ESSAYS IN BANKRUPTCY AND FIRM FINANCE

By

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ABSTRACT OF THE DISSERTATION

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This dissertation investigates the role that capital market imperfections play in shaping the behavior of firms along several dimensions: capital structure, investment policies, bankruptcy decisions and life-cycle dynamics. The dissertation puts together two separate but closely related papers, both of which are concerned with bankruptcy and firm financing under asymmetric information and limited enforcement.

In Chapter 2, I present a model of firm finance that encompasses imperfect investor protection, risk aversion and costly state verification. Imperfect investor protection is introduced through the limited liability clause of the financial contract, and captures the maximum fraction of returns that the investor can seize from the entrepreneur. A positive lower bound on consumption then interacts with entrepreneurial risk aversion in non-trivial ways. I characterize optimal contracts and study the conditions under which standard debt is optimal. Under suitable assumptions about the structure of the problem, standard debt contracts (SDCs) are optimal if and only if investor protection is sufficiently low. On the other hand, low investor protection results in higher funding costs and bankruptcy probabilities. In my setting, this implies that

when SDCs are optimal, lowering investor protection reduces the entrepreneur's welfare. Numerical examples show that moderate changes in investor protection can have large effects on the terms of the contract and on the entrepreneur's welfare. Finally, I study the role of leverage and consider the welfare consequences suboptimally implementing standard debt contracts.

In Chapter 3 I study firm dynamics and industry equilibrium when firms under financial distress face a non-trivial choice between alternative bankruptcy procedures. Given limited commitment and asymmetric information, financial contracts specify default, renegotiation and reorganization policies. Default occurs in equilibrium and leads to either liquidation or renegotiation. Renegotiation entails a redistribution of social surplus, while reorganization takes the form of enhanced creditor monitoring. Firms with better contract histories are less likely to default, but, conditional on default, firms with better outside options successfully renegotiate, in line with the empirical evidence. Unless monitoring is too costly, renegotiation leads to reorganization, which resembles actual bankruptcy practice. I calibrate the model to match certain aspects of the data on bankruptcy and firm dynamics in the U.S. My counterfactual experiments show that, compared with an economy with liquidation only, the rehabilitation of firms (renegotiation and reorganization) has a sizable negative effect on exit rates and size dispersion, and positive effects on average size and productivity.

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CHAPTER 1

INTRODUCTION

Since at least Bagehot (1873) and Schumpeter (1911) financial market development has been recognized as a first-order requirement for an efficient allocation of resources. Importantly, well-functioning financial markets allow for an efficient matching of credit with entrepreneurial talent, and enhance firms' capacity to invest in projects that boost their productivity. Given that productivity growth is the main driver of long term growth in living standards [Hall and Jones (1999)], financial development policies are an important part of the economic development agenda.

A key driver of financial development is the severity of information and enforcement frictions that agents must face when designing optimal financial arrangements to fund productive activities. The two papers of this dissertation are concerned with the varios ways in which investors are effectively protected against opportunistic behavior from entrepreneurs. My main contribution to the field is to show that when the mechanisms for protecting investors against private information and limited commitment are precarious, we can expect to observe higher funding costs, higher rates of bankruptcy and misallocation of capital and entrepreneurial talent.

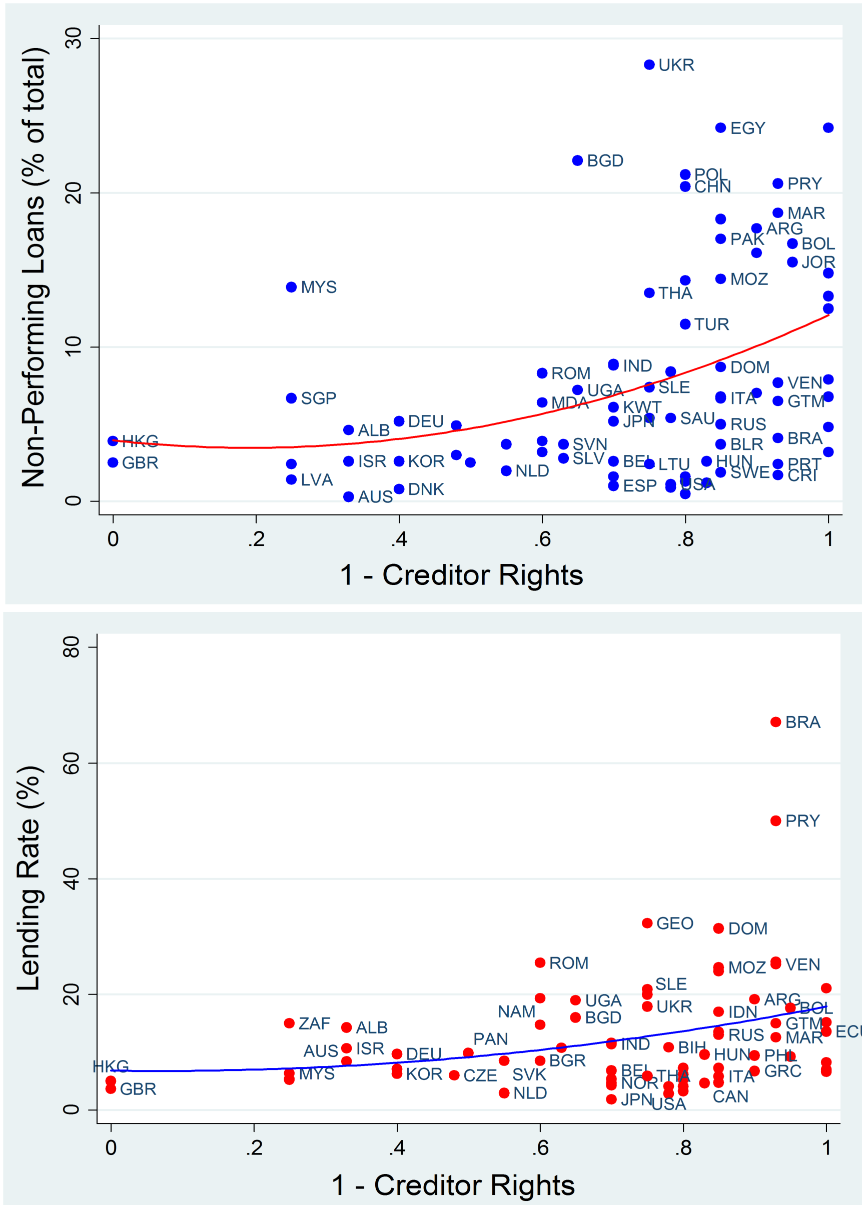
In the first chapter of this dissertation entittled "**Investor Protection and Optimal Contracts Under Risk Aversion and Costly State Verification**", I develop a static, partial equilibrium model of financial contracting encompassing entrepreneurial risk aversion, limited enforcement and costly state verification (CSV). I account for investor protection through the maximum fraction of returns that the investor can recover. The motivation for this additional source of imperfect investor protection comes from the legal codes and the extant evidence on bankruptcy settlements. Indeed, many countries have introduced legislation aimed at limiting the fraction of income and property that creditors can seize from borrowers.

The most widely used measure of investor protection -a creditor rights index that ranges from 0 ("weak") to 4 ("strong") introduced by La Porta, Lopez-de Silanes, Shleifer and Vishny (1997)- suggests that protection to financiers is low on average, 1.8 in a sample of developed and developing countries, and varies widely across countries. Using an updated version of this index, one obtains that only nine out of a sample of 133 countries can be said to have "strong" creditor rights (i.e. have a score of 4), while 21 exhibit "weak" creditor rights (i.e., have a score of 0).

A first look at the relationship between the above mentioned measure of investor protection and financial market outcomes is suggestive indeed. Figure 1.1 below presents data on creditor rights [as defined by La Porta et al. (1997)], delinquency rates and lending raes for a cross-section of 81 developed and developing countries.

The positive relationship between the inverse of creditor rights and interest rates may not be too surprising. After all, if creditors are not well protected inside bankruptcy, interest rates must be such that their no-default payoff compensates for the lower expected return conditional on default. Perhaps what is less obvious is the reason why this relationship should be non-linear as suggested by the upper panel of Figure 1.1. Most interesting of all is the positive and seemingly non-linear relationship between the inverse of creditor rights and delinquency rates -which in turn are a good predictor of bankruptcy rates. As shall be seen below, the model presented in Chapter 2 is able to account for this positive non-linear relationship between creditor rights and financial market outcomes (see Figure 2.2).

A natural question to ask after the above analysis is why poor investor protection persists in some countries? The model in Chapter 2 also provides an answer to this



source: Own calculations based on WDI (World Bank) and Djankov, McLiesh and Shleifer (JFE, 2007)

Figure 1.1. Investor protection and financial market outcomes

question. If markets are sufficiently incomplete such that debt contracts are the only instrument available, risk-averse borrowers would greatly suffer in the case of bankruptcy with perfect creditor protection. Therefore, for a sufficiently high degree of risk aversion, imperfect investor protection results optimal (see Proposition 4 of Section 2.2.5 and the right panel of Figure 2.2). Finally, the model of Chapter 2 also provides a rationale for the empirical regularity that higher investor protection is associated with higher leverage [Pereira and Ferreira (2011), Cheng and Shiu (2007)].

The second chapter of the dissertation entitled "**Bankruptcy Choice with Endogenous Financial Constraints**" develops a dynamic, industry equilibrium model of firm dynamics in which default occurs in equilibrium, and defaulting firms face a non-trivial choice between liquidation, renegotiation and reorganization. In contrast to most existing models of firm dynamics [e.g., Clementi and Hopenhayn (2006)] where liquidation is the only alternative to deal with financially distressed firms, the option to renegotiate and reorganize is well alligned with modern corporate bankruptcy practice.

Indeed, recent empirical evidence suggests that alternatives to liquidation are becoming popular in bankruptcy codes. In the U.S., Chapter 11 bankruptcy is a widely used procedure that, in contrast to Chapter 7 bankruptcy where firms are liquidated and its assets are sold, allows for an exchange of securities under a formally proposed reorganization plan. During the 2008-2009 recession in the U.S., over 33% of bankruptcy filings were for Chapter 11, and at least 66% of total business failures underwent some type of formal or informal reorganization process.

Outside the U.S. alternatives to liquidation are also important resolution mechanisms for troubled firms. In the sample of 35 economies used by Claessens and Klapper (2005) to assess the relative use of bankruptcy, 33 countries had laws permitting both liquidation and restructuring of distressed firms. However, survey data from the World Bank's Doing Business suggest that viable businesses are likely to be liquidated rather than reorganized in 115 out of 182 countries. Furthermore, Figure 1.2 above shows that while recovery rates from all procedures show a positive association with (log) real GDP per capita, recovery rates during reorganization appear to be

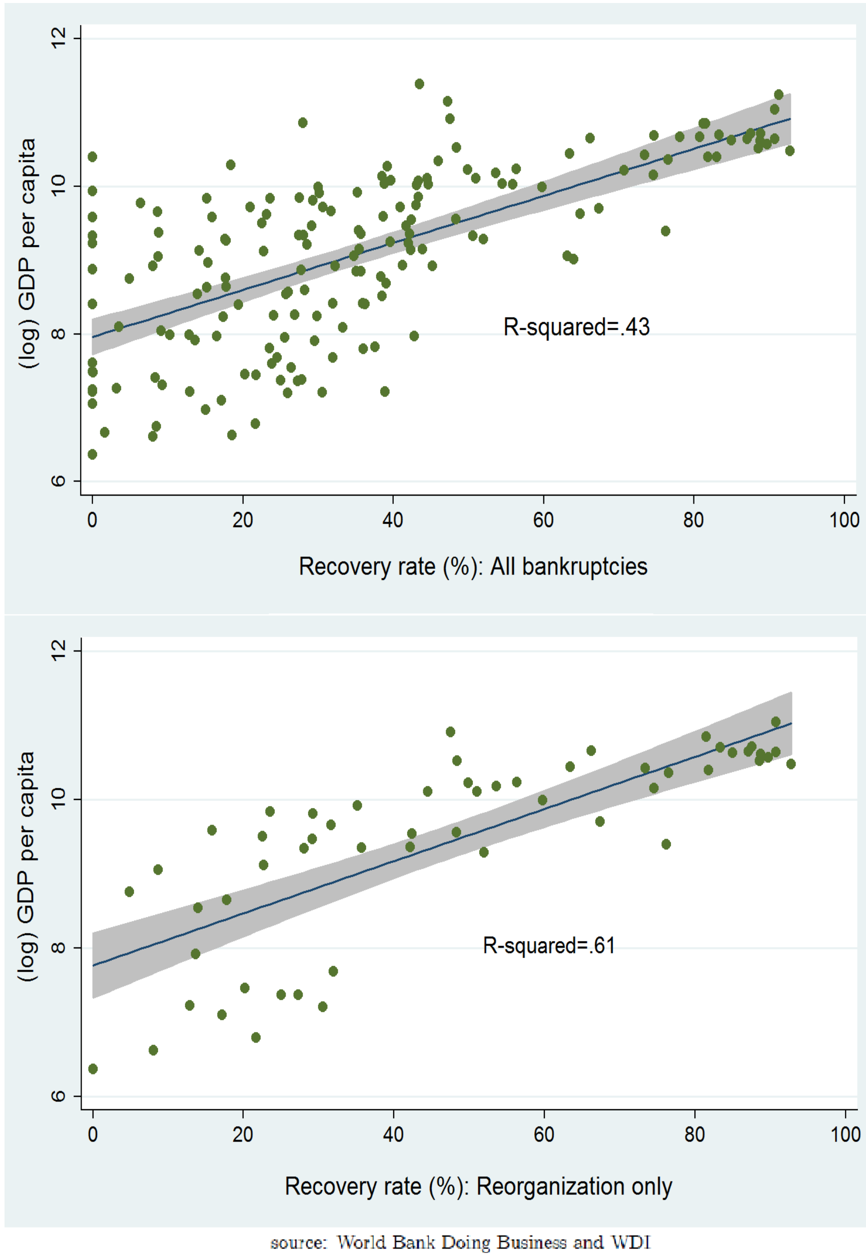


Figure 1.2. Recovery rates, reorganization and real GDP per capita

the important link between investor protection, financial development and economic success.

The model presented in Chapter 3 of this dissertation provides a rationale for the empirical regularity from Figure 1.2. In the model, if bankruptcy costs are sufficiently low the renegotiation-reorganization options may enhance financial selection. That is, if properly designed, corporate bankruptcy law may act as a filetring device, liquidating firms which are inherently less productive and reorganizing more productive ones. This in turn has the potential to reduce firm exit while at the same time increase average firm productivtity, which, as was mentioned before is the main driver of long term economic development.

CHAPTER 2

INVESTOR PROTECTION AND OPTIMAL CONTRACTS UNDER RISK AVERSION AND COSTLY STATE VERIFICATION

2.1. Introduction

Scholars and policymakers seem to agree on the fact that good investor protection, broadly defined, bolsters financial development and economic growth.[1] Yet, available survey data suggests that the degree of investor protection varies widely across countries and is particularly low in developing economies [Djankov, Hart, McLiesh and Shleifer (2008)]. Even in developed countries like the U.S., court data shows that investors are able to recover only a fraction of their investment once a firm files for bankruptcy [Bris, Welch and Zhu (2006)].

Along with investor protection issues, recent research [e.g. Panousi and Papanikolaou (2012), Lewellen (2006)] has revisited the old working assumption of risk-neutral entrepreneurs and the consequences of risk-averse behavior for firm investment and financing decisions. For instance, if ownership is disperse or shareholders have well-diversified portfolios, and if the manager's compensation scheme is independent of the firm's returns, the issues of firm finance and manager's insurance can be studied separately (i.e. we can model the firm as a relatively risk-neutral agent). While these may be plausible features of large firms, small and medium businesses are characterized precisely by the opposite: Managers are typically owners and have limited access to

[1]A recent survey of the literature on the links between financial development and growth can be found in Fernandez and Tamayo (2014).

hedging instruments. This inverse relationship between firm size and risk attitudes is formalized in the entrepreneurial decreasing absolute risk aversion (DARA) hypothesis of Cressy (2000) and supported by the empirical evidence collected by Fang and Nofsinger (2009) and Schmid, Ampenberger, Kaserer and Achleitner (2008).

In this paper, I explore the roles of risk aversion and imperfect investor protection in jointly determining the terms of optimal financial contracts when there is asymmetric information. The starting point of our analysis is the celebrated costly state verification (CSV) model of Townsend (1979) in which an entrepreneur and an investor design a financial contract to fund a risky project.

This basic CSV model is hereby extended to explicitly study the role of investor protection.[2] In particular, I follow Krasa, Sharma and Villamil (2008) and account for investor protection through the maximum fraction of returns that the investor can recover.[3] The motivation for this additional source of imperfect investor protection comes from the legal codes and the extant evidence on bankruptcy settlements. Indeed, many countries have introduced legislation aimed at limiting the fraction of income and property that creditors can seize from borrowers (e.g., Chapter 7 exemptions in the U.S.).[4]

In contrast to the model in Krasa et al. (2008), however, I retain the assumption that the entrepreneur is risk-averse as in Townsend's original model. Thus, I provide a more general framework that is amenable to comparisons with recent iterations of

[2]Townsend's model has been extended in several dimensions and is now the workhorse of dynamic macroeconomic models with financial frictions. A partial list of extensions is: economies with production (Gale and Hellwig (1985)), heterogeneous borrowers (Williamson (1987)), multiple investors (Winton (1995)), limited commitment (Krasa and Villamil (2000)), non-contractible verification (Hvide and Leite (2010)), and imperfect monitoring (Greenwood, Sanchez and Wang (2010)). The use of CSV in macroeconomics was launched by Bernanke and Gertler (1989).

[3]In related but independent work, Galindo and Micco (2005) also model investor protection in this fashion. A Our model contrasts with Sevcik (2012) where protection is captured by monitoring costs only. See also Castro, Clementi and MacDonald (2004).

[4]As pointed by Galindo and Micco (2004), the problem of limited creditor protection is widespread in developing countries, and particularly acute in Latin America.

the CSV model [e.g., Winton (1995)] and to other recent macro models of firm finance that include risk aversion [e.g., Smith and Wang (2006)].

I seek to answer two main sets of questions. First, how do optimal contracts look like in a CSV model that encompasses risk aversion and imperfect investor protection? Under which conditions is a standard debt contract (SDC) optimal? Secondly, how do the terms of the optimal contract change as the level of investor protection varies? what are the effects of such variations on the entrepreneur's welfare?

My results suggest that optimal contracts may be cataloged in three families: (i) standard debt contracts, (ii) debt-like contracts with continuous repayment functions and (iii) discontinuous debt-like contracts. This contrasts with the risk-neutral borrower case where only SDCs are optimal and with the risk-averse borrower case under full investor protection where the second family of contracts is not present.

For some popular specifications of preferences, I show that a SDC can never be optimal if investor protection is perfect. Moreover, I establish that under certain assumptions about the structure of the problem, a SDC is optimal if only if investor protection is sufficiently low. These assumptions are more easily satisfied when verification costs are relatively high, investor protection is relatively low or when the entrepreneur's degree of risk aversion is relatively low.

My results also show that in a continuous contract, the cost of funds and the probability of bankruptcy are decreasing in the level of investor protection, extending the main comparative statics result found in Krasa et al. (2008). I also show how this result can be extended to a fully dynamic setting. When a SDC is optimal, my model implies that the borrower's welfare increases with the level of investor protection, but may increase or decrease with it if a SDC is implemented suboptimally.

To illustrate my analytical results, I parametrize the static model so as to reflect recent available estimates of bankruptcy costs and recovery rates. I show that lowering investor protection can have considerable quantitative effects on the terms of the contract and on the entrepreneur's welfare.

I set up the contracting problem in section 2, first considering the symmetric information case, and then proceeding to characterize optimal contracts under CSV. Some extensions to the benchmark model are provided in section 3. In section 4 I present some numerical simulations. Section 5 concludes.

2.2. The contracting problem

In this section, I lay out a static CSV model in which investor protection plays an explicit role. The model is a blend of the ideas on CSV with risk aversion presented in Townsend (1979), and the notion of imperfect investor found in Krasa et al. (2008).

2.2.1. Physical environment

The environment I consider is one with a risk neutral investor and a risk averse entrepreneur who owns a production technology. Operating the technology (i.e. starting a project) requires investing one unit of input and I assume that the entrepreneur has only $0 \leq (1-b) < 1$ units. Thus to start the project, she must raise b from the investor who for simplicity is assumed to have zero opportunity cost.[5] The entrepreneur has no outside investment alternatives.

After project returns are realized, the entrepreneur repays $R(\hat{s}, s)$ to the investor where s is the true state of nature and $\hat{s}$ is what the entrepreneur reports as the state. Under private information, only the entrepreneur can costlessly observe the state of

[5] As in Krasa et al. (2008), b captures the fraction of *inputs* (i.e., working capital) that are financed by the investor. For simplicity, our analysis abstracts from any setup investment or fixed costs, which can be interpreted in two ways: either setup investments/costs are zero or they are fully recovered under any contract. When nonzero setup costs are brought into the analysis (and subject to the legal constraints introduced below), our results will hold as long as imperfect investor protection impairs the distribution of cash flows more than the distributions of the initial investment [see Gennaioli and Rossi (2010)].

nature so the investor would either rely on reports or verify them by paying a cost $0 \leq \gamma < 1$, and may penalize the entrepreneur if she misrepresents. Throughout the paper, I restrict the attention to full commitment environments and to pure strategy equilibria.[6] Since my model is static, I also abstract from the difference between default, liquidation or reorganization, and consider all these as bankruptcy situations.

Both agents are expected utility maximizers. In particular, the risk averse agent values consumption according to the function $u(\cdot)$ which satisfies:

Assumption 1. *u is $\mathbb{C}^2$ with $u' > 0$ and $u'' < 0$.*

Project returns are stochastic and equal to the state of nature, which is itself a continuous random variable (r.v.) S with twice continuously differentiable CDF $H(\cdot)$, $dH(\cdot) = h(\cdot)$. The support of S is assumed to be bounded, $\Sigma = [\underline{s}, \bar{s}] \subseteq \mathbb{R}_+$, such that if s is a particular realization of the state, $0 < \underline{s} \leq s \leq \bar{s}$ and $\infty > h(s) > 0$.[7] The distribution and support of S are common knowledge.

Imperfect investor protection is introduced through the limited liability clauses of the contract, much in the spirit of Krasa et al. (2008). In particular, I assume:

Assumption 2. *The legal system is such that, in any contract, after realization s, the entrepreneur is bound to repay at most $(1-\eta)s$ with $\eta \in [0,1]$.*

Notice that this limited liability clause also reflects the fact that the production technology is deemed useless to the investor without the entrepreneur. Next I describe the contracting problem under symmetric information, and then, I study the more interesting case of private information.

[6] We assume full commitment since we want to study in isolation the effects of *exogenous* imperfect investor protection (e.g., estate exemptions in Chapter 7). For a CSV model in which stochastic verification is optimal see Hvide and Leite (2010).

[7] The assumption that $\underline{s} > 0$ is also in Townsend (1979) and allows us to better study reward functions that satisfy Inada conditions.

2.2.2. The case of symmetric information

In order to establish a benchmark, I first consider the problem under symmetric information (i.e. $\gamma = 0$). The sequence of events is straightforward: the entrepreneur raises b from the investor, invests one unit of the input and, when a return s is realized, makes a payment to the investor according to the agreed-upon schedule $R(s)$.

There are a number of ways in which this contracting problem can be specified. Here I proceed in the tradition of Townsend (1979), Gale and Hellwig (1985), and Williamson (1987), and maximize the expected utility of the entrepreneur subject to the investor receiving at least her reservation utility.[8] Accordingly, the optimal contract may be found by solving:

$$\max_{R(\cdot)} \mathbb{E}u\left[S - R(S)\right] \tag{FB.1}$$

$$s.t. : \mathbb{E}R(S) \geq b \tag{FB.2}$$

$$(1-\eta)\, s \geq R(s) \geq 0 \quad \forall\ s \in \Sigma \tag{FB.3}$$

The first constraint is the individual rationality one for the investor (lender); given risk-neutrality, it specifies that she must at least break even in expectation. The second and third are limited liability constraints (LLCs). Under such an environment, the optimal repayment function satisfies a constrained-optimal risk-sharing rule $\{\lambda^* - u'[s - R^*(s)]\}\, h(s) = \mu_1^*(s) - \mu_2^*(s)$, where $\mu_1^*(\cdot), \mu_2^*(\cdot)$ are, respectively, the (optimal value) multipliers for the first and second constraints in (FB.3) and λ^*,

[8]For an appropriately chosen weight vector, the solution to the program in (FB.2)-(FB.3) is also a solution to a problem that maximizes the weighted average of the payoffs of the match subject to individual rationality constraints (IRC) for both. For a setup that maximizes the investor's payoff subject to the entrepreneur's IRC see Krasa et al. (2008).

is the multiplier for the constraint (**FB.2**).[9] When the solution to this problem is interior, risk-neutrality implies that the investor provides full insurance to the entrepreneur whose consumption is constant. In any event, it is easy to see that the optimal contract has a substantial equity component to it. Having characterized the optimal benchmark contract, I now turn to the issue of costly state verification.

2.2.3. Costly state verification

Suppose now that the entrepreneur can costlessly observe project returns but the investor must pay a cost $0 < \gamma < 1$ for using the legal system to verify returns. Then, a reporting strategy for the entrepreneur maps the state of nature into reports, $\{\hat{s}(s)\}_{s\in\Sigma}$, and a verification region, B, will now be part of the contract. The sequence of decisions and events is as follows:

Entrepreneur obtains b, invests 1 unit of input	$\longrightarrow$	$s \in \Sigma$ is realized, entrepreneur reports $\hat{s}(s)$, investor verifies if $\hat{s} \in B$	$\longrightarrow$	Entrepreneur repays $R(\hat{s}, s)$ to investor

As is standard in the CSV literature, I assume that if the entrepreneur is indifferent between truth-telling and misreporting, she will report truthfully. I can now define a contract under private information:

Definition 1. *A contract under CSV is a pair* $\{R(\cdot,\cdot), B\}$ *where* $R(\hat{s}, s)$ *is what the entrepreneur repays when the state of nature is* s *and she reports state* $\hat{s}$*, and*

[9] Throughout the paper, inequalities and equations involving random variables are assumed to hold almost everywhere (a.e.). We also assume that it is legitimate to differentiate under the integral sign.

$B \subseteq \Sigma$ is the set of reports $\hat{s}$ for which the investor uses the legal system to verify returns.

