EXPLORING IN-SERVICE TEACHERS' RECOGNITION OF STUDENT REASONING IN A SEMESTER-LONG GRADUATE COURSE

BY

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ABSTRACT OF THE DISSERTATION

Exploring In-service Teachers' Recognition of Student Reasoning in a Semester-Long Graduate Course

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In 1989, the National Research Council recommended "a shift from teaching routine procedures to developing mathematical reasoning". Since that time, professional development programs (Santagata 2009; Jacobs, Lamb, & Phillip 2010, Bell, Wilson, Higgins & McCoach, 2010) have attempted to increase teachers' knowledge for recognizing student reasoning and supporting students in their developing mathematical reasoning. As evidenced by international comparisons (PISA 2008) the shift to developing mathematical reasoning has yet to occur in the USA on a large scale. The purpose of this study is to trace teachers' developing knowledge in a graduate mathematics-education course that has a goal that the participating teachers grow in their recognition of student reasoning. The research questions guiding the study are:

1. What forms of reasoning do middle-school mathematics teachers identify from the following:
    a. Their own solutions to a series of mathematical tasks during a PD intervention;
    b. Their students' solutions to the same mathematical tasks implemented in their own classrooms;
    c. Students' solutions working on the same or similar mathematical tasks from assigned VMC videos, and
    d. Teachers' pre and post-test responses concerning the forms of reasoning used by fourth grade students to solve mathematical tasks in the Gang of Four VMC video?

2. What changes, if any, can be identified in teachers' beliefs about learning or teaching mathematics?
3. What pedagogical moves or strategies are used by the instructor to facilitate the teachers' construction of knowledge about mathematical reasoning as the teachers:
    a. Work on tasks in a combinatorics strand
    b. Study student reasoning from video, and
    c. Analyze samples of their own students' written work the tasks?

Data for the study include video data of teachers working on mathematics tasks and sharing samples of student work, text of teachers' discussions in an online forum, and final projects consisting of samples of students' collected written work, with teachers' reflections about the student work, the tasks in general, and the teacher's own assessment of their implementation of the tasks in each of three cycles. Video data of course meetings were transcribed and verified. All data were coded using Dedoose, and coded data were analyzed to identify patterns and trends relative to the research questions.

Findings suggest that videos provided formative experiences for teachers. Solution arguments based on inductive reasoning and reasoning by cases were frequently referred to by teachers using the names of the students in videos who made the arguments. Findings also suggest that the course instructor effectively modeled the type of teacher interactions necessary to engage teachers in justifying solutions to mathematical tasks and developing convincing arguments.

This study adds to a body of research involving professional development initiatives that attempt to help teachers attend to student reasoning. Its findings may be of value to designers of professional development initiatives, particularly those that seek to improve teachers' recognition of the variety of forms of reasoning their students employ.

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1 STATEMENT OF THE PROBLEM

People of many perspectives agree that reasoning is a valuable, perhaps essential, skill that all students need opportunities to develop. In 1989, the National Research Council recommended "a shift from teaching routine procedures to developing mathematical reasoning". Since that time, others have continued to claim that students need to develop reasoning skills. Citing ample research on the skill of mathematical reasoning, the NCTM Principles and Standards for School Mathematics (2000) included reasoning and proof as one of its 5 process standards. In their Research Companion to the Principles and Standards of School Mathematics, a challenge for teachers is presented:

> An important task for educators is to develop meaningful ways to help students make the transition to formal proof from their early experiences with reasoning, explaining, and justifying
>
> (Yackel & Hanna in Kilpatrick, Martin & Schifter, 2003, p. 234)

More recently, The Partnership for 21st Century Skills has included reasoning and critical thinking as components in its list of "Learning and Innovation Skills," skills that they claim are essential for success in the 21st century (P21, 2009).

As a result of these and other documents, there has been a greater focus on reasoning in state and national curriculum documents. In 1996, the state of New Jersey required that all students develop mathematical reasoning abilities (NJDOE, 1996). Reasoning was also included on the 2004 edition of the New Jersey Core Curriculum Content Standards, under section 4.5 "Mathematical Processes." The Common Core State Standards Initiative includes reasoning in its list of standards for mathematical practice (CCSSI, 2010). There are eight standards of mathematical practice. Of these eight standards, three refer specifically to reasoning: (2) reason abstractly and quantitatively; (3) construct viable arguments and critique the reasoning of others; and (8) look for and express regularity in repeated reasoning. The standards of mathematical practice were designed to align with leading research in mathematics education. References to the NCTM process standards and NRC documents are included in the description of these standards of mathematical practice.

Reasoning is described as an essential skill on many mathematical standards documents, but among educators, there is some confusion in regards to what is meant by "reasoning" in the classroom. As Wilhelm (2014) notes, many teachers implement cognitively demanding reasoning tasks, but reduce the cognitive demands through lowered expectations or increased scaffolding. In many cases, teachers are not well prepared to assess and interpret the reasoning of the children in their classroom (Bell, Wilson, Higgins & McCoach, 2010). The common core standards of mathematical practice include descriptions and some examples, but these are not sufficient to provide teachers with a deep understanding of what reasoning in the mathematics classroom looks like, or how to foster students' development of mathematical reasoning.

In New Jersey, researchers at Rutgers have been conducting studies of children's mathematical learning and ways of reasoning. For example, in a longitudinal case study that is currently in its 27th year, a cohort of students was given challenging mathematics tasks and ample time to work, reason, interact, and reflect. From this study and other cross sectional studies, over 4500 hours of video data have been recorded, and many dissertations and research publications have resulted from analyzing an array of data that is both vast in scope and deep in detail. Several of these publications have expanded the mathematics education community's understanding of student reasoning. (Maher & Martino, 1996; Sran, 2009; Maher, 2005; Maher & Yankelewitz, 2010; Muter, 1999; Maher & Muter, 2010; Mueller, 2007; Yankelewitz, 2009; Maher, Powell, & Uptegrove, 2010)

Two NSF grant funded initiatives, New Jersey Partnership for Excellence in Middle School Mathematics [1](NJ PEMSM) and the Video Mosaic Collaborative[2] (VMC)(Award DRL-0822204) make research about student reasoning accessible to in-service teachers. In particular, one of the courses in the NJ PEMSM sequence, "Lesson Study on Student Reasoning" used classroom practice, video data from the longitudinal study (made accessible to teachers through the VMC) and several scholarly articles to enhance practicing teachers' attention to and understanding of student reasoning. This course grew out of similarly structured professional development interventions. These interventions were held in different school districts in New Jersey for several years (Maher, Landis & Palius, 2010). The intervention model evolved over the years, and in the NJ PEMSM intervention was run as a semester long graduate-level course for practicing teachers. An analysis of data from many of the interventions indicates that the course prepares teachers to better attend to the forms of student reasoning that occur as they and the students they study seek to solve, explain and justify their solutions to the mathematical tasks (Maher, Palius, Maher, Hmelo-Silver, & Sigley, 2014). In this study, one implementation of this course will be described in detail to trace the effect it had on participants' recognition of student reasoning and beliefs about teaching and learning as measured by certain pre and post assessments and an analysis of teacher behaviors. In particular, the following research questions guide the present study:

1. What forms of reasoning do middle-school mathematics teachers identify from the following:
    a. Their own solutions to a series of mathematical tasks during a PD intervention;
    b. Their students' solutions to the same mathematical tasks implemented in their own classrooms;

[1] The New Jersey Partnership for Excellence in Middle School Mathematics is a research and development project sponsored by the National Science Foundation (Award DUE-0934079), directed by A. Cohen, Rutgers University. We gratefully acknowledge the support from the National Science Foundation and note that the views expressed in this paper are those of the author and not necessarily those of the NSF.
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   c. Students' solutions working on the same or similar mathematical tasks from assigned VMC videos, and
   d. Teachers' pre and post-test responses concerning the forms of reasoning used by fourth grade students to solve mathematical tasks in the Gang of Four VMC video?
2. What changes, if any, can be identified in teachers' beliefs about learning or teaching mathematics?
3. What pedagogical moves or strategies are used by the instructor to facilitate the teachers' construction of knowledge about mathematical reasoning as the teachers:
   a. Work on tasks in a combinatorics strand
   b. Study student reasoning from video, and
   c. Analyze samples of their own students' written work the tasks?

The exploration of these research questions yields results that may be beneficial to future professional development initiatives. A case study format is used to describe this intervention in order to gain a better understanding of the growth, if any, of teachers' attending to student reasoning and their beliefs about learning and teaching. Selected events in the course are analyzed, and interpretations regarding the effect of these events on the participants' beliefs are offered. Based on the findings, it is anticipated that instructors can make better informed choices about the tasks and videos to include in their interventions.

2 LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.1 Introduction

This chapter is organized into two sections. The first is a review of the literature regarding mathematics education reforms, professional development interventions, and the longitudinal study conducted at Rutgers. The second is a theoretical framework that situates the current study within research regarding mathematical reasoning and professional development.

2.2 Literature Review

In this section, international comparisons of mathematics classrooms and student achievement are briefly described and related to reform efforts in mathematics education. Subsequently, several professional development programs, which aim to address the concerns raised by research and international comparisons, are described. Following that is a review of the research that was done as part of the longitudinal study at Rutgers. This research forms the foundation of the NJ PEMSM intervention, and is therefore relevant to the current study.

2.2.1 Mathematics Education

In an era of data driven educational reform, many sources suggest mathematics education in American schools could be improved. Numerous international tests (TIMSS 1995, 1999; PISA 2003) project lamentable performance about American students' achievement in mathematics compared with other countries. Several researchers have compared the typical teaching styles of American teachers to the teaching styles typical of other countries. For example, Liping Ma (1999) compared the instructional strategies of mathematics teachers in China with strategies of American mathematics teachers. She determined that the teachers in China had a “profound understanding of fundamental mathematics”

(PUFM) (p. 120) and were able to address mathematical content in a way that aligned with the students' prior mathematical understanding (p. 115). She also found that the American group studied had more formal mathematical training than their Chinese teacher counterparts, and yet did not have as much mathematical knowledge for teaching, and often taught mathematical concepts as rules to be followed and applied in specific contexts under specific conditions. Ma's findings make a case for providing teachers with the opportunity to attend specifically to the reasoning of their students, so that teachers may teach in a way that better aligns with their students' current reasoning.

Stigler and Hiebert (1999) compared mathematics instruction in America to mathematics instruction in Germany and Japan. Their data were gathered from the video data collected during the TIMSS (1995) study. They found that the mathematical content taught in Germany was much more complex than that in America. Comparing the United States to Japan, they determined that students in Japanese classrooms were given more opportunities to solve problems on their own and share their solution strategies with the class. In conclusion, they advocated for more teacher collaboration, and in particular, the lesson study approach, to draw out mathematical content in the context of problems that require the students to develop mathematical reasoning skills.

At the same time as these international comparisons and subsequent calls for reform have been made, American research in mathematics education has begun to attend to these issues. The research paradigms that underscored studies seem to divide the mathematics education scholars into two camps. As a result, debates that came to be known as the "Math Wars" ensued. Davis, et al. report that one camp relied on observable results from children's mathematical work, to make inferences that could be made from more rigorous testing and more explicit instruction (Davis, Maher, & Noddings, 1990). Davis, et al. were opponents of this behavioral view of mathematics learning. Since the 1970s, research examining the intricacies of children's thought processes became more pervasive. The idea was that mathematics instruction could be improved by working with a student's ways of thinking while he/she was engaged in mathematical problem solving. The difference in perspectives is complex and the movement will be examined in the paragraphs that follow.

Erlwanger (1972) drew attention to the flaws inherent in a personalized and purely behavioral mathematics instruction program. His series of interviews with a student named Benny revealed a stunning paradox. Benny was deemed a success according to the instructional program in which he participated. He worked hard, and was progressing rapidly through the program. However, conversations with Benny about the understanding behind his solutions revealed that he was convinced that mathematics was a set of arbitrary rules that someone had developed and had to be memorized. Because of this belief, Benny took each example with which he was presented and generalized it (often incorrectly) into a new rule. His rule-making efforts led him to believe that 2/10 was equal to 1.2 despite agreeing that 2/10 was less than one and 1.2 was greater than one.

In an interview with a student who had successfully completed a fifth-grade mathematics program, Maher and Alston (1989) identified a situation in which the student, Van Chu, was given a problem in which she was required to determine a fraction of some quantity already expressed in fraction form. Van Chu used pattern blocks to solve the problem successfully. She was also able to demonstrate her solution with a picture. But when the student was asked to express her solution in written mathematical symbols, she struggled. Earlier in her fifth grade school year, she had been taught how to add, subtract, multiply and divide fractions, but at the time of the interview she could not make the connection between the problem with which she was presented and the mathematical operations. After several failed attempts, she expressed the problem with division, and applied the division algorithm with an error to get the answer she knew she needed. This student was a good student, and was able to reason well, as evidenced by her successful completion of the problem when using pictures or pattern blocks. On the other hand, she had significant difficulties when attempting to apply a routine procedure to a practical problem.

These two studies give some insight into the difficulties students face when they are taught mathematical procedures devoid of meaning and are not given the opportunities to develop a deeper understanding of the mathematics. Both studies present rather problematic situations and as such, are an important contribution to the literature. On a more positive side, concurrent studies have demonstrated the success students have in solving problems when the students themselves construct meaning and build mathematical ideas. Several of these are described below.

In 1992, after studying the work of a student named Sandy, researchers Pirie and Kieren developed a model to study the process by which students reason through unfamiliar problems. Sandy's class had explored fractions of the ½ family. Pirie and Kieren asked him about fractions with other denominators, and based on Sandy's solutions to the problems they presented, Pirie and Kieren were able to encourage Sandy to use his understanding of fractions to develop a procedure for adding fractions with unlike denominators. This was a remarkable achievement, but the real benefit was that the researchers were able to trace Sandy's understanding using their model, and could explain the process by which Sandy developed his understanding of the situation. Their model proposes that students construct images or mental representations, and once those images become established as exemplars, students can identify relationships between those images. Once a student has identified those properties, or relationships, they can formalize those properties or relationships and test them in other settings. Pirie and Kieren's research provided evidence that it is possible for teachers to infer what students are thinking as they solve problems. Also, it provided a model that could be used by others to describe a student's growth in understanding while solving novel problems.

Subsequent to the publication of the Pirie and Kieren article, Robert B. Davis (1994), a proponent of discovery learning, encouraged teachers to focus on the reasoning and understanding of their students. He proposed that teachers should be concerned with discerning the mental representations their students use to represent their ideas in problem solving. Davis, working collaboratively with Carolyn Maher in a number of studies, gave examples of how students develop meaning using their own representations to solve and justify solutions to problems. A key component of these problems is an emphasis on sense making and reasoning. Several of their examples, in which students built sophisticated mathematical ideas as a result of the reasoning they used while working on the problem, are described below.

Maher and Davis (1996) report on the precursors to proof that a fourth grade student developed as a result of building mental representations of fractions through his work with Cuisenaire rods. Through communication with his classmates and the teacher, David had developed the idea that some numbers can be divided in half evenly, while others cannot. The student's development of the notion of even and odd is of value, but even more valuable is manner in which he expressed his ideas to his classmates. David used Cuisenaire rods to express the mathematical concept of upper and lower bounds. This technique of mathematical proof is expected of college level students, and is rarely explored in primary or secondary schools. To see a nine-year old student using this form of reasoning may lead some to assume he was exceptional. Certainly his reasoning was. However, the same task has been replicated with students in other classrooms who also support their solution using upper and lower bounds as a form of reasoning. For example, the same set of tasks was given to sixth grade students in an urban setting. These students developed, as an independent community of learners, the same strategies for justifying their responses to the tasks (Mueller, Yankelewitz & Maher, 2010). In fact, Francisco and Maher (2005) have shown that certain tasks tend to elicit specific forms of reasoning in not only students, but also their teachers.

Both Maher and Martino (1996) and Sran (2009) have studied the reasoning that elementary school students developed while working on a combinatorics problem. These students investigated the problem of how many towers of a specific height can be built when there are Unifix cube blocks of two colors to choose from. Both students answered the question for the case of 4 and 5-tall towers, but the two students used different forms of reasoning while working on the problems. As a result, the process by which each student developed an understanding of the general case of the towers problem was different. Both students did develop a sophisticated understanding of the problem, providing evidence that when students are required to use their reasoning skills, they can learn much without being explicitly taught solution methods.

Alston, Davis, Maher and Martino (1994) provide a similar example of student reasoning. Brandon, a fourth grader, had worked on the towers problem as described above. He also worked on a problem, which invited him to determine the number of pizzas that could be made if four different toppings were available. While explaining his solution to the pizza problem with a researcher, Brandon mentioned that it seemed similar to the tower problem. The researcher pursued this idea and asked Brandon to justify his conjecture. As a result of both the researcher's careful questioning and Brandon's own reasoning, he was able to understand why the solution to both problems was structurally the same. In so doing, Brandon was also able to recognize an isomorphic relationship between the two problems by showing a one to one correspondence between each of the towers he had built and each of the pizzas he listed. Greer and Harel (1998) compare Brandon's recognition of an isomorphism to the work of the great French mathematician, Poincaire. (p. 6) Brandon's example illustrates the kind of thinking students are capable of when provided the opportunity.

Steencken and Maher (2003) report on the reasoning students used when working on problems involving fractions. Prior to the intervention, the students had no formal instruction in school about operating with fractions and were invited to explore fraction ideas by building models with Cuisenaire rods. Through the careful questioning of teacher researchers, the students developed ideas about comparing fractions, equivalence, and ordering of fractions. The students, collaborating and reasoning from rod models, invented rules for fraction operations. Steencken (2001) and Schmeelk (2010) further document the work of this cohort of students as they made the transition from thinking of fractions as operators to thinking of fractions as numbers. Bulgar (2002) studied the students as they invented methods of dividing fractions. Yankelewitz (2009) identified the variety of forms of reasoning and argumentation that students used as they explored fraction concepts and used manipulatives to model number concepts. They provide evidence that, given the proper mathematical tasks, young students can develop "meaningfully constructed" (Maher & Yankelewitz, 2010) mathematical representations.

From these examples, it is clear that American students can, and do reason mathematically while solving rich problems in a wide variety of mathematical contexts. With the recent research that has been carried out and the corresponding abundant examples of student reasoning, it follows that we must respond to the claims asserted by Ma (1999) and Stigler and Hiebert (1999). An appropriate response must activate the benefits of the research in actual classrooms and support teachers in analyzing their students' reasoning on complex mathematics tasks, in a collaborative environment. In the professional literature, examples of such intervention programs are described. These examples provide information about successful professional development interventions, in addition to suggesting possibilities for future interventions. Several of these interventions are described below.

2.2.2 Professional Development and Mathematical Reasoning

Critical thinking and mathematical reasoning are skills that the NCTM recommends for all math teachers to promote in their classrooms. Mathematics education research suggests methods and teaching styles that can support students in developing this kind of reasoning. Professional developers are beginning to incorporate this knowledge in their work with teachers so that mathematical reasoning can be promoted in classrooms. In order to better understand the context of these interventions, it is important to understand some of the history of research in professional development. In the mid-1990s, a call to reform the professional development experience was issued. Lieberman (1995) summarizes some important ideas from this challenge:

> What everyone appears to want for students -- a wide array of learning opportunities that engage students in experiencing, creating, and solving real problems, using their own experiences, and working with others -- is for some reason denied to teachers when they are the learners.
>
> (np 1995)

Lieberman (1995) also describes some successful professional development programs which, since the time of her publication, have become even more pervasive. In fact, following her call to reform, articles have been published with the purpose of describing professional development programs so they can be evaluated or emulated. Three examples follow.

Santagata (2009) has worked extensively with teachers in "low performing" schools. Although she does not define "low performing," she reviewed the literature to characterize some of the views of the population of teachers with whom she has been working. In particular, there are four characteristics that frequently arise in the literature. The teachers:

1. Ascribe poor student performance to external factors such as the economic conditions, violence in the students' neighborhoods, or lack of parental reinforcement.
2. Underestimate their students' abilities, especially in regards to higher order thinking.
3. Are required to teach subjects for which they have not been properly trained.
4. Are forced, by federal and state legislation, to focus primarily on improving their students' scores on standardized tests.

She has reported on the results of her interventions in several articles (Santagata, 2009; Santagata, 2010; Santagata & Guarino, 2010) and has described (2009) the nature of her intervention in detail. The participants were given specific questions relating to instructional techniques and student understanding, in addition to videos of classroom lessons to examine. The participants then answered the questions, using computer software to cite specific instances in the videos as references for their responses to the questions. Subsequent to their individual work on the questions, the participants met as a single group and discussed both the videos and their responses to the questions.

Santagata (2009) provides some insight into aspects of the intervention that made it work well, but more useful for our purposes is her description of the difficulties that the participants experienced during the first year of the intervention. She describes three recurring characteristics of the difficulties:

1. Participants' lack of a deep understanding of the mathematical concepts.
2. Participants' inability to assess student understanding
3. Participants' inability to assess student reasoning to a higher degree than whether or not the student's response was the correct answer.

These three difficulties are all worth examining in the context of the Rutgers intervention, but the third difficulty is particularly worth examining, because the development of teachers' ability to attend and evaluate student reasoning is one of the stated goals of the Rutgers intervention. This being the case, the teachers in the study were required to complete pre and post assessments specifically designed to determine the degree to which they (the teachers) attend to and evaluate student reasoning. Maher, Palius, Maher, Hmelo-Silver and Sigley (2014) analyzed teachers' assessment data over many instances of the Rutgers intervention. They determined that there was an increase in participants' ability to identify student reasoning.

In addition to promoting a deeper analysis of the three difficulties mentioned above, Santagata proposes that similar studies be conducted in other schools. Santagata (2009) suggests that her framework for video-based professional development can be used to provide professional development to other teachers. Because the NJ PEMSM intervention uses video data, and requires participants to collaborate in a way similar to that described by Santagata (2009), the outcomes of the NJ PEMSM intervention can be compared to those described by Santagata. The analysis of the Rutgers NJ PEMSM intervention will add to the research on professional development for mathematics teachers.

Jacobs, Lamb and Phillip (2010) conducted a thorough analysis of teachers' varying abilities to notice children's mathematical thinking. Although their participants were teachers of K-4 mathematics, the results are telling enough to be worth discussing.

Jacobs presents the results of a study of a cross section of 131 teachers of mathematics. In their study of the teachers, Jacobs et al. considered three components of professional noticing of student mathematical thinking:

1. The attention given to the strategies students employ in solving a problem
2. Interpreting the nature of students' understanding as a result of examining their (the students') solutions.
3. The teachers' reactions, in terms of questions, guidance, or further mathematical tasks that are proposed in response to the solutions examined.

The teachers studied were separated into four groups:

1. Prospective teachers
2. Initial participants
3. Advancing participants
4. Emerging teacher leaders

All of the initial (group 2) and advancing participants (group 3), as well as the emerging teacher leaders (group 4) were teachers with at least 4 years of teaching experience. Teachers in these groups were all participants in a multi-year sustained professional development program, consisting of about 5 full-day workshops per year. The distinguishing feature among teachers in these groups was the amount of experience they had in this professional development program. Teachers in group 2 had not been previously involved in the professional development program, but were about to begin. Teachers in group 3 had been in the program for two years, and teachers in group 4 had been in the program for four years. The prospective teachers (group 1) were undergraduate students enrolled in their first mathematics education course.

Teachers completed activities that required them to (1) attend to children's problem solving strategies, (2) interpret those children's understanding of the mathematics, and (3) decide how to respond appropriately to the child. Teachers' responses to these three components of professional noticing were scored based on the teachers' ability to engage with the child's mathematical thinking. For each group of teachers, mean scores were calculated and compared across groups. In general, the study showed a positive correlation between the number of years in the professional development program and the accuracy and detail in the responses provided to the prompts.

In regards to the question of how well each group attended to the solution strategy of the students (component 1), the authors reported an increase between the means of groups 1 and 2, as well as in the means of groups 2 and 3. There was not a significant increase in the means of groups 3 and 4, which the authors described as a result of the fact that the scores for group 3 had closely approached the maximum score. As a result of this, the authors claimed that with two years of professional development, the participants were able to attend correctly to student reasoning.

Throughout all group pairings, the authors found an increase in evidence that teachers could engage in interpreting student reasoning (component 2).

Two important points arose from the data analysis for the teachers' engagement with children's mathematical thinking while deciding how to construct a hypothetical response to the student. The first was that the scores for groups 1 (preservice teachers) and 2 (initial participants in the intervention with at least four years of experience) were both low and were not significantly different. The authors determined that teaching experience alone was not enough to help teachers respond appropriately to the reasoning and misconceptions of students. Comparing group 2 to group 3 and comparing group 3 to group 4 both revealed an increase in teacher engagement with children's mathematical thinking, which demonstrated that, in the case of the professional development program in which teachers were enrolled, more years of professional development led to an increase in engagement with student's mathematical thinking when constructing a response for the student.

Jacobs et. al. (2010) acknowledged that all their results were based on the analysis of one specific professional development intervention. They suggested that further research be done in other professional development settings to determine the generalizability of the results. Based on their findings, Jacobs et. al. (2010) hypothesize that, among the three components of professional noticing that they have studied, attending to student reasoning strategies is most likely to be overlooked in professional development programs (p. 194). They suspect that most professional developers incorrectly assume adults can attend to student reasoning. Whether or not this is the case in general, the NJ PEMSM intervention does not make this assumption, and designed to provide teachers with the opportunity to attend to and engage with student reasoning. It will be well worth investigating both the nature of the NJ PEMSM intervention and the results of the participants' assessments. The current study of the NJ PEMSM intervention will add to this research base, expanding the literature on the subject of student reasoning in mathematics education.

Bell and her colleagues (2010) have conducted a similar study, which collected data from several instances of the "Developing Mathematical Ideas" (DMI) professional development program. The DMI programs are divided by content area. For example, one program consisting of several sessions will focus solely on geometry, while another will focus on number and operation content. Each DMI program is run by a facilitator, who follows a guidebook and presents the participants with video segments and facilitates a discussion. The participants have a workbook, which contains questions about the video segments and includes space for participant reflection.

The data for this study were collected from experimental and control groups at sites across the country which were identified as having been using the DMI program for several years. The experimental group consisted of participants in the DMI program. The control group consisted of teachers at the same sites who were not participants in the DMI program. Both groups were given pre and post assessments designed to assess their Mathematical Knowledge for Teaching, as well as their knowledge of content and students, and knowledge of content and teaching (Ball 2008). These assessments included both a multiple choice and a short answer portion.

As a result of the data analysis, the authors concluded that participants in the DMI program "demonstrated learning on questions that cover specialized content knowledge, knowledge of content and students and knowledge of content and teaching" (p. 503). The first of these three topics, "specialized content knowledge" was not assessed by Jacobs. The other two topics relate to the participant's ability to assess student understanding and determine appropriate reactions to a student based on an understanding of the student's understanding, both of which were assessed by Jacobs.

These three studies of professional development interventions have a common theme. All focus on the teachers' understanding of student reasoning. The current study will add to the growing body of literature describing professional development efforts that focus on supporting teachers in developing deeper understanding student reasoning, and using that understanding to construct responses (written or verbal) to student work.

2.2.3 Professional Development and Mathematical Discourse

One thing is certain: In order for teachers to understand and evaluate the reasoning of their students, the students must communicate that reasoning. Teachers play a vital role in the classroom, ensuring that students have the opportunity to share their reasoning, and assisting students in developing norms of communication. Chapin, O'Connor and Anderson (2003) described "Talk Moves" that support students' mathematical discourse. Herbel-Eisenmann, Steele, and Cirillo (2013) developed a framework for professional development that was built on the "Talk Moves" concept and focused on assisting secondary mathematics teachers in promoting mathematical discourse in their classrooms.

Herbel-Eisenmann, Steele, and Cirillo (2013) describe a set of "Teacher Discourse Moves" that support classroom discourse. The six teacher discourse moves can be separated into three categories. The first category includes the teacher discourse moves of "waiting" and "inviting student participation." These teacher discourse moves make possible the sharing of student ideas. The second category of teacher discourse moves includes "revoicing" and "probing a student's thinking" These teacher discourse moves involve the teacher interacting with students and modeling norms of mathematical discourse. The third category of teacher discourse moves includes "asking students to revoice" and "creating opportunities to engage with another's reasoning." These two teacher discourse moves create opportunities for students to interact with the ideas of their peers.

The professional development program developed by Herbel-Eisenmann, Steele, and Cirillo involves teachers identifying the teacher discourse moves in transcripts of actual classes. After the teacher discourse moves are identified and defined, the participant teachers record samples of their own teaching to whether and how any of the teacher discourse moves are used in their own classrooms. Then teachers are encouraged to develop lesson plans that include the intentional use of teacher discourse moves at various points in the lesson.

Other researchers have outlined a process by which mathematical discussions can be organized to meet the instructional goals of lessons in an age of standards-based accountability. Smith and Stein (2011) describe "Practices for Orchestrating Productive Mathematical Discussions." These practices are different from the "Teacher Discourse Moves" in that the practices provide a framework for planning and delivering problem based lessons. The practices involve "anticipating" strategies that students may use to solve a problem, "monitoring" or checking in with groups of students to better understand their reasoning, "selecting" samples of work to be discussed, "sequencing" the solutions to emphasize those strategies most aligned with the lesson goal, and "connecting" the strategies by asking students to compare strategies. The practices of "anticipating," "selecting," and "sequencing" are largely accomplished by the teacher, but the practices of "monitoring" and "connecting" provide opportunities for the teacher to use the discourse moves defined by "Herbel-Eisenmann, Steele, and Cirillo (2013)

2.2.4 The longitudinal Study

In New Jersey, researchers at Rutgers have been conducting a longitudinal case study that was funded by the National Science Foundation[3]. The Longitudinal study is currently in its 27th year, and follows a cohort of students who were given challenging mathematics tasks and ample time to work, reason, interact, and reflect. Several of the publications based on this data have expanded the mathematics education community's understanding of student reasoning. For example, Maher and Martino (1996) examined the development of mathematical reasoning strategies of one of the students, Stephanie, over the course of two years during which time she developed an argument by contradiction and an argument by cases. They examined her solutions to the "Towers Problem" and clarified the process by which she developed an "elegant" proof of her solution to the problem. For those unfamiliar with the problem, the "Towers Problem" presents the students with two different colored sets of unifix cubes and invites them to find the number of distinct towers of a specific height that could be made using blocks available from the two sets. Stephanie organized the towers by the number of blocks of a given color that they contained. A similar analysis was more recently conducted that followed the work

[3] The longitudinal study is a research and development project sponsored by the following National Science Foundation awards MDR-9053597, directed by Robert B. Davis, Rutgers University, REC-9814846, directed by Carolyn A. Maher, Rutgers University, REC-0309062, directed by Carolyn A. Maher, Rutgers University, and DRL-0723475, directed by Carolyn A. Maher, Rutgers University. We gratefully acknowledge the support from the National Science Foundation and note that the views expressed in this paper are those of the author and not necessarily those of the NSF.

of another student, Milin, who was working on the same problem and justified his solution by an inductive argument. Milin was able to show that the number of towers of a given height (say n) was always twice as much as the number of towers of the next smaller height (say n-1). He was able to demonstrate this idea to his classmates, showing how the towers of height 3 could be built from the set of towers of height 2. Several of his classmates took ownership of the idea and were able to show how the set of towers 4 tall could be built from the set of towers 3 tall. Sran's (2009) analysis demonstrated the different kinds of reasoning that students utilize when solving the same problem.

Video data collected from the longitudinal study includes a conversation in which Stephanie, Milin, and two other students discussed solutions to the towers problem. This piece of video data is known as the "Gang of Four" video. In it, both Stephanie and Milin make arguments for the completeness of their solutions tor towers of height three. Because this video contains a variety of forms of reasoning, it is used in the pre and post assessments that teachers in the NJ PEMSM intervention took. The Gang of Four video and other video data from the longitudinal study is accessible through the Video Mosaic Collaborative (VMC).

Several other tasks from this longitudinal study have also been analyzed. Maher and Yankelewitz (2010) examined the work of the same cohort in second and third grade as they worked on a problem that required them to systematically list possible combinations of articles of clothing. The reasoning strategies that the students developed in working on this task served as the foundation upon which they developed such strategies as "fixing a variable" while they considered cases in counting problems. Muter (1999) studied some of these students in high school as they worked on a challenge extending the towers task, which was posed by one of the students in the group[4]. In response to the challenge, the students used the same reasoning strategies of fixing variables and considering different cases of the

[4] This problem is referred to as Ankur's Challenge. The statement of the task is included in Appendix B.

solution. These students were able to communicate their mathematical ideas effectively, and shared different strategies for solving the problem. (Maher & Muter, 2010)

2.3 Theoretical Framework

2.3.1 Mathematical Reasoning

Historically, reasoning in mathematics has been highly connected to the construct of a mathematical proof. The role of reasoning in mathematical proof has been analyzed by many researchers and theoreticians. Hersh (1994) describes two of the roles of proof, convincing and explaining. These roles make different requirements of the reader. In the former, the reader takes a critical role and in the latter, the reader merely attempts to understand what is being explained. Oner, (2009) describes how proofs may aid in generalizing and verifying mathematical explorations. Others support this view (e.g., Bell, 1976; DeVillers, 1990).

The distinction between mathematical reasoning as explaining and mathematical reasoning as providing a convincing argument may be artificial. As Weber (2010) notes, many mathematicians use mathematical proofs to gain a deeper insight into a particular mathematical construct. The data from the longitudinal study supports Weber's point. Take for example David's work[5] on determining which numbers could not be divided evenly by two (Maher & Davis, 1996). This argument by upper and lower bounds served the dual purposes of explaining a process to peers, and providing a convincing argument that the number associated with a Cuisenaire rod was not divisible by 2.

Von Glasersfeld (1990) notes that, since the development of a constructivist view on mathematics education in the late 1980s and early 1990s, researchers have come to observe and value reasoning as the process by which a student can learn. His thesis is that it is through the building up and

[5] David's work was described in section 2.2.1

connecting of ideas that new knowledge is constructed. Studies of the longitudinal data by Maher and Martino (1996), Sran (2009), and Reynolds (2005) support this view. Maher and Martino (1996) documented Stephanie's development of mathematical reasoning over two years as she worked on the towers problem. Sran (2009) analyzed Millin's process of developing and refining an inductive argument as a solution to the towers problem. Reynolds (2005) explored fourth-grade students' development of conjectures regarding odd and even numbers, the nature of fractions (as opposed to counting numbers), and the relative size of different fraction models. In each of these studies, children came to a deeper mathematical awareness, not by being given information from a teacher, but by working with mathematical ideas and building concrete models to express and support solutions, communicating justifications to others and testing any mathematical conjectures they may have posed.

Maher and Martino (1996) described Stephanie's development of proof-like reasoning as she worked on the tower problem in grades 3 through 5. Maher and Martino described three distinct organizational structures of this student's work.

1. Unorganized Work- In grade 3, the student initially used a "guess-and-check" strategy to find all towers, and did not describe a systematic or organized method for generating towers.

2. Local Organizations- On the following day, a researcher asked the student to justify whether she had found all the towers. In constructing a justification, the student described pairs of towers as being "opposites" or "cousins". These patterns allowed the student to group towers together, and provided a method for generating certain towers, but it did not help the student justify that all towers had been built.

3. Global Organizations- In grade 4, after recognizing the inefficiency of using patterns such as opposites and cousins, the student developed a form of "proof by cases" of her solution to the towers problem.

This solution organized all the towers by the number of white cubes in each tower. For each number of white cubes, the student was able to demonstrate that all possible towers had been built.

In the three examples above, it is worth noting that the student did not initially organize her solution, but began to consider organizations in order to justify, or explain why her solution was complete. Maher and Martino (2000) noted that students do not naturally build justifications (or generalizations), but are often satisfied proposing a solution to a problem. They describe the role of the teacher as estimating a student's progress in the development of mathematical reasoning and asking questions that encourage a student to refine or extend his or her reasoning. Maher and Martino describe questions teachers can ask that encourage students to construct mathematical justifications, or generalize their thinking to explain mathematical connections between classes of problems. (p. 57) The work of Maher and Martino highlights the interactions required to support students in developing mathematical reasoning and proof-like arguments. Two essential components to these interactions are the teacher's ability to interpret his or her students' reasoning, and the asking of questions that prompt students to justify the completeness of their solution.

2.3.2 Expectations of Professional Development

Characteristics of quality professional development programs have been documented. The literature in both mathematics education and professional development suggests a set of guidelines outlined below that characterize quality professional development for mathematics educators.

An effective program (hereafter referred to as an intervention) should have a wide timespan (Battey & Franke 2008; Bell et al., 2010; Franke et al., 2001). Franke et al. describe an intervention that lasted three years. The benefit to having such a long-term intervention was that even four years after the intervention, several of the participants still made use of the understandings they developed as a result of the intervention. The intervention studied by Bell et al. (2010) included participants who were involved in the intervention for four years. In several areas of assessment, these teachers exhibited significant gains over their peers who were involved in the program for two years. Battey and Franke (2008) determined that time was an important factor in affording participants the opportunity to adjust their professional identities. Teachers need time with new ideas to come to terms with them and develop ownership. Once ownership is established, the ideas are more likely to be applied in the classroom.

Also, it is recommended, that facilitators (teacher educators) should model the instructional strategies they expect the participants (practicing teachers) to use in the classroom (Lieberman, 1995; Maher et al., 2010). Lieberman describes the contradiction in which an educational consultant stands at a podium and tells an audience of seated teachers that they need to use more interactive instructional methods. Maher et al. (2010) proposed that a critical aspect of their program was that the teachers were involved in completing the same mathematical tasks they were expected to implement in their classroom. In order for this to happen, the facilitator could model the teacher's role as the participants become learners. As a result, the participants directly experience the teaching approach that has been modeled. Thus it can be replicated by the teachers in their own classrooms. Putnam and Borko (2000) made similar claims about the value of requiring teachers to engage with the mathematical tasks, as if they were students, prior to the implementation of the mathematical tasks in their classrooms. They claim such activities help to situate the knowledge of the participants within the context of their own classrooms, which in turn improves implementation fidelity. According to Ball, Thames, and Phelps, (2008) the participants' engagement with the mathematics tasks they are to implement in their classrooms further develops their Mathematical Knowledge for Teaching (MKT), a form of Knowledge Schulman (1986) considered invaluable in educators.

Another recommendation is that there be ample opportunities for participant reflection and discussion (Battey & Franke, 2008; Bell et al., 2010; Franke et al., 2001; Lieberman, 1995; Maher et al., 2010). In examining identities, Battey and Franke claim that the discussion among participants in which thoughts, feedback and concerns with the material are shared can greatly assist the incorporation of new ideas into the participant's identity as a teacher. Franke and colleagues (2001) determined that participant discussion of mathematical education ideas was one of the factors that determined whether or not the participants continued to grow as a result of the intervention in which they had participated. Lieberman (1995) described a few initiatives in which teachers in one school or district participated as a cohort in professional development programs. One such program was the Primary Language Record, which assists teachers in recognizing key aspects of literacy. The Foxfire Network was one of the other programs. In it, the participating teachers met to practice and discuss new instructional techniques. She proposed that the professional discussions that arose in these settings were of great value and suggested that through these discussions, new ideas would be shared and creative solutions to problems would arise.

The semester–long course in the NJ PEMSM intervention meets several of the criteria for effective professional development. Because the course is also part of an advanced degree program, it can be considered part of a professional development program with a wider timespan. The fact that the NJ PEMSM intervention meets these criteria makes it a good candidate for study.

3 DESIGN OF THE STUDY

3.1 Background

The NJ PEMSM intervention aims to share some of the findings from the Longitudinal Study with preservice and in-service teachers. The intervention uses publications, as well as video data of students working on and discussing mathematics problems. The videos used in the intervention were collected from the Rutgers longitudinal study funded by the National Science Foundation (Awards MDR-9053597, REC-9814846, REC-0309062, and DRL-0723475). The videos have been compiled, edited, transcribed and made available over the world wide web though the Video Mosaic Repository, a Rutgers initiative funded in part by the National Science Foundation (Award DLR-0822204).

The Video Mosaic Repository collection contains video data in other mathematical domains: algebra, geometry, fractions, probability, and pre-calculus. For example, fraction task also elicit valuable examples of student reasoning. Steencken (2001), Reynolds (2005), Mueller (2007), Yankelewitz (2009), and Schmeelk (2010) examined the reasoning of students as they developed ideas about fractions through a series of explorations involving Cuisenaire rods. Muller studied sixth graders in an after school program in an urban setting, while the others studied fourth graders in a suburban/rural setting. All found that students made reasonable claims about fractions through the use of the Cuisenaire rods. Students were found to argue by direct reasoning as well as presenting cases, induction, contradiction and upper-lower bounds.

The course studied in this dissertation, funded in part by the NJ PEMSM project, makes use of tasks, video data from cross sectional and longitudinal studies ,and scholarly articles to enhance practicing teachers' attention to and understanding of student reasoning. This course grew out of an intervention begun in a New Jersey school district (Maher, Landis & Palius, 2010). The intervention model has developed over the years, and in its current form is a one semester graduate-level course.

There are similarities between the Rutgers model and the UC Assessment Project described by Putnam and Borko (2000). The Rutgers model is composed of four stages, three of which are included in the UC Assessment Project. In Stage 1, participating teachers work on the mathematical tasks that they will be implementing in their own classrooms. The rationale for this is that the teachers develop an understanding of the underlying mathematical concepts (thus enhancing their subject matter knowledge) while also gaining first-hand experience with mathematical reasoning. In Stage 2, the teachers view, analyze and discuss videos of children working on the same or similar mathematical tasks. It is this second stage that differentiates the Rutgers model from the one described by Putnam and Borko (2000). The videos shown in this Stage 2 emphasize the degree of sophistication in reasoning ability that students of many ages and from a variety of backgrounds are capable of demonstrating. The purpose of this stage is to develop the attention that the teachers give to the students' mathematical reasoning. In Stage 3 of the intervention, the teachers implement the mathematical tasks in their own classrooms. Stage 4 of the intervention is the teachers' discussion of their experiences during Stage 3. Stages 3 and 4 together serve to provide teachers with the opportunity to recognize, share and discuss the forms of reasoning that were elicited during the students' work on the mathematical task (Maher et al. 2010). The intervention being studied includes 3 cycles, each of which introduces one or more mathematical tasks and contains all the stages described above.

3.2 Definitions

The following definitions will be used to describe people and work used in this study:

Teachers- The teachers enrolled in the fall 2010 course, Lesson Study on Student Reasoning

Instructor- The teacher educator who facilitates discussions, assigns tasks, readings, and videos.

Current Students- Students of the teachers enrolled in the course.

Research Students- Students whose problem solving can be observed from video or whose work can be studied through assigned readings.

Intervention- The section of the course "Lesson Study on Student Reasoning" being studied. The intervention makes use of combinatorics tasks in each cycle. The statements of each task are available in Appendix B. For reference purposes, the structure of the intervention is described by the table below:

Cycle 1: 4-tall towers problem, selecting from 2 colors Predictions for 3-tall and 5-tall towers	Stage 1: Teachers work on tasks.
	Stage 2: Teachers observe video or read literature involving research students working on the tasks.
	Stage 3: Teachers implement the tasks in their classrooms with their students.
	Stage 4: Teachers discuss samples of student work from Stage 3.
Cycle 2: Pizza problem, selecting from 4 toppings 5-tall towers problem, selecting from 2 colors	Stage 1: Teachers work on tasks.
	Stage 2: Teachers observe video or read literature involving research students working on the tasks.
	Stage 3: Teachers implement the tasks in their classrooms with their students.
	Stage 4: Teachers discuss samples of student work from Stage 3.
Cycle 2: 3-tall towers problem, selecting from 3 colors Ankur's Challenge (4-tall towers problem, selecting from 3 colors: Each tower must contain all three colors)	Stage 1: Teachers work on tasks.
	Stage 2: Teachers observe video or read literature involving research students working on the tasks.
	Stage 3: Teachers implement the tasks in their classrooms with their students.
	Stage 4: Teachers discuss samples of student work from Stage 3.

Table 3.1 Structure of the intervention

3.3 Lesson Study on Student Reasoning

In this section, the course "Lesson Study on Student Reasoning" is described. The section includes a description of the expectations for each stage in the cycles, a description of the tasks implemented in each cycle, and a timeline of events in the cycle.

3.3.1 Tasks

As previously noted, the intervention is composed of three cycles. Each cycle consists of the teachers working on a set of mathematical tasks, reading and discussing research and scholarly articles about the tasks, watching video(s) of students working on the mathematical tasks, implementing the task(s) in their own classrooms, and discussing the problem solving of their own students in their written descriptions of solutions. In each cycle, all the work focuses on a particular set of mathematical tasks. Those tasks and the cycles in which they are used appear below.

In cycle 1, the teachers worked on a series of counting problems that required building towers of varying heights with unifix cubes ™ available in 2 colors. They began with towers 4-tall, selecting from two colors. The cycle included extensions involving predicting solutions to the 5-tall and 3-tall towers problems. In cycle 2, the teachers worked on the pizza problem, choosing from 4 toppings. The teachers also worked on the 5-tall towers problem, selecting from 2 colors. In cycle 3, the teachers worked on the 3-tall towers problem, selecting from 3 colors. The cycle included determining the number of 4-tall towers that can be made when selecting from 3 colors with the requirement that each tower contains at least one block of each color, referred to as Ankur's Challenge since the 10th grader, Ankur, created the task and posed it to his group The statements of each task used in this intervention are included in Appendix B.

3.3.2 Expectations by Stage

Teachers' work in the intervention was distributed across meetings (both on-campus at Rutgers, and at the teachers' schools), an online discussion forum (eCollege), and the teachers' classrooms. The type of work expected of teachers varied by the stage of the intervention as well as by medium of the work. Expectations for each stage are described below.

Teachers' work in stage 1 of each cycle took place at meetings. The first of these meetings was on campus at Rutgers, and subsequent meetings were held at a school in Old Bridge. In stage 1, teachers worked in groups to complete the tasks for the given cycle. Teachers were expected to work together, share their thinking with their peers, and agree on a solution to the task. Teachers were then required to develop a convincing argument for the completeness of their solution, present it to the other groups, and discuss the arguments of each group.

The Bulk of teachers' work in stage 2 occurred in the online discussion forum. Each week of the intervention, up to the week of November 18, teachers were required to read research literature and/or view video describing research students' work on the tasks. After reviewing the research, teachers were presented with discussion questions relating to the research. Each week, teachers were expected to construct at least one original response and comment on the responses of at least two other teachers.

Teachers' work in stage 3 occurred in the teachers classrooms. Teachers were expected to implement the tasks for a given cycle in at least one of their classes, reflect on the implementation, and select samples of student work to share with the other teachers in the intervention.

Teacher's work in stage 4 was occurred in meetings as well as in the online discussion forum. Discussion questions in the forum were posted after teachers implemented the tasks in their classrooms. At the beginning of each at-school meeting, teachers shared samples of student work they found notable. For each sample presented, teachers were expected to describe the reasoning the student used to develop a solution to the problem.

3.3.3 Timeline

Events in the intervention took place in on-campus meetings, regional meetings, the teachers' classrooms, and the online discussion forum. Teachers met for two on-campus meetings, one at the beginning of the intervention, and one at the end. These meetings included teachers from all four sections of the course "Lesson Study on Student Reasoning". Teachers in each section met for smaller reginal meetings. There were three of these meetings in the intervention. Teachers' week-by-week work was done in online discussions. The table below indicates the organization of activities in the intervention. The full syllabus for the intervention is available in Appendix A.

Date	Cycle/Stage	Activity
Prior to 9/11/2010	NA	Complete Reasoning pre-assessment Complete beliefs inventory pre-assessment
9/11/2010	Cycle 1 Stage 1	First on-campus meeting Complete the 4-Tall towers problem selecting from 2 colors
9/16/2010 to 9/30/2010	Cycle 1 Stage 2, Cycle 1 Stage 3	Participate in weekly online discussions Describe research student work Implement the Cycle 1 tasks in classrooms Discuss implementation of Cycle 1 tasks
10/07/2010	Cycle 1 Stage 4 Cycle 2 Stage 1	First regional meeting Teachers share current student work on Cycle 1 tasks Teachers work on cycle 2 tasks
10/14/2010 to 10/22/2010	Cycle 2 Stage 2, Cycle 2 Stage 3	Participate in weekly online discussions Describe research student work Implement the Cycle 2 tasks in classrooms Discuss implementation of Cycle 2 tasks
10/28/2010	Cycle 2 Stage 4 Cycle 3 Stage 1	Second regional meeting Teachers share current student work on Cycle 2 tasks Teachers work on cycle 3 tasks
11/04/2010 to 11/11/2010	Cycle 3 Stage 2, Cycle 3 Stage 3	Participate in weekly online discussions Describe research student work Implement the Cycle 3 tasks in classrooms Discuss implementation of Cycle 3 tasks
11/18/2010	Cycle 3 Stage 4	Third regional meeting Teachers share current student work on Cycle 3 tasks

		Teachers review guidelines for final projects
12/04/2010	NA	Final on-campus meeting Teacher focus group interviews Teachers hand in final projects Teachers in all sections of the course discuss their experiences

Table 3.2 Course Timeline

3.4 Subjects

There are seven teachers in the intervention. Three teach in Old Bridge and have hour-long mathematics classes. Four teach in Sayreville and have 40-minute mathematics classes. Old Bridge and Sayreville are both suburbs of New York City, located near South Amboy, NJ. According to 2010 census data, the racial makeup of Old Bridge is 74% White, 6% Black or African American, 14% Asian. Latinos of any race make up 11% of the population. The median household income in Old Bridge is $82,640. According to 2010 census data the racial makeup of Sayreville is 67% White, 11% Black or African American, 16% Asian. Latinos of any race make up 12% of the population. The median household income in Sayreville is $71,808. See table below for information about the district and grade taught by each teacher.

Name	District	Grade
Mitch	Old Bridge	8
Kate	Old Bridge	7
Sally	Old Bridge	7
Angela	Sayreville	7
Connie	Sayreville	6
Rich	Sayreville	6
Justin	Sayreville	Special Education 6-8

Table 3.3 Participating Teachers

3.5 Data Sources

The intervention draws on nine sources of data.

1. These are the problem solutions of the teachers, as identified in the transcripts included in Appendix D.
2. These are the selected problem solutions of the students of the teachers, produced when the teachers implemented the tasks in their classrooms. These selected problem solutions are identified in the transcripts included in Appendix D. Additional problem solutions are included in teachers' final projects, available in Appendix C.
3. These data are teachers' reactions to representative student solutions
4. These data are teachers' written reflection on their own learning throughout the span of the intervention. The teachers' final projects containing these reflections are included in Appendix C.
5. Three times during the course, the teachers met in person after school. Videos of these after school meetings have been recorded. These videos include the teachers' group work on the combinatorics problems that they will subsequently use in their classrooms, the teachers' discussion of their current students' reasoning as the students worked on the tasks the teachers had completed at their previous meeting, and the instructor's interactions with the teachers during both the problem-solving sessions and the discussions of the implementation of the tasks in their own classrooms. These video recorded observations constitute another source of data.
6. An online discussion board for the course provides another source of data. These discussion threads provide information about the teachers' reactions to both the assigned videos and the assigned readings. In addition, they provide insight into the teachers' reactions to the

work of their students. This source of data will form a complement to the video data. The ideas that arise from the discussion threads may have been the result of more reflection and introspection than the ideas that arose in the spur of the moment in the group meeting sessions.

7. Another source is a transcript of an interview with the instructor. The interview made use of a general interview guide (Patton, 1992) which served as an interview protocol (Creswell, 2007) and allowed the instructor freedom in responses and topics. (The interview protocol and a transcript of the interview are included in Appendix E). The instructor has been implementing interventions based on the Rutgers model for more than three years, so a general guide proved more beneficial than a standardized interview. The freedom to follow up on some of the instructors' responses proved quite beneficial.
8. This data source is a transcript of a focus group interview with the teachers. The focus group interview was held at the final meeting of the teachers on December 4.The interview was held prior to the discussion portion of the on-campus meeting in which teachers from all four sections of the course discussed their overall impressions of their experiences in the course. (The focus group interview transcript is included in appendix D).
9. Pre- and post-assessment data of (a) recognition of student reasoning from the Gang of Four video and (b) teacher beliefs about learning and teaching constitute the source of data. The assessments were scored to trace changes, if any, prior to and after the intervention. The pre- and post-assessments were scored together in a double blind setting. In order to standardize the results across the different scorers, each assessment was scored by two different researchers. Inter-rater reliability was assessed and found to be 90% or greater.

These nine sources constitute the data collected for this study. These data were examined with a blend of the constructivist and systematic paradigms as defined by Creswell and Miller (2000). The details of this process are described below.

3.6 Data Collection

Data directly involving the teachers were collected throughout the intervention. The sections below describe the collection of video data, online discussions, and final projects.

3.6.1 Video Data

Four videos were recorded during this intervention. Three of the videos involve after school meetings, which took place on October 7, October 28 and November 18. One of the videos involves the focus group which took place on December 4. In all cases, the videos were videoed, transcribed and verified. Transcripts of all of these videos are available in appendix D.

3.6.2 Discussion Threads

During weeks two through ten of the intervention, the teachers were assigned discussion questions through the eCollege course site. Each week, the teachers were invited to write a response to the questions, and comment on the response of at least one other teacher. The discussion questions required teachers to elaborate on required reading, videos of student work, the implementation of tasks in the teachers' classrooms, or the teachers work to complete one of the tasks. After the intervention was completed, the discussions were downloaded from the eCollege site. A list of the discussion questions, and the discussion threads for each week are included in appendix E

3.6.3 Assignments

The teachers were assigned a final project, which was due at the December 4 meeting. The assignments required the teachers to reflect on the three task cycles and choose, for each cycle, three examples of student work; one that impressed the teacher, one that surprised the teacher, and one that concerned the teacher. The teachers' final projects are included in Appendix C.

3.7 Methods and Coding

For each research question in this study, a coding scheme was developed in collaboration with a team of researchers. Transcripts of video, records of online discussions, and participants' final projects were imported into Dedoose and coded using each coding scheme. The details of each coding scheme, including relevant definitions are described below.

3.7.1 Examining Reasoning

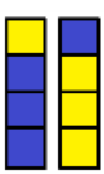
A key component of this study is the analysis of the forms of reasoning identified by teachers as they discuss solutions to combinatorics tasks. These solutions may come from a variety of sources: The teacher's own work (identified as "teacher"), work done by one or more students of a teacher (identified as "student"), work done by students in the longitudinal study and recorded as video (identified as "video"), or work done by students in the longitudinal study and recorded in research literature (identified as "literature"). For each solution discussed in the intervention, the teacher describing the solution as well as the source of the solution (teacher, student, video, literature) was recorded to allow for analysis across teachers and sources.

A coding scheme was developed to organize and describe the teachers' analysis of student reasoning. The coding scheme was developed in collaboration with other researchers who analyzed similar interventions. In this section, the coding scheme is described and related to the research upon which the scheme was developed.

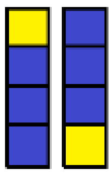
When presenting samples of student or teacher work, various characteristics of a given sample are described and analyzed as follows:

1. **Heuristic/ Strategy**: This characteristic describes the method by which the work was organized in building a solution. Codes for identifying types of strategies and heuristics based on this body of research were developed in collaboration with other researchers analyzing similar tasks making use of common heuristics and strategies used in solving combinatorics problems that have been identified from the research literature (Maher and Martino 1996, Maher, Sran, and Yankelewitz, 2011). Names for heuristics and strategies arose from students' work on the towers problems, but in some cases the strategies can be applied to pizzas as well. The heuristic or strategy used was recorded as fitting one of the following types:
   a. **Guess and Check**-The strategy of guess and check involves first guessing a solution then testing that the solution is correct. Students can be observed using the guess and check method when building towers or listing pizzas in a random order and then double-checking for duplicate towers or pizza toppings (Maher & Martino, 1996).

   b. **Opposites-** The opposite of a tower in two colors is a tower of the same height where each position holds the opposite color of the first tower. For example, a 4-tall tower with yellow, blue, blue, blue and one with blue, yellow, yellow, yellow are opposites. (Maher, Sran &Yankelewitz, 2011) This strategy can be applied to pizzas as well. For example two pizzas, one with peppers and pepperoni, and the other

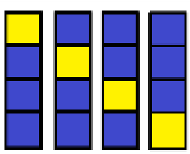

Example of Opposites

with sausage and mushrooms could be considered opposites because there is no topping shared by both pizzas, and all of the toppings that appear on one pizza do not appear in the other.



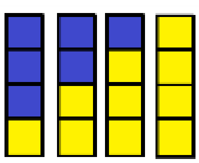
Example of Cousins

c. **Cousins-** Two towers are said to be cousins if one tower can be flipped to form the second tower. For example, a 4-tall tower with yellow, blue, blue, blue and a tower with blue, blue, blue, yellow are cousins (Maher & Martino, 1996)



Example of the elevator pattern

d. **Elevator-** The elevator pattern is used when finding all possible towers containing one cube of one color and the remaining cubes of the other color. The single colored cube is placed in the first position of the first tower. To create a second tower, the cube is then moved to the second position. The cube is continuously lowered one position to create new towers until it is placed in the final position (Maher, Sran &Yankelewitz, 2011). This strategy can appear in the pizza problem as well. For example, when a student lists all the pizzas with only one topping in some systematic fashion.



Example of the staircase pattern

e. **Staircase-** The staircase pattern is named as such due to its resemblance to a staircase. In towers of two colors, the first tower begins with the first three positions as the same color followed by the 2nd color in the last position. In each new tower, the number of cubes of the 2nd color increases from the bottom by one cube until the final tower is a solid tower of that color (Maher, Sran &Yankelewitz, 2011). This strategy can appear in the pizza problem, for example when a student starts with a one topping pizza, and successively adds toppings to identify new pizzas.

f. **Controlling for Variables-** Controlling for variables is a method in which one variable is held constant while adjusting another variable. An example of this when building towers is when one color of the tower is held constant in one position while the color arrangements in all other positions are varied (Maher & Martino, 1996).

g. **Other-** Any strategy or heuristic other than those previously defined.

2. **Representation** - This characteristic describes the format used to monitor progress or describe a solution. Maher (2011) lists some common representations (physical objects, words, and symbols) and describes how existing representations are elaborated upon or related to new representations. To analyze the development of representations in this intervention, representations used by students or teachers were recorded as fitting one of the following types:
    a. **Manipulatives**- Tangible objects used by students or teachers to help them solve the mathematical tasks. While the objects mostly used in the study included unifix cubes™, other tangible items may be used.
    b. **Drawings-** Pictures or diagrams used by students or teachers to help them solve the mathematical tasks. These may include tree diagrams.
    c. **Charts-** Any graphic form or table used to represent a student's or teacher's work.
    d. **Symbols-** Numbers, letters, or any other symbols (including written words) that are used to help students or teachers represent their work.
    e. **Gestures-** Using hands to indicate (with the intent to help represent a student's or teacher's work).
3. **Form of Argument**: This characteristic describes the structure of the argument used to justify that a solution set is complete accounting for all possible elements fitting the task criteria. Initial definitions of argument type were developed by Wright (2015, personal correspondence). The definitions were then discussed and evaluated by a team of

researchers (Maher, Wright, Cipriani, Krupnik, and McGowan). The form of argument was recorded as fitting one of the following types:

a. **Case Argument-** In a proof by cases, a statement is proved by proving all of the smaller subsets of statements that make up the whole. For example, the solution to the Four-tall Tower Task when selecting from two colors (i.e. blue and yellow) can be justified by separating the towers into cases using a characteristic of the tower. One such characteristic is the number of cubes of a specific color that the towers contain. In this situation, the towers can be broken down into 5 cases; (1) towers containing 0 yellow, (2) towers containing 1 yellow, (3) towers containing 2 yellow, (4) towers containing 3 yellow and (5) towers containing 4 yellow. A complete argument by cases would include justifications that (1) the cases describe the entire set of four-tall towers when selecting from two colors (2) all towers fitting each case have been identified and (3) no towers can be described by more than one of the cases.

b. Inductive Argument- In an inductive argument, the particular solution is considered to be an extension of an initial problem. To make an inductive argument,(1) an initial case is identified and a solution is presented. (2) The relationship between one case's solution and the subsequent case's solution is assumed to hold up to some arbitrary point. (3) It is demonstrated in a general way that the solution can be extended beyond the arbitrary point identified in step 2.

The general solution to the Towers Task,2^n where 2 represents the number of colors selected from and *n* represents the height of the tower, can be proved through an inductive argument.

The first step is to prove the result is true for a basis case (often *n = 0 or n =*

1). In the case of towers, we prove the basis case n=1 or towers of one cube in height. Since there are only two cubes from which to select, i.e. yellow or blue, there are only two towers that can be built. $2^1 = 2$. Thus, the justification is established for the case, n=1.

In the second step, an inductive hypothesis is made. The inductive hypothesis assumes the result of step 1 is true for *n=k*. Therefore, it is assumed that the total number of different towers of height *k* is 2^k. In the third step, this assumption is used to prove the next case (*n = k+1*). The total number of towers that are *k + 1* tall can be found by placing another cube on the top of each of the 2^k towers that are *k* tall. That additional cube can take on one of the two colors, e.g., yellow or blue. Therefore, for each of the existing 2^k towers, two new towers of height k+1 can be created; one with a yellow cube added to the top and one with a blue cube added to the top. Therefore, the total number of towers that can be created of height *k* + 1 is $2^k \cdot 2 = 2^k \cdot 2^1 = 2^{k+1}$. Thus, the argument is made for the case of *n = k+1*.

The provision of an induction argument coded in this research of the general solution 2^n includes the basis step (n=1) in which a teacher (or student) describes that the total number of 1-tall towers created when selecting from two colors is 2, i.e. one of only blue and one of only yellow. The second step is less formal but describes that the total number of towers of a given height can be found by placing either a yellow or blue cube on the top of all of the towers of the previous height, therefore doubling the total number of towers created in the previous height.

c. **Recursion-** Recursion is defined as an operation on one or more preceding elements according to a rule or formula involving a finite number of steps (Merriam-Webster, 2015). An example of recursive reasoning can be seen in one possible solution of the 4-topping Pizza with Halves problem. The total number of 4-topping combinations is 2^4 or 16, thus there are 16 different whole 4-topping pizzas (same topping(s) on each side). When determining the total number of 4-topping pizzas in which the two sides of the pizza are not the same, a recursive calculation can be used. First choose one topping on one side, i.e. plain, leaving 15 remaining toppings for the other side. Next choose a different topping for one side, i.e. pepperoni. Again there are 15 toppings for the remaining side but one would create a duplicate from the previous set, thus only 14 remaining toppings can be used. Choose a third topping for one side, i.e. peppers. Again there are 15 toppings for the remaining side but two would create a duplicate from the two previous sets, thus only 13 remaining toppings can be used and so on. Each new set of pizzas can be found by subtracting one from the previous set. The total number of different 4-topping pizzas that can be created is the sum of 1 through 16.

d. **Contradiction-** When a situation arises that is inconsistent or contrary to known or inherent facts, a contradiction has been reached. In the 4-tall Tower Problem, when selecting from two colors, (e.g., yellow and blue), a proof by contradiction can be used to prove the total number of towers that can be built in the case of exactly one yellow cube. The yellow cube can be placed in either first, second, third or fourth position. If other towers can be built with one yellow cube, the yellow cube would have to be in a different position, say, the fifth position. Placing a cube in the fifth position would require the tower to be a height of at least five. This is a contradiction of the requirement that the tower has a height four.

e. **Rule-** Features of a given task may be used to identify numbers and perform calculations leading to a solution. In that case, the work is justified with a procedure or "rule", which is a statement that relates the mathematical operations to features of the problem. For example, in the 4-tall towers problem, selecting from two colors, a student may incorrectly claim that $4^2 = 16$ makes sense as a solution because there are four blocks in each tower, and two colors to choose from.

4. **Teacher Evaluation:** In addition to recording the forms of reasoning identified by teachers as they progressed through the intervention, this study aims to identify which arguments (if any) were found convincing.
    a. **Convincing -** When a teacher made a claim that a particular argument was convincing, that argument was recorded as "convincing" for that teacher.
    b. **Not convincing -**When a teacher made a claim that a particular argument was not convincing, that argument was recorded as "not convincing" for that teacher. In some instances, the teacher provided a reason as to why the argument was not convincing. Instances in which the teacher claimed the argument was not convincing because it was incomplete will be coded as "incomplete." Instances in which the teacher claimed the argument was not convincing because it was not a valid argument will be coded as "invalid."
5. **Researcher Evaluation:** In order to gain a truer picture of each teacher's recognition of forms of reasoning, it was necessary to identify missed opportunities, or situations in which

teachers may have failed to recognize a particular form of reasoning. In order to identify these situations, the researcher evaluated each form of reasoning presented or discussed by the teachers. This evaluation was done using codes identical to those used in the "Teacher Evaluation" section- with one exception. The Researcher Evaluation includes an additional code "Undetailed Description" This code is applied to indicate situations in which there is not enough information about the particular argument to allow a code of "Convincing" or "Not Convincing" to be applied.

This coding scheme was developed by a team of researchers, each studying different course interventions, in collaboration with their advisor. The coding scheme that evolved was applied in three of the four stages (stages 1, 2, and 4) for each of the three cycles of the intervention.

In Stage 1, the coding scheme was used to identify teachers' reasoning as they worked on tasks. In Stage 2, it was used to record the forms of reasoning identified and described by teachers on the discussion boards as they reviewed research literature and videos of student working on tasks. In Stage 4, the coding structure was used to record the teacher's recognition and evaluation of student reasoning when samples of student work were presented for in-person discussion.

3.7.2 Examining Instructor Moves

This study also examines the pedagogical moves or strategies used by the instructor throughout the intervention to facilitate the teachers' construction of knowledge about mathematical reasoning cycles (research question 3). The instructor engaged with teachers during three of the four stages in the intervention. The instructor was present at the after school meetings in which teachers worked on strands of tasks (Stage 1) and discussed samples of their students' work (Stage 4). The instructor also maintained a presence on the online discussion as teachers engaged in postings attending to readings, videos of research students' reasoning, and their students' solutions to problems (Stage 2). Instructor pedagogical moves were identified by stage and cycle to allow for the analysis of patterns by phase, or general trends across the span of the intervention.

A coding scheme was developed to describe the type of pedagogical moves or strategies used by the instructor. The coding scheme was developed in collaboration with other researchers who analyzed parallel interventions. In this section, the coding scheme for instructor pedagogical moves is described, and related to the research which acted as a framework for the coding.

Codes for instructor moves were developed based on the work of Maher (2011), Maher and Martino (1999), Smith and Stein (2011), Marzano, (2011), and Herbel-Eisenmann et al. (2013). The codes were organized into two groups. One group of codes describes the representation used by the instructor to communicate ideas. The other group of codes describes a variety of forms of pedagogical practice.

The set of codes describing the representations used by the instructor is identical to the set of representational codes described in section 3.6.1 Examining Reasoning. Instructor representations were coded to identify the use of the following representations: (1) manipulatives, (2) drawings, including tree diagrams, (3) charts, (4) symbols, and (5) gestures.

3.7.2.1 Definitions

The set of codes describing the forms of instructor pedagogical practice were developed to identify practices promoted in professional development literature. These codes are defined below.

1. **Anticipating**: Predicting teachers' or students' behaviors or strategies while working on a mathematical task. (Smith & Stein, 2011)

2. **Monitoring**: Checking for teachers' understanding as they are working on the task. The instructor monitors to make decisions about which solutions or strategies to make public without direct interaction. (Smith & Stein, 2011)
3. **Selecting**: Choosing to share a particular teacher's work. (Smith & Stein, 2011)
4. **Sequencing**: Asking for teachers' work to be presented in a certain order as opposed to allowing teachers to choose the order of work shared. (Smith & Stein, 2011)
5. **Motivating**: Celebrating students' or teachers' work through praise or encouragement. Marzano (2011)
6. **Waiting**: Pausing to allow time for teachers to process and then respond to questions posed by the instructor or another teacher. (Herbel-Eisenmann, Steele, & Cirillo, 2013)
7. **Inviting**: Soliciting multiple solution strategies, often with the goal of "making diverse solutions available for public consideration" or "including multiple students in the discussion. (Herbel-Eisenmann et al., 2013, p. 183)
8. **Revoicing**: "Restating or rephrasing a teacher's contribution." (Herbel-Eisenmann et al., 2013, p. 183)

In addition to these codes characterizing actions, a set of codes identifying the types of questions the instructor posed was developed. Maher and Martino (1999) and Herbel-Eisenmann et al. (2013) described several purposes of teacher questioning. This work informed the set of codes identifying the types of questions that the instructor asked of the teachers. For the purposes of analysis, the question types described by Maher and Martino (1999) as well as those described by Herbel-Eisenmann et al. (2013) were reinterpreted to refer to instructor-to-teacher questions. For those familiar with the "Five practices for Orchestrating Productive Mathematics Discussions," (Smith & Stein 2011) it is worth noting that the practice of "Connecting" is covered within this set of questioning codes. The codes identifying the type of instructor questioning are defined below.

1. **Explanation**: Questions that invite a teacher or group of teachers to describe what they are doing or did. Explanation questions might be used while teachers are working on a task, in contrast to describing a completed task. (Maher and Martino, 1999)

2. **Justification**: Questions that elicit how the teachers are convinced that the solution is correct. (Maher and Martino, 1999)

3. **Generalization**: Questions that invite teachers to consider a similar problem with the goal of encouraging them to consider patterns that suggest a solution to the original problem. For example, by considering building towers of different heights, with different color choices, students can begin to consider how the height of a tower might be related to the number of color choices in finding the total number of towers that can be made. (Maher and Martino, 1999, p. 65)

4. **Connection**: Questions that invite teachers to consider whether they can identify similar problems, and if so, to describe similarities and/or differences. (Maher and Martino, 1999)

5. **Probing**: Questions that invite teachers "to elaborate on particular ideas." (Herbel-Eisenmann et al., 2013, p. 183) For the purposes of this study, "probing" will be distinguished from "inviting." "Probing" will refer to situations in which one particular teacher is invited to elaborate on his or her particular idea, whereas "inviting" will refer to situations in which the question is asked in a way to encourage many teachers to respond.

6. **Other Solution**: Questions that make public to other teachers various solutions. (Maher and Martino, 1999) For the purposes of this study, "Other Solutions" will be used to describe the

first time a particular solution is presented, but not for each time the solution is mentioned by the instructor.

9. **Summary:**

The coding scheme for analyzing instructor moves is based on research in mathematical discourse (Maher and Martino 1999; Smith & Stein 2011; Herbel-Eisenmann et al. 2013). This coding scheme was developed by a team of researchers, each studying different course interventions, in collaboration with their advisor. This coding scheme was used to gather information in three of the four stages of each cycle. In stage 1, it was used to identify the instructor's interaction with teachers as they worked on mathematical tasks. In stage 2, it was used to identify the instructor's interaction with teachers as they discussed samples of student work from the research literature. In stage 4, this coding structure was used to identify the instructor's interaction with teachers as they discussed examples of their students' work.

3.7.3 Examining Beliefs

It was also a goal of the study to monitor the stability of teachers' beliefs about learning and teaching mathematics (research question 2). Data regarding teacher beliefs were collected from a Beliefs Inventory Assessment, administered before and after the intervention and from teacher claims during the intervention. All of the data sources (videos of regional meetings, online discussions, final projects) were examined for knowledge about teacher beliefs. The methods for analyzing these data are described below.

3.7.3.1 Beliefs Inventory

Teachers completed a Belief Inventory prior to and at the completion of the intervention. The Inventory included 34 items, of which 22 were related to the intervention and linked with changes in teacher beliefs in analyses of the intervention model (Maher, Landis, and Palius 2010; Maher, Palius, and Mueller 2010). These were used to examine the stability of teacher beliefs over time. Some of the belief items were presented as statements consistent with current National Council of Teacher s of Mathematics (NCTM) Standards, while others were presented as statements inconsistent with those standards. In the list of questions below, the statements inconsistent with current standards are indicated with an asterisk.

Q1 - Learners generally understand more mathematics than their teachers or parents expect

Q2 - Teachers should make sure that students know the correct procedure for solving a problem

Q4 - It's helpful to encourage student-to-student talking during math activities.

*Q5 - Math is primarily about learning the procedures.

*Q6 - Students will get confused if you show them more than one way to solve a problem.

Q7 - All students are capable of working on complex math tasks.

Q9 - If students learn math concepts before they learn the procedures, they are more likely to understand the concepts.

*Q10 - Manipulatives should only be used with students who don't learn from the textbook.

*Q11 - Young children must master math facts before starting to solve problems.

*Q13 - Only really smart students are capable of working on complex math tasks.

Q15 - Learners generally have more flexible solution strategies than their teachers or parents expect.

*Q17 - Manipulatives cannot be used to justify a solution to a problem.

Q18 - Learners can solve problems in novel ways before being taught to solve such problems.

Q19 - Understanding math concepts is more powerful than memorizing procedures.

Q21 - If students learn math concepts before procedures, they are more likely to understand the procedures when they learn them.

*Q23 - Collaborative learning is effective only for those students who actually talk during group work.

Q24 - Students should be corrected by the teacher if their answers are incorrect.

Q28 - Learning a step-by-step approach is helpful for slow learners.

*Q29 - Only the most talented students can learn math with understanding.

*Q30 - The idea that students are responsible for their own learning does not work in practice.

Q31 - Teachers need to adjust math instruction to accommodate a range of student abilities.

*Q32 - Teacher questioning of students' solutions tends to undermine students' confidence.

Some of the questions refer to similar beliefs. For example, questions 10 and 17 relate to beliefs about the use of manipulatives in mathematics classes. For the purposes of analyzing beliefs, the questions were grouped into the following categories:

Expectations and Student Abilities: Q1, Q7, *Q13, *Q29

Mathematical Discourse: Q4, *Q23

Concepts and Procedures: Q2, *Q5, Q9, *Q11, Q18, Q19, Q21,

Manipulatives: *Q10, *Q17

Student and Teacher Roles: Q24, *Q30, *Q32

Differentiated Instruction: *Q6, Q15, Q28, Q31

Teachers completed the beliefs inventory assessments by rating each statement on a 5-point Likert scale. Teacher responses were recorded as "Consistent", "Inconsistent" or "Undecided" in relation to the educational standard described in each item. Teacher ratings of "3" (neutral) were coded as "Undecided". Teacher ratings expressing agreement with statements consistent with standards, as well as ratings expressing disagreement with statements inconsistent with standards were coded as

"Consistent". Teacher ratings expressing disagreement with statements consistent with standards, as well as ratings expressing agreement with statements inconsistent with standards were coded as "Inconsistent". The use of these codes allowed for the exploration of trends in teachers' beliefs relative to the standards expressed in the beliefs assessments.

3.7.3.2 Intervention Data

Codes were developed to relate teacher claims made during the intervention to each of the question categories described above. Additional codes identifying beliefs as pertaining more generally to the topics of learning and teaching mathematics were developed. Teacher statements may have been coded with question category codes as well as topic codes. Each belief statement was coded for its relationship to the NCTM Standards presented by the beliefs inventory assessments. Statements were coded as inconsistent, consistent, or undecided with the Standards. The criteria for establishing whether beliefs statements in each question category or topic are consistent or inconsistent with standards presented by the beliefs assessments are described below. Any statement in which teachers described a topic or question category, but not in a way that clearly aligned or conflicted with the Standards was coded as undecided.

Expectations and Student Abilities:

Statements indicating lower expectations for some learners, of that only some students are capable of mathematical success will be marked as inconsistent with standards.

Statements indicating beliefs that all students are capable of mathematical success will be marked as consistent with standards.

Mathematical Discourse:

Statements claiming that student mathematical discourse is not valuable, or that mathematical discourse is only valuable to students actively discussing the mathematics will be marked as inconsistent with standards.

Statements claiming that mathematical discourse is valuable for all students will be marked as consistent with standards.

Concepts and Procedures:

Statements claiming that mathematics is more about procedures than concepts will be marked as inconsistent with standards.

Statements claiming that concepts and procedures are both important in mathematics will be marked as consistent with standards.

Manipulatives:

Statements claiming that manipulatives have a limited value or are only useful for certain learners will be marked as inconsistent with standards.

Statements claiming that manipulatives are valuable for all learners, particularly as reasoning and communication tools, will be marked as consistent with standards.

Student and Teacher Roles:

Statements claiming that the teacher is the sole authority in the classroom will be marked as inconsistent with standards.

Statements claiming that students can have mathematical authority, particularly be making and supporting claims, will be marked as consistent with standards.

Differentiated Instruction:

Statements claiming that all students learn the same way, and that teachers do not need to accommodate a range of student abilities will be marked as inconsistent with standards.

Statements claiming that teachers do need to accommodate a range of student abilities will be marked as consistent with standards.

Learning:

Statements claiming that students learn mathematics through direct instruction as a set of rules or procedures will be marked as inconsistent with standards.

Statements claiming that students can take ownership of their learning, or that students can learn from their peers will be marked as consistent with standards.

Teaching:

Statements claiming that the teacher must be the authority in the classroom, or that the teacher should tell students how to solve problems before the students interact with those problems will be marked as inconsistent with standards.

Statements claiming that the teacher can assist students in sharing and refining mathematical ideas, without being the sole authority in the classroom will be marked as consistent with standards.

3.7.3.3 Summary

Data regarding teacher beliefs were collected from pre- and post-assessments, as well as from teacher statements during the intervention. The data from the intervention were coded as pertaining to learning or teaching mathematics. Additionally, the data were coded by category to allow for comparison to beliefs pre- and post-assessment items. Each statement was identified as consistent, inconsistent, or undecided in regard to the standards established by the Beliefs Inventory.

The data regarding teacher beliefs were analyzed by teacher as well as by cycle and phase. Patterns in question categories and trends in beliefs about the topics of learning and teaching mathematics are described. These data are then compared to the data from the pre and post Beliefs Inventory.

METHODS AND RESULTS

In this study, the analysis of the research questions is distributed across five chapters. Chapter four is a detailed description of the events of the intervention. Chapter five is an analysis of teacher recognition of student reasoning (research question 1). Chapter six is an analysis of instructor moves to facilitate teacher learning about student reasoning (research question 2). Chapter seven is an analysis of teacher stability of beliefs regarding learning and teaching mathematics (research question 3). Chapter eight presents narratives describing each teacher's experience in the intervention.

4 ANALYSIS OF EVENTS

In this chapter, the events of the intervention are described. For meetings, the tasks that teachers worked on, the teachers reasoning on the tasks, the samples of work that teachers discussed, and the teachers' evaluation of those work samples are described. In all meetings, the teachers worked on tasks in the same three groups. Rich, Connie and Angela are referred to as Group 1. Kate and Sally are referred to as Group 2. Justin and Mitch worked are referred to as Group 3. In the weekly discussions, participants were required to answer each of the posted questions, and respond to at least one post from a fellow teacher for each question. For weekly discussions, the sources of data referenced in the discussions are described and key points from the teachers' discussion are summarized.

4.1 First On-Campus Meeting

Prior to this meeting, teachers completed the beliefs inventory pre-assessment, as well as the reasoning pre-assessment. At this meeting, teachers met with their instructors to review the syllabus and course expectations. Teachers also worked on the 4-tall towers problem, selecting from 2 colors.

4.2 Week 2 Discussion (9/16)

In this discussion, teachers were asked to share reflections on their work on the 4-tall towers problem, which they had worked on at the September 11 on-campus meeting. Teachers were also asked to make predictions regarding how their students would approach the problem. Additionally, teachers were asked to read "Representations as Tools for Building Arguments" (Maher & Yankelewitz 2011), chapter 3 of Combinatorics and Reasoning (Maher, Powell, & Uptegrove, 2011) and compare the work of two students, Stephanie and Dana, working on the Shirts and Pants problem, first as second graders and then, five months later as third graders. The Shirts and Pants problem required the research students to identify the number of outfits that could be made from three different colored shirts and two different colored pants. Additionally, teachers were asked to view videos of third grade research students working on the 4-tall towers problem, selecting from two colors, as well as videos of the research students' predictions of the solution to the 3-tall towers problem, selecting from two colors.

In the discussion, teachers were asked to describe whether they were convinced by Stephanie and Dana's arguments in the 4-tall tower problem, and describe what insights they gained from listening to the girls' predictions for the number of 3-tall towers that could be built.

Angela, Connie, Justin, Kate, and Rich were impressed by Stephanie and Dana's justification (9/16 Discussion). They did not find it completely convincing and indicated that they expected more of their students who were middle school, but expressed that it was good reasoning for a third grader. All of the teachers described Milin's inductive argument from a video in which he was explaining his reasoning for building towers, 3 tall, selecting from 2 colors. When discussing strategies the teachers expected their students to use to solve the 4-tall tower problem, teachers did not expect students to use the strategy of finding opposites. One teacher anticipated that her students might use a guess and check strategy to construct towers.

4.3 Week 3 Discussion (9/23)

In preparation for the week 3 discussion, teachers were asked watch video of Stephanie and Dana as they worked on the 4-tall towers problem, selecting from two colors. In this video, Stephanie and Dana developed the notions of “families” and “cousins” and used these notions in addition to the concept of opposites to organize the towers in their solution. Teachers were also asked to read “Towers: Schemes, Strategies, and Arguments” (Maher, Sran, and Yankelewitz, 2011), chapter 4 of Combinatorics and Reasoning (Maher, Powell, & Uptegrove, 2011) which describes Stephanie's development of a cases argument, and Milin's development of an inductive argument, as solutions to the 4-tall towers problem, selecting from 2 colors. Stephanie's cases argument sorted the set of 3-tall towers into groups based on the number red and blue cubes in each tower. Milin claimed that two 1-tall towers can be made when selecting from two colors, and that there are two ways to construct towers of height $n+1$ from each tower of height n. Milin used this inductive argument to predict the number of 6-tall towers that can be made when selecting from two colors.

In the discussion, teachers were asked by the instructor to compare Stephanie and Dana's work on the 4-tall towers problem as fourth graders to their work on the same problem as third graders, and to decide which of their arguments were convincing. In these early weeks of the intervention, teachers attended to non-mathematical behaviors of students such as about the personality, behavior or presentation styles of the students in the video. “If I were Dana, I would want a new partner. Stephanie is overbearing!” (Kate 9/23 Discussion line 23) Four teachers made statements about Stephanie's personality.

Kate indicated “I did not find much difference in how they approached the problems from third to fourth grade.” (9/23 Discussion line 18) Connie found the grouping systems used by the students to be more sophisticated. (Families and Cousins, rather than opposites and guess and check) All teachers described components of Milin's inductive argument.

4.4 Week 4 Discussion (9/30)

In preparation for this discussion, teachers were asked to read "Building an Inductive Argument" (Maher, Sran, & Yankelewitz, 2010), chapter 5 of Combinatorics and Reasoning (Maher, Powell, & Uptegrove, 2011). Maher, Sran, and Yankelewitz (2010). This chapter describes how different students made sense of and took ownership of the inductive argument presented by Milin. Teachers were also asked to watch video of research student, Milin, as he shared his inductive reasoning with his classmates.

In this discussion, teachers were asked to reflect on their students' solutions to the 4-tall towers problem. They were also asked whether they found Milin's inductive argument to be convincing. Kate wished to know more about the longitudinal study and the events leading up to Milin's argument. Mitch indicated that he was expecting a more formal proof than Milin's argument, while Justin appreciated Milin's reasoning as describing a method that could be used by others to solve similar problems. Angela and Kate described Milin's argument in terms of the doubling pattern across towers and expressed a desire to have students work through a set of towers problems starting with 1-tall towers and progressing through towers of increasing height: "I feel like we set our students up to be confused because we started them with the four-tall towers and didn't allow them to use the blocks to see a connection between, 1, 2,3 and 5-tall towers" (Kate 9/30 Discussion line 185). "I completely agree with you about the connection from the 1-tall through the 5-tall towers" (Angela 9/30 Discussion Line 210).

4.5 10/7 Meeting

At the beginning of the meeting, teachers shared samples of their students' work on the 4-tall towers problem, and discussed the students' organization of the towers in their solutions. The instructor invited teachers to share samples of student work. For each sample shared, the instructor invited other teachers to identify the organization of the solution, before the teacher sharing the sample described it. This discussion of student work on the 4-tall problem marked the conclusion of cycle 1. After that, teachers began cycle 2, and worked in groups on the 5-tall towers problem, selecting from 2 colors and the pizza problem, selecting from 4 toppings. After working on the tasks, teachers shared their solutions. Samples of student work are shared below, followed by descriptions of the teachers' work on the tasks.

4.5.1 Discussion of Student Work

Kate shared the following sample of student work (10/7 Meeting transcript 1 of 3, line 204):

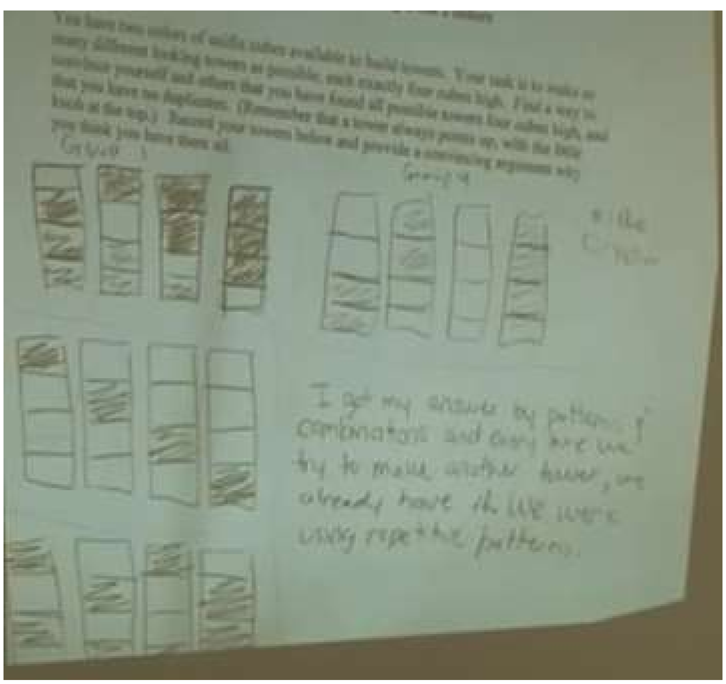


Figure 4.5.1.1 Kate's Cycle 1 student work sample 1

Teachers identified the student's use of the heuristics of organizing towers in patterns of opposites and elevators in tower groups 1 and 2. This cohort frequently used the word "staircases" to describe the elevator pattern. Teachers disagreed about the student's method of organizing groups 3 and 4. One teacher claimed that the first two towers in group 4 should be included in group 3. Another teacher hypothesized that group 3 included towers that had separated colors, and group 4 had colors that were kept together. No teachers made claims about whether they were convinced by this student's organization.

Kate also shared a sample of student work (10/7 Meeting transcript 1 of 3, line 299):

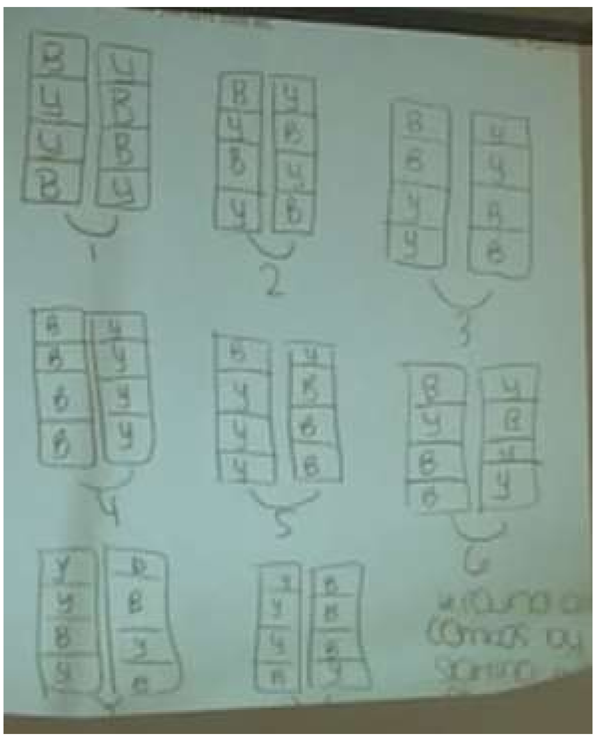


Figure 4.5.1.2 Kate's Cycle 1 Student work sample 2.

All teachers agreed that this student used the concept of opposites to find new towers. Kate shared that this student had a strong verbal argument for the completeness of her solution, but noted that the written argument was incomplete.

Mitch also shared this sample of student work (10/7 Meeting transcript 1 of 3, line 398):

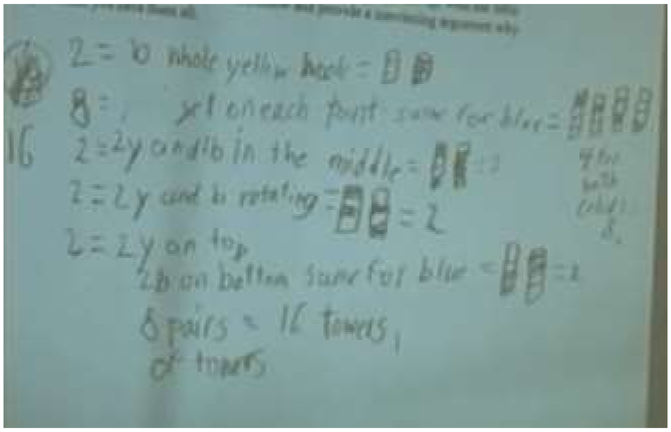


Figure 4.5.1.3 Mitch's Cycle 1 student work sample 1.

Teachers identified this sample of work as following a cases argument. They noted that the student considered towers of all one color, followed by towers with 3 blocks of one color and 1 block of the other color, organized in an elevator pattern. Participants noted that the final three groups described by the student described the case of 2 blocks of one color and 2 blocks of the other color. In discussing this student's work, Mitch acknowledged that he had difficulty getting the student to provide a written argument for the completeness of his solution. The instructor provided some suggestions for encouraging students to record their solutions.

Rich shared this sample of student work (10/7 Meeting transcript 1 of 3, line 493):

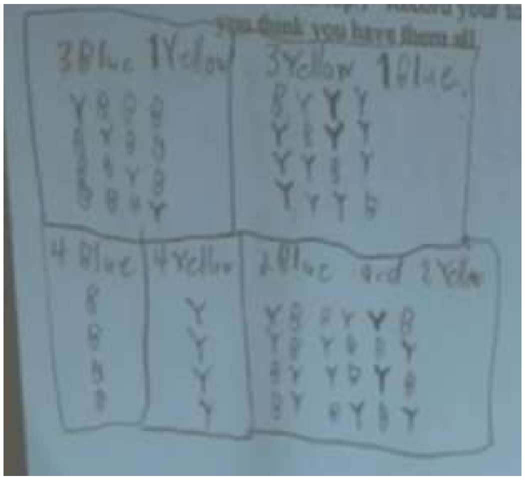


Figure 4.5.1.4 Rich's Cycle 1 student work sample 1

Connie identified this student's work as organized by cases describing the number of blocks of each color. She noted the elevator pattern in the 3 blue 1 yellow case, and in the 3 yellow 1 blue case. Connie also identified the student's use of opposite pairs in the 2 blue and 2 yellow case.

Angela shared this sample of student work (10/7 Meeting transcript 1 of 3, line 529):

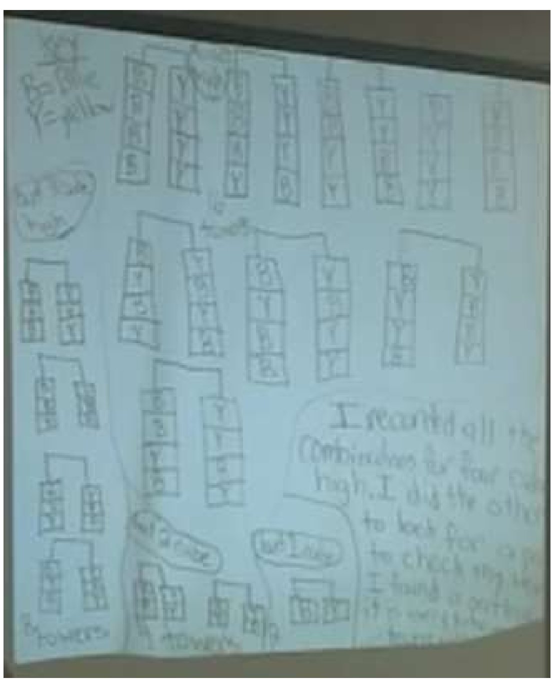


Figure 4.5.1.5 Angela's Cycle 1 student work sample 1

Rich compared this student's work to the form of reasoning used by Milin. He and other teachers noticed that the student showed towers of heights 1, 2, 3 and 4 in this sample. According to Angela, the student drew the sixteen 4-tall towers first, and constructed the towers of other heights as a way of checking his work. The instructor used this sample of work to discuss a concern expressed by some teachers, that the students should be required to construct towers of height 1, followed by height 2, and so on. The teacher noted that the student who produced this solution came up with the method as a justification of his solution to the 4-tall towers problem.

Justin shared this sample of student work (10/7 Meeting transcript 1 of 3, line 602):

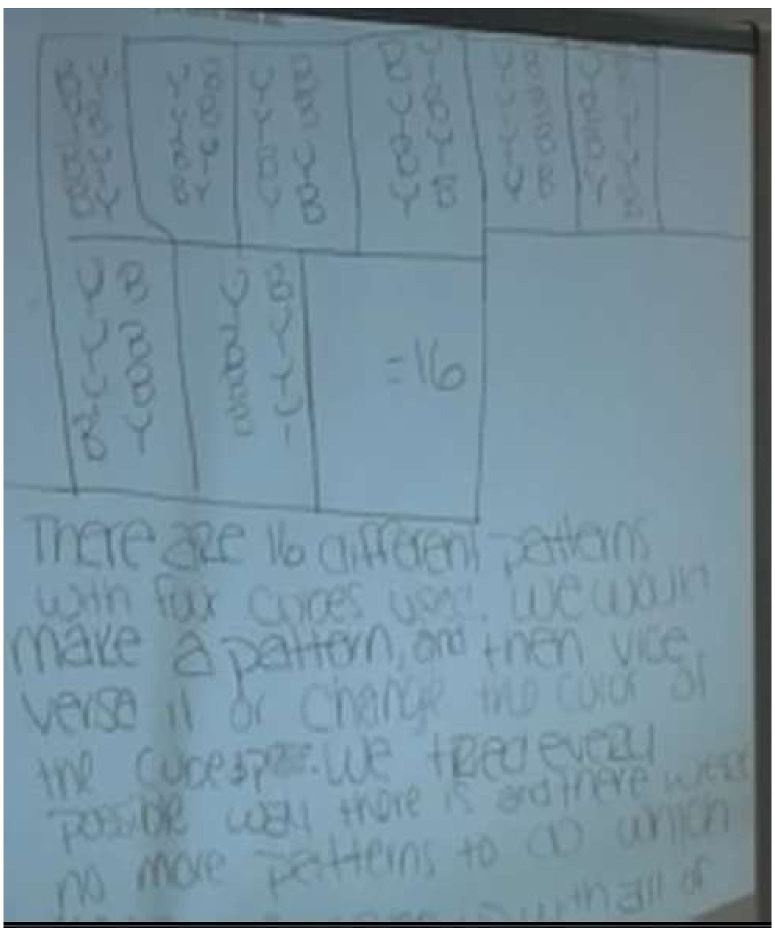


Figure 4.5.1.6 Justin's Cycle 1 student work sample 1

Teachers identified from Justin's work sample the student's use of opposites. Justin explained that the student developed her own language and referred to the action of constructing a tower's opposites as a "vice verse"-ing the tower. Justin also shared the work of this student's partner, in order to compare the notation each student used (10/7 Meeting transcript 1 of 3, line 639).

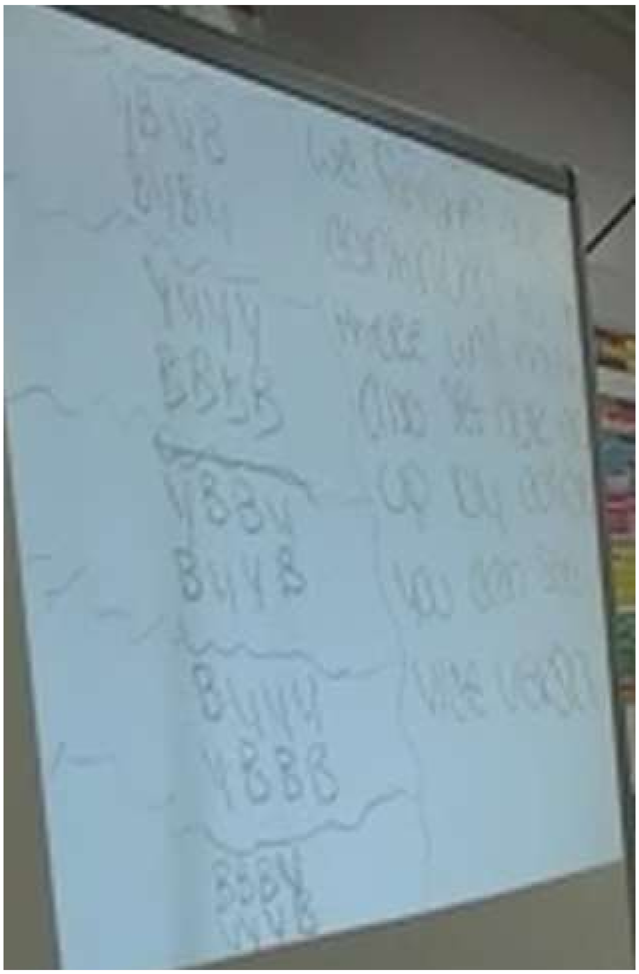


Figure 4.5.1.7 Justin's Cycle 1 student work sample 2

Teachers pointed out that both students used symbols to represent towers, rather than pictures of the actual towers. They also pointed out that one student used vertical strings of letters to represent each tower, and that the other student used horizontal strings of letters to represent each tower.

Connie shared this sample of student work (10/7 Meeting transcript 1 of 3, line 680):

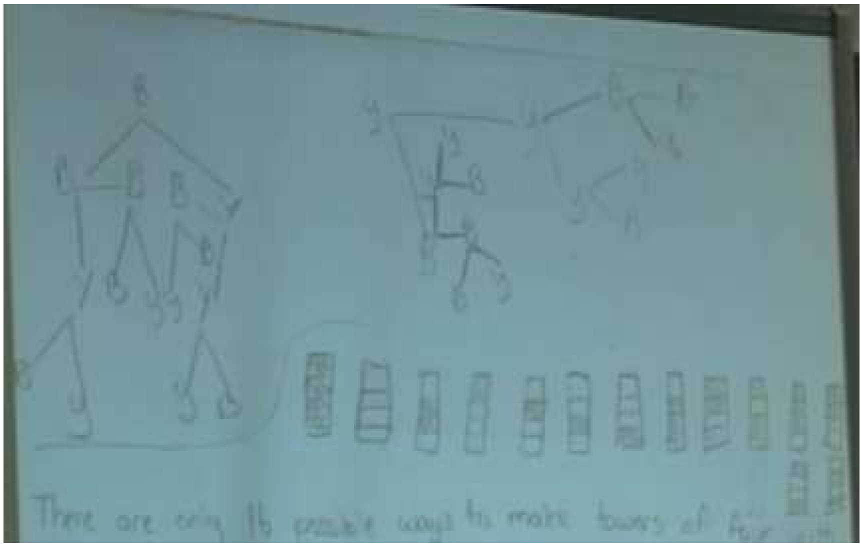


Figure 4.5.1.8 Connie's Cycle 1 student work sample 1

In the student work sample, the student claimed to have constructed all the towers she could, and then used the tree diagram as a way of checking her work. The student did not explicitly connect each path on the tree diagram with a tower.

Sally shared this example of student work (10/7 Meeting transcript 1 of 3, line 740):

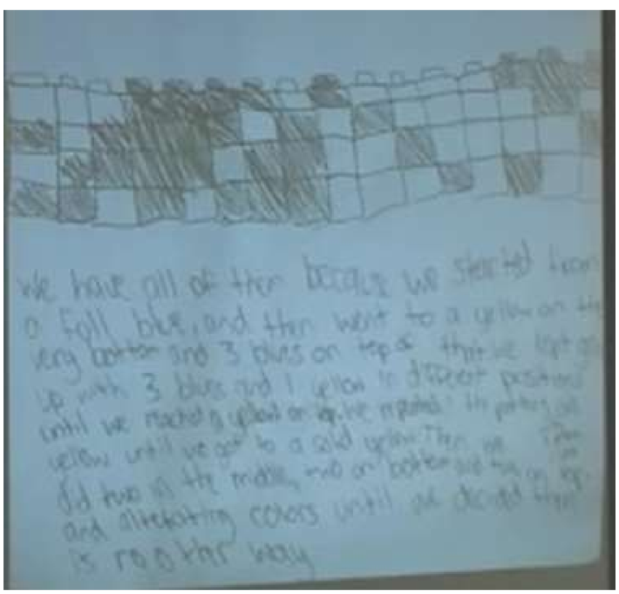


Figure 4.5.1.9 Sally's Cycle 1 student work sample 1

In the discussion, the teachers pointed out the symmetry in the student's solution and identified the pattern of elevators. They seemed to have had difficulty identifying the overall organization of this student's solution, while Rich referred to the symmetry as making a butterfly wing pattern.

4.5.2 The 5-Tall Tower Problem

Teachers then worked on the 5-tall towers problem, selecting from two colors. Group 1 used elevators and opposite pairs to construct the following set of towers (10-7 Meeting transcript 2 of 3, line 248):

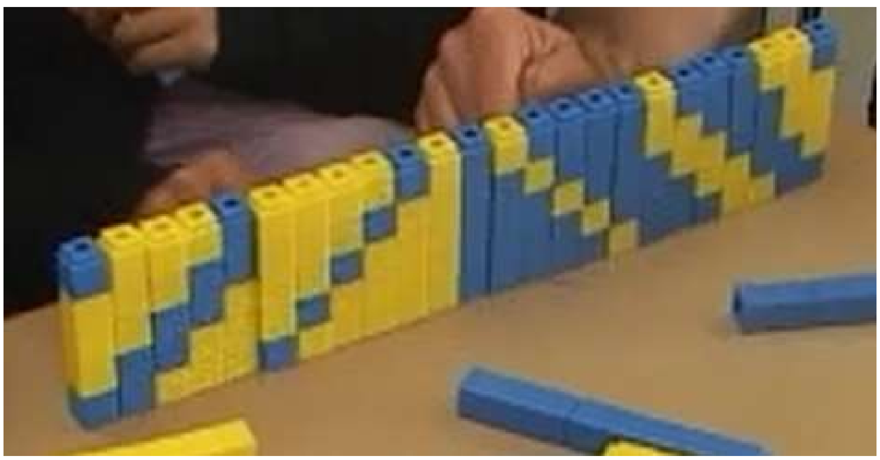


Figure 4.5.2.1 Group 1 5-tall towers organization 1

Two of the group members began to express concerns that this pattern would yield duplicates, and may not account for all towers containing two blocks of a given color. One group member constructed a tree diagram to describe the solution. Based on the tree diagram, the group decided to organize the towers in a new way, controlling for the color on the bottom of the tower. The group's revised organization is provided in the figure below. (10-7 Meeting transcript 2 of 3, line 805)

Figure 4.5.2.2 Group 1 5-tall towers organization 2

The members of group 1 indicated that this organization represented half of the towers in the solution. The opposites of these 16 towers would complete the solution.

Group 2 organized their towers into cases based on the number of blocks of each color. They used an elevator pattern initially, and extended it to the case of 2 yellow and 3 blue. In this case, they also used controlling of variables to make the following organization of towers (10/7 Meeting transcript 2 of 3, line 1059):

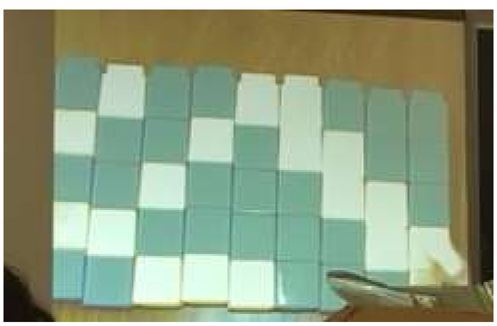


Figure 4.5.2.3 Group 2 5-tall towers organization (2 yellow 3 blue)

Group 3 organized their towers by the number of each color block. They developed a cases argument to describe the completeness of their solution. The group used an elevator pattern to describe the 4 blue 1 yellow case and the 4 yellow 1 blue case. An example of this is shown in the figure below. (10/7 Meeting transcript 2 of 3, line 958)

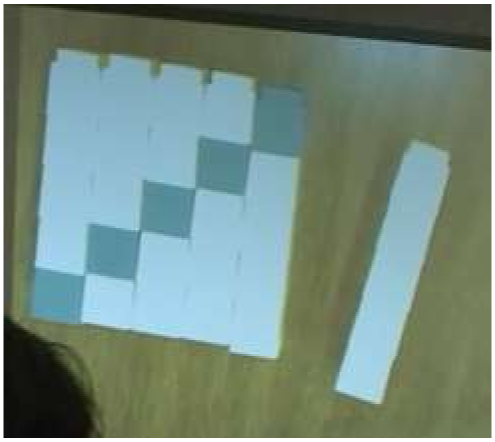


Figure 4.5.2.4 Group 3 5-tall towers organization (4 yellow 1 blue and all yellow)

The group used a recursive pattern, based on elevators to describe the 3 yellow 2 blue case, and recognized that the 3 blue 2 yellow case could follow the same argument, with opposite towers. (10/7 Meeting transcript 2 of 3, line 869)

Figure 4.5.2.5 Group 3 5-tall towers organization (3 yellow 2 blue)

4.5.3 The Pizza Problem

After sharing solutions to the 5-tall towers problem, selecting from 2 colors, teachers worked on the pizza problem. A statement of the pizza problem is available in Appendix B. All groups claimed that 16 different pizzas can be made when selecting from four toppings that can be applied to the entire pizza (no halves). All groups used a cases argument to justify their solution to the pizza problem. Each group listed the number of 1-topping pizzas, 2-topping pizzas, 3-topping pizzas, a pizza with all four toppings and a plain pizza.

At this meeting, Rich mentioned that he felt Brandon's description of the isomorphic relationship between the pizza problem, with 4 toppings, and the 4-tall towers problem, selecting from 2 colors was the result of leading questions from the researcher (10/7 Meeting transcript 3 of 3 line 208). This started a discussion about teacher questioning. The instructor claimed that the researchers' questions were not leading, and suggested that Rich watch the interview again. From this point on, throughout the intervention teachers made note of their own ability to ask questions. Several of these instances are described below, in the relevant subsections.

4.6 Week 5 Discussion (10/7)

In preparation for this week's discussion, teachers were asked to read "Making Pizzas: Reasoning by Cases and Recursion" (Maher & Yankelewitz 2011), chapter 6 of Combinatorics and Reasoning (Maher, Powell, & Uptegrove, 2011), which describes student work on several pizza problems, the original problem with 4 toppings, and different toppings allowed on each half, a simpler problem with 4 toppings that could be applied to the whole pizza, and two additional problems which required teachers to allow for two different types of crusts in both of the previous problems. Teachers were also asked to watch a video of a researcher interviewing a student, Brandon, about his solution to the pizza problem, selecting from 4 toppings. In this interview, when asked by the researcher if the problem reminded him of any other that he had worked on, Brandon responded that the pizza problem reminded him of the 4-tall towers problem, selecting from 2 colors. In exploring the relationship, the video shows Brandon recognizing the structural similarity of the solution to the two problems.

In this discussion, teachers were asked to consider the reasoning demonstrated by students as they worked on the pizza problem. They were also asked to discuss the researcher's style of questioning of Brandon during the interview.

All teachers were impressed with the researcher's interviewing style. Angela claimed "I was so amazed with the questioning technique of the interviewer." (10-7 Discussion line 105) Teachers wanted to emulate her questioning technique, pointing to her use of wait time, not inferring Brandon's thinking, asking questions about specific elements of his diagram, and keeping questions open ("in any way") were all noted by the teachers. The teachers were impressed by Brandon's reasoning and later referred to some of their students as "Brandons." Also during this week, students' work on the pizza problem was discussed as an example of recursive reasoning. Teachers found this to be a more useful technique than cases (1 topping, 2 toppings, and 3 toppings) for this problem.

4.7 Week 6 discussion (10/14)

In preparation for this week's discussion, teachers were asked to read "Brandon's Proof and Isomorphism" (Maher & Martino, 1998) a chapter describing and analyzing the video teachers watched in the previous week, in which a researcher interviewed a student, Brandon, about his solution to the 4-topping pizza problem, without halves.

In this discussion, teachers were asked to share questions used to help students think more deeply about mathematical ideas. They were also asked to compare student strategies for solving the 4-tall and 5-tall towers problems. It is worth noting that only one of the teachers (Justin) posted an original response sharing questions used to help students think more deeply about mathematical ideas. Justin reported an admiration for open ended questions, which he described as not pushing students in a particular direction, but rather create a space for students to explore and share ideas. Four other teachers commented on Justin's response, and two teachers did not address the question at all. Teachers seemed more readily to discuss the work of their own students. Connie and Angela remarked that they had a few students share their work prior to the implementation of the 5-tall tower task. They noticed that some students adopted similar strategies. Connie said that she would normally be disappointed in a student copying the work of another, but since the students could explain the method they were using, she was impressed.

4.8 Week 7 Discussion (10/21)

In preparation for this week's discussion, teachers were asked to implement the 4-topping pizza problem, without halves in their classrooms.

In this discussion, teachers were asked to describe their students' work on the pizza problem, and share whether any students responded to the question "Does this problem remind you of any other?" Sally's responses to the prompts showed attention to student reasoning. Connie described a student's argument based on cases of initial toppings. The student made lists of all the pizzas with pepperoni, all the pizzas with mushrooms, all the pizzas with peppers, and all the pizzas with sausage. In response, Sally asked where in the lists a pizza with pepperoni and sausage should belong (10/21 Discussion line 229). She also noted that Brandon's organizational system for the pizza problem made it easy to visually relate his solution to the set of 4-tall towers. Based on the representation used by some students, she indicated that she was not surprised that many students did not make a connection between towers and pizzas (10/21 Discussion line 274).

4.9 10-28 Meeting

At the beginning of the meeting, teachers shared samples of their students' work on the pizza problem, and the 5-tall towers problem, selecting from 2 colors. The instructor invited teachers to share samples of student work. For each sample shared, the instructor invited other teachers to identify the organization of the solution, before the teacher sharing the sample described it. This discussion of student work marked the end of cycle 2. After the discussion, teachers began stage 1 of cycle 3. They worked on the 3-tall towers problem, selecting from 3 colors and Ankur's Challenge, a special case of the 4-tall towers problem, selecting from 3 colors. Ankur's Challenge required that the number of 4-tall towers that can be made selecting from three colors, under the condition that all towers must contain a block of each of the possible colors be determined.

4.9.1 Discussion of student work

Rich presented the following sample of student work (10/28 Meeting transcript 1 of 2, line 21):

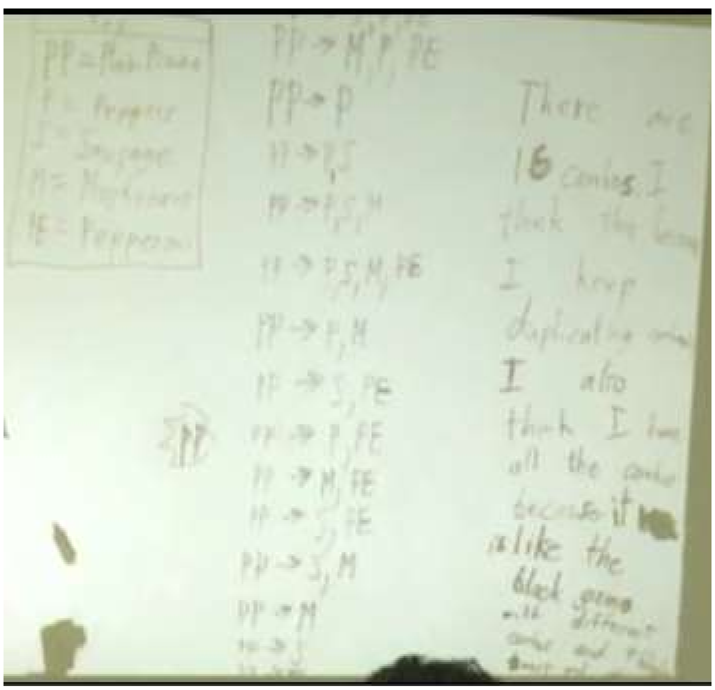


Figure 4.9.1.1 Rich's Cycle 2 student work sample 1

Rich claimed that the student's organization of toppings followed a staircase pattern similar to the work that was produced in working on the 4 and 5-tall towers problem.

Rich then shared these students' solution to the 5-tall towers problem, selecting from 2 colors. (10/28 Meeting transcript 1 of 2, line 29)

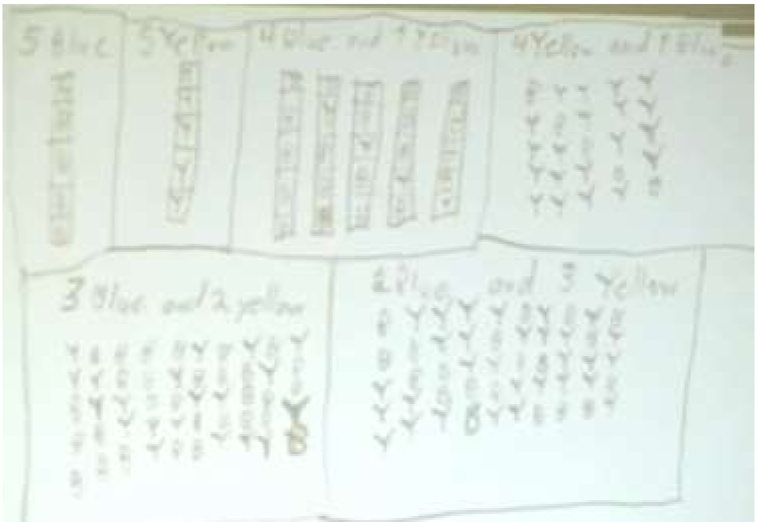


Figure 4.9.1.2 Rich's Cycle 2 student work sample 2.

Rich suggested that this group of students built their solution to the 5-tall towers problem from their previous solution to the 4-tall towers problem, by placing blocks on top of previously constructed towers. He noted that the group did not provide an argument for the completeness of the cases: 3 blue and 2 yellow, and 2 blue and 3 yellow cubes. He indicated that he did not want to push the students too hard to justify these cases, knowing that they would be working on the pizza problem later. Rich also indicated that the students provided a convincing description of the isomorphism between the pizza problem and the 4-tall towers problem, but questioned that there may not be sufficient written information to justify whether students really understood the isomorphism.

Connie shared the following example of student work (10/28 Meeting transcript 1 of 2, line 119):

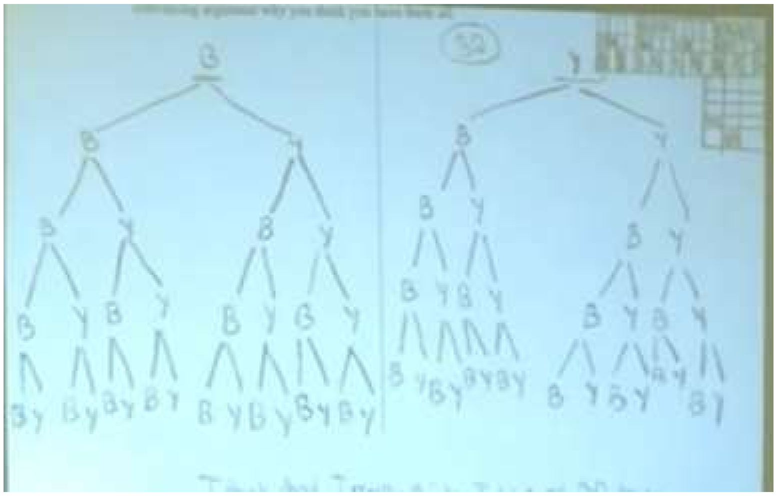


Figure 4.9.1.3 Connie's Cycle 2 Student work sample 1

Connie stated that the students immediately constructed a tree diagram to determine a solution to the problem. Justin noticed that the structure of the tree diagram could support an inductive argument, as either a blue or a yellow block is added to the existing set of blocks. Connie claimed that the student's written argument was not convincing, even though her work was easy to follow, and the tree diagram aligned with the tower solutions.

Connie also shared a student's solution to the pizza problem, which was organized into cases by the number of toppings (10/28 Meeting transcript 1 of 2, line 153). Connie noted that this student's solution was the same as the solutions the teachers developed at their last meeting.

Kate shared the following sample of student work (10/28 Meeting transcript 1 of 2, line 198):

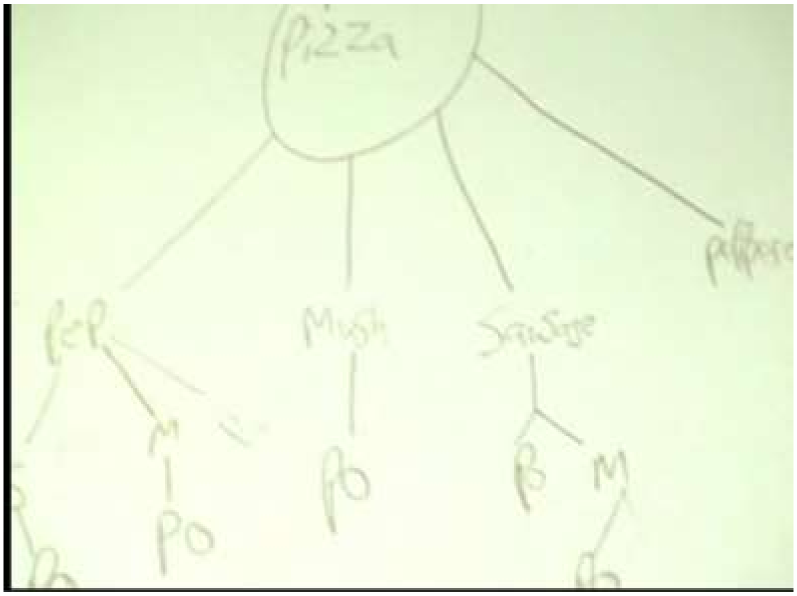


Figure 4.9.1.4 Kate's Cycle 2 student work sample 1

Kate recognized this student's solution as correct but had difficulty interpreting the tree diagram in which the nodes are added to the plain pizza to represent 1-topping pizzas, and nodes are added to the 1-topping pizza nodes to represent 2-topping pizzas. Nodes are not added to 1-topping pizzas if they will result in a 2-topping pizza that was already identified. Nodes are then added to 2-topping pizza nodes to represent 3-topping pizzas. Students in this group claimed that this problem reminded them of the Shirts and Pants problem, as well as the Towers problem, but did not elaborate further. Kate briefly shared a second example of student work on the pizza problem, in which the solution was written as a list, following the familiar 1-topping, 2-topping, 3-topping 4-topping organizational structure (10/28 Meeting transcript 1 of 2, line 263).

Kate also shared this third example of student work (10/28 Meeting transcript 1 of 2, line 281):

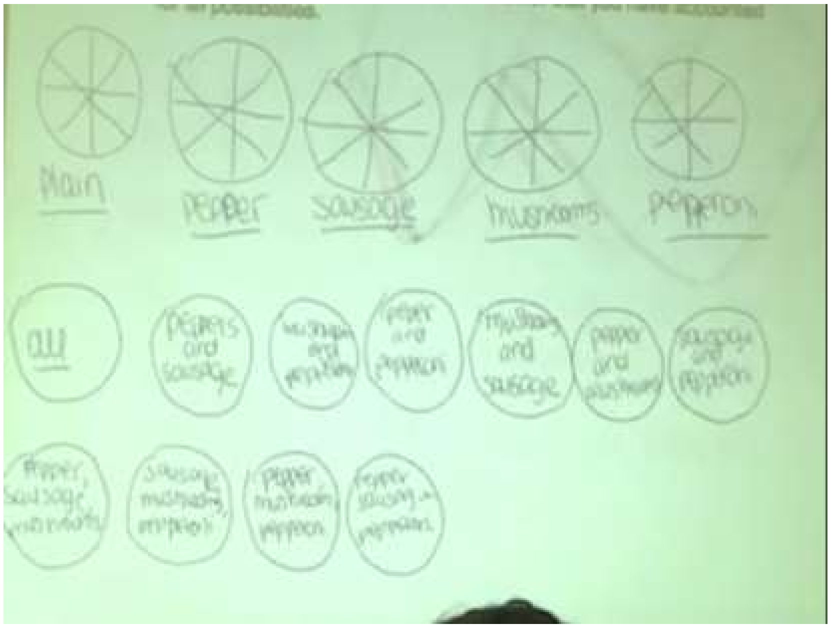


Figure 4.9.1.5 Kate's Cycle 2 student work sample 3

Sally claimed that the student's organizational structure reminded her of compliments, and Rich agreed. Teachers noted that the two topping pizzas appeared to be grouped as pairs that exhaust the set of four toppings. For example, one pair of pizzas exhausting the toppings was (peppers and sausage, mushroom and pepperoni) and another pair was (pepper and mushroom, sausage and pepperoni).

Justin shared the following sample of student work (10/28 Meeting transcript 1 of 2, line 316):

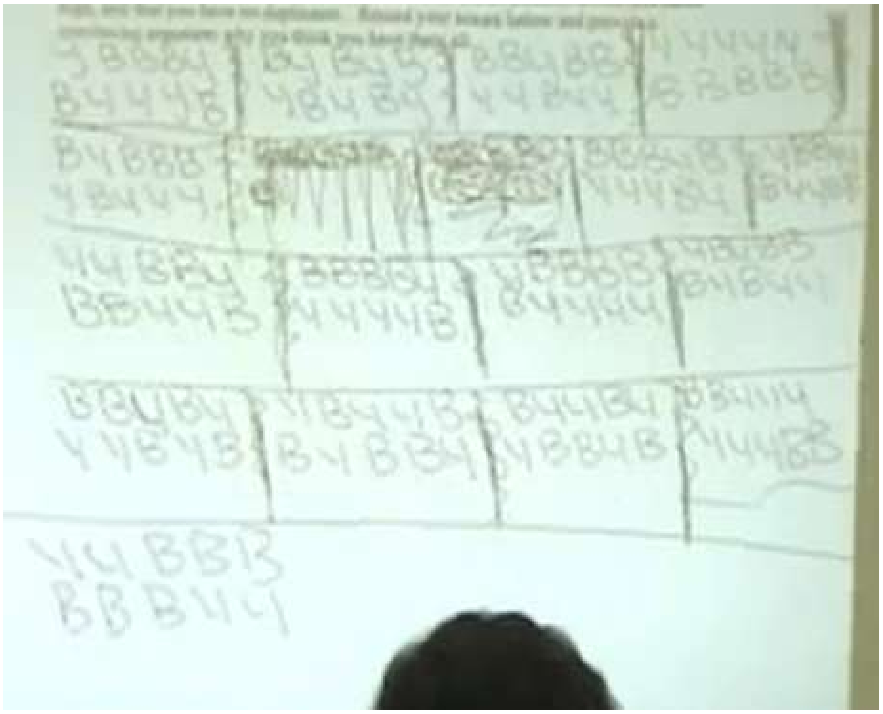


Figure 4.9.1.6 Justin's Cycle 2 student work sample 1

This student claimed that the solution to the 5-tall towers problem selecting from 2 colors contained 32 towers. The student noted that this was double the number of 4-tall towers that could be made selecting from 2 colors. Justin described the student's reasoning process as it transitioned from guess and check to a more systematic approach. He claimed students were able to find 24 towers by guessing and checking. After that, students used the concepts of cousins and opposite pairs to identify remaining towers.

Justin also shared the following example of student work (10/28 Meeting transcript 1 of 2, line 423):

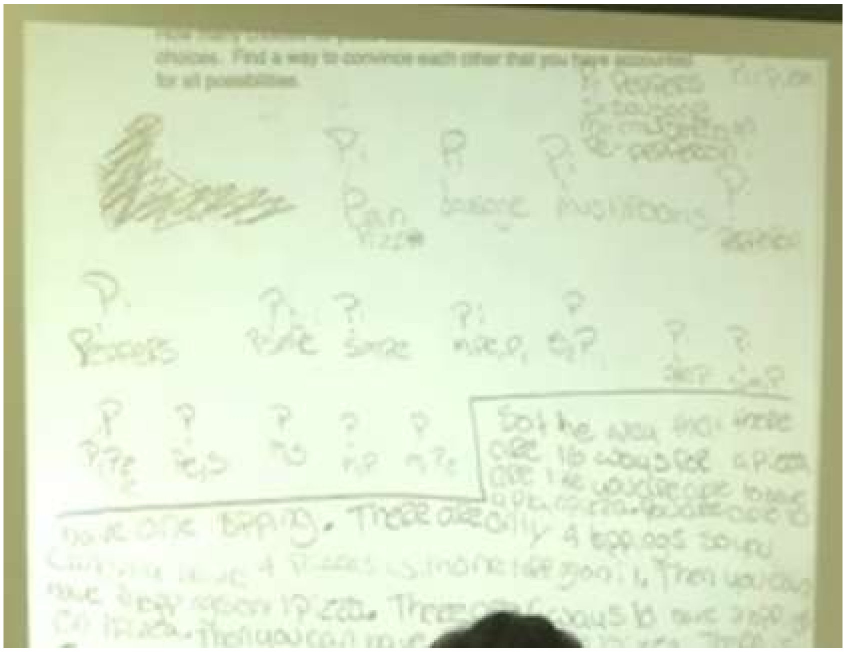


Figure 4.9.1.7 Justin's Cycle 2 student work sample 2

Jarret noted that the students determined the number of 2-topping pizzas last. Teachers appreciated the notation used in the problem but reported that they found it difficult to follow.

Sally shared the following sample of student work (10/28 Meeting transcript 1 of 2, line 535):

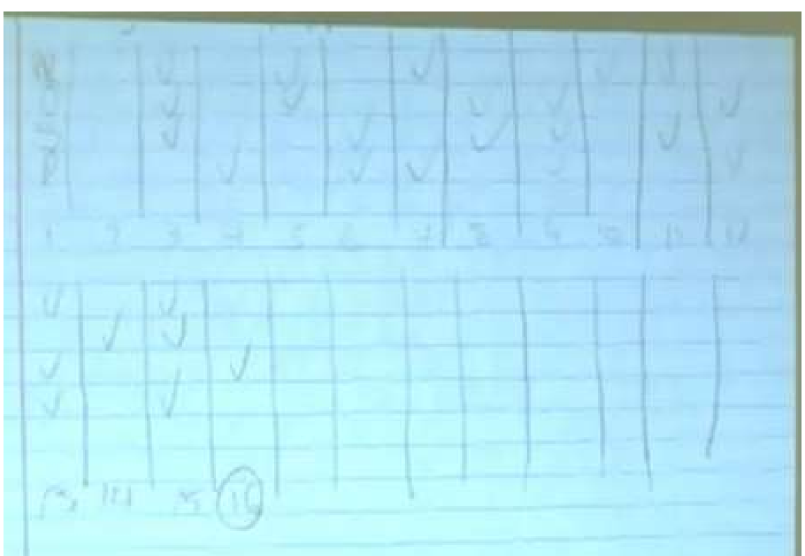


Figure 4.9.1.8 Sally's Cycle 2 student work sample 1

Sally claimed that she was shocked by this student's solution because it reminded her of Brandon's solution. The student identified 16 pizzas, and used a complement structure to identify pizzas. Each pair of pizzas uses all four toppings exactly once. Sally asked the student if this problem reminded her of any other problem, but the student did not claim that it did.

Sally also shared this example of student work (10/28 Meeting transcript 1 of 2, line 600):

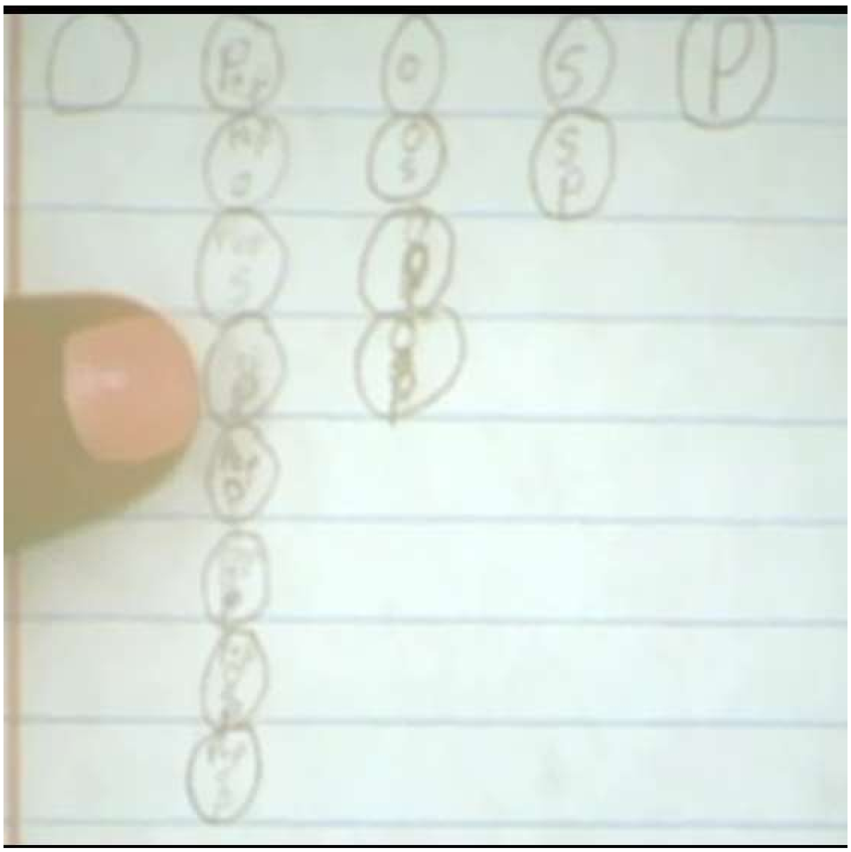


Figure 4.9.1.9 Sally's Cycle 2 student work sample 2

This student organized their pizzas into cases by the initial topping. Pepperoni was listed first, followed by onions (this teacher replaced "mushrooms" with "onions" in the problem) then sausage, and finally peppers.

Mitch shared an example of student work on the pizza problem (10/28 Meeting transcript 1 of 2, line 624). This student organized pizzas into cases by the number of toppings, but also controlled for a variable in the case of 2-topping pizzas, fixing one topping as the base, and systematically adding toppings to the identified base topping. Mitch said that he asked this student if they recognized a connection to other problems. The student claimed the problem was similar to the towers problem "because they were looking for patterns in both."

Mitch also shared this example of student work (10/28 Meeting transcript 1 of 2, line 663):

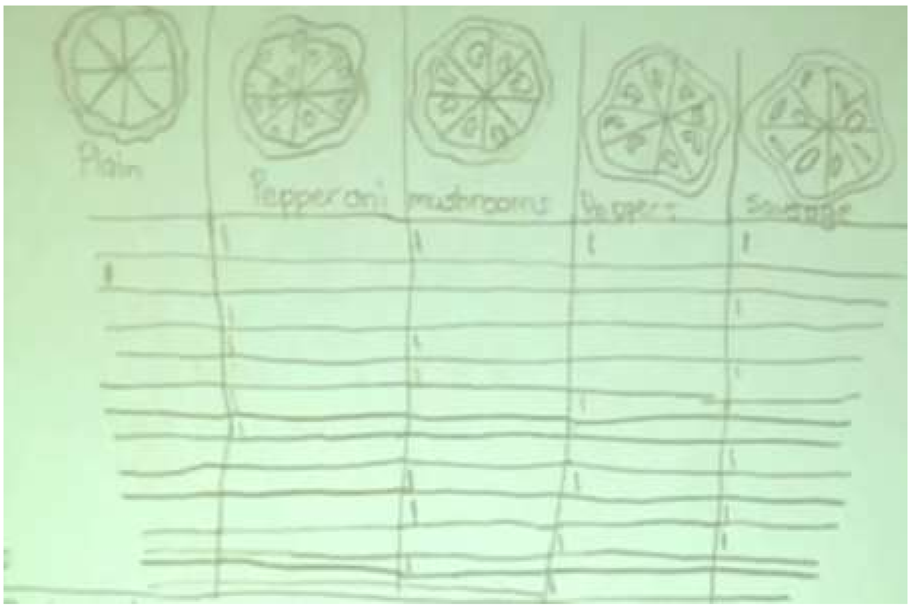


Figure 4.9.1.10 Mitch's Cycle 2 student work sample 2

Teachers noted that this student's work was organized similar to Brandon's, but also recognized that the student did not account for one of the three topping pizzas.

In this session, teachers recognized that some of their students picked up ideas from their classmates (10/28 Meeting transcript 1 of 2, line 738). This led to a discussion about whether this constitutes "stealing" of ideas. Several teachers mentioned that they shared student work and noticed that in later tasks, students were more likely to use a strategy that another student had previously shared. The students who adopted the strategy acknowledged that it was a method that made sense to them, and was better than ones they had previously used.

4.9.2 The 3-Tall Towers Problem, Selecting from 3 Colors

Group 1 used an inductive method to construct the towers. One color was identified as a base color, and 1-tall towers of the base color were made. New towers of height 2 were created by adding blocks of each possible color to these 1-tall towers. Towers of height 3 were then created by adding blocks of each possible color to the 2-tall towers. This process was repeated for each of the three possible base colors.

Group 2 initially organized their towers by cases based on the color at the top of the tower, but later organized their towers by the color at the bottom of the tower.

Group 3 organized their towers by cases based on the color at the bottom of the tower. These cases were then organized by the color at the middle (second position) of the tower.

All three groups developed organizations of completed towers that were similar in appearance. The teachers described their organization for the case of towers with a blue cube on the bottom and agreed that there would be nine additional towers with a yellow cube on the bottom, and nine additional towers with a red cube on the bottom, for a total of 27 towers. An example of the teachers' solution to the blue bottom towers is provided below. (10/28 Meeting transcript 2 of 2, line 154)

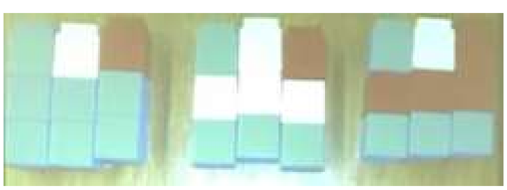


Figure 4.9.2.1 Teacher solution to the 3-tall towers problem (blue base shown).

4.9.3 Ankur's Challenge

All groups initially attempted to extend their solutions to the 3-tall tower problem, selecting from 3 colors to Ankur's challenge.

Group 1 initially attempted to add blocks to the top or bottom of their existing towers, with the condition that after adding blocks, the new towers would have one of each color. The group had difficulty keeping track of towers and decided to only add cubes to the bottom of the existing towers. Eventually, this group identified 37 towers, but two of the three group members were not convinced by this solution and attempted to group towers and identify duplicates. (10/28 Meeting transcript 2 of 2, line 405)

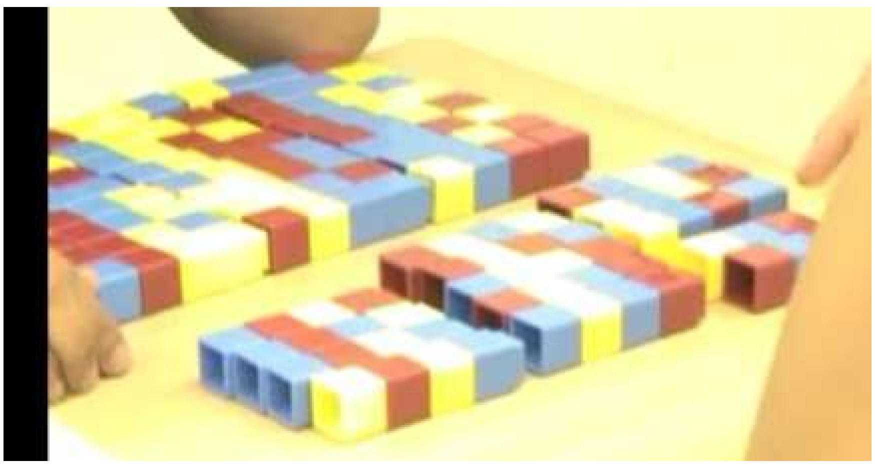


Figure 4.9.3.1 Group 1's towers for Ankur's Challenge

Group 2 began by placing blocks on the top of their existing 3-tall towers, selecting from 3 colors. Eventually the group decided that the towers they constructed could be flipped as well, and attempted to identify the new towers but found duplicates. One of the group members constructed a tree diagram, which only included towers with at least one of each color. This group identified 12 towers with a blue base and claimed that there would be a total of 36 towers, 12 for each base color. (10/28 Meeting transcript 2 of 2, line 1129)

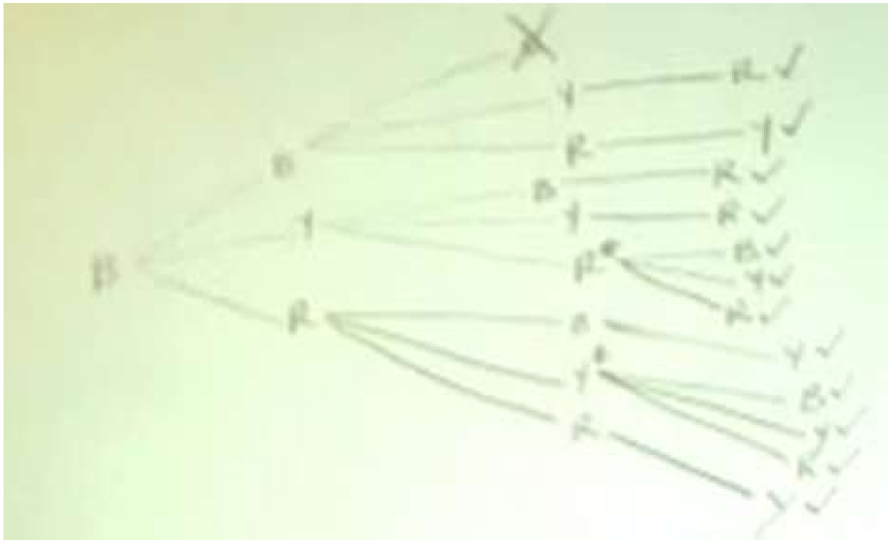


Figure 4.9.3.2 Group 2's tree diagram for Ankur's Challenge

Group 3 identified 24 towers from the solution to the 3-tall towers problem that could be built into solutions for Ankur's challenge. They had removed the 3 solid-colored towers. This group organized the 24 remaining towers into cases, depending on the color blocks that could be added to make these towers satisfy the criteria of Ankur's Challenge. Towers were identified as requiring a red block, a blue block, a yellow block, or requiring any color block. Six towers were identified in each of these sets. The group claimed that a block could be place on the top or the bottom of each tower. The group identified three sets of 12 towers each; 12 towers that could be made by adding a red block to the top or bottom of the 6 towers requiring a red block, 12 towers that could be made by adding a blue block to the top or bottom of the 6 towers requiring a blue block, and 12 towers that could be made by adding a yellow block to the top or bottom of the 6 towers requiring a yellow block. The group also claimed 36 additional towers could be made by adding blocks of any color to the 6 towers requiring any color block. Group 3 claimed the solution to Ankur's Challenge was 72 towers. The figures below show two components of this group's solution. (10/28 Meeting transcript 2 of 2, line 1085)

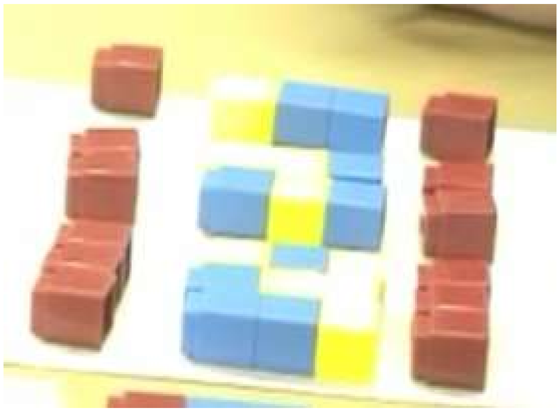


Figure 4.9.3.3 Group 3's collection of 6 towers requiring a red block

Figure 4.9.3.4 Group 3's collection of 6 towers requiring any color block

At the end of this session, Group 3 shared their tree diagram. All groups agreed that 36 towers was the correct solution. Two of the members in Group 1 claimed to have identified the duplicate tower in the set of towers they had constructed.

4.10 Week 8 Discussion (10/28)

In preparation for this discussion, participants were asked to watch the video "Romina's Proof" Which outlines the work of two groups of research students as they attempted to solve Ankur's Challenge. In the video, Romina recognizes that, in each tower of the solution, there must be exactly two blocks with the same color. She then characterizes cases of towers based on the location of pairs of blocks with the same color. As Romina describes her argument to her peers, she refines her work, and gradually develops an elegant proof. Two other research students in the video attempted to reach the solution in a different way. They determined the total number of 4-tall towers, selecting from 3 colors to be 81, then they attempted to remove from this set of 81 towers the towers that did not have at least one of each color block.

In this discussion, the teachers were asked to consider the different approaches taken to solve Ankur's Challenge. The teachers appreciated the elegance of Romina's proof shown in the video. Connie provided a synopsis of the proof, and the other teachers felt she did a good job summarizing Romina's reasoning. The teachers also understood the logic of Mitch and Ankur's proof, and thought it was a good strategy, despite the fact that it did not yield them a correct solution. Kate and Sally used the same strategy in the meeting on the 28th. Kate planned to review students work with the class before presenting the next task.

4.11 Week 9 Discussion (11/4)

In preparation for this week's discussion, participants read "Responding to Ankur's Challenge: Co-construction of Argument Leading to Proof" (Maher & Muter 2011), chapter 8 of Combinatorics and Reasoning (Maher, Powell, & Uptegrove, 2011). This chapter described the situation in the video "Romina's Proof" and analyzed the process by which students constructed the argument which eventually led to Romina's proof of the solution.

