SEGMENTED LABOR MARKETS AND MONETARY POLICY

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ABSTRACT OF THE DISSERTATION

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This dissertation examines the impact of a segmented labor market on aggregate dynamics and discusses optimal monetary policy. The first chapter investigates whether differentials in labor market variables in segmented labor markets have an aggregate effect. I find a mechanism by which a segmented labor market model generates stickier aggregate nominal wages and thus more volatile output, employment ratio and unemployment rate. In the second chapter, I estimate the extended version of the model using a typical Bayesian estimation method in which the model incorporates several features that are common in medium-scale New Keynesian DSGE Models. The estimation results confirm the results obtained by the calibrated model of the first chapter. In particular, the estimates for the labor supply and demand elasticity of low-skilled workers are greater than those of high-skilled workers. In the third chapter, I discuss an optimal monetary policy, taking into account income inequality. The model shows that a tight monetary policy leads to an increase in income inequality. This increase in inequality induces stickier aggregate nominal wages. I also find that the income inequality poses a policy trade-off with traditional objectives. A quantitative analysis shows that a monetary policy that concerned aggregate variables only causes a larger welfare loss after idiosyncratic productivity shocks.

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Dedication

I dedicate this dissertation to my beloved wife, Hyunhee, for her unconditional support during these many years of research and to our son, Jacob Yun-Gun, for the joy he brought to our life. I am also grateful to my dearest parents for their endless support. I must also thank my in-laws who have given me their fullest support.

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Chapter 1

Introduction

In this dissertation, I attempt to shed light on the impact of segmented labor markets on aggregate dynamics and discuss optimal monetary policy. According to the Current Population Survey data, there are substantial differences in labor market variables across demographic subgroups. In particular, the low-skilled workers' (defined as workers with less than a Bachelor's degree) unemployment rate is greater and more volatile than that of high-skilled workers (defined as workers with Bachelor's degree or above). On the other hand, low-skilled wages are relatively more stable than high-skilled wages.

The first chapter investigates whether such differentials in labor market variables have an aggregate effect based on a two-sector New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model. I find a mechanism ("strategic complementarity" in wage setting) by which a segmented labor market model generates stickier aggregate nominal wages and thus more volatile output, employment ratio and unemployment rate. The greater labor demand and supply elasticity of low-skilled workers lead to a flatter low-skilled wage Phillips curve, and create a sectoral difference in the wage adjustment after an economic shock. Therefore, the wage premium, which is defined as the gap between the average logged high-skilled wage and the average logged low-skilled wage, declines whenever aggregate wages are under downward pressure. Once the wage premium falls, firms substitute high-skilled workers for low-skilled workers due to the relatively lower price of high-skilled workers. More demand for high-skilled workers makes high-skilled wages bounce back after the initial decrease, and drags low-skilled wages down further. However, these two opposite forces on aggregate wages are dominated by the high-skilled wage adjustment because the high-skilled wages are more flexible, and consequently, the aggregate wages do not decrease as much as they do under a single labor market model. As a result, aggregate output, employment ratio and unemployment rates become more volatile under the segmented labor market model.

In the second chapter, I estimate the extended version of the model using a typical Bayesian estimation technique. The model incorporates several features that are common in medium-scale New Keynesian DSGE Models, such as price and wage indexation and consumption habit formation. In order for the segmented labor market model to have an aggregate implication, it is crucial that the composite elasticity of labor supply and demand for low-skilled workers is greater than that of high-skilled workers. In addition, the elasticity of substitution between different skilled workers must be greater than one in order to account for the more volatile employment ratio of low-skilled workers. Accordingly, one main objective of the second chapter is to confirm that the estimates of the parameters which are associated with the labor market are consistent with the calibrated values that I obtained in the first chapter based on the macroeconomic structural model. The data set of the labor market variables used in the estimation is constructed based on the Merged Outgoing Rotation Group of Current Population Survey on the period of [1984Q1, 2007Q4]. The estimation results indicate that labor demand and supply elasticity of low-skilled workers are much larger than those of high-skilled workers. Thus, the wage premium decreases after a negative demand shock, which causes stickier aggregate wages as shown in the analysis of the first chapter. In addition, the posterior distribution indicates that different skilled workers are highly substitutable so that the negative demand shock also induces a greater labor income gap across sectors.

In the third chapter, I discuss an optimal monetary policy taking into account income inequality which arises from the segmented labor market with the different labor elasticities across sectors. The model assumes that labor supply elasticities are the same across sector, but low-skilled workers have limited access to financial markets instead. This assumption simplifies the model in obtaining the loss function of a central bank. However, the labor demand elasticity of low-skilled workers is still greater than that of high-skilled workers. Thus, the aggregate implications of the segmented labor markets discussed in the first chapter are still applicable to this model. In particular, an economic shock causes a variation in the wage premium, and the wage premium is negatively related to the income inequality. The model shows that a contractionary monetary policy brings about an increase in income (and consumption) inequality, which, in turn, causes stickier aggregate nominal wages via the strategic complementarity in wage setting. In addition, I found that the income inequality poses a policy trade-off with traditional objectives such as the price inflation and the output gap. This implies that a central bank cannot achieve the first best allocation even under the flexible prices and wages. Moreover, a quantitative analysis shows that a monetary policy that is only concerned aggregate variables only causes about a 5% larger welfare loss after idiosyncratic productivity shocks compared to the case where the central bank takes into account the variations in the inequality. I conclude that optimal monetary policy requires the consideration of inequality variation.

Chapter 2

Aggregate implications of the segmented labor model

2.1 Introduction

Recently, the Federal Open Market Committee (FOMC) made statements about the future path of interest rates to achieve its goal: Maximum employment and price stability. Even though some economists are concerned that the continued high level of monetary accommodation raised the risks of financial imbalances, the FOMC have decided that a policy of exceptionally low interest rates is appropriate as long as the unemployment rate remains above 6.5%. Given that a central bank sets policy rates that are contingent on unemployment rate, is the average unemployment rate for the economy the right measure? Or should the central bank focus on the unemployment rate for a particular demographic group or industry? In this paper, I attempt to explain whether heterogeneity in labor markets matters for monetary policy.

Many studies have already pointed out that there is a significant heterogeneity in labor markets. For instance, among others, Farber (2005) points out that job loss rates are closely related to unemployment rates and the job loss rates have much stronger cyclical pattern for less educated workers than for more educated workers. Elsby et al. (2010) also demonstrates that average unemployment rate is decreasing in education and relative change in unemployment rate is much higher in less educated population, especially, during the Great Recession. In fact, as Figures 2.1 shows that unemployment rates have moved quite differently across over the education level.[1] In particular, unemployment rates of workers with less than high school education began to rise from 7.3% in October 2007, and peaked in November 2010 reaching up to 15.7% while the

[1]The unemployment rates are quite different across industries, occupation within the industries as well according to the CPS data.

average unemployment rate of workers who has at least Bachelor's degree increased from 2.1% to 5.1% over the same period. Furthermore, as Hoynes et al. (2012) noted that labor market effects of the business cycle is not uniform across the demographic gruops, and the significant differences have been stable for more than 4 decades. They also note that the difference in the cyclicality between industries can be explained by the differences of the share of certain demographic groups; the demographic groups that exhibit larger cyclical variation are more likely to be employed in the industries with greater exposure to cycles. Rogengren (2013) also mentioned in his speech that "construction and durable goods industries have provided jobs for some with less educational attainment and such industries, as interest-sensitive sectors, would directly affected by monetary policy."[2]

In spite of the evident differential in labor markets, however, monetary policy considering heterogeneous labor market is not sufficiently studied. Accordingly, in the present paper, I build a structural model based on Galí (2011) to account especially for the heterogeneity in unemployment rates and then analyze the aggregate implications arising from it. A growing number of researchers have tried to incorporate labor market frictions to New Keynesian framework through search and matching theory in which the wage is determined by Nash bargaining between households and the firms. However, such models generally yield too much volatility in wages. Although many papers adopted real rigidities in a wage setting or introduced wage indexation in order to produce staggered wage behavior, the model then easily become too complicated to get a solution analytically. Galí (2011) builds a model within a standard New Keynesian framework following Erceg et al. (2000)'s specification on the labor market. The Galí (2011) model is convenient and appropriate for optimal monetary policy with unemployment in that it takes the unemployment explicitly into account as an observable variable. It allows us to measure the welfare relevant output gap based on the observable variables. I consider the educational attainment as the origin of the heterogeneity in labor market. This is because education level that a worker attained is closely related to his/her occupation, and the portion an occupation affects the unemployment

[2]http://www.bostonfed.org/news/speeches/rosengren/2013/011513/011513text.pdf

fluctuation of the industries[3].

I assume that there are two monopolistic labor markets and each worker is assigned to one of the two markets according to their education level. The workers with high education level, called high-skilled workers, are less substitutable than low-skilled workers. This assumption implies that high-skilled workers have higher natural markup than low-skilled workers, and hence, the average wage of high-skilled workers is greater than those of low-skilled workers in equilibrium. In addition, I assume that labor supply elasticity for low-skilled workers is larger than high-skilled workers. Intuitively, low-skilled workers with a lower wage have to spend greater fraction of their wage on living costs, and with limited access to financial market, they become sensitive to changes in their wage. On the other hand, since high-skilled workers with higher wages have better opportunities to access financial markets, high-skilled workers have much superior capacity to offset the fluctuation in their wage. Thus, their labor supply is relatively stable compared to low-skilled workers. The difference in the labor elasticities across sector has a significant impact on the differentials in the sectoral unemployment rates. In particular, smaller labor demand and supply elasticity of high-skilled workers lead to a steeper wage Phillips curve and hence much less volatile response of unemployment rate given a change in the nominal wages. Furthermore, the difference in the slope of the wage Phillips curves induces a variation in the wage-premium, aggregate wage, and in turn, it affects aggregate dynamics of the other endogenous variables in response to economic shocks. In fact, the wage premium moves procyclically in response to a negative demand shock, which makes monetary policy more effective in stabilizing output fluctuation and inflation under the segmented labor market than under a single labor market model. Therefore, understanding the heterogeneity in labor market would be crucial to determine appropriate policy rate.

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[3]However, the heterogeneity also can be introduced through any other factors generating different labor supply elasticities such as gender, race or age

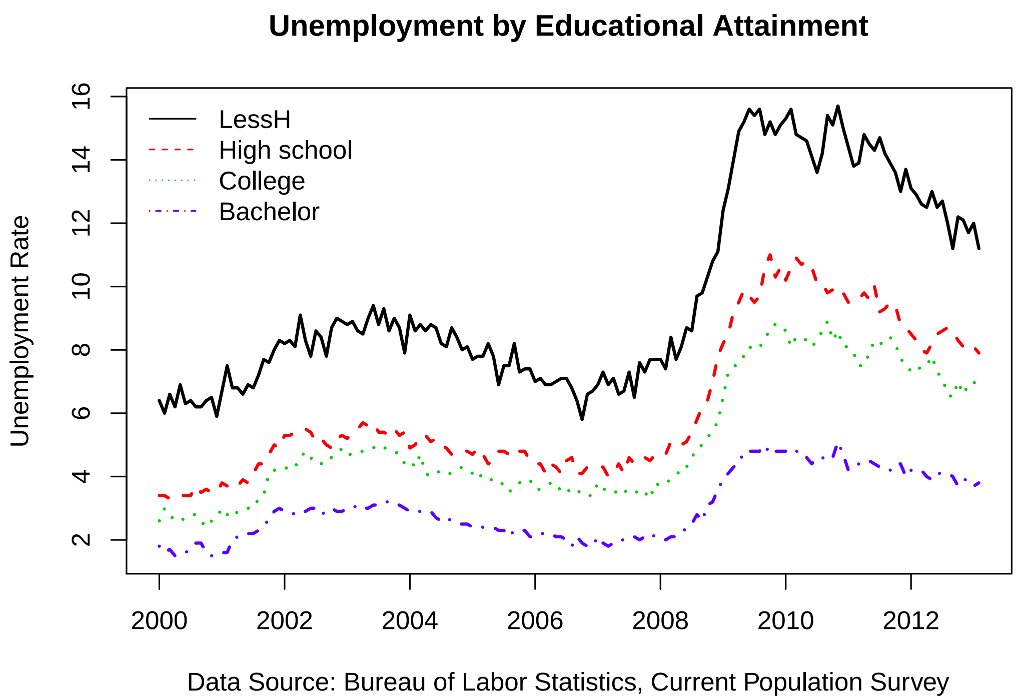

Figure 2.1: Unemployment rates by Educational Attainment (Seasonally Adjusted)

2.1.1 Empirical Evidence

In this subsection, I examine the effect of a monetary policy shock on labor market differentials in unemployment rates, employment ratios, and real wages through the VAR analysis based on micro data which covers from 1984Q1 to 2008Q3. I use the "Merged Outgoing Rotation Group (MORG) of the Current Population Survey (CPS)" which provides monthly information on about 50 variables including employment status, level of education, and weekly earnings. Using this, I obtained quarterly data of unemployment rates and employment to population ratio for each labor market and the wage-premium that are defined as the gap between average logged high-skilled wage and average logged low-skilled wage. In addition, I obtained quarterly real GDP per capita from (A939RX0Q048SBEA), Inflation from the percentage change in implicit price deflator (GDPDEF), and nominal interest rate from the effective Federal Fund Rate (FEDFUNDS) of Federal Reserve Economic Database (FRED) maintained at

Federal Reserve Bank of St. Louis. Finally, I set the vector of endogenous variables as:

$$Y_t = \begin{bmatrix} y_t & u_t^H & u_t^L & n_t^R & w_t^R & \pi_t & i_t \end{bmatrix}$$

where u_t^H and u_t^L are the unemployment rates for high-skilled workers and low-skilled workers respectively, $n_t^R (\equiv n_t^H - n_t^L)$ denotes employment gap across sector, and w_t^R denotes wage-premium.[4] The variables, y_t, π_t and i_t denote logged real GDP per capita, percentage change of GDP deflator and the effective Federal fund rate (FFR) respectively. Once I constructed 7 quarterly time series data, firstly, I build seasonally adjusted labor market time series data using X-12 and then remove the trend from all the variables using HP filter and finally obtain the variables in percentage change term. In order to determine the lag order p, I compare the four types of criteria; Akaike Information criterion (AIC), Hannan-Quinn (HQ) criterion, Schwarz criterion (SC), and Akaike's Final Prediction Error (FPE). HQ and SC indicate p = 1 is appropriate while AIC and FPE suggest p = 10 and p = 9 respectively. The VAR model in this section is estimated using p = 1 and is given by:

$$Y_t = AY_{t-1} + u_t$$

where A is coefficient matrix, and u_t is an unobservable error term assumed zero-mean independent white noise with $u_t \sim \mathcal{NID}(0, \Sigma_u)$. Using Cholesky decomposition, I obtained the Impulse response functions to a one percentage standard deviation increase in FFR and is plotted in Figure 2.2. The red lines show the 95% bootstrap confidence interval with 1000 runs. As I expected, the negative demand shock causes decrease in output and inflation. More importantly, both unemployment rates rise while low-skilled unemployment rate increases more than twice of high-skilled workers in response to contractionary monetary shock. The wage-premium rises initially but persistently falls indicating lagged procyclical in response to monetary shock as in Khalifa (2013).

[4] ω^R is defined as $\log(W_t^H) - \log(W_t^L)$ and W^H and W^L are logged average weekly earnings per worker. For instance, $W^H = \dfrac{\sum_i \text{weekly earning}_i^H \times \text{earning weight}_i^H}{\sum_i \text{high-skilled population weight}_i}$

Employment gap increases by more than 20%. A relatively small decrease in wage premium and larger increase in employment gap implies the higher income gap between high-skilled workers and low-skilled workers. This result is consistent with the inequality literature such as Coibion et al. (2012) and Gornemann et al. (2012)

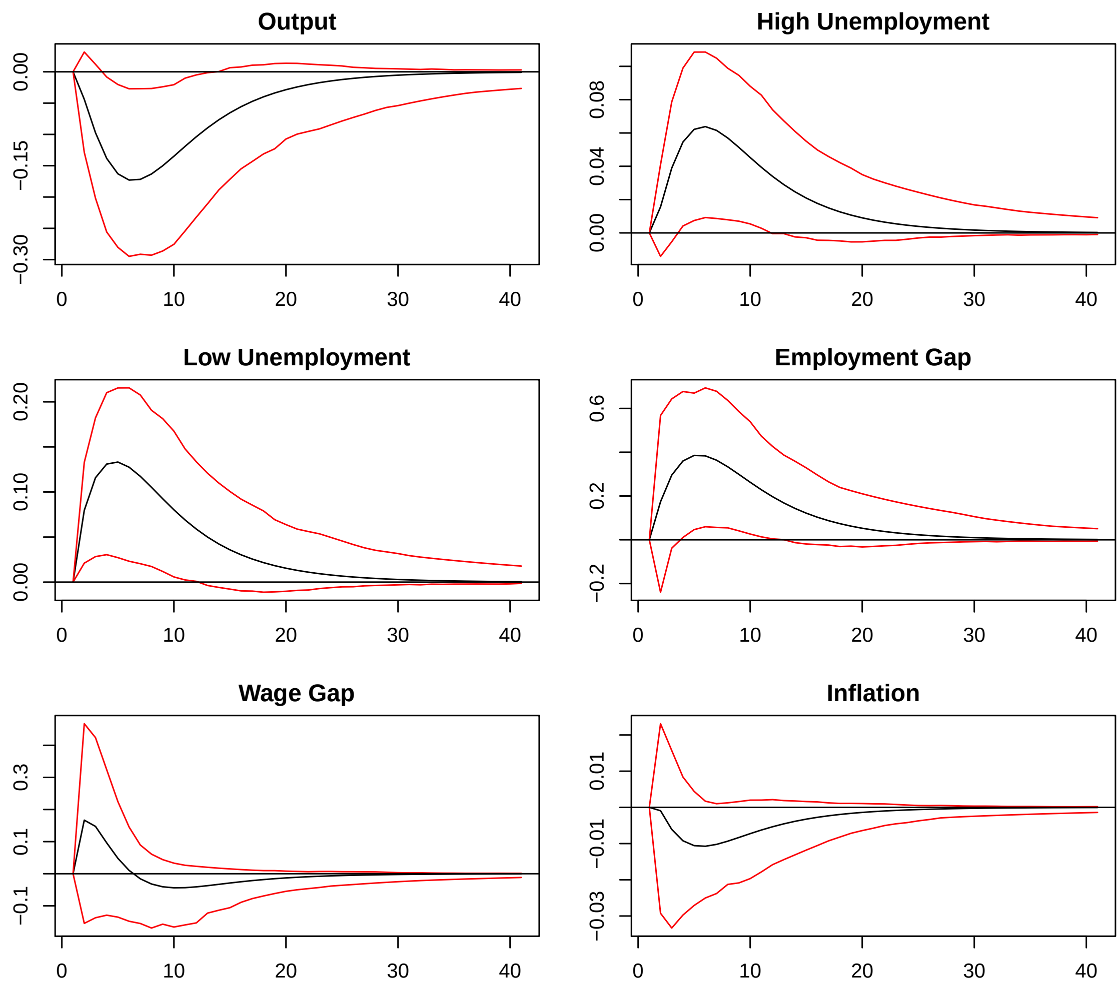

Figure 2.2: VAR(1) IRFs to the monetary shock

2.1.2 Related Literature

To understand the implications of a monetary policy shock, many researchers already have taken the empirical analysis with different data set. Carpenter and Rodgers III (2004) conduct VAR analysis based on the data obtained from BLS's October Supplement of the Current Population Survey which covers a period from 1948 to 2002. They argue that monetary policy have a disproportionate effect on the unemployment rate of low-skilled workers noting that contractionary monetary policy lowers the employment-population ratios of minorities and less-skilled by increasing their unemployment rates. Williams (2004) also points out that there is a substantial differential monetary policy impact based on VAR analysis. In particular, he argues that positive monetary policy shock has affected much more the construction and the durable manufacturing sectors than service sector. He notes as well that because the majority of the workers in manufacturing and construction are designated as PCR and OPL[5], these occupations have experienced significantly higher cyclical unemployment rates than workers in the MP and TSA[6] categories.

Recently, Khalifa (2013) also takes VAR analysis with the data from "Outgoing Rotation Group of the Current Population Survey" for the period 1979 to 2004, and shows that a positive shock to Fed funds rate causes a drop in the inflation rate, and in real gross domestic product as well as a lagged decline in the skill premium. In addition, he builds a model in order to explain the persistent aggregate unemployment rate and more persistent unemployment rate of low-skilled workers than high-skilled workers using search and matching model. He argues that downgrading of high-skilled workers forces low-skilled workers to be unemployed during economic downturn and this mechanism contributes to the persistence of unemployment. The model assumes that the high-skilled workers have more efficient matching rate than low-skilled workers so that the high-skilled can crowd out the low-skilled occupation when they compete. Important weaknesses of his model are that, in contrast to the data, the impulse responses based on his model not only show that unemployment rate for the high-skilled are more volatile than for the low-skilled but also fail to explain staggered wages.

However, the model developed in present paper overcomes both of the weaknesses. I am motivated from the micro evidence that high-skilled workers have less Frisch elasticity than low skilled workers. Blanchard and Katz (1997) assert that "unskilled

[5]PCR and OPL are the acronyms respectively for Production / Craft / Repairers and Operators / Fabricators / Laborers

[6]MP and TSA are the acronyms respectively for Manager / Professional and Technical / Sales / Administrative

individuals have significantly higher labor supply elasticities than skilled individuals, and thus, a fall in labor demand in economic downturn will have a larger impact on the employment prospects of less-skilled workers." In fact, this is confirmed by Fiorito and Zanella (2012) and Kimball and Shapiro (2008). Kimball and Shapiro (2008) estimate labor supply elasticities for individuals with different educational attainment using survey data from the Health and Retirement Study and find that individuals with college degrees have substantially lower labor supply elasticities than individuals with some college education or no college education.[7] On the other hand, Fiorito and Zanella (2012) estimate the micro/macro Frisch elasticity based on Panel Study of Income Dynamics (PSID) data of 1968 - 1997 waves. Especially, showing that the elasticity for the subgroups including the low-educated workers are much elastic than others, they argue that the aggregate elasticity is decreasing sharply with education. Moreover, they point out that the correlation between wages and the volatility of net flow (the difference between entry and exit), which accounts for extensive margin on aggregate labor supply, is much larger for without college degree groups than college degree group. Although the estimates are different from each other, both papers confirm that high-skilled workers have lower Frisch elasticity.

2.1.3 Outline

I describe the model in detail at section 2.2, and I illustrate aggregation and the equilibrium of the model in section 2.3. In section 2.4, I calibrate model and obtain impulse responses to both of the productivity shock and monetary policy shock. I discuss about the estimation results in section 5. Finally, I give a conclusion in section 2.5.

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[7] Gourio and Noual (2009) also show that even though most workers are inelastic, aggregate elasticity can be large depending on the size of marginal workers whose wage are close to reservation wage using National Longitudinal Survey of Youth 1979 (NLSY79) data. Their estimates of Frisch elasticity as of July 1985 are 1.13, 1.22, and 2.05 for college graduates, high-school graduates, and high-school dropouts respectively.

2.2 Model

I extend Galí (2011) so that each household comprised of a continuum of workers and they are belong to one of the two sectors according to their education levels. I make three assumptions to simplify the analysis. First, a worker is prohibited to cross the sector (sectoral immobility), and thus, there are two distinct monopolistic labor markets. Second, population share of a sector is constant, that is, a fixed fraction of total population is always belongs to a sector. Third, I also assume there is only one good producing sector in which a firm hires workers from both of the labor markets, and aggregate it into homogeneous labor as an input factor. By this, the model can be generalized with heterogeneous good producing sectors with different technologies of aggregation; some firms need more high-skilled workers relative to other firms; or the elasticity of substitution between high-skilled workers and low-skilled workers could vary across the sectors. I then introduce heterogeneity through the elasticity of labor supply and the elasticity of substitution between workers within a sector; elasticities of high-skilled sector differ from those of low-skilled sector. Furthermore, I assume indivisible labor so that all the labor variations occur only at extensive margin.

2.2.1 Firms and Price setting

Monopolistic competitive firms hire workers from both labor markets and then aggregate it with CES technology into homogeneous effective labor input. Each firm produces a differentiated good $z \in [0, 1]$ using a production function which is given by:

$$
\begin{aligned}
& Y_t(z) = A_t \left(H_t(z)\right)^{1-\alpha} \\
& \quad \text{where} \quad H_t(z) = \left[\gamma^{\frac{1}{\eta}} \left(N_t^H(z)\right)^{\frac{\eta-1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}} \left(N_t^L(z)\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \\
& \quad \text{and} \quad N_t^j(z) \equiv \left(\int_0^1 N_t^j(i,z)^{\frac{\epsilon_w^j-1}{\epsilon_w^j}}\, di\right)^{\frac{\epsilon_w^j}{\epsilon_w^j-1}} \quad \text{for} \quad j \in (H, L)
\end{aligned}
$$

where $H_t(z)$ is the homogeneous effective labor input of firm z obtained by labor aggregation technology; $\alpha < 1$ denotes decreasing returns to scale parameter of effective labor by which firms face increasing marginal cost when production expands through more employment; η is elasticity of substitution[8] between high-skilled ($N_t^H(z)$) and low-skilled labor ($N_t^L(z)$); γ is the parameter governing the relative income share and it becomes relative income share itself when $\eta = 1$, that is, when production function is a Cobb-Douglas; ε_w^j is labor elasticity of substitution within the corresponding sector $j \in (L, H)$ and I would defer further explanation of this parameter to wage determination section. A_t is an exogenous technology parameter which is assumed that $a_t \equiv logA_t$ and $a_t = \rho_a a_{t-1} + \varepsilon_t^a$ where $\rho_a \in [0,1]$ and ε_t^a is a white noise process with zero mean and variance σ_a^2. Nominal marginal cost can be derived from the optimality condition for a representative firm with respect to labor input:[9]

$$\Psi_t(z) = \frac{W_t}{(1-\alpha)\left(H_t(z)\right)^{-\alpha}} \quad \text{where} \quad W_t \equiv \left(\gamma \left(W_t^H\right)^{1-\eta} + (1-\gamma)\left(W_t^L\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}$$

where W_t^H and W_t^L are the average nominal wage for high-skilled workers and low-skilled workers respectively. In addition, firm's cost minimization problem, taking wages as given, implies the set of labor demand schedules:[10]

$$N_t^j(i,z) = \left(\frac{W_t^j(i)}{W_t^j}\right)^{-\epsilon_w^j} N_t^j(z) \quad \text{and} \quad N_t^j(z) = \gamma \left(\frac{W_t^j}{W_t}\right)^{-\eta} H_t(z) \quad \text{where j} \in (H, L)$$

for all $i \in [0,1]$ and for all $z \in [0,1]$. I introduce nominal rigidities in price through the Calvo (1983) pricing; only $1-\theta_p$ fraction of the firms can choose their optimal price, P_t^*, in period t to maximize their profit subject to the sequence of demand schedule constraint $Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon_p} C_{t+k}$, for $k = 0, 1, 2, ...$ Where $Y_{t+k|t}$ denotes output at time $t+k$ of a firm that last reset its price in period t. Then optimality condition for

[8] as $\eta \to 0$, production function become Leontief in which low-skilled workers and high-skilled workers used only with fixed proportions; as $\eta \to \infty$, workers with both education level are perfectly substitutable, and hence, only low-skilled workers are used unless the average wages are the same for both skilled workers; as $\eta \to 1$, it become Cobb-Douglas production function in which income share is fixed with γ

[9] See Appendix B.1.2 for the details of derivation

[10] See Appendix B.1.2 for the details of derivation

the firm is given by

$$\sum_{k=0}^{\infty} \theta_p^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \left(P^* - \mathcal{M}^p \Psi_{t+k|t} \right) \right\} = 0$$

where $Q_{t,t+k} \equiv \beta^k \left(\frac{C_t}{C_{t+k}} \right)^{\sigma} \frac{P_t}{P_{t+k}}$ is the relevant stochastic discount factor for nominal payoffs in period $t+k$, $\Psi_{t+k|t} \equiv \frac{W_t}{(1-\alpha) A_{t+k} H_{t+k|t}^{-\alpha}}$ is the nominal marginal cost in period $t+k$ of producing quantity $Y_{t+k|t}$ and $\mathcal{M}^p \equiv \frac{\epsilon_p}{\epsilon_p - 1}$ is the desired or frictionless price markup over the marginal cost. Log-linearization of the optimality condition around the zero inflation steady state yields

$$p_t^* = \mu^p + (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_p)^k E_t \left\{ \psi_{t+k|t} \right\} \tag{2.1}$$

Note that lower case variables in present paper denote the log-deviation of the variables from the steady state and $\mu^p \equiv \log(\mathcal{M}^p)$. Define the average nominal marginal cost as $\Psi_{t+k} \equiv \frac{W_{t+k}}{(1-\alpha) Y_{t+k} / H_{t+k}}$. Noting that first order log approximation of aggregate relation $y_t = a_t + (1-\alpha) h_t$, it follows that

$$\begin{aligned} \psi_{t+k|t} &= \psi_{t+k} + \alpha \left(h_{t+k|t} - h_{t+k} \right) \\ &= \psi_{t+k} - \frac{\alpha \epsilon_p}{1-\alpha} \left(p_t^* - p_{t+k} \right) \end{aligned} \tag{2.2}$$

Price inflation equation can be derived by combining (2,1), (2,2) and the log-linearized price index, $p_t = \theta_p p_{t-1} + (1-\theta_p) p_t^*$:

$$\pi_t^p = \beta E_t \left\{ \pi_{t+1}^p \right\} - \lambda_p \left(\mu_t^p - \mu^p \right) \tag{2.3}$$

where $\pi_t^p \equiv p_t - p_{t-1}$ is wage inflation, $\mu_t^p \equiv p_t - \psi_t$ denotes the log average price markup which is the same as negative real marginal cost, and $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} \frac{(1-\alpha)}{(1-\alpha+\alpha\epsilon_p)} > 0$.

2.2.2 Household

There are large number of identical households which are comprised of a continuum of members for each labor market represented by the unit indexed by $i \in [0, 1]$. The index $i \in [0,1]$ indicates the type of labor service in which a given household member is specialized in a given sector. The household members are heterogeneous in that labor elasticities of workers are different across the sectors and the workers are assigned to a sector depending on the level of skill they have.[11] I assume that there are two level of skill, $j \in (L,H)$, by which the members are categorized as low skilled workers and high skilled workers respectively. Each of agents assumed to consume the same level of goods and services independently of their employment status and skill level, so it assumed full risk sharing within a household. Representative household maximize household's discounted lifetime utility (2.4) subject to budget constraint (2.5). While labor demand, $N_t^j(i)$, are determined by the aggregation of firm's labor demand decisions and allocated uniformly across households, workers choose their optimal wages, $W_t^j(i)$. Therefore, both $W_t^j(i)$ and $N_t^j(i)$ are taken as given by each individual household. Each household's discounted lifetime utility is given by:

$$\sum_{t=0}^{\infty} \beta^t U(C_t, N_t^j(i)) = \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - s \int_0^1 \frac{\left(N_t^H(i)\right)^{1+\frac{1}{\varphi_H}}}{1+\frac{1}{\varphi_H}} di - (1-s) \int_0^1 \frac{\left(N_t^L(i)\right)^{1+\frac{1}{\varphi_L}}}{1+\frac{1}{\varphi_L}} di \right] \tag{2.4}$$

where s[12] is the ratio of the high-skilled population to the total population and is assumed to be constant. $C_t \equiv \left(\int_0^1 C_t(z)^{\frac{\epsilon_p - 1}{\epsilon_p}} dz \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$ and $C_t(z)$ is the quantity consumed of good z, for $i \in [0,1]$ and $N_t^j(i) \in [0,1]$ is the fraction of members specialized in type i labor in $j \in (L,H)$ sector who are employed in period t. φ_j denotes the Frisch labor supply elasticity of j skilled worker. Blanchard and Katz (1997) argue that since the wages of low- skilled workers are close to their reservation wage (whereas the wages of high-skilled workers is much higher than their reservation wage), high-skilled workers has much flatter labor supply than low-skilled worker. Taking this point of view, I assume that parameter for Frisch elasticity, $\varphi_H < \varphi_L$, that is, high-skilled workers would have higher marginal rate of substitution than low-skilled worker at the same level of

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[11] Again, they also can be classified by gender or race

[12] when all variables are measured in per capita term, $\frac{N_t^H}{Pop_t} = \frac{Pop_t^H}{Pop_t} \frac{L_t^H}{Pop_t^H} \frac{N_t^H}{L_t^H} = s \frac{N_t^H}{Pop_t^H}$ where $j \in (H,L)$. Therefore, N_t^j and L_t^j can be interpreted as Employment to Population Ratio and participation rate respectively as explained in later.

employment. Households budget constraint is given by:

$$\int_0^1 P_t(z)C_t(z)dz + Q_tB_t \leq B_{t-1} + s\int_0^1 W_t^H(i)N_t^H(i)di + (1-s)\int_0^1 W_t^L(i)N_t^L(i)di + \Pi_t \tag{2.5}$$

where $P_t(z)$ is the price of good z , $W_t^j(i)$ is the nominal wage for type i labor with $j \in (L, H)$ skill, B_t represents purchases of nominally riskless one-period discount bond paying one monetary unit, Q_t is the price of that bond, and Π_t is a lump-sum component of income at time t. The first order conditions for the maximization problem subject to the budget constraint give the consumption Euler equation:

$$Q_t = \beta E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}} \tag{2.6}$$

Optimal demand for each good resulting from utility maximization takes the familiar form:[13]

$$C_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\epsilon_p} C_t$$

where $P_t \equiv \left(\int_0^1 P_t(z)^{1-\epsilon_p}\right)^{\frac{1}{1-\epsilon_p}}$ denotes the price index for final goods.

2.2.3 Wage Determination

Analogous to the good market, the labor markets are monopolistic competitive, and wages are determined by Calvo pricing. I assume that the Calvo parameters are the same, $\theta_w = \theta_w^H = \theta_w^L$, following Barattieri et al. (2014). In addition, I assume that high-skilled workers are not easily substituted by others relative to low-skilled workers; $\varepsilon_w^H < \varepsilon_w^L$. It implies that the markup of high-skilled workers in wage setting is greater than that of low-skilled workers, and it also makes sure that high-skilled wage is larger than low-skilled wage on average with the assumption on Frisch elasticities. As I explain later, these two elasticities are closely related to divergent unemployment rates. For each labor market, only $1-\theta_w$ fraction of workers can re-optimize their wage. When re-optimizing their wage in period t, workers choose a wage W^{j*}, where again $j \in (L, H)$, in order to

[13]Details for derivation in Appendix B.1.1

maximize their households' utility taking as given all aggregate variables, including the aggregate wage index $W_t^j \equiv \left(\int_0^1 W_t^j(i)^{1-\epsilon_w^j} di\right)^{\frac{1}{1-\epsilon_w^j}}$. The optimal wage setting rule for j-skilled workers can be obtained from the maximization problem subject to the budget constraint and the corresponding labor demand schedule which is determined by firms:

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ N_{t+k|t}^j C_{t+k}^{-\sigma} \left(\frac{W^{j*}}{P_{t+k}} - \mathcal{M}^j MRS_{t+k|t}^j \right) \right\} = 0 \tag{2.7}$$

where $N_{t+k|t}^j$ denotes the aggregate quantity demanded in period t+k of high-skilled workers whose wage was last reset in period t. $MRS_{t+k|t}^j \equiv sC_{t+k}^{\sigma}(N_{t+k|t}^j)^{\frac{1}{\varphi_j}}$ is the period t+k marginal rate of substitution between consumption and employment for a high-skilled worker whose wage is reset in period t, and $\mathcal{M}^j \left(\equiv \frac{\epsilon_w^j}{\epsilon_w^j - 1}\right)$ is the desired or frictionless wage markup and $\mu^j \equiv \log \mathcal{M}^H$. The first order log approximation of (7) around the zero inflation steady states gives the optimal wage equation as the markup plus weighted average of future price-adjusted marginal rate of substitution:

$$w_t^{j*} = \mu^j + (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ mrs_{t+k|t}^j + p_{t+k} \right\} \tag{2.8}$$

Defining the j-skilled sector's average marginal rate of substitution as $MRS_t^j \equiv sC_t^{\sigma}(N_t^j)^{\frac{1}{\varphi_j}}$, the marginal rate of substitution of each individual in j-skilled sector can be written in terms of the relationship between the average marginal rate of substitution and the relative wage:[14]

$$\begin{aligned} mrs_{t+k|t}^j &= mrs_{t+k}^j + \frac{1}{\varphi_j}\left(n_{t+k|t}^j - n_{t+k}^j\right) \\ &= mrs_{t+k}^j - \frac{\epsilon_w^j}{\varphi_j}\left(w_t^{j*} - w_{t+k}^j\right) \end{aligned} \tag{2.9}$$

---

[14]where $N_t^j \equiv \int_0^1 N_t^j(i)di$ is the sector aggregate employment rate

Finally, combining (2.8), (2.9) and the log-linearized form of aggregate wage index, I obtain j-skilled wage Phillips curve as:[15]

$$\pi_t^j = \beta E_t \left\{ \pi_{t+1}^j \right\} - \lambda_w^j \left(\mu_t^j - \mu^j \right) \tag{2.10}$$

where $\pi^j \equiv w_t^j - w_{t-1}^j$ is the j-skilled wage inflation, $\mu_t^j \equiv w_t^j - p_t - mrs_t^j$ denotes the log average j-skilled wage markup, and $\lambda_w^j \equiv \frac{\Theta}{1+\frac{\epsilon_w^j}{\varphi_j}} > 0$ where $\Theta \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w}$.

2.2.4 Unemployment

Since the idea for introducing the unemployment is exactly the same as in Galí (2011), I will briefly summarize his idea then explain further about heterogeneity. An individual will be willing to work in period t if and only if real wage for his labor type exceeds his disutility of labor. Thus the marginal high-skilled supplier of type i labor, $L_t^j(i)$, is given by

$$\frac{W_t^j(i)}{P_t} = C_t^\sigma (L_t^j(i))^{\frac{1}{\varphi_j}}$$

Define the aggregate labor force (or participation rate) as $L_t^j \equiv \int_0^1 L_t^j(i)di$, then the first order approximations gives the log-linearized estimate relation:

$$w_t^j - p_t = \sigma c_t + \frac{1}{\varphi_j} l_t^j$$

The unemployment rate u_t can be written as the log difference between the labor force and employment:[16]

$$u_t^j \equiv l_t^j - n_t^j$$

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[15] $w_t^j = \theta_w w_{t-1}^j + (1-\theta_w) w_t^{j*}$

[16] $u_t^j = 1 - \frac{N_t^j}{L_t^j} \Rightarrow -u_t^j \approx \log(1-u_t^j) = n_t^j - l_t^j$. Note, in efficient steady state, all labor force has to be hired ($l = n$, that is, $u = 0$); and government subsidies impose symmetric labor market ($l^H = l^L = n^H = n^L$). Moreover, note that I assume that share of the high-skilled population is constant s. Then, $L_t = sL_t^H + (1-s)L_t^L$ and $N_t = sN_t^H + (1-s)N_t^L$. $\hat{u}_t \approx \hat{l}_t - \hat{n}_t = s\hat{u}_t^H + (1-s)\hat{u}_t^L$.

Noting that real wage is the markup over the marginal rate of substitution, $\mu_t^j \equiv \left(w_t^j - p_t\right) - mrs_t^j = \left(w_t^j - p_t\right) - \left(\sigma c_t + \frac{1}{\varphi_j} n_t^j\right)$, the unemployment can be written as:

$$u_t^j = \varphi_j \mu_t^j \tag{2.11}$$

Therefore, as Galí (2011) mentioned, (2.11) implies that unemployment fluctuations are a consequence of variations in the wage markup. The presence of market power in wage setting reflected in the wage markup ($\mu^j > 0$) implies the existence of positive unemployment even in the absence of wage rigidities unless labor supply is absolutely inelastic: $u^{jn} = \varphi_j \mu^j > 0$, if $\varphi_j \neq 0$ and $\epsilon_w^j \neq 1$. Note that natural unemployment, u^{jn}, is increasing in labor supply elasticity, φ_j, and wage markup μ^j (or decreasing in labor demand elasticity, ε_w^j). In order to understand the relative unemployment fluctuation, one needs to consider the ratio of the labor demand to supply elasticity. This is because the effect of markup fluctuation is highly depending on labor supply elasticity. The ratio is the key factor accounting for the differentials in unemployment rates. Recall the assumption I made above ($\varepsilon_w^H < \varepsilon_w^L$ and $\varphi_H < \varphi_L$). For instance, when labor supply elasticities are equal for both workers regardless their skill level but elasticities of substitution are kept as in the assumption, natural rate of unemployment for high-skilled worker would be greater than for low-skilled workers due to the greater wage markup. On the other hand, if labor demand elasticities (elasticity of substitution between workers within a sector) are the same for both labor markets and the Frisch elasticity of low-skilled workers exceeds that of high-skilled workers, then low-skilled workers would have higher natural rate of unemployment. Finally, combining (2.10) with (4.10), I derived the sectoral New Keynesian wage Phillips Curve:

$$\pi_t^j = \beta E_t \left\{\pi_{t+1}^j\right\} - \frac{\Theta}{\varphi_j + \epsilon_w^j} \left(u_t^j - u^{j,n}\right) \tag{2.12}$$

The slope of the Phillips curve for each labor market also depends on both elasticities. In contrast to the case for unemployment, the slope is decreasing in both elasticities. Therefore, high-skilled workers face steeper wage Phillips curve under the assumption, accordingly, high-skilled nominal wages are much volatile than low-skilled nominal wages in unemployment fluctuation as shown in data.

