Description: Discovering Equivalent Fractions and Introducing Fraction Notation (classroom view) **Date: 1993-10-04** **Location: Colts Neck Elementary School** **Researcher: Professor Carolyn Maher**	**Transcriber(s): Yankelewitz, Dina** **Verifier(s): Yedman, Madeline** **Date Transcribed: Spring 2009** **Page: 1 of 22**

Line	Time	Speaker	Transcript
7.0.1	2:33 S	T/R 1:	Good morning! Are you all as awake as I am?
7.0.2	2:39	Meredith:	Yeah.
7.0.3	2:40	T/R 1:	I don't know if that is good or bad, Meredith. Let me shut this [the overhead projector fan] off. [Holding up Mark's diorama] I was thinking when I was looking at Mark's model, and I noticed many of you made models, also for projects for another class. I was thinking about this model because we were talking about models the other day, weren't we. Remember that? And I, remember I asked you to think about something about the models that you built. Remember what I asked you to think about? Does anyone remember, Andrew?
7.0.4	3:12	Andrew:	Um, is one-half bigger, uh is one half bigger than one fourth, by how much?
7.0.5	3:21	T/R 1:	Does anyone else remember anything in our discussion about models? Andrew remembered something. Is your hand up Audra?
7.0.6		Audra:	No.
7.0.7	3:32	T/R 1:	Audra's yawning. Does anyone remember anything in our discussion about models? We talked about models, we asked some questions about them. Think for a minute. Do you remember Meredith?
7.0.8	3:45	Meredith:	Um, what's bigger, one half or one quarter and by how much?
7.0.9	3:50	T/R 1:	That's what Andrew said. Right, which is bigger. But we also were talking about models in general. We asked ourselves some questions about models. Did you all build the same model?
7.0.10	4:04	Students:	No.
7.0.11	4:04	T/R 1:	To answer that question?
7.0.12	4:05	Students:	No, no.
7.0.13	4:07	T/R 1:	Some of you built different models. [Erik raises his hand] Erik?
7.0.14	4:10	Erik:	Some of us built the same models and some of us built different.
7.0.15	4:12	T/R 1:	Some of you built different models, and I asked you a question about that. Do you remember?
7.0.16	4:18	Erik:	[Raising his hand] Oh!

7.0.17	4:19	T/R 1:	Erik?
7.0.18	4:20	Erik:	Could you get different answers
7.0.19		Michael:	Using barred models?
7.0.20		Erik:	if you use different models.
7.0.21	4:27	T/R 1:	Yeah, can you get different answers, right Michael and Erik? - if you use different models. What did you think? How many of you thought you shouldn't get different answers? [Some hands are raised] How many of you are not sure? [Few more hands are raised] It's okay not to be sure. Have you been thinking about that at all since then? Maybe not much. Michael, have you been thinking about that a little bit?
7.0.22	4:48	Michael:	Um, well, I figured that it couldn't be because our answer that we got, me and Brian, was that it was bigger by one fourth because it will always take two, it will always take four quarters to equal up
7.0.23	5:03	Erik:	Yeah, because four is an even number and you can divide it by two.
7.0.24	5:04	Michael:	In half
7.0.25	5:08	Erik:	So there will always be one fourth and two fourths, three fourths, four fourths and two fourths is always going to be a half, a half in fourths.
7.0.26	5:16	T/R 1:	What do you think about that? There are a lot of good ideas in what you are saying. [Picking up Mark's diorama] I was thinking that maybe it would help you, it sort of helped me to look at Marks' model. Sometimes it helps to look at a model that's a little different. Maybe this is a model that doesn't use the Cuisenaire rods, but in a sense it's a model. Um, I found out a little bit from Mark about a book he read, he was telling me. This [pointing into diorama] was supposed to be a sea monster and this was supposed to be [again pointing into the diorama] two friends. And I looked at, I looked at what he built here[still pointing to diorama] to represent some of the story and I thought by looking at this model that I couldn't really tell of the boy and the girl who was taller by looking at them, I wasn't really sure, and I didn't know really if Mark cared about that. But I looked at the sea monster, okay, and I looked at the boat, okay, and I was thinking about their sizes a little bit, right? What are you, why are you smiling about Mark?

7.0.27	6:27	Mark:	Uh, well I wasn't thinking about the sizes. I made the sea monster bigger than the boat.
7.0.28	6:33	T/R 1:	Did you want the sea monster to be bigger than the boat?
7.0.29	6:36	Mark:	No.
7.0.30	6:36	T/R 1:	You really didn't. What about the boat and the children?
7.0.31	6:42	Mark:	Those too. The children are bigger.
7.0.32	6:43	T/R 1:	The children are bigger than the boat. Did you want that? [Mark, still smiling, shakes his head sideways, indicating negation.] No.
7.0.33	6:48	Michael:	Maybe he was trying to focus on the children and instead of just the boat.
7.0.34	6:52	Erik:	Yeah, he was probably trying maybe to make them look bigger, like you're looking at the children, not the boat
7.0.35	6:55	T/R 1:	I, I
7.0.36	6:58	Erik:	like he doesn't, he just put the boat in
7.0.37	7:01	T/R 1:	Yeah
7.0.38	7:03	Erik:	Cause they're at the dock. Yeah, but he wasn't just focusing on the boat.
7.0.39	7:04	T/R 1:	Maybe the boat wasn’t intended to be so close, but that he could make it, you know, the dock, not as far out as those things. Or maybe he didn't think about it, all those things. That wasn’t what he was focusing on, but I think suppose changed this, suppose we took this story and made it a math problem. Suppose we changed it for a different purpose. And I said to all of you, I want you to go and make me a model of two children, right, and they're sitting at a dock and they're fishing, and they just caught a fish, right? Let's not make it a sea monster and let's change it a little bit, they're fishing and then their boat is docked somewhere, do you understand? If I asked you to do that and it mattered now what sizes they were. What would you expect to be the largest object and the next and the next? What would you expect if you were really worrying about the size, you had two children at a dock and you have a boat and you have a fish, now we’re not going to go with sea monsters. Mark?
7.0.40	8:09	Mark:	Uh, the boat's the biggest.
7.0.41	8:10	T/R 1:	The boat's the biggest. Do you agree?
7.0.42	8:12	Students:	Mm, hmm [nods of affirmation from various students].
7.0.43	8:13	T/R 1:	You think the boat's the biggest. okay.

