TOWARDS RESILIENCY WITH MICRO-GRIDS: PORTFOLIO OPTIMIZATION AND INVESTMENT UNDER UNCERTAINTY

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ABSTRACT OF THE DISSERTATION

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Energy security and sustained supply of power are critical for community welfare and economic growth. In the face of the increased frequency and intensity of extreme weather conditions which can result in power grid outage, the value of micro-grids to improve the communities' power reliability and resiliency is becoming more important. Micro-grids capability to operate in islanded mode in stressed-out conditions, dramatically decreases the economic loss of critical infrastructure in power shortage occasions. More wide-spread participation of micro-grids in the wholesale energy market in near future, makes the development of new investment models necessary. However, market and price risks in short term and long term along with risk factors' impacts shall be taken into consideration in development of new investment models.

This work proposes a set of models and tools to address different problems associated with micro-grid assets including optimal portfolio selection, investment and financing in both community and a sample critical infrastructure (i.e. wastewater treatment plant) levels. The models account for short-term operational volatilities and long-term market uncertainties. A number of analytical methodologies and financial concepts have been adopted to develop the aforementioned models as follows.

- Capital budgeting planning and portfolio optimization models with Monte Carlo stochastic scenario generation are applied to derive the optimal investment decision for a portfolio of micro-grid assets considering risk factors and multiple sources of uncertainties.
- Real Option theory, Monte Carlo simulation and stochastic optimization techniques are applied to obtain optimal modularized investment decisions for hydrogen tri-generation systems in wastewater treatment facilities, considering multiple sources of uncertainty.
- Public Private Partnership (PPP) financing concept coupled with investment horizon approach are applied to estimate public and private parties' revenue shares from a community-level micro-grid project over the course of assets' lifetime considering their optimal operation under uncertainty.

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1 INTRODUCTION

1.1 Objectives

This thesis intends to address and tackle the following problems.

- Development of a framework for sustainable and resilient planning for a portfolio of micro-grid assets, which enforces hedging mechanisms for risks due to infrastructure failure attributed to natural disasters or major technical issues. The considered micro-grid portfolio includes purchase from the grid, wind turbine, photovoltaic cell, combined heat and Power (CHP), electricity storage and gas-fired generation.
- Taking into consideration regional risk factors in the micro-grid long-term planning which alters the micro-grid optimal portfolio selection and capital budgeting decisions.
- Optimal incremental investment decisions in recently introduced “hydrogen tri-generation” distributed energy generation system and hydrogen dispensing hardware for energy consumptive wastewater treatment plants, using Monte Carlo simulation for compound real options. The model considers stochasticity in capital costs of hydrogen tri-generation and hydrogen dispensing hardware, hydrogen price and natural gas price. Operational savings from hydrogen tri-generation and onsite hydrogen dispensing system are estimated based on optimal control of these assets with stochastic electricity demand and electricity spot price.
- Development of a Public-Private Partnership (PPP) financing model for micro-grids in which public entity transfers responsibility and risk of designing, building, operating and maintaining (DBOM) of the project to the private sector while maintaining the project

ownership. We are specifically interested to demonstrate how both public and private parties can collaborate and have profit shares from large scale micro-grid projects.

1.2 Brief Overview of Thesis Accomplishments

In chapter 2, we developed a comprehensive framework for micro-grid long-term investment which takes into consideration regional risk factors in planning decisions. To ensure feasibility of these plans, long-term and short-term planning problems of a micro-grid are merged into a single framework with resiliency, economics and reduced carbon footprints as main drivers. The model accounts for short-term savings, costs and penalties that are weighted according to the priorities of existing or planned industrial, residential and commercial sectors within the community in stressed out occasions. It is demonstrated that the proposed model is most valuable once applied for regions with high vulnerability to extreme conditions which results in power failure or/and with many critical sectors.

Chapter 3 extends the current state of art in compound real option approach to incremental investment in hydrogen tri-generation and onsite hydrogen dispensing systems for a wastewater treatment plant. Hydrogen tri-generation which is one of the most recent developments in the context of integrated conversion systems can be fed by a mixture of natural gas price and waste biogas generated from wastewater treatment processes. The overall savings are estimated using a stochastic operation optimization model developed for hydrogen tri-generation systems that takes into account stochastcity of the facility's heat and power demands and electricity spot price. From a practical point of view, the proposed model could have enormous impact on how decisions are made for large-scale facilities, such as wastewater treatment plants, where, on one hand, the owner must be concerned with power resiliency of the facility specifically in rare events of extremely high impacts, and on the other hand, be able to generate maximum revenue under normal operating conditions.

In chapter 4, we develop a Public-Private Partnership (PPP) financing model for micro-grids in which public entity transfers responsibility and risk of designing, building, operating and maintaining (DBOM) of the project to the private sector while maintaining the project ownership. The private sector owns the micro-grid's revenue till the investment horizon; however, the revenue ownership will be transferred to the public entity after the investment horizon till a finite after-horizon period. Public entity incentivizes the private sector by providing an initial senior debt opportunity (through issuing zero coupon municipal bonds) and possibility of annual junior debts. Here, the (DBOM – PPP) financing model is merged with micro-grid short-term operation optimization into a single framework under host of short-term and long-term stochastic variables. An illustrative example is presented in which optimal financial activities and optimal micro-grid incremental portfolio over the course of investment horizon are defined for a vulnerable community located in 100-year flood zone region of city of Hoboken in New Jersey. From a practical point of view, the proposed model could have enormous impact on building a collaborative environment for public and private entities, which will facilitate implementation of micro-grid projects.

1.3 Synopsis of Contributions

1.3.1 Chapter 2: Incorporating Risk in Micro-Grid Portfolio Optimization and Planning

In this chapter we develop a power outage risk based comprehensive framework for micro-grid long-term capital planning and portfolio optimization. The presented work extends the current state of art in micro-grid planning and control as follows: (i) Incorporating resiliency as an objective in micro-grid planning and present risk-based estimate of micro-grid saving which includes monetary value of enhanced resiliency, (ii) Long-term capital budgeting plan and incremental portfolio selection for micro-grids incorporating regional power failure risk (iii) Merging short-term and long-term micro-grid planning problems in a single and decomposable framework.

1.3.2 Chapter 3: Investment in Hydrogen Tri-generation for Wastewater Treatment Plants under Uncertainties

In this chapter, we propose a methodology for optimal investment in hydrogen tri-generation and onsite hydrogen dispensing system for a wastewater treatment plant. Investment in hydrogen tri-generation has the objective of increasing the power resiliency of the facility, however, the expansion option is considered to provide additional source of revenue. The problem is formulated as a compound real option each investment stage under two sources of uncertainty. This work extends the current state of investment modeling within the context of hydrogen tri-generation distributed energy generation by considering: (i) Modular investment plan for hydrogen tri-generation and dispensing systems, (ii) Multiple sources of uncertainties along with more realistic probability distributions, (iii) Optimal operation of hydrogen tri-generation is considered, which results in more realistic saving estimation.

1.3.3 Chapter 4: Public Private Partnership (PPP) Financing Model for Micro-Grids

Since the focus of many recent researches in the micro-grids domain has been the micro-grid's design, implementation and operation, there is a lack in comprehensive models which solve the problem of financing such projects. Many of recent micro-grid projects have not been expanded from pilot scale to massive scale capable of supplying considerable portion of the communities' power demands, due to existence of no clear financial plan that takes the project from the financing step out to the end of the micro-grid's lifetime. In this chapter, we extend the current state of art in Public Private Partnership (PPP) financing approach to community scale micro-grid projects. The developed (PPP) model which takes into account all operational and financial modeling details, can be a starting point for a collaborative environment for public and private parties that eases the implementation of large scale micro-grid projects.

1.4 Motivation

Energy security and sustained supply of power are critical for community welfare and economic growth. The aging power grid and higher than normal frequency of occurrence of natural disasters in recent years and their subsequent high societal costs have been driving many communities to re-evaluate their energy infrastructure plans and investment policies. Some communities and towns, especially in coastal regions, are turning to micro-grids to ensure that at the time of power outage, they are able to provide power to, at least, critical functions and services in their communities. This trend demonstrates the potentials of micro-grids to increase communities' power resiliency. Micro-grid is a concept characterized by low voltage distribution network, micro generators, loads and storage devices with locally coordinated functions []. A typical micro-grid includes renewable (solar PVs and wind turbines) and non-renewable generation assets (e.g., gas-fired generation) [5]. There are already many studies in the literature on the topic of micro-grid planning and control. However, we are not aware of any prior work, which interlinks the micro-grid configuration and operation to the level of protection and resiliency that it can provide to its community under stressed out conditions. In order to incorporate resiliency in the micro-gird planning objective, regional power failure risk factors shall be necessarily taken into consideration. With this background in mind, we are particularly interested to develop tools to optimally incorporate regional power failure risk characteristics in micro-grid long-term portfolio selection and capital budgeting plans. In addition, we take into consideration uncertainty in natural gas price, micro-grid assets capital costs of investment, electricity spot price, heat and electricity demands and forecasts of renewable energy resources.

Narrowing down the subject of community resiliency to the level of critical infrastructure power reliability, we consider wastewater treatment plants among critical infrastructures for energy resiliency for the following reasons.

i. Wastewater treatment facilities are necessarily located close to the waterfront to discharge the treated wastewater and also for more efficient sludge handling. This waterfront geographical requirement increases likelihood of being affected and damaged during extreme weather conditions.
ii. High economic loss and possible health problems in case of wastewater treatment plants temporary shutdown
iii. Generation of considerable amount of waste energy through treatment processes

Recently introduced hydrogen tri-generation systems which are fed by a mixture of natural gas and waste biogas from treatment processes can significantly enhance the resiliency of wastewater treatment plants and provide additional source of revenue [2]. Along with profit from onsite supplying heat and power demands, additional profit could be achieved from hydrogen tri-generation through onsite hydrogen dispensing for local hydrogen vehicles use. We are particularly motivated to build necessary tools to optimally make investment decisions including timing and investment thresholds to invest in hydrogen tri-generation and hydrogen dispensing systems considering various sources of uncertainty rising from capital costs of hydrogen tri-generation and onsite hydrogen dispensing hardware, hydrogen price and natural gas price.

In spite of many ongoing research works on control, planning and investment in micro-grids, the answer to the practical question of "Where shall the funding come from for these large scale projects?" is still not clear. High cost and risk of investment in micro-grid assets and lack of proper regularization are barriers towards wide-spread installation of micro-grids. However, collaboration between public and private sectors through Public Private Partnership (PPP) contracts could pave the road for micro-grids. The advantages of such agreements are making the project a possibility in the first place and sooner completion of the project as well as transfer of risk from public entity to the private sector over the life of the project In these contracts, public sector provides some incentives for the private sector such as loans proportionate to the level of risk the implementer bears, reduction in loan fees or/and transparent communication, collaboration and less political behavior to carry out the project. On the other hand, we can benefit from the private sector's expertise and experience in carrying out such large scale projects. The project revenue will be split between two parties based on the contract.

1.5 Brief Introduction to Micro-Grids

Micro-grid is a localized grouping of electricity micro generators, loads and storage devices that normally operate connected to the power grid. A typical micro-grid includes wind turbines, solar panels, fuel cells, gas fired generators or other generation resources. Micro-grid can be isolated from the macro power grid and run on its own resources. Optimal operation of micro-grids in average normal conditions decreases the cost of supplying energy demands and has the potential to generate revenue. It is no secret that micro-grids can also increase the power resiliency of communities by continuous operation in stressed out conditions in which the grid is disconnected due to extreme environmental conditions or technical issues.

Micro-grids can significantly change the electricity market dynamics in future, since these assets can act as either generator or customer in the power market. As a generator, micro-grid can sell electricity in the wholesale electricity market if it makes economic sense. On the hand, micro-grid can buy electricity from the power grid if its internal generation cannot satisfy power demand. Although micro-grids reduce the volatility to high peak demands, these systems add their own risks to the market. For this reason, new modeling approaches and tools are required to capture the behavior of micro-grids with respect to the power grid.

1.6 Brief Introduction to Hydrogen Tri-generation

Hydrogen tri-generation which is an efficient high temperature fuel cell capable of simultaneous production of heat, electricity and hydrogen is one of the most recent developments in the context of integrated conversion systems. Hydrogen tri-generation system has internal reformers and heat from electricity-production efficiency losses is used to produce hydrogen. Most important benefits of hydrogen tri-generation are as below.

i. Near zero emission
ii. Co-generated hydrogen is a more valuable product than thermal energy
iii. Efficient generation of hydrogen and electricity reduces the overall operating cost
iv. Can be fed by a mixture of fuels such as natural gas and waste biogas generated in wastewater treatment plants
v. Hydrogen tri-generation might be a solution to overcome the challenge of initial investment cost of hydrogen early infrastructure deployment
vi. Hydrogen scales well for long storage times and large amounts of energy

Although, hydrogen tri-generation is in its infancy and is not fully commercialized, it seems that such a technology can play an important role in energy market as a cost effective and reliable distributed resource of energy in near future.

2 INCORPORATING RISK IN MICRO-GRID PORTFLIO OPTIMIZATION AND PLANNING

It is no secret that micro-grids increase the power resiliency of communities by continuous operation in stressed out conditions in which the grid is disconnected due to extreme environmental conditions or technical issues. In recently developed “Sustainable and Resilient Community Planning” concept, micro-grids are considered to ensure power availability for critical functions at the times of power outage. Optimal development of such a planning framework requires considering power failure risk in micro-grid long-term decisions. In this chapter, we develop a framework for resilient and sustainable planning of micro-grids with hedging mechanisms for risks due to infrastructure failures. To ensure feasibility of these plans, micro-grid’s long-term and short-term planning problems are merged into a single framework with resiliency, economics and reduced carbon footprints as main drivers. The model accounts for short-term savings, costs and penalties that are weighted according to the priorities of existing residential and commercial sectors within the community in stressed out occasions. The presented work extends the current literature as follows: (i) Incorporating resiliency as an objective in micro-grid planning, (ii) Long-term capital budgeting plan and incremental portfolio selection for micro-grids considering power failure risk (iii) Merging short-term and long-term micro-grid planning problems in a single framework.

2.1 Introduction

Energy security and sustained supply of power are critical for community welfare and economic growth. The aging power grid and higher than normal frequency of occurrence of natural disasters in recent years and their subsequent high societal costs have been driving many communities to re-evaluate their energy infrastructure plans and investment policies. Some communities and towns, especially in coastal regions, are turning to micro-grids to ensure that at the time of power outage, they are able to provide power to, at least, critical functions and services in their communities [1]. This trend demonstrates the potentials of micro-grids to increase communities' power resiliency. With energy resiliency as an objective, the policy makers and planners are slowly but surely turning to adopting the concept of "Sustainable and Resilient Community Planning". A sustainable and resilient community, by definition, is a community, which is capable of fast recovering from and of mitigating the economic and societal costs of natural or human-made disasters [2]. Micro-grid planning for such a community will utilize risk-based monetary value of enhanced resiliency along with normal operational savings for its optimization. To ensure that these plans are economically feasible and sustainable, planning decisions must ensure that both short-term and long-term operations of the community's micro-grid assets are optimized. In this chapter, we develop such a framework for resilience and sustainability that simultaneously optimizes short-term day-to-day volatile operation (under normal and stressed out conditions) of the community's micro-grid assets, and investment decisions that are subject to long-term uncertainties and power failure risks. We merge the problems of long-term and short-term planning into a single framework with resiliency, economics and reduced carbon footprints at the core. This framework enforces hedging mechanisms for risks due to infrastructure failures, market and price risks in short-term and long-term, load fluctuations, and such externalities as weather. At the same time, every community may pledge to reduce risks of its own macro-grid by providing power to its surrounding communities at the peak times or at the times of natural disasters. The model accounts for short-term savings, costs and penalties that are weighed according to the priorities of existing or planned industrial, residential and commercial sectors within the community in stressed out occasions. The model provides capital plans that are subject to day-to-day optimal operation of the energy assets within the community, including the optimal allocation of energy resources over average normal and stressed out conditions. We will demonstrate that this investment approach usually incurs more capital investment on assets, but results in more micro-grid overall savings and higher cash flow position for the investors. We also demonstrate that the differences in optimal portfolios and capital budgeting decisions for micro-grids with and without resiliency criterion become more significant in more risky regions with high probability of power failure or subsequent economic loss. This work builds on the basis of our earlier works [3] on micro-grid planning and control, with the caveat that the concept of resiliency was not considered there and that the overall optimization was decomposed into two separate problems [4]. The decomposability advantage of the aggregate operation optimization and capital budgeting model presented in this work makes this model a generic framework which could be applied for various applications of distributed energy resources with different objectives.

There are already many studies in the literature on the topic of micro-grid planning and control. A typical micro-grid in these studies includes renewable (solar PVs and wind turbines) and non-renewable generation assets (e.g., gas-fired generation), each with its own underlying uncertainties [5]. However, we are not aware of any prior work, which interlinks the micro-grid configuration and operation to the level of protection and resiliency that it can provide to its community under stressed out conditions. This observation is also echoed by [6] which described Distributed Energy Resources Customer Adoption Model (DER-CAM) developed by Lawrence Berkley National Lab [7]. DER-CAM software links the operation schedule of distributed generation equipment on hourly basis to the optimal investment choice decisions. However, it neglects underlying stochasticity and solves the scheduling and investment problem as a non-stochastic mixed Integer Linear Optimization Program (MILP). DER-CAM lacks an investment model, which can evaluate incremental capacity installment decisions. As reported in [6], DER-CAM will incorporate the concept of power reliability and resiliency in micro-grid planning in the future. In addition to DER-CAM, Hybrid Optimization Model for Electric Renewable (HOMER) [8] tool designed by the National Renewable Energy Laboratory interlinks the configuration and operation of the micro-grids. However, this model does not include an operational optimization; different design configurations are evaluated by comparing their operating benefits and investment costs. The long-term resiliency consideration has not been incorporated in this tool yet. [9] formulates the similar problem as a deterministic nonlinear integer program which minimizes the sum of the total capital, operational, maintenance and replacement costs of a stand-alone micro-grid including photovoltaic cell, wind turbine, battery storage and fuel cell. The approach is similar to ours; it merges operation and investment models into a single framework while only considering deterministic impacts of resiliency measured on the basis of total hours of power outage. Moreover, the model does not consider incremental capacity installment of micro-grid assets and does not provide a capital budgeting plan. [10] focuses on deterministic optimal sizing and operation of standalone wind turbine and battery storage as well as tilt and capacity of photovoltaic panels with reliability as a constraint to meet. The underlying concept in this study differs from ours in terms of long-term planning perspective and resiliency consideration in the modeling approach. [11] considers reliability in standalone micro-grid from perspective of impact of stochastic generation on reliability. In this study, instead of modeling the generation and load independently, a combined generation-to-load ratio is modeled as a Markov process which illustrates the impact of stochastic behavior on reliability in a transparent manner. The reliability benefits of increasing local generation capacity are also evaluated in this research. [12] extends this work by introducing an evaluation methodology for islanded micro-grids with stochastic resources, to examine the influence of supply-to-load correlation on reliability.

The rest of the chapter is organized as follows. In section (2.2) problem statement and preliminaries are provided. Section (2.3) presents the problem formulation. In section (2.4), illustrative results and sensitivity experiments are presented. Eventually, conclusion and future work are presented in section (2.5).

2.2 Problem Statement and Preliminaries

We assume a micro-grid asset portfolio that includes photovoltaic cells (PV), wind turbines (WT), combined heat and power (CHP), gas fired generation (GF), electricity storage (ST) and purchase form power grid. Micro-grid power generation along with electricity purchased from the grid supply the community's power demand. The option to sell-back the excess electricity to the grid is not considered. Heat generated by boiler and CHP supply the community's heat demand. The problem of interest is to construct a planning and capital budgeting model of micro-girds, which accounts for risk factors that are driven by geographical, weather and market conditions of the community that these micro-grids are intended to serve. Both short-term and long-term risk factors and volatilities must be taken into account. In particular, the results from the model must clearly demonstrate sensitivity of the micro-grid design to the risks factors that are attributed to its community, in particular, to its resiliency requirements.

To evaluate the feasibility and sustainability of planning decisions, we simultaneously optimize short–term operation (under normal and stressed out conditions) of the community's distributed energy resources and long-term investment decisions in a single framework. The model accounts for short-term savings, costs and penalties that are weighed according to the resiliency priorities set for various community sectors. Under both normal and stressed out conditions, the objective for optimization is to maximize the savings from micro-grid operation; the savings are characterized differently under the two conditions. The micro-grid saving in average normal condition is the cost of supplying electricity and heat demands of the community without micro-grid less the same cost with the micro-grid. However, the micro-grid saving in stressed out conditions is the incurred loss to the community without micro-grid less the incurred loss with micro-grid. The overall micro-grid saving is the aggregation of these two saving terms. Minimizing the micro-grid operation cost maximizes the saving amount in the average normal conditions. However, minimizing the incurred economic loss of unsupplied electricity via maximal onsite power generation and optimal allocation of the available power among different sectors maximizes the saving amount in the stressed out conditions. The long-term investment planning problem particularly aims at what capacity of each resource, if any, at each time period should be purchased within a planning horizon. This is a capital budgeting problem over the investment horizon and investor has the option to either invest on micro-grid assets or invest on an alternative investment with fixed rate of return. The objective is to maximize the cash flow at the end of horizon, which includes cash flow at the end of horizon and beyond the horizon projected investment cash flows. By developing such a comprehensive single framework, the loop between long-term and short-term planning is closed and optimal operation and long-term planning decisions are interrelated.

Having the problem stated, Table 2-1 presents the nomenclature of this work.

Table 2-1 : Nomenclature

Indices	
Y	Year
J	Scenario
T	Time of day
S	Sector
M	Micro-grid asset
H	Stressed out days
Parameters	
J^*	Number of scenarios
S^*	Number of sectors
M^*	Number of micro-grid assets
ϕ	Number of stressed out days
τ	Investment horizon
gwt	WT power generation
$wt\_o\&m$	WT operation and maintenance cost
$wssq$	Squared wind speed
$wssqmax$	Maximum operational wind speed square
$wssqmin$	Minimum operational wind speed square
gwt_dis	WT power generation in stressed out conditions
gpv	PV power generation
$pv\_o\&m$	PV operation and maintenance cost
SI	Solar radiation intensity
gpv_dis	PV power generation in stressed out conditions
ggf	GF power generation
$cfuel$	Natural gas price
$Gthr$	GF gas to heat ratio
ggf_dis	GF power generation in stressed out conditions
$gchp$	CHP power generation
Fur	CHP fuel utility ratio
$gchp\_heat$	CHP heat generation
$gchp\_total$	CHP total generation
chp_total_eff	CHP total efficiency
phr	CHP power to heat ratio
$gchp\_dis$	CHP power generation in stressed out condition
$charge$	ST charge
$discharge$	ST discharge
$stavail$	Available electricity in storage
$stdur$	Electricity storage duration

$charge\_dis$	ST charge in stressed out condition
$discharge\_dis$	ST discharge in stressed out condition
$gboiler$	Boiler heat generation
$boiler\_eff$	Boiler efficiency
$boiler\_o\&m$	Boiler operation and maintenance cost
$spirce$	Electricity spot price
$gbuy$	Electricity purchase from the grid
$demand$	Total electricity demand
$heat\_demand$	Total electricity demand
$demand\_sector$	Each sector electricity demand
$unsp$	Total unsupplied electricity demand in stressed out condition
sf_dis	Micro-grid electricity generation in stressed out condition
$loss$	Economic loss of total unsupplied electricity
$unsp\_sector$	Each sector unsupplied power demand in stressed out condition
$loss\_sector$	Each sector economic loss of 1 (MW) unsupplied power, ($)
$loss\_nomg$	Economic loss without micro-grid
$loss\_mg$	Economic loss with micro-grid
$pvcap$	PV capacity
$wtcap$	WT capacity
$stcap$	ST capacity
$gfcap$	GF capacity
$chpcap$	CHP capacity
$boilercap$	Boiler capacity
$asset\_capital$	Capital cost per unit capacity of each asset
$asset\_exp$	Added capacity of each asset in each year
$asset\_capital$	Capital cost per unit capacity of each asset
pv_lp	Rent of land for PV installation
wt_lp	Rent of land for WT installation
$borrow\_limit$	Borrow limit
B	Borrowed fund
$B.fc$	Finance charge on borrowed fund
FC	Finance charge
FT	Finance term
$saving\_oper\_normal$	Annual expected micro-grid saving in average normal conditions
$saving\_oper\_stress$	Annual expected micro-grid saving in stressed out conditions
$saving$	Annual expected saving of micro-grid
$acc\_saving_{(\tau)}$	Accumulated saving at τ
$net\_cf_{(\tau)}$	Net cash flow at τ
$\widehat{net\_cf}_{(\tau)}$	Beyond horizon cash flow
$\widehat{saving}_{(\tau)}$	Beyond horizon saving

$\widehat{B.fc}_{(\tau)}$	Beyond horizon finance charge
$cost\_nomg$	Total yearly cost without micro-grid
$cost\_mg$	Total yearly cost with micro-grid
operation_cost	Annual expected operation cost of micro-grid
capital_cost	Annual expected capital cost of micro-grid
RCA	Invested cash plus its return in an alternative investment
C	Cash spent to purchase assets
CA	Cash spent on alternative investment
bhp	Beyond horizon period

2.2.1 Distributed Generation Components

The power generated by photo voltaic cell (PV) is conserved by following equation.

$$gpv = pvcap * pv_{constant} * SI \quad (2.1)$$

where gpv is power in watt, $pvcap$ is the PV capacity, $pv_{constant}$ is a constant related to PV type and SI is the solar radiation. The power generated by wind turbine is given by:

$$gwt = wt_{eff} * wtcap * wssq \quad (2.2)$$

where gwt is the generated power in watt, wt_{eff} is the wind turbine efficiency as a function of with wind speed (obtained from wind turbine efficiency curve) , $wtcap$ is the wind turbine capacity and $wssq$ is wind speed squared. Gas fired power generation system provide reliable power flow and unlike wind turbine and PV cells, is a controllable power generation asset. Battery storage operation is fundamentally based on economic gain through charging and discharging based on electricity spot prices. Battery storage is mainly used for peak shaving and demand charge avoidance. Combined heat and power (CHP) is another asset in the micro-grid designs which is capable of efficient bi-generation of heat and power. The heat to electricity generation ratio is defined by the "*Power to Heat Ratio*" related to CHP type.