2.3 Aggregation and Equilibrium

I assume that the government can eliminate the wage markups by appropriate subsidies (τ) to guarantee that $\mathcal{M}^j(1-\tau^j) = 1$ for $j \in (L, H)$, and $\mathcal{M}^p(1-\tau^p) = 1$, and hence, $W = W^H = W^L$ in the efficient steady state, then log-linearized wage index can be written:[17]

$$w_t = sw_t^H + (1-s)w_t^L$$

2.3.1 Resource Constraint and Euler Equation

Because of absence of investment and government spending in closed economy, all outputs produced by each firms are consumed. Therefore, market clearing condition is $C_t(z) = Y_t(z)$ for all $z \in [0,1]$, and hence, $C_t = Y_t$. Euler equation (2.6) can be written as

$$Q_t = \beta E_t \left(\frac{Y_{t+1}}{Y_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}}$$

and its log-linear approximation from the zero inflation steady state can be expressed as

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}\left\{i_t - E_t \pi_{t+1}^p\right\}$$

where $i_t = -q_t$.[18]

2.3.2 Labor Market Equilibrium

Aggregate labor supply from all households must be equal to firm's aggregate labor demand for each labor market in equilibrium. Accordingly, the labor market clearing

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[17] $w_t = \gamma\left(\frac{W^H}{W}\right)^{1-\eta} w_t^H + (1-\gamma)\left(\frac{W^L}{W}\right)^{1-\eta} w_t^L = \left(\frac{W^H N^H}{WN}\right) w_t^H + \left(\frac{W^L N^L}{WN}\right) w_t^L = sw_t^H + (1-s)w_t^L$

[18] I define $Q_t \equiv \frac{1}{R_t} = \frac{1}{1+i_t}$

conditions are following:[19]

$$N_t^j = \int_0^1 N_t^j(z)dz = \int_0^1 \int_0^1 N_t^j(i,z)didz \quad = \gamma_j \Delta_t^j \Delta_t^j \left(\frac{W_t^j}{W_t}\right)^{-\eta} \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}}$$

Note that Δ_t^P, Δ_t^j for $j \in (H, L)$ are measures for the price, the high skilled, and the low skilled wage dispersion respectively, and those can be approximated to 1 up to first order.[20] The log-linearization of the employment in each sector around the steady state can be written:

$$n_t^j = -\eta\left(w_t^j - w_t\right) + \frac{1}{1-\alpha}\left(y_t - a_t\right) \tag{2.13}$$

Furthermore, defining H_t subjects to $Y_t = A_t H_t^{1-\alpha}$, log-linearized effective labor can be derived as[21]:

$$h_t = \frac{1}{1-\alpha}\left(y_t - a_t\right) = \gamma n_t^H + (1-\gamma) n_t^L$$

It is clear from (2.13) that when the wage-premium increases, firms more demand low-skilled workers substituting high-skilled workers. In homogeneous labor market model, the employment rate only depends on aggregate demand (output), whereas, with two sectors in labor market, the employment affected by the wage-premium (or relative wage) as well; Aggregate shock affects both low-skilled and high-skilled employment directly and this affects the wages for both market. However, the sizes of the wage adjustment in both sectors are different due to different slope of wage Phillips curve arising from the different elasticities. It causes the change in wage-premium. The change in the wage-premium has asymmetric (depending on γ) and opposite impact on the labor

[19] See Appendix B.1.2 for the details of derivation

[20] $\Delta^P \approx 1 + \frac{1}{2}\left(\frac{\varepsilon_p}{1-\alpha}\right)\left(\frac{1-\alpha+\alpha\varepsilon_p}{1-\alpha}\right) Var_z\{p_t(z)\}$, $\Delta_t^H \approx 1 + \frac{\varepsilon_w^H}{2} Var_z\left\{w_t^H(z)\right\}$ and $\Delta_t^L \approx 1 + \frac{\varepsilon_w^L}{2} Var_z\left\{w_t^L(z)\right\}$ and details for second order log approximations are in Appendix C

[21] Log-linearized effective labor can be written $h_t(z) = \gamma^{\frac{1}{\eta}}\left(\frac{N^H}{H}\right)^{\frac{\eta-1}{\eta}} n_t^H(z) + (1-\gamma)^{\frac{1}{\eta}}\left(\frac{N^L}{H}\right)^{\frac{\eta-1}{\eta}} n_t^L$. However, since I assume that the government subsidies make $W^H = W^L = W$ in steady state, and it is simplified to $h_t(z) = \gamma n_t^H(z) + (1-\gamma) n_t^L(z)$ because $\frac{N^H}{H} = \gamma\left(\frac{W^H}{W}\right)^{1-\eta} = \gamma$, In addition, defining $Y_t = A_t H_t^{1-\alpha}$, I obtain first order approximation of aggregate effective labor as $h_t = \gamma n_t^H + (1-\gamma) n_t^L$.

markets. Note that even though aggregate shock brings about disproportionate effect on labor markets, it has no direct effect on aggregate output. This is because firms can produce the same level of homogeneous effective labor input by changing the proportion of the inputs (high-skilled and low-skilled workers). However, the differential in wage response to aggregate shock influences aggregate wage inflation and it affects price inflation and thus aggregate level of output. I will discuss this channel with the wage-premium in next subsection.

2.3.3 Aggregate Dynamics under the segmented labor market

In this subsection, I discuss the model dynamics under the segmented labor market and contrast that under the standard (single) labor market model. To this end, I derive the aggregate wage Phillips curve by taking weighted average of the two sectoral wage Phillips curves:[22]

$$\pi_t^w = \beta E_t \left\{ \pi_{t+1}^w \right\} + \kappa_w y_t - \left(\gamma \lambda_w^H \tilde{\omega}_t^H + (1-\gamma)\lambda_w^L \tilde{\omega}_w^L \right) - \eta\gamma(1-\gamma)\Theta \left(\chi_H - \chi_L \right) \tilde{\omega}^R \tag{2.14}$$

where $\chi_j \equiv \frac{1}{\varphi_j + \varepsilon_w^j}$ denotes inverse of composite elasticity for $j \in (L, H)$ sector and $\kappa_w > 0$ denotes the weighted average slope of the two sectoral wage Phillips curve. Noting that composite elasticity of high-skilled workers is always less than that of low-skilled workers, $\chi_H > \chi_L$, it is straightforward that the aggregate wage Phillips curve has two negative endogenous shift terms related to weighted sum of real wages and the wage-premium. The endogenous shift term arising from wage-premium brings about important channel for aggregate dynamics. This is because the two endogenous shifts could be offset each other depending on the direction of real wages and the wage-premium; if both terms have the same direction the shift would be amplified, otherwise it would be offset. For example, suppose a shock forces nominal wage to decrease. High-skilled wage will fall more than low-skilled wage inducing decrease in the wage-premium. After positive technology shock leading increase in real wage, the effect of

[22]It also can be expressed as: $\pi_t^w = \beta E_t \left\{ \pi_{t+1}^w \right\} - \Theta \left\{ \frac{\gamma \hat{u}_t^H}{\chi_H} + \frac{(1-\gamma)\hat{u}_t^L}{\chi_L} \right\}$

endogenous shift will be offset. On the other hand, negative demand shock will push real wage down, and the effect will be amplified. Therefore, the directions of these two endogenous variables are important in order to understand aggregate dynamics. Similarly, Lee (2011) discussed that aggregate Phillips curve has endogenous shift term arising from changes in relative price and it makes aggregate Phillips curve flatter than homogeneous model assuming the heterogeneity in good producing sectors in that each sector has different frequencies of price adjustment. Another important thing of aggregate wage Phillips curve is the effect of γ. In present model, obviously, the slope of the aggregate wage Phillips curve become steeper as γ increase. However, the magnitude of the endogenous shift is also depending on the value of γ; if $\gamma = 1$ ($\gamma = 0$), the curve will be steeper (flatter) but the endogenous shift arising from the wage-premium will disappear implying homogeneous labor market. Note that under homogeneous labor market, that is, when $\varphi_H = \varphi_L = \varphi$ and $\varepsilon_w^H = \varepsilon_w^L = \varepsilon_w$, the aggregate wage Phillips curve coincides with Galí (2011). Aggregate elasticity can be directly calibrated once I have sectoral elasticities, it allows us to compare heterogeneous labor market model with homogeneous labor market model. I explain more about the aggregate dynamics later with the impulse responses.

2.4 Quantitative Analysis

2.4.1 Calibration

I adopt decreasing return to labor parameter $\alpha = 0.17$ from Galí et al. (2011).[23] I set $\eta = 2.6$, adopting the mean value of plausible range of this parameter suggested

[23] It is also close to the calibrated value (0.13) of Greenwood et al. (1997)

by Mollick (2011).[24] In order to obtain γ, I compute relative income share of high-skilled household based on the "Merged Outgoing Rotation Group"(MORG) of Current Population Survey (CPS) which covers from 1979Q1 to 2008Q3. I then obtain $\gamma = 0.52$.[25] I assume relatively risk averse households by setting the intertemporal elasticity of consumption; $\sigma = 2$. Noting that the natural unemployment rate is determined by the ratio between desired wage markup and Frisch elasticity, $u^j = \varphi_j \mu^j$, I compute combinations of the values of φ_j and ε_w^j, which is consistent with average unemployment rates of the data. I then set $\varphi_H = 0.125$, $\varphi_L = 0.56$, $\epsilon_w^H = 4$, and $\varepsilon_w^L = 7.5$. This implies 33.3% (15.3%) wage markup for the high-skilled (low-skilled) workers. Then aggregate wage markup is approximately 25% as in Galí (2011). In doing this, I consider natural rate of unemployment for each labor market as average unemployment which is obtained from MORG of CPS data I mentioned above; $u^{Ln} = 0.079, u^{Hn} = 0.037$. I set $\varepsilon_p = 10$ implying price markup at steady state is about 11%. I set $\theta_p = 0.75$ following Nakamura and Steinsson (2008) which means that an average duration of price is 4 quarters. Barattieri et al. (2014) argue that there is little evidence of heterogeneity in frequency of wage adjustment across industries and occupations, and they estimate the Calvo parameter as 0.822 implying that only 17.8% of hourly workers experienced wage change in a quarter. As usual, β is set to 0.99. Finally, I set policy response parameter to inflation, ϕ_π, and output gap, ϕ_y, is 1.5 and 0.125 respectively. Table 2.1 summarize the parameters, descriptions, and the values.

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[24] Krusell et al. (2000a) estimates CES production function and shows capital structures' share of income at 0.117 and elasticities of substitution 1.67 between skilled and unskilled labor. The elasticity of substitution parameter η is somewhat controversial since the production function does not take capital as an input in the present model. In addition, there are broad ranges of estimates: Raval (2011) estimates CES production function with factor augmenting technology and shows that the estimate of elasticity of substitution between skilled and unskilled labors is 2.43. Lee and Wolpin (2010) also estimate the elasticity between composite capital-white-collar workers and pink or blue collar workers in the service (good) sector of 1.21 (1.34). Mollick (2011) estimates the elasticity using enrollment ratios and government expenditures on education as instrument variable, and argues that the plausible elasticities vary over 2.00 to 3.21. I also get IRFs by setting different values for η. The main result is not affected by different values of η.

[25] Note that, I consider that W_t^H is average weekly earnings of high-skilled workers who attains at least Bachelor's degree and and W_t^L is that of low-skilled workers whose educational attainment is less than Bachelor's degree. γ is used for the weight when I obtain the aggregate variables from the sectoral variables in present paper.

Parameter	Description	Value
α	Decreasing returns to labor	0.17
η	Labor elasticity of Substitution across the sector	2.6
γ	Relative income share	0.52
σ	Inetertemporal Elasticity of consumption	2
φ_H	Frisch elasticity of High-skilled labors	0.125
φ_L	Frisch elasticity of Low-skilled labors	0.56
ε_w^H	Elasticity of Substitution among High-skilled labors	4
ε_w^L	Elasticity of Substitution among Low-skilled labors	7.5
ε_p	Elasticity of Substitution among goods	10
θ_p	Calvo parameter of price rigidity	0.75
θ_w	Calvo parameter of wage rigidity	0.822
β	Discount factor	0.99
ϕ_p	Inflation coefficient in policy rule	1.5
ϕ_y	Output coefficient in policy rule	0.125
ρ_a	Persistent parameter of technology shock	0.9
ρ_ν	Persistent parameter of monetary policy shock	0.9

Table 2.1: Parameter Calibration

2.4.2 Impulse Responses

In doing quantitative analysis, I assume that the central bank sets nominal interest rate following Taylor rule:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)\left(\phi_\pi \pi_t^p + \phi_y y_t\right) + \nu_t$$

where $\rho_i \in (0, 1)$ is persistent parameter of monetary policy, and ν_t is a white noise with zero mean and standard deviation, σ_ν. I then build a linear system of equations with two exogenous shocks; aggregate technology shock, a_t, and monetary policy shock, ν_t.[26] I use the 'Dynare' to solve the model as well as to obtain impulse responses. Figure 2.3 and Figure 2.4 show the dynamic responses of basis endogenous variables (output, price inflation, wage-premium, employment gap, labor income gap and unemployment rate gap across sectors) to each shock based on the calibrated model. Figure 2.3 displays the impulse responses to a 1 percent increase in the technology shock. The aggregate variables such as output, price inflation, real wage, employment and unemployment rate

[26] see appendix A.1 for the system of equations

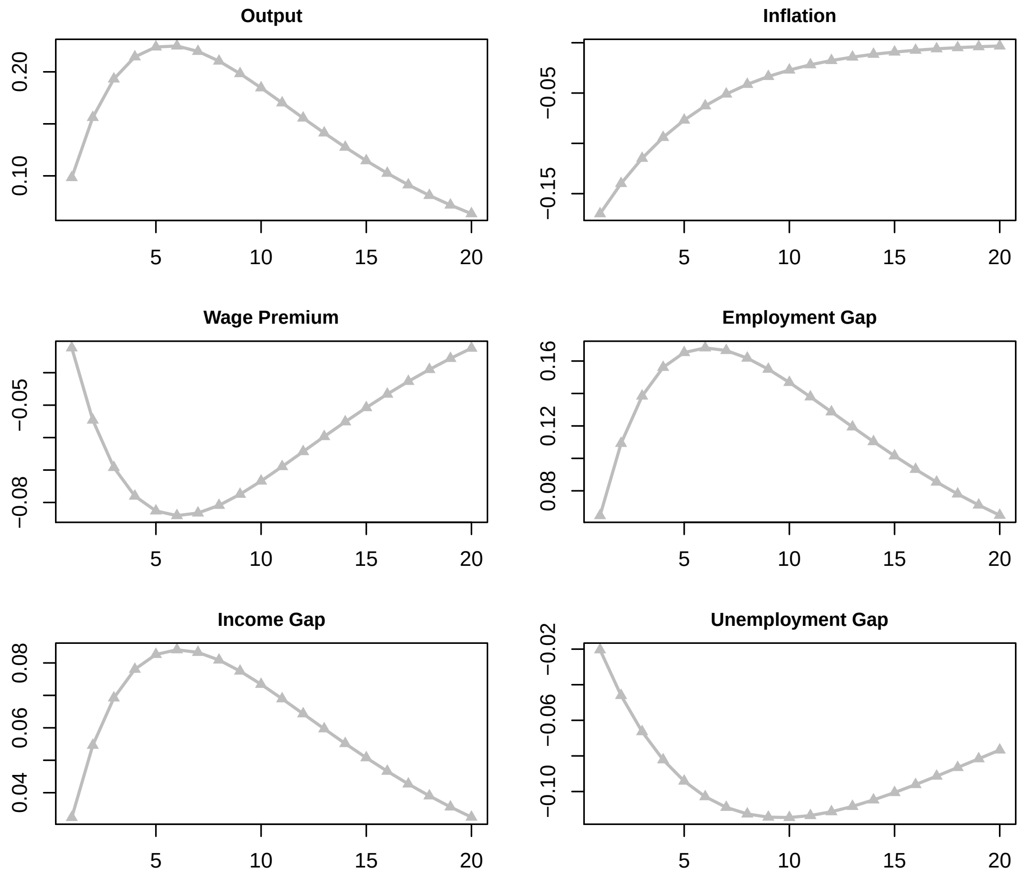

Figure 2.3: IRF to the Technology shock

moves as in Galí (2011). In particular, output increases and inflation falls (countercyclical price inflation) in response to the supply shock. The employment rates decrease whereas the unemployment rates increase in both sectors. These results are consistent with Canova et al. (2010) and Barnichon (2010) respectively, although it is in contrast to the predictions of Real Business Cycle model, such as a standard search and matching model.[27] An intuition behind this is that, firms with upgraded labor aggregation technology have more efficient internal labor allocation and they can reduce unnecessary employment raising unemployment. Nominal wages decrease as a consequence of increase in unemployment rate in both sectors. However, real wages rise because of

[27]Canova et al. (2010) argue that the data with an appropriate treatment for the long cycles in hours suggest that per-capita hours fall in response to neutral shocks and increase in response to investment shocks. Barnichon (2010) finds the empirical evidence of short-run increase in unemployment after technology shock

further decline in inflation than nominal wages as in Galí (2011) and those increase gradually due to the nominal rigidities.

Since, more importantly, this paper is intended to account for the heterogeneity in the labor market, I do not plot most of the aggregate variables, and I display the gaps in the sectoral variables to highlight the differentials instead. The high-skilled real wage rises less than low-skilled real wage. This is because the high-skilled nominal wage falls more than that of low-skilled workers due to the steeper wage Phillips curve, and thus it is less covered by decreased inflation. This result is consistent with Lagakos and Ordonez (2011) in which the authors argue that real wages in low-skilled sectors respond relatively more to the productivity shock than high-skilled sectors. As a result, the wage-premium falls in response to an increase in productivity. This result is noteworthy because the wage-premium generates important channel for the differentials in employment and unemployment rates as (2.13); A reduced wage-premium implies a decline in firms' relative cost to carry with high-skilled workers and hence it induces a greater demand of the firms for high-skilled workers so that employment gap across sectors increases. As a consequence, the low-skilled unemployment rate increases more than the high-skilled unemployment rate due to further decline in low-skilled employment. In addition, a widened employment gap dominates reduced wage premium given elastic substitution between two different skilled workers, and consequently the income gap rises. Figure 2.4 shows the impulse responses to 25 basis point increase in monetary policy shock, ν_t, implying that 1 percentage point rise in the (annualized) nominal rate. Again, the aggregate variables moves in the same directions as in Galí (2011). Both employment ratios decline substantially along with output (resulting from the decline in consumption due to higher interest rate) in response to the contractionary monetary policy. In addition, both price inflation and the real wage decline at this time. This co-movement in output, real wages, and inflation in response to the demand shock is well known in the literature. The labor force tends to rises for both markets due to the negative wealth effect, consequently, the unemployment rates increase by more than the falls in corresponding employment.

The wage-premium decreases due to relatively flexible high-skilled nominal wages;

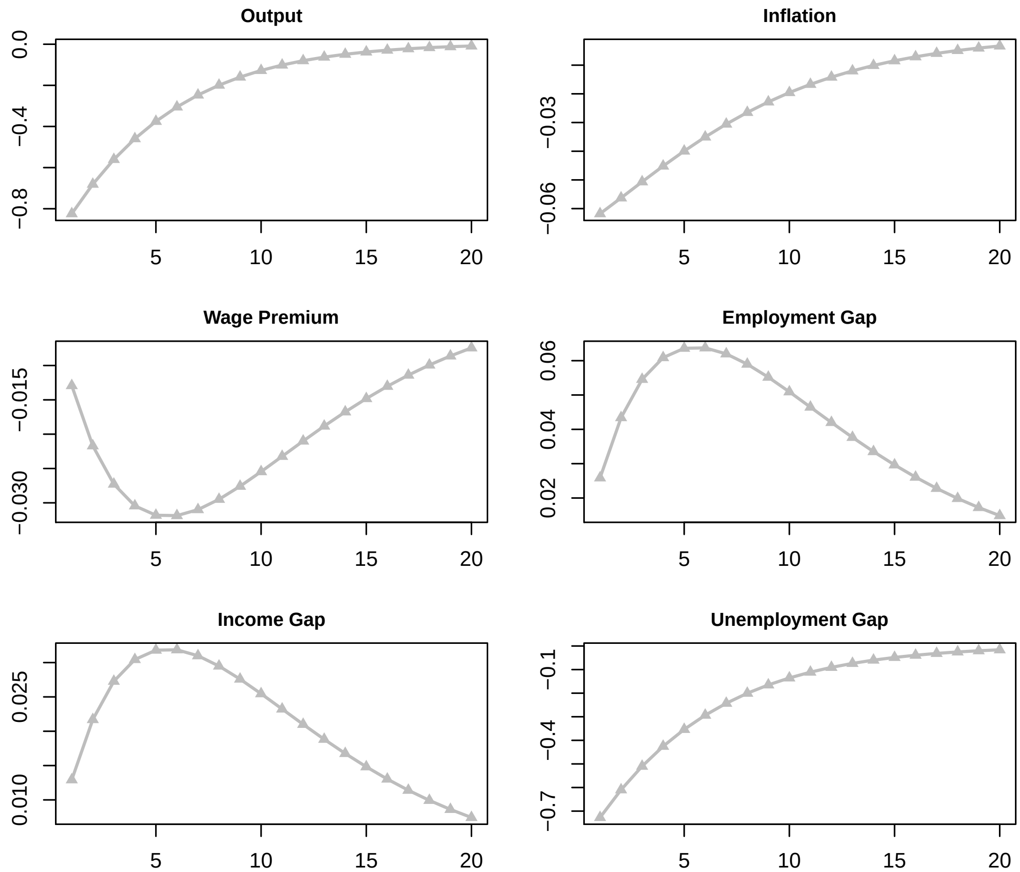

Figure 2.4: IRF to the monetary shock

when nominal wage decreases, high-skilled nominal wage drops more than low-skilled nominal wages. Given the reduced wage premium, firms start to substitute high-skilled workers for low-skilled workers, and in turn, low-skilled employment declines more than high-skilled employment. Accordingly, the low-skilled unemployment rate increases more and becomes more volatile. This is enhanced by a greater labor supply elasticity of low-skilled workers. Barnichon and Figura (2010) note that "attachment to the labor force is counter-cyclical, with workers more likely to join/stay in the labor force during the recession". However, the magnitude in participation rate fluctuate is too large in comparison to the VAR analysis. Galí et al. (2011) introduces smoothing parameter for the disutility of labor supply generating the smaller wealth effect in the short run and muted labor supply response.[28] All the qualitative results are consistent with the

[28]However, the unemployment differential can be generated without heterogeneity in labor supply

VAR analysis; much larger increase in unemployment rate and participation rate of low-skilled workers than high-skilled workers; procyclical wage-premium conditional on monetary policy shock.

2.4.3 Aggregate Dynamics

In this subsection, I discuss about the aggregate implication of the model by comparing the aggregate impulse responses of the segmented (heterogeneous) labor market model with those of the single (homogeneous) labor market model under basis calibration. In doing this, I compute the aggregate Frisch labor supply elasticity, φ, and aggregate elasticity of substitution between workers, ε_w, for the single labor market model to be consistent with the average value (5.3%) of aggregate unemployment rate. I set $\varphi = 0.25$ and $\varepsilon_w = 4$ which imply that workers have 25.6% wage markup similar to the Gali's calibration. I also target to match the slope of the aggregate wage Phillips curve with that of single labor market model up to three decimal places. Obviously, aggregate dynamics depend not only on the direction of the endogenous shift terms but also on the slope of the wage Phillips curve (and hence γ). Thus, such a calibration allows me to see the effect of the wage premium, which is not considered in the single labor market model, only on the aggregate dynamics. As I discussed above, nominal wages decrease due to the higher unemployment after both positive productivity and negative monetary policy shock under basis calibration. These shock reduces *output gap* and hence wage-premium. As in 2.14, the decrease in wage-premium leads to an upward shift of aggregate wage Phillips curve, and it mitigates downward pressure of higher unemployment on nominal wages. Consequently, aggregate wages become stickier and aggregate unemployment rate responds to output gap more sensitively. Figure 2.5 and A.3 show Impulse responses of the segmented labor market model with different values of γ together with those of the single labor market model. First of all, the IRFs confirm that unemployment increases more while the difference in the real wages decrease under

---

elasticity. I present Figure A.1 and Figure A.2 for employment and unemployment gap under the assumption the workers in both sector have the same labor supply elasticity: $\varphi_H = \varphi_L = 0.25$. It shows that low-skilled employment/unemployment rates are much more persistent as well.

the segmented labor market model in response to both positive technology and negative monetary policy shock. This is because real wage increase (falls) more (less) in response to technology (monetary policy) shock due to the degree of decrease in inflation. Again, since aggregate nominal wage cannot fall sufficiently due to the upward shift caused by the wage premium, nominal wage moves as if stickier wage under the segmented labor market model. Accordingly, real marginal cost for the firms increase (decrease) more (less) and thus output difference becomes negative after a positive technology shock (negative monetary policy shock).

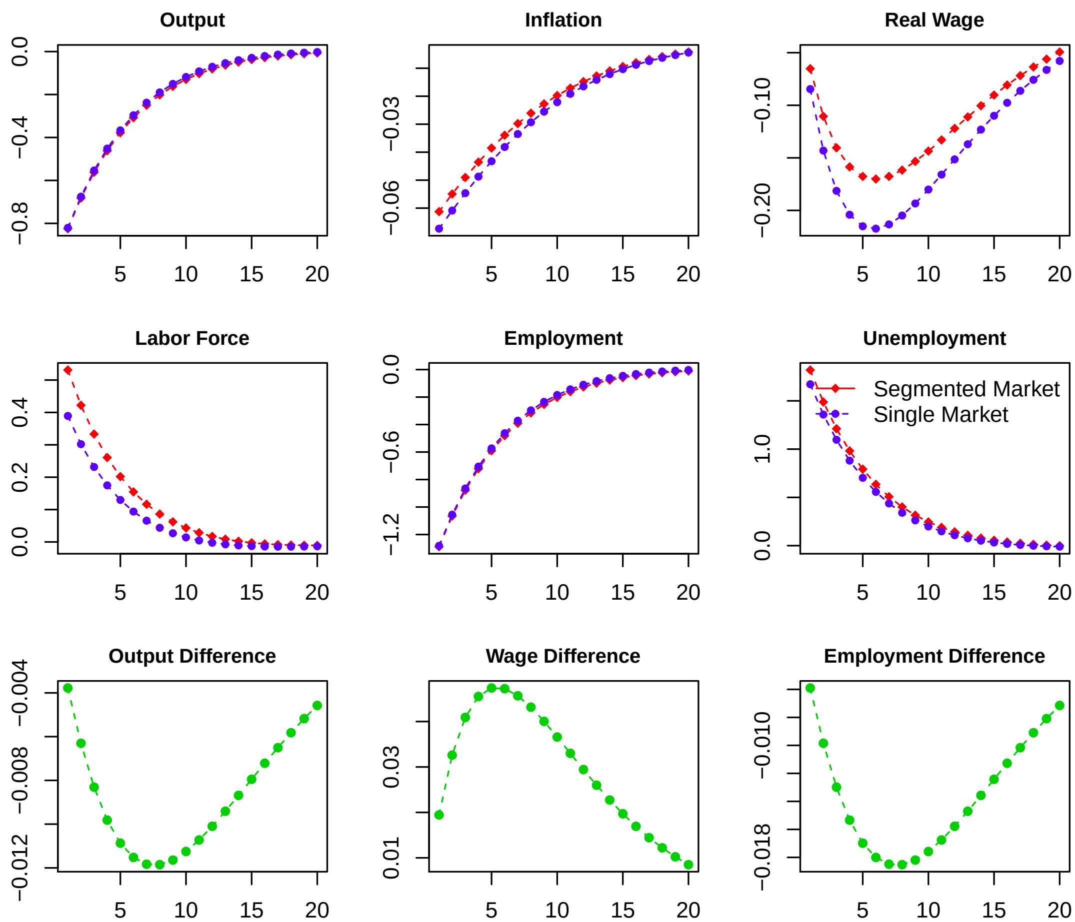

Figure 2.5: Aggregate Impulse Responses to monetary policy shock

2.5 Conclusion

In this paper, I build a New Keynesian model in order to account for the heterogeneity in labor market variables over the workers with different demand and supply labor elasticities. I split household members into two groups; high-skilled workers and low-skilled workers. I assume that low-skilled workers are more substitutable than high-skilled workers, $\varepsilon_w^H < \varepsilon_w^L$, and hence have smaller markups, $\mu^H > \mu^L$ and lower average wage. Higher unemployment rate prevailed in low-skilled workers is explained by greater Frisch labor supply elasticity than high-skilled workers. In addition, more elastic labor supply and demand of low-skilled workers induce a flatter wage Phillips curve so that low-skilled unemployment rate responses more sensitively to wage inflation. In addition, a flatter wage Phillips curve implies effectively stickier low-skilled wages, and consequently, it induces a procyclical wage premium. On the other hand, greater labor demand elasticity makes low-skilled employment more vulnerable to an economic shock, and hence, the employment gap across sectors moves counter-cyclically. If elasticity of substitution between different skilled workers is sufficiently large as in the literature, $\eta > 1$, a decline in the wage premium leads to a greater employment gap, and thus, income gap widens. I calibrate the model with “Merged Outgoing Rotation Group” of Current Population Survey data. I define “high-skilled workers” when an individual has Bachelor’s degree or above and “low-skilled workers” otherwise. I then obtain impulse responses based on the model. It shows that unemployment rate fluctuations vary considerably across the sectors especially in response to the demand shock which confirms the empirical evidence. In addition, endogenous shift caused by the change in wage-premium prevents nominal wages from fully adjusting after both positive technology and negative demand shock, and thus, firms marginal cost and production. Therefore, output, employment ratio, and unemployment rate becomes more volatile under the segmented labor market model. Therefore, if labor market is segmented so that the procyclical wage premium takes an important role on the aggregate dynamics, policy makers should be cautious in that monetary policy might be more effective than they have thought.

Chapter 3

Segmented labor markets in an estimated NK model

3.1 Introduction

Although the differentials in labor market variables at the business cycle frequency have been recognized by many papers, the effect of such differentials on the aggregate dynamics has not been studied sufficiently. I propose a mechanism (*strategic complementarity* in wage setting) by which aggregate fluctuation is amplified in response to aggregate and idiosyncratic shocks. In order to account for the differentials in real wages and unemployment within the mechanism, it is important that elasticity of substitution (or labor demand elasticity) within a labor sector and the Frisch elasticity (or labor supply elasticity) are different across sectors. This is because the parameters determine the slope of the wage Phillips curve. The slope of the Phillips curve is important for the propagation of shocks and determines the unemployment-inflation tradeoff faced by policymakers. For instance, a flatter wage Phillips curve implies a less decline in wage inflation given a rise in unemployment. In other words, real wages with a flatter wage Phillips curve are effectively stickier than those of other sector. Lichter et al. (2014b) among others show that elasticity of labor demand for the unskilled (high-skilled) labor is significantly higher (lower) than for the overall workforce by conducting a comprehensive meta-regression analysis. In addition, Fiorito and Zanella (2012), for instance, show that estimate the Frisch elasticity of subgroup including the low-educated workers is much greater than other groups. The present paper is intended to confirm those empirical findings based on a structural model within a standard the New Keynesian framework. If the labor demand and supply elasticity are different across sectors, it is natural to expect that the gap in consumption and labor market variables across sectors changes in response to an economic shock. Accordingly, consumption and income inequality would change after an economic shock, especially after monetary policy shock. The second objective of this paper is to investigate the contribution of shocks in the dynamics of such gaps across sectors in consumptions and labor market variables and to derive a policy implication from the estimation results.

I modify Galí et al. (2011) to account for the heterogeneity in labor market variables. There are two labor markets instead of one (single) labor market; one for high-skilled workers and the other for low-skilled workers. Those two types of workers are different only in the labor supply and demand elasticity. In addition, I introduce bond-holding cost in order to avoid singularity problem as Schmitt-Grohé and Uribe (2003) discussed. However, it also takes into account the heterogeneity in accessibility to the financial market across sectors. Firms need to hire both high and low skilled workers and aggregate it into a homogeneous production input. I maintain most features of Galí et al. (2011) in that I consider nominal rigidities through the Calvo pricing with indexation, time-varying markups, risk-premium on the government bond price, and consumption habit formation. I simplified their model in that I do not consider capital in production, and thus, investment. Instead, I consider diminishing return on labor input.

I use a typical Bayesian estimation method to estimate the model. I linearize a system of the optimality and equilibrium equations to obtain the solution to the model in the form of linear state space model. I then use the Kalman filter, which is known for optimal projection of state variables within the class of linear model, to compute the likelihood. As the posterior distribution is proportional to the likelihood multiplied by prior distribution, I set priors following Del Negro and Schorfheide (2008) and Galí et al. (2011). Finally, I characterize the posterior distribution of the parameters using Random Walk Metropolis-Hasting Algorithm. I burn-in the draws until the algorithms are converged and adjust jump-scale parameter to target 23% of acceptance rate.

The estimation result confirms that labor demand and supply elasticity for low-skilled workers are greater than those of high-skilled workers. The estimate for the elasticity of substitution across sectors is sufficiently larger to generate greater income gap after monetary policy shock. The posterior distribution indicates that nominal interest rate is quite persistent and the monetary policy has been strongly responded to the inflation. However, the consumption habit parameter and the concavity parameter of production function are surprisingly low. This implies that heterogeneity in consumption can explain persistent aggregate consumption and the production function is almost linear in aggregate labor input.

3.2 Model

I extend the model developed in Chapter I in several dimensions. First, I relax perfect consumption risk-sharing assumption across the labor sector by introducing bond-holding cost that depends on their labor income. Consequently, heterogeneity in labor supply decisions across sector is influenced not only by differentials in employment level but also by differentials in consumption. In addition, the model allows us to discuss consumption and income gaps (inequality) across sectors. The rest of specification of the model is similar to Smets and Wouters (2007) in that the model features consumption habit formation, Calvo pricing with prices and wages indexation. There are 8 exogenous shocks including risk premium shock, price and wage markup shocks, and two sectoral productivity shocks.

3.2.1 Firms and Price setting

Monopolistic competitive firms hire workers from both labor markets and then aggregate it with CES technology into homogeneous effective labor input. Each firm produces a differentiated good $z \in [0,1]$ using a production function which is given by:

$$
\begin{aligned}
&Y_t(z) = A_t\left(H_t(z)\right)^{1-\alpha} \\
&\quad \text{where} \quad H_t(z) = \left[\gamma_H^{\frac{1}{\eta}}\left(A_t^H N_t^H(z)\right)^{\frac{\eta-1}{\eta}} + \gamma_L^{\frac{1}{\eta}}\left(A_t^L N_t^L(z)\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \\
&\quad \text{and} \quad N_t^j(z) \equiv \left(\int_0^1 N_t^j(i,z)^{\frac{1}{1+\lambda_{w,t}^j}} di\right)^{1+\lambda_{w,t}^j} \quad \text{for} \quad j \in (H,L)
\end{aligned}
$$

where $H_t(z)$ is the homogeneous effective labor input of firm z obtained by labor aggregation technology; $\alpha < 1$ denotes decreasing returns to scale parameter of effective labor by which firms face increasing marginal cost when production expands through more employment; η is elasticity of substitution between high-skilled ($N_t^H(z)$) and low-skilled labor ($N_t^L(z)$).[1] The parameter γ determines the relative income share and it becomes relative income share itself when $\eta = 1$, that is, when production function is a Cobb-Douglas; $1 + \lambda_{w,t}^j$ is time varying desired wage markup in j labor sector and $\ln \lambda_{w,t}^j$ is assumed to be follow AR(1) process:[2]

$$\ln \lambda_{w,t}^j = (1-\rho_w^j) \ln \lambda_w^j + \rho_w^j \ln \lambda_{w,t-1}^j + \sigma_w^j \varepsilon_{w,t}^j \quad \text{for } \ j \in (H, L)$$

where $\rho_w^j \in (0,1)$, $\lambda_w^j > 0$, $\sigma_w^j > 0$, and $\varepsilon_{w,t}^j \sim \mathcal{NID}(0,1)$. A_t is an exogenous technology parameter which is assumed that $a_t \equiv logA_t$ and $a_t = \rho_a a_{t-1} + \varepsilon_t^a$ where $\rho_a \in [0,1]$ and ε_t^a is a white noise process with zero mean and variance σ_a^2. Similarly, A_t^j for $j \in \{H, L\}$ denotes idiosyncratic productivity shock in j sector. The average nominal marginal cost can be derived from the optimality condition for a representative firm with respect to labor input[3]:

$$\Psi_t(z) = \frac{\varpi_t}{A_t} \quad \text{where} \quad \varpi_t^j \equiv \frac{W_t^j}{A_t^j} \ \text{ and } \ \varpi_t = \left(\gamma_H \left(\varpi_t^H\right)^{1-\eta} + \gamma_L \left(\varpi_t^L\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}$$

where W_t^H and W_t^L are the average nominal wage for high-skilled workers and low-skilled workers respectively. In addition, firm's cost minimization problem, taking wages

[1] as $\eta \to 0$, production function become Leontief in which low-skilled workers and high-skilled workers used only with fixed proportions; as $\eta \to \infty$, workers with both education level are perfectly substitutable, and hence, only low-skilled workers are used unless the average wages are the same for both skilled workers; as $\eta \to 1$, it become Cobb-Douglas production function in which income share is fixed with γ

[2] I also use $\mathcal{M}_t^j$ as the j-sector time-varying desired wage markup for sake of simplicity in the derivation of the wage Phillips curve. In addition, $\mathcal{M}_t^j$ can be written in terms of stochastic sectoral wage elasticity ($\epsilon_{w,t}^j$): $\mathcal{M}_t^j = \frac{\epsilon_{w,t}^j}{\epsilon_{w,t}^j - 1}$

[3] See Appendix B.1.2 for the details of derivation

as given, implies the set of labor demand schedules[4]:

$$N_t^j(i,z) = \left(\frac{W_t^j(i)}{W_t^j}\right)^{-\left(1+\frac{1}{\lambda_{w,t}^j}\right)} N_t^j(z) \quad \text{and} \quad N_t^j(z) = \gamma_j \left(\frac{\varpi_t^j}{\varpi_t}\right)^{-\eta} \frac{H_t(z)}{A_t^j} \quad \text{where j} \in (H, L)$$

for all $i \in [0,1]$ and for all $z \in [0,1]$.

There is competitive firm that aggregate intermediate goods using CES technology and produces the final good:

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{1}{1+\lambda_{p,t}}}\right]^{1+\lambda_t^p}$$

where $1+\lambda_{p,t}$ is the time varying price markup, and $\ln \lambda_{p,t}$ evolves according to the AR(1) process:

$$\ln \lambda_t^p = (1-\rho_p)\ln \lambda_p + \rho_p \ln \lambda_{p,t-1} + \sigma_p \varepsilon_{p,t}$$

where $\rho_p \in (0,1)$, $\ln \lambda_p > 0$, $\sigma_p > 0$ and $\varepsilon_{p,t} \sim \mathcal{NID}(0,1)$.