In this discussion, teachers reflected further on Romina's proof. In particular, they were asked to consider the value in giving students multiple opportunities to explain and write about their ideas. Mitch, Kate and Connie stated that it was important to give students time to organize their thoughts and explain their reasoning. They indicated that student thinking was constrained by the class schedule.

Some students spent much available time on the task itself, and had little time remaining in the period to organize their thoughts into written statements. (11/4 Discussion, line 98)

4.12 Week 10 Discussion (11/11)

In preparation for this discussion, teachers were asked to implement the 3-tall towers problem, selecting from 3 different color cubes in their classrooms.

In this discussion, teachers were asked to describe the strategies used by their students to solve the 3-tall towers selecting from 3 colors, and if applicable, Ankur's Challenge. Angela described a cases argument used by one group of students in her class. She described the number of towers in each group but did not identify the characteristics of towers in each group. Angela also noted that she shared her strategy for solving the problem with her class. Kate, Sally, and Connie were all impressed with student reasoning on Ankur's Challenge.

4.13 11-18 Meeting

At this meeting, teachers shared samples of student work on the 3-tall towers problem, selecting from 3 colors, and Ankur's Challenge.

4.13.1 Discussion of student work

Rich shared the following sample of student work (11/18 Meeting transcript 1 of 2, line 63):

Figure 4.13.1.1 Rich's Cycle 3 student work sample 1

Teachers recognized that these towers were organized by the color of the top cube and recognized that some pairs of towers in each row indicated a switch of the bottom two blocks in each tower.

Rich also shared the following sample of student work (11/18 Meeting transcript 1 of 2, line 395):

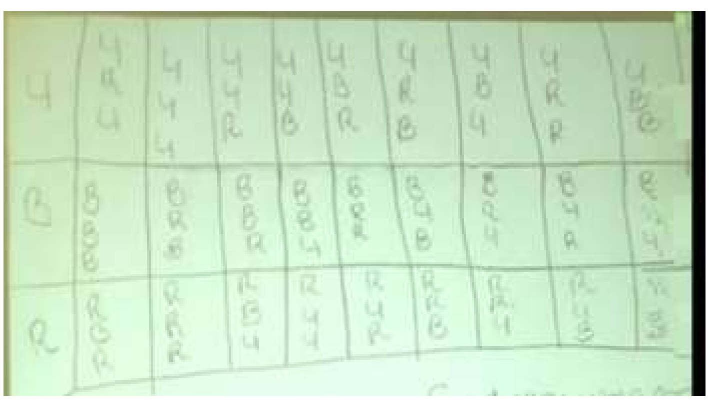


Figure 4.13.1.2 Rich's Cycle 3 student work sample 2

Rich read this student's solution:

> We believe that we found all possible combos. We have found twenty-seven combinations. There might be more than twenty-seven, but we believe that there's twenty seven. First we got one color. I got red. Allie got yellow, and Aliyah got blue. We started with easier combinations, like all blue, reds, and yellows. Then we add all the

> colors like red blue yellow, then we reversed it, red yellow blue. Then to record the blocks, we each got one color and we passed them to our right when we were done with them. This is the order we used to find the possible combinations with the blocks.
>
> (11/18 Meeting transcript 1 of 2, lines 397 – 404)

Rich indicated this was an improvement in his students' writing, but the Instructor suggested that students should be encouraged to attempt to justify their solutions, and not just describe how they reached the solution.

Angela also shared an example (11/18 Meeting transcript 1 of 2, line 459) of student work on the 3-tall towers problem, selecting from 3 colors, in which the solution was organized into 5 groups of towers: a group with 3 towers of a single color, three groups of 6 towers with two colors each (red & yellow, yellow & blue, red & blue) and one group of 6 towers with all three colors in each tower. Several participants claimed that they had students who used a similar organization.

Angela shared another example of student work on the 3-tall towers problem, selecting from 3 colors. In this other example, the student also identified 4 groups of 6 towers and one group of 3 towers. This student referred to his groups of 6 towers as "different tops", "different middles", "different bottoms", and "all three colors." Angela had difficulty interpreting these group names, but other teachers helped identify each of the groups shown below with one of the names provided by the student. (11/18 Meeting transcript 1 of 2, line 527)

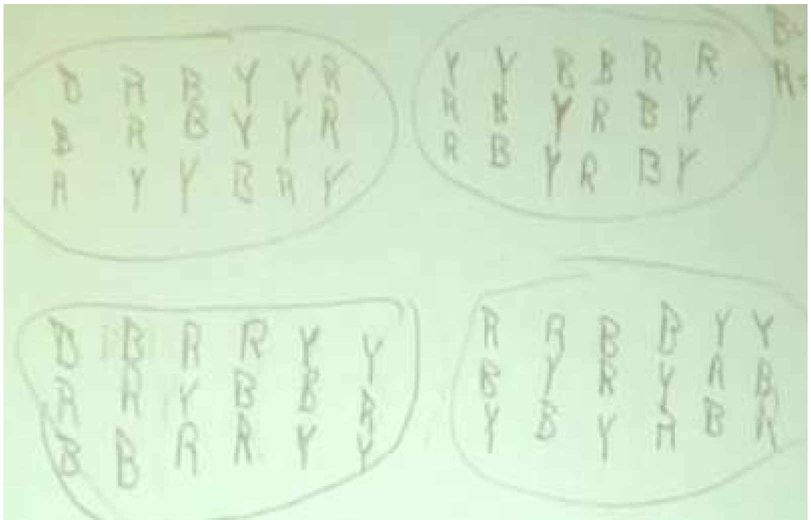


Figure 4.13.1.3 Angela's Cycle 3 student work sample 2 (clockwise from top left: different tops, different bottoms different middles, and all three colors)

Sally shared the following sample of student work (11/18 Meeting transcript 1 of 2, line 672):

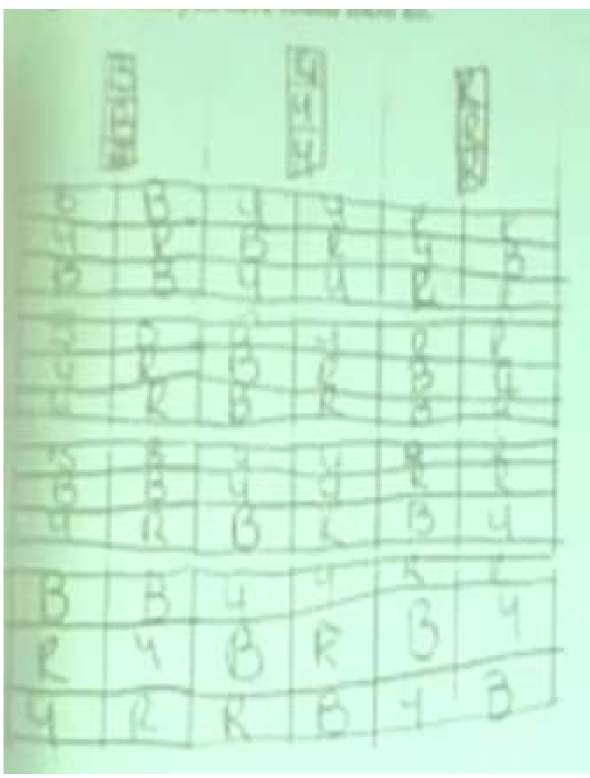


Figure 4.13.1.4 Sally's Cycle 3 student work sample 1

Teachers identified this student's use of controlling for variables, holding the top and bottom colors constant in first row of this solution, and holding the bottom two blocks constant in the second row of this solution. Sally also shared the student's solution to Ankur's Challenge, and noted that the student organized this solution into cases based on the colors of blocks in the top two levels of each tower. A mathematician attending this meeting identified duplicate towers in this student's solution. (11/18 Meeting transcript 1 of 2, lines 749-859)

Sally shared a third example of student work (11/18 Meeting transcript 1 of 2, line 873):

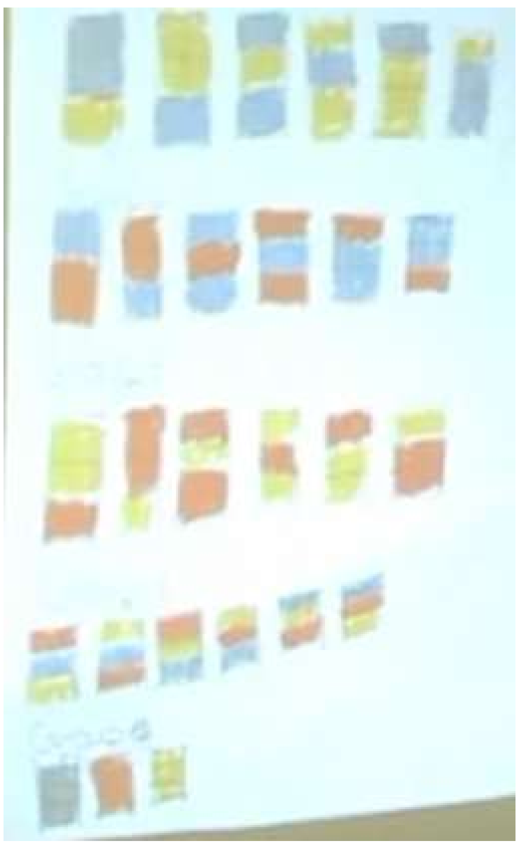


Figure 4.13.1.5 Sally's Cycle 3 student work sample 3

Sally noted that this solution was similar to the solution in the first sample that Angela shared, but wanted to share the student's work because of the towers the student drew to represent the solution. Kate noticed that within each of the 2-color tower groups, the students appeared to group towers as opposites. The student's justification of the solution described the five groups of towers, but did not provide a convincing argument that all towers in groups 1 through 4 were accounted for.

Kate shared an example of student work on the 3-tall towers problem that was similar to Angela's second example. Towers were organized into 5 groups: 6 towers with different tops, 6 towers with different middles, 6 towers with different bottoms, 6 towers with one of each color, and 3 solid towers. Kate also shared this student's solution to Ankur's challenge. (11/18 Meeting transcript 1 of 2, line 980) The student made three groups of towers, those with 2 red blocks, those with 2 blue blocks and those with 3 yellow blocks. A portion of the student's solution (the case of 2 yellow blocks) is shown in the figure below. Teachers appreciated the organization of the towers.

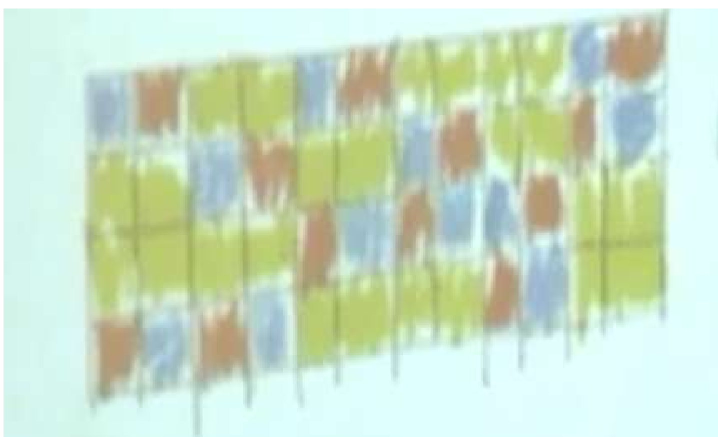


Figure 4.13.1.6 Kate's Cycle 3 student work sample 2

Kate shared two more examples of student work on Ankur's challenge. One student determined that eighty one, 4-tall towers could be made selecting from 3 colors and carefully identified the towers that did not have one of each color. Another student attempted to develop a formula to determine the solution (11/18 Meeting transcript 2 of 2, line 1199). This student claimed: "You can pick 3 [colors] in 2 spots, but for two spots you can pick 2 [colors]. So three times three times two times two equals thirty-six." The instructor invited teachers to explain this student's justification, and asked Angela to construct a tower and relate it to the student's solution. Kate and Rich were impressed with this student's development of a formula.

Connie shared the following sample of student work (11/18 Meeting transcript 2 of 2, line 74):

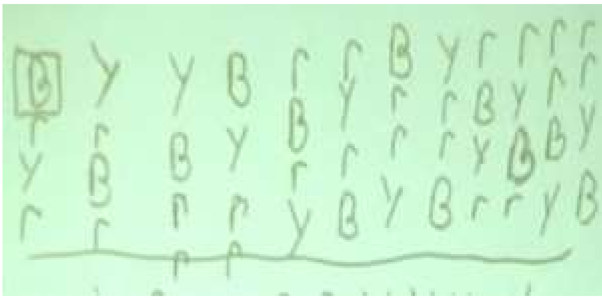


Figure 4.13.1.7 Connie's Cycle 3 student work sample 1

Connie compared this student's work to the work of one of Kate's students. Both students made groups of towers with a duplicated color. This student described his three groups as containing 2 red, 2 blue, and 2 yellow cubes.

Connie shared another example of student work on Ankur's Challenge. She compared this student's work to the work teachers had done while working on Ankur's Challenge. The student constructed a tree diagram and counted the branches that would describe towers with at least one block of each color. The student indicated that, for each starting color on the tree diagram, 12 possible towers could be created, and claimed that 36 towers satisfy the criteria of Ankur's Challenge. The student's tree diagram is shown in the figure below. (11/18 Meeting transcript 2 of 2, line 103)

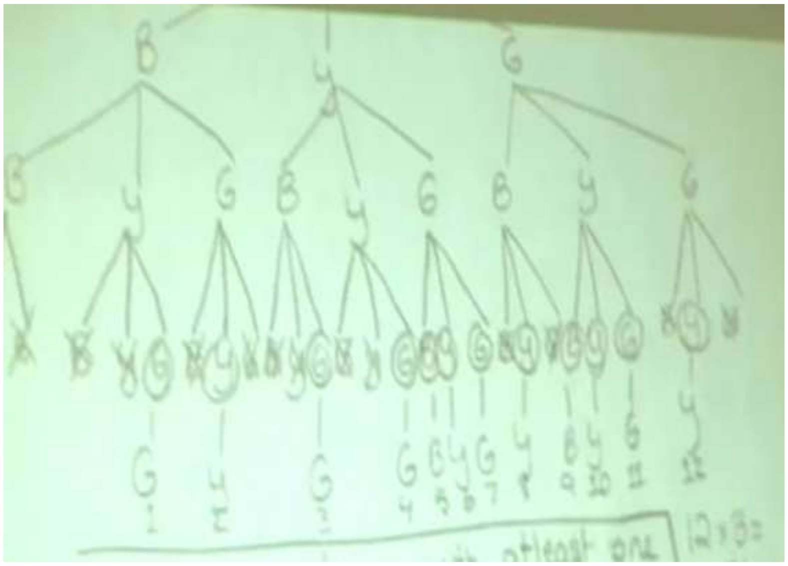


Figure 4.13.1.8 Connie's Cycle 3 student work sample 2

Mitch shared the following example of student work (11/18 Meeting transcript 2 of 2, line 144):

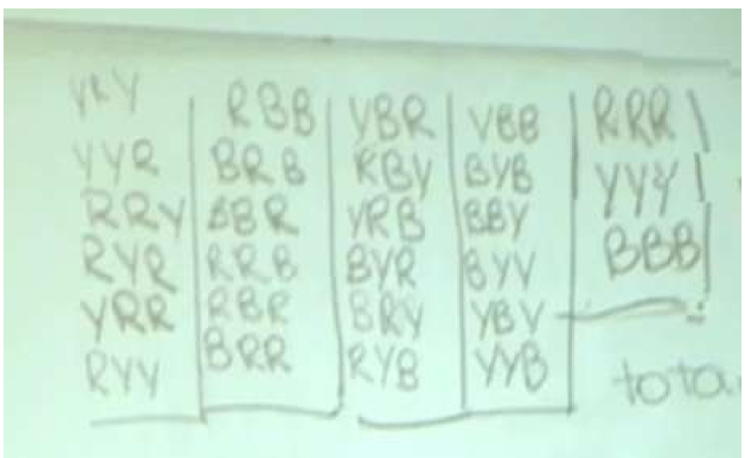


Figure 4.13.1.9 Mitch's Cycle 3 student work sample 1

Teachers identified the initial grouping of towers by pairs of colors, with one group containing all three colors. Teachers also identified an elevator pattern within the organization of each of the 4 groups of six towers. This student also began a solution to Ankur's Challenge, and identified that some color must be duplicated in each of the towers satisfying Ankur's challenge, but did not have time to complete the task.

Justin shared the following example of student work, which teachers compared to similar organizations shared by other teachers. (11/18 Meeting transcript 2 of 2, line 471)

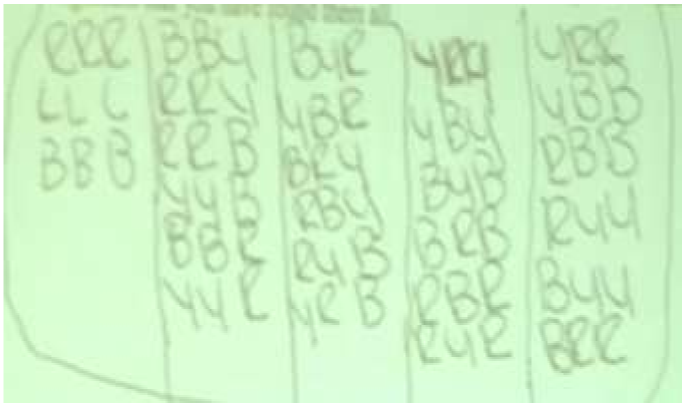


Figure 4.13.1.10 Justin's Cycle 3 student work sample 1

At the end of the meeting, Rich shared this example of student work on Ankur's Challenge (11/18 Meeting transcript 2 of 2, line 653):

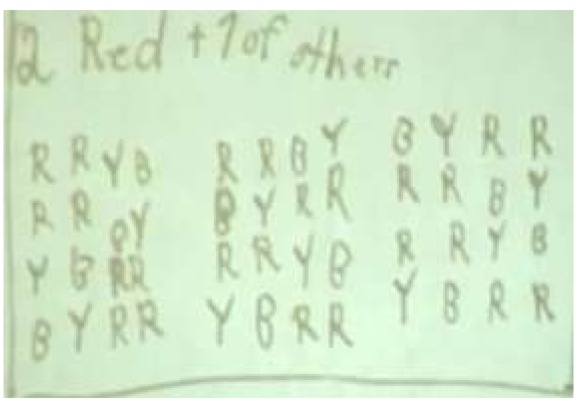


Figure 4.13.1.11 Rich's Cycle 3 student work sample 3 (2 red blocks)

The student noted that there must be two blocks of a given color in each of the towers that satisfy the conditions of Ankur's challenge problem. Teachers compared this recognition to Romina's reasoning on the problem. Rich noticed that students used a concept of opposite pairs to construct towers within each group.

4.14 12-4 Focus Group and Regional Meeting

In preparation for the reginal meeting, a focus group interview of the teachers was conducted. The teachers shared the parts of the intervention they found most memorable. Three teachers thought the video of Brandon was the most memorable part of the intervention. One teacher thought Romina's proof was the most memorable. One teacher remembered that although students could correctly predict solutions to the problems, they could not explain how they made the prediction. One teacher was most struck by the background knowledge and creativity that research students bring to a problem. He cited the example of the student who did not include one of the combinations in the shirts and pants problem because the outfit did not match. One teacher found the work of one of her students to be most memorable. This teacher's current student solved Ankur's Challenge in a way similar to Romina's proof.

Teachers then attended the reginal meeting, in which participating teachers from all four sections of the fall 2010 course "Lesson Study on Student Reasoning" met for a discussion. At this regional meeting, teachers from the other sections shared their memorable experiences. Teachers in the intervention being studied did not actively participate in this discussion.

4.15 Summary

The regional meetings afforded the opportunity for teachers to get to know each other and interact while solving problems. The online discussions allowed for thoughtful interactions because immediate responses were not necessary. Together, the face-to-face and virtual interactions allowed teachers to share their experience with the material. In several instances, one of the teachers shared an experience which other teachers considered novel and of value.

5 ANALYSIS OF REASONING

In this chapter, the forms of reasoning identified by teachers are analyzed. This chapter contains three primary sections; the analysis of teachers' recognition of reasoning in arguments, the analysis of teachers' recognition of reasoning in strategies and heuristics, and an analysis of the claims of teachers regarding convincing arguments.

5.1 Reasoning in Arguments

Throughout the intervention, all teachers described examples of cases arguments, recursive arguments, and inductive arguments. Cases arguments were most prevalent overall. Four of the seven teachers used arguments by contradiction while working on tasks in cycles two or three. No teachers shared examples of student work that included arguments by contradiction. This information is summarized in table 5.1.

Argument	Working on Tasks	Research Student Work	Teachers' Student Work	Total Teachers Referencing
Cases	7	7	7	7
Induction	7	7	3	7
Recursion	6	4	3	7
Contradiction	4	0	0	4
Rule	0	2	2	3
Other	0	1	3	4

Table 5.1 Number of teachers referencing strategies, by source

In this section, the arguments identified by teachers in cycles 1, 2, and 3 are discussed. The relationship between the data collected in the cycles and the data collected in the reasoning assessments is then analyzed.

5.1.1 Argument data from Cycles

Teachers identified case-based, inductive, and recursive arguments in all sources, but the frequency of each argument varied for each source. The data are summarized in table 5.2 and are described in detail below.

Argument	Working on Tasks	Research Student work	Teachers' Student Work
Cases	36	15	49
Induction	20	14	6
Recursion	10	4	4
Contradiction	4	0	0
Rule or Formula	0	2	2
Other	0	1	3

Table 5.2 Argument frequency by source

While working in groups on the tasks in cycles 2 and 3, all teachers used elements of arguments based on cases, induction, and recursion. Cases arguments were used a total of 36 times. Inductive arguments were used a total of 20 times, and recursive arguments were used a total of 10 times. No teachers justified their work with a rule or formula, or argument other than those listed.

When discussing student work samples from the research, all teachers described arguments based on cases and induction. Case arguments were described a total of 15 times. Inductive arguments were described a total of 14 times. Four of the seven teachers described recursive arguments. Additionally, two teachers described arguments based on a rule or formula, and one teacher described an argument other than those listed.

When discussing the student work provided by a participating teacher, all teachers described case-based arguments. Case-based arguments were described a total of 49 times. Three teachers shared a total of six inductive arguments. Three teachers also described a total of four recursive arguments.

Two teachers identified student work samples that included arguments based on a rule or formula, and three teachers identified student work samples that included arguments other than those listed.

Four teachers used an argument by contradiction to respond to questions posed by the instructor, and this form of argument was briefly discussed, (10/7 Meeting transcript 2 of 3, line 507; 10/28 Meeting transcript 2 of 2, line 1010; 10/28 Meeting transcript 2 of 2, line 1182) but no teachers described this form of argument when discussing student work samples from the research, or the work of teachers' students.

It is worth noting that the "Gang of 4" video used in the pre and post assessments for teachers' recognition of student reasoning and the related research was a focal point of this intervention. Teachers discussed Stephanie's case argument and Milin's inductive argument for the 4-tall towers selecting from two colors. These two exemplars were referenced throughout the intervention. It may be of benefit to identify strong exemplars of a recursive arguments, or arguments by contradiction and include a discussion of those exemplars in future instances of the intervention.

It is also worth noting that case arguments took a variety of forms. For example, in the pizza problem some arguments were organized by the number of toppings on each pizza, while others were organized by the type of topping. Teachers successfully identified the overall structure of case arguments. The instructor modeled questioning techniques to push students to justify the completeness of cases, and encouraged teachers have students justify the completeness of at least one case in a solution.

5.1.2 Relationship to Reasoning Assessments

In this cohort, many teachers identified cases and inductive arguments in the pre-assessment. Two of the teachers did not describe elements of inductive arguments on the reasoning pre-assessment.

All teachers described inductive arguments in the post assessment. Five of the seven teachers offered complete descriptions of Milin's inductive argument. In the post-assessment, all teachers made claims that aspects of an inductive argument were convincing. The two teachers who did not describe aspects of an inductive argument in the pre-assessment did describe aspects of inductive arguments when discussing Milin's strategy. In the online discussion, both of these teachers claimed that Milin's Inductive argument was convincing. Moreover, both of these teachers used an inductive approach to complete the 3-tall towers task, selecting from three colors.

Two of the teachers did not describe elements of cases arguments on the reasoning pre-assessment. These teachers used cases arguments to complete tasks in cycles 2 and 3, and identified cases arguments in the reasoning post-assessment. Both of these teachers claimed that cases arguments presented by their own students were convincing. All teachers were able to identify case arguments by the end of the intervention.

Two case arguments were presented in the assessments. These case arguments are compared in table 5.3. In the pre-assessment, five teachers described aspects of Stephanie's case argument, and two teachers did not describe any aspects of Stephanie's cases argument. In the pre-assessment, no teachers offered complete descriptions of the alternative cases argument. Aspects of an alternative case argument were described by three of the teachers. In the combined scoring of the pre-and post-assessments, six teachers completely described Stephanie's case argument, and three teachers completely described the alternative cases argument.

		Pre-Assessment	Post-Assessment
Stephanie's Cases Argument	Partial Description	3	1
	Complete Description	2	5
Alternative Cases Argument	Partial Description	3	3
	Complete Description	0	3
Convincing or Not	Convincing	2	4
	Not Convincing	0	2

Table 5.3 Comparison of cases arguments in reasoning assessments

In the post-assessment, teachers appeared more comfortable making claims about case arguments. In the pre-assessment, two of the teachers claimed aspects of the case argument were convincing. In the post-assessment, four of the teachers claimed aspects of the case argument were convincing. Of the remaining three teachers, two made claims that aspects of the case argument were not convincing. In the intervention, the three teachers who did not identify case arguments presented in the post-assessment as convincing did make claims that certain other case arguments were convincing. The intervention appeared to successfully demonstrate examples of convincing case arguments.

No data regarding recursive arguments were collected in the reasoning assessments. Recursive arguments were discussed during the intervention. Three teachers described student work on the pizza problem as having a recursive structure. One teacher described students' work on the 4-tall towers problem, selecting from 2 colors. This teacher claimed "I never would have recognized recursive patterns and the idea of holding a constant in the towers had they not been addressed during our meeting sessions." (Sally, Final Project p. 33)

5.2 Strategies and Heuristics

Throughout all phases of the intervention, the teachers developed, defined and recognized organizational strategies and heuristics which could be used to complete tasks, and in some cases lead to justifications of solutions. The frequency of each strategy or heuristic is included in table 5.4 and is described below.

Strategy/Heuristic	Working on Tasks	Research Student Work	Teachers' Student Work
Controlling a Variable	60	4	30
Elevator	15	3	9
Opposites	17	19	38
Cousins	1	2	4
Generalize to Specialize	1	5	4
Guess and Check	3	10	9
Staircase	3	9	8
Other	6	3	2

Table 5.4 Frequency of strategies by source

Teachers used a variety of strategies and heuristics to determine solutions to the tasks presented in this intervention. All teachers used the strategies of controlling for variables, constructing opposite pairs, and identifying an elevator pattern. Controlling a variable was the most common strategy used. Teachers used this strategy a total of 60 times. Teachers used the elevator strategy 15 times and constructed opposite pairs 17 times. Other strategies were less common. One teacher used the "cousins" strategy and the "generalizing to specialize" strategy. Three other teachers used "guess and check" and the "staircase" pattern. Six teachers used a strategy other than those listed.

The strategies and heuristics that teachers noted in the literature tended to differ from the strategies the teachers used to complete the tasks. Four of the teachers described "controlling a variable" and two described the elevator pattern. The elevator pattern was described in the research a total of three times. All teachers, however, did describe students' use of opposite pairs. This strategy was mentioned 19 times. Six of the teachers mentioned that students used "guess and check" to determine a solution to one of the tasks (mentioned 10 times). Three of the six teachers who recognized guess and check in the research did not use it to complete the tasks. Six of the teachers also mentioned that students used "staircases" to determine a solution to one of the tasks (mentioned nine times). Three of these six teachers did not use "staircases" to complete any of the tasks. Five teachers described the "generalizing to specialize" strategy, and none of these teachers had used this strategy to complete a task. Two teachers described "cousins" and neither of those teachers had used this strategy to complete a task. One teacher described three strategies other than those listed.

The samples of student work that teachers shared generally represented a blend of the strategies that teachers used to complete the tasks and the strategies that teachers identified in the research. All teachers described samples of student work that involved "controlling for a variable" and "opposites". Samples demonstrating controlling for a variable were shared 30 times. Samples demonstrating opposite pairs were shared 38 times. Three teachers described samples of student work that involved "cousins". A total of four samples involving cousins were shared. Five of the teachers described samples of student work that involved "elevators". A total of nine samples involving the elevator pattern were shared. Two teachers, neither of whom used the "generalizing to specialize" strategy, described it in four samples of student work. Five teachers described a total of nine samples of student work involving "guess and check". Two of these teachers did not use this strategy in their own work. Four teachers described a total of 8 samples of student work that used the staircase pattern. Two of these teachers did not use this strategy in their own work. Two teachers described samples of student work involving a strategy other than those listed.

5.2.1 Of Note:

Teachers recognized some strategies other than ones they used to complete the tasks. Although only one teacher used the "cousins" strategy, it was described by five of the teachers in the discussions of student work samples. Two teachers described "cousins" when discussing student work samples from the research. Three teachers (including the one who used the strategy) described "cousins" when discussing the work of a teacher's student.

Five teachers described the "generalizing to specialize" strategy. This strategy was described as teachers discussed Mike and Ankur's approach to the Ankur's Challenge problem. Two of the teachers who described "generalizing to specialize" also shared samples of their own students' work that used this strategy. The one teacher who did use this strategy to complete a task did not mention it while discussing student work.

Although many teachers used opposite pairs and elevator or staircase patterns within their own cases arguments, not many teachers claimed these strategies were convincing in the assessments. Only one teacher claimed opposites were convincing. Two teachers claimed that patterns such as elevator or staircase were convincing.

5.2.2 Relationship between Strategies and Arguments

Many samples of work were coded with both argument codes and strategy codes. In this section, relationships between strategies used to construct solutions and arguments used to justify the completeness of those solutions are discussed. First, trends within specific tasks are discussed. After that, trends across all work samples throughout the intervention are considered.

5.2.2.1 Relationships by Task

Teachers worked in three groups on the 5-tall towers problem, selecting from 2 colors. All groups used a controlling for variables strategy and all groups presented case arguments to justify their solutions. Two of the three groups organized their cases by the number of cubes of one color in each tower. These groups also used recursive arguments to describe the completeness of the 2 blue, 3 yellow case. One of these groups built their recursive argument from an elevator pattern in the towers.

Teachers also worked in three groups on the pizza problem. The only co-occurring strategy and argument codes for this task were "cases" and "controlling for variables."

Work on the 3-tall towers problem, selecting from 3 colors generated the most co-occurring codes of any task. Controlling for variables was the most prevalent strategy. This strategy was connected to a cases argument in 10 instances, and an inductive argument in 1 instance. The group that used controlling for variables to make an inductive argument considered the height of the tower a variable. Other strategies also led to case arguments in the 3-tall towers problem. Opposite pairs were used in four instances to lead to a cases argument. Elevators and cousins each led to one case argument.

In Ankur's Challenge, controlling for a variable led to three case arguments and two inductive arguments. In the case arguments, the variable was the one color that would appear twice in each tower. In the inductive arguments, the teachers considered the variable to be the different options for the fourth cube that could be added to 3-tall towers, selecting from 3 colors.

5.2.2.2 General Relationships

Controlling for variable was the most common strategy, and case arguments were the most common arguments. It is not surprising that the most frequent co-occurring codes were cases and controlling for variable. The strategy of controlling for variables also led to several instances of recursive and inductive arguments. Other strategies were frequently used to construct case arguments. Cousins, elevators, opposites, and staircases all were more frequently associated with case arguments than other arguments. Elevator, opposites, and staircases were less frequently associated with recursive arguments. The data regarding co-occurring strategy and argument codes is summarized in table 5.5.

	Cases	Induction	Recursion	Contradiction	Rule	Other
Controlling a Variable	42	7	7	1	0	0
Cousins	3	0	0	0	1	0
Elevator	8	0	1	0	0	0
Generalizing to Specialize	0	0	0	0	0	1
Guess and Check	2	0	1	0	0	0
Opposites	15	0	1	0	1	1
Staircase	3	0	1	0	0	0
Other	0	3	1	0	0	0

Table 5.5 Frequency of argument and strategy co-occurrence

5.3 Analysis of Arguments Claimed as Convincing

It is worth noting the context in which teachers made claims about arguments or strategies being convincing. (See table 5.3) While working on tasks, only two teachers made claims that they were not convinced by another teacher's argument. A total of three claims of this type were made. Teachers appeared slightly more comfortable making claims about the work of their students. Four teachers made a total of 10 claims that a student's argument was convincing. Four teachers made a total of 7 claims that a student's argument was not convincing. When discussing samples of student work from the research, teachers seemed most willing to make claims about whether an argument was convincing. All teachers made claims that some argument presented in the research was convincing. A total of 20 claims were made that some argument from the research was convincing. All teachers also made claims that some argument presented in the research was not convincing. A total of 15 claims were made that some argument from the research was not convincing. It may be that the instructor's discussion questions required teachers to make these claims, but it may also be that teachers were more comfortable making claims about whether an argument was convincing in the online setting.

Claim	Working on Tasks	Teachers' Student Work	Research Student Work
Convincing	0	10	20
Not Convincing	3	7	15

Table 5.6 Frequency table for claims about arguments

Teachers most frequently claimed that case arguments were convincing. A total of 15 claims were made that a given case argument was convincing. Of those arguments, four were deemed incomplete by the researcher. As noted previously, teachers did not consistently verify that each case in a case argument was completely accounted for with no repeating elements across the cases.

Teachers also frequently claimed that inductive arguments were convincing. A total of 11 claims were made that inductive arguments were convincing. None of these arguments were flagged by the researcher as being incomplete.

In total, teachers made four other claims that arguments were convincing. One of these claims was based on a rule that a student had defined. This argument was determined to be invalid by the researcher.

6 ANALYSIS OF TEACHER MOVES

The instructor's use of the teacher moves varied by stage, and generally increased across the three cycles. The instructor's use of teacher moves also varied depending on the medium of communication. In the online discussions, the instructor almost exclusively used "motivating" statements. In this section, trends in teacher moves are described by stage and by cycle.

6.1 Teacher Moves by Stage

Each cycle of the intervention consisted of four stages: (1) teachers working on mathematical tasks, (2) teachers reading research literature and observing videos of students working on those tasks, (3) teachers implementing the tasks in their own classrooms, and (4) teachers discussing the results of implementing the tasks in their classrooms. Due to practical limitations of conducting research in public schools, no data were collected during stage 3, but the data for the other stages are summarized below.

While teachers worked on tasks, the instructor monitored teachers and praised their work as a form of motivation. The instructor used explanation questions and probing questions to better understand teachers' thinking, and used of revoicing to verify that she understood that teacher's thinking. Once a group claimed to have completed a task, the instructor asked teachers to justify their work. At times, the instructor would select examples of the teachers' work and request that the teachers share a particular strategy with the class. During this stage, the instructor also used other teacher moves, but these other moves were used much less frequently.

Teachers' analysis and discussion of research occurred primarily through the online discussion boards. The instructor participated in these discussions, and read teachers' posts, but she chose to foster communication by thanking teachers for sharing ideas, and praising teachers and students for their ingenuity. In this stage, motivation was the most common teacher move, and none of the questioning techniques were used. The instructor wanted to encourage the teachers to interact with each other in the online setting. In her words: "What I tried to do… not butt into their conversations. Because I wanted to give them a chance to, not only talk, but to talk to each other." (Personal Interview 11/09/10)

While teachers discussed student work, the instructor took the role of facilitator. She frequently invited teachers to share samples of student work, and would use probing questions to require teachers to engage with each student's strategy. The instructor used the teacher moves of "inviting" and "waiting" to promote conversations among the group members. "Waiting" was paired with "inviting" nine times in this stage. "Waiting" was also paired with questions for interaction with "Other Solutions" nine times in this stage. The instructor used "revoicing" in this stage for two purposes; sometimes to clarify statements made by students or teachers, and other times to introduce a common language. For example, in response to a teacher's description of a student's organization, the instructor said: "But what you're saying is you see a double control for variables". The instructor often found something to praise in each sample of student work, and frequently made motivational statements as student work was shared.

The table below summarizes the use of teacher moves by stage:

Teacher Move	Stage 1: Working on Tasks	Stage 2: Reviewing Literature and Video	Stage 3: Implementing Tasks	Stage 4: Discussing Implementation
Anticipating	4	1	0	8
Inviting	16	0	0	58
Monitoring	18	0	0	3
Motivating	32	19	0	64
Revoicing	20	0	0	15
Selecting	7	0	0	1
Sequencing	0	0	0	0
Waiting	3	0	0	19
Question Types				
Q: Other Solution	4	0	0	32
Q: Explanation	34	0	0	9
Q: Probing	49	0	0	23
Q: Justification	22	0	0	6
Q: Connection	3	0	0	3
Q: Generalization	1	0	0	2

Table 6.1Teacher Moves by stage

6.2 Teacher Moves by Cycle

The frequency of teacher moves generally increased throughout the intervention. One explanation for the lower frequency of teacher moves in cycle 1 is that no data were collected in stage 1 of cycle 1. There are no counts of "monitoring" in Cycle 1, because this teacher move primarily occurred during stage 1. Similarly, the instructor frequently asked "explanation", "probing", and "justification" questions during stage 1 of each cycle. Comparing the data from stages 2 and 4, in all cycles, it appears that the instructor's use of questioning techniques was more consistent throughout the intervention. In general, the instructor's use of motivational statements did increase throughout the intervention. The data regarding teacher moves by cycle are presented in the two tables below.

Teacher Move	Cycle 1	Cycle 2	Cycle 3
Anticipating	4	6	3
Inviting	17	20	37
Monitoring	0	6	15
Motivating	7	40	68
Revoicing	4	10	21
Selecting	0	4	4
Sequencing	0	0	0
Waiting	4	8	10
Question Types			
Q: Other Solution	10	13	13
Q: Explanation	3	18	22
Q: Probing	4	35	33
Q: Justification	1	7	20
Q: Connection	0	6	0
Q: Generalization	0	1	2

Table 6.2 Teacher Moves by Cycle (Including stage 1)

Teacher Move	Cycle 1	Cycle 2	Cycle 3
Anticipating	4	3	2
Inviting	17	14	27
Monitoring	0	0	0
Motivating	7	24	55
Revoicing	4	2	9
Selecting	0	0	1
Sequencing	0	0	0
Waiting	4	7	8
Question Types			
Q: Other Solution	10	11	11
Q: Explanation	3	1	5
Q: Probing	4	14	5
Q: Justification	1	0	5
Q: Connection	0	3	0
Q: Generalization	0	1	1

Table 6.3 Teacher Moves by Cycle (Excluding stage 1)

Questions for describing a connection between two problems were noted only during Cycle 2 since it was during this cycle that teachers worked on the pizza problem. In this cycle, the teachers watched the Brandon Interview and read "Brandon's Proof and Isomorphism" (Maher and Martino 1998). The instructor asked the teachers whether the pizza problem was similar to any other tasks in this intervention. It is worth noting that none of the teachers in this intervention described a connection between the pizza problem and the 4-tall towers task. All teachers were impressed by Brandon's identification of the similarity of structure in the solution to the two problems, later referring to Brandon when describing the fact that students of all perceived ability levels are capable of deep mathematical reasoning.

The instructor modeled the use of questioning techniques throughout the intervention. The types of questions asked generally depended on the stage of the cycle. The instructor primarily used probing questions, questions for explanation, and questions for justification of a solution while teachers were working on tasks. While teachers were describing samples of student work, the instructor primarily used questions that required teachers to consider other solutions, as well as probing questions.

The modeling of other teacher moves also depended on the stage of the cycle. While teachers worked on tasks, the instructor modeled the techniques of monitoring, inviting, motivating and revoicing. While sharing samples of student work, the instructor modeled the techniques of inviting, motivating, revoicing, and waiting. Motivating was the teacher move most used in the intervention.

6.3 Instructor Representations Used

The instructor primarily referred to existing samples of work, either those of teachers or students. However, two instances of the instructor using a particular representation were identified in the data. Both of these instances occurred during the 11/18 group meeting. In one instance, the instructor used the unifix cubes to identify a "staircase" pattern for a visitor. In the other instance, the instructor used the unifix cubes to make an example tower for teachers to reference when making claims about a formula developed by one student as a solution to the Ankur's Challenge problem. In both of these examples, the instructor used unifix cubes to construct a physical, visible representation of a concept that the group was discussing.

7 ANALYIS OF BELIEFS

The third goal of this research is to identify what changes, if any, occurred in teachers' beliefs regarding learning, teaching, or mathematics. Teachers in this intervention completed two beliefs inventories; one pre-assessment and one post-assessment. The beliefs pre-assessment provides a baseline for understanding teachers' initial beliefs, but due to the number of items in the beliefs inventory (22), the inventory results are of limited generalizability. Pre-assessment results for this cohort indicate that teachers' beliefs were relatively well aligned (greater than 50% aligned) with the standards expressed by the inventories. The table below summarizes teachers' scores. In each cell, the first number represents the assessment score as the percent of questions for which that teacher actively agreed with the standard. The second number represents the assessment score as the percent of questions for which that teacher did not disagree with the standard. In short, the second percentage includes the statements for which the teacher claimed "undecided" as well as the statements for wich the teacher's response aligned with the standard.

Teacher	Pre-Assessment	Post-Assessment
Angela	55/82	63/82
Rich	95/100	95/95
Connie	86/91	91/95
Kate	82/90	91/95
Justin	82/95	91/91
Mitch	86/86	77/86
Sally	82/95	77/100

Table 7.1 Beliefs Inventory scores by teacher

Due to the relatively high percentage of alignment with standards in the pre-test and limited sample size of assessment questions, a more fine-grained study of each teacher's beliefs will be necessary. Information regarding beliefs will be further examined, first by teacher, and then by cycle, and finally by phase.

7.1 Beliefs by Teacher

For each teacher, the results of the beliefs inventories will be described. Instances of possible change in beliefs based on the assessment data will be noted. Beliefs data from throughout the intervention will be summarized and related to the beliefs assessment data.

7.1.1 Kate

The table below summarizes Kate's pre- and post-assessment results. Questions are grouped by category. In each cell, the numbers represent the total number of questions for which Kate scored consistent with the Standards, and the percentage represents the percentage of questions in that category for which Kate scored consistent with the Standards.

Question Category	Pre-Assessment	Post-Assessment
Expectations and Abilities	2 (50%)	4 (100%)
Mathematical Discourse	2 (100%)	1 (50%)
Concepts and Procedures	4 (57%)	6 (86%)
Manipulatives	2 (100%)	2 (100%)
Student and Teacher Roles	3 (100%)	3 (100%)
Differentiated Instruction	4 (100%)	4 (100%)

Table 7.2 Kate's Beliefs inventory results by question category

Based solely on the beliefs inventory assessments, Kate's beliefs regarding student expectations, mathematical discourse, and concepts and procedures may have shown some change.

Throughout the intervention, Kate made a total of 17 claims that were consistent with the Standards described in the beliefs inventories. Kate made one claim that was inconsistent with the Standards described in the beliefs inventories. She also made two claims that were undecided in relation to the Standards described in the beliefs inventories. Kate's one inconsistent claim was characterized as relating to "Teaching" as well as "Concepts and Procedures". Her claim was that starting with 4-tall towers tricked students, and that in order to identify the doubling pattern, students should have started by constructing smaller towers. This claim led to a discussion about the purpose of engaging students in constructing solutions to the problems; that is, discovering patterns and relationships. Later in the intervention, Kate appeared to focus more on the arguments students provided with their solutions. In her final project, she claimed of her students: "Once they realized that I was looking more at how they went about coming up with their answer than the actual answer itself, they seemed more willing to spend more time thinking more deeply about the problem." Data from the intervention support the claim that Kate's beliefs regarding concepts and procedures changed over the course of the intervention.

Data from the intervention also support the claim that Kate's beliefs regarding expectations and student abilities changed over the course of the intervention. In her final project, Kate cites an example of being surprised by student's ability to justify his solution: "I was happy with this because I thought his partner was the stronger member of the group but he was able to show that he understood the reasoning used to come up with his answer" (Final Project).

In total, Kate made three claims indicating beliefs consistent with the Standards regarding mathematical discourse. There is no evidence of claims made that were inconsistent with the Standards regarding mathematical discourse. Aside from the assessment data, there is not evidence of a change in beliefs regarding mathematical discourse.

Although not reflected in the responses to the beliefs assessment inventory data, there is other evidence that Kate shifted some beliefs regarding student and teacher roles. In particular, Kate indicated a desire to improve her questioning techniques. When discussing her implementation of the 4-tall towers problem, she claimed: "And that's my other thing, I honestly at this point, I don't know if I feel qualified to be questioning them because I don't know what I'm supposed to be looking for, I feel like." (10/14 Meeting transcript 1 of 3 line 222). Later, in her final project, Kate claimed: "I feel my ability to ask delving questions improved by these tasks so I was able to pull more from my students forcing them to think more deeply about what they were doing." (Kate Final Project p. 52)

Summary

Kate's beliefs regarding expectations and student abilities, as well as concepts and procedures appeared to change over the course of the intervention as indicated by her responses to observing researcher moves from videos. Also, Kate's beliefs regarding the teacher's role in probing students to understand their reasoning appears to have changed as well.

7.1.2 Angela

The table below summarizes Angela's pre-and post-assessment results. Questions are grouped by category. In each cell, the numbers represent the total number of questions for which Angela scored consistent with the Standards, and the percentage represents the percentage of questions in that category for which Angela scored consistent with the Standards.

Question Category	Pre-Assessment	Post-Assessment
Expectations and Abilities	3 (75%)	3 (75%)
Mathematical Discourse	2 (100%)	1 (50%)
Concepts and Procedures	3 (43%)	5 (71%)
Manipulatives	2 (100%)	2 (100%)
Student and Teacher Roles	1 (33%)	1 (33%)
Differentiated Instruction	1 (25%)	2 (50%)

Table 7.3 Angela's Beliefs inventory results by question category

Based solely on the beliefs inventory assessments, Angela's beliefs regarding mathematical discourse, concepts and procedures, and differentiated instruction may have shown some change.

Throughout the intervention, Angela made a total of seven claims that were consistent with the Standards described in the beliefs inventories. She also made five claims that were inconsistent with the Standards described in the beliefs inventories.

Angela made no claims consistent with Standards regarding concepts and procedures during the intervention. However, early in the intervention, Angela claimed "I feel that with a good method of organizing being pointed out to them, more of my students would have been successful with this task." (9/30 Discussion, line 286) Reflecting on the intervention overall, Angela claimed:

> My students really improved upon their organization of the way they were building the towers; they relied less on the term "random" and were able to show why they built the towers in the order that they did. I believe this truly contributed to the strength of their arguments.
>
> (Angela Final Project p. 22)

Although Angela made no claims that indicated a change in beliefs regarding teaching procedures before having students work on problems, her recognition of improvement in student arguments may support the assessment data indicating a change in beliefs regarding concepts and procedures.

Early in the intervention, Angela made two claims that were consistent with Standards regarding differentiated instruction. After watching the gang of four video, she recognized a variety of accurate methods of justifying a solution to the 4-tall towers problem. She also noted that it would be interesting to see the variety of ways her students attempt the tasks in this intervention. Angela also made one claim that was inconsistent with the Standards regarding differentiation. She expressed surprise at student solutions that used different methods than she had been taught (10/28 Discussion). Data from the intervention supports the beliefs assessment data demonstrating limited change in Angela's beliefs regarding differentiation.

Angela made two claims regarding mathematical discourse. In one, she recognized the value of students discussing their solutions, and claimed that this discussion led to improved arguments. In the other claim, Angela expressed concern at students "stealing" ideas from their peers (10/28 Group Meeting Transcript 1 of 2). This concern may explain why Angela claimed that "Collaborative learning is effective only for those students who actually talk during group work." Data from the intervention may support the claim that Angela's beliefs regarding mathematical discourse did change during the intervention.

Summary

Angela's beliefs assessments indicated changes in three question categories. The assessment data may indicate that beliefs regarding concepts and procedures, and differentiated instruction became more aligned with Standards over the course of the intervention. The assessment data also may indicate that beliefs regarding mathematical discourse became less aligned with standards over the course of the intervention. Examples from the intervention support claims that beliefs in each of these three question categories did change.

7.1.3 Rich

The table below summarizes Rich's pre-and post-assessment results. Questions are grouped by category. In each cell, the numbers represent the total number of questions for which Rich scored consistent with the Standards, and the percentage represents the percentage of questions in that category for which Rich scored consistent with the Standards.

Question Category	Pre-Assessment	Post-Assessment
Expectations and Abilities	4 (100%)	4 (100%)
Mathematical Discourse	2 (100%)	2 (100%)
Concepts and Procedures	7 (100%)	6 (86%)
Manipulatives	2 (100%)	2 (100%)
Student and Teacher Roles	3 (100%)	3 (100%)
Differentiated Instruction	4 (100%)	4 (100%)

Table 7.4 Rich's Beliefs inventory results by question category

Rich's beliefs assessment scores were rather high. Based solely on the inventory assessments, Rich's beliefs regarding concepts and procedures may have shown some change over the course of the intervention.

Throughout the intervention, Rich made a total of 13 claims that were consistent with the Standards described in the beliefs inventory. He also made three claims that were inconsistent with the Standards and made two claims that could not be marked as consistent or inconsistent with the Standards.

The majority of Rich's claims related to student and teacher roles. He made seven claims consistent with Standards regarding student and teacher roles, and one claim that was undecided in regard to student and teacher roles. In the "undecided" claim, Rich noted that his approach to teaching changed, and had a stronger relationship with his students, but did not provide enough detail about these changes to be able to record whether this change aligned with the Standards regarding student and teacher roles.

In the post-assessment, Rich agreed with the claim "Math is primarily about learning the procedures." Rich may have agreed with this claim for a variety of reasons, but during the intervention he made three claims regarding concepts and procedures that were aligned with Standards. In one of those claims he stated: "I used to think the answer was the most important component of problem solving. However, sometimes the process and reasoning was more important" (Final Project)

Rich made two claims that were not consistent with Standards involving expectations and student abilities. In one claim, Rich stated that he did not expect student work would "come close" to Romina's proof for the solution to Ankur's challenge. It is worth noting that Rich did share a student's solution to Ankur's challenge that was similar in structure to Romina's proof. In another claim, Rich stated that he did not want to push a recently declassified student to provide a more complete argument (11/18 Group Meeting Transcript 1 of 2). Although Rich's assessment scores for expectations and abilities were consistently high, he did make claims during the intervention indicating beliefs inconsistent with the Standards.

Summary

Rich's beliefs assessment scores were high. Based on claims he made during the intervention, it is possible that Rich's beliefs regarding student and teacher roles did change over the course of the intervention. It is also possible that Rich's assessment results regarding expectations and abilities did not accurately represent his beliefs.

7.1.4 Connie

The table below summarizes Connie's pre-and post-assessment results. Questions are grouped by category. In each cell, the numbers represent the total number of questions for which Connie scored consistent with the Standards, and the percentage represents the percentage of questions in that category for which Connie scored consistent with the standards.

Question Category	Pre-Assessment	Post-Assessment
Expectations and Abilities	3 (75%)	4 (100%)
Mathematical Discourse	2 (100%)	2 (100%)
Concepts and Procedures	6 (86%)	6 (86%)
Manipulatives	2 (100%)	2 (100%)
Student and Teacher Roles	2 (67%)	2 (67%)
Differentiated Instruction	4 (100%)	4 (100%)

Table 7.5 Connie's Beliefs inventory results by question category

Based solely on the Beliefs Inventory, Connie's beliefs regarding expectations and abilities, and student and teacher roles may have shown some change.

Throughout the intervention, Connie made a total of 17 claims that were consistent with the Standards described in the beliefs inventories. She made two claims that were inconsistent with the Standards in the beliefs inventories. Connie made seven claims that could not be coded as consistent or inconsistent with the Standards.

Connie made four claims regarding expectations and abilities. None of these claims were coded as consistent or inconsistent with the Standards set in the beliefs assessment, but in those claims, Connie notes being surprised or impressed by students' work during cycle three of the intervention. These data support the claim that Connie's beliefs regarding expectations and student abilities changed over the course of the intervention.

Connie made seven claims that were consistent with Standards regarding student and teacher roles during the intervention. She made no claims that were inconsistent or undecided in relation to the Standards.

Although Connie scored 100% consistent with the Standard regarding student use of manipulative tools in both the pre-and post-assessments, there is evidence of a change in her beliefs regarding their appropriate use over the course of the intervention. Initially, Connie indicated that she did not expect students to use the manipulatives, and made no claims about the value of manipulatives. When discussing Milin's inductive strategy for building towers, Connie notes how the manipulatives were a valuable communication tool: "However, once he starts building the towers after being prompted to do so, it becomes quite clear that he is supporting his idea of multiplication to help solve for how many towers…" (9/30 Discussion, lines 34-36). This change may not have been durable, however. In her final project, Connie references manipulatives twice. In both instances, she claims that students enjoyed working with manipulatives, but does not make any claims about their value as a tool for organization or communication.

Summary

There is some evidence to support the claim that Connie's beliefs regarding expectations and student abilities did change over the course of the intervention. There is some evidence that Connie's beliefs regarding manipulatives did change over the course of the intervention, but this change may not have been durable.

7.1.5 Justin

The table below summarizes Justin's pre-and post-assessment results. Questions are grouped by category. In each cell, the numbers represent the total number of questions for which Justin scored consistent with the Standards, and the percentage represents the percentage of questions in that category for which Justin scored consistent with the Standards.

Question Category	Pre-Assessment	Post-Assessment
Expectations and Abilities	3 (75%)	3 (75%)
Mathematical Discourse	2 (100%)	2 (100%)
Concepts and Procedures	6 (86%)	6 (86%)
Manipulatives	2 (100%)	2 (100%)
Student and Teacher Roles	2 (67%)	3 (100%)
Differentiated Instruction	3 (75%)	4 (100%)

Table 7.6 Justin's Beliefs inventory results by question category

Based solely on the beliefs inventory assessments, Justin's beliefs regarding student and teacher roles, and differentiated instruction may have shown some change.

Throughout the intervention, Jarret made a total of 19 claims that were consistent with the Standards described in the beliefs inventory. He made one claim that was inconsistent with the Standards, and five claims that were neither consistent nor inconsistent with the Standards.

Justin made a total of 11 claims regarding student and teacher roles during the intervention. All of these claims were consistent with the Standards described in the beliefs inventories. A statement Justin made in the final meeting of the intervention can explain this change.

> Like, I would propose the question to the students. Like "Are you guys convinced that she's right?" You know? Or "he's right?" And how are or how aren't. And they would ask me "Is this problem right?" And I'm saying "I don't know. Ask your peers. Ask your

> peers." Then they would actually have to prove out mathematically, what they did and explaining each step. And then some students would catch up, like on a step that they did right or did well. And then they could say that "No, that's not correct. And this is why…" And that sort of thing
>
> (12/4 Focus Group, line 331)

Justin appears to have become comfortable letting students correct their peers.

During the intervention, Justin made two claims regarding differentiated instruction. Both of these claims were coded as consistent with the Standards expressed in the Beliefs Inventory. In his final project, Justin described his increased awareness of the value of each student's unique set or mathematical abilities.

> Students come to the "Mathematical Table" with a unique system of how they make sense of their environment. All people possess a level of individuality that they use to solve problems and interpret situations with. I had no idea how much of their thoughts and uniqueness is used and could be used in the math class.
>
> (Justin Final Project p. 36)

Although Justin scored 100% consistent with the Standards regarding manipulatives in the pre- and post-assessments, there is some evidence of a change in his beliefs regarding the value of manipulatives. Justin described his students as being "intrigued at the novelty of the unifix cubes" (Final Project p. 13) but did not describe their value as a tool for organization or reasoning. When describing examples of student work from the research, however, Justin did note the value of manipulatives and physical representations. He described Milin's construction of towers as a justification for multiplying the total number of towers by two each time a new level was added. Justin also described how Ankur's drawing of the pizzas in the pizza problem with halves led another student to better understand the solution to that problem.

Summary

There is evidence that Justin's beliefs regarding differentiated instruction and student and teacher roles did change over the course of the intervention. This evidence supports the results of the Beliefs assessments.

7.1.6 Mitch

The table below summarizes Mitch's pre-and post-assessment results. Questions are grouped by category. In each cell, the numbers represent the total number of questions for which Mitch scored consistent with the Standards, and the percentage represents the percentage of questions in that category for which Mitch scored consistent with the Standards.

Question Category	Pre-Assessment	Post-Assessment
Expectations and Abilities	3 (75%)	3 (75%)
Mathematical Discourse	2 (100%)	1 (50%)
Concepts and Procedures	7 (100%)	7 (100%)
Manipulatives	2 (100%)	2 (100%)
Student and Teacher Roles	1 (33%)	1 (33%)
Differentiated Instruction	4 (100%)	3 (75%)

Table 7.7 Mitch's Beliefs inventory results by question category

Based solely on the beliefs inventory assessments, Mitch's beliefs regarding mathematical discourse and differentiated instruction may have shown some change.

Throughout the intervention, Mitch made a total of 21 claims that were consistent with the Standards described in the beliefs Inventory. He made one claim that was inconsistent and one claim that was undecided in regard to the Standards described in the inventories.

During the intervention, Mitch made three claims regarding mathematical discourse. One of these claims was coded as inconsistent with the Standards in the beliefs inventory. The other two claims were coded as consistent with the Standards in the beliefs inventory. The inconsistent claim was made prior to the two consistent claims. Although Mitch disagreed with the statement "It's helpful to encourage student-to-student talking during math activities" on the post-assessment, he shared this story in his final project.

> I'm really happy specifically about Matt's explanation. He says "After a while, Max noticed a pattern. I wasn't sure, but then he showed me how one combo could mean the same for a similar color scheme." He provides an example and explains why he was convinced. This is exactly the type of collaboration I was looking for on this project.
> (Mitch Final Project p. 38)

Data from the intervention does not support the claim that Mitch's beliefs regarding mathematical discourse became less consistent with the Standards over the course of the intervention.

During the intervention, Mitch made three claims regarding differentiated instruction. All three claims were consistent with the Standards described in the Beliefs Inventory. Mitch disagreed with the statement, "learning a step-by-step approach is helpful for slow learners" in the post-assessment, but he may have interpreted the statement differently. There is not sufficient data from the intervention to support the claim that Mitch's beliefs regarding differentiated instruction changed over the course of the intervention.

Summary

Mitch's assessment results indicate a relatively high consistency with the standards described in the beliefs assessments. Data from the intervention also indicate a high consistency with those Standards. There is not sufficient data from the intervention to support claims that Mitch's beliefs changed over the course of the intervention.

7.1.7 Sally

The table below summarizes Sally's pre-and post-assessment results. Questions are grouped by category. In each cell, the numbers represent the total number of questions for which Sally scored consistent with the Standards, and the percentage represents the percentage of questions in that category for which Sally scored consistent with the Standards.

Question Category	Pre-Assessment	Post-Assessment
Expectations and Abilities	3 (75%)	3 (75%)
Mathematical Discourse	2 (100%)	1 (50%)
Concepts and Procedures	7 (100%)	6 (86%)
Manipulatives	2 (100%)	2 (100%)
Student and Teacher Roles	2 (67%)	2 (67%)
Differentiated Instruction	2 (50%)	3 (75%)

Table 7.8 Sally's Beliefs inventory results by question category

Based solely on the beliefs inventory assessments, Sally's beliefs regarding mathematical discourse, as well as concepts and procedures may have changed. Both of these beliefs appeared to become less consistent with the Standards in the beliefs assessments. Sally's beliefs regarding differentiated instruction appeared to become more consistent with the standards in the beliefs assessments. It is worth noting that Sally never scored "inconsistent" with the standard on items in the post-assessment. All of her results were either coded as "consistent" or "undecided".

Throughout the intervention, Sally made a total of 13 claims that were consistent with the Standards described in the beliefs inventories. She also made one claim that was inconsistent with the Standards described in the beliefs inventories.

During the intervention, Sally made three claims regarding mathematical discourse. Two of these claims were consistent with the standards described in the beliefs inventory assessments, but the first claim was inconsistent. Sally expressed concern about students using solutions that other students had provided. By the end of the intervention, Sally appeared more comfortable with the idea of students sharing solution strategies and learning from their peers.

> By the end of the implementation of the third cycle of tasks, I was able to see growth, or a refining of thought processes of my students. Students used ideas from previous tasks

> and expanded upon them. Students also took ideas from other students who shared ideas of recursion and holding a constant in order to achieve the correct answers
> (Sally Final Project, p. 32)

Although the beliefs inventory assessment data appears to indicate that Sally's beliefs regarding mathematical discourse became less consistent over the course of the intervention, data from the intervention supports the claim that her beliers regarding mathematical discourse became more consistent over the course of the intervention.

Sally made one claim regarding concepts and procedures during the intervention. This claim was coded as consistent with the Standards described in the beliefs inventory. There is not sufficient evidence from the intervention to support the claim that Sally's beliefs regarding concepts and procedures changed over the course of the intervention.

During the intervention, Sally made two claims regarding differentiated instruction. Both of these claims were coded as consistent with the standards described in the beliefs inventory. In her final reflection, Sally claimed: "I also learned that there are countless representations and methods for finding answers to one problem, and they are all valuable, especially to make connections in order to deepen understanding." Data from the intervention supports the claim that Sally's beliefs regarding differentiated instruction did change over the course of the intervention.

Summary

Sally's response of "undecided" on several of the post-assessment items led to an apparent change in consistency with the standards described in the beliefs assessments. Data from the intervention demonstrates an increase in consistency with Standards regarding mathematical discourse and differentiated instruction.

7.1.8 Summary

For each teacher, beliefs inventory assessment results were compared to coded claims from the intervention data. Some of these data supported claims of a teacher's change in beliefs. Other times, the data indicated possible changes in beliefs that were not captured in the assessment results.

7.2 Beliefs by Cycle

The following table organizes beliefs statements by all teachers in the intervention by cycle.

Cycle	Consistent	Inconsistent	Undecided
Cycle 1	22 (~71%)	8 (~26%)	1 (~3%)
Cycle 2	21 (~81%)	3 (~12%)	2 (~8%)
Cycle 3	25 (~78%)	3 (~9%)	4 (~17%)
Final Projects	33 (~73%)	0 (0%)	12 (~27%)

Table 7.9 Beliefs Statements by Cycle

When organized by cycle, the beliefs data reveal a consistent decrease in the number of claims made that were inconsistent with the standards presented in the assessment. In the final projects, teachers made many claims that could not be identified as consistent or inconsistent with standards. For example, Connie claimed: "My students loved working with manipulatives! I think it helped motivate them to try and get the correct answer." This statement did not appear to describe manipulatives as having limited value, but it did not describe manipulatives as a valuable tool for reasoning. Because of the number of "undecided" statements, the lack of inconsistent statements in the teachers' final projects is a more meaningful number than the relative percentage of "consistent" claims made in the final projects.

7.3 Beliefs by Stage

Sorting teachers' beliefs claims by stage yields insight into the settings in which beliefs are most easily expressed. Table 4.3.3a shows the frequency of claims sorted by stage. (Note: Stage 3 is not included on the table. There is no data for that stage). The data in the table shows that teachers made the most claims about beliefs while analyzing and discussing research. The instructor's use of discussion questions that drew out teacher beliefs is a likely explanation for the relatively high number of claims about beliefs made during this stage of the intervention.

Belief Category	Stage 1: Working on Tasks	Stage 2: Reviewing Literature and Video	Stage 4: Discussing Implementation
Learning	2	15	6
Teaching	3	23	8
Mathematics	0	3	1
Expectations and Abilities	0	7	6
Mathematical Discourse	0	8	7
Concepts and Procedures	0	7	6
Manipulatives	0	3	3
Student and Teacher Roles	4	24	9
Differentiated Instruction	1	3	7

Table 7.10 Frequency of beliefs statements by Stage.

The data reveal that discussion of research literature and video led to a significant increase in claims regarding beliefs about student and teacher roles, as well as learning and teaching. There are several interesting features of these claims. The majority of claims in all three categories were consistent with the Standards presented in the beliefs inventory assessments. Table 7.3.2 indicates the frequency of consistency codes applied to claims in these categories. It is worth noting that all of the claims coded as "inconsistent" with the Standards described in the beliefs assessment occurred during the week of 9/30, relatively early in the intervention. Also, the claims coded as "undecided" in regard to the standards described in the beliefs assessment occurred during the month of October, during the middle third of the intervention. In the second stage of each cycle, there was a decrease in the number of "inconsistent" and "undecided" claims regarding beliefs about learning, teaching and student and teacher riles over the course of the intervention

Category	Consistent	Inconsistent	Undecided
Learning	11	2	2
Teaching	19	4	0
Student and Teacher Roles	22	1	1

Table 7.11 Consistency of beliefs statements

7.4 Summary

Data regarding teacher beliefs was coded for its relationship to the Standards described in the beliefs inventory. These data were examined in several ways. Sorting the beliefs data by teacher allowed for an interpretation of each teacher's beliefs assessment, based on question categories formed by grouping questions from the beliefs inventory. For some teachers, data from the intervention supported the assessment results. For some teachers, the intervention data indicated possible changes in beliefs that were note captured by the beliefs inventory assessments. Sorting beliefs data by cycle revealed a decrease in the number of "inconsistent" claims over the course of the intervention. Sorting the beliefs data by stage revealed the teachers' discussion of research as a fruitful source of claims regarding beliefs about learning, teaching, and student and teacher roles. Moreover, claims in these categories appeared to become more consistent with the standards described in the beliefs inventory assessments.

8 TEACHER NARRATIVES

In this section teacher narratives, organized by groups, are presented. Although the same videos were observed, the same chapters were assigned to be read, and the same tasks were implemented, the teachers were observed to attend to different concepts, ideas, or actions. The following section describes each teacher's experience within the intervention. These narratives were constructed form the data included in the eCollege discussion threads, the transcripts of regional meetings, and the teachers' final projects.

The narratives that follow, are organized by teacher groups who worked on the same tasks together, in each cycle. Rich, Connie and Angela are referred to as Group 1. Kate and Sally are referred to as Group 2. Justin and Mitch worked together as Group 3. A detailed narrative is included for one teacher in each group: Connie, Kate, and Justin.

8.1 Connie

Connie, a sixth grade teacher from Sayreville, demonstrated a transition from belief in a direct instruction model to a belief that students could learn from each other while discussing their approaches to a problem. Events in this transition are described in the section that follows with a description of Connie's attention to student reasoning.

8.1.1 Connie's Beliefs

After completing the pre-assessments and working on the 4-tall towers task at Rutgers, Connie indicated that she did not expect her students to use the strategy of constructing opposites to develop a solution to the 2-tall towers task. In a discussion on the online forum, in response to another teacher's claim that students would not use the idea of opposites to construct towers, Connie stated: "I agree with you about not thinking that my students would use the idea of 'opposites'."(9-16 Discussion). However, Connie did mention being interested in understanding the variety of solution methods for a problem, indicating, "It's always interesting to see how many ways a problem can be solved." (9/16 Discussion).

After implementing the towers task in her class, Connie was surprised that many of her students did use an opposites strategy to construct a solution. She reported, "I was very surprised when I saw how my students built towers that were 4 high. I would say about 80-90% of them used the idea of 'opposites'." (9/23 Discussion)." Connie was not convinced by this form of argument, however and reported, "They did not really come up with a conclusive argument about why they definitely had the correct answer and that there were no more towers." (9/23 Discussion)

After implementing the 4-tall towers task, and noting that students' arguments were generally not convincing, Connie remarked: "I do believe that if I had introduced this activity after teaching the problem-solving strategy of 'make an organized list', my students would have developed an exhaustive way of determining that they had reached the correct amount of towers without duplicates." (9/30 Discussion).

Connie was one of the first teachers to recognize the value in having students share solutions with their classmates. After watching a video clip in which Milin explained his strategy to Michelle, she cited Michelle's ability to continue with the strategy as evidence that she truly understood Milin's strategy.

> He [Milin] seemed to convince Michelle too. At first, Michelle didn't seem to understand, but once she started seeing what Milan was doing, she seemed to catch on. She was even able to share with Stephanie and Matt what Milan's train of thought was. This demonstrates true understanding when she was able to discuss the argument with other students and convince them successfully.

(9/30 Discussion)

At the first after school meeting on 10/7, Connie identified cases arguments, and described examples of student work as using elevators and opposites (10/7 Meeting transcript 1 of 3, lines 495-497). Connie worked on the 5-tall towers task, selecting from two colors, with Rich and Angela. Rich took the lead, but Connie and Angela expressed a concern that Rich's method would yield duplicates (10-7 Meeting transcript 2 of 3, line 248). Connie made an attempt to follow along with Rich's reasoning and continued to make towers following his strategy.

During the week of 10/7, the teachers viewed a video of a researcher interviewing a student, Brandon, about his solution to the pizza problem, selecting from 4 toppings. Connie appeared to be impressed, both by the student's reasoning and by the researcher's skill as a questioner. In a response to another student's post in the online discussion, Connie claimed:

> I think waiting for the students to develop their own thoughts and ideas is very interesting. I also think you are right about creating a positive and comfortable environment for the students is important too! When students do not feel nervous about the correctness of their answers, they feel more confident about exploring their ideas. I definitely will be using more of the techniques I saw in this video with my students during the 5-tall tower problem.
>
> (10/7 Discussion)

Connie appeared to have used some of these questioning techniques in her classroom implementation of the 5-tall towers problem. Connie claimed: "my questioning techniques were much better this time and I could see how the students responses were more thought out." (10/14 Discussion).

In describing the work of her class on the 5-tall towers problem, Connie stated that she first shared two examples of student work with the class; one which used opposites and one which used a staircase pattern. (The instructor had recommended that the teachers share student solutions to the 4-tall towers problem prior to the implementation of the 5-tall towers problem.) Connie noticed that more students were using the staircase method when attempting to solve the 5-tall towers problem. Additionally, she reported that these students appeared to be better able to describe the strategy and said it made sense to them.

> I do think it is students' tendency to try and follow examples of other work that they see. Students who may have felt lost last time with developing solutions, may have felt more comfortable in following some of the examples they saw. I saw that with almost every single group. When I walked around the classroom, I saw several students duplicating ideas from the few examples I went over last week (one showing the "staircase" and one using a tree diagram). I normally might have been disappointed that they simply copied ideas from other students instead of developing their own. However, they were able to explain why they were using those ideas. I thought that it showed growth for them.
>
> (10/14 Discussion, lines 426-434)

During the meeting on October 28, a discussion arose in which some teachers referred to the adopting of another students' strategy as "stealing". Connie also expressed concern, and in sharing her experience (the use of staircases on 5-tall towers) with the cohort said:

> One caught it and the rest they did pairs, but I did hear when I walked around and said "Oh why did you choose to do a staircase?" and they said "Well, now I finally, I understand like, how they did that" but I said "Can you apply that to you other ones?" and they, some had a little difficulty. I mean, I don't know, is it ever a bad thing? I don't know.
>
> (10/28 Meeting Transcript 1 of 2 line 763)

At this meeting, Connie shared an example of student work on the 5-tall towers problem which made use of a tree diagram, and noted that this representation supported an inductive approach. (10/28 Meeting transcript 1 of 2, line 119) Despite agreeing with the student's solution, and noting that the work was easy to follow, Connie noted that the student's written argument was not convincing.

Connie took the lead in her group when working on the 3-tall towers problem, selecting from 3 colors. She followed an inductive approach and justified her thinking for her groupmates. (10/28 Meeting Transcript 1 of 2, lines 848-926). When her group transitioned to Ankur's Challenge, they attempted to add blocks to the towers they had made as solutions to the 3-tall towers problem, selecting from 3 colors. Using this approach the group had identified 37 as the number of towers that satisfied Ankur's condition that the towers be 4-blocks tall, selecting from 3 colors and have at least one block of each color. Although Rich was comfortable with this solution, Connie and Angela continued to attempt to find ways of organizing the towers to identify duplicates, missing towers, or justify the completeness of their solution. (10/28 Meeting Transcript 2 of 2, lines 425-832).

When describing Romina's approach to Ankur's challenge, Connie noted that timing constraints often impose limits on students' abilities to describe and refine their reasoning. She also described the value in allowing students to revisit and reconstruct their justifications:

> I know that even for us as adults, the first time writing things down in a timed period doesn't allow for complete thoughts to be represented. It can be difficult for students to explain their reasoning and even more difficult for them to write them down. Giving them more time can allow them to clarify their explanation, and give them more insight on how to improve their demonstration of their reasoning. It is evident that more time allowed Romina to better explain her thought process. She was a little frustrated at first trying to explain her reasoning. She even said she needed to collect her thoughts. However, the more times she had to explain to her fellow classmates, the better she was able to demonstrate her correct explanation.
>
> (11/04 Discussion)

After implementing the 3-tall towers problem, selecting from 3 colors in her class, Connie claimed:

> I also definitely saw pairs this time talking more than before. I feel that they felt more confident after being exposed to these types of problems several times and both people in the pair felt more comfortable discussing their reasoning. I really liked when I would go over and they would be discussing and debating which method was correct and why. I was surprised that so many of my students got the answer of 27 towers for this problem! Overall, their methods and reasoning have gotten better!

(11/11 Discussion)

Connie demonstrated a change in beliefs about learning and teaching. In her description of student work on the 4-tall towers problem, Connie recognized that most students were able to find all 16 towers, but that their arguments were not convincing. She claimed that the students would have done a better job if they had been taught a lesson on making organized lists. In the week 10 discussion, she mentioned that students were able to give better arguments, and appreciated hearing students debate which methods were more productive. "both people in the pair felt more comfortable discussing their reasoning. I really liked when I would go over and they would be discussing and debating which method was correct and why." (11/11 Discussion) Connie noted this improvement in her final project as well, reporting, "implementing these tasks has significantly improved my students' mathematical learning, reasoning, and communication skills."(Final Project lines 473-476) She also remarked in her final project, "I held back from helping the students too much. I allowed them to do more of the talking, which I found they really liked." (KF Final Project P. 17)

Several times throughout the intervention, Connie mentioned that her students worked on the intervention tasks without being taught the strategy of making an organized list (9/30 Discussion, 10/7 Discussion, Final Project). During the focus group meeting before the final regional meeting and discussion, Connie described the result of waiting to teach the strategy. "And I actually taught it one week after last cycle. Like the last cycle's tasks. And they did wonderfully with it, better than any of my other years ever...They were like 'This is so easy'" (12-04 Focus Group, Lines 285-291)

8.1.2 Connie's Attention to Reasoning

In addition to demonstrating a change in beliefs about learning and teaching mathematics, Connie also demonstrated growth in her attention to student reasoning throughout the intervention. In her descriptions of from week 2, she made vague claims about the student reasoning.

> In 2nd grade, all three students drew pictures with letters to represent the colors of the shirts and pants. Stephanie and Dana tried to list out the combinations and for one reason or another they forgot to include the 6th combination. Michael drew pictures, but just matched the particular color shirt with the same color pants and did not go any further. He included a color that was not even there for the color of pants (yellow). From the explanation, it seemed like Dana was the only one who was on the right track of reasoning to get a correct answer of 6 combinations, except for the fact that she didn't think a yellow shirt would go with white pants.
>
> (9/16 Discussion)

Her description of Milin's strategy in week 4 was more detailed and included a reference to the way in which it was accepted by other students. Connie's summary of Brandon's work was also detailed. In addition to the summary, Connie described the environment which the interviewer of Brandon created as being necessary for students to feel comfortable sharing their reasoning.

> I think wait time is very important. Often, teachers do try to finish off thoughts for students. I think waiting for the students to develop their own thoughts and ideas is very interesting. I also think you are right about creating a positive and comfortable environment for the students is important too! When students do not feel nervous about the correctness of their answers, they feel more confident about exploring their ideas.
>
> (10/7 Discussion)

In this description, Connie also recognized a tendency for teachers to try and complete the thoughts of their students, rather than hear what they have to say. In the final example the cohort studied, Romina's proof to Ankur's Challenge, Connie gave a detailed description of Romina's reasoning.

> She knew that in towers that were 4-tall choosing from 3 different colors that in each tower she would have a duplicate of one of the colors. She chose to stick with one color that would have a double, at a time. She designated 1's to represent where the duplicated color's blocks would be in the 4-tall tower. She used the idea of location of those duplicated colors to organize her options. She started with the duplicated colors being in the 1st and 2nd position with the remaining two blocks having a different color option. Then she went on to move the 1's to the 1st and 3rd, then 1st and 4th, 2nd and 3rd, 2nd and 4th, and then 3rd and 4th positions, resulting in 6 different towers. She knew that the other two blocks could have two options(she used x's and o's) so she multiplied her 6 possible tower possibilities by 2 to get all the possible towers with that one color being used twice. She then multiplied her answer of 12 by 3 in order to get all the towers with the other colors being used twice.

(10/28 Discussion)

Over the course of the intervention, Connie showed attention to the details of students' reasoning, both from the videos she observed and in her classroom practice. Attending to the strategies used by her students in their problem solving, her beliefs about what students could do, under conditions she described as "waiting for the students to develop their own thoughts" (10/7 Discussion) showed indications of higher expectations for student successful problem solving.

8.2 Angela

Angela, a seventh grade teacher from Sayreville, had high initial expectations for her students' approach to the problems. After observing student work, she was surprised that the students did not begin with a systematic approach (9/23 Discussion, 10/28 Discussion). She put effort into helping her students develop stronger arguments, (11/11 Discussion, Final Project).

Angela appeared to have a relatively strong, albeit procedural approach to doing mathematics. She expected her students to approach the tasks in the same way that adults using a procedure would. In the videos of third-grade student work on the towers problem, Angela was not convinced by the reasoning demonstrated. She considered Stephanie's 3rd grade work on the towers problem to be based on trial and error, and expected more sophisticated arguments from her middle-school students: "This is how I would expect a 3rd grader to respond. When it comes to our middle schoolers, I would expect a more in depth proof of why there are only 16 combinations" (9/16 Discussion). Angela had little tolerance for following approaches that were not convincing to her. In the October 7th meeting, her groupmates attempted the 5-tall towers problem using a system of opposites. Angela chose to pursue her own solution working with a tree diagram, rather than work with the rest of her group. When sharing her work at the meeting, she claimed: "We kind of started like they were working and I was working in my notebook, and I, they were working on a strategy like Jared and Mitch were working on, and I just, I don't know, I just couldn't see it" (10/7 Group Meeting Transcript 2 of 3, Line 1187).

Angela did not expect her students to use opposites, because it was not a concept taught to them.

> My original thought in class was that my students, just like yours, would not use the concept of "opposites." I personally have not spoken to them about this concept in depth, and neither have they brought it up on their own from previous knowledge during any of the tasks in my classroom. I will find it very interesting to see their differing approaches to this task, and if they do use the concept of "opposite."
> (9/16 Discussion)

When discussing her implementation of the 4-tall towers task at the first after school meeting, she claimed: "Yeah, they did [use opposites]. I find that funny, because I was never taught that way" (10/28 Discussion). After implementing the 4-tall towers problem in her class, she was surprised to see that most of her students described opposites, or a guess and check method in their solutions (9/23 Discussion). Angela was not convinced by the reasoning of most of her students, and claimed: "I feel that with a good method of organizing being pointed out to them, more of my students would have been successful with this task" (9/30 Discussion). In her final project, Angela noted her students' improvements in constructing a reasonable argument. She credits this to the fact that she showed examples of good student reasoning from the previous task prior to implementing the next task. For example, before implementing the task requiring students to build 5-tall towers, she showed a piece of student work on the 4-tall towers problem. The student used a staircase pattern to organize towers. Angela described the students' work in the online discussion. "I had a pair of students who used the staircase method of organization to build their towers present their answer and explanation in class and we discussed briefly how organization was so important to this task. We spoke about how it was a sure way to know you were systematically building each tower" (10/14 Discussion). Angela noticed that some students used this pattern and that in general, students were more organized in their approach to the problem.

After implementing the 3-tall towers task, selecting from 3 colors, Ashley shared her solution with the class. In the online forum, she claimed: "After my students were done with this task, I shared with them my strategy of solving this problem. Many of them said they thought that strategy would have helped them organize their towers better to see and prove to themselves that there were no more towers" (11/11 Discussion).

In the pre-assessments, Angela commented that none of the students seemed to make a plan to approach the problem before they began working on it. She expected her 7th grade students to use tree diagrams or combinations to make accurate predictions of the solutions to the 3-tall and 5-tall towers problems. When Angela noticed that her students did not begin the first task cycle with a plan, she changed the requirements of the second two task cycles by specifically requiring students to make a prediction of the solution before working on the task. "As they started I just went around and I said 'I don't want an explanation. How many do you predict? How many do you predict?'" (11/18 Group Meeting Transcript 1 of 2 line 446).

Angela's focus on procedures may have been a detriment to her ability to evaluate student reasoning. In the discussion thread for week three, she describes Milin's argument and considers it an example of "solving a simpler problem" (9/23 Discussion). She does not describe Stephanie's cases argument at all. Her focus on Milin's inductive strategy at the expense of a cases argument is made evident by the fact that she required students to solve the towers problem for towers 1-tall and 2-tall as a "do now" exercise in early October, before the implementation of the 5-tall towers problem (9/30 Discussion). Later in the intervention, Angela noticed more forms of reasoning. After reading "Making Pizzas: Reasoning by Cases and Recursion" (Maher, Sran, & Yankelewitz, 2011) , she described Group 1's use of a cases argument and described the structure of cases by number of topping as similar in structure to Stephanie's cases argument in the 4-tall towers problem. Angela also noted that Group 2 used a recursive argument but did not describe it in detail "Group # 2 used the recursive argument justification. The group kept listing pizzas until they found duplicates" (10/07 Discussion). She also gave a detailed description of the solution of Ankur's problem by Romina and group when it was presented:

> Romina first found that in the towers 4 high containing at least one of each color, there had to be a duplicate of one of the colors. That left the 2 remaining spaces for the other 2 colors; one of each of them to ensure the requirements were being met. She then came up with a diagram to represent the 6 different towers and the position of the cube whose color was to be duplicated in the tower. She placed the remaining colors in the remaining spots. She ten knew that the 2 remaining colors could be switched with one another to form a completely different tower, in essence doubling the amount of towers in her diagram. This is why she multiplied by 2 when finding her solution to Ankur's challenge.
>
> (11/04 Discussion)

Angela noted that her students did not respond to her questioning on the 4-tall towers problem. "When questioning them, the students seemed to think they had been doing something wrong" (9/30 Discussion). After watching the interview of Brandon. She described her desire to emulate the interviewer's technique.

> "I either feel that I give too much information to lead them to an answer or not enough information, and do not get the response I intended from my students. I hope to someday be able to be a skilled questioner, just as much so as the interviewer. She got as much information from Brandon as she intended without feeding him too much information or guiding.
>
> (10/07 Discussion)

8.3 Rich

Rich, a 6th grade teacher from Sayreville, was a caring teacher. He had a tendency to mix behavioral assessments with his assessments of student reasoning. (10/7 Group Meeting Transcript 1 of 3, 11/18 Group Meeting Transcript 1 of 3) While evaluating student reasoning, he may have given his students too much credit, accepting incomplete arguments as convincing. (10/28 Transcript 1 of 2 lines 54-60) At times, though, he demonstrated a keen ability to follow the organization of student work. (10/7 Meeting Transcript 1 of 3, line 529)

Rich was the only teacher to state that he expected his students to use the concept of opposites while working on the 4-tall towers problem "I think some of your students may surprise you and use the idea of opposites to support their work" (9/16 Discussion). In total, four teachers made statements that they didn't expect their students to use opposites. The other two teachers did not mention opposites at all in their predictions of how their students would work on the 4-tall towers problem. Rich was also one of the two teachers who appreciated Stephanie and Dana's use of "families" and "cousins" as a more sophisticated version of opposites while also acknowledging that their use would not guarantee the completeness of their solutions: "Their method does not eliminate the potential for a missing combination" (9/16 Discussion).

Although Rich did claim that some student reasoning was not convincing, he may have been generous in his evaluation of student reasoning. In the week 7 discussion thread, Rich described the student responses to his question of whether the pizza problem reminded them of any other problem.

> I think they were trying to tell me what they thought I wanted to hear. However, one pair of students said because it was four toppings and there were four blocks and each topping represented a specific color. Another pair of students said because 16 was the solution for both. I feel this was insightful. I did not expect any of them to see the connection. I am happy that four students made the connection and the rest of my students had a good time exploring math.
>
> (10/21Discussion, lines 30-35)

In the group meetings, Rich was able to describe patterns in student work, even at times when the teacher presenting the work was not sure how the student had organized his or her work. For example, In the October 7 meeting, Angela had difficulty following a student's representation, but Rich noticed it as a "butterfly wing" organization of opposites (10/7 Group Meeting Transcript 1 of 3, Line 807).

Sometimes, Rich's demeanor led to lower expectations for students. In the October 28 meeting, he shared a piece of student work in which the student described a method of organizing 5-tall towers with 4 blocks of one color and 1 block of another color. The instructor then asked if the students had developed an argument for towers in which there were 2 blocks of one color and 3 blocks of the other color. Rich's response was as follows:

> The three and two, they really didn't, and I didn't want to push them at this point. I was really happy I have to be honest, that they got the 32 combinations. Then they were able to help others, and even helping take apart blocks and put them in the bags, so they were really able to do good. I didn't want to give them any follow up questions to that, knowing the pizza question was coming.
>
> (10/28 Meeting transcript 1 of 2 lines 58-60)

Through working on the tasks posed in the intervention, Rich came to a deeper understanding of mathematics. He had used patterns of opposite towers to approach the 5-tall towers problem (10/7 Group Meeting Transcript 1 of 3). Some of his group members demonstrated frustration with his methods "Can you explain to me, like this was just random in my head" (1-/7 Group Meting Discussion, Line 331). When the task of constructing 3-tall towers selecting from three colors was presented, his groupmates argued for a more systematic approach. After following their reasoning, and attempting to build the towers according to the pattern proposed by his groupmates, Rich described his experience: "I understand what we're doing, it's just not the natural way I would do it. Which is okay" (10/28 Group Meeting Transcript 1 of 2, Line 941).

Rich's perception of his role as a teacher appeared to change throughout the course of the intervention. After admitting that he did not have students share their solutions to the 4-tall towers problem before they attempted the 5-tall towers problem, he decided to share strategies to these problems. He also felt that he had not asked students enough eliciting questions during their work on the earlier tasks, but noted an improvement in later tasks: "My questioning was better this time around and I was able to get them to describe what they actually doing" (10/14 Discussion). By the end of the intervention, Rich was energized in his role as a facilitator. He eagerly shared information about a PLC experience in which he modeled a problem based lesson for a fellow teacher (Focus Group Transcript, Lines 342 - 350). Rich also remarked that, in the following school year, he would like to begin his classes with the tasks implemented in this intervention. He claimed that these tasks would be good at establishing the tone of his class and would familiarize students with his role as a facilitator: "And it gets them comfortable with you being a facilitator in the classroom." (Focus Group Transcript).

8.4 Kate

Kate, a seventh grade teacher from Old Bridge, showed evidence of giving increased attention to student reasoning, a change in beliefs about student abilities, and a consistent dedication to improving her practice as a teacher. Kate acknowledged progress in her abilities as a teacher and noted progress in her students as well (Final Project, 11/18, 11/04 Discussion). Events in this transition are described in the section that follows.

8.4.1 Kate's Attention to Student Reasoning

Kate's first post of the year demonstrated that she valued the structure of the intervention. After admitting that she did not follow the reasoning presented by some students in the videos, she claimed that working on the same task gave her a frame of reference, from which she could make sense of the examples of reasoning.

> I did not follow the logic behind some of the students' explanations until I actually did the task myself and started realizing that I was grouping the towers using the same

> method. That really showed me that there is a tremendous difference in watching someone participate in the activity and in doing it yourself. The "experience" is important!
>
> (9/16 Discussion)

Initially, Kate gave undetailed descriptions of student reasoning. When describing the work of two research students, Stephanie and Dana, she claimed: "I did not find much difference in how they approached the problems from third to fourth grade. Both times they seemed to randomly try combinations and then they started checking for repeats" (9/23 Discussion). Kate's descriptions of her student work in cycle 1 were also undetailed. "Most of my students used the "opposites" argument, if that is one. I was not delighted with any reasoning that my students verbalized or put in writing." (9/30 Discussion)

In cycle 1, Kate compared forms of reasoning and noted similarities, but did not describe details. Kate described one current student's work as being similar to Milin's (9/23 Discussion). Kate also evaluated student arguments in cycle 1. She expressed a concern about an incorrect generalization of the 4-tall towers problem, but did not describe the reasoning behind the student's solution to the towers problem. "I feel like we tricked them with the 4-tall towers because if we had started with 3-tall towers they would have realized that 3 x 3 doesn't work because they would have built 8 towers. I think this would have made them think harder to develop a pattern" (9/30 Discussion).

Kate found Milin's argument very convincing, and when students shared reasoning that was not based on a doubling pattern, Kate stated that she felt the students had been tricked by the 4-tall towers problem: "I feel like we set our students up to be confused because we started them with the four---tall towers and didn't allow them to use the blocks to see a connection between, 1, 2,3 and 5---tall towers" (9/30 Discussion). Kate's claim was that starting with 4-tall towers tricked students, and that in order to identify the doubling pattern, students should have started by constructing smaller towers. This claim led to a discussion about the purpose of engaging students in constructing solutions to the problems; that is, discovering patterns and relationships.

In cycle 2, Kate gave slightly more detailed descriptions of student work. "It seemed to me that the recursive argument that group two used really helped to keep them organized. Their systematic way of approaching the problem helped them to be sure that they had no duplicates and found all possible combinations" (10/7 Discussion). She also compared two student strategies that she thought were both correct. "There was nothing wrong with group one using the proof by cases, however there was a better chance of coming up with duplicates, so that was something they had to watch out for." (10/07 Discussion)

After the cycle 2 implementation, Kate described a variety of student approaches to the problem. She compared students' work on the 4-tall towers task to their work on the 5-tall towers problem:

> Most of my students used the "opposites" argument when they built the 4-tall towers, this was still a popular strategy but it seemed to be a last resort. Many groups made a staircase pattern and its opposite and then resorted to opposite pairs. One or two groups controlled for a variable or tried a recursive argument
> (10/14 Discussion)

In the second afterschool meeting, Kate described her interactions with some students to support their reasoning on the 5-tall towers problem: "I said to them 'If you're just going to keep doing opposites, I don't think you're going to get anywhere. You're going to have to organize better'" (10/28 Discussion). She also noted that she perceived an improvement in student reasoning: "I have seen growth in my students' thought process since the first task with the 4-tall towers" (10/28 Discussion). In her final project, Kate also described her students' improvements during cycle 2: "I started to see my students notice patterns and use more organized strategies to arrive at an answer. Their verbal arguments got better and their written justifications came along as well" (Final Project Cycle 2).

In cycle 3, Kate demonstrated increased attention to student reasoning, and provided more detailed descriptions of student reasoning. Kate's description of Romina's proof is included below:

> Initially Romina came up with all the positions that the one duplicate color could be located in, this gave her six to start with. Since three colors had to be used; that left the two remaining colors in each of the spots left. She multiplied by 2 to account for the possibility of 2 different colors being in each of the positions that remained. 6 x 2 = 12. There are twelve different towers that can be built when one of the colors remains in duplicate positions and the other two colors alternate positions. Romina was then able to generalize and extend her pattern for the possibility of all three colors being in the duplicate position
>
> (11/04 Discussion)

When contrasted with Kate's description of Stephanie and Dana's work (9/23 Discussion) this description of a research student's reasoning indicates much more detail and suggests a more detailed attention to student reasoning.

Kate claimed to see continued improvement in student reasoning on the cycle 3 tasks. "I had several students use the same method of controlling for a variable on the bottom and the top. I was very pleased to see this; I feel like my students have come a long way also! They are organizing their towers with more thought than just "opposites", which is what they started with" (11/11 Discussion).

In her final project, Kate described her students' progress during the intervention:

> Most groups had progressed from the simple argument of "opposites" to more sophisticated arguments which included, controlling for one or more variables and using a recursive argument… I think the best part about this last set of tasks is that the students started to see their own progress; they started to feel good about thinking more mathematically so they were willing to try different strategies
>
> (Final Project Cycle 3)

Kate's final project also contains evidence or her increased attention to student reasoning. In the project, she revisited student work from cycle 1. This time she identified more detailed reasoning. Two examples are included below:

> I did not recall any of my groups talking about anything more sophisticated than "opposites" during the first task. When I went back through my students' work and saw the beautiful organization of her drawings and the write-up using the word "staircase", I was thrilled. She clearly has some insight into developing a pattern.
>
> (Final Project)

> I was impressed by this students' reasoning because I feel like she had the beginnings of the idea to control for a variable. Her drawings look simply like opposites, but I feel her explanation shows a little more insight than that. Especially if you look at her prediction for the 5-tall towers; she talks about the different combinations in a generic form, 3 and 2, 2 and 3, 1 and 4, 4 and 1, 5 and 0. She doesn't need to think of actual colors, but it appears that she already has a picture of combinations in her head. Impressive for the first task!
>
> (Final Project)

Kate identified many examples of student reasoning, and identified several student arguments that were incomplete (Final Project), but she may have been too willing to accept one student's development of a formula as a solution to Ankur's Challenge. This student presented a numerical expression equivalent to 36 in their solution to the Ankur's Challenge problem. This student did not show any towers on her paper, and had various expressions written on her page in addition to 3 x 3 x 2 x 2. Kate described her work as follows:

> I was most surprised with this student above all others. She seems to be an average math student who doesn't say much in class. She was the only one of my students that even thought to attempt this problem by trying to come up with a formula. She went through a few different ideas before she came up with one that seemed to work. Her explanation makes sense, "you can pick from three colors in two spots, and in the last two spots you can only choose from two colors." 3 x 3 x 2 x 2 = 36 The mathematician at our last meeting said she wasn't sure this would hold up, but I think it is a sound argument that both surprised and impressed me! I am proud of her mathematical reasoning.
>
> (KK Final Project P. 51)

Kate's example serves as a warning against allowing similar interventions to focus on written mathematical expressions as evidence of student reasoning. In the case of this task, the use of towers (whether physical or drawn) assists teachers in focusing on the reasoning or strategies rather than the numerical value of the solution.

Despite the example above, Kate did demonstrate an increase in attention to student reasoning during the intervention. She identified cases, recursive, and inductive arguments and described examples of student reasoning in detail.

8.4.2 Kate's Role as a Teacher

After observing student work in cycle 1, Kate expressed a disappointment in her students' work, but she maintained responsibility for the quality not meeting her expectations. In her final project, she described this feeling: "I had expected more from my students. I was disappointed with their lack of strategies and I was also disappointed with my lack of questioning skills. I didn't ask the right questions to pull information out of them" (Final Project Cycle 1). Kate expressed concern with her ability to get her students to communicate their reasoning. She claimed: "I honestly at this point, I don't know if I feel qualified to be questioning them because I don't know what I'm supposed to be looking for" (10/07 Group Meeting Transcript 1 of 3).

After watching the researcher's interview of Brandon, she commented on the researcher's ability as a questioner: "Her answer was not leading in any specific direction, which allowed Brandon to keep thinking for himself. I'm afraid that my questions are too leading. I need to learn to be more like Amy [the researcher]" (10/07 Discussion).

Kate used this feeling of a deficit to spur her growth. When discussing the "Brandon's Proof and Isomorphism" (Maher & Martino, 1998), Justin shared a set of questions he typically uses to get his students thinking. Kate indicated a desire to improve her own questioning techniques: "Being a good questioner is definitely a skill that can be developed, but it takes practice and thought. This course is making me more aware of how I question my students. I need to make sure I am not asking leading questions. The open-ended questions you mentioned are great" (10/14 Discussion).

In cycle 3, Kate noted that her efforts at questioning her students and focusing on their reasoning were yielding results. When discussing the implementation, she claimed: "Their verbal arguments are getting stronger, my students are starting to understand what I mean when I say, 'Justify your answer'" (11/11 Discussion).

Reflecting on her own practice during cycle 3, Kate claimed: "I feel my ability to ask delving questions improved by these tasks so I was able to pull more from my students forcing them to think more deeply about what they were doing" (Final Project).

Kate also noted the relationship between her improved questioning and attention to student reasoning, and the quality of work provided by her students. In her final reflection, she claimed:

> At first I think my students were so concerned with getting the right answer that they didn't allow themselves time to really explore the problem. Once they realized that I was looking more at how they went about coming up with their answer than the actual answer itself, they seemed more willing to spend more time thinking more deeply about the problem.
>
> (Final Project)

In addition to improving her questioning techniques and attending more to student reasoning, Kate made intentional efforts to have students revisit the tasks they had previously completed. After studying Romina's solution to Ankur's Challenge, Kate indicated plans to have students review some of their peers' solutions to other tasks. "I am going to review some student answers from the 5-tall towers with my classes before I do the 3-tall with 3 colors and Ankur's Challenge. I am hoping they start to form some ideas about the patterns they have seen in the other towers problems and relate this to the next task" (10/28 Discussion). In a later discussion, Kate indicated the value in having students revisit problems: "Every time we have students verbalize, write or think further about a problem it gives them a chance to internalize the problem and get a better feel for it. As you mentioned, they can change their focus from the numerical answer to focusing on the justification and reason for their answer" (11/04 Discussion). Kate also referred to her own experience teaching multiple classes per day to describe the value in allowing students to revisit problems and rebuild their solutions: "I tend to make deeper connections as the day goes on, or explain in more depth because I've noticed something that I hadn't before" (11/04 Discussion).

Before the final on-campus meeting, Kate described how Romina's proof made an impression on her:

> That made me realize I really need to give them that time to repeat it, do it again, and explain it again, Not just, it's a one-time thing, 'good, that's what you did.' We need to give them time to get that thought process going so that they can organize it more and. The more exposure they have to it, the better.
>
> (Focus Group Transcript).

8.4.3 Kate's Beliefs

Kate demonstrated an increasing attention to student reasoning (9/23 Discussion, 10/14 Discussion, 11/04 Discussion) but at times she expressed a bias towards accepting arguments with formal mathematical symbols without the same attention to detail she applied to other student arguments (11/18 Group Meeting Transcript 1 of 3, Final Project). Throughout the intervention, Kate expressed a desire to improve her practice as a teacher, and described her own progress as well as the progress of her students as they developed mathematical reasoning (10/07 Group Meeting Transcript 1 of 3, 10/14 Discussion, 11/11 Discussion, Final Project). In her final project, Kate described the value of the tasks implemented in this intervention:

> What was so enlightening about these activities is that I got to see into some of my students reasoning in a way that I never would have without these tasks. Several of my "Basic Skills" students impressed me the most with their reasoning and I would not have

> expected that. This just goes to show what children are capable of if we let them explore and don't put limitations on them.
>
> (Final Project)

8.5 Sally

Sally, a seventh grade teacher from Old Bridge, had a strict set of expectations for convincing arguments. She made predictions about her students' work in cycle 1:

> I don't think my students will use that [opposites], and I know not all of them will use the recursive argument of the one color that changes in a different part each time, but I'm interested to see the other types of arguments that the students have and how I can challenge them to really prove it to me beyond a doubt.
>
> (9/16 Discussion)

In describing the implementation of the task in her classroom, Sally stated that students were able to find all the towers, but she was not convinced by any of their reasoning (9/23 Discussion). She acknowledged that she wanted to see students use cases or recursive arguments. "The recursive pattern with the block or set of blocks changing spots while keeping all others controlled has me thinking that if I don't see student reasoning similar to that, it isn't convincing enough" (9/30 Discussion). Sally described the work of students who organized their towers in a staircase pattern, or grouped by the number of each color (4 blue, 3 blue and 1 red, etc.) but she did not find these arguments convincing (9/30 Discussion). She also described a concern about not successfully supporting students in developing arguments. "I fear that he could be a student that comes to depend on learning and memorizing formulas and not attaching any meaning to them. I had a difficult time asking him things to encourage a further thought process or further explanation from him" (Final Project).

Sally had a critical eye for student reasoning. When describing student work in cycle 1, she noted that a student's cases argument was incomplete.

> Other students were able to group and categorize their towers into groups, but had trouble describing each group. Most groups described the categories according to the number of one color that each tower contained, but none talked about keeping a certain level the same color and switching another level one by one.
>
> (9/30 Discussion)

In the discussion for cycle 2, one of the teachers described a student's approach to the pizza problem, in which the student made combinations based on a different starting topping (e.g., all the pizzas that have pepperoni, all the pizzas that have peppers, etc.). Sally commented on this student's solution: "I am wondering if Michele had duplicates under each heading, for example, would the 'pepperoni and sausage' pizza go under the Pepperoni column, the Sausage column, or both and then eventually be crossed out in one spot?" (10/21 Discussion).

In the final meeting, Sally demonstrated keen attention to student reasoning. She claimed that she did not notice the students' strategy at first, because it wasn't written, but after looking at the towers, she noticed that they were organized in a recursive pattern and acknowledged that the students talked about how they moved blocks in a progression to build more towers (12/04 Focus Group Transcript, Lines 180 – 184). Sally also described her impression of the instructional methods promoted in the intervention. "Often times we just like show them how to do it, like a tree diagram, or, you know, we show them the way. But I think it's so much stronger if you just present them with the problem and have them invent their own way" (Focus Group Transcript).

8.6 Justin

Justin, a 6-8 special-needs teacher from Sayreville indicated an understanding that his students could develop mathematical arguments through interactions with their peers. He also showed an appreciation for the students' unique approaches to solving problems. Events in Justin's development supporting this growth are described below.

8.6.1 Justin's Attention to Reasoning

In the online discussion forum, Justin described the work he did on the 4-tall towers problem at the first on-campus meeting. In particular, he noted that prior to working on the task, he did not think it was possible to construct a thoroughly convincing description of the solution. Justin claimed:

> Many of the reasoning and diagrams were well constructed however, none would have me thoroughly convinced that all [the towers] were represented. The responses would always leave me with the supplementary task of lending my own understanding of the problem to acquiesce to the participant's display of reasonable understanding of the task. It was not until numerous amounts of feedback from my partner, encouragement from our instructors, time and the most accurate of verbiage, that I had totally convinced MYSELF that we had come up with the most convincing position to satisfy anyone. The truth of the matter is that everyone had come to the conclusion that there were sixteen varied towers 4-tall. However, the reasoning that there were only 16 and how one had come to that position widely differed from group to group. The language of no blue, one blue, two blue, three blue and four blue absolutely blew me away! … The many feelings of constructing a novel (to me) and thoroughly convincing position was the experience that has set the bar of expectation for my students. To conclude, I will push my students to really think and not be satisfied with them just trying to satisfy the task like I had begun to do...
>
> (9/16 Discussion)

As will be demonstrated below, Justin did support his students in developing convincing arguments, and recognized arguments as valuable based on their ability to clearly communicate ideas to peers.

When teachers discussed Milin's description of an inductive argument as a solution to the 4-tall towers problem, Mitch, an 8th grade teacher claimed that Milin had not given an argument, and that the instructor in the video appeared to be attempting to guide Milin into "making a convincing argument" (9/30 Discussion). Justin responded to Mitch's claim, describing the interviewer's role as more of a facilitator in the conversation among students. Justin claimed: "The aid of the instructor's prompts to push and prod was just an effective educator pulling on her student to not let him settle for a halfhearted attempt at convincing his partner" (9/30 Discussion). In this discussion, Justin described the value in having Milin construct actual towers in support of his argument. Justin claimed that he found Milin's inductive argument expressed in the research video somewhat difficult to follow, and recognized in one of the videos that Michelle, a student in the video episode, also did. Justin continued making a comparison about the claims that both he and Michelle were able to pick up on Milin's strategy once they saw him actually build the towers he was describing. Justin indicated, "When Milan physically created the towers by placing each color to each existing tower his idea of multiplying each height tower level by two was made crystal clear" (9/30 Discussion). While still appreciating the value of words, Justin recognized that manipulatives can be an effective tool for communication as well as reasoning.

When working on the 5-tall towers problem, selecting from 2 colors, Justin and Mitch organized their towers into cases based on the number of blocks of each color. They used an elevator pattern initially, and extended it to the case of 2 yellow and 3 blue. In this case, they also used controlling of variables to make organize their towers (10/7 Meeting transcript 2 of 3, line 1059).

Justin noted the use of visual representations in Mike, Ankur and Romina's work on the pizza problem, with halves. He claimed that the less formal visual representation of pizzas allowed the students to recognize a truth about the problem which may not have been apparent if students approached the problem in a purely symbolic fashion.

> Our sophistication--perhaps--leads us away from tangible proofs, to abstract writings with linear connections. The one student Ankur drew up his pizzas and from his drawings Romina was able to exclaim, "It's the same thing if you turn it around!" This moment may not have ever happened if it weren't for the drawings of Mike and Ankur. (10/07 Discussion)

Prior to his implementation of the 5-tall towers problem, Justin claimed, "Since the last time [implementing the 4-tall towers problem], I have challenged my students on almost every class work problem//question to provide convincing arguments to their classmates and myself that their public responses are correct" (10/14 Discussion). Justin also described the types of questions he intended to pose to his students, indicating: "I will be sure to not ask leading questions. I will attempt to pose questions that will illicit deeper mathematical thinking" (10/14 Discussion).

When discussing the classroom implementation of the pizza problem, Justin recognized that two of his students began the task (the pizza problem) in the same way he did (using a tree diagram) but later switched to a representation that was used by other teachers in the intervention (a chart).

> Many of my students began to create a tree diagram but about 10 minutes into the tasks had supplemented their tree diagram with a hybrid chart/tree. It seemed that they really did not trust the tree diagram to provide them with an exact representation of the answer. In our regional class, I provided a tree diagram to convince myself that my answer was thorough. However, when I saw how my partner and Connie's teamed responded with a chart, I thought that their description was far more sophisticated and convincing. I now empathize with my students at their choice for picking the greater, more convincing tool.
>
> (10/21 Discussion)

Justin appreciated the students' transition because they switched to a representation that he found more convincing than his own.

When teachers shared samples of student work on the pizza problem, Justin noticed a student's erroneous work and hypothesized that the student worked alone on the task (10/28 Group Meeting Transcript 1 of 2 Line 706). It was verified that the student had worked alone, and Justin described how he reached his hypothesis: "…if you work with someone else, they could challenge you. When I ask them, 'What do you think?' you know, they would then chat. Then when they were chatting, I knew they were on the right path. Then I would go on to the next one." (10/28 Group Meeting Transcript 1 of 2 Lines 710-715)

At the second afterschool meeting, Justin and Mitch completed the 3-tall towers problem, selecting from 3 colors. They also worked on Ankur's Challenge. Justin and Mitch identified the towers in their solution to the 3-tall towers problem that could be built into towers satisfying Ankur's challenge:

"So then what we did was, we put them in groups of, groups that were missing a yellow, that needed to have a yellow, groups that needed to have a red, and groups that needed to have a blue" (10/28 Group Meeting Transcript 2 of 2, Line 1078). Justin and Mike identified 12 towers that a blue block could be added to, to fit the criteria for Ankur's Challenge. Likewise, they identified 12 towers that a red block could be added to, and 12 towers that a yellow block could be added to. Justin and Mitch then claimed that these additional blocks could be added to the top or the bottom of the existing towers and decided that 72 towers could be made 4 blocks tall, selecting from 3 colors, with the criteria that the towers contain at least one block of each color. It was not until another group presented their solution that Justin and Mitch recognized the error in their thinking.

After reading about Romina's proof as a solution to Ankur's Challenge, Justin noted the value in co-constructing an argument by trying to convince a peer.

> Romina benefitted from her partner's suggestions. When a person attempts to prove their thought of methodology they embark on a difficult path. To prove or convince someone that you're completely right entails a commitment until that person is an absolute believer. So, Romina's drive and commitment to her solution is evidenced in her motivation to totally convince everyone that there is only 36 possible towers and no duplicates to Ankur's challenge.
>
> (11/04 Discussion)

Justin implemented the 3-tall towers problem, selecting from 3 colors, in his classroom. He described his interaction with one of the groups:

> As the writer [Justin] went to each table, it was evident that Thomas and his partner had physically constructed all of the towers but still needed a convincing statement. As the writer probed to be convinced, Thomas began to group his towers in a manner that made it easier to follow his train of thought. Thomas had done an impressive job convincing the instructor [Justin] that all towers were accounted for.
>
> (Final Project)

Justin also described how the students in another group transitioned from a guess and check strategy to a more organized cases approach. Although Justin was able to follow the students' reasoning, he did not find it convincing.

> I believe that they have found reasonable success with guess and check methods from their past. They rely heavy on it! I'm not sure as to why they believe this is a convincing strategy. However, I have observed that once they have reached a point where they are approximately 3/4 complete in satisfying the task, their strategies change--which brings me some modicum of joy. One group of students (young ladies) began to group the towers together in some patterned arrangement. "We organized our cubes in 5 groups all with 6 *towers* and one *group* with 3 solids...we organized them in color coded groups...for example all double-top *of one* color we put together." (Not written) This pair of students would categorize "patterned arrangements" and notice which group of towers they had failed to construct. They would easily (seemingly) recognize an isolated tower and used a constant color (whether on top of the tower or at the base) and alternate the remaining color to create a pair within their group of six. They understood that each group had to have six towers in order to be complete. With this method they were able to figure out that they had accounted for 27 towers which they believed were the maximum. I'm kind of dying to show them what I believe a convincing argument is!
> (11/11 Discussion)

At the final after school meeting on 11/18, Justin shared with the group that he pushed his students to make clear justifications of their work. "I was saying like, you know, like 'I don't want to guess, like tell me what you're doing'" (11/18 Meeting Transcript 2 of 2, Lines 488-490). In addition to supporting his students in describing and justifying their work, Justin encouraged his students to evaluate the reasoning of their peers. In the focus group before the final on-campus meeting, Justin remarked:

> I would propose the question to the students. Like "Are you guys convinced that she's right?" You know? Or "he's right?" And how are or how aren't. And they would ask me "Is this problem right?" And I'm saying "I don't know. Ask your peers. Ask your peers." Then they would actually have to prove out mathematically, what they did and explaining each step. And then some students would catch up, like on a step that they did right or did well. And then they could say that "No, that's not correct. And this is why…" And that sort of thing. So just every day math that we did. Like I had brought in this concept of proving and convincing. And like on student reasoning, so that was helpful in our projects, I actually think. Like throughout the year I'm going to just keep going back to that type of thing. I'm not saying what's right or what's wrong.
> (12/04 Focus Group Transcript lines 327-337)

This statement indicates that Justin did not see himself as the source of mathematical authority in the classroom. In reflecting on the value of having students work together in the development of their reasoning, Justin claimed: "The moments that I allowed my students with room to bring forth their reasoning, I tapped into young mathematicians." (Final Project)

8.6.2 Student Uniqueness

From early on in the intervention, Justin was struck by the unique set of experiences students bring to the table when they work on mathematics tasks. After reading about research students' work on the Shirts and Pants Problem, Justin noted Dana's claim that "yellow does not go with white" (9/30 Discussion) as an example of the unique background information students bring to a problem. He claimed Dana's solution to the Shirts and Pants problem in which the yellow and white combination was rejected represented a more "authentic" solution (9/23 Discussion). He was concerned that "Educators, curriculum and standardized test perhaps moves students away from cultural influence, authentic real world application and individual uniqueness." Justin referred to "uniqueness" several times throughout the intervention (9/23 Discussion, 10/04 Focus Group Transcript, Final Project) In the focus group before the final on-campus meeting, Justin described the student work on the Shirts and Pants Problem as an example of his claim that "the most compelling part to me was that students bring their own individuality to mathematics" (12/04 Focus Group line 125-127).

When Justin implemented the 4-tall towers problem, selecting from 2 colors in his classroom, he suggested students use a particular method to represent the towers on their paper. One of his groups of students developed a different method of recording their towers. These students used the letters "B" and "Y" to represent the blocks in the towers, rather than squares as Justin had recommended. The unique representation developed by these students appeared to have a formative effect on Justin. In his final reflection, he described the work of these students.

> In error, the instructor [Justin] provided an example of "Simple" notation whereas, an empty square represented a yellow cube and a shaded in square represented a blue cube when documenting the towers in pen // pencil. Angel used B - for a blue cube and Y -to represent a yellow cube. This was an original idea that came from her creativity, which was impressive.
>
> (Final Project)

Justin also noted that one of the students in this group used the term "vice versa" to describe opposite towers. He indicated, "I overheard one of my students continuously repeat the words, "Vice versed it." It was comical. I inquired about what she had meant by "vice versed it" and she exclaimed, "do the opposite or reverse" (9/30 Discussion). This student used the phrase and included an example of its meaning in her written justification of the solution. (Final Project)

Justin responded to a post by another teacher who noted that a student used the words "positive" and "negative" to describe opposite towers. In his response, Justin said "It's quite interesting how students create their own mathematical language to represent a thought or pattern for their justification" (9/30 Discussion). The insight, an important one in encouraging students to express the ideas without having yet learned the mathematics register, has important implications for future practice.

> Students come to the "Mathematical Table" with a unique system of how they make sense of their environment. All people possess a level of individuality that they use to solve problems and interpret situations with. I had no idea how much of their thoughts and uniqueness is used and could be used in the math class.
>
> (Final Project)

Throughout the intervention, Justin noted the value of using manipulatives (9/30 Discussion), images (10/14 Discussion), and non-standard language (9/30 Discussion) in developing arguments. Justin did not accept all student work as convincing. In his final project, describing a student's work that was correct but not well recorded and justified in the submission. Justin claimed: "in the field of Mathematical Studies, there is a component of documentation that is necessary. Construction and experimentation are the fun aspects of math. However, the documentation of the findings and solutions is paramount" (Final Project). Justin appears to have recognized a variety of forms of reasoning that students could use to construct mathematically convincing arguments and shows a deep understanding of the process of student development of building these ways of reasoning.

8.7 Mitch

Through the intervention, Mitch, the eighth grade teacher from Old Bridge, became a thoughtful facilitator. He demonstrated a belief that students could learn from each other(10/21 Discussion, 10/28 Discussion, Final Project), and a desire to have students follow through with their own reasoning and see where "their line of thinking takes them, because I think it is very important for students to arrive at answers by themselves" (10/14 Discussion). He also noted the value in having students share and revise their arguments: "If they are asked to go back and write or explain their findings, now they might not be so set on finding any more possible pizza combinations or tower combinations, but instead they can now focus on the justification and reason for their answer" (11/04 Discussion). Mitch continued to express a consideration for supporting his students in discussing their work:

> I would have also had them go back after doing the first task and had them explain their answers again to give them more practice explaining and flushing out their arguments. I think that if they had a little more practice going back and talking about their strategies and solutions they would get a better understanding of how to give a convincing argument.
>
> (Final Project)

Mitch demonstrated an expectation that students would incorrectly use numerical expressions to explain solutions if given the opportunity. He made a prediction about his students' work in cycle 1.

> I have done combination problems with some of my students and they have been quick to try to find an easy way to figure out a problem and generalize that answer or method for other problems (which is often unsuccessful). In doing so, they sometimes struggle to explain how or why they think this answer or method is right and instead just come up with a general rule to use. This being the case, I think they will try to do something

> similar for the 3-tall and 5-tall towers. After realizing that the answer for the 4-tall tower is 16 (although they may not yet fully understand why this is the case), I believe they will understand that 16 is 4 times 4, and simply apply this rule for the 3-tall and 5-tall towers. I expect them to tell me that because the answer for the 4-tall tower is 16, then the 3-tall tower must be 9 and the 5 tall tower must be 25.
>
> (9/16 Discussion)

When the cohort studied Milin's inductive argument, Mitch compared Milin's work to that of his students: "While Milin was spending his time trying to understand the problem, my students were trying to get the correct formula or strategy that would give them the correct answer and then tried to see if it would match up with what they got when they listed their responses" (9/23 Discussion). Towards the end of the intervention, when another teacher asked how he had addressed the combinatorics topics in the school curriculum, Mitch stated that he had held off on teaching those topics until after all of the intervention task cycles had been completed. "I actually have not gone over combinations with my enriched class and I was intentionally not going to even mention combinations at all until we have finished all of these tasks for the reasons that you [Kate] talk about" (11/11 Discussion). Mitch said that he chose to do this so that students would not get distracted from the tasks, by attempting to fit them into a specific problem category and calculate a solution before actually working on the task as stated.

Mitch's perception of his role as a teacher appears to have changed over the course of the intervention. He claimed to have analyzed the video of the researcher's interview with Brandon more than any of the previous videos. He was impressed by her interview style and noted that her responses to Brandon's statements encouraged him (Brandon) to continue and follow through with his thinking. In another post about the interviewer's style, Mitch made a statement which demonstrated his position in the transition.

> I'm trying to walk the thin line between leading them in the right direction without giving them the answer. On another level, sometimes I just want to see where their line

> of thinking takes them, because I think it is very important for students to arrive at answers themselves.
>
> (10/21 Discussion, lines 224-227)

Later, in week 9, Mitch demonstrated a greater comfort in letting his students take ownership, follow their reasoning, and share their thinking.

> I have many students who were very eager to tell me and others about what they did and I even had some students who wanted to share their strategies with the whole class. I think students really take pride when they are able to work towards solving a problem like this.
>
> (11/04 Discussion, lines 424 - 427)

Mitch had high expectations of his students. He often claimed their arguments were not convincing (9/23 Discussion, 9/30 Discussion), but he appreciated the reasoning. When describing students work on cycle 3, Mitch claimed:

> I think that the session with the three tall towers with three colors was probably the best session out of all of the ones that we did because students were able to decide on strategies as a group almost right away and then worked together as a group much better. Students felt confident in their arguments and were able to explain them verbally and on paper.
>
> (Final Project)

Mitch expressed a desire to show his students some of the videos from the intervention. When describing the video of Romina's proof, Mike noted: "I was actually wondering if at any point it would be a good idea to let students see an explanation like this to see if they understand it and can appreciate a higher level method like this" (10/28 Discussion). He pursued this idea again at the "I think I would also like to, the videos that we saw. I think I'd like to, if we're even allowed to show those to the kids. Because even when we did this problem for the first time. And, you know, they would say 'is it, or how do you know?' or 'Convince us'" (Focus Group Transcript). He felt that the students would appreciate seeing other students developing convincing arguments.

8.8 Summary

Some general common experiences emerge from the narratives above. Most teachers did appreciate the researcher's interviewing of Brandon, and noted that her example helped them to pose better questions to their students. Most teachers also referred positively to examples of students discussing mathematics in their classroom. These instances demonstrate changes in the teachers' beliefs about learning and teaching.

Teachers also demonstrated an increase in the precision with which they described student reasoning. While vague words such as "pattern" or "method" were used to describe students' reasoning processes at the beginning of the intervention, complete and detailed descriptions of student's reasoning were given by the end. The increase in precision demonstrates a change in the teachers' attention to student reasoning.

9 FINDINGS

The objective of this study is to provide a detailed account of a semester-long intervention that was run as a course for practicing teachers. In particular, three research questions (as indicated in Chapter 1) guided the study:

1. What forms of reasoning do middle-school mathematics teachers identify from the following:
    a. Their own solutions to a series of mathematical tasks during a PD intervention;
    b. Their students' solutions to the same mathematical tasks implemented in their own classrooms;
    c. Students' solutions working on the same or similar mathematical tasks from assigned VMC videos, and
    d. Teachers' pre and post-test responses concerning the forms of reasoning used by fourth grade students to solve mathematical tasks in the Gang of Four VMC video?
2. What changes, if any, can be identified in teachers' beliefs about learning or teaching mathematics?
3. What pedagogical moves or strategies are used by the instructor to facilitate the teachers' construction of knowledge about mathematical reasoning as the teachers:
    a. Work on tasks in a combinatorics strand
    b. Study student reasoning from video, and
    c. Analyze samples of their own students' written work the tasks?

Findings relevant to each research question are presented below. The findings regarding teacher recognition of student reasoning are presented first, followed by the findings relating to teacher moves and subsequently the findings regarding teacher beliefs. Each section below contains a summary of findings and suggestions based on those findings.

9.1 Teacher Recognition of Student Reasoning

In this section, key findings from the analysis of reasoning are discussed and potential implications are described. Early parts of this chapter describe findings related to specific examples of research student work. Later parts relate the findings of the chapter to other studies.

9.1.1 Inductive and Case Arguments: The "Gang of Four"

In this instance of the intervention, teachers frequently referred to components of the "Gang of Four" video. Both Milin's inductive argument and Stephanie's case argument became exemplars for this cohort.

Whether it was due to targeted discussion questions regarding Milin's inductive strategy, or general discussions of inductive strategies, the intervention appeared to successfully establish the criteria for identifying a convincing inductive argument. Two of the three groups of teachers used inductive approaches to complete the 3-tall towers task, selecting from two colors (10/28 meeting transcript 1 of 2). Additionally, the two teachers who did not recognize Milin's inductive argument in the pre-assessment described it in the online discussions and even used an inductive strategy to complete tasks later in the intervention.

Teachers described cases arguments more frequently than any other type of argument, and multiple solution cases were described for each task. Teachers demonstrated the knowledge to identify an initial organization of cases, but data regarding teachers' knowledge to justify the completeness of a solution based on the completeness of each case in a solution set is limited. The instructor modeled probing questions and encouraged teachers to ask these questions of their students. There is not sufficient evidence in the student work samples, or discussions to indicate whether teachers consistently asked these questions of their students. It may be beneficial in future instances of the intervention to include specific discussion questions regarding student justification of the completeness of a case argument.

In this intervention, teachers' recognition of cases and inductive arguments appeared to increase overall. The data from this intervention indicate that analysis and discussion of research literature and video was effective in exposing teachers to these strategies (See section 5.1). It is possible that the juxtaposition of Milin's argument with Stephanie's argument may have caused these teachers to claim Stephanie's argument was not convincing. Or, it may be that Stephanie's partitioning the category of the case of two blue cubes into two blues "stuck together' and two blues "separated" was unexpected and perhaps lacked elegance. In the intervention, the three teachers who did not claim identify case arguments presented in the post-assessment were as convincing did make claims that certain other case arguments were convincing.

9.1.2 Recursive Arguments

All teachers either used or described recursive arguments during cycle two of the intervention (See section 5.1.1). It was during cycle two that teachers read "Making Pizzas: Reasoning by Cases and Recursion" (Maher & Yankelewitz, 2011). Despite reading this chapter, teachers did not refer to a particular sample of student work as an exemplar for this argument, as they did for the inductive argument (Milin) and the case argument (Sally and later Romina). It may be worth identifying a video clip with a strong exemplar of a recursive argument, and including examples of recursive arguments in the reasoning pre and post assessments. Teachers in this cohort all recognized components of case and inductive arguments on the reasoning pre-assessment, so the inclusion of recursive arguments in the assessments may increase the opportunities for teachers to show growth on the reasoning assessments.

9.1.3 Arguments by Contradiction

Teachers used components of arguments by contradiction when prompted by the instructor (10/7 Meeting transcript 2 of 3, line 507; 10/28 Meeting transcript 2 of 2, line 1010; 10/28 Meeting transcript 2 of 2, line 1182). No teachers described samples of student work that used arguments by contradiction. It may be that the tasks in this intervention were not well suited to arguments by contradiction. It may also be the case that arguments by contradiction, although used, were not given significant attention. For example, none of the readings in this instance of the intervention focused on a student's argument by contradiction, and none of the online discussion questions involved arguments by contradiction.

9.1.4 Evaluation of Arguments

Teachers evaluated the claims of other teachers and students, but the majority of evaluations regarding whether an argument was convincing referred to the arguments of research students, and occurred in the online discussions (See section 5.3). It may be that the instructor's discussion questions required teachers to make these claims, but it may also be that teachers were more comfortable making claims about whether an argument was convincing in the online setting. In order to encourage teacher consideration of the degree to which an argument is convincing, future instances of the intervention could include targeted questions requiring that they consider whether each solution presented includes a convincing justification. One of the recommendations for further research is to examine the settings in which teachers report most comfort making claims about whether a solution contains a convincing justification.

9.1.5 Relation to Other Studies

This intervention is similar in design to professional development interventions studied by Santagata (2009). Describing her study, Santagata (2009) noted teachers' inability to assess student reasoning to a higher degree than whether or not the student's response was the correct answer. The current study indicates that teachers did have the knowledge to assess student reasoning to a higher degree than evaluating correctness, but also indicates room for improvement. As noted above, teachers could identify portions of case arguments, but did not generally identify or construct complete case arguments (11-18 meeting transcript 1 of 2, lines 397 – 404). In some instances, teachers identified an initial organization of towers, but failed to notice duplicates (10-28 transcript 2 of 2, line 405) (11-18 transcript 1 of 2, lines 749-859). The case of Kate's student (section 4.1.12, 11-18 transcript 2 of 2, line 1199) demonstrates an example of an incorrect formula yielding a correct solution, and in this case, several teachers identified this incorrect formula as "convincing". The situation provides an example of why it is important to focus on the reasoning, devoid of answer or mathematical expression. Teachers may be as eager to "formula grab" as students.

Jacobs et al. (2010) hypothesized attending to student reasoning strategies is most likely to be overlooked in professional development programs. Attending to student reasoning was a stated goal of the intervention being studied, and teachers demonstrated some increase in attention to student reasoning. While not supporting the hypothesis of Jacobs et al. (2010), the findings of this study may indicate the value of challenging teachers to attend to student reasoning in professional development programs.

9.2 Teacher Moves

In this section, findings resulting from the teacher moves analysis are summarized, followed by findings resulting from key events in the intervention.

9.2.1 Findings from Teacher Moves Analysis

The instructor in this intervention modeled many of the behaviors recommended in the research literature (Herbel-Eisenmann, Steele, and Cirillo 2013, Smith and Stein 2011, Martino and Maher 1999). In all stages, the most frequent teacher move was that of motivating teachers. Motivational statements aside, the instructor's actions and questions varied based on the stage. (See section 6.1)

While teachers worked on tasks, the instructor monitored teachers' progress and invited them to share ideas with their group members. She also used revoicing to check her own understanding of a teacher's work. The instructor asked probing questions, questions requiring teachers to explain their thinking, and questions requiring teachers to justify their solution. The instructor's use of questioning assisted teachers in organizing solutions and justifications for each task.

While teachers discussed samples of student work, the instructor invited teachers into the conversation, made use of wait time, and used revoicing to clarify statements and introduce vocabulary. In this stage, the instructor primarily asked probing questions and questions to facilitate awareness of other solutions. The instructor's use of teacher moves helped to facilitate deep discussions of student work. In particular, the instructor modeled techniques of inviting, motivating, revoicing, and waiting.

Teachers were active on the online discussion forums in this intervention. The instructor's choice of open-ended questions may have helped teachers feel comfortable sharing their thoughts. In the online discussions, the instructor's primary interaction was to offer motivating statements in response to teachers' posts.

9.2.2 Findings from Key Events

The instructor in this intervention was deeply familiar with the research base of this intervention. Her familiarity with the longitudinal study enabled her to respond effectively to several issues that arose in the intervention.

9.2.2.1 Scaffolded Instruction

After analyzing Milin's development of an inductive process to solve towers problems, selecting from two colors, teachers claimed "I feel like we set our students up to be confused because we started them with the four-tall towers and didn't allow them to use the blocks to see a connection between, 1, 2,3 and 5-tall towers" (9/30 Discussion, line 185). "I completely agree with you about the connection from the 1-tall through the 5-tall towers" (9/30 Discussion, line 210). These teachers expressed a desire to organize students' instructional experience in a way that may promote an inductive argument over other forms of reasoning. The instructor generally remained passive in the online discussion, but she responded quickly to these claims. "We do not believe that you start students by building towers 1-tall, 2-tall, 3-tall, 4-tall and then they see this pattern. That's not what we're trying to do - that's a programmed way of proceeding. We'll talk more about this at our meeting tomorrow." (9/30 Discussion, line 200). At the meeting, the instructor and the teachers discussed the value in having students discover patterns and develop justifications for solutions, rather than showing them a procedural pattern (10/7 Meeting transcript 1 of 3, lines 45-105).

9.2.2.2 Non-Leading Questioning

At the 10/7 meeting, Rich mentioned that he felt Brandon's description of the isomorphic relationship between the pizza problem, selecting from 4 toppings, and the 4-tall towers problem, selecting from 2 colors was the result of leading questions from the researcher (10/7 Meeting transcript

3 of 3, line 252). This started a discussion about teacher questioning. The instructor challenged that the researcher's questions were not leading, and suggested that Rich watch the interview again. After that meeting, teachers described the skill with which the researcher conducted the interview. Connie claimed "The researcher in the Brandon video is definitely a skillful questioner." (10/7 Discussion, line 275) Mitch followed up on this claim, noting:

> When she started asking him about how this problem reminded him of any other problems she was sure not to lead him even when he asked a question and wanted to know if she was talking about the way that he solved it. Instead of just saying yes or telling him what she wanted him to talk about she replied, "In any way" to let him talk about what he wanted to.
>
> (10/7 Discussion, Lines 321-326)

The instructor's familiarity with the longitudinal study, and in particular, the video of Brandon's interview enabled her to respond quickly and effectively to a claim that required further study.

9.2.2.3 Learning VS Stealing

In the 10/28 meeting, teachers reported that some of their students picked up ideas from their classmates (10/28 Meeting transcript 1 of 2, line 741-782). This led to a discussion about whether this constitutes "stealing" of ideas. Some teachers expressed a concern for students using the ideas of their peers, particularly when they were mimicking a student's procedure without understanding. Several teachers mentioned that it was there practice to share other student work and noticed that in later tasks, students were more likely to use a strategy from another student that had previously been shared. They indicated that students who adopted the strategy acknowledged that it was a method that made sense to them, and was better than ones they had previously used. Teachers later referred jokingly to "stealing" ideas of their peers (10/28 transcript 1 of 2, line 183). Mitch, who had initially expressed concern over "stealing", commented in his final project:

> I'm really happy specifically about Matt's explanation. He says "After a while, Max noticed a pattern. I wasn't sure, but then he showed me how one combo could mean

> the same for a similar color scheme." He provides an example and explains why he was convinced. This is exactly the type of collaboration I was looking for on this project
> (Mitch Final Project p. 38)

9.2.3 Summary

The instructor's quick responses the issues raised by participating teachers maintained a focus on the goals of the intervention, and may have influenced teacher beliefs about learning and teaching mathematics. It is hypothesized that the instructor's familiarity with the longitudinal study and connectedness to the philosophy underlying both the longitudinal study and this intervention assisted her in making these quick responses.

It was also valuable to have an experienced instructor leading the intervention. That way when specific misdirecting concerns arose, the instructor was able to redirect attention to the purpose of the intervention. For example, it was important that the instructor entered the conversation when teachers who viewed Milin's inductive proof felt their students had been tricked and wanted to have them construct towers of heights 1, 2, and 3 before constructing 4-tall towers selecting from two colors.

9.3 Teacher Beliefs

In this section, findings resulting from the teacher moves analysis are summarized, followed by descriptions of possible relationships among the findings.

9.3.1 Findings from Beliefs Analysis

Considering the beliefs of the entire cohort, it appears that, over the course of the intervention, beliefs in general became more aligned with the standards presented in the beliefs assessments. The percent of beliefs inconsistent with the standards relative to the total number of beliefs statements made decreased from one cycle to the next, including the final projects (See section 7.2).

In the online discussions, claims were made regarding learning, teaching, and student and teacher roles. Over the course of the intervention, beliefs in these three categories were shown to become increasingly consistent with the standards presented in the beliefs assessment (See section 7.2).

The intervention data showed that six of the seven teachers demonstrated a change in beliefs relative to the standards presented in the beliefs inventory. Categories for which a change in beliefs was noted varied by teacher, but the category of student and teacher roles was the category for which the greatest number of teachers demonstrated a change in beliefs. Evidence was found indicating that three teachers' beliefs regarding student and teacher roles did change over the course of the intervention.

9.3.2 Relationships in Findings

Some of these changes in beliefs may be related to the teacher moves modeled and discussed by the instructor. Throughout the intervention, the instructor modeled questioning techniques that promoted student and teacher roles aligned with Standards. This includes the group discussion on questioning (10/7 Meeting transcript 1 of 3, line 252) and on the "stealing" of ideas (10/28 Meeting transcript 1 of 2, line 741-782).

Teachers may have recognized that quality of student work is not solely based on their judgment of a student's mathematical ability, suggesting that certain conditions for learning need consideration. After reading "Responding to Ankur's Challenge: Co-construction of Argument Leading to Proof" (Maher & Muter, 2011), teachers expressed the value in giving students time dedicated to the construction and refinement of arguments. As Connie noted:

> ...the first time writing things down in a timed period doesn't allow for complete thoughts to be represented. It can be difficult for students to explain their reasoning and even more difficult for them to write them down. Giving them more time can allow them to clarify their explanation, and give them more insight on how to improve their demonstration of their reasoning. It is evident that more time allowed Romina to better explain her thought process. She was a little frustrated at first trying to explain her

> reasoning. She even said she needed to collect her thoughts. However, the more times she had to explain to her fellow classmates, the better she was able to demonstrate her correct explanation.
>
> (11/4 Discussion, line 98-106)

Connie and other teachers reported on the difficulty students have constructing an argument within the limited time of a classroom period, and may have recognized that claims about student ability based on such data alone are not truly accurate.

10 CONCLUSIONS

In this chapter, implications for professional development initiatives are described, followed by a description of limitations of the study, and suggestions for future research.

10.1 Implications

The analysis of this intervention demonstrates that teachers could use and identify forms of reasoning that were not included on the reasoning assessments. Teachers used recursive arguments, as well as arguments by contradiction. Teachers work on the tasks prepared them to consider forms of reasoning used by research students and their own current students (9/16 Discussion). The inclusion of examples of these forms of reasoning in assessments may allow for greater teacher growth to be recorded regarding recognition of student reasoning.

The strategies and heuristics that teachers noted in the literature tended to differ from the strategies the teachers themselves used to complete the tasks. Despite reading about and discussing inductive arguments and recursive arguments, teachers used case arguments more frequently than any other form of argument. Teachers frequently referred to Milin, Brandon, and Romina when describing examples of their own students' reasoning. For future interventions, it may be valuable to identify examples of video and literature regarding a particular student's use of recursive arguments or arguments by contradiction to solve a task. Developers of interventions following similar structures should attempt to identify student exemplars of the concepts they are addressing.

The instructor's use of teacher moves was well aligned with current expectations of teachers. Examples of the instructor's interactions with teachers in this intervention could be used in training future instructors for other interventions, or more generally for professional development of teachers.

Some change in teacher beliefs regarding learning, teaching, and student and teacher roles accompanied the intervention. The beliefs inventory assessments were able to capture limited information, but a more in-depth analysis of teacher beliefs using coded data revealed more changes in teacher beliefs. In similar interventions, invitations for teachers to voice their beliefs early on may encourage more discussion and dialogue about existing practices and how these practices conflict with new ideas and approaches that are introduced and encouraged.

10.2 Limitations

A cohort of seven teachers participated in the intervention. While the small sample size enabled detailed analysis of video and other data, the results are not generalizable. These data, combined with the data from interventions with other cohorts, offer the opportunity to study in detail the process by which change in teacher knowledge and beliefs occurs. However, differences due to instructors, who had the opportunity to select tasks, readings, video samples, and discussion questions, might be a factor worth studying.

Data regarding student reasoning while working on tasks was largely collected through the lens of the teachers describing the work. Collection of video of students working on these tasks will allow for more careful analysis of teacher attention to student reasoning.

This intervention was designed as a semester-long graduate course. The work involved was intensive. Teachers had to make time to implement tasks in their classrooms, meet with the instructor, and travel to group meetings. Teachers' districts understood the commitments required and were highly supportive of the participating teachers. Teachers were also required to complete work outside of school hours, which included studying videos and readings, as well as participating in online discussions. In this intervention, teachers were compensated with course credit. Similarly structured interventions that are not able to provide compensation to teachers, or do not have a high degree of support from participating districts may not yield similar results.

10.3 Suggestions for Further Study

This study provided detailed information about a single instance of an intervention. The study was limited in scope, and its findings are not generalizable to other instances of the intervention. However, It may be useful to examine teacher changes in recognizing student reasoning and beliefs about teaching and learning with other implementations of the intervention to see what effects, if any, are durable and independent of instructor and cohort. This may aid in determining which, if any of the findings generalize beyond this particular instance of the intervention.

It is suggested that in future implementations, data regarding teacher's implementation of the tasks in their own classrooms be collected. If teachers are given the opportunity to discuss and reflect on their implementation of these tasks with specific examples of their interactions with students, they may be better able to identify opportunities to ask probing questions, or questions that require students to justify a part of their solution.

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Wilhelm, A. G. (2014). Mathematics Teachers' Enactment of Cognitively Demanding Tasks: Investigating Links to Teachers' Knowledge and Conceptions. *Journal For Research In Mathematics Education*, *45*(5), 636-674.

Yackel, E., & Hanna, G. (2003). Reasoning and proof. In J. Kilpatrick, W. G. Martin & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 227-236). Reston, VA: National Council of Teachers of Mathematics.

Yankelewitz, D. (2009). *The development of mathematical reasoning in elementary school students' exploration of fraction ideas*

Yankelewitz, D., Mueller, M., & Maher, C. A. (2010). A task that elicits reasoning: A dual analysis. *Journal of Mathematical Behavior, 29*(2), 76-85. doi:10.1016/j.jmathb.2010.02.002

APPENDICES

Appendix A Course Syllabus

Landis: Topics in Math Education: Lesson Study on Reasoning

Fall 2010 – Hybrid Course Course number: 15:254:599, Section 81

Syllabus for Old Bridge/Sayreville (Central Region)

HYBRID COURSE (Index # 17676)

On-Campus Meeting Dates: 9/11, 12/4

Saturdays, 10:00 am -12:30 pm, GSE Room 30

Regional Meeting Dates: 10/7, 10/28, 11/18

Thursdays, 3:15 – 5:45 pm

Old Bridge Carl Sandburg Middle School

In-District Classroom Visits: 9/23, 10/28, 11/18

CONTACT INFO

Course Instructor Old Bridge & Sayreville	Judy Landis	jlandis@rci.rutgers.edu	(732) 830-4731

Individual Meeting Opportunities

By appointment in district on the classroom visitation and/or regional meeting days

Course Overview

This course is designed as a practical research-based set of experiences focusing on the development of reasoning and justification. Participants will engage in a variety of activities that blend in-person, on-campus meetings, smaller regional sessions, and in-school implementations with interactions done asynchronously online through a course web site.

The on-campus and regional activities will include working in small groups on a series of mathematical problem-solving tasks, discussing possible modifications for specific classroom use, and sharing the actual experiences and student work resulting from follow-up implementations by each of the participants.

The online course work will include reading assignments that relate to each of the problem tasks within the overall focus of students' reasoning and justification. Online course assignments will also include video clips of children engaged in solving the same or similar problem tasks as those introduced in the group sessions. Each assignment will include guiding questions to elicit small group reflection and discussion of the readings and their relevance to learning and teaching.

Particular emphases for each assignment will be on the mathematics, children's learning, and conditions of the learning environment. Examples will be selected from the content strand of counting and combinatorics, from early years through high school, and participants will be expected to consider implications drawn from their own practice in light of research for instruction and NCTM Standards.

As one component of the course, each participant will complete assessments (pre and post) for measuring the impact of course activities in the focal mathematical strand on what you notice and how you describe what you observe in a video episode and a set of student products, as well as on participant beliefs about learning and teaching math. Completing the assessments is not optional; it is a course requirement. However, each participant will be given a consent form about whether assessments can be among those analyzed for ongoing research.

COURSE REQUIREMENTS

You are invited to be an active participant in the class through small group work in the general and regional meetings and through web-based discussions, classroom implementations, projects, and writing. Successful completion of the course requires that you engage in all activities and submit all assignments. You are required to:

1. Complete all pre- and post-assessments.
2. Attend all on-campus and regional sessions.
3. Actively participate in online discussions as you engage with assignments (readings and videos) and respond to guiding questions as posted on the eCompanion course website. You are required to make at least one original posting and respond to at least two group member postings per week.
4. Be knowledgeable of all the assigned readings and video clip viewings.
5. Complete an *Individual / Group Research Poster Project*. Individually, or working with a partner, participants will complete a summary narrative of their implementations of the problem tasks with their students. This narrative, accompanied by student work and other artifacts from the terms activities are to become a poster that will be shared with everyone in the three regions at the December 4 final meeting.
6. Complete a *reflective assessment* of your work in the course. This will be the final assignment and due on December 13. You should reflect on your knowledge of the mathematics, research on how students learn, and implications for teaching with regard to NCTM Standards. You may review your postings on the course web site and notes from problem solving and sharing of solutions as you develop your reflective assessment, which should be about one to two pages in length.

You will be evaluated on your work products for the individual / group research poster project, completion of all pre and post assessments, and your participation both in person and on line.