2.2.2 Dynamics of Uncertainty

Our approach accounts for existing short-term volatilities and long-term uncertainties. Long-term uncertain variables in the model are: (i) Natural gas price, (ii) WT, ST and PV capital costs of investment. Short-term volatile variables in the model are: (i) Heat and electricity demands of the community, (ii) Electricity spot price, (iii) Wind speed, (iv) Solar radiation.

The long-term behavior of natural gas prices has been studied extensively in the literature [13], [14], [15], [16]. Natural gas price is modeled by either mean reverting or non-mean reverting stochastic processes. Geometric Brownian Motion (GBM) is the most applied non mean-reverting stochastic process [17] and stationary Ornstein-Uhlenbeck (OU) Brownian motion is the most popular mean reverting stochastic process to model natural gas price [18]. We model natural gas price by mean reverting (OU) process. (OU) is a special case of a Hull-White-Vasicek (HV) model with constant volatility. The constant volatility guarantees the process tendency to return to its global mean. The general Hull-White-Vasicek (HV) model is in form of:

$$dS_t = \Delta\,(L - S_t)dt + V_t\,dW_t \quad (2.3)$$

where S_t is vector of process variables, Δ is mean reversion speeds (the rate of mean reversion), L is mean reversion levels (long-run mean or level), V is an instantaneous volatility rate matrix (constant in case of (OU) model) and dW_t is a Brownian motion vector [19]. (OU) process is the analogue of the AR (1) model in the continuous space. In case of natural gas price, the reversion rate and mean level are calculated from the coefficients of a linear fit between the log natural prices and their first difference scaled by the time interval parameter. Thereafter, using the defined parameters of (OU) model, several sample paths of natural gas price are generated using Monte Carlo sampling technique. In this technique, random vectors of a parameter from given probability distribution are generated repeatedly. For large number of generated scenarios, the sampled discrete probability distribution approaches the initial continuous distribution [20].

It is generally believed that the prices of technology for photovoltaic cells (PV), wind turbine (WT) and storage (ST) are decreasing. Therefore, we assume that the underlying processes follow a decreasing trend and assign a binomial probability mass function to the annual rate of decrease.

$$asset\_capital_{\,(Y)} = \psi * asset\_capital_{\,(Y-1)} \qquad for\ \{WT, PV\ and\ ST\} \quad (2.4)$$

where

$$\psi = \begin{cases} \psi_1 & with\ probablity\ of\ P_1 \\ \psi_2 & with\ probability\ of\ P_2 \end{cases} \qquad (2.5)$$

As stated earlier, micro-grid is subject to four short-term variations namely; variations in demand, solar intensity, wind speed and electricity spot price. Hourly wind speed and solar intensity are multivariate normal variables with negative correlation. The popular approach to sample from a multivariate normal distribution is the Cholesky decomposition. In this method, a positive definite Hermitian matrix is decomposed into a product of a lower triangular matrix and its conjugate transpose. This technique is very useful for numerical solution and Monte Carlo simulations [21]. Applying this technique, several paths of correlated solar intensity and wind speed can be generated. Moreover, the short term cost saving of the distributed generation assets is highly dependent on the grid electricity price. It is assumed that the peak electricity price on each day is correlated to natural gas price as follows [4]:

$$Peak\ electrcity\ price = Natural\ gas\ price * Ratio * \epsilon_{Grid} \quad (2.6)$$

where $Ratio$ accounts for transmission cost at grid level and ϵ_{Grid} is the grid average rate for electricity generation from natural gas. Assuming a daily profile for daily electricity spot price as a percentage of peak prices, hourly electricity prices over the course of a day is obtained.

2.3 Problem Formulation

This section is organized as follows. First, the short-term operation optimization formulation is presented which includes operation in average normal days and stressed out conditions. Then, we explain how we link the short-term and long-term decision variables. Finally, we present the capital budgeting formulation.

2.3.1 Short-term Operation Optimization

In order to calculate micro-grid savings in average normal days, we consider similar days over a year which is an abstraction of reality and it only simplifies our computations. Therefore, the yearly saving from normal operation is simply (365 - ϕ) times the daily saving (ϕ is the number of days in a year in which stressed out conditions are experienced). For normal day operation optimization, the objective is to maximize the expected micro-grid savings which is the expectation of difference between cost of supplying power and heat demands with and without micro-grid. Expected cost of supplying heat and power without micro-grid is the expected summation of cost to supply heat and power demands of the community by boiler and electricity from the grid over a period of T hours (i.e. 24 hours). Therefore, the annual expected cost without micro-grid is as follows.

$$cost_{nomg_{(Y)}} = prob * \left(365 - \phi_{(Y)}\right) * \sum_{J=1}^{J^*} \sum_{T=1}^{T^*} demand_{(Y,J,T)} * sprice_{(Y,J,T)} \quad (2.7)$$

$$+ \ demand\_heat_{(Y,J,T)} * \left(\frac{cfuel_{(Y)}}{boiler_{eff}}\right)$$

where $prob$ is the probability of each scenario equals to $\left({}^{1}/_{Number\ of\ scenarios}\right)$ on the discrete sampling plan. The expected cost of micro-grid in average normal days is the expected summation of micro-grid cost to satisfy heat and electricity demands of the community over a period of T hours (i.e. 24 hours). The cost of electricity and heat to satisfy the demands has the following components.

i. Purchase cost of electricity from the grid, $sprice_{(Y,J,T)}$

ii. Cost of electricity generation from gas fired, $cfuel_{(Y)}$

iii. Cost of electricity and heat generation by CHP , $\frac{cfuel_{(Y)}}{chp\_total\_eff}$

iv. Cost of heat generation by boiler, $\frac{cfuel_{(Y)}}{boielr\_eff}$

v. Operation cost of WT, $wt\_o\&m\ _{(Y)}$

vi. Operation cost of PV , $pv\_o\&m_{(Y)}$

Therefore, the annual micro-grid expected cost is as follows.

$$operation\_cost_{(Y)} = prob * (365 - \phi_{(Y)}) * \sum_{J=1}^{J^*} \sum_{T=1}^{T^*} (ggf_{(Y,J,T)} * cfuel_{(Y)} + gridbuy_{(Y,J,T)} * sprice_{(Y,J,T)} + gwt_{(Y,J,T)} * wt\_o\&m\ _{(Y)} + gpv_{(Y,J,T)} * pv\_o\&m_{(Y)} + gboiler_{(Y,J,T)} * \left(\frac{cfuel_{(Y)}}{boielr_{eff}}\right) + gchp_{total(Y,J,T)} * \left(\frac{cfuel_{(Y)}}{chp_{total_{eff}}}\right) \qquad (2.8)$$

Therefore, the expected yearly micro-grid operational savings in normal conditions is as below.

$$saving\_oper\_normal_{(Y)} = cost\_nomg_{(Y)} - operation_{cost\,(Y)} \qquad (2.9)$$

Constraints

The operation optimization of the micro-grid is subject to several constraints. For energy balance at year$Y$, time period T and scenario J , the total micro-grid power output consisting of battery discharge, WT generation, GF generation, PV and CHP generation less the battery charge must collectively satisfy the demand.

$$gwt_{(Y,J,T)} + gpv_{(Y,J,T)} + ggf_{(Y,J,T)} + gbuy_{(Y,J,T)} + discharge_{(Y,J,T)} - charge_{(Y,J,T)} \geq demand_{(Y,J,T)} \quad (2.10)$$

Heat demand of the community must be supplied by boiler and CHP generated heat.

$$gboiler_{(Y,J,T)} + gchp\_heat_{(Y,J,T)} \geq heat\_demand_{(Y,J,T)} \quad (2.11)$$

There are also operational constraints for the micro-grid assets. Power generation from WT is restricted by the operational range of the equipment. WT operates for a range of wind speeds. Two binary variables are defined namely; "wt_min" (equals 1 if wind speed is upper than the lower operational limit; 0 otherwise) and "wt_max" (equals 1 if wind speed is lower than upper operational limit; 0 otherwise). We use "*Big-M*" method to define when both "wt_min" and "wt_max" equal one. "*Big-M*" method is a variation of the simplex method designed for solving problems encompassing either "less-than" or "greater-than" constraints. The "*Big-M*" is assumed to be an extremely large number associated with artificial variables represented by "$bigM$". [22]

<u>Lower operational limit</u>

$$wssq_{(Y,J,T)} - wssqmin < bigM * wt_{\mathrm{m}(Y,J,T)} \quad (2.12)$$

$$wssqmin - wssq_{(Y,J,T)} < bigM * \left(1 - wt\_min_{(Y,J,T)}\right) \quad (2.13)$$

<u>Upper operational limit</u>

$$wssq_{(Y,J,T)} - wssqmax < bigM * \left(1 - wt\_max_{(Y,J,T)}\right) \quad (2.14)$$

$$wssqmax - wssq_{(Y,J,T)} < bigM * wt\_max_{(Y,J,T)} \quad (2.15)$$

We also introduce "$wt_{indicator}$" binary indicator as below.

$$wt_{indicator\ (Y,J,T)} = \ wt\_min_{(Y,J,T)} + wt\_max_{(Y,J,T)} - 1 \qquad (2.16)$$

$wt_{indicator\ (Y,J,T)}$ is WT availability at year Y, time T and scenario J; $wt_{indicator\ (Y,J,T)} = 1$ if wind speed is in the operational range, otherwise $wt_{indicator\ (Y,J,T)} = 0$. Therefore, the WT power generation is constrained as below.

$$wt_{indicator(Y,J,T)} * \left(wt_{eff} * wtcap_{(Y,J)} * wssq_{(Y,J,T)}\right) \leq \ gwt_{(Y,J,T)} \ \leq \ wt_{indicator(Y,J,T)} * \left(wt_{eff} * wtcap_{(Y,J)} * wssq_{(Y,J,T)}\right) \qquad (2.17)$$

We also need a set of constraints for the electricity storage device. Charge and discharge of electricity can be very beneficial when electricity spot price varies during different times of a day. Available electricity in battery at end of each time period (i.e. Hour) is conserved by:

$$stavail(_{(Y,J,T)} = \ charge_{(Y,J,T)} - discharge_{(Y,J,T)} + stavail_{(Y,J,T-1)} \quad for\ T > 1 \quad (2.18)$$

Charge of battery cannot exceed battery capacity less available electricity in the storage left from previous hour of operation.

$$charge_{(Y,J,T)} \leq \frac{(\ stcap_{(Y,J,T)} * stdur\ ) - stavail_{(Y,J,T-1)}}{stdur} \qquad for\ \ T > 1 \qquad (2.19)$$

Also discharge cannot exceed the amount of electricity available in the battery.

$$discharge_{(Y,J,T)} \leq \frac{stavail_{(Y,J,T-1)}}{stdur} \qquad for\ \ T > 1 \quad (2.20)$$

Eventually, the amount of electricity stored in battery cannot exceed the maximum energy limit.

$$stavail_{(Y,J,T)} \leq \ stcap_{(Y,J,T)} * stdur \quad (2.21)$$

Regarding the CHP operation, hourly electricity generation is correlated to heat generation as follows [23].

$$gchp_{(Y,J,T)} = \ gchp\_heat_{(Y,J,T)} * phr \quad (2.22)$$

In addition, the sum of electricity and heat generation must be equal to the total CHP power output.

$$gchp_{(Y,J,T)} + \; gchp\_heat_{(Y,J,T)} = \; gchp\_total_{(Y,J,T)} \quad (2.23)$$

Now, we proceed to the micro-grid operation optimization in stressed out conditions in which power grid is totally disconnected. We assume that the number of days with stressed out conditions and duration of each grid outage are random variables with known distribution and parameters. Therefore, for random number ϕ of days in each year, grid is disconnected for a random number of hours. In order to simulate such a condition, we define a binary variable ($stress_{(H,Y,J,T)}$) for hour T of H-th stressed out day in scenario J and year Y that take 1 when the grid is out and 0 otherwise. Therefore, the micro-grid saving is maximized by minimization of micro-grid cost (as explained before) when $stress_{(H,Y,J,T)}$ is 1 (i.e. normal condition) and by minimization of micro-grid loss when $stress_{(H,Y,J,T)}$ is 0 (i.e. stressed out condition). The expected micro-grid savings in stressed out conditions is defined as below.

$$saving\_oper\_stress_{(Y)} = \; loss\_nomg_{(Y)} - \; loss_{mg_{(Y)}} \quad (2.24)$$

where

$$loss\_nomg_{(Y)} = prob * \; \sum_{H=1}^{\phi(Y)} \sum_{S=1}^{S^*} \sum_{J=1}^{J^*} \sum_{T=1}^{T^*} demand\_sector_{(S,Y,J,T)} * \; loss\_sector_{(S,Y,J,T)}$$

(2.25)

where $demand\_sector_{(S,Y,J,T)}$ is the demand of sector S, in year Y, scenario J, time T and in H-th stressed out day. Moreover,

$$loss\_mg_{(Y)} = prob * \; \sum_{H=1}^{\phi(Y)} \sum_{S=1}^{S^*} \sum_{J=1}^{J^*} \sum_{T=1}^{T^*} unsp\_sector_{(S,Y,J,T)} * \; loss\_sector_{(S,Y,J,T)} \; (2.26)$$

where $unsp\_sector_{(S,Y,J,T)}$ is the unsupplied demand of sector S, in year Y, scenario J, time T and in H-th stressed out day. In order to minimize the $loss\_mg$, micro-grid power generation and battery discharge is maximized and the available power is allocated among community sectors based on the criticality of their operation. Each sector's operation criticality is defined as the economic loss of one unit of unsupplied power. For hours of grid disconnection, the amount of micro-grid power is as follows.

$$sf\_dis_{(Y,J,T)} = gpv\_dis_{(Y,J,T)} + gwt\_dis_{(Y,J,T)} + ggf\_dis_{(Y,J,T)} + gchp\_dis_{(Y,J,T)} + discharge\_dis_{(Y,J,T)} - charge\_dis_{(Y,J,T)} \quad (2.27)$$

Therefore, the amount of unserved power is calculated as follows.

$$unsp_{(Y,J,T)} = demand_{(Y,J,T)} - sf\_dis_{(Y,J,T)} \quad (2.28)$$

In stressed out conditions, each asset's operational constraints are similar to those in average normal days. However, the constraint of satisfying power and heat demand does not hold. Allocating the available power based on sectors criticality decreases the total economic loss of unsupplied power. We assume constant economic loss of unsupplied unit power for each sector. The total amount of unsupplied power in each hour is the sum of all sectors unsupplied power.

$$unsp_{(Y,J,T)} = \sum_{s=1}^{S^*} unsp\_sector_{(S,Y,J,T)} \quad (2.29)$$

Unsupplied demand of each sector cannot exceed that sector's demand.

$$unsp\_sector_{(S,Y,J,T)} \leq demand\_sector_{(S,Y,J,T)} \quad (2.30)$$

Before, proceeding to next section, we present the expected operational savings of micro-grid asset in each year as below.

$$saving_{(Y)} = saving\_oper\_normal_{(Y)} + saving\_oper\_stress_{(Y)} \quad (2.31)$$

Loop between Short term and Long term Decisions

In order to link short-term operational decisions to long-term planning following constraints must hold which ensure proper linkage between capacity (i.e. long term decision variable) and hourly operation of each asset in both average normal and stressed out conditions (i.e. short term decision variable).

Average normal days

$$gpv_{(Y,J,T)} = \; pvcap_{(Y,J)} * pv_{constant} \; * SI_{(Y,J,T)} \quad (2.32)$$

$$gwt_{(Y,J,T)} = \; wt_{eff} * \; wtcap_{(Y,J)} * \; wssq_{(Y,J,T)} \quad (2.33)$$

$$gboiler_{\,(Y,J,T)} \leq \; boilercap_{(Y,J)} \quad (2.34)$$

$$ggf_{\,(Y,J,T)} \leq \; gfcap_{(Y,J)} \quad (2.35)$$

$$gchp_{(Y,J,T)} \; \leq \; chpcap_{(Y,J)} \quad (2.36)$$

Stressed-out conditions

$$gpv\_dis_{(Y,J,T)} = \; pvcap_{(Y,J)} * pv_{constant} \; * SI_{(Y,J,T)} \quad (2.37)$$

$$gwt\_dis_{\_(Y,J,T)} = \; wt_{eff} * \; wtcap_{(Y,J)} * \; wssq_{(Y,J,T)} \quad (2.38)$$

$$gboiler\_dis_{\,(Y,J,T)} \leq \; boilercap(_{(Y,J)} \quad (2.39)$$

$$ggf\_dis_{\,(Y,J,T)} \leq \; gfcap_{(Y,J)} \quad (2.40)$$

$$gchp\_dis_{(Y,J,T)} \; \leq \; chpcap_{(Y,J)} \quad (2.41)$$

2.3.2 Capital Budgeting

Capital budgeting (or investment appraisal) is a finance terminology, which aims at deciding whether or not to undertake an investment project. In other words, it is the process of allocating resources for major capital or investment. In this section, we are aiming at finding out optimal capital capacity of each asset to be purchased and installed in each year. Net Present Value (NPV) is a common approach to decide whether to undertake a project or to refuse it. The basic idea is to maximize the discounted value of the net cash flow stream over the course of investment horizon. Once dealing with multiple investment options, calculating the discount factor in (NPV) approach is not straight forward. In addition, criticism to the (NPV) approach has been introduced once considering outside investments. Investment horizon approach is an alternative method to evaluate the economic value of investment projects. In the horizon models, the objective function to be maximized is accumulated cash flow at the time of horizon plus the value of post horizon cash flows [24]. A number of conceptual issues such as proper choice of discount factor could be avoided using this approach. In this work investment horizon approach is applied to evaluate the capital budgeting problem and by incremental investment decisions, micro-grid portfolio is constructed during the course of investment horizon. We assume that at each time period, borrowing opportunity with constant finance charge for a fixed finance period exists and borrowed fund must be invested only on the micro-grid assets. It is also assumed that any available cash in each period could be either invested in alternative investment opportunity or spent to purchase assets. The following preliminary assumptions are made to develop the model.

i. Cash inflow from the micro-grid's saving will be added to the available cash in the same period.
ii. Beyond the horizon period is assumed to be finite (15 years).
iii. Each asset will be available at the purchase time period.
iv. There would be initial cash available in the first period
v. We assume a fixed maximum borrowing limit at each time period.
vi. Required land for WT and PV installation is rented with fixed fee per unit of area.
vii. Rate of return for investment in the alternative investment opportunity is less than the finance charges.

As stated earlier, the objective is to maximize the end of horizon cash flow plus the horizon time value of any cash flows beyond the horizon. The end of horizon net cash flow is calculated as below.

$$net\_cf_{(\tau)} = \left(saving_{(\tau)} + RCA_{(\tau)} + B_{(\tau)} - B.fc_{(\tau)} - C_{(\tau)} - CA_{(\tau)} \right) \quad (2.42)$$

where:

$saving_{(\tau)}$: Micro-grid saving at the end of horizon

$RCA_{(\tau)}$: Invested cash plus return on invested cash in an alternative investment in the previous period

$B_{(\tau)}$: Borrowed fund in the investment horizon period

$B.fc_{(\tau)}$: Finance charge on the borrowed fund at the horizon

$C_{(\tau)}$: Cash spent to purchase asset at horizon

$CA_{(\tau)}$: Cash spent on alternative investment at horizon

Finance charge of the borrowed fund at each period is calculated as below.

$$B.fc_{(Y)} = B_{(Y-1)} * (1 + FT) \quad (2.43)$$

Return on cash invested in an alternative investment opportunity in the previous period is as below.

$$RCA_{(Y)} = CA_{(Y-1)} * (1 + ROI) \quad (2.44)$$

Beyond horizon cash flows include discounted cumulative savings from micro-grid ($\widehat{saving}_{(\tau)}$) from the after horizon time period until the end of finite beyond horizon period.

$$\widehat{saving}_{(\tau)} = \sum_{Y=1}^{bhp} \frac{Saving_{(\tau+1)}}{(1 + DF)^{Y}} \quad (2.45)$$

The finance charge of fund borrowed in last period ($\widehat{borrow}\_fc_{(\tau)}$) and return on cash invested in the alternative investment in the last period ($\widehat{RCA}_{(\tau)}$). Each of these terms is calculated below.

$$\widehat{borrow}.fc_{(\tau)} = \frac{borrow.fc\ (\tau+1)}{(1+DF)} \quad (2.46)$$

$$\widehat{RCA}_{(\tau)} = \frac{RCA\ (\tau+1)}{(1+DF)} \quad (2.47)$$

Where *DF* is discount factor which is computed as a weighted average of debt and equity financing.

Therefore, beyond horizon cash flows is as follows.

$$\widehat{net_{cf}}_{(\tau)} = \widehat{saving}_{(\tau)} + \widehat{RCA}_{(\tau)} - \widehat{borrow}.fc_{(\tau)} \quad (2.48)$$

Consequently, the objective function is as below.

$$Max\ \{\ net_{cf}{}_{(\tau)} + \widehat{net_{cf}}_{(\tau)}\} \quad (2.49)$$

<u>Constraints</u>

Borrowed fund at each period cannot exceed a pre-defined limit.

$$borrow_{(Y)} \leq borrow\_limit \quad (2.50)$$

There is a constraint for cash invested in alternative investment based on the availability of cash:

$$CA_{(1)} \leq IC_0 - C_{(1)} \quad for\ Y = 1 \quad (2.51)$$

$$CA_{(Y)} \leq RCA_{(Y)} + saving_{(Y)} - C_{(Y)} \quad for\ Y \neq 1 \quad (2.52)$$

Borrows funds are not allowed to be invested in the alternative investment. Therefore, the total investment on micro-grid in each time period equals to borrowed fund plus the amount spent from available cash:

$$capital\_cost_{(Y)} = B_{(Y)} + C_{(Y)} \quad for\ 1 \leq Y \leq \tau \quad (2.53)$$

where

$$capital\_cost_{(Y)} = \left[\sum_{M=1}^{M^*} asset\_capital_{(M.Y)} * \ asset\_exp_{(M,Y)}\right] + wt\_lp_{(Y)} + \ pv\_lp_{(Y)} \quad (2.54)$$

where $asset\_capital$ is the capital cost of unit capacity of each micro-grid asset in year Y, $asset\_exp$ is the added capacity of each asset in year Y , wt_lp and pv_lp are the rental fees for the land to install WT and PV respectively.

As mentioned earlier, it is assumed that assets purchased at each time period is active at the same time period. Therefore, the available capacity of each asset time *T* and scenario *J* and year *Y* is as follows.

$$asset\_cap_{(M,Y,J)} = \ asset\_cap_{(M,Y,J)} + \ asset\_exp_{(M,Y,J)} \qquad for \ \ 1 < T \leq \tau \quad (2.55)$$

$$asset\_cap_{(M,1,J)} = \ asset\_exp_{(M,1,J)} \qquad for\, Y = \ 1 \qquad (2.56)$$

We also assume that the PV and WT capacities are restricted with the available land to install assets. Therefore,

$$pvcap_{(Y)} * unit.pv.land \ + \ \ wtcap_{(Y)} * unit.wt.land \leq avail.land \quad (2.57)$$

Moreover, the capacities of CHP and gas fired assets are restricted. We assume that each of these assets shall not provide more than half of the maximum power demand of the community.

To apply such a model and also account for uncertainty in our problem; the above illustrated investment horizon model is solved under different stochastic scenarios of natural gas price and capital costs of PV, ST and WT over the investment horizon. The overall model is solved as a mixed-integer non-linear stochastic optimization problem. Lingo optimization software is used to solve the problem and obtain global optimal solution.

2.4 Illustrative Results

Here, we intend to show the following factors through a number of numerical experiments.

i. Dependency of micro-grid incremental optimal portfolios on risk factors (i.e. probability and consequences of power outage)

ii. Dependency of optimal capital budgeting plans on risk characteristics

iii. Higher value for proposed model in more risky regions

iv. Impact of stochastic process applied to model natural gas prices over the course of investment horizon

This section is organized as follows: first, necessary inputs to set up the model will be explained. Second, numerical results for sensitivity experiments will be provided.

2.4.1 Input Data

In order to generate paths of hourly electricity spot price, it is assumed that the peak electricity price on each day is correlated to natural gas price as explained earlier. By assuming a profile for daily electricity spot price as a percentage of peak prices, we obtain the hourly electricity prices over the course of a day. Figure 2-1 shows the daily spot price profile as a percentage of daily electricity peak prices.

