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MULTI-OBJECTIVE GENERATION EXPANSION PLANNING CONSIDERING
UNCERTAINTY AND MODELING WITH THE PARETO UNCERTAINTY INDEX

by

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ABSTRACT OF THE DISSERTATION

Multi-Objective Generation Expansion Planning
Considering Uncertainty and Modeling with the Pareto
Uncertainty Index

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Many real life optimization problems are multi-objective problems where objectives under consideration usually conflict with each other and they are also stochastic due to inherent uncertainties. The electricity Generation Expansion Planning (GEP) problem is an example of such problems in which the goal is to expand the electric power network with new power plant investments including renewable resources. Decisions are made where and when to build new power plants and which technology to choose for new investments. Objectives can include but are not limited to minimization of the cost and pollutant emissions and maximization of reliability. There are inherent uncertainties in the GEP problem due to climate change, demand increase, fuel prices, technological progress and many other aspects that have to be considered. Some of these uncertainties directly affect the objective functions and some affect the constraint sets in the optimization model.

In this study, a new uncertainty metric, the Pareto Uncertainty Index (PUI), is presented. The PUI includes uncertainty as part of the Pareto optimality concept so that

the decision or policy maker can observe the uncertainty of Pareto optimal solutions. Using the PUI approach for objective function uncertainties and chance constrained programming or scenarios for constraint set uncertainties, a new multi-objective stochastic genetic algorithm, the Pareto Uncertain Genetic Algorithm (PUGA), is presented in this research, as well. In contrast with the other multi-objective genetic algorithms and classical methods, PUGA can incorporate both the multi-objective and stochastic aspects of problem solving without any transformation. A new post-Pareto pruning approach that reduces the number of Pareto optimal solutions to a smaller practical set is also included in PUGA with the help of the uncertainty information preserved in the PUI. Furthermore, this uncertainty information is used for risk assessments of solutions depending on the risk preferences of decision makers. The PUI and PUGA concepts are demonstrated and tested on several problems including the US Northeast region generation expansion planning (NEGEP) problem.

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LIST OF FREQUENTLY USED ACRONYMS

PUI	Pareto Uncertainty Index
rPUI	Risk Adjusted Pareto Uncertainty Index
PUGA	Pareto Uncertainty Genetic Algorithm
GEP	Generation Expansion Planning
NEGEP	Northeast United States Generation Expansion Planning
NSGA-II	Fast Elitist Non-dominated Sorting Genetic Algorithm

1. Introduction

The major question today for electric power is no longer market design - regulation versus deregulation. Rather, it is fuel choice. Whatever the setup in different parts of the country, the United States faces the same question about the future of its electricity supply as do many other countries: What kind of generation to build? This struggle over fuel choice is not just about meeting today's needs, but also how to meet expected growth in demand and new environmental objectives. Coal, nuclear power, and natural gas will all be part of the picture, both in the United States and around the world. Each, however, comes with its own constraints.

Daniel Yergin, The Quest

This dissertation focuses on multi-objective stochastic optimization problems, particularly the electricity Generation Expansion Planning (GEP) problem, and presents a new uncertainty index and a new multi-objective stochastic genetic algorithm to solve such problems.

The GEP problem consists of the expansion of the electricity generation network to satisfy the future electricity demand with new energy investments. The GEP problems start with an existing power network. The existing network will be insufficient in the future due to new policies, demand increase and/or existing unit retirements. Therefore, the existing system should be expanded by new sources in order to provide economic and reliable energy supply in the future. There are four main group of technologies that can be added to the system; generation units, transmission lines, distribution lines and smart grid technologies [1]. Decisions have to be made concerning the location and timing of the investments and selection of technologies over a specified multi-decade planning horizon.

Many real life optimization problems are multi-objective problems in which objectives under consideration often conflict with each other. Objectives in the GEP, for example, may include minimization of cost, minimization of pollutant emissions,

maximization of reliability and maximization of energy security and independency. It is important for a decision maker to observe the trade-offs of objectives. However, most of the multi-objective optimization problems as well as the most commonly studied GEP problems are typically transformed to the single objective equivalents where the problem is formulated with only one objective. Another common solution methodology for multi-objective optimization problems is to determine a Pareto optimal set which provides a trade-off curve of conflicting objectives. Although this approach preserves the multi-objective aspect of the problem, it does not include the uncertainty aspect.

Real life engineering and optimization problems are also stochastic due to inherent uncertainties. For instance, uncertainties in climate change, electricity demand increase, fuel prices, and technological progress are examples of inherent uncertainties that have to be considered in the GEP problem. Although stochastic programming problems are frequently encountered in practice, most models assume deterministic conditions especially when the problem is modeled to be multi-objective. Otherwise, the problem is first aggregated to obtain a single objective stochastic model, which is then solved by a stochastic programming approach. This kind of transformation maintains the randomness and uncertainty but eliminates the multi-objective aspect of the problem.

This dissertation presents a new uncertainty index and a new multi-objective stochastic genetic algorithm to overcome the transformations where either the multi-objective or the uncertainty aspect is eliminated. The research contributions of the dissertation are summarized next.

1.1 Research Contribution

This research results in several distinct research contributions. As Gorenstin et al. [2] states, it is necessary to reformulate the use of expected values and single objective modeling. The use of expected values, i.e., the deterministic assumptions, does not capture the uncertainty. The use of only one scalar measure (usually cost), i.e., single objective modeling, is not adequate to represent conflicting objectives such as power production and environmental impacts in the GEP. Therefore, a new uncertainty index, the Pareto Uncertainty Index (PUI), and a new multi-objective stochastic genetic algorithm, the Pareto Uncertain Genetic Algorithm (PUGA) are presented in this research to solve multi-objective stochastic optimization problems. There are several research contributions of this research, some of them are related to the GEP problem domain and some to the single and multi-objective stochastic optimization processes. These research contributions are summarized as follows:

- Sources of uncertainty in the GEP problem are identified and classified into epistemic and aleatory uncertainty types. Effects of different types of uncertainty on the variables of the GEP model are also identified for optimization modeling purposes. These contributions are useful to model the uncertainty in the planning process. They also assist in providing transparency in decision-making.
- The GEP problem is modeled and solved as a multi-objective stochastic problem. This allows a more realistic representation of the problem and the results of the optimization. The need for transforming the problem is eliminated.

- The concept of Pareto optimality is extended from a purely deterministic framework to a more realistic stochastic framework.
- A new uncertainty index, the PUI, is presented which can be incorporated in the problem solving methods to solve single or multi-objective optimization problems without ignoring uncertainty.
- A new multi-objective stochastic genetic algorithm, PUGA, is presented to find stochastic - Pareto optimal solutions of multi-objective stochastic optimization problems with the ability of observing uncertainty information of solutions.
- Risk measures are incorporated in the PUI and PUGA so that the risk preference of the decision maker can affect the solution process. This is important for both the GEP problem domain and the optimization processes.
- Post-Pareto analysis is also integrated in the optimization process. The probability information preserved in the PUI is utilized to make a preference decision between solutions on the same front using the risk preference of the decision maker. Thus, obtaining a smaller subset of the preferred solutions from a large Pareto optimal set and the evaluation and interpretation of the results can be accomplished within the optimization process.

1.2 Dissertation Organization

The dissertation is organized as follows:

In Chapter 2, the GEP problem is first explained and formulated. Single objective least cost and multi-objective GEP models in the literature are reviewed. Second, the uncertainty in GEP is introduced. GEP models that consider uncertainty are surveyed and

aleatory-epistemic uncertainty classification is explained. Afterwards, a general review of multi-objective optimization is provided. Classical methods and evolutionary algorithms, particularly multi-objective genetic algorithms, are reviewed. Later, uncertainty in multi-objective optimization and inadequacy of present methods are discussed. Finally, post-Pareto pruning is introduced.

In Chapter 3, the PUI is presented. The formulations of the PUI for dependent and independent objectives are explained. The risk adjusted PUI (rPUI), where a risk measure is employed to incorporate the risk preference of the decision maker in the solution process, is introduced. Formulations of the PUI and the rPUI for discrete cases and scenarios are also explained.

In Chapter 4, PUGA is presented. First, design components of PUGA are explained and algorithms for these components are provided. Second, the main loop of PUGA is described and the algorithm is provided. Two modified test problems are employed to demonstrate the algorithm and to test the performance of PUGA.

In Chapter 5, a multi-objective stochastic optimization model for the power generation expansion planning of the Northeast region of the United States is presented. Aleatory and epistemic uncertainty modeling of this problem is explained. Scenarios, input data and assumptions for the NEGEP model is provided and the model is solved to find Pareto solutions for the cost and CO₂ emissions objectives. To conclude the chapter, the results, comparisons and conclusions are presented.

Finally, in Chapter 6, general conclusions of the dissertation and future research opportunities are discussed.

2. Background and Literature Review

This dissertation involves multi-objective stochastic optimization problems and presents a new uncertainty index and a new multi-objective stochastic genetic algorithm to solve such problems, particularly the GEP problem. In this section, the GEP problem is first explained and formulated. Single objective least cost and multi-objective GEP models in literature are reviewed. Second, the uncertainty in GEP is introduced. GEP models that consider uncertainty are surveyed and aleatory-epistemic uncertainty classification is explained. Afterwards, a general review of multi-objective optimization is provided. Classical methods and evolutionary algorithms, particularly multi-objective genetic algorithms, are reviewed. Later, uncertainty in multi-objective optimization and inadequacy of present methods are discussed. Finally, post-Pareto pruning is introduced.

2.1 Generation Expansion Planning Problem

The GEP problem pertains to the expansion of the electricity generation network to satisfy the future electricity demand with new energy investments. Decisions have to be made concerning the location and timing of the investment and selection of technology over a specified multi-decade planning horizon. Objectives may include minimization of cost, minimization of pollutant emissions, maximization of reliability, and maximization of energy security and independency. Some commonly used constraints are energy demand constraints, capacity constraints, investment and budget constraints, environmental regulations and renewable portfolio standards. There are inherent uncertainties in this problem due to climate change, demand increase, fuel prices, technological progress and many other aspects that have to be considered. The GEP is one of the most challenging real life multi-objective stochastic optimization problems.

2.1.1 Problem Formulation and Definitions

In practice, the GEP is a multi-objective stochastic problem. However, the most commonly studied GEP problems are typically deterministic cost minimization problems where the problem is formulated with only one objective and uncertainty is neglected. Some researchers convert other objectives to the cost objective. For example, instead of minimizing emissions directly, a cost or penalty function is used for emissions and this function is added to the cost objective as another cost component. Other researchers use the *weighted sum method* where all objectives are summed into a single objective by assigning weights to each. Other classical methods (see Section 2.3.2) are also used for converting the multi-objective GEP problem to a single objective formulation.

The needs of utility planners for GEP optimization models have changed in response to environmental concerns, increased competition, and growing uncertainty. Some of the new needs include:

- The need to explicitly examine trade-offs among objectives (multi-objective formulation)
- The need to recognize uncertainty (stochastic formulation) [3].

Hobbs [3] presents a detailed review about the incorporation of transmission costs, the treatment of multiple objectives, methods for including uncertainty, and the use of resource planning models in a competitive environment. Hobbs' review also provides a basic mixed integer linear program formulation for single objective GEP problem. Formulations and a literature review for single objective GEP models are presented in Section 2.1.2.

Table 1. Nomenclature for multi-objective stochastic formulation of GEP

Indices and Sets	
i, I	Generation units
y, Y	Years of planning horizon
u	Years of planning horizon (u is an alias for y for modeling)
t, T	Time periods in a year
r, R	Regions
k, K	Emissions gases (CO ₂ , SO ₂ , NO _x)
n, N	Renewable generation units (subset of generation units I)
Decision Variables	
$x_{y,t,r,i}$	Generation amount of generation unit i in region r in time period t in year y (MWh)
$s_{y,r,i}$	Investment amount of generation unit i in region r in year y (MW)
Random Parameters (Stochastic Inputs)	
$\tilde{c}_{y,i}$	Generation variable cost for generation unit i in year y (\$/MWh)
$\tilde{h}_{y,i}$	Fixed operation and maintenance cost for new generation unit i in year y (\$/MW)
$\tilde{a}_{y,i}$	Investment cost for generation unit i in year y (\$/MW)
$\tilde{g}_{y,i}$	Fixed operation and maintenance cost for existing generation unit i in year y (\$/MW)
$\tilde{Y}_{i,k}$	Amount of emissions gas k from generation unit i (lbs/MWh)
$\tilde{w}_{t,i}$	Derate factor of generation unit i in time period t
$\tilde{p}_{t,r,i}$	Availability factor for generation unit i in region r in time period t
$\tilde{d}_{y,t,r}$	Demand in region r in time period t in year y (MWh)
$\tilde{d}_{y,r}^p$	Peak demand in year y in region r (MWh)
\tilde{m}_r	Reserve margin percentage (rate) for region r
Deterministic Parameters (Deterministic Inputs)	
ira	Interest rate
$\chi_{y,r,i}$	Forced new capacity of generation unit i in region r in year y (MW)
$\mu_{r,i}$	Initial capacity of generation unit i in region r at the beginning (MW)
$\dagger_{y,r,i}$	Forced retirement capacity of generation unit i in region r in year y (MW)
f_t	Hours in time period t
$S_{y,r}^{\min}$	Annual minimum percentage of renewable generation requirement in region r in year y
$S_{y,r,n}$	Minimum percentage of renewable generation requirement from renewable generation unit n in region r in year y
$u_{y,r,i}$	Yearly construction limit of generation unit i in region r in year y (MW)

GEP problems start with an existing power network. The existing network will be insufficient in the future due to new policies, demand increase and/or existing unit retirements. Therefore, the existing system should be expanded by adding new technologies in order to provide an economic and reliable energy supply in the future. There are four main groups of technologies can be added to the system: generation units, transmission lines, distribution lines and smart grid technologies [1]. The formulation presented in this section focuses on generation expansion only. In some papers (e.g., [4]), transmission lines are also considered together with generation units. A few researchers also consider demand side management as in [5], and distributed generation units and smart grid technologies as expansion options as in [6].

An example model of the GEP problem is formulated here with two objectives (minimization of cost and minimization of emissions). The economic uncertainties of investment cost and generation variable cost, uncertainties in technological progress for operation and maintenance and amount of pollutant emissions and climate change uncertainties for derate factor, availability factor, total annual and regional demand, peak demand and reserve margin are included in the formulation. Table 1 explains the nomenclature for this multi-objective stochastic formulation.

The first objective function (1) is to minimize the net present value of the total cost, which consists of generation cost (2), investment cost (3), and fixed operation and maintenance cost (4).

$$\min \text{Cost} = \sum_y \left\{ (1+ira)^{-y} \times (C_{y,1} + C_{y,2} + C_{y,3}) \right\} \quad (1)$$

$$C_{y,1} = \sum_t \sum_r \sum_i x_{y,t,r,i} \tilde{c}_{y,i} \quad (2)$$

$$C_{y,2} = \sum_r \sum_t s_{y,r,i} \tilde{a}_{y,i} \quad (3)$$

$$C_{y,3} = \sum_r \sum_i \left(\sum_{u=1}^y (s_{u,r,i} + x_{y,r,i}) \right) \tilde{h}_{y,i} + \sum_r \sum_i \left(\#_{r,i} - \sum_{u=1}^y \dagger_{y,r,i} \right) \tilde{g}_{y,i} \quad (4)$$

The second objective function (5) is to minimize the total pollutant emissions such as CO₂, SO₂ and NO_x.

$$\min Emissions = \sum_{y \in Y} \sum_{t \in T} \sum_{r \in R} \sum_{i \in I} x_{y,t,r,i} \tilde{y}_{i,k} \quad (5)$$

Equations (6) to (12) are the constraints of the model. Equation (6) is the supply-demand constraint that ensures that the total generation from all units satisfies the total annual demand. Equation (7) is the capacity constraint that states that the model cannot generate electricity over the built capacity. Equation (8) is the reserve margin requirement that is a reliability necessity and ensures that the built capacity is greater than or equal to the peak demand capacity plus the reserve margin capacity. Equations (9) and (10) represent the Renewable Portfolio Standards (RPS). Equation (9) satisfies the annual minimum percentage of renewable generation requirement and, Equation (10) satisfies the minimum percentage of renewable generation requirement from each renewable generation unit such as solar, wind, biomass, etc. Equation (11) represents the annual construction limit of each generation unit, and Equation (12) is the non-negativity constraint.

$$\sum_{i \in I} x_{y,t,r,i} = \tilde{d}_{y,t,r} \quad \forall y, t, r \quad (6)$$

$$x_{y,t,r,i} \leq \left(\#_{r,i} + \sum_{u=1}^y s_{u,r,i} \right) \times \tilde{w}_{t,i} f_t \tilde{p}_{t,r,i} \quad \forall y, t, r, i \quad (7)$$

$$\sum_{i \in I} \#_{r,i} + \sum_{u=1}^y \sum_{i \in I} s_{u,r,i} \geq \tilde{d}_{y,r}^p \tilde{m}_r \quad \forall y, r \quad (8)$$

$$\sum_{t \in T} \sum_{r \in R} \sum_{n \in N} x_{y,t,r,n} \geq s_{y,r}^{\min} \sum_{t \in T} \tilde{d}_{y,t,r} \quad \forall y, r \quad (9)$$

$$\sum_{t \in T} \sum_{r \in R} x_{y,t,r,i} \geq s_{y,r,i} \sum_{t \in T} \tilde{d}_{y,t,r} \quad \forall y, r, i \quad (10)$$

$$s_{y,r,i} \leq u_{y,r,i} \quad \forall y, r, i \quad (11)$$

$$x_{y,t,r,i}, s_{y,r,i} \geq 0 \quad \forall y, t, r, i \quad (12)$$

2.1.2 Single Objective Least cost GEP Models

GEP research often focuses on minimizing the cost that satisfies predefined targets of emissions, reliability and other important aspects of the problem by modeling them as constraints. Therefore, single objective least cost GEP formulation is the same with the above multi-objective formulation with only one difference. The second objective of the multi-objective formulation becomes either part of the cost objective (13) as another cost component (14) or as a constraint that limits the annual emissions amount of each pollutant (15).

If the emissions objective becomes a part of the cost objective, then the single objective function that minimizes the net present value of total cost is

$$\min \text{Cost} = \sum_y \left\{ (1 + ira)^{-y} \times (C_{y,1} + C_{y,2} + C_{y,3} + C_{y,4}) \right\} \quad (13)$$

where $C_{y,4}$ is the total cost for emissions of all types of pollutants:

$$C_{y,4} = \sum_{y \in Y} \sum_{t \in T} \sum_{r \in R} \sum_{i \in I} x_{y,t,r,i} \tilde{y}_{i,k} e_k \quad (14)$$

where e_k (\$/ton) is the cost of pollutant type k per ton emitted while generating electricity.

If the modeler decides to convert the emissions objective to a constraint, then the cost objective function stays the same as in the multi-objective formulation but a new constraint is added to the constraint set as

$$\sum_{i \in T} \sum_{r \in R} \sum_{i \in I} x_{y,t,r,i} \tilde{y}_{i,k} \leq L_{y,k} \quad \forall y, i, k \quad (15)$$

where $L_{y,k}$ is the annual emissions limit of pollutant type k in year y .

GEP researches in the literature are often least cost single objective models. Even when other objectives such as minimization of emissions or maximization of reliability are added to the model, the problem is generally modeled as a single objective one by transforming multiple objectives to a single objective. The methods for transforming a multi-objective problem into a single objective one are discussed in Section 2.3.2.

One of the earliest works where a least cost GEP is solved is by Masse and Gibrat [7]. They use linear programming for the French electric power industry. In their paper, the economy of uncertainty is converted into an economy of certainty by the use of safety margins over and above the probable values.

Anderson [8] provides a survey for such earlier work on least cost GEP models. First the investment problem is formulated in cost minimization form, and then the various approaches used to find optimum solutions are reviewed. There are three classes reviewed: marginal analysis, marginal analysis using simulation models, and global models. All the formulations presented in this survey are deterministic.

Later, Beglari and Laughton [9] describe a mathematical programming model for the economic planning of generation and transmission systems. They solve the least cost expansion plan for generation units and transmission lines where the objective is to minimize the total capital cost and operations cost.

Sawey and Zinn [10] also provide a linear mixed integer program for the planning of generation and transmission systems. The objective function is the minimization of the net present value of the capital investment cost associated with the construction of power plants and transmission lines plus the operating costs of the system. The model provides construction expansion schedules for power plants and transmission lines. Additionally, an approximate operating schedule for plants and transmission lines is provided.

Noonan and Giglio [11] formulate the GEP as a large-scale, chance constrained, mixed integer program. The solution algorithm employs Benders' partitioning principle. The objective function is to minimize the investment and operations cost. They use chance constraints that ensure that the probability that annual peak demand for each year will not be satisfied must be less than or equal to some specified level of risk. They provide an equivalent deterministic constraint for this chance constraint when the probability function of available capacity, at peak demand hour minus peak demand is normal.

Sherali et al. [12] propose a branch-and-bound algorithm to solve the least cost GEP problem that considers the option of investing in non-dispatchable or renewable energy sources. They consider discrete capacity expansion options for plants.

Later GEP research are reviewed by Kagiannas [13] focusing on the shift of markets from monopoly to competition. They first review the GEP models developed for a centralized monopolistic electricity system. Afterwards they emphasize the need for new techniques for GEP under the wholesale power competition.

Sirikum and Techanitisawad [14] present an application of genetic algorithms for solving the GEP problem. The problem is formulated into a mixed integer nonlinear

program that determines the most economical investment plan for additional thermal power generating units over a planning horizon, subject to the requirements of power demands, power capacities, loss of load probability (LOLP) levels, locations, and environmental limitations. They model the demand side management (DSM) program such that if the DSM program is implemented, it provides a power saving by the efficient energy using equipment.

Kannan et al. [15] provide an application and comparison survey of meta-heuristic techniques to the GEP problem and all papers surveyed treat the GEP as a single objective problem.

2.1.3 Multi-Objective GEP Models

GEP may involve trade-offs encompassing a wide range of economic, financial, social, security, independency and environmental criteria. Because of the size of the electric utility industry, the extent of its environmental impacts, and many public agencies who oversee it, multiple objectives have become a fact of life for decision makers in the electric generation industry [16]. For instance, renewable portfolio standards involve trade-offs between cost, reliability, system security and transmission investments.

One way of addressing multi-criteria problems is to quantify the decision maker priorities. Such methods in general are called *Multi-Criteria Decision Making* (MCDM) and include additive utility functions, the Analytical Hierarchy Process (AHP) and goal programming. In MCDM methods, the chosen objectives or criteria are compared and alternatives are ranked by each decision maker or interest group. An example application of that type is presented by Karni et al. [17], who had 19 policy makers use more than

one method to rank alternative electricity pricing policies. One of the problems with MCDM methods is that most decision makers are unsure of their priorities when a decision involves a unique problem along with strongly held and conflicting values [3]. Another major obstacle is limited access to key decision makers and interest group representatives.

Multi-objective optimization, on the other hand, helps to display the trade-offs between objectives and quantify value judgments. A general discussion about multi-objective optimization and methods are presented in Section 2.3.

Climaco et al. [18] present a multi-objective linear programming (MOLP) model for GEP that considers three objective functions: net present value of the expansion plans, reliability of the supply system, and environmental impacts. There are three categories of constraints: load requirements, operational restrictions and budget. Three generating technologies are considered for power system expansion: oil, nuclear and coal.

Mavrotas et al. [19] solve single period GEP with a new approach based on a mixed 0-1 multi-objective linear programming (MOLP) model for the Greek electricity generation industry where the objectives are to minimize cost and SO₂ emissions. The core of the model is a branch and bound algorithm, which has been modified for the multi-objective case and is capable of generating the whole set of efficient solutions.

Antunes et al. [5] present a multi-objective mixed integer linear programming (MOMILP) model for electricity generation expansion planning that allows integer values of investment decisions. This avoids the problem of converting continuous capacity values to integer values in a post-processing phase. The model considers three objectives: total expansion cost, the environmental impact associated with the installed

power capacity, and the environmental impact associated with the energy generation. Demand-side management (DSM) is also considered as an option in the planning process.

Meza et al. [20] propose a model with four objectives. The first objective is to minimize the investment, operation and transmission cost. The second objective is to minimize the amount of carbon dioxide (CO₂) emissions. The third objective function is to minimize the imported fuel, which is calculated by fuel used in each year by the corresponding forecasted price. The last objective function is to minimize the energy price risks. The approach to solve the problem is based on multi-objective linear programming to obtain the set of non-dominated solutions and the AHP to select the best alternative.

Meza et al. [4] solve a single period GEP problem where they minimize the same objective functions as in Meza et al. [20]. However, in this study they include the Kirchoff's second law into the model, making the problem nonlinear. Therefore, the GEP problem is solved to determine the number of generating units, the number of new circuits on the network and the voltage angle at each node. The proposed solution framework first determines the set of non-dominated solutions via a multi-objective evolutionary programming method based on a multi-objective genetic algorithm and then utilizes a clustering algorithm to decrease the number of alternatives considered for the alternative ranking process with the AHP.

Tekiner et al. [1] propose an approach to the GEP problem to minimize multiple objectives, such as cost and pollutant emissions over a long term planning horizon and explicitly consider the operational dispatching decisions and the availability of the system components over the planning horizon. Monte-Carlo simulation is used to generate

numerous scenarios based on the component availabilities and anticipated demand for energy. The problem is formulated as a mixed integer linear program, and optimal solutions are found based on the simulated scenarios with a combined objective function considering the multiple problem objectives. The different objectives are combined using dimensionless weights and a Pareto front is determined by varying these weights.

Tekiner et al. [6] later examine how the availability of smart grid technologies change and improve the electric power system generation expansion plans. The model specifically considers the improvement in the distribution system and the shift in the demand from peak to off-peak time periods. To consider the reliability of the system, unmet demand is added as a cost in the objective function. Monte Carlo simulation is used to generate component availability scenarios. The optimization problem is solved to find optimum expansion solutions considering these scenarios.

Murugan et al. [21] present an application of the Elitist Non-dominated Sorting Genetic Algorithm version II (NSGA-II) to solve the multi-objective generation expansion planning (GEP) problem. The GEP problem in the paper is considered as a bi-objective problem. The first objective is the minimization of investment cost and the second objective is the minimization of outage cost (or maximization of reliability). As they state in the paper, GEP has been treated as a multi-objective problem in very few papers and conventional techniques have been used to solve the problem. There are just a few applications of evolutionary algorithms used for the multi-objective GEP.

Nara [22] provides a general survey for state-of-the-art research of modern heuristics applications to power systems and notes that genetic algorithms are applied in many areas, and further development is expected.

Table 2 summarizes the reviewed multi-objective GEP models in this section providing the objectives that researchers included in their models and their contribution to the multi-objective GEP modeling.

Table 2. Reviewed multi-objective GEP models

Paper	Objectives	Contribution to the GEP
Climaco [18]	Cost, reliability and environmental impact	MOLP
Mavrotas [19]	Cost and SO ₂ emissions	Mixed 0-1 MOLP
Antunes [5]	Cost, environmental impact of capacity addition and generation	MOMILP
Meza [20]	Cost, CO ₂ emissions, imported fuel, energy price risk	AHP to select the best alternative
Meza [4]	Cost, CO ₂ emissions, imported fuel, energy price risk	A multi-objective evolutionary algorithm application
Tekiner [1]	Cost and air emissions	Considers the operational dispatching decisions and the availability of the system components over the planning horizon
Tekiner [6]	Cost and air emissions	Investigates the effects of smart grid technologies.
Murugan [21]	Cost, reliability	NSGA-II application

2.2 Uncertainty in the GEP

Uncertainty in power systems modeling is depicted in Figure 1 (adapted from [23]). As this figure shows, the level of uncertainty in GEP is very high relative to other

research areas because of its spatial dimensions and time scale. Therefore, it becomes necessary to introduce in the decision making process a systematic and consistent treatment of the various sources of uncertainty.

In the following subsections, first a literature review of the GEP models which consider uncertainty is provided, second the sources of uncertainty in the GEP modeling are summarized, and later, aleatory and epistemic uncertainties are introduced, as a way of categorization of uncertainties in general, and uncertainties of GEP as aleatory or epistemic are classified.

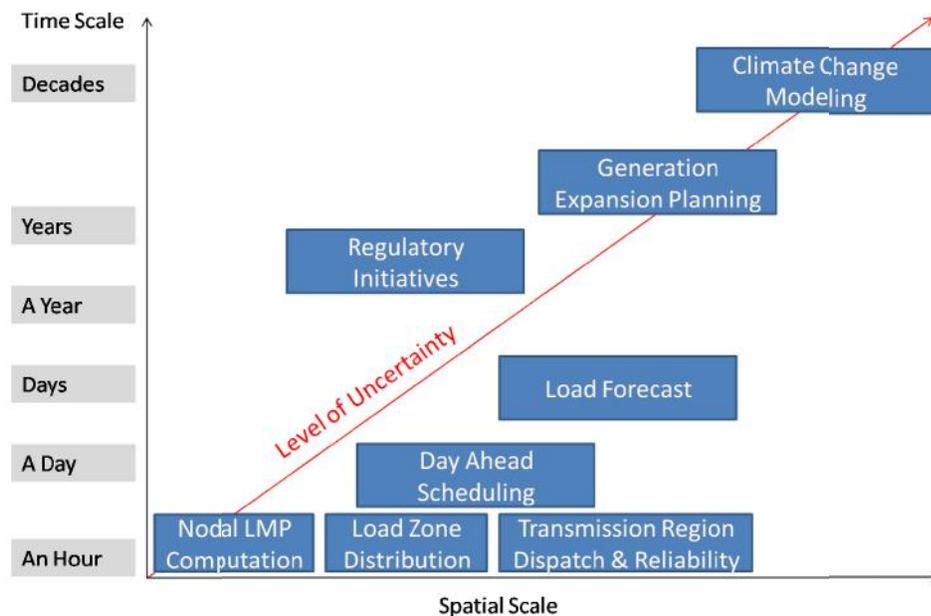


Figure 1. Uncertainty in power systems modeling (adapted from [23])

2.2.1 GEP Models Considering Uncertainty

The issues of uncertainty and risk associated with the GEP are mainly addressed by stochastic optimization, decision analysis, trade-off analysis and fuzzy set theory. Sahinidis [24] reviews theory and methodology that have been developed to cope with the complexity of optimization problems under uncertainty. The review discusses and

contrasts the classical recourse-based stochastic programming, robust stochastic programming, probabilistic (chance constraint) programming, fuzzy programming, and stochastic dynamic programming.

One of the first applications of stochastic programming to the GEP problem is presented in Dapkus and Bowe [25]. They formulate the GEP problem as a stochastic dynamic programming problem where they consider the uncertainties in demand, the commercialization date of new technologies and the possible loss of existing nuclear capacity due to an accident, regularity action or lack of fuel. Contingency plans are established which consider how uncertainty is resolved over time. These contingency plans are for decision makers to wait until uncertainty is resolved before committing to construction.

Mo et al. [26] also describe a method for handling uncertainties in GEP problems based on stochastic dynamic programming. The method is based on splitting up the problem into operation and expansion problems. Uncertainties of demand, water inflow for hydropower, fuel prices and investment costs are included in both the expansion problem and the operation problem.

Gorenstin et al. [2] describe a methodology that draws upon three classes of techniques. Decomposition and stochastic optimization provide the basic framework, and allow an implicit representation of alternative investment strategies. Decision analysis is used to represent the dynamic aspects of decision making as uncertainties are resolved over time and trade-off analysis is used to select expansion strategies. Uncertainties of inflows to hydro plants and load forecasts are considered in their case study.

Escudero et al. [27] use scenarios to characterize the uncertainty. Solutions are obtained for each scenario and then those individual solutions are aggregated to yield generation expansion planning that either minimizes the regret of wrong decisions, or minimizes the expected cost of the expansion plan. Uncertainties included are investment and operation costs, energy demand, economic environment, generation availability, and operation life.

Felder [28] integrates financial theory and methods in electricity resource planning using risk adjusted discount rates (RADR) and options theory (OT). It is claimed in the paper that by correctly using RADR and OT and understanding their limitations, decision makers can improve their ability to value risk properly in power plant projects and integrated resource plans.

Marin and Salmeron [29] present a stochastic optimization model under capacity deterioration and demand uncertainty. The goal of the model is to provide an initial generation plan for short periods of the planning horizon that might be adequately modified in real time assuming penalties in the operation cost. Uncertainty is modeled under the assumption that the demand is a random vector. The cost of the risk associated with decisions, that may need some tuning in the future, is included in the objective function. The proposed scheme to solve the nonlinear stochastic optimization model is generalized Benders' decomposition.

Dhar [30] introduces the concept of a fuzzy algorithm based on fuzzy sets in the power system long-range planning decision analysis. In the process of decision analysis, both quantitative and linguistic measures are used to quantify the utility of an alternative. The final objectives, the system states and constraints are not sharply defined and are

fuzzy in nature. Dhar claims that the decision maker implicitly equates the system imprecision or fuzziness with randomness because of unavailability of data. Therefore the goal of the paper is to emphasize the need to differentiate between randomness and fuzziness. By fuzziness, it is meant a type of imprecision which is associated with fuzzy sets. This claim is indicative of the differentiation between aleatory and epistemic uncertainties but the paper does not explicitly mention this classification.

Table 3. Reviewed models that include uncertainty in the GEP

Paper	Considered Uncertainties	Contribution to the GEP
Dapkus and Bowe [25]	Demand, the commercialization date of new technologies and the possible loss of existing nuclear capacity due to the accident, regularity action or lack of fuel	A stochastic dynamic programming model
Mo et al. [26]	Demand, water inflow for hydropower, fuel prices and investment costs	A stochastic dynamic programming model
Gorenstin et al. [2]	Inflows to hydro plants and load (demand) forecast	A model with combination of stochastic optimization, decision analysis and tradeoff analysis
Escudero et al. [27]	Investment and operation costs, demand, economic environment, generation availability and book life	Uses scenarios to characterize the uncertainty
Marin and Salmeron [29]	Capacity deterioration and demand	Nonlinear stochastic optimization model with Generalized Benders' decomposition
Dhar [30]	The final objectives, the system states and constraints are not sharply defined and are fuzzy.	The concept of fuzzy algorithm based on fuzzy sets
Torabi and Madadi [31]	Demand, reserve margin, transmission loss, generation unit capacity, costs and emissions are modeled with fuzzy numbers with triangular possibility distributions.	A fuzzy multi-objective mixed integer linear programming model

Torabi and Madadi [31] propose a fuzzy multi-objective mixed integer linear programming model (FMOMILP) for integrated power generation and transmission expansion planning problem. Uncertainties are modeled by appropriate fuzzy numbers with triangular possibility distributions. As such, for each fuzzy parameter, they consider three prominent values i.e., the pessimistic, optimistic, and the most likely values based on considering both available objective data and subjective data quoted by the field experts. However, they convert the MOMILP model into an equivalent single-objective MILP and solve this single-objective model. If the decision maker is satisfied with the solution, the algorithm stops. Otherwise, another solution has to be provided by changing the value of fuzzy parameters. Hence, the method is not very different than interactive classical methods discussed in Section 2.3.2 except using the fuzzy set theory.

Reviewed models in this section are summarized in Table 3. The models that explicitly consider uncertainty in the GEP problem have the issue of neglecting the multi-objective aspect of the problem. For the discussion of issues of uncertainty modeling in multi-objective optimization refer to Section 2.4.

2.2.2 Sources of Uncertainty in the GEP

Hobbs [3] lists some uncertainties that utility companies face in resource planning. This list is also viable in terms of the GEP problem and it is provided in Table 4. Gorenstin et al. [2] also list some sources of uncertainty which includes load growth rates, fuel costs, construction time, economic growth, environmental constraints, interest rates and financial constraints. In their case study, however, they only consider uncertainties of inflows to hydro plants and load forecast. Similarly, Mo et al. [26]

include uncertainties of energy demand, water inflow for hydropower, fuel prices and investment costs to their model.

Table 4. Sources of uncertainty in utility resource planning

<p><u>Market / demand uncertainties</u></p> <p>Load growth</p> <p>Price elasticity</p> <p>Markets for off-system sales and purchases</p> <p>Competition with non-electric fuels</p>
<p><u>Resource uncertainties</u></p> <p>Technological developments</p> <p>Availability and initial costs of resource options</p> <p>Construction times</p> <p>Fuel prices and emissions allowance prices</p> <p>Generating unit availability</p> <p>Climate change</p> <p>Water supplies</p> <p>Amount and dependability of nonutility generation</p> <p>Customer response to Demand-Side Management (DSM) programs</p> <p>Dependability and persistence of DSM</p>
<p><u>Legal and economic uncertainties</u></p> <p>Inflation, interest rates and economic growth</p> <p>Government policies concerning ratemaking and cost recovery</p> <p>Environmental regulations</p> <p>Municipalization / government takeover</p> <p>Public concerns</p>

In this dissertation, the focus is on uncertainties due to climate change, economic variations and technological development. Considered GEP problem parameters are peak and annual total demand, costs, reserve margin, availability and efficiency of power

plants (capacity and derate factors) and emissions rates of technologies. In our model we classify uncertainties as follows:

- Uncertainties due to climate change
 - Uncertainty in peak demand
 - Uncertainty in total demand
 - Uncertainty in reserve margin
 - Uncertainty in efficiency of power plants
- Economic uncertainties
 - Uncertainty in investment cost
 - Uncertainty in generation cost
- Uncertainties due to technological development
 - Uncertainty in fixed operation and maintenance cost
 - Uncertainty in availability factor
 - Uncertainty in derate factor
 - Uncertainty in emissions rate of technologies

Although variables are listed separately according to the most influential uncertainty, in reality every variable can be affected by more than one type of uncertainty. Figure 2 shows these multiple effects on GEP modeling parameters for the model of this study more explicitly.

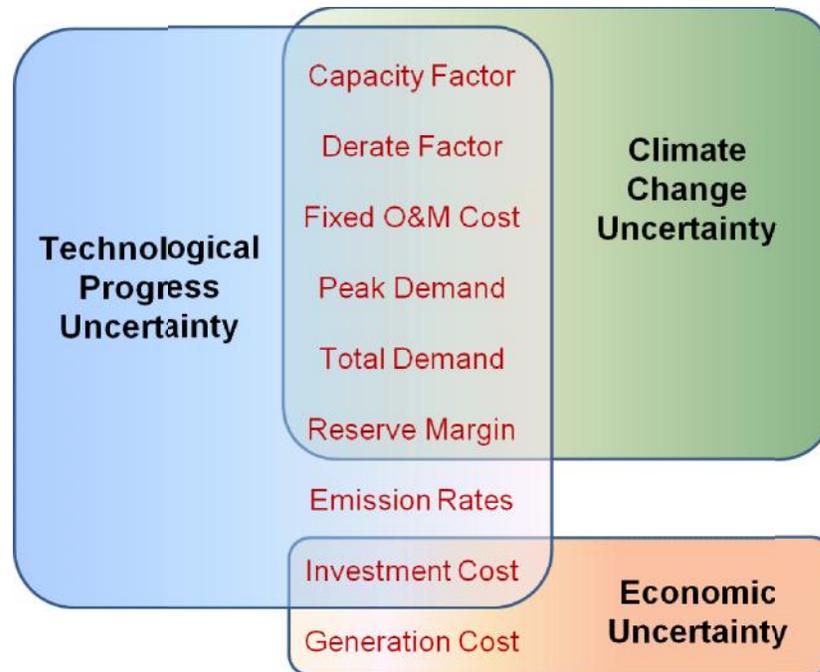


Figure 2. Effects of different types of uncertainty on variables of the GEP model

In the following subsections, these uncertainties and their impacts on GEP parameters of the model of this study and also the uncertainty of the impacts on the GEP problem are briefly discussed.

2.2.2.1 Climate Change Uncertainty

Assessing future benefits, risks and costs of different electricity options could be improved by integrating climate change into electricity and energy planning [32]. However, potential impacts of climate change and extreme weather on the energy sector are hard to predict and have many uncertainties due to a number of factors, such as insufficient data, models that are not yet able to represent the interactions and interdependencies of multiple stresses, and incomplete understanding of physical climate mechanisms. Furthermore, climate impacts tend to be localized and can be very difficult to predict [32].

According to the *Energy Sector Vulnerabilities to Climate Change and Extreme Weather* report of the US Department of Energy [33], at least three major climate trends are relevant to the energy sector:

- Increasing air and water temperatures
- Decreasing water availability in some regions and seasons
- Increasing intensity and frequency of extreme events, such as storms and flooding.

Chandramowli and Felder [34] provide a comprehensive review about the research where the impact of these climate trends on electricity systems and markets are studied. According to their review, the system-wide impact on renewable and thermal power generation, economic modeling of extreme weather events, transmission and coastal energy infrastructure vulnerabilities, long-term modeling and the effects on modeling outcomes are still uncertain.

In the model proposed in this study, among the three major climate trends listed above, increasing air temperature affects peak and total demand; increasing water temperature affects efficiency of thermal power plants and transmission system capacity; decreasing water availability affects hydropower units; increasing intensity and frequency of extreme weather events affect reserve margin and availability of power plants and transmission lines.

Although impacts of climate change on the energy sector and the electricity generation is a popular research area, uncertainty of climate change impact on the electricity generation expansion is not sufficiently studied. To capture the range of scientific uncertainty inherent in future projections of climate change, researchers usually

use combinations of scenarios from different atmosphere–ocean general circulation model (AOGCM) simulations by the Intergovernmental Panel on Climate Change (IPCC) Special Report on Emissions Scenarios [35]. The scenarios provide the basis for future assessments of climate change and possible response strategies. Using these scenarios, researchers try to calculate or quantify the impact and range of climate change. For example, Miller et al. [36] state that peak electricity demand and temperatures in California are strongly correlated. For temperatures above 28°C (82°F), California peak electricity demand exhibits a linear increase with temperature increase. For the uncertainty part, they provide the range of residential peak electricity demand increase between 2.8% and 10% under different scenarios. In another study by Gorenstin [2], uncertainties due to the annual demand growth are considered as a binary tree representing the different growth rates, resulting in 16 scenarios and 31 decision nodes for the 5-year study. Scenario 9 is the forecasted demand growth for this period used in the expansion planning studies. The other scenarios were built using small variations around the annual increase rate.

Li [37] proposes a linear programming based least cost robust optimization model where all the variables that are directly or indirectly impacted by the climate change are defined. Different scenarios are presented to model the possible outcomes of future weather. Each discrete climate scenario is a realization of a set of random variables over the planning horizon. Climate scenarios have three major climate variables namely; temperature, precipitation and extreme events. These climate variables correspond to six sets of GEP parameters: demand, peak demand, availability factor, transmission capacity, reserve margin and derate factor . The robust model finds a compromise solution that is

good for all scenarios, which avoids the possible risk associated with a poor decision that is only beneficial for several particular scenarios.

2.2.2.2 Economic Uncertainties

Sometimes the cost of an investment is very uncertain, particularly for large projects that take considerable time to build. An example is a nuclear power plant, for which total construction costs are hard to predict due to both engineering and regulatory uncertainties [38]. With projects that take time to complete, two different kinds of uncertainty arise: *technical uncertainty* and *input costs uncertainty* ([39] and [38]). Technical uncertainty relates to the physical difficulty of completing a project. This kind of uncertainty affects the quantification of cost uncertainty because of the calculation of the net present value of investment and uncertainty in interest rates.

Input costs uncertainty arises when the prices of labor, land, and materials needed to build a project fluctuate unpredictably, or when unpredictable changes in government regulations change the cost of construction [38]. Uncertainty in government regulations or uncertainty in decision making is not discussed in this dissertation, because the aim of models such as the one built in this dissertation is to help decision or policy makers to be more certain of their decisions or regulations. On the other hand, input costs uncertainty of investment is one of the most important parts of the model, because it directly affects one of the objective functions: minimization of cost.

Other than investment cost uncertainty, there is also operating cost uncertainty in GEP models. Operating cost includes fuel costs and fixed operation and maintenance costs and these costs and their uncertainty vary by type of technology. For example, the uncertainty of natural gas prices is very crucial to the investment decision of natural gas

fired technologies. On the other hand, renewable technologies such as wind or solar have no fuel costs so their operating cost uncertainty is very low.

The determination of operating cost of each of the expansion alternatives can be calculated by simulation. For example, Booth [40] uses a method of production costing based on probabilistic simulation methods which is combined with a dynamic programming formulation of the problem in order to treat uncertainty. This technique combines several factors subject to uncertainty in a calculation of both the probability of loss of load and the expected value of the energy production of the various stations, and thus, the total system operating cost.

2.2.2.3 Uncertainties Due to Technological Development

Uncertainty about the future rate and direction of technological change is often an important part of engineering problems. Technological change creates new opportunities and/or constraints and it has a meaningful effect on cost analysis of future investments. The environmental impact of social and economic activity is greatly affected by the rate and direction of technological change. New technologies may either create or mitigate pollution, and many environmental problems and policy responses are evaluated over time frames in which the cumulative impact of technological changes is likely to be large. However, technological advancement is difficult to project beyond a 10-year timeline because of the large uncertainties pertaining to the rate of discovery, evaluation, and social adaptation of new technologies [41]. Heal and Millner [42] give an example of coal which provided 52% of the US electric power in 2008 and decreased to 38% in 2013. As they stated, such a change would have been hard to anticipate as recently as ten years before this observation.

Measuring the effects of environmental technological change is equally if not more challenging, because innovation and market penetration of new technologies and the environmental policy to encourage adoption of environmental technology should be separately identified. As Popp et al. [43] state, one of the most difficult questions remaining in aggregate energy-economic modeling is the appropriate treatment of technological change, particularly for analyzing long-term environmental and resource problems.

Technological progress has direct effects on the availability factor, derate factor and emissions rate of power plants in our model, which makes related input variables uncertain. It can also indirectly affect peak demand and total demand if smart grid and demand side management options are included in the modeling. Furthermore, if technological development in the reliability area is considered, then reserve margin can be reduced beyond its forecast.

2.2.3 Aleatory and Epistemic Uncertainty

While many sources of uncertainty may exist, a way of categorization is to classify uncertainty into two categories: aleatory or epistemic. Epistemic uncertainty is one that is presumed as being caused by lack of knowledge (or data). Uncertainties are characterized as epistemic, if the modeler sees a possibility to reduce them by gathering more data or by refining models [44]. Epistemic uncertainty is knowledge based and therefore can be reduced by better information. But that information or knowledge may be not available to the modeler at the time of modeling.

Aleatory uncertainty is one that is presumed to be the intrinsic randomness of a phenomenon. Uncertainties are categorized as aleatory if the modeler does not foresee the

possibility of reducing them [44]. Aleatory uncertainty arises because the study system can potentially behave in many different ways. Faber [45] states that the uncertainty associated with a model concerning the future transforms from a mixture of aleatory and epistemic uncertainty to a purely epistemic uncertainty when the modeled phenomenon is observed. Furthermore, a model dominated by epistemic uncertainties has the potential for reducing the uncertainties by updating.

2.2.4 Importance of Aleatory and Epistemic Uncertainty Distinction

Aven [46] and Paté-Cornell [47] state that classical decision theorists have often taken the position that the distinction between aleatory and epistemic uncertainties is unnecessary. The different probabilities for both types of uncertainties can be combined for decision-making purposes as if all uncertainties were of the same nature. Dubois [48], however, expresses that if the imperfect information about inputs or parameters are all represented by probability distributions, then the resulting distribution of the output can hardly be properly interpreted. The part of the resulting variance due to epistemic uncertainty that could be reduced is unclear. Aven [46] also claims that one should acknowledge that the full scope of uncertainties cannot be transformed to a mathematical formula using probabilities. By skipping the distinction and directly using probabilities, important uncertainty aspects could easily be overlooked or truncated, meaning that potential surprises could be left unconsidered. By classifying uncertainties beyond probabilities, models are able to give the decision-makers more informative and detailed risk description.

On the other hand, there are many occasions where the distinction is helpful and needed. Paté-Cornell [47], for example, asserts that decision-makers may need or ask for

a full display of the magnitudes and the sources of uncertainties before making an informed judgment.

Kiureghian and Ditlevsen [44] argue that the nature of uncertainties and how one addresses them depends on the context and application, so the distinction between aleatory and epistemic uncertainties is determined by modeling choices. The distinction is useful for identifying sources of uncertainty that can be reduced in the near-term, i.e., without waiting for major advances to occur in scientific knowledge, and in developing sound risk and reliability models. The distinction is also important from the viewpoint of transparency in decision-making, since it then becomes clear as to which reducible uncertainties have been left unreduced.

As Faber [45] concludes, a central role for engineers is to provide a basis for decision making in situations where uncertainties are present. The key point, as Aven [49] states, is to guarantee that uncertainties are taken into account in a way that the information and knowledge relevant for the problem are represented as precisely as possible. A planning process that ignores or underestimates uncertainty may yield plans that perform disappointingly under circumstances other than the narrow ones considered when developing the plan [3].

2.2.5 Mathematical Representations of Aleatory and Epistemic Uncertainties

Climate change, future technological progress and the long-run global economy are a few examples where incomplete knowledge of fundamental phenomena dominates uncertainties in the problem. These are fundamentally irreducible uncertainties because the uncertainties about these issues are not readily quantified and expressed in probabilistic terms. Paté-Cornell [47] argues that epistemic uncertainties are sometimes

ignored, especially in public policy studies of sensitive issues. By contrast to epistemic uncertainty, aleatory uncertainty is more easily integrated in mathematical models because there is sufficient historical data to model by probabilistic distributions. According to Paté-Cornell [47] the problem is that available samples are insufficient to represent exactly the phenomenon of interest.

Aleatory and epistemic uncertainties can be represented and analyzed by different mathematical methods depending on the available information. In the situations where the uncertainty of the variables is mainly due to inherent randomness and there is sufficient information to assign probability distributions and estimate their parameters, probabilistic modeling is preferred [50]. When there is limited or insufficient information to determine parameters for probability distributions, fuzzy and possibility distributions or scenarios can be suitable for alternative representations of epistemic uncertainties.

Helton et al. [51] provide a comprehensive review where the use of several uncertainty representations are explored and compared. The review summarizes probability theory, evidence theory, possibility theory and interval analysis. Problems that involve both aleatory and epistemic uncertainty are solved with different formulations of the uncertainty in the parameters. In these problems, aleatory uncertainty is represented with probability theory, and representations of epistemic uncertainty with probability theory, evidence theory, and possibility theory are demonstrated.

Joint propagation and aggregation of uncertainties under probability and possibility theories have been studied by various researchers. Baudrit et al. [52], for example, represent random variability (aleatory uncertainty) by probability distribution functions, and imprecision (epistemic uncertainty) by possibility distributions. A hybrid method is

then used to jointly propagate probabilistic and possibilistic uncertainty which produces results in the form of a random fuzzy interval. Baraldi and Zio [53] extend this framework to event tree analysis and propose a hybrid approach for the propagation of mixed uncertainty information where the Monte Carlo technique is combined with fuzzy set theory. To process the uncertainty of the probabilistic and possibilistic variables, Monte Carlo sampling and fuzzy interval analysis are used, respectively. Flage et al. [54] use a similar framework with fault tree analysis and possibility-probability (probability-possibility) transformations to jointly propagate uncertainties within purely probabilistic and possibilistic settings.

The review by Helton et al. [51] discusses very important issues for different representations of uncertainties, which can be summarized as follows:

- Separation of aleatory and epistemic uncertainties is a major conceptual and computational challenge.
- The uncertainty characterization is a very challenging part of a practical analysis because most practical analyses require expert review and assessment process to convert available information into a mathematical form.
- Multiple uncertainty characterizations must be aggregated into a single characterization and this aggregation must be consistent with the supplied uncertainty information and characterizations.
- The cost of the model evaluations for propagation and aggregation of uncertainties are often very expensive in real analyses.
- A meaningful interpretation of results has to be based on underlying uncertainty representations.

