable to identify a contrast between the predictions and observations, they gained confidence in their model.

VI.3 Contrasting Cases Types

The third research question we set out to answer was: What are the patterns in physics experts' use of contrasting cases? In order to answer this question, we identified critical events during the problem solving process. We analyzed critical events and categorized critical events that contained contrasting cases based on the objects that were being contrasted. We found that the objects being contrasting were epistemologically significant. The types of contrasting cases we identified were: contrasting observational experiments, contrasting hypotheses, contrasting assumptions, contrasting testing experiments, contrasting predictions vs. testing experiments, and contrasting assumptions vs. testing experiments. By analyzing how frequently each of these contrasting cases occurred, we found most critical events could be categorized by four of the six types of contrasting cases. These four were contrasting observational experiments, contrasting testing experiments, contrasting predictions vs. testing experiments, and contrasting assumptions vs. testing experiments. It appears that these four contrasting cases may be more important to the overall problem solving process of physics experts than the other two types.

VI.4 Contrasting Cases and the ISLE Cycle

The fourth and final research question we wanted to answer was: How can we link experts' use of contrasting cases to more complex epistemological processes? To understand the specific role that these contrasting cases play in the inquiry processes of physics experts requires that we look at processes through which knowledge is constructed in science or scientific epistemology. Many philosophers of science have struggled with identifying a clear cut "science process" and there appears to be little consensus as to whether or not there is any specific process that is common to scientific inquiry (Etkina & Van Heuvelen, 2007). However, by evaluating research related to the history of physics, the philosophy of science, and the nature of science, it is possible to find several cross-cutting features which most would agree are at the heart of scientific inquiry: empirical evidence, inductive, hypothetico-deductive and analogical reasoning, coherence of ideas, the testability of ideas, and collegiality (Etkina & Van Heuvelen, 2007). Etkina and Van Heuvelen proposed a learning cycle model to incorporate these elements into student learning of physics (the ISLE cycle). In the ISLE cycle students develop new ideas working together and progressing from observing simple experiments, to identifying patterns, to devising causal or mechanistic explanations for those patters and finally testing those explanations experimentally. The experimental testing involves designing a new experiment, making predictions of its outcome based on different explanations, conducting the experiment and comparing the outcome to the predictions. The above elements form the elements of the cycle that repeat many times for different concepts that the students have to construct. It appears that our findings of contrasting cases show that we can map steps of experts' problem solving to either contrasts of different elements of the cycle or contrasts of the different aspects of the same element.

Our finding is in agreement with the work of Poklinek-Cancula et al. (2015) who used the ISLE framework to analyze what experts do when they solve the light cone problem. They found that physicists do engage in processes that resemble the steps of the ISLE cycle and their sequence roughly resembles the sequence of the steps proposed by the cycle (although it is much more complicated). The difference between their study and the current study is that they used the ISLE framework to design their coding scheme. We, on the contrary, were interested in epistemological resources that experts used. It appears that while we were not using ISLE framework to analyze how experts solved two experimental problems, the crucial elements of the cycle appeared through the coding of the contrasting cases.

Our previous analysis of C and R's problem solving session using variation theory can help to shed light on the apparent relationship between contrasting cases and the ISLE cycle. In the analysis, we described how C and R used contrasting cases to identify patterns of variation and that these patterns helped identify focal points, or critical features, for the group's model of the light cone. This describes why the contrasting cases in observational experiments are critical. Without them, the experts would be unable to identify patterns. Contrasting hypotheses, while not prevalent in C and R, was used by other groups of experts to compare potential identifications of critical aspects and critical features. Contrasting hypotheses allowed experts to consider the effect different critical features would have on the phenomenon and determine which seemed most probable. Contrasting testing experiments plays a similar role. The experts, having identified a potential critical feature of the phenomenon, discuss how that feature would manifest itself in different scenarios. This can either serve the purpose of determining whether or not the critical feature seems reasonable based on its predictions or produce predictions for different experiments that can later be performed to test the validity of this critical feature. Contrasting assumptions help the experts to determine if there are any non-critical features that might influence the predictions their model makes.

After the experts developed their model, they continually checked to see if their model was correct by contrasting the predictions and assumptions of their model against the actual phenomenon itself, embodying both the importance of testing experiments and the analysis of any assumptions in the ISLE cycle. According to variation theory, at this stage the experts were developing an understanding of the parts of the phenomenon the phenomenon and how those parts fit into a coherent whole. Contrasting predictions and assumptions of the model that the experts were building with the phenomenon itself in the form of testing experiments and assumption analysis allows the experts to determine whether or not the experts are building a model which accurately represents reality. Without these contrasts, experts would never be able to determine if the models they were building were accurate.

VII. Implications for Instruction

The evidence from the experts' problem solving sessions shows that they use contrasting cases in multiple ways. One implication this has for instruction is that students need have an opportunity to use contrasting cases in ways similar to that of experts. Simply having students contrast observations is not enough to have them develop more sophisticated epistemological processes that rely on other types of contrasting cases. To be able to help students develop the multitude of contrasting cases they will need, students first need to develop an understanding of scientific hypotheses, predictions, assumptions, and experiments so that they can contrast these entities. In other words, students need to develop an understanding of the epistemology of science. One way to do this is to engage students in the ISLE cycle. Throughout the ISLE cycle, students are given the opportunity to perform observation experiments, develop hypotheses, analyze assumptions, and develop testing experiments. Giving our students these opportunities will emphasize the role of these epistemological entities play in the learning process. While it is unclear whether or not expert's use of contrasting cases develops as a result of their use of processes similar to that represented in the ISLE cycle or whether use of processes similar to the ISLE cycle develops from a tacit understanding of the importance of contrasting cases, it is clear that the ISLE cycle and contrasting cases are connected. The direction of this connection is the subject for future studies.

Chapter 5

Summary

In this final section, I will revisit each study, show how the results of each study helped me answer the questions that were proposed in the introduction, discuss implications for instructions, talk about new questions that have arisen from the results of these studies, and end with concluding remarks.

I. Resource Activation Patterns during Conceptual Breakthroughs

The first research question was: Are there any patterns in the way that professional physicists activate and combine different resources during critical moments of the problem solving process?

Answering this question required that I first address the more preliminary issue of reliably identifying instances during which critical moments occur. To do this I developed criteria that needed to be met in order for a passage of text that was picked out of the transcript to be coded as critical, i.e. showing evidence of a conceptual breakthrough. The criteria were as follows:

- A conceptual breakthrough should happen through reasoning and not just through observations.