As usual, the revelation principle allows me to focus only on direct revelation mechanisms that are incentive compatible, and to identify the reports set with Σ. Thus, the optimal contract for the case of CSV can be obtained by solving:

$$\max_{\{B,R(\cdot)\}} \mathbb{E}u\left[S - R(S,S)\right] \tag{P.1}$$

$$s.t.\ \mathbb{E}R(S,S) - \gamma \int_{s\in B} dH(s) \geq b \tag{P.2}$$

$$u\left[s - R(s,s)\right] \geq u\left[s - R(\hat{s},s)\right] \quad \forall\ s, \hat{s} \in \Sigma \tag{P.3}$$

$$(1-\eta)\, s \geq R(s,s) \geq 0 \ \ \forall\ s \in \Sigma \tag{P.4}$$

$$R(s,s') = R(s,s'')\ \ \forall\ s \notin B \text{ and } s'', s' \in \Sigma \tag{P.5}$$

$$B \subseteq \Sigma \tag{P.6}$$

The first constraint is the individual rationality one; it tells us that the investor should at least break even in expectation after expected verification costs. The second set of constraints imposes incentive compatibility (ICC), requiring that the entrepreneur (or borrower) prefers to report truthfully in every state. The third and fourth constraints are the LLCs and the fifth constraint requires that unverified payments depend only on the report (UPC). In order to have an interesting problem, I introduce:

Assumption 3. $\mathbb{E}(S) - \gamma > b > \underline{s}$.

This assumption is sufficient to ensure that, at least for some values of η, an optimal contract will give neither the manager nor the investor all of the firm's returns. Notice that when $\eta = 0$ the model effectively reduces to that in Townsend (1979), and when $u(c) = c$ (i.e. linear preferences) I have the CSV version of Krasa et al. (2008).[10] Thus, the model considered here contains in it some of the popular versions of the CSV framework while specifying an explicit role for investor protection.

2.2.4. Optimal contracts

I now characterize optimal financial contracts under risk aversion, imperfect investor protection and costly state verification. I do so in steps: first I introduce a series of lemmas that partially characterize the optimal contract and then I rewrite the problem in a more convenient way that allows for an explicit solution (formal proofs can be found in the appendix).

Lemma 1. *In the optimal contract, for all $\hat{s} \notin B$, the repayment function is constant, i.e., $R(\hat{s}, s) = \bar{R}$ for some constant $\bar{R}$.*

Lemma 2. *Under the optimal contract, in the verification region the repayment function is given by $\hat{R}(s)$, with $\hat{R}(s) < \bar{R}$ a.e. and $\hat{R}(s) \leq \bar{R}$ everywhere.*

The first result above is a straightforward implication of the unverified payments constraint, while the second mainly follows from incentive compatibility (and the optimality requirement). A more interesting result, which is a simple extension of Proposition 3.2 in Townsend (1979) to the case of imperfect investor protection is:

Lemma 3. *In the optimal contract, B is a lower interval.*

Corollary 1. *If $B \neq \varnothing$, $\exists\, x \in \Sigma$ such that $\hat{s} \leq x \Rightarrow \hat{s} \in B$, and $\hat{s} > x \Rightarrow \hat{s} \notin B$.*

[10] Krasa et al. (2008) consider a model that imposes sequential rationality in the players' strategies (the equilibrium contract must be a PBE) so the CSV is a special case of their model.

I can summarize the findings of lemmas 1-3 by the optimal repayment rule:

$$R(\hat{s}, s) = \begin{cases} \hat{R}(s) \leq \bar{R}, & \text{if } \hat{s} \leq x \\ \bar{R}, & \text{if } \hat{s} > x \end{cases}$$

Under such rule, incentive compatibility is always satisfied and the constraints (**P**.**3**) can be replaced. Moreover, the limited liability constraints can also be replaced and the contracting problem can be reformulated as:

$$\text{(PP.1)} \qquad \max_{\{\bar{R}, \hat{R}(\cdot), x\}} \int_{\underline{s}}^{x} u[s - \hat{R}(s)] dH(s) + \int_{x}^{\bar{s}} u(s - \bar{R}) dH(s)$$

$$\text{(PP.2)} \qquad s.t. \int_{\underline{s}}^{x} \hat{R}(s)\, dH(s) + \bar{R}[1 - H(x)] - \gamma H(x) \geq b$$

$$\text{(PP.3)} \qquad (1-\eta)\, x \geq \bar{R}, \qquad (1-\eta)\, s \geq \hat{R}(s) \geq 0 \ \ \forall\ s \leq x$$

Necessary and sufficient conditions to find a solution for the program in (**PP**.**1**)-(**PP**.**3**) are presented in the appendix as (2.3)-(2.9), and lead to the following families of optimal contracts:

Theorem 1. *Suppose that* $\Phi^* = \{\bar{R}^*, \hat{R}^*(\cdot), x^*\}$ *solves (*(**PP**.**1**)*-(*(**PP**.**3**)*). Then:*

i) *Either* Φ^* *is the SDC of Gale and Hellwig (1985):* $\hat{R}^*(x^*) = (1-\eta)\, x^* = \bar{R}^*$*,*

ii) *or* Φ^* *is debt-like with* $s \mapsto \hat{R}^*(s)$ *continuous and* $\hat{R}^{*\prime}(s) = 1$ *for some* s*,*

iii) *or* Φ^* *is debt-like but discontinuous at* x^* *with* $\hat{R}^{*\prime}(s) = 1$ *for some* $s \leq x^*$*,* $\hat{R}^*(s) = 0$ *for some* $s \leq x^*$*, and* $\bar{R}^* > \hat{R}^*(x^*)$*.*

The proof of Theorem 1 is a simple application of the Maximum Principle and Arrow's Sufficiency Theorem. To gain some intuition about the general form of the optimal contract, first consider the case of perfect investor protection (i.e., $\eta = 0$). In that case, the left panel of Figure 1 shows that the optimal contract either is a SDC or dictates zero repayment for some low states of nature and is discontinuous at

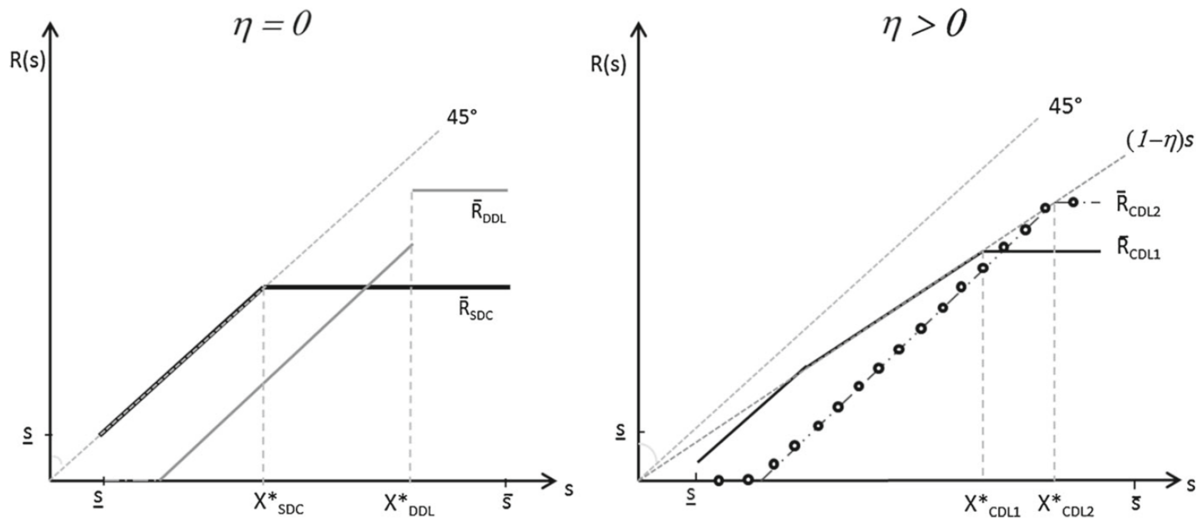


Figure 2.1. Optimal contracts and imperfect investor protection

the verification threshold. The zero repayment region is optimal if the entrepreneur's marginal utility is very high at low levels of consumption (e.g., CRRA). Notice also in the left panel of Figure 1 that when investor protection is perfect, payments to the investor, whenever they are positive, increase one-to-one with returns.

Now consider the case of imperfect investor protection. In that case, a third family of contracts is available in addition to the two already described; the right panel of Figure 1 depicts two alternative contracts belonging to this third family. Since imperfect investor protection lowers the slope of the (upper) LLC, payments to the investor may increase one-to-one with returns before limited liability binds. In fact, it is possible that the optimal contract dictates zero repayment for low returns but remains continuous at the verification threshold. The following remark will be useful below:

Remark 1. *When the optimal contract is continuous, the cutoff x^* pins down the probability of verification, $H(x^*)$, and the implied cost of funds $(1-\eta)x^* = \bar{R}^*$.*

I am now ready to extend one of the main results of Krasa et al. (2008) to the case of a risk averse borrower and to general, continuous, optimal contracts:[11]

Proposition 1. *Whenever the optimal contract is continuous and $\eta \in (0,1)$, the probability of verification and the implied cost of funds are increasing in η and in γ.*

The proof of Proposition 1 is a straightforward exercise of comparative statics, and presents us with a trade-off of reducing investor protection. Given that the optimal contract is continuous, a lower level of investor protection will benefit the entrepreneur in (some or all of) the low states of nature, but will result in a higher probability of verification (legal bankruptcy) and a higher implicit cost of funds. When the optimal contract is discontinuous, the LLCs may not bind and therefore changes in η may leave the terms of the contract unaffected.

Since the optimality of SDCs is a classic question in the CSV literature and the use of such contracts is pervasive in practice, I now explore the role that the different parameters play in satisfying the conditions required for their optimality. Notice that the proof of Theorem 1 shows that if λ^* denotes the optimal value of the multiplier on (**PP.2**), a SDC is optimal if and only if:

$$\lambda^* = \frac{(1-\eta)\int_{x^*}^{\bar{s}} u'\left[s-(1-\eta)\,x^*\right]dH\left(s\right)}{(1-\eta)\left[1-H\left(x^*\right)\right]-\gamma h\left(x^*\right)} > u'\left(\eta \underline{s}\right) \tag{2.1}$$

Precise comparative statics results are difficult to obtain from condition (2.1) without imposing more structure on the problem because λ^* depends directly and indirectly on the parameters through x^*, and some of these effects have opposite signs. However, I am able to extract the following corollary from Theorem 1:

Corollary 2. *If $\eta = 0$ and $\lim_{c\to 0} u'\left(c\right) = \infty$, a SDC is never optimal.*

[11] In Krasa et al. (2008), risk neutrality (of the borrower) implies that a SDC is the only optimal contract so their comparative statics results naturally apply to SDCs only.

In other words, under such specification of preferences, standard debt can be optimal only if $\eta > 0$. Functions satisfying this type of "Inada condition" include, for instance, the constant elasticity of substitution (CES) family with elasticity parameter greater than or equal to one (e.g. Cobb-Douglas), and the constant relative risk aversion (CRRA) function, one of the most widely used in macroeconomics.

I now introduce additional assumptions in order to provide a sharper characterization of optimal contracts. In particular, for each $\gamma \in (0,1)$ define $\underline{x}^* = x^*(0,\gamma)$ as the solution to the problem parametrized by γ and $\eta = 0$. Notice that $\underline{x}^*$ is the solution to the problem originally posed by Townsend (1979) and is henceforth treated parametrically. Moreover, for each (γ, η) Proposition 1 ensures that $\underline{x}^* \leq x^*(\eta,\gamma)$ whenever the contract is continuous. Likewise, define:

$$\bar{u}' \equiv \frac{\int_{\underline{x}^*}^{\bar{s}} u'\left[s - \underline{x}^*\right] dH(s)}{1 - H(\underline{x}^*) - \gamma h(\underline{x}^*)}$$

as the marginal utility of financing in the problem originally studied by Townsend (1979) (i.e., when $\eta = 0$). I now introduce the following assumption:

Assumption 4. $\lim_{c\to 0} u'(c) > \bar{u}'$

This is a relatively weak assumption; it obviously holds for any utility function satisfying the "Inada condition" $\lim_{c\to 0} u'(c) = \infty$ and for functions exhibiting constant absolute risk aversion (CARA) if the coefficient of ARA is not too low. Next, let $\Psi(s) = h(s)/[1 - H(s)]$ denote the hazard function at s, and introduce the following additional assumption:

Assumption 5. *At least one of the following holds:*

A. $h'(s) \geq 0 \; \forall \; s \in \Sigma$

B. $\frac{\Psi'(s)}{\Psi(s)} \geq \frac{1-\eta}{\gamma} \left\{ \frac{u'(\eta z) - u'[\bar{s} - (1-\eta)s]}{u'(\eta \underline{s})} \right\} \forall \ s \in \Sigma$

Assumption 5-A is satisfied, for instance, by the uniform distribution. By requiring that the distribution of returns satisfies this condition, no further restrictions are needed. On the other hand, Assumption 5-B places less stringent conditions upon the distribution of returns, but requires a joint assumption about η, γ and risk aversion.

Heuristically, Assumption 5-B would be difficult to satisfy only if the problem features a combination of high degree of entrepreneur's risk aversion, high investor protection and low verification costs. For instance, if the entrepreneur is risk neutral, or if she is fully protected in bankruptcy ($\eta = 1$), Assumption 5-B simply requires that the hazard function is increasing.[12] In section 2.4.3 we show that for our benchmark parametrization of $u(\cdot)$ and $h(\cdot)$, Assumption 5-B translates into lower bounds for investor protection and verification costs that are satisfied by most available estimates.

We can now provide the following condition for the optimality of a SDC:

Proposition 2. *When assumptions 4-5 are satisfied, there exists η^* such that: for $\eta \leq \eta^*$ a SDC is not optimal, and for $\eta > \eta^*$, a SDC is optimal if a solution exists.*

Proposition 2 tells us that, under assumptions 4-5, standard debt is optimal if and only if investor protection is sufficiently low. The intuition for this result is simple; when the entrepreneur is guaranteed a sufficiently large amount of consumption in bankruptcy, the limited liability constraints bind in every state and the standard debt contract becomes optimal. The qualification at the end of the proposition is required because, as illustrated in the parametric example of section 2.4, if investor protection is sufficiently low, the constraint set given by **PP.2** is empty (see Figure 4). An

[12]This condition is satisfied, for example, by any monotonically increasing transformation of the normal distribution and by most relevant instances of the log-normal distribution.

immediate consequence of Proposition 2 is that, as the entrepreneur becomes more risk averse, the set of values of η for which a SDC is optimal shrinks:

Corollary 3. η^* *is increasing in the entrepreneur's degree of risk aversion.*

The results recorded in Theorem 1, Proposition 2 and corollaries 2-3 relate to the existing literature in at least two ways. First, they complement the capital structure-theoretical argument that higher values of distributable assets (i.e., physical assets plus output) should favor debt as the choice contractual arrangement [Williamson (1988)]. In my model, it is true that for a *given* level of η, a higher minimum output $\underline{s}$ implies that condition (2.1) is more easily satisfied, facilitating the optimality of a SDC. However, as the fraction of output that the entrepreneur receives in bankruptcy decreases, the conditions under which debt is optimal become harder to satisfy.

Secondly, my results can be seen as theoretical support for the findings of Acharya, Amihud and Litov (2011) who suggest a demand-side effect in the market for debt: if investors are well protected in debt contracts, risk-averse entrepreneurs are heavily damaged in the case of bankruptcy, and would therefore find debt less attractive.[13] I should emphasize that this result has no bearing with the *aggregate* level of firm finance, but merely with the relative appeal of different financial contracts (i.e. the capital structure).

[13]Among the first to consider seriously the posibility that strong investor protection may discourage debt issuance are Rajan and Zingales (1995) although their analysis was far from conclusive. More recent studies provide sharper inference on the negative relationship between strong creditor rights and firm debt (e.g., Acharya et al. (2011), Ghoul, O., Cho and Suh (2012)). This is in contrast with the supply-side hypothesis of credit provision bourne out of the strong association observed in the data between investor protection and measures such as credit-to-GDP (see, e.g., La Porta, Lopez-de Silanes, Shleifer and Vishny (1998) and Djankov, McLiesh and Shleifer (2007)).

2.2.5. Investor protection, SDCs and entrepreneur's welfare

I now consider the welfare effects of changes in the level of investor protection when SDCs are implemented.[14]. Define the vector $\boldsymbol{\theta} = \{b, \gamma, \eta, \rho, \boldsymbol{\psi}\}$ where ρ captures the borrower's degree of risk aversion and $\boldsymbol{\psi}$ contains the parameters of the distribution $H(\cdot)$. Since a cut-off value x completely characterizes the SDC, I can reformulate the contracting problem as:

$$v(\boldsymbol{\theta}) = \max_{x \in \Sigma} \int_{\underline{s}}^{x} u(\eta s)\, dH(s) + \int_{x}^{\bar{s}} u[s - (1-\eta)x]\, dH(s) \tag{D.1}$$

subject to:

$$b \leq \int_{\underline{s}}^{x} (1-\eta)\, s dH(s) - \gamma H(x) + (1-\eta)\, x\, [1 - H(x)] \tag{D.2}$$

Once again, notice that Assumption 3 ensures that at least for some values of $\eta \in [0,1]$, the constraint set given by (D.2) is non-empty. The following proposition analyzes the net effect on the borrower's welfare of an increase in η :

Proposition 3. *When a SDC is optimal, the borrower's welfare is decreasing in* η.

Debt contracts are frequently observed in practice even though the analysis presented here suggests that the conditions for their optimality can sometimes be fairly strict. Moreover, since SDCs are simple to understand and enforce, it is conceivable that policymakers or regulators may find these arrangements attractive even if they are suboptimal from the standpoint of the contracting parties. Thus, I now consider the welfare effects of implementing SDCs when they are not optimal:

[14]The effect of increasing γ on the borrower's welfare is always negative and is independent of optimality considerations so we concentrate on η.

Proposition 4. *Suppose that a SDC is implemented but not optimal. Then the borrower's welfare may increase or decrease with η.*

The intuition of Proposition 4 is simple: when condition (2.1) is not satisfied, a SDC gives the borrower too little consumption smoothing (across states) compared to what would be optimal. Therefore, since the partial effect of η (on utility) is greater in the bankruptcy states, rising η gives the borrower more consumption in these states and reduces the gap between optimal and actual consumption smoothing.

2.3. Extensions: investor risk-aversion and dynamic contracts

The foregoing analysis assumes that the investor is risk neutral and that contracts last only one period. In this section I relax these assumptions, revisit the issue of optimal contracts and provide some comparative statics results.

2.3.1. Optimal contracts with a risk-averse investor

Suppose as in Winton (1995) that the investor's utility from consumption c and verification effort (cost) γ is given by $W(c) - \gamma$, where $W' > 0,\ W'' < 0$.[15] The first order of business is to modify Assumption 3 as:

Assumption 6. $\mathbb{E}W[\,(S)] - \gamma > W(b) > W(\underline{s})$.

Clearly, all of the results from lemmas (1)-(3) continue to hold. Thus I need only modify the problem (**PP**.1)-(**PP**.3) by replacing the IRC (**PP**.2) with:

$$\textbf{(PPA.2)} \qquad \int_{\underline{s}}^{x} W[\hat{R}(s)]dH(s) + W(\bar{R})[1 - H(x)] - \gamma H(x) \geq W(b)$$

Let ρ_e, ρ_i denote, respectively, the degrees of risk aversion of the entrepreneur and the investor. Then a condition analogous to (2.1) for the optimality of SDCs follows:

[15] Separability of preferences over consumption and effort/cost is a common assumption in the asymmetric information literature.

Lemma 4. *For the problem (PP.1) subject to (PPA.2)-(PP.3) a SDC is optimal iff:*

$$\lambda^{*}=\frac{(1-\eta)\int_{x^{*}}^{\bar{s}}u'\left[s-(1-\eta)\,x^{*}\right]dH\left(s\right)}{(1-\eta)\,W'[(1-\eta)\,x^{*}]\left[1-H\left(x^{*}\right)\right]-\gamma h\left(x^{*}\right)}>\frac{u'\left(\eta s^{*}\right)}{W'[(1-\eta)\,s^{*}]}\tag{2.2}$$

where $s^{*}=\underline{s}$ *if* $\rho_e>\rho_i$ *while* $s^{*}=x^{*}$ *if* $\rho_e<\rho_i$.

Notice that for the special case of CRRA preferences and when both parties are equally risk averse, condition (2.2) reduces to $\lambda^{*}>u'[\eta/(1-\eta)]$. The natural extension of Proposition 1 trivially follows:

Proposition 5. *Suppose that* $\rho_e>0$. *Then, whenever the optimal contract is continuous and* $\eta\in(0,1)$, *the probability of verification and the implied cost of funds are increasing in* η *and in* γ. *Furthermore,* $\frac{dx^{*}(\eta)}{d\eta}$ *and* $\frac{dx^{*}(\gamma)}{d\gamma}$ *are lower (higher) when* $\rho_e>0$ *if* $W'[(1-\eta)\,x^{*}]$ *is greater (smaller) than one.*

The proof of Proposition 5 shows that if the investor is sufficiently risk averse, the effects of investor protection and verification costs on bankruptcy rates and interest rates are dampened. Next, I provide a comparative statics result that follows a change in the investor's degree of risk aversion:

Proposition 6. *For each* ρ_e *there exists* $\hat{\gamma}\left(\rho_i\right)$ *such that if* $\gamma>\hat{\gamma}\left(\rho_i\right)$, $\rho_i'<\rho_i$ *implies* $x^{*}(\rho_i')>x^{*}\left(\rho_i\right)$ *whenever a SDC is optimal.*

In words, Proposition 6 tells us that so long as verification costs are not too low, the probability of verification and the implied cost of funds decrease as the investor's degree of risk aversion increases. To understand the intuition behind this result, recall that outside the verification region the investor is guaranteed a constant level of consumption. As she becomes more risk averse, a smoother consumption profile becomes more attractive and can be achieved by shrinking the verification set.

2.3.2. Dynamic CSV and imperfect investor protection

In this section I revert to the case of a risk-neutral investor but allow contracts to be long-term in nature. While a complete study of dynamic CSV is beyond the scope of this paper, here I show that in a simplified version of the repeated contracting problem, a result analogous to the first part of Proposition 1 exists. To this end I assume throughout this section (unless otherwise stated) that $\eta \in (0, 1)$.

Assume that time is discrete and runs forever. For simplicity, assume that S has finite support $\tilde{\Sigma} = [s_1, s_2, ...s_N]$ with $0 < s_1 < s_2... < s_N$, and that returns are i.i.d. across periods with $\Pr(S = s_i) = \pi_i \in (0, 1)$ and $\sum_{i=1}^{N} \pi_i = 1$. Furthermore, assume that there is no storage across periods, and that the entrepreneur has no outside saving or borrowing opportunities. Finally, assume that agents discount the future at a common rate $\beta \in (0, 1)$.

Since this problem has been studied elsewhere [e.g. in Wang (2005), Monnet and Quintin (2005)] and the inclusion of imperfect investor protection poses no major technical challenges, a detailed exposition of the dynamic contracting problem is relegated to the Appendix. Here I simply note that, under some mild conditions (shown to hold in the Appendix), the problem can be reduced to a simple static variational problem where the entrepreneur's expected utility, v, is used as a state variable.[16]

Define $\underline{v} = \frac{1}{1-\beta} u(\eta s_1)$ and $\bar{v} = \sup u(c) / (1 - \beta)$, respectively as the minimum and maximum attainable expected (lifetime) utility by the borrower. Define also w as the one-period-ahead expected utility that a contract "promises" the entrepreneur. Let $\mathcal{J} : [\underline{v}, \bar{v}] \to \mathbb{R}$ denote the value that the optimal contract delivers to the lender.

[16]Heuristically, these conditions are that the entrepreneur's continuation payoffs induced by a contract after each history are common knowledge, and that a boundedness condition on the continuation payoffs is satisfied. Green (1987) and Spear and Srivastava (1987) were the first to show that under these conditions there exists a recursive formulation for the maximization problem faced by the principal in models of repeated moral hazard.

Then $\mathcal{J}$ and the optimal contract $\{B^*(v), R^*(v, s_i), w^*(v, s_i), s_i \in \tilde{\Sigma}, v \in [\underline{v}, \bar{v}]\}$ can be characterized recursively by a functional equation that maximizes the lender's expected payoff subject to incentive compatibility, feasibility and a promise keeping constraint (PKC) (see equations (2.19)-(2.24) in Appendix A2).

Instead of trying to fully characterize optimal contracts, which is difficult task without adding more structure to the problem, I focus on the verification set, and look for a result that is the dynamic analog of Proposition 1. To do so, I first reproduce a key proposition found in Wang (2005) for the case of $\eta = 0$:

Proposition 7 (Wang (2005))**.** *Assume that* $\underline{v} > -\infty$. *Then there exists* $\hat{v} \in [\underline{v}, \bar{v}]$, *such that* $\{s_1, s_2, ..., s_{N-1}\} \subseteq B^*(v) \; \forall \; v \leq \hat{v}$.

Wang's result implies that after the entrepreneur's promised utility reaches a lower threshold, the optimal contract specifies that the she is monitored in all income levels except the highest. Such result is familiar in the firm financing literature and is analogous to e.g., that of Clementi and Hopenhayn (2006), in which the investment project is liquidated for sure after a sufficiently long sequence of low realizations of returns. For my purposes, I return to the case of $\eta > 0$ and write the threshold found above as $\hat{v}(\eta)$. Now I can state my result:

Proposition 8. *Suppose that* $\eta_1, \eta_2 \in (0, 1)$. *Then* $\eta_2 > \eta_1 \Rightarrow \hat{v}(\eta_2) > \hat{v}(\eta_1)$.

Thus, a lower level of investor protection expands the set of states $[\underline{v}, \hat{v}(\eta)]$ under which the borrower is fully monitored (except for s_N). Additionally, since both agents have the same discount rate, $[\underline{v}, \hat{v}(\eta)]$ is a set of absorbing states so that $v \leq \hat{v}(\eta)$ resembles bankruptcy. That is, once $v \leq \hat{v}(\eta)$, the borrower is fully monitored forever.

Notice that Proposition 7 depends crucially upon the assumption that $\underline{v} > -\infty$. In particular, because in Wang (2005) $\eta = 0$, his result does not admit reward functions that are unbounded from below (e.g., CRRA). However, in my model, the fact that $\eta > 0$ (along with $s_1 > 0$) implies that $\underline{v} > -\infty$ even for the case of $\lim_{c\to 0} u(c) = -\infty$. This observation bears some resemblance with that of Corollary 2 above.

2.4. Quantitative analysis

I now study the quantitative implications of the static model studied in sections 2.2.1-2.2.5. Our main goal is to illustrate Proposition 1 and the results of section 2.2.5, so I focus on SDCs without loss of generality. That is, I solve the problem (**D.1**)-(**D.2**) and then test whether condition (2.1) holds under various parametrizations. Special attention is given to the relationship between the degree of risk aversion and the remaining parameters of the model. I also consider the relationship between investor protection and leverage, and the quantitative effects of suboptimally implementing SDCs.