COURSE OUTLINE AND ASSIGNMENTS

ONLINE ASSIGNMENT prior to Sept. 11	**Activities:** Complete pre-assessments using the eCompanion course web site. These assessments must be completed prior to the on-campus class session on Sept. 11th.
9/11/2010 ON-CAMPUS	**Class Activities:** Introduction to the course; Engage in 4-tall Towers selecting from 2 colors problem-solving task, with problem extensions and focused discussion about representations. Review syllabus and discuss course requirements.
9/16/2010 ON-LINE	**All teachers will implement Task I in their classrooms between 9/13 and 9/27.** **On-line Activities:** Respond to the guiding questions to be posted online for engagement in threaded discussion about the various towers problem-solving

	tasks and related videos and readings. **Assigned Reading:** Maher, C.A., Powell, A.B. & Uptegrove, E. (Eds) (in press) *Combinatorics and reasoning: Representing, justifying and building isomorphisms.* Chapter 3 **Videos:** Clips to be posted on course web site: PUP Math, Stephanie and Dana, grade 3; Meredith Removes the Top Cube
9/23/2010 ON-LINE	**All teachers will implement Task I in their classrooms between 9/13 and 9/27.** **In-school activities:** Implementation of Towers problems in teachers' classrooms. Instructor will implement task with teacher in one of the districts on 9/23. If possible, other teachers will be released to observe and debrief. **On-line Activities:** Respond to the guiding questions to be posted online for engagement in threaded discussion about the various towers problem-solving tasks and related videos and readings. **Assigned Reading:** Maher, C.A., Powell, A.B. & Uptegrove, E. (Eds) (in press) *Combinatorics and reasoning: Representing, justifying and building isomorphisms;* Chapter 4 **Videos:** Clips to be posted on course web site: PUP Math, Stephanie and Dana, grade 4
9/30/2010 ON-LINE	**On-line Activities:** Respond to the guiding questions to be posted online for engagement in threaded discussion about the various towers problem-solving tasks and related videos and readings. **Assigned Reading:** Maher, C.A., Powell, A.B. & Uptegrove, E. (Eds) (in press) *Combinatorics and reasoning: Representing, justifying and building isomorphisms.* Chapter 5 **Videos:** Clips to be posted on course web site: Milan Shares His Inductive Argument
10/07/2010 FIRST REGIONAL GROUP	**All teachers implement Task II in their classrooms between Oct 8 and Oct 22.**

MEETING	**Group Activities:** Share classroom experiences and student work from Task I. Engage in a pizza problem task: pizzas, selecting from 4 toppings. Share how solutions were found and examine representations used in problem solving. Consider how these tasks might be used in classroom instruction. **Study Video**: Brandon Invents Isomorphism Share observations/ impressions of video. **On-line Activities:** Respond to the guiding questions posted online for engagement in threaded discussion about the assigned readings and videos. **Assigned Reading:** Maher, C.A., Powell, A.B. & Uptegrove, E. (Eds) (in press) *Combinatorics and reasoning: Representing, justifying and building isomorphisms.* Chapter 6
10/14/2010 ON-LINE	**All teachers implement Task II in their classrooms between Oct 8 and Oct 22.** **On-line Activities:** Respond to the guiding questions to be posted online for engagement in threaded discussion about the second task and related videos and readings. **Assigned Reading:** Maher, C. A. & Martino, A. (1998). "Brandon's Proof and Isomorphism". In C. A. Maher, *Can teachers help children make convincing arguments? A glimpse into the process.* Rio de Janeiro, Brazil: Universidade Santa Ursula.
10/21/2010 ON-LINE	**Online Activities:** Respond to the guiding questions to be posted online about student work and implementation experiences from Task II: Pizza problem.
10/28/2010 SECOND REGIONAL GROUP MEETING	**In-school activities:** Classroom implementation of Pizza problem. Instructor supporting in one of the districts. If possible, other teachers released to observe and debrief. Share classroom implementation experiences and student work from Task II. Engage in Task III: building 3-tall towers, selecting from 3 colors, and extension problem, Ankur's Challenge. Share how solutions were found and examine

	representations used in problem solving. Consider how these tasks might be used in classroom instruction. **Online Activities:** Respond to the guiding questions to be posted online for engagement in threaded discussion about ideas from Towers with 3 colors and Ankur's Challenge, with focus on reasoning and proof in mathematics. **Video:** Romina's Proof
11/04/2010 ON-LINE	**All teachers implement Task III in their classrooms between Oct 29 and November 10.** **On-line Activities:** Respond to the guiding questions to be posted online for engagement in threaded discussion about the assigned reading. **Assigned Reading:** Maher, C.A., Powell, A.B. & Uptegrove, E. (Eds) (in press) *Combinatorics and reasoning: Representing, justifying and building isomorphisms.* Chapter 8
11/11/2010 ON-LINE	**All teachers implement Task III in their classrooms between Oct 29 and November 10.** **Online Activities:** Respond online to guiding questions about the implementations and student work from the first two tasks.
11/18/2010 THIRD REGIONAL GROUP MEETING	**Group Activity:** Share classroom experiences and student work from Task III. Form groups for Research poster projects. Discuss guidelines for preparing poster presentations for December 4th on-campus meeting. **Assignments –** Guidelines to be posted for preparing research poster projects. Begin work on poster projects.
11/25/2010 ON-LINE	**Assignment:** Preparation of Research Poster Projects.
12/04/2010 ON-CAMPUS	**Class Activity:** Share Research Poster Project Reports. **Assignment:** Reflective narrative.

12/13/2010 ON-LINE	Reflective Narrative Due

Notes about reading assignments:

Assigned readings will be made available through the eCompanion site for this course.

* Maher, C. A., Powell, A. B. & Uptegrove, E. (Eds.), (in press). *Combinatorics and reasoning: Representing, justifying and building isomorphisms.* Springer Publishers.

Readings from the above-listed book are being made available, however the book is still in press and must not be cited.

As a general guideline for engaging in online discussions, we offer a few words on "Netiquete".

This is drawn from Palloff, R. M., & Pratt, K. (1999). *Building learning communities in cyberspace*. San Francisco: Jossey-Bass, p. 101.

a. Check the discussion frequently and respond appropriately and on the subject.

b. Focus on one subject per message and use pertinent, informative, and not-too-long subject titles

c. Capitalize words only to highlight a point or for titles. Capitalizing otherwise is generally viewed as SHOUTING.

d. Be professional and careful with your online interaction

e. Cite all quotes, references, and sources.

f. When posting a long message, it is generally considered courteous to warn readers at the beginning of the message that is a lengthy post.

g. It is inappropriate to forward someone else's message(s) without their permission.

h. Use humor carefully. The absence of face-to-face cues can be misinterpreted as angry, antagonistic criticism.

Appendix B Task Statements

Student: ____________________ Date: ____________________

School: ____________________ Teacher: ____________________

Other Group Members: ____

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

Student: ___________________________ Date: _____________________

School: ___________________________ Teacher: ___________________

Other Group Members: ____

Building Towers 2 Colors Extension

Without building them, make a prediction about a solution for finding all possible 3-tall towers, selecting from 2 colors. Do you think there will be more, fewer, or the same number of possible towers as you found for towers that were 4 cubes high? Why do you think that?

Without building them, make a prediction about a solution for finding all possible 5-tall towers, selecting from 2 colors. Do you think there will be more, fewer, or the same number of possible towers as you found for towers that were 4 cubes high? Why do you think that?

Student: ___________________________ Date: _____________________

School: ___________________________ Teacher: ___________________

Other Group Members: ____

Building 5-tall towers, selecting from 2 colors

You have two different colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly five cubes high. Find a way to convince yourself and others that you have found all possible towers five cubes high, and that you have no duplicates. Record your towers below and provide a convincing argument why you think you have them all.

Name: ____________________________ Date: ________

Partner: ___________________ Teacher: ___________________

The Pizza Problem

Capri Pizza has asked you to help design a form to keep track of certain pizza choices. They offer a standard "plain" pizza with cheese and tomato sauce. A customer can then select from the following toppings: peppers, sausage, mushrooms, and pepperoni. How many choices for pizza does a customer have? List all possible choices. Find a way to convince each other that you have accounted for all possibilities.

Student: ____________________ Date: ____________________

School: ____________________ Teacher: ____________________

Other Group Members: ____

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

Student: ___________________________ Date: _____________________

School: ___________________________ Teacher: ___________________

Other Group Members: ____

BUILDING TOWERS THREE COLORS EXTENSION

Find all possible towers that are four cubes tall, selecting from cubes available in three different colors so that the resulting towers have at least one of each color. In the space below, show your solution and provide a convincing argument that you have found them all.

Appendix C Final Projects

Statement of Final Project assignment

15:254:599, Section 81 Central Region
TOPICS IN MATH EDUCATION: LESSON STUDY ON REASONING

FINAL PROJECT – To be handed in at our **December 4th** meeting at Rutgers

We had three cycles of tasks this semester:
Cycle I: Towers 4-tall, selecting from 2 colors; Predicting Towers 3-tall and 5-tall, selecting from two colors
Cycle II: Towers 5-tall, selecting from 2 colors; Pizza Problem, selecting from 4-toppings
Cycle III: Towers 3-tall, electing from 3 colors; Ankur's challenge

For the final project, you are to prepare a booklet with actual samples of your students' work. You should select samples of student work from each of the three cycles that (1) impressed you as an interesting example of reasoning, (2) surprised you because of the strategy or representation selected, or as an impressive product from an unlikely student, and (3) concerned you about the student's struggles to understand the mathematical ideas.

Your booklet should have a cover page with the name of our course, your name, school, and date. Prior to each piece of student work, have a page showing the statement of the task, the grade and class the student was in, the time of your math period, whether the student is regular or special ed, the number of students in that class, and any other pertinent information, i.e. inclusion class, self-contained class, etc.

Be sure the student work that you include is easily read – sometimes work done in pencil does not copy well. Whenever possible, please include original copies of student work in your booklet. Write a reflective description for each piece of student work that you include– explaining why the work impressed, surprised, or concerned you.

At the end of each cycle, include a paragraph about the intervention implementation - what you learned about your students, what was good about the session, what could have been better, and what you would do differently if you had the opportunity to do it again.

Conclude your booklet with a one-page reflection that focuses on what you observed about your students' reasoning and your role as facilitator over the course of the three cycles. Questions to address:

- What did you learn about the mathematics?
- What did you learn about your students' reasoning and mathematical thinking?
- What, if anything, emerged from implementing the three cycles of tasks – for your students, and for you as teacher – i.e. connections made, deepening of understanding, etc.?

Note:
Post-Assessments will be done on line. They will be posted after the December 4th meeting and will be **due on December 13.**

Justin

TOPICS IN MATHEMATICES EDUCATION:

LESSON STUDY ON REASONING

15:254:599 Section 81 Central Region
Professor Judith Landis

Submitted by: Justin

December 4th, 2010

THE ACHIEVEMENT PROGRAM

The Achievement Program is predominantly a self-contained program that consists of a professional team of two Teachers, a School-Counselor as well as a Para-Professional. The program was thought of and made manifest by the Sayreville Middle School Principal, Donna Jakubik. Ms. Jakubik envisioned a program that could service students who were academically at risk and/or behaviorally on the verge of being expelled from continual poor behavioral choices, both in and out of the school. The Achievement Program seeks to promote goal oriented students that realize their potential. The students have opportunities to learn about their strengths. They are given strategies to manage their emotions, develop and achieve goals, resolve conflicts non-violently, and experience a high quality of education.

The program educates Regular and Special Education students but follows the regular education curriculum. Currently, there are fourteen students split into a 6th & 7th grade group and an 8th grade group. There are nine 6th and 7th graders, and five 8th graders. Math is taught fifth period to the 6th & 7th grade group from 10:45 to 11:30. The 8th grade group is taught Algebra I our last academic period from 1:00 to 1:40 pm.

Student: ____________________________ Date: ______________________

School: ____________________________ Teacher: ____________________

Other Group Members:____

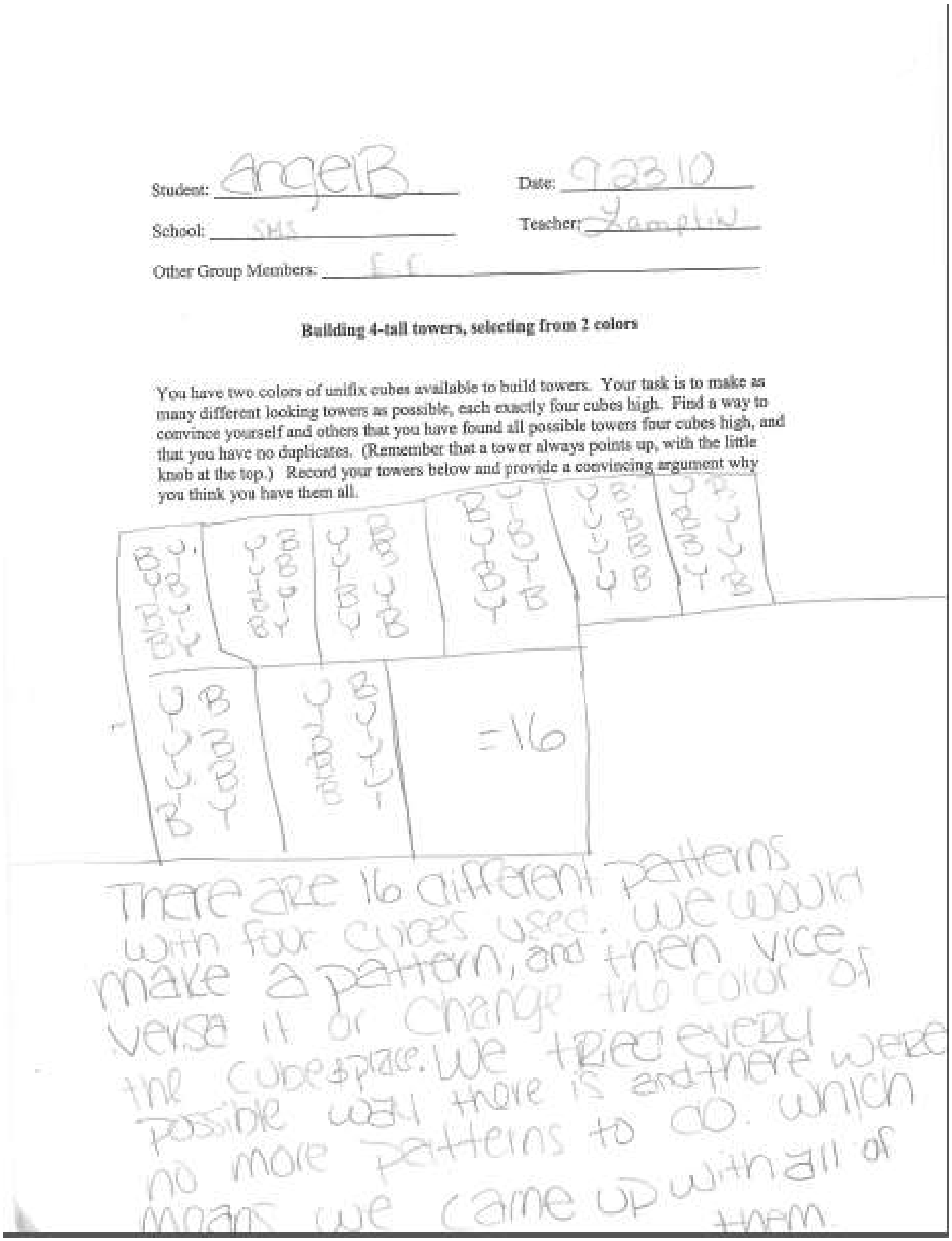
Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why

you think you have them all.

STUDENT:ANGEL

Angel is an 8th grade student who has repeated the seventh grade. Although she is a regular education student, Angel has experienced several losses that lead to gaps in her education. This is Angel's second year in the program. She has experienced many accomplishments since she began. Her meticulously organized notes is a proven tool that helps her mathematically, even though she, "Hates math!" I beg to differ.


Student: AngelB.
Date: 9.23.10
School: SMS
Teacher: Lampkin
Other Group Members: E.F.

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

=16

There are 16 different patterns with four cubes used. We would make a pattern, and then vice versa it or change the color of the cubes place. We tried every possible way there is and there were no more patterns to do. which means we came up with all of them.

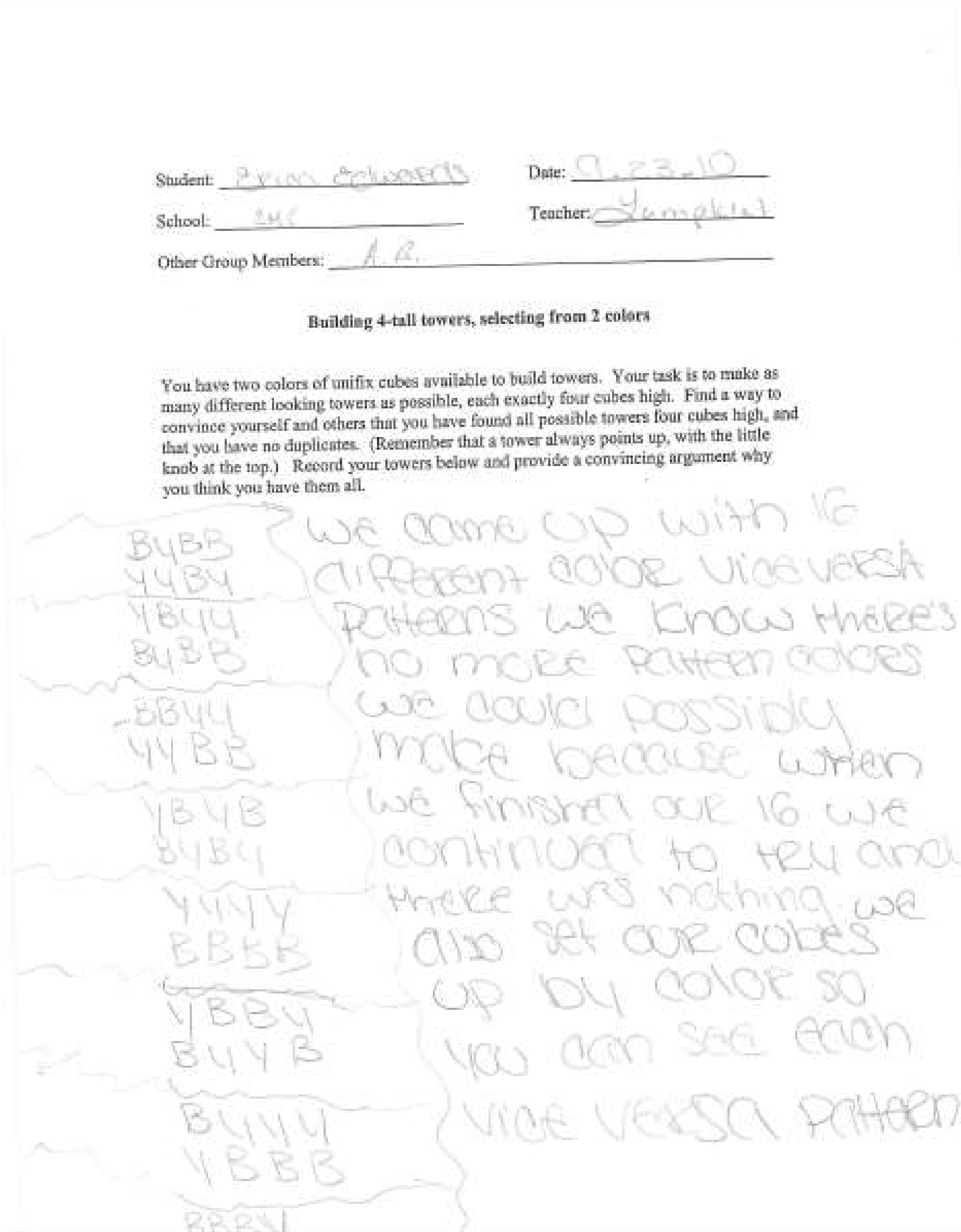
Figure C.1.1 Angel's Cycle 1 Work

IMPRESSIVE EXAMPLE OF REASONING

Angel was most vocal and proactive with her suggestions in the dyad. She would offer and implement her ideas with passive support from her partner. Yet, being the leader, Angel was able to provide some convincing strategies that lead their group to a correct solution to the tower task "Building 4-tall towers, selecting from 2 cubes." I was impressed with Angel's notation of the blocks as well as her creation of logical language that helped her best describe her process of providing all possible towers. In error, the instructor provided an example of "Simple" notation whereas, an empty square represented a yellow cube and a shaded in square represented a blue cube when documenting the towers in pen // pencil. Angel used B — for a blue cube and Y —to represent a yellow cube. This was an original idea that came from her creativity, which was impressive. Lastly, her process of identifying all possible towers by "Vice versa" it was interesting. She even provided a definition of "Vice versa it" meaning to change the color of the cube and its place. This convinced me that she was sincere about making a valiant attempt at expressing her mathematical reasoning just how she thought and created the towers.

STUDENT: ERICA

Erica had been academically unmotivated the past two years. She failed the 6th grade twice and was in desperate need of professional intervention. Fifteen in the 6th grade was not a motivating option. As a result, she was socially promoted to the 8th grade, contingent upon her entry into the Achievement Program. With maturity, ownership, academic support and goal development, Erica has morphed into an "A-Student" in mathematics.



Student: Bryan Edwards Date: 9-23-10

School: SMS Teacher: [illegible]

Other Group Members: A. R.

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

BYBB
YYBY
YBYY
BYBB
BBYY
YYBB
YBYB
BYBY
YYYY
BBBB
YBBY
BYYB
BYYY
YBBB
BBBY

We came up with 16 different color vice versa patterns we know there's no more pattern colors we could possibly make because when we finished our 16 we continued to try and there was nothing we also set our cubes up by color so you can see each vice versa pattern.

Figure C.1.2 Erica's Cycle 1 Work

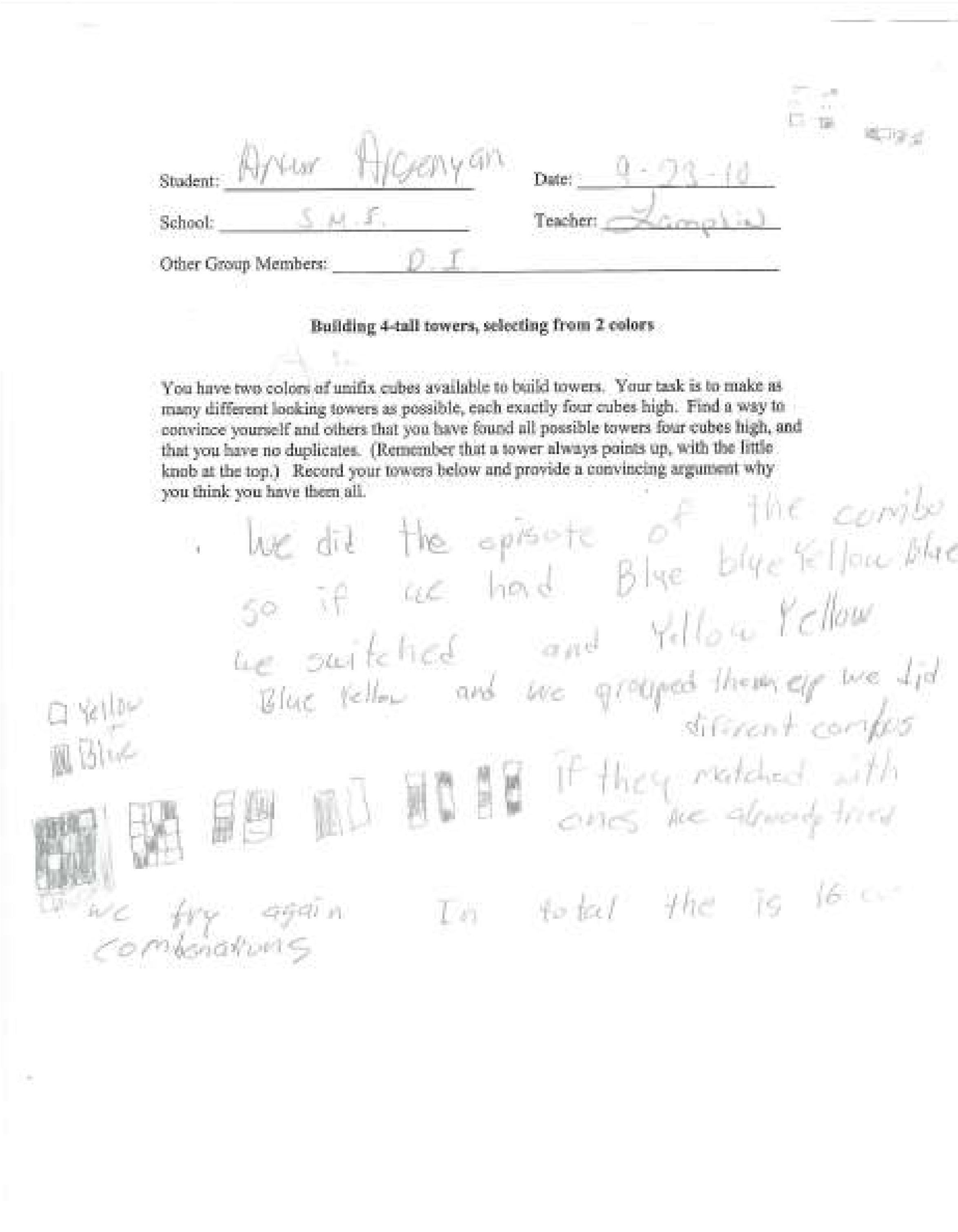
SURPRISED ME BECAUSE OF THE REPRESENTATION SELECTED

Erica's write up really surprised me. September 23rd, approximately two weeks into the school year with limited knowledge about Erica, the instructor perused this record of towers for several moments. I was intrigued by Erica's representation of towers. Her paper was the only one of all other students that had the towers documented horizontally. All others had documented their records of towers vertically. In the last moments of examining her paper I then knew that each and every student has the capacity to invent and or express their mathematical uniqueness in ways in which I would not want to limit. I knew then that I must remove myself (my thoughts of math and the way in which I thought and solved problems) and allow my students the opportunity to express their own mathematical style that made sense to them.

Erica's paper taught me this lesson very early in the year.

STUDENT: ARTUR

Artur is an 8th grade student turning sixteen at the end of January. He has experienced major setbacks both academically and personally. Artur would sit in class unengaged and non-productive. Many of times Artur would not even attend school. Artur entered the Achievement Program mid-year of '09-10 and by the end he had developed more of a positive attitude regarding school. This year "King Artavazd" is a competitive math student that rarely misses school and welcomes the rigor of Algebra I concepts.



Student: Artur Arsenyan Date: 9-23-10

School: S.M.S. Teacher: Lamphin

Other Group Members: D.I.

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

we did the opisote of the combo Blue blue Yellow Blue so if we had Blue and Yellow Yellow we switched and Blue Yellow and we grouped them up we did different combos

Yellow

Blue

if they matched with ones we already tried we try again In total the is 16 co combinations

Figure C.1.3 Artur's Cycle 1 Work

OF CONCERN: A STUDENT'S STRUGGLE

Artur and his partner's physical constructing of 16 towers were quite impressive. They were able to point out several interesting discoveries as to how and why they believed that their construction of 16 towers was the limit. However, when it came time to record the towers AND provide a convincing argument as to why he thinks they have them all, Artur's literary skills showed some of his educational limitations. The instructor understands mathematics as a language that helps people understand and better organize complexities in life. When the instructor read Artur's write up, it became a concern that all of his beautiful thoughts may not be captured just as he constructed and considered.

The initial meeting of teachers at Rutgers was a powerful experience. Prior to our implementation of the task I did not think there was a legitimate way of convincing others and myself that all possible tower combinations would be accounted for. With the assistance of my partner and prodding from our instructors we had truly come up with a strong case that was convincing to others that all towers were represented.

REFLECTION OF CYCLE ONE

The session was a success due to my level of motivation to implement as well as my student's intrigued at the novelty of the Unifix cubes. The students feed off of my energy towards this assignment. I really pushed the students that the write up portion was the most important component and I had them take notes on their thought process and strategies while constructing the towers. Encouraging the students to take notes, really worked out well as evidenced in their write ups. On the other hand, I would have not provided my students with a notation key for recording their towers. Erica and Angel's unique documentation was neat and I would have wanted to see what the males in the classroom would have come up with, had it not been for my help.

Building 5-tall towers, selecting from 2 colors

You have two different colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly five cubes high. Find a way to convince yourself and others that you have found all possible towers five cubes high, and that you have no duplicates. Record your towers below and provide a convincing argument why you think you have them all.

Student: [illegible] Date: 10.28.10

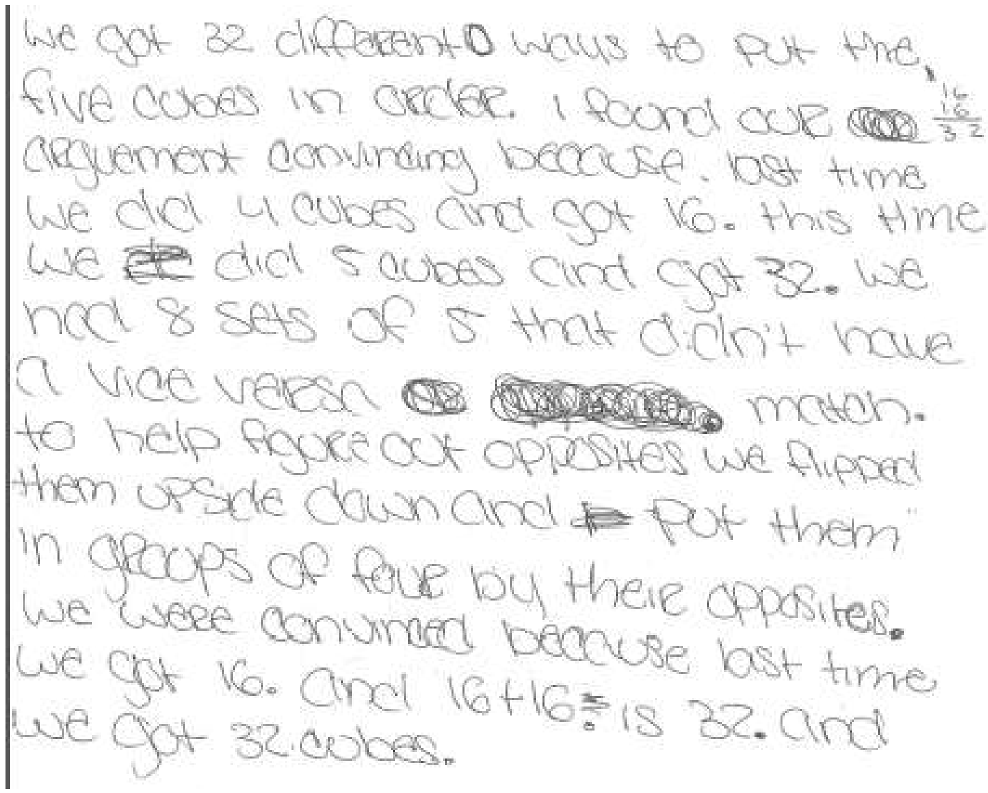
School: ______________________ Teacher: ______________________

Other Group Members: ____

Building 5-tall towers, selecting from 2 colors

You have two different colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly five cubes high. Find a way to convince yourself and others that you have found all possible towers five cubes high, and that you have no duplicates. Record your towers below and provide a convincing argument why you think you have them all.

YBBBY BYYYB	BYBYB YBYBY	BBYBB YYBYY	YYYYY BBBBB	
BYBBB YBYYY	[illegible]	[illegible]	BBBYB YYYBY	YBBBY BYYBB
YYBBY BBYYB	BBBBY YYYYB	YBBBB BYYYY	YBYBB BYBYY	
BBYBY YYBYB	YBYYB BYBBY	BYYBY YBBYB	BBYYY YYYBB	
YYBBB BBBYY				


we got 32 different ways to put the five cubes in order. I found our arguement convincing because. last time we did 4 cubes and got 16. this time we did 5 cubes and got 32. we had 8 sets of 5 that didn't have a vice versa match. to help figure out opposites we flipped them upside down and put them in groups of four by their opposites. we were convinced because last time we got 16. and 16+16= is 32. and we got 32 cubes.

16
16
32

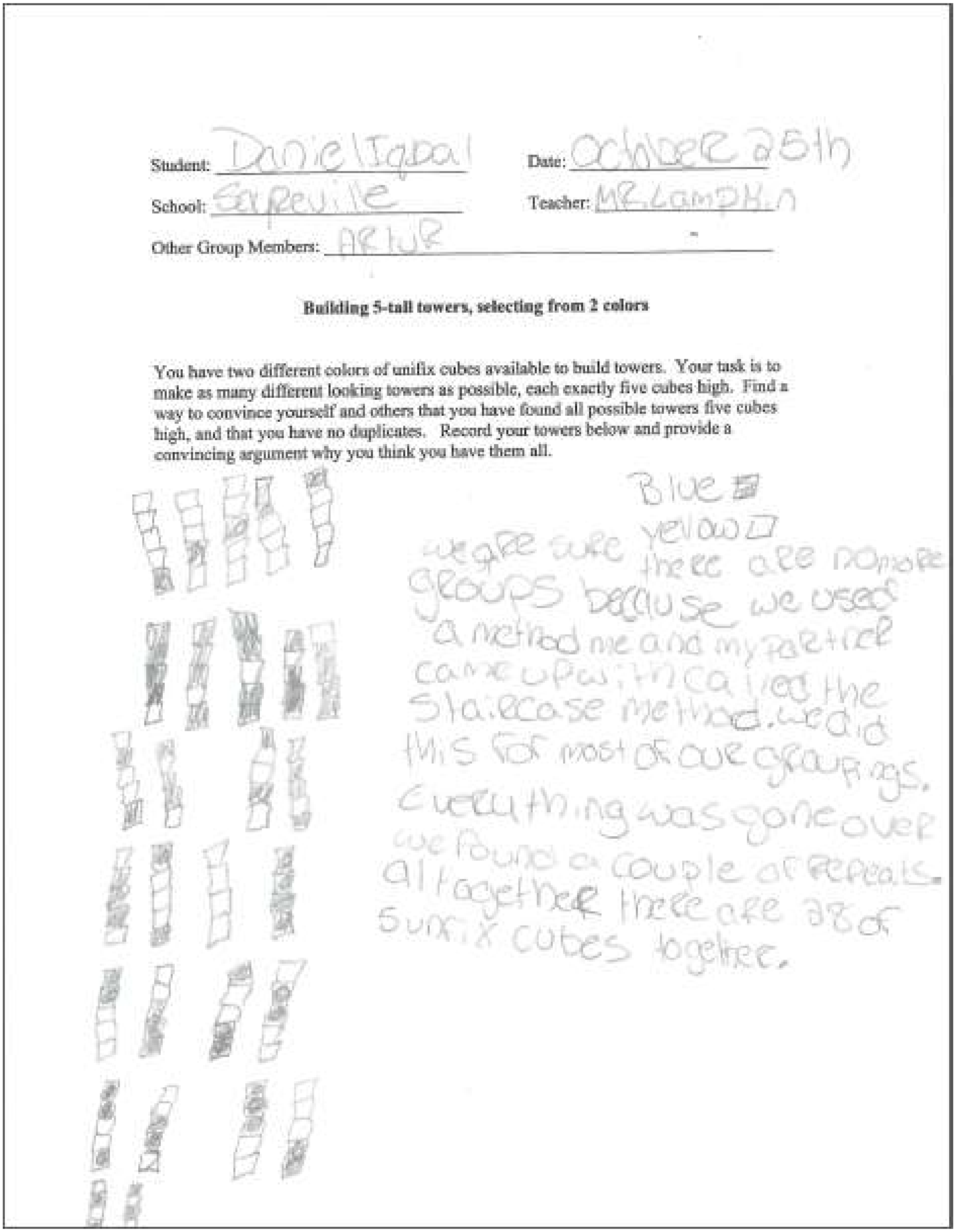
Figure C.1.4 Erica's Cycle 2 Work

IMPRESSIVE EXAMPLE OF REASONING

Erica's documentation of her convincing argument had improved greatly compared to her initial implementation. The reasoning that made this work most impressive was the section of the write up where Erica alluded to the former task of 4-tall towers, selecting from two colors and this task comparatively. On her paper there is an addition computation of 16 + 16 = 32. She then writes that she is convinced that her answer is 32 because, "Last time we did 4 cubes and got 16... this time we did 5 cubes and got 32." Towards the end of her write up she continued with this though and stated, "We were convinced because last time we got 16. And 16 + 16. Is 32. And we got 32 cubes." It is the writer's belief that she indeed believed in this connection as her proof or evidence that her response was accurate.

STUDENT: DANIEL

Last year Daniel missed lots of school days due to hospitalizations and frequent suspensions. Subsequently, Dan failed his core subjects but was promoted to the 8th grade by satisfying summer school requirements. At the conclusion of last year '09-10, Daniel initiated a conversation with the School Counselor to join the Achievement Program as an academic and behavior intervention. With persistence and pride, Dan has become the most mathematically astute student in the class.



Student: Daniel Iqbal Date: October 25th

School: Sayreville Teacher: MR. Lampkin

Other Group Members: ARTUR

Building 5-tall towers, selecting from 2 colors

You have two different colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly five cubes high. Find a way to convince yourself and others that you have found all possible towers five cubes high, and that you have no duplicates. Record your towers below and provide a convincing argument why you think you have them all.

Blue

yellow

we are sure there are no more groups because we used a method me and my partner came up with called the staircase method. we did this for most of our groupings. everything was gone over we found a couple of repeats. altogether there are 28 of 5 unifix cubes together.

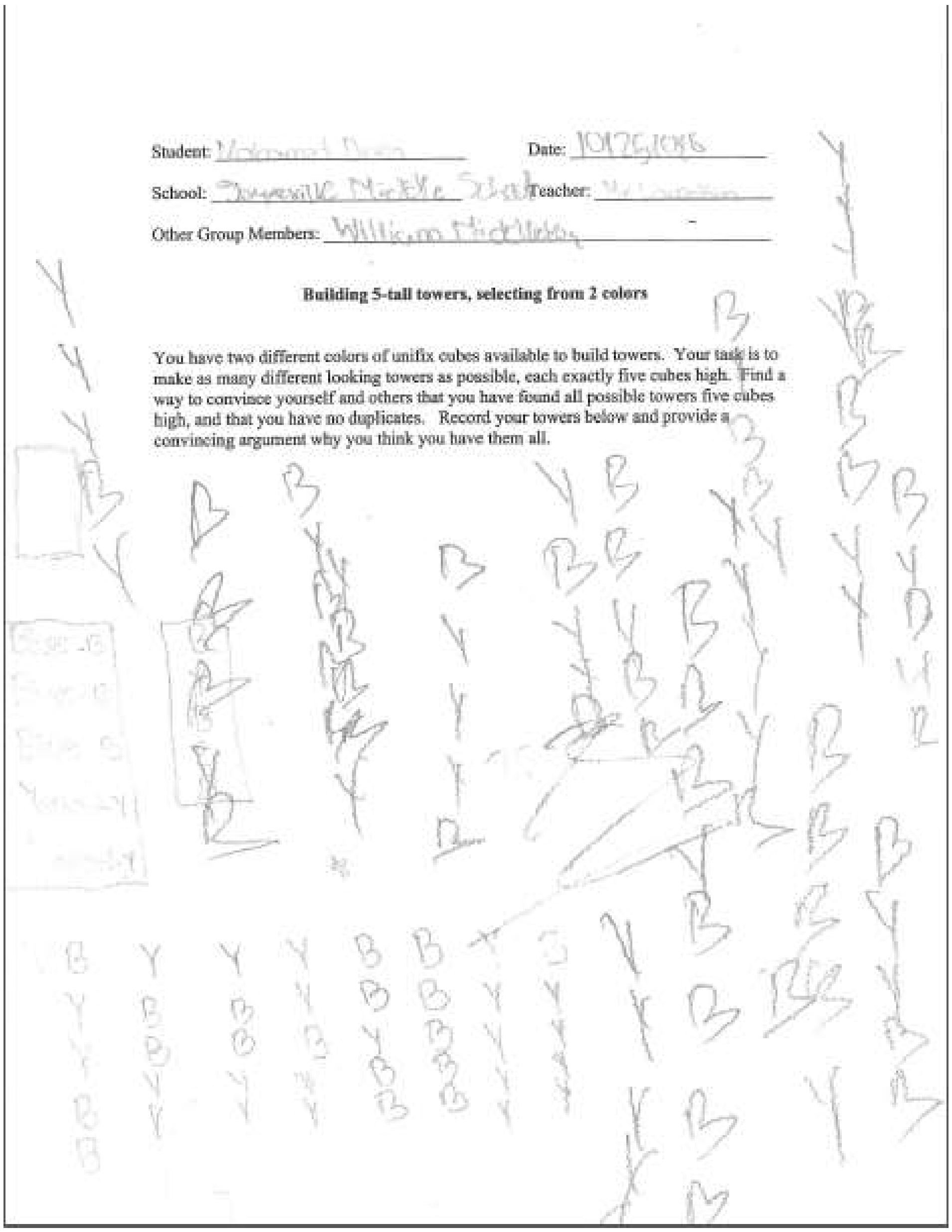
Figure C.1.5 Daniel's Cycle 2 Work

SURPRISED ME BECAUSE OF THE STRATEGY SELECTED

As the instructor, one is privy to observing multiple strategies with varied nuisances of thoughts and verbiage. There were instances where the writer doubted the student's level of engagement. There would be struggling to get them to record a convincing argument. Some of the students would resist, whine and complain. Others would simply write the loosest interpretation of the actual process of work performed. The moment the writer examined Dan's write up, it was realized that they were performing and well vested. "We are sure there are no more groups because we used a method me and my partner came up with called the Staircase Method. We did this for most of our groupings." Dan's partner documented this similar thought in his write up. This was surprising because other "proofs" the writer observed used similar wording. Daniel identified this staircase method that he and his partner "Came up with" as the convincing strategy. The surprise is that the writer never heard this strategy mentioned during the construction of the towers, yet it was reflected in their write up.

STUDENT: MOHAMED

Mohamed is a 7th grade student who is classified as E.D. (Emotionally Disturbed). He spent more than half of the 2009-2010 school year being home schooled because he could not conform to the school's behavior standards. Mathematically, Mohamed's skills are that of a third grader.


Student: Date:
School: Teacher:
Other Group Members:

Building 5-tall towers, selecting from 2 colors

You have two different colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly five cubes high. Find a way to convince yourself and others that you have found all possible towers five cubes high, and that you have no duplicates. Record your towers below and provide a convincing argument why you think you have them all.

Figure C.1.6 Mohamed's Cycle 2 Work

STRUGGLES TO UNDERSTAND THE MATHEMATICAL IDEAS

Mohamed is the most meticulous writer the instructor has ever seen. He spends a great deal of time carefully writing each letter to each word. The construction of towers exercise went rather well. He and his partner seemingly had worked well together without issue. When Mohamed was instructed to record the thoughts and actions of the task, he began at his normal detailed way as seen in the lower left-hand comer of the attached document. All of a sudden-it is assumed-Mohamed became incensed with frustration and began to record the towers on his paper in a frantic manner. There still was plenty of time left in the period and many of the other students had not completed their write ups. By examining his paper, it seems that his learning disability was triggered by the rigor or duration of the task. This was evident through his paper that he lost the concentration to complete the task with care. His paper reminds me of the Matrix movie with all of the letters moving at a rapid pace vertically. Perhaps, after a half of an hour, this math task must have looked like a bunch of letters.

REFLECTION ON CYCLE TWO

I had both of my groups implement the 5-tall tower task and the Pizza Problem.

However, I had the ability to provide my 8th grade group with an hour of time, whereas my 6th and 7th grade group only had approximately 35 minutes. Without altering my students schedule the 8th graders have an extra twenty minutes of time before their dismissal. My 8th grade students performed well on this task. But I feel as if my 6th and 7th grade students were more engaged in the process this time but had to be rushed. I would have liked to have my younger group work a little longer to see them thoroughly complete this assignment without rushing them along. I was beginning to see some interesting work but had to cut them short. I would have extended my time for having the blocks with my colleagues and given the students another day to complete the task with care. I realized that this was unfair and perhaps damaging to them mathematically by causing unwarranted stress due to time constraints.

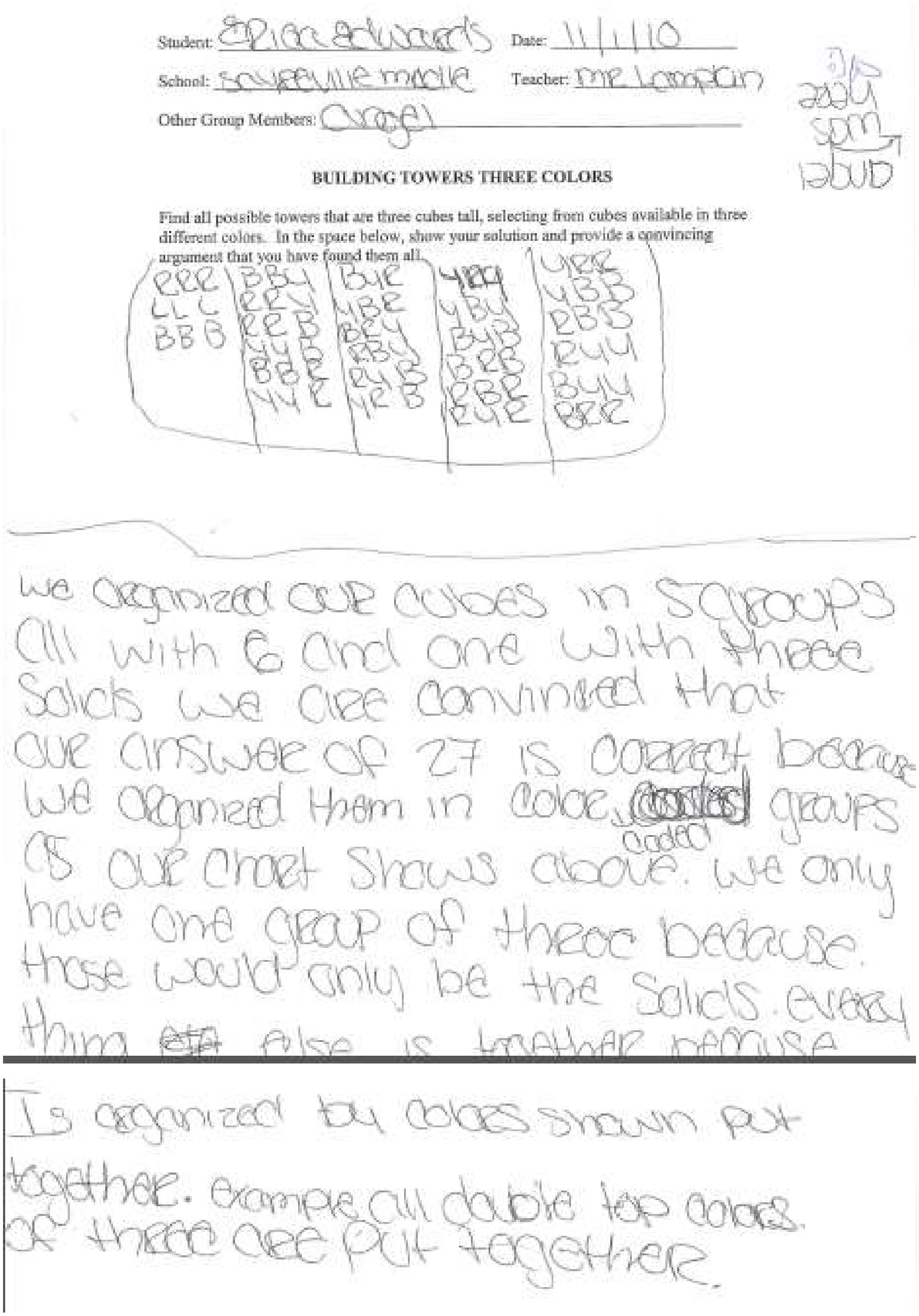
Student: ____________________________ Date: ______________________

School: ____________________________ Teacher: ____________________

Other Group Members: ___________________________

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.



Student: Erica edwards Date: 11/1/10

School: Sayreville middle Teacher: Mr Lampkin

Other Group Members: Angel

3/6
22
5pm
12bud

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

RRR	BBY	BYR	YRR	YRR
LLL	RRY	YBR	YBY	YBB
BBB	RRB	BRY	BYB	RBB
	YYB	RBY	BRB	RYY
	BBR	RYB	RBR	BYY
	YYR	YRB	RYR	BRR

we organized our cubes in 5 groups all with 6 and one with three Solids we are convinced that our answer of 27 is correct because we organized them in color coded groups as our chart shows above. we only have one group of three because those would only be the Solids. every thing else is together because

Is organized by colors shown put together. example all double top colors of three are put together.

Figure C.1.7 Erica's Cycle 3 Work

IMPRESSIVE EXAMPLE OF REASONING

On the back of Erica's paper, she expressed and example of how her partner and her grouped their towers together. "All double top colors of three are put together." Erica's strategy of collecting all of the towers worked as follows. They would randomly construct as many towers as they could. Then, guess and check to see if there are more creations they could construct. Their strategy would enter at this point. They would group all of the towers together by color tops. The impressive thought to this is found in her recording of the towers. The four groups of towers, six in each, all have a top or bottom constant that helped them account for all of the towers. They knew that if they controlled for a constant that there were only two alternative positions, thus two towers. As a result, it was obvious which towers had not been created. By the end of cycle three, Erica and her partner had mastered this approach of constructing all of the towers.

STUDENT: THOMAS

Thomas is a 6th grader who recently transferred to the Sayreville Middle School from out of district. Tom has literally been in six different districts in the last five years. His I.E.P. (Individualized Educational Plan) indicates major gaps in education as well as a dual learning disability. Thomas was considered academically and behaviorally at risk prior to his entry into the program. Currently, Thomas is performing well mathematically and considered "Smart" by his peers.

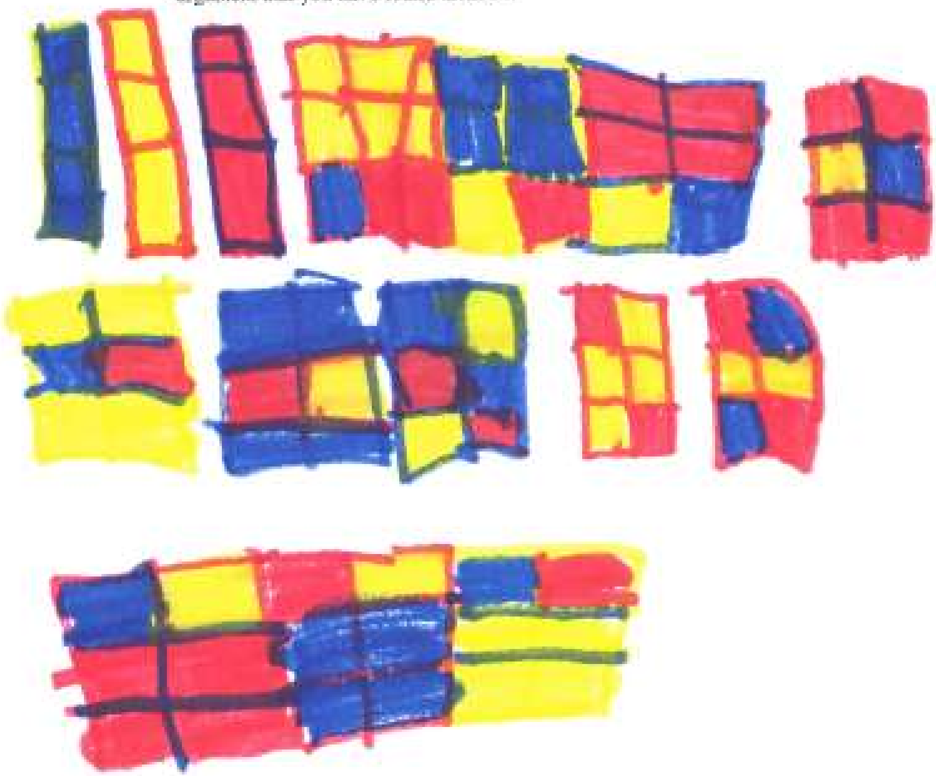
Student: Tom Date: 11/1/10

School: Teacher: Lumpkin

Other Group Members: Maigone

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.



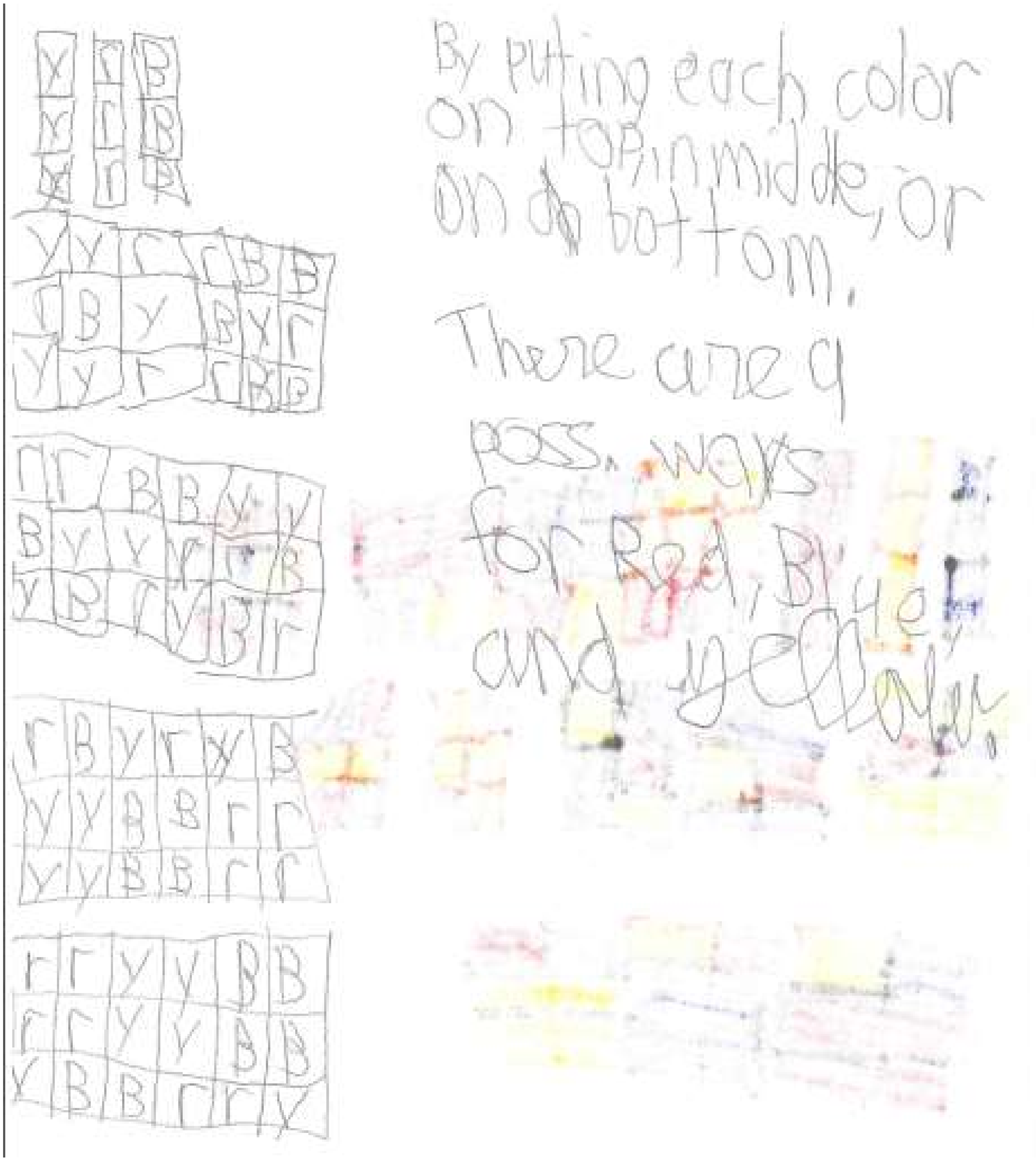
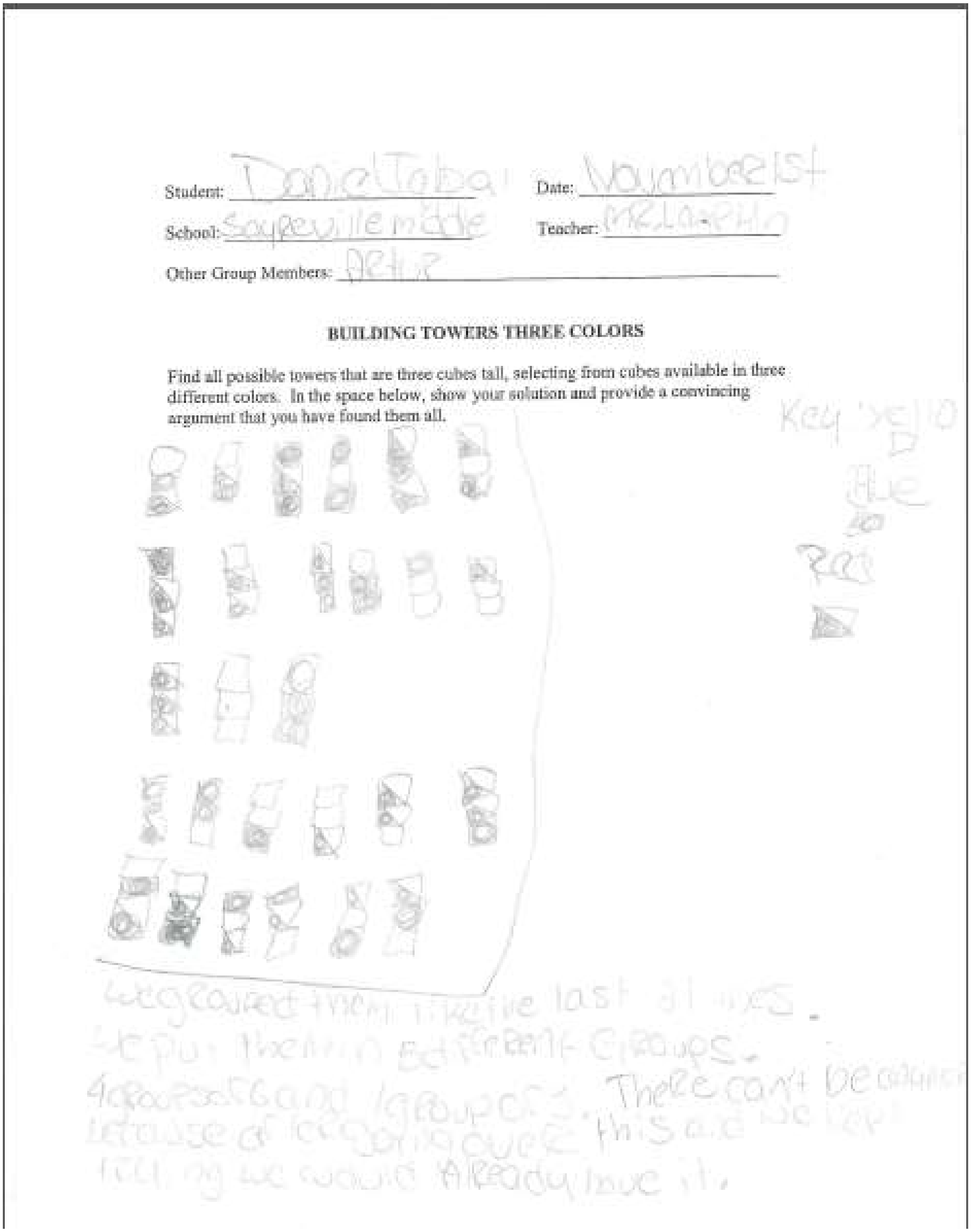


Figure C.1.8 Thomas's Cycle 3 Work

IMPRESSIVE PRODUCT FROM AN UNLIKELY STUDENT

The front of Tom's paper illustrated an account of all towers he constructed with color coded pictures. As the writer went to each table, it was evident that Thomas and his partner had physically constructed all of the towers but still needed a convincing statement. As the writer probed to be convinced, Thomas began to group his towers in a manner that made it easier to follow his train of thought. Thomas had done an impressive job convincing the instructor that all towers were accounted for. As a result, he recorded all of his towers again on the back of his sheet in a more convincing way. Thomas began to write up his statement but had run out of time. It is the writer's belief that Thomas would have documented a more solid response if there was more time.



Student: ______________ Date: ______________

School: ______________ Teacher: ______________

Other Group Members: ______________

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

Figure C.1.9 Daniel's Cycle 3 Work

STUDENT'S STRUGGLE WITH THE MATHEMATICAL IDEAS

By the end of cycle three, some of the students lost the zeal to want to satisfy these tasks with care. The concerning part, illustrated in Daniel's paper, is his lack of documentation. The write up portion became more of an obstacle than the construction of the towers. This is concerning because in the field of Mathematical Studies, there is a component of documentation that is necessary. Construction and experimentation are the fun aspects of math. However, the documentation of the findings and solutions is paramount. As illustrated in my most astute student's work, the fervor and follow through had been lost.

REFLECTION OF CYCLE THREE

The dyad of girls really worked well throughout all three cycles. They required limited probing and direction. However, I found myself so intrigued by their strategies and thoughts that I remained at their station longer than the other groups. My male group would have benefitted from this critical time. I feel like I could have poked and prodded more at their argument. My Para-Professional spent most of the time at their station and did not have the same level of training with questioning tools and savvy that I had. It is evidenced in their last work that they had not progressed in their write up regarding reasoning and a more thorough strategy of convincing. I was encouraged by the Professor to spend perhaps a lengthy period of time with one group at a time, and the following session move around to another group, in contrary to dotting from group to group only providing them with snippets of probing and questioning.

OVERALL REFLECTION

Students come to the "Mathematical Table" with a unique system of how they make sense of their environment. All people posses a level of individuality that they use to solve problems and interpret situations with. I had no idea how much of their thoughts and uniqueness is used and could be used in the math class. This was made evident by focusing on student reasoning. In the early readings, I learned how a second grade female student, when solving a combinatory problem, stated that "white pants and a yellow shirt" did not match (go together). This piece of information allowed me opportunities to engage my students in a way in which their perspectives and paradigms would be exposed and experienced.

I learned that my students were creative! I know that mathematics is an ongoing, progressive discipline that encourages innovation and is constantly evolving. The moments that I allowed my students with room to bring forth their reasoning, I tapped into young mathematicians. My students began to invent words that helped them better describe their thought processes. They creative novel notations and documented information far differently than what I had been used to, and moreover, it made lots of sense! I was impressed at what training in student reasoning can unveil.

Lastly, I realized that through the study of reasoning, that I could apply these same techniques in every aspect of learning. The proposals to have someone provide a convincing argument in any circumstance is powerful. It pulls at a person's inner thoughts and exposes a deeper level of thinking and engagement. The students then take more ownership in the educational process and strive to present a more diligent and thorough explanation for themselves. To my surprise, the students enjoy this dialogue of convincing and supportive theories regarding most topics including math.

Mitch

Topics in Math Education:

Lesson Study on Reasoning

Mitch

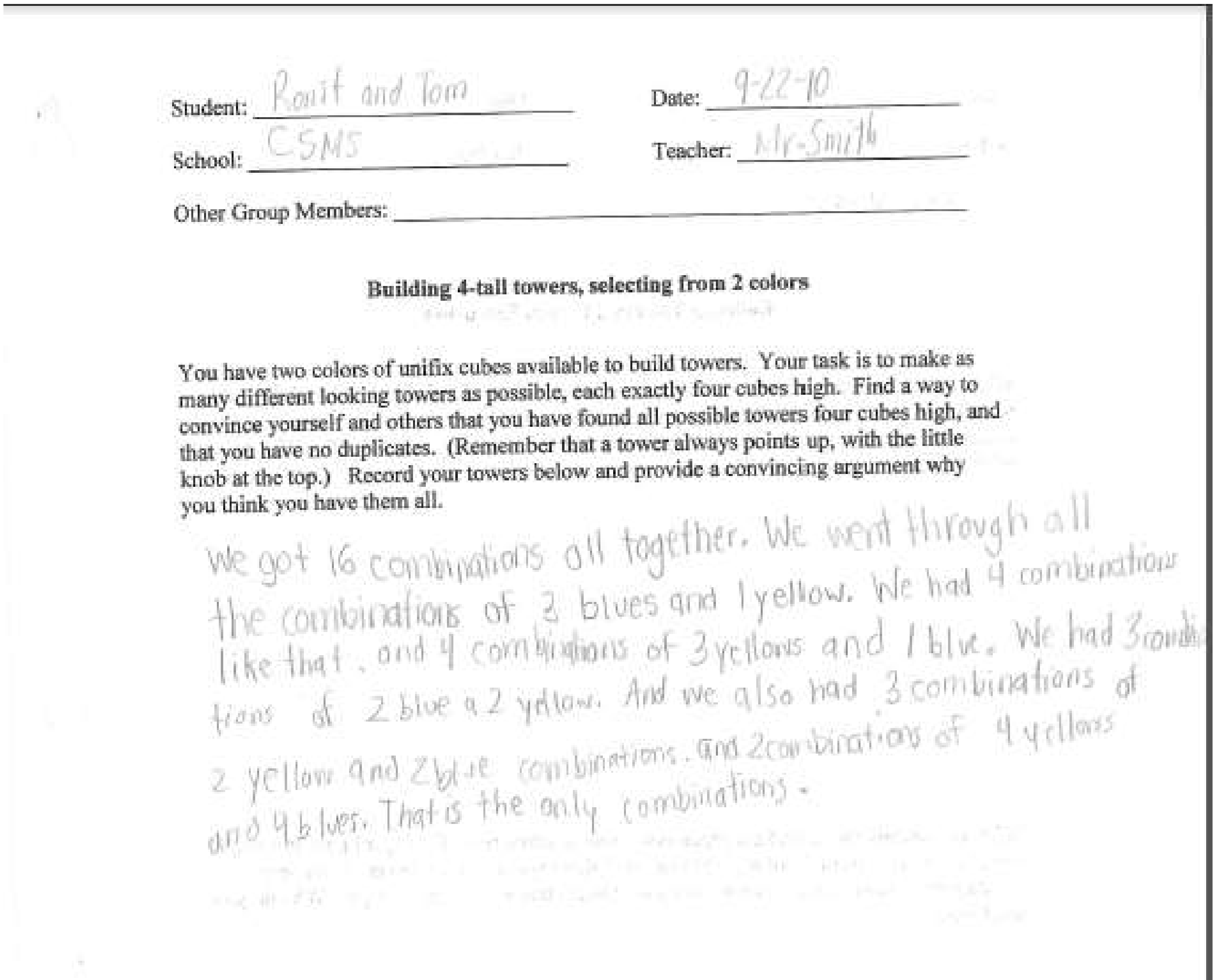
December 4,

2010

Cycle I: Towers 4-tall, selecting from 2 colors; Predicting Towers 3-tall and 5-tall, selecting from two colors

Example 1: Student work that impressed me as an interesting example of reasoning: Ronit

Statement of the task	Building 4-tall towers, selecting from 2 colors: You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that youhave no duplicates. (Remember that a tower always points up, with the little knob at the top). Record your towers below andprovide a convincing argument why you think you have them
Grade	8
Class	Pre-Algebra, Inclusion Class
Regular/Special Ed	Regular Ed
Number of students in class	21



Student: Ronit and Tom Date: 9-22-10

School: CSMS Teacher: Mr. Smith

Other Group Members:

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

We got 16 combinations all together. We went through all the combinations of 3 blues and 1 yellow. We had 4 combinations like that, and 4 combinations of 3 yellows and 1 blue. We had 3 combinations of 2 blue a 2 yellow. And we also had 3 combinations of 2 yellow and 2 blue combinations. and 2 combinations of 4 yellows and 4 blues. That is the only combinations.

Figure C.2.1 Ronit's Cycle 1 Work

Explanation: Why Ronit's Work is Impressive

Ronit's work impressed me because of his ability to describe his overall understanding of this assignment. He was able to provide a convincing argument and do so clearly and thoroughly. His use of combinations and his explicit discussion of each one clearly demonstrates that he understood the object of the assignment. Not only this, but his work did not discuss the very prevalent use of "opposites," which is something that many of my other students utilized. This means that Ronit's explanation is even more valuable in that it demonstrates a type of learning and understanding that did not necessarily exist among many other students in the class.

Cycle I: **Towers 4-tall, selecting from 2 colors; Predicting Towers 3-tall and 5-tall, selecting from two colors**

Example 2: Student work that surprised me because of the strategy/representation selected OR Student work that is an impressive product from an unlikely student: Susan and Dhara

Statement of the task	Building 4-tall towers, selecting from 2 colors: You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top). Record your towers below and provide a convincing argument why you think you
Grade	8
Class	Pre-Algebra
Regular/Special Ed	Regular Ed
Number of students in class	21

Student: Dhara Patel Date: ____________

School: ____________ Teacher: ____________

Other Group Members: ____________

Building Towers 2 Colors Extension

Without building them, make a prediction about a solution for finding all possible 3-tall towers, selecting from 2 colors. Do you think there will be more, fewer, or the same number of possible towers as you found for towers that were 4 cubes high? Why do you think that?

1) yyy 8) ybb They will be less possible towers then towers of 4 cubes.

2) bbb

3) yby

4) byb

5) yyb

6) byy

7) bby

Without building them, make a prediction about a solution for finding all possible 5-tall towers, selecting from 2 colors. Do you think there will be more, fewer, or the same number of possible towers as you found for towers that were 4 cubes high? Why do you think that?

1)
2)
3)

We thought that since we divided 18÷2 in the prevois promblem, we could mullple by 2.

Student: Dhara Patel Date: 7/21/10

School: Carl Sandburg Teacher: Mr. Smith

Other Group Members: Susan.

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

1) byyy
2) yyyb
3) yyyy
4) bbbb
5) ybyy
6) bybb
7) yyby
8) bbyb
9) bbby
10) yyyb
11) yybb
12) bbyy
13) byyb
14) ybby
15) ybyb
16) byby

We think that there are 16 different possibleities for making different looking towers, each exactly four cubes high. Our stagery was that I would make a combination and my pathner would make the oppisite of my design. After that we started moving the colored blocks down. For example, we had ybyy & bybb, then we moved down the colors & got yyby & bbyb.

Student: Isai Zaitsev Date: 9-22-10

School: Teacher:

Other Group Members:

Building Towers 2 Colors Extension

Without building them, make a prediction about a solution for finding all possible 3-tall towers, selecting from 2 colors. Do you think there will be more, fewer, or the same number of possible towers as you found for towers that were 4 cubes high? Why do you think that?

1 yyy
2 bbb
3 yby
4 byb
5 yyb
6 byy
7 bby
8 ybb

We thought there would be half less than 4 cubes high. So we left 8 empty spaces and used the same strategy from before and concluded that since there are only 3, we can only make 8 combinations

Without building them, make a prediction about a solution for finding all possible 5-tall towers, selecting from 2 colors. Do you think there will be more, fewer, or the same number of possible towers as you found for towers that were 4 cubes high? Why do you think that?

We came up with the conclusion that if dividing by 2 from 16 (orig. Answer) we got all the possibilties for the 3-cube high, so we multiplied by 2 on 16 and got 32. We think this is the answer, because before since it was only 3 cubes, we eliminated a few 4-cube possibilties and got 8, further pushing our strategy. So by saying 32 is the answer, we added 2 cubes per group, with this we think is all the combinations

Student: Susan Zaitz Date: 9-22-10

School: ____________ Teacher: ____________

Other Group Members: Dhara

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

1 b, y, y, y
2 y, b, b, b
3 y, y, y, y
4 b, b, b, b
5 y, b, y y
6 b, y, b b
7 y, y, b, y
8 b, b, y, b
9 b, b, b, y
10 y, y, y, b
11 y, y, b b
12 b, b, y, y
13 b, y, y, b
14 y, b, b, y
15 y, b, y, b
16 b, y, b, y

We believe we have found them all because we did the combinations in groups. I would do, for example, b, y, y, y and then my partner would do the opposite, y, b, b, b. Using this strategy we came up with 16 combinations and then went through the cubes, switching the colors back and forth until we concluded there were no more possibilities

Continuing this strategy, we would go down with the colors, going from b, y, y, y to y, b, y, y then y, y, b, y and finally y, y, y, b. We used this to pair the groups together with their color opposites

Figure C.2.2 Susan's and Dhara's Cycle 1 Work

Explanation: Why Susan and Dhara's Work Surprised and Impressed Me

Susan and Dhara worked together very well during this project. I was impressed by the extremely collaborative nature of their work. The girls explained that one of them would first make the combinations in groups that they saw, and then the other, based on what the first one came up with, would find the opposite combinations. This is a great way to work together in a group, and it helps each student to really understand each aspect of the assignment. By working together in this way, the girls were each able to work on and perfect first finding combinations and then finding their opposites. This eventually led them to the correct answer of 16. Once they found this answer, each of them was also able to thoroughly and accurately articulate how and why they came to their answer. By doing so, they made a convincing argument and proved their understanding. I was very impressed with this way of doing things, as it is not what some of the other students did. I also really enjoyed the fact that each girl worked so well with each other and that they were able to be a part of each aspect of the assignment. I was proud that they came up with this strategy on their own and that it helped improve their understanding, eventually leading them to the correct answer.

Cycle I: Towers 4-tall, selecting from 2 colors; Predicting Towers 3-tall and 5-tall, selecting from two colors

Example 3: Student work that concerned me about the student's to understand the mathematical ideas affiliated with this project: Stephen

Statement of the task	Building 4-tall towers, selecting from 2 colors: You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top). Record your towers below and provide a convincing argument why you think you
Grade	8
Class	Pre-Algebra
Regular/Special Ed	Regular Ed
Number of students in class	21

Student: Stephen Carroll Date: ______________

School: ______________ Teacher: ______________

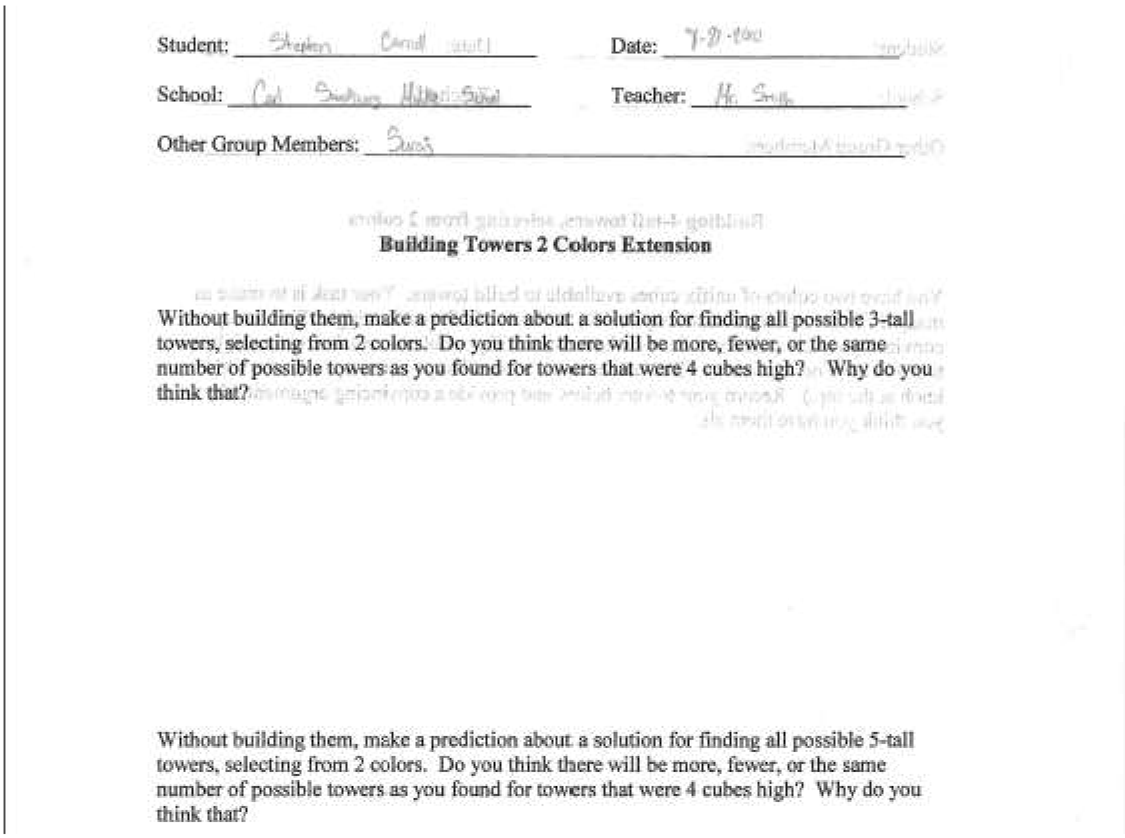
Other Group Members: ______________

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

1. YBBB
2. BBBY
3. BYBB
4. BBYB ⟸ The total number of combinations is 13.
5. BYYB
6. YYBB
7. BBYY
8. BBBB
9. YYYB
10. YBYB
11. BYBY
12. YBYY
13. YYBY

I am convinced that I have done all of the combinations because I have made every combination that is possible and listed it above.



Student: ______ Date: ______

School: ______ Teacher: ______

Other Group Members: ______

Building Towers 2 Colors Extension

Without building them, make a prediction about a solution for finding all possible 3-tall towers, selecting from 2 colors. Do you think there will be more, fewer, or the same number of possible towers as you found for towers that were 4 cubes high? Why do you think that?

Without building them, make a prediction about a solution for finding all possible 5-tall towers, selecting from 2 colors. Do you think there will be more, fewer, or the same number of possible towers as you found for towers that were 4 cubes high? Why do you think that?

Figure C.2.3 Stephen's Cycle 1 Work

Explanation: Why Stephen's Work Concerned Me

Stephen's work is concerning on a number of levels. Not only is his answer incorrect, but his explanation is incomplete. This suggests that he does not understand the material or the concepts involved in this project. He did not ask for help, yet he is not able to provide a convincing argument, Instead, he only says "I am convinced that I have done all of the combinations because I have made every combination that is possible and listed it above." This explanation does not account for the possibility that he could have missed combinations, nor does it try to employ any logical reasoning or explanation as to why he knows that these are all, and the only combinations available. His work here points to some greater issues about problem solving that are concerning.

Cycle I Conclusion

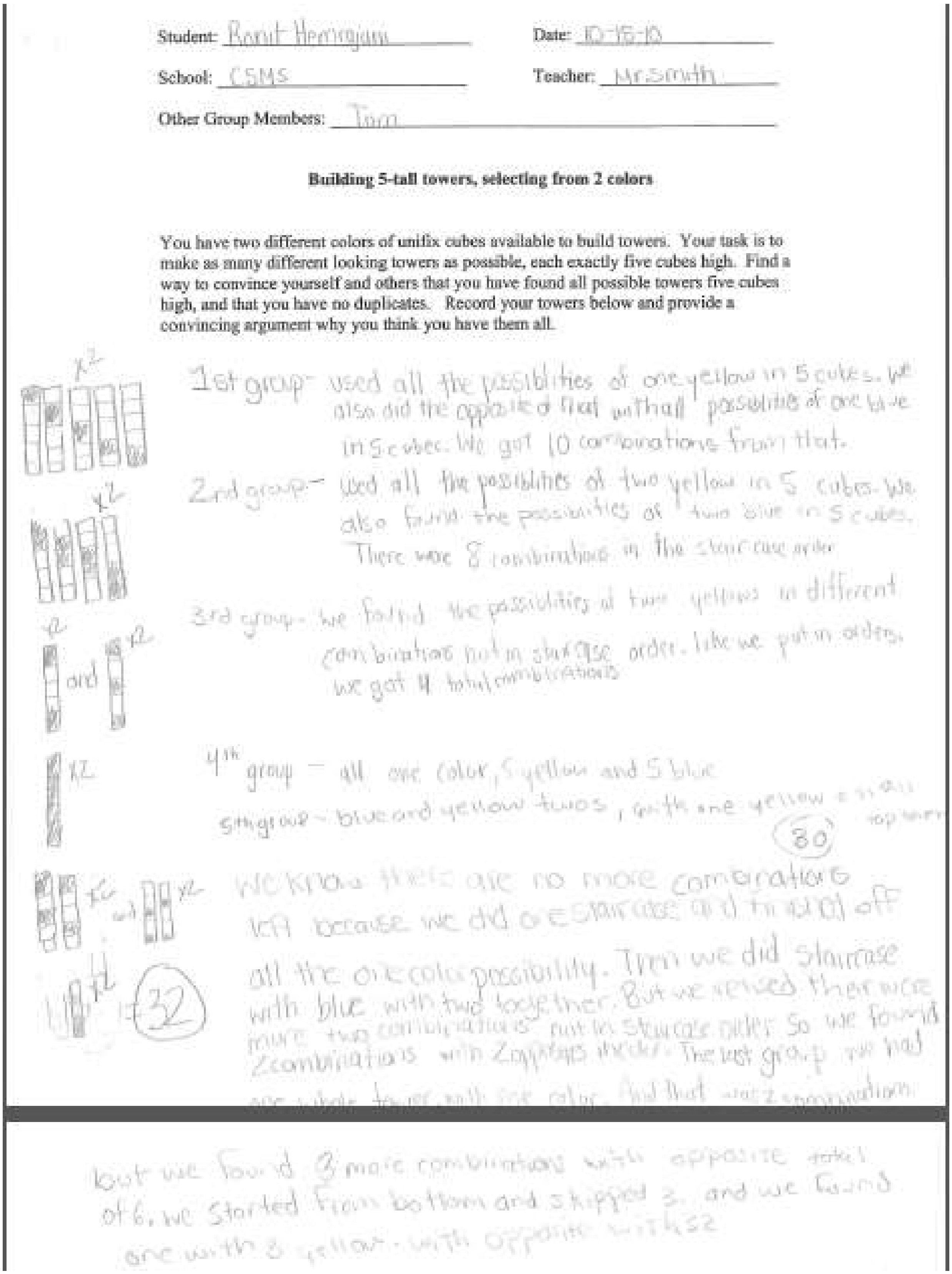
One thing that I learned about my students is that although they are very good at coming up with creative ways to solve problems like the ones in this cycle, they were not very good at writing and explaining what they did. I noticed that what students told me that they were doing and how they were arranging their towers was not what they were writing down on their papers. Even after I encouraged many different groups to write down what they had already explained to me, they had trouble putting their explanations into words or just wanted to draw pictures. I did think it was very impressive to see how students were able to group the towers together and then come up with different ways to figure out if they had created

all of the possible towers. If l had to do this session again, I might have spent a few minutes the day before talking about what makes something a convincing argument, but without mentioning anything about this specific problem.

Cycle II: Towers 5-tall, selecting from 2 colors; Pizza Problem, selecting from 4-toppings

Example 1: Student work that impressed me as an interesting example of reasoning: Ronit

Statement of the task	Building 5-tall towers, selecting from two colors: Your have two different colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, eachexactly five cubes high. Find a way to convince yourself and others that you have found all possible towers five cubes high, and that you have no duplicates. Record your towers below and
Grade	8
Class	Pre-Algebra
Regular/Special Ed	Regular Ed
Number of students in class	21



Student: Ronit Hemrajani Date: 10-15-10

School: CSMS Teacher: Mr. Smith

Other Group Members: Tom

Building 5-tall towers, selecting from 2 colors

You have two different colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly five cubes high. Find a way to convince yourself and others that you have found all possible towers five cubes high, and that you have no duplicates. Record your towers below and provide a convincing argument why you think you have them all.

x2

1st group- used all the possiblities of one yellow in 5 cubes. We also did the opposite of that with all possiblities of one blue in 5 cubes. We got 10 combinations from that.

x2

2nd group- Used all the possiblities of two yellow in 5 cubes. We also found the possiblities of two blue in 5 cubes. There were 8 combinations in the staircase order

x2 and x2

3rd group- we found the possiblities of two yellows in different combinations not in staircase order. like we put in orders. we got 4 total combinations

x2

4th group - all one color, 5 yellow and 5 blue

5th group - blue and yellow twos, with one yellow

30

x2 and x2

x2 32

We know there are no more combinations left because we did one staircase and found all off all the one color possibility. Then we did staircase with blue with two together. But we noticed there were more two combinations not in staircase order. So we found 2 combinations with Zigzags inside. The last group we had one whole tower with one color. And that was 2 combinations

but we found 3 more combinations with opposite total of 6. we started from bottom and skipped 3. and we found one with 3 yellow. with opposite with 2

Figure C.2.4 Ronit's Cycle 2 Work

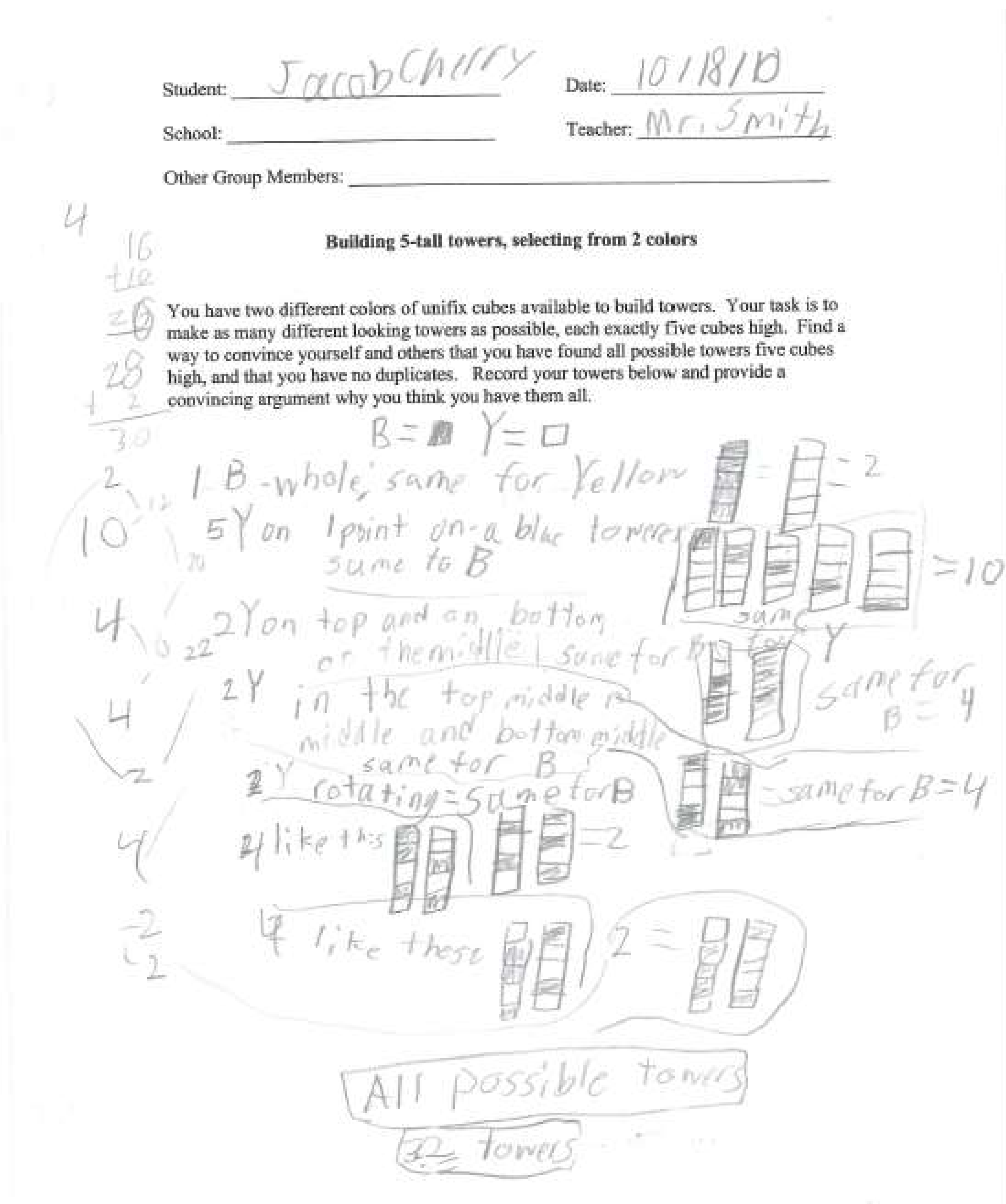
Explanation: Why Ronit's Work is Impressive

Normally, I wouldn't like to use Ronit's work twice in this booklet; however, his explanation in this problem is exceptional. He is organized and thorough in his explanation and his work is almost entirely convincing. He combines the usage of actual diagrams to visualize the answer to this problem, and then uses words to describe what is happening in these diagrams. He also employs multiple methods, which complement each other, such as the stair case method in addition to his drawings. He also uses the idea of grouping, something that many other students did not think of, in order to find the solution to this problem. In working by groups, Ronit is able to identify each combination while keeping track of what he has already said and what he has not yet accounted for in a neat and easy to understand way. He is methodical and meticulous in his execution of this problem and his answer is comprehensive and complete. Although he is my better students, this is beyond what I might have thought he was capable of, and therefore his work and explanation is extremely impressive.

Cycle II: Towers 5-tall, selecting from 2 colors; Pizza Problem, selecting from 4-toppings

Example 2: Student work that surprised me because of the strategy/representation selected OR Student work that is an impressive product from an unlikely student: Jacob

Statement of the task	Building 5-tall towers, selecting from two colors: Your have two different colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly five cubes high. Find a way to convince yourself and others that you have found all possible towers five cubes high, and that you have no duplicates. Record your towers below
Grade	8
Class	Pre-Algebra, Inclusion Class
Regular/Special Ed	Regular Ed
Number of students in class	21



Student: Jacob Cherry Date: 10/18/10

School: ______ Teacher: Mr. Smith

Other Group Members: ______

Building 5-tall towers, selecting from 2 colors

You have two different colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly five cubes high. Find a way to convince yourself and others that you have found all possible towers five cubes high, and that you have no duplicates. Record your towers below and provide a convincing argument why you think you have them all.

B = ■ Y = □

1 B - whole, same for Yellow = 2

5 Y on 1 point on a blue tower same to B = 10

2 Y on top and on bottom or the middle, same for B same for Y

2 Y in the top middle middle and bottom middle same for B same for B = 4

2 Y rotating = same for B = same for B = 4

2 like this = 2

4 like these 2 =

All possible towers

32 towers

4, 16, +16, 28, 28, +2, 30, 2, 10, 4, 4, 2, 4, 2, 2

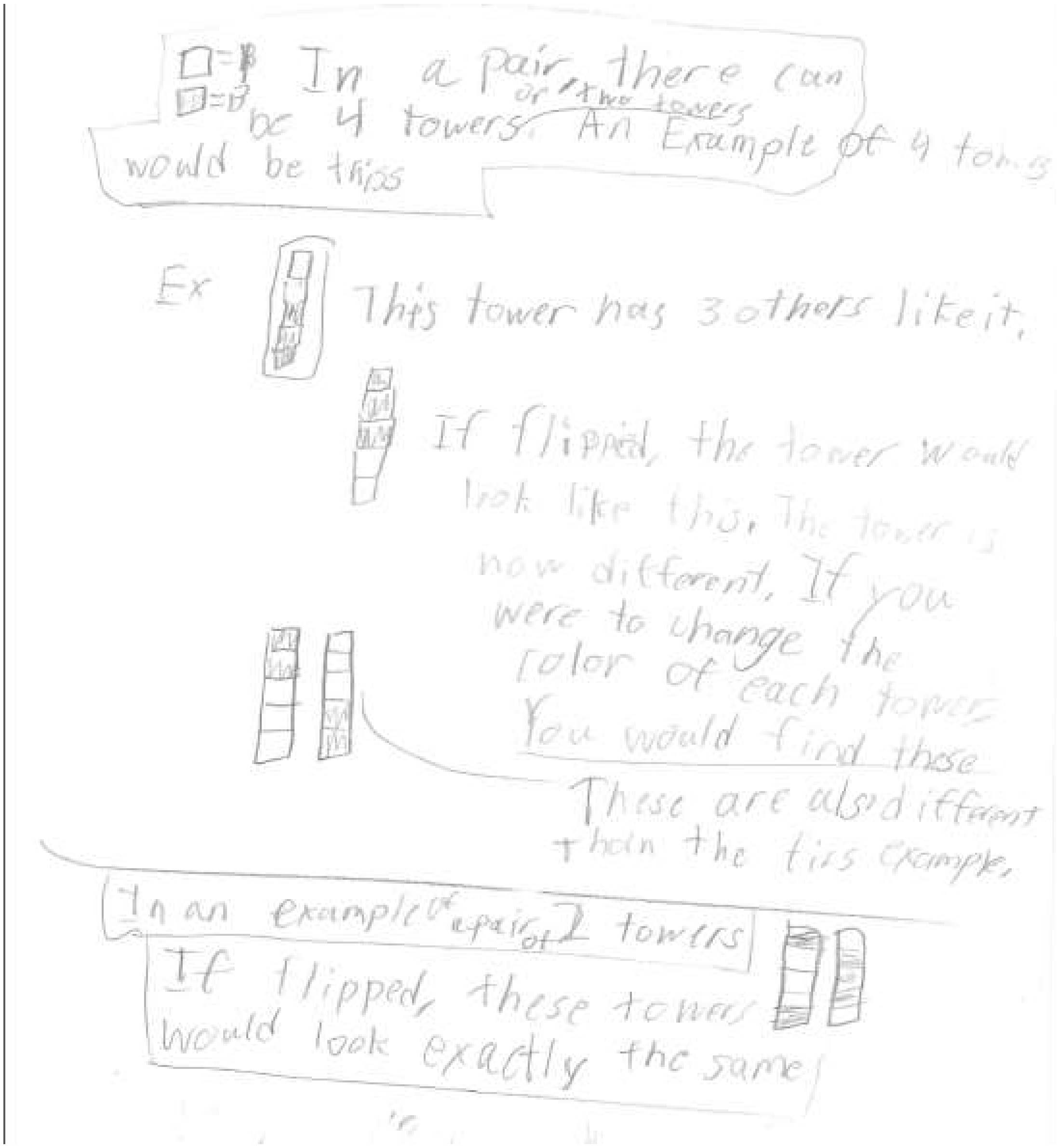


Figure C.2.5 Jacob's Cycle 2 Work

Explanation: Why Jacob's Work Surprised and Impressed Me

Jacob is a good student, but he is not always at the front of the pack. The way that he solved this problem, though, was very surprising and encouraging. Although his work may look a bit sloppy, his ideas are all on the paper and many of them mimic the "grouping" method that Ronit used. I am impressed by his choice to come up with a method of labeling each color ——a filled in square is blue and a blank square is yellow. Coming up with this method of labeling helped him to draw each tower, but what is even more impressive is that he also chose to label each tower with

words as well. This shows a greater level of understanding than I would have expected. Additionally, if you flip to the back of the page, he uses a detailed explanation in addition to examples of that explanation to really demonstrate that he understands the problem. These examples also help to give more proof to his answer and provide a convincing argument to his method of reasoning. I really like that Jacob took the time to do all this. It shows that he put a lot of work into the problem and that he really understands what is going on here. While I might have guessed that he would have gotten the right answer, I would not have expected the level of explanation and the type of explanation he provides, so I am very happy and impressed by his work.

Cycle II: Towers 5-tall, selecting from 2 colors; Pizza Problem, selecting from 4-toppings Example 3: Student work that concerned me about the student's to understand the mathematical ideas affiliated with this project: Marcella and Rachel

Statement of the task	Building 5-tall towers, selecting from two colors: Your have two different colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly five cubes high. Find a way to convince yourself and others that you have found all possible towers five cubes high, and that you have no duplicates. Record your towers below and provide a convincing argument why you think you have them
Grade	8
Class	Pre-Algebra, Inclusion Class
Regular/Special Ed	Regular Ed
Number of students in class	21

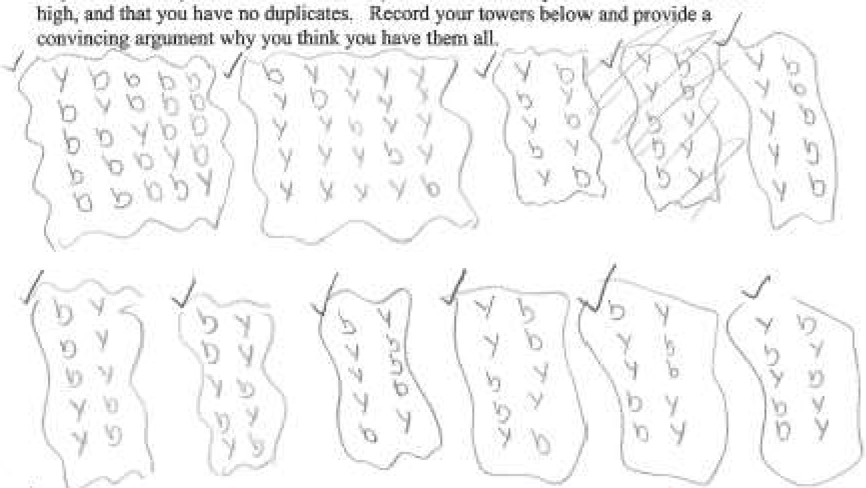
Student: marcella bergamo Date: October 18 2010

School: Sandburg middle Teacher: mr. Smith

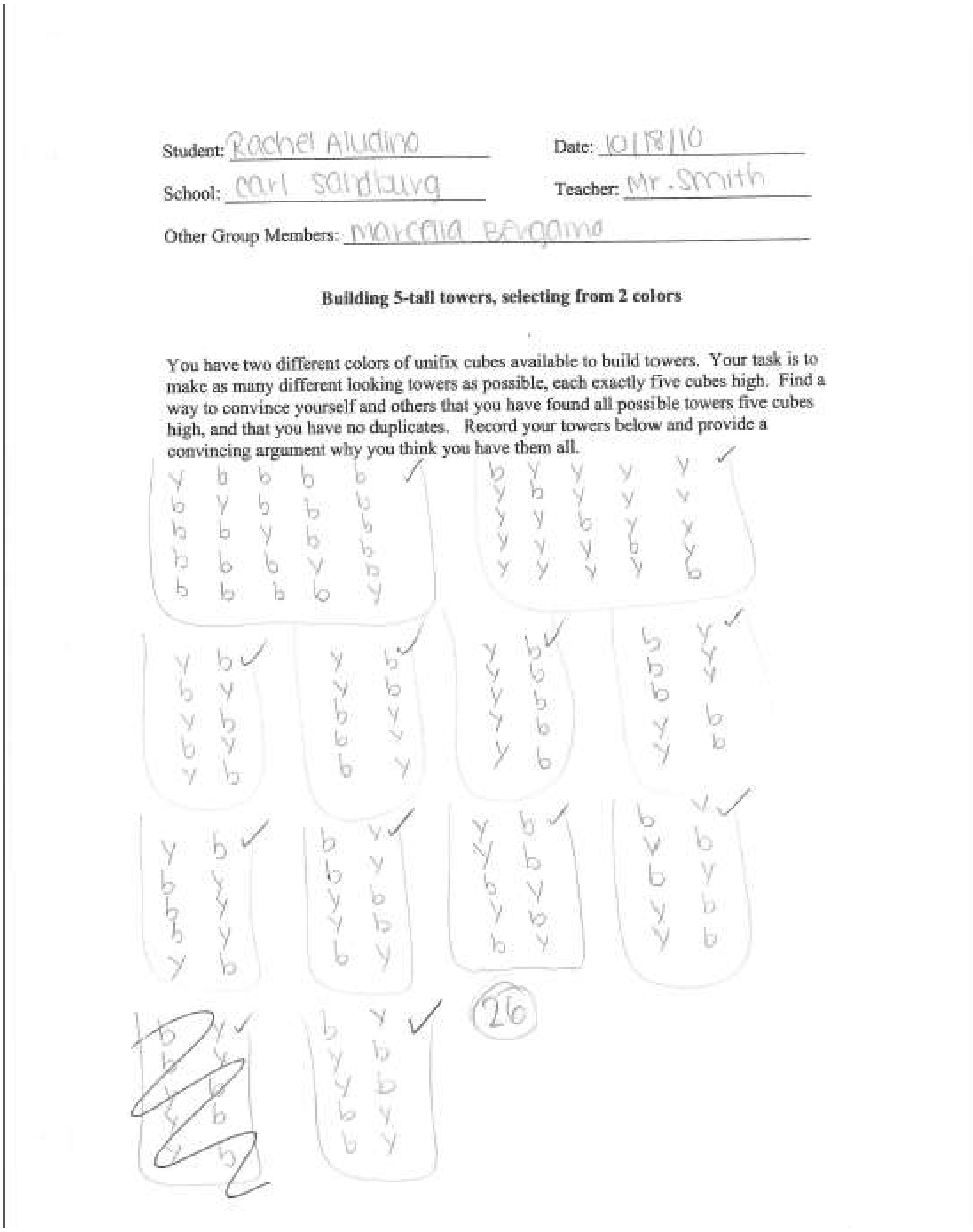
Other Group Members: Rachel Aludino

Building 5-tall towers, selecting from 2 colors

You have two different colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly five cubes high. Find a way to convince yourself and others that you have found all possible towers five cubes high, and that you have no duplicates. Record your towers below and provide a convincing argument why you think you have them all.



28 towers I got I got this by making opposites and making stair cases. I did this because it was just easier and simple to understand.



Student: Rachel Aludino Date: 10/18/10

School: Carl Sandburg Teacher: Mr. Smith

Other Group Members: Marcella Bergamo

Building 5-tall towers, selecting from 2 colors

You have two different colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly five cubes high. Find a way to convince yourself and others that you have found all possible towers five cubes high, and that you have no duplicates. Record your towers below and provide a convincing argument why you think you have them all.

26

Figure C.2.6 Marcella's and Rachel's Cycle 2 Work

Explanation: Why Marcella and Rachel's Work Concerned Me

Marcella and Rachel's work concerns me. Neither of them provided very good examples or reasons for why they came to the conclusions that they did, and Rachel does not provide any explanation at all beyond listing out the letters for each separately-colored tower. Marcella's work, even though it does provide an explanation, is perhaps a bit more concerning. She simply says that by doing the problem in the way they did it (which is not entirely clear because of their lack of explanation), "it was just easier and simple to understand." This is extremely troubling because both girls are good students and did not seem to grasp this assignment. They worked in a group together but did not come to the same answer at the end, suggesting that they either were not working together or that they really did not understand. It seems to me that students often want a "quick fix" on how to solve problems, but I have learned that providing such "quick fixes" (if they exist) often makes students simply memorize them and hinders them from learning the concepts involved with the problem in the first place. I am concerned that this is what Marcella and Rachel did during this problem, as their work shows that they certainly did not understand it.

Cycle II **Conclusion**

In this session I saw some improvement in students being able to explain their reasoning and I also noticed that many students stuck with the same type of strategy that they used for the previous task with 4tall towers. Some students used the "opposites" method, and most students who did for the first task tried the same thing for this one. Some students started with this method of opposites and eventually changed their grouping or used a different method, but they were very quick to go back to the way that they were thinking for the first task. I think more students overall were coming up with organized strategies for putting towers into groups and I did not see as many students just making random towers and then checking to see if they had already made that tower. I still did not have too many groups that were definitely confident in their answer. I also had some groups that were very confident that they had the correct answer when they still had some towers missing. I did see that students were writing more with their explanations, but not many of them were able to convince me, or even themselves of their answer.
If I had to do this again, I would not have changed the actual activity itself, but instead I would have had students not only look back at the work that some of the groups had done, but I would have also had them go back after doing the first task and had them explain their answers again to give them more practice explaining and flushing out their arguments. I think that if they had a little more practice going back and talking about their strategies and solutions they would get a better understanding of how to give a convincing argument.

Cycle III: Towers 3-tall, electing from 3 colors; Ankur's challenge

Example 1: Student work that impressed me as an interesting example of reasoning: Olympia

Statement of the task	Building Towers Three Colors: Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.
Grade	8
Class	Pre-Algebra, Inclusion Class
Regular/Special Ed	Regular Ed
Number of students in class	21

Student: Olympia Chelchowski Date: 11-18-10

School: CSMS Teacher: Mr. Smith

Other Group Members: Kelsi & megan

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

we found out that the total for the blocks is 27. We were convinced that this is the right amount because we found all the staircases and then we found the opposite of them so there is no more groups possible.

YRY	RBB	YBR	YBB	RRR
YYR	BRB	RBY	BYB	YYY
RRY	BBR	YRB	BBY	BBB
RYR	RRB	BYR	BYY	
YRR	RBR	BRY	YBY	
RYY	BRR	RYB	YYB	

total: 27

Student: Kelsi [illegible]

Date: 11/18

School: CSMS

Teacher: Mr. Smith

Other Group Members: Olympia

BUILDING TOWERS THREE COLORS EXTENSION

Find all possible towers that are four cubes tall, selecting from cubes available in three different colors so that the resulting towers have at least one of each color. In the space below, show your solution and provide a convincing argument that you have found them all.

There has to be two of one color because there is only three color with a tower of four cubes. We did the stair case because you can see the color slide the the case. Throughfore the two other colors change from top to bottom.

Step 1

YYRB
BYYR
RBYY

Step 2

YYBR
RYYB
BRYY

After the stair case we still wouldn't have all the combinations so there would be anothe group called the mixed up color which would look like this.

YBRY

Figure C.2.7 Olympia's and Kelsi's Cycle 2 Work

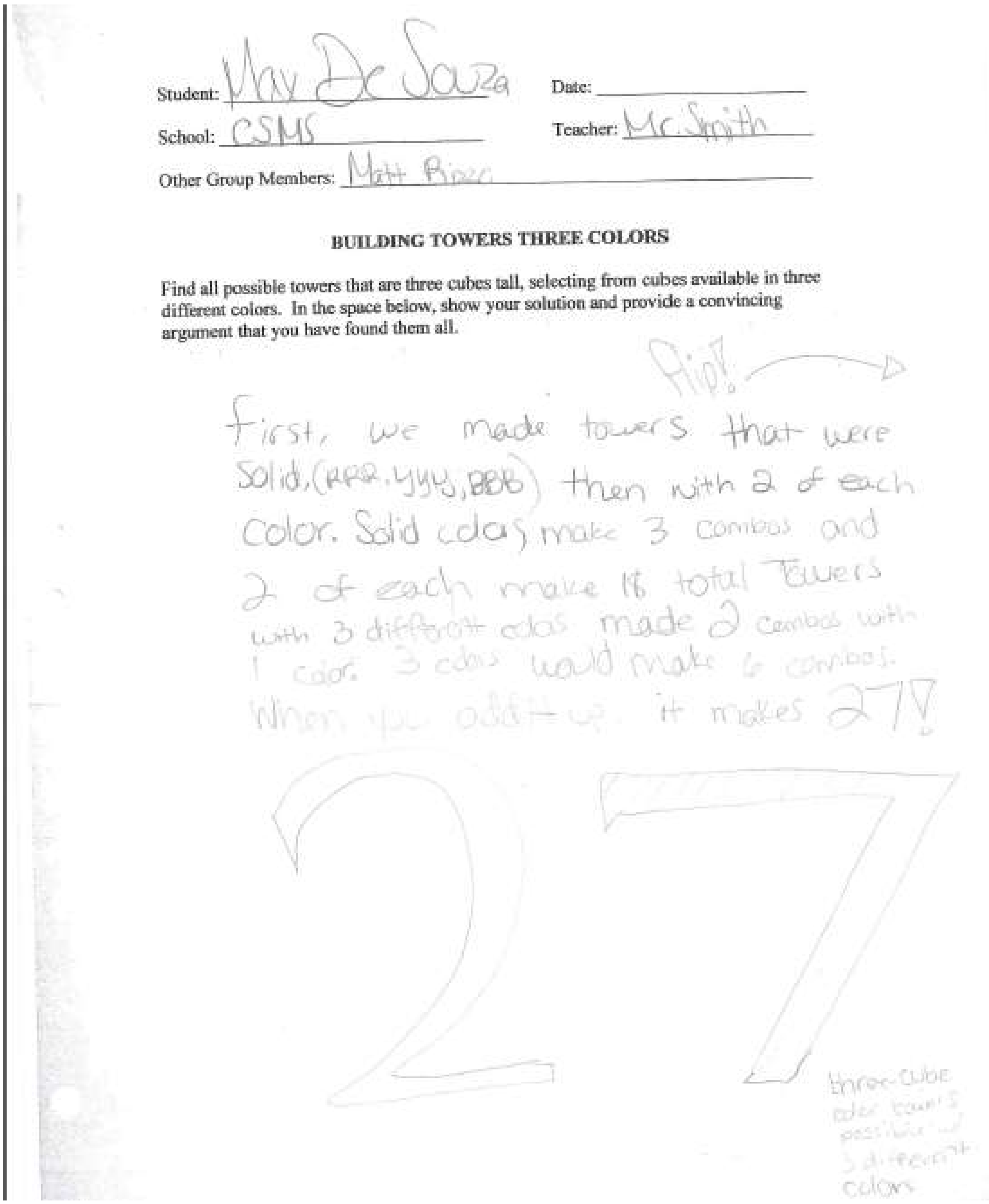
Explanation: Why Olympia's Work is Impressive

Olympia's work is great because it is clear, concise and convincing. She talks about using two methods: the staircase method and then finding opposites. I was quite happy with her work in finding both of these. It is clear that she understands this problem and her reasoning is accurate and convincing. In addition to the written portion of her explanation, she provides groupings that depict each of the combinations clearly and accurately. I am very happy with her work and the fact that it shows that she really understood the processes involved in figuring out this problem.

Cycle III: Towers 3-tall, electing from 3 colors; Ankur's challenge

Example 2: Student work that surprised me because of the strategy/representation selected OR Student work that is an impressive product from an unlikely student: Max and Matt

Statement of the task	Building Towers Three Colors: Find all possible towers that arethree cubes tall, selecting from cubes available in three differentcolors. In the space below, show your solution and provide a convincing argument that you have found them all.
Grade	8
Class	Pre-Algebra
Regular/Special Ed	Regular Ed
Number of students in class	21



Student: May De Souza Date: ______

School: CSMS Teacher: Mr. Smith

Other Group Members: Matt Rizzo

BUILDING TOWERS THREE COLORS

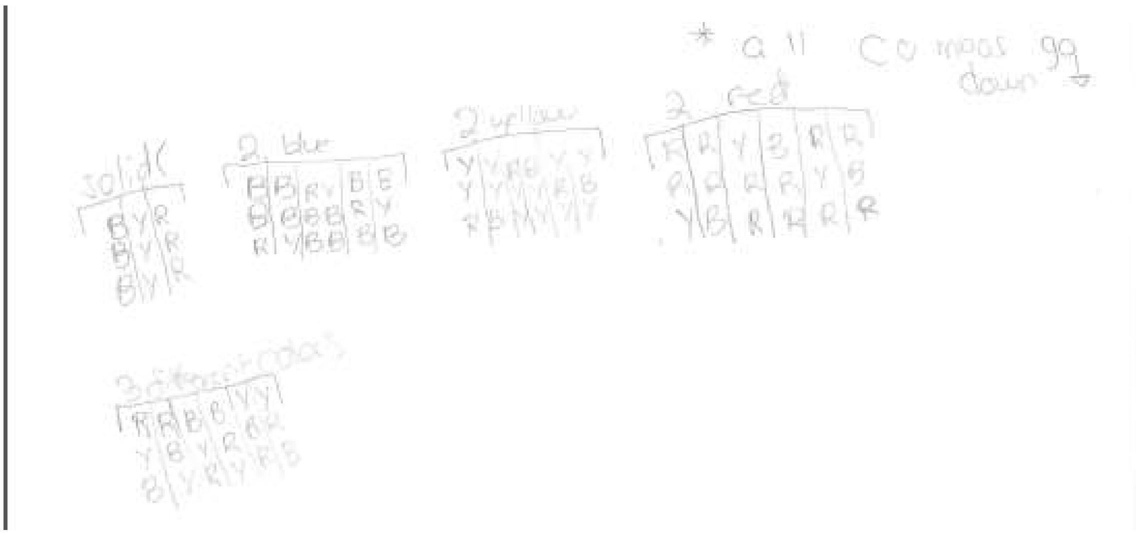
Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

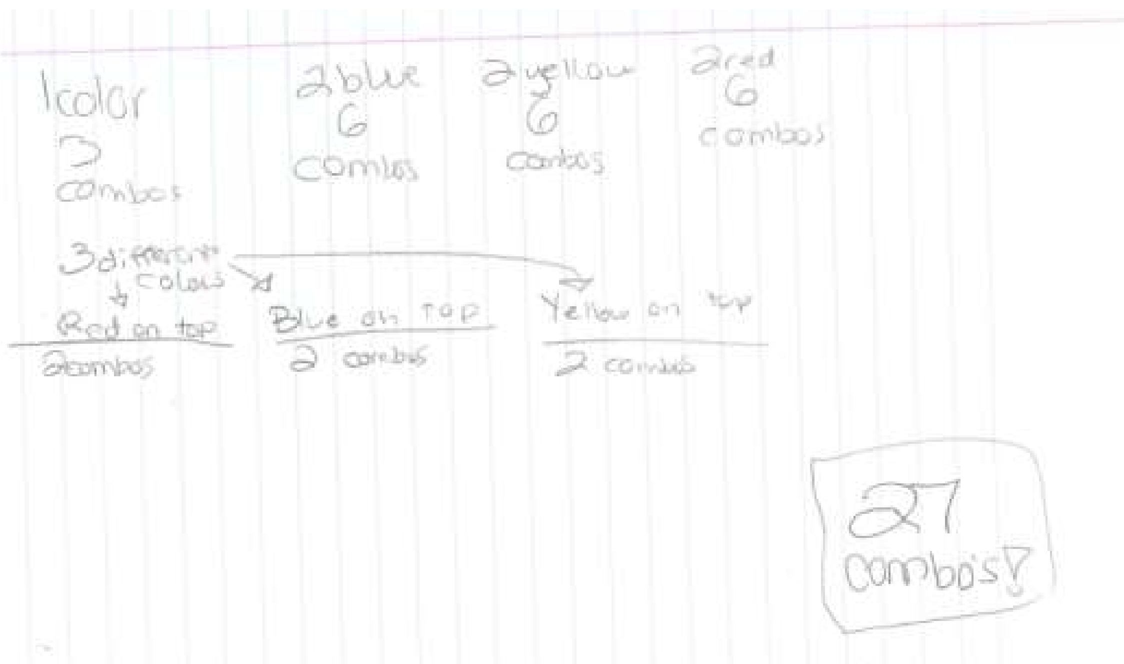
Flip!

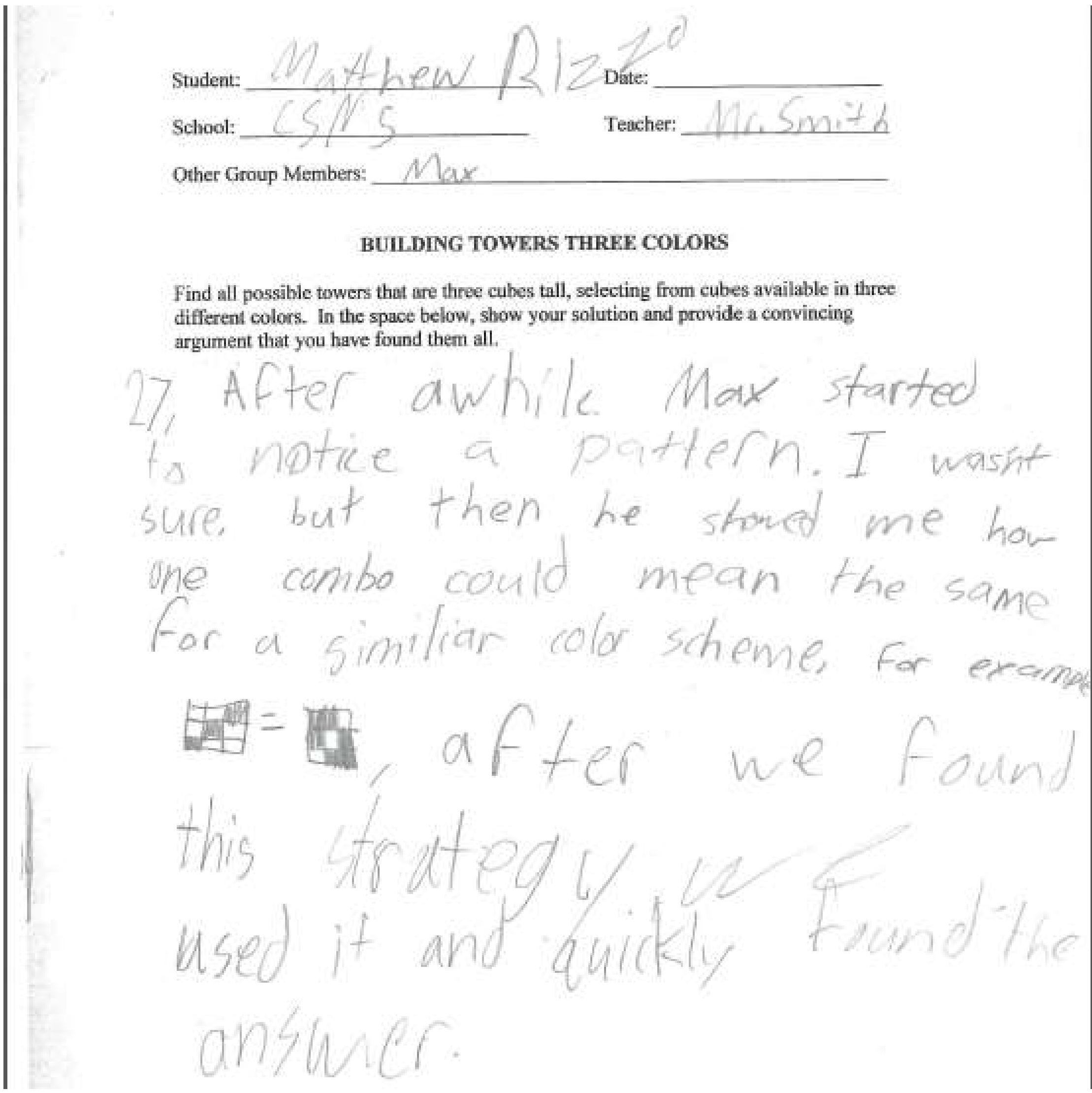
First, we made towers that were Solid, (RRR, YYY, BBB) then with 2 of each color. Solid colors make 3 combos and 2 of each make 18 total towers with 3 different colors made 2 combos with 1 color. 3 colors would make 6 combos. When you add it up, it makes 27!

27

three-cube color tower's possible w/ 3 different colors


solid
2 blue
2 yellow
2 red
3 different colors


1color
3
combos
2blue
6
combos
2yellow
6
combos
2red
6
combos
3different colors
Red on top
2combos
Blue on top
2 combos
Yellow on top
2 combos
27
combos!



Student: Matthew Rizzo Date: ____________

School: CSMS Teacher: Mr. Smith

Other Group Members: Max

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

27, After awhile Max started to notice a pattern. I wasnt sure, but then he showed me how one combo could mean the same for a similiar color scheme, For example = , after we found this strategy we used it and quickly found the answer.

Figure C.2.8 Max's and Matt's Cycle 2 Work

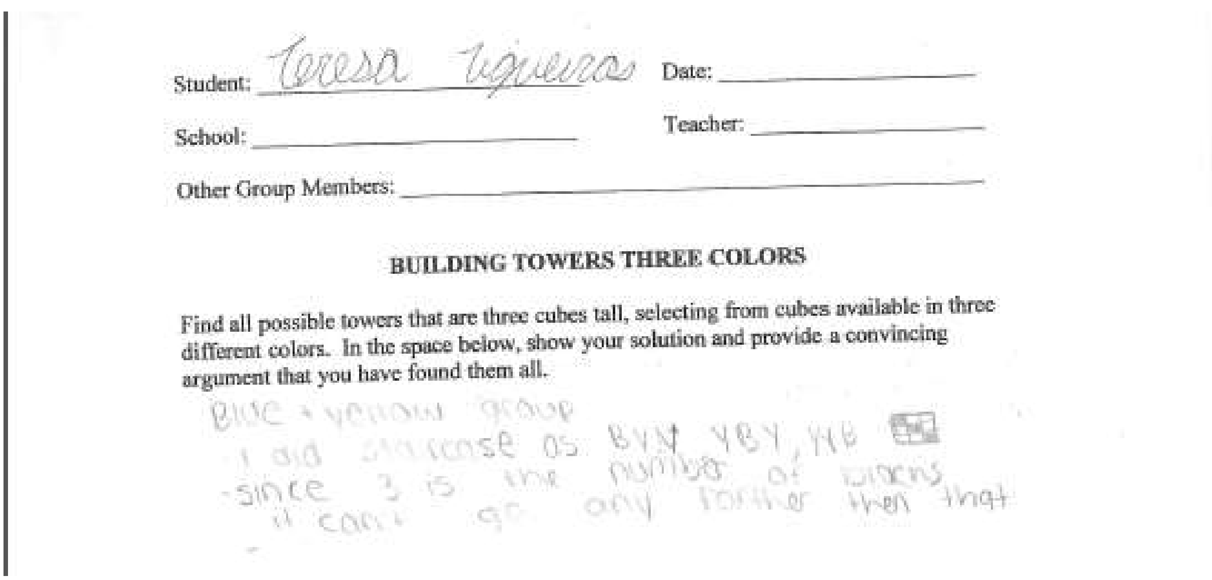
Explanation: Why Max and Matt's Work Surprised and Impressed Me

I am really excited about Max and Matt's work on this problem. Max is one of my better students who, as you can see by his work, was very enthusiastic about working with the towers. I am always happy when students are enthusiastic, but what really impressed me was his ability to work with Matt, who is not one of my better students. I'm really happy specifically about Matt's explanation. He says, "After a while, Max started to notice a pattern. I wasn't sure, but then he showed me how one combo could mean the same for a similar color scheme." He provides an example and then explains why he was convinced. This is exactly the type of collaboration that I was looking for on this project, and I'm really happy that a better student was able to explain and relate to a student who usually struggles. Both students demonstrated that they completely understood the problem and provided convincing arguments, which, as a teacher, is really exciting to see.

Cycle III: Towers 3-tall, electing from 3 colors; Ankur's challenge

Example 3: Student work that concerned me about the student's to understand the mathematical ideas affiliated with this project Marcella and Rachel

Statement of the task	Building Towers Three Colors: Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.
Grade	8
Class	Pre-Algebra
Regular/Special Ed	Regular Ed
Number of students in class	21


Student: Teresa Figueroa Date:
School: Teacher:
Other Group Members:

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

Blue + yellow group
I did increase as BYB YBY, YYB
-since 3 is the number of blocks
it can't go any farther then that

Figure C.2.9 Theresa's Cycle 3 Work

Explanation: Why Teresa's Work Concerned Me

Teresa's work is concerning. Her explanation is incomplete and she does not provide an answer to the problem. She makes a small attempt to give and explanation but does not go into any detail about what she did to solve the problem. She only used a visual pattern and she did not make any connections to any of the problem solving skills some of the other students attempted, even if they were not correct. It is concerning in that the work she did here does not show an understanding of the material nor does it provide a convincing argument. This could potentially have implications for her greater understanding of problems like this, and therefore is a bit troubling.

Cycle III Conclusion

In the third cycle students were much more confident in their answers and did a much better job of giving convincing arguments. I think that the session with the three tall towers with three colors was probably the best session out of all of the ones that we did because students were able to decide on strategies as a group almost right away and then worked together as a group much better. Students felt confident in their arguments and were able to explain them verbally and on paper. I was disappointed that I was not able to have many groups try Ankur's challenge in the same class period that they did the three tall tower. I think they would be a lot more successful if they were able to try Ankur's challenge right after doing the three tall tower because they were already in the right frame of mind to successfully solve the problem. If I was to do this again I might try to get two back-to-back 45 minute periods to see if and how their reasoning evolved as they had more time.

Final Reflection

I really enjoyed working on these problems to see how it made sense mathematically to come to the total number of combinations. Some of the most powerful things that I learned were from watching the videos of students work. I really enjoyed the explanation that one the students in one of the first videos that talked about how he took his original answers for the three tall towers and talked about how there were two different options because of the two different colors. I learned that my students were able to do very creative ways of thinking that came to them almost intuitively without thinking about how it connected to any formula or mathematical topic. Many of the students who struggled the most at first were the students who tried to guess what type of problem this was (a counting principal problem, or factorial) and then tried to get an answer that met their explanation. I was surprised by how many students were not certain about a strategy at first but were able to come up with one by seeing some type of pattern or were able to come up with a grouping that made sense to them. One thing that I learned from this is that many of the students that I did not consider to be very good at math were able to come up with many very impressive ways of solving these problems. When they were allowed to just read the problem

and come up with a solution without any specific directions or limitations they were able to demonstrate their ability to solve a complex problem. I realized that I need to give my students more opportunities to "do" math by giving them a problem and allowing them to come up with their own way of thinking to solve it and then going back and talking about not only there solutions but how it relates to a certain mathematical topic.

Kate
15:254:599

TOPICS IN MATH EDUCATION: LESSON STUDY ONREASONING FINAL PROJECT

KATE

DECEMBER 2010

CYCLE I

Towers 4-tall, selecting from 2 colors

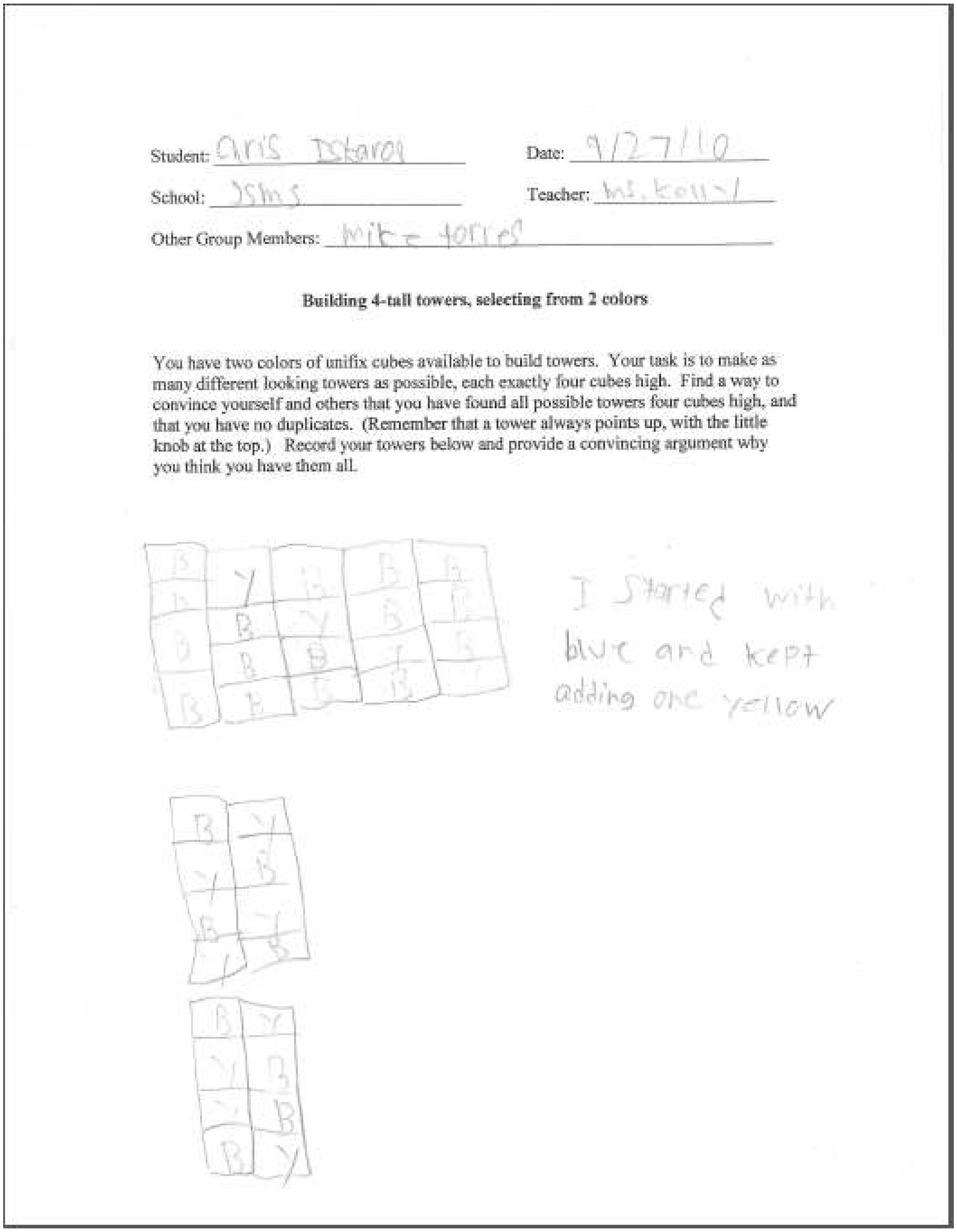
Predicting Towers 3-tall and 5-tall, Selecting from two colors

STUDENT WORK THAT CONCERNED ME

Teacher: Mrs. Kelly/ Mrs. Clark, Inclusion Class, 45 minutes in length # of students: 16

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.



Student: Chris Dstaroq Date: 9/27/10

School: JSMS Teacher: Ms. Kelly

Other Group Members: Mike Torres

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

I Started with blue and kept adding one yellow

Student: ____________________ Date: ____________________

School: ____________________ Teacher: ____________________

Other Group Members: ____

Building Towers 2 Colors Extension

Without building them, make a prediction about a solution for finding all possible 3-tall towers, selecting from 2 colors. Do you think there will be more, fewer, or the same number of possible towers as you found for towers that were 4 cubes high? Why do you think that?

9

Without building them, make a prediction about a solution for finding all possible 5-tall towers, selecting from 2 colors. Do you think there will be more, fewer, or the same number of possible towers as you found for towers that were 4 cubes high? Why do you think that?

25 towers

Figure C.3.1 Chris's Cycle 1 work

Cycle I- Concerned

What concerns me about this student is that he didn't make a connection between what hedid with all blue and one yellow to trying it with all yellow and one blue. His initial idealooks strong, it resembles a staircase but there is no follow through. The fact that this pairof boys was content with finding nine towers is disconcerting to me. This was one of myonly pairs of students that did not find all sixteen towers.

It also appears that he made no connection to building the 4-tall towers and his prediction for the 3-tall towers. His prediction his for 3-tall towers was nine; the same answer he recorded for the 4-tall towers. If he put any logical thought into what he was putting onpaper I would think he would have realized the taller tower should have more possibilities than the shorter one.

This student tends to have an apathetic attitude in class so I'm not sure if my concern stems from his lack of interest or his possible lack of ability.

STUDENT WORK THAT SURPRISED ME

Teacher: Mrs. Kelly

# of students: 14 Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as manydifferent looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.)Record your towers below and provide a convincing argument why you think you have themall

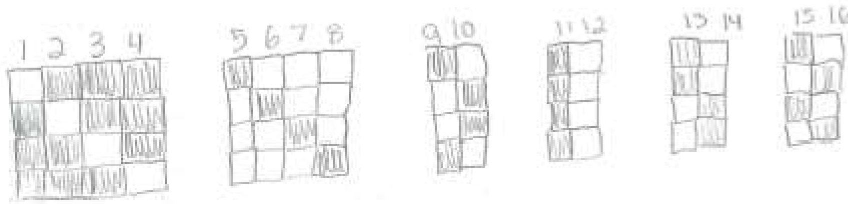
Student: Allyson Hathaway Date: 9/27/10

School: JSMS Teacher: Mrs. Kelly

Other Group Members: Kristen Celini and Sarah Kayden

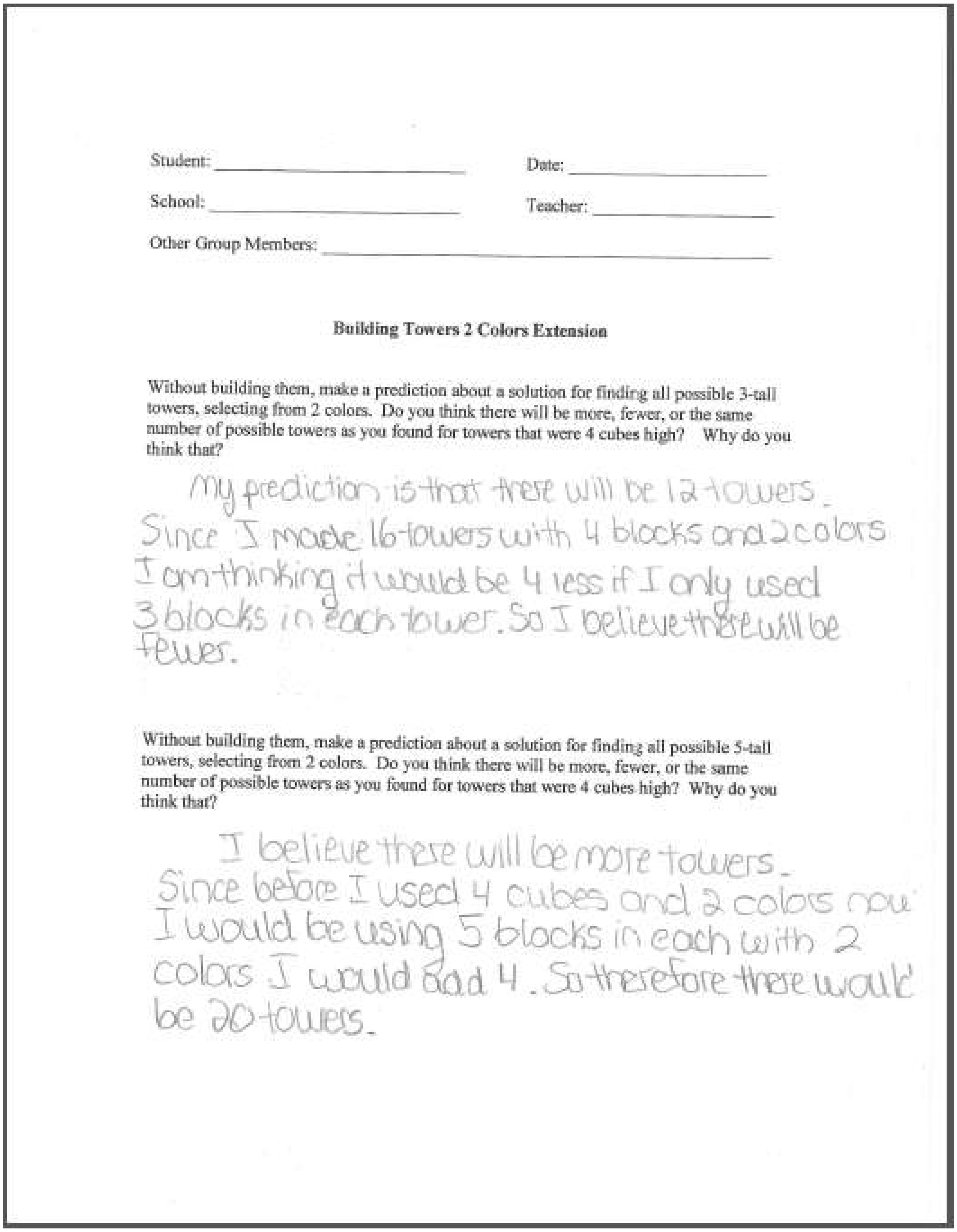
Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.



I got sixteen towers total and I orgainized them in a certain way to prove I got them. You will notice that for towers 1-8 they go dow diagnoly almost like a staircase. For tower numbers 9-16 you can see they are paired up where each tower is the same but I used different colored blocks so they are opposite which makes them different.

- blue
- yellow



Student: ____________ Date: ____________

School: ____________ Teacher: ____________

Other Group Members: ____________

Building Towers 2 Colors Extension

Without building them, make a prediction about a solution for finding all possible 3-tall towers, selecting from 2 colors. Do you think there will be more, fewer, or the same number of possible towers as you found for towers that were 4 cubes high? Why do you think that?

My prediction is that there will be 12 towers. Since I made 16 towers with 4 blocks and 2 colors I am thinking it would be 4 less if I only used 3 blocks in each tower. So I believe there will be fewer.

Without building them, make a prediction about a solution for finding all possible 5-tall towers, selecting from 2 colors. Do you think there will be more, fewer, or the same number of possible towers as you found for towers that were 4 cubes high? Why do you think that?

I believe there will be more towers. Since before I used 4 cubes and 2 colors now I would be using 5 blocks in each with 2 colors I would add 4. So therefore there would be 20 towers.

Figure C.3.2 Alyson's Cycle 1 Work

Cycle I- Surprised

The reason I was surprised by this student is because I did not recall any of my groups talking about anything more sophisticated than "opposites" during the first task. When I went back through my students' work and saw the beautiful organization of her drawings and the write-up using the word "staircase", I was thrilled. She clearly has some insight into developing a pattern.

STUDENT WORK THAT IMPRESSED ME

Teacher: Mrs. Kelly

# of students: 14 Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

Student: Sanjana Pemmaraju Date: 9/27/10

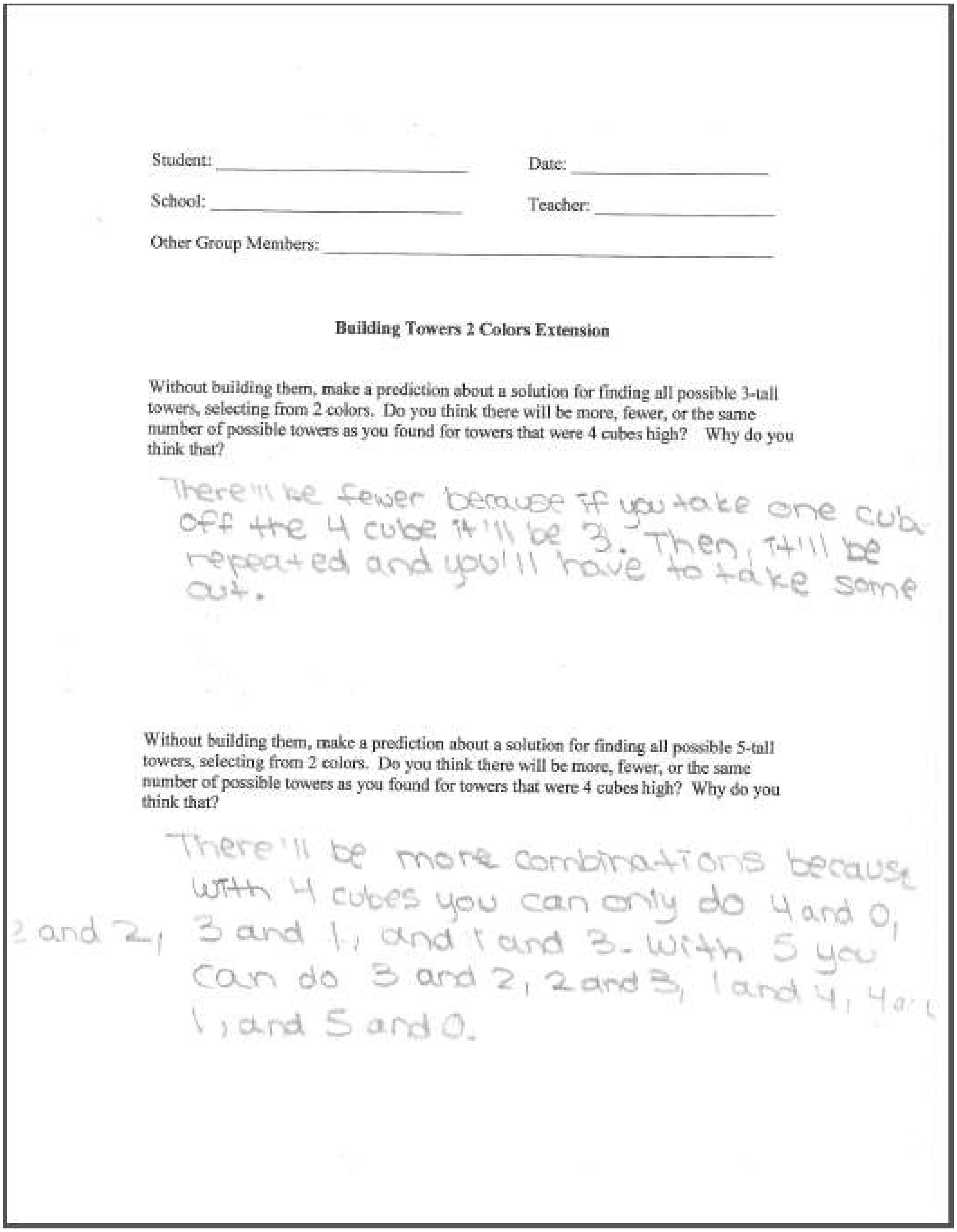
School: Jonas Salk Middle School Teacher: Ms. Kelly

Other Group Members: Anne Marie

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

We put all blue and all yellow. Then we did each color twice. First we alternated and then we did two on the top and two on the bottom. Next, we did 3 blues and 1 yellow. We put the one yellow on the bottom, the second to the bottom, then second bottom, then second to the top, then the top. We did the same with 3 yellows and 1 blue. Then we did 1 yellow on the bottom and one on the top. Then I did the same for blue.



Student: ____________ Date: ____________

School: ____________ Teacher: ____________

Other Group Members: ____________

Building Towers 2 Colors Extension

Without building them, make a prediction about a solution for finding all possible 3-tall towers, selecting from 2 colors. Do you think there will be more, fewer, or the same number of possible towers as you found for towers that were 4 cubes high? Why do you think that?

There'll be fewer because if you take one cube off the 4 cube it'll be 3. Then, it'll be repeated and you'll have to take some out.

Without building them, make a prediction about a solution for finding all possible 5-tall towers, selecting from 2 colors. Do you think there will be more, fewer, or the same number of possible towers as you found for towers that were 4 cubes high? Why do you think that?

There'll be more combinations because with 4 cubes you can only do 4 and 0, 2 and 2, 3 and 1, and 1 and 3. With 5 you can do 3 and 2, 2 and 3, 1 and 4, 4 and 1, and 5 and 0.

Figure C.3.3 Sanjana's Cycle 1 Work

Cycle I- Impressed

I was impressed by this students' reasoning because I feel like she had the beginnings of the idea to control for a variable. Her drawings look simply like opposites, but I feel her explanation shows a little more insight than that. Especially if you look at her prediction for the 5-tall towers; she talks about the different combinations in a generic form, 3 and 2,2 and 3, 1 and 4, 4 and 1, 5 and 0. She doesn't need to think of actual colors, but it

appears that she already has a picture of combinations in her head. Impressive for the first task!

Cycle I Implementation

I must admit that I was quite concerned about my students' mathematical reasoning after the implementation of the first task. Having watched the videos of second and third graders performing the same task, I had expected more from my students. I was disappointed with their lack of strategies and I was also disappointed with my lack of questioning skills. I didn't ask the right questions to pull information out of them.

If I have the opportunity to do this task again I will make sure to stay away from using the word "combinations". I must have used that word in my questioning because almost every student used it in their write up of the task. I feel I put a preconceived notion in their heads by using that term; in the future I would stick to using the words "towers" or "possibilities".

CYCLE II

Towers 5-tall, selecting from 2 colors

Pizza Problem, selecting from 4-toppings

STUDENT WORK THAT CONCERNED ME

Teacher: Mrs. Kelly

of students: 13

The Pizza Problem

Capri Pizza has asked you to help design a form to keep track of certain pizza choices. They offer a standard "plain" pizza with cheese and tomato sauce. A customer can then select from the following toppings: peppers, sausage, mushrooms, and pepperoni.