I introduce nominal rigidities in price following Calvo (1983) pricing with partial indexation, that is, only $1-\theta_p$ fraction of the firms can choose their optimal price, P_t^* and θ_p fraction of the firms adjust their price according to an indexation rule: $P_{t+1}(z) = P_t(z)\left(\frac{P_t}{P_{t-1}}\right)^{\iota_p}$ in each period t. Firms profit maximization problem subject to the sequence of demand schedule constraint $Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\iota_p}\right)^{-\left(1+\frac{1}{\lambda_{p,t}}\right)} C_{t+k}$, for $k = 0,1,2,...$ lead to the optimality condition for the firm:

$$\sum_{k=0}^{\infty} \theta_p^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \left(P_t^* \left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\iota_p} - \mathcal{M}_t^p \Psi_{t+k|t} \right) \right\} = 0$$

where $Y_{t+k|t}$ denotes output at time $t+k$ of a firm that last reset its price in period t, $Q_{t,t+k} \equiv \beta^k \left(\frac{\widetilde{C}_{t+k}}{\widetilde{C}_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$ is the relevant stochastic discount factor for nominal payoffs in period $t+k$, $\Psi_{t+k|t} \equiv \frac{\varpi_t}{(1-\alpha)A_{t+k}H_{t+k|t}^{-\alpha}}$ is the nominal marginal cost in period $t+k$ of producing quantity $Y_{t+k|t}$ and $\mathcal{M}_t^p \,(\equiv 1+\lambda_t^p)$ is the desired or frictionless price markup over the marginal cost. Log-linearization of the optimality condition around the zero

[4]See Appendix B.1.2 for the details of derivation

inflation steady state yields

$$\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ p_t^* + \iota_p \left(p_{t+k-1} - p_{t-1} \right) - \mu_{t+k}^{np} - \psi_{t+k|t} \right\} = 0 \tag{3.1}$$

Note that lower case variables in present paper denote the log-deviation of the variables from the steady state and $\mu_t^{np} \equiv \log(\mathcal{M}_t^p) \approx \left(\frac{\lambda_p}{1+\lambda_p}\right) \ln \lambda_{p,t}$. Define the average nominal marginal cost as $\Psi_{t+k} \equiv \frac{W_{t+k}}{(1-\alpha)Y_{t+k}/H_{t+k}}$. Noting that first order log approximation of aggregate relation $y_t = a_t + (1-\alpha)h_t$, it follows that

$$\begin{aligned} \psi_{t+k|t} &= \psi_{t+k} + \alpha \left(h_{t+k|t} - h_{t+k} \right) \\ &= \psi_{t+k} - \frac{\alpha \epsilon_p}{1-\alpha} \left(p_t^* - p_{t+k} + \iota_p \left(p_{t+k-1} - p_{t-1} \right) \right) \end{aligned} \tag{3.2}$$

Price inflation equation can be derived by combining (3.1), (3.2) and the log-linearized price index, $p_t = (1-\theta_p)p_t^* + \theta_p p_{t-1} + \theta_p \iota_p \pi_{t-1}$:

$$\pi_t^p = \beta E_t \left\{ \pi_{t+1}^p - \iota_p \pi_t^P \right\} + \iota_p \pi_{t-1}^p - \lambda_p \left(\mu_t^p - \mu_t^{np} \right) \tag{3.3}$$

where $\pi_t^p \equiv p_t - p_{t-1}$ is wage inflation, $\mu_t^p \equiv p_t - \psi_t$ denotes the log average price markup which is the same as negative real marginal cost, and $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} \frac{(1-\alpha)}{(1-\alpha+\alpha\epsilon_p)} > 0$.

3.2.2 Household

Households are classified into two groups depending on the members' skill level; high-skilled household if its members supply labor in high-skilled labor sector, and low-skilled household if its members supply labor in low-skilled labor sector. There are large number of identical households in each labor sector which are comprised of a continuum of members for each labor market represented by the unit indexed by $i \in [0,1]$. $i \in [0,1]$ indicates the type of labor service in which a given household member is specialized in a given sector. All member of a given household are assumed to consume the same level of goods and services independently of their employment status and skill level, so it assumed full risk sharing within a household. Representative household maximize household's discounted lifetime utility (3.4) subject to budget constraint (4.2). While labor demand, $N_t^j(i)$, are determined by the aggregation of firm's labor demand decisions and allocated uniformly across households, workers choose their optimal wages, $W_t^j(i)$. Therefore, both $W_t^j(i)$ and $N_t^j(i)$ are taken as given by each individual household. Each household's discounted lifetime utility is given by:

$$\sum_{t=0}^{\infty}\beta^t U(C_t^j, N_t^j(i)) = \sum_{t=0}^{\infty}\beta^t \left[\frac{\widetilde{C_t^j}^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{\left(N_t^j(i)\right)^{1+\varphi_j}}{1+\varphi_j} di\right] \quad \text{for } j \in (H, L) \quad (3.4)$$

where $\widetilde{C_t^j} = C_t^j - h\bar{C}_{t-1}^j$ where h is habit persistent parameter and $\bar{C}_{t-1}$ is the aggregate consumption at time $t-1$ of j sector. $C_t^j \equiv \left(\int_0^1 C_t^j(z)^{\frac{1}{1+\lambda_p}} dz\right)^{1+\lambda_p}$ and $C_t^j(z)$ is the quantity consumed of good z of j-skilled household and $N_t^j(i) \in [0, 1]$ is the fraction of members specialized in type i labor in $j \in (H, L)$ sector who are employed in period t.[5] The parameter φ_j denotes the inverse of the Frisch labor supply elasticity of workers in j sector. Blanchard and Katz (1997) argue that since the wages of low- skilled workers are close to their reservation wage (whereas the wages of high-skilled workers is much higher than their reservation wage), high-skilled workers has much flatter labor supply than low-skilled worker. Taking this point of view, I assume that parameter for inverse of Frisch elasticity, $\varphi_H > \varphi_L$, that is, high-skilled workers would have higher marginal rate of substitution than low-skilled worker at the same level of employment. Households budget constraint is given by:

$$\int_0^1 P_t(z)C_t(z)dz + e^{b_t}Q_tB_t + \phi\log\left(1 + s_j - \frac{\mathcal{X}_t^j}{Y_t}\right)B_t \leq B_{t-1} + \int_0^1 W_t^H(i)N_t^H(i)di + \Pi_t$$

where $P_t(z)$ is the price of good z , $W_t^j(i)$ is the nominal wage for type i labor with $j \in (H, L)$ skill, B_t represents purchases of nominally riskless one-period discount bond paying one monetary unit, Q_t is the price of that bond, and Π_t is a lump-sum component of income at time t. e^{b_t} is the risk premium in the return to bonds and is assumed to

[5]Optimal demand for each good resulting from utility maximization takes the familiar form: $C_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\left(1+\frac{1}{\lambda_{p,t}}\right)} C_t$ where $P_t \equiv \left(\int_0^1 P_t(z)^{-\frac{1}{\lambda_{p,t}}}\right)^{\frac{1}{-\lambda_{p,t}}}$ denotes the price index for final goods. Details for derivation in Appendix B.1.1

follow AR(1) process:

$$b_t = \rho_b b_{t-1} + \sigma_b \varepsilon_t^b$$

where $\rho_b \in (0,1)$, $\sigma_b > 0$, and $\varepsilon_t^b \sim \mathcal{NID}(0,1)$. I assume that bond-holding cost is decreasing in relative average sectoral income to total income. The lower income leads to a greater bond-holding cost taking into account that low income household tends to hold less financial asset in which $\mathcal{X}_t^j$ is the average nominal income of j-skilled households, s_j is the relative income share of j-skilled households at the steady state, and ϕ is the scale parameter of the bond-holding cost. The first order conditions for the maximization problem subject to the budget constraint give the consumption Euler equation:

$$e^{b_t} Q_t + \phi \log\left(1 + s - \frac{\mathcal{X}_t^j}{Y_t}\right) = \beta E_t \left(\frac{\widetilde{C}_{t+1}}{\widetilde{C}_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}} \tag{3.5}$$

3.2.3 Wage Determination

Analogous to the good market, the labor markets are monopolistic competitive, and wages are determined by Calvo pricing. I assume that the Calvo parameters are the same, $\theta_w = \theta_w^H = \theta_w^L$, following Barattieri et al. (2014). In addition, I assume that high-skilled workers are not easily substituted by others relative to low-skilled workers; $\varepsilon_w^H < \varepsilon_w^L$. It implies that the markup of high-skilled workers in wage setting is greater than that of low-skilled workers, and it also makes sure that high-skilled wage is larger than low-skilled wage on average (at the steady state). For each labor market, only $1-\theta_w$ fraction of workers can re-optimize their wage. When re-optimizing their wage in period t, workers choose a wage W^{j*}, where again $j \in (H, L)$, in order to maximize their households' utility taking as given all aggregate variables, including the aggregate wage index $W_t^j \equiv \left(\int_0^1 W_t^j(i)^{1-\epsilon_w^j} di\right)^{\frac{1}{1-\epsilon_w^j}}$. The optimal wage setting rule for j-skilled workers can be obtained from the maximization problem subject to the budget constraint and the corresponding labor demand schedule which is determined by firms:

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ N_{t+k|t}^j \left(\widetilde{C}_{t+k}^j\right)^{-\sigma} \left(\frac{W^{j*}}{P_{t+k}} - \mathcal{M}_t^j MRS_{t+k|t}^j\right)\right\} = 0$$

where $N^j_{t+k|t}$ denotes the aggregate quantity demanded in period t+k of j-skilled workers whose wage was last reset in period t. $MRS^j_{t+k|t} \equiv \left(\widetilde{C}^j_{t+k}\right)^{-\sigma}\left(N^j_{t+k|t}\right)^{\varphi_j}$ is the period t+k marginal rate of substitution between consumption and employment for a j-skilled worker whose wage is reset in period t, and $\mathcal{M}^j_t \left(\equiv 1+\lambda^j_{w,t}\right)$ is the desired or frictionless wage markup. The first order log approximation of this wage setting rule around the zero inflation steady states gives the optimal wage equation as the markup plus weighted average of future price-adjusted marginal rate of substitution:

$$\sum_{k=0}^{\infty}\left(\beta\theta_w\right)^k E_t\left\{w^{j*}_t - p_{t+k} + \iota_w\left(p_{t+k-1}-p_{t-1}\right) - \mu^{nj}_{t+k} - mrs^j_{t+k|t}\right\} = 0 \tag{3.6}$$

where $\mu^{nj}_t = \log\mathcal{M}^j_t \approx \frac{\lambda^j_w}{1+\lambda^j_w}\ln\lambda^j_{w,t}$ is log of desired wage markup. Defining the j-skilled sector's average marginal rate of substitution as $MRS^j_t \equiv \left(\widetilde{C}^j_t\right)^{\sigma}\left(N^j_t\right)^{\varphi_j}$, the marginal rate of substitution of each individual in j-skilled sector can be written in terms of the relationship between the average marginal rate of substitution and the relative wage:[6]

$$\begin{aligned} mrs^j_{t+k|t} &= mrs^j_{t+k} + \varphi_j\left(n^j_{t+k|t} - n^j_{t+k}\right) \\ &= mrs^j_{t+k} - \epsilon^j_w\varphi_j\left(w^{j*}_t - w^j_{t+k}\right) \end{aligned} \tag{3.7}$$

Finally, combining (3.6), (4.7) and the log-linearized form of aggregate wage index[7], I obtain j-skilled wage Phillips curve as:[8]

$$\pi^j_t = \beta E_t\left\{\pi^j_{t+1} - \iota_w\pi^p_t\right\} + \iota_w\pi^p_{t-1} - \kappa_j\left(\mu^j_t - \mu^{nj}_t\right) \tag{3.8}$$

where $\pi^j \equiv w^j_t - w^j_{t-1}$ is the j-skilled wage inflation, $\mu^j_t \equiv w^j_t - p_t - mrs^j_t$ denotes the log average j-skilled wage markup, and $\kappa_j \equiv \frac{\Theta}{1+\epsilon^j_w\varphi_j} > 0$ where $\Theta \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w}$.

---

[6] where $N^j_t \equiv \int_0^1 N^j_t(i)di$ is the sector aggregate employment rate

[7] $w^j_t = \theta_w w^j_{t-1} + \left(1-\theta_w\right)w^{j*}_t$

[8] See the Appendix B.1.4 for the detailed derivation

3.2.4 Unemployment

I define unemployment following Galí (2011). An individual will be willing to work in period t if and only if real wage for his labor type exceeds his disutility of labor. Thus the marginal j-skilled supplier of type i labor, $L_t^j(i)$, is given by

$$\frac{W_t^j(i)}{P_t} = \left(C_t^j\right)_t^{\sigma} \left(L_t^j(i)\right)^{\varphi_j}$$

Define the aggregate labor force (or participation rate) as $L_t^j \equiv \int_0^1 L_t^j(i)di$, then the first order approximations gives the log-linearized estimate relation:

$$w_t^j - p_t = \sigma c_t^j + \varphi_j l_t^j$$

The unemployment rate u_t can be written as the log difference between the labor force and employment:

$$u_t^j \equiv l_t^j - n_t^j$$

Noting that real wage is the markup over the marginal rate of substitution, $\mu_t^j \equiv \left(w_t^j - p_t\right) - mrs_t^j = \left(w_t^j - p_t\right) - \left(\sigma c_t^j + \varphi_j n_t^j + \xi_t\right)$, the unemployment can be written as:

$$\mu_t^j = \varphi_j u_t^j \tag{3.9}$$

Combining (3.8) with (4.10), I derived the j sector New Keynesian Wage Phillips Curve:

$$\pi_t^j = \beta E_t \left\{\pi_{t+1}^j - \iota_w \pi_t^p\right\} + \iota_w \pi_{t-1}^p - \kappa_j \varphi_j \left(u_t^j - u_t^{nj}\right) \tag{3.10}$$

3.2.5 The Government

The government sets the nominal interest rates according to the Taylor rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}_t}\right)^{\phi_y}\right]^{1-\rho_R} exp\left(\sigma_R \varepsilon_t^R\right)$$

The variable R denotes the steady state gross nominal interest rate, ρ_R is the smoothing parameter of monetary policy, $\bar{\pi}$ is the steady state inflation, which is assumed to be zero in this paper, and $\bar{Y}$ is the steady state output, and $\varepsilon_t^R \sim \mathcal{NID}(0,1)$ is the monetary policy shock, and $\sigma^R > 0$ is the size of monetary policy shock. In addition, I assume that the government collects the bond-holding cost from household of a given sector and evenly distribute to the households and no government spending. Thus, the government budget constraint is given by:

$$B_{t-1} = e^{b_t} Q_t B_t + T_t + \phi \left\{ \gamma \log\left(1 + s - \frac{\mathcal{X}_t^H}{Y_t}\right) + (1-\gamma)\log\left(1 + (1-s) - \frac{\mathcal{X}_t^L}{Y_t}\right) \right\} B_t$$

3.3 Aggregation and Equilibrium

I assume that the government can eliminate the wage markups by appropriate subsidies (τ) to guarantee that $\mathcal{M}^j(1-\tau^j) = 1$ for $j \in (H, L)$, and $\mathcal{M}^p(1-\tau^p) = 1$, and hence, $W = W^H = W^L$ in the efficient steady state, then log-linearized wage index can be written:[9]

$$w_t = s w_t^H + (1-s) w_t^L$$

3.3.1 Resource Constraint and Euler Equation

Because of absence of investment and government spending in a closed economy, all outputs produced by each firms are consumed. Therefore, market clearing condition is $\gamma C_t^H(z) + (1-\gamma) C_t^L(z) = C_t(z) = Y_t(z)$ for all $z \in [0,1]$, and hence, $C_t = Y_t$. Consumption Euler equation for j-skilled households is now log-linearized as:

$$c_t^j = \frac{1}{1+\rho_c} E_t c_{t+1}^j + \frac{\rho_c}{1+\rho_c} c_{t-1}^j - \frac{1-\rho_c}{\sigma(1+\rho_c)} \left\{ i_t - E_t \pi_{t+1}^p - b_t \right\} + \frac{s\phi(1-\rho_c)}{\beta\sigma(1+\rho_c)} \left(\chi_t^j - y_t \right) \tag{3.11}$$

---

[9] $w_t = \gamma \left(\frac{W^H}{W}\right)^{1-\eta} w_t^H + (1-\gamma)\left(\frac{W^L}{W}\right)^{1-\eta} w_t^L = \frac{W^H N^H}{WH} w_t^H + \frac{W^L N^L}{WH} w_t^L = s w_t^H + (1-s) w_t^L$ where s is the relative high-skilled income share at the steady state.

where χ_t^j is the log of j-skilled income, and $i_t(= -q_t)$ is nominal interest rate on risk-free bond.[10] The log-linearized resource constraint is:

$$y_t = sc_t^H + (1-s)c_t^L$$

3.3.2 Labor Market Equilibrium

Aggregate labor supply from all households must be equal to firm's aggregate labor demand for each labor market in equilibrium. Accordingly, the sector labor market clearing conditions for sector $j \in (H, L)$ are following:[11]

$$N_t^j = \int_0^1 N_t^j(z)dz = \int_0^1 \int_0^1 N_t^j(i,z)didz \quad = \gamma_j \Delta_t^j \Delta_t^P \left(\frac{\varpi_t^j}{\varpi_t}\right)^{-\eta} \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \left(\frac{1}{A_t^j}\right)$$

where $\varpi_t^j \equiv \frac{W_t^j}{A_t^j}$. Note that Δ_t^P, Δ_t^H and Δ^L are measures for the price, the high skilled, and the low skilled wage dispersion respectively, and those can be approximated to 1 up to first order.[12] Log-linearization of the employment in each sector around steady state can be written:

$$n_t^H = -\eta(1-s)\left(\omega^R - a_t^R - a_t\right) + \frac{1}{1-\alpha}(y_t - a_t) - a_t^H$$
$$n_t^L = \eta s\left(\omega_t^R - a_t^R - a_t\right) + \frac{1}{1-\alpha}(y_t - a_t) - a_t^L$$

where ω_t^R is the real wage gap (or premium) between average high-skilled wages and low-skilled wages and $a_t^R = a_t^H - a_t^L$.

3.4 Bayesian Estimation

In this section, I estimate the model described above following the conventional Bayesian estimation method for the DSGE model.

[10] $Q_t = \frac{1}{R_t} = \frac{1}{1+i_t}$

[11] See Appendix A.4 for the details of derivation

[12] $\Delta^P \approx 1 + \frac{1}{2}\left(\frac{\varepsilon_p}{1-\alpha}\right)\left(\frac{1-\alpha+\alpha\varepsilon_p}{1-\alpha}\right) Var_z\left\{p_t(z)\right\}$, $\Delta_t^H \approx 1 + \frac{\varepsilon_w^H}{2} Var_z\left\{w_t^H(z)\right\}$ and $\Delta_t^L \approx 1 + \frac{\varepsilon_w^L}{2} Var_z\left\{w_t^L(z)\right\}$ and details for second order log approximations are in Appendix B.1.2

3.4.1 State space representation of the model

Since the optimality conditions obtained above are non-linear equations, the first step in a typical Bayesian estimation of DSGE model is to linearize those equations to construct a system of expectational stochastic difference equation as below:[13]

$$E_t \{\mathcal{F}(D_{t+1}, D_t, U_{t+1}, U_t)\} = 0$$

where D_t is the vector of predetermined (state) variables and U_t is a vector of non-predetermined (control) variables that are defined as:

$$D_t = \left[c_{t-1}^H \;\; c_{t-1}^L \;\; \omega_{t-1}^H \;\; \omega_{t-1}^L \;\; R_{t-1} \;\; \pi_{t-1} \;\; \pi_{t-1}^H \;\; \pi_{t-1}^L \;\; a_t \;\; a_t^H \;\; a_t^L \;\; b_t \;\; \xi_t \;\; \lambda_{p,t} \;\; \lambda_{w,t}^H \;\; \lambda_{w,t}^L\right]'$$
$$U_t = \left[y_t \;\; c_t^H \;\; c_t^L \;\; n_t^H \;\; n_t^L \;\; \omega_w^H \;\; \omega_t^L \;\; \pi_t \;\; \pi_t^H \;\; \pi_t^L \;\; R_t\right]'$$

By introducing expectation error terms, the solution of the model can be written as a typical linear state space model:

$$D_t = AD_{t-1} + B\zeta_t$$
$$U_t = CD_t$$

where A, B, and C are matrices that are nonlinear functions of the structural parameters of the model, and $\zeta_t = \left[\varepsilon_{a,t} \;\; \varepsilon_{a,t}^H \;\; \varepsilon_{a,t}^L \;\; \varepsilon_t^b \;\; \varepsilon_t^\xi \;\; \varepsilon_{p,t} \;\; \varepsilon_{w,t}^H \;\; \varepsilon_{w,t}^L \;\; \varepsilon_t^R\right]'$ is the vector of structural innovations. Combining these two vectors and construct expanded state vector: $\mathbb{S}_t = [U_t' \;\; D_t']'$ Now, the state space representation of the model can be written as:

$$\mathbb{S}_t = \mathbb{F}\mathbb{S}_{t-1} + \mathbb{Q}\zeta_t \quad \zeta_t \sim \mathcal{NID}(0, I_m) \tag{3.12}$$

$$\mathbb{Y}_t = \mathbb{M} + \mathbb{H}\mathbb{S}_t + \mathbb{D}\varepsilon_t \quad \varepsilon_t \sim \mathcal{NID}(0, I_n) \tag{3.13}$$

[13]The system of linearized equations for present paper are in Appendix B.2

where $\mathbb{Y}_t$ denotes the vector of observables at time t; $\mathbb{F}$ and $\mathbb{Q}$ are function of the matrices A, B, and C; $\mathbb{M}$ is a vector required to match the means of the observed data; the matrix $\mathbb{H}$ relates the model's definitions with the data; the number m and n is the dimension of state vector and the vector of observables.

3.4.2 The Kalman filter and likelihood of the model

As the Kalman filter is known for optimal forecasts of state variables given an information set of observed macro time series, the next step is to construct the likelihood of the model using the Kalman filter. The likelihood function is obtained by Kalman filter as follows:

Let $\mathbb{S}_0 \sim \mathcal{N}(\mathbb{S}_0, P_0)$

- $\mathbb{S}_t|\mathbb{Y}^{t-1} \sim \mathcal{N}(\hat{\mathbb{S}}_{t|t-1}, P_{t|t-1})$ where $\hat{\mathbb{S}}_{t|t-1} = \mathbb{F}\mathbb{S}_{t-1}$, and $P_{t|t-1} = \mathbb{F}P_{t-1}\mathbb{F}' + \mathbb{Q}\Sigma_\varepsilon\mathbb{Q}'$
- $\mathbb{Y}_t|\mathbb{S}_t, \mathbb{Y}^{t-1} \sim \mathcal{N}(\mathbb{H}\mathbb{S}_t, \mathbb{D}\Sigma_\varepsilon\mathbb{D}')$
- $\mathbb{Y}_t|\mathbb{Y}^{t-1} \sim \mathcal{N}(\hat{\mathbb{Y}}_{t|t-1}, F_{t|t-1})$ where $\hat{\mathbb{Y}}_{t|t-1} = \mathbb{H}\hat{\mathbb{S}}_{t|t-1}$, and $F_{t|t-1} = \mathbb{H}P_{t|t-1}\mathbb{H}' + \mathbb{D}\Sigma_\varepsilon\mathbb{D}'$.
- Joint distribution
$$\begin{bmatrix} \mathbb{S}_t \\ \mathbb{Y}_t \end{bmatrix} |\mathbb{Y}^{t-1} \sim \mathcal{N}\left(\begin{bmatrix} \hat{\mathbb{S}}_{t|t-1} \\ \hat{\mathbb{Y}}_{t|t-1} \end{bmatrix}, \begin{bmatrix} P_{t|t-1} & P_{t|t-1}\mathbb{Q}' \\ \mathbb{Q}P'_{t|t-1} & F_{t|t-1} \end{bmatrix}\right)$$

From the joint distribution, I get $\mathbb{S}_t|\mathbb{Y}_t, \mathbb{Y}^{t-1}$, then finally, then calculate likelihood function Ł$(\mathbb{Y}_t|\Theta)$ with Bayes Theorem.

$$\mathcal{L}(\mathbb{Y}_t|\Theta) = (2\pi)^{-\frac{mT}{2}} \left(\prod_{t=1}^{T} |F_{t|t-1}|\right)^{-\frac{1}{2}} exp\left\{-\frac{1}{2}\sum_{t=1}^{T} \nu_t F_{t|t-1}\nu_t'\right\} \tag{3.14}$$

where $\nu_t = \mathbb{Y}_t - \hat{\mathbb{Y}}_{t|t-1} = \mathbb{B}(\mathbb{S}_t - \mathbb{S}_{t|t-1}) + u_t$.

3.4.3 Estimation

I then need to set priors because the likelihood multiplied by the prior is proportional to the posterior as in Bayes Theorem:

$$\mathcal{P}\left(\Theta|\mathcal{Y}^T\right) \propto \mathcal{L}\left(\mathcal{Y}^T|\Theta\right)\mathcal{P}\left(\Theta\right)$$

where Θ is the vector of model parameters, and $\mathcal{Y}^T$ is the length T of sample data, $\mathcal{Y}^T \equiv \{\mathbb{Y}_0, \mathbb{Y}_1, \cdots, \mathbb{Y}_t\}$. Finally, the posterior distribution of the model parameters in Θ is characterized using Random Walk Metropolis-Hasting Algorithm (RWMH). I obtain 3 chains of 3 million draws. To remove the effect of initial value of the parameters and to make sure the convergence of RWMH, I burn-in 2,400,000 draws. The acceptance rates are 24.58%, 24.61%, and 24.75%.

Prior Distribution

Analogous to Del Negro and Schorfheide (2008), I categorize the model parameters, Θ into three groups; 1) A set of fixed or calibrated parameters that are problematic for estimation, Θ_1; 2) A set of structural parameters on preference, technologies, and market structure, $\Theta_{2,endo}$; 3) A set of parameters associated with exogenous shock processes, $\Theta_{2,exo}$. The elements of the vector of parameters are follows:

$$\begin{aligned}
\Theta_1 &= [s \ \ \beta]' \\
\Theta_{2,endo} &= \left[\sigma \ \ \eta \ \ \varphi_H \ \ \varphi_L \ \ \epsilon_p \ \ \epsilon_w^H \ \ \epsilon_w^L \ \ \phi \ \ \alpha \ \ \theta_p \ \ \theta_w \ \ \iota_p \ \ \iota_H \ \ \iota_L \ \ \rho_R \ \ \phi_y \ \ \phi_\pi\right]' \\
\Theta_{2,exo} &= \left[\rho_a \ \ \rho_a^H \ \ \rho_a^L \ \ \rho_b \ \ \rho_\xi \ \ \rho_p \ \ \rho_w^H \ \ \rho_w^L \ \ \eta_p \ \ \eta_H \ \ \eta_L \ \ \sigma_a \ \ \sigma_a^H \ \ \sigma_a^L \ \ \sigma_b \ \ \sigma_\xi \ \ \sigma_p \ \ \sigma_w^H \ \ \sigma_w^L \ \ \sigma_R\right]'
\end{aligned}$$

First, I take average high-skilled income share over the period of 1984Q1 $\sim$ 2008Q3 for the high-skilled income share, s, and obtain the discount factor, β, by assuming 4% steady state annual interest rate. For monopoly power in the good market, ϵ_p, and nominal rigidities, θ_p, I just adopt standard values in the literature since those parameters are hardly estimated in present model. Therefore, the calibration of Θ_1

results in

$$\Theta_1 = [s \;\; \beta]' = [0.52 \;\; 0.99]'$$

Table 3.4 summarizes the prior distributions and provides implied 95 percent probability intervals. Following Galí et al. (2011), I assumed the variances of the exogenous shock processes to follow Uniform distribution rather than Inverse-Gamma. The Inverse-Gamma distribution covers larger domain but is needed to put more weight on a certain range that requires some information on the parameters. The prior distribution of the smoothing parameters (ρ_c, ρ_i),and wage and price indexation parameters (ι_p, ι_H, and ι_L) are assumed to follow Beta distribution with 0.5 and 0.2 for mean and standard deviation respectively which is a quite loose prior distribution reflecting my uncertainty on those parameters. I also impose Beta distribution for the AR(1) coefficient and MA(1) coefficient with lower mean 0.4 and the smaller standard deviation 0.1 to avoid extreme values. I use Gamma distribution for demand (ϵ_w^H and ϵ_w^L) elasticities to impose positive estimated values with a wide range. In addition, I draw priors from Inverse-Gamma distribution for labor supply (φ_H and φ_L). However, the mean of those parameters are somewhat smaller than the calibrated values in the previous chapter. I made this adjustment because, in most of the trials with different prior means, posterior means for those parameters fall into much smaller values than the calibrated one. Since the relative size of those parameters are crucial to discuss aggregate dynamics, I set somewhat loose prior for those parameters allowing the priors to be overlapped. The prior on elasticity of substitution across sector (η) is drawn from the Normal distribution so that 95% intervals of this parameter is consistent with Mollick (2011)'s suggestion. The scale parameter of bond-holding cost (ϕ) relies on the Normal distribution with mean zero and standard deviation 0.01 implying that 95% draws from the distribution falls in between -0.026 and 0.026. I follow the specification of Del Negro and Schorfheide (2008) for monetary policy reaction coefficients (ϕ_π, ϕ_y). The priors on nominal price and wage stickiness is assumed to be Beta distribution with mean 0.7 and standard deviation 0.1 so that 90% of the draws fall in between 0.4 and 0.9 which implies average duration of prices and wages are from 5 month to 10 quarters. I impose Normal distribution for the intertemporal elasticity parameter with mean 10 and standard deviation 1 allowing to have relatively large values as in Guvenen (2006).[14]

The Randon Walk Matropolis-Hasting (RWMH) algorithm

There is one more step to go because the posterior distribution mostly does not have closed form. Therefore, I should get only empirical distribution for the posterior such as MCMC algorithm. In doing this, I use Random Walk proposal density so called Random Walk Metropolis-Hastings algorithm: Assuming there exists ergodic distribution for $\mathcal{P}(\Theta|\mathbb{Y}_t)$, we can use MH algorithm accepting the draws which gives higher likelihood based on specific acceptance rate. The RWMH algorithm to draw a chain $\left\{\Theta^{(i)}\right\}_{i=1}^{M}$ from $\mathcal{P}(\Theta)$is as follows:
For $i = 1, \cdots, M$

1. Initialization: set $i = 0$ and initial $\Theta^{(0)}$. Solve the model for $\Theta^{(0)}$ and compute $f(\cdot,\cdot|\Theta^{(0)})$ and $g(\cdot,\cdot|\Theta^{(0)})$ in equation (3.12) and (3.13) respectively. Then evaluate $\mathcal{P}(\Theta^{(0)})$ and $\mathcal{L}(\mathbb{Y}_t|\Theta^{(0)})$ from (3.14). Set $i = i + 1$.

2. Proposal draw: get a proposal draw $\Theta^{(i^*)} = \Theta^{(i-1)} + \varepsilon_i$, where $\varepsilon_i \sim \mathcal{N}(0, \Sigma_\varepsilon)$.

    - Σ_ε is a scaling matrix ; when acceptance rate is too high we can scale up it by multiplying constant.

    - A recommended optimal acceptance rate is around 23%

3. Solving the model: solve the model for $\Theta^{(i^*)}$ and compute $f(\cdot,\cdot|\Theta^{(i^*)})$ and $g(\cdot,\cdot|\Theta^{(i^*)})$ from (3.12) and (3.13) respectively building new state space representation.

4. Evaluating the proposal: Evaluate $p(\Theta^{(i^*)})$ and $L(\mathbb{Y}_t|\Theta^{(i^*)})$ from (3.14)

5. Accept/Reject: Draw $\rho_i \sim \mathcal{U}(0,1)$. If $\rho_i \leq \dfrac{L(\mathbb{Y}_t|\Theta^{(i^*)})p(\Theta^{(i^*)})}{L(\mathbb{Y}_t|\Theta^{(i-1)})p(\Theta^{(i-1)})}$ set $\Theta^{(i)} = \Theta^{(i^*)}$ otherwise $\Theta^{(i)} = \Theta^{(i-1)}$. If $i \leq M$ set $i = i + 1$ and go to 2. Otherwise stop.

[14] Many researchers find that intertermporal elasticity of consumption for non-stockholders differs widely from that of stockholders which is close to unity and Guvenen (2006) choose 10 so that workers have substantially risk averse than the capitalist.

Once I get the draws, I can approximate expected value of function $h(\Theta)$ by $\frac{1}{M}\sum_{i=1}^{M} h(\Theta^{(i)})$.

3.4.4 DATA

The data set for the labor market variables is obtained from the "Merged Outgoing Rotation Group" of the Current Population Survey which is available at National Bureau of Economic Research[15] over the period of [1984Q1, 2008Q3]. Although the data is available from 1979Q1, I exclude the data prior to 1984Q1 to avoid a structural break known as the Great Moderation. Researches on the Great Moderation point out that the volatility of many macroeconomic variables sharply declines since 1984. In addition, I exclude the data after 2008Q3 because the effective federal funds rate is considered to hit the zero lower bound causing non-linearity in nominal interest rate.

CPS

The Current Population Survey (CPS) is a monthly survey of about 60,000 households. An adult (non-institutional population) at each household is asked to report on the activities of all other persons in the household. Each household entering the CPS is administered 4 monthly interviews, then ignored for 8 months, then interviewed again for 4 more months. If the occupants of a dwelling unit move, they are not followed; rather the new occupants of the unit are interviewed. Merged Outgoing Rotation Groups (MORG) is extracts of the Basic Monthly Data. Since 1979 only households in months 4 and 8 have been asked their usual weekly earnings/usual weekly hours. These are the outgoing rotation groups, and each year the BLS gathers all these interviews together into a single Merged Outgoing Rotation Group File. This data set is appropriate for the analysis in present paper since it includes about 50 variables selected relate to employment. In particular, it includes weekly earnings, highest educational attainment as well as their employment status.

First, I extracted relevant variables only: "Interview calendar month", "highest

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[15] More details of the DATA is available at http://www.nber.org/morg/docs/cpsx.pdf

graded attended", "employment status recodes last week", "weekly earnings", "Earnings weight for all races", "Final weight", "age", and "class of workers". I summarize the variables and the corresponding definitions in Table 3.1.

Variables	Definition
intmonth	Interview calendar Month
gradeat / grade92	Highest grade attended/ Highest completed degree
esr / lfsr89 / lfsr94	Employment status recode/ Labor force status recode
earnwke	Earnings per week
earnwt	Earning Weight
weight	Final Weight
age	age
class / class94	Class of workers

Table 3.1: Extracted Variables

Interview calendar month takes the values from 1 to 12 corresponding January to December. For the highest grade attended variable, the value coded for education is one more than the actual grade in the original BLS coding for 1979-1988, so 13 was coded for a person who has at least started the senior year of high school. In 1989-1991 the actual grade is coded, without adding one. So that senior in high school is coded as 12 in the later system. However, for the data set I used here, it is adjusted based on the latter system. For the period from Jan. 1992 to Dec. 2007, I used "Highest grade completed" (grade92); "What is the highest level of school ... has completed or highest degree received? Rumor has it that a labor economist who estimated wage equations for 1991 and 1992 without noticing the difference in the CPS education measure was surprised only by the change in the constant term.

High and low skilled workers

To construct an appropriate data set, firstly, I classify total population into two groups based on their education level: the high-skilled and the low-skilled. The criterion that I used to make these subgroups is in Table 3.2; I consider the person who has at least Bachelor's degree as a high-skilled worker and otherwise the workers are defined as low-skilled workers.

Definition	Bachelor's Degree and above	
Period	High skilled	Low skilled
1979 $\sim$ 1991	$16 \leq$ gradeat ≤ 19	$1 \leq$ gradeat ≤ 15
1992 $\sim$ 2007	$43 \leq$ grade92 ≤ 46	$31 \leq$ grade92 ≤ 42

Table 3.2: Ranges for high and low education levels

Employment Status

Once I categorize the population as above, I split each subgroup into the employed workers and the unemployed. In doing this, I used "employment status recodes last week" named as "esr" for the period of Jan. 79 $\sim$ Dec. 88. This is later called the Labor Force Status Recode (lfsr89 and lfsr94). When NAs appears (no information on the employment status), I abstracted those individuals from the data set. I gave the details in Table 3.3.

Description	esr (79$\sim$89)	lfsr89 (89$\sim$93)	lfsr94 (94$\sim$07)	Status
Working	1	1	1	E
With a job not at work	2	2	2	E
Looking	3	3	4	U
Layoff	-	4	3	U
Housework	4	-	-	NILF
School	5	-	-	NILF
Unable to work/Disabled	6	-	6	NILF
Working without pay	-	5	-	NILF
Unavailable for work	-	6	-	NILF
Other(Includes Retired)	7	7	5,7	NILF

Table 3.3: Employment Status Record

Unemployment

I used "Final weight" to calculate the Employment-Population Ratio (EPR), participation rates and the unemployment rates of each labor market which are corresponding to the employment (N_t), the labor force (L_t) and the unemployment rate (u_t) in the model. The sum of the Final Weights (weight) in each monthly survey is the US non-institutional population (Pop 16+). The outgoing rotation group includes one-fourth of that population. So one single month MORG file is one-fourth the population 16 years of age and over, and a year of MORG would sum to 3 times that population. However, my interest is on the ratio of the population, these facts do not have any effect on the result. I obtained monthly unemployment rate by definition: $\frac{U}{E+U}$; U and E is just sum of the corresponding weight for each month and then take the average to obtain quarterly data.

Real Wages

In constructing series of the real wage data, I filter out self-employed workers from the data set by "class of workers" (class / class94). I then define the nominal wage as the logged weighted average weekly earnings of employed workers in each labor sector using "earnings per week" (earnwke) and Earning weight (earnwt). Since sum of earning weight is equal to the total population each month, I multiply "earnwke" by "earnwt" and then divide it by the sum of the earning weight.[16] I take the average of three consecutive months to obtain quarterly data. Finally, I subtract logged Implicit Price Deflator (GDPDEF) from the nominal wage series to get real wages. I use the result for W_t^H and W_t^L as average real wage (earnings) of each sector. In doing this, I used weekly earnings itself rather than hourly earnings which could be obtained from weekly earnings/hours worked per week. This is because I focus on the labor market adjustment only at the extensive margin. Again, I take the average of three consecutive months to obtain quarterly wage series.

Inequality Measure

The employment (wage) gap is obtained by log difference between high-skilled employment (wage) and low-skilled employment (wage). The income gap is sum of the employment gap and the wage gap.

Seasonal Adjustment

All of data obtained above are seasonally adjusted by X-12 equipped in *Eviews*.

[16]For each sector $j \in \{H, L\}$, I compute monthly weighted average weekly earnings: $W_t^j = \sum_i \frac{earnwke(i) \times earnwt(i)}{\sum_i earnwt(i)}$ where i is respondent index. I then take log

Federal Reserve Economic Database

All the other data set is obtained from the FRED which is maintained by Federal Reserve Bank at St. Louise over the same period as CPS data (Jan. 1984 $\sim$ Sept. 2008). I use real Gross Domestic Product per capita (A939RX0Q048SBEA) for "output", and the effective Federal Fund Rate (FEDFUNDS) for "nominal interest rate". Inflation rate is obtained as the percentage change of implicit price deflator (GDPDF). The date is compiled as following order: output, high-skilled unemployment, low-skilled unemployment, high-skilled employment, low-skilled employment, wage gap, inflation, and the effective federal fund rate. All the series are HP-filtered.

3.4.5 Estimation Results

Table 3.4 reports the posterior distributions and the priors for the structural parameters and the parameters associated with the exogenous shock process. In particular, I report 90% probability intervals as well as mean and standard deviation of the priors and 90% of Highest Probability Density (HPD) Interval with posterior mean and mode. In general, the data are informative on the parameters in that posterior distributions have smaller standard error relative to the prior distribution with similar means.

Since the paper has intended to show that monetary policy has an effect on income inequality via its effect on wage premium, the parameters of labor supply and demand is crucial. The estimation result indicates that labor demand elasticity of low-skilled workers is significantly larger than that of high-skilled workers in that posterior mean of ε_w^L (7.398) is greater than that of ε_w^H (4.295) and 90% HPD is not even overlapped. The posterior distributions confirm the greater labor supply elasticity of low-skilled workers but the parameters are much smaller than the calibrated values; $\varphi_H \in (0.4571, 0.6387)$, $\varphi_L \in (0.3096, 0.4359)$. This might be because heterogeneity in labor supply decision is already captured by the bond-holding transaction cost depending relative income. As noted earlier, the greater labor elasticities lead to a flatter wage Phillips curve, and, in turn, more volatile unemployment rate and stable wages. In addition, this results imply that about 30% wage markup and 15% wage markup for high-skilled workers and low-skilled workers respectively. However, the posterior mean of labor elasticity also implies the greater natural unemployment rate for high-skilled workers mainly caused by higher wage markup. The 90% HPD for the elasticity of substitution across sector, $\eta \in (3.011, 3.685)$, is consistent with the calibrated model in that greater than 1 but is much larger than Mollick (2011). This result means that the wage premium has very strong effect on the income inequality.