In conclusion, as Aven [49] states, any method of uncertainty representation and propagation must address a number of very practical issues such as complete and faithful representation of available knowledge and information, cost of the analysis, and the confidence that the decision maker gains from the analysis and the results.

2.2.6 Aleatory and Epistemic Uncertainties in Power Systems and the GEP

There is no previous research that has focused on aleatory and epistemic uncertainty identification and classification in GEP models. Furthermore, there is only a very limited amount of research about such classification in power systems planning and optimization in general. In one of the studies, Li and Zio [50] address the issues of identifying, classifying and representing the epistemic and aleatory uncertainties in distributed generation systems. Probability and possibility distributions are used to model the aleatory and epistemic uncertainties, respectively. Five components of the distributed system, solar generators, wind turbines, electrical vehicle (EV) aggregation, transformers and load are considered as uncertain parameters. Uncertainties in operation parameters of wind and solar units and power output of EV aggregation are represented by possibility distributions, mainly because the type of information available for these variables is expert judgments. Uncertainties in solar irradiation, wind speed, power fluctuations, time to failure for transformers and load are represented by probability distributions because there is sufficient historical data to define distributions and parameters of distributions.

Two different propagations are introduced for modeling probabilistic and possibilistic uncertainties. The first technique is an algorithm of joint propagation of two types of uncertainty. Repeated Monte Carlo Sampling (MCS) is used to process the uncertainty in probabilistic variables as the outer loop of the algorithm and fuzzy interval analysis is

used for treating the uncertainty in possibilistic variables as the inner loop of the algorithm. The second technique presented for modeling probabilistic and possibilistic uncertainties is the pure probabilistic propagation where the possibilistic distributions are converted into probability density functions by normalization. In the numerical case study the effects and comparison of the joint propagation and the pure probabilistic approach is demonstrated and discussed. According to the numerical example results, the cumulative distribution by the pure probabilistic method lies within the belief and plausibility functions obtained by the joint propagation approach. It is claimed that the imprecision in the parameters is explicitly reflected by the gap between the belief and plausibility functions. It is also implied that incorporating the imprecision due to incomplete knowledge, can be relevant for the decision maker.

In another power systems related work, Billinton and Huang [55] illustrate the differences of aleatory and epistemic uncertainty by incorporating the implications of these uncertainties in power system reliability evaluation. They examine the impacts of load forecast uncertainty, wind power and their interactive effects on the system reliability. Although future load growth uncertainty is defined as an epistemic uncertainty, it is represented with a normal distribution whose parameters are estimated from past data. The aleatory uncertainty associated with the annual loss of load index is generated by Monte Carlo simulation using various load uncertainty standard deviations. An Auto-Regressive Moving Average (ARMA) time series model is built for wind power using the past data. The uncertainty in wind power model is due to the variability of the wind speed which is modeled as a normal white noise process. The load forecast uncertainty and wind model are incorporated in the simulation process. The effects of

load forecast uncertainty and wind power on the test system are presented, and in conclusion, it is asserted that better load forecasting techniques can reduce the effects of both aleatory and epistemic uncertainty.

Although research on aleatory and epistemic uncertainty distinction in power systems and GEP problems is minimal, uncertainties in parameters of power systems are studied extensively. One of the most studied stochastic variables in the power systems modeling is the load (demand) uncertainty. There are several factors that affect the electricity demand and forecasts in modeling such as weather variability due to climate change, cost variability due to economic uncertainty and forecast model imprecision due to lack of knowledge. Tiabrat and Eua-arporn [56] discuss the application of the normal distribution for a load distribution model because there are several factors having impact on the forecasted demand. However, it is also important to investigate the historical data of long-term load forecasts in the study region. For example, Tiabrat and Eua-arporn [56] in their case study for Thailand find out that most of the forecasted peak load is always higher than the actual peak load. Therefore, a Weibull distribution function is chosen. Marin and Salmeron [29] also uses a statistical preprocessing to derive a probability distribution representation of the demand. Zhai et al. [57] use a Gauss-Markov random sequence load model to model the uncertain load where the hourly load assumes a normal probability distribution. Billinton and Huang [58] discuss that the most common practice is to describe the load uncertainty by a normal distribution with a given standard deviation. They represent the load uncertainty with a normal distribution whose parameters are estimated from past data.

Economic uncertainty is another important uncertainty source in power systems. Therefore, costs are modeled as stochastic variables by many researchers and several methods are proposed to manage the cost uncertainty. Siriruk and Valenzuela [59] asserts that the cost of a specific fuel type (coal, natural gas, etc.) is assumed to be a continuous random variable which has an associated probability density function. They use past data of coal and natural gas to fit a probability density function and a lognormal distribution is selected to represent the price of coal and the price of natural gas. The cost of uranium and hydro are not notably volatile in their case study so they are assumed to be constant. Torabi and Madadi [31] model fuel cost, maintenance cost, transmission cost and outage cost by appropriate fuzzy numbers with triangular possibility distributions. Wang [60] constructs a set of scenarios that represent the evolution of cost uncertainty. Shan [61] models fuel costs as discretely distributed random variables and a rolling two-stage approach is applied to solve the stochastic recourse problem. In long-term planning such as a GEP problem, it is also reasonable to use normal distributions for the variability of costs because an increase or a decrease is likely to occur on either side of the expected cost in the long-term.

Other stochastic variables of the GEP problem such as reserve margin, capacity and derate factors and emissions amounts are mostly affected by several sources as shown in Figure 2. Furthermore, technological progress and climate change uncertainty, which have the main effects on reserve margin, capacity and derate factors and emissions amount of technologies, are hard to predict and have unpredictable variability due to a number of factors. Therefore these uncertainties can be classified as epistemic uncertainty and can be modeled by possibility distributions or scenarios. Torabi and Madadi [31]

model reserve margin, transmission loss and generation unit capacity, forced outage rate, and amount of emitted pollutant by appropriate fuzzy numbers with triangular possibility distributions. They consider three values for the fuzzy numbers of triangular possibility distributions, i.e., the pessimistic, optimistic, and the most likely values based on considering both available objective data and subjective data quoted by the field experts. This method is similar to defining pessimistic, optimistic, and the most likely scenarios for future realizations. Escudero et al. [27] directly use scenarios to characterize the uncertainty in investment and operation costs, energy demand, economic environment, generation availability, and operation life of power plants. Li [37] defines all the variables that directly or indirectly impacted by the climate change and different scenarios are presented to cover the possible outcomes of future weather variability. Each discrete climate scenario is a realization of a set of random variables over the planning horizon. Climate scenarios have three major climate variables namely; temperature, precipitation and extreme events. These climate variables correspond to six sets of GEP parameters: demand, peak demand, availability factor, transmission capacity, reserve margin and derate factor .

A GEP model for the Northeast US where the stochastic parameters are classified according to their uncertainty type, i.e., aleatory or epistemic, is proposed in Section 5. Uncertainty representation of each parameter is also discussed.

2.3 Multi-Objective Optimization

Optimization is the process of selecting the best solution for a problem, subject to some constraints, from some set of available alternatives. An optimization model seeks to find values of the decision variables that optimize (maximize or minimize) an objective

function among the set of all values for the decision variables that satisfy the given constraints [62].

An objective function is the performance measure to be maximized or minimized (e.g., maximize profit, minimize cost). Decision variables are the set of values that represent the decisions to be made and implemented (e.g., type of technology to invest in, number of plants to be built). Constraints are limitations or requirements on the set of allowable decisions (e.g., budget limitations). Mathematical notation of an optimization model is as follows:

$$\begin{aligned}
 \min / \max \quad & f_m(\mathbf{x}) && m = 1, 2, \dots, M \\
 \text{subject to} \quad & g_j(\mathbf{x}) \geq 0 && j = 1, 2, \dots, J \\
 & h_k(\mathbf{x}) = 0 && k = 1, 2, \dots, K \\
 & x_i^{(L)} \leq x_i \leq x_i^{(U)} && i = 1, 2, \dots, n
 \end{aligned} \tag{16}$$

where,

- $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$ is the vector of the n decision variables.
- $f_m(\mathbf{x}_i)$ is the m^{th} objective function evaluated at solution vector \mathbf{x}_i .
- M is the number of objectives.
- J and K are the number of inequality and equality constraints.
- $x_i^{(L)}$ and $x_i^{(U)}$ are respectively the lower and upper bounds for each decision variable x_i .

In the above optimization model when the number of objectives is equal to one ($M = 1$), the problem is called a *single objective optimization*.

However, most real life optimization problems are multi-objective problems in which objectives under consideration often conflict with each other. When an optimization problem involves more than one objective function ($M \geq 2$), the task of finding optimum

solution(s) is known as *multi-objective optimization*. In multi-objective optimization problems, the decision maker wants to obtain more than one goal. For example, in the GEP problem, minimization of cost is a very important objective but increasing the reliability and minimizing emissions of pollutants are also crucial. These two objectives require spending which conflicts with minimization of the cost objective.

In multi-objective optimization problems, there is usually not a single optimal solution, but there is a set of solutions as a result of the optimization process. Any optimization process first produces *the objective space* as shown in Figure 3. Each value of each objective function evaluated at different decision variable vectors is reflected in the objective space with one-by-one correspondence. Then the set of solutions, which provides a trade-off curve of conflicting objectives for the multi-objective optimization problem, is determined. This set is called *the Pareto optimal set* and it is determined by using *the Pareto dominance* concept.

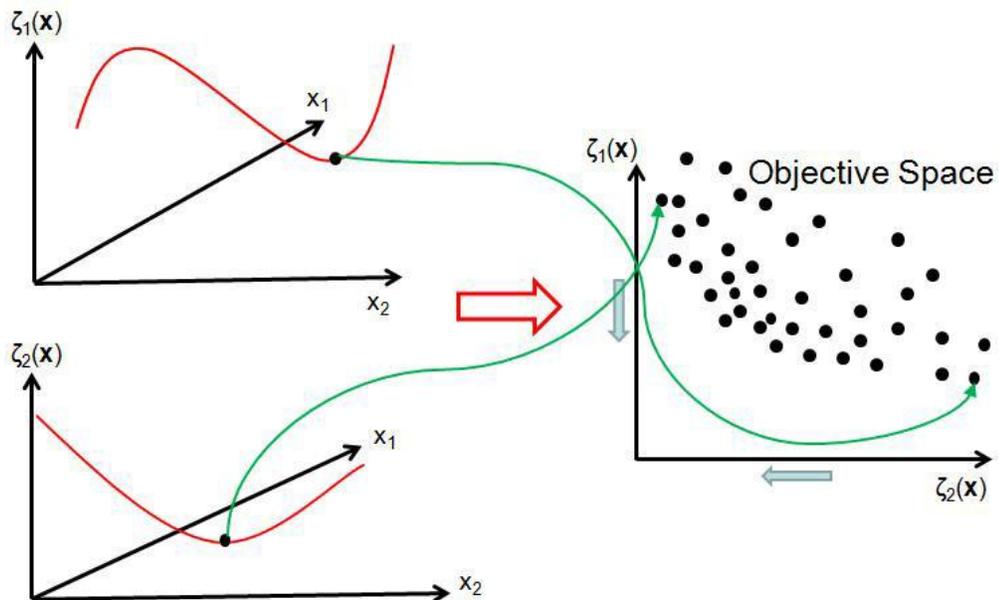


Figure 3. One-by-one correspondence between the decision space and the objective space

2.3.1 Pareto Optimality

For multi-objective optimization problems, a common solution methodology is to determine a Pareto optimal set which provides a trade-off curve of conflicting objectives. Solutions in the objective space are compared in pairs according to the Pareto dominance concept to determine the Pareto optimal set. Moving from one point to another on the trade-off curve (or in the Pareto optimal set) means losing in one objective of the problem in return for gaining in another objective for competing objective functions (Figure 4).

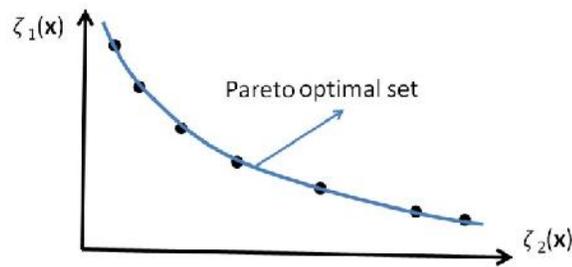


Figure 4. Pareto optimal set gives a trade-off curve of conflicting objectives. Every point is an expected solution to the multi-objective optimization problem

2.3.1.1 The Pareto Dominance Concept

Pareto optimality uses the concept of dominance. For a minimization problem, a solution \mathbf{x}_i said to dominate another solution \mathbf{x}_j , $\mathbf{x}_i \succ \mathbf{x}_j$, if these two conditions are both satisfied:

1. The solution \mathbf{x}_i is no worse than \mathbf{x}_j for all M objective functions, that is,

$$f'_m(\mathbf{x}_i) \leq f'_m(\mathbf{x}_j) \text{ for all } m = 1, 2, \dots, M$$

2. The solution \mathbf{x}_i is strictly better than \mathbf{x}_j for at least one objective function, that is,

$$f'_m(\mathbf{x}_i) < f'_m(\mathbf{x}_j) \text{ for at least one } m \in \{1, 2, \dots, M\}$$

where $f_m(\mathbf{x}_i)$ is the m^{th} objective function evaluated at solution vector \mathbf{x}_i and M is the number of objective functions.

For instance, in Figure 5 (all objectives are to be minimized), Solution 1 dominates Solution 3 because, even though they have the same value for objective function 1, Solution 1 is a better solution than Solution 3 in terms of objective function 2. On the other hand, there is no solution that satisfies the above conditions compared to Solution 2. Pairwise comparisons between Solutions 1 and 2, and Solutions 2 and 3 show that Solution 2 is not dominated by 1 or 3, and also Solution 2 dominates neither 1 nor 3. Therefore, Solutions 1 and 2 are called non-dominated Solutions and they are in the Pareto optimal set. Solution 3 is called a dominated solution and it is not in the Pareto optimal set.

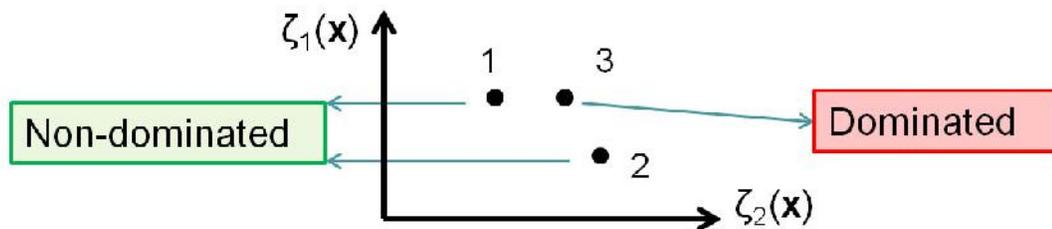


Figure 5. The concept of dominance for a minimization problem

Using the concept of dominance, solutions in the objective space can be compared in pairs to find the Pareto optimal set, which provides a trade-off curve of conflicting objectives (Figure 3). Moving from one point to another on the trade-off curve (or in the Pareto optimal set) means losing in one objective of the problem in return for gaining in the other objective.

There are two important tasks while finding the Pareto optimal set in multi-objective optimization: convergence and diversity. Convergence is finding the closest Pareto front

to the true Pareto front of the problem which is often unknown. Diversity means that solutions in the best-known Pareto optimal set should be uniformly distributed and diverse over the Pareto front in order to provide the decision-maker a true picture of trade-offs and the diversity of solutions.

Many algorithms exist involving multiple objectives to determine the Pareto optimal set. The majority of these methods transform multiple objectives into a single objective function. Thus, most of these classical methods do not treat multi-objective optimization any differently than single objective optimization. Furthermore, these classical optimization methods can at best find one solution, and thereby making those methods inconvenient to solve multi-objective problems [63].

2.3.2 Classical Methods

In this section some of the commonly used classical methods for handling multi-objective optimization problems are described. Following Deb's explanation [63], these methods are referred to as classical methods, mainly to distinguish them from evolutionary algorithms, which are discussed in the following section.

Miettinen [64], classified the classical methods in the following four types:

1. **No-preferences methods** do not assume any information about the importance of objectives but a heuristic is used to find a single optimal solution.
2. **Posteriori methods** use preference information of each objective and iteratively generate a set of Pareto optimal solutions.
3. **A priori methods** use more information about the preference of the objectives and usually find one preferred Pareto optimal solution.

4. **Interactive methods** use the preference information progressively during the optimization process.

The following presented classical methods are outlined in the order of increasing use of preference information.

2.3.2.1 Weighted Sum Method

This method combines a set of objectives into a single objective by pre-multiplying each objective with a user supplied weight as,

$$\begin{aligned} \min \quad & \sum_{m=1}^M w_m f_m(\mathbf{x}) \\ \text{s.t.} \quad & \mathbf{x} \in X \end{aligned} \quad (17)$$

where $\sum_m w_m = 1$, $w_m \in [0,1]$, $\forall m \in \{1, \dots, M\}$ and X represent the feasible set.

It is likely that in an optimization problem different objectives take different orders of magnitude. For example, cost of a product may vary between 100 to 1000 dollars but the amount of wasted material may vary between 0.01 to 0.1 m³. Therefore, there is a need to scale the objectives appropriately to make them equally important [63]. This is called *normalization*. This procedure is a drawback for the weighted sum method because it may affect the solution quality [1]. Other disadvantages of the weighted sum method are as follows:

- There is a need to define weights for objectives.
- In nonlinear problems, uniformity/diversity of Pareto set is not guaranteed.
- More tests are needed to know whether the solution is truly optimal (convergence).

- The method fails to find some Pareto optimal solutions in a non-convex objective space.

On the other hand this method has some advantages such as:

- It is very simple.
- It is easy to implement.
- For convex Pareto optimal fronts, it guarantees finding solutions on the entire set.

2.3.2.2 The ϵ -Constraint Method

This method selects one of the objectives and restricts the rest within user-specified values. v_m represents an upper bound of the value of m^{th} objective function (f_m) and not need necessarily to mean a small value close to zero [65].

$$\begin{aligned}
 \min \quad & f_{\tilde{m}}(\mathbf{x}) \\
 \text{s.t.} \quad & f_m(\mathbf{x}) \leq v_m \quad m = 1, 2, \dots, M \text{ and } m \neq \tilde{m} \\
 & g_j(\mathbf{x}) \geq 0 \quad j = 1, 2, \dots, J \\
 & h_k(\mathbf{x}) = 0 \quad k = 1, 2, \dots, K
 \end{aligned} \tag{18}$$

Mavrotas [66] proposes a novel version of the method (augmented ϵ -constraint method – AUGMECON) that avoids the production of weakly Pareto optimal solutions and accelerates the whole process by avoiding redundant iterations. An interactive approach that is based on AUGMECON and eventually results in the most preferred Pareto optimal solution is also proposed in the paper.

Advantages of the ϵ -constraint method are:

- It is simple.
- It is easy to implement.
- It can be used for convex or non-convex or discrete objective spaces.

Some disadvantages of this method are as follows:

- Solution of the problem largely depends on chosen w -vector and it has to be chosen carefully to find feasible solutions.
- More objectives in the problem mean more elements in w -vector which means more information is needed from the user.

2.3.2.3 Weighted Metric Methods

These methods combine multiple objectives into single objective using weighted metrics, l_p and w_m .

$$\begin{aligned} \min \quad & l_p(\mathbf{x}) = \left(\sum_{m=1}^M w_m \left| f_m(\mathbf{x}) - z_m^* \right|^p \right)^{1/p} \\ \text{s.t.} \quad & g_j(\mathbf{x}) \geq 0 \quad j = 1, 2, \dots, J \\ & h_k(\mathbf{x}) = 0 \quad k = 1, 2, \dots, K \end{aligned} \quad (19)$$

where z_m^* is the ideal solution to the m^{th} objective function of the problem.

When $p = 1$, this equation is simply the weighted sum method. When $p = 2$, a weighted Euclidean distance of any point in the objective space from the ideal point is minimized. When a very large p is used, problem becomes minimization of the largest deviation $\left| f_m(\mathbf{x}) - z_m^* \right|$. This is called *the weighted Tchebycheff* problem.

The weighted Tchebycheff method guarantees to find each and every Pareto optimal solution. However, as p increases the problem becomes non-differentiable. Normalization is needed in metric weighted methods, which requires the knowledge of minimum and maximum function values of each objective. Furthermore ideal solutions z_m^* are required and all objectives are needed to be independently optimized.

2.3.2.4 Value (Utility) Function Method

The user provides a mathematical value function relating all objectives. The value function must be valid over the entire feasible search space and must be strongly decreasing: The preference of a solution must increase if one of the objective function values is decreased while keeping the others the same [63]. The task is to maximize the value function:

$$\begin{aligned} \max \quad & U(\mathbf{x}) \\ \text{s.t.} \quad & g_j(\mathbf{x}) \geq 0 \quad j=1,2,\dots,J \\ & h_k(\mathbf{x}) = 0 \quad k=1,2,\dots,K \end{aligned} \quad (20)$$

where $\mathbf{x} = (x_1(\mathbf{x}), x_2(\mathbf{x}), \dots, x_M(\mathbf{x}))^T$.

Again, this method is simple, easy to implement and it can be used with a discrete set of feasible solutions. However, the method depends entirely on the value function and there is a danger of using an over-simplified function. It is also a drawback of the method that the value function must be valid over the entire feasible search space.

2.3.2.5 Goal Programming Methods

Goal programming was first introduced by Charnes et al. [67] and gained popularity after the works of Lee [68] and Ignizio [69].

The main idea in these methods is to find predefined target solutions which can be different from optimal solutions. If there are no solutions for the targets, then the task is to find solutions which minimize deviations from the targets. Deviation variables n and p are used for different target types as,

$$\begin{array}{ll}
\text{Less than or equal to} & f'_m(\mathbf{x}) \leq t_m & f'_m(\mathbf{x}) - p_m \leq t_m \\
\text{Greater than or equal to} & f'_m(\mathbf{x}) \geq t_m & f'_m(\mathbf{x}) + n_m \geq t_m \\
\text{Equal to} & f'_m(\mathbf{x}) = t_m & f'_m(\mathbf{x}) - p_m + n_m = t_m \\
\text{Within a range} & f'_m(\mathbf{x}) \in [t_m^l, t_m^u] & f'_m(\mathbf{x}) - p_m \leq t_m^l \text{ and } f'_m(\mathbf{x}) + n_m \geq t_m^u
\end{array} \tag{21}$$

In general, the objective of goal programming methods is to minimize the deviation variables n and p .

When targets are used as constraints and objectives are combined in a composite objective function with deviations, it is called *weighted goal programming* as indicated in Equation (22).

$$\begin{array}{ll}
\min & \sum_{m=1}^M (\Gamma_m p_m + S_m n_m) \\
\text{s.t.} & f'_m(\mathbf{x}) - p_m + n_m = t_m \quad m = 1, 2, \dots, M \\
& \mathbf{x} \in X \\
& p_m, n_m \geq 0 \quad m = 1, 2, \dots, M
\end{array} \tag{22}$$

When objectives are prioritized and goal programming problem is solved sequentially for each objective, the method is called the *lexicographic goal programming*. In the first level of the method all other objectives are ignored and the prioritized objective function is optimized. In the second level the next prioritized objective function is optimized considering the set of optimal solutions of the first objective function. The method continues in this fashion until all the objective functions are optimized.

When the maximum deviation (d) in any goal from the target is minimized, the method is called the *min-max goal programming*, as follows:

$$\begin{array}{ll}
\min & d \\
\text{s.t.} & \Gamma_m p_m + S_m n_m \leq d \quad m = 1, 2, \dots, M \\
& f'_m(\mathbf{x}) - p_m + n_m = t_m \quad m = 1, 2, \dots, M \\
& \mathbf{x} \in X \\
& p_m, n_m \geq 0 \quad m = 1, 2, \dots, M
\end{array} \tag{23}$$

2.3.2.6 Review of Classical Methods

The classical methods convert a multi-objective problem into a single objective problem. Convergence to the Pareto optimal set is their strength. Methods are simple and easy to implement. However, only one Pareto optimal solution can be found in one simulation run of a classical method and not every simulation run produces different solutions. There can be identical solutions with different parameters. Not all Pareto optimal solutions can be found by some methods in non-convex problems. Some problem knowledge is required, such as weights or target values or value functions. Furthermore, classical methods may not produce a uniformly spaced Pareto optimal set, which means diversity may not be satisfied.

2.3.3 Multi-Objective Evolutionary Algorithms

Evolutionary algorithms are stochastic search methods that simulate the process of natural evolution to solve problems with a complex objective space. They are inspired on the survival of the fittest principle of the natural evolution.

In an evolutionary algorithm a population of solutions is processed in every generation (or iteration) and this feature gives an evolutionary algorithm a tremendous advantage for its use in multi-objective optimization because convergence can be achieved in only one iteration [63]. Furthermore, evolutionary algorithms have special operators that preserve the diversity in the Pareto optimal set, which is a drawback of most of the classical methods. In addition to these advantages, evolutionary algorithms also specifically address the following difficulties of classical methods:

- In classical methods, the convergence to an optimal solution depends on the chosen initial value. Evolutionary algorithms do not need initial values of problems.
- Most classical methods tend to converge to a suboptimal solution. Evolutionary algorithms have special operators to avoid this problem.
- A classical method efficient in solving one problem may not be efficient in solving another problem. Evolutionary algorithms, however, are capable of dealing with any problem.
- Some classical methods are not efficient with problems having a discrete search space, but evolutionary algorithms are applicable to any kind of problem.
- Classical methods cannot be efficiently used on parallel machines. Evolutionary algorithms do not have such disadvantage.

Table 5. Basic evolutionary algorithm form [70]

1	<i>Initialize</i> the population with random individuals
2	<i>Evaluate</i> each individual
3	Repeat
4	<i>Select</i> parents
5	<i>Recombine</i> pairs of parents
6	<i>Mutate</i> the resulting offspring
7	<i>Evaluate</i> new individuals
8	<i>Select</i> individuals for the next generation
9	Until a termination condition is satisfied

A generic form of a basic evolutionary algorithm is shown in Table 5. Every iteration of the algorithm corresponds to a generation, where a population of candidate solutions to a given optimization problem, called individuals, is capable of reproducing and is subject to genetic variations followed by the environmental pressure that causes natural selection

(survival of the fittest). New solutions are created by applying recombination, that combines two or more selected individuals (the so-called parents) to produce one or more new individuals (the children or offspring), and mutation, that allows the appearance of new traits in the offspring to promote diversity. The fitness (how good the solutions are) of the resulting solutions is evaluated and a suitable selection strategy is then applied to determine which solutions are to be maintained into the next generation. As a termination condition, a predefined number of generations (or function evaluations) of simulated evolutionary process is usually used, or some more complex stopping criteria can be applied [70]. For example, Figure 6 shows the flow chart of a genetic algorithm.

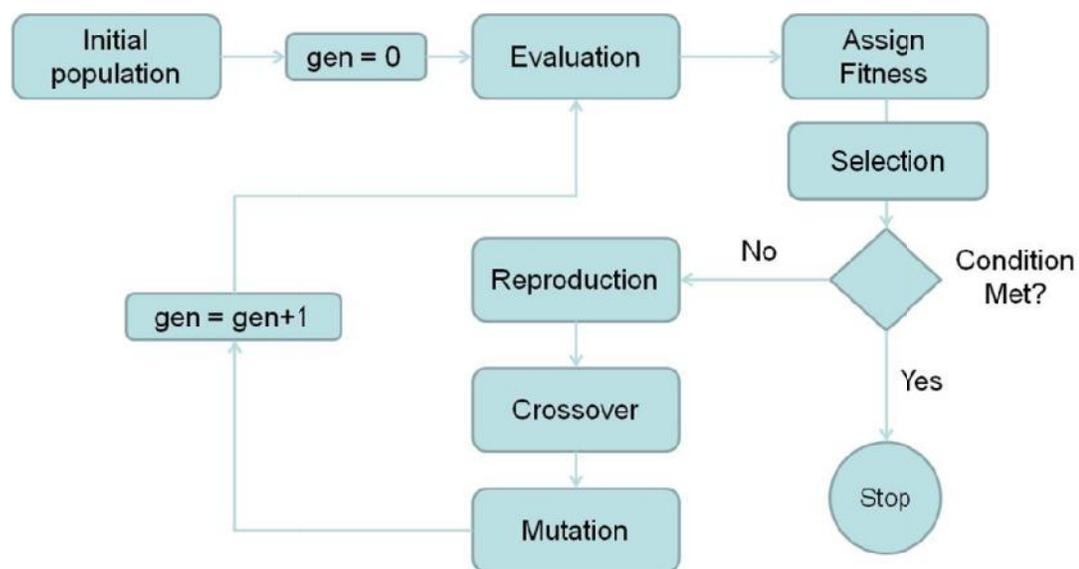


Figure 6. Flow chart of a genetic algorithm

Multi-objective evolutionary algorithms are one of the developing evolutionary algorithms. An excellent overview of current issues, algorithms, and existing systems in this area is presented in [63]. Another very good survey about meta-heuristics in general is [70]. Section 3 of the survey is devoted to evolutionary algorithms.

Jones et al. [70] reviewed 115 articles concerned with the theory and application of multi-objective meta-heuristics and reported that 90% of the approaches to multi-

objective optimization aimed to approximate the true Pareto front for the underlying problem. A majority of these used a meta-heuristic technique, and 70% of all meta-heuristics approaches were based on evolutionary approaches, primarily genetic algorithms.

There are several comparison studies of multi-objective evolutionary algorithms. References [71] and [72] compares eight algorithms on six test problems. Knowles and Corne [73] compare thirteen algorithms on a set of six test problems. In [74] and [75] three different algorithms are compared on nine test problems. In all these comparisons genetic algorithms are reported to outperform other evolutionary algorithms.

Multi-objective genetic algorithms have been the most popular heuristic approach to multi-objective design and optimization problems because they are population-based approaches and have the ability to find a set of multiple non-dominated solutions in a single run. The ability of a genetic algorithm to simultaneously search different regions of an objective space makes it possible to find a diverse set of solutions for difficult problems with non-convex, discontinuous, and multi-modal objective spaces. The crossover operator of a genetic algorithm may exploit structures of good solutions with respect to different objectives to create new non-dominated solutions in unexplored parts of the Pareto front. In addition, most multi-objective genetic algorithms do not require the user to prioritize, scale, or weigh objectives [76].

2.3.4 Multi-Objective Genetic Algorithms

In this section, basic principles and design issues of genetic algorithms first for single objective and then for multi-objective versions are discussed and a literature review for multi-objective genetic algorithms is presented.

2.3.4.1 Components of Genetic Algorithms

The concept of genetic algorithm was first conceived by Holland [77]. Genetic algorithms are search heuristics that mimic the process of natural evolution. In its general form, a genetic algorithm works as follows (also Figure 6):

- An initial population of individuals (solutions) is generated.
- At every generation, the individuals are evaluated and a fitness value is assigned.
- *Selection* operator selects the fittest individuals.
- If the stopping criterion is met the algorithm stops and gives the result.
- If not, new generation is created by operators such as *mutation* and *crossover*.

Crossover operator is mainly responsible for the convergence to the optimal solution(s) and *mutation* operator keeps diversity in the population.

Encoding

The first design issue of genetic algorithm is how to represent a solution. A solution vector of the optimization problem is called an *individual* or a *chromosome* in the genetic algorithm. These chromosomes are made of units called *genes*. Each gene controls one or more features of the chromosome. Normally, a chromosome corresponds to a unique solution in the solution space. This requires a mapping mechanism between the solution space and the chromosomes. This mapping is called an *encoding*. In fact, genetic algorithm works on the encoding of a problem, not on the problem itself [76].

There are many different ways of encoding. In the original implementation by Holland [77], binary coding was used. There are also permutation encoding, value encoding, and tree encoding. These encoding schemes are discussed in [78].

Fitness Assignment and Selection

Genetic algorithms are based on the idea of survival of the fittest. Therefore, the algorithm assigns a fitness value to all individuals at every generation and selects the fittest individuals for the creation of the next generation. Chromosomes are first decoded (reverse of encoding) and a fitness value is assigned according to the objective and constraint functions. In the absence of constraints, the fitness is the solution's objective function value. For the violated constraints a penalty value is often added to the fitness value.

The *selection* operator is intended to eliminate bad solutions and improve the average quality of the population while keeping the population size the same. This is achieved by giving a higher quality individual a higher probability of selection. There exist a number of methods for selection operator. Some common methods are *roulette wheel selection*, *tournament selection*, *proportionate selection*, *ranking selection*, *stochastic universal sampling selection*, and *steady-state selection*. Explanations and examples of these selection operators can be found in [63] and [78].

Elitism

Elitism in the context of single objective genetic algorithm means that the best solution found so far during the search always survives to the next generation. In this respect, all non-dominated solutions discovered by a multi-objective genetic algorithm are considered elite solutions. Early multi-objective genetic algorithm did not use elitism. However, multi-objective genetic algorithms using elitist strategies tend to outperform their non-elitist counterparts. Elitism can rapidly increase the performance of the algorithm because it prevents the loss of the best found solution(s).

Crossover and Mutation

The creation of new individuals is performed by crossover and mutation operators. These are the most important parts of the genetic algorithm because the performance of the algorithm is directly influenced by them.

Crossover operator selects two individuals from the previous generation (*parents*) and some portion of the solution vectors (*genes*) are exchanged between the individuals to create a new individual (*offspring*) for the next generation. For example in Figure 7, two binary coded individuals, I_1 and I_2 , are selected and left side genes of I_1 from the dotted line and right side genes of I_2 from the dotted line are used to form the new individual, I_{1-2} .

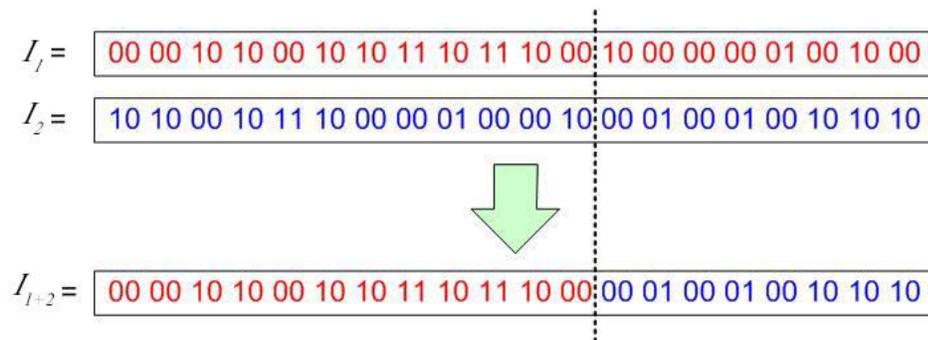


Figure 7. An example for crossover operator

The crossover operator is mainly responsible for the convergence to the optimal solution(s) because the new individual is often better than parent solutions. Although creating a better individual is not always true for all crossover operations in the algorithm, the chance of creating better individuals is very high because higher quality individuals have a higher probability to be selected as parents.

The mutation operator works on only one parent individual from the previous generation and changes some genes of the parent individual to create a new offspring

individual for the next generation (Figure 8). Mutation operator keeps diversity in the population and prevents premature convergence. Different crossover and mutation operator types for different encoding schemes are presented in [78].

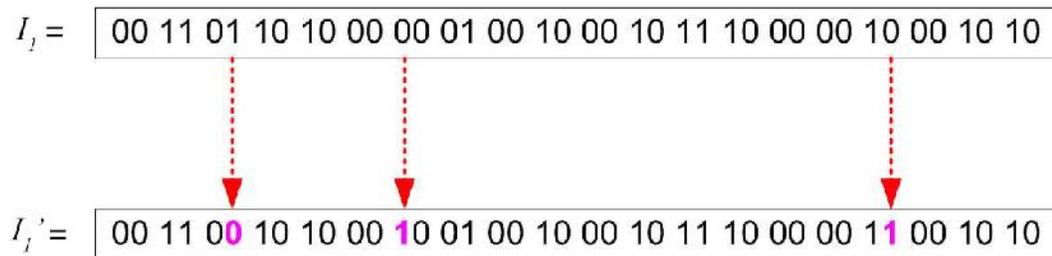


Figure 8. An example for mutation operator

2.3.4.2 Differences between Single- and Multi-Objective Genetic Algorithms

Being a population-based approach, genetic algorithms are well suited to solve multi-objective optimization problems. A generic single-objective genetic algorithm can be modified to find a set of multiple non-dominated solutions (the Pareto set) in a single run [76]. Multi-objective genetic algorithms also use the same operators mentioned above. Crossover and mutation operators do the same work for multi-objective genetic algorithms. Elitism is used in several of them.

Multi-objective genetic algorithms differ from single objective genetic algorithms in the way the selection is made and the fitness is assigned to each solution in the population. The concept of dominance is implemented for selection of individuals. Methods such as weighted sum approaches, altering objective functions and Pareto-ranking approaches are proposed for selection and fitness assignment. Multi-objective genetic algorithms introduce different methods to promote solution diversity. Most commonly used methods are fitness sharing, crowding distance method and cell-based density method.

2.3.4.3 State-of-the-Art Multi-Objective Genetic Algorithms

The first real implementation of a multi-objective genetic algorithm (Vector Evaluated Genetic Algorithm, VEGA) was suggested by Schaffer [79]. VEGA is a modified genetic algorithm with selection, crossover and mutation operators which performs independent selection cycles according to each objective. Population at every generation is randomly divided into M equal sized subpopulations (M is the number of objectives). Each solution in each subpopulation is assigned a fitness value based on each objective function. Then all subpopulations are combined and crossover and mutation are applied. VEGA is easy to implement and computationally efficient. The major drawback is that the population tends to converge to solutions which are superior in one objective, but poor at others.

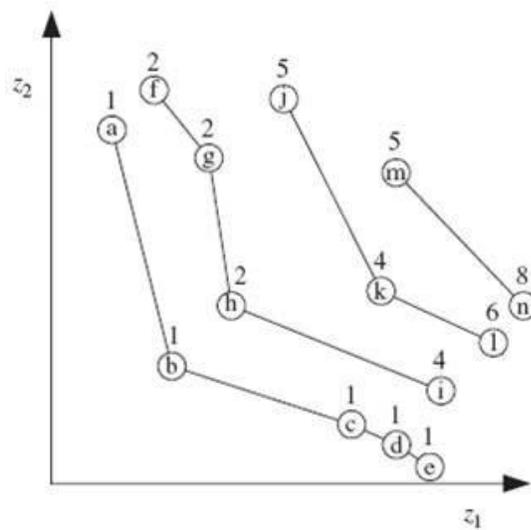


Figure 9. MOGA ranking method (adapted from Konak et al. [76])

MOGA (Multi-Objective Genetic Algorithm) [80] is the first multi-objective genetic algorithm that explicitly used Pareto-based ranking and niching techniques together. Niching is a neighborhood formation scheme based on the Euclidean distances between every solution pair, and it is used to maintain diversity. In MOGA, each solution

(individual) is ranked according to its degree of dominance. Ranking of a solution equals one plus the number of solutions that it is dominated by (Figure 9). This scheme penalizes solutions located in the regions of the objective function space which are dominated (covered) by densely populated sections of the Pareto front. MOGA is also a simple extension of single objective genetic algorithm but it is usually slow at convergence due to the niche size parameter.

Niched Pareto Genetic Algorithm (NPGA) [81] also uses the Pareto dominance concept to solve the multi-objective problems. This method differs from the other methods in the selection operator. NPGA uses the binary tournament selection. First, two individuals are randomly selected as candidates from the parent population and they are compared with solutions from a subpopulation of size T_{dom} from the parent population. If only one of the individuals is dominated by any other individual from the subpopulation, then the other candidate (non-dominated one) is selected for reproduction. If both candidates are dominated or non-dominated then specialized fitness sharing is performed and the candidate with the lower niche count is selected.

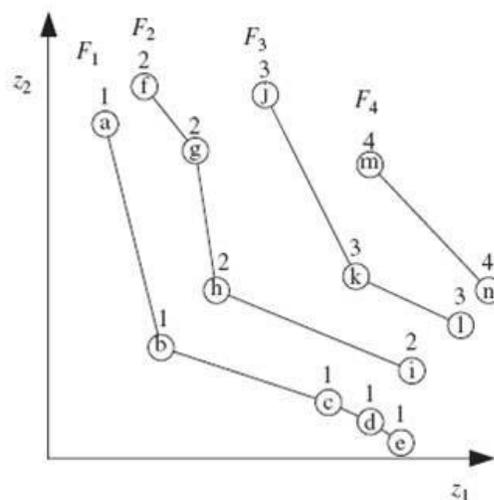


Figure 10. NSGA ranking method (adapted from Konak et al. [76])

NSGA (Non-dominated Sorting Genetic Algorithm) [82] is another algorithm that uses the dominance in ranking and a niche method for selection. It varies in the manner in which the selection operator works. The first step of NSGA is to sort the population according to non-dominance concept. The population is classified into non-dominated fronts (F_i). The same ranked individuals are assigned to the same front (Figure 10). After the fronts are determined, starting from the best (first) front, fitness functions are assigned to each individual in each front. Niche count of each individual is also calculated. Using fitness and niche count, shared fitness for each individual is calculated. Once the shared fitness is assigned, the population is reproduced according to the shared fitness values.

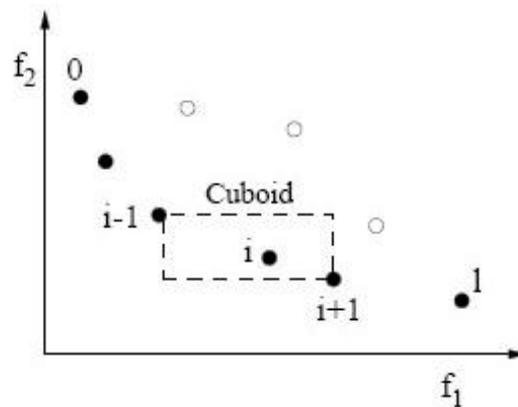


Figure 11. NSGA-II crowding distance method (adapted from Deb et al. [76])

NSGA-II (Fast Elitist Non-dominated Sorting Genetic Algorithm) [83] is an improved version of the NSGA. In most aspects, this algorithm does not have much similarity with the original NSGA version. NSGA-II does not have the problems of using the shared fitness function which requires appropriate selection of the sharing parameter. The algorithm first ranks the population and identifies non-dominated fronts (F_i). For each objective function, the algorithm sorts the individuals in each front F_i in the ascending order and it assigns crowding distance values to each of the individuals, which

is a measure of population density around the individual as shown in Figure 11. The crowding distance approach aims to obtain a uniform spread of solutions along the best-known Pareto front without using a fitness sharing parameter. The main advantage of the crowding distance approach is that it is computed without requiring a user-defined parameter. In NSGA-II, the crowding distance measure is used as a tiebreaker in a selection technique called the crowded tournament selection operator. In this method two solutions are randomly selected. If the solutions are in the same non-dominated front, then the solution with a higher crowding distance is the winner. Otherwise, the solution with the lowest rank is selected [76].

Table 6 gives a list of mentioned multi-objective genetic algorithms with their advantages and disadvantages. Konak et al. [76] provides a very informative tutorial about multi-objective optimization using genetic algorithms. In the paper, design issues and components of multi-objective genetic algorithms are explained and different algorithms are compared according to their approach to fitness functions, diversity mechanisms, elitism and constraint handling techniques.

Table 6. List of reviewed multi-objective genetic algorithms

Algorithm	Description	Advantages	Disadvantages
VEGA	Fractions of succeeding populations are selected based on separate objective performance.	Straightforward implementation	Tend to converge to the extreme of each objective
MOGA	Incorporates niching and ranking.	Simple extension of single objective GA	Usually slow convergence Problems related to niche size parameter

NPGA	Specialized Pareto domination tournaments.	Very simple selection process with tournament selection	Problems related to niche size parameter Extra parameter for tournament selection.
NSGA	Assigns and shares dummy fitness in each front.	Fast convergence	Problems related to niche size parameter
NSGA-II	Uses elitism and a crowded comparison operator that keeps diversity without specifying any additional parameters.	Single parameter (N) Well tested Efficient	Crowding distance works in objective space only.

2.4 Uncertainty in Multi-Objective Optimization

Although multi-objective stochastic programming problems are frequently encountered in practice, the literature on solution methodologies accounting for the uncertainty is still in its nascence [84]. References [85] and [86] point out that the resolution of multi-objective stochastic problem involves two kinds of transformations namely; the multi-objective transformation and the stochastic transformation.

In many problems there are random constraints in addition to the random objectives. Therefore, the first step for both transformations is addressing the random constraints. Some researchers, as Ben Abdelaziz et al. [86], consider that random constraints can be viewed as extra random objectives. Teghem et al. [87], for instance, consider the cost of violation of these random constraints as an extra objective to be minimized. Other authors, as Ben Abdelaziz et al. [88], require that the constraints as in the chance constrained programming need to be satisfied with a certain probability, where there are threshold values of constraints specified by the decision maker. All the constraints can also be transformed at the same time by considering the joint chance constrained

programming approach. After transforming the random constraints, a deterministic feasible set is obtained [89]. Once the constraints are addressed, the second step is the transformation of random objectives with the multi-objective transformation or the stochastic transformation.

In the multi-objective transformation, as in [87], [90], and [91], the problem is transformed as a multi-objective deterministic optimization problem and is generally solved by means of interactive methods. The methodology follows a two-phase approach where, in the first phase, the stochastic multiple objectives are converted into deterministic equivalents based on the minimum expectation and variance efficiency concepts (Figure 12). The second phase solves the deterministic multi objective problem, using a Pareto generation methodology, which aims at generating the entire Pareto surface of multi-objective programming problems [84]. This kind of transformation eliminates the randomness and uncertainty.

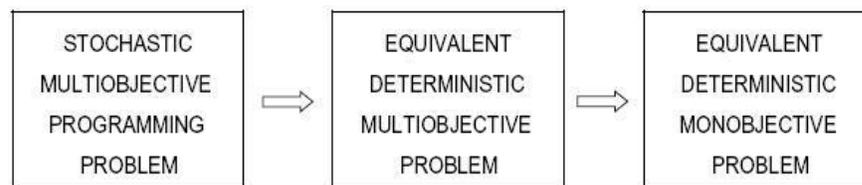


Figure 12. Steps for obtaining efficient solutions in the multi-objective approach [92]

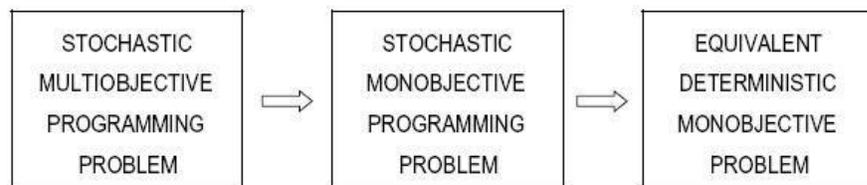


Figure 13. Steps for obtaining efficient solutions in the stochastic approach [92]

In the stochastic transformation, as in the stochastic goal programming proposed in [85], the multi-objective problem is first aggregated to obtain a single objective stochastic program, which is solved by a stochastic programming approach (Figure 13). This kind

of transformation keeps the randomness and uncertainty but eliminates the multi-objective aspect of the problem.

In both transformations at least one of the important aspects of the multi-objective stochastic problem is eliminated. The problem after transformation is either a multi-objective deterministic problem or a single objective stochastic problem. If the multi-objective transformation is preferred, then the result is a Pareto optimal set but without any knowledge of uncertainty. If the stochastic transformation is used, then the result is a single optimal solution. Furthermore, the role of the modelers in understanding the environment of the problem and their participation in the resolution process are very important in providing good solutions to the original problem [89].

Ben Abdelaziz [89] formulates the multi-objective stochastic problem as follows:

$$\begin{aligned}
 & \min/ \max \quad ' (\check{S}, \mathbf{x}) = ('_1(\check{S}, \mathbf{x}), '_2(\check{S}, \mathbf{x}), \dots, '_M(\check{S}, \mathbf{x})) \\
 & \text{s.t} \quad \mathbf{x} \in X = \tilde{g}_j(\check{S}, \mathbf{x}) \leq \tilde{b}_j(\check{S}) \quad j = 1, \dots, K \\
 & \quad \quad \mathbf{x} \in D, \check{S} \in \Omega
 \end{aligned} \tag{24}$$

where

- M objectives $'_1(\check{S}, \mathbf{x}), '_2(\check{S}, \mathbf{x}), \dots, '_M(\check{S}, \mathbf{x})$ are random
- \mathbf{x} is a solution in the objective space and it is an n -dimensional vector consisting of n decision variables: $\mathbf{x} = (x_1, \dots, x_n)$
- feasible set $X(\check{S})$ is random
- constraints \tilde{g}_j and the parameters \tilde{b}_j are random
- Ω is a probability space
- D is a deterministic convex set.

According to Abdelaziz [89], when solving the multi-objective stochastic problem, one can be observing the behavior of the Pareto set of the obtained deterministic multi-objective problem for the different values of the state of the nature \tilde{S} . This kind of distributional problem is not well addressed in the literature as in general [85], and from a decision point-of-view, the need is to generate a *here and now* solution without addressing the distributional problem. Abdelaziz [89] claims that a decision maker is rarely interested in knowing how the Pareto set changes depending on the occurrence of \tilde{S} .

In contrast, it is very important for a decision maker to have the knowledge of uncertainty in the Pareto set. The following three example cases illustrate why it is important to observe the uncertainty of each solution in the objective space. All cases are examples of minimization problems.

Case 1: In Figure 14, Solutions 1 and 2 are non-dominated, and Solution 3 is dominated (by Solution 1), but this is only strictly true if these results are deterministic. Compare this example with the one presented in Figure 15. In Figure 15, the uncertainty is depicted by clouds around the solutions. A large cloud means that the solution has a high level of uncertainty. In Figure 15, Solution 1 has a high level of uncertainty whereas Solution 3 has a lower level of uncertainty. All the solutions have a probability of being dominated by other solutions. If the decision maker prefers to observe expected values of results, then the preference of the solutions would be the same as in the deterministic case (Figure 14), and Solution 3 would not be presented in the Pareto optimal set. However, if the decision maker is interested in uncertainty or risk in these solutions, then the dominance relation between solutions is not deterministic, and not always obvious or

intuitive. The probability of Solution 3 being dominated by Solution 1 becomes very low. Therefore, it should be presented to the decision maker as an alternative, i.e., it should take its place in the Pareto optimal set.

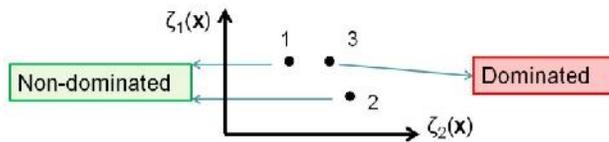


Figure 14. Deterministic example for Case 1

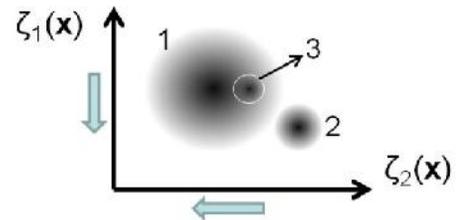


Figure 15. Stochastic example for Case 1

Case 2: In Figure 16, Solution 1 clearly dominates Solution 2 since Solution 1 has better values for both objectives 1 and 2. In Figure 17, however, the concept of domination is not clear any more. Solution 1 has a higher level of uncertainty compared to Solution 2. Again, if the decision maker prefers to observe expected values, then the preference of the solutions would be the same as in the deterministic case, and Solution 2 would not be presented in the Pareto optimal set. However, when the uncertainty is included, the probability of Solution 2 being dominated by Solution 1 becomes very low, which gives it a probability to be a preferable solution.