How many choices for pizza does a customer have? List all possible choices. Find a way to convince each other that you have accounted for all possibilities.

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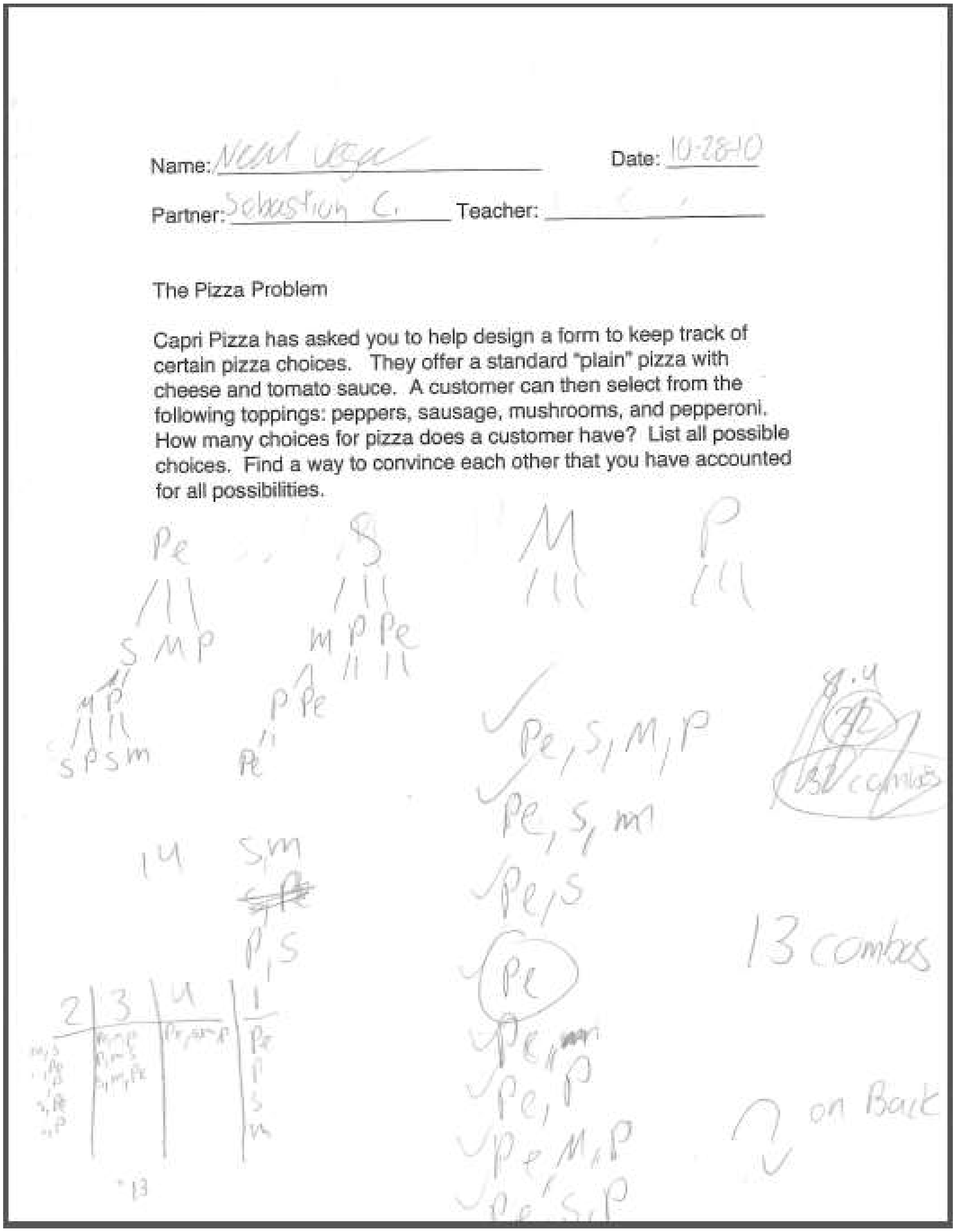


Name: ____________ Date: 10-28-10

Partner: Sebastian C. Teacher: ____________

The Pizza Problem

Capri Pizza has asked you to help design a form to keep track of certain pizza choices. They offer a standard "plain" pizza with cheese and tomato sauce. A customer can then select from the following toppings: peppers, sausage, mushrooms, and pepperoni. How many choices for pizza does a customer have? List all possible choices. Find a way to convince each other that you have accounted for all possibilities.

13 combos

on Back

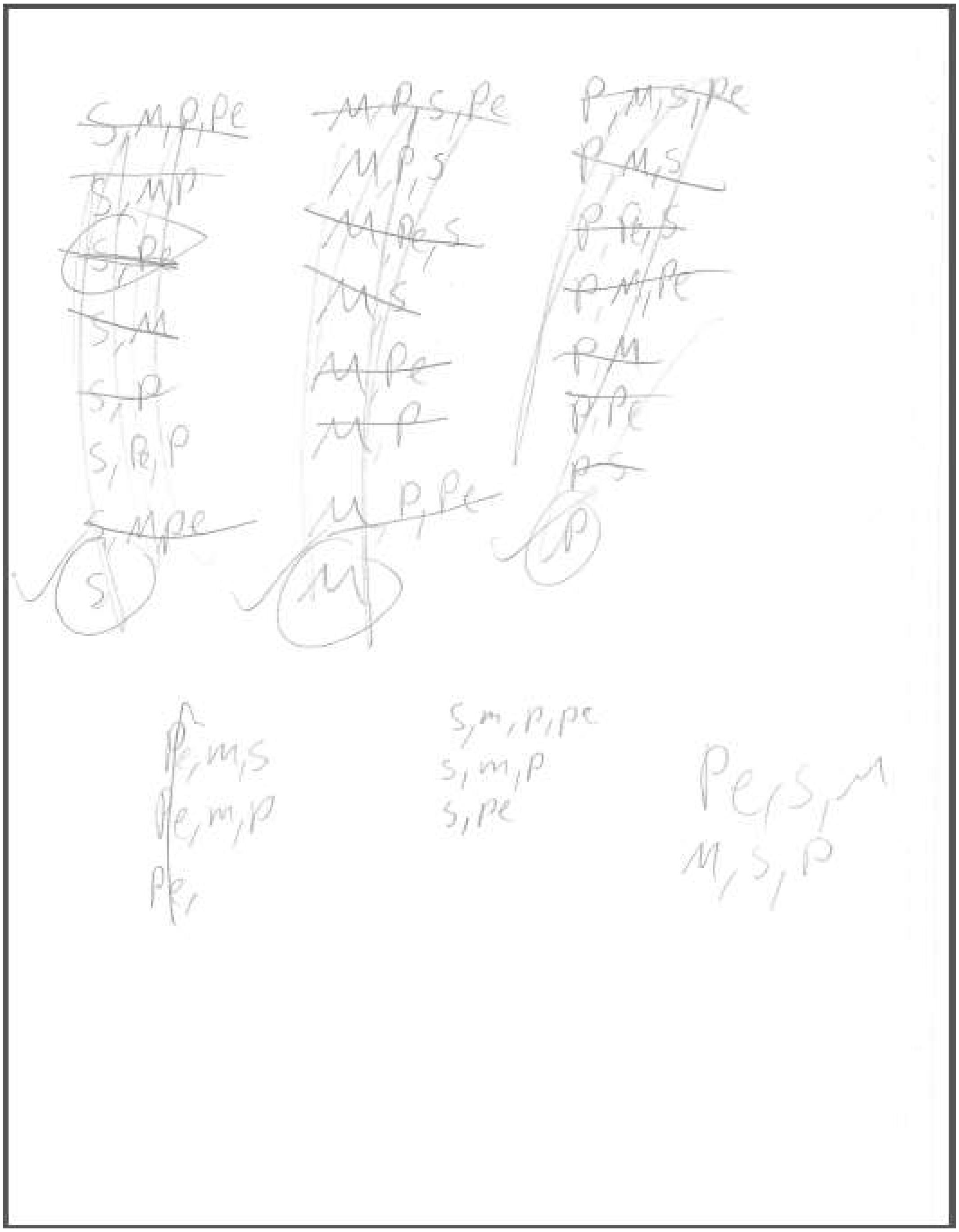


Figure C.3.4 Neal's Cycle 2 Work

Cycle II- Concerned (Pizza Problem)

This particular student and his partner have struggled with each of the tasks. Neither ofthem seems to have much of a mathematical mind for the abstract, they seem to do muchbetter with concrete concepts. My specific concern with this student is that he somehowcame up with 32 combinations, he multiplied 4 times 8. When I asked where he got thosenumbers he couldn't justify the eight, but he continued to list 32 combinations on theback of the sheet. I asked him and his partner if there were any duplicates and it took avery long time for them to decide that pepperoni and sausage was the same as sausageand pepperoni. It took a lot of questioning on my part to have them realize they had duplicates. They decided to start from scratch and were only able to come up with 13 combinations. At some point he attempted a tree diagram but abandoned that idea. To go from 32 combinations to 13 showed me that he didn't use a system to eliminate the duplicates. I feel there is no connection from one train of thought to another, causing confusion throughout the tasks.

STUDENT WORK THAT SURPRISED ME

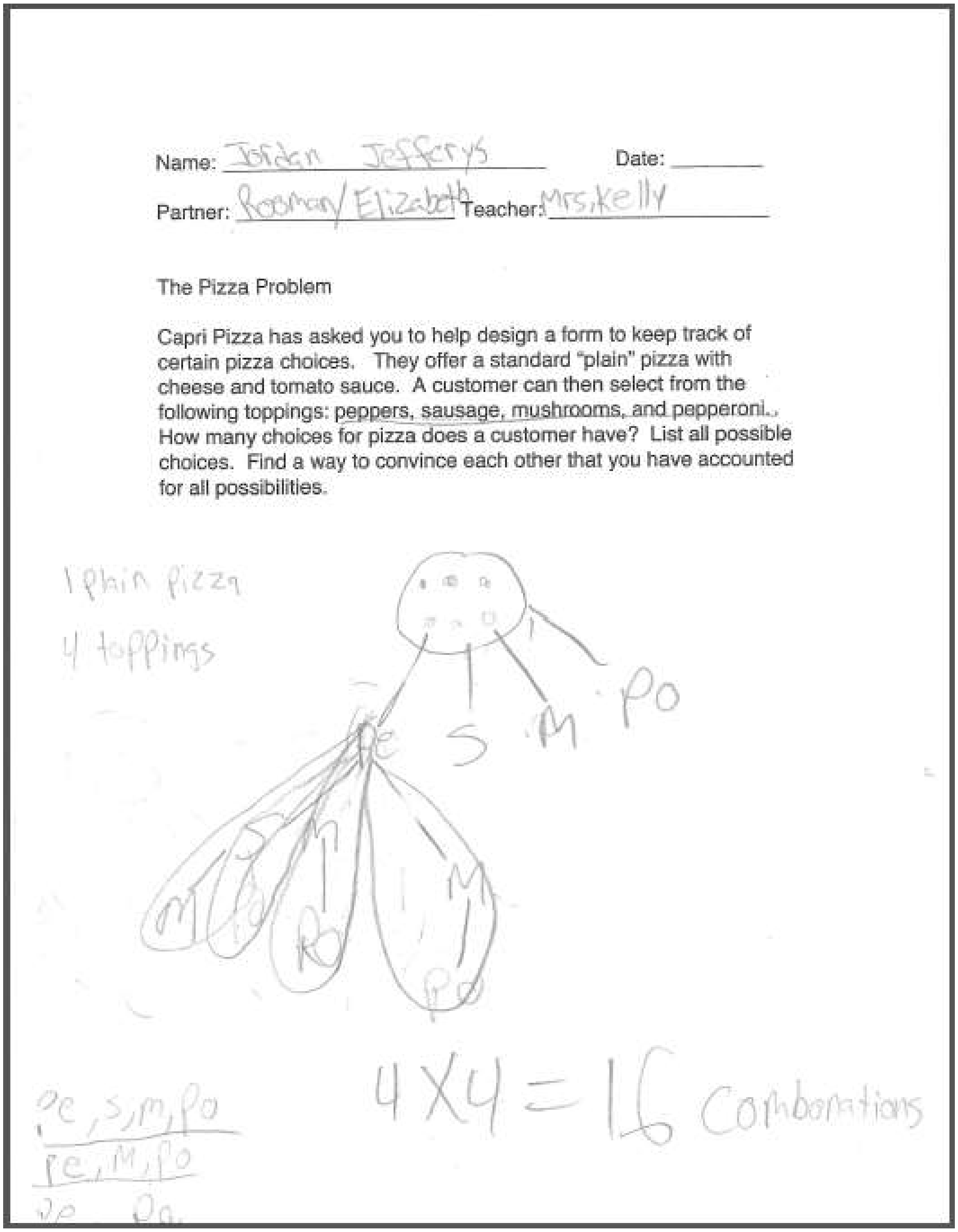
Teacher: Mrs. Kelly

of students: 13

The Pizza Problem

Capri Pizza has asked you to help design a form to keep track of certain pizza choices. They offer a standard "plain" pizza with cheese and tomato sauce. A customer can then select from the following toppings: peppers, sausage, mushrooms, and pepperoni.

How many choices for pizza does a customer have? List all possible choices. Find a way to convince each other that you have accounted for all possibilities.



Name: Jordan Jefferys Date: ________

Partner: Rooman/ Elizabeth Teacher: Mrs, Kelly

The Pizza Problem

Capri Pizza has asked you to help design a form to keep track of certain pizza choices. They offer a standard "plain" pizza with cheese and tomato sauce. A customer can then select from the following toppings: peppers, sausage, mushrooms, and pepperoni. How many choices for pizza does a customer have? List all possible choices. Find a way to convince each other that you have accounted for all possibilities.

1 plain pizza

4 toppings

Pe S M Po

$4 \times 4 = 16$ Combinations

Pe, S, M, Po

Pe, M, Po

Pe Po

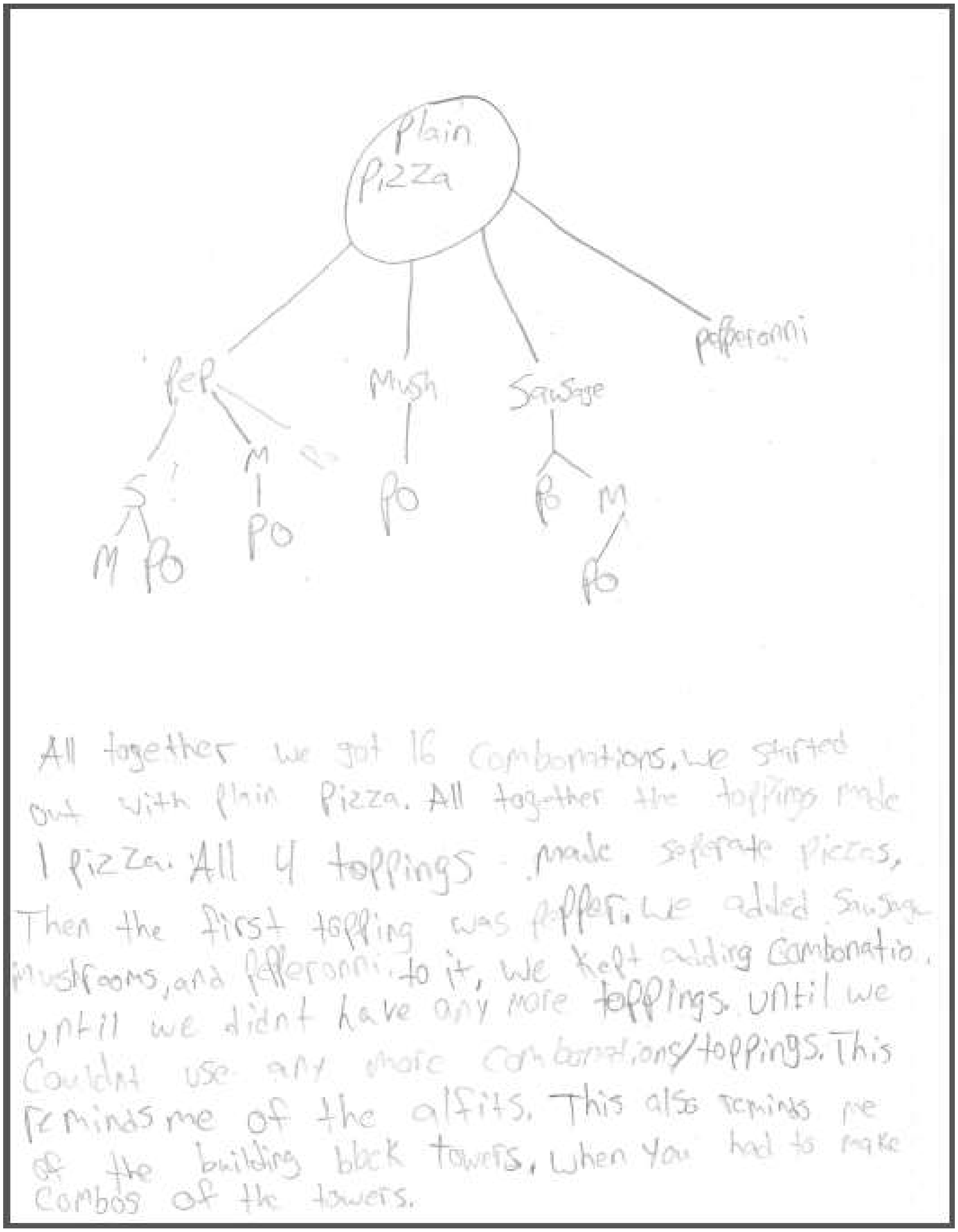


Figure C.3.5 Jordan's Cycle 2 Work

Cycle II- Surprised (Pizza Problem)

This student surprised me because his group was able to come up with a tree diagram that was able to be followed. A few other groups attempted a tree diagram but were unable tomake it work for them. This student was able to identify all 16 combinations using thetree diagram on the back of his paper. After some questioning he also made an interesting observation that as you used each topping there was always one choice less for the nextset of combinations; he explained this in response to my question of," Why doesn't pepperoni have anything corning from it?" I was happy with this because I thought hispartner was the stronger member of the group but he was able to show that he understoodthe reasoning used to come up with his answer. I was also happy to see that he said the task reminded him problem involving "outfits" and "building block towers". He definitely made some nice connections!

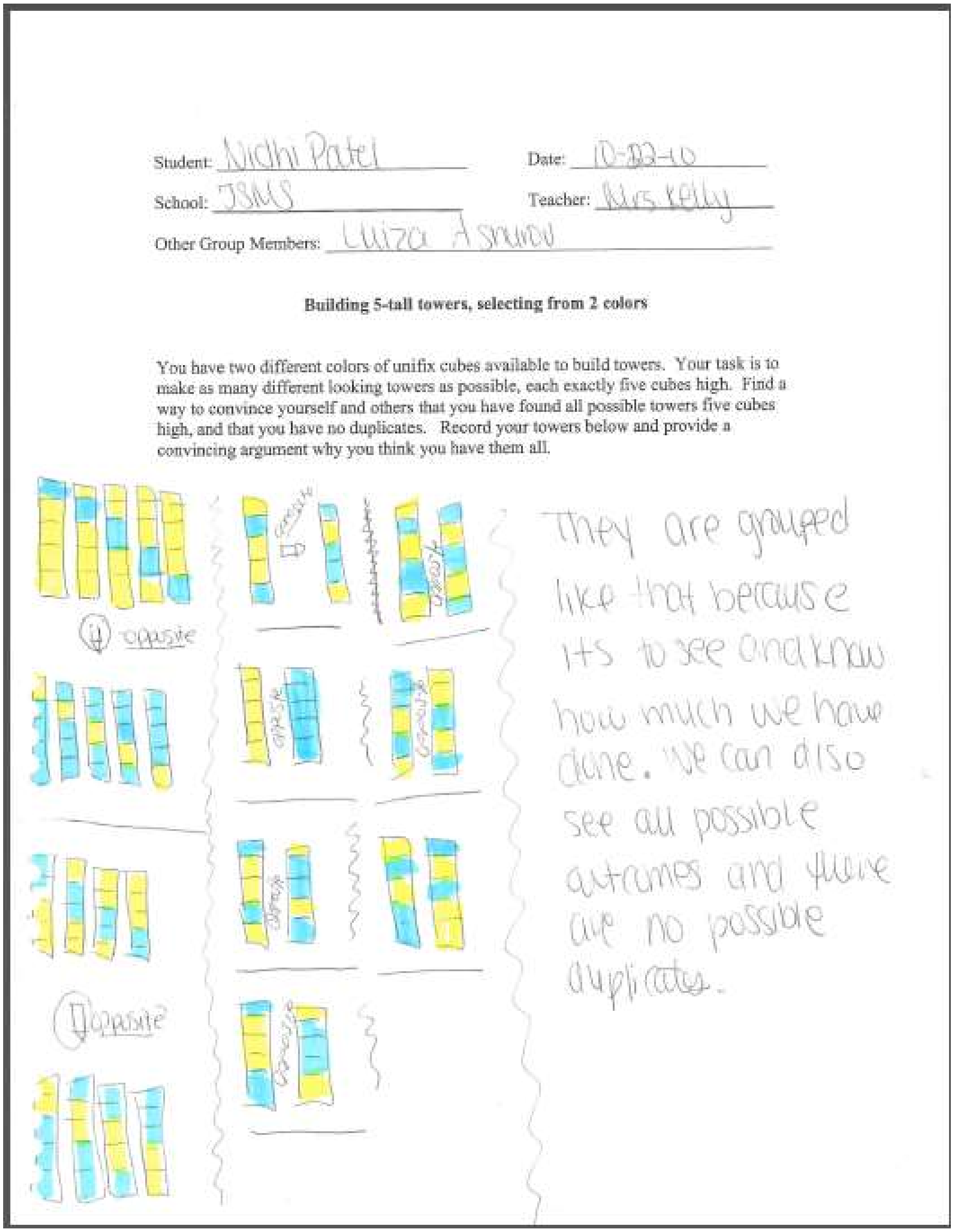
STUDENT WORK THAT IMPRESSED ME

Teacher: Mrs. Kelly

of students: 13 (of 45 minutes in length each)

Building 5-tall towers, selecting from 2 colors

You have two different colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly five cubes high. Find a way to convince yourself and others that you have found all possible towers five cubes high, and that you have no duplicates. Record your towers below and provide a convincing argument why you think you have them all.



Student: Nidhi Patel Date: 10-22-10

School: JSMS Teacher: Mrs Kelly

Other Group Members: Uliza Asrurov

Building 5-tall towers, selecting from 2 colors

You have two different colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly five cubes high. Find a way to convince yourself and others that you have found all possible towers five cubes high, and that you have no duplicates. Record your towers below and provide a convincing argument why you think you have them all.

opposite

opposite

They are grouped like that because it's to see and know how much we have done. We can also see all possible outcomes and there are no possible duplicates.

Figure C.3.6 Nidhi's Cycle 2 Work

Cycle II- Impressed (5-tall)

This particular student worked with the same partner for all of the tasks. I saw such progress in their organization from the first task to the second. The first task was nothing more than guess and check, although the written justification of the second task was not super convincing, the arrangement of the towers shows nice organization. She controlled the movement of one blue through yellow towers, one yellow through blue towers then she repeated the same pattern moving two of the same color down the towers. This showed me that she was getting a sense of developing a pattern.

STUDENT WORK THAT IMPRESSED ME

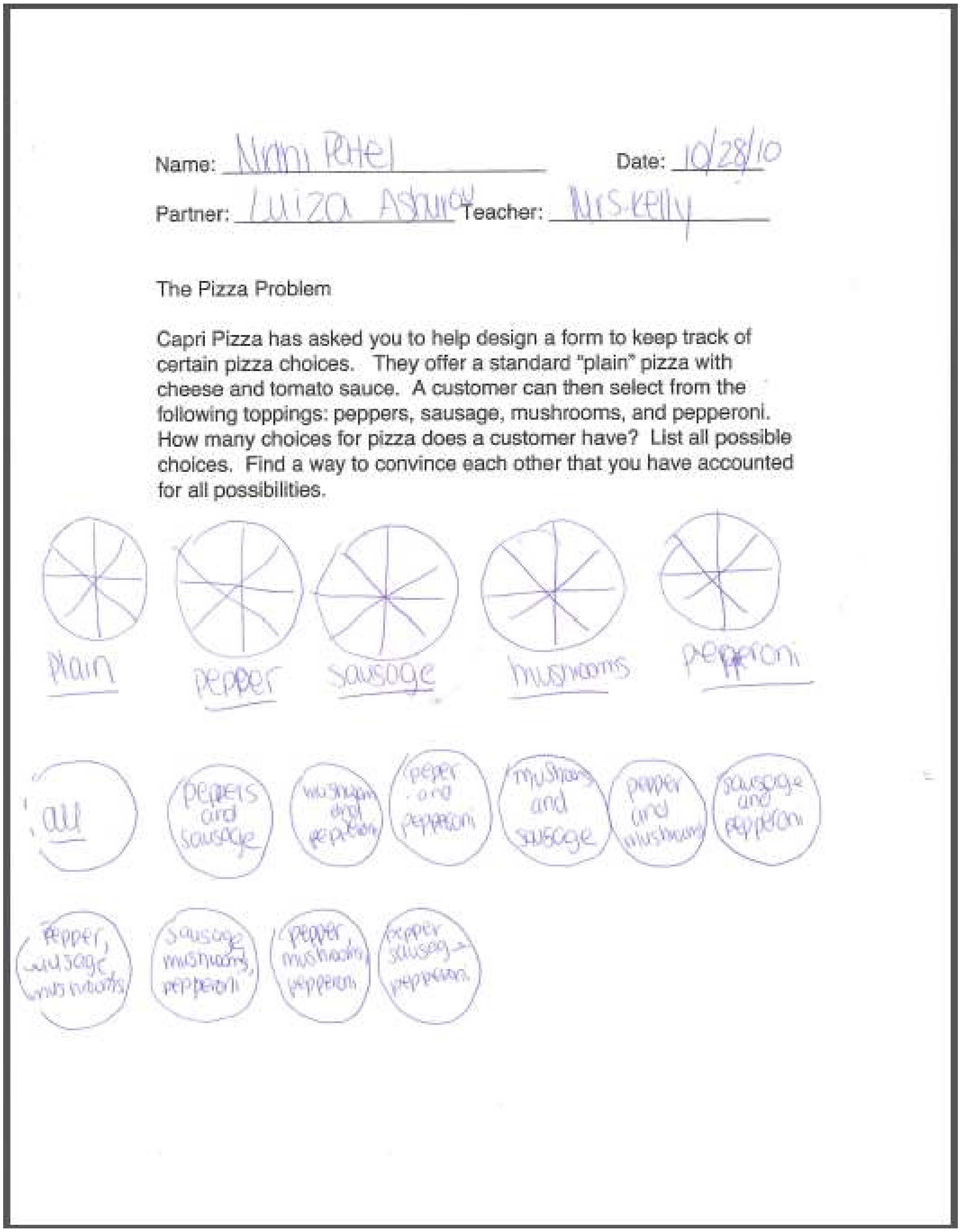
Teacher: Mrs. Kelly

# of students: 13

The Pizza Problem

Capri Pizza has asked you to help design a form to keep track of certain pizza choices. They offer a standard "plain" pizza with cheese and tomato sauce. A customer can then select from the following toppings: peppers, sausage, mushrooms, and pepperoni.

How many choices for pizza does a customer have? List all possible choices. Find a way to convince each other that you have accounted for all possibilities.

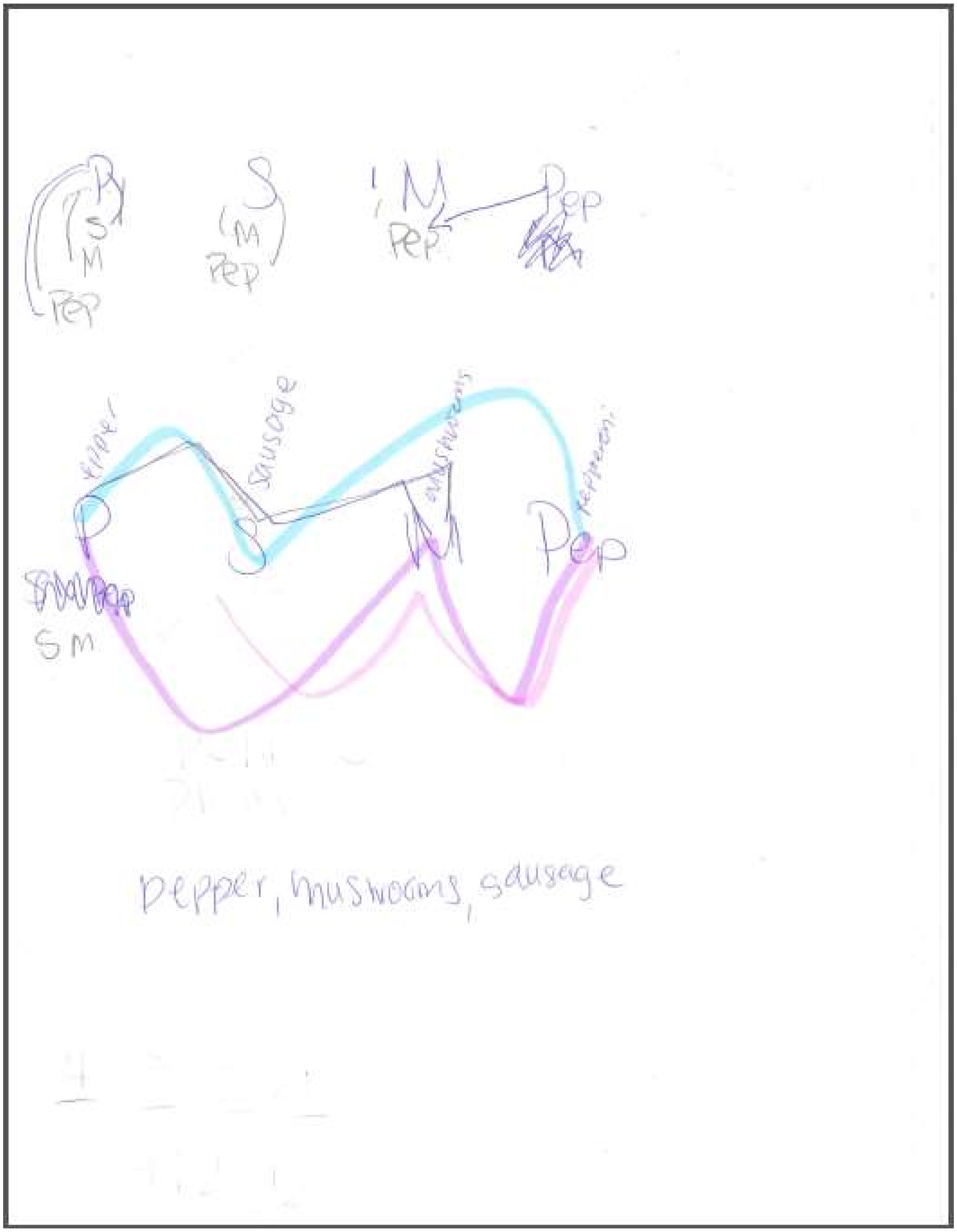


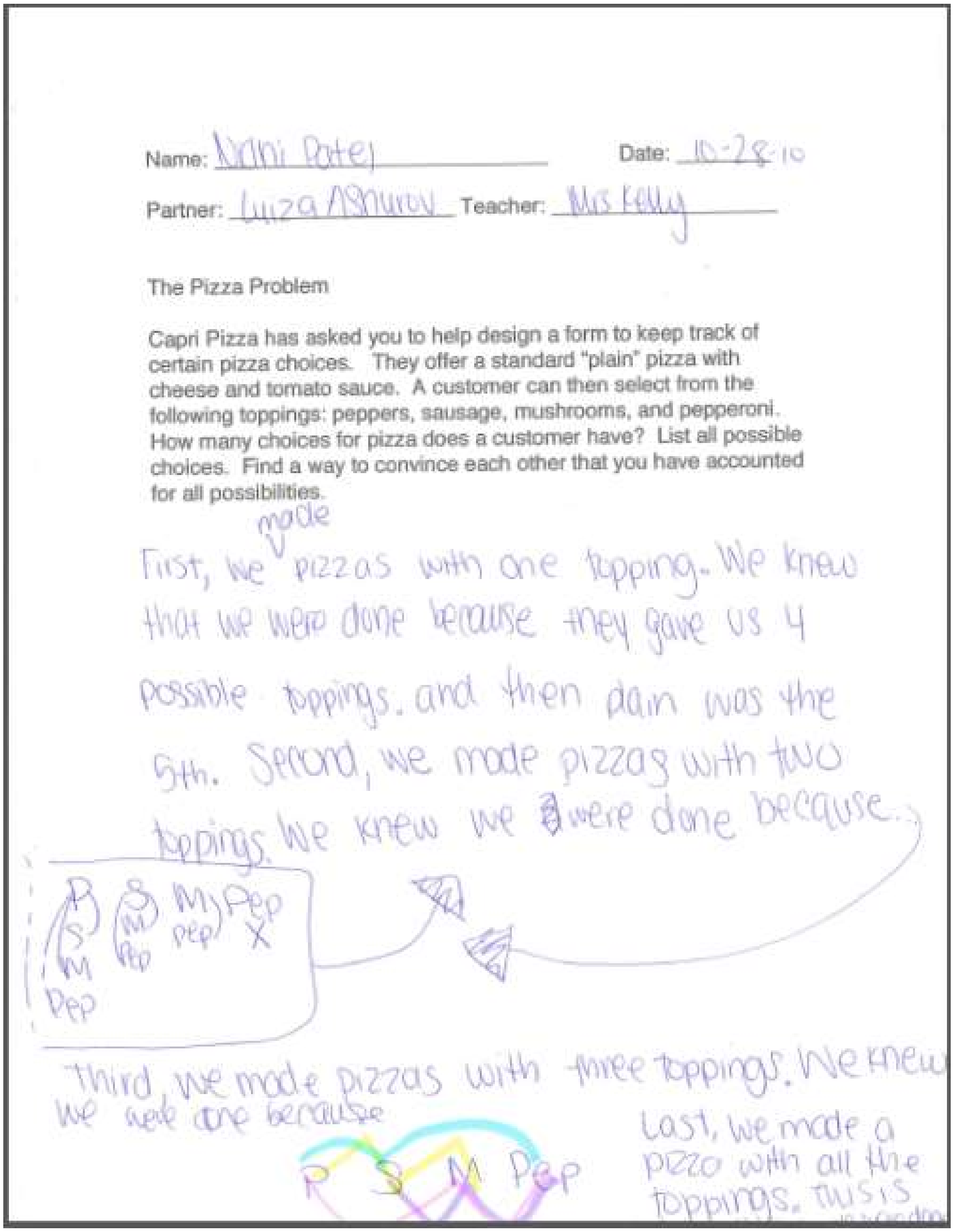
Name: Nidhi Patel Date: 10/28/10

Partner: Luiza Ashurov Teacher: Mrs. Kelly

The Pizza Problem

Capri Pizza has asked you to help design a form to keep track of certain pizza choices. They offer a standard "plain" pizza with cheese and tomato sauce. A customer can then select from the following toppings: peppers, sausage, mushrooms, and pepperoni. How many choices for pizza does a customer have? List all possible choices. Find a way to convince each other that you have accounted for all possibilities.


Pep
S
M
Pep
S M
sausage
Pep
pepper, mushrooms, sausage



Name: Nani Patel Date: 10-28-10

Partner: Luiza Ashurov Teacher: Mrs Kelly

The Pizza Problem

Capri Pizza has asked you to help design a form to keep track of certain pizza choices. They offer a standard "plain" pizza with cheese and tomato sauce. A customer can then select from the following toppings: peppers, sausage, mushrooms, and pepperoni. How many choices for pizza does a customer have? List all possible choices. Find a way to convince each other that you have accounted for all possibilities.

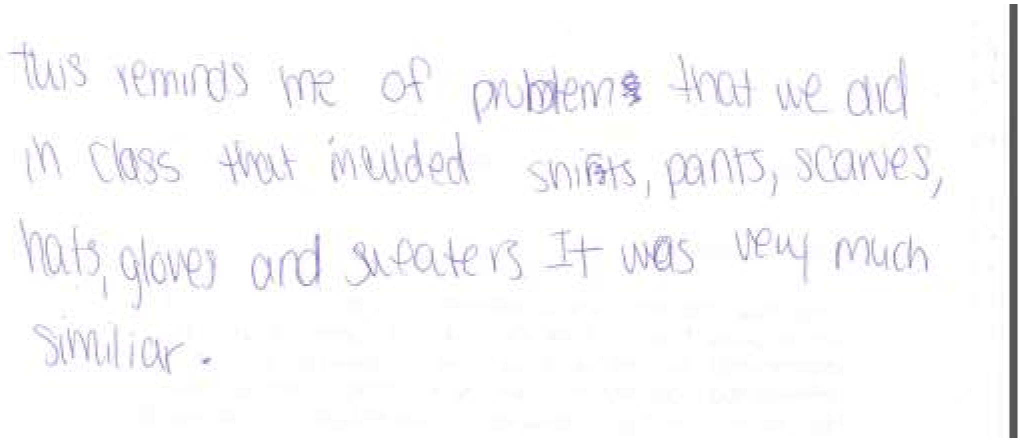
First, we made pizzas with one topping. We knew that we were done because they gave us 4 possible toppings. and then plain was the 5th. Second, we made pizzas with two toppings. We knew we were done because...

P S M Pep
S M Pep X
M Pep
Pep

Third, we made pizzas with three toppings. We knew we were done because

P S M Pep

Last, we made a pizza with all the toppings. This is



this reminds me of problems that we did in Class that included shirts, pants, scarves, hats, gloves and sweaters It was very much similiar.

Figure C.3.7 Nidhi's Cycle 2 Work (Pizza)

Cycle II- Impressed (Pizza Problem)

This is the second time I am using this student's work, and again the reason is because she has come so far since the first task. The organization of her pizzas looks random atfirst but at a second glance there was a system. She arranged her pizzas in a very organized fashion, using the order of her single topping pizzas to help with selecting two and three toppings. She was also able to justify how she knew she had all possible combinations by explaining the diagram that she created and color coded. I am quiteimpressed with her progress.

Cycle II Implementation

I felt much better about the implementation of the 5-tall towers and the pizza problem.

I started to see my students notice patterns and use more organized strategies to arrive atan answer. Their verbal arguments got better and their written justifications came alongas well. I feel that most students were more enthusiastic about the pizza problem becauseit was something they could relate to, where as building the towers is a little abstract forsome of them. Time was a concern with the 5-tall task; one forty-five minute period wasnot enough for students to build, justify verbally, and record. There is no question thatthese tasks require a minimum of an hour. I taped the towers together but I felt like some

of my groups never regained their original train of thought that brought them to their final organization of towers.

CYCLE III

Towers 3-tall, selecting from 3 colors

Ankur's challenge

STUDENT WORK THAT CONCERNED ME

Teacher: Mrs. Kelly/ Mrs. Clark

of students: 16

(took the extension problem home to complete)

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

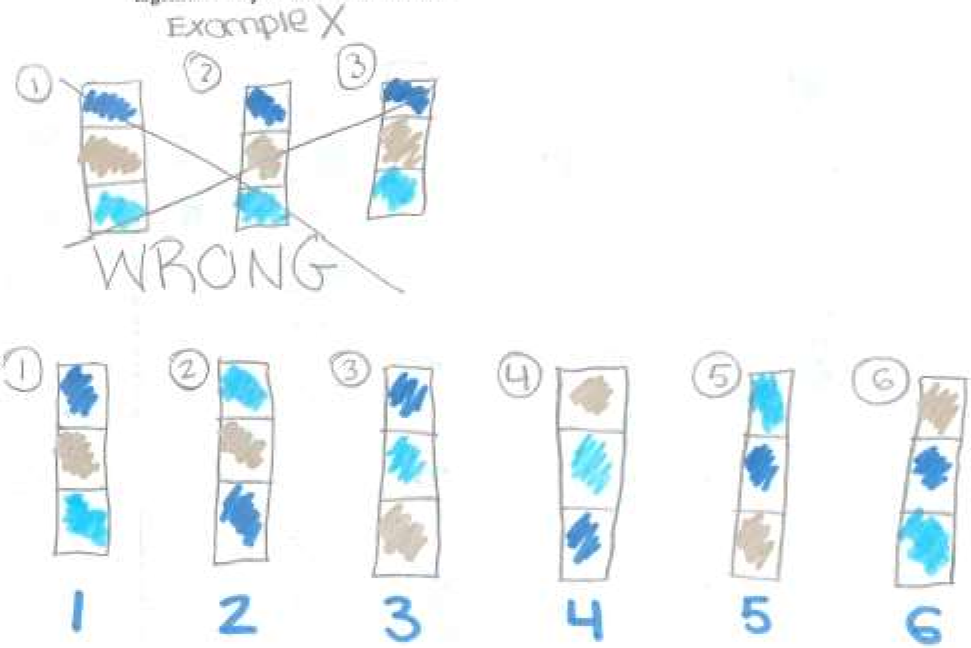
Student: Suebia Rizvi Date: 11-16-10

School: JSNS Teacher: Mrs. Kelly & Mrs. Clark

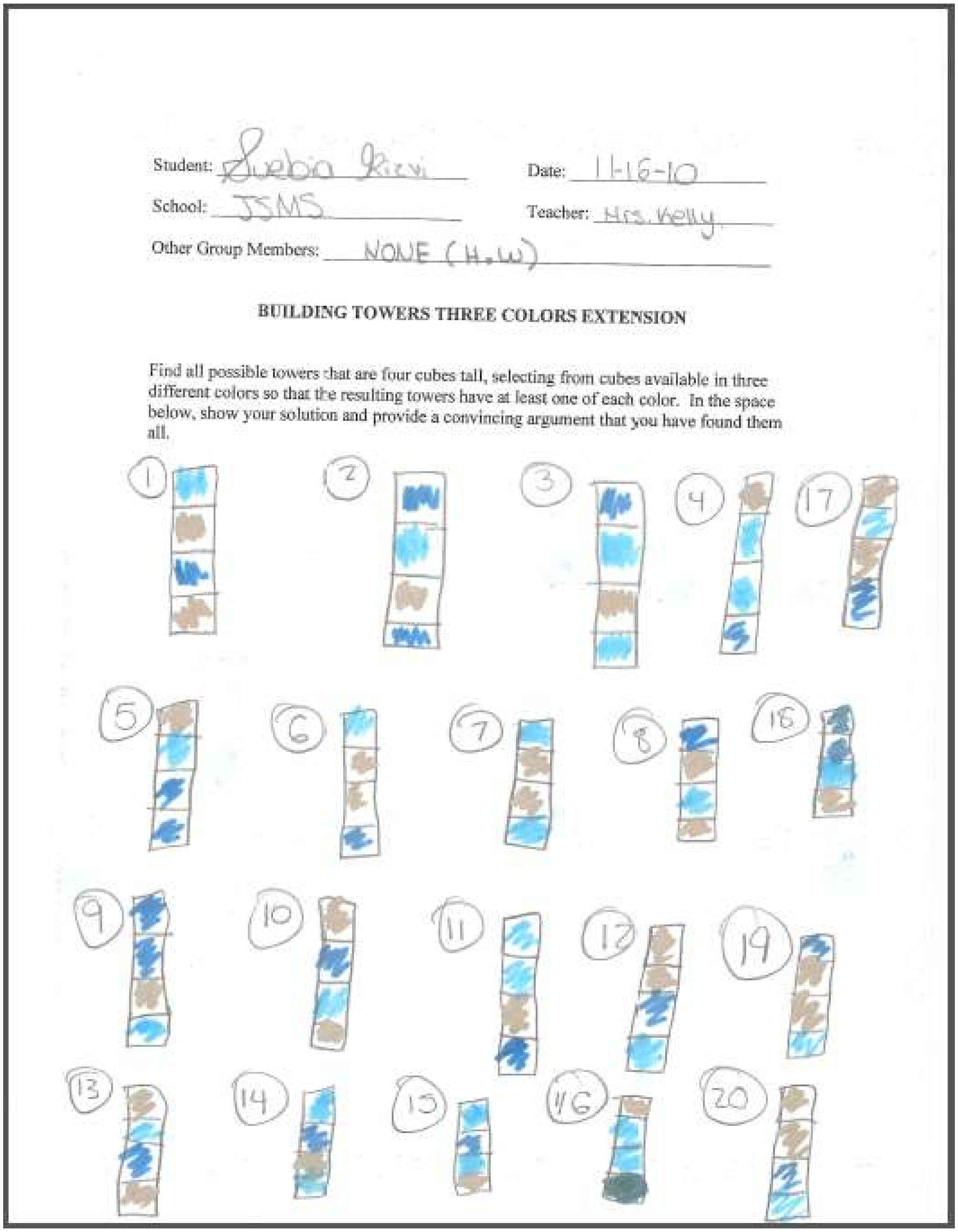
Other Group Members: Jenny Lewis

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.



6



Student: Suebia Rizvi Date: 11-16-10

School: JSMS Teacher: Mrs. Kelly

Other Group Members: NONE (H.W)

BUILDING TOWERS THREE COLORS EXTENSION

Find all possible towers that are four cubes tall, selecting from cubes available in three different colors so that the resulting towers have at least one of each color. In the space below, show your solution and provide a convincing argument that you have found them all.

Figure C.3.8 Suebia's Cycle 3 Work

Cycle III- Concerned

I am most concerned about this student's reasoning above all others because there seemsto be no connection to this task and the others. I was stunned that she only came up withsix possibilities. In looking at her first example that she crossed out, she made the same exact tower three times. I can't believe she even bothered to draw them; I would have thought once she made all three and saw that they were exactly the same she would haverealized that they were duplicates before she drew them. It just dawned on me that perhaps she didn't realize they were all the same until she drew them. I am just surprisedthat she didn't think back to the other tasks and at least come up with three of one colorfor each of the colors. I would expect this type of answer from a second or third graderwho had not had experience with the unifix cubes before. I this point, I am concerned about more than just a lack of mathematical reasoning.

STUDENT WORK THAT SURPRISED ME

Teacher: Mrs. Kelly

# of students: 14

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all

Student: Jessica [illegible] Date: 11/16/10

School: ______________ Teacher: Kelly

Other Group Members: ______________

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

[illegible]

Y	R	B	Y	B	R
R	Y	Y	B	R	B
Y	R	B	Y	B	R

G3

Y	B	R
Y	B	R
Y	B	R

G2

B	Y	R	B	R	Y
Y	B	B	R	Y	R
R	R	Y	Y	B	B

G4

Y	Y	R	R	B	B
Y	Y	R	R	B	B
B	R	B	Y	R	Y

G5

R	B	R	Y	B	Y
B	R	Y	R	Y	B
B	R	Y	R	Y	B

- For the first group we split two of the same colors and split one color in the middle.
- The second group we put each of the three different colors with 2 red on the bottom, 2 yellow and 2 blue and then put the other two colors not used on top.
- For the third group we put 1 group of yellow solids, one group of red solids and one group of blue solids.
- For the fourth group we put two of the same colors on top and a different color on the bottom.
- The fifth group we took two of the same colors on the bottom and a different color on top.

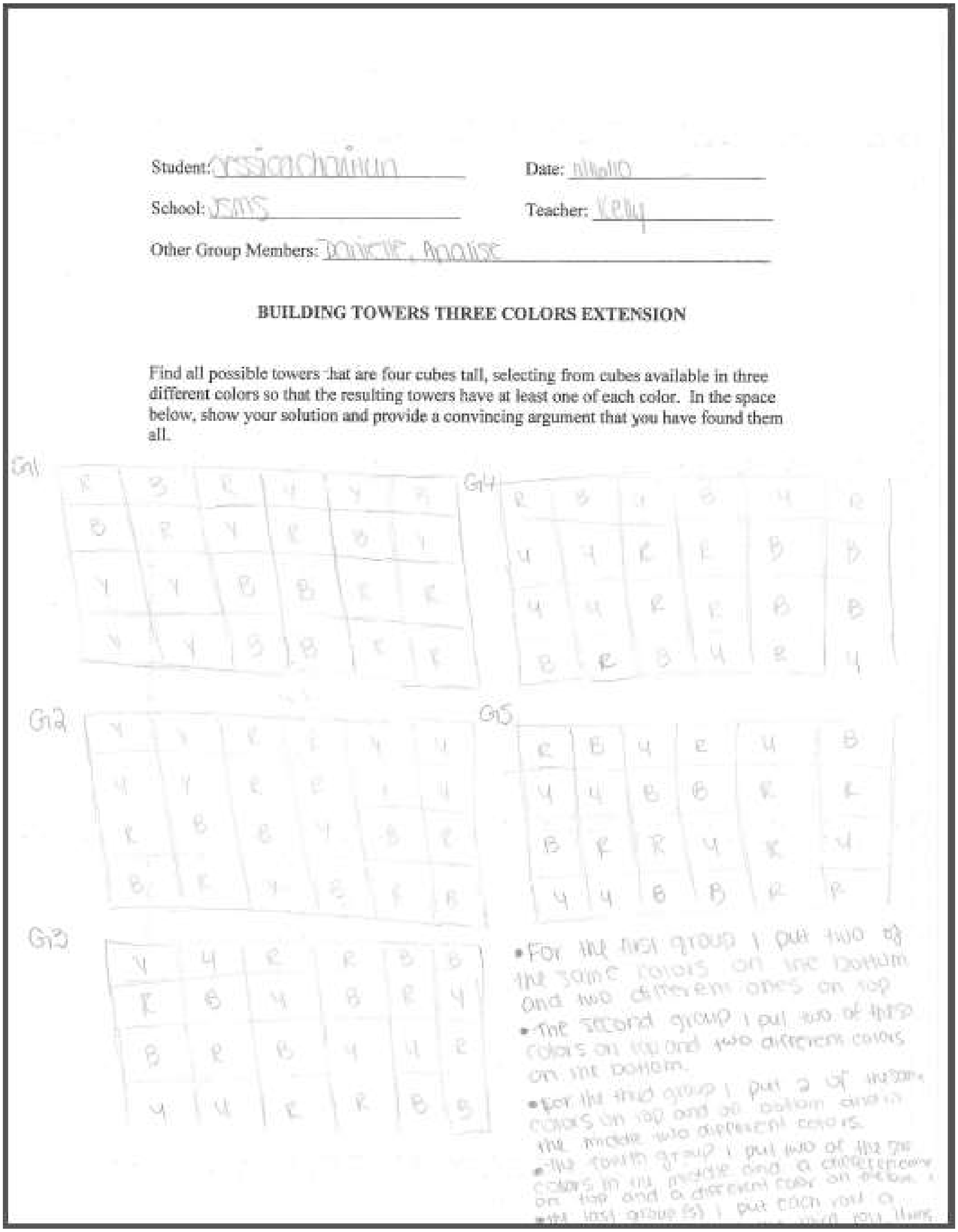


Figure C.3.9 Jessica's Cycle 3 Work

Cycle III- Surprised

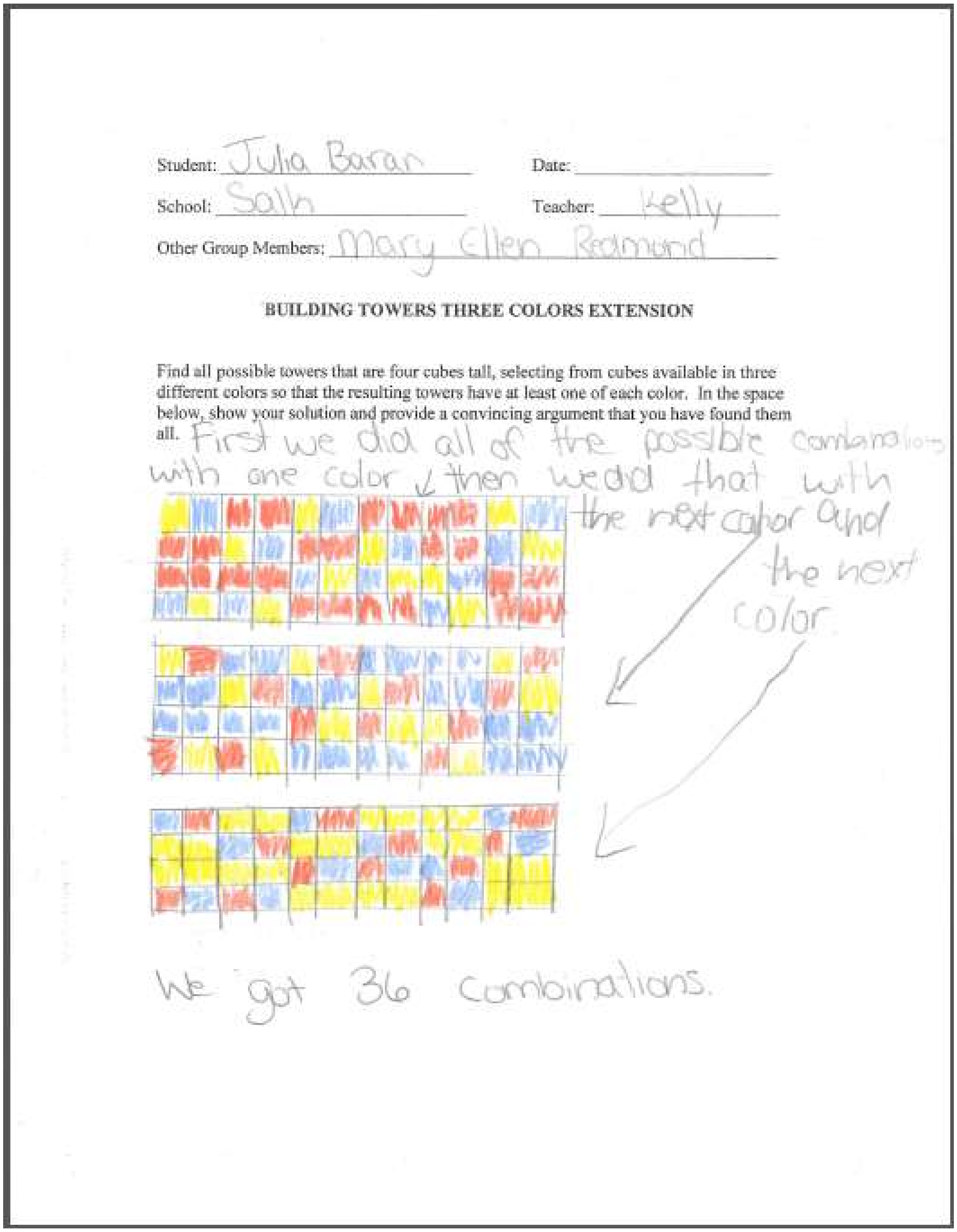
I was so pleased with the organization and justification of this student's work. She is aBasic Skills student who's first two tasks did not go beyond the "opposites" argument. This just goes to show that when we expose students to different ways of thinking itgreatly expands what they are capable of. Even if her partner was the driving force behind the organization of the towers, she was able to put into words how the towerswere grouped. This seemed to help her gain her own understanding.

STUDENT WORK THAT IMPRESSED ME

School: Jonas Salk Middle School **Teacher:** Mrs. Kelly

BUILDING TOWERS THREE COLORS EXTENSION

Find all possible towers that are four cubes tall, selecting from cubes available in three different colors so that the resulting towers have at least one of each color. In the space below, show your solution and provide a convincing argument that you have found them all.



Student: Julia Baran Date: ______

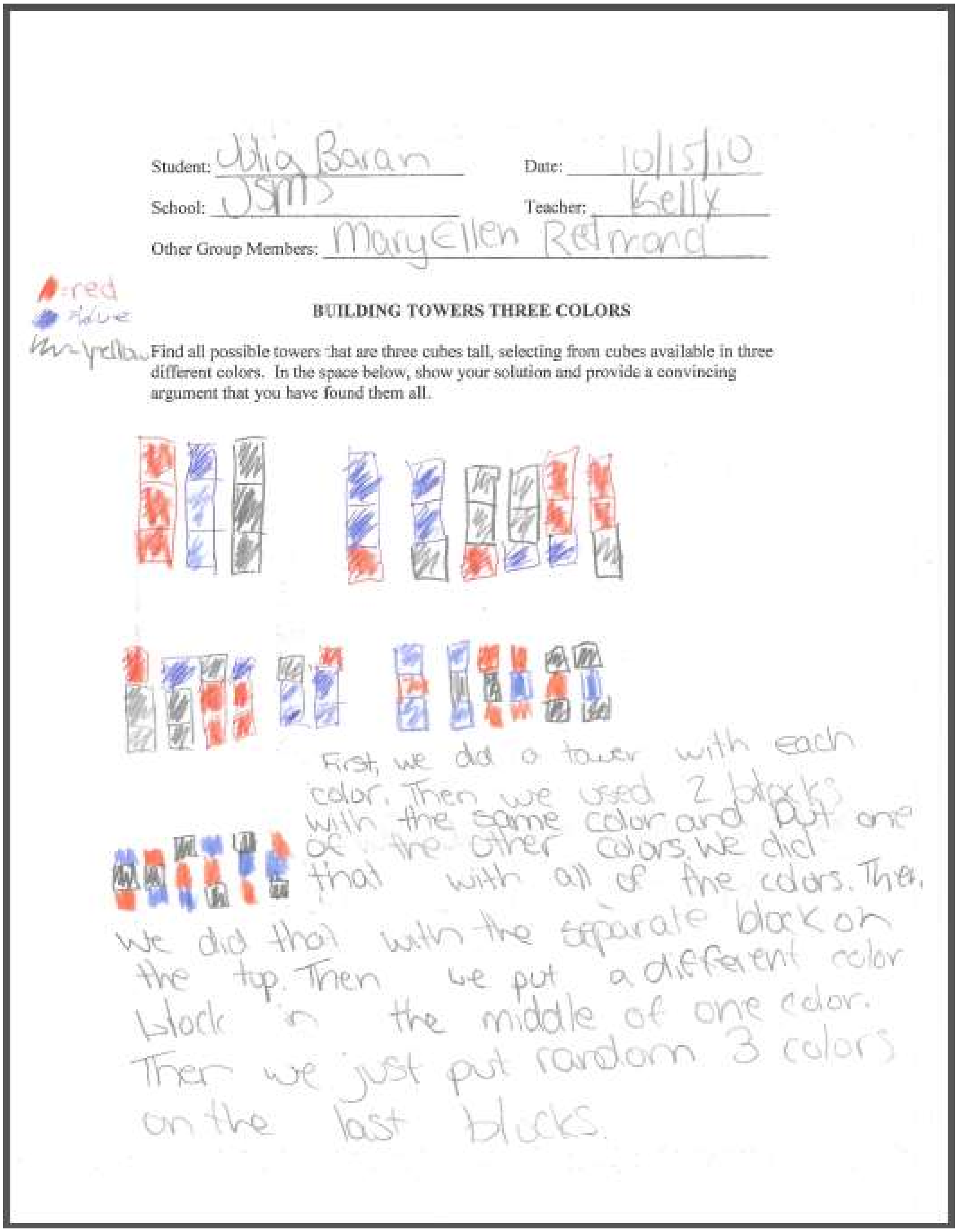
School: Salk Teacher: Kelly

Other Group Members: Mary Ellen Redmond

BUILDING TOWERS THREE COLORS EXTENSION

Find all possible towers that are four cubes tall, selecting from cubes available in three different colors so that the resulting towers have at least one of each color. In the space below, show your solution and provide a convincing argument that you have found them all. First we did all of the possible combinations with one color then we did that with the next color and the next color.

We got 36 combinations.



Student: Julia Baran Date: 10/15/10

School: JSMS Teacher: Kelly

Other Group Members: MaryEllen Redmond

= red
= blue
= yellow

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

First, we did a tower with each color. Then we used 2 blocks with the same color and put one of the other colors. We did that with all of the colors. Then, we did that with the separate block on the top. Then we put a different color block in the middle of one color. Then we just put random 3 colors on the last blocks.

Figure C.3.10 Julia's Cycle 3 Work

Cycle III- Impressed (Ankur's Challenge)

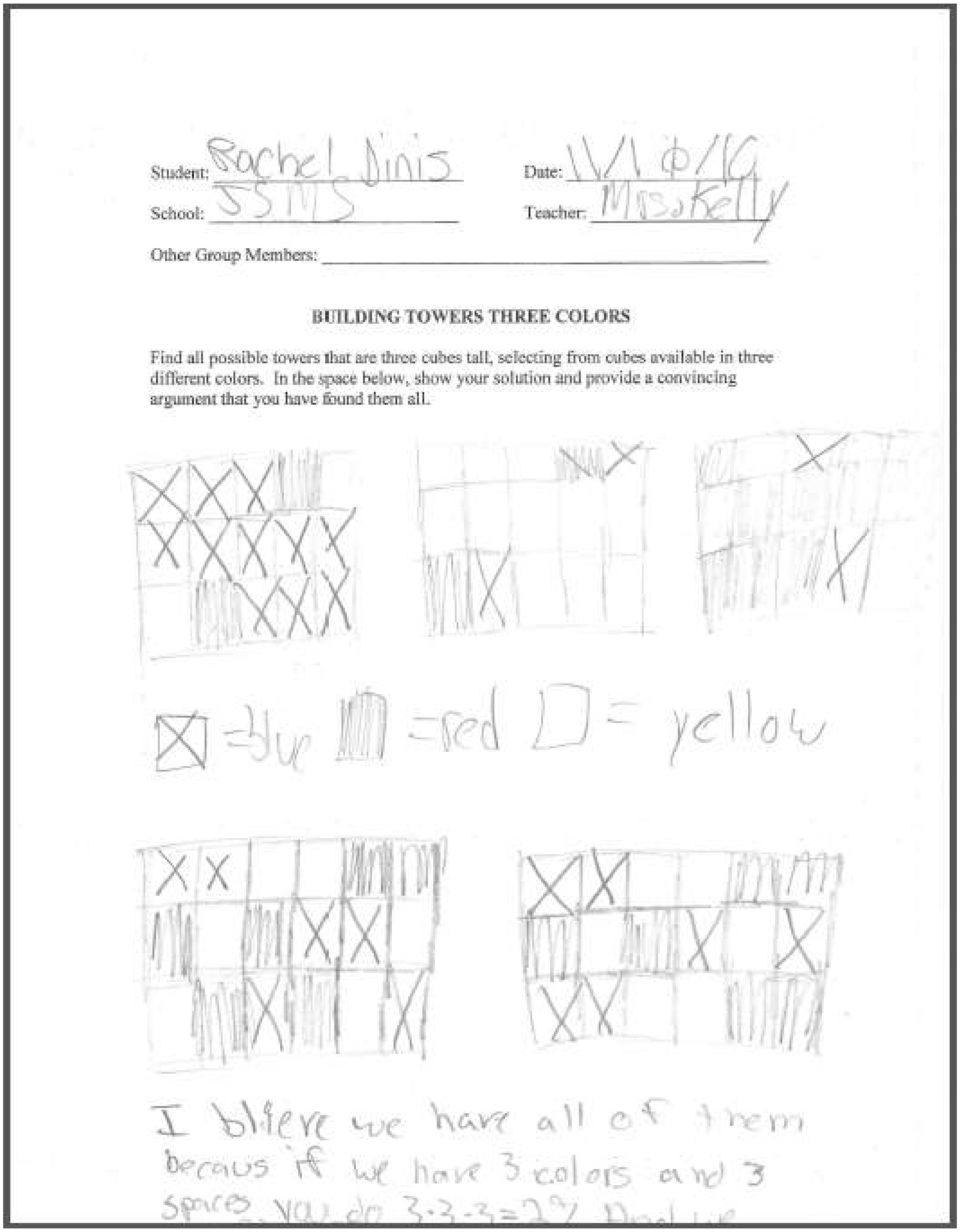
Not only was I impressed with the organization of this student's response but the speed with which she arrived at an answer was incredible. The written justification is lacking but you can see such organization in her diagram. She was able to verbally explain to me exactly what she did and how she was sure that she accounted for all the possible towers. She built off her organization from the three-tall towers, selecting from three colors. I was truly impressed with how effortlessly she seemed to come up with the correct solution; it was like she could see the picture of the towers in her head before she even built them. An excellent mathematical thinker!

STUDENT WORK THAT SURPRISED ME

Teacher: Mrs. Kelly/ Mrs. Clark # **of students:** 15

BUILDING TOWERS THREE COLORS EXTENSION

Find all possible towers that are four cubes tall, selecting from cubes available in three different colors so that the resulting towers have at least one of each color. In the space below, show your solution and provide a convincing argument that you have found them all.



Student: Rachel Dinis Date: 11/10/16

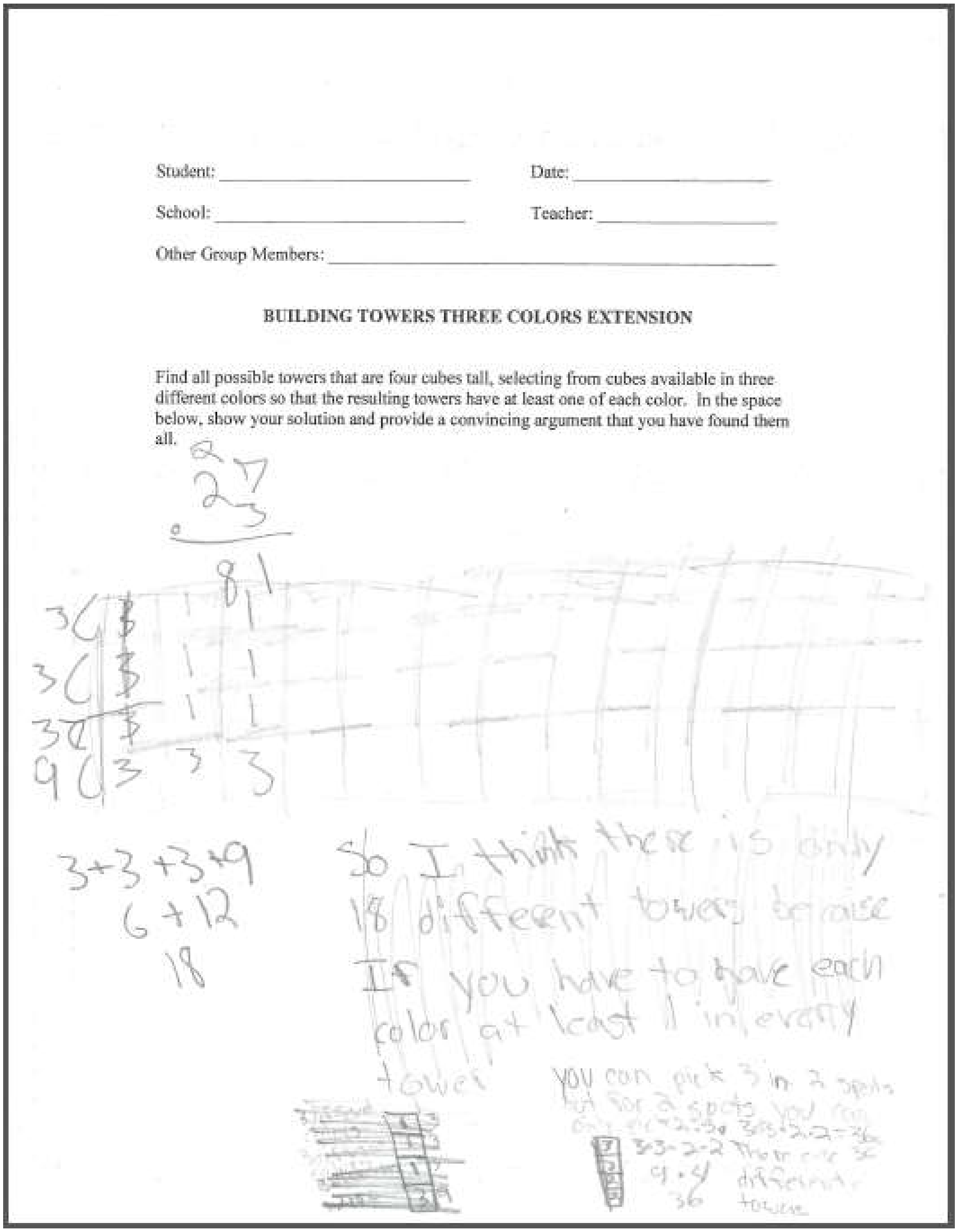
School: JSMS Teacher: Miss Kelly

Other Group Members: ______

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

I blieve we have all of them becaus if we have 3 colors and 3 spaces you do 3·3·3=27 And we



Student: ______________________ Date: ________________

School: ______________________ Teacher: ________________

Other Group Members: ____

BUILDING TOWERS THREE COLORS EXTENSION

Find all possible towers that are four cubes tall, selecting from cubes available in three different colors so that the resulting towers have at least one of each color. In the space below, show your solution and provide a convincing argument that you have found them all.

27

3+3+3+9

6+12

18

So I think there is only 18 different towers because If you have to have each color at least 1 in every tower

Figure C.3.11 Rachel's Cycle 3 Work

Cycle III- Surprised (Ankur's Challenge)

I was most surprised with this student above all others. She seems to be an average math student who doesn't say much in class. She was the only one of my students that even thought to attempt this problem by trying to come up with a formula. She went through a few different ideas before she came up with one that seemed to work. Her explanation makes sense, "you can pick from three colors in two spots, and in the last two spots you can only choose from two colors." 3 x 3 x 2 x 2 = 36

The mathematician at our last meeting said she wasn't sure this would hold up but I think it is a sound argument that both surprised and impressed me! I am proud of her mathematical reasoning.

Cycle III Implementation

I was thrilled with the progress that I saw from most of my students by the time we did towers 3-tall, selecting from 3 colors and Ankur's Challenge. Most groups had progressed from the simple argument of "opposites" to more sophisticated arguments which included, controlling for one or more variables and using a recursive argument. I feel my ability to ask delving questions improved by these tasks so I was able to pull more from my students forcing them to think more deeply about what they were doing. Time was again an issue, and in several classes I had to assign Ankur's Challenge for homework which probably broke their concentration and train of thought. I was very proud of several of my students who did a fantastic job with Ankur's Challenge. They used better reasoning than I had used when I performed the task!

I think the best part about this last set of tasks is that the students started to see their own progress; they started to feel good about thinking more mathematically so they were willing to try different strategies.

Reflection

This has been an eye opening experience for both me and my students. We have both come so far in our ability to reason mathematically, or in my case how I perceive mathematical reasoning as it relates to mathematical ability. I was so concerned about my students after the first task but I realized that no one had ever asked them to think like

that before. Sheer exposure to the types of tasks we asked them to perform through this class helped them to develop their own strategies for how they reason their way through a problem. It is clear that the more of these types of activities that students engage in the better developed their mathematical reasoning will become. I am impressed with my students' progress in just a few short months. Imagine if we asked them to think like this all the time?!

At first I think my students were so concerned with getting the right answer that they didn't allow themselves time to really explore the problem. Once they realized that I was looking more at how they went about coming up with their answer than the actual answer itself, they seemed more willing to spend more time thinking more deeply about the problem.

What was so enlightening about these activities is that I got to see into some of my students reasoning in a way that I never would have without these tasks. Several of my "Basic Skills" students impressed me the most with their reasoning and I would not have expected that. This just goes to show what children are capable of if we let them explore and don't put limitations on them.

The mind is an incredible thing and when we have students partake in activities that allow them to think it expands what they will be able to achieve in the future!

Sally

Topics in Math Education: Lesson Study on Reasoning

15:254:599, Section81

Sally

Central Region

Old Bridge, NJ

December 4, 2010

Cycle1 Task:

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates.

Grade 7

Class Size: 27 students

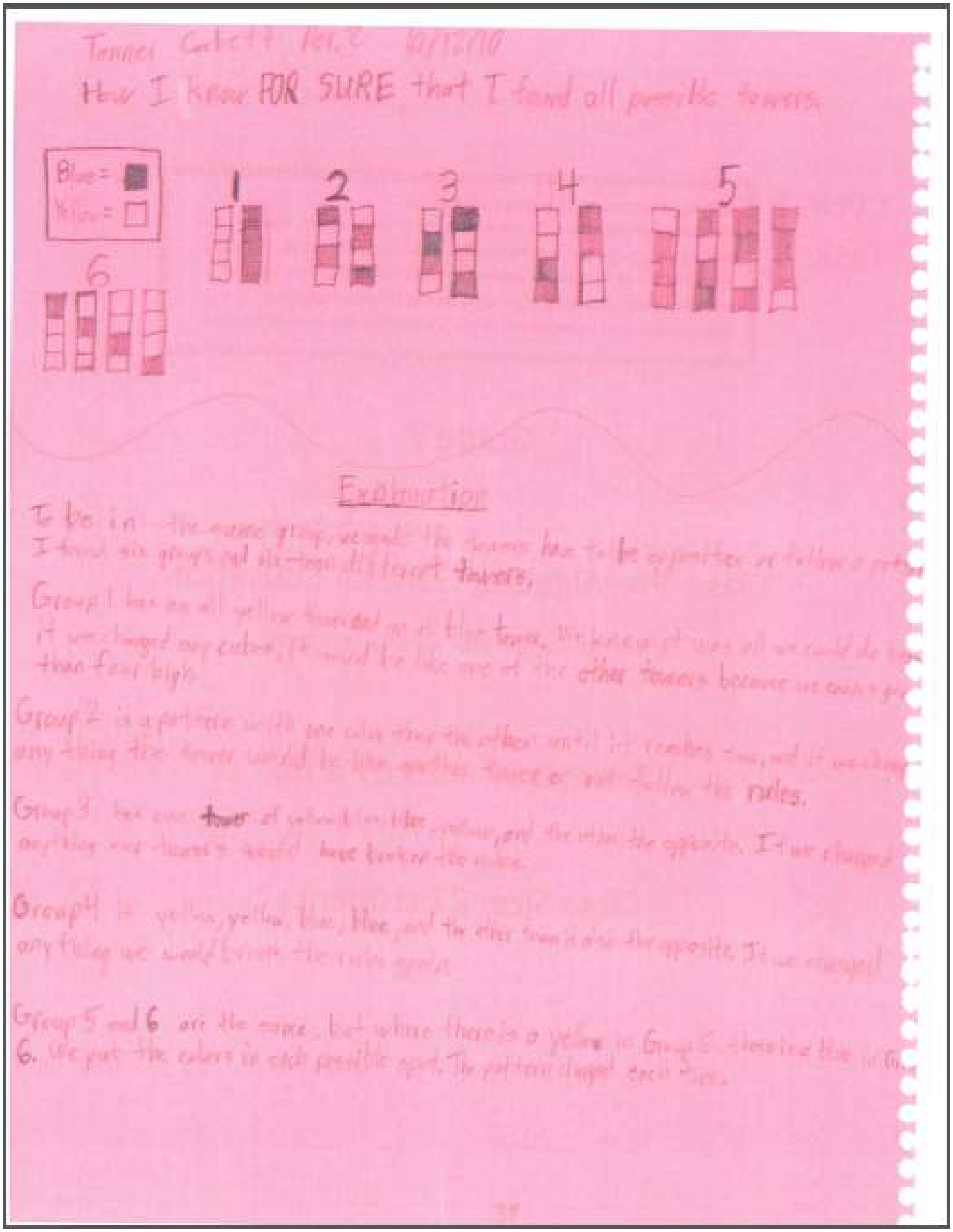


Figure C.4.1 Tanner's Cycle 1 Work

Tanner **surprised** me because of the way he intertwined the idea of opposites, patterns, and rules.

As I noticed in preassessments we've had as well as the majority of my own students in the classroom opted to create their own definition of "opposites" and to continue to use that term throughout, I tended to be immediately not convinced by students who claimed "we found all pairs of opposites." To me, "opposites"were a way to duplicate a tower that was found by trial and error, to make a couple of towers that were unrelated to the previous ones that they had come up with, and since students could use the concept of "opposites" to put the towers into many groups of two, it made the students feel organized and strategic. I was somewhat disappointed when students' only proof of finding all the towers was "we found all that wecould find, and their opposites."

Tanner's reasoning was the exception to the way that I felt about opposites. He found different patterns and made rules from them. For example, one pattern was same color on the ends, and other color in the two middle spots. Since there were only two colors, there were only two towers that fit the criteria for that rule. If they used the two colors to create a different tower, the rule would have been broken. Tanner and his partner found all possible patterns for two yellows and two blues together: 3 different pattern rules, producing 6 towers. Although there was no constant held and no evident recursive pattern throughout the solution, my students as well as me were thoroughly convinced of Tanner's solution.

Cycle1 Task:

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates.

Grade7

Class Size: 27 students

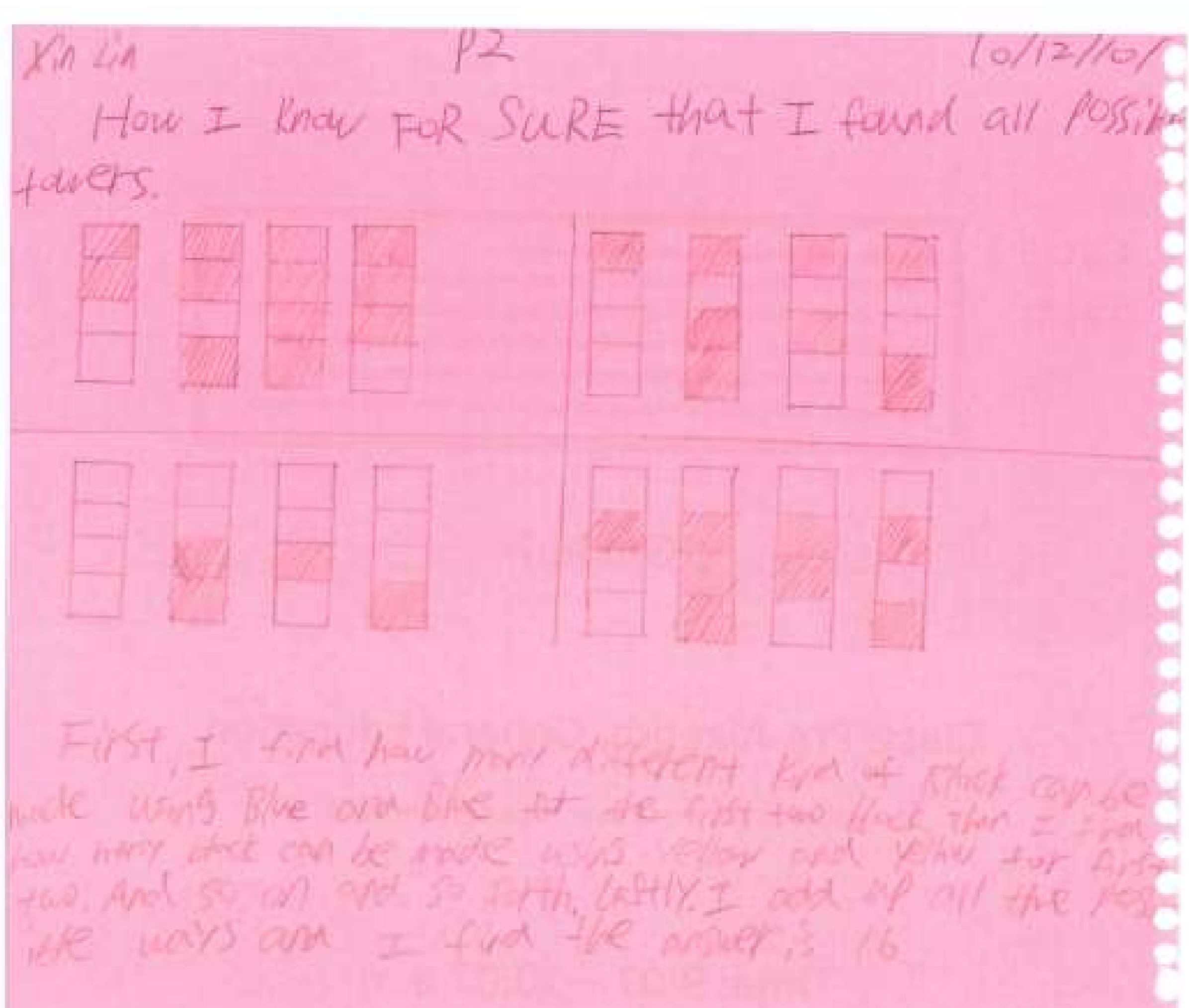


Figure C.4.2 Xin's Cycle 1 Work

Xin **impressed** me because he was the only student who held a double constant, which created four groups of towers containing similarities within groups(top two cubes) and similarities across groups (bottom two cubes).

When we as teachers originally underwent the task of creating towers four tall, selecting from two colors, I had never held a constant or a double constant and changed the rest of the tower. I'm not sure whythat way of thinking appears to be so rare, but I feel that it gives a much clearer and simpler proof. Xin really didn't need to explain much when he presented his rationale. The groups he created spoke for themselves.

When there are two colors held constant on the top of the tower, there are only four possible options for

what the bottom two colors can be, and since there are only four types of combinations that are held constantcan be, four groups of four is the result. If HE was a student that told me "I know there are 16 towers becauseyou do four times four," I would believe him. I would also love to see how he could apply that thinking to thetowers with three colors.

Cycle 1 Task:

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.)

Grade 7

Class Size: 27 students

Nicole Anderson 10/12/10

How I Know For Sure That I Found All Possible Towers

Me and my partner found every tower possible. We would make a design, then flip it around. Example. Y, Y, B, B then we would do B, B, Y, Y. we also made blue towers traveling up the tower. We had blue as the top cube then in another the second, then third, then fourth. Then after that we flipped it around with yellow traveling up. In all we had 16 towers and in each row we had 4 yellow cubes and 4 blue cubes. If u push the towers together u would see this. In addition, when u multiply 4 blue cubes times 4 yellow cubes you get 16 towers.

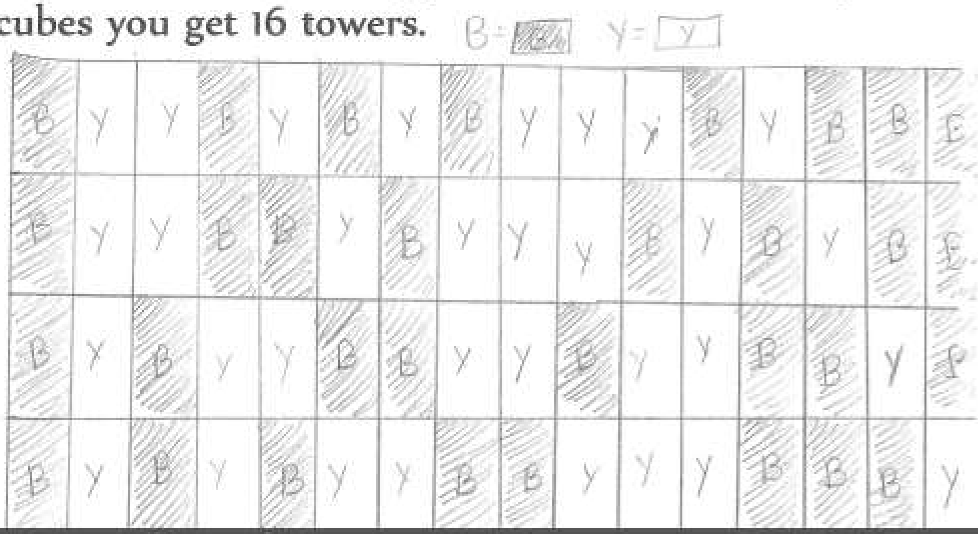


Figure C.4.3 Nicole's Cycle 1 Work

Nicole **concerned** me because she went too far when she wrote her explanation. She looked at the numbers too much rather than relying on her own justification.

Nicole was able to find all of the 16 towers by using a similar thinking that Tanner had used. She used a recursive pattern in the towers that had three cubes of one color and one cube of the other color. I think she may have felt that her proving explanation was incomplete and therefore tried to count things and use number sentences to verify her answer. Her last sentence says that you must multiply four yellow cubes times four blue cubes, but it is difficult for me to see why that statement makes sense. I think she may have been looking for too many rules rather than proving her number of towers in another way. Perhaps if similar tasks such as this one are given to her, she will break the habit of looking for formulas or rules and inventing them without a rationale.

Cycle 1Task Implementation Reflection:

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.)

As a result of the first tower task implementation, I was able to learn that my students work well together when I assign pairs to work together. Pairing the students with similar ability classmates definitely made certain students rise to the challenge and become more outspoken or more like leaders, so Ithought that was great. I also learned that some students are systematic and some are not as systematic and organized as I would hope for. I thought the groupings that they created and the convincing arguments could have been better, but I was pleased by the fact that all students found the sixteen towers. What I would do differently next time I implement this task is to request to have a document camera available, as well as colored pencils so that the students could share some of their strategies immediately. Iwould also make it so that they didn't have to convince me verbally before I allow them to explain their logic on paper. Then I would choose good examples of convincing arguments so that the students could see how to improve their own arguments in thefuture

Cycle 2 Task:

The Pizza Problem

Capri Pizza has asked you to help design a form to keep track of certain pizza choices. They offer a standard "plain" pizza with cheese and tomato sauce. A customer can then select from the following toppings: peppers, sausage, onions, and pepperoni. How many choices for pizza does a customer have? List all possible choices. Find a way to convince each other that you have accounted for all possibilities.

Grade 7

Class Size: 27 students

Rowen **surprised** me because of the representation she selected, but also because when I asked her to redo the diagram for me so she could explain it as she drew, she created something similar, but not the same.

Rowen's strategy for solving the pizza problem looked just like the way Brandon, the student in the video that we had seen, had explained his solution to the four-tall towers problem with the 1's and the 0's. Rowen, instead put check marks and empty boxes to represent how she found her solution. This was a strategy very unique from any other student's solution, and it looked to me ask if she used the same idea of opposites, or, from my perspective, complements, to find new combinations of toppings. However, when I asked her to show me her thinking again, she used more of a recursive representation in her sequence. Both times though, she was able to gather a total of 16 different pizzas.

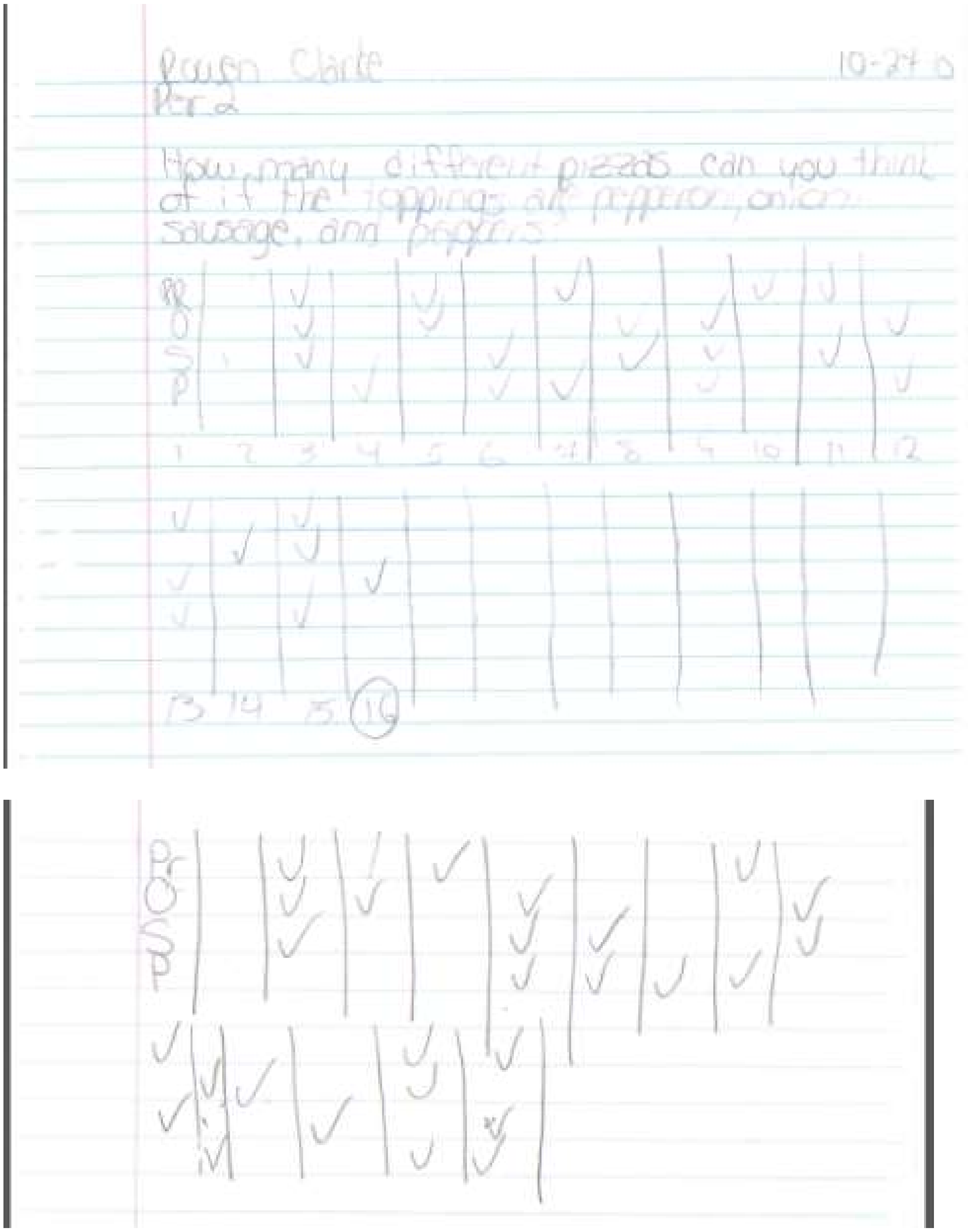


Figure C.4.4 Rowen's Cycle 2 Work

Cycle 2 Task:

The Pizza Problem

Capri Pizza has asked you to help design a form to keep track of certain pizza choices. They offer a standard "plain" pizza with cheese and tomato sauce. A customer can then select from the following toppings: peppers, sausage, onions, and pepperoni. How many choices for pizza does a customer have? List all possible choices. Find a way to convince each other that you have accounted for all possibilities.

Grade 7

Class Size: 27 students

Tanner **impressed** me because his solution is very easy to follow, simple, and leaves little room for misinterpretation.

He started with a plain pizza, then lists all the possible pizzas with pepperoni on them in the next column, and proceeds to all the possible pizzas with onion (but not pepperoni) on them, and then the "sausage" column (without any onion or pepperoni toppings), and finally the last possible pizza with only peppers. Ithought his solution was neat because with the exception of the plain pizza, each column is half of the column before it, and he explained to the class that if all columns were equal there would be many duplicates. When I solved the problem, I classified groups as "1topping," "2 topping," "3 topping," "4 topping," and "no topping" pizzas, and Ithought that would be the way that most students solved it. I didn't anticipate students to classify pizzas in terms of the main topping (so to speak) on it. Ifeel that Tanner's visual would connect very well to a tree diagram.

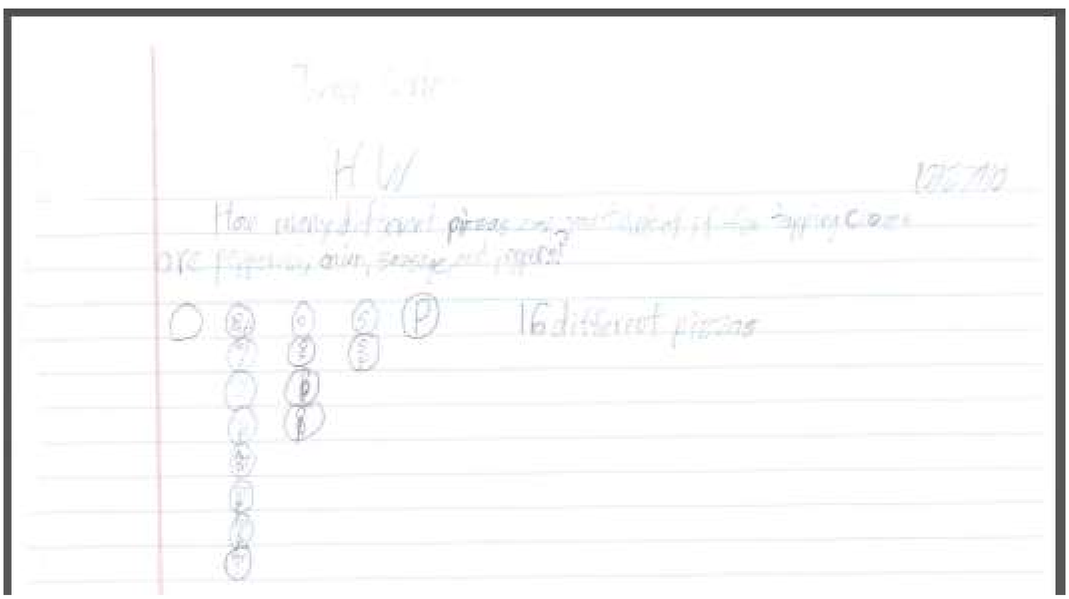


Figure C.4.5 Tanner's Cycle 2 Work

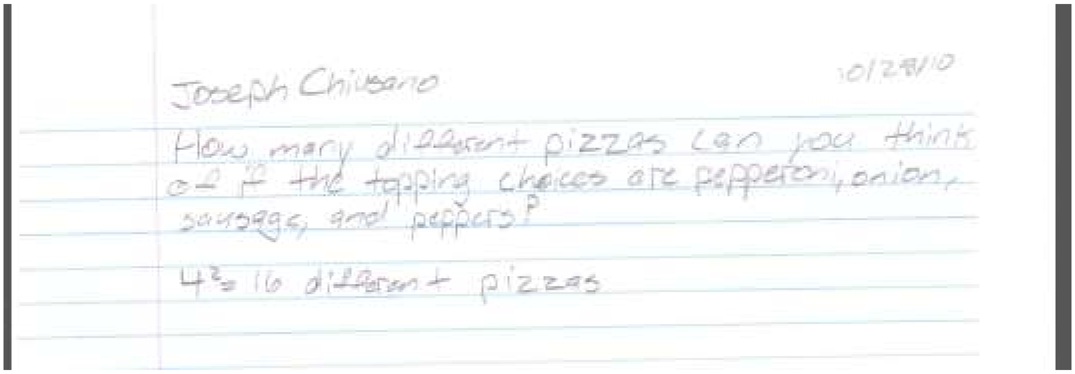
Cycle 2 Task:

The Pizza Problem

Capri Pizza has asked you to help design a form to keep track of certain pizza choices. They offer a standard "plain" pizza with cheese and tomato sauce. A customer can then select from the following toppings: peppers, sausage, onions, and pepperoni. How many choices for pizza does a customer have? List all possible choices. Find a way to convince each other that you have accounted for all possibilities.

Grade 7

Class Size: 27 students



Joseph Chiusano 10/29/10

How many different pizzas can you think of if the topping choices are pepperoni, onion, sausage, and peppers?

$4^2 = 16$ different pizzas

Figure C.4.6 Joseph's Cycle 2 Work

Joseph **concerned** me because he seems to struggle with the mathematical ideas of solving the problem using diagrams or some other visual model of representing ideas.

Even after discussion with his classmates and prompting from me, he had replies such as "I just know it's four times four," and "There are four toppings so it's four to the second power." He had no tangible evidence or logical rationale to back up his answer, and my concern is that if he can't make any connection here, he won't be able to generalize ideas or make connections from this problem to any other one. I fear that he could be a student that comes to depend on learning and memorizing formulas and not attaching any meaning to them. I had a difficult time asking him things to encourage a further thought process or further explanation from him.

Cycle 2 Task Implementation Reflection:

The Pizza Problem

Capri Pizza has asked you to help design a form to keep track of certain pizza choices. They offer a standard "plain" pizza with cheese and tomato sauce. A customer can then select from the following toppings: peppers, sausage, onions, and pepperoni. How many choices for pizza does a customer have? List all possible choices. Find a way to convince each other that you have accounted for

As a result of the task implemented in cycle two, I learned that my students like to solve problems in the quickest way possible, which results in some mistakes and sometimes an inability to articulate thought when asked to recall what the student did to achieve a solution. Some students who attempted the pizza problem as well as towers 5-tall, selecting from two colors, rushed through the tasks and wanted to move on and be done with it even if the answer wasn't correct. I struggled with the fact that I wanted them to use reasoning and prove to themselves if they were correct. Due to the fact that they wanted me to tell them whether they were right or wrong, they were not inclined to proving themselves wrong or right. I still am not sure what to do in the situation where the student wants to just be done with the problem, but I don't want to tell them that they can't be done because they aren't correct and have an insufficient explanation. The positive point, however, was that many students did find the correct answers in strategic ways. Next time, I would change the end result of the problem. I would create a rubric to determine the students grade on the task, since the students are so grade oriented and this task is relatively divergent.

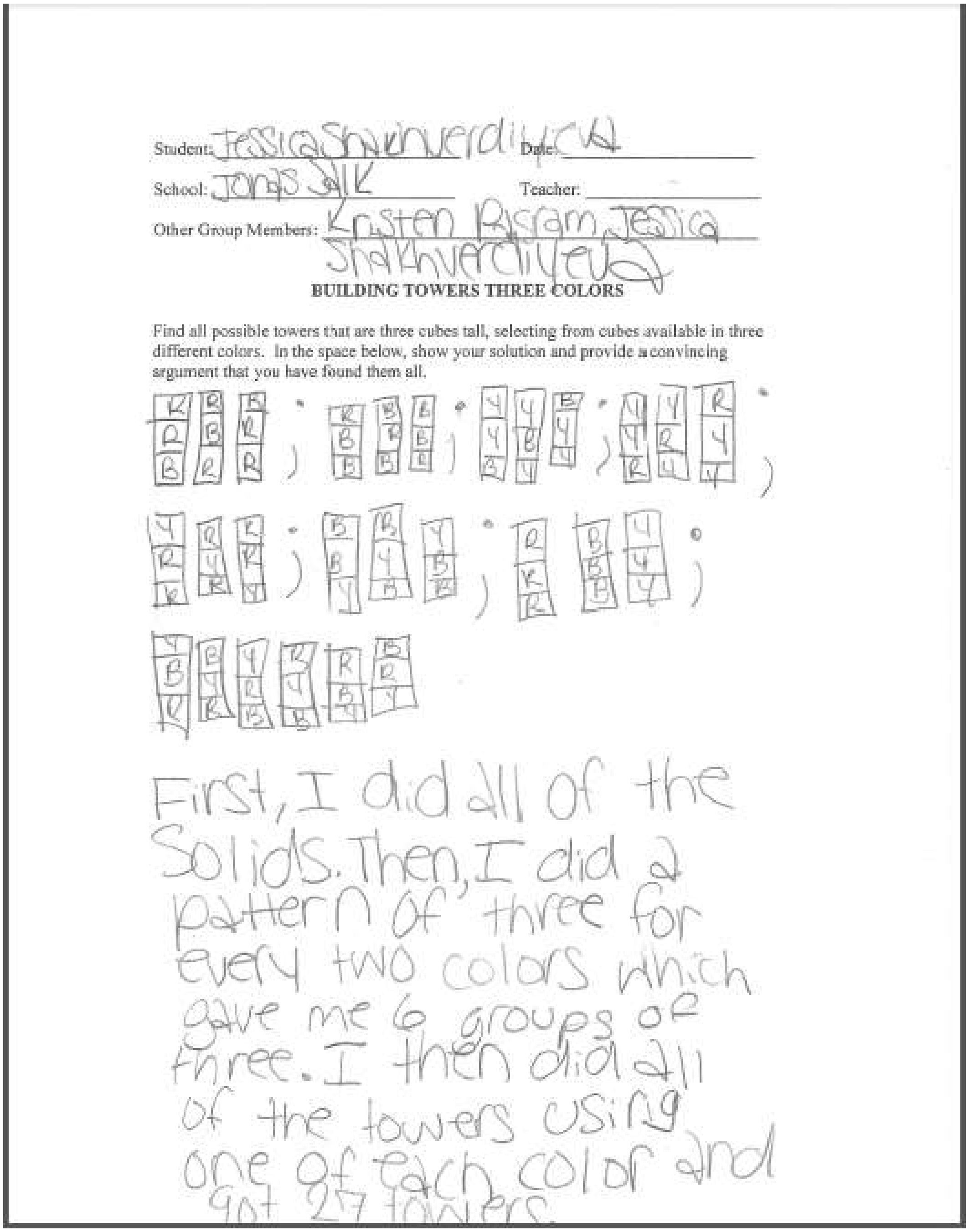
Cycle 3 Task:

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

Grade 7

Class Size: 27 students



Student: Jessica Shakhverdiyeva Date:

School: Jonas Salk Teacher:

Other Group Members: Kristen Risram, Jessica Shakhverdiyeva

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

First, I did all of the Solids. Then, I did a pattern of three for every two colors which gave me 6 groups of three. I then did all of the towers using one of each color and got 27 towers.

Figure C.4.7 Jessica's Cycle 3 Work

Jessica **impressed** me because she had the most unique strategy with the groups that she created.

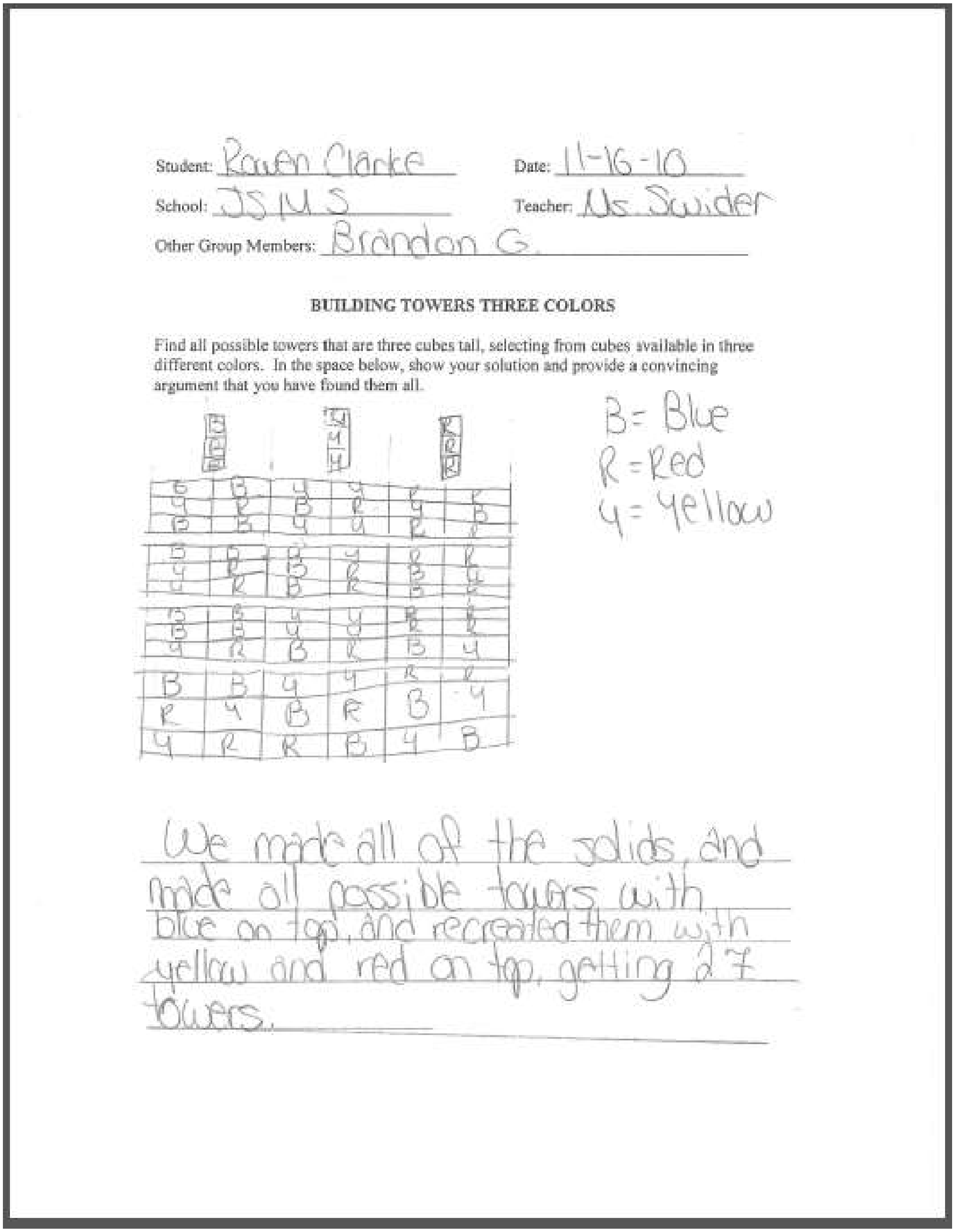
Jessica looked classified the tower groups in terms of how many colors were used, and how many of each color were in the tower. Her first group was all one color; there were only three towers: all red, all yellow, and all blue. From that point, she mixed two colors and kept the number of each color constant. For example, she found all the ways to put two blue cubes with one red cube: BBR, BRB, RBB. Since there are five more ways to mix two colors (two reds and one blue, two reds and one yellow, two yellows and one red, two yellows and one blue, and two blues and one yellow), she had six groups of three. Finally, she saw that all three colors could be combined, and there were six ways to do that: YBR, BYR, YRB, RYB, RBY, RYB. Altogether, she came up with 27 towers. What I thought was so neat about this is that the number three comes up over and over in different ways, and it is easy to justify each part and to be sure that there are no towers missing at the end.

Cycle 3 Task:

BUILDING TOWERS THREE COLORS Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

Grade 7

Class Size: 27 students



Student: Rowen Clarke Date: 11-16-10

School: JSMS Teacher: Ms Swider

Other Group Members: Brandon G.

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

B	B	Y	Y	R	R
Y	R	B	R	Y	B
B	B	Y	Y	R	[illegible]
B	B	B	Y	R	R
Y	R	B	R	B	Y
Y	R	B	R	B	[illegible]
B	B	Y	Y	R	R
B	B	Y	Y	R	R
Y	R	B	R	B	Y
B	B	Y	Y	R	R
R	Y	B	R	B	Y
Y	R	R	B	Y	B

B = Blue
R = Red
Y = Yellow

We made all of the solids, and made all possible towers with blue on top, and recreated them with yellow and red on top, getting 27 towers.

Figure C.4.8 Rowen's Cycle 3 Work

Rowen **surprised** me because her diagram was an interesting representation to try to follow. However, once she proved that all of her towers with blue on topwere the only ones able to be created, her justification is impossible to be debated.

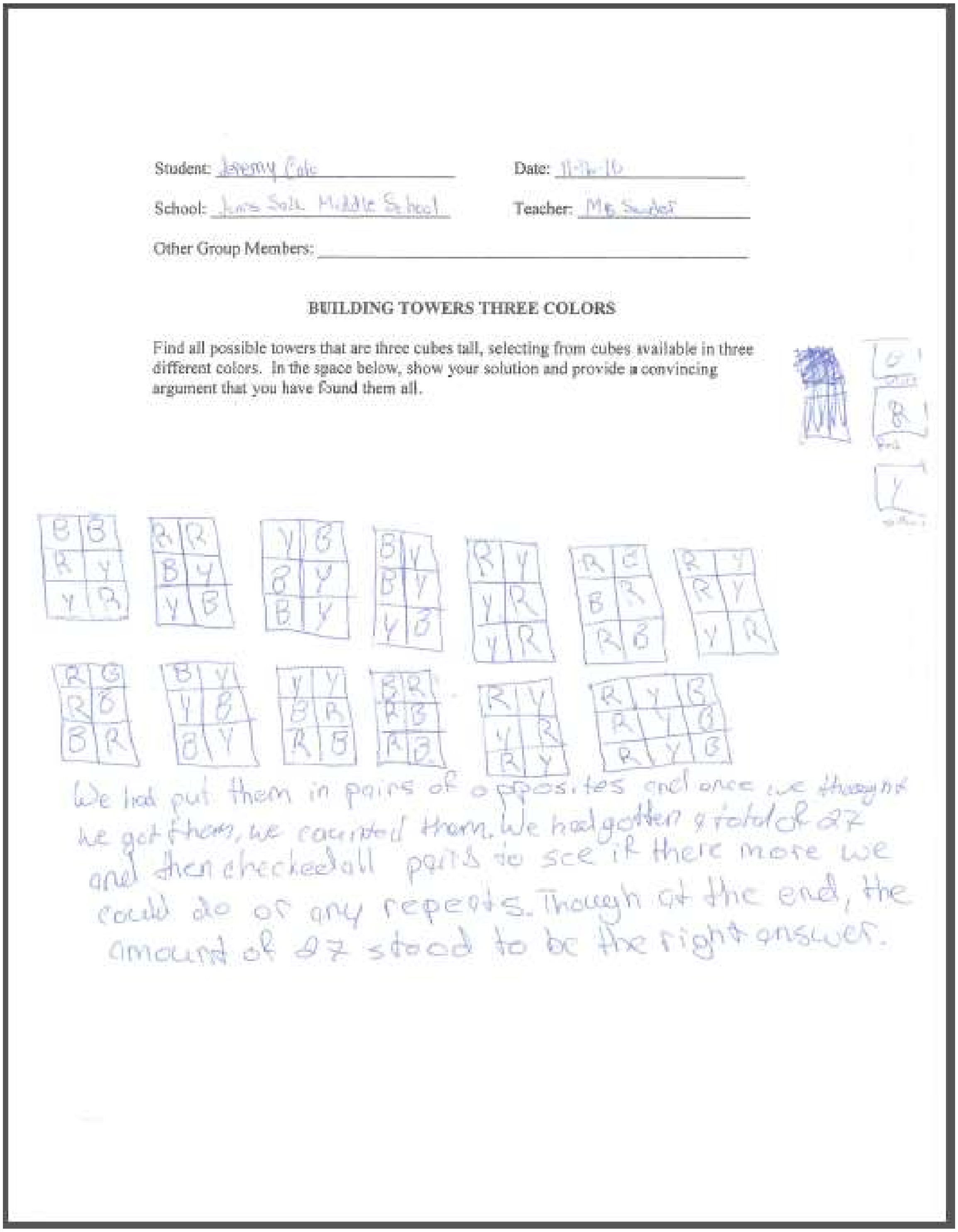
Rowen stated that she first created all the "solid" towers. She then took the solid blue tower and made a group using the top cube as the constant, the top blue cube. She came up with 8 different towers,then added the solid blue tower, coming up with 9 total "blue cube on top" towers. She then determined thatthis should be able to be repeated with the yellow cube on top and the red cube on top. She found all the new towers according to her finding, and she was sure that she found all the towers, since there were only eight possible "different bottoms," or combinations of two cubes to put on the bottom when the top is constant (when you don't consider the "all solid" cube).

Cycle3 Task:

BUILDING TOWERS THREE COLORS Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

Grade 7

Class Size: 27 students



Student: Jeremy Cole Date: 11-16-16

School: [illegible] Middle School Teacher: Ms. [illegible]

Other Group Members:

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

We had put them in pairs of opposites and once we thought we got them, we counted them. We had gotten a total of 27 and then checked all pairs to see if there more we could do or any repeats. Though at the end, the amount of 27 stood to be the right answer.

Figure C.4.9 Jeremy's Cycle 3 Work

Jeremy **concerned** me because he has no organization system and states that "27 stood to be the right answer."

As previously stated, my feeling about students using the concept of "opposites" to justify the fact that they found all towers, is not a positive feeling. I agree that "opposites" will help create more towers, but I do not thing it is a means to a proven solution. Jeremy basically said that they found towers, counted twenty- seven, checked for duplicates or checked "to see if there was more we could do," and then thought twenty- seven was the answer. There was no real justification in the process and no thought about why they would get duplicates if they made any more towers. I feel that maybe if Jeremy had presented and had been questioned by the student audience, he might be able to articulate himself better and therefore come up with a stronger rationale.

Cycle 3 Task Implementation Reflection:

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

By the end of the implementation of the third cycle of tasks, I was able to see a growth, or a refining of thought processes of my students. Students used ideas from previous tasks and expanded upon them.

Students also took ideas from other students who shared ideas of recursion and holding a constant in order to achieve the correct answers. Students organization of towers into groups and their articulation of the group descriptions were much more clear and easy to understand. I was pleased to see in this session that students were less unsure about how to tackle the problem; many students started following a strategy immediately after given the task. Many students were sure that they achieved the correct solution when they did because they had stronger organization strategies. I feel that perhaps changing the groups might have benefitted the students, because I think that many pairs had one student who lead the group while the other student just helped. I think refreshing the groups might have been a good idea. When I implement this sequence of tasks next time, I would change the pairs, and also use a rubric in order to grade the end result of the student explanations.

Lesson Study Final Reflection

During the course of the lesson study, I gained new perspectives and understandings regarding the mathematics involved, my students' thinking and reasoning, and I was able to recognize a deepening of understanding of my students about how to justify and explain themselves, and of myself in analyzing the student work and enkindling students' thinking. Regarding the mathematics, I learned two additional ways to prove that there cannot be more towers made as a result of our analysis of different approaches to the towers problems. I never would have recognized recursive patterns and the idea of holding a constant in the towers problems had they not been addressed during our meeting sessions. I also learned that there are countless representations and methods for finding answers to one problem, and they are all valuable, especially to make connections in order to deepen understanding. Making concrete connections from tangible objects to actual numbers and new formulas can happen from the towers task, and I think that it is important for students to try to make those connections in order to encourage a mindset in which the students always look for the connection.

Regarding my students' reasoning and mathematical thinking, I learned several things. The first thing I learned came from the moment when they had to explain their thinking and justify their answers on paper.

Many students, though they were able to come up with an answer to the towers tasks quickly and knew there could be no more combinations, had a very difficult time justifying their solution to prove that there were no more combinations. I felt that it was a fairly difficult task for them to justify, but I still thought that they wouldbe able to at least explain their method for finding the answer to the initial towers task. However, in the first task that was implemented, many students had trouble explaining how they organized their towers and how they created more. It was interesting to me to see that they couldn't think about how they began the task effectively enough to describe their thinking on paper. By the time the third task was implemented, however, I felt that the students' explanations became much stronger regarding the strategies that they used as well as the way that they described their methods and organization. Some of the students were even able to fully convince me that they were correct through their explanation and diagrams. I saw a major improvement in the thinking and written justifications of my students.

A few things emerged from implementing the three cycles of tasks. What stands out most is the deepening of my understanding between two problems: the fact that there is a connection between the pizza problem and the towers problem. I have done the pizza problem numerous times before, but I never realized that it can be connected to a problem such as the towers task, because I think of the pizza problem as an "order doesn't matter" problem, which would be classified as combinations. Thinking about the towers problem, I perceive it as an "order matters" problem, which would be permutations. I'm not so much of a formula person, but thinking of the problem as permutations or combinations helps me realize what my list of solutions should look like. With the visual of the 1's and 0's seen in the video about Brandon, I can see how they are related when each pizza topping gets "a level," so to speak. My understanding of combinations was deepened just by using the Unifix cubes. To be able to put the different choices on top of an unfinished tower solidifies the necessity for multiplying by, for example, 2, when there are two colors to choose from. I feel that a good extension for the entire activity with the towers would be to show students how to create a tree diagram and use the Fundamental Counting Principle as a short cut to find the solution. I would like to see how students can generalize the idea to other problems that can be solved in a similar manner.

Connie

Topics in MathEducation:

Lesson Study onReasoning

15:254:599, Section 81

Central Region

Connie

Sayreville

December 4, 2010

Cycle I- Task 1

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

6th Grade Problem Solving Class

40 minute class length (task done over 2 days)

11:30am -12:10pm

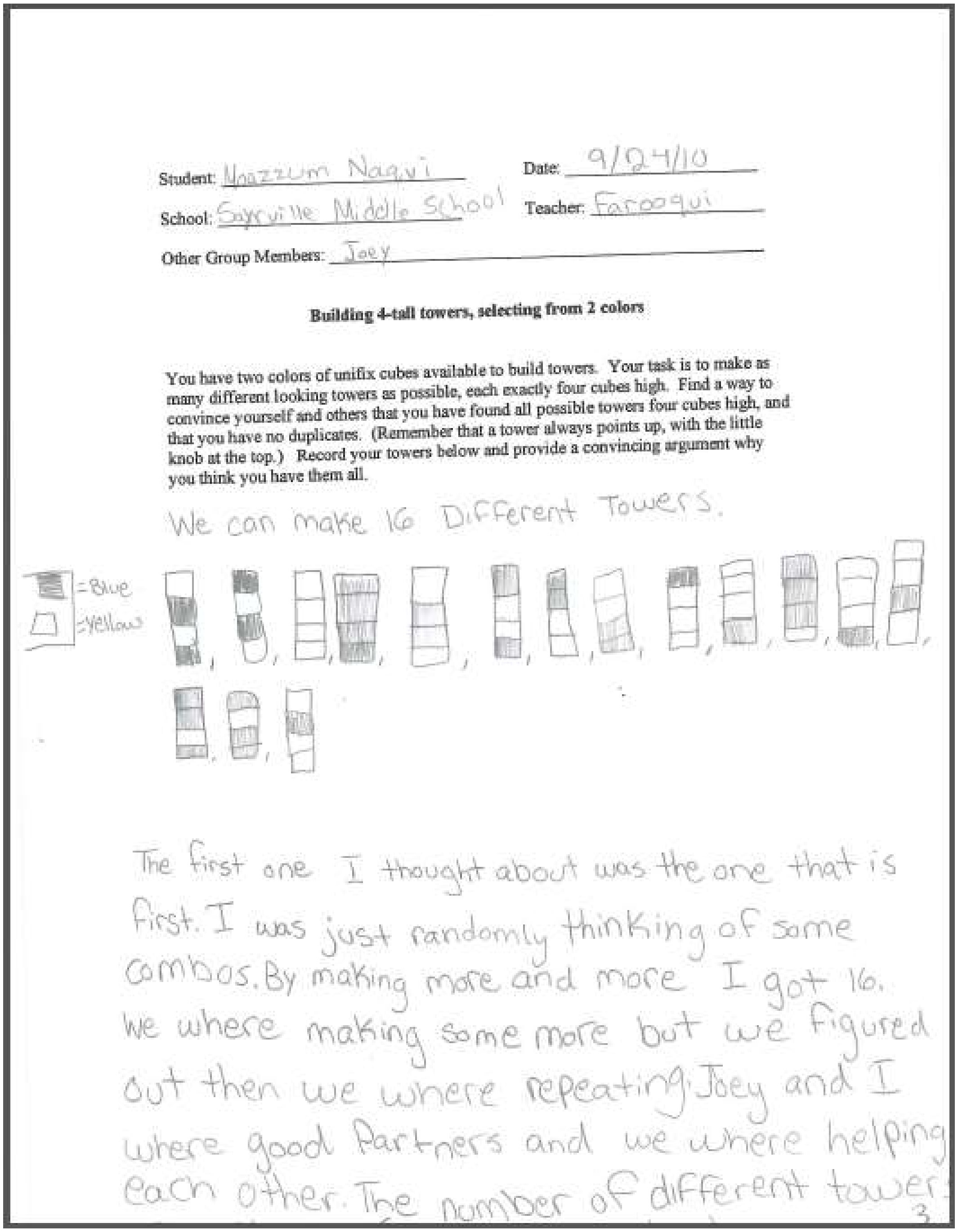
Regular Ed student

23 students

(Mixed ability class-students not tracked in 6th grade)

Moazzum

I found Moazzum's work to be very interesting. Even after watching the video clips, I still had my doubts that any students would utilize the idea of "opposites" when they attempted to solve this problem- solving task. When reading Moazzum's explanation, he does not even appear to recognize that there was a method to the way he was building his towers. Even when I went over to speak to Moazzum and his partner, Joey, they told me they were "randomly" making new towers. It is interesting to see that they did in fact have a method in the order that they were building towers, which is evident in Moazzum's drawing of the towers. They did come up with the correct answer of 16 towers after I had gone over to them and questioned some of their towers and their methodology.



Student: Moazzum Naqvi Date: 9/24/10

School: Sayreville Middle School Teacher: Farooqui

Other Group Members: Joey

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

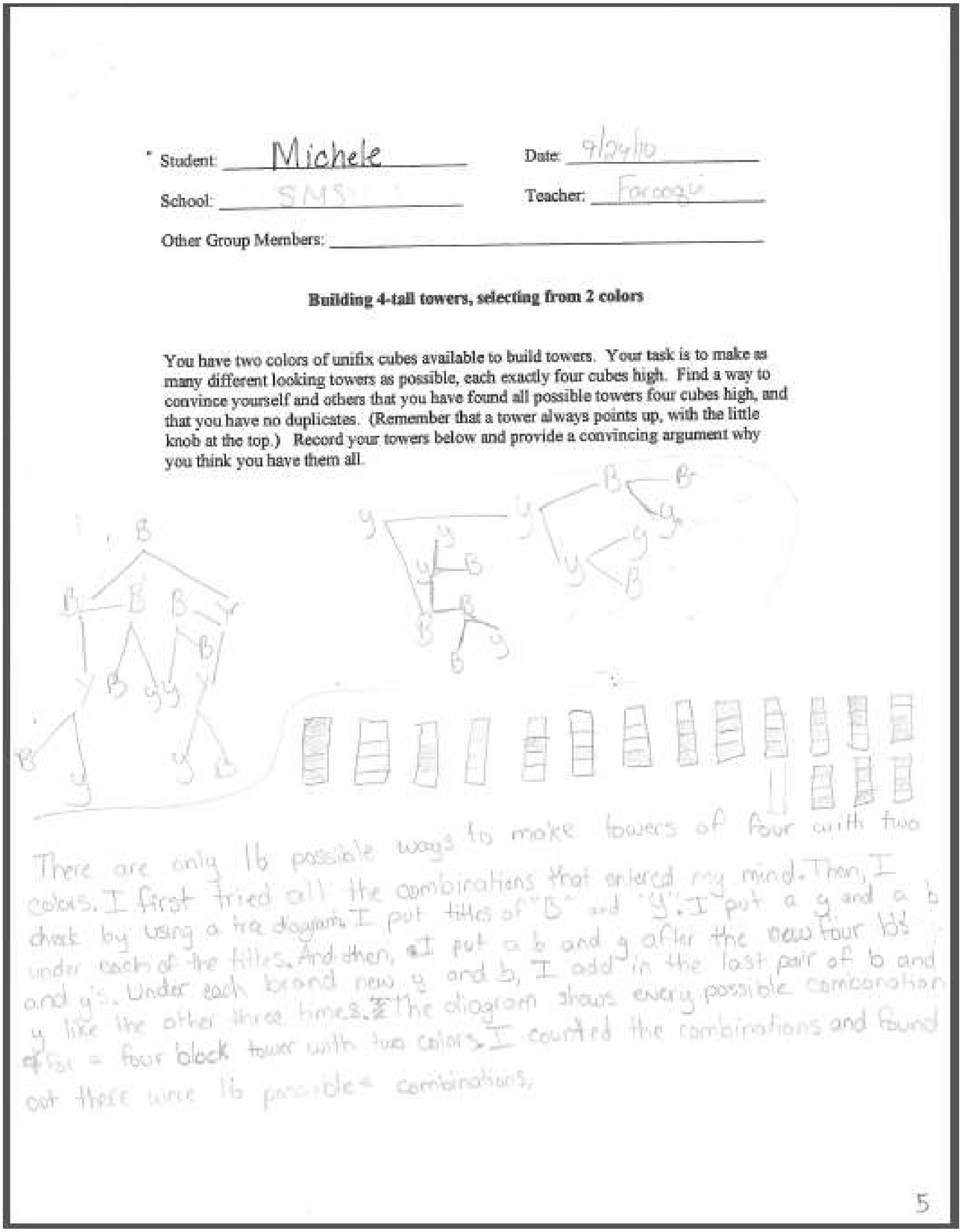
We can make 16 Different Towers.

The first one I thought about was the one that is first. I was just randomly thinking of some combos. By making more and more I got 16. We where making some more but we figured out then we where repeating. Joey and I where good Partners and we where helping each other. The number of different towers

Figure C.5.1 Moazzum's Cycle 1 Work

Michele

Michele's work and explanation surprised me for this problem. I had not taught combinations, tree diagrams, or the strategy of making an organized list yet in problem solving. When I walked over to Michele's group, I was surprised to see that even before they started building towers, Michele was having her group draw out a tree diagram. I had asked Michele why she chose to draw out the tree diagram and she had said because she wanted to see all the possibilities out on paper first. She and her other partners started building the towers and then crossing off the ones they had built on the tree diagram in their notebook, to ensure they did not duplicate one of their towers when they were building. This group was the only group that chose to utilize a tree diagram in solving this problem, so I found it to be very surprising and pleasing to see a different idea!



Student: Michele Date: 9/24/10
School: SMS Teacher: Faroogui
Other Group Members: ____

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

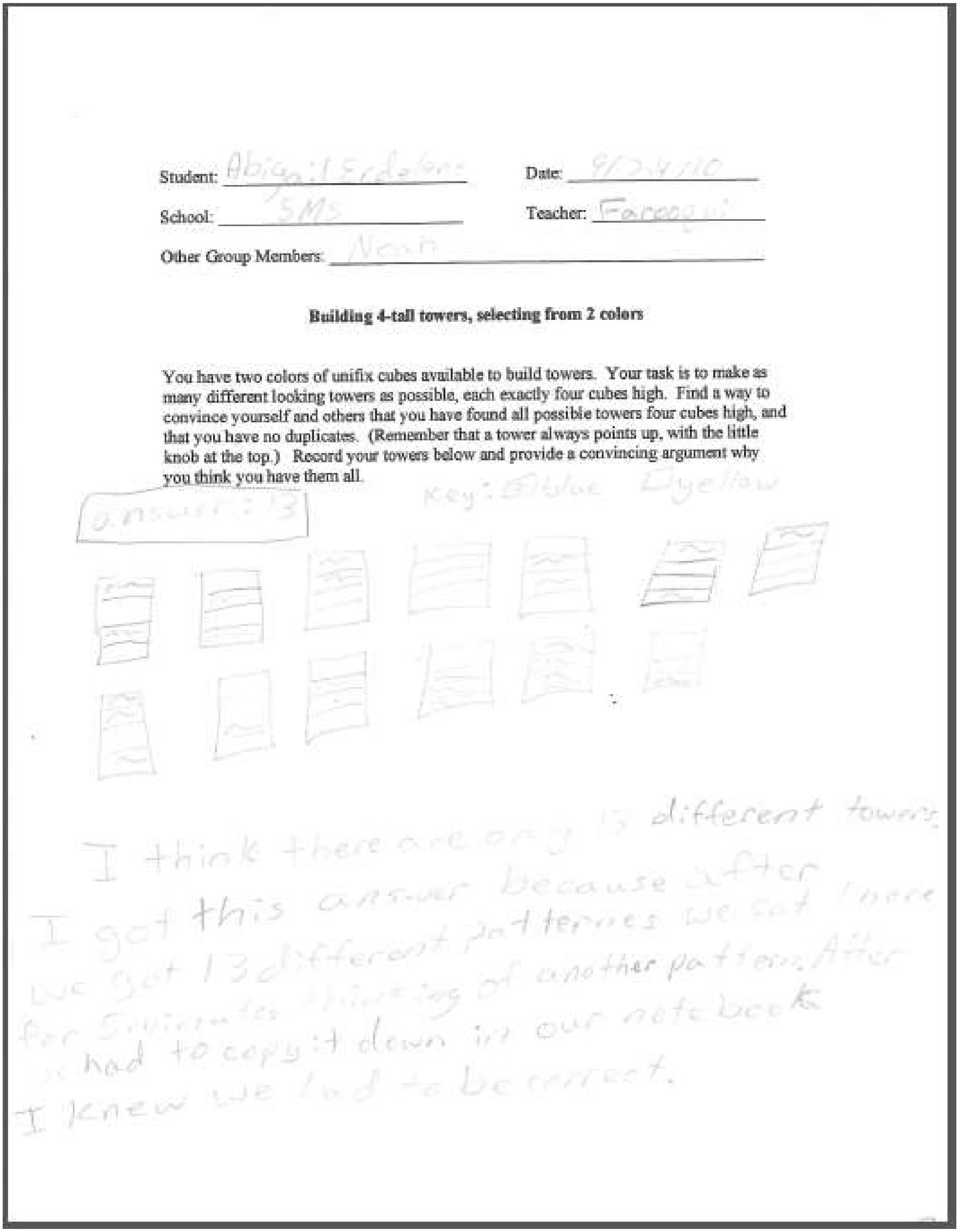
There are only 16 possible ways to make towers of four with two
colors. I first tried all the combinations that entered my mind. Then, I
check by using a tree diagram. I put titles of "B" and "Y". I put a y and a b
under each of the titles. And then, I put a b and y after the new four b's
and y's. Under each brand new y and b, I add in the last pair of b and
y like the other three times. The diagram shows every possible combination
of four block tower with two colors. I counted the combinations and found
out there were 16 possible combinations.

5

Figure C.5.2 Michele's Cycle 1 Work

Abigail

Abigail's work and explanation concerned me for this problem. Most of the groups who started solving this problem utilized some sort of method for organizing their towers, even if it was simply the idea of "opposites". However, when I walked over to Abigail and her partner, they truly were just coming up with random towers. They were building towers that they did not already build, and then they would sit there and think of more. When they came to point where they started making duplicates, they kept trying. After some time, they seemed to give up and were determined that they had the correct answer of 13 towers. They, like many groups, had difficulty explaining why and how they knew that they had the correct answer. She just said that they knew they had them all because every time they tried to build another one, they were coming up with duplicates. It concerned me that this group did not come up with any way of organization when building the towers.



Student: Abigail [illegible] Date: 9/24/10

School: SMS Teacher: Farooqui

Other Group Members: Noah

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

answer: 13

Key: blue yellow

I think there are only 13 different towers. I got this answer because after we got 13 different patternes we sat there for 5 minutes thinking of another pattern. After we had to copy it down in our notebook I knew we had to be correct.

Figure C.5.3 Abigail's Cycle 1 Work

Cycle I Intervention Implementation:

I found it extremely interesting to see how my students approached this problem. I, like some of my colleagues, did not predict that the students would utilize the idea of "opposites" when solving this problem. I would say about 80% of my students used some form of the idea of opposites. I was really surprised! Other students approached the problem with pure randomness. In addition, I did have one student surprise me with using a tree diagram to help solve this problem and one student surprise me with using a "staircase" method. I liked having my students use manipulatives with this problem because they really seemed to enjoy it. I would have liked to better help my students with the proper questioning techniques that I had learned later on in this course.

Cycle II-Task 2

The Pizza Problem

Capri Pizza has asked you to help design a form to keep track of certain pizza choices. They offer a standard "plain" pizza with cheese and tomato sauce. A customer can then select from the following toppings: peppers, sausage, mushrooms, and pepperoni. How many choices for pizza does a customer have? List all possible choices. Find a way to convince each other that you have accounted for all possibilities.

6th Grade Problem Solving Class

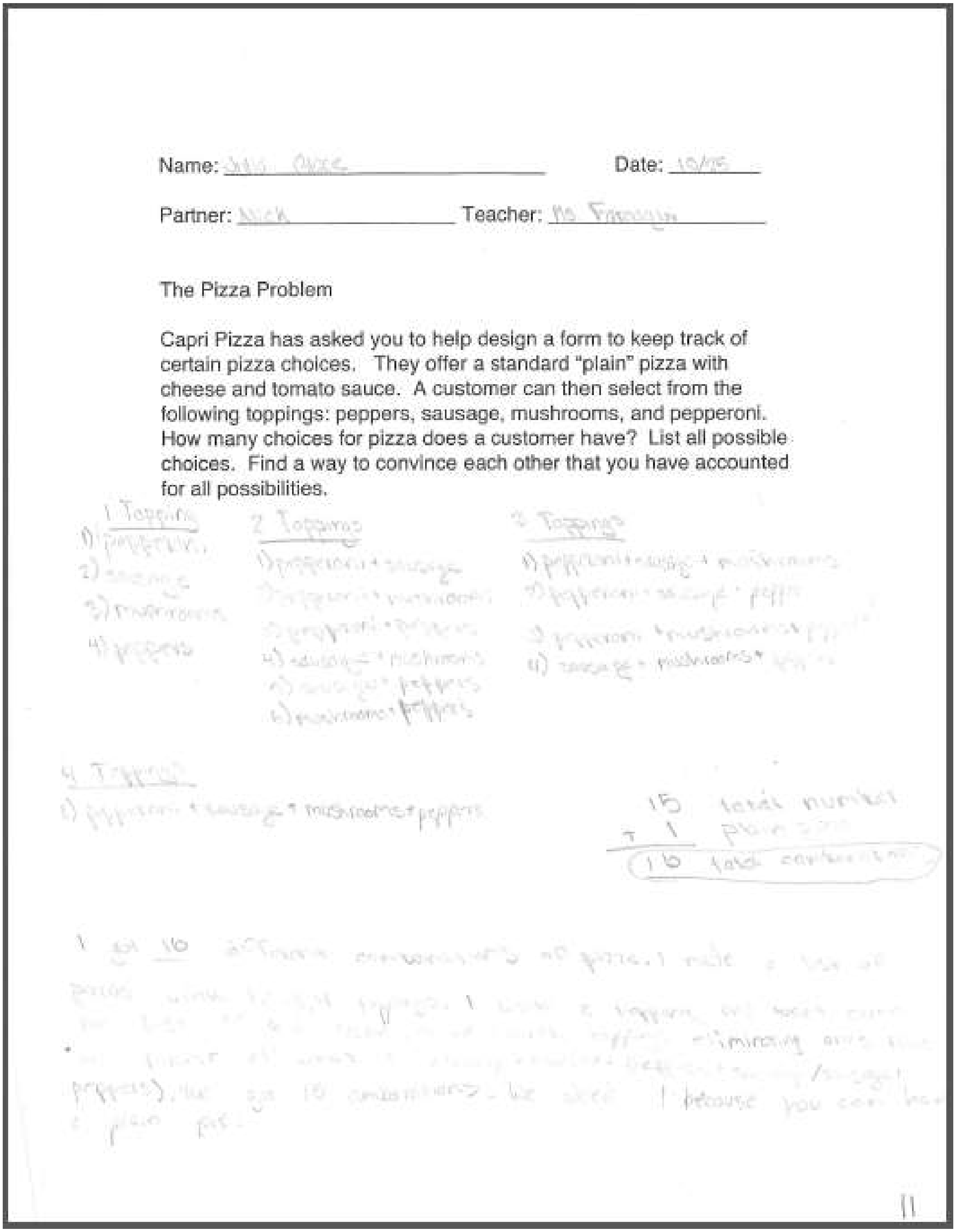
40 minute class length (task done over 2 days)

11:30arn-12:lOpm Regular Ed student 23 students

(Mixed ability class-students not tracked in 6th grade)

Julia

Julia's solution impressed me very much! After reading the problem, she started labeling columns on her paper with I-topping, 2-toppings, 3-toppings, and 4-toppings. In the column with 1 topping, she simply listed all the options of pizzas for 1 topping. Under the 2-topping column, she went in order from her I-topping list, and labeled all the possible combinations (pepperoni & sausage, pepperoni & mushrooms, pepperoni & peppers, etc). Under the 3-topping column she also went in order and labeled the possible combinations (pepperoni & sausage & mushrooms, pepperoni & sausage & peppers, pepperoni & mushrooms & peppers, etc). Under the 4-topping column, there is only one possible combination of toppings. She impressed me because of how well-organized she was with labeling her columns and going in order under each column as to make sure she did not miss any combination of toppings. She solved this the exact way I did, and she is in 6th grade! I had not taught the strategy of making an organized list, and she already knew that she had to be organized to help ensure she did not make a mistake. She even remembered the plain pizza option to get her answer of 16 possibilities of pizzas.



Name: ______________ Date: ______

Partner: ______________ Teacher: ______________

The Pizza Problem

Capri Pizza has asked you to help design a form to keep track of certain pizza choices. They offer a standard "plain" pizza with cheese and tomato sauce. A customer can then select from the following toppings: peppers, sausage, mushrooms, and pepperoni. How many choices for pizza does a customer have? List all possible choices. Find a way to convince each other that you have accounted for all possibilities.

1 Topping

2 Toppings

3 Toppings

4 Toppings

1) pepperoni + sausage + mushrooms + peppers

15 total number

+ 1 plain

16 total combinations

Figure C.5.4 Julia's Cycle 2 Work

Michele

Michele's work surprised me in the way she approached the problem. Michele had work that impressed me in Cycle I's tasks. Similarly to Julia, she organized the toppings under 4 columns.

However, the way she organized the toppings was much different, which was very surprising to me.

Michele organized the toppings by starting with all possibilities of pizzas with peppers (1, 2, or 3 toppings). She then labeled all the pizzas with mushrooms, making sure not to use any combinations that contain peppers, which she had already exhausted. She then did the same with sausage and pepperoni. She remembered to account for the pizzas with everything and with no toppings. I never would have thought to organize the toppings in this way! It was really interesting to see how she decided to organize her answer, and how she was able to explain her work clearly. I would only suggest to her to include a key to differentiate between peppers and pepperoni.

Name: Michele Philip Date: 10/25/10

Partner: Mohsin Teacher: Mrs. Farooqui

The Pizza Problem

Capri Pizza has asked you to help design a form to keep track of certain pizza choices. They offer a standard "plain" pizza with cheese and tomato sauce. A customer can then select from the following toppings: peppers, sausage, mushrooms, and pepperoni. How many choices for pizza does a customer have? List all possible choices. Find a way to convince each other that you have accounted for all possibilities.

pizza with nothing
pizza with everything > 2 possibilities

P	M	S	PO
P,S -3	M -10	S -14	PO -16
P,M -4	M,S -11	~~S,PO~~	
P,PO -5	M,PO -12	S,PO -15	
P,M,S -6	M,S,PO -13		
P,M,PO -7			
P,S,PO -8			
P -9			

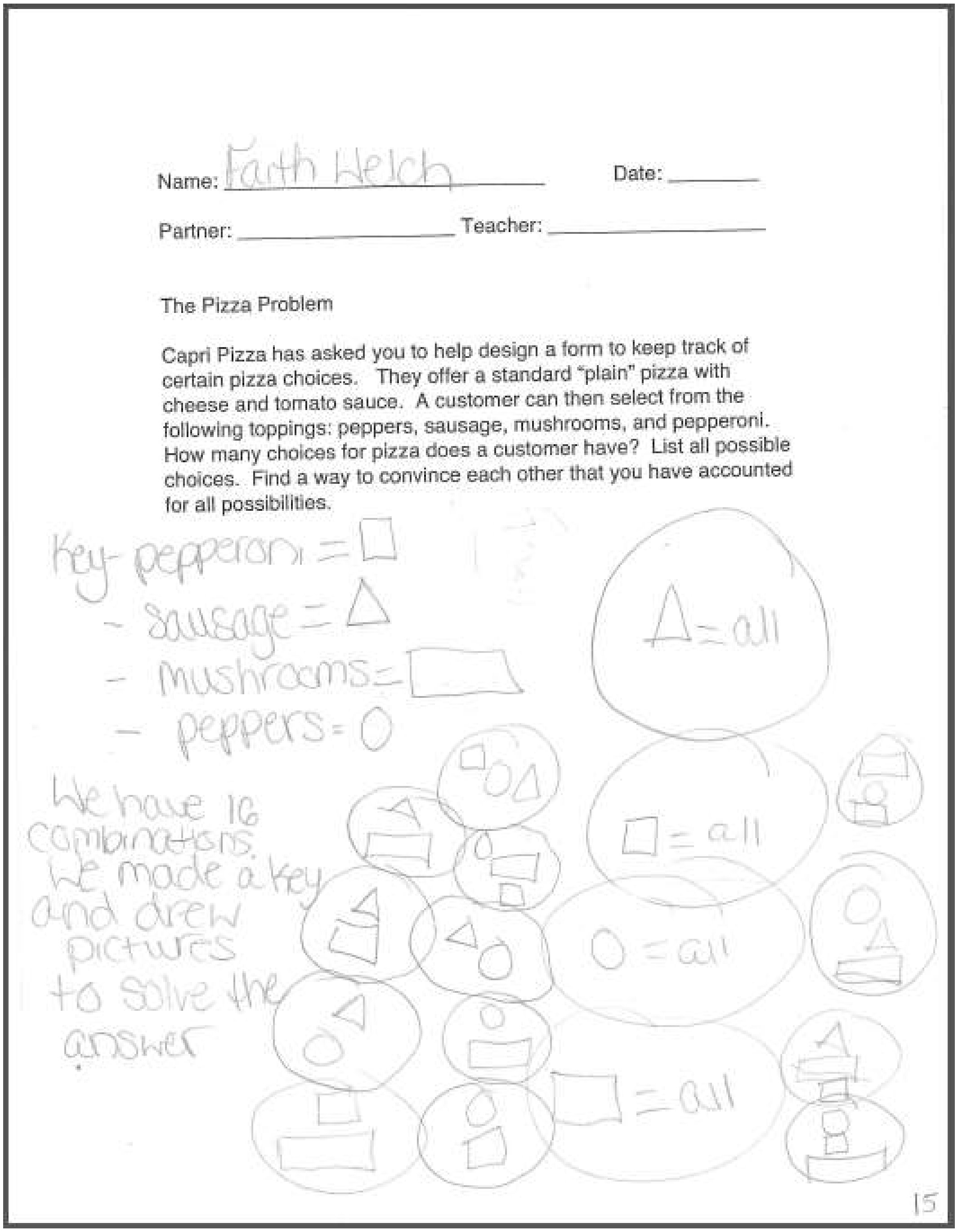
There are sixten possible combinations of pizza. First, we listed all the toppings with pepper, making sure not to repeat any combinations. We did the same with the the toppings with the mushrooms but made sure not to use pepper. We did the same for sausag, avoiding using peppers or mushrooms. for pepperoni, we could not combine it with any other topping because all the possible combinations using pepperoni were alredy mentioned. We also added the pie with nothing on it, and the pie with everything on it.

13

Figure C.5.5 Michele's Cycle 2 Work

Faith

I was concerned with Faith's work on this problem. At first, when I looked at her work I liked how she made a key with shapes for each topping. It appeared she even had the correct answer. However, after analyzing her solution and her work, I realized that there was a lack of organization and her work was incorrect. She did not have the larger round circles (representing pizzas) with their toppings in respective organized columns according to the amount of toppings like I had seen many of my students do. She also duplicated one of her pizzas three times (the one with a square, rectangle, and circle for toppings) and forgot the one plain pizza and the one with everything on it. She counted 16 pizzas, but on her paper there are actually 17 drawn. Her explanation is not complete because she did not explain her reasoning on why she drew those particular pizzas and in which order she decided to draw the pizzas. It showed me that she still needed some help and experience with these types of problems.


