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| Line 7.0.1 7.0.2 7.0.3 | Time 2:33 S 2:39 2:40 | Speaker T/R 1: Meredith: T/R 1: | Transcript Good morning! Are you all as awake as I am? Yeah. I don't know if that is good or bad, Meredith. Let me shut this [the overhead projector fan] off. [Holding up Mark's diorama] I was thinking when I was looking at Mark's model, and I noticed many of you made models, also for projects for another class. I was thinking about this model because we |
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| | | | were talking about models the other day, weren't we. Remember that? And I, remember I asked you to think about something about the models that you built. Remember what I asked you to think about? Does anyone remember, Andrew? |
| 7.0.4 | 3:12 | Andrew: | Um, is one-half bigger, uh is one half bigger than one fourth, by how much? |
| 7.0.5 | 3:21 | T/R 1: | Does anyone else remember anything in our discussion about models? Andrew remembered something. Is your hand up Audra? |
| 7.0.6 | | Audra: | No. |
| 7.0.7 | 3:32 | T/R 1: | Audra's yawning. Does anyone remember anything in our discussion about models? We talked about models, we asked some questions about them. Think for a minute. Do you remember Meredith? |
| 7.0.8 | 3:45 | Meredith: | Um, what's bigger, one half or one quarter and by how much? |
| 7.0.9 | 3:50 | T/R 1: | That's what Andrew said. Right, which is bigger. But we also were talking about models in general. We asked ourselves some questions about models. Did you all build the same model? |
| 7.0.10 | 4:04 | Students: | No. |
| 7.0.11 | 4:04 | T/R 1: | To answer that question? |
| 7.0.12 | 4:05 | Students: | No, no. |
| 7.0.13 | 4:07 | T/R 1: | Some of you built different models. [Erik raises his hand] Erik? |
| 7.0.14 | 4:10 | Erik: | Some of us built the same models and some of us built different. |

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| 7.0.15 | 4:12 | T/R 1: | Some of you built different models, and I asked you a |
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| 7.0.16 | 1.10 | Erik: | question about that. Do you remember? |
| 7.0.10 | | T/R 1: | [Raising his hand] Oh! Erik? |
| 7.0.17 | | Erik: | Could you get different answers |
| 7.0.19 | 7.20 | Michael: | Using barred models? |
| 7.0.20 | | Erik: | if you use different models. |
| 7.0.20 | 4:27 | T/R 1: | Yeah, can you get different answers, right Michael and Erik? - if you use different models. What did you think? How many of you thought you shouldn't get different answers? [Some hands are raised] How many of you are not sure? [Few more hands are raised] It's okay not to be sure. Have you been thinking about that at all since then? Maybe not much. Michael, have you been thinking about that a little bit? |
| 7.0.22 | 4:48 | Michael: | Um, well, I figured that it couldn't be because our answer that we got, me and Brian, was that it was bigger by one fourth because it will always take two, it will always take four quarters to equal up |
| 7.0.23 | 5:03 | Erik: | Yeah, because four is an even number and you can divide it by two. |
| 7.0.24 | 5:04 | Michael: | In half |
| 7.0.25 | 5:08 | Erik: | So there will always be one fourth and two fourths, three fourths, four fourths and two fourths is always going to be a half, a half in fourths. |
| 7.0.26 | 5:16 | T/R 1: | What do you think about that? There are a lot of good ideas in what you are saying. [Picking up Mark's diorama] I was thinking that maybe it would help you, it sort of helped me to look at Marks' model. Sometimes it helps to look at a model that's a little different. Maybe this is a model that doesn't use the Cuisenaire rods, but in a sense it's a model. Um, I found out a little bit from Mark about a book he read, he was telling me. This [pointing into diorama] was supposed to be a sea monster and this was supposed to be [again pointing into the diorama] two friends. And I looked at, I looked at what he built here[still pointing to diorama] to represent some of the story and I thought by looking at this model that I couldn't really tell of the boy and the girl who was taller by looking at them, I wasn't really sure, and I didn't know really if Mark |

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| | | | cared about that. But I looked at the sea monster, okay, and I |
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| | | | looked at the boat, okay, and I was thinking about their sizes |
| | | | a little bit, right? What are you, why are you smiling about |
| | | | Mark? |
| 7.0.27 | 6:27 | Mark: | Uh, well I wasn't thinking about the sizes. I made the sea |
| | | | monster bigger than the boat. |
| 7.0.28 | 6:33 | T/R 1: | Did you want the sea monster to be bigger than the boat? |
| 7.0.29 | 6:36 | Mark: | No. |
| 7.0.30 | 6:36 | T/R 1: | You really didn't. What about the boat and the children? |
| 7.0.31 | 6:42 | Mark: | Those too. The children are bigger. |
| 7.0.32 | 6:43 | T/R 1: | The children are bigger than the boat. Did you want that? |
| | | | [Mark, still smiling, shakes his head sideways, indicating |
| | | | negation.] No. |
| 7.0.33 | 6:48 | Michael: | Maybe he was trying to focus on the children and instead of |
| | | | just the boat. |
| 7.0.34 | 6:52 | Erik: | Yeah, he was probably trying maybe to make them look |
| | | | bigger, like you're looking at the children, not the boat |
| 7.0.35 | 6:55 | T/R 1: | I, I |
| 7.0.36 | 6:58 | Erik: | like he doesn't, he just put the boat in |
| 7.0.37 | | T/R 1: | Yeah |
| 7.0.38 | 7:03 | Erik: | Cause they're at the dock. Yeah, but he wasn't just focusing |
| | | | on the boat. |
| 7.0.39 | 7:04 | T/R 1: | Maybe the boat wasn't intended to be so close, but that he |
| | | | could make it, you know, the dock, not as far out as those |
| | | | things. Or maybe he didn't think about it, all those things. |
| | | | That wasn't what he was focusing on, but I think suppose |
| | | | changed this, suppose we took this story and made it a math |
| | | | problem. Suppose we changed it for a different purpose. And |
| | | | I said to all of you, I want you to go and make me a model of |
| | | | two children, right, and they're sitting at a dock and they're |
| | | | fishing, and they just caught a fish, right? Let's not make it a |
| | | | sea monster and let's change it a little bit, they're fishing and |
| | | | then their boat is docked somewhere, do you understand? If I |
| | | | asked you to do that and it mattered now what sizes they |
| | | | were. What would you expect to be the largest object and the |
| | | | next and the next? What would you expect if you were really |
| | | | |
| | | | worrying about the size, you had two children at a dock and |