7.0.44	8:15	Mark:	And then the children, um, and then the fish.
7.0.45	8:20	T/R 1:	Would be the smallest. You all agree with that?
7.0.46	8:23	Students:	Um, hmm.
7.0.47	8:24	T/R 1:	Ok. Is there anyone who disagrees with that? Now suppose we said okay. We're all agreeing that that's our assignment and you're all supposed to go home and do that. Would you all make the same model?
7.0.48	8:37	Students:	No
7.0.49	8:38	T/R 1:	But, now what would be the same about all of your models? What wouldn't change about all of your models, Beth?
7.0.50	8:45	Beth:	We'd have the same idea.
7.0.51	8:46	T/R 1:	And what's that idea that would be the same?
7.0.52	8:48	Beth:	That two people, fish and...[inaudible]... and the boat [inaudible]
7.0.53	8:58	T/R 1:	Okay, is that enough? Because we have that here.
7.0.54	9:02	Beth:	Their sizes are, the boat's biggest and then the children
7.0.55	9:12	T/R 1:	And then, and then the fish. Okay so you agree with that, that there are certain things that all of you would have in your model. You have these four principal players right or things. The boat, two children and you have the fish. What size will you make them will that necessarily be the same? Meredith?
7.0.56	9:33	Meredith:	Well, maybe not because everybody can't have the same because they don't have, they're not like copying each other
7.0.57	9:41	T/R 1:	Yeah. You make your children. some of you might use little dolls or something, right or bigger dolls, or
7.0.58	9:47	Meredith:	You're not measuring the same.
7.0.59	9:48	T/R 1:	You wouldn't measure them the same. But one thing that would be the same is the relative, one thing you have to be careful each of you in your own models would be the sizes in relation to the other sizes, right? And if somebody came in now with a fish bigger than the boy, [laughing] that would have missed the point, right?
7.0.60	10:06	Michael:	No, because a fish could be bigger than a boy.
7.0.61	10:10	T/R 1:	That's true, ok, that's true. But we really mean two children fishing at a little dock, not out in the ocean somewhere where we expect the fish to be smaller, but you're right, you're absolutely right Michael, it could be. But we'd have to agree on some things, on some constraints, here. Obviously if we changed it and we were deep sea fishing right, and we could

			be catching some whales or whatever. Some very, very big fish. That would change things. Now, what does that have to do with the models you made and some of the comments that Michael and Erik made about the models you made? What does that have to say about it? Or anything? Thank you very much Mark [returns his diorama to Mark at his desk]. I'm glad I saw that lovely model. Does it have anything to do with the models we make in order to make an argument. Would you expect one model to come up with something different than the other? Would it look different?
7.0.62	11:13	Students:	Yes
7.0.63	11:15	T/R 1:	Maybe. Would the relationships that you're suppose to show change?
7.0.64	11:16	Students:	No
7.0.65	11:18	T/R 1:	No. And that's the important thing to remember. That your model that you make should not be changing, right, your argument. But suppose Mark had his model and suppose Danielle made a model, ok? And Danielle decided to make a very little model okay a little tiny model? She doesn't like to carry big things to school. And let's suppose that Audra made a big model, right? She got some help. Could I take the fish in Danielle's model, the little fish in Danielle model, and swap it, or let's take Audra's big fish, can I put it in Danielle's little box. No. Well, it depends on how big the fish is.
7.0.66	12:18	T/R 1:	Danielle's little box is really a little box, so, it's, um, you know, about this size [she holds horizontally a thermos bottle approximately 10 inches long]... and Audra's is like that [with her hands she shapes in air a box approximately two feet by two and one half feet] and so Audra's fish is maybe about this big [she holds two pens together in a straight line as these dashes are formed --] and Danielle's fish is about this big [she holds her thumb and forefinger approximately one inch apart]. Would it be okay to put Audra's fish in Danielle's box? No.
7.0.67	12:44	Michael:	It would look like a shrimp!
7.0.68	12:46	T/R 1:	It would look like a shrimp. Why wouldn't it be okay? What would probably happen if you did that? Graham?
7.0.69	12:52	Graham:	Well it wouldn't fit.