Name: Faith Welch Date: ________

Partner: ________________ Teacher: ________________

The Pizza Problem

Capri Pizza has asked you to help design a form to keep track of certain pizza choices. They offer a standard "plain" pizza with cheese and tomato sauce. A customer can then select from the following toppings: peppers, sausage, mushrooms, and pepperoni. How many choices for pizza does a customer have? List all possible choices. Find a way to convince each other that you have accounted for all possibilities.

Key- pepperoni = □
- sausage = △
- mushrooms = ▭
- peppers = ○

We have 16 combinations. We made a key and drew pictures to solve the answer

△ = all

□ = all

○ = all

▭ = all

15

Figure C.5.6 Faith's Cycle 2 Work

Cycle II Intervention Implementation:

I selected students' work from the Pizza Problem because I really liked the variety of approaches that I was able to see on their papers. The other task from this Cycle (Towers 5 high selecting from 2 colors) was a little more difficult for my students and I did not have a large variety of work that I could have selected from. I noticed that many students were getting frustrated with the towers 5-tall problem because their lack of organization led them to repeatedly create duplicates. Only a few pairs of students said they were building towers completely randomly, which was much fewer than during Cycle I. Many of my students stated that they liked solving the Pizza Problem. Many more students attained the right answer and showed a better understanding of the Pizza Problem compared to problems in Cycle I. I felt that going over samples of student work from Cycle I helped students for the tasks in Cycle II. I was able to see their growth from their work and also from their explanations when I walked around the room. I also found that my questioning techniques were better because I held back from helping the students too much. I allowed them to do more of the talking, which I found that they really liked!

Cycle III-Task 1

Towers 3-tall, Selecting from 3 colors

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

Cycle III-Task2

Ankur's Challenge-Towers 4 tall, Selecting from 3 colors extension

Find all possible towers that are four cubes tall, selecting from cubes available in three different colors so that the resulting towers have at least one of each color. In the space below, show your solution and provide a convincing argument that you have found them all.

6th Grade Problem Solving Class

40 minute class length (task done over 2 days)

(Mixed ability class-students not tracked in 6th grade)

Michele

Michele's work really impressed me on Ankur's challenge (Cycle III-Task 2 extension)! She was the only student during Cycle I to use a tree diagram to solve the problem. She felt very comfortable using the tree diagram for the towers problem from Cycle 2 as well as Cycle 3's first task. Due to the specific criteria of having one of each color in the tower 4-tall the extension problem, I did not think she was going to use a tree diagram, but she did. When I walked over to her, she was able to explain how a tree diagram would work even for the specific criteria stated in the problem. She stated that she would draw a tree diagram for one color as the first cube in the tower, and cross off all the towers that did not have the requirement of having at least one of each color. She came up with 12 total possible towers with one specific color as the first cube, and multiplied it by 3 for the three colors. She came up with an answer of 36 possible towers. She achieved the right answer with wonderful reasoning that was shown in her explanation! Her approach was similar to Sally and Kate's approach during our time working on the problem together in class. I was thoroughly impressed!

Student: Michele Philip Date: November 12, 2010

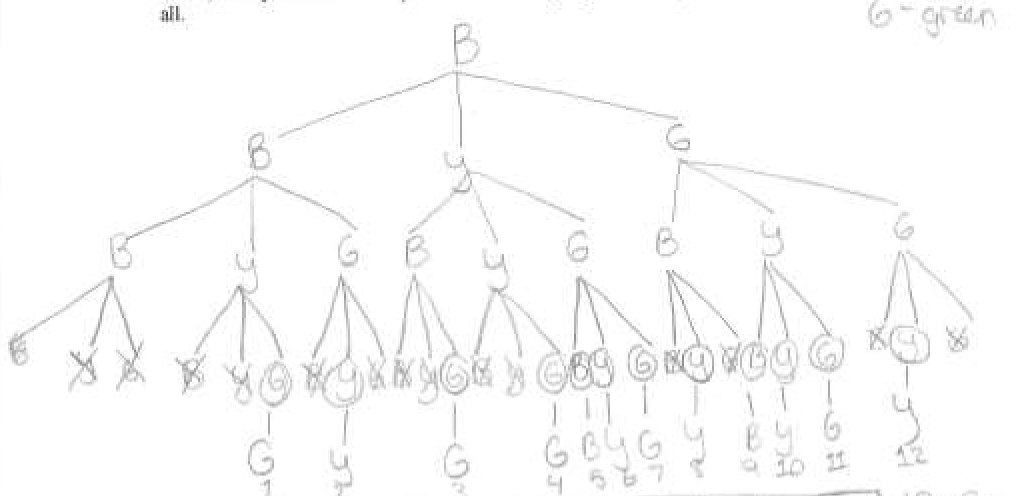
School: Sayreville Middle School Teacher: Miss Farooqui

Other Group Members: Mohsin, Allan

BUILDING TOWERS THREE COLORS EXTENSION

Find all possible towers that are four cubes tall, selecting from cubes available in three different colors so that the resulting towers have at least one of each color. In the space below, show your solution and provide a convincing argument that you have found them all.

B - blue
Y - yellow
G - green



12 x 3 = 36

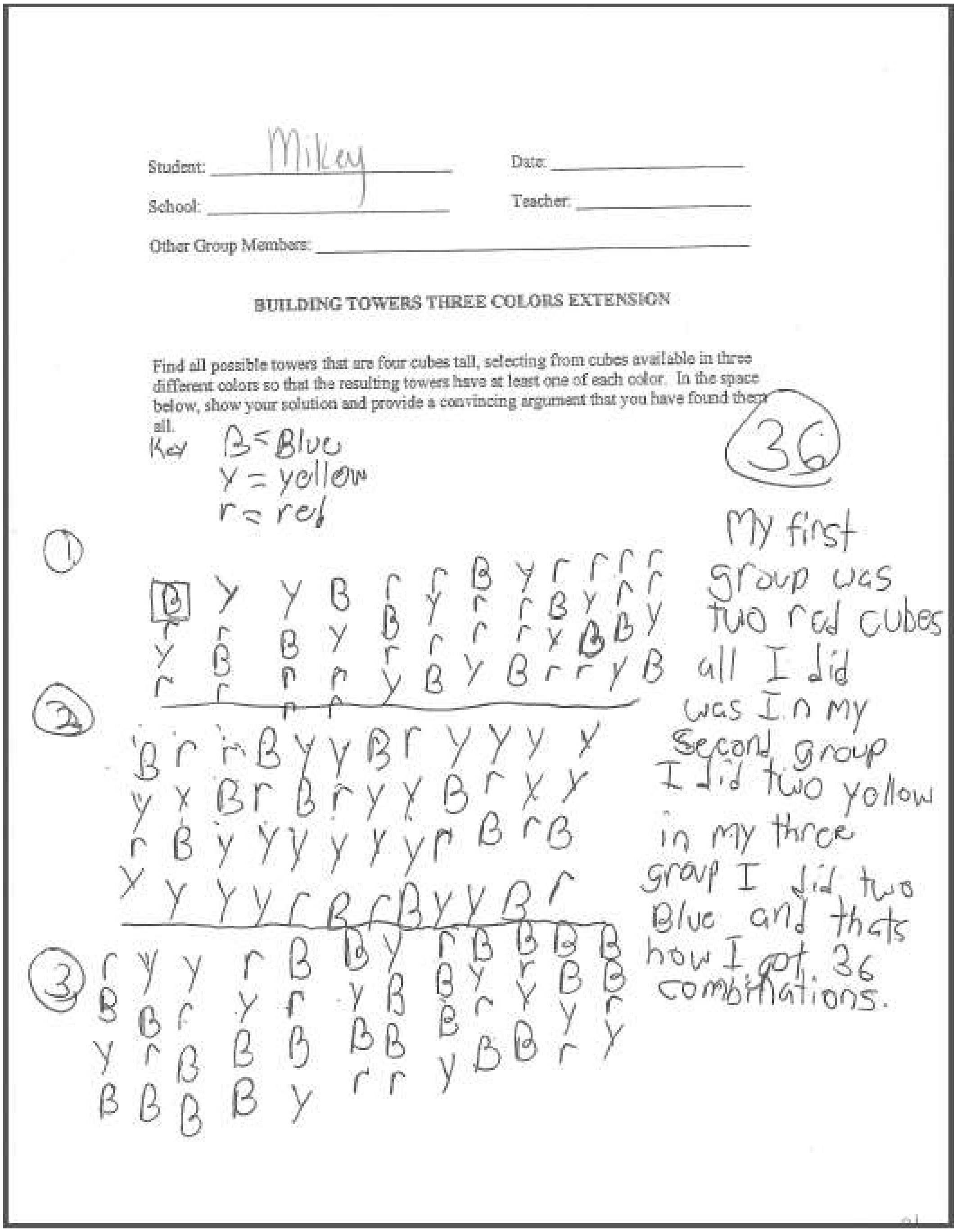
There are 36 possible towers with atleast one of each of the three colors in 4 blocks. First, I draw a tree diagram to show all possible combonations. Then, I crossed out each possible tower that didn't have atleast one of each color, and circled the towers that did. I counted all the circled towers and I multiplied by three for the other colors, getting 36 towers.

19

Figure C.5.7 Michele's Cycle 3 Work

Mitchy

Mitchy's work was definitely the one that I knew I would select for surprising me! Mitchy is a student who I had to separate from another student due to his poor behavior. I found that once I separated him from his partner, he was able to focus better. Many students did not have an opportunity to attempt to solve Ankurs's challenge (Cycle III-Task 2). Therefore I was impressed that he tried it and was one of the few who got the correct answer and by himself! I had the opportunity to speak to him when he was working on this task. I asked him how he was building the towers, and he said he knew that there would have to be 2 of one color if you have to select from 3 colors and the towers have to be 4 cubes tall. He built all the towers with 2 reds, then 2 yellows, and then 2 blues separating them into 3 distinct groups. He kept the color that was going to appear twice in the tower in the same location and switched the two positions of the other 2 colors to create 2 towers. This is similar to what Romina did in her proof in the video clip we watched. He controlled for variables, holding the location of the color, which appeared twice constant and switching the other two colors. He did not write out a thorough explanation, but he was able to orally state exactly how he came to build his towers and the reasoning behind it. I was very happily surprised at his work!



Student: Mikey Date:

School: Teacher:

Other Group Members:

BUILDING TOWERS THREE COLORS EXTENSION

Find all possible towers that are four cubes tall, selecting from cubes available in three different colors so that the resulting towers have at least one of each color. In the space below, show your solution and provide a convincing argument that you have found them all.

Key B = Blue
y = yellow
r = red

36

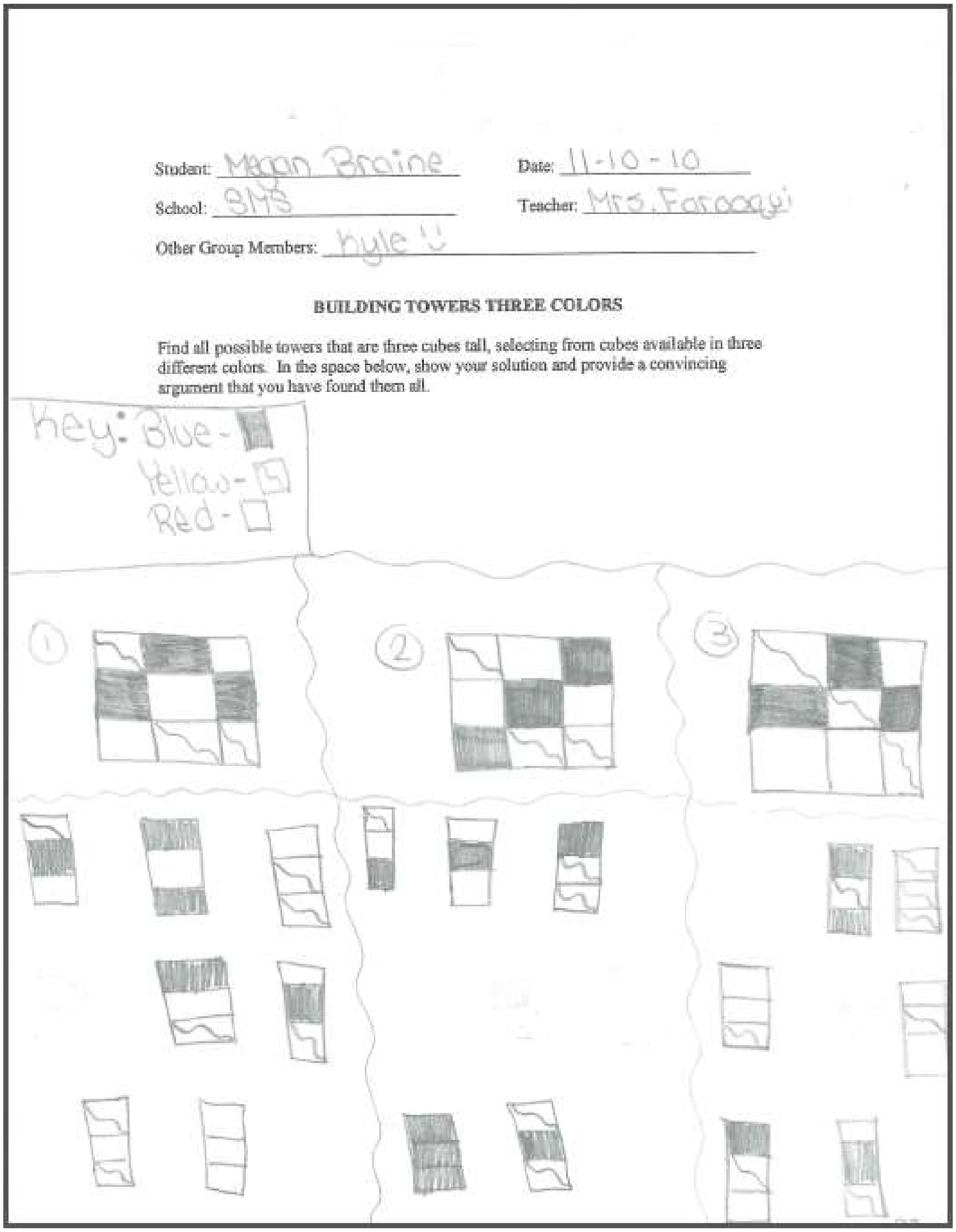
My first group was two red cubes all I did was I n my second group I did two yollow in my three group I did two Blue and thats how I got 36 combinations.

Figure C.5.8 Mitchy's Cycle 3 Work

Megan

Megan is a good student in math and problem solving because she works hard at understanding the concepts. Her work for Task 1 of Cycle III (Towers 3 tall, Selecting from 3 colors), concerned me

because I believed that she would have had a more strongly organized method for building her towers. I did like how she came up with a clear key and that she decided to split her towers into groups. I cannot really see much of a pattern for her grouping besides the first three groups of towers, which utilize a "staircase" method. In her explanation on the back of her paper, she explains that she and her partner split the "staircases" into the separate towers underneath each group. However, I do not see consistency in how she placed specific towers in the designated groups. There were duplicates and ones left out of her solution. It seemed like she was very confused still when I went over to her and her partner. She did not seem to get how she her partner was trying to solve the problem, so I know she was trying to do it on her own. I hoped she would have demonstrated more growth in solving problems such as these, but I was able to see growth at a point after the task when the students learned the making an organized list strategy in my class.


Student: Megan Braine Date: 11-10-10

School: SMS Teacher: Mrs. Farooqui

Other Group Members: Kyle :)

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

Our anwser came out to be 18.
We used the stair case methid with the cubes.
Then split the staircase into indiviqual parts, each had 3 different colors. Thats how our anwser came to be 18!

Figure C.5.9 Megan's Cycle 3 Work

Cycle III Intervention Implementation:

During the implementation of the tasks of Cycle III, I was very pleased to see that most of my students felt more comfortable with these types of problems. I was very surprised to see that many students successfully completed Task 1 of this Cycle! I was so incredibly impressed by the two students who were successfully able to complete Ankur's challenge! We, as teachers, did not come to the correct answer right away, so it was amazing to see Michele and Mitchy have a clear and definitive method in solving the problem. I really liked seeing that my students as a whole had improved their reasoning and that they had better explanations of their ideas. My questioning continued to improve for Cycle III, which I could see because I allowed my students to talk more. It was evident that they were able to see how organization is critical in attaining an answer that was correct.

Class Reflection

After the first day of class, I was very interested in finding out how a class based on such specific task-based word problems would benefit my students and myself as an educator. My 6th grade classes this year are made up of a variety of mathematical abilities. In our middle school, students are not tracked, which can make it difficult for teacher. Open-ended activities, such as the tasks that we were asked to implement, allow the teachers in our 6th grade math classes an opportunity to see a wide array of ideas, methods, and solutions. I was excited to see where my students would go with these tasks.

My students had an opportunity to gain experience in improving their mathematical thinking and reasoning. I saw how my students had difficulty in organizing and explaining their work, especially in the beginning. Their growth in becoming aware of how important organization is in their answer, as well as how important it is to provide a clear explanation of their work, was evident by the end of the three cycles. They could see even more, that unless you can definitively prove that your method is completely exhaustive, there is not enough evidence that the answer is correct. My students loved working with manipulatives! I think it helped motivate them to try to get the correct answer. I was happy to see my students helping each other understand and learn the mathematics behind these problems. I witnessed many different ideas and methods on how to solve these tasks, which was wonderful!

My idea of what a competent answer to a word problem has changed over this course. I learned how important a clearly developed explanation is in supporting written work. I learned how essential it is to ask the proper questions and provide a supportive environment for students to allow their thoughts to flow. Through my experiences this past semester, it was reinforced that it not necessary to push students into doing work a certain way. My role as a facilitator in the classroom was strengthened from what I have learned this semester.

These particular series of cycles and tasks actually have directly improved my students' understanding of the strategy of making an organized list. I taught the strategy of making an organized list one week after the last task was implemented. My students repeatedly told me that they found this strategy very easy, and even related problems that we were doing to some of the tasks they completed during the cycles! I know students' abilities change from year to year so it is hard to compare, but I can say that I feel implementing these tasks has significantly improved my students mathematical learning, reasoning, and communication skills.

Angela

Final Project- Topics in Math Education: Lesson Study on Reasoning

Angela

Sayreville

December 4, 2010

Cycle I – Task

Student______________________________Date:______________________

School: ____________________________Teacher: ---------_

Other Group Members: _________________

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

About the Class

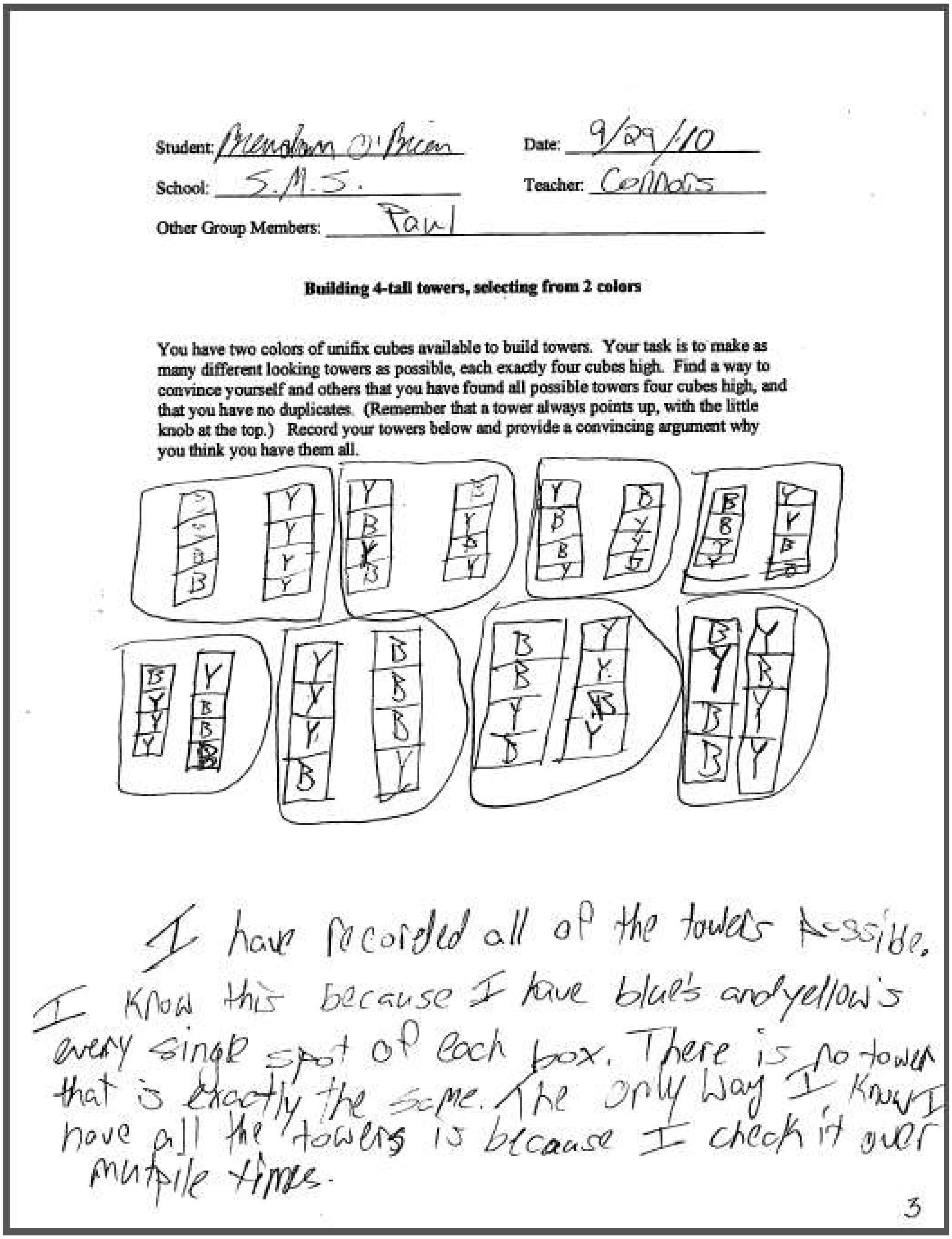
This task was conducted in a regular-level, 7th grade classroom. The students were placed in regular- level math and problem solving according to their results on the NJASK test from 6th grade. All of these students tested at a proficient or partially-proficient level on the mathematics portion of the NJASK test. I conducted this activity over the course of 2 days, since our problem solving class is only 40-minutes. My students had approximately 60-65 minutes total over the two days to complete the task. This class has 21 students in it.

Impressed, Surprised, or Concerned - Cycle I

Brendan's work impressed me. In his work he showed he grouped his towers by opposites. I had anticipated that not many of my students would use the concept of "opposites." I really liked how he circled the opposites to clearly show the reader what this concept means. Though I am not sure where he had seen this concept before, this impressed me because before seeing the videos in class of the students completing the task in this way, I would not have done so in the same manner myself.

Erika's work surprised me. She knew before completing the task that there were going to be 16 total towers. When I asked her to explain how she knew this, she could not provide any proof. In the end of her argument, she states that "if there is only 2 colors and 4 cubes high made you can only use 16." Though grammatically incorrect, this was the first inkling from Erika that she knew how to connect the concept of combinations with the tower problems.

Brandon's work concerned me. Him and his partner were only able to find 12 towers. I felt that this was because they really did not have a systematic way of organizing their towers to create new ones. I felt that if they were to use a concept like "opposites" or "staircase" then they could have seen that there are more possible towers than what they had found.



Student: Brendan O'Brien Date: 9/29/10

School: S.M.S. Teacher: Connors

Other Group Members: Paul

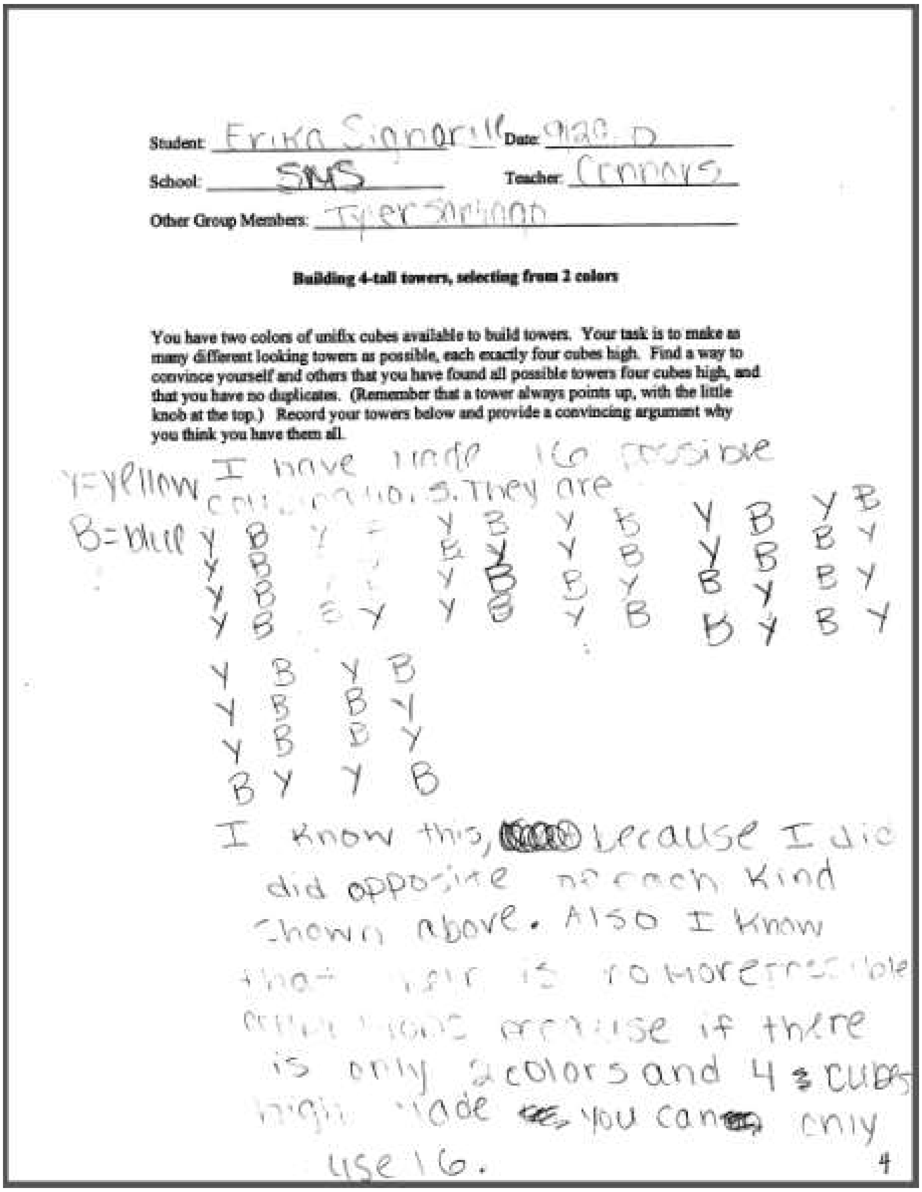
Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

I have recorded all of the towers possible. I know this because I have blue's and yellow's every single spot of each box. There is no tower that is exactly the same. The only way I know I have all the towers is because I check it over mutpile times.

3

Figure C.6.1 Brendan's Cycle 1 Work



Student: Erika Signorill Date: 9/28 D

School: SMS Teacher: Connors

Other Group Members: Tyler Sarhand

Building 4-tall towers, selecting from 2 colors

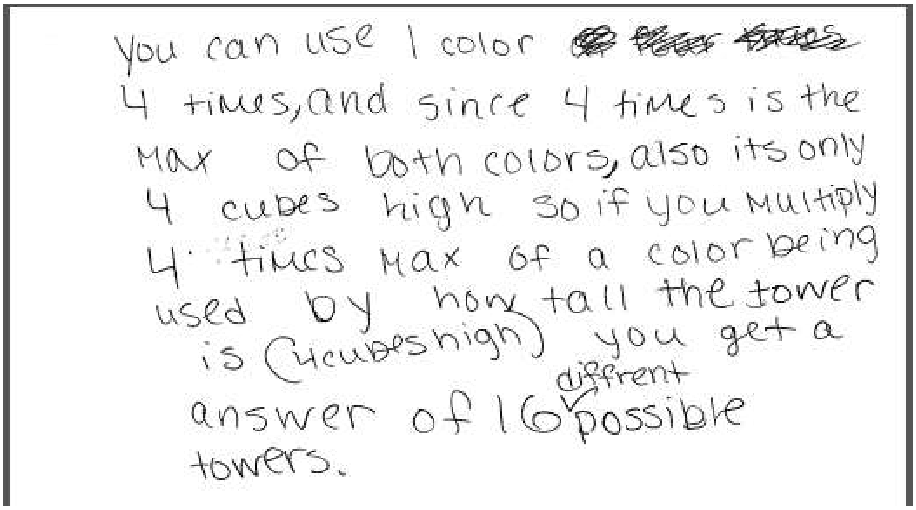
You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

Y=yellow I have made 16 possible combinations. They are

B=blue

I know this, because I did opposite of each kind shown above. Also I know that there is no more possible combinations because if there is only 2 colors and 4 cubes high made you can only use 16.

4



you can use 1 color
4 times, and since 4 times is the
max of both colors, also its only
4 cubes high so if you multiply
4 times max of a color being
used by how tall the tower
is (4 cubes high) you get a
answer of 16 diffrent possible
towers.

Figure C.6.2 Erika's Cycle 1 Work

Student: Brandon Santos Date: 9/29/10

School: SMS Teacher: Conners

Other Group Members: Maria

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

There are only 12 combination. I can't make any more combination because when I was trying to make another combination it came out the same thing like the others. I could not think of do another combinations. Every time I make a combination there's always something that is the same.

1. b,b,b,b
2. y,b,b,y
3. b,y,y,b
4. y,b,y,b
5. b,y,b,y
6. y,y,b,y
7. y,y,b,b
8. b,b,b,y
9. y,y,b,b
10. b,b,y,y
11. y,y,y,y
12. b,b,y,b

5

Figure C.6.3 Brandon's Cycle 1 Work

Intervention Implementation - Cycle I

During this first cycle, I learned that some of my students were very confident in their strategies to solve this problem. I also learned that they needed help to prove their work. I believe that what students think is convincing is different that what us as adults find to be convincing. May of the ways they argued their case did not have any mathematical proof. A lot of their reasons had to do with, "I know there are 16 towers because I tried and couldn't find anymore." To me, I don't find this convincing unless you can tell me the method that you used to build the towers, ensuring that there was some systematic way of doing so that didn't allow for duplicates or missed towers.

During this first cycle, I had my students complete the task without the task paper; keeping notes in their notebook for the first class period while the task was displayed on my SmartBoard. On the second day, I distributed the papers for them to complete. I felt this was a little rushed, and revised this methods for the tasks to come.

Cycle II –Tasks

Student: __________ Date: _______

School: ____________________________ Teacher: ________

Other Group Members: ————————————————————————————

Building 5-tall towers, selecting from
2 colors.

You have two different colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly five cubes high. Find a way to convince yourself and others that you have found all possible towers five cubes high, and that you have no duplicates. Record your towers below and provide a convincing argument why you think you have them all.

The Pizza Problem

Capri Pizza has asked you to help design a form to keep track of certain pizza choices. They offer a standard "plain" pizza with cheese and tomato sauce. A customer can then select from the

following toppings: peppers, sausage, mushrooms, and pepperoni. How many choices for pizza does a customer have? List all possible choices. Find a way to convince each other that you have accounted for all possibilities.

About the Class

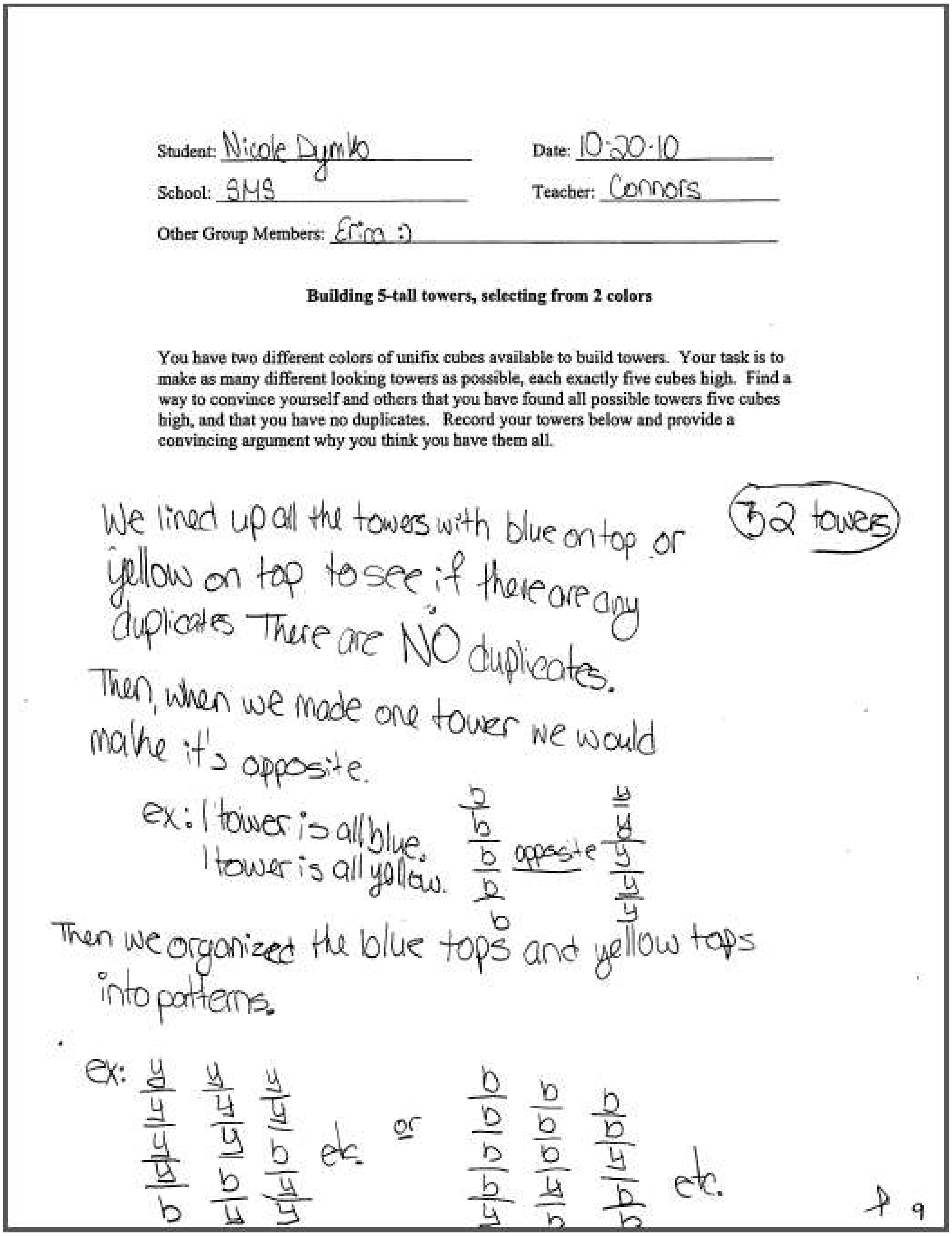
This task was conducted in a regular-level, 7th grade classroom. The students were placed in regular- level math and problem solving according to their results on the NJASK test from 6th grade. All of these students tested at a proficient or partially-proficient level on the mathematics portion of the NJASK test. I conducted this activity over the course of 2 days, since our problem solving class is only 40-minutes. My students had approximately 60-65 minutes total over the two days to complete the task. This class has 21 students in it.

Impressed,Surprised,orConcerned-CycleII

Nicole's work impressed me. Her and her partner's work was the first of its kind in terms of organizing the way they built the towers. She explains in her work that they grouped the towers by holding the cube on the top constant. From her work, you can also see that they used the staircase method. When they would build a new yellow top tower, they would immediately build its opposite blue top towers and place it in its appropriate place. I think this method allowed the girls to more clearly believe that they had gotten the correct amount of towers, and that there were no more possible.

Zaineb's work surprised me. Though she and her partner were able to get the correct answer of 32 towers, there is not much in terms of organization of the way the towers were built. I cannot see that they used any particular method, such as staircase or opposites, even though in her explanation she tells me "every time we found a combination we reversed it doing the opposite." The use of her strong words in her explanation surprised me. I wasn't convinced after reading her explanation that she had found the correct amount of towers, but it seems that she had definitely convinced herself. She had never used such strong words, like "I know for a fact."

Mayee's work concerned me. At first glance, you may believe that Mayee has gotten the correct answer to the pizza problem. Upon further examination, it seems that somehow Mayee and her partner knew by some method other that what they had used, that there had to be a total of 16 pizzas. In her work you can clearly see that she listed only 16 pizzas, but there are many duplicates. I think her prior knowledge knowing that there was a connection between this problem and the towers problems, whether conscious or not, was a hindrance to understanding what the problem was truly asking.



Student: Nicole Dymko Date: 10-20-10

School: SMS Teacher: Connors

Other Group Members: Erin :)

Building 5-tall towers, selecting from 2 colors

You have two different colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly five cubes high. Find a way to convince yourself and others that you have found all possible towers five cubes high, and that you have no duplicates. Record your towers below and provide a convincing argument why you think you have them all.

We lined up all the towers with blue on top or yellow on top to see if there are any duplicates There are NO duplicates.

32 towers

Then, when we made one tower we would make it's opposite.

ex: 1 tower is all blue. 1 tower is all yellow.

b b b b b opposite y y y y y

Then we organized the blue tops and yellow tops into patterns.

ex: y y y y b   y y y b y   y y y b y etc. or b b b b y   b b b y b   b b y b b etc.

9

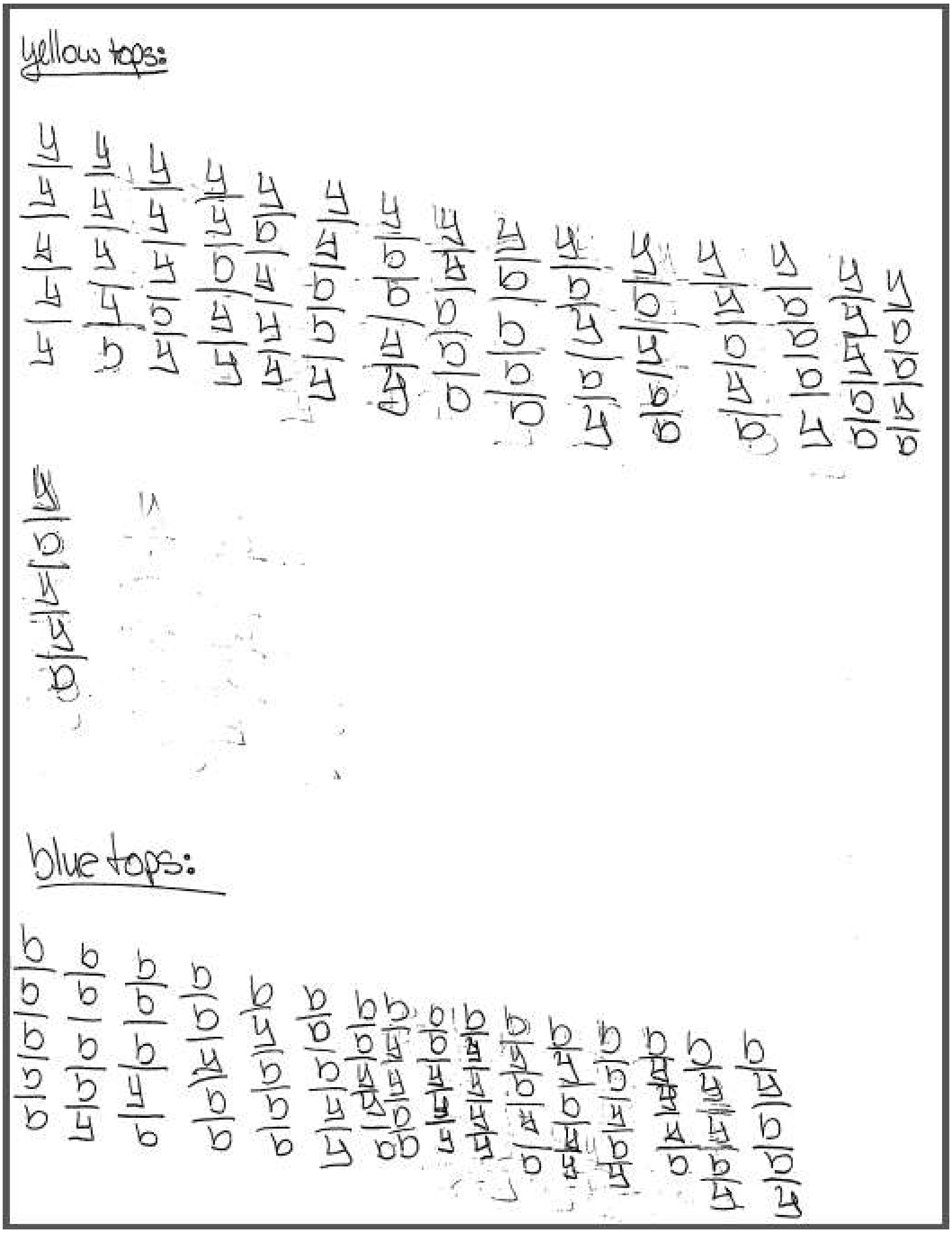


Figure C.6.4 Nicole's Cycle 2 Work

Student: Zaineb Nagvi Date: 10/20/10

School: SMS Teacher: Ms. Connors

Other Group Members: Andrea

Building 5-tall towers, selecting from 2 colors

You have two different colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly five cubes high. Find a way to convince yourself and others that you have found all possible towers five cubes high, and that you have no duplicates. Record your towers below and provide a convincing argument why you think you have them all.

Top to down

5 yellow
5 blue
4 yellow, 1 blue
1 yellow, 1 blue, 1 yellow, 1 blue, 1 yellow
1 blue, 1 yellow, 1 blue, 1 yellow, 1 blue
3 blue, 2 yellow
3 yellow, 2 blue
2 blue, 1 yellow, 1 blue
2 yellow, 1 blue, 2 yellow
2 yellow, 2 blue, 1 yellow
3 blue, 1 yellow, 1 blue
3 yellow, 1 blue, 1 yellow
2 blue, 1 yellow, 1 blue
1 blue, 1 yellow, 3 blue
1 blue, 4 yellow
1 yellow, 4 blue
1 yellow, 3 blue, 1 yellow
1 blue, 3 yellow, 1 blue
4 blue, 1 yellow
2 yellow, 1 blue, 1 yellow
2 yellow, 3 blue
2 blue, 3 yellow

2 blue, 1 yellow, 1 blue
1 yellow, 1 blue, 1 yellow, 2 blue
1 yellow, 2 blue, 2 yellow
1 blue, 2 yellow, 1 blue
1 blue, 1 yellow, 1 blue, 2 yellow
1 blue, 1 yellow, 2 blue, 1 yellow
1 yellow, 1 blue, 2 yellow, 1 blue
1 blue, 2 yellow, 1 blue, 1 yellow
1 yellow, 2 blue, 1 yellow, 1 blue
1 blue, 1 yellow, 1 blue, 2 yellow

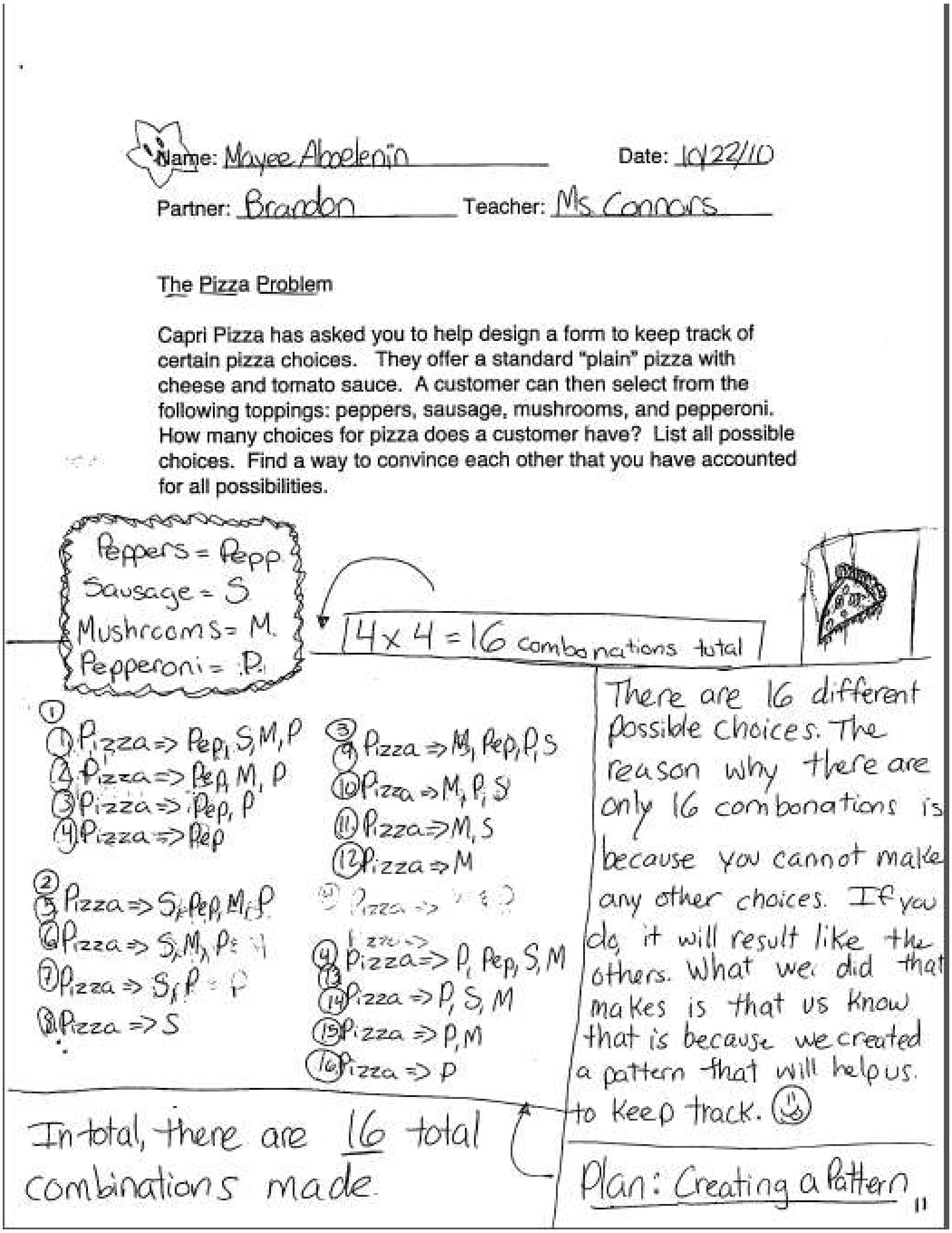
Total 32

10

Argument In Back

They're 32 combonations. I know for a fact that these are all the possible kinds. There are no more and no less than 32. Everytime we found a combonation we reversed it doing the opposite. Once we could not reverse anymore, we knew that we were in the end. Definately 32 is the number to describe how many possible combonations there are.

Figure C.6.5 Zaineb's Cycle 2 Work



Name: Mayee Abelenin Date: 10/22/10

Partner: Brandon Teacher: Ms. Connors

The Pizza Problem

Capri Pizza has asked you to help design a form to keep track of certain pizza choices. They offer a standard "plain" pizza with cheese and tomato sauce. A customer can then select from the following toppings: peppers, sausage, mushrooms, and pepperoni. How many choices for pizza does a customer have? List all possible choices. Find a way to convince each other that you have accounted for all possibilities.

Peppers = Pepp
Sausage = S
Mushrooms = M.
Pepperoni = P.

4 x 4 = 16 combonations total

1
1 Pizza => Pep, S, M, P
2 Pizza => Pep, M, P
3 Pizza => Pep, P
4 Pizza => Pep

2
5 Pizza => S, Pep, M, P
6 Pizza => S, M, P
7 Pizza => S, P
8 Pizza => S

3
9 Pizza => M, Pep, P, S
10 Pizza => M, P, S
11 Pizza => M, S
12 Pizza => M

4
13 Pizza => P, Pep, S, M
14 Pizza => P, S, M
15 Pizza => P, M
16 Pizza => P

There are 16 different possible choices. The reason why there are only 16 combonations is because you cannot make any other choices. If you do, it will result like the others. What we did that makes is that us know that is because we created a pattern that will help us to keep track.

In total, there are 16 total combinations made.

Plan: Creating a Pattern

Figure C.6.6 Mayee's Cycle 2 Work

Intervention Implementation - Cycle II

During this second cycle, I felt that my students were much more confident in the task itself; not asking as many questions and doing much more discovery themselves and with their partners. I certainly feel that they went into the task of building the 4 high towers with more confidence, but being that it was more work that the towers 3 high, at times I could see their frustration. Most of the students had not finished the task at the end of day 1, so I taped up their towers in the same groupings as they had built them. When they came to class on the second day, this taping method which I thought was going to work so well, turned out not so great, as the students were not careful when removing the tape and many of the towers detached themselves from the groups they were placed in or the cubes even got separated from one another. This caused for some confusion, as if they did not record as they built the towers during the first day, they had to think back and almost use more time to try to figure out what the tape had ruined.

During the pizza task, which most of the students started on the second day, I felt very confident in their answers. I contributed this to the real life application of the combinations idea. I truly believe they were able to tackle this task with more efficiency and confidence because they could understand why it was being asked. Though my students had many questions, such as "is a pizza with peppers and sausage the same as a pizza with sausage and peppers?" and "is a plain pizza an option here?", afterthey truly understood all of the details, they answered and understood this problem very successfully.

During this task, I had distributed the task papers immediately, in contrast to how I conducted the first task. I think this worked better for my students, as many of they took notes from the beginning on the task paper.

Cycle III — Task

School:____________________________Teacher: _____

Other Group Members: ____________________________________

BUILDING TOWERS
THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

About the Class

This task was conducted in a regular-level, 7th grade classroom. The students were placed in regular- level math and problem solving according to their results on the NJASK test from 6th grade. All of these students tested at a proficient or partially-proficient level on the mathematics portion of the NJASK test. I conducted this activity over the course of 2 days, since our problem solving class is only 40-minutes. My students had approximately 60-65 minutes total over the two days to complete the task. This class has 21 students in it.

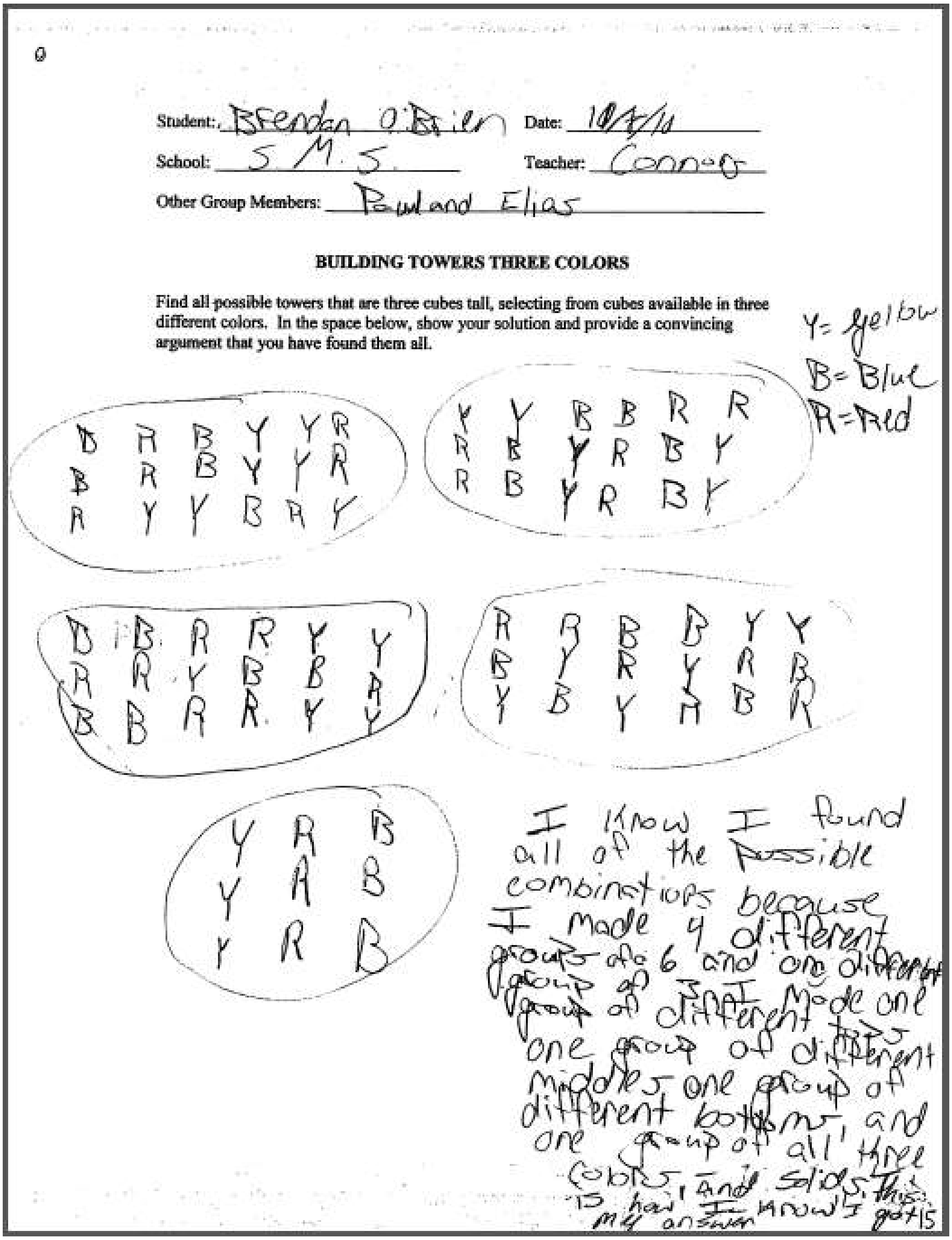
Impressed, Surprised, or Concerned - Cycle III

Brendan's work impressed me. Prior to starting this cycle, I had went around to each group and asked the students to predict how many towers they thought they were going to be able to make.

Brendan immediately guessed that he was going to be able to make 27 towers. His guess was met with opposition in his group; his other group member had told him that he thought they were going to be able to make an even number of towers, since the method in which they were building them showed 6 in each group. Brendan stuck by his answer, however. Inhis work, you can see that they built 4 groups of 6 towers; these are organized by placing two of the same color at the top (1), two of the same color on the bottom (1), two of the same color matching on the top and bottom (1), and one mixed group (1), and the last group is the solid group (1). The way he explained this initially puzzled me, but after looking at it in class with the rest of our classmates, I fully understand his organization, which impresses me.

Mayee's work surprised me. She also organized building the towers by placing them into 5 groups: one group of solid colors, which she calls singles, one group of red and yellows, one group of red and blues, one group of blue and yellows, and one group of all three colors. I think that her and her partner have come a long way from where they began this case study. Their organization skills have enabled their argument to be much stronger than where they had begun with building the 4 tall towers. She uses the word "opposite" to describe the building of her towers. Though it is hard to see what she means by this since there are 3 colors this time, if you examine her work carefully, you can see that she must be referring to building the towers with opposing colors in the last two or first two positions.

Nadir's work concerned me. Him and his partner were only able to come up with 19 different towers, even though they stated they found 21 towers in their explanation. I think this is because they did not have a strong way of organizing the way they were building their towers. Within the total of 60 minutes of class time to work on this activity, I expected them to be able to come up with more combinations.


Student: Brendan O'Brien Date: 10/4/10
School: S.M.S. Teacher: Connor
Other Group Members: Paul and Elias

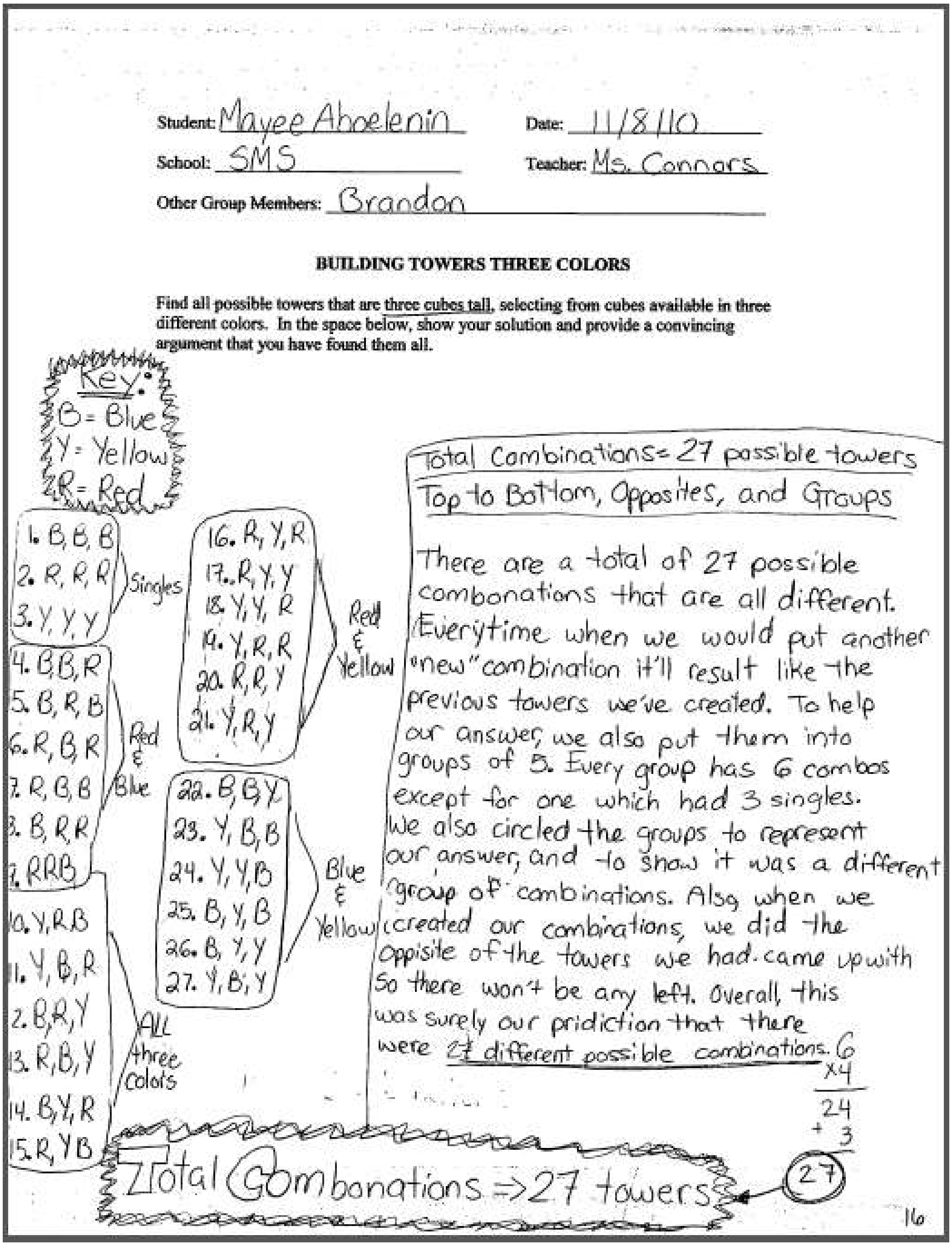
BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

Y= yellow
B= Blue
R=Red

I know I found all of the possible combinations because I made 4 different groups of 6 and one different group of 3 I made one group of different tops one group of different middles one group of different bottoms, and one group of all three colors, and solids. This is how I know I got 15 my answer

Figure C.6.7 Brendan's Cycle 3 Work



Student: Mayee Aboelenin Date: 11/8/10
School: SMS Teacher: Ms. Connors
Other Group Members: Brandon

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

Key:
B = Blue
Y = Yellow
R = Red

1. B, B, B
2. R, R, R
3. Y, Y, Y
Singles

4. B, B, R
5. B, R, B
6. R, B, R
7. R, B, B
8. B, R, R
9. R, R, B
Red & Blue

10. Y, R, B
11. Y, B, R
12. B, R, Y
13. R, B, Y
14. B, Y, R
15. R, Y, B
All three colors

16. R, Y, R
17. R, Y, Y
18. Y, Y, R
19. Y, R, R
20. R, R, Y
21. Y, R, Y
Red & Yellow

22. B, B, Y
23. Y, B, B
24. Y, Y, B
25. B, Y, B
26. B, Y, Y
27. Y, B, Y
Blue & Yellow

Total Combinations = 27 possible towers
Top to Bottom, Opposites, and Groups

There are a total of 27 possible combonations that are all different. Everytime when we would put another "new" combination it'll result like the previous towers we've created. To help our answer, we also put them into groups of 5. Every group has 6 combos except for one which had 3 singles. We also circled the groups to represent our answer, and to show it was a different group of combinations. Also when we created our combinations, we did the oppisite of the towers we had came up with so there won't be any left. Overall, this was surely our prediction that there were 27 different possible combinations.

$$\begin{array}{r} 6 \\ \times 4 \\ \hline 24 \\ +\ 3 \\ \hline 27 \end{array}$$

Total Combonations ⇒ 27 towers

16

Figure C.6.8 Mayee's Cycle 3 Work

Student: Nadir Shatter Date: 11/8/10

School: SMS Teacher: Connors

Other Group Members: Jasmine Urrutia

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

r,r,r-r,y,y-r,r,b-r,r,y-r,y,b-r,b,b-

b,b,b-b,y,y-b,b,y-b,b,r-b,y,b-b,r,r

y,y,y-y,r,r-y,y,b-y,b,y-y,r,y-y,b,b-y,y,r-y,b,

There are 21 diffrant conbinations. if there are any more it will repeat. we did ours like opposites and then we lined them up by the last color thats how we no theres no more conbinations

17

Figure C.6.9 Nadir's Cycle 3 Work

Intervention Implementation -Cycle III

During this last cycle, my students really impressed me. I think their investigation skills have really improved and I was able to identify that most of them were able to reason the number of towers they were going to be able to build prior to actually building them. I asked each group to predict how many towers they thought they would be able to make, and most of the groups either had a correct guess of 27 or something close to that amount.

My students really improved upon their organization of the way they were building the towers; they relied less on the term "random" and were able to show why they built the towers in the order that they did. I believe this truly contributed to the strength of their arguments.

If I could do this last session over, I would have loved to have given my students more time so that more of them could have been able to try Ankur's challenge problem. Only two groups were able to even attempt this challenge, and both groups were not able to finish. I had to tell the students to predict how many towers they would be able to make so that they could hand in some complete thoughts.

Reflection

Over the course of the nearly three months that I conducted these various activities with my class, I learned a lot about the way my students think, mathematically speaking. I think that most of my students have developed a system to organize their work so that they can prove to themselves that they have exhausted all options for combination problems. This was certainly not the case in the beginning. Most of my students used a system of "randomly" making towers and then creating their opposites. Soon after I showed my students an example of student work that did not use this method, they quickly got away from it. I think they realized that it is hard to convince another person that you have found all options by simply telling them that you randomly made towers for 40 minutes and just haven't been able to come up with any different ones. By coming up with a system to explain and/or show someone, your work can be so much more convincing.

During the activities, I really felt that my students were most engaged during the "pizza problem." They entered this task most confident out of all of the tasks. I attribute this to the real-life application of the concept of combinations. Although not all of my students were able to successfully complete the task and they had various questions about it prior to starting, my students really put great effort into it. I believe they could see why someone would ask them this question. It is very important for students to see the real-world application in math. Though the tower problems really showed me the students reasoning skills, I feel it is hard for them to see why someone would be asking them such a question.

As an instructor, I usually feel it is necessary to concentrate ideas in class on one particular topic at a time. With these activities, I was forced to depart from that line of thinking, as I was not completing a unit on combinations at the time. I think it is important that students be able to complete questions in a variety of topics in math at any one time. After all, life isn't organized into neat little topics, so when students encounter these in real life, this will better prepare them.

Rich

Topics in Math Education: Lesson Study on Reasoning Final Project.

Rich

Sayreville Middle School December 4, 2010

Student: ___________________________ Date: ______________________

School:___________________________ Teacher: ____________________

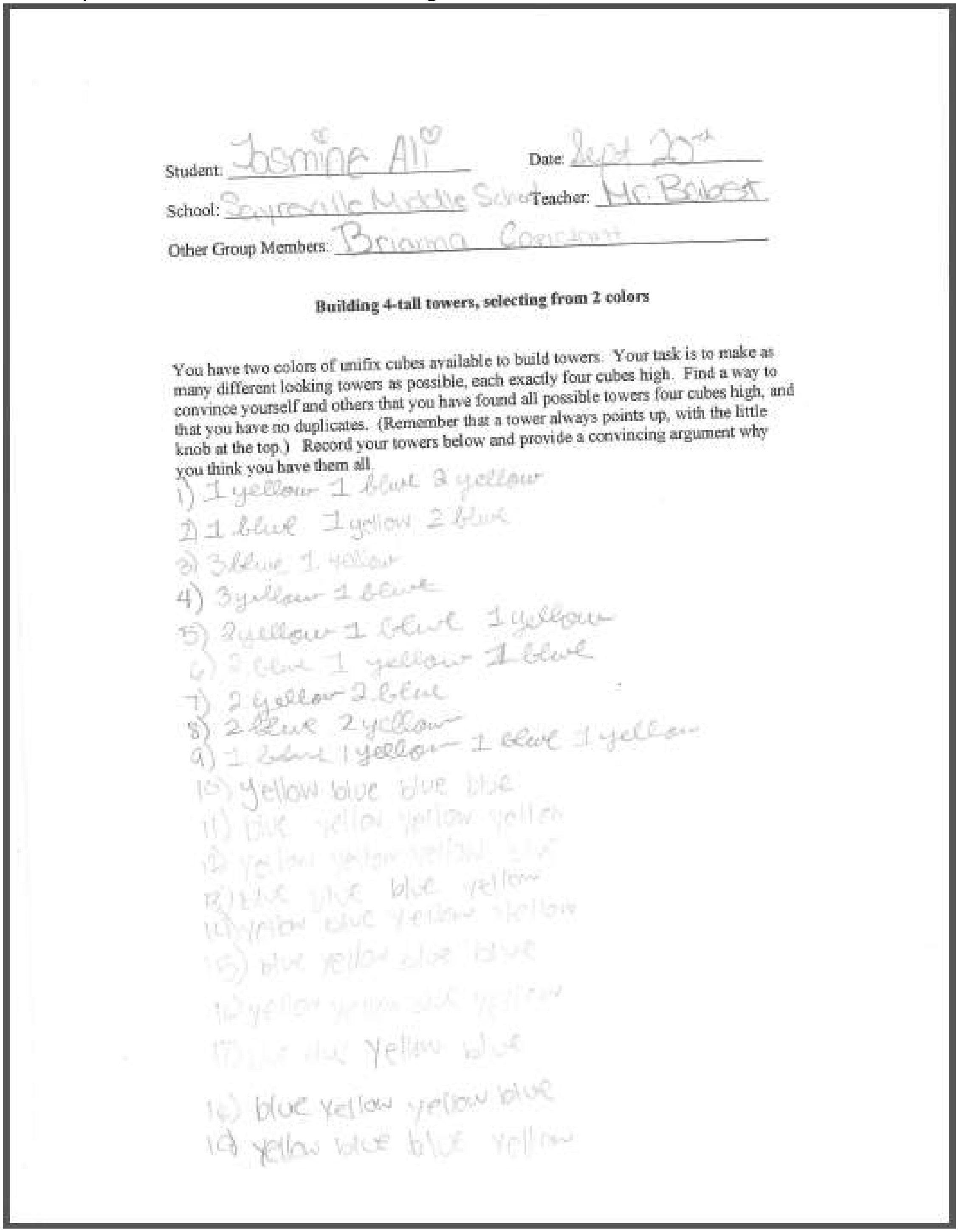
Other Group Members:____

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why

you think you have them all.

Jasmine is a sixth grade math student and completed the task during her problem solving class. The problem solving class meets for forty minutes and she worked on the task for one and a half class sessions totaling sixty minutes. The class is heterogeneously grouped and has twenty-two students. Jasmine is a regular education student.



Student: Jasmine Ali Date: Sept 20th

School: Sayreville Middle School Teacher: Mr. Balbest

Other Group Members: Brianna Constant

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

1) 1 yellow 1 blue 2 yellow
2) 1 blue 1 yellow 2 blue
3) 3 blue 1 yellow
4) 3 yellow 1 blue
5) 2 yellow 1 blue 1 yellow
6) 2 blue 1 yellow 1 blue
7) 2 yellow 2 blue
8) 2 blue 2 yellow
9) 1 blue 1 yellow 1 blue 1 yellow
10) yellow blue blue blue
11) blue yellow yellow yellow
12) yellow yellow yellow blue
13) blue blue blue yellow
14) yellow blue yellow yellow
15) blue yellow blue blue
16) yellow yellow blue yellow
17) blue blue yellow blue
18) blue yellow yellow blue
19) yellow blue blue yellow

Figure C.7.1 Jasmine's Cycle 1 Work

Jasmine's solution concerned me because she was struggling to understand the mathematical concepts needed to complete the task. She was not able to explain her method or the process sheused in her attempt to solve the problem. There was no evidence of a systematic attempt to finish the task. There were duplicates and missing towers that proves she was struggling or her methodwas flawed. On the positive side, it appears that Jasmine was looking for opposites to completethe task. However, she was unable to eliminate her duplicates or arrange her towers to develop aconvincing argument that she found all of the possible towers.

Student: ____________________________ Date: ______________________

School: ____________________________ Teacher: ____________________

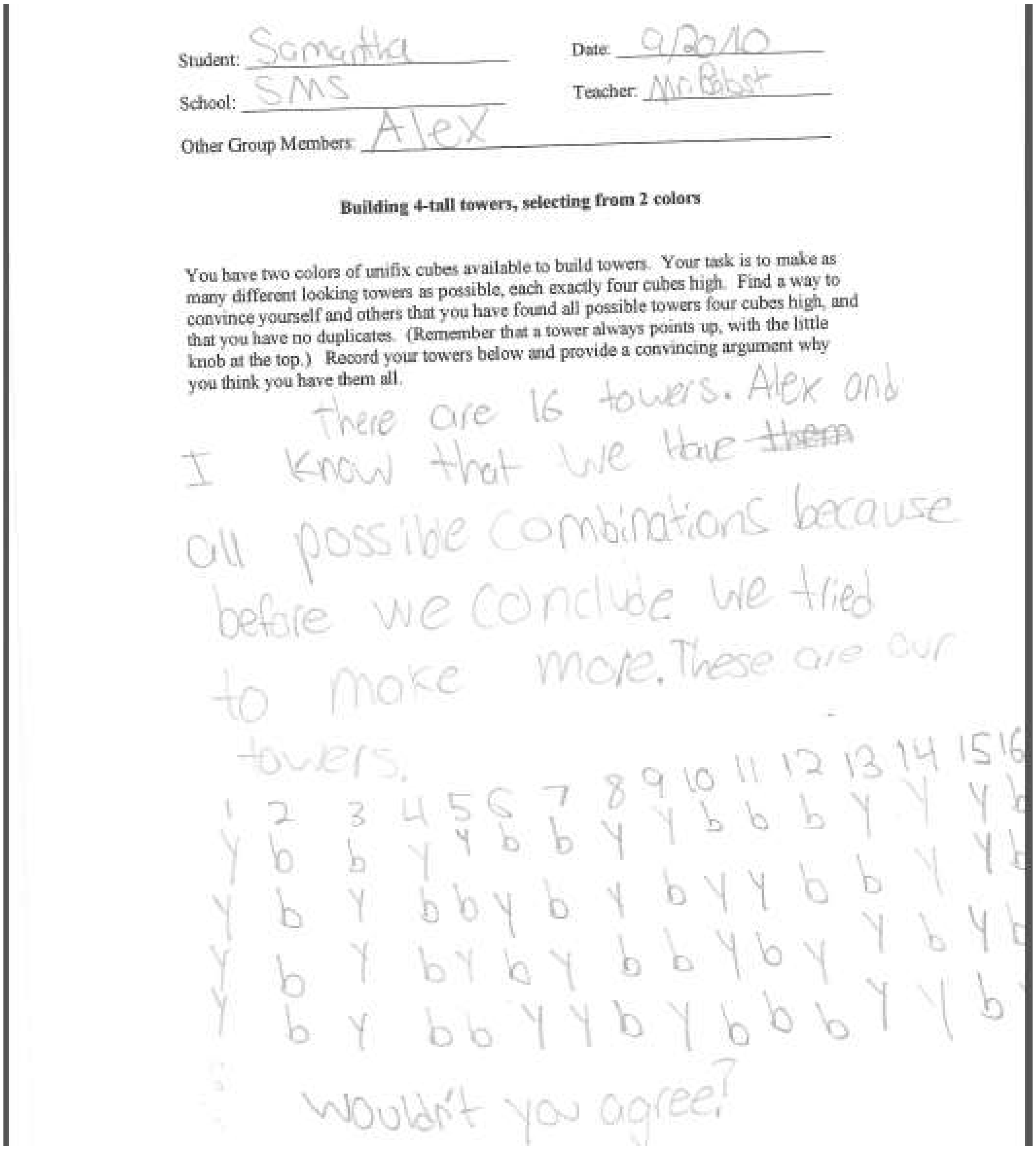
Other Group Members: ____

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why

you think you have them all.

Samantha is a sixth grade math student and completed the task during her problem solving class. The problem solving class meets for forty minutes and she worked on the task for one and a half class sessions totaling sixty minutes. The class is heterogeneously grouped and has twenty-two students. Samantha is a regular education student.



Student: Samantha Date: 9/20/10
School: SMS Teacher: Mr Babst
Other Group Members: Alex

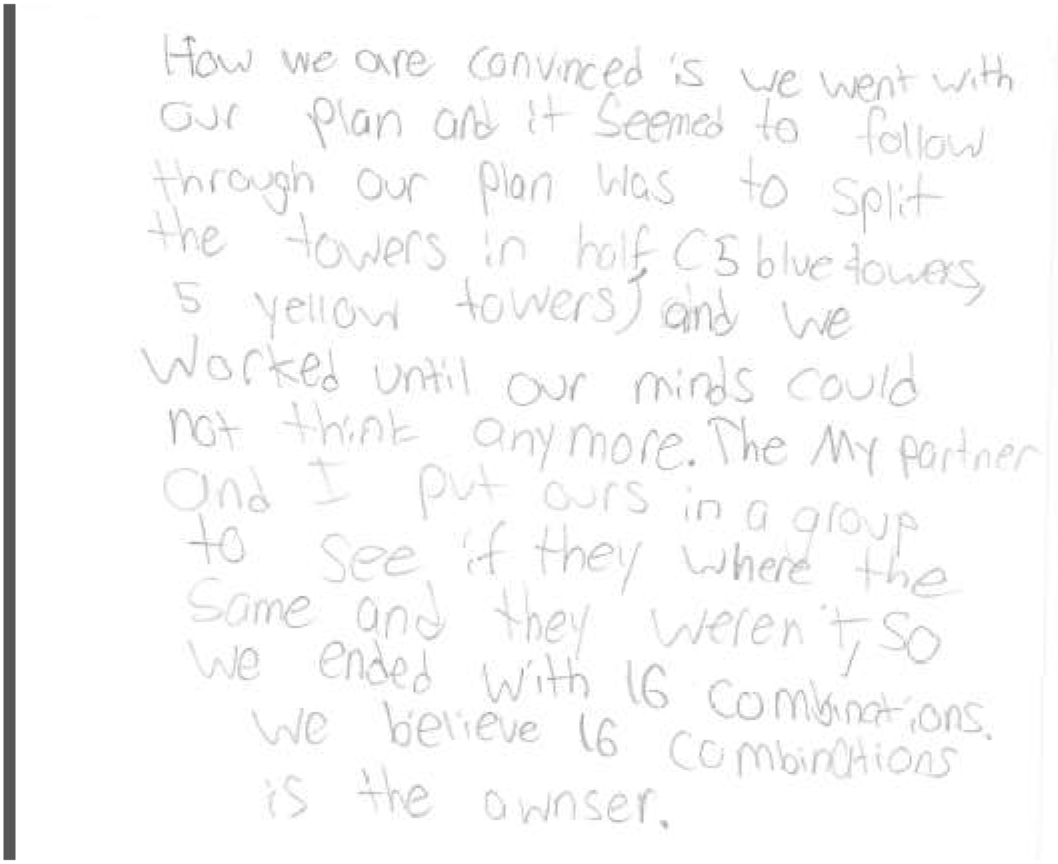
Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

There are 16 towers. Alex and I know that we have ~~them~~ all possible combinations because before we conclude we tried to make more. These are our towers.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Y	b	b	Y	Y	b	b	Y	Y	b	b	b	Y	Y	Y	b
Y	b	Y	b	b	Y	b	Y	b	Y	Y	b	b	Y	Y	b
Y	b	Y	b	Y	b	Y	b	b	Y	b	Y	Y	b	Y	b
Y	b	Y	b	b	Y	Y	b	Y	b	b	b	Y	Y	b	[illegible]

Wouldn't you agree?



How we are convinced is we went with our plan and it seemed to follow through our plan was to split the towers in half (5 blue towers, 5 yellow towers) and we worked until our minds could not think anymore. The My partner and I put ours in a group to see if they where the same and they weren't, so we ended with 16 combinations. We believe 16 combinations is the awnser.

Figure C.7.2 Samantha's Cycle 1 Work

Samantha's response surprised me for several reasons. The first reason was that she completed the task relatively fast. She also used a combination of strategies to complete the assignment. She used the idea of opposites in conjunction with the quantity of a specific color. She attempted toexplain her results and did an adequate job. She demonstrated an understanding of themathematical concepts needed to complete the task. I was surprised with how well she completed the assignment considering it was her first time attempting a problem like this one.

Student: ____________________________ Date: ______________________

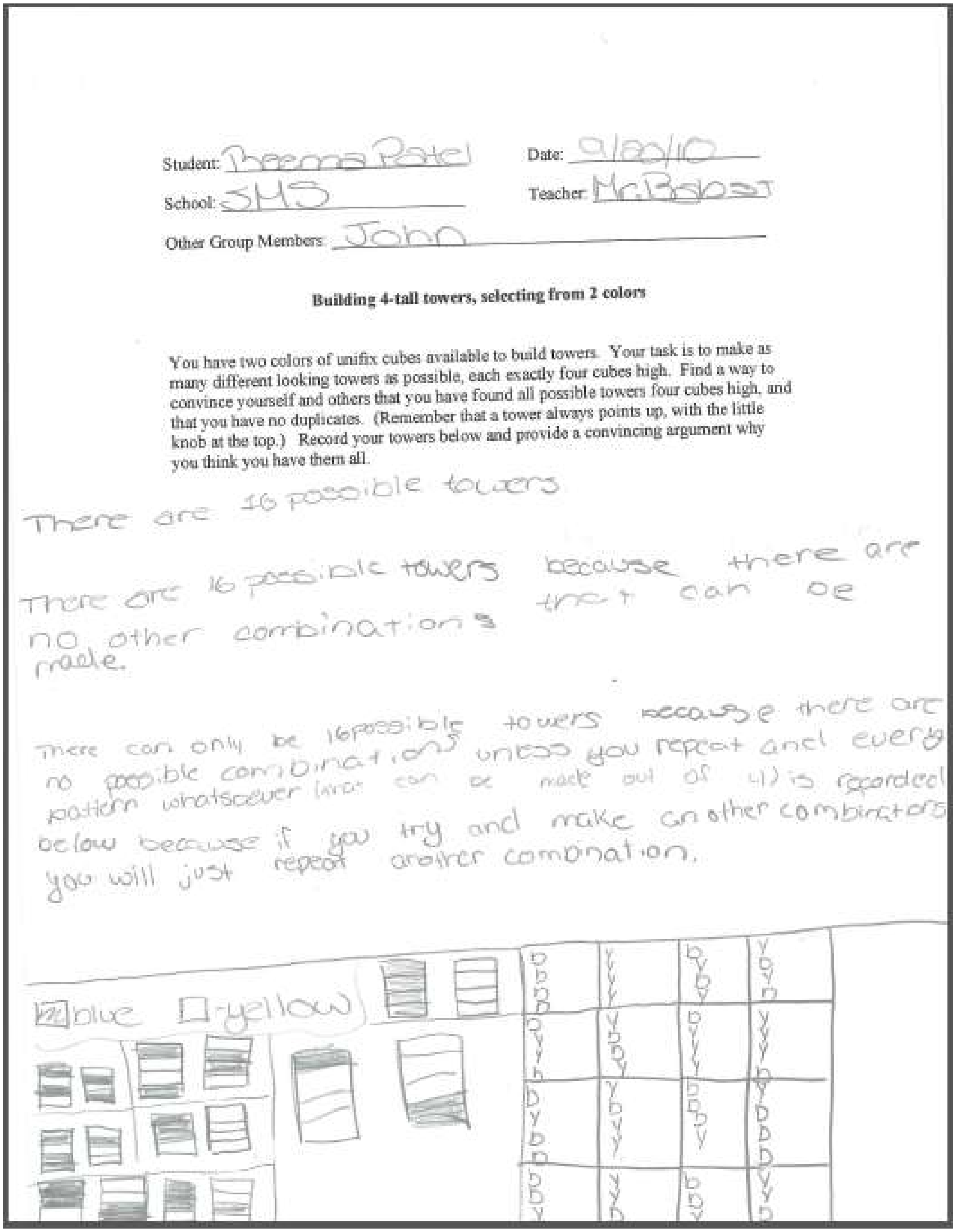
School: ____________________________ Teacher: ____________________

Other Group Members:____

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

Reema is a sixth grade math student and completed the task during her problem solving class. The problem solving class meets for forty minutes and she worked on the task for one and a half class sessions totaling sixty minutes. The class is heterogeneously grouped and has twenty-two students. Reema is a regular education student.



Student: Reema Patel Date: 9/20/10

School: SMS Teacher: Mr. Bobst

Other Group Members: John

Building 4-tall towers, selecting from 2 colors

You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates. (Remember that a tower always points up, with the little knob at the top.) Record your towers below and provide a convincing argument why you think you have them all.

There are 16 possible towers

There are 16 possible towers because there are no other combinations that can be made.

There can only be 16 possible towers because there are no possible combinations unless you repeat and every pattern whatsoever (that can be made out of 4) is recorded below because if you try and make another combinations you will just repeat another combination.

blue yellow

Figure C.7.3 Reema's Cycle 1 Work

Reema' s solution impressed me for different reasons. One reason being, she was able to explain how the towers would repeat because her pattern was complete because every time she attempted to continue her pattern, she explained there was a repetition. She also appeared to use a controlcolor of blue, yellow and then blue, blue while simultaneously creating the opposite tower. She did not organize them by the number of colors used and was still able to correctly complete the task. For the first attempt of a problem such as this one, she did a terrific job by showingsufficient evidence of the mathematic concepts required to solve this problem..

The intervention implementation of constructing four-tall towers from two colors was a rewarding experience for me. I learned how well my students could critically think in order to solve a problem. I also learned that my students' intuition allowed them to demonstrate their mathematical knowledge, and they currently have a great deal of problem solving skills. I sawhow much fun my students were having working together on this task. I discovered that my students could verbalize what they were doing but had a difficult time communicating theprocess in their written explanation. I was nervous about splitting the task time into two thirty minute segments, and probably might have rushed my students. I was also concerned about asking too many or not enough questions to my students. When I assign this task again, I willallow my students to work and not let the time factor get in the way of their task at hand becausein hindsight the time was not an issue. Overall, I was pleased to see how the majority of my students were able to correctly complete the task but was disappointed in their written explanations.

Student: ______________________________ Date: ________________________

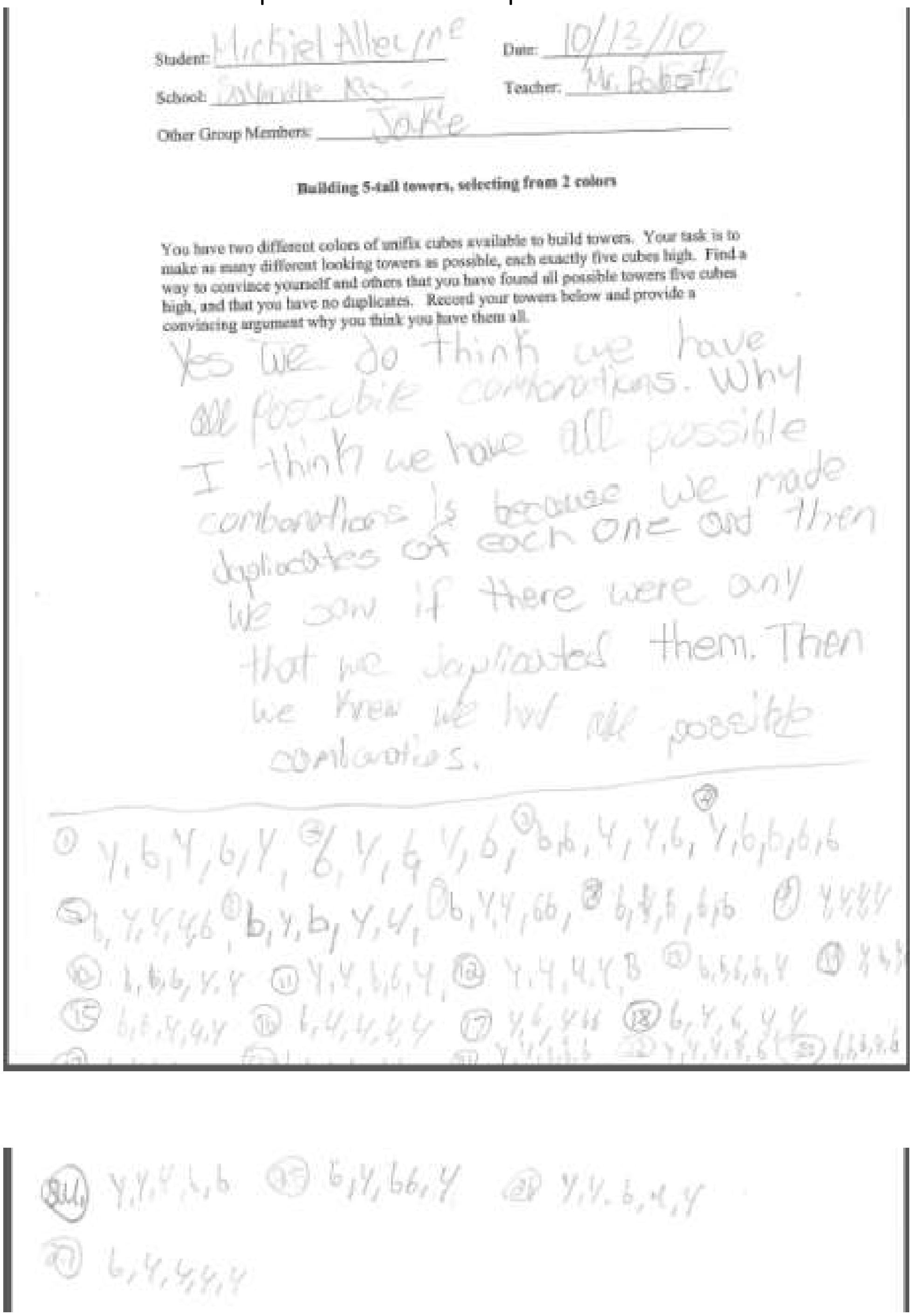
School: ______________________________ Teacher: ______________________

Other Group Members: ____

Building 5-tall towers, selecting from 2 colors

You have two different colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly five cubes high. Find a way to convince yourself and others that you have found all possible towers five cubes high, and that you have no duplicates. Record your towers below and provide a convincing argument why you think you have them all.

Mickiel is a sixth grade math student and completed the task during his problem solving class. The problem solving class meets for forty minutes and he worked on the task for one and a half class sessions totaling sixty minutes. The class is heterogeneously grouped and has twenty-two students. Mickiel is a regular education student who struggles sometimes with mathematical processes and concepts.


Student: Mickiel Allegrine Date: 10/13/10
School: Teacher: Mr. Roberts
Other Group Members: Jake

Building 5-tall towers, selecting from 2 colors

You have two different colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly five cubes high. Find a way to convince yourself and others that you have found all possible towers five cubes high, and that you have no duplicates. Record your towers below and provide a convincing argument why you think you have them all.

Yes we do think we have all possible combonations. Why I think we have all possible combonations is because we made duplicates of each one and then we saw if there were any that we duplicated them. Then we knew we had all possible combonations.

Figure C.7.4 Mickiel's Cycle 2 Work

Both Mickiel's solution and his clear lack of understanding of the mathematical concepts needed to complete this problem concerned me. In his written explanation he used the word "duplicates" (the first time it appears) instead of the word opposites. I think he was truly confused because he only used the word duplicate because in the past, some of the students in the class used the term opposites to describe their solution to a similar problem. He clearly does not use the strategy of opposites in his construction of his towers. He only has twenty-seven combinations and there are some duplicates including b,y,b,b,b. I do not feel he understood the mathematical concepts being presented in this problem. He gave it a good attempt and will develop as the year progresses.

Name: _________ Date:______________ Partner: ________________________________Teacher:_________________________

The Pizza Problem

Capri Pizza has asked you to help design a form to keep track of certain pizza choices. They offer a standard "plain" pizza with cheese and tomato sauce. A customer can then select from the following toppings: peppers, sausage, mushrooms, and pepperoni. How many choices for pizza does a customer have? List all possible choices. Find a way to convince each other that you have accounted for all possibilities.

Amber is a sixth grade math student and completed the task during her problem solving class. The problem solving class meets for forty minutes and she worked on the task for one and a half class sessions totaling sixty minutes. The class is heterogeneously grouped and has twenty-two students. Amber is a regular education student.

Name: Amber Walsh Date: 10/22/10

Partner: Jake Teacher: Mr. Babst

The Pizza Problem

Capri Pizza has asked you to help design a form to keep track of certain pizza choices. They offer a standard "plain" pizza with cheese and tomato sauce. A customer can then select from the following toppings: peppers, sausage, mushrooms, and pepperoni. How many choices for pizza does a customer have? List all possible choices. Find a way to convince each other that you have accounted for all possibilities.

16 combonations

Pizza types

1) Peppers
2) sausage
3) mushrooms
4.) pepperoni
5.) peppers + Sausage
6) Peppers Mushrooms
7.) Peppers pepperoni
8) sausage + Mushrooms
9.) Sausage + Pepperoni
10) Mushrooms + pepperoni
11.) Plain pizza
12.) pepper + sausage + Mushrooms
13.) Peppers + sausage + Pepperoni
14.) Pepperoni + Sausage + Mushroom Mushrooms
16) Peppers + Sausage + Mushrooms + pepperoni

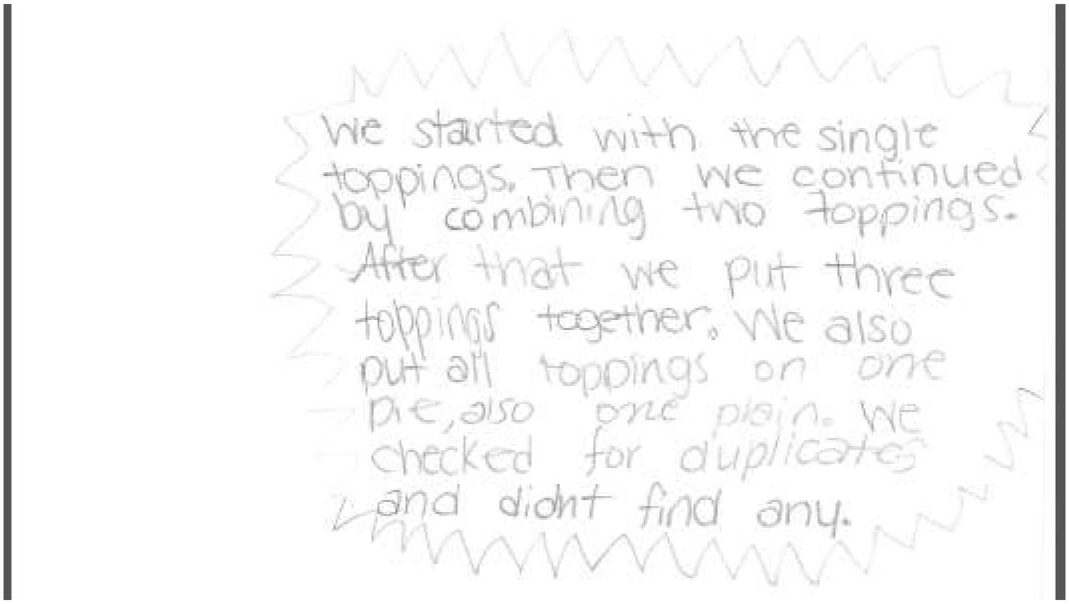


Figure C.7.5 Amber's Cycle 2 Work

Amber's response surprised me because she had a difficult time with both the four tall and twocolor problem and the five tall and two color task. She did an exceptional job completing the pizza problem. Her plan to start with one topping, then creating two topping combinations and then three topping combinations demonstrated a full understanding of the mathematical concepts needed to complete the assignment. I was surprised that she remembered to include the plainpizza and the pizza with all four toppings because some of my students omitted them. Her progress from the other two problems surprised me. I think by solving the pizza problem she was provided an opportunity to complete a "real world" scenario allowing her to show her ability tosuccessfully complete the task on hand.

Student: ___________________________ Date: _____________________

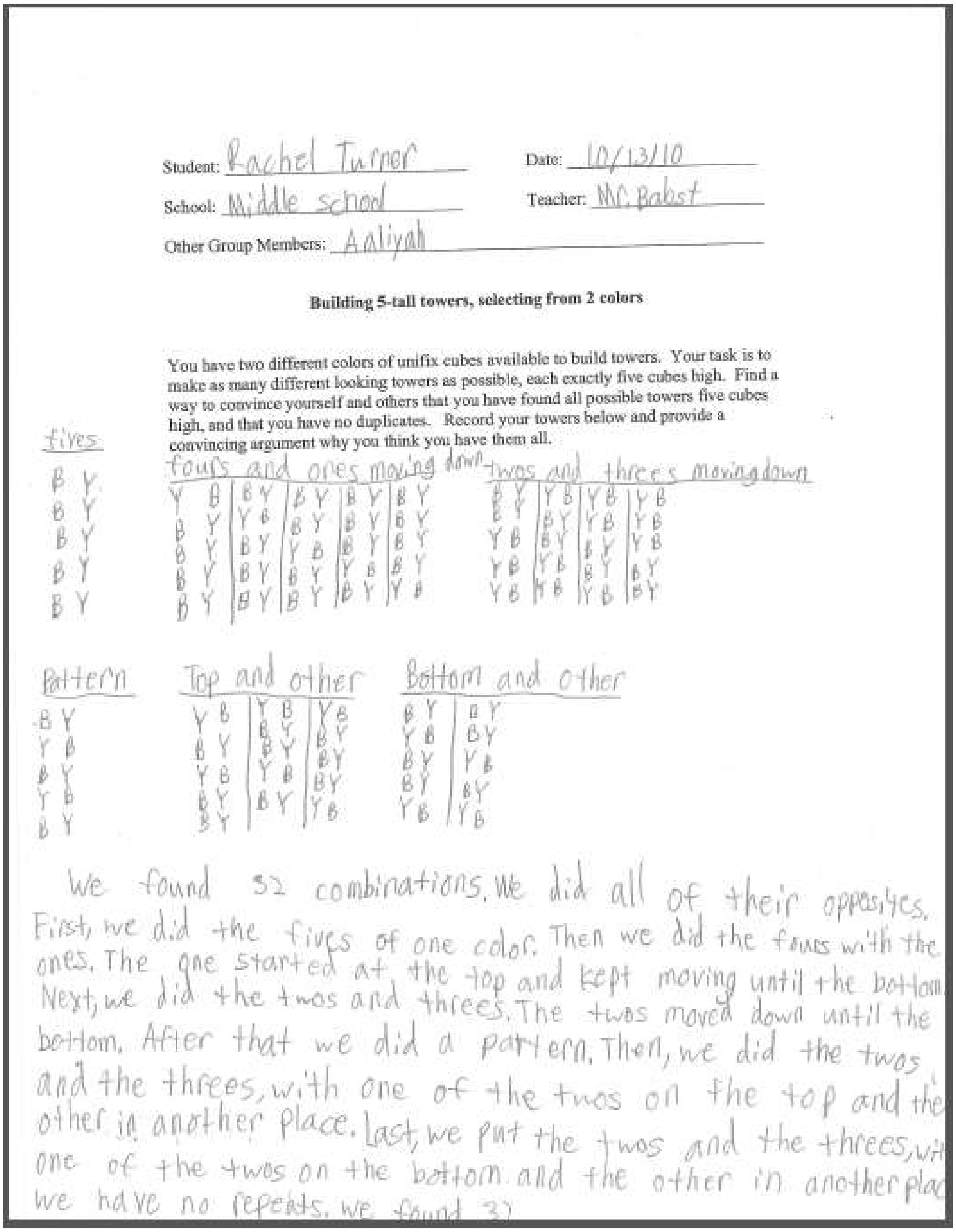
School: ___________________________ Teacher: ___________________

Other Group Members:____

Building 5-tall towers, selecting from 2 colors

You have two different colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly five cubes high. Find a way to convince yourself and others that you have found all possible towers five cubes high, and that you have no duplicates. Record your towers below and provide a convincing argument why you think you have them all.

Rachel is a sixth grade math student and completed the task during her problem solving class. Theproblem solving class meets for forty minutes and she worked on the task for one and a half classsessions totaling sixty minutes. The class is heterogeneously grouped and has twenty-two students. Rachel is a regular education student who is a participant of the gifted and talented program.


Student: Rachel Turner Date: 10/13/10

School: Middle school Teacher: Mr Babst

Other Group Members: Aaliyah

Building 5-tall towers, selecting from 2 colors

You have two different colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly five cubes high. Find a way to convince yourself and others that you have found all possible towers five cubes high, and that you have no duplicates. Record your towers below and provide a convincing argument why you think you have them all.

fives

fours and ones moving down

twos and threes moving down

Pattern

Top and other

Bottom and other

We found 32 combinations. We did all of their opposites. First, we did the fives of one color. Then we did the fours with the ones. The one started at the top and kept moving until the bottom. Next, we did the twos and threes. The twos moved down until the bottom. After that we did a pattern. Then, we did the twos and the threes, with one of the twos on the top and the other in another place. last, we put the twos and the threes, with one of the twos on the bottom and the other in another plac we have no repeats. we found 32

Figure C.7.6 Rachel's Cycle 2 Work

Rachel's response impressed me. She was able to complete the task by using the staircase method in conjunction with building opposite towers. She was able to vividly describe exactly what she did and completely described her pattern. Her organization of the towers was creative and much different from the way I organized mine. She clearly understood the task at hand and correctly used various strategies to master the assignment. Her solution was outstanding and impressive.

Cycle II's implementation provided an opportunity for me to learn some information about my students. I discovered how some of my students were successful and others were obviously confused but all of them worked diligently wanting to solve the problems. None of my students gave up. I noticed when I was asking my students questions during the implementation of the pizza problem some of my questioning was leading my students too much. Next time I have to be more cautious. During the five tower two color task my questioning was acceptable and not leading in any way. However, I needed to instruct my students to write down their process as they working on the towers and not waiting until they were finished. I think my students made a personal connection to pizza problem because it was a "real life application" scenario that they could have related to easier than the tower task. Even though they related better to the pizza problem, many of my students missed the plain pizza, a pizza with all toppings and some forgot a pizza can have three toppings on it. My students thoroughly enjoyed both problems and there was an overall improvement in their strategies and their written explanation. I also noticed how my questioning skills improved from cycle one.

Student: ___________________________ Date: _____________________

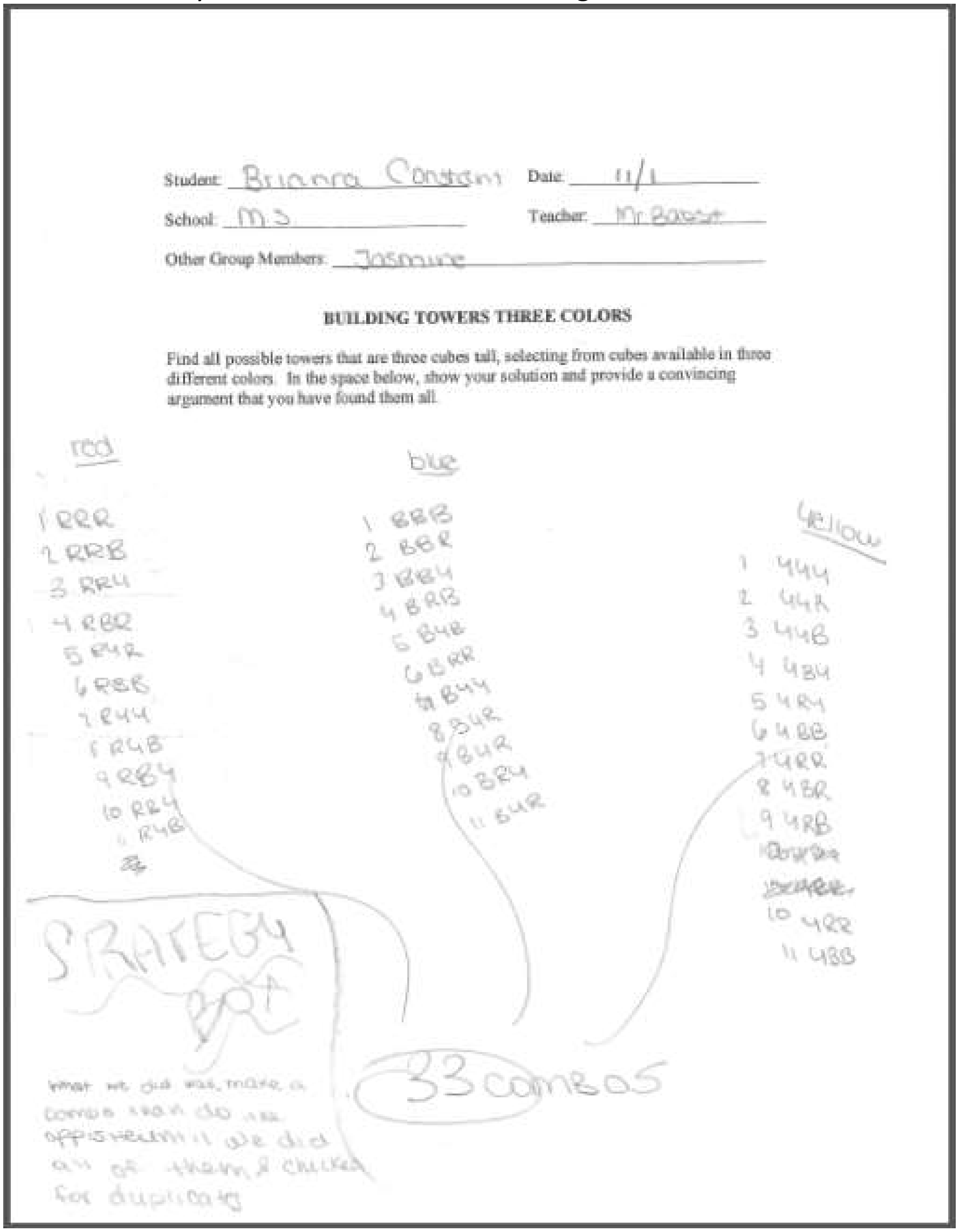
School: ___________________________ Teacher: ______

Other Group Members: ____

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

Brianna is a sixth grade math student and completed the task during her problem solving class. The problem solving class meets for forty minutes and she worked on the task for one and a half class sessions totaling sixty minutes. The class is heterogeneously grouped and has twenty-two students. Brianna is a regular education student.



Student: Brianna Constant Date: 11/1

School: M.S. Teacher: Mr Bassett

Other Group Members: Jasmine

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

red
1 RRR
2 RRB
3 RRY
4 RBR
5 RYR
6 RBB
7 RYY
8 RYB
9 RBY
10 RRY
11 RYB

blue
1 BBB
2 BBR
3 BBY
4 BRB
5 BYB
6 BRR
7 BYY
8 BYR
9 BYR
10 BRY
11 BYR

yellow
1 YYY
2 YYR
3 YYB
4 YBY
5 YRY
6 YBB
7 YRR
8 YBR
9 YRB
10 YRR
11 YBB

STRATEGY

what we did was make a combo then do the opposite. we did all of them & checked for duplicates

33 combos

Figure C.7.7 Brianna's Cycle 3 Work

Brianna's response concerned me for a couple of reasons. The first reason was she had duplicates in her construction of towers. This concerned me because her plan was valid but she was not able to see how using opposites would lead her to constructing several duplications of towers. I think if she arranged the towers differently she would have noticed the duplicates. It also concerned me because I wondered if she rushed to a conclusion which lead her to an incorrect response or just could not identify the duplicates. It also concerned me because her written explanation was not descriptive enough. I see how she duplicated the red, yellow and blue towers and wondered if she saw they would duplicate two more times using her strategy. I think with a little more timeshe would have completely completed the task correctly.

Student: ___________________________ Date: _____________________

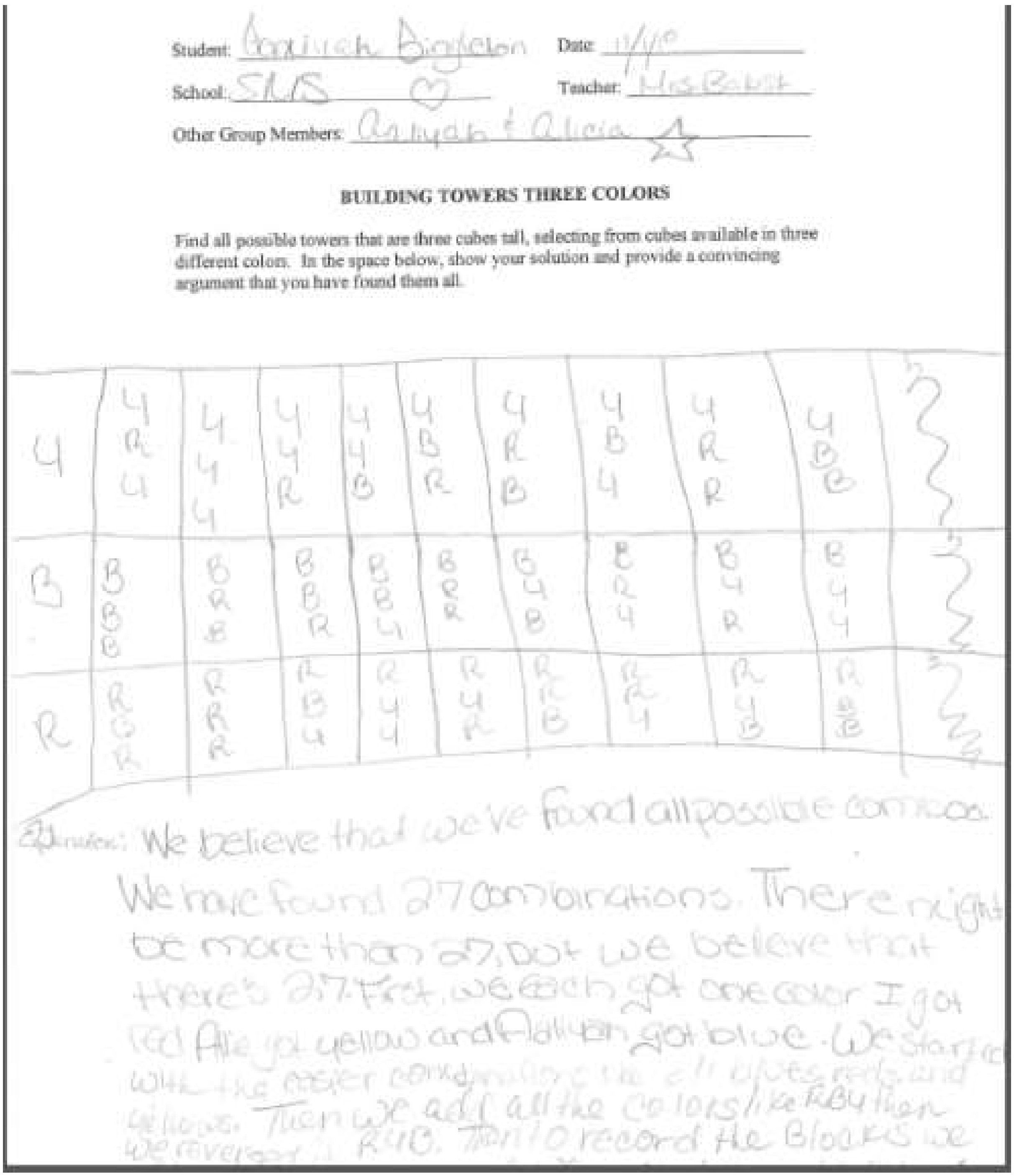
School: ___________________________ Teacher: _____________________

Other Group Members: ____

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

Gemiyah is a sixth grade math student and completed the task during her problem solving class. The problem solving class meets for forty minutes and she worked on the task for one and a half classsessions totaling sixty minutes. The class is heterogeneously grouped and has twenty-two students.Gemiyah is a regular education student.



Student: Date:
School: Teacher:
Other Group Members:

BUILDING TOWERS THREE COLORS

Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. In the space below, show your solution and provide a convincing argument that you have found them all.

Explanation: We believe that we've found all possible combos. We have found 27 combinations. There might be more than 27, but we believe that there's 27. First, we each got one color I got red Allie got yellow and Aaliyah got blue. We started with the easier combinations like all blues, reds, and yellows. Then we add all the colors like RBY then we reversed it RYB. Then to record the Blocks we

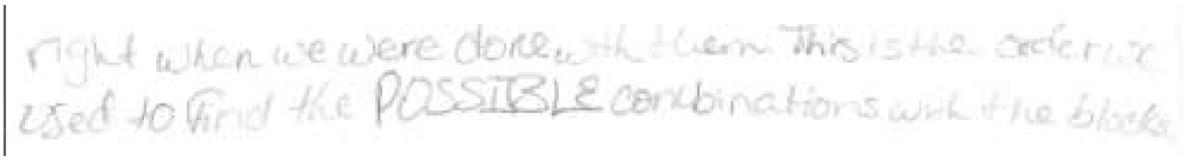


Figure C.7.8 Gemiyah's Cycle 3 Work

Gemiyah's response surprised me because she had a difficult time solving the prior tasks. I think it was shrewd for the group of three to divide the towers into three groups of colors. One of herstrategies was holding a specific color as a constant. She also created opposite towers from thebottom two colors and by doing this a duplicate would not occur. Her written explanation wasacceptable and a tremendous improvement from the prior tasks. She surprised me by the greatstrides she made from the first task that she attempted.

Student: ____________________ Date: ____________________

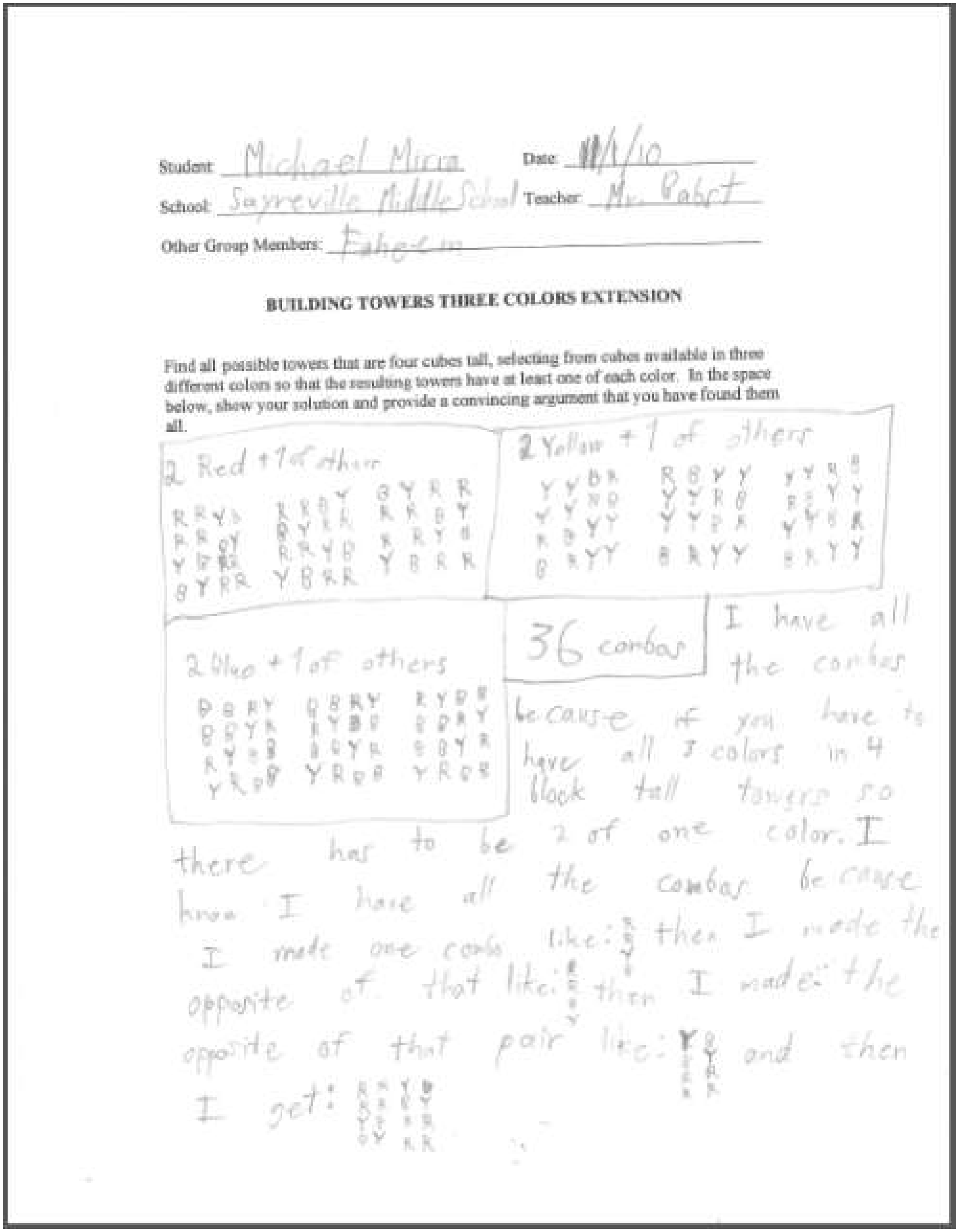
School: ____________________ Teacher: _ _

Other Group Members: ______________________________

BUILDING TOWERS THREE COLORS EXTENSION

Find all possible towers that are four cubes tall, selecting from cubes available in three different colors so that the resulting towers have at least one of each color. In the space below, show your solution and provide a convincing argument that you have found them all.

Michael is a sixth grade math student and completed the task during his problem solving class. The problem solving class meets for forty minutes and he worked on the task for one and a half classsessions totaling sixty minutes. The class is heterogeneously grouped and has twenty-two students. Michael is a regular education student who was able to complete each task quickly and accurately.


Student: Michael Mirra Date: 11/1/10

School: Sayreville Middle School Teacher: Mr. Pabst

Other Group Members: Faheem

BUILDING TOWERS THREE COLORS EXTENSION

Find all possible towers that are four cubes tall, selecting from cubes available in three different colors so that the resulting towers have at least one of each color. In the space below, show your solution and provide a convincing argument that you have found them all.

2 Red + 1 of others

2 Yellow + 1 of others

2 Blue + 1 of others

36 combos

I have all the combos because if you have to have all 3 colors in 4 block tall towers so there has to be 2 of one color. I know I have all the combos because I made one combo like: then I made the opposite of that like: then I made the opposite of that pair like: and then I get:

Figure C.7.9 Michael's Cycle 3 Work

Michael's solution impressed me for many reasons. The first reason was he finished the assignment in approximately ten minutes. He was able to accurately solve this problem and eachof the other problems we worked on in a timely manner. He provided a descriptive explanation supporting his work. He realized he needed to have exactly two of the same color and of each ofthe other colors in order to satisfy the criteria of this task. His strategy of dividing the towers into sections of two of the same color and one of each of the other colors was ingenious. Michael's solution was outstanding and he demonstrated a true understanding of the mathematical processneeded to successfully complete this challenging task on hand.

Cycle III was a positive experience for both myself and my students. My students had fun working on the tasks. They were more confident and comfortable as they were completing the assignments. Their strategies were more practical than some of the strategies they used on the prior assignments. Their written explanations were more descriptive, and their solutions were accurate. My questioning skills during the task were more effective than the ones I use during Cycle I and II. I needed to make some changes to one group of students because of absenteeism.I had no other option and would try to prevent switching groups in the future.

During this course my personal knowledge of mathematics increased significantly. I gained a stronger understanding of how my students' reason based upon their mathematical thinking. By implementing these tasks, I established a stronger relationship with my students. Since implementing these tasks, I saw a noticeable difference to myapproach in teaching lessons not related to this course.

The videos and readings that I read throughout this course increased my knowledge of mathematical concepts. I saw new and different strategies such as; the staircase method; holding a constant; or the idea of grouping familiestogether to use when solving problems. I found out how by working systematically, a correct solution can occur. Iwatched and read about real students thought processes and in return I now know different ways to solve problems. I usedto think the answer was the most important component of problem solving. However, sometimes the process and reasoning was more important. Especially in Stephanie's idea on how colors clashed and could not be an acceptable combination.

As I coached my students through the tasks, I felt like I was making a personal connection to them. I listened to their plans, their ideas and found out their methods were unique and clever. They had more knowledge than I ever anticipated. I discovered how much fun learning could be for them and I had a wonderful time as well. I learned not to help them too much because they can figure out solutions but even more importantly, I learned when they needed my experience to guide them. At the conclusion of the course I had a hard time deciding what responses were disappointing,because I enjoyed interpreting all of them. I found validity to each approach that my students used and even though the conclusion may have been incorrect, I felt true learning was taking place.

My students were recently learning the divisibility rules for seven and eleven. I had them try to discover the rules on their own in small groups before I explained the rules to them. I noticed myself being a true facilitator. I asked them the correct questions and allowed them to work. They discovered patterns, practiced the four operations of mathematics, worked cooperatively to achieve a common goal and they felt a sense of pride as I cheered them on. Even though a handful of my students were correct, I feel they all learned something about mathematics during the lesson. It was amazing to see what my students can do when provided the opportunity.

Appendix D Transcripts

10/7 Meeting transcript 1 of 3

Title: 10/7 Judy's Class 1 of 3
Location: Oldbridge
Date: 10/07/2010
Length: 59:42
Transcribed by: Will McGowan April 2012
Verified by: Maddie Yedman

Line	Time	Speaker	
1.			It was a separate, a separate course
2.			Oh is it? interesting, interesting.
3.			Each is 40 minutes.
4.	0:08	JL	Oh, that's good too, put the length of your class, your math class. Okay, Mitch, we're going to let you catch up. Oh you're going to be doing this on a card for me.
5.		Mitch	
6.		S	Haha
7.		JL	Everyone will know about it. That's a good idea.
8.			
9.			Well you know, you can actually hand that in, Mitch. If you don't want to rewrite it.
10.		Mitch	No, its alright
11.		JL	Alright. Then what I'd like you to do, some of you had a different job before your current job. Okay, so if you taught at this school or you were in business, what I'd like you to do is tell me, what you did before your current job. If you were teaching, what were you teaching? If you were in business, what did you do?
12.		KK	Can it be anything?
13.		JL	And how many years was that? Okay? Good. And when Justin comes, he'll catch up. Is Cindy coming too? Cindy is?
14.		CM	She's going to be late.
15.		JL	Oh, okay
16.		CM	She may get lost, she tends to get lost.
17.		JL	Okay, what I was asking for, your name, your school your

			grade, what you are teaching, what content. Also, are they regular or special ed students. Is it an inclusion class. Is it, you know, tracked in some way, or is it just a heterogeneous group of students? And, how many years you've been teaching. And then did you do something before this present job? And, if so, what was it? Was it teaching somewhere else? How many years were you teaching there? What were you teaching there? Or, were you in business? Okay? Excellent
18.		JL	Okay, now we have some guests today. Some of them look familiar to you. I think, from the summer, but I don't think you know Jim.
19.		CM	He was there that Saturday.
20.		JL	Oh, you were? Okay. Well you know everybody so far. You may not know Cindy, though, because Cindy is coming.
21.		CM	She was there. She was there at the first meeting.
22.		JL	Oh, you met everybody. Okay, so they're going to be here, and they are going to be… If they want to work with us, they can. If they want to visit and listen to what's going on, they can. We are actually talking first about the student work that your children did when they did towers four tall. Then we are actually going to be engaging in tasks. We have a lot to do in a very short amount of time, so I think we're going to… I'll be talking fast, but hopefully we'll have enough time to really share and ask questions. And celebrate what your children did and maybe question where we go from here. Okay? So
23.		KK	Do we need blocks?
24.		JL	Yes, we do.
25.		KK	I have a bunch in my car. Do you want me to
26.		JL	Well, Mitch has
27.		M	Well, I have some
28.		JL	Yeah, Mitch was good. Hello Justin, come on in
29.		CM	Nice to meet you
30.		JL	Well, yes, Mitch is, was really good. He helped me set up everything in his room. And by the way this is what I was talking about. What I called "ELMO" The projection camera. The technology we are going to use today Is really really neat. Because we're going to put your students' work underneath it. It will come up here on the

			smart board and we'll all be able to see, and it is so much easier to use than an overhead projector. If that's what you're currently using in your schools. So if they ask you "What do you want?" It's this camera that you want. A projection camera. Alright?
31.	3:42	JL	Okay. Let's go first to talk about what happened when you did your four tall… Oh no, we have to do one other thing. If you see Rich here, he's carrying a camera. And Rich is going to be videotaping the session today because they want to study the intervention, what we're doing, what I'm doing with you as teachers so I don't have a problem with him taping. I'm going to ignore him. I want you to ignore him too, but Rutgers needs a release saying that you're okay with the taping. Okay? And it's just to study the intervention. It's not to study you. It's to study me. Okay, so if you can fill this out, and then Rich will take these back to Rutgers, and then I don't know where they put them. Is it some secret file?
32.		RB	Yeah
33.		CM	No, they won't
34.		JL	Here you go.
35.		RB	Oh, thank you.
36.		JL	Okay
37.		A	Thank you.
38.		JL	Okay. Alright, so we're getting all the basics taken care of. Right? How many sixes are there? You need this to copy. You got it.
39.		A	Did you have any that got sixteen?
40.		RB	No. I had some that got twelve. I had some that had … and actually these two have in common. In may hands and they didn't even realize it.
41.			
42.		JL	Uh uh
43.		M	It's under Justin
44.		JL	Oh, It's underneath. Okay. That is. It's fine. Thank you.
45.	5:47	JL	Okay Why don't we, While you are and you can just, I'll collect them as we go along. You were doing the four tall towers with your children. You brought papers from your class. You kind of thought about what you wanted to share. Alright? Hopefully you picked things that you

			found interesting for whatever reason. Maybe you thought it was much different solution than you saw from most of your children. Maybe it was a solution that you didn't understand and you'd like us to kind of look at it and talk about it. But for whatever reason you found that as a solution that you wanted to bring to us. Alright? And I hope you gave it some thought because we're not going to have time to talk about everyone's paper but we do want to talk about some of your children's papers. So while you're fishing through your papers to get the paper you want to be talking about, Talk to me about how did it go when you did the four tall towers with your students. Were they engaged? Were they confused? Kulsom.
46.		KF	They really liked it. They had a really good time, you know. It was something that was hands on. It was good, but I, I know when we did this, when we did what our kids would do.
47.		JL	Yes
48.		KF	I did not think that they would do opposites at all, because I'm just so used to teaching, like make an organized list and a tree diagram so I just really didn't see that. Eighty percent did opposites.
49.		JL	And what grade was that?
50.		KF	Sixth grade
51.		JL	And were they a heterogeneous group? Or
52.		KF	Yes
53.		JL	They were. Okay That's interesting. How about the rest of you? What did you find in your students? Did they
54.		RB	I found that my students, as I circulated around the classroom, they knew what they were doing, and they knew their plan wasn't as systematic as like <Millin> Milan, but they did have a plan they explained it to me really well, but when it came time to writing down their explanations they did a pretty poor job.
55.		JL	How many of your children had a hard time writing? (Most hands up) How many of you had a hard time writing when we asked you to write at Rutgers September 11th?
56.			Yeah
57.		JL	It's easy to, It's easy to get a solution. It's harder to get a convincing argument, and its tough to write it out. How

			many of your children had trouble recording their towers, let alone writing the convincing argument?
58.		J	Yeah
59.		JL	Sometimes that's hard for youngsters to do. Alright? But you will see and I can guarantee it, because I've done this in many schools and in many different classrooms. They will get better at not only getting solutions but being able to record them, and also they'll get better and be able to be systematic and give you some kind of an argument. And it will be interesting today to see whether you are more systematic than you were when we did our first problem September 11th. Okay, who would like to put up some student work? Let's take a look.
60.		M	Inaudible
61.		JL	Okay
62.		M	Just has one more time.
63.		JL	Okay. This is new technology. I didn't know that he had this camera in his building. So I think it's great. He's going to now hoard it.
64.		KK	Yeah
65.		JL	Because the other teachers don't know it's in the building either. He's going to, uh
66.		M	They will.
67.		JL	They will soon, but we won't tell them. Because it's a gold mine. This is probably one of the best tools for a teacher that you could have.
68.		M	There's
69.		JL	If you can get it to work.
70.		M	It was working all day
71.		JL	Hahaha. Okay, thank you, Justin
72.			Do you have to turn it off?
73.		M	I did, I have to. He was like, don't do that
74.		CM	Why, is that…
75.		M	No, it's just he doesn't want anybody using it if they haven't like… Show them how it works.
76.		JL	Thanks. Okay. Now, what this tool shall do is if we put the student work down, it will project it right up on the screen. Which will be really, really neat. What we have to

			do, and what probably you have to do is if you're using an overhead.
77.	10:00	JL	It's cumbersome, You have to first take the student's papers and make them into transparencies and then stick them on the overhead. It doesn't happen quickly. This, the students could finish their work, we could put it right up. While Mitch is getting it to work, you haven't yet talked about this problem with the children, have you? Have you discussed their solutions?
78.		RB	No
79.			
80.		JL	No, because you didn't really have time in that period. You basically had just enough time to barely get the problem done. After today, I would like you to spend a little bit of time. Again, don't do everyone's paper, because you're going to bore them to tears if eighty percent of the class used one strategy, you're not going to put up, you know, all those papers, but you're going to go through the papers and you're going to handpick things that were interesting. Things that you think would be good to be shared. And then you're going to let the children who did the work do the sharing. If you have an overhead projector, you might have to help them by getting a transparency, and if they have trouble negotiating where their work is on the overhead, because it's sometimes tricky to find if you go left, right, up down,
81.		KK	Mm hm
82.		JL	You might have to help point as they are talking. But you're going to spend not a whole period, but maybe fifteen minutes letting students share their work. And you're going to do that before they do towers five tall. Alright?
83.		RB	Okay
84.		JL	We are going to, which will be the next task and there will be another task after that, which deals with pizza. Okay? And we're going to do both those today, in this block of time.
85.	11:38	JL	Okay, let's see. How are we doing? Is it coming?
86.		M	It's
87.		JL	Let's

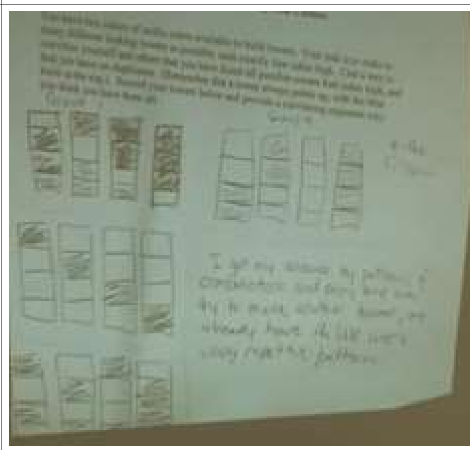
88.		M	It's there. It's just, there's something wrong with the. Let me try this.
89.		JL	Okay.
90.		M	Let me put the light on so we can see it better.
91.		R	Oh, maybe it's zoomed in too much?
92.		CM	Rich can you help him?
93.			Cut
94.	11:59	CM	You want them to keep it. We have a little, there's a story you can tell your students. This problem can get increasingly very complex. To really deal with mathematical concepts that are, you know, college level.
95.		KK	Mhm
96.		CM	The actual foundation for the later reasoning comes here and you build on it. So what's really important here, you all need to remember: It's not whether or not they got the answer or not it's what strategies they are using. Okay, what heuristics, what patterns they are noticing and what's helping them be sure. I'm just going to tell one other story because I can't help this. We were in Judy's Ms. Landis's school, and there was this third grader in her class that was working on this. And so one of the children said "Well, how could you ever be sure that you found them all?" and so one student said "Well you can never, never be sure, oh yeah, maybe you could, if you asked God."
97.		CM	You remember that one.
98.			
99.		JL	I do remember , yeah, yeah
100.		CM	So what I'm saying is you, they're at different points. And certain things they are sure of. What are you sure of? Judy's going to go through all that.
101.		JL	Absolutely, and if you read, and I know Justin did. The comments that I put on yesterday, on the website. It should sound very similar to what Carolyn just said. Again, we are really interested in the mathematical thinking of these children. And, if you trust that they are thinkers and give them an environment where they can show you they are thinkers, you're going to be really surprised at how far they can go. We're going, okay yes, Justin.

102.		J	Okay, well the, when they were in the second grade as well, that's when they first started challenging them with that skill of reasoning.
103.		JL	That's right, with shirts and pants.
104.		J	With shirts and pants.
105.		JL	And that's what I wrote on that site as well. That was the first problem they had as second graders, and absolutely, that was when they were getting to look as combinations, but it was the same idea of: "Show us how you're going to do it." We're not going to teach you how to get the combinations of shirts and pants. We want to see what you can do, and then we want you to convince us that you have them all. And the more you ask children to convince you and to explain and to show strategies and to… They will get better and better at it. And you know all your state testing wants them doing that. They want them writing, they want them to be mathematical thinkers. They don't want them just giving the solutions.
106.	14:33	JL	Oh my, the expert can't do it. We're in trouble.
107.		M	Yeah, we're in trouble
108.		JL	How about our handy dandy overhead that's over there? Haha
109.		CM	We don't have overheads.
110.		JL	Well, we could do that. Or, the other thing we could do is, we need to see what you're talking about so the other thing is, the other thing we could do is we could Xerox the student paper we're talking about. So let's give… How much more time do you think you need to make it work?
111.			Give me, haha
112.		M	No pressure
113.		JL	Haha
114.			I'll bring down the other one
115.		JL	Oh thank you so much
116.		CM	Okay another one!
117.			Disconnect that.
118.		JL	He's the man to know. He's the man, Mitch.
119.			But I want to know what happened to my camera
120.		JL	Uh oh, Mitch, you're in for it

121.		CM	Hidden treasure
122.			
123.		JL	Okay, when it works, it's a very good tool. Okay, in the meantime, so that we don't waste time, let's talk about what some of your children did. Let's talk about what kind of strategies did they use to solve the four- towers four tall problem? Justin?
124.		J	Most of my students they began using the guess and check method.
125.		JL	Okay,
126.		J	They were, they would stack four up and then if Kathleen was my partner, I say, well, you know "look at this and do something different."
127.		JL	Okay
128.		J	Let's say they got to around like twelve.
129.		JL	Yep
130.		J	And then they got more challenging of course because then they started to make duplicates more often. So then at that point what I saw was a different strategy that they had to lean on.
131.		JL	Okay
132.		J	And it was not just like, you put some together, I put some together to see if it doesn't match what we have already.
133.		JL	Okay
134.		J	So at that point, I saw them start to actually like try to group the things together, you know, the towers together.
135.		JL	They started to form groups?
136.		J	Yes, so there's two groups of girls. They form like the opposites so they had like eight pairs
137.		JL	Okay
138.		J	Of towers.
139.		JL	Okay
140.		J	And my fellas, they started to actually put things together like a tower four high, elevator staircase.
141.		JL	Oh, Okay
142.	16:53	J	And I asked them, I asked "Why are you doing those things?" So, that's when some of their reasoning or some

			strategies started to come out.
143.		JL	Okay
144.		J	Actually, but initially it was guess and check.
145.		JL	Okay
146.		J	I asked them, were they convinced, like "How could you convince me you have all of them?"
147.		JL	Yes
148.		J	And, the girls, what they would do, would try to make more, to
149.		JL	Okay
150.		J	Until like twenty minutes went by, and they said that they were exhausted.
151.		JL	Right
152.		J	And that that was it. They were finished.
153.		JL	Right. They can't find any more.
154.		J	Yeah, they can't find any more.
155.		JL	Did you hear that from any of your children
156.		S	Yes
157.		JL	I think I have them all because we're looking and looking can't find any more.
158.		S	I've spent ten minutes on it.
159.		JL	Right, Okay, yeah. And that is an argument. Is it convincing?
160.		J	No
161.		JL	Not too convincing, but it is an argument. And at one point it is okay, and if you think you know your children, and as the year goes on, you'll know them even more. If you believe that they've reached their limit, they've gone as far as they can go. Don't frustrate the heck out of them, let it be. You get them to record what they have, you get them to write their argument down. And that's where they are right now. They are going to be at different places now. And you don't want them all to end up at the same place because they are starting at different places. What other…
162.		S	There we go
163.		JL	Oh, look at this! Yay! Isn't this a great tool?

164.			Yeah
165.			This is
166.		JL	She was a teacher
167.		RB	Can you get one for our room? Where do you have more of these?
168.			Do you want to know where the storage closet is, so you can drive your car past?
169.		JL	This is absolutely great. Seriously, if your parent groups, I don't know in your school, if the parents like to buy gifts, but
170.			Oh yeah, this would be a great gift
171.		JL	Not for an individual person, but a gift for the school
172.			Yeah after thirty five years
173.		JL	Do the parents in Old Bridge do that?
174.			Yep
175.		JL	Okay, Sayreville?
176.		RB	No, our budget hasn't passed in thirty five years.
177.		JL	No, no, no, no
178.		RB	No, they won't buy anything.
179.		JL	And the PTA or the PTO
180.			No
181.		JL	Okay
182.			Yeah, our PTA would do this
183.		JL	So the Okay, then this what you want to ask for. And I don't think it's all that expensive, is it?
184.			No, they're not too bad.
185.			Okay
186.			But don't forget, you still need to have an LCD or some kind of projection unit to go with it.
187.	19:05	JL	Right, Right. But I'll tell you. Once you have this and you've used it, it's going to be very hard not to have it. Look how great that is. Okay, Mr. Smith's work. Is that the one you really wanted up?
188.		M	Um, it is now.
189.			We'll talk about what happened in the morning.
190.		M	I'll fix it, figure it out.

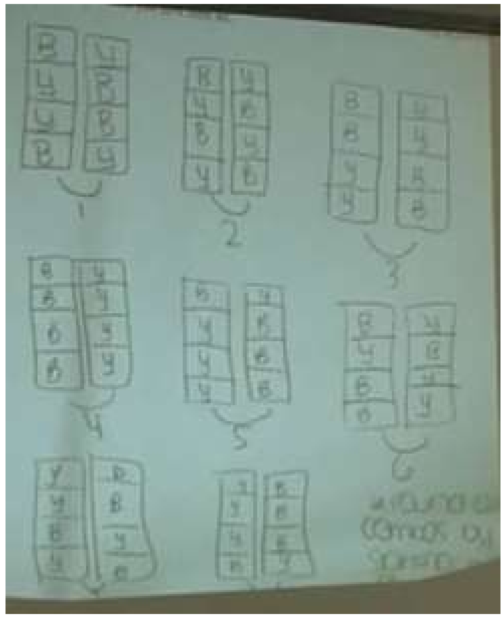
191.		JL	He's a good guy, don't be hard on him. Thank you for helping.
192.			You're welcome, take care
193.		JL	Okay. That's great.
194.			Lock them both up tonight, though.
195.		M	Okay
196.		JL	Mitch, if you want to take a minute to look, does someone know which student work they really want to talk about?
197.			
198.		JL	Okay, let's put one up. Kate? And what you want to do next time is really think ahead which student work you really want to be talking about. Because then that will let you not just pick something out of the pile, but it will be something that you really think would be interesting to share. Oh, let's see. Okay, sometimes the student writing is hard to
199.		KK	Too light?
200.		JL	See, let's see, back it down
201.			Because the desk looks, brown is yellow now.
202.		JL	There we go, yeah. Okay, great.
203.		M	
204.			 Kate's cycle 1 student work sample 1
205.		JL	What we're going to do, we're going to play detective. We're not going to have Kate talk about the work until you look at it and see if you can figure out what this student did, okay? Maybe we'll put it all up if we can. Let's see.

206.		KK	I just want to say, I did this in all of my classes
207.		JL	Okay
208.		KK	And, this is the class that you're coming in to. Which is why I picked some work from there because I thought you might want to go with that.
209.		JL	Good, okay
210.		KK	But, I noticed the first time I did it with this class, that was my first time doing it.
211.		JL	Yes
212.		KK	I think I must have used the word instead of how many towers, I used the word "combinations" because I noticed that every single one of them had the "combinations" in there, and I know that's not necessarily what they were thinking, so that made me realize later on in the day, that they shouldn't be using that word,
213.		JL	Okay
214.		KK	And I kind of changed how I, I started saying "Towers" instead.
215.	21:13	JL	Okay
216.		KK	Instead of "How many combinations did you come up with?" "How many towers?"
217.		JL	Okay good. And you know what?
218.		KK	So Just
219.		JL	I think we are teachers, and we're going to, as the students are engaging, you're going to get better and better at what you say, how you question
220.		KK	What you say, yeah.
221.		JL	And how you hold back, right?
222.		KK	And that's my other thing, I honestly at this point, I don't know if I feel qualified to be questioning them because I don't know what I'm supposed to be looking for, I feel like.
223.		JL	Well, after today, I'm hoping you will feel a little bit more secure, alright?
224.		KK	Okay
225.		JL	Okay, but I want you to take a look at what this student did. And see if you can figure out how they grouped their towers. Because they have four groups.

226.	22:05	RB	It makes sense until the third or fourth group
227.			Yeah
228.			Okay, what makes sense in one and two?
229.		RB	They went with three, I would imagine, blues in the first group, and then three yellows in the second group.
230.		JL	Okay
231.		RB	Then they went to two yellows and two blues in group three, but carries over into group four. Those two towers in the top, the top left of group four should be down with group three and then because they have four of each. Then it would have made
232.		S	No, I disagree
233.		RB	Oh, okay.
234.		S	I feel like, I think the difference between group four and group three is that in four, they are keeping all the groups together, and in group three, they are allowing them to be separate, not the last one.
235.		RB	But in group three, the third one in and the fourth one
236.		S	The fourth one
237.		RB	They are different. They are together. Two yellows
238.		S	Well
239.		RB	The other is two blues together.
240.		S	Well the top and the bottom are what they are defining as "Separated"
241.		KK	Look at the blues. There's
242.		S	Because it's not continued to… You're saying
243.		KK	Except till the end one.
244.		JL	These two is what he's pointing at
245.		S	Right
246.		JL	And what you're saying that the top and bottom are separated.
247.		S	Right
248.		JL	So that if there's any
249.			
250.		KK	They have exhausted all of the
251.		JL	That. And you know, sometimes it's neat…

252.		KK	Combinations where they couldn't be connected, so they had to connect the last two.
253.		JL	And you know, this is the way they saw it.
254.		KK	Right
255.		JL	So, even if we wanted to impose how we would group it, it's not necessarily the best thing to do. Okay? How about their, we said we'd talked about this, didn't we? <reading> "I got my answer by a pattern. Any time we tried to make another, we had it. We were using repetitive patterns."
256.		KK	Yeah, I found the word "repetitive patterns" kind of interesting.
257.		JL	Okay,
258.		KK	I don't know where it came from, or what she means by "repetitive" patterns.
259.		JL	Okay
260.		KK	I guess that they kept getting duplicates, that's what
261.		JL	And I would guess that's what they meant too.
262.		KK	Right.
263.		JL	Again, we can make guesses, about what children mean.
264.		KK	Right
265.		JL	But again, they're guesses. The only way we could know for sure what they meant by repetitive patterns is if we had an opportunity to ask them, okay?
266.		KK	Right
267.		JL	Alright, strategy here, aright. We have another paper to look at. Want to talk? Sure.
268.			
269.		KK	So this one was different. That was a, what most did
270.		JL	Most did that. Interesting!
271.		KK	Most did that
272.		JL	And this is what grade?
273.			Did she use the word opposites?
274.		JL	No she did not.
275.		KK	She did not. Most of them did use opposites.
276.		JL	Because she did. What they really were, were they

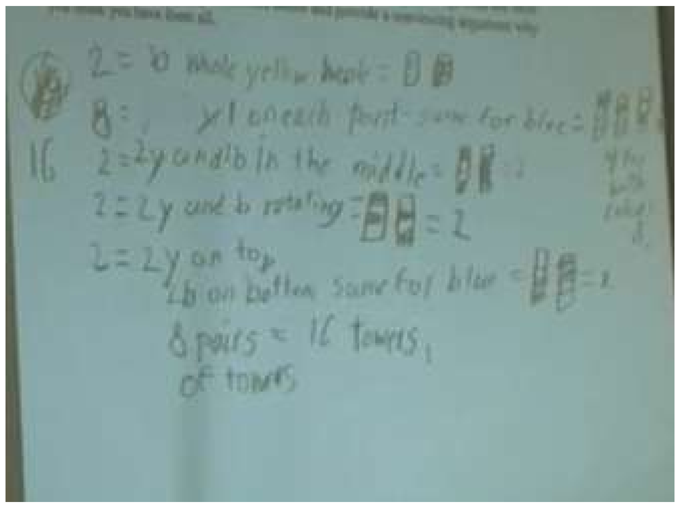
			making opposite towers? Oh my gosh.
277.		KK	No this one wasn't. look at this. This one comes in…
278.		JL	It does, it has to focus, but
279.			
280.		JL	Were they using opposites here?
281.			Uh, no not, they don't seem to be grouped by opposites.
282.		JL	They're not, really
283.		RB	No, it's not, no
284.		JL	Alright, so if most of your class did opposites, this would be a good thing to show because it was something different?
285.		RB	Mm hm
286.		JL	How many of your youngsters did four groups of four?
287.		J	Yeah, that's what I, that's the question I was going to ask
288.		JL	Yeah
289.		J	Was that purposely done? Or did they just like try and illustrate it close together? Or they purposely had four
290.		KK	Well, they labeled it "group one, group two, group three, group four"
291.		JL	Yeah
292.		KK	So this one did eight
293.		JL	Let's put it up
294.		KK	Yeah this one did eight separate
295.		JL	I would say, Justin, that when you have the grouping
296.		KK	This is eighth grade, they're all eighth
297.		JL	On their desk, the towers and the unifix cubes. You want to say to them "I want you to put on this paper, exactly the way you have them grouped on the desk, I want it on the paper. Because when you take away those unifix cubes at the end of the period, I want to remember what you did, because it was so neat." Okay? Alright, here's another thing. Here we go
298.	25:42	KK	Now, I would imagine these are opposites.