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| | | | you have a boat and you have a fish, now we're not going to |
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| | | | go with sea monsters. Mark? |
| 7.0.40 | 8:09 | Mark: | Uh, the boat's the biggest. |
| 7.0.41 | 8:10 | T/R 1: | The boat's the biggest. Do you agree? |
| 7.0.42 | 8:12 | Students: | Mm, hmm [nods of affirmation from various students]. |
| 7.0.43 | 8:13 | T/R 1: | You think the boat's the biggest. okay. |
| 7.0.44 | 8:15 | Mark: | And then the children, um, and then the fish. |
| 7.0.45 | 8:20 | T/R 1: | Would be the smallest. You all agree with that? |
| 7.0.46 | | Students: | Um, hmm. |
| 7.0.47 | 8:24 | T/R 1: | Ok. Is there anyone who disagrees with that? Now suppose |
| | | | we said okay. We're all agreeing that that's our assignment |
| | | | and you're all supposed to go home and do that. Would you |
| | | | all make the same model? |
| 7.0.48 | | Students: | No |
| 7.0.49 | 8:38 | T/R 1: | But, now what would be the same about all of your models? |
| 7050 | 0.45 | | What wouldn't change about all of your models, Beth? |
| 7.0.50 | | Beth: | We'd have the same idea. |
| 7.0.51 | | T/R 1: | And what's that idea that would be the same? |
| 7.0.52 | 8:48 | Beth: | That two people, fish and[inaudible] and the boat |
| 7052 | 0.50 | T/D 1. | [inaudible] |
| 7.0.53 | | T/R 1: | Okay, is that enough? Because we have that here. |
| 7.0.54 | | Beth: | Their sizes are, the boat's biggest and then the children |
| 7.0.55 | 9:12 | T/R 1: | And then, and then the fish. Okay so you agree with that, that |
| | | | there are certain things that all of you would have in your model. You have these four principal players right or things |
| | | | model. You have these four principal players right or things. The boat, two children and you have the fish. What size will |
| | | | you make them will that necessarily be the same? Meredith? |
| 7.0.56 | 0.33 | Meredith | Well, maybe not because everybody can't have the same |
| 7.0.50 | 1.55 | Micrountil. | because they don't have, they're not like copying each other |
| 7.0.57 | 9· 4 1 | T/R 1: | Yeah. You make your children. some of you might use little |
| 1.0.57 | 2.11 | 1/10 11 | dolls or something, right or bigger dolls, or |
| 7.0.58 | 9.47 | Meredith | You're not measuring the same. |
| 7.0.59 | | T/R 1: | You wouldn't measure them the same. But one thing that |
| 110109 | 2110 | 1/1011 | would be the same is the relative, one thing you have to be |
| | | | careful each of you in your own models would be the sizes in |
| | | | relation to the other sizes, right? And if somebody came in |
| | | | now with a fish bigger than the boy, [laughing] that would |
| | | | have missed the point, right? |
| | | | 1 / U |

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| 7.0.60 7.0.61 | 10:06 10:10 | Michael: T/R 1: | No, because a fish could be bigger than a boy. That's true, ok, that's true. But we really mean two children fishing at a little dock, not out in the ocean somewhere where we expect the fish to be smaller, but you're right, you're absolutely right Michael, it could be. But we'd have to agree on some things, on some constraints, here. Obviously if we changed it and we were deep sea fishing right, and we could be catching some whales or whatever. Some very, very big fish. That would change things. Now, what does that have to do with the models you made and some of the comments that Michael and Erik made about the models you made? What does that have to say about it? Or anything? Thank you very much Mark [returns his diorama to Mark at his desk]. I'm glad I saw that lovely model. Does it have anything to do with the models we make in order to make an argument. Would you expect one model to come up with something different than the other? Would it look different? |
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| 7.0.62 | 11:13 | Students: | Yes |
| 7.0.63 | 11:15 | T/R 1: | Maybe. Would the relationships that you're suppose to show change? |
| 7.0.64 | 11:16 | Students: | No |
| 7.0.65 | 11:18 | T/R 1: | No. And that's the important thing to remember. That your model that you make should not be changing, right, your argument. But suppose Mark had his model and suppose Danielle made a model, ok? And Danielle decided to make a very little model okay a little tiny model? She doesn't like to carry big things to school. And let's suppose that Audra made a big model, right? She got some help. Could I take the fish in Danielle's model, the little fish in Danielle model, and swap it, or let's take Audra's big fish, can I put it in Danielle's little box. No. Well, it depends on how big the fish is. |
| 7.0.66 | 12:18 | T/R 1: | Danielle's little box is really a little box, so, it's, um, you know, about this size [she holds horizontally a thermos bottle approximately 10 inches long] and Audra's is like that [with her hands she shapes in air a box approximately two feet by two and one half feet] and so Audra's fish is maybe about this big [she holds two pens together in a straight line as these dashes are formed] and Danielle's fish is about this |