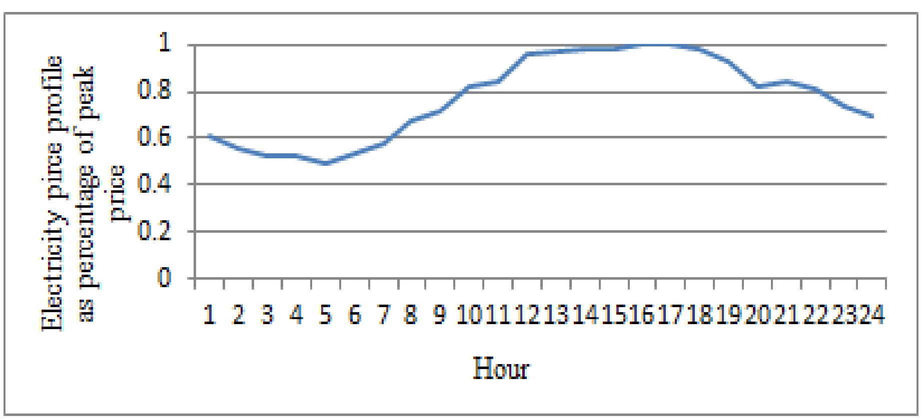

Figure 2-1 : Electricity spot price profile as percentage of peak price

Wind speed and solar radiation are correlate random variables with negative correlation. Solar radiation and wind speed data is extracted from the National Renewable Energy Laboratory datasets for a sample day in 2005 [25]. It is assumed that the correlation matrix is a follows.

Table 2-2 : Solar intensity and wind speed correlation matrix

	Solar intensity	**Wind speed**
Solar intensity	1	-0.5
Wind speed	-0.5	1

Figure 2-2 represents a sample path of normalized wind speed and solar radiation along a sample day.

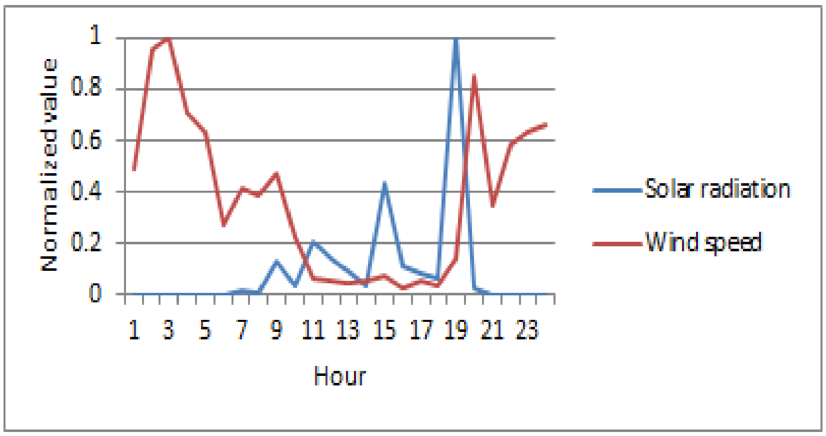

Figure 2-2 : Sample Monte Carlo simulated path for correlated wind speed and solar radiation

Natural gas price is modeled using (OU) mean reverting stochastic process. The reversion rate and mean level are calculated from the coefficients of a linear fit between the log natural prices and their first difference scaled by the time interval parameter. Using historical data, the (OU) model parameters are defined and presented in Table 2-3.

Table 2-3 : (OU) process parameters for natural gas price

Volatility	0.74
Mean reversion	1.71
Mean reversion speed	1.77

Gas fired, boiler and CHP investment costs are considered to be deterministic. Table 2-4 presents the annual investment cost of these assets over the course of investment horizon (i.e. 4 years).

Table 2-4 : Deterministic investment costs of gas fired, CHP and boiler

	Year 1	Year 2	Year 3	Year 4
Gas fired (S/MW)	100,000	100,100	100,200	100,300
CHP ($/MW)	1,200,000	1,210,000	1,220,000	1,230,000
Boiler ($/MW)	600	700	800	900

The parameters of binomial probability mass function assigned to PV, WT and ST capital costs decline rate are presented in Table 2-5.

Table 2-5 : Parameters of Binomial distributions

Asset	ψ_1	ψ_2	P_1	P_2
PV	0.8	0.6	0.33	0.67
WT	0.8	0.6	0.33	0.67
Storage	0.8	0.6	0.33	0.67

Land needed for unit capacity (MW) of PV and WT installment are 4 and 10 acres respectively. The rent of land is assumed to be 10000 ($\frac{\$}{acre}$).

We assume that state space of the number of days with stressed out conditions is {0, 1, and 2} with a trinomial probability mass function as follows.

$$\phi_{(Y)} = \begin{cases} 0 & with\ probablity\ of \quad 0.33 \\ 1 & with\ probability\ of \quad 0.33 \\ 2 & with\ probability\ of \quad 0.33 \end{cases} \qquad (2.58)$$

In addition, the total power demand of the community is broken down to the constituent sectors' demand. We rank the sectors based on criticality for optimal power allocation is stressed out conditions. Table 2-6 presents the sectors criticality ranking.

Table 2-6 : Sectors criticality ranking

Rank	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Sectors	Health	Office	Retail	Education	Warehouse	Residential	Leisure

In addition, financial parameters applied in the model are as below.

Table 2-7 : Financial parameters

τ (year)	4
ROI	0.02
FC	0.04
FT(year)	1
$\mathbf{borrow\_limit}$ (M$)	3
$\mathbf{IC_0}$ (M$)	1
$\mathbf{avail.land(acre)}$	25
lifetime (year)	15

2.4.2 Illustrative Example (I): Micro-Grids with and without Resiliency Criterion

Not taking into account the economic value of resiliency caused by micro-grids underestimates the value of these assets specifically in risky regions. Considering resiliency criterion in micro-grid planning results in higher savings and hence alters the long-term investment decisions including incremental micro-grid optimal portfolios and capital budgeting decisions. Figure 2-3 demonstrates how average optimal incremental micro-grid portfolios differ for two micro-grids with resiliency criterion (MG-I) and without resiliency criterion (MG-II).

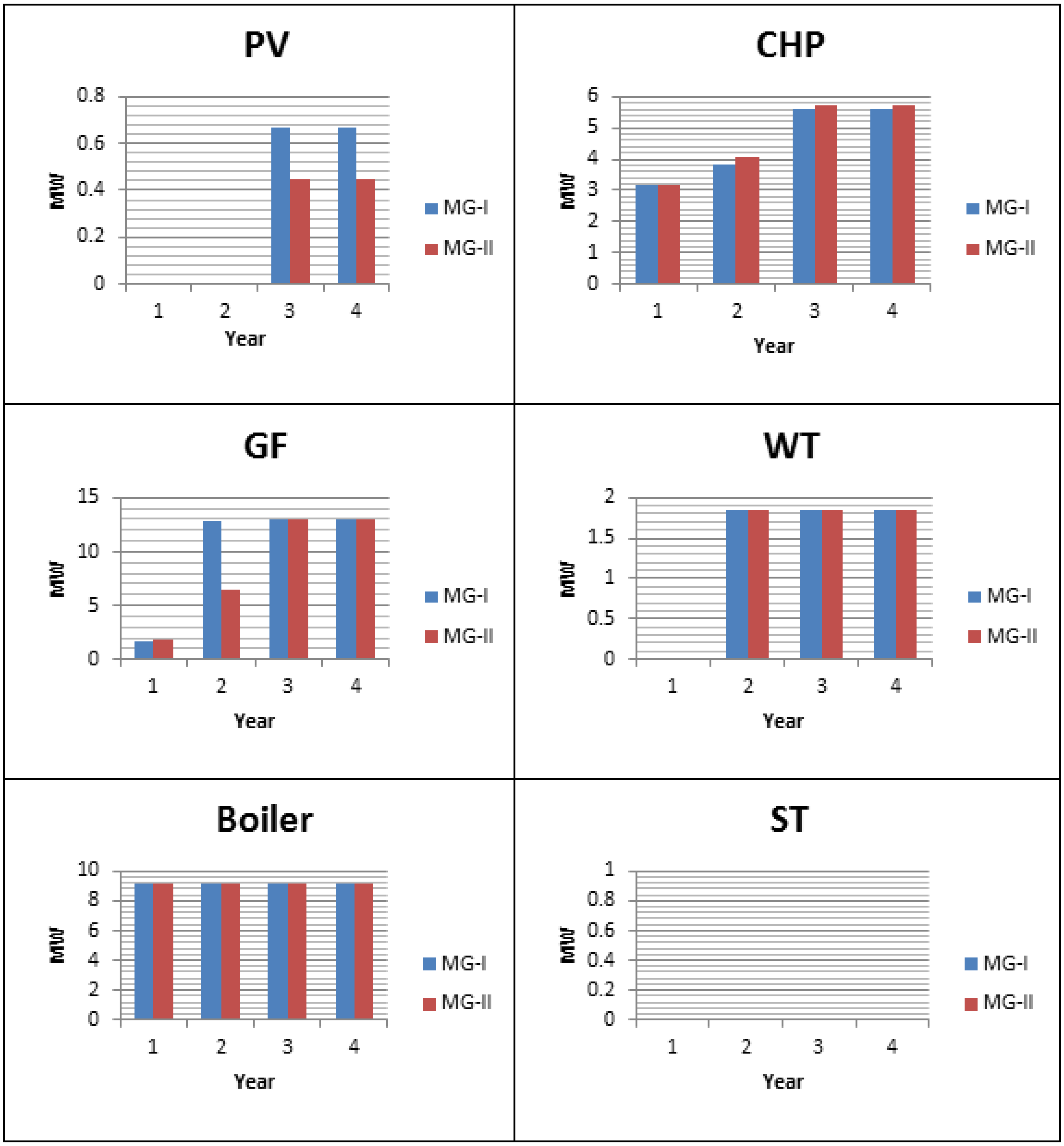

Figure 2-3 : Micro-grid optimal incremental portfolios with resiliency criterion (MG-I) and without resiliency criterion (MG-II)

Comparing assets incremental capacities for micro-grids MG-I and MG-II, following observations are made.

i. In MG-I, higher capacities of PV are purchased.

ii. In MG-I, higher capacities of GF are purchased in first two years. However, in both cases maximum allowable capacities of GF are installed in year 3.

iii. In MG-II, tiny higher capacities of CHP are installed.

iv. Higher overall capacity of assets is installed in MG-I to reduce the loss of unsupplied power in stressed out conditions.

We are also interested to see how capital budgeting decisions (i.e. average financial activities including borrowing fund, investment in alternative investment and yearly spent cash) are different for MG-I and MG-II micro-grids. Figure 2-4 represents the average financial activities along with expected savings for MG-I and MG-II over the course of investment horizon.

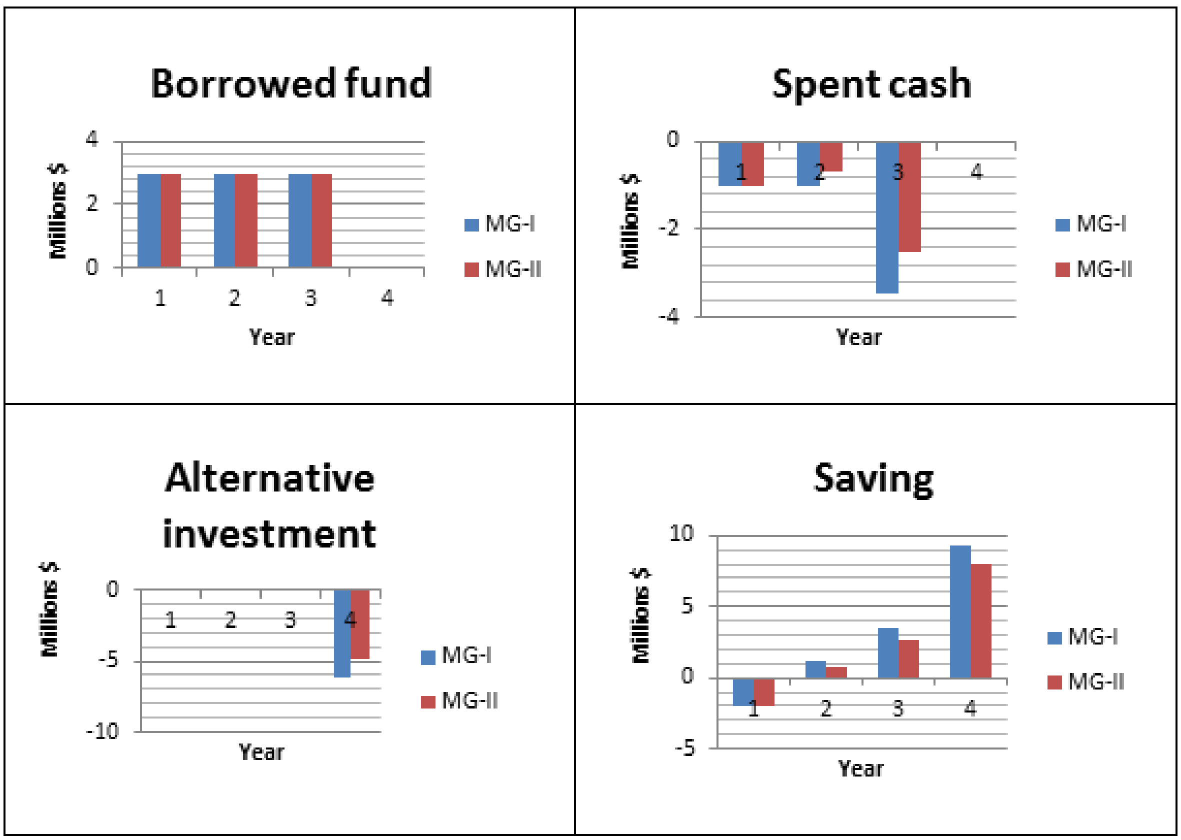

Figure 2-4 : Average financial activity over the course of investment horizon (MG-I and MG-II)

Comparing the average financial activities for MG-I and MG-II following observations are made.

i. More cash is invested in MG-I to purchase higher capacities of micro-grid assets.

ii. Higher savings with MG-I compared to MG-II in spite of higher cost of capital for MG-I. Higher savings are acquired for MG-I due to inclusion of avoided loss in stresses out conditions in calculation of savings.

iii. Higher savings with MG-I let the investors to invest more in alternative investment in year 4 to increase the cash in-flow. In order to compare the financial values of MG-I and MG-II, the expected cash flow positions at the end of horizon (including beyond horizon projected cash flow) for both cases are presented in Figure 2-5.

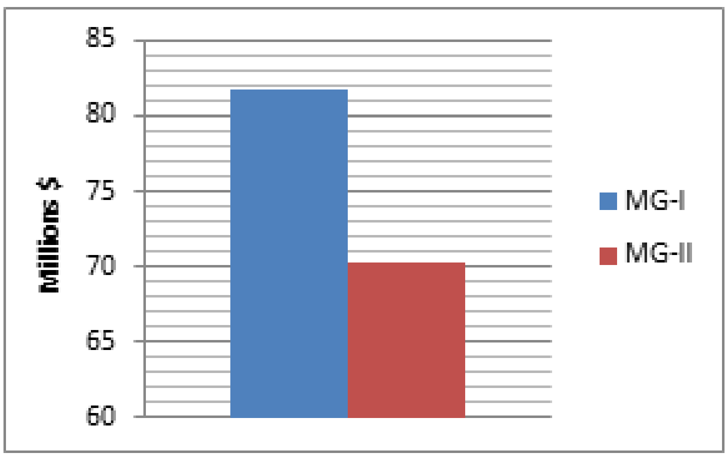

Figure 2-5 : Expected cash flow position at τ (including beyond horizon projected cash flow) (MG-I and MG-II)

2.4.3 Illustrative Example (II): Impact of Risk Characteristics

According to the risk based structure of the model, we conjecture higher value for this model in more risky regions in which value of added resiliency by micro-grid is significant. Therefore, using this model is most valuable for investments in vulnerable geographic regions (such as coastal towns) and/or regions where many critical sectors (e.g. hospitals and offices) are located. In this section, we check the sensitivity of our model to risk factors (i.e. consequent economic loss and probability of power failure). In order to examine our model results sensitivity to loss due to power outage, we define three categories of loss values for sectors in the community as presented in Table 2-8. The loss values are defined based on the range of estimated direct costs of outage for different sectors customers available in US Environmental Protection Agency (EPA) report [26].

Table 2-8 : Loss values of unsupplied unit power for different sectors

	Loss categories and loss values $\left(\frac{Loss\ (\$)}{Unsupplied\ power(kWh)}\right)$		
Sector	L_1	L_2	L_3
Health	22	44	66
Office	20	40	60
Retail	15	30	45
Education	10	20	30
Warehouse	10	20	30
Residential	5	10	15
Leisure	5	10	15

By increasing the loss index from 1 to 3, power grid outage incurs higher economic loss to the community sectors. Figure 2-6 demonstrates the average micro-grid incremental capacities for different loss scenarios.

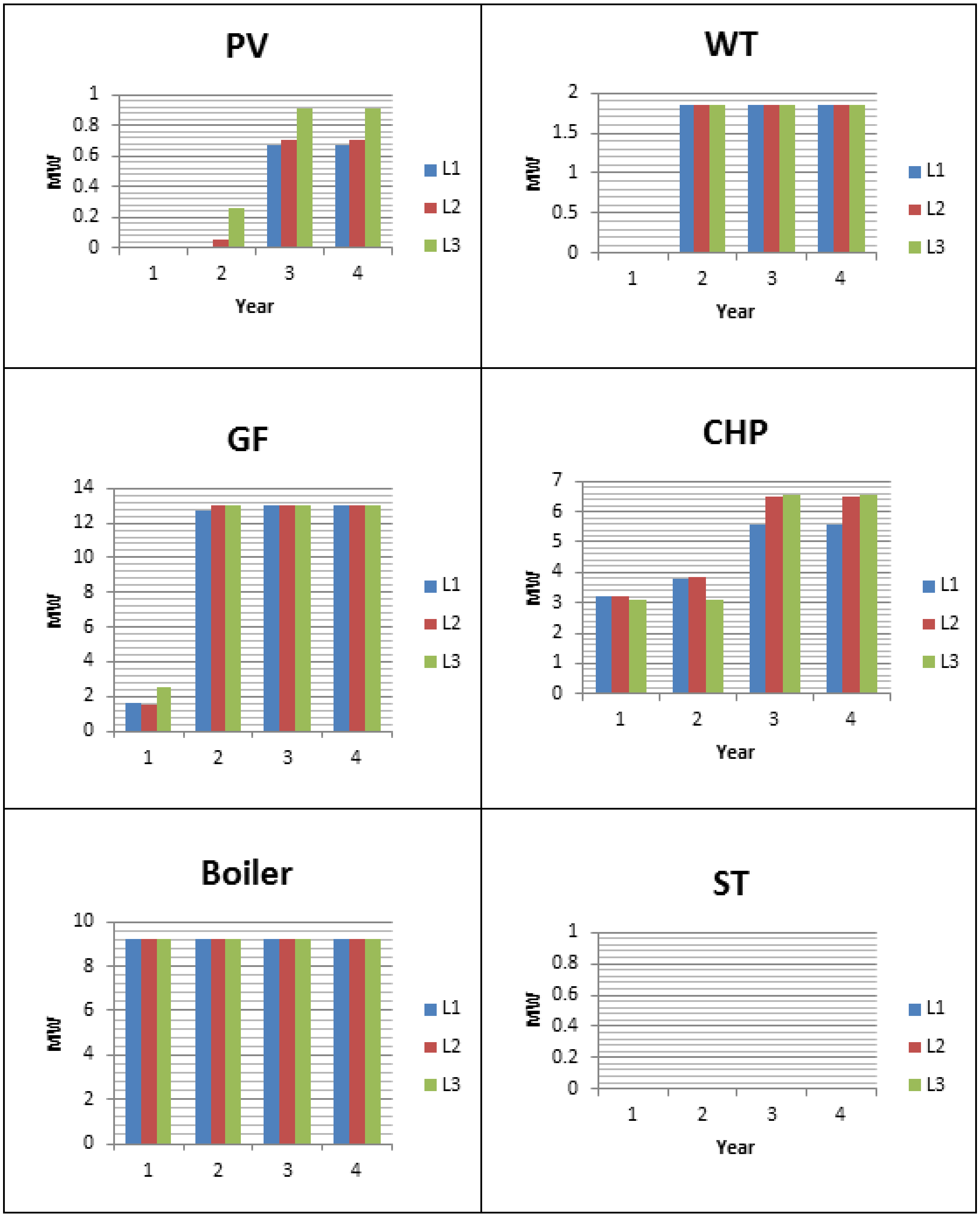

Figure 2-6 : Average micro-grid assets optimal capacity for different loss categories

Following observations are made from Figure 6.

i. Higher loss incurs investment on higher overall capacity of micro-grid assets over the course of investment horizon.

ii. Higher loss values makes investment in GF more attractive and in CHP less attractive in initial two years of investment.

iii. Higher loss values forces the investors to install higher capacities of PV. Additional PV capacity mitigates the loss incurred by power outage specifically in middle hours of day when solar radiation and PV efficiency are maximized.

Moreover, Figure 2-7 demonstrates the annual borrowed fund, investment in alternative investment and micro-grid annual saving considering different loss categories.

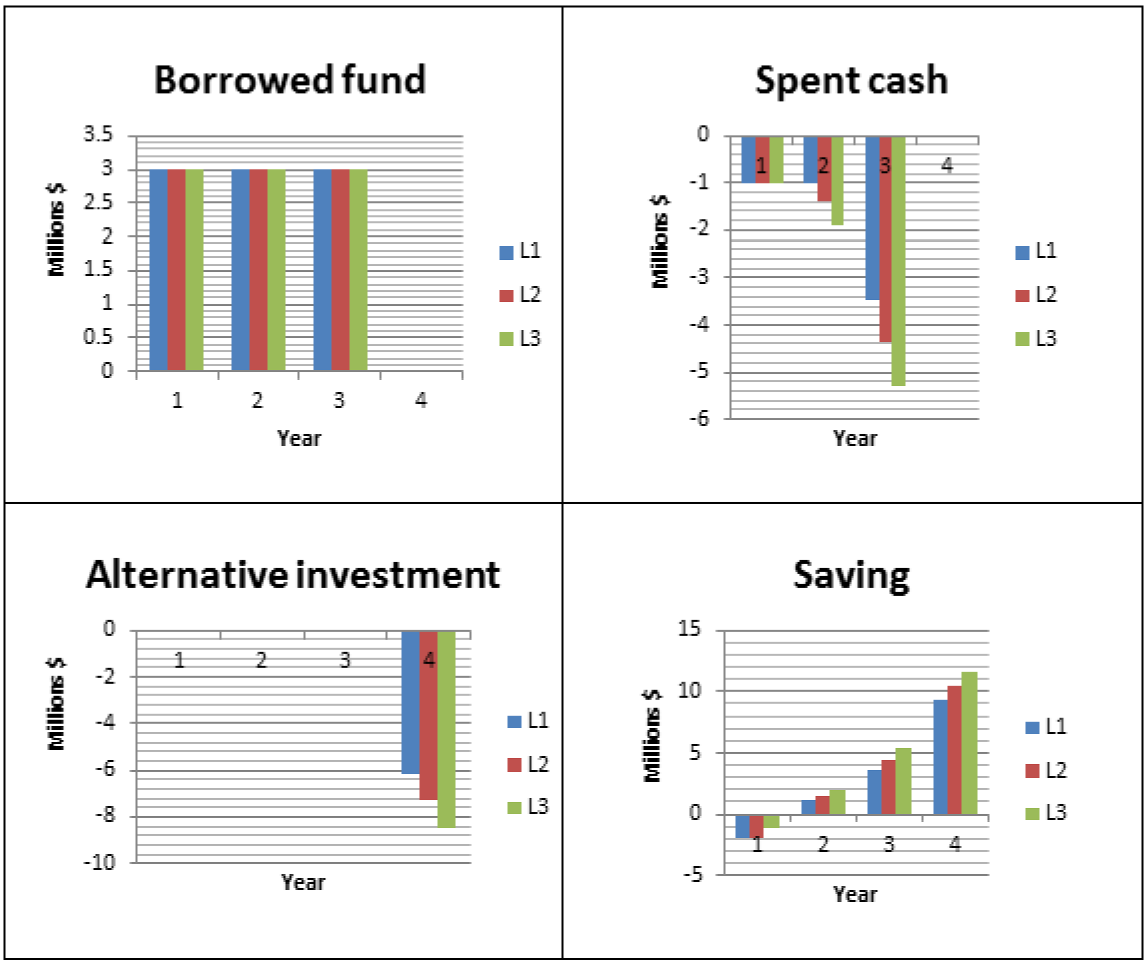

Figure 2-7 : Average annual borrowed fund, investment in alternative and micro-grid savings for different loss categories

Following observations are made.

i. Higher loss yields in more savings from micro-grid.

ii. More cash is spent to purchase assets when loss is higher,

iii. Highest micro-grid saving at τ and highest investment in alternative project is observed for L_3 category.

Now, we proceed to examine the sensitivity of our model results to the probability of power failure which can also be interpreted as regional vulnerability to extreme conditions which yields to power outage. We assume two scenarios (V_1 and V_2) for probabilities of stressed out days in each year as follows.

Table 2-9 : Vulnerability scenarios

	Vulnerability scenarios	
	$\boldsymbol{V_1}$	$\boldsymbol{V_2}$
$\mathbf{Pr}(\boldsymbol{\phi}_{(Y)} = \mathbf{0})$	0.66	0.165
$\mathbf{Pr}(\boldsymbol{\phi}_{(Y)} = \mathbf{1})$	0.165	0.165
$\mathbf{Pr}(\boldsymbol{\phi}_{(Y)} = \mathbf{2})$	0.165	0.66

Vulnerability scenario V_2 can be interpreted as a community located in a coastal area, while scenario V_1 could represent a not coastal region. The loss category for both scenarios is L_3. Figure 2-8 demonstrates the average micro-grid incremental capacities for V_1 and V_2 regions.