- A critical moment should propose or answer a question that advances the experts understanding of the problem.

In order to test these criteria I:

- Picked passages of text that captured full exchanges between experts.
- Identified the questions that the experts were answering during these passages.
- Asked another researcher to determine whether or not the selected passages met the above criteria.

In this way I was able to establish an inter-rater agreement of 82% before discussion and 96% after discussion. Once I established that my criteria for critical events were reliable, I was able to proceed with identification of critical events.

To determine whether or not the experts were using resources during critical events in any particular way, I broke down the experts' reasoning, identified resources that the experts were using, and classified these resources as p-prims, higher level physics based resources, and epistemological resources. By analyzing critical events for the light cone problem, I found that critical events were much more likely to have all three different types of resources than non-critical events. Specifically, we found that 88% of all critical events had all three types of resources while only 53% of non-critical events showed the same pattern. Moreover, I found that all events had evidence of epistemological resources. When I analyzed critical events for the solar cell problem in the same way, I found that only 74% of critical events had all three types of resources. This prompted me to take a deeper look at the critical events. I found that by classifying critical events based on the types of questions that the experts were answering, I could pick out physics critical events and non-physics critical events. When I grouped the events in this way, I found that 84% of physics critical events showed evidence of all three types of resources, while only 30% of non-physics critical events did.

By comparing these percentages to the patterns found by Richards (2013) in a similar investigation of problem solving by pre-service physics teachers, I was able to answer the second research question: How do the patterns of resource activation during professional physicists' problem solving compare to the patterns of resource activation during pre-service physics teachers' problem solving? I found physics experts' patterns of resource activation were similar to those of pre-service teachers when only physics related conceptual breakthroughs in experts' work were considered. The pre-service teachers did not have to deal with non-physics questions because their experience was more structured, as they learned about solar cells in the context of a lesson taught by Richards.

To try to understand the significance of this pattern, I investigated the content of a critical event in detail. I found evidence that epistemological resources helped focus the experts' attention on particular p-prims. These p-prims were then used to justify higher level conceptual physics resources (diSessa, 1993), apparently blending their intuitive and formal knowledge to build a deeper understanding of the problem (Kuo et al., 2013).

II. Reliably Identifying Epistemological Resources

To gain a deeper understanding of expert physicists' problem solving knowledge, I investigated the epistemological resources they used during critical moments. In order to ensure the integrity of our analysis, I first had to answer our third research question: How can we reliably identify epistemological resources?

In order to do this I developed the following process. First, I coded critical events for resources. I performed a preliminary analysis by looking at individual epistemological resources and determining the fraction of critical events in which each epistemological resource was found. To narrow the focus, enabling me to continue with a more rigorous analysis, I selected resources that were found in at least 5% of all critical events. This narrowed my analysis to the 19 most common epistemological resources. For each resource, I looked at all the critical events that a resource was in and identified common features across the critical events for that resource. I used the common features to develop criteria for deductively identifying these resources. Together with another researcher we used these criteria to code 10% of all critical events for a single resource, discussed our choices, and refined the criteria based on discussions. We then coded another 20% of critical events to determine the reliability of the refined criteria. The process was repeated for each of the 19 resources. In this way, we developed reliable criteria for each of the 19 most common epistemological resources. The least reliable of these criteria resulted in a 77% inter-rater agreement, while many others were higher. On average, the criteria produced coding that resulted in 84% inter-rater agreement.

III. Epistemological Resources of Physics Experts

After establishing reliable criteria for identifying epistemological resources, I could move on to answering our fourth research question: What epistemological resources do professional physicists use when solving challenging, novel problems? Are there certain resources that experts tend to use more often than others?

In the previous preliminary analysis of critical events, I identified 46 different epistemological resources that the experts used. Out of these 46, 19 epistemological resources were found in at least 5% of all critical events during preliminary analysis. These epistemological resources were:

- Analogical reasoning
- Attention to novelty
- Causal reasoning
- Consistency
- Contrasting Cases
- Deductive reasoning
- Experimentation
- Hypothetico-deductive reasoning
- Inductive reasoning
- Knowledge from direct observation
- Limitations of models
- Mathematical reasoning
- Mechanistic Reasoning
- Multiple representations
- Peer cognitive awareness
- Personal cognitive awareness
- Plausibility
- Relative value of knowledge
- Sense making

To determine if the experts used any of these resources more often than others I recoded all critical events using the refined criteria I developed for deductively identifying the individual epistemological resources. This allowed me to figure out how often the experts used each epistemological resource by finding the fraction of critical events each resource was coded in. I found that different resources were used with varying frequency. All recoded resources were found in greater than 20% of all critical events, but three resources, contrasting cases, causal reasoning, and consistency, were found in greater than 50% of all critical events. Contrasting cases was the most frequently used epistemological resource, found in 67% of all critical events.

IV. Importance of Contrasting Cases used by Physics Experts

After establishing that contrasting cases was the epistemological resource that experts used most during critical moments, I could proceed to analyze experts' use of contrasting cases in order to answer the fifth research question: If there are resources that are used more frequently than others, how do the experts use these resources to develop an understanding of the problem they are solving? I answered this question by first conducting an in depth analysis to determine how a single group, C and R solving the light cone problem, used contrasting cases and then I looked for patterns across all groups of experts' use of contrasting cases during critical moments.

To aid in the analysis of C and R's problem solving session, I analyzed their use of contrasting cases using variation theory (Marton & Booth, 1997; Marton & Tsui, 2004; Ling, 2012). I found that experts C and R used contrasting cases throughout the problem solving process when they were making observations, building models, and refining and testing models. Variation theory helped to understand the role that contrasting cases played during these phases of problem solving. As the experts made observations, contrasting cases helped draw their attention to important critical aspects of the phenomenon and helped them to determine whether or not these aspects were relevant. Once the experts began to identify relevant critical features of the phenomenon, they built models to represent the phenomenon. Critical features, identified through contrasting cases, were at the focal points of these models. As the experts built these models they contrasted the models with the actual phenomenon they were trying to explain. These contrasts helped the experts make sure that they properly identified and represented critical features in their model and helped the experts discover new critical features that needed to be incorporated into their model to fully represent the phenomenon.