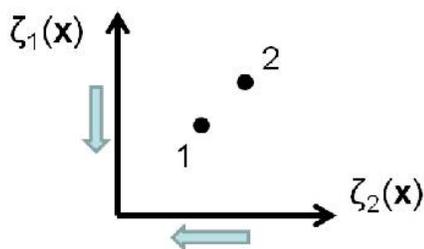


Figure 16. Deterministic example for Case 2

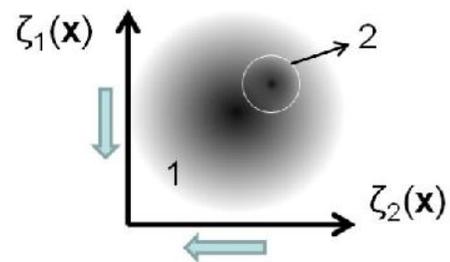


Figure 17. Stochastic example for Case 2

Cases 1 and 2 show the importance of knowing the uncertainty, particularly for a risk-averse decision maker. A risk-averse decision maker would be interested in worst case

scenarios and expected results would not reveal the risk of the solutions to the decision maker.

The following example case involves a risk-seeking decision maker and the concept of *regret*. Making poor decisions under uncertainty can lead to disappointment that alternative choices would have been preferable, which is called *regret*. If the decision maker is willing to take the risk for more gain or wants to minimize the regret, then again, the uncertainty of solutions gains importance.

Case 3: Compare Figure 16 and Figure 18. As in Case 2, Solution 1 clearly dominates Solution 2 since Solution 1 has better values for both objectives 1 and 2 in the minimization problem (Figure 16). In Figure 18, the concept of domination is not clear any more. Solution 2 has a higher level of uncertainty compared to Solution 1 but this is not a problem for a risk-seeking decision maker. In fact, it is an opportunity. There is a probability that Solution 2 will provide better results than Solution 1. This also means that there is a probability that the decision maker will regret the decision if Solution 2 happens to be a better choice.

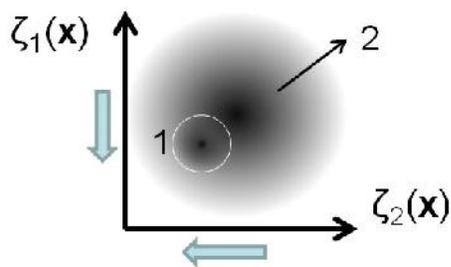


Figure 18. Stochastic example for Case 3

The above discussion about transformation of stochastic multi-objective problems and example cases illustrate why it is important to observe the uncertainty of each solution in the objective space. They demonstrate that there is a need for a new approach

to solve stochastic multi-objective problems without a transformation which also keeps the uncertainty information of Pareto solutions. Therefore, a new metric, the PUI is presented in this research. This metric enables a direct approach to solve a stochastic multi-objective problem without any transformation of the problem, and it also provides the opportunity to the decision maker to observe the uncertainty of solutions in the Pareto set. Later the PUI is used in PUGA, a new multi-objective stochastic genetic algorithm that is also presented in this research, to solve multi-objective stochastic optimization problems.

2.5 Post-Pareto Pruning

Multi-objective optimization techniques often yield a very large number of non-dominated Pareto optimal solutions, which makes the selection of one single best solution very difficult, especially as the number of objectives increase. Eliminating the less satisfactory trade-offs and reducing the number of alternatives is not a simple task. Although, several methods for solving multi-objective optimization problems have been developed and studied, little prior work has been done on the evaluation of results obtained in multi-objective optimization. This selection stage is often referred as post-Pareto optimality. The two main objectives of the post-Pareto optimality analysis are *i*) to obtain a smaller sub-set of preferred solutions from the large Pareto optimal set, and *ii*) the evaluation and interpretation of the results obtained from any optimization method [78]. Taboada et al. [93] propose following two methods to reduce or limit the size of the Pareto optimal set, 1) pruning by using non-numerical objective function ranking preferences method, and 2) pruning by using data clustering. The first method is a pseudo-ranking scheme that helps decision makers select solutions that reflect their

objective function priorities. In the second approach, data mining clustering techniques are used to group the data by using the k -means algorithm to find clusters of similar solutions. This provides the decision maker with just k general solutions to choose from. With this second method, from the clustered Pareto optimal set, they attempt to find solutions which are likely to be more relevant to the decision maker.

Aguirre and Taboada [94] introduce the dynamic self organizing tree algorithm as a method to perform post-Pareto analysis. This algorithm offers two main advantages. There is no need to provide an initial number of clusters, and at each hierarchical level, the algorithm optimizes the number of clusters, and can reassign data from previous hierarchical levels in order to rearrange misclustered data.

Kulturel-Konak et al. [95] use a tabu search meta-heuristic approach to initially find the entire Pareto optimal front, and then, Monte-Carlo simulation to provide the decision maker with a pruned and prioritized set of Pareto optimal solutions based on user-defined objective function preferences.

In this research, finding the Pareto optimal set and pruning for post-Pareto analysis are incorporated in one approach. PUGA uses the PUI to calculate the domination probability of two solutions on the same Pareto front. The algorithm can use this probability information to make a preference decision between solutions on the same front using the risk preference of the decision maker. Thus, obtaining a smaller subset of preferred solutions from a large Pareto optimal set and the evaluation and interpretation of the results can be accomplished within the optimization process.

3. Pareto Uncertainty Index (PUI)

A new metric, the Pareto Uncertainty Index (PUI), is presented in this section. This metric enables a direct approach to solve a multi-objective stochastic problem without any transformation of the problem, and it also provides the opportunity to the decision maker to observe the uncertainty of solutions in the Pareto set. For a discussion of how uncertainty is modeled in multi-objective optimization and why it is important, see Section 2.4.

3.1 PUI Formulation

A stochastic multi-objective problem can be formulated as follows:

$$\begin{aligned}
 & \min/ \max \quad ' (\check{S}, \mathbf{x}) = ('_1(\check{S}, \mathbf{x}), '_2(\check{S}, \mathbf{x}), \dots, '_M(\check{S}, \mathbf{x})) \\
 & \text{s.t} \quad \quad \quad g_j(\check{S}, \mathbf{x}) \leq b_j(\check{S}) \quad \quad \quad j = 1, \dots, K \\
 & \quad \quad \quad \mathbf{x} \in D, \check{S} \in \Omega
 \end{aligned} \tag{25}$$

where

- M objectives $'_1(\check{S}, \mathbf{x}), '_2(\check{S}, \mathbf{x}), \dots, '_M(\check{S}, \mathbf{x})$ are random.
- \mathbf{x} is a solution in the objective space and it is an d -dimensional vector consisting of d decision variables: $\mathbf{x} = (x_1, \dots, x_d)$
- feasible set $X(\check{S})$ is random.
- parameters of constraints $g_j(\check{S})$ and $b_j(\check{S})$ are random
- Ω is a probability space
- D is a deterministic convex set.

The first step for calculating PUI_j , the Pareto Uncertainty Index of solution \mathbf{x}_j , is calculating the probability that other solutions dominate the considered solution \mathbf{x}_j . Solution \mathbf{x}_j is the vector of decision variables such that $\mathbf{x}_j = (x_1, x_2, x_3, \dots, x_d)$, where d is the number of decision variables. $\Pr\{\mathbf{x}_i \succ \mathbf{x}_j\}$, the probability that solution \mathbf{x}_i dominates solution \mathbf{x}_j , can be formulated as in (26). $\Pr\{\mathbf{x}_i \succ \mathbf{x}_j\}$ is the combination of probabilities that solution \mathbf{x}_i has a better objective function value than solution \mathbf{x}_j for all M objective functions.

$$\Pr\{\mathbf{x}_i \succ \mathbf{x}_j\} = \Pr\{f'_{1,\mathbf{x}_i} < f'_{1,\mathbf{x}_j} \cap f'_{2,\mathbf{x}_i} < f'_{2,\mathbf{x}_j} \cap \dots \cap f'_{M,\mathbf{x}_i} < f'_{M,\mathbf{x}_j}\} \quad (26)$$

- $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_i, \dots, \mathbf{x}_j, \dots, \mathbf{x}_n$ are solutions in the objective space.
- f'_{m,\mathbf{x}_i} is the value of m^{th} objective function evaluated at i^{th} solution, \mathbf{x}_i .

For continuous dependent objective functions, $\Pr\{\mathbf{x}_i \succ \mathbf{x}_j\}$ can be calculated with several integration operations of joint probability density function for all objectives evaluated at \mathbf{x}_j , $f_{1,2,\dots,M} (z_1, z_2, \dots, z_M | \mathbf{x}_j)$; and joint cumulative density function for all objectives evaluated at \mathbf{x}_i , $F_{1,2,\dots,M} (z_1, z_2, \dots, z_M | \mathbf{x}_i)$. Formulation of $\Pr\{\mathbf{x}_i \succ \mathbf{x}_j\}$ for N dependent objective functions is presented in Equation (27).

The probability that a solution \mathbf{x}_i dominates a solution \mathbf{x}_j can be used in different ways to form different designs of the PUI. We propose four different designs of the PUI:

1. Cumulative PUI
2. Pairwise PUI
3. Neighborhood PUI
4. Preference PUI

$$\begin{aligned}
\Pr\{\mathbf{x}_i \succ \mathbf{x}_j\} &= \Pr\{z'_{1,x_i} < z'_{1,x_j} \cap \dots \cap z'_{M,x_i} < z'_{M,x_j}\} \\
&= \int_{z'_{1,x_j}} \dots \int_{z'_{M,x_j}} \Pr\{z'_{1,x_i} < z'_{1,x_j} \cap \dots \cap z'_{M,x_i} < z'_{M,x_j} \mid z'_{1,x_j} = z_{1,x_j}, \dots, z'_{M,x_j} = z_{M,x_j}\} \\
&\quad \times f'_{1,\dots,M}(z_1, \dots, z_M \mid \mathbf{x}_j) d'_{1,x_j} \dots d'_{M,x_j} \\
&= \int_{z'_{1,x_j}} \dots \int_{z'_{M,x_j}} \Pr\{z'_{1,x_i} < z_{1,x_j} \cap \dots \cap z'_{M,x_i} < z_{M,x_j}\} f'_{1,\dots,M}(z_1, \dots, z_M \mid \mathbf{x}_j) d'_{1,x_j} \dots d'_{M,x_j} \quad (27) \\
&= \int_{z'_{1,x_j}} \dots \int_{z'_{M,x_j}} \left[\int_0^{z_{1,x_j}} \dots \int_0^{z_{M,x_j}} f'_{1,\dots,M}(z_1, \dots, z_M \mid \mathbf{x}_i) d'_{1,x_i} \dots d'_{M,x_i} \right] \\
&\quad \times f'_{1,\dots,M}(z_1, \dots, z_M \mid \mathbf{x}_j) d'_{1,x_j} \dots d'_{M,x_j} \\
&= \int_{z'_{1,x_j}} \dots \int_{z'_{M,x_j}} F_{1,\dots,M}(z_1, \dots, z_M \mid \mathbf{x}_i) f'_{1,\dots,M}(z_1, \dots, z_M \mid \mathbf{x}_j) d'_{1,x_j} \dots d'_{M,x_j}
\end{aligned}$$

Cumulative PUI utilizes the sum of domination probabilities of all solutions in the objective space. Therefore, the PUI of solution \mathbf{x}_j is calculated as in Equation (28).

$$\begin{aligned}
PUI_j &= \sum_{i=1, i \neq j}^n \Pr\{\mathbf{x}_i \succ \mathbf{x}_j\} \\
&= \sum_{i=1, i \neq j}^n \Pr\{z'_{1,x_i} < z'_{1,x_j} \cap \dots \cap z'_{M,x_i} < z'_{M,x_j}\} \quad (28) \\
&= \sum_{i=1, i \neq j}^n \left[\int_{z'_{1,x_j}} \dots \int_{z'_{M,x_j}} F_{1,\dots,M}(z_1, \dots, z_M \mid \mathbf{x}_i) f'_{1,\dots,M}(z_1, \dots, z_M \mid \mathbf{x}_j) d'_{1,x_j} \dots d'_{M,x_j} \right]
\end{aligned}$$

If objectives are independent from each other, then there is no need for integration over joint probability functions and $\Pr\{\mathbf{x}_i \succ \mathbf{x}_j\}$ is the multiplication of probabilities that solution \mathbf{x}_i has a better objective function value than solution \mathbf{x}_j for all objective functions. Therefore, PUI_j for independent objective functions can be calculated as in Equation (29).

$$\begin{aligned}
PUI_j &= \sum_{i=1, i \neq j}^n \Pr\{\mathbf{x}_i \succ \mathbf{x}_j\} \\
&= \sum_{i=1, i \neq j}^n \Pr\{f_{1, \mathbf{x}_i} < f_{1, \mathbf{x}_j} \cap f_{2, \mathbf{x}_i} < f_{2, \mathbf{x}_j} \cap \dots \cap f_{M, \mathbf{x}_i} < f_{M, \mathbf{x}_j}\} \\
&= \sum_{i=1, i \neq j}^n \prod_{m=1}^M \Pr\{f_{m, \mathbf{x}_i} < f_{m, \mathbf{x}_j}\} \\
&= \sum_{i=1, i \neq j}^n \prod_{m=1}^M \int_{f_{m, \mathbf{x}_j}} F_m(z_m | \mathbf{x}_i) f_m(z_m | \mathbf{x}_j) dz_m
\end{aligned} \tag{29}$$

For example, if all of the objective functions are distributed as independent normal distributions, then the calculation is carried out as in Equation (30) where $\tilde{x}_i^{(m)}$ is the mean of the m^{th} objective function evaluated at the i^{th} solution and $\dagger_{x_i^{(m)}}$ is the standard deviation of the m^{th} objective function evaluated at the i^{th} solution.

$$\begin{aligned}
PUI_j &= \sum_{i=1, i \neq j}^n \Pr\{\mathbf{x}_i \succ \mathbf{x}_j\} \\
&= \sum_{i=1, i \neq j}^n \prod_{m=1}^M \Pr\{f_{m, \mathbf{x}_i} < f_{m, \mathbf{x}_j}\} \\
&= \sum_{i=1, i \neq j}^n \prod_{m=1}^M \left(1 - \frac{\tilde{x}_i^{(m)} - \tilde{x}_j^{(m)}}{\sqrt{\dagger_{x_i^{(m)}}^2 + \dagger_{x_j^{(m)}}^2}} \right)
\end{aligned} \tag{30}$$

An easier way to explain the calculation of the cumulative PUI for a solution is to give an example of a problem with two objective functions. For a bi-objective problem a matrix as in Table 7 can be formed by calculating the probabilities in each cell. \Pr_{iq} or $\Pr\{\mathbf{x}_i \succ \mathbf{x}_q\}$ is the probability that solution i dominates solution q . Then, for each solution, row sums give the PUI of the solution as in (31).

$$PUI_q = \Pr_{1q} + \Pr_{2q} + \Pr_{3q} + \dots \tag{31}$$

Table 7. Calculation of cumulative PUI for a bi-objective optimization problem

	\mathbf{x}_1	\mathbf{x}_2	\mathbf{x}_3	\mathbf{x}_4	...	\mathbf{x}_q	PUI
\mathbf{x}_1	-	Pr_{21}	Pr_{31}	Pr_{41}	...	Pr_{q1}	$PUI_1 = \text{sum of the first row}$
\mathbf{x}_2	Pr_{12}	-	Pr_{23}	Pr_{42}	...	Pr_{q2}	$PUI_2 = \text{sum of the second row}$
\mathbf{x}_3	Pr_{13}	Pr_{23}	-	Pr_{43}	...	Pr_{q3}	$PUI_3 = \text{sum of the third row}$
\mathbf{x}_4	Pr_{14}	Pr_{24}	Pr_{34}	-	...	Pr_{q4}	$PUI_4 = \text{sum of the fourth row}$
.
.
.
\mathbf{x}_q	$\text{Pr}(\mathbf{x}_1 \succ \mathbf{x}_q)$	Pr_{2q}	Pr_{3q}	Pr_{4q}	...	-	$PUI_q = \text{sum of the } q^{\text{th}} \text{ row}$

Pairwise PUI utilizes the sum of domination probabilities of only the closest neighbor solutions for each objective function in the objective space. This is similar to the crowding distance idea of NSGA-II [74] but it includes the stochastic nature of the problem into calculation. Crowding distance is of NSGA-II explained in Section 4.1.2.

Pairwise PUI of a solution \mathbf{x}_j is calculated as in Equation (32) where M is the number of objective functions, $\mathbf{x}_{m,j-1}$ and $\mathbf{x}_{m,j+1}$ are the closest neighbor solutions for m^{th} objective function.

$$PUI_j = \sum_{m=1}^M \text{Pr}\{\mathbf{x}_{m,j-1} \succ \mathbf{x}_j\} + \text{Pr}\{\mathbf{x}_{m,j+1} \succ \mathbf{x}_j\} \quad (32)$$

Neighborhood PUI utilizes the sum of domination probabilities of the solutions in the same neighborhood. For this design of PUI, the objective space is partitioned into neighborhoods similar to cells of RDGA shown in Figure 19 [96]. The neighborhood approach is explained in Section 4.1.6.

Neighborhood PUI for a solution \mathbf{x}_j is calculated as in Equation (33) where C_k is the neighborhood of solution \mathbf{x}_j and i is a neighbor solution in the same neighborhood.

$$PUI_j = \sum_{i \in C_k} \text{Pr}\{\mathbf{x}_i \succ \mathbf{x}_j\} \quad (33)$$

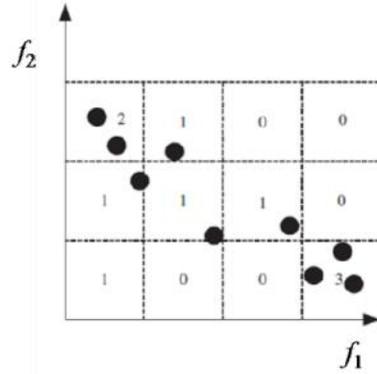


Figure 19. RDGA cell-based density approach

Preference PUI utilizes the domination probabilities to assign a preference value to each solution. For a minimization problem, a solution \mathbf{x}_i is said to dominate another solution \mathbf{x}_j , $\mathbf{x}_i \succ \mathbf{x}_j$, if these two conditions are both satisfied:

1. The solution \mathbf{x}_i is no worse than \mathbf{x}_j for all M objective functions, that is,

$$f'_m(\mathbf{x}_i) \leq f'_m(\mathbf{x}_j) \text{ for all } m = 1, 2, \dots, M$$

2. The solution \mathbf{x}_i is strictly better than \mathbf{x}_j for at least one objective function, that is,

$$f'_m(\mathbf{x}_i) < f'_m(\mathbf{x}_j) \text{ for at least one } m \in \{1, 2, \dots, M\}$$

where $f'_m(\mathbf{x}_i)$ is the m^{th} objective function evaluated at solution vector \mathbf{x}_i and M is the number of objective functions. This is the Pareto dominance concept where the uncertainty is neglected. We call this design the *expected domination*. This is similar to the ranking design of NSGA-II.

A solution \mathbf{x}_i is said to dominate another solution \mathbf{x}_j probabilistically if the probability that solution \mathbf{x}_i dominates solution \mathbf{x}_j is greater than the probability that solution \mathbf{x}_j dominates solution \mathbf{x}_i , that is, $\Pr\{\mathbf{x}_i \succ \mathbf{x}_j\} > \Pr\{\mathbf{x}_j \succ \mathbf{x}_i\}$.

If a solution \mathbf{x}_i dominates a solution \mathbf{x}_j both expectedly and probabilistically, then \mathbf{x}_i dominates \mathbf{x}_j and it is also preferred over \mathbf{x}_j . If a solution \mathbf{x}_i dominates a solution \mathbf{x}_j expectedly but not probabilistically, then there is no dominance relation between these two solutions, that is, they are on the same Pareto front, but \mathbf{x}_j is preferred over \mathbf{x}_i . Preference PUI design is explained in Table 8.

Table 8. Preference PUI

Expected Domination (No Probability)	Stochastic Domination	Final Decision of Dominance	Preference of the solution
<i>i dominates j</i>	<i>i dominates j</i>	<i>i dominates j</i>	<i>i is preferred</i>
<i>i dominates j</i>	<i>j dominates i</i>	No domination between i and j	<i>j is preferred</i>
<i>j dominates i</i>	<i>i dominates j</i>	No domination between i and j	<i>i is preferred</i>
<i>j dominates i</i>	<i>j dominates i</i>	<i>j dominates i</i>	<i>j is preferred</i>

As a fifth option, preference PUI and neighborhood PUI designs can be incorporated as described below:

1. Partition the objective space into neighborhoods.
2. Calculate the PUI of every solution i in a neighborhood by summing all the probabilities that each neighbor solution of solution i in its neighborhood dominates solution i .
3. Prefer the solution with the smallest PUI in each neighborhood.

These PUI interpretations are used in different designs of the Pareto Uncertain Genetic Algorithm (PUGA) in Section 4. Different designs can be used in different steps of an algorithm when it is more appropriate to use a specific design. For example for the

post-Pareto analysis, neighborhood design is more relevant but for the selection operator of a genetic algorithm cumulative PUI design can give better results.

3.2 Risk Adjusted Pareto Uncertainty Index (rPUI)

A risk measure can be incorporated in the PUI formulation so that the risk can also be analyzed in the objective space, and the risk preference of the decision maker can affect the selection of Pareto solutions. The risk function is included into the cumulative PUI formulation as in (34). Other designs of the PUI can also have the risk measure similar to the cumulative PUI example provided here.

$$\begin{aligned}
rPUI_j &= \sum_{i=1, i \neq j}^n \mathfrak{R}(\mathbf{z}_j) \Pr\{\mathbf{x}_i \succ \mathbf{x}_j\} \\
&= \sum_{i=1, i \neq j}^n \mathfrak{R}(\mathbf{z}_j) \Pr\{x_{1,x_i} < x_{1,x_j} \cap x_{2,x_i} < x_{2,x_j} \cap \dots \cap x_{M,x_i} < x_{M,x_j}\} \\
&= \sum_{i=1, i \neq j}^n \prod_{m=1}^M \mathfrak{R}(z_{m,j}) \Pr\{x_{m,x_i} < x_{m,x_j}\} \\
&= \sum_{i=1, i \neq j}^n \prod_{m=1}^M \int_{x_{m,x_j}}^{\infty} \mathfrak{R}(z_{m,j}) F_{m,x_i}(x_{m,x_i} < z_{m,j} \mid x_{m,x_j} = z_{m,j}) f_{m,x_j}(z_{m,j}) dz_{m,x_j}
\end{aligned} \tag{34}$$

where $\mathbf{z}_j = (z_{1,j}, z_{2,j}, \dots, z_{M,j})$ and $\mathfrak{R}(\mathbf{z}_j)$ is calculated as:

$$\begin{aligned}
\mathfrak{R}(\mathbf{z}_j) &= \mathfrak{R}(z_{1,j}) \mathfrak{R}(z_{2,j}) \dots \mathfrak{R}(z_{M,j}) \\
&= \prod_{m=1}^M \mathfrak{R}(z_{m,j})
\end{aligned} \tag{35}$$

where $\mathfrak{R}(z_{m,j})$ is a risk function and shifts the probability according to the risk preference of the decision maker. $\mathfrak{R}(z_{m,j})$ risk function is formulated as in (36), and the risk adjusted PUI, rPUI, is calculated as in (37).

$$\mathfrak{R}(z_{m,j}) = \exp \left(\dots_m \left(\frac{z_{m,j} - E \left[\begin{matrix} ' \\ m, x_j \end{matrix} \right] }{E \left[\begin{matrix} ' \\ m, x_j \end{matrix} \right]} \right) \right) \quad (36)$$

$$\begin{aligned} rPUI_j &= \sum_{i=1, i \neq j}^n \mathfrak{R}(\mathbf{z}_j) \Pr\{\mathbf{x}_i \succ \mathbf{x}_j\} \\ &= \sum_{i=1, i \neq j}^n \prod_{m=1}^M \int_{\begin{matrix} ' \\ m, x_j \end{matrix}} \exp \left(\dots_m \left(\frac{z_{m,j} - E \left[\begin{matrix} ' \\ m, x_j \end{matrix} \right] }{E \left[\begin{matrix} ' \\ m, x_j \end{matrix} \right]} \right) \right) F'_{m,i} (\begin{matrix} ' \\ m, x_i \end{matrix} < z_{m,j}) f'_{m,x_j} (z_{m,j}) \mathrm{d} z_{m,j} \end{aligned} \quad (37)$$

where \dots_m is the risk measure for the m^{th} objective function and designates the growth and decay effect on the probability. When \dots_m is 0 (zero) the PUI is risk-neutral for the considered objective function. If the decision maker is risk-averse, then \dots_m takes values larger than 0. Therefore, a solution which has less probability to have higher values than the expected value for a minimization objective is favored over a solution which has higher probability to have higher values for the same objective function. If the decision maker is risk-seeking, then \dots_m takes values smaller than zero. In this case, a solution which has higher probability to provide lower values than the expected value for a minimization objective is favored. The risk function adopted in this research works effectively; however, other risk function forms could be used as well.

In any case (risk-neutral/seeking/averse), the smaller PUI or rPUI shows a better solution than others. Following example cases are provided to explain the workings of the risk adjusted PUI for three solutions in the objective space with two objective functions.

3.2.1 Case 1: Risk-Averse Decision Maker

Let there be three solutions in the objective space of a bi-objective (cost and emissions) minimization problem. Figure 20a shows solutions when the uncertainty is

neglected. Assume solutions are all normally distributed random variables. Their expected values and variances are provided in Table 9. Figure 20b shows expected values of solutions (squares) and their 3 -distances from expected values (ellipses).

Table 9. Expected values and variances for risk-averse case

Solutions	Expected Values [Cost, Emissions]	Variances [Cost, Emissions]
1	[3075,160]	[700,80]
2	[3085,165]	[150,15]
3	[3090,155]	[700,90]

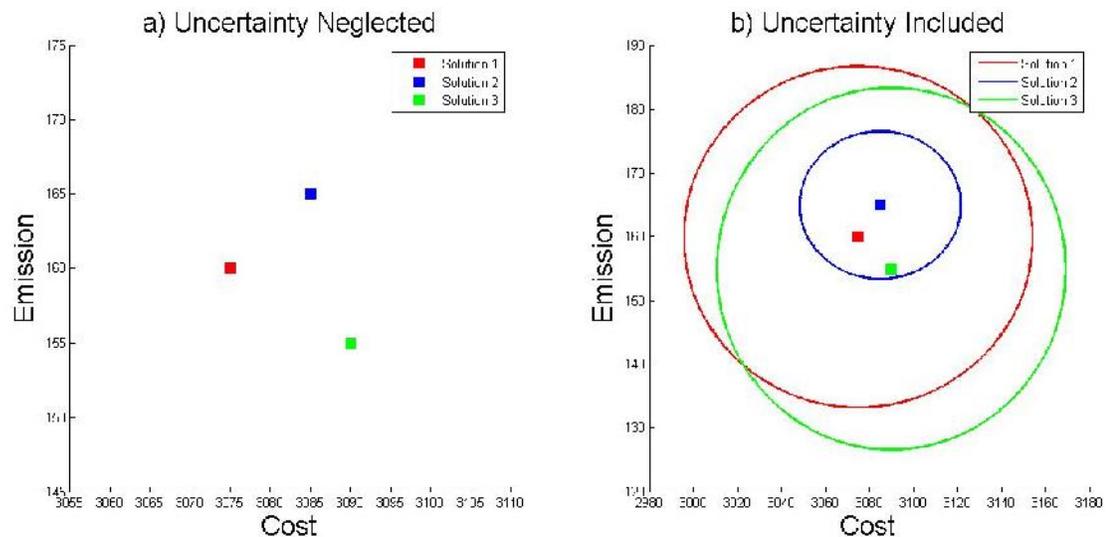


Figure 20. Risk-averse case. a) Uncertainty neglected results. b) Uncertainty included results

When uncertainty is neglected (Figure 20a), Solution 2 (blue colored in figures) is a dominated solution which means it is not considered as an alternative in the final Pareto set. However, it has a lower variance and a lower probability to have high cost and high emissions in worse case realizations compared to the other solutions (Figure 20b). It might be more favorable to a risk-averse decision maker and should be presented in the Pareto set. Therefore, its Pareto order with rPUI is lower than the order with PUI. The PUI and rPUI of solutions and their respective Pareto orders are presented in Table 10.

Table 10. PUI and rPUI values for risk-averse case ($\sigma_1 = 15$, $\sigma_2 = 15$)

Solutions	PUI	Pareto order with PUI	rPUI	Pareto order with rPUI
1	0.3347	Second	0.8045	First
2	0.8022	Third	0.9176	Second
3	0.3234	First	1.0716	Third

3.2.2 Case 2: Risk-Seeking Decision Maker

In this case, assume that the variance of Solution 2 is higher and the variance of Solution 1 is lower as shown in Table 11. Although Solution 1 in this case is a safer (more certain) solution for a risk-averse decision maker, Solution 2 may provide better opportunities for a risk-seeking decision maker, if optimistic future projections are realized.

Table 11. Expected values and variances for risk-seeking case

Solutions	Expected Values [Cost, Emissions]	Variances [Cost, Emissions]
1	[3075,160]	[1000,100]
2	[3085,165]	[7000,800]
3	[3090,155]	[5000,500]

When uncertainty is neglected Figure 21a, Solution 2 is a dominated solution which means it is not considered as an alternative in the final Pareto set. However, it provides better opportunities in better case realizations because it has a higher probability to have lower cost and emissions compared to the other solutions (Figure 21b). Therefore, its rPUI is calculated to be lower than the others, so it might be more favorable to a risk-seeking decision maker and should be presented in the Pareto set. The PUI and rPUI of solutions and their respective Pareto orders are presented in Table 12.

Table 12. PUI and rPUI values for risk-seeking case ($\sigma_1 = -6$, $\sigma_2 = -6$)

Solutions	PUI	Pareto order with PUI	rPUI	Pareto order with rPUI
1	0.4434	First	0.4162	Third
2	0.6018	Third	0.3974	Second
3	0.4442	Second	0.2711	First

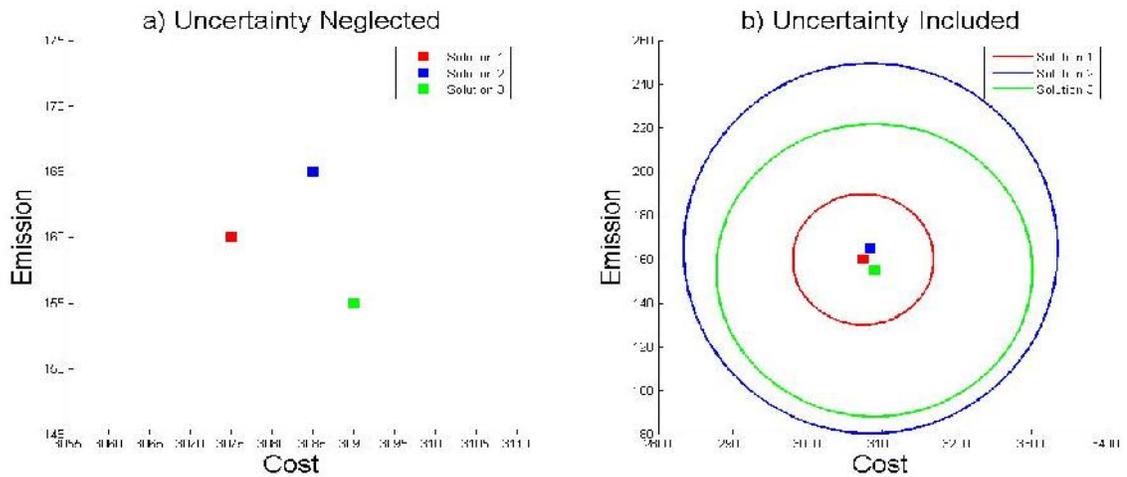


Figure 21. Risk-seeking case. a) Uncertainty neglected results. b) Uncertainty included results

3.2.3 Case 3: Same Expected Values and Different Variances

Assume there are two solutions which have the same expected value but different variances. The expected values and variances for the solutions are provided in Table 13. This may not generally be a very realistic case but provides additional explanations and demonstrations for working of the rPUI.

Table 13. Expected values and variances for Case 3

Solutions	Expected Values [Cost, Emissions]	Variances [Cost, Emissions]
1	[3075,1000]	[400,200]
2	[3075,1000]	[100,50]

Figure 22 shows the expected value of solutions as a black square. Different colored ellipses are the 3-sigma-distances from expected values. Solution 2 has a higher variance so its ellipse (blue colored) is wider.

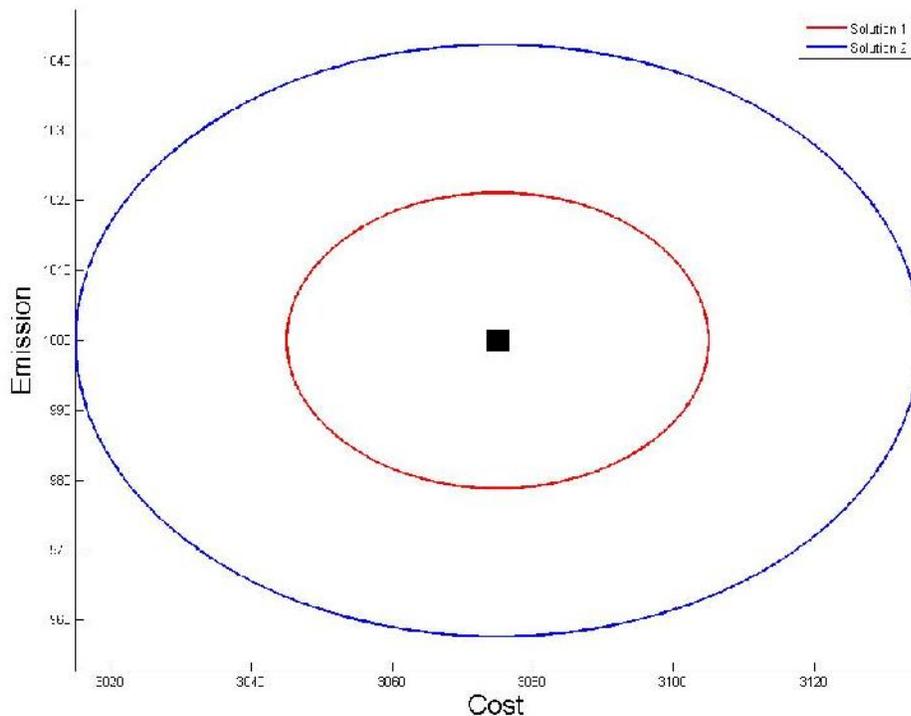


Figure 22. Solutions with same expected value different variance

Table 14 shows different risk-averse and risk-seeking m settings for two solutions and their objectives. When $m = 0$ for both objectives, it is the risk-neutral case and the result is the PUI with no risk adjustment. For other settings, the shaded cells show the better solution for that m setting.

When $\alpha_1 = \alpha_2 = -1$, i.e., risk-seeking settings in both objectives, or risk-neutral in one objective and risk-seeking in the other, Solution 2 is a better choice because it may offer better results for a risk-seeking decision maker. On the contrary, when $\alpha_1 = \alpha_2 = 1$, i.e., risk-averse settings in both objectives, or risk-neutral in one objective and risk-averse in the other, Solution 1 is a better choice because it is a safer option.

Table 14. Different m settings for two solutions of Case 3

	Sol.	$m = 0$ (PUI)	$\alpha_1 = 0$ $\alpha_2 = 1$	$\alpha_1 = 0$, $\alpha_2 = -1$	$\alpha_1 = 1$ $\alpha_2 = 0$	$\alpha_1 = -1$ $\alpha_2 = 0$	$\alpha_1 = 1$ $\alpha_2 = 1$	$\alpha_1 = -1$ $\alpha_2 = -1$
rPUI	1	0.2500	0.2506	0.2494	0.2503	0.2497	0.2509	0.2490
	2	0.2500	0.2525	0.2475	0.2512	0.2488	0.2537	0.2464

Table 15 shows the rPUI results when the m setting is risk-seeking in one objective and risk-averse in the other. When the setting is risk-seeking in objective 2 and risk-averse in objective 1, Solution 2 is a better solution unless risk-aversion is very strong in objective 1 ($\alpha_1 = 10$, $\alpha_2 = -1$).

Table 15. Risk-averse vs. risk-seeking for Case 3

	Sol.	$\alpha_1 = 1$, $\alpha_2 = -1$	$\alpha_1 = 1$, $\alpha_2 = -10$	$\alpha_1 = 10$, $\alpha_2 = -10$	$\alpha_1 = 10$, $\alpha_2 = -1$
rPUI	1	0.2497	0.2446	0.2473	0.2524
	2	0.2487	0.2282	0.2381	0.2595
	Sol.	$\alpha_1 = -1$, $\alpha_2 = 1$	$\alpha_1 = -1$, $\alpha_2 = 10$	$\alpha_1 = -10$, $\alpha_2 = 10$	$\alpha_1 = -10$, $\alpha_2 = 1$
rPUI	1	0.2503	0.2566	0.2541	0.2479
	2	0.2514	0.2766	0.2656	0.2414

Similarly, when the setting is risk-seeking in objective 1 and risk-averse in objective 2, Solution 1 is a better solution unless risk-seeking is very strong in objective 2 ($\alpha_1 = -10$, $\alpha_2 = 1$). This is due to sensitivity of objectives to the risk. Objective 2 in this example case is more sensitive to risk setting, and its m value determines the outcome unless m value of objective 1 is significantly higher. Objective 2 is more sensitive to the risk because its variance/expectation ratio is lower compared to objective 1.

3.2.4 Case 4: Same Variance/Expectation Ratio

Assume there are two solutions which have the same expected value and different variances similar to Case 3 but variance/expectation ratio is the same for both objectives.

The expected values and variances for the solutions are provided in Table 16. Again, this may not often be a realistic case but it is easier to compare how different m settings change the Pareto order of solutions with their rPUI values.

Table 16. Expected values and variances for Case 4

Solutions	Expected Values [Cost, Emissions]	Variances [Cost, Emissions]
1	[3000, 3000]	[300, 300]
2	[3000, 3000]	[3000, 3000]

Figure 23a shows the expected value of solutions as a black square. Different colored circles are the 3 σ -distances from the expected values. Solution 2 has a higher variance so its circle (blue colored) is wider. Figure 23b is provided for visual explanation of different variance/expectation ratio. Compare parts *a* and *b* of Figure 23 for different variance/expectation ratios. When the ratio is different as in part *b*, 3 σ -distances from the expected values are ellipses rather than circles.

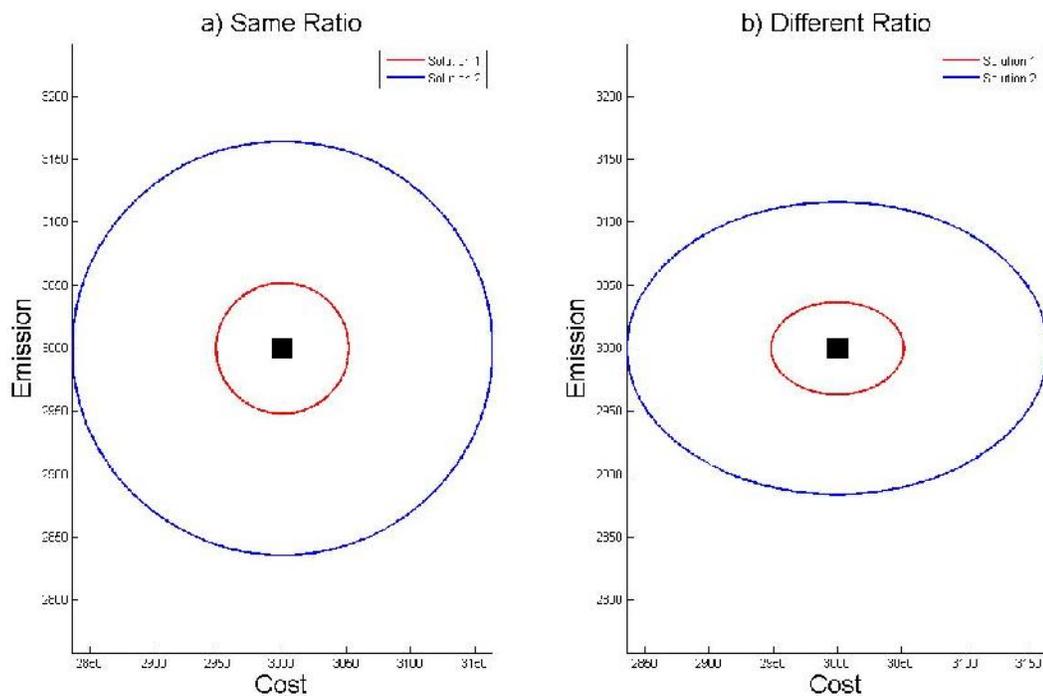


Figure 23. Same and different variance/expectation ratios

Table 17 shows the rPUI results for Case 4. When $\alpha_1 = \alpha_2 = 1$, risk-averse in both objectives, Solution 1 is a safer choice. When $\alpha_1 = \alpha_2 = -1$, risk-seeking in both objectives, Solution 2 is a better choice. When $\alpha_1 = 1$ and $\alpha_2 = -1$ or $\alpha_1 = -1$ and $\alpha_2 = 1$, risk-seeking in one objective and risk-averse in the other, the result is the same as the risk-neutral case because neither of the solutions is better than the other. If risk-seeking effect is larger in one of the objectives ($\alpha_1 = -2, \alpha_2 = 1$) then Solution 2 is again a better option. If risk-aversion effect is larger in one of the objectives ($\alpha_1 = 2, \alpha_2 = -1$) then Solution 1 is a better option.

Table 17. Risk-averse vs. risk-seeking for Case 4

	Sol.	$\alpha_1 = 1, \alpha_2 = 1$	$\alpha_1 = -1, \alpha_2 = -1$	$\alpha_1 = 1, \alpha_2 = -1$	$\alpha_1 = -1, \alpha_2 = 1$	$\alpha_1 = -2, \alpha_2 = 1$	$\alpha_1 = 2, \alpha_2 = -1$
rPUI	1	0.2507	0.2493	0.2500	0.2500	0.2497	0.2504
	2	0.2570	0.2432	0.2500	0.2500	0.2466	0.2536

3.2.5 Different α_m Settings

The risk measure α_m in the risk function $\mathfrak{R}(z_{m,j})$ effects how much the probability is shifted towards the risk-averse or risk-seeking aspects of a solution. Table 18 shows a comparison of rPUI values for Case 1 (the risk-averse case) with different α_m values. When α_m is zero for both objective functions, rPUI is equal to PUI value, i.e. it becomes a risk-neutral case.

In Table 18, only α_m value settings in the shaded columns change the Pareto order of solutions. The last column of Table 18 shows $\alpha_1 = -15$ and $\alpha_2 = 15$ case where decision maker is risk-seeking in objective 1 but risk-averse in objective 2. This setting gives the same order with the previous settings where decision maker is risk-averse or neutral in objective 1 but strictly risk-averse in objective 2. The Pareto order of solutions does not

change when $m_1 = 15$ and $m_2 = -15$, risk-averse in objective 1 but risk-seeking in objective 2. From these experiments it can be observed that the second objective function (emissions) is more sensitive to risk-aversion and the first objective function (cost) is less sensitive.

Table 18. Different m values for risk-averse case

Solutions		$m = 0$ (PUI)	$m_1 = 0,$ $m_2 = 1$	$m_1 = 1,$ $m_2 = 0$	$m_1 = 5,$ $m_2 = 5$	$m_1 = 10,$ $m_2 = 10$
rPUI	1	0.3347	0.3467	0.3370	0.4234	0.5672
	2	0.8022	0.8054	0.8032	0.8279	0.8658
	3	0.3234	0.3431	0.3247	0.4544	0.6775
Solutions		$m_1 = 15,$ $m_2 = -15$	$m_1 = 15,$ $m_2 = 15$	$m_1 = 15,$ $m_2 = 0$	$m_1 = 0,$ $m_2 = 15$	$m_1 = -15,$ $m_2 = 15$
rPUI	1	0.2886	0.8045	0.3736	0.7192	0.6488
	2	0.8195	0.9176	0.8184	0.8994	0.8845
	3	0.1870	1.0716	0.3452	1.0012	0.9455

Table 19 shows a comparison of rPUI values for Case 2 (the risk-seeking case) with different m values. Once again, when m is zero for both objective functions, rPUI is equal to the PUI value, i.e. it becomes a risk-neutral case. In Table 19, only m value settings in the shaded columns change the Pareto order of solutions. When the setting is risk-neutral or risk-averse for one of the objective functions, the Pareto order of the solutions does not change. The order only changes only when the setting is risk-seeking and m are higher than 5 for both objective functions.

In general, the value of m determines the magnitude of decision maker's risk preference. Assigning a higher value means the decision maker is more risk-averse, and assigning a lower value means the decision maker is more risk-seeking. The decision maker or the modeler of a problem can run experimental simulations to understand the

effect of different settings and find the appropriate risk settings (m). This task is not computationally expensive if an evolutionary algorithm is used. On the other hand, weighting methods similar to Multi-Criteria Decision Making (MCDM) techniques can be used to determine the appropriate risk preference. For example, in the Analytic Hierarchy Process (AHP) [97], an important task of the decision makers is to determine the weight to be given to each criterion in making the choice. A meaningful and objective numerical value on each of the criteria is set in the process.

Table 19. Different m values for risk-seeking case

		$m = 0$ (PUI)	$m_1 = -1,$ $m_2 = -1$	$m_1 = -4,$ $m_2 = -4$	$m_1 = -5,$ $m_2 = -5$	$m_1 = -6,$ $m_2 = -6$
rPUI	1	0.3347	0.4348	0.4189	0.4168	0.4162
	2	0.8022	0.5400	0.4293	0.4100	0.3974
	3	0.3234	0.3965	0.3040	0.2853	0.2711
		$m_1 = 0,$ $m_2 = -6$	$m_1 = -6,$ $m_2 = 0$	$m_1 = -6,$ $m_2 = 6$	$m_1 = 6,$ $m_2 = -6$	$m_1 = -6,$ $m_2 = 15$
rPUI	1	0.4245	0.4348	0.5163	0.4344	0.8513
	2	0.4388	0.5446	1.3659	0.4917	24.937
	3	0.2908	0.4135	0.9851	0.3157	9.6542

3.3 Discrete PUI and rPUI

For some problems, the variables are naturally discrete, and there are other instances, where it is convenient to approximate a continuous distribution with random selected outcomes using Monte Carlo simulations. If probability distributions are not available for objective function uncertainties, then simulation methods can be used to provide discrete points for each solution to represent the uncertainty. When there are discrete points for

each solution, the cumulative PUI is formulated as in (38). Other designs of the PUI can be modified for discrete case similar to the cumulative PUI example provided here.

$$\begin{aligned}
PUI_j &= \sum_{i=1, i \neq j}^n \Pr\{\mathbf{x}_i \succ \mathbf{x}_j\} \\
&= \sum_{i=1, i \neq j}^n \Pr\{ '_{1, \mathbf{x}_i} < '_{1, \mathbf{x}_j} \cap '_{2, \mathbf{x}_i} < '_{2, \mathbf{x}_j} \cap \dots \cap '_{N, \mathbf{x}_i} < '_{N, \mathbf{x}_j} \} \\
&= \sum_{i=1, i \neq j}^n \prod_{m=1}^M \Pr\{ '_{m, \mathbf{x}_i} < '_{m, \mathbf{x}_j} \} \\
&= \sum_{i=1, i \neq j}^n \prod_{m=1}^M \left[\left(\sum_{jd=1}^D \sum_{id=1}^D \Psi('_{m, \mathbf{x}_i}^{id} < '_{m, \mathbf{x}_j}^{jd}) \right) / D^2 \right]
\end{aligned} \tag{38}$$

where D is the number of discrete points for each solution. id and jd are the indices for discrete points of solutions i and j , respectively. $'_{m, \mathbf{x}_i}^{id}$ is the m^{th} objective function value of id^{th} discrete point of solution i . $\Psi('_{m, \mathbf{x}_i}^{id} < '_{m, \mathbf{x}_j}^{jd})$ is a binary function where it takes 1 if $'_{m, \mathbf{x}_i}^{id} < '_{m, \mathbf{x}_j}^{jd}$ is true and 0 else.

When risk adjustment is necessary, the rPUI is used and the continuous rPUI formula is as follows:

$$\begin{aligned}
rPUI_j &= \sum_{i=1, i \neq j}^n \mathfrak{R}(\mathbf{z}_j) \Pr\{\mathbf{x}_i \succ \mathbf{x}_j\} \\
&= \sum_{i=1, i \neq j}^n \mathfrak{R}(\mathbf{z}_j) \Pr\{ '_{1, \mathbf{x}_i} < '_{1, \mathbf{x}_j} \cap '_{2, \mathbf{x}_i} < '_{2, \mathbf{x}_j} \cap \dots \cap '_{N, \mathbf{x}_i} < '_{N, \mathbf{x}_j} \} \\
&= \sum_{i=1, i \neq j}^n \prod_{m=1}^M \mathfrak{R}(z_{m,j}) \Pr\{ '_{m, \mathbf{x}_i} < '_{m, \mathbf{x}_j} \} \\
&= \sum_{i=1, i \neq j}^n \prod_{m=1}^M \int_{'_{m, \mathbf{x}_j}} \exp \left(\dots_m \left(\frac{z_{m,j} - E['_{m, \mathbf{x}_j}]}{E['_{m, \mathbf{x}_j}]} \right) \right) \Pr\{ '_{m, \mathbf{x}_i} < '_{m, \mathbf{x}_j} \} d z_{m, \mathbf{x}_j}
\end{aligned} \tag{39}$$

where

$$\Pr\{ '_{m, \mathbf{x}_i} < '_{m, \mathbf{x}_j} \} = F_{'_{m, \mathbf{x}_i}}('_{m, \mathbf{x}_i} < z_{m,j} \mid '_{m, \mathbf{x}_j} = z_{m,j}) f_{'_{m, \mathbf{x}_j}}(z_{m,j}) \tag{40}$$

When we have discrete points for each solution we can formulate the rPUI as follows:

$$\begin{aligned}
rPUI_j &= \sum_{i=1, i \neq j}^n \mathfrak{R}(\mathbf{z}_j) \Pr\{\mathbf{x}_i \succ \mathbf{x}_j\} \\
&= \sum_{i=1, i \neq j}^n \mathfrak{R}(\mathbf{z}_j) \Pr\{ '_{1, \mathbf{x}_i} < '_{1, \mathbf{x}_j} \cap '_{2, \mathbf{x}_i} < '_{2, \mathbf{x}_j} \cap \dots \cap '_{N, \mathbf{x}_i} < '_{N, \mathbf{x}_j} \} \\
&= \sum_{i=1, i \neq j}^n \prod_{m=1}^M \mathfrak{R}(z_{m,j}) \Pr\{ '_{m, \mathbf{x}_i} < '_{m, \mathbf{x}_j} \} \\
&= \sum_{i=1, i \neq j}^n \prod_{m=1}^M \left[\left(\sum_{jd=1}^D \mathfrak{R}('_{m, \mathbf{x}_j}^{jd}) \sum_{id=1}^D \Psi('_{m, \mathbf{x}_i}^{id} < '_{m, \mathbf{x}_j}^{jd}) \right) / D^2 \right]
\end{aligned} \tag{41}$$

where $\mathfrak{R}('_{m, \mathbf{x}_j}^{jd})$ for the discrete rPUI is formulated as follows:

$$\mathfrak{R}(z_{m,j}) = \exp \left(\dots_m \left(\frac{ '_{m, \mathbf{x}_j}^{jd} - E['_{m, \mathbf{x}_j}]}{E['_{m, \mathbf{x}_j}]} \right) \right) \tag{42}$$

3.3.1 Comparison of Discrete and Continuous PUI and rPUI

Let there be three solutions in the objective space of a bi-objective minimization problem. Assume solution objective functions ($'_{m, \mathbf{x}_i}$) are all normally distributed random variables. Their expected values and variances are provided in Table 20.