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| | | | big [she holds her thumb and forefinger approximately one |
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| | | | inch apart]. Would it be okay to put Audra's fish in Danielle's |
| | | | box? No. |
| 7.0.67 | 12:44 | Michael: | It would look like a shrimp! |
| 7.0.68 | 12:46 | T/R 1: | It would look like a shrimp. Why wouldn't it be okay? What would probably happen if you did that? Graham? |
| 7.0.69 | 12:52 | Graham: | Well it wouldn't fit. |
| 7.0.70 | 12:54 | T/R 1: | It wouldn't fit in it. That's exactly right, it probably wouldn't even fit in. Maybe it would but it might not, right? And what would happen, Meredith? |
| 7.0.71 | 13:02 | Meredith: | Well you could put the Audra's fish and you could put |
| | | | Danielle's fish into Audra's box, because it's small and it could fit in. |
| 7.0.72 | 13:14 | T/R 1: | It could be a shrimp [laughing] |
| 7.0.73 | 13:19 | Meredith: | but you can't put Audra's fish into Danielle's box because it's |
| | | | [the box] too small |
| 7.0.74 | 13:27 | T/R 1: | Ok, it raises some interesting questions doesn't it? We're sort |
| | | | of, you know, making up some hypothetical things and imagining some things. But do you get the idea? That once you've built your model and you decide what you are going to call one, right? You've chosen to make your other principal players in relationship to that one, right? So in this case if, if your one is going to be the size of this little stage, if you like [gestures in the air a rectangle approximately one and one half feet by one foot], your players are made the boy the girl the fish the boat in relationship to this stage isn't it. |
| 7.0.75 | 14:06 | Erik: | Mm, hmm [agreeing] |
| 7.0.76 | 14:07 | T/R 1: | But if you've made your one a much bigger stage, if you like[gestures a rectangle approximately three feet by two feet] your players are going to be in relationship to that stage, isn't that right? And as long as you stay within your stage, right, you show your relationships and if they may or may not work when you switch stages right? And that's like switching candy bars right? Isn't that right? |
| 7.0.77 | 14:24 | Erik: | Yup. |
| 7.0.78 | 14:33 | T/R 1: | So I want you to think about that for models. Would you expect if you were building a brand new model that what you showed to be true with your first model, should it still work? |

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| | | | Should it still work with the new model, the relationships you showed with your old model? Would you expect it to work if |
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| 7070 | 1447 | N 11-1 | your- |
| 7.0.79 | | | Maybe, maybe. |
| 7.0.80 | 14:50 | T/R 1: | [As Michael is shaking his head side to side in negation] |
| | | | Michael changed his mind, he doesn't expect it to work |
| | | | Before he said it should work, and now he saying it may not work. So tell me what you're thinking. |
| 7.0.81 | 15:02 | Michael: | Well, your old model, say your old model, you decided it was |
| 7.0.81 | 13.02 | WIICHACI. | too little and you couldn't see all the figures in it. So you |
| | | | make a bigger model and you try to take the fish from that |
| | | | little model because you decide that you don't want to make |
| | | | another one, you put it in and you wouldn't be able to see it |
| | | | there. |
| 7.0.82 | 15:19 | T/R 1: | Okay, but that's not my question now. Suppose in your little |
| | | | stage you showed the people and the boat and the fish, right? |
| | | | And you showed the fish were smaller than the people who |
| | | | were smaller than the boat. Right? |
| 7.0.83 | | Michael : | |
| 7.0.84 | | T/R 1: | Would you expect, let's say in Audra's model, which is a |
| | | | different model that her fish was smaller than the people and |
| 7.0.85 | 15:44 | Michael: | smaller than the boat? You'd have bigger people, bigger boat and a bigger fish. |
| 7.0.85 | 15:50 | T/R 1: | But should those relationships hold? |
| 7.0.87 | 15:50 | Michael: | Yeah. |
| 7.0.88 | 15:52 | Others: | Yes [simultaneous to Michael's reply] |
| 7.0.89 | | T/R 1: | Yeah is that right? |
| 7.0.90 | 15:55 | Michael: | Yeah. |
| 7.0.91 | 15:55 | T/R 1: | Or if we had sort of a medium size model like Mark's and he |
| | | | were trying to make these fit, would you expect the fish to be |
| | | | smaller than the people than the boat? |
| 7.0.92 | 16:04 | Michael: | Yeah. |
| 7.0.93 | 16:05 | T/R 1: | So in each of your models would have those relationships |
| | 4 < 2 = | a 1 | holding, right? |
| 7.0.94 | 16:07 | Students: | Yeah. |
| 7.0.95 | 16:08 | T/R 1: | But they wouldn't all be built the same way and they wouldn't |
| | | | all be the same size |

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| 7.0.96 | 16:11 | Erik: | So it'd be, it's standard that the fish would be smaller than the boat and the people, except the fish would be different sized and the people different sized and the boat different sized. |
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| 7.0.97 | 16:20 | T/R 1: | Right. Is that like what you're doing when you make models to compare fractions? |
| 7.0.98 | 16:23 | Students: | Yeah. |
| 7.0.99 | 16:24 | T/R 1: | In what way is it the same or different? [some students raise their hands] Meredith? |
| 7.0.100 | 16:28 | Meredith: | Well if you have the same question asked and you do it right then you're going to wind up with the same answer and some of the models could be bigger and some of them could be smaller. |
| 7.0.101 | 16:40 | T/R 1: | What do the rest of you think? How many of you agree with what Meredith said? [some hands are raised]How many of you disagree? [no additional hands raised, at least in what was visible] How many of you are still not sure? [more hands are raised] You know we have to help the people who are not sure to understand. they don't disagree, but they're still not following. Can someone help? Let's talk about this a little bit more to help them? Who wants to give it a try? [Meredith's hand goes up] Or the people who aren't sure want to tell us what they are confused about. Do you want to talk a little bit? Audra? Jackie? What bothers you and then maybe the people here will try to help, ok? Do you know what the question is? What do you think the question is? |
| 7.0.102 | 17:33 | Jacquelyn: | Um, is the mod- is different models bigger than others and um |
| 7.0.103 | 17.44 | T/R 1: | Do you want to say that one more time? |
| 7.0.103 | | | You can use different models that are, they're the same. |
| 7.0.104 | | T/R 1: | Is that the question? What do you think, Audra, is that your |
| | | 1/K 1. | question? You think it's a different question? Maybe we are answering a different question. Let's see what Audra thinks the question is and then we can hear from those of you who can try to help. |
| 7.0.106 | 18:06 | Audra: | [hesitantly] It's that we um, it's about, ah, there are different sizes of, just like the candy bar that we did before. Um, you were asking, um if we thought what sizes can fit into each |

