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ESSAYS ON INFORMATION, LIQUIDITY AND FINANCIAL FRICTIONS

by

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ABSTRACT OF THE DISSERTATION

Essays on Information, Liquidity and Financial Frictions

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This dissertation seeks to understand how financial frictions arise and how they can affect the economy, and explores the implications of financial frictions for monetary policy during crises. Specifically, Chapter 2 and 3 study the endogenous nature of information asymmetry and explore its implications for financial markets and the macro economy. Chapter 4 studies the potential side effects of large scale asset purchase by central banks.

In Chapter 2, I study a dynamic economy in which the information on asset quality is asymmetric and the degree of information asymmetry endogenously varies with the macro-economy, which amplifies the effects of shocks. In the model, firms hold assets of heterogeneous quality and borrow for operating expenses. Production is subject to idiosyncratic shocks, which may force the firms to liquidate their assets to pay off debts. Firms are initially uninformed of the qualities of their assets, but they can acquire private information on their own assets at a cost. Private information is individually beneficial, but it creates a lemons problem that lowers market liquidity and distorts economic decisions. Adverse shocks trigger private information acquisition, which exacerbates the lemons problem. As results, market liquidity drops and economic activity declines. The model can generate larger fluctuations in financial and macroeconomic variables than an otherwise the same model with the level of information asymmetry being fixed.

In Chapter 3, I provide a possible explanation for the countercyclical movements in the measures of asset return volatility. In the model, external financing is costly due to the information asymmetry between borrowers and lenders. When the borrowers' financial conditions are worsened, the costs of external financing rise. Borrowers respond by increasing their transparency to outside investors to mitigate information asymmetry, which helps reduce the external financing cost. As a result, returns on external financing instruments disperse and fluctuate more as more information is disclosed, leading to increases in the cross sectional dispersion and the time series volatility of returns. This model can generate countercyclical dispersion, volatility in returns and external finance premium, with correlation coefficients between pairs of these measures quantitatively in line with the data.

In Chapter 4, I explore the potential side effects of central bank asset purchase. In the model, commercial banks and shadow banks hold liquid assets as part of their operations. Asset purchases by the central bank decreases the supply of liquid assets that shadow banks can directly hold. When commercial banks do not face binding leverage constraints, shadow banks respond by increasing their deposits in or credit lines from commercial banks and central bank asset purchases are neutral. In the presence of a binding leverage constraint, however, asset purchases create distortions that decrease shadow banks' liquidity holdings and their lending. While conventional wisdom says that central bank asset purchases should be expansionary, I show that central bank asset purchases are necessarily contractionary when the level of bank reserves is high.

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Chapter 1

Introduction

The financial crisis of 2007-2008 is a stark reminder of financial frictions' ability to amplify and propagate shocks and focused attention on the role of central banks in managing crises. My dissertation seeks to understand how financial frictions arise and how they affect the macro economy, and explores the implications of financial frictions for monetary policy during crises.

The first part of this dissertation studies how financial frictions arise and the macro economic implications of financial frictions. Specifically, it focuses on the endogenous nature of information asymmetry and explores its implications for financial markets and the macro economy. Economists have long believed that information asymmetry in asset markets plays a key role in amplifying and propagating shocks. In most existing studies, the level of information asymmetry is exogenously given. However, adverse shocks may trigger private information acquisition that changes the information environment (Dang et al., 2012). In Chapter 2, I study a dynamic economy in which the information on asset quality is asymmetric and the level of information asymmetry endogenously varies over the business cycle. The dynamics of information environment creates a fragility mechanism that amplifies aggregate shocks. In Chapter 3, I focus on the information asymmetry between corporate insiders and outside investors. It studies how the transparency of firms to outside investors varies with the firms' financial conditions and provides a possible explanation for the countercyclical movements in the dispersion and the volatility of stock returns. The second part of my dissertation studies the implications of financial frictions for monetary policy in crises. In particular, it focuses on liquidity management and the role of central banks in supplying liquid assets. In Chapter 4, I study how central bank asset purchases affect the economy by changing the volume of reserve balances in the banking system and the supply of liquid assets in markets. In contrast to conventional wisdom, I show that central bank asset purchases are necessarily contractionary when the level of bank reserves is high.

In Chapter 2, entitled **Endogenous Lemons Market and Information Cycles**, I study an economy in which the information on asset quality is asymmetric and the degree of information asymmetry endogenously respond to aggregate shocks. In the model, firms hold assets of heterogeneous quality and borrow for operating expenses. Production is subject to idiosyncratic shocks, which may force the firms to liquidate their assets to pay off debts. Firms are initially uninformed of the qualities of their assets. However, they can produce private information on their own assets at a cost, which leads to asymmetric information in the asset market and creates a lemons problem. The lemons problem in turn generates an endogenous liquidation cost, i.e. the cost to sell high quality assets at the lemons price.

Privation information acquisition is individually beneficial, but socially inefficient. At the centre of this mechanism lies an externality. Firms that produce private information fail to internalize the adverse effects of private information on market liquidity. Without any private information, participants in the market are symmetrically uninformed and market liquidity is perfect. As results, firms would borrow and hire more. I show that if firms were restricted from producing private information, the allocations of this model is identical to that of the perfect information economy.

The amount of private information acquired is determined by balancing the trade-off between information cost and the individual benefits from private information. Negative shocks shift the trade-off towards information acquisition. It leads to counter-cyclical movements in the level of information asymmetry, which in turn amplify the effects of aggregate shocks. I show that this model can generate larger fluctuations in financial and macroeconomic variables than an otherwise the same model with the level of information asymmetry being fixed.

Chapter 3, entitled **Endogenous Dispersion and Volatility in Asset Returns**, focuses on the information asymmetry between corporate insiders and outside investors and provides a possible explanation for the countercyclical movements in the dispersion and the volatility of stock returns. In the model, borrowers use both internal and external funds to invest. Borrowers have private information about the investment outcomes and the verification of the outcomes is costly for the lenders. External financing is costly due to the information asymmetry between borrowers and lenders. Borrowers have access to a technology that can increase their transparency to outside investors at a cost. When the borrowers' financial conditions are worsened, the costs of external financing rise. Borrowers respond by increasing their transparency to outside investors to mitigate information asymmetry, which helps reduce the external financing cost. As a result, returns on external financing instruments disperse and fluctuate more as more information is disclosed, leading to increases in the cross sectional dispersion and the time series volatility of returns. This model can generate countercyclical dispersion, volatility in returns and external finance premium, with correlation coefficients between pairs of these measures quantitatively in line with the data.

In Chapter 4, entitled **Contractionary Effects of Central Bank Asset Purchases: The Other Side of QE**, I study the potential side effects of central bank asset purchase. The large-scale asset purchases made by the Federal Reserve and other central banks during recent years have greatly changed the composition of liquid assets available to private investors. These purchases have created trillions of dollars in bank reserves that can only be held by commercial banks and have re-moved an equal quantity of Treasury securities and other safe assets from markets. While these actions are designed to promote economic activity by lowering long-term interest rates, the reduction in the supply of safe assets that non-banks can directly hold may have unintended consequences. In this chapter, I build a model to study these side effects of central bank asset purchases. In the model, commercial banks and shadow banks hold liquid assets as part of their operations. Asset purchases by the central bank decreases the supply of liquid assets that shadow banks can directly hold. When commercial banks do not face binding leverage constraints, shadow banks respond by increasing their deposits in or credit lines from commercial banks and central bank asset purchases are neutral. In the presence of a binding leverage constraint, however, asset purchases create distortions that decrease shadow banks' liquidity holdings and their lending. While conventional wisdom says that central bank asset purchases should be expansionary, I show that central bank asset purchases are necessarily contractionary when the level of bank reserves is high.

Chapter 2

Endogenous Lemons Markets and Information Cycles

2.1 Introduction

Financial markets exhibit fragility where "small shocks can have disproportionately large effects[1]." Information asymmetry plays an important role in why financial markets are fragile. Many studies have been done to explain how information asymmetry makes financial markets sensitive to shocks, while taking the information environment as given. However, the incentives to obtain information and thus the information environment may endogenously vary with macro-economic states (Gorton and Ordonez, 2014a,b). In this paper, I study a dynamic economy in which the information on asset quality is asymmetric and the level of the information asymmetry endogenously respond to shocks. The dynamics of information environment creates a fragility mechanism that amplifies aggregate shocks. The model can generate larger fluctuations in financial and macroeconomic variables than an otherwise the same model with the level of information asymmetry being fixed. .

In one relevant line of research on financial friction, macro-economists embed information asymmetry into dynamic macroeconomic models, allowing them to examine how asymmetric information is related to real variables. These papers incorporate different types of information asymmetry into to dynamic general equilibrium models, which includes costly state verification (Bernanke and Gertler, 1989; Carlstrom and Fuerst, 1997; Bernanke et al., 1999; Motto et al., 2010) and adverse selection (Eisfeldt, 2004; Kurlat, 2013; Bigio et al., 2011). Of particular importance, then, is an analysis of the information environments on which these models are based. Gilchrist et al. (2013) studies the effects of uncertainty shock (Bloom, 2009), that is, a second moment productivity shock, in a model using a cost state verification approach (Townsend, 1979). Ordoñez (2012) endogenizes the information environment by introducing the learning process of Veldkamp (2005) into an environment with the same information friction. My paper contributes to this literature by analysing a model which features endogenously determined asymmetric information about asset quality and liquidity driven business cycles.

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[1] Allen and Gale (2004) defines financial fragility as large effects from small shocks.

In the model, firms hold capital of different qualities and produce consumption goods by combing capital and labour. Firms borrow for the labour expenses before production takes place and the individual productivity is subject to idiosyncratic shocks. When a firm's idiosyncratic output is insufficient to repay its debts, the firm has to liquidate its capital to pay off the debts. Initially, firms are uninformed of the qualities of their capital. However, firms can produce private information on their own capital at a cost, which allows them to select low quality capital to sell when liquidation happens. Private information acquisition leads to asymmetric information in the market for capital and creates a lemons problem. The lemons problem then generates an endogenous liquidation cost, i.e. the cost to sell high quality capital at the lemons price. To avoid being forced to liquidate high quality capital in the lemons market, firms borrow and hire less in the first place.

Privation information acquisition is individually beneficial, but socially inefficient. At the centre of this mechanism lies an externality. Firms that produce private information fail to internalize the adverse effects of private information on market liquidity. Without private information acquisition, participants in the market are symmetrically uninformed. When a firm liquidates, it just randomly picks up some units of its capital to sell. Thus, the market price should reflect the average quality of capital held by the firms and thus the market liquidity is perfect. As results, firms would borrow and hire more. I show that if firms were restricted from producing private information, the allocation of this model would be identical to that of the perfect information economy.

The amount of private information produced is determined by balancing the trade off between the information cost and the individual benefits from private information. Negative shocks shift the trade-off towards information acquisition. This leads to counter-cyclical movements in the level of information asymmetry, which in turn amplify the effects of aggregate shocks.

I focuses on three shocks: the standard RBC technology shock, the capital quality shock and the income dispersion shock. A negative RBC technology shock decreases the overall output of firms, which increases their probability of liquidation. The firms then produce more private information on their capital goods so that they can select low quality capital to sell when liquidation happens. However, private information acquisition worsens the lemons problem and asset price drops further. A capital quality shock exogenously changes the capital quality. It can be compared to the collateral quality shock studied in Gorton and Ordonez (2014a,b). When a negative shock hits, the quality of some units of the capital in economy becomes lower. Though the overall decrease in the capital quality is public information, which piece of capital is affected is initially unknown. It gives firms more incentives to distinguish their capital of different qualities so that they can sell the low quality capital they hold. Thus, firms produce more private information on their capital, which exacerbates the lemons problem and lowers the asset price further. An income dispersion shock is an exogenous change in the dispersion of idiosyncratic productivities. This shock determines the degree to which output is dispersed across firms, which is similar to the uncertainty shock studied in Bloom (2009) and Gilchrist et al. (2013). When the income dispersion is higher, the firms face a higher chance of liquidation. The firms respond by producing more private information about the qualities of their capital, which allows them to select low quality capital to sell when liquidation occurs. However, more private information exacerbates the lemons problem in the asset market, which lowers market liquidity. In the simulation exercises, I compare the impulse responses of this model with those of an otherwise the same economy in which the information choices of firms are fixed at the steady state level (i.e. the level of information asymmetry is fixed). My model can generate larger fluctuations in financial and macro-economic variables than the model with fixed level of information asymmetry.

In the model, additional (private) information is only individually beneficial and has no social value, which may seem surprising. Actually, different types of information can affect an economy in very different ways. Some information leads to improvements in productive arrangement and could thus be socially beneficial (Veldkamp, 2005; Ordoñez, 2012). For example, with more information on their farmland, farmers can choose different crops to grow on different pieces of farmlands according to their agricultural conditions.[2] However, some information can be purely distributive and does not lead to any improvement in productive arrangements (Hirshleifer, 1971). For example, additional information on the amounts of gold underneath each unit of land only reassigns values to the land and does not increases the total quantity of gold in the economy. When this type of information is privately produced, the information production is not only socially wasteful, but actually leads to information asymmetry that can distort the economic decisions. This paper focuses the production of the (private) information that has only distributive effects.

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[2] In some set-ups, this type of information is a public good and the excess of social over private value of

Financial markets exhibit pro-cyclical liquidity. The mechanism in this model that gives rise to pro-cyclical liquidity is different from those in the standard models of the lemons market. Here, liquidity is measured by the cost of transferring future revenues of average quality capital into liquid funds[3]. When this cost is higher, liquidity is lower. In Kiyotaki and Moore (2012), the liquidity of assets is exogenously determined by the resell-ability constraint. In Eisfeldt (2004), Kurlat (2013) and Bigio et al. (2011), market liquidity is pro-cyclical since the lemons problem is alleviated in good times. For example, in Eisfeldt (2004) and Kurlat (2013), when investment efficiency increases, agents have more incentives to liquidate old capital and use the proceeds to produce new capital. As a result, more high quality capital is sold, which thereby increases the asset price and liquidity. In these models, the degree of information asymmetry is given and liquidity is determined by the amount of trade due to the reasons other than private information (Eisfeldt, 2004). In my model, the level of information asymmetry endogenously respond to shocks and moves counter-cyclically, which leads to pro-cyclical market liquidity. I show that my model can generate larger fluctuations than an otherwise the same economy with fixed information environment.

In the standard models of the market for lemons, asset price and market liquidity increase when the size of liquidation becomes larger, since more high quality assets are sold. However, a negative aggregate shock can also result in large-scale liquidation. Uhlig (2010) summarized the 2008 crisis as following, “The affected banks sell their questionable assets at market prices to meet withdraw... the current market prices are below fundamental values.” In the standard models of the market for lemons, such large-scale liquidation should

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it actually leads to underinvestment, rather than over-investment, in the production of information (Fajgelbaum et al., 2014).

[3] Eisfeldt (2004) proposed a similar measure for liquidity.

increase the average quality of assets sold in the market and thus asset price, since more high quality assets are sold. In my model, the severity of lemons problem is endogenously determined. An adverse aggregate shock leads to large-scale liquidation, which tends to improve market liquidity (as in standard models); however, it triggers information acquisition as well, which exacerbates the asymmetric information problem and decreases liquidity.

Private information about capital enables firms to distinguish among capital of different qualities, which leads to a dispersion of privately-observed capital quality. The dispersion of privately-observed capital quality measures the extent to which firms can tell apart their capital of different qualities and the degree of information asymmetry. Recently, Bigio et al. (2011) studies a dynamic macro-economic model with the lemons problem. He shows that increases in the capital quality dispersion is a meaningful source of business cycles through liquidity mechanism.[4] As in Bigio et al. (2011), the dispersion of privately-observed capital quality in my model also measures the degree of information asymmetry. However, in my model, the dispersion of privately-observed capital quality is determined by agents' information choice, while in Bigio et al. (2011), it is exogenously given.

Literature.

This paper is related to the recent literature that study the endogenous nature of information asymmetry. Gorton and Ordonez (2014a,b) argue that adverse shocks trigger information acquisition, which leads to less aggregate borrowing. In their model, assets are used as collateral that relaxes borrowing constraints. Individual productivity is diminishing in scale and thus the shadow value of relaxing borrowing constraint is diminishing with the value of collateral. A small negative shock that triggers information acquisition increases the dispersion of collateral value among agents, which lowers the aggregate productivity. Dang et al. (2009, 2012) show how private information acquisition can impede trading. Their papers shed light on security design by showing that debt is the optimal financial contract that minimizes the information sensitivity of assets returns. Another reason why information can be endogenous is that the cost or the limited capability of information processing leads to "rational inattention" (Sims, 2003, 2005). This theory suggests that even freely available information might be unused due to the individual's limited processing capability. Recently, Beltran et al. (2013) empirically show that the collapse of ABS, CDOs, MBS markets in the crisis was due to the sudden rise in the information asymmetry between sellers and buyers in these markets. In this paper, I embed the endogenous information mechanism in to a macro-dynamic model and studies how it amplifies and propagates shocks.

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[4] In Bigio et al. (2011), capital quality is private information of the capital owner. The information asymmetry becomes more severe when the capital quality dispersion increases. He shows that the exogenous fluctuation of capital quality dispersion is a meaningful source of business cycles.

This paper builds on the literature studying liquidity and the lemons problem in macroeconomic settings. Eisfeldt (2004) presents a model in which agents sell the ongoing projects to finance new projects in a market with information asymmetry. Kiyotaki and Moore (2012) and Del Negro et al. (2010) study a business cycle model with random arrival of investment opportunity and borrowing constraint. In their models, market liquidity is exogenously given. Kurlat (2013) studies a general equilibrium model in which entrepreneurs, with heterogeneous investment efficiency, trade old capital in a market with lemons problem. Bigio et al. (2011) studies a macro economic model in which agents liquidate capital to get liquid funds that can relax financial constraints on investment and employment. In all these models, the information environment is exogenous and the level of information asymmetry is given. However, in my model, the information environment is endogenously determined and varies with the aggregate states.

The rest parts of the paper is organized as follows. Section 2 describes the set up of the model. The model is solved in two steps. Section 3 presents the solution of the intra-temporal optimization problem and describes the key mechanism of this model. Section 4 presents the solution of the inter-temporal optimization problem. Section 5 presents the externalities in this endogenous information mechanism: firms that produce private information fail to internalize the negative effects of private information on asset price. I show that if firms are not allowed to produce information, the allocation of the model would be identical to that of the perfect information economy. Section 6 describes the amplification mechanism. Section 7 presents the simulation results. Section 8 concludes.

2.2 The Model

Time is discrete and infinite, and $t = 0, 1, 2, ...$ There are two goods: consumption goods, which is the numeraire, and capital goods.

Demography. The economy is populated by workers, households and firms. All populations are normalized to one. The economy is modelled in a way such that the distribution of agents have no effect on the aggregate outcomes. Workers only supply labour and consume. They cannot save and are subject to a static budget constraint: $C_t^w = w_t L_t$. Workers supply labour according to the elastic supply schedule $w_t = L_t^{\frac{1}{\nu}}$, where ν is the Fischer elasticity.

Households own the firms and receive dividends from firms in each period. Their period utility function is logarithm, $U(C_t) = log(C_t)$, and their discount factor is β. Households do not save either and consume whatever they receive in that period: $C_t = D_t$, where D_t is the aggregate dividends received from firms. The only role of households is to provide the stochastic discount factor, that is, $\beta U^{'}(C_{t+1})/U(C_t)$.

Firms hold capital goods and produce consumption goods by combing labour and capital. They make dividend and investment decisions. Let d_t denote dividends delivered by an individual firm in period t. A firm maximizes the expected value of its life time dividends, which is given by:

$$E_t \sum_{i=0}^{+\infty} \beta^i \frac{U^{'}(C_{t+i})}{U(C_t)} d_{t+i}$$

The optimization of a firm is done in two steps. Within a period, the firm starts with capital stock k and maximizes its wealth at the end of the period, denoted by W, which is the sum of the firm's profits and the value of its capital. Then, the firm allocates its wealth W between dividends d_t and capital stock tomorrow k_{t+1}.

Technology. Firms produce consumption goods using capital and labour, and are subject to idiosyncratic productivity shocks v. The production technology of firms is Cobb-Douglas, which is given by $y_t = v_t^j a_t f(k_t, l_t)$, where k_t and l_t are the amounts of capital and labour inputs, a_t denotes the aggregate technology shock and v_t^j is the idiosyncratic productivity of firm j. The idiosyncratic shock v is i.i.d. across firms and over time, which follows a log normal distribution and has a mean of one. Let $F(.)$ and $f(.)$ denote the c.d.f. and p.d.f. of v respectively. The standard deviation of the idiosyncratic productivity v, i.e. σ_t, which is an exogenous aggregate state, governs the income dispersion across firms.

Firms produce capital goods using consumption goods. In each period, firms are hit by idiosyncratic investment efficiency shocks θ. The firm j that is assigned with an idiosyncratic investment efficiency θ_t^j can transform a unit consumption goods into θ_t^j units of capital. With probability τ, $\theta_t^j = 1$, and with probability $1-\tau$, $\theta_t^j = \bar{\theta}$, where $\bar{\theta} > 1$.

Heterogeneous Capital and Information. Capital is infinitely divisible. A unit of capital consists of a continuum of pieces. After production, a fraction ρ of a firm's capital becomes low-quality capital, i.e. "lemons". The other units of the firm's capital remain the same after production and are thus called "non-lemons". A unit of lemon capital is not totally valueless. In a unit of lemon capital, some of the pieces are worn out during production. Specifically, I assume that a fraction $1-\lambda_t$, with $\lambda_t \in (0,1)$, of the pieces in a unit of lemon capital are worn-out after production ends. Thus, $1-\lambda_t$ denotes the depreciation rate of lemon capital, where λ_t is an exogenous aggregate state. The worn-out pieces in the lemon capital do not disappear immediately. They only disappear at the end of the period. After the worn-out pieces disappear, a unit of lemon capital shrinks to λ_t units and thus capital quality is automatically revealed.

Initially, no one, even the firm who own these capital, can distinguish lemon capital from non-lemon capital until the end of the period when the worn-out pieces automatically disappear.[5] To obtain information on capital quality, a firm can install "wear sensor" on its capital. For a unit of capital with wear sensor, the sensor emits an informative message s about the quality of the unit of capital it was attached to, when production ends. If the unit of capital becomes lemon, $s = s_L$; otherwise, $s = s_H$. For a unit of capital without wear sensor, the owner always receives the same message $s = s_M$ when production ends, no matter it becomes lemon or not. All these signals s are privately observable to the capital owners.[6]

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[5] How much private information asset owners have can be determined by a variety of economic factors. In Akerlof (1970), the car owner can obtain information about the car's quality simply by owning the car for a length of time. It implies that the owner of the car can produce private information at no cost. However, in reality, capital can have very complex structure such that even the owner cannot easily investigate its quality. One can also think that capital in this model refers those complicate financial instruments. The complex structure of the underlying assets of these securities makes it difficult for the holder to figure out their true values.

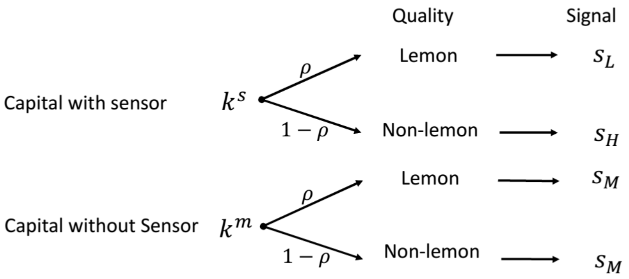

Figure 2.1: Private Information Production

Let k_t^s be the units of capital with wear sensor and k_t^m be the units of capital without sensor. To install wear sensor on k_t^s units of capital, μk_t^s, with $\mu \in (0,1)$, units of capital will be spent.[7] When the production ends, a fraction ρ of firms' capital become lemons and firms observe the signals of wear sensors. As results, k_t^s is separated into to two groups: the units that become lemons, denoted by k_t^l, and those units that remain the same, i.e. non-lemons, denoted by k_t^n. In addition, $k_t^l = \rho k_t^s$ and $k_t^n = (1-\rho)k_t^s$ and thus k_t^s is called "separable capital". For a unit of k_t^m, the owner cannot tell whether or not it becomes lemon during production. Thus, k_t^m is called "mixed capital". After production, mixed capital k_t^m is a mixture of lemons and non-lemons. In sum, a firm holds three types of capital after production: lemons k_t^l, non-lemons k_t^u, and mixed capital k_t^m.

Timing. There are three stages within a period. In the first stage, firms produce consumption goods using capital and labour, which is called *production stage.* In the second stage, firms transform consumption goods into capital, which is called *investment stage.* In the third stage, firms deliver dividends and households consume, which is called *consumption*

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[6] In a line of corporate finance literature, economists study security design and optimal capital structure in an environment where there are heterogeneous assets being subject to certain information friction to different extents. For example, Boyd and Smith (1998, 1999) study the optimal capital structure in an economy where there are two types of projects; one being subject to and one not being subject to a costly state verification problem. In this paper, I study this type of heterogeneity of assets in a macro-economic model and explore its implication on the information environment.

[7] Here, the parameter of information cost measures the level of opacity. In financial market, opacity can facilitate trading by reducing information asymmetry. Dang et al. (2009, 2012); Gorton and Ordonez (2014a,b) have studied on how the opacity of assets can improve liquidity via reducing information acquisition. At security level, opacity refers to the extent to which the structure of the underlying assets of a security makes it difficult for people to figure out its true value. As will be shown later, when the information cost is high enough, firms do not produce private information and the externalities problem is eliminated. In this case, the prices and allocations of the model replicate those of a perfect information economy.

stage.

At the beginning of the production stage, the aggregate shocks hit the economy. Then, firms choose labour inputs l. Let w be the market wage rate. The contract signed between workers and firms are incomplete. The firm promises to pay the workers a fixed amount wl in the second stage in any situation. Then, firms make information decisions; they choose how many sensors to install on their capital. After that, production takes place and the idiosyncratic productivity shocks v hit the economy. At the end of the stage, firms receive signals from the wear sensors.

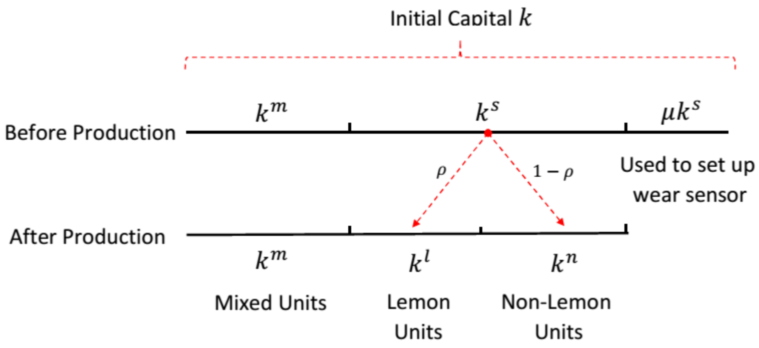

Figure 2.2: The Composition of Capital before and after Production

At the beginning of the investment stage, firms are hit by idiosyncratic investment efficiency shocks θ. Then, firms trade old capital in an anonymous and non exclusive market, and produce new capital goods. In the second stage, selling old capital is the only financial transaction allowed. For the firms that are hit by low idiosyncratic productivity shock and do not have enough output to repay the loan, i.e.$y < wl$, they have to sell capital in order to pay off their debts. At the end of the investment stage, the wear-out parts in the lemon capital disappear and capital quality is automatically revealed. However, the wage bill wl should be paid and the production of new capital should be done before that.

In the consumption stage, firms deliver dividends and households consume. Since firms are hit by different idiosyncratic shocks in the fist two stages, firms may hold different amounts of capital goods and consumption goods at the end of the second stage. To avoid tracking the distribution of the firms, I make a simplifying assumption: firms can trade capital goods again in the third stage after capital quality is automatically revealed (with perfect information). Note that the production of consumption goods and new capital is already done in the first two stages. Thus, in the third stage, firms only reallocate capital and consumption goods, while the total amounts of capital and consumption goods in the economy are fixed. At the end of the stage 3, all the consumption goods will be delivered to households as dividends and all the capital will be held to the next period. However, when individual firm choose dividends d_t and capital stock it holds for the next period k_{t+1}, the relative price of k_{t+1} in terms of consumption goods, denoted by q_t, is determined.

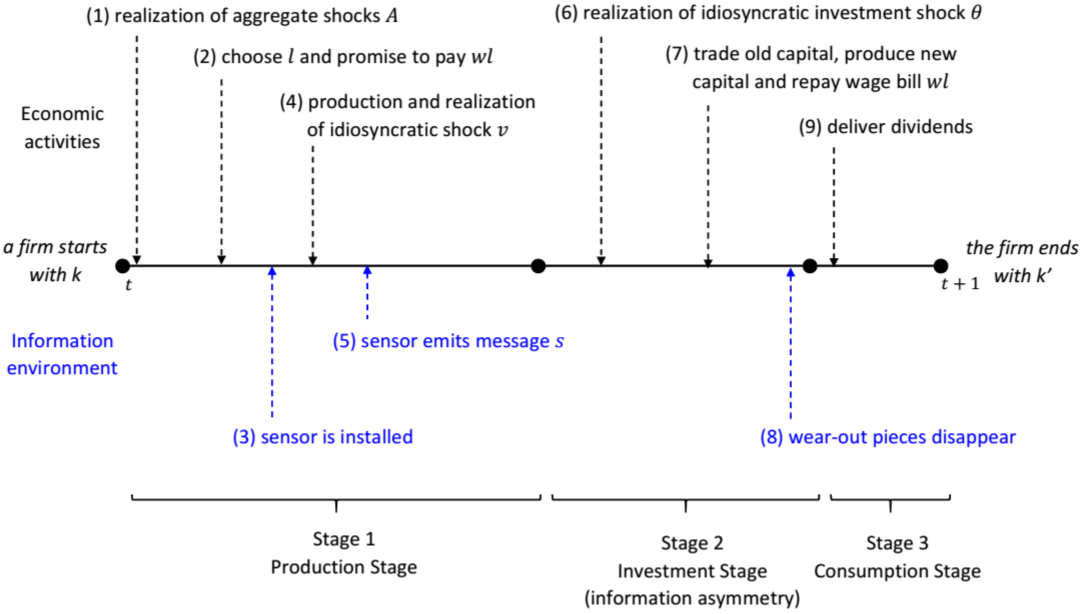
(1) realization of aggregate shocks A
(2) choose l and promise to pay wl
(4) production and realization of idiosyncratic shock v
(6) realization of idiosyncratic investment shock θ
(7) trade old capital, produce new capital and repay wage bill wl
(9) deliver dividends
Economic activities
a firm starts with k
t
$t+1$
the firm ends with k'
Information environment
(5) sensor emits message s
(3) sensor is installed
(8) wear-out pieces disappear
Stage 1
Production Stage
Stage 2
Investment Stage
(information asymmetry)
Stage 3
Consumption Stage

Figure 2.3: Timing

Shocks. There are three aggregate shocks: income dispersion shock σ_t, capital quality shock λ_t and the standard RBC technology shock a_t. Income dispersion shock σ_t governs the volatility of the idiosyncratic productivity v. It has no effect on the aggregate output, but it determines the extent to which output is dispersed across firms. Capital quality shock λ_t governs the depreciation rate of lemon capital; in a unit of lemon capital, a fraction $1-\lambda_t$ of the pieces is worn-out during production. When a negative shock hits, the quality of lemon capital becomes even lower. Standard technology shock a_t governs the overall capability of producing consumption goods.

Let A_t be the vector that summarizes all the shocks, i.e. $A_t = (\sigma_t, \lambda_t, a_t)$. A Markov process for A_t evolves according to the transitory probability Π. The only endogenous aggregate states in this model is K_t. Let Z_t be the vector that summarises all the aggregate states, then $Z_t = (K_t, a_t, \sigma_t, \lambda_t)$.

2.3 Information Acquisition and Lemons Market

As mentioned earlier, firms maximize the expected value the lifetime dividend in two steps. In the first step, firms maximize their wealth at the end of the investment stage, that is, W (intra-temporal optimization). In the second step, firms allocate their wealth W between dividends d_t and capital for the next period k_{t+1} (inter-temporal optimization). In this section, I focus on the intra-temporal optimization of firms, which is about how firms' decisions on information production, employment and investment are made, while taking the real price of capital as given. In the next section, I will turn to the inter-temporal optimization of firms and endogenize capital price.

The economy is modelled in a way such that the distribution of firms does not have any effects on the aggregate outcomes. Thus, I temporarily drop the subscripts for convenience. I solve this partial equilibrium problem in two steps. First, I solve the optimization problem in the investment stage. Then, I turn back to the optimization problem in the production stage.

2.3.1 Optimization in the Investment Stage

In the investment stage, firms trade old capital, repay wage bills and produce new capital. A firm maximizes its value at the end of the investment stage, which equals the sum of cash and the value of capital it holds.

In the production stage, firms produce consumption goods by using capital and labour. When the production stage ends, firms receive messages from the wear sensor and then their capital are separated into three groups. At the beginning of the investment stage, firms start with lemon k^l, non-lemon k^n, mixed capital k^m and consumption goods y. The individual states of a firm in the investment stage is summarized by the vector (k, Z, x), where $Z = (K, a, \lambda, \sigma)$ is the vector of the aggregate states and $x = (v, \theta)$ is the vector of the idiosyncratic shocks that hit the firm.

A firm chooses the amounts of capital it sells and buys. Let k_s^l, k_s^n, k_s^m denote the amounts of lemon, non-lemon and mixed capital the firm sells and k_b denote the amount of old capital it buys from market. In addition, the firm chooses the amount of consumption goods i to be used in the production of new capital, and cash profits it holds when the second stage ends, i.e. m.

Let k^e be the amount of capital the firm holds when the investment stage ends (i.e. after the wear-out pieces disappear). At the end of the investment stage, the wear-out pieces in lemon capital disappear and a unit of lemon capital shrinks to λ units. For the mixed capital k^m, a fraction ρ of it are lemons, and a unit of k^m thus shrinks to $\bar{\lambda}$ units on average at the end of the investment stage, where $\bar{\lambda} \equiv \rho\lambda + (1-\rho)$. Let ρ_m denote the ratio of lemon capital to the total capital transacted in the market. Thus, a unit of old capital bought from market k_b shrinks to λ_m units on average as the second stage ends, where $\lambda_m \equiv \rho_m\lambda + (1-\rho_m)$ units.

Let q be the exogenous price of k^e, which is determined in the third stage. Let p be the market price of old capital. Within a period, the firm maximizes its total wealth W, which is the sum of cash profits m and the value of capital it holds at the end of the second stage, i.e. $m + qk^e$.

Problem 1 *Given p and q, the firm maximizes its value by choosing $\left\{k_s^l, k_s^n, k_s^m, k_b, i, m\right\}$:*

$$W(k, Z, x) = \max_{\left\{k_s^l, k_s^n, k_s^m, k_b, i, m\right\}} m + qk^e$$

subject to:

$$k^e = i\theta + \lambda(k^l - k_s^l) + (k^n - k_s^n) + \bar{\lambda}\left(k^m - k_s^m\right) + \lambda_m k_b$$

$$i + m + pk_b + wl = p(k_s^l + k_s^n + k_s^m) + y$$

$$i \geq 0, \quad m \geq 0, \quad k^l \geq k_s^l \geq 0, \quad k^n \geq k_s^n \geq 0, \quad k^m \geq k_s^m \geq 0, \quad k_b \geq 0$$

Here, k^e denotes the amount of capital the firm holds at the end of the investment stage. It is the sum of three parts: (i) the new capital produced, $i\theta$, (ii) the effective units in the old capital that the firm keeps, $\lambda(k^l - k_s^l) + (k^n - k_s^n) + \bar{\lambda}(k^m - k_s^m)$, (note that there are λ, 1 and $\bar{\lambda}$ units of effective capital in each unit of lemon, non-lemon and mixed capital), and (iii) the effective units in the old capital that the firm buys from market, $\lambda_m k_b$. The second constraint is the firm's budget constraint. The available goods of the firm equals its total output y plus the revenue it receives from selling old capital. These are used for producing new capital i, cash profits m, purchasing of old capital pk_b and repaying wage bill wl. The last constraint states that investment i, dividend d, the sales and purchases of old capital should be non negative, and the sales of old capital is limited by how much old capital the firm owns.

Solution

After the wage bill wl is paid, for a unit of consumption goods, the firm can keeps it as cash profits m or use it to produce or buy capital. If the firm chooses to keep the unit of consumption goods as cash profits m, then the value of a unit of consumption goods is one. If the firm chooses to spend the consumption goods to produce capital, the value of the unit of consumption goods is $q\theta$. If the firm use it to purchase old capital from the market, its value is then $(q/p)\lambda_m$. Let $V_m(\theta)$ denote the value of a unit of consumption goods. It is thus given by,

$$V_m(\theta) = \max\left\{1, q\theta, q\left(\frac{1}{p}\right)\lambda_m\right\}$$

After the wage bill wl is paid, for a unit of old capital, the firm can either liquidate or keep it. If the firm keeps the old capital, the values of a unit of lemon, non-lemon and mixed capital are $q\lambda$, q and $q\bar{\lambda}$ respectively. If the firm sells the unit of old capital, its value is $pV_m(\theta)$. Let $V_k^l(\theta)$, $V_k^n(\theta)$ and $V_k^m(\theta)$ denote the value of a unit of lemon, non-lemon and mixed capital. Thus, they are given by:

$$V_k^l(\theta) = max\left\{q\lambda, pV_m(\theta)\right\}$$

$$V_k^n(\theta) = max\left\{q, pV_m(\theta)\right\}$$

$$V_k^m(\theta) = max\left\{q\bar{\lambda}, pV_m(\theta)\right\}$$

In the market for old capital, the sellers are the firms that are forced to liquidate capital by negative idiosyncratic shock v and those firms that have high investment efficiency $\theta = \bar{\theta}$. The firms with low investment efficiency are the buyers. How old capital is priced depends on the sizes of the sellers of the buyers. When the size of the firms with high investment efficiency is sufficiently large, the price of capital is determined merely by the amount of cash available in the market, that is, "cash-in-the-market" pricing. When this happens, market price does not reflect the average quality of capital sold in the market. The buyers cannot bid up the price since they do not have enough cash. To focus on the lemons problem, I focus on the equilibria where "cash-in-the-market" pricing never happens and the price of old capital always reflects the average quality of capital sold in the market. Thus, I assume that τ is large enough such that the firms with low investment efficiency $\theta = 1$ are always the marginal producers of new capital.