7.0.70	12:54	T/R 1:	It wouldn't fit in it. That's exactly right, it probably wouldn't even fit in. Maybe it would but it might not, right? And what would happen, Meredith?
7.0.71	13:02	Meredith:	Well you could put the Audra's fish and you could put Danielle's fish into Audra's box, because it's small and it could fit in.
7.0.72	13:14	T/R 1:	It could be a shrimp [laughing]
7.0.73	13:19	Meredith:	but you can't put Audra's fish into Danielle's box because it's [the box] too small
7.0.74	13:27	T/R 1:	Ok, it raises some interesting questions doesn't it? We're sort of, you know, making up some hypothetical things and imagining some things. But do you get the idea? That once you've built your model and you decide what you are going to call one, right? You've chosen to make your other principal players in relationship to that one, right? So in this case if, if your one is going to be the size of this little stage, if you like [gestures in the air a rectangle approximately one and one half feet by one foot], your players are made the boy the girl the fish the boat in relationship to this stage isn't it.
7.0.75	14:06	Erik:	Mm, hmm [agreeing]
7.0.76	14:07	T/R 1:	But if you've made your one a much bigger stage, if you like[gestures a rectangle approximately three feet by two feet] your players are going to be in relationship to that stage, isn't that right? And as long as you stay within your stage, right, you show your relationships and if they may or may not work when you switch stages right? And that's like switching candy bars right? Isn't that right?
7.0.77	14:24	Erik:	Yup.
7.0.78	14:33	T/R 1:	So I want you to think about that for models. Would you expect if you were building a brand new model that what you showed to be true with your first model, should it still work? Should it still work with the new model, the relationships you showed with your old model? Would you expect it to work if your-
7.0.79	14:45	Meredith:	Maybe, maybe.
7.0.80	14:50	T/R 1:	[As Michael is shaking his head side to side in negation] Michael changed his mind, he doesn't expect it to work Before he said it should work, and now he saying it may not work. So tell me what you're thinking.

7.0.81	15:02	Michael:	Well, your old model, say your old model, you decided it was too little and you couldn't see all the figures in it. So you make a bigger model and you try to take the fish from that little model because you decide that you don't want to make another one, you put it in and you wouldn't be able to see it there.
7.0.82	15:19	T/R 1:	Okay, but that's not my question now. Suppose in your little stage you showed the people and the boat and the fish, right? And you showed the fish were smaller than the people who were smaller than the boat. Right?
7.0.83		Michael :	Yeah, mmm, hmm [agreeing]
7.0.84		T/R 1:	Would you expect, let's say in Audra's model, which is a different model that her fish was smaller than the people and smaller than the boat?
7.0.85	15:44	Michael:	You'd have bigger people, bigger boat and a bigger fish.
7.0.86	15:50	T/R 1:	But should those relationships hold?
7.0.87	15:51	Michael:	Yeah.
7.0.88	15:52	Others:	Yes [simultaneous to Michael's reply]
7.0.89	15:54	T/R 1:	Yeah is that right?
7.0.90	15:55	Michael:	Yeah.
7.0.91	15:55	T/R 1:	Or if we had sort of a medium size model like Mark's and he were trying to make these fit, would you expect the fish to be smaller than the people than the boat?
7.0.92	16:04	Michael:	Yeah.
7.0.93	16:05	T/R 1:	So in each of your models would have those relationships holding, right?
7.0.94	16:07	Students:	Yeah.
7.0.95	16:08	T/R 1:	But they wouldn't all be built the same way and they wouldn't all be the same size
7.0.96	16:11	Erik:	So it'd be, it's standard that the fish would be smaller than the boat and the people, except the fish would be different sized and the people different sized and the boat different sized.
7.0.97	16:20	T/R 1:	Right. Is that like what you're doing when you make models to compare fractions?
7.0.98	16:23	Students:	Yeah.
7.0.99	16:24	T/R 1:	In what way is it the same or different? [some students raise their hands] Meredith?
7.0.100	16:28	Meredith:	Well if you have the same question asked and you do it right then you're going to wind up with the same answer and some

			of the models could be bigger and some of them could be smaller.
7.0.101	16:40	T/R 1:	What do the rest of you think? How many of you agree with what Meredith said? [some hands are raised]How many of you disagree? [no additional hands raised, at least in what was visible] How many of you are still not sure? [more hands are raised] You know we have to help the people who are not sure to understand. they don't disagree, but they're still not following. Can someone help? Let's talk about this a little bit more to help them? Who wants to give it a try? [Meredith's hand goes up] Or the people who aren't sure want to tell us what they are confused about. Do you want to talk a little bit? Audra? Jackie? What bothers you and then maybe the people here will try to help, ok? Do you know what the question is? What do you think the question is?
7.0.102	17:33	Jacquelyn:	Um, is the mod- is different models bigger than others and um…
7.0.103	17:44	T/R 1:	Do you want to say that one more time?
7.0.104	17:47	Jacquelyn:	You can use different models that are, they're the same.
7.0.105	17:51	T/R 1:	Is that the question? What do you think, Audra, is that your question? You think it's a different question? Maybe we are answering a different question. Let's see what Audra thinks the question is and then we can hear from those of you who can try to help.
7.0.106	18:06	Audra:	[hesitantly] It's that we um, it's about, ah, there are different sizes of, just like the candy bar that we did before. Um, you were asking, um if we thought what sizes can fit into each box, what sizes should be that we are going to get confused that the fish can fit into a box.
7.0.107	18:48	T/R 1:	Who else is confused, what you think the problem is? There are a some other people who are confused, or aren't sure. Laura? Are you in this category of not being sure? [Laura nods affirmatively] What do you think the question is?
7.0.108	19:04	Laura:	I'm not sure.
7.0.109	19:06	T/R 1:	You're not sure what the questions is. Okay, well, that's a start. Maybe if we got the question, if we understood the question, that might help us. Who's going to try with the question? [Erik's hand is raised.] Go ahead Erik, give it a try, because we also have some people coming in