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| | | box, what sizes should be that we are going to get confused |
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| | | that the fish can fit into a box. |
| 7.0.107 18:48 | T/R 1: | Who else is confused, what you think the problem is? There |
| | | are a some other people who are confused, or aren't sure. |
| | | Laura? Are you in this category of not being sure? [Laura |
| | | nods affirmatively] What do you think the question is? |
| 7.0.108 19:04 | Laura: | I'm not sure. |
| 7.0.109 19:06 | T/R 1: | You're not sure what the questions is. Okay, well, that's a |
| | | start. Maybe if we got the question, if we understood the |
| | | question, that might help us. Who's going to try with the |
| | | question? [Erik's hand is raised.] Go ahead Erik, give it a try, |
| | | because we also have some people coming in |
| 7.0.110 19:18 | Erik: | It has to do with the model that Mark made. Can the fish, the |
| | | fish should be smaller than the people in the boat, but the |
| | | people should be bigger than the boat, or, no, they should be |
| | | bigger than the fish, but they shouldn't be bigger than the |
| | | boat either. And how does that, how do those models [pause] |
| | | help us understand the models we're building [models of |
| 7 0 1 1 1 10 50 | TT/D 1 | fractions built with Cuisenaire rods]? |
| 7.0.111 19:52 | T/R 1: | Jackie? Michael do you want to add to that? |
| 7.0.112 19:59 | Michael: | Well, it's sort of like um, you can't, the fish has to be smaller then the needle and the needle have to be smaller than the |
| | | than the people and the people have to be smaller than the |
| | | boat, cause the people have to go in the boat and the people |
| | | have to be able to pull the fish out of the water and if it was bigger then it they might have a little trouble getting it out |
| | | bigger than it they might have a little trouble getting it out. |
| | | [laughter] So um, so then, um, its sort of like so, that just helps us understand what we're talking about with the |
| | | Cuisenaire rods when we are using different sized boxes to |
| | | make different sized, um, halves and quarters, um, but, |
| | | they're basically you can call it the same thing as you would |
| | | then just the small one with the small one if you call the box |
| | | a whole, and the boat a half it would equal a quarter. You |
| | | could still do that in Audra's model or any box. |
| 7.0.113 21:11 | T/R 1: | Does that help Laura, Audra, or would you like to ask |
| | | Michael a question? Does anybody want to add to that? |
| | | We've heard from Michael and we've heard from Erik. |
| | | Meredith, you were going to say something earlier? |
| | | [Meredith mutters]. Oh, it was said already? |
| | | |

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| 7.0.114 21:28 7.0.115 21:29 | Meredith: T/R 1: | Yes. Does anybody want to add to that? Sarah, Beth, okay, well it's something to think about isn't it, as we make, uh, different models. Um I remember that you wrote about the models that you worked on and I, I'm looking forward to reading them and, um, knowing more about they way you think about them. Let's try a different one. Ok, let's try a different one. Let's see what happens here. So this is the problem I would like you to think about. I'm wondering which is bigger, one half or two thirds. [pauses] Now before you model it you might think in your head, before you begin to model it what you is bigger and if so, if one is bigger, by how much. Why don't you work with your partner and see if you can figure it out. |
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| 7.0.117 | | [David builds a model with one orange, two yellow, and a purple] |
| 7.0.118 22:40 | David: | Try the purples. Get third purples. It doesn't work, try the greens |
| 7.0.119 22:51 | Meredith: | What was it? Two thirds? |
| 7.0.120 22:55 | David: | It would be like brown or something like that. |
| 7.0.121 22:58 | Meredith: | |
| 7.0.122 23:01 | David: | We're not doing the one third, we're doing two thirds. That is one third |
| 7.0.123 23:10 | Meredith: | First we've got to find out what a third of it is. What's a third of an orange? |
| 7.0.124 23:15 | David: | One third? [He places two green rods instead of the purple rod] |
| 7.0.125 23:33 | Meredith: | What's third of an orange? Let's start a different model. [She begins to make a different model] The green. The green, half of it is the light green [David places a third light green rod next to the original two] |
| 7.0.126 23:45 | David: | [Demolishes his original model and begins to build the same as Meredith] Alright, yeah, I was thinking of that way before |
| 7.0.127 23:48 | Meredith: | And you can take the take the red, and the light green, and put it up to it [Meredith has a model of one dark green rod, two light green rods, and three red rods], it's, she asked, is one half bigger than, what did she ask? What did she ask? |
| 7.0.128 | David: | She asked, which is bigger, one half or two thirds? |

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| 7.0.129 24:16 | Meredith: | One half or two thirds? Now take six of the ones [she takes six white rods] |
|---------------|--------------------------|--|
| 7.0.130 24:23 | David: | Yeah, I know, and put 'em up to there, and that would be one |
| | | sixth. Hey, wait a minute, hey wait, maybe that's it, yeah it's |
| | | bigger by one sixth |
| 7.0.131 24:40 | T/R 1: | [To Beth and Sarah] What do you think? Which is bigger? |
| 7.0.132 24:42 | Beth: | One half [Beth's model is the same as Meredith's] |
| 7.0.133 24:44 | David: | I think one half is |
| 7.0.134 24:48 | | This is one half [pointing to the light green rod], this is a half |
| | | and this is a third [pointing to the red rod], and a half is one |
| | | unit more than [she places a white rod next to the red rod] |
| 7.0.135 24:58 | T/R 1: | Could you tell me the problem I asked you to solve? |
| 7.0.136 25:01 | Sarah: | How much, is one half, what? |
| 7.0.137 25:18 | Beth: | Oh, by two thirds, it's if two thirds is bigger than one half, because we did, we did one third |
| 7.0.138 25:26 | T/R 1: | You compared one third and a half and what was the |
| | | question I asked you to solve? |
| 7.0.139 25:31 | Beth: | Two thirds and a half |
| 7.0.140 25:31 | T/R 1: | So now you did two problems, ok. So, ok, so the question is, |
| | | what's the questions, Sarah, that I asked you to solve? |
| 7.0.141 25:41 | Sarah: | You wanted us to figure out if one half or two thirds is larger |
| 7.0.142 25:48 | T/R 1: | Ok, and by how much? |
| 7.0.143 25:52 | Beth: | [Beth puts a white rod next to the light green rod and |
| | | compares it to the two red rods] It's one |
| 7.0.144 26:09 | T/R 1 : | Yes, David and Meredith? What do you have? |
| 7.0.145 26:11 | Meredith and David: Well | |
| 7.0.146 | David: | we think |
| 7.0.147 | | two thirds |
| 7.0.148 | | and David: is bigger than |
| 7.0.149 26:15 | T/R 1: | 0 |
| 7.0.150 26:16 | Meredith: | one half by one sixth. Cause if you put six ones up to a |
| | | whole |
| 7.0.151 26:27 | David: | dark green |
| 7.0.152 26:28 | | If you put it up to a whole |
| 7.0.153 26:28 | T/R 1: | I'm sorry, what's the number name for dark green |
| 7.0.154 26:32 | | Meredith: One |
| 7.0.155 26:33 | T/R 1: | Ok. |
| 7.0.156 26:35 | Meredith: | And you put six ones up to the dark green |