Assumption 1 *Assume that firms with* $\theta = 1$ *are always the marginal producers of new capital in equilibrium.*

Under Assumption 1, the real price of capital q has to be equal to one in equilibrium. If $q > 1$, all the consumption goods will be used to produce new capital and the consumption of households drop to zero. Then, the marginal value of consumption rises and the real price of capital q has to drop.[8] If $q < 1$, firms with low investment efficiency $\theta = 1$ won't produce any new capital then. When $q = 1$, firms with $\theta = 1$ is indifferent between producing new capital and keeping cash profits that will be used to pay dividends in the third stage.

Lemma 1 *Under Assumption 1,* $q = 1$ *always holds in equilibrium.*

In addition, under assumption 1, firms with low investment efficiency $\theta = 1$ should be indifferent between producing capital and buying capital goods from the market. It follows that $p = \lambda_m$. Note that the average quality of capital sold in the market λ_m cannot be lower than λ or higher than $\bar{\lambda}$. Under Assumption 1, it follows that $p \in [\lambda, \bar{\lambda}]$.

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[8] When the consumption of households is zero, the marginal utility of consumption becomes infinitely large and the value of current consumption increases to infinity. Then, the relative price of capital in terms consumption goods q will drop.

Lemma 2 *Under Assumption 1, $p = \lambda_m$ holds and the average quality of capital sold in the market, λ_m, should be between λ and $\bar{\lambda}$.*

Lemma 2 implies that firms with $\theta = 1$ always sell out lemons and keep mixed and non-lemon units if they could. They only sell mixed or non-lemon capital when they need cash to repay wage bill. In such cases, they will liquidate the mixed capital first and then the non-lemon capital.

Firms with $\theta = \bar{\theta}$ would sell mixed and non-lemons capital when $\bar{\theta}$ is high enough. I assume that $\bar{\theta}\lambda > 1$ and thus firms with $\theta = \bar{\theta}$ always sell all its old capital and use the proceeds to produce new capital.[9]

Assumption 2 $\bar{\theta}\lambda > 1$

By combing the budget and non-negative constraints, the solution to problem 2 is:

Proposition 1 *Given p and q, the optimal decisions in the investment stage are given by*

	high efficiency firm $\theta = \bar{\theta}$	low efficiency firm $\theta = 1$
k_s^l	k^l	k^l
k_s^n	k^n	$max\{0, min\{k^n, (wl - y - pk^l - pk^m)/p\}\}$
k_s^m	k^m	$max\{0, min\{k^m, (wl - y - pk^l)/p\}\}$
k_b	0	non-negative
m	0	non-negative
i	$max\{0, p(k^l + k^n + k^m) + y - wl\}$	non-negative

Table 2.1: The Optimal Decision Rules in the Investment Stage

Ex-ante value of Cash and Capital. Let $\hat{V}_k^l$, $\hat{V}_k^n$, $\hat{V}_k^m$ and $\hat{V}_m$ be the ex-ante value of a unit of lemon, non-lemon, mixed capital and consumption goods before the realization of idiosyncratic investment efficiency shock θ. Note that under assumption 1, $q = 1$ and

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[9] Note that p cannot be less than λ. Thus, $p\bar{\theta} > \lambda\bar{\theta} > 1$, and it is always optimal for firms with $\theta = \bar{\theta}$ to liquidate all its capital and use the proceeds to produce new capital.

$p = \lambda_m$ (Lemma 2). Thus, by combing the optimal decision rules, one can obtain

$$\hat{V}_k^l = \tau\lambda_m + (1-\tau)\lambda_m\bar{\theta}$$

$$\hat{V}_k^n = \tau + (1-\tau)\lambda_m\bar{\theta}$$

$$\hat{V}_k^m = \tau\bar{\lambda} + (1-\tau)\lambda_m\bar{\theta}$$

$$\hat{V}_m = \tau + (1-\tau)\bar{\theta}$$

Individual Value of Private Information. Let $\hat{V}_k^s$ denote the ex-ante value of a unit of separable capital k^s. It follows that $\hat{V}_k^s = \rho\hat{V}_k^l + (1-\rho)\hat{V}_k^m$, and, obviously, $\hat{V}_k^s - \hat{V}_k^m > 0$. The difference between $\hat{V}_k^s$ and $\hat{V}_k^m$ represents the individual value of private information. It is easy to see where the discrepancy comes from: for the firms with low investment efficiency $\theta = 1$, they always keep all the mixed capital k^m; however, for a unit of separable capital k^s, they only keep it when it is non-lemon.

2.3.2 Optimization in the Production Stage

In the production stage, firms produce consumption goods using capital and labour. A firm makes information decision and chooses labour inputs to maximizes the expected value of the firm. I solve the firm's optimization problem in two steps. First, I solve for the optimal choice of labour input, given the information choice. Second, I solve for the optimal information choice.

Before production, a firm makes information decision by allocating its capital between mixed capital k^m and separable capital k^s. For each unit of separable capital k^s, the firm spent μ units of capital to install the wear sensor. Define γ to be the share of k^s in the capital used in production. It is given by

$$\gamma \equiv \frac{k^s}{k^s + k^m}$$

In what follows, γ is to be used as a measurement for the level of information asymmetry. Define ϕ to be the share of capital the firm spent to install the wear sensor: $\phi = \mu k^s/k$. The above definition equation implies that ϕ can be written as a function of γ: $\phi = \phi(\gamma) = \frac{\gamma}{\mu^{-1}+\gamma}$.

Given the information choice γ and initial capital k, the amounts of mixed and separable capital are given by:

$$k^s = \gamma(1-\phi)k$$
$$k^m = (1-\gamma)(1-\phi)k$$

The firm chooses labour inputs before the realization of idiosyncratic shock v and promises to pay wl after production ends. when the idiosyncratic output of the firm is not enough to repay the wage bill wl, it has to liquidate its capital. The firm will always liquidate lemons k^l first; however, when the liquidation exceeds certain threshold, the firm is forced to liquidate other capital goods (e.g. mixed capital and non-lemon capital), whose value can be higher than the market price. Let $\bar{v}$ be the threshold of v, such that the firm has to start liquidate mixed capital if $v < \bar{v}$. The critical level of v is defined by

$$\bar{v}a\left[(k^s + k^m)^\alpha l^{1-\alpha}\right] + pk^l = wl$$

Let n denote the labour-to-capital ratio, $n = l/(k^s + k^m)$. The above equation can be written as:

$$\bar{v}an^{1-\alpha} + p\gamma\rho = wn$$

Without any friction, the first best labour-to-capital ratio n is chosen such that the marginal product of labour is equal to the wage rate. Let n^*_{best} denote the first best choice of n. Thus, it is never optimal to choose a n that is larger than n^*_{best}.

Given the choice of n, a higher γ lowers the liquidation threshold $\bar{v}$. When γ is large enough, the amount of funds the firm can get by selling lemons is enough to repay the wage bill wn for any realization of v. Then, the liquidation threshold $\bar{v}$ becomes irrelevant. Define γ_{max} to be the threshold for γ and when $\gamma > \gamma_{max}$ the amount of funds the firm can get by selling lemons is enough to repay the firm's wage bill when it chooses $n = n^*_{best}$. To ensure the existence of interior solution, I assume that $\gamma_{max} < 1$.

Assumption 3 *Assume that* γ_{max}*, which is defined as* $\gamma_{max} \equiv (wn^*_{best})/(\rho p)$*, is always less* *than 1 in equilibrium.*

Assumption 3 implies that firms never liquidates non-lemon capital k^n. By liquidating its lemon k^l and mixed capital k^m, the firm can always get enough funds to repay the wage bill even when it chooses n^*_{best}. In addition, under assumption 3, the firm always have enough funds to repay the wage bill when it chooses a n with $n \leq n^*_{best}$. Thus, the firm never defaults on the wage bill.

Lemma 3 *Under Assumption 3, the firms can always fully pay its wage bill without liquidating its non-lemon capital goods.*

In the production stage, the firm chooses labour inputs l, and how much sensor to install k^s, to maximize its expected wealth at the end of the investment stage:

$$\begin{aligned}\hat{W}(k,Z) = \max_{\{l,k^s\}} \int_{\bar{v}}^{+\infty} & \left\{ \left[va(k-\mu k_s)^\alpha l^{1-\alpha} + pk^l - wl \right] \hat{V}_m + k^m \hat{V}_k^m + k^n \hat{V}_k^n \right\} dF(v) \\ & + \int_0^{\bar{v}} \left\{ \left[k_m - \frac{wl - va(k-\mu k_s)^\alpha l^{1-\alpha} - pk^l}{p} \right] \hat{V}_k^m + k^n \hat{V}_k^n \right\} dF(v)\end{aligned} \tag{2.1}$$

The first term is the expected value of the firm, when $v \geq \bar{v}$. In this case, the firm only sells the lemon capital. The second term is the firm's expected value when $v < \bar{v}$, and the firm is forced to liquidate mixed capital.

Note that $\hat{W}(k,Z)$ is linear in k. Firms actually maximize the expected return on k. Let $\hat{r}(Z)$ denote the maximized return on capital k and thus $\hat{W}(k,Z) = \hat{r}(Z)k$.

Note that a fraction $\phi(\gamma)$ of initial capital stock k is used to install the wear sensor. Let $\pi(\gamma|Z)$ be the maximized return on the capital that remains after the installation of sensor, i.e. $[1-\phi(\gamma)]k$. Thus, $\hat{r}(Z) = [1-\phi(\gamma)]\pi(\gamma|Z)$. The optimization problem of the firm in the production stage can be solved in two steps. In the first step, given the information choice γ, I solve for $\pi(\gamma|Z)$. In the second step, I solve for $\hat{r}(Z)$.

In the first step, given the information choice γ, the firm chooses optimal labour-to-capital ratio n. The optimization problem can be written as:

Problem 2 *Given the asset price p, the wage rate w and the information choice γ, the firm solves*

$$\pi(\gamma|Z) = \max_{n} \underbrace{\left(an^{1-\alpha} - wn\right)\hat{V}_m}_{\text{I. operating profits}} + \underbrace{\left[(1-\gamma)\hat{V}_k^m + \gamma\hat{V}_k^s\right]}_{\text{II. value of capital}} - \underbrace{\left(\hat{V}_k^m - p\hat{V}_m\right)\frac{an^{1-\alpha}}{p}\int_0^{\bar{v}}(\bar{v}-v)\,dF(v)}_{\text{III. liquidation costs}} \tag{2.2}$$

and

$$\bar{v}an^{1-\alpha} + p\gamma\rho = wn$$

In the firm's objective function, the first term represents the expected profits from production, the second term is the expected value of capital and the third term is the expected liquidation costs, i.e. the costs incurred when the firm is forced to liquidate mixed capital in the lemons market.

The first order condition is given by

$$\underbrace{\left[a(1-\alpha)n^{-\alpha} - w\right]}_{\text{wedge between MPL and } w} \hat{V}_m = \underbrace{\left(\frac{\hat{V}_k^m}{p} - \hat{V}_m\right)\left[wF(\bar{v}) - a(1-\alpha)n^{-\alpha}\int_0^{\bar{v}} v dF(v)\right]}_{\text{marginal} \uparrow \text{ in liquidation costs}} \tag{2.3}$$

In the above equation, the LHS is the marginal benefit of hiring one more unit of labour, and the RHS shows the marginal increase in the liquidation costs. Note that when $\hat{V}_k^m > p\hat{V}_m$, the LHS is positive and the marginal product of labour is higher than the wage rate. The liquidation cost creates a wedge between wage rate and the marginal product of labour.

In the second step, I solve for the optimal choice of information, γ. The optimization problem is given by

Problem 3 *Given the asset price p and the wage rate w, the firm solves*

$$\hat{r}(Z) = \max_{\gamma}\left[1 - \phi(\gamma)\right]\pi(\gamma|Z)$$

The first order condition is given by:

$$\frac{\pi'(\gamma|Z)}{\pi(\gamma|Z)} = \frac{\phi'(\gamma)}{1-\phi(\gamma)} \tag{2.4}$$

The above condition says that when the optimal γ is chosen, the marginal increase in the information cost (in percentage) should be equal to the marginal increase in the profits (in percentage).

Marginal Benefit of Private Information. The marginal benefit of private information is measured by the marginal increase in π when γ is increased. Using envelop theory, it is given by

$$\pi'(\gamma|Z) = \underbrace{\left(\hat{V}_k^s - \hat{V}_k^m\right)}_{\text{I. } \uparrow \text{ in value of capital}} + \underbrace{\rho\left(\hat{V}_k^m - p\hat{V}_m\right)F(\bar{v})}_{\text{II. } \downarrow \text{ in liquidation cost}} \tag{2.5}$$

Given the asset price p and the firms' choices of n and γ, additional information can benefit the firm in two ways. First, the firm can take advantage of private information when it trades old capital in the asset market, which increases the capital value. Second, the additional private information helps the firm avoid selling mixed capital when liquidation happens, which decreases the liquidation threshold $\bar{v}$ and thus the liquidation costs. However, the marginal benefit of private information is decreasing with γ.

Lemma 4 *Given the asset price p and the wage rate w, for γ_1 and γ_2, with $\gamma_1 < \gamma_2$ and $\gamma_1, \gamma_2 \in [0, \gamma_{max}]$, $\pi'(\gamma_1)/\pi(\gamma_1) > \pi'(\gamma_2)/\pi(\gamma_2)$.*

2.3.3 Asset Market Equilibrium

In this partial equilibrium model, the only market is the market for old capitals. Under Assumption 1, cash-in-market pricing never happens and the price p always reflects the average quality of capital sold in the market, i.e. $p = \lambda_m$. Given the asset price p, firms choose n and γ. Note that all the firms are ex-ante the same, so they choose the same n and γ. The average quality of old capital in the market, λ_m, is then determined by the firms' choices of n and γ. In equilibrium, the asset price should be consistent with the average quality of capital sold in the market, i.e. $p = \lambda_m$.

In the investment stage, firms with low investment efficiency $\theta = 1$ sell all the lemons and keep mixed and non-lemon capital after the wage bills are paid. Note that the idiosyncratic shocks are i.i.d. across firms and are uncorrelated with the firms' initial capital k. The total amount of lemon capital sold by firms with low investment efficiency $\theta = 1$ is $\tau\rho\gamma(1-\phi)K$ and the amount of mixed capital k^m sold by these firms is $p^{-1}an^{1-\alpha}\int_0^{\bar{v}}(\bar{v}-v)dF(v)(1-\phi)K$. Under the assumption 2, firms with high investment efficiency $\theta = \bar{\theta}$ always sell all their capital. Thus, the total amount of capital sold by these firms is $(1-\tau)K$, and the average quality of the capital sold is $\bar{\lambda}$.

In sum, given the firms' choice of n and γ, the total amount of effective units, i.e. the units of capital that will remain after the investment stage ends, sold in the market is:

$$\left\{\tau\rho\gamma\lambda + \tau\left[\frac{an^{1-\alpha}}{p}\int_0^{\bar{v}}(\bar{v}-v)dF(v)\right]\bar{\lambda} + (1-\tau)\bar{\lambda}\right\}(1-\phi)K$$

In the above expression, the sum of the first two terms is the total amount of effective units sold by the firms with low investment efficiency $\theta = 1$: the first term shows the amount of effective units that are in the lemon capital sold by these firms and the second term shows the amount of effective units that are in the mixed capital sold by these firms. The third term shows the amount of effective units sold by the firms with high investment efficiency θ.

In addition, the total amount of old capital sold in the market is:

$$\left\{\tau\rho\gamma + \tau\left[\frac{an^{1-\alpha}}{p}\int_0^{\bar{v}}(\bar{v}-v)dF(v)\right] + (1-\tau)\right\}(1-\phi)K$$

In the above expressions, the first term shows the amounts of lemon capital sold by the firms with $\theta = 1$, and the second term shows the amount of mixed capital sold by them. The third term shows the amount of capital sold by the firms with high investment efficiency. They always sell all their capital.

Thus, the equilibrium condition is given by

$$p = \lambda_m \equiv \frac{\tau\rho\gamma\lambda + \tau\left[\frac{an^{1-\alpha}}{p}\int_0^{\bar{v}}(\bar{v}-v)dF(v)\right]\bar{\lambda} + (1-\tau)\bar{\lambda}}{\tau\rho\gamma + \tau\left[\frac{an^{1-\alpha}}{p}\int_0^{\bar{v}}(\bar{v}-v)dF(v)\right] + (1-\tau)} \tag{2.6}$$

Liquidity and Investment Efficiency. In the model, firms with $\theta = \bar{\theta}$ have higher investment efficiency. When a larger share of new capital is produced by these firms, the aggregate investment efficiency would be higher. Since the only way to obtain funds in the investment stage is selling old capital, the amount of funds the firms with high investment efficiency can obtain and use in the production of new capital depends on the price of old capital. The aggregate investment efficiency is thus related to the liquidity of the asset market. When the level of information rises, more lemons are sold and thus the average quality of capital in the market λ_m and asset price p drop. As results, the firms with high investment efficiency obtain less funds by selling old capital and thus produce less new capital goods. Then, the aggregate investment efficiency and capital accumulation decline.

2.4 Inter-Temporal Optimization

In the consumption stage, firms allocate their wealth W between dividends d_t and capital for the next period k_{t+1}. The production of consumption goods and new capital is done in the first two stages. Thus, the aggregate amount of capital for the next period is determined in the previous stages. However, when individual firms choose dividends d_t and capital for the next period k_{t+1}, the relative price of k in terms of consumption goods, i.e. q, is determined. Note that the capital quality is automatically revealed at the end of the second stage. In the third stage, the firms trade capital with perfect information.

At the end of the second stage, firms has wealth W which consists of m units of cash profits and the k^e units of capital, $W(k, Z, x) = m + qk^e$. Note that W is linear in k, $W(k_t, Z_t, x_t) = r(Z_t, x_t)k_t$, where the gross return function $r(Z_t, x_t)$ is obtained from the optimization in the first two stages.

In the third stage, the firm solves the following optimization problem:

Problem 4 *Given the gross return function $r(Z_t, x_t)$, the firm solves:*

$$\max_{\{d_t, k_{t+1}\}} E_t \sum_{i=0}^{+\infty} \beta^i \frac{U'(C_{t+i})}{U'(C_t)} d_{t+i}$$

subject to

$$d_t + q_t k_{t+1} = r(Z_t, x_t) k_t$$

The first order condition is given by

$$q_t = \beta E_t \left[r(Z_{t+1}, x_{t+1}) \frac{U'(C_{t+1})}{U'(C_t)} \right]$$

Under assumption 1, the marginal producers of new capital are the firms with low investment efficiency $\theta = 1$. Thus, $q_t \geq 1$ in equilibrium; otherwise, firms with $\theta = 1$ won't produce any new capital. However, if $q_t > 1$, firms would use more of the consumption goods to produce new capital in the second stage. Thus, the expected return on capital $E_t[r(Z_{t+1})]$ drops and the marginal utility of consumption $U'(C_t)$ rises. As a results, q_t has to drops to one. Finally, q_t has to be equal to one in equilibrium (Lemma 1).

Note that the idiosyncratic shock x_t are i.i.d. across time and are uncorrelated with aggregate states Z_t. Thus, $\hat{r}(Z_t) = E_t[r(Z_t, x_t)]$. Note that $\hat{r}(Z_t)$ is also the aggregate capital (gross) return. Then, the first order condition can be written as:

$$1 = \beta E_t \left[\hat{r}(Z_{t+1}) \frac{U'(C_{t+1})}{U'(C_t)} \right]$$

Due to the logarithm preference, the aggregate consumption is always a fraction $(1-\beta)$ of the total wealth. The aggregate consumption is given by

$$C_t = (1-\beta)\hat{r}(Z_t)K_t \tag{2.7}$$

Lemma 5 *The aggregate consumption is* $C_t = (1-\beta)\hat{r}(Z_t)K_t$ *and the capital (gross) return function* $\hat{r}(Z_t)$ *is obtained by solving the intra-temporal optimization problems in the first two stages.*

The aggregate capital accumulation is then

$$K_{t+1} = \beta\hat{r}(Z_t)K_t \tag{2.8}$$

At the end of the consumption stage, all the capital goods firms hold at the end of the second stage is to be carried into the next period and all the cash profits are delivered to the households as dividends. Thus,

$$\int_0^1 m_t(j)dj = \int_0^1 d_t(j)dj = C_t \qquad \textit{and} \qquad \int_0^1 k^e_{t+1}(j)dj = \int_0^1 k_{t+1}(j)dj = K_{t+1}$$

In the second stage, since the firms with low investment efficiency are indifferent between producing new capital and keeping cash profits, the amount of dividends d delivered by an individual firm is undetermined. However, at the aggregate level, the amount of dividends is determined by the above condition.

2.5 Private Information and Externalities

Private information is not only costly but also distorts economic decisions. The lemons problems leads to an endogenous liquidation cost, that is, the cost to sell good assets at low price. The firms which produce private information fail to internalize the negative effects of private information on the asset price. In this section, I show that the allocation would be the same as that of an economy with perfect information if firms are restricted from producing private information.

Symmetrically Uninformed. Suppose that the information cost μ is high enough that it is never optimal for the firms to produce private information. See Appendix A for a detail description on how information costs can affect firms' information choice. Thus, all the capital that exist in the economy is mixed capital k^m and thus $\gamma = 0$. Since there is only mixed capital being transacted in the asset market, under assumption 1, the price of old capital is $p = \bar{\lambda}$ and $q = 1$.

The optimization in the investment stage becomes trivial. Under assumption 2, firms with high investment efficiency $\theta = \bar{\theta}$ sell all their old capital and use the proceeds in addition to its operating profits to produce new capital. Firms with low investment efficiency $\theta = 1$ are actually indifferent between selling or keeping mixed capital when $p = \bar{\lambda}$ and $q = 1$. It follows that $\hat{V}_m = \tau + (1-\tau)\bar{\theta}$ and $\hat{V}_k^m = \tau\bar{\lambda} + (1-\tau)\bar{\lambda}\bar{\theta}$. Thus, the optimization problem 2 and 3 are reduced to

$$\hat{r}(Z) = \max_n \left(an^{1-\alpha} - wn\right)\hat{V}_m + \hat{V}_k^m$$

Since $p\hat{V}_m = \hat{V}_k^m$, the liquidation cost, that is, the third term in equation (2.2), drops to zero. Thus, firms would hire labour till the marginal product of labour equals the wage rate, i.e. $n^* = n^*_{best}$.

Perfect Information. Consider an economy described in the section 2, but now assume that the"wear out" pieces disappear at the end of the first stage. Thus, a firm starts with capital k will end up with $\bar{\lambda}k$ units of non-lemon capital k^n at the end of the first stage. In the second stage, only non-lemon capital goods are transacted in the market. Under assumption 1, the price of capital is one, i.e. $p = q = 1$.

In the investment stage, under assumption 2, firms with high investment efficiency $\theta = \bar{\theta}$ sell all their capital and use the proceeds in addition to its operating profits to produce new capital. Firms with low investment efficiency $\theta = 1$ are actually indifferent between selling or keeping capital when $p = q = 1$. Thus, the ex-ante value of a unit of consumption goods and a unit of capital are $\hat{V}_m = \tau + (1-\tau)\bar{\theta}$ and $\hat{V}_k^n = \tau + (1-\tau)\bar{\theta}$. Thus, optimization problem 2 and 3 are reduced to

$$\hat{r}(Z) = \max_n \left(an^{1-\alpha} - wn\right)\hat{V}_m + \bar{\lambda}\hat{V}_k^n$$

Since $p\hat{V}_m = \hat{V}_k^m$, the liquidation cost, that is, the third term in equation (2.2), drops to zero. Firms would hire labour till the marginal product of labour equals the wage rate, i.e. $n^* = n^*_{best}$.

Equivalence. It is easy to see that in the two economies, the allocations are the same. Firms choose $n = n^*_{best}$ and has the same expected return $\hat{r}(Z)$. Thus, the aggregate consumption and capital accumulation in the two economies are the same according to equation 2.7 and 2.8. The intuition is that when the firms are not allowed to produce private information, they can only randomly pick up some units of their capital to sell. Thus, the lemon and non-lemon capital goods are "bundled" in the transaction and the adverse selection problem is eliminated.

Proposition 2 *When the information cost μ is high enough, firms do not produce private information and the allocation of the economy is identical to that of the perfect information economy.*

2.6 Aggregate Shocks and Amplification

Since the households have log utility function and no wage income, the equilibrium conditions are actually static. To gain some intuition about how aggregate shocks affect the economy, this section analyses the effects of aggregate shocks while taking the aggregate states $\{a, \lambda, \sigma\}$ as parameters. I study how the equilibrium changes when these "parameters" are different.

In this analysis, I focus on three aggregate shocks, standard technology shock, capital quality shock and income dispersion shock. To highlight the endogenous mechanism, I compare my model with an otherwise the same model with the firms' information choice being fixed at its steady state level.

Technology Shock. Consider a standard RBC technology shock that affects the consumption goods producing capability. It affects the equilibrium through its effects on the marginal product of labour. A negative technology shock lowers the marginal product of labour (the LHS of equation 2.3). At the same time, it raises the liquidation threshold $\bar{v}$, since with less output the chance for a firm to liquidate mixed capital increases. As a result, it increases the liquidation cost (the RHS of equation 2.3). The marginal costs of labour then becomes higher than its marginal benefits. Other things being equal, the firm would decrease labour inputs and increases private information production. If firms' information choices are fixed at its steady state level, i.e. $\gamma = \gamma_{ss}$, the level of information asymmetry is fixed. When firms decrease labour inputs due to decrease in aggregate productivity, the wage bills wl drop and thus firms liquidate less in the second stage. Note that when firms start to liquidate, they always sell the lemon capital first. When firm liquidate less, less mixed capital and non-lemon capital would be sold and thus the average quality of capital goods in the market declines. This leads to a mild decrease in the asset price. In addition, when the liquidation costs (term III in equation 2.2) rise, the marginal benefit of private information (the RHS of equation 2.4) becomes relatively higher than the cost. If the firm can freely make information choice, it would like to increase its private information production γ to lower liquidation cost (the term II in equation 2.5). However, more private information worsens the lemons problem; more lemons are sold and the assets price decrease furthermore. When asset price is lower, liquidation cost rises. To avoid selling mixed capital at the low price, firms decrease their labour inputs furthermore. Figure 2.6 shows how the equilibrium changes for different value of technology shock a. As it shows, firms increase the private information production when a drop, and the drops in asset price and labour inputs in the endogenous information economy are larger than those in the fixed information economy. .

Capital Quality Shock. Consider a capital quality shock that affects the depreciation rate of lemon capital $1-\lambda$. A negative shock decreases λ. It represents an observed economy-wide loss in the value of some assets; however, which assets suffered the loss is unknown. Other things being equal, firms may have more incentives to produce information in order to locate the affected assets.

Lemma 6 *Given the asset price* p*, the wage rate* w *and the information choice* γ*, for two depreciation rates,* λ_1 *and* λ_2*, with* $1 > \lambda_2 > \lambda_1 > 0$*, let* $\pi_1(\gamma)$ *and* $\pi_2(\gamma)$ *be the corresponding profit functions when* λ *equals* λ_1 *and* λ_2 *respectively. Then,* $\pi_2'(\gamma)/\pi_2(\gamma) > \pi_1'(\gamma)/\pi_1(\gamma)$ *for any* γ*, with* $\gamma \in [0, \gamma_{max}]$*.*

When a negative capital quality shock hits, the quality of low-quality capital goods (lemons) becomes even lower. When firms' information choices are fixed at the steady state level, asset price drops only due to the decline in the overall quality of capital, and the fraction of lemon capital in total capital sold in market being kept almost the same. However, if firms can freely make information choice, the shock would firstly increases the marginal benefits of private information (the RHS of equation 2.4). Given the asset price p, when the lemons are more different from the non-lemons, firms could gain more by locating and selling the lemons (the term I in equation 2.5). Other things being equal, firms have more incentives to distinguish them from the non-lemons. However, more private information worsens the lemons problem. The asset price drops further due to more lemons are sold in the market. It increases the liquidation costs and decreases the labour inputs further. As Figure 2.6 shows, firms increase the private information production when λ drops. In addition, the drops in asset price and labour inputs in the endogenous information economy are larger than those in the fixed information economy.

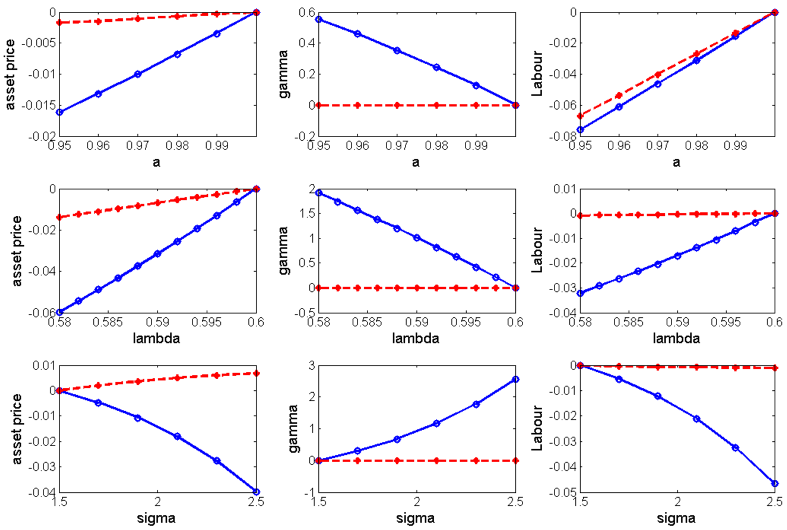

Figure 2.4: Comparative Statics

(Solid: Endogenous Information; Dashed: Fixed Information)

Income Dispersion Shock. The idiosyncratic productivity shock v disperses output among the firms, which gives rise to their potential needs for liquidation. An income dispersion shock is an exogenous change in the distribution of v. This shock is also named "uncertainty shock" and studied in literature, such as, Bloom (2009) and Gilchrist et al. (2013). An increase in the income dispersion leads to more information acquisition. The intuition is straight forward. When the firm faces higher chance to liquidate her assets, she has more incentives to tell apart assets of different equalities, so that she can avoid selling good assets when liquidation happens.

I specify the level of income dispersion in the following way. For two density functions, $f_1(v)$ and $f_2(v)$, if both of them are increasing in v and $f_2(v) > f_1(v)$ hold for the relevant values of v, then the level of income dispersion is higher when v follows the distribution $f_2(v)$.

Lemma 7 *Assume that* $\bar{v}_1$ *solves* $\frac{\rho\lambda+(1-\rho)\theta}{\rho(\bar{\lambda}-\lambda)} = \frac{(\bar{v}_1)^2}{\alpha} f(\bar{v}_1)$. *Let* $f_1(v)$ *and* $f_2(v)$ *be two probability density functions. Assume that* $f_1(v)$ *and* $f_2(v)$ *are increasing in* v *and* $f_2(v) > f_1(v)$ *holds for any* v *on* $[0, v_1]$.

Given the asset price p, *the wage rate* w *and the information choice* γ, *let* $\pi_1(\gamma)$ *and* $\pi_2(\gamma)$ *be the corresponding profit functions when the p.d.f. of* v *follows* $f_1(v)$ *and* $f_2(v)$

respectively. Then, $\pi_2'(\gamma)/\pi_2(\gamma) > \pi_1'(\gamma)/\pi_1(\gamma)$ *for any* γ *on* $[0, \gamma_{max}]$.

A negative shock increases the income dispersion. It changes the equilibrium through its effects on liquidation risk. An increase in income dispersion raises the uncertainty of firms on how much capital goods they have to liquidate in order to pay off wage bills. If firms' information choices are fixed at the steady state level, the amount of separable capital and thus the amount of lemon capital is fixed. Firms face a higher chance to be forced to sell mixed capital goods when income dispersion is higher. But more liquidation actually increases the average quality of assets, since more mixed and non-lemon capital goods are sold. The assets price slightly goes up as a result. The labour inputs slightly drop due to the increased liquidation risk. However, if firms can freely make information choice, the marginal benefit of private information (the RHS of equation 2.4)increases when the income dispersion is high. Other things being equal, firms produce more private information so that they can avoid selling mixed capital when liquidation happens, which helps lower the liquidation cost (the term II in equation 2.5). More private information worsens the lemons problem and asset price drops further since more lemons are sold. Thus, labour inputs significantly drops. Figure 2.6 shows how the equilibrium changes for different value of σ. As it shows, firms increase the private information production as σ rises. In addition, the drops in both asset price and labour inputs in the endogenous information economy are larger than those in the fixed information economy.

2.7 Simulations

This section analyses the qualitative business cycle patterns generated by shocks through the endogenous information mechanism. In this section, I focus on three shocks: standard technology shock, capital quality shock and income dispersion shock. To highlight the key mechanism described in the previous section, I compare my model with an otherwise the same model with the level of information asymmetry being fixed at its steady state.

Parameters. The length of a period is one year. The discount factor of households β is set to 0.94. The production is Cobb-Douglas and the capital share α is set to 0.25. The Frisch elasticity of labour supply ν is set to 2. I set the share of low efficiency firm τ

to 0.86 in order to guarantee that firms with low investment efficiency always have enough cash to buy all old capital goods from those with high investment efficiency, and thus "cash-in-the-market" pricing never happens. Thus, asset price always reflects the average quality of capital in the market. The parameter ρ is set to 0.5 and thus the amounts of lemon and non-lemon capital goods are equal. The investment efficiency of high efficiency firm $\bar{\theta}$ and the depreciation rate of lemon capital $1-\lambda_{ss}$ at steady state are set to 1.5 and 0.4 respectively, which leads to a steady state consumption-to-GDP ratio of 0.83. The information cost μ is set to 0.078, which is close to those used in literature. The steady state aggregate productivity a is set to 1.

Parameter	Value	Notes
β	0.94	discount factor
α	0.25	capital share
ν	0.5	Frisch elasticity of 2
τ	0.86	fraction of firms with $\theta = 1$
ρ	0.5	fraction of lemon at ss
$\bar{\theta}$	1.5	investment efficiency (high)
λ_{ss}	0.6	depreciation rate of lemon capital
μ	0.078	information cost
σ_{ss}	1.5	std of idiosyncratic shock $log(v)$ at ss
μ_{ss}	$-\sigma_{ss}^2/2$	mean of idiosyncratic shock $log(v)$
a_{ss}	1	steady state aggregate productivity

Model Generated Moments. The table below compares the model generated moments. The shock process is assumed to be i.i.d. over time. To highlight the endogenous information mechanism, I compare my model with an otherwise the same model with firms' information decisions being fixed at the steady state level. I compute the standard deviations of output Y, employment L, asset price p and TFP for the both models and compare their relative sizes. Note that in this model, TFP is endogenous and different from technology shock a_t. TFP depends on both technology shock a_t and firms information choices, i.e.

the amount of capital goods that are used in the production of information, rather than the production of consumption goods.

In the table, $\{\sigma_{e,.}\}$ are the standard deviations of the variables from the endogenous information model, while $\{\sigma_{f,.}\}$ are the standard deviations of the variables from the fixed information model. The first three columns show the relative sizes of standard deviations when there is only the standard technology shock a_t, the capital quality shock λ_t and the income dispersion shock σ_t. The last column reports the relative sizes of standard deviations, when there are all the three shocks.

		Shocks			
	Variables	a_t	λ_t	σ_t	$\{a_t, \lambda_t, \sigma_t\}$
$\sigma_{e,y}/\sigma_{f,y}$	output	1.09	3.23	4.10	1.20
$\sigma_{e,l}/\sigma_{f,l}$	labour	1.14	4.79	2.95	1.40
$\sigma_{e,p}/\sigma_{f,p}$	asset price	10.1	4.81	1.83	4.85
$\sigma_{e,a}/\sigma_{f,a}$	TFP	1.03	NA	NA	1.03

Table 2.2: Model Generated Moments

As shown in the Table 2.2, the endogenous information model can generate larger fluctuations in macro economic and financial variables than the fixed information model. For the standard RBC technology shock, the relative sizes of the standard deviations of output, labour and TFP are lower than the relative size of the standard deviation of asset price. The reason is that the fluctuations in output, labour and TFP are largely driven by the shock process; however, the fluctuations in asset price is much larger due to the endogenous changes in the information environment. When there is only the capital quality shock, the endogenous information mechanism significantly amplifies the effects of shock and leads to larger fluctuations in all the variables. For the income dispersion shock, the amplification effects of the endogenous information mechanism are also significant.

When firms' information choices are fixed at the steady state level, adverse shocks only increase the reallocation of wealth across firms. Since firms cannot produce additional private information, adverse shock won't increase information asymmetry and worsen the lemons problem further, and thus it has limited aggregate effects. When firms can produce private information, adverse shocks trigger private information production, which changes the information environment and thus leads to much larger fluctuations in financial market and macro-economic variables.

Impulse Responses. I compute the dynamic examples of the responses of the economy to three different aggregate shocks: the standard RBC technology shock, the capital quality shock and the income dispersion shock. In order to highlight the importance of the endogenous information mechanism, I compare the impulse responses of my model to those of an otherwise the same economy with the information choice being fixed at the steady state level.

The economy start from the steady state and is hit by a shock in the second period. The solid (blue) lines show the responses of the model, while the dashed (red) lines show the responses of an otherwise the same economy with the information choice being fixed at its steady state ($\gamma = \gamma_{ss}$).