Table 20. Expected values and variances for risk-averse case

Solutions	Expected Values [Cost, Emissions]	Variances [Cost, Emissions]
1	[3075,160]	[700,80]
2	[3085,165]	[150,15]
3	[3090,155]	[700,90]

Using Monte Carlo simulation and the same values for expectation and variance, we generate D number of discrete points for each solution and compare results of continuous and discrete PUI formulas for the same solutions. Table 21, Table 22 and Table 23 show

the comparison between continuous and discrete PUI and rPUI for 1000, 500 and 100 discrete points, respectively.

Table 21. Comparison of continuous and discrete PUI and rPUI
($D = 1000$, $n_1 = 15$, $n_2 = 15$)

Solutions	PUI			rPUI		
	Continuous	Discrete	Error	Continuous	Discrete	Error
1	0.3347	0.3269	2.33%	0.8046	0.7298	9.30%
2	0.8023	0.7940	1.03%	0.9176	0.9223	0.51%
3	0.3234	0.3371	4.24%	1.0716	1.1501	7.33%

Table 22. Comparison of continuous and discrete PUI and rPUI
($D = 500$, $n_1 = 15$, $n_2 = 15$)

Solutions	PUI			rPUI		
	Continuous	Discrete	Error	Continuous	Discrete	Error
1	0.3347	0.3453	3.17%	0.8046	0.7917	1.60%
2	0.8023	0.7795	2.84%	0.9176	0.8712	5.06%
3	0.3234	0.3455	6.83%	1.0716	1.3578	26.71%

Table 23. Comparison of continuous and discrete PUI and rPUI
($D = 100$, $n_1 = 15$, $n_2 = 15$)

Solutions	PUI			rPUI		
	Continuous	Discrete	Error	Continuous	Discrete	Error
1	0.3347	0.2922	12.70%	0.8046	0.6643	17.44%
2	0.8023	0.8471	5.58%	0.9176	1.0120	10.29%
3	0.3234	0.3576	10.58%	1.0716	1.2043	12.38%

The following figures depict the expected values, 3 -distances and discrete points for solutions in the objective space. Squares are the expected values of solutions and ellipses are 3 -distance from the expectations. Asterisks (*) are the discrete points generated for each solution. Figure 24, Figure 25 and Figure 26 show the comparison between

continuous and discrete representation of solutions for 1000, 500 and 100 discrete points, respectively.

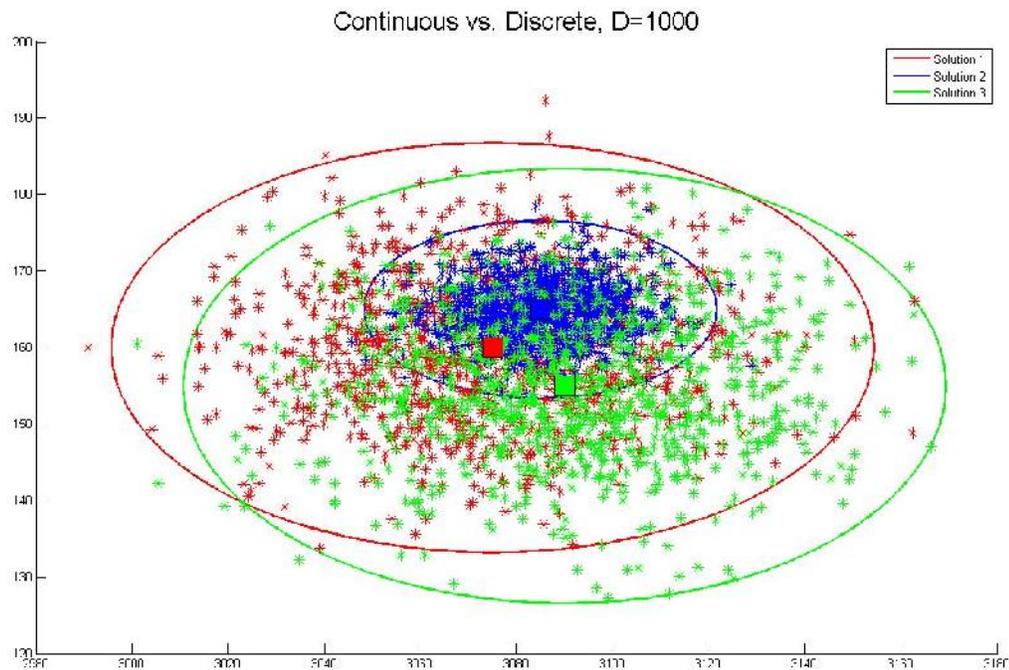


Figure 24. Continuous and discrete representation of solutions when $D = 1000$

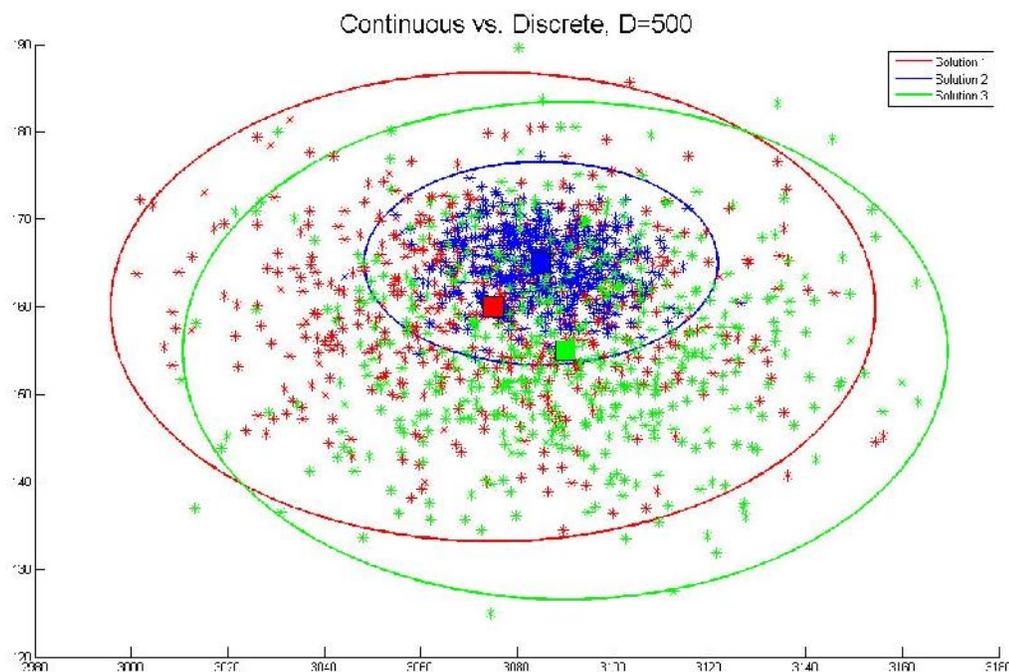


Figure 25. Continuous and discrete representation of solutions when $D = 500$

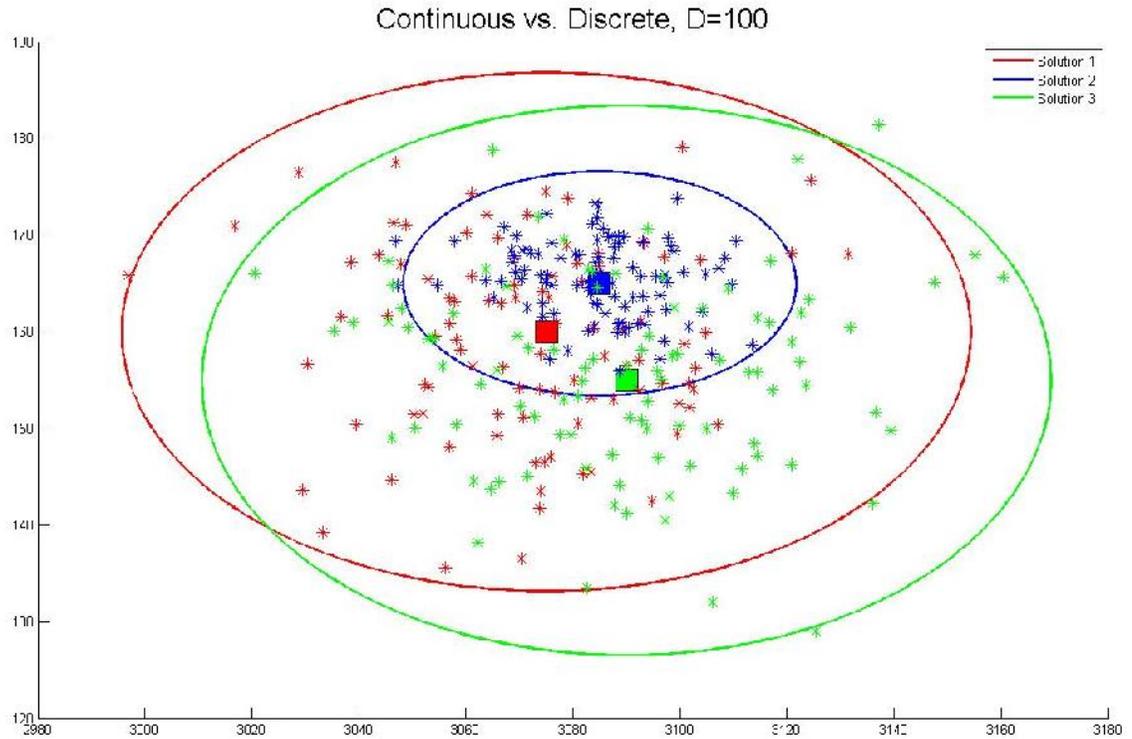


Figure 26. Continuous and discrete representation of solutions when $D = 100$

The more discrete points that are used to represent the uncertainty, the more accurate the PUI and rPUI are. Using discrete points, especially in large numbers, may increase the computation time but it is very useful when underlying uncertainty is from a combination of different probability distributions or dependency of objective functions or coefficients exists. As discussed in the dependent PUI formulations, if objectives are dependent, then the PUI formulation requires multivariate integration. This might be impractical or inefficient. However, simulating the randomness and using the discrete PUI can make it very simple and effective to consider dependent random coefficients of the objective functions. Section 4.5.8 presents a test problem where the discrete PUI is used.

3.4 PUI and rPUI with Scenarios

If probability distributions are not available for objective function uncertainties and simulation methods are computationally expensive, then scenarios can be used to represent the uncertainty. Similar to the discrete case we can formulate the cumulative PUI with scenarios and their respective probabilities as in (43). Other designs of PUI can be modified for scenario case similar to the cumulative PUI example provided here.

$$\begin{aligned}
PUI_j &= \sum_{i=1, i \neq j}^n \Pr\{\mathbf{x}_i \succ \mathbf{x}_j\} \\
&= \sum_{i=1, i \neq j}^n \Pr\{f'_{1, \mathbf{x}_i} < f'_{1, \mathbf{x}_j} \cap f'_{2, \mathbf{x}_i} < f'_{2, \mathbf{x}_j} \cap \dots \cap f'_{N, \mathbf{x}_i} < f'_{N, \mathbf{x}_j}\} \\
&= \sum_{i=1, i \neq j}^n \prod_{m=1}^M \Pr\{f'_{m, \mathbf{x}_i} < f'_{m, \mathbf{x}_j}\} \\
&= \sum_{i=1, i \neq j}^n \prod_{m=1}^M \sum_{s=1}^S \Psi(f'^s_{m, \mathbf{x}_i} < f'^s_{m, \mathbf{x}_j}) \Pr(s)
\end{aligned} \tag{43}$$

where S is the number of scenarios and $\Pr(s)$ is the probability of scenario s occurring.

f'^s_{m, \mathbf{x}_i} is the m^{th} objective function value of solution i for scenario s . $\Psi(f'^s_{m, \mathbf{x}_i} < f'^s_{m, \mathbf{x}_j})$ is a binary function equal to 1 if $f'^s_{m, \mathbf{x}_i} < f'^s_{m, \mathbf{x}_j}$ is true and 0 else.

We can also formulate the rPUI with scenarios as follows:

$$\begin{aligned}
rPUI_j &= \sum_{i=1, i \neq j}^n \Re(\mathbf{z}_j) \Pr\{\mathbf{x}_i \succ \mathbf{x}_j\} \\
&= \sum_{i=1, i \neq j}^n \Re(\mathbf{z}_j) \Pr\{f'_{1, \mathbf{x}_i} < f'_{1, \mathbf{x}_j} \cap f'_{2, \mathbf{x}_i} < f'_{2, \mathbf{x}_j} \cap \dots \cap f'_{N, \mathbf{x}_i} < f'_{N, \mathbf{x}_j}\} \\
&= \sum_{i=1, i \neq j}^n \prod_{m=1}^M \Re(z_{m, j}) \Pr\{f'_{m, \mathbf{x}_i} < f'_{m, \mathbf{x}_j}\} \\
&= \sum_{i=1, i \neq j}^n \prod_{m=1}^M \sum_{s=1}^S \Re(f'^s_{m, \mathbf{x}_j}) \Psi(f'^s_{m, \mathbf{x}_i} < f'^s_{m, \mathbf{x}_j}) \Pr(s)
\end{aligned} \tag{44}$$

where $\Re(f'^s_{m, \mathbf{x}_j})$ for the rPUI with scenarios is formulated as follows:

$$\mathfrak{R}(z_{m,j}) = \exp \left(\dots_m \left(\frac{z_{m,j} - E[z_{m,j}]}{E[z_{m,j}]} \right) \right) \quad (45)$$

3.5 Summary

A new metric, the Pareto Uncertainty Index (PUI), is presented in this section. This metric enables a direct approach to solve a multi-objective stochastic problem without any transformation of the problem, and it also provides the opportunity to the decision maker to observe the uncertainty of solutions in the Pareto set.

We first explain the PUI formulation for continuous objective functions without a risk preference. Objective functions can be dependent or independent and PUI formulations are presented for each case. There can be several ways to form different designs of the PUI. We propose four different designs which can be used for different purposes in a search algorithm. Later, the risk adjusted PUI (rPUI) formulation is explained and tested with different cases.

PUI can also be formulated for simulated discrete values or scenarios. Formulations for these are also explained and discrete PUI is compared with the continuous cases.

In the next chapter, a new multi-objective genetic algorithm, the Pareto Uncertainty Genetic Algorithm (PUGA), is presented. PUGA incorporates the PUI models and provides a direct approach to solve a stochastic multi-objective problem without any transformation of the problem.

4. Pareto Uncertain Genetic Algorithm (PUGA)

Genetic algorithm is a search heuristic that mimics the process of natural evolution. Being a population-based approach, genetic algorithms are well suited to solve the multi-objective optimization problems. A generic single-objective genetic algorithm can be modified to find a set of multiple non-dominated solutions (the Pareto set) in a single run [76]. Multi-objective genetic algorithms differ from single objective genetic algorithms in the way the selection is made and the fitness is assigned to each solution in the population. The concept of dominance is implemented for selection of individuals.

Multi-objective genetic algorithms introduce different methods to promote solution diversity. Most commonly used methods are fitness sharing, crowding distance method and cell-based density method. State-of-the-art multi-objective genetic algorithms are introduced in Section 2.3.4.3. All of the introduced genetic algorithms are for multi-objective problems but they do not consider the stochastic nature of real world problems. As we have discussed in Section 2.4, uncertainty in multi-objective optimization, the multi-objective stochastic problem is transformed into a multi-objective deterministic problem and then solved by genetic algorithms. The trade-off of objectives (Pareto front) can be captured but the randomness and uncertainty are eliminated. However, it is very important for a decision maker to have the knowledge of uncertainty in the Pareto set. There are three example cases in Section 2.4 to illustrate why it is important to observe the uncertainty of each solution in the objective space.

In this section, a new multi-objective genetic algorithm is presented: the Pareto Uncertainty Genetic Algorithm (PUGA). PUGA incorporates the Pareto Uncertainty Index (PUI) models explained in Section 3 and provides a direct approach to solve a

stochastic multi-objective problem without any transformation of the problem, and it also gives the opportunity to the decision maker to observe the uncertainty of solutions in the Pareto set.

4.1 Design Components of PUGA

In the next subsections, design components of PUGA are explained in detail. Components of a generic genetic algorithm are explained in Section 2.3.4.1. In its general form, a genetic algorithm works as follows and as depicted in Figure 27.

- An initial population of individuals (solutions) is generated.
- At every generation, the individuals are evaluated and a fitness value is assigned.
- *Selection* operator selects the fittest individuals.
- If the stopping criterion is met the algorithm stops and gives the result.
- If not, a new generation is created by operators such as *mutation* and *crossover*.

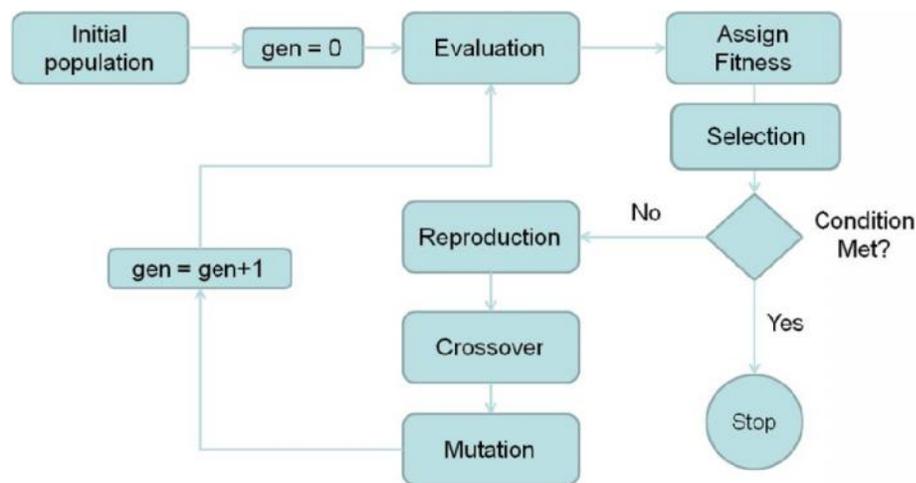


Figure 27. Flow chart of a genetic algorithm

4.1.1 Pareto Probability Sorting

A multi-objective optimization technique should achieve the closest possible Pareto front to the true Pareto front. This is called the convergence to the Pareto optimal set.

Pareto ranking approaches are one of the most commonly used techniques in multi-objective genetic algorithms. Examples of the multi-objective genetic algorithms that use a kind of Pareto-ranking approaches are: MOGA [80], PAES [73], NSGA [82] and NSGA-II [83]. Pareto-ranking approaches use the concept of an expected Pareto dominance idea (Section 2.3.1) to rank solutions in the objective space where only the expected values of objective functions are considered, thus uncertainty is neglected. Solutions are ranked according to a dominance rule, and then they are ranked accordingly. The actual objective function value is not a fitness criterion after this type of ranking procedure. When all objectives are to be minimized, a lower rank corresponds to a better solution. We will assume minimization problems in all sections to follow.

The sorting mechanism of PUGA uses one of the PUI formulations explained in Section 3.1, so that the probability is included in the Pareto-ranking. There can be several sorting designs using PUI formulations. We propose four different designs in this study. Table 24 shows the main differences of proposed Pareto ranking designs of PUGA.

Table 24. Main differences of Pareto ranking designs of PUGA

	Ranking Design
Design 1	<ul style="list-style-type: none"> • First check expected domination • PUI calculated pairwise with dominated solutions • Rerank with stochastic domination
Design 2	<ul style="list-style-type: none"> • No domination scheme • PUI calculated cumulatively • Order by PUI only
Design 3	<ul style="list-style-type: none"> • First check expected domination • PUI calculated in neighborhoods • Preference by stochastic domination
Design 4	<ul style="list-style-type: none"> • First check expected domination • PUI calculated pairwise with closest solutions • Preference by stochastic domination

For a minimization problem, a solution \mathbf{x}_i said to dominate another solution \mathbf{x}_j , $\mathbf{x}_i \succ \mathbf{x}_j$, if these two conditions are both satisfied:

1. The solution \mathbf{x}_i is no worse than \mathbf{x}_j for all M objective functions, that is,

$$f'_m(\mathbf{x}_i) \leq f'_m(\mathbf{x}_j) \text{ for all } m = 1, 2, \dots, M$$

2. The solution \mathbf{x}_i is strictly better than \mathbf{x}_j for at least one objective function, that is,

$$f'_m(\mathbf{x}_i) < f'_m(\mathbf{x}_j) \text{ for at least one } m \in \{1, 2, \dots, M\}$$

where $f'_m(\mathbf{x}_i)$ is the m^{th} objective function evaluated at solution vector \mathbf{x}_i and M is the number of objective functions. This is the Pareto dominance concept where the uncertainty is neglected. We call this design the *expected domination*. This is similar to the ranking design of NSGA-II [74].

For the expected domination, algorithm finds the domination count (n), which is the number of solutions which dominate the solution i for each solution in the population. When we compare two solutions for their domination relationship, we first consider their expected domination to each other. If the two solutions i and j do not expectedly dominate each other, then both of them keep their n value as zero (not changed), that is they are both non-dominated to each other in the expected domination sort. While counting the number of solutions (n) which dominate the solution we also keep a set of solutions that the solution dominates (S_i).

In PUGA **design 1**, if one of the solutions expectedly dominate the other one, then we calculate the probability that solution i dominates solution j (Pr_{ij}) and the probability that solution j dominates solution i (Pr_{ji}). If Pr_{ij} is greater than Pr_{ji} , then we say solution i

dominates solution j and n_j value of solution j is increased by one. Otherwise (if $\Pr_{ji} > \Pr_{ij}$), we say solution j dominates solution i and n_i value of solution i is increased by one.

The rest of the sorting works similar to NSGA-II. All solutions with domination count of zero are designated in the first non-dominated front. For each solution in the first front, we visit each member of its set S_i and reduce its domination count n_j by one. If the domination count n_j becomes zero for any member, it is put in a separate list which defines the second non-dominated front. This procedure is continued until all fronts are identified. The algorithm for Pareto Probability Sorting with ranking design 1 is as follows:

```

For each  $i \in P$ 
   $S_i = \emptyset$  and  $n_i = 0$ 
  For each  $j \in P$  and  $j \neq i$ 
    If  $\mathbf{x}_i \succ \mathbf{x}_j$  or  $\mathbf{x}_j \succ \mathbf{x}_i$ 
      If  $\Pr\{\mathbf{x}_i \succ \mathbf{x}_j\} > \Pr\{\mathbf{x}_j \succ \mathbf{x}_i\}$ 
         $S_i = S_i \cup j$  and  $n_j = n_j + 1$ 
      Else if  $\Pr\{\mathbf{x}_j \succ \mathbf{x}_i\} > \Pr\{\mathbf{x}_i \succ \mathbf{x}_j\}$ 
         $S_j = S_j \cup i$  and  $n_i = n_i + 1$ 
  End  $j$  loop
  If  $n_i = 0$ 
     $i_{rank} = 1$  and  $F_1 = F_1 \cup \{i\}$ 
     $r = 1$ 
  While  $F_r \neq \emptyset$ 
     $Q = \emptyset$ 
    For each  $i \in F_r$ 
      For each  $j \in S_i$ 
         $n_j = n_j - 1$ 
        If  $n_j = 0$  then  $j_{rank} = r + 1$  and  $Q = Q \cup \{j\}$ 
      End  $j$  loop
    End  $i$  loop
     $r = r + 1$ 
   $F_r = Q$ 

```

End while loop

Calculation of probabilities increases the complexity of the procedure, particularly in the beginning of the algorithm when many solutions may expectedly dominate many other solutions, but by doing so we eliminate the situation in example cases mentioned in Section 2.4. For example in Figure 29 compared to Figure 28, the concept of domination is not clear any more. Solution 1 has a higher level of uncertainty compared to Solution 2. If the decision maker prefers to observe expected values, then the preference of the solutions would be the same as in the deterministic case, and Solution 2 would not be presented in the Pareto optimal set. However, when the uncertainty is included, the probability of Solution 2 being dominated by Solution 1 becomes low, which gives it a probability to be a preferable solution.

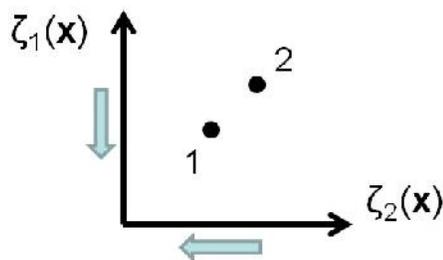


Figure 28. Deterministic example

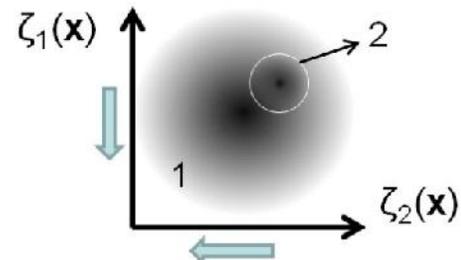


Figure 29. Stochastic example

In the second ranking design (**design 2**), there is no domination scheme. The algorithm uses the cumulative PUI to sort all the solutions in the population. The solution with the smallest PUI value is the best in the population. This design tends to eliminate the diversity of solutions and promotes a single solution. Thus, it is better for single objective genetic algorithms. The algorithm for Pareto Probability Sorting with ranking design 2 is as follows:

For each $i \in P$

$$PUI_i = \sum_{j=1, i \neq j}^n \Pr\{\mathbf{x}_j \succ \mathbf{x}_i\}$$
End i loop
Sort and rank ($i \in P, PUI_i$)

In **design 3**, the algorithm first utilizes the expected domination, similar to design 1. Then the PUI is calculated in neighborhoods. The neighborhood approach is explained in Section 4.1.6. Ranking is based on the expected domination but in each neighborhood the solution with the smallest PUI value is the preferred solution of the neighborhood. The solutions are sorted based on their rankings first and then PUI values. The smallest ranking solution with the smallest PUI value is the best solution of the population. This scheme is very helpful in operators for extraction of the best solutions, selection of parent solutions for reproduction and the Pareto pruning in post-Pareto analysis.

Design 4 differs from the previous design with PUI calculation. In this design, pairwise PUI calculation is utilized, that is, the PUI is the sum of domination probabilities of only the closest neighbor solutions for each objective function. Again, the solutions are sorted based on their rankings first and then PUI values. The smallest ranking solution with the smallest PUI value is the best solution of the population.

4.1.2 Preservation of Uncertainty Information and Diversity

Maintaining diversity is one of the most important issues in multi-objective optimization. In order to provide the decision maker a true representation of the trade-offs of the objectives, Pareto solutions should be uniformly distributed over the Pareto front. NSGA-II uses the *crowding distance* approach for diversity preservation. The average distance of two points on either side of a solution i along each of the objectives is

calculated. The higher this value is, the better the diversity of solution i . The crowding distance computation is applicable to more than two objectives.

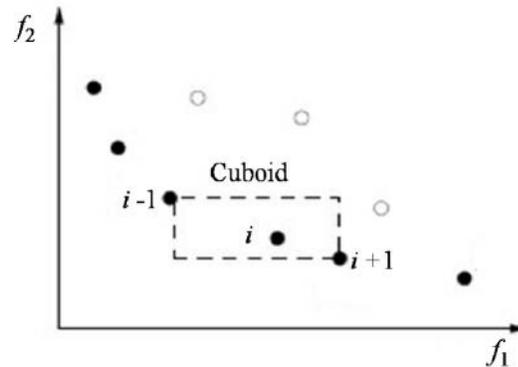


Figure 30. NSGA-II crowding distance method (adapted from Deb et al. [76])

For each objective function, the algorithm sorts the individuals in each front F_r in the ascending order and it assigns crowding distance values to each of the individuals, as shown in Figure 30. Although Figure 30 illustrates the crowding distance computation for two objectives, the procedure is applicable to more than two objectives as well. The main advantage of the crowding distance approach is that it is computed without requiring a user-defined parameter. In NSGA-II, the crowding distance measure is used as a tiebreaker in a selection technique called the crowded tournament selection operator. The algorithm randomly selects two solutions; if the solutions are in the same non-dominated front, the solution with a higher crowding distance is the winner. Otherwise, the solution with the lowest rank is selected [76]. This approach considers only the expected values of objective functions. Therefore, uncertainty is neglected.

PUGA, on the other hand, uses one of the PUI methods to preserve the diversity and uncertainty information of solutions. The PUI of a solution, PUI_i , is used to compare solutions in the same front. The solution with a smaller PUI is a more diverse and less

uncertain solution so it has a higher probability to be selected for the next generation or the last Pareto front.

4.1.3 Elitism

Elitism means that the best solutions found so far during the search always survives to the next generation. In this respect, all non-dominated solutions discovered by a multi-objective genetic algorithm are considered elite solutions [76].

PUGA uses the same strategy as NSGA-II and combines the parent population and the new offspring population, and then the combined population is sorted according to the Pareto probability sorting. This approach makes sure that all the non-dominated solutions are included in the next population. The algorithm for elitism is as follows:

Q_t : Offspring population from reproduction
 Combine parents and offspring $R_t = P_t \cup Q_t$
 Sort the combined solutions into fronts $F = \text{sort}(R_t)$
 Initialize next generation population $P_{t+1} = \emptyset$ and ranks $r = 1$
 Until the population size N is reached: $|P_{t+1}| + |F_r| \leq N$
 $P_{t+1} = P_{t+1} \cup F_r [1:(N - |P_{t+1}|)]$: Choose the best $(N - |P_{t+1}|)$ members of the front F_r
 $r = r + 1$
 End

4.1.4 Constraint Handling

Most optimization problems include constraints that must be satisfied. Constraints are often treated as deterministic but they usually include stochastic elements. PUGA can accommodate both deterministic and stochastic constraints.

4.1.4.1 Deterministic Constraints

Deterministic constraints can be efficiently handled with the approach proposed for constrained NSGA-II. For each solution in the population, the algorithm checks all

constraints and stores the number of violations and sum of all violations of each solution.

When comparing two solutions, the algorithm uses the following rules:

- If both solutions are feasible choose the better ranked solution, and if they have the same rank choose the less crowded solution, i.e., that is the solution with higher crowding distance value.
- If one solution is feasible and the other is not, then choose the feasible solution.
- If both solutions are infeasible, then choose the solution with smaller overall constraint violation.

PUGA uses the same rules if the constraints are assumed as deterministic. Instead of the crowding distance value, the PUI value can also be used for the first rule.

4.1.4.2 Constraints with Uncertainty

For constraints with uncertainty we present a method that is based on chance constraint method. PUGA checks all constraints for each solution depending on their probability of satisfying the constraints over a user-defined risk measure r_i for each constraint i . For example, let the optimization problem is subject to below constraints:

$$\mathbf{a}^1\mathbf{x} \leq b^1, \mathbf{a}^2\mathbf{x} = b^2, \mathbf{a}^3\mathbf{x} \geq b^3$$

The decision maker defines r_i for each constraint and the constraints are converted to the following, respectively:

$$\Pr\{\mathbf{a}^1\mathbf{x} \leq b^1\} \geq r_1, \Pr\{\mathbf{a}^2\mathbf{x} = b^2\} \geq r_2, \Pr\{\mathbf{a}^3\mathbf{x} \geq b^3\} \geq r_3$$

If a solution satisfies all r_i , then it is stored as a feasible solution. If it does not satisfy any of the constraints, then the number of violations and sum of all violations are stored.

In the selection process, PUGA uses the same rules described above to compare two solutions.

The constraint handling technique based on chance constraint method and the user-defined α_i values are also used as a risk measure in PUGA. For instance, a risk-seeking decision maker may choose a smaller value for α_i , so that the algorithm provides solutions with more relaxed constraints. On the contrary, a risk-averse decision maker may choose a bigger value for α_i , and the algorithm provides solutions with strictly satisfied constraints.

For every constraint in the model, α_i can be defined separately, so that every constraint can be considered, however the decision maker prefers. If the decision maker prefers expected values for constraints only, then PUGA can easily be converted to a deterministic constraint handling algorithm.

4.1.5 Twin Solution Elimination

Genetic algorithms sometimes produce the same solutions in the same generation which reduces the effectiveness of the process and diversity of the population. These solutions which have the exact same decision variable vectors are called twin solutions. PUGA checks all solutions with others in the population and if there are twins of a solution, then twin solutions are punished by increasing their rankings to the last front rank. The algorithm for twin solution elimination is as follows:

```

For  $i = 1$  to  $N-1$ 
  For  $j = i+1$  to  $N$ 
    If  $\mathbf{x}_i = \mathbf{x}_j$ 
       $\mathbf{x}_j(\text{rank}) = \max(F_r)$ 
    End  $j$  loop
  End  $i$  loop

```

4.1.6 Neighborhood Mechanism

Haiming and Yen [96] present a cell-based density approach in Rank-Density Based Genetic Algorithm (RDGA). RDGA cell-based density approach is depicted in Figure 31. A modified version of this approach is used to improve the diversity and post-Pareto pruning performance of PUGA.

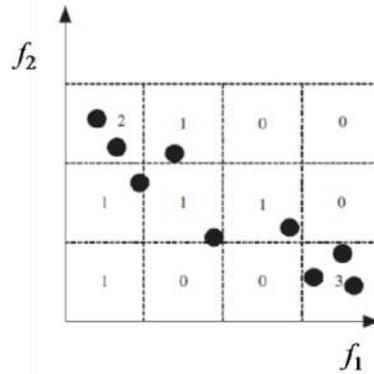


Figure 31. RDGA cell-based density approach

In every generation, the neighborhood approach in PUGA first creates neighborhoods by dividing the range of the current objective space based on user given number of neighborhoods for each objective function (K_m). To form neighborhoods, PUGA calculates the neighborhood width (d_m):

$$d_m = \frac{\max f_m(\mathbf{x}) - \min f_m(\mathbf{x})}{K_m} \quad (46)$$

where d_m is the width of the neighborhood in the m^{th} dimension, K_m denotes the number of neighborhoods designated for the m^{th} dimension. $\max f_m(\mathbf{x})$ and $\min f_m(\mathbf{x})$ are maximum and minimum fitness values of the m^{th} objective function (dimension).

Second, each individual of the population searches for its neighborhood and considers the other individuals who share the same neighborhood as its neighbors. Then, the PUI

for each individual of each neighborhood is calculated. The algorithm for the neighborhood is as follows:

```

For  $m = 1$  to  $M$ 
   $d_m = \frac{\max f_m(\mathbf{x}) - \min f_m(\mathbf{x})}{K_m}$ 
End  $m$  loop
For  $i = 1$  to  $N$ 
  For  $m = 1$  to  $M$ 
     $idx_m = \text{round}\left(\frac{f_m(\mathbf{x}_i) - \min f_m(\mathbf{x})}{d_m}\right)$ 
  End  $m$  loop
   $neighborhood_i = \mathbf{id}\mathbf{x}$ 
End  $i$  loop

```

4.1.7 Post-Pareto Pruning

While PUGA uses the PUI to find diverse and less uncertain solutions, it calculates the domination probability of solutions. The algorithm can use this probability information to make a preference decision between two solutions on the same front so that the *preference* of a solution i can be determined for post-Pareto pruning. The algorithm for *preference* calculation of PUGA is as follows:

```

Set of solutions in the Front  $r$ :  $\Gamma = \{F_r\}$ 
Number of solutions in the non-dominated set  $\Gamma$ :  $l = |\Gamma|$ 
For  $i = 1$  to  $l-1$ 
  If  $PUI_{i+1} > PUI_i$ 
     $pref_i = pref_{i+1}$ 
  Else  $pref_{i+1} = pref_{i+1} + 1$ 
End  $i$  loop

```

4.2 Main Loop of PUGA

Genetic algorithms start the process with an initial population. The initial population can be randomly created in the objective space or it can be provided by the user. If the user provides an initial population, then the performance of algorithm increases because

convergence would be faster. The initial population can be created by one of the classical methods discussed in Section 2.3.2. Operators of the main loop are explained in the next subsections.

4.2.1 Evaluation Operator

After creation of initial population, each solution is sent to *evaluation operator* to compute its expected values of objective functions and variances of objective functions, feasibility and violation of constraints if it is infeasible. The initial population is then sorted according to the *Pareto Probability Sorting* mechanisms described in Section 4.1.1. This step concludes the initialization and the main loop starts and works until the stopping criterion is met. The algorithm for evaluation operator is as follows:

```

For  $i = 1$  to  $N$ 
   $E[\mu_k(P_i)]$  for all  $k$ 
   $\text{Var}[\mu_k(P_i)]$  for all  $k$ 
   $P_i(\text{feasibility})$ 
  If  $P_i(\text{feasibility}) = 0$ 
     $P_i(\text{violation})$ 
  End if
End  $i$  loop

```

4.2.2 Selection Operator

The first step of the main loop is to select parent solutions to create the next generation offspring solutions. There are several techniques proposed for selection operator. PUGA uses *tournament selection* technique to increase the selection probability of better ranked and less uncertain solutions. The algorithm for selection operator with tournament selection technique is as follows:

```

For  $i = 1$  to  $N$ 
  Randomly choose  $C_1$  and  $C_2$  as candidates for next population  $Q$ 
  If  $PUI_1 > PUI_2$  select candidate 1:  $Q_i = C_1$ 
  Else  $Q_i = C_2$ 
End  $i$  loop

```

4.2.3 Crossover and Mutation Operators

After selecting parent solutions, the algorithm utilizes *crossover* and *mutation* operators to create an offspring population Q_t of size N . PUGA uses the *intermediate crossover* [98] which creates offspring solutions by taking a weighted average of the parents. User can specify the weights by a single parameter, *ratio*, which can be a scalar or a row vector of length number of variables. The operator creates the offspring from $parent_1$ and $parent_2$ using the following formula:

$$Q_i = parent_1 + rand \times ratio \times (parent_2 - parent_1)$$

where *rand* is a random number uniformly distributed between 0 and 1.

If all the entries of *ratio* lie in the range [0, 1], the offspring produced are within the hypercube defined by placing the parents at opposite vertices. If *ratio* is not in that range, the children might lie outside the hypercube. If *ratio* is a scalar, then all the children lie on the line between the parents.

All of the individuals in the population would be processed by the crossover operator, but only *crossover fraction* of all variables would have the crossover operator applied. Although default value of *crossover fraction* is set to $2/numVar$, where *numVar* is the number of variables of the model, it can also be defined by the user.

PUGA uses the *Gaussian mutation* [98] which creates offspring solutions by adding a random number taken from a Gaussian distribution with mean zero (0) to each entry (decision variable) of the parent vector. The standard deviation of this distribution is

determined by the parameters *scale* and *shrink*. Mutation operator uses the following equations:

$$Q_i = \text{parent} + ss \times \text{randn} \times (ub - lb)$$

$$ss = \text{scale} \times (1 - \text{shrink} \times \text{currGen} / \text{maxGen})$$

The *scale* parameter determines the standard deviation at the first generation. The *shrink* parameter controls how the standard deviation shrinks as generations go by. *ub* and *lb* are upper and lower bounds of decision variables, respectively. *randn* is a normally distributed random variable.

As in the crossover operator, all of the individuals in the population would be processed by the mutation operator but only *mutation fraction* of all variables would have the mutation operator applied. Although default value of *mutation fraction* is set to $2/\text{numVar}$, it can also be defined by the user.

4.2.4 Evaluation, Elitism and Sorting of the New Generation

When the creation of the offspring population is completed, the algorithm evaluates the offspring population using the *evaluation operator* explained in Section 4.2.1.

PUGA uses the same strategy for elitism as NSGA-II and combines the parent population and the new offspring population, and then this combined population is sorted according to Pareto probability sorting. This approach makes sure that all non-dominated solutions are included in the next population. Pareto probability sorting and elitism are also explained in Section 4.1.1 and Section 4.1.3, respectively.

4.2.5 Extraction of the Best Solutions

PUGA first sorts all solutions in the combined population, R_t , of size $2N$ according to their front ranks and PUI. The best N solutions of the combined population, R_t , with smallest front ranks and PUI values are extracted as the next generation population, P_{t+1} .

If the stopping criterion is not met, then the generation counter is increased by one and the main loop of PUGA continues to generate another set of solutions. The stopping criterion of PUGA is the number of maximum generations defined by the user.

4.2.6 Algorithm for PUGA Main Loop

PUGA follows the main loop below while the generation counter (t) is less than the maximum number of generations provided by the user. When the main loop reaches to the maximum number of generations the algorithm outputs the results in the user defined format.

```

While  $t < maxgen$ 
   $t = t + 1$ 
   $Q_t = select(P_t)$ 
   $Q_t = crossover(Q_t)$ 
   $Q_t = mutation(Q_t)$ 
   $Q_t = evaluate(Q_t)$ 
   $R_t = P_t \cup Q_t$ 
   $R_t = sort(R_t)$ 
   $P_{t+1} = extract(R_t)$ 
End loop

```

4.3 PUGA designs

Using different PUI designs (explained in Section 3.1) in different algorithm operators mentioned above, we define four PUGA designs and test them. Table 25 shows the main differences of operators in these PUGA designs. Mainly the way that the PUI is

used in ranking and selecting solutions are different. Extraction operator uses the given PUI design with the expected domination scheme. Only the second design does not have domination approach. Instead, it completely depends on the cumulative PUI design for all related operators.

Table 25. PUGA designs

	Pareto Sorting	Extraction (Order of ranking)	Selection (Tournament Criteria)
Design 1	<ol style="list-style-type: none"> 1. First check expected domination 2. PUI calculated pairwise with dominated solutions 3. Rerank with stochastic domination 	<ol style="list-style-type: none"> 1. Rank 2. Pairwise PUI 	Pairwise PUI
Design 2	<ol style="list-style-type: none"> 1. No domination scheme 2. PUI calculated cumulatively 3. Order by PUI only 	Cumulative PUI	Cumulative PUI
Design 3	<ol style="list-style-type: none"> 1. First check expected domination 2. PUI calculated in neighborhoods 3. Preference by stochastic domination 	<ol style="list-style-type: none"> 1. Rank 2. Preference 3. Neighborhood PUI 	Neighborhood PUI
Design 4	<ol style="list-style-type: none"> 1. First check expected domination 2. PUI calculated pairwise with closest solutions (Preference PUI design) 3. Preference by stochastic domination 	<ol style="list-style-type: none"> 1. Rank 2. Preference 3. Pairwise PUI 	Pairwise PUI

4.4 CONSTR Test Problem

To test Pareto convergence and diversity performance of PUGA, a test problem, CONSTR, from Deb et al. [68] is used in this section. The CONSTR problem is used by Deb et al. [68] to test constraint handling of NSGA-II and the objective functions are as follows:

$$\min f_1(\mathbf{x}) = x_1 \quad (47)$$

$$\min f_2(\mathbf{x}) = \frac{1+x_2}{x_1} \quad (48)$$

Variable bounds are $x_1 \in [0.1, 1.0]$ and $x_2 \in [0, 5]$, and the model is subject to the following constraints:

$$x_2 + 9x_1 \geq 6 \quad (49)$$

$$-x_2 + 9x_1 \geq 1 \quad (50)$$

In this problem, a part of the unconstrained Pareto optimal region is not feasible. Thus, the resulting constrained Pareto optimal region is a concatenation of the first constraint boundary and some part of the unconstrained Pareto optimal region (Figure 32).

Deb et al. [83] use the simulated binary crossover (SBX) operator and polynomial mutation for real-coded NSGA-II. The crossover probability is 0.9 and the mutation probability is $1/\text{numVar}$, where numVar is the number of decision variables. Distribution indexes for crossover and mutation operators are 20 and 100, respectively. The population size is 100 and maximum number of generations is 500. This rather large number of generations is chosen by Deb et al. [83] to investigate if the spread in solutions can be maintained for a large number of generations. Figure 32 shows the feasible region and obtained non-dominated solutions with NSGA-II on the CONSTR test problem.

Since PUGA is an entirely new concept that solves multi-objective stochastic problems without any transformation, test problems that are used in literature are not adequate to test PUGA. Therefore, modified versions of problems are used to demonstrate the workings of PUGA. The CONSTR test problem is modified by assuming

uncertainty in its objective function coefficients and constants. The modified objective functions are as follows:

$$\min f_1(\mathbf{x}) = \tilde{c}_1 x_1 \quad (51)$$

$$\min f_2(\mathbf{x}) = \frac{\tilde{c}_2 + \tilde{c}_3 x_2}{x_1} \quad (52)$$

where c_i are all random variables and are normally distributed with mean of 1, and standard deviation of 0.001. Decision variable bounds are $x_1 \in [0.1, 1.0]$ and $x_2 \in [0, 5]$, and the model is subject to the same constraint set as the original model.

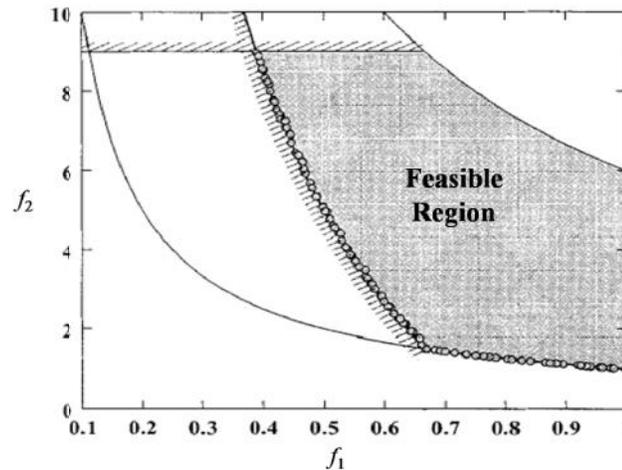


Figure 32. Feasible region and obtained non-dominated solutions with NSGA-II on CONSTR test problem (adapted from [83])

PUGA uses the intermediate crossover [98] and the Gaussian mutation [98] for real-coded decision variables. PUGA operator parameters are explained in Section 4.2.3. The population size and maximum number of generations for CONSTR problem are the same with NSGA-II, 100 and 500, respectively.

Figure 33 shows only PUGA results on CONSTR test problem and Figure 34 shows PUGA and NSGA-II results together for comparison. It is clear that PUGA performs as well as NSGA-II in terms of converging to the true Pareto optimal front and also in terms

of maintaining a diverse population of non-dominated solutions. According to Deb et al. [83], NSGA-II obtains a reasonably good spread of solutions as early as 200 generations. Although PUGA is slower due to the PUI probability calculations, it obtains a good spread of solutions as early as 80 generations. Additionally, in contrast to other optimization methods, PUGA has the ability to incorporate uncertainty.

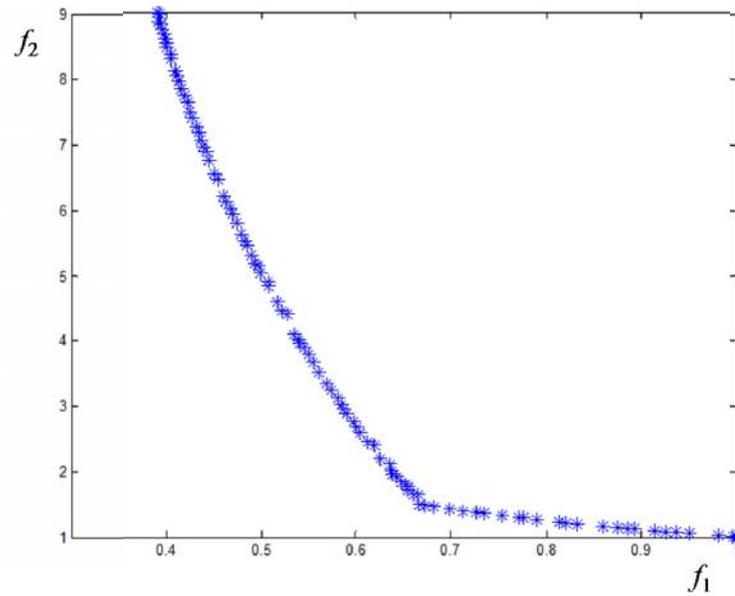


Figure 33. Obtained non-dominated solutions with PUGA on CONSTR test problem

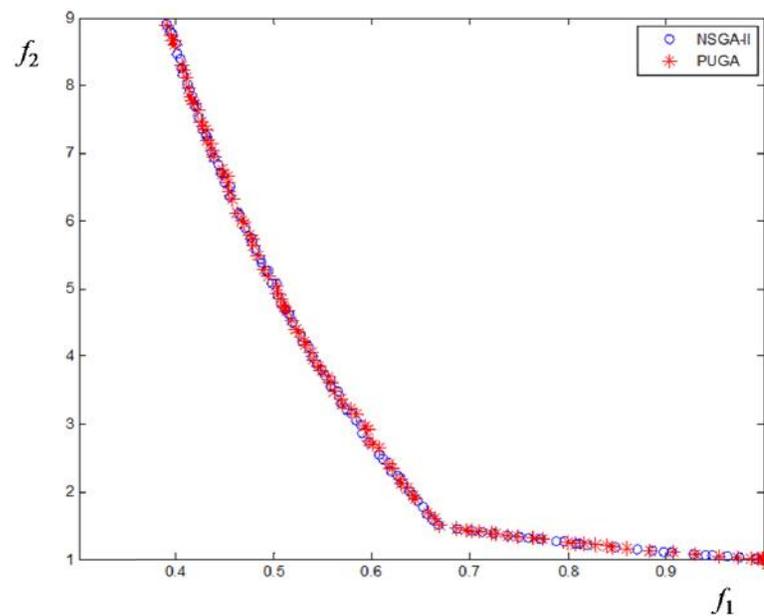


Figure 34. Obtained non-dominated solutions with NSGA-II and PUGA on CONSTR test problem

4.5 Power Generation Test Problem

The test problem in this section is originally a simplified power generation problem excerpted from [66]. Four types of power generation units are considered in a region, namely, lignite fired, oil fired, natural gas fired and units exploiting renewable energy sources (RES) which are mostly small hydro and wind. The power generation characteristics of these units are shown in Table 26.

Table 26. Power generation characteristics

Production units, p	Lignite	Oil	Natural Gas	RES
Maximum production per year (GWh), cap_p	31,000	15,000	22,000	10,000
Cost of production (\$/MWh), $c_{1,p}$	30	75	60	90
CO ₂ emissions coefficient (t/MWh), $c_{2,p}$	1.44	0.72	0.45	0

The yearly demand is 64,000 GWh and is characterized by a load duration curve which can be divided into three types of loads: base load (60%), medium load (30%) and peak load (10%). The lignite fired units can be used only for base and middle load, the oil fired units for middle and peak load, the RES units for base and peak load and the natural gas fired units for all type of loads. Two objective functions are considered: the minimization of production cost and the minimization of CO₂ emissions.

Table 27. Standard deviations of power generation characteristics

Production units, p	Lignite	Oil	Natural Gas	RES
Maximum production per year (GWh)	3,100	1,500	2,200	1,000
Cost of production (\$/MWh)	0.6	3.6	3.6	0.6
CO ₂ emissions coefficient (t/MWh)	0.23	0.01	0.01	0

In the original problem, everything is assumed to be deterministic, which ignores uncertainty and cannot accurately represent the problem. For the modification of the

problem, uncertainty is assumed for cost coefficients of production, CO₂ emissions coefficients, maximum production of power generation units and demand. For PUGA test problem, these deterministic coefficients and constants of the original problem are assumed to be random variables with a normal distribution and values provided in Table 26 are used as the expected values of these random variables. Standard deviations of the random variables are provided in Table 27. The expected values and standard deviations of three types of loads are provided in Table 28.