I complemented the analysis of C and R's use of contrasting cases by looking for patterns across all groups. I found that it was possible to categorize the different ways that experts used contrasting cases during critical moments by focusing on different elements of scientific epistemology that were being contrasted. I identified the following types of contrasting cases:

- Contrasting observational experiments
- Contrasting hypotheses
- Contrasting assumptions
- Contrasting testing experiments
- Contrasting predictions vs. testing experiments
- Contrasting assumptions vs. testing experiments.

These different types of contrasting cases match the elements of the ISLE cycle (Etkina & Van Heuvelen, 2007), one of the approaches to learning physics that focuses on science practices. Out of the different types of contrasting cases, experts used contrasting hypotheses and contrasting assumptions relatively infrequently, while the other types of contrasting cases accounted for 89% of contrasting cases.

V. Implications for Instruction

From my research findings it is possible to draw the following conclusions about implications for instruction.

First, the finding that p-prims, higher level physics based resources, and epistemological resources are all interconnected during critical moments means that we need to give students a chance to reconcile their intuitive knowledge with the more formal knowledge that they are learning about in physics class. If they are not given this opportunity, they will be less likely to make a conceptual breakthrough. Encouraging this reconciliation requires instructional interventions designed specifically with an emphasis on epistemological goals (Elby, 2001).

Second, as experts frequently use a variety of epistemological resources during critical moments, we must design instruction with the development of a multitude of epistemological resources in mind. Physics problem solving requires many different tools. If a student lacks the required tool for a particular part of a problem, she/he will be unable to make any progress. While experts do use a variety of epistemological resources, there are some that they used more frequently than others. Instructors should place added emphasis on these most frequently used resources, especially contrasting cases and design lessons to help students develop their use of contrasting cases.

Third, students should be taught to use contrasting cases in the different ways similar to the ones that experts follow. This work is not the first to point to the importance of contrasting cases. Several instructional methods that use contrasting cases (Bransford & Schwartz, 1998; Schwartz et al., 2011) have focused on the use of contrasting observations. Our analysis shows that physics experts use contrasting cases not only when they are making observations, but also when they are building and testing models of phenomena. Instruction needs to focus on all types of contrasting cases in order to help students become more expert-like. Different types of contrasting cases rely on a student's ability to develop hypotheses, identify assumptions, make predictions, and develop observational and testing experiments. Fostering these abilities can be achieved by teaching students physics through the ISLE cycle (Etkina & Van Heuvelen, 2007) and emphasizing the role that these entities play during the learning process.

VI. Future Research Directions

The three studies have helped me answer many important questions about how expert physicists deal with novel, challenging problems and direct our attention to future possible research questions.

Now that we know that expert physicists' reasoning during critical moments can be characterized by p-prims, higher level physics based resources, and epistemological resources, we need to figure out how to encourage students to reason this way. Doing so will require answering the following questions: How often do students reason using p-prims, higher level physics based resources, and epistemological resources simultaneously? What are the necessary features of instruction that promotes reconciliation of intuitive knowledge with formal knowledge while utilizing epistemological resources?

The importance of certain epistemological resources and the variety of resources that expert physicists use lead us to ask the following questions. How can we create instruction that effectively emphasizes and develops the multitude of epistemological resources that expert physicists use? The importance of contrasting cases itself means that this resource deserves much future attention. Studies need to focus on students' use of contrasting cases and should look to answer the following questions. Under what conditions will students use contrasting cases spontaneously? To what extent do they spontaneously use contrasting cases in the way that expert physicists use contrasting cases? What conditions promote productive use of contrasting cases? What conditions promote unproductive use of contrasting cases? What are the necessary features of instruction that will help students develop their ability to use contrasting cases and other important epistemological resources?

VII. Concluding Remarks: Contributions to the PER field

In this dissertation I have used the resource-based model of cognition to conduct several studies which investigate how expert physicists construct new knowledge. My dissertation contains the following original contributions to the field of Physics Education Research (PER):

- In order to conduct my studies, I developed two ways of establishing reliability:
    - Established a way to reliably identify critical moments, or reasoned conceptual breakthroughs.
    - Established a way to reliably code for epistemological resources.
- By looking at several different types of resources, I was able to find a resource-based pattern in expert reasoning that was not known before.
- By analyzing how p-prims, physics based resources, and epistemological resources were being used during one of these critical moments, I showed that epistemological resources act as a control mechanism while the

experts reconcile intuitive and formal physics knowledge to make a conceptual breakthrough.

- I identified epistemological resources that experts commonly use during critical moments.
- I found that experts used different epistemological resources with varying frequency, with contrasting cases being the resource that the experts used most often.
- Using variation theory, I was able to show that contrasting cases played varied roles during the observing, model building, and model testing phases of the problem solving process.
- By looking at contrasting cases used by all groups of experts, I was able to show that the experts' contrasts are significant from a scientific epistemology perspective.

These studies enabled us to discover some of the ways in which physics experts excel at solving novel, challenging physics problems, offered many insights that can be used to improve physics instruction, and helped to uncover important questions that will need to be answered in the future.

Appendix A: Rationale for Coding of Critical Event

Below we present our rationale for coding the critical event presented in section 3.IV.2.a. The goal of this presentation is to help the reader understand the reasoning we used when we coded participants' utterances for specific resources. First, we provide the context surrounding this critical event. Then, we explain how we coded each resource.

Table A.1 Example of critical event coded for resources

C: Yeah. What this diagram (1) is ignoring though is what happens at the paper. Right? (2,3,4) R: Yeah. C: What happens when the laser hits the paper? (5) We don't – do we get – we don't get the exact same beam coming back, right? (6) Cause that's what you were trying to look at before, I think. (7) Just sort of looking at what (8,9,10) – R: Yeah. Somehow it spreads out (11) in a very well defined way. (12) Uh…in the paper. C: Ok, so what is this paper doing (13)? Beam hits it, the paper is – the paper is what? Optically rough (14)? Is that a term we can use? C and R: Haha. C: Why not? R: It's not a mirror. (15,16) C: That's true. I'll point it away from you. R: Heheh. C: So the laser hits the paper and has to be diffused (17) somewhere, right? It has to…it scatters (18). I mean if we – The paper is definitely rough (19).	1. Multiple Representations 2. Contrasting Cases 3. Hypothetico-deductive Reasoning 4. Limitations of Models 5. Causal Reasoning 6. Something's Changing 7. Peer Cognitive Awareness 8. Experimentation 9. Inductive Reasoning 10. Knowledge from direct observation 11. Spreading 12. Concentration and/or localization 13. Mechanistic Reasoning 14. Roughness 15. Analogical Reasoning 16. Mirrors 17. Spreading 18. Scattering 19. Roughness

A.1 Context Surrounding Critical Event

Prior to the above critical event, C and R have made several observations of the light cone using different materials beneath the container of water. From these observations, they have concluded that the light cone phenomenon is related to the interface beneath the container. This prompts them to construct a Snell's Law type of diagram focusing on what happens to the laser beam at each optical interface at the bottom of the container. The diagram that they have drawn and discuss in this critical event is shown in Figure A.1.