299.			
300.		JL	Are they? Take a look everyone. Are these opposites?
301.		J	Yeah
302.		KK	What they would consider, yes I guess so
303.		JL	Alright, okay
304.		RB	Yes
305.		JL	Alright, everyone thinks they are opposites, and this was a strategy
306.		KK	That she used let me see the word
307.		JL	Okay, looks a little bright
308.		KK	Oh, this is like the overhead "We found all"
309.		JL	"We found all the combos by starting with four of the same color and each time" Forget this "And each time" I can't even read it. Maybe you can read it.
310.		KK	Now she had a great verbal argument
311.		JL	Uh huh
312.		KK	And it didn't follow through at all in what she wrote here.
313.		JL	Okay
314.		KK	Four of the same color, and each time taking, taking one away? Taking one away
315.		JL	It's hard to read what she wrote
316.		KK	It's kind of squeezed into the corner. Taking one away
317.		JL	Okay

318.		KK	So she didn't use the word "opposites,"
319.		JL	Okay
320.		KK	And, I don't think she used the word "opposites" with me. Yet, she grouped them by opposites, but she had that starting with four, taking one away and replacing it with a different color. Which I thought was interesting, not, I didn't hear that from anybody else.
321.		JL	Okay, and can you see what this child meant by "starting with four and taking one away"? What did she mean?
322.		S	I guess that they didn't start with the four blue
323.			Four
324.			Yeah, right?
325.		JL	Starting with which group?
326.	27:13	KK	Four
327.		JL	Four, okay. And move over one. And what did she?
328.		KK	They took one blue away and replaced it, took one yellow away, and replaced it with the other color. And she took two away
329.		JL	I'm not sure
330.		KK	But then going from six they moved it down?
331.		JL	So they're saying something can anyone see what we're, where they are saying what they did, or see what they did, that "We started with four of the same color, and then we took one away."?
332.		CM	Yeah
333.		JL	Okay, Carolyn, where is it?
334.		CM	Well, Justin might see it first
335.		J	Well, I was, my response to them, would be something different.
336.		JL	Okay
337.		J	Just going back to what you said before, that they could have presented, they could have worked on it a certain way and
338.		KK	And wrote it up
339.		J	And come up with, and wrote it up. Something
340.	28:00	JL	And that's possible. And that sometimes does happen, but Carolyn thinks she sees here.

341.		CM	Well, I'm not sure
342.		JL	Oh
343.		CM	But look at number four, right?
344.		JL	Here's number four
345.		CM	They're all blue on the left, and you go to five and the top blue one of the right one becomes a yellow.
346.		S	Right
347.		JL	Okay
348.		CM	Now, go to number six, right? And the second blue one
349.			Now moves down one
350.			It goes down one. And essentially, she's doing a little of controlling for variables, but she's not really worrying about the consistency.
351.		JL	Right, Right
352.		CM	And conformity
353.		JL	Sure
354.		CM	In her head, she's taking them all the same and changing them
355.		KK	And I want to say
356.		CM	Changing one and then changing the other
357.		KK	If I remember correctly, she was verbalizing that to me. To take like blue
358.		CM	
359.		KK	…and
360.		CM	Well, as a first time
361.		KK	And that was my opposite. I thought that they would write better, like I think I write better than I speak, because you're speaking off the cuff, and you're, you know, you're not organizing your thoughts, and when you go to write it down, you could organize it, and present it better.
362.		CM	And they didn't have a lot of time to write it down
363.		KK	No, they didn't
364.		CM	They didn't have a lot of time
365.		JL	And an hour is what they had in your school. And an hour isn't even a lot of time.

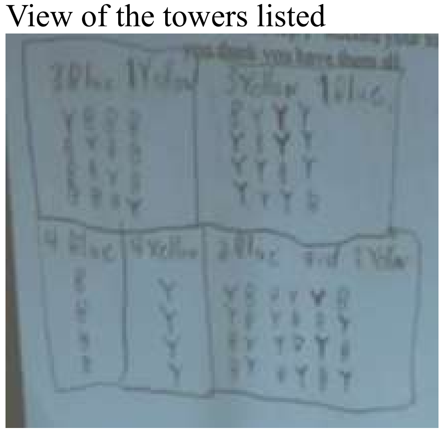
366.		KK	No
367.		JL	Carolyn said Kennilworth had an hour and a half.
368.			Right
369.			So, I think that, and Sayreville has what, forty minutes?
370.		A	And I feel like that's
371.		RB	They did it over two classes
372.		JL	You did it over two classes, for you, good
373.		A	About thirty minutes, the problem
374.		JL	You have to, that's good
375.		A	I noticed with some of my answers.
376.		JL	Yes, you like to talk about it?
377.		A	I said take off the first day, and then we'll write the draft the next day.
378.		JL	Absolutely
379.		A	And I'm like "but you got eighteen yesterday" and they're like "no but it's sixteen" and I'm like "what do you mean?"
380.		JL	Exactly, but you know, I think now that is one thing that happens when you go over two days. The other thing that happens is students kind of have a hard time getting back to where they left off.
381.			Yeah, exactly,
382.			and that's why, but that's what you have to do to do the problem in forty minutes. It's impossible
383.		A	No, no it really is.
384.		RB	What I did was I
385.		JL	Let's get another work up here.
386.		KK	Somebody else? Okay, sure.
387.		RB	Had my kids put them in a zip lock bag, and the next day they got the same ones
388.		JL	Is Mitch up there?
389.		RB	The ziplock, they put the work in it, so when they came back
390.		JL	Good
391.		RB	Right where they left off.
392.		JL	Good. And did you tape the towers so that they had the

			groups that they had? With masking tape?
393.		RB	No, I didn't, I didn't think to do it, but next time, I will
394.		JL	Do it next time, absolutely. Because sometimes you could actually put the towers in the bag, but they don't remember how they were grouped.
395.			Remembered
396.			Sure. Okay, this will darken, wait, but don't touch. It will do it. It has to focus, and we'll see the result. If we break another machine, your friend will never let you have another one. Okay, let's see, it darkens by itself.
397.		M	Oh, right.
398.			
399.	30:25	JL	Okay, now before Mitch talks, I want you to look and read what this student did. Mitch, maybe you could read it to them if it's hard for them to read it. What does it say?
400.		KK	Just tell me what that says: "two equal"
401.		M	It says, "Two equals 'b' whole 'yellow' whole" One blue whole and then a whole yellow
402.		KK	Oh, alright
403.		M	And then the eight, it's yellow one at each point, so at each position
404.			One on each
405.		JL	Okay
406.		M	And then he draws, I think, and then. Well, I'll let you figure out the rest
407.		CM	Yeah, well you're doing a good job.
408.		JL	Now you are, keep going. Yeah.

409.		M	And then the two "y" and "b" rotating and the two "y" and "b" in the middle, the two spaces and the two rotating
410.			That's interesting
411.		JL	I love his language, don't you?
412.		KK	Yeah, the rotating, yeah.
413.		JL	And what's the last one?
414.		M	The last one is two yellow on the bottom, and just kind of you know
415.		JL	Uh huh
416.		M	Two grouped together, but on the bottom, and then on the top. But
417.		JL	This is a good one to talk about because I bet no one else has something like this
418.			No
419.			Am I correct?
420.	31:29	KK	Yeah, not at all.
421.		JL	This is quite unusual.
422.		A	It's interesting
423.		JL	I haven't seen it before. Okay, I want, when you have figured it out, I want you to talk about it. What do you think this child did? His name is Jacob.
424.		M	Yeah
425.		JL	And what grade is Jacob?
426.		M	Eighth grade.
427.		JL	Eighth grade. Regular ed?
428.		M	Regular ed.
429.		JL	Regular. Okay, anyone have any ideas, even if you don't have the whole thing figured out. What kind of strategy, what kind of groupings is he doing?
430.		S	My, my view might be biased, but.
431.		JL	Okay
432.		S	Because I feel that's the way I look at it. But, the beginning is all four of the one color.
433.		JL	Alright, uh huh.
434.		S	And then, the second row is three of one color, one of another.

435.		JL	Okay
436.		S	And he exhausts that.
437.		JL	And he actually has four for both colors. Right?
438.		S	Right
439.		JL	He's showing you one do the same for the others
440.		S	He's showing, right.
441.		JL	Okay
442.		S	And then, the rest is all the different ways you could do two and two.
443.		JL	Okay, so he has his two in the middle he has the "rotating," he calls that. Did your kids call that anything else?
444.		S	Checkerboard
445.		JL	Checkerboard, okay, so then some call it candy cane, barber pole, I've heard all kinds of ways to call that, but rotating is kind of cute. And what did he do on the bottom?
446.		M	That's the interesting part. It says eight pairs, sixteen towers. I think he just wanted to get, because there's a lot, well, not a lot, but there's a bunch of kids that when they started, they had everything matched up.
447.		JL	Yep
448.		M	In twos, you know separated. But then when he started to realize that like some of them go into groups,
449.		JL	Uh huh
450.		M	He put them instead of four groups of four, like some people did
451.		JL	Uh huh
452.		M	He, the main ones were just like the, you know, the elevator type pattern
453.		JL	Okay
454.		M	And it's the other ones he has separated into pairs of, two different pairs of two
455.		JL	But he actually has them, grouped as very specially doesn't he, that's why those pairs there are two of each color.
456.		M	Yeah

457.		JL	Neat, very neat. Did he write anything about what he…
458.		M	He didn't, I couldn't. This was the most I could get him to
459.		JL	Okay
460.		M	I tried, and I kept saying "can you write that down?" And he kept doing, like you know, just visual. And showed it like this
461.		JL	Okay, okay
462.		M	Um
463.		JL	What you're going to do, when he works on the towers five tall.
464.		M	Mmhm
465.		JL	You're going to say to him "Are there any more towers?" "How do you get the towers, how do you know you have them all?" "How can you convince me there aren't any more?" and then when he says something, you say "Write that down."
466.		M	Mmhm
467.		JL	Okay? And sometimes, it's very hard for them, but if you actually get them to write
468.		M	That's what I had to do to get him to do this.
469.		JL	Good, well this is great. What you did was great, because I think this is really neat, what he did. And you can actually see, what it looks like he's keeping track as he's finding more towers.
470.			Uh huh, yeah
471.		JL	Right
472.		M	Well
473.			Ten
474.		JL	I assume that's what he was
475.		M	Well, he was, he was more of a guess and check.
476.		JL	Mm hm
477.		M	I think he kind of came to this after the fact, a lot of it.
478.		JL	Okay
479.		M	Because he thought it was fourteen at first.
480.		JL	Okay

481.		M	And when I started asking him "well, can you prove, you know"
482.		JL	Uh huh
483.		M	That's when I started getting him to write this stuff down, and then after the fact,
484.		JL	Uh huh
485.		M	He kind of, you know, justified why he had, why he thought he had all of them
486.		JL	Okay, could we have another paper? This is neat. Another one, somebody. Come on up with it. You own it.
487.	35:01	RB	Well actually Michael does, but
488.		JL	Michael does, okay. When you do the problems, the next two problems with the children, make sure you are going into a class that has done this first problem. Don't pick another class to go to. Because we really are hoping they are going to build upon what they've done with towers four tall. Okay, It's going to take time. It will come up, you've got to be patient. It's like magic, it's like a magician. There's nothing there and the all of a sudden, it appears.
489.		KK	Hopefully
490.		JL	It will, it will. We have to love and trust.
491.			I see something
492.			It's coming, it's coming. Here it comes. Okay, now this is going to be hard for us to read, just because, sometimes the students write a lot, small, light. Let's look, before we read, let's look at how they grouped and figure out what they did.
493.	36:06		View of the towers listed 
494.		JL	Have you figured it out, what did he see, what did he do?

			Kulsom
495.		K	Well, that's what, the one of the papers I have has too. They're kind of the same thing. They have the staircase.
496.		JL	Uh huh
497.		K	Three of one color and then the one of the other. They grouped those two together, but then all, all four blue and all four yellow, but then the two blue and the two yellow, they kind of did as opposite pairs of opposites kind of next to each other
498.		JL	Okay
499.		K	The fourth group.
500.		JL	Let's see if that's what they did. Can you read to us what they wrote?
501.		RB	It says: "I think we have all of them because we made each other tower combos but we always end up with duplicating other towers. We made opposites of every combo and we can't think of any more combos that we didn't already make, The only way to not duplicate towers is to use more, or less than four blocks."
502.		JL	Interesting, interesting thought. Was that before or after you asked them to make a prediction?
503.		RB	They did the prediction the second day, so these were pretty.
504.		JL	It was before?
505.		RB	These were before.
506.		JL	Okay, interesting, interesting thought. What, did they have any rhyme or reason for how they grouped the two blue and two yellow?
507.		RB	Um ,they were working really well together, these boys. One of them is actually special ed. They actually, I have him without an in class support teacher.
508.		JL	Okay
509.		RB	On that, but he
510.		JL	One was a special ed youngster, the other was a regular ed youngster.
511.		RB	Mm hm, but he's in my class with no in class support.
512.		JL	Okay
513.		RB	He does not have any modifications for math

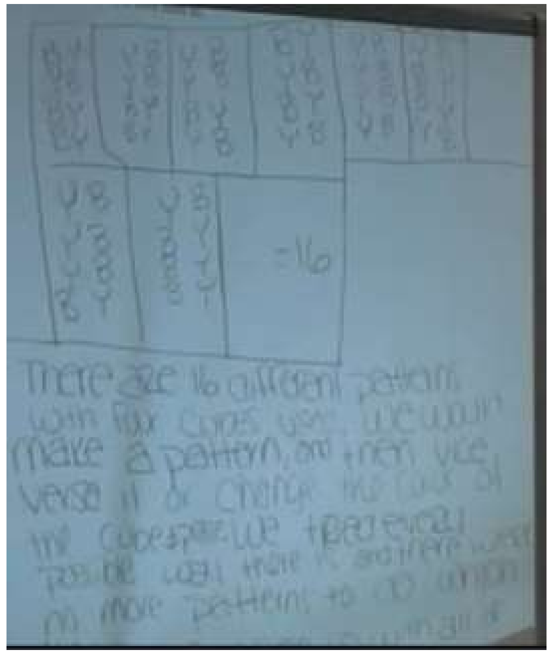
514.		JL	Okay
515.		RB	And they really seemed to have been working well together.
516.		JL	Good
517.		RB	And I think it was the special ed boy who actually came up with this strategy.
518.		JL	Okay, good.
519.		RB	Versus, and then he said "well" and they kind of worked together in an organized fashion. And it was more than a lot of my students who just jumped in, and was making, like guess and check
520.		JL	Sure, sure. Okay, well, neat. Alright, let's get another paper up. Um, I think you're going to be surprised Barrie… come on up, Angela. Barriers are going to be broken, not only special ed barriers, but if your school is tracking, which I know some of your schools do. Go ahead, let's see the paper, give it time to warm up. I have seen really, really neat solutions from not just your best and brightest, but from your children who were deemed to be in the lower track. In fact, you're going to meet Brandon. Brandon's one of mine.
521.			He is
522.		JL	Well, he's not my personal child, but he was in my school, at Colt's Neck where I was principal. And you're going to be watching a video of Brandon, this week. You, did you see it already?
523.		M	Mm hm
524.		RB	I watched it today.
525.		JL	Okay, well, it's a fascinating, You're going to probably have to watch it more than once, because what Brandon does, did is actually brilliant. And it may take you more than once to hear what he's doing.
526.		KK	He was special ed?
527.		JL	No, he was, he didn't have, it was a regular ed class, but he was on the lowest track. They had tracking. When I first came to Colt's Neck, they had still tracking for both regular ed and special ed and immediately, when I came in, I got rid of the tracks for my reading language arts. Because it is absolutely absurd to think that you can't have a good conversation about what they're reading with a heterogeneous group. And then once the teachers got

			used to that, I got rid of the math tracks. Because Brandon was placed in the lowest track, by the way the teachers grouped, but you're going to see him, and I think you won't believe that they could put him into the lowest track. Today, Brandon is a veterinarian, okay?
528.		RB	Hm
529.			
530.		JL	Oh my gosh! There's a lot on this paper.
531.			Mm hm. And then it goes onto the back.
532.	40:00	JL	Okay, so let's see. Before we go on to the back, take a look at how these children recorded the towers. How are they grouping?
533.		RB	That looks like <Milin> Milan's strategy.
534.		JL	Yeah
535.		RB	Where he started with one cube, and he built on the one cube
536.		A	Yeah, I know, that was interesting.
537.		RB	and then he went to the three cubes and then he went to the four cubes
538.			He didn't give them
539.		JL	Ah, okay
540.		KK	He didn't begin with the twos, he backtracked until he hit
541.		A	I teach four classes. I teach three classes of advanced kids, and this is the one regular class. And before, this kid actually, because he got moved into my advanced class.
542.		KK	Oh, wow
543.		A	So, to me when I read this was one of the most poignant

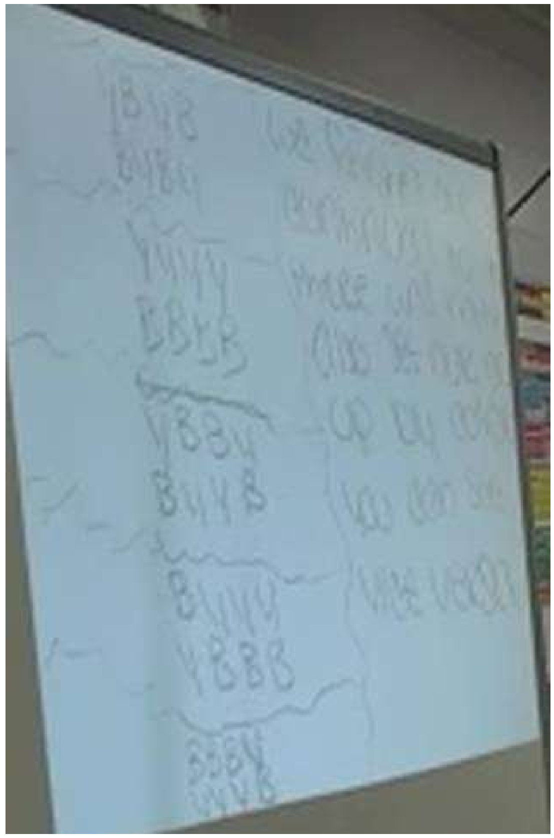
			in my class. This is one that I get …
544.		JL	So, what did he do first? He built the towers up there. And then he says "but one cube…"
545.		A	He did draw all of these first, and then he went back.
546.		KK	The ones on the back? Oh
547.		A	He did sixteen first, and then, when I said "make sure you claim, make sure you're proving to everyone"
548.		JL	Yep
549.		A	He went back to the one, two, and three groups.
550.		JL	Isn't that neat? Huh? Isn't that neat? So he's kind of going back and he's trying to, now this is different. Someone in this group asked me "wouldn't it be a good idea to start them with towers one tall, and then go to towers two tall, and then"
551.		KK	Exactly, I was questioning it.
552.		JL	Okay, and that's not what this happened here. Teacher didn't do that at all. Teacher gave problem, child decided. If we build, he came up with it
553.		KK	Right
554.		JL	…He came up with, "Well, let me go back and think of a one tall tower, what would it look like? And what would it look like two tall? What would it look like three tall? and then what would it look like four tall?" If you do a, if you as teacher say we're going to start and do one tall, everyone, now let's all do two tall. What, it's like a prescription.
555.			There's no thinking
556.		JL	We're actually, you know telling them what to do, and we're prescribing what they should be doing and how they do it. It doesn't give them the opportunity to develop their own heuristics and strategies and solving. What did he say, Why don't you read it.
557.		A	He had a pretty nice explanation.
558.		JL	Okay
559.		A	It says: "I recorded all the combinations for four cubes high. I did the others to look for a pattern to check my work. I found a pattern and it is every time you make towers with one more than the last ones, you multiply by two to get the total number of towers"

560.		JL	Stop right there. Stop a minute, is that beautiful?
561.		RB	Mm hm
562.		JL	That is amazing.
563.		A	"To get the number of towers for the present one. Since every tower has an opposite tower" in parenthesis "has a blue or a yellow opposites, there is always an even number of towers. This means if you have" and he draws a diagram, or like one tower and its opposite, so that for whoever's reading it, it is clear
564.		JL	Uh huh
565.		A	It says: "the opposite of this, it's blue on the bottom, is yellow on the bottom. I checked to make sure no repeats were there and that I had every possible tower."
566.		JL	Is that amazing?
567.		CM	That's great
568.		JL	I have the chills. And this was, and good for you, for moving him up to the class.
569.		A	I didn't move him, he got moved.
570.		JL	Oh
571.		A	But, I mean it's clear to me. I don't have any power in that department.
572.		JL	But you probably made a recommendation.
573.		A	I mean, he should
574.		JL	I would say so, I would say so. He's he looks like a Brandon. So, and there are Brandons in your room, by the way. So, you know. That's really neat.
575.		A	Yeah. It was nice.
576.		JL	Let's get another
577.		KK	And what grade is that? Seventh, Ash?
578.		A	Right
579.	43:20	JL	Seventh grade. Let's go
580.		RB	Wow, not eighth. He would have finished like that.
581.		JL	Um, you know, we
582.			I didn't have enough
583.		JL	Have found that it is not by grade, that you can make a prediction, and it's not by the level of the class, if they're leveled. You can find stuff like this happening.

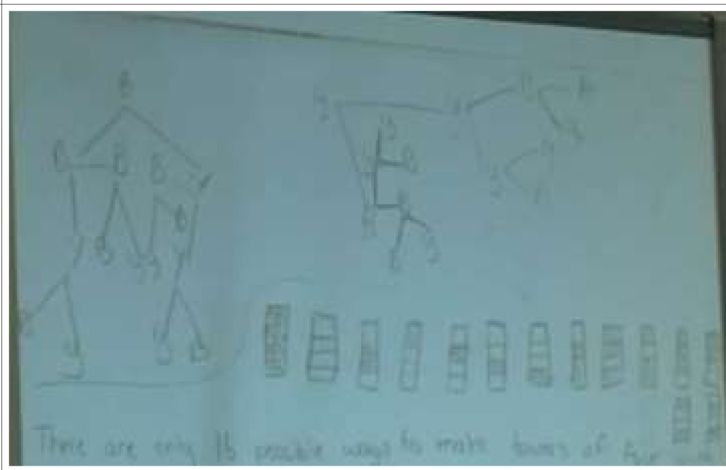
584.		KK	Every day?
585.		JL	It's how we give them an opportunity to be thinkers. Because they are thinkers. It's just sometimes we don't give them the opportunity to show us that they are thinking. <softly> I would like a copy of that can I see it a second
586.		A	Sure
587.		JL	Because that was beautiful. We'll Xerox it before we go.
588.		A	Okay
589.		JL	Okay. Alright, let's look at what Justin's youngsters did.
590.		J	So, what
591.		JL	Don't tell us, we've got to look.
592.		J	Okay
593.			Haha
594.	43:57	JL	Because sometimes, we take a while to decipher it.
595.		J	I just noticed that I should have picked another one
596.		JL	Do you want a different one.
597.		CM	He has two. He has the partner's also
598.		JL	Is the partner's good? Is the partner's different than this, the recording
599.		J	Alright, let's see.
600.		JL	Well, let's do this and then the partner's
601.		J	Yes

602.			
603.		JL	Okay, so what did this youngster do? How did he organize his towers?
604.		J	Oh, it was a she.
605.		JL	What did she do?
606.		J	Yes
607.		JL	Okay. I can tell by the handwriting.
608.		J	Yes, bubbles
609.		JL	Someone in the group. How are they organized?
610.			Opposites
611.		KK	Well, it looks like opposites
612.		JL	It does look like opposites, doesn't it? And, read to us what they said.
613.		J	Okay, well, she writes: "there are sixteen different patterns with four cubes used. We would make a pattern and then vice versa"
614.		JL	Haha "vice versa" good.
615.		J	Yeah, "or change the color of the cubes in place. We tried every possible way there is and there were no more patterns to do. Which means we came up with them all."
616.		JL	Okay, so they're convinced.
617.		J	Yes
618.		JL	So they "vice vers-ed" it.

619.		J	Yes.
620.		JL	And that's, You chuckled at that.
621.		J	Yes, haha
622.		JL	And you know, children inventing language, it's really neat to see what they say, as long as we can figure out what they're saying, it's really okay. I mean, these are middle school children, right? So, it's better that they understand what they are doing. If they can give language to it, that's good.
623.		CM	Don't you all love how they use the notation? Notice how they're not even drawing all the towers.
624.		JL	Yep, absolutely.
625.		CM	They have gone past the physical model, they've gone past the picture of the physical model.
626.		JL	Absolutely
627.		CM	And now they're using their own notation.
628.		JL	Absolutely,
629.		CM	That's what was interesting with the other student,
630.		JL	Uh huh
631.		CM	Because it isn't quite the same representation and notation, is it Justin? One is
632.		JL	Could we see the other one?
633.		CM	One is vertical
634.		JL	Oh
635.		CM	The other one
636.		J	Horizontal
637.		JL	Yeah, these actually could look like the towers, right?
638.		CM	Yes, but look at the other one.

639.			
640.		JL	But see what the partner did. And that's why we want both people to record. No look at this the partner is also using letters to show the color of the cube, but they're going horizontal, okay.
641.		J	Now, it was interesting, because I kind of fed them some stuff, like I had put on the board. Because they said, like "how do we record the towers?" I said you have to record them. So what I did was write like a box, a square, and I just shaded one and left the other one
642.		JL	Oh, you gave them a hint
643.		J	But now, the fellas I had, they used
644.		JL	Oh, okay
645.		J	They used what I had on the board.
646.		JL	Okay, they chose not to
647.		J	And you know, yeah, oddly enough they just
648.		JL	Good, good, good
649.		J	Did their own thing, without being
650.		JL	Excellent, and I think it's really nice when you let children have the opportunity to invent how they want to

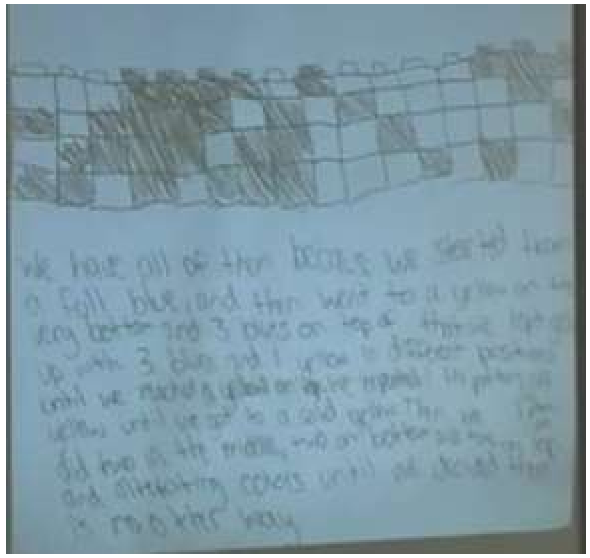
			record their towers. Did any of you actually use the words "blue" and "yellow?" Did any of your students write out the words?
651.			Um, I think, yeah
652.		JL	Now, okay
653.			That's what I have
654.		JL	Because sometimes they do that as well. Neat, thank you, Justin. Do we have another paper? Another class? Good Kulsom, It's Kulsom, Kulsom
655.		K	Kulsom, yeah. Like "Wholesome" but with a "K"
656.		JL	I remember you talking about it. Right, that's something to help me remember. Kulsom, okay
657.		K	Yeah
658.		JL	Give it time to warm up, turn it around
659.		K	Yeah
660.		JL	Got to go this way, it will warm up and again you're going to look at what Kulsom's students did and figure it out, and then she's going to talk about it. Magic. How many of your children used factor trees? Anybody? Yeah,, we're going to see one here. Okay. Lots of different ways to record. Hi Cindy
661.	48:05	CH	So much for being unobtrusive.
662.		JL	It's okay, we saw you come in, welcome. Okay, this a great tool, we were saying, to show the student work without using an overhead. It just takes a little time to warm up. See, now the top warmed up. Looks like it's never going to come. Here it comes, here it comes, magic. No, it doesn't want to come. Well, it is written in pencil
663.		KF	Here it comes
664.		JL	Okay, it is coming, okay, we'll be patient. Because the printed words, the ink. Okay, good. When you have your students working, in your class, if you don't have this tool, to share the work, What you're going to need to do is, once you've picked what student work you want to be shared with the class, you're going to have to make overheads for them.
665.		KF	Slide it?
666.		J	Yeah

667.		JL	Okay
668.		J	Forward, and then I think that's the. I think it picks up on the black ink I'm assuming.
669.		JL	I don't know. This one doesn't want to pick up. What a shame, because it's a neat paper. It's a neat paper?
670.		RB	What if you were to cover the ink with like a piece of paper on the top, then like would that pick up the…
671.		JL	We'll see. Okay, we're going to trick him, huh is that what we're doing? We're tricking technology. Let's see if it works. Yeah, pencil is hard to come up. We also don't want students erasing their work. Sometimes it's we want to let them work in pen or a black, not a magic marker, it's too thin, but like a black felt tip pen, if you have one of those in your room. Or even a ballpoint pen. Because work like this doesn't want to be picked up for whatever reason.
672.		CM	Before in research, we would always use
673.		JL	Oh, ah there we go
674.	50:00	RB	See, you tricked it
675.		JL	Oh, look at that. Okay.
676.			There we go
677.		JL	It got scared I was going to take it off.
678.		RB	She did say that.
679.		CM	It didn't like the trees
680.			
681.		JL	No, It didn't like, but I liked it, Okay, take a look at the tree diagram, also look at the unifix cubes there. There are two different ways of showing what they did. Look at the trees and see if you can figure out what they are

			doing.
682.		S	I see the trees are a little crooked.
683.		JL	The trees are crooked. Pick one of the trees and see if you could figure it out. Because it's neat what they did.
684.	5102	S	Okay, Even though it's crooked, it still works
685.		JL	Yeah, you have
686.		KF	It does follow
687.		JL	You, point and talk to them about it. Maybe we can help them decipher. Yeah, it's easier, it's actually easier to point. I'll hold it down so it doesn't move.
688.		K	Do you have a pen, have a pen or a pencil?
689.		JL	Okay, perfect
690.		K	I guess, even though it's crooked, you can
691.		JL	Yep
692.		K	Have a blue, and then you could go to blue or yellow, and then this blue
693.		JL	Uh huh
694.		K	Can go to a yellow or a blue.
695.		KK	Oh, I see.
696.		JL	Yep
697.		K	And then, I don't know is this connected to…
698.		S	Yep, and
699.		K	This yellow could go to a blue or a yellow.
700.		JL	Uh huh
701.		K	This could go to a blue or yellow
702.		JL	Uh huh
703.		K	So even though it's not the typical way we see a tree diagram, you could go that way.
704.		JL	Mm hm
705.		K	She had asked me, she said "Can I use a tree diagram?" I said, you can use whatever you need to do
706.		JL	Right
707.		K	To try to show us your explanation. So she drew the towers first.
708.		JL	Okay

709.		K	And she, as I was walking around, she did that.
710.		JL	Okay
711.		K	And the extension problem, she used a diagram also to do the three tall towers.
712.		JL	Okay
713.		K	But she didn't have enough time, they didn't have enough time to do the second problem.
714.		JL	Okay, what did she write? You want to read that?
715.		K	She wrote: "There are only sixteen possible ways to make towers of four with the colors. I first tried all the combinations that entered my mind. And I checked by using a tree diagram. I put titles of 'B' and 'Y' I put a 'Y' and a 'B' under each of the titles. And then I put a 'B' and 'Y' after the new four 'B's and 'Y's. Under each brand new 'Y' and 'B' I added in the last pair of 'B's and 'Y's like the other three times. The diagram shows every possible combination for a four block tower with two colors. I counted the combinations and found out there were sixteen possible combinations"
716.		JL	Now, when you look at her tree diagram, she doesn't really have labeled how many towers or how many combinations she has.
717.		K	Right
718.		JL	So that would be something, I've actually seen kids do tree diagrams and actually show you which of the towers, and you can see.
719.		K	I'd like to share this one.
720.		JL	Okay, this is interesting. This is, remember, we told them, they shouldn't be solving.
721.		K	Right
722.	53:00	JL	But this
723.		K	This kid, she wanted to
724.		JL	But she wanted to, when we made predictions for what do you think towers three tall would be. Fewer or more towers than you had for four tall? And she decided even though she was told not to solve it, she decided she would go ahead and figure it out.
725.		K	And this time, I guess she used, like one is the one color and then two is the second color.

726.		JL	So she chose, isn't that neat. And that's interesting too, the way that she is coding her stuff. Right, she's saying it doesn't matter what the colors are, this is your first color, this is your second color, and then from that first color, you can either put on top if it the second color or the first color. And then on top of the second color, you could even put a first color or a second color. That's very interesting. Very neat. I don't think I've seen that before. Isn't it fascinating, how you can do this a million times and keep finding new stuff. Okay, and she said there will be fewer possible combinations because there is a shorter amount of blocks in the tower. Most of your kids said that, I think you wrote to me on eCollege, yes?
727.			Right
728.		JL	Okay, very nice, thank you. Who else hasn't shared?
729.		S	I haven't gone
730.		JL	Come on
731.		S	I was going to pick other ones, but you guys kind of showed the same things. So I looked for a different one.
732.		JL	And it's good not to, and it's good not to keep showing the same thing when your kids do it. If you keel showing the same thing, they're going to doze out on you.
733.		S	This one is interesting.
734.		JL	Good. We like interesting
735.		S	You came to my class and
736.		JL	Yes
737.	54:30	S	Told me to comment on one group, instead of using the word "opposites" they chose to talk about towers by saying the "positive" and the "negative" option of it. So that was interesting, but she didn't actually write about it.
738.		JL	Okay
739.		S	On the paper. This is something totally different.

740.			
741.		JL	Okay, so let's look at the towers that are built there and see if you can figure out. It looks like one big line of towers. See if you can figure out if there's a strategy that was used to go from the first tower to the second and so on.
742.	55:15	JL	Carolyn sees it, but we're not letting her tell us
743.		CM	I'm not telling
744.		JL	Okay, We want one of us to see something that is really neat. And it, look, you like those little towers
745.			At the top
746.		JL	Little top. That's cute?
747.		CM	They have the chimneys
748.		JL	Yeah, so you really know that that's the top of the towers. Angela, do you see something?
749.		A	I guess I'm misinterpreting it. Because I read
750.		S	The bottom?
751.		A	The explanation
752.		JL	Okay
753.		A	And to me, I can't follow that explanation. Like they said they started with all blue,
754.		JL	Okay
755.		A	And I'm trying to find the all blue
756.		KF	That's the fourth one in
757.		S	

758.		A	Yeah, I feel like I'm like.
759.		JL	Okay
760.		A	Right, that's why "Where did she get this?"
761.		S	Yeah, haha
762.		A	Now I'm like "okay, I don't get it"
763.		JL	Okay, well, let's say if they started here, they didn't draw it as the start.
764.		A	Yeah, maybe they didn't have enough room and they went on the other side.
765.		JL	Where did they go after this tower?
766.		S	They did have it standing like that, though on the, their desk.
767.		JL	Okay, this is exactly the formation?
768.		S	Yes, yes
769.		JL	Okay
770.			Four, three
771.		JL	And then they went to a yellow on the very bottom and three blues on the top. Which is that one?
772.		S	Right next to it.
773.		JL	That's
774.			On the right
775.		JL	This one
776.		S	Yeah.
777.		JL	Okay
778.		KK	So they did four, three, three, three. So they did the full color and like the staircase color
779.		JL	Okay, so you see the staircase, your elevator
780.			
781.		JL	Okay, let's see where it is. It's here, right?
782.		KK	Yeah
783.		JL	Okay, and then going down. And I'm going the wrong way. Okay, this one, this… This one, this one, this one, this one!
784.		S	Right
785.		JL	Okay, then what?

786.		KK	Then, the oppo… oh, where's the all yellow?
787.		JL	There
788.		KK	It's all the way on the other side.
789.		JL	All the way over there.
790.		KK	It almost seems like they did that first and then over
791.		RB	They did all the towers over there
792.		KK	A mirror image of it
793.		JL	Uh huh
794.		KK	Right.
795.		JL	Uh huh, okay, so then you have the three yellow and the one blue.
796.		KK	Uh huh
797.		JL	Okay. What else?
798.		S	So yeah, they really did start, and then they went all the way to get to the yellow. Or
799.		JL	This solid yellow?
800.		KK	And then did combinations of the two, I guess.
801.		S	They matched it, it's like, it's like
802.		A	It is, like what Rich said, the mirror image.
803.			Yeah
804.		JL	What do you mean, "The mirror image"? talk to us about that
805.		A	These two have that
806.		KF	Like those and those. Go ahead, Rich
807.		RB	I guess like a butterfly wing.
808.		JL	Okay,
809.		RB	Well, I guess I kind of heard Carolyn say,
810.		JL	You heard "symmetry"
811.		RB	I heard symmetry, but if you look at both sides, starting with the three blues and the one yellow. Then you have the three yellows and the one blue.
812.		A	Yeah, I see it.
813.		RB	And then you work your opposites on the way out.
814.		KK	And come out, yeah.

815.		RB	And it's a mirror image, so
816.		KK	Yeah
817.		RB	But if you were to literally flip one over on top of the other
818.		JL	Okay
819.		RB	They would line up as opposites
820.		JL	Okay, so the middle is the symmetry is when you go out from the middle to either side.
821.		RB	Yes
822.		JL	And that's pretty neat
823.		KK	Yeah
824.		JL	Isn't that
825.		KK	That is interesting
826.		JL	Pretty neat
827.		J	I think something too, that when I look at this
828.		JL	Yep
829.		J	I look at this and I kind of go back to the reasoning of one of my other students.
830.		JL	Uh huh
831.		J	Because he, like one student I had, of course it was two pairs but one guy grouped them up a certain way, but the other guy was a little overbearing
832.		JL	Okay
833.		J	Like, said: "No let's group it up this way"
834.		JL	Okay
835.		J	But you see how, if you go to the left
836.		JL	From where? Go to the left from where?
837.		KK	Of the center, probably
838.		J	Go to the left of, or no, starting from all the way to the left of. All the way left. Like you see how it goes with the elevator almost, with the dark block and then two dark blocks
839.		JL	Uh huh
840.		J	And then
841.		JL	Ah

842.		J	Like, almost like a staircase, but we know it's not the staircase method. And then the other side kind of goes with the same type of symmetry, the staircase. So that they almost did like the two, like the four rows of staircase as like patterns in their groupings. Then added them together.
843.		JL	Okay, so you called it "symmetry" you called it, What did you call it?
844.		RB	The mirror
845.		JL	I liked it
846.		RB	A mirror image
847.		JL	A mirror image
848.		CM	Butterfly, I liked that one
849.		JL	Butterfly, really neat, huh. Thank you so much, this is really neat. There's a lot of neat stuff, huh? Who else has to go? Everyone went? Excellent. What we're going to do now, and remember, we're not taking any break because we want to get done for Rich to leave, to go to his next commitment. But if you feel you need to a take a break and use the ladies' room, mens' room, or take some brownie feel free to. So what we're going to do
850.			
851.	59:34	JL	Towers five tall and you're going to work in pairs or as a triplet. Okay? So let's get some unifix cubes and begin.

10/7 Meeting transcript 2 of 3

Title: 10/7 Judy's Class 2 of 3
Location: Oldbridge
Date: 10/07/2010
Length: 01:35:55
Transcribed by: Will McGowan May 2012
Verified by: Maddie Yedman

Line	Time	Speaker	
1.		RB	How come it's got a Yankees emblem on it?
2.			
3.		A	It's my good luck charm
4.			I hate these bags
5.		M	Uh, no
6.		JL	She's part of the grant. Carolyn is the Grant.
7.			Cindy is, Cindy and Carolyn are the grant
8.			
9.			Probably, probably Well, Linda, Linda
10.	00:32	RB	We should have thirty two
11.		A	Right
12.		RB	It's like this: one, two, three, four, five
13.		K	So, how would you want to start this?
14.		RB	All the same I guess
15.		K	Alright
16.		RB	How's that?
17.		K	It's good.
18.		JL	Your partner's
19.		KK	She went ahead
20.		RB	Now one more?
21.		JL	You're going to, absolutely
22.		RB	And now with four
23.		JL	And I'm coming to do
24.		K	You'll do four blues one yellow?
25.		RB	Four blues one yellow.
26.		JL	In that case, she may not hold

27.		BR	You do four yellows, one blue?
28.		A	How are you guys organizing? Could you tell me?
29.		RB	Okay, we did five of the same.
30.		A	Okay
31.		K	Let's move, let's move this
32.		A	And it's opposite or no?
33.		RB	Help yourself
34.		K	Yes
35.		RB	I'm still doing four and one
36.		JL	Just in case you need it
37.		KF	Okay
38.		A	Thank you
39.		K	And then just put it in the second position
40.			He was testing
41.		RB	Hm, and then make one at a time
42.			I think you're safe
43.		RB	Whoa
44.		K	Haha
45.		RB	Don't go too fast.
46.		K	Okay, sorry
47.			I want to see, like
48.			Okay
49.			October eighteenth,
50.		KF	Now we'll do blue in a third position
51.			I'll send those emails to you. I should just give them to you, rather than
52.			Right.
53.			Putting it in the mail
54.			Okay
55.		JL	Okay, when you feel that you have a convincing argument, invite me over. But I'll keep circulating, and any of our guests that would like to circulate and see what's going on, feel free to. Okay?
56.			What's we're supposed to

57.			I feel like
58.		JL	If you want to look and see what's going on,
59.		A	She needs us to personally
60.		RB	Okay
61.		A	Seems so much
62.		RB	We'll stop right here. They're all organized. So Angela can catch up.
63.			Yeah
64.		RB	Okay, Ashely, what we did, so far.
65.			You want to see here?
66.	2:00	RB	We started with five blues and she started with five yellows. Then we went with one yellow,
67.		K	See this butterfly technique?
68.		A	Yeah
69.		RB	Well
70.		K	Staircase
71.		RB	Okay
72.		A	Okay, so now we have all of the towers
73.		RB	With one
74.		A	That are either solid
75.		RB	Or with one
76.		A	Or four of the same color.
77.		RB	Yes, so now we're going to go
78.		A	So now we're going to work with three?
79.		K	Yeah
80.		RB	Well, you. Yeah, I would go three, and two
81.		A	And how are these, like so, starting with three on the bottom, is that how we're starting?
82.		RB	Three on the bottom, yes.
83.		A	Okay. And then solid two on top?
84.		RB	Well, three and then they'll go, the two.
85.		A	And then
86.		RB	The two in the middle.
87.		K	Oh, wait

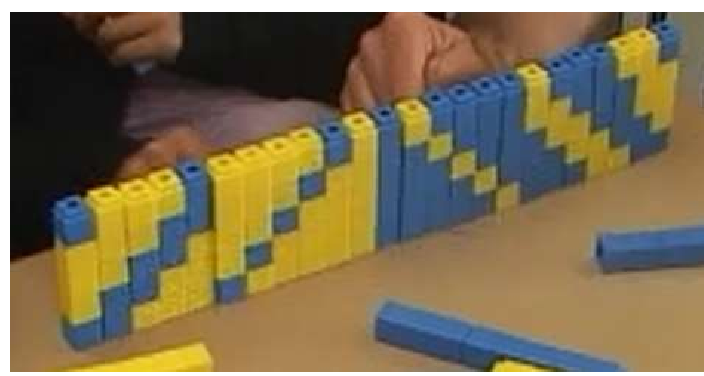
88.		RB	Is that good?
89.		K	You're doing something else, I would put
90.		A	Could put this on the bottom? Bottom, bottom, bottom, bottom here, or?
91.		K	I would, I would s…
92.		RB	Yeah
93.		A	Oh, yeah
94.		RB	That will work too.
95.		K	I would not, I would not
96.		A	Do it. I don't know if I would
97.		K	See, how we moved this one in all the directions? Now we should start with like, two
98.		RB	Yeah
99.		K	Two blues
100.		A	From this
101.		RB	Look, we went to one yellow here.
102.		K	Right
103.		RB	Okay? Now we go to two yellows.
104.		K	Oh, you want to move it down?
105.		RB	Yes. See?
106.		K	Okay, let's move it down. Okay, he wants to move these two progressively down.
107.		A	So, two there?
108.		K	Right
109.		A	One there.
110.		RB	See?
111.		A	So, let me
112.		RB	See this. See, now we'll go three yellows, and you'll go three blues.
113.	3:26	K	Yeah, but remember Rich,
114.		RB	Oh, wait hold on, hold on.
115.		K	Remember how now, this two, one will be towards the bottom
116.		RB	Yes

117.		K	And then one will get pulled back to the top.
118.			
119.		K	Okay, you don't want to do that.
120.		RB	Oh, I see what you're saying.
121.		JL	Tell me again, what did you just say?
122.		K	Like I was saying, since these two, right?
123.		JL	Okay
124.		K	Now you want to move those two down to the next level.
125.		JL	Okay
126.		K	So one would be on the bottom, and then one would get forced back to the top, since you can't have
127.		RB	Or do you want to continue with this pattern and then go back to that?
128.		JL	Why, why would that happen?
129.	4:00	K	Well, I'm saying, if you kept going down, you can't obviously, there's only one more place for this… Oh wait a second.
130.		RB	Yeah, look.
131.		K	Oh, sorry, that's not what I want.
132.		JL	Is that what you wanted to do?
133.		RB	Two blues, three yellows.
134.		K	That's what I, that's what I thought, that's what I meant, sorry
135.		JL	Ha haha
136.		RB	Two blues three yellows.
137.		K	The next part, the next part.
138.		JL	Okay
139.		RB	You got this?
140.		K	Okay, sorry, sorry, sorry.
141.		RB	Okay
142.		JL	Okay, so that, now that… So what did you say after that, though? You did say something.
143.		K	I'll take it back. I don't know. Haha
144.		RB	Now she said that
145.		K	I was, I was saying that

146.		RB	Now you want to split them.
147.		K	That I would split it because
148.		JL	How will you split that?
149.		RB	A yellow on the bottom, a yellow on the top. Three in the middle.
150.		K	Well, because there is two. This one would have
151.		RB	So she would go a blue on the bottom, a blue on the top, so she's making
152.		A	Why?
153.		K	Because now these two, right?
154.		RB	Because
155.		A	Because this is moving?
156.		K	Yeah
157.		RB	We're, we're still working with two yellows.
158.		A	So it's moving down, down, down, down
159.		K	Okay
160.		RB	And we're still working with two yellows, okay. So we're good now?
161.		A	And now what? Move this down one?
162.		RB	One on the
163.		A	Put the one yellow on top, and have this
164.		RB	One on the. One here, two here now. Wait, no
165.		JL	Are you doing the mirror image of what she just did?
166.		RB	I guess
167.			Yeah, he's working on the blue side
168.			Okay, aah
169.			So we split it
170.		A	The original blue side
171.		RB	So now we split it
172.		JL	That was the blue side okay
173.		RB	So, wait a minute
174.		JL	The original blue side
175.	5:00	RB	Wait, yellow on the top. That's what we're starting
176.		K	Yeah, but look, look, look. Now this blue got moved to

			the next part, it would be this one again.
177.		RB	Okay
178.		JL	So now what?
179.		K	Well I guess
180.		JL	Wait, but Rich, do you agree?
181.		RB	No I don't because, because, I think we could have.
182.			I don't know if, I don't
183.		RB	I think we could have something like this, hold on
184.		JL	You better say again
185.		RB	Yellow,
186.		JL	Convince him
187.		RB	Blue
188.		K	But we've been constantly, move this, these two blues, right?
189.		RB	But couldn't we have this? And now two blues below it?
190.		K	Yeah but, I'm just saying, If we're continuing like this train of thought right here, like these two, these two, these two, these two, these two. Now this one went down here, and this one went to the top.
191.		RB	Yes
192.		K	And now we move this one back up to the top we would have this one again. Like, do you see what I'm saying?
193.		A	Yes, I do
194.		RB	Okay
195.		K	So
196.		JL	Do you agree?
197.		RB	Yeah, I agree
198.		A	Okay, so
199.		JL	Okay
200.		A	If that's the case, then where do you progress to next?
201.		RB	Now we go three yellows, to follow the pattern. Because we had, we had, she would go three blues. I would go three yellows, because this is one yellow
202.		A	This is
203.		RB	Two yellows, now three yellows on the top.

204.		A	So, in essence, I'm like. I guess I'm thinking about this. Okay, four yellows, three yellows, so I'm moving to two yellows?
205.	6:01	K	Yes
206.		A	Okay
207.		K	Or, three blues
208.		RB	And
209.		A	And how do you start it
210.		K	Them on the top, top.
211.		RB	Three yellows on the top
212.		A	Two yellows on the top?
213.		RB	Three yellows
214.		K	Three yellows on the top, because, see: Here we started with the top. One
215.		A	I totally don't think of it like this.
216.		JL	Well, Angela is not following
217.		A	Yes
218.		JL	So if you guys want to convince her
219.		RB	Yes. Hey, watch Angela
220.		JL	Talk to her about it
221.		RB	Angela, look. Okay, let's start over. We started here, with all. Then we went with one yellow,
222.		K	Or one blue
223.		RB	One yellow on the top. She went one blue on the top
224.		A	Got it. And then
225.		A	It moves down.
226.		RB	My one yellow went one down.
227.		A	I totally understand that part.
228.		RB	Then my yellow, her blue moved one down.
229.		A	Yep, got it, got it, got it.
230.		RB	Okay, so after we have exhausted all the yellows like that, we went two yellows at the top. She went two blues. So then the two blues went down one position.
231.		A	I totally understand where we're going from here
232.		RB	Okay, Okay, so and then it goes down. But then it needs

			to be separated. One blue on the top for you, one blue on the bottom for you with the three in the middle. Okay, and then Kulsom said that it would bump back up…
233.		A	Yes, and I understand that.
234.		RB	Okay. So we exhausted all the twos, so now we're on threes. So we went one yellow.
235.		A	Wait. Did we exhaust all the twos?
236.		RB	Yes, um
237.		A	No, right? Because what about when it's split up?
238.		K	When
239.		K	That's where I get to
240.		A	Because that's where I get confused
241.		RB	We're going to get to that
242.		A	When?
243.		K	When do you want to get to it?
244.		A	See, to me
245.		JL	Haha
246.		RB	I think we're going to get to it real soon, because
247.		A	To me, I feel like I need to follow such a systematic approach
248.			
249.	7:20	RB	Okay, Angela, We're going to get to it now, because She's actually making the opposites of what I'm making. Look what happened. I have the three yellows here, where they're separated.
250.		A	So I guess, what is my justification for moving from here to splitting it.
251.		RB	We actually made a duplicate. We, I
252.		JL	What is the duplicate that you just made

253.		RB	We just made a duplicate right now
254.		JL	Oh, oh look at that
255.		RB	See,
256.		JL	Ah, interesting
257.		RB	We just made a duplicate, so we will get to that
258.		K	Wait, why did
259.		RB	Because I moved down one position. I can't go down one position. I need to go down two positions to avoid that duplicate.
260.		KF	But you can't
261.		RB	Nope
262.		JL	Because you have that one there
263.		RB	So that's a duplicate as well
264.		K	No, wait, I'm not really understanding this
265.		RB	Okay, look. Kulsom, watch what happens. If we follow the pattern,
266.		K	Adding more
267.		RB	These three bump down one. It's a duplicate right there. If I follow it one more, where I put the three on the bottom, and the two on the top, it's a duplicate to this. So we're done with the threes.
268.		K	Or, now like what Angela said
269.		A	Put mine in
270.		K	Let's put them, let's move them, like they're not separated right here, at all. Like these two blues.
271.		A	Yeah
272.		JL	Show him what you mean
273.		K	Like, I
274.		JL	Okay
275.		A	I mean like this.
276.		K	Yeah
277.		JL	And what do you think of that. Rich?
278.		RB	I think it's going to, I think that's going to duplicate one of mine.
279.		JL	Well

280.		RB	Eventually, eventually.
281.		JL	Let's see if it does. Does it?
282.		RB	Not yet, not yet
283.		JL	Not yet
284.		A	Can you build an opposite- Whoa! I'm sorry
285.		JL	You know what you might do?
286.		A	Lay them flat
287.		JL	If that, it is easier so they don't just keep falling
288.		K	No, I'm just Klutzy
289.		A	No, no, it's a good idea.
290.		RB	I like the wall
291.		JL	You know what I'm saying? You like the wall
292.		RB	Yeah
293.		JL	Then you keep the wall
294.		RB	I like them vertically
295.		JL	Okay
296.		K	Let's move them.
297.		A	That's why I'm trying not to
298.		JL	Move them up so they don't flip over, because that's… Okay.
299.		RB	So I have to make the opposite of that, so it's blue
300.		KF	Sorry. Thank you
301.		JL	Okay
302.		RB	a second
303.		K	It's not
304.		RB	Position
305.		A	Is that blue?
306.		JL	Move this out of the way
307.		A	I can't follow.
308.		RB	Okay
309.		JL	The desk is slanted.
310.		RB	So that's the opposite of this.
311.		K	I'm going to move this back here.

312.		A	Yeah, no? Yeah, that's the exact same as this
313.		K	How many do we have so far?
314.		A	That is the exact same as this one.
315.		RB	Oh, Is that. Wait. Let me see that again, so I
316.		A	It was like this, but the problem is I have to put it back to where
317.		KF	It was this.
318.		A	It was, so let me build that one.
319.		K	Where was it?
320.		A	But, I can't justify in my head
321.		RB	You have to move that one down
322.		A	Why it was like this.
323.		K	Can I have more blue
324.		JL	Oh, I'm sorry,
325.		K	It's okay
326.		JL	I thought I was helping.
327.		RB	So, let me go again. Let me see what she made, I gotta make the opposite of that, or did I?
328.		K	This needs to be moved over, right?
329.		A	No, you made the same as me. I don't
330.		RB	So, I need, I need to see that. So I need blue, yellow, blue, blue, yellow. There, I just made the opposite
331.		A	Can you explain to me, like this was just random in my head
332.		RB	So, we need to shift this down
333.		A	So now we're keeping the one on the bottom, it stationary?
334.	10:00	RB	So now we need this, yeah, the one on the bottom would be stationary. So now we need this. I shifted down one more because I still have the two.
335.		A	But you shifted down one more.
336.		RB	I shifted the
337.		A	But you added a blue
338.		RB	This yellow
339.		A	One more

340.		RB	I left the yellow at the bottom alone. I'm not moving these.
341.		A	But you didn't really shift down, you actually added
342.		RB	I shifted the top. No, they're still there
343.		A	No, because what about this one, Rich? I'm thinking about this one right here <builds tower>
344.		RB	Mm hm. That's what I just made. That's the opposite of the one I made.
345.		A	Oh,
346.		RB	Yep, because the, your blue stayed the same
347.		A	Oh, so
348.		RB	My yellows stayed the same.
349.		A	Yes, yes, yes. Okay, I'm comparing
350.		RB	And now, if I go one more down, I can't because that would duplicate um, this one.
351.		JL	What do you mean "one more down"?
352.		RB	If I move this yellow down to this position,
353.		JL	Okay
354.		RB	I change the position, I switch the blue with the yellow
355.		K	The yellow would be
356.		RB	I just duplicated this one.
357.		JL	Okay, so
358.		RB	So I can't
359.		JL	You don't want to duplicate it.
360.		RB	No.
361.		JL	Okay
362.		RB	So I've exhausted all of the twos at this point. And they've exhausted all the twos over there.
363.		JL	Do you agree with that?
364.	11:00	RB	Angela, if you were to bump this down and switch the two, like move that…
365.		A	I, if I were to bump this down and move the yellow, switch these two positions, yes
366.		RB	You would get a duplicate.
367.		A	I would have a duplicate. I totally agree with it.

368.		RB	Yes
369.		JL	Okay
370.		A	I have to think about. Why did you guys do this?
371.		K	I don't know, I guess for me it makes sense, like to put it in each position to exhaust the, It's like all those options are done if I have a one yellow and move it into
372.		A	Why there? Why not the bottom up
373.		K	Just
374.		A	Preference?
375.		K	I guess
376.		RB	Well, I was doing the opposite of what she was doing.
377.		K	No, but she's like "Why did we start at the top, and not the bottom?"
378.		JL	Does it make a difference? If she started at the bottom?
379.		A	No
380.		JL	Okay
381.		A	I guess, I don't think like this at all.
382.		JL	Okay, So is it, You would have, you would have done this grouping by starting the yellow at the bottom? Is that what you're saying?
383.		A	No,
384.		JL	What would you have done?
385.		A	I'm saying, like I'm, I drew a tree diagram.
386.		JL	Oh,
387.		A	That's the way I did it last time, before we started, and that's the way like,
388.		JL	So you,
389.		A	Whenever someone asks me that, like I always think in that
390.		JL	You think tree diagrams
391.		A	I think
392.		JL	Now, when you did tree diagram, how many towers did you find?
393.		A	Thirty-two
394.		JL	Okay and you're hoping they're going to get thirty two.

395.		A	Uh huh, and I'm trying to understand a different way of thinking.
396.		JL	Good, good.
397.		A	Because when I attempted this when, on September eleventh, I did the same exact thing
398.		JL	Okay, okay
399.		A	Because it justifies it in my head. If I start with a blue, I could have a blue or yellow
400.		JL	Okay, and it's real important to
401.		RB	Because you write these out.
402.		JL	What you guys did is very good. You do
403.		RB	Figure out these as we're working.
404.		JL	You do want to understand it
405.		A	I do.
406.	12:22	JL	This is not the way you think about it. But you do want to understand what they're thinking about, and hopefully, they're going to want to understand how you're thinking about it, Okay?
407.		RB	I think we're going to have some duplicates here with the fours.
408.		JL	So, let's see, so they're still going with this strategy. See if you can, If you're understanding it, if you can help them
409.		K	Do we have this already, Rich? Three blues, no, right? No. So this is one for three blues
410.		RB	No, yes. So you need one of that for the yellows.
411.		JL	So you took away that one, why did you take that away?
412.		RB	Because we didn't get there yet
413.		JL	Oh, Oh
414.		K	I just looked at it, I just moved it.
415.		JL	I see, I see.
416.		A	And that goes here?
417.	13:11	K	Two, four, six, eight, ten twelve, fourteen, sixteen, eighteen, twenty, twenty-two, twenty-four, twenty-six, twenty-eight,
418.		RB	Okay

419.		K	And we still have to do the ones with four.
420.		RB	Okay, so I would have one yellow, and four blues.
421.		A	And now you're just starting this from the top? And then just shifting it down?
422.		RB	Here's some yellows for you.
423.		A	Thank you
424.		RB	We're going to shift it down but
425.		JL	You're not convinced about what's going on
426.		A	No, I don't know
427.		RB	Not yet, let's see
428.		JL	No, she doesn't look like she's happy with what's going on.
429.		A	Ha ha
430.		JL	You've got to talk up, talk up.
431.		RB	I think we're going to duplicate here
432.		K	We are too, we are duplicating
433.		RB	This duplicates your this.
434.		K	Because the one over here is four over here
435.		A	This duplicate this
436.		RB	This duplicates this
437.		JL	What did you say?
438.		K	Like, his one yellow and four blues,
439.		JL	Right
440.		K	Is my one blue and four yellows, so
441.		RB	Yes, so we're duplicating it
442.		K	Duplicating those
443.		JL	Okay
444.		AA	Hey Judy.
445.		JL	Yes
446.		AA	We're going to go
447.		JL	You're going, okay
448.		AA	And enjoying you all
449.		JL	Absolutely,
450.			How are you planning, haha

451.		JL	Yeah
452.			It is a shame
453.			Yeah, oh well,
454.		AA	The lesson study days, visit across
455.		JL	Yeah
456.		AA	The groups
457.		JL	Sure, but I think that we're happy that it's working the way it is. Thank you for coming. You're staying?
458.		CH	Yes
459.		JL	Perfect. Perfect, perfect. They have a really interesting strategy
460.		AA	Thank you all
461.		JL	If you want to watch what they're doing.
462.		CH	Okay, yeah
463.		RB	We're not able to complete it
464.			Ha ha ha
465.		JL	Oh, you're not done yet?
466.		K	No
467.		JL	That's the hard part.
468.		A	Oh, God, this is like scaring me to anticipate what this is going to look like.
469.		RB	We need thirty-two. We know we need thirty two.
470.		A	I counted twenty-nine
471.		K	Why?
472.		A	Because we had one duplicate, remember
473.		RB	No, no, no, no. We started systematically.
474.		K	Remember when Rich and me built that same one?
475.		RB	We did. We did start systematically.
476.		A	Can we, oh.
477.		K	This is a duplicate right here.
478.		RB	Is it?
479.		K	It's one. Yeah. We have twenty eight so far.
480.		RB	Twenty-eight. And we need four more.
481.		K	Thirty-two. We did all the combinations with one, like all

			four combinations, like all four, three, two,
482.		A	We have to look at these again.
483.		K	Yeah
484.		RB	The twos?
485.		A	Yes
486.		RS	Where's the opposite for this one?
487.		K	This one.
488.		RS	That's the opposite of that?
489.		K	Oh
490.		RB	Oh
491.		K	My gosh
492.		RB	We have to take one of them away.
493.		K	Ha ha
494.		A	See, now there's my question: Why does that happen? See, I can't like
495.	15:36		Judy moves to Mitch and Jared's group.
496.	15:43	JL	You have twenty two?
497.		M	We'll try to figure out how many should be in each group. Like with one blue, two blues, three blues,
498.		JL	Okay, okay
499.		M	And
500.		JL	Right
501.		J	So count again. So we got one
502.		JL	How many should be in this group?
503.		M	Five
504.		JL	I see five, could there be a sixth one in this group?
505.		M	No.
506.		JL	Why not?
507.		M	Because every position, If this is only one blue, there's only one. There's five positions it could be in. So it's got to be in all five positions.
508.		JL	Okay, and that to you it's a trivial question. In fact, you're probably saying "What is she, off the wall, that she's asking this question"
509.		M	Right, okay

510.		JL	But you're going to ask that. Your children build this. You're going to say to them: "Could there be another in this group?"
511.		M	Right
512.		JL	How about a blue cube in the sixth position? And hopefully your kid's going to answer, what?
513.		M	It, then it would be six high.
514.		JL	Then it would be six high. Okay, so trivial for you, but important to ask of your children.
515.		M	Right
516.		JL	Okay
517.		J	Oh, they're perfect, because there's going to be ten. Would there be ten like this too?
518.		M	Uh, two high?
519.		JL	Ten like what?
520.		J	Five high? Um, no, ten.
521.		M	With two blocks, with two blues?
522.		J	With, three blues because it would be the opposite of this. So it would have to be ten three blues.
523.		M	It, If you're saying this is correct, then yeah.
524.		J	I'm exhausted
525.		M	Okay
526.		JL	You think you have all
527.		M	Yeah, so then, and you're, you got ten of these, we should have ten of the others.
528.		J	Yes
529.		M	We should have ten, ten, twenty
530.		J	Thirty two
531.		JL	You have all the, you built what, what are these?
532.		J	This is two blue
533.		JL	All towers with two blue
534.		J	Yes
535.		JL	And you think you have them all?
536.		J	I know I have them all.
537.		JL	Well, how can you convince me you have them all.

538.		J	Alright. Okay, so, Well, for one.
539.		JL	Okay
540.		J	For one, with the. I know that there's thirty two. total blocks
541.		JL	Okay
542.		J	And, I could reason by saying that there's the inductive type of reasoning as well
543.		JL	Okay
544.		J	Or you know, tree diagrams, I know that there's going to , like mathematically
545.	18:00	JL	So you're working backwards, okay. But I'm not convinced, How does that tell you you have these all?
546.		J	I'm glad you asked
547.		JL	Okay
548.		J	Um, okay, I definitely know, I know that there's no blue, that there's definitely no blue
549.		JL	What do you mean, "no blue"?
550.		J	Okay, here.
551.		JL	Ah, this is "no blue"
552.		J	Yes
553.		JL	Okay
554.		J	And I know tat this is exhausted as well. There's going to be, that's five
555.		JL	Okay
556.		J	There's going to be an opposite of those five
557.		JL	Okay
558.		J	Which gives us ten, well, if you count them together, there's going to be eleven.
559.		JL	What, Where will your eleven come from?
560.		J	I'm saying, so here, is one, is no blue, one blue. If we fast forward a little bit,
561.		JL	Right, right
562.		J	I can make this a four blue
563.		JL	Okay
564.		J	Which is the opposite of this

565.		JL	I'm following you
566.		J	Right, so that's eleven.
567.		JL	Oh
568.		J	We know that there's going to be a five blue, that's twelve
569.		JL	Oh okay.
570.		J	I come over here, and I have ten
571.		JL	Right
572.		J	That's twenty-two
573.		JL	Right
574.		J	And I know that me doing these, from me doing them,
575.		JL	Right
576.		J	With the controlling the variables
577.		JL	Yes
578.		J	Of each setting
579.		JL	Okay
580.		J	There's going to be another ten opposite this pattern. So this is two blue
581.		JL	Right
582.		J	I know that there's going to be an opposite pattern of three blue, but it's two yellow
583.		JL	Got it, got it. I'm following you. What I don't really, what I'm not convinced is, why you think
584.		M	Right
585.		JL	If you didn't know thirty two was the total you were aiming for
586.		J	Got it, Okay
587.		JL	Why is this all that there are?
588.		J	Yes
589.		JL	Okay?
590.		J	So, I know that I'm doing two blue. Start off with these for now.
591.		JL	You can help him.
592.		M	I'd rather make my own.
593.			Haha

594.		M	Still working on it
595.		J	But we're going to talk next
596.		M	We will, we will
597.		J	Like, once we get, He's
598.		M	I just want to, I just want to get all mine built
599.		JL	Okay, let's he wants to build his so that he has exactly two yellow.
600.		M	Right
601.		JL	I'm going to let him build.
602.		J	Then we could talk about
603.		JL	I'm going to let you two, good, good
604.		J	How you feel you have exhausted
605.		JL	Absolutely
606.		J	And then we'll build the
607.		JL	Convince each other
608.		J	Yes
609.		JL	And then call me back. Okay?
610.		J	Beautiful
611.	20:11		Switched groups
612.		S	Alright, so are these all
613.		KK	The combinations of three blue and two yellow
614.		S	I think we could check it by kind of blocking the rows. Or not. Because like
615.		KK	We have an odd number, so we're not.
616.		S	Yeah
617.		JL	Now, why does an odd number bother you?
618.		KK	Well because there should be, Maybe so, maybe not. They could because… Wait, this is yellow over here.
619.		JL	Kate, why did, why does an odd number, like you said, "Nine, that's not right." Why?
620.		S	Well, no because some of the patterns will overlap
621.		JL	Say it again.
622.		S	Some of the patterns will overlap, though.
623.		KK	Well, I'm thinking overall,

624.		JL	Right
625.		KK	I need to have a total of an even number.
626.		JL	Okay
627.		KK	Because there's two of each one.
628.		JL	There's two of each one, What does that mean?
629.		KK	For every tower, it has an opposite.
630.		JL	Okay, Okay
631.		KK	Which means there's two of each,
632.		JL	Okay
633.		KK	So I know we're going to end up with an even number
634.		JL	Okay, you're thinking it will be an even number, so why does nine bother you?
635.		KK	Because it's odd.
636.		JL	Yeah, but nine is
637.		KK	Oh, but eighteen doubled. Okay, nevermind
638.		JL	It doesn't bother you?
639.		KK	No
640.		S	Can we put this one over here?
641.		KK	Really, you can put it any way you want.
642.		S	Because I feel like this one beside the wall
643.		JL	Why did you want to move it?
644.		KK	Oh, you're moving them up?
645.		S	Because I'm trying to make sure that we have, like, accounted for. So this is where the two stick together and it goes up.
646.		KK	They're two together, two together, two together, two together.
647.		S	Then
648.		KK	Then we split them.
649.		S	Then we left the yellow on the bottom.
650.		KK	This moved down, this moved down, this moved down. And then
651.		S	And then that would be the one
652.		KK	That right there

653.		S	To finish it
654.		KK	That would be
655.		S	So that overlaps, kind of.
656.		KK	Right
657.		S	It, in our second, uh, yeah, I guess pattern. And then,
658.		KK	So you think we're done here
659.		S	This is where we keep the yellow on the bottom. What if we keep the yellow in the middle? In the, this one, and alternate where it goes here.
660.		KK	Okay
661.		S	So here's where we have one yellow
662.		KK	Now, don't we have some of that?
663.		S	In the middle, and one on top. And then we could do one with. Oh, I'm taking this one apart.
664.		KK	Here's
665.		S	The yellow in the middle
666.		JL	Do you need more cubes?
667.		S	And then one yellow here.
668.		KK	Oh, I have that one.
669.		S	Okay, then
670.		KK	So, we're keeping that?
671.		S	Yeah, and then the one, the only one left where it could either be here, or here is already over there.
672.		KK	We already have
673.		S	Because they're together.
674.		KK	Right.
675.		S	So then, if we keep the yellow in the middle and alternate spots, you have middle bottom, middle second spot, middle fourth spot, and then do you have middle and top? Like all
676.		KK	No, I don't think we do, Oh.
677.		S	Like, all
678.		KK	Wait, all blue?
679.		S	So it will be two blues on the bottom a yellow in the middle, and then a blue and a yellow on top. Oh yeah, it's this one, duh.

680.		KK	Ha ha
681.		JL	Ha ha
682.		S	Hah okay, so now we've got all the ones where we've controlled the yellow in the middle.
683.		KK	The yellow in the middle, through. And then we did this, did we, the yellow a second one? No. We need to check that.
684.		S	Right. So, this would be this one and the top, and then this one and then the middle
685.		KK	Below it
686.		S	And then this one and that one, and then this one and the bottom.
687.		KK	This one and the bottom. Okay
688.		S	Okay, what about,
689.		KK	What about this one and the top?
690.		S	Yes
691.		KK	Okay, we have that one too. Okay. And we have
692.		S	And since, yeah, and we have all the ones where the top is controlled, because top second top middle, top fourth
693.		JL	Point to it up here, so I can see. This is top second?
694.	24:00	S	Yeah
695.		JL	But what else is?
696.		S	Top second
697.		JL	Good
698.		S	Top middle
699.		JL	Okay
700.		S	Top fourth
701.		KK	Top fourth
702.		JL	Oh
703.		S	And then top bottom.
704.		KK	Top fifth
705.		JL	I can follow you, okay
706.		S	Okay, so we have all the ones with three blues and two yellows
707.	24:12	KK	Three blues

708.		JL	Are you sure
709.		KK	Yes
710.		S	Yes
711.		KK	Because we checked every spot that had every color
712.		JL	Okay, Okay And how many did you get?
713.		KK	We're, not every color. We checked every spot for the three, for the yellow.
714.		JL	Okay, how many did you end up with?
715.		S	Nine? Ten.
716.		KK	Six, seven, eight, nine, ten. See that was bothering me!
717.		JL	Ha ha
718.		S	Ha ha
719.		JL	Do you feel better now?
720.		KK	Yes
721.		JL	You do, okay
722.		S	But it's weird because, like you have a visual thing until you get, like here.
723.		KK	I know and then it changes. And then to continue your visual, you can move one to the other end. And it gives you a different visual, which is confusing. Right?
724.		S	We almost, I feel like almost a good picture would be to duplicate the ones but then put a line through them. You know what I mean?
725.		KK	Say that again.
726.	25:00	S	Like, you know how when you're like
727.		KK	Like the picture when you're writing them?
728.		S	Right, Like, you know how there's a visual pattern up to here,
729.		KK	Right
730.		S	But to keep the visual pattern you'd have to duplicate. But I feel like it would almost be easier to follow if you do duplicate them, and then just put a line through them. I don't know. I like see, I don't know. I write slow.
731.		KK	Right, right, right, right. I see what you're saying. I see what you're saying.
732.		S	But, so now should we match it up with like the

			opposites?
733.		KK	Yeah. So we got two where's this one. Two, two.
734.			Camera changes
735.	25:31	RB	No, just take away all five, because it was confusing with
736.		A	Because we said, "Let's do all"
737.		RB	Yes, yeah so you took away all five of, this is how we, we went in order. That was a
738.		A	So then what did I do? I did that one?
739.		RB	No. We um,
740.		K	Which one did we do?
741.		RB	We went to the other side then.
742.		K	Yeah we did. We started going on this
743.		RB	Yep, we did, we started going to the other side of the two lines
744.		A	With two blues
745.		RB	So, wait. Let's move these. Let's move these. Let's move these, and then we went to the other side of the staircase, because Kulsom said let's work on the other side. No, no! That's fine, and then these came to the other side.
746.		A	How
747.		RB	We started separating them.
748.		A	What for?
749.		K	I want to say
750.		A	What was the organization?
751.		RB	Blue on the top, blue on the bottom again. It was blue on the top, blue in the bottom, and blue in the middle.
752.		A	But why?
753.		RB	To keep them
754.		K	These are
755.		RB	Yes.
756.		A	I don't know if I see that. I don't know
757.		JL	Look at this interesting looking thing here
758.		A	Well, we can't
759.		JL	This looks different
760.		A	We came up with something that worked,

761.		JL	Okay
762.		A	But we're trying to go back and figure out why.
763.		K	This is not why
764.		JL	Now, what did you do? It looks different than when I was here last
765.		A	It is
766.		JL	Tell me why you wanted to change that
767.		A	I told them I couldn't grasp that.
768.		JL	Okay, so
769.		A	So I said "If I were to do this"
770.		JL	Good
771.		A	"without looking at this"
772.		JL	Good, good.
773.		A	What I would do, I would start with like say for example, let's build all of the one blue on the bottom.
774.		JL	Okay, okay.
775.		A	What we did. Well, we started with that. And then we got stuck for a little bit.
776.		JL	Okay
777.		A	So it's like
778.		JL	So these are all blue on the bottom?
779.		RB	Yeah
780.		A	Yes, well these are all. Yes. And it comes
781.		RB	These are one, one blues.
782.		A	And then we did like subsections.
783.		JL	Okay
784.		A	These have exactly one blue on the bottom.
785.		JL	Okay
786.		A	So we started here
787.		JL	Got it.
788.		A	And then we got stuck, and we're all just sitting here staring.
789.		JL	Okay
790.		RB	Wait, let me

791.	27:15	A	Let's build the twos.
792.		JL	Ah, Okay.
793.		A	And then threes, fours
794.		JL	Okay
795.		A	And then look
796.		JL	Isn't that interesting.
797.		A	And then we had
798.		RB	For the, for the missing ones
799.		JL	Five
800.		A	Yeah. And we had to come back and come back to the ones with the one on the bottom. And we figured out the missing ones, but
801.		JL	So do you have them all now?
802.		RB	Yes
803.		A	Yes, but
804.		JL	How many do you have right here?
805.			
806.		A	Sixteen
807.		JL	You have sixteen
808.		RB	Yeah, but, and then the opposites would make the thirty two.
809.		JL	You get the thirty two. Now this is very interesting. Now you are doing controlling for variables. Remember, we talked about that. A very neat strategy. Maybe we didn't talk about it.
810.		RB	I don't think we did.
811.		JL	When you, when you hold a row constant, you are controlling variables. And that's what they do in algebra, don't they? So this is a really neat strategy. You'll see kids doing this as well. Sometimes they'll control the solid blue line on the bottom. Sometimes they'll have blue tops.

812.		A	Mm hm
813.		JL	Okay? But that, this is really neat because what you have done also is you now have a double blue on the bottom. So now your constant is the double blue. Here's your constant is the triple blue, and now you have the four blue, and you have the five blue. Now my question to you is "How do you convince me you have all the ones that have exactly one blue on the bottom?"
814.		RB	Kulsom's our genius at this
815.		K	Ah, no
816.		RB	Yes you are
817.		K	No, yeah right. It was Angela's idea.
818.		RB	Yes
819.		A	But I'm still trying to come up with how we organized.
820.		JL	Okay
821.		RB	Well, we couldn't convince you of our strategy, so we had to come up with another strategy.
822.		A	True
823.		JL	Okay
824.		K	Which I like this, I wouldn't have thought of it this way.
825.		RB	Yeah, we wouldn't have thought of it this way either.
826.		A	See, but if you really go and think about it
827.		JL	This is really neat. This is very
828.		RB	Kulsom and I were on our own
829.		A	This is actually a tree diagram
830.		RB	We were having a hard time convincing her.
831.		A	You know what I mean. Like, in my head, for some reason
832.		JL	Okay
833.		A	I am tree diagrammed crazy. Because think about it: In a tree diagram, there's first block is always blue. So to me, I'm like "Alright, let's start blue."
834.		JL	Ah, So that's, but yours are different than a tree diagram, because it's a double blue, a triple blue
835.		K	Yeah
836.		A	Yeah, true, true.

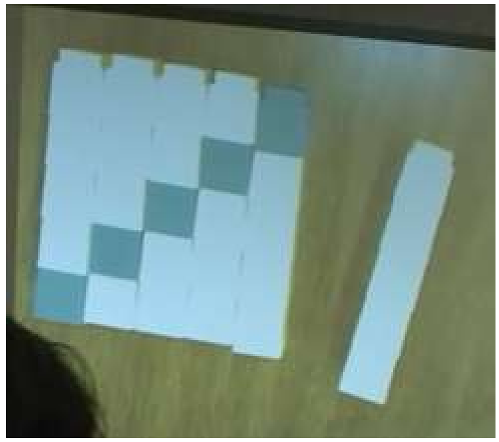
837.		JL	But just, look at this, okay? Just look at the single blue. Convince me that you have all the towers that are possible with a single blue bottom. I'm going to give you a minute to think about it. Talk to each other, convince each other, you may have to rearrange it, or you may be able to do it the way it's sitting, but I need a convincing argument that there aren't any more towers that have a blue bottom.
838.		K	Alright. Okay, let's do this. Alright, so let's go where we need a blue, obviously a blue and a yellow. And then there's a blue in this spot, right?
839.		RB	Mm hm
840.		K	Blue yellow, blue yellow, blue yellow, blue
841.		A	Perfect, Right?
842.		K	Sorry
843.		A	That's good. And this one too, or no?
844.		K	Um, Is there any more that there's blue yellow, and then there's another yellow here? No, right?
845.		A	No.
846.		K	Okay, so then here blue, right, and then
847.		A	Now what are you doing?
848.		K	I'm just trying to figure out like how we can move this one up like, I'm just trying to see.
849.		A	You typically think staircase
850.		A	I want to move into each position I possible can be in. And this will go. I'm just trying to see, like if anything makes sense here. All the ones with blue yellow blue,
851.	30:20		Camera moves
852.		J	This is how I started off Saying this is how I did it at first
853.		JL	Okay, yeah, okay.
854.		J	But, the strength of his argument was pretty much, was a lot stronger.
855.	30:33	JL	Okay
856.		J	Mitch, you want to explain how, like can you use this and explain?
857.		M	Yeah
858.		J	Your reasoning

859.		JL	Okay
860.		M	Because originally, I had three blues. He was doing two blues and I had three blues.
861.		JL	Okay
862.		M	But honestly, I just thought of it because it's the opposite.
863.		JL	Okay
864.		M	You know, I thought of it as just "figure out two yellows"
865.		JL	Sure
866.		M	Then I would have the same amount as him, and I would have the opposite.
867.		JL	Okay. And they would match up.
868.		J	Like, can you organize them with this, with these towers?
869.			
870.		M	Oh, so with, with two blues, what I did was
871.		JL	Okay
872.		M	I started with all the blues on the bottom.
873.		JL	And that's a neat strategy
874.		J	Yeah
875.		JL	It's holding a constant
876.		M	Right, yeah.
877.		JL	Oaky, your kids will do that. You're gonna see that strategy they're doing. Sometimes they'll hold the blue on the bottom. Sometimes they'll put the blue on the top. But holding a constant, really something very important
878.		M	Yes, right.
879.		JL	When they study algebra, they hold constants. Right?
880.		M	Right, so then the first I did kind of like the elevator technique

881.		JL	Oh
882.		M	Until it got
883.		JL	I see
884.		M	Because, If you're taking this one out,
885.		JL	I got it
886.		M	There's only one blue left
887.		JL	I got it
888.		M	And there's only four spaces, four positions.
889.		JL	Isn't that neat, Okay
890.		J	I thought it was
891.		JL	Pretty convincing. Okay
892.		M	Now, the second group to make sure that we don't have any duplicates,
893.		JL	Okay, okay
894.		M	I just start with the second position
895.		JL	Okay, got it.
896.		M	But since you know this bottom spot has already been taken,
897.		JL	Okay
898.		M	You can just, did I pick the right one?
899.		J	Yeah
900.		M	No, I want to start with this guy.
901.		J	Okay
902.		JL	Okay
903.		M	So, we start with this and then we just kind of do the elevator again.
904.		JL	Oh, so you're again keeping the two together the way you had it here.
905.		M	Right
906.		JL	And then how did you go from here to here?
907.		M	Because, like I said, since that second spot has a blue,
908.		JL	Oh
909.		M	We know it can't be in this spot anymore, so in that second group

910.		JL	Okay
911.		M	There's only basically three spots that are left.
912.		JL	So you actually are holding a double constant aren't you?
913.		M	Yes
914.		J	Mm hm
915.		JL	Pretty nifty
916.		M	And then the third spot, we start with this
917.		JL	Yep
918.		M	And again, it can't be below because those have already been taken up.
919.		JL	Got it, got it.
920.		M	So then you just keep that constant, and then there's only one other space for it to go.
921.		JL	Okay
922.		M	This one in the fourth spot, there's only one other spot.
923.		JL	Oh, isn't that pretty nifty. I like
924.		J	Yeah, beautiful.
925.		JL	I like. You know what guys, I hate to do this, but I don't want to run out of time. So you are doing absolutely amazing things, and I don't know if you realize it, but do you remember September eleventh, when you worked on towers? And do you remember what kind of strategies you had then? Not quite as sophisticated as what you're doing today. You are doing neat stuff today. Every group is doing neat stuff. Now we're going to show you something that's really nifty. We're not going to write anything, because we're saving time. We're going to take your unifix cubes, put them underneath this elmo. And they're going to come up on this screen, and we're going to be able to share our strategies that way. Okay? So, what I want you to do, is keep your groups the way they are, Okay? And carefully carry over your groups to this table. Let's start with Jared and Mitch. Okay?
926.		M	Do you want me to turn the camera on?
927.		JL	I think that would be a good idea. I think it's on.
928.		M	Well, It's probably not
929.		JL	Okay
930.		M	Because it's getting warm

931.		J	Okay
932.		M	Basically, what we did was we thought of them as blues and started off with no blues, then one blue then two blues. In those categories, so at first we got
933.		J	Can you see them?
934.		JL	Okay, they looked at, they'll turn color in a minute, like magic. Again, make sure that you're putting up the stuff in the order you had it. And each of you did something differently, so really watch what the other groups did. Amazing stuff you're doing, that's very, very exciting.
935.		J	So here it is, so Is this one now moved in?
936.		JL	It's kind of upside down. Can you turn it around? Because I see the chimneys. That's bothering me
937.		J	Okay
938.		JL	That's good, okay, is that the chimney?
939.		KK	Yeah
940.		J	So do you want the chimneys this way?
941.		JL	That way. Chimney on top
942.			Perfect
943.		JL	Doesn't that make you feel better?
944.		J	Um,
945.			Ha
946.		JL	It makes me feel better.
947.		J	Okay, so we just thought, like to keep ourself organized,
948.			Yes
949.		J	That we would go with a strategy that me and Kulsom had done before,
950.		K	Yes, we did.
951.		J	But a strategy that I'm comfortable with, was that having no blue, So this right here,
952.		JL	Oh
953.		J	We're convinced that there's no blue there, right?
954.			Yes
955.		JL	We're convinced. We are so convinced.
956.		J	Absolutely, there beautiful. And then, we got to this, the second, our second grouping

957.		JL	Yes
958.		J	Where we have just one blue, and
959.			
960.		JL	You know at the end, I'm going to interrupt a minute, You're going to think it's a crazy question, but it IS a question you should be asking your children. If they make that group and you say "How do you know you have them all?" And you ask them "Why can't you put a blue in the sixth position?" Okay, and they will come back saying "Oh, come on, give me a break, the tower's five tall." But ask that question, it's an important question. Okay
961.		J	Alright, so that's
962.		JL	Those are, we got those two we're convinced
963.		M	For the two tall
964.		JL	Upside down
965.		M	The next one we did was with the two tall, two blues
966.		JL	Okay
967.		M	And the first thing I started off with
968.		JL	A little low
969.		M	They're going to be backwards the whole time.
970.		JL	Isn't that a great machine, that you can do that?
971.		M	I really should be able to just flip this.
972.		JL	Oh, now it's upside down again.
973.		M	Alright now. So basically what I did
974.		JL	Good
975.		M	For the two blues was, I started with, I put them in different groups. The first one has, I kind of the bottom

			position. I was going to see how many different ways I could do with the bottom position having blue. And then basically, since there's only the four other spaces, where the second blue could go.
976.		JL	Yep
977.		M	So we've exhausted all the possibilities with blue on the bottom.
978.		JL	Isn't that neat
979.			Mm hm
980.		JL	So what they did here, and I saw it in this group too, they held a constant, okay? Sometimes your kids will hold a constant on the top cube, sometimes they'll hold the bottom cube, but it's a great strategy, because they made a simpler problem. Now they only have towers four tall, and they could put a single blue cube in each spot.
981.		M	And then again, just so we weren't, we didn't have to worry about doubles.
982.		JL	Good
983.		M	We put it in the second spot. And now we're looking at the second position having all blues. You know that on the first one's already been taken, so you've already used the blue and a blue right here. So now what you're doing is there's only. Hold on a second, I should probably change this right here.
984.		K	Yeah, that's what I did
985.		M	There's only three different spots that it's left. So with this second position, now I only have a group of three.
986.		JL	Now, watch what they did. They have a double constant, they have, Oh look, they turn yellow. Magic! They have the yellow, in the bottom position. They have the blue in the second to the bottom position. They have a double constant. Really neat. Again, they have a blue in each spot.
987.		M	And then the next group is going to start off with two yellows on the bottom and the third one
988.		JL	Oh, look how they are yellow now.
989.		M	So the third position is blue. And, like I said, Since we've already used the bottom two positions.
990.		JL	Yep
991.		M	Since we already have the blue and the blue there, you

			just start from here and then there's only two other positions where blue can go.
992.		JL	Is that nifty.
993.			Mm hm
994.		JL	So now they're controlling for variables in three spots.
995.		KK	Yeah
996.		JL	These three spots
997.		M	And then the last one, there's only one. Since we're in the fourth spot, and like I said, it's all these positions have already, I don't know a better way to explain it, just been used
998.		JL	Okay
999.		M	And there's only one spot right there
1000.		JL	Isn't that nifty?
1001.		M	And then, Jared's going to talk about how the next, the three blues is basically
1002.		J	Well, yeah, I mean. This pretty much goes with the same idea. We had no blue, one blue, this is the two blue
1003.		JL	Yep
1004.		J	And the three blue is pretty similar because it's two yellow,
1005.		JL	Exactly
1006.		J	So it's kind of hidden. So but this three yellow, we kind of did the same thing where we controlled the variable, with four here. And then we just went up with the two controlled variables, and so forth and so on. And that was really interesting, I liked how Mitch, like what we, our strategy was "alright I exhausted my ten" and then Mitch said he got his ten
1007.		JL	Yep
1008.		J	And then Mitch explained to me his strategy, and I really appreciated it, I thought it was a more thorough and convincing, so that just, to know we could just flip and do the opposite.
1009.		JL	Uh huh
1010.		J	And the same thing for the, for the four blue. Four blue just looked the opposite of the one blue
1011.		JL	Exactly, so it's what color you're focusing on

1012.		M	Right
1013.		J	Yes
1014.		JL	So if your focus was on blue, what would that be called, what you just held up?
1015.		J	This is four blue
1016.		JL	That would be four blue
1017.		J	Yes
1018.		JL	Okay, where would be no blue?
1019.		J	No blue is the yellow.
1020.		KK	They did have that up.
1021.		M	It was
1022.		JL	Isn't that neat really, really neat guys. Very, very nice
1023.		M	I was actually supposed to do the three blue,
1024.		JL	Yep
1025.		M	But after about like two seconds, I just decided I was going to do two yellow instead.
1026.		JL	Okay
1027.		M	I just thought of it as two yellow
1028.	40:00	JL	Okay, okay
1029.		M	Then we knew it just had to kind of double, you know because they all had those opposites.
1030.		JL	Very good. Okay, that's great. Let's get up one of the other groups. Who would like to go first? They have neat stuff, both of you. So one of you go up.
1031.		RB	We'd like Sally to go
1032.		S	Alright
1033.		JL	Okay
1034.		S	Touch your nose
1035.		RB	She always has some neat stuff
1036.		JL	I think he means he doesn't want to go
1037.		K	That's funny because we were doing that
1038.		JL	Yes
1039.		K	With the opposites
1040.		A	That's why I thought that's actually what we were doing

1041.		JL	Oh, okay
1042.		A	But I could follow that, because we were doing that
1043.		JL	Well, we're going to get another follow here, and you know what, the very, very important thing, I can tell you, when I was an assistant principal, in Holmdel, my job was to also teach two math classes a day. In grade three for gifted students. Enrichment math it was called. They were brilliant, those kids. There were times that I had to really sit back to try and follow what they were doing, because it was so different than the way I was thinking about it. And when I could follow them, they had unbelievable solutions that I went, "Oh, my God! That's a neater way to think about it than what I was thinking." It was my best teaching, when I was able to get outside of my mind and get into theirs. So it's a hard thing to do. What you want to do is not only get into your students minds. You want your students to get into each other's minds so that they understand, go ahead. They understand what's going on. Oh look at this!
1044.		S	So immediately, we made the two solids.
1045.		JL	Okay
1046.		S	And then I said to Kate "I'll do the four to one"
1047.		JL	You're familiar with that, you're comfortable with that?
1048.		S	Yeah
1049.		JL	Okay
1050.		S	So these are all the four to ones, and
1051.		JL	Okay, so you had them both in the four yellow, one blue; four blue, one yellow
1052.		S	Yes
1053.		JL	Okay
1054.		S	And we know that the one block has been in all the positions,
1055.		JL	Okay
1056.		S	So those are all the possibilities for that.
1057.		JL	Okay, I think we're convinced, yes?
1058.		RB	Mm hm
1059.		JL	Okay
1060.		KK	Okay, so I took on the two to three ratio

1061.			
1062.		S	Ha ha ha
1063.		JL	Okay
1064.			Which took a lot longer
1065.		JL	The two to three, a little bit harder to convince
1066.		KK	Yes
1067.		JL	But you did a good job. Tell them what you told me.
1068.		KK	Um, if you
1069.		S	I can help you
1070.		KK	Remember what I did
1071.		JL	Okay
1072.		KK	Okay, so we started initially doing
1073.		S	You did those, right?
1074.		KK	Yeah, I did those on the end, so I did the three blue at the top, and the
1075.		JL	Point to those, point to the one you're talking about.
1076.		KK	There
1077.		JL	Okay
1078.		KK	I moved it up one, I was concentrating on three blue and taking the two yellow and keeping them together and moving them up the pattern
1079.		JL	Okay
1080.		KK	So I got two yellow
1081.		JL	Yep
1082.		KK	And then I moved them up to the next position
1083.		JL	Good
1084.		KK	And then the next position
1085.		JK	Yep
1086.		KK	And then the next position
1087.		JK	Okay
1088.		KK	And then since we couldn't go higher, I split them and bring the one yellow down to the bottom.
1089.		JL	Everyone follow that? That is kind of like a recursive argument, where you're moving it into all of the positions. Okay

1090.		KK	And then we took, I left the
1091.		JL	Why didn't you do that again?
1092.		S	We knew we couldn't leave them together, because, right, because these were all the ones where we had the two yellows, like stuck together
1093.		KK	So we knew we would have to split it
1094.		JL	So if you move this this one down, what would happen?
1095.		S	It would have been that one there
1096.		KK	It would have been that one again
1097.		JL	Okay, because you already have it.
1098.		KK	And we did that, I think
1099.		JL	Okay
1100.		KK	Or we at least looked at it
1101.		S	Yeah, we
1102.		KK	So we knew after we went two together, we had to split them
1103.		JL	Okay
1104.		KK	So then we kept the one at the bottom, and then we moved back the other way.
1105.		JL	Okay
1106.		KK	So we kept the yellow here, and the new moved the yellow down here one position.
1107.		JL	Okay
1108.		KK	Kept the yellow on the bottom moved it down here
1109.		JL	Okay
1110.		KK	If I had moved it down again, like you had just pointed out, I would be back to here.
1111.		JL	Everyone following? That's very beautiful. What you're saying makes a whole lot of sense, you're being very systematic. Okay, keep going.
1112.		KK	So, then we decided to move (I can't remember) the yellow up to the second position?
1113.		S	We, I think we said
1114.		KK	Let me move these over now, so you can see them.
1115.		JL	Good, good.
1116.		S	Right, we said that if we move this one down, we would

			already have two together
1117.		KK	Right, two yellow.
1118.		S	Again
1119.		JL	And you don't want "stuck together"
1120.		S	So then we said
1121.		KK	And we did all the bottoms
1122.		JL	Okay
1123.		KK	Right, because we have the four on the bottom. We did the original one which you can't see,
1124.		S	Yeah
1125.		JL	Okay
1126.		KK	And so we said "let's move the yellow up to the second position."
1127.		JL	Okay
1128.		S	So yeah, we controlled that
1129.		JL	You're controlling for variables, yep.
1130.		KK	That one
1131.		JL	Okay
1132.		KK	And, (Why did we go to the top?)
1133.		S	I guess maybe because we were following the same thing from here, Like
1134.		KK	Oh, maybe, yeah. Down again. We had moved these up, so we were going to start with that same pattern, moving the yellow down each position. The second
1135.		JL	Okay, why didn't you have another one here? Like you could have had another yellow here, and then go bum, bum, bum. Why didn't you do that?
1136.		S	Because that would have been this one.
1137.		JL	Okay, because remember the yellows can't be touching.
1138.		KK	Right, because we already exhausted all of those.
1139.		JL	Because they're already touching over there.
1140.		KK	Yes
1141.		JL	Okay, you following? Okay, everyone following, because it's some, this is different than what you guys were doing. And then what's this thing?
1142.	45:03	S	So,

1143.		JL	The last tower.
1144.		S	So, so we had the yellow controlled at the bottom.
1145.		JL	Good
1146.		S	Then we had it controlled at the second
1147.		JL	Good
1148.		S	Level, and then we decided to control it at the third
1149.		JL	Good
1150.		S	Level.
1151.		JL	Okay, and why didn't you put one here?
1152.		S	Because then they would be touching again.
1153.		KK	Touching again
1154.		JL	They would be touching again and you already have it, so they put it up here. And how many towers did you get that were in this?
1155.		S	Ten.
1156.		KK	Ten.
1157.		JL	Ten, and that made you happy, because they had nine,
1158.		KK	We had nine for a really long time, and I knew
1159.		JL	It's like Mitch had nine and Jared would not have liked nine either, right? Okay. That's great
1160.		S	Oh! We also noticed something.
1161.		JL	Yeah
1162.		S	Is that, because we were like counting the tops for some reason.
1163.		JL	Yep
1164.		S	Because we wanted to be sure to have the same number
1165.		KK	Oh yeah, this is interesting.
1166.		S	Of cubes in each row.
1167.		JL	Did you this by your, did you do this by yourself? Because your supervisor was finding that they worked?
1168.		S	She tried to steer us in a different direction.
1169.		JL	Oh ,did she try? Okay.
1170.		KK	She wasn't happy with the fact that they were different, but we kind of told her the reason why they were different.

1171.		JL	Okay, I see what she was saying, though, this is, um, what is her name?
1172.		KK	Jean
1173.		S	Jean
1174.		JL	Jean. When Jean Cur was here, she said "look at this. In this row, if you go across, left to right, and just counting the blue, they have one, two, three, four, five, six blue." Second row, one, two, three, four, five, six.
1175.		S	Well, she didn't go down, but She said in the top one, there were six blue and only four yellow
1176.		JL	But she was saying that every row had six blue
1177.		KK	Right
1178.		JL	And therefore four yellow. Very interesting. We're not going to talk about it right now, but I'm saying it's really interesting. And just the way it bothered you to get nine towers, if you were Jean, and you saw that this had six and four, six and four, six and four, and then this one now had five and five, you would go "Ew"
1179.		KK	It's the way you look at it
1180.		JL	It's a way to bother, it's a way to cause, "do I have a problem here?" So that's, so that's interesting. Okay, we got to get to this group, and we got to get to the last problem because you're going to be doing it with your children and I don't want you doing it with your children until you do it. Okay. You are doing phenomenal stuff today. Do you notice how this is so much more sophisticated than what you did when we met in September? Yes?
1181.		J	Like Mitch said something like, to know that there's, to first find out that there's thirty two,
1182.		JL	It is helpful
1183.		KK	I think the kids are going to have trouble.
1184.		JL	But, however, sometimes it gets you into trouble. Because, if you were a child who predicted five tall is five times five, twenty five. You may be happy to stop at twenty-five. Until you find the twenty-sixth tower, and then you go "Ew" I have a little agita
1185.		KK	Right
1186.		JL	So sometimes the mathematics is helpful when the mathematics is correct. But if the kids have the wrong

			mathematics, it's not helpful. Okay? Alright. We have a very interesting strategy here. Different than what you two groups did. So watch carefully. Really neat again. And the idea is you want to get to really follow other people's thinking. Okay
1187.		A	Okay, we started with, We kind of started like they were working and I was working in my notebook, and I, they were working on a strategy like Jared and Mitch were working on, and I just, I don't know, I just couldn't see it.
1188.		JL	Mm hm
1189.		A	So I was like, I typically like to write something down before I do it. So I said, If I were going to do this without paper, I would probably do something like "Let's build all the ones that are blue on the bottom first." Keeping blue on the bottom.
1190.		JL	Mm hm
1191.		A	From there, what we did was we built all of the towers that are exactly one blue on the bottom.
1192.		JL	Okay, so where are the ones that are one blue on the bottom?
1193.			Here
1194.		JL	There they are. Isn't that nifty? They said "We're going to build every tower that has one blue on the bottom." Now, how do you know you have them all?
1195.		A	This is what we were just working on.
1196.		K	We were trying to develop the reasoning to prove it, but
1197.		JL	Okay. Okay.
1198.		K	Basically we had controlled for our variable here too.
1199.		JL	Mm hm
1200.		K	We had, we did ones with exactly one blue. So we did blue yellow blue,
1201.		JL	Mm
1202.		K	And blue yellow yellow
1203.		JL	Mm hm
1204.		K	So we made those
1205.		A	And then we know that
1206.		JL	Oh
1207.		A	All of the positions on top

1208.		JL	Now, look at that. If these are being controlled. Look at the top. These are both the same. Blue yellow blue. What could the top be, blue and yellow, or what's this up here?
1209.		K	Yellow yellow,
1210.		JL	And what's this?
1211.		K	Yellow
1212.		JL	Is that the only two ways it could be?
1213.		A	No
1214.		K	But that's what we were trying to organize. This is what we were trying to figure out, like organize.
1215.		JL	Okay
1216.		K	Like, you're saying that this should be next to?
1217.		A	Well, why did you build it like that? You had a reason why you put it like that. We went back and tried to organize these to try and come up with like another.
1218.		JL	Okay, okay
1219.		K	But finding the thinking that worked. Like,
1220.		A	You had it like this? Did you or no?
1221.		JL	What if you looked
1222.		A	There is the reason, you would know. Like, don't say it should be another way. You thought about it, so
1223.		JL	Alright so, Angela is insistent
1224.		A	She just put it together and had it
1225.		JL	Okay, so tell us what you have. And look how fleeting it could be you have it, and then it's gone, Right?
1226.		A	See, maybe I would think of it more like this.
1227.		JL	Ah
1228.		A	Or maybe even like this.
1229.		JL	And tell us why. Angela.
1230.		A	Again, again these are all blue yellow blue.
1231.		KK	Yes
1232.		A	This one is blue yellow,
1233.		K	Yellow blue
1234.		JL	Is there any other way to put these up here? If it's a blue and a yellow, or a yellow and a blue

1235.		K	No
1236.		JL	No, unless they're just solids. Solid blue, solid yellows. So does this make sense to people?
1237.		KK	Yes
1238.		S	Mm hm
1239.		JL	Okay
1240.		A	And then that means, that's the opposite
1241.		JL	Now this is very, very convincing. And look how many they controlled. They controlled the bottom three rows, didn't they? Neat. Okay
1242.		K	Then what we built next was all the twos. These are all of the twos, on the bottom
1243.		A	Exactly two
1244.		JL	Put them, push them so we can see them all. Okay, now look how neat they, Okay, no you can put them all on.
1245.			Oh, all
1246.		RB	These are the only ones
1247.		M	The other way
1248.		JL	You only want to show us half?
1249.		A	These are only twos
1250.		JL	Now, you see what they did? Before they had a solid one blue on the bottom. Now they're doing a double blue. And are they in order?
1251.			Yeah
1252.		JL	Okay
1253.		KK	That's the same as what you just did before, right?
1254.		JL	Okay
1255.		A	Yeah, we switched that, yeah
1256.		JL	Okay, so you're happy with that?
1257.		K	Yeah
1258.		JL	Okay
1259.		KK	And then the alternate of each
1260.		JL	Okay, how about that one? How many blues on the bottom?
1261.		K	Exactly three.

1262.		JL	Exactly three blues on the bottom, and again. Oh, here comes some more. Oh wait a second, is that the only two?
1263.		K	Yeah
1264.		JL	Those are the only two.
1265.		K	Exactly three
1266.		JL	Can anyone. Go back to the twos. Can anyone think of another way to get two, where are they
1267.			Is that it, the two?
1268.		JL	No, the three.
1269.		A	The three
1270.		JL	The three. The three blues on the bottom. Is there another way to put three blues on the bottom?
1271.		A	Like "exactly" three?
1272.		JL	Could, yeah, could, you have a double yellow, you have a blue yellow, could you have a blue yellow?
1273.			No because you would have four blues on the bottom.
1274.		JL	Okay, good. And again, these are questions that you're going to want to ask the kids. Because they're going to have to think. It may be obvious to you, but yeah, put a blue in that spot, it would become four blue. It may be obvious to you, but you want them to think about it and verbalize it. Okay so then you had four blue on the bottom?
1275.			Yeah, oh sorry.
1276.		JL	Okay
1277.			I thought we were done.
1278.		JL	They did it. Wasn't that a neat strategy?
1279.		KK	Yeah
1280.		J	I want to see it again, actually, impressive.
1281.		S	You could email
1282.		J	Well, then maybe I could play with it, like at um seven o'clock.
1283.		JL	To be convinced, you want to see it again, and that's okay. But that what you did is a lot of controlling of variables, and really neat. All of you were good. What I'm going to do is not ask for you, at this point to do anything with writing your towers. Right, it's a shame but

			we're not going to have time for that. What would rather you do is, I'm going to give you the next problem which is a pizza problem. No cubes are used. You're going to be working this problem with your partners on paper, Okay. So I'm going to give each of you a paper, and I'll turn around and I'll be the unifix cleaner-upper, so you can be the pizza problem doer.

10/7 Meeting transcript 3 of 3

Title: 10/7 Judy's Class 3 of 3
Location: Oldbridge
Date: 10/07/2010
Length: 00:21:56
Transcribed by: Will McGowan May 2012
Verified by: Maddie Yedman

Line	Time	Speaker	
1.	00:14	JL	Okay, good
2.		J	Who should I thank if I appreciate?
3.		JL	Me, me. I thought people would be hungry.
4.	00:33	A	Well, no. Actually I was thinking, would you organize this like, plain pizza, one topping, two toppings, three toppings four toppings
5.		K	Okay
6.		RB	Well, I don't want to answer yet. I saw a good solution today that was kind of.
7.		A	Well then don't tell us. Don't tell us.
8.		K	That's what I would do, that's what I would do.
9.		RB	No, I won't. Because that would just.
10.		A	All with one
11.		RB	I would want to do it the same way he did it. It was kind of leading them into something I know the answer to. And that's not a good.
12.		JL	Well, why don't you think of it a different way than Brandon did.
13.		K	I would do an organized list.
14.		A	Alright, I'll do a list. Tell me how you would organize this list.
15.		RB	I probably would have done it
16.		JL	Okay
17.		RB	Very similarly
18.		JL	Okay.
19.		K	PL is plain, right? Because there's peppers and the pepperoni.
20.		RB	I wouldn't have made the staircase.
21.		JL	Okay
22.		RB	Or anything like that, but I probably would have

23.			P is peppers S equals sausage
24.		RB	Mushrooms
25.		A	I'm so hungry right now. Ha ha
26.		K	M is mushrooms
27.		JL	I know, this is a bad one, a hard problem to work on when you're hungry.
28.		K	And P, Oh great! There's pepperoni and peppers.
29.		A	So do PP is pepperoni. I don't know, or PR, I don't know.
30.		K	Fine, PR is pepperoni.
31.		A	Mushrooms
32.		JL	And that was done on purpose.
33.		RB	You could do peppers as green. Assume they're green.
34.		JL	Angela
35.		RB	Do G for peppers
36.		JL	It was done on purpose, so you have to invent notation
37.		A	Okay. PR is pepperoni.
38.		K	That was your idea.
39.		A	My favorite.
40.		K	Okay, so this is how, with my kids, and I would always do organized lists. So just plain, right. So just plain
41.		A	Then you would do plain
42.		K	Just peppers.
43.		A	Plain with peppers, plain with sausage,
44.		K	Well, how about just peppers? All the ones with just one topping?
45.		A	Oh, I was pairing them up like plain with peppers, I don't know why.
46.		K	I would do, like when I do two. I would do that when I do two. Second
47.		A	I don't understand what you mean.
48.		RB	She's right. Because, you would have a plain, but once you put peppers on it, it's no longer plain. So that would be zeroes.
49.		K	I'm confused.
50.		RB	Not zeroes, this would be like, all, like nothing on it.
51.		K	No toppings.

52.		RB	Yeah, now
53.		K	Okay, fine. I see what you're saying. So this would be no toppings.
54.		A	So, is what you're saying, and the next one could be
55.		K	All the ones with
56.		A	We could eliminate the word plain, because it's automatically will
57.		RB	The next one would be either peppers or
58.		A	Plain
59.		K	Right
60.		RB	The next one would be either peppers
61.		K	So just peppers,
62.		RB	Or just sausage,
63.		K	Or just sausage,
64.		RB	Just mushrooms,
65.		K	Just mushrooms,
66.		RB	Or just peperoni.
67.		A	Just pepperoni. So could I just keep going like this? One topping
68.		K	Yeah
69.		RB	Yes. So there's four one toppings. Very good
70.		A	And now,
71.		K	Two I would just do
72.		A	I would just do, Wait, wait, wait, wait. I hate when people go ahead of me, I get really scared.
73.		A	Now we go two?
74.		K	Now you're going to go P and S, P and M, P and R
75.		A	Two toppings
76.		RB	Mm hm
77.	2:44	K	And then I would move. Okay, I'll wait.
78.		A	Wait, wait
79.		K	Okay, I'll wait I got it
80.		A	P, PR
81.		K	P, oh, wait, what did I do?

82.		A	Right
83.		K	P and PR, right.
84.		RB	Plain sausage, plain mushroom
85.		K	Now I do this: Sausage and Mushroom, Sausage and Pepperoni. And the Mushrooms and Pepperoni.
86.		A	I do diagrams with them in class. Not as complex, but we've done.
87.		K	So this is all the two toppings.
88.		A	So, we've gotten all of them?
89.		K	Yep. Because we just went in order, like P S, P M, PR. So should I be putting, yeah, I guess I should be putting commas in between here. Okay, and then three toppings, that would be Peppers,
90.		A	Wait, wait, wait, wait. So now, would you start with
91.		K	I would do
92.		A	Let's pair peppers and Sausage?
93.		K	I would do this, yeah. P S M, P S PR
94.		A	Wait, my question. I have a question.
95.		K	Yep
96.		A	Can you double up on things, or no?
97.		K	What do you mean double up on things?
98.		RB	No, no , you're making it too complicated.
99.		A	No? No? It has to be two toppings? Ha ha
100.		RB	Yeah, you wouldn't have extra peppers or extra mushrooms.
101.		A	You never know. I would .
102.		RB	No, I know.
103.		A	Not mushrooms, ew. Ha ha. Okay, so would you pair up peppers and and sausage and pair them with the other two first?
104.		K	Yes, yep.
105.		A	Okay.
106.		K	The other one. Peppers and sausage with, oh! Yeah, yeah. I see what you're saying.
107.		A	Peppers Sausage Mushrooms. Peppers Sausage Pepperoni. Right?
108.	4:01	K	Mm hm

109.		A	Okay, then where do you go?
110.		K	Then Sausage Mushrooms Pepperoni. And these are all the ones with three toppings, Right?
111.		RB	Yeah, there's three.
112.		K	Three.
113.		RB	And then we have one with all four.
114.		A	Yep
115.		RB	Peppers Sausage Mushrooms and Pepperoni. One, two, three, four, five, six, seven, eight, nine, eleven, twelve, thirteen, fourteen, fifteen. We're missing one.
116.		K	Why do I have?
117.		RB	We're missing one of the three.
118.		A	Oh. One, two, three, four. Oh, Pepper oh, Peppers Mushrooms Pepperoni.
119.		K	Yep. Peppers Mushrooms Pepperoni.
120.		RB	There we go.
121.		K	Yeah
122.		A	Why'd you miss that? Why did we miss that?
123.		K	Because, like, in order we just did this: We did P S M, and we did P S PR, and then we went straight to S M PR, but we didn't, we never did, um, P M PR. Like we did, we never went with all the ones with P first. This, this, this. This, this, this. But also this, this, this. We never like it.
124.		A	Like, I'm trying to think in my head if I would do this, like with, like I could have pepperoni
125.		K	Sixteen?
126.		RB	Sixteen, yes.
127.		JL	Are you comfortable, do you think that's all?
128.		RB	Yes
129.		K	Yeah, I think so.
130.		JL	Can you convince me?
131.		RB	Yes
132.		K	I think so.
133.		JL	Okay
134.		RB	Okay
135.		JL	I'm ready. Who's going to be the convincer?

136.		K	Rich
137.		RB	We, we
138.		JL	Kulsom
139.		RB	Okay
140.		JL	Kulsom, you can do it. Let her go.
141.		RB	Okay.
142.		JL	Because you guys kind of did the other one. Okay.
143.		K	Alright, so with no toppings, right?
144.		JL	Got it.
145.		K	You have just a plain pizza.
146.		JL	Yes
147.		K	Then all the ways with just one topping,
148.		JL	Okay
149.		K	Like, just peppers, just sausage, just mushroom,
150.		JL	Yep.
151.		K	Okay. And two toppings.
152.		JL	I'm convinced.
153.		K	Okay
154.		JL	Keep going.
155.		K	So two toppings, I would start with peppers. I would do Peppers and Sausage, Peppers and Mushrooms, Peppers and Pepperoni.
156.		JL	Okay.
157.		K	I'm done with all the ones with, like I started with this one first. Peppers first.
158.		JL	Oh
159.		K	And then I went to Sausage.
160.		JL	Oh
161.		K	And Mushrooms Sausage and Pepperonis
162.		JL	Good
163.		K	Okay, and then the last one is mushroom and pepperoni. Like I went, I
164.		JL	Okay, so you did pepper. How come you have three with peppers, but you only have two with sausage?
165.		K	Because you would repeat it if you did, a sausage and peppers

			again, because you already have peppers and sausage.
166.	6:17	JL	Oh, Okay, and when they put the stuff on the pizza, it doesn't matter the order.
167.		K	It doesn't matter, yeah.
168.		JL	Okay, Good. I'm following.
169.		K	And then in this one, the three toppings would kind of follow the same way, like,
170.		JL	Okay
171.		K	But we realized we missed one, so like. So this should really be above this.
172.		JL	Okay
173.		K	Like Peppers Sausage Mushrooms,
174.		JL	Okay
175.		K	Peppers Sausage Pepperoni,
176.		JL	Okay
177.		K	And it should be then Peppers Mushrooms Pepperoni.
178.		JL	Okay
179.		K	But we missed that one
180.		JL	Okay, that's okay. Uh huh
181.		K	And then Sausage Mushrooms Pepperoni.
182.		JL	Okay, and again there are no more with sausage?
183.		K	No because I had every other combination Sausage Peppers Pepperoni was already in there
184.		JL	Oh, Okay. Okay, so it's used, so you're not going to repeat it. Okay I got it
185.		K	Right, and then all four toppings just one way, Right? All of them together.
186.		JL	I get it. That's really neat. Very systematic, very organized.
187.		K	That's the way, yeah, that's the way we would do it in class, when we teach them like organized lists.
188.		JL	Okay now does this problem remind you of anything that you've done before?
189.		K	Towers.
190.		JL	Which towers?
191.		K	The four tall, right?

192.		JL	I don't know. She's asking me.
193.			Ha ha
194.		RB	But I won't answer you either.
195.		JL	And he already
196.		RB	I saw the video, I kind of
197.		JL	He already looked at the video, so he
198.		RB	We were allowed to, it was posted.
199.		JL	Yes you did, yes it is.
200.		RB	And I like to give my folks thing early so they can comment on me.
201.		JL	It is absolutely fine that you did it. But it's this answering this question
202.		RB	Sure
203.		JL	Probably is not fine if you looked at it. What you're going to see when you look at Brandon. He actually makes a connection between the
204.		K	This
205.		JL	Four tall towers and the Pizza problem with four toppings and I'm telling you what he does is amazing
206.		K	Amazing.
207.		JL	Absolutely amazing.
208.		RB	He's led in the right direction really nicely.
209.		JL	I don't know if he was lead.
210.		RB	No?
211.		JL	I don't believe so.
212.		RB	I don't think he was told what to say.
213.		JL	Right
214.		RB	I don't mean "lead" that way.
215.		JL	Yeah
216.		RB	They asked the, I called it scaffolding. Scaffolding. They scaffold him correctly to bring about his knowledge and make that connection
217.		JL	No, he
218.		RB	They helped him bridge.
219.		JL	Not true.

220.		RB	No?
221.		JL	He made that connection without being led at all.
222.		K	Two with S's
223.		JL	Amy Martino was a researcher, the one who did the questioning,
224.		RB	Okay
225.		JL	Was a researcher at Rutgers, she was so good I snatched her and hired her as one of my teachers.
226.		RB	She did a great job.
227.		JL	She did an amazing job, but she did not lead him. That was all in his head.
228.		RB	I guess it's the wrong.
229.		JL	What she did was she helped him explain it.
230.		RB	Yes. Yes, yes, yes. That's what I mean was like for example, she goes "does it remind you of any other problem?"
231.		JL	Yes.
232.		RB	And he thought about it. And then he said, "Yes, the." And then he started. He's the one that was putting the
233.		JL	Amazing, what he did
234.		RB	Yes, yes
235.		JL	So. Okay, this is great. I'm going to let you finish this, and then I'm going to ask you to share one of your papers. Okay? So decide who's going to share.
236.		A	No it's not. It's messy now.
237.		A	Yours is the neatest, Kulsom.
238.		JL	Okay, everyone is done?
239.	8:57	JL	Everybody, this problem you all did basically the same way. You all did it kind of like looking at plain pizza, one topping pizzas, two topping pizzas. Right? You kind of did it by cases. They're still talking because they started separate. So let's give them a minute. You can get your unifix cubes back in the bag while we're waiting. And, Angela, what did I do with that paper that I was going to Xerox?
240.		A	It was sitting right here.
241.		JL	I know. What did you do?
242.		A	Oh. I didn't take it back. I didn't,
243.		JL	I wonder if I moved it.

244.		K	Is it in that pile?
245.		JL	I did. I moved it.
246.		A	Okay
247.		JL	So before you leave, I'll make a copy, so I can give you back. Okay? Alright. Okay, let's see. You got a lot. Maybe some of them came from
248.		A	Would you like me, what I plan to do, to share these responses, I'm going to scan them because I have a smartboard and nothing.
249.		JL	Oh that's neat.
250.		A	Would you rather me scan it and send you a copy of it?
251.		JL	I would like to have the original.
252.		A	Okay
253.		JL	I really would
254.		A	No problem
255.		JL	So if you want to scan it, and then give me the original next time, that's okay too.
256.		A	Okay.
257.		JL	Okay, I'm fine with that.
258.		A	Okay definitely, that works.
259.		JL	Okay. That's good, that's good. Are you finding the tree easier, or harder?
260.		S	Harder.
261.		JL	Oh, because you have to get rid of duplicates.
262.		S	This is my four toppings.
263.		JL	Yeah
264.		S	And then I had figured these would be all my three toppings.
265.		JL	Oh my, okay.
266.		S	But then I stopped when I got to three high.
267.		JL	Not so easy, though.
268.		S	Uh uh
269.		JL	To keep track, it's kind of complicated.
270.		KK	It was easier this way.
271.		JL	Was it
272.		KK	And I thought a tree diagram would be easier. I think.

273.		JL	Okay, okay, you found it easier this way. Good.
274.		KK	Mm hm
275.	10:48	JL	Well, you know, you did the same thing that they did, and I'm not sure what Mitch and Jared are doing, let me check.
276.		M	This one, I went over to the next one.
277.		JL	Mitch and Jared. You also did one topping, two toppings, three toppings. Yep.
278.		M	We did
279.		JL	You did, good. Angela, Kulsom and Rich. Can you show one of your paper up? We're only going to do one because all of you basically did it the same way. Okay. So we're going to let you talk about what you did.
280.	11:22	A	Kulsom, this is yours so,
281.		JL	Okay
282.		K	So enthusiastic
283.		A	No, your paper was the neatest.
284.		JL	Look how neat that is.
285.		S	Beautiful, mine's a mess.
286.		K	So we started off, if there was no toppings, right? There's only one way to have no toppings. One plain pizza.
287.		JL	Good.
288.		K	And then to get like just one topping, you could have just peppers, just sausage, just mushrooms or just pepperoni.
289.		JL	Got it.
290.		K	So there's four ways.
291.		JL	Got it.
292.		K	Two toppings, we kind of just went in order of the letters and of the symbols
293.		JL	Of the problem, okay
294.		K	So peppers and sausage, peppers and mushrooms, peppers and pepperoni. Then we went to the second item, which is sausage. Sausage and mushrooms, sausage and pepperoni, and then we were asked "Well, how come you didn't put sausage with peppers again?" and well, we already had that as our first item. And we're talking about pizza and it doesn't really matter, you know it's still sausage and peppers.
295.		JL	Okay, that pesky instructor kept asking these questions. And

			you're going to be pesky in your class.
296.		K	Yes
297.		JL	And what I was saying was, and look how nicely they are holding a constant again. Their peppers are being held constant, and they're combining with the other ingredients, so I said, "When you got to sausage, how come you only have two ways to group the sausage?" And very nicely, I was told:
298.		K	Sausage and peppers was the same thing as peppers and sausage, so…
299.		JL	And it's already there. Okay, and then there was only one with mushrooms. Notice how they had to invent notation. It is not an accident that this problem was written with two "P" ingredients. It's going to force your children to give two, if they assign P to peppers, they have to think of what they're going to assign to pepperoni. Now, some of you have "PP" and that, in middle school, they may be giggling up a storm, but if they come up with "PP" then that's what pepperoni is. "PR" is safer.
300.		K	Yeah
301.		JL	Keep going.
302.		K	And then for three we kind of have the same thing. We went in order. Peppers first: the peppers sausage mushroom, peppers sausage pepperoni, and then if you look, we kind of made a mistake and realized we missed one with peppers first. We went peppers then to mushrooms and peperoni. And then we had done then sausage mushrooms and pepperoni. So we kind of said, "Oh, there's one more. We missed one"
303.	13:30	JL	Okay, so they found it, and now it's here. Now the last one?
304.		K	It's just all four toppings which is one of each.
305.		JL	Okay, we got sixteen combos. All of you got sixteen combos?
306.			Mm hm
307.		JL	Does this problem remind you of anything else that you've done? What does it remind you of, Jared? You're shaking your head.
308.		J	Oh yeah, the four towers, but more, like, what was really interesting was the idea of no toppings, and then one topping, two toppings, how we kind of organized
309.		JL	Uh huh
310.		J	Our cubes.
311.		JL	Yep

312.		J	So, like once we did it that way, it was like, "Alright, that's very similar to the four tall towers."
313.		JL	Four tall towers selecting from two colors. When you look at your video which is posted,
314.		RB	Yes
315.		JL	You will see that Brandon. My little Brandon, my brilliant Brandon actually made a brilliant argument, and showed how those two problems are connected. It's amazing what he did. You will not see it with too many people, whether they be children or whether they be grown-ups, or whether they be graduate students or whatever. But I mean, what he did was brilliant. You may not follow him the first time.
316.		KK	I was just going to say, I'm not seeing
317.		JL	Okay, I want you
318.		KK	I know sixteen is the answer, but
319.		JL	I want you to watch Brandon's solution.
320.		KK	Okay
321.		JL	And Amy Martino is his interviewer. Amy Martino was a grad student working with Carolyn Maher. I was the principal at Colts Neck. They were coming into my school. I saw the potential in Amy and I fell in love with Amy and I convinced Amy to come and be one of my teachers. So Amy became a fourth grade teacher in my school, but when she interviewed Brandon, she was a researcher with Carolyn Maher at Rutgers. She is probably one of the best interviewers that are around. She knows just how to ask the right question without leading, without giving too much information. She's brilliant. And what I actually asked you to do is watch her questioning, Because
322.		KK	That's where I feel like I'm lacking
323.		JL	Well, all of us,
324.		KK	I don't know what to ask guide them the right way.
325.		JL	I think, I don't think there are too many of us that start out as "Amy"s. Amy developed as well. She started out strong, but she got stronger. So what you're going to do is don't be hard on yourself. You're at the beginning of this process.
326.	15:58	KK	Okay
327.		JL	You are going to get better as an interviewer. You are going to get better. Look at you today, look at how you problem solved today. Your strategies were amazing. All the different things you were doing. Holding constants, and you know, forming your

			groups in a very systematic way, and having very convincing arguments. How many of you on September eleventh had convincing arguments, or had such a systematic way of approaching.
328.			Yeah
329.		JL	Some of you did, but not many, Right?
330.			Mm
331.			So, I'm saying that there is tremendous growth, and I anticipate you will see growth in your children as well. Now remember, they are not starting where you started, so they're not going to end up where you ended up today. But that's okay. What you're going to do is you're going to let them go as far as they can go. This is your process that you are going to do. Step one, you're going to take ten or fifteen minutes one day, and you're going to go over their solutions from towers four tall. After that, on a different day, you're going to let them build towers five tall. If you have a forty minute math period,
332.		A	Mm hm
333.		JL	You're going to take masking tape and group their towers with masking tape. Put them in plastic bags so when they come out the next day, they can start from where they left off. If you have the hour class, you're going to use the full hour to let them get their solution and record their solution. Okay? And their convincing argument. I bet, not all, but many of them will have a better convincing argument this time. Or pieces of a better convincing argument. So you're going to let them.
334.		KK	More organization
335.	17:31	JL	More organization. You're going to let them go as far as they can. If you have two kids can't go any further, do not frustrate them. Let them stop where they've gone as far as they can. Let them record what they have, their argument. Okay? Remember, in the videos of the kids. They didn't all find the solution at the beginning of third grade. But when they revisited the problem, they were better. And your kids will be better at five tall just because they've done four tall.
336.	18:04	JL	On a third day, separate. You're finished now with five tall towers. You're going to do the pizza problem. No manipulatives. Give them the paper. They're going to work right on the paper with their partner. If your partners were terrific, Keep them.
337.		A	Should I have them work with the same person each time
338.		JL	If they were terrific. Okay? If you had a bomb, of a partnership,

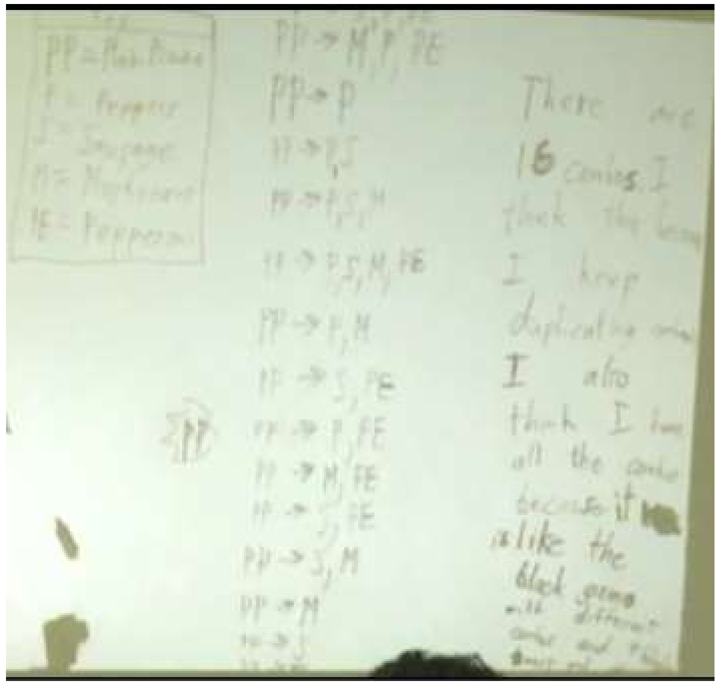
			break it up. You know the kids better now. You've had them another few weeks. So, if they need to be broken up personality wise, or because one took the whole show and the other one just copied, break them up. If they were good, keep them together. But it should be like ability. Alright, any questions?
339.		J	Yes
340.	18:44	JL	Yes
341.		J	Now, like, how much do we push them, on the four, like when we go over the four tall towers, like how much do we push on their reasoning?
342.		JL	How much do you push them?
343.		J	Yeah, because, so
344.		JL	Bye bye, Rich.
345.		J	Take care, Rob.
346.		RB	See you all later.
347.		J	Like, we're going to share their solutions, and then like what's the expectation tonight?
348.		KK	Just expose them to it, or?
349.		JL	You are trying to get them to do as much as they can some of your kids will do exactly what you did. They will have exactly,
350.		S	Oh, what are you asking?
351.		JL	He's saying "How much do you push them?" Is that the question?
352.		J	When going over the four tall towers.
353.		JL	Yes.
354.		J	When I showed them, now this is what you did, I explained it, and?
355.		JL	That's, Okay, when they're going over their four tall towers, you are picking just maybe two or three samples of student work. Because you want to do it in about fifteen minutes. You don't want it to take a whole period. You are picking representative samples of work that you think will be interesting, the way we did today here, we didn't go over all your students' work. We went over some. Let them explain their work. Okay? You might have to help them if it's on an overhead and they can't point to stuff. As they're talking, you may be their pointer, okay? But let them explain their work. If they didn't get a complete beautiful solution that was so elegant that, that's okay. Let them explain what they did. Okay? When that's

			over then you're going to in another class, do five tall towers. Okay? And in a third class, you're going to do the pizza problem. And before you come to meet with me at the end of the month. I think it's October twenty-eighth. I want you to look through the student work and pick a few samples from both the pizza problem and the five tall towers. And you're going to share those. Think ahead what you want to pick to share.
356.	20:37	JL	Now, if a child says to you, I am "Oh, this reminds me of those blocks." Because they may. Then you could say "oh really, what do you mean? How does it remind you?"
357.		A	Mm hm
358.		JL	"Why don't you write for me how it reminds you?"
359.		J	Pizza problem
360.		JL	They may say it. Okay. If you have a kid that's done real early, a group with the pizza problem. You might say to them, like I said to you. "Does this remind you of anything else that you've ever, you know done?" And if they say "Not really." Drop it. And if they say "Oh yeah, the tower that tower problem. The four tall tower." Ask them to write how. Okay? And then we'll talk about it. It should be fun.
361.	21:19	KK	How far apart should the five tall and the pizza problem be?
362.		JL	I'm letting you decide. You have about a two week period, so you figure out when it fits into your teaching, okay?
363.		KK	Like you're going to come in and see me on the twenty-eighth.
364.		JL	I'm coming on the day
365.		KK	That's the day that we have our,
366.		JL	And the day I come, we're going to do the pizza problem.
367.		KK	Okay, that's what I want to know.
368.		JL	So you will have already
369.		KK	It should be that we did the five tall already.
370.		JL	Absolutely. And what we'll do, if I can meet you a few minutes after, I don't know if that's possible
371.		KK	Mm hm, uh probably
372.		JL	If it's possible, you and I could pick which student work we want to share that afternoon
373.		KK	Alright
374.		JL	Guys, you were terrific. You really,really were good. Okay.

10/28 Meeting transcript 1 of 2

Title: Old Bridge Review
Location: Jonas Salk?
Date: 10/28/2010
Length: 01:35:55
Transcribed by: Will McGowan on October 2010
Verified by: Maddie Yedman

Line	Time	Speaker	
1.	0:01	JL	Not True
2.		RB	Exactly
3.		JL	Kate, Speak to that.
4.		KK	Yeah, Um, I did the pizza problem today and they made no connection to the blocks at all, but they made connections from third or fourth grade where they had a shirt and pants combinations
5.		RB	Hm really
6.		KK	Where they had ice cream and ice cream cone combinations
7.			Right, yeah
8.		KK	Which I thought was pretty remarkable
9.			Absolutely
10.		KK	Not the blocks based on combinations
11.		JL	They saw other things
12.		JL	Exactly. If they saw it as a combinatorics problem where you were making combinations, they made that link.
13.		JL	So lets see what did they do
14.		KK	Yeah
15.			
16.		JL	Let's study what they did with how they formed their pizzas. Let's take a look and see if we can figure it out before we let Rich talk.
17.			Is there something above the one that's there?
18.			
19.		RB	Trying to get it centered
20.			That's okay. Is that mushroom?

21.			
22.		RB	I was tossed between this one and a different one, when I did try to pick work. But I wanted to show this one because they were pretty consistent throughout the process I think
23.			It looks like kind of random
24.			Is there a pattern I'm not seeing?
25.			I also
26.		JL	Alright, so you're not seeing a systematic approach as they're listing it. Rich, what do you see?
27.		RB	I see him grouping it the same exact way. I don't want to, well, I guess I'll put this over it
28.		JL	OK

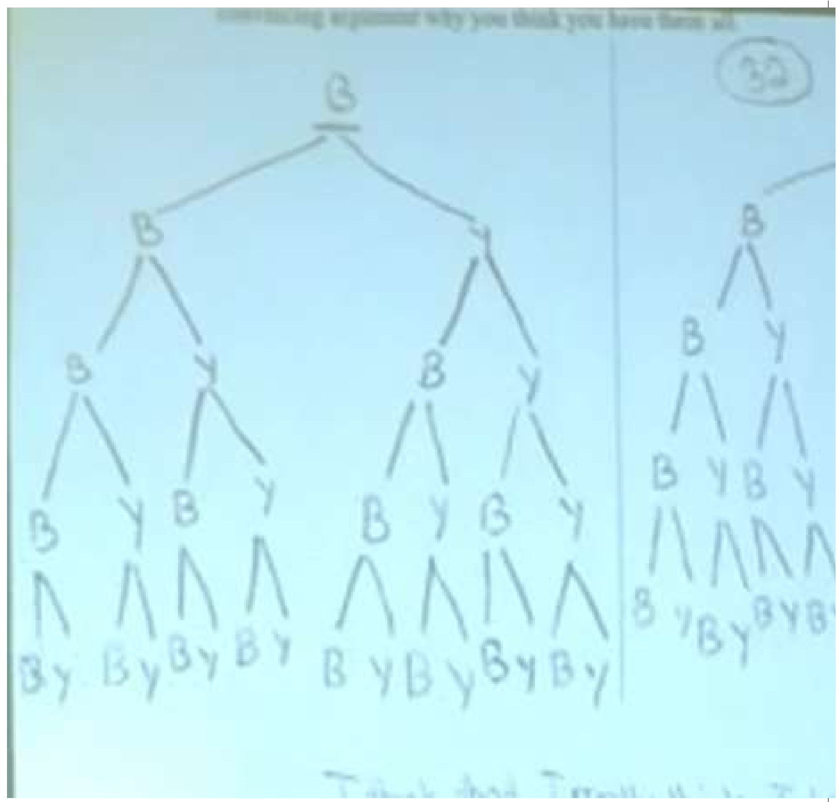
29.			
30.		RB	That's the way he grouped it. After taking a really close look at the pizza toppings he went with the one. Then he went with the four blue one yellow, that would be your mushrooms versus your other toppings. For that. And this group of students even with the four towers. They were doing the same exact thing. Um, let me just take this off now.
31.		JL	OK
32.		RB	Ok they were starting with the peppers, peppers sausage, peppers mushroom ok
33.	2:14	JL	Is that peppers or plain pizza?
34.		RB	Plain, I'm sorry, plain plain and sausage, sausage and, and so they kept it constant.
35.			Ok yeah
36.			Yeah I was looking at the top
37.			Those two don't really go, but I was thinking
38.		RB	Ok, I think a little bit
39.			They ran out of room and went to the top.
40.		RB	Possibly, but that's kind of what they did. They were grouping it together, um, as far as the. I can visualize it, because like I saw what they were doing in class. It was kind of like the staircase method, I guess on that. Because with the five towers and I know I keep switching it
41.		JL	That's ok
42.		RB	They were doing, they grouped the 4 towers in the exact same way. Five blue, five yellow, and then they found the four blue and the four yellow, and when you look at it, it's yellow blue blue blue, blue yellow, then the yellow drops down one and

			you can see the staircase for the five then they did the four yellow with the one blue. The opposite, then they did the three blue and the two yellow: yellow yellow blue blue blue, then the yellow shifted down and you could obviously see the staircase. Then they separated the yellows in those. So they got something to work, and they were able to solve all three problems relatively quickly. And they were the only ones to make a true connection to the pizza with the…
43.		JL	And is that were they made the connection?
44.		RB	They made the connection
45.		JL	What they wrote there, what does that say, can you read it for us?
46.		RB	We did this first, before the pizza.
47.		JL	Oh, ok, what did they write there, if we build…
48.		RB	If we build other towers, we end up copying others. Now they were going back to four towers high.
49.		JL	Oh, ok
50.		RB	What they did was, they actually tried to remember what they did for the four towers, and just built the one block on top of each of them.
51.		KK	I think that's really impressive
52.		RB	That's what these two boys did
53.		KK	If that's what they really did
54.		RB	Well, this student up here, you can't see his name. He was actually declassified so we actually have some classification. Well, he was classified but is no longer classified. And he said, I know we have all the four blue and one yellow because there is only one yellow and five blocks in all so they can only be five of these combos and we did the opposite for the four yellow and one blue combos so we have all of those too.
55.		JL	Ok, so what they are doing is trying to convince you of the four and one combinations
56.		RB	Yes
57.		JL	Do they ever do anything to the three and twos? Because that's a harder convincing job.
58.		RB	The three and two, they really didn't, and I didn't want to push them at this point. I was really happy I have to be honest, that they got the 32 combinations
59.		JL	Yeah, it's impressive

60.		RB	Then they were able to help others, and even helping take apart blocks and put them in the bags, so they were really able to do good. I didn't want to give them any follow up questions to that, knowing the pizza question was coming
61.		JL	You didn't do it the same day though, did you?
62.		RB	No, I did them about a week apart
63.		JL	Ok
64.		RB	A week and a half apart actually. So, and then we went back to this and there are 16 combos I think because I keep duplicating combos. I think I have all the combos because it is like the block game, with different combos of four block towers and they are similar to I did then. So they actually saw how these represented, I guess how the colors in their own mind.
65.	5:40	JL	Did they at any point explain to you how they were similar? Did they do that, because they didn't write it.
66.		RB	They did verbally tell me.
67.		JL	They did
68.		RB	They did say that the towers were four high and there were four toppings, and then one topping would represent the one color. They kind of explained it that way. I understood what they were saying and they were able to explain it verbally. And again, in the translation, writing it down here where they ran into the problem. And I wouldn't even call it a problem, because it's clear. And I think that they did a wonderful job. And they were able to solve this quickly.
69.		JL	Uh huh
70.		RB	The pizza problem, they weren't able to do as quick
71.		JL	Ok
72.		RB	This was, I think, record time
73.		JL	That is impressive. Now, I'm asking everyone in the room. Look at their connecting these four tall towers with the pizza problem. Are you convinced that they see the isomorphism? Why or why not?
74.		KK	Not with what's there.
75.		JL	ok
76.		RB	If you spoke to them, you would have seen that they made the connection.
77.		JL	And that often happens, I mean there are times that the kid gives you much more when you are talking with them and

			interacting with them, but on paper this would not be convincing if you read it, because we weren't there and we didn't see
78.		RB	No
79.		JL	So what you want to do as you go through the next set of tasks with the children you want to get them to try, if they really are making a wonderful connection, and verbally telling you, you want to get them to get it on paper.
80.		RB	Alright
81.		JL	Ok, you want to push them to get it on paper because then that way, you could actually show them "look this is, I'm not imagining this is what they were thinking, This is that they were thinking "
82.		RB	Uh huh, This is really what they were thinking.
83.		JL	Because they are telling you, they're telling you, they are showing all their work.
84.		RB	Uh huh
85.		JL	Alright, that's great though, That's very very nice, Thank you. Who else? OK
86.		KF	Actually bring up both at the same time
87.	7:31	RB	Justin, I was closing my class out and that's the class that, I'm sorry, that's very, they need every minute.
88.			
89.		JL	Did you have trouble getting here?
90.		J	Well, actually, so, what happened was I was rushing and I locked myself out of the room
91.		JL	You've got problems, Justin.
92.		All	Haha
93.		JL	But you drove today
94.		J	Yeah, and, you know, I want to apologize to
95.		JL	Ok
96.		J	The class, just, I just want to apologize.
97.		JL	We're glad you're here, we're glad you're here. OK, let's go, what do we have?
98.		RB	You can sit on that side if you want.
99.		All	Haha
100.		RB	No, I'm just kidding, Justin.