2.4.1. Benchmark parametrization

2.4.1.1. Functional forms and non-bankruptcy parameters. Throughout the quantitative analysis I assume risk neutrality for the investor and use $u^e(c) = (c^{1-\rho} - 1)/1-\rho$ for the entrepreneur's payoff function. For my benchmark parametrization I choose a risk aversion coefficient $\rho = 0.25$. This specification of preferences satisfies Assumption 1 and can deliver SDCs as optimal contracts over a range of values for η (see section 2.4). Moreover, this function satisfies $\lim_{c\to 0} u'(c) = \infty$ implying that a SDC would never be optimal if investor protection were perfect ($\eta = 0$). For the distribution function I choose to follow Krasa et al. (2008) and assume $S \sim N(\mu, \sigma^2)$. Table 1 summarizes my choices.

As for the parameters of the density function $h(\cdot)$, I set $\mu = 1.1$ and $\sigma = 0.18$ which are slightly higher than the average and standard deviation of real returns on

Function	Form
investor's payoff	$u^{i}(c) = c$
entrepreneur's payoff	$u^{e}(c) = \frac{c^{1-\rho}-1}{1-\rho}$
distribution	$h(s) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{s-\mu}{\sigma}\right)^{2}\right\}$

Table 2.1. Functional forms

the S&P500. As in Krasa et al. (2008), these somewhat higher figures account for the fact that I am considering an individual firm rather than an index. Finally, I set $b = 0.57$ which is in between the asset:equity ratio of 2:1 required by loans from the Small Business Administration (SBA) and the 2.45:1 mean leverage ratio reported by Kalemli-Ozcan, Sorensen and Yesiltas (2011) for non-financial firms in the U.S. for 2004-2009.

2.4.1.2. Bankruptcy related parameters. The two key parameters for our quantitative exercise are γ which captures bankruptcy costs and η which measures the level of debtor protection. I build my baseline parametrization following the findings of a recent and influential paper by Bris et al. (2006) who report on costs of bankruptcy and recovery rates for what their consider "the largest and most comprehensive sample of (U.S.) corporate bankruptcies assembled for an academic paper".

Importantly for my purposes, Bris et al. (2006) present lower and upper bounds for both parameters, under what they label, respectively, as an "optimistic" and a "reported-only" basis. For the average η, the authors report a range between 0.2 and 0.83, while for the average γ, they report a range between 38% and 80% of

Parameter	Value	Source
γ	0.35	Bris et al. (2006)
b	0.573	Kalemli-Ozcan et al. (2011)
η	0.27	Bris et al. (2006)
μ	1.1	S&P500 and Krasa et al. (2008)
σ	0.18	S&P500 and Krasa et al. (2008)

Table 2.2. Parameter values

distributed post-bankruptcy assets.[17,18] I choose $\gamma = 0.35$ which, as explained below, yields bankruptcy costs as a fraction of assets within the latter range. Finally, with this configuration of parameter values and functional forms, I obtain a threshold $\eta^* = 0.27$ above which a SDC is optimal. Since this value lies within the range reported by Bris et al. (2006), I set it as our benchmark and then conduct sensitivity analysis.[19] My baseline parametrization is summarized in Table 2.

2.4.2. Results of the benchmark parametrization

Here, I present some numerical results from the solution to the contracting problem under the baseline parametrization. My baseline scenario yields a cutoff value $x^* =$

[17]Given the heterogeneity of bankruptcy cases, we would very much like to work with the median rather than the average. However, averages allow us to recover some figures from the original data and ensure internal consistency which the median does not. For instance, in Table III of Bris et al. (2006) the average optimistic recovery rate *before* expenses is 80% of total assets. This is consistent with the average recovery rate of 51% *after* expenses reported on their table XIII, and with the average costs being 38% and 8% of bankrupcty assets and total assets, respectively. On the other hand their reported median recovery rate before expenses is 38%, and after expenses is 70%; something altogether problematic for our purposes.

[18]Interestingly, the parameter values for the benchmark specification in Krasa et al. (2008) lie outside these ranges as they use $\eta = 0.1$ and $\gamma = 0.1$, implying bankruptcy costs of 25% of distributed assets.

[19]An alternative source of estimates for η is Blazy, Petey and Weil (2010) who repor recovery rates (η) between 76% (senior creditors) and 10% (junior creditors) in Germany and between 31% and 6% in the U.K.

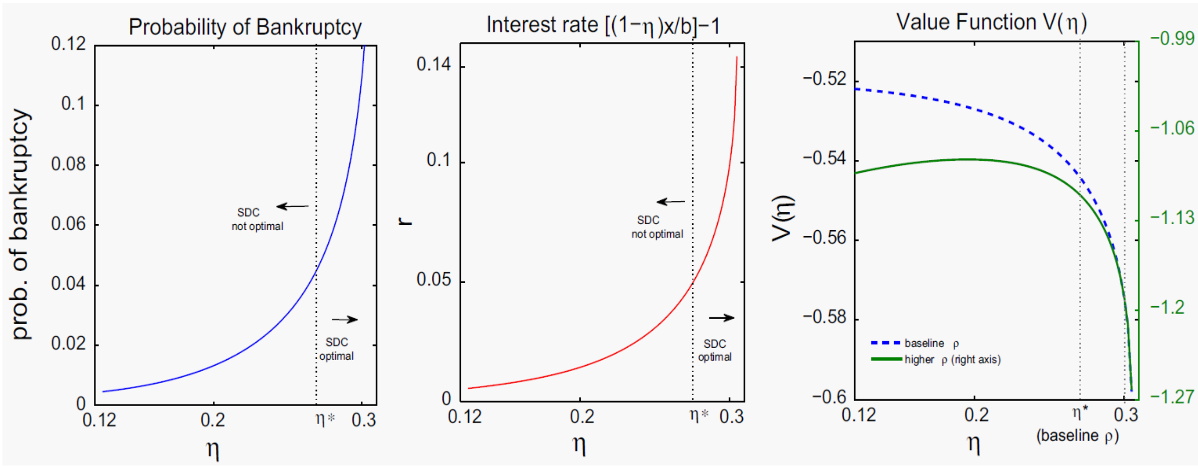


Figure 2.2. Comparative statics

0.82 which in turn results in a bankruptcy probability of 4.9%. Next, knowing x^* I can compute expected *distributed* bankruptcy assets:

$$(1-\eta)\,\mathbb{E}\left[s \mid s \leq x^*\right] = (1-\eta)\,\frac{\int_{\underline{s}}^{x^*} sh\,(s)\,ds}{\int_{\underline{s}}^{x^*} h\,(s)\,ds} = 0.74$$

and bankruptcy costs as a fraction of such assets: $\gamma\left\{(1-\eta)\,\mathbb{E}\left[s \mid s \leq x^*\right]\right\}^{-1} = 0.48$, which lies inside the range reported by Bris et al. (2006). Finally, the benchmark exercise yields a real rate of interest of $(1-\eta)\,x^*b^{-1} - 1 = 5.04\%$, slightly below the average interest rate on small loans reported by the SBA.

I now carry out some simple comparative statics in a neighborhood of the baseline scenario. In particular, I study how the terms of the contract and the borrower's welfare change as our parameter of interest, η, varies. The comparative statics exercise is limited, however, by the fact that optimality of SDC requires a tight IRC (see condition (2.1)). That is, I are able to vary the level of investor protection only up to about 12% before the constraint set given by (**D.2**) becomes empty.

The three panels of Figure 2.2 are simply an illustration of propositions 1 and 3. They show, for instance, that a decrease of 11% in the level of investor protection (i.e. from $\eta = 0.27$ to $\eta = 0.3$) virtually doubles the probability of bankruptcy and the cost of funds (from around 5% to 10% in both cases), which in turn lowers the borrower's value function by more than 5%.

Thus, moderate decreases in creditor protection have substantial quantitative effects on the terms of the contract and the welfare of the borrower. One must keep in mind, however, that such dramatic responses to modest changes in investor protection are largely driven by the tightness of the constraint (**D.2**), which in turn is required by condition (2.1). In other words, given that a SDC is only optimal when the marginal value of financing is very high, it is not surprising that the borrower is willing to incur in increasingly higher costs of funding (via higher interest rates and higher bankruptcy probabilities).

Figure 2.2 also presents comparative statics results for the case in which a SDC is suboptimally implemented. As shown by Proposition 2 this happens when η takes values to the left of $\eta^* = 0.27$. Naturally, the results from Proposition 1 carry over since the contract remains continuous. However, the rightmost panel of Figure 2.2 illustrates the conclusion of Proposition 4 that, when a SDC is suboptimally implemented, the entrepreneur's welfare may increase or decrease with η. In fact for a value of $\rho = 1.5$, such that a SDC is optimal only for $\eta \geq 0.3$, the borrower's value function peaks around $\eta = 0.2$. In a sense, this may help rationalize the existence of imperfect investor protection (i.e., $\eta > 0$) in the presence of risk aversion and incomplete financial markets.

2.4.3. Risk aversion and optimality of standard debt

In the previous subsection, I chose a parametrization that guaranteed the optimality of a SDC. I now test whether this condition holds for alternative combinations of the parameters. In particular, I study the role of γ, η and ρ in satisfying (2.1), while

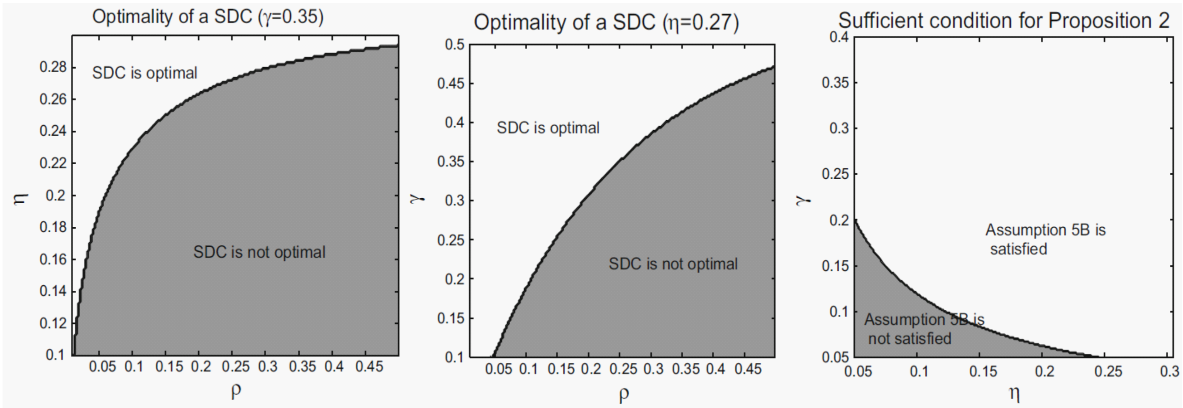


Figure 2.3. When is a SDC optimal?

keeping μ, σ, b as in the baseline exercise. I also study the range of values γ, η for which Assumption 5-B is statisfied. The results are presented in Figure 2.3.

These results illustrate the highly nonlinear relationships between η and ρ, on the one hand, and γ and ρ on the other, in delivering conditions for the optimality of SDCs. The effect is particularly dramatic in the case of (η, ρ): for (2.1) to be satisfied even at a relatively low value $\rho = 0.2$, it is already required that the borrower keeps at least 25% of the assets in the event of bankruptcy. In fact, given $\gamma = 0.35$ and $b = 0.57$, any value of $\rho > 0.4$ requires that $\eta > 0.3$ for a SDC to be optimal.[20] As shown in the next subsection, with these values of γ, b, the constraint set given by IRC (**D.2**) is empty for any $\eta \geq 0.31$.

The left panel of Figure 2.3 also illustrates the result recorded in Proposition 2. That is, for each ρ, the values of η for which a SDC is optimal comprise an upper interval. On the other hand, the right panel shows that (with ρ, σ, μ, b as in the baseline) Assumption 5-B -used in the proof of Proposition 2- is statisfied whenever $\gamma \geq 0.16$ and $\eta \geq 0.24$.

[20]For instance, $u(c) = \ln c$ requires $\eta \geq 0.3064$, resulting in a very tight constraint, with $\lambda^* \geq 18.3$.

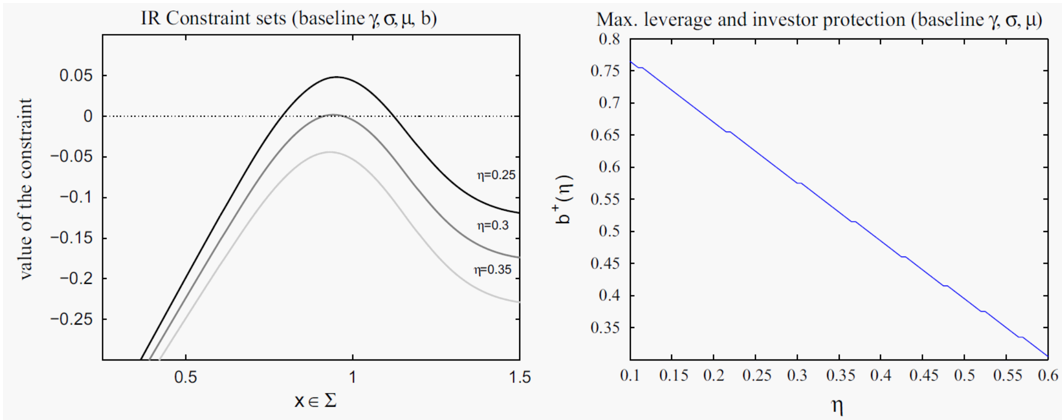


Figure 2.4. The IR-constraint set, maximum leverage and investor protection

2.4.4. Investor protection and leverage

What if we want to consider the case of $\eta \geq 0.31$? This requires alternative values for the remaining parameters. Here, I analyze in particular the relationship between leverage and investor protection. Although this exercise is independent of risk aversion, the issue has not been addressed by the CSV literature, and therefore, I pursue it here. I want to find out how b, which measures the fraction of debt, needs to be varied if we want to consider σ, γ, μ as in the baseline parametrization and $\eta \geq 0.31$.

In the right panel of Figure 4 I have defined $b^{+}(\eta)$ as the maximum fraction of debt (inverse of the leverage ratio) consistent with η, such that the constraint set given by (D.2) is non-empty. My model, thus, gives some analytical background to the recent empirical evidence provided by Pereira and Ferreira (2011) and Cheng and Shiu (2007) who look at panel data regressions and conclude that firms in countries with better creditor protection have higher leverage.

2.5. Concluding remarks

Building upon the existing literature, in this paper, I have presented a simple theory of debt when there is costly verification, imperfect investor protection and a risk averse entrepreneur. These features make my model more amenable to informationally opaque, small and medium sized firms with concentrated ownership or contingent compensation schemes.

Since much of the theoretical literature on investor protection focuses on monitoring costs (γ), a natural avenue for future research is the study of our main parameter of interest, η. Two extensions that come to mind are allowing for η to be either uncertain when financial contracts are signed (i.e. stochastic) and/or endogenously determined by the contracting parties. This last extension would clearly have political economy ramifications, another promising area of research.

2.6. Appendix I: Proofs

2.6.1. Proof of Theorem 1

Proof of Lemma 1. The UPC implies that unverified payments must only depend on the message, that is, $R(\hat{s}, s) = R(\hat{s})$ for any $\hat{s} \notin B$. Therefore, the entrepreneur will choose $\tilde{s} = \arg\min_{\hat{s} \notin B} R(\hat{s})$ so the contract may as well set $R(\tilde{s}) = \bar{R}$. $\square$

Proof of Lemma 2. For any $\hat{s}, s \in B$, the assumption $u' > 0$ implies that a necessary and sufficient condition for (P.3) to be satisfied is that $R(\hat{s}, s) = R(s, s)$. Hence, in the verification region $R(\hat{s}, s) = \hat{R}(s)$ for some $\hat{R}(\cdot)$. Now, for $s \in B$, $\hat{R}(s) > \bar{R}$ can never be optimal since in this case the entrepreneur will prefer to misreport and pay $\bar{R}$ (the ICC is not satisfied). Next, if $\hat{R}(s) = \bar{R}$ *on a set of positive measure*, then the investor will inefficiently pay verification costs when she does not need to so the contract cannot be optimal. This implies that $\hat{R}(s) = \bar{R}$ can hold only for a zero-measure event (i.e. a single point). Therefore, $\hat{R}(s) < \bar{R}$ a.e. and $\hat{R}(s) \leq \bar{R}$ everywhere. $\square$

Proof of Lemma 3. I first show that B is a connected set. This part of the proof is constructive and is a special case of item (iii) of Proposition 1 in Winton (1995). Without loss of generality, suppose that the contract has as verification set a disjoint interval $B = [\underline{s}, x] \cup [s_1, s_2]$ for some $\bar{s} > s_2 > s_1 > x > \underline{s}$, and repayment $\hat{R}(s)$ for $\hat{s} \in B$ and $R(\hat{s}, s) = \bar{R}$ for $\hat{s} \notin B$. The investor's payoff from this contract is:

$$V = \int\limits_{\underline{s}}^{x} \hat{R}(s)\, dH(s) + \int\limits_{s_1}^{s_2} \hat{R}(s)\, dH(s) + \bar{R} - (\bar{R} - \gamma)\left[H(s_2) - H(s_1) + H(x)\right]$$

and the entrepreneur's payoff from the contract is given by:

$$U = \int\limits_{\underline{s}}^{x} u[s - \hat{R}(s)]dH(s) + \int\limits_{x}^{s_1} [us - \bar{R}]dH(s) + \int\limits_{s_1}^{s_2} u[s - \hat{R}(s)]dH(s) + \int\limits_{s_2}^{\bar{s}} u[s - \bar{R}]dH(s)$$

Now, incentive compatibility then requires that for $s \in [s_1, s_2]$, $\bar{R} \geq \hat{R}(s)$. If $\bar{R} = \hat{R}(s)$ there is nothing to prove so suppose that $\bar{R} > \hat{R}(s)$. Now construct a new contract $(\triangle)$. To do so, notice that $\bar{R} > \hat{R}(s)$ implies $\int_{\underline{s}}^{x} \hat{R}(s)\, dH(s) + \bar{R}[1 - H(x)] - \gamma H(x) > V$. Hence, there exists a contract with $B^{\triangle} = [\underline{s}, x]$, $R^{\triangle}(s, s) = \hat{R}(s)$ for $\hat{s} \in B^{\triangle}$ and $R^{\triangle} \in [\hat{R}(s), \bar{R})$ satisfying:

$$V^{\triangle} = \int_{\underline{s}}^{x} \hat{R}(s)\, dH(s) + R^{\triangle}[1 - H(x)] - \gamma H(x) = V$$

Such a contract is feasible since in the initial contract, for $s \in [s_1, s_2]$, $(1 - \eta)s \geq \hat{R}(s) > \bar{R} > R^{\triangle}$. It is also incentive compatible since the repayment function is constant $\forall\ \hat{s} \notin B$ and satisfies $R^{\triangle} \geq \hat{R}(s)$. Under such a contract, the concavity of u guarantees that:

$$U^{\triangle} = \int_{\underline{s}}^{x} u\left[s - \hat{R}(s)\right] dH(s) + \int_{x}^{\bar{s}} u\left[s - R^{\triangle}\right] dH(s) \geq U$$

Thus, I have found a contract that is feasible, incentive compatible and that weakly improves the entrepreneur's welfare, while leaving the investor as well off. Summarizing, when the contract specifies B as a disjoint interval, the contract fails to be optimal. I now show that B is in fact a *lower* interval. It suffices to show that $B \neq \varnothing \Rightarrow \underline{s} \in B$ and I proceed by contradiction. Suppose that $B \neq \varnothing$ but $\underline{s} \notin B$. Since $\underline{s} \notin B$, we have $R(\underline{s}, s) = \bar{R}$, while incentive compatibility requires $\bar{R} \geq \hat{R}(s)$. On the other hand, limited liability requires $(1-\eta)\underline{s} \geq \bar{R}$. Since $(1-\eta)\underline{s} \leq (1-\eta) s \ \forall \ s \in \Sigma$, it follows that $\bar{R} = \hat{R}(s) = (1-\eta)\underline{s}$ which in turn implies that $B = \varnothing$, a contradiction. □

Proof of Theorem 1. For the reformulated problem (**PP.1**)-(**PP.3**) Lemma 5 below shows that the constraint qualification holds. Thus, the problem is equivalent to problem (43) on page 102 of Caputo (2005) with no differential constraints. The Maximum Principle [e.g. Theorem 4.4 in of Caputo (2005)] then implies that there exist constants $\lambda > 0$, $\phi \geq 0$ and nonnegative, continuous functions $\mu_1(\cdot)$, $\mu_2(\cdot)$ such that the following conditions hold:

$$-\mu_1(s) = \left\{u'\left[s - \hat{R}(s)\right] - \lambda\right\} h(s) - \mu_2(s) \quad \forall \ s \leq x \tag{2.3}$$

$$-\phi = \int_x^{\bar{s}} u'\left[s - \bar{R}\right] dH(s) - \lambda\left[1 - H(x)\right] \tag{2.4}$$

$$\frac{-\phi(1-\eta)}{h(x)} = u\left[x - \hat{R}(x)\right] - u\left[x - \bar{R}\right] + \lambda\left[\hat{R}(x) - \bar{R} - \gamma\right] \tag{2.5}$$

along with complementary slackness conditions:

$$0 = \lambda \left\{ \int_{\underline{s}}^{x} \hat{R}(s)\, dH(s) + \bar{R}\,[1 - H(x)] - \gamma H(x) - b \right\} \tag{2.6}$$

$$0 = \phi \left[(1-\eta)\, x - \bar{R}\right] \tag{2.7}$$

$$0 = \mu_1(s) \left[(1-\eta)\, s - \hat{R}(s)\right] \tag{2.8}$$

$$0 = \mu_2(s)\, \hat{R}(s) \tag{2.9}$$

where $\lambda, \phi, \mu_1(\cdot), \mu_2(\cdot)$ are, respectively, the multipliers on the IRC and LLCs. Now suppose that the seven-tuple $\{\bar{R}^*, \hat{R}^*(\cdot), x^*, \lambda^*, \phi^*, \mu_1^*(\cdot), \mu_2^*(\cdot)\}$ is a solution to the system comprising (2.3)-(2.9). Then the triplet $\{\bar{R}^*, \hat{R}^*(\cdot), x^*\}$ achieves the unique maximum of (**PP**.1). To see this, notice that the constraint set is convex and the "maximized Hamiltonian" of the problem above $H(s, \hat{R}(s), \bar{R}^*)$ is strictly concave in $\hat{R}$ for every $s \in \Sigma$. Thus, Arrow's Sufficiency Theorem [see, e.g. Theorem 6.4 in Caputo (2005)] immediately applies. I now classify optimal contracts into families:

i) First, the optimal contract is standard debt if and only if $\mu_1^*(s) > 0\ \forall\ s \leq x^*$. In turn, $\mu_1^*(s) > 0 \Leftrightarrow u'(\eta \underline{s}) < \lambda^*$. To see this, suppose that $\lambda^* > u'(\eta \underline{s})$. Then $\lambda^* > u'(\eta s)\ \forall\ s \leq x^*$. But $\hat{R}^*(s) \leq (1-\eta)\, s\ \forall\ s \leq x^*$ so we have that $\lambda^* > u'(\eta \underline{s}) > u'(\eta s) = u'(s - (1-\eta)\, s) \geq u'[s - \hat{R}^*(s)]\ \forall\ s \leq x^*$. By (2.3) this means that $0 > \frac{\mu_2^*(s) - \mu_1^*(s)}{h(s)}$ which implies $\mu_1^*(s) > 0$. Then (2.8) implies $\hat{R}^*(s) = (1-\eta)\, s\ \forall\ s \leq x^*$. Limited liability then implies that $(1-\eta)\, x^* \geq \bar{R}^* \geq \hat{R}^*(x^*) = (1-\eta)\, x^*$ and the SDC is optimal. To see that the converse is true, suppose that a SDC is optimal. Then $\hat{R}^*(s) = (1-\eta)\, s$ $\forall\ s \leq x^*$ and $\bar{R}^* = (1-\eta)\, x^*$. By (2.8) this implies $\mu_1^*(s) > 0$ which clearly means that $\mu_2^*(s) = 0$. In turn, this implies, by (2.3), that $\forall\ s \leq x^*$:

$0 > u'[s - \hat{R}^*(s)] - \lambda^* = u'(\eta s) - \lambda^*$ since the SDC is optimal. In turn, given $u'(\eta s) \leq u'(\eta \underline{s})$ this implies that $\lambda^* > u'(\eta \underline{s})$.

ii) Now suppose that $\mu_1^*(s) = 0$ for some $s < x^*$. As long as $\mu_1^*(x^*) > 0$ the contract is continuous since $\hat{R}^*(x^*) = (1-\eta)x^* = \bar{R}^*$. Using again condition (2.3) we know $\mu_1^*(s) = 0 \Leftrightarrow u'(\eta s) \geq \lambda^*$. There are two cases to consider. First suppose that $\mu_2^*(s) = 0 \; \forall \; s \leq x^*$ which holds iff $\lambda^* > u'(s)$. Then the optimal contract specifies that $\hat{R}^*(s) > 0 \; \forall \; s$, $\hat{R}^*(s) = s - u'^{(-1)}(\lambda^*)$ whenever $u'(\eta s) \geq \lambda^* > u'(s)$ and $\hat{R}^*(s) = (1-\eta)s$ when $\lambda^* > u'(\eta s)$. Next suppose that $\mu_2^*(s) > 0$ for some s which implies that $s < u'^{(-1)}(\lambda^*)$. Then the optimal contract specifies $\hat{R}^*(s) = 0$ whenever $u'(s) > \lambda^*$, $\hat{R}^*(s) = s - u'^{(-1)}(\lambda^*)$ whenever $u'(\eta s) \geq \lambda^* > u'(s)$ and $\hat{R}^*(s) = (1-\eta)s$ as long as $\lambda^* > u'(\eta s)$. That $\hat{R}^{*\prime}(s) = 1$ for some s follows immediately from $\hat{R}^*(s) = s - u'^{(-1)}(\lambda^*)$ since λ^* is unique and independent of s.

iii) Finally, suppose that $\mu_1^*(s) = 0 \; \forall \; s \leq x^*$ and $\mu_2^*(s) = 0$ for some $s \leq x^*$. Then (2.3) implies that $(1-\eta)x^* > \hat{R}^*(x^*) = x^* - u'^{(-1)}(\lambda^*)$. Thus, LLC and Lemma 2 imply that the optimal contract is discontinuous, i.e., $\bar{R}^* > \hat{R}^*(x^*)$. To see this, suppose that it is continuous and find a contradiction. Continuity implies $\hat{R}^*(x^*) = \bar{R}^* < (1-\eta)x^*$ which in turn implies that $\phi = 0$. But then (2.5) implies that $0 = -\lambda\gamma$. This is true only if $\lambda = 0$ but then (2.4) requires $\int_{x^*}^{\bar{s}} u'\left[s - \bar{R}^*\right] dH(s) = 0$, a contradiction since $\bar{R}^* < (1-\eta)x^*$.