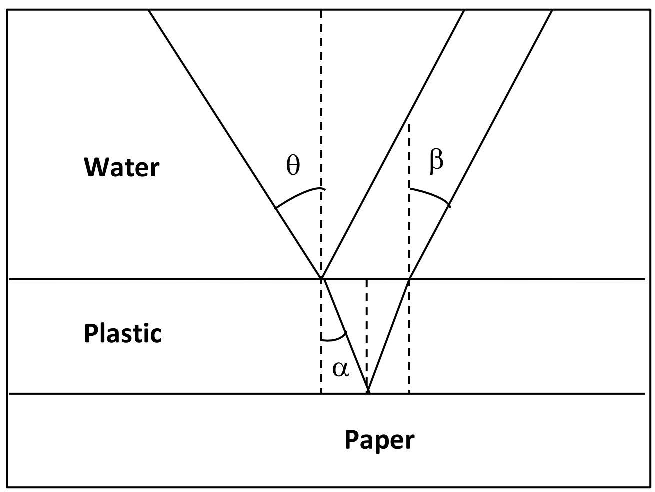


Figure A.1 The diagram that experts C and R have drawn on the board at the start of the critical event.

Shortly before the critical event, R has performed an observational experiment that the experts discuss during the critical event. In this experiment, R removed the white piece of paper from underneath the container and shined the laser beam directly at it. This led R to observe, "…when you shine the light off just the paper…you can see it, um, reflecting off at a lot of different angles." At the time, this observation was not addressed further, but it becomes a focal point of this critical event.

A.2 Rationale for Coding of Individual Resources

Below we explain our rationale for the coding of each individual resource. We discuss our coding in the same order that the resources were listed in the critical event.

1. The first resource we coded for was multiple representations. We coded for multiple representations because the experts were reasoning using the diagram shown in Figure A.1.
2. The second resource that we coded for was contrasting cases. In this event, C is discussing the diagram of the light cone in Figure A.1. He also discusses R's previous observation that when the laser beam hits the paper it spreads out. This is what he is referring to when he says, "…that's what you were trying to look at before…" His comparison between the two is evident when he says that the diagram ignores what happens when the laser hits the paper, that they "…don't get the exact same beam coming back..." as they have drawn in the diagram.
3. The third resource that we coded for was hypothetico-deductive reasoning. In our criteria for hypothetico-deductive reasoning, we said that four things should be identified: an observation, a hypothesis, a prediction based on that hypothesis, and a conclusion about how the observation relates to the prediction. In this critical event, the observation that is being discussed is R's previous observation that when the laser beam hits paper, it spreads out. The hypothesis being analyzed is the hypothesis implicit in the diagram that C and R have constructed: when light scatters off paper, it reflects as though it were scattering off a mirror. The prediction is manifested in the diagram where C and R have drawn the laser beam so that the angle of reflection of the laser beam off the paper is the same as the

angle of incidence. C compares R's observation to the diagram when he says that "…they don't get the same laser beam coming back…" He makes this statement to justify his conclusion that the diagram, and hence their prediction, is ignoring what happens when the laser hits the paper.

4. The fourth resource we coded for was the limitations of models. In this case, the model is the diagram in Figure A.1. In this diagram, the laser beam follows the law of reflection at every optical interface. C states that there is a mismatch between this model and the phenomenon and suggests that the model needs modification when he says the "diagram is ignoring…what happens at the paper."
5. In this event C and R discuss what happens when the laser beam hits the paper (the cause). The effect is that "…they don't get the same laser beam coming back…" It is clear that these two events are linked because C's statement about the effect is an answer to the question, "What happens when the laser hits the paper?" The fact that he says the laser beam "hits" the paper satisfies the final criterion for causal reasoning.
6. When C and R say that they "…don't get the exact same beam coming back…" it's clear that they are referencing the fact that the laser beam is somehow changing. This caused us to code for the conceptual resource something's changing.
7. The seventh resource we coded for was peer cognitive awareness. In this critical event, C explicitly mentions makes reference to R's previous observation.
8. When C brings up the fact that they "…don't get the exact same beam coming back…" and later says to R "…that's what you were trying to look at before, I

think," he is explicitly addressing the results of an experiment that R personally did. This satisfies the second criterion for experimentation.

9. The ninth resource we coded in this critical event was inductive reasoning. R's previous observation that the laser appears to reflect off the paper at lots of different angles is the observational premise that C references when he says "Cause that's what you were trying to look at before…" This statement also shows that he uses this observation to justify his conclusions that "…they don't get the same laser beam coming back…" and that the diagram ignores what happens when the laser hits the paper.
10. C's reference to what R was "trying to look at before" and the fact that he is referencing the experiment R personally performed to gather knowledge caused us to code this event for knowledge from direct observation.
11. We coded R's specification that when the laser beam hits the paper, "Somehow it spreads out…" as evidence that he was using some kind of conceptual resources related to things spreading out.
12. R's further specification that the laser spreads out in a "well-defined way" was somewhat contrary to his previous statement about the laser spreading out. We coded this statement as showing evidence that the phenomenon of spreading was somehow concentrated or localized.
13. When C asks "so what is this paper doing?" he is beginning to search for a process which links the cause (the laser beam hitting the paper) and the effect (that the laser beam does not come back exactly the same) that he has previously identified. Through the discussion after this question he develops an argument

which links this cause and effect through the process of scattering off the rough paper, showing evidence of mechanistic reasoning.