7.0.110	19:18	Erik:	It has to do with the model that Mark made. Can the fish, the fish should be smaller than the people in the boat, but the people should be bigger than the boat, or, no, they should be bigger than the fish, but they shouldn't be bigger than the boat either. And how does that, how do those models [pause] help us understand the models we're building [models of fractions built with Cuisenaire rods]?
7.0.111	19:52	T/R 1:	Jackie? Michael do you want to add to that?
7.0.112	19:59	Michael:	Well, it's sort of like um, you can't, the fish has to be smaller than the people and the people have to be smaller than the boat, cause the people have to go in the boat and the people have to be able to pull the fish out of the water and if it was bigger than it they might have a little trouble getting it out. [laughter] So um, so then, um, its sort of like so, that just helps us understand what we're talking about with the Cuisenaire rods when we are using different sized boxes to make different sized, um, halves and quarters, um, but, they're basically you can call it the same thing as you would then just the small one with the small one if you call the box a whole, and the boat a half it would equal a quarter. You could still do that in Audra's model or any box.
7.0.113	21:11	T/R 1:	Does that help Laura, Audra, or would you like to ask Michael a question? Does anybody want to add to that? We've heard from Michael and we've heard from Erik. Meredith, you were going to say something earlier? [Meredith mutters]. Oh, it was said already?
7.0.114	21:28	Meredith:	Yes.
7.0.115	21:29	T/R 1:	Does anybody want to add to that? Sarah, Beth, okay, well it's something to think about isn't it, as we make, uh, different models. Um I remember that you wrote about the models that you worked on and I, I'm looking forward to reading them and, um, knowing more about they way you think about them. Let's try a different one. Ok, let's try a different one. Let's see what happens here. So this is the problem I would like you to think about. I'm wondering which is bigger, one half or two thirds. [pauses] Now before you model it you might think in your head, before you begin to model it what you is bigger and if so, if one is bigger, by how much. Why

			don't you work with your partner and see if you can figure it out.
7.0.234	24:46 F	Erik:	One half, where's the dark green, one half or two thirds.
7.0.235		Alan:	This time you [inaudible]
7.0.236		Erik:	This time I what?
7.0.237		Alan:	Two thirds are bigger. Look
7.0.238		Erik:	Exactly
7.0.239		Alan:	Two thirds are bigger by one sixth. And one half is one bigger than one third by one sixth. But also, making a train model,
7.0.240		Erik:	Oh no
7.0.241		Alan:	Create a chain reaction using the theory of relativities
7.0.242		Erik:	Ok, it's bigger by
7.0.243		Alan:	Who's using up all the twosies?
7.0.244		Erik:	It can't be done. Can't be done.
7.0.245		Alan:	A half is not bigger than two thirds.
7.0.246		Erik:	Oh this is the exact-
7.0.247		Alan:	This is one half
7.0.248		Erik:	This is the exact same problem we had before except it's one third, remember?
7.0.249		Alan:	It's only one sixth
7.0.250		Erik:	This is easy. One half is larger than one third but smaller
7.0.251		Alan:	It's still one sixth
7.0.252		Erik:	Of course. It's larger by one little sixth. [looks for pencil] Ok. There! I did it.
7.0.253		Alan:	I did it. I know another way to figure it out. Create a balance.
7.0.254		Erik:	Make the balance like this.
7.0.255		Alan:	This would be a half, this would be two thirds. Determine which is bigger. Two thirds are bigger.
7.0.256		Erik:	But you have to do it like this. Ok, here we go. Ok, now, one half, uh, give me two more reds please. Two thirds. Let me support this. Perfect! It stays! It's equal.
7.0.257		Alan:	No, they're not equal. Look here. Those are halves
7.0.258		Erik:	It's equal
7.0.259		Alan:	These are thirds
7.0.260		Erik:	The balance is equal. But if I do it like this, with the orange, it's very, very different. Two thirds is bigger.
7.0.261		Alan:	Ok. Look. These are two thirds. Which is bigger? See? This is bigger [uses train model].