Most of the structural parameters are consistent with the calibrated model. The posterior distribution also indicates quite sticky nominal wages in that 90% HPD of $\theta_w \in (0.596, 0.867)$ with posterior mean 0.733, which is a little bit lower than Barattieri et al. (2014). Nominal rigidity parameter for aggregate price, $\theta_p \in (0.545, 0.903)$ with mean 0.728 is very close to the calibrated value in the literature. In addition, the results support Taylor principle as the posterior mean of ϕ_π is 1.917 and $\phi_y = 0.153$ in that nominal interest rate increase about 2 percent in response to 1 percent increase in inflation. In addition, nominal interest rate is persistent in that the smoothing parameter ρ_i is in between 0.733 and 0.822.

On the other hand, there are some estimates of parameters that are substantially different from the calibrated value as well. For instance, the posterior distribution results show a little bit smaller consumption habit formation, $\rho_c \in (0.117, 0.312)$ in contrast to Galí et al. (2011)'s result which supports substantial consumption habit formation. This might imply that heterogeneity in consumption across households account for aggregate consumption smoothing behavior. The intertemporal elasticity of substitution, $\sigma \in (9.083, 12.111)$, is also much larger than unity which is very close to the one for non-stockholders in Guvenen (2006). The HPD intervals of diminishing return parameter, $\alpha \in (0.043, 0.131)$, is much smaller than the calibrated value, and it is even lower than the estimated value (0.17) of Galí et al. (2011). This means that the production function is almost linear in homogeneous labor input.

The AR(1) coefficients of the high-skilled productivity shock, the aggregate labor supply shock and the risk premium shock are relatively persistent and those shocks account for the most of the error variances of endogenous variables in the long-run. On the other hand, AR(1) coefficient of aggregate productivity shock and price and wage markup shocks are relatively small. The estimated standard deviations of the aggregate labor supply shock is very larger than other shocks.

Figure 3.5 shows the Bayesian impulse responses of 7 variables to monetary policy shock; output, wage premium, price inflation, employment gap, unemployment gap, consumption gap, and labor income gap. In response to a contractionary monetary policy shock, IRFs are pretty much the same as the calibrated one; Output, wage premium and price inflation and unemployment gap decrease while employment gap, consumption gap and labor income gap rise. This result shows that a tight monetary policy has a persistent effect on consumption and income inequality as in the literature, and that inequality channel of monetary policy may have important role in aggregate dynamics. In addition, it is interesting that consumption gap increases less than income gap. This suggests that bond-holding cost function rather than limited asset market participation can replicate the empirical findings on inequality. As I discuss in the next chapter, the assumption that low-skilled workers are completely excluded from the financial market induces much more volatile consumption inequality which is in contrast with Gornemann et al. (2012) and Coibion et al. (2012)

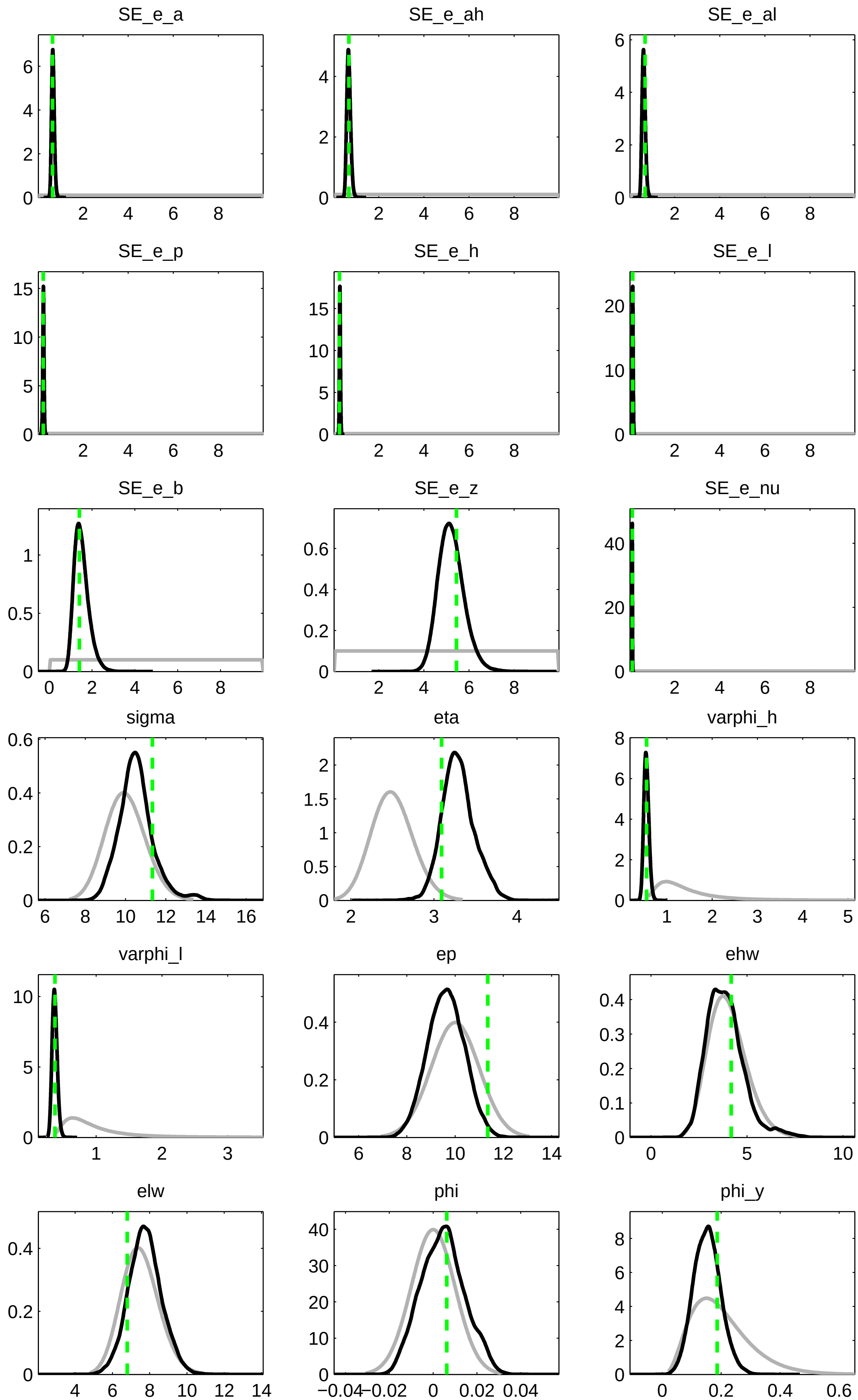
3.6 Conclusion

The heterogeneity in labor supply and demand elasticity across sectors with segmented labor markets causes a *strategic complementarity* in wage setting and thus leads to a greater fluctuation in output, employment ratio and unemployment rates. In other words, greater labor elasticity induces a flatter wage Phillips curve implying stickier nominal wages. When workers from different sectors are substitutable, firms are willing to hire relatively cheaper workers by substituting relatively expensive workers. In response to a negative demand shock, this firm's reaction affects relative marginal rate of substitution between consumption and leisure, and hence, desired wages for workers in opposite direction. Therefore, two sectoral wages will be adjusted in opposite direction after the initial impact of the shock. Consequently, the aggregate wage becomes stickier in a segmented labor market model than it would be in a single labor market model. Therefore, in order for the mechanism I introduced in the previous chapter to be effective, it is necessary to confirm the differences in the labor supply and demand elasticities across sectors in a structural macroeconomic model. Even though empirical microeconomic papers have already shown that those labor supply and demand elasticities are decreasing in educational attainment, a few, if not no, papers has attempted to estimate those parameters in a structural macroeconomic model. The contribution of this paper to the literature is that the present paper provides the empirical evidence of such heterogeneity in a New Keynesian framework, and show how it affects aggregate dynamics in response to an economic shock.

I estimate the model introducing 9 structural shocks; aggregate productivity shock, aggregate labor supply shock, aggregate price markup shock, two sectoral productivity shocks, two sectoral wage markup shocks, risk-premium shock, and the monetary policy shock. In order to avoid singularity problem, which is common in a two-sector DSGE model, I incorporate bond-holding cost as Schmitt-Grohé and Uribe (2003) suggested. The estimation results confirm the result of the calibrated model and the model replicates the stylized facts obtained from the empirical analysis; the procyclical wage premium and income gaps across sector; the greater volatility of low-skilled employment ratio and unemployment rate. The estimation results show that very weak consumption habit formation, low decreasing return on effective labor, and more elastic substitution between different skilled workers. Monetary policy reaction coefficients are consistent with the estimates of the literature and supportive to the Taylor rule. The Bayesian impulse responses show that the wage premium declines after a tight monetary policy. The employment gap and unemployment gap moves counter-cyclically in response to the contractionary monetary policy shock.

para.	Prior Distribution: prior	mean	std.dev	5%	95%	Posterior Distribution: Mean	Mode	90%HPD	
σ	$\mathcal{N}$	10	1	7.4230	12.5802	10.6873	11.3363	9.0827	12.1112
η	$\mathcal{N}$	2.5	0.25	1.8562	3.1442	3.3365	3.0915	3.0107	3.6846
φ_H	$\mathcal{IG}$	1.5	1	0.4246	6.4080	0.5511	0.5534	0.4571	0.6387
φ_L	$\mathcal{IG}$	1	0.7	0.2728	4.4417	0.374	0.3751	0.3096	0.4359
ε_p	$\mathcal{N}$	10	1	7.4230	12.5802	9.8052	11.3528	8.6141	10.9463
ε_w^H	Γ	4	1	2.8850	7.4160	4.2947	4.1765	2.6226	5.6935
ε_w^L	Γ	7.5	1	5.1703	10.3322	7.3979	6.7953	6.016	8.622
ϕ	$\mathcal{N}$	0	0.01	-0.0257	0.0257	0.0064	0.0062	-0.0087	0.0221
ϕ_y	Γ	0.2	0.1	0.1004	0.3253	0.1529	0.1863	0.0823	0.222
ϕ_π	Γ	2	0.25	0.5682	2.8287	1.9165	1.9809	1.6523	2.1655
α	$\mathcal{N}$	0.3	0.1	0.2182	0.3813	0.0876	0.0967	0.0428	0.131
θ_p	$\mathcal{B}$	0.7	0.1	0.4183	0.9103	0.7275	0.7412	0.5453	0.9028
θ_w	$\mathcal{B}$	0.7	0.1	0.4183	0.9103	0.7316	0.7811	0.5961	0.8669
ι_p	$\mathcal{B}$	0.5	0.2	0.1720	0.8323	0.7507	0.7004	0.5265	0.9694
ι_H	$\mathcal{B}$	0.5	0.2	0.1720	0.8323	0.9384	0.9073	0.8881	0.9929
ι_L	$\mathcal{B}$	0.5	0.2	0.1720	0.8323	0.1374	0.0849	0.0264	0.2408
ρ_i	$\mathcal{B}$	0.5	0.2	0.1720	0.8323	0.8049	0.8116	0.7607	0.8507
ρ_c	$\mathcal{B}$	0.5	0.2	0.1720	0.8323	0.2173	0.1652	0.1168	0.3123
ρ_a	$\mathcal{B}$	0.4	0.1	0.1652	0.6666	0.3628	0.3961	0.2639	0.4594
ρ_a^H	$\mathcal{B}$	0.4	0.1	0.1652	0.6666	0.921	0.9219	0.9199	0.9219
ρ_a^L	$\mathcal{B}$	0.4	0.1	0.1652	0.6666	0.1413	0.1935	0.0726	0.2064
ρ_{μ^p}	$\mathcal{B}$	0.4	0.1	0.1652	0.6666	0.3173	0.4457	0.2001	0.4339
ρ_{μ^H}	$\mathcal{B}$	0.4	0.1	0.1652	0.6666	0.2937	0.288	0.19	0.4019
ρ_{μ^L}	$\mathcal{B}$	0.4	0.1	0.1652	0.6666	0.4164	0.4121	0.2965	0.5624
ρ_b	$\mathcal{B}$	0.4	0.1	0.1652	0.6666	0.7827	0.7982	0.7335	0.8328
ρ_z	$\mathcal{B}$	0.4	0.1	0.1652	0.6666	0.7542	0.7688	0.7019	0.8072
η^p	$\mathcal{B}$	0.4	0.1	0.1652	0.6666	0.5112	0.7625	0.3955	0.6171
η^H	$\mathcal{B}$	0.4	0.1	0.1652	0.6666	0.5873	0.5357	0.5323	0.6462
η^L	$\mathcal{B}$	0.4	0.1	0.1652	0.6666	0.3132	0.2947	0.194	0.4294
σ_a	$\mathcal{U}$	5	2.88	0.5081	9.5126	0.659	0.6472	0.5617	0.7553
σ_a^H	$\mathcal{U}$	5	2.88	0.5081	9.5126	0.655	0.6714	0.5214	0.7863
σ_a^L	$\mathcal{U}$	5	2.88	0.5081	9.5126	0.614	0.6815	0.4998	0.7278
σ_{μ^p}	$\mathcal{U}$	5	2.88	0.5081	9.5126	0.2326	0.2342	0.1834	0.2838
σ_{μ^H}	$\mathcal{U}$	5	2.88	0.5081	9.5126	0.274	0.2516	0.2363	0.3111
σ_{μ^L}	$\mathcal{U}$	5	2.88	0.5081	9.5126	0.1406	0.1364	0.111	0.17
σ_b	$\mathcal{U}$	5	2.88	0.5081	9.5126	1.548	1.412	1.0154	2.0616
σ_z	$\mathcal{U}$	5	2.88	0.5081	9.5126	5.3093	5.4431	4.3688	6.2529
σ_ν	$\mathcal{U}$	5	2.88	0.5081	9.5126	0.1167	0.1153	0.1023	0.1306

Table 3.4: Prior and Posterior Distribution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ehw
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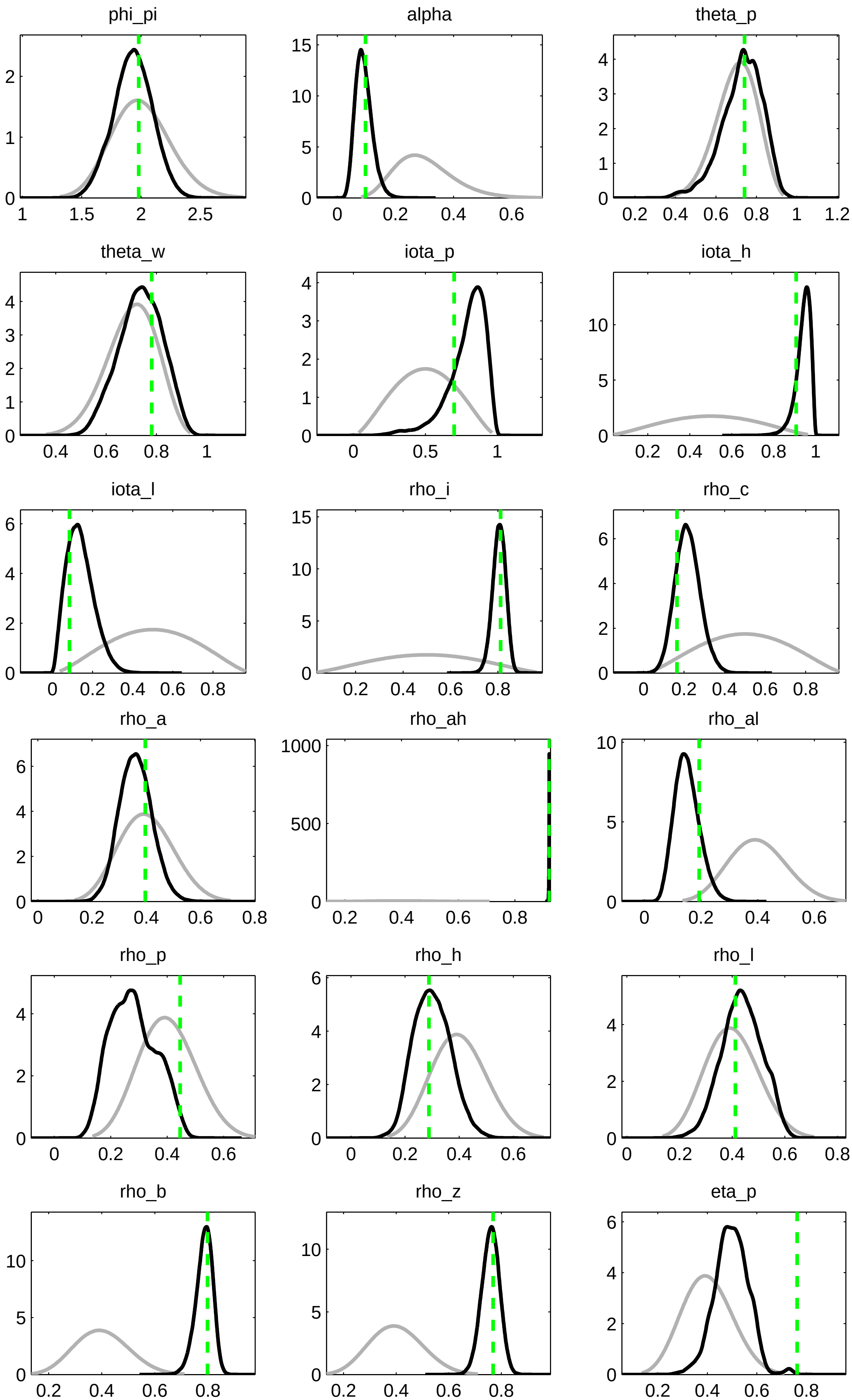
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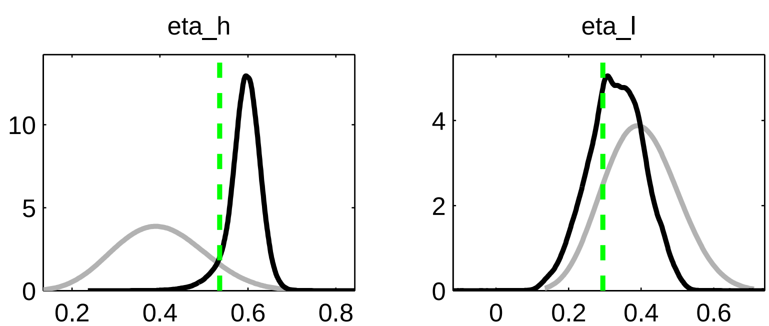
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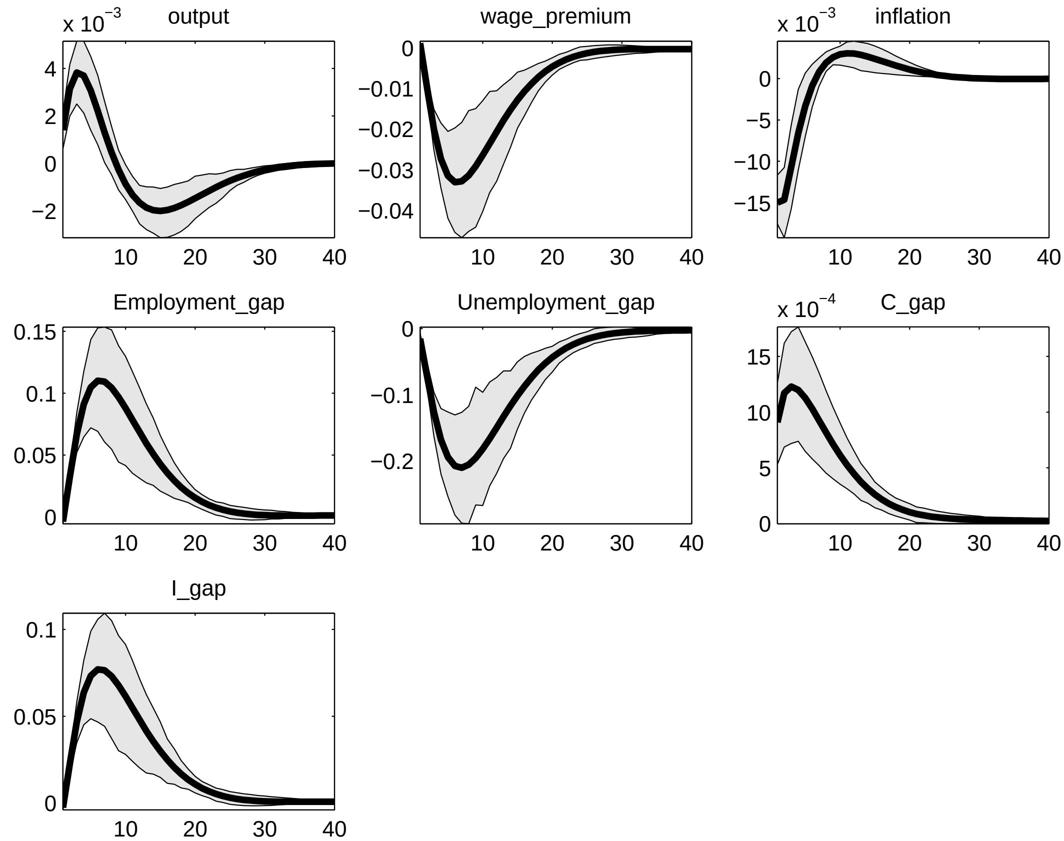
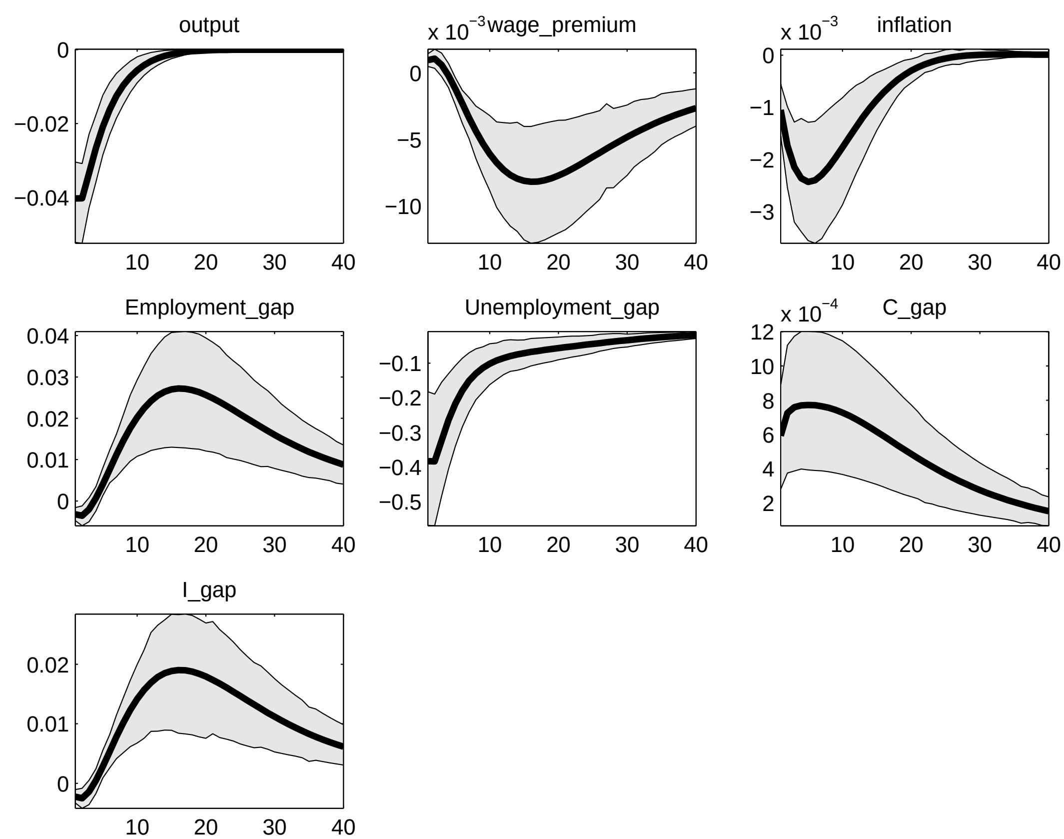
3.5 Bayesian Impulse Responses



(a) Bayesian IRFs to Productivity shock



(b) Bayesian IRFs to Monetary policy shock

Chapter 4

Inequality and optimal monetary policy

"*Because monetary policy is transmitted through many channels, direct and indirect and because households differ in many respects (with regard to socio-demographic factors, such as age and education, as well as economic variables, such as income, wealth, employment status and housing status) monetary policy does not affect all households in the same way. . . . it is not only the extent of income and wealth shocks that affects consumers welfare, but also the fluctuation in their consumption expenditure. All households are not equal in this respect. Some of them are able to insure against wealth shocks and can thus mitigate the adverse consequences of such shocks for their well-being. But poorer households have limited or no access to the financial system (let alone to financial markets) and do not have adequate buffers in the form of precautionary savings. Consequently, their consumption and welfare are particularly vulnerable to adverse shocks. Even if all households were hit by negative shocks to the same extent, poorer, less-insured households would suffer from more volatile consumption and lower welfare.*"

Benoît Coeuré in "What can monetary policy do about inequality?" Oct. 17. 2012

4.1 Introduction

Since the recent financial crisis, inequality has become a hotly debated issue once again not only for economists but also for the public at large. Data indicate that earning inequality has grown rapidly over the past three decades and it tends to become even more pronounced in a recession.[1] This upward trending and counter-cyclical inequality has already been recognized and studied by many economists. The empirical and theoretical literature have shown that inequality is a cause and a consequence of macroeconomic volatility at the same time.[2]

In the present paper, I try to propose a new mechanism through which a monetary policy influences aggregate dynamics via inequality variation in a context of optimal monetary policy. Thus, the two objectives of this paper are to investigate the following: 1) how inequality variation affects aggregate dynamics and consequently social welfare; and 2) whether such effect of inequality variation matters for optimal monetary policy design.

Before constructing a model to meet those objectives, I consider some necessary prerequisites to an appropriate model: the model should be able to account for, at least, three salient features of data associated with income inequality:[3] The first is the higher volatility in unemployment of less-educated (or lower income) households than in high-educated (or higher income) households; The second is that, as Pourpourides (2011) and Champagne and Kurmann (2013) note, wages for high-educated workers are more volatile than those for less-educated workers, while employment is less volatile for high-educated workers than less-educated workers;[4] The third is that a contractionary monetary policy shock increases income and consumption inequality (Coibion et al. (2012) and Gornemann et al. (2012)).

Obviously, a representative agent model or single labor market model cannot explain such differentials in labor market variables and the dynamics of inequality. Therefore, in an attempt to account for these stylized facts, I introduce heterogeneity into a standard

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[1] See Krueger et al. (2010) and Heathcote et al. (2010)

[2] See Breen and Garcia-Penalosa (2005), Fitoussi and Saraceno (2010), Ghiglino and Venditti (2011), Stiglitz (2012), Kumhof et al. (2013), and Dosi et al. (2013)

[3] Note that although income inequality itself arises from various income sources, the Consumer Expenditure Survey (CEX) data shows labor income is the largest contributor to total income for most households. Therefore, for the sake of simplicity, I focus only on employment and wages which consist of labor income and derive some stylized facts on those variables from the literature.

[4] Similarly, Heathcote et al. (2010) finds that earning dynamics of households in the upper end of the income distribution are driven by changes in wages while changes in hours play a central role in the earning dynamics at the lower end of the income distribution.

New Keynesian DSGE model used in Erceg et al. (2000) by assuming segmented labor markets and the Limited Asset Market Participation (hereafter LAMP). These two features of the model are well supported by empirical micro-evidences but also introduce heterogeneity in a relatively simple and tractable way.

In the baseline model, there are two types of households, one supplying high-skilled labor and the other supplying low-skilled labor. In the model, labor markets are segmented for different skilled workers.[5] Even though I classify the households into high-skilled and low-skilled, they are different only in regards to labor demand and supply elasticities. I assume that the low-skilled workers are more substitutable than the high-skilled workers in production, and hence, the demand for low-skilled workers is more susceptible to a change in low-skilled wages. This assumption is consistent with empirical findings in the literature such as Lichter et al. (2014a), in which the authors show that there is a significant heterogeneity in labor demand elasticity and, in particular, labor demand for unskilled workers and workers with atypical contracts is more responsive to wage rate changes.

I also introduce heterogeneity in the labor supply side by assuming LAMP following Galí et al. (2007) and Furlanetto (2011). For the sake of simplicity and tractability, I assume that the low-skilled households are also the households that have limited access to the financial market. These households are therefore not able to smooth their consumption through financial assets. Because of this restriction, their consumption and labor supply are more susceptible to income changes. Intuitively, low-skilled workers with a lower wage have to spend a greater fraction of their earnings on living costs. If financial transactions require some cost or financial intermediaries require a high standard for their financial services, it is relatively more difficult to gain access to financial markets for the low-skilled households. Therefore, low-skilled households' labor supply becomes more sensitive to changes in their wages. On the other hand, since high-skilled workers with higher wages have better opportunities to access financial markets, high-skilled workers have a much superior capacity to offset the fluctuation in their wages.

[5] I use high-skilled workers, Ricardian agents, and financially included agents inter-changeably.

Thus, their consumption and labor supply are relatively stable compared to low-skilled workers.

These assumptions about labor markets structure enable the model to generate variations in income and consumption inequality across sectors after an economic shock and to show how the variations in inequality magnify a macroeconomic volatility. In particular, the greater elasticity in both labor supply and demand for low-skilled workers leads to a flatter low-skilled wage Phillips curve even if nominal rigidities are the same across sectors.[6] The effectively stickier low-skilled wages, in turn, induce a more volatile more volatile employment and unemployment rates for low-skilled workers. Furthermore, the difference in such real rigidities generates a variation in wage premium defined as the gap between average high-skilled wages and average low-skilled wages, which, in turn, brings about strategic complementarities in wage setting resulting in stickier adjustment of aggregate nominal wages and greater fluctuation of real variables such as output, employment and unemployment.[7] This is because changes in the wage premium force firms to substitute relatively cheap workers for expensive workers, and that raises (lowers) marginal rates of substitution between consumption and labor and thus wages for relatively cheap (expensive) workers. However, these two competing forces on aggregate nominal wage are dominated by high-skilled wages because low-skilled wages are effectively stickier than high-skilled wages. As a consequence, both high-skilled and low-skilled wages initially decrease after negative demand shock but high-skilled wages bounce back somewhat in response to the fall in the wage premium whereas low-skilled wages decrease further. Thus, aggregate wages cannot decrease as much as they do under the single labor market model, which causes more volatile real variables. An endogenous shift term in the aggregate wage Phillips curve captures this indirect effect of shocks on the aggregate wage. In addition, given that the elasticity of substitution

[6]The greater elasticities of labor demand and supply for low-skilled workers imply that low-skilled unemployment fluctuate more given a change in wages; conversely, low-skilled wages are relatively stable given a change in unemployment. Thus, the slope of the wage Phillips curve that describes the relationship between the wage inflation rate and unemployment rate becomes flatter as the elasticities increase.

[7]A difference in the size of change in wages across sectors gives rise to sectoral wages mutually reinforcing one another.

across sectors is greater than 1, as found in the literature, a variation in the wage premium results in more variation in employment gap between two different skilled households, and hence, labor income inequality changes in the opposite direction of the wage premium.

The paper then studies optimal monetary policy based on the Central Bank's welfare loss function which is obtained from the second order approximation to the weighted average of households' life-time utility function as in Bilbiie (2008). A Central Bank's target variables include consumption and income inequality as well as standard objectives (price and wage inflation, and the output gap from its efficient level). Galí (2011)'s specification of unemployment is adopted so that the output gap can be transformed into unemployment. This specification helps to express the Central Bank's loss function in terms of observable variables only. As is well-known in the literature, when an economy features both price and wage stickiness, the Central Bank cannot achieve efficient equilibrium and thus suffers from a substantive welfare loss. In this paper, I find that inequality poses an additional policy trade-off with output gap after idiosyncratic productivity shocks even with flexible wages. Accordingly, the first best allocation is not attainable when sectoral productivity shocks hit the economy. In other words, inflation targeting is not an optimal policy even under the flexible wages in contrast to Erceg et al. (2000). I therefore conclude a Central Bank may need to take into account consumption and income inequality when constructing monetary policy.

Finally, I conduct counter-factual experiments in which the Central Bank sets its optimal monetary policy as if the true economy is different from the baseline model. I consider three different scenarios: 1) the central bank recognizes sticky wages but not segmented labor market; 2) the central bank recognizes segmented labor markets but not sticky wages; and 3) the central bank recognizes neither sticky wages nor segmented labor market. The results indicate that when the Central Bank ignores labor market segmentation and, consequently, inequality, the welfare losses are significantly larger than those of the baseline model, even if the Central Bank recognizes wage stickiness.

The remaining sections are organized as follows: In Section 4.2, I review the literature on the disproportionate effect of monetary policy and LAMP. Section 4.3 describes the model in detail and Section 4.4 gives equilibrium and market clearing conditions, In Section 4.5, I define consumption and income inequality and discuss the aggregate dynamics of the model. I conduct numerical simulations in Section 4.6 to show the disproportionate effect of monetary policy and heterogeneity in labor market dynamics. In Section 4.7, I contemplate an optimal monetary policy design that takes into account inequality with a welfare analysis, and provide concluding remarks in Section 4.8.

4.2 Related Literature

As noted above, inequality has been mostly ignored in monetary policy design in spite of its disproportionate effects of monetary policy. Rather, the literature has focused on the relationship itself between monetary policy and inequality. In particular, Carpenter and Rodgers III (2004) shows that a contractionary monetary policy lowers the employment-population ratios of minorities and less-skilled households and raises their unemployment rates. Breen and Garcia-Penalosa (2005) show that high volatility of monetary policy has been shown to result in high-output volatility and assert that, as a regression tax, higher inflation causes a greater inequality. Recently, Coibion et al. (2012) have also studied the effects of monetary policy shock using micro-level data on income and consumption, and found that contractionary monetary policy actions have systematically increased inequality in the U.S. since 1980. Romer and Romer (1999) also empirically analyzes the influence of monetary policy on inequality and show that an expansionary monetary policy lowers inequality temporarily by boosting the economy. However, they also argue that higher inflation after the expansionary monetary policy shock would lead to a tight monetary policy resulting in a rise in unemployment, which would, in turn, offset the temporary positive effect on inequality.

The theoretical literature has focused on the relationship between inflation and wealth distribution (and hence inequality) over the long-run. Among others Albanesi (2007) demonstrates that inflation is positively related to income inequality due to the relative vulnerability to inflation of low income households. Williamson (2008) addresses the monetary policy effect on an economy with segmented financial and goods markets. He argues that contractionary monetary policy shocks reallocate wealth from those connected to the financial market toward the unconnected agents, and that, therefore, consumption and income inequality fall after the shocks. However, there are very few papers in which the authors study the disproportionate effect of monetary policy and inequality at a business cycle frequency. Dosi et al. (2013) discuss the relationship between income inequality and monetary policy using an agent-based Keynesian model. In particular, they find a non-linearity of monetary policy impact and argue that a contractionary monetary policy lead a more "unequal" economies. Gornemann et al. (2012) build a structural model in a New Keynesian framework with search and matching friction and find that contractionary monetary policy shocks lead to a pronounced increase in earnings, income, wealth, and consumption heterogeneity. However, even though their model features a richer environment considering various income sources, they do not discuss the effect of inequality on optimal monetary policy design.

In addition, Aghion et al. (1999) shows that unequal access to investment opportunity leads to greater fluctuations in real variables and argues that counter-cyclical fiscal policy is quite effective in stabilizing economy. Similarly, the LAMP framework has been used mostly for analyzing the effect of fiscal policy on aggregate output. Among others, Furlanetto (2011) extends the basic LAMP model by considering segmented labor markets. He argues that a common wage and employment is suboptimal for both Ricardian and Rule-of-Thumb agents. That is because their consumption behaviors are different in response to an exogenous shock, which creates a variation in the relative marginal rate of substitution between consumption and labor, and thus, a gap in the desired wage across the sectors. Therefore, the common wage (and hours) assumption is to exclude both agents from a mutually beneficial trade. In contrast, there are a couple of papers that use LAMP to investigate optimal monetary policy; Ascari et al. (2011) and Areosa and Areosa (2006). However, neither paper is able to model income inequality and the dynamics of labor market variables due to their own assumptions: the single labor market in the earlier paper and the Cobb-Douglas production function with flexible wages in the later one. The LAMP framework with segmented labor market and staggered wages allows me to discuss consumption and income inequality in two ways: the effect of inequality on optimal monetary policy and monetary policy's impact on inequality.

4.3 Model

There are two different types of skilled households with each type of household consisting of a continuum of workers supplying labor to the corresponding skilled labor market. Wages are set by representative unions for each type of workers in segmented markets. There is a continuum of monopolistic competitive firms producing differentiated goods and they determine the price given the wages and aggregate demand. In the benchmark model, a Central Bank sets a nominal interest rate following a "Taylor-type" rule.

To simplify the model, I make three assumptions. The first is sectoral immobility; Workers are prohibited from crossing from the low-skilled labor market to the high-skilled labor market and vice versa. The second is a constant population share of a sector. Thus the relative size of each labor sector remains constant over time. Third, I also assume that there is only one good producing sector in which a monopolistic firm hires both high-skilled and low-skilled workers and both skilled workers are aggregated into one homogeneous effective labor input and used to produce differentiated goods.

4.3.1 Household

I assume that there are two levels of skill, $j \in \{H, L\}$, by which the households are categorized as high skilled or low skilled households. For j-skilled households, there are a large number of identical households which are comprised of a continuum of members represented by the unit indexed by $i \in [0, 1]$. The index $i \in [0, 1]$ indicates the type of labor service in which a given household member is specialized in a sector. I also assume that a constant fraction, s, of the total population are high-skilled workers and $1 - s$ fraction of population are the low-skilled workers in every period. These two types of households are heterogeneous in two dimensions: first, low-skilled workers are more substitutable than high-skilled workers so that the labor elasticity of substitution between the low-skilled workers is greater than that between the high-skilled workers. In other words, demand for the low-skilled workers is more responsive to changes in wage; second, low-skilled workers are limited in their access to financial markets so that they cannot smooth their consumption using financial assets. That is, they use up all the disposable income in every period.

Representative households of j-skilled households maximize their discounted lifetime utility (4.1) subject to budget constraint (4.2) for $j \in \{H, L\}$. While labor demand, $N_t^j(i)$, is determined by the aggregation of firm's labor demand decisions and allocated uniformly across j-skilled households, workers choose their optimal wages, $W_t^j(i)$. Therefore, both $W_t^j(i)$ and $N_t^j(i)$ are taken as given by each household. Each household's discounted lifetime utility in j sector is given by:

$$\sum_{t=0}^{\infty} \beta^t U(C_t^j, N_t^j(i); \chi_t) = \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(C_t^j\right)^{1-\sigma}}{1-\sigma} - \chi_t \int_0^1 \frac{\left(N_t^j(i)\right)^{1+\varphi}}{1+\varphi} di \right] \tag{4.1}$$

where the variable χ_t is a aggregate labor supply shock following AR(1) process in log ($\log \chi \equiv \xi$), $\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_t^\xi$, and $\varepsilon_t^\xi \sim \mathcal{N}(0, \sigma_\xi^2)$. The parameter σ is the inverse of intertemporal elasticity of substitution, and the parameter φ denotes the inverse of the Frisch labor supply elasticity of workers which is common for all types of workers. Aggregate consumption of a representative household with j-skill is given by

$$C_t^j \equiv \left(\int_0^1 C_t^j(z)^{\frac{\epsilon_p - 1}{\epsilon_p}} dz \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$$

where $C_t^j(z)$ is the quantity consumed of good z by a j-skilled household, ε_p is the elasticity of substitution between two differentiated goods, and $N_t^j(i)$ for $i \in [0, 1]$ is the fraction of members specialized in type i labor in each j-skilled household who are employed in period t.[8] and the parameter ε_p is the elasticity of substitution over differentiated goods. The high-skilled households budget constraint is given by:

$$\int_0^1 P_t(z) C_t^H(z) dz + Q_t B_t \leq B_{t-1} + \int_0^1 W_t^H(i) N_t^H(i) di + \Pi_t \tag{4.2}$$

[8]when all variables are measured in per capita term, $\frac{N_t^H}{Pop_t} = \frac{Pop_t^H}{Pop_t} \frac{N_t^H}{Pop_t^H} = s \frac{N_t^H}{Pop_t^H}$. Therefore, N_t^j where $j \in \{H, L\}$ can be interpreted as Employment to Population Ratio and participation rate respectively as explained in later.

where $P_t(z)$ is the price of good z, $W_t^H(i)$ is the nominal wage for type i high-skilled labor, B_t represents purchases of nominally riskless one-period discount bonds paying one monetary unit, Q_t is the price of that bond, and Π_t is a lump-sum component of income at time t.