14. We coded C's direct attention to the possible roughness of the paper as evidence that some conceptual resources related roughness were being used.
15. In this critical event C is reasoning about the properties of paper. As he is doing so, R reminds him that paper "is not a mirror." In this exchange, the paper is the target and the mirror is the analog. R's comment indicates that the properties of mirrors should not be transferred to paper.
16. We coded the utterance of the word mirror as evidence that R was using resources for understanding mirrors and specifically surfaces which obey the law of reflection. While just the utterance of this word itself may not be sufficient evidence to support this, there are several other factors that contribute to this coding. First, the diagram in Figure A.1 shows scattering which obeys the law of reflection. Furthermore, the dialogue before this statement supports that the experts are reasoning using the physics specific resources related to mirrors. This is most clear when C states, "We don't – do we get – we don't get the exact same beam coming back, right?"
17. C's attention to the diffuse nature of the cone was coded as further evidence of the experts using resources related to spreading.
18. While C's statement "…it scatters" could be viewed as further evidence of him using resources related to spreading, we coded this as evidence of the more physics specific phenomenon of scattering. We coded it this way because C uses the statement that the laser beam scatters as clarification to his previous statement.

The statement that the light "has to be diffused somewhere" is a generic statement about what the light is doing while "it scatters" adds significance by articulating this idea in a physics specific way.

19. Again, we coded C's direct attention to the possible roughness of the paper as evidence that some conceptual resources related to roughness were being used.

Appendix B: Additional Epistemological Resources

B.1 Multiple Representations

Using multiple representations basically means using some means of representation other than verbal representation. Different types of representations include mathematical equations, graphs, sketches, and diagrams. Reasoning through multiple representations is a tool that not only helps students learn a subject better, but different representations have also helped further advance the scientific community (Van Heuvelen, 1991; Van Heuvelen & Zou, 2001; Etkina & Van Heuvelen, 2007). To be coded as having evidence of multiple representations, the critical event only needed to satisfy the single following criteria:

1. Evidence of some type of representation of information that is not verbal communication (such as a diagram, graph, or mathematical equation) being used or discussed in some way.

Below is a critical event which was coded for multiple representations. In this critical event, experts S and P are discussing their predictions related to the second part of the solar cell problem. Prior to this, they had developed predictions for almost all of the different coverage scenarios, but were unsure about their prediction for the scenario where the solar cells are connected in opposition (anti-series) with one of the solar cells completely covered, while the other is completely uncovered. Previously, they believed that it should be similar to the case where the solar cells are connected in series and covered the same way. However, after they construct a circuit diagram, they remembered that the equivalent circuit for a solar cell incorporates a diode. In this critical event, P states that he believes this should change their prediction.

> S: ... I think in this case (series) these two (one covered and one uncovered) will be the same with these two (anti-series case).
> P: That's what I thought.
> S: Yeah.
> P: But from the electric equivalent circuit there's a diode here.
> S: A diode? Where?
> P: If you represent the solar cell with an equivalent circuit –
> S: A what?
> P: You have a current source. Well, this is just an electrical diagram. The diode and the shunt resistor –

The alternative representation being used in this critical event is a circuit diagram and most specifically, the equivalent circuit representation of a solar cell. This is most clearly evidence by P's statement where he describes the equivalent circuit for a solar cell.

B.2 Limitations of Models

Building models is an important part of inquiry in physics research and physics instruction (Etkina, Warren, & Gentile, 2006). One important aspect of model building is understanding that models are only representations of reality and therefore have limitations which put bounds on their applicability. In order for a critical event to be coded for evidence of the resource "limitations of models" it needed to have evidence of both of the following criteria:

1. A model.
2. Some statement that references a mismatch between the model and the current situation. This can be:
    a. A statement that the model is inappropriate for this situation.
    b. A statement that the model is only an approximation of the situation.

c. A statement that the model needs modification to match the current situation.

The critical event below shows an instance where A and D show evidence of understanding the limitations of models. In this critical event, A and D are working on the second part of the solar cell problem. Previously, to better understand the situation, A and D have drawn a circuit diagram on the board, which they have been using to reason about the problem. In this diagram, the solar cells are represented with a circle with an arrow inside, representing a constant current generator. In the critical event, A and D make some suggestions of how they can improve this representation.

> D: Hmm. But if it's, if it's actually a solar cell, right, isn't there some sort of, it looks like that? (adds a resistor in parallel to the current generator)
> A: Yeah and there's a diode in there somewhere.
> D: Yeah. There's a diode in there.

At the beginning of the event, D identifies the model when he says "if it's actually a solar cell…" The "it" being referred to in this statement is the circuit diagram drawn on the board. This becomes more transparent when D says "it looks like that?" and proceeds to make a change to the circuit diagram. This entire first statement serves to satisfy element c of the second criteria. From this statement it is clear that the model of a constant current generator that A and D have been using needs modification if it can adequately represent a solar cell. The modifications that A and D propose are adding both a resistor and a diode to the circuit element.

B.3 Mechanistic Reasoning

As described before, a framework for analyzing mechanistic reasoning has been described in detail by Russ et al. (2008). A key addition to the reasoning process which differentiates mechanistic reasoning from strictly causal reasoning is the inclusion of a process that can link cause and effect phenomenon. We have included that as our primary criteria for identifying mechanistic reasoning. In order for a critical event to be coded as evidence of mechanistic reasoning, all three of the following criteria needed to be satisfied.

1. An initial phenomenon.
2. A final phenomenon.
3. A process linking the initial and final phenomenon.

The following critical event shows an example of an instance where experts S and P used mechanistic reasoning. In this critical event, experts S and P are developing explanations which can explain the shape of the graphs in the first part of the solar cell problem.

> P: … And then you start covering, so what happens if you do this way or this way? (Cover the solar cell from the long or short side) It doesn't have to be exactly like this, but it seems from the graph that once you go like that (gestures as if coming in from the short side), you effectively disable certain tiny cells, which are in series, which kind of become huge resistors for the total current.
> S: Yeah. Mhmm.
> P: So you would expect that for this kind of experiment…
> S: Yeah. The resistors –
> P: You're gonna have everything. Illuminated area is equal to the total area.
> S: Mhmm.
> P: And then you just block some part and make it very resistive and you see a fast decay. And in the second case you go like this (gestures as if coming in from the long side). And uh the arrangement is so that they still get, each one is still getting some light.

In the critical event, P's explanation of how covering the solar cell from the short side would lead to a fast decay is the example of mechanistic reasoning in this critical event. The initial phenomenon criteria is satisfied when P says "once you go like that" and gestures as if he is covering the solar cell from the top side. The final phenomenon is that "you see a fast decay." A process that provides a link between these two is given by P when he says, "once you go like that, you effectively disable certain tiny cells, which are in series, which kind of become huge resistors for the total current."