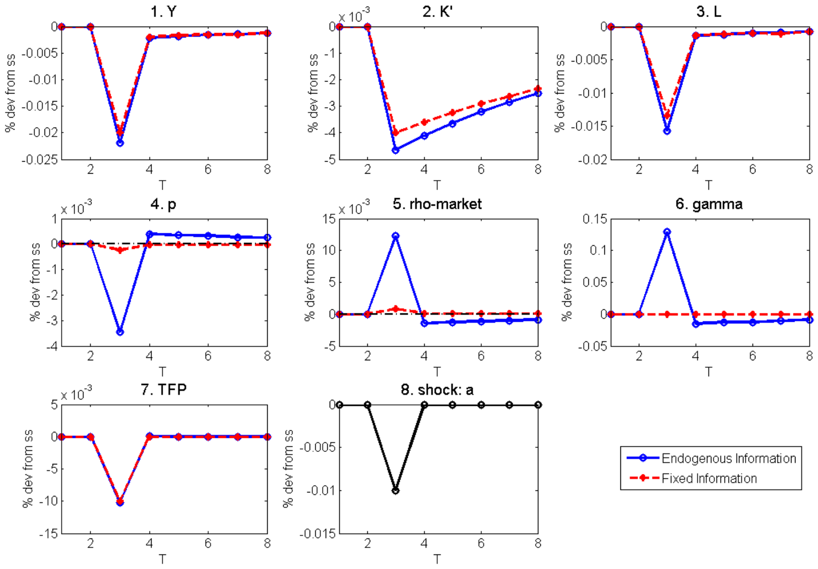

Figure 2.5: Transitory Technology Shock

(Solid: Endogenous Information; Dashed: Fixed Information)

Figure 2.5 shows the responses of the economy to a standard RBC technology shock. Panel 1 2, and 3 show the responses of aggregate variables, i.e. output, capital stock (for the next period) and employment. All the macro variables decrease sharply when the shock hits and stays under the steady state in the following periods due to low capital stock. Since the drop in Y and L are largely driven by the shock process, the difference between two cases is not very significant. Panel 2 shows the response of (next period) capital stock. The drop in capital stock is larger when the information environment is endogenous, which illustrates the amplification effects of the endogenous information environment. Negative shock triggers private information acquisition, which worsens the lemons problem and lowers the asset price further. Note that the only way for the firms with high investment efficiency to finance their investment in new capital is to sell their old capital. Thus, when the asset price is low firms with high investment efficiency get less funds from selling out their old capital. Thus, they produce less new capital, which leads to slow capital accumulation.

Panel 4, 5 and 6 show the responses of the financial market to the shock. Panel 4 shows the response of capital price. Panel 5 shows the responses of the fraction of lemons capital in the total capital transacted in the market. Panel 6 shows the level of information asymmetry, that is, the share of separable capital goods k^s. When the shock hits, financial market variables are more sensitive to shocks when the information environment is endogenous. However, after the shock dies out, the asset price rises above its steady state, and the ratio of lemon capital in market ρ_m and the level of information asymmetry γ drop below its steady state. The information environment is actually improved during the recovery periods. The reason is that when the aggregate capital stock drops, the wage rate declines and the marginal product of capital goods increases. The opportunity cost of information then rises, which leads to less information production and improvement in the information environment. Panel 7 shows the responses of the total factor productivity (TFP) and Panel 8 shows the exogenous technology shock a. In this model, TFP is different from a. A drop in the TFP can be due to two reasons:(1) a negative technology shock a and (2) more capital is used in information production rather than consumption goods production. For the standard technology shock, the drop in the TFP is largely driven by the shock process, and thus the difference between the two lines is not significant.

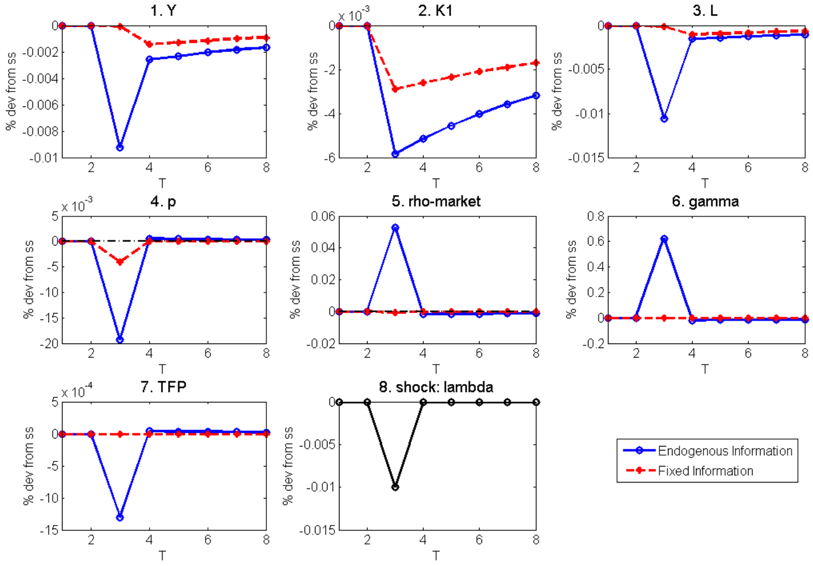

Figure 2.6: Transitory Capital Quality Shock

(Solid: Endogenous Information; Dashed: Fixed Information)

Figure 2.6 shows the responses of economy to a capital quality shock, i.e. an exogenous increase in the depreciation rate of lemon capital $1 - \lambda$. Note that in the model non-lemon capital are those do not depreciate during production, but in a unit of lemon capital, a fraction $1 - \lambda$ of its pieces depreciate. When a negative capital quality shock hits, the quality of lemon capital becomes lower. The drops in macro variables, i.e. output, capital stock and employment, are much larger when the information environment is endogenous. When the value of lemon capital becomes lower, given the asset price p, firms thus have more incentives to distinguish their lemon capital from the non-lemons. Asset prices drop in both economies due to the lower overall capital quality. However, asset price drop more in the economy with endogenous information environment, since firms produce more private information and thus more lemons are sold in the market, as shown in the panel 4-6. As show in the panel 7, when the information environment is fixed, TFP is constant; however, when the information environment is endogenous, the TFP drops since more capital is used in the private information production rather than the production of consumption goods. The panel 8 shows the path of shock λ.

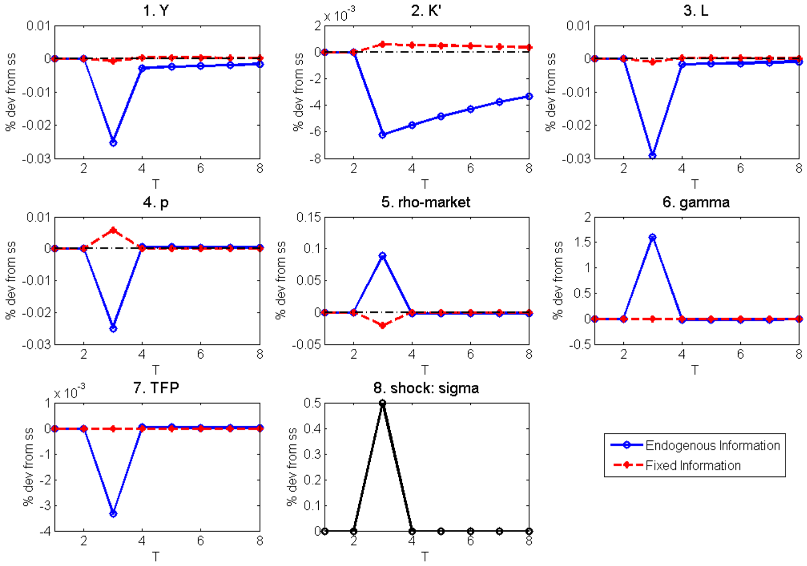

Figure 2.7: Transitory Income Dispersion Shock

(Solid: Endogenous Information; Dashed: Fixed Information)

Figure 2.7 shows the responses of the economy to an income dispersion shock, i.e. an exogenous increase in the volatility of idiosyncratic shock v. An income dispersion shock does not change the aggregate output, but disperses the output among the firms. This increases the liquidation risks faced by the firms, i.e. the uncertainty on how much capital the firm has to sell in order to repay its wage bill increases. When the level of information asymmetry is fixed, higher income dispersion increases the liquidation in the second stage. This actually increases the average quality of capital in the market since more mixed and non-lemon capital is sold. As results, asset price slightly rise as show in the panel 5. The capital accumulation increases as shown in the panel 2. The reason is that firms with high investment efficiency invest more, since they can get more funds by selling their old capital when the asset price is high. However, when the firms can freely make information choices, they produce more private information and the level of information asymmetry rises as show in the panel 6. This leads to drop in asset price (panel 4), since more lemons are sold in the market (panel 5). When the asset price drops, firms with high investment efficiency produce less new capital, since they can obtain less funds by selling out old capital goods.

As results, the capital accumulation drops (panel 2), leading to recession.

2.8 Final Remarks

This paper explores the macro-economic implications of the endogenous changes of information environment triggered by aggregate shocks. In the model, information about asset quality is asymmetric and the degree of information asymmetry is endogenously determined and moves counter-cyclically. The dynamics of the information environment creates a fragility mechanism that amplifies aggregate shocks.

At the centre of the endogenous asymmetric information problem lies an externality. Firms that acquire private information cannot internalize the adverse effects of private information on asset price and market liquidity. The externality becomes stronger when negative aggregate shocks hit the economy. When the dispersion of idiosyncratic output increases, individual firms are more likely to be forced to liquidate. As results, the individual benefit of private information increases and this externality becomes stronger. This happens as well when a negative aggregate shock decreases the value of some assets in the economy, while which assets suffered from the loss is unknown.

What should the government do to correct this problem? When the information costs is small and neglectable, the government may want to force both sides, i.e. both the sellers and the buyers, to disclose and process the information. When information cost is not neglectable, the government can regulate the information production and processing in a way such that the information environment does not vary with the aggregate states.

The endogenous information mechanism may be embed into dynamic settings in different ways. In this model, agents make information decisions period by period and there is no persistency in the information environment. Explore the macro-economic implications of an endogenous information environment with persistency is a question for my future research.

2.9 Appendix A: Information Costs and Opacity

When do the firms produce private information? Firms choose whether or not to produce private information on their capital by comparing the marginal benefits and costs of information (equation 2.4). When the information cost is extremely high, firms would not produce any information and the information about capital quality becomes symmetric. High information cost leads to an equilibrium with no information acquisition and perfect market liquidity. This result is in line with the idea that asset opacity helps improve its liquidity by preventing people from producing private information on it.

In this model, the information choices of firms depend on aggregate states. Recall that there are four elements in the aggregate states vector Z, which include one endogenous state, capital stock K, and three aggregate shocks, $\{a, \sigma, \lambda\}$. Given the aggregate capital stock K and information cost μ, firms' information choices depend on the realization of aggregate shocks. Suppose that the economy is initially in the no information production regime. When the aggregate productivity a drops, the marginal product of capital and the opportunity cost of information production decreases. Firms may want to use their capital to produce wear sensor. When the economy is hit by a negative capital quality shock, the depreciation rate of lemon capital, $1-\lambda$, increases, and the quality of lemon capital becomes even lower. Firms then may want to distinguish the lemons from the non-lemons, so that they can sell the lemons in the market.

Actually, given the aggregate states $\{K, a, \lambda, \sigma\}$, there is a threshold of μ such that when the information cost is higher than the threshold value, firms do not produce any private information in equilibrium. Given the aggregate capital stock K, the labour supply function $w = L^{1/\nu}$, and the aggregate productivity a, let n^*_{best} be the first best choice of labour-to-capital ratio. Given the realization of capital quality shock λ, then $\bar{\lambda}$, $\bar{\lambda} = \rho\lambda + (1-\rho)$, is the overall quality of capital in the economy. Firms do not produce any private information in equilibrium if the following condition holds:

$$\mu > \frac{(1-\rho)\tau(1-\bar{\lambda})}{\left[\alpha a \left(n^*_{best}\right)^{1-\alpha} + \bar{\lambda}\right]\left[\tau + (1-\tau)\bar{\theta}\right]} \tag{2.9}$$

Lemma 8 *Given the aggregate states $\{K, a, \lambda, \sigma\}$, let n^*_{best} be the first best labour-to-capital ratio and $\bar{\lambda}$, with $\bar{\lambda} = \rho\lambda + (1-\rho)$, be the overall quality of capital in the economy. Firms do not produce any private information in equilibrium if the equation 2.9 holds.*

Suppose that the economy is initially in the no information production regime, that is, firms do not produce any private information in equilibrium. How a negative shock changes the information environment depends on the level of the information cost μ. When the information cost μ is small, a small shock can leads to a sudden change in the information environment (i.e. equation 2.9 does not hold any more); firms suddenly start to produce private information on their capital and the economy moves into the information acquisition regime. When the information cost μ is large, only a large shock may change the information environment. When the information cost μ is sufficiently large, firms will never produce private information.

In the following numerical exercise, I suppose that the economy is initially in the steady state of benchmark model. For the different pairs of $\{a, \lambda\}$ (note that σ is irrelevant in this analysis, since it is not in the condition for no information production, the equation 2.9), I solve for the equilibrium. Note that with log preference, the equilibrium conditions of the model are actually static. Figure 2.9 shows the firms' information choices given the pair $\{a, \lambda\}$. In each panel, firms produces private information when $\{a, \lambda\}$ lies in the blue (dark) area, and do not produce information when it lies in the yellow (light) area. As shown below, when I increase the information cost μ, the information production area shrinks.

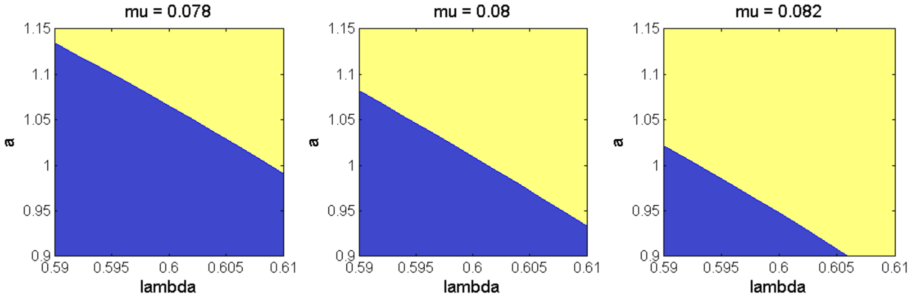

Figure 2.8: Information Costs

(Blue Area: Information Acquisition; Yellow Area: No Information Acquisition)

2.10 Appendix B: Proof

2.10.1 Proof of Lemma 1

Lemma 1 says that when firms with low investment efficiency $\theta = 1$ are the marginal producers of new capital, then $q_t = 1$ in equilibrium. If $q_t < 1$, firms with low investment efficiency $\theta = 1$ won't produce any new capital.

If $q_t > 1$, all the consumption goods will be used to produce new capital. Thus, the consumption of households in the stage 3 drop to zero. As a result, the marginal utility of consumption $U'(C_t)$ rises to infinity, which would pushes down the price of capital q. Finally, q_t has to drop to one. When $q = 1$, firms with low investment efficiency $\theta = 1$ are then indifferent between producing new capital and keeping cash profits that will be used to pay dividends in the third stage.

2.10.2 Proof of Lemma 2

Lemma 2 says that the average quality of capital sold in the market, λ_m, should be between λ and $\bar{\lambda}$. First, the capital sold in the market is a mixture of lemons and non-lemons. The average quality of capital sold in the market cannot be lower than the quality of lemon capital and thus $\lambda_m \geq \lambda$. Second, note that when firms start to liquidate, they always sell low quality capital first and high quality capital last. Thus, the average quality of capital in the market cannot be higher than the average quality of capital hold by firms, which implies $\lambda_m \leq \bar{\lambda}$.

2.10.3 Proof of Lemma 3

Lemma 3 says that when γ_{max}, defined as $\gamma_{max} \equiv (wn^*_{best})/(\rho p)$, is less than one, firms can always fully pay its wage bill without liquidating its non-lemon capital goods. By rearranging the definition equation, we can obtain $\rho\gamma_{max} = (wn^*_{best})p^{-1}$. The RHS of the equation represents the amount of capital the firm has to sell in order to repay wl when it is hit by the worst idiosyncratic shock, i.e. $v = 0$. The LHS is the amount of lemon capital it has.

First, note that it is never optimal to choose a labour-to-capital ratio that is larger than its first best n^*_{best}. Thus, $n \leq n^*_{best}$ should hold. Second, note that the sum of the lemon and the mixed capital is $\rho\gamma + (1-\gamma)$, which is decreasing in γ. Third, when $\gamma = 1$, the amount of the lemon capital is ρ and the amount of mixed capital is 0. Since $\gamma_{max} < 1$, it follows that $\rho > (wn^*_{best})p^{-1}$ holds under the assumption 3. It means that the firm can always repay its wage bill without liquidating non-lemons even when it chooses $n = n^*_{best}$ and $\gamma = 1$. Finally, since the sum of the lemon capital and the mixed capital would be larger if the firm chooses a γ that is less than 1 and the wage bill would be smaller if the firm chooses a n that is less than n^*_{best}, thus the firm can always repay its wage bill without liquidating its non-lemon capital when $\gamma < 1$ and $n < n^*_{best}$.

2.10.4 Proof of Lemma 4, 6, and 7

Note that in the first stage, the firm solves the following optimization problem:

$$\pi(\gamma) = \max_n \underbrace{\left(an^{1-\alpha} - wn\right)\hat{V}_m}_{\text{I. operating profits}} + \underbrace{\left[(1-\gamma)\hat{V}^m_k + \gamma\hat{V}^s_k\right]}_{\text{II. value of capital}} \\ \underbrace{- \left(\hat{V}^m_k - p\hat{V}_m\right)\frac{an^{1-\alpha}}{p}\int_0^{\bar{v}} (\bar{v} - v)\,dF(v)}_{\text{III. liquidation costs}} \tag{2.10}$$

subject to

$$\bar{v} = \left(\frac{w}{a}\right)n^{\alpha} - \gamma\left(\frac{\rho p}{a}\right)n^{\alpha-1} \tag{2.11}$$

Let $z(n|\gamma)$ be the firm's expected gross return on capital when it chooses n, given the information choice γ. Thus, $z(n|\gamma) \leq \pi(\gamma)$, and $z(n^*|\gamma) = \pi(\gamma)$ holds only when n^* is the solution of problem 2. Define $G(\bar{v}) \equiv \int_0^{\bar{v}} vdF(v)$. The first order condition of problem 2 is given by:

$$\underbrace{\left[a(1-\alpha)n^{-\alpha} - w\right]}_{\text{wedge between MPL and } w} \hat{V}_m = \underbrace{\left(\frac{\hat{V}^m_k}{p} - \hat{V}_m\right)\left[wF(\bar{v}) - a(1-\alpha)n^{-\alpha}G(\bar{v})\right]}_{\text{marginal liquidation costs}} \tag{2.12}$$

It is easy to show that the LHS (RHS) of equation 2.12 is decreasing (increasing) in n

The marginal benefit of information is measured by the marginal increase in π with γ. Using envelop theory, it is given by

$$\pi'(\gamma) = \underbrace{\left(\hat{V}_k^s - \hat{V}_k^m\right)}_{\text{I. } \uparrow \text{ in capital value}} + \underbrace{\rho\left(\hat{V}_k^m - p\hat{V}_m\right)F(\bar{v})}_{\text{II. } \downarrow \text{ in liquidation cost}} \tag{2.13}$$

Proof of Lemma 4

Given the asset price p and the wage rate w, for any γ_1 and γ_2, with $0 < \gamma_1 < \gamma_2 < \gamma_{max}$, let n_1 and n_2 be the solutions of problem 2, and $\bar{v}_1$ and $\bar{v}_2$ be the corresponding optimal thresholds, when γ equals γ_1 and γ_2 respectively.

Assume that initially $\gamma = \gamma_1$ and $n = n_1$. Then, γ is increased from γ_1 to γ_2:

(1) First, $n_2 > n_1$ should hold. An increase in γ lowers the liquidation threshold $\bar{v}$ (according to equation 2.11). The RHS of equation 2.12, which is a function of n, shifts downward. (note that the RHS is decreasing in $\bar{v}$) Then, the RHS of equation 2.12 then becomes lower than the LHS. Since the LHS (RHS) of equation 2.12 is decreasing (increasing) in n, n has to increase for them to be equal. It follows that $n_2 > n_1$.

(2) Second, $\bar{v}_2 < \bar{v}_1$ should hold. Suppose that n_2 is increased to a level so that $\bar{v}_2 = \bar{v}_1$. Note that when $\bar{v}_2 = \bar{v}_1$, $n_2 > n_1$. The RHS of equation 2.12 then rises above its initial level (when $\gamma = \gamma_1$ and $n = n_1$), while the LHS drops below its initial level, and thus the LHS is lower than the RHS. For the two sides to be equal, n should decrease, which implies $\bar{v}_2 < \bar{v}_1$ has to hold. It follows that $F(\bar{v}_2) < F(\bar{v}_1)$. According to equation 2.13, $\pi'(\gamma_2) < \pi'(\gamma_1)$.

(3) Third, $z(n_1|\gamma_2) > \pi(\gamma_1)$ should hold. Given that $n = n_1$, when γ is increased, the value of capital (II in equation 2.10) increases, and the liquidation cost (III in equation 2.10) decreases. Thus, $z(n_1|\gamma_2) > \pi(\gamma_1)$. It follows that $\pi(\gamma_2) \geq z(n_1|\gamma_2) > \pi(\gamma_1)$.

Thus, $\pi'(\gamma_1)/\pi(\gamma_1) > \pi'(\gamma_2)/\pi(\gamma_2)$.

Proof of Lemma 6

For any λ_1 and λ_2, with $0 < \lambda_1 < \lambda_2$, given the asset price p, the wage rate w and the information choice γ, let n_1 and n_2 be the solutions of the problem 2, $\bar{v}_1$ and $\bar{v}_2$ be the corresponding optimal liquidation thresholds, $\pi_1(\gamma)$ and $\pi_2(\gamma)$ be the corresponding profit function, when λ equals λ_1 and λ_2 respectively.

Given the prices, an increase in λ actually only increases the value of the mixed capital $\hat{V}_k^m$. Let $\hat{V}_{k,1}^m$ and $\hat{V}_{k,2}^m$ be the value of the mixed capital, when λ equals λ_1 and λ_2 respectively.

Assume that initially $\lambda = \lambda_1$ and $n = n_1$. Then, λ is increased from λ_1 to λ_2:

(1) First, $n_2 < n_1$ should hold. Assume that initially $n = n_1$, an increase in λ increases $\hat{V}_k^m$ and thus the liquidation cost (the RHS of equation 2.12) increases. The RHS of equation 2.12 becomes higher than the LHS. Since the LHS (RHS) of equation 2.12 is decreasing (increasing) in n. Thus, n has to decrease for them to be equal. It follows that $n_2 < n_1$ and that $\bar{v}_2 < \bar{v}_1$ and $F(\bar{v}_2) < F(\bar{v}_1)$.

(2) Note that $\rho F(\bar{v}) - 1 < 0$. Thus, the marginal benefit of information $\pi'(\gamma)$ is decreasing in $\hat{V}_k^m$ and increasing in $F(\bar{v})$. Since $F(\bar{v}_2) < F(\bar{v}_1)$ and $\hat{V}_{k,2}^m > \hat{V}_{k,1}^m$, thus $\pi_2'(\gamma) < \pi_1'(\gamma)$. [10]

(3)Third, $z_2(n_1|\gamma) > \pi_1(\gamma)$ should hold. Given that $n = n_1$, when λ increases, both the capital value (II in equation 2.10) and the liquidation cost (III in equation 2.10) increases. However, the former increases more than the later.[11] Thus, $z_2(n_1|\gamma) > \pi_1(\gamma)$ holds. It follows that $\pi_2(\gamma) \geq z_2(n_1|\gamma) > \pi_1(\gamma)$.

Thus, $\pi'(\gamma_1)/\pi(\gamma_1) > \pi'(\gamma_2)/\pi(\gamma_2)$.

Proof of Lemma 7

Assume that $\bar{v}_1$ solves:

$$\frac{\rho\lambda + (1-\rho)\theta}{\rho(\bar{\lambda} - \lambda)} = \frac{(\bar{v}_1)^2}{\alpha} f(\bar{v}_1)$$

Let $f_1(v)$ and $f_2(v)$ be two probability density functions. Assume $f_1(v)$ and $f_2(v)$ are increasing in v and $f_2(v) > f_1(v)$ hold for the any v in $[0, min\{v_{max}^*, v_1\}]$.

Given the asset price p, the wage rate w and firms' information choice γ, let n_1 and n_2 be the solutions of problem 2, $\bar{v}_1$ and $\bar{v}_2$ be the corresponding optimal thresholds, $\pi_1(\gamma)$ and $\pi_2(\gamma)$ be the corresponding profits function, when v follows $f_1(v)$ and $f_2(v)$ respectively.

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[10] Note that $[\rho F(\bar{v}_2) - 1]\, \hat{V}_{k,2}^m < [\rho F(\bar{v}_2) - 1]\, \hat{V}_{k,1}^m < [\rho F(\bar{v}_1) - 1]\, \hat{V}_{k,1}^m$.

[11] Note that the following has to hold under assumption 3: $an^{1-\alpha} \int_0^{\bar{v}} (\bar{v} - v)\, dF(v) < an^{1-\alpha}\bar{v} = wn - \rho\gamma p < (1-\gamma)p$. Thus, the increase in the capital value is higher than the increase in the liquidation cost.

Define $H(n)$ and $R(n)$:

$$H(n) = a(1-\alpha)n^{-\alpha}\left[G(\bar{v}) + \hat{V}_m\left(\frac{\hat{V}_k^m}{p} - \hat{V}_m\right)^{-1}\right]$$

$$R(n) = w\left[F(\bar{v}) + \hat{V}_m\left(\frac{\hat{V}_k^m}{p} - \hat{V}_m\right)^{-1}\right]$$

Thus, the first order condition (equation 2.12) implies that $H(n^*) = R(n^*)$ holds if n^* solves problem 2. Let $H_1(n)$ and $R_1(n)$, and, $H_2(n)$ and $R_2(n)$, denote function $H(.)$ and $R(.)$, when the p.d.f. of v is $f_1(v)$ and $f_2(v)$ respectively.

Define $G(\bar{v}) \equiv \int_0^{\bar{v}} vdF(v)$ and let $F(.)$ be the c.d.f. of v. Let $G_1(v)$ and $F_1(v)$, and, $G_2(v)$ and $F_2(v)$, denote function $G(.)$ and $F(.)$, when the p.d.f. of v is $f_1(v)$ and $f_2(v)$ respectively. Obviously, for any $\bar{v}$ on $[0, min\{v^*_{max}, v_1\}]$, $F_1(\bar{v}) < F_2(\bar{v})$, $G_1(\bar{v}) < G_2(\bar{v})$ and $F_1(\bar{v}) - G_1(\bar{v}) < F_2(\bar{v}) - G_2(\bar{v})$ hold.

(1) When v lies in $[0, min\{v^*_{max}, v_1\}]$, $H(n)$ is decreasing in n. To see this,[12]

$$\begin{aligned} H'(n) &= -\alpha a n^{-\alpha-1}\left[G(\bar{v}) + \hat{V}_m\left(\frac{\hat{V}_k^m}{p} - \hat{V}_m\right)^{-1}\right] + an^{-\alpha}\bar{v}f(\bar{v})\frac{\partial\bar{v}}{n} \\ &< -an^{-\alpha-1}\left[\hat{V}_m\left(\frac{\hat{V}_k^m}{p} - \hat{V}_m\right)^{-1} - n\bar{v}f(\bar{v})\frac{\partial\bar{v}}{n}\right] \\ &< -an^{-\alpha-1}\left[\frac{\rho\lambda + (1-\rho)\theta}{\rho(\bar{\lambda}-\lambda)} - \frac{\bar{v}^2}{\alpha}f(\bar{v})\right] \end{aligned}$$

Note that $v^2 f(v)$ is increasing in v on $[0, min\{v^*_{max}, v_1\}]$. Thus, $H'(n) < 0$ should hold when $v < \bar{v}_1$.

(2) Note that $H(n)$ is decreasing in n and $R(n)$ is increasing in n. When the distribution of v is changed from $f_1(v)$ to $f_2(v)$, both $H(n)$ and $R(n)$ are shifted upward. The intersection of the two curves should be higher: let n_2 be the new intersection of these two curves, then $R_2(n_2) > R_1(n_1)$ and $H_2(n_2) > H_1(n_1)$. It follows that $F(\bar{v}_2) > F(\bar{v}_1)$ and $\pi_2'(\gamma) > \pi_1'(\gamma)$.

(3)Third, $\pi_2(\gamma) < z_1(n_2|\gamma)$ should hold. Given that $n = n_2$, the liquidation cost (III in equation 2.10) decreases as the p.d.f. of v is changed from $f_2(v)$ to $f_1(v)$. Thus, $\pi_2(\gamma) <$

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[12] Note that $\hat{V}_m(\frac{\hat{V}_k^m}{p} - \hat{V}_m)^{-1}$ is increasing in p and $p \in [\lambda, \bar{\lambda}]$, thus it has to be larger than $\frac{\rho\lambda+(1-\rho)\theta}{\rho(\bar{\lambda}-\lambda)}$.

$z_1(n_2|\gamma)$ holds. It follows that $\pi_2(\gamma) < z_1(n_1|\gamma) \leq \pi_1(\gamma)$.

Thus, $\pi_2'(\gamma)/\pi_2(\gamma) > \pi_1'(\gamma)/\pi_1(\gamma)$.

2.10.5 Proof of Lemma 8

Suppose that there in no firm producing private information. Then, the asset price should reflects the average quality of capital in this economy, i.e. $p = \bar{\lambda}$. The problem 2 is reduced to:

$$\pi(\gamma = 0) = \max_{n} \left(an^{1-\alpha} - wn\right) \hat{V}_m + \hat{V}_k^m = \left[\alpha a \left(n_{best}^*\right)^{1-\alpha} + \bar{\lambda}\right] \left[\tau + (1-\tau)\bar{\theta}\right]$$

In addition, the marginal benefit of private information at $\gamma = 0$ is

$$\pi'(\gamma = 0) = \hat{V}_k^s - \hat{V}_k^m = (1-\rho)\tau(1-\bar{\lambda})$$

For the firms not to produce any private information, the information cost μ should be large enough, such that the following condition holds:

$$\frac{\pi'(\gamma = 0)}{\pi(\gamma = 0)} < \frac{\phi'(\gamma = 0)}{1 - \phi(\gamma = 0)} = \mu \tag{2.14}$$

Chapter 3

Endogenous Dispersion and Volatility in Asset Returns

3.1 Introduction

Measures of stock market volatility, such as, the cross sectional dispersion of individual stock returns and the time series volatility of market returns, tend to rise in bad times. Recently, this robust empirical pattern has attracted growing attention from economists. Cross sectional dispersion and time series market volatility are typically used as measures for "uncertainties". The significant countercyclical movements of these two measures are often used as evidence supporting the notion that uncertainty shock, i.e. exogenous increase in uncertainty, negatively impacts aggregate economic activities and leads to recessions. Bloom (2009) estimated a series of VARs and shows that an uncertainty shock (i.e. a shock to market volatility index) leads to a rapid fall of industrial production and employment. A similar VAR exercise is done in Gilchrist, Sim and Zakrajšek (2010), which shows that an uncertainty shock (i.e. a shock to stock return volatility) leads to a widening spread between interest rates on corporate and Treasury bonds and decreases in output and investment. This strand of research focuses on the impacts of increased uncertainties on aggregate economic activities: how exogenous increases in uncertainties, propagated by various frictions, discourage economy activities.[1]

Though stock market volatility measures are widely used as proxies for uncertainty, the reason why they move counter-cyclically might not have been fully explained by existing studies. In this paper, I explore the viability of an alternative explanation for the countercyclical movements of dispersion and market volatility. I provide a mechanism through which a negative first moment shock that worsens financial conditions leads to increases in the dispersion and the market volatility in stock returns. The main idea is that when the financial condition of the borrower is worsened, the likelihood and the expected costs of (ex post) auditing rises. Then it is optimal for the borrower to distribute more risks to the lenders, which can help lower the expected costs of (ex post) auditing, but may lead to increases in the dispersion and the market volatility in asset returns.

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[1] Examples are Arellano, Bai, and Kehoe (2010), Bloom (2009), Gilchrist, Sim and Zakrajšek (2010), to name a few.

I explore this hypothesis in the context of financial contracting. The asset I focus on refers to outside equity[2]. I start with a simple model with costly state verification problem (CSV hereafter), but assume that both "ex-ante monitoring" and "ex-post monitoring" are allowed. Ex-post monitoring refers to the monitoring which happens when the borrower cannot repay the promised amount, that is, when bankruptcy happens[3], while ex-ante monitoring refers to setting up a mechanism in advance which can credibly force the borrower to reveal the true investment outcome to the lender[4]. In addition, the agents are allowed to make use of combinations of the two monitoring technologies: a borrower can split his investment into two parts and use different monitoring technologies for different parts. As in conventional CSV models, the cost for ex-post monitoring is a fraction of unobservable residual. Symmetrically, a fraction of initial investment has to be spent to set up the ex-ante monitoring mechanism[5]. Ex-ante monitoring is costly, but it makes it possible to write the financial contract contingent upon the information it reveals. It can help mitigate the problems of information asymmetry and reduces the expected (ex post) monitoring costs[6]. If ex ante monitoring is the only monitoring technology adopted, the information asymmetry is completely eliminated. The optimal contract is standard equity with return perfectly correlated to investment outcome. If both monitoring technologies are used, the optimal contract resembles outside equity and its repayment depends on the observable performance of investment which is revealed by ex ante monitoring. The more intensively ex-ante monitoring is used, the more equity-like the optimal contract becomes.

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[2] As will be shown, the optimal contract resembles outside equity and its return depends on the observable performance of investment.

[3] Ex-post monitoring refers to the activity of the lender who verifies the ex post investment outcome. Ex-post monitoring can be interpreted as costly (ex post) auditing or costly bankruptcy, which is exactly like that available in standard CSV set-ups. In traditional CSV models, such as Townsend (1979) and Gale and Hellwig (1985), ex post monitoring is the only available monitoring technology.

[4] In practice, public firms disclose their private information in order to satisfy the requirements of regulation or fund providers. Firms also voluntarily disclose their private information when they have strong incentives to mitigate the problems of information asymmetry and reduce the cost of external financing, for example, before making equity offerings. See Healy and Palepu (2001) for a summary of researches on the incentives and the consequences of information disclosure.

[5] An example of ex-ante monitoring costs can be the cost of enhancing the credibility of information disclosed by firms. For example, lenders may require the firm to hire an independent auditor as a condition of lending.

[6] Mitigating the problems of information asymmetry and reducing external financing costs is an important incentive of firms to disclose information, Healy and Palepu (2001).

To finance for his investment, a borrower not only borrows from lenders, but also invests his own wealth into the project, which is called “net worth”. As in conventional CSV framework, the optimal contract maximizes the borrower’s expected profits while guaranteeing the lender with an expected return which is not less than his opportunity cost. The optimal contract actually minimizes the expected cost of monitoring. When the financial condition of the borrower is worsened, the likelihood and the expected cost of bankruptcy (ex post monitoring) increases. In such a situation, the optimal contract that minimizes monitoring costs calls for more intensive use of ex ante monitoring and requires the repayment to be contingent on the information it makes known. The financial contract thus becomes more equity like and the lenders take more aggregate and idiosyncratic risks. As a result, market volatility (cross sectional dispersion) increases, when more aggregate (idiosyncratic) risks are distributed to the lenders. The model shows how the financial condition of the borrower affects the way risks are distributed to the borrower and the lender, and then how it determines the dispersion and the market volatility in asset returns. It also shows how the marginal costs of ex ante and ex post monitoring much equal. This mechanism differs from risk sharing in the sense that it is not about how the borrower and the lender smooth their consumption. Actually, in the model both the borrower and the lender are assumed to be risk neutral. However, it is optimal to distributed more risks to the lender for a borrower with worsened financial condition, since it helps reduce bankruptcy (ex post monitoring) cost.

Secondly, the observed cross sectional dispersion depends not only on how much idiosyncratic risks are distributed to the lenders, but also the dispersion in ex ante asset returns. In this paper, the heterogeneity in borrowers’ wealth is introduced into the model. Non financially constrained borrowers make the same financial decisions even though their wealth levels are different, while financially constrained borrowers make different financial decisions based on their wealth levels. The wealth levels of financially constrained borrowers determine their degrees of being financially constrained. A negative shock, which shifts the wealth distribution to the left, increases the fraction of financially constrained borrowers.

Consequently, the heterogeneity in borrowers' degrees of being financially constrained increases, leading to increases in the heterogeneity in borrowers' financial decisions. Ex-ante asset returns that are determined by borrowers' financial decisions thus become more dispersed, which also leads to increase in the dispersion in ex post asset returns. Gilchrist et al. (2013) documented a sharp increase in the dispersion in firms' borrowing costs after 2007, which reflects the increased heterogeneity in firms' degrees of being financially constrained in the crisis.

In sum, the cross sectional dispersion and the market volatility in asset returns rise following a negative first moment shock due to two reasons. First, a negative shock worsens the borrower's financial condition and increases the likelihood and the expected cost of bankruptcy (ex post monitoring). To minimize the bankruptcy cost, ex ante monitoring is used more intensively and the optimal contract becomes more equity like. As a result, the lenders take more risks, which leads to increases in cross sectional dispersion and market volatility. Second, a negative shock that shifts the borrowers' wealth distribution to the left increases the fraction of financially constrained borrowers, who make their financial decisions based on their wealth levels. This leads to increases in the heterogeneity in financial decisions and the dispersion in ex ante returns. Then, the dispersion in ex post asset returns rises as a result.