The first order conditions for the maximization problem subject to the budget constraint give the high-skilled consumption Euler equation:

$$Q_t = \beta E_t \left(\frac{C_{t+1}^H}{C_t^H} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \tag{4.3}$$

Low-skilled households have the same utility function as high-skilled households, (4.1), but they do not face an intertemporal consumption decision because they are not able to hold bond in this simple model. Rather, they consume all the disposable income in each period:

$$\int_0^1 P_t(z) C_t^L(z) dz = \int_0^1 W_t^L(i) N_t^L(i) di \tag{4.4}$$

In addition, optimal demand for each good resulting from utility maximization takes the familiar form:

$$C_t^j(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\epsilon_p} C_t^j$$

for $j \in \{H, L\}$ where $P_t \equiv \left(\int_0^1 P_t(z)^{1-\epsilon_p} dz \right)^{\frac{1}{1-\epsilon_p}}$ denotes the price index for final goods.

4.3.2 Wage Determination

The labor markets are monopolistically competitive, and wages are determined by the representative unions. Nominal rigidities in wages are introduced through Calvo (1983) pricing; For each labor market, only $1 - \theta_w$ fraction of workers can re-optimize their wage. When re-optimizing their wage in period t, workers choose a wage W^{j*}, where again $j \in \{H, L\}$, in order to maximize their households' utility taking all aggregate variables including the aggregate wage index as given.[9] I assume that the Calvo parameters are the same, $\theta_w = \theta_w^H = \theta_w^L$, following Barattieri et al. (2014). In addition, I assume that high-skilled workers are not easily substituted by others relative to low-skilled workers; $\varepsilon_w^H < \varepsilon_w^L$. This assumption implies that the markup of high-skilled workers in wage setting is greater than that of low-skilled workers, and it also assure that the high-skilled wage is larger than the low-skilled wage on average for a given Frisch elasticity. As will be explained in section 4.3.4, these two elasticities are closely related to divergent unemployment rates. The optimal wage setting rule for j-skilled workers for $j \in \{H, L\}$ can be obtained from the maximization problem subject to the budget constraint and the corresponding labor demand schedule determined by firms:

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ N_{t+k|t}^j \left(C_{t+k}^j\right)^{-\sigma} \left(\frac{W^{j*}}{P_{t+k}} - \mathcal{M}_t^j MRS_{t+k|t}^j \right) \right\} = 0 \tag{4.5}$$

where $N_{t+k|t}^j$ denotes the aggregate quantity demanded in period $t+k$ of j-skilled workers whose wage was last reset in period t. Here, $MRS_{t+k|t}^j \equiv \chi_t \left(C_{t+k}^j\right)^{\sigma} \left(N_{t+k|t}^j\right)^{\varphi}$ is the period $t+k$ marginal rate of substitution between consumption and labor for a high-skilled worker whose wage is reset in period t, and $\varepsilon_{w,t}^j$ is the elasticity of substitution between two different types of workers in j sector, $\mathcal{M}_t^j \left(\equiv \frac{\varepsilon_{w,t}^j}{\varepsilon_{w,t}^j - 1} \right)$ is the desired or frictionless wage markup and $\mu_t^{nj} \equiv \log \mathcal{M}_t^j$. The first order log approximation of (4.5) around the zero inflation steady states gives the optimal wage equation as following:

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ w_t^{j^*} - p_{t+k} - \mu_{t+k}^{nj} - mrs_{t+k|t}^j \right\} = 0 \tag{4.6}$$

Defining the j-skilled sector's average marginal rate of substitution as $MRS_t^j \equiv \chi_t \left(C_t^j\right)^{\sigma} \left(N_t^j\right)^{\varphi}$, the marginal rate of substitution of each individual in the sector can be written in terms of the relationship between the average marginal rate of substitution and the relative

[9] Aggregate wage for j-skilled workers is given by $W_t^j \equiv \left(\int_0^1 W_t^j(i)^{1-\epsilon_w^j} di \right)^{\frac{1}{1-\epsilon_w^j}}$ and aggregate nominal wage is defined as $W_t \equiv \left(\gamma_H \left(W_t^H\right)^{1-\eta} + \gamma_L \left(W_t^L\right)^{1-\eta} \right)^{\frac{1}{1-\eta}}$.

wage.[10]

$$\begin{aligned} mrs^j_{t+k|t} &= mrs^j_{t+k} + \varphi\left(n^j_{t+k|t} - n^j_{t+k}\right) \\ &= mrs^j_{t+k} - \epsilon^j_w \varphi\left(w^{j*}_t - w^j_{t+k}\right) \end{aligned} \tag{4.7}$$

Finally, combining (4.6), (4.7) and the log-linearized form of the aggregate wage index, I obtain the j-skilled wage Phillips curve as:[11]

$$\pi^j_t = \beta E_t\left\{\pi^j_{t+1}\right\} - \kappa^j_w\left(\mu^j_t - \mu^{nj}_t\right) \tag{4.8}$$

where $\pi^j \equiv w^j_t - w^j_{t-1}$ is the j-skilled wage inflation, μ^{nj}_t is the wage markup shock of j-skilled workers, and $\mu^j_t \equiv w^j_t - p_t - mrs^j_t$ denotes the log average j-skilled wage markup and $\kappa^j_w \equiv \frac{\Theta}{1+\epsilon^j_w\varphi} > 0$ where $\Theta \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w} > 0$.[12]

4.3.3 Firms and Price Determination

Monopolistically competitive firms hire workers from both labor markets and then aggregate these workers with CES technology into homogeneous effective labor input. Each firm produces a differentiated good $z \in [0,1]$ using a production function which is given by:

$$\begin{aligned} &Y_t(z) = A_t H_t(z) \\ &\quad \text{where } \; H_t(z) = \left[\gamma_H^{\frac{1}{\eta}}\left(N^H_t(z)\right)^{\frac{\eta-1}{\eta}} + \gamma_L^{\frac{1}{\eta}}\left(N^L_t(z)\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \\ &\quad \text{and } \; N^j_t(z) \equiv \left(\int_0^1 N^j_t(i,z)^{\frac{\varepsilon^j_{w,t}-1}{\varepsilon^j_{w,t}}}\, di\right)^{\frac{\varepsilon^j_{w,t}}{\varepsilon^j_{w,t}-1}} \end{aligned}$$

where $H_t(z)$ is the homogeneous effective labor input of firm z obtained by labor aggregation technology; η is elasticity of substitution between high-skilled $(N^H_t(z))$ and

[10] where $N^j_t \equiv \int_0^1 N^j_t(i)di$ is the sector j aggregate employment rate.

[11] $w^H_t = \theta_w w^H_{t-1} + (1-\theta_w)\, w^{H*}_t$.

[12] See Galí (2011) for the detailed derivation.

low-skilled labor ($N_t^L(z)$); γ_j is a parameter governing the relative income share of j-skilled of labor; $\varepsilon_{w,t}^j$ is the labor elasticity of substitution with in the corresponding sector $j \in \{L, H\}$ as I mentioned above. The variable A_t is an exogenous technology process which is assumed that $a_t \equiv \log A_t$ and $a_t = \rho_a a_{t-1} + \varepsilon_t^a$ where $\rho_a \in (0,1)$ and ε_t^a is a white noise process with a zero mean and variance σ_a^2. The firm's cost minimization problem, taking wages and aggregate demand as given, implies the following set of labor demand schedules:

$$N_t^j(i,z) = \left(\frac{W_t^j(i)}{W_t^j}\right)^{-\varepsilon_w^j} N_t^j(z) \text{ and } N_t^j(z) = \gamma_j \left(\frac{W_t^j}{W_t}\right)^{-\eta} H_t(z) \text{ where } j \in \{L, H\}$$

for all $i \in [0,1]$ and for all $z \in [0,1]$.[13] I introduce nominal rigidities in price through the Calvo (1983) pricing. Firms' profit maximization problem subject to the sequence of demand schedule constraint $Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon_p} C_{t+k}$, for $k = 0, 1, 2, \ldots$ leads to the optimality condition for the firm:

$$\sum_{k=0}^{\infty} \theta_p^k E_t \left\{Q_{t,t+k} Y_{t+k|t} \left(P_t^* - \mathcal{M}^p \Psi_{t+k|t}\right)\right\} = 0$$

where $Y_{t+k|t}$ denotes output at time $t+k$ of a firm that last reset its price in period t, $Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$ is the relevant stochastic discount factor for nominal payoffs in period $t+k$, $\Psi_{t+k|t} \equiv \frac{W_{t+k}}{A_{t+k}}$ is the nominal marginal cost in period $t+k$ of producing quantity $Y_{t+k|t}$ and $\mathcal{M}^p \equiv \frac{\epsilon_p}{\epsilon_p - 1}$ is the desired or frictionless price markup over the marginal cost. Log-linearization of the optimality condition around the zero inflation steady state yields

$$\sum_{k=0}^{\infty} (\beta\theta_p)^k E_t \left\{p_t^* - \psi_{t+k|t}\right\} = 0$$

Note that lower case variables denote the log-deviation of the variables from the steady state. The price inflation equation can be derived using the log-linearized price index, $p_t = (1-\theta_p)p_t^* + \theta_p p_{t-1}$:

$$\pi_t^p = \beta E_t \left\{\pi_{t+1}^p\right\} + \kappa_p mc_t \tag{4.9}$$

[13] Log-linearized employment rate of i-type of j-skilled labor is given by $n_t^j(i) = -\varepsilon_w^j \left(\omega_t^j(i) - \omega_t^j\right) + n_t^j$ and average j-skilled employment rate is given by $n_t^j = -\eta\left(\omega_t^j - \omega_t\right) + y_t - a_t$

where $\pi_t^p \equiv p_t - p_{t-1}$ is wage inflation, mc_t denotes average real marginal cost, $mc_t = \omega_t - a_t\,(\equiv \widetilde{\omega}_t)$, and $\kappa_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} > 0$.

4.3.4 Unemployment

I define sectoral unemployment rates following Galí (2011). An individual will be willing to work in period t if and only if the real wage for his labor type exceeds his disutility of labor. Thus the marginal j-skilled supplier of type i labor, $L_t^j(i)$, is given by

$$\frac{W_t^j(i)}{P_t} = \chi_t \left(C_t^j\right)^\sigma \left(L_t^j(i)\right)^\varphi$$

Define the aggregate labor force (or participation rate) as $L_t^j \equiv \int_0^1 L_t^j(i)di$, then the first order approximations gives the log-linearized estimate relation:

$$w_t^j - p_t = \sigma c_t^j + \varphi l_t^j + \xi_t$$

The unemployment rate u_t^j can be written as the log difference between the labor force and employment:[14]

$$u_t^j \equiv l_t^j - n_t^j$$

Noting that real wage is the markup over the marginal rate of substitution, $\mu_t^j \equiv \left(w_t^j - p_t\right) - mrs_t^j = \left(w_t^j - p_t\right) - \left(\sigma c_t^j + \varphi n_t^j + \xi_t\right)$, the unemployment rate can be written as:

$$\mu_t^j = \varphi u_t^j \tag{4.10}$$

Therefore, as Galí (2011) noted, (4.10) implies that unemployment fluctuations are a consequence of variations in the wage markup. Finally, combining (4.8) with (4.10), I derived the sectoral New Keynesian wage Phillips Curve:

$$\pi_t^j = \beta E_t\left\{\pi_{t+1}^j\right\} - \kappa_j\varphi\left(u_t^j - u_t^{nj}\right) \tag{4.11}$$

[14] $u_t^j = 1 - \frac{N_t^j}{L_t^j} \Rightarrow -u_t^j \approx \log(1-u_t^j) = n_t^j - l_t^j$. Note, in efficient steady state, all labor force has to be hired ($l = n$, that is, $u = 0$); and government subsidies impose symmetric labor market ($l^H = l^L = n^H = n^L$).

The slope becomes flatter as labor demand elasticity increases. Therefore, high-skilled workers face a steeper wage Phillips curve, and accordingly, high-skilled nominal wages are more volatile than low-skilled nominal wages in response to unemployment fluctuation. Conversely, low-skilled unemployment is more volatile given changes in wage, which is consistent with the empirical findings shown in Pourpourides (2011) and Champagne and Kurmann (2013).

4.3.5 Government

The government budget constraint is:

$$P_t G_t + B_{t-1} = Q_t B_t + T_t$$

where T_t is the lump-sum tax from high-skilled household after subsidies which are used to eliminate desired markups on price and wages. I assume that government spending, G_t, is zero at any period.

4.4 Equilibrium and Market Clearing

4.4.1 Steady States

I consider the zero inflation efficient steady states. I assume that government can eliminate markups in both goods and labor markets by giving appropriate subsidies.[15] I also assume that the government does not issue government bonds in the steady states. This guarantees that the wages and the consumptions are the same for any type of workers in the steady states.[16] Note that aggregate consumption is now given by

$$C_t = sC_t^H + (1-s)C_t^L$$

[15] That is $\mathcal{M}^j\left(1-\tau^j\right)=1$ for $j \in \{P, H, L\}$ where τ^j is the subsidies for j market.

[16] Log-linearization of aggregate wage index is given by $w_t = \frac{W^H N^H}{WH} w_t^H + \frac{W^L N^L}{WH} w_t^L = s w_t^H + (1-s) w_t^L$. Since steady state wages are the same, the relative labor income of each sector equals its population share. In addition, given the same wages with zero bond-holding, all the workers enjoy the same level of consumption.

and log-linearized as $c_t = sc_t^H + (1-s)c_t^L$. Thus, in the steady state, I obtain

$$C^H = C^L = C = Y = H = N^H = N^L$$

4.4.2 Labor Market Equilibrium

Since the labor markets are segmented, aggregate j-skilled labor supply must be equal to the firm's aggregate labor demand for j-skilled labor in equilibrium for $j \in \{H, L\}$. Accordingly, the j-skilled labor market clearing conditions are the following:

$$N_t^j = \int_0^1 N_t^j(z)dz = \int_0^1 \int_0^1 N_t^j(i,z)didz \quad = \gamma_j \Delta_t^j \Delta_t^P \left(\frac{W_t^j}{W_t}\right)^{-\eta} \frac{Y_t}{A_t}$$

Note that Δ_t^P, Δ_t^H and Δ^L are measures for the price, high skilled, and low skilled wage dispersion respectively, and can be approximated to 1 up to the first order.[17] Log-linearization of the employment in each sector around steady state can be written:

$$n_t^j = -\eta\left(\omega_t^j - \omega_t\right) + (y_t - a_t) \tag{4.12}$$

Note that the sectoral employment rates are affected by not only aggregate demand but also by the relative wage, and thus, the wage premium. This is important because the effect of a variation in the wage premium affects sectoral employments in opposite way, and therefore, generates differentials in employment rates across sectors. For instance, although both employment rates decrease initially in response to a positive technology shock, high-skilled employment decreases less than low-skilled employment due to the decrease in wage premium; the greater elasticity of substitution between low-skilled workers than between high-skilled workers induces more volatile low-skilled employment and effectively stickier wages for low-skilled workers, and thus, the shock results in a decline in the wage premium. Moreover, when the two different skilled workers are highly substitutable, a change in the wage premium leads to a greater

[17] $\Delta^P \equiv \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon_p} dz \approx 1 + \frac{\varepsilon_p}{2} Var_z\left\{p_t(z)\right\}$, $\Delta_t^j \equiv \int_0^1 \left(\frac{W_t^j(i)}{W_t^j}\right)^{-\varepsilon_w^j} di \approx 1 + \frac{\varepsilon_w^j}{2} Var_i\left\{w_t^j(i)\right\}$ for $j \in \{H, L\}$ Details for the second order log approximations see Galí (2011).

gap between sectoral employment rates. Therefore, aggregate productivity shock that lowers the wage premium causes greater inequality.

4.4.3 Resource Constraint and Consumption Euler Equation

Because of the absence of investment and government spending in a closed economy, all outputs produced by each firms are consumed. Therefore, the market clearing condition is $C_t(z) = Y_t(z)$ for all $z \in [0,1]$, and hence, $C_t = Y_t$. From (4.3), the log-linearized high-skilled consumption Euler equation is given by:

$$c_t^H = E_t c_{t+1}^H - \frac{1}{\sigma}\left\{i_t - E_t \pi_{t+1}^p\right\} \tag{4.13}$$

where $i_t(= -q_t)$ is the nominal interest rate on a risk-free bond.[18] and low-skilled consumption is just equal to low-skilled worker's labor income:

$$c_t^L = \omega_t^L + n_t^L \tag{4.14}$$

where ω_t^L is the average real wage for the low-skilled workers. Noting that $c_t^H = \frac{c_t-(1-s)c_t^L}{s}$, good market clearing condition, and (4.12), I derive aggregate consumption Euler equation as:

$$\begin{aligned} c_t &= E_t c_{t+1} - \frac{s}{\sigma}\left\{i_t - E_t \pi_{t+1}^p\right\} - (1-s)\left\{\Delta E_t c_{t+1}^L\right\} \\ y_t &= E_t y_{t+1} - \frac{1}{\sigma}\left\{i_t - E_t \pi_{t+1}^p\right\} - \frac{1-s}{s}\left\{(1-\eta s)\,\Delta E_t \omega_{t+1}^L + \eta s \Delta E_t \omega_{t+1}^H - \Delta E_t a_{t+1}\right\} \end{aligned} \tag{4.15}$$

4.5 Inequality and aggregate dynamics

When an economy is efficient, the wages are determined at the level at which the marginal rate of substitution equals the marginal product of labor in any given period. Using this condition, I obtained the efficient level of output and the interest rate that

[18] $Q_t = \frac{1}{R_t} = \frac{1}{1+i_t}$.

makes output gap equal to zero in equilibrium.[19] I now define a variable $\widetilde{X}_t \equiv X_t - X_t^E$ as the difference from its efficient level.

4.5.1 Consumption and Income Inequality

In this subsection, I study the relationship between inequality and wages. I measure the consumption inequality by the Gini coefficient. The households are divided into two skilled groups and I assumed perfect risk sharing within a household. As all the members of a household therefore enjoy the same level of consumption, the economy has only two types of consumption level and so the Gini coefficient is given by $\mathcal{G}_t^c = (1-s)\left\{1 - \frac{C_t^L}{C_t}\right\}$ and is approximated as:[20]

$$\mathcal{G}_t^c \approx -(1-s)\left(c_t^L - c_t\right) = -(1-s)\left(\widetilde{\omega}_t + (\eta-1)s\widetilde{\omega}_t^R\right) \tag{4.16}$$

Similarly, I define labor income inequality by the Gini coefficient, $\mathcal{G}_t^I = (1-s)\left(1 - \frac{\mathcal{X}_t^L}{\mathcal{X}_t}\right)$ where $\mathcal{X}_t$ is the economy's average labor income or total payment of the economy.[21]

$$\mathcal{G}_t^I \approx -(1-s)\left(\widehat{\mathcal{X}}_t^L - \widehat{\mathcal{X}_t}\right) = -s(1-s)(\eta-1)\widetilde{\omega}_t^R \tag{4.17}$$

If the production function is Cobb-Douglas, that is $\eta = 1$, the relative income share is constant and hence income inequality is fixed over time. If two different skilled workers are close to complementary inputs ($\eta < 1$), then income inequality moves along with wage premium because relative employment does not change as much as wage premium. However, if two inputs are highly substitutable ($\eta > 1$), relative employment variation dominates a change in wage premium, and hence, income inequality moves in the opposite direction of the wage premium. Moreover, the greater η implies the stronger effect of wage premium on income inequality.[22]

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[19] See Appendix C.1 for the details.

[20] $c_t^L - c_t = \omega_t^L + n_t^L - y_t = \widetilde{\omega}_t^L + a_t + a_t^L + \left(\eta s \widetilde{\omega}_t^R + y_t - a_t - a_t^L\right) - y_t = \widetilde{\omega}_t^L + \eta s \widetilde{\omega}_t^R = \widetilde{\omega} + (\eta-1)s\widetilde{\omega}_t^R$.

[21] See Appendix **??** for details.

[22] In an extreme case in which $\eta = 0$, production function becomes Leontief production function, that is two different skilled workers are perfect complements, income inequality only depends on the wage premium. In other extreme case in which $\eta = \infty$, the two different workers are perfect substitute, and

4.5.2 IS Curve

Combining (4.15) and (4.16), I derive the economy's IS curve as

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left\{i_t - E_t \pi^p_{t+1} - r^E_t\right\} + \frac{1}{s}\Delta \mathcal{G}^c_{t+1} \tag{4.18}$$

where $y^E_t = \frac{1+\varphi}{\sigma+\varphi}a_t - \frac{1}{\sigma+\varphi}\xi_t$, $x_t = y_t - y^E_t$, and $r^E_t = \sigma \Delta y^E_{t+1}$. The IS curve differs from that of standard LAMP model due to the extra term associated with consumption inequality, which comes from the imperfect risk-sharing across households.[23] When all the agents are not financially excluded, that is when $s = 1$, the IS curve becomes the one that is in a standard NK model. Since labor markets are segmented and households do not perfectly share the risk (the labor income shock), consumption responses are different after a real interest rate change. As will be explained later, low-skilled workers who are prohibited from holding bonds only respond to their labor income change rather than an interest rate change. Therefore, the impact of monetary policy on output is weakened by the presence of Non-Ricardian households.[24] The last term captures this channel.

4.5.3 Wage Phillips Curves

Note that wage markups can be expressed as the difference between the real wage and the marginal rate of substitution. Solving for markups in terms of wages, I obtain the aggregate wage Phillips curve which is a convex combination of two sectoral wage Phillips curves[25] as in (C.8). In doing this, I assume log utility function, $\sigma = 1$, to simplify the equation, which allows me to focus only on the relationship between inequality and the macroeconomic volatility.The aggregate wage Phillips curve is then

even a very tiny deviation of the wage premium from its efficient level makes one sector takes all.

[23] It is actually very similar to Ascari et al. (2011) but the extra term is now associated with consumption inequality rather than a real wage gap.

[24] In present paper, I mean low skilled workers (households) by both financially excluded agents and Non-Ricardian agents.

[25] See the Appendix C.3 for details.

given by:

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \kappa_w (1+\varphi) x_t - \kappa_w^H \widetilde{\omega}_t + \Upsilon \mathcal{G}_t^I + \epsilon_t \tag{4.19}$$

where $\Upsilon \equiv \Theta \frac{\eta(1+\varphi)}{\eta-1} \left\{ \frac{1}{1+\varepsilon_w^H \varphi} - \frac{1}{1+\varepsilon_w^L \varphi} \right\}$ and ϵ_t is the composition of wage markup shocks.[26] Since the (absolute) slope of the curve is decreasing in labor demand elasticity, it can clearly be seen that the slope will be steeper as the population share of high-skilled workers increases. The aggregate wage Phillips curve differs from the standard one due to the presence of the endogenous shift term related to the wage-premium, and hence income inequality. This endogenous shift term brings about more sluggish aggregate nominal wage, and thus, more volatile macroeconomic variables. For instance, suppose that when an economy is hit by a negative demand shock, then output decreases and unemployment rates increase, thereby pushing the nominal wage to fall. However, the high-skilled nominal wage falls more than the low-skilled nominal wage because the latter is effectively stickier. This causes a decrease in the wage-premium and strategic complementarities in wage setting. Once the wage premium decreases, firms start to substitute high-skilled workers for low-skilled workers that raises high-skilled employment and lowers low-skilled employment, which, in turn, causes a higher marginal rate of substitution between consumption and labor of high-skilled workers and lower that of low-skilled workers. As a result, high-skilled wages bounce back while low-skilled wages decrease further. Thus, the strategic complementarities dampen the decrease in high-skilled wages and amplify the decrease in low-skilled wages.[27] However, since the magnitude of high-skilled wages adjustment is greater than that of low-skilled wages, the aggregate wage is influenced by the changes in high-skilled wages. Therefore, the net effect of the cross-sector income effect is to generate slower aggregate wage adjustments. On the other hand, if the elasticity of substitution across sectors is sufficiently high, $\eta > 1$, income inequality widens because a change in employment gap is larger

[26] $\epsilon_t \equiv s\kappa_w^H \mu_t^{nH} + (1-s)\kappa_w^L \mu_t^{nL}$.

[27] Lee (2011) builds a NK model based on firm specific labors and heterogeneity in firms' price setting frequency and discusses the aggregate effect of the heterogeneity. He argues that heterogeneity in price rigidities across sectors creates cross-sector income effect and hence strategic complementarities. As a result, the aggregate Phillips curve has endogenous shift terms arising from the heterogeneity and this term causes stickier aggregate price adjustment than the homogeneous model due to changes in relative price in response to aggregate shocks.

than that of the wage premium. Similarly, in response to a positive technology shock, unemployment rates increase (due to the more efficient labor aggregation technology) putting downward pressure on nominal wages. Again, high-skilled wages decrease more than those of low-skilled workers, thereby reducing the wage premium. Note that when $\varepsilon_w^H = \varepsilon_w^L = \varepsilon_w$, the aggregate wage Phillips curve coincides with Galí (2011) in which neither wage premium nor income inequality has a significant role in aggregate wage dynamics. Therefore, inequality does not require any policy intervention when all workers face the same labor market condition even if the markets are segmented.

4.6 Quantitative Analysis

To complete the model, I additionally define equations for the wage dynamics as:

$$\begin{aligned} \widetilde{\omega}_t^L &= \widetilde{\omega}_{t-1}^L + \pi_t^L - \pi_t^p - \Delta\omega_t^E \\ \widetilde{\omega}_t^H &= \widetilde{\omega}_{t-1}^H + \pi_t^H - \pi_t^p - \Delta\omega_t^E \\ \widetilde{\omega}_t &= s\widetilde{\omega}_t^H + (1-s)\widetilde{\omega}_t^L \end{aligned} \tag{4.20}$$

where $\omega_t^E = a_t$. In addition, for the benchmark model, I assume the Central Bank sets a nominal interest rate following a Taylor rule type of monetary policy responding to inflation and output gap.

$$i_t = \phi_\pi \pi_t + \phi_x x_t + \nu_t \tag{4.21}$$

where ν_t is a monetary policy shock following AR(1) process, $\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_t^\nu$ and $\varepsilon_t^\nu \sim \mathcal{N}(0, \sigma_\nu^2)$.

4.6.1 Calibration

I set discount factor β to 0.99 for a 4% annual nominal interest rate. I assume relative risk averse agents by setting intertemporal elasticity of consumption parameter, σ, to 2. I compute the historical average high-skilled labor income share, s, from CPS data to be around 0.53. The price rigidity parameter, θ_p is set to 0.75 which implies that the average duration of price is one year. I adopt the wage rigidity parameter from

Barattieri et al. (2014). Their estimate for the parameter is 0.822 implying that only 17.8% of hourly workers experienced a wage change in a quarter. Moreover, as they find little evidence of heterogeneity in wage adjustment frequency across the sectors, I set the same Calvo parameter for both labor markets. Elasticity of substitution between differentiated goods (ε_p) and labor parameters (ε_w^H and ε_w^L) are set to, 9, 3.8, and 6.2 implying 12.5%, 36% and 19% markups, respectively. This imply that the average (or aggregate) wage markup is about 25% which is the value estimated by **?**. I adopt the value for the elasticity of substitution between different skilled workers (η) from **?** who estimate the elasticity of labor substitution across education levels and argues that the plausible value varies over 2.00 to 3.21.[28] The inverse of Frisch elasticity is set to 5 to be consistent with 5% average unemployment (natural rate of unemployment). Finally, following Christiano et al. (2010), the standard deviation of technology shock, labor supply shock, and monetary policy are set to 0.62, 0.24, and 0.13 respectively.

Table 4.1: Baseline Calibration

Parameter	Description	Value
β	Discount factor	0.99
σ	Intertemporal elasticity of consumption	2
s	High-skilled income share	0.53
θ_p	Calvo parameter for price adjustment	0.75
θ_w	Calvo parameter for wage adjustment	0.822
ε_p	Elasticity of substitution between differentiated goods	9
η	Elasticity of substitution between different skilled workers	2.43
ε_w^H	Elasticity of substitution between high-skilled workers	3.8
ε_w^L	Elasticity of substitution between low-skilled workers	6.2
φ	Inverse of the Frisch labor supply elasticity	5
ϕ_π	Inflation reaction coefficient of monetary policy	1.5
ϕ_y	output gap reaction coefficient of monetary policy	0.2
σ_a	Standard deviation of aggregate technology shock	0.62
σ_ξ	Standard deviation of labor supply shock	0.24
σ_ν	Standard deviation of monetary policy shock	0.13

[28] Previous studies in the literature such as Katz and Murphy (1992) and Krusell et al. (2000b) estimate elasticity of substitution between skilled workers and unskilled workers as 1.67 and 1.41 respectively. The estimates are somewhat lower than **?**'s estimate but still greater than 1.

4.6.2 Dynamic Responses

Monetary policy shock

Figure 4.1 shows the dynamic responses of sectoral variables and inequality measures to an increase in nominal interest rate by one standard deviation. This rise in the interest rate initially lowers only high-skilled consumption because low-skilled consumption is not affected by nominal interest rate but by their labor income. The decline in high-skilled consumption induces weaker aggregate demand (and thus lower output gap) as well as lower demand for both high-skilled and low-skilled labor. Consequently, the shock reduces employment and raises unemployment rates pushing aggregate nominal wage down. Real wages decline as well even though price inflation moves procyclically. "Talyor type" rule of monetary policy responds to this disinflation (and drop in output gap) lowering nominal interest rate. Therefore, the real interest rate declines thereafter, and high-skilled consumption is recovered gradually. On the other hand, low-skilled consumption fall as well because of drop in both employment and real wages. Labor force participation rates rise due to the negative wealth effect, and thus, unemployment rates rise more than the decrease in employment to population ratio. However, about two third of the increases in unemployment can be attributed to the decrease in employment which is in line with Erceg and Levin (2013)'s findings that unemployment rates mostly influenced by employment-to-population ratio as labor force participation rate is acyclical.

The monetary policy shock also has a disproportionate effect on labor market variables. Note that when nominal wages are under downward pressure, high-skilled wages drop more because of low-skilled wages are stickier than high-skilled wages due the greater labor demand elasticity. In other words, since the high-skilled wage Phillips curve is steeper, high-skilled nominal wages respond more sensitively to a change in unemployment, and this leads to a decrease in the wage premium. Even though low-skilled real wages decrease much less than high-skilled real wages, labor income for low-skilled workers actually decreases more than that for high-skilled workers due to greater drops in employment. This is because firms substitute high-skilled workers for low-skilled workers in response to the drop in the wage premium making even further decrease in low skilled workers. Moreover, the size of the rise in the employment gap is larger than that of drop in the wage premium because the two different skilled workers are highly substitutable given the parameterization, ($\eta > 1$). As a consequence, labor income inequality rises after the tightening of monetary policy. This is consistent with the findings in Pourpourides (2011) in which the author takes U.S. data from 1979 to 2003 and shows that high-skilled wages are more volatile than those of low-skilled workers, while high-skilled employment is relatively more stable than that of low-skilled workers.[29]

In addition, high-skilled consumption decreases less than low-skilled consumption as high-skilled workers smooth their consumption, consequently, consumption inequality increases after the contractionary monetary policy shock. This result is consistent with empirical evidence obtained by Coibion et al. (2012). They intensively studied the effect of monetary policy on various measure of inequality and argue that "contractionary monetary policy shocks are associated with higher levels of economic inequality". However, according to their simulation, earning inequality decreases initially for about 2 years and the volatility of earning inequality is greater than consumption inequality for all measure of inequality. This discrepancy might arise from the absence of other sources of income because consumption relies on total income rather than on labor earnings only. For instance, as Coibion et al. (2012) noted, the labor income share of total income is larger for the higher quantiles in income distribution. This finding implies that the lower quantiles would reduce consumption relatively less after a negative labor income shock than they would when labor income is a unique source of income. More importantly, monetary policy impact on inequality is quite persistent as it depends on wage variation.

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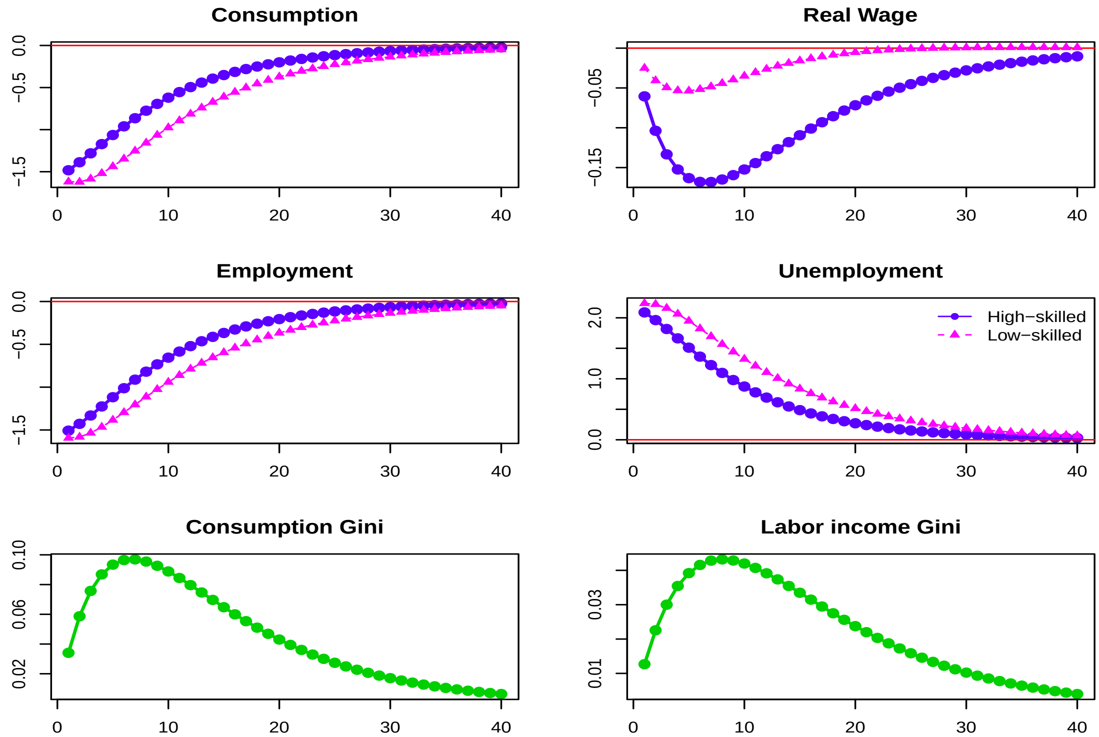
[29] More recently, Champagne and Kurmann (2013) analyze the wage data of CPS in various dimension and find "substantial heterogeneity in how the absolute volatility of hourly wages of different worker groups changes over time. The largest increases in volatility occur for skilled workers (with a college degree) that are either male and young or middle-aged or salaried." As a byproduct, they also show that volatility of skilled wages is greater than that of unskilled wages in any given period and at any decomposition except for older workers (aged $60 - 70$).

Figure 4.1: Dynamic responses to the positive monetary policy shock



High-skilled productivity shock

Figure C.2 in Appendix displays impulse responses of the same variables to a positive high-skilled productivity shock. The shock widens marginal productivity gap between high-skilled and low-skilled workers, and therefore, firms increase high-skilled workers and reduce low-skilled employment. This puts upward pressure on high-skilled nominal wage and downward pressure on low-skilled nominal wages and hence raises both the wage premium and income inequality. However, as the magnitude of the changes in high-skilled wages is much larger, high-skilled employment increases less relative to the decrease in low-skilled employment. Consequently, high-skilled unemployment falls somewhat while low-skilled unemployment increase substantially. The greater high-skilled productivity leads to a decline in real marginal cost and lower inflation. The inflation falls enough to push up even the low-skilled real wages. A Taylor type rule of monetary policy forces the nominal interest rate to fall in response to the drop in inflation and results in a rise in high-skilled consumption. However, low-skilled labor income decreases because of the huge drop in employment and small increase in real wages. Consequently, low-skilled consumption decreases. This result is consistent with Heathcote et al. (2010) in that low-skilled earnings dynamics are dominated by employment fluctuation. Therefore, a high-skilled productivity shock causes a rise in both consumption and labor income inequality.

High-skilled wage markup shock

I also consider the high-skilled wage markup shock as another idiosyncratic shock. When the high-skilled markup rises, the high-skilled nominal wage jumps up immediately, and the wage premium rises. As a consequence, firms reduce high-skilled employment by substituting low-skilled workers, which raises the low-skilled nominal wage in contrast to the case of high-skilled productivity shock. The increase in both high-skilled and low-skilled wages induces higher inflation. Consequently, the central bank raises nominal interest rate to stabilize such a rise in inflation resulting in a higher real interest rate, followed, in turn, by a drop in high skilled consumption. Again, since high-skilled wages are relatively flexible, there is only a small amount of decrease (increase) in employment (unemployment) of high-skilled workers. However, the relatively stickier low-skilled nominal wage induces two opposite consequences. On one hand, the strong increase in inflation overturns the muted increase in nominal wages and brings about a slight decline in real wages for low-skilled workers. On the other hand, relatively more staggered wages motivate firms to demand more low-skilled workers. Accordingly, low-skilled employment increases, which leads to a drop in unemployment. Given high substitutability of labor across sectors, ($\eta > 1$), a small decrease in relative wages leads to a larger increase low-skilled labor demand, and, as a result, low-skilled labor income actually increases and thus consumption increase as well. Moreover, sufficiently large increase in low-skilled consumption dominates the decrease in high-skilled consumption, and therefore, aggregate output increases. In sum, all of aggregate output, inflation, and real wages increase, whereas both consumption and income inequality falls, as illustrated in Figure C.4.

4.6.3 The Role of Labor Market Assumption

As noted in Introduction, the labor market assumptions are important in discussing the disproportionate effect of monetary policy and the dynamics of inequality. Figure C.6 plots dynamic responses of aggregate variables and inequality measures under two alternative labor market assumptions in comparison to the baseline model. Under the single labor market assumption, the differences in consumption occur because of LAMP. However, employments are identical so that all the workers face the same labor demand and wages even though their willingness to work is different. Thus, we cannot say anything about labor income inequality and the wage premium under this assumption. Consequently, the aggregate wage falls more than the baseline model because there is no strategic complementarities in wage setting without income inequality. Accordingly, marginal cost and inflation decrease more as well. If workers are homogeneous with segmented labor markets, the wage difference occurs only due to the financial friction (LAMP) which affects the consumption level and marginal rate of substitution.[30] However, the slope of sectoral wage Phillips curves are the same so that wage premium does not have an impact on aggregate wage inflation. Furthermore, since financially excluded workers have a relatively strong incentive to work (due to lower aggregate demand) under downward pressure on wages, their employment decreases less (because labors are the same from the perspective of the firms) than workers in other sectors. Therefore, income inequality decreases in response to a contractionary monetary policy shock which is in contrast with the empirical evidence the literature have shown.

4.7 Optimal Monetary Policy

In the previous chapter, I showed that monetary policy has a disproportionate effect on labor market variables and eventually causes changes in consumption and labor income inequality. In this section, I approach the same phenomenon from the opposite direction by asking if inequality and heterogeneous response of households with different

[30] No difference in skills and market power implies no difference in elasticities of substitution between workers within a sector

characteristics affect optimal monetary policy design. To this end, I derive the welfare loss function of the economy, and contemplate the optimal monetary policy. Following Bilbiie (2008), I assume that the social planner maximizes the convex combination of the utilities of the two types of households, weighted by the mass of agents of each type:

$$\mathcal{W}_t = \left\{ s \left(U(C_t^H) - V(N_t^H) \right) + (1-s) \left(U(C_t^L) - V(N_t^L) \right) \right\} \tag{4.22}$$

A Central Bank's loss function is obtained by the second order approximation of the welfare around the efficient steady state as in Woodford (2003)[31]:

$$\mathcal{L} = \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left\{ (\sigma + \varphi) x_t^2 + \frac{\varepsilon_p}{\kappa_p} (\pi_t^p)^2 + \frac{s\varepsilon_w^H}{\kappa_w^H} \left(\pi_t^H\right)^2 + \frac{(1-s)\varepsilon_w^L}{\kappa_w^L} \left(\pi_t^L\right)^2 + \psi_c \left(\mathcal{G}_t^c\right)_t^2 + \psi_I \left(\mathcal{G}_t^I\right)^2 \right\}$$

where $\psi_c \equiv \frac{(\sigma-1)}{s(1-s)}$ and $\psi_I \equiv \frac{(1+\varphi)}{s(1-s)} \left(\frac{\eta}{\eta-1}\right)^2 > 0$. Because prices and wages are sticky, any change in those variables causes inefficient dispersion in prices and wages and hence inefficient output. This inefficiency is captured by inflation and output gap terms in loss function. Obviously, as prices and wages get stickier (κ_p and $\kappa_w^j \to 0$), the Central Bank puts more weight on the corresponding inflation. In comparison to the standard model, the loss function has two additional terms; consumption and income Gini coefficient which are associated with LAMP and segmented labor market respectively. Therefore, so long as the financial and labor market are segmented, changes in wages affect inequality and hence a welfare loss.[32] If high skilled labor is equally substitutable for the low skilled workers, a Central Bank does not need to be concerned about sectoral wage inflations, but aggregate wage inflation matters for welfare loss. As elasticity of substitution across labor sectors becomes larger within a plausible range of parameter suggested by **?**, loss from income inequality gets smaller. This occurs because when workers are perfectly substitutable, firms can fully accommodate a shock in relative wages (and hence income inequality) by substituting workers with different skills. Therefore, the effect of the shock on output distortion will be negligible. As population

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[31] see Appendix C.4 for details.