7.0.262	Erik:	Well, one half...
7.0.263	Alan:	Erik,
7.0.264	Erik:	Yeah?
7.0.265	Alan:	Look. This is two thirds.
7.0.266	Erik:	Yeah, I know.
7.0.267	Alan:	That is one half. Which is bigger, the two thirds or the half?
7.0.268	Erik:	Two thirds. Of course!
7.0.269	Alan:	You're right!
7.0.270	Erik:	Now I can easily make a train model.
7.0.271	Alan:	You can easily quarter it.
7.0.272	Erik:	Could I have the purples? Thank you, three purples, that's all I needed.
7.0.273	Alan:	We still haven't [inaudible]
7.0.274	Erik:	What? Dark green! Oh no, that's a black. Let's see, where's another dark green, where's another dark green, ah! There we go!
7.0.275	T/R 1:	Gentlemen? What do you think?
7.0.276	Alan:	He used up my example.
7.0.277	Erik:	I have it right here!
7.0.278	T/R 1:	Ok, is it possible to make another example, Alan?
7.0.279	Alan:	Yeah I guess.
7.0.280	T/R 1:	Would it still work?
7.0.281	Alan:	Yeah.
7.0.282	T/R 1:	You're sure it would work?
7.0.283	Erik:	Just like we did! Two after the other can be third-
7.0.284	T/R 1:	By the way, which is bigger?
7.0.285	Alan:	Ok. We figured out by taking out
7.0.286	Erik:	Because if you have, we figured that, well, let me just see, right here, both models we have the halves and the thirds. Like, it was like the other problem, it was one half and one third. And we explained it, we said that one half was bigger than one third but smaller than two thirds. Like up here, there's one half right there, and there's the thirds, there's the second third
7.0.287	T/R 1:	By how much?
7.0.288	Erik:	One sixth.
7.0.289	T/R 1:	But one half and two thirds.
7.0.290	Erik:	One- oh that's exactly, that's exactly what we meant. These are two thirds and that's one half

7.0.291		Alan:	With one of the thirds, it would be a sixth. But if you added one, it would still be one sixth.
7.0.292		T/R 1:	Ok, could you write it up and any others you can find, gentlemen? And be ready-
7.0.293		Erik:	What do we do, just diagram them?
7.0.294		Alan:	Yeah, just diagram them
7.0.295		Erik:	Just diagram which ones we did?
7.0.296		Alan:	Yeah we just
7.0.297		Erik:	And then write about them.
7.0.298		Alan:	Yeah, I'll just, we just have to diagram one of 'em
7.0.299		Erik:	No, I'll diagram both
7.0.300		Alan:	Yeah, same here. I diagrammed both on the one third bigger than one half by how much, you know? I did both on that one. I did those over the weekend.
7.0.301		Erik:	I'm going to trace them just to get the exact size. You're writing?
7.0.302	36:13 F	Danielle:	Well, we've got, we've got that whole, this is the whole, we have the three thirds, and we then the half
7.0.303		T/R 3:	And what we supposed to figure out after we did that?
7.0.304		Danielle:	Which is bigger a half or two thirds?
7.0.305		T/R 3:	Oh, I want to know. Is it still the same or does it change when your model changes?
7.0.306		Danielle:	Two thirds is still bigger.
7.0.307		T/R 3:	How much? [Danielle begins to line up white rods] Let's line 'em up. Two thirds is bigger, but now I want to know by how much. Can you figure that? [Gregory passes white rods to Danielle. Talk about getting white rods] You need some more whites. Uh, how many more do you think you need? A bunch? Takes a lot, doesn't it? How many do you think…
7.0.308		Danielle:	Eighteen
7.0.309		T/R 3:	Hmm. So how much larger?
7.0.310		Danielle:	It's bigger
7.0.311		Gregory:	One eighteenth
7.0.312		T/R 3:	[Danielle begins to dismantle her model to show the comparison] You can use some more of these if you want
7.0.313			[T/R 1 talks to class about writing about more than one solution]
7.0.314		Danielle	It's bigger by three eighteenths.

7.0.315	T/R 3:	My goodness, tell me, help me remember what it was over there.
7.0.316	Danielle:	It was bigger by one, one sixth.
7.0.317	T/R 3:	Ok, so does that mean we have a different answer? No? This is different from the other one or the same?
7.0.318	Danielle:	It's different in a way and it's the same in a way
7.0.319	T/R 3:	How's it different and how's it the same?
7.0.320	Danielle:	Well, it's the same because the half is smaller and it's different because, um, this one, it only ta- the little box are only um, two three four, there's only six of them and here's there's eighteen, and this, the thirds are bigger by three eighteenths
7.0.321	T/R 3:	You mean, yeah, the two thirds are bigger by three eighteenths
7.0.322	Danielle:	and the two thirds over here is bigger by one sixth
7.0.323	T/R 3:	Mmm hmm. And so you think that you get a different answer if you have different models? As to how much bigger? I agree with you, you're saying that two thirds is still bigger, but it it bigger by a different thing?
7.0.324	Danielle:	Well, [long pause]
7.0.325	T/R 3:	[to Gregory] You're still looking for another way to do it? We found one way over here, we found this way, it seems to me there ought to be something in between, is that what you're thinking? Hmm, I wonder if there's another way. Hmm, she used the orange and the brown, is there something smaller than the brown that you could put together that would work, no add onto the orange? She added the brown to the end of the orange and that got hers to work. This, is there something smaller than this brown that would work attached to this? You tried that one, it didn't work. Let's try this one and see if it can work. Why don't you try the orange and the red. [to Danielle] I'm still concerned about, about whether the three eighteenths is a different answer from the one sixth. You said here that if you have two thirds and a half, oh, there, you said over here [to Gregory] now you have to see if you can do it with thirds, is that right? [to Danielle] Hmm. Look, we have a different model over here, even. So now we have three. I wonder if it's going to be the same as yours, or if it's going to be the same as this one. Is two thirds still