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| 7.0.157 26:39 | T/R 1: | Hold on, I'm a little confused. Tell me again. Six ones? You called this [dark green] one? What are you calling these [white]? |
|---------------|-----------|--|
| 7.0.158 26:45 | Meredith: | One sixth |
| 7.0.159 26:46 | T/R 1: | One sixth. |
| 7.0.160 26:46 | David: | And then these, this would be |
| 7.0.161 26:50 | Meredith: | We're calling them each sixths, |
| 7.0.162 26:51 | T/R 1: | Ok |
| 7.0.163 26:54 | Meredith: | So there's six sixths |
| 7.0.164 26:54 | David: | This would be [red] one third, and this [light green] is one |
| | | half of dark green, and then it would be bigger by one sixth, |
| | | because |
| 7.0.165 27:01 | T/R 1: | Do you both agree with that? |
| 7.0.166 27:02 | Meredith: | Yeah, mm hmm. |
| 7.0.167 27:03 | T/R 1: | Ok, could you write that up? Uh, let me get you some paper, |
| | | I want you to write that up. And see if you can make me - if |
| | | it works for other models because some students don't |
| | | believe that it works for other models and I think you two |
| | | believe that it works for other models |
| 7.0.168 27:15 | | Mmm hmm, yeah |
| 7.0.169 27:16 | T/R 1: | So can you try to find some so that you can try to convince |
| | | them that it should work for other models, and come up with |
| | | a way of explaining it to the class, ok? That if it works, if you |
| | | really believe it, that if it works for one it works for others, |
| 7.0.170 | | and then write it up, let me get you some paper. |
| 7.0.170 | | [David builds a model of an orange and two yellow rods |
| | | again, then tries to line up purples, discards them, and tries |
| 7.0.171 27:39 | Maradith | green] It's not good for brown [David and Meredith get up to get |
| 1.0.171 27.39 | Mereurin. | pens] |
| 7.0.172 28:18 | David: | I'll make another |
| 7.0.172 28:18 | | Oh! Purple |
| 7.0.174 28:36 | David: | I already tried purple. I just tried it, it doesn't work. |
| 7.0.175 28:41 | | Oh, dark - this green. Oh! We need halves! [pause] Let's try |
| | | brown, oh no. |
| 7.0.176 29:05 | David: | [Holding the blue rod] It's an uneven number, it's an odd |
| | | number |
| | | |

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| 7.0.177 29:13 | Meredith: | Oh! [She looks at a model from another table] Orange and red! Connected. Remember? Orange and red connected [Meredith builds a train of orange and red] And then you take, let's see, what's, the two dark greens and three purples |
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| 7.0.178 29:59 | T/R 3: | [Talking to Gregory and Danielle] What was this? |
| 7.0.179 30:00 | Gregory: | One half [Gregory's model is a dark green rod, two light |
| 1001179 20000 | 0109017. | greens, three reds, and six white rods] |
| 7.0.180 | T/R 3: | This is one half. And so - [BREAK IN TAPE] |
| 7.0.181 30:18 | T/R 3: | Is there some other way you can show using some rods or |
| | | anything? Is there another model you could build to show the |
| | | same thing? Which is bigger, two thirds or one half? |
| 7.0.182 | David: | That's the one half |
| 7.0.183 | Meredith: | No. The question is, what's bigger, one half- |
| 7.0.184 | David: | I know, but then we're going to put this up and put the other |
| | | red, I just don't have the one whole |
| 7.0.185 | Meredith: | You don't understand. Seven, ten, I need them. Go get |
| | | another box. [Meredith finishes her second model] |
| 7.0.186 32:36 | T/R 2: | [approaches Meredith and David] Ok I see you had your |
| | | hands up over here? |
| 7.0.187 32:37 | Meredith: | Yeah |
| 7.0.188 32:41 | T/R 2: | Let me come around and see what you're doing. |
| 7.0.189 | Meredith: | We found two answers |
| 7.0.190 | David: | Well, I have three. |
| 7.0.191 | T/R 2: | You have two solutions three. Ok, let me hear about one of |
| | | these models. Here, which one do you want to tell me about? |
| 7.0.192 32:47 | Meredith: | 6 |
| | | with one orange rod and one red rod which represents her |
| | | unit. Beneath the train she has 2 dark green rods; below, she |
| | | has 3 purple rods; and again, below, she has 12 white rods] |
| 7.0.193 32:49 | T/R 2: | Yeah, that's an interesting looking one, tell me about it. |
| 7.0.194 32:53 | | Now if you call this rod one |
| 7.0.195 32:55 | T/R 2: | The orange and red together? |
| 7.0.196 32:56 | Meredith: | Yeah, and you take the two dark green rods, those are the halves [She takes a dark green rod and a purple rod from her model and places the purple rod beneath the dark green.] And you take two thirds, and put it up to it, and you take two sixths, it's bigger than two sixths. [She puts two white |