B.4 Mathematical Reasoning

Mathematics plays a very important role in physics. As a result, we looked to identify instances when the experts were using some kind of mathematical reasoning during a critical event. There were several different things the experts could do that would cause us to code the critical event as having evidence of mathematical reasoning. If an event was coded as having evidence of mathematical reasoning, it satisfied at least one of the following criteria:

1. Talking about a mathematical equation.
2. Manipulating an equation.
3. Using a graph.
4. Discussing a mathematical relationship between variables.
5. Discussing the mathematical form of a relationship.

An excerpt taken from a critical event which was coded as having evidence of mathematical reasoning is shown below. In this critical event experts S and P are working on the first part of the solar cell problem. P has previously remarked that the solar cells must be wired so that the individual elements within the solar cell are in series, but did not elaborate. The critical event begins with D asking P to explain how he knows that the elements in the solar cell must be in series.

> D: So how do you know that they're connected in series?
> P: Well from the graph because in the series connection in order to – you should see this kind of connection. So for each one its efficiency for individual cell is proportional to the relative illuminated area. Right? So if you cover it completely

> with the screen it becomes a huge resistor and it doesn't help at all. But if you cover it like this, [from the long side, partially covering each] its efficiency drops by 10% and 10% for this guy, 10% for this guy. So overall, a series connection would be a sum of contribution from each one and it would go down linearly for this case [from the long side, each partially covered] and somehow very rapidly for this one [from the short side, entire cells fully covered].

In this critical event, P very clearly satisfies two of the criteria for mathematical reasoning. First, P immediately references the graph as the source of his idea that the individual elements are in series. Later, P elaborates by describing a model which he is using to reason and then is able to develop a mathematical form for the current graph based on his model, satisfying the fifth criteria for mathematical reasoning as well.

B.5 Plausibility

Coming to a correct solution to a problem often relies on first brainstorming possible solutions and then coming up with some way to test the correctness of these potential solutions to find out which works best. The first part of this relies on resources for developing many plausible ideas without knowing whether or not these ideas are correct. The plausibility code was developed to determine how often experts came up with ideas and acknowledged that these ideas may not be completely correct. In order to be classified as having evidence of the plausibility code, both of the following criteria needed to be satisfied.

1. Discussion of an idea/model/hypothesis.
2. An individual making an explicit indication that they have not made a finite decision about whether the idea is right or wrong.

The critical event below is an example of an instance when experts were proposing a plausible explanation. In this critical event, experts B and D are working on solving the light cone problem. Expert B explains an idea about how the light could deviate from a specular reflection.

> B: Well, so, I mean, this has a non-zero thickness right? The bottom, the plastic container.
> D: Yeah.
> B: So it could be some sort of internal, uh – So we have many, many reflections and transmissions going through that.
> D: Uh huh.
> B: Right? So like, it enters via the top surface and then kind of bounces around inside the small plastic container and then it could come out at different angles.

In this critical event, B discusses his model starting with, "So we have many, many reflections…" and ending with "…and then it could come out at different angles." This satisfies the first of the two criteria. Before he discusses this model he says "So it could be…" This is an explicit acknowledgement by expert B that he is not certain whether or not this idea is correct. Therefore, the statement satisfies the second criteria. With both criteria satisfied, this event can be coded as having evidence of plausibility.

B.6 Inductive Reasoning

Inductive reasoning is a form of logic which individuals use when they draw conclusions based on observations that they make. An individual reasoning inductively would begin a line of reasoning with observations acting as premises and then develop conclusions from these premises. Using observations as a starting point is one of the primary differences between inductive and deductive reasoning. Another important difference between inductive and deductive reasoning is that the conclusions of deductive reasoning will always be true as long as the premises are true and the individual uses proper logic to reach the conclusion. When reasoning inductively, it is possible to reach incorrect conclusions even if your premises are true as a result of an improper generalization. When we coded a critical event for inductive reasoning all of the following criteria needed to be identified:

1. A statement based on observations that acts as a premise.
2. A conclusion derived from that premise.
3. A linguistic link showing evidence that the conclusion is based on the premise.

In the excerpt below, all three of these criteria were satisfied. This critical event finds experts C and R at an early stage in their efforts to understand the light cone phenomenon. Up to this point, they have not identified a possible locale to focus their attention. Just prior to this critical event, they have just observed the light cone at many different angles of incidence of the laser beam. In this critical event, they discuss how observations that they have made of the light cone at many different angles of incidence of the laser beam lead them to propose a possible locale.

> R: … The fact that the angle of the cone doesn't change [at various incident angles of the laser] makes me think that it [the light cone] has something to do with the, the –
> C: Interfaces?
> R: Yeah the interface since the –
> C: Interfaces isn't changing. Interfaces aren't changing.
> R: Mhmm.

R's first statement, that the "angle of the cone doesn't change" is the observation that acts as the premise in this line of inductive reasoning. The conclusion that C and R both reach is that the interfaces play some part in the overall phenomenon, evidenced by the agreement they reach in the middle of the passage. Finally, R links these two with the phrase "The fact that…makes me think that…"

B.7 Knowledge from Direct Observation

While more complex epistemological resources, such as inductive and hypothetico-deductive reasoning processes can be used by experts while solving novel physics problems, it is also possible that they make use of simpler processes as epistemological resources as well. Knowledge from direct observation is one such simple resource that is likely available from an early age to young children as a way of understanding how an individual can come to know something (Hammer & Elby, 2002). The following criteria were developed to capture moments when individuals appeared to make an effort to gain knowledge through observing a phenomenon directly, convey knowledge through showing someone else directly, or show some evidence that they believed observing a phenomenon directly was important. For this code, only one of the following criteria needed to be satisfied in order for the passage to be coded as having evidence of knowledge from direct observation.

1. A reference to "seeing" something happening in the experiment.
2. A reference to "look" at the experiment.
3. Evidence of prolonged eye contact with the experiment as a statement about the experiment is being made related to observations.
4. Reference to data that has been gathered or an experiment that has been performed and the fact that it was personally gathered and observed by the participants.

The following piece of dialogue is a critical event that was coded as having evidence of knowledge from direct observation. In this critical event, experts B and D are working towards solving the light cone problem. Prior to this critical event, expert's B and D were brainstorming possible causes for the light cone, with D writing ideas on the board while B was continually experimenting. The critical event begins with B stating that he believes the light cone's shape is unaffected by his manipulations. D asks him to explain, and B does so by directly showing him.