		bigger, Greg, is two thirds still bigger than a half, on this model too, or did it change? [they get another box of rods] Ok, Danielle, what do you think about this time?
7.0.326	Danielle:	Well, um, two thirds
7.0.327	T/R 3:	What is two thirds? Can you build a two thirds and a one half for him separate so we can then compare?
7.0.328	Danielle:	Here's the two thirds, and here's the half
7.0.329	T/R 3:	What's the difference?
7.0.330	Danielle:	and it's bigger by two [counts Gregory's white rods] twelfths. It's, um, it's bigger by two twelfths
7.0.331	T/R 3:	Oh, so is he getting a different answer from that, too, or are they the same? How are the answers, I don't understand, what do you think about this?
7.0.332	Danielle:	One, two three
7.0.333	T/R 3:	Over here it was how much?
7.0.334	Danielle:	This one was bigger by three eighteenths
7.0.335	T/R 3:	And this one?
7.0.336	Danielle:	Was bigger by… how much?
7.0.337	Gregory:	Two twelfths. One two three four five six seven eight nine ten eleven twelve.
7.0.338	T/R 3:	And your original one was
7.0.339	Danielle:	It was bigger by one sixth.
7.0.340	T/R 3:	Oh, so what do you think?
7.0.341	Danielle:	I think they're all different, but then all the same. Cause they're the same because the thirds are always bigger than the half
7.0.342	T/R 3:	The two thirds are always bigger than the half?
7.0.343	Danielle:	And, um, and they're different because these are all, the whites
7.0.344	Gregory:	they're different sizes
7.0.345	Danielle:	They're all different, like one, two, uh three, they're different. So they're different like that.
7.0.346	T/R 3:	Mmm hmm. Is there any other way that you could show that difference here than with the whites? It's the only way you could show it there, isn't it? I don't mean for you to change your model, I mean, is there any other way that you could show me what that difference looks like without using the whites? Or this difference here?
7.0.347	Danielle:	You could use a light green

7.0.348	T/R 3:	What would that be?
7.0.349	Danielle:	That would be one [starts to line up light green] That would be one sixth.
7.0.350	T/R 3:	Hmm. And what did you say it was over here, with the little one?
7.0.351	Danielle:	Um, that's one sixth
7.0.352	T/R 3:	Mmm hmm. So if you used the light green
7.0.353	Danielle:	It could be one sixth
7.0.354	T/R 3:	It could be one sixth. And if you used the whites
7.0.355	Danielle:	It would be three eighteenths
7.0.356	T/R 3:	Mmm hmm. What about for this one?
7.0.357	Danielle:	What problem-
7.0.358	T/R 3:	It was this one here [pointing to Gregory's model using the orange and red]. Uh, Gregory, I want you to watch and see if you agree with what Danielle is doing here. [Danielle lines up red rods on Gregory's model]
7.0.359	Danielle:	[After lining up and counting six red rods, Danielle shows that he two white rods that show the difference between one half and two thirds is equal in length to the one red rod] And then that would be one sixth too.
7.0.360	T/R 3:	Mmm, over each of 'em?
7.0.361	Danielle:	That would be one sixth, that would be one sixth, and that one would be one sixth.
7.0.362	T/R 3:	But you have, had two, two different names for the answer if you did it this way it was
7.0.363	Danielle:	It was two twelfths
7.0.364	T/R 3:	And, and, uh, Gregory, for this one over here, where she had the three, what was the name for that one?
7.0.365	Gregory:	Three eighteenths.
7.0.366	T/R 3:	Yeah, it was.
7.0.447	T/R 1:	[Jackie, Erin, and Jessica have built two models at the OHP] I'm going to have to have to stop you for a minute, I know that, I hate to do this because I know you're working all so hard, but I would like to spend ten minutes, uh, just have us think about a few things, and you can finish this, is that ok, Mrs. Phillips if they can finish writing this up for us when we come back on Wed? Ok, so you really have today and maybe some time tomorrow to finish writing this up. I, I would like all of you though to sort of give me your attention for a

			minute, um, because I'm wondering about a few things. I need you to help me straighten out some things in my head, and if you can help me straighten them out in my head, you may be helping out other people straighten them out also. So I'm going to ask you some, some important questions, are you all listening to my questions? If you could stop what you're doing for a moment, I know it's hard, and listen to my questions. How many of you made one model and absolutely are convinced, that you know by your model, which is bigger one half or two thirds? How many of you did that with one model and you are absolutely convince with your model you know which is bigger, one half or two thirds. Would you please raise your hand if you made one model, you could have made more than one but you made at least one. If you made at least one model, girls, and you're absolutely convinced [All visible hands raised] No one could persuade you otherwise that you know which is bigger, one half or two thirds. Alright, so tell me, which is bigger?
7.0.448		Students:	Two thirds.
7.0.449	47:59	T/R 1:	Again?
7.0.450	48:36	Students:	Two thirds!!
7.0.451		T/R 1:	And you also know how much bigger. How many of you are convinced you know how much bigger and no one can persuade you otherwise that two thirds is not only bigger than a half but it is how much bigger?
7.0.452		Students:	One sixth.
7.0.453	48:40	T/R 1:	How many of you believe one sixth? [All visible hands raised] That's what I thought Walking around I thought that…that is what I believe that everyone has done. How many of you made a second model?
7.0.454		Meredith:	Oh, Oh! [eagerly]
7.0.455		T/R 1:	You could have made more than two, but you made at least two models. And in your second model you got a different answer. You got an entirely different answer, you no longer have two thirds bigger than one half, you showed something else. Are you watching? Some of you got a second model that showed something different. Meredith? Let's listen to what Meredith says. Girls [Jackie, Erin, and Jessica], why don't we stop that for just a moment and then we'll make