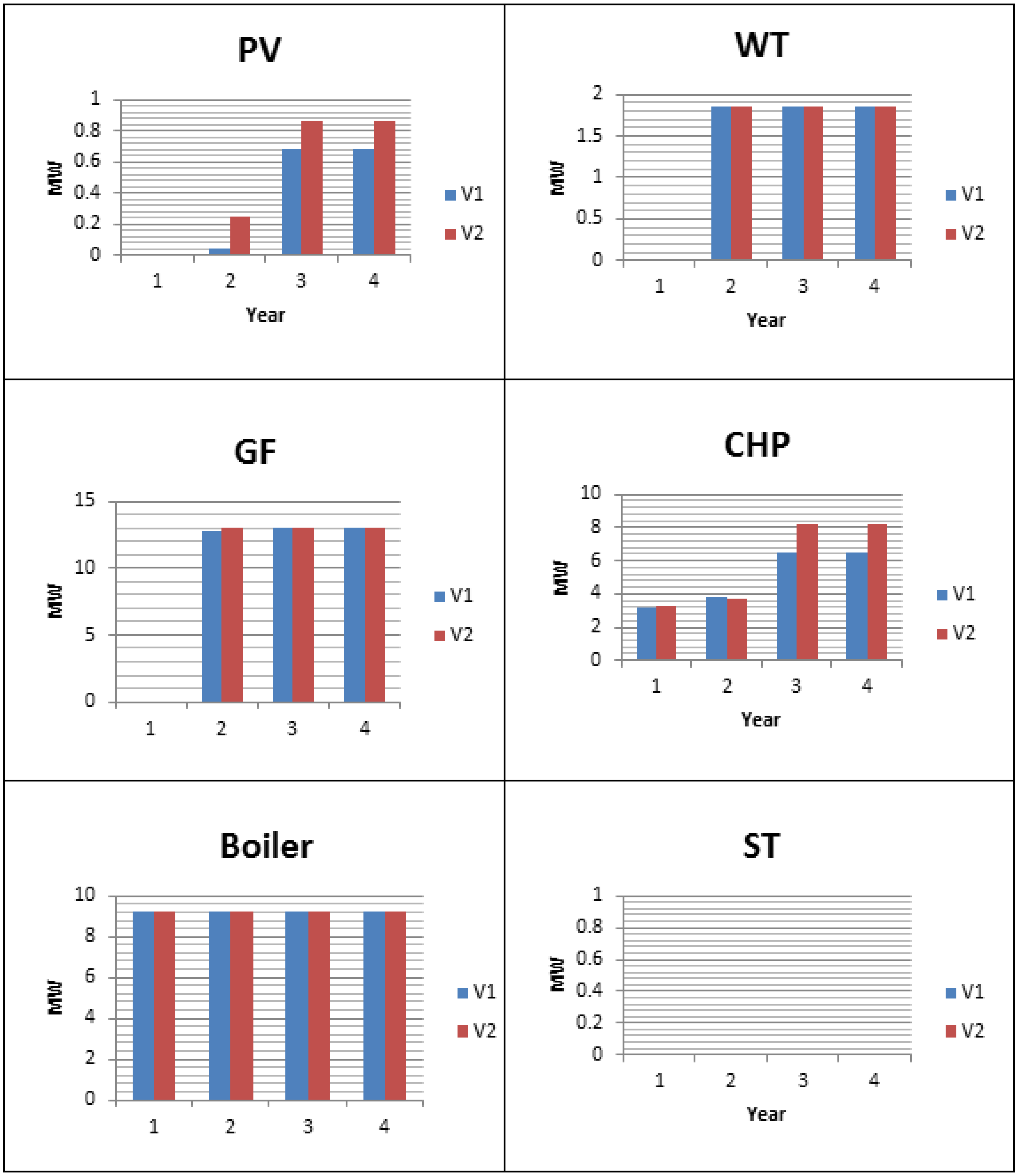

Figure 2-8 : Average micro-grid assets optimal capacity for different vulnerability scenarios

Comparing the micro-grids assets incremental capacities in V_1 and V_2 regions, following observation is made.

i. Higher capacity of PV and CHP capacities are installed for the more vulnerable region.

ii. Tiny higher capacity of GF is installed in more vulnerable region at year 2.

The average financial activities for the two vulnerability scenarios are demonstrated in Figure 2-9.

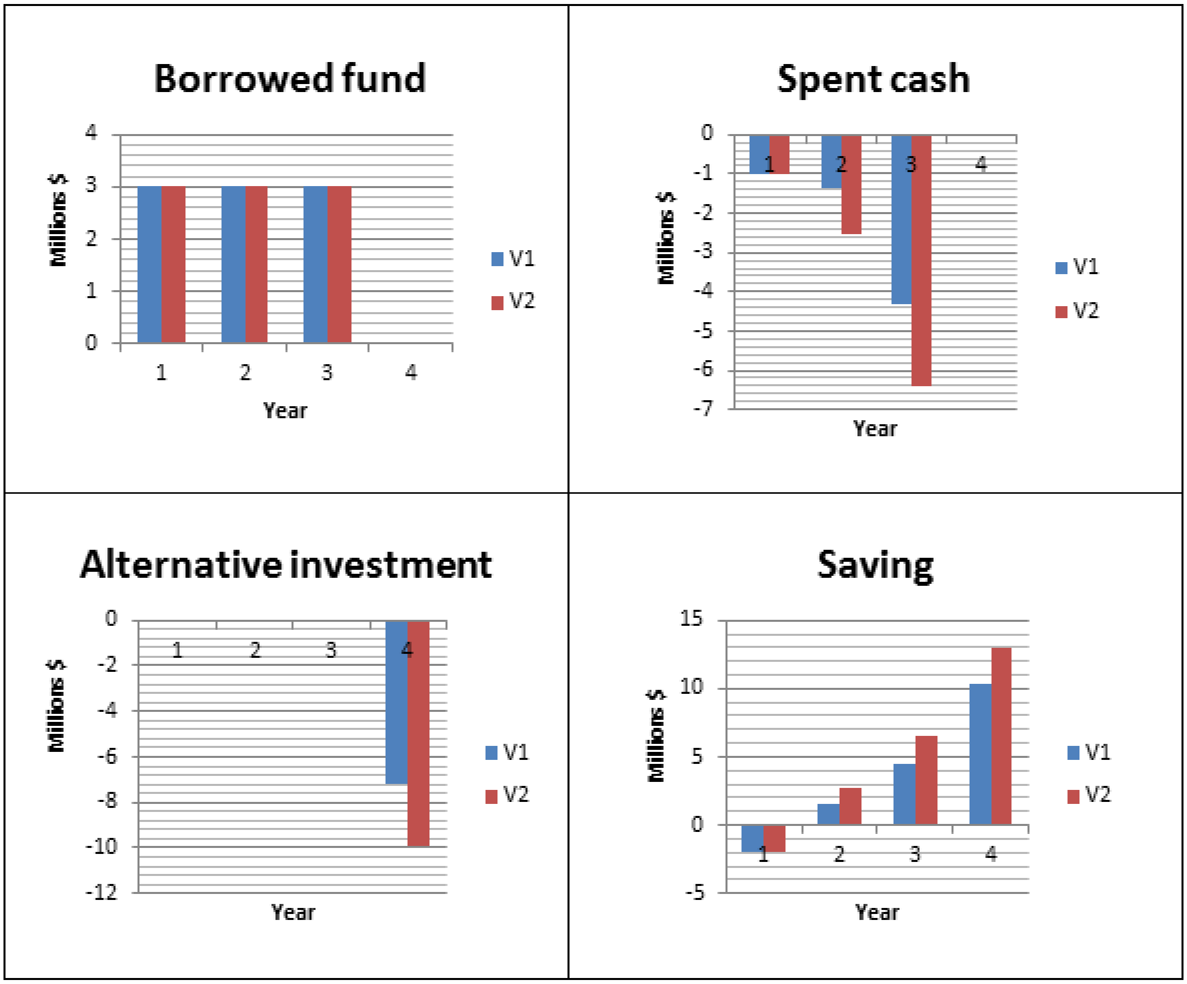

Figure 2-9 : Average financial activity over the course of investment horizon (regions V1 and V2)

Following observations are made.

i. Higher vulnerability (i.e. V_2) yields in more savings from micro-grid.

ii. More cash is spent to purchase assets in V_2,

iii. Highest micro-grid saving and highest investment in alternative project is observed for V_2.

2.4.4 Illustrative Example (III): Impact of Natural Gas Price Stochastic Process (Mean-reverting vs. Non Mean-reverting)

Natural gas price plays an important role in our investment results and modeling this uncertain variable with different stochastic processes, modifies the long-term decisions. We already modeled the natural gas price with mean reverting stochastic (OU) process. In this section, we model the natural gas price by non-mean reverting Geometric Brownian Motion (GBM) process with deterministic positive mean growth rate and random volatility. The GBM process satisfies the following stochastic differential Equation.

$$dS_t = \mu\, S_t + \sigma S_t\, dW_t \quad (2.59)$$

where S_t is natural gas price ($/mmBtu), μ is natural gas yearly drift, σ is the natural gas yearly volatility and W_t is standard Brownian Motion and $dW_t = e\sqrt{dt}$ and e is standard normally distributed. We assume that natural price follows a (GBM) with following parameters.

Table 2-10 : GBM parameters

Natural gas price drift (μ)	0.06
Natural gas price volatility (σ)	0.04

We are interested to see how modeling natural gas price with (GBM) and (OU) processes, differs the micro-grid assets optimal capacities. In this section, we make the following assumption; (i) investment region is V_1 (ii) loss category is L_3 (iii) electricity price and natural gas price are independent. Figure 2-10 demonstrates the average micro-grid incremental capacities for (GBM) and (OU) processes used to model natural gas price.

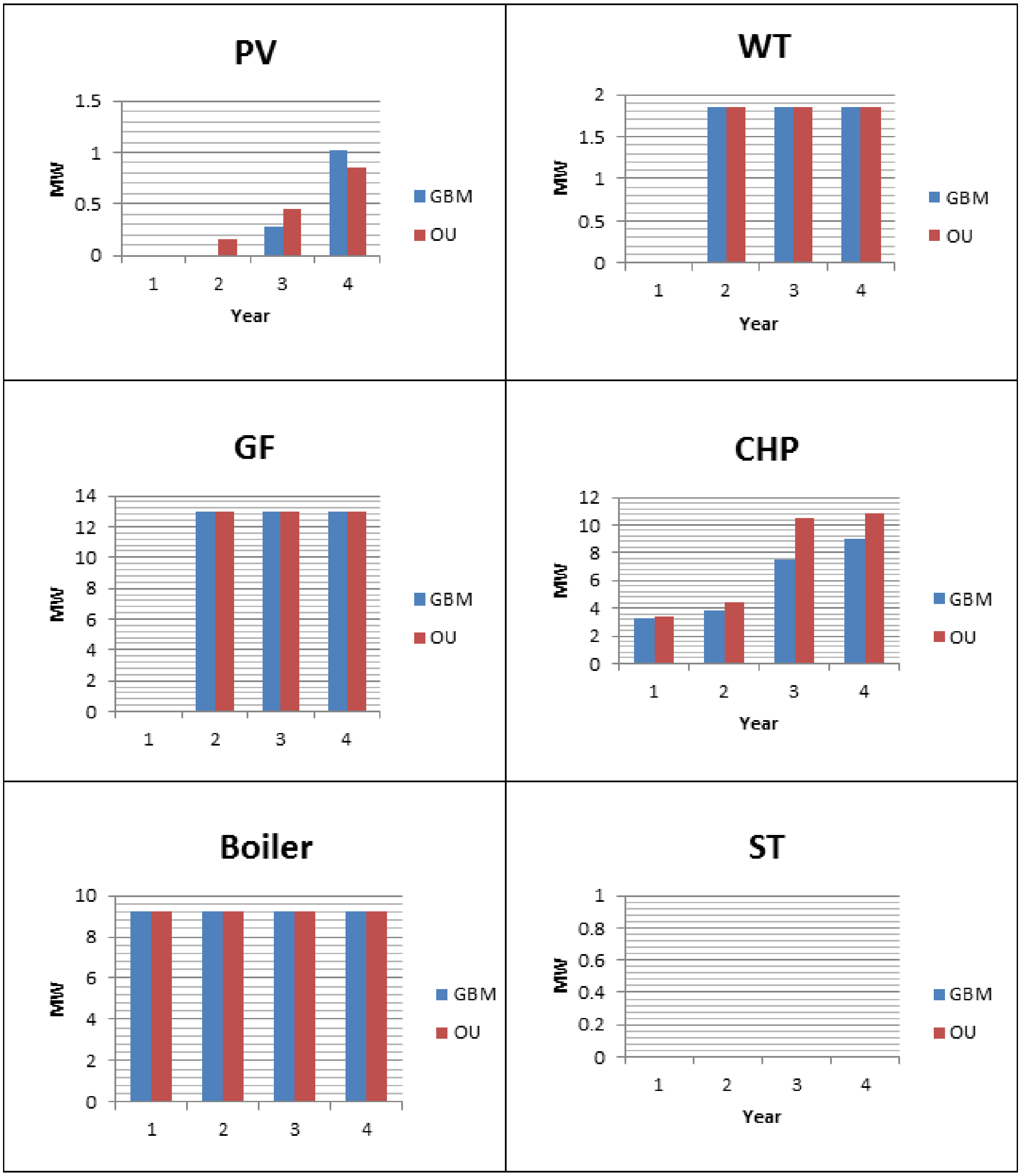

Figure 2-10 : Average micro-grid assets optimal capacity; gas price modeled by (OU) and (GBM)

Comparing the micro-grids assets incremental capacities, following observations are made.

i. Modeling natural gas price by (GBM) process with positive mean growth rate increases attraction of PV and decreases attraction of gas fueled CHP due to higher input fuel cost.

ii. GF has similar capacities in both cases, which can be justified by GF low capital cost.

Moreover, the average financial activities are presented in Figure 2-11.

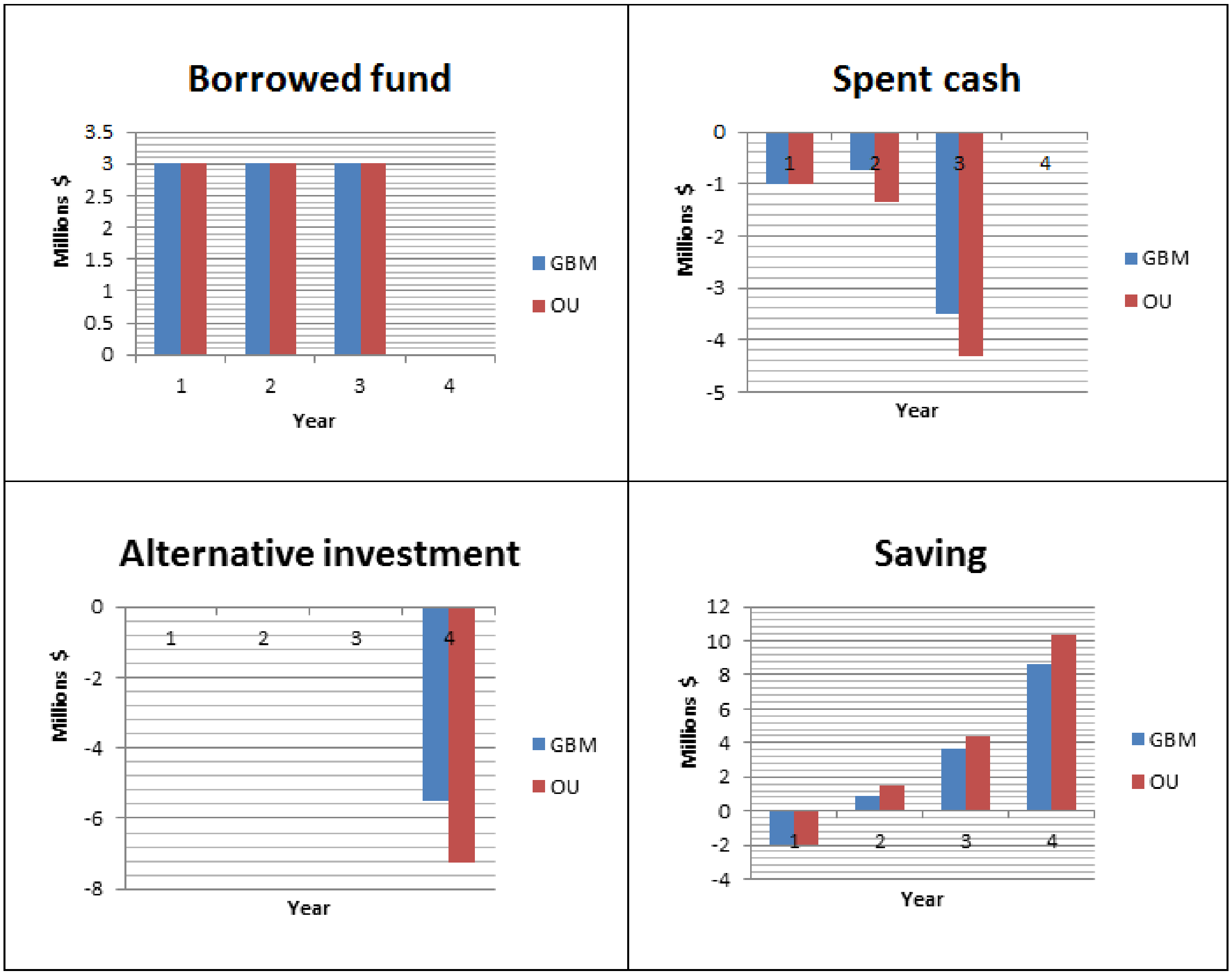

Figure 2-11 : Average financial activity over course of investment horizon; natural gas price modeled by (OU) and (GBM)

Comparing the results for (OU) and (GBM) processes, the following observations are made.

i. Due to the independency between gas and electricity prices in this section, savings from micro-grid decreases when average gas price has increasing trend (GBM process).

ii. More cash is spent and more funds are invested in alternative project when gas price is modeled by (OU).

2.5 Conclusion

In this work, a framework was developed for planning decisions for a portfolio of micro-grid assets, which enforces hedging mechanisms for risks due to infrastructure failure. The proposed framework uniquely merges and optimizes short-term day-to-day volatile operation (under normal and stresses out conditions) of the community's distributed energy resources, and long-term investment decisions that are subject to price and market uncertainties and power failure risks. It was shown that the model is more valuable once applying for regions with higher risk of power failure. Sensitivity of capital budgeting decisions and incremental micro-grid portfolio capacity enhancements to the probability of power failure and economic loss due to power outage are demonstrated. Moreover, it was shown that using mean reverting (OU) and non-mean reverting (GBM) stochastic processes to model natural gas price yields in different micro-grid assets capacity selection and cash flow position at the investment horizon.

3 Investment in Hydrogen Tri-generation for Wastewater Treatment Plants under Uncertainties

In this article, we present a compound real option model for investment in hydrogen tri-generation and onsite hydrogen dispensing systems for a wastewater treatment plant under price and market uncertainties. The ultimate objective is to determine optimal timing and investment thresholds to exercise initial and subsequent options such that the total savings are maximized. Initial option includes investment in a 1.4 (MW) Molten Carbonate Fuel Cell (MCFC) fed by mixture of waste biogas from anaerobic digestion and natural gas, along with auxiliary equipment. Produced hydrogen in MCFC via internal reforming, is recovered from the exhaust gas stream using Pressure Swing Adsorption (PSA) purification technology. Therefore the expansion option includes investment in hydrogen compression, storage and dispensing (CSD) systems which creates additional revenue by selling hydrogen onsite in retail price. This work extends current state of investment modeling within the context of hydrogen tri-generation by considering: (i) Modular investment plan for hydrogen tri-generation and dispensing systems, (ii) Multiple sources of uncertainties along with more realistic probability distributions, (iii) Optimal operation of hydrogen tri-generation is considered, which results in realistic saving estimation.

3.1 Introduction

Wastewater treatment facilities are necessarily located close to the waterfront in order to discharge the treated wastewater and also for more efficient sludge handling. This waterfront geographical requirement increases likelihood of being affected and damaged during extreme weather conditions. It is no secret that during the super-storm Sandy, many wastewater treatment plants were shut down for several days due to power failure, creating major safety problems for the surrounding communities [27]. Along with high risk of failure during extreme weather conditions, other factors such as need for reliable power supply for continues normal operation and generation of considerable amount of waste energy through treatment processes especially anaerobic digestion are the main motivations to consider wastewater treatment plants for energy resiliency. As demonstrated in a number of recent research works, hydrogen tri-generation technology can be a proper solution towards achieving these goals, and can significantly enhance resiliency of wastewater treatment plants and provide additional source of revenue [28]. Hydrogen tri-generation system which simultaneously produces heat, power and hydrogen can be fed by a mixture of natural gas and facility's waste products. Along with profit from onsite supplying heat and power demands, additional profit could be achieved through onsite hydrogen dispensing for local hydrogen vehicles use. Therefore, hydrogen tri-generation might be also a solution to overcome the challenge of initial investment cost of hydrogen early infrastructure deployment [29]. Given the aforementioned distributed energy generation asset, this article develops tools and models that can be used to optimally plan for investing in energy resilient wastewater treatment facilities that can also generate revenue. The particular problem addressed in this chapter is to define the optimal investment timing and thresholds to invest in a hydrogen tri-generation system and option for expansion to provide onsite hydrogen dispensing. The model optimally enforces hedging mechanisms for risks against existing market and price uncertainties, under optimally planned operation of assets.

Since hydrogen tri-generation technology is in its infancy, there is a lack of comprehensive research on capital planning and investment in these assets. Li and Ogden [30] developed an analytic tool to identify the optimal design and evaluate the economic and environmental performance of hydrogen tri-generation systems for home and neighborhood scale hydrogen refueling, and power and heat demand supply. Their approach for economic evaluation of the hydrogen tri-generation is to estimate the levelized cost of one energy product such as electricity while calculating the value of other energy products based on market price. The levelized cost of electricity is the cost of each power unit that would be incurred during the hydrogen tri-generation system life time. In order to find out when a tri-generation system is comparative with conventional systems, the levelized cost of electricity is compared to the price of grid electricity. Li and Ogden [31] extend their work by modeling operation, capacity design and economic analysis of commercial building tri-generation systems for big box store businesses.

Moreover, the U.S. Department of Energy (DOE) has been carrying out research on financial benefits of deploying tri-generation in large scale facilities other than wastewater treatment plants such as hotels and university campuses. A released report of this study includes spread sheet software to deterministically calculate the Net Present Value (NPV) of deploying tri-generation to supply heat and power demands of energy intensive sectors along with hydrogen demand of the local hydrogen vehicles [31, 32]. Their model does not account for any type of short-term volatilities or long-term uncertainties.

In this work, we decompose the similar investment problem and formulate it as a two stage risk based investment plan and solve for optimal timing of each investment stage using compound real option approach by which you can account for uncertainties. The first stage investment aims at self-sustainability and power resiliency of the wastewater treatment facility; however, the objective of the subsequent expansion investment is to create additional revenue stream via onsite hydrogen dispensing for transportation or other applications. The first option includes investment in Molten Carbonate Fuel Cell (MCFC) as a common hydrogen tri-generation system without utilizing hydrogen by-product in a downstream operation. In this stage, along with MCFC stacks, required auxiliary equipment such as hydrogen purification, safety, mechanical, electrical and piping systems are installed. The second option (i.e. expansion option) includes investment in an onsite hydrogen refueling station, which uses hydrogen by-product of the plant to generate additional revenue. In this stage, compression, storage and dispensing (CSD) hardware is added to the facility by which the produced hydrogen will be sold onsite at retail price. Significant difference between hydrogen retail price (as a system output) and natural gas price (as a system input), makes utilization of natural gas to generate hydrogen economically profitable in spite of additional operational cost and natural gas feedstock cost. The initial and expansion options exercise timing is dependent on behavior of stochastic variables and investment options are exercised once the stochastic variables are most favorable to the investor. Initial option exercise time is assumed to be dependent on two stochastic variables as shown below.

[X_1: Natural Gas Price, X_2: MCFC technology cost]

MCFC technology cost includes MCFC stacks cost along with required auxiliary equipment costs. The expansion option exercise time depends on the two following stochastic variables.

[$X_1^{'}$: Hydrogen Price, $X_2^{'}$: Expansion capital cost (CSD hardware)]

Hydrogen price and natural gas price are factors which drive the operational dynamics. By formulating the problem as a compound real option, we are particularly interested in the perpetual American option with exchange a bundle of "*n*" stochastic costs against a bundle of "*m*" assets in both initial and expansion options. The right time to exercise the so called option is classified as an (*n*, *m*) exchange problem [33]. The (1,1) exchange model also referred as "the price and cost uncertainty" was initially developed by McDonald and Siegel [34]. Using the same notation, our initial investment problem is an extended (1,1) exchange problem, where we seek to determine the right time to exercise the investment option with one stochastic cost (i.e. MCFC capital cost in our case), and one stochastic project value (i.e. operational savings of micro-grid in our case). Note that the value of the project is driven by natural gas price, thus it is stochastic. The expansion problem is also an (1,1) exchange where we seek to determine the right time to exercise the expansion option with one stochastic cost (i.e. expansion capital cost in our case), and one stochastic variable (i.e. hydrogen price in our case) defining the project value. In the exchange (1,1) problem, the investment trigger is presented by a line in a 2-D space of stochastic variables (i.e. natural gas price and MCFC technology capital cost for initial option and hydrogen price and expansion capital cost for expansion option in our case).

In order to solve the real option problems, three methods are commonly used: finite difference methods which directly deal with PDE's (e.g. [35]), lattice methods (e.g. [36]), and Monte Carlo simulation based methods (e.g. [37]). Based on [38], closed-form solutions for real option embedded in capital budgeting problems are rarely available. On the other hand, lattice methods for real option valuation suffer the curse of dimensionality. Therefore, simulation is commonly applied to solve the real option problems. Longstaff and Schwartz [39] proposed a simulation based approach for simple real option evaluation namely Least Squares Monte Carlo (LSM). Gamba [38] developed an extension to LSM approach to solve the complex real options with many interacting options. This approach can be applied to solve three main categories of multi-option problems namely mutually exclusive, compound and independent options. In this work, we adopt this methodology to solve our compound real option problem.