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| | | rods next to the dark green rod] And in this one, it you take this [Indicating a white rod] |
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| 7.0.197 33:28 | T/R 2: | Can we go back to that one again? |
| 7.0.198 33:30 | | I mean it's bigger than one tenth, I mean twelfth, one twelfth, |
| 10.170 22.20 | 1.1010annii. | one twelfth. [She puts the white rods back to make a train of |
| | | 12.] |
| 7.0.199 33:34 | T/R 2: | How does that work? I'm confused about that. I'm confused |
| | | about the little white rods, I am following you right up to that |
| | | point. |
| 7.0.200 33:41 | Meredith: | If you put the white rods up to here [She moves the orange |
| | | and red train under the train of 12 white rods], there's twelve |
| | | of them, and then you call them twelfths, because there are |
| | | twelve of them. |
| 7.0.201 33:52 | T/R 2: | All right, okay. |
| 7.0.202 33:55 | Meredith: | |
| | | and then you put it up to the thirds [She moves two white |
| | | rods over to her model representing two thirds and one half] |
| 7.0.203 34:12 | T/R 2: | and it is bigger by two tens two twelfths. By two twelfths, okay. |
| 7.0.203 34:12 | | If you use this model [Meredith refers to the original model |
| 7.0.20+ 54.15 | Mercului. | in which 1 dark green rod represent her unit. Beneath them, |
| | | she placed 3 red rods, then 6 white rods, and then one dark |
| | | green rod.] |
| 7.0.205 | T/R 2: | Uh hmm. |
| 7.0.206 | Meredith: | And if you use this model [referring to the 1 dark green rod |
| | | as unit model] |
| 7.0.207 | T/R 2: | Uh hmm. |
| 7.0.208 | Meredith: | |
| | | her original model] And you call these [white rods] sixths, |
| | | and you put this one [white rod] up to it [light green rod] and |
| | | it is bigger than one sixth. [Meredith, with her original |
| | | model, has indicated that the difference between one half - |
| | | length of 1 light green rod- and two thirds -length of a train |
| 7.0.209 34:34 | T/R 2: | of two red rods- is one sixth -the length of a white rod.] |
| 1.0.207 54.54 | $1/\mathbb{R} \ \mathbb{Z}$. | Okay, so here it was bigger by two twelfths and here it was bigger by one sixth. |
| 7.0.210 34:37 | Meredith: | |

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| 7.0.211 34:28 | T/R 2: | That's interesting. Could we call the difference between the two thirds and the one half in this model [a train of orange and red as the unit] another number name besides two twelfths? |
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| 7.0.212 34:53 7.0.213 34:56 | Meredith: T/R 2: | Um, yeah, well, maybe You said two of those little white ones were two twelfths, right? |
| 7.0.214 35:02 | Meredith: | - |
| 7.0.215 35:19 | T/R 2: | Oh, that's interesting, that's kind of interesting, then, so if you then used the reds to describe the difference, you could call this one sixth, the difference. |
| 7.0.216 35: 7.0.217 | Meredith: T/R 2: | Uh hmm. And over here [Meredith's second model with dark green as |
| | | the unit.] one of the whites you say is one sixth? |
| 7.0.218 | Meredith: | |
| 7.0.219 35:37 | T/R 2: | Oh, that's interesting, two different models. Okay. David, did you say you had another one? These are wonderful. |
| 7.0.220 35:41 | David: | What I did was the purple was one and then these [red] were the two thirds, no wait a minute, I think this one was [David originally has a model of one purple, two reds, and a train of light green and white] |
| 7.0.221 | T/R 1: | [To class, as David is remodeling and speaking] Now if you're writing about your solution, I'm hoping you're writing about more than one model, if you found more than one, and I'd also like you to answer the question, does it hold up for different models, is that what you expected? Why or why not? |
| 7.0.222 | | [David's model is now similar to Meredith's original model] Alright, the dark green is one, and then the red is two thirds, and then the light green is one half, and then the white to the green is one sixth, so two thirds is bigger by one sixth |
| 7.0.223 36:35 | T/R 2: | Ok, very nice |

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| 7.0.224 36:37 | David: | And then I had the same thing down here [points to his model of one dark green, two light greens and a train of yellow and white], and that shows that shows that the light green is, I think I messed this up a little, ok, I think it's like this [puts down and orange instead of the dark green], and then the light green was two thirds, um, two thirds |
|--------------------|--------|--|
| 7.0.225 37:05 | T/R 2: | How would you prove that - that the light green is two thirds of the orange? |
| 7.0.226 37:10 | David: | Like this, and that [puts a third light green down] |
| 7.0.227 37:17 | T/R 2: | Is that the same length? |
| 7.0.228 37:27 | David: | Ok [shows that it isn't and laughs]. Alright. |
| 7.0.229 37:30 | T/R 2: | Well, that model is all botched up. I wonder if you modify it to make a model that would work. Well, this one worked, right? [points to the model with the orange and red train as one] With an orange and a red. |
| 7.0.230 37:38 | David: | Alright, then on this one, with the orange and the red, and then this [purple] is two thirds and that's [dark green] one half, and then this is bigger by one sixth |
| 7.0.231 37:48 | T/R 2: | Ok, do you think there are any more models? You've already come up with two that I've seen that work. I'll let you think about this some more and if you come up with any more I want you to call me back. Um, while you're thinking about it, you may want to take some time to report what you've done. Before you even begin you may want to put your models down. But I want you to continue to think about that question, both of you, ok, if there's any more models that will work. |
| 7.0.232 7.0.447 | T/R 1: | [Meredith and David begin to draw their models] [Jackie, Erin, and Jessica have built two models at the OHP] I'm going to have to have to stop you for a minute, I know that, I hate to do this because I know you're working all so hard, but I would like to spend ten minutes, uh, just have us think about a few things, and you can finish this, is that ok, Mrs. Phillips if they can finish writing this up for us when we come back on Wed? Ok, so you really have today and maybe some time tomorrow to finish writing this up. I, I would like all of you though to sort of give me your attention for a minute, um, because I'm wondering about a few things. I |