To provide a quantitative assessment of the mechanism described above, the benchmark model is extended to a dynamic framework with infinite horizons, which is calibrated at steady state. The calibrated model fed with a first moment shock is capable of generating patterns of the dispersions in ex ante and ex post asset returns, market volatility and financial condition that are comparable to the data. The model has implications that are consistent with the following observations on stock market: (1) cross sectional dispersion and market volatility are counter cyclical and highly correlated to each other; (2) cross sectional dispersion and market volatility are positively correlated to financial condition indicators. The model generated correlation coefficients between pairs of these measures are quantitatively in line with the data counterparts[7]. In particular, the model predicts a significant correlation between the dispersion in ex post returns and market volatility, which is consistent with the commonplace observation documented in literature, including Campbell and Lettau (1999) and Bloom (2009), among others. An impulse response exercise shows that after a first moment shock which has no persistence at all (zero persistence), the increases in dispersion and market volatility are significant and persistent. This is because that the cross sectional dispersion and the market volatility in asset returns are largely determined by financial conditions and it takes time for the worsened financial conditions to recover. Thus, the responses of dispersion and market volatility are persistent even if the shock is not.

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[7] See Table 1 and 2 for details.

As mentioned, the countercyclical movements of stock market volatility measures are often used as measures for uncertainties. This paper provides another possible explanation for the observed countercyclical movements of dispersion and market volatility. This does not necessarily diminish the importance of stock market volatility measures as uncertainty proxies. The main goal of this paper is to explore another possible explanation for this phenomenon. The model focuses on the borrower side, while assuming that lenders have a linear utility function and a constant opportunity cost. In particular, I focus on the relationship between the cross sectional dispersion/market volatility in asset returns and the financial conditions of borrowers. In Gilchrist, Sim and Zakrajšek (2010), they use stock return volatility as proxy for uncertainty and examined the important role played by financial condition in propagating uncertainty shock (i.e. shock to stock return volatility) to the broader economy in a VAR exercise. However, in this paper, I look at the relationship between dispersion/volatility and financial conditions from a different angle: dispersion and market volatility increase due to the worsened financial conditions of borrowers, which are caused by a negative first moment shock. If the mechanism proposed in this paper is correct, then the dispersion and the market volatility in stock returns also provide information on firms' financial conditions.

This paper entertains a similar casual direction as that in endogenous uncertainty papers, including Tian (2012) and Bachmann and Moscarini (2011) among others, which argue that a negative first moment shock may cause riskier behaviours of firms, leading to increases in uncertainties. Tian (2012) studies the risk taking choices of firms and predicts that small firms tend to take more risks and the fraction of risky firms rises in recessions, leading to countercyclical productivity dispersion. Bachmann and Moscarini (2011) studies the price setting of firms when the information about the elasticity of demand is imperfect. It predicts that bad economic times are good times for a firm to do price experiment to gain information on its market power and thus price dispersion increases during recessions. In their theories, the total risks in the whole economy rises in bad time, since firms create more idiosyncratic risks when a negative shock shifts them closer to the exit point where their value functions are locally convex. However, in this paper, I focus on how the investment risks are distributed to different groups of agents, while the total risks within the economy are unchanged.

This model also predicts significant correlation between the cross sectional dispersion and the market volatility in asset returns. This significant empirical pattern is well documented, but might not have been clearly explained by existing studies on uncertainty shocks. An increase in idiosyncratic risks, i.e. risks that can be reduced or eliminated through diversification, can indeed increase the cross sectional dispersion, but not necessarily the aggregate volatility. If there exists certain linkage between aggregate and idiosyncratic risks, what is it and how does it translate into the observed correlation between cross sectional dispersion and aggregate volatility? In this paper, I look at this phenomenon from a different viewpoint. Since the amounts of idiosyncratic and aggregate risks distributed to the lenders are both determined by the borrowers' financial conditions, thus dispersion and market volatility tend to move together.

The modelling of this paper is closely related to Boyd and Smith (1997), which studied the optimal capital structure in a similar CSV framework with two technologies, with one being subject to and one not being subject to the costly state verification problem. However, there are three main differences between the two papers: (1) The objectives of the two papers are very different. Their paper attempts to provide an endogenous mechanism that explains the use of equity and debt as financing tools, while this paper focuses on how the optimal contract determines the way risks are distributed to the borrower and the lender[8]. (2) Their paper focuses on how the optimal capital structure changes with monitoring costs, while this paper introduces the net worth of borrowers and focuses on how the optimal contract changes with the borrowers' financial conditions. (3) This model is also novel in its information structure. This paper includes aggregate risk and studies the market volatility in asset returns, while their work did not. In this paper, I assume that borrowers and lenders are unable to directly observe the realized aggregate state (even if monitoring happens) before repayment process is over. They can only observe the individual investment return, which consists of an aggregate component and an idiosyncratic component. The idiosyncratic components hide the aggregate component from them[9]. In such an environment, how the aggregate risks to be distributed to the borrower and the lender is determined by the financial condition of the borrower.

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[8] Actually, papers on security design and capital structure are closely related to each other. The former views the optimal contract as an optimal mechanism for mitigating frictions, while the later takes standard debt and equity contracts as given and focuses on the optimal mix of them (Allen and Winton 1994).

This paper explores a different implication of the CSV framework. The CSV framework has been the workhorse of macro economists for many years. Recently, there are growing literatures on "financial accelerator" which embed the costly state verification problem into DSGE models, such as, Bernanke and Gertler (1989), Carlstrom and Fuerst (1997) and Bernanke et al. (1999), to name a few. These papers focus mainly on how the financial friction, i.e. the CSV problem, amplifies real shocks and leads to fluctuations in investment, employment and output. Though this paper also takes the CSV problem as starting point, it focuses on the role played by financial friction in determining how risks are distributed to borrowers and lenders, and how this leads to fluctuations of the dispersion and the market volatility in asset returns over business cycles. When borrowers and lenders are risk neutral, it is the monitoring technology and bankruptcy cost minimization, rather than risk sharing, that determines how risks are distributed to the borrower and the lender. This could be another implication of costly state verification framework, which is less explored by economists.

The rest of this paper is organized as following. Section 2 describes the stylized facts on the countercyclical movements of the dispersions in ex ante and ex post stock returns, market volatility and financial condition indicators. Section 3 introduces a simple two period model which shows the key mechanism and implications. Section 4 contains the full model with infinite horizon and heterogeneous agents, which is calibrated and simulated. Section 5 concludes.

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[9] In reality, stock returns are contingent on the performances of firms, but not the performance of aggregate economy.

3.2 Some Empirical Facts

The countercyclical movements of the cross sectional dispersion and the market volatility in stock returns are well documented facts. In addition, cross sectional dispersion and market volatility are highly correlated to financial condition indicators. Campbell and Lettau (1999) and Campbell et al. (2001) showed that the dispersion and the market volatility of stock returns are countercyclical. Bloom (2009) shows that stock market volatility index, i.e. VXO, is countercyclical, and co-varies positively with the cross sectional dispersion of firm stock returns and firm profits. Gilchrist, Sim and Zakrajšek (2010) shows that stock return volatility is countercyclical and highly correlated with financial condition indicator (credit spread) at both firm and aggregate levels.

This paper studies how a negative first moment shock leads to increases in the cross sectional dispersion and the market volatility in asset returns via worsened financial conditions. I focus on the following measures: (1) the cross sectional dispersion in ex-post returns, i.e. the cross sectional standard deviation of actual stock returns; (2) market volatility, i.e. the implied volatility of stock market index; (3) financial condition indicator; (4) the cross sectional dispersion in ex-ante returns, i.e. the cross sectional standard deviation of expected stock returns.

Figure 1 illustrates the countercyclical movements of dispersion, volatility and financial conditions. The first panel of Figure 1 plots the dispersions in ex post (line with cycles) and ex ante (line with crosses) stock returns, as deviation from trend, where the trend is approximated using the average before 2006. The data are monthly stock returns from 2003 to 2012. The basket of selected stocks is the same as that in Bloom (2009). The series of ex ante stock returns are obtained by fitting the time series returns of each stock into an AR(1) model. The second panel of Figure 1 plots the VIX index (a popular measure of implied market volatility), as deviation from trend, where the trend is approximated using the average before 2006. The third panel of Figure 1 plots the Chicago Fed National

Financial Condition Index. The shaded bars indicate official NBER recessions.

Table 1 and 2 report the correlation coefficients of pairs of these measures. Table 1 reports the correlation coefficients of the cross sectional dispersion in actual stock returns and other measures, while Table 2 reports the correlation coefficients of market volatility and other measures. For each pair, there are three columns, where the first one is the correlation coefficient calculated using the data between 1985 M01-2012 M12 and the second one calculated using the data between 2003 M01-2012 M12. The basket of selected stocks is the same as that in Bloom (2009). Dispersion in ex-post and ex-ante stock returns are the cross sectional standard deviation of actual and expected stock returns. The series of expected stock returns are obtained by fitting the time series returns of each stock into an AR(1) model. The market volatility is VXO index. The financial condition is Chicago Fed National Financial Condition Index. To link my model to data, I also calculated these correlation coefficients using model simulated data, which are shown in the third columns. For the model simulated data, dispersion in ex-post and ex-ante returns are the cross sectional standard deviation of ex-post and ex-ante asset returns (conditional on that bankruptcy does not happen). The market volatility is the volatility of weighted average returns. The financial condition indicator is the spread between the marginal return of investment and borrowing interest rate [10].

3.3 A Simple Model

To highlight the key mechanism, I start with a tractable two period model with costly state verification problem. Some unessential features of the full model are removed and this section mainly focuses on the optimal financial contracting and the borrower's optimal choice of net worth in the settings with information friction between borrowers and lenders.

3.3.1 Setup

Environment. Time is discrete and there are two periods $t = 0, 1$. There is only one goods, which can be consumed or invested.

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[10] A detailed description of these measures can be found in section 3.4.4

Demography and Endowments. Agents are divided into two groups, which are borrowers and lenders. The measure of borrowers (lenders) is normalized to a unit. Each lender gets one unit of goods as endowment at date 0. Each borrower is identified by a number j, with $j \in [0, 1]$. Borrower j is endowed with w_j units of goods at date 0. The wealth distribution is denoted by a cumulative density function, $\varPhi(w)$. For convenience, I temporarily drop the borrower's index j in this section.

Preference. Lenders consume only at the second period and are risk neutral. Borrowers may consume at date 0 and/or date 1, and discount their consumption at date 1 by β. The objective of the borrower is to maximize his expected consumption in the two periods, $c + \beta E(c')$, where c and c' are consumption of the borrower at date 0 and 1 respectively.

Investment Technologies. All the borrowers and lenders have accesses to a storage technology that yields R units of goods tomorrow for each unit goods stored today, where R is exogenously given. Thus, R is the opportunity cost of lenders. In addition, each borrower is endowed with an investment opportunity which comes in discrete, non-divisible units, called "project". To start a project, a borrower has to invest *one* unit (assumed to be fixed) of goods. The borrower can invest in his own project. The amount of goods invested in the project by the borrower himself is called "net worth", which is denoted by n, with $n \in [0, w]$. The rest of the funds needed to start the project, $1 - n$, is borrowed from lenders. To borrow from lenders, the borrower signs a financial contract, denoted by $\mathcal{A}$, with the lenders.

Costly State Verification and Monitoring. There are two monitoring technologies [11]: ex ante monitoring and ex post monitoring. Ex-post monitoring happens only when the promised payment is not made and the lender takes all the residual, i.e. when bankruptcy happens. Ex-ante monitoring refers to setting up a mechanism at date 0, i.e. before the realization of investment revenue, which can credibly force the borrower to disclose his ex post private information, i.e. the actual investment revenue, to the lender at date 1. This institutional setting is close to that of a large firm, which issues public equity on stock market. As will be shown, the optimal financial contract signed between borrowers and lenders in this environment resembles outside equity; the repayment to lenders is contingent on the observable performance of the borrower.

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[11] This is very similar to the settings in Boyd and Smith (1997), which studied a similar CSV framework with two technologies, with one being subject to and one not being subject costly state verification problem.

When a project is started, the borrower can invest in two production technologies, technology z and u. The returns of these two technologies are both stochastic and have the same distribution. The only difference between the two technologies is that ex ante monitoring is only applicable to investment in technology z, while ex post monitoring is only applicable to investment in technology u. Individual production technology is a linear function. Let (z, u) be the individual shocks of the borrower. Technology z produces $e^z R_k$ units of goods tomorrow for each unit invested today and technology u produces $e^u R_k$ units of goods tomorrow for each unit invested today. Here, R_k is the aggregate rate of return, which is exogenously given and known in advance, and e^z and e^u are the individual shocks. When investing in technology z, a fraction $1 - \rho$ of initial investment is used to set up the ex ante monitoring at date 0, i.e. the cost of information disclosure[12]. The net (ex post) return of investment in technology z is $\rho e^z R_k$. However, the realized return of investment in technology z, i.e. $\rho e^z R_k$, is observable to both the borrower and the lender. The ex-ante monitoring reveals the realized state z of the borrower to the lender. When investing in technology u, ex ante technology is not applicable. The net return of investment in technology u is $e^u R_k$, which is only observable to the borrower. It can be observed by the lender only by bearing a cost. For the lender to observe u, he has to pay a cost which is a fraction μ of the unobservable residual, i.e. $\mu e^u R_k$.

The cost of ex ante monitoring is represented by $1 - \rho$. When ex ante monitoring is very costly, for example, $1 - \rho > \mu$, technology u is strictly preferred to technology z. Thus, the costly state verification problem becomes the textbook one and the optimal contract is standard debt: the borrower repays the lender a fixed amount if it is feasible and the lender takes all the residual when the promised amount is not paid. The lender does not take any risk, except for default risk, while the borrower takes all risks. When ex ante monitoring is free, i.e. $1 - \rho = 0$, technology z is strictly preferred to technology u. Thus the information asymmetry is eliminated and the optimal contract is pure equity. Since the repayment is perfectly correlated to the investment outcome, the lender takes more risks. When the ex ante monitoring cost, $1-\rho$, lies between 0 and μ, the optimal contract should be something between pure debt and pure equity. The risks of investment are distributed to both the borrower and the lender. The co-existence of ex-ante monitoring and ex-post monitoring potentially allows a variety of schemes to distribute risks to borrowers and lenders. To ensure that there is a trade-off between the two technologies, I assume that $0 < 1-\rho < \mu$.

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[12] For example, lenders may require the borrower to hire an independent auditor, who can enhance the credibility of information the borrower discloses, as a condition of lending.

Assumption 1 $0 < 1-\rho < \mu$

The problem of multiple monitoring is not taken into account in this model. One can think that there is a delegated intermediary between the borrower and the lender called "banker". Bankers are perfect competitive and earn zero profits by transferring funds from lenders to borrowers. There is information friction between borrowers and bankers, but no additional friction between lenders and bankers. Bankers collect repayments from borrowers and distribute them to lenders. When the borrower cannot pay the promised amount, the ex post monitoring is done by the delegated banker. Each banker signs a financial contract with only one borrower and bankers do not communicate with each other.

Risks. At date 1, the individual shocks (z, u) are randomly drawn. The individual shocks, i.e. (z, u), consist of two separate uncorrelated random components: an aggregate component s and idiosyncratic components, $(\varepsilon_z, \varepsilon_u)$:

$$z = s + \sigma_\varepsilon \varepsilon_z \qquad and \qquad u = s + \sigma_\varepsilon \varepsilon_u$$

where s denotes the aggregate component and ε_z and ε_u are the two idiosyncratic components which are i.i.d. across borrowers and have standard normal distributions. The sizes of the idiosyncratic components are σ_ε. The idiosyncratic components, ε_z, ε_u, and aggregate state s are independent. In addition, for the same borrower, ε_z and ε_u are independent of each other. Finally, the aggregate component s has a normal distribution with mean 0 and variance σ_s^2, i.e. $s \sim \mathcal{N}(0, \sigma_s^2)$.

A novel element of this model is its information structure. I assume that neither the borrower nor the lender can observe the aggregate state s directly before the repayment process is over, even if monitoring happens. Monitoring only reveals the individual states

(z, u), but not the aggregate state s. The idiosyncratic components ε_z and ε_u hide from the borrower and the lender the aggregate state s. The borrower's states, (z, u), can be high for one of two reasons: the aggregate state s is high, and/or the idiosyncratic components, $(\varepsilon_z, \varepsilon_u)$, are high.

Since the individual states z and u have the same aggregate component s, z and u are not independent of each other. They are independent only conditional on s, since their idiosyncratic components ε_z and ε_u are independent. Thus, the realization of z provides some information on the distribution of u. Since s and z have normal distributions and share the same random component, the joint distribution of s and z is given by:

$$\left[\begin{array}{c} s \\ z \end{array}\right] \sim N\left(\left[\begin{array}{c} 0 \\ 0 \end{array}\right], \left[\begin{array}{cc} \sigma_s^2 & \sigma_s^2 \\ \sigma_s^2 & \sigma_s^2 + \sigma_\varepsilon^2 \end{array}\right]\right)$$

Conditional on z, the conditional distribution of s is given by:

$$s|z \sim \mathcal{N}(\kappa z, \sigma_{s|z}^2)$$

with

$$\kappa = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_\varepsilon^2} \quad and \quad \sigma_{s|z}^2 = \frac{\sigma_s^2 \sigma_\varepsilon^2}{\sigma_s^2 + \sigma_\varepsilon^2}$$

Note that $u = s + \sigma_\varepsilon \varepsilon_u$. The distribution of u conditional on z is thus $u|z \sim N(\kappa z, \sigma_{u|z}^2)$, where $\sigma_{u|z}^2 = \sigma_{s|z}^2 + \sigma_\varepsilon^2$. Let $g(z)$ and $f(u)$ denote the density functions of z and u. Let $f(u|z)$ and $F(u|z)$ denote the density and cumulative density functions of u conditional on z.

3.3.2 The Borrower' Problem

The borrower is endowed with initial wealth w at date 0 and invests n, with $n \in [0, w]$ and $n \leq 1$, units into the project, which is called net worth. To start a project which needs an unit of goods as initial investment, the borrower borrows $1 - n$ units of goods from lenders. Once a project is started, the borrower has accesses to two different technologies: technology z and u, which are described in the last subsection. Let α be the share of funds invested in the technology z and $1 - \alpha$ be the share invested in the technology u. Thus, the realized investment revenue, $[\alpha \rho e^z + (1 - \alpha) e^u] R_k$, has two parts: an observable part, $\alpha \rho e^z R_k$ and an unobservable part, $(1 - \alpha) e^u R_k$.

Given the net worth n, the borrower borrows $1 - n$ from the lender. He signs a financial contract, denoted by $\mathcal{A}$, with the lender, which specifies (1) the investment plan, i.e. how the funds to be allocated between technology z and u (the value of α) and (2) repayment and (ex post) monitoring rules, which specifies how the repayment depends on the investment outcome and conditions under which (ex post) monitoring takes place. The borrower's wealth at date 1, w', equals the profits he earns from operating the project, which depends on the financial contract $\mathcal{A}$ and the realized individual shocks, (z', u'), i.e. $w' = \pi(\mathcal{A}, z', u')$, where π is a function that maps financial contract and realized individual shocks into the borrower's profits.

Given the initial wealth w endowed, the borrower chooses net worth n and the financial contract, $\mathcal{A}$, to maximize his expected consumption in the two periods. Thus,the borrower's optimization problem is given by:

Problem I (The Borrower's Problem)

$$V(w) = \max_{\{n \in [0,w], \mathcal{A} \in \Omega(n)\}} c + \beta E(c') \tag{3.1}$$

subject to

$$0 \leq c \leq w - n \qquad 0 \leq c' \leq w' \qquad w' = \pi(\mathcal{A}, z', u')$$

The borrower chooses the amount of net worth he puts in his project, i.e. n, and the financial contract he signs with the lender, $\mathcal{A} \in \Omega(n)$, to maximize his expected consumption, where $\Omega(n)$ is the set of possible financial contracts given the net worth n. The borrower's choices on n and $\mathcal{A}$ is called the borrower's *financial decision.*

Note that the first two constraints always bind and thus $c = w - n$ and $c' = w'$. The borrower's problem is to be solved in two steps. First, taking the borrower's net worth n as given, solve for the optimal financial contract, $\mathcal{A}^*$, with $\mathcal{A}^* \in \Omega(n)$, which maximizes his expected profits, $E(w')$, where $w' = \pi(\mathcal{A}, z', u')$ and π is a function that maps financial contract and realized shocks to the borrower's profits. This is the optimal financial contracting problem:

Problem I.1 (Optimal Financial Contracting):

$$\pi^*(n) = \max_{\mathcal{A} \in \Omega(n)} E(w') \qquad s.t. \qquad w' = \pi(\mathcal{A}, z', u')$$

In the first step, I solve for the optimal contract $\mathcal{A}^*$ that maximizes the borrower's expected profits and then obtain the profit function $\pi^*(n)$, which maps the net worth n into the borrower's expected profits.

Second, with the obtained profit function $\pi^*(n)$, I will turn to the optimal choice of net worth n^*:

Problem I.2 (Optimal Choice of Net Worth)

$$V(w) = \max_{n}(w - n) + \beta\pi^*(n)$$

Thus, the solution to the borrower's problem (*Problem I*), n^* and $\mathcal{A}^*$, is obtained by solving *Problem I.1* and *Problem I.2*.

3.3.3 Financial Contract

In this subsection, I focus only on the optimal financial contracting problem: given the borrower's net worth n, solve for the optimal contract $\mathcal{A}^*$, which maximizes the borrower's expected profits $E(w')$ (*Problem I.1*). In the next subsection, I will turn to the optimal choice of net worth, n^* (*Problem I.2*).

As mentioned, given the borrower's net worth n, the financial contract signed between the borrower and the lender specifies: (1) investment plan (α): how the funds to be allocated between technology z and u. (2) repayment and (ex post) monitoring rules. The repayment and monitoring rules are functions of the borrower's states, i.e. (z, u). Since z is observable to both the borrower and the lender, the repayment rules can always be contingent on z. The repayments can be contingent on u only when ex post monitoring happens. Let $A(z)$ denote the (ex post) monitoring region and $A^c(z) = (-\infty, +\infty) - A(z)$ denote the non (ex post) monitoring region, given the realization of z. Let $x(z)$ be the promised repayment to the lender in the non monitoring region and $r(z, u)$ be the repayment when ex post monitoring takes place.

In sum, a financial contract is a set of contract terms, which specifies investment plans, repayment and monitoring rules, i.e. $\mathcal{A} = \{\alpha, x(z), r(z,u), A(z)\}$. I focus on a set of financial contracts, which are feasible, incentive compatible and assure the lender with an expected return no less than their opportunity cost, R. In addition, short sale is not allowed, i.e. $0 \leq \alpha \leq 1$ must hold. Thus, we have four constraints on the financial contracts:

Feasibility. The financial contract should be feasible. Under the assumption of limited liability, neither the borrower nor the lender can get a negative return:

$$\begin{aligned} 0 \leq x(z) &\leq [\alpha\rho e^z + (1-\alpha)e^u]R_k \\ 0 \leq r(z,u) &\leq [\alpha\rho e^z + (1-\alpha)e^u]R_k \end{aligned} \tag{3.2}$$

Incentive Compatibility. The financial contract should also be incentive compatible, such that the borrower never has incentive to cheat. It requires that the repayment in the monitoring region is no more than that in the non monitoring region:

$$x(z) \geq r(z,u) \quad for\ all\ z \tag{3.3}$$

Expected Return Constraint. The lender should get an expected return no less than his opportunity cost R. Let $\psi(z,u)$ denote the cost of ex post monitoring. The expected return constraint is given by:

$$\int_{-\infty}^{\infty} \left\{ \int_{u \in A^c(z)} x(z) f(u|z) du + \int_{u \in A(z)} [r(z,u) - \psi(z,u)] f(u|z) du \right\} g(z) dz \geq (1-n)R \tag{3.4}$$

No Short Sale. I assume that the borrower cannot "short sell" investment in project and only allocates a non-negative amount of resources to either of the two investment technologies. Thus, $0 \leq \alpha \leq 1$. In this subsection, I temporarily assume that this constraint never binds. In the next subsection, I will explain why this is true.

Let $\Omega(n) \equiv \{\{\alpha, x(z), r(z,u), A(z)\} | (2)-(4)\ \mathit{hold}\}$ be the set of financial contracts, which satisfy constraints (2)-(4). Given the net worth the borrower puts in the project, the optimal contract $\mathcal{A}$ is chosen from $\Omega(n)$ to maximize the borrower's expected profits.

Given the borrower's choice of n, the financial contract $\mathcal{A}$ maximizes his expected profits, $E(w')$. The optimal financial contracting problem (*Problem I.1*) can be written as:

Problem I.1.1 Optimal Financial Contracting

$$\pi^*(n) = \max_{\mathcal{A}\in\Omega(n)} E[\alpha\rho e^z + (1-\alpha)e^u]R_k - (1-n)R - \int_{u\in A(z)} \psi(z,u)f(u|z)du \tag{3.5}$$

Given the interest rates R and R_k, and the net worth n, the optimal contract maximizes (5) subject to (2)-(4).

Three steps of solving for the optimal contract. Given the borrower's net worth n, the optimal contract is to be solved for in three steps: (1) First, given the investment plan α, solve the optimization problem for each state z; (2) Second, solve the optimization problem across state z, under the given α; (3) Finally, solve for the optimal α.

Given the borrower's net worth n, if the expected revenue from investment in technology z is large enough to pay his debt, i.e. $(1-n)R \leq \alpha\rho E(e^z)R_k$, there exists a trivial solution for this financial contracting problem. Note that $1-n$ is the amount borrowed from the lender and R is the lender's opportunity cost. Thus, the expected repayment the borrower should assure the lender with, i.e. $(1-n)R$, is less than the expected value of the observable revenue, $\alpha\rho E(e^z)R_k$. Then, the borrower can simply assure the lender with a fraction $\gamma \leq 1$ of the observable revenue $\alpha\rho e^z R_k$, such that, $(1-n)R = \gamma\alpha\rho E(e^z)R_k$. In such cases, no ex post monitoring is needed. However, as will be shown in the next subsection, this actually never happens. The reason is that the marginal cost of ex post monitoring is nearly zero when $(1-n)R \leq \alpha\rho E(e^z)R_k$ and it is optimal for the borrower to reduce α in such situations. This will be discussed in detail in the next subsection.

When $(1-n)R > \alpha\rho E(e^z)R_k$, then $x(z) \geq \alpha\rho e^z R_k$ should hold at least for some states of z. Note that $x(z)$ is the promised payment outside the monitoring region and $\alpha\rho e^z R_k$ is the observable revenue. Thus, $x(z) - \alpha\rho e^z R_k$ is the difference between the promised repayment and the observable revenue. For the states of z where $x(z) \geq \alpha\rho e^z R_k$, it is helpful to define:

$$u_z^c = ln\left[\frac{x(z) - \alpha\rho e^z R_k}{(1-\alpha)R_k}\right] \tag{3.6}$$

where u_z^c is the critical value of u, given the state z, and if $u < u_z^c$ the repayment $x(z)$ will be infeasible. If $x(z) = \alpha\rho e^z R_k$, $u_z^c = -\infty$.

Intuitively, given the investment plan α and the realization of observable state z, it is actually a standard CSV problem with only one unobservable state. The optimal contract is a standard debt contract: the borrower repays a pre-determined amount, i.e. $x(z)$, when it is feasible; otherwise, ex post monitoring happens and the lender takes away the residual. However, the promised repayment outside the monitoring region, i.e. $x(z)$, may depend on the realization of observable state z.

Lemma 1 *When $0 \leq \alpha \leq 1$ does not bind and $(1-n)R > \alpha\rho E(e^z)R_k$, the optimal contract has:*

(1) $x(z) \geq \alpha\rho e^z R_k$ for all z

(2) $A(z) = (-\infty, u_z^c)$ for all z

(3) If $u \in A(z)$, then $r(z,u) = [\alpha\rho e^z + (1-\alpha)e^u]R_k$

The proof of Lemma 1 can be found in Appendix 1. Given investment plan α and the realization of z, the optimization problem is similar to that in the conventional CSV models. Lemma 1 asserts that ex post monitoring never happens if it is feasible to repay $x(z)$. When the promised repayment $x(z)$ is infeasible, ex post monitoring happens and the lender takes away all the residual.

Now, the first step is done.

Note that the cost of ex post monitoring $\psi(z,u)$ is fraction μ of the unobservable residual, i.e. $\psi(z,u) = \mu(1-\alpha)e^u R_k$. By combining the results in *Lemma 1*, the optimization problem (*Problem I.1.1*) can be reduced to:

Problem I.1.2 Optimal Financial contracting

$$\max_{\{u_z^c,\alpha\}} E[\alpha\rho e^z + (1-\alpha)e^u]R_k - (1-n)R - \mu(1-\alpha)R_k \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{u_z^c} e^u f(u|z)du\right] g(z)dz$$

subject to:

$$R_k \int_{-\infty}^{+\infty} \left\{\alpha\rho e^z + (1-\alpha)\left[e^{u_z^c}(1-F(u_z^c|z)) + (1-\mu)\int_{-\infty}^{u_z^c} e^u f(u|z)du\right]\right\} g(z)dz = R(1-n)$$

In *Problem I.1.2*, the choices of the borrower are reduced to $\{u_z^c, \alpha\}$.

For the revenue of investment in technology u, it is helpful to define $\mathit{\Gamma}(u_z^c|z)$ to be the *lender's share* and $\mathit{\Psi}(u_z^c|z)$ to be *dead weight loss* of ex post monitoring, given the realization of z:

$$\mathit{\Gamma}(u_z^c|z) \equiv e^{u_z^c}[1 - F(u_z^c|z)] + \int_{-\infty}^{u_z^c} e^u f(u|z) du \tag{3.7}$$

$$\mathit{\Psi}(u_z^c|z) \equiv \mu \int_{-\infty}^{u_z^c} e^u f(u|z) du \tag{3.8}$$

The borrower chooses the critical u_z^c for each z, and α to maximize his expected profits.

Optimal choice of u_z^c (Problem I.1.2). The first order condition for u_z^c is given by:

$$\frac{1}{1+\lambda} = \frac{\mathit{\Gamma}'(u_z^c|z) - \mathit{\Psi}'(u_z^c|z)}{\mathit{\Gamma}'(u_z^c|z)} \tag{3.9}$$

where λ is the Lagrangian multiplier related to the expected return constraint.

Marginal Rate of Transformation. Here, $\frac{1}{1+\lambda}$ is the *marginal rate of transformation*[13], and henceforth MRT: to increase the expected repayment to the lender by one unit, it requires $1 + \lambda$ unites decrease in the borrower's expected profit. Thus, λ is the loss rate of transformation: to in crease the lender's expected revenue by one unit, the expected cost of (ex post) monitoring increase by λ units.

Price of Net Worth. Note that the price or marginal return of net worth is given by $(1 + \lambda)R$. When $\lambda > 0$, that is, the MRT is less than 1, the inside funds is more valuable than the outside funds due to financial friction.

According to equation (9), when MRT is low and transformation loss is high, the marginal return of net worth is high. The borrower chooses different u_z^c across z, such that the MRT is kept constant across z. This means that the borrower chooses u_z^c to smooth the ex post monitoring cost across z.

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[13] This concept was used in Labadie 1998.

Note that

$$\Gamma'(u_z^c|z) = e^{u_z^c}[1 - F(u_z^c|z)] \tag{3.10}$$

$$\Psi'(u_z^c|z) = \mu e^{u_z^c} f(u_z^c|z)dz \tag{3.11}$$

Here, $f(u|z)$ and $F(u|z)$ denote the p.d.f. and c.d.f. of u conditional on z. Since the distribution of u condition on z is $u|z \sim \mathcal{N}(\kappa z, \sigma_{u|z}^2)$[14], then $(u - \kappa z) \sim \mathcal{N}(0, \sigma_{u|z}^2)$. Note that $\sigma_{u|z}^2 = \frac{\sigma_s^2\sigma_\varepsilon^2}{\sigma_s^2+\sigma_\varepsilon^2} + \sigma_\varepsilon^2$ does not depend on the realization of z. Let $\tilde{f}(u-\kappa z)$ and $\tilde{F}(u-\kappa z)$ denote the p.d.f. and c.d.f. of $u - \kappa z$. Then $F(u_z^c|z) = \tilde{F}(u_z^c - \kappa z)$ and $f(u_z^c|z) = \tilde{f}(u - \kappa z)$. Then equation (9) can be re-written as:

$$\frac{1}{1+\lambda} = 1 - \frac{\mu \tilde{f}(u_z^c - \kappa z)}{1 - \tilde{F}(u_z^c - \kappa z)} \tag{3.12}$$

Note that $\frac{1}{1+\lambda}$ is monotonically decreasing in $u_z^c - \kappa z$, at least over the relevant values of $u_z^c - \kappa z$ [15]. Thus, there exists an unique $u_0 = u_z^c - \kappa z$, such that,

$$\frac{1}{1+\lambda} = 1 - \frac{\mu \tilde{f}(u_0)}{1 - \tilde{F}(u_0)} \tag{3.13}$$

Now, we have finished the second step of the proof. The borrower chooses the critical value u_z^c of u, defined by (6), according to the realization of z, i.e. $u_z^c = u_0 + \kappa z$. It has two parts, u_0 and κz. Let us call u_0 unconditional critical value, as opposed to critical value u_z^c. Since the individual states u and z have the same aggregate component and the realization of z updates the distribution of u, thus the borrower adjusts the critical value u_z^c according to the realization of z in order to smooth the bankruptcy cost across z.

Now, the second step is done.

Lemma 2 *(Optimal Choice of u_z^c, Problem 1.1.2)*
The borrower chooses critical values u_z^c to smooth the bankruptcy cost across z. The rules for u_z^c is given by: $u_z^c = u_0 + \kappa z$ for all z..

Optimal choice of α (Problem I.1.2). Define $\hat{\Gamma}(u_0)$ to be the expected lender's share and $\hat{\Psi}(u_0)$ to be the expected dead weight loss of ex post monitoring. Using the results in *Lemma 2*, $\hat{\Gamma}(u_0)$ and $\hat{\Psi}(u_0)$ are given by:

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[14] See section 3.1

[15] Since $u_z^c - \kappa z$ has normal distribution, $\frac{1}{1+\lambda}$ is actually monotonically decreasing over $(-\infty, +\infty)$.

$$\hat{\Gamma}(u_0) = \int_{-\infty}^{+\infty} \Gamma(u_z^c|z)g(z)dz = E(e^{\kappa z})\left\{e^{u_0}[1-\tilde{F}(u_0)] + \int_{-\infty}^{u_0} e^t \tilde{f}(t)dt\right\}$$
$$\hat{\Psi}(u_0) = \int_{-\infty}^{+\infty} \Psi(u_z^c|z)g(z)dz = \mu E(e^{\kappa z})\int_{-\infty}^{u_0} e^t \tilde{f}(t)dt$$

Then, the expected return constraint in *Problem I.1.2* can be written as:

$$(1-n)R = \alpha\rho E(e^z)R_k + (1-\alpha)(\hat{\Gamma}(u_0) - \hat{\Psi}(u_0))R_k \tag{3.14}$$

The first order condition for α *(Problem I.1.2)* is thus given by:

$$\rho E(e^z) = \frac{1}{1+\lambda}[E(e^u) - \hat{\Gamma}(u_0)] + [\hat{\Gamma}(u_0) - \hat{\Psi}(u_0)] \tag{3.15}$$

Now, the third step is done.

At the beginning of this section, I mentioned that $(1-n)R > \alpha\rho E(e^z)$ always holds and now here is the reason why this is true. Let us assume that for a given α, $(1-n)R = \alpha\rho E(e^z)$. When the observable revenue, $\alpha\rho E(e^z)$, is enough to repay the lender, it is optimal for the borrower to use observable revenue to repay and keep the unobservable for himself. For the expected constraint (14) to hold, the unconditional critical value u_0 drops to $-\infty$. In such situation, the *marginal rate of transformation* (13) rises to 1, which means the marginal cost of transferring revenue of investment in technology u to the lender is zero. Thus, it is optimal for the borrower to invest more in technology u, which has a higher net return.

To see this, note that the expected lender's share $\hat{\Gamma}(u_0)$ and dead weight loss of ex post monitoring $\hat{\Psi}(u_0)$ drop to zero, when u_0 drops to $-\infty$. Note that $E(e^u) = E(e^z)$ and $\rho < 1$. The RHS of equation (15) equals $E(e^u)$, which is larger than the LHS, $\rho E(e^z)$. Thus, it is optimal for the borrower to lower the amount invested in technology z and invest more on technology u by reducing α, which leads to $(1-n)R > \alpha\rho E(e^z)$.

Now, I have solved the optimal financial contracting problem (*Problem I.1.1*) under the assumption that $0 \leq \alpha \leq 1$ does not bind. In the next subsection, I will show why this is true when the parameter values are appropriately assigned. By combing *Lemma*1 and all the other results obtained in this subsection, the solution to *Problem I.1.1* are summarized in *Proposition 1.*

Proposition 1 *(Problem I.1.1)*

Given the net worth n *and that* $0 \le \alpha \le 1$ *does not bind, the optimal financial contract* $\mathcal{A}^*$ *is specified by:*

(1) $x(z) = [\alpha\rho e^{z} + (1-\alpha)e^{u_0+\kappa z}]R_k$

(2) $r(z,u) = [\alpha\rho e^{z} + (1-\alpha)e^{u}]R_k$

(3) $A(z) = (-\infty, u_0 + \kappa z)$

where $\{u_0, \alpha, \lambda\}$ *are determined by (13),(14) and (15).*

3.3.4 Static Analysis

In the subsection 3.3, I solved the optimal financial contracting problem (*Problem I.1.1*) under the assumption that the no short sale constraint, i.e. $0 \le \alpha \le 1$, does not bind. The results are summarized in *Proposition 1.*

This subsection begins with describing the trade off between ex ante and ex post monitoring. Then, it describes the borrower's optimal choice of α and explains why the no short sale constraint $0 \le \alpha \le 1$ does not bind if parameter values are appropriately assigned. It also discusses on how the choice of α determines the way risks are distributed to the lenders. In addition, it explains how the borrower chooses net worth n optimally and how a negative shock increases the heterogeneity in financial decisions. Finally, it gives formal definitions of dispersion and volatility in the model context and shows how they change with aggregate economic conditions.