[32] Recall that consumption inequality and income inequality are derived from the weighted sum of two sectoral wages and wage premium respectively.

is distributed equally into two sectors (as $s \to \frac{1}{2}$), inequality measures become less important.

4.7.1 A new policy trade-off with inequality

It is well known that equilibrium with flexible prices and wages is not attainable if both prices and wages are sticky unless the natural wage is constant. There exists, that is, a policy trade-off between three standard target variables; output gap, price inflation and wage inflation. As stated above, without nominal rigidities in wage, wage inflations do not affect welfare, and thus the Central Bank only needs to be concerned with the variations of inequality in addition to output gap and price inflation. If there is no variation in inequality, a strict inflation targeting rule leads to an efficient equilibrium by achieving zero inflation and output gap simultaneously. However, inequality that arises from the differential in wage and consumption across labor sectors introduces a new trade-off so that a Central Bank cannot achieve the first best allocation even if wages are flexible. To distinguish the role of inequality in optimal monetary policy from the standard one that arises from nominal rigidities in wages, I consider an economy with flexible wages in this subsection. Equilibrium wages are determined at a marginal rate of substitution between consumption and labor supply for each type of household in any given period under flexible wages. Subtracting the low-skilled equilibrium wage from the high-skilled equilibrium wage, I obtain a relationship between the output gap and income inequality.[33] If I assume log-utility, $\sigma = 1$, then the equation is simplified further:

$$\mathcal{G}_t^I = (1-s)(\eta-1)x_t + \frac{s(1-s)(\eta-1)}{\eta}(a_t^H - a_t^L) \tag{4.23}$$

where I use $\widetilde{\omega}_t = (\sigma + \varphi)x_t$ which is obtained from the convex combination of two sectoral wages weighted by their population share. The equation (4.23) shows that a Central Bank is not able to completely stabilize both income inequality and output gap at the same time after idiosyncratic productivity shocks even under flexible wages.

[33] $\widetilde{\omega}_t^R = \frac{\sigma}{1+\varphi\eta}c_t^R - \frac{1+\varphi}{1+\varphi\eta}\left(a_t^H - a_t^L\right)$ and $c_t^R = -\frac{1}{s}\left(\widetilde{\omega}_t + s(\eta-1)\widetilde{\omega}_t^R\right)$.

4.7.2 Dynamic response under optimal monetary policy

This section explores optimal monetary policy under full commitment in which a Central Bank minimizes the loss function above, (C.20), subject to the given constraints.[34] I compare the dynamic responses of endogenous variables under optimal monetary policy with those under the Taylor rule to see how they differ in response to aggregate as well as idiosyncratic shocks.

A positive technology shock

Figure 4.2 plots impulse responses to a positive aggregate technology shock. In response to the technology shock, optimal monetary policy lowers the nominal interest rate substantially, allowing output and high-skilled consumption to increase more than those variables under Taylor rule. Accordingly, the optimal monetary policy encourages firms to demand more workers as opposed to the Taylor rule. However, the shock does not cause a variation in the relative marginal productivity across sectors, and thus, the changes in employment rates are almost the same for both sectors initially. The increase in labor demand induces higher wages for both high-skilled and low-skilled workers. Once the nominal wage increases, however, the wage premium increases due to effectively stickier low-skilled nominal wages, and in turn low-skilled employment rises more than high skilled employment. As a result, income inequality falls opposite to that under Taylor rule. In addition, low-skilled workers are more willing to supply their labor relative to high-skilled workers and thus the low-skilled unemployment rate slightly increases while the high-skilled unemployment actually decreases, which is in stark contrast with huge increases in both high-skilled and low-skilled unemployment rates observed under the Taylor rule. On the other hand, the optimal monetary policy, which is much more accommodative, leads to a greater disinflation and a larger increase in real wages. Therefore, labor income for both high and low skilled workers increases, and accordingly, low-skilled consumption rises as well.

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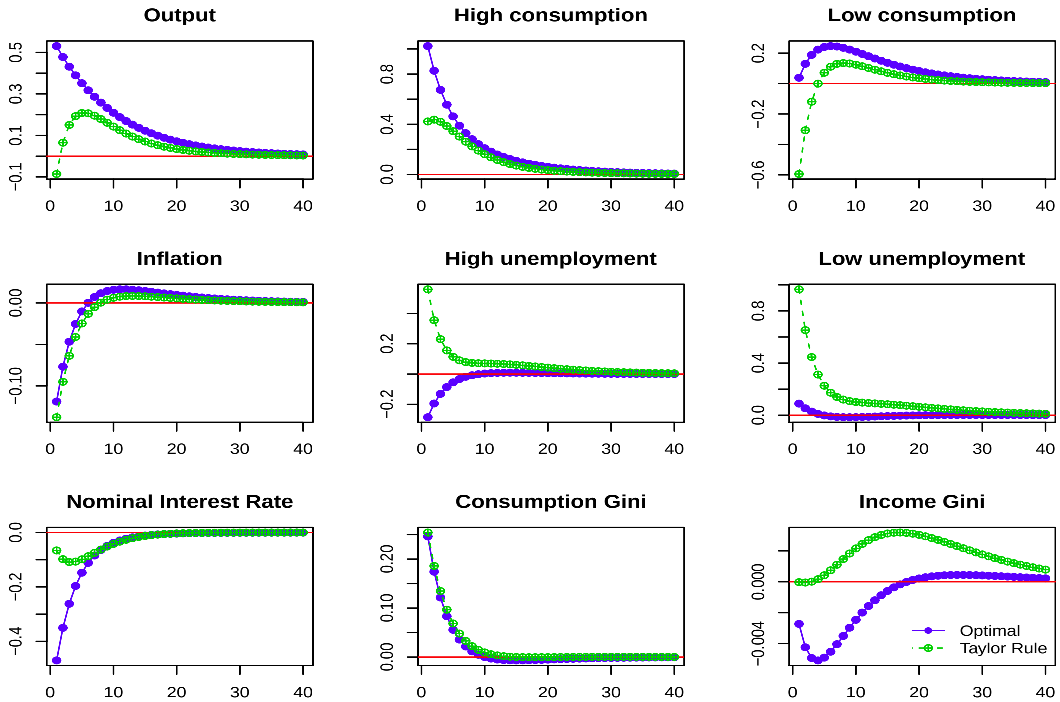
[34] Details for the constraints and the first order conditions are provided in Appendix C.5.

Figure 4.2: Dynamic responses to the positive technology shock



A high-skilled wage markup shock

Figure C.9 shows the response of the same variables to a positive high-skilled wage markup shock. An exogenous increase in high-skilled wages induces a sharp decrease in high-skilled employment, and consequently high-skilled unemployment increases. A rise in real wages also leads to a higher marginal cost and inflation. However, since optimal monetary policy again aggressively responds to this change, the increase inflation is muted resulting in a higher real interest rate. Consequently, the high-skilled consumption decreases substantially. Such a huge drop in high-skilled consumption causes a drop in aggregate output followed by lower demand for low-skilled workers. Accordingly, low-skilled unemployment rather increases somewhat in contrast to that under Taylor rule. This, in turn, pushes low-skilled wages down, and as a result, the labor income for both high and low skilled workers decreases. However, a substantial drop in high-skilled employment induces lower income inequality. The optimal monetary policy in response to this idiosyncratic shock becomes remarkable in comparison to aggregate shock in that the optimal monetary policy generates effectively different path of inequality; the dynamic responses of inequality are muted under an optimal monetary policy relative to under the Taylor rule.

A positive high-skilled productivity shock

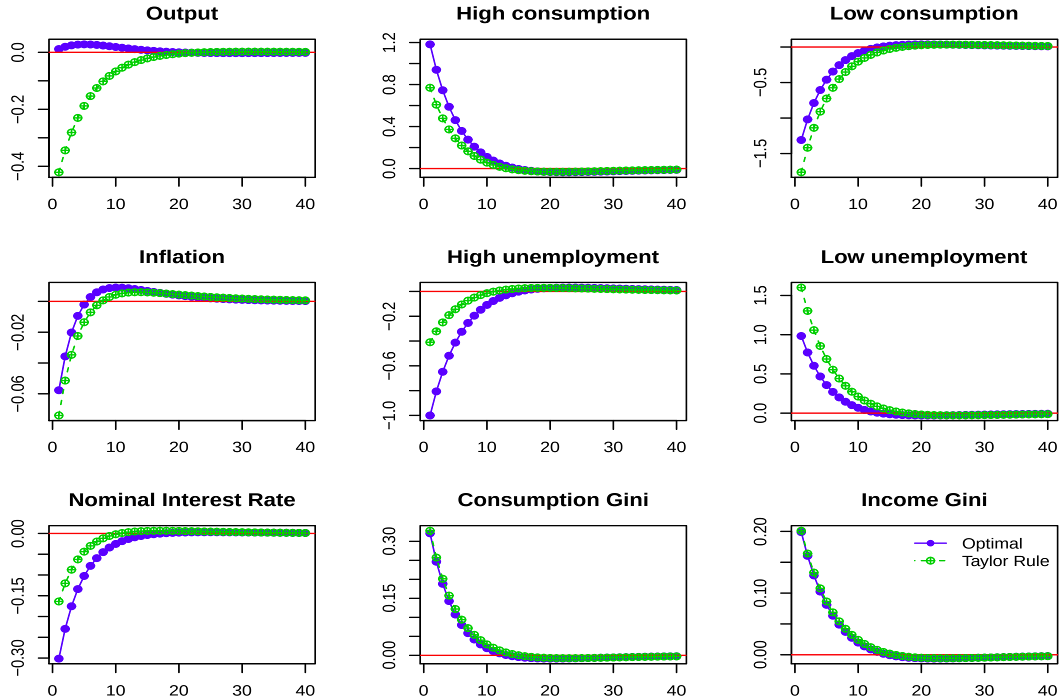
Figure 4.3 displays the dynamic responses of the same variables. Firms demand more high-skilled workers immediately in response to the idiosyncratic shock by substituting low-skilled workers. Consequently, high-skilled employment increases while low skilled employment decreases. However, the magnitude of the rise in high-skilled employment is diminished by the substantive increase in wages. On the other hand, low-skilled employment falls substantially due to stickier low-skilled wages, and therefore, aggregate employment decreases. The optimal monetary policy is highly accommodative to this shock again. As a result, high-skilled consumption increases more dominating decrease in low-skilled consumption because of larger decrease in real interest rate, which, in turn, leads to a slight increase in aggregate demand and real wages. Low-skilled consumption decrease due to a drop in low-skilled income, however, the size is much less than that under Taylor rule. The greater increase in consumption and employment for high-skilled workers induces more drop in high-skilled unemployment, and the less severe drop in low-skilled unemployment causes less rise in low-skilled unemployment. Therefore, aggregate unemployment remains virtually unchanged in contrast to the significant increase in unemployment under the Taylor rule.

4.7.3 Counter-factual experiments

In this section, I examine aggregate dynamics under three hypothetical scenarios in comparison to those of the present model (benchmark model).[35] In the benchmark model, a Central Bank sets an optimal interest rate, considering segmented labor markets under sticky wages. In Scenario 1, a Central Bank considers a single labor market in which the representative union sets a desired wage for each type of workers regardless of their skill level, but the wages are sticky. In Scenario 2, a Central Bank is aware of

[35]See Appendix C.6 for a detailed description of each scenario.

Figure 4.3: Dynamic responses to the high-skilled productivity shock



the segmented labor market, but considers the wages flexible. In Scenario 3, a central bank considers a single labor market with flexible wages. All the Central Banks admit that some fraction of the total population is financially excluded.

Scenario 1: Single labor market with staggered wages

If workers are homogeneous with respect to the demand elasticity and the two different skilled workers are perfect substitute then loss function is simplified to the one in Ascari et al. (2011). In this scenario, workers are different only in financial accessibility and face the same wages and labor demand. Since the representative unions of i-type of workers who maximize weighted average of both skilled life-time utility, all i-type workers face the same labor demand and wages regardless of their skill level. A Central Bank now cares only aggregate real wage gap arising from LAMP rather than sectoral wage gaps or inequality measures. As Ascari et al. (2011) noted, LAMP does not affects optimal monetary policy design, because consumption inequality only shows up in demand equation, (4.18), and supply block of the economy is not influenced by the income inequality, in fact, there is no income inequality in this scenario. In a special case when $\sigma = 1$, a Central Bank's loss function and the constraints are exactly the same as the one in Erceg et al. (2000). Evidently, if all agents are able to smooth consumption by holding bonds, that is $s = 1$, the loss function collapses to one in Erceg et al. (2000) as well.

Scenario 2: Segmented labor market with flexible wage case

When the economy approaches to a flexible wages, $\theta_w \to 0$ and $\frac{\varepsilon_w^j}{\kappa_w^j} \to 0$, welfare losses from wage dispersion become negligible and hence terms associated with wage inflations disappear. However, presence of Rule-of-Thumb agents (low-skilled workers) still matters for welfare loss because LAMP imposes different marginal rate of substitution across different skilled workers, wages, and consumption, in turn, which causes consumption and income inequality variation. In addition, since marginal cost (real wage gap in baseline model) become a proportional to output gap, inflation is directly affected by output fluctuation via (4.9). Therefore, the trade-off explained above takes place, and a Central Bank need to allow output gap variations in the face of changes in inequality. As a special case, when production function is Cobb-Douglas ($\eta = 1$), income inequality becomes constant, and thus, a Central Bank consider wage premium rather than income inequality. Therefore, an idiosyncratic productivity shock causes a trade-off between output gap and wage premium. In this case, the larger steady state high-skilled income share implies the more weight on wage premium variation.

Scenario 3: Single labor market with flexible wages

If there is a single labor market and the wages are flexible, the loss function is exactly the same as the standard New Keynesian model with LAMP such as Bilbiie (2008). In this case, welfare loss comes from price and output variation only. The existence of low-skilled workers who are financially constrained suggest a Central Bank to put more weight on output relative to standard NK model, since these workers are affected directly by output fluctuation but not inflation. If there is no cost-push shock, then a Central Bank can impose zero inflation and output gap by strict inflation targeting and the economy achieves the first best allocation.

Welfare analysis

As I expected, the welfare losses under the optimal monetary policy when a Central Bank ignores (or is not aware of) inequality are much greater than that of the baseline model. Table 4.2 reports the relative welfare loss of the first scenario (single labor market) in comparison to the baseline model after aggregate and idiosyncratic productivity shocks as well as sectoral wage markup shocks. If a Central Bank ignores inequality by focusing only on the aggregate variables, welfare loss is 0.87% higher than the baseline model after aggregate technology shock. Since aggregate shock does not cause trade-off with inequality, the extra losses come only from the stickier aggregate nominal wage adjustment caused by income inequality. As Figure C.10 shows, aggregate dynamics of endogenous variables under different policies are not distinguishable after aggregation technology shock; they are different only if the Central Bank thinks differently on the wage stickiness. However, the optimal monetary policy induces significantly larger welfare loss in response to idiosyncratic shocks when a central bank do not care of inequality variation. The welfare losses are 5.19% and 5.16% greater than the benchmark model after positive high-skilled and low-skilled productivity shock respectively. Similarly, the loss are 1.8% and 1% larger after high-skilled and low-skilled markup shock respectively.

Table 4.2: Relative Welfare Losses

Scenario	Productivity shock			Markup shock	
	Aggregate	High-skilled	Low-skilled	High-skilled	Low-skilled
Baseline	1	1	1	1	1
Single labor market	1.000873	1.051882	1.051621	1.017812	1.010268

4.8 Conclusion

As data indicate, households with different characteristics behave very differently over the business cycle and income inequality moves countercyclically in response to monetary policy shock. In this paper, I have shown that segmented labor markets with limited asset market participation can account for the differentials in labor market variables and the dynamics of inequality. Any economic shock that affects the relative wage results in a variation in income inequality and the change in inequality amplifies aggregate dynamics through strategic complementarities in wage setting. In particular, a contractionary monetary policy, which has a disproportionate effect on labor market variables, lowers the wage premium and thus raises income inequality. A variation in income inequality enhances the stickiness of aggregate wage adjustments and leads to greater fluctuations in macroeconomic variables such as output, employment, and unemployment. Welfare analysis based on the central bank's loss function, which is obtained from the weighted average of households' life time utility, suggests that a Central Bank needs to react more aggressively to an output gap relative to the standard Taylor rule. In addition, when a Central Bank ignores heterogeneity in the labor market and thus inequality, its optimal monetary policy causes substantive welfare losses relative to those under the benchmark model in which a Central Bank takes into account inequality variation.

Finally, according to Reis (2013), "if financial stability is to be included as a separate goal for the Central Bank, it must pass certain tests: 1) there must be a measurable definition of financial stability, 2) there has to be a convincing case that monetary policy can achieve the target of bringing about a more stable financial system, and 3) financial stability must pose a trade-off with the other two goals, creating situations where prices and activity are stable but financial instability justifies a change in policy that potentially leads to a recession or causes inflation to exceed its target." Even though, financial stability might not be an appropriate target variable for a Central Bank as Reis (2013) mentioned, income inequality fulfills those three criteria. Income inequality is measurable, causes substantive welfare loss, and poses a trade-off with output gap. Thus, a Central Bank should pay attention to income inequality in addition to inflation and the output gap.

There are some useful extensions of the model. First, a fraction of the population who is excluded from the financial markets can vary over the business cycle. In a recession, more people will have limited access to the financial market, and, therefore, low skilled unemployment becomes more vulnerable to an economic shock. Accordingly, variations in income inequality will be larger, and more attention to inequality by a Central Bank will therefore be required. Second, even though labor income is the largest contributor to total income for households, other income sources may also affect households' behavior. For example, an expansionary monetary policy that lowers nominal interest rate raises asset prices. However, since high-skilled workers whose average labor income is larger than low-skilled workers tend to hold more financial assets, the income gap between two different types of households will be widened. In addition, when nominal interest rates hit the zero lower bound, an unconventional monetary policy intended to boost the economy may cause wider income inequality as shown in Saiki and Frost (2014). Therefore, other income sources such as capital income might have a significant impact that mitigates the positive effect of expansionary monetary policy on inequality and the result would enhance the portfolio channel of the monetary policy.

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Appendix A

Aggregate implications of segmented labor market

A.1 Linearized System of Equations

- $y_t = E_t y_{t+1} - \frac{1}{\sigma} \left\{ i_t - E_t \pi_{t+1}^p \right\}$
- $\pi_t^p = \beta E_t \left\{ \pi_{t+1}^p \right\} + \lambda_p mc_t$
- $mc_t = \omega_t - \frac{1}{1-\alpha} a_t + \frac{\alpha}{1-\alpha} y_t$
- $\omega_t = \omega_t^H - \omega_t^L$
- $\omega_t^H = \omega_{t-1}^H + \pi_t^H - \pi_t^p$
- $\omega_t^L = \omega_{t-1}^L + \pi_t^L - \pi_t^p$
- $\pi_t^H = \beta E_t \left\{ \pi_{t+1}^H \right\} - \frac{\Theta}{\epsilon_w^H + \varphi_H} u_t^H$
- $\pi_t^L = \beta E_t \left\{ \pi_{t+1}^L \right\} - \frac{\Theta}{\epsilon_w^L + \varphi_L} u_t^L$
- $u_t^H = \varphi_H \mu_t^H$
- $u_t^L = \varphi_L \mu_t^L$
- $\mu_t^H = \omega_t^H - \sigma y_t - \frac{1}{\varphi_H} n_t^H - \xi_t^H$
- $\mu_t^L = \omega_t^L - \sigma y_t - \frac{1}{\varphi_L} n_t^L - \xi_t^L$
- $n_t^H = -\eta(1-\gamma) \left(\omega_t^H - \omega_t^L \right) + \frac{1}{1-\alpha} \left(y_t - a_t \right)$
- $n_t^L = \eta\gamma \left(\omega_t^H - \omega_t^L \right) + \frac{1}{1-\alpha} \left(y_t - a_t \right)$
- $i_t = \rho_i i_{t-1} + (1-\rho_i) \left\{ \phi_\pi \pi_t^p + \phi_y y_t \right\} + \nu_t$
- $a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_t^a$

- $\xi_t^H = \rho_H \xi_{t-1}^H + \sigma_H \varepsilon_t^H$
- $\xi_t^L = \rho_L \xi_{t-1}^L + \sigma_L \varepsilon_t^L$
- $\nu_t = \sigma_\nu \varepsilon_t^R$

A.2 IRFs with the same labor supply elasticity, $\varphi_H = \varphi_L$

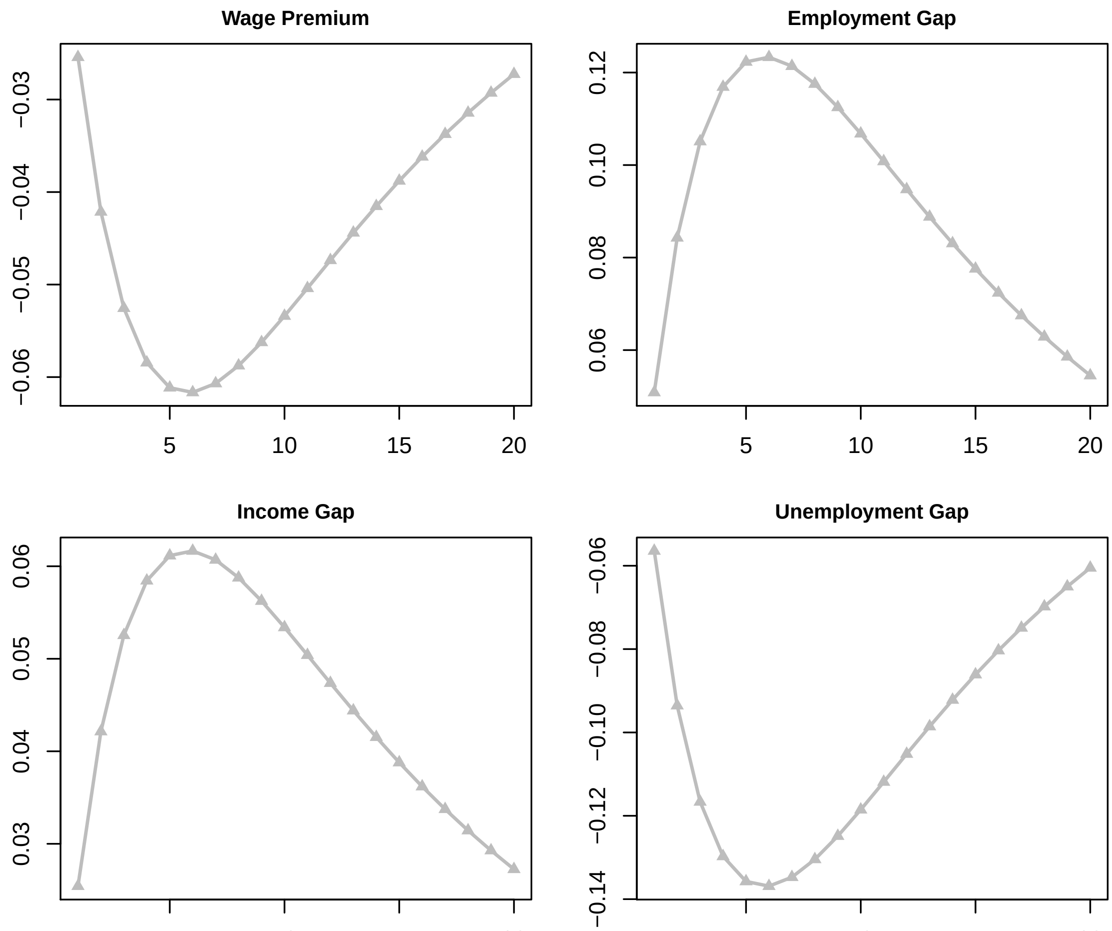


Figure A.1: Aggregate Impulse Responses to productivity shock

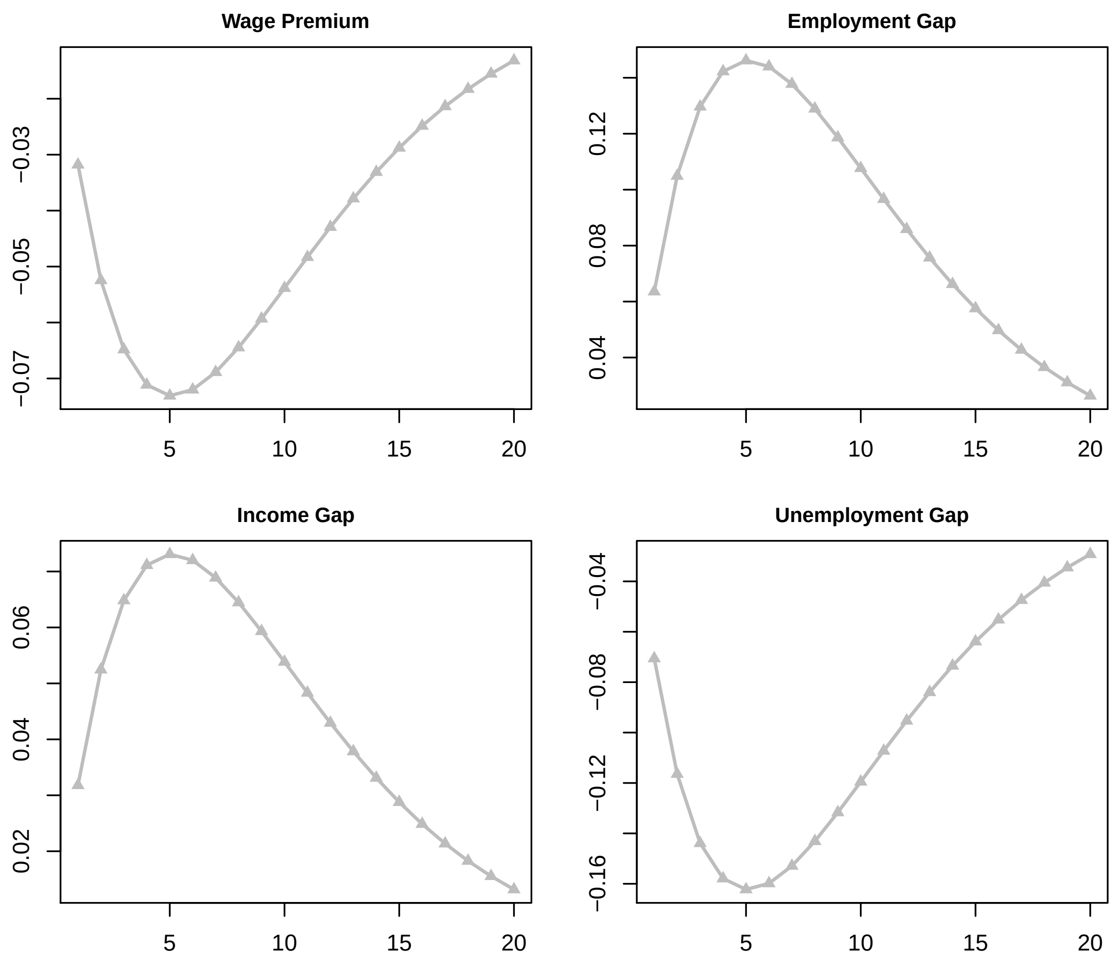


Figure A.2: Aggregate Impulse Responses to Monetary policy shock

A.3 Segmented vs. Single Labor Market

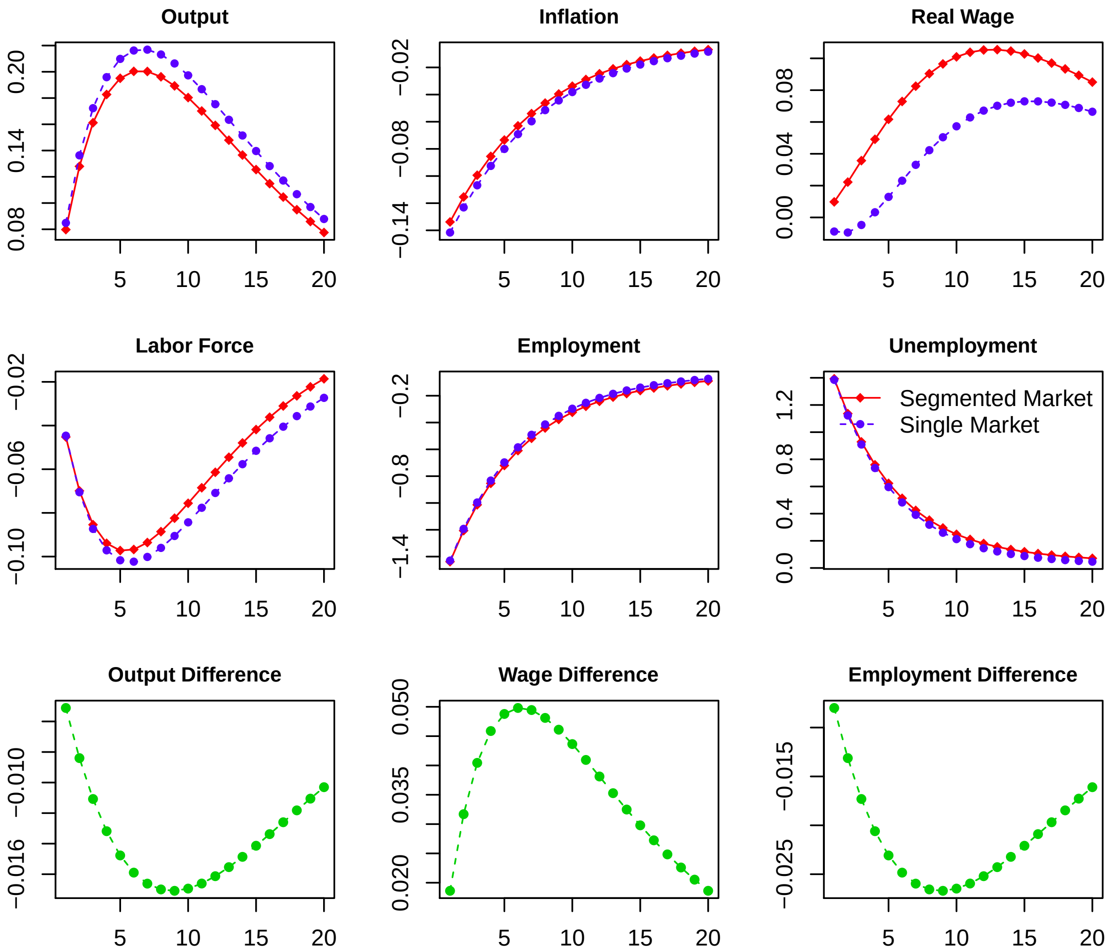


Figure A.3: Aggregate Impulse Responses to productivity shock

Appendix B

Segmented labor markets in an estimated NK model

B.1 Derivations

B.1.1 Consumption

Cost Minimization problem given consumption level Z_t

$$\pounds = \left(\int_0^1 C_t^{\frac{\varepsilon_p - 1}{\varepsilon_p}}(z)dz\right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}} + \lambda\left\{Z_t - \int_0^1 P_t(z)C_t(z)dz\right\}$$

First order condition of the problem is given by

$$\left(\frac{\varepsilon_p}{\varepsilon_p - 1}\right) C_t^{\frac{1}{\varepsilon_p}} \left(\frac{\varepsilon_p - 1}{\varepsilon_p}\right) C_t^{-\frac{1}{\varepsilon_t}}(z) = \left(\frac{C_t(z)}{C_t}\right)^{-\frac{1}{\varepsilon_p}} = \lambda P_t(z)$$

Using symmetry between differentiated goods, it follows

$$C_t(z) = \left(\frac{P_t(z)}{P_t(x)}\right)^{-\varepsilon_p} C_t(x)$$

$$\Rightarrow \int_0^1 P_t(z)C_t(z)dz = Z_t = \int_0^1 P_t^{1-\varepsilon_p}(z) \cdot P_t^{\varepsilon_p}(x)C_t(x)$$

$$\Rightarrow C_t(x) = \frac{Z_t}{P_t^{1-\varepsilon_p}} \cdot P_t^{-\varepsilon_p}(x) \tag{B.1}$$

Now, from the definition of aggregate consumption,

$$C_t = \left(\int_0^1 C_t^{\frac{\varepsilon_p - 1}{\varepsilon_p}}(x)dz\right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}} = \frac{Z_t}{P_t^{1-\varepsilon_p}}\left(\int_0^1 P_t^{-\varepsilon_p \frac{\varepsilon_p - 1}{\varepsilon_p}}(x)dx\right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}} = \frac{Z_t}{P_t^{1-\varepsilon_p}} P_t^{-\varepsilon_t}$$

$$\Rightarrow Z_t = \int_0^1 P_t(z)C_t(z)dz = P_t C_t \tag{B.2}$$

Finally, we get $C_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon_p} C_t$ combining (B.1) and (B.2).

B.1.2 Labor demand and supply

Labor Demand Schedule

$$\min_{N_t^H, N_t^L} W_t^H N_t^H + W_t^L N_t^L \quad \text{s.t. } H_t \geq \left[\gamma_H^{\frac{1}{\eta}} \left(A_t^H N_t^H\right)^{\frac{\eta-1}{\eta}} + \gamma_L^{\frac{1}{\eta}} \left(A_t^L N_t^L\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

$$\text{F.O.C:} \quad \varpi_t^j \equiv \frac{W_t^j}{A_t^j} = \lambda_t \left\{\gamma_j^{\frac{1}{\eta}} \left(\frac{H_t}{A_t^j N_t^j}\right)^{\frac{1}{\eta}}\right\} \quad \text{for } j \in \{H, L\}$$

Convex combination of the wages after taking $1-\eta$ power and using the definition of H_t, I get

$$\left[\gamma_H \left(\varpi_t^H\right)^{1-\eta} + \gamma_L \left(\varpi_t^L\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} = \left[\gamma_H \left(\frac{W_t^H}{A_t^H}\right)^{1-\eta} + \gamma_L \left(\frac{W_t^L}{A_t^L}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} = \lambda_t \equiv \varpi_t$$

replace λ_t in F.O.C

$$\begin{aligned}
N_t^j = \gamma_j \left(\frac{\varpi_t^j}{\varpi_t}\right)^{-\eta} \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \frac{1}{A_t^j} &\Rightarrow n_t^j \approx -\eta \left(\widehat{\varpi}_t^j - \widehat{\varpi}_t\right) + \frac{y_t - a_t}{1-\alpha} - a_t^j \\
&\Rightarrow n_t^H = -\eta \left(1-s\right) \left(\omega_t^R - a_t^R\right) + \frac{y_t - a_t}{1-\alpha} - a_t^H \\
&\Rightarrow n_t^H - n_t^L \equiv n_t^R = -\eta \left(\omega_t^R - a_t^R\right) - a_t^R = -\eta \omega_t^R + \left(\eta - 1\right) a_t^R
\end{aligned}$$

Marginal Cost

$$\min_{N_t^H, N_t^L} W_t^H N_t^H + W_t^L N_t^L \quad \text{s.t. } Y_t \geq A_t \left[\gamma_H^{\frac{1}{\eta}} \left(A_t^H N_t^H\right)^{\frac{\eta-1}{\eta}} + \gamma_L^{\frac{1}{\eta}} \left(A_t^L N_t^L\right)^{\frac{\eta-1}{\eta}}\right]^{1-\alpha}$$

$$\text{F.O.C:} \quad \varpi_t^j = \lambda_t \left(1-\alpha\right) A_t H_t^{-\alpha} \left\{\gamma^{\frac{1}{\eta}} \left(\frac{H_t}{A_t^j N_t^j}\right)^{\frac{1}{\eta}}\right\} \quad \text{for } j \in \{H, L\}$$

As is above,

$$\gamma_j \left(\varpi_t^j\right)^{1-\eta} = \lambda_t^{1-\eta} \left(\left(1-\alpha\right) A_t H_t^{-\alpha}\right)_t^{1-\eta} \left\{ \gamma_j^{\frac{1}{\eta}} \left(\frac{H_t}{A_t^j N_t^j}\right)^{\frac{1-\eta}{\eta}} \right\} \quad \text{for } \; j \in \{H, L\}$$

$$\begin{aligned}
\Rightarrow \left(\frac{\varpi_t}{(1-\alpha)A_t H_t^{-\alpha}}\right)^{1-\eta} &= \lambda_t^{1-\eta} \Rightarrow mc_t = \widehat{\varpi}_t - a_t + \alpha h_t = \widehat{\varpi}_t - a_t + \frac{\alpha}{1-\alpha}\left(y_t - a_t\right) \\
&\Rightarrow \underbrace{s w_t^H + (1-s) w_t^L}_{\equiv \; w_t} - \underbrace{s a_t^H + (1-s) a_t^L}_{\equiv \; a_t^c} - \frac{a_t - \alpha y_t}{1-\alpha} \\
&= \omega_t - a_t^c - \frac{1}{1-\alpha} a_t + \frac{\alpha}{1-\alpha} y_t
\end{aligned}$$

which is obtained by a convex combination with weight of population share.