This chapter is organized as follows. Section (3-2) presents a brief description of hydrogen tri-generation systems. In section (3-3) problem statement and preliminaries are provided. In section (3-4) dynamics of uncertainty along with details of calculation of hydrogen tri-generation operational saving are presented. Section (3-5) presents details of the investment approach.

Moreover, an illustrative example and sensitivity experiments are presented in sections (3-6) and (3-7). Eventually, conclusion is presented in section (3-8).

3.1.1 Brief Description of Hydrogen Tri-generation

Hydrogen tri-generation which is an efficient high temperature fuel cell capable of simultaneous production of heat, electricity and hydrogen is one of the most recent developments in the context of integrated conversion systems. Key benefits of hydrogen tri-generation include, (i) Near zero emission, (ii) Co-generated hydrogen is a more valuable product than thermal energy, (iii) Efficient generation of hydrogen and electricity reduces the overall operating cost, (iv) Can be fed by a mixture of fuels such as natural gas and waste biogas generated in wastewater treatment plants. Molten Carbonate Fuel Cell (MCFC) is a recent hydrogen tri-generation system introduced to the market [40]. The MCFC has internal reformers and heat from electricity-production efficiency losses is used to produce hydrogen. Hydrogen produced via internal reforming which is not utilized to generate electricity exits the anode in the exhaust stream. This stream contains CO, H_2 and H_2O. The stream is cooled by using Pressure Swing Adsorption (PSA) exhaust and enters the shift catalyst which recovers about 75% of the hydrogen in the gas stream. Then hydrogen is compressed and stored for mainly transportation application. In spite of availability of other hydrogen separation technologies (e.g. electrochemical hydrogen pumping), we consider PSA according to its expected use in first hydrogen tri-generation systems and also market availability [41]. Figure 3-1 schematically demonstrates the concept of MCFC hydrogen tri-generation systems.

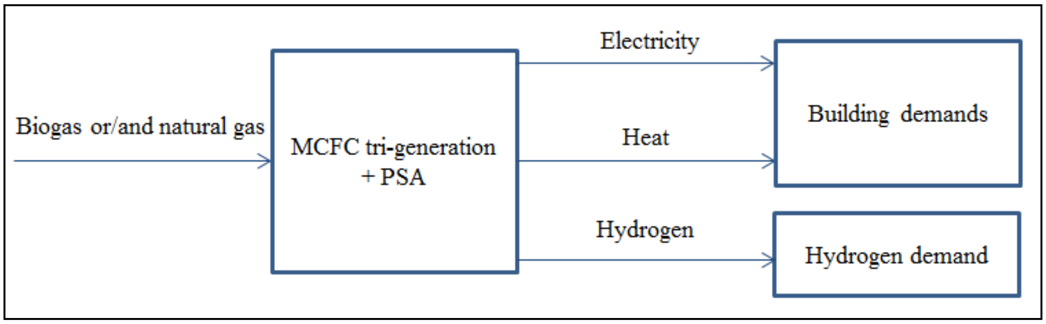

Figure 3-1 : Schematic of MCFC hydrogen tri-generation concept

Moreover, Figure 3-2 is a generalized schematic of the MCFC system including detailed processes.

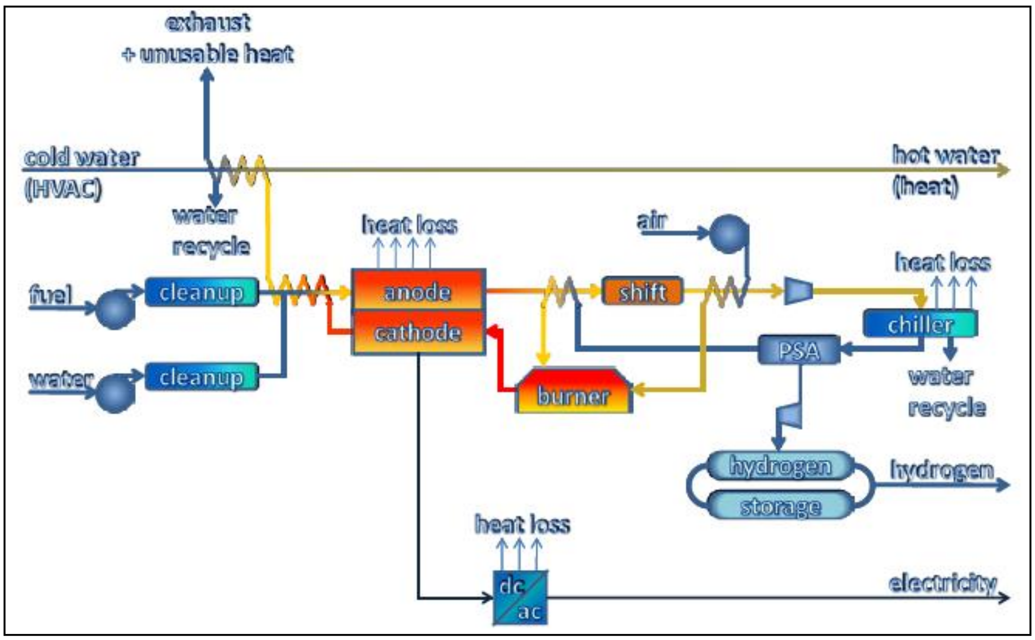

Figure 3-2 : Schematic of modeled MCFC system with detailed processes [41]

3.2 Problem Statement and Preliminaries

In this work, we assume that the facility's electricity demand is supplied by grid and hydrogen tri-generation system. Moreover, the option to sell back the electricity to the grid exists. Boiler and hydrogen tri-generation supply heat demand of the facility. Generated hydrogen is sold onsite at retail price, if hydrogen dispensing hardware is installed; otherwise, hydrogen is transported to demand point and sold at wholesale market price. We find that the optimal investment strategy for such a hydrogen tri-generation system and onsite hydrogen dispensing hardware for a wastewater treatment plant under several sources of uncertainty. We consider the investment as a two stage plan including initial investment option (i.e. purchasing and installation of hydrogen tri-generation and required auxiliary equipment) and expansion option (i.e. purchasing and installation of onsite hydrogen dispensing system). The initial and expansion investment options are categorized as exchange (1,1) problems each under two sources of uncertainties. It is assumed that the capacity configurations of initial and expansion investments are parametrically fixed. However, according to the stage-wise structure of the proposed solution approach, the model is capable to consider capacity enhancement of hydrogen tri-generation as another subsequent option if required. The proposed investment approach, particularly defines optimal investment timing and thresholds to exercise the investment options with pre-defined capacities based on facility and hydrogen demands.

Table 3-1 presents the nomenclature of this work.

Table 3-1 : Nomenclature

Indices	
Y	Year
J	Scenario
T	Time of day
Parameters	
J^*	Number of scenarios
Y^*	Investment horizon
gp	Natural gas price ($/MWh)
$h2\_rp$	Hydrogen retail price ($/MWh)
Exp_Cost	Expansion (Onsite refueling station) capital cost ($)
$MCFC\_Cost$	MCFC stack capital cost ($)
$Saving_{annual}$	Yearly saving of onsite generation ($)
$Saving_{lifetime}$	Cumulative discounted savings during life time of assets ($)

r	Risk free rate of return
gp^*	Natural gas price trigger threshold ($/mmBtu)
$MCFC\_Stack\_Cost^*$	MCFC stack cost trigger threshold (S)
$h2\_rp^*$	Hydrogen price trigger threshold ($/MWh)
Exp_Cost^*	Expansion (Onsite refueling station) capital cost trigger threshold ($)
$ElecDemand$	Electricity demand (MWh)
$HeatDemand$	Heat demand (MWh)
$Sprice$	Electricity spot price ($/MWh)
$Sbprice$	Electricity sell back price ($/MWh)
$ngInput$	Inlet natural gas to MCFC (MWh)
$MCFCouput$	Cumulative power output of MCFC (MWh)
$MCFo\&m$	MCFC operation and maintenance cost ($/MWh)
$H2ouput$	Hydrogen output of MCFC (MWh)
$BoilerEff$	Boiler efficiency
$BoilerGen$	Boiler heat generation (MWh)
Gb	Electricity bought from grid (MWh)
Sb	Electricity sold back to grid (MWh)
ng_inlet	Input natural gas (MWh)
ng_norm	Input natural gas used for normal operation (MWh)
ng_excess	Input natural gas used for hydrogen over-production (MWh)
$MCFC_M$	Maximum energy required for normal operation (MWh)
$OverH2P\_inlet$	Energy used for hydrogen over-production (MWh)
Bg_inlet	Input biogas (MWh)
$MCFC\_norm\_inlet$	Input energy to MCFC for normal operation (MWh)
AH	Heat before conversion to heat and hydrogen (MWh)
$MCFCeg$	MCFC electricity generation (MWh)
$MCFCheat$	MCFC heat generation (MWh)
$MCFCh2$	MCFC hydrogen production (MWh)
$MCFCh2Op$	MCFC hydrogen over-production (MWh)
$MCFCcap$	MCFC capacity (MW)
$H2\_dc$	Hydrogen storage discharge (MWh)
$H2\_c$	Hydrogen storage charge (MWh)
ω	Hydrogen over production efficiency
λ	Maximum hydrogen over-production to hydrogen production ratio
LF	MCFC loading fraction
θ	Heat to hydrogen conversion efficiency
ζ	Maximum fraction of heat convertible to hydrogen
$ACeff$	MCFC power efficiency
$TOTeff$	MCFC total efficiency

3.2.1 Dynamics of Uncertainty

There are four sources of long term uncertainty in our model namely (i) Natural gas price, (ii) Hydrogen price, (iii) MCFC stack capital cost, and (iv) Expansion capital cost including hydrogen compression, storage and dispensing (CSD) system. Modeling the long-term behavior of natural gas price has been extensively studied in the literature. Commonly speaking, natural gas price is modeled by Geometric Brownian Motion (GBM) stochastic process with deterministic mean and random volatility. GBM model exploits the tendency of price to revert around a long-term average cost of production. In order to model the price, either one or two factor schemes can be applied. In the two-factor scheme, the short-term variation of price is also included and therefore a more accurate form of historical data is presented. However, [42] demonstrates that the one factor model (GBM) is accurate enough for long-term investment decisions. Also [43] argues that long-term factors are more effective elements in the long-term decisions. The GBM process to model natural gas price satisfies the following stochastic differential Equation.

$$dS_t = \mu\, S_t + \sigma S_t\, dW_t \quad (3.1)$$

where S_t is natural gas price (\$/mmBtu), μ is natural gas yearly drift, σ is the natural gas yearly volatility and W_t is standard Brownian Motion and $dW_t = e\sqrt{dt}$ and e is standard normally distributed. In order to estimate the process parameters, the model is fitted to the historical data and a linear regression is fitted to the logarithm of the natural gas price and its first difference to find out the drift and volatility values of the GBM process.

Hydrogen price is another stochastic variable in the model. Hydrogen price can also be modeled by Geometric Brownian Motion (GBM) process with increasing trend [44]. Since we did not have access to proper real data, we made our best guesstimates on the drift and volatility of the hydrogen

GBM process. The capital cost of investment on recently developed Molten Carbonate fuel cell (MCFC) and hydrogen refueling station hardware are subject to great uncertainty. Due to lack of data on capital cost of these assets, we assume a decreasing trend according to a GBM. Monte Carlo sampling technique is applied to generate samples of the stochastic variables.

Moreover, it is assumed that the distributed energy system is subject to two sources of short-term variation: (i) variation in electricity spot price, (ii) variation in demand. It is assumed that the peak electricity price on each day is correlated to natural gas price as follows.

$$Peak\ electrcity\ price\ = Natural\ gas\ price * Ratio * \ \epsilon_{Grid} \quad (3.2)$$

where *Ratio* accounts for transmission cost at grid level and ϵ_{Grid} is the grid average rate for electricity generation from natural gas. Assuming a daily profile for daily electricity spot price as a percentage of peak prices, hourly electricity prices over the course of a day is obtained. Figure 3-3 shows the electricity spot price profile as percentage of peak price.

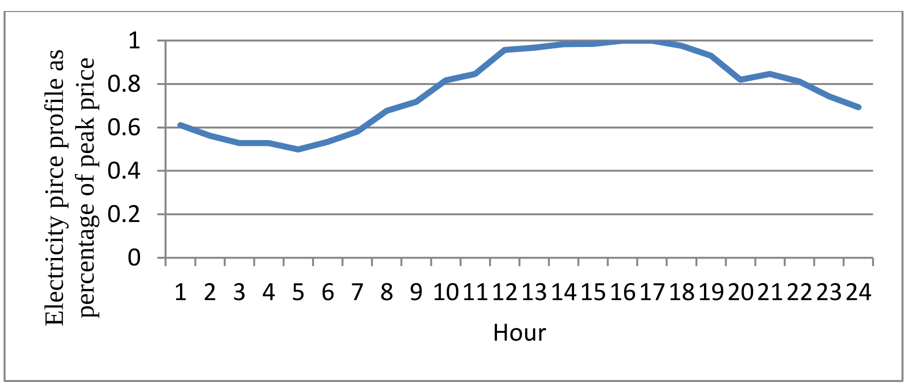

Figure 3-3 : electricity spot price profile as percentage of peak price

Moreover, it is assumed that electricity and heat demands of the facility in each hour have distributions with known mean and variance.

3.3 Operational Saving Estimation: Stochastic Operation Optimization Model

Savings are estimated for two modes; hydrogen tri-generation with and without onsite hydrogen dispensing. The difference of these two modes is selling procedure of the generated hydrogen. Once the initial investment option is exercised, the produced hydrogen will be transported to the demand point by truck and sold at wholesale market price. Hydrogen delivery cost is the fuel and labor cost of driving to and from the station [45]. However, by exercising the expansion option, hydrogen will be sold onsite at retail price. Thereafter, the operational saving for hydrogen dispensing system is the difference between these two saving values. In order to calculate the savings, a short term operation optimization framework is developed based on "*Fuel Cell Power*" model released by National Renewable Energy Laboratory (NREL). The "*Fuel Cell Power*" model simulations for tri-generation systems are created using a two-step process. In the first step, thermodynamically correct tri-generation system designs are developed using ASPEN Plus (i.e. optimization software for chemical processes). Then these models are used to develop simplified linear models of system performance. It has been demonstrated that under certain conditions, "*Fuel Cell Power"* results approximate the ASPEN Plus results [41]. In our work, the "*Fuel Cell Power*" model is modified to a generic stochastic operation optimization model. We assume that MCFC could be fed with a mixture of biogas and natural gas and the optimization model is solved under different scenarios of demand and prices of hydrogen, natural gas and electricity which are generated using Monte Carlo sampling technique. Thereafter, the path-wise savings of the system is calculated and fed to the investment model. In order to calculate operational savings, we consider similar days over a year which is an abstraction of reality and it only simplifies our computations. Therefore, the yearly saving is simply 365 times the daily saving.

As stated earlier, MCFC systems capable of production of electricity, heat and hydrogen can be fed by natural gas and/or waste biogas. The capacity of MCFC system could be selected such that the generated biogas would be sufficient for its operation. However, we select MCFC with a higher capacity which is close to the average daily power demand of the plant under consideration to be fed by mixture of biogas, and natural gas according to the following reasons.

i. Our main objective of installing MCFC is energy resiliency and power reliability of wastewater treatment plants, especially in stressed out conditions (i.e. natural disasters/technical issues occurrence which results in power grid failure). Therefore we select the MCFC capacity such that the produced electricity maximally supplies the plant's electricity demands. Typically, the amount of generated biogas in the wastewater treatment plants is not sufficient to satisfy electricity demands of the plant if solely used in MCFC system. Hence, we consider natural gas as another component in the feedstock for the MCFC system. It is also worth noting that MCFC systems operate with higher efficiency once the loading fraction is higher, therefore the addition of natural gas to the system inputs, increases the overall efficiency.

ii. According to existence of the opportunity to sell electricity back to the grid, utilizing natural gas for extra electricity generation and selling it back to the grid, is economically profitable. It is worth noting that assuming positive correlation between the natural gas price, peak electricity price, consequently hourly electricity purchase and sell-back prices maintains a profit margin for natural gas usage in MCFC regardless of natural gas price fluctuations.

iii. From the hydrogen economy point of view, although extra hydrogen generation with higher capacity MCFC has very low profit margin if sold in wholesale price; however, hydrogen becomes a very economically profitable output from the MCFC system once the onsite hydrogen dispensing is installed. Onsite hydrogen dispensing systems creates the opportunity to sell hydrogen in retail price which is much higher than natural gas price.

The power efficiency of the MCFC system is a function of loading fraction and is defined as the ratio of generated power to inlet fuel power. Moreover, the total efficiency of the MCFC system is defined as the aggregate outlet heat and power divided by the total energy inlet. The system is also capable of hydrogen over-production which is achieved by inletting extra fuel. The efficiency of hydrogen over-production is defined as the ratio of hydrogen overproduction to the excess fuel input.

The inlet natural gas is expressed as aggregation of two terms namely "ng_excess" and ng_norm" that define the amount of natural gas input utilized for the normal operation and hydrogen overproduction respectively.

$$ng\_inlet_{(Y,J,T)} = ng\_norm_{(Y,J,T)} + ng\_excess_{(Y,J,T)} \quad (3.3)$$

$$Y = 1, \dots, Y^* \; \& \; J = 1, \dots, J^* \; \& \; T = 1, \dots, 24$$

Generally, two operational conditions could occur as below.

1 - Available energy from biogas exceeds the maximum energy required for normal operation;

(i) MCFC works in its maximum fuel input mode:

$$MCFC\_norm\_inlet_{(Y,J,T)} = MCFC_M \quad (3.4)$$

$$Y = 1, \dots, Y^* \; \& \; J = 1, \dots, J^* \; \& \; T = 1, \dots, 24$$

where $MCFC_M$ is the maximum energy required for normal operation.

(ii) No natural gas is utilized for normal operation:

$$ng\_norm_{(Y,J,T)} = 0 \quad (3.5)$$

$$Y = 1, \dots, Y^* \;\&\; J = 1, \dots, J^* \;\&\; T = 1, \dots, 24$$

(iii) Excess biogas energy and natural gas are available for hydrogen overproduction:

$$OverH2P\_inlet_{(Y,J,T)} = \; Bg\_inelt_{(Y,J,T)} - MCFC_M + \; ng\_excess_{(Y,J,T)} \quad (3.6)$$

$$Y = 1, \dots, Y^* \;\&\; J = 1, \dots, J^* \;\&\; T = 1, \dots, 24$$

2 - Available energy from biogas is less than the energy required for normal operation;

(i) Mixture of biogas and natural gas are fed to MCFC for normal operation:

$$MCFC\_norm\_inlet_{(Y,J,T)} \; = \; Bg\_inlet_{(Y,J,T)} + \; ng\_norm_{(Y,J,T)} \quad (3.7)$$

$$Y = 1, \dots, Y^* \;\&\; J = 1, \dots, J^* \;\&\; T = 1, \dots, 24$$

(ii) Hydrogen overproduction will take place if excess natural gas is fed to the system:

$$OverH2P\_inlet_{(Y,J,T)} = ng\_excess_{(Y,J,T)} \quad (3.8)$$

$$Y = 1, \dots, Y^* \& J = 1, \dots, J^* \;\&\; T = 1, \dots, 24$$

In this work the capacity of MCFC is selected such that its maximum input fuel is always greater than the generated waste biogas. Therefore, natural gas is the main input fuel and case (1) never happens. In both cases (1) and (2), $MCFC\_inlet$ is the amount of energy that is used just for normal operation, hence should not be greater than the maximum energy input to the system for normal operation which is specified by the manufacturer.

$$MCFC\_norm\_inlet_{(Y,J,T)} \leq \; MCFC_M \qquad (3.9)$$

$$Y = 1, \dots, Y^* \;\&\; J = 1, \dots, J^* \;\&\; T = 1, \dots, 24$$

The MCFC system can overproduce hydrogen, if the system is already in the full capacity mode. To ensure that the system will not overproduce hydrogen when it is not in the full capacity mode the following constraints are added using “Big M” method. The “Big M” method is a variation of the simplex method designed for solving problems encompassing either “less-than” or “greater-than" constraints. The "Big M" is assumed to be an extremely large number associated with artificial variables represented by M. More details of this method are presented in chapter 2.

$$ng\_norm_{(Y,J,T)} - \left[MCFC_M - Bg_{inlet\,(Y,J,T)} \right] \leq bin_{(Y,J,T)} * M \qquad (3.10)$$

$$ng\_norm_{(Y,J,T)} - \left[MCFC_M - Bg_{inlet\,(Y,J,T)} \right] > \left(bin_{(Y,J,T)} - 1 \right) * M \qquad (3.11)$$

$$Y = 1, \dots, Y^* \;\&\; J = 1, \dots, J^* \;\&\; T = 1, \dots, 24$$

where bin is a binary variable and M is big positive number. The binary variable bin takes zero once the system is in normal operation mode and takes one when system is in hydrogen over-production mode. Using bin and adding the following constraint, the energy from natural gas used for hydrogen overproduction would be zero if system is working under full capacity mode.

$$ng\_excess_{(Y,J,T)} \leq bin_{(Y,J,T)} * M \qquad (3.12)$$

$$Y = 1, \dots, Y^* \;\&\; J = 1, \dots, J^* \;\&\; T = 1, \dots, 24$$

The amount of hydrogen over-production is calculated by multiplying the available energy for hydrogen over-production in hydrogen over-production efficiency (ω).

$$MCFCh2Op_{(Y,J,T)} = OverH2P\_inlet_{(Y,J,T)} * \omega \qquad (3.13)$$

$$Y = 1, \dots, Y^* \;\&\; J = 1, \dots, J^* \;\&\; T = 1, \dots, 24$$

Hydrogen overproduction is constrained based on the fuel cell specifications. Hydrogen over-production cannot exceed a defined ratio of produced hydrogen (λ). Therefore,

$$MCFCh2Op_{(Y,J,T)} \leq \lambda * MCFCh2_{(Y,J,T)} \qquad (3.14)$$

$$Y = 1, \dots, Y^* \;\&\; J = 1, \dots, J^* \;\&\; T = 1, \dots, 24$$

where λ is the maximum ratio of hydrogen over-production to production ratio.

Since the MCFC power and overall efficiency at each time step is dependent on the portion of the full capacity which is loaded, we calculate the loading fraction (*LF*) as below.

$$LF_{(Y,J,T)} = \frac{MCFC\_norm\_inlet_{(Y,J,T)}}{MCFC_M} \quad (3.15)$$

$$Y = 1, \dots, Y^* \;\&\; J = 1, \dots, J^* \;\&\; T = 1, \dots, 24$$

By defining *LF*, overall and power efficacies could be extracted from the tables provided by "*Fuel Cell Power*" model (i.e. overall and power efficacies vs. loading fraction). Then MCFC electricity generation is calculated as below.

$$MCFCeg_{(Y,J,T)} = ACeff_{(Y,J,T)} * MCFC_{norm_{inlet}\,(Y,J,T)} \quad (3.16)$$

$$Y = 1, \dots, Y^* \;\&\; J = 1, \dots, J^* \;\&\; T = 1, \dots, 24$$

Moreover, according to fuel cell operational constraints, generated electricity in each time step cannot vary more than a defined limit from previous time step electricity production. The electricity variation limit is defined by multiplication of a ratio namely "AC Response Time (ACRT)" in the MCFC capacity. Therefore,

$$MCFCeg_{(Y,J,T)} \leq MCFCeg_{(Y,J,T-1)} + ACRT * MCFCcap \quad (3.17)$$

$$MCFCeg_{(Y,J,T)} \geq MCFCeg_{(Y,J,T-1)} - ACRT * MCFCcap \quad (3.18)$$

$$Y = 1, \dots, Y^* \;\&\; J = 1, \dots, J^* \;\&\; T = 2, \dots, 24$$

By calculating the MCFC electricity generation and overall efficiency of the system, the amount of available energy (AH) which can be either outlet as heat or used to generate hydrogen in fuel cell is defined.

$$AH_{(Y,J,T)} = \left[TOTeff_{(Y,J,T)} * MCFC_{norm_{inlet\,(Y,J,T)}}\right] - MCFCeg_{(Y,J,T)} \quad (3.19)$$

$$Y = 1, \dots, Y^* \;\&\; J = 1, \dots, J^* \;\&\; T = 1, \dots, 24$$

According to "*Fuel Cell Power*" model, there is a maximum fraction of heat convertible to hydrogen (ζ). Therefore,

$$\frac{MCFCh2_{(Y,J,T)}}{\theta} \leq AH_{(Y,J,T)} * \zeta \qquad (3.20)$$

$$Y = 1, \dots, Y^* \;\&\; J = 1, \dots, J^* \;\&\; T = 1, \dots, 24$$

where θ is the heat to hydrogen conversion efficiency and ζ is the maximum fraction of heat convertible to hydrogen.