Table 28. Expected values and standard deviations of loads

Loads, l	Base	Medium	Peak
Expected	38400	19200	6400
Standard Deviation	3840	1920	640

Mavrotas [66] solves the deterministic problem with the augmented ϵ -constraint method. Expected Pareto front from the augmented ϵ -constraint method is presented in Figure 35 and these solutions are used as the initial population for PUGA test problem to increase the performance of the algorithm.

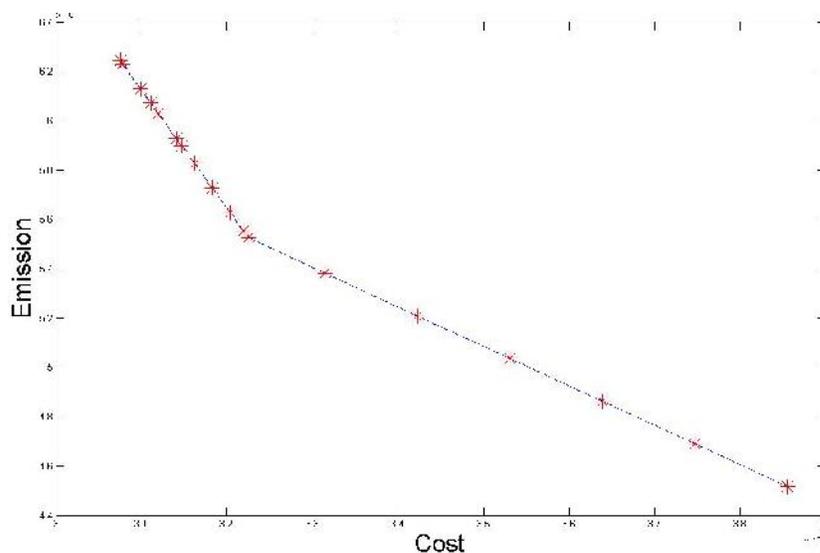


Figure 35. Test problem results from augmented ϵ -constraint method

4.5.1 Multi-objective stochastic model of the test problem

Two objective functions considered in the problem are the minimization of production cost and the minimization of CO₂ emissions. There is uncertainty in both of the objective functions due to the randomness of their coefficients. Each coefficient is a random variable with a normal distribution. All random variables have the tilde symbol (~) over them. The expected values and standard deviations of all random variables are presented in Table 26, Table 27 and Table 28.

$$\min \text{cost} = \sum_p \sum_l \tilde{c}_{1,p} x_{p,l} \quad (53)$$

$$\min \text{emissions} = \sum_p \sum_l \tilde{c}_{2,p} x_{p,l} \quad (54)$$

where $\tilde{c}_{k,p}$ is the coefficient of decision variable $x_{p,l}$ in objective function k . There are 12 decision variables in the problem which are all amounts of electricity production per unit p in load type l . For example, $x_{1,1}$, $x_{1,2}$, and $x_{1,3}$ are amounts of electricity production of lignite unit in base, medium and peak loads, respectively.

The model is subject to the following constraints:

$$\sum_l x_{p,l} \leq \tilde{c}ap_p \text{ for each unit } p \quad (55)$$

$$\sum_p x_{p,l} \geq \tilde{load}_l \text{ for each load } l \quad (56)$$

$$x_{1,3}, x_{2,1}, \text{ and } x_{4,2} = 0 \quad (57)$$

$$x_{p,l} \geq 0 \quad \forall p \text{ and } l \quad (58)$$

Equation (55) is the capacity limit for each production unit p and $\tilde{c}ap_p$ is a normal random variable which is the capacity limit of production unit p . Equation (56) is the demand constraint and \tilde{load}_l is a normal random variable which is the demand amount

that has to be satisfied in load l . Equation (57) is the availability of some units. The lignite fired units can be used only for base and middle load, so $x_{1,3}$ is 0, the oil fired units are only available for middle and peak load, so $x_{2,1}$ is 0, the RES units can be used only for base and peak load, so $x_{4,2}$ is 0. Finally, Equation (58) is the positive variable constraint, that is, all decision variables have to be greater than or equal to zero.

4.5.2 Calculation of probabilities

All random variables in the test problem are assumed to be normal random variables with a mean and a standard deviation. Objective functions and constraints are treated as independent functions, that is, there is no covariance considered in the test problem. When PUGA calculates a probability that a solution dominates another solution, it first obtains the mean and variance of every solution in every objective function. For example, let \mathbf{x}_i and \mathbf{x}_j be two solutions in the objective space to be compared. PUGA calculates the mean, $E[{}'_k(\mathbf{x}_i)]$, and the variance, $\text{Var}[{}'_k(\mathbf{x}_i)]$, of objective function k evaluated at solution vector $\mathbf{x}_i = (x_1, x_2, x_3, \dots)$ as follows:

$$E[{}'_k(\mathbf{x}_i)] = \sim_{c_{k,1}} x_1 + \sim_{c_{k,2}} x_2 + \sim_{c_{k,3}} x_3 + \dots \quad (59)$$

$$\text{Var}[{}'_k(\mathbf{x}_i)] = (\dagger_{c_{k,1}} x_1)^2 + (\dagger_{c_{k,2}} x_2)^2 + (\dagger_{c_{k,3}} x_3)^2 + \dots \quad (60)$$

where $\sim_{c_{k,1}}$ is the expected value of the first coefficient of objective function k and $\dagger_{c_{k,1}}$ is the standard deviation of the first coefficient of objective function k .

The second step is to calculate $\Pr\{\mathbf{x}_i \succ \mathbf{x}_j\}$, that is, the probability that solution \mathbf{x}_i dominates solution \mathbf{x}_j , using Equation (61):

$$\begin{aligned}
& \Pr\{\mathbf{x}_i \succ \mathbf{x}_j\} \\
&= \int_{-\infty}^{\infty} \Pr\left\{t_{1,x_i} < t_{1,x_j} \mid t_{1,x_j} = z_{1,x_j}\right\} f_{t_1}(z_1 | \mathbf{x}_j) d'_{1,x_j} \\
&\times \int_{-\infty}^{\infty} \Pr\left\{t_{2,x_i} < t_{2,x_j} \mid t_{2,x_j} = z_{2,x_j}\right\} f_{t_2}(z_2 | \mathbf{x}_j) d'_{2,x_j} \\
&= \int_{-\infty}^{\infty} F_{t_1}(z_1 | \mathbf{x}_j) f_{t_1}(z_1 | \mathbf{x}_j) d'_{1,x_j} \times \int_{-\infty}^{\infty} F_{t_2}(z_2 | \mathbf{x}_j) f_{t_2}(z_2 | \mathbf{x}_j) d'_{2,x_j} \\
&= \Phi\left(\frac{E[t_{1,x_i}] - E[t_{1,x_j}] - 0}{\sqrt{\text{Var}[t_{1,x_i}] + \text{Var}[t_{1,x_j}]}}\right) \times \Phi\left(\frac{E[t_{2,x_i}] - E[t_{2,x_j}] - 0}{\sqrt{\text{Var}[t_{2,x_i}] + \text{Var}[t_{2,x_j}]}}\right)
\end{aligned} \tag{61}$$

When the problem includes uncertain constraints, chance constraint model is used to calculate the probability that the constraint is satisfied. For example, a constraint in the form of $\mathbf{a}^1 \mathbf{x} \leq b^1$ with a random right hand side b^1 is converted into $\Pr\{\mathbf{a}^1 \mathbf{x} \leq b^1\} \geq r_1$. Then, $\Pr\{\mathbf{a}^1 \mathbf{x} \leq b^1\}$ is calculated as:

$$\begin{aligned}
\Pr\{\mathbf{a}^1 \mathbf{x} \leq b^1\} &= \Pr\{b^1 \geq \mathbf{a}^1 \mathbf{x}\} \\
&= \Phi\left(\frac{E[b^1] - \mathbf{a}^1 \mathbf{x}}{\sqrt{\text{Var}[b^1]}}\right)
\end{aligned} \tag{62}$$

If this probability is bigger than the user-defined r_1 value, then the solution \mathbf{x}_i is considered as a feasible solution. Note that when there are two or more constraints, the solution has to satisfy all r_i values for all constraints to be counted as a feasible solution.

4.5.3 Algorithm Parameters

There are some parameters that are used for genetic algorithm such as the number of maximum generations or the crossover rate. The parameters used in PUGA for the test problem are as follows:

- Maximum Generations: 500
- Population Size : 40

- Crossover rate: All of the individuals in the population would be processed by a crossover operator, but only *crossover fraction* of all variables would do crossover. Default value of *crossover fraction* is set to $2/numVar$ ($numVar$ = Number of decision variables). There are 12 decision variables in the test problem.
- Mutation rate: As in the crossover operator, all of the individuals in the population would be processed by the mutation operator but only *mutation fraction* of all variables would do mutation. Default value of *mutation fraction* is set to $2/numVar$.

4.5.4 Design tests with stochastic objective function coefficients

PUGA designs are first tested with random objective function coefficients while uncertainty of constraints is ignored, that is, expected values of constraints are used. The risk measure is set to risk-neutral, that is, $\alpha_1 = 0$ and $\alpha_2 = 0$. Neighborhood number for design 3 is set to 10 for each objective function so the objective space is divided into 100 cells in total.

Design 1 gives similar results to the no uncertainty (deterministic assumption) case (Figure 36) but it provides a few more solutions that are expectedly dominated but are not dominated probabilistically. These solutions are similar to the cases we discussed in Section 2.4 and they are the reason we implement PUI to include uncertainty in the algorithm.

Design 2 tends to eliminate the diversity of solutions and promotes a few solutions over all (Figure 37). Thus, it might be better for single objective genetic algorithms. However, the design automatically prunes solutions which is, in general, applied as a

post-Pareto analysis technique. Therefore, this design can be useful for some multi-objective problems, too.

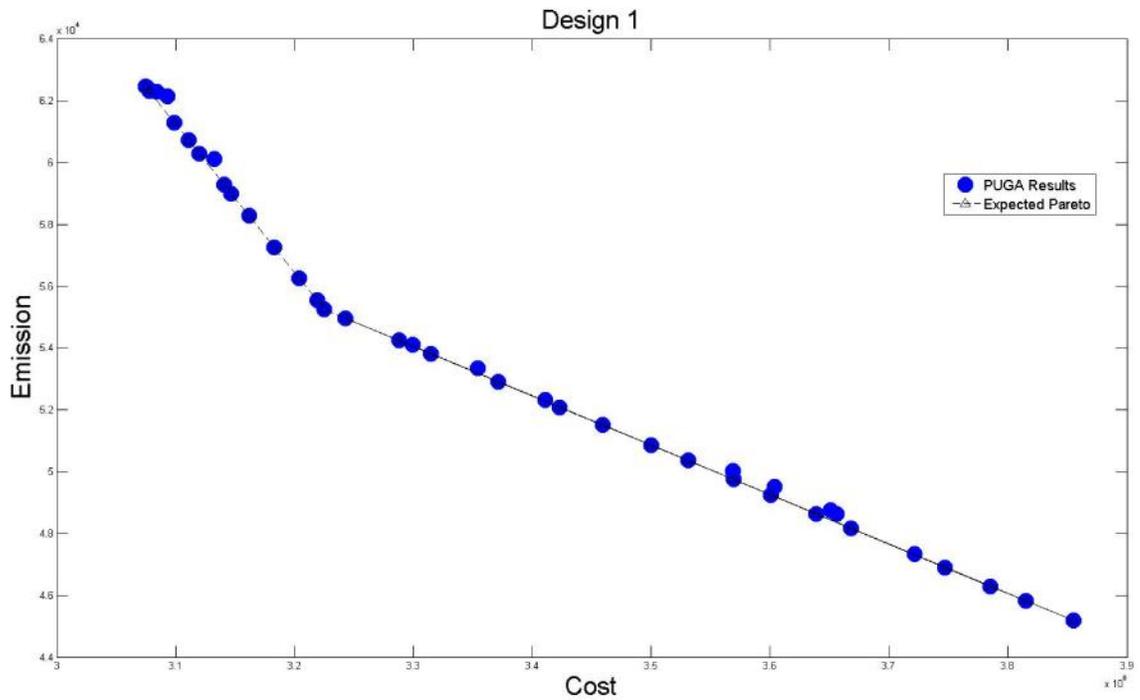


Figure 36. PUGA design 1 Results

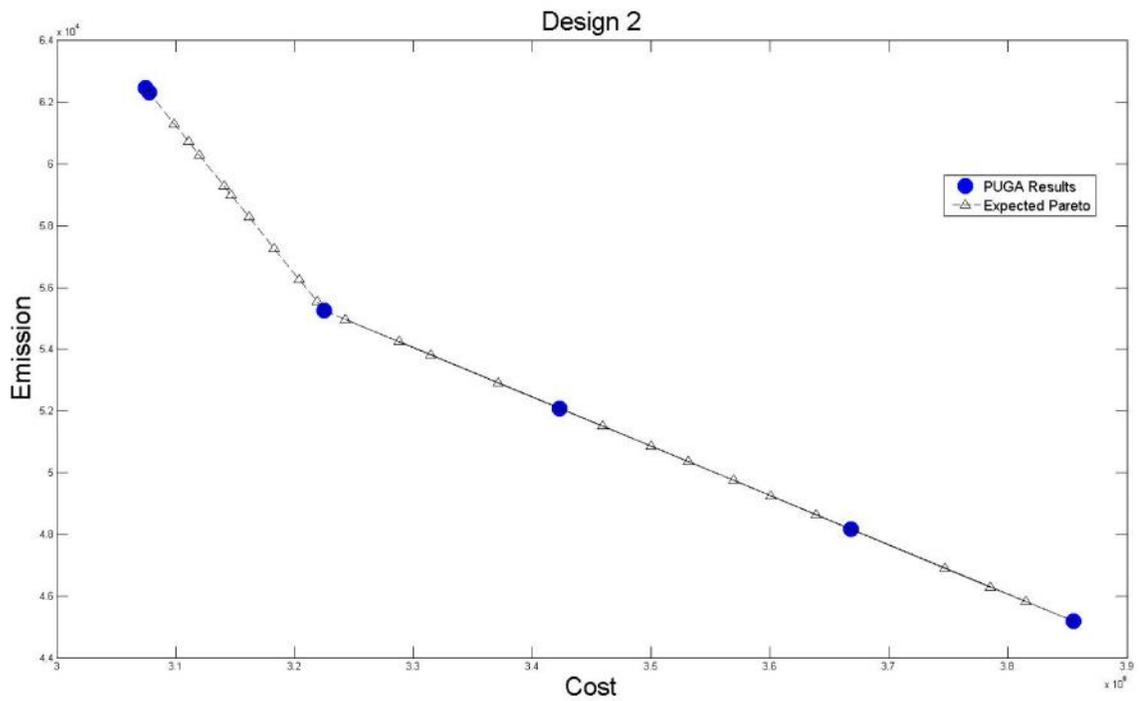


Figure 37. PUGA design 2 Results

Design 3 also gives similar results to the no uncertainty (deterministic assumption) case (Figure 38) but it provides more solutions that are expectedly dominated but are not dominated probabilistically. Furthermore, neighborhood approach provides selected solutions (red star solutions in the figure) for each neighborhood which is helpful for post-Pareto analysis. Neighborhood approach also increases the performance of the algorithm and helps with operators for extraction of the best solutions and selection of parent solutions for reproduction. Overall, this design is the best option for the test problem.

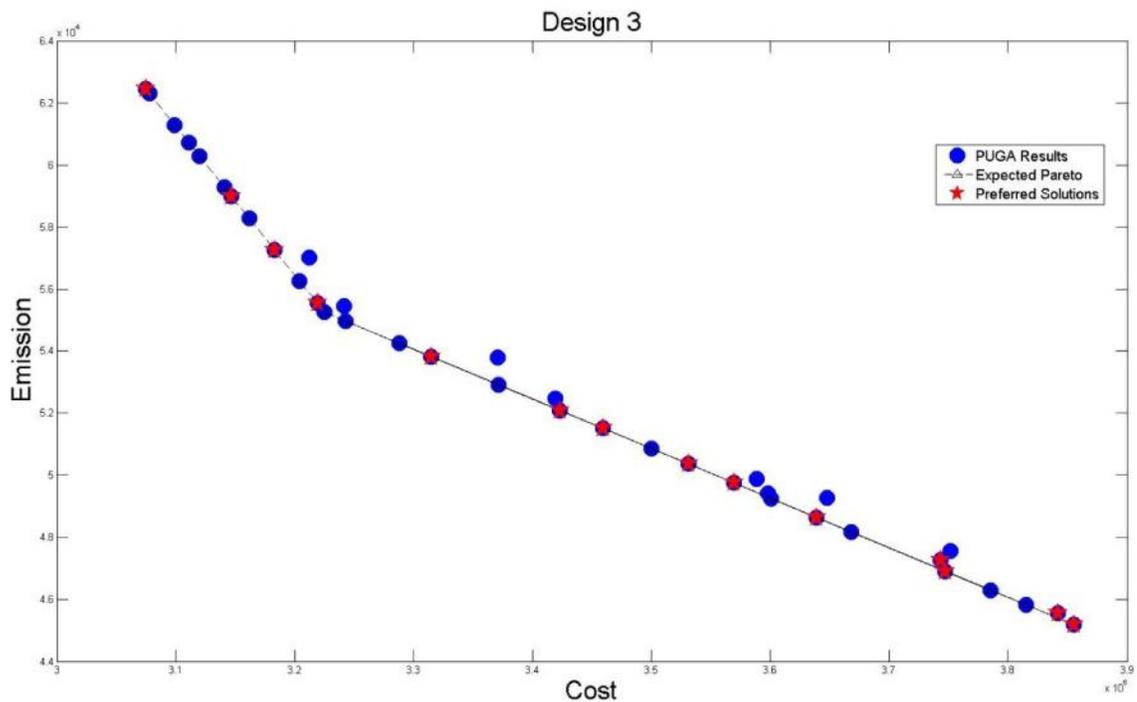


Figure 38. PUGA design 3 Results

Design 4 is similar to designs 1 and 3 with respect to Pareto results. However, the algorithm tends to focus on a particular side of the trade-off curve where the solutions are less uncertain. This is clearly seen with the preferred solutions chosen by the pairwise PUI calculation. Figure 39 shows the top 5 preferred solutions with red stars and they are all on the cost minimization side of the Pareto set. Design 4 is not very helpful for post-

Pareto analysis but gives a good idea about where solutions are less uncertain in the objective space.

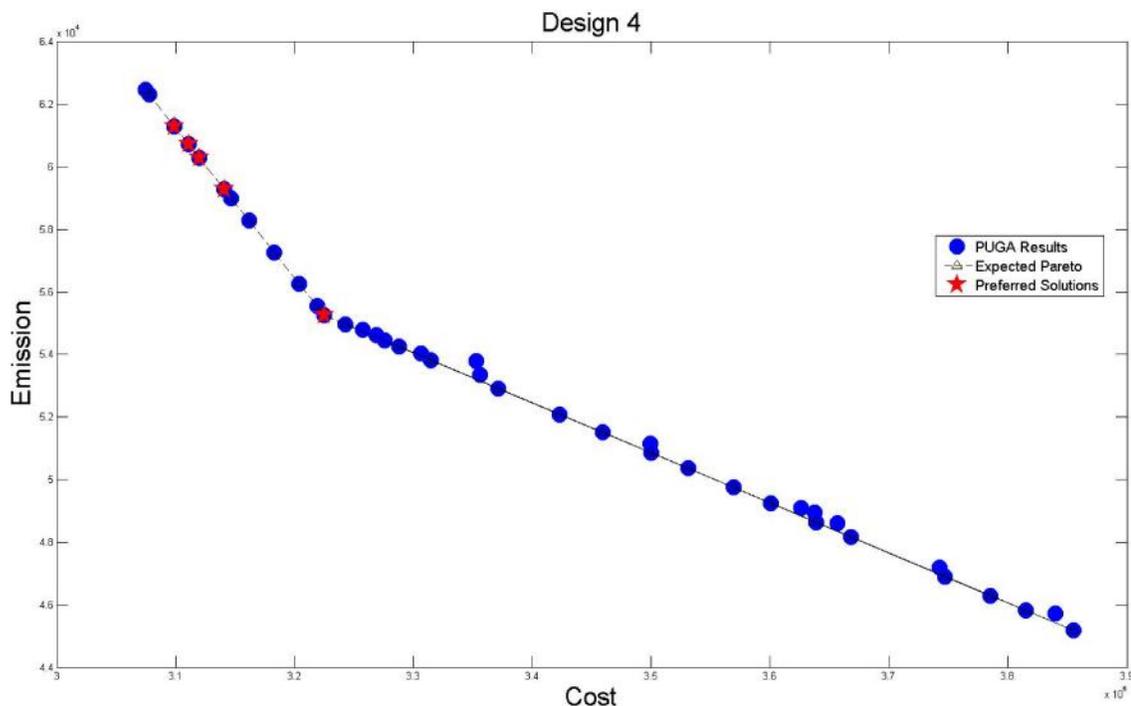


Figure 39. PUGA design 3 Results

4.5.5 Risk Measure Tests

In order to test how the risk measure of rPUI (risk adjusted PUI) affects solutions, we modify the test problem once again to equalize the expected value of the cost of production and emissions for lignite and renewable sources while diverging their standard deviations. We assume that there will be a new lignite technology with carbon capture in the future with low expected emissions but the emissions uncertainty is high. We also assume that there will be a new renewable technology with low expected cost but high cost uncertainty. Table 29, Table 30 and Table 31 show the new modified characteristics for lignite and renewable sources (RES).

Table 29. Modified characteristics (expected values)

Production units, p	Lignite	RES
Expected cost of production (\$/MWh)	30	30
Expected CO ₂ emissions	0	0

Table 30. Modified characteristics (Standard deviation of costs)

	Lignite	Oil	Gas	RES
Base	0.1	3.6	3.6	4
Middle	0.5	3.6	3.6	8
Peak	1	3.6	3.6	12

Table 31. Modified characteristics (Standard deviation of emissions)

	Lignite	Oil	Gas	RES
Base	0.2	0.1	0.1	0
Middle	0.4	0.1	0.1	0
Peak	0.8	0.1	0.1	0

In this case, if the problem is treated without uncertainty, then there are solutions with the same expected value but different decisions. For example, the cost and emissions of two solutions in Table 33 are the same but this can be achieved by two different vectors of decision variables by varying generation decisions from lignite and renewable sources (RES) as seen in the table. Furthermore, the uncertainty levels of these two solutions in each objective function are different from each other (Table 32).

Table 32. Uncertainty levels of two solutions

	Solution 1	Solution 2
Expected Cost	2,625,000	2,625,000
Expected Emissions	10,620	10,620
Cost standard deviation	139,340	175,340
Emissions standard deviation	16,180	14,180

Table 33. Solutions with same objective values, but different decisions

	Solution 1			Solution 2		
	Base	Middle	Peak	Base	Middle	Peak
Lignite	5,400	19,200	6,400	15,400	9,200	6,400
Oil	1,000	0	0	1,000	0	0
Gas	22,000	0	0	22,000	0	0
RES	10,000	0	0	0	10,000	0

Table 34. Comparison of solutions by rPUI

Risk measure	rPUI of Solution 1	rPUI of Solution 2	Selected
$\alpha_1 = 0, \alpha_2 = 0$	0.2500	0.2500	1 or 2
$\alpha_1 = 1, \alpha_2 = 0$	0.2569	0.2610	1
$\alpha_1 = 0, \alpha_2 = 1$	1.3945	0.9883	2
$\alpha_1 = 1, \alpha_2 = 1$	1.4333	1.0318	2
$\alpha_1 = -1, \alpha_2 = 0$	0.2438	0.2401	2
$\alpha_1 = 0, \alpha_2 = -1$	0.2010	0.2310	1
$\alpha_1 = -1, \alpha_2 = -1$	0.1960	0.2218	1

An algorithm which neglects uncertainty treats these two solutions the same, that is, they are equally likely to be selected. However, if the uncertainty is taken into consideration the choice of the decision maker would change. For example, a decision maker who is risk-averse for the cost objective would prefer Solution 1 since its uncertainty is lower than Solution 2. Table 34 gives the comparison of two solutions by their rPUI values. α_i is the risk measure for i^{th} objective function. If α_i is set to zero then the risk measure is neutral. When α_i is greater than zero it is risk-averse, and when it is less than zero it is risk-seeking. When the setting is risk-neutral for both objectives there is no difference among two solutions so there is no selection of one over the other. When

the second objective is set to more risk-averse or both objectives are equally risk-averse, then Solution 2 is selected. Otherwise, Solution 1 is more preferable.

Example above demonstrates how the selection of two solutions change when they have the same expected value for both objective functions but different uncertainty level. We further test PUGA with the original assumptions of the test problem and different risk preferences. PUGA design 3 is used for this test and two different risk preference sets, risk-averse ($\alpha = [15 \ 15]$) and risk-seeking ($\alpha = [-15 \ -15]$), are used.

Figure 40 and Figure 41 show the Pareto fronts for the risk-averse and risk-seeking cases. Blue dots are full Pareto results and red stars are the rPUI-selected solutions for neighborhoods. Figure 42 and Figure 43 show the decision changes between these rPUI-selected solutions. In each graph solutions are ordered by minimization of cost. The number in the x -axis shows the rPUI order of the solution. For example, Solution 8 in Figure 42 is the solution with the minimum cost value among rPUI-selected solutions for the risk-averse case but its rPUI order is the eighth.

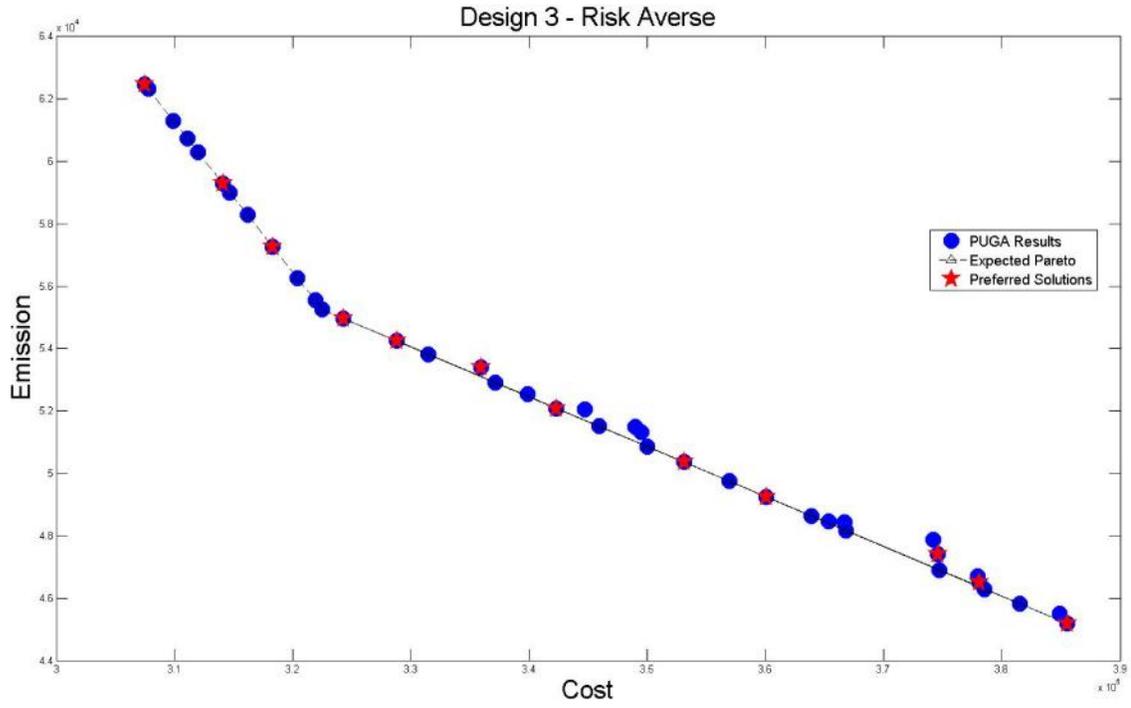


Figure 40. Risk-averse PUGA results

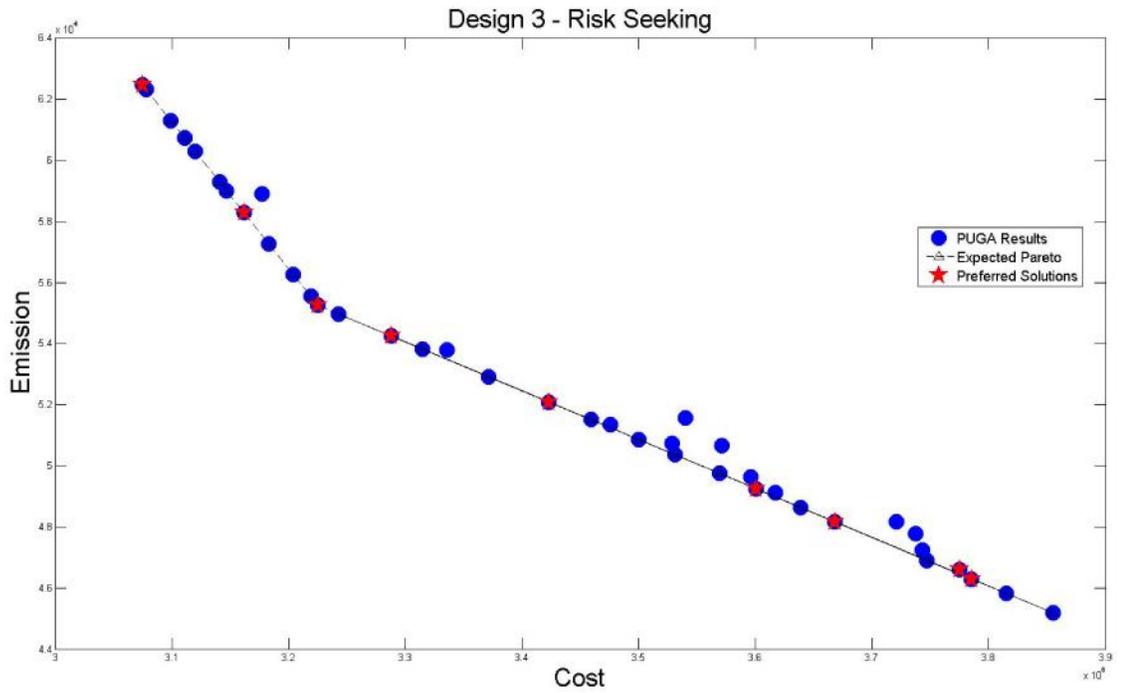


Figure 41. Risk-seeking PUGA results

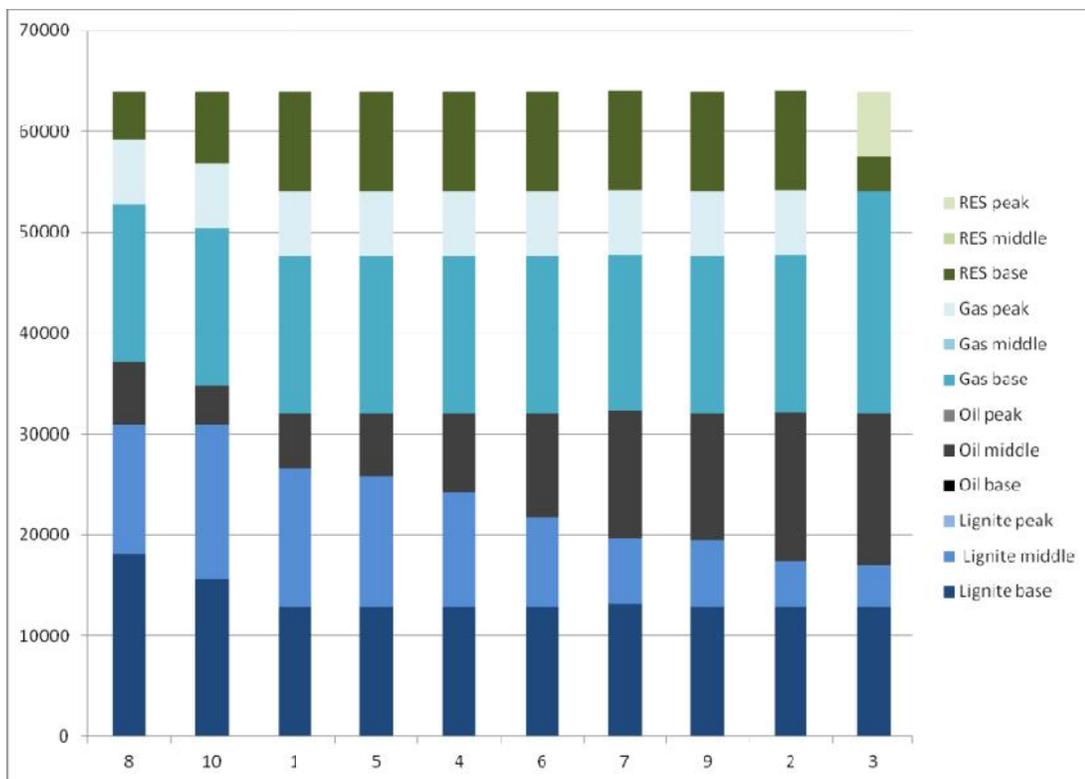


Figure 42. Risk-averse rPUI-selected solutions

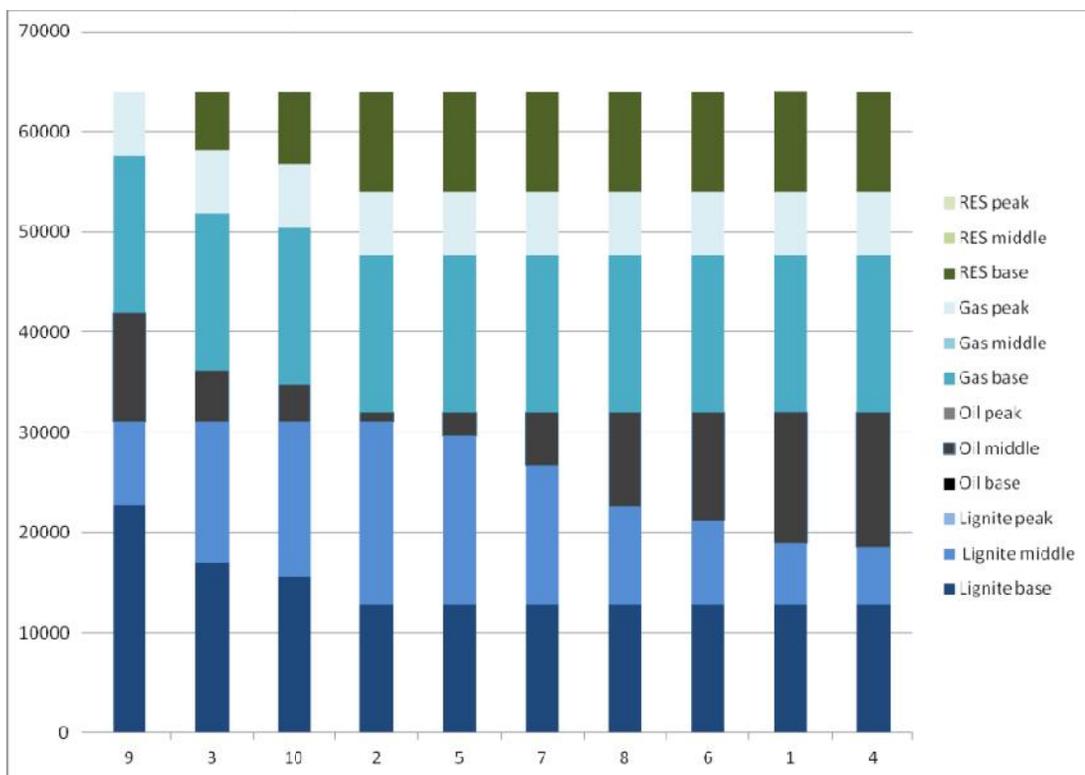


Figure 43. Risk-seeking rPUI-selected solutions

4.5.6 Test with stochastic constraints

Constraints in the test problem are assumed as deterministic constraints. The test problem model is subject to the following constraints:

$$\sum_l x_{p,l} \leq \tilde{c}ap_p \text{ for each unit } p \quad (63)$$

$$\sum_p x_{p,l} \geq \tilde{load}_l \text{ for each load } l \quad (64)$$

$$x_{1,3}, x_{2,1}, \text{ and } x_{4,2} = 0 \quad (65)$$

$$x_{p,l} \geq 0 \quad \forall p \text{ and } l \quad (66)$$

In this section PUGA is tested with constraints where uncertainty is included. Only Equations (63) and (64) have stochastic elements. The maximum production of unit p per year ($\tilde{c}ap_p$) and demand in load type l (\tilde{load}_l) are random variables with normal distributions. Values for their means and standard deviations are presented in Table 26, Table 27 and Table 28.

For the test problem Equations (63) and (64) are converted to the following forms:

$$\Pr\{\sum_l x_{p,l} - \tilde{c}ap_p \leq 0\} > r_1 \text{ for each unit } p \quad (67)$$

$$\Pr\{\sum_p x_{p,l} - \tilde{load}_l \geq 0\} > r_2 \text{ for each load } l \quad (68)$$

The user defined r_i values mean that the probability of satisfying constraint i is greater than r_i . This can also be used as a risk measure. For instance, a risk-seeking decision maker may choose a smaller value for r_i , so that the algorithm provides solutions with more relaxed constraints. On the contrary, a risk-averse decision maker may choose a larger value for r_i , and the algorithm provides solutions with strictly satisfied constraints.

In this test problem, all stochastic elements of the constraints are random variables with normal distributions. Therefore assigning 0.5 for a β_i provides the same results as in the deterministic assumption case. From another point-of-view, in deterministic assumption the probability of satisfying a constraint is only 0.5. For this test problem, a risk-neutral decision maker may assume the problem is deterministic or assign 0.5 for β_i values; a risk-seeking decision maker may choose a smaller value than 0.5 for β_i values; and a risk-averse decision maker may choose a bigger value than 0.5. The algorithm is tested in cases where β_i values are changed for Equations (67) and (68).

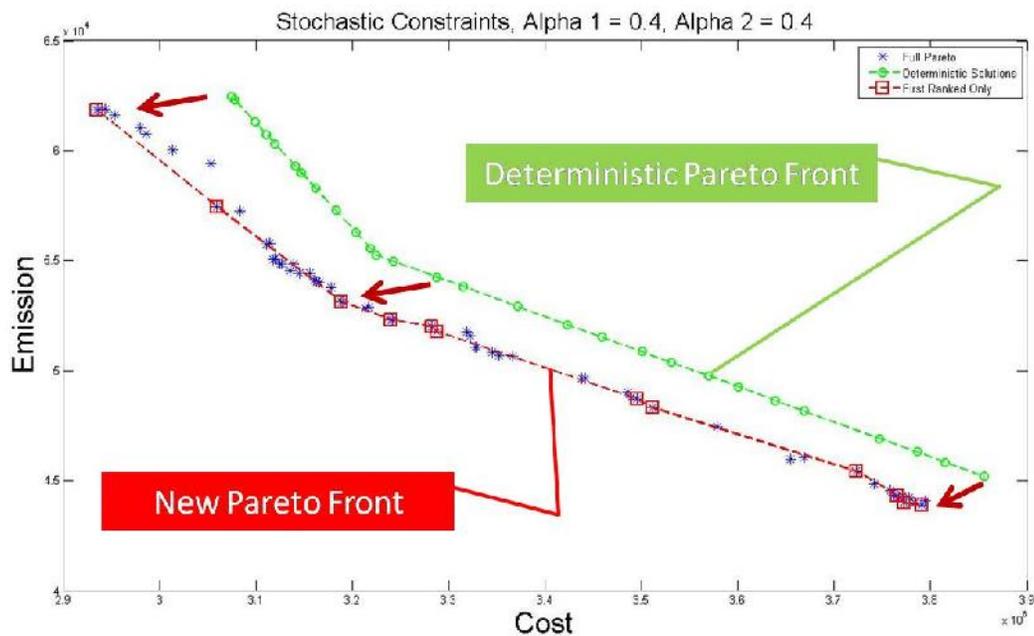


Figure 44. Test results for Case 1, $\beta_1 = \beta_2 = 0.4$

Case 1: $\beta_1 = \beta_2 = 0.4$

The user defined β_i values mean that the probability of satisfying constraint i is greater than 0.4. Test results for the case are presented in Figure 44. The green line shows the deterministic Pareto front and green circles are initial solutions provided to the algorithm which are also the deterministic assumption results. The red dotted line is the new Pareto front for the test case where $\beta_1 = \beta_2 = 0.4$ and red squared solutions are the

preferred solutions for post-Pareto pruning. When constraints are relaxed, the Pareto front extends to lower values for both objective functions. This is shown by red arrows in the figure.

Table 35. Comparison of two solutions which minimizes the cost for deterministic and stochastic case $\alpha_1 = \alpha_2 = 0.4$

		Deterministic	Stochastic
Base	Lignite	22,800	21,900
	Oil	0	0
	Gas	15,600	15,523
	RES	0	13
Middle	Lignite	8,200	9,740
	Oil	11,000	8,998
	Gas	0	0
	RES	0	0
Peak	Lignite	0	0
	Oil	0	0
	Gas	6,400	6,302
	RES	0	0
Total	Lignite	31,000	31,640
	Oil	11,000	8,998
	Gas	22,000	21,825
	RES	0	13
	Production	64,000	62,477
	Cost	3,075,000	2,934,744
	Emissions	62,460	61,862

Table 35 shows the comparison of the two extreme solutions where the cost is minimized for the deterministic case and the stochastic test case where $\alpha_1 = \alpha_2 = 0.4$. Total production is reduced because the demand constraint is relaxed. This reduces both the cost and emissions objective. In total, the gas and oil usage is reduced but the lignite usage is increased. Lignite is used more than its expected limit because the limit of maximum production per unit is relaxed by $\alpha_1 = 0.4$.

Case 2: $\alpha_1 = \alpha_2 = 0.45$

Test results for the case are presented in Figure 45. The dotted green line shows the deterministic Pareto front and green circles are initial solutions provided to the algorithm which are also deterministic case results. The solid blue line is the new Pareto front for the test case where $\alpha_1 = \alpha_2 = 0.45$ and red squared solutions are the preferred solutions for post-Pareto pruning. The dotted black line is the Pareto front of the previous case where $\alpha_1 = \alpha_2 = 0.4$. Constraints are less relaxed compared to the previous case but the Pareto front still can extend to lower values for both objective functions.

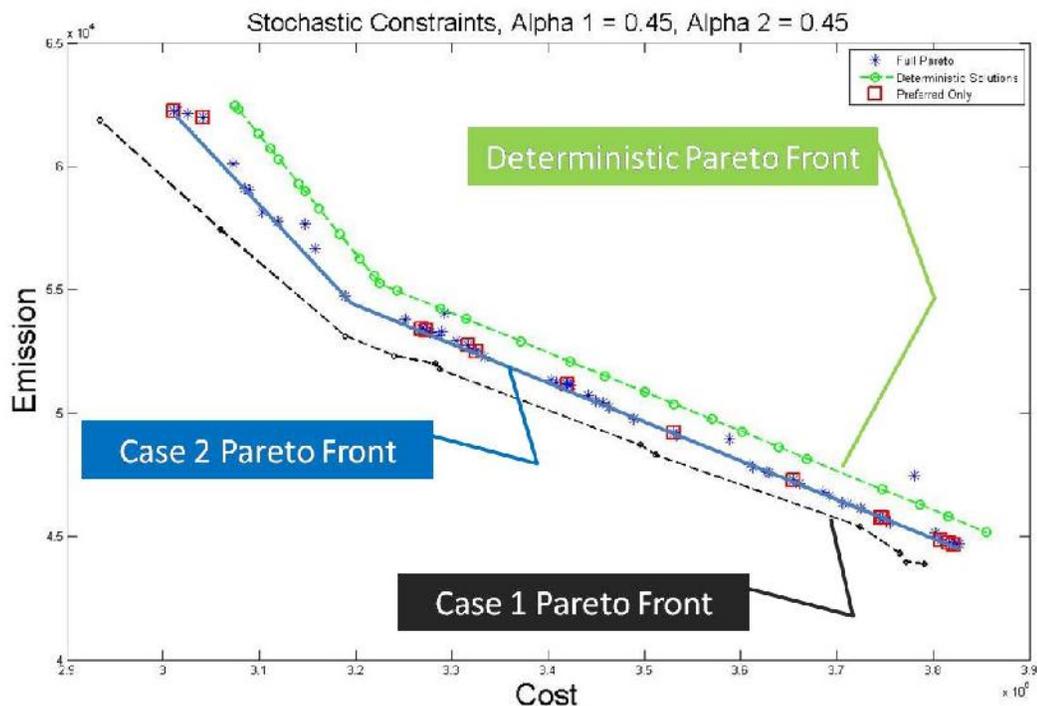


Figure 45. Test results for Case 2, $\alpha_1 = \alpha_2 = 0.45$

Table 36 shows the comparison of the three extreme solutions where the emissions objective is minimized for the deterministic case and two stochastic test cases where $\alpha_1 = \alpha_2 = 0.4$ and $\alpha_1 = \alpha_2 = 0.45$. Total production is reduced in stochastic cases because the demand constraint is relaxed. This reduces both the cost and emissions objective. In

general, production from all units in all load types is reduced but usage of renewable sources in peak load is slightly increased in case 1.

Table 36. Comparison of three solutions which minimizes the emissions for deterministic and stochastic case $\alpha_1 = \alpha_2 = 0.4$ and stochastic case $\alpha_1 = \alpha_2 = 0.4$ 5

		Deterministic	Case 1	Case 2
Base	Lignite	12,800	12,263	12,548
	Oil	0	0	0
	Gas	22,000	21,539	21,893
	RES	3,600	3,762	3,507
Middle	Lignite	4,200	4,200	4,200
	Oil	15,000	14,558	14,843
	Gas	0	0	0
	RES	0	0	0
Peak	Lignite	0	0	0
	Oil	0	0	0
	Gas	0	0	0
	RES	6,400	6,376	6,400
Total	Lignite	17,000	16,463	16,748
	Oil	15,000	14,558	14,843
	Gas	22,000	21,539	21,893
	RES	10,000	10,138	9,907
	Production	64,000	62,699	63,390
	Cost	3,855,000	3,790,529	3,820,820
	Emissions	45,180	43,881	44,655

Case 3: $\alpha_1 = 1$ and $\alpha_2 = 0.4$

In this case only the constraint set related to the load types is relaxed ($\alpha_2 = 0.4$). The maximum production limit is assumed to be deterministic (when $\alpha_1 = 1$, the algorithm treats the first constraint set as deterministic). Test results for the case are presented in Figure 46. The dotted green line shows the deterministic Pareto front and green circles are initial solutions provided to the algorithm which are also the deterministic case results. The solid blue line is the new Pareto front for the test case where $\alpha_1 = 1$ and $\alpha_2 =$

0.4 and red squared solutions are the preferred solutions for post-Pareto pruning. The dotted black line is the Pareto front of Case 1 where $\alpha_1 = \alpha_2 = 0.4$.

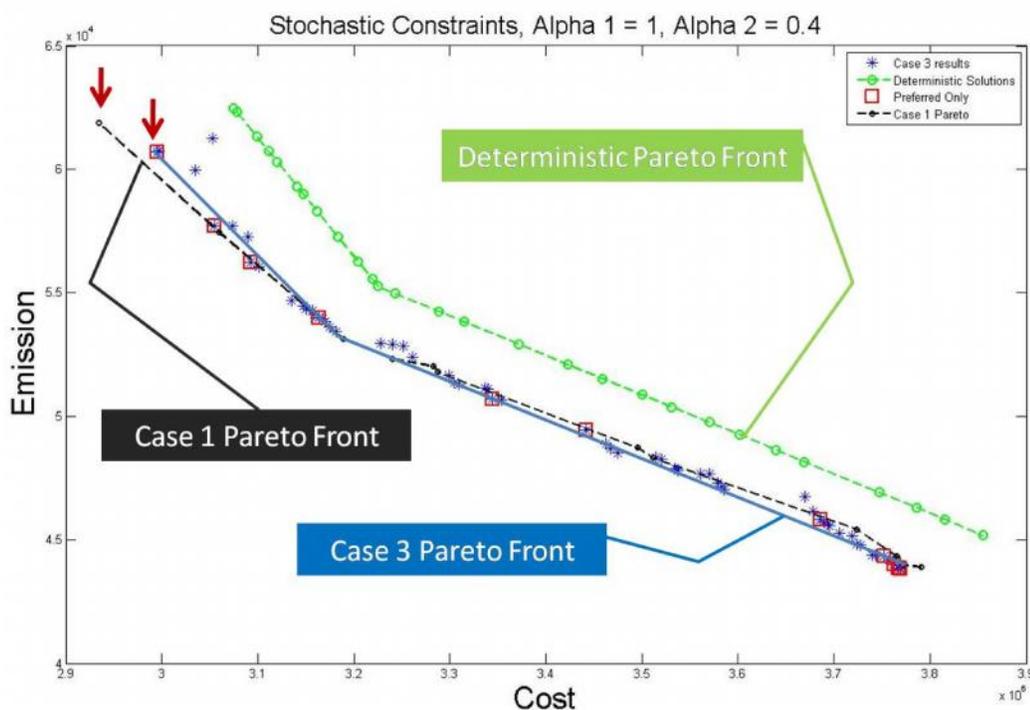


Figure 46. Test results for Case 3, $\alpha_1 = 1$, $\alpha_2 = 0.4$

Table 37 shows the comparison of the two extreme solutions, shown by red arrows in Figure 46, where the cost is minimized for Case 3 where $\alpha_1 = 1$ and $\alpha_2 = 0.4$, and Case 1 where $\alpha_1 = \alpha_2 = 0.4$. Total production is reduced in these cases because the demand constraint is relaxed. This reduces both the cost and emissions objective. In Case 1, the model could use more lignite than it is allowed in the deterministic case but in Case 3 this constraints is assumed to be deterministic. Therefore usage of lignite is reduced and substituted by oil and renewable sources.