			some more. Listen, listen. You [Meredith] think you showed something else in your second model? [Meredith goes to the overhead projector and places twelve white rods beneath the two dark green rods in the girls' second model - two of the other girls help her find and position the white rods] I am really confused. I have no ideas what Meredith is going to do. Because I thought I understood this and I thought she was going to tell me she got the same answer. Did you think that? And now she's telling me no. [Meredith is smiling.] I'm going to get so confused. You are all going to have to help me. [Pause.] How many of you built a second model that looks like that model up there that Meredith is fiddling with? How many of you have a model that looks like that? [Many hands are raised.] By the way, what was one in that model? What did you call one in that model? Amy?
7.0.456		Amy:	Ah, the orange and red.
7.0.457		T/R 1:	How many of you called 'one' orange and red in that model? Yeah, you did that model. Did you get to have two thirds bigger than a half?
7.0.458	50:53	Michael:	No. [Michael raises his hand, shaking his head from side to side, signifying dissent.]
7.0.459	50:56	T/R 1:	Amy got two thirds bigger than a half in that model, how many of you got two thirds bigger than a half in that model, where the orange and the red were one. Michael didn't, Meredith did. You didn't Michael?
7.0.460	51:04	Michael:	No, they can't do that. [He begins to stand.] Because um, the, the two thirds are bigger than the half by a red. So they can't use those whites to show it.
7.0.461	51:08	T/R 1:	Oh, but you're saying that, you're saying that two thirds, what's a third?
7.0.462	51:21	Michael:	A third is the purple [He begins to approach the overhead projector.]
7.0.463	51:31	T/R 1:	And what's two thirds? Just tell us.
7.0.464	51:37	Michael:	[He returns to his seat.] Um, two thirds is two purples.
7.0.465	51:39	T/R 1:	Did you all do that? Did you get two thirds to be two purples? [She addresses Michael] And what did you get to be one half?
7.0.466	51:42	Michael:	Uh, dark green.

7.0.467	51:43	T/R 1:	[She addresses the class.] Did you get dark green to be a half? [Mutterings of assent occur.] And you got two thirds to be bigger than one half?
7.0.468	51:49	Michael:	[Politely impatient] Yes.
7.0.469	51:51	T/R 1:	By how much?
7.0.470	51:56	Michael:	[Deliberately, again almost impatiently] By one sixth.
7.0.471	51:57	Meredith:	Or, or two twelfths.
7.0.472	51:58	Michael:	[Shaking his head sideways] No.
7.0.473	52:00	:	[Mutterings in the classroom of no.]
7.0.474	52:02	T/R 1:	Tell us Meredith. Aha! How many of you got one sixth? [Most hands are raised.] And what rod did you use to represent one sixth? What color rod?
7.0.475		Students:	Red.
7.0.476	52:04	T/R 1:	How many of you used a red rod to represent one sixth in that model and you showed it was bigger by one sixth? And Meredith says she did it a little differently and she didn't get one sixth. And what did you get Meredith?
7.0.477	52:15	Meredith:	Two twelfths.
7.0.478	52:15	T/R 1:	What do you think about that?
7.0.479	52:26	Students:	Well, in a way. No. Uh, uh [negatively].
7.0.480	52:29	T/R 1:	Well, she showed it's bigger by the two whites, she shows two whites bigger.
7.0.481	52:32	Michael:	Yeah, but then she would have to call the two whites together one sixth.
7.0.482	52:35	Erik:	Yeah, exactly.
7.0.483	52:40	Michael:	She's calling the whites, one white one sixth.
7.0.484	52:44	Erik:	Yeah, she said
7.0.485	52:45	T/R 1:	She's calling one white one sixth?
7.0.486	52:46	Meredith:	No I'm not, I'm calling it one twelfth.
7.0.487	52:50	T/R 1:	She's calling one white one twelfth.
7.0.488	52:52	Erik:	Yeah, but see just the whites together. That'd be right, it would be two twelfths. But you have to combine them. You can't call them, you can call them separately, but you could also call them combined and if you combine them it would be uh, one sixth.
7.0.489	52:53	T/R 1:	Ok, but she didn't combine them and she's calling the two whites together, again, Meredith?
7.0.490	52:53	Meredith:	One twelfth, two twelfths.