Similar to electricity hourly production, hourly hydrogen generation is constrained by a limit which is defined by multiplication of a ratio namely "Hydrogen Response Time (HRT)" in the MCFC capacity. Therefore,

$$MCFCh2_{(Y,J,T)} \leq MCFCh2_{(Y,J,T-1)} + (HRT * MCFCcap) \qquad (3.21)$$

$$MCFCh2_{(Y,J,T)} \geq MCFCh2_{(Y,J,T-1)} - (HRT * MCFCcap) \qquad (3.22)$$

$$Y = 1, \dots, Y^* \;\&\; J = 1, \dots, J^* \;\&\; T = 2, \dots, 24$$

Therefore, the available energy can be expressed as aggregation of heat and hydrogen produced as below.

$$AH_{(Y,J,T)} = \frac{MCFCh2_{(Y,J,T)}}{\theta} + MCFCheat_{(Y,J,T)} \qquad (3.23)$$

$$Y = 1, \dots, Y^* \;\&\; J = 1, \dots, J^* \;\&\; T = 1, \dots, 24$$

Moreover, the hourly generated electricity by MCFC along with the electricity bought from the grid must meet the demand plus any possible electricity sellback to the grid.

$$MCFCeg_{(Y,J,T)} + \; Gb_{(Y,J,T)} \geq \; Sb_{(Y,J,T)} + ElecDemand_{(Y,J,T)} \quad (3.24)$$

$$Y = 1, \dots, Y^* \;\&\; J = 1, \dots, J^* \;\&\; T = 1, \dots, 24$$

Boiler and heat from MCFC must meet the heat demand of the facility.

$$MCFCheat_{(Y,J,T)} + BoilerGen_{(Y,J,T)} \geq \; HeatDemand_{(Y,J,T)} \quad (3.25)$$

$$Y = 1, \dots, Y^* \;\&\; J = 1, \dots, J^* \;\&\; T = 1, \dots, 24$$

In addition, once the hydrogen dispensing system is installed, the generated hydrogen at each time step could be either used to fuel hydrogen vehicles or charged to the hydrogen storage. Stored hydrogen could also be discharged to fuel hydrogen vehicles if required. Therefore,

$$MCFCh2_{(Y,J,T)} + \; MCFCh2Op_{(Y,J,T)} + H2_{dc\,(Y,J,T)} \geq \; H2_{c\,(Y,J,T)} \; + H2Demand_{(Y,J,T)} \quad (3.26)$$

$$Y = 1, \dots, Y^* \;\&\; J = 1, \dots, J^* \;\&\; T = 1, \dots, 24$$

If hydrogen refueling station is not in place, there is no constraint to meet hydrogen demand.

The initial investment path-wise annual saving ($Saving_{annual_{(Y,J)}}$) is defined as the difference between savings with and without hydrogen tri-generation system in each scenario.

$$Saving_{annual_{(Y,J)}} = 365 * \sum_{T=1}^{24} \left(C_{nh,(Y,J,T)} - \; C_{wh,(Y,J,T)} \right) \quad (3.27)$$

where C_{nh} is the total cost of supplying heat and power demands with no hydrogen tri-generation capacity and C_{wh} is the total cost of supplying heat and demand by hydrogen tri-generation capacity less the revenue from selling hydrogen and selling back electricity to the grid. Mathematically speaking $C_{nh(Y,J,T)}$ is as calculated as below.

$$C_{nh(Y,J,T)} = ElecDemand_{(Y,J,T)} * Sprice_{(Y,J,T)} + HeatDemand_{(Y,J,T)} * \, {}^{gp_{(Y,J)}}\!/_{BoilerEff}$$

$$Y = 1, \dots, Y^* \ \& \ J = 1, \dots, J^* \ \& \ T = 1, \dots, 24 \qquad (3.28)$$

Also, C_{wh} in scenario J and time T and Year Y is defined as below.

$$C_{wh,(Y,J,T)} = \ ngInput_{(Y,J,T)} * \ gp_{(Y,J)} - \ Sb_{(Y,J,T)} * \ Sbprice_{(Y,J,T)} + \ Gb_{(Y,J,T)} * \ Sprice_{(Y,J,T)} + MCFCouput_{(Y,J,T)} * \ MCFCo\&m - H2output_{(Y,J,T)} * \ H2Price_{(Y,J,T)} \ + BoilerGen_{(Y,J,T)} * \, {}^{gp_{(Y,J)}}\!/_{BoilerEff} \quad Y = 1, \dots, Y^* \ \& \ J = 1, \dots, J^* \ \& \ T = 1, \dots, 24 \qquad (3.29)$$

Where $MCFCo\&m$ includes MCFC and hydrogen purification operation and maintenance cost [46]. Moreover:

$$MCFCouput_{(Y,J,T)} = MCFCeg_{(Y,J,T)} + MCFCh2_{(Y,J,T)} + MCFCh2Op_{(Y,J,T)} + MCFCheat_{(Y,J,T)} \qquad (3.30)$$

$$H2output_{(Y,J,T)} = \ MCFCh2_{(Y,J,T)} + \ MCFCh2Op_{(Y,J,T)} \qquad (31)$$

$$Y = 1, \dots, Y^* \ \& \ J = 1, \dots, J^* \ \& \ T = 1, \dots, 24$$

Following similar logic for calculation of savings, the expansion investment path-wise annual saving is defined as the difference between savings with and without a hydrogen dispensing system in each scenario. In the case of initial investment the produced hydrogen is sold as whole sale price; however, by the addition of an onsite dispensing system hydrogen is sold at a retail price. Therefore, in the initial case the $H2Price$ equals the wholesale hydrogen price less the transport cost; however, in the expanded case, the $H2Price$ equals the hydrogen retail price.

The path-wise initial and expansion savings along with simulated sample paths of the initial and expansion capital costs are used in the long-term investment model which defines optimal timing to invest in the assets. In the investment model, the optimal investment timing strategy is the one which maximizes the discounted cumulative cash flows over the life time of the assets. Hence, the present value of hydrogen tri-generation and onsite hydrogen dispensing savings during their lifetimes at the time of investment is calculated by discounting the savings cash flow with a discount rate of r, as shown below.

$$Saving_{lifetime} = \sum_{Y=1}^{lifetime} \frac{Saving_{annual}}{(1+r)^{Y-1}} \quad (3.32)$$

where $Saving_{lifetime}$ is the present value of total savings during lifetime of the asset. We use this formula for both initial and expansion investments.

3.4 Monte Carlo Simulation for Real Option

This section includes a short synopsis of real option and methodology, followed by representation of the algorithm. Since traditional Net Present Value (NPV) approach is not capable to handle uncertainty, the real option approach adopted from option theory in finance is applied to solve capital budgeting problems under uncertainty. Formally speaking, “Real option” is the right (option) to undertake certain business initiatives such as contracting an investment, abandoning, expanding and deferring [46]. The main advantage of real option approach is the opportunity to delay an investment by which more information for better decisions would be revealed. Therefore, the investors will make the investment if the stochastic variables values are favorable, otherwise the option to invest would not be exercised [47, 48].

Longstaff and Schwartz introduced Least Squares Monte Carlo (LSM) technique to solve the real option problems by simulation [40]. Using this method, at each exercise time, the option holder decides whether to keep the option alive or to immediately exercise the option. The exercise strategy is determined by comparing the immediate payoff from exercising the option and the conditional expectation of payoff from keeping the option alive. The conditional expectation is estimated from the cross sectional information in the simulated paths using least squares regression. To do so, the subsequent realized payoffs from continuation are regressed on the values of state variables. This function is then applied to determine the conditional expectation of continuation at each exercise time. Backward dynamic programming is applied to solve for the optimal investment timing. The calculation is carried out for each sample path and conditional distributions of investment thresholds are determined.

According to the composite structure of our investment strategy, simple real option models are not applicable and the investment problem can be categorized as a compound real option. Compound real options are combination of real options, where an exercise of a real option opens another real option and are mostly used in research and development [49]. In order to solve for compound real options, Gamba [38] presents an extension to the Least Squares Monte Carlo (LSM) approach which is based on the concept that in the compound real option, the value of the initial claim also depends on the value of the subsequent one.

3.4.1 Methodology

Assume that there are two states variables defined by $X = (X_{gp}, X_{MCFC\_Stack\_Cost})$, where X_{gp} is the natural gas price and $X_{MCFC\_Stack\_Cost}$ is the MCFC stack capital cost. The investor has the option to invest in hydrogen tri-generation, with maturity date of T and payoff $\Pi\ (T, X_t)$. Let $F\ (T, X_t)$ be the value of the option at $t\ \leq T$. In case of the American option we have:

$$F(t, X_t) = \ max_{\zeta \epsilon \Gamma(t,T)} \left\{ e^{-r(\zeta - t)} E_t^* [\Pi(\zeta, X_\zeta)] \right\} \qquad (3.33)$$

where $E_t^*[.]$ is the expectation conditional on available information at time step *t* and $\Gamma(t,T)$ is the set of possible stopping times. Given the valuation problem of an American claim contingent on *X* and expiration date of *T*, an approximation of the value is defined by selecting an integer number *N* and dividing the time span [0,T] to *N* intervals, with the length of an interval equal to $\Delta t = T/N$. Afterwards, we generate *K* simulated paths of stochastic process $\{X_t\}$ and denote $X_t(\omega)$ the value of the process at time *t* along the ω-th path and $\zeta(\omega)$ the path-wise stopping time with respect to the information generated by $\{X_t\}$. The objective is to find the optimal exercise date restricted to the following set.

$$\{t_0 = 0, t_1 = \Delta t, \dots, t_N = N\Delta t\} \quad (3.34)$$

The optimal policy regarding exercising the option is determined using backward dynamic programming. If at time t_N, along the path ω, the option is still alive, the decision is made by comparing the payoff $\Pi(t_N, X_t(\omega))$ with $(t, X_t(\omega))$. In case of our problem, the initial option payoff is defined as the discounted saving of hydrogen tri-generation over the life time of the asset minus the initial capital cost (i.e. cost of MCFC stacks and required auxiliary equipment).

$$\Pi\ (t_n, X_t(\omega)\ ) = Saving_{lifetime} - X_{MCFC\_Stack\_Cost_t}(\omega) \quad (3.35)$$

Likewise, expansion investment payoff is defined as the discounted savings from onsite hydrogen dispensing over the life time of the system less the initial capital cost of investment. Expansion investment payoff will be used later in the expansion investment payoff consideration. Thereafter, the stopping time is defined as below.

$$\zeta = inf\ \{t | F(t, X_t) = \Pi(t, X_t)\} \quad (3.36)$$

Using the Bellman equation $F(t, X_t)$ is calculated as below.

$$F(t, X_t) = \max\{\Pi(t_n, X_{t_n}), e^{-r(t_{n+1}-t_n)} E^*_{t_n}[F(t_{n+1}, X_{t_{n+1}})]\} \quad (3.37)$$

The path-wise optimal policy restricted to given dates can be computed by comparing the continuation value.

$$\theta(t_n, X_{t_n}) = e^{-r(t_{n+1}-t_n)} E^*_{t_n}\left[F\left(t_{n+1}, X_{t_{n+1}} \middle| F_{t_n}\right)\right] \quad (3.38)$$

with the payoff $\Pi(t_n, X_{t_n})$. For the case of our problem, $\theta(t_n, X_{t_n})$ is the expected value of savings less capital cost of investment, if the investor invests in one of the following years of t_n. The decision rule to find out the stopping time for a simple real option problem denoted $\zeta(\omega)$ at t_n on the ω -th simulated path is as follows.

$$\text{If } \theta\,(t_n, X_{t_n}(\omega)\,) \leq \Pi\,(t_n, X_{t_n}(\omega)\,) \text{ the } \zeta\,(\omega) = t_n \quad (3.39)$$

At $t_n = T$, since the option is expiring, $\theta\left(t_n, X_{t_n}\right) = 0$ and the rule is to exercise the option if the payoff is positive. At any t_n, the optimal stopping time is found by recursively applying the decision rule in (3.38), from $t_n = T$ back to t_n. If at some previous step of this procedure, $(\omega) > t_n$, and condition (3.38) holds at the current step, then the stopping time along the path is updated: $\zeta\,(\omega) = t_n$. At $t_n = 0$, when the optimal stopping times along all paths are determined, the value of the option is estimated by averaging the path-wise values:

$$F\,(0, X) = \frac{1}{K}\sum_{\omega=1}^{K} e^{-r\zeta(\omega)}\,\Pi\left(\zeta(\omega), X_{\zeta(\omega)}(\omega)\right) \quad (3.40)$$

In order to find the continuation value at (t, X_t) , we apply Least Square Monte Carlo method. The intuition behind this method is as follows: if at the option is still available, the continuation value is the expectation, conditional on the information available at that date, of future optimal payoffs from the contingent claim. Let $\Pi\ (t,s,\zeta,\omega)$ be the cash flow from optimal exercise of the option at time s (with respect to the stopping time (ω)), conditional on not being exercised at $t < s$, along the ω-path .Hence:

$$\Pi(t,s,\zeta,\omega) = \begin{cases} \Pi\big(s, X_s(\omega)\big) & if\ s\ = \zeta(\omega) \\ 0 & if\ s\ \neq \zeta(\omega) \end{cases} \qquad (3.41)$$

The continuation value at t_n is the present value of all future expected cash flows from the contingent claim:

$$\theta\ \big(t_n, X_{t_n}\big) = E^*_{t_n}\left[\sum_{i=1+n}^{N} e^{-r(t_i-t_n)}\Pi(t_n,t_i,\zeta,.)\right] \qquad (3.42)$$

In order to calculate the expected conditional continuation value, future realized payoffs are regressed on state variables. Taking this approach, the expectation of continuation at each exercise time for initial investment in our problem is as follows.

$$\begin{aligned} &E\ \big[Continuation_t \big| gp_{(t-1)}, MCFC\_Stack\_Cost_{(t-1)}\big] \\ &\qquad = \beta_{0,t-1} + \beta_{1,t-1} gp_{(t-1)} + \beta_{2,t-1} MCFC\_Stack\_Cost_{(t-1)} + \beta_{3,t-1} {gp_{(t-1)}}^2 \\ &\qquad + \beta_{4,t-1} {MCFC\_Stack\_Cost_{(t-1)}}^2 \qquad (3.43) \end{aligned}$$

where gp and $MCFC\_Stack\_Cost$, are gas price and MCFC stack capital cost respectively.

For the case of our problem with one embedded interdependent option, we use the following algorithm. We apply the described method to determine the optimal investment timing for the second option in each path. Therefore, the two state variables are X_{h2_rp} (Hydrogen price) and X_{Exp_Cost} (Expansion capital cost). Thereafter, we carry out all the explained steps. We also define the expectation of continuation at each exercise time for expansion investment as below.

$$E\left[Continuation_t \middle| h2\_rp_{(t-1)}, Exp\_Cost_{(t-1)}\right] = \alpha_{0,t-1} + \alpha_{1,t-1} h2\_rp_{(t-1)} + \alpha_{2,t-1} Exp\_Cost_{(t-1)} + \alpha_{3,t-1} h2\_rp_{(t-1)}{}^2 + \alpha_{4,t-1} Exp\_Cost_{(t-1)}{}^2 \quad (3.44)$$

where $h2\_rp$ and Exp_Cost are hydrogen retail price and expansion investment cost respectively. Thereafter, we determine the optimal stopping time in each path for the initial option using Bellman equation as follows.

$$F_1(t_n, X_{t_n}) = \max\left\{\Pi_1(t_n, X_{t_n}) + F_2(t_n, X_{t_n}), e^{-r(t_{n+1}-t_n)} E^*_{t_n}[F_1(t_{n+1}, X_{t_{n+1}})]\right\} \quad (3.45)$$

where F_1 is the value of the initial option, F_2 is the value of the second option and Π_1 is the payoff of initial investment option. We then compute the stopping time $\zeta_1(w)$ for initial investment option at t_n on the ω-th path is as follows.

$$\text{If} \quad \theta_1(t_n, X_{t_n}(w) \leq \Pi_1\left(t_n, X_{t_n}(w)\right) + F_2\left(t_n, X_{t_n}(w)\right) \quad then \quad \zeta_1(w) = t_n \quad (3.46)$$

where θ_1 is the continuation value and ζ_1 is the stopping time for the initial option.

3.5 Illustrative Example

In this, section we present a numerical example to demonstrate how the investment model works. This section is organized as follows; firstly, we present the input parameters and data, secondly, we present the results of the investment model.

3.5.1 Input Data

According to the Molten Carbonate Fuel Cell manufacturer, MCFC systems are available in 350 kW (DFC-300), 1.4 MW (DFC-1500) and 2.8 MW (DFC-3000) capacities [50]. We select 1.4 (MW) MCFC which is close to the average daily power demand of the facility under consideration. The hydrogen dispensing hardware includes compression, storage and dispensing. We consider

0.5 (MW) hydrogen storage and 2 dispensers. More details of required auxiliary instruments are available in "*Fuel Cell Power*" model report [32, 41]. In order to find out the initial costs of MCFC stack, auxiliary equipment and hydrogen dispensing equipment, data presented in Table 3-2 has been used.

Table 3-2: MCFC and hydrogen dispensing equipment capital costs breakdown [31]

Component	Cost ($)	Notes
MC fuel cell	$179256.2 * 0.85 * P_{max}^{0.33}$	$P_{max}(kW)$ is rated maximum power of the FC stack, 0.85 is production volume discount factor
Heating system	$200 * P_{max}$	$P_{max}(kW)$ is rated maximum power of the MCFC stack
H_2 purification	$87058 * (\frac{H_{purif}}{4.167})^{0.5} * (\frac{x}{10})^{\frac{ln(0.85)}{ln(2)}}$	H_{purif} is the H_2 purification rate in kg h^{-1}, 0.5 is a size scaling factor, and 0.85 reflects production volume discount
Storage system	$1026 * H_{Storage}{}^{1.081} * a$	$H_{Storage}$ is hydrogen stored in kg, a = 0.95 is production volume discount factor
Compressor	$26913 * P_{Compressor}{}^{0.5202} * b$	$P_{Compressor}$ is the Compressor flow rate in kg h^{-1}, b = 0.91 is production volume discount factor
Dispenser	$55073 * n * c$	n is the number of dispenser, c = 0.77 is production volume discount factor
Electrical equipment	$170.63 * P_{max}$	$P_{max}(kW)$ is rated maximum power of the MCFC stack
Safety equipment	$40 * P_{max}$	$P_{max}(kW)$ is rated maximum power of the MCFC stack
Mechanical and piping	$80 * P_{max}$	$P_{max}(kW)$ is rated maximum power of the MCFC stack

The parameters of GBM processes for stochastic variables are either estimated from historical data (dataset at Henry Hub from 2000 to 2008 is used to estimate GBM parameters for natural gas price) or based on our best guesstimates. Table 3-3 gives the initial value along with annual drift and volatility of stochastic processes.

Table 3-3 : Stochastic parameters of GBM processes

	Gas price	Hydrogen price	MCFC stack cost	Expansion cost
Initial value	7 ($/mmBtu)	365 ($/Mwh)	1450000 ($)	561616 ($)
Drift (μ)	0.045	0.045	-0.03	-0.03
Volatility (σ)	0.2	0.5	0.03	0.03

Similar to "*Fuel Cell Power*" model, we assume a deterministic daily profile for hydrogen demand as percentage of total daily demand for standard high volume refueling as shown in Figure 3-4 [45]. We also assume no annual increase in the total daily demand over the course of investment horizon. The profile is as below.

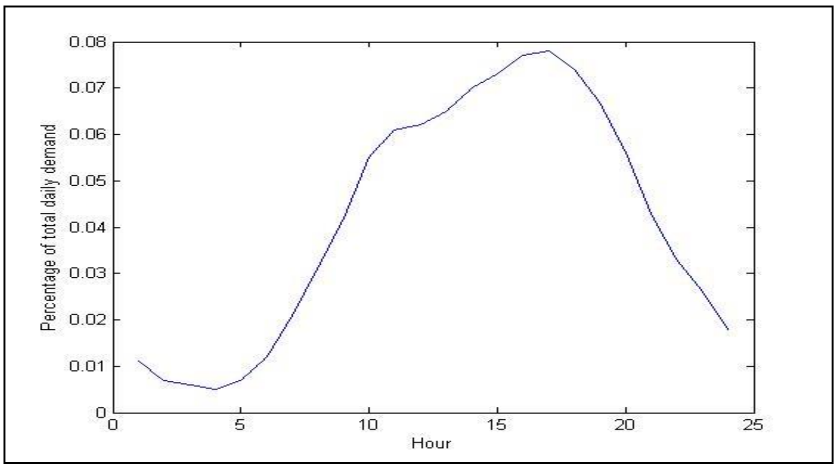

Figure 3-4 : Daily hydrogen demand as percentage of total daily demand

3.5.2 Results

The probability of exercising the initial investment option in each year ($Pr_{initial}$) over the course of planning horizon (i.e.4 years) is obtained using the optimal investment timing results over the Monte Carlo simulated paths. According to the general rule of option analysis, higher uncertainty results in exercising the option with delay in higher number of Monte Carlo sample paths and vice versa. $Pr_{initial}$ results are presented in Table 3-4.

Table 3-4 : Probabilities of exercising the initial option

	Year 1	Year 2	Year 3	Year 4
$Pr_{initial}$	0.22	0.31	0.47	0

The probability of exercising the expansion option conditional on exercising the initial option in each year is presented in Table 3-5.

Table 3-5 : Probabilities of exercising the expansion option

	Year 2	Year 3	Year 4
Expansion|initial @ year 1	0.04	0.12	0.75
Expansion|initial @ year 2	0	0.12	0.79
Expansion|initial @ year 3	0	0	0.98

High probability of exercising the expansion option in year 4 regardless of initial option exercise year, demonstrates a high magnitude of uncertainty in exercising the expansion option which results in a delay in investment. The impact of level of uncertainty on optimal investment timing will be demonstrated using sensitivity analysis experiments in section 7. Moreover, Figure 3-5 represents the linear relationship between natural gas price and MCFC stack cost trigger thresholds.

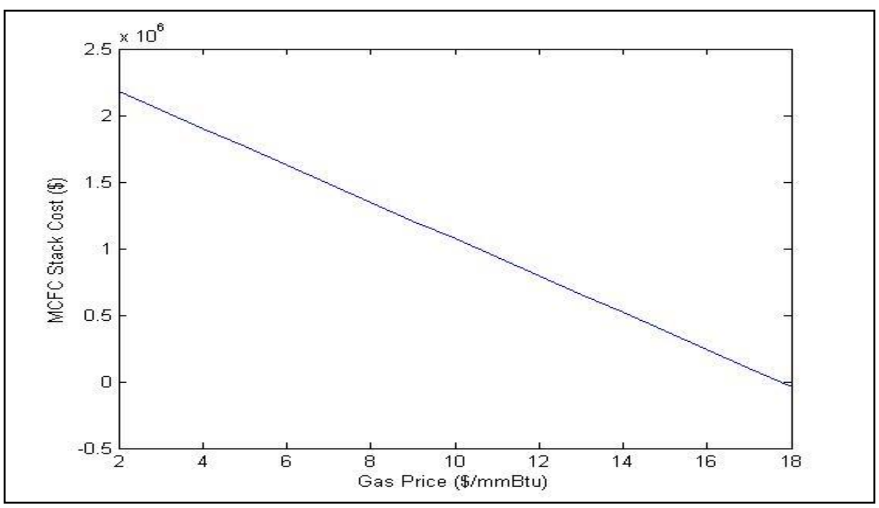

Figure 3-5 : Thresholds line to exercise the initial investment option

Proper investment timing is when two stochastic variables intercept under the threshold line. Similar plots can be generated for expansion thresholds conditional on initial option exercise at years 1, 2 and 3. The expectation of optimal thresholds is calculated from conditional distribution function presented below.

$$E(X^*) = \sum_{i=1}^{4} E(X^* | \zeta = 1) P(\zeta = i) \quad (3.47)$$

where X^* is the threshold value for stochastic variable X in the initial and expansion investments. Table 3-6 represents the expected thresholds to exercise the initial option.

Table 3-6: Initial options trigger thresholds

E [$\mathbf{gp}^*$]	7.25 ($/mmBtu)
E [$\mathbf{MCFC\_Stack\_Cost}^*$]	1377507.23 ($)

In addition, Figure 3-6 shows the expected thresholds to exercise the expansions option conditional on exercising the initial option.

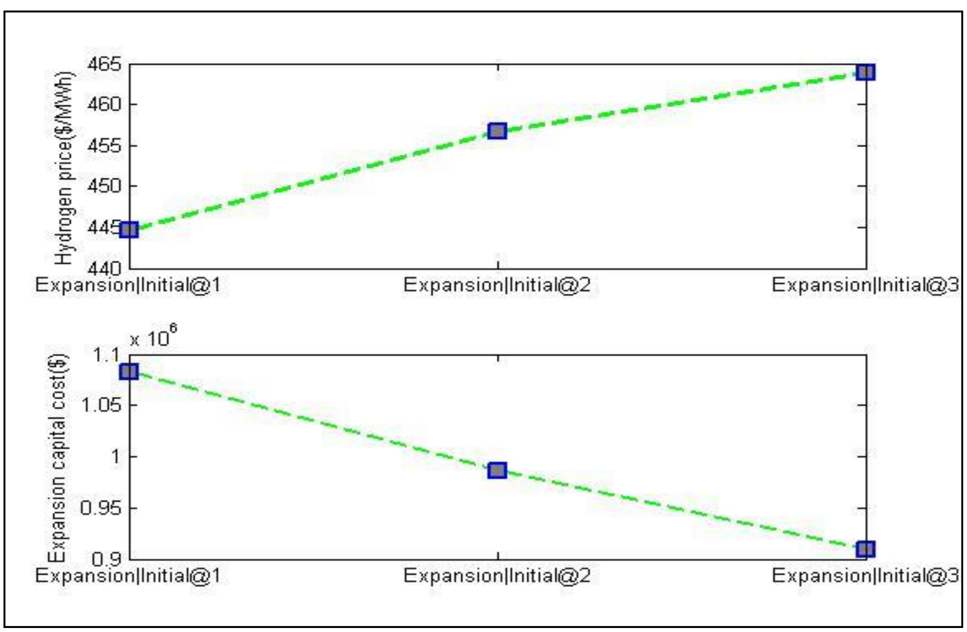

Figure 3-6 : Expected thresholds to exercise the expansion investment option

We observe lower trigger value for lower expansion capital costs and higher lower trigger value for hydrogen price as delay in investment is increased. This observation is in line with decreasing trend of expansion capital cost and increasing trend of hydrogen price.