Table 37. Comparison of two solutions which minimizes the cost for Case 1 where $\alpha_1 = \alpha_2 = 0.4$ and Case 3 where $\alpha_1 = 1$ and $\alpha_2 = 0.4$

		Case 1	Case 3
Base	Lignite	21,900	21,105
	Oil	0	0
	Gas	15,523	15,352
	RES	13	986
Middle	Lignite	9,740	9,750
	Oil	8,998	9,004
	Gas	0	0
	RES	0	0
Peak	Lignite	0	0
	Oil	0	0
	Gas	6,302	6,400
	RES	0	0
Total	Lignite	31,640	30,855
	Oil	8,998	9,004
	Gas	21,825	21,752
	RES	13	986
	Production	62,477	62,597
	Cost	2,934,744	2,994,785
	Emissions	61,862	60,702

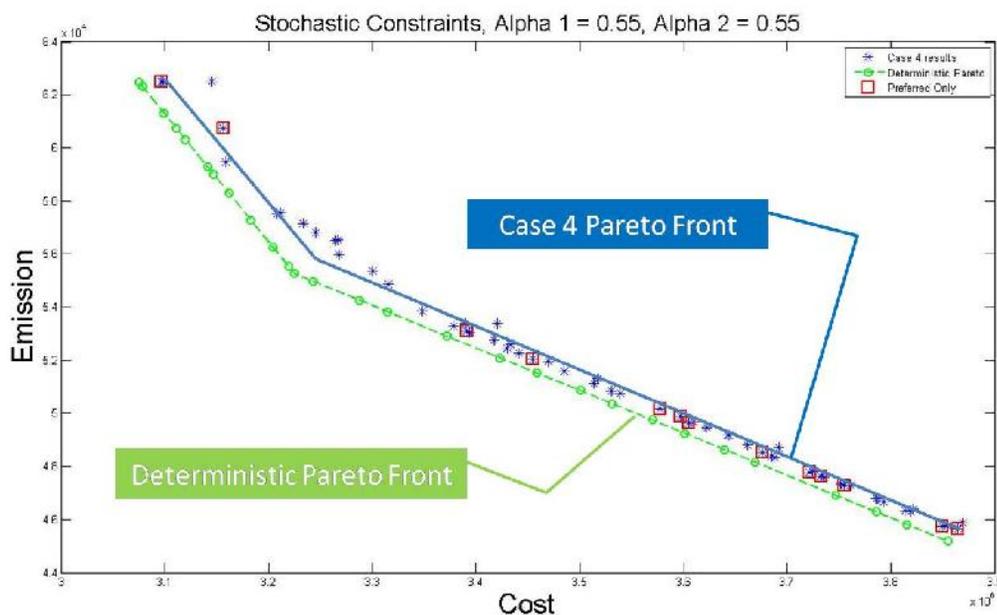


Figure 47. Test results for Case 4, $\alpha_1 = \alpha_2 = 0.55$

Case 4: $\alpha_1 = \alpha_2 = 0.55$

In this case α values are set to 0.55 which is higher than the risk-neutral case $\alpha_i = 0.5$. This means that the probability of satisfying the constraints is stricter, and the decision maker is risk-averse. Test results for the case are presented in Figure 47. The dotted green line shows the deterministic Pareto front and green circles are initial solutions provided to the algorithm which are also deterministic case results. The solid blue line is the new Pareto front for the test case where $\alpha_1 = \alpha_2 = 0.55$ and red squared solutions are the preferred solutions for post-Pareto pruning.

Table 38 shows the comparison of the two extreme solutions where the cost is minimized for the deterministic case and Case 4 where $\alpha_1 = \alpha_2 = 0.55$. Total production is increased in Case 4 because the demand constraint is stricter. The model uses more renewable sources in the base load and more lignite and oil in the middle load to satisfy the increased demand constraint. In total the production is increased 250 GWh.

Table 38. Comparison of solutions which minimizes the cost for the deterministic case and Case 4 where $\alpha_1 = \alpha_2 = 0.55$

		Deterministic	Case 4
Base	Lignite	22,800	22,600
	Oil	0	0
	Gas	15,600	15,600
	RES	0	200
Middle	Lignite	8,200	8,400
	Oil	11,000	11,050
	Gas	0	0
	RES	0	0
Peak	Lignite	0	0
	Oil	0	0
	Gas	6,400	6,400
	RES	0	0

Total	Lignite	31,000	31,000
	Oil	11,000	11,050
	Gas	22,000	22,000
	RES	0	200
	Production	64,000	64,250
	Cost	3,075,000	3,096,729
	Emissions	62,460	62,496

4.5.7 Test with three objective functions

The test problem excerpted from [66] originally has three objective functions. So far we simplified this problem into two objectives for other test purposes. In this section we consider the third objective, too. The third objective of the problem is to minimize the external dependence, that is, to minimize the generation amount from oil and natural gas units:

$$\min \text{dependence} = \sum_l (\tilde{c}_{3,oil} x_{oil,l} + \tilde{c}_{3,gas} x_{gas,l}) \quad (69)$$

where $\tilde{c}_{3,p}$ are artificial coefficients of the third objective function for oil and natural gas units and added arbitrarily just to include some level of uncertainty for test purposes. $\tilde{c}_{3,p}$ are assumed to be normally distributed with an expected value of 1 and standard deviation of 0.01.

Pareto results for the three objective function test obtained by PUGA design 3 are presented in Figure 48 and Figure 49 for the risk-averse ($\gamma = [15 \ 15 \ 15]$) and the risk-seeking ($\gamma = [-15 \ -15 \ -15]$) cases, respectively. As in the bi-objective tests, PUGA finds extra solutions depending on the risk preference that were not considered in the expected case and it also eliminates some expectedly non-dominated solutions in favor of newly found more preferred solutions.

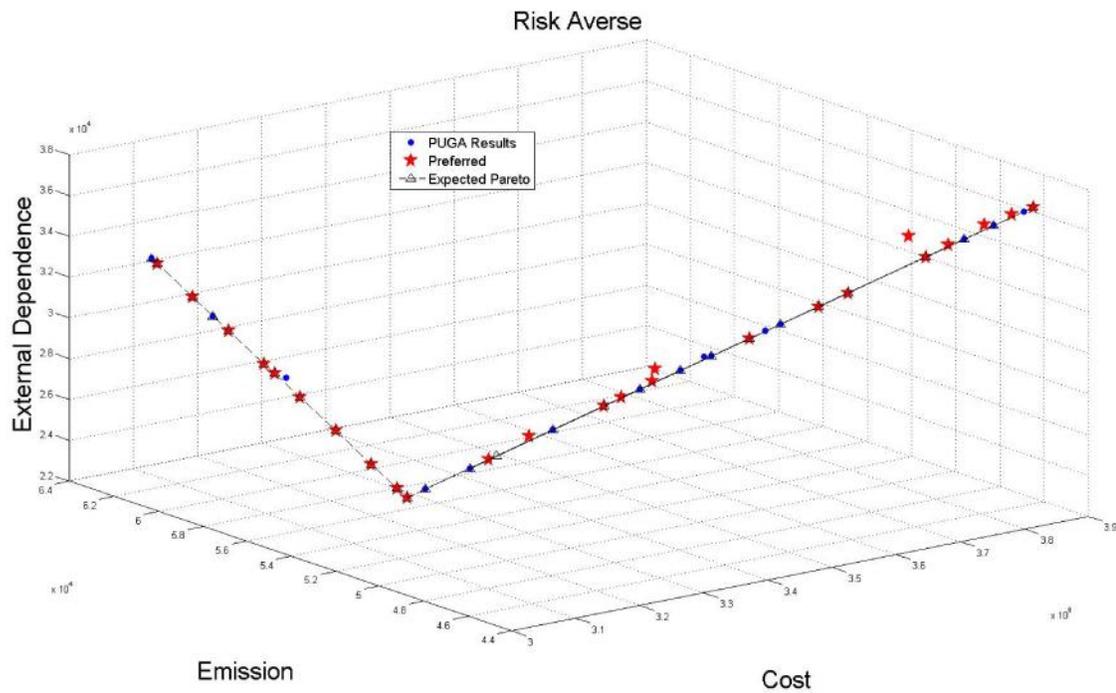


Figure 48. Risk-averse results with 3 objectives

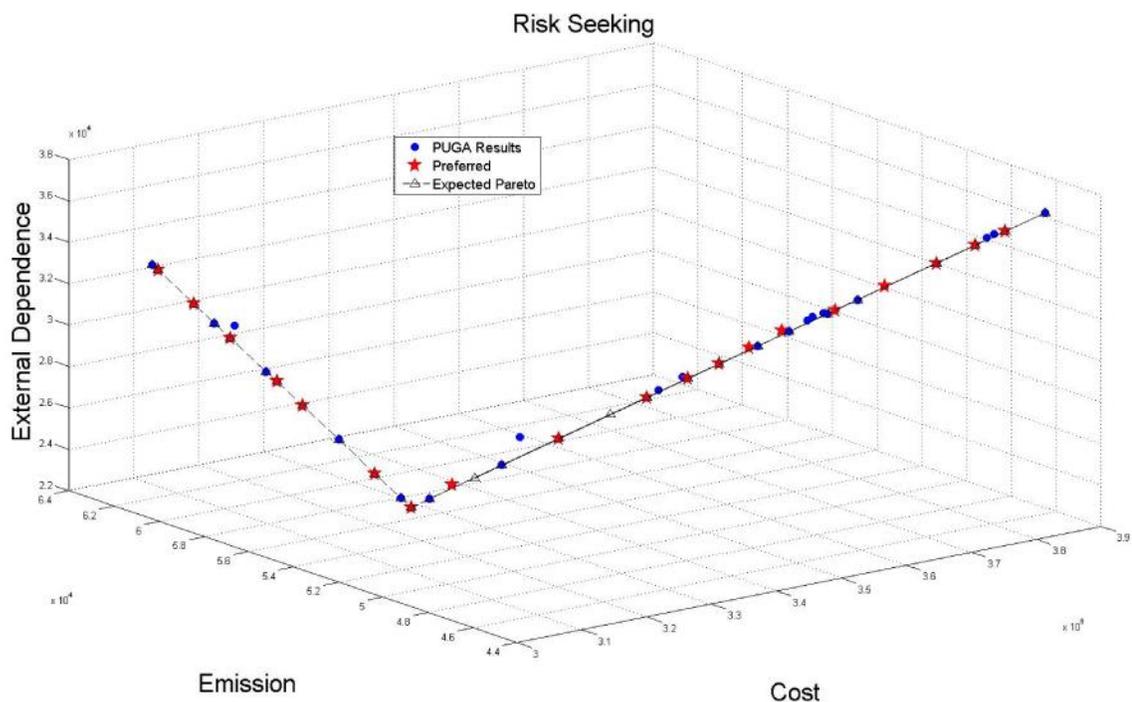


Figure 49. Risk-seeking results with 3 objectives

4.5.8 Test with dependent objective coefficients and different distributions

Two objective functions considered in the test problem are the minimization of production cost and the minimization of CO₂ emissions:

$$\min cost = \sum_p \sum_l \tilde{c}_{1,p} x_{p,l} \quad (70)$$

$$\min emissions = \sum_p \sum_l \tilde{c}_{2,p} x_{p,l} \quad (71)$$

where $\tilde{c}_{k,p}$ is the coefficient of decision variable $x_{p,l}$ in objective function k . There is uncertainty in both of the objective functions due to the randomness of their coefficients. In all of the previous tests a general assumption is that all the coefficients in the objective functions are normally distributed independent random variables.

Table 39. Probability distributions of objective function coefficients

Coefficients of Cost rate per MW generation		Distribution	Parameters
Lignite	$c_{1,lignite}$	Gamma	(51, 0.6)
Oil	$c_{1,oil}$	Normal	(75, 3.6)
Natural Gas	$c_{1,gas}$	Uniform	(55, 65)
RES	$c_{1,RES}$	Normal	(90, 0.6)
Coefficients of Emissions rate per MW generation		Distribution	Parameters
Lignite	$c_{2,lignite}$	Gamma	(6.275, 0.23)
Oil	$c_{2,oil}$	Weibull	(0.83, 2.5)
Natural Gas	$c_{2,gas}$	Uniform	(0.3, 0.6)
RES	$c_{2,RES}$	N/A (Constant)	0

In this section, PUGA, which uses the discrete version of the PUI, is tested with dependent objective coefficients that have different probability distributions. Each coefficient is a random variable with its probability distribution shown in Table 39. The

probability distributions and their parameters are randomized for the test problem. The correlation matrix for indicated dependencies is provided in Table 40.

Table 40. Correlation matrix of objective function coefficients

		Lignite		Oil		Gas		RES	
		Cost	Emis.	Cost	Emis.	Cost	Emis.	Cost	Emis.
Lignite	Cost	1	-0.5	0.4	0	-0.2	0	0	0
	Emissions	-0.5	1	0	0	0	0	0	0
Oil	Cost	0.4	0	1	0	-0.8	0	0	0
	Emissions	0	0	0	1	0	0	0	0
Gas	Cost	-0.2	0	-0.8	0	1	0	0	0
	Emissions	0	0	0	0	0	1	0	0
RES	Cost	0	0	0	0	0	0	1	0
	Emissions	0	0	0	0	0	0	0	1

The discrete version of the PUI uses discrete points generated for each solution in the objective space. Discrete points represent the randomness of each solution. For the test in this section, we use a copula-based simulation technique to generate discrete points for each solution. The details of this simulation technique is presented in [99]. PUGA design 3 with the discrete PUI is employed for the test case. The discrete PUI formulation is explained in Section 3.3.

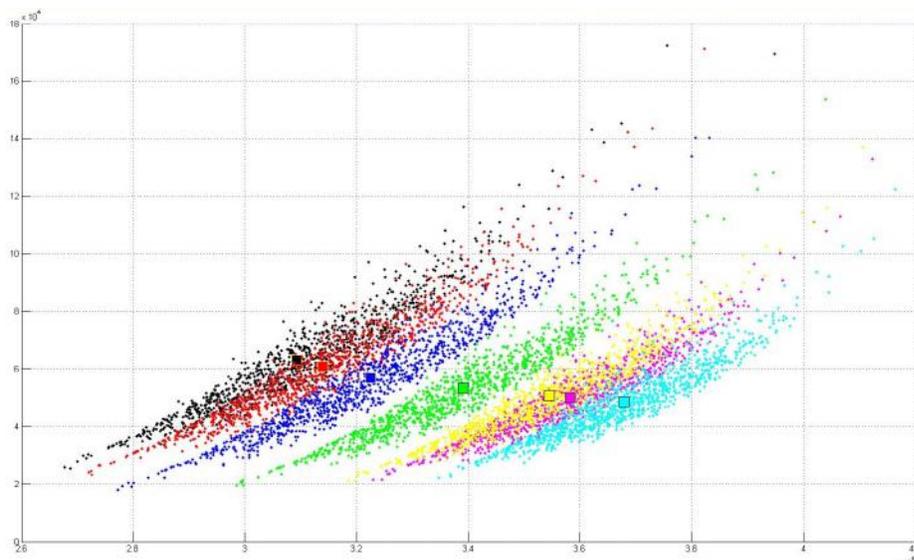


Figure 50. Discrete points generated to represent the randomness

Figure 50 shows the expected values and discrete points generated for seven Pareto solutions. Squares in the figure are the expected values of solutions and smaller dotted points are discrete points generated to represent the randomness of each solution. If parameters of the problem are assumed to be deterministic, then the high level of uncertainty is neglected which can be a significant factor for the decision making process.

Figure 51 shows the expected values and discrete points generated for dependent case and independent normal case for the same solution. The dependent case has the same probability distribution assumptions of this section. Independent normal case considers that all objective function coefficients are independent and normally distributed as in previous sections. The importance of representing the uncertainty as realistic as possible is obvious from this comparison. Simulating discrete points to represent the uncertainty and using the discrete PUI formulation in PUGA help to overcome the difficulties of representing the uncertainty and provide an effective approach for stochastic multi-objective problems with complicated relationships among the problem parameters.

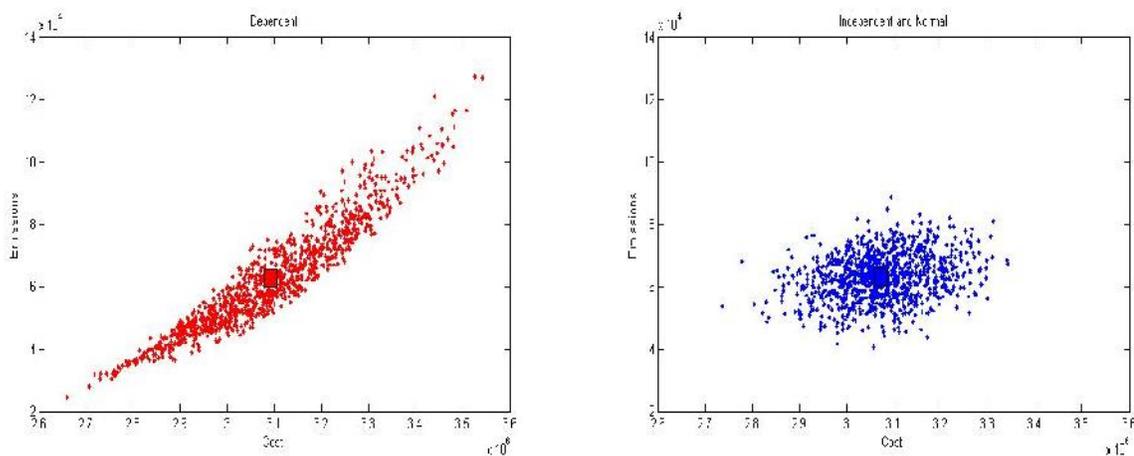


Figure 51. Dependent case (left) and independent normal case (right) for the same solution

As discussed in Section 3, if the objectives are dependent, the PUI formulation needs several integrations. This might be impractical or very inefficient for an algorithm to run

several generations. However, simulating the randomness and using the discrete PUI in PUGA makes it very simple and effective to consider dependent random coefficients of the objective functions. For the test problem in this section, neighborhood partitioning is set to 100 and 1000 discrete points are generated for each solution of population size of 40 for each generation. PUGA completed 500 generations in 700 seconds. This is a fairly good performance considering the information gained by including uncertainty of the dependent coefficients. Test results for the risk-averse and the risk-seeking cases are presented in Figure 52 and Figure 53, respectively.

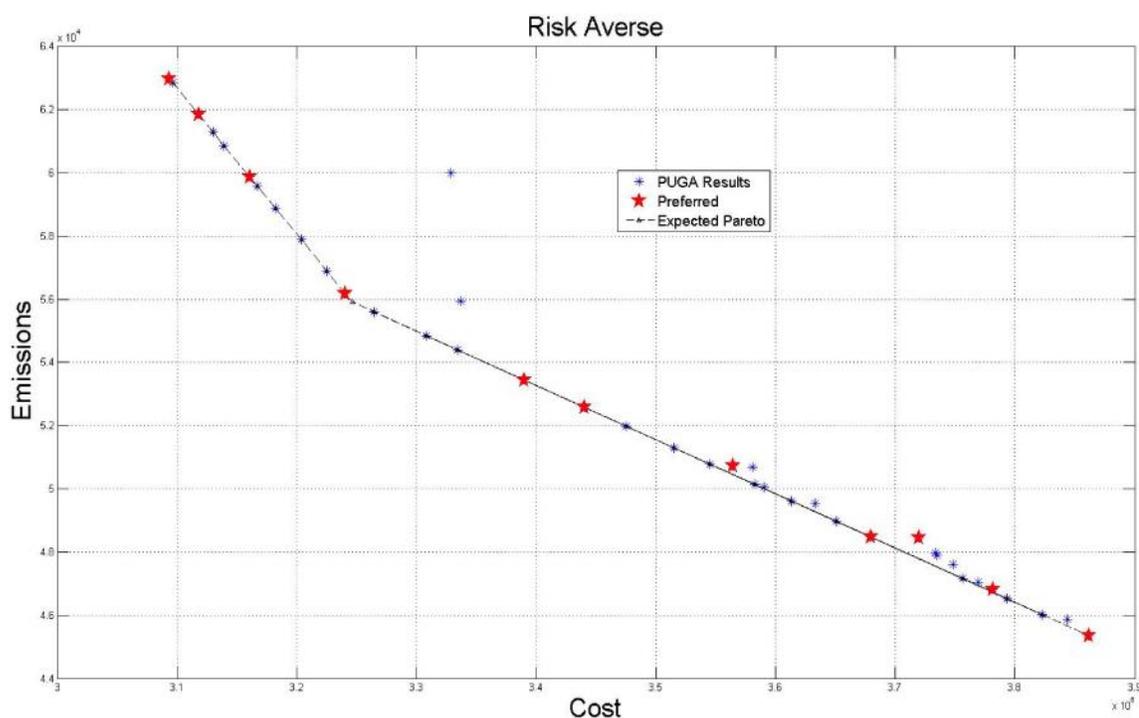


Figure 52. Risk-averse results for dependent case

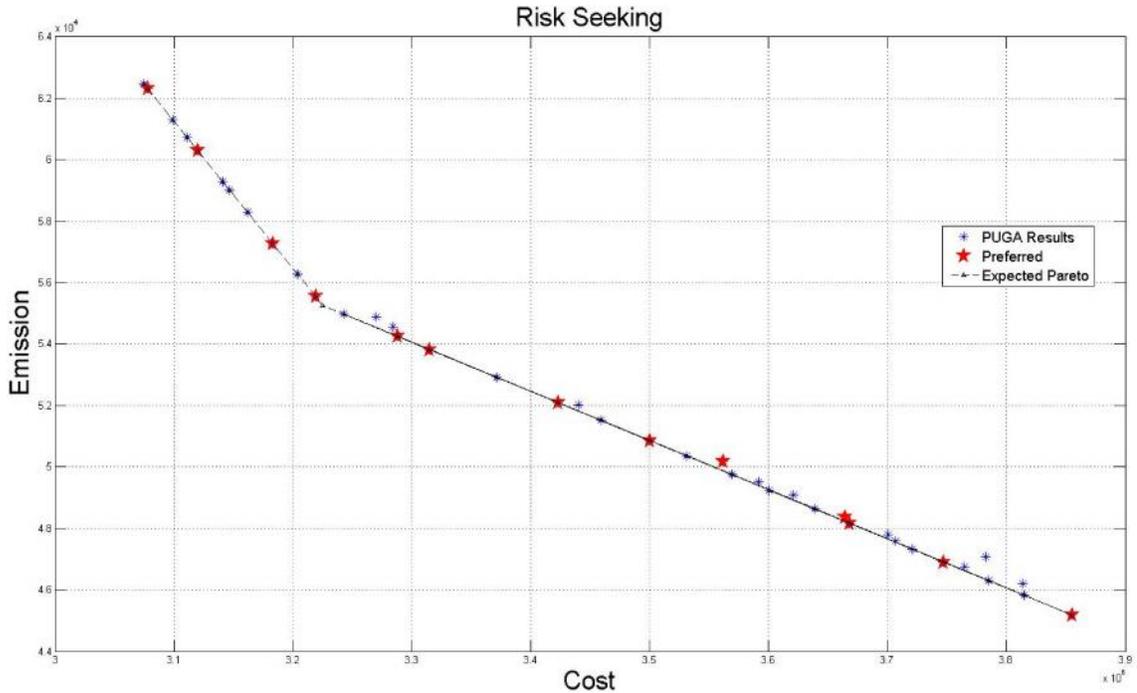


Figure 53. Risk-seeking results for dependent case

4.6 Summary

Many optimization problems are multi-objective and stochastic in practice; however, multi-objective models often use deterministic assumptions, and when uncertainty is considered models are transformed to single objective equivalents.

In this section, PUGA, a new multi-objective stochastic genetic algorithm is presented. PUGA incorporates the PUI model (explained in Section 3) and provides a direct approach to solve a multi-objective stochastic problem without any transformation of the problem. The main architecture of PUGA is built on general multi-objective genetic algorithm design components such as Pareto ranking or elitism but preservation of uncertainty information is accomplished by incorporating the PUI model so that the algorithm gives the opportunity to the decision maker to observe the uncertainty of solutions in the Pareto optimal set. Furthermore, PUGA handles stochastic constraints as well as deterministic ones with a method that is based on a chance constraint model. This

type of constraint handling technique based on the chance constraint method and the user-defined δ_i values are also used as a risk measure. In addition to Pareto optimality, PUGA uses the domination probability of solutions provided by the PUI model to make a preference decision between solutions as part of post-Pareto analysis.

PUGA is tested with two modified problems first to test the performance and second to demonstrate the workings of PUGA. Results from a simplified power generation problem excerpted from [66] show that considering uncertainty changes the Pareto front and the preference of solutions in the Pareto front. The modified CONSTR test problem results, on the other hand, show that PUGA performs as well as NSGA-II in terms of converging to the true Pareto optimal front and also in terms of maintaining a diverse population of non-dominated solutions.

5. The US Northeast Generation Expansion Planning (NEGEP)

The first objective of this chapter is to provide a multi-objective stochastic optimization model for the power generation expansion planning of the Northeast region of the United States presented in Section 5.1. This model also defines stochastic coefficients and parameters of the problem. The stochastic parameters are classified according to their uncertainty type. Aleatory and epistemic uncertainty modeling of the problem is explained in Section 5.2. The stochastic nature of the problem is due to the uncertainties in the economy, climate change and technological progress. Uncertainties due to the economy are included by modeling cost parameters as random variables with normal distributions. Uncertainties due to climate change and technological progress are modeled via scenarios. There are six scenarios: a reference scenario and five alternative scenarios considering climate change and technological progress. Section 5.3 gives details of the US Northeast GEP (NEGEP) model and the reference scenario where climate change effects are considered insignificant with respect to the beginning of the planning horizon and technological progress is assumed to be the business-as-usual, that is, there is only normal progress. Section 5.4 explains how the remaining five scenarios are defined. The scenarios consider two technological levels, namely, reference (business-as-usual) and advanced progress levels. The amount of climate change is considered in three different levels, namely, reference (no significant climate change), medium change and high change (Table 41). Objective function cost and emissions parameters are adjusted in every scenario so that the dependency of objective functions is assured. There is a trade-off between the number of scenarios and complexity. The number of scenarios can be increased by including more levels of climate change and

technological progress but it also increases the complexity of the problem and computation time.

Table 41. Scenarios

Climate Change	Technological Progress	
	Normal Progress	Advanced
No Change	Reference (Scenario 1)	Scenario 4
Medium	Scenario 2	Scenario 5
High	Scenario 3	Scenario 6

The second objective of this chapter is to provide input data and assumptions for the NEGEP model. The data of this problem is collected from various sources, mainly from reports of Eastern Interconnection Planning Collaborative (EIPC) [100], National Renewable Energy Laboratory (NREL) [101], Environmental Protection Agency (EPA) [102], Energy Information Administration (EIA) [103]. Reference scenario modeling, assumptions and data are based on the base scenario of [37]. Other scenarios and assumptions are based on literature review and expert judgments [104-108].

The final objective of the chapter is to solve the NEGEP problem to find Pareto solutions for cost and CO₂ emissions objectives. PUGA design 3 (with neighborhood approach) is used to solve the problem. Sections 5.8, 5.9, 5.10 and 5.11 provide various results and comparisons of these results for the NEGEP model.

5.1 Multi-objective Stochastic NEGEP

The NEGEP model used in this chapter is similar to the example of the multi-objective stochastic model provided in Section 2.1.1 and details of the model equations can be found in that section. Two objectives are considered in the NEGEP model. The first objective function is to minimize the net present value of total cost, which consists of

generation cost, investment cost, and fixed operation and maintenance cost. The second objective function is to minimize the total CO₂ emissions.

Decision variables in the NEGEP model are generation amount (MWh) of generation units in all regions in each time period of every year of the planning horizon, investment amount (integer units) of generation units in all regions in every year of the planning horizon, and transmission amount (MWh) between regions in each time period of every year of the planning horizon. Generation and transmission decisions are continuous variables and investment decisions are integer variables, and thus, the NEGEP model is a Mixed Integer Linear Program (MILP).

The NEGEP model considers the supply-demand constraint that ensures that the model satisfies the demand of each region in each time period with generation from all units and transmission from other regions. There is also the capacity constraint which states that the model cannot generate electricity over the built capacity. The reserve margin requirement is modeled as the third constraint. This constraint is a necessity of reliability and ensures that the built capacity is greater than or equal to the peak demand capacity plus the capacity added due to the reserve margin requirement. The annual and total construction limit of each generation unit, transmission limit between regions and the non-negativity of generation, investment and transmission decision variables are ensured by upper and lower bounds in genetic algorithm modeling, therefore, they are not modeled as constraints in the NEGEP model.

5.2 Aleatory and Epistemic Uncertainty Modeling in NEGEP

In this section the stochastic parameters of the NEGEP model are classified according to their uncertainty type, i.e. aleatory or epistemic. Uncertainty representation of each

parameter is also discussed. Classification and representation of considered parameters (peak and annual total demand, cost coefficients, reserve margin, capacity and derate factors and emissions rates of technologies) are summarized in Table 42 and they are explained in the following sub-sections.

Table 42. Classification and representation of stochastic NEGEP parameters

GEP Variable	Symbol	Uncertainty Type	Representation
Peak demand	\tilde{d}_{yr}^p	Epistemic	Scenarios
Reserve margin	\tilde{m}_r	Epistemic	Scenarios
Availability factor	\tilde{p}_{tri}	Epistemic	Scenarios
Derate factor	\tilde{w}_{ti}	Epistemic	Scenarios
Emissions amount of technologies	\tilde{y}_{ik}	Epistemic	Normal Distribution
Operation cost	$\tilde{h}_{y,i}$	Aleatory	Normal Distribution
Maintenance cost	$\tilde{g}_{y,i}$	Aleatory	Normal Distribution
Investment cost	$\tilde{a}_{y,i}$	Aleatory	Normal Distribution
Generation cost	$\tilde{c}_{y,i}$	Aleatory	Normal Distribution
Total demand	\tilde{d}_{ytr}	Aleatory	Scenarios

5.2.1 Parameters with aleatory uncertainty

Total demand of each region in each time period (\tilde{d}_{ytr}): There is sufficient historical data for total annual demand and its variability so it is reasonable to assume a probability density function for annual demand. On the other hand, annual demand uncertainty affects constraints, and from the modeling aspect, it is often easier to model the constraint

uncertainties with scenarios. In this study uncertainty of total yearly demand of each region is modeled in scenarios.

Fixed operation cost ($\tilde{h}_{y,i}$), *maintenance cost* ($\tilde{g}_{y,i}$), *investment cost* ($\tilde{a}_{y,i}$), *generation cost* ($\tilde{c}_{y,i}$): Costs are often assumed to be continuous random variables, which have associated probability density functions. The use of a symmetrical distribution is justifiable for the variability of costs in long-term planning because an increase or a decrease is likely to occur on either side of the expected cost in the long-term. Therefore costs are modeled with normal distribution in this study.

5.2.2 Parameters with epistemic uncertainty

Peak demand (\tilde{d}_{yr}^p): Although annual total demand is classified as aleatory uncertainty and can be modeled with probability distributions, peak demand is classified as epistemic uncertainty because changes in peak demand are highly affected by weather variability and climate change and these are hard to predict, and modeling their variability with probability distributions is not very realistic. For example, The PJM (Pennsylvania - New Jersey - Maryland Interconnection) demand curve for January 7, 2014, was 35,000 MW higher than typical of a January peak load [109]. Figure 54 shows the difference between the expectation and the realization of the daily demand and peak demand for the PJM on January 7, 2014. On this particular day the weather was unexpectedly cold due to a phenomenon called *polar vortex* and the demand for electricity due to heating needs was also unexpectedly high. This kind of unexpected extreme weather and climate events make the associated uncertainty of system parameters epistemic.

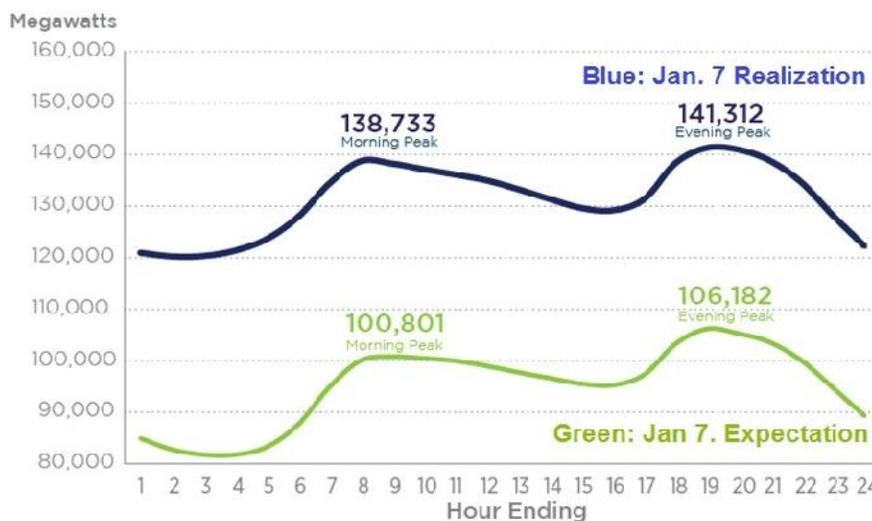


Figure 54. PJM peak demand expectation (green line) and realization (blue line) on January 7, 2014

Reserve margin (\tilde{m}_r), *availability factor* (\tilde{p}_{tri}) and *derate factor* (\tilde{w}_{ti}): Reserve margin is the percent of excess capacity to maintain reliability in electric power systems. For instance, a reserve margin of 15% means that available generation capacity is 15% more than the expected peak demand. Reserve margin is a parameter to be assigned for each region.

Availability factor is the potential availability of each generation unit, as an upper bound of generation output [37]. For example, a wind turbine with availability factor of 20% means at most 20% of the time the wind turbine can work due to insufficient wind power or other conditions.

Derate factor is a reduction rate in the available capacity of an electric generating unit, commonly due to a system or equipment modification or environmental, operational, or reliability considerations [103]. For example, the onshore class 3 wind turbine (WT_on3) has a derate factor of 0.1781 during summer peak in NEISO, which means at most 17.81% of time the wind turbine can work due to insufficient wind power or other conditions [37].

These NEGEP parameters have uncertainty mainly due to technological development and climate change. For example, extreme climate events are likely to increase transmission and generation losses in the system. Therefore, there is uncertainty in reserve margin due to the climate change. Uncertainty of technological progress and climate change is difficult to forecast and they have unpredictable variability due to a number of volatile factors. Hence, these uncertainties are classified as epistemic uncertainties and uncertainty of the associated variables are modeled by scenarios.

Emissions amount of technologies (\tilde{y}_{ik}): Only CO₂ emissions from different types of generation technologies are considered in the NEGEP model. CO₂ emissions of technologies are dependent on the technology used and the uncertainty associated with the parameter is due to the uncertainty in the technological development in the future. As discussed before, the uncertainty of technological progress is hard to forecast. Thus, the uncertainty is classified as epistemic but for modeling purposes and to provide explicit information about uncertainty and risk of the Pareto solutions to the decision maker, this parameter is modeled with a normal distribution.

5.3 Reference Scenario

Reference scenario modeling, assumptions and data are based on the base scenario in [37]. The planning horizon is from 2015 to 2040. The year is divided into three seasons (summer, winter, and spring/fall) and the related time periods in the model are summer-peak, summer-offpeak, winter-peak, winter-offpeak, shoulder-peak and shoulder-offpeak. The summer is defined from May through September. The winter includes December, January and February. Spring and fall is combined and named as shoulder in the study,

which includes March, April, October, and November. The corresponding number of hours in the time periods in a year is provided in Table 43.

Table 43. Hours in time periods

Summer-peak	1749
Summer-offpeak	1923
Shoulder-peak	1394
Shoulder-offpeak	1534
Winter-peak	1029
Winter-offpeak	1131

The regions considered in this study are the same with [37]: NEISO, NYISO_A-F, NYISO_G-I, NYISO_J-K, PJM_E, PJM_ROM, PJM_ROR. These regions are the Regional Transmission Organizations (RTO) in the Eastern Interconnection grid of North America, namely, NEISO-Independent System Operator of New England, NYISO-New York Independent System Operator, PJM-PJM Interconnection. New York state is divided into eleven sub-regions (A-K), we consider Upstate (A-F), Lower Hudson Valley (G-I), New York City/Long Island (J-K) as three big sub-regions. We consider PJM Eastern Mid-Atlantic Area Council (NJ, DE, east MD), PJM Rest of Mid-Atlantic Area Council (east PA, DC, west MD), PJM Rest of Regional Transmission Operator (north IL, OH, west PA, west MD, WV, VA, east NC) as three big sub-regions.

The existing, new and renewable generation technology types are shown in Table 55 in Appendix I. This study uses the same technology types with [37]. Table 56 and Table 57 in Appendix I summarize the technical and cost characteristics of generation technologies for the reference scenario, respectively. It is assumed that existing or already planned and under construction (forced new) generation plant investment costs are sunk

and are not included in the result. Investment costs are not listed for some technologies that are not allowed to be invested in the future. Costs and emissions rates are assumed to be normal random variables and expected values of parameters and outage rates are from [37]. Standard deviations are calculated based on data and assumptions of several references [37;100;101;104-108] and expert judgments. Table 58 in Appendix I gives the existing capacity at the beginning of the planning horizon. The information is gathered from [37;100].

Table 44 and Table 45 present the electricity demands and peak demands at the beginning of the planning horizon and projected growth rates for the reference scenario, respectively. The data for demand and peak demand for the reference scenario is the same with base scenario of [37] and the data is mainly from [100].

Table 44. Electricity demands (GWh) and peak demands (MW) in 2010

Time Periods	NEISO	NYISO_A-F	NYISO_G-I	NYISO_J-K	PJM_E	PJM_ROM	PJM_ROR
Summer-peak	30,115	14,710	5,048	19,759	38,078	35,124	124,635
Summer-offpeak	23,953	12,918	4,001	15,185	29,507	28,614	102,943
Shoulder-peak	20,773	10,908	3,266	12,162	23,305	23,731	84,720
Shoulder-offpeak	18,014	9,985	2,858	10,495	20,819	21,511	78,294
Winter-peak	16,628	8,687	2,601	9,229	18,642	19,527	68,549
Winter-offpeak	14,608	8,021	2,329	8,196	16,948	17,856	63,614
Peak (MW)	26,043	11,455	4,356	17,030	32,910	27,332	99,146

Table 45. Electricity load growth rate

		NEISO	NYISOA-F	NYISO_G-I	NYISO_J-K	PJM_E	PJM_ROM	PJM_ROR
Annual load growth	2010-2020	0.23%	0.2%	0.14%	0.39%	-0.98%	0.86%	0.4%
	2021-2040	0%	0.51%	0.85%	0.88%	0.67%	0.67%	0.61%
Peak load growth	2010-2020	0.49%	0.1%	-0.09%	0.1%	-0.92%	0.71%	0.42%
	2021-2040	0.12%	0.51%	0.85%	0.88%	0.67%	0.67%	0.61%

Derate factor is a discount rate in the available capacity of an electric generating unit, commonly due to a system or equipment modification or environmental, operational, or reliability considerations [103]. For example, the onshore class 3 wind turbine has a

availability factor of 0.1781 during summer peak in NEISO, which means that 17.81% of time the wind turbine can work due to insufficient wind power or other conditions [37]. Derate factors for wind turbines in NEISO region are provided in Table 46 as an example. Data for derate factors of all generation units are derived from and can be found in [100;101].

Table 46. NEISO wind turbine derate factors

NEISO	WT	WT_on3	WT_on4	WT_off
summer-peak	19.50%	17.81%	21.18%	41.85%
summer-offpeak	25.81%	23.31%	28.31%	40.46%
shoulder-peak	31.96%	28.95%	34.96%	44.63%
shoulder-offpeak	33.18%	29.78%	36.57%	44.24%
winter-peak	42.00%	37.92%	46.07%	54.28%
winter-offpeak	41.83%	37.83%	45.83%	52.06%

Electric power systems have excess capacity available than may be required to maintain reliability. This is called the reserve margin and it is the ratio of capacity minus peak demand over peak demand, where capacity is the expected maximum available supply. For instance, a reserve margin of 0.15 means available generation capacity must be 15% more than the expected peak demand [103]. Reserve margin requirements for each region can be found in [100].

Transmission within regions is also considered but transmission to and from Canada and other states are neglected, and transmission losses are not included. Transmission capacity within regions is provided in Table 47 [100]. Transmission capacity remains constant throughout the planning horizon, that is, there will be no new transmission line investment.

Table 47. Transmission capacity (MW)

Regions	NEISO	NYISO_A-F	NYISO_G-I	NYISO_J-K	PJM_E	PJM_ROM	PJM_ROR
NEISO	-	600	600	430	-	-	-
NYISO_A-F	600	-	4,250	-	-	1,000	-
NYISO_G-I	600	1,999	-	6,130	1,500	-	-
NYISO_J-K	-	-	1,999	-	-	-	-
PJM_E	-	-	500	330	-	8,000	-
PJM_ROM	-	2,000	-	-	8,000	-	8,000
PJM_ROR	-	-	-	-	-	8,000	-

5.4 Climate Change and Technological Progress Scenarios

The remaining five scenarios are formed as in Table 48. There are two technological levels considered in this study, namely, reference (business-as-usual) and advanced progress levels. Climate change is considered in three different levels, namely, reference (no significant climate change), medium change and high change levels.

Table 48. NEGEP Scenarios

Climate Change	Technological Progress	
	Normal Progress	Advanced
No Change	Reference (Scenario 1)	Scenario 4
Medium	Scenario 2	Scenario 5
High	Scenario 3	Scenario 6

Reference scenario is not a frozen scenario but it considers the business-as-usual case for technological progress where there is no significant climate change expected in the future. Scenarios 2 and 3 assume normal (business-as-usual) progress in technology but medium level and high level changes in climate, respectively. Scenarios 4, 5 and 6 assume advanced progress in technology but climate change levels differ; being no significant change, medium level and high level changes, respectively.

Climate change effects on power systems are studied in [37] reviewing a wide range of literature. Three major climate variables (temperature, precipitation and extreme weather events) are identified to have effects on six parameters of the generation expansion model, namely, demand, peak demand, derate factor, transmission capacity, reserve margin and outage rate. This study uses the results and assumptions of [37] to define climate change effects of Scenarios 2, 3, 5 and 6. For example, Table 59 in Appendix I presents the percentage change in climate variables corresponding to the considered climate change levels in Scenarios 2, 3, 5 and 6 with respect to the beginning of the planning horizon. Table 60 in Appendix I presents the percentage change in GEP parameters corresponding to the considered climate change levels in Scenarios 2, 3, 5 and 6 with respect to the beginning of the planning horizon.

Table 49. Yearly percentage change in GEP parameters with respect to reference case due to advanced level technological progress

Unit Type	Investment Cost	Fixed O&M Cost	Emissions rate	Derate
CC	-0.75%	-0.75%	-0.55%	0.55%
CT	-0.75%	-0.75%	-0.55%	0.55%
GEO	0.00%	0.32%	-	0.32%
LFG	-0.75%	-0.75%	-0.55%	0.55%
NU	-0.18%	-0.18%	-	-
PV	-1.47%	-0.37%	-	2.00%
Wind Turbines	-0.36%	-0.12%	-	1.73%
IGCC	-0.75%	-0.75%	-0.31%	0.31%
IGCC_seq	-0.75%	-0.75%	-0.31%	0.31%
AC	-0.75%	-0.75%	-0.31%	0.31%
Biomass	-0.38%	-0.38%	-0.32%	0.32%

In this study, costs and efficiencies of generation units are considered to be affected by the technological progress. An increase in efficiency of a generation unit increases the

derate factor and decreases the emissions rate of the unit. Yearly percentage change in parameters with respect to the reference case due to advanced level technological progress is presented in Table 49. Data and assumptions are based on [104-108].

5.5 Solution and Decision Process of PUGA with Scenarios

The solution and decision process of PUGA with the given scenarios in the previous sections and formed decision profiles and robust Pareto fronts (RPF) in the next sections are explained in the process chart given in Figure 55.

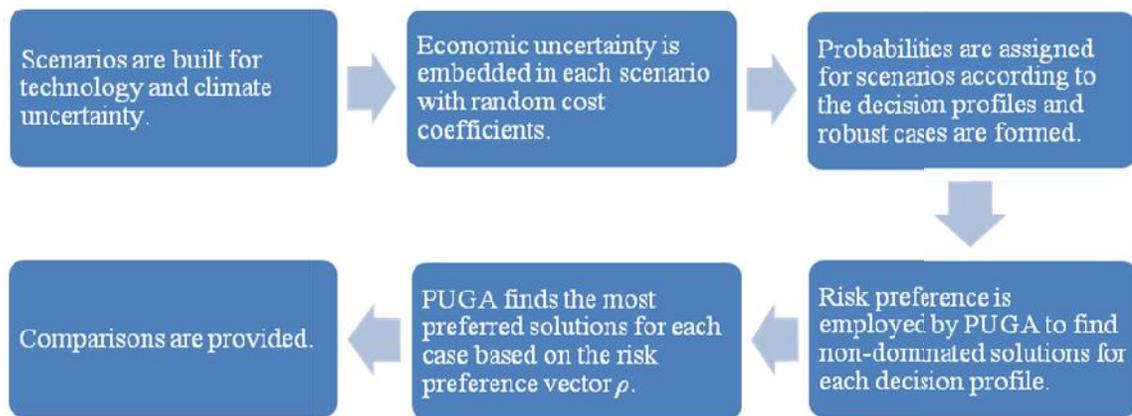


Figure 55. Solution and Decision Process of PUGA with Scenarios

Uncertainties due to climate change and technological progress are modeled via scenarios. Uncertainties due to the economy are included by modeling cost parameters as random variables with normal distributions. Objective function cost and emissions coefficients are adjusted in every scenario so that each scenario is internally consistent. For example, both the cost coefficients and emissions coefficients of objective functions are adjusted in advanced technology scenarios so that they are both lower than normal technological progress scenarios. PUGA finds non-dominated Pareto solutions for each decision profile after forming robust cases according to the given probabilities of scenarios and risk preferences of each decision profile. Comparisons of results are

provided between decision profiles and different risk preference cases in the next sections.

5.6 Decision Profiles

Three decision maker profiles are formed to provide an easy comparison of results of different preferences of risk. The first decision profile is risk-neutral where the reference scenario (S1) is assumed to occur in the future and the decision maker is risk-neutral for both objectives; cost and emissions. Therefore the risk preference is set to zero for both objectives, $\mathbf{w} = [0 \ 0]$. The risk-averse decision profile assumes higher probability for the worst case scenario (S3) and risk-averse in both objectives, that is, the risk preference vector $\mathbf{w} = [15 \ 15]$. The risk-seeking decision profile assumes higher probability for the best case scenario (S4) and risk-seeking in both objectives, that is, the risk preference vector $\mathbf{w} = [-15 \ -15]$. These decision profiles are created only for easy presentation purposes and can be extended to any risk preference and scenario combination.

5.7 Robust Pareto Fronts

The risk-averse decision profile assumes higher probability for the worst case scenario (S3) and the risk-seeking decision profile assumes higher probability for the best case scenario (S4). Therefore two compromise and robust Pareto fronts (RPF) S7 and S8 are formed for each decision profile. Table 50 presents the assumed probabilities for each case. Robust Pareto fronts are created to provide robust and compromise solutions that consider all of the scenarios.

Objective function and constraint coefficients and parameters are calculated using the given probabilities in Table 50 and equations (72) and (73).

$$\min f_m = \sum_i \sum_s x_i c_{m,i,s} p_s \quad (72)$$

$$\sum_i \sum_s p_s a_{i,j,s} x_i \leq \sum_s p_s b_{j,s} \quad \forall j \quad (73)$$

where f_m is the m^{th} objective function, x_i is the i^{th} decision variable, $c_{m,i,s}$ is the i^{th} coefficient of the m^{th} objective function in scenario s , p_s is the probability of scenario s , $a_{i,j,s}$ is the i^{th} coefficient of the j^{th} constraint in scenario s and $b_{j,s}$ is the right-hand side parameter of the j^{th} constraint in scenario s .

Table 50. Probability of scenarios for RPF S7 and S8

Scenarios	Probabilities for RPF S7	Probabilities for RPF S8
1	0.1	0.1
2	0.1	0.1
3	0.5	0.1
4	0.1	0.5
5	0.1	0.1
6	0.1	0.1

5.8 No Risk Preference Case

In this section results for the no risk preference case are presented, that is, uncertainty in the objective space is neglected but scenarios are still in consideration. Table 51 shows the objective function values and comparisons for the cost minimization solutions for all scenarios and robust Pareto fronts. Cost increases due to climate change effects in scenarios 2, 3 and 5 but decreases in scenarios 4 and 6 due to technology advancement. Emissions are only decreased in advanced technology scenarios 4, 5 and 6.

Table 51. Cost and emissions comparisons of cost minimization solutions for all scenarios

	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Cost (\$)	6.392E+11	6.493E+11	6.810E+11	6.315E+11
Emissions (lbs)	4.168E+13	4.194E+13	4.262E+13	3.945E+13
Cost Change from Scenario 1	0.00%	1.57%	6.54%	-1.20%
Emissions Change from Scenario 1	0.00%	0.62%	2.25%	-5.36%
Cost Variance	1.844E+21	1.861E+21	1.887E+21	1.747E+21
Emissions Variance	1.791E+25	1.821E+25	1.896E+25	1.627E+25
	Scenario 5	Scenario 6	RPF S7	RPF S8
Cost (\$)	6.405E+11	6.711E+11	6.626E+11	6.431E+11
Emissions (lbs)	3.906E+13	3.874E+13	4.068E+13	4.002E+13
Cost Change from Scenario 1	0.20%	4.98%	3.65%	0.60%
Emissions Change from Scenario 1	-6.30%	-7.07%	-2.41%	-3.99%
Cost Variance	1.710E+21	1.647E+21	1.774E+21	1.758E+21
Emissions Variance	1.593E+25	1.568E+25	1.715E+25	1.666E+25

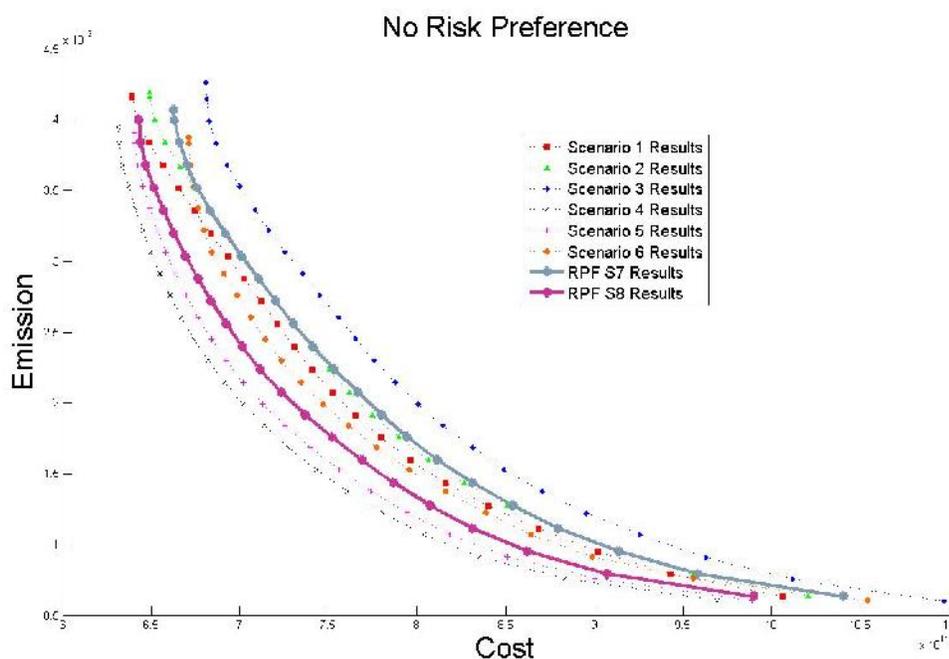


Figure 56. Pareto front comparison of all scenarios (no risk preference)

Figure 56 presents non-dominated solutions for all scenarios for the no risk preference case. Scenario 3 gives the highest cost and solutions for both objectives because it is the worst case scenario and scenario 4 is the best case scenario and gives the lowest cost and emissions solutions for both objectives.

Figure 57 and Figure 58 present minimization of cost solution generation and investment results and comparisons for all scenarios and robust Pareto fronts for the no risk preference case, respectively. Figure 59 and Figure 60 shows more detailed generation and investment comparisons for specific generation units between scenarios and robust Pareto fronts. Generation from coal decreases and generation from wind increases from scenario 1 through scenario 6 with the help of technological advancement in renewable energy sources and due to the efficiency decreasing effects of climate change on thermal technologies.