$\square$

Lemma 5. *The rank constraint qualification (RCQ) holds in problem (**PP**.1)-(**PP**.3).*

Proof. In an optimal contract, constraint (**PP.2**) will bind. The remaining constraints may or may not bind depending on the form of the optimal contract. However, a key observation is that $(1-\eta)\, s \geq \hat{R}^*(s)$ and $\hat{R}^*(s) \geq 0$ cannot both be binding at a given s. That the RCQ is satisfied can now be seen by noticing that (**a**) at least one control is present in each of the binding constraints, and (**b**) the number of binding constraints at any given s is at most three, (weakly) less than the number of controls in the problem. In fact, if the solution to the problem (**PP.1**)-(**PP.3**) belongs to family *(i)* of contracts in Proposition 1, $(1-\eta)\, x^* = \bar{R}^*$ and $(1-\eta)\, s = \hat{R}^*(s)\ \forall\ s$. The Jacobian matrix of all active constraints, $\mathbf{J}_i(s, x^*, \bar{R}^*, \hat{R}^*(s))$, in such case is therefore (omitting the arguments):

$$\mathbf{J}_i = \begin{bmatrix} h(x^*)\,[\hat{R}^*(x^*) - \bar{R}^* - \gamma] & 1 - H(x^*) & h(s) \\ 1-\eta & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \equiv \begin{bmatrix} J_i^1 \\ J_i^2 \\ J_i^3 \end{bmatrix}$$

and one can verify that $\mathbf{det}\,(\mathbf{J}_i) = -h(x^*)\,\gamma - (1-\eta)\,[1 - H(x^*)] \neq 0$ since $\eta, \gamma \in (0,1)$ and $h(x^*) > 0$. Let $\mathbf{J}_{ii}$ and $\mathbf{J}_{iii}$ denote the Jacobian matrices of all active constraints when the optimal contract belongs, respectively, to families *(ii)* and *(iii)* in Theorem 1 (again, omitting the arguments). If the optimal contract belongs to family *(ii)*, there are two cases to consider. First, if $\hat{R}^*(s) > 0\ \forall\ s$, $(1-\eta)\, x^* = \bar{R}^*$ and $(1-\eta)\, s = \hat{R}^*(s)$ whenever $s \in (s^\Diamond, x^*]$ for some threshold $s^\Diamond > \underline{s}$, then $\mathbf{J}_{ii} = \mathbf{J}_i$ for $s > s^\Diamond$ and $\mathbf{J}_{ii} = [\ J_i^1 \quad J_i^2\ ]'$ for $s \in \left[\underline{s}, s^\Diamond\right]$; in the latter case $\mathbf{rank}\,(\mathbf{J}_{ii}) = 2$ since $\hat{R}^*(x^*) \leq \bar{R}^* \Rightarrow h(x^*)\,[\hat{R}^*(x^*) - \bar{R}^* - \gamma] - (1-\eta)\,[1 - H(x^*)] \neq 0$ so that at least one of the 2×2 submatrices of $\mathbf{J}_{ii}$ has non-zero determinant. Secondly, if $(1-\eta)\, x^* = \bar{R}^*$, $(1-\eta)\, s = \hat{R}^*(s)$ whenever $s \in (s^\Delta, x^*]$ for some threshold $s^\Delta \leq x^*$ and $\hat{R}^*(s) = 0$ whenever $s \in [\underline{s}, s^\nabla]$ for some threshold $\underline{s} < s^\nabla < s^\Delta$, then $\mathbf{J}_{ii} = \mathbf{J}_i$ for $s > s^\Delta$, $\mathbf{J}_{ii} = [\ J_i^1 \quad J_i^2\ ]'$ for $s \in (s^\nabla, s^\Delta]$ and $\mathbf{J}_{ii} = [\ J_i^1 \quad J_i^2 \quad -J_i^3\ ]'$ for $s \in [\underline{s}, s^\nabla]$; in the latter case, $\mathbf{det}\left(\mathbf{J}_{ii}\right)=h\left(x^{*}\right)\gamma+\left(1-\eta\right)\left[1-H\left(x^{*}\right)\right]\neq0$. The condition also holds if the optimal contract belongs to family *(iii)* in Theorem 1 since $\mathbf{J}_{iii}=\left[\begin{array}{cc} J_{i}^{1} & -J_{i}^{3}\end{array}\right]'$ and $H\left(x^{*}\right)<1$ implies that at least one of the 2×2 submatrices of $\mathbf{J}_{iii}$ has non-zero determinant. $\square$

2.6.2. Proof of propositions 1 - 6

Proof of Proposition 1. First, notice that continuity of the contract implies that $\hat{R}\left(x\right)=\bar{R}=\left(1-\eta\right)x^{*}$ so that condition (**PP.2**) can be written as:

$$\int_{\underline{s}}^{x^{*}}\hat{R}\left(s\right)dH\left(s\right)-\gamma H\left(x^{*}\right)+\left(1-\eta\right)x^{*}\left[1-H\left(x^{*}\right)\right]=b \tag{2.10}$$

Next, write $x^{*}\left(\eta\right)$ to explicitly account for the dependence of x^{*} on η and totally differentiate (2.10) w.r.t. η while solving for $\frac{dx^{*}\left(\eta\right)}{d\eta}$:

$$\frac{dx^{*}\left(\eta\right)}{d\eta}=\frac{x^{*}\left(\eta\right)\left[1-H\left(x^{*}\left(\eta\right)\right)\right]}{\left(1-\eta\right)\left[1-H\left(x^{*}\left(\eta\right)\right)\right]-\gamma h\left(x^{*}\left(\eta\right)\right)} \tag{2.11}$$

Now, $\left(1-\eta\right)\int_{x^{*}}^{\bar{s}}u'\left[s-\left(1-\eta\right)x^{*}\right]dH\left(s\right)=\lambda^{*}\left\{\left(1-\eta\right)\left[1-H\left(x^{*}\right)\right]-\gamma h\left(x^{*}\right)\right\}$ and $\lambda^{*}>0$ together imply that $\frac{dx^{*}}{d\eta}>0$ and the first part of the proposition follows. Now write $x^{*}\left(\gamma\right)$ and differentiate (2.10) w.r.t. γ while solving for $\frac{dx^{*}\left(\gamma\right)}{d\gamma}$:

$$\frac{dx^{*}\left(\gamma\right)}{d\gamma}=\frac{H\left(x^{*}\left(\gamma\right)\right)}{\left(1-\eta\right)\left[1-H\left(x^{*}\left(\gamma\right)\right)\right]-\gamma h\left(x^{*}\left(\gamma\right)\right)}>0$$

$\square$

Proof of Proposition 2. Recall that a SDC is optimal iff $\lambda^{*}>u'\left(\eta\underline{s}\right)$ or, equivalently, iff:

$$C\left(\eta\right)\equiv\frac{\int_{x^{*}}^{\bar{s}}u'\left[s-\left(1-\eta\right)x^{*}\right]dH\left(s\right)}{\left[1-H\left(x^{*}\right)\right]}-u'\left(\eta\underline{s}\right)+u'\left(\eta\underline{s}\right)\frac{\gamma}{1-\eta}\Psi\left(x^{*}\right)>0$$

We will therefore show that $C(\eta) > 0$ if and only if $\eta > \eta^*$ for some $\eta^* > 0$. Notice that $\lim_{\eta \mapsto 1} C(\eta) > 0$ while Assumption 4 ensures that $\lim_{\eta \mapsto 0} C(\eta) < 0$. By the continuity of $\eta \mapsto C(\eta)$, it suffices to show that $C(\eta) > 0 \Rightarrow C'(\eta) > 0$. To show this, suppose that $C(\eta) > 0$ and differentiate:

(2.12)

$$\begin{aligned}
C'(\eta) = & \frac{\int_{x^*}^{\bar{s}} u''\left[s-(1-\eta)x^*\right]dH(s)}{1-H(x^*)}\left\{x^* - (1-\eta)\frac{dx^*}{d\eta}\right\} \\
& - u''(\eta \underline{s})\,\underline{s}\left\{1 - \frac{\Psi(x^*)\gamma}{1-\eta}\right\} + u'(\eta \underline{s})\frac{\Psi(x^*)\gamma}{(1-\eta)^2} \\
& + \frac{dx^*}{d\eta}\left\{\frac{\Psi(x^*)\int_{x^*}^{\bar{s}} u'\left[s-(1-\eta)x^*\right]dH(s)}{1-H(x^*)} + \frac{\Psi'(x^*)\,u'(\eta \underline{s})\gamma}{1-\eta} - u'(\eta x^*)\Psi(x^*)\right\}
\end{aligned}$$

The expression inside braces in the first term is negative. To see this, notice that SDC optimal implies:

$$(1-\eta)\frac{dx^*}{d\eta} = \frac{(1-\eta)\left\{\int_{\underline{s}}^{x^*} s\,dH(s) + x^*\left[1-H(x^*)\right]\right\}}{(1-\eta)\left[1-H(x^*)\right] - \gamma h(x^*)} > \frac{x^*}{1 - \frac{\Psi(x^*)\gamma}{1-\eta}}$$

and $\lambda^* > 0$ requires $1-\eta > \Psi(x^*)\gamma$, which in turn means that $x^* - (1-\eta)\,dx^*/d\eta < 0$. Next, $1-\eta > \Psi(x^*)\gamma$ also implies that the second term in (2.12) is positive. Lastly, notice that the expression inside braces in the third line of (2.12) is also positive. This is true by construction when Assumption 5-B holds. To see that this is also true if Assumption 5-A holds instead, rewrite as:

$$\Psi(x^*)\left\{\frac{\int_{x^*}^{\bar{s}} u'\left[s-(1-\eta)x^*\right]dH(s)}{1-H(x^*)} + \frac{u'(\eta \underline{s})\gamma}{1-\eta}\Psi(x^*) - u'(\eta x^*)\right\} + \frac{u'(\eta \underline{s})\gamma}{1-\eta}\frac{h'(x^*)}{1-H(x^*)}$$

Clearly, $C(\eta) > 0$ and $h'(x^*) \geq 0$ guarantee that this last expression is positive. Hence, we conclude that $C(\eta) > 0 \Rightarrow C'(\eta) > 0$ and the statement of the proposition follows. $\square$

Proof of Corollary 3. Let ρ_e denote the entrepreneur's degree of risk aversion and suppose that for (η, ρ_e') a SDC is optimal. Notice that since a SDC is optimal, (**PP**.2) becomes (**D**.2) and since verification costs are deadweight losses, x^* is the smallest x such that (**D**.2) binds. Moreover, (**D**.2) does not depend on ρ_e, which implies $dx^*(\rho_e')/d\rho_e = 0$. Next, recall that for any ρ_e, $\lambda^*(\eta) > u'(\eta\underline{s}) \Leftrightarrow \eta > \eta^*$ and notice that (2.1), $dx^*(\rho_e')/d\rho_e = 0$ together imply that $D(\eta, \rho_e') = \lambda^*(\eta, \rho_e')/u'(\eta\underline{s})$ is decreasing in ρ_e for each η. Hence, we may write the threshold found in Proposition 2 as $\eta^*(\rho_e')$ so that $\eta > \eta^*(\rho_e') \Rightarrow \lambda^*(\eta, \rho_e') > u'(\eta\underline{s})$. To complete the proof, choose $\eta_\varepsilon = \eta^*(\rho_e') + \varepsilon$ for $\varepsilon > 0$ arbitrarily small. Then $\rho_e'' > \rho_e' \Rightarrow D(\eta_\varepsilon, \rho_e') > 0 \geq D(\eta_\varepsilon, \rho_e'')$. That is, a SDCs is optimal for $(\eta_\varepsilon, \rho_e')$ but not for $(\eta_\varepsilon, \rho_e'')$. □

Proof of Proposition 3. Let $\boldsymbol{\theta}_{-\eta} = \{b, \gamma, \rho_e, \boldsymbol{\psi}\}$. Applying the Envelope theorem to (**D**.1)-(**D**.2):

$$\frac{dv(\boldsymbol{\theta}_{-\eta};\eta)}{d\eta} = \int_{\underline{s}}^{x^*} [u'(\eta s) - \lambda^*]\, s dH(s) + x^* \left\{ \int_{x^*}^{\bar{s}} u'[s - (1-\eta)x^*]\, dH(s) - \lambda^*[1 - H(x^*)] \right\}$$

using the expression for λ^* and after a minor algebraic manipulation we get:

$$\frac{dv(\boldsymbol{\theta}_{-\eta};\eta)}{d\eta} = \int_{\underline{s}}^{x^*} [u'(\eta s) - \lambda^*]\, s dH(s) - x^* \frac{\gamma h(x^*) \int_{x^*}^{\bar{s}} u'[s-(1-\eta)x^*]\, dH(s)}{[1-H(x^*)](1-\eta) - \gamma h(x^*)}$$

now, the last term above is clearly negative since $\lambda^* > 0 \Rightarrow [1 - H(x^*)](1-\eta) - \gamma h(x^*) > 0$. Moreover, the first term is also negative since the optimality of SDCs implies $\forall\ s \leq x^*$, $u'(\eta s) < \lambda$. Thus, we conclude that $dv(\boldsymbol{\theta}_{-\eta};\eta)/d\eta < 0$. □

Proof of Proposition 4. It suffices to show that: $\int_{\underline{s}}^{x^*} [u'(\eta s) - \lambda]\, s dH(s) > x^* \frac{\gamma h(x^*)\lambda^*}{(1-\eta)}$ for some parametrization that satisfies assumptions 1-3 and violates condition (2.1).

Consider the case used in Corollary 2, that is, suppose that $\lim_{c\to 0} u'(c) = \infty$ and $\eta = 0$. Then obviously $\lambda^* < u'(\eta \underline{s})$ and assumptions 1-3 are satisfied but $\int_{\underline{s}}^{x^*} [u'(\eta s)]\, s dH(s) > \lambda^* [H(x^*) + x^* \gamma h(x^*)]$. $\square$

Proof of Lemma 4. The complete system of necessary and sufficient conditions is given by (2.7)-(2.9) and:

$$\text{(2.13)} \quad -\mu_1(s) = \left\{ u'\left[s - \hat{R}(s)\right] - \lambda W'\left[\hat{R}(s)\right] \right\} h(s) - \mu_2(s) \quad \forall\ s \leq x$$

$$\text{(2.14)} \quad -\phi = \int_x^{\bar{s}} u'\left[s - \bar{R}\right] dH(s) - \lambda W'\left(\bar{R}\right)[1 - H(x)]$$

$$\text{(2.15)} \quad \frac{-\phi(1-\eta)}{h(x)} = u\left[x - \hat{R}(x)\right] - u\left[x - \bar{R}\right] + \lambda \left\{ W\left[\hat{R}(x)\right] - W\left(\bar{R}\right) - \gamma \right\}$$

$$\text{(2.16)} \quad 0 = \lambda \left\{ \int_{\underline{s}}^{x} W\left[\hat{R}(s)\right] dH(s) + W\left(\bar{R}\right)[1 - H(x)] - \gamma H(x) - W(b) \right\}$$

An argument identical to that found in the proof of Theorem 1-(i) shows that a SDC is optimal iff:

$$\lambda^* = \frac{(1-\eta)\int_{x^*}^{\bar{s}} u'\left[s - (1-\eta)x^*\right] dH(s)}{(1-\eta) W'[(1-\eta)x^*]\left[1 - H(x^*)\right] - \gamma h(x^*)} > u'(\eta s)/W'[(1-\eta)s]\ \forall\ s \leq x^*$$

The proof is completed by noticing that

$$\rho_e > \rho_i \Rightarrow u'(\eta \underline{s})/W'[(1-\eta)\underline{s}] \geq u'(\eta s)/W'[(1-\eta)s] \forall\ s \leq x^*$$

while

$$\rho_e < \rho_i \Rightarrow u'(\eta x^*)/W'[(1-\eta)x^*] \geq u'(\eta s)/W'[(1-\eta)s]\ \forall\ s \leq x^*.$$

$\square$

Proof of Proposition 5. Differentiate:

$$\int_{\underline{s}}^{x^*} W\left[\hat{R}(s)\right] dH(s) - \gamma H(x^*) + W[(1-\eta)x^*][1-H(x^*)]$$

w.r.t. γ and η to obtain

$$\frac{dx^*(\eta)}{d\eta} = \frac{x^*(\eta)[1-H(x^*(\eta))]W'[(1-\eta)x^*]}{(1-\eta)W'[(1-\eta)x^*][1-H(x^*(\eta))] - \gamma h(x^*(\eta))}$$

$$\frac{dx^*(\gamma)}{d\gamma} = \frac{H(x^*(\gamma))}{(1-\eta)W'[(1-\eta)x^*][1-H(x^*(\eta))] - \gamma h(x^*(\eta))}$$

and, as before, $\lambda^* > 0$ implies that $\frac{dx^*(\eta)}{d\eta}, \frac{dx^*(\gamma)}{d\gamma} > 0$. The last statement of the proposition trivially follows by replacing $W'[(1-\eta)x^*] \lesseqgtr 1$ and comparing the results with the proof of Proposition 1. $\square$

Proof of Proposition 6. It suffices to show that $x^*(\rho_i) < x^*(0)$ for $\rho_i > 0$. To do so, first define $\mathbb{E}(C_x) = \int_{\underline{s}}^{x}(1-\eta)s dH(s) + (1-\eta)x[1-H(x)]$. Next, notice that $W(\cdot)$ concave implies that the function $\Omega(\gamma) = W[\mathbb{E}(C_x)] - \gamma H(x) - W[\mathbb{E}(C_x) - \gamma H(x)]$ is non-negative, continuous and increasing for each x. In fact, $\Omega(0) = 0$ and $\lim_{\gamma\to\infty}\Omega(\gamma) = \infty$. Hence, for each ρ_i, $\exists\ \hat{\gamma}(\rho_i)$ such that $\gamma \geq \hat{\gamma}(\rho_i) \Rightarrow \Omega(\gamma) > W[\mathbb{E}(C_x)] - \mathbb{E}[W(C_x)]\ \forall\ x$. Thus, for a given $\rho_i > 0$ and $\gamma \geq \hat{\gamma}(\rho_i)$ we have: $\mathbb{E}\left[W\left(C_{x^*(0)}\right)\right] - \gamma H(x^*(0)) > W\left[\mathbb{E}\left(C_{x^*(0)}\right) - \gamma H(x^*(0))\right] = W(b)$, where the last inequality follows from the observation that $x^*(0)$ must satisfy $\mathbb{E}\left(C_{x^*(0)}\right) - \gamma H(x^*(0)) = b$. Therefore, at $x^*(0)$, $F(x^*(0)) \equiv \mathbb{E}\left[W\left(C_{x^*(0)}\right)\right] - \gamma H(x^*(0)) - W(b) > 0$ and the IR constraint is slack. Next, notice that, in general, $x^* = \min\{x \mid F(x) = 0\}$ which in turn means that $x < x^* \Rightarrow F(x) < 0$. Therefore, it must be that $x^*(0) > x^*(\rho_i)$. $\square$

2.6.3. The dynamic contracting problem and propositions 7 - 8

In this section I spell out the details of the dynamic extension of the contracting problem. As in the static case, I assume that the entrepreneur has access to a technology but lacks enough funds to operate it so she must enter a (long-term) contract with an investor. I also maintain the assumption of two-sided commitment. In addition to the modifications mentioned in the first two paragraphs of Section 2.3.2, I allow for the transfers to the investor to be positive or negative and assume that he has instant access to a credit market. Thus, the lower LLC is relaxed to a negative number, $-s_0$ with $s_0 > 0$.

At time $t = 0$ the investor makes a take-it-or-leave-it offer to the entrepreneur with the terms of a financial contract. Each t is associated with a history of events $h^t = \{h_1, h_2, ..., h_t\} \in \mathcal{H}^t$ where $\mathcal{H}^t$ is the set of all possible such histories and without loss of generality $h^t = \emptyset \; \forall \, t \leq 0$. Under CSV, histories include all past announcements by the entrepreneur and the list of previous periods in which verification took place. That is, the typical component of a particular history is a pair $h_t = \{\hat{s}_t, q_t\}$ where $\hat{s}_t$ is what the entrepreneur reports as the state and $q_t = 1$, if monitoring occurred in period t, and $q_t = 0$ otherwise.[21] Moreover, under CSV the contract also includes $B\left(h^{t-1}\right)$, a set of states in which the lender verifies after observing history h^{t-1}. In the symmetric information case $(\gamma = 0)$, $h^t = s^t = \{s_1, s_2, ..., s_t\}$. In order to proceed to the formulation of the problem, I need some definitions:

Definition 2. *A reporting strategy for the entrepreneur,* $\hat{\mathfrak{Z}}$, *is a sequence of functions that maps histories up to* t *into reports of the state, i.e.:* $\hat{\mathfrak{Z}} = \left\{\hat{s}_t\left(h^t\right)\right\}_{t=1}^{\infty} = \left\{\hat{s}_t\left(h^{t-1}, s_t\right)\right\}_{t=1}^{\infty}$.

[21] In the current environment the revelation principle still holds and therefore we can reduce the message space to the set $\tilde{\Sigma}$ without loss of generality. A formal proof be found in Monnet and Quintin (2002).

Let $\mathbf{S}$ denote the set of all possible reporting strategies. As in the static problem, the entrepreneur will not misrepresent in the verification region so that $q_t = 1 \Rightarrow h_t = \{s_t, 1\}$ and $q_t = 0 \Rightarrow h_t = \{\hat{s}_t, 0\}$. Next, let $\mathfrak{z}$ be the strategy in which the agent always reports truthfully.

Definition 3. *A verification strategy for the lender is a sequence of set-valued mappings* $\{B_t(h^{t-1})\}_{t=1}^{\infty}$ *assigning to each history* h^{t-1} *a verification region, i.e., a set of states for which verification occurs.*

The set B_t is the natural time-varying extension of the set B in section 2.3. Next, define:

Definition 4. *A dynamic CSV contract* $\mathbf{\Phi}= \{B_t(h^{t-1}), R_t(h^t)\}_{t=1}^{\infty}$ *is a sequence of mappings assigning current period verification strategies and repayments to each history.*

Notice that $B_t(h^{t-1}) \subseteq \tilde{\Sigma}$ depends upon the history of events up to $t-1$, as verification decisions are independent of the current period realization of the state. On the other hand, $R_t(h^t)$ is contingent on the current realization of the state and therefore depends upon the history up to t. Let $\mu(B_t(h^{t-1}))$ be the probability with which verification takes place after history h^{t-1} when the principal uses strategy B_t. That is, $\mu(B_t(h^{t-1})) = \sum_{i \mid s_i \in B_t(h^{t-1})} \pi_i$. Next, define the lender's expected discounted payoff from the strategies in the subgame starting after h^t given a contract $\mathbf{\Phi}$ and a reporting strategy $\hat{\mathfrak{z}}$:

$$\mathcal{Q}_t\left(h^t, \hat{\mathfrak{z}}, \mathbf{\Phi}\right) = \mathbb{E}_t \sum_{\tau=t+1}^{\infty} \beta^{\tau-t-1}\left\{R_\tau(h^{\tau-1}, \hat{s}_\tau\left(h^{\tau-1}, s_\tau\right), q_\tau) - \mu\left(B_\tau\left(h^{\tau-1}\right)\right)\gamma\right\}$$

where the expectation is conditional on h^t taken with respect to the probability measure that the reporting strategy and verification policy implicitly induce on $\mathcal{H}^t$.

On the other hand, the entrepreneur's payoff is:

$$V_t\left(h^t, \hat{\mathfrak{z}}, \mathbf{\Phi}\right) = \mathbb{E}_t \sum_{\tau=t+1}^{\infty} \beta^{\tau-t-1} u\left[s_\tau - R_\tau(h^{\tau-1}, \hat{s}_\tau\left(h^{\tau-1}, s_\tau\right), q_\tau)\right]$$

Let $\mathcal{Q}_1(h^0, \hat{\mathfrak{z}}, \mathbf{\Phi}\ ), V_1(h^0, \hat{\mathfrak{z}}, \mathbf{\Phi})$ be the parties' payoffs at the beginning of time 1. I can now define incentive compatible and feasible contracts:

Definition 5. *A contract* $\mathbf{\Phi}$ *is incentive compatible if* $\forall\ h^t$ *and* $\forall\ \hat{\mathfrak{z}} \in \mathbf{S}$

$$\begin{aligned} V_t\left(h^t, \mathfrak{z}, \mathbf{\Phi}\right) &= \mathbb{E}_t \sum_{\tau=t+1}^{\infty} \beta^{\tau-t-1} u\left[s_\tau - R_\tau(h^{\tau-1}, s_\tau, q_\tau)\right] \\ &\geq \mathbb{E}_t \sum_{\tau=t+1}^{\infty} \beta^{\tau-t-1} u\left[s_\tau - R_\tau(h^{\tau-1}, \hat{s}_\tau\left(h^{\tau-1}, s_\tau\right), q_\tau)\right] = V_t(h^t, \hat{\mathfrak{z}}, \mathbf{\Phi}) \end{aligned} \tag{2.17}$$

Definition 6. *A contract is feasible if,* $\forall\ h^t,\ R_t(h^t) \geq -s_0,$

$$R_t(h^t) \leq (1-\eta)\, \hat{s}_t \text{ if } q_t = 0 \text{ and } R_t(h^t) \leq (1-\eta)\, s_t \text{ if } q_t = 1. \tag{2.18}$$

Notice that conditions (2.18) imply that $\forall\ h^t$, $V_t(h^t, \mathfrak{z}, \mathbf{\Phi}) \geq \frac{1}{1-\beta} \sum_{i=1}^{N} \pi_i u\left(\eta s_i\right) = \underline{v}$. Now let $\mathcal{V}$ be the set of all entrepreneur's expected discounted payoffs, v, that can be generated by a contract satisfying (2.17)-(2.18) and $V_1\left(h^0, \mathfrak{z}, \mathbf{\Phi}\right) = v$. Let $\bar{v} = \sup u\left(c\right) / \left(1-\beta\right) = \frac{1}{1-\beta} \sum_{i=1}^{N} \pi_i u\left[s_0 + s_i\right]$ be the maximum attainable expected payoff for the entrepreneur. Statement (i) of Lemma 2 in Wang (2005) (pp. 902) established that $\mathcal{V} = [\underline{v}, \bar{v}]$ when $\eta = 0$. This result can be trivially extended for the case of $\eta > 0$. Next, for each $v \in [\underline{v}, \bar{v}]$, an optimal contract maximizes the value obtained by the investor among all incentive-compatible and feasible contracts that deliver an initial value v to the entrepreneur. This defines a frontier of values $\mathcal{J}\left(v\right) = \max\left\{\mathcal{J} \mid \exists\ \mathbf{\Phi} \text{ such that } V_1(h^0, \mathfrak{z}, \mathbf{\Phi}) = v \text{ and } \mathcal{Q}_1(h^0, \mathfrak{z}, \mathbf{\Phi}\ ) = \mathcal{J}\right\}.$

Under mild condition shown to hold below, the extensive form contract has an equivalent recursive representation $\left\{B(v), R\left(v, s_i\right), w\left(v, s_i\right), s_i \in \tilde{\Sigma}, v \in [\underline{v}, \bar{v}]\right\}$ where

$R(v, s_i)$ is the repayment schedule and $w(v, s_i)$ is next period "promised" utility when the current expected payoff for the borrower is v and the state of nature is s_i. Hence, the optimal contract is found by solving:

$$\mathcal{J}(v) = \max_{R(v,\cdot),w(v,\cdot),B(v)} \left\{ \sum_{i=1}^{N} \pi_i \left[R(v, s_i) + \beta \mathcal{J}(w(v, s_i)) \right] - \sum_{i|s_i \in B(v)} \pi_i \gamma \right\} \tag{2.19}$$

subject to :

(2.20)

$$u(s_i - R(v, s_i)) + \beta w(v, s_i) \geq u(s_i - R(v, s_j)) + \beta w(v, s_j) \;\; \forall\, s \in \tilde{\Sigma}, \;\forall\, s_j \notin B(v)$$

$$w(v, s_i) \in [\underline{v}, \bar{v}] \;\; \forall\, s_i \in \tilde{\Sigma} \tag{2.21}$$

$$v = \sum_{i=1}^{N} \pi_i \left[u(s_i - R(v, s_i)) + \beta w(v, s_i) \right] \tag{2.22}$$

$$-s_0 \leq R(v, s_i) \leq (1 - \eta) s_i \;\; \forall\, s_i \in \tilde{\Sigma} \tag{2.23}$$

$$B(v) \subseteq \tilde{\Sigma} \tag{2.24}$$

where (2.20) require temporary incentive compatibility (t.i.c.), (2.23) are the modified LLCs and (2.22) is the so-called promise-keeping constraint (PKC). The functions $\{R(v, \cdot), w(v, \cdot), B(v)\}$ should be though of as policy rules in the sense that they are invariant functions of the state variable v.