> B: Because I'm saying there's an independence of the cone shape.
> D: (sits down and focuses on experiment) You think there's an independence of the cone's shape?
> B: Yeah so see, watch this, right. (begins shining the laser into the water, scanning through various angles) The cone is much wider, right, than the incident and reflected angle.
> D: Ok.
> B: Right. But then if we do this way, it doesn't really seem to change….

There are several parts of this critical event which show evidence of knowledge from direct observation. The first, most clear-cut piece of evidence is when B says to D "Yeah so see, watch this, right." B then proceeds to perform the experiment while D watches. In this statement, B specifically says to D "see" and "watch this." On top of this, throughout the majority of the critical event, B and D are both making eye contact with the experiment while talking about the observations they are making. As a note, B simply could have told D that when he changed the angle of the laser beam the cone did not change, but felt that it would be more convincing if he were to actually show this to D first hand.

B.8 Attention to Novelty

One aspect of expertise is the ability perceive important elements of a scenario that novices do not (deGroot, 1965; Bransford et al., 2000). For example, expert chess players are able to identify tactically significant positions of chess pieces much more effectively than novices (Chase & Simon, 1973). As a consequence of this, we were interested in identifying when experts made mention of something catching their attention. We found that this happened three different ways which we incorporated into our criteria for this resource. The criteria were that a critical event should show evidence of:

1. Discussion of some intuitive expectation not being met.
2. Explicitly mentioning that something is interesting.
3. Identifying that something is confusing.

A critical event need not satisfy all three of these criteria, but rather only needed to satisfy one of the above criteria to be coded as having evidence of the attention to novelty resource. Below, we present a critical event where we found evidence of the experts paying attention to novelty. Prior to the critical event, A and D were trying to gain a better understanding of how the solar cell used in the first part of the solar cell problem was constructed. In the critical event, D notices that it appears like a metal strip connects both the negative and positive terminals within the solar cell and A and D discuss this.

> D: I'm just, I'm just a little confused about why it looks like the ones [metal strips] on top go everywhere. Like it goes, I mean, to the positive and negative terminals.
> A: Well it must, this must be insulated here, there must not be a connection there.
> D: Hmm. Cause it looks like the…yeah it actually looks like both of them go everywhere. No, no. That's this guy. So this guy's for the negative…I don't, that looks like metal to me.
> A: It does look like metal, but it can't be. I mean there's no way they would have those things in electrical contact with each other.

Both the first and the third criteria are met in this critical event. First, D explicitly states, "I'm just a little confused about why it looks like the ones on top go everywhere." D's use of the word confusing in this utterance shows clear evidence that he finds something confusing. Furthermore, it is clear from A's statement, "…there's no way they would have those things in electrical contact with each other," that there is some expectation that positive and negative terminals of a circuit element should not be electrically connected. A makes this statement in response to D's previous statement which can also be interpreted as him voicing concern that this expectation, that positive and negative terminals not be connected, is not being met.

B.9 Analogical Reasoning

Analogical reasoning is an important tool for both scientists (Hesse, 1966; Hoffmann, 1980; Dunbar, 1995) and educators (Glynn, Law, & Gibson, 1994; Glynn, Duit, & Thiele, 1995). As defined by Glynn, Duit, and Thiele, an analogy is a way of transferring ideas about something you are familiar with, an analog, to something you are unfamiliar with, the target. A comparison between the two, called mapping, allows an individual to draw out common features between the analog and the target. When looking for evidence of analogical reasoning, we looked for evidence of all three of these features, an analog, a target, and some form of mapping between the two. Specifically, the criteria we established that must be satisfied for a critical event to be coded as analogical reasoning are:

1. Can identify a physical scenario or object that serves as the analog.
2. Can identify a physical scenario or object that serves as the target.
3. Presence of a phrase that links the two in such a way that meaning is transferred from source to target without saying that the target literally is the source. The phrase can also be negative and link the two in such a way that meaning cannot be transferred.

For a critical event to be coded as having evidence of analogical reasoning, all three of the above criteria must be met. Below is an excerpt from a critical event that was coded as showing evidence of analogical reasoning. In this critical event, two experts, A and D, are working on the first part of the solar cell problem. At this point they are discussing their understanding of what happens when you cover the solar cell from the long side.

> D: So, you're, you assume that current is proportional to area and as you start to decrease the area you're going to decrease the current.
> A: In that situation, yeah. Because then it's basically like, as long as we're going this way and we're effecting all the cells equally, then it's basically like making the cell smaller.

In this critical event, A shows evidence of analogical reasoning. First he talks about the situation that they're discussing, "…we're going this way and we're effecting all the cells equally." This situation serves as the target that the experts are trying to better understand. The utterance, "making the cell smaller," acts as the analog in this critical event and the phrase "then it's basically like" serves to link the analog with the target.

B.10 Experimentation

Experimentation is an important part of any scientific endeavor. In order to determine whether or not experimentation was important during critical events, the following criteria were developed to evidence of experimentation. For a critical event to be coded for experimentation, it did not need to satisfy all of the following criteria, but did need to satisfy at least one.

1. Actually carrying out an experiment.

2. Directly talking about a hypothesis that was supported or disproven by performing some experiment or the results from personally doing an experiment.
3. Discussing a new potential experiment that they could do and how you could test your hypothesis from the experiment with intent or evidence of taking data.

The following excerpt was coded for evidence of experimentation. In this excerpt, M and P were working on the second half of the solar cell problem. They had previously made predictions for the different arrangements and coverings for two solar cells connected together. They had been shown the actual measurements for the various scenarios and were trying to figure out why there were discrepancies between their predictions and the measurements they had been shown. In the critical event below, they are trying to figure out a way to better understand the solar cells they are working with.

> M: Uh, my concern is that if we do this experiment right? With all internal and external resistances given, can we detect how the voltage for a single cell depends on uh this coverage sideways? Uh, so, can we please turn on the light? (Begins setting up some of the equipment) Just to see…That's the one that we played with, right? Ok so this is voltage. (performs initial side to side experiment) It changes.

In this critical event, M proposes and performs an experiment. In this experiment, a solar cell is connected to a voltmeter and the potential difference across the solar cell is observed as the solar cell is covered from the side at various covering percentages. In the video, it is also very clear that M actually performs the experiment. This satisfies the first criteria for coding a critical event as having evidence of experimentation.