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| | | need you to help me straighten out some things in my head, and if you can help me straighten them out in my head, you may be helping out other people straighten them out also. So I'm going to ask you some, some important questions, are you all listening to my questions? If you could stop what you're doing for a moment, I know it's hard, and listen to my questions. How many of you made one model and absolutely are convinced, that you know by your model, which is bigger one half or two thirds? How many of you did that with one model and you are absolutely convince with your model you know which is bigger, one half or two thirds. Would you please raise your hand if you made one model, you could have made more than one but you made at least one. If you made at least one model, girls, and you're absolutely convinced [All visible hands raised] No one could persuade you otherwise that you know which is bigger, one half or two thirds. Alright, so tell me, which is bigger? |
|---------------|-----------|--|
| 7.0.448 | Students: | Two thirds. |
| 7.0.449 47:59 | T/R 1: | Again? |
| 7.0.450 48:36 | Students: | Two thirds!! |
| 7.0.451 | T/R 1: | And you also know how much bigger. How many of you are convinced you know how much bigger and no one can persuade you otherwise that two thirds is not only bigger than a half but it is how much bigger? |
| 7.0.452 | Students: | One sixth. |
| 7.0.453 48:40 | T/R 1: | How many of you believe one sixth? [All visible hands raised] That's what I thought Walking around I thought thatthat is what I believe that everyone has done. How many of you made a second model? |
| 7.0.454 | Meredith: | Oh, Oh! [eagerly] |
| 7.0.455 | T/R 1: | You could have made more than two, but you made at least two models. And in your second model you got a different answer. You got an entirely different answer, you no longer have two thirds bigger than one half, you showed something else. Are you watching? Some of you got a second model that showed something different. Meredith? Let's listen to what Meredith says. Girls [Jackie, Erin, and Jessica], why don't we stop that for just a moment and then we'll make |

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| | | some more. Listen, listen. You [Meredith] think you showed something else in your second model? [Meredith goes to the overhead projector and places twelve white rods beneath the two dark green rods in the girls' second model - two of the other girls help her find and position the white rods] I am really confused. I have no ideas what Meredith is going to do. Because I thought I understood this and I thought she was going to tell me she got the same answer. Did you think that? And now she's telling me no. [Meredith is smiling.] I'm going to get so confused. You are all going to have to help me. [Pause.] How many of you built a second model that looks like that model up there that Meredith is fiddling with? How many of you have a model that looks like that? [Many hands are raised.] By the way, what was one in that model? What did you call one in that model? Amy? |
|---------------|----------|--|
| 7.0.456 | Amy: | Ah, the orange and red. |
| 7.0.457 | T/R 1: | How many of you called 'one' orange and red in that model? |
| | | Yeah, you did that model. Did you get to have two thirds |
| | | bigger than a half? |
| 7.0.458 50:53 | Michael: | No. [Michael raises his hand, shaking his head from side to side, signifying dissent.] |
| 7.0.459 50:56 | T/R 1: | Amy got two thirds bigger than a half in that model, how many of you got two thirds bigger than a half in that model, where the orange and the red were one. Michael didn't, Meredith did. You didn't Michael? |
| 7.0.460 51:04 | Michael: | No, they can't do that. [He begins to stand.] Because um, the, the two thirds are bigger than the half by a red. So they can't use those whites to show it. |
| 7.0.461 51:08 | T/R 1: | Oh, but you're saying that, you're saying that two thirds, what's a third? |
| 7.0.462 51:21 | Michael: | A third is the purple [He begins to approach the overhead projector.] |
| 7.0.463 51:31 | T/R 1: | And what's two thirds? Just tell us. |
| 7.0.464 51:37 | Michael: | [He returns to his seat.] Um, two thirds is two purples. |
| 7.0.465 51:39 | T/R 1: | Did you all do that? Did you get two thirds to be two |
| | | purples? [She addresses Michael] And what did you get to be one half? |
| 7.0.466 51:42 | Michael: | Uh, dark green. |
| | | |

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| 7.0.467 51:43 | T/R 1: | [She addresses the class.] Did you get dark green to be a half? [Mutterings of assent occur.] And you got two thirds to |
|---------------|-----------|---|
| | | be bigger than one half? |
| 7.0.468 51:49 | Michael: | [Politely impatient] Yes. |
| 7.0.469 51:51 | T/R 1: | By how much? |
| 7.0.470 51:56 | Michael: | [Deliberately, again almost impatiently] By one sixth. |
| 7.0.471 51:57 | Meredith: | Or, or two twelfths. |
| 7.0.472 51:58 | Michael: | [Shaking his head sideways] No. |
| 7.0.473 52:00 | : | [Mutterings in the classroom of no.] |
| 7.0.474 52:02 | T/R 1: | Tell us Meredith. Aha! How many of you got one sixth? |
| | | [Most hands are raised.] And what rod did you use to |
| | | represent one sixth? What color rod? |
| 7.0.475 | Students: | Red. |
| 7.0.476 52:04 | T/R 1: | How many of you used a red rod to represent one sixth in |
| | | that model and you showed it was bigger by one sixth? And |
| | | Meredith says she did it a little differently and she didn't get |
| | | one sixth. And what did you get Meredith? |
| 7.0.477 52:15 | Meredith: | Two twelfths. |
| 7.0.478 52:15 | T/R 1: | What do you think about that? |
| 7.0.479 52:26 | Students: | Well, in a way. No. Uh, uh [negatively]. |
| 7.0.480 52:29 | T/R 1: | Well, she showed it's bigger by the two whites, she shows |
| | | two whites bigger. |
| 7.0.481 52:32 | Michael: | Yeah, but then she would have to call the two whites together |
| | | one sixth. |
| 7.0.482 52:35 | Erik: | Yeah, exactly. |
| 7.0.483 52:40 | Michael: | She's calling the whites, one white one sixth. |
| 7.0.484 52:44 | Erik: | Yeah, she said |
| 7.0.485 52:45 | T/R 1: | She's calling one white one sixth? |
| 7.0.486 52:46 | Meredith: | ý U |
| 7.0.487 52:50 | T/R 1: | She's calling one white one twelfth. |
| 7.0.488 52:52 | Erik: | Yeah, but see just the whites together. That'd be right, it |
| | | would be two twelfths. But you have to combine them. You |
| | | can't call them, you can call them separately, but you could |
| | | also call them combined and if you combine them it would |
| | | be uh, one sixth. |
| 7.0.489 52:53 | T/R 1: | Ok, but she didn't combine them and she's calling the two |
| | | whites together, again, Meredith? |
| 7.0.490 52:53 | Meredith: | One twelfth, two twelfths. |
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| 7.0.491 53:11 | T/R 1: | Two twelfths. She's calling [Michael, still seated, shakes his head sideways in dissent and fingers some rods.] Do you all agree that one white has a, the number name for the white rod is one twelfth? Someone told me that when I was walking around, it might have been Audra. And some other people told me a white would be one twelfth? Is that true? And two white rods would be |
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| 7.0.492 53:15 | Students: | Two twelfths. |
| 7.0.493 53:17 | T/R 1: | Two twelfths. And one red would be |
| 7.0.494 53:31 | Students: | One sixth. |
| 7.0.495 53:32 | T/R 1: | One sixth. So, so what is Meredith saying here? |
| 7.0.496 53:35 | Meredith: | There's two answers. |
| 7.0.497 53:37 | T/R 1: | Are there two answers? |
| 7.0.498 53:41 | Michael: | [simultaneously with Erik] No, they're the same answer. |
| 7.0.499 53:43 | Erik: | No, they're the exact same thing, except she, she took the red and divided it into half, she divided it into halves, into half and called, and called each half one twelfth. They're the exact same answer except they're just in two parts. |
| 7.0.500 53:44 | | [Note all written notation will be enclosed in [] as it is recorded by the teacher] |
| 7.0.501 53:47 | T/R 1: | [Joins the four girls at the overhead projector] Let me write this down. This, what you are saying here is so important, here. Let me see if I can write this down. You're saying that you're calling the red, you're giving red the number name, right? The length of the red, right? We'll give it the number name, what did you say? |
| 7.0.502 | Students: | One sixth. |
| 7.0.503 54:02 | T/R 1: | [R one sixth] One sixth. And two whites, can I write two 'w ' for two whites? |
| 7.0.504_54:23 | Students: | Yeah. |
| 7.0.505 54:25 | T/R 1: | And you're calling two whites |
| 7.0.506 54:29 | Students: | Two twelfths. |
| 7.0.507 54:31 | T/R 1: | [2W two twelfths] Two twelfths. But what Erik just told me, right?, is something about red and white. |
| 7.0.508 54:33 | Erik: | Yeah. A red, one red equals, one red rod up here, one red equals two of the white ones. |
| 7.0.509 54:36 | T/R 1: | [1R = 2W] So we're talking about the length of the red rod, the length of the red rod is the same as the length of the two |