Aggregate Labor Supply

$$\begin{aligned}
N_t^H &\equiv \int_0^1 \int_0^1 N_t^H(i,z) di dz \\
&= \int_0^1 N_t^H(z) \int_0^1 \frac{N_t^H(i,z)}{N_t^H(z)} di dz \quad = \int_0^1 N_t^H(z) \underbrace{\int_0^1 \left(\frac{\varpi_t^H(i)}{\varpi_t^H}\right)^{-\varepsilon_t^H} di}_{\equiv \Delta_t^H} dz \\
&= \Delta_t^H \int_0^1 \frac{H_t(z)}{A_t^H} \left(\frac{N_t^H(z)}{H_t(z)}\right) dz \quad = \quad \Delta_t^H \int_0^1 \frac{H_t(z)}{A_t^H} \gamma \left(\frac{\varpi_t^H}{\varpi_t}\right)^{-\eta} dz \\
&= \gamma \Delta_t^H \left(\frac{\varpi_t^H}{\varpi_t}\right)^{-\eta} \frac{1}{A_t^H} \int_0^1 \left(\frac{Y_t(z)}{A_t}\right)^{\frac{1}{1-\alpha}} dz \quad = \gamma \Delta_t^H \left(\frac{\varpi_t^H}{\varpi_t}\right)^{-\eta} \frac{1}{A_t^H} \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \underbrace{\int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\frac{\varepsilon_p}{1-\alpha}} dz}_{\equiv \; \Delta_t^P} \\
&= \gamma \Delta_t^H \Delta_t^P \left(\frac{\varpi_t^H}{\varpi_t}\right)^{-\eta} \frac{1}{A_t^H} \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}}
\end{aligned}$$

Similarly, $N_t^L = (1-\gamma)\Delta_t^L \Delta_t^P \left(\dfrac{\varpi_t^L}{\varpi_t}\right)^{-\eta} \frac{1}{A_t^L} \left(\dfrac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}}$ where $\Delta_t^L \equiv \int_0^1 \left(\frac{\varpi_t^L(i)}{\varpi_t^L}\right)^{-\varepsilon_t^L} di$

Then, log-linearized from of these two equations can be derived as:

$$
\begin{aligned}
n_t^H &\approx -\eta\left(w_t^H - w_t\right) + \frac{1}{1-\alpha}\left(y_t - a_t\right) - a_t^H \\
&= -\eta\left(\hat{\varpi}_t^H - \left(s\hat{\varpi}_t^H - (1-s)\hat{\varpi}_t^L\right)\right) + \frac{1}{1-\alpha}\left(y_t - a_t\right) - a_t^H \\
&= -\eta\left(1-s\right)\underbrace{\left(\hat{\varpi}_t^H - \hat{\varpi}_t^L\right)}_{\equiv\ \hat{\varpi}_t^R} + \frac{1}{1-\alpha}\left(y_t - a_t\right) - a_t^H \\
&= -\eta\left(1-s\right)\left(\omega^R - a^H + a^L\right) + \frac{1}{1-\alpha}\left(y_t - a_t\right) - a_t^L \\
n_t^L &\approx \eta s\left(\omega^R - a^H + a^L\right) + \frac{1}{1-\alpha}\left(y_t - a_t\right) - a_t^L
\end{aligned}
$$

B.1.3 Optimal Pricing and the price NKPC

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta_p^k E_t \left\{ Q_{t,t+k} \left(P_t^* \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\iota_p} \right) Y_{t+k|t} - \Psi_{t+k}\left(Y_{t+k|t}\right) \right\} \tag{B.3}$$

$$s.t. Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\iota_p} \right)^{-\epsilon_{p,t}} C_{t+k}$$

F.O.C; $$\sum_{k=0}^{\infty} \theta_p^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \left(P_t^* \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\iota_p} - \mathcal{M}_{t+k}^p \psi_{t+k|t} \right) \right\} = 0$$

$$\Rightarrow \sum_{k=0}^{\infty} \theta_p^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} P_t^* \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\iota_p} \right\} = \sum_{k=0}^{\infty} \theta_p^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \mathcal{M}_{t+k}^p \psi_{t+k|t} \right\}$$

$$\text{where} \quad \mathcal{M}_{t+k}^p \equiv \frac{\epsilon_{p,t+k}}{\epsilon_{p,t+k} - 1} \quad \text{and} \quad \psi_{t+k|t} = \Psi'_{t+k}\left(Y_{t+k|t}\right)$$

Log-linearization;

LHS; $$\sum_{k=0}^{\infty} (\beta\theta_p)^k YP \left(\frac{P}{P} \right)^{\iota_p} E_t \left\{ \widehat{q_{t,t+k}} + \widehat{y_{t+k|t}} + \widehat{p_t^*} + \iota_p \left(\widehat{p_{t+k-1}} - \widehat{p_{t-1}} \right) \right\}$$

RHS; $$\sum_{k=0}^{\infty} (\beta\theta_p)^k YP \left(\frac{P}{P} \right)^{\iota_p} E_t \left\{ \widehat{q_{t,t+k}} + \widehat{y_{t+k|t}} + \widehat{\mu_{t+k}^{np}} + \widehat{mc_{t+k|t}} + \widehat{p_{t+k}} \right\} \left(\because \psi_{t+k|t} = MC_{t+k|t} P_{t+k} \right)$$

Rearrange (LHS - RHS = 0);

$$\sum_{k=0}^{\infty} (\beta\theta_p)^k E_t \left\{ \widehat{p_t^*} + \iota_p \left(\widehat{p_{t+k-1}} - \widehat{p_{t-1}} \right) - \widehat{\mu_{t+k}^{np}} - \widehat{mc_{t+k|t}} - \widehat{p_{t+k}} \right\} = 0$$

$$\Rightarrow \sum_{k=0}^{\infty} (\beta\theta_p)^k E_t \left\{ \underbrace{\left(\frac{1-\alpha+\alpha\epsilon_p}{1-\alpha} \right)}_{\equiv \Gamma} \left(\widehat{p_t^*} + \iota_p \left(\widehat{p_{t+k-1}} - \widehat{p_{t-1}} \right) - \widehat{p_{t+k}} \right) - \widehat{\mu_{t+k}^{np}} - \widehat{mc_{t+k}} \right\} = 0$$

$$\left(\because \widehat{mc_{t+k|t}} = \widehat{mc_{t+k}} + \frac{\alpha}{1-\alpha} \left(\widehat{y_{t+k|t}} - \widehat{y_{t+k}} \right) = \widehat{mc_{t+k}} - \frac{\alpha\epsilon_p}{1-\alpha} \left(\widehat{p_t^*} - \widehat{p_{t+k}} + \iota_p \left(\widehat{p_{t+k-1}} - \widehat{p_{t-1}} \right) \right) \right)$$

$$\Rightarrow \widehat{p_t^*} - \iota_p \widehat{p_{t-1}} = (1-\beta\theta_p)\left(\widehat{p_t} - \iota_p \widehat{p_{t-1}} \right) + \frac{1-\beta\theta}{\Gamma} \left(\widehat{mc_t} + \widehat{\mu_t^{np}} \right) + \beta\theta_p E_t \left(\widehat{p_{t+1}^*} - \iota_p \widehat{p_t} \right)$$

Log-linearizing price index, $(P_t)^{1-\epsilon_{p,t}} \equiv (1-\theta_p)(P_t^*)^{(1-\epsilon_{p,t})} + \theta_p \left(P_{t-1} \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\iota_p} \right)^{1-\epsilon_{p,t}}$, we get $\widehat{p_t^*} = \frac{1}{1-\theta_p}\widehat{p}_t - \frac{\theta_p}{1-\theta_p}\widehat{p}_{t-1} - \frac{\theta_p \iota_p}{1-\theta_p}\pi_{t-1}$. Finally, we obtain

$$\pi_t^p = \beta E_t \left\{ \pi_{t+1}^p - \iota_p \pi_t^p \right\} + \iota_p \pi_{t-1}^p - \lambda_p \left(\mu_t^p - \mu_t^{np} \right) \quad \text{where } \lambda_p = \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p \Gamma}$$

B.1.4 Wage determination and the wage NKPC

1. High-skilled workers

$$\max_{W_t^*} \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left[\left(\frac{\widetilde{C_{t+k}}}{1-\sigma} \right)^{1-\sigma} - \Theta_{t+k} \left\{ s\chi_t^H \int_0^1 \frac{N_{t+k|t}^{H^{1+\varphi_H}}}{1+\varphi_H} di + (1-s)\chi_t^L \int_0^1 \frac{N_{t+k|t}^{L^{1+\varphi_L}}}{1+\varphi_L} di \right\} \right]$$

$$s.t. \quad N_{t+k|t}^H = \left(\frac{W_t^{H*}}{W_{t+k}^H} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\iota_w} \right)^{-\epsilon_{w,t}^H} N_{t+k}^H$$

$$P_{t+k}C_{t+k} + e^{\varepsilon_{t+k}^b} Q_{t+k}B_{t+k} = B_{t+k-1} + s \int_0^1 W_t^{*H} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\iota_w} N_{t+k|t}^H di + (1-s) W_{t+k}^L N_{t+k}^L$$

F.O.C;

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left[\widetilde{C}_{t+k}^{-\sigma} N_{t+k|t}^H \left\{ \frac{W_t^{H*}}{P_{t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\iota_w} - \mathcal{M}_t^H MRS_{t+k|t}^H \right\} \right] = 0$$

$$\text{where} \quad MRS_{t+k|t}^H = \chi_t^H Z_{t+k}^{\sigma} N_{t+K|t}^{H\varphi_H} \quad \text{and} \quad \mathcal{M}_t^H = \frac{\epsilon_{w,t}^H}{\epsilon_{w,t}^H - 1}$$

Log-linearization;

$$\text{LHS;} \quad \sum_{k=0}^{\infty} (\beta\theta_w)^k C^{-\sigma} N \frac{W}{P} \left(\frac{P}{P} \right)^{\iota_w} E_t \left\{ -\sigma \widehat{\widetilde{c}_{t+k}} + \widehat{n_{t+k|t}} + \widehat{w_t^{H*}} - \widehat{p_{t+k}} + \iota_p \left(\widehat{p_{t+k-1}} - \widehat{p_{t-1}} \right) \right\}$$

$$\text{RHS;} \quad \sum_{k=0}^{\infty} (\beta\theta_w)^k C^{-\sigma} N M^H MRS^H E_t \left\{ -\sigma \widehat{\widetilde{c}_{t+k}} + \widehat{n_{t+k|t}} + \widehat{\mu_{t+k}^{nw}} + \widehat{mrs_{t+k|t}^H} \right\}$$

Rearrange (LHS - RHS = 0, $\frac{W}{P} = M^H MRS^H$ at steady state);

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ \widehat{w_t^{H*}} - \widehat{p_{t+k}} + \iota_w \left(\widehat{p_{t+k-1}} - \widehat{p_{t-1}} \right) - \widehat{\mu_{t+k}^{nH}} - \widehat{mrs_{t+k|t}^H} \right\} = 0$$

$$\Rightarrow \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ \left(1 + \varepsilon_w^H \varphi_H\right) \left(\widehat{w_t^{H*}} + \iota_w \left(\widehat{p_{t+k-1}} - \widehat{p_{t-1}} \right) \right) - \widehat{p_{t+k}} - \varepsilon_w^H \varphi_H \widehat{w_{t+k}} - \widehat{\mu_{t+k}^{nH}} - \widehat{mrs_{t+k}^H} \right\} = 0$$

$$\left(\because \widehat{mrs_{t+k|t}^H} = \widehat{mrs_{t+k}^H} - \varphi_H \left(\widehat{n_{t+k|t}} - \widehat{n_{t+k}} \right) = \widehat{mrs_{t+k}^H} - (\varepsilon_w \varphi_H) \left(\widehat{w_t^{H*}} - \widehat{w_{t+k}} + \iota_w \left(\widehat{p_{t+k-1}} - \widehat{p_{t-1}} \right) \right) \right)$$

$$\Rightarrow \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ \left(1 + \varepsilon_w^H \varphi_H\right) \left(\widehat{w_t^{H*}} + \iota_w \left(\widehat{p_{t+k-1}} - \widehat{p_{t-1}} \right) \right) + \mu_t^H - \left(1 + \varepsilon_w^H \varphi_H\right) \widehat{w_{t+k}} - \widehat{\mu_{t+k}^{nH}} \right\} = 0$$

$$\left(\text{where} \quad \mu_t^H \equiv \widehat{w_{t+k}} - \widehat{p_{t+k}} - \widehat{mrs_{t+k}^H}\right)$$

$$\Rightarrow \widehat{w_t^{H*}} - \iota_p \widehat{p_{t-1}} = (1 - \beta\theta_p) \left(\widehat{w_t} - \iota_p \widehat{p_{t-1}} \right) + \frac{1 - \beta\theta}{1 + \varepsilon_w \varphi_H} \left(\widehat{\mu_t^H} - \widehat{\mu_t^{nH}} \right) + \beta\theta_p E_t \left(\widehat{w_{t+1}^{H*}} - \iota_p \widehat{p}_t \right)$$

Log-linearizing price index, $\left(W_t^H\right)^{1-\epsilon_{w,t}^H} \equiv (1-\theta_w)\left(W_t^{H*}\right)^{(1-\epsilon_{w,t}^H)} + \theta_w \left(W_{t-1}^H \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\iota_w}\right)^{1-\epsilon_{w,t}^H}$, we get $\widehat{w_t^{H*}} = \frac{1}{1-\theta_w}\widehat{w}_t - \frac{\theta_w}{1-\theta_p}\widehat{w}_{t-1} - \frac{\theta_w \iota_w}{1-\theta_w}\pi_{t-1}$. Finally, we obtain

$$\pi_t^H = \beta E_t \left\{\pi_{t+1}^H - \iota_w \pi_t^p\right\} + \iota_w \pi_{t-1}^p - \lambda_H \left(\mu_t^H - \mu_t^{nH}\right) \quad \text{where } \lambda_H = \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p\left(1+\varepsilon_w^H \varphi_H\right)}$$

Similarly, we can obtain New Keynesian wage Phillips curve for the low-skilled workers as:

$$\pi_t^L = \beta E_t \left\{\pi_{t+1}^L - \iota_w \pi_t^p\right\} + \iota_w \pi_{t-1}^p - \lambda_L \left(\mu_t^L - \mu_t^{nL}\right) \quad \text{where } \lambda_L = \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p\left(1+\varepsilon_w^L \varphi_L\right)}$$

B.2 A system of Linearized equations

Exogenous shocks

1. $a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_t^a$: Aggregate technology shock
2. $a_t^H = \rho_a^H a_{t-1}^H + \sigma_a^H \varepsilon_{a,t}^H$: High-skilled productivity shock
3. $a_t^L = \rho_a^L a_{t-1}^L + \sigma_a^L \varepsilon_{a,t}^L$: Low-skilled productivity shock
4. $b_t = \rho_b b_{t-1} + \sigma_b \varepsilon_t^b$: Risk premium shock
5. $\xi_t = \rho_\xi \xi_{t-1} + \sigma_\xi \varepsilon_t^\xi$: Aggregate labor supply shock
6. $\mu_{p,t}^n = \rho_p \mu_{p,t-1}^n + \sigma_p \varepsilon_{p,t}$: Price markup shock
7. $\mu_{w,t}^{nH} = \rho_w^H \mu_{w,t-1}^{nH} + \sigma_w^H \varepsilon_{w,t}^H$: High-skilled wage markup shock
8. $\mu_{w,t}^{nL} = \rho_w^L \mu_{w,t-1}^{nL} + \sigma_w^L \varepsilon_{w,t}^L$: Low-skilled wage markup shock
9. $\nu_t = \sigma_R \varepsilon_t^R$: Monetary policy shock

Structural Equations

1. $y_t = \gamma c_t^H + (1-\gamma) c_t^L$
2. $c_t^H = \frac{1}{1+\rho_c} E_t c_{t+1}^H + \frac{\rho_c}{1+\rho_c} c_{t-1}^H - \frac{1-\rho_c}{\sigma(1+\rho_c)}(i_t - E_t \pi_{t+1}^p - b_t) - \frac{\phi s(1-\rho_c)}{\beta\sigma(1+\rho_c)}\left(\hat{\mathcal{X}}_t^H - y_t\right)$

3. $c_t^L = \frac{1}{1+\rho_c} E_t c_{t+1}^L + \frac{\rho_c}{1+\rho_c} c_{t-1}^L - \frac{1-\rho_c}{\sigma(1+\rho_c)} (i_t - E_t \pi_{t+1}^p - b_t) - \frac{\phi(1-s)(1-\rho_c)}{\beta\sigma(1+\rho_c)} \left(\hat{\mathcal{X}}_t^L - y_t\right)$

4. $\pi_t^p = \iota_p \pi_{t-1}^p + \beta E_t \left(\pi_{t+1}^p - \iota_p \pi_t^p\right) - \lambda_p \mu_t^p + \mu_{p,t}^n$

5. $\mu_t^p = \frac{a_t}{1-\alpha} - \frac{\alpha}{1-\alpha} y_t - \omega_t + s a_t^H + (1-s) a_t^L$

6. $\pi_t^H = \iota_H \pi_{t-1}^p + \beta E_t \left(\pi_{t+1}^H - \iota_H \pi_t^p\right) - \lambda_H \mu_{w,t}^H + \mu_{w,t}^{nH}$

7. $\pi_t^L = \iota_L \pi_{t-1}^p + \beta E_t \left(\pi_{t+1}^L - \iota_L \pi_t^p\right) - \lambda_L \mu_{w,t}^L + \mu_{w,t}^{nL}$

8. $\mu_{w,t}^H = \omega_t^H - \sigma c_t^H - \varphi_H n_t^H - \xi_t$

9. $\mu_{w,t}^L = \omega_t^L - \sigma c_t^L - \varphi_L n_t^L - \xi_t$

10. $n_t^H = -\eta(1-s)\left(\omega_t^R - a_t^R\right) + \frac{y_t - a_t}{1-\alpha} - a_t^H$

11. $n_t^L = \eta s \left(\omega_t^R - a_t^R\right) + \frac{y_t - a_t}{1-\alpha} - a_t^L$

12. $\varphi_H u_t^H = \mu_{w,t}^H$

13. $\varphi_L u_t^L = \mu_{w,t}^L$

14. $\omega_t^H = \omega_{t-1}^H + \pi_t^H - \pi_t^p$

15. $\omega_t^L = \omega_{t-1}^L + \pi_t^L - \pi_t^p$

16. $\omega_t = s\omega_t^H + (1-s)\omega_t^L$

17. $\omega_t^R = \omega_t^H - \omega_t^L$

18. $i_t = \rho_i i_{t-1} + (1-\rho_i)\left(\phi_\pi \pi_t^p + \phi_y y_t\right) + \hat{\nu}_t$

Appendix C

Inequality and optimal monetary policy

C.1 Efficient Equilibrium

When wage markups are gone, both labor markets become identical, and all the workers get the same wages, and hence, enjoy the same level of consumption as if representative agents. Thus, the marginal product of labor and marginal rate of substitution for workers of different skill levels are the same. I then define efficient equilibrium condition as:

$$\begin{aligned} mpn_t &= \omega_t = mrs_t \\ a_t &= \sigma y_t + \varphi h_t + \xi_t = (\sigma + \varphi) y_t - \varphi a_t + \xi_t \\ y_t^E &= \frac{1+\varphi}{\sigma+\varphi} a_t - \frac{1}{\sigma+\varphi} \xi_t \quad \text{and} \quad \omega_t^E = a_t \end{aligned} \tag{C.1}$$

C.2 Household's total labor income

Each household consists of a continuum of workers, and the total income of the household is just the sum of each worker's labor income. Therefore, j-skilled household's total income is written by

$$
\begin{aligned}
\mathcal{X}_t^j &= \int_0^1 \omega_t^j(i) \int_0^1 N^j(i,z) dz di = \int_0^1 \omega_t^j(i) \int_0^1 N_t^j(z) \frac{N_t^j(i,z)}{N_t^j(z)} dz di \\
&= \int_0^1 \omega_t^j(i) \int_0^1 N_t^j(z) \left(\frac{\omega_t^j(i)}{\omega_t^j} \right)^{-\varepsilon_w^j} dz di = \omega_t^j \underbrace{\int_0^1 \left(\frac{\omega_t^j(i)}{\omega_t^j} \right)^{1-\varepsilon_w^j} di}_{=\ 1} \int_0^1 N_t^j(z) dz \\
&= \omega_t^j \int_0^1 H_t(z) \left(\frac{N_t^j(z)}{H_t(z)} \right) dz = \omega_t^j \int_0^1 H_t(z) \gamma \left(\frac{\omega_t^j}{\omega_t} \right)^{\eta} dz \\
&= \gamma \omega_t^j \left(\frac{\omega_t^j}{\omega_t} \right)^{-\eta} \frac{Y_t}{A_t} \underbrace{\int_0^1 \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon_p} dz}_{\Delta_t^p} \\
&\approx \omega_t^j - \eta \left(\omega_t^j - \omega_t \right) + y_t - a_t
\end{aligned} \tag{C.2}
$$

for $j \in \{H, L\}$ and where Δ_t^p is measure for the price dispersion and is second order term. Since the Gini coefficient for income $\mathcal{G}_t^I$ is given by $(1-s)\left(1 - \frac{\mathcal{X}_t^L}{\mathcal{X}_t}\right)$, it is approximated as $-(1-s)\left(\widehat{\mathcal{X}}_t^j - \widehat{\mathcal{X}}_t\right)$:

$$
\begin{aligned}
& -(1-s)\left(\widehat{\mathcal{X}}_t^j - \widehat{\mathcal{X}}_t\right) \\
= & -(1-s)\left[\omega_t^L + \eta s \omega_t^R + y_t - a_t - \left\{ s\left(\omega_t^H - \eta(1-s)\omega_t^R + y_t - a_t\right) + (1-s)\left(\omega_t^L + \eta s \omega_t^R + y_t - a_t\right)\right\}\right] \\
= & \ -s(1-s)(\eta - 1)\,\omega_t^R
\end{aligned} \tag{C.3}
$$

C.3 Wage Phillips curve

C.3.1 High-skilled wage Phillips Curve ($\kappa_w^H \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varepsilon_w^H\varphi)}$)

$$\begin{aligned}
\pi_t^H &= \beta E_t \pi_{t+1}^H - \kappa_w^H \mu_t^H + \mu_t^{Hn} \\
\Rightarrow \mu_t^H &= \omega_t^H - \sigma c_t^H - \varphi n_t^H - \xi_t \\
&= \omega_t^H - \sigma\left(\frac{c_t - (1-s)c_t^L}{s}\right) - \varphi n_t^H - \xi_t = \omega_t^H - \frac{\sigma}{s}c_t + \frac{\sigma(1-s)}{s}c_t^L - \varphi n_t^H - \xi_t \\
&= \omega_t^H - \frac{\sigma}{s}c_t + \frac{\sigma(1-s)}{s}\left(\omega_t^L + n_t^L\right) - \varphi\left\{-\eta(1-s)(\omega_t^H - \omega_t^L) + y_t - a_t\right\} - \xi_t \\
&= \underbrace{(1+\eta(1-s)(\sigma+\varphi))}_{\equiv\lambda_H^H}\omega_t^H - \underbrace{\left(\eta(1-s)(\sigma+\varphi) - \frac{\sigma(1-s)}{s}\right)}_{\equiv\lambda_H^L}\omega_t^L \\
&+ \left(-\frac{\sigma}{s} + \frac{\sigma(1-s)}{s} - \varphi\right)y_t + \left(-\frac{\sigma(1-s)}{s} + \varphi\right)a_t - \xi_t \\
&= -(\sigma+\varphi)y_t + \left(-\frac{\sigma(1-s)}{s} + \varphi\right)a_t + \lambda_H^H\omega_t^H - \lambda_H^L\omega_t^L - \xi_t \\
&= -(\sigma+\varphi)x_t + \lambda_H^H\widetilde{\omega}_t^H - \lambda_H^L\widetilde{\omega}_t^L \\
\Rightarrow \pi_t^H &= \beta E_t\pi_{t+1}^H + \kappa_w^H(\sigma+\varphi)x_t - \kappa_w^H\lambda_H^H\widetilde{\omega}_t^H + \kappa_w^H\lambda_H^L\widetilde{\omega}_t^L + \mu_t^{Hn} \qquad \text{(C.4)}
\end{aligned}$$

This also can be written in terms of relative consumption and wage

$$= \beta E_t\pi_{t+1}^H + \kappa_w^H(\sigma+\varphi)x_t + \kappa_w^H\left((\sigma+\varphi)\eta s - \varphi\eta\right)\widetilde{\omega}_t^R + \kappa_w^H\sigma c_t^R - \kappa_w^H\left(\widetilde{\omega}_t^H - \sigma\widetilde{\omega}_t^L\right) + \mu_t^{Hn} \qquad \text{(C.5)}$$

C.3.2 Low-skilled wage Phillips Curve

$$
\begin{aligned}
&\pi_t^L = \beta E_t \pi_{t+1}^L - \kappa_w^L \mu_t^L + \mu_t^{Ln} \\
\Rightarrow\ &\mu_t^L = \omega_t^L - mrs_t^L = \omega_t^L - \sigma c_t^L - \varphi n_t^L - \xi_t \\
&\quad = \omega_t^L - \sigma(\omega_t^L + n_t^L) - \varphi n_t^L - \xi_t = (1-\sigma)\omega_t^L - (\sigma+\varphi) n_t^L - \xi_t \\
&\quad = (1-\sigma)\omega_t^L - (\sigma+\varphi)\left(\eta s\left(\omega_t^H - \omega_t^L\right) + y_t - a_t\right) - \xi_t \\
&\quad = -(\sigma+\varphi)\eta s \omega_t^H + \left((1-\sigma) + (\sigma+\varphi)\eta s\right)\omega_t^L - (\sigma+\varphi)(y_t - a_t) - \xi_t \\
&\quad = -\left\{\left((\sigma-1) - (\sigma+\varphi)\eta s\right)\widetilde{\omega}_t^L + (\sigma+\varphi)\eta s \widetilde{\omega}_t^H\right\} - (\sigma+\varphi) x_t \\
\Rightarrow\ &\pi_t^L = \beta E_t \pi_{t+1}^L + \kappa_w^L(\sigma+\varphi)x_t + \kappa_w^L(\sigma+\varphi)\eta s \widetilde{\omega}_t^H - \kappa_w^L\left((\sigma+\varphi)\eta s - (\sigma-1)\right)\widetilde{\omega}_t^L + \mu_t^{Ln}
\end{aligned}
\tag{C.6}
$$

$$
\text{or,}\quad \pi_t^L = \beta E_t \pi_{t+1}^L + \kappa_w^L(\sigma+\varphi)x_t + \kappa_w^L(\sigma+\varphi)\eta s \widetilde{\omega}_t^R + \kappa_w^L(\sigma-1)\widetilde{\omega}_t^L + \mu_t^{Ln} \tag{C.7}
$$

C.3.3 Aggregate wage Phillips curve

By definition, I aggregate wage Phillips is a convex combination of two sectoral wage Phillips curves weighted by corresponding population share.

$$
\begin{aligned}
\pi_t^w &= s\pi_t^H + (1-s)\pi_t^L \\
&= \beta \pi_{t+1}^w + \underbrace{\left(s\kappa_w^H + (1-s)\kappa_w^L\right)}_{\equiv\ \kappa_w}(\sigma+\varphi)\,x_t - \left(\kappa_w^H - \kappa_w^L\right)\eta s(1-s)\,(\sigma+\varphi)\,\widetilde{\omega}_t^R \\
&\quad - \left(\kappa_w^H - \kappa_w^L\right)(1-s)\sigma\widetilde{\omega}_t^L - \left(s\kappa_w^H\widetilde{\omega}_t^H + (1-s)\kappa_w^L\widetilde{\omega}_t^L\right) + \underbrace{\left(s\mu_t^{Hn} + (1-s)\mu_t^{Ln}\right)}_{\equiv\ \epsilon_t} \\
\text{(and if } \sigma = 1\text{)}\quad &= \beta\pi_{t+1}^w + \kappa_w\,(1+\varphi)\,x_t + \underbrace{\left(\kappa_w^H - \kappa_w^L\right)\frac{\eta\,(1+\varphi)}{\eta-1}}_{\equiv\ \Upsilon}\mathcal{G}_t^I - \kappa_w^H\widetilde{\omega}_t + \epsilon_t
\end{aligned}
\tag{C.8}
$$

C.3.4 Special Case with log utility and homogeneous labor ($\sigma = 1$, $\varepsilon_w^H = \varepsilon_w^L$)

If $\varepsilon_w^H = \varepsilon_w^L$, (C.5) is then simplified further,

$$
\pi_t^w = \beta\pi_{t+1}^w + \kappa_w\,(1+\varphi)\,x_t - \kappa_w\widetilde{\omega}_t + \epsilon_t \tag{C.9}
$$

and subtracting (C.4) from (C.2) I obtain,

$$\widetilde{\omega}_t^R = \psi_R \widetilde{\omega}_{t-1}^R + \beta \psi_R E_t \widetilde{\omega}_{t+1}^R + \psi_R \kappa_w c_t^R + \epsilon_t^D \tag{C.10}$$

where $\psi_R \equiv \frac{1}{1+\beta+\kappa_w(1+\varphi\eta)}$ and $\epsilon_t^D \equiv \psi_R \left(\mu_t^{Hn} - \mu_t^{Ln}\right)$

C.4 Utility-based Loss Function (Woodford (2003))

$$W_t \equiv \left\{ s \left(U(C_t^H) - V(N_t^H) \right) + (1-s) \left(U(C_t^L) - V(N_t^L) \right) \right\}$$

- Efficient Steady State:

$$\begin{aligned} C(z) = C = Y \text{ and } \frac{Y}{H} = A = 1 \Rightarrow C = H \\ C = C^H = C^L, \quad H = N^H = N^L \\ \frac{W}{P} = MRS = MPN \Leftrightarrow \frac{W}{P} = C^\sigma H^\varphi = A = 1 \Rightarrow U_c = V_N \end{aligned}$$

- Utility from Consumption $\left(\text{for a variable X, } \frac{X_t - X}{X} \approx x_t + \frac{1}{2} x_t^2\right)$

$$\begin{aligned} U(C_t^j) &\approx U(C) + U_c C \left(\frac{C_t^j - C}{C} \right) + \frac{U_{cc} C^2}{2} \left(\frac{C_t^j - C}{C} \right)^2 + \mathcal{O}\left(||\zeta||\right)^3 \\ U(C_t^j) - U(C) &\approx U_c C \left\{ \left(\frac{C_t^j - C}{C} \right) + \frac{U_{cc} C}{2U_c} \left(\frac{C_t^j - C}{C} \right)^2 \right\} + \mathcal{O}\left(||\zeta||\right)^3 \\ &\approx U_c C \left\{ c_t^j + \frac{1}{2} \left(c_t^j \right)^2 - \frac{\sigma}{2} \left(c_t^j \right)^2 \right\} + \mathcal{O}\left(||\zeta||\right)^3 \end{aligned} \tag{C.11}$$

- Disutility from Labor Supply

$$
\begin{aligned}
V(N_t^j) &\approx V(N) + V_j N \left(\frac{N_t^j - N}{N} \right) + \frac{V_{jj} N^2}{2} \left(\frac{N_t^j - N}{N} \right)^2 \\
&\quad + V_\chi (\chi_t - 1) + V_{\chi N} N \left(\frac{\chi_t - 1}{1} \right) \left(\frac{N_t^j - N}{N} \right) + V_{\chi\chi} (\chi_t - 1)^2 + \mathcal{O}\left(||\zeta||^3 \right) \\
V(N_t^j) - V(N) &\approx V_j N \left\{ \left(\frac{N_t^j - N}{N} \right) + \frac{V_{jj} N}{2 V_j} \left(\frac{N_t^j - N}{N} \right)^2 + \left(\frac{N_t^j - N}{N} \right) \xi_t \right\} + t.i.p + \mathcal{O}\left(||\zeta||^3 \right) \\
&\approx V_j N \left\{ n_t^j + \frac{1}{2} \left(n_t^j \right)^2 + \frac{\varphi}{2} \left(n_t^j \right)^2 + n_t^j \xi_t \right\} + t.i.p + \mathcal{O}\left(||\zeta||^3 \right)
\end{aligned}
\tag{C.12}
$$

- Combine (C.11) and (C.12)

$$
\frac{W_t - W}{U_c C} \approx \left[\left\{ c_t + \frac{1-\sigma}{2} \left(s (c_t^H)^2 + (1-s)(c_t^L)^2 \right) \right\} - \left\{ h_t + \frac{1+\varphi}{2} \left(s (n_t^H)^2 + (1-s)(n_t^L)^2 \right) + h_t \xi_t \right\} \right]
\tag{C.13}
$$

where $U_c C = V_j N$, $c_t = s c_t^H + (1-s) c_t^L$, and $h_t = s n_t^H + (1-s) n_t^L$

- $s \left(c_t^H \right)^2 + (1-s) \left(c_t^L \right)^2$

$$
c_t^L = \omega_t^L + n_t^L = \omega_t^L + \eta s \omega_t^R + \widehat{\Delta}_t^L + \widehat{\Delta}_t^p + y_t - a_t = \widehat{\Delta}_t^L + \widehat{\Delta}_t^p + \underbrace{(1-\eta s)\omega_t^L + \eta s \omega_t^H}_{\equiv \Phi_t} + y_t - a_t
$$

$$
\left(c_t^L \right)^2 = \Phi_t^2 + y_t^2 + 2\Phi_t y_t - 2\Phi_t a_t - 2 y_t a_t + t.i.p + \mathcal{O}\left(||\zeta||^3 \right)
\tag{C.14}
$$

$$
\begin{aligned}
c_t^H &= \frac{c_t-(1-s)c_t^L}{s} = \frac{1}{s}c_t - \frac{1-s}{s}c_t^L \\
\left(c_t^H\right)^2 &= \left(\frac{1}{s}\right)^2 c_t^2 + \left(\frac{1-s}{s}\right)^2 \left(c_t^L\right)^2 - 2\left(\frac{1}{s}\frac{1-s}{s}\right) c_t c_t^L \\
&= \left(\frac{1}{s}\right)^2 c_t^2 + \left(\frac{1-s}{s}\right)^2 \left(\Phi_t^2 + y_t^2 + 2\Phi_t y_t - 2\Phi_t a_t - 2y_t a_t\right) - 2\left(\frac{1-s}{s^2}\right) c_t \left(\Phi_t + y_t - a_t\right) \\
&= \underbrace{\left(\frac{1}{s^2} + \left(\frac{1-s}{s}\right)^2 - 2\frac{1-s}{s^2}\right)}_{=1} y_t^2 + \left(\frac{1-s}{s}\right)^2 \left(\Phi_t^2 - 2\Phi_t a_t\right) \\
&+ 2\underbrace{\left(\left(\frac{1-s}{s}\right)^2 - \frac{1-s}{s^2}\right)}_{=-(1-s)/s} \left(\Phi_t y_t - y_t a_t\right)
\end{aligned}
$$

$$
\begin{aligned}
&\Rightarrow s\left(c_t^H\right)^2 + (1-s)\left(c_t^L\right)^2 \\
&\quad = y_t^2 + \left((1-s) + \frac{(1-s)^2}{s}\right)\left(\Phi_t^2 - 2\Phi_t a_t\right) + 2\left((1-s)-(1-s)\right)\left(\Phi_t y_t - y_t a_t\right) \\
&\quad = y_t^2 + \frac{1-s}{s}\left(\Phi_t^2 - 2\Phi_t a_t\right) \qquad \text{(C.15)}
\end{aligned}
$$

- $sn_t^H + (1-s)n_t^L$

$$
\begin{aligned}
&\Rightarrow s\left(\widehat{\Delta}_t^H + \widehat{\Delta}_t^p - s(1-s)\omega_t^R + y_t - a_t\right) + (1-s)\left(\widehat{\Delta}_t^L + \widehat{\Delta}_t^p + \eta s\omega_t^R + y_t - a_t\right) \\
&\quad = \underbrace{s\widehat{\Delta}_t^H + (1-s)\widehat{\Delta}_t^L}_{\equiv\widehat{\Delta}_t^w} + \widehat{\Delta}_t^p + y_t - a_t \qquad \text{(C.16)}
\end{aligned}
$$

- $s\left(n_t^H\right)^2 + (1-s)\left(n_t^L\right)^2$

$$
\begin{aligned}
&\Rightarrow s\left\{(\eta(1-s))^2\left(\omega_t^R\right)^2 + y_t^2 - 2\eta(1-s)\omega_t^R y_t + 2\eta(1-s)\omega_t^R a_t - 2y_t a_t\right\} \\
&\quad + (1-s)\left\{(\eta s)^2\left(\omega_t^R\right)^2 + y_t^2 + 2\eta s\omega_t^R y_t - 2\eta s\omega_t^R a_t - 2y_t a_t\right\} \\
&\quad = \eta^2 s(1-s)\left(\omega_t^R\right)^2 + y_t^2 - 2y_t a_t \qquad \text{(C.17)}
\end{aligned}
$$

- substituting (C.15), (C.16) and (C.17) for (C.13) we get,

$$\frac{W_t - W}{U_c C} \approx -\frac{1}{2}\left(\begin{array}{c} 2\left(\widehat{\Delta}_t^w + \widehat{\Delta}_t^p\right) + (\sigma+\varphi)\, y_t^2 - (1+\varphi)\, 2y_t a_t + 2y_t \xi_t \\ \\ +\frac{(\sigma-1)(1-s)}{s}\left(\Phi_t - a_t\right)^2 + (1+\varphi)\eta^2 s(1-s)\left(\omega_t^R\right)^2 \end{array}\right) + t.i.p + \mathcal{O}\left(||\zeta||^3\right) \tag{C.18}$$

where $(\sigma+\varphi) y_t y_t^E = ((1+\varphi)a_t - \xi_t)\, y_t$ and $\Phi_t^E = a_t$ and $\omega_t^{R,E} = 0$. Now, replacing inequality measure,(4.16) and (4.17), for the last two terms, I get

$$\frac{W_t - W}{U_c C} \approx -\frac{1}{2}\left(\begin{array}{c} 2\left(\widehat{\Delta}_t^w + \widehat{\Delta}_t^p\right) + (\sigma+\varphi)\, y_t^2 - (1+\varphi)\, 2y_t a_t + 2y_t \xi_t \\ \\ +\underbrace{\frac{(\sigma-1)}{s(1-s)}}_{\psi_c}\left(\mathcal{G}_t^c\right)^2 + \underbrace{\frac{(1+\varphi)}{s(1-s)}\left(\frac{\eta}{1-\eta}\right)^2}_{\psi_I}\left(\mathcal{G}_t^I\right)^2 \end{array}\right) + t.i.p + \mathcal{O}\left(||\zeta||^3\right) \tag{C.19}$$

In addition, $\widehat{\Delta}_t^P \approx \frac{\varepsilon_p}{2} Var_z \{p_t(z)\}$ and $\widehat{\Delta}_t^j \approx \frac{\varepsilon_w^j}{2} Var_i \left\{w_t^j(i)\right\}$ for $j \in \{H, L\}$ and can be expressed in terms of corresponding inflation.

$$\sum_{t=0}^{\infty} \beta^t Var_p \{p_t(z)\} = \frac{\theta_p}{(1-\theta_p)(1-\beta\theta_p)} \sum_{t=0}^{\infty} \beta^t \left(\pi_t^p\right)^2$$

Similarly, $$\sum_{t=0}^{\infty} \beta^t Var_i \left\{w_t^j(i)\right\} = \frac{\theta_w}{(1-\theta_w)(1-\beta\theta_w)} \sum_{t=0}^{\infty} \beta^t \left(\pi_t^j\right)^2 \quad \text{for } j \in \{H, L\}$$

Finally, we obtain the Central Bank's loss function as

$$L = \frac{1}{2}\sum_{t=0}^{\infty} \beta^t \left\{ \frac{\varepsilon_p}{\kappa_p}\left(\pi_t^p\right)^2 + (\sigma+\varphi)\, x_t^2 + \frac{s\varepsilon_w^H}{\kappa_w^H}\left(\pi_t^H\right)^2 + \frac{(1-s)\varepsilon_w^L}{\kappa_w^L}\left(\pi_t^L\right)^2 + \psi_c \left(\mathcal{G}_t^c\right)^2 + \psi_I \left(\mathcal{G}_t^I\right)^2 \right\} \tag{C.20}$$

C.5 Optimal monetary policy under commitment

Minimize (C.20) subject to (4.9) (C.4),(C.6) and (4.20), in addition to inequality measures, (4.16) and (4.17), with corresponding Lagrange multiplier $\phi_{i,t}$ for $i = 1, 2, 3, \cdots, 8$

and for $t = 0, 1, 2, \cdots$

w.r.t.	First Order Conditions
x_t	$(\sigma+\varphi)\, x_t - \kappa_w^H (\sigma+\varphi)\, \phi_{2,t} - \kappa_w^L (\sigma+\varphi)\, \phi_{3,t} = 0$
π_t^p	$\frac{\varepsilon_p}{\kappa_p} \pi_t^p + \Delta\phi_{1,t} + \phi_{4,t} + \phi_{5,t} = 0$
π_t^H	$\frac{s\varepsilon_w^H}{\kappa_w^H} \pi_t^H + \Delta\phi_{2,t} - \phi_{4,t} = 0$
π_t^L	$\frac{(1-s)\varepsilon_w^L}{\kappa_w^L} \pi_t^L + \Delta\phi_{3,t} - \phi_{5,t} = 0$
$\widetilde{\omega}_t^H$	$\kappa_w^H \lambda_H^H \phi_{2,t} - \kappa_w^L (\sigma+\varphi)\, \eta s \phi_{3,t} + \phi_{5,t} - \beta\phi_{5,t+1} - s\phi_{6,t} + \eta s(1-s)\phi_{7,t} - s(1-s)(1-\eta)\phi_{8,t} = 0$
$\widetilde{\omega}_t^L$	$-\kappa_w^H \lambda_H^L \phi_{2,t} + \kappa_w^L \left((\sigma+\varphi)\eta s - (\sigma-1)\right)\phi_{3,t} + \phi_{5,t} - \beta\phi_{5,t+1} - (1-s)\phi_{6,t}$ $+(1-s)(1-\eta s)\phi_{7,t} + s(1-s)(1-\eta)\phi_{8,t} = 0$
$\widetilde{\omega}_t$	$-\kappa_p \phi_{1,t} + \phi_{6,t} = 0$
$\mathcal{G}_t^c$	$\psi_c \mathcal{G}_t^c + \phi_{7,t} = 0$
$\mathcal{G}_t^I$	$\psi_I \mathcal{G}_t^I + \phi_{8,t} = 0$

C.6 Counter-factual Experiment

C.6.1 Scenario 1: sticky wage + single labor market model

$$\mathcal{L} = \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left\{ (\sigma+\varphi)\, x_t^2 + \frac{\varepsilon_p}{\kappa_p} \left(\pi_t^p\right)^2 + \frac{\varepsilon_w}{\kappa_w} \left(\pi_t^w\right)^2 + \frac{(\sigma-1)(1-s)}{s} \left(\widetilde{\omega}_t\right)^2 \right\}$$

subject to

$$\pi_t^p = \beta E_t \pi_{t+1}^p + \kappa_p \widetilde{\omega}_t$$

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \kappa_w (\sigma+\varphi)\, x_t - \kappa_w \widetilde{\omega}_t$$

$$\widetilde{\omega}_t = \widetilde{\omega}_{t-1} + \pi_t^w - \pi_t^p - \Delta\omega_t^E$$

w.r.t.	First Order Conditions
x_t	$(\sigma+\varphi)\, x_t - \kappa_w (\sigma+\varphi)\, \phi_{2,t} = 0$
π_t^p	$\frac{\varepsilon_p}{\kappa_p} \pi_t^p + \Delta\phi_{1,t} + \phi_{3,t} = 0$
π_t^w	$\frac{\varepsilon_w}{\kappa_w} \pi_t^w + \Delta\phi_{2,t} - \phi_{3,t} = 0$
$\widetilde{\omega}_t$	$\frac{(\sigma-1)(1-s)}{s} \widetilde{\omega}_t - \kappa_p \phi_{1,t} + \kappa_w \phi_{2,t} - \beta\phi_{3,t+1} + \phi_{3,t} = 0$

C.6.2 Scenario 2: flexible wage + segmented labor market model

$$L=\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left\{\frac{\varepsilon_p}{\kappa_p}\left(\pi_t^p\right)^2+\left(\sigma+\varphi\right)x_t^2+\psi_c\left(\mathcal{G}_t^c\right)^2+\psi_I\left(\mathcal{G}_t^I\right)^2\right\}$$

subject to

$$\pi_t^p=\beta E_t\pi_{t+1}^p+\kappa_p\left(\sigma+\varphi\right)x_t$$

with (4.20), in addition to inequality measures, (4.16) and (4.17)

w.r.t.	First Order Conditions
x_t	$(\sigma+\varphi)\,x_t-\kappa_p(\sigma+\varphi)\phi_{1,t}=0$
π_t^p	$\frac{\varepsilon_p}{\kappa_p}\pi_t^p+\Delta\phi_{1,t}+\phi_{2,t}+\phi_{3,t}=0$
$\widetilde{\omega}_t^H$	$\phi_{2,t}-\beta\phi_{2,t+1}+\eta s(1-s)\phi_{4,t}-s(1-s)(1-\eta)\phi_{5,t}=0$
$\widetilde{\omega}_t^L$	$\phi_{3,t}-\beta\phi_{3,t+1}+(1-s)(1-\eta s)\phi_{4,t}+s(1-s)(1-\eta)\phi_{5,t}=0$
$\mathcal{G}_t^c$	$\psi_c\mathcal{G}_t^c+\phi_{4,t}=0$
$\mathcal{G}_t^I$	$\psi_I\mathcal{G}_t^I+\phi_{5,t}=0$

C.6.3 Scenario 3: flexible wage + single labor market Model

$$\mathcal{L}=\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left\{\left(\sigma+\varphi\right)x_t^2+\frac{\varepsilon_p}{\kappa_p}\left(\pi_t^p\right)^2\right\}$$

subject to

$$\pi_t^p=\beta E_t\pi_{t+1}^p+\kappa_p\left(\sigma+\varphi\right)x_t$$

w.r.t.	First Order Conditions
x_t	$(\sigma+\varphi)\,x_t-\kappa_p\left(\sigma+\varphi\right)\phi_{1,t}=0$
π_t^p	$\frac{\varepsilon_p}{\kappa_p}\pi_t^p+\Delta\phi_{1,t}=0$

C.7 An extension with idiosyncratic productivity shock

C.7.1 Labor Demand

$$\min_{N_t^H, N_t^L} W_t^H N_t^H + W_t^L N_t^L \quad \text{s.t. } H_t \geq \left[\gamma_H^{\frac{1}{\eta}} \left(A_t^H N_t^H\right)^{\frac{\eta-1}{\eta}} + \gamma_L^{\frac{1}{\eta}} \left(A_t^L N_t^L\right)^{\frac{\eta-1}{\eta}}\right]$$

$$\text{F.O.C:} \quad \varpi_t^j \equiv \frac{W_t^j}{A_t^j} = \lambda_t \left\{ \gamma^{\frac{1}{\eta}} \left(\frac{H_t}{A_t^j N_t^j} \right)^{\frac{1}{\eta}} \right\} \quad \text{for } j \in \{H, L\}$$

Convex combination of the wages after taking $1-\eta$ power and using the definition of H_t, I get

$$\left[\gamma \left(\varpi_t^H\right)^{1-\eta} + (1-\gamma)\left(\varpi_t^L\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} = \left[\gamma \left(\frac{W_t^H}{A_t^H}\right)^{1-\eta} + (1-\gamma)\left(\frac{W_t^L}{A_t^L}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} = \lambda_t \equiv \varpi_t$$

replace λ_t in F.O.C

$$N_t^j = \gamma_j \left(\frac{\varpi_t^j}{\varpi_t}\right)^{-\eta} \frac{Y_t}{A_t A_t^j} \Rightarrow n_t^H \approx -\eta\left(\widehat{\varpi}_t^j - \widehat{\varpi}_t\right) + y_t - a_t - a_t^j$$

$$\Rightarrow n_t^H - n_t^L \equiv n_t^R = -\eta\left(\omega_t^R - a_t^R\right) - a_t^R = -\eta\omega_t^R - (1+\eta)a_t^R$$

C.7.2 Marginal Cost

$$\min_{N_t^H, N_t^L} W_t^H N_t^H + W_t^L N_t^L \quad \text{s.t. } Y_t \geq A_t \left[\gamma_H^{\frac{1}{\eta}} \left(A_t^H N_t^H\right)^{\frac{\eta-1}{\eta}} + \gamma_L^{\frac{1}{\eta}} \left(A_t^L N_t^L\right)^{\frac{\eta-1}{\eta}}\right]$$

$$\text{F.O.C:} \quad \varpi_t^j = \lambda_t A_t \left\{ \gamma^{\frac{1}{\eta}} \left(\frac{H_t}{A_t^j N_t^j} \right)^{\frac{1}{\eta}} \right\} \quad \text{for } j \in \{H, L\}$$

Similarly,

$$\gamma_j \left(\varpi_t^j\right)^{1-\eta} = \lambda_t^{1-\eta} A_t^{1-\eta} \left\{ \gamma_j^{\frac{1}{\eta}} \left(\frac{j_t}{A_t^j N_t^j} \right)^{\frac{1-\eta}{\eta}} \right\} \quad \text{for } \ j \in \{H, L\}$$

$$\Rightarrow \left(\frac{\varpi_t}{A_t}\right)^{1-\eta} = \lambda_t^{1-\eta} \Rightarrow mc_t = \widehat{\varpi}_t - a_t \Rightarrow \underbrace{sw_t^H + (1-s)w_t^L}_{\equiv \ w_t} - \underbrace{sa_t^H + (1-s)a_t^L}_{\equiv \ a_t^c} - a_t$$

which is obtained by a convex combination with weight of population share.