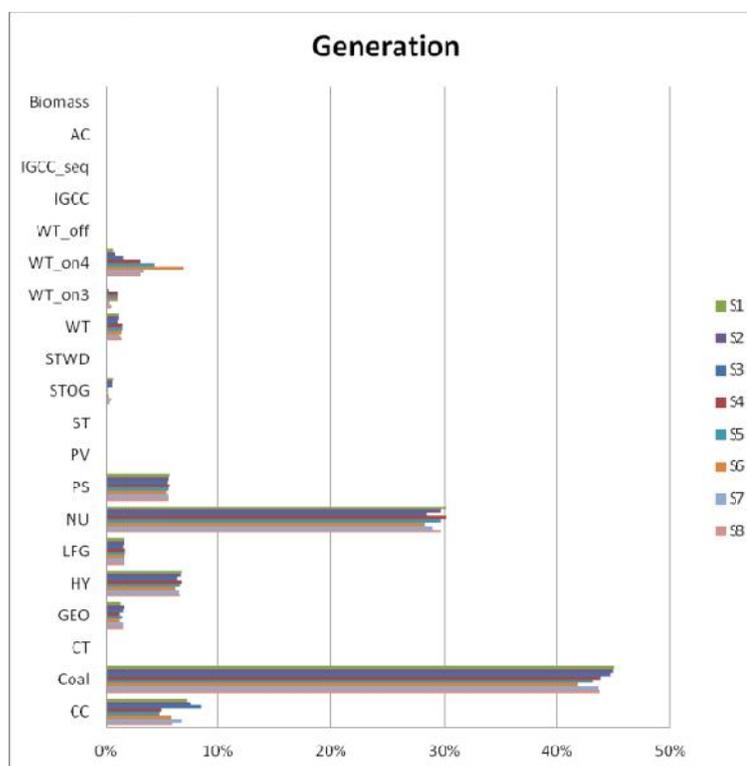


Figure 57. Generation levels of cost minimization solutions for all scenarios and RPF S7 and S8

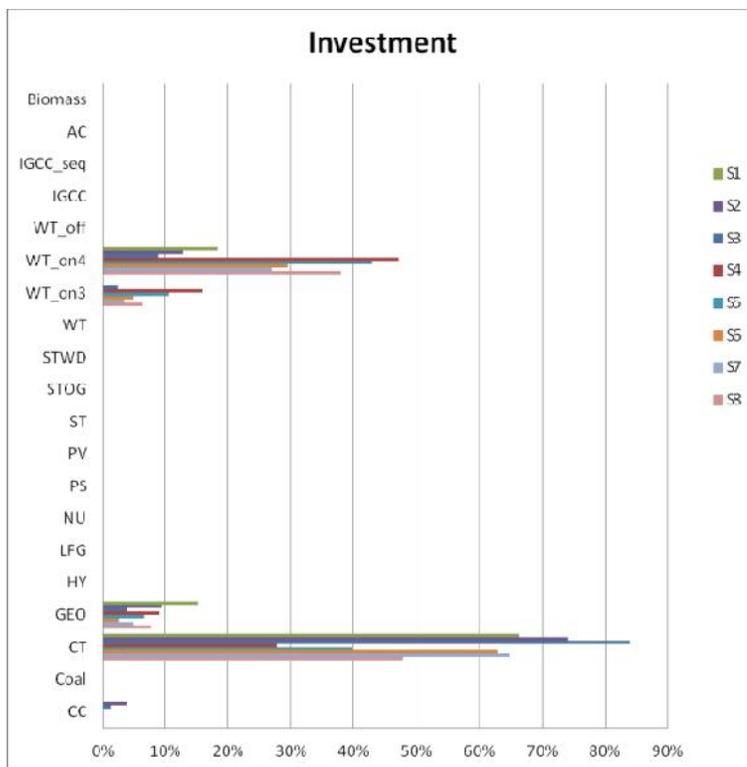


Figure 58. Investment levels of cost minimization solutions for all scenarios and RPF S7 and S8

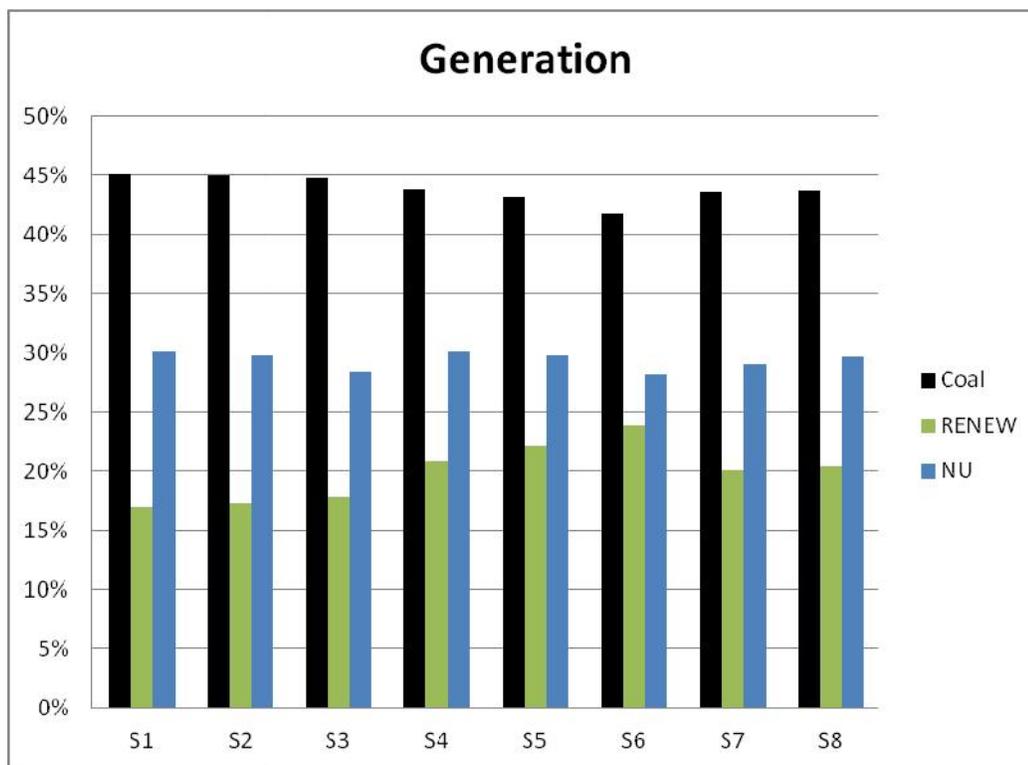


Figure 59. Coal, renewable and nuclear generation levels for all scenarios and RPF S7 and S8

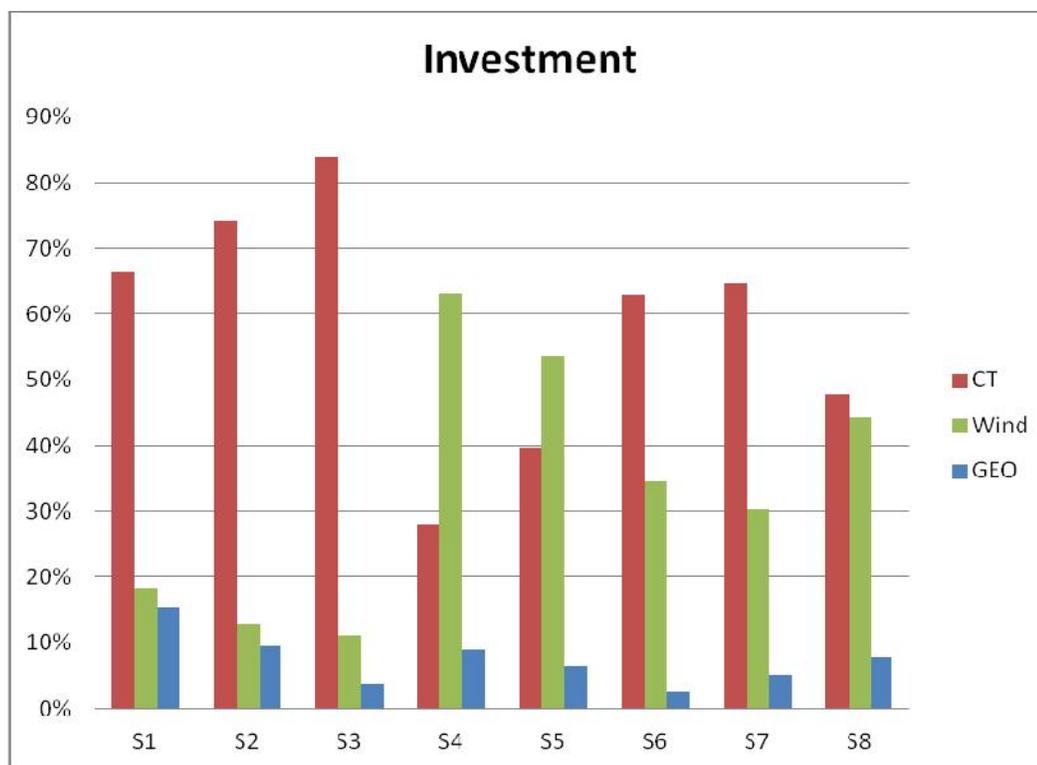


Figure 60. Combustion turbine, wind and geothermal units investment levels for all scenarios and RPF S7 and S8

5.9 Comparison of Decision Profiles

In this section, results and comparisons of decision profiles are presented. Decision profiles are created as demonstrations and can be extended to any risk preference and scenario combination. In a sense, a certain scenario or a robust Pareto front is associated with a risk preference vector to test how the decision of investment and generation changes among different profiles. This section provides results for three decision profiles, namely, risk-neutral, risk-averse and risk-seeking decision profiles.

Figure 61 shows the non-dominated solutions for decision profiles. Although risk-neutral decision profile does not consider any risk preference, PUGA still sorts all solutions according to their PUI values to provide more information for the decision

process. Solutions with star markers in the figure are the neighborhood risk selected solutions for each decision profile.

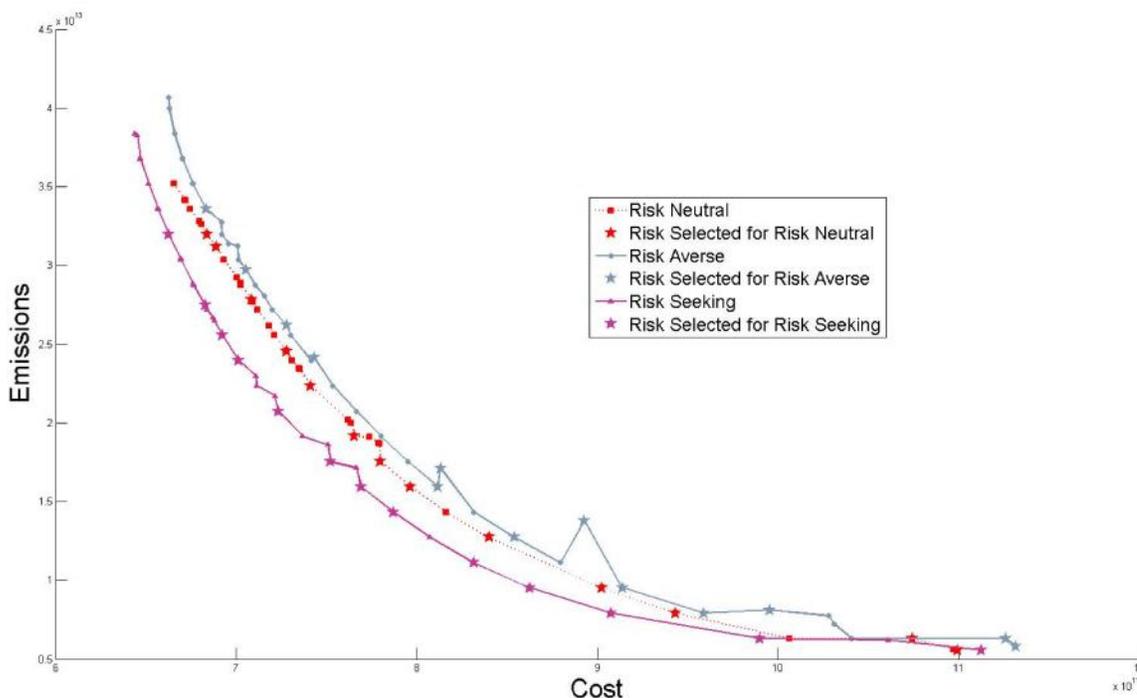


Figure 61. Pareto fronts for different decision profile results

Figure 62 shows the minimization of cost solutions for reference scenario (S1) and robust Pareto fronts S7 and S8 and the top risk selected solutions for each decision profile on the objective space. When risk preferences are taken into account, PUGA favors solutions on the emissions side of the trade of curve, that is, solutions with high cost but low emissions. Table 52 presents cost and emissions comparisons of the minimization of cost solutions and the top risk selected solutions. The risk-averse decision profile selects a solution with lower uncertainty for both cost and emissions although the expected cost is higher than the solution selected in the risk-neutral case. On the other hand, risk-seeking decision profile favors a solution with lower expected cost. Furthermore, cost variance is high which means that cost can be even lower in the future, so taking risk in favor of cost and ignoring the high emissions level compared to the risk-averse case.

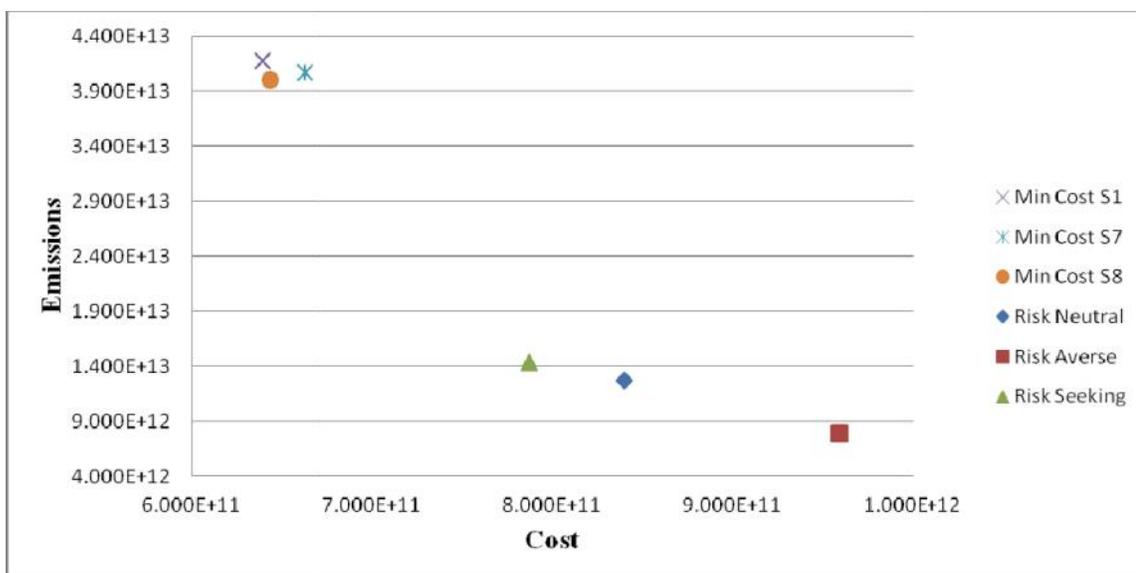


Figure 62. Minimization of cost and top risk selected solutions for each decision profile

Table 52. Cost and emissions comparisons of top risk selected solutions for each decision profile

<i>Minimization of cost solutions</i>	Min Cost S1	Min Cost RPF S7	Min Cost RPF S8
Cost	6.392E+11	6.626E+11	6.431E+11
Emissions	4.168E+13	4.068E+13	4.002E+13
Cost Change from Risk Neutral	-23.89%	-21.11%	-23.44%
Emissions Change from Risk Neutral	227.45%	219.56%	214.40%
Cost Standard Deviation	4.294E+10	4.212E+10	4.192E+10
Emissions Standard Deviation	4.232E+12	4.142E+12	4.082E+12
<i>Decision Profiles</i>	Risk Neutral	Risk Averse	Risk Seeking
Cost	8.399E+11	9.587E+11	7.870E+11
Emissions	1.273E+13	7.920E+12	1.433E+13
Cost Change from Risk Neutral	0.00%	14.14%	-6.31%
Emissions Change from Risk Neutral	0.00%	-37.78%	12.59%
Cost Standard Deviation	1.414E+10	8.576E+09	1.906E+10
Emissions Standard Deviation	2.421E+12	1.838E+12	1.840E+12

Figure 63 and Figure 64 present the generation and investment levels of top risk selected solutions for each decision profile, respectively. Nuclear investment and generation decrease both in risk-averse and risk-seeking profiles compared to risk-neutral profile. Generation from renewable sources and investment in renewable technologies, however, increase with inclusion of risk preference. Figure 65 shows this change for nuclear and wind investment levels.

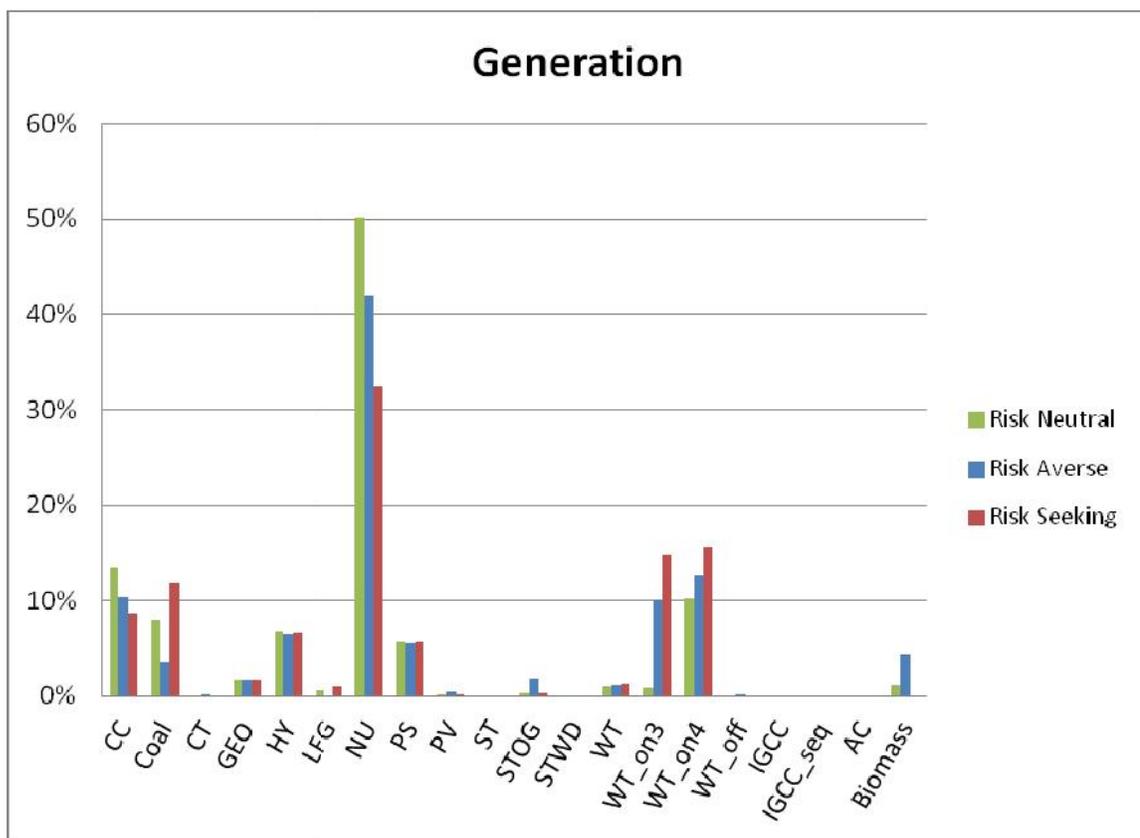


Figure 63. Generation levels of top risk selected solutions for each decision profile

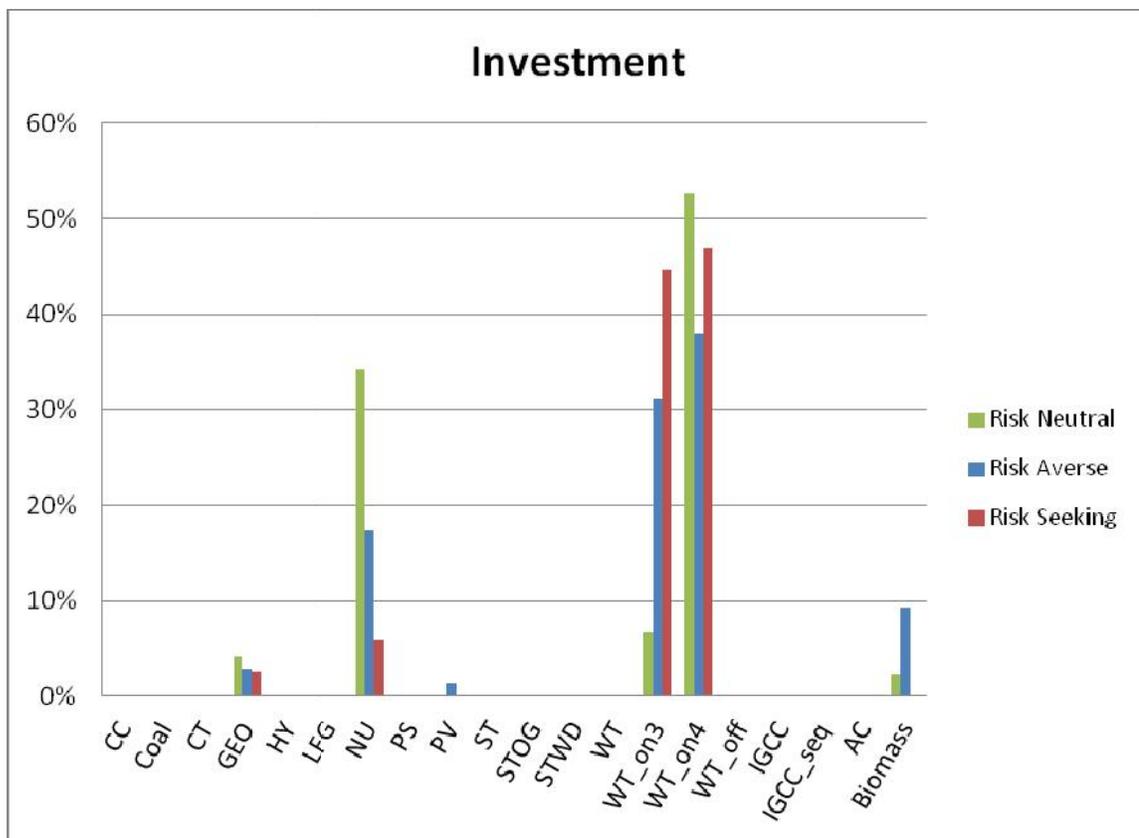


Figure 64. Investment levels of top risk selected solutions for each decision profile

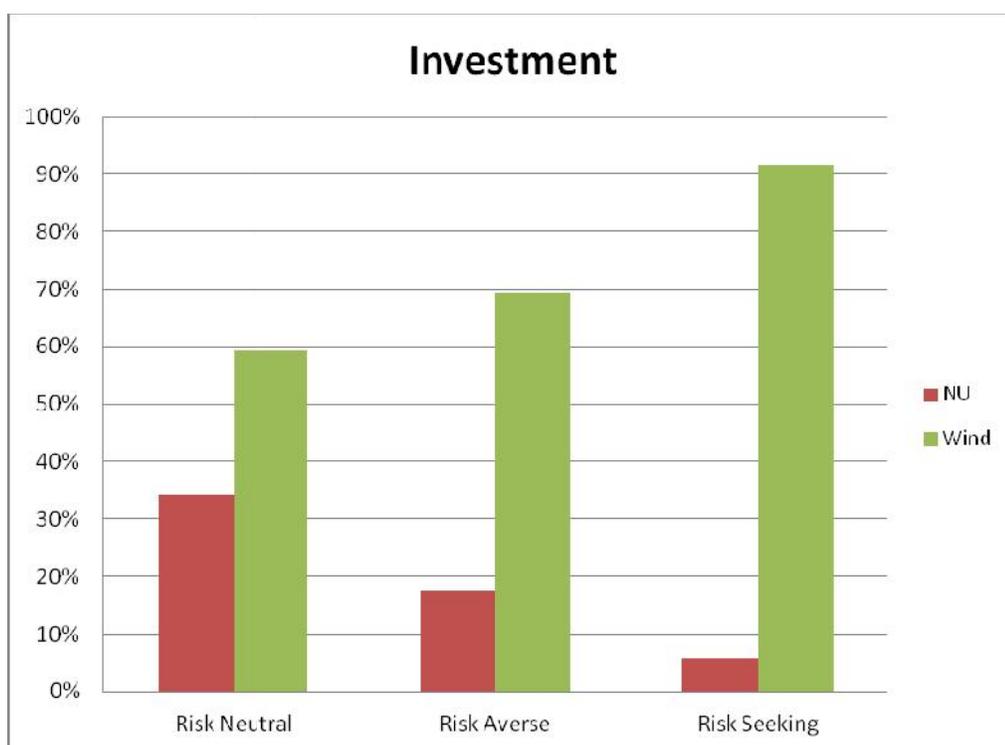


Figure 65. Nuclear and wind investment levels of top risk selected solutions for each decision profile

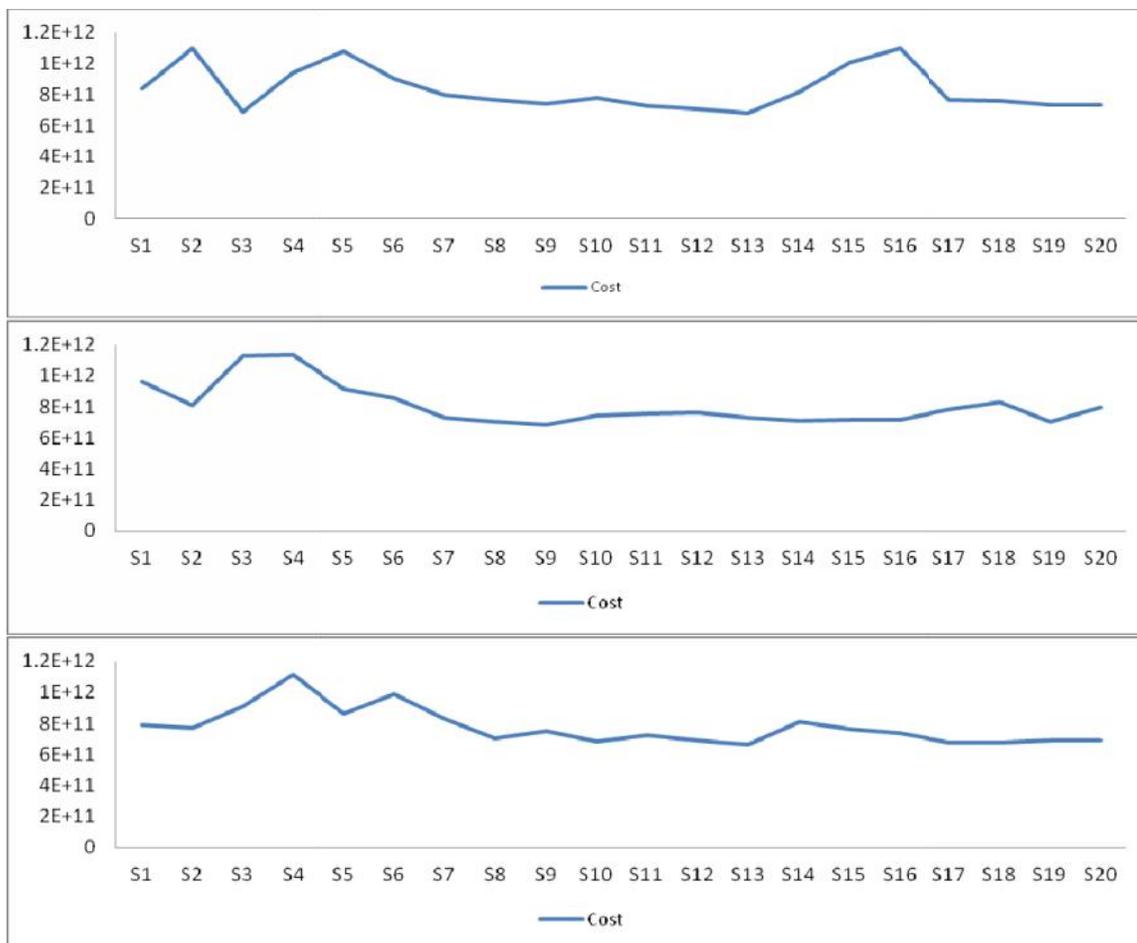


Figure 66. Change in cost between top 20 risk selected solutions of the risk-neutral, risk-averse and risk-seeking decision profiles

Figure 66 displays changes in cost between top 20 risk selected solutions of the risk-neutral, risk-averse and risk-seeking decision profiles. Figure 67 is similar but displays emissions change. These graphs show the competing nature of cost and emissions objectives of the problem.

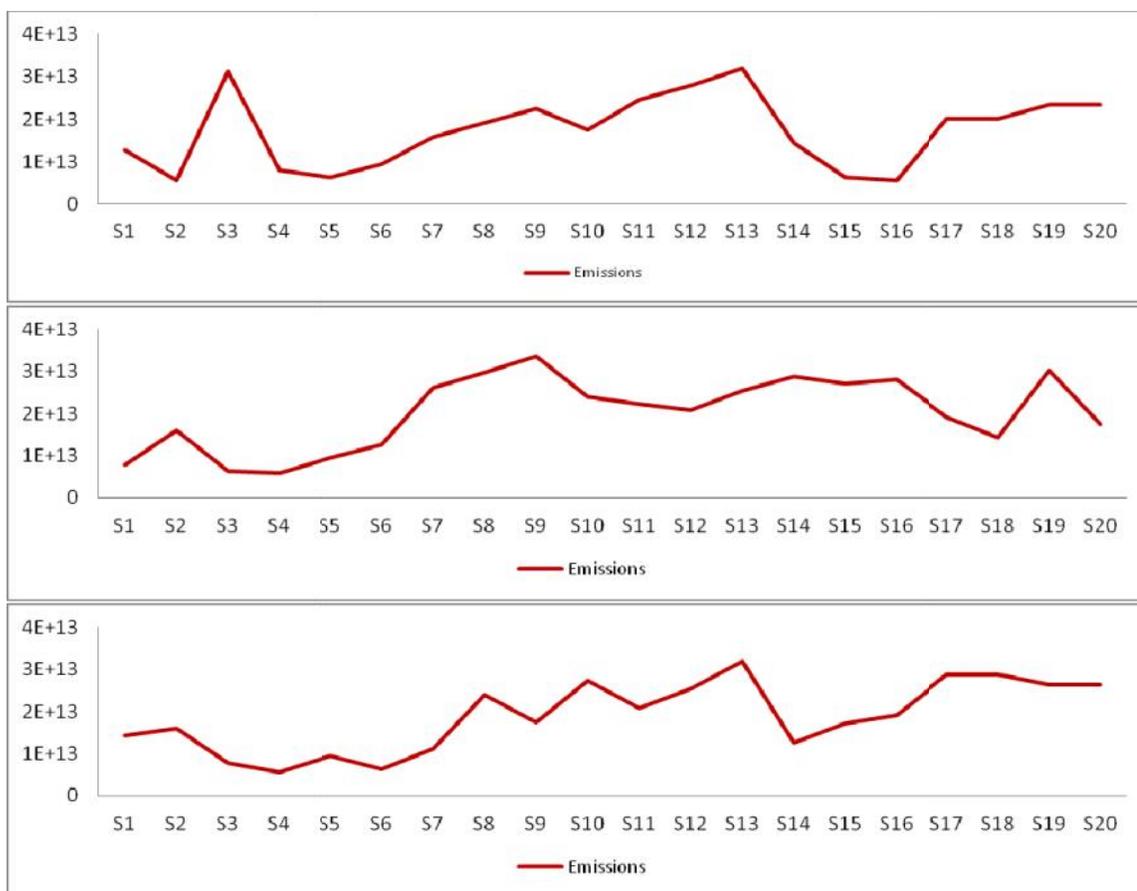


Figure 67. Change in emissions between top 20 risk selected solutions of the risk-neutral, risk-averse and risk-seeking decision profiles

Figure 68 is a comparison of coefficients of variation (CV) for cost and emissions objectives, and it shows that uncertainty level and expected values of both objectives are competing. When this figure is compared to Figure 66 and Figure 67, it is observed that if the expected value of an objective function is minimized, then the uncertainty level increases.

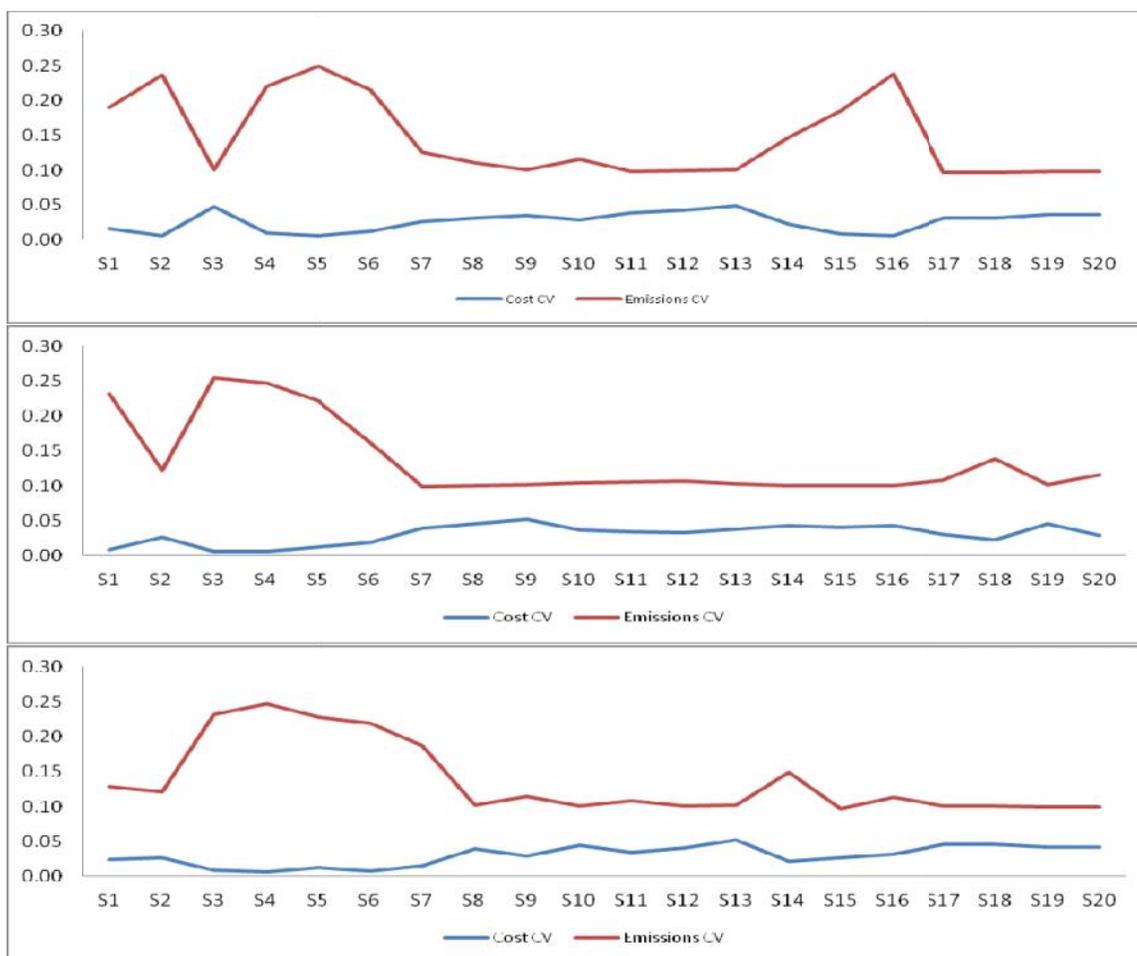


Figure 68. Comparison of coefficients of variation (CV) for cost and emissions objectives

Figure 69 and Figure 70 display the change in generation and investment levels of units between top 20 risk selected solutions of the risk-neutral, risk-averse and risk-seeking decision profiles. In the risk-neutral profile, reference scenario is assumed to be occurring in the future. Therefore, generation levels of nuclear for all solutions are the highest. However, generation from renewable sources is always below 30% of the total generation. Generation from wind units is the lowest in the risk-neutral profile. For investment decisions, the risk-neutral profile has the highest level of nuclear investment. Renewable source investments in the risk-neutral profile are less than the others.

In the risk-averse profile, nuclear generation is still high but generation from renewable sources is over 30% of the total generation for all solutions. Generation from wind in average doubles the levels of the risk-neutral profile. The major difference from the risk-neutral profile is in investment levels. Nuclear investment on average is cut in half and renewable investment increased by 30% on average. Renewable sources, mainly wind units, make more than 80% of the total investment in almost all solutions.

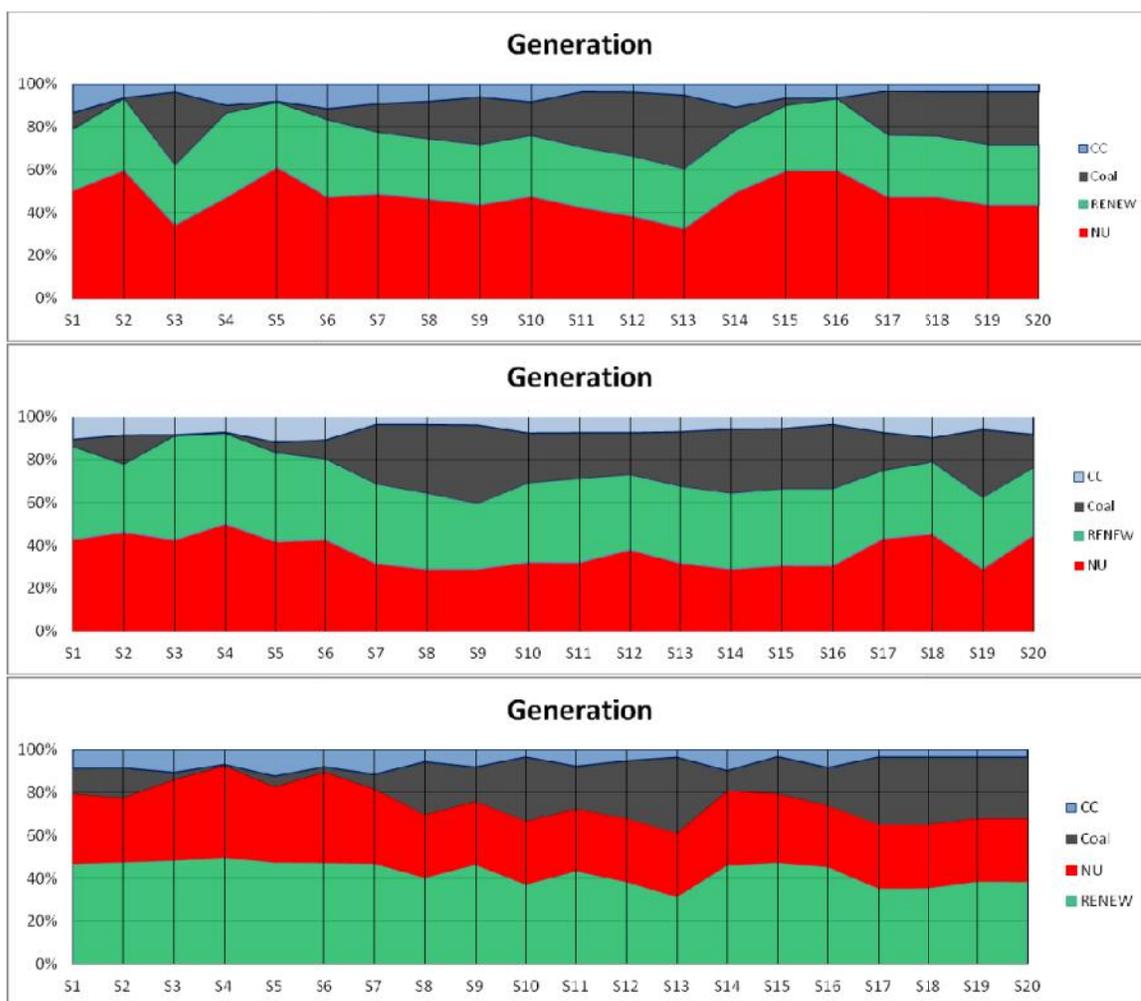


Figure 69. Change in generation levels of units between top 20 risk selected solutions of the risk-neutral, risk-averse and risk-seeking decision profiles.

In the risk-seeking profile, nuclear generation is decreased further to 30% of the total generation. Renewable sources surpass the nuclear generation in all solutions and make

up almost half of the total generation. Generation from wind units in average is 25% of the total. Nuclear investment in the risk-seeking profile is almost reduced to zero. More than 90% of all investment is from renewable sources for the most of the risk selected solutions. Share of wind units in renewable sources are the highest of all profiles, comprising almost all of the renewable investment.

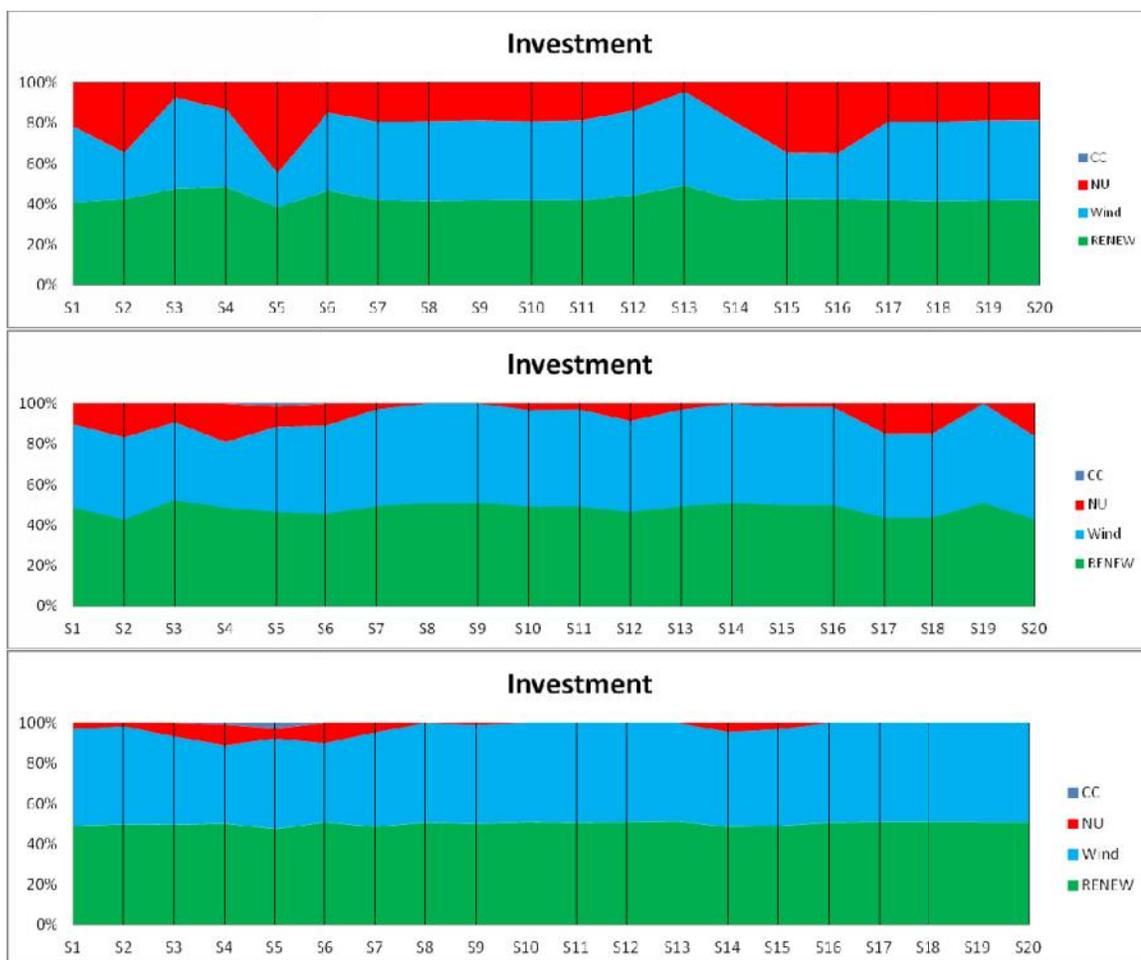


Figure 70. Change in investment levels of units between top 20 risk selected solutions of the risk-neutral, risk-averse and risk-seeking decision profiles.

In all profiles, coal and CC (combined cycle natural gas) have steady shares of generation levels. CC is used at 10% on average. Coal is not usually chosen for the top five or six solutions of any risk profile but in most of the solutions it has up to a 30% share of the total generation. Share of coal tends to decrease with risk ranking of

solutions. Neither coal nor CC is preferred for investment in any risk profiles. Generation and investment are dominated by nuclear and wind units in all profiles.

5.10 Scenario Comparisons

In this section, all scenarios are compared according to the risk preferences of both objectives. In the previous section, certain scenarios are assumed to be realized in the future and decision profiles are tested. In this section, we fix the risk preference vector and vary the scenarios. The risk-averse preference compares all scenarios when the risk preference vector is set to risk aversion for both objective functions, that is, $\mathbf{w} = [15 \ 15]$. The risk-seeking preference compares all scenarios when the risk preference vector is set to risk seeking for both objective functions, that is, $\mathbf{w} = [-15 \ -15]$.

5.10.1 All scenarios with risk-averse preference, $\mathbf{w} = [15 \ 15]$

When $\mathbf{w} = [15 \ 15]$, risk preference is set to risk-averse in both objectives, cost and CO₂ emissions. In this section, all scenarios are compared with $\mathbf{w} = [15 \ 15]$. Figure 71 presents non-dominated and neighborhood risk selected solutions for all scenarios for the risk-averse preference. In the figure, there are 40 Pareto and 10 risk selected solutions for each scenario, and in total 320 solutions. It is impractical to compare them all, even with neighborhood pruning. Therefore, we confine our comparison to the top risk selected solutions of each scenario. The top risk selected solutions are the solutions with the lowest PUI value among the 40 Pareto solutions of each scenario.

Figure 72 and Figure 73 present generation and investment results and comparisons of the top risk selected solutions for all scenarios and robust Pareto fronts. Figure 74 and Figure 75 show more detailed generation and investment comparisons for specific generation units between scenarios and robust Pareto fronts to easily observe the change

in major decisions. Generation and investment decisions mainly change between nuclear and renewable sources due to the technological progress assumptions of scenarios. Scenarios and Pareto front with advanced technologies (S4, S5, S6 and S8) have higher levels of renewable generation and investment. However, generation from nuclear, and investment in nuclear is lower in those scenarios. As soon as the cheap and efficient renewables are available, they replace the nuclear investment and generation. Wind investment in particular is almost the only investment option (over 90% of total) when advanced technology is available. Generation from coal units, on the other hand, remains at similar levels through all scenarios.