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| 7.0.510 54:43 7.0.511 55:03 | Students: Erik: | white rods? [On the overhead projector, Meredith builds a model with one red rod as the base and places two white rods directly above it.] Is that true? Do you all agree to that? Yeah. Yes. And since she's calling a white rod one twelfth and the other white rod one twelfth and the red rod is really one sixth. But, when she calls them two twelfths, the two twelfths are actually just two white rods put together to equal a red, so it should be really, it's really one sixth. Because two whites, two whites |
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| 7.0.512 55:12 | T/R 1: | She says one white is a twelfth $[1/12]$ and then if you put it together with another one twelfth $[+1/12]$, she's saying you get two twelfths $[=$ two twelfths]. |
| 7.0.513 55:12 | Erik: | And it's one sixth, it's one sixth. |
| 7.0.514 55:39 | T/R 1: | And you're saying if you have, if you take one half that's all right? [1/2] If you're taking one half of one sixth [of one sixth], you're saying you get one twelfth [= 1/12] You're saying that. That's the two things I'm hearing. Right? And you're saying that [one sixth], the length of one sixth is the same as the length of two twelfths. [= two twelfths] Is that what you are saying? |
| 7.0.515 55:47 | Erik: | Yeah. |
| 7.0.516 55:48 | T/R 1: | All those things, are they true? |
| 7.0.517 56:15 | Erik: | Yeah. But I don't really think you could call, call them two twelfths because two twelfths equal exactly to the same size as one sixth. Well, if you want to you could call them, I guess. But I think it would be easier just to call them one sixth, then wouldn't want to exactly call them one twelfth and another twelfth. I'd just call them one sixth. Therefore I think you just really call them one sixth. |
| 7.0.518 56:16 | Student: | Well, maybe you can call them |
| 7.0.519 56:18 | Erik: | Well you can call them, if you want to, but |
| 7.0.520 56:46 | T/R 1: | Well, we have different number names for these rods |
| 7.0.521 56:47 | Student: | they're not different |
| 7.0.522 56:49 | Brian: | There's just half of one, there's just half of one. |
| 7.0.523 56:53 | T/R 1: | So you're saying that one half of the one sixth is another way of saying one twelfth. |
| 7.0.524 57:00 | Brian: | They're just two answers. |

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| 7.0.525 57:01 | T/R 1: | Well, you're saying if you took a twelfth, a rod that has length one twelfth, and another rod that has length one twelfth and put them together, right? That rod would have length two twelfths. Isn't that what you said? |
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| 7.0.526 57:09 | Jessica: | What Erik said is that two whites equal one red, so it would be the exact same thing. |
| 7.0.527 57:09 | T/R 1: | Or a rod that has length one sixth, that would be the red one in this problem, would also have length two twelfths. Is that what you said when you talk about the lengths of the rods? So are all of these [pointing to the recorded notations on the overhead projector] true statements? |
| 7.0.528 57:19 | Students: | Yeah. |
| 7.0.529 57:26 | T/R 1: | That's amazing. Look at all the fancy mathematics you're doing, that's amazing. That's something for us to think about, ok? So Meredith is still saying that, "I don't disagree with you when I say that it's a red bigger in this model," right? |
| 7.0.530 57:38 | Meredith: | Um, hmm. |
| 7.0.531 57:39 | T/R 1: | I'm just going to give this red a different number name. I could give it the number name one sixth, if I think about it when I compare it to the rod I call one, the orange and red, I could give it the number name one sixth. Or, if I'm thinking about the white rods, right? I could give it the number name two twelfths. And that's very interesting. Does that contradict what you're doing? Or does it still work, what you're doing? It still works, Meredith thinks. That's something to think about, isn't it? That's very interesting, thank you for sharing that. Well, I think we've run out of time. Um, there's a lot of things to write about. We have a whole lot of new ideas, don't we? I really hope that you write to me and show me as many models as you can. That you will also, you will also, think about, in your models. What is different about each of those models? Write a statement about each of those models that makes it a different models? Is that a good question, Meredith? |
| 7.0.532 57:55 | Meredith: | Mm, hmm. |

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7.0.533 57:56 T/R 1: You can think about that question and write to me about it, I'd really like to know what you're thinking... What is different and what is alike. I can't wait to read what you write to me
7.0.534 59:19 End of class