The Trade-off between Ex-ante and Ex-post Monitoring

According to *Proposition 1*, the optimal financial contract can be characterized by a set of terms, i.e. $\{u_0, \alpha, \lambda\}$, which are determined by equations (13), (14) and (15). The optimal u_0 and λ, are determined by (13) and (15). The optimal u_0 and λ do not depend on the net worth n. The optimal α is then determined by the expected return constraint (14) after u_0

and λ. A change in net worth n only changes the optimal choice of α, while the optimal u_0 and λ remain the same. By the same token, a change in aggregate return R_k also changes the optimal choice of α only, while the optimal u_0 and λ are kept the same.

The intuition for this result is as follows: the ex-ante monitoring has a linear cost, i.e. $1-\rho$, which is determined by the value of parameter ρ, while the ex-post monitoring cost is non-linear, which increases in the unconditional critical value u_0. When $0 \leq \alpha \leq 1$ does not bind, the borrower can substitutes between the two technologies freely. Thus, the marginal cost of ex-ante and ex-post monitoring should be equal; otherwise, it is optimal to substitute between the use of the two technologies. Then, the ex ante monitoring cost, i.e. $1-\rho$, pins down the optimal values of u_0 and thus λ.

Lemma 3 *When $0 \leq \alpha \leq 1$ does not bind, a change in net worth n or aggregate return R_k only changes the optimal allocation of funds between the two technologies, i.e. the optimal α, while the optimal choices of u_0 and λ are kept the same.*

To see this, let us take a look at how the borrower chooses α and allocates funds between the two technologies. For the revenue from a unit of investment in technology z, the borrower can transfer all of it to the lender without additional monitoring. Thus, this part of revenue, $\rho E(e^z)R_k$, is *pledgeable* to the lender. For the revenue from a unit of investment in technology u, the borrower transfers only a part of it to the lender and keeps the rest. Note that $\hat{\Gamma}(u_0)$ is the expected lender's share and $\hat{\Psi}(u_0)$ is the expected dead weight loss of bankruptcy. The *pledgeable* part is thus $[\hat{\Gamma}(u_0) - \hat{\Psi}(u_0)]R_k$. The rest part, i.e. $[E(e^u) - \hat{\Gamma}(u_0)]R_k$, is to be kept by the borrower and is *non-pledgeable*. To increase the *pledgeable* part, the borrower has to increase the unconditional critical value u_0, but this will increase the cost of bankruptcy and lowers the *marginal rate of transform* (13)[16].

Intuitively, the borrower should allocate funds between technology z and u so that the marginal return on the two technologies are equal; otherwise, it is always optimal to substituting between them. By multiplying both sides of equation (15) by R_k, we can obtain:

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[16] The *pleadgeable* part of revenue from investment in technology u, i.e. $[\hat{\Gamma}(u_0) - \hat{\Psi}(u_0)]R_k$, may decrease in u_0 when u_0 is very large, as shown in Bernanke et al. (1999). Actually, $[\hat{\Gamma}(u_0) - \hat{\Psi}(u_0)]R_k$ is concave function of u_0 and it reached its maximum at $u_0 = u_0^m$. When $u_0 > u_0^m$, $[\hat{\Gamma}(u_0) - \hat{\Psi}(u_0)]R_k$ is decreasing in u_0. However, when parameters are set appropriately, u_0 never exceeds u_0^m in equilibrium.

$$\underbrace{\rho E(e^z)R_k}_{pledgeable} = \frac{1}{1+\lambda}\underbrace{[E(e^u) - \hat{\Gamma}(u_0)]R_k}_{non-pledgeable} + \underbrace{[\hat{\Gamma}(u_0) - \hat{\Psi}(u_0)]R_k}_{pledgeable} \tag{3.16}$$

The left hand side of above equation is the marginal return of investment in technology z, which is completely *pledgeable*. The right hand side is the marginal return of investment in technology u, which has two parts, a *pledgeable* part and *non-pledgeable* part. The *non-pledgeable* revenue is less valuable than the *pledgeable* revenue, so its value is adjusted by the *marginal transform rate*, $\frac{1}{1+\lambda}$. The marginal return of investing in technology z is constant. However, the marginal return of investing in technology u is decreasing in u_0. When u_0 increases, the dead weight loss of monitoring, $\hat{\Psi}(u_0)$, increases and MRT decreases. The optimal α is chosen such that the marginal returns of technology z and u are equal, i.e. equation (15) holds. This shows how the borrower balance the trade off between ex ante and ex post monitoring.

The marginal returns of investment in technology z and u are shown in the Figure 2. Note that for an α, there exists a u_0 such that the expected return constraint (14) holds. When α is lower than its optimal level, the unconditional critical value u_0 has to increase above its optimal value for the expected return constraint (14) to hold. Then the marginal return of investment in technology u (RHS of 16) decreases and it is optimal for the borrower to invest less in technology u, i.e. raise α. When α is higher than its optimal level, u_0 drops. Then, the marginal return of investment in technology u (RHS of 16) increases and it is optimal for the borrower to increase investment in technology u, i.e. reduce α. An increase in the borrower's net worth shifts the curve of marginal return of investment in technology u to the left and thus decreases the optimal α.

Optimal Choice of α and Dispersion and Volatility in Asset Returns

All the analysis above depends on the assumption that $0 \leq \alpha \leq 1$ does not bind. Here, I will show why this is true if the parameter values are assigned appropriately. By re-arranging the expected return constraint (14), we have:

$$1 - \alpha = \frac{\rho E(e^z)R_k - R + nR}{\rho E(e^z)R_k - [\hat{\Gamma}(u_0) - \hat{\Psi}(u_0)]R_k} \tag{3.17}$$

Note that according to *Lemma 3*, when $0 \leq \alpha \leq 1$ does not bind, a change in net worth n only changes α, while u_0 and λ are kept the same. Thus, the share invested in technology u, $1-\alpha$, is linearly increasing in the net worth n. When the net worth is high, the borrower invests more in technology u which has a higher net return but worsens the agency problem; when the net worth is low, the borrower invests more in technology z, which has a lower net return but can help mitigate the agency problem (due to ex ante monitoring). This equation shows how the borrower chooses between ex ante and ex post monitoring given different levels of net worth.

The net worth, n, ranges from 0 to 1. Thus, α may hits its bounds when the net worth is extremely small or large. When the borrower puts little funds in the project, i.e. n is low, α may hit its upper bound 1. According to equation (17), to prevent α from going above 1, $\rho E(e^z)R_k - R > 0$ must hold.

Assumption 2 *ρ is large enough such that $\rho E(e^z)R_k - R > 0$ holds.*

According to *Assumption 2*, even if the borrower puts nothing in the project, i.e. $n = 0$, it is not optimal to invest only in the technology z. Thus, $\alpha \leq 1$ never binds.

According to (17), when the net worth n is large, α may hit its lower bound 0. There exists a critical value of net worth, n_{max}, such that $\alpha \geq 0$ binds when $n > n_{max}$. According to (17), the n_{max} is given by:

$$n_{max} = 1 - [\hat{\Gamma}(u_0) - \hat{\Psi}(u_0)]\frac{R_k}{R} \tag{3.18}$$

When $0 \leq \alpha$ does not bind, the optimal u_0 and λ are pinned down by the parameter ρ, i.e. the cost of ex ante monitoring. Thus, the marginal return of net worth, $(1+\lambda)R$, does not change with the amount of net worth or aggregate return. What will happen when $0 \leq \alpha$ binds? When α hits its lower bound, the borrower cannot freely substitute between the two technologies, even if the marginal return of investment in technology z, i.e. the LHS of (16), is less than the marginal return of investment in technology u, i.e. the RHS of (16). The first order condition of α (15), is then replaced by constraint $\alpha = 0$. When $n > n_{max}$, $0 \leq \alpha$ binds. For the expect return constraint (14) to hold, an increase in net worth leads to a decrease in u_0. As a results, the *marginal rate of transformation* increases (13) and then the marginal return of net worth, i.e. $(1+\lambda)R$, decreases.

Lemma 4 *There exists a n_{max} such that:*

(i) when $n \leq n_{max}$, the marginal return on net worth, i.e. $(1+\lambda)R$, does not change with net worth n or aggregate return R_k;

(ii) when $n > n_{max}$, $\alpha \geq 0$ binds and the marginal return of net worth, i.e. $(1+\lambda)R$, becomes decreasing in net worth n.

I assume that the inverse of discount factor of borrowers, i.e. β^{-1}, exactly equals to the marginal return of net worth when $0 \leq \alpha \leq 1$ does not bind, i.e. $(1+\lambda)R$, where λ is determined by (13)-(15). When $n > n_{max}$, the marginal return of net worth is less than the inverse of discount factor. Thus, it is never optimal for the borrower to invest more than n_{max} in his own project and then $0 \leq \alpha$ never binds.

Assumption 3

(a) $\beta^{-1} = (1+\lambda)R$, where $(1+\lambda)R$ is the marginal return of net worth when $0 \leq \alpha \leq 1$ does not bind.

(b) When it is indifferent between investing in the project and consuming, the borrower chooses to invest.

In sum, under *Assumption 2* and *3*, the no short sale constraint, i.e. $0 \leq \alpha \leq 1$ never binds.

Lemma 5 *Under assumption 2 and 3, the no short sale constraint, i.e. $0 \leq \alpha \leq 1$ never binds.*

Risks Distributed to Lenders. Now, let us turn to one of the key questions of this paper: how aggregate and idiosyncratic risks are distributed to the borrower and the lender? How the dispersion and the market volatility in asset returns depend on the borrower's net worth?

According to *Proposition 1*, the repayment to the lender outside the (ex post) monitoring region is given by:

$$x(z) = [\alpha\rho e^{z} + (1-\alpha)e^{u_0+\kappa z}]R_k \tag{3.19}$$

Note that $\kappa < 1$ and that the first part of $x(z)$, i.e. ρe^z, is more volatile than its second part, i.e. $e^{u_0+\kappa z}$. Also note that $z = s + \sigma_\varepsilon \varepsilon_z$ consists of an aggregate component s and an idiosyncratic component ε_z. When the borrower's net worth n is low, the *marginal transformation rate* decreases and bankruptcy cost increases. Thus, the borrower increases the share of funds invested in technology z, i.e. increases the use of ex ante monitoring, which can help mitigate the agency problem and lower bankruptcy cost (17). This increases α and $x(z)$ becomes more volatile. As a results, the optimal contract becomes more equity like in the sense that its payment becomes more correlated to the observable performance of the borrower. Consequently, the lender takes more idiosyncratic and aggregate risks and the dispersion and the market volatility in asset returns increase.

Optimal Choice of n and Heterogeneity in Financial Decisions

According to *Lemma 4* and *Assumption 3*, when the initial wealth of the borrower w is larger than n_{max}, the borrower only invests n_{max} units of goods in his project and consumes the rest, since the marginal return of net worth is less than his marginal utility of consumption when $n > n_{max}$. When the initial wealth of the borrower w is less than n_{max}, he invests all his wealth into his project. These results are summarized in *Proposition 2*.

Proposition 2 *(Problem I.2, Optimal Choices of Net Worth n)*
Given the borrower's initial wealth w:
(a) if $w \leq n_{max}$, the borrower invests all his wealth into his project, i.e. $n = w$;
(b) if $w > n_{max}$, the borrower invests n_{max} units of goods into his project, i.e. $n = n_{max}$, and consumes the rest, i.e. $c = w - n_{max}$.
where n_{max} is defined by Equation (19).

Thus, the borrower with initial wealth $w > n_{max}$ is "*non-financial constrained*" in the sense that the marginal return of investing his wealth in his project is less than the marginal utility of consumption.

Definition 1 *Borrower j is non financially constrained if and only if the marginal return of investing his wealth in his own project is less than the marginal utility of consumption, i.e. $w_j > n_{max}$.*

Non financially constrained borrowers always make the same financial decisions, invests n_{max} in their projects, and chooses the same investment plan, $i.e.\alpha$. However, the amounts invested as net worth by the financially constrained borrowers are determined by their wealth, i.e. $n_j = w_j$. Thus, they choose different n_j and α_j, based on their wealth level w_j. The financial contracts they sign with the lenders depend on their initial wealth levels.

As shown in Figure 4, when a negative shock shifts the borrowers' wealth distribution to the left, the fraction of financial constrained borrowers increases. This will increases the heterogeneity in financial decisions, which finally leads to increase in the dispersion in ex post return.

Countercyclical Dispersion and Volatility

Before analysing how the cross sectional dispersion and the market volatility in asset returns change with individual and aggregate financial conditions, it is helpful to formally define these measures we are interested in. The simple two period model does not have business cycle features. Instead, I will analyse the comparative statics that mimics different times of business cycles. In the next section, I will describe the full model with infinite horizons and business cycle features.

Risks. For a borrower indexed by j, with $j \in [0, 1]$, the individual shocks, denoted by (z_j, u_j), consist of two components: an aggregate components s and idiosyncratic components $(\varepsilon_{z,j}, \varepsilon_{u,j})$:

$$z_j = s + \sigma_\varepsilon \varepsilon_{z,j} \quad and \quad u_j = s + \sigma_\varepsilon \varepsilon_{u,j}$$

The idiosyncratic components $(\varepsilon_{z,j}, \varepsilon_{u,j})$ are i.i.d. across agents and for any borrower j, $\varepsilon_{z,j}$ and $\varepsilon_{u,j}$ are independent and have the standard normal distribution. Then, the individual shocks (z_j, u_j) are i.i.d. across borrowers conditional on aggregate state s. The aggregate state s has a normal distribution, with mean 0 and variance σ_s^2, i.e. $s \sim \mathcal{N}(0, \sigma_s^2)$. Thus, σ_ε and σ_s represents the sizes of idiosyncratic and aggregate risks respectively.

Bankruptcy over the Business Cycle. As mentioned, the bankruptcy (ex post monitoring) occurs when the realization of the unobservable shock u is below its critical value u_z^c. Given the aggregate shock s, define $B(s) \equiv \{j \ |u_j > u_{z_j}^c\}$ to be the set of borrowers for whom bankruptcy, i.e. ex post monitoring, does not take place. The critical value for bankruptcy is given by $u_z^c = u_0 + \kappa z$. Using the definitions of z_j and u_j, the set for non-bankrupt borrowers can be written as:

$$B(s) \equiv \{j \mid u_j > u_{z_j}^c\} = \{j \mid \varepsilon_{u,j} - \kappa \varepsilon_{z,j} > u_0 - (1-\kappa)s\}$$

According to *Lemma 3*, the borrower's choice of unconditional critical value u_0 does not depend on the borrower's net worth n_j. In other words, all the borrowers choose the same u_0. In addition, $\varepsilon_{z,j}$ and $\varepsilon_{u,j}$ are i.i.d. across borrowers. The linear combination of them two, $\varepsilon_{u,j} - \kappa\varepsilon_{z,j}$, are also i.i.d. across borrowers. Thus, each individual borrower actually faces the same incidence of bankruptcy. Since $\kappa < 1$, when the aggregate state s is low, $B(s)$ becomes a larger set and more borrowers go to bankruptcy.

Let $q(s)$ denote the bankruptcy rate given the aggregate state s. Then the bankruptcy rate is given by:

$$q(s) = 1 - \int_{j \in B(s)} 1 dj$$

Let *Prob*(.) denote the probability function. The bankruptcy rate $q(s)$ can be written as (assuming that law of large number holds):

$$q(s) = 1 - Prob(\varepsilon_u - \kappa\varepsilon_z > u_0 - (1-\kappa)s) \tag{3.20}$$

When the aggregate state s is low, the bankruptcy rate $q(s)$ increases.

Dispersion in Ex post Return. Given the aggregate state s, let $r_{j,s}$ be the ex post return of the asset issued by borrower j, conditional on that ex post monitoring does not happen, i.e. with $j \in B(s)$. Then, $r_{j,s}$ is given by:

$$r_{j,s} = \frac{\left[\alpha_j \rho e^{z_j} + (1-\alpha_j) e^{u_0 + \kappa z_j}\right] R_k}{1 - n_j} \tag{3.21}$$

where α_j denotes the investment plan of borrower j and $1 - n_j$ is the total amount of funds borrowed by the borrower j.

The cross sectional dispersion in ex post asset returns is the standard deviation of ex post assets returns (conditional on that ex post monitoring does not take place). Given the aggregate state s, let $\sigma_{p|s}$ denote the cross sectional dispersion in asset returns. It is given by:

$$\sigma^2_{p|s} = \frac{1}{\int_{j\in B(s)} 1dj}\left(\int_{j\in B(s)} (r_{j,s}-\bar{r}_s)^2 dj\right) \tag{3.22}$$

where $B(s)$ is the set of indexes of borrowers for whom ex post monitoring does not occur, $r_{j,s}$ is the actual return on the asset issued by borrower j and $\bar{r}_s$ is the mean of $r_{j,s}$.

Given the aggregate state s, let $\hat{r}_{j,s} \equiv E\left[r_{j,s}|j \in B(s), s\right]$ be the expected return of asset issued by borrower j, conditional on that bankruptcy does not happen, i.e. $u > u^c_z$. Thus,

$$\hat{r}_{j,s} = E\left[r_{j,s}|u_j > u^c_{z_j}, s\right] = \frac{\left\{\alpha_j \rho E\left[e^{z_j}|u_j > u^c_{z_j}, s\right] + (1-\alpha_j)E\left[e^{u^c_{z_j}}|u_j > u^c_{z_j}, s\right]\right\} R_k}{1-n_j}$$

As mentioned, the individual shocks (z_j, u_j) are i.i.d. across agents conditional on aggregate state s. In addition, the borrowers choose the same critical values, u_0 and $\{u^c_z\}$, where $u^c_z = u_0 + \kappa z$. Let $\tilde{E}$ be the conditional expectation operator, that is, $\tilde{E}(.|s) = E(.|u_j > u^c_{z_j}, s)$. Then, $\hat{r}_{j,s}$ can be re-written as:

$$\hat{r}_{j,s} = \frac{\left[\alpha_j \rho \tilde{E}(e^z|s) + (1-\alpha_j)\tilde{E}(e^{u^c_z}|s)\right] R_k}{1-n_j} \tag{3.23}$$

Given the aggregate state s, $\hat{r}_{j,s}$ actually depends only on the financial decisions, i.e. n_j and α_j.

Given the aggregate state s, the difference between the actual return of asset j $r_{j,s}$ and the market return $\bar{r}_s$ can be decomposed into two parts: $r_{j,s} - \hat{r}_{j,s}$ and $\hat{r}_{j,s} - \bar{r}_s$. The first part is related to the idiosyncratic risk distributed to the asset holders, while the second part reflects the heterogeneity in borrowers' financial decisions. The dispersion in ex post returns can be decomposed into two parts in the similar way[17]:

$$\sigma^2_{p|s} = \frac{1}{\int_{j\in B(s)} 1dj}\left(\underbrace{\int_{j\in B(s)} (r_{j,s}-\hat{r}_{j,s})^2 dj}_{\textit{idiosyncratic risk}} + \underbrace{\int_{j\in B(s)} (\hat{r}_{j,s}-\bar{r}_s)^2 dj}_{\textit{heterogeneity}}\right) \tag{3.24}$$

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[17] The expectation of cross term $(r_{j,s} - \hat{r}_{j,s})(\hat{r}_{j,s} - \bar{r}_s)$ is zero.

When a negative shock shifts the wealth distribution of borrowers to the left and worsens the borrowers' financial conditions, the dispersion in ex post returns $\sigma_{p|s}$ increases due to two reasons. First, given the aggregate state s, the assets issued by the non-financially constrained borrowers always have the same $\hat{r}_{j,s}$, while the assets issued by financially constrained borrowers have different $\hat{r}_{j,s}$. When more borrowers become financially constrained, the heterogeneity in financial decisions increases, which leads to increases in the second component of the dispersion in ex post returns $\sigma_{p|s}$. In addition, for financially constrained borrowers, the negative shock decreases their wealth and thus the net worth they can invest in their projects. This leads to increases in the use of ex ante monitoring and α. As a result, more idiosyncratic risks are distributed to asset holders, leading to increases in the first component of the dispersion in ex post returns $\sigma_{p|s}$.

Dispersion in Ex ante Return. Let $\hat{r}_j = E(\hat{r}_{j,s})$ be the expected return on the assets issued by borrower j conditional on that ex post monitoring does not take place, and let $\bar{r}$ be the average of $\hat{r}_j$. Then, the dispersion in ex ante returns is defined by:

$$\sigma_h^2 = \int_0^1 (\hat{r}_j - \bar{r})^2 dj \tag{3.25}$$

As mentioned, the ex ante asset return depends only on the borrower's financial decisions. Thus, the dispersion in $\hat{r}_j$ reflects the heterogeneity in borrowers' financial decisions.

Market Volatility. The market volatility σ_m^2 is the ex-ante variance of the market index. Let $r_{m,s}$ be the ex post market index of asset returns given the aggregate state s. Let ω_j be the weight of the asset j in the market. Thus, we have:

$$r_{m,s} = \int_{j \in B(s)} \omega_j r_{j,s} dj \quad with \quad \omega_j = \frac{1 - n_j}{\int_{j \in B(s)} (1 - n_j) dj} \tag{3.26}$$

Note that all borrowers face the same incidence of bankruptcy, since their choices on critical value u_0 are the same. Thus n_j and α_j have the same distribution on $B(s)$ as on the entire population $[0, 1]$. Thus, the means of n_j and α_j on $B(s)$ and the entire population $[0, 1]$ are equal. Let $\bar{n}$ and $\bar{\alpha}$ denote the mean of n_j and α_j. Then, $r_{m,s}$ can be written as:

$$r_{m,s} = \frac{\left[\bar{\alpha}\tilde{E}(e^{z}|s) + (1-\bar{\alpha})\tilde{E}(e^{u_z^c}|s)\right] R_k}{1-\bar{n}} \tag{3.27}$$

where $\tilde{E}(.|s)$ is the conditional expectation operator given the aggregate state s and that ex post monitoring does not happen, i.e. $u > u_z^c$. Thus, the market volatility is given by:

$$\sigma_m^2 = var(r_{m,s}) \tag{3.28}$$

Note that $\tilde{E}(e^{z}|s) = e^{s}\tilde{E}(e^{\sigma_\varepsilon \varepsilon_z}|s)$ and $\tilde{E}(e^{u_z^c}|s) = e^{u_0+\kappa s}\tilde{E}(e^{\kappa\sigma_\varepsilon \varepsilon_z}|s)$ and that $\kappa < 1$. The first component of $r_{m,s}$ is more volatile than its second component, since e^{s} is more volatile than $e^{u_0+\kappa s}$. When the average net worth $\bar{n}$ decreases, the average $\bar{\alpha}$ increases, which then tends to increase the market volatility, $var(r_{m,s})$.

The market volatility is determined by the aggregate financial condition, represented by $\bar{\alpha}$ and $\bar{n}$, and measures the aggregate risks distributed to the lenders.

3.4 The Full Model

The simple two period model illustrates the key mechanism of the full model in a tractable manner. In this section, the full model with infinite horizons and business cycle features is built to examine the quantitative performance of the mechanism.

3.4.1 Setup

Time is discrete and $t = 0, 1, 2....$ There are two groups of agents: borrowers and lenders. The population of borrowers (lenders) is normalized to one. Borrowers can live for infinite periods while lenders only live for two periods. In each period, there are young and old generations of lenders. Each lender is endowed with one unit of goods when he is born. Lenders only consume when they are old and are risk neutral. All the borrowers and the lenders have accesses to a storage technology which produces R units of goods tomorrow for each unit stored today. Thus, R is the opportunity cost of lenders. R is exogenously given and assumed to be constant over time.

In each period, each borrower is endowed with a project. To start the project, the borrower has to invest one unit of goods. The project can be financed by the borrower himself (net worth) and/or the lenders (external funds). To borrow from the lenders, the borrower signs a financial contract $\mathcal{A}$ with the lenders. The settings for the costly state verification problem and the optimal financial contracting problem between the borrower and the lender are exactly like those in the two period model.

At the end of the current period, given the wealth level w, the borrower makes decisions on consumption c, investment in project (net worth) n and financial contract signed with the lender $\mathcal{A}$. At the beginning of the next period, the individual shocks (z', u') are randomly drawn. Then, the borrower and the lender split the realized investment revenue according to the financial contract. If bankruptcy, i.e. ex post monitoring, does not happen, the borrower earns a positive profits, which is also the wealth he has in that period, w'. If ex post monitoring takes place, the borrower gets nothing and has zero wealth in that period. Let $\varPhi(w)$ denote the cross sectional c.d.f. of borrowers' wealth and $\phi(w)$ denote the density of point w.

One important difference between the full model and the simple two period model is that in the full model the aggregate return R_k is endogenously determined by the aggregate capital formation of the economy, i.e. the gross investment minus the fraction of resources spent to set up ex ante monitoring. Here, I reintroduce the borrower's index j for aggregation. Let α_j and $1 - \alpha_j$ denote the amounts invested in technology z and u by borrower j in the current period. The aggregate capital formation K and aggregate return R_k are given by:

$$K = \int_0^1 [\rho\alpha_j + (1 - \alpha_j)]dj \quad \textit{and} \quad R_k = AK^{\delta - 1} \tag{3.29}$$

where $\delta < 1$ and A is a constant. The spread between R_k, i.e. the return of investment, and R, i.e. the borrowing cost, captures the aggregate financial condition.

The aggregate output is then given by:

$$Y = \int_0^1 \left[\rho e^{z_j}\alpha_j + e^{u_j}(1 - \alpha_j)\right] R_k dj = e^s \tilde{A} K^\delta \tag{3.30}$$

where $\tilde{A}$, with $\tilde{A} = Ae^{\sigma_{\varepsilon}^2/2}$, is a constant, and e^s is the aggregate productivity shock[18].

As in the simple two period model, the aggregate shock s has a normal distribution, $s \sim N(0, \sigma_s^2)$ and is i.i.d. over time. The idiosyncratic random components, i.e. $\varepsilon_{z,j}$ and $\varepsilon_{u,j}$, are i.i.d. across borrowers and over time, and have standard normal distributions. They are also independent of the aggregate shock s. For the same borrower j, $\varepsilon_{z,j}$ and $\varepsilon_{u,j}$ are independent of each other. Thus, the shocks do not have any persistence at all at both individual and aggregate levels.

3.4.2 Individual Decisions and Aggregate Dynamics

The borrowers' decisions. The borrower's dynamic programming problem can be written recursively. A borrower is characterized by his individual wealth w and a vector θ that summarizes aggregate states, which include an exogenous state s and two endogenous states: aggregate return R_k and the borrower's wealth distribution $\varPhi(w)$.

The borrower maximizes his life time (expected) consumption, which is discounted by β. The optimization problem of a borrower is given by:

Problem II The Borrower's Problem

$$V(w,\theta) = \max_{\{n\in[0,w],\mathcal{A}\in\Omega(n,\theta)\}} \left\{c + \beta E[V(w',\theta')|\theta]\right\} \tag{3.31}$$

subject to

$$0 \le c \le w - n \qquad \textit{and} \qquad w' = \pi(\mathcal{A}, z', u')$$

Given the current wealth level w and the aggregate states θ, the borrower chooses to invest n, with $n \in [0, w]$, units of goods in his project and consumes the rest of his wealth, i.e. $c = w - n$. Given the net worth n and the aggregate states θ, the borrower chooses the financial contract he signs with the lender, $\mathcal{A}$, from set $\Omega(n,\theta)$, i.e. the set of all financial contracts which satisfy feasibility, incentive compatibility and expected return constraints.

The borrower's wealth in the next period, i.e.w', is the profits he earns via operating the project, which depends on the financial contract $\mathcal{A}$ and the realization of individual shocks (z', u') in the next period. The borrower's wealth in the next period is given by $w' = \pi(\mathcal{A}, z', u')$, where π is a function that maps the financial contract $\mathcal{A}$ and the realization of individual shocks (z', u') into the borrower's profits.

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[18] Note that $z_j = s + \sigma_{\varepsilon}\varepsilon_{z,j}$ and $u_j = s + \sigma_{\varepsilon}\varepsilon_{u,j}$ and that the idiosyncratic risks (i.e. $\varepsilon_{z,j}$ and $\varepsilon_{u,j}$) are exogenous and do not depend on borrowers' wealth levels and choices of α_j. Thus, $\varepsilon_{z,j}$ and $\varepsilon_{u,j}$ are independent of α_j.

The solution to the borrower's problem is a set of policy functions $\{n(w, \theta), \mathcal{A}(w, \theta)\}$, where $n(w, \theta)$ is the choice of net worth and $\mathcal{A}(w, \theta)$ is the choice of financial contract. For this simple model, the solution of this dynamic programming problem can be derived manually. See Appendix 2 for detail. The solved value function is given by:

$$V(w, \theta) = w + h \quad with \quad h_t = \sum_{i=0}^{\infty} \beta^i E_t \left[\rho E(e^z) \left(\frac{R_{k,t+i}}{R} \right) - 1 \right] \tag{3.32}$$

The value function consists of two parts: the borrower's current wealth w and the present value of his life time endowments h_t. Note that $\left[\rho E(e^z) \left(\frac{R_{k,t+i}}{R} \right) - 1 \right]$ is the value of the access to project in period $t+i$ and thus h_t is the present value of all the accesses to projects in the borrower's life time[19].

The borrower's decision rules are actually very similar to those in the simple two period model. To see this, substitute (32) into (31), then *Problem II* can be reduced to:

$$\max_{\{n \in [0,w], \mathcal{A} \in \Omega(n,\theta)\}} c + \beta E(w') + \beta E(h')$$

subject to

$$0 \leq c \leq w - n \qquad and \qquad w' = \pi(\mathcal{A}, z', u')$$

Note that $E(h')$ enters in the value function as a lump sum term which the borrower takes as given. The borrower actually maximizes the sum of consumption today and his expected profits in the next period, i.e. $c + \beta E(w')$ and his decisions do not depend on future aggregate returns, i.e. $\{R_{k,t+i}\}$ with $i > 0$. The optimization problem faced by the borrower is exactly like that in the simple two period model (*Problem I*). The borrower's decision rules are similar to those in the simple two period model too, which are summarized in *Proposition 3*.

Proposition 3 *(Problem II)*

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[19] See Appendix 2 for detail.

a. (Optimal financial contract) Given the aggregate return R_k *and the borrower's net worth* n*, the optimal financial contract* $\mathcal{A}^* = \{\alpha, x(z), r(z,u), A(z)\}$ *is specified by:*

(1) $x(z) = [\alpha \rho e^z + (1-\alpha) e^{u_0 + \kappa z}] R_k$

(2) $r(z,u) = [\alpha \rho e^z + (1-\alpha) e^u] R_k$

(3) $A(z) = (-\infty, u_0 + \kappa z)$

where $\{\alpha, u_0, \lambda\}$ *are determined by equation (13),(14) and (15).*

b. (Optimal choices of net worth n) Given the aggregate return R_k *and the borrower's wealth* w*:*

(1) if $w \leq n_{max}$*, the borrower invests all his wealth into his project, i.e.* $n = w$*;*

(2) if $w > n_{max}$*, the borrower invests* n_{max} *units of goods into his project, i.e.* $n = n_{max}$*, and consumes the rest, i.e.* $c = w - n_{max}$*.*

where the critical value n_{max}*, defined by (18), depends on the aggregate return* R_k*.*

An borrower makes decisions on the net worth, n, and the financial contract, $\mathcal{A}$, he signs with lenders. Actually, the only aggregate state that is relevant to an individual borrower's decisions is aggregate return R_k: First, the borrower chooses the amount he invests in his own project based on his wealth level. The decision rule is given by: $n = \max\{n_{max}, w\}$, where the critical value of net worth n_{max}, defined by (18), depends on the aggregate return R_k. Second, the financial contract is characterized by a vector $\mathcal{A} = \{\alpha, u_0, \lambda\}$. The choices of u_0 and λ do not depend on the net worth n or aggregate return R_k (*Lemma 3*). The borrowers only choose different α, based on their wealth levels and aggregate return R_k. The decision rule for α is given by (17). The choosing of financial contract $\mathcal{A}$ can be reduced to the choosing of α. Thus, the policy functions of the borrowers can be written as: $\{n(w, R_k), \alpha(w, R_k)\}$

Aggregate Dynamics. The aggregate state vector θ includes an exogenous aggregate state s, and two endogenous states, which are the aggregate return R_k and the wealth distribution of borrowers, $\phi(w)$. Given the borrower's decision rules, $\{n(w, R_k), \alpha(w, R_k)\}$, now we can turn to the laws of motion of the aggregate states. First, note that the aggregate shock s is i.i.d. over time and its stochastic path is exogenously given.

Second, the aggregate state that is relevant to the borrowers' decisions is the aggregate return R_k. The borrowers' financial decisions depend on their wealth levels and the aggregate return R_k: the choices of $\alpha = \alpha(w, R_k)$ depends on the aggregate return R_k (17) and the critical value of net worth n, i.e. $n_{max} = n_{max}(R_k)$, depends on R_k (18). In addition, the aggregate return R_k depends on the aggregate capital formation K, i.e. the aggregate gross investment minus the fraction spent to set up ex ante monitoring (29). Thus, equilibrium (29) requires:

$$R_k = A\left[\int_0^{+\infty} [\rho\alpha(w, R_k) + (1 - \alpha(w, R_k))]d\Phi(w)\right]^{(\delta-1)} \tag{3.33}$$

The aggregate return R_k is a function of the borrowers' wealth distribution, $\Phi(w)$. Let $X(.)$ denote the function that maps wealth distribution to the aggregate return R_k, thus, $R_k = X(\Phi(w))$. The path of aggregate return, R_k, is then determined by that of wealth distribution $\Phi(w)$.

Third, given the borrowers' decision rules, the wealth distribution tomorrow depends on the realization of future aggregate state s' and the wealth distribution today:

$$\phi(w'|s') = \int_0^{+\infty} Prob\{\pi(\alpha, z', u') = w'|s'\}d\Phi(w)$$

where $\phi(w'|s')$ is the density at w' given the aggregate state s'. The evolution of the distribution of borrowers' wealth is denoted by: $\Phi(w') = \Theta(\Phi(w), s')$.

3.4.3 Numerical Experiment

The simple two period model illustrates the key mechanism and shows how the financial condition of the borrower determines the way risks are distributed to the borrower and the lender. When the borrower's financial condition is worsened, the optimal financial contract calls for more intensive use of ex ante monitoring and becomes more equity like. Thus, more aggregate and idiosyncratic risks are distributed to the lenders, leading to increases in the dispersion and the market volatility in asset returns. In addition, a negative shock shifts the wealth distribution to the left and increases the fraction of financially constrained borrowers, who make financial decisions based on their wealth levels. As a result, the heterogeneity in financial decisions and the dispersion in ex ante asset returns increases, leading to increase in the dispersion in ex post asset returns also. This mechanism remains at work in the full model with infinite horizons.

To examine the dynamics of an economy with the above mechanism, I extend the simple two period model to a model with infinite horizons. In this subsection, a numerical exercise is done to show how a first moment shock can lead to countercyclical dynamics of the dispersions in ex post and ex ante asset returns, market volatility and financial conditions. The four measures I focus on in this numerical exercise are the dispersions in ex post and ex ante asset returns, market volatility, and financial condition, i.e. external finance premium. The definition of them can be found in the section 3.4.4.

Calibration. For aggregate and idiosyncratic risks, I follow Bloom (2009) and assume that the aggregate and the idiosyncratic risks have the same scales, i.e. $\sigma_s = \sigma_\varepsilon$. In addition, σ_s and σ_ε are set to 0.1, so that the steady state cross sectional dispersion (in ex-post returns) is 0.066, roughly the same as the its data counterpart in 2006, which is 0.077. The opportunity cost of the lender, R, is normalized to 1 and assumed to be time invariant. The parameter for ex ante monitoring cost, ρ, is set around 0.976. The ex post monitoring cost μ is set to 0.2. Thus, the steady state default rate is roughly 2 %. For the parameters associated with the aggregate return function (29), A is set to 1 and δ is set to 0.25, such that the steady state aggregate return is 1.05 and the steady state external finance premium is 5%. The steady state wealth distribution is obtained by simulating the model without aggregate fluctuations. Calibrated parameters are summarized in Table 3.

Generated Correlations. Table 1 (Table 2) shows the correlation coefficients between the dispersion in ex post asset returns (the time series volatility of market index) and other measures. The values are in line with the data counterparts.

The correlation coefficient between market volatility and financial condition is almost one. This is due to the set up of the model. Note that in the model, financial condition is represented by external financing premium, i.e. $R_k - R$. The aggregate return R_k is endogenously determined by how the borrowers allocate their resources between technology z and u, that is, the average α (29). According to its definition (section 3.4.4), market volatility also depends on the average α. Thus, these two measures are almost perfectly correlated.

Impulse Response. The shock is a negative first moment aggregate shock. In the first period, $s_1 = -2\sigma_s$, where σ_s is the standard deviation of the aggregate shock s. After that, $s_t = 0$ for $t = 2, 3, 4....$ As mentioned, the shocks do not have any persistence at all.

The upper panel of Figure 5 plots the impulse response of the spread between aggregate return R_k and lender's opportunity cost R. This spread is interpreted as "external finance premium", that is, the spread between marginal return of investment and borrowing interest rates, and captures the aggregate financial condition. In the model, the borrowing interest rates equals the lender's opportunity cost which is assumed to be fixed at R over time. However, when borrowers' financial conditions are worsened, to reduce the expected cost of bankruptcy, borrowers depend more on ex ante monitoring and allocate more funds in technology z, which helps mitigate the agency problem. Since more initial investment is spent to set up ex ante monitoring, the capital formation decreases and then the aggregate return R_k increases (29).

The lower panel of Figure 5 shows the impulse responses of the cross sectional dispersions in ex post and ex ante asset returns and market volatility. The detailed definitions of these measures can be found in the section 3.4.4. Note that market volatility and the dispersion in ex ante asset returns are ex ante measures which are predetermined. They do not response to the shock immediately.

As shown in Figure 5, the dispersion in ex post asset returns temporarily drops below its steady state value in the first period. The reason is that when the negative productivity shock hits the economy, all the asset returns drop sharply, which lowers the cross sectional standard deviation. In the second period, asset returns rise immediately, since the persistence of the shock process is zero.