B.11 Sense Making

Another important aspect of metacognition is sense making which is in turn an important part of building a solid understanding of a topic. As a result, we wanted to look for moments where the experts identified whether or not an idea they were discussing made sense or not. To identify these moments we used the following criteria:

1. Discussion about some idea/model/hypothesis.
2. A statement related to whether or not that idea/model/hypothesis does, doesn't, or should somehow make sense.

The excerpt from the critical event below was coded as showing evidence of sense making. In this critical event, experts A and D are trying to solve part 1 of the solar cell problem. They've previously identified that the construction of the solar cell is important. During this excerpt, A and D notice that there is a silver strip which appears to run from the positive terminal on the solar cell, across the tops of all the elements within the solar cell, and connects to the negative terminal of the solar cell. They then discuss this.

> D: I'm just, I'm just a little confused about why it looks like the ones on top go everywhere. Like it goes, I mean, to the positive and negative terminals.
> A: Well it must, this must be insulated here, there must not be a connection there.
> D: Hmm. Cause it looks like the…yeah it actually looks like both of them go everywhere. No, no. That's this guy. So this guy's for the negative…I don't, that looks like metal to me.
> A: It does look like metal, but it can't be. I mean there's no way they would have those things in electrical contact with each other.
> D: No it doesn't make sense, but I'm just wondering, how can we make sense of it?

The excerpt starts with D stating the model that he has inferred from his observations of the solar cell when he says, "…it looks like the ones on top go everywhere. Like it goes…to the positive and negative terminals." This is the model under discussion that satisfies the first criteria for this code. At the end of the excerpt, D addresses whether or not he feels this model makes sense when he states, “Not it doesn’t make sense, but I’m just wondering, how can we make sense of it?” This satisfies the second criteria for this code.

B.12 Hypothetico-Deductive Reasoning

Hypothetico-deductive reasoning is a reasoning process at the heart of many scientific discoveries, including Galileo’s discovery of Jupiter’s moons (Lawson, 2002). By analyzing Galileo’s report of his discovery of Jupiter’s moons, Lawson is able to identify some key features about Galileo’s reasoning process which fit the hypothetico-deductive reasoning process. Lawson describes the key features of Galileo’s reasoning process as:

> “(1) making a puzzling observation, (2) formulating a causal question, (3) formulating one or more hypotheses, (4) using a hypothesis and an imagined test to generate expected results/predictions, (5) making actual observations and comparing them with the expected observations, and (6) drawing conclusions as to the extent to which the initial hypotheses have or have not been supported.” (Lawson, p. 9, 2002)

To determine whether or not a critical event shows evidence of hypothetico-deductive reasoning, we have condensed and adapted the following criteria from Lawson’s model that must all be identified:

1. An observation that is being discussed.
2. A hypothesis under analysis.
3. A prediction based on that hypothesis.
4. A conclusion about how the observations relate to that prediction.

Below is an example of an excerpt from a critical event that shows evidence of hypothetico-deductive reasoning. Prior to this critical event, C and R have been developing a model to explain the light cone using a critical angle argument. Their idea was that after light is scattered off the paper and re-enters the container, the transition from one optical medium to another upon entering into the water would cause certain incident rays which were beyond the critical angle to be cutoff and would not re-enter the water. In this critical event, R objects to this model.

> R: Right. I'm just trying to figure out why, um, because the way we set this up –
> C: Mhmm.
> R: You should be able to get, um, light coming off in all directions still. Through the water instead of just some specific angle.
> …
> C: You have some point and it's going to emit, scattering and emitting, in all directions to make this halo, right?
> R: Mhmm.
> C: But beyond this crit – whatever, uh, if we say this is the normal – there's some critical angle. So anything beyond that you won't see, these will just get reflected back.
> R: Right, but the angle's very close to that you should get something coming off almost parallel to the –
> C: Oh! I see what you're saying.
> R: To the surface of the plastic. Which we're not seeing.
> C: So you're saying we need to – Yeah. Very very close – if this is the critical angle then you should see something like that –
> R: Yeah.
> C: That, that, and that. So we should be getting a, uh…
> R: We should still get –
> C: Hemisphere instead of a cone. That's a good point.

In the excerpt, the light cone itself is the observation that satisfies the first criteria. While this goes unmentioned for most of the excerpt, at the end C remarks "So we should be getting a…hemisphere instead of a cone." The hypothesis under analysis is stated by C when he says, "…there's some critical angle. So anything beyond that you won't see, these will just get reflected back." The prediction based on this hypothesis is that "you should get something coming off almost parallel to the…surface of the plastic." Which is immediately followed with, "Which we're not seeing." This statement identifies that the predictions do not match the observations.

B.13 Relative Value of Knowledge

One way in which experts are different from novices is that they have "conditionalized" their knowledge (Simon, 1980; Glaser, 1992, Bransford et al., 2000). This means that they not only have accumulated a vast storage of knowledge, but also that they have an understanding of the contexts in which their knowledge is appropriate. They understand that different pieces of their knowledge are valuable in different situations. In this study, we attempted to identify moments where this conditionalization of knowledge was addressed explicitly by the experts as it related to the value of a certain idea when solving the problem at hand. We decided that the following criteria should be to identified to classify a moment under this category:

1. Discussion of an idea/model/hypothesis.
2. An individual making some comment about the importance or usefulness of that idea.

The following excerpt is taken from a critical event that was coded for evidence of the relative value of knowledge. Prior to this critical event, experts M and P were trying to address discrepancies between their model's predictions for part 2 of the solar cell problem and the results to the experiment as it had been presented to them by the interviewers. Immediately prior, they've introduced the idea that they can model the solar cells as current sources. In this critical event, they make a comment about the usefulness of this idea.

> M: … So what you're saying is that, uh, I'm not sure that how I need to treat two current sources connected in series. The current source is nothing but the voltage source with a very large internal resistance. Roughly speaking. Uh if you want to maintain current, right?
> E: Mhmm, mhmm.
> M: The internal resistance much greater than the load.

E: Circuit, right.
M: So I'm not sure that this is a constructive approach in this case.

In this critical event, M begins by discussing the model of the solar cells as "two current sources connected in series." This is the idea that satisfies the first criteria for this code. After discussing this idea for a bit, he states at the end of the critical event, "I'm not sure that this is a constructive approach in this case." This indication that he's not convinced that treating a solar cell as a current source is productive satisfies the second criteria for this code.

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