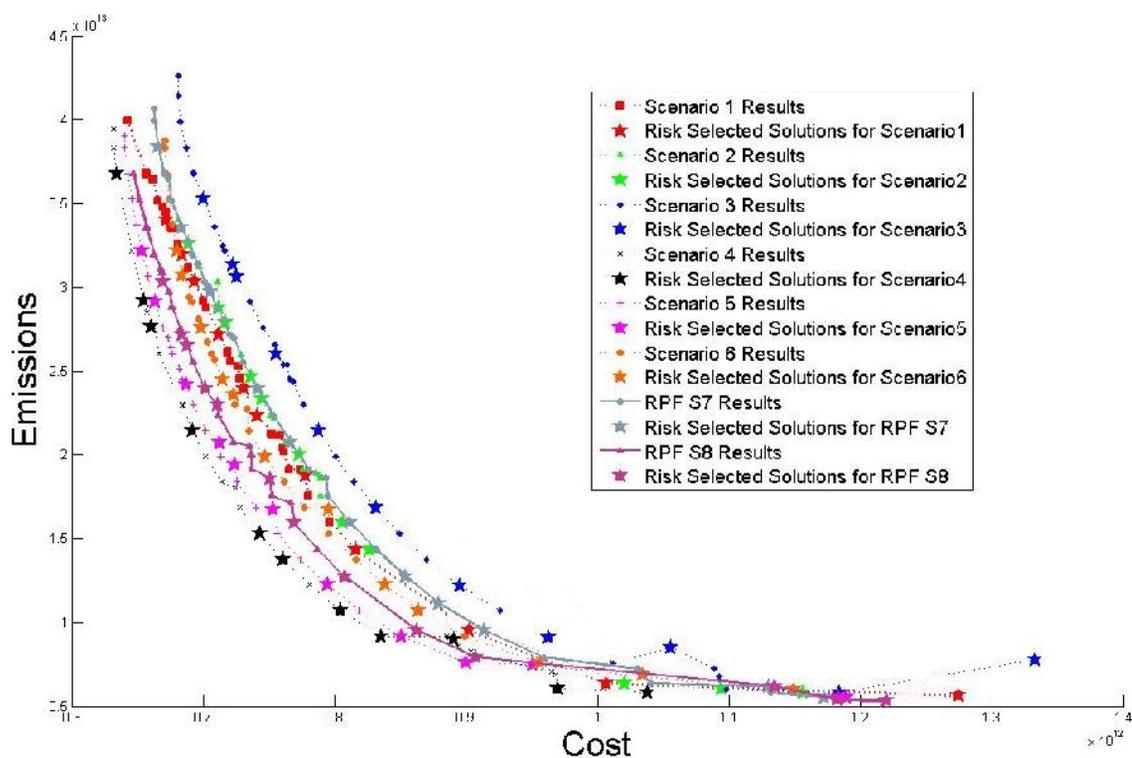


Figure 71. Pareto front comparison of all scenarios, = [15 15]

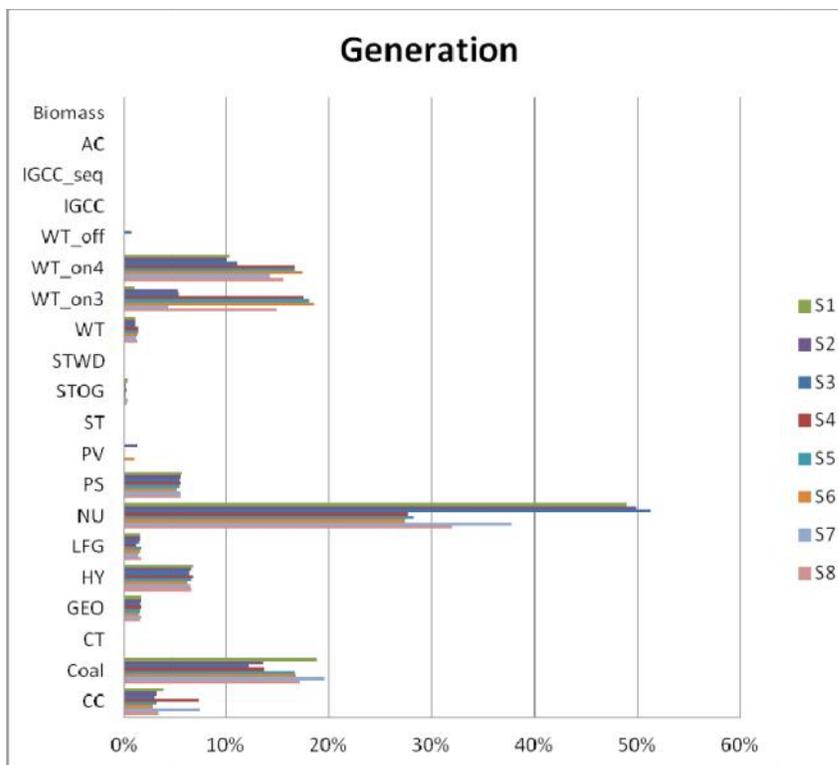


Figure 72. Generation levels of the top risk selected solutions for all scenarios and RPF S7 and S8 = [15 15]

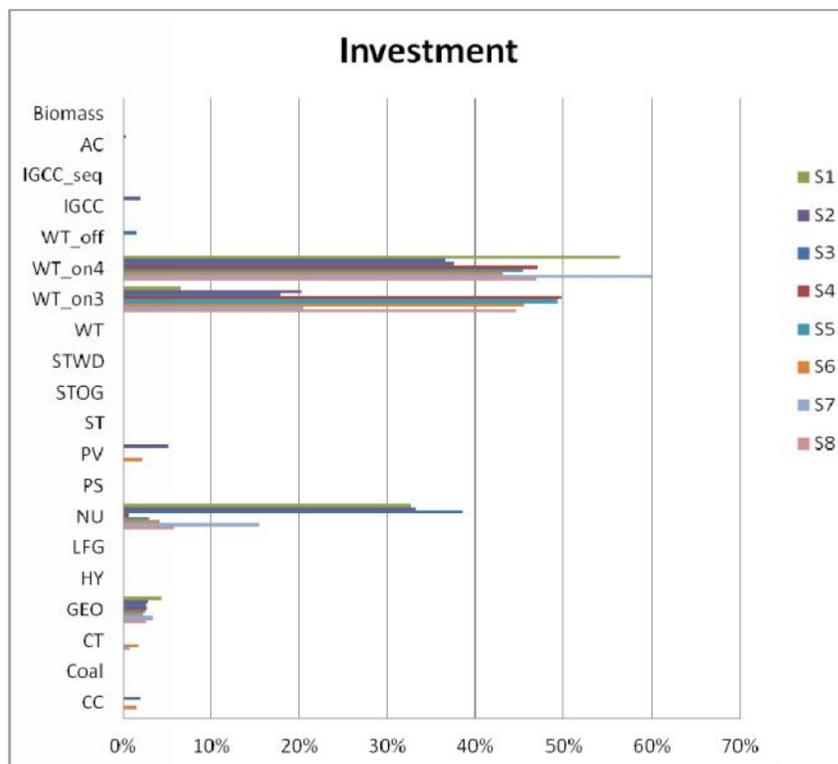


Figure 73. Investment levels of the top risk selected solutions for all scenarios and RPF S7 and S8 = [15 15]

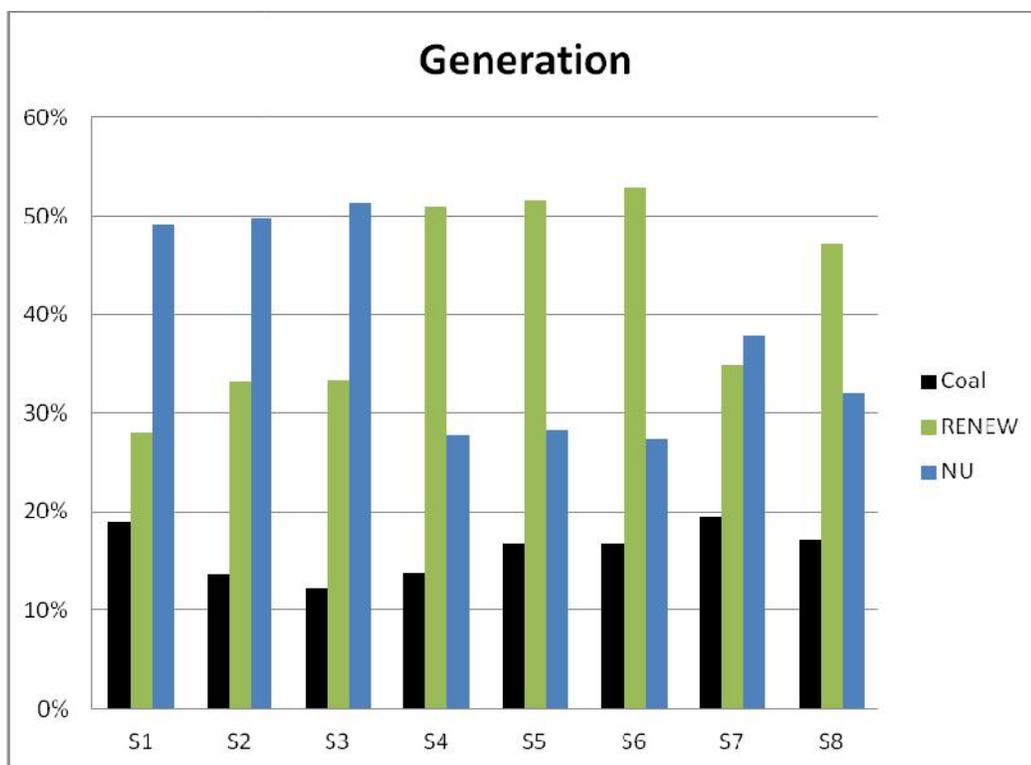


Figure 74. Coal, renewable and nuclear generation levels of the top risk selected solutions for all scenarios and RPF S7 and S8, = [15 15]

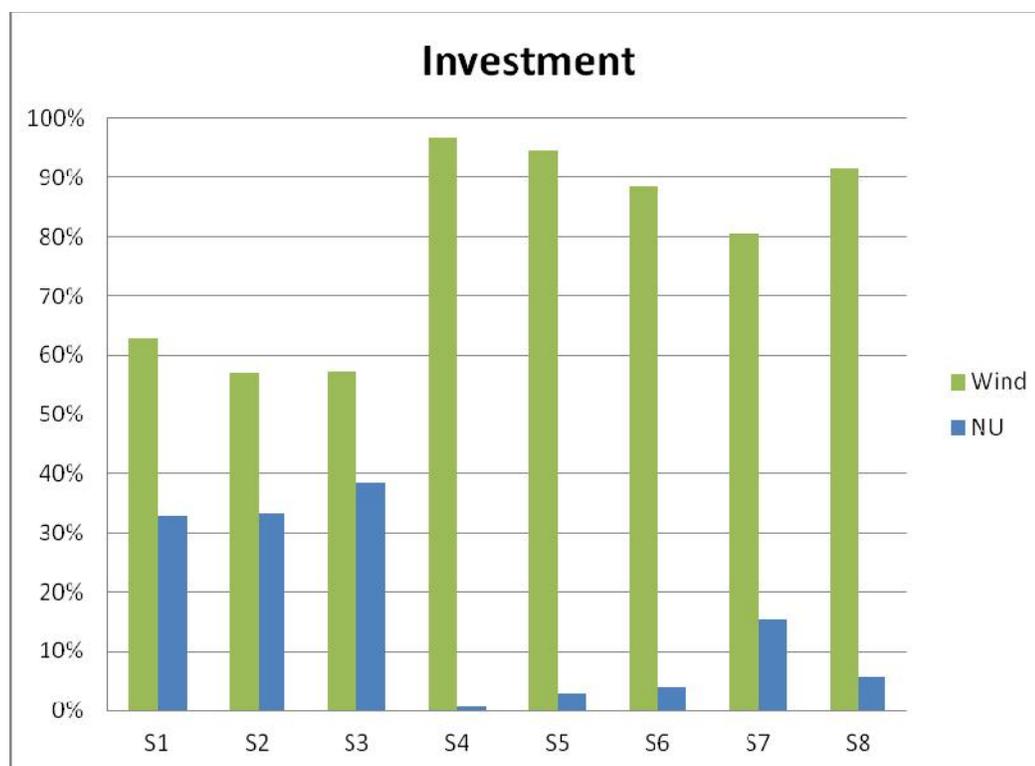


Figure 75. Nuclear and wind investment levels of the top risk selected solutions for all scenarios and RPF S7 and S8, = [15 15]

5.10.2 All scenarios with risk-seeking preference, $\alpha = [-15 \ -15]$

When $\alpha = [-15 \ -15]$, the risk preference is set to risk-seeking in both objectives, cost and CO₂ emissions. In this section, all scenarios are compared with $\alpha = [-15 \ -15]$. Figure 76 presents non-dominated and neighborhood risk selected solutions for all scenarios for the risk-seeking preference. Again, we confine our comparison to the top risk selected solutions of each scenario. The top risk selected solutions are the solutions with the lowest PUI value among the 40 Pareto solutions of each scenario.

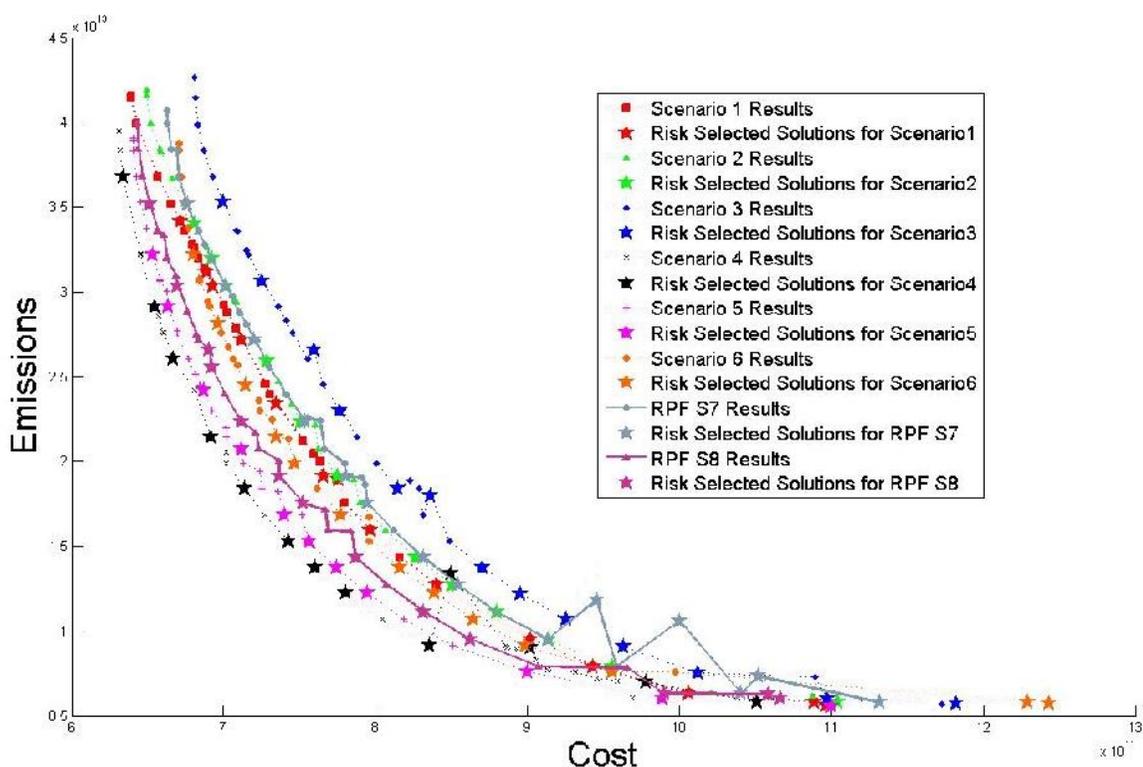


Figure 76. Pareto front comparison of all scenarios, $\alpha = [-15 \ -15]$

Figure 77 and Figure 78 present generation and investment results and comparisons of the top risk selected solutions for all scenarios and robust Pareto fronts. Figure 79 and Figure 80 show more detailed generation and investment comparisons for specific generation units between scenarios and robust Pareto fronts to easily observe the change

in major decisions. Similar to the previous section results, generation and investment decisions mainly change between nuclear and renewable sources due to the technological progress assumptions of scenarios. Generation from coal units in the risk-seeking preference again remains at similar levels (below 10%) through all scenarios but it is lower compared to the risk-averse case results (average 15%).

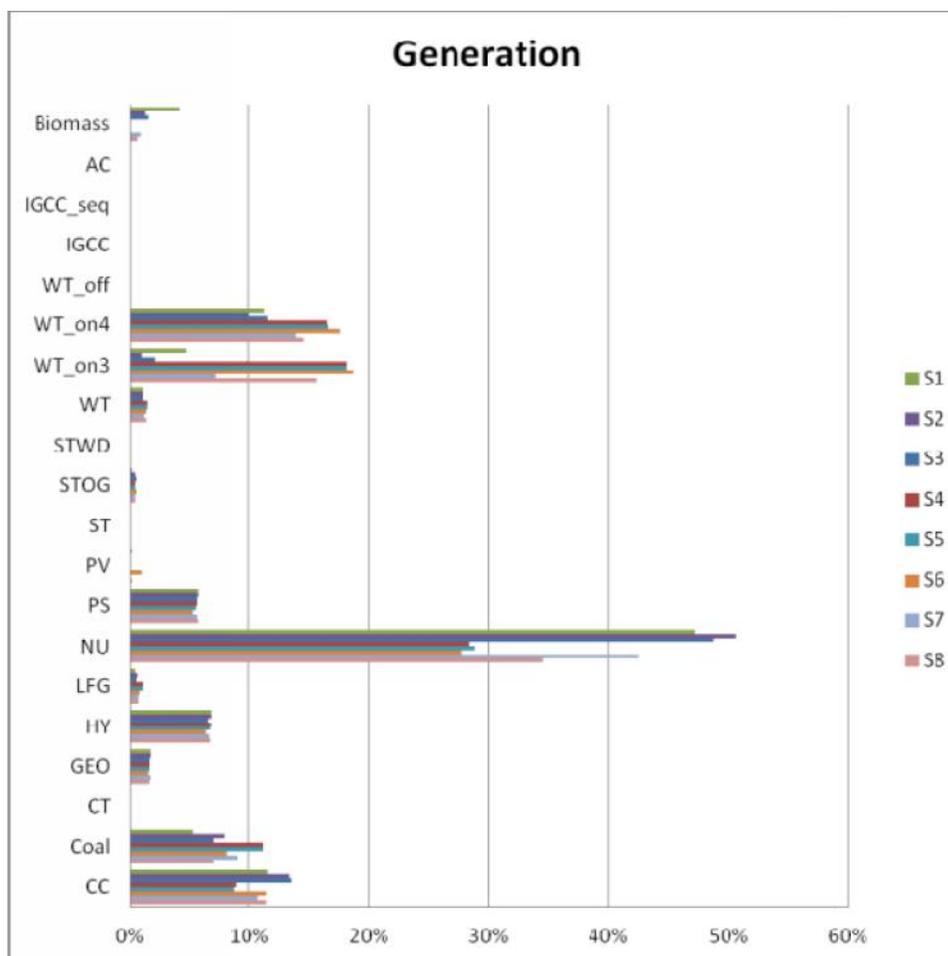


Figure 77. Generation levels of the top risk selected solutions for all scenarios and RPF S7 and S8
= [-15 -15]

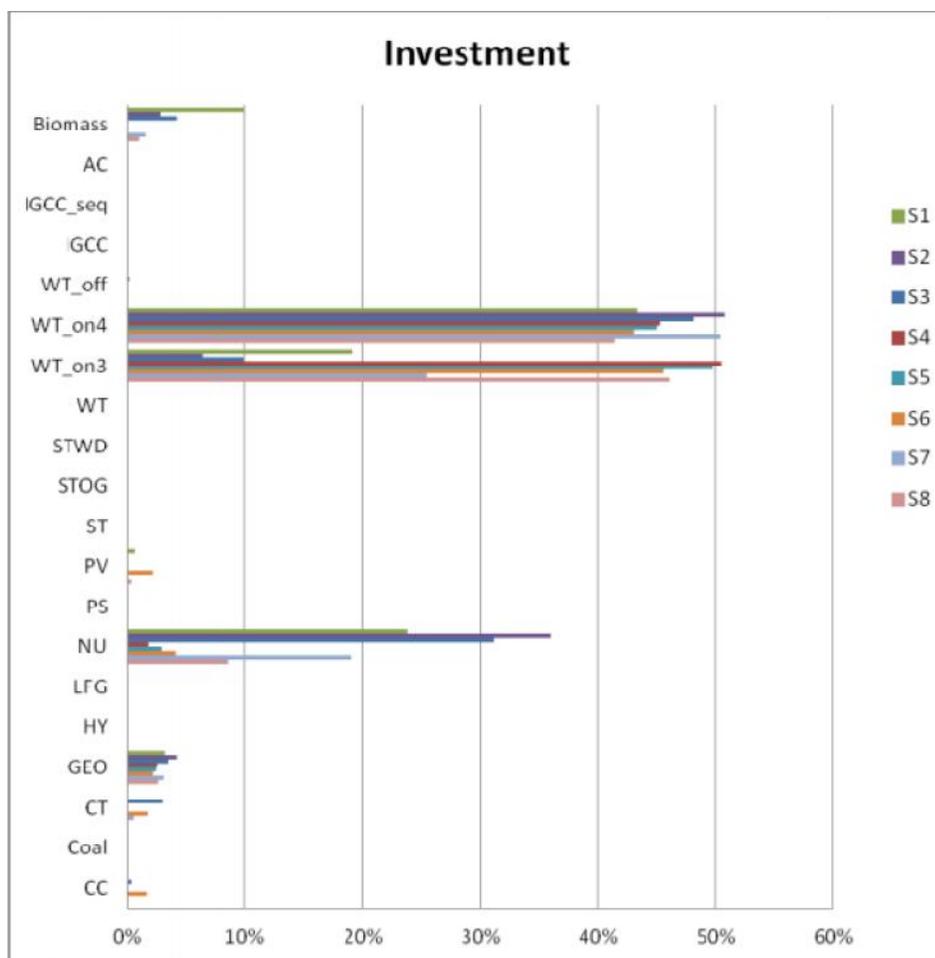


Figure 78. Investment levels of the top risk selected solutions for all scenarios and RPF S7 and S8
= [-15 -15]

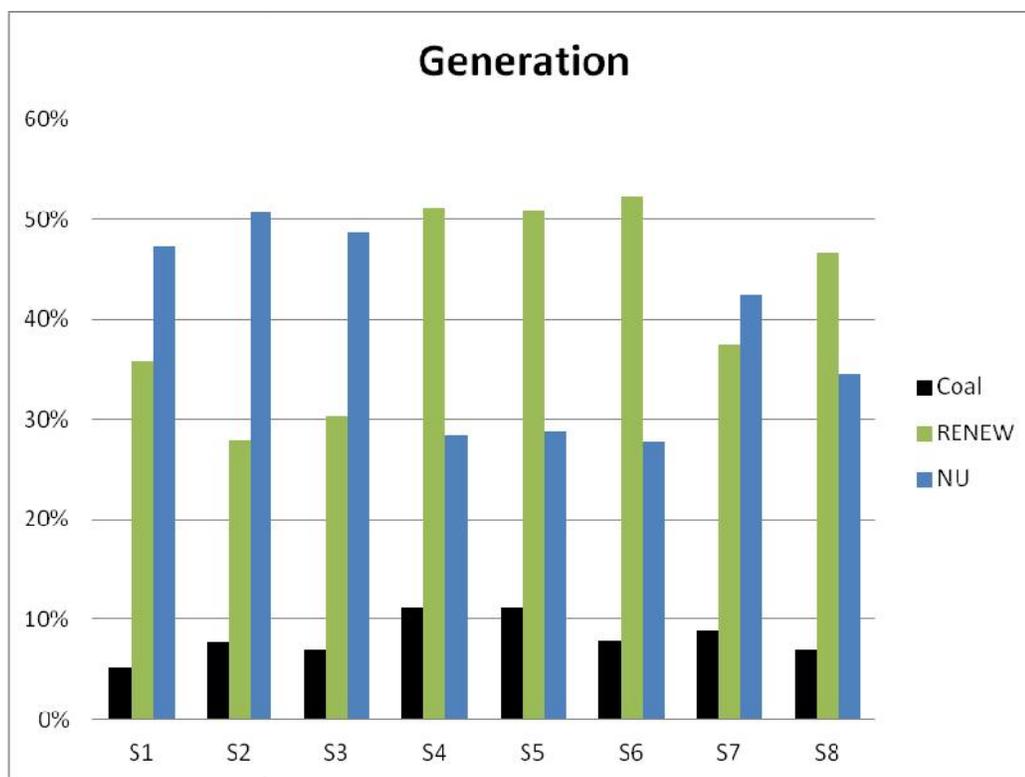


Figure 79. Coal, renewable and nuclear generation levels of the top risk selected solutions for all scenarios and RPF S7 and S8, = [-15 -15]

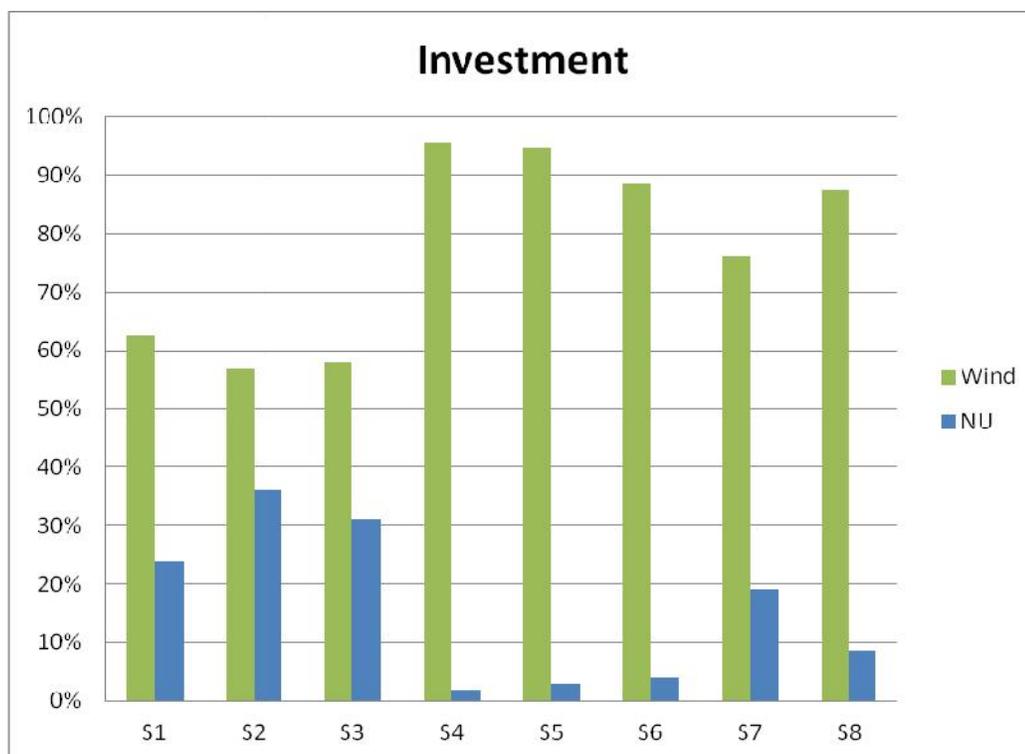


Figure 80. Nuclear and wind investment levels of the top risk selected solutions for all scenarios and RPF S7 and S8, = [-15 -15]

5.11 Risk Preference Comparisons on the Robust Pareto Front S7

In the previous sections, first, decision profiles are compared where a certain scenario or a robust Pareto front is assumed to be realized in the future, and later risk preferences are kept constant while scenarios are compared. In this section risk preferences of objective functions are compared on the robust Pareto front S7 which is a compromise trade-off curve of all scenarios with the probability of the worst case scenario (S3) being the highest. Assumed probabilities of scenarios to form robust Pareto front S7 are shown in Table 53. The objective in this section is to keep the robust Pareto front S7 constant and to vary the risk preference vector to test how the selection of a solution on the Pareto front and decision of investment and generation changes.

Table 53. Probability of scenarios for RPF S7

Scenarios	Probability
1	0.1
2	0.1
3	0.5
4	0.1
5	0.1
6	0.1

Figure 81 shows the top risk selected solutions for different risk preference vectors. When the risk preference of an objective function is greater than zero, risk preference is on the risk-averse side and when it is less than zero risk preference is on the risk-seeking side. For example, $\lambda = [15 \ -15]$ means preference is risk-averse for the cost objective and it is risk-seeking for the emissions objective.

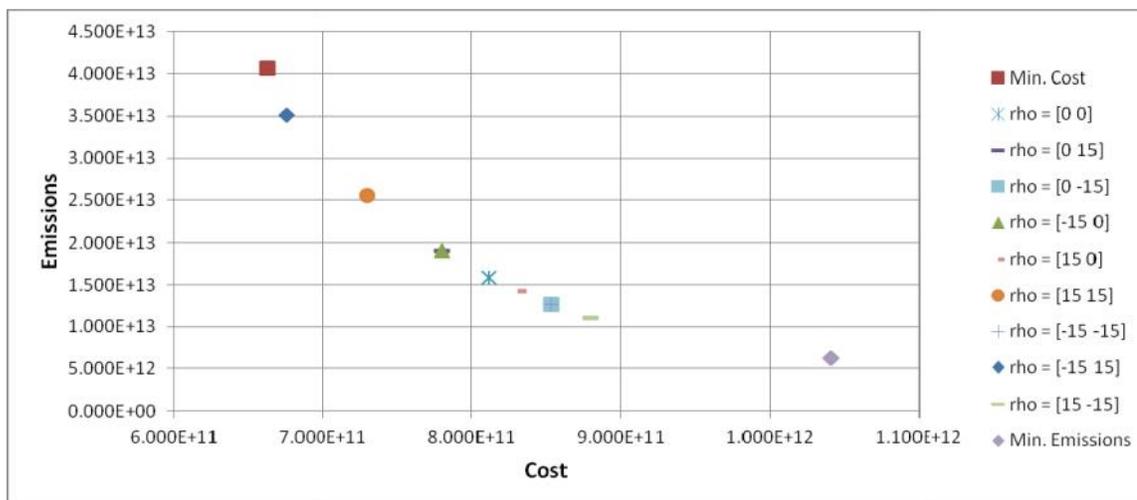


Figure 81. Risk Preference Comparisons on the RPF S7

When we only consider the minimization of cost neglecting the uncertainty and the emissions objective, the cost is minimized subject to constraint sets, and since the objectives are competing, the emissions objective is at its highest value (Table 54). Coal generation and combustion turbine investment have their highest share when cost is minimized (Figure 82 and Figure 83). Renewable generation and investment, and nuclear generation are at their lowest level and there is no nuclear investment in this case. These are expected results because generation from coal and combustion turbine investment are the cheapest options and nuclear investment is the most expensive one.

When $\rho = [0 \ 0]$, the risk preference is neutral for both objectives. PUGA does not evaluate the PUI values based on a risk preference but it still sorts solutions based on the stochastic domination, that is, it still selects the least dominated solution on the Pareto front but without considering the risk preference. The top selected solution of this case ($\rho = [0 \ 0]$) is a balance solution between the cost and the emissions objectives (Figure 81) in the vicinity of averages of the cost and emissions minimization solutions (Table 54).

Objective values and standard deviations of the selected solutions are provided in Table 54. In general, the cost minimization solution has the highest variability. The

emissions minimization solution, on the other hand, has the lowest variability, but cost objective variances for all solutions are always lower than emissions objective and the risk preference favors solutions with less variability when it is set to risk-averse and it favors solutions with higher variability or lower expected value. For example, when $\alpha = [15 \ -15]$, the risk preference is risk-averse for the cost objective and risk-seeking in the emissions objective. The top selected solution of this case is the closest one among others to the emissions minimization solution which also has the smallest variability for cost emissions. On the other hand, when $\alpha = [-15 \ 15]$, the risk preference is risk-averse for the emissions objective and risk-seeking in the cost objective. This time, the top selected solution is the closest one among others to the cost minimization solution.

Table 54. Risk Preference Comparisons of Objectives for the RPF S7

<i>Risk Preference Comparisons on S7</i>			Cost	Emissions	Cost Standard Deviation	Emissions Standard Deviation	Cost Change from Minimized Cost	Emissions Change from Minimized Cost
Min. Cost			6.626E+11	4.068E+13	4.212E+10	4.142E+12	0.00%	0.00%
	0	0	8.117E+11	1.594E+13	2.116E+10	1.944E+12	22.51%	-60.82%
	0	15	7.801E+11	1.914E+13	2.367E+10	2.067E+12	17.73%	-52.94%
	0	-15	8.540E+11	1.273E+13	1.630E+10	2.053E+12	28.89%	-68.71%
	-15	0	7.801E+11	1.914E+13	2.367E+10	2.067E+12	17.73%	-52.94%
	15	0	8.315E+11	1.433E+13	1.920E+10	1.991E+12	25.49%	-64.77%
	15	15	7.301E+11	2.556E+13	2.799E+10	2.625E+12	10.19%	-37.18%
	-15	-15	8.540E+11	1.273E+13	1.630E+10	2.053E+12	28.89%	-68.71%
	-15	15	6.759E+11	3.521E+13	3.713E+10	3.566E+12	2.01%	-13.46%
	15	-15	8.794E+11	1.113E+13	1.104E+10	2.286E+12	32.73%	-72.65%
Min. Emissions			1.041E+12	6.317E+12	8.002E+09	1.303E+12	57.06%	-84.47%

Generation levels of the selected solutions are shown in Figure 82. A more detailed comparison for generation percentages of the risk selected solutions at selected risk preference levels and the cost minimization solution is presented in Figure 84. The main decision changes for generation levels are of nuclear, coal and renewable units (mainly wind) while combined cycle generation levels are always below 10%. Coal is utilized

more when the algorithm focuses on minimizing the cost objective or accepts risk to minimize it. Nuclear levels are higher in balance solutions such as $\rho = [0 \ 0]$ or $\rho = [-15 \ -15]$ cases. Renewable generation is a strong alternative to nuclear and coal especially when the algorithm seeks possibilities to minimize emissions objective (when $\rho = -15$ for the emissions objective function).

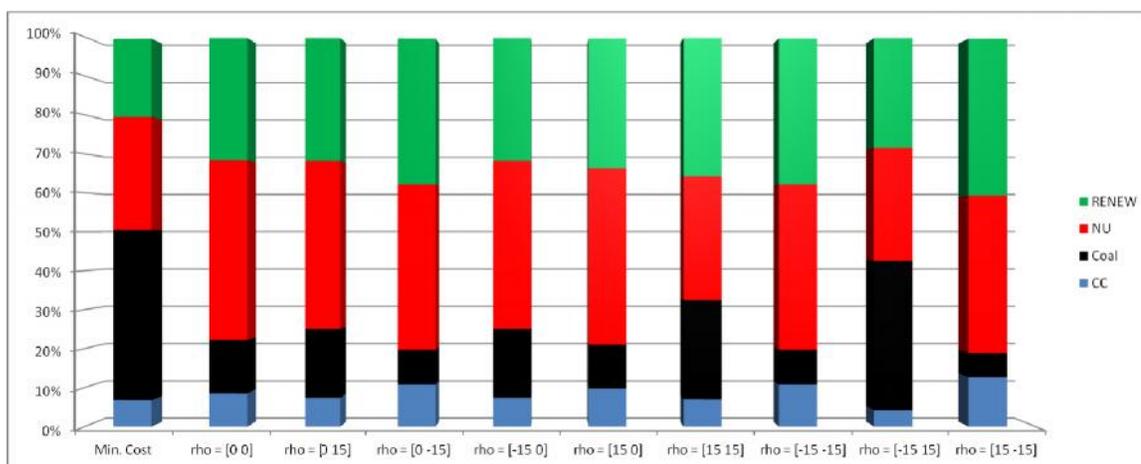


Figure 82. Coal, nuclear, renewable and combined cycle generation levels for the RPF S7 at different risk preference levels

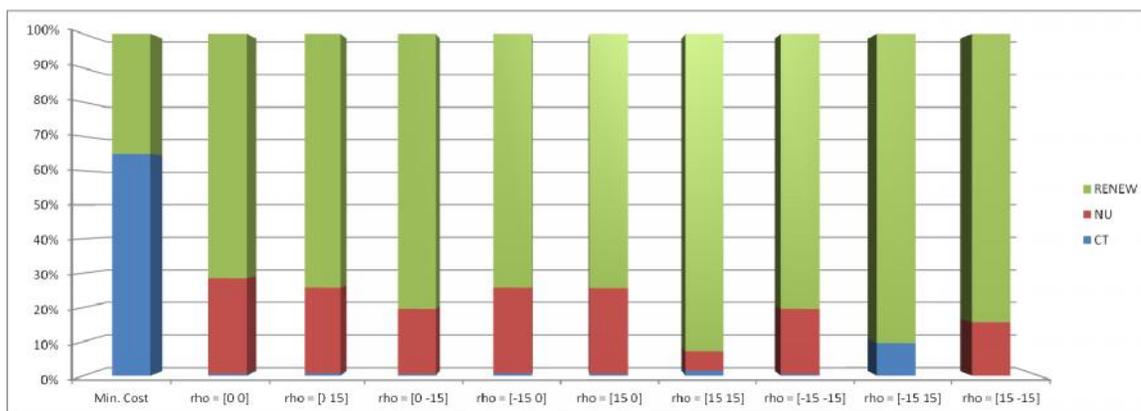


Figure 83. Combustion turbine, nuclear and renewable investment levels for the RPF S7 at different risk preference levels

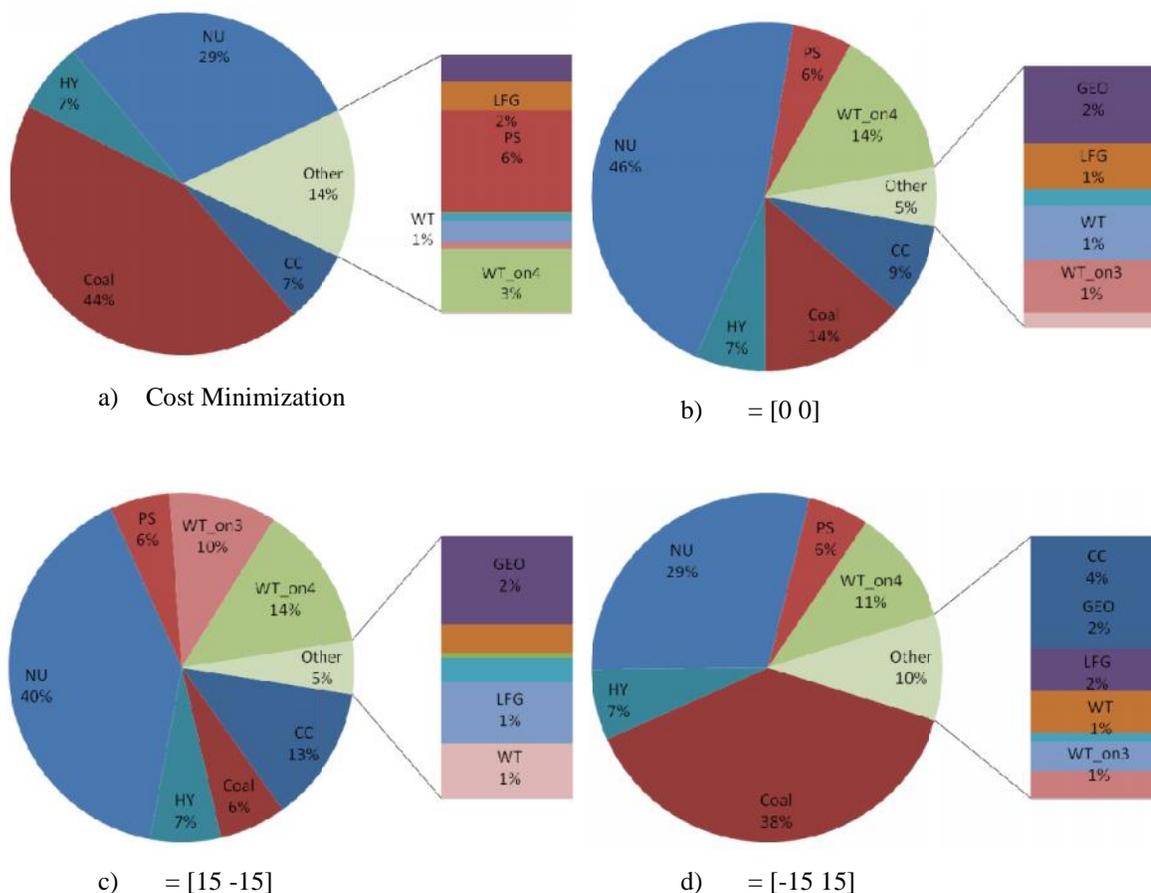


Figure 84. Generation percentages of the risk selected solutions at different risk preference levels and the cost minimization solution

Investment levels of the selected solutions are shown in Figure 83. Investment level of combustion turbine (CT at 65%) in the cost minimization solution is considerable especially when its level of generation is not that high (7%). The algorithm invests highly in CT because of the reserve margin constraint and it is the cheapest available option for investment. CT capacity is needed for reliability reasons. On the other hand, investment levels of renewable units are remarkable in the top risk selected solutions at selected risk preference levels. The cost objective increases (between 2% to 33%) in these solutions but uncertainties in both objectives and emissions objective decrease significantly (Table 54) with the help of renewable and nuclear investment and generation.

5.12 Conclusion

This chapter provides a multi-objective stochastic optimization model, input data and assumptions for the model, uncertainty modeling and results and comparison of results for the power generation expansion planning of the Northeast region of the United States. The model (NEGEP) is constructed with basic constraint sets such as demand-supply equality or reserve margin requirement. Detailed constraint sets such as Renewable Portfolio Standards (RPS) or emissions caps are not included, so that the Pareto set can be observed as broad as possible; because these constraints would prune the feasible region if applied. On the other hand, the main objective of this chapter is to examine the effects of uncertainty and risk associated with it rather than to provide specific solutions, although the model is constructed as realistic as possible with real data and rational and systematic assumptions. The practical aspect of the model can be further improved by working with decision or policy makers and incorporating their input and modifying the model for their interest.

Electricity generation network expansion planning already confronts several complicated economic, environmental and technological challenges and the impact of these challenges will only increase in the future. Inherent uncertainties in economic and technological development and climate change make it much harder to determine planning decisions today. As it is seen throughout the chapter, the amount of information is extensive when compared to the single-objective or deterministic assumption cases. It might seem to be easier to make such assumptions for the decision or policy making processes. However, the amount of lost information, especially from the uncertainty

perspective, has to be considered. Ignoring uncertainty while modeling these planning problems may cause greater risks.

Results of this chapter suggest that the extent of capacity addition is expected to be nuclear and especially renewable units when uncertainty is taken into consideration. Mitigating pollutant emissions also necessitates emissions-free electricity generation from nuclear or renewable units. Investing in clean and renewable energy technologies to cost effectively satisfy the electricity demand while maintaining the reliability of the network requires more technological advancement and also regulation and policy encouragement.

On the other hand, technologies such as advanced coal units with carbon capture and sequestration (CCS) will need to attain even higher advances in technology and discounts in costs to be employed. The high costs and slow advancement of their technology prevent CCS units to be preferred in large scale and long term planning.

6. Conclusions and Future Research

This dissertation focuses on multi-objective stochastic optimization problems, particularly the electricity Generation Expansion Planning (GEP) problem. Although multi-objective stochastic programming problems are frequently encountered in practice, there is not much research that focuses on solution methodologies incorporating the uncertainty of multi-objective problems. Current solution methods of multi-objective stochastic problem involve two kinds of transformations, namely the multi-objective transformation and the stochastic transformation, where either the multi-objective aspect or the stochastic aspect of the problem is neglected or greatly simplified.

This research presents a new uncertainty metric, the Pareto Uncertainty Index (PUI), and a new multi-objective stochastic genetic algorithm, the Pareto Uncertainty Genetic Algorithm (PUGA), which incorporates the PUI, to solve multi-objective stochastic optimization problems without any transformation of the problem. These new methods also provide the opportunity to the decision maker to observe the uncertainty of solutions. The PUI can also be used in single objective optimization methods or other multi-objective evolutionary algorithms; however, this study does not include these prospects.

The Generation Expansion Planning (GEP) problem, being a multi-objective stochastic problem, may include many objectives such as minimization of cost, minimization of pollutant emissions, maximization of reliability and maximization of energy security and independence. The GEP problem already confronts several complicated economic, environmental and technological challenges and the impact of these challenges will only increase in the future. Inherent uncertainties in economic and technological development and climate change make it much harder to determine

planning decisions today. Thus, the GEP is one of the most difficult real life multi-objective stochastic optimization problems, and it is an exceptional problem domain for this study.

One of the main objectives of this dissertation is to provide a multi-objective stochastic optimization model for the power generation expansion planning of the Northeast region of the United States. The presented model (NEGEP) also defines stochastic coefficients and parameters of the problem. The stochastic parameters are classified according to their uncertainty type, i.e., aleatory and epistemic uncertainty. Input data and assumptions for the NEGEP model are also provided. The data of the problem, scenario modeling and assumptions are established with the help of various reports, literature reviews and expert judgments. Finally the NEGEP problem is solved to find Pareto solutions for cost and CO₂ emissions objectives. Results of NEGEP problem suggest that the extent of capacity addition is expected to be nuclear and especially renewable units when uncertainty is taken into consideration. Mitigating pollutant emissions also necessitates emissions-free electricity generation from nuclear or renewable units. Investing in clean and renewable energy technologies to cost effectively satisfy the electricity demand while maintaining the reliability of the network requires more technological advancement and also regulation and policy encouragement.

In the next two sections research contributions of this dissertation are summarized and future research opportunities are outlined.

6.1 Research Contribution

This research results in several distinct research contributions. For multi-objective stochastic problems, it is necessary to address the uncertainty and account for risk

attitudes while preserving multi-objective aspect and representing tradeoffs of conflicting objectives such as power production cost and environmental impacts in the GEP. Therefore, a new uncertainty metric, the Pareto Uncertainty Index (PUI), and a new multi-objective stochastic genetic algorithm, the Pareto Uncertain Genetic Algorithm (PUGA) are presented in this research. Among the two approaches, there are several research contributions of this research, some of them being related to the GEP problem domain and some to the single and multi-objective stochastic optimization processes. These research contributions are summarized as follows:

- Sources of uncertainty in the GEP problem are identified and classified into epistemic and aleatory uncertainty types. Effects of different types of uncertainty on the variables of the GEP model are also identified for optimization modeling purposes. These contributions are useful to model the uncertainty in the planning process. They also provide transparency in decision-making.
- The GEP problem is modeled and solved as a multi-objective stochastic problem. This allows a more realistic representation of the problem and the results of the optimization. The need for transforming the problem is eliminated.
- The concept of Pareto optimality is extended from a purely deterministic framework to a more realistic stochastic framework.
- A new uncertainty index, the PUI, is presented which can be incorporated in problem solving methods to solve single or multi-objective optimization problems without ignoring uncertainty.

- A new multi-objective stochastic genetic algorithm, PUGA, is presented to find Pareto optimal solutions of multi-objective stochastic optimization problems with the ability of observing uncertainty information of solutions.
- Risk measures are incorporated in the PUI and PUGA so that the risk preference of the decision maker can affect the solution process. This is important for both the GEP problem domain and the stochastic optimization processes.
- Post-Pareto analysis is also integrated in the optimization process. The probability information preserved in the PUI is utilized to make a preference decision between solutions on the same front using the risk preference of the decision maker. Thus, obtaining a smaller subset of the preferred solutions from a large Pareto optimal set and the evaluation and interpretation of the results can be accomplished within the optimization process.

6.2 Future Research

This research focuses on multi-objective stochastic problems, particularly the GEP problem, and results in several contributions but these research topics and opportunities are extensive.

Although most of the real life engineering and policy problems are inherently multi-objective and stochastic in practice, solution methodologies accounting for the uncertainty in multi-objective problems are still very few in numbers and they are in their developing phase. As a future research task, The PUI and PUGA presented in this dissertation, for example, can be investigated in different real life problems than the GEP and the test problems. They are also bound to be compared with other similar

methodologies. Even though the PUI is initially offered for multi-objective formulations it can also be applied to single objective methodologies.

There is already a large collection of research focused on the GEP problems. However, there are still areas to be investigated, especially in multi-objective and uncertainty modeling of the problem. For instance, this dissertation incorporates cost and emissions objectives and uncertainties in economic and technological development and climate change effects, however, reliability and resiliency aspects are not examined fully. Especially the climate change effects on electricity grid resiliency and the uncertainty arises from this issue are very important subjects to investigate.

Uncertainty classification and representation are fairly well studied in the literature but their application in modeling, especially in multi-objective optimization is rare. This dissertation, for the first time, attempted to classify the GEP uncertainties into aleatory and epistemic uncertainty types and utilized their representation in the optimization model. Yet, the representation and modeling can be improved by employing possibility and probability distributions that represent the uncertainty better for problem parameters. Especially for the NEGEP model there is a need to further investigate the uncertainty of parameters and their mathematical modeling. Furthermore, the practical aspect of the model can be improved by working with decision or policy makers and incorporating their input and modifying the model for their interest.

Appendix I

Table 55. Generation technologies

	Generation Type	Description	
Existing	CC	Combined Cycle - Natural Gas	Nonrenewable
	Coal	Steam Turbine - Coal	
	CT	Combustion Turbine - Natural Gas or Oil	
	NU	Nuclear	
	STOG	Steam Turbine - Oil/Gas	
	STWD	Steam Turbine - Wood	
	GEO	Geothermal	Renewable
	HY	Hydro - Conventional	
	LFG	Landfill Gas	
	PS	Hydro - Pumped Storage	
	PV	Solar - Photovoltaic	
	ST	Solar - Solar Thermal/Solar Power	
	WT	Wind Turbine onshore	
	New	WT_on3	
WT_on4		Wind Turbine (onshore class 4+ wind)	
WT_off		Wind Turbine offshore	
Biomass		Biomass	
IGCC		Integrated Gasification Combined Cycle	
IGCC_seq		IGCC with carbon capture/sequestration	
AC		Advanced or Pulverized Coal	

WT_on3 and WT_on4 are both onshore wind turbine technologies but have different target wind resources (depending on the wind power, wind can be divided into different classes, class 3 and class 4+ wind are considered in this study), they are only distinguished for new generation units.

Table 56. Technical characteristics of all generation technologies

Type	Outage rate	Expected emissions rates (lbs/MWh)			Standard Deviation of emissions rates		
		SO ₂	NO _x	CO ₂	SO ₂	NO _x	CO ₂
CC	6.1%	0.1	1.7	1,135	0.04	0.71	475.64
Coal	6.5%	13	6	2,249	1.43	0.66	247.42
CT	9%	0.66	2.9	1,565	0.28	1.22	655.84
GEO	13%	0	0	0	0.00	0.00	0.00
HY	4.9%	0	0	0	0.00	0.00	0.00
LFG	5%	0.8	5.4	2,988	0.09	0.59	328.72
NU	3.2%	0	0	0	0.00	0.00	0.00
PS	4%	0	0	0	0.00	0.00	0.00
PV	60%	0	0	0	0.00	0.00	0.00
ST	1%	0	0	0	0.00	0.00	0.00
STOG	6.7%	3	2.4	1,325	0.33	0.26	145.77
STWD	10%	3	4	1562	0.33	0.44	171.84
WT	0%	0	0	0	0.00	0.00	0.00
WT_on3	0%	0	0	0	0.00	0.00	0.00
WT_on4	0%	0	0	0	0.00	0.00	0.00
WT_off	0%	0	0	0	0.00	0.00	0.00
IGCC	8%	0.13	0.4	1,540	0.05	0.17	645.36
IGCC_seq	8%	0.13	0.4	154	0.05	0.17	64.54
AC	6%	0.13	1.6	1,540	0.03	0.42	407.07
Biomass	7.5%	28.6	11	0	3.15	1.21	0.00

Table 57. Cost characteristics of all generation technologies

Type	Expected Investment cost 2010\$/kW	Standard Deviation of Investment cost	Expected Fixed O&M Cost (2010\$/kW)		Standard Deviation of Fixed O&M Cost		Expected Variable cost 2010\$/MWh	Standard Deviation of Variable cost
			Existing	New	Existing	New		
CC	1,035	57.50	29.68	14.39	3.30	1.60	47.45	3.95
Coal	-	-	48.22	-	19.03	-	28.63	14.60
CT	711	39.50	-	6.7	0.74	0.74	78.43	6.54
GEO	4,163	231.28	89.76	84.27	4.68	4.68	0	0
HY	-	-	14.24	-	0.79	-	0	0
LFG	2,525	238.24	120.65	120.33	11.35	11.35	0	0
NU	5,615	1,334.33	112.77	88.75	42.54	33.48	12.06	1.01
PS	-	-	23.74	-	1.32	-	5.98	0.50
PV	4,777	1,239.92	14.66	16.7	10.08	11.48	0	0
ST	4,714	1,223.57	60.32	64	41.46	43.99	0	0
STOG	-	-	37.15	-	4.13	-	58.82	4.90
STWD	-	-	32.05	-	3.56	-	78.43	6.54
WT	-	-	34.22	28.07	14.69	12.05	0	0
WT_on3	2,460	584.02	34.22	28.07	14.69	12.05	0	0
WT_on4	2,460	584.02	34.22	28.07	14.69	12.05	0	0
WT_off	5,997	1,423.73	-	53.33	-	22.89	0	0
IGCC	3,262	181.22	-	48.9	-	5.43	44.12	3.68
IGCC_seq	5,389	299.39	-	69.3	-	7.70	53.04	4.42
AC	2,885	581.95	-	29.67	-	9.03	30.1	16.71
Biomass	3,901	368.06	-	100.5	-	9.48	41.47	3.91

Table 58. Existing generation capacity at the beginning of the planning horizon (MW)

Type	NEISO	NYISO_A-F	NYISO_G-I	NYISO_J-K	PJM_E	PJM_ROM	PJM_ROR
CC	11,463	3,594	1,157	3,658	7,649	3,986	10,542
Coal	2,570	2,252	369		3,853	16,381	59,868
CT	2,384	260	152	4,948	6,899	3,555	21,073
HY	1,933	4,395	32		258	1,236	1,604
LFG	532	166	64	124	462	338	482
NU	4,645	3,197	2,045		8,472	5,036	20,000
PS	1,674	1,412			400	1,513	3,081
PV	2				22	4	24
STOG	6,236	1,701	2,431	6,799	3,252	4,109	2,122
STWD	609	86				70	194
WT	202	1,283			10	731	2,597

Table 59. Climate change effects on climate parameters

Climate Change Level	Temperature	Precipitation		Extreme events frequency
	Annual	Annual	Summer	
High	+4.4°C	+12%	-4%	+300%
Medium	+1.7°C	+4%	+1%	+75%

Table 60. Climate change effects on GEP parameters

		Climate Change Level	
		High	Medium
Additional demand growth	Summer	+0.24%	+0.092%
	Shoulder (spring/fall)	+0.19%	+0.075%
	Winter	-0.2%	-0.078%
Additional peak demand growth		+0.56%	+0.175%
Additional reserve margin requirement		+0.6%	+0.15%
Additional maintenance time		+1.2%	+0.3%
Additional transmission loss		+0.31%	+0.096%
Additional summer derate factor decrease	Coal, STOG, STWD, AC	-0.209%	-
	CT	-0.257%	-
	CC, IGCC, IGCC_seq	-0.099%	-
	GEO, LFG, Biomass	-0.249%	-
	PV	-0.057%	-
	NU	-0.284%	-
	ST	-0.297%	-
Additional shoulder (spring/fall) and winter derate factor decrease	Coal, STOG, STWD, AC	-0.009%	-0.003%
	CT	-0.057%	-0.022%
	CC, IGCC, IGCC_seq	-0.035%	-0.014%
	GEO, LFG, Biomass	-0.009%	-0.003%
	NU	-0.044%	-0.017%
	PV	-0.057%	-0.022%
	ST	-0.057%	-0.022%

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