The dispersion in ex post returns rises sharply in the second period and slowly comes back to its steady state after that. The increase in the dispersion in ex post returns is due to two reasons: (i) when borrowers' financial conditions are worsened, the optimal contract distributes more risks to the lenders (the first component of equation 24); (ii) when the fraction of financially constrained borrowers increases, the heterogeneity in financial decisions increases, which leads to increase in the dispersion in ex ante asset returns (the second component of equation 24). As shown in Figure 5, the dispersion in ex ante asset returns increases since the second period as expected, which reflects the increase in the heterogeneity in financial decisions.

Finally, note that all the responses are persistent even if the shocks do not have any persistence at all. The reason is that dispersion and market volatility are largely determined by the financial conditions and it takes time for the financial conditions to recover after the negative shock.

3.5 Conclusion

The cross sectional dispersion and the market volatility in stock returns, which are countercyclical, are usually used as measures for uncertainties. However, exogenous change in uncertainties might not be the only reason for the countercyclical dispersion and market volatility. This paper provides a possible mechanism through which standard first moment shocks can generate countercyclical movements of dispersion and market volatility.

I incorporate a costly state verification problem into a dynamic framework. The model deviates from classical CSV frameworks by allowing ex ante monitoring, which refers to setting up a mechanism in advance that can credibly force the borrower to disclose the true investment outcome to the lender, in addition to the standard ex post monitoring that is available in conventional CSV settings. Ex ante monitoring is costly, but can mitigate the information friction. By this feature, the optimal contract resembles outside equity.

In the model, the dispersion and the market volatility in asset returns move countercyclically due to two reasons: First, when the borrower's financial condition is worsened, the expected cost of bankruptcy, i.e. ex post monitoring, increases. The optimal contract that minimizes monitoring costs calls for more intensive use of ex ante monitoring and require the repayment be contingent on the information it reveals. As a result, the optimal contract becomes more equity like and the lender takes more aggregate and idiosyncratic risks, leading to increases in dispersion and market volatility.

In addition, non financially constrained borrowers make the same financial decisions even though their wealth levels are different, while financially constrained borrowers make different financial decisions based on their wealth levels. A negative shock increases the fraction of financially constrained borrowers and leads to increase in the heterogeneity in financial decisions. Ex ante asset returns, which are determined by borrowers' financial decisions, become more dispersed, which then leads to increase in the dispersion in ex post returns.

The model predicts counter cyclical movements of the cross sectional dispersions in ex post and ex ante asset returns, market volatility and financial condition. The model, which is calibrated at steady state, is capable of generating correlation coefficients between pairs of these measures that are quantitatively in line with the data. A negative first moment shock that is not persistent at all (zero persistent) can cause persistent increases in dispersion and market volatility. In this paper, it is financial conditions that determine how risks are distributed to the borrowers and the lenders, and thus the dispersion and the market volatility in asset returns. Since it takes time for worsened financial conditions to recover, the increases in dispersion and market volatility can be persistent even if the shock is not.

3.6 Appendix

A1. Proof of Lemma 1

Let us start from the (2) and (3) of the Lemma 1. For any z, such that $x(z) \geq \alpha\rho e^z R_k$, there is a critical value of u, i.e. u_z^c, defined by (6). First, note that if $u < u_z^c$ the repayment $x(z)$ is not feasible. Then, $(-\infty, u_z^c) \subseteq A(z)$, since when $u < u_z^c$, it is impossible for the borrower to fully repay the promised amount, $x(z)$. Suppose that there is $u_1 \in A(z)$ and $u_1 > u_z^c$. Note that the repayment $x(z)$ is feasible at $u = u_1$. Thus, it is optimal to reduce $x(z)$ and take u_1 out of monitoring region, while keeping the expected repayment to lender unchanged. Thus, $A(z) \subseteq (-\infty, u_z^c)$. In sum, $A(z) = (-\infty, u_z^c)$.

Second, if for any $u \in A(z)$, $r(z,u) < [\alpha\rho e^z + (1-\alpha)e^u]R_k$, it is optimal to increase $r(z,u)$ to $[\alpha\rho e^z + (1-\alpha)e^u]R_k$ and reduce $x(z)$ and also the monitoring region $A(z)$, while keeping the expected repayment to lender unchanged. Thus, $r(z,u) = [\alpha\rho e^z + (1-\alpha)e^u]R_k$ must hold.

Third, now assume that for some z_1, $x(z_1) < \alpha\rho e^{z_1} R_k$. It is always optimal to increase

$x(z_1)$ to $\alpha\rho e^{z_1}R_k$ and reduce $x(z)$ and also the monitoring region $A(z)$ for the other states of z, while keeping the expected repayment to lender unchanged. Thus, $x(z) \geq \alpha\rho e^{z}R_k$ for any z.

A2. Solve the dynamic programming problem

To solve the borrower's optimization problem, let us begin with reviewing the borrower's decision rules in the simple two period model. The borrower's wealth in the second period w' is the profits he earns via operating the project. The borrower maximizes the sum of his current assumption and the expected value of his consumption (wealth) in the second period discounted by β, i.e. $c+\ \beta E(w')$.

The borrower makes decisions on the amount he invests in his own project, i.e. net worth n, and the financial contract he signs with the lender, $\mathcal{A}$. Using the results obtained in the section 3, (5), (14) and (15), the expected profits the borrower earns by operating the project can be written as:

$$\pi^*(n) = \left[\rho E(e^z)\left(\frac{R_k}{R}\right) - 1 + n\right](1+\lambda)R \tag{3.34}$$

The expected profit has two components. The first component, $\rho E(e^z)\left(R_k/R\right) - 1$, is the value of the access to the project, that is, the profits the borrower can earn even if he puts noting in the project. The access to the project is an endowment to the borrower. The second component is the value of the net worth n. According to *Lemma 3*, the price (marginal return) of net worth, i.e. $(1+\lambda)R$, is constant when $0 \leq \alpha \leq 1$ does not bind. Thus, the expected profits π^* is linear in n. By substituting (34) into (1), we obtain:

$$V(w) = c + \beta\pi^*(n) \tag{3.35}$$

Note that $\beta^{-1} = (1+\lambda)R$ (*Assumption 3*) and $c = w - n$. Thus, we have:

$$V(w) = w + \left[\rho E(e^z)\left(\frac{R_k}{R}\right) - 1\right] \tag{3.36}$$

The maximized expected utility of a borrower with initial wealth w, i.e. $V(w)$ consists of two parts; the value of the endowed initial wealth w and the value of the access to the project $\left[\frac{\rho E(e^z)R_k}{R} - 1\right]$. Actually, as will be shown, in the full model, the value function is very similar to the above equation.

In the full model, a borrower is characterized by (w, θ), where w is the borrower's wealth level and θ is a vector that summarize all the aggregate states. The aggregate state vector θ includes an exogenous state s, and two endogenous states, which are aggregate return R_k and borrowers' wealth distribution $\varPhi(w)$.

At the end of each period, a borrower makes the decision on consumption and investment. Let c be the consumption of the borrower at the end of current period, thus $n = w - c$ is the amount of net worth he puts in his project. Then, he signs a financial contract $\mathcal{A}$, with $\mathcal{A} \in \Omega(n, \theta)$, with lender, where $\Omega(n, \theta)$ is the set of all possible financial contracts which satisfies feasibility, incentive compatibility and expected return constraints, given the net worth n and the aggregate states θ. The wealth tomorrow w' depends on today's financial contract and the realization of individual states (z', u'), which evolves according to $w' = h(\mathcal{A}, z', u')$, where π is a function that maps the financial contract and the realized individual shocks into the borrower's profits.

The borrower's problem is given by:

Problem II:

$$V(w, \theta) = \max_{\{n \in [0,w], \mathcal{A} \in \Omega(n,\theta)\}} \left\{c + \beta E[V(w', \theta') | \theta]\right\}$$

subject to:

$$0 \leq c \leq w - n \qquad \textit{and} \qquad w' = \pi(\mathcal{A}, z', u') \tag{3.37}$$

Actually, for this model it is possible to solve this dynamic programming problem manually.

Guess: Let us start with a guess: $V_0(w, \theta) = w$.

Iteration 1: Then, the borrower's optimization problem becomes:

$$V_1(w, \theta) = \max_{\{n \in [0,w], \mathcal{A} \in \Omega(n,\theta)\}} \left\{c + \beta E(w' | \theta)\right\}$$

subject to:

$$0 \leq c \leq w - n \qquad \textit{and} \qquad w' = \pi(\mathcal{A}, z', u')$$

The borrower maximizes the sum of his current consumption and the expected value of his wealth tomorrow discounted by β, i.e. $c + \beta E(w'|\theta)$. It is exactly the same as the optimization problem faced by the borrower in the simple two period model (*Problem I*). The only difference is that all the borrower's decisions depend on the aggregate states, which is summarized by vector θ. Note that shocks, including individual shocks (z, u) and aggregate shock s, are i.i.d. over time, which do not depend on the current states. The only aggregate state that is relevant to the borrower's decisions is the aggregate return R_k, which is taken as a parameter in the simple two period model. However, a change in R_k won't change the borrower's decisions rules as long as $\rho E(e^z)R_k - R > 0$ still holds (*Assumption 2*)[20].

Note that the individual shocks z and u are i.i.d. over time and thus for any θ, $E(e^z|\theta) = E(e^z)$. Define $\hat{e}^z = E(e^z)$. Using the results we obtained in the section 3 (Equation 36), we have:

$$V_1(w, \theta) = w + \left[\rho \hat{e}^z \left(\frac{R_k}{R}\right) - 1\right]$$

Iteration 2: Thus, for the next iteration over V_i, the optimization problem becomes:

$$V_2(w, \theta) = \max_{\{n \in [0,w], \mathcal{A} \in \Omega(n,\theta)\}} \left\{c + \beta E(w'|\theta)\right\} + \beta E\left[\rho \hat{e}^z \left(\frac{R_k'}{R}\right) - 1|\theta\right]$$

subject to:

$$0 \leq c \leq w - n \qquad \textit{and} \qquad w' = \pi(\mathcal{A}, z', u')$$

Note that the second part of V_2 is the value of the access to the project, which enters in the value function as a lump sum term and is taken as given by the borrower. The borrower's decisions do not depend on the future aggregate states R_k'. The borrower is only maximizing the sum of his consumption today and the expected wealth tomorrow discounted by β, i.e. $c + \beta E(w'|\theta)$, which is exactly like that in the two period model.

Similarly, the borrower's optimal choice of n and $\mathcal{A}$ are also the same as those in the two period model. Using the results in section 3 (Equation 36), V_2 is then given by,

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[20] Note that in the full model, aggregate return R_k is determined by capital formation K. Also note that there is an upper bound for K: $K \leq 1$. When all the borrowers invest only in technology u, K reaches its maximum. Thus, the parameters for the aggregate return function, i.e. $R_k = AK^{\delta-1}$, can be chosen such that $\rho E(e^z)R_k - R > 0$ holds even when $K = 1$.

$$V_2(w,\theta) = w + \left[\rho\hat{e}^z\left(\frac{R_k}{R}\right) - 1\right] + \beta E\left[\rho\hat{e}^z\left(\frac{R'_k}{R}\right) - 1|\theta\right]$$

Solution: By iterating over V_i, we can obtain the value function:

$$V(w,\theta) = w + h \quad with \quad h_t = E_t\sum_{i=0}^{\infty}\beta^i\left[\rho\hat{e}^z\left(\frac{R_{k,t+i}}{R}\right) - 1\right]$$

Table 3.1: Correlations between Dispersion and Others

Independent Variable	Dependent Variable: Dispersion in Ex-post Return								
	1	2	3	4	5	6	7	8	9
Market volatility	0.6666	0.7280	0.7365						
	(0.0408)	(0.0631)	(0.0276)						
Financial condition				0.4845	0.7704	0.7378			
				(0.0479)	(0.0587)	(0.0276)			
Disp. in ex-ante return							0.8080	0.8596	0.6749
							(0.0324)	(0.0477)	(0.0302)
Time span	85-12	03-12	model	85-12	03-12	model	85-12	03-12	model
R-square	0.4443	0.5300	0.5426	0.2347	0.5936	0.5443	0.6518	0.7353	0.4555
Observation	336	120	600	336	120	600	336	120	600

[1] Each column reports the coefficient from regressing the dispersion in ex-post return on the explanatory variables listed in the first column. All variables are normalized to have a standard deviation of 1. The standard deviation is reported in the brackets below.

[2] For each pair, the first two correlation coefficients are calculated using the data between 1985 M01-2012 M12 and the data between 2003 M01-2012 M12. Dispersion in ex-post and ex-ante stock returns are the cross sectional standard deviation of actual and expected stock returns. The series of expected stock returns are obtained by fitting the time series returns of each stock into an AR(1) model. The market volatility is VXO index. The financial condition is Chicago Fed National Financial Condition Index.

[3] The third correlation coefficient reported is calculated using model simulated data. Dispersion in ex-post and ex-ante returns are the cross sectional standard deviation of ex-post and ex-ante stock returns (conditional on that bankruptcy does not happen). The market volatility is the ex-ante volatility of weighted average returns. The financial condition indicator is the spread between marginal return of investment and borrowing interest rate.

Table 3.2: Correlations between Volatility and Others

Independent Variable	Dependent Variable: Volatility								
	1	2	3	4	5	6	7	8	9
Disp. in ex-post return	0.6666	0.7280	0.7365						
	(0.0408)	(0.0631)	(0.0276)						
Financial condition				0.7305	0.8787	0.9998			
				(0.0374)	(0.0439)	(0.0006)			
Disp. in ex-ante return							0.6612	0.7273	0.8605
							(0.0412)	(0.0639)	(0.0208)
Time span	85-12	03-12	model	85-12	03-12	model	85-12	03-12	model
R-square	0.4443	0.5300	0.5426	0.5336	0.7721	0.9998	0.4361	0.5251	0.7406
Observation	336	120	600	336	120	600	336	120	600

[1] Each column reports the coefficient from regressing the market volatility on the explanatory variables listed in the first column. All variables are normalized to have a standard deviation of 1. The standard deviation is reported in the brackets below.

[2] For each pair, the first two correlation coefficients are calculated using the data between 1985 M01-2012 M12 and the data between 2003 M01-2012 M12. Dispersion in ex-post and ex-ante stock returns are the cross sectional standard deviation of actual and expected stock returns. The series of expected stock returns are obtained by fitting the time series returns of each stock into an AR(1) model. The market volatility is VXO index. The financial condition is Chicago Fed National Financial Condition Index.

[3] The third correlation coefficient reported is calculated using model simulated data. Dispersion in ex-post and ex-ante returns are the cross sectional standard deviation of ex-post and ex-ante stock returns (conditional on that bankruptcy does not happen). The market volatility is the ex-ante volatility of weighted average returns. The financial condition indicator is the spread between marginal return of investment and borrowing interest rate.

Table 3.3: Parameters

Parameters		
	Value	Description
σ_s	0.100	Aggregate uncertainty
σ_ε	0.100	Idiosyncratic uncertainty
R	1.000	Rate of return of the storage technology
ρ	0.976	Ex-ante monitoring cost
μ	0.200	Ex-post monitoring cost
A	1.000	Aggregate return function
δ	0.250	Aggregate return function

Figure 3.1: Dispersion, Volatility and Financial Condition

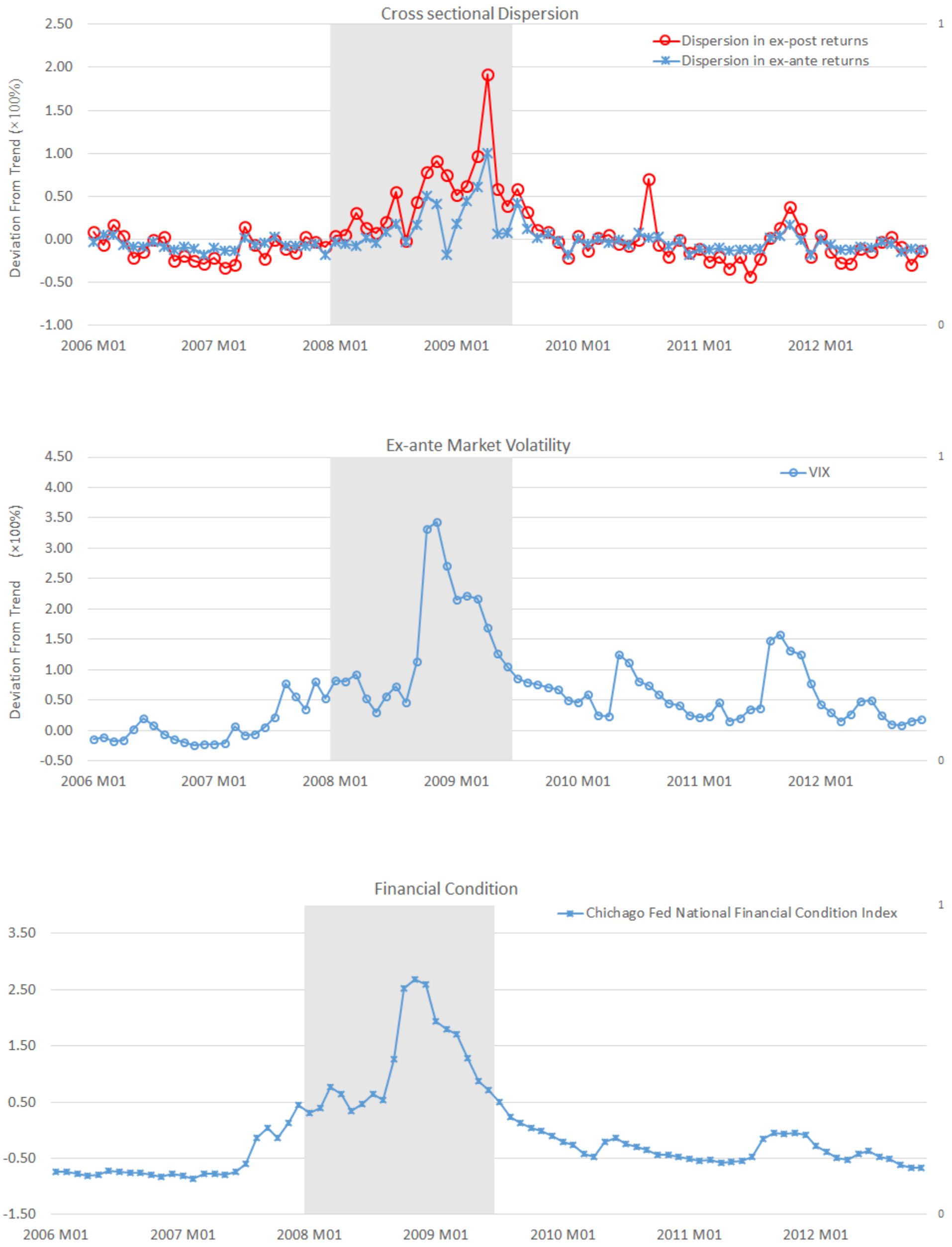
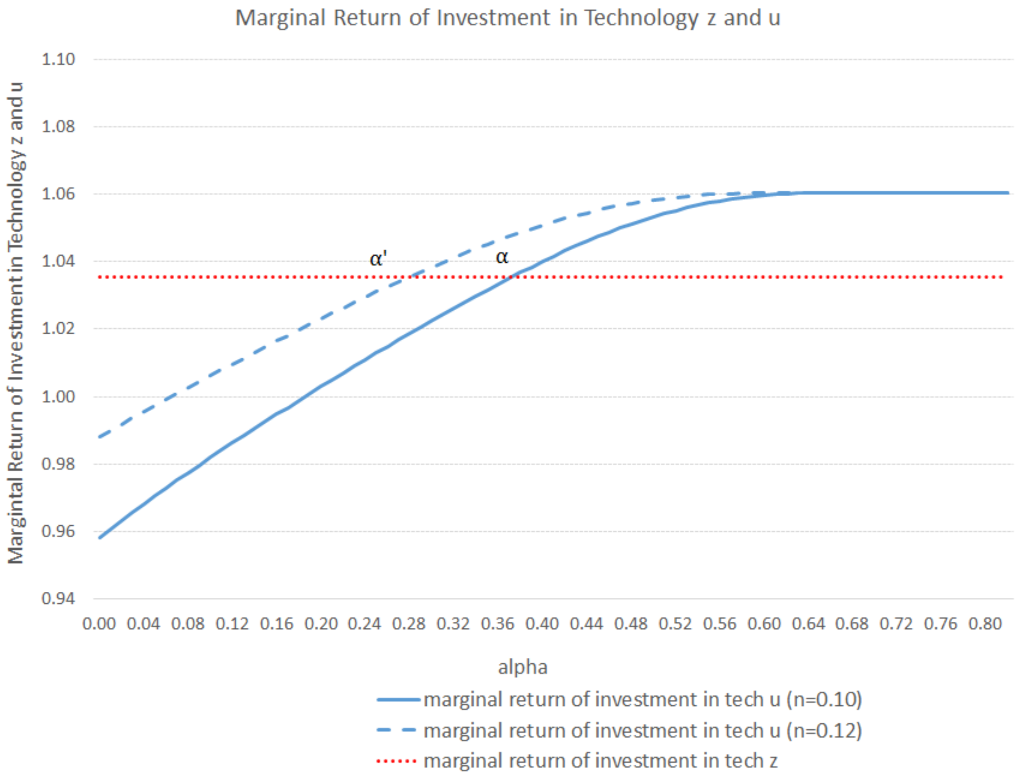
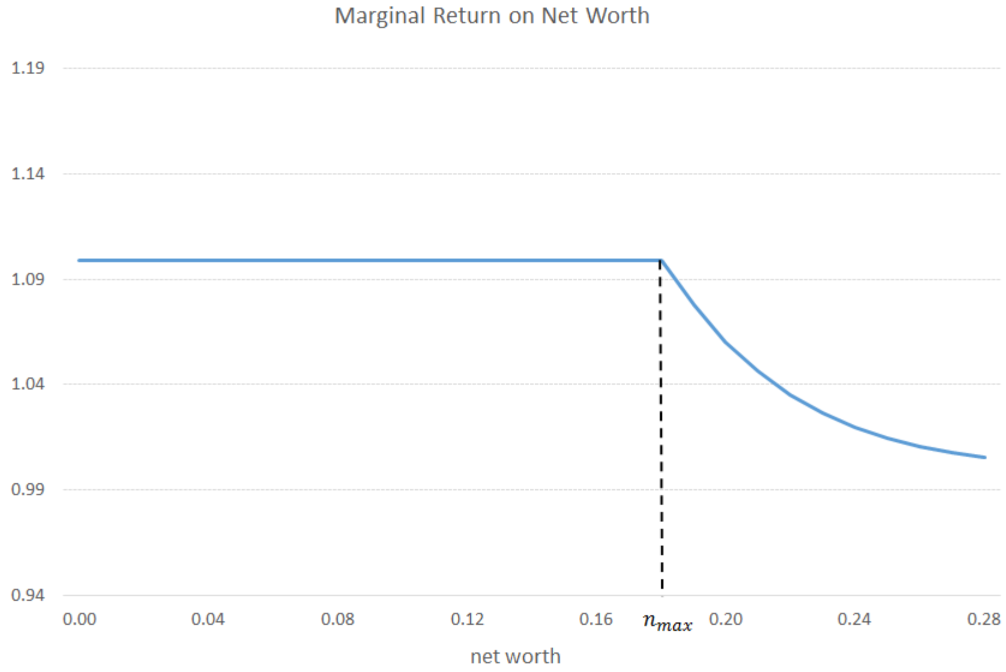


Figure 3.2: Marginal Return of Investment in Technology z and u



When α is equal to its optimal value, the marginal returns of investment in technology z and u are equal. When α increases, u_0 has to decrease for the expected return constraint (14) to hold. When α increases toward 1, u_0 moves to $-\infty$, the MRT decreases to 1 and the marginal return of investment in technology u converges to $E(e^u)R_k$. The dotted (red) line shows the marginal return of investment in technology z, while the solid line shows the marginal return of investment in technology u. An increase in the borrower's net worth shifts the curve of marginal return of investment in technology u to the left and thus decreases the optimal α.

Figure 3.3: Marginal Return of Net Worth



When $n > n_{max}$, the no short sale constraint $0 \leq \alpha$ binds. The unconditional critical value u_0 decreases and then the marginal return of net worth $(1 + \lambda)R$ drops.

Figure 3.4: Negative Wealth Shock and Heterogeneity in Financial Decisions

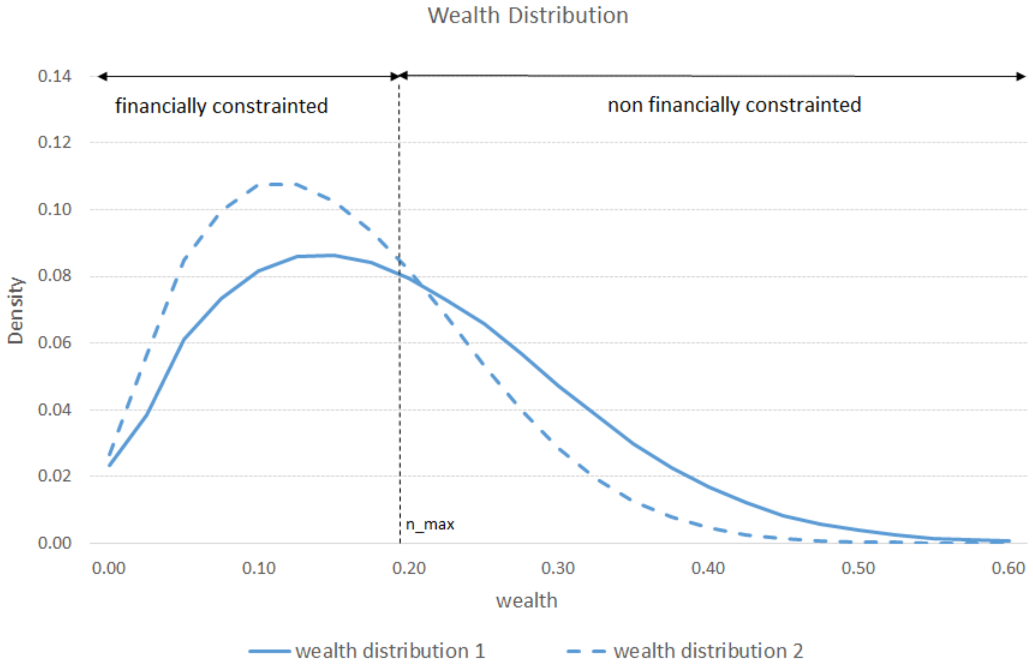
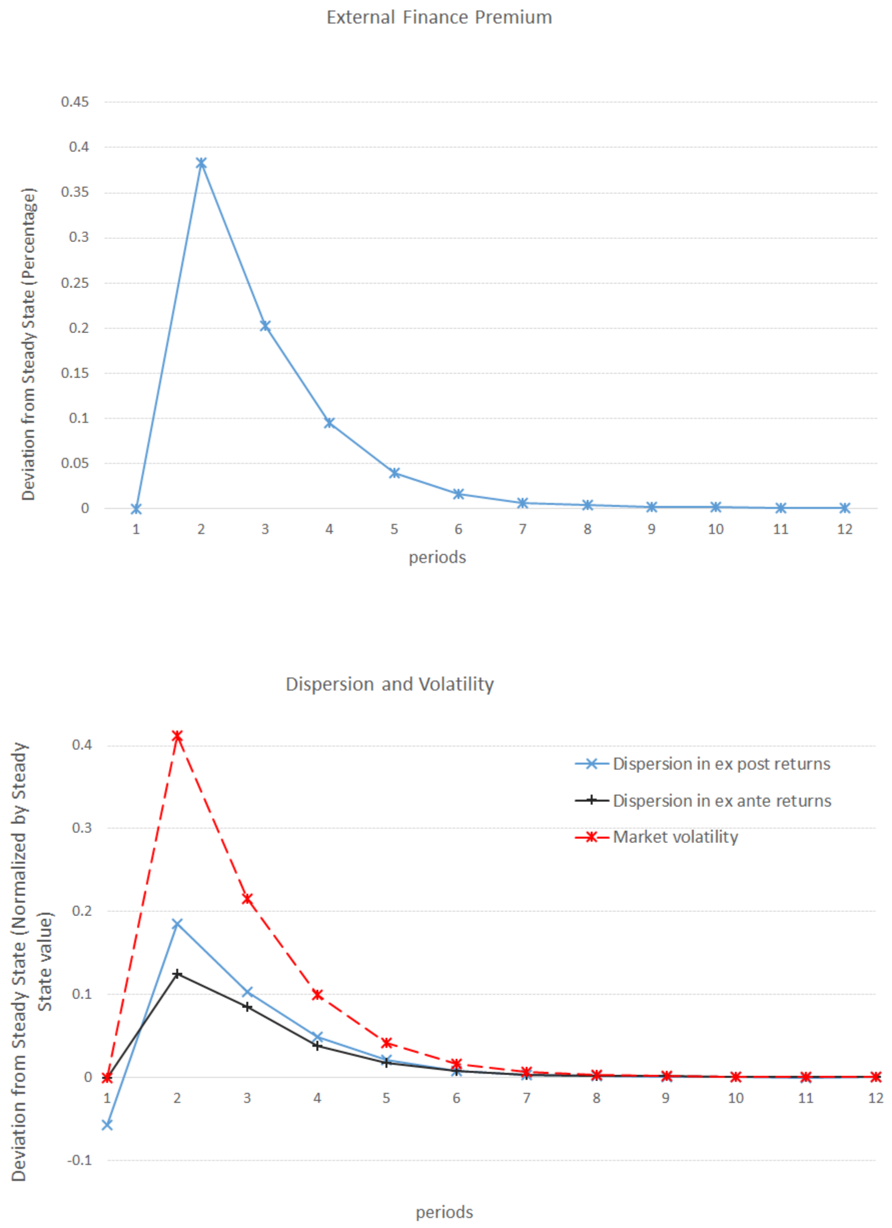


Figure 3.5: Impulse Response after a Negative First Moment Shock



Chapter 4

Contractionary Effects of Central Bank Asset Purchases: The Other Side of QE

4.1 Introduction

After crisis, the large scale assets purchasing (LSAP) by Fed has greatly changed its balance sheet and the composition of assets available in the open market. First, trillions of safe and liquid assets, such as, treasury bills, bonds and AA+ assets disappears into the Fed's portfolio. Second, bank reserves, which is only reachable by banks, increases by trillions. Bank reserves and treasury bills and bonds are almost equally safe, but they are different in their marketability: bank reserves can only be held by banks, while anyone can hold treasury bills and bonds, which means that treasury bills and bonds are more liquid than bank reserves in this regard.

Recently, there has been increasing concerns that by buying huge amounts of safe assets and issuing bank reserves, the Fed may be actually contracting[1]. The claims that central bank asset purchases can have contractionary effects runs counter to conventional wisdom, but the reasoning behind is easy to understand when we are aware of the fact that nowadays banking and money creation no longer works in the textbook way. The textbook banking framework requires a stable "money multiplier", which allows the central bank to affect credit and money creation by changing the supply of bank reserves. But nowadays in the US, a large part of credit is created by shadow banks, the lending of which are financed by creation of liabilities that are not subject to the reserve requirement. Today, required reserve ratio (RRR) constraints of commercial banks do not bind[2] and the major reason for commercial banks to hold bank reserves could simply be that they need to keep some liquid assets in their portfolio. Treasury bills and bonds play a very similar role in the shadow banking sector as bank reserves do in the commercial banking sector. When the central bank purchases safe assets using bank reserves, the supply of safe assets that are available to the shadow banks decreases, which have contractionary effects on the shadow banks.

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[1] For example, Kessler (2013) argues that the Fed's purchases of treasuries lead to shortage of collateral in the repo market. Stella (2013) argues that nowadays bank reserves are no longer connected with money via "monetary multiplier" and buying huge amounts of treasuries and issuing bank reserves will not expand credit and money supply, but may lead to shortage of liquid assets available to non-banks.

[2] Bennett and Peristiani (2002).

However, is the difference between the marketabilities of bank reserves and treasury bills overstated? Is there any evidence for the limited accessibility of bank reserves for shadow banks? The relative prices of bank reserves and short term treasury bills could be used as an indicator. Bank reserves and short term treasury bills are equally safe, and bank reserves is even slightly more liquid than treasury bills. When the interest rate paid on bank reserves is higher than the interest rate short term treasury bills bear, this could imply the limited accessibility of bank reserves for shadow banks. Shadow banks could not directly achieve bank reserves, even though they are cheaper. If the central bank does not pay interest on bank reserves, bank reserves is cheaper than short term treasury bills when the nominal rate paid on treasury bills is negative. Contrary to our common beliefs, nominal rates do drop below zero sometimes[3]. If the central bank pays interest on bank reserves, bank reserve is cheaper when the interest paid on bank reserves is higher than that paid on short term treasury bills. In the US, the market interest rate on the three month treasury bills have been lower than the interest rate paid on bank reserves since late 2009.

Despite these concerns, little formal work has been conducted to analyse such contractionary effects of central bank asset purchases. The objective of this paper is to build a theoretical framework to examine these “side effects” of open market operation (OMO) and large scale assets purchasing (LSAP). It shows that central bank assets purchases could be ineffective and even contractionary in certain situations and also gives what these “situations” are. In this paper, I study an environment with idiosyncratic liquidity shock as in Holmström and Tirole (1998) and Tirole (2011). There are three periods. At date 0, the initial investment in a variable-sized project that pays off at date 2 is made. At date 1, the project is hit by an idiosyncratic liquidity shock. To continue the project, an additional investment must be made, which is a random fraction of the initial investment at date 0; otherwise, the project is abandoned and delivers nothing at date 2. The liquidity needs at date 1 can be met in two ways: (i) by raising funds at date 1 using the project as collateral, i.e. “finance as you go”, borrowing when needs arise, or (ii) by holding liquid assets at date 0, i.e. “liquidity hoarding”, holding liquid assets to survive a liquidity shock. However, the date 1 value of the project is not fully pledgeable. The banker can borrow against the date 1 value of his project, but subject to a positive haircut. If the banker does not hold any liquid assets at date 1, he may have to give up the project even when the date 1 net value of the project is positive, which leads to demand for liquid assets at date 0. Different from Holmström and Tirole (1998) and Tirole (2011), I introduce two different types of public liquidities, treasury bills and bank reserves, which mature at date 1, into this environment. The central bank conducts monetary policy via open market operations, that is, buying treasury bills using bank reserves, which only changes the composition of the supply of public liquidities, but not the total quantity. In addition, I assume there are two groups of bankers, which are shadow and commercial bankers, which are different only in two ways. First, all the bankers can hold treasury bills, while only commercial bankers can hold bank reserves directly, and, second, commercial bankers have to hold enough bank reserves to satisfy the reserve requirement. Bankers are endowed with certain amount of net worth at date 0 and also issue one and two period debts to raise outside funds, but subject to a leverage constraint.

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[3] For example, Anderson and Liu (2013) documents that nominal interest rates on several European government treasury bills turned negative in mid 2012. Fleming and Garbade (2004) documented negative rates on certain US treasury security repo in late 2003

The key characteristics of this environment is the different marketabilities of bank reserves and treasury bills. It is true that commercial banks can issue one period deposit using bank reserves as the underlying assets, and then shadow banks can indirectly hold bank reserves by holding the one period deposits issued by commercial banks. However, issuing bank deposit tightens the commercial banks’ leverage constraint when it binds, and commercial banks charge a premium when issuing one period deposits. Thus, even if there is plenty of bank reserves, the shadow banks may not be able to easily achieve them even indirectly due to the commercial banks’ leverage constraint.

I begin the analysis by characterizing the equilibriums in the absence of leverage constraint. In the absence of leverage constraint, the compositions of the supply of public liquidities is irrelevant to the resource allocation of the economy and central bank asset purchases is neutral. Thus, the Modigliani-Miller theorem of open market operation (Wallace, 1981) holds. The reason behind is quiet straight forward. In the absence of leverage constraint, there is no friction for commercial banks to make arbitrage between liquidities of different marketabilities. Note that shadow banks can directly hold treasury bills and indirectly hold bank reserves by holding one period deposits issued by commercial banks. In the absence of leverage constraint, commercial banks issue one period deposit using bank reserves as the underlying assets, whenever the price of bank reserves is lower. Thus, bank reserves cannot be cheaper than treasury bills in equilibrium. In addition, the required reserve ratio constraint of commercial banks does not bind, due to "regulatory arbitrage". When the required reserve ratio constraint binds, the commercial banks' cost to intermediate funds is higher than that of shadow banks. Then, commercial banks shrink their balance sheets until the required reserve ratio constraint does not bind and more funds are intermediated by shadow banks. In this case, central bank asset purchases only changes the allocation of investment and liquidities across commercial and shadow banks, but has no effect on the allocation of the economy.

In the presence of leverage constraint, there is friction for commercial banks to make arbitrage between bank reserves and one period deposits, and thus shadow banks cannot easily achieve bank reserves even indirectly. In such environment, the composition of the supply of public liquidities matters. Central bank asset purchases has different effects in different regions.

i. When the level of bank reserves is low, the RRR constraint binds. Bank reserves are more expensive than treasury bills, and the nominal interest rate is positive. Commercial banks hold both bank reserves and treasury bills, while shadow banks only hold treasury bills. In this region, central bank asset purchases have expansionary effects through monetary multiplier.

ii. When the level of bank reserves is high, the RRR constraint does not bind, and bank reserves and treasury bills are perfect substitutes for commercial banks. Bank reserves are as expensive as treasury bills, which implies that the nominal rate on treasury bills is zero. In this region, central bank asset purchases are neutral, which only changes the composition of commercial banks' liquid asset portfolio.

iii. When the level of bank reserve is very high, the RRR constraint does not bind, and treasury bills become more expensive relative to bank reserves, which implies that the nominal rate on treasury bills turns negative. Commercial banks only hold bank reserves, and shadow banks hold treasury bills and one period deposits issued by commercial banks. In this region, central bank asset purchases further reduces the supply of liquid assets achievable for shadow banks, and thus are contractionary.