C.7.3 Efficient level of output

$$MRS_t^j = \chi_t \left(C_t^j\right)^{\sigma} \left(N_t^j\right)^{\varphi} = \frac{W_t^j}{P_t} = \gamma_j^{\frac{1}{\eta}} A_t A_t^j \left(\frac{H_t}{A_t^j N_t^j}\right)^{\frac{1}{\eta}} = MPN_t^j$$

$$\Rightarrow \sigma c_t^j + \varphi n_t^j + \xi_t = \omega_t^j = a_t + a_t^j + \frac{1}{\eta}\left(h_t - a_t^j - n_t^j\right)$$

Convex combination of the two sectoral efficient conditions gives

$$\sigma y_t + \varphi\left(sn_t^H + (1-s)n_t^L\right) + \xi_t = a_t + a_t^c + \frac{1}{\eta} \underbrace{\left(h_t - s\left(a_t^H + n_t^h\right) - (1-s)\left(a_t^L + n_t^L\right)\right)}_{=0}$$

$$\sigma y_t + \varphi\left(y_t - a_t - a_t^c\right) + \xi_t = a_t + a_t^c$$

$$y_t^E = \frac{1+\varphi}{\sigma+\varphi}\left(a_t + a_t^c\right) - \frac{1}{\sigma+\varphi}\xi_t$$

$$w_t^E = a_t + a_t^c \quad \text{and} \quad w_t^{H,E} = a_t + a_t^H \quad \& \quad w_t^{L,E} = a_t + a_t^L$$

$$w_t^{R,E} = a_t^H - a_t^L$$

C.7.4 Log-linearized Equations

1. $x_t = E_t x_{t+1} - \frac{1}{\sigma}\left\{i_t - E_t\pi_{t+1}^P - r_t^E\right\} + \frac{1}{s}\Delta\mathcal{G}_{t+1}^c$ (IS curve)

2. $\pi_t^p = \beta E_t \pi_{t+1}^P + \kappa_p \widetilde{\omega}_t$ (NKPC)

3. $\pi_t^H = \beta E_t \pi_{t+1}^H + \kappa_w^H (\sigma+\varphi) x_t - \kappa_w^H \lambda_H^H \widetilde{\omega}_t^H + \kappa_w^H \lambda_H^L \widetilde{\omega}_t^L + \mu_t^H$ (high-skilled NKWPC)

4. $\pi_t^L = \beta E_t \pi_{t+1}^L + \kappa_w^L (\sigma+\varphi) x_t + \kappa_w^L \lambda_L^H \widetilde{\omega}_t^H - \kappa_w^L \lambda_L^L \widetilde{\omega}_t^L + \mu_t^L$ (low-skilled NKWPC)

5. $\mathcal{G}_t^c = -(1-s)\left(\eta s \widetilde{\omega}_t^H + (1-\eta s)\widetilde{\omega}_t^L\right)$ (Consumption Gini coefficient)

6. $\widetilde{\omega}_t^H = \widetilde{\omega}_{t-1}^H + \pi_t^H - \pi_t^p - \Delta\omega_t^{H,E}$

7. $\widetilde{\omega}_t^L = \widetilde{\omega}_{t-1}^L + \pi_t^L - \pi_t^p - \Delta\omega_t^{L,E}$

8. $\widetilde{\omega}_t = s\widetilde{\omega}_t^H + (1-s)\widetilde{\omega}_t^L$

9. $i_t = \rho_i i_{t-1} + (1-\rho_i)\left(\phi_\pi \pi_t^p + \phi_x x_t\right) + \nu_t$ (Monetary policy rule)

10. $x_t = y_t - y_t^E$

11. $y_t^E = \frac{1+\varphi}{\sigma+\varphi}\left(a_t + a_t^c\right) - \frac{1}{\sigma+\varphi}\xi_t$

12. $r_t^E = \sigma\Delta y_{t+1}^E$

13. $\omega_t^{H,E} = a_t + a_t^H$

14. $\omega_t^{L,E} = a_t + a_t^L$

15. $a_t = \rho_a a_{t-1} + \epsilon_t^a$ (Labor aggregation technology shock)

16. $a_t^H = \rho_{a,H} a_{t-1}^H + \epsilon_t^{a,H}$ (Labor aggregation technology shock)

17. $a_t^L = \rho_{a,L} a_{t-1}^L + \epsilon_t^{a,L}$ (high-skilled productivity shock)

18. $\mu_t^H = \rho_H \mu_{t-1}^H + \epsilon_t^H$ (low-skilled productivity shock)

19. $\mu_t^L = \rho_L \mu_{t-1}^L + \epsilon_t^L$ (low-skilled wage markup shock)

20. $\nu_t = \rho_\nu \nu_{t-1} + \epsilon_t^\nu$ (Monetary policy shock)

21. $\xi_t = \rho_\xi \xi_{t-1} + \epsilon_t^\xi$ (Labor Supply shock)

C.8 Additional Figures

Figure C.1: Dynamic responses to the positive technology shock

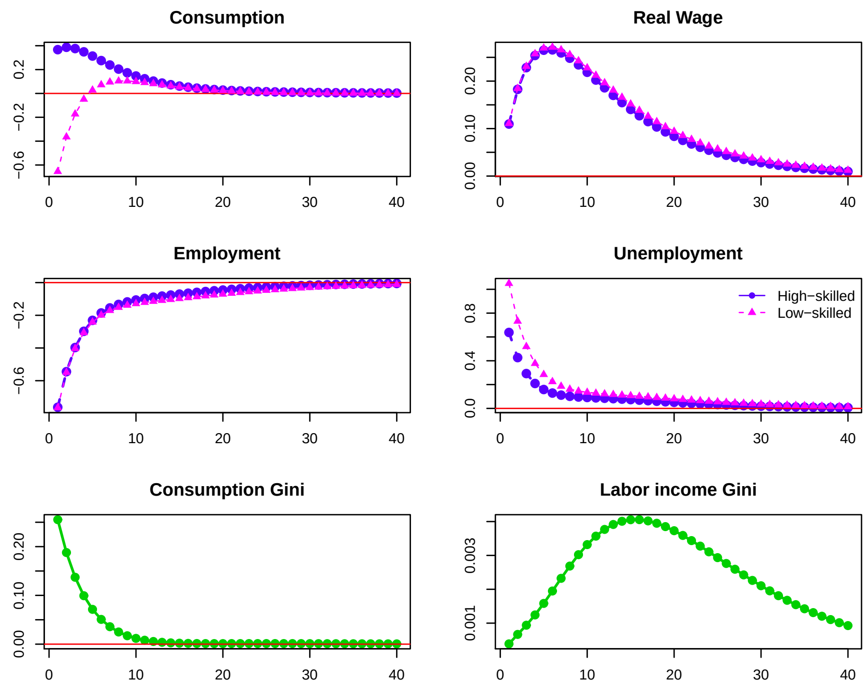


Table C.1: Relative Welfare Losses

Scenario	Productivity shock			Markup shock	
	Aggregate	High-skilled	Low-skilled	High-skilled	Low-skilled
Baseline	1	1	1	1	1
1	1.000873	1.051882	1.051621	1.017812	1.010268
2	30.951561	1.542624	4.387239	1.093651	1.079710
3	30.677522	2.246138	3.020883	1.022344	1.025460

Figure C.2: Dynamic responses to the positive high-skilled productivity shock

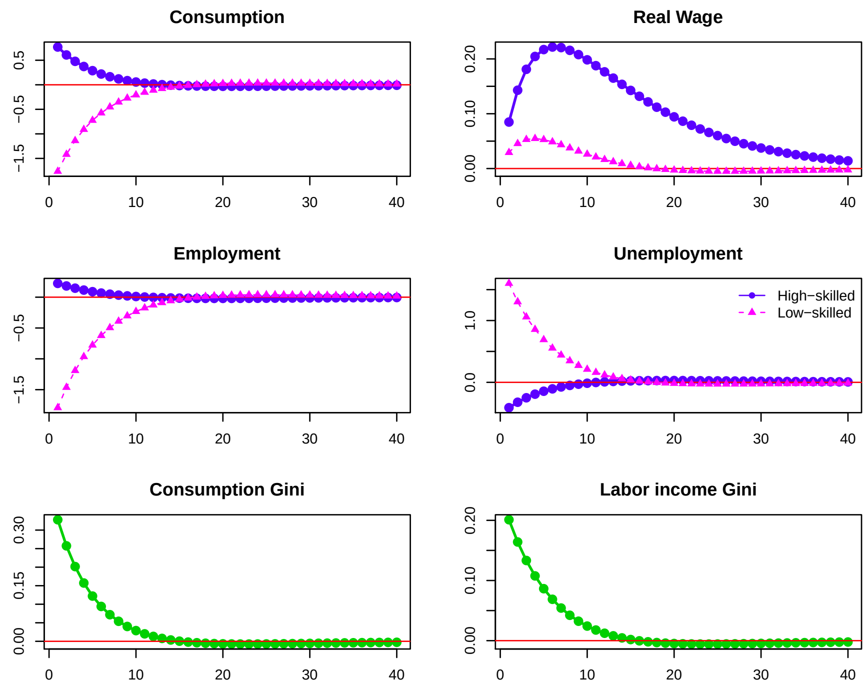


Figure C.3: Dynamic responses to the positive low-skilled technology shock

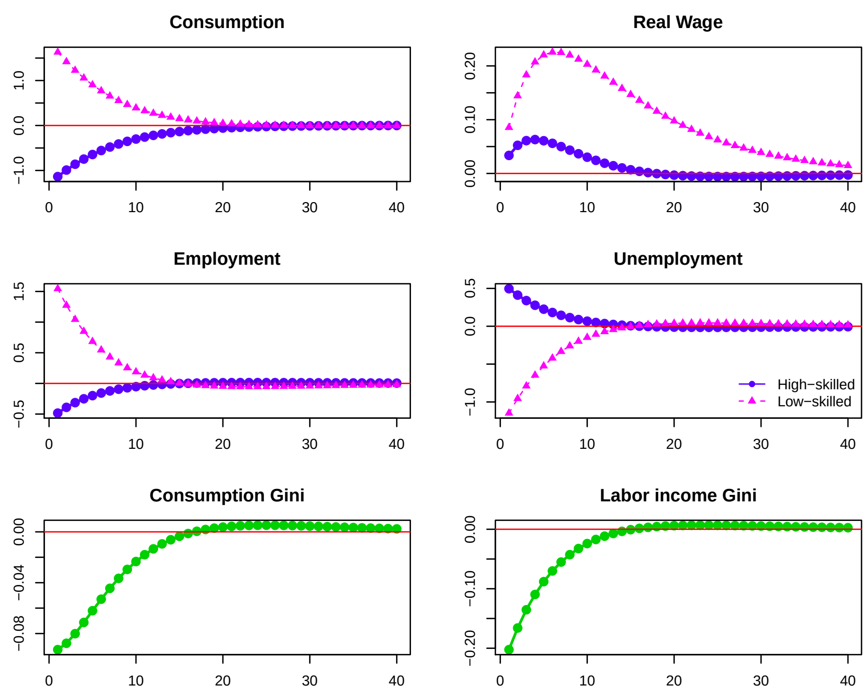


Figure C.4: High skilled wage markup shock

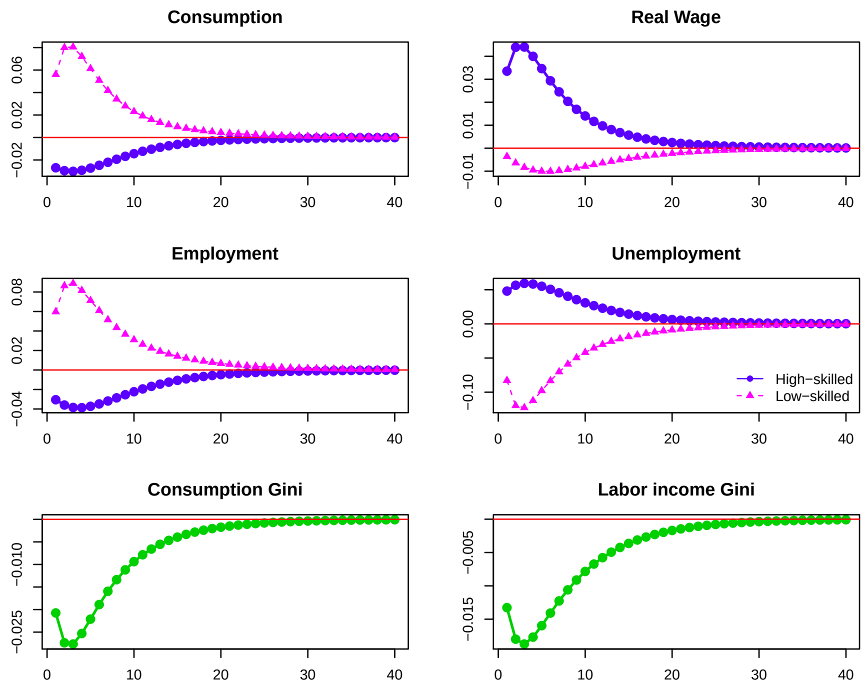


Figure C.5: Low skilled wage markup shock

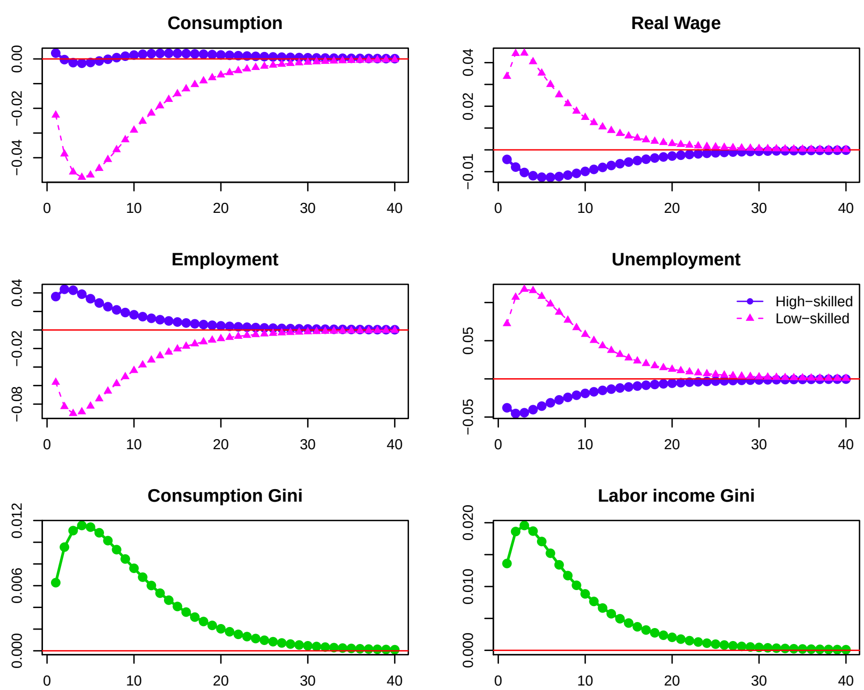
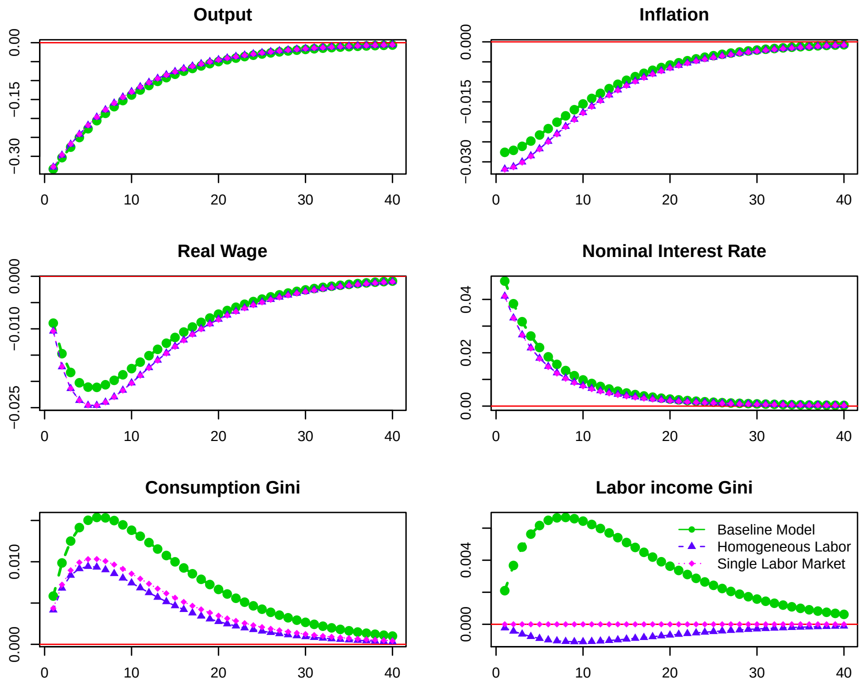
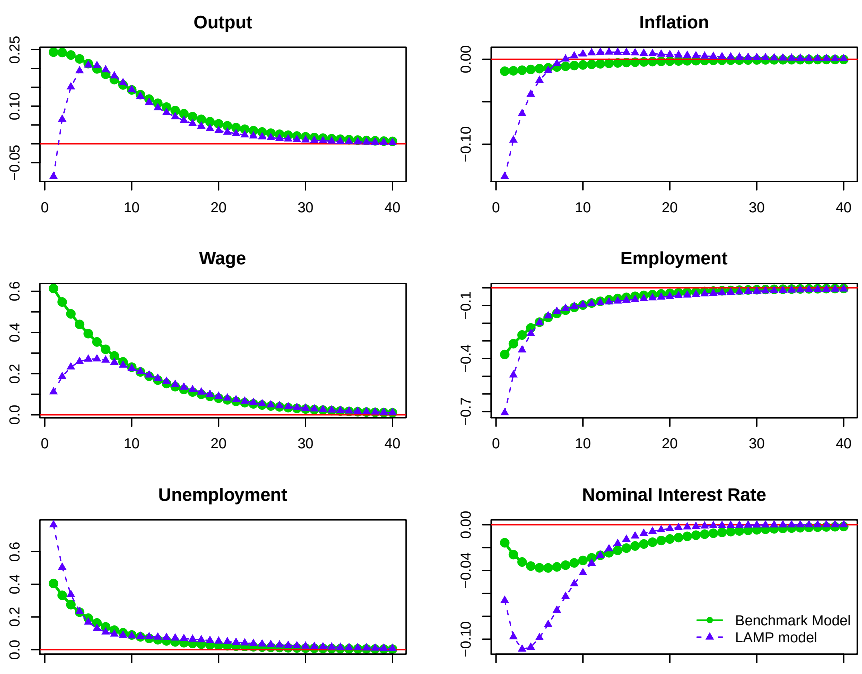


Figure C.6: Role of labor market assumption: IRFs to a Monetary policy shock



NOTE: Homogeneous labor market implies the same labor demand elasticity ($\varepsilon_w^H = \varepsilon_w^L$). The single labor market features the same labor demand and wages in addition to homogeneous labor

Figure C.7: Dynamic responses to positive technology shock



Note: I consider Galí (2011) as a benchmark model which is a standard NK model with staggered wages

Figure C.8: Dynamic responses to the low-skilled productivity shock

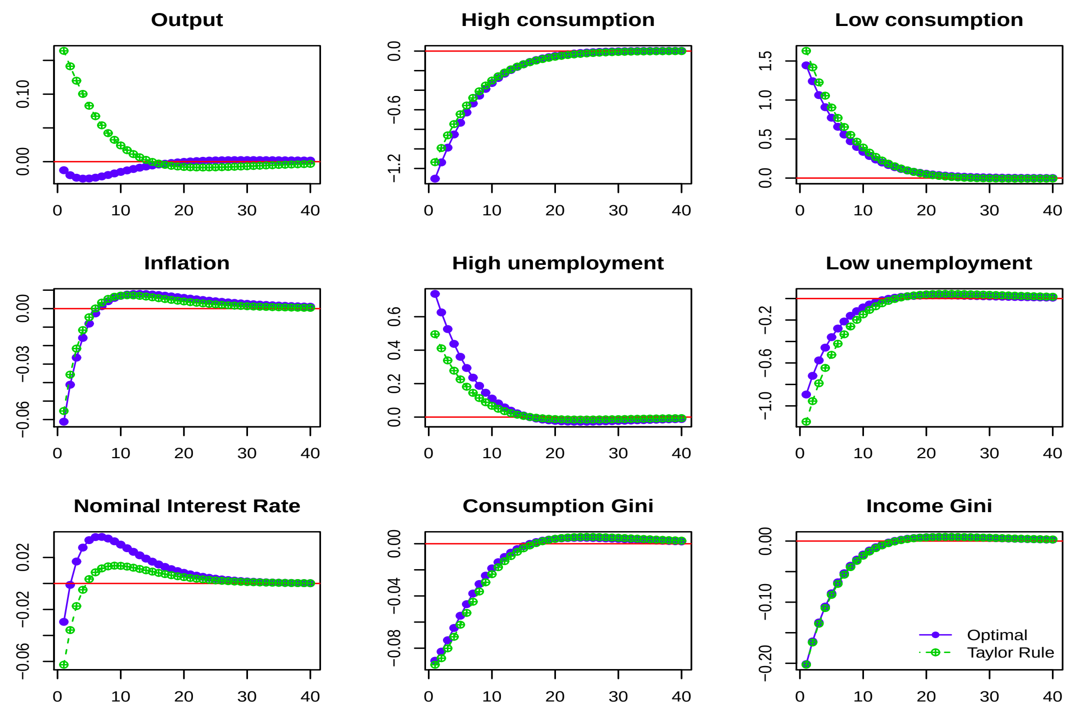


Figure C.9: Dynamic responses to the high-skilled wage markup shock

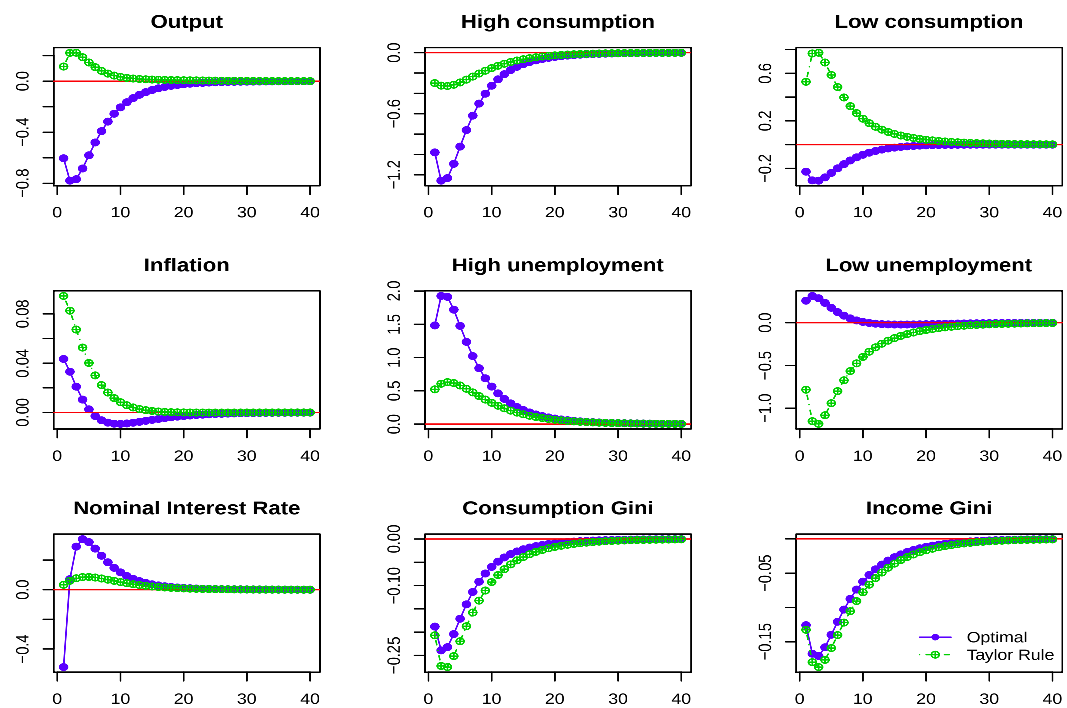
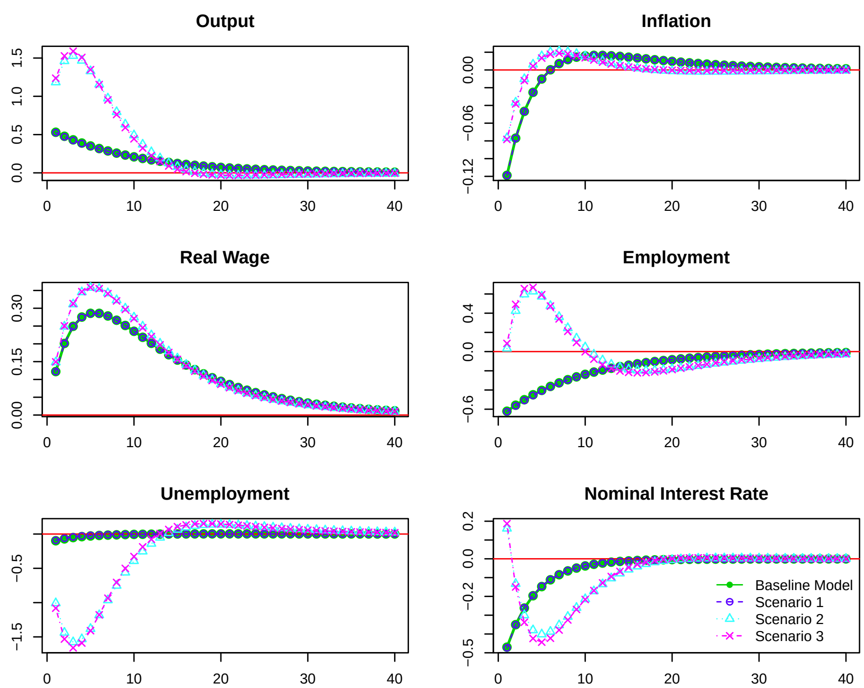
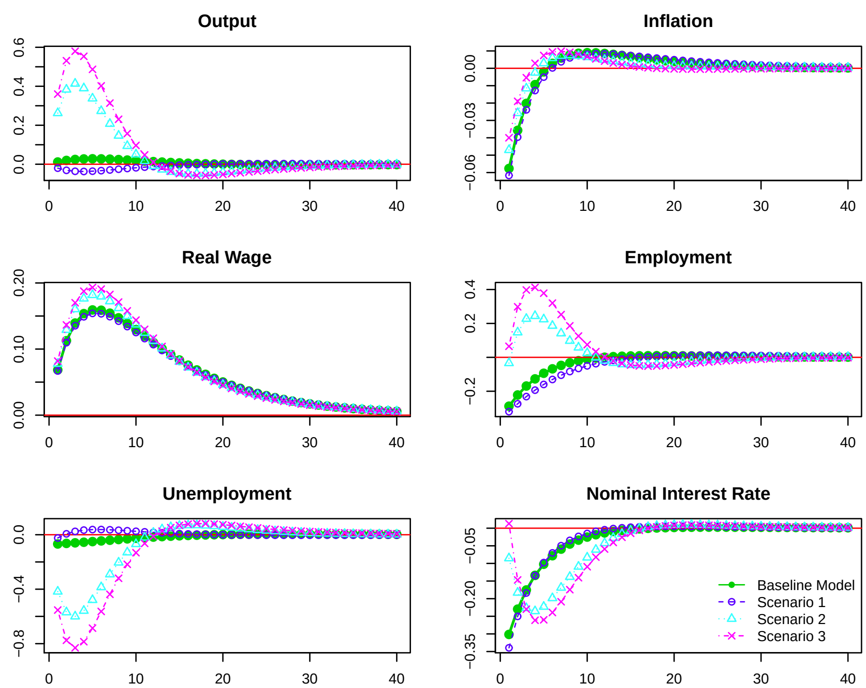


Figure C.10: Optimal responses to the positive technology shock



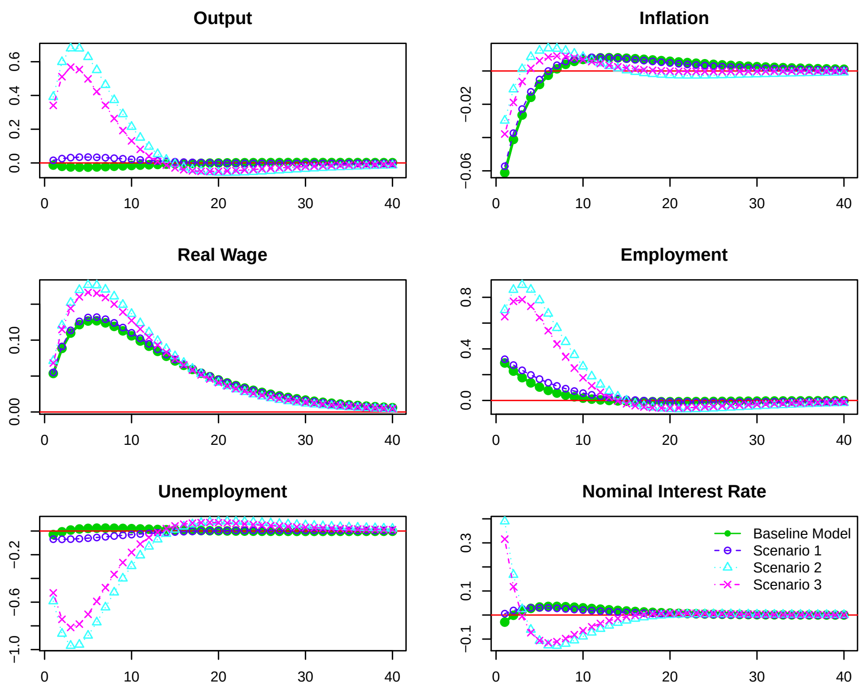
Note: Baseline: Segmented labor market with staggered wages; Scenario 1: Single labor market with staggered wages;
Scenario 2: Segmented labor market with flexible wages; Scenario 3: Single labor market with flexible wages

Figure C.11: Optimal responses to the positive high-skilled productivity shock



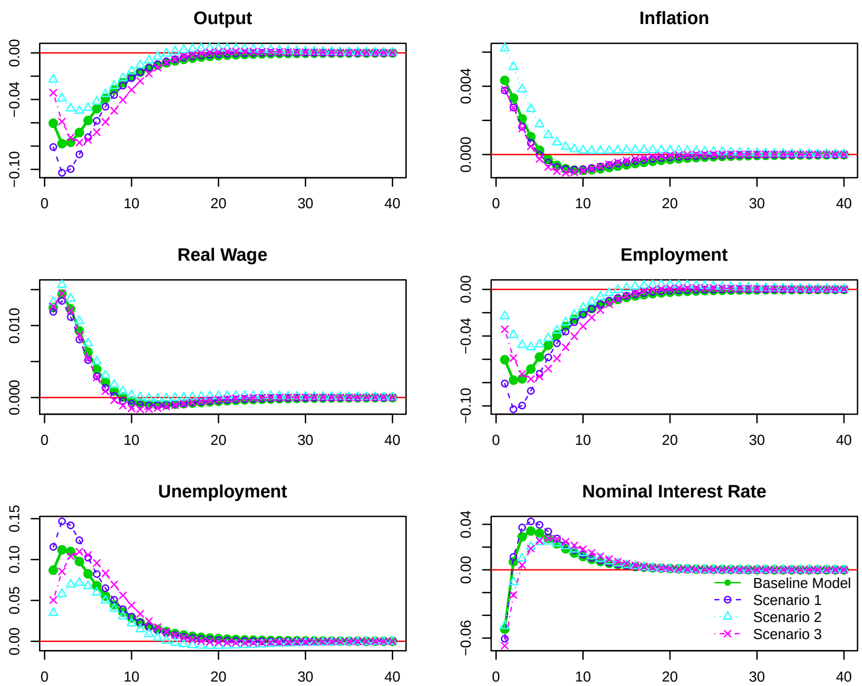
Note: Baseline: Segmented labor market with staggered wages; Scenario 1: Single labor market with staggered wages;
Scenario 2: Segmented labor market with flexible wages; Scenario 3: Single labor market with flexible wages

Figure C.12: Optimal responses to the positive low-skilled productivity shock



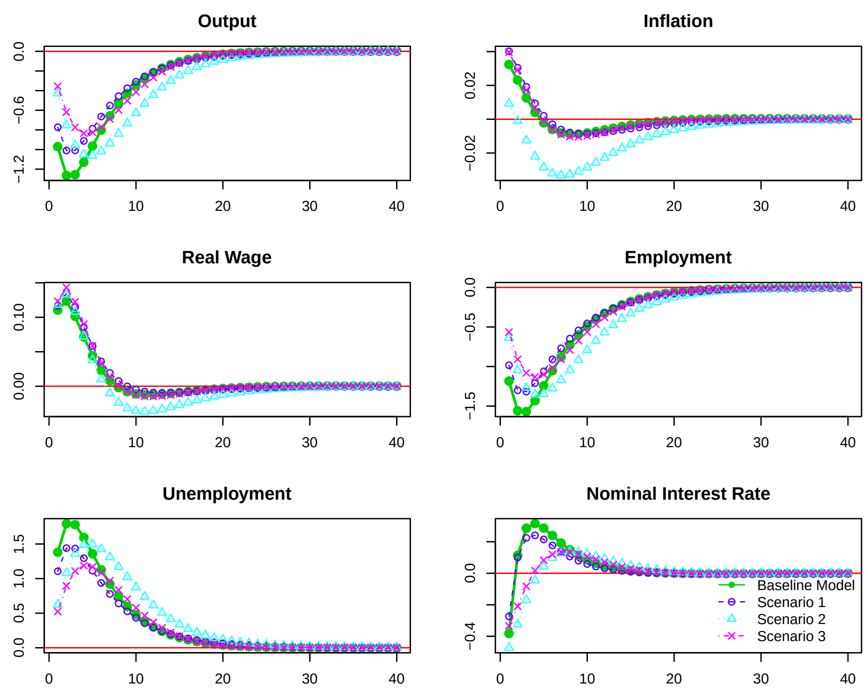
Note: Baseline: Segmented labor market with staggered wages; Scenario 1: Single labor market with staggered wages;
Scenario 2: Segmented labor market with flexible wages; Scenario 3: Single labor market with flexible wages

Figure C.13: Optimal responses to the positive high-skilled wage markup shock



Note: Baseline: Segmented labor market with staggered wages; Scenario 1: Single labor market with staggered wages;
Scenario 2: Segmented labor market with flexible wages; Scenario 3: Single labor market with flexible wages

Figure C.14: Optimal responses to the positive low-skilled wage markup shock



Note: Baseline: Segmented labor market with staggered wages; Scenario 1: Single labor market with staggered wages;
Scenario 2: Segmented labor market with flexible wages; Scenario 3: Single labor market with flexible wages