There are two main requirements for (2.19)-(2.24) to be an equivalent formulation to the sequential problem. The first and most obvious one is that, if the entrepreneur's continuation utility is to be considered a candidate for summarizing history, her preferences over continuation contracts must be common knowledge after any history. This condition is satisfied in the current problem given my assumption that the entrepreneur cannot save or engage in side trades.

The second condition is that continuation payoffs lie in a bounded set.[22] This condition is satisfied since $\frac{1}{1-\beta}u\left(\eta s_1\right) = \underline{v} > -\infty$ while $\frac{1}{1-\beta}\sum_{i=1}^{N}\pi_i u\left[s_0 - (1-\eta)s_i\right] = \bar{v} < \infty$. Notice that since I are not assuming $\lim_{c\to 0} u\left(c\right) > -\infty$, it is essential that $\eta \in (0,1)$ and $s_1 > 0$. Under this boundedness condition, one can show that, if for a given v_0, an allocation $\{B_t(h^{t-1}), R_t(h^t)\}_{t=1}^{\infty}$ is generated recursively by the policy rules, then the allocation delivers the promised utility v_0. To see this, define $c_t\left(h^t\right) = s_t - R_t(h^t)$ and iterate on the PKC (2.22) to get:

$$v_0 = \sum_{t=0}^{T} \beta^T \mathbb{E}_0 u(c_t(h^t)) + \beta^T \mathbb{E}_0 w_T(h^T), \quad \forall\ T$$

so that $v_0 = \sum_{t=0}^{\infty} \beta^T \mathbb{E}_0 u(c_t(h^t)) + \lim_{T\to\infty} \beta^T \mathbb{E}_0 w_T(h^T) = \sum_{t=0}^{\infty} \beta^T \mathbb{E}_0 u(c_t(h^t))$, where the last equality follows since $w_T \in [\underline{v}, \bar{v}] \Rightarrow \lim_{T\to\infty} \beta^T \mathbb{E}_0 w_T(h^T) = 0$. Under these conditions, Lemma 2 in Green (1987) implies that t.i.c. is equivalent to incentive compatibility in the sense of (2.17). This justifies the use of a recursive formulation, from which it is easy to obtain propositions 7 and 8.

Proof of Proposition 7. See Wang (2005), page 915. □

Proof of Proposition 8. Suppose that the borrower has been promised $\underline{v}$. It is enough to show that if $s_{N-1} \notin B^*\left(\underline{v}\right)$ the contract fails to be optimal. Suppose that $s_{N-1} \notin B^*\left(\underline{v}\right)$; and suppose that the borrower uses a strategy that calls her to report s_{N-1} when when she observes s_N. Then the PKC is necessarily violated, for, the

[22] This is actually a sufficient condition. In fact, the necessary condition is $\lim_{t\to\infty} \beta^t \mathbb{E}_0 w_t\left(h^t\right) = 0$, which is a condition equivalent to (7) in Theorem 9.2 of Stokey, Lucas and Prescott (1989) (pp. 246).

minimum expected utility that the borrower can obtain is

$$\sum_{i=1}^{N-2} \pi_i u(\eta s_i) + \sum_{i=N-1}^{N} \pi_i u\left[s_i - R^*\left(\underline{v}, s_{N-1}\right)\right] + \beta w^*\left(\underline{v}, s_{N-1}\right).$$

But, since we know that $w^*\left(\underline{v}, s_{N-1}\right) \geq \underline{v}$ and $R^*\left(\underline{v}, s_{N-1}\right) \leq (1-\eta)\, s_{N-1}$ (by (2.23)), this lower bound cannot be less than:

$$\sum_{i=1}^{N-1} \pi_i u(\eta s_i) + \pi_N u\left[s_N - (1-\eta)\, s_{N-1}\right] + \beta \underline{v} > \sum_{i=1}^{N} \pi_i u(\eta s_i) + \beta \underline{v} = \underline{v}$$

So define the left hand side of the first inequality to be the threshold $\hat{v}(\eta)$. Clearly then, for any $v \leq \hat{v}(\eta)$, $\{s_1, s_2, ..., s_{N-1}\} \nsubseteq B^*(v)$ is not incentive compatible. It is now easy to see that $\eta \mapsto \hat{v}(\eta)$ is increasing so that the statement of the proposition follows. $\square$

CHAPTER 3

BANKRUPTCY CHOICE WITH ENDOGENOUS FINANCIAL CONSTRAINTS

3.1. Introduction

Models of financial frictions and firm dynamics typically ignore the possibility that troubled firms are rehabilitated.[1] However, recent data from both developed and developing countries strongly suggest that alternatives to liquidation are important resolution mechanisms for financially distressed firms. In the U.S., according to data from bankruptcy courts and Dun & Bradstreet, about 25% of formal bankruptcy filings that followed the 2008 financial crisis were for Chapter 11,[2] while two thirds of overall business failures (including informal bankruptcy and private workouts) were resolved under some reorganization procedure (see Table B1 and Figure B1 in Appendix B). Moreover, the World Bank's 2012 Doing Business reports that 21 economies have introduced or improved (in- and/or out-of-court) reorganization or debt restructuring proceedings since 2005, including: Spain in 2009, Italy and Austria in 2010, Denmark and South Africa in 2011, and more recently, Germany.

What are the potential consequences of this recent spread in the use of alternative bankruptcy procedures? What is the role of institutional factors in producing different outcomes from alternative bankruptcy procedures? This paper provides answers to

[1] Recent exceptions are Corbae and D'Erasmo (2014), Peri (2014) and Senkal (2013).

[2] Chapter 7 of the U.S. Bankruptcy Code governs the process of liquidation in which the assets of a corporation are sold either piecemeal or as a going concern. Alternatives to liquidation are Chapters 11 and 13 of the Code, under which an exchange of securities is formally proposed in a reorganization plan.

such questions by extending a simple model of endogenous financial constraints with liquidation, to allow for the possibility of default, renegotiation and reorganization.

In the model presented here, an entrepreneur is endowed with a risky project and signs a contract with a bank in order to invest and realize project returns. However, financial contracts are constrained by moral hazard and limited commitment. In particular, in every period the entrepreneur receives a random outside option and cannot credibly commit not to exercise it. A key feature of the model is that the outside option is observed only by the entrepreneur and therefore actual default occurs along the equilibrium path.

If the entrepreneur defaults, the parties may renegotiate the contract or liquidate the firm. Renegotiation is immediate and results in a redistribution of the social surplus. If the entrepreneur does not default, then she privately observes project returns and may divert cash flows at some cost. The cost of diversion in turn depends on the quality of the monitoring technology used, which is itself a decision variable. That is, in every period the parties choose between a high quality, high cost and low quality low cost monitoring technology.

The high quality monitoring technology shares some features with formal (e.g., Chapter 11) or informal reorganization procedures. First, the use of this alternative is costly as are all cases of reorganization, where dismissing management entails learning costs and payments to trustees, accountants or courts are made. Second, it allows creditors to exert tighter control over the firm's revenues, which is one of the purposes of most reorganization cases (see Section 3.2.2). Finally, in equilibrium, this costly financing technology will only be used when the firm is under financial distress -after experiencing a long sequence of bad revenue shocks- but before deciding to liquidate.

I embed the financial contracting model described above into a standard industry equilibrium framework and then calibrate it to conduct a quantitative analysis. The baseline calibration seeks to match certain features of the U.S. data on bankruptcy and firm finance. In particular, I match quite well the exit rate, the frequency of renegotiation (Chapter 11) relative to liquidation (Chapter 7), and the average size of firms in renegotiation relative to the average size of all firms in financial distress (including liquidation and reorganization).

I use the calibrated version of the model to conduct a simple counterfactual exercise: a comparison with an economy where liquidation is the only way to deal with financially distressed firms. My results suggest that the richer bankruptcy framework that allows for renegotiation and reorganization significantly reduces firm exit (thus increasing firm age). It also increases average (and aggregate) firm size and decreases size dispersion.

The paper is organized as follows. The next section summarizes the empirical literature on dynamic contracts and bankruptcy procedures, whose main findings motivate my theory. Section three highlights my contribution to the literature. In section four I presents and solve the contracting problems which are then embedded into the industry equilibrium framework in section five. Section six contains the quantitative analysis, and section seven concludes.

3.2. An overview of the empirical evidence and related literature

My theory of bankruptcy and firm finance is built on two premises: the widespread use of dynamic credit arrangements and the importance of renegotiation and reorganization as alternatives to liquidation. In what follows I survey the main findings from the specialized literature devoted to each of these aspects. I also present a detailed description of where I stand in comparison with other contributions to the literature on financial frictions and firm dynamics.

3.2.1. Dynamic credit relationships

From a conceptual standpoint, long-term lending relationships can be an efficient way of overcoming informational asymmetries. In particular, the use of long-term contracts minimizes the cost of providing incentives for borrowers to reveal information through intertemporal transfers that are not available in short-term interactions (Boot (2000)).

Consistent with these theoretical arguments, available evidence for the U.S. does suggest that borrowers suffering from greater information asymmetry (e.g., small, non rated firms) are more likely to use long term credit relationships for funding their operation (Bharath, Dahiya, Saunders and Srinivasan (2007)). In fact, there is sound evidence that the duration of the bank-lender relationship positively affects the availability of credit, and, specially for young firms, is positively associated with intertemporal smoothing of contract terms (Petersen and Rajan (1994)). Overall, repeated borrowing seems to be a particularly attractive feature for firms with severe informational that would otherwise face higher financing costs (Bharath, Dahiya, Saunders and Srinivasan (2011)).

The recent financial crisis has provided another illustration of the benefits associated with enduring credit relationships. In a study of Italian firms, Bolton, Freixas, Gambacorta and Mistrulli (2013) report that banks specializing in long-term relationships offered more favorable continuation-lending terms to their client firms in response to the crisis. This is consistent with earlier evidence presented in Elsas and Krahnen (1998), according to which, German banks engaging in long-term lending relationships do provide some kind of liquidity insurance in situations of unexpected deterioration of borrower creditworthiness.

Given its above mentioned benefits, it is perhaps not surprising to find that long term financing is widely used by firms in the U.S. and elsewhere. In a panel of over 90,000 non-financial firms, collected by Custodio, Ferreira and Laureano (2013) for the period 1976-2008, the average fraction of debt with maturities over three and five years was 43% and 28%, respectively. Furthermore, the average maturity of syndicated loans for the period 1987-2008 was 4.15 years. But credit relationships often last longer than a single loan. For instance, in a sample of Belgian firms, Degryse and Cayseele (2000) report that the average duration of a loan is 2.4 years while the average length of relationship with the current lender is about 7.9 years.

The nature and purpose of long-term financing is directly related to the extent to which state contingency can be truthfully reported under asymmetric information. Inter-temporal transfers and information acquisition imply therefore that state contingency will indeed be a defining feature of dynamic credit relationships. In a recent paper, Roberts and Sufi (2009) provide compelling evidence that this is the case. For a sample of 1,000 credit agreements between U.S. firms and financial institutions, these authors show that 90% of long-term debt contracts are renegotiated prior to maturity but less than 18% of these renegotiated episodes are associated with a covenant violation or payment default. Renegotiation of terms in the absence of severe financial distress or default suggests that long-term arrangements can be viewed as state-contingent contracts.[3]

3.2.2. Alternative bankruptcy procedures: costs and benefits

Alternatives to liquidation are important resolution mechanisms for financially distressed firms. In order to avoid liquidation, a firm under severe financial distress must

[3]Naturally, long term contracts usually have quite some built-in state contingency whenever contract terms are linked to observables (e.g., adjustable rates, loan-to-value ratios).

do at least one of the following. Either it must raise cash through asset sales, operating improvements, and new financing; or it must negotiate with its creditors to reduce or postpone interest and principal payments on the debt. Of course, firms typically do both and the two are usually related to one another; as pointed out by Aghion and Bolton (1992) creditors are willing to renegotiate or write-off a fraction of their claims only if they can be credibly protected against borrowers' future opportunistic behavior.

Debt restructuring is a well known feature of bankruptcy procedures such as Chapter 11 in the U.S. The standard practice is that an exchange of securities is formally proposed by debtors[4] and may or may not be accepted by claimholders. Restructuring plans tend to have higher probability of success when the main creditors are banks, who appear to be more willing to renegotiate (lengthen) maturities and accept reductions of interest and principal (Gilson et al. (1990)). Furthermore, Roberts and Sufi (2009) show that renegotiations are more likely to result in favorable terms for the borrower when the latter has access to relatively inexpensive alternative sources of funds and thus the threat of exiting the relationship is credible. In their own words this illustrates how "outside options can generate surplus under the initial terms of the contract and lead to renegotiation".

Renegotiation and restructuring of debt are typically part of a broader reorganization plan aimed at improving operational efficiency and securing additional sources of cash. In fact, the most prominent way in which firms obtain new financing from

[4]Filing for Chapter 11 is not always the exclusive right of stockholders. Creditors may file an "involuntary" Chapter 11 if they can demonstrate that the firm has been delinquent on its debt. For more details on the procedures under Chapter 11, see, for instance, Gilson, Kose and Lang (1990).

creditors (and usually improve its operations in the process) is by realigning the interests of managers and creditors.[5] Detailed studies of reorganization cases provide ample support to the notion that under such arrangements creditors often condition renegotiations on the replacement of incumbent management and board of directors (Gilson (1990), Jostarndt and Sautner (2008), Ayotte and Morrison (2009)).[6]

In addition to management/directors dismissal, there is ample evidence that creditors exert control by influencing distressed firms' capital expenditures, leverage and shareholder payouts policies through stringent debt covenants (Nini, Smith and Sufi (2012)). Indeed, fresh financing provided under Chapter 11 usually comes with stringent restrictions from creditors (Bharath, Panchapegesan and Werner (2013)).[7]

In practice, formal restructuring procedures offer several benefits to both debtors and creditors. First, provisions such as "debtor in possession" (DIP) offers a solution to the debt overhang problem, as new lenders are senior to all other claimants, except secured creditors. In fact, DIP lenders rarely fail to be fully repaid, which increases the troubled firm's chances of raising new cash (Gilson (2012)). In addition to DIP financing, automatic stay clauses ensure that the firm can continue its operations

[5]Many other sources of operational improvements are used during formal corporate reorganizations (e.g., asset sales, layoffs, changes in business segment; see Wruck (1990)). We focus here in the agency problems of the firm since this is the key motivating factor behind our mdoelling choices.

[6]For the U.S., Gilson (1990) finds that in a sample of 111 firms that were formally or privately reorganized between 1979 and 1985, on average 55% and 57% of the incumbent board directors and CEOs, respectively, were replaced over the course of reorganization. Ayotte and Morrison (2009) find that in a sample of firms filing for Chapter 11 during 2001, about 70% of CEOs are replaced right before the start of a reorganization process. For a sample of 267 German firms in the 1996-2004 period Jostarndt and Sautner (2008) find that, following debt restructurings and informal reorganizations, ownership representation by outside investors doubles, and four years into the reorganization process only 14% and 22% of incumbent directors and CEOs remain in office. In his study of 94 publicly traded businesses that exited Chapter 11 in 2002, Baird and Rasmussen (2003) concludes that "creditors use their powers to remove managers in whom they have lost conffidence, [and] replace the board of directors".

[7]These restrictions are directly aimed at influencing the corporate governance of the firm in bankruptcy, and often include specific covenants relating to board seats, asset sales, and even the transfer of control.

without the risk of "race against the assets" by its creditors. Recent evidence suggests that declines in operating income are reversed in the two quarters immediately following the Chapter 11 filing, and that this is mostly associated with improved investment policy (Kalay, Singhal and Tashjian (2007)).

Bankruptcy procedures are, of course, subject to sizable costs that are often the deciding factor in the course of action. Bris et al. (2006) is the most recent attempt to measure bankruptcy costs in the U.S.[8] That paper shows that bankruptcy costs are largely dependent upon firm size, which is not surprising since the administrative protocols of Chapter 11 virtually guarantee that, in larger and more complex cases, such costs will rapidly escalate (Gilson (2012)).[9] Bris et al. (2006) also show that, taken together, direct and indirect bankruptcy costs result in recovery rates under liquidation that are on, average close to 50%, much lower than under reorganization where this average is closer to 80%. These results coincide remarkably well with the cross-country survey data presented in Djankov et al. (2008) for high income countries. The picture in low-and-middle income countries is more grim, as recovery rates are expected to be, on average, less than 25% and 48% for liquidation and reorganization, respectively.

3.2.3. Related literature: financial frictions and firm dynamics

This paper draws heavily on the insights from the literature on financial frictions and firm dynamics.[10] Among the first contributions to this literature is the paper by Cooley and Quadrini (2001) in which the external finance premium is used to explain

[8]Earlier attempts include Warner (1977) and Altman (1984).

[9]A simple ilustration of this feature of Chapter 11 is that when a firm files for such bankruptcy procedure, it must pay a fixed filing fee for each and every one of its subsidiaries.

[10]The flourishing of this literature owes a large debt to the seminal papers of Jovanovic (1982) and Hopenhayn (1992) which developed the basic theory of firm dynamics that we still use today.

certain empirical regularities of firm dynamics such as age and size dependence.[11] In that paper, however, the sources of financial frictions and firm exit are not modelled explicitly as in the current paper.

The papers by Albuquerque and Hopenhayn (2004), Quadrini (2004), Clementi and Hopenhayn (2006) and Hopenhayn and Werning (2008, 2006) study the roles of limited commitment and moral hazard in generating financial constraints and producing non-trivial firm dynamics.[12] The microfounded financial frictions embedded in these models create endogenous liquidation and are able to produce firms whose size and profits increase with age while their growth and hazard rates endogenously decrease with it. Quadrini (2004) and Clementi and Hopenhayn (2006) study risky investment under asymmetric information but abstract from the possibility of default. Albuquerque and Hopenhayn (2004) considers the threat of default but perfectly symmetric information implies that actual default does not occur in equilibrium. Finally, Hopenhayn and Werning (2008, 2006) build models with limited commitment and unobservable outside options in which default occurs in equilibrium. In contrast, my model includes private information on both firm returns and outside options while also allowing for the possibility of contract renegotiation and costly mitigation of agency problems.

Firm dynamics models have also been used to study the role financial market institutions in explaining cross-country differences in firm size, productivity and firm growth. Arellano, Bai and Zhang (2012) adopt an incomplete markets framework in

[11]Size dependence corresponds to the observation that, conditional on age, the dynamics of firms (growth, volatility of growth, job creation, job destruction, and exit) are negatively related to the size of firms. On the other hand, age dependence refers to the observation that, conditional on size, the dynamics of firms are negatively related to the age of firms

[12]While Albuquerque and Hopenhayn (2004) consider the option of default, enforcement constraints are added to the contracting problem to ensure that default never occurs in equilibrium.

which default-risk interacts with direct lending costs to explain the observed cross-country variations in average firm size, leverage and growth as a function of differences in the level of financial development. Exit in this model is completely exogenous and bankruptcy procedures are not part of the analysis. Rodriguez-Delgado (2010) extends this same framework to consider costly liquidation and to show that high costs associated with liquidation not only increase the cost of external finance but also create significant drops in aggregate productivity. Alternatives to liquidation are not considered in this model, however.

Broadly speaking, the papers surveyed above consider one of the two following environments: firms under financial distress either default and renegotiate their debt, or they are liquidated and exit the market irreversibly. Recently Senkal (2013) and Corbae and D'Erasmo (2014) considered an environment in which liquidation and renegotiation coexist in an otherwise standard firm dynamics framework. In particular, these papers take an incomplete markets approach in which firms under distress can either file for liquidation or renegotiate their debt with creditors in order to continue operating. A similar approach is taken by Peri (2014) who models reorganization as the combination of debt renegotiation and an uncertain period in which the firm is unable to issue debt or distribute dividends. However, all three papers abstract from asymmetric information and, accordingly, the only goal of a restructuring procedure is to reduce the amount of firm debt. By contrast, my model captures both aspects of rehabilitation procedures summarized in the previous section: debt renegotiation and firm reorganization through reduced agency.

Finally this paper is also related to a large body of research on the issues of financial distress, security design and corporate control.[13] In the theoretical strand of this literature, White (1994) first introduced the notion of corporate bankruptcy as a

[13] A comprehensive survey of this literature can be found in Senbet and Wang (2012).

filtering device: bankruptcy law should be designed so as to force inefficient firms into liquidation and allow efficient firms to be rehabilitated.[14] Recent country studies from developing countries which carried out bankruptcy reform have provided empirical support for this proposition (Gine and Love (2010), Araujo, Ferreira and Funchal (2012), and Lim and Hee Hahn (2004)). Taken together, these empirical studies show that lowering bankruptcy costs and enhancing creditor protection improves the separation between persistently underperforming firms (that should be liquidated) and firms with temporary setbacks (that should undergo reorganization).

3.3. The model economy

I now present a theory of firm finance in which contracting parties are presented with different alternatives to deal with the possibility of financial distress. After describing the environment and main workings of the model, I introduce ex-ante project heterogeneity and then embed the contracting model into a standard industry equilibrium framework.

3.3.1. Preferences and technology

The entrepreneur has access to a project characterized by a production technology $F : \{0, 1\} \times R_+ \to R_+$ which combines working capital inputs, $k_t \in R_+$, with project-level productivity z_t. More specifically, $F(k_t, z_t) = z_t f(k_t)$, with $f(0) = 0, f' > 0, f'' < 0$ and $\lim_{k\to 0} f'(k) = \infty$, $\lim_{k\to\infty} f'(k) = 0$, while z_t is the realization of an i.i.d. random variable Z with support $\{z_L, z_H\} = \{0, 1\}$ and $\Pr(Z = z_H = 1) = \pi$. The project requires, in addition to $k_{t\geq 0}$, an initial set-up cost I_0. The entrepreneur has wealth $M < I_0$ so, to operate the technology, she must enter a financial contract with a bank with deep pockets. We refer to projects that are successfully initiated

[14]However, White (1994, 1989) also points out that in the presence of imperfect information this filtering mechanism is deemed to operate with error at best.

as "firms". In each period in which the firm operates and returns z_t are realized, the bank expects a repayment from the entrepreneur, τ_t, per period.

Both the entrepreneur (e) and the bank (b) are risk-neutral, discount cash flows at a common rate $\beta \in (0,1)$, and seek to maximize the expected present value of dividends:

$$\left\{ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t d_t^j \right\}$$

for $j = e, b$. The following assumption is introduced to guarantee the existence of a firm:[15]

Assumption 7. $\{\pi f[(f^{(-1)})'(1/\pi)] - [f^{(-1)}]'(1/\pi)\}(1-\beta)^{-1} > I_0$

3.3.2. First-best

Under symmetric information and perfect enforcement, this problem is trivial enough: the properties of $f(\cdot)$ imply that there exists a unique:

$$\tilde{k} = \arg\max_k [f(k)\mathbb{E}Z - k] = \arg\max_k [\pi f(k) - k],$$

which is referred to the first-best level of capital. A planner facing no information or commitment constraints, and concerned with maximizing social surplus only, will choose $k_t = \tilde{k}\ \forall\ t \geq 0$. Thus, the first-best value of the firm is given by $\tilde{W} = [\pi f(\tilde{k}) - \tilde{k}]/(1-\beta)$, with $\tilde{V} = \pi f(\tilde{k})/(1-\beta)$ being the lifetime expected value accruing to the entrepreneur. The solution to the first-best problem, therefore, implies that all entrepreneurs are able to borrow the first-best level of capital $\tilde{k}$ and, once started, the firm will never grow, shrink or exit.

[15] In what follows, the terms "entrepreneur" and "firm" will be used interchangeably.

3.3.3. Contracts under private information and limited commitment

The problem becomes interesting when private information and limited commitment are introduced as follows. At the beginning of each period, the entrepreneur can leave the project and take an outside option s which is itself the realization of an i.i.d. random variable with support $S = [\underline{s}, \bar{s}]$ and differentiable cdf $G(s)$. As in Hopenhayn and Werning (2008), s is private information to the entrepreneur. The entrepreneur will therefore take her outside option ("default") whenever s is higher than the value that she can expect from continuing with the project given the current terms of the contract. Let $x_t = 1$ if the entrepreneur decided to default (after observing s_t) in period t and $x_t = 0$ otherwise.