The idea that central bank asset purchases can have contractionary effect seems running counter to the conventional wisdom. However, this argument won't be surprising in an environment where commercial banking and shadow banking are equally important in terms of channelling funds and the required reserve ratio constraint does not bind, which could actually be how banking system looks like nowadays.

In the presence of leverage constraint, issuing one period deposit tightens the leverage constraint of commercial banks and then commercial banks charges a premium when they issuing one period deposits using bank reserves as the underlying assets. Thus, shadow banks are not able to indirectly hold bank reserves costlessly. When central bank pays interests on bank reserves, it tends to make the relative price of bank reserves even cheaper and thus arbitraging between bank reserves and one period deposits becomes more profitable. This reduces the commercial banks' cost of issuing one period deposits and also help keeping the nominal interest rate on treasury bills positive[4]. However, when commercial banks issue more one period deposits holding bank reserves as the underlying assets, it "crowds out" their investment in projects.

Literature Review

There are growing literatures on the effects and the side effects of QE. In many different settings that have been studied, QE is effective in terms of reducing long term rate[5]. However,

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[4] Keister et al. (2008) and Ennis and Keister (2008) point out that to continue targeting its policy rate when there is large bank reserves, Fed started to pay interests on bank reserves which allows it to increases the supply of bank reserve without driving interest rate below its target. In this model, paying interests on bank reserves has the similar effects.

[5] Bernanke (2010) thinks it works and his argument is mainly based on a fundamental financial friction: market segmentation. When market segmentation exists, there is friction to arbitrage across short and long term assets, which allows the central bank to change the relative price of short and long term assets by manipulating their relative supplies. Williamson (2013) thinks the short term public debts have a greater degree of pledgeability than the long term government debts and central bank assets purchases increases the total value of collateralizable wealth.

QE has its side effects, for example, large bank reserves. During the recent crisis, banks seem to have demand for bank reserves for precautionary reasons[6]. However, after 2009, the high level of bank reserves is mainly determined by the Fed's large scale asset purchasing, rather than banks' precautionary demand[7]. Nowadays, the huge amount of bank reserves can easily satisfy any precautionary demand of banks (Martin et al., 2013).

Recently, there are growing concerns about the undesirable effects of large bank reserves, and, interestingly, they are running in opposite directions. Some economists worry that the large bank reserves will finally lead to expansive bank lending. In contrast, some think the large bank reserves could have contractionary effect on bank lending. For example, Martin et al. (2013) shows that large bank reserves has no effect on bank lending in a frictionless environment and may distort the bank's lending decisions in the presence of balance sheet cost frictions. This paper looks at the "other side" of this issue. When central bank aggressively purchases safe assets, the stock of bank reserves, which are only available for interbank trades, rose by trillions, on one hand, and the same amount of marketable AA+ assets, which are available to all, disappear into the Fed's portfolio, on the other hand. The LSAP by Fed actually is squeezing achievable liquidities out of the shadow banking system. This paper focuses on this side effect of LSAP.

For the rest of this paper, section 2 describes the environment of the model. Section 3 studies the different equilibriums under different monetary policies, and how central bank asset purchasing can have very different effects in different regions. Section 4 concludes the paper.

4.2 Model

The analysis is based on a simplified version of Tirole and Holmström(1998). In this section, I describe the basic elements of the model.

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[6] For analysis of banks' precautionary demand for bank reserves, see Acharya and Merrouche (2013), Ashcraft et al. (2011) and Berrospide (2013)

[7] Keister and McAndrews (2009) points out that the high level of reserves is simply of a by-product of the Fed's recent lending facilities and asset purchases programs.

4.2.1 The Physical Environment

Time is discrete, $t = 0, 1, 2$. There is only one goods which can be consumed or invested. There are two groups of agents, which are households and bankers. Both groups have the same population, which is normalized to one. The preference of all the agents is given by: $u(c) = c_0 + c_1 + c_2$, where c_i denotes the household's consumption at date i.

A household is endowed with w units of goods at date 0 and w' units of goods at date 1. I assume that w and w' are large enough to finance all investments and tax. The bankers are divided into two groups which have the same size. The first group of bankers are called **commercial bankers** and each of them starts a financial intermediary, called commercial bank, at date 0. The second group of bankers are called **shadow bankers** and each of them starts a financial intermediary, called shadow bank, at date 0. Each banker is endowed with a units of goods at date 0.

There are two technologies. All the bankers have access to invest in project at date 0. To start a project, a banker initially invests k units of goods at date 0. At date 1, a liquidity shock v hits the banker, which is distributed on $[0, \bar{v}]$ with a cumulative distribution function $F(.)$ and a probability density function $f(.)$. The banker has to invest additional vk units of goods to continue the project; otherwise, the project is abandoned. The project returns $r_k k$ at date 2 if the project is continued at date 1 and nothing if it is abandoned at date 1. The public sector can invest in a storage technology, which returns one unit of goods at date 1 for each unit invested at date 0. The maximum amount the public sector can invest in the storage technology is $\bar{l}$.

4.2.2 Financing and Refinancing

The public sector issues **treasury bills**, denoted by b, and **bank reserves**, denoted by e, to finance its investment in the storage technology. Each unit of b and e has a face value of one and repays one unit of goods at date 1. The supply of b and e is exogenously determined by the public sector. Let $\bar{b}$ and $\bar{e}$ denote the supply of treasury bills and bank reserves. Since the public sector can only invests $\bar{l}$ units of goods in the storage technology at date 0, thus $\bar{b} + \bar{e} = \bar{l}$. Let q and p denote the prices of b and e in terms of consumption goods at date 0.

The loss (profits) of issuing b and e is covered by a lump sum tax (subsidy) on households at date 0. Both b and e are referred as the **public liquidities**. The key characteristic of this model is that the public liquidities are heterogeneous in their accessibilities for agents in the economy. Commercial bankers and households can directly hold b and e, while shadow bankers can only directly hold b.

To finance his investment, the banker issues **deposits** in addition to his net worth a at date 0. A banker can issue two types of deposits: d_1, which matures at date 1, and d_2, which matures at date 2. Each unit of d_1 and d_2 has a face value of one and repays one unit of goods when it matures. Let z_1 and z_2 denote the prices of d_1 and d_2 in terms of consumption goods at date 0. If the banker cannot repay the depositors the promised amount, the bank goes into bankruptcy and the depositors take away whatever is left in the bank. If the banker goes into bankruptcy at date 1, the holders of d_1 get repaid before the holders of d_2. Deposits d_1 and d_2 can be held by both the bankers and the households. Let d_1 and d_2 denote the amounts of one period and two period debts issued by the banker. Assume that the banker can diversify his holdings of deposits issued by other bankers and let d_1^m and d_2^m denote the market portfolio of one period and two period deposits held by the banker.

At date 1, the banker has to reinvest vk to continue his project. After the realization of v, a **refinancing market** opens. In the refinancing market, the lenders are households and the bankers with extra liquidities. The borrowers are the bankers who need to borrow to continue their projects. To finance the reinvestment, the banker can either sells his liquid assets or use his project (physical asset) as collateral to borrow in the refinancing market. I assume that bankers cannot use the two period deposits issued by other banks, d_2^m, as collateral to borrow money in the refinancing market. Thus, "privately-produced" liquidity is ruled out. Otherwise, bankers can "produce" liquid assets by issuing and holding two period deposits at date 0 and then borrowing against these deposits in the refinancing market at date 1. However, as will be shown in the later section, in the presence of leverage constraints, allowing for "privately produced" liquidity only slightly change the results of the model.

Let $\hat{v}$ denote the cut off rule of the banker: if $v \leq \hat{v}$, the banker continues the project, and if $v > \hat{v}$, the banker abandons the project. Let r_l be the interest rate in the refinancing market. Assume that the maximum amount the banker can borrow against his project is $\theta\left(r_k/r_l\right)k$, with $0 < \theta < 1$. Here, $1-\theta$ denotes a positive haircut. Note that the first best cut off rule of v is r_k/r_l: it is optimal to continue the project whenever $v \leq (r_k/r_l)$. However, the first best cut off scheme cannot be implemented if the banker only use his project as collateral to borrow in the refinancing market when $\theta < 1$. The limited pledgeability of project gives the banker incentives to hold liquid assets at date 0. To make the analysis more interesting, I assume that when $r_l = 1$, the net value of the project is positive if the first best cut off scheme is implemented ($\hat{v} = r_k$), but negative if the banker only use the project as collateral to borrow in the refinancing market ($\hat{v} = \theta r_k$).

Assumption 1 $\int_0^{r_k}\left(r_k - v\right)f(v)dv - 1 > 0 > \int_0^{\theta r_k}\left(r_k - v\right)f(v)dv - 1$

Thus, when $r_l = 1$, if the banker does not hold any liquidities as buffer to the liquidity shock, the net value the project is negative.

Limited Commitment. To introduce **leverage constraint**, I assume that at the beginning of date 1, the banker has a chance to divert a fraction ρ of assets in the bank, before the one period debt d_1 is repaid and the liquidity shock v realizes. After the banker diverts, the bank is taken over by the depositors. After the banker diverts, the project is destroyed; however, the remaining liquid assets can be recovered. After the bank is taken over, the holders of d_1 are repaid first and then the holders of d_2 take away whatever remains.

4.2.3 Monetary Policy

At date 0, the central bank buys treasury bills b using bank reserves e. The central bank acts as a market maker who buys or sells b using bank reserves e at its target price for any amount. However, the total supply of public liquidities should satisfy $\bar{b} + \bar{e} = \bar{l}$. Thus, open market operation only changes the composition of the public liquidities, while its total quantity is kept constant.

Note that both b and e repays one unit of goods at date 1. The **nominal** face value of b, that is, its value in terms of e at date 1, is one. Also note that at date 0, the real prices of b and e are q and p respectively. The **nominal** price of b, that is, the relative price of b in terms of e at date 0, is thus q/p. Let i denote the **nominal interest rate** paid on b, which is thus given by:

$$1 + i = \frac{p}{q}$$

Here, the nominal interest rate i measures the relative price of e and b. The central bank affects the nominal interest rate i by choosing the supply of b and e.

Note that bank reserves are more expensive than treasury bills when $p/q > 1$ and $i > 0$. Thus, a positive nominal interest rate implies a premium for e. In this model, this premium stems from that holding bank reserves loosens the required reserve ratio constraint of commercial banks when it binds. When the required reserve ratio constraint does not bind and b and e are perfect substitutes for commercial banks, then $q = p$ and the nominal interest rate i drops to zero.

4.2.4 Optimization by Bankers

I temporarily drop the superscripts of bankers for convenience. At date 0, a banker may invest on project k, treasury bills b, bank reserves e and deposits issued by other bankers ($e = 0$ for shadow bankers). To finance his investment, the banker issues deposits in addition to his net worth a. Note that the two period deposits issued by other bankers, d_2^m, cannot be used as collateral to borrow in the refinancing market and thus bankers won't get any benefit by holding d_2^m, no matter whether the leverage constraint binds or not. Thus, bankers do not cross hold two period deposits and $d_2^m = 0$ in equilibrium. In the later section, I will extend the model by allowing the bankers to borrow against two period deposits d_2^m. As will be shown, this only slightly change the results when the banker's leverage constraint binds. At date 0, the balance sheet of a banker is given by

$$k + qb + pe + z_1 d_1^m = z_1 d_1 + z_2 d_2 + a \tag{4.1}$$

where q, p, z_1 and z_2 are prices of b, e, d_1 and d_2, in terms of goods at date 0, respectively. Here, $e = 0$ for shadow bankers.

At date 1, a banker's net holdings of liquidities equals to his holdings of one period assets, b, e and d_1^m, minus his one period debt, d_1, which is called the banker's "liquidity buffer". To continue the project the banker has to reinvest vk units of goods. Let $\hat{v}$ to be the cut off rule of the banker: when $v \leq \hat{v}$ the banker continues its project; otherwise, the project is abandoned. To implement this cut off rule, the banker should hold enough liquidity buffer and the following should hold:

$$\hat{v}k - (b + e + d_1^m - d_1) \leq \theta \left(\frac{r_k}{r_l} \right) k \tag{4.2}$$

where the LHS is the maximum amount the banker may need to borrow given the cut off rule $v = \hat{v}$, and the RHS is the maximum amount the bank can borrow against his project. The inequality (4.2) is referred as the **collateral constraint**.

If the banker holds a non-negative amount of liquidity buffer, i.e. $b+e+d_1^m-d_1 \geq 0$, the one period deposit issued by this banker d_1 is risk free, since it can be always fully repaid regardless of the realization of v; however, if the banker holds a negative liquidity buffer, i.e. $b + e + d_1^m - d_1 < 0$, the banker defaults on d_1 whenever $v > \hat{v}$. However, as will be shown, banker never holds a negative buffer and the one period deposit d_1 is endogenously **risk free**. The intuition is that when a banker holds negative liquidity buffer, the cut off $\hat{v}$ is even less than θr_k and the marginal benefit of holding liquidity buffer is extremely large. If the banker were issuing d_1 while holding a negative liquidity buffer, the price he asks would be extremely high, at which the demand is zero.

Lemma 4.2.1 *In any equilibrium, bankers hold non-negative liquidity buffer and the one period deposits issued by bankers are endogenously risk free.*

Also note that the interest rate in refinancing market is pinned down at 1 by households, who are the marginal lenders.

Lemma 4.2.2 *In any equilibrium, the refinancing rate is pinned down by households at 1, i.e.* $r_l = 1$.

At date 2, the expected gross revenue of the bank is $y(v) = \mathbb{1}_{v \geq \hat{v}}(r_k - vr_l)k + (b + e + d_1^m - d_1)r_l$, where $\mathbb{1}_{v \geq \hat{v}} = 1$ when $v \geq \hat{v}$ and 0 otherwise. The ex post repayments to the holders of d_2 is $min\{d_2, y(v)\}$. Note that the expected return on the two period deposit d_2 should be one in equilibrium, which implies:

$$\int_0^{\bar{v}} min\{d_2, y(v)\} f(v) dv = z_2 d_2$$

Define $F(\hat{v}) \equiv \int_0^{\hat{v}} f(v)dv$, $g(v) \equiv vf(v)$, and $G(\hat{v}) \equiv \int_0^{\hat{v}} g(v)dv$. The gross return of the bank at date 2 is: $[F(\hat{v})r_k - G(\hat{v})r_l]\, k + (b + e + d_1^m - d_1)\, r_l$. The banker's expected profits is given by:

$$[F(\hat{v})r_k - G(\hat{v})r_l]\, k + (b + e + d_1^m - d_1)\, r_l - z_2 d_2 - a \tag{4.3}$$

Note that the diverting is never efficient, since in equilibrium the expected return of the project is higher than ρ: $F(\hat{v})r_k - G(\hat{v})r_l \geq 1 > \rho$. To ensure that the banker will not divert at date 1, his expected revenue should not be less than the revenue he can get by diverting:

$$[F(\hat{v})r_k - G(\hat{v})r_l]\, k - r_l(b + e + d_1^m - d_1) - z_2 d_2 \geq \rho r_l(k + e + b + d_1^m) \tag{4.4}$$

where the LHS is the banker's expected return if he does not divert, while the RHS is the return to the banker if he diverts in date 1. The above inequality represents the **leverage constraint** faced by bankers.

Shadow bankers' optimization problem. The shadow banker chooses k, b, d_1^m, d_1, $\hat{v}$ to maximize his expected profits subject to the collateral constraint (cc), the leverage constraint (lc) and the non negative constraint (nc):

$$\max\, [F(\hat{v})r_k - G(\hat{v})r_l - 1]\, k + (r_l - q)b + (r_l - z_1)(d_1^m - d_1) \tag{4.5}$$

subject to

$$cc: \quad \hat{v}k - (b + d_1^m - d_1) \leq \theta\, (r_k/r_l)\, k \tag{4.6}$$

$$lc: \quad \left\{ \begin{array}{l} [1 - (F(\hat{v})r_k - G(\hat{v})r_l - \rho r_l)]\, k + [q - r_l(1-\rho)]\, b \\ + [z_1 - r_l(1-\rho)]\, d_1^m + (r_l - z_1)d_1 \end{array} \right\} \leq a \tag{4.7}$$

$$nc: \quad k, b, d_1, d_1^m \geq 0$$

where the objective function (4.5) is obtained by combing (4.1) and (4.3), the leverage constraint (4.7) is obtained by combining (4.1) and (4.4).

Let μ and λ be the Lagrangian multipliers associated with the collateral constraint (4.6) and the leverage constraint (4.7). Here, $1+\lambda$ represents the price of the internal funds and λ represents the external funding premium. In addition, μ is interpreted as the price of liquidity service. Define $\tilde{\lambda} \equiv \frac{\lambda}{1+\lambda}$ and $\tilde{\mu} \equiv \frac{\mu}{1+\lambda}$ to be the **external funding premium** and the **price of liquidity services** in terms of internal funds. The FOCs are thus given by

$$k: \quad F(\hat{v})r_k - G(\hat{v})r_l - 1 = \tilde{\lambda}\rho r_l + \tilde{\mu}\left[\hat{v} - \theta\left(r_k/r_l\right)\right] \tag{4.8}$$

$$b: \quad q - r_l = \tilde{\mu} - \tilde{\lambda}\rho r_l \tag{4.9}$$

$$d_1^m: \quad z_1 - r_l = \tilde{\mu} - \tilde{\lambda}\rho r_l \tag{4.10}$$

$$d_1: \quad z_1 - r_l = \tilde{\mu} \tag{4.11}$$

$$\hat{v}: \quad f(\hat{v})r_k - g(\hat{v})r_l = \tilde{\mu} \tag{4.12}$$

Commercial bankers' optimization problem. The commercial banker's problem is basically the same as that of shadow bankers. The only differences are that the commercial banker can hold bank reserves e and they face the required reserve ration (RRR) constraint. Let rr denote the required reserve ratio and thus the RRR constraint is given by

$$rr(z_1 d_1 + z_2 d_2) \leq pe \tag{4.13}$$

A commercial banker chooses k, b, e, d_1^m, d_1, $\hat{v}$ to maximize his expected profits subject to the collateral constraint, the RRR constraint, the leverage constraint and the non negative constraint:

$$\max\left[F(\hat{v})r_k - G(\hat{v})r_l - 1\right]k + (r_l - q)b + (r_l - p)e + (r_l - z_1)(d_1^m - d_1) \tag{4.14}$$

subject to

$$cc: \quad \hat{v}k - (b + e + d_1^m - d_1) \leq \theta \left(r_k / r_l \right) k \tag{4.15}$$

$$rc: \quad k + qb + z_1 d_1^m - a \leq \phi pe \tag{4.16}$$

$$lc: \quad \left\{ \begin{array}{l} \left[1 - \left(F(\hat{v}) r_k - G(\hat{v}) r_l - \rho r_l \right) \right] k + \left[q - r_l(1-\rho) \right] b \\ + \left[p - r_l(1-\rho) \right] e + \left[z_1 - r_l(1-\rho) \right] d_1^m + (r_l - z_1) d_1 \end{array} \right\} \leq a \tag{4.17}$$

$$nc: \quad k, b, e, d_1^m, d_1 \geq 0$$

where $\phi \equiv \frac{1-rr}{rr}$. Here, the leverage constraint (4.17) is obtained by combining (4.4) and balance sheet equality (4.1), the RRR constraint (4.16) is obtained by coming (4.13) and (4.1).

Let μ, η and λ be the Lagrangian multipliers associated with the collateral constraint (4.15), the RRR constraint (4.16) and the leverage constraint (4.17), the following should hold in equilibrium:

$$k: \quad F(\hat{v}) r_k - G(\hat{v}) r_l - 1 = \tilde{\lambda} \rho r_l + \tilde{\mu}(\hat{v} - \theta s) + \tilde{\eta} \tag{4.18}$$

$$b: \quad q - r_l = \tilde{\mu} - \tilde{\lambda} \rho r_l - \tilde{\eta} q \tag{4.19}$$

$$d_1^m: \quad z_1 - r_l = \tilde{\mu} - \tilde{\lambda} \rho r_l - \tilde{\eta} z_1 \tag{4.20}$$

$$d_1: \quad z_1 - r_l = \tilde{\mu} \tag{4.21}$$

$$e: \quad p - r_l = \tilde{\mu} - \tilde{\lambda} \rho r_l + \tilde{\eta} \phi p \tag{4.22}$$

$$\hat{v}: \quad f(\hat{v}) r_k - g(\hat{v}) r_l = \tilde{\mu} \tag{4.23}$$

where $\tilde{\lambda} = \frac{\lambda}{1+\lambda}$, $\tilde{\mu} = \frac{\mu}{1+\lambda}$ and $\tilde{\eta} = \frac{\eta}{1+\lambda}$.

4.2.5 Equilibrium under Perfect Commitment

I analyse separately the cases in which bankers optimize with/without leverage constraint. In this subsection, I assume perfect commitment: the bankers cannot divert resources from depositors and the banker's leverage is not constrained. As will be shown, in this environment, the composition of the supply of the public liquidities is irrelevant. The intuition is that the commercial banks can make profitable arbitrage between one period deposits and bank reserves via issuing one period deposits using bank reserves as the underlying assets. Thus, the composition of the supply of public liquidities is irrelevant with the allocation of liquidities across banks.

In the absence of leverage constraint, the Lagrangian multiplier associated with leverage constraint should be zero, i.e. $\lambda = 0$. In addition, $q = p$ and RRR does not bind. Commercial bankers hold bank reserves for shadow bankers by issuing one period deposits using bank reserves as the underlying assets. This won't tighten the RRR constraint of the commercial bank even if it binds, since one hundred percent of the one period deposits it issues are held as reserves. Thus, $z_1 = q \leq p$ should hold in the equilibrium; otherwise, the commercial banks can make arbitrage between d_1 and e by issuing one period deposits using bank reserves as the underlying assets. Actually, the RRR constraint of the commercial banks does not bind. The intuition is that when the RRR constraint binds, the commercial banker are forced to hold more bank reserves than what they would hold if there were not RRR constraint to satisfy the reserve requirement, leading to an additional cost for the commercial banks to invest. The commercial banker reduces his investment until the RRR constraint does not bind.

Lemma 4.2.3 *In the absence of leverage constraint, $p = q$ and the RRR constraint does not bind in equilibrium.*

Note that the Lagrangian multiplier associated with the leverage constraint and RRR constraint are zero, i.e. $\tilde{\lambda} = \tilde{\eta} = 0$. Thus, the optimization problems faced by shadow bankers and commercial bankers are actually the same. Since both shadow bankers and commercial bankers choose the same k and $\hat{v}$, I drop the superscripts of bankers for convenience. By combing the first order conditions of k and $\hat{v}$, i.e. (4.8) and (4.12), or (4.18) and (4.23), and $\tilde{\lambda} = \tilde{\eta} = 0$, we can have

$$[F(\hat{v})r_k - G(\hat{v}) - 1] - [f(\hat{v})r_k - g(\hat{v})]\,(\hat{v} - \theta r_k) = 0 \tag{4.24}$$

To rule out multiple equilibrium, I assume that $f(\hat{v})r_k - g(\hat{v})$ is monotonically decreasing in $\hat{v}$ on $[\theta r_k, r_k]$, e.g. when v is uniformly distributed.

Assumption 2 *$f(\hat{v})r_k - g(\hat{v})$ is monotonically decreasing in $\hat{v}$ on $[\theta r_k, r_k]$.*

Thus, there exists an unique $\hat{v}$ that solves (4.24). In addition, under Assumption 1, the $\hat{v}$ that solves (4.24) satisfies $\hat{v} \in [\theta r_k, r_k]$. Since $\hat{v} > \theta r_k$, all the bankers hold a positive liquidity buffer and thus the one period deposits issued by bankers are endogenously **risk free**.

By combing the first order conditions of b, e, and d_1^m, the liquidity premium is given by:

$$q - 1 = p - 1 = z_1 - 1 = f(\hat{v})r_k - g(\hat{v}) \tag{4.25}$$

Since the price of liquidities are larger than 1, households won't hold any liquid assets in equilibrium. All the public liquidities $\bar{l}$ are held by bankers. Note that all the bankers choose the same cut off rule $\hat{v}$. Using the collateral constraint, we can have

$$k = \frac{\bar{l}}{\hat{v} - \theta r_k} \tag{4.26}$$

Thus, the total investment in project k depends only on the total supply of the public liquidities and the composition of the supply of public liquidities have no effect on the resource allocation.

Proposition 4.2.1 *In the absence of leverage constraint, the composition of the supply of the public liquidity is irrelevant to the resource allocation. All the bankers make the same decisions on k and $\hat{v}$, where $\hat{v}$ solves* (4.24) *and k is then given by* (4.26).

4.3 The Effects of Central Bank Asset Purchases

When the bankers' leverage is unconstrained, the composition of the public liquidities is irrelevant. This section analyses the cases where bankers' leverage are constrained and compares different cases in which central bank asset purchasing has very different effects on the economy. One important feature of the economy with binding leverage constraint is that commercial banks cannot costlessly make arbitrage between one period deposits and bank reserves by issuing one period deposits using bank reserves as the underlying assets, since this will tighten its leverage constraint. **Notation:** from this section, I use different superscripts to represent bankers and households. Let "s" denote shadow bankers, "c" denote commercial bankers and "h" denote households.

4.3.1 Neutral Region

I start with the case when the prices of b and e are both equal to one, i.e. $q = p = 1$, and the RRR constraint does not bind. When $q = p = 1$ and the RRR constraint does not binds, treasury bills and bank reserves are perfect substitutes for the commercial banks. Since the prices of bank reserves e and treasury bills b are equal, there is no room for the commercial banks to make arbitrage by issuing one period debts using bank reserves as the underlying assets, i.e. $d_1^{m,c} = 0$. When the central bank buying treasury bills using bank reserves, this will change nothing but the commercial banks' portfolio of liquid assets. One can think this region as capturing the characteristics of the **liquidity trap**: when the economy is in this region, the nominal interest rates i drops to zero and the asset purchasing by central bank will not further lower the nominal interest rate. In this region, central bank asset purchases does not have any effect on the economy and thus it is referred as the **neutral region**.

Choices of Bankers. When the RRR constraint does not bind and $\tilde{\eta} = 0$, the optimization problems faced by shadow bankers and commercial bankers are essentially the same. **Notation:** since the shadow bankers and the commercial bankers make the same decisions in this region, there is no need to introduce the superscripts of bankers in this subsection. Let variables with superscript * to denote the variables in the **neutral region**. I use k^* and $\hat{v}^*$ to denote the investment decisions of shadow bankers and commercial bankers, which are to be used as benchmark later on.

When $q = p = 1$, the leverage constraint should bind, since when the price of liquidities are one, the net cost of holding liquidities is zero and bankers will invest as much as possible until the leverage constraint binds. Given that $r_l = 1$ and $\tilde{\eta} = 0$, by combing the first order conditions of k and $\hat{v}$, (4.8) and (4.12), or (4.18) and (4.23), we can have

$$F(\hat{v}^*)r_k - G(\hat{v}^*) - 1 - [f(\hat{v}^*)r_k - g(\hat{v}^*)]\,(\hat{v}^* - \kappa r_k + 1) = 0 \tag{4.27}$$

Under assumption 1 and 2, the LHS is monotonically increasing in $\hat{v}^*$ and there is an unique $\hat{v}^*$, with $\hat{v}^* \in [\theta r_k, r_k]$, such that the above equation holds. In addition, the price of liquidity service $\tilde{\mu}^*$ in this region is given by:

$$\tilde{\mu}^* = f(\hat{v}^*) r_k - g(\hat{v}^*) \tag{4.28}$$

Given that $r_l = 1$, $\tilde{\eta} = 0$ and $d_1 = d_1^m = 0$, the collateral and the leverage constraints of shadow banks and commercial banks can be written as:

$$cc: \qquad \hat{v}k - (b + e) = \theta r_k k$$

$$lc: \qquad [1 - (F(\hat{v}) r_k - G(\hat{v}) - \rho)]\, k + \rho(b + e) = a$$

where $e = 0$ for shadow bankers. Define $\psi_1(\hat{v})$ to be the fraction of investment in project k that is collateralized by net worth a and $\psi_2(\hat{v})$ to be the ratio of liquid assets $b + e$ to k. Thus, $\psi_1(\hat{v})$ and $\psi_2(\hat{v})$ are given by

$$\psi_1(\hat{v}) \equiv 1 - (F(\hat{v}) r_k - G(\hat{v}) r_l - \rho r_l) \tag{4.29}$$

$$\psi_2(\hat{v}) \equiv \hat{v} - \theta\,(r_k / r_l) \tag{4.30}$$

By combing the collateral and leverage constraints we can have

$$k^* = \frac{a}{\psi_1^* + \rho \psi_2^*} \tag{4.31}$$

$$b^s = b^c + e^c = \frac{a \psi_2^*}{\psi_1^* + \rho \psi_2^*} \tag{4.32}$$

where $\psi_1^* = \psi_1(\hat{v}^*)$ and $\psi_2^* = \psi_2(\hat{v}^*)$ with $\hat{v}^*$ solves (4.27).

Proposition 4.3.1 *When $q = p = 1$ and RRR constraint does not bind, all the bankers choose the same $\{k, \hat{v}\}$, where $\hat{v}$ solves* (4.27) *and k is then given by* (4.31).

4.3.2 Expansionary Region

If $p > q = 1$ and the commercial banks hold both treasury bills and bank reserves, bank reserves is relatively more expensive than the treasury bills, which are held only by commercial banks. The RRR constraint of commercial banks binds. Note that commercial banks would hold only treasury bills if there were no reserve requirement. The binding RRR constraint implies that commercial banks are forced to hold bank reserves, a more expensive public liquidities, to satisfy the reserve requirement. When $p > q = 1$, the shadow banks hold only treasury bills. Since the price of bank reserves e is higher than that of treasury bills b, there is no room for the commercial banks to make arbitrage by issuing one period debts using bank reserves as the underlying assets, i.e. $d_1^{m,c} = 0$. When the central bank buying treasury bills using bank reserves, it increases the supply of bank reserves, lowers the price of bank reserves p and thus has expansionary effects on the commercial banks.

One can think of this region as capturing how the monetary policy work in usual times. When the economy is in this region, the relative price of treasury bills in terms of bank reserves is less than one and thus the nominal interest rate i is positive. The central bank asset purchases increase the supply of bank reserves and the relative price of treasury bills, leading to decrease in the nominal interest rate. This region is referred as **expansionary region**.

Choices of Shadow Bankers. When $p > q = 1$, the shadow bankers hold treasury bills only. The shadow bankers' optimization problem is actually the same as the one they faces in the neutral region. Thus, their decisions are exactly the same as in the neutral regime as well, i.e. $k^s = k^*$, $b^s = \psi_2(\hat{v}^*)k^*$ and $\hat{v}^s = \hat{v}^*$.

Choices of Commercial Bankers. Given that $q = 1$ and $r_l = 1$, by combing the commercial banks' FOCs of $\hat{v}$, k and b, we can see that the cut off rule $\hat{v}^*$ solves (4.27) as in the neutral region. The commercial banks chooses the same cut off of v as in the neutral regime, i.e. $\hat{v}^c = \hat{v}^*$. Given that $b^c > 0$, the collateral constraint, the RRR constraint and the leverage constraint of commercial banks can be written as:

$$\begin{aligned}
cc: \quad & \hat{v}^c k^c - (b^c + e^c) = \theta r_k k^c \\
rc: \quad & k^c + b^c - a \leq \phi p e^c \\
lc: \quad & \left[1 - (F(\hat{v}^c) r_k - G(\hat{v}^c) - \rho)\right] k^c + \rho(b^c + e^c) = a
\end{aligned}$$

By combing the above three conditions, the commercial banker's investment in project k^c is given by:

$$k^c = \frac{a}{\psi_1^* + \rho\psi_2^* + \left(1 - \frac{1}{p}\right) rr\left[1 - \psi_1^* + (1-\rho)\psi_2^*\right]} \tag{4.33}$$

$$e^c = \frac{rr\left[1 - \psi_1^* + (1-\rho)\psi_2^*\right] a}{p\left(\psi_1^* + \rho\psi_2^*\right) + (p-1) rr\left[1 - \psi_1^* + (1-\rho)\psi_2^*\right]} \tag{4.34}$$

where $\psi_1^* = \psi_1(\hat{v}^*)$ and $\psi_2^* = \psi_2(\hat{v}^*)$.

Proposition 4.3.2 *When $p > q = 1$ and commercial banks hold both treasury bills and bank reserves:*

(a) the shadow bankers chooses the same k^s and $\hat{v}^s$ as in the neutral region, i.e. $k^s = k^*$, $\hat{v}^s = \hat{v}^*$.

(b) the commercial bankers choose the same cut off rule as in the neutral regime, i.e. $\hat{v}^c = \hat{v}^*$, while their decisions on k^c is then given by (4.33)*.*

Discussion. Note that both k^c and e^c are decreasing in p. Also, note that in this region all the bank reserves are held by commercial banks. Central bank asset purchases increases the supply of bank reserves, lowers its price p and the nominal interest rate i, and thus have expansionary effects on commercial banks.

4.3.3 Contractionary Region

If $q > p = 1$ and the RRR constraint does not bind, the commercial banks only hold bank reserves and all the treasury bills are held by the shadow banks. In the absence of leverage constraint, $q > p$ cannot happen in equilibrium, since the commercial banks can make profitable arbitrage by issuing one period deposits while holding bank reserves as the underlying assets. However, when the leverage of the commercial banks is constrained, issuing one period deposit tightens its leverage constraint. Thus the commercial banks charge a premium when they issue the one period deposits.

As will be shown, if commercial bank were to issue one period deposits, the price they would charge is $1 + \tilde{\mu}^*$, where $\tilde{\mu}^*$ is given by (4.28). Actually, q cannot be higher than $1+\tilde{\mu}^*$ in equilibrium and the commercial bankers are willing to supply any positive amount of one period deposits at price $1 + \tilde{\mu}^*$. The intuition is that when $q = 1 + \tilde{\mu}^*$, investing in project is as profitable as making arbitrage between one period deposits and bank reserves for the commercial banks. Thus, when $1 + \tilde{\mu}^* > q$, shadow banks only hold treasury bills. The demand for one period deposit is still zero. When $1 + \tilde{\mu}^* = q$, the shadow banks hold a positive amount of one period deposits issued by the commercial bank.

One can think of this region as capturing what is happening now after rounds of QE. In this region, since the relative price of b in terms of e is larger than one, thus the nominal interest rate i is negative. This happened in Europe and also may be happening in the US now. Nowadays, in the US the interest rates paid on bank reserves is higher than that on three month treasury bills, which implies that treasury bills is relatively more expensive than bank reserves. In this region, the asset purchasing by central bank can have contractionary effects via reducing the supply of liquidities accessible for shadow banks. This region is referred as **contractionary region**.

Case 1: $1 + \tilde{\mu}^* > q > p = 1$, where $\tilde{\mu}^*$ is the price of liquidity service in the neutral region (4.28). In this case, the commercial bankers hold only bank reserves and do not issue any one period deposit.

Choices of Shadow Bankers. The shadow bankers' decisions are different from that in the neutral region. Given that $r_l = 1$, by combing the FOCs of k and $\hat{v}$, we can have

$$F(\hat{v}^s)r_k - G(\hat{v}^s) - 1 - [f(\hat{v})r_k - g(\hat{v}^s)](\hat{v}^s - \theta r_k + 1) = 1 - q \tag{4.35}$$

Note that the LHS of the above equation is increasing in $\hat{v}^s$. Thus, when $q > 1$ the cut off rule $\hat{v}^s$ is lower than that in the neutral region, i.e. $\hat{v}^s < \hat{v}^*$.

When $q \leq 1 + \tilde{\mu}^*$, the shadow banks' collateral constraint and leverage constraint bind.

Actually, the leverage constraint dose not bind only when the price q is equal to or higher than the liquidity price in the perfect commitment equilibrium which is surely higher than $1+\tilde{\mu}^*$ [8]. Given that $r_l = 1$ and $d_1^s = d_1^{m,s} = 0$, the collateral constraint and leverage constraint of the shadow banks can be written as:

$$cc: \qquad \hat{v}^s k^s - b^s = \theta r_k k^s$$

$$lc: \qquad [1-(F(\hat{v}^s)r_k - G(\hat{v}^s) - \rho)]\,k^s + [q-(1-\rho)]b^s = a$$

By combing the above two equations we can obtain:

$$k^s = \frac{a}{\psi_1(\hat{v}^s) + [q-(1-\rho)]\psi_2(\hat{v}^s)} \tag{4.36}$$

$$b^s = \frac{a\psi_2(\hat{v}^s)}{\psi_1(\hat{v}^s) + [q-(1-\rho)]\psi_2(\hat{v}^s)} \tag{4.37}$$

where $\hat{v}^s$ is determined by (4.35).

Choices of Commercial Bankers. The optimization problem of the commercial bankers are exactly the same as that in the neutral region. Thus, the commercial banker's decisions on k^c and $\hat{v}^c$ are exactly the same as in the neutral region as well, i.e. $k^c = k^*$ and $\hat{v}^c = \hat{v}^*$.

Proposition 4.3.3 *When $1+\tilde{\mu}^* > q > p = 1$ and the RRR constraint does not bind:*
(a) the shadow bankers do not hold one period deposits issued by commercial banks and choose k^s and $\hat{v}^s$ where $\hat{v}^s$ solves (4.35) *and k^s is then given by* (4.36)
(b) the commercial bankers choose the same k^c and $\hat{v}^c$ as in the neutral region, i.e. $k^c = k^*$, $\hat{v}^c = \hat{v}^*$

Discussion. In this case, all the treasury bills are held by shadow bankers, $b^s = \bar{b}$, and they do not hold one period deposits issued by commercial banks. When the central bank buying treasury bills using bank reserves, this decreases the supply of treasury bills $\bar{b}$, that is, the only liquidity accessible for the shadow banks. Note that b^s is decreasing [9] in q. Assets purchasing in this regime has contractionary effects on shadow banks, since it reduces the supply of liquidities accessible for them. This also increases the relative price of treasury bills in terms of bank reserves and thus lead to a further decrease in the nominal interest rate.