3.6 Sensitivity Analysis

In this section, we validate our model through a number of sensitivity experiments. A general result of the real option evaluation is that higher volatility and uncertainty results in higher value of waiting for more information about the uncertainty. Therefore, higher uncertainty increases the expected investment threshold of stochastic variables with increasing trend and decreases the expected exercise threshold of stochastic variables with decreasing trend [51].

3.6.1 Impact of MCFC and Expansion Capital Costs Decline Rate

We are interested in examining the sensitivity of investment thresholds to MCFC and expansion the capital costs decline rates according to immaturity of these technologies. Our conjecture is to delay investment decisions by increasing the absolute value of decline rates of both MCFC and expansion capital costs. Figure 3-7 represents MCFC cost expected thresholds for different levels of annual decline rate (i.e. -0.09, -0.07, -0.05 and -0.03).

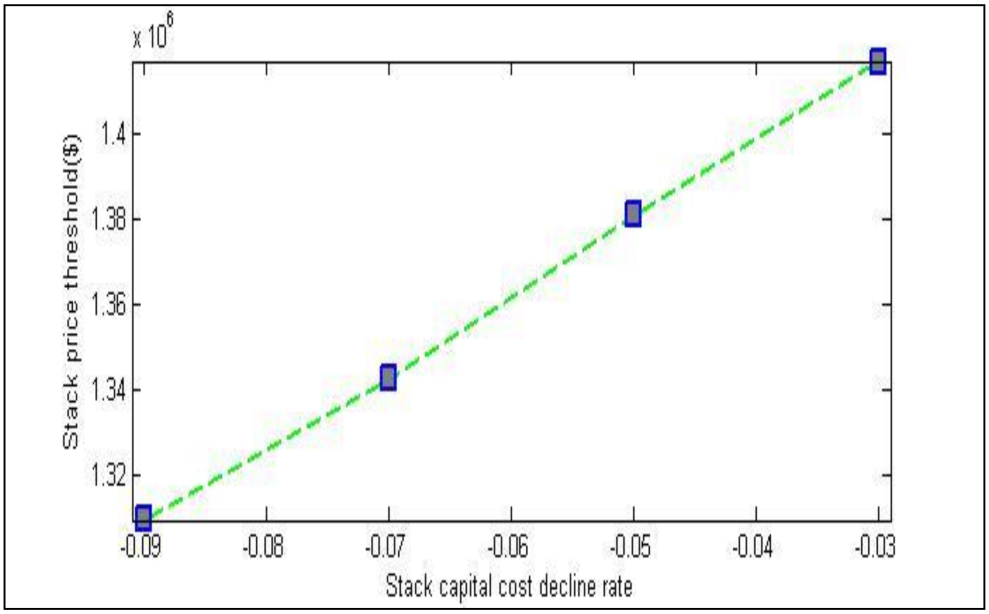

Figure 3-7 : MCFC cost expected threshold sensitivity to decline rate

In addition, Figure 3-8 shows the expansion costs expected thresholds conditional on initial investment, for different levels of expansion cost decline rate.

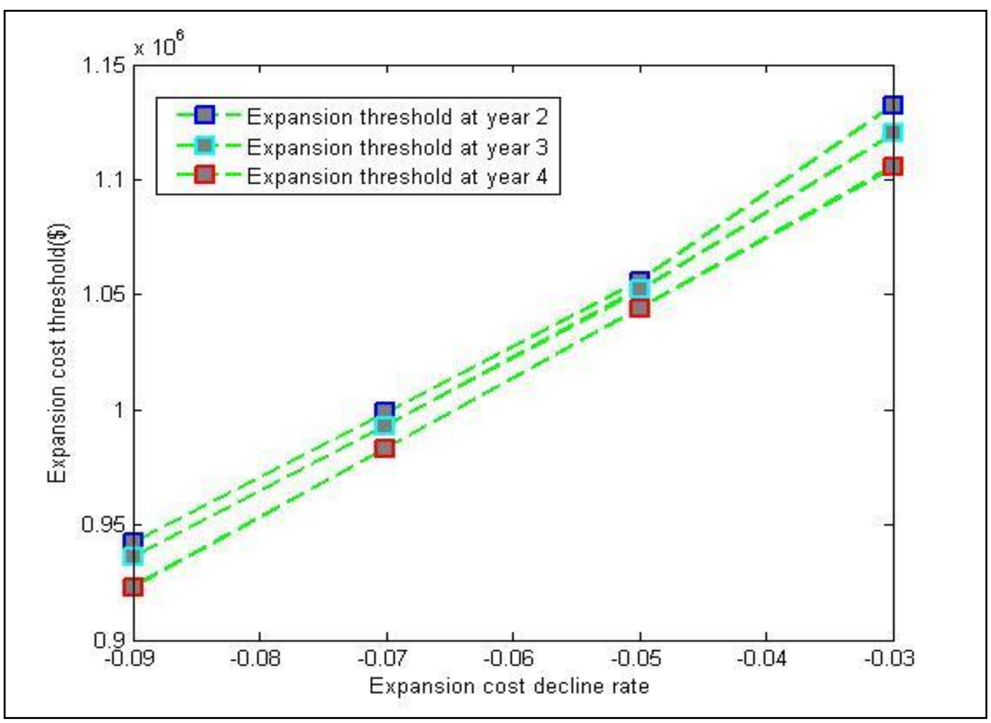

Figure 3-8 : Expansion cost expected threshold sensitivity to decline rate

Therefore, by increasing the absolute value of MCFC stack cost and expansion capital costs decline rates, investment thresholds decrease, meaning that the decision maker shall wait for lower cost to invest.

3.6.2 Impact of MCFC Cost Volatility

MCFC cost volatility is a crucial factor in our initial investment timing decision. Our conjecture is delay in initial option exercise once the volatility of MCFC capital cost increases. Figure 3- 9 shows how the initial investment decisions are modified for three levels of MCFC stack cost volatility (i.e. 0.01, 0.05 and 0.1).

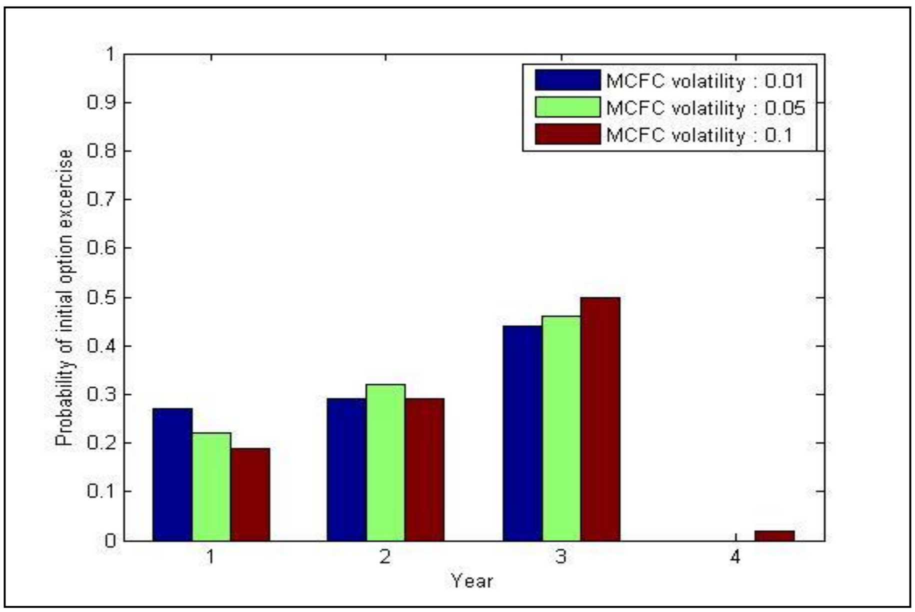

Figure 3-9 : Initial investment decision sensitivity to MCFC cost volatility

As demonstrated, by increasing the MCFC cost volatility, waiting for more information and therefore delay in investment becomes more significant. In addition, delay in initial investment results in lower MCFC cost and higher gas price thresholds according to decreasing MCFC expected cost and increasing natural gas expected price. Figure 3-10 shows how MCFC capital cost volatility impacts the expected investment thresholds.

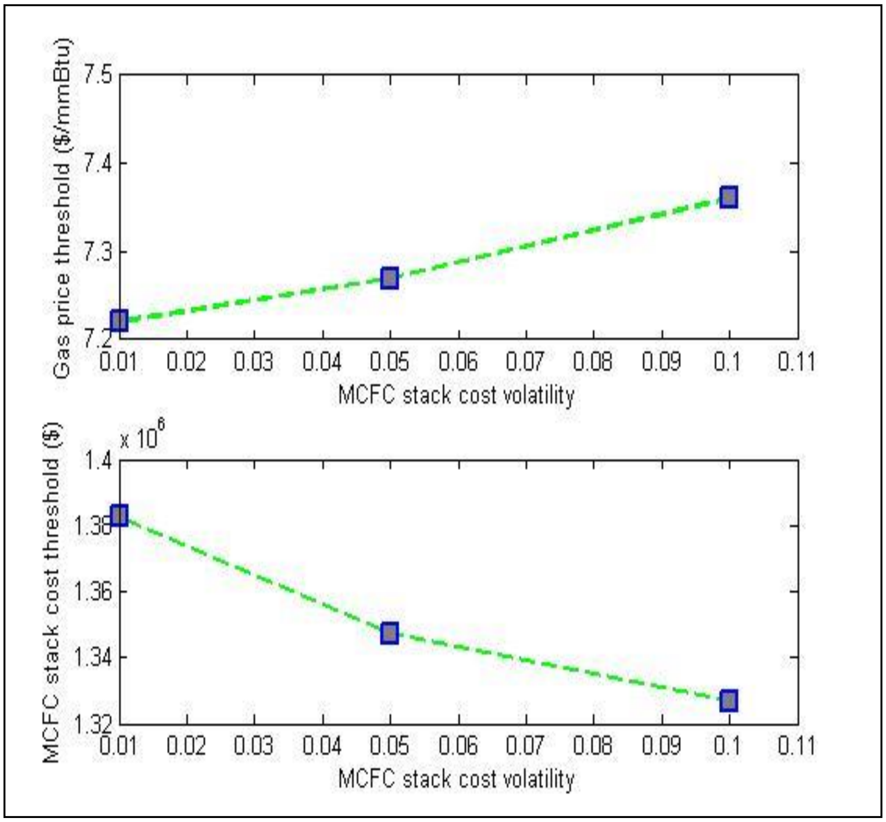

Figure 3-10 : Initial investment expected thresholds sensitivity to MCFC stack cost volatility

3.6.3 Impact of Hydrogen Price Volatility

According to the immaturity of hydrogen generation technologies and the uncertainty of the hydrogen future market, we are interested in demonstrating how uncertain hydrogen price influences the timing of investment in onsite hydrogen dispensing systems. We examine the impact of hydrogen price volatility on expansion investment timing for three levels of hydrogen price volatility (i.e. 0.1, 0.3 and 0.5). Figure 3-11 shows that delay in expansion investment becomes more significant by increasing hydrogen price volatility, which is in line with the general rule of real option evaluation that higher volatility results in delayed investment.

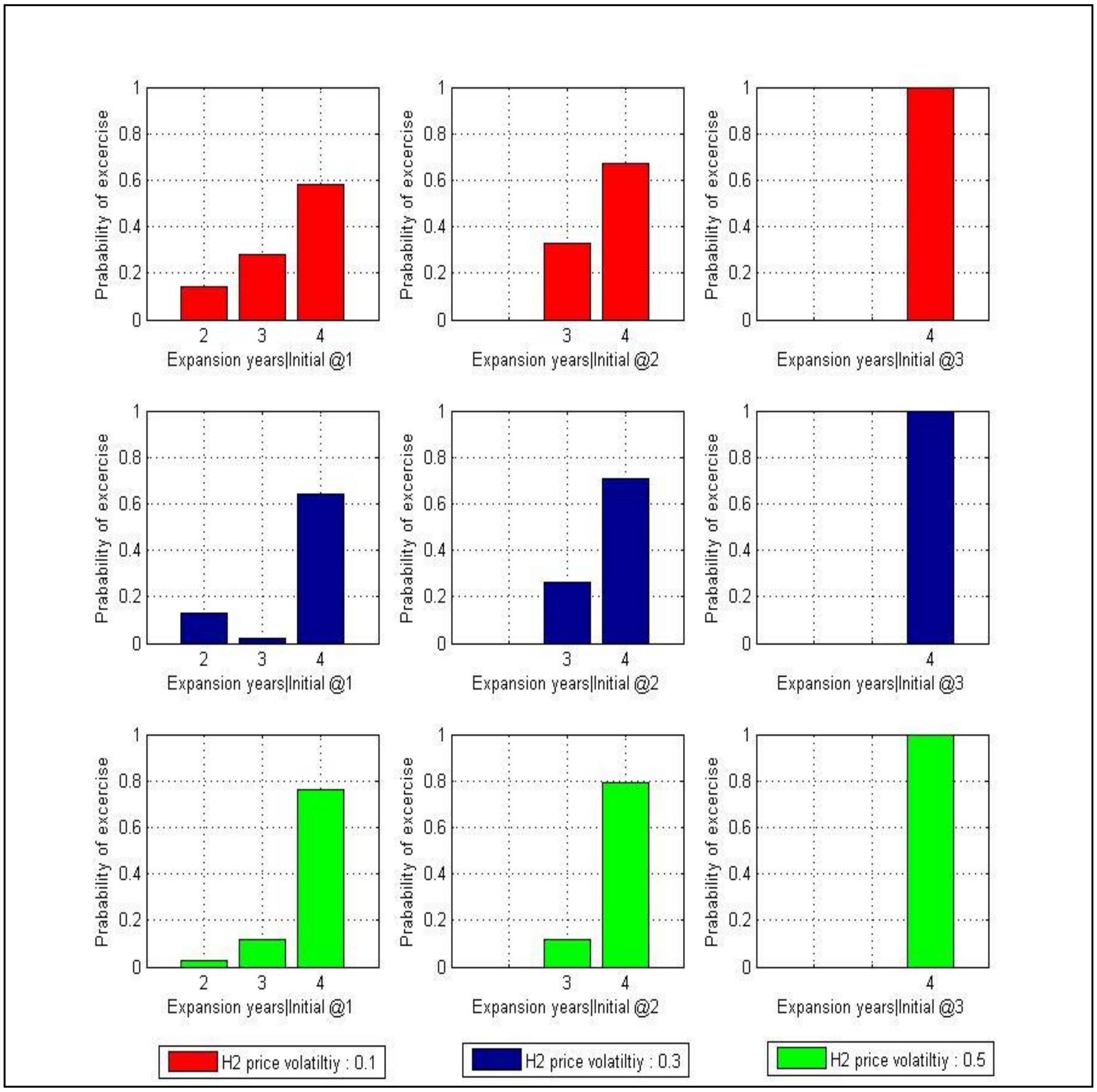

Figure 3-11 : Expansion investment decision sensitivity to hydrogen price volatility

Delay in expansion investment results in lower expansion cost and a higher hydrogen price expected thresholds according to decreasing expansion expected cost and increasing hydrogen expected price. Expected thresholds to exercise the expansion option for aforementioned levels of hydrogen volatility are presented in Figure 3-12.

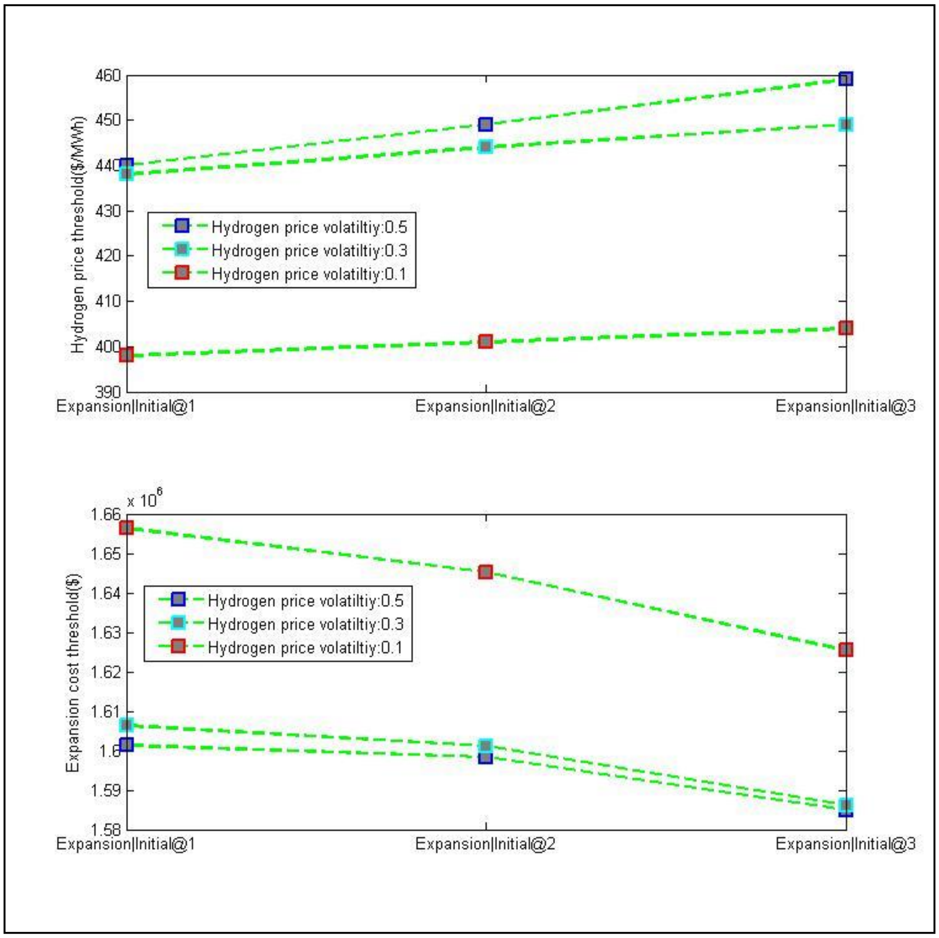

Figure 3-12 : Expansion investment expected thresholds sensitivity to hydrogen price volatility

We are also interested to see how expansion investment uncertainty impacts the initial option exercise timing. We intuitively anticipate that the initial investment option exercise date is brought forward once the hydrogen price volatility is lower which causes a less volatile expansion investment payoff. Our anticipation is based on the fact that the expansion option payoff is taken into consideration in defining the optimal timing in order to exercise the initial option. Figure 3-13 shows how delay in initial investment becomes less significant by decreasing the hydrogen volatility.

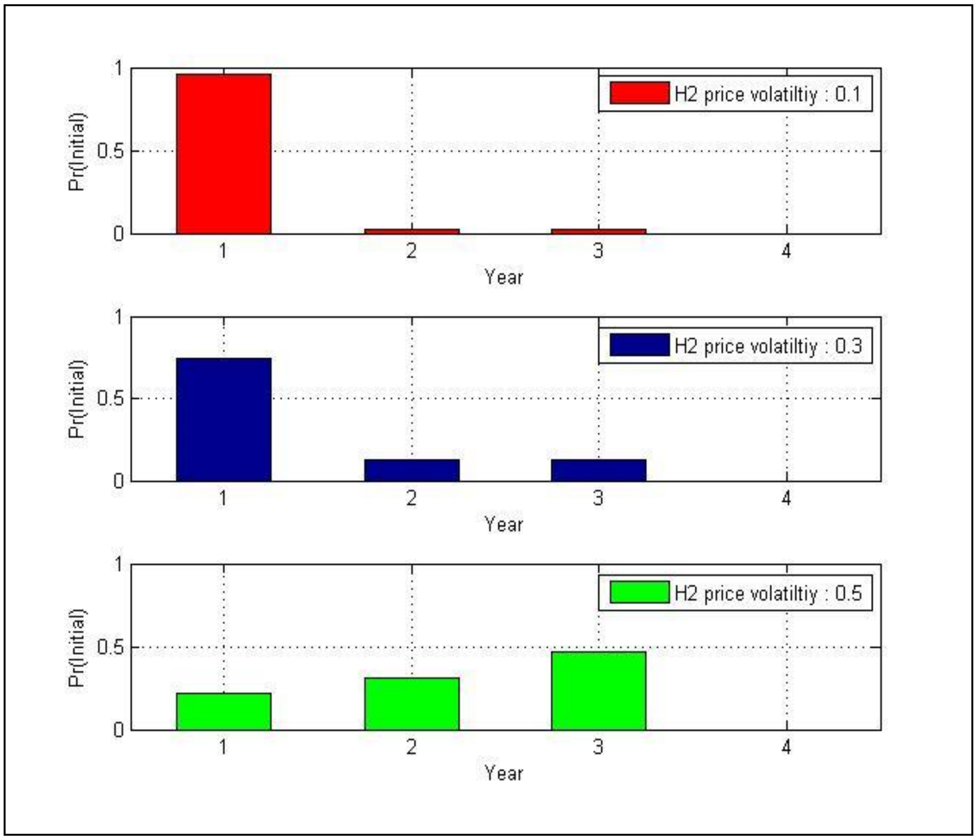

Figure 3-13 : Initial investment decision sensitivity to hydrogen price volatility

3.6.4 Impact of Natural Gas Price Volatility

We are interested to investigate the impact of natural gas price volatility on the initial investment thresholds. Our conjecture is delay in investment by increasing the natural gas price volatility. Delay in initial investment results in triggering the initial investment option in lower MCFC stack cost and higher natural gas price as shown in Figure 3-14.

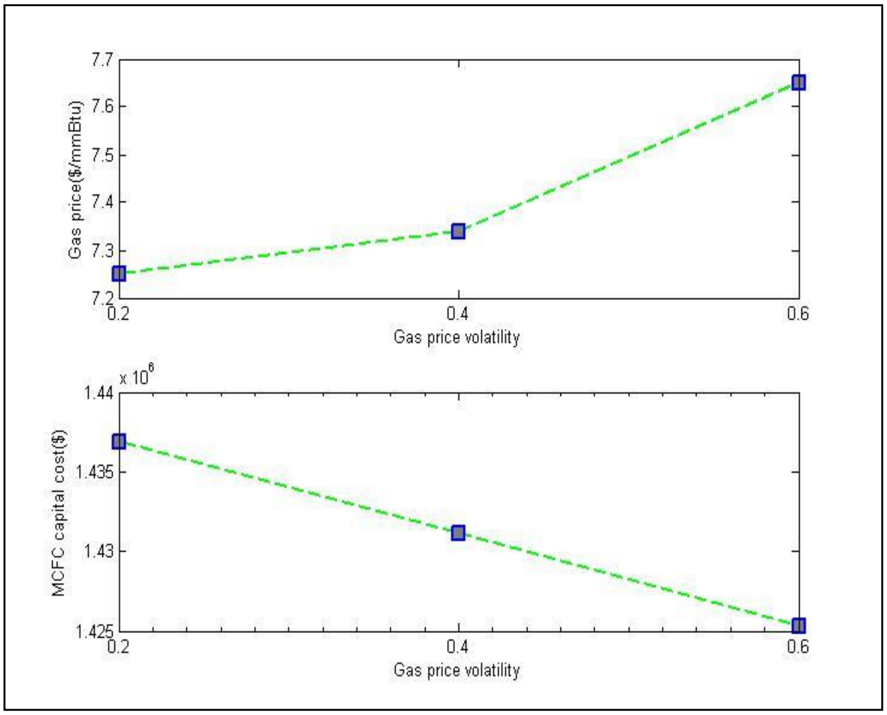

Figure 3-14 : Initial investment expected thresholds sensitivity to natural gas price volatility

3.6.5 Impact of Hydrogen Demand Increase Rate

In the previous cases hydrogen demand is assumed to be a constant over the investment horizon. However, in order to investigate the impact of hydrogen demand increase rate on the investment decisions, we define three levels for annual total hydrogen demand increase rate (i.e. 0.1, 0.2 and 0.3). Figure 3-15 shows that by increasing the annual hydrogen demand increase rate, delay in the expansion investments become more significant.