7.0.491	53:11	T/R 1:	Two twelfths. She's calling [Michael, still seated, shakes his head sideways in dissent and fingers some rods.] Do you all agree that one white has a, the number name for the white rod is one twelfth? Someone told me that when I was walking around, it might have been Audra. And some other people told me a white would be one twelfth? Is that true? And two white rods would be
7.0.492	53:15	Students:	Two twelfths.
7.0.493	53:17	T/R 1:	Two twelfths. And one red would be
7.0.494	53:31	Students:	One sixth.
7.0.495	53:32	T/R 1:	One sixth. So, so what is Meredith saying here?
7.0.496	53:35	Meredith:	There's two answers.
7.0.497	53:37	T/R 1:	Are there two answers?
7.0.498	53:41	Michael:	[simultaneously with Erik] No, they're the same answer.
7.0.499	53:43	Erik:	No, they're the exact same thing, except she, she took the red and divided it into half, she divided it into halves, into half and called, and called each half one twelfth. They're the exact same answer except they're just in two parts.
7.0.500	53:44		[Note all written notation will be enclosed in […] as it is recorded by the teacher]
7.0.501	53:47	T/R 1:	[Joins the four girls at the overhead projector] Let me write this down. This, what you are saying here is so important, here. Let me see if I can write this down. You're saying that you're calling the red, you're giving red the number name, right? The length of the red, right? We'll give it the number name, what did you say?
7.0.502		Students:	One sixth.
7.0.503	54:02	T/R 1:	[R one sixth] One sixth. And two whites, can I write two 'w ' for two whites?
7.0.504	54:23	Students:	Yeah.
7.0.505	54:25	T/R 1:	And you're calling two whites
7.0.506	54:29	Students:	Two twelfths.
7.0.507	54:31	T/R 1:	[2W two twelfths] Two twelfths. But what Erik just told me, right?, is something about red and white.
7.0.508	54:33	Erik:	Yeah. A red, one red equals, one red rod up here, one red equals two of the white ones.
7.0.509	54:36	T/R 1:	[1R = 2W] So we're talking about the length of the red rod, the length of the red rod is the same as the length of the two white rods? [On the overhead projector, Meredith builds a

			model with one red rod as the base and places two white rods directly above it.] Is that true? Do you all agree to that?
7.0.510	54:43	Students:	Yeah. Yes.
7.0.511	55:03	Erik:	And since she's calling a white rod one twelfth and the other white rod one twelfth and the red rod is really one sixth. But, when she calls them two twelfths, the two twelfths are actually just two white rods put together to equal a red, so it should be really, it's really one sixth. Because two whites, two whites
7.0.512	55:12	T/R 1:	She says one white is a twelfth [1/12] and then if you put it together with another one twelfth [+1/12], she's saying you get two twelfths [= two twelfths].
7.0.513	55:12	Erik:	And it's one sixth, it's one sixth.
7.0.514	55:39	T/R 1:	And you're saying if you have, if you take one half that's all right? [1/2] If you're taking one half of one sixth [of one sixth], you're saying you get one twelfth [= 1/12] You're saying that. That's the two things I'm hearing. Right? And you're saying that [one sixth], the length of one sixth is the same as the length of two twelfths. [= two twelfths] Is that what you are saying?
7.0.515	55:47	Erik:	Yeah.
7.0.516	55:48	T/R 1:	All those things, are they true?
7.0.517	56:15	Erik:	Yeah. But I don't really think you could call, call them two twelfths because two twelfths equal exactly to the same size as one sixth. Well, if you want to you could call them, I guess. But I think it would be easier just to call them one sixth, then wouldn't want to exactly call them one twelfth and another twelfth. I'd just call them one sixth. Therefore I think you just really call them one sixth.
7.0.518	56:16	Student:	Well, maybe you can call them
7.0.519	56:18	Erik:	Well you can call them, if you want to, but
7.0.520	56:46	T/R 1:	Well, we have different number names for these rods
7.0.521	56:47	Student:	they’re not different
7.0.522	56:49	Brian:	There's just half of one, there's just half of one.
7.0.523	56:53	T/R 1:	So you're saying that one half of the one sixth is another way of saying one twelfth.
7.0.524	57:00	Brian:	They're just two answers.
7.0.525	57:01	T/R 1:	Well, you're saying if you took a twelfth, a rod that has length one twelfth, and another rod that has length one

			twelfth and put them together, right? That rod would have length two twelfths. Isn't that what you said?
7.0.526	57:09	Jessica:	What Erik said is that two whites equal one red, so it would be the exact same thing.
7.0.527	57:09	T/R 1:	Or a rod that has length one sixth, that would be the red one in this problem, would also have length two twelfths. Is that what you said when you talk about the lengths of the rods? So are all of these [pointing to the recorded notations on the overhead projector] true statements?
7.0.528	57:19	Students:	Yeah.
7.0.529	57:26	T/R 1:	That's amazing. Look at all the fancy mathematics you're doing, that's amazing. That's something for us to think about, ok? So Meredith is still saying that, "I don't disagree with you when I say that it's a red bigger in this model," right?
7.0.530	57:38	Meredith:	Um, hmm.
7.0.531	57:39	T/R 1:	I'm just going to give this red a different number name. I could give it the number name one sixth, if I think about it when I compare it to the rod I call one, the orange and red, I could give it the number name one sixth. Or, if I'm thinking about the white rods, right? I could give it the number name two twelfths. And that's very interesting. Does that contradict what you're doing? Or does it still work, what you're doing? It still works, Meredith thinks. That's something to think about, isn't it? That's very interesting, thank you for sharing that. Well, I think we've run out of time. Um, there's a lot of things to write about. We have a whole lot of new ideas, don't we? I really hope that you write to me about your different models and I hope when you write to me and show me as many models as you can. That you will also, you will also, think about, in your models. What is different about each of those models? Write a statement about each of those models that makes it a different model, okay? And then, what is alike about all of those models? Is that a good question, Meredith?
7.0.532	57:55	Meredith:	Mm, hmm.
7.0.533	57:56	T/R 1:	You can think about that question and write to me about it, I'd really like to know what you're thinking… What is different and what is alike. I can't wait to read what you write to me

7.0.534 59:19 End of class