Notice that the unobservable nature of the outside option implies that contract terms may not depend upon s as it does in other models of limited commitment (Thomas and Worrall (1994), Albuquerque and Hopenhayn (2004)). There, contracts are designed so as to avoid actual default. In contrast, given the current information structure, default will happen along the equilibrium path as in Hopenhayn and Werning (2008).[16]

If the entrepreneur defaults and leaves the relationship, she gets to keep s, and the firm is liquidated. In such case, the lender appropriates the scrap value of the project, Δ. Alternatively, the parties may pay verification and renegotiation costs, θ, and renegotiate the original contract. Let $\ell_t = 1$ if the firm was liquidated upon default and $\ell_t = 0$ if the parties renegotiated. I assume that in case of liquidation, both parties receive their payoff at the beginning of the following period.

If the entrepreneur does not default, the parties then face an investment decision under asymmetric information since returns z_t are not observed by the bank. That

[16] For a sovereign debt model with unobservable outside option and production shocks, see Aguiar and Amador (2014), section 4.1.

is, the entrepreneur can misreport actual returns to the bank and divert cash flows at some cost ρ^i. Such cost in turn depends on the monitoring technology used which is a decision variable at the beginning of the period (after observing the default decision). In particular, $\rho^r > 0$ if the parties decide to use a high-quality monitoring technology which costs a fraction μ_r of the firm's returns and $\rho^u = 0$ if they use a low-quality monitoring technology which is costless. Let $u_t = 0$ if the parties decide to use the high-quality monitoring technology in t and $u_t = 1$ otherwise. In what follows -and for reasons that will become apparent shortly- I refer to the case in which $u_t = 0$ as "reorganization"[17] and label the case of $u_t = 1$ as "undistressed".

Private information implies that the terms of the contract must depend upon the agent's report, $\hat{z}_t$, rather than on z_t itself. While the agent's reporting strategy may be arbitrarily complicated, the Revelation Principle can be invoked to identify the support of Z with the set of admissible reports. To avoid any indeterminacies and for simplicity it is assumed that when indifferent the agent will not divert cash flows and that costs θ, μ_r are borne by the bank.

To complete the formal statement of the problem, let $s_t = s_t$ if there is renegotiation ($x_t = 1$ and $\ell_t = 0$) and $s_t = \emptyset$ if $x_t = 0$ or $\ell_t = 1$. Let $\tilde{h}^{t-1} = (h^{t-1}, x_t)$ denote the interim public history after the default decision has been made. For notational convenience, I label choices made after observing $\tilde{h}^{t-1}$ with a subscript $i_t \in \{u, r\}$, where $i_t = r$ ("reorganization") if $u_t = 0$ so the firm is financed under the high-quality monitoring technology and $i_t = u$ ("undistressed") if $u_t = 1$ so the firm operates under the low-quality financing technology. That is, in what follows, I write, e.g.,

[17] As mentioned in the introduction, this costly hi-quality monitoring technology shares some features with formal or informal reorganization procedures. First, the use of this alternative is costly as are most (all) cases of reorganization, where dismissing management entails learning costs and payments to trustees, accountants or courts are made. Second, it allows creditors to exert tighter control over the firm's revenues which is one of the purposes of most reorganization cases. Finally, in equilibrium, this reduced agency path will only be taken when the firm is under financial distress -after having experienced a long enough sequence of bad revenue shocks- but before deciding on liquidation.

$k_t\left(h^{t-1},0,\mathfrak{u}_t\right)$ as $k_t^i\left(h^{t-1}\right)$. Furthermore, I adopt the convention $\hat{z}_t^i=k_t^i=\tau_t^i=u_t=\emptyset$ if $x_t=1$, $\ell_t=\emptyset$ if $x_t=0$. Public histories are given by $h^t=\{h^{t-1},h_t\}\in H^t$ where $h_t=\left\{x_t,\mathfrak{u}_t,\ell_t,\mathfrak{s}_t,[k_t^i,\hat{z}_t^i,\tau_t^i]_{i=u,k}\right\}$. The timing of events is depicted in Figure 3.1.

A contract, σ, is a collection of functions specifying a reorganization - undistressed policy, u_t, liquidation-renegotiation decisions, ℓ_t, capital inputs, k_t^i, and repayments to the bank, τ_t^i : $\sigma=\left\{x_t\left(h^{t-1}\right),\mathfrak{u}_t(\tilde{h}^{t-1}),\ell_t(\tilde{h}^{t-1},\mathfrak{s}_t),[k_t^i(\tilde{h}^{t-1}),\tau_t^i(\tilde{h}^{t-1})]_{i=u,r}\right\}_{t=0}^{\infty}$. This contract implicitly defines an equity value for the firm V_t and the long-term debt level or value to the bank B_t. The equity value for the firm gives the discounted sum of future dividends whereas the long-term debt or value to the lender gives the discounted cash flows to the lender. Thus, the total asset value after history h^t is defined by $W_t=V_t+B_t$. As in Spear and Srivastava (1987), V_t effectively summarizes all information provided by history up to $t-1$, and can be used as state variable in a recursive formulation of the contracting problem. The points $(\mathsf{B}\left(V\right),V)$ trace the Pareto frontier and $W\left(V\right)=B\left(V\right)+V$ is usually referred to as the "value of the firm".

I will characterize contracts recursively by specifying value functions at the different decision stages within a period. Working backwards, consider first the problem of a firm which has not defaulted in the current period $(x_t=0)$ and is being financed under monitoring quality i. This problem can be written recursively as follows:

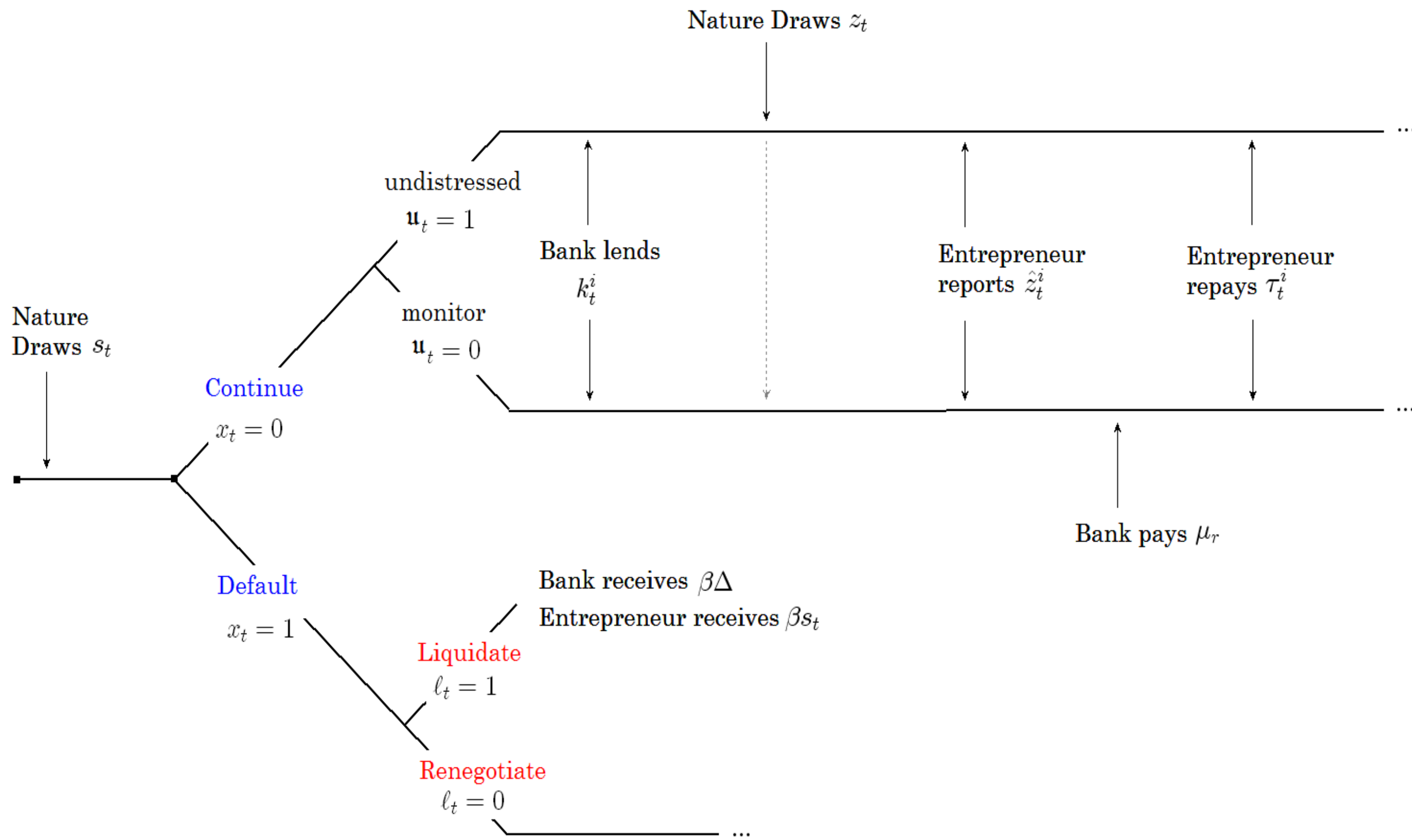


Figure 3.1. Timing of events within a period

$$(\widehat{\mathbf{PP}})\quad \widehat{\mathrm{W}}_i\left(V_c^i\right)=\max_{k^i,\tau^i,V_H^i,V_L^i}\pi\left(1-\mu_i\right)f\left(k^i\right)-k^i+\beta\left[\pi\mathrm{W}\left(V_H^i\right)+\left(1-\pi\right)\mathrm{W}\left(V_L^i\right)\right]$$

$$s.t.:f\left(k^i\right)+\beta\left(V_H^i-V_L^i\right)\geq\tau^i+(1-\rho^i)f\left(k^i\right),$$

$$V_c^i=\pi\left(f\left(k^i\right)-\tau^i\right)+\beta\left[\pi V_H^i+\left(1-\pi\right)V_L^i\right],$$

$$f\left(k^i\right)\geq\tau^i\ \text{ and }\ V_H^i,V_L^i\geq 0$$

In ($\widehat{\mathbf{PP}}$), $\mu_u=0$ and V_z^i, $z=H,L$, is the firm's value of equity beginning the following period after a revenue shock z has been reported. Moreover, this formulation of the problem already uses the fact that from limited liability $z_L^i=0\Rightarrow\tau_L^i=0$ and hence τ_H^i can be written as τ^i. The first constraint in ($\widehat{\mathbf{PP}}$) requires that contracts are incentive compatible. Here, I make use of the fact that a realization of $z_L^i=0$ will never result in the agent reporting z_H^i and thus only one incentive constraint is required. The second constraint in ($\widehat{\mathbf{PP}}$) imposes individual rationality (the so-called promise-keeping constraint) while constraints $f\left(k^i\right)-\tau^i,V_H^i,V_L^i\geq 0$ capture the entrepreneur's limited liability.

Next, given $x=0$ the problem of choosing monitoring quality is given by:[18]

$$(\mathbf{PP})\qquad \mathrm{W}^c\left(V_c\right)=\max_{V_c^u,V_c^r,\mathfrak{u}}\mathfrak{u}\widehat{\mathrm{W}}_u\left(V_c^u\right)+(1-\mathfrak{u})\widehat{\mathrm{W}}_r\left(V_c^r\right)$$

$$s.t.:V_c=\mathfrak{u}V_c^u+(1-\mathfrak{u})V_c^r\text{ and }V_c^u,V_c^r\geq 0$$

where $\widehat{\mathrm{W}}_i\left(V_c^i\right),\ i=u,r$ satisfy ($\widehat{\mathbf{PP}}$). Now, consider the problem of the match when $x_t=1$. At this point I assume that the entrepreneur will prefer to renegotiate and keep the firm if she gets at least her outside option in the renegotiated contract. That

[18]Hereafter, the dependence of the policy functions on equity is supressed and we write e.g., $x\left(s,V\right)$ as $x\left(s\right)$. Ocasionally we revert to e.g., $x\left(s,V\right)$ when characterizing these policies as functions of V.

is, I assume that the bank has all the bargaining power. Then the decision whether to liquidate the firm or to continue with the relationship by renegotiating the original contract, for a given realization s, solves:

$$(\mathbf{PD}) \qquad \mathrm{W}_d(s) = \max_{\ell(s)\in\{0,1\}} \ell(s)\,\mathrm{W}_\ell(s) + [1-\ell(s)](1-\theta)\,\beta\mathrm{W}(s)$$

where $\mathsf{W}_\ell(s) = \beta(\Delta + s)$ and I have used the fact that, upon default, the entrepreneur is indifferent between renegotiation and liquidation, so that s is the appropriate argument for $W(\cdot)$. Then the optimal default policy can be found as the solution to:

$$\begin{aligned} \mathrm{W}(V) &= \max_{x(s)\in\{0,1\}} \mathrm{W}^c(V_c)\int_{\mathcal{S}}[1-x(s)]\,dG(s) + \int_{\mathcal{S}} x(s)\,\mathrm{W}_d(s)\,dG(s) && (\mathbf{P}) \\ s.t. &: \quad V = V_c\int_{\mathcal{S}}[1-x(s)]dG(s) + \beta\int_{\mathcal{S}} x(s)\,s\,dG(s) \\ &: \quad x(s) = \begin{cases} 1, & \text{if } \beta s > V_c \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

where $\mathsf{W}^c(V_c)$ and $\mathsf{W}_d(s)$ in the objective function of (**P**) satisfy, respectively, (**PP**) and (**PD**) and the promise-keeping constraint already uses the fact that upon default the entrepreneur receives exactly the value of her outside option.

3.3.4. Optimal contracts and the value of the firm

I begin the characterization of optimal contracts by studying the solution to problems ($\widehat{\mathbf{PP}}$)-(**PP**). That is, I first consider optimal policies when $x = 0$. Notice that for each $i = u, r$, the problem in ($\widehat{\mathbf{PP}}$) is virtually identical to that of Clementi and Hopenhayn (2006) (henceforth CH) and hence their main results apply. In particular, capital advancement policy satisfies $k^i(V^i) < \tilde{k}^i = \arg\max_k [\pi(1-\mu_i) f(k^i) - k^i]$ as long as $V^i \leq \tilde{V}^i = \pi(1-\rho_i) f(\tilde{k}^i)/(1-\beta)$; that is, the firm is borrowing constrained. Along with risk neutrality, this implies that allowing the equity value V^i to reach the threshold $\tilde{V}^i$ in the shortest possible time is optimal, i.e., $V^i \leq \tilde{V}^i$ implies $f(k^i) = \tau^i$. This allows capital to increase with equity values so that the endogenous financing constraints tend to relax as the firm's equity grows. Finally, for $V^i \leq \tilde{V}^i$ future equity values satisfy $V_L^i(V^i) < V^i < V_H^i(V^i)$, implying that the firm's equity value increases with a good shock and decreases with a bad shock ("cash-flow sensitivity"). These results allow us to reduce ($\widehat{\mathbf{PP}}$) to unidimensional maximization problems:

$$\begin{aligned} \widehat{\mathrm{W}}_i\left(V_c^i\right) &= \max_{k^i} \pi\left(1-\mu_i\right) f(k^i) - k^i + \beta\left\{\pi \mathrm{W}\left[\frac{V_c^i + (1-\pi)\left(1-\rho^i\right) f(k^i)}{\beta}\right]\right. \qquad (\widehat{\mathbf{P3}}) \\ &\left. + (1-\pi)\,\mathrm{W}\left[\frac{V_c^i - \pi(1-\rho^i) f(k^i)}{\beta}\right]\right\} \end{aligned}$$

Notice that for $i = u$, and using the notation and definitions of the previous paragraph, one has that $\tilde{k} = \tilde{k}^u$, $\tilde{\mathsf{W}} = \tilde{\mathsf{W}}^u$ and $\tilde{V} = \tilde{V}^u$. Denote by $\tilde{\mathsf{W}}^r$ the value of a firm that is currently under "reorganization" and operated at scale $\tilde{k}^r$. For the reminder of the paper, and given that $\mu_u = \rho^u = 0$, I write $\mu = \mu_r$ and $\rho = \rho^r$. Then:

Lemma 6. $\mu > 0 \Rightarrow \tilde{\mathsf{W}}^r < \tilde{\mathsf{W}}$.

Proof. See appendix A. □

Lemma 6 establishes that for large enough values of equity, leaving the firm undistressed is optimal. Finding conditions under which $u = 0$ is optimal requires some more work:

Proposition 9. *There exist $\Delta, \mu, \rho \in (0,1)$ with $\mu_r < \rho^r$ such that for some $0 < V_{\mathcal{R}} < \tilde{V}^r$, $V_c < V_{\mathcal{R}} \Rightarrow \mathsf{W}^c(V_c) = \widehat{\mathrm{W}}_r(V_c^r)$ and $V_c \geq V_{\mathcal{R}} \Rightarrow \mathsf{W}^c(V_c) = \widehat{\mathrm{W}}_u(V_c^u)$*

Proof. See Appendix A. □

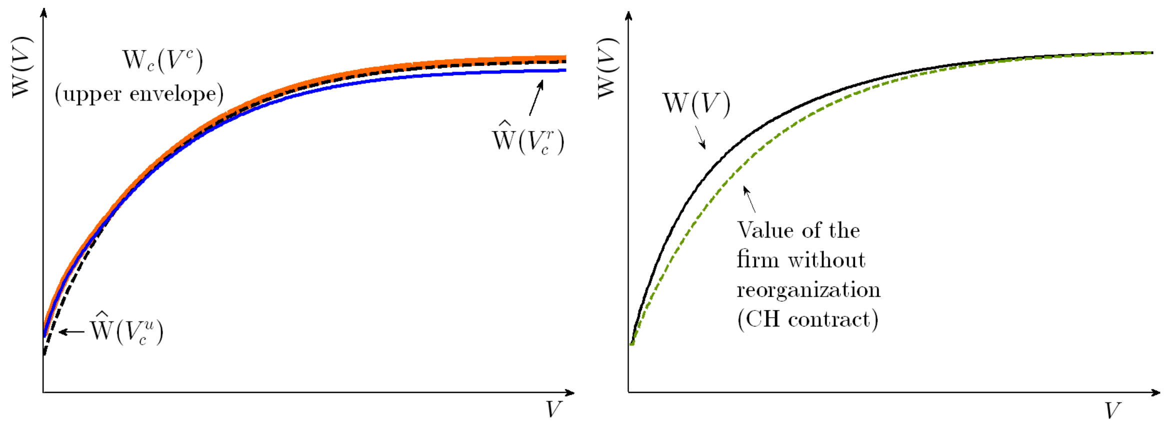


Figure 3.2. Reorganization and the value of the firm

Heuristically, Proposition 9 says that reorganization is optimal for intermediate values of equity and the firm is left undistressed if equity is large enough. The content and intuition for Proposition 9 can be seen graphically in the left panel of Figure 3.2, where the function $\mathsf{W}^c\left(V_c\right)$ is shown as the upper envelope of the functions $\widehat{\mathrm{W}}_u\left(V_c^u\right)$ and $\widehat{\mathrm{W}}_r\left(V_c^r\right)$. An immediate consequence of the proposition is that for some combinations of parameters, the value of the firm is higher when reorganization is an option than when only liquidation is available as in CH (see right panel of Figure 3).

Figure 3.2 traces the value of the firm as a function of continuation (i.e., no-liquidation) equity V_c. In the region to the right of $V_{\mathcal{R}}$ the firm is undistressed but may be financially constrained. In turn, the equity region in which the firm is financially distressed (left of $V_{\mathcal{R}}$) can be divided into liquidation and reorganization. The right panel of Figure 3.2 compares the value of the firm under a contract which allows for costly high quality monitoring, with a contract in which only the low quality monitoring technology is available (i.e., the contract found in CH).

Next I add the following assumption in order to provide a sharper characterization of the default and liquidation policies:

Assumption 8. *Suppose now that* $s \sim U[0, \bar{s}]$ *with* $\bar{s} < \tilde{\mathsf{W}}^u$

The final statement in Assumption 8 implies that the outside opportunity is never more valuable than the total value of the project, if the latter were operated at full efficiency. Confronted with any contract, the entrepreneur will employ a reservation policy, taking any outside opportunities above some threshold s_d and rejecting the rest. That is, for each V, $s \geq s_d \Rightarrow x(s, V) = 1$ and $s < s_d \Rightarrow x(s, V) = 0$. The entrepreneur's lifetime utility evolves according to:

$$V = \int_0^{\bar{s}} \max\{\beta s, V_c\} \, dG(s) = V_c \int_0^{s_d} dG(s) + \beta \int_{s_d}^{\bar{s}} s dG(s)$$

It is easy to see that if $V \geq \beta \bar{s}$ the entrepreneur will be better off by staying in the relationship regardless of the realization of s. For $V < \beta \bar{s}$ the following proposition characterizes the default policy in the optimal contract:

Proposition 10. *Suppose that Assumption 8 is satisfied. Then in the optimal contract, for* $V < \beta \bar{s}$*, the default threshold,* $s_d(V)$*, is weakly increasing in* V*. Moreover, there exists* $V^+ > \beta E(s)$ *such that for* $V \geq V^+$*,* $s_d(V)$ *is strictly increasing in* V*.*

Proof. See Appendix A. □

The intuition of Proposition 10 is straightforward: as the value delivered by the original contract increases, the entrepreneur requires a higher realization of the outside option to be tempted to default. However, the truncation of the equity domain introduced by the risk of default imposes a lower bound on the default threshold, hence the weak qualification at the beginning of the statement. An immediate result from this proposition is:

Corollary 4. *There exists* $V^{-} > \beta E(s)$ *such that for* $V \geq V^{+}$ *the optimal working capital policy is* $k = 0$

Proof. See Appendix A. □

That is, for small enough values of equity, the firm can survive into the following period without capital advancement (inaction). This will happen if the realization of the outside option is very low and the entrepreneur does not default even knowing that the contract promises her little equity. Again, because the lack of commitment truncates the equity domain from below, no positive level of capital can satisfy the incentive constraint (recall that $k > 0$ requires $V_L < V$).

I now turn to characterizing the liquidation decision. Recall that the entrepreneur is indifferent between the liquidation/renegotiation outcome as she is assured a payoff βs. If the contract is renegotiated, the value of the firm (after proportional renegotiation costs are paid) is given by $\beta(1-\theta)\mathsf{W}(s)$. In such case, the bank receives $\beta(1-\theta)\mathsf{W}(s) - \beta s$, while if the firm is liquidated, the bank receives $\beta\Delta$. Thus, the liquidation threshold solves:

$$\mathsf{W}(s_\ell) = \frac{\Delta + s_\ell}{1-\theta} \tag{3.1}$$

Naturally, if θ is too large, equation (3.1) will not have a solution which, once again, points to the role of bankruptcy costs in shaping renegotiation/liquidation decisions. Unfortunately, low renegotiation costs are not enough to find s_ℓ as equation (3.1) may not have a unique solution. The following assumption introduces a sufficient condition for s_ℓ to be unique and allows us to provide a sharper characterization of the liquidation decision:

Assumption 9. $\mathsf{W}(\bar{s}) > \frac{\Delta+\bar{s}}{1-\theta}$

When assumption (9) is satisfied, the bank will find it optimal to renegotiate if s is sufficiently large and liquidate otherwise. I summarize our previous discussion in the following proposition:

Proposition 11. *Suppose that assumptions 8 and 9 are satisfied. Then $s_\ell(V)$ is unique and satisfies $s_\ell(V) \geq s_d(V)$.*

This result is in line with the evidence discussed in section 3.2 according to which firms with better outside options in the form of alternative financing (e.g., through equity) are more likely to successfully renegotiate their contracts with creditors. With the results from propositions (10)-(11) at hand, the problem in (**P**) can be conveniently reformulated, for $V \in [V_+, \bar{s}]$, as:

$$\begin{aligned} \mathrm{W}(V) &= \mathrm{W}^c(V_c)\int_0^{s_d} dG(s) + \beta\int_{s_d}^{s_\ell}[\Delta + s]\,dG(s) + \beta(1-\theta)\int_{s_\ell}^{\bar{s}}\mathrm{W}(s)\,dG(s) \qquad (\mathbf{PU}) \\ &= \frac{1}{\bar{s}}\left\{s_d\mathrm{W}^c(V_c) + \beta\Delta(s_\ell - s_d) + \frac{\beta}{2}\left(s_\ell^2 - s_d^2\right) + \beta(1-\theta)\int_{s_\ell}^{\bar{s}}\mathrm{W}(s)\,dG(s)\right\} \end{aligned}$$

where, $\mathrm{W}^c(V_c)$ solves $(\widehat{\mathbf{P3}})$, s_ℓ solves (3.1) and $s_d = \max\{\beta E(s), \sqrt{2\bar{s}V\beta^{-1} - \bar{s}^2}\}$. The results from Propositions (10)-(11) can be seen graphically in the left panel of Figure 3.3 where I have depicted the optimal default, liquidation and renegotiation policies for a given parametrization, as well as the value of the firm when there is the possibility of default and renegotiation.

The right panel of Figure 3.3 illustrates how the default risk affects the value of the firm. Importantly for my purposes the figure shows that, while the contract with the reorganization option strictly dominates the CH contract, this may or may not be true for the contract with a default option. In particular, the CH contract may

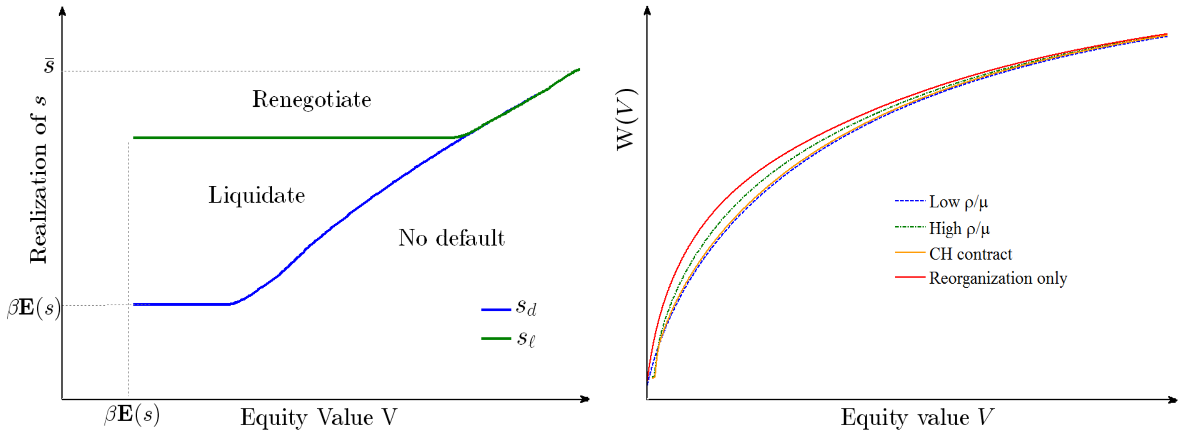


Figure 3.3. Default policies and the value of alternative contracts

dominate the contract with default if the ratio ρ/μ is too low, i.e., if the benefit of reorganization is low relative to its cost.