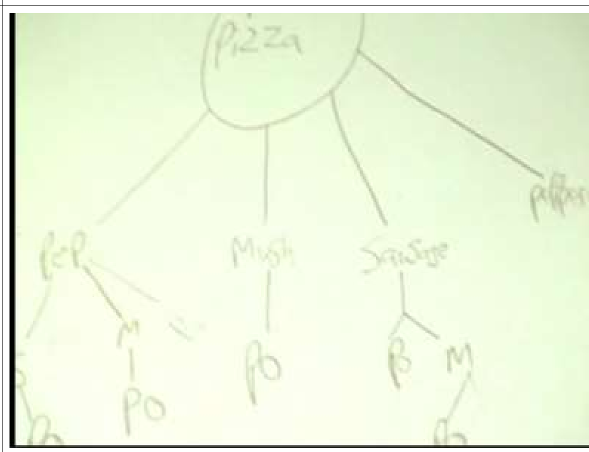
101.		KK	We'll take him
102.			We'll take him
103.			Anytime
104.		JL	OK, so what, this is
105.			Wow, this is very interesting
106.			Is this the same girl from last time?
107.		KF	It's the same
108.		KF	There's a trio that work together
109.			Uh huh
110.		KF	Since there's an odd number. And this is one of the kids in that trio
111.			
112.			Wow
113.		KF	So, I This is what they wanted to do first, and then they drew all the thirty two towers
114.		JL	Wow, they were able to read their tree diagram
115.		KF	It looks like it, because if you look here, right, you have b, b, b, b then they did
116.			Oh, wow, that's cool, I mean they have the chart over there. Yeah
117.			I think it looks like it would be quick for them
118.		KF	Yeah

119.			
120.		JL	Very neat. What, what does this kind of remind you of? They way they started on… Look, just look to the left side. They started, they did the blue block, and then on it, they put a blue and yellow. And on the blue, they put
121.		KF	A blue or a yellow, yeah.
122.		JL	So, what does that remind you of? Anything?
123.		J	It looks like Milin's, I forgot how you named it, but that was like Milin's
124.		JL	Yeah, Yeah it was like Milin's
125.			Yes
126.		JL	Remember the inductive argument?
127.		many	Yeah, mm hm
128.		JL	Where you have, when you're starting with a color, blue; you have two choices you put on top of it. You're dealing with two colors, and only two. So it could either become a blue blue or a blue yellow. OK? And similarly on, so that's great.
129.		KF	They just felt very, they just right away, they wanted to build a tree diagram
130.		JL	Yeah, and I bet if you ask them, when they are doing it, solving it this way, I bet if you ask them, these kids might be able to

			see that this has nothing to do with five times five
131.		KF	Right, right
132.		JL	But they may see this as a doubling effect as you move, as you keep increasing the height of the tower, you double the number of the towers you get. So let's see, what did they write?
133.		KF	I don't think it's that convincing because
134.		JL	Ok
135.		KF	I think they wrote
136.		KK	It's a shame because it's so clear
137.		KF	I know
138.		KK	By what they wrote, that they know
139.	10:18	KF	(reading) I think I really think that I have all 32 towers. I think this because I use a tree diagram to see all of the combinations. Then I copied all my combinations on the sheet of paper. I also checked to see if I had all the towers a few times. This is why I think I have all the towers. Even though they could verbally, like Rich said, say it. Then, when they write it, I don't know if they feel rushed, or they
140.		JL	It's hard
141.		KF	Don't want to take the time to write everything out.
142.		JL	Now, it's also hard to do
143.			Yeah
144.		JL	I mean even if they had the time, it's very hard to do.
145.		KF	It is hard, it's been so long.
146.		JL	So they did follow, I see that the first tower is five blues. What should the next tower be? Blue blue…
147.		KF	It should be blue blue blue blue yellow
148.		JL	So is it? And it is! So they actually could read their tree diagram. Sometimes, what happens is, is the children make a tree diagram, and they don't even know what they have. They don't even know where the towers are in that
149.		KF	That's what well, blue blue blue yellow blue. That's what they did there. So they did copy it.
150.		JL	Very Nice
151.		KF	And then, Here I just picked two. This one, that was a different girl. So, she and her partner decided to group them this way. And they chose to do all the combinations with one topping,

			two toppings, three toppings, and then four toppings. What she did was, she put fifteen total plus one plain pizza.
152.		JL	Okay, okay
153.		KF	What she wrote was "I got 16 different combinations of pizza, because I made a list of combinations of one, two, three and four toppings. I wrote a topping and then went down the list to add a second, third or fourth topping, eliminating ones that are reversed because we already had peppers and sausage" oh, they're parenthesis
154.		KK	An example
155.		KF	Yep. "we got fifteen combinations and we added one because you could have a plain pie."
156.		JL	Very neat
157.		KF	It's kind of like the way we solved.
158.	12:15	JL	It is the way. Many of you when we did it here, solved it just in the same way; holding constants underneath your two topping pizzas and three topping pizzas. I have a question: Some of you in your ecollege dialogue were saying that children didn't see the difference between peppers and sausage and sausage and peppers. They saw that as two different pizzas. Did any of you have that in here?
159.		RB	I had
160.		KF	They asked.
161.		RB	I actually explained to them. I explained it going into it. I said "If you have a pizza with sausage and peppers, and you could have the same pizza as peppers and sausage." I wanted to take that out. I don't know if that was a bad thing to do…
162.		JL	Probably not, yeah, probably was a bad thing. And the reason is that is just giving too much information.
163.		RB	Mm hm
164.		JL	Because that would be interesting to see if they really believed they were two different pizzas. Sometimes order does matter, doesn't it?
165.		KK	Right, right
166.		JL	But in this particular case
167.		KF	It's not, right
168.		JL	When you put the toppings on the pizza, when you put the sausage on the pizza first, and then the peppers, it's not. And what you did, as teachers we want to help children, we want to

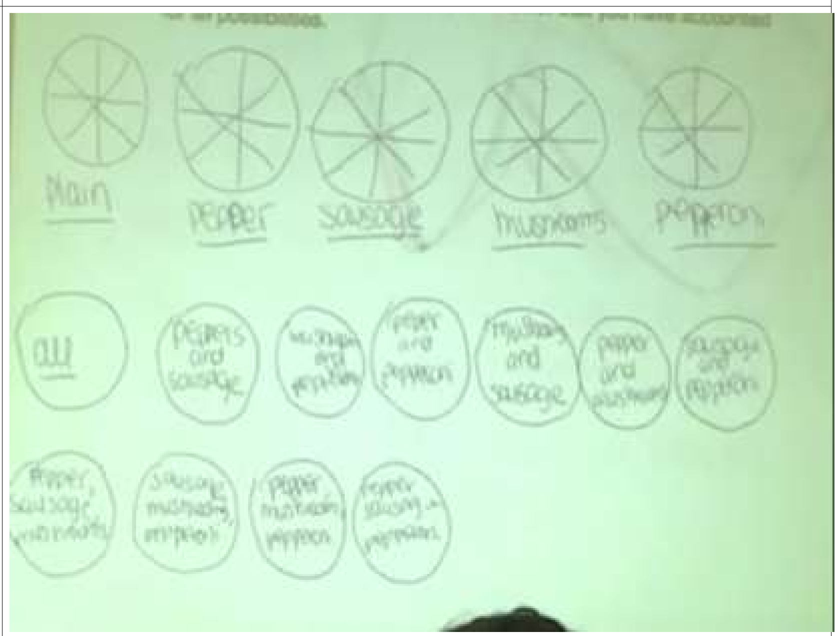
			lead children, we want them to be successful. So that's a normal thing. I still, after many many years of being in classrooms and working with teachers and children, I have to bite my tongue because I want to lead. You have to hold back. Okay, giving too much information is not necessarily a good thing.
169.		RB	They actually asked, I think two of the groups had asked, and then I said "Okay, everybody listen" and I just kind of…
170.		JL	What else could Rich have said? If they were asking?
171.			I would have said "Well, if you ordered it, would it be the same thing?"
172.		JL	Okay, or what do you think? Or you know, "How do you feel about that?" you know "Talk to your partner and see if you can come to an agreement."
173.		KK	I just today had students that had thirty two, they had thirty two combinations and the interesting thing was, what I said to them
174.		JL	This was the pizza problem, by the way, that they had thirty two for.
175.		KK	Right, they had thirty two for the pizza problem and I said "That's interesting" I said "Well, look at this one here, we have sausage and pepper, and pepper and sausage, what do you think?" And they honestly couldn't tell me whether they were. So then, when one of the pair said "I think it would be the same thing" and then, then they went back to rework everything and they came up with eleven. And I was like "Oh my God"
176.			Wow
177.		KK	You had everything and then some, and then they only ended up with eleven.
178.		JL	Awesome
179.		KK	And that to me was really bizarre
180.		JL	So but you know when they go over this problem and then you have someone going up like your student that says "well I'm not going to put peppers and sausage because sausage and peppers is the same thing" They may hear it, if they're ready to, they may hear it and go "Oh!"
181.		KK	Yeah
182.		JL	"Of course that makes sense, how silly" you know? Or they may not be ready to hear it and it may go in one ear and out the other and it won't bother them at all.

183.		KF	I don't want to take too much time, but this was interesting, this was the girl of the tree diagram form last time
184.		JL	Yep
185.		KF	She organized it, just a very different way. She listed all the ones with peppers, like that, peppers; all the ones with mushrooms, sausage and the pepperoni
186.		JL	Neat
187.		KF	And she just said "For pepperoni, you could not combine with any other topping because all the possible combinations using pepperoni were already mentioned. We also added the pie with nothing on it and that would be one." So it just
188.			Yeah
189.		JL	And that is interesting, isn't it? That that's the way she saw it, and she was able to start with the peppers and do all the pizzas that had peppers. It didn't matter whether they had one topping two toppings, three toppings
190.			Yeah
191.			Very nice
192.		KK	Yeah, that is interesting, That's kind of how, actually if you don't mind
193.	15:37	JL	Okay, sure sure
194.		KK	Their tree diagram
195.		JL	Okay
196.		KK	Kind of follows that same thing
197.		JL	Okay, alright, pull down just a little, perfect. Let it just darken a little, it will. It will darken if you give it time.
198.			
199.		KK	So this group was also a trio, and this was the only one of the three that could make the details up, and they were able to

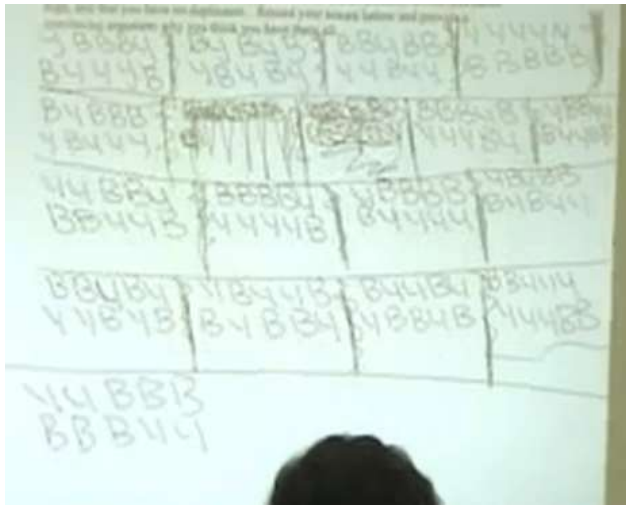
			explain to me in words, although I didn't really see exactly what they had written down. But the interesting thing was, and they were able to verbalize it to me; That, first of all their one plain pizza, and then they did tell me that this would be four, one of each. And then they had started with pepperoni, and they had the four choices off pepperoni, three off sausage.
200.		JL	So go back, okay, the first line down would be what kind of pizza?
201.		KK	One of each.
202.		JL	One topping
203.		KK	One pepperoni, one mushroom, sausage oh, I'm sorry peppers.
204.		JL	Yep, okay
205.		KK	And then they kind of built off from there, so this one was two toppings with pepperoni, pepperoni and sausage, pepperoni and mushroom, peperoni and …
206.		JL	No that's peppers, it's peppers
207.		KK	Oh peppers
208.		JL	And sausage
209.		KK	I'm sorry, peppers and sausage, peppers and mushroom, peppers and pepperoni. Mushrooms and
210.		JL	Pepperoni
211.		KK	Pepperoni, peppers
212.		JL	No, "PO" is pepperoni
213.		KK	Oh, now I'm confused though, wait. Yeah, okay. Sausage and pepperoni
214.		JL	Yep
215.		KK	Sausage and mushroom
216.		JL	Good
217.		KK	And then because they had already done it, just like that last list you had, they had nothing ever connected to pepperoni because they used it in all the other situations.
218.		JL	Good
219.		KK	So if you look at the tree diagram, it follows the same thing. The most here because they started with that. Then sausage, I don't know why they skipped over, but. Then two then nothing.
220.		JL	Show them the two topping pizzas just trace the lines so that they can see where they are.

221.		KK	This is peppers, right
222.		JL	Right
223.		KK	So peppers and sausage,
224.		JL	That's one
225.		KK	peppers and mushrooms,
226.		JL	Two
227.		KK	peppers and pepperoni
228.		JL	Three
229.		KK	Then
230.		JL	Keep going
231.		KK	Mushroom and
232.		JL	Pepperoni
233.		KK	Pepperoni
234.		JL	Four
235.		KK	Sausage and pepperoni
236.		JL	Five
237.		KK	Sausage and mushroom
238.		JL	Six. Is that pretty neat? So that was an easy and usually the tree diagram, that gets kids into trouble, right
239.		KK	Yeah
240.		JL	Because they ended up with such a mess they didn't know what they had
241.			That's what I saw
242.		JL	But here, these students really understood, what their tree diagram helped them see the different pizzas they got sixteen pizzas and what did they write? What's down below? Okay
243.		KK	"All together we got sixteen combinations. We started out with plain pizza. All together the toppings made one pizza." So that's the four. "All together the toppings made separate pizzas" That's the first four. "Then the first topping was pepper. We added sausage, mushrooms and pepperoni to it. We kept adding combinations until we didn't have any more toppings. Until we couldn't use any more combinations/toppings. This reminds me of the"
244.		JL	Outfits
245.		KK	Alfits, alright

246.		All	Haha
247.		KK	I think that might be it
248.		JL	It is outfits.
249.		KK	"This also reminds me of" OH! "the building block towers. When you count the combos of the towers." Wow, I didn't even notice that part.
250.		JL	And we were rushed. This was at the end of the class today.
251.		KK	Yeah
252.		JL	So if there were more time, a good question would have been, "How does it relate to the towers?"
253.		KK	How does it relate? Yeah. That's it
254.		JL	How does it relate? How does it remind you? Why do you think there's a connection?
255.		KK	So, I thought that was interesting
256.		JL	Mm hm
257.		KK	Way that they represented the tree diagram,
258.		JL	Mm hm
259.		KK	Which tied into Kulsom's because it followed that same
260.		JL	Yes
261.		KK	Four, three, two, one thing
262.		JL	Yeah
263.		KK	This one, they kind of controlled for the variable. Which is basically exactly what Sally and I did when we did it. I thought it was interesting that they did "plain with peppers, plain…"
264.		S	They all wanted to do that.
265.		KK	Isn't that funny?
266.		S	A lot of them.
267.		JL	Well, I guess they're saying to you "You can't have peppers, you have to have a pizza with peppers on it."
268.		KK	Yeah
269.		JL	Right
270.		S	Yeah, I guess so
271.		KK	So, they did. You know, they have four choices. Then they did everything with peppers. And then Sausage
272.		JL	Right, okay and then what's the next one down? On the next side

273.		KK	Mushrooms and peppers?
274.		JL	Okay, we can't see it yet. So it should be their sixth two topping pizza. And it probably is. Is it?
275.		KK	Yeah, it is.
276.		JL	Okay, we believe it. What's the feat?
277.		KK	And the very last one was super interesting.
278.		JL	Yes, and what we're going to do. We decided, Kate and I, is we're going to let you try and be the detective and figure out what this child did. Okay? Because it is interesting. Give it a minute to come up, and see if you can figure out, were they systematic? Did they have an organized approach to doing it? Was it not… Was this the one?
279.		KK	Yep
280.		JL	Okay it was the one, yes it was. Okay
281.			
282.		RB	Mm hm
283.		JL	Oh, you see it?
284.		RB	Yeah, I see it.
285.		JL	Okay, well, give everyone else a minute, because you're too fast.
286.			Haha
287.			Okay
288.		J	Okay
289.		JL	You see something too?

290.		A	It looks like they used complements, kind of.
291.		JL	"Complements" what does that mean?
292.		JL	Um, haha.
293.		A	If you look at the whole as all our toppings,
294.		JL	Right
295.		A	Like, they had the two on the left. Then the next ones go on the right.
296.		JL	Oh
297.		A	Then the next ones go on the outside.
298.		JL	Can you point out what she's…
299.		KK	Yeah
300.		JL	Is that what you saw, Rich?
301.		RB	Yeah, yeah they go for the peppers and the sausage. Then they go
302.		KK	Because when we first, when we looked at it
303.		RB	Mushrooms and pepperoni
304.		KK	We were like "Oh my God" they were all over the place, how would they do it. But she's right. First they did all of their, obviously, one topping. They did the all. Then they did peppers and sausage, mushrooms and pepperoni. And then they went from the outside. Peppers and pepperoni, mushrooms and sausage. And then they did, like every other. Peppers and mushrooms and then pepperoni…
305.		JL	Isn't that neat?
306.		KK	Right, just in how they saw it?
307.		JL	Yeah
308.		KK	And then the three, they want in a row. "Peppers, sausage, mushrooms" Then they skipped. They did "sausage, mushrooms, pepperoni" And then they skipped the sausage, right, so "pepper, mushroom, pepperoni"
309.		JL	Now you do it
310.		KK	"peppers, sausage and pepperoni"
311.		JL	Isn't that neat?
312.		KK	It was very systematic. Once you, to how they took it. So that was, that was very interesting.
313.		JL	Very good. Thank you

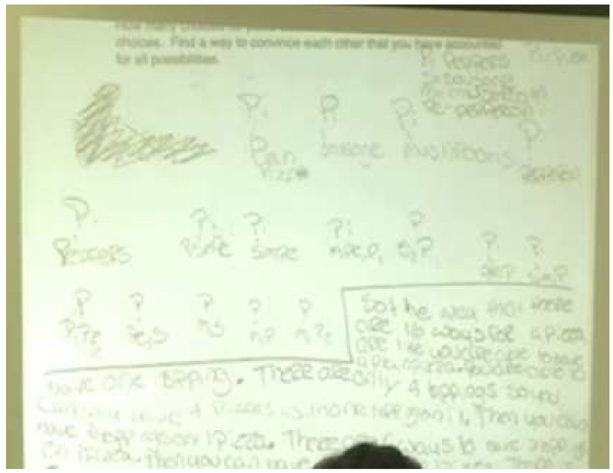
314.		KK	Sure
315.		JL	Okay, do we have another person to share some work? Okay, good, Justin? Interesting what kids do, huh? And when we first looked at it, we had three eyes looking at it, at this paper after the kids left. And we said, "oh, it looks like they're all over the place." Until we looked at it more closely and we said "mm, this is really quite systematic, quite organized." Okay, let's give it a minute to come up. Isn't this an amazing machine?
316.			
317.		J	Now, this student, now some of my students, some of my students, they are academically at risk.
318.		JL	Okay
319.		J	This student, she failed the sixth grade twice. But they moved her up to the eighth grade because she was fifteen.
320.		JL	So she skipped seventh grade.
321.		J	Yeah
322.			It's so funny.
323.			Why?!
324.		A	As soon as I saw this, I knew who the student was, because I had her last year.
325.		JL	By the handwriting?
326.		A	Yeah, I could tell by the handwriting
327.		JL	Wow, wow
328.		J	So this is how she kind of, this is how she wrote it up.
329.		JL	Okay
330.			
331.		J	I thought that that was interesting

332.			Yeah
333.		J	Her group actually did it vertically. This is how she did horizontally
334.		JL	Okay
335.		J	Which was interesting because my brain doesn't work that way
336.		JL	But she, it worked for her, obviously. Did she find thirty two?
337.		J	She found thirty two. So yes.
338.		JL	Were there duplicates? Or, you don't know?
339.		J	No, they were official.
340.		JL	They were each unique. Okay
341.		J	Yes
342.		JL	What did she write? Want to read it to us?
343.	23:53	J	So this is interesting. She, when I was asking, and probing them for convincing arguments, she wrote "We got thirty-two ways to put the five cubes in order. I found our argument convincing because last time we did four cubes and got sixteen. This time we did five cubes and got thirty two. We had eight sets of five that didn't have a vice versa match."
344.		JL	That's so, stop a minute.
345.		All	Hahaha
346.		JL	Justin. "Last time we did four cubes and got sixteen. This time we did five cubes and got thirty two." What is she talking about?
347.		S	They did four cubes high, and now you could put a yellow or a blue on top of that and then.
348.		JL	So she is seeing the doubling effect.
349.		KK	Yeah, she is.
350.		JL	Isn't it. You see, I think she belongs in the eighth grade. She belongs in the eighth grade. I'm glad they skipped her because that's really neat.
351.			Yeah
352.		JL	How many of your children saw?
353.			None
354.		J	None of my students ever saw that
355.		JL	Okay
356.		J	And the fact that she actually said that was, I was just like "oh

			wait a minute"
357.		JL	Wow, neat, so isn't that something? Keep going.
358.		J	So I was like "she needed to put that in" but anyway. So yeah, "Eight sets of five that didn't have a vice versa match."
359.		JL	What does she mean by that? Stop. What do you think she means by that? "We had eight sets of five that didn't have a vice versa match."
360.		RB	No opposite?
361.		A	Maybe she built them all and noticed that, okay these match, and then there were eight of them that didn't because then she said "We couldn't flip them upside down."
362.		J	What happened was that someone, certain towers, like they started with all blue. If you flip it upside down, it's still gonna be all blue. But the way they were able to get thirty two was that they began, towards the end, to flip cubes over. To see if they had that particular cube.
363.		KK	Right, pattern, yeah
364.		J	If they didn't have that particular cube, tower, I mean. They would create it. So, that was their…
365.		JL	Was that the vice versa?
366.			So if it didn't have an opposite?
367.		J	Yes
368.		S	So if it has like a vertical symmetry, like three blue in the middle with a yellow on top, and a yellow on the bottom, that would be the one that doesn't have a vice versa?
369.		J	Exactly.
370.		S	Oh, okay
371.		J	So what really did was they did guess and check all the way until they got to like twenty-four or so. And then they did opposites.
372.		JL	So they found twenty four by guessing and checking.
373.		J	And then they got to twenty eight or thirty
374.		KK	Twenty eight is when mine started having trouble
375.		J	By doing opposites
376.		JL	Okay
377.		J	And then to get thirty two, they flipped each of them to see if they had any pairs like that. And that's kind of how she wrote it up. "We flipped them upside down and put them in groups of

			four." Which is interesting too. So they had each opposite pair originally, and then I asked them could they get more by considering to organize them, or is there a way they could organize them to get to even more.
378.		JL	Right
379.		J	So they had them in pairs, but then when they flipped them upside down, they could group those pairs into fours. Groups of four
380.		JL	Groups of four, so did they have eight groups of four?
381.		J	No
382.		JL	No? okay
383.		J	They didn't have eight groups of four because some of them didn't
384.		KK	There was no "vice versas"
385.		J	Right
386.		JL	So, do they show you what their towers look like? Do they have a picture of their towers?
387.		J	On the front
388.		JL	Okay, before we go to the, okay oh, I see it, alright. So let's see. Let's just look a minute. Do we see how they grouped them?
389.		A	Oh, the fours are going across.
390.		JL	They're going
391.			Wow, that's interesting
392.		JL	But it looks like the top left and the second row left are together. There's a line before you go to the next two, so how are they grouping their towers?
393.		J	Are you asking this class?
394.		JL	Well, I was asking the class.
395.		KF?	Is that what she means those first four and the next one, are those the groups that don't have an opposite?
396.		J	Yes
397.		KF	Those are the five groups of eight she's talking about?
398.		J	Yes
399.			Oh, one two three four five
400.		J	I believe so, I believe that's how

401.		S	I don't know what she said; five groups of eight or eight groups of five? What did she say, eight groups of five?
402.		KK	Yeah, yeah, yeah
403.		J	Eight groups of five. So the ones on the top were the ones that you couldn't flip
404.		JL	What do you think Mitch?
405.		M	Yeah, because
406.			There's towers, blocks
407.			Eight blocks towers
408.			The first was
409.			Eight towers symmetrical.
410.		JL	So, but she has them in pairs, isn't that interesting? So it's quite impressive that she is pairing her towers, okay, and they're not in an organized way, like three blue and two yellow, or four blue and one yellow. They're not in terms of cases, right?
411.			Mm hm
412.		JL	But yet, they found their thirty two, and yet she's able to see a doubling effect. Flip it again and look at her last sentence that she writes. "We got sixteen. And sixteen and sixteen is thirty two. And we got thirty two cubes." What I would be curious to know is, what?
413.		KF	If she, the next, if she could figure out six tall.
414.		JL	Yeah, yeah. Because if she really understood the doubling effect. You would then say to her "Well, can you imagine, don't build them, but what do you think you're going to get if I asked you to build towers six tall?" And if she really understands that it's doubling then she'll be able to tell you sixty four, okay? And you might do that, you might, you know revisit it with her. But that, that's fascinating, you know? Very neat, very neat.
415.		S	I would also like to see if they could say, like, why they think it's doubling also.
416.		JL	Absolutely, that would be a good question to add.
417.		J	They couldn't, they couldn't provide me with that. Like why
418.		S	Right, yeah
419.		J	Like how for five towers tall, and they did to thirty two towers
420.		JL	Right, right

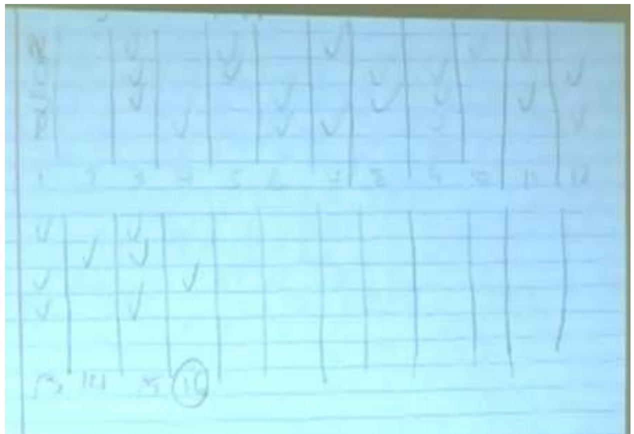
421.		J	So
422.		JL	She saw the doubling, but she didn't know why. But it would be interesting to pose that question, absolutely. Okay, so what else do we have?
423.			
424.	30:00	J	Okay, so this is Damien and Artur. So, Again. But oh well, anyway so. This is the peppers and pizza toppings.
425.		S	Yes, yes
426.		J	So, now Damien actually did his notation a little differently.
427.		S	Uh huh
428.		J	But he also, he had said that they were groupings, instead of toppings. Things like that. But what I thought was interesting, were Damien's pictures. He tried to start off with the, with the tree diagram and then it kind of morphed into what he has here.
429.		JL	Okay
430.		J	So, first he started off with five pizzas across. He said five one topping and he included the plain as one topping.
431.		JL	Many students did that. Did any of yours?
432.		KK	Yeah
433.		JL	Okay
434.		J	So he did peppers, and the rest of the five one toppings, I mean four toppings.
435.		JL	Right
436.		J	So then he went from, he went from one topping, yeah to three, and then I think to maybe four. And then I said "Well look, you have, you have one topping and then you move to three. Why would you, like what was that about?" And again I'm not sure how far I led him to it, by saying that, but then I was like "look

			at it, look some more." And they both, him and Artur, they both looked, they both went from one to three to four. So one topping, then to all toppings, then to three toppings.
437.		JL	Right
438.		J	And then they were able to get from ten to sixteen, by the two toppings afterwards.
439.		JL	The two toppings came last.
440.	31:58	J	Yes.
441.		JL	Okay. How many two toppings did they get?
442.		J	Um, six.
443.		JL	Okay, and it says it down there. It says "There were six ways to make two toppings on a pizza."
444.		J	Where, are you reading that?
445.		JL	I'm reading what they wrote. "There are six ways to have two toppings on one pizza. Then you can have three toppings on a pizza. There is five combos that have three toppings on a pizza." Interesting, they had five three topping pizzas. Do they actually show you five?
446.		J	No, I don't think so. Five combos that have three.
447.		JL	Have three toppings on a pizza. So it looks like they are thinking there are five three topping pizzas. Let's see if we can see which they are. Point to them, Justin, to help us.
448.		J	So, this is one
449.		All	That's one
450.			No, no that's
451.			That's four toppings
452.			That's four
453.			Okay
454.			So it looks like he went from one to four.
455.		KK	That looks like, that one's four also, though.
456.			It looks confusing
457.			No
458.			Oh, Pe, P alright
459.		J	So he went from four, I mean one to four and then to three. So he has one, two. So sausage, mushroom, pepperoni
460.		JL	So that's two

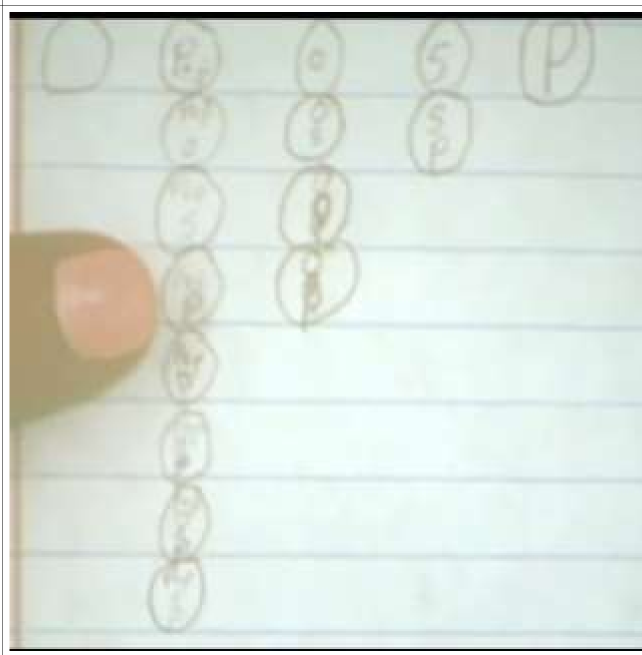
461.		J	Mushroom, pepperoni, peppers
462.		JL	Three
463.			And skip over
464.		JL	So skip it. Is that a two or a three that you're pointing at?
465.			That's sausage.
466.		A	I think it's a p squeezed in there, no?
467.		J	That's sausage, pepperoni and peppers.
468.		JL	So that's a three. And four
469.		J	Sausage, mushroom and peppers
470.		JL	So he does have five. Is there a duplicate?
471.			I only see four.
472.			It's four, there's four.
473.		JL	Well, let's see. I think you pointed to
474.		J	That's one
475.		JL	That's two, so that's one, two.
476.		J	One, Two, Three and Four.
477.		JL	Okay, so he does have, he is saying five though, in his write up.
478.			Yeah
479.		J	I'm guessing that he counted this. But this was, this was supposed to be down here.
480.		JL	Okay
481.		J	This one.
482.		JL	Okay, okay. Possibly, you know, and I think their code; and this is neat. He is giving you a code. But sometimes their codes get them into trouble because the- And this was done intentionally, to have peppers and pepperoni as two possibilities, forces children to invent notation. But sometimes it's hard to read what they are writing, especially with the commas, and the p, i, or pl is plain, and the comma kind of, it's hard to say but. But they had a very, you're right they did go, they didn't go in order, but they had a systematic way to do it didn't they?
483.		M	Mm hm
484.		J	What was happening was, they kept challenging, they would draw questions They hadn't come up with a lot of the combos

485.		JL	Right
486.		J	So, and this, some of them had, they had duplicates. So they were talking to each other with duplicate ones.
487.		JL	Okay
488.		J	This one said "Well, what do you have?" "Well I have this." You know, so they were going back and forth. And they made a lot of, you know, corrections and erasures, actually.
489.		JL	And you want to try and encourage kids. They really don't like to leave something that they're not happy with. Encourage them to just put one line through it and keep going. Rather than erasing it, because then you could see what they did. Because sometimes, they are going down a path that really isn't a bad path and you can see how they change direction. Okay, great. Thank you, Justin. Who else?
490.		S	I'll go.
491.		JL	Okay
492.		S	Ben was about the thirty two, the five tall towers. But I only had two groups who got it.
493.		JL	Okay
494.		S	And because I didn't want to lead them, I
495.		JL	Sure
496.		S	I didn't say the total.
497.		JL	And, you know, remember: Are we looking to see that they got thirty two? No.
498.		S	Right
499.		JL	We are looking to see what kind of strategies are they using to solve this problem. Because they could have good strategies, Sally, even if they don't get the answer.
500.		S	Right, right. The thing is that they kept going for like the pairs of opposites
501.		JL	Okay
502.		S	They would do the, the diagonal, the two groups of diagonal. But then they would do the pairs of opposites, and they would only get like twenty or thirty.
503.		JL	Sure, sure
504.		S	And with the diagonal
505.		KK	With the diagonals, I think that threw them off, because there's five.

506.		JL	Uh huh
507.		KK	And they kept thinking "It should be pairs"
508.		JL	Oh
509.		KK	You know, they would think of that even pairs thing.
510.		JL	Okay
511.		KK	And there were five of this. I mean they did get better at the groupings.
512.		JL	Right
513.		KK	Because of those two
514.		JL	Uh huh
515.		KK	Sets of ten. It gave them ten. You're right, after that, they just. I said to them "If you're just going to keep doing opposites, I don't think you're going to get anywhere. You're going to have to organize better."
516.		JL	Right
517.		KK	Organize better
518.		JL	And that's true except for which, which of the kids that did the pairs and got it? Justin's
519.			Justin's kid
520.		KK	But they did pairs and opposites
521.		S	Yeah
522.			Which is
523.		JL	But even, even doing pairs and flips and the vice versa or whatever you want to call it, it still is very very hard to keep track of
524.		KK	Yes
525.		JL	If you have it already, don't you have it? Is it a duplicate? Isn't it? So let's see what they did. What about this.
526.		S	Okay, I have to say that, um, I'm biased against mushrooms, so I changed the problem. And I substituted onions.
527.		All	Haha
528.		S	So I'm sorry.
529.		JL	You're kidding, I'm glad you did not take away the peppers
530.		S	I mean, yeah, I decided to leave that.
531.		JL	Okay
532.		S	So, sorry

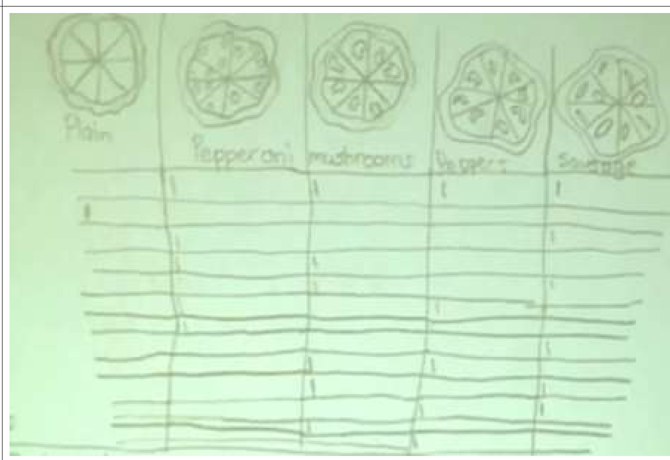
533.		KK	Sally, you're crazy.
534.		S	I wouldn't change anything like that. Okay, so this: I was shocked when I saw that she did this because it reminded me of what Brandon did.
535.			
536.		K	Yes
537.		J	Oh, wow
538.		S	And, um, I, I went a cross looking, and I feel like she really didn't use the same format as him, but it was kind of the same idea.
539.		JL	So she used checks and blanks?
540.			Well that's something I want
541.			Is there something over the top?
542.		S	It's just, it's just the problem, yeah
543.		JL	Okay, so she used checks and blanks. Brandon used
544.		S	Zeroes and ones, yeah
545.		JL	Okay
546.		JL	And, so she got sixteen pizzas?
547.		S	Mm hm
548.		JL	Okay, can we see, did she go, like her first pizza she started?
549.		S	She listed the four toppings,
550.		JL	Okay
551.		S	And then I noticed she named it, like that was the first pizza. And I do think, you can pick up on what she did.
552.		JL	Okay
553.		S	Like as you go across.
554.		JL	It's so

555.		JL	Like she didn't go in a particular order.
556.		M	One everything, one nothing
557.			Mm hm
558.		S	She,
559.		KK	It's one check then three checks, then back to three and then two. Am I seeing it?
560.		S	But there is, there is sort of a connection.
561.		JL	There is. Let's see if they can find it.
562.		S	Okay
563.		JL	Look at her, go from left to right, okay. Go from Pizza two, its labeled with a two, what is that one?
564.		KK	It's just sausage
565.		S	Plain
566.		JL	Plain, right
567.		KK	Oh, there's no check on that one, I'm sorry
568.		JL	Okay, what was pizza one?
569.		RB	Everything
570.		JL	Everything, okay, what is pizza three?
571.		KK	She got one, um, three toppings
572.		JL	Right, three toppings and?
573.		J	No toppings
574.		JL	And then you have one topping, right?
575.		J	And the one topping is…
576.		JL	And what she's actually doing, it looks like what Sally?
577.		RB	And four is
578.		S	The, like the complement idea, where almost like opposites, when you build towers. Like they would invert the colors
579.		JL	Uh huh
580.		KK	Three and one, two and two. That's separated
581.		RB	Every two columns equals four
582.		S	Right
583.		RB	And if you put them together, you would have the four, the vertical.
584.		S	Exactly

585.		JL	Okay, okay, and it would. Now it would be interesting to see what these towers looked like when she actually set them up. Did she build towers, and this is her recording?
586.		S	This was
587.		RB	Pizza
588.		JL	This was pizza?!
589.		KK	This was pizza, yeah, right. This was pizza.
590.		JL	You're right, I'm confused. You're right. This is the pizza problem, okay. Very very interesting, that's
591.		S	So, yeah, that's why I had made my comment that, and um this is funny because I was saying that Brandon had such a way of doing it that he recognized the connection with the towers.
592.		JL	Yes
593.		S	And I feel like she would have been the one to make that connection, and she didn't for whatever reason.
594.		JL	Okay, okay. And here she, she got her sixteen, and I wonder if, if she. Did you ask her if this reminded her of anything?
595.		S	And she didn't.
596.		JL	No, okay, okay. Very interesting
597.			So
598.		JL	Huh. Alright, that was that
599.		S	And this was kind of like what we've seen before. But he used more of a picture method. He writes really small.
600.			

601.		JL	He does. You're going to have to help us.
602.		S	Okay, so here's the plain
603.		JL	Okay
604.		S	And then here, kind of like, I guess I would call this the pepperoni, or maybe the pepper column. And he matched pepperoni with every possible thing. So this is the one pepperoni. This is pepperoni onion, pepperoni sausage, peperoni peppers. And then this one is pepperoni onion sausage, pepperoni onion peppers, this is all four and then this is pepperoni sausage peppers.
605.		JL	Pretty neat
606.		S	And so he
607.		JL	He's now seeing each column is going to get shorter, why?
608.			He didn't start
609.		S	Because he already used up pepperoni.
610.		JL	That's right. He already has them listed in the pepperoni column. Um, who had this written in a different format?
611.		A	I did and he did
612.			Yeah
613.		JL	And that is that is a neat way to organize, isn't it
614.			Yeah
615.		S	Yeah, and I thought it was cool that he did it in like a circle, because that makes me feel like he would have been the student that already knew that like onion sausage is the same as sausage and onion.
616.		JL	Okay
617.		S	So, and so this is his onion column, this is his sausage column, and then the peppers.
618.		JL	Do you notice anything about the columns? You notice any patterns?
619.		S	They become half, starting with
620.		JL	That interesting, huh. There were eight in that first column, and then there were four and then there were two. Interesting, huh. Okay great, thank you Sally. Okay, who else? Mitch.
621.		M	Um, this first one, I should probably get his work for the towers too, but this is the pizza one.
622.		JL	Okay

623.		M	And he split it into different groups.
624.		JL	Okay, so let's see what he did. Give us a minute to look. "The works"
625.		All	Whoa, haha
626.		M	That is the works, is it not?
627.		KK	Plain (on its own) that is good
628.		JL	Okay, sausage combos, mushroom combos
629.		S	Oh yeah, that's interesting
630.		RB	Actually it's pretty nice how he followed the directions because you have to make an order form.
631.		JL	Right
632.		KK	And that's what he did
633.		RB	That's what the directions said.
634.		S	You're right it said that
635.		RB	And
636.			Nobody
637.		JL	Yep
638.		RB	And, it's pretty interesting because if somebody called up on the phone
639.		JL	Absolutely
640.		RB	It would be very easy
641.		JL	Absolutely
642.		RB	To check that off
643.		JL	Very neat
644.		RB	I know that's not how we solved it, but that's still important.
645.		JL	It could be a nice order form, you're right absolutely. And what does he write? Can you read it to us?
646.		M	It says there's sixteen choices for pizza. I am positive my answer is correct because I went through single, double, triple and the works of every combo. I also took the one topping from the singles and working from there in a downward pattern chose any combos.
647.		JL	What does he mean? You're shaking your head, Kate. What does he mean?
648.		KK	He basically controlled for a variable.
649.		JL	Okay, and he did control for his variable.

650.			Yeah
651.		JL	Right
652.		KK	All the way through, yeah
653.		JL	Very, very neat, isn't that?
654.		M	He was one of the only kids. When he got the thirty-two problem, he controlled for a variable also.
655.		S	He did
656.		M	He put them in groups of one blue, two blues three blues, and he put it in groups of two blue, he actually moved the blocks down each time.
657.		JL	So he had a recursive argument
658.		M	Uh huh, and when I asked him if he saw a connection, the only thing he really said though was "yes because we were looking for patterns and because we put them in groups."
659.		JL	And many children see that they are both combination problems. Alright, and that's okay, I mean, that's at least nice for them to know, but it's not, um. It's not that they are saying they see the structure, the mathematical structure of the problem is identical. That's hard to do. Not many adults do it, forget about middle school kids. Alright, we did find though, in old bridge and toms river there actually were two children who saw the isomorphism. Which was amazing, amazing. Two Brandons in those districts, amazing, huh? So but I think that's, this is a remarkable paper, is it not?
660.			Yeah, really
661.		JL	Really nice. Good, what else do you have for us?
662.		M	This is another one that kind of did, kind of a lot like what Brandon did on his.
663.			

664.		JL	Okay
665.		M	Um, she doesn't get the right answer.
666.		All	Haha
667.		JL	It's okay, we're not worried about right answers
668.		M	I know
669.		JL	What did she forget?
670.		KK	The all?
671.		JL	No
672.			She didn't forget the plain
673.		M	One of the threes
674.			On
675.			Oh
676.			One of the threes
677.			She forgot a three,
678.			Threes
679.			Three toppings
680.	45:18	S	I don't see any
681.		RB	Pepperonis peppers and mushrooms
682.		JL	If you look at the bottom
683.	45:25	RB	Pepperoni peppers and mushrooms
684.			Um hmm
685.		S	Yeah you're right
686.		JL	So now, if you look at her chart, isn't that cute, she has little pizzas drawn in
687.		KF	With the squiggle stuff, yeah
688.		All	Haha
689.		JL	But if you look at her first pizza, What is her first pizza?
690.			All
691.		JL	All, okay, because she's going across, right?
692.			Mm hm
693.		JL	And the second pizza?
694.			Plain
695.		JL	Third pizza?

696.			Pepperoni and… sausage
697.		JL	Sausage, yeah so it's a two topping pizza, and then she has?
698.			Pepperoni, mushroom,
699.		JL	Okay, and then she has?
700.		A	She switches it to mushrooms and sausage.
701.		JL	So she isn't quite holding it constant, is she?
702.		M	Right
703.			No
704.			No
705.		JL	No, okay, so it's easy to slip something and not see all the combinations when you aren't holding a constant. But that, that's a neat solution, and I think that it's kind of neat what she did. Great, alright. Any other things that you wanted to share about what your students did?
706.		J	Yeah, just that, did she work with someone else?
707.		M	No she worked by herself.
708.		J	That's what I thought. I really, I totally thought that.
709.		JL	Why did you think, that, why?
710.	46:42	J	Because if you work with someone else, they could challenge you.
711.		JL	Okay
712.		J	Like
713.		KK	Like uh huh
714.		JL	Okay
715.		J	Yeah, like my students, when I ask them, "What do you think?" you know, they would then chat. Then when they were chatting, I knew they were on the right path. Then I would go on to the next one.
716.		JL	Okay
717.		J	Then I would come back, and they would say, well I did this, this and that.
718.		JL	Okay, good
719.		J	But I really thought that she probably didn't work with anybody, because of one, the detail, but two, that she missed that point. And somebody could have said, "well, since we were doing this way, this organized way, we're good." You know, "Why not just do this one and that one, then this one and

			that one"
720.		JL	Right, right. Now you know, when you, was there a reason why she worked alone? Is she a child that doesn't work well with others?
721.		M	She's very shy, she doesn't speak a lot, um, When I asked her if she saw a connection between this and the other one she just (shakes head) not at all, but
722.		JL	But, I mean you could be very vocal and not see a connection either.
723.		M	Yeah
724.		JL	I would encourage, for the reasons that Justin is saying, I would encourage you, unless you have a child who would behaviorally be a mess with another kid
725.		M	Mm hm
726.		JL	I would encourage them to work with someone. It might get her to talk more, if she
727.		M	She did on the first one and she came up with just pairs
728.		JL	Okay
729.		M	She would just basically guess and check, and she had like a million pairs.
730.		JL	Okay, okay, was she talking with the other child?
731.		M	Not that much.
732.		JL	Okay, what you might need to do for a child like that is help them learn how to communicate
733.		M	Mm hm
734.		JL	And what it might be is when you walk over to. If you pair her carefully, and you walk over to her and her partner, you might be asking questions that help them talk to each other. You know, "Can you explain to your partner what you're doing?" "Can you, ah, tell me if you understand what your partner is doing?" Things like that. There is real reason why you want them paired, okay. One reason is, there's only one of you, and if you have a lot of individuals, you're not going to get around. We had three adults today, in a wonderful classroom.
735.		KK	And there were still kids that were sitting
736.		JL	And it was a classroom of fourteen?
737.	48:50	KK	Thirteen.
738.		JL	Thirteen students, and we had three adults, and we still didn't

			get to everyone, you know when we needed to. So I'm saying for you to be able to hear, you want to have them paired. The other reason is, if you have what Justin said, if you have kids who can feed off each other, they can kind of say "Well what about this?" or "I don't know what you're doing." Or "I'm thinking of it this way, what do you think?" They can learn from each other. Okay, and the third reason is when you go over and eavesdrop, you can hear them talking. I don't think because she won't be talking to herself out loud.
739.		S	Haha
740.		JL	Okay, unusual for a solo kid to be talking out loud.
741.	49:33	M	What I saw a lot of was, a lot of kids stole other groups ideas.
742.			Yeah
743.		JL	That's okay, is it bad? Is that bad when they steal other ideas that they hear?
744.		S	I would think so
745.		M	I think sometimes
746.		JL	You do
747.		S	Honestly
748.		JL	Okay
749.		A	Why are you looking at their answer?
750.		JL	But what do they hear
751.		KK	The pizza or something with five
752.		JL	What if they hear a way
753.			Staircase problem
754.		A	I don't care about the answer, I don't care about what answer you get. Can you back it up?
755.		KK	Right, um hm
756.		JL	And I could, what were you going to say
757.		KF	Well, back to what Kate was saying, We went over the um, four tall tower problem, on the overhead
758.		KF	With a couple kids, and I put them on the overhead
759.		JL	Yes
760.		KF	And so when they saw that, like the next week with the five tall towers.
761.		KF	The ones I used, I showed the staircase method, but the one positive thing, and he said that too, right

762.		RB	Yeah
763.		KF	One caught it and the rest they did pairs, but I did hear when I walked around and said "Oh why did you choose to do a staircase?" and they said "Well, now I finally, I understand like, how they did that" but I said "Can you apply that to you other ones?" and they, some had a little difficulty. I mean, I don't know is it ever a bad thing? I don't know.
764.	50:43	JL	Okay, well I'll say I'll take the other position. If you can learn from what someone else is doing, Hallelujiah!
765.		M	Mm hm
766.		JL	If you really don't understand, you aren't even going to hear what they are saying. I've had classrooms where I've gone into. Fourth Grade classrooms. One teacher, in fact, Amy Martino. You saw her interviewing Brandon. She was a teacher in my building and I would watch, as she would let children explain their work to one another. And they would be listening, they would hear what the child said. A few in the class went "Oh! I'm going to use Nicole's strategy next time. I'm going to, I like what she did. I like the way she did her notation. I'm gonna use her's you know, what she did." Other kids "Psst!" In one ear, out the other. They really didn't hear it. You know. They were in the same room. The same words were spoken so unless they're ready to hear, they're not going to "steal" by your words, okay? Stealing is, probably makes it sound like a bad thing. It really isn't. Like, if you were here, and you saw a neat strategy, or a neat way of organizing, as we're talking. Wouldn't you want to use it?
767.		KK	As long as I understood it
768.		JL	Absolutely
769.		A	But I feel like a lot of the middle school students, they want the easy route. Like for example, when we did the four, the four tall towers and I explained it to them, I mean one group was younger, and they had like two towers next to each other, completely not opposites, so I'm kind of like "What is this?"
770.		JL	Okay
771.		A	And they were like "well, they have to be four tall towers" And I'm like "Yeah, why do you have them next to each other?" "Well, isn't that what we're supposed to do?" And I'm like "Well, they're doing it" And I'm like "…"
772.		JL	That's different, okay
773.		A	They had no idea what it meant, they were like "Well, they're doing it" so I feel like a lot of middle school students

774.		KK	Opposites
775.			
776.		JL	That, that would be a different situation. And I agree with you, that doesn't sound so good. If they don't understand, and they're doing something it won't make sense. In Kate's room today, it was quite interesting. We saw a lot of kids who had a teacher, or several teachers in the past, who drummed in to their head some kind of rule about how you get the answer for combinations by using a symbolic statement, you don't even have to build, so here, four tall , four pizzas. "Ah you have four toppings, just multiply it by itself and there's your answer, wow!" Okay, so they remember the teacher telling them it was four times four and that's the answer. And in fact, one child said it was four times four and then you had to add the one plain pizza. So your answer should be seventeen. And I said "well, wait a second, you didn't get seventeen you got sixteen." And they said "oh, it's four times four *times* one." So I'm saying, in other words, they played with the, you know their numbers and they played with their operations till they forced it to become the answer that they you know, knew that they had gotten. But I'm saying that there are ways for kids to hear and learn from another when they are ready. They really will not hear and make sense of something unless they understand it.
777.		KK	Mm hm
778.		JL	So, I think that you want to continue to let children share and it might be as they are working on the problem, you don't want them to be eavesdropping on another group, okay. I think that's what you're saying. But when they share their solutions, you want them listening. And if they don't hear it, it means they're not ready.
779.		S	I think also though, like with this particular one, if they were to take an idea, a strategy to solve the four towers and apply it to the towers five high, then that's fine because you're applying that idea, but if. But I feel like if they're taking someone's idea for this particular thing, my thing is I want them to develop their own strategies.
780.		JL	Okay, okay
781.		S	So When you're taking someone else's idea, how is that going to help you for the next unrelated task when you're not, kind of like, challenging yourself to create your own ideas
782.		JL	Right, yeah and I understand what you're saying, and I think when they're working with a partner they should be working

			with the partner as opposed to eavesdropping on everyone else in the room so I don't think we're disagreeing.
783.	54:45	JL	Okay, what I'd like you to do is, clear your desks. Find a partner, or a triple, for one of you. And we do have unifix cubes in the room, is that right?
784.		M	Yeah, on the ledge
785.		JL	Excellent, okay what we're gonna do now… Find a partner you like working with, because we're going to be doing a problem, as an extension problem that's quite challenging. So you want to work with someone that you like to work with. Is the third color, Mitch?
786.		M	Uh, what's that?
787.		JL	Is the third color underneath?
788.		M	Yeah.
789.		JL	Okay, good, okay. (side conversations and directions to the restroom)
790.		JL	Alright, we kept the same groups. This is who you worked with last time, right? Yeah. Another thing to tell you, when you're grouping your children for this next set of tasks. If they worked really well with a partner this time, keep them together. If they really talked, I don't mean behaviorally, but if they actually talked with each other and developed a solution as a team, keep them together. Okay? If you had a team that was a bomb for whatever reason, separate them and regroup. But you don't have to regroup everyone, just keep those together that worked, and regroup those that didn't work, okay? Alright, now you're going to have two problems. The first one that we're going to do is what you're going to be doing with your students, alright? It is a problem that involves three colors and you're going to be building towers. And this one, everyone is going to do, okay? The second problem that we're going to do is an extension problem not everyone's going to get to it, okay? But for those students of yours that whiz through this and are finished in record speed. Challenge them, after they get a convincing argument on this problem, challenge them to the extension problem, which is quite challenging, and you'll see when we do it here, okay? But in the meantime, I want you to just see if you can, read the problem, and then talk, tell me, and tell the group what is
791.		A	Thank you
792.		JL	You're welcome. Tell the group what you're task is.
793.	58:00	JL	Okay, who can say what you're supposed to do?

794.		RB	Three, three colors, three towers high. All the combinations of the three colors three towers high.
795.		JL	Okay, alright? Begin. And a convincing argument, okay, that you have them all.
796.			*Groups working*
797.	58:52	RB	And then the opposite is orange.
798.		A	I've never thought opposites
799.		JL	You never would have thought of it?
800.		A	Never.
801.		JL	Oh, okay, but your students did, right
802.		A	Yeah, they did. I find that funny, because I was never taught that way.
803.		JL	That is something children do, but is it very convincing?
804.		A	None of my kids were convincing.
805.		JL	Yeah, as you get more towers that are taller, it's very hard to use the opposite strategy and find a way to.
806.		RB	Two blues and a yellow.
807.		JL	You need to react to him, he's doing all the work.
808.		KF	Yeah, I know, I wouldn't do it this way.
809.		JL	You wouldn't?
810.		KF	No
811.		JL	So talk to him, tell him what you would do.
812.		KF	I feel like I would want it to, like say we're building them one tall
813.		RB	Okay
814.		KF	Then we would take it to three tall, right? So say, okay now let's do all the possibilities for two, starting with blue. Blue blue, blue red, blue yellow. Right? Do you know what I'm saying, so far we did three, or no?
815.		RB	No.
816.		KF	Okay, we want them to get to be three high, right?
817.		RB	Sure
818.		KF	So, say we're starting like this, and see there's one blue here? And then blue blue, blue yellow, blue red.
819.		RB	Okay
820.		KF	Do you understand what I'm saying?

821.		JL	I understand what you're saying, so
822.		KF	Okay, he doesn't really.
823.		JL	Yes he does
824.		KF	Okay
825.		JL	It's just he doesn't know where you're going. Is that what it is?
826.		RB	No, I'm not a tree diagram person.
827.		KF	Okay, so then.
828.		JL	Look how this is going, and then you're working by yourself. This is an independent worker.
829.		A	Last time, I couldn't understand
830.		JL	Okay, you couldn't understand what they're doing.
831.		RB	She confused us last time. She
832.		A	You confused me.
833.		JL	Well, we're going to have you two, if she wants to work independently, it's okay. We're going to have you two work together, but then I want her to explain what she did, to you. Okay, go ahead.
834.		KF	So at this point, like let's say I had one b, I went like, b b, b yellow, b red.
835.		RB	Um hm
836.		KF	So now, that one other branch can have blue blue blue, or blue blue yellow, or blue blue red.
837.		JL	Where are all your towers going? Are you taking them apart to build new ones?
838.		KF	Well, since we had three high. I'm just showing the concept of where we're going.
839.		JL	You're showing, I think it would be easier to follow the concept if you left the, didn't you have a third tower here? Did it disappear?
840.		RB	That's what I'm wondering, what happened?
841.		KF	Do you want me to start making a whole new one?
842.		JL	Yeah, while, if you're solving the problem, and you're trying to show him what you're doing, this do you follow what she's doing? It's tough.
843.		RB	I did at first, but now I'm lost
844.		KF	Alright, hold on, okay

845.		RB	She lost me
846.		JL	Okay
847.		RB	I understand the concept, I understand where she's going
848.		KF	Alright, so it has to be three high, right?
849.		RB	Uh huh
850.		KF	So, All the ones I started with blue first.
851.			
852.		RB	Blue on the bottom,
853.		RB	Blue, blue, blue
854.		KF	So then if I'm building them to be three high, then my three options would be blue with blue, blue with yellow, blue with red.
855.		RB	I understand that, and then you lost me from there.
856.		KF	So then, each of, so, like I'd have three options for my third level tower.
857.		RB	So shouldn't we build one that's two high, two more that are two high, to fit the
858.		KF	Where?
859.		RB	Those three options, okay.
860.		JL	And maybe it would help him to see, she did that
861.		KF	Yeah
862.		KF	Like those three options
863.		RB	Okay
864.		KF	Blue blue blue, and then blue blue yellow, and the blue blue red. Right?
865.		RB	Okay
866.		KF	Then with this one three options, so then I could have
867.		RB	But what's with this blue blue one?
868.		KF	Nothing, it's just to show you, it's just to show you.
869.		RB	We're done with that.
870.			Oh, this is, this is not gonna count?
871.			No
872.			Okay, and why?
873.			Because it's only two high.

874.			Oh, only two high. So it doesn't count, okay
875.			So then, we're still going through the other options.
876.		RB	We're still going with the blue on the bottom, but then the other one is the blue with the yellow?
877.		KF	Right
878.		RB	Okay
879.		KF	So then I have, so I have yeah, blue. And what's the three options that could be on top?
880.		RB	Then I could have a blue blue yellow.
881.		KF	No
882.		RB	No, blue yellow blue
883.		KF	One second, so blue yellow blue, right and then no.
884.		RB	No that one, I'm trying to follow you
885.		KF	So then yes
886.		RB	Blue yellow blue, and then we would have blue yellow yellow.
887.		KF	Okay, yep
888.			And then red.
889.			No uh
890.		RB	No?
891.		KF	Yeah, it's fine. Yep, and blue yellow red. Blue yellow, right
892.			Okay
893.		JL	Do you follow what she's doing?
894.		RB	So far, I'm going along good.
895.		KF	And now I'm gonna do my options with this one
896.		RB	The blue with the red
897.		KF	So blue red
898.		RB	So blue red yellow
899.		KF	Well, blue red blue first, right?
900.		RB	Does it have to be or?
901.		KF	Well, I don't know. I'm just trying to keep track of (points at arrangement of colors in towers)
902.		RB	Uh huh
903.			Right or no?
904.			So blue red blue

905.			We're pathetic
906.		RB	And then blue red yellow
907.		KF	Yeah
908.		RB	And then that's as far as we could go with that.
909.		KF	No
910.		RB	Or, no then blue red red
911.		KF	Yeah
912.		JL	Ah, okay see he's got, he's following you.
913.		RB	So
914.		JL	And now what
915.		KF	And now I'm convinced that I have all the ones that started with blue on the bottom.
916.		RB	Yes and now we would go to either yellow, or red in the bottom.
917.		KF	Correct, yellow or red, so let's go yellow on the bottom
918.		RB	So let's do yellow on the bottom
919.		JL	So you like her, the way she's doing?
920.		RB	No, no no
921.		JL	He doesn't like it
922.		RB	I'm going along with it, I'm going along with it.
923.		JL	But he's going to follow it.
924.		RB	I'm a team player
925.	1:04:03	KF	It's okay if you don't like what I'm doing, but do you know? Do you see that…
926.		RB	I see what she's doing
927.		KF	That I definitely would get all of them this way.
928.		RB	I'm not in my comfort zone, but it's okay
929.		KF	Let's do both ways
930.		RB	No
931.		KF	Here let me finish this, and then do you want to explain to me?
932.		RB	No no no no no. I want to help you do this
933.		KF	Okay
934.		RB	So we can do this, so now we have, so I can understand this if one of my students does it. Yellow yellow yellow,

935.			Thread one RB and KF
936.	1:04:22	KF	Sure Okay
937.		RB	So now we could go, yellow, no no. Okay. We're doing yellow on the bottom
938.		KF	Right
939.		RB	Okay, so yellow yellow yellow
940.		KF	Yep
941.		RB	So now we could have, yellow yellow yellow, and then you're we want to follow this where the next color is yellow, so yellow yellow blue. Okay and now it's going to be yellow yellow red. If we follow that same pattern. Yellow, yellow red, okay? And now we have that. Now, we're still with the yellows on the bottom. Okay, yellow yellow and now we're going to go with the red. So we're going to go yellow red yellow Right? We're going to go yellow red blue, okay? And then we're going to go still with yellow; yellow red red right? I understand what we're doing, I it's just not the natural way I would do it. Which is okay. You like it?
942.		KF	Yes
943.		RB	Okay, so now I'm going to do all reds on the bottom. More so free of the yellow yellow yellow. Yellow, okay now I see yellow blue yellow. Okay But then it's yellow blue red. Then its yellow blue blue. These are all the blues
944.	1:06:35	RB	So now we have three of reds that follows our pattern. Okay. So now we're going to put red from the bottom up. We're going to have three two talls from this point. So it's red red blue, that's what we'll do. Red red blue. Then we go red red yellow. Because it's gonna be the opposite of yellow red red. Okay, so now we go one red on the bottom again, and then instead of a red we switch over to yellow. It's, I'm, We're flipping these two around. So it's really yellow. Red yellow yellow, and then red yellow red, red yellow blue. I'll work back to the, well no, that should be red. So three reds on the bottom so now it's red blue red, It's red blue yellow, red blue blue. We've got red blue yellow there, so it's red blue blue,
945.		KF	Now let's go through the way you were doing them. Because I want to see what you were doing
946.		RB	That's okay
947.	1:08:15	KF	No, its not
948.		RB	It's fine, I like it

949.		KF	It's not like a bad idea
950.			KK Strand
951.	1:04:26	JL	Talk to me about; it looks like you did this first and now you're doing this? Tell me, how did you do this?
952.		KK	We decided to do all the possibilities with blue on top.
953.		JL	Oh neat
954.		KK	So we did all blue, two blue with the yellow bottom, two blue with the red bottom
955.		JL	Okay,
956.		KK	Then we did, we kept the blue on either end and showed the combinations
957.		JL	Okay, got it
958.		KK	And then we got rid of blue. And we shift that over so she's at the top
959.		JL	Okay
960.		KK	Two of these
961.		JL	Okay, neat very neat, so do you think that these are all the blue tops that there are? Because how many are there? how many?
962.		KK	No
963.		JL	How many are there?
964.		KK	Nine
965.		JL	You don't think there's ten?
966.		KK	No
967.		JL	How about you?
968.		S	Haha, well I think we're done, because we did account for all
969.		JL	Well, keep going and then you're going to convince me why you think it's nine.
970.	1:05:35	KK	Okay
971.			MJ Strand
972.	1:05:38	JL	Alright, let's see what's going on here
973.		M	We're just making the kind of like the chart
974.		JL	So talk to me about what this is here, what is that tree diagram, it looks like?
975.		J	Um, well, pretty much we started with blue in the basis
976.		JL	Interesting, so you're keeping a constant on the bottom?

977.		M	Right, exhausted all the ways
978.		JL	How do I know it's all of them? How do you know it's all of them?
979.		M	Because every spot is only three different colors that could be there.
980.		JL	Talk to. I don't want to look at this it's confusing. Talk to me about what you have going on with the cubes. How do I know these are all the possibilities for towers that have blue on the bottom? Because that's what you're telling me, right?
981.		M	Right
982.		JL	Okay, convince me.
983.		M	Well, these are all. We started out with blues on the bottom.
984.		JL	Okay
985.		M	So, if we have a blue and a red, there's going to be one other space
986.		JL	Aah
987.		M	These are all with blue on the bottom, red on the second one, and there's only three possible colors that could go on the next spot.
988.		JL	Okay. That's pretty neat, how about these?
989.		J	We pretty much did the same thing. Like if our bottom color's blue, then we do a yellow constant.
990.		JL	Yeah
991.			Inaudible
992.	1:07:46	JL	So you were working with two constants. Isn't that nifty, right? Not only a single constant, a double constant. When we talk about it, I want you to share that, okay?
993.			Sure
994.			Very neat, so are you thinking these are the blue bottom ones? You have nine. Is there a tenth, could there be a tenth one?
995.			No
996.			Why?
997.			Because we have- and this is constant down here, that's not going to change.
998.			Right
999.			So this would be all the different ways for red being the center, yellow being the center and blue being the center.

1000.			Okay
1001.			There's nothing else you could for the -
1002.			So I bet I know where you're going from here.
1003.			Right? So keep going
1004.	1:08:40	RB	I understood once we got going, it's just that I'm out of my comfort zone, but that's okay.
1005.		JL	Well, you know, we
1006.		RB	We make our kids do that, and that's good
1007.		JL	We are going to now reverse it.
1008.		RB	Oh, that's what she wanted.
1009.		JL	We're going to do it your way. Well, now, so this is what you did?
1010.		RB	We did that.
1011.		JL	Oh interesting! This kind of looks like what they did. So now what did, talk to me just about the blues, because if you can convince me of the blue bottoms, I'll be convinced about the others.
1012.		KF	You want to do it? Like I told it to you, you tell it to her?
1013.		RB	Okay, well first we started with all blues
1014.		JL	On the bottom?
1015.		RB	Yes.
1016.		JL	Okay
1017.		RB	And our control for that was all blues on the bottom.
1018.		JL	And Ashely, I want you to…
1019.		A	I did the same thing actually
1020.		JL	Oh you did?
1021.		A	I drew a tree diagram to go by
1022.		JL	Isn't that neat, okay.
1023.		RB	We worked alone this time. She went by herself
1024.		JL	Okay, but so keep going because I'm not sure I…
1025.		RB	So then we went from the blue blue blue
1026.		JL	Yep
1027.		RB	To the blue blue yellow, which was the two blues on the bottom
1028.		JL	Okay

1029.		RB	And then we went to the blue blue red, which would still be the two blues on the bottom, but now the red.
1030.		JL	Okay now, Question: Could there be another member in this group?
1031.		RB	No
1032.		JL	Why not?
1033.		RB	Because, when you have three and then the two, you could only have yellow or red as the option. Because when you repeat the color, it's back to blue. It would have to be blue blue blue.
1034.		A	That's how, that's how I thought of it. Like I built reds first. Now I knew that I could only make three of that because if I placed another layer of red, there had to be three because it had to be either blue or red or yellow on top. So that's how I thought of all this.
1035.		JL	So, what you did here is the same as what they did. So this is the red bottom group?
1036.		A	Yeah
1037.		JL	Okay, and this is not only red bottom, but what else is…
1038.		A	Red middle
1039.		JL	Red middle, and this is?
1040.		A	Red yellow. So it's kind of like I'm controlling for the constant with the first two levels.
1041.		JL	Yes you are, so it's kind of like you're doing a double constant
1042.		A	Mm hm
1043.		JL	Pretty neat stuff
1044.		JL	And that's what you did. Isn't that neat?
1045.		RB	Mm hm
1046.		JL	Are you liking it better, or are you still are not comfortable?
1047.		RB	It's fine, I'm okay with it it
1048.		KF	You know what's funny, while we were doing this
1049.		JL	Yeah
1050.		KF	We were like let me just do it, because I know what you are doing, and then like
1051.		RB	She explained it to me and then I understood it
1052.	1:10:49	JL	Ok

1053.		KF	Then he still wants to say, I'm going to do the opposite. He likes opposites
1054.		JL	Okay, well
1055.		RB	But the thing is, but that's what I would want my students to do. To like to work off of each other
1056.		JL	And that's fine
1057.		RB	And that's why we're paired up with them, and sometimes a good combination works. So now I understand this
1058.		JL	Okay
1059.		RB	Prior to this, I really didn't understand tree diagrams
1060.		JL	But are you
1061.		RB	I would stay away from them
1062.		JL	Well, this doesn't even, you don't even have to have a tree diagram. You could do this without a tree diagram.
1063.		RB	It's the same concepts
1064.		JL	Absolutely
1065.		RB	And it made a lot of sense and the first time, for the blues I really wasn't getting it. But
1066.		KF	Because we started with one, I think maybe?
1067.		RB	But then, I think it was because she was going with one and not building the towers.
1068.		JL	Okay
1069.		KF	I just wanted to explain to you, like why I was getting the three options with blue on the bottom. I meant like you know blue blue yellow.
1070.		RB	Yes
1071.		KF	This was the start of
1072.		RB	Yeah, and once we got started, I was on a roll
1073.		JL	Okay
1074.		RB	I was on a run. I probably could have beat her in that race.
1075.		JL	Now, no racing
1076.		KF	Haha
1077.		JL	What is a neat thing about this strategy is that, I know you did double constants, but you actually have nine in a group. So if students are trying to do a blue bottom group, and then a yellow bottom group and then they get eight, they get worried

			and ask where's that other, right? So this is a good thing. What I want you to do is write me a convincing argument, okay? And
1078.		JL	Actually, the tree diagram doesn't help me see this
1079.		RB	No, that's, the blocks help.
1080.		JL	Well because, I would have to, figure out
1081.		KF	This would
1082.		JL	In other words, I don't know how many you got here, unless I…
1083.		RB	In going back to here, when we want to go again with the next color that's blue again so you can't repeat it. Because there's only three options, so you have only blue or yellow or red. So then, in the next one, would have to be blue. So therefore there can't be any more repeats in this. And then here we have blue on the bottom, and it follows that same argument.
1084.		JL	Okay, so you're feeling good about this now?
1085.		RB	I definitely feel better about it. We'll see
1086.		KF	Haha
1087.		JL	You did good, you got a gold star. So he's drawing his towers.
1088.		RB	I'm drawing these towers.
1089.		JL	That's what I want you to do. I just want to know what they are, okay? And because it is hard to see here.
1090.		A	Sure, yeah yeah yeah.
1091.		JL	Okay, and you have a convincing argument?
1092.		A	Ah, I could work on it a little.
1093.		JL	Good

10/28 Meeting transcript 2 of 2

Title: 10-28 Oldbridge 2
Location: Oldbridge
Date: 10/28/2010
Length: 52:21
Transcribed by: Will McGowan
Verified by: Maddie Yedman

Line	Time	Speaker	
1.		JL	…figure it out and see is your, is my tree diagram, when you figure out what the nine towers are you want them to draw the towers, ok
2.		J	Draw it, or write it or something?
3.		JL	Or write it, absolutely. Show me on paper what the towers look like. Absolutely. I need a convincing argument, so let's get one down on paper, okay?
4.		M, J	Mm hm
5.		JL	Why do you think you have them all?
6.		KF	She's okay, I can't remember her, but she's okay.
7.		JL	Okay we are getting a convincing argument down because we want to get to the challenging problem.
8.		RB	Kulsom's doing explaining this for us, she did a good job.
9.		KF	No he did a great job, once I, once I showed him. I want to know his way, though.
10.		JL	No, is this called, what did you say, "stealing" Is this stealing? Did you steal her ideas?
11.		RB	No, no. She taught me. There's a difference.
12.		JL	She taught you, oh, ok that would be
13.		KF	
14.		RB	But the problem with stealing, and you have to be careful, like in my class, I posted this. A lot of the kids were saying it was random, because they heard one group of students said it was random.
15.		JL	What was random?
16.		RB	When they were creating the towers.
17.		JL	Okay
18.		RB	I said, "So how did you do this?" "Oh, it was random" "Okay, so let me ask you this question: What if you just broke up all hundred of these blocks and you just put four together, that's

			random." "No that's what we did"
19.		JL	Okay
20.		RB	I said "Well that would be random, so what did you do?" I asked them those Justin style questions as I call them with the.
21.		JL	Well, you know, the better you get at questioning, and the more experience they have at being required to explain and… The better they will get.
22.		RB	And they were, and they said "Well, it wasn't really random. We were working with one block at a time." And I said "Well, you didn't tell me that. You said it was random. Random is taking all the blocks and… "
23.		JL	So when they were able to, and maybe next time they won't need so much of you saying to them, well if it wasn't random, what if. They may be better able to explain what they did.
24.		RB	Possibly
25.		JL	They will get better at it. I can promise you they will get better. And hopefully in time, they will even get good at writing. Writing is harder than verbally. But you want them to also be able to also get on print what they did and why they think they have them all.
26.		RB	Well, they struggle with that. I even put up a very convincing…
27.		JL	Can you do it? Let's get on with it. Give me a convincing argument in print.
28.		A	I'm trying to think more, because I don't know if I'm clear. Needs convincing, this. But I feel like this like above it. It's like
29.		JL	Then we controlled for the first two blocks on each tower make the top of each one of the
30.		A	Make the top, each one of the
31.		JL	Of the three colors. That's okay, and you know, sometimes when children do this and they are not sure if it's clear. If they can show you, you know, Look at this picture, or something like that. So let's see if this helps.
32.		A	Yeah, it does. Because it's three for each of the
33.		JL	Because if I saw this and saw this I would understand exactly what you meant.
34.		A	Uh huh
35.		JL	If I just read this I would probably go "What?"
36.		A	Yeah, exactly

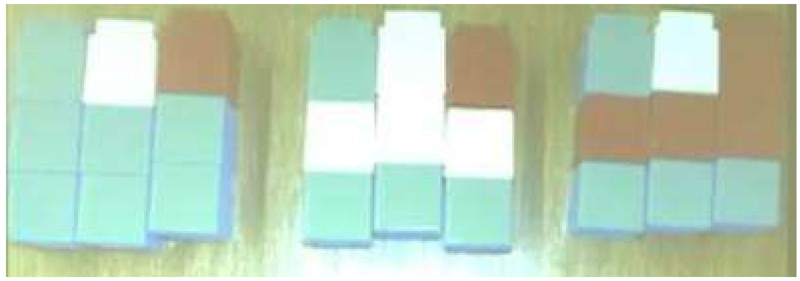
37.		JL	But this is very clear. And you want them to do the same thing. You want them to have a picture of what their towers look like and some kind of explanation. And so you said "Each group had nine, had nine, three for each of the bottom two colors." Three for each? Oh okay. So double constants, okay
38.		A	So nine groups and then there's three for each.
39.		JL	Yeah, okay. Which and since
40.		A	That's where I, That's where I, Haha
41.		JL	Keep going
42.		A	Okay
43.		JL	Because I think this with this will give you a nice convincing argument.
44.		A	Okay
45.	3:44	JL	Okay. How are you guys doing? Convincing Arguments?
46.		J	For each color. On top of each for the second row
47.		M	Uh huh
48.		JL	Are you having trouble with the words?
49.		J	We're going …
50.		JL	Okay, if you were having trouble with words; very helpful is to have a picture of your towers to go along with the words. Let's say, if someone doesn't really follow your words, they can look at your picture and say, "aah! That's he means" okay? You won't have to guess. You won't have to, come up with what the student is meaning. It will be right on paper. Now you would probably not want a student to give you this as their solution. Because it forces you to figure out what their towers are. You don't want to be coming up with anything on your own. You want to know what they are coming up with. You want to, like Justin… No, he didn't either! Okay, you would want to see the towers looking like this
51.		A	I feel like there's a huge difference from a sixth grader to a seventh grader.
52.		RB	Possibly, I found my students really want to do well on this.
53.		A	Mine, no
54.		RB	It probably depends which ones you have
55.		A	I can't say the same, my kids go "This is a pain, why do we have to do this?"
56.		A	I like my kids, but they're very lazy

57.			
58.			
59.			
60.		M	Like on the other side? That's one of the problems I do admit. This is not enough for them to write on. A lot of times they have to go on the back.
61.		JL	That's okay
62.		M	And what I try
63.		JL	You don't like the flipping?
64.		M	No, what they do a lot of times is they condense their writing to fit.
65.		JL	Because it's little.
66.		M	Like one kid, the one that I put up there
67.		M	He redid that.
68.		JL	Okay
69.		M	Because he didn't have enough room.
70.		JL	Okay
71.		M	He asked for another one to try to put it all on one thing.
72.		JL	If you feel that it's stopping them from writing more, Give them a blank piece of paper with this.
73.		M	That's what going to do next time.
74.		JL	Because then they don't have to go like this, like this like this (flipping the paper over) like this like this which is hard. Okay?
75.		M	Well, because, this is usually what they just.
76.		JL	Okay
77.		M	They work out, like how we did this.
78.		JL	Okay, that's fine, that's fine.
79.		M	Like after that and some of the kids were actually erasing stuff.
80.		JL	Don't, no erasing, right.
81.		M	Right, and so that's why because for the one kid, she just fit it in as small as she could. But then I felt it kind of affected what she was writing.
82.		JL	Okay. So you can definitely give them another paper. The only thing you're going to want to do is make sure they put their name and their partner's name on the next paper.
83.		M	Right

84.		JL	Because you don't want it to get disconnected. Okay
85.	6:00	JL	Alright Just about five more minutes because I really want to share some of what you did. And what you want to do is be writing that convincing argument. Because you want to see how hard it is for the students, right? It's not easy. But sometimes, what will help them is their picture of their towers together with their explanation, convincing argument, as a complete package.
86.		KF	…This is what I have so far: There are three options for each, I want to say "subset" but
87.		JL	Sometimes if you can say (points) this: If someone understands this, that has convinced you of the argument, you don't have to do this and this.
88.		KF	Okay
89.		JL	Okay. So if you can just get it for one group of nine.
90.		KF	Okay
91.		JL	Not easy, though. I think it's good for you to see how hard it is.
92.	7:10	JL	Neat solutions, you all did the same thing, so that's really neat.
93.	7:26	JL	I'm going to borrow these for a second. We're going to actually, can I do that?
94.		RB	I was pretty much doing the same thing. It would have been just working them differently. It would have worked out though. I was doing the same thing.
95.		KF	
96.		RB	
97.		A	
98.		RB	All those reds would have been together, and all those yellows. It's the same thing.
99.		KK	Oh, right, bathrooms are over there.
100.	8:12	JL	Yeah, as you have to use the facilities, just go. Because we're not going to break because then that way, we can, you know, get done on time. But feel free to leave when you have to.
101.		RB	Okay, thank you.
102.		JL	Okay, let's see. Is this going to go on by itself, or is it
103.		M	Um, not plugged in, that's always a problem.
104.			
105.			
106.			

107.	8:41	JL	Did all of you finish your convincing argument?
108.			Yeah
109.			No
110.		A	I got messed up. My first sentence, I lost my train of thought
111.		JL	Okay
112.		A	I could add to my diagram, but I don't know what I stopped for, and now my "since," "Since" what?
113.		JL	And that's why, when Sayreville tells the kids to come back the next day, it's very hard.
114.			Yeah
115.		JL	It's very hard. How do you remember when, you're saying,
116.		A	I did a tape them this time
117.		JL	Good, good
118.		A	That was an issue because when I untaped them
119.		JL	They flew apart, aw
120.		RB	I really pushed for them. I had the stuff set up for them when they came in, and I really pushed for them to get to at least this point, so at least they were able to do it backwards, because the next day, tomorrow, would I be able to?
121.		JL	You know it's horrible that you are forced with the high school schedule in the middle school, right?
122.		RB	Well, we also teach six periods.
123.		JL	Ah, okay
124.		RB	I don't know if that's common, but what I've seen, it's not.
125.		JL	Um, it's not common to have a middle school with a high school schedule, I can tell you that. But you need to get the administrators on board.
126.		A	They used to have, they used to have teams.
127.		JL	Did they?
128.		A	But they got rid of that about what, seven years ago.
129.		JL	I'm sure they are doing it for economic reasons is what I think. Yeah?
130.			Yes
131.			Which is sad. In these times, that's having it not work.
132.	10:05	JL	Okay, you know what we want to do? Let's take just one more minute, and then I want you to just see, if you haven't been able

			to write the convincing argument, let's see if you can get the convincing argument verbally. Because I think you have a pretty convincing argument all of you, and you have a very systematic way to solve this. A very very neat solution. So it should be easy to provide a convincing argument. Maybe we can all do it, Okay? Let's just wait for Sally and Kate to get back, and then we will, we will do it.
133.		A	I think the door's locked
134.		JL	Oh, they're not coming back, Okay. We didn't want to leave you out. You can leave it open.
135.		KK	Yes, Whose is this?
136.		JL	This is what you all did. It looks like the same thing of what you all did. This is two of the three groups. The third one just does not fit on the screen. That's two of the three groups. But you all did the same way. You all did it the same way. Did I put it, the diagram wrong?
137.		RB	No, it's right
138.		M	Do you want me to move stuff?
139.		JL	Yeah how did you do it, when I moved it, I think I messed up the order. How did you all do it? Just put up nine.
140.		M	Alright
141.		JL	Just put up nine, okay,
142.		RB	And then the bottom of every one, the bottom of every row.
143.		JL	That's right, Okay. And that is how, you know what, you can just as easily have your, looking at the top as a constant.
144.		KK	That's what we did.
145.		JL	You guys did that, but you didn't like it.
146.		KK	No
147.		JL	And, you switched it to the bottom.
148.		KK	Yes
149.		Jl	But there's no reason, you might have kids who like to do the top.
150.		KK	Right,
151.		JL	Do the top down
152.		KK	The top down, or the bottom up, right.
153.		JL	It would work just as fine. Okay, let's begin, and Kate will join us. This is what you guys did, and it's beautiful. Why is it beautiful? Talk to me about what you guys did.

154.			
155.		RB	Well, I
156.		A	It is clear, right?
157.		JL	It is clear.
158.		RB	The argument
159.		JL	Okay
160.		RB	We controlled the first set of three blocks by using two blues on the bottom. And as you can see, there's two blues on the bottom.
161.		JL	Okay
162.		RB	So then on the top, that leaves us with three different combinations: either blue, or yellow, or red.
163.		JL	Right
164.		RB	And then, if we wanted other combinations, it would either repeat blue or yellow or red. So then we went on.
165.		JL	So that was the only way to do the first set of three. Second set of three?
166.		RB	Second set of three, we had a blue and a yellow on the bottom. Okay, same as the first set because you can't have a blue blue again. Because we already have that. So now we have a blue and a yellow on the bottom.
167.		JL	Okay, good
168.		RB	Same thing on the top: It could either be blue, yellow, or red.
169.		JL	Okay
170.		RB	And if it's another color on top, it would have to repeat. So therefore, we have all the combinations without repeating.
171.		JL	Okay
172.		RB	So then we took the blue and the red.
173.		JL	Right
174.	13:00	RB	Because that's the third combination that you could have as either blue blue, or blue yellow, or it could be blue red. Now you can't do blue blue again, or blue yellow again. It can only

			be blue and red.
175.		JL	Okay
176.		RB	So then on the top, there's a choice of three colors again, which is blue, yellow, or red, and if you repeat it again, then you would have a blue on the top, or a yellow on top, or a red on top, and we could stop there. So now, if we look at the two control groups again, It could either be the blue blue, blue yellow, or blue red, there's no other possible combination.
177.		JL	Are you convinced?
178.			Mm hm
179.		RB	And if it worked for that, it would work for the other two sets.
180.		JL	Absolutely. And if your kids can convince you of this group of nine
181.		RB	Thank you Kulsom
182.		JL	You don't have to check, yeah. Isn't that neat. Now it's no longer her strategy.
183.		RB	No, I was using it, I'm stealing it.I stole it
184.		JL	It's equally yours, right?
185.		RB	I "stole" it
186.		KF	That's why almost to show why it would be twenty seven as the answer. Is when I was showing him the one block, and it has three options. For one block.
187.		JL	Ah
188.		KF	For the three for each would be nine for two high
189.		A	Twenty seven for three high.
190.		JL	Isn't that interesting, so you were
191.		KF	I was trying to show him.
192.		JL	You were trying to look at that, look at those numbers again, so what numbers are you thinking of.
193.		KF	For this problem, it's twenty seven
194.		JL	It is, the answer is twenty seven. There are twenty seven towers
195.			Right
196.		JL	But how did we get it?
197.		KK	I was thinking three to the third.
198.		JL	Three to the third power
199.		KK	Three to the n

200.		RB	Three cubed
201.		KK	Three choices for each of the three positions.
202.		JL	Right, so that's pretty neat. Very nice job. Double constant, Double constant, double constant really neat, and then the only thing left is what you have. So really nice job. You ready for the tough one?
203.		A	Sure
204.		KK	Bring it on!
205.		JL	Bring it on! Okay! Now this was done, developed by a student, okay. Do you know when you're working on a problem, and the kids say, "Well, what if we try this instead?" and they make up a new problem?
206.		RB	No
207.		JL	Okay, well, Ankur is a tenth grader from Kenilworth who was in that longitudinal study. And they were working on towers. With three colors and Ankur came up with this challenge, which is the extension. Okay, what I want you to do is read it, and before you begin it with your partners, I want you to first figure our "What the heck is this challenge?" Okay?
208.	15:20	A	Thank you
209.		JL	You're Welcome. I'm going to give back your towers.
210.	15:38	JL	What does Ankur, when you've read it, talk to me about what he's asking you to do.
211.		KF	He's kind of making it a little bit more specific. It has to have at least one of each color.
212.		JL	Okay, has to, each tower has to have at least one of each color, and what else?
213.		KF	Four high and selecting from three colors.
214.		JL	Okay, you know what the challenge is?
215.		M	Mm hm
216.	15:56	JL	Begin
217.		RB	Using blocks
218.		JL	Oh yeah, oh yeah. Definitely want to use the unifix cubes. We can actually shut this down, okay? Thank you.
219.		JL	You're doing so well, I don't want to touch it
220.		M	It's really not that bad, you know, just let it adjust
221.		JL	It's good

222.	16:20	RB	With them on the bottom three different colors, but suppose we had a red here, a yellow here, and a red here. We would have three different options
223.		KF	What about blue blue blue?
224.		A	We need more
225.			It's a pill of warmth
226.		RB	Actually, though the three tall won't work. We can get rid of it.
227.		KF	Right
228.			If we put blue blue blue.
229.		RB	We can get rid of those blue blue and yellow yellow.
230.		KF	Can't we just add one color?
231.		RB	No because it won't be three high.
232.		KF	It has to be four high.
233.		RB	It has to be four high?
234.		KF	That would be four cubes tall.
235.		RB	So we could get rid of the three yellows.
236.			That's like
237.		KF	We can't get rid of it, though because we have to add on one more.
238.		RB	No, no we can because if we add one, then it's not three colors.
239.		KF	It says it has to have at least one of… Oh! I see, I see
240.		A	I know, It's like easier when he's talking.
241.		KF	Now I know what you're saying.
242.		RB	These are no good.
243.		JL	Okay, so now you got it. And that's a real important point.
244.			Yeah
245.			Okay
246.		RB	We don't need that. Okay? So now what we can do, is we can do the yellow, add it to that so. And just keep adding to what we have
247.			Do you think we're going to get all of them?
248.			I think we're going to get it. I think that this is our possible answer, okay?
249.		RB	It can't be more than this. It definitely can't be more than this.
250.		JL	Why not?

251.		RB	Because it can only be three colors.
252.		JL	Right
253.		RB	And we took two away. Actually, if anything, it could be less. And we're going to see that in a minute.
254.		KF	So this one, add red?
255.		RB	Yes, we need a red.
256.		JL	That's interesting. What do you think about what he's saying, Angela?
257.		A	I'm, I'm not sure.
258.		JL	You're not sure.
259.		KF	I'm not convinced. It might be, but I'm not convinced.
260.		RB	We'll see in a minute.
261.		JL	Okay
262.		KF	So what would we add to this one?
263.		RB	That one you would add…
264.		KF	A blue, a red…
265.		RB	A blue or a yellow.
266.		A	Can we add
267.		RB	A blue a red or a yellow.
268.		KF	One of each? We should build another one, right?
269.		A	We should build three more
270.		KF	Yeah
271.		A	Yes, that's what I think too.
272.		RB	Let's build a blue
273.		KF	Yeah, I got it.
274.		RB	We need three on that, okay? That's good, you guys are
275.			What is it?
276.			Red yellow blue, red yellow blue
277.	18:14	RB	And then put one of each on the bottom. So I would do a red, then a yellow then a blue on the bottom. Here you go Connie.
278.			Okay
279.		RB	Red and blue, red and blue. Okay? No no no, it will work, it will work
280.		A	It will work?

281.		RB	A red on top.
282.		A	On top?
283.		RB	You can only go
284.		A	On top? You could also go on the bottom
285.		RB	You could also go on the bottom.
286.		A	Wait wait, Are we actually adding to ones that we just made?
287.		RB	Because I think we're going to be
288.		A	It could be the opposite.
289.		RB	Let's not do opposites, it's getting confusing. So let's… No those are complete
290.		A	He said to add red to the bottom, so that's what I'm doing. So these are the same thing as this, or no, they're different?
291.		KF	Blue blue yellow like this…
292.	i	RB	Yes, but I think when we get down here. Hold on, hold on
293.			It's so weird
294.			Them at the the top, but then down here
295.		KF	I don't want to add to the top, let's only do the bottom.
296.		A	Only do the bottom.
297.		RB	Only from the bottom
298.		A	Did we add to the top or bottom here
299.			No bottom
300.			So we're still
301.			Yeah
302.			This is the original tower, red yellow blue
303.		RB	Yeah, All these duplicates you're talking about will not be made down here.
304.		A	And that's what we're saying. We need more of these, right?
305.		RB	No, no, Angela because when we get down here
306.			How about this one.
307.	19:55	RB	Same thing this one
308.	20:00		
309.		KF	I'm convinced of that.
310.		RB	Okay
311.		A	These ones, this add anything. This one I'm convinced.

312.		RB	We did different ones because
313.		KF	Right, yeah
314.		JL	What are you convinced? I…
315.		KF	This was our original tower of three. We only had one of them
316.			We only had one
317.		JL	Okay
318.			So
319.		JL	Oh, cloned it
320.			Three colors, so we had three options
321.		JL	Cloned it and then put one of each
322.		KF	On the bottom
323.		A	We were discussing each of these.
324.		RB	And if we went with different combinations, we would have them when we get down here.
325.		JL	Oh, I see
326.		RB	Possibly, so
327.		JL	I see
328.		RB	It's a work in progress.
329.		KF	Ashely, is this one doubling as that?
330.		A	Yes
331.		JL	Okay, okay
332.		A	We're only adding to the bottom on these.
333.		RB	We're only adding to the bottom
334.		JL	Got it
335.		A	So we can't do yellow and the red.
336.		RB	No
337.		A	Because that would be not the same color.
338.		KF	Okay, so this was here. It has…
339.		A	It has to be red
340.		KF	It has to be red
341.		RB	It has to be red
342.		KF	Okay
343.		A	This one has to be yellow

344.		KF	Correct
345.		RB	Yep
346.		A	Yellow is the only option.
347.		JL	How come you didn't make clones of these the way you made clones here?
348.		A	Because
349.		RB	Because here we made. They'll repeat down here
350.		A	We wouldn't have three of.
351.			
352.		JL	Ah, would not, why not?
353.		KF	Because it has to only be four high.
354.		A	If we added a blue here
355.		RB	Four tall three colors
356.		A	If we added a blue here
357.		JL	Ahh so you needed to add, it started as a blue red blue?
358.		KF	Yeah
359.			Okay I got you.
360.		A	This one was like this, we could only do yellow. This one's like this, we need to clone it.
361.		KF	Yeah
362.		JL	Oh, so when it has all three you're cloning it.
363.		KF	Angela, we need to do a yellow on the bottom.
364.		JL	And look at that, you're all happy with the strategy.
365.		KF	Yeah, I know. I wasn't at first, but I like it. I like it
366.		A	He slowed, once he slowed down
367.		KF	We told him to take a deep breath, I was like "Let's stop a second and"
368.		JL	Stop a second, okay
369.	21:27	A	Blue then
370.		RB	It's yellow red blue. Then we go red. Yellow …
371.		KF	Only counting red. We do?
372.		RB	We took all these red. No we don't. We could add red to the bottom of that. And that's it.
373.		A	Okay. That's the only option with blue, how about this. Only a

			blue.
374.		RB	Only a blue
375.		A	Because it will repeat.
376.		RB	Yep
377.		A	Is that a repeat? No.
378.		RB	No. It won't be None of these will repeat.
379.		KF	Why, why wouldn't we clone this?
380.		RB	No, this one has three.
381.		A	That's the only way. Only way we can add to one that has two colors.
382.		RB	
383.		A	
384.	22:29	RB	Make sure we build, none of those are duplicates.
385.		A	None of those are. Blue red, blue yellow.
386.		RB	Correct
387.		A	
388.		RB	That may not be
389.		KF	None of them will
390.		RB	Are you convinced we won't have duplicates?
391.		KF	No, I'm not. If we do a blue and a red here?
392.		A	So yellow red blue red? That's only a red right?
393.		RB	Yep
394.		KF	This one I add only a yellow.
395.		RB	Mm
396.		KF	This one only a blue.
397.		RB	Only a blue here.
398.		A	It's so much easier to cut these
399.		A	What is it, yellow?
400.		KF	Yellow blue red blue
401.		RB	Here's a yellow blue
402.			You do red, right?
403.		RB	There's a yellow and a blue
404.		A	And we need a yellow on the bottom

405.			
406.	23:39	KF	We're convinced.
407.		RB	And now we're going to clone it and then go with the yellow blue and the red, and then the red, Angela. There we go. And now we're convinced. Let's
408.		KF	Yay, now we're done.
409.		JL	You're convinced this is the set.
410.		KF	Yeah
411.		RB	We're convinced. That's it.
412.		JL	How many did you get?
413.		KF	(counting) twelve
414.		RB	Thirty six, I would say.
415.		KF	Yeah, thirty six
416.		RB	Thirty six.
417.		KF	Thirty five is what we have.
418.		RB	That's okay. It's okay to have thirty five.
419.		JL	What do you have, thirty seven?
420.		RB	I don't know. We're going to actually count them up.
421.		KF	Fourteen, sixteen, eighteen, twenty, twenty two, twenty four twenty five.
422.		RB	It's okay to have twenty five.
423.		KF	Twenty seven, twenty nine, thirty one
424.		A	
425.		KF	…Thirty seven
426.		RB	Thirty seven. It's okay to have thirty seven.
427.		JL	Thirty seven so you have how many in this group?

428.		KF	Well, thirteen
429.		A	Well, that's not really how we grouped them. I wouldn't say
430.		JL	How you grouped, oh so you don't really have them in groups. But you have thirty seven, and that doesn't bother you?
431.		RB	No, it doesn't bother me at all.
432.		JL	It doesn't bother Rich, but it does bother Fulsom.
433.		KF	No but I
434.		JL	Am I pronouncing it right?
435.		KF	Kulsom
436.		JL	Oh Kulsom, Kulsom
437.		RB	There's only three. Three colors four high. So you're not going to have. If it was all fours, then it would be an even number, but we had to take those two out at the beginning, remember?
438.		JL	What two out?
439.		RB	Well, remember. We took three out at the beginning, which would be thirty. Because we had three blues,
440.		JL	Okay
441.		RB	We had three blues, three yellows, and three reds which are three groups that we took out.
442.		JL	Okay, you know what I'm going to ask of you guys? Because I'm not as comfortable as you that you have the solution.
443.		RB	Oh
444.	25:00	JL	Okay? I'm not sure, maybe you do, but I'm not convinced. Okay? What I'd like to ask you is: You think this is the solution. Can you arrange it differently to make a convincing argument?
445.		RB	Um
446.		A	I feel like grouped with this we can, sure.
447.		RB	Grouped with
448.		JL	Grouped with this you can, what do you mean?
449.		A	If I were to draw, I think we can do it.
450.		RB	We used Milan's argument.
451.		JL	She thinks she can do it. She thinks she can do it, so let's do it.
452.		KF	If we use those
453.		A	This diagram, I think that's what we did
454.		KF	We used preexisting towers.

455.		RB	I don't know about that.
456.		JL	Well, why don't you use the towers that you, If you think this is the solution,
457.		A	Maybe not, no
458.		RB	I don't think we
459.		JL	You don't think that's going to help?
460.		RB	No
461.		JL	Well, how are you going to convince me then, that thirty seven is the right number?
462.		RB	This goes back to Milan's argument. No I'm sorry, Millin's argument.
463.			Because really
464.			No, I'm talking about the student. And what he said is When you have a really convincing argument for three high, you only have one more possible combination for the block. So by working with what we have.
465.		JL	Right, okay
466.		RB	We need to add to the bottom of each tower either a red or a yellow or a blue.
467.		JL	Okay
468.		RB	If we keep it, this which I'm hearing is three colors four high
469.		JL	Right
470.		RB	This is Some of them couldn't have been built, so
471.		JL	Okay
472.		RB	So they were eliminated. The three blues, we had to eliminate them. That's a possible way to do it, Okay?
473.		JL	You can use that.
474.		RB	So then when we had the three separate colors, we had to clone them.
475.		JL	Right
476.		RB	Because on the bottom,
477.		JL	I saw you do that.
478.		RB	On the bottom, it could either be yellow, blue or red.
479.		JL	Right
480.		RB	And before we clone them, these over here
481.		JL	Yep

482.		RB	We covered all the avenues for when it started out as all the blue on the bottom. Now all the blue on the bottom is the blue in the middle.
483.		JL	Okay
484.		A	I think you want us to organize it a little so we have these ones here.
485.		JL	I don't want you to do anything that you don't want to do.
486.		RB	So now we have all the blue as the second color.
487.		JL	Okay, oh
488.		RB	With the exception of, okay, with the exception of that one, but I don't know why.
489.		JL	So you have, oh, you
490.		RB	But
491.		JL	Are these all here
492.		RB	But these have to be stitched.
493.		JL	Oh
494.		RB	See that one and this one have to be switched.
495.		KF	Why
496.		RB	Because its blue and that's yellow. That one yellow's in the middle. Same as that one.
497.		JL	Interesting
498.		KF	Then why was it like that originally?
499.		JL	He's, now, but he's saying that
500.		RB	Now we have all the blue and yellows like that
501.		JL	They might not have been like that, but he's saying the second…
502.		RB	Second row of blue
503.		KF	Okay
504.		JL	Blue, the second's all yellows
505.		RB	All yellows
506.		JL	And then this one
507.		RB	And it should have been Angela because, remember we had all the blue on the bottom for the first set?
508.		A	Oh, yeah, yeah, yeah
509.		RB	So now with all the yellows

510.		A	That messed it up
511.		RB	And now here we have all the reds.
512.		A	I got it
513.		JL	Now I have a question for you.
514.		RB	Yes
515.		JL	Are you happy with these three groups?
516.		RB	Now I am.
517.		JL	Are you happy?
518.		A	I don't know, why would they be uneven? Are they? Two, four, six, eight, ten twelve, thirteen. Twelve, twelve and thirteen, or what is this?
519.		KF	Twelve. There's thirteen up here.
520.		RB	Because, I'll explain it to you. I'll explain it to you why. The reason for this is one of the towers has to be the three colors only once. It could only be used that one time. Because if you do…
521.		A	What do you mean by that?
522.		KF	I don't know
523.		A	Explain to me what you mean.
524.		RB	See this tower. This was our first tower that we started with.
525.		A	Got it
526.		RB	The blue blue and the red. In order to have two different colors, no matter which set of blocks its in, this is your all four. This is your four solids. It's really not four solids. This is the only combination that you could have with the.
527.		JL	You've got to look at him with that look.
528.		KF	No,
529.		JL	She's saying I don't follow.
530.		A	Yeah, that's not telling me why that's not thirteen.
531.		RB	Because
532.		JL	Oh, look at this, what are?
533.			Maria
534.			I'm going to
535.		JL	Oh it's fine. I'm glad you're here.
536.		RB	This is the extra one. That's the plain pizza.

537.		JL	We're working on Ankur's challenge
538.			Oh, awesome
539.		KF	
540.		RB	I'm going to compare it to the pizza.
541.		KF	This is the plain pizza.
542.		RB	That's your plain pizza. Or that's your all four. That's the extra one.
543.		KF	Why is this not all four?
544.		RB	We already made the combinations down here. This is kind of the opposite of that.
545.		A	This is going off on a tangent that I don't want to go down.
546.		JL	He's taking, tell him. Rich, you've got someone not happy with you.
547.		A	Rich, I understand the groupings, and I understand why we have these. I don't understand why we have thirteen here, twelve here and twelve here. Because each one of these have, like to me, these are the same. I see this and I see this and I see this, okay?
548.		RB	I think it was your problem, you can't shake your own hand. With the hand shake problem. You can't shake your own hand. You're going
549.		A	Okay, so why is it thirteen?
550.		RB	Because there's, it's three, three blocks three colors four high. That takes out the, that takes out any possibility. In other words "can't shake your own hand" It's three. If it was five towers high, then I think we would get an even number, I don't know.
551.		JL	Haha
552.		A	I don't know, I'm still not convinced.
553.		KF	Can we double check
554.		JL	Okay, you have someone who is so not convinced. How are you doing on this?
555.		KF	I'm not,
556.		RB	Let's regroup them.
557.		KF	I'm really trying to, like, think about what he's saying, really understand it.
558.		A	Are you confused about thirteen twelve and twelve?
559.		KF	Yeah, I'm just trying to understand what he's saying, like what

			does that mean, like.
560.		RB	I might have done a bad job of explaining, but
561.		A	No, its not that you're doing a bad job, it's not that at all.
562.		JL	What I would say, ok, what I would say since you're not convinced and you're not convinced of why thirteen twelve twelve.
563.		A	It would be different if it was like thirteen twelve eleven.
564.		JL	So can you try, maybe you should be talking to Rich about why you think this can't possibly be.
565.		KF	Let me ask you that, why is there…
566.		A	I think these all kind of go like in a pattern, do you know what I mean, like in a way?
567.		RB	Well I don't know about the pattern.
568.		A	Like maybe these two
569.		RB	I think it has to do with the method that we set up the blocks.
570.		KF	Like here
571.		JL	They're doing something.
572.			The other kind
573.		JL	Follow what they're doing
574.		RB	Okay
575.		A	This one doesn't
576.		JL	Because they were not convinced with your argument.
577.		KF	Yeah
578.		A	Whereas the yellow. But this is the yellow.
579.		RB	In other words.
580.		JL	So what are they doing, Rich? They look like they are doing something.
581.		RB	They're rearranging them into groups of four.
582.		JL	Are they? Are you rearranging
583.		KF	No, no, we're
584.		JL	Tell him what you're doing.
585.		A	We're making, like, triplets.
586.		JL	Triplets
587.		A	Okay, this one kind of looks like this one, kind of looks like this one.

588.		JL	You're saying that these three are triplets, why are they triplets?
589.		A	Because they have like two of the solid in the middle.
590.		JL	Okay
591.		A	So do these, and then, right, is that?
592.		KF	We have red yellow blue, and then these are bookends.
593.		RB	I see what you're doing
594.		KF	Like ends of the puzzle, with each color is bookends? See what I'm saying?
595.		JL	So you have triplets, okay what else is going on?
596.		KF	Here and this would be here, right?
597.		RB	Yes
598.		JL	Oh, she moved it, interesting, okay.
599.		KF	Yeah
600.		JL	Okay, why did you move that one over?
601.		KF	Because here's two, two and two
602.		JL	You want the double on the bottom.
603.		KF	And this one?
604.		JL	You agree Rich?
605.		RB	I agree, I know what's going to happen, there will be one left
606.		A	This, this this this
607.		KF	Yeah
608.		RB	Triplicate, because there will be two duplicates. For the other two.
609.		JL	You think so?
610.		RB	That's what I think
611.		JL	You better watch what they're doing
612.		RB	I'm watching, I'm watching, I could be wrong, I'm wrong all the time.
613.		JL	Okay ha
614.		A	Blue red blue
615.		RB	There's nothing wrong with being wrong.
616.		JL	No, nothing at all?
617.		A	Let's put this one aside.
618.		KF	Okay, two on top

619.		RB	Two on top again
620.		A	Where's the blue on top?
621.	31:50	KF	Here
622.		RB	Yep, it looks like a duplicate.
623.		KF	No, no its not a duplicate.
624.		RB	If you were to have the third, it would duplicate.
625.		KF	Why?
626.		RB	Let's build it.
627.		JL	Well, let's finish what you're doing.
628.		RB	Okay.
629.		A	Because I feel like there's a pattern we're going by, that's not like stated, so
630.		JL	Yet, see you had no trouble finding this as a group.
631.		RB	Those were clones.
632.		JL	Okay. Well, they're not clones.
633.		A	No, they're not.
634.		RB	No
635.		JL	They're not.
636.		A	These were clones.
637.		RB	Those were clones
638.		JL	Okay, why were these three a group?
639.		KF	I got not really sure
640.		JL	Verbalize it. Yes you do, yes you do. Verbalize it. You put them together, you must have a good reason.
641.		KF	This one, these would be a pair because there's red on top?
642.		A	Is red on top? Or
643.		RB	Doesn't make a difference at this point, you're running by colors.
644.		JL	You put these three together.
645.		KF	Yeah
646.		JL	I want to know why.
647.		KF	Because I feel like, blue yellow blue; yellow blue yellow; red yellow red. Oh this shouldn't, red yellow red
648.		A	I don't know, I don't know why they're together.

649.		KF	I don't know. It's alternating.
650.	32:47	A	Like these make sense to me. Let's go back to here.
651.		KF	Okay
652.		A	Two blue, two yellow, two red. Okay, got it.
653.		KF	Okay. Want to try something else that we're sure of first? This has to go with this.
654.			Yes, two together.
655.			Off Screen, but Audible
656.			We have this is two on top.
657.			
658.			Is there anything else that is two on top?
659.			Why is there not two blue on top?
660.			I don't know
661.		JL	Okay, how many do you have now?
662.			
663.			Because the two blue on top is right here.
664.			But we should have two of them.
665.			No but, no but what happens is, either you have the red or the blue
666.			Stop and look, two yellow two red two blue
667.			But that's that's you would get duplicates. It would be a duplicate
668.			Could we have all three colors without duplicates with blue on the bottom.
669.			Could two blue a yellow and a red be together?
670.			No
671.			Show me where that is already, right?
672.			So two blue
673.			Two blue a yellow and a red
674.			But then they
675.			No, two blue a yellow and a red is not there.
676.			I think you're right
677.			Two blue, a yellow,
678.			And a red

679.			And a red
680.			That would give us this one.
681.			The red
682.			Then we could reorganize them.
683.			Yeah, okay
684.			Is there something else that you can see
685.			This is one of the towers we cloned.
686.			Yeah, but we should we should have had a blue blue red. We did
687.			Here
688.			Where did this one go off to, then?
689.			I don't know.
690.			How did that happen? It's really nowhere, like it's nowhere to be found?
691.			Just like two blues on top, this one got lost in the shuffle somehow.
692.			Alright so
693.			Yellow, blue, red
694.			How would we organize these?
695.			Well, okay, um Dr. Landis, we had these not grouped this way, and she said, well how about you find another way that could be more convincing. So we're just trying to see, to do that for ourselves.
696.			Oh, okay
697.			We're not sure, this isn't exactly the best
698.			This isn't a definite way
699.			We had thirty seven. We thought it was weird to have thirty seven.
700.			But now we have thirty eight
701.			But now we have thirty eight
702.			Now we have thirty eight and I'd like to be more comfortable with that.
703.			I like an even number.
704.			But this one doesn't?
705.	35:51		No, it doesn't.
706.			When you say group, what do you mean?

707.	35:58		We used this this problem here to isolate the variable.
708.			We kept adding to the bottom of the tower
709.			Like for example this one. It already has one of each color, so you could add a blue, a yellow or a red. So you could add a blue on the bottom of this one, a yellow on this
710.			Now it seems we have one missing, so now, we need one more missing
711.			So let's see what it might be. So let's
712.			So it was two colors. Which one did we just use?
713.			So we just did
714.			It was two blues on top. Let's put two reds on top
715.			That's two reds.
716.			Is there another one, another possibility with two reds on top?
717.			Yes, of course there is, right?
718.			Two reds, two yellows
719.			Oh, there is, right here.
720.			That's it
721.			Okay, how about two yellows on top?
722.			Two reds?
723.			Two yellows on top. How many two yellows on top are there?
724.			I think there's a lot
725.			And then you can't have any more there.
726.			So that really makes it
727.			Why thirteen thirteen twelve?
728.			Because we can't get the blue, well,
729.			You got, you've got to show me.
730.			Two here
731.			Here's red, here's red and blue on top, right
732.			Red blue yellow
733.			I'm just trying to find all the ones that have three different colors on top. Per se, all the ones that have three different colors on top.
734.			So what's up with this one?
735.			There's a red blue yellow yellow.

736.			On the bottom, on the bottom
737.			
738.	38:03	JL	Um, What I would like to do, Um, I see you are still working, so I'd hate to kind of cut you short, but I also don't want to let it go on too long. Let's do five more minutes so you can kind of finish your thinking. We have some very interesting things going on here. Um, it's quite a complex problem, isn't it?
739.		RB	Well, we found one more.
740.			Are they all the same?
741.			You found another one?
742.			Um, no, they are not doing the same things. They have a different solution, different solution path, and so do they.
743.			
744.	38:34	JL	So tell me. Five more minutes, and then we're going to talk.
745.		A	Well, now we found another one in this group that we were missing, so we're at thirty eight right now. And now we're really not convinced because there's thirteen in this group, thirteen in this group and twelve in this group.
746.		JL	Haha. And that bothers you again.
747.		A	Yes
748.		JL	Because you want all the groups to be the same.
749.		A	Not necessarily the same.
750.		RB	It doesn't bother me.
751.		JL	Haha
752.		A	We need some type of pattern, though.
753.		KF	I want them to be the same
754.		A	Like if it were thirteen twelve eleven, I could totally see some justification there.
755.		KF	Right
756.		A	But thirteen thirteen and twelve doesn't make sense.
757.		JL	So it does bother her.
758.		KF	Yeah
759.		JL	It does bother Angela
760.		RB	You can't shake your own hand, I mean. There's going to be one that can't because.
761.		A	But then why would these two be the same?

762.		KF	Yeah
763.		RB	Why would what two?
764.		JL	Why Which two
765.		A	Why would these two groups be the same. Okay, you have two the same and one different
766.		JL	Okay
767.		RB	Because of the third color would be the third repetition.
768.		A	Well, could we find where that repetition would be?
769.		KF	
770.		RB	We just did it, we just tried a couple.
771.		JL	We have a question here. What he's saying. Can these three be a group?
772.		KF	Can they be a group, sure.
773.		JL	Well, no no no, why could they be a group?
774.		A	We have two, two , two
775.		KF	Alternating on the, well no. I'm just trying to see because nothing else alternates
776.		A	Yeah
777.		RB	This alternates
778.		KF	This alternates too, kind of like this?
779.		A	But there's not a yellow in the middle. See, there's a yellow in all these.
780.		KF	See, Angela, it was similar. We want this to all be blue
781.		A	I'm trying to represent, because then we can justify, like this is thirteen, this is thirteen, this
782.		KF	Oh, I got you.
783.		A	This goes here, this goes here
784.		KF	Okay
785.		RB	If we grouped them maybe differently, then maybe we would have.
786.		KF	So, like, but can we look at what we have left and say what's missing right here?
787.		JL	Oh
788.		A	Exactly. That's kind of what I thought.
789.		KF	What's missing, what's here that's not here.

790.		A	Can we have yellow, red, blue yellow?
791.		KF	Yellow, red, blue yellow?
792.		A	Yeah, it's right next to it.
793.		JL	Haha
794.		A	And yellow red blue blue?
795.		KF	It's over here, yellow red blue blue. Do we want to make that, or no?
796.		A	I'm just trying to double check these last ones, like double check these and try to think of… Blue red blue, it's the only option.
797.		KF	Yeah
798.		A	Red, yellow, blue
799.		RB	Do we have red?
800.		A	Red on top?
801.		RB	Red yellow blue red?
802.		A	Red yellow blue, red on the bottom?
803.		KF	Red yellow blue red
804.		RB	Yep
805.		A	How about red yellow blue blue?
806.		JL	Just separate these so you can see that you have.
807.		KF	Yeah, yeah, yeah. That's definitely it, so yeah, right here.
808.		A	Let's look at, go through these again, because I feel like.
809.		RB	Actually, Ashely, If we were to group these differently, You're looking for something like, where its like thirteen twelve and eleven you would be happy or twelve, twelve twelve you would be happy. This doesn't necessarily have to be divisible by three or combinations. It doesn't necessarily need to, and it might not work out that way, but if we were to group these differently, we may see something that you're actually looking for. That you don't see, because I'm pretty convinced that we went, that if we create any more, we're going to duplicate.
810.		KF	Wait, I have a question. Do we have a red blue yellow, red.
811.		RB	Red
812.		KF	Red blue yellow red
813.		RB	Red, Red blue yellow. But the yellows are here.
814.		KF	Red blue yellow red. Red blue

815.		RB	Red blue yellow blue
816.		KF	Red blue yellow red
817.		RB	Red
818.		KF	Red blue yellow red
819.		RB	Red blue yellow red.
820.		KF	That's yellow
821.		RB	Oh
822.		KF	Looking for red
823.		RB	Oh
824.		KF	We do have that.
825.		RB	Red
826.		KF	Oh yeah
827.		RB	Red blue yellow red. We do have it.
828.		JL	Ah. Did you find a duplicate?
829.		RB	No
830.		KF	No
831.		RB	She was asking to see it
832.		JL	What was the red blue yellow, oh. You were looking for the red blue yellow red.
833.			
834.			
835.			
836.			
837.			Strand 2
838.	16:35	S	It doesn't matter what the duplicated color is, so if we can figure out the number of these that's three high, that you need to add one more color. Then you just multiply that by three. Because then we could put
839.		KK	Right. So three high of just one of each color block.
840.		S	That would be eighteen. One two three four five six. Times three. Okay so, I actually like putting them together. It's like soothing
841.			
842.			
843.		KK	My kids don't think like that all.

844.	18:10	JL	What are you guys up to?
845.		KK	We're getting rid of all the other stuff. We're doing
846.		JL	So, you're starting from scratch?
847.			Yeah, well we don't know yet
848.		JL	Oh
849.		S	The towers that have one red and have one of each color
850.		JL	Oh,
851.		S	Because then you could add any one and it doesn't matter.
852.		JL	Oh, okay.
853.		K	The fourth one, the fourth one doesn't matter, so…
854.		JL	Okay
855.		S	And then we'll take that and multiply it by three.
856.		JL	Interesting, and what would that give you?
857.		S	All the different possible ones that satisfy that.
858.		KK	Three different ones on top
859.		JL	So what do you think the answer is going to be?
860.			Eighteen.
861.		JL	Eighteen, interesting. Keep working, Keep working.
862.		S	Are we right?
863.		JL	Oh!
864.		KK	You never ask that question.
865.		JL	Ha ha ha
866.		S	That's what I tell my students all the time. "Can't you just tell me!"
867.		JL	And what do you say to them? "Why don't you try it out?"
868.	19:00		That's it.
869.		S	No, I don't think so, because look. Here you're saying, If I were to put red on that one.
870.	20:10	KK	So, here's my blue yellow reds. So we're going to add… Oh, you're right, you're right. So wait.
871.		S	Yeah, that one would have been kind of like that.
872.		KK	Let's do that.
873.		S	So this would be blue, red yellow. I know it's hard and it's annoying…

874.	21:30	KK	It's so funny, I say one thing and I laugh.
875.		JL	Are we done?
876.		S	Okay, so here's our blue and yellows
877.		KK	On these we controlled
878.		JL	What do you mean you controlled here?
879.		KK	We took all the three highs that we had
880.		JL	Three high, okay.
881.		KK	That had one of each
882.		JL	One of each
883.		KK	In all the different positions
884.		JL	Okay
885.		KK	Like only adding three different choices only on the top
886.		JL	Okay
887.		KK	At the top
888.			
889.		S	Uh huh
890.		KK	Oh my God, you are fast!
891.		S	Haha
892.		JL	So in other words, you had this and cloned it.
893.		KK	Right
894.		S	Yeah
895.		JL	I got it
896.		KK	And now we're putting ones on top
897.		JL	I got it
898.		S	You're really evil, hahaha
899.		JL	What is that?
900.		S	You called it a clone. I like the word clone because that's what I feel like I'm doing, kind of.
901.		JL	Okay, haha
902.		S	It's a whole family
903.		JL	And you have the originals, so you know what we started with?
904.		KK	Yes
905.	22:26	JL	Okay, I'm following you.

906.			
907.	24:34	S	Oh my gosh, It's thirty six. Oh my gosh! Oh because we were to the bottom three, then we can reverse it…. Like you were saying that the bottom three have to be different ones.
908.		KK	But like with these, it could be the top ones.
909.		S	Wait, no, because if you turn this one over, then that would be, this one. Hold on, I have an idea. If we flip the ones that have two of the same on top. Each one of the one of each really would be on the bottom, so the only way that, like this one has one on the top so, but this one doesn't. So if you flip this one, you'll get this tower. And if you flip this one, this one here, you get a tower. Because these three, on the top we controlled for the variable. You know what I mean? If we flipped these two we'll get a new one. Do you see why I'm
910.		KK	
911.		S	You have one of each, if I cover up the bottom. The ones that I have left, that can't have one or two on the top
912.	26:57	KK	Okay, so now, we need to…
913.			
914.		KK	I'm having trouble verbalizing… Our first one was awesome, and we did steal
915.	28:50	S	This one goes with these
916.	30:08	S	I think the last thing we could do to check, and maybe make sure would be If we decided we could flip. It started from, these two were the originals. And then this one was oh, this flipped. No because we had yellow, blue blue, I mean yellow blue yellow. So it was one that… this was the one that could not be flipped. Yeah so this one was flipped…
917.		KK	The original
918.		S	If you separate them. Alright, I'm going to start. It's the top part, the top that if you don't have the color. So that's why we were able to.
919.		KK	Oh, right, right, right.
920.		S	Because it didn't exist, it was only the top three. Because if you take the ones like we took, you know that you'll find it somewhere else. It's the original, you built that out, so… The other ones that don't have the top all built in, don't have, we wouldn't have started them all that way. So we'll have six of these on top
921.			

922.			
923.		KK	We realized our mistake.
924.		JL	What was our mistake?
925.			
926.	33:05	KK	Because we had only accounted for a different one of the top.
927.		JL	Okay, how many do you have now?
928.		KK	Thirty
929.		JL	Thirty, You think that's it?
930.		S	Yeah
931.		JL	Are you sure
932.		S	Well the way that we got the new ones, Was by, like, saying "Alright, we can turn this upside down" Because we started by making the bottoms
933.		JL	Right
934.		S	One of the each.
935.		JL	Okay
936.		S	And we did all possible "one of each bottoms"
937.		JL	Okay
938.		S	And just added the three different colors to the top.
939.		JL	Okay
940.		S	But then, like with this one
941.		JL	Alright.
942.		S	It has one of each on the bottom, but then if you cover up the bottom, it only has one of each on the top, so
943.		KK	It doesn't have one of each
944.		S	Oh yeah, so it doesn't have one of each on the bottom
945.		JL	Okay
946.		S	So we said "Well, if we flip it, do we have it anywhere else?" And we don't, and the reason is because we started with the bottom three, making sure that they had one of each.
947.		JL	Okay
948.		S	So here we have that extra spot. We can "not have" one of each and put one color on top. So we got new towers.
949.		JL	Okay, so you found more than eighteen and you got up to thirty.
950.		S	Uh huh

951.		JL	But you're not sure, or you are sure?
952.		KK	I think we're sure because the one, one of the three from each one couldn't flip. Because if we flipped it,
953.		JL	Right
954.		KK	We already had that because it had one of each.
955.		JL	I'm not totally convinced that you found them all.
956.		KK	Okay
957.		JL	Are you convinced?
958.		S	I think the other way we could check is by determining how many total four tall towers there are,
959.		JL	Ooh
960.		S	And then taking all the ones that don't have one of
961.		JL	What do you mean? What do you mean? That's interesting.
962.		S	So we would do three to the fourth power.
963.		JL	Ah
964.		S	So that would be eighty-one.
965.		JL	So why is it eighty one
966.		S	Because a lot of them wouldn't have one of each color.
967.		JL	Oh, so you have, you know three to the fourth would give you four tall towers.
968.		S	Uh huh
969.		JL	You know this is not going to … How are you going to get rid of them?
970.		S	We have to, I would do a tree diagram.
971.	35:51	JL	Well with this, would you have any clue how to do it? You know it's going to be less than eighty-one. You know it's going to be more than eighteen. The question is how many, How many?
972.		KK	Alright, so if you think that…
973.	39:28	KK	Top and bottom, or middle two, and combinations of
974.		S	I think it is, because this is the only possible way they could be: Split, split and then together, together, together. So that's six times six. It's just weird to get different combinations, like yellow blue, yellow red, blue red. Yellow blue, yellow red, blue red.
975.		KK	So six times three is going to be eighteen.

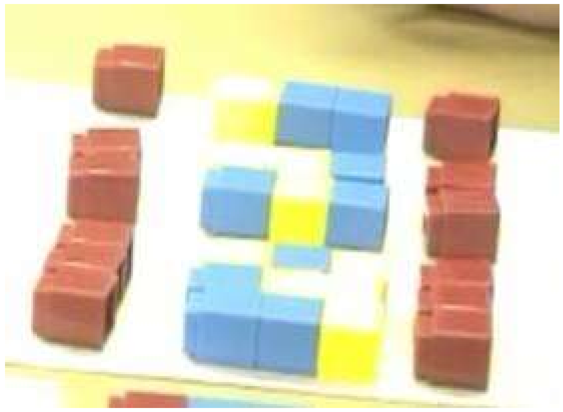
976.		S	Yeah, I think we would have it then.
977.			
978.			
979.			Strand 3
980.			
981.	17:29	M	So what you're saying, out of all of these… All blues, all yellows
982.			
983.			
984.			
985.	18:30	J	We had twenty-four. So we put all blues on the bottom of the twenty four. Recreate these twenty four with all yellow on the bottom. You could also recreate them with all red on the bottom.
986.			
987.	19:02	JL	Oh my gosh, what do we have here?
988.		M	So
989.		J	We're still brainstorming, but I'm thinking we have twenty four here.
990.		JL	And were these the original ones?
991.		M	Yes, minus.
992.		JL	But you had twenty seven. Oh, you got rid of them. Why did you get rid of these?
993.		J	Because in order to have, if you add any colors to these, there's still two. You got to have three colors.
994.		JL	Got it. Got it. Okay.
995.		J	So if you had two colors here. You would have to add blue all the way across here. So we controlled for that variable. Recreate another twenty four, put all yellow at the bottom. And we go through once more and put red at the bottom.
996.		JL	Interesting, okay
997.			
998.		JL	So is that what you're brainstorming?
999.		J	Yeah, that's what we're considering doing.
1000.		JL	Okay
1001.		J	But we're thinking through it
1002.		JL	You're thinking through it. That's good, thinking is good.

1003.	20.05	M	So, if I put this on the bottom and I have blue blue blue, that's different than if I had…
1004.		M	There's twenty-four and I put all blue on the bottom. So it's forty-eight.
1005.		J	If you add that and that, it's twenty-four towers.
1006.		M	Right
1007.			
1008.			
1009.			
1010.	21:07	J	So if you add that and that, that's twenty-four combos. And then if you add a blue on top. You can't add a third blue.
1011.		M	This is, there's already three here
1012.		J	So we have two options for blues. Then we have two more.
1013.		M	Well, so did you put blues all across? And then if we put them at the top. That would give you another, different ones. You're saying "Is there another way we could put"
1014.		J	Like a yellow, and then two blue, but that could be three, that could potentially be three blues over there?
1015.		M	What do you mean? You can't have three blues. Because you have to have all three. So, that's actually a problem. So then all these ones. …on the bottom, this isn't going to help. It's just going to add to that. So what we've got to do
1016.			
1017.			
1018.			
1019.	22:28		Okay, how are we doing here? Did you do your thinking?
1020.		M	Still working on it
1021.		JL	Okay, what did you come up with, because
1022.		J	Well, Mitch just suggested that there would be a difference it we could not only stay with the blues on the bottom, but then add the blues on the top. But then he just said "Well, if we do just blues at the bottom, then the ones that would be left would have to be out of it because, you know, there's only… You add that other two colors "
1023.		JL	Okay, so you would be eliminating some. If you put a blue here, you're telling me you would be eliminating this tower.
1024.		J	Yes

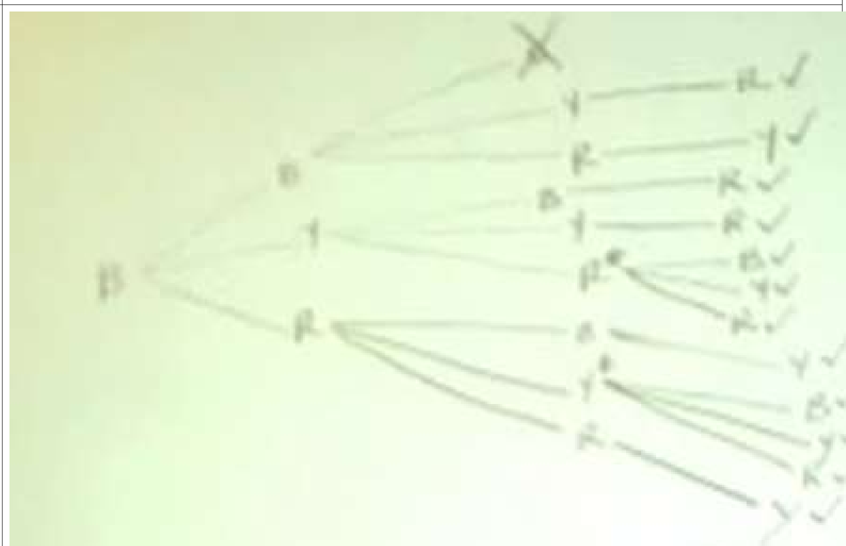
1025.		JL	Okay, I, That makes sense. So now what are you going to do? You have got to do something because we're going to run out of time. So, make your decision.
1026.		J	Haha
1027.		JL	You've been thinking about this, right? What's your decision, what are you going to do?
1028.		J	Alright, We um, can we have two more minutes?
1029.	23:33	JL	Absolutely.
1030.		J	Beautiful
1031.		M	So Let's organize all the ones that are missing a blue and all that are missing a yellow. We need a yellow.
1032.	25:05	M	So, do you have all the ones … that… because I'm only doing yellows. This is all the different ways you could have yellow on the bottom. This is all the different ways you could have blue. We are controlling for variables
1033.			There's only one other place for the yellows… We're looking at the middle two positions. None of these can be duplicates.
1034.		J	I know that
1035.		M	They're also when we add it to the bottom… So
1036.	30:00	J	You know what,
1037.		M	If you add it to the top that's… If you add it to the bottom, that's another set of twelve. Over here, I have another six on the top, six on the bottom, which is twelve.
1038.		J	But you just did three
1039.		M	Right, this is. so we have thirty-six. We have three groups of twelve. This one right here, see, this is already… So we could add blues on top, blues on bottom. This is, it's going to be three more groups of twelve. Because with blue on top, blue on bottom. This was six, so there's nine, another six which is twelve. We could also add yellows So in total, that gives us three more, which his twelve. I probably would have to build them… About how we want to go about this one.
1040.		J	Like you started before… If you create towers. You could, all possibilities of two colored towers. Alright, you had two colors that were three tall.
1041.	32:22	M	This because, we already had our last one. Controlled for that position. So keeping… Every single, like we have yellows on the bottom. Yellows in the second position, there's yellos in the third position. So we've got every single

1042.		J	And we know that its twenty-four. So we have, are sure that it's twenty four.
1043.		M	Yes, because we have the original twenty-four. And then all we did was add on to the top and bottom. So I know, you know I'm confident there's no duplicates.
1044.		J	So it's perfect, because we could convince anybody in that group
1045.		M	I don't know
1046.		J	But first that there's twenty-seven to start
1047.		M	Right
1048.		J	Three tall three colors Subtract out all those all red, all blue all yellow, because they have to have three.
1049.		M	Right
1050.		J	So then most people would react less to that. There's no tricks up our sleeves there. Then we reorganized them.
1051.		M	Right
1052.		J	Once We reorganized them, we reorganized it with colors of one
1053.		M	Right
1054.		J	Colors of two colors, and now, and look, like, Even though we have this, like you said, we controlled for this. Now that we have that, we don't have to sort of modify this type of strategy, because we have twenty four.
1055.		M	Right
1056.		J	Unless we separated it into colors of two's even though I think that would be, you add the last color to the top or to the bottom.
1057.		M	Right. How would you, I think the best way to visually show this would be to do what you did. Like I just kind of got lazy and… But we're not going to build all of them, like I would not. I would do this, this
1058.		J	It's awesome though, this…. Is like you separated the colors.
1059.		M	Right, the variables.
1060.		J	So then there's two colors
1061.		M	Right
1062.		J	And then there are groups of all three colors. The groups, three groups of two colors.
1063.		M	Right, What I realized, we would have had. We would have had duplicates. Like we would have had this one because it has all blues and yellows, so we have to put a red.

1064.		J	Yes
1065.		M	So
1066.		J	So we have to add onto our theme, because first we just thought we had to…Three groups of two colors and then those groups.
1067.	35:55	JL	Ah, what do we got here?
1068.		M	Well, like we talked about, we were going to add to the bottom and to the top. We kept the original twenty-four that we had.
1069.		JL	And the twenty four were minus the three solids?
1070.		J	Yes
1071.		JL	Okay
1072.		M	So the three solids were out because we couldn't have them.
1073.		JL	I understand
1074.		M	Okay, so with the twenty-four, we're confident that there's no other combinations for the middle group here. So these three, so all these middle groups as you can see here,
1075.		JL	Okay
1076.		M	There's no duplicates, because we know, we already went through all them. So there's no other combinations for four different positions.
1077.		JL	Okay
1078.		M	So then what we did was, we put them in groups of, groups that were missing a yellow, that needed to have a yellow, groups that needed to have a red, and groups that needed to have a blue. We actually got to make top and bottom towers.
1079.		JL	Okay
1080.	37:00	M	…that already had three colors, so them that was missing just the yellows, you could put on the top or on the bottom. So basically, We put them all on the bottom.
1081.		JL	Okay
1082.		M	Or put them all on the top. So there's twelve right here.
1083.		JL	There's twelve there, yeah.
1084.		M	So then the ones that are missing a red, that's going to be the same thing, I just didn't build it yet, but it's top and bottom.

1085.			
1086.		JL	So you think you're going to find twelve of those.
1087.		M	So the twelve, twelve, twelve and this one since it already has three, We're not only going to add blue on the top or bottom, we could add yellow on top and bottom, red on top and bottom
1088.			
1089.		JL	Okay
1090.		M	So this is going to be thirty-six, seventy-two. So we have twelve, twelve, twelve, thirty-six. And then this alone with the three different groups is going to give us another thirty-six. So we'll have seventy-two.
1091.		JL	Seventy-two, Are you convinced?
1092.		J	Absolutely
1093.		JL	Haha! I'm laughing because, you're not, your face doesn't say it. Your words are saying "yes", but your face is saying "I'm not so sure"
1094.		J	No no no
1095.	37:59	JL	Are you convinced? I don't believe either of you! Okay, we're going to be talking about it.
1096.			
1097.	41:00	J	But that's convincing, though.
1098.		S	What! No you don't

1099.		J	We do
1100.		S	What! No way, you have to have duplicates. There's no way!
1101.		J	There's no duplicates.
1102.		M	Show me.
1103.		J	Sally, stop laughing
1104.		J	This is all we have with blue at the bottom.
1105.		S	Why can't you
1106.		M	I'll show you guys later
1107.			
1108.			
1109.			
1110.			Back together as a single group
1111.	42:26	JL	Um, Are you ready to talk as a group?
1112.		RB	Yeah, we should.
1113.		JL	I think we should, okay. Because I think some of you are seeing the wrong colors now.
1114.		A	I was too
1115.		JL	Which means we need to talk, alright. Um Let's get back together, Okay. Why don't we
1116.		M	You want this on?
1117.		JL	Yeah, we want this on for sure. If your kids were still working, you would never ever ever do what I'm doing now. Is stop the thinking and whatever, but for time purposes we are going to get back together as a group. Um Before we begin: You had different strategies which were fascinating, very interesting. How many did you end up finding?
1118.		M	Seventy-two
1119.		JL	How many did you end up finding?
1120.		A	Thirty-Eight
1121.		JL	How many did you end up finding?
1122.		KK	We're at thirty.
1123.		JL	They're at thirty.
1124.		S	But I think there are thirty-six though, with a tree diagram.
1125.		JL	You think you got with a tree diagram.
1126.		KK	Um hm

1127.		S	But,
1128.		JL	What do you mean a tree diagram? Is it on paper?
1129.		S	Yeah
1130.		JL	Can you put that up for a minute? Remember that other problem that we did? The extension of the one we just did. Where they didn't put, Ankur didn't put a restriction. It didn't matter that you didn't have to have all three colors in each tower? And do you remember what we got for a solution? Towers three tall choosing from three colors. What did we get? In the last problem.
1131.			
1132.		KF	Twenty-seven.
1133.		JL	We got twenty-seven. And remember you told me it was three to the…
1134.			Third
1135.		JL	Third. Now, okay. Sally and Kate, very interesting, said we think this solution might be three to the fourth if, was it you who said it?
1136.		KK	Yes
1137.		JL	If what?
1138.		KK	Three to the fourth, if you could use three in each of the four places.
1139.		JL	Say that again?
1140.		A	That's that's
1141.		KK	It would be three to the fourth
1142.		JL	Right

1143.		KK	If we could use all three colors in every position.
1144.		JL	Ah Okay, so another way of saying it. When would the solution be three to the fourth? What would we have to, what, how would we change the problem.
1145.		RB	If it wasn't
1146.		A	That criteria
1147.			Four tall
1148.		JL	Four tall
1149.		RB	That's it, three colors four tall
1150.		JL	Choosing from three colors, no restrictions that every tower had to have one of each color. Okay? So they said, they know.
1151.		S	The maximum was eighty-one, but
1152.		JL	Okay, so they know that it's not going to be eighty one.
1153.		KK	It's not
1154.		JL	Why won't it be eighty-one? Why won't the answer be eighty-one?
1155.		A	Because you have to eliminate for when there's not three colors in that tower.
1156.		JL	Okay. Alright? So it's going to be less than eighty one. So so far you're all in the running, right you all have less than eighty-one.
1157.		A	Yeah
1158.		JL	Let's see the tree diagram.
1159.		S	Okay. Do you want me to throw it up on the board because.
1160.		JL	Yeah, no up here. It's not coming up?
1161.		M	It's having a hard time.
1162.		JL	It will come up
1163.		M	It's in night view.
1164.		JL	Yay, let's just center it.
1165.		S	Alright, so.
1166.		JL	And what you're going to do is exactly what you have. Do not worry about starting with yellow or red.
1167.		S	Okay
1168.		JL	If you can convince us of this, we will all be convinced of the other two. Is that correct?
1169.			Mm hm

1170.		S	Well, I'm going to just tell you how I, why I stopped here.
1171.		JL	Okay.
1172.		S	So I figured you can start on the bottom having a blue a yellow or a red.
1173.		JL	Okay
1174.		S	And then on top of each of those you could put blue yellow red, blue yellow red, blue yellow red
1175.		JL	okay
1176.		S	So, I have all my possibilities for my first two layers.
1177.		JL	Got it
1178.		S	Then I said "I want to keep going BYR"
1179.		JL	Go down, just bring it down a little so we can see, okay.
1180.		S	Okay, so then I said, If I do a blue, if I have a blue, then a blue, and then another blue
1181.		JL	Right
1182.		S	Then I don't have room for both of the other colors. So I stopped there.
1183.		JL	Okay.
1184.		S	And I said that's a dead end.
1185.		JL	Okay, good.
1186.		S	And then I said "If I do blue blue yellow, the fourth one has to be a red." So I put a check mark there for a possibility.
1187.		JL	You have one tower
1188.		S	Yep. So then I said, "If I do blue blue red, the other one has to be a yellow" So I'm eliminating, I don't have to do the full tree diagram.
1189.		JL	Got it
1190.		S	So then I do blue yellow blue, then the last one has to be a red. I need that for the color. Blue yellow yellow, the last one has to be a red
1191.		JL	Okay
1192.		S	Blue yellow red, and then I have my three colors, so I put a star there, because I can put any other color on top of that.
1193.		JL	Because you already have three colors. So, and that's kind of what you guys were doing. Good, okay.
1194.		S	So then I kind of went to the red branch, for the second

1195.		JL	Good
1196.		S	Blue red blue
1197.		JL	Right
1198.		S	So the fourth one has to be a yellow.
1199.		JL	Okay
1200.		S	And every check mark represents a different tower
1201.		JL	Okay
1202.		S	And the n blue red yellow
1203.		JL	Right
1204.		S	Again, I can attach any three on top of that.
1205.		JL	Good.
1206.		S	And then I said "Blue red red" and the fourth one has to be a yellow.
1207.		JL	Good. How many check marks do you have?
1208.		S	I think I counted twelve. One, two, three ,four, five, six, seven, eight, nine, ten, eleven, twelve.
1209.		JL	Excellent, so you have twelve towers
1210.		S	With blue on the bottom
1211.		JL	That you could make with blue on the bottom.
1212.		S	
1213.		JL	Can you guess Rich, how many towers they're going to have with yellow on the bottom?
1214.		RB	Twelve
1215.		JL	And similarly with yellow on the bottom. So Angela's concern that she really wanted to have.
1216.		A	I just found two duplicates.
1217.		KF	We just found two duplicates we have thirty six
1218.		S	Oh
1219.			Twelve, twelve, twelve
1220.		JL	So that's really really nifty. Okay
1221.			About what we missed.
1222.		JL	What are you, you know what, we're not going to worry about it now, I'm glad though. If you guys could start putting them back in the bag
1223.		KF	In tens

1224.		JL	Yeah
1225.		KK	Oh, I don't want to.
1226.		JL	While you're putting them back in tens, I know you're all so conscientious. I know that some of you are going to bring cubes home tonight, and you'll be doing it at home before, okay, before you go to bed tonight.
1227.		KF	It was bothering me that it was an odd number
1228.		JL	You're right, good good.
1229.		JL	Okay, so what I want you to do though it listen, when you do it with the kids, okay, these problems are online, they're on ecollege so you have clean paper for both the towers three tall choosing from three colors, and the Ankur's challenge. Which is his extension. You are going to start the whole class on towers three tall, choosing from three colors. Okay? You are going to let them build, you're going to give them the paper, then when they have a convincing argument, to record their towers and write their convincing argument. Okay? When they are done with that, if they get done. That's when you're going to give them Ankur's Challenge. Do not rush to give them Ankur's Challenge because, do you see how long it took you? So If you have an hour, some will get to start it, some absolutely won't. If you have forty-five minutes, forget it, right? You're going to have to go into a second day.
1230.		KF	Forty minutes
1231.		JL	Forty minutes is even worse. Okay, especially with the cubes.
1232.		KF	By the time you get set up,
1233.		JL	Absolutely.
1234.		KF	You want another hour for this?
1235.		JL	I would let it spill into
1236.		KF	The next day
1237.		JL	Absolutely. But if they get to build first, let them convince you that they have all the towers two tall, three different colors, then you give them the paper and let them record their solution, and write their convincing argument, okay? If they can't find them all, do not lead them to find them all. Let them write what they have. Let them write as convincing an argument as they can come up with. They will be in all different places some of them might be to three to the cube, uh third power. Ask them what those threes mean, Okay? Some of them might think of three to the fourth power for Ankur's challenge and know that they then have to take away from the eighty-one. Ask them what that

			means, Okay? But you will have some kids in tenth grade, we have a girl named Romina. You are going to watch a video of Romina solving this. When you see Romina's solution to Ankur's challenge, you are going to go "<gasps> oh my, is this brilliant!" Okay, because wait until you see what she did, and it actually got called Romina's proof because it was so beautiful, Okay? So you may have to watch Romina explaining how she's solved Ankur's challenge more than once. Don't be afraid to rewind it and watch it again because it really is beautiful. And then you're going to say, "Oh, why didn't I think of that" Okay, Alright, terrific, you guys are doing a good job.
1238.		KF	So I wonder, like for us
1239.		JL	Yes
1240.		KF	Like why did it make it
1241.			
1242.		JL	And you know, for me it was very hard not to ask you to check for duplicates. I really held back.
1243.			
1244.			Because it would have been easy to get you out of your agony, because both of you were in agony. He didn't he was all into it, he had no problem, he was into handshakes, He was into all this
1245.			Yeah
1246.			But you guys had a
1247.			Well, we found them
1248.			And I think though that you found it you feel better.
1249.			Kulsom said "Yes!"
1250.			I definitely feel better
1251.			Once I found one
1252.			It was easy to see once she found one
1253.			Yeah
1254.			That's good, and the way you're talking to each other, the way you're, I can see that your questioning of each other is so much more sophisticated than what it was
1255.			Right
1256.			You were actually able to challenge that and say that doesn't make sense to me, and that's great.
1257.			Yeah
1258.			And you want your kids doing the same stuff

1259.			Do you think what Rich meant was just like
1260.			I don't you know I'm so, I wasn't here enough to
1261.			We used our existing, our twenty four towers
1262.		JL	Which was, what you did was brilliant, it was a nice strategy. Something went wrong, I don't know what. Because I wasn't sitting here watching you.
1263.		KF	Yeah
1264.		JL	But I wouldn't worry about it.
1265.		KF	Yeah, yeah
1266.		JL	That you found them was
1267.		KF	I won't lose sleep over it tonight.
1268.		JL	Don't lose sleep over it. Okay. Guys you did a great job, you really did.

11/18 Meeting transcript 1 of 2

Title: 11/18 Oldbridge-1of2
Location: Oldbridge
Date: 11/18/2010
Length: 00:56:05
Transcribed by: Will McGowan June 2012
Verified by: Madeline Yedman August 2012

Line	Time	Speaker	
1.	00:00	JL	Time to talk about your student work. And to talk about the final project. And we can do one of two things: We can either take a break, which we haven't been doing. Or, if you would rather not take a break, and you know, just go out when you have to use the facilities, or get a drink, or whatever. We could end a little earlier. So what's your preference?
2.		RB	Earlier
3.		K	Yeah
4.		S	Yeah earlier is fine, ha ha.
5.		KK	I know you want to get out of here.
6.		JL	So then
7.		RB	No, not want. Need.
8.		JL	That's what, yeah. That's what we'll do. Okay. We are going to have a bunch of visitors today. Our first visitor that you haven't met yet is Mary Schwartz.
9.		K	Hi
10.		M2	Hey
11.		RB	Did she bring food?
12.		M	Ha ha
13.		JL	Did Mary bring food?
14.		M2	No I did not.
15.		JL	No. She said yes! Did you bring food?
16.		RB	Actually I did.
17.		JL	Where is it?
18.		RB	Well, I brought cupcakes the last day of class over the summer.
19.		JL	No, no, no, no
20.		RB	Well, I know.