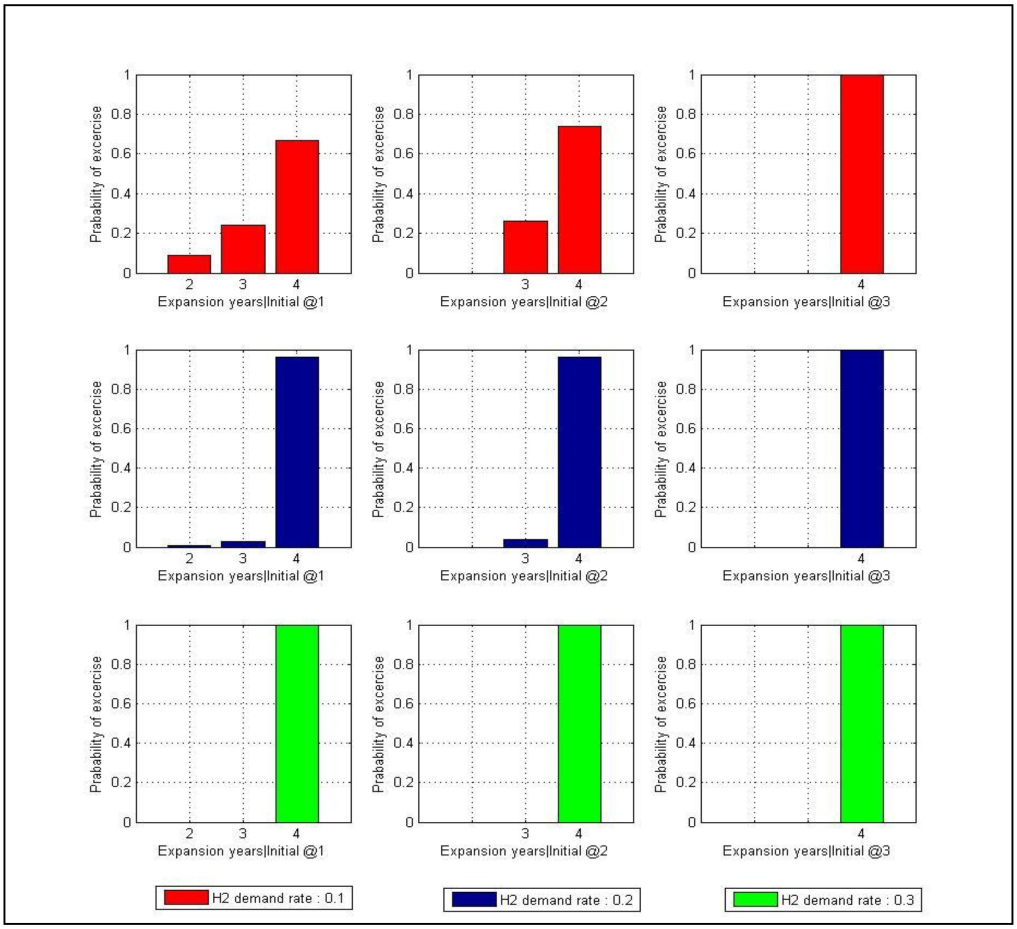

Figure 3-15 : Expansion investment decisions sensitivity to hydrogen demand annual increase rate

This observation indicates the value of real option in hedging the risk of larger projects in the face of uncertainty. Once the hydrogen demand rate increases, impact of hydrogen price volatility is amplified. Therefore, the investment becomes more risky which results in postposing the investment.

3.7 Conclusion and Future Work

The work presented in this article tackles the problem of optimal two stage investment in hydrogen tri-generation with fixed capacity and addition of onsite hydrogen dispensing systems under uncertainties. To handle multiple uncertain variables, a simulation based approach with least squares regression is applied. The sensitivity of investment decisions to a number of parameters are examined and the results are in line with the general result of the real option that delay in investments becomes more significant as volatility increases.

This work can be extended by relaxing the assumption of fixed capacity of MCFC system. Thereafter, incremental investment in MCFC system can be taken into consideration as another subsequent option. This modification enables us to enhance the capacity of MCFC by addition of the required number of MCFC stacks to the existing system, if it is economically attractive. In this manner, installation of the initial MCFC system provides the option to expand the system in future.

4 Public-Private Partnership (PPP) Financing Model for Micro-Grids

In this chapter, we develop a Public-Private Partnership (PPP) financing model for micro-grids in which public entity transfers responsibility and risk of designing, building, operating and maintaining (DBOM) of the project to the private sector while maintaining the project ownership. The private sector owns the micro-grid's revenue till the investment horizon; however, the revenue ownership will be transferred to the public entity after the investment horizon till a finite after-horizon period. Public entity incentivizes the private sector by providing an initial senior debt opportunity (through issuing zero coupon municipal bonds) and possibility of annual junior debts. Here, the (DBOM – PPP) financing model is merged with micro-grid short-term operation optimization into a single framework under host of short-term and long-term stochastic variables. An illustrative example is presented in which optimal financial activities and optimal micro-grid incremental portfolio over the course of investment horizon are defined for a vulnerable community located in 100-year flood zone region of city of Hoboken in New Jersey. From a practical point of view, the proposed model could have enormous impact on building a collaborative environment for public and private entities, which will facilitate implementation of micro-grid projects.

4.1 Introduction

Optimal operation of micro-grids in average normal conditions decreases the cost of supplying energy demands of the communities and has the potential to generate revenue. It is no secret that micro-grids can also increase the power resiliency of communities by continuous operation in stressed out conditions in which the power grid is disconnected due to extreme environmental conditions or technical issues. With this background, micro-grid projects have gained popularity in long-term plans for "Sustainable and Resilient Communities". A sustainable and resilient community by definition is a community, which is structurally developed to mitigate the economic and societal cost of disasters, and also have the capability to recover quickly [52]. Since the focus of many recent researches has been on the micro-grid's design, implementation and operation, there is a lack in comprehensive models which solve the problem of financing such projects. Many of recent micro-grid projects have not been expanded from pilot scale to massive scale capable of supplying considerable portion of the communities' power demands, due to existence of no clear financial plan that takes the project from the financing step out to the end of the micro-grid's lifetime. In this chapter, we are aiming to develop a long-term financing plan for incremental investment in community level micro-grids, which is based on the Public-Private Partnership (PPP) financing model. The (PPP) is a business relationship between a private sector entity and a government agency for the purpose of carrying out a project which will serve the public [53]. These contractual agreements are used to finance, build, operate and maintain large scale projects such as wastewater treatment plants, public transportation networks and convention centers. The advantages of such agreements are making the project a possibility in the first place and sooner completion of the project as well as transfer of risk from public entity to the private sector over the life of the project [54]. The (PPP) contracts come in a wide range of forms which are basically different in the degree of involvement of the private entity in the project. In this research, we take DBOM (Design- Build- Operate- Maintain) type of (PPP) models abbreviated as (DBOM-PPP) for micro-gird projects financing in which the private sector is responsible for design, build, operate and maintain the project over the course of a specified period, however public sector maintains the ownership of the project [55]. These project components are procured from the private entity in a single contract with financing secured by public sector. In all (PPP) contracts, public sector provides some incentives for the private sector such as loans proportionate to the level of risk the implementer bears, reduction in loan fees or/and transparent communication, collaboration and less political behavior [56]. We consider the public sector's incentive in our model in the form of a "Senior debt" in initial year and possibility of annual "Junior debts" over the course of investment horizon [57]. The rest of the chapter is organized as follows. In section (4.2), brief explanation of the developed model is presented. In section (4.3), the model is demonstrated through an illustrative example. Finally, conclusion is presented in section (4.4).

4.2 Public Private Partnership (PPP) model for Micro-Grids

We will use the recent micro-grid's operation and (DBOM–PPP) financing model as in chapter 2. In our (DBOM–PPP) model, the public sector provides a specified amount of fund for the private entity in the initial year of contract in the form of "Senior debt" through issuing sufficient number of zero coupon municipal bonds with face value F and maturity at the investment horizon. In addition, public sector provides the private sector opportunity to borrow defined amount of funds in form of "Junior debts" with fixed rate of return in each year. "Junior debt" is either unsecured (i.e. do not require collateral behind the debts) or has a lower priority than of other debts claim on the same asset. It is worth noting that municipal bonds are tax exempted bonds issued by entities such as states, cities, counties, special-purpose districts or any other governmental entity under the state level, with the purpose of financing large scale infrastructural projects which could include micro-grid. Municipal bonds are categorized as (I) General obligation bonds and (II) Revenue Bonds. In our model, we consider Revenue bonds in which the principal and interest rate are secured by the revenue from the project (i.e. micro-gird). Therefore, in order to build our long-term (DBOM –PPP) finance model, we need to accurately estimate annual savings/revenue from micro-grid operation under operation optimality condition. In our model, we merge the micro-grid short-term operation optimization and the (DBOM–PPP) financing model into a single framework under a host of short-term and long-term stochastic variables and solve it as a mixed integer stochastic optimization problem. The micro-grid's operation optimization accounts for short-term savings, costs and penalties that are weighed according to the priorities of existing or planned residential and commercial sectors within the community in stressed out occasions. Hence, micro-grid saving is calculated in both average normal and stressed out conditions (i.e. power gird outage) with the objective of savings maximization. The power allocation between different sectors in case of stressed out conditions is based on criticality rank of each sector. The (DBOM –PPP) model is on the basis of cash flow reflecting the actual outflows and inflows of monetary values. The (DBOM –PPP) is formulated as a stochastic model for financial planning which defines the following:

i. Optimal annual average financial activities over the course of investment horizon.
ii. Optimal annual micro-grid incremental portfolio under capacity constraints and available area limitation for micro-grid assets.

The objective of (DBOM –PPP) is to maximize the end of horizon cash flow plus the horizon time value of beyond the horizon cash flows till a finite beyond horizon period. We assume that the private entity can use the cash inflow resulting from micro-grid's operational savings along with the senior debt and junior debts to purchase micro-grid assets. The private sector owns the micro-grid's revenue till the investment horizon; however, the revenue ownership will be transferred to the public entity after the investment horizon till a finite after horizon period. The (DBOM –PPP) model captures the long-term market and price uncertainties such as natural gas price and capital cost of micro-grid assets. Taking Monte Carlo simulation approach, several sample path realizations over the course of project investment horizon are generated, and the deterministic (DBOM –PPP) is solved for each sample path and expected results are estimated.

4.3 Illustrative Example

In this section, we are aiming to demonstrate how the (DBOM –PPP) model works for a community level micro-grid project through an illustrative example. We assume that micro-grid portfolio includes gas fired generators (GF), combined heat and power (CHP), photovoltaic cells (PV), electricity storage (ST), wind turbine (WT) and boiler. Therefore, we are considering this portfolio to supply as much as possible of electricity and heat demands of the community under study. We select the 100-year flood zone region (i.e. region with 0.01 probability of flooding in each year) in city of Hoboken, New Jersey, for micro-grid implementation. This specific region is selected according to the following reasons: (I) High vulnerability to extreme weather condition and power gird outage; as this region was extremely affected by super-storm Sandy in 2012, (II) Two super critical sectors (i.e. Health service and Information technology) are located in this region whose economic loss due to power outage are significant. In addition this region has high density of residential and retail units, which can be classified as medium critical sectors. Figure 4-1 shows the 100-year flood zone area in Hoboken along with land use classification in this region extracted using GIS (Geographic Information System) land use data.

Figure 4-1 : Land use classification in 100-year flood zone, Hoboken, NJ

Table 4-1 shows approximate number of units and total roof area of each sector in the region. Roof area can be partially used for installation of PV panels.

Table 4-1 : Sectors' quantity and total area

Sector	Number of units	Total roof area (acre)
Health service	10	11.38
Information technology (IT)	21	9.37
Retail	72	98.77
Leisure	8	28.69
Residential	1500	339.71

We also make the following assumptions regarding available area to install renewable power generation assets in our micro-grid portfolio:

i. Half of the roof area of each sector can be used to install PV panels.

ii. WT installation is not practical in this region, since area under study is located in a dense municipal region with no available land to install massive wind turbine hardware.

Sine, we do not have access to hourly heat and electricity demand data for each of the units in this region, we use daily profiles for typical health service, information technology (IT), retail, leisure and residential sectors, as shown in Figure 4-2 [58, 59 and 60].

Figure 4-2 : Typical electricity demand profile for each sector

We approximate the heat demand (supplied by gas fueled boiler and CHP) to electricity demand ratio for all sectors to be 0.4 [61]. We also assume that natural gas prices follow a Geometric Brownian Motion (GBM) with drift and volatility estimated using historical data (Henry Hub from 2000 to 2008) as demonstrated in Table 4-2.

Table 4-2 : Parameters of natural gas price GBM process

Natural gas initial price ($/mmBtu)	Drift	Volatility
7	0.045	0.2

PV, ST and WT capital costs are assumed to be stochastic. We assume a decreasing trend and assign a binomial probability mass function to the rate (ψ) by which the capital cost decreases in each year. Table 4-3 demonstrates the parameters of assigned binomial probability mass functions.

Table 4-3 : Parameters of Binomial distributions

Asset	ψ_1	ψ_2	Probablity (ψ_1)	Probablity (ψ_2)
PV	0.8	0.6	0.33	0.67
WT	0.8	0.6	0.33	0.67
ST	0.8	0.6	0.33	0.67

GF, CHP and boiler investment costs are considered to be deterministic according to maturity of these technologies. Table 4-4 shows the annual investment cost of these assets over the course of investment horizon.

Table 4-4: Deterministic investment costs of GF, CHP and boiler over the course of investment horizon

	Year 0	Year 1	Year 2	Year 3
GF (S/MW)	100,000	100,100	100,200	100,300
CHP ($/MW)	1,200,000	1,210,000	1,220,000	1,230,000
Boiler ($/MW)	600	700	800	900

As stated earlier, we assume that the public entity provides a senior debt to the private sector via issuing sufficient number of municipal bonds (i.e. 500 in our case). In order to price zero-coupon floating rate municipal bond with maturity at investment horizon with face value F , we use the binomial lattice method which is a popular approach to model one-factor Markov processes [35]. The binomial lattice weakly converges to Geometric Brownian Motion (GBM) stochastic process. In the binomial lattice, a binomial step of Δt is considered such that u and d are multipliers associated to up and down movements of variable in each step with risk neutral probabilities of p and $1 - p$ respectively. We start with constructing a binomial lattice structure for the floating short-term interest rate. The risk neutral probabilities are set to 0.5, and each step's up and down multipliers are set to be1.25 and 0.9 respectively. Short term interest rate binomial lattice is demonstrated in Figure 4-3.

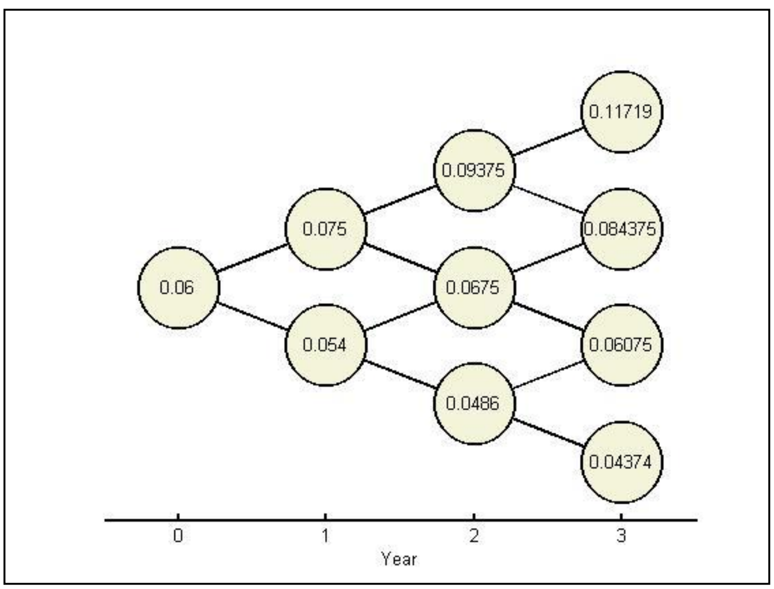

Figure 4-3 : Short term interest rate lattice

Thereafter, we construct a binomial lattice for zero coupon municipal bond with terminal states' values all equal to the bond's face value. Working backwards on the binomial lattice and using risk neutral probabilities and floating interest rates, price of the zero coupon bond is calculated.

Applying this methodology, we calculate the price of a municipal bond with maturity at year 3 and face value of $5,000 (i.e. common face value of municipal bonds) to be $4145.61 as demonstrated in Figure 4-4.

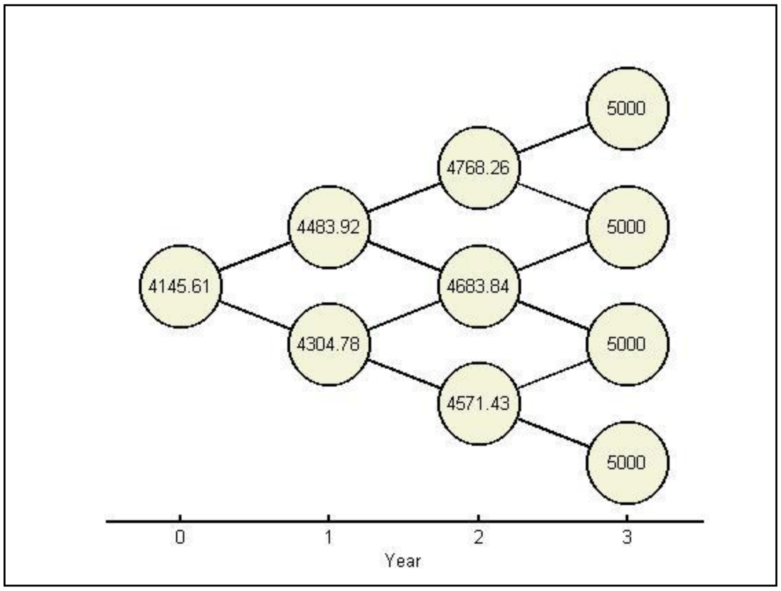

Figure 4-4 : Municipal bond pricing ($) lattice

We assume that the annual junior debt is also limited to be no more than 2M$. We also assume that the public entity has a revenue stream from micro-grid operation for a finite after-horizon period (i.e.15 years). Now, we proceed to the results of the devolved (DBOM –PPP) model. The average financial activities (i.e. annual junior debts, initial senior debt, annual cash from micro-grid savings spent to purchase micro-grid assets along with annual micro-grid savings) of the private sector in the context of (DBOM-PPP) model are presented in Figure 4-5.

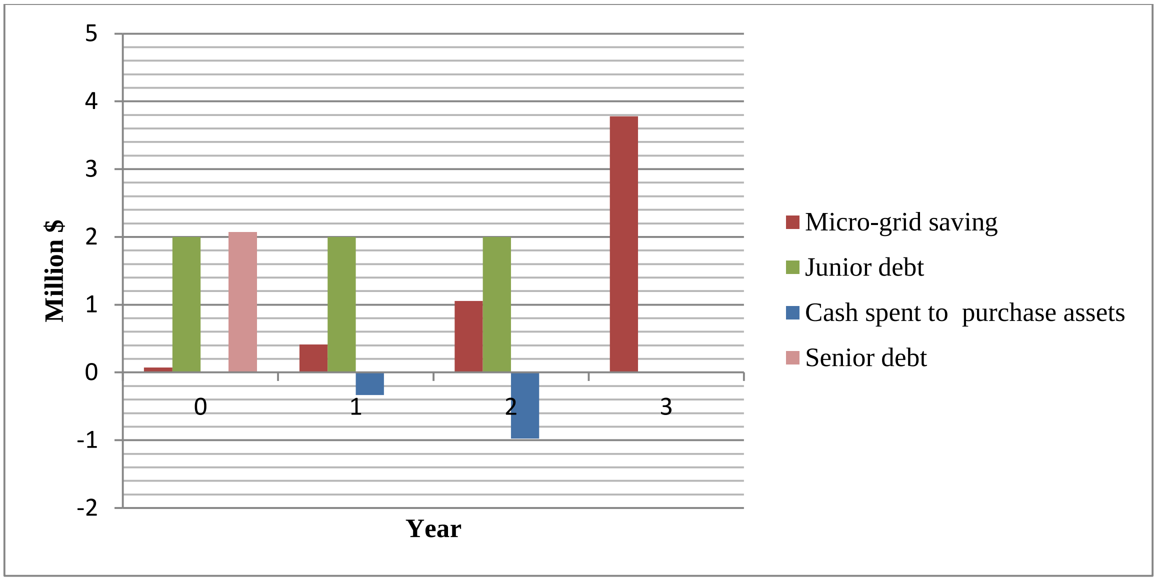

Figure 4-5 : Optimal annual financial activities averaged over all scenarios

The private sector is using the fund supplied by the public entity in the form of a senior debt along with the junior debts and cash inflows gained from micro-grid savings to install assets and maximize the cash flow position of both public and private entities in the investment horizon. The private sector cash flow position at the investment horizon includes the net cash flow at horizon. However, according to the fact that the public entity governs the project revenue right after the investment horizon, its cash flow position at the investment horizon includes beyond the horizon discounted cash inflows minus the issued municipal bonds principal payments to the bond holders (i.e. principal payment of 500 issued bonds with 5000$ face value). Table 4-5 presents the public and private entities cash flow position at the investment horizon.

Table 4-5: Public and private entities cash flow at the investment horizon

Entity	Expected Cash flow position (M$) at the investment horizon
Public	28.78
Private	1.69

Results show how both public and private sectors can considerably benefit from a (PPP) financing model. We are also interested to define the optimal micro-grid incremental portfolio in each year. Figure 4-6, demonstrates the optimal micro-grid portfolio averaged over all scenarios.

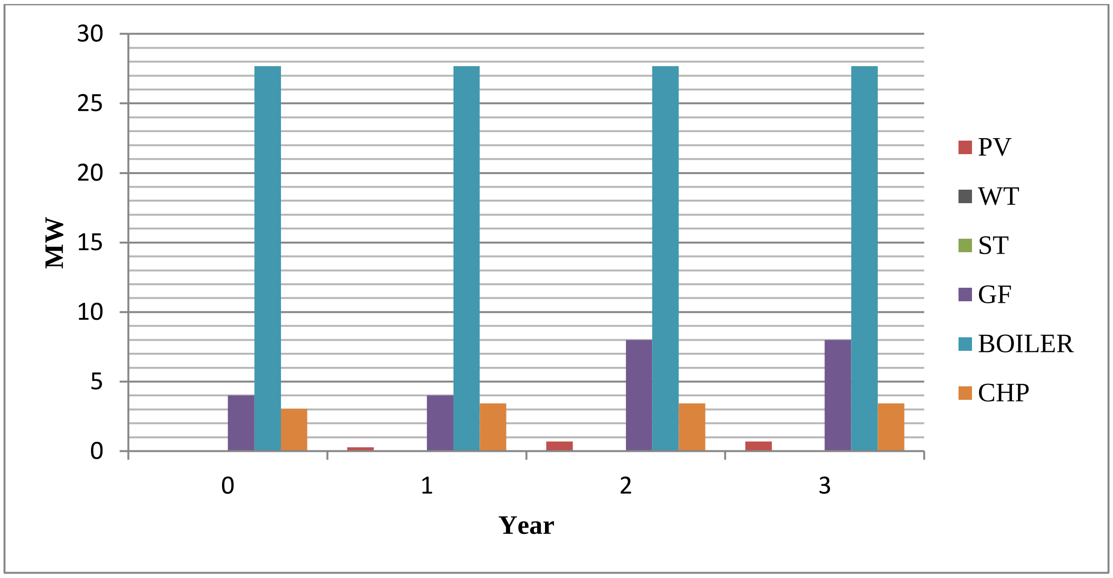

Figure 4-6 : Optimal incremental portfolio averaged over all scenarios

The results show that investing on GF and CHP are most desirable among micro-grid electricity generator assets, which can be explained by low capital cost of GF and high efficiency of CHP in producing both heat and electricity. In addition, small investment on PV and no investment on ST can be justified by high capital cost of these assets. Investment in WT is not possible, according to spatial limitations. Boiler with very low capital cost is also in place to supply heat demands.

4.4 Conclusion

The model results clearly show how a Public-Private Partnership (PPP) model can be utilized to finance and implement a micro-grid project while both private and public entities' profits are significant. The developed (DBOM-PPP) model guarantees a revenue stream for the public entity over the course of a finite horizon, while provides an opportunity for the private sector to apply its expertise and generate revenue for its own. Development of such a (PPP) model can be a forward step towards closer collaboration of public and private entities for more widespread micro-grid implementation.

5 APPLICATIONS AND FUTURE WORK

The results from chapter 2, 3 and 4 can be integrated into a single framework/software for portfolio selection, capital budgeting, operation optimization and investment strategy of traditional micro-grid assets along with hydrogen tri-generation systems.

In this work, we developed analytics to make optimal decisions for a portfolio of micro-grid assets taking into consideration the regional risk factors in chapter 2. Chapter 3 focuses on possibility of installation of hydrogen tri-generation systems in a sample critical infrastructure (i.e. wastewater treatment plant). In chapter 4, a micro-grid project financing scheme based on Public Private Partnership is developed. We shall notice that in designing analytical tools to support decision making in planning of distributed energy generation, many elements must be modeled in tandem. The next section, gives our vision on the future extensions of this work.

5.1 FUTURE WORK

5.2 Enhancement of micro-grid's portfolio (Chapter 2)

In the near future, micro-grid's activities will not be limited to local power supply but also micro-grids can provide power to the surrounding communities at the peak times or at the times of natural disasters. Moreover, interaction between micro-grids to maximize the economic benefits will be considered. Having this said, presented model in chapter 2 can be extended to include multiple micro-grids which interact with each other especially in the times of stressed out conditions to lower the overall loss of power outage. In addition, we can expand the micro-grid's portfolio to consider recent technologies such as thermal storage and tri-generation.

5.3 Enhancement of demand side model (Chapter 2)

In chapter 2, we assumed that different sectors of the sample community have exactly similar power demands in either average normal or stressed out condition. This assumption is not in line with available historical data which shows that the overall demand is stressed out occasions is lower than that in average normal conditions. We can relax this assumption and develop more advanced demand model which captures the dynamics of demand in stressed out occasions.

5.4 Enhancement of Investment model (Chapter 3)

As previously stated, a practical extension of real option model introduced in chapter 3 is to relax the assumption of parametrically fixed initial (i.e. investment on hydrogen tri-generation) capacity. Thereafter, the expansion of hydrogen tri-generation can be included as a subsequent option in the compound real option structure.

5.5 Enhancement of hydrogen vehicles demand model (Chapter 3)

In chapter 3, a fixed daily profile for hydrogen vehicles demand is assumed. A more complicated model could be developed to replace the current simple model. This requires a model which is able to capture the dynamics of hydrogen vehicles demand taking into consideration several economic, geographical and demographic variables. Such model will result in more realistic decision in the investment side.

5.6 Enhancement of Public Private Partnership (PPP) model (Chapter 4)

The PPP model developed in chapter 4 is a Design, Build. Operating and Maintaining (DBOM) contract. Possible extension of this work includes comparison of different PPP contracts for micro-grid projects and selecting the most economical one. The model could be also extended from the financing side by considering different types of bonds which yield to different revenue streams from the project.

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