3.3.5. Heterogeneous projects

Suppose now that in every period a continuum of entrepreneurs are born, each of which has access to exactly one project of average productivity π (recall $\pi = \Pr(Z=1)$), which is drawn from a time-invariant distribution $\Gamma(\pi)$ with support Π. After project types are drawn, they become public information so that banks offer each entrepreneur a contract indexed by her type π. Accordingly, the value and policy functions are now written e.g., $\mathsf{W}(V;\pi)$.

As a first approximation to the effect of project heterogeneity, consider the simple case in which there is neither default nor reorganization (i.e., $\bar{s} = 0$ and $\mu_r = \infty$), but the firm can be liquidated at the beginning of each period (the CH contract). A deterministic liquidation rule will specify a cutoff value of equity $V_\ell(\pi)$ such that for $V \leq V_\ell(\pi)$ the firm is liquidated. I will next show that even if there is neither renegotiation nor reorganization, a financial contract that relies on intertemporal incentives for truthful reporting induces selection along the productivity dimension:

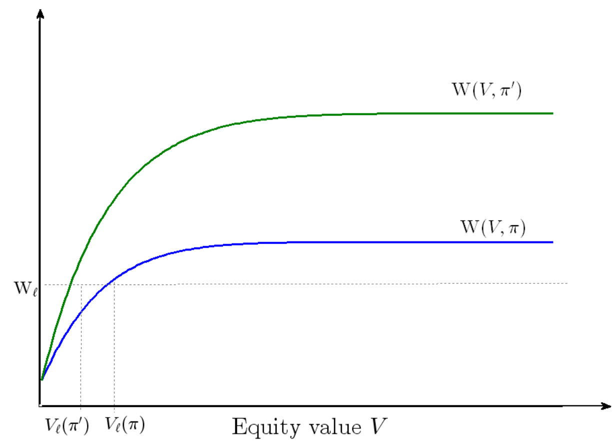


Figure 3.4. Financial selection

Proposition 12. *Suppose that $\pi' > \pi$. Then $V_\ell(\pi') < V_\ell(\pi)$.*

Proof. See Appendix. □

Figure 3.4 illustrates the content of Proposition 12: A project with higher average productivity will face a smaller liquidation set. Since a firm reaches the liquidation set only after experiencing enough bad realizations of the revenue shock, and such low realizations are more likely with lower π, firms with higher average productivity face a lower unconditional probability of exit. In other words, the contract exhibits *financial selection*.

This intuition can be carried over to the contracting problem of section 3.3.3. As shown in Figure 3.2, the reorganization option increases the slope of the value function at the origin. With heterogeneous projects, this effect is compounded so that projects with higher average productivity disproportionately benefit from the reorganization option and financial selection is enhanced. This issue will be pursued further in the quantitative exercise (see Figure 3.6).

3.3.6. Industry equilibrium

In order to conduct a meaningful quantitative exercise, the contracting problems studied above are embedded into a standard industry equilibrium framework. The details of the industry follow closely those found in Li (2010).

Incumbent firms behave competitively, taking prices in output (p) market as given. Aggregate demand for the product is given by the inverse demand function, $p = D(Q)$, where the function D is continuous, strictly decreasing, and satisfies $\lim_{Q\to\infty} D(Q) = 0$ and $0 < \lim_{Q\to 0} D(Q) < \infty$. Notice that the output price is now a state variable and therefore value functions and policy functions depend upon it, for instance, $\mathsf{W}(V; p)$.

I assume that entrepreneurs must pay the setup cost I_0 before drawing a project from the invariant distribution $\Gamma(\pi)$. Therefore, the value entitlement to an entrepreneur upon entry, V_0, is such that -given competition- banks break even in expectation:

$$\text{(P0)} \qquad \max_{V_0 \geq 0} V_0$$

$$\text{(3.2)} \qquad s.t. \int_{\Pi} \mathrm{B}(V_0, \pi; p)\, d\Gamma(\pi) \geq I_0 - M$$

That is, entrepreneurs are offered the "average" initial equity though all other contract terms are project specific.[19] If a solution to (**P0**) exists,[20] denoted as $V_0(p)$, (3.2) is binding, i.e., $\int B(V_0(p), \pi; p)\, d\Gamma(\pi) = \int \mathsf{W}(V_0(p), \pi; p)\, d\Gamma(\pi) - V_0(p) = I_0 - M$. Once the initial equity value $V_0(p)$ is determined, the evolution of a new firm's equity value during its life cycle is completely regulated by the (project-specific) optimal financial contract.

To generate an invariant distribution of firms, one must allow for some exogenous exit. Otherwise, for each cohort of new entrants there will be some firms that reach

[19] That is, after paying the initial set-up cost, the entrepreneur's project type becomes common knowledge.
[20] A solution to this problem may not exist if $I_0 - M$ is too large.

the unconstrained status and never exit. This would imply that the total mass of firms grows over time without bound. Hence, it is hereby assumed that, in every period, entrepreneurs are assumed to face a time-invariant exogenous probability $1-\eta$ of surviving into next period. For simplicity, I assume that if the entrepreneur dies exogenously, the firm becomes obsolete and the bank's payoff is zero.[21]

The state of the industry can be described by the distribution of firms over equity-type pairs (V,π). Let $\psi_t(V,\pi;p)$ denote the distribution of incumbent firms after (endogenous and exogenous) liquidation has taken place and let E_{t+1} stand for the mass of new entrants at the beginning of $t+1$. The law of motion for ψ_t can be written as:

$$\psi_{t+1} = \mathcal{T}^*\left(\psi_t, E_{t+1}; p\right) \tag{D}$$

The expression for the mapping $T^*(\cdot)$ is derived in Appendix 2 for the economy with default and renegotiation. This is the more general economy and definitions are easily obtained for the other cases by applying suitable changes. I am now in a position to define a stationary recursive competitive equilibrium:

Definition 7. *A stationary recursive competitive equilibrium for the industry consists of output Q^* and price $p^* \geq 0$; policy functions for default, liquidation and reorganization decisions $\left[\ell(s,V,\pi;p^*), x(s,V,\pi;p^*)\right]_{s\in\mathcal{S}}$, $u(V,\pi;p^*)$, for continuation, liquidation values $V_c(V,\pi;p^*)$, $V_\ell(V,\pi;p)$, and for capital, repayments and continuation values, $\left[V_c^i(V,\pi;p^*), k^i(V_c^i,\pi;p^*), \tau^i(V_c^i,\pi;p^*), V_H^i(V_c^i,\pi;p^*), V_L^i(V_c^i,\pi;p^*)\right]_{i=r,u}$;*

[21] This way, exogenous exit merely implies a lower discount rate $\hat{\beta} = \beta(1-\eta)$ and does not require modifying the contracting problems of the previous sections.

value functions $\mathsf{W}(V,\pi;p^*)$, $\mathsf{W}_d(s,V_\ell,\pi;p^*)$, $\mathsf{W}^c(V_c,\pi;p^*)$, $[\widehat{\mathrm{W}}_i(V_c^i,\pi;p^*))]_{i=r,u}$*; a measure of incumbent firms* ψ^* *and a mass of entrants* E^* *such that:*

(i). *The value and policy functions solve (*$\widehat{\mathbf{PP}}$*), (***PP***), (***PD***), and (***P***)*[22]

(ii). $p^* = D(Q^*)$ *and* $Q^* = \int \pi f(k(V,\pi;p^*))\,\psi(dV,d\pi;p^*)$

(iii). $V_0(p^*) \leq M$ *solves (***P0***) with equality if* $E^* > 0$

(iv). $\psi_t = \psi^*$ *and* $E_t = E^*$ *for all* t *solve (***D***)*

Condition (i) states that all players must optimize while condition (ii) requires goods market clearing. Condition (iii) is the free entry condition for firms; when $E^* > 0$ then $V_0(p^*) = M$ and $\int \mathsf{W}(V_0(p^*),\pi;p^*)\,d\Gamma(\pi) = I_0$, which pins down p^* in a stationary equilibrium with positive entry.

3.4. Quantitative analysis

I now calibrate the model presented above to match some salient features of the U.S. economy. With a reasonably realistic calibration at hand, I then conduct counterfactual experiments aimed at assessing the quantitative importance of reorganization and renegotiation.

3.4.1. Baseline calibration

My baseline calibration is aimed at matching certain aspects of the U.S. economy. Given the nonlinearities of our model, it is not possible to precisely match parameters to moments. However, the mechanics of the model clearly indicates which are the key parameters for each set of moments. Below is a brief description of our calibration strategy and Table 1 presents a summary of the parameter values.

The model period is a quarter and the price of output is normalized to unity. The discount factor is set at $\beta = 0.97$ so as to match the average annual return of the S&P

[22] With suitable changes in notation to include π, p.

Parameter	**Description**	**Value**
β	Discount factor	0.97
$f(k)$	Production function	Ak^{α}
α	Returns to scale	0.88
Δ	Liquidation value (bank)	3.0
$\bar{s}$	Outside option upper bound	2.5
θ	Renegotiation costs	0.34
$\Gamma(\pi)$	Distribution of project types	$1 - R(\pi)^{-\zeta}$
Π	Support of $\Gamma(\cdot)$	[0.5,0.58]
ζ	Tail parameter for $\Gamma(\cdot)$	4.45
V_0	Initial equity of new firms	3.728
η	Exogenous exit	0.01
μ	Cost of reducing agency	0.05
ρ	Cost of diverting cash flows	0.4

Table 3.1. Baseline calibration

500 over the period 1928-2013, which is 11.5%. As much of the finance literature, I view this as a better measure of the bank's opportunity cost than the risk-free interest rate often used in the RBC literature. I parametrize the production function as $f(k) = Ak^{\alpha}$ and choose $\alpha = 0.88$ for the returns to scale parameter following the calibration for the U.S. economy found in Li (2010). In his numerical exercises, Quadrini (2004) uses a value of 0.85 citing empirical work by Atkeson, Khan and Ohanian (1996), while Clementi and Palazzo (2013) use a value of 0.8.

Following the literature on financial frictions and heterogeneous firms, we parametrize the distribution of project types as a Pareto distribution: $\Gamma(\pi) = 1 - R(\pi)^{-\zeta}$ where $R(\pi) \geq 1$ denotes the rank of π in the distribution. For the tail parameter we use $\zeta = 4.45$. In quantitative exercises similar to mine, Antunes, Cavalcanti and Villamil (2008) and Buera and Shin (2013) use $\zeta = 4.43$ and $\zeta = 4.15$, respectively. The support of the distribution is a less straightforward choice. In their numerical computations Clementi and Hopenhayn (2006) use a value of $\pi = 0.5$ while Li (2010) calibrates this parameter to 0.58. I therefore choose a Π that spans the entire interval between these two values found in the literature; that is, I set $\Pi = [0.5, 0.58]$.

Next, given α, Π, ζ and β, the upper bound for the total value of a liquidated firm, $\bar{s} + \Delta$, is the key determinant (along with renegotiation costs) of the endogenous exit rate since it drives both incentives to liquidate and the initial equity of new firms, V_0. I choose $\bar{s} + \Delta = 5.5$ and take the exogenous exit rate $\eta = 0.01$ from Quadrini (2004) so as to match the average annual exit rate of firms in the U.S., estimated to be between 5.5% and 4.5% by Lee and Mukoyama (2013) and Dunne, Roberts and Samuelson (1988), respectively. In turn, I set $\Delta = 3$ so that the lender receives, on average 65% of the total value of the liquidated firm in line with available evidence on the fraction of bankruptcy assets that are distributed to creditors in Chapter 7 cases (see Bris et al. (2006)).

Given my assumption about the financing of new firms (i.e., problem (**P0**)), the choices made so far help me pin down the initial amount to be financed $I_0 - M = 4.3$, which gives me $V_0 = 3.728$. This in turn results in a ratio of working capital of new firms relative to incumbent firm of 18.6% which lies between the 17.4% reported by OECD (2001) for entering firms in manufacturing, and the 19.5% found in Haltiwanger, Jarmin and Miranda (2013) for young firms (1-10 years). The resulting stationary distribution of constrained firms (along the working capital dimension) bears

Moment	**Value**	**Data**
Exit rate	5.1%	4.5%-5.5%
Relative size of entrants (working capital)	18.6%	17.5%-19.8%
Assets of Ch.11 / assets of all distressed	63.4%	66.2%
Frequency of Ch.11 relative to Ch. 7	21.7%	24.5%
Ch. 11 recovery rate (pre-fees)	110.9%	107%
Ch. 11 recovery rate (post-fees)	74%	69%-90%

Table 3.2. Moments and data

some resemblance with the U.S. distribution of firms by employment (see upper right panel of Figure 3.5 below).[23]

Finally, the key parameters of interest -those associated with bankruptcy procedures- are chosen so as to match some features of the data reported in empirical studies of financially distressed firms. The cost of renegotiation, θ, is set to 0.34 so I can approximate the frequency of Chapter 11 relative to Chapter 7. Next, given the data presented in the introduction (see also table B1 in Appendix B) showing that out-of-court reorganizations account for a large fraction of total business failures, I want to allow for reorganization to be optimal even beyond the Chapter 11 (renegotiation+reorganization) option. Accordingly, the net benefit of reorganization, captured by the ratio ρ/μ is set to approximate the ratio of total assets $(\mathrm{B}_t + V_t)$ of firms in

[23]The distribution of U.S. firms is notorious for exhibiting signifficant positive skeweness (i.e., toward small firms) and a long right tail; see e.g., Luttmer (2007).

Chapter 11 to all firms under reorganization which Chatterjee, Dhillon and Ramirez (1996) report to be close to 66%.[24]

As seen in Table 3.2, the results from my benchmark calibration also help me approximate some features of the U.S. corporate bankruptcy outcomes found in the data. In particular, I compute Chapter 11 recovery rates as the bank's value after renegotiation, $B\left(s_t\right)$, divided by the bank's value prior to default, $B(V_{t-1})$. This gives me $B\left(s_t\right)/B\left(V_{t-1}\right) = 110.9\%$ before bankruptcy costs, and of $\left(1-\theta\right)B\left(s_t\right)/B\left(V_{t-1}\right) = 74\%$ after costs. These figures conform well with those reported by Bris et al. (2006) of 107% and 69%-90.1%, respectively.

Firm dynamics. I now describe the model's implications for firm growth and survival, and compare them with the empirical evidence. The dynamics of firms resulting from the baseline calibration are presented in Figure 3.5.

The first thing to notice is that the model studied here is consistent with the observation made elsewhere that firm age is positively correlated with size (lower-left panel) and negatively correlated with the variance of growth (lower-right panel). Secondly, the model is also consistent with the observation that exit hazard rates increase at the early stages of the firm and decrease systematically thereafter (Bruderl, Preisendorfer and Ziegler (1992)). To give a sense of the quantitative performance of the model, it is useful to note that estimates for the U.S. imply that around 75% of firms survive through their first three years of operation, while almost half of them have failed by their sixth year (Headd and Kirchhoff (2009)).

[24] Unfortunately, we cannot match the frequency of Chapter 11 relative to out-of-court reorganization since our parametrization implies that new firms have equity below the reorganization threshold.

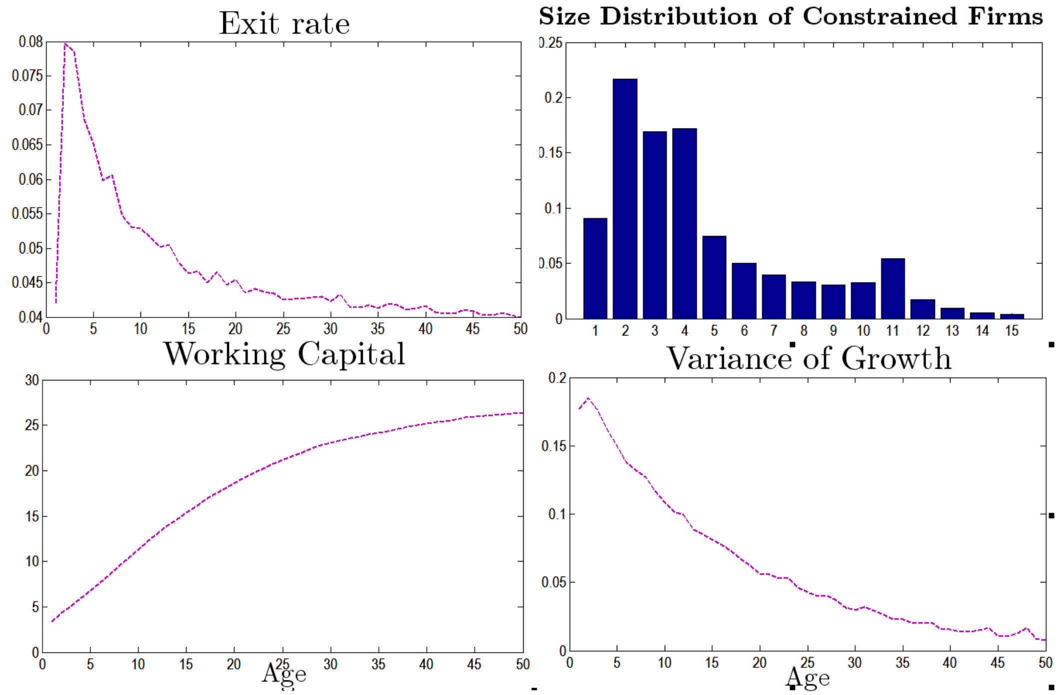


Figure 3.5. Firm dynamics

3.4.2. Counterfactual experiment

The counterfactual exercise that I conduct is aimed at quantifying the effects of allowing for renegotiation and reorganization as an alternative to liquidation. To this end, I use the CH contract as the data generation process for a large number of firms and compare the results with those obtained under the contract with renegotiation and reorganization. Table 3.3 presents the results of this comparison.

The most noteworthy consequence of not having the option to rehabilitate troubled firms is that the exit rate increases by over a quarter. This is perhaps not surprising as some of the firms that would otherwise renegotiate and survive are liquidated. This has a corresponding effect on age; the average firm in the liquidation-only economy is over four years younger. Table 3.3 also shows that in the economy with renegotiation and reorganization the average firm is modestly larger and the dispersion of asset

Moment	Baseline	Liquidation only
Exit rate	5.1%	6.36%
Relative size of entrants (working capital)	18.6%	19.3%
Average firm age (years)	40.6	36.2
Average firm productivity	0.51	0.505
Average firm size (capital)	6.65	6.54
Size dispersion (Coeff. of variation)	0.74	0.77

Table 3.3. Counterfactual: Moments and data

size is moderately lower. The mechanism behind this result can be seen in Figure 3.6 which tracks the average firm productivity in both economies. While there is some selection under both contracts (as shown by Proposition 12), selection is modestly enhanced by the contract with renegotiation and reorganization.

3.5. Concluding remarks

In this paper I have presented an industry equilibrium-dynamic contracts approach to corporate default, liquidation and renegotiation. The model presented here is able to predict the patterns of default and renegotiation observed in actual economies. Furthermore, the model performs well quantitatively, as it allows me to reproduce certain moments of the U.S. distribution of firms while fiting the data on corporate bankruptcy outcomes, specially recovery rates. A novel feature of the model is that it is able to account for the empirical regularity that, when outside opportunities for the firm are promising, borrowing entrepreneurs are more likely to successfully renegotiate their debt (rather than liquidate the firm) if they were to default.

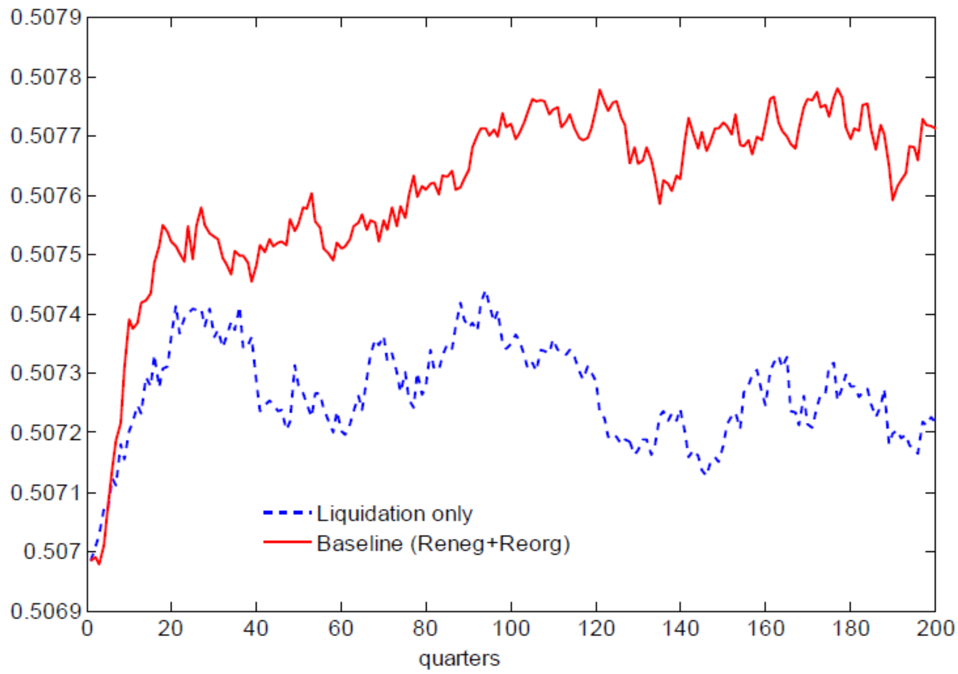


Figure 3.6. Average productivity and financial selection

The framework presented here allows one to think about some of the issues arising from corporate bankruptcy design in an orderly fashion. Extending this framework -and in fact developing a research agenda about the details underlying corporate bankruptcy law- is of foremost importance considering that every crisis brings about a wave of bankruptcies, and that such waves tend to precede comprehensive reforms to bankruptcy codes.

The model remains unsatisfactory in a number of dimensions, however, and I see lots of potential for future research. Perhaps the most attractive route for improving this framework is to allow for heterogeneous firms to solve an entry problem that depends on their type. In this case, the costs and benefits from alternative bankruptcy procedures would be anticipated by potential entrants, which will allow for selection along the entry decision.

3.6. Appendix 2

3.6.1. Proofs and derivations

Proof of Lemma 6. A firm currently under reorganization, and operating at $\tilde{k}^r$ can either become undistressed next period with probability π, or remain under reorganization and have the chance to become undistressed in the following period:

$$\begin{aligned}\tilde{\mathrm{W}}_r &= \pi(1-\mu) f(\tilde{k}^r) - \tilde{k}^r + \beta\left[\pi\tilde{\mathrm{W}} + (1-\pi)\tilde{\mathrm{W}}_r\right] \\ &= \frac{\beta\pi\tilde{\mathrm{W}} + \pi(1-\mu) f(\tilde{k}^r) - \tilde{k}^r}{1-\beta(1-\pi)}\end{aligned} \tag{3.3}$$

since $\frac{\beta\pi}{1-\beta(1-\pi)} < 1$, it suffices to show that:

$$\frac{\pi(1-\mu) f(\tilde{k}^r) - \tilde{k}^r}{1-\beta(1-\pi)} < \left[1 - \frac{\beta\pi}{1-\beta(1-\pi)}\right]\tilde{\mathrm{W}}$$

some tedious algebra and $\tilde{W} = \frac{\pi f(\tilde{k}) - \tilde{k}}{1-\beta}$ reduces this to showing that:

$$\pi f(\tilde{k}) - \tilde{k} > \pi(1-\mu) f(\tilde{k}^r) - \tilde{k}^r$$

which holds given the strict concavity of $f(\cdot)$ and $\mu > 0$. □

Proof of Proposition 9. First notice that $\widehat{\mathrm{W}}_i(V^i) > \widehat{\mathrm{W}}_{j\neq i}\left(V^{j\neq i}\right)$ implies $V = V^i$ and $W(V) = \widehat{\mathrm{W}}_i(V^i) = \widehat{\mathrm{W}}_i(V)$. Now, $\widehat{\mathrm{W}}_u(V) > \widehat{\mathrm{W}}_r(V)$ for V large enough follows from continuity and Lemma 6, while $\widehat{\mathrm{W}}_u(V) > W_\ell$ is true by Assumption 7 and continuity. Next, to show that $\widehat{\mathrm{W}}_r(V) > \widehat{\mathrm{W}}_u(V)$ for some V, first define $W_u^*(V)$ as the value of a firm that cannot be liquidated or reorganized and $W_r^*(V)$ as the value of a firm that always operates under reorganization and cannot be liquidated. Results analogous to those of the CH contract hold for each of these sub-problems, so one has that $\tau_*^i = f(k_*^i) = \frac{\beta\left(V_{H*}^i - V_{L*}^i\right)}{1-\rho^i}$, and therefore $V = \beta\left[\pi V_{H*}^i + (1-\pi) V_{L*}^i\right]$ for $i = u, r$.