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[8] To see this, comparing (4.24) and (4.27). It is obvious that the cut off rule $\hat{v}$ in the perfect commitment equilibrium is lower than $\hat{v}^*$. Under assumption 2, the price of liquidity service μ in the perfect commitment equilibrium is higher than $\tilde{\mu}^*$.

[9] To see this, note that $\hat{v}^s$ is decreasing in q (4.35), $\psi_1()$ is decreasing in $\hat{v}^s$ and $\psi_2()$ is increasing in $\hat{v}^s$.

Case 2: $1 + \tilde{\mu}^* = q > p = 1$, where $\tilde{\mu}^*$ is the price of liquidity service in the neutral region (4.28). In this case, the shadow bankers are indirectly holding bank reserves via holding the one period deposits issued by commercial bankers.

Choices of Shadow Bankers. The decisions of the shadow bankers are actually the same as in case 1; however, now the price of liquidity is $1 + \tilde{\mu}^*$ and they hold both b and $d_1^{m,c}$:

$$k^s = \frac{a}{\psi_1(\hat{v}^s) + [\tilde{\mu}^* - (1-\rho)]\psi_2(\hat{v}^s)} \tag{4.38}$$

$$b^s + d_1^{m,c} = \frac{a\psi_2(\hat{v}^s)}{\psi_1(\hat{v}^s) + [\tilde{\mu}^* - (1-\rho)]\psi_2(\hat{v}^s)} \tag{4.39}$$

where $\hat{v}^s$ solves

$$F(\hat{v}^s)r_k - G(\hat{v}^s) - 1 - [f(\hat{v})r_k - g(\hat{v}^s)]\,(\hat{v}^s - \theta r_k + 1) = 1 - \tilde{\mu}^* \tag{4.40}$$

Choices of Commercial Bankers. When $q = 1+\tilde{\mu}^*$, the optimization problem faced by the commercial bankers is very similar as that in the neutral region; however, in this case, $d_1^c \geq 0$ does not bind. The collateral constraint and the leverage constraint of the commercial banks bind as in the neutral region. The commercial banks choose the same cut off rule as in the neutral regime, i.e, $\hat{v}^c = \hat{v}^*$. Given that $r_l = 1$, $d_1^c > 0$ and $d_1^{m,c} = 0$, the collateral constraint and the leverage constraint of the commercial banks can be written as:

$$cc: \quad \hat{v}^* k^c - (e^c - d_1^c) \leq \theta r_k k^c$$

$$lc: \quad [1 - (F(\hat{v}^*)r_k - G(\hat{v}^*) - \rho)]\,k^c + \rho e^c - \tilde{\mu}^* d_1^c \leq a$$

By combing the two above equations, we can see that the commercial banker's investment in project is

$$k^c = \frac{a - (\rho - \tilde{\mu}^*)\, d_1^c}{\psi_1^* + \rho\psi_2^*} \tag{4.41}$$

$$e^c = \frac{a\psi_2^* - (\rho - \tilde{\mu}^*)\, d_1^c \psi_2^*}{\psi_1^* + \rho\psi_2^*} + d_1^c \tag{4.42}$$

where $\psi_1^* = \psi_1(\hat{v}^*)$ and $\psi_2^* = \psi_2(\hat{v}^*)$.

Proposition 4.3.4 *When* $1 + \tilde{\mu}^* = q > p = 1$ *and the RRR constraint does not bind:*
(a) the shadow banks choose $\hat{v}^s$ *which solves* (4.40) *and* k^s *is then given by* (4.38).
(b) the commercial banks choose the same cut off rule as in the neutral region, i.e. $\hat{v}^c = \hat{v}^*$*; they are willing to issue any positive amount of one period deposit* d_1^c *at price* $1 + \tilde{\mu}^*$ *and their investment in project* k^c *is then given by* (4.41).

Discussion. In this case, all the treasury bills are held by the shadow bankers, $b^s = \bar{b}$. When $q = z_1$, one period deposits and treasury bills are perfect substitutes for the shadow bankers. The commercial bankers are willing to supply any amount of one period deposits at price $z_1 = 1 + \tilde{\mu}^*$, since investing in project is as profitable as making arbitrage between one period deposits and bank reserves. When the central bank buying treasury bills using bank reserves, the supply of treasury bills decreases and the shadow bankers' demand for one period deposits increases. To issue more one period deposits, the commercial banks hold more bank reserves as the underlying assets, which "crowd out" their investment in project. Thus, assets purchasing by central bank is contractionary in this region.

4.3.4 Monetary Policy and Equilibrium

The central bank conducts monetary policy via open market operation, i.e. exchange bank reserves e for treasury bills b. Note that the total supply of public liquidities is fixed at $\bar{l}$. The open market operation only changes the composition of the supply of public liquidities. I use $\bar{e}$ as the indicator of the monetary policy and $\bar{b}$ is automatically determined after $\bar{e}$, given that $\bar{l} = \bar{e} + \bar{b}$. In this section, I analyse how the monetary policy works differently in the different regions and where are the boundaries of these regions.

In this section, I focus the case where all the three regions mentioned above exist. Thus, we need two more assumptions. First, assume that the required reserves ratio rr is small enough such that the RRR constraint does not bind when $q = p = 1$. In addition, the total supply of public liquidities $\bar{l}$ should be large enough relative to the net worth a, such that it is possible to find out a combination of $\bar{b}$ and $\bar{e}$, with $\bar{l} = \bar{b} + \bar{e}$, such that $q = p = 1$ in equilibrium.

Assumption 3 $1 - \psi_1(\hat{v}^*) \leq (\rho + \phi)\psi_2(\hat{v}^*)$, *where* $\phi = \frac{1-rr}{rr}$ *and* $\hat{v}^*$ *is the cut off rule in the neutral region.*

Assumption 4 $\frac{\bar{l}}{a} \geq \frac{\psi_2(\hat{v}^*)}{\psi_1(\hat{v}^*)+\rho\psi_2(\hat{v}^*)}$, *where* $\hat{v}^*$ *is the cut off rule in the neutral region.*

The two above assumptions guarantee that there exists a neutral region, in which treasury bills and bank reserves become perfect substitutes for commercial banks, between the expansionary and contractionary regions.

Neutral Region. When $q = p = 1$ and the RRR constraint does not bind, treasury bills b and bank reserves e are perfect substitutes for commercial bankers. Let e_1 denote the minimum amount of bank reserves the commercial banker holds to satisfy the reserve requirement and thus $e^c - e_1$ is the excess reserves held by the commercial bankers. Using the RRR constraint (4.16), we can obtain that the minimum amount of bank reserves the commercial bankers hold in the neutral region is:

$$e_1 = rr\left[(1 + \psi_2(\hat{v}^*))\, k^* - a\right] \tag{4.43}$$

If $q = p = 1$, the supply of e cannot be less than e_1, i.e. $\bar{e} \geq e_1$; otherwise, p increases and the RRR constraint starts to bind.

Similarly, when $q = p = 1$, the supply of b should be more than the shadow banker's demand for treasury bills, i.e. $\psi_2(\hat{v}^*)k^*$, so that the marginal buyer of treasury bills is households; otherwise, q start increases. Define

$$e_2 \equiv \bar{l} - \psi_2(\hat{v}^*)k^*$$

When $\bar{e} > e_2$, the supply of b is not enough for $q = p = 1$ to hold in equilibrium.

Thus, e_1 and e_2 are the lower and upper bounds of the "neutral region". When

$\bar{e} \in [e_1, e_2]$, $q = p = 1$ and the central banks' open market operation is neutral and the composition of the supply of public liquidities is irrelevant.

Lemma 4.3.1 *When $\bar{e} \in [e_1, e_2]$, the nominal interest rate i is zero and central bank asset purchasing is neutral and the composition of the supply of public liquidities is irrelevant.*

Expansionary Region. When $\bar{e} < e_1$, the supply of bank reserves is not enough for $q = p = 1$ to hold. In equilibrium, $p > 1$ and the RRR constraint binds. Central bank asset purchasing does not have any contractionary effect on shadow bankers; it decrease only the amount of treasury bills held by households. However, this increases the supply of bank reserves, lowers its price and thus increases the commercial banks' investment in the project.

Proposition 4.3.5 *When $\bar{e} < e_1$ the nominal interest rate i is positive and central bank asset purchasing increases the supply of bank reserves, lowers nominal interest rate and have expansionary effects on the economy.*

Contractionary Region I. Note that when $1 + \tilde{\mu}^* > q > p = 1$ the commercial banks do not issue one period deposits and the price of bank reserves p is pinned down at one by households who are the marginal buyers. Increase in $\bar{e}$ does not have any expansionary effect on commercial bankers; the increased bank reserves from assets purchasing flows out of the banking sector and are finally held by households. However, it decreases the supply of treasury bills $\bar{b}$. Note that only shadow bankers hold treasury bills b when $p > 1$. The demand for treasury bills is decreasing in its price p (4.37). A decrease in the supply of b increases its price and thus the shadow banks' costs of holding liquidity buffer, which is contractionary. Define e_3 to be the critical point of $\bar{e}$ at which the price of treasury bills q reaches $1 + \tilde{\mu}^*$ (using (4.39)):

$$\bar{l} - e_3 = \frac{a\psi_2(\hat{v}^s)}{\psi_1(\hat{v}^s) + [\tilde{\mu}^* - (1 - \rho)]\psi_2(\hat{v}^s)} \tag{4.44}$$

Thus, the region $[e_2, e_3]$ is referred as contractionary region I.

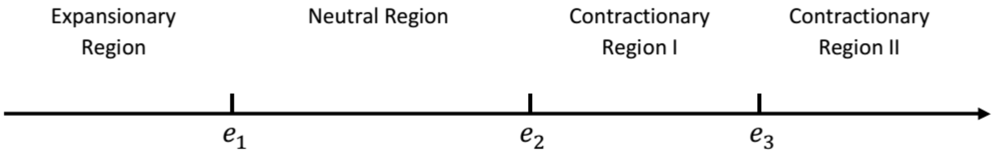
Lemma 4.3.2 *When $\bar{e} \in [e_2, e_3]$, the nominal interest i rate is negative, but commercial banks do not make arbitrage between one period deposits and bank reserves. Central bank* *asset purchasing decreases the supply of treasury bills, the only liquidity accessible for shadow banks, decreases nominal interest rate and have contractionary effects on the economy.*

Contractionary Region II When $1+\tilde{\mu}^* = q > p = 1$, the commercial banks are willing to issue any amount of one period deposits d_1^c at a constant premium $\tilde{\mu}^*$, since investing in project is as profitable as making arbitrage between one period deposits and bank reserves. However, to make this arbitrage, the commercial banks have to hold bank reserves as the underlying assets, which will "crowd out" the commercial bankers' investment in the project.

Given that $z_1 = q = 1 + \tilde{\mu}^*$, the shadow bankers' demand for liquid assets, i.e. the sum of one period deposit and treasury bills, is constant. Their demand for one period deposits increases when the supply of treasury bills $\bar{b}$ decreases. Thus, an decrease in the supply of $\bar{b}$ increases the demand and the supply of one period deposits.

Lemma 4.3.3 *When $\bar{e} > e_3$, the nominal interest rate i is negative and commercial banks are willing to issue any positive amount of one period deposits at price $1+\tilde{\mu}^*$. Central bank's assets purchasing does not further increases the price of treasury bills or lowers the nominal interest rates. However, it decreases the supply of treasury bills $\bar{b}$ and increase the demand for one period deposits. When issuing more one period deposits, commercial banks have to hold more bank reserves as the underlying assets, which "crowds out" their investment in project.*

In sum, under assumption 3 and 4, central bank asset purchasing can have different effects in different regions. The results are summarized in the proposition below:



Proposition 4.3.6 *Under assumption 3 and 4, there exists e_1, e_2, e_3, with $e_1, e_2, e_3 \in (0, \bar{l})$, such that:*

(a) when $\bar{e} < e_1$*, the RRR constraint binds and the nominal interest rate is positive. Central bank asset purchasing lowers the nominal interest rate and is expansionary;*

(b) when $\bar{e} \in [e_1, e_2]$*, the RRR constraint does not bind and the nominal interest rate is zero. Central bank asset purchasing dose not change the nominal interest rate and is neutral;*

(c) when $\bar{e} \in [e_2, e_3]$*, the RRR constraint does not bind and the nominal interest rate is negative. Central bank asset purchasing lowers the nominal interest rate and is contractionary;*

(d) when $\bar{e} > e_3$*, the RRR constraint does not bind and the nominal interest rate is negative. Central bank asset purchasing does not further lowers the nominal interest rate and is contractionary;*

4.3.5 A Numerical Example

This subsection gives a numerical example, which can help us understand the results of the model. I assume the liquidity shock v is distributed on $[0, \bar{v}]$, with $\bar{v} = 2$ and a probability density function $f(v) = 1 - 0.5v$. The value of parameters are fixed at $(r_k, \theta, \rho, a, rr, \bar{l}) = (1.8, 0.5, 0.5, 1, 0.1, 2)$.

Liquidities Prices and Nominal Interest Rates

Figure 1 shows the real prices of b and e in terms of date 0 goods when the supplies of bank reserves e and treasury bills b are varied. Figure 2 shows the nominal interest rates paid on treasury bills when the supplies of e and b are varied. Note that the sum of b and e is fixed at $\bar{l}$. Central bank asset purchases increase the supply of e, while decreasing the supply of b. Also note that nominal interest rates is given by, $i = \frac{p}{q} - 1$, which shows the relative price between bank reserves and treasury bills. Since the nominal interest rate paid on bank reserves is zero, thus a positive nominal rate on treasury bills implies that bank reserves are relatively more expensive than treasury bills.

In the expansionary region, the RRR constraint of commercial banks binds, and bank reserves are relatively more expensive than treasury bills, that is, $p > q$, and thus the nominal interest rate on treasury bills is positive. In this region, central bank asset purchases increase the supply of bank reserves, lowering their relative price and thus the nominal interest rate on b. In the neutral region, the RRR constraint does not bind, and bank reserves and treasury bills are perfect substitutes for commercial banks. Thus, b and e are equally expensive, i.e. $p = q$, and the nominal interest rates on b is zero. In this region, central bank asset purchases does not change the prices of bank reserves and treasury bills, and thus cannot lowers the nominal interest rate furthermore.

However, if the central bank continues to purchase treasury bills, the economy goes into contractionary regions. In the contractionary region I, treasury bills becomes relatively more expensive than bank reserves, and the nominal interest rate turns negative. The premium on treasury bills represents the limited accessibility of bank reserves for shadow banks. Though commercial banks can hold bank reserves for shadow banks by issuing one period deposits, liquid assets achievable for shadow banks, using bank reserves as the underlying assets, they charge a premium when issuing such short term deposits since it tightens their leverage constraints. In the contractionary region I, the premium on liquidity achievable for shadow banks, i.e. $q - p$, is not large enough to compensate the commercial banks' costs of issuing short term deposits. In this region, shadow banks do not indirectly hold bank reserves via commercial banks. In addition, central bank asset purchases only further reduces the supply of treasury bills and pushes up their price. In the contractionary region II, the premium on liquidity achievable for shadow banks is large enough to compensate commercial banks' costs of issuing one period deposits. In this region, shadow banks indirectly hold bank reserves by holding the one period deposits issued by commercial banks. Central bank asset purchases does not raise treasury bills' price further, since commercial banks are willing to hold bank reserves for shadow banks for any amount at the equilibrium price.

Effects of Central Bank Asset Pruchases

Figure 3 shows the net values of project, that is, $[F(\hat{v})r_k - G(\hat{v})r_l - 1]\,k$, of shadow and commercial banks, when the supplies of e and b are varied. The net value of project depends on not only the initial investment k, but also the banks' liquidity hoarding and cut off rule $\hat{v}$. In the expansionary region, central bank asset purchasing increases the supply of bank reserves and lowers the price of e, which encourages commercial banks' investment in project. In the neutral region, central bank asset purchasing are neutral and has no effect on banks' investment decisions. In the contractionary region I, central bank asset purchasing reduces the supply of treasury bills and increases its price, which distorts the shadow banks' decisions on investment. Shadow banks lower their cut off rule

$\hat{v}$ since the liquidity achievable for them are expensive. In the contractionary region II, commercial banks hold bank reserves for shadow banks by issuing one period deposits using bank reserves as the underlying assets. Central bank asset purchases decreases the supply of treasury bills, which increases shadow banks' demand for one period deposits issued by commercial banks. However, to issue one period deposits, commercial banks have to hold bank reserves as the underlying assets, which "crowds out" the investment in projects in the presence of leverage constraint. Thus, central bank assets purchase in this region decreases commercial banks' investment in projects.

Allocation of Liquidities

Figure 4 shows the allocation of public liquidities. Note that commercial banks and households can hold both treasury bills and bank reserves, while shadow banks can only hold treasury bills. In the expansionary region, bank reserves are more expensive relative to treasury bills, $p > q$. All the bank reserves are held by commercial banks. Treasury bills are held by commercial banks, shadow banks, and households. The price of treasury bills is pinned down at one by households, who are the marginal buyers. In the neutral region, treasury bills and bank reserves are perfect substitutes for commercial banks. Commercial banks hold both bank reserves and treasury bills and shadow banks only hold treasury bills. In the contractionary region I, treasury bills become more expensive relative to bank reserves, $q > p$; however, shadow banks do not indirectly hold bank reserves via commercial banks. All the treasury bills are held by shadow banks. The price of bank reserves is pinned down at one by households, who are the marginal buyers. In the contractionary region II, commercial banks hold bank reserves for shadow banks by issuing one period deposits using bank reserves as the underlying assets. Shadow banks hold all the treasury bills and indirectly hold bank reserves via holding one period deposits issued by commercial banks.

4.4 Final Remarks

This paper studies how central bank asset purchases can have contractionary effects on the economy by changing the volume of reserve balances in the banking system and the supply of liquid assets in markets. Central bank asset purchases increases the bank reserves, on one hand; it decreases the supply of safe bonds in the markets, on the other hand. Since bank reserves can only be directly held by commercial banks, while safe bonds can by be held by both commercial and shadow banks. The safe bonds could be more liquid than bank reserves in this sense. Thus, the large-scale asset purchases by central banks that are designed to lower the long-term rates could be potentially contractionary. In contrast to conventional wisdom, I show that when the level of bank reserves is high, central bank asset purchases are necessarily contractionary.

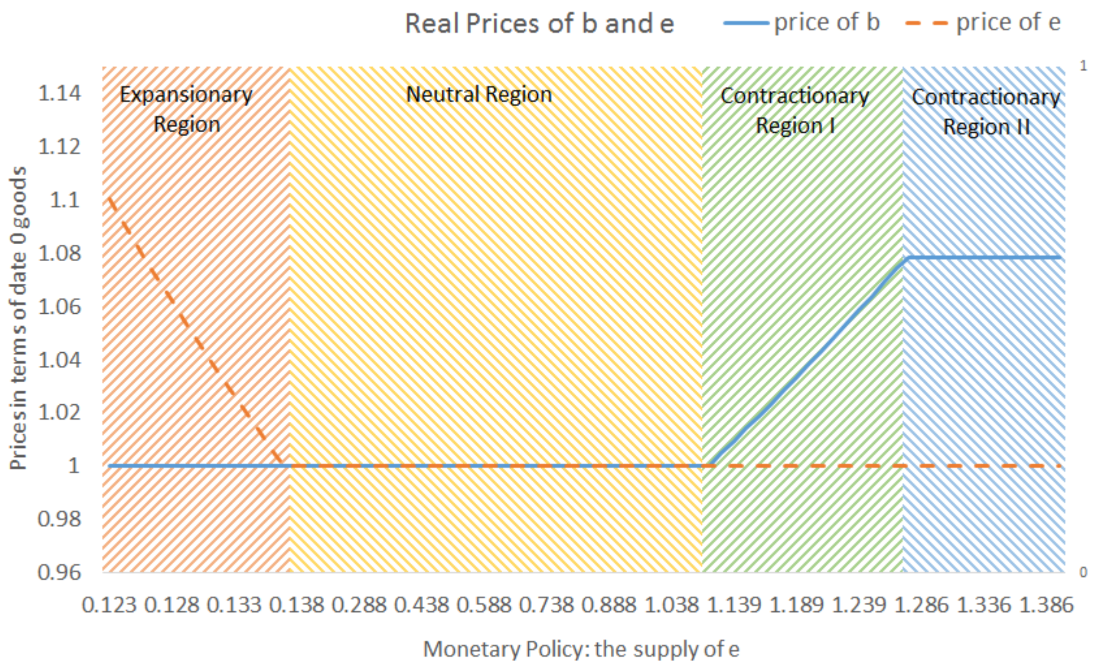


Figure 4.1: Real prices of treasury bills and bank reserves

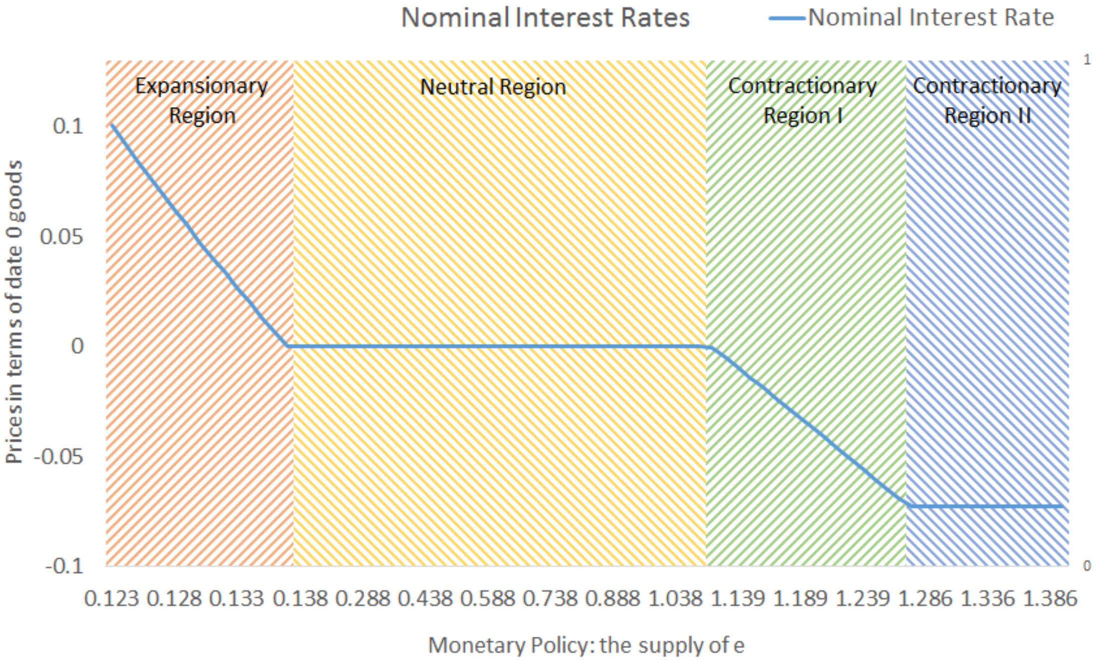


Figure 4.2: Nominal interest rates on treasury bills

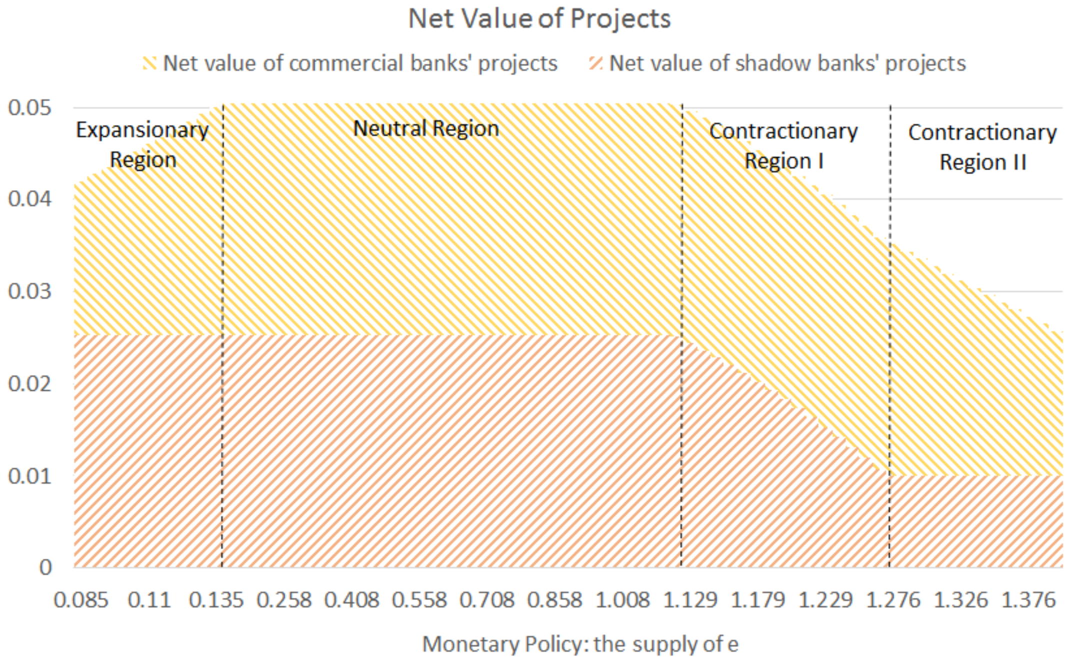


Figure 4.3: Net values of projects

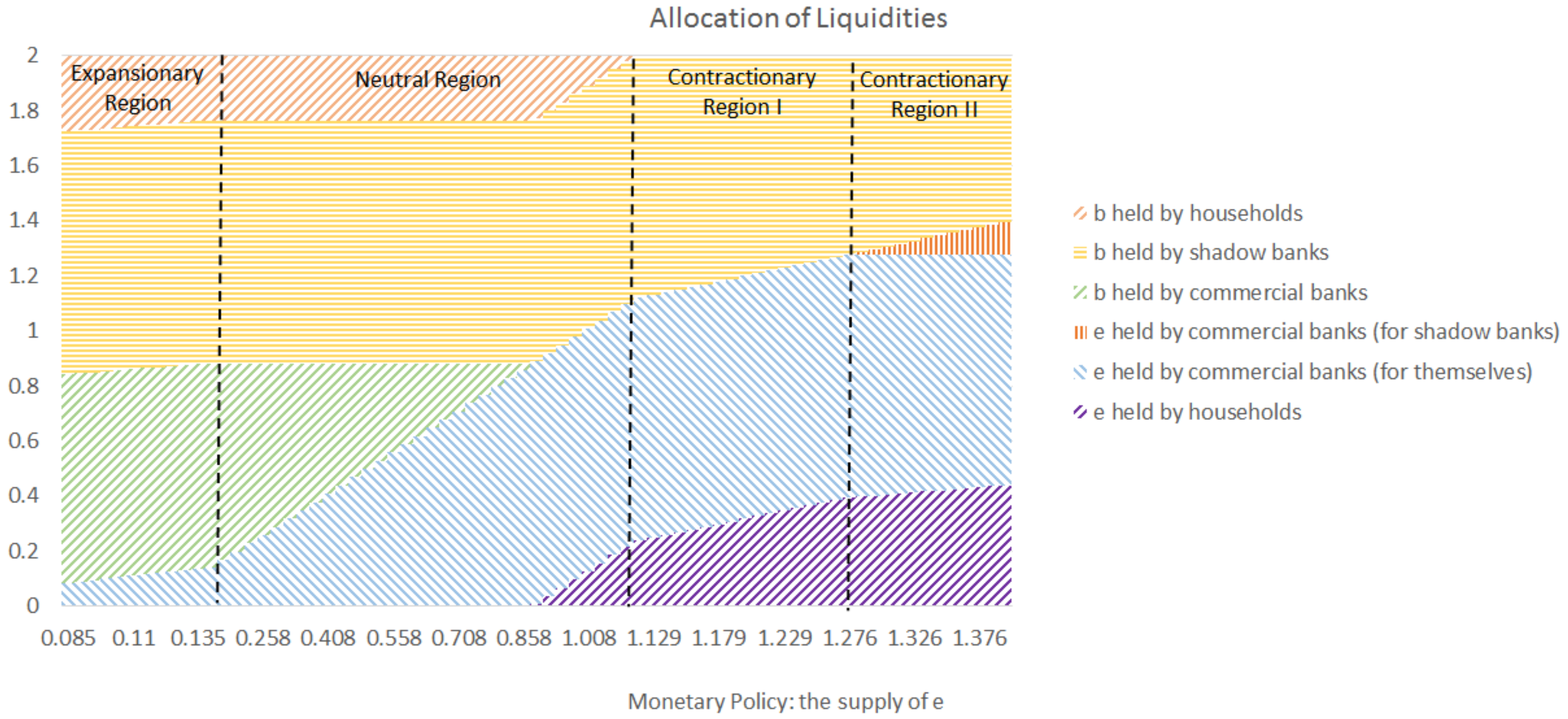


Figure 4.4: Allocation of liquidities

Bibliography

Viral V Acharya and Ouarda Merrouche. Precautionary hoarding of liquidity and interbank markets: evidence from the subprime crisis. *Review of Finance*, 17(1):107–160, 2013.

George A Akerlof. The market for" lemons": Quality uncertainty and the market mechanism. *The quarterly journal of economics*, pages 488–500, 1970.

Franklin Allen and Douglas Gale. Financial fragility, liquidity, and asset prices. *Journal of the European Economic Association*, 2(6):1015–1048, 2004.

Franklin Allen and Andrew Winton. *Corporate financial structure, incentives and optimal contracting*. Rodney L. White Center for Financial Research, 1994.

Richard G. Anderson and Yang Liu. How low can you go? negative interest rates and investors' flight to safety. *The Regional Economist*, (Jan), 2013.

Cristina Arellano, Yan Bai, and Patrick Kehoe. Financial markets and fluctuations in uncertainty. *Federal Reserve Bank of Minneapolis Working Paper*, 2010.

Adam Ashcraft, James McAndrews, and David Skeie. Precautionary reserves and the interbank market. *Journal of Money, Credit and Banking*, 43(s2):311–348, 2011.

Rudiger Bachmann and Giuseppe Moscarini. Business cycles and endogenous uncertainty. *manuscript, Yale University, July*, 2011.

Daniel Beltran, Larry Cordell, and Charles Thomas. Asymmetric information and the death of abs cdos. *FRB International Finance Discussion Paper*, (1075), 2013.

Paul Bennett and Stavros Peristiani. Are us reserve requirements still binding. *Federal Reserve Bank of New York Economic Policy Review*, 8(1):53–68, 2002.

Ben Bernanke and Mark Gertler. Agency costs, net worth, and business fluctuations. *american economic review*, 79(1):14–31, 1989.

Ben S Bernanke. The economic outlook and monetary policy. In *Speech at the Federal Reserve Bank of Kansas City Economic Symposium, Jackson Hole, Wyoming*, volume 27, 2010.

Ben S Bernanke, Mark Gertler, and Simon Gilchrist. The financial accelerator in a quantitative business cycle framework. *Handbook of macroeconomics*, 1:1341–1393, 1999.

Jose Berrospide. Bank liquidity hoarding and the financial crisis: an empirical evaluation. Finance and Economics Discussion Series 2013-03, Board of Governors of the Federal Reserve System (U.S.), 2013.

Saki Bigio et al. Endogenous liquidity and the business cycle. *Manuscript, New York University*, 2011.

Nicholas Bloom. The impact of uncertainty shocks. *econometrica*, 77(3):623–685, 2009.

Nicholas Bloom, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J Terry. Really uncertain business cycles. Technical report, National Bureau of Economic Research, 2012.

John H Boyd and Bruce D Smith. The evolution of debt and equity markets in economic development. *Economic Theory*, 12(3):519–560, 1998.

John H Boyd and Bruce D Smith. The use of debt and equity in optimal financial contracts. *Journal of Financial Intermediation*, 8(4):270–316, 1999.

John Y Campbell and Martin Lettau. Dispersion and volatility in stock returns: An empirical investigation. Technical report, National Bureau of Economic Research, 1999.

John Y Campbell, Martin Lettau, Burton G Malkiel, and Yexiao Xu. Have individual stocks become more volatile? an empirical exploration of idiosyncratic risk. *The Journal of Finance*, 56(1):1–43, 2001.

Charles T Carlstrom and Timothy S Fuerst. Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis. *The American Economic Review*, pages 893–910, 1997.

Tri Vi Dang, Gary Gorton, and Bengt Holmstrom. Opacity and the optimality of debt for liquidity provision. *Manuscript Yale University*, 2009.

Tri Vi Dang, Gary Gorton, and Bengt Holmström. Ignorance, debt and financial crises. *Unpublished, Yale SOM*, 2012.

Marco Del Negro, Gauti Eggertsson, Andrea Ferrero, and Nobuhiro Kiyotaki. The great escape? a quantitative evaluation of the fed's non-standard policies. *unpublished, Federal Reserve Bank of New York*, 2010.

Andrea L Eisfeldt. Endogenous liquidity in asset markets. *The Journal of Finance*, 59(1): 1–30, 2004.

Huberto M Ennis and Todd Keister. Understanding monetary policy implementation. *Federal Reserve Bank of Richmond Economic Quarterly*, 94(3):235–263, 2008.

Pablo Fajgelbaum, Edouard Schaal, and Mathieu Taschereau-Dumouchel. Uncertainty traps. Technical report, National Bureau of Economic Research, 2014.

Michael J Fleming and Kenneth D Garbade. Repurchase agreements with negative interest rates. *Federal Reserve Bank of New York Current Issues in Economics and Finance*, 10 (5):1–7, 2004.

Douglas Gale and Martin Hellwig. Incentive-compatible debt contracts: The one-period problem. *The Review of Economic Studies*, 52(4):647–663, 1985.

Simon Gilchrist and Egon Zakrajšek. Credit spreads and business cycle fluctuations. Technical report, National Bureau of Economic Research, 2011.

Simon Gilchrist, Vladimir Yankov, and Egon Zakrajšek. Credit market shocks and economic fluctuations: Evidence from corporate bond and stock markets. *Journal of Monetary Economics*, 56(4):471–493, 2009.

Simon Gilchrist, Jae W Sim, and Egon Zakrajsek. Uncertainty, financial frictions, and investment dynamics. In *2010 Meeting Papers*, volume 1285, 2010.

Simon Gilchrist, Jae W Sim, and Egon Zakrajšek. Misallocation and financial market frictions: Some direct evidence from the dispersion in borrowing costs. *Review of Economic Dynamics*, 16(1):159–176, 2013.

Gary Gorton and Guillermo Ordonez. Collateral crises. *American Economic Review*, 104 (2):343–78, 2014a.

Gary Gorton and Guillermo Ordonez. Crises and Productivity in Good Booms and in Bad Booms. PIER Working Paper Archive 14-008, Penn Institute for Economic Research, Department of Economics, University of Pennsylvania, February 2014b.

Paul M Healy and Krishna G Palepu. Information asymmetry, corporate disclosure, and the capital markets: A review of the empirical disclosure literature. *Journal of accounting and economics*, 31(1):405–440, 2001.

Jack Hirshleifer. The private and social value of information and the reward to inventive activity. *The American Economic Review*, pages 561–574, 1971.

Bengt Holmström and Jean Tirole. Private and public supply of liquidity. *Journal of Political Economy*, 106(1):1–40, 1998.

Todd Keister and James McAndrews. Why are banks holding so many excess reserves? *Current Issues in Economics and Finance*, 15(8), 2009.

Todd Keister, Antoine Martin, and James McAndrews. Divorcing money from monetary policy. *Federal Reserve Bank of New York Economic Policy Review*, 14(2):41–56, 2008.

Andy Kessler. The fed squeezes the shadow-banking system. *Wall Street Journal*, 2013. online.wsj.com.

Nobuhiro Kiyotaki and John Moore. Liquidity, business cycles, and monetary policy. Technical report, National Bureau of Economic Research, 2012.

Pablo Kurlat. Lemons markets and the transmission of aggregate shocks. *The American Economic Review*, 103(4):1463–1489, 2013.

Pamela Labadie. Aggregate fluctuations, financial constraints and risk sharing. *Economic Theory*, 12(3):621–648, 1998.

Antoine Martin, James McAndrews, and David Skeie. Bank lending in times of large bank reserves. Technical report, Staff Report, Federal Reserve Bank of New York, 2013.

Roberto Motto, Massimo Rostagno, and Lawrence J Christiano. Financial factors in economic fluctuations. In *2010 Meeting Papers*, number 141. Society for Economic Dynamics, 2010.

Guillermo Ordoñez. The asymmetric effects of financial frictions. Technical report, National Bureau of Economic Research, 2012.

Christopher A Sims. Implications of rational inattention. *Journal of monetary Economics*, 50(3):665–690, 2003.

Christopher A Sims. Rational inattention: a research agenda. Technical report, Discussion paper Series 1/Volkswirtschaftliches Forschungszentrum der Deutschen Bundesbank, 2005.

Peter Stella. Exit-path implications for collateral chains. *Vox-Eu*, 2013. www.voxeu.org.

Can Tian. Riskiness choice and endogenous productivity dispersion over the business cycle. Technical report, Penn Institute for Economic Research, Department of Economics, University of Pennsylvania, 2012.

Jean Tirole. Illiquidity and all its friends. *Journal of Economic Literature*, 49(2):287–325, June 2011.

Robert M Townsend. Optimal contracts and competitive markets with costly state verification. *Journal of Economic theory*, 21(2):265–293, 1979.

Harald Uhlig. A model of a systemic bank run. *Journal of Monetary Economics*, 57(1): 78–96, 2010.

Laura L Veldkamp. Slow boom, sudden crash. *Journal of Economic Theory*, 124(2):230–257, 2005.

Neil Wallace. A modigliani-miller theorem for open-market operations. *The American Economic Review*, pages 267–274, 1981.

J. Williams. *Ex-ante Monitoring, Ex-post Asymmetry, and Optimal Securities.* University of British Columbia, Faculty of Commerce and Business Administration, 1989.

Stephen D Williamson. Scarce collateral, the term premium, and quantitative easing. Technical report, working paper, 2013.