Now $W_u^*(V), W_r^*(V)$ can be written:

$$\text{(3.4)} \qquad \mathrm{W}_u^*(V) = \max_{V_*^{1H}, V_*^{1L} \geq 0} V - \beta V_{L*}^u - f^{-1}\left(\frac{V - \beta V_{L*}^u}{\pi}\right) + \beta \mathbb{E} \mathrm{W}_u^*(V)$$
$$s.t. \quad : \quad V = \beta\left[\pi V_{H*}^i + (1-\pi) V_{L*}^u\right] = \beta \mathbb{E} V_*^u$$

and:

$$\text{(3.5)} \quad \mathrm{W}_r^*(V) = \max_{V_*^{0H}, V_*^{0L} \geq 0} \frac{(1-\mu)(V - \beta V_{L*}^r)}{1-\rho} - f^{-1}\left[\frac{V - \beta V_{L*}^r}{(1-\rho_0)\pi}\right] + \beta \mathbb{E} \mathrm{W}_r^*(V)$$
$$s.t. \quad : \quad V = \beta\left[\pi V_{H*}^r + (1-\pi) V_{L*}^r\right] = \beta \mathbb{E} V_*^r$$

clearly the functions $W_u^*(V), W_r^*(V)$ are increasing, concave and differentiable so the Envelope Theorem applies and:

$$\frac{d\mathrm{W}_u^*(V)}{dV} = 1 - \frac{1}{\pi}\left[f^{-1}\left(\frac{V - \beta V_*^{1L}}{\pi}\right)\right]'$$

where $\left[f^{-1}(y)\right]' = \frac{df^{-1}(y)}{dy}$, while:

$$\frac{d\mathrm{W}_r^*(V)}{dV} = \frac{1-\mu}{1-\rho} - \frac{1}{(1-\rho)\pi}\left[f^{-1}\left(\frac{V - \beta V_{L*}^r}{(1-\rho)\pi}\right)\right]'$$

Now $\rho > \mu$, $V = 0 \Rightarrow V_{L*}^i = 0$ and $\left[f^{(-1)}(0)\right]' = \frac{df^{-1}(0)}{dk} = 0$ together imply that $\frac{d\mathrm{W}_*^r(0)}{dV} > \frac{d\mathrm{W}_u^*(0)}{dV}$. By continuity of the value functions, $\exists!\ V^{++}$ such that $\frac{d\mathrm{W}_u^*(V)}{dV} < \frac{d\mathrm{W}_r^*(V)}{dV}\ \forall\ V \in (0, V^{++})$. Given $W_u^*(0) = W_r^*(0) = 0$, it can be concluded that $W_u^*(V) < W_r^*(V)\ \forall\ V \in (0, V^{++})$. Next, let $W_u^\Delta(V)$ denote the value of a firm that is currently undistressed and can be liquidated with scrap value Δ, but cannot be reorganized. Define $W_r^\Delta(V)$ analogously for a firm currently under reorganization. Clearly, $\lim_{\Delta\to 0} W_u^\Delta(V) = W_u^*(V)$ and $\lim_{\Delta\to 0} W_r^\Delta(V) = W_r^*(V)$. Then continuity ensures that $\exists!\ \Delta^S$ and V^S such that $\Delta \in \left(0, \Delta^S\right) \Rightarrow W_r^\Delta(V) > W_u^\Delta(V)$ for $V \in \left(0, V^S\right)$. It remains to show that $W_r^\Delta(V) > W_u^\Delta(V)$ for some V is sufficient for

$\widehat{\mathrm{W}}_r(V) > \widehat{\mathrm{W}}_u(V)$ to hold for some V. To see this, suppose otherwise (and find a contradiction). That is, suppose $W_r^{\Delta}(V) > W_u^{\Delta}(V)$ for some V but $\widehat{\mathrm{W}}_r(V) \leq \widehat{\mathrm{W}}_u(V)\ \forall\ V$. Then $\widehat{\mathrm{W}}_u(V) \geq \widehat{\mathrm{W}}_r(V) \forall\ V \Rightarrow \widehat{\mathrm{W}}_u(V) = W_u^{\Delta}(V)$. On the other hand, it must be true that $\widehat{\mathrm{W}}_r(V) \geq W_r^{\Delta}(V)$ since a policy of never leaving the firm undistressed is clearly feasible and incentive compatible for the problem in $\widehat{\mathrm{W}}_r(V)$. In other words, $\widehat{\mathrm{W}}_u(V) = W_u^{\Delta}(V) \geq \widehat{\mathrm{W}}_r(V) \geq W_r^{\Delta}(V) \forall\ V$, a contradiction since we have already shown that $W_r^{\Delta}(V) > W_u^{\Delta}(V)$ for some V. $\square$

Proof of Proposition 10. Notice that the entrepreneur will default if and only if $\beta s > V_c$, so the indifference point is $\beta s_d = V_c$. Notice also that $V \geq \beta E(s) = \frac{\beta \bar{s}}{2}$ since a strategy of defaulting for all realizations of s is always feasible for the entrepreneur, and such strategy gives her exactly $\beta E(s)$. This obviously implies $V_L^i, V_H^i \geq \beta E(s)$. Since $V_c \geq V_c^i = \beta\left[\pi V_H^i + (1-\pi) V_L^i\right]$, it follows that $V_c \geq \beta^2 E(s)$.Thus we require $\beta s_d \geq \beta^2 E(s) \Rightarrow s_d \geq \beta E(s)$ so we can find $s_d = \max\{\beta E(s), \sqrt{2\bar{s}V\beta^{-1} - \bar{s}^2}\}$. Obviously, $V = \beta E(s)$ implies $2\bar{s}V\beta^{-1} - \bar{s}^2 = 0$ in which case $s_d = \beta E(s)$. Hence, $\exists\ V^+ > \beta E(s)$ such that $s_d = 2\bar{s}V\beta^{-1} - \bar{s}^2$ and the second part of the proposition follows. $\square$

Proof of Corollary 10. From the last proof, we know that if $V \leq \frac{\beta \bar{s}}{2}\left(\frac{\beta^2}{4}+1\right)$ then $s_d = \beta E(s)$. This in turn implies that $\beta^2 E(s) = V_c$ which in turn means that $V_L^i = V_H^i = \beta^2 E(s)$. Thus, the binding ICC $\beta\left(V_H^i - V_L^i\right) = (1-\rho^i) f\left(k^i\right)$ implies $k^i = 0$. $\square$

Proof of Proposition 12. First, notice that using the result $f(k) = \tau$ it follows that for $V < \tilde{V}(\pi)$:

$$\frac{\partial \widehat{\mathrm{W}}(V;\pi)}{\partial \pi} = f(k) + \beta\pi\left[\mathrm{W}(V_H;\pi) - \mathrm{W}(V_L;\pi)\right](V_H - V_L) \geq 0$$

where the inequality is ensured by $W(V_H;\pi) \geq W(V_L;\pi)$ and $V_H \geq V_L$. Next, notice that $\frac{\partial \widehat{\mathrm{W}}(V;\pi)}{\partial V}$ is increasing in π. To see this, differentiate () w.r.t. its first argument:

$$\frac{\partial \widehat{\mathrm{W}}(V;\pi)}{\partial V} = 1 - \frac{1}{\pi}\left[f^{-1}\left(\frac{V - \beta V^L}{\pi}\right)\right]' = 1 - \frac{1}{\pi}\left\{\frac{1}{f'\left[f^{-1}\left(\frac{V-\beta V^L}{\pi}\right)\right]}\right\}$$

where $\left[f^{-1}(y)\right]' = \frac{df^{-1}(y)}{dy}$ and the last equality is by the Inverse Function Theorem. Now, f increasing implies f^{-1} is increasing. Therefore, $f^{-1}\left(\frac{V-\beta V^L}{\pi}\right)$ decreases with π. Moreover, f concave implies that $f'\left[f^{-1}\left(\frac{V-\beta V^L}{\pi}\right)\right]$ increases with π, which in turn means that the term in braces decreases with π. Summarizing, one has that $\pi' > \pi$ implies $\widehat{\mathrm{W}}(V;\pi') > \widehat{\mathrm{W}}(V;\pi)$ and $\frac{\partial \widehat{\mathrm{W}}(V;\pi')}{\partial V} > \frac{\partial \widehat{\mathrm{W}}(V;\pi)}{\partial V}$. Since $\widehat{\mathrm{W}}(0;\pi') > \widehat{\mathrm{W}}(0;\pi) = \beta\Delta$, this establishes that $V_\ell(\pi') < V_\ell(\pi)$. □

Invariant distribution of firms. Let $\phi(V,\pi) = 1$ if $V \in E$ and $\pi \in Q$, and zero otherwise. Then ψ_t satisfies the law of motion:

$$\begin{aligned}
\psi_{t+1}(\mathcal{E},\mathcal{Q};p) \quad = \quad & (1-\eta)\int \pi \sum_{i=u,r}\sum_{j=u,r}\left\{\phi\left[V_c^j\left(V_H^i(V,\pi;p),\pi;p\right),\pi\right]\times\right. \\
& \left.\left[1-s_\ell\left(V_H^i(V,\pi;p),\pi;p\right)\left(1-s_d\left(V_H^i(V,\pi;p);p\right)\right)/\bar{s}\right]\right\}\psi_t(dV,d\pi;p^*) \\
& +(1-\eta)\int(1-\pi)\sum_{i=u,r}\sum_{j=u,r}\left\{\phi\left[V_c^j\left(V_L^i(V,\pi;p),\pi;p\right),\pi\right]\times\right. \\
& \left.\left[1-s_\ell\left(V_L^i(V,\pi;p),\pi;p\right)\left(1-s_d\left(V_L^i(V,\pi;p);p\right)\right)/\bar{s}\right]\right\}\psi_t(dV,d\pi;p^*) \\
& +(1-\eta)\int\sum_{j=u,r}\left\{\phi\left[V_c^j\left(V_\ell(V,\pi;p),\pi;p\right),\pi\right]\times\right. \\
& \left.\left[1-s_\ell\left(V_\ell(V,\pi;p),\pi;p\right)\left(1-s_d\left(V_\ell(V,\pi;p);p\right)\right)/\bar{s}\right]\right\}\psi_t(dV,d\pi;p^*) \\
& +E_{t+1}\int\phi\left[V_0(p),\pi\right]d\Gamma(\pi)
\end{aligned}$$

The first four lines add up all the firms that have not defaulted in t, and who survive exogenous exit and endogenous liquidation in $t+1$. The next two lines add up all the firms that defaulted (and renegotiated) in t and survive exogenous exit and endogenous liquidation in $t+1$. The final line accounts for new entrants (whose type is drawn from $\Gamma(\pi)$). Notice that V_{t+1} for the firms that did not default depends on the realization of z_t while for the defaulted firms does not (i.e., $V_{t+1} = V_\ell$). Notice also that we have used the fact that $s_d(V,\pi;p) = s_d(V;p)$ which follows directly from Proposition 10.

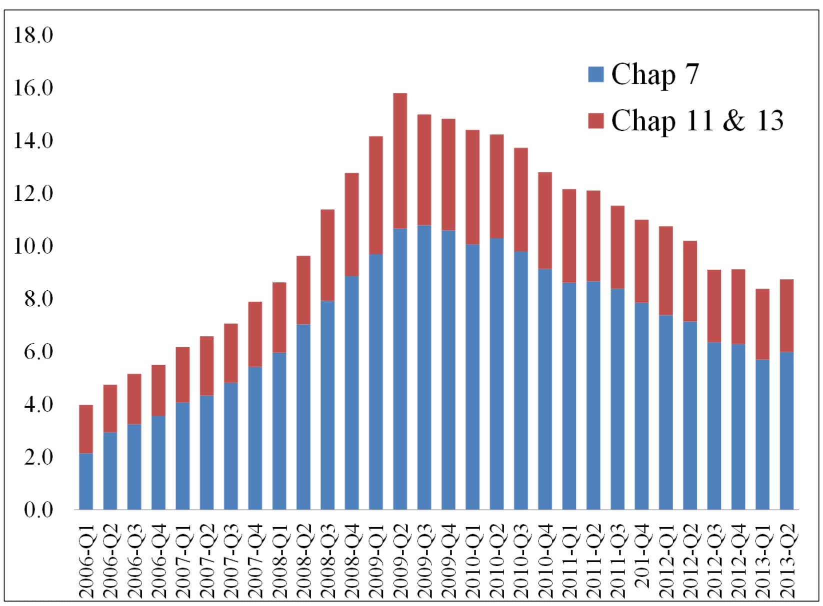
3.6.2. Additional figures

Table B1. U.S. Business failures Jul/2008-Jun/2009

Liquidations	Reorganizations	
Chapter 7	Chapter 11	Private workouts
37,174	14,480	42,835

Source: uscourts.gov and Dun & Bradstreet

Figure B1. Quarterly formal bankruptcy filings in the U.S.



Source: uscourts.gov

References

Acharya, V.V., Y. Amihud, and L. Litov, "Creditor rights and corporate risk-taking," *Journal of Financial Economics*, 2011, *102 (1)*, 150–166.

Aghion, P. and P. Bolton, "An Incomplete Contracts Approach to Financial Contracting," *Review of Economic Studies*, 1992, *59*, 473–94.

Aguiar, M. and M. Amador, "Sovereign Debt," in E. Helpman, K. Rogoff, and G. Gopinath, eds., *Handbook of International Economics*, Vol. 4 Elsevier 647-687 2014.

Albuquerque, R. and H. A. Hopenhayn, "Optimal Lending Contracts and Firm Dynamics," *Review of Economic Studies*, 2004, *71*, 285–315.

Altman, E., "A further empirical investigation of the bankruptcy cost question," *Journal of Finance*, 1984, *39*, 1067–1089.

Antunes, A., T. Cavalcanti, and A. Villamil, "The effect of financial repression and enforcement on entrepreneurship and development," *Journal of Monetary Economics*, 2008, *55*, 278–297.

Araujo, A., R. Ferreira, and B. Funchal, "The Brazilian bankruptcy law experience," *Journal of Corporate Finance*, 2012, *18(4)*, 994–1004.

Arellano, C., Y. Bai, and J. Zhang, "Firm dynamics and financial development," *Journal of Monetary Economics*, 2012, *59*, 533–549.

Atkeson, A., A. Khan, and L. Ohanian, "Are data on industry evolution and gross job turnover relevant for macroeconomics?," *Carnegie-Rochester Conference Series on Public Policy*, 1996, *44 (1)*, 216–250.

Ayotte, K. and E. Morrison, "Creditor Control and Conflict in Chapter 11," *Journal of Legal Analysis*, 2009, *1(2)*, 511–540.

Bagehot, W., *Lombard street*, 1962, Irwin, Homewood, IL, 1873.

Baird, D. and R. Rasmussen, "Chapter 11 at twilight," *Stanford Law Review, ,673*, 2003, *56*, 673–699.

Bernanke, B. and M. Gertler, "Agency costs, net worth, and business fluctuations," *American Economic Review*, 1989, *79(1)*, 14–31.

Bharath, S., S. Dahiya, A. Saunders, and A. Srinivasan, "So what do I get? The bank's view of lending relationships," *Journal of Financial Economics*, 2007, *85*, 368–419.

______, ______, ______, and ______, "Lending relationships and loan contract terms," *Review of Financial Studies*, 2011, *24(4)*, 1141–1203.

______, V. Panchapegesan, and I. Werner, "The changing nature of Chapter 11," *IIM Bangalore Research Paper No. 461*, 2013.

Blazy, R., J. Petey, and L. Weil, "Can bankruptcy codes create value? Evidence from creditorsâĂŹ recoveries in France, Germany, and the UK," *Working Paper, Laboratoire de Recherche en Gestion et Economie (LaRGE), UniversitÃľ de*

Strasbourg (France)., 2010.

Bolton, P., X. Freixas, F. Gambacorta, and P. Mistrulli, "Relationship and Transaction Lending in a Crisis," *BIS Working Paper 417*, 2013.

Boot, A., "Relationship banking: What do we know?," *Journal of Financial Intermediation*, 2000, *9*, 7–25.

Bris, A., I. Welch, and N. Zhu, "The costs of bankruptcy: Chapter 7 liquidation versus Chapter 11 reorganization," *Journal of Finance*, 2006, *61(3)*, 1253–1303.

Bruderl, J., P. Preisendorfer, and R. Ziegler, "Survival Chances of Newly Founded Business Organizations," *American Sociological Review*, 1992, *57(2)*, 227–242.

Buera, F. and Y. Shin, "Financial frictions and the persistence of history: a quantitative exploration," *Journal of Political Economy*, 2013, *121*, 221–272.

Caputo, M., *Foundations of Dynamic Economic Analysis: Optimal Control Theory and Applications*, Cambridge University Press, 2005.

Castro, R., G. Clementi, and G. MacDonald, "Investor protection, optimal incentives, and economic growth," *Quarterly Journal of Economics*, 2004, *119(3)*, 1131–1175.

Chatterjee, S., U. Dhillon, and G. Ramirez, "Resolution of financial distress: debt restructurings via chapter 11, prepackaged bankruptcies, and workouts," *Financial Management*, 1996, *25(1)*, 5–18.

Cheng, S. and C. Shiu, "Investor protection and capital structure: International evidence," *Journal of Multinational Financial Management*, 2007, *17*, 30–44.

Claessens, S. and L. Klapper, "Bankruptcy around the world: explanations of its relative use," *American Law and Economics Review*, 2005, *7*, 253–283.

Clementi, G. and B. Palazzo, "Entry, Exit, Firm Dynamics and Aggregate Fluctuations," *NBER Working Paper No. 19217*, 2013.

_______ and H. Hopenhayn, "A A theory of financing constraint and firm dynamics," *Quarterly Journal of Economics*, 2006, *121*, 229–265.

Cooley, T. and V. Quadrini, "Financial markets and firm dynamics," *American Economic Review*, 2001, *91(5)*, 1286–1310.

Corbae, D. and P. D'Erasmo, "Reorganization or exit: bankruptcy decisions and firm dynamics," *manuscript*, 2014.

Cressy, R., "Credit rationing or entrepreneurial risk aversion? An alternative explanation for the Evans and Jovanovic finding," *Economic Letters*, 2000, *66*, 235–240.

Custodio, C., M. Ferreira, and L. Laureano, "Why are US firms using more short-term debt?," *Journal of Financial Economics*, 2013, *108*, 182–212.

Degryse, H. and P.V. Cayseele, "Relationship lending within a bank-based system: evidence from european small business data," *Journal of Financial Intermediation*, 2000, *9*, 90–109.

Djankov, S., C. McLiesh, and A. Shleifer, "Private credit in 129 countries," *Journal of Financial Economics*, 2007, *84*, 299–329.

_______, O. Hart, C. McLiesh, and A. Shleifer, "Debt enforcement around the world," *Journal of Political Economy*, 2008, *116 (6)*, 1105–1149.

Dunne, T., M.J. Roberts, and L. Samuelson, "Patterns of Firm Entry and Exit in U.S. Manufacturing Industries," *Rand Journal of Economics*, 1988, *19*, 495–515.

Elsas, R. and J.P. Krahnen, "Is relationship lending special? Evidence from credit-Âőle data in Germany," *Journal of Banking and Finance*, 1998, *22*, 1283–1316.

Fang, H. and J. Nofsinger, "Risk aversion, entrepreneurial risk, and portfolio Selection," *Journal of Entrepreneurial Finance*, 2009, *13*, 25–55.

Fernandez, A. and C. Tamayo, "From institutions to financial development and growth: what are the links?," *IADB Working Paper*, 2014, *565.*

Gale, D. and M. Hellwig, "Incentive compatible debt contracts: the one period problem," *Review of Economic Studies*, 1985, *52(4)*, 647–663.

Galindo, A. and A. Micco, "Creditor protection and financial markets: Empirical evidence from Latin America," *Economic Review, FRB of Atlanta*, 2004, *2nd Quarter*, 29–37.

______ **and** ______, "Creditor protection and credit volatility," *Research Department Working Paper No. 528, Inter-American Development Bank*, 2005.

Gennaioli, N. and S. Rossi, "Judicial Discretion in Corporate Bankruptcy," *Review of Financial Studies*, 2010, *23*, 4078–4114.

Ghoul, S.E., Guedhami O., S. Cho, and J. Suh, "Creditor rights and capital structure: Evidence from international data," in "Preceedings of the Annual Meetings, European Financial Management Association" 2012.

Gilson, S., "Bankruptcy, Boards, Banks, and Blockholders," *Journal of Financial Economics*, 1990, *27*, 355–387.

______, "Coming through in a crisis: how chapter 11 and the debt restructuring industry are helping to revive the U.S. economy," *Journal of Applied Corporate Finance*, 2012, *24*, 23–35.

______**, J. Kose, and L. Lang**, "Troubled debt restructurings: An empirical study of private reorganization of firms in default," *Journal of Financial Economics*, 1990, *27*, 315–353.

Gine, X. and I. Love, "Do reorganization costs matter for efficiency? evidence from a bankruptcy reform in colombia," *Journal of Law and Economics*, 2010, *53(4)*, 833–864.

Green, E., "Lending and the smoothing of uninsurable income," in E. Prescott and N. Wallace, eds., *Contractual Arrangements for International Trade*, University of Minnesota Press 1987.

Greenwood, J., J.M. Sanchez, and C. Wang, "Financing development: The role of information costs," *American Economic Review*, 2010, *100*, 1875–1891.

Hall, R. and C. Jones, "Why Do Some Countries Produce So Much More Output per Worker Than Others?," *Quarterly Journal Of Economics*, 1999, *114(1)*, 83–116.

Haltiwanger, J., R. Jarmin, and J. Miranda, "Who Creates Jobs? Small versus Large versus Young," *The Review of Economics and Statistics*, 2013, *95(2)*, 347–361.

Headd, B. and B. Kirchhoff, "The Growth, Decline and Survival of Small Businesses: An Exploratory Study of Life Cycles," *Journal of Small Business Management*, 2009, *47(4)*, 531–550.

Hopenhayn, H., "Entry, Exit and Firm Dynamics in Long-Run Equilibrium," *Econometrica*, 1992, *60*, 1127–1150.

______ **and I. Werning**, "Equilibrium Default with Limited Commitment," *SED Conference Papers*, 2006.

——— and ———, "Equilibrium Default," *Unpublished Manuscript*, 2008.

Hvide, H.K. and T.E. Leite, "Optimal debt contracts under costly enforcement," *Economic Theory*, 2010, *44(1)*, 149–165.

Jostarndt, P. and Z. Sautner, "Financial distress, corporate control, and management turnover," *Journal of Banking and Finance*, 2008, *32(10)*, 2188–2204.

Jovanovic, B., "Selection and the evolution of insdustry," *Econometrica*, 1982, *50(3)*, 649–670.

Kalay, A., R. Singhal, and E. Tashjian, "Is Chapter 11 costly?," *Journal of Financial Economics*, 2007, *84*, 772–796.

Kalemli-Ozcan, S., B. Sorensen, and S. Yesiltas, "Leverage across firms, banks and countries," *NBER Working Paper No. 17354*, 2011.

Krasa, S. and A.P. Villamil, "Optimal contracts when enforcement Is a decision variable," *Econometrica*, 2000, *68(1)*, 119–134.

———, T. Sharma, and A.P. Villamil, "Bankruptcy and firm finance," *Economic Theory*, 2008, *36*, 239–266.

Lee, Y and T. Mukoyama, "Entry, exit, and plant-level dynamics over the business cycle," *Working Paper 0718*, 2013, *Federal Reserve Bank of Cleveland.*

Lewellen, K., "Financing decisions when managers are risk averse," *Journal of Financial Economics*, 2006, *82*, 551–589.

Li, S., "Employmentflowswithendogenousfinancingconstraints," *The B.E. Journal of Macroeconomics*, 2010, *10(1)*, Article 20 (Contributions).

Lim, Y. and C. Hee Hahn, "Bankruptcy policy reform and total factor productivity dynamics in korea: evidence from microdata," in "Growth and Productivity in East Asia: NBER-East Asia Seminar" University of Chicago Press 2004.

Luttmer, E., "Selection, growth and the size distribution of firms," *Quarterly Journal of Economics*, 2007, *122 (3)*, 1103–1144.

Monnet, C. and E. Quintin, "Optimal contracts in a dynamic costly state verification model," *ECB Working Paper No. 126*, 2002.

——— and ———, "Optimal contracts in a dynamic costly state verification model," *Economic Theory*, 2005, *26*, 867–885.

Nini, G., D. Smith, and A. Sufi, "Creditor control rights, corporate governance, and firm value," *Review of Financial Studies*, 2012, *26*, 1713–1761.

OECD, "Productivity and firm dynamics: evidence from microdata," *OECD Economic Outlook*, 2001, *69(1)*, 209–223.

Panousi, V. and D. Papanikolaou, "Investment, idiosyncratic risk, and ownership," *Journal of Finance*, 2012, *67*, 1113–1148.

Pereira, P.F. and M.A. Ferreira, "Capital structure and law around the world," *Journal of Multinational Financial Management*, 2011, *21*, 119–150.

Peri, A., "A Quantitative Theory of Corporate Bankruptcy," *unpublished manuscript*, 2014.

Petersen, M. and R. Rajan, "The benefits of lending relationships: Evidence from small business data," *Journal of Finance*, 1994, *49*, 1367–1400.

Porta, R. La, F. Lopez de Silanes, A. Shleifer, and R. W. Vishny, "Legal determinants of external finance," *Journal of Finance*, 1997, *52(30)*, 1131–50.

———, ———, ———, and ———, "Law and finance," *Journal of Political Economy*, 1998, *106(6)*, 1113–1155.

Quadrini, V., "Investment and liquidation in renegotiation-proof contracts with moral hazard," *Journal of Monetary Economics*, 2004, *51(4)*, 713 751.

Rajan, R. and L. Zingales, "What do we know about capital structure? some evidence from international data," *Journal of Finance*, 1995, *50*, 1421–1460.

Roberts, M. and A. Sufi, "Renegotiation of financial contracts: Evidence from private credit agreements," *Journal of Financial Economics*, 2009, *93*, 159–184.

Rodriguez-Delgado, J.D., "Bankruptcy and Firm Dynamics: The Case of the Missing Firms," *IMF Working Paper WP/10/41*, 2010.

Schmid, T., M. Ampenberger, C. Kaserer, and A-K. Achleitner, "Family firms, agency costs and risk aversion: empirical evidence from diversification and hedging decisions," *CEFS Working Paper No. 2008-13*, 2008.

Schumpeter, J., *The theory of economic development: an inquiry into profits, capital, credit, interest and the business cycle*, 1934, Harvard University Press, Cambridge, MA„ 1911.

Senbet, L. and T. Wang, "Corporate Financial Distress and Bankruptcy: A Survey," *Foundations and Trends in Finance*, 2012, *7(3)*, 159–288.

Senkal, A., "Misallocation and Recovery Rates," *University of Virginia*, 2013.

Sevcik, P., "Financial contracts and the political economy of investor protection," *American Economic Journal: Macroeconomics*, 2012, *4*, 163–197.

Smith, A. and C. Wang, "Dynamic credit relationships in general equilibrium," *Jorunal of Monetary Economics*, 2006, *53*, 847–877.

Spear, S. and S. Srivastava, "On repeated moral hazard with discounting," *Review of Economic Studies*, 1987, *54*, 599–617.

Stokey, N., R.E. Lucas, and E. Prescott, *Recursive methods in economic dynamics*, Harvard University Press, 1989.

Thomas, J. and T. Worrall, "Foreign direct investment and the risk of expropriation," *Review of Economic Studies*, 1994, *61*, 81–108.

Townsend, R., "Optimal contracts and competitive markets with costly state verification," *Journal of Economic Theory*, 1979, *21(2)*, 265–293.

Wang, C., "Dynamic Costly State Verification," *Economic Theory*, 2005, *25*, 887–916.

Warner, J., "Bankruptcy cost: Some evidence," *Journal of Finance*, 1977, *32*, 337–347.

White, M., "The corporate bankruptcy decision," *Journal of Economic Perspectives*, 1989, *3*, 129–151.

———, "Corporate bankruptcy as a filtering device: Chapter 11 reorganizations and out-of-court debt restructurings," *Journal of Law, Economics, and Organization*, 1994, *10*, 268–295.

Williamson, O., "Corporate finance and corporate governance," *Journal of Finance*, 1988, *43*, 567–591.

Williamson, S., "Costly monitoring, loan contracts, and equilibrium credit rationing," *Quarterly Journal of Economics*, 1987, *102*, 135–146.

Winton, A., "Costly state verification and multiple investors: The role of seniority," *Review of Financial Studies*, 1995, *8(1)*, 91–123.

Wruck, K.H., "Financial distress, reorganization, and organizational efficiency," *Journal of Financial Economics*, 1990, *27*, 419–444.