21.		JL	It doesn't count.
22.		RB	So it's Kulsom's turn.
23.		K	Oh, yeah?
24.		JL	It should have been tonight, when I was here.
25.		RB	I should have, I'll bake for our, I'll bake something for our final shindig on the fourth.
26.		JL	The only thing is, at our final thing on the fourth. I won't be there.
27.			Aw,
28.		KK	You're kidding?
29.		JL	No. I'm not kidding. But Carolyn Maher and Alice Alston will be there. And so will, Will will be there. And Jonathan who is following my other sessions in Toms River and in Long Branch. He'll be there. And you will be having a good time talking, you know sharing the whole semester with your final project which will be due that day.
30.	1:30	JL	Good, you're on, you're almost on time.
31.		RB	Almost
32.		JL	Good.
33.			Ha ha
34.		JL	We're glad you're here. We're just talking about this evening, okay?
35.		J	Okay.
36.		JL	So, you will hand in your final projects on the fourth, and you'll have cupcakes complements of,
37.		RB	Or cookies or something.
38.		JL	Or cookies.
39.		RB	I'll make something.
40.		JL	And if ever you have a class with me again, I expect some cookies or cupcakes.
41.		RB	Okay
42.			Ha ha
43.		JL	But seriously, you have done a phenomenal job. I have been very impressed with your online responses. And when you talk to each other, and the way you really are much more observant of what your students are doing. You actually are seeing some neat things happening. They have come a long

			way. Haven't they?
44.			Mm hm
45.		JL	And I think you in turn as a facilitator have really come a long way since, it's been a very short semester. I mean, it was only a couple of months ago when we began. And things have definitely. Things worked for your students and you. I think you really are seeing some neat things happening. And you are very, very aware of what funny things, or what's good thinking. And your questioning is getting better. And I really think you are able to question without leading. And that's a hard thing to do. So I think you are going to continue to get better. And I'm hoping that this kind of reasoning, and this kind of teaching is not just going to be when you do these problems, but when you teach anything. And I know, it can be done with any mathematics. As your teaching to get kids to really be thoughtful. So I think it's great. So, that's your compliment. And we're going to start off by letting you now, Justin, Mary Schwartz is here tonight
46.		J	Hi there
47.		JL	We will have some other guests coming from Rutgers. Do you know Lynn Ginsberg?
48.		RB	Yes.
49.			Uh huh
50.		JL	Okay, Lynn said she's coming. And she's bringing "someone."
51.			Ha ha
52.		JL	I don't know who "Someone" is.
53.			Ha ha
54.		S	Hope she does.
55.		JL	But I said "Sure!" You know, why not? And who knows who else will come. Maybe your supervisor.
56.		KK	Oh, from Oldbridge?
57.		JL	Uh huh. Jean might show up. The principal from this building may come after his meeting. So I think it's great. I think the more administrators we can get interested in what you're doing in your classrooms, the better it will be for you. Because then they'll understand what you're doing. Because what you're doing is neat. Okay, so right now, let's start sharing. Oh, and let me talk about what this night's all about. We're going to share first. Okay? And we're going to really take time first to share the, um, the regular problem. Three

			towers choosing from three colors. And then, how many of you got a chance to do Ankur's Challenge? A couple of you. We'll let you share that. Okay. I know, and not everyone did, and that's Okay. I would rather you have spent enough time to let the students really show you what they have done on the regular problem before you went on to the extension problem. But if you did the extension problem, boy would we love to see what your students did. And today we had some amazing stuff happening in Mitch's classroom. And that was in an eighth grade inclusion classroom. And some special ed girls. One special ed girl, right, one was regular ed?
58.		M	Yeah.
59.		JL	Didn't get to complete the, Ankur's Challenge. But I bet if we gave them more time, and if they weren't worried about missing their lunch
60.			Ha ha
61.		JL	I bet they could have. Because they really had a very neat strategy. You're going to get to see the beginning of their thoughts on that. When we are done with sharing, we are going to talk about the final project that you're doing. And make it very clearly spelled out so that you you're not guessing what I'm looking for but you'll really know. And I think you're going to have fun doing it. Because it's going to give you a chance to reflect on your students' work over the course of the three cycles. Okay? And then we're going to break early because we didn't take a break. But feel free to get up if you have to, and use the facilities. Okay, who's going first?
62.	5:43	RB	I have one actually.
63.			
64.		JL	Okay.
65.		RB	I chose this one because it was my thought. Because these girls,
66.		JL	Okay

67.		RB	Stole my thunder.
68.		KK	Ha ha
69.		JL	They "Stole" it?
70.		RB	Yeah. I mean, I called them "Justin." Because Justin always seems to
71.		S	Ha ha
72.		RB	Steal my thunder.
73.			Ha ha
74.		J	Oh, wow
75.		RB	He's pretty detailed, always. They um.
76.		J	Man
77.		JL	Can we, can we adjust the
78.		M	So we can see all of it.
79.		JL	The document camera so we see the whole screen? Ah.
80.		RB	I'm not going to tell you what they did. Because that's the fun of it.
81.			Oh
82.		JL	Okay, We're going to try and figure it out.
83.		RB	Oh, you'll know right away.
84.		JL	Well then give us time to first figure it out. Because
85.		RB	Oh yeah. It might take a moment for the text. We'll leave it like that.
86.		JL	Yeah, and it should get a little clearer as it focuses. Whoa!
87.	6:33	M	It should be
88.		JL	Okay, so we're going to let it focus. It looks like an eye test. "Can you see this?"
89.		S	Ha ha
90.		A	Yeah
91.		RB	Actually, I can. Because I need glasses.
92.		JL	Ha ha
93.		RB	All of a sudden. I used to have good vision.
94.		JL	Okay, it can, Mitch, is that going to get clearer?
95.		M	Did it?
96.			No

97.		JL	Yeah, it's hard.
98.		M	If it doesn't, I can borrow something.
99.		JL	Okay.
100.		S	Ha ha
101.		JL	Yeah, because
102.		RB	You got pretty good with this, Mitch.
103.		JL	Mitch, actually is good with the technology. I recommend strongly, while your other teachers in your school don't know how to use the technology, grab it.
104.		S	Yeah.
105.		JL	Use it. Because once they learn how to use this ah, technology.. You're not going to be able to get your hands on it. It's going to always be busy. Okay, take your time to read what they did, and let's see if we can figure it out.
106.		RB	No cheating.
107.		JL	Oh, okay.
108.			What does it say over there?
109.		JL	It says "red, yellow, blue" Is that what it says?
110.		RB	No, red, Okay, I'll help you with that one just so we're all on the same
111.			On the left
112.		RB	Red, red, red
113.		KK	No, no, no
114.			To the left
115.		KK	Just the very left.
116.		RB	Left. Red, yellow, yellow.
117.			No, no, no.
118.		S	The red section
119.			To the left, the left.
120.		S	Red section.
121.		RB	Oh
122.		JL	Red section
123.		KK	Section
124.		JL	Okay
125.		RB	Red section, yellow section blue section.

126.		JL	Okay
127.		S	So those are the tops.
128.		M	Are they all of them red tops?
129.		KK	Right. All red tops.
130.		M	All yellows, all blues.
131.		JL	Right.
132.		K	And then they did red with one of each, and then they reversed it.
133.		KK	And then red red yellow, red red blue. And then they separated red.
134.		JL	Really, really look at it. Think, and then we're going to ask people to say what they see. Looking at the student's work.
135.		S	Are they putting in the middle where it's red red yellow, red red blue
136.		JL	Yeah
137.		S	Then red blue red, red blue blue?
138.		JL	Yes.
139.		S	They held the, like the middle one constant.
140.		JL	Oh
141.		S	Red red, red red.
142.		JL	Oh you're seeing good stuff. Okay. Can you, Rich, point out what, what's being said?
143.	8:25	RB	Well, I was speaking to the girls,
144.		JL	Yeah
145.		RB	And they were working on this. They had the control of the red on the top.
146.		S	Yes.
147.		RB	That they did. And their idea was to convince me.
148.		JL	Okay
149.		RB	And, uh. And they were pretty verbal with it and they did a great job. Then they said "With the next color, it could either be red, blue, or yellow." So, they made that the constant for the next one. And then they kept changing. So, it's kind of like, a staircase in a way. Because they went red blue yellow, then red red blue, blue yellow yellow. Then they continued it on the bottom.

150.		JL	Slow down, I 'm not following.
151.		KK	Yeah me neither.
152.		RB	Let me put their work
153.		JL	Yeah, just jump right in. don't uh,
154.		RB	Okay.
155.		JL	Okay.
156.		RB	They took the combinations, they organized them from the top block.
157.		JL	MM hm
158.		RB	And then put in which the top color was, okay?
159.		JL	Mm hm
160.		RB	So that's how they categorized it at first. Then they put as many combinations together making sure there were no duplicates. And they put them in patterns. And what they named. But the pattern was a little different than what we did. We went red here. Then red here. We did the two reds.
161.		JL	Okay
162.		RB	They organized it different by going the red, the blue, the yellow.
163.		JL	Okay
164.	9:43	RB	And but when they rearranged it, that's where they came into the problem. They put it back to record their data.
165.		JL	When they put it, in other words, what they recorded was not how their towers looked?
166.		RB	No, no. This is how their towers looked.
167.		JL	Okay
168.		RB	But when they originally did it, they flipped a couple. As they did the recording.
169.		JL	Okay, okay.
170.		RB	And it's pretty much consistent with all three colors. And they did a nice job with this one.
171.		JL	Okay.
172.		RB	They did it starting at the top. And then this group
173.		JL	Wait, wait. Go back because that's an interesting paper. I don't want to leave it. And Sally, you were seeing stuff. And I think I saw what you saw as well. Say again what you saw.

174.		S	Well, I thought that where it starts with red red yellow,
175.		RB	Mm hm
176.		S	And then red red blue,
177.		JL	Right
178.		S	I thought that they were controlling the top and the middle, and that it was flipping the bottom.
179.		JL	Uh huh
180.		S	But then I guess since they already did red red red, they didn't need to record that third one.
181.		JL	Correct.
182.		S	And then when it goes to red blue red, and then red blue blue, that was the same idea.
183.		JL	Right
184.		S	Except I don't see
185.		KK	Red blue yellow
186.		S	Red blue yellow was already done. Towards the left, right?
187.		KK	Oh, right.
188.		A	Okay
189.		S	So they had to
190.		JL	Wait, yes.
191.		S	The second one over there.
192.		JL	Right, right.
193.		S	So
194.		JL	But what you're saying is you're seeing a double control for variables,
195.		S	Uh huh
196.		JL	On the first and second positions.
197.		S	Yeah, yeah.
198.		JL	And then taking the third possibility and putting it in both spots.
199.		S	Yeah
200.		JL	And then you've exhausted with a double red top,
201.		S	Uh huh
202.		JL	Using all three colors.

203.		KK	Yeah
204.		JL	What could happen. Especially since the red yellow red is here.
205.		S	Yeah.
206.		JL	Okay. What else did people see?
207.		M	I think it's interesting how they have, like a lot of kids,
208.		JL	Uh huh
209.		M	Will right, they have to do like a red red red,
210.		JL	Uh huh, uh huh
211.		M	And then they say well let's do one with all three colors.
212.		JL	Yep
213.		M	Like,
214.		JL	Yep
215.		M	Some of them even recognize that you couldn't put back into that pattern over there, but if it, like, looks different than the other ones. They will kind of put it separate.
216.		JL	Are you talking about the ones that are one of each.
217.		M	Well, yeah, like Sally said,
218.		JL	Uh huh
219.		M	Red yellow blue
220.		S	Yes
221.		JL	Yep
222.		M	You could put that in with, you know, the red yellow. You know, you could put that at the end.
223.		JL	Yes you could.
224.		M	Because you're doing red yellow and red yellow.
225.		JL	Over here. They could have had, that's right, Mitch.
226.		M	But because that one has all three different colors.
227.		JL	That's right they didn't see it,
228.		M	They kind of
229.		JL	I think you're right.
230.		M	You know.
231.		JL	Okay.
232.		M	And I see a lot of kids doing, if they see one that either looks

			very different,
233.		JL	Right
234.		M	Or if they say "Well, let's do one with all different colors."
235.		JL	Yep, that's this one here.
236.		KK	Mine do the same thing,
237.		JL	Yeah
238.		KK	They put three of those different color ones on a
239.		JL	Separate, separate
240.		KK	Yeah. In a league by themselves.
241.		JL	And usually, they come to be the last ones they find, right?
242.		KK	Yes, uh huh
243.		JL	Now, not always because, Mitch was it your class? Today where all the… Or no, It was, it was um, Toms River. When I was in Linda Kofak's class. They looked for the one of each color first. Which I find quite interesting.
244.		KK	Yeah.
245.		S	I think they like that because it's colorful.
246.		JL	Maybe, maybe. Okay.
247.		RB	And one of the reasons why I picked this specific problem was these two girls,
248.		JL	Uh huh
249.		RB	They were the ones that with the first problem,
250.		JL	Yep
251.		RB	Had, had no clue.
252.		JL	Okay
253.		RB	And with the second, they had an idea of
254.		JL	Welcome
255.		RB	Random, and they kept saying "Random"
256.		S	Yes
257.		RB	And "Random" and I really liked the way that they were more, that they were able to communicate it better, in this problem.
258.		JL	Systematically. Uh huh
259.		RB	Very systematic
260.		JL	Nice

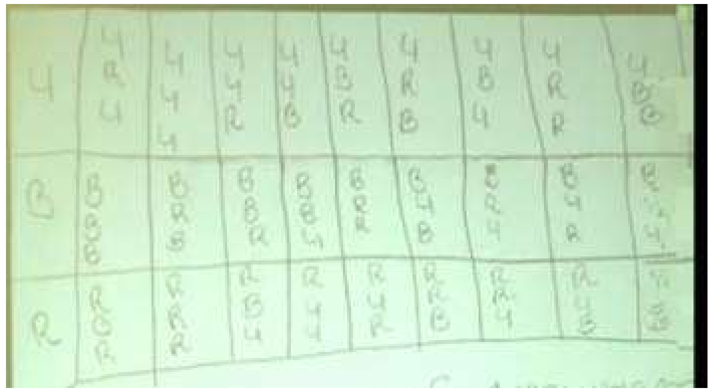
261.		RB	And you could see their process. And they did flip one or two around.
262.		JL	Okay
263.		RB	And I happened to have noticed that.
264.		JL	It is
265.		RB	But they did an excellent job on this.
266.		JL	Oh, I would definitely say,
267.		RB	Big improvement from, from the first one that they did,
268.		JL	Right
269.		RB	To this one.
270.		JL	Right, and who was this grade, what?
271.		RB	Sixth.
272.		JL	Sixth grade, amazing.
273.		RB	Mm hm
274.		JL	And what level, is it leveled?
275.		RB	No.
276.		JL	No. Okay, Sayreville's
277.	13:22	RB	No, each
278.		JL	All mushed sixth grade. Just sixth grade. Okay.
279.		RB	These are two students who are probably, I would say high average.
280.		JL	Okay, good ,good. And you did this over one day or two?
281.		RB	Um
282.		JL	Was that the short class
283.		RB	We did this, um, over two days.
284.		JL	Okay
285.		RB	But they finished. Most of the students finished this the first day,
286.		JL	Good.
287.		RB	And then they recorded.
288.		JL	Good.
289.		RB	And then the second day was to just to finalize. And I had one student even finish the Ankur problem even within the forty minutes. With this problem as well.

290.	13:50	JL	Wow' that's great.
291.		RB	So
292.		JL	That's great.
293.		RB	I have their work, but.
294.		JL	Justin had a comment.
295.		RB	Yes.
296.		J	Rob, did they, like that red section. They first started off just doing red themselves, like?
297.		RB	No, at, this took them a while. This was a work in progress. At first they didn't really know where to go. They really didn't. But then they said "Wait, let's go with all the red on top." And that's what they did. And then they said "Let's do a different color in the middle, and then on the bottom." And then as they were talking to me as well, they were saying it won't duplicate here. It can't. There's no way we can get another red unless we go one tower higher.
298.		S	Yeah
299.		JL	Ah
300.		RB	That's what they verbalized.
301.		JL	Neat
302.		RB	To me. But I really enjoyed, uh
303.		JL	Neat, very neat. This is wonderful.
304.		S	What I think is neat about this too, is like after
305.		JL	Yep.
306.		S	After they did like say all the red tops,
307.		JL	Yes
308.		S	They didn't just switch out the top red,
309.		RB	No.
310.		S	And put in top yellow.
311.		JL	Yep
312.		S	They kind of like repeated their thinking
313.		JL	Uh huh
314.		S	But with like
315.		RB	That's why I called it kind of staircase method as well. Because they didn't go the red with the red, and then the red with the red with the red.

316.		S	Right
317.	15:01	RB	The red with the red with the bottom, then the yellow, then the blue. They didn't do it, and you saw it laid out on paper, it doesn't really do it justice.
318.		JL	When you see the,
319.		RB	You see the colors.
320.		JL	Okay.
321.		RB	And it's set up this way.
322.		JL	Uh huh
323.		RB	You can definitely see the pattern that they were looking for.
324.		JL	Very nice, very, very nice work. Justin.
325.		J	So, like, they first started off, kind of like trading towers, and then they grouped them into reds on tops?
326.		RB	They were looking for opposites. Because that seemed to have been the big thing with, with my class.
327.		JL	Uh huh
328.		RB	They wanted to look for opposites, because the first two groups that presented,
329.		JL	Okay
330.		RB	That class. They said that they solved it by using opposites.
331.		JL	Okay
332.		RB	And then another students said they grouped it by the amount of towers they were going to get per color. Which we're going to see, they were the ones who did Ankur's
333.		JL	Oh, Ankur
334.		RB	That's the one that's consistent with throughout.
335.		JL	Right,
336.		RB	But they saw that the opposites weren't really working for this
337.		JL	Yep
338.		RB	Because three towers three colors high. And now you're dealing with a variable with three
339.		JL	Yep
340.		RB	Instead of just two.
341.		JL	Okay.
342.		RB	So once they saw that wasn't working, I guess their instinct kicked in. And they decided then to control for red. And then

			control for yellow, and control for blue.
343.		JL	Very neat. Very, very neat. Before we go on, Welcome, Linda.
344.	16:06	A	Hi
345.		JL	Do you know Mary?
346.		A	No.
347.		JL	Mary Schwartz was a student of Carolyn's long, long ago in the eighties.
348.		A	Oh
349.		JL	And I don't think I know you. You are?
350.			
351.		A	This is Sunika Batook
352.		SB	Batuk
353.		A	And she's actually a mathematician, who's working with us on our research
354.		JL	Excellent
355.		A	Side,
356.		JL	Good
357.		A	And she wanted to see what you folks were doing.
358.		JL	Okay
359.		A	And I wanted to see what was going on
360.		JL	Yeah.
361.		A	Because I've heard nice things about it.
362.		JL	This is, this is phenomenal
363.		A	Like the other day
364.		JL	And just began, Okay?
365.		A	Thanks for letting us come and work.
366.		JL	Well, we are happy that you are here.
367.		KK	Sure
368.		JL	And what, Just so that you know what we are doing, this, the problem that the students were working on, were building towers three tall, selecting from three colors. Okay, so this is the first solution we talked about. Okay, any more for us?
369.	17:04	RB	Um, I have another one that's similar.
370.		JL	Okay,

371.		RB	Um, which I was pretty impressed with as well.
372.		JL	Okay.
373.		RB	They used almost the same strategy. And this was a group of three students.
374.		JL	Okay.
375.		RB	Again, this was another group who, with the last problem was, they kept saying "Random" I kept saying "Random is you put all the blocks on and you just keep making towers." And that wasn't what they were doing, and they were able to actually come up with this solution. I was very proud of the three girls. Because they're not, I'm not gonna. They are not strong students and they were having a difficult time with the other two. But they found this one much simpler than the, uh, the four towers.
376.		JL	And, you know what? Do you remember what, um the teachers in my school said of Brandon? Do you remember Brandon was in the lowest math group?
377.		RB	Yes.
378.		KK	Right, okay
379.	18:08	JL	So I think we have to rethink what is our definition of a strong student. Because some times what students show you with just a pencil paper task isn't really what they can do. And sometimes what they show you when you give them a short amount of time to do something, is not what they can do.
380.		RB	But they struggled with the other two problems.
381.		JL	Okay,
382.		RB	In the pizza problem too.
383.		JL	Uh huh, good.
384.		RB	They struggled too. So obviously, they did learn something from it because they were able to complete the task.
385.		JL	Excellent.
386.		RB	Now, we don't group, but they would be at the lower end of the spectrum
387.		JL	Okay
388.		RB	Whereas the other two
389.		JL	Okay
390.		RB	were above average, but they're not honor students by any means. And then these are not lower level students.

391.		JL	Okay
392.		RB	But they do struggle sometimes.
393.		JL	Okay
394.		RB	And they were able to solve the problem, and I was pretty impressed with their writing as well.
395.			
396.		JL	Can you read us what they wrote?
397.		RB	Mm hm. We believe that we found all possible combos. We have found twenty-seven combinations. There might be more than twenty-seven, but we believe
398.			Ha ha
399.		RB	That there's twenty seven.
400.			Ha ha
401.		RB	First we got one color. I got red. Allie got yellow, and Aliyah got blue. We started with easier combinations, like all blue, reds, and yellows. Then we add all the colors like red blue yellow, then we reversed it, red yellow blue. Then to record the blocks, we each got one color and we passed them to our,
402.			Ha ha
403.		JL	Let's see if we can get this.
404.	19:30	RB	To our right. When we were done with them. This is the order we used to find the possible combinations with the blocks. They were very thorough with their explanation and I appreciated that. Because I found that my students really struggled in the beginning with communicating this and I think that was a frustration I put up on, on the threads, each week. That they were having difficulty communicating. But these three girls obviously didn't, and I told them as I was asking questions, "Remember that when you write your explanation because that's really good."
405.		JL	Right

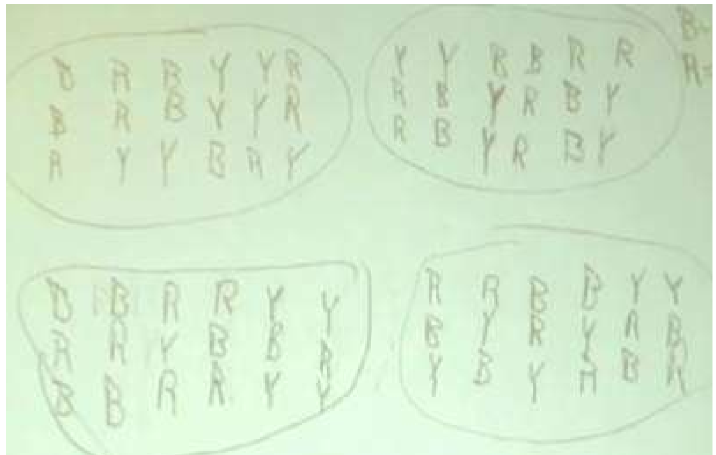
406.		RB	And they did. And I was, I was
407.		JL	So they're getting better at writing. And what you really want to push students to do is not just write about, like say, the order that they did things, You want them to say "How do you know that those are all towers there are that have a yellow top? How can you convince me?" And so they're going to get better and better at that convincing part. But you see that they're already getting better at being more systematic in the solving.
408.		RB	Mm hm
409.		JL	And I think that the more problems you do, they actually do build upon what came earlier. And it doesn't have to be earlier, the day before. It could be earlier in the semester.
410.	20:29	RB	And they second guessed the twenty-seven because I had a student with like thirty-four.
411.		JL	Right
412.		RB	And I wasn't giving them the solutions, I definitely was not.
413.		JL	Oh, so you're getting better at holding back, huh?
414.		RB	And I didn't point out duplicates to the thirty-fours either.
415.		JL	Okay, good.
416.		RB	So I wasn't doing that.
417.		JL	Okay
418.		RB	Okay, and those were two that I found interesting.
419.		JL	Good. Thank you, thank you. Who's next?
420.	20:48	A	I'll go.
421.		S	Ha ha
422.		JL	That was good.
423.		JL	Judy Landis, Nice to meet you
424.		SB	Nice to meet you.
425.	21:05	JL	Okay
426.		RB	I'm going to do it with one class.
427.		JL	You're okay
428.		A	I chose
429.		KK	This was sixth grade?
430.		A	No, this was seventh grade.
431.		JL	Okay.

432.		A	Um, I only did this with actually the level ones, the highest class. I only did all of these with my level two class. There's one of them. So this is interesting for me.
433.		JL	Oh
434.		A	This was a group of students who last time was able to answer the pizza problem correctly. Almost as if this could predict the answer, but when they wrote down the pizzas there were duplicates.
435.		JL	Oh
436.		A	Like, I'm like "You know how many there were."
437.		JL	Oh, okay, yeah
438.		A	And that's one of the ones I'm trying to
439.		JL	Okay
440.		A	With them I went over it
441.		JL	Okay
442.		A	"Did anyone see anything wrong here? We'll agree that the answer is right"
443.		JL	Mm
444.		A	But they're like "That's a duplicate, and that's a duplicate, and that's a duplicate."
445.		JL	Uh huh
446.		A	So this time, when we did this problem, as they started I just went around and I said "I don't want an explanation. How many do you predict? How many do you predict?"
447.		JL	Interesting. What did they say?
448.	22:00	A	Um, I'm going to get to that. That was the next one.
449.		JL	Okay.
450.		A	Most of them were like, "uh thirty"
451.		JL	So, just before you go talking about this, I want you to look at the student work because this is done differently. Different organization than Rich's students'
452.		S	Oh, that's how this girl,
453.		KK	Me too,
454.		S	That I was going to show did it.
455.		KK	Yeah, this one too.
456.		JL	Okay.

457.		A	And I'm not sure if her explanation is one hundred percent,
458.		JL	Well
459.		JL	Take a look first, everybody. How did they organize? Well, they actually are telling you so you don't have to figure it out
460.		A	Yeah, that's nice
461.		JL	That's pretty nice, isn't it?
462.			Mm hm
463.		JL	Huh?
464.			Mm hm
465.		JL	So who can say? What did they tell you? How did they organize?
466.	22:38	S	In groups according to the color.
467.		JL	Exactly. Okay, and they have a red and blue group, a red and yellow group, a blue and yellow group and an all three color group. Different organization. Really neat too.
468.		A	What I found interesting about them is when you look at these groups with the two colors,
469.		JL	Yep
470.		A	There's really no consistency, like for example, this one starts blue blue red.
471.		JL	Right
472.		A	You would think maybe this would start like red red yellow. Because they start
473.		JL	Uh huh, uh huh
474.		A	But there's really no, like strict consistency.
475.		JL	Okay
476.		A	And sometimes it looks like they started making opposites, for example when you look at seventeen and eighteen.
477.		JL	Right
478.		A	Like, or maybe flip them
479.		JL	Right
480.		A	But like,
481.		JL	Right
482.		A	I really don't see like a complete consistency here.
483.		JL	Going group to group.

484.		A	Yes
485.		JL	Okay, and were they working as a pair? And they found all those groups together?
486.		A	Yes
487.		JL	Okay
488.		A	This was actually a very quiet group, but they worked really well together.
489.		JL	It looks like they did great work. And again, what, ah, what did they write? What was their, read to us.
490.		A	So they say "There are a total of twenty-seven possible combinations that are all different. Every time when we would put another 'new' combination, it will result like the previous towers we've created. To help our answer, we also put them into groups of five." Which I guess should be six. So I guess what she's saying is that there's five groups.
491.		JL	Okay.
492.		A	"Every group has the six combos"
493.		JL	Alright, yeah
494.		A	"Except for one which had three singles. We also circled the groups to represent our answer, and to show it was a different group of combinations. Also, when we created our combinations we did the opposite of the towers we had came <sic> up with, so there won't be any left. Overall this was surely our prediction that there were twenty-seven possible."
495.			Ha ha
496.		A	So, you could tell that I asked them to predict.
497.		JL	And that's okay. Now, the next thing that you would want to push them to, is what? What would be the next question you'd want to say?
498.		A	I still, like, I struggle with my class. Like "I'm not convinced."
499.		JL	Good, good. And you might want to say "Can you convince me of the red and yellow group? That you have all possible red and yellow towers."
500.		A	Like Rich said,
501.		JL	Yep
502.		A	About saying "Random"
503.		JL	Yep

504.		A	I feel like that was such a struggle.
505.		JL	yeah
506.		A	For me
507.		JL	Okay
508.		A	And it's very difficult.
509.		JL	Uh huh
510.		A	I came down from my class, and I was like "I feel like you'll believe anything at this point." I said "The sky outside is pink." They're like "Okay"
511.		JL	Ha ha
512.		A	And I'm like "you need to, like, convince people of stuff."
513.		JL	Uh huh, uh huh.
514.		A	And I don't know
515.		JL	They're going to get better at it. This is, isn't this a lot better than were they were in like, Right? So like I'm saying, because I could remember you telling me two tasks ago, that they weren't writing. And they weren't really coming up with it. A, you know, a systematic approach and all this kind of stuff. So I think they have come light years,
516.		A	I agree, I agree.
517.		JL	Now, when you're looking at, and look how neat they have a key. They have, in case you don't know that blue is b
518.		S	Ha ha
519.		KK	That's so
520.		JL	Because, you know, you are the teacher, and teacher's "Duh" you know?
521.		S	Ha ha
522.		JL	So isn't that nice, that they're using notation, and they're, they're telling you what the notation means. Um, and they have the groups by color, and it's neat. Very nice. What else do you have?
523.		A	Okay, I just wanted to show you this one more.
524.		JL	Okay
525.		A	This was a group of three boys, one who actually wasn't with us. He's out of district, he missed the first two tasks,
526.		JL	Okay

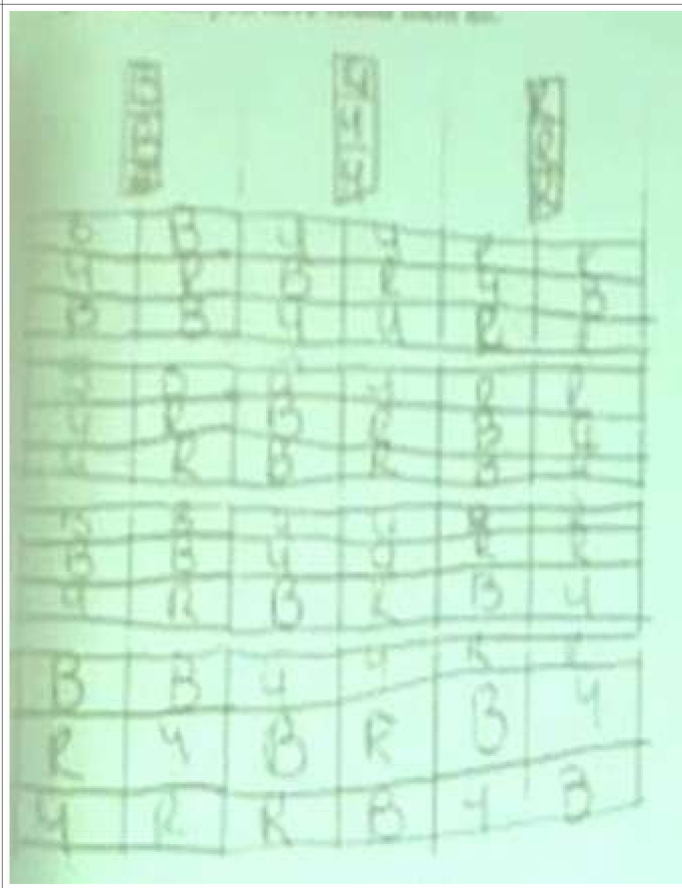
527.			
528.		A	So jumping into it, he was like "What the heck is going on?"
529.		JL	Uh huh
530.	25:46	A	This group, when asked to predict, this gentleman was so strong it's twenty-seven. He's pretty bright,
531.		JL	And why did he think that?
532.		A	And I said to him "Why?"
533.		JL	Yeah
534.		A	"I don't know, I think there's twenty-seven." And I, he wouldn't go further than that.
535.		JL	Okay, okay
536.		A	They, the boy who joined us from out of district was like "I think there's more than twenty-seven."
537.		JL	Uh huh
538.		A	And he was like very, very adamant about it.
539.		JL	Uh huh. Okay.
540.		A	So they were going back and forth.
541.		JL	Okay
542.	26:07	A	He was adamant that there was more than twenty seven, because they started building these groups of six first. He's like "How could there be twenty-seven? We have groups of six."
543.		JL	Ah
544.		A	It is going to be an even amount.
545.		JL	Okay
546.		A	And, Brendan, whose paper this is.
547.		JL	Yes.

548.		A	He was like "No, it's got to be twenty-seven. We have this group of three."
549.		JL	Okay
550.		A	There's not going to be six in that group.
551.		JL	Okay interesting.
552.		A	So ,this paper's really interesting if you read the, I was saying to Kulsom before. If you read the, explanation, I don't know if I understand what he's talking about.
553.		JL	Let's see if we can figure it out as a group.
554.		A	Okay, he was the one who was adamant there was twenty-seven. He was very sure about himself.
555.		JL	Sure
556.		A	He found them.
557.		JL	Okay
558.		A	And he convinced the other one by the second day.
559.		JL	Okay
560.		A	Okay. "I know I found all the possible combinations because I made four different groups of six and one different group of three. I made one group of different tops, one group of different middles, one group of different bottoms, and one group of all three colors." That's the part that I don't
561.		JL	Okay, well, look the all three colors, I think we get.
562.		A	Well, yeah
563.		JL	But what does he mean by "One group of different tops, one group of different middles, one group of different bottoms?"
564.		RB	I um, I think they just grouped them differently. They did it the same way, like that my student did it.
565.		JL	Yeah
566.		RB	But they just grouped them differently. Instead of controlling, uh, them to be all reds. They controlled it for different ones on the top. Um, they just happened to group them differently. So they're not all the, all nine of the reds together. They're not all nine of the yellows together
567.		JL	No, they're not.
568.		A	No, but can you tell me, like
569.		JL	What he means
570.		KK	But he's saying he did it that way.

571.		RB	Yeah
572.		A	That's where I'm like, I don't know.
573.		KK	He's saying he did that
574.		A	I don't understand.
575.		KK	But its not, that's not how it's shown.
576.		JL	"One group with different tops, one group with different middles, one group with different bottoms." Where's the, where
577.		A	When, when I see
578.		JL	Yeah, sure,
579.		SB	May I say something for a minute.
580.		JL	Absolutely
581.		SB	In the first group, you notice that the only color that's different is the bottom one. So he's got blue blue red.
582.		A	Yes
583.		KK	Right, right, right
584.		A	I did notice that too. He's got blue blue yellow, yellow yellow blue..
585.		KK	Oh! Maybe he's saying he changed the bottom
586.		SB	So he's saying the only different color in that tower is the bottom
587.		KK	Is on the bottom.
588.		JL	And that could very well be.
589.		KK	I was looking for where it was the same.
590.		SB	And in the next group, the only different group is the top one, because
591.		KK	I get it
592.		SB	Red on the bottom to match one another
593.		KK	Right.
594.		A	Yeah
595.		SB	So that's a really cool way of organizing it.
596.		JL	Keep going, keep going.
597.		SB	So that's what I see.
598.		KK	What's the third?
599.		S	And on that one he kept the middle

600.		SB	The middles are different.
601.		S	Outside.
602.		A2	But the top and bottom are the same.
603.		JL	Oh, Okay.
604.		KK	You're right, you're right.
605.		A	Blue blue, red red, yellow yellow, yellow yellow.
606.		KK	And the same thing there in the middle. The top and the bottom are the same? No , no no. Yeah, I don't know.
607.		M	Or maybe
608.		A2	Or that third group the top and bottom are the same.
609.		JL	What about those two
610.		KK	What about the fourth group?
611.		A2	Unless it's the bottom
612.		S	The fourth group is all three colors.
613.		JL	It's all of them
614.		KK	Oh all three colors.
615.		JL	Yeah
616.		KK	Okay
617.		JL	Isn't that neat. Thank you
618.		KK	Because all
619.		JL	That is neat.
620.		KK	Everything he did with red top, blue and yellow, yellow blue.
621.		JL	Uh huh
622.		KK	Blue blue tops,
623.		JL	Yeah
624.		KK	Red and yellow, yellow and red
625.		JL	So now
626.			Oh
627.		KK	Yellow tops
628.		S	But this is the kid that was sure of twenty seven before he even made them?
629.		RB	No. Is there a duplicate?
630.	28:53	JL	Yes, there is, but and I don't think it's a duplicate. I just think that it in writing,

631.		M	Writing
632.		JL	He did it. Did everyone see this?
633.		A	Yeah.
634.		KK	I know we asked
635.		JL	Okay, alright. So if the middles were going to be different, one of them should have been what?
636.			Yellow
637.		M	Right
638.		A	And that's why you see it on the,
639.		JL	Okay
640.		A	Person trying to prove him wrong
641.			They were missing the typo
642.		KK	Yeah, yeah
643.		JL	And I really think, you know when students record, they could have the tower right in front of them, but
644.		KK	Transferring
645.		JL	To get it down on. And so that could be a very careless error. And I, I really believe they really knew since these are all,
646.		RB	Mm hm
647.		JL	That they knew that also was a yellow in the middle of one of them. Uh, that's neat. Isn't it?
648.		S	Yeah
649.		KK	Yeah
650.		JL	Different organization, but really, really neat.
651.		A	I didn't see that. That's funny.
652.		KK	I was looking for all the same except the one piece
653.		JL	Yeah. That's the
654.		RB	That's completely different.
655.		JL	Great. We're going to leave it up so everyone has time to
656.		A	Okay, got it.
657.		JL	Okay, who's next?
658.		S	Alright.
659.		JL	Neat stuff
660.	29:46	S	I'll do this one.

661.		JL	Don't kids surprise you, what they can do?
662.		S	This wasn't, um, I also wanted to connect it to the extension that she did.
663.		JL	Okay
664.		S	Because I think that she did, the way she did the regular.
665.		JL	Absolutely, absolutely.
666.		S	Really helped her.
667.		JL	And Mitch's going to do that too.
668.		S	So,
669.		JL	Okay. Give it time to warm up. And you will do that. Do you want to come here so you can point to whatever you want to point to? Let's see if I can get it to just do. Sometimes it just takes time. But their writing is teeny, teeny tiny, on this one.
670.		S	Ha ha
671.		JL	So, you may have to help us. Alright.
672.			
673.	30:27	S	Okay, Alright so, this is BBB.
674.		JL	Right

675.		S	This is YYY. This is RRR.
676.		JL	Okay
677.		S	Um, and then it's BYB, BRB.
678.		JL	Okay so there are your towers.
679.		S	Okay
680.		JL	Right
681.		S	And then BYY and then it's BRR. And then it's BBY, BBR
682.		JL	Okay
683.		S	BRY, BYR.
684.		JL	Okay.
685.		S	And
686.		JL	Ah, Okay.
687.		S	So this is like one section, I think.
688.		JL	And did they
689.			Organized
690.		JL	And when they did it, did they work vertically, I mean, is that, on the desk?
691.		S	I didn't, um, I didn't watch her,
692.		JL	Okay
693.		S	Do the diagram, um, but I let them use rulers when they were doing their diagram. So,
694.		JL	Okay, and that's fine. Take a minute to look at this, though. This is a neat organization too. And see if you can figure out what these students did.
695.		K	It seems like they just had the, each color by themselves first, on the top, sorry, my voice is,
696.		JL	That's okay.
697.		K	And then they held the top constant,
698.		JL	Yep
699.		K	Just letting the second and third finish it.
700.		JL	Point out what she's showing, because that is. That does look like what they did.
701.		S	Um, So like here, the top and the bottom are both blue.
702.		K	Yep
703.		S	So that was constant and she switched the middle.

704.		JL	Uh huh
705.		S	Here, um
706.		KK	The top is constant.
707.		S	She has only the top,
708.		JL	Right
709.		S	With, I guess a double yellow and a double red.
710.		JL	Uh huh
711.		S	And then this one she kept both of the top ones blue,
712.		JL	Right
713.		S	And she switched the bottom.
714.		JL	Yep
715.		S	This one she kept, um, the
716.		JL	Top
717.		S	Top and then red yellow, yellow red.
718.		JL	Yeah, neat.
719.		S	And then she carried that idea through for yellow, and for red.
720.		JL	Isn't that neat, huh? Very nice organization. Justin?
721.		J	Like some of that's really interesting, and I found with my group was that none of the kids did it like I'd do.
722.		JL	Uh huh, right.
723.			Ha ha
724.		JL	Okay, okay.
725.		J	And I'm baffled by it. And in these things where they don't think like me.
726.		JL	Yeah
727.		J	Like, for instance,
728.		JL	Yeah
729.		J	Like, the blue blue blue,
730.		JL	Yes.
731.		J	Like, my next step would have been, you know, blue blue, you know, yellow or red.
732.		JL	Okay, okay.
733.		J	Like, the two, those two.
734.		JL	Okay

735.		J	But none of my students
736.		JL	Okay
737.		J	And none of the, what I've just seen up here, is in that order.
738.	3:00	JL	Would have had it your way. And you want to know the best fun you're going to have as teacher, is when they don't think like you, and when you try and get into their mind and understand how they're coming about what they are doing. I don't know if I told you, when I taught third and fourth grade, enrichment math, it was the best teaching in my life. Because those students, they were whippersnappers. I had to work hard to understand what they were talking about. And sometimes, the solutions they came up with were so much more elegant and beautiful than what I was thinking. So the best teaching you will have, is you, if you can really understand the way that they're thinking about something. That's neat. Okay, very nice. I'd like a copy of that one too.
739.		S	Okay
740.		JL	Good.
741.	33:33	S	Can I do the extension?
742.		JL	Yes you can, absolutely.
743.		S	She has the same kind of format here.
744.		JL	Oh, my gosh, look at that chart. Okay.
745.		S	Ha ha
746.		JL	Now this, for our guests. What this problem is, is Ankur's Challenge. Which was, a tenth grader at Kenilworth, where Carolyn's study was, the longitudinal study. And this tenth grader, as he was working on the problem we just talked about, said "I have an idea. I have another problem we can work on." And he came up with this challenge. Which was, now build the towers four tall, selecting from three colors. So, also, each tower has to have at least one of each color.
747.			Mm
748.		JL	Different problem. A little bit harder than the one that they had been working on. And talk to us now.
749.	34:22	S	Okay. So what I think is interesting, is you can see at the bottom, that she just kind of took her ruler and did all her lines.
750.		JL	Yep
751.		S	I guess, for planning, and she wasn't sure what she was going to come up with.

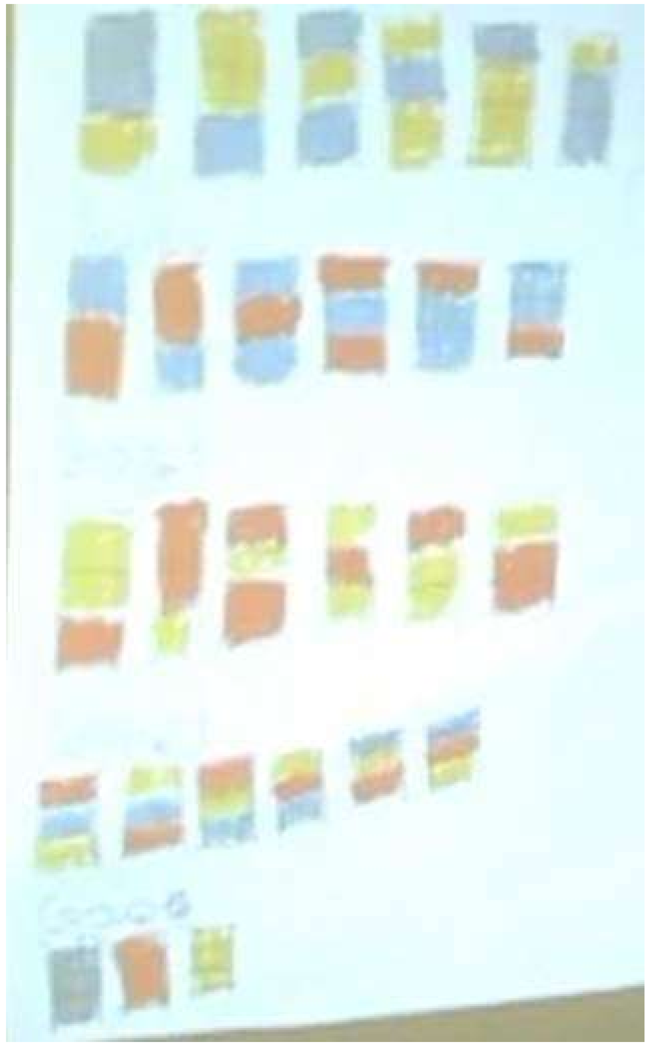
752.		JL	Okay
753.		S	But then, so this is like her blue column.
754.			Whoa.
755.		S	And it starts with blue blue, and then red yellow, and then yellow red.
756.		JL	Okay
757.		S	And then, blue red blue yellow. Blue red yellow blue. So she held the top two constant,
758.		JL	Uh huh
759.		S	Switched the bottom.
760.		JL	Uh huh
761.		S	And do far we have double blues.
762.		JL	Yep, yep
763.		S	Here, here, here. Again we have blue and a blue
764.		JL	Yep
765.		S	But she held the top at blue yellow.
766.		JL	Yep
767.		S	And then just switched the bottom two.
768.		JL	Uh huh.
769.		S	And then this one, she has blue yellow red yellow, blue red yellow red.
770.		JL	And why is that under the blue column.
771.		S	Um, I guess because the top is blue.
772.		JL	Okay
773.		S	Because over here she talks about the top.
774.		JL	Okay
775.		S	So, she also, I don't know. She did something with the yellow yellow, red red here.
776.		JL	Okay
777.		S	And then here again you have the blue tops.
778.		JL	Yep
779.		S	And it's yellow yellow blue. Oh this looks like
780.		JL	Yellows in the middle, yellow
781.		S	Double

782.		JL	Double yellow in the middle.
783.		S	Oh, yeah, this is red.
784.		KK	That's an "R" yeah.
785.		S	This one is blue.
786.		JL	Yeah. And those "R"s and "B"s are hard to tell apart.
787.		S	So then you have blues on top again, and red red blue, and red red yellow.
788.		JL	Okay
789.		S	Um, so I guess she thought that was it for the blue tops.
790.		JL	Okay
791.		S	And then she followed that same thought process through with yellow tops and red tops.
792.		JL	Very interesting way of organizing, isn't it? Justin?
793.	36:07	J	Like, did she have a system of how she got to this? Um, like did you, did you inquire about that? Or they just knew, or that was their system?
794.		S	I
795.		JL	Did they organize it differently first? Is what I think he is saying. And then come to this?
796.		J	Yeah
797.		S	Well, you know what? With this problem, a lot of them didn't use the blocks. Like they just, I just let them think about it on paper, and try and do it on paper. Some of them didn't have time for it. It was in the class. They did this when they were in the class.
798.		JL	Uh huh
799.		S	So, I think that what she did is what she took this and she kind of built it off that. And I have a feeling she might have gotten the notion of how, um, Romina did the double colors.
800.		JL	Uh huh
801.		S	And figured that out. Because I had another boy who, the way that it looked like with him,
802.		JL	Uh huh
803.		S	It looks like he figured out if he could find all the ways of putting a double color.
804.		JL	Okay, great. Very interesting organization. Yeah.
805.		SB	Looks like on the bottom left it doesn't have a yellow. Is it

			supposed to have all three colors?
806.		J	Yes.
807.		JL	The bottom left
808.		S	Oh! It should.
809.		JL	Is that the
810.		J	You're so perceptive
811.		SB	Sorry
812.		KK	Or it might be. There should be a yellow there.
813.		S	Oh yeah, another
814.		SB	I think she should have
815.			This was the
816.		RB	The red before that
817.		A	Red yellow red
818.		RB	No blue with the red.
819.		A	R and B, R and B?
820.		S	Blue red, maybe it's supposed to be blue red and then this is
821.		RB	Yellow?
822.		S	Yellow red
823.		RB	Right
824.		S	Because, maybe I think
825.		KK	It's red yellow yellow red, Right?
826.		JL	Yeah, that has to be
827.		S	A yellow one
828.		JL	There, you're right. There has to be at least one of each color in each tower. That was the, that was one of the requirements.
829.		S	Mm hm
830.		JL	So, you know, it's really interesting, when you're… It's still a neat strategy and what's nice is, they have groups where they have twelve in each group, right?
831.			Yeah
832.		JL	Twelve towers. Which sometimes students will come up with twelve in one group, eleven, remember I held the thing up?
833.			Yeah
834.		JL	Eleven in another group. And maybe twelve in the next group.

			And they'd go "Hmm, that's bothering me, because it really should be that they're all the same." So here they did get them all the same, but you're right. You do have to go back, and again we don't know whether it was a careless error in their recording,
835.		SB	Actually, it's repeated.
836.		J	It is all of them are
837.		KK	All of them
838.		S	It is
839.		SB	Bottom ones, they're all
840.		KK	Oh, so you're right, she doesn't have
841.		SB	Yellow red yellow.
842.		KK	Any of the third color
843.		JL	Okay
844.		J	And the one on top too, like on some of them don't fit the requirements.
845.		RB	Yeah, red yellow yellow red.
846.		JL	Okay
847.		S	Where?
848.		JL	So really, what we want to do. Now, even though sometimes students might
849.		RB	Oh, Justin. Over to the right there's two as well.
850.		JL	Yeah. Even though students sometimes
851.		JL	Like using just pencil and paper
852.		S	Uh huh
853.		JL	I really encourage you to let them
854.		KK	Build it
855.		JL	use the manipulative, because I really think they wouldn't make that mistake. It would shout out at them if they didn't have all three colors.
856.		S	Right
857.		JL	Right, and then they record it. It takes longer but I think it is powerful to do. Thank you, Justin.
858.		J	No problem.
859.		JL	Alright, so, but I, but that's a neat way of organizing. And again we don't really know for sure. Did they really make

			mistakes or was it careless, or what's the story here, but you're right. We don't really have a solution there that works. Anything else? Any other work that you brought to share, or no, Sally?
860.		S	I do, but it's it's pretty similar to hers.
861.		JL	Is it the extension also?
862.		S	Um,
863.		JL	Or is it the regular problem?
864.		S	Oh yeah, I have the extension here.
865.		JL	Well, whichever one you want. If it's just an extension, by itself, hold it.
866.		S	Okay
867.		JL	Do you have anything more of just the regular problem? Three towers three tall,
868.		S	Yeah
869.		JL	Okay
870.		S	I liked this one, um, but again, it was very similar to
871.		JL	Okay
872.		S	To other people's

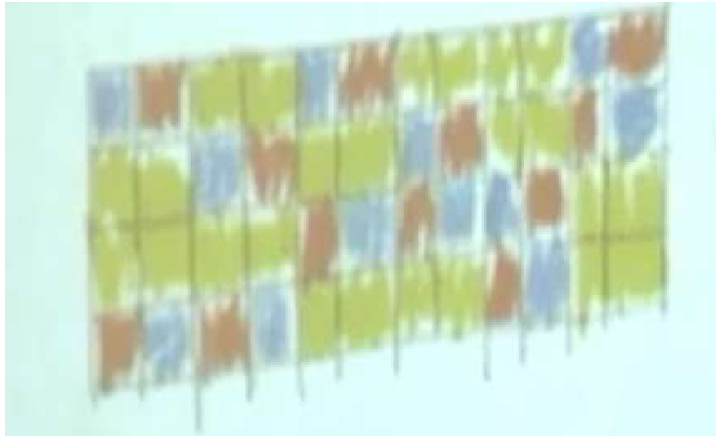
873.			
874.		JL	But look how their recording. That's different, huh. Okay, so they actually got some colored crayons and they showed you what the towers looked like. They have four groups of six, a group of three,
875.		KK	They did by two colors to. And somebody else did that, right?
876.	40:16	JL	Yes. We saw it, um, in another paper. And here they are focusing on color. And why are those groups, like um, the way they are?
877.		KK	It looks like they reversed, they did opposites.
878.		JL	It does look like they
879.		KK	Like they did their two yellows, then reds with two yellows.
880.		JL	Okay, mm hm
881.		KK	One blue
882.		JL	Okay yeah. Within the

883.		KK	Split blue with the yellow
884.		JL	Mm hm
885.		KK	In the middle. Split yellow, blue so they could.
886.		JL	Yep, it does look like within a group of blue yellow, they did opposite towers.
887.		KK	Mm hm
888.		JL	But really neat, huh? Now here again, we would want them telling us, how do we know that, Did they write them down?
889.		S	Yeah, on the back
890.		JL	Oh, good. Excellent. Oh how neat. Look how that, they draw lines. Oh my gosh these kids are amazing.
891.		S	Ha ha
892.		JL	Read to us what they wrote, Sally.
893.		S	"I made six groups to show that twenty-seven is the most I could get. Group one I used two colors, blue and yellow. The most I could make with blue and yellow going three units high is six. (That's the most possible)"
894.		KK	Ha ha
895.		S	"Then I used blue and red. The most I could make is six. I know that because if you're using two colors, three units high, three times two equals six. Also, if I try and make more it would be a duplicate of another one. This is what I did for groups one through four. Group number five I did all yellow, all blue, and all red. All together I have twenty-seven. This is reasonable because it is divisible by the number three."
896.			Hm
897.		JL	Isn't that interesting? Yeah.
898.		S	This is, this was my enriched class.
899.		JL	Okay,
900.		S	So they're constantly looking for
901.		JL	For the mathematics?
902.		S	Yes.
903.		JL	Try to connect the math and that's a good thing to do. And I wonder, why do they think it has to be divisible by three? Do we know? Because I heard that in Toms River,
904.		KK	Really
905.		JL	And in Long Branch.

906.		S	Well, you have two different threes, you have three units high, and you have three colors also.
907.		JL	Okay
908.		S	So,
909.		JL	So they may have some kind of connection with this three thing. I know that, um, some of the students in, I think it was Long Branch, said that, um, "We need an even number. It can't be twenty-seven."
910.			Mm hm
911.		JL	Why, because there's a tower and an opposite. It has to be an even number. And then when they found twenty-seven, they said "Ah! Divisible by three, so we're okay now." You know? But, you know, I think that they are starting to make connections. Did any of your students do three to the third? Did any of them?
912.		KK	My enriched class did, but I had done combinations with them.
913.		JL	Okay
914.		KK	As a lesson in my book.
915.		JL	Okay, okay. And could they make
916.		KK	And they made that connection
917.		JL	Could they connect what those threes were?
918.		KK	Yes, I actually have
919.		JL	Good.
920.		KK	An example of one.
921.		JL	That will be fun to see.
922.		KK	Yeah.
923.		JL	So, you know, eventually, you do want them to connect math to this concrete and pictorial work. But you know, you don't want them to, And sometimes they're going to come up with we saw equations earlier in the year that made no sense, right? But they got the right number, because they could plug on those numbers and force them to be the right number.
924.		KK	Mm hm
925.	43:07	JL	That's great. Thank you. Who else?
926.		KK	I'll go if you want me to show that
927.		JL	Sure,

928.		KK	That one.
929.		JL	Absolutely.
930.		KK	Um, this is, was the regular one. With my enriched class.
931.		JL	Turn it around, there you go. Okay.
932.		KK	And they controlled for two variables.
933.		JL	Okay, so give us a minute to look at it.
934.		KK	Sorry.
935.		JL	Alright, that's okay. So look at their groups. Figure out how they grouped, and then tell us, before we get Kate to talk to us.
936.	43:58	M	Two colors on the top, same color on the bottom.
937.		JL	Okay. Two colors on the top, you're meaning two stuck together?
938.		M	Yeah
939.		JL	Okay, two on the bottom?
940.		M	And then one on the top, one on the bottom.
941.		JL	Ah, they split them apart.
942.		KK	See where they split them.
943.		JL	Right? Remember Stephanie's, split apart?
944.		KK	Mm hm, yeah.
945.		JL	Isn't that interesting? Very nice controlling, um for variables there. Huh? And then when they have them split apart, the middle, they took the other
946.		KK	Two alternate
947.		JL	Colors and they put them in. Very neat.
948.		KK	So we found this to be very organized.
949.		JL	Yeah, very, very.
950.		KK	And they did the, um, what's the last one?
951.		A	Over there?
952.		RB	One of each.
953.		KK	Oh, one of each color.
954.		JL	Yeah, yeah.
955.		KK	One of each color. But they still controlled that,
956.			Yeah

957.		KK	Because with the same one is in the middle. And they did the opposite top and bottom.
958.		JL	Uh huh, uh huh.
959.			Mm hm
960.		KK	Same one is in the middle, they did the opposite top.
961.		JL	Aha, very nice.
962.		KK	And there, they did the same two in the middle.
963.		JL	Very nice.
964.		KK	Opposite top and bottom.
965.		JL	Did any of your students focus, let's say, on red? And keep the red both on the top, then both in the middle, then both in the bottom? Because I saw that in, uh, Long Branch and Toms River.
966.			For Ankur's Challenge?
967.		JL	I should say. For Ankur's Challenge? Okay, good. Neat, very neat. I'd like a copy of that one.
968.		KK	Sure, sure. And then this, their extension on the back was.
969.		JL	Okay, Oh!
970.		KK	Is beautiful. They did the exact same thing.
971.		JL	Okay, give us a minute to look at this one. Neat how they made their recording. Note, they didn't want to mess around with it. Really coloring the box, so they kind of scribbled the color. And it works. Uh, problem with this, and I caution you, when you do your final project, and you're going to include student work, don't Xerox this on a black and white machine,
972.		KK	Mm hm
973.		JL	Because then we won't see what the towers are. So I'm going to ask you to please use
974.		KK	Color copier
975.		JL	The original.
976.		KK	Oh, okay,
977.		JL	Well, if you have a color copier, that's probably, perfect. But otherwise use the original student work.
978.		KK	Uh huh
979.		JL	Okay, so who knows what they did?

980.			
981.		K	The ones that I had do the same thing.
982.		JL	Okay
983.		K	The first group is all, it looks like the student did one color duplicated in his tower.
984.		JL	Yep
985.		K	So that one's all two reds.
986.		JL	Good, good.
987.		K	In the next group two blues, and then two yellows in the third group.
988.		JL	Okay
989.		K	I thought it was pretty cool.
990.		KK	Yeah
991.		JL	Isn't that neat?
992.		KK	What's weird is that
993.		K	And we didn't
994.		KK	I know, I know
995.		JL	And isn't it neat,
996.		S	I know, that's what I was thinking about.
997.		JL	I'm impressed, how about you?
998.			Yeah
999.		KK	I thought this was brilliant.
1000.		JL	Very, very neat.
1001.		RB	Two reds, two yellows
1002.		KK	Yeah, yeah. And they did the same thing.
1003.		JL	Yeah

1004.		KK	Organized. Like two together, they alternated top and bottom.
1005.		JL	Yeah
1006.		KK	They split them alternated, uh, second and fourth alternated.
1007.		JL	Yeah, uh huh.
1008.		KK	They had a complete system. And they followed it down with.
1009.		S	Yeah
1010.		KK	Right down. Once they had the first.
1011.		JL	Really really neat.
1012.		KK	Which was really good.
1013.		JL	Now, what you really want to do is, when they do stuff like this, even though we know, when we look at this, what they did. You would ask them. You want to ask them, and you want them t say "Well, what did you do in this group that convinced you, you had everything?"
1014.	46:46	KK	And verbally,
1015.		JL	Yeah
1016.		KK	They did so good.
1017.		JL	Yes
1018.		KK	And I was so disappointed that that's what they wrote.
1019.		JL	You're going to, you going to keep pushing them
1020.		KK	Mm hm.
1021.		JL	Um, because they will get better and they really don't have to explain for each group.
1022.		KK	Right
1023.		JL	If they can explain for the top group.
1024.		KK	I think they'll be able to
1025.		JL	You can
1026.		KK	The partner
1027.		JL	Yeah
1028.		KK	But the partner did an explanation for each group.
1029.		JL	Yeah
1030.		KK	But it was the same exact thing.
1031.		JL	Okay
1032.		KK	So she didn't really expand

1033.		JL	Sure
1034.		KK	But they had told me when I taught them.
1035.		JL	Sure
1036.		KK	We knew there had to be two of each color, so
1037.		JL	Yeah
1038.		KK	So we put the two colors in all positions
1039.		JL	Yes
1040.		KK	And alternate with the blue and the yellow. Is what they told me.
1041.		JL	Yes.
1042.		KK	But it's not what they wrote.
1043.		JL	Wouldn't that be neat? Now sometimes you have to actually say "Wow! That's neat! Write it down."
1044.		KK	Right
1045.		JL	And before you leave them,
1046.		KK	Sure.
1047.		JL	You actually say, "Write it down." You know? You're not, yeah.
1048.		SB	I just had a question.
1049.		JL	Sure.
1050.		SB	Did they, it looks to me as I'm reading this that what they did to get from the top set to the middle set is exchange the roles of blue and yellow.
1051.		KK	Yes
1052.		JL	Right.
1053.		SB	So, did they say that?
1054.		KK	They did say that. Yes, exactly. And then they did with yellow. They did the same thing. The partner has it in more detail. But we did the same thing where the reds were with the blues and change the yellow and red. Or whatever.
1055.		JL	Isn't that neat.
1056.		KK	Yeah, so that one's neat.
1057.		JL	So the partner's writing is more than what this one was?
1058.		KK	Yeah.
1059.		JL	Yeah.

1060.		KK	I think so. I thought I had it
1061.		JL	Yeah
1062.		KK	Her picture was pretty.
1063.		JL	Okay. It is a pretty picture. Good. I want that too.
1064.		KK	Um, okay. It's
1065.		JL	And if you're going to give me copies of the, the work that's in color. If you could scan it, into your computer, and then just email me. That would be great. Because most schools don't have a color copier.
1066.	48:16	RB	No scanners either.
1067.		JL	You don't have scanners?
1068.		K	No.
1069.		JL	Okay.
1070.		A	We have a scanner.
1071.		RB	Do we?
1072.		JL	Oh,
1073.		JL	Okay
1074.		A	By the library.
1075.		RB	I'll have to find it.
1076.		JL	Keep going.
1077.		KK	This was the extension of,
1078.		JL	Yep
1079.		KK	Again, an enriched class.
1080.		JL	Okay
1081.		KK	And I actually wrote about this this week. I thought it was interesting. They did kind of what Sally and I did. We figured there was eighty-one,
1082.		JL	Oh
1083.		KK	If there were no conditions.
1084.		JL	Yep, yep. Good.
1085.		KK	And then kind of backtracked. And they made tick marks for every time they realized that they shouldn't duplicate something
1086.		JL	Oh, okay.
1087.		KK	So she says the total was

1088.		JL	How did they know the forty-five? Where did they get that the forty five had to be subtracted from the eighty-one?
1089.		KK	Well, they kept track of things that they said they couldn't use.
1090.		JL	Okay. So what are they showing you there? Just the thirty six?
1091.		KK	Those are the ones that can. I believe.
1092.		JL	That can, oh. Okay. Isn't that interesting?
1093.		J	So they, they made all eighty-one towers?
1094.		KK	No. I don't think so, I don't think they made all eighty-one.
1095.		M	Okay
1096.		KK	But, while he, he was making. She was recording, I think he was making ones that they didn't think they should have. To make sure that. So I guess in effect, they may have made all eighty-one. At some point.
1097.		RB	Hard to believe they didn't make duplicates of the duplicates.
1098.		JL	What do you mean?
1099.		RB	If, if, if they were just making the tick marks,
1100.		JL	Yeah
1101.		KK	Right.
1102.		RB	To
1103.		KK	And I'm not one hundred percent sure
1104.		RB	And, and how do they know that they didn't make like
1105.		KK	A duplicate somewhere.
1106.		RB	Like a yellow blue yellow red, and then make yellow blue yellow red again later on, like maybe?
1107.		KK	Well, they were making. They built. No wait, wait wait. Let me think about it, because. They
1108.		RB	Did they make little
1109.		KK	The tick marks were for what they couldn't have. So they, they were, they went and realized, you know, we can't have all four
1110.		RB	Okay
1111.		KK	Of the color. And you can't have three and ones.
1112.		RB	Okay
1113.		KK	And he was making those, and she was making. They, maybe. They must have made all eighty-one at some point.
1114.		RB	Okay

1115.		KK	They did
1116.		RB	I understand what you're saying
1117.		KK	Right
1118.		RB	So instead of just, uh, holding it to the three, they made four, four blue. And then they would make the three, three blues one yellow.
1119.		KK	And then go back again
1120.		RB	Three blues one red
1121.		KK	Count that. Yes.
1122.		RB	And, and so they have even more towers
1123.		KK	And they were taking those apart.
1124.		RB	That's a lot of work.
1125.	50:16	JL	That is a lot of work. Isn't it. I think, didn't some of you try that?
1126.		KK	We, well, yeah
1127.		JL	You guys did?
1128.		KK	We did it like a round
1129.		JL	Sally and Kate
1130.		S	Do that
1131.		JL	You were trying to start with what it would be three to the fourth, and then work backwards to what it would be if you had the restriction.
1132.		RB	And would it, were they convinced that the eighty one was the correct answer of possible combinations with, um
1133.		KK	Yes.
1134.		RB	With, okay.
1135.		KK	But again, this is my enriched class, so they know combinations.
1136.		RB	So with, instead of, so with
1137.		KK	You can see they did it up there
1138.		RB	Mm hm
1139.		KK	Three three three three choices for four
1140.		RB	So if it's four colors, no, but with three colors, four high, but your solution would be eighty-one.
1141.		KK	The eighty-one

1142.		RB	They were convinced with that.
1143.		KK	Yes
1144.		RB	So from there, they worked backwards
1145.		KK	Backwards
1146.		RB	And eliminated the ones that it couldn't be.
1147.		JL	Right, that's right.
1148.		RB	Okay, Okay
1149.		A	That's
1150.		JL	And last time when we were here, I think, uh, you were the only group, Sally and Kate,
1151.		KK	Worked backwards
1152.		JL	Were using that strategy. Right? Because no other group used that strategy.
1153.			Mm hm
1154.		S	Yeah
1155.		JL	You did?
1156.		RB	No
1157.			At first, though.
1158.		S	Well, we were, we didn't get the right answer though.
1159.		JL	No, no
1160.		S	I know, we were like
1161.		JL	We're going to go back
1162.		S	Yeah
1163.		JL	Okay, well it's good.
1164.		KK	That's what their strategy was.
1165.		JL	Neat. Very nice.
1166.		KK	And then I just want to quickly show this last one.
1167.		JL	Sure, Okay.
1168.		KK	This to me was Romina's explanation.
1169.		JL	Talk to us about it.
1170.		KK	Regular student,
1171.		JL	Yeah
1172.		KK	Went through a lot of different stuff.
1173.		JL	Okay

1174.		KK	Um, She
1175.		JL	Three plus three plus three plus nine
1176.		KK	I don't know exactly
1177.		JL	Could be somewhere
1178.		KK	Yeah, I don't know where that,
1179.		JL	You don't know what that is. Okay.
1180.		KK	And it looked like she was starting to try to build
1181.		JL	Okay
1182.		KK	And then at some point here, she had,
1183.		JL	Isn't it horrible when they cross things out?
1184.		KK	Yes. She had three three two one.
1185.		JL	Okay
1186.		S	Ha
1187.		KK	She had three three two one
1188.		JL	Okay
1189.		KK	And I said "So what's that?" But when she, It looks like she erased it here too
1190.			Yeah
1191.		KK	When I was there, she actually calculated three three two two. Three times three times two times two.
1192.		JL	And where did the three three two two come in?
1193.		KK	Well,
1194.		JL	Come from?
1195.		KK	She was saying that three
1196.		JL	She wrote something there.
1197.		KK	Yeah. You can see
1198.		JL	What did she write?
1199.		KK	"You can pick three in two spots, out for two spots?" Wait.
1200.		JL	But, but, is that a but
1201.		KK	Oh, but for two spots you can pick two. So three times three times two times two equals thirty-six.
1202.		JL	Now, why would she say that? Why did she say that?
1203.		KK	That's what she got for separate towers.
1204.		JL	Who can figure out what she means?

1205.		K	Because of the criteria. You can't have all three colors.
1206.		KK	Right, so
1207.		JL	Where are those unifix cubes? Can I have
1208.		KK	Okay, two of the spots
1209.		JL	Yep
1210.		KK	You can do all three,
1211.		JL	Yeah
1212.		KK	Once you use those, that one color
1213.		JL	Right
1214.		KK	Twice, you can only use
1215.			That's great
1216.		KK	Two colors here and two colors here
1217.		JL	Okay. So what I want you to, Angela, can you build for us a tower? Okay.
1218.		S	I think she means like you can put all three different colors in it.
1219.		KK	Yes.
1220.		JL	She does
1221.		S	In like five
1222.		JL	She does
1223.		KK	Exactly
1224.		M	Right
1225.		KK	And then you could, once you do that,
1226.		S	But you would leave the ones on the bottom left you can only make
1227.		KK	Two more
1228.		S	Two towers from that
1229.		KK	Yeah
1230.		JL	Not make all towers
1231.		SB	But if you do blue red, then you still have three choices left in the third spot.
1232.		JL	Are you asking about the tower?
1233.			Yeah, well, okay
1234.		KK	This doesn't show all the different combinations,

1235.		JL	Right, right
1236.		KK	But it shows that in two spots, you can use all three colors,
1237.		JL	That's right, that's right
1238.		KK	And then the other two spots, it leaves two colors to choose from.
1239.		JL	That's right, that's right.
1240.	53:24	KK	So I was amazed at that.
1241.		JL	Yeah, that is pretty neat. So in other words, she's saying "In my first spot, I can pick any color I want."
1242.		SB	Right.
1243.		JL	I could pick my red or my yellow or my blue,
1244.		KK	Any one of my three
1245.		JL	Right? In my second spot, I still could pick any color I want
1246.		KK	So
1247.		JL	Because I know that in a tower of four tall,
1248.		KK	I'm still going to have that second color.
1249.		JL	I'm going to have to have two of one color.
1250.		S	Uh huh
1251.		JL	So, so far, I'm good. Now I go to my third spot. Can I choose red, yellow or blue for it?
1252.		KK	No
1253.		S	No
1254.		KK	Because if one of them already is
1255.		JL	Okay, so two left
1256.		KK	Two left.
1257.		JL	Because if I had a yellow over here,
1258.		RB	Oh right.
1259.		JL	I would only have at most two colors in this tower.
1260.		S	That's like the license plate problem
1261.		JL	Yes it is. Yes
1262.		SB	The mathematical problem with that is, flip it over. So she has three choices for the bottom
1263.		JL	Mm hm, yep.
1264.		SB	Three choices for the second.

1265.		SB	She still has three choices for the third one then.
1266.		KK	No
1267.		JL	No, no
1268.		SB	Yes, because look
1269.		JL	Why
1270.		SB	She could have cho- no, flip it over.
1271.		JL	Flip it
1272.		SB	So, one blue
1273.		JL	Mm hm
1274.		KK	But it's
1275.		SB	Two red, the third could be red or yellow or blue.
1276.		KK	But she's not in that position.
1277.		JL	Yeah, yeah
1278.		KK	She's not considering positions
1279.		JL	No, no. She's saying, this
1280.		KK	I don't think she thought that far at all. I would almost say it was a fluke
1281.		JL	Yeah, yeah.
1282.		SB	Because when you turn,
1283.		KK	But it makes sense
1284.		JL	Well,
1285.		RB	Maybe it wasn't a fluke.
1286.		JL	Well I have to
1287.		KK	No, no, not that. But she does have to explain it.
1288.		RB	I believe it
1289.		KK	Three and three
1290.		RB	They knew what they were doing.
1291.		KK	Two spots, right? But for two spots, you can only pick two.
1292.		RB	I don't think that was a fluke. I think they really knew what they were doing mathematically.
1293.		KK	Right
1294.		RB	And actually, I'm going to take your word, powerful.
1295.			Mm hm
1296.		RB	That's that, that's really powerful what

1297.		JL	Okay
1298.		RB	That the students were able to come up with that
1299.		JL	Okay
1300.		RB	And to come up with like, I guess an equation
1301.		JL	Mm hm
1302.		RB	To solve this problem.
1303.	54:55	JL	Okay, now what I would say to you, is "We are guessing, all of us."
1304.		KK	Right, what she thought.
1305.		JL	What she thought
1306.		KK	Right
1307.		JL	And the only way we would know for sure, is if we went back to her and said "What does that mean? That you can pick three in two spots but then you can only pick two" uh, you know, from two possible colors in the other spots.
1308.		KK	Interesting
1309.		JL	What do you mean? You'd have to go to those, to that student and get her to explain.
1310.		KK	Right, right.
1311.		JL	But I think it's a good guess
1312.		KK	Right
1313.		RB	Hm
1314.		JL	That she is saying that once you use,
1315.		KK	But, but if you look the way she says it, it does seem like she understands it.
1316.		RB	Mm hm
1317.		JL	Maybe that's
1318.		KK	Three in two spots, which is true
1319.		JL	Yes, yes.
1320.		KK	But, for two spots you can only pick two.
1321.		JL	Right, Right. But it would be, if you have time, it would be interesting to ask her.
1322.		KK	Yeah, I definitely can.
1323.		JL	Because the only way we know what was in her head, is if we get to ask her.

1324.		KK	Right.
1325.		JL	Okay, Good.
1326.		A	If we could look at her blocks
1327.		KK	And her
1328.		A	You could ask "What do you mean?"
1329.		S	Ha ha
1330.		A	Because it's easier, like you know?
1331.		KK	Yeah Right, right. Her new partner,
1332.		JL	Yeah, yeah
1333.		KK	Was their first day in class from Sayreville middle school.
1334.		JL	Wow!
1335.		KK	Just moved here.
1336.		RB	Who was it?
1337.		KK	Numal?
1338.		RB	Who
1339.		KK	Numal?
1340.		RB	Oh,
1341.		JL	They don't know who Numal is.
1342.		KK	She
1343.		A	Was she advanced?
1344.		KK	No, no, this was regulars
1345.		JL	Regulars, so that's pretty neat.
1346.		KK	Yeah, yeah.
1347.		JL	So, uh huh, thank you.
1348.		KK	Sure
1349.		JL	Okay, who's next?

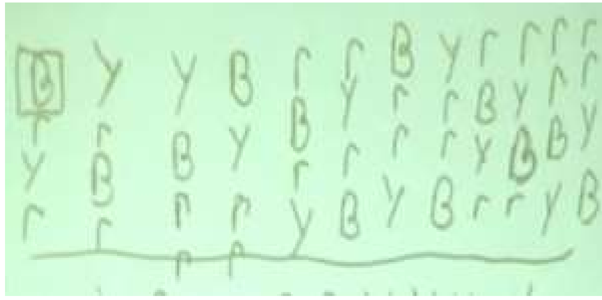
11/18 Meeting transcript 2 of 2

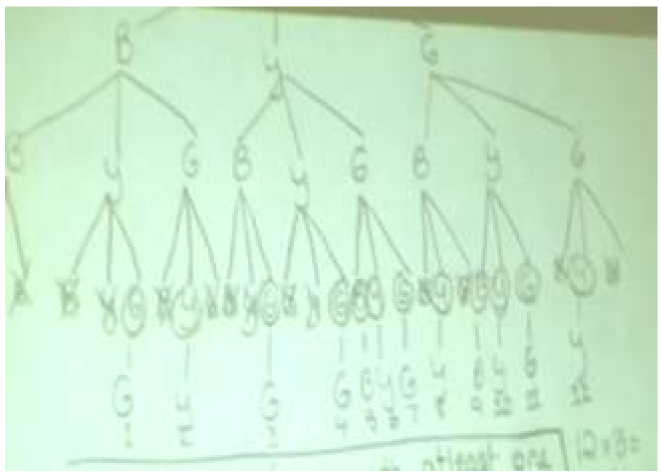
Title: 11/18 Oldbridge-2of2
Location: Oldbridge
Date: 11/18/2010
Length: 01:00:20
Transcribed by: Will McGowan June 2012
Verified by: Madeline Yedman August 2012

Line	Time	Speaker	
1.	0:00	S	Um, the top middle looks like they, My students call it "moving." Like they take the top one
2.		JL	Alright
3.		S	And put it on the bottom.
4.		JL	Okay
5.		S	And it bumps everything up. And then, like, see how it's blue red yellow,
6.		JL	Yep
7.		S	So if you take the blue off, put it on the bottom,
8.		JL	Okay
9.		S	It pushes up the yellow and the red,
10.		JL	What did we call that?
11.		S	So there's the yellow at the top.
12.		J	Recursion.
13.		JL	A recursion.
14.		S	Oh, yeah
15.		JL	A recursive argument. Right?
16.		S	Yeah
17.		J	Recursive
18.		JL	So they may have gotten their towers that way. What else may have, how else could they have gotten their groups? Take a look at just that second group that is right here. What do you see about this group? Because I saw this in, um, Mitch's class. I think someone did.
19.		M	The staircase, or
20.		JL	Yeah. Do you see that?
21.		S	Oh. Yeah.
22.		JL	Staircase. Okay?

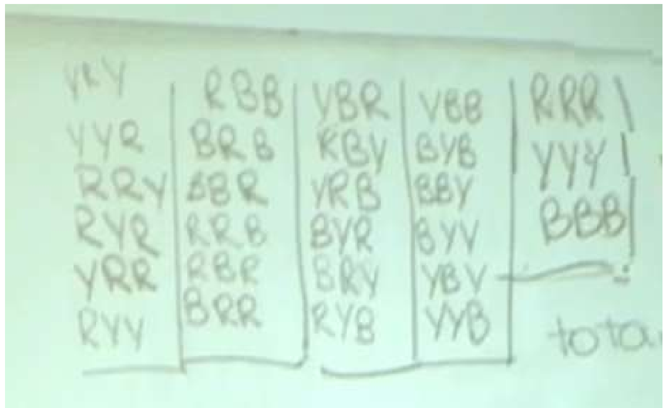
23.		KK	I know a lot of kids do that.
24.		JL	Alright. And is there a staircase in the other groups?
25.		KK	Yeah.
26.		M	Yes.
27.		KK	Blue blue blue,
28.		JL	Yep, there's a staircase of blue. Moving the blue into the different positions. How about in this one?
29.		A	Yeah
30.		K	That one, no.
31.		JL	Blue blue blue, going down.
32.		KK	Oh, going the other way. Opposite way.
33.		JL	Okay. So it's a
34.		KK	And then yellow yellow yellow.
35.		JL	Yep, so
36.		KK	And then with red.
37.		JL	It looks, and we're not really sure how they formed their groups. But it looks like they, and some of Mitch's kids actually said "This worked for us last time, we're going to try it again." You know, yeah.
38.		J	That's awesome.
39.		JL	And what did they say?
40.		K	"I believe there's only twenty seven towers you can make by stacking three cubes of three colors. I can prove this by: One, when you try to get more, all were repeats. Second, we made graphs, so there's only one blue and two reds or only one red and two blues. When we made one graph, we did the opposite. Example, two reds and one," I guess that's, "two reds and one blue."
41.		JL	Uh huh
42.		K	"Two blues and one red. We finished, there were nine graphs with three in each column and row which is twenty-seven towers. That's why I believe we have all the towers."
43.		JL	Very nice writing, isn't it?
44.			Mm hm
45.		JL	Now, isn't that interesting. Here, they're taking a group and getting the opposite. Okay? It's not a tower and an opposite. it's a group of towers and an opposite.

46.			Mm hm
47.		JL	What do you think of it?
48.	2:11	RB	If you go back to the front page, um, they would also show that the three cubed, three times three times three. They would also show that it would equal the twenty seven as well.
49.		KK	Oh yeah, that's true. The way they did it.
50.		RB	Three times three times three. So that's pretty interesting too.
51.		JL	Mm hm
52.		RB	So it reinforces. Alright, and maybe that, that's the way my mind thinks. But what I really liked, and I didn't notice this in mine until tonight,
53.			Mm hm
54.		RB	Where it's six times, uh six times four, plus the three. And the three are the yellow yellow yellow, red red red, and blue blue blue. I never looked at it that way.
55.		KK	No, me either
56.		JL	Okay
57.		RB	No
58.		KK	I saw the kids do that.
59.		JL	Yep
60.		RB	And
61.		JL	Uh huh, uh huh.
62.		RB	And tonight, that actually makes a lot of sense.
63.		JL	Okay
64.		RB	You do have your even number, but because
65.		KK	Four groups of six
66.		RB	Mm hm
67.		JL	Uh huh.
68.		RB	Your four groups of, mm hm.
69.		JL	Yep. We've seen a bunch of different organizations tonight, haven't we? Uh huh.
70.		RB	But mathematically, this reinforces the three times three times three.
71.		K	And then
72.		JL	Very nice, anything else?

73.		K	Yeah. I have this.
74.			
75.		JL	Okay
76.	3:04	K	There was just two really interesting ones here.
77.		JL	Okay
78.		K	This is the one similar to Kate's.
79.		JL	So this is the extension problem?
80.		K	This is the extension.
81.		JL	Okay
82.		K	And this is actually interesting, because this boy. I had to separate him because he was causing trouble with his partner the first, thing.
83.		JL	Okay
84.		K	So he was working by himself.
85.		JL	Okay
86.		K	When he was on his own, he kind of focused.
87.		JL	Okay,
88.		K	And, so he, I mean basically did the same exact thing, Kate,
89.		KK	Mm hm, mm hm
90.		K	Your students did. It's just "My first group was two red cubes and all I did was in my second group I did two yellow, in my third group," it says "I did two yellow in my third group"
91.		JL	Two blue
92.		K	Two blues and that's how I got thirty-six combinations. But,
93.		JL	Okay
94.		K	So he did the same type of thing.
95.		JL	Very neat, very neat. Now isn't it interesting how sometimes, we said it's always, we think better, to work with someone, but here a student was very successful, was having trouble working with someone. By himself, he was okay.

96.		K	And he was, in the class,
97.		JL	Okay
98.		K	In that particular class, he was the only one who got this problem.
99.		JL	Okay. You know, that's why as teacher, you can never make a hard, fast rule.
100.			Uh huh
101.		JL	That always, something has to happen. Because you're working with children, you're working with people. And they always are going to have outliers in what you think is the way that it has to be. So your flexibility is good.
102.		K	And, there's one other one real quick.
103.			
104.		JL	Okay.
105.		K	It's similar to what Sally was trying to do when she was explaining this to Kate.
106.		JL	Okay
107.		K	The tree diagram people
108.		JL	Oh
109.		S	Oh
110.		K	They, um, did the tree diagram, and they knew that they had to eliminate, they had to eliminate some,
111.		JL	Oh
112.		K	Because they didn't satisfy the requirements.
113.		JL	Good
114.		K	So,
115.		JL	Good.

116.		K	I thought it was really cool, because she and her partner both did this way, and
117.		JL	Very neat, and you can see that they eliminated, over here. And why did they eliminate this part of the tree diagram?
118.		K	It didn't satisfy,
119.		M	Because it
120.		S	Three blues
121.		K	Yes.
122.		JL	Anything off this
123.		K	Right
124.		JL	Is not going to work because already, you have too, too many blues.
125.		K	Mm hm
126.		JL	You're never going to get all three colors in the tower four tall if you have three blues. So that, that part of the tree is gone.
127.		K	And she explained it,
128.		JL	Yeah
129.	5:02	K	She said, "There are thirty-six possible towers that use one of each of the three colors and four blocks. First, I used a tree diagram, to show all possible combinations. Then I crossed out each possible tower that didn't have at least one of each color."
130.		JL	Great
131.		K	"And circled the towers and counted the circled towers, and I multiplied by three for the other colors, getting thirty-six towers."
132.		JL	Isn't that nifty?
133.		K	Really
134.		JL	So she knew she didn't have to worry about all three colors. She's focusing on what color here?
135.		KK	Blue
136.		JL	Blue. Very neat
137.		JL	I'd like a copy of that.
138.		K	Sure.
139.		JL	Very nice.
140.		K	And her partner did it really nice, and drew out all the blocks too, so

141.		JL	Ah,
142.		K	Ha ha
143.		JL	Very nice. Very, very nice. Don't they do amazing stuff, huh? Wow, Okay. Next, Mitch's going. Okay
144.			
145.		M	I'm going to cover up the explanation.
146.		JL	Okay, Mitch is going to make you work. Ha. He's covering up the explanation. Okay, so, and by the way, I was really impressed. I was in Mitch's class today. It was an inclusion classroom. Okay, that means it was special ed and regular ed and in this school, the way they do the inclusion, is they don't have, What are the other things called?
147.		M	Well, there's
148.		JL	The other
149.		M	We have the enriched class, we have
150.		JL	It's not enriched.
151.		M	The advanced class, so
152.		JL	And it's not advanced, you know, I'm thinking of,
153.		K	What's "enriched?"
154.		JL	Oh, no, no, no. this class, they have tracks. So it's not enriched, it's not advanced.
155.		M	It's not
156.		A	What does "enriched" mean?
157.		M	It's the highest top level.
158.		RB	It's a level, yeah.
159.		A	Enriched is the top?
160.		JL	Yeah
161.		M	It's like level one, two, three.

162.		KK	Enriched is like,
163.		M	It's the
164.		JL	Regular?
165.		M	No.
166.		RB	Honors
167.		S	A little bit above average.
168.		M	Ha ha. That's
169.		A	How many different levels can there be?
170.		KK	Three.
171.		JL	Three. So the top level,
172.		S	Honors Algebra, or you have
173.		JL	From the top.
174.		S	Top level. So like,
175.		KK	Enriched
176.		S	Okay
177.		M	Which is from
178.		A	Normal
179.		KK	Yes
180.		M	Correct
181.		S	The kids that are in honors are two grade levels ahead.
182.		KK	A little bit above average. And then regular. Even our inclusion are in regular class.
183.		JL	Regular, yeah
184.		KK	Okay
185.		A	Oh
186.		JL	It makes me feel like we're talking about bread. We have enriched.
187.		S	Basic Skills is a separate class.
188.		RB	So, it's like level one, two and three, basically.
189.			Yeah
190.		KK	But is your three below average?
191.		RB	We have honors and regular. So, instead of having honors and regular, it would be like splitting the honors kids into two tiers. You have your honors, and then I guess like your above

			average.
192.			Uh huh
193.		JL	Right
194.		S	Yeah
195.		RB	And then just
196.		KK	Everybody else.
197.		RB	The rest of the population.
198.		S	Ha ha
199.		JL	But, your special ed is all in the regular class.
200.		M	Mm hm
201.		S	Yeah
202.		KK	Uh huh.
203.		JL	So, your kind of class here with how many special ed, how many regular ed?
204.		M	There's eight special ed, and today there's only, other kids were absent, so there's maybe ten regular ed.
205.		JL	Yeah, so that, that really is a tough class to teach in terms of abilities, in terms of special needs. And there was a special ed teacher in the room. And Mitch's going to talk a little bit about how, when you have an inclusion class, How you might have the special ed teacher work to help you with those children. Go ahead, so, let's look at this. He wants you to figure out what those kids did.
206.	7:56	JL	Anyone?
207.		K	It looks like it's yellows and reds,
208.			Right
209.		K	Blue and reds, and then all three colors. And then the third column, blues and yellows and then the fourth column
210.		JL	And then all three. We're going to
211.		KK	All three
212.		JL	<Phone rings> That's me, ignore it. Okay, so what you have there. They're focusing on color. What else do you see, anything? Is there any organization within the grouping?
213.		KK	Red, no. Yellow red
214.		A	There I, like I see
215.		KK	Red blue yellow.

216.		A	Like, in the first,
217.		RB	There's
218.		A	How, it's like staircase.
219.		RB	Staircase
220.		M	Yeah, I don't think
221.		A	But then there's one group that's not.
222.		M	Right, I think when they did this. And there was another group like this.
223.		JL	Uh huh
224.		M	When they recorded it, it's
225.		A	It got messed up.
226.		M	They're not very good at recording, like
227.		JL	Yeah
228.		M	And they had it all in order, but they didn't write it down. I didn't realize it until, you know, I was just looking at it recently.
229.		A	So is that last group supposed to be, like staircases?
230.		M	But I, yeah, it's supplied to be like, you look at, um. So she said, "We found out the total for the blocks it twenty-seven. We were convinced that this was the right amount because we found all the staircases."
231.	9:00	JL	Oh,
232.		M	We found the opposite of them. So there is no more groups possible.
233.		JL	Idea
234.		M	So like, if you look at, like this last one right here. They had a staircase of like, yellow yellow yellow, and then they also had the blue blue blue,
235.			Mm hm
236.		M	You know. So they were kind of almost opposite staircases.
237.		RB	Mm hm
238.		JL	Mm
239.		M	This one, the first one. When I'm looking at it, it doesn't, you know, like this one right here, you could see, uh
240.		A	Yellow yellow yellow
241.		RB	Red red
242.		M	She kind of does it backwards here. "red red red"

243.		JL	Uh huh
244.		M	And then "blue blue blue"
245.		JL	That's right.
246.		A	But the first one is too.
247.		S	It looks like the bottom.
248.		RB	Red red red, yellow yellow yellow.
249.		S	Red yellow yellow, That has to go on top.
250.		M	So, it's not as easy to see when you first look at it. But they did.
251.		JL	Okay
252.		M	It was pretty much just staircases, and they were convinced they couldn't make any more staircases. And, this girl especially, who kind of grouped them in four, four, four, four. And then also, the last group of three.
253.		JL	Uh huh, you mean "six, six, six ,six"
254.		M	Yeah.
255.		JL	Four groups of six.
256.		M	Yeah
257.		JL	And then a group of three
258.		M	Oh, right.
259.		JL	And this was
260.		M	That was Olivia.
261.		JL	This was the regular ed youngster who was paired with a special ed youngster.
262.		M	Right.
263.	10:00	JL	Um, and they worked really well together. Because their math ability was pretty similar.
264.		M	When they, they didn't get to finish
265.		JL	Uh huh
266.		M	So this next one is their extension problem. And they didn't get to finish, because they were all trying to do all of this in a one hour period.
267.		JL	Yeah
268.		M	She got to start and I'm just going to read a bit.
269.		JL	Sure, sure
270.		M	It says "There has to be two of one color because there, because

			there's only three colors with a tower of four cubes. We did the staircase because you can see the colors slide…" uh
271.		JL	Down, like she's thinking
272.			Bottom two
273.		M	Right
274.		JL	Down
275.		KK	Yeah
276.		M	"Throughout the two other colors change from the bottom to the top. Therefore, Throughfore"
277.		S	Therefore Ha ha
278.		M	Right
279.		JL	This was a special ed youngster by the way.
280.		M	"The two other colors change from top to bottom." And then it was funny because they only got to start this. So we kind of asked them what they were
281.		JL	Stop before you go on.
282.		M	Okay
283.		JL	Because I think you and I understand it because we were there.
284.		M	Right
285.		JL	It might be hard for you to follow, so see if you can look at their step one, step two. They're going to help you, Okay? So look at step one. Their towers are going left to right.
286.		KK	Oh, across, right.
287.		JL	They're horizontal, Okay?
288.		S	Oh, so they moved the yellow
289.		KK	They doubled it.
290.			That's cute.
291.		JL	Right, so you see how the double yellow
292.		KK	Move
293.		JL	Is sliding?
294.		KK	Mm hm
295.		JL	Okay and. Now this is real interesting. This is a youngster who has no trouble seeing that the tower, if it's written horizontally.
296.			Yeah
297.		JL	But for some people, that would be very hard. Um, but so she

			has a tower that's yellow yellow red blue, blue yellow yellow red, red blue yellow yellow. So your double yellow is first at the top of the tower,
298.		S	Mm hm
299.		JL	Then in the middle two positions and then in the bottom two positions. And then in step two Um, She…
300.		KK	She kept them but reversed the yellow, and um
301.		J	Reversed
302.		JL	The blue and red
303.		KK	The blue and red.
304.		JL	Isn't that neat?
305.		KK	Yeah, that is cool.
306.		JL	This is a youngster. What do you think she got in math? Is she a good thinker?
307.		J	Yes
308.		JL	I would say she's a pretty good thinker. What do you think she got in math?
309.		RB	A D
310.		JL	What did she get?
311.		M	I believe it was a c plus.
312.		JL	C plus, c plus.
313.		RB	Mm
314.		JL	Because I said to her, "Wow! You're a good math thinker." And she, I said, "I bet you like math?" And she said "I do." I said, "I bet you do good in math." She said, "C plus?"
315.			Ha
316.		JL	I said, "Mm, I bet you could do better than that." Because her thinking, this is phenomenal. And this is the special ed youngster of the group. It was not the regular ed youngster that wrote this. Okay? So then what?
317.		M	She actually, because I had questions, but we were running out of time, and
318.		JL	Yeah
319.		M	I asked her because they were the ones who did this, you know this one that I just showed
320.		JL	Mm hm, yeah.

321.		M	How they used all staircases. And I said, "Are you going to be able to use that method to find all of them? If you just keep doing staircases like this, are you going to find all possible combinations?"
322.		JL	Mm hm
323.		M	And then the regular ed girl said "Yes. If I just keep doing that,"
324.		JL	Mm hm
325.		M	But it was funny because the other girl already had this one built, and kind of like threw it down and said, "Well what about this one."
326.			Mm
327.		M	So we said, "Well, what are you going to do with that?"
328.		JL	Uh huh
329.		M	You know, "Are you going to" So she kind of said. "Well, we're going to make another group and call it the mixed up group, the mixed up color"
330.		JL	Mixed up color.
331.		M	So, she also knew, she recognized, whereas the other girl thought that if you just do that with all the different colors,
332.		JL	Uh huh
333.		M	Like, keep two blues together
334.		S	She didn't realize it had to be split.
335.		M	Right. She didn't realize there was going to be one was mixed up. So, and like I said, they didn't have time to go through this,
336.		S	Yeah
337.		JL	Right
338.		M	But she recognized that they couldn't just,
339.		JL	That that would not give you the whole solution
340.			Yeah
341.		M	So
342.	13:29	JL	You have a comment, go ahead. What is it?
343.		SB	I'm sorry, because I'm not sure I should keep commenting so much,
344.		JL	Absolutely,
345.		SB	You sure it's okay?
346.		JL	Absolutely. You're a part of the group. Go ahead.

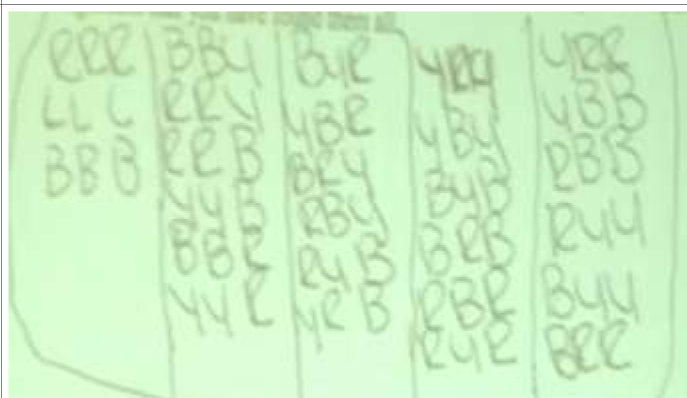
347.		SB	Really?
348.		JL	Yeah.
349.		SB	Because I'm also noticing, I thought with staircases, that you test everything, I don't know, I mean that's the term you're using as though it's a technical term, but.
350.		JL	No, no, no. It's
351.		SB	Shouldn't the red blue be in the same order, in that first block? Like, shouldn't it be, red blue, red yellow yellow blue?, red blue yellow yellow? If all you're doing is moving the yellow? She starts
352.		JL	You're saying
353.		SB	With
354.		JL	You're, she, yeah. And I don't think, um,
355.		SB	Is that not right?
356.		JL	You're saying, "Should it" you want it to be super systematic, and super controlled.
357.		SB	Well, I'm just worried about how they deal with the double counting.
358.		JL	Well,
359.		SB	If they're using the staircase as a way of organizing, they're double counting.
360.		S	I think what they
361.		SB	Flipping the second one too, but that's like a extra way
362.		S	I think, like for the first one, it says "yellow yellow red blue." I think what they did was they popped a blue off the end and like
363.		KK	Put it back on top
364.		S	Stuck it on the other side.
365.		M	Right.
366.		SB	Well, that's what they mean by the, oh, that's
367.		S	And that, like, it's like
368.		M	Right
369.		S	Yeah
370.		SB	Okay
371.		RB	And then had been like the red red
372.			Middle
373.		RB	Uh, red red yellow blue

374.		S	Yeah
375.		RB	Might be separate. And then in step four would be,
376.		A2	Oh, so then why wouldn't you? If you have to move yellow around, you could make the fourth one, you could make that one that's in the bottom spot by itself, like
377.		M	Mm
378.		A2	By pulling the yellow around to the front.
379.		JL	Now, and
380.		KK	You could, but a lot of times, the don't see it. If they're concentrating on the double yellow,
381.		M	Right
382.		SB	Yeah
383.		KK	They don't even see that,
384.		M	Right
385.		KK	Which is so interesting.
386.		SB	Okay
387.		JL	And I'm not sure that they used a recursive argument in making the towers. Um, I
388.		SB	I just didn't know what you meant by "staircase." I thought you meant you move the yellow.
389.		JL	No, they. I didn't mean anything, they meant it.
390.		M	These students, yeah
391.		SB	Do they mean moving that one
392.		M	It differs
393.		JL	Where are those cubes?
394.		K	I think Angela has them
395.		M	They, see, they may mean it looks like a staircase
396.		JL	Angela, can I have them? Thank you
397.		S	It's like a student invented term
398.		JL	Yeah
399.		SB	I see. So it's like
400.		M	So it means different
401.		S	So they all mean different things.
402.		JL	Yeah, yeah

403.		SB	Alright
404.		JL	And actually, um, they. This did happen very quickly, and I'm not sure they were using a recursive argument. You could have used a recursive argument.
405.		SB	Mm hm
406.		JL	I'm not sure that they did. But I think all that they were really doing was saying "I'm not focusing on anything but double yellow."
407.			Mm
408.			Uh huh, Okay.
409.		JL	Okay?
410.		KK	Only the double yellow
411.		JL	And I'm moving my double yellow so that it's in the top two position, the middle two positions, and the bottom two positions. Like that.
412.		SB	And they're not really worrying about what the red and blue are doing?
413.		M	Mm hm
414.		JL	No. They're just saying that I have to have a red and blue in each tower. And it doesn't really matter where. But then down in here, I'm going to account,
415.			Mm hm
416.		JL	I'm going to make everything else.
417.			Explain
418.		JL	Now, I'm going to account for it.
419.		KK	Switch the red and blue from where they were.
420.		JL	So,
421.		KK	First, second
422.		JL	So here, you have red and blue. Here you better have blue and red.
423.	16:00	A	But I think you may be right, they may have done it, especially if they have moved the cubes
424.		JL	Yep
425.		A	By taking the top one off and moving it onto the bottom.
426.		JL	They might have.
427.		A	

428.		JL	We didn't have, I didn't see them, did you?
429.		M	Right, I
430.		JL	It was so quick, and I'm saying, a lot of this she was thinking in her head. And this is why I think this was a phenomenal mathematical thinker. This was the only group that got to even try the Ankur's Challenge. And the only group who really this one student who got to say, "No, this isn't going to help us find all the towers." Because you're not going to have towers that look like this where they're not stuck together, the double. Okay? So she actually had a piece of Romina in her, Right?
431.		KK	Mm hm, mm hm
432.		JL	She had a piece of, of a lot of stuff in her, but I think she is a super mathematical thinker. And Mitch's challenge is going to be to get her to see herself as a good mathematical thinker and be a good math student in math class. Um, I told him he should go back and really say how impressed I was with what she did.
433.			Mm
434.		JL	Because she doesn't have a very good image of herself as a mathematical student. Okay.
435.		M	So
436.		JL	That's great. I'd like a copy of,
437.		M	Yep
438.		JL	Of her work. Anything else?
439.	17:07	M	Um, just a little bit of what we were talking about with the special ed students.
440.		JL	Okay. Yes.
441.		M	There was one group where
442.		JL	Yes.
443.		M	This was all I got out of them. This was the special ed group.
444.		JL	Yeah.
445.		M	And they got the correct answer for the three tall one. And every time you would go and ask them, and our special ed teacher would say, "Well, what do you have here? How did you get this?" And they would explain it to you.
446.		JL	Mm
447.		M	But then when you ask them to write it down, they didn't write down all different combinations. And after, it was weird because they would have a group of, they explained: "Well, first

			we're going to do all of the reds on top. Then we're going to do all, like, reds on bottom." So they were only doing like two colors together on top, two on the bottom. When you would come back, they would be all over the place.
448.		JL	Mm hm
449.		M	Their group that they did was scattered everywhere.
450.		KK	I had that.
451.		M	They didn't record, they didn't record anything.
452.		JL	Mm hm
453.		M	So, I tried to get them to you know, go back and said "Well, what did you do?" And they said, "I don't know."
454.		JL	Yeah.
455.		M	So, it's a real challenge for these guys.
456.		JL	What's your thought about the special ed teacher? How well do you use her? Possibly.
457.		M	So what we were saying is for the next time, maybe having her stay with one special ed group and just kind of helping them go through the writing. And then maybe for the next time after that, she'll just go to another special ed group. Um, because they had trouble so much with being able to put it down on paper,
458.		JL	Mm
459.		M	Being able to explain it. Even verbally, you know. When I said, "Well what did you guys do before?" You know "Can you explain, you know, how you were doing that?" With the, they even struggled with that.
460.		JL	Mm hm
461.		M	But the, they always have a good strategy. They always, I think for almost every one they've come up with the correct answer.
462.		JL	Mm hm
463.		M	They just struggle with this kind of stuff.
464.		JL	So you know, sometimes we think "get to everyone in the room" As teacher and as special ed teacher. It's not essential that you see every group every time. In fact, you're one teacher in a room where you might have ten, twelve, fourteen different groups. Depending on how large your class is.
465.		M	Mm hm
466.		JL	It is impossible to think that you're going to get to see fourteen groups and their reasoning. So if you focus on a subset. Maybe

			a third of that and really see what the thinking is and the reasoning is of those five groups. Then the next time you do it, with a problem, make sure you get to five different groups. Okay? And the next time, with the groups that you haven't seen yet. Similarly the special ed teacher. Had a bunch, what were there? How many special ed kids?
467.		M	It was eight.
468.		JL	Eight. That's a lot of special ed kids in a classroom. But, so she tried to spread evenly. Right, we do, we want to be fair, to all the special ed kids, but as soon as she left this group,
469.			Mm hm
470.		JL	Nothing happened. So my feeling is, maybe she'll get to two of the four special ed groups. And really stay long enough to get them to where they can, not, you're writing for them, but you're helping them, uh, verbalize and say, you know, "Write that down." Because sometimes once they get started, they really can move, uh, further. That's great. Thank you. Okay, next? Justin. Okay.
471.			
472.	20:07	J	I think, like to note what you just said, um, like writing down, like that makes a lot of sense because my, like what I was telling them was "Write afterwards."
473.		JL	Mm
474.		J	But maybe by writing notes of how they, you know, how they got to, you know, that one particular thing.
475.		JL	Well, as they're explaining to you, like, because remember they have to convince you before they're going to write. So as they're convincing you and they say something that's brilliant, "Write it down." Okay.
476.		JL	Alright
477.		JL	Everyone, let's look at what this pair of students did. When you think you know, tell us. Just shout it out.

478.	21:03	J	They did the, they did, uh, three colors and two colors together. And, uh, so they did the six, the four groups of six with the one group of three. So the first one was all three colors. Then it was blue blue yellow, red red yellow, red red blue, yellow yellow blue. See how you got two of the colors together
479.		JL	Mm hm
480.		J	And then, and then the next one they did one of each color.
481.		JL	Mm hm
482.		J	And then they flipped it, they took the opposites. And then, and then they split them out. Like, the reds on the outside, the blues on the outside, and then the yellows on the outside.
483.		JL	Okay?
484.		RB	And then the last one, they did the opposites of the first group. Yellow blue blue, yellow red red. Those two are switched, though.
485.		JL	Okay. And what did they, what did they write? Read to us Justin.
486.	21:54	J	Okay, "We organized our cubes in five groups all with six and one with three solids. We are convinced that our answer of twenty-seven is correct. Because we organized them in the color-coded groups." Or, in co, "We organized them in color coded groups," uh "as our chart shows above. We only have one group of three because those would only be the solids. Everything else is together because" I guess "it is organized by colors shown put together. For example," And this is why
487.		JL	Uh huh
488.		J	I was saying, like, you know, like "I don't want to guess." Like, "Tell me,"
489.		JL	Good. Good.
490.		J	What you're doing.
491.	22:44	JL	Excellent. And that's good, to push them. To say "What do you mean?" And they did. Huh?
492.		J	So then when I came back
493.		JL	Good.
494.		J	The only thing
495.		JL	Okay, okay
496.		J	They put down was "For example, all double top colors of three are put together."

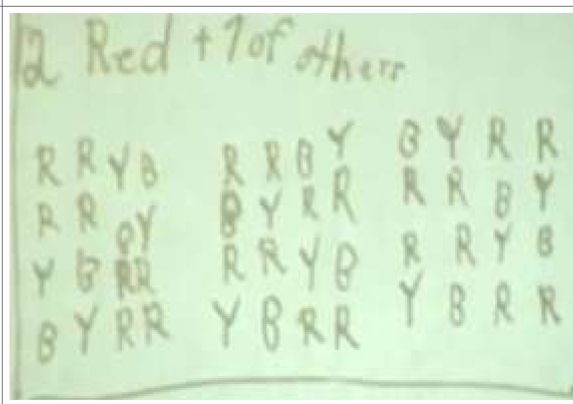
497.		JL	So it's a start.
498.		J	Yeah, it is.
499.		JL	And it's a nice start, right?
500.		J	I would, uh, I was happy. Yes, that's a start.
501.			Ha ha
502.		JL	Okay. Do you remember? It was not so easy for you guys?
503.		J	Yes
504.		JL	To really write convincing arguments. And, you know, to really explain what you were doing. But I'm saying that now they're showing you that there, this one group had double, the double color was at the top. So let's say it's this one. Alright?
505.		KK	Yeah
506.	23:32	JL	The next thing would be is "Well how do you know there are only six of them with a double color at the top?" So, in other words, you're taking steps towards getting a convincing argument. And you're not, And they may not be ready to go there. They may go "Oh, we just know." Right?
507.		M	Mm hm
508.		JL	"Because we made others and we got duplicates and that's it. So stop bothering me, and" Right?
509.		J	Yes, a fight. Now, it was interesting. Like if sometimes I say "Well, for instance, the blue"
510.		JL	Yeah
511.		J	Blue blue yellow
512.		JL	Yes
513.		J	And then blue blue red
514.		JL	Yes
515.		J	Like, I'll say um, "Well, um, what do you think about this? If I shift these two and put them together?" They say "No, Mr. Nick. This way they're separated." I'm like "That doesn't. like how do you? Like, how does that happen?"
516.			Ha ha
517.		J	I don't understand that.
518.		JL	You mean, in other words, you wanted them to put this one right below this one?
519.		J	Yes!
520.			Ha

521.		JL	That would have made you feel better.
522.		J	Yes because,
523.		JL	Okay
524.		S	Ha ha
525.		J	You kind, like, alright, you did two blue tops,
526.		JL	Right.
527.		J	Right, on top, and that was all you could do else is yellow and red.
528.		JL	Okay
529.		J	And that's all you could do.
530.		JL	Right. Right, right.
531.		J	And, um,
532.		JL	Right.
533.		J	Now, one other student did say that from another group.
534.		JL	Uh huh
535.		J	Like, you know, "This is the only thing you could do here"
536.		JL	Right
537.		J	"Is just another yellow or a red" or something like that.
538.		JL	Right, right.
539.		J	And I said "Very good."
540.		JL	Uh huh
541.		J	And, you know, "Keep on going." And that sort of thing.
542.		JL	Okay.
543.		J	So, but now, how these young ladies, they love the, this creating, you know, like, creating different, um
544.		JL	Towers
545.		J	Different towers.
546.		JL	Yeah
547.		J	Then they would do opposites,
548.		JL	Uh huh
549.		J	And then, but what they did to come up with twenty-seven was, they grouped them, and that sort of thing.
550.		JL	Okay.
551.		J	So,

552.		JL	Okay. Very nice, And you know we really want kids to, if we think it's easier in our heads, we think it's easier in their heads. And you know, sometimes, you have to, um on this would have been still where they have to convince you that they have all possible towers in here. Um, that we see six of them and you're saying, "I have a way, as teacher, to organize it." But they may convince you, but not in the way that you're thinking. Okay?
553.	25:33	J	Okay. And what she was saying was that, "Here I have this: two yellows, two blues, and this." And they
554.		JL	Oh
555.		J	Those things made more sense.
556.		JL	Okay, isn't that interesting? Isn't that interesting?
557.		J	And that type of thing.
558.		KK	Okay, yeah, look at that
559.		J	And then, uh, also
560.		JL	Stop a minute
561.		J	Oh, okay.
562.		JL	Let's all look at that because I was thinking the way Justin was thinking.
563.		KK	Yeah, me too.
564.		JL	And they're saying, "Well, look, I don't want to move that tower. Because if I move that tower, I will no longer have my two yellows here, my two blues and my two reds. And I've, what I've swapped is the blue yellow, the red yellow, and the blue red." Isn't that interesting?
565.		J	And then they even, they kept going along
566.		JL	Very interesting.
567.		J	Each group. Here,
568.		JL	Okay
569.			I didn't notice
570.		J	And, uh, YBY
571.		JL	Yeah
572.		J	And then
573.		JL	And they actually told you that, isn't that interesting?
574.		J	Here, YRR, YBB
575.		JL	Yeah. Now how many of you caught that?

576.		RB	No.
577.		JL	You did? Okay. I didn't catch it.
578.		SB	I feel as
579.		JL	Because I really
580.		SB	Like reading left to right, and like reading right to left,
581.		JL	Okay, yeah
582.		SB	Like, in a way,
583.		JL	Okay
584.		SB	You're reading left to right…
585.		JL	Or, did. It's not even reading, it was
586.		SB	Or
587.		JL	He was thinking that
588.		SB	Because they're towers.
589.		KK	We're going top to bottom, oh bottom to top, yeah. Right
590.		JL	And it's, it's just a different way of organizing. And I was thinking the same way you were.
591.		KK	Mm hm
592.		JL	Like "Oh, yeah I would have moved that up." But no wonder they said "No."
593.		J	Yeah
594.		JL	Because that's not the way they saw it.
595.		J	I had
596.		JL	Isn't that neat?
597.		J	Like, ha ah
598.		JL	Good for them for not moving it because the teacher said. Okay. I'd like a copy of that.
599.		J	Okay
600.		JL	Good.
601.	26:58	J	And, um, this, uh, this is, the other group, but, um.
602.		JL	Okay.
603.		J	Now this is, um
604.		JL	Now this is getting hard for us because we can't see the three colors. Wow.
605.		M	The key.

606.		A	No, there's a key.
607.		JL	Oh, there is a key.
608.		A	The red
609.		JL	Oh my gosh. Is that kind of cute?
610.		J	Funny, yes.
611.		JL	Look at their red.
612.			Oh, whoa
613.			Half of a box.
614.		JL	A half a box.
615.			Ha
616.		JL	Okay, so let's see if you can. Now kids come up with ingenious codes, don't they? Their keys are quite interesting? Alright so let's, it's a little hard to read. Um,
617.			Yeah
618.		JL	Why don't you talk to us about it?
619.	27:39	J	Alright, now, ha ha. Now, um, this group, like, this group of students,
620.		JL	Yes
621.		J	They, um, they were boys. And they, uh, their explanation was like, was great to me.
622.		JL	Mm hm
623.		J	But they only came up with twenty six groups. Um, like twenty-six sets of,
624.		JL	Oh, okay.
625.		J	Of combinations.
626.		JL	Okay.
627.		J	Um, but actually it's
628.			Did they see it?
629.		J	Well, there's twenty-seven written.
630.		SB	Well, they found twenty-seven pictures.
631.		J	Yeah, there's twenty-seven written,
632.		JL	Right
633.		J	But they were saying that there's only twenty-six.
634.		JL	Ah

635.		J	And, what was interesting was that they were talking like, "staircase talk" the whole time.
636.		JL	Okay
637.		J	And the way they set them us was "Staircase"
638.		JL	Uh huh.
639.		J	Um, but then the way they wrote them out was interesting.
640.		JL	Very interesting. I don't know if it's so good. But it's interesting. Right? There was a really messed up way of, that the kids were. They had a code, they had like a star and a zero and a this. And they really had a very complicated key for the way they recorded. Sometimes their recordings are very interesting, because they tell you a lot. And sometimes, they're really very helpful. But so, I'm not sure this is helpful. Um, it's hard to read.
641.		JL	But thank you that's great. Good. Okay.
642.		J	Ha ha Alright.
643.		JL	Did everyone get to share, or is someone still needing to share?
644.	29:06	RB	Uh, I just wanted to show one of Ankur's Challenge, and then
645.		JL	Absolutely.
646.		RB	This is just
647.		JL	Good
648.		RB	It's consistent with what, with um, and I've showed this student's work.
649.		JL	Absolutely
650.		RB	Pair of students' work every time.
651.		JL	Oh, graphs.
652.		RB	I'm just going to put it up. I'll let you enjoy it.
653.			
654.		JL	Okay.

655.		RB	Because we've all been enjoying
656.		JL	Okay
657.		RB	It's like
658.		JL	Alright, so here.
659.		RB	I had the pleasure of seeing it firsthand.
660.		JL	Oh, you did?
661.		RB	And it took them fifteen minutes, I mean, not even.
662.		JL	Okay, okay, great.
663.		RB	I mean, they just
664.		JL	Okay, so look carefully before we talk about it. Uh huh. Mm
665.		RB	I also felt that their explanations have gotten better too.
666.		JL	Good. Good.
667.		RB	This group.
668.		JL	Absolutely
669.	30:02	S	So the top left is double reds.
670.		JL	Yep
671.		S	And on the end. Like top end, then bottom end.
672.		JL	Uh huh
673.		S	And then middle one is double reds that are separated
674.		JL	Yep
675.		S	With one of them being on either the top end or the bottom end.
676.		JL	Uh huh
677.		S	And then the third square is double reds, either in the middle or the outside.
678.		JL	Good
679.		S	With the,
680.		JL	Yep. That's what they did.
681.			Organized
682.		JL	Yep. Very systematic, isn't it?
683.			Mm hm
684.		JL	Very ,very neat. Talk to us about what they wrote.
685.		RB	Um, Okay. It's
686.		JL	Read it.

687.		RB	It's consistent with the other,
688.		JL	Okay
689.		RB	With, with, the uh, with the three colors, three, uh three high.
690.		JL	Good.
691.		RB	It's consistent. And they've been consistent throughout.
692.		JL	Okay
693.		RB	Except on this one they've actually used the concept of opposites they said "I know I have all the combos, because if you have to have all three colors and four blocks tall towers, so there has to be two of one color. I know I have all the combos because I made one combo like red red yellow blue, then I made the opposite, that like; red red blue yellow. Then I made the opposite of that pair like; yellow blue red red, blue yellow red red. And then I get: red red yellow blue, red red blue yellow, yellow blue red red, blue yellow red red."
694.		JL	Okay, so. Let's go back. Because they're saying a lot there. Do you see a touch of Romina?
695.	31:40	RB	Mm hm
696.		JL	Right, what's the touch of Romina?
697.		KK	The double color and then switching the spots.
698.		JL	That they knew that it has to be in a four tall tower with one of each color, there has to be at least two of one color. Okay? And then they're actually showing you why they think they have them all. And it's pretty nice argument.
699.		RB	It, it is pretty convincing
700.		JL	Mm hm
701.		RB	And they, they finished all of the challenges relatively quickly.
702.		JL	Mm hm
703.		RB	And, their explanations their written explanations haven't been that strong. But they finally have it.
704.		JL	They
705.		RB	I think that they
706.		JL	Yeah
707.		RB	That they put the stamp on
708.		JL	Yep.
709.		RB	On mastering it. The only thing I could have asked for was more of the, uh, formulas, I guess. But I've seen some of the

			other students do, but.
710.		JL	Well, you know, like
711.			It's
712.		JL	I think that, it's, we're in a process
713.		RB	Mm hm
714.		JL	We're in a, uh, a journey that's going to be continued.
715.		RB	Mm hm
716.		JL	So I think that you're going to continue to see growth. The more that you expect this kind of thinking and reasoning, and sharing of their thinking and reasoning. Now, if you're looking here. So they have the double red on top, and they're showing the bottom can either be yellow and blue, or blue and yellow.
717.		RB	Mm hm
718.		JL	So they're exhausting all the possibilities. And it's nice. They're really explaining it to you. So, as to why they think they have them all. Very nice. Yeah.
719.		SB	Just something, as a mathematician, that I notice. They're using the word opposite in two completely different ways.
720.		S	Yes, yes.
721.		SB	And to me,
722.		S	Yes.
723.		SB	This is like nascent in linear algebra.
724.		RB	Mm hm
725.		JL	Okay
726.		SB	Do you, do you see what I'm saying?
727.		JL	Mm hm, mm hm.
728.		SB	I mean they're doing this transformation
729.		RB	Mm hm
730.		SB	Among, basically, the last two. And then they're doing the transformation of the blocks
731.		JL	Yes
732.		SB	Of two
733.		JL	Yes, yep.
734.		SB	And that's like
735.		RB	I don't want to disturb, but, uh, this one's definitely a higher student, and was working with somebody who was just

			declassified from special ed, so. But and, the one that was just declassified is, I don't want to say a "mathematical genius", but is. But he's definitely higher level with math as well, so.
736.		SB	Right
737.		JL	But what you're saying is
738.		SB	But, I mean,
739.		JL	Yeah
740.		SB	What's interesting to point out to them
741.		JL	Sure.
742.		SB	How differently they are using the same word.
743.		JL	Yes.
744.		RB	Mm hm
745.		JL	Yes. And that happens a lot.
746.		SB	And how that's something that
747.		JL	Yes
748.		SB	I mean, I don't know how and when you guys choose to deal with choosing a language.
749.		JL	You have
750.		RB	Mm hm
751.		JL	It should be dealt with.
752.		SB	It's sort of like, in one sense, to be opposite, it should mean this.
753.		JL	That's right.
754.		SB	And in another sense it should mean
755.		JL	That's right
756.		SB	And just, is there really truly an opposite when you have three colors? Like the meaning of "opposite"
757.		RB	Mm hm
758.		SB	Is completely unambiguous,
759.		JL	Yes
760.		SB	When you have two colors.
761.		JL	Uh huh
762.		RB	You're right.
763.		SB	But what happens when you have three colors is no longer unambiguous.

764.		JL	Right.
765.		SB	It's no longer an unambiguous meaning.
766.		RB	Mm hm
767.		SB	And so that's a really interesting discussion to get into
768.		JL	Absolutely, absolutely.
769.		SB	Because we have this instinct towards inverse.
770.		JL	Yeah
771.		SB	You know, and we mean so many different things by it.
772.		JL	Yep
773.		SB	In mathematics throughout. And it's a perfect opportunity to say "See how we use this one word"
774.		JL	Yes
775.		SB	"to mean so many different things."
776.		JL	Sure.
777.		SB	"But we need to be clear and communicate with each other."
778.		JL	About what we mean.
779.		SB	At which point, are we using the word?
780.		RB	Sure
781.		SB	And this is a perfect opportunity
782.		JL	Absolutely
783.		RB	Mm hm
784.		K	That would be a nice conversation.
785.		JL	And when that, when they actually talk about their solutions. Because if you haven't done that yet, you should.
786.		SB	Mm hm
787.		JL	You should give them time to share. And I can promise you, that not everyone in your class is going to follow this. And that's okay. Alright? Because if they're not ready to really understand the reasoning of this, it's going to go in one ear and out the other, and it will be fine. But definitely the conversation of, of how you're using the word, the "opposite" word, in different meanings, it can become confusing.
788.		RB	Sure.
789.		JL	Um, and I think that in Kenilworth, when they were trying to get opposites and then they were doing "flips"
790.		KK	That's what my kids called it.

791.		JL	Okay
792.		KK	Opposites and flips
793.		JL	Okay, yeah. So they
794.		SB	Oh, so you gave them two different words
795.		JL	No no no!
796.		KK	They gave me two different words. I didn't give them anything.
797.		SB	Oh, your class.
798.		KK	Yeah
799.		JL	We actually don't give them the language.
800.		KK	Yeah
801.		JL	They invent it. In fact, there were some kids who called it, that these were um, "opposites" and these were "cousins"
802.		KK	Yes
803.		JL	When it was flipped. You know, so I'm saying that they, they invent language, but when they're using the same word to mean more than one thing,
804.		RB	Mm hm
805.	35:32	JL	We have to help them
806.		SB	Invent a new word, right.
807.		JL	Like "Well, what do you want it to mean?" Mm hm. Good.
808.			Yeah
809.		JL	Excellent. Alright, um, really nice stuff. Really. I'd like a copy of that, okay?
810.		RB	Okay.
811.		JL	Um, what I would like to do is let you know that I have seen unbelievable stuff from your kids this time. I think you would agree. And I, I don't think it's just that the kids are getting better. I think that you're providing them with a better environment, where they can do this. You're not leading them, you're listening to them. You're trying to get them to explain their reasoning. Your questioning has gotten much better. And when you write in, um, our little dialogue online on eCollege, you have better postings because you are really focusing on the thinking and reasoning of your students. Much, much more than you did, uh, just a few months ago. So, I compliment you, and I think you should be very proud of what your students are doing as well. Alright?
812.	36:32	JL	So, knowing that, we can turn this off. And I am going to give

			out a paper that will tell you very, very carefully what the final project should look like, and again I think you 're going to have fun doing this. Remember the syllabus said something about a poster?
813.			Mm hm
814.		JL	No, we're not doing posters. It's too much busywork to get a big piece of oaktag, and we're not meaning to do that. So you can just make, like a pamphlet, brochure. Okay? And let's give you this.
815.		KK	Mm hm
816.		RB	Thank you.
817.		JL	Yep, you're welcome. Alright. And what you will have here is a real clear.
818.			Thank you
819.		JL	That's yours. Let's see, I've got
820.			I can share
821.		JL	Good. Excellent. Okay, what you're going to do is see that you have December fourth is your deadline. Okay? That's when you're coming to Rutgers, um, for our final, ah, you know meeting. Those that were here early know that I will not be there on the fourth, Okay? But Carolyn Maher will be there. Uh, Alice Alston will be there. Will is going to be there. Um, Jonathan Flint, who has been following me in the other two districts, he will be there. So there be plenty of people there. And what you will be doing is handing this in to Carolyn Maher, so that I can get it, and I will be reading these before I get your grades. Okay? So real important that you really work hard on this, because phenomenal stuff has been done by you, uh, this semester. Okay? And phenomenal stuff has been done by your students. And this gives you a chance to show it, alright? So what you're going to do is,
822.	38:22	JL	There were three cycles this semester, Okay? The first cycle, and I kind of spelled them out for you so you remember what they were because it was like years ago when we did cycle one. That was with towers four tall, selecting from two colors. And predicting towers three tall and five tall selecting from two colors. Cycle two was towers five tall selecting from two colors, and the pizza problem, selecting form four toppings. Cycle three was towers three tall selecting from three colors and Ankur's Challenge. Okay? So, for the final project, you're going to prepare a booklet with actual samples of your students' work, Okay? Um, you are going to be selecting samples of

			student work from each of the three cycles, Okay? You don't have to do every task. You're going to be picking three samples of student work from each cycle, Okay? One sample of student work is going to be something that impressed you, as an interesting sample of reasoning. A second piece of student work is going to be something that surprised you because of the strategy or the representation selected. Or, as an impressive product from an unlikely student. Okay? Mitch might pick the student, special ed student in class today who doesn't do well in class, and yet she did a phenomenal job. So that would be something that surprised him.
823.	39:55	JL	Or you might, the third piece is a piece of student work that concerned you because the student struggles to understand the mathematical ideas. Okay? They really are showing you they have no idea what makes sense mathematically. Alright, so you are going to pick three pieces of student work for each of the cycles.
824.		J	So like nine all together.
825.		JL	Nine altogether. Now, if you say to me, "I can't limit it to nine, I have ten! I have I have another one I really, really want to put in." Do it. Okay? Um, but you have to have at least nine. Okay? So if you want to put in ten eleven twelve samples of student work over the three cycles, that's fine. But, this is not supposed to be a, you know something that's going to take you years to do. It's supposed to be something where it's going to get you to reflect. And that's, that's the beauty of this. Okay?
826.	40:50	JL	Now, you're going to want your, your booklet to look polished. You are graduate students. You are getting your masters' program in leadership. So have a cover page of your booklet. Put the name of our course on it. Your name, the school you work in, and the date. And then prior to each piece of student work, you're going to have a page showing the statement of the task, Okay?
827.	41:16	JL	So in other words, you're going to just have what the problem was that the students were working on. Um, and you can pull that right off of eCollege, Okay? Because I have all the tasks on there for you. And I also need you to fill me in, before each piece of student work. Some information about, like the, um, student. The grade and the class the student was in, the time.
828.	41:44	JL	Now, when I say time of your math period, don't tell me ten thirty in the morning. I really mean how long is your math period. Okay? I should have said the length of your math period. Okay? Um, You're going to want to tell me whether the student was regular or special ed. The number of students in

			that class. And any other pertinent, pertinent information that you think would help me understand the setting. Like was it an inclusion class? Was it a self-contained class? Is it the enriched class? And don't assume I know what "enriched" means. So just really explain to me what the class is all about. Okay?
829.		RB	I've got a question.
830.	42:22	JL	Sure.
831.		RB	And this is a poor math question.
832.		JL	Sure.
833.		RB	So, let's say we do like the first, the towers four high problem.
834.		JL	Okay
835.		RB	So we have the paragraph describing that.
836.		JL	Yes
837.		RB	Would we put the information about the students just below that?
838.		JL	You could put it on another piece of paper.
839.		RB	On another sheet, so
840.		JL	Yes
841.		RB	So introducing each one of these nine, or ten, or eleven, or twelve, we should have the problem task just printed out. And then, in a second page on that
842.		JL	Yes
843.		RB	Before their work, explaining about the student and the math class. Now, um, if it's through the, throughout, like, uh, consistent throughout, because I used the same class. Still give the same information for each, each of them?
844.		JL	Exactly, exactly. Just copy it over again.
845.		RB	Yeah, I know that's not a big deal.
846.		JL	Because it's. Otherwise it will be very hard for me to go back and forth to say,
847.		RB	Mm hm
848.		JL	"Oh, is this the same class?" or "What was the" and then flipping back and forth through your booklet. So before each piece of student work, you want to write that description of what the class looked like. What the setting looked like.
849.		RB	So, by using what we've learned, it's at least like twenty-seven pages now.

850.		JL	Ha ha ha
851.		RB	Ha
852.	43:25	JL	Oh, okay. Alright, now,
853.		RB	I figured I would complicate things.
854.		JL	Let's see. Okay, The student work has to be easily read. Okay? So that means work that's done in pencil sometimes doesn't copy good. Work that's in color, definitely doesn't copy good on a black and white copier. So when, I encourage you, use the original copies of student work. Because it will make your paper more powerful if we can really see what the student work looked like. Okay?
855.	43:57	JL	And what you're going to do is: For each piece of student work that you include, you're going to, uh, put a reflective piece saying "Why did this work impress you?" "Why did it surprise you?" Or "Why did it concern you?" Okay? So in other words, when you put in a piece of student work. Remember for each cycle, you're going to be putting in three student work. You're going to be putting in one that impressed you. One that surprised you, one that concerned you. Alright? Now, when you put in one that impressed you, I want to know "Why did it impress you?" When you're putting in the one that surprised you "Why?" Why did it surprise you? And, concern you, "How?" Okay?
856.	44:40	JL	At the end of each cycle, you're going to include a paragraph about the intervention implementation. What you learned about your students, what was good about the session, what could have been better, and what you would do differently if you had the opportunity to do it again. Alright, so it's a reflection. Not always do things go well. Um, I've had classes that I've taught and I said "Oh my God, what was I thinking? It didn't work!" And then I reflected and said "Well what could I have done differently?" So that the next time I do it, it would work.
857.	45:18	JL	So you're going to be writing a reflection. You're not doing it for each student. You're doing it for each cycle, Okay? So you're going to have three reflections. Okay? And then the conclusion of your booklet is one page. And you're going to reflect on what you observed about your students' reasoning. And what your role as the facilitator over the course of the three cycles.
858.	45:41	JL	Do not do more than a page on that. Okay? In a page I would like you to address these questions: "What did you learn about the mathematics?" Because remember, we were students. We did all these tasks before we even brought them to our students.

			Okay? So you're going to say "What did you learn about the math, yourself?" "What did you learn about your students' reasoning and mathematical thinking?" "And what, if anything, emerged from implementing the three tasks, cycles of tasks? Both for your students, and for you as teacher." Uh, you might talk about connections that were made, or deepening of understanding, or whatever else. Okay? Any question? Yeah, Kulsom.
859.	46:22	K	Um, after, after the, I think it's paragraph. It says "Students' work"
860.		JL	Yeah
861.		K	"Whenever possible, please put original copies of student work."
862.		JL	Yeah
863.		K	"Write a reflective description" uh, "for each piece about either why it impressed, surprised or concerned you." Is that a paragraph also, or longer?
864.		JL	Just a paragraph.
865.		K	Okay.
866.	46:39	JL	Quick. You know, because I, you may think it's real obvious why it impressed you. And I'm not going to be sure. And I can't ask you, so you're going to have to write it for me. I want to get into your head and what made you think it was impressive. Or, "What concerned you about, uh, you know, the student's work?"
867.		K	Okay
868.		JL	Alright?
869.	47:00	RB	So, before each one, we have the, the actual assignment. Then we have about a paragraph, just the class, the time of the class. Or a couple sentences or a paragraph.
870.		JL	Mm hm
871.		RB	And then at the end, we kind of talk about why it surprised us, or, okay, that's about a paragraph.
872.		JL	Mm hm
873.		RB	Then at the end of each cycle, there would be three. A group of three.
874.		JL	You got it.
875.		RB	We would go with about a paragraph
876.		JL	Right

877.		RB	Of just reflection of that one cycle.
878.		JL	Ha ha. Yes
879.		RB	Okay, so you do that three times.
880.		JL	Yes.
881.		RB	And after that, now at the very end, we have a conclusion answering these questions, no more than a page.
882.		JL	You got it.
883.		RB	Thirty six.
884.		JL	Ha ha
885.		RB	No, I'm just kidding.
886.			Ha ha
887.		RB	Cover page makes thirty-seven.
888.		JL	And yeah, and I think what you really, really want to do. And it's not, you don't have to make it real fat. What you want to do is make it very thoughtful. That's what I'm looking for. I'm looking to see you reflecting, over the course of the semester so that you are actually. And I think you've been doing that. I don't think this is going to be hard for you. Because I think you actually have been really looking at your student work. A lot of the student work you've shared here might become part of your booklets. It might be that you get a chance to look through the rest of your student work and say "Oh my God, I wish I had shared this, because this would have been a perfect example."
889.	48:19	JL	So pick what you want three student work for each of the cycles, okay? And again, don't pick all, ones that impressed you. You want an impressed, surprised and concerned.
890.		RB	Can we keep this, the same student throughout, like cycle one
891.		JL	If you want to
892.		RB	So,
893.		JL	If you want to.
894.		RB	Okay
895.		JL	I'm not, you don't have to though.
896.		RB	But we could if we wanted to.
897.		JL	You are going to do whatever makes sense to you.
898.		RB	Okay.
899.		JL	Okay? You actually can see growth, and the reason why you work with the same classes over the course of the semester is so

			you could see growth.
900.		RB	Mm hm
901.		JL	Okay, so if you choose to use the same student's work over the, all three, that's fine. You don't have to, Okay? Um, in the syllabus, you had originally seen, I had written about having a reflection paper at the end. This is going to be included in here.
902.		RB	Okay
903.		JL	It made no sense to me to have the reflection separate from this, Okay? So when you do this, this will have everything that will be required for the semester. So this will be handed in December fourth.
904.	49:21	JL	After December fourth, on eCollege will be the post-assessments. Because remember, this was a grant, and remember that we're trying to see growth, and we're trying to see.
905.		RB	Mm hm
906.		JL	And remember, you took those pre-assessments at the beginning?
907.		KK	Mm hm
908.		JL	Well, there's going to be post-assessments. And they're going to be online, just the way you did them online before. They'll be posted after the December fourth meeting. And you will have that week to do the post-assessments. Get them done that week. Because I'm going to want to be assigning grades and I've been told I cannot assign any grades unless the post-assessments are done. Okay? So don't let that get in the way. That's not going to count towards your grade, but it's, it has to be done in order to get a grade, Okay?
909.	50:07	RB	I have another question.
910.		JL	Sure
911.		RB	Not that I don't like talking to everybody in this room, I really do.
912.		JL	Ha ha
913.		RB	I thoroughly enjoyed it. People I don't get to see every day. Are our threads, uh, done for the semester, or no?
914.		JL	Yes.
915.		RB	So we really don't need to, uh.
916.		JL	No, that's not true
917.		RB	Okay.

918.		JL	Okay, good question. He's saying "Are our threads done?" In terms of posting, an original response, and a reaction to two of your classmates each, you know, time each week. That's done.
919.		RB	That was my question.
920.		JL	Alright, however, I am keeping the threaded discussion up because you may, while you're working on this paper, may say "Oh my God. I really wanted, I don't know, I don't understand."
921.		RB	Mm hm, Okay.
922.		JL	So you, it's a way for you to talk to me, or you might want to talk to each other. But you are not required to post anything, and you are not required to respond to postings. I will, though. I will be looking at eCollege.
923.		KK	And what you said over the last week that you posted, You had posted the assessment, I saw it said that.
924.		JL	Yeah, It's there.
925.		KK	And you will post the, that's where we would post, under that?
926.		JL	Yeah. Exactly right, that's right
927.		KK	Okay.
928.		JL	That's right, and you'll have it next week too. You'll have another eCollege posting that says "Keep working on these papers." Okay? You know keep, you know, finishing because they are due December fourth. And then I'll have another place for you to post.
929.		KK	Okay.
930.		JL	That's your opportunity to ask questions. And, if you want to talk to each other, you can. Okay? While you're working on it, you might say, "Oh my God, I'm having trouble doing blah blah blah blah. Are you having trouble?" And then if you have a nice colleague, they may say to you "Well this is what I'm doing, and it's working." Okay? So, definitely, eCollege is still up and the way for us to talk is still up. I will be checking it religiously. Okay, I, I really have fun looking at it. Um, and I will be there to answer if you have any questions of me. Okay? Uh yes? Kulsom.
931.	51:54	K	Are we going to get information on registering for the next course?
932.		JL	You know, they asked me that in the other class too. And that's a Marjory question.
933.		K	Okay.
934.		JL	Okay

935.		A	And, I for the only thing I know, about that is, since I think everybody is still a non-matriculated student,
936.		JL	Ah
937.		A	You have to wait a week, or something later than the matriculated students
938.		K	Okay
939.		A	Which, in your case, doesn't matter. Because classes are not going to be closed, or anything like that.
940.		K	Okay.
941.		A	Um, you have to get a number from Marjory again,
942.		K	Okay
943.		A	To get into that next class. The next class it Yot's class
944.		RB	Mm hm
945.		A	It will meet Thursday night
946.		RB	Mm
947.		A	Thursday nights?
948.		RB	Mm hm, Thursday nights, yes.
949.	52:33	A	Yeah. Um, at Rutgers. Um, and so you should get some kind of, maybe Marjory will send out a note that now it's time to register.
950.		JL	But it's okay
951.		K	If you still have to get accepted to Rutgers.
952.		RB	I want to stay together.
953.		KK	Did you get, did you find out?
954.		K	I didn't yet.
955.		KK	I just looked today, it said, "No decision."
956.		RB	I applied back in September.
957.		K	Yeah
958.		RB	For the grad school.
959.		JL	If you have any questions, about the course,
960.		RB	Marjory would
961.		JL	Next course
962.		K	Ask Marjory.
963.		A	She's the person to ask, and I know it's still in the process because they, they don't exactly do it rolling. They, so the fact

			that you sent it in September
964.		RB	It makes no difference
965.		A	Probably doesn't make at all
966.		RB	They get a stack this high.
967.		A	Yeah. And so I know they're going through them. Now, for people who have applied for the masters or
968.		RB	Okay
969.		A	My understanding, for people who are applying for.
970.		JL	That's good Linda's here, because we didn't have you Tuesday.
971.		A	Yeah
972.		JL	So I said "Ask Marjory."
973.		A	Yeah, well, I find out stuff like this from Marjory, so.
974.		JL	Good.
975.		RB	
976.	53:26	JL	And, um, the next group that's going to take this class, that I think next fall?
977.		A	Yes.
978.		JL	Okay, okay. Good, good. I ,
979.		A	Ann,
980.		JL	Yep
981.		A	Ann and, um, Ken from here and
982.		JL	Oh, okay, good.
983.		A	And Who.
984.		JL	Okay, good.
985.		A	He's going to take it next fall.
986.		JL	Good. Excellent.
987.		A	After the summer instead of
988.		JL	Good.
989.		A	You folks taking it. So that he can get that done too.
990.		JL	Good, good.
991.		A	And, uh, because you were in Long Branch and Toms River, too?
992.		JL	Yeah.
993.		A	That person from the summer. Whose name right now I blanked

			on, O'Keefe,
994.		RB	Jackie
995.		A	Jackie. Was she able to.
996.		JL	No, she never came. She never came.
997.		A	No.
998.		KK	She's had, she's had back problems.
999.		A	She dropped, so yeah.
1000.		JL	So she might come in next fall too?
1001.		A	Yeah
1002.		JL	Good. Excellent.
1003.		A	That was her intent.
1004.		JL	Oh, okay.
1005.		A	She couldn't do it this often.
1006.		JL	Term. That's great.
1007.		A	So.
1008.		JL	Okay, that's good, that's good,
1009.		A	Are you all done?
1010.		JL	Yeah
1011.		A	Could I say one thing.
1012.		JL	Sure.
1013.		A	As you all know, um, this class has been part of a research project, as you know. And from NJ PEMS, PEMS own, I call it PEM. Somebody at the graduate school of education calls it PMS.
1014.			Ha ha
1015.		KK	Oh, so that's not a bonus if we put we were a fellow for it.
1016.			Ha ha
1017.		A	You guys might be,
1018.			Ha ha
1019.		A	But, you know, we have a research component as well. That we are required, you know, I mean most of our project is really about, you know helping teachers gain content knowledge and all the rest of it. But we do have a research component that we are getting moving. And so, some of you may have received emails, asking, uh requesting time to interview you about different things. There are going to be about three or four

			different studies going on.
1020.		JL	Mm hm
1021.		A	And so we hope you'll cooperate, and we're trying to figure out a way to get people paid a nominal amount of money for your time. Including those folks who have already been gracious enough to talk to Catherine. Um so, we just want to let you know. And so that's going to be coming and Sunita's going to be one of the people who in January will probably be trying to get in touch with people.
1022.		SB	And I'm going to be coming to Yot's class, so I can ask you in person as well, but I might be sending out some emails in late January, but.
1023.		JL	Excellent, good.
1024.		A	So we really appreciate it because you know.
1025.		RB	Mm hm
1026.		A	Part, part of what we have is the evaluation and all those tests that you had to do. Um, and that's the formal evaluation part
1027.		JL	Mm hm, mm hm.
1028.		A	You know, but then we also have some other things
1029.		JL	Mm hm
1030.		A	Teachers and faculty member. We're interviewing, Sunita's been interviewing all the faculty members who have been involved in teaching during the summer, or will be teaching next summer. You know it's part of their reasoning and their thinking about these things
1031.		JL	Mm hm
1032.		A	And so she may ask similar questions to participants, and
1033.		JL	That's great.
1034.		A	Maybe to some students as well.
1035.		JL	Yeah, very good.
1036.		A	Yeah.
1037.		JL	Sounds exciting.
1038.		A	So it's thanks.
1039.		JL	Yeah, good.
1040.		L	It won't be big money, but we'll try
1041.			Ha ha
1042.			We have to figure out how to do it through the university.

1043.	56:35	JL	Were there questions that you had? Yes, Justin?
1044.		KK	Oh, you were one?
1045.		SB	She would not even tell me who it was, and I said I promised everyone anonymity, so.
1046.		JL	Oh, Okay, okay. Justin has a question.
1047.	56:48	J	Just like, for instance, some of the students who really did, like who performed, like, you know, unsatisfactory. I would say like initially,
1048.		JL	Mm hm
1049.		J	And like I wasn't impressed.
1050.		JL	Right.
1051.		J	You know, so
1052.		JL	Right
1053.		J	But I just keep, like say I had two. Because I had a short amount of group to choose from.
1054.		JL	How many, your, self-contained classes? How many students?
1055.		J	Sort of like a self-contained, because we do,
1056.		JL	Yeah
1057.		J	We go with the treatment program
1058.		JL	Okay
1059.		J	But for the kids that I actually, did have that hour with, it was those four students.
1060.		JL	Oh, that's very little.
1061.		J	Yes
1062.		JL	Yeah, you don't have a whole lot of choice. And that's, you only did a group of one class of four students?
1063.		J	Well, my other math class was forty minutes.
1064.		JL	Yeah
1065.		J	But they were
1066.		JL	Yeah
1067.		J	They were sixth and seventh grade. But they were, um, like I didn't put a lot of, like, not a lot of time effort, but their behaviors were whacky.
1068.		JL	Okay.
1069.		J	From the start. Because that's the type of population I deal with.

1070.		JL	Sure, sure, sure.
1071.		J	The achievement pilgrimage.
1072.		JL	Sure, sure.
1073.		J	Yeah, they. You know, they're whacky a little bit.
1074.		JL	Yeah
1075.		J	You know, so, uh like some of them weren't that impressive initially.
1076.		JL	Okay, okay.
1077.		J	Do I, do I?
1078.		JL	So you're saying, out of four it might be hard for you to find in cycle one an impressive paper. Is that what I'm hearing?
1079.		J	For me, yes.
1080.			Ha ha
1081.		JL	Okay.
1082.			Ha ha
1083.		JL	Now, this is Justin, only Justin,
1084.		S	Yes
1085.		JL	Because most of you have more than four students. Is that correct? This does not hold for the rest of you.
1086.		JL	Look carefully. I'm talking to you. Look carefully. Because it would be nice to see if you had a paper that impressed you. And again, "Impress" doesn't mean they had
1087.		KK	Everything right.
1088.		JL	Picture perfect, everything wonderful. It might be that impressed you because they held a constant. Or they had the start a neat strategy. Uh, they started to be systematic. And everybody else was way off, or, you know. So I'm saying "Really look for an impressive paper, in addition to the others. Okay?" If you can't find it, in the cycle one. Then do, out of the four students, put two in one of the other categories. Okay? So it was impressed you, surprised you, concerned you.
1089.		J	Concerned.
1090.	59:03	JL	And, I hope you don't have all "concerns"
1091.			Ha ha
1092.		JL	Okay?
1093.		S	Ha ha

1094.		JL	And hopefully, as you get to, uh cycle two and cycle three, I'm hoping you're going to see impressed.
1095.		J	Oh, yes.
1096.		JL	Okay
1097.		J	Well, yeah
1098.		JL	Okay, That's all we want.
1099.		A	Justin's got the hardest, like, group.
1100.		JL	It sounds like it.
1101.		A	He does, yeah, he does.
1102.		JL	Sounds like it.
1103.		RB	He's
1104.		JL	And good for you, good for you. Some, you know, Anyone can teach the bright.
1105.		A	He does.
1106.		JL	Right? Yeah. Anyone can teach kids that want to learn.
1107.			Mm hm
1108.		JL	That are self-motivated, that are bright. But it's the real teacher that can teach kids that are tough. Right? Yeah.
1109.		RB	That's him right there.
1110.		JL	And I think you all do, you know. In different degrees, have kids that are you know, challenges. And that you really have to work hard to get them to, to move in the direction you want to go. But, you know, I, I again compliment you all. It has been a real pleasure working with you.
1111.		Everyone	Thank you
1112.			You too.
1113.		JL	Okay, you're done.
1114.		RB	Thank you
1115.			Alright.
1116.		JL	Okay, alright.
1117.		J	Thanks a lot.
1118.		JL	You're welcome, you're very welcome.
1119.		RB	I would have, I would have. If I had known, there would be cupcakes, and

12/04 Focus Group transcript

Title: 12/04 Preparation OldbridgeSayreville
Location: Rutgers GSE
Date: 12/04/2010
Length: 32:17
Transcribed by: Will McGowan May 2012
Verified by: Maddie Yedman

Line	Time	Speaker	
1.		WM	Do you have the assignment?
2.			No
3.		WM	Are they in the room? Well, I think, yeah, take that. If you don't have it, go get it real quick. Because that will be good to, good to have.
4.	0:17	WM	So, we are going to be, I guess around 10:45, we're going to be going back into that room and sharing all the stuff, uh that you feel like sharing. Until we run out of time, mostly, and then you've got to do your assessment. But what we want to do is take this time right now, to get yourself basically prepared, so like, you know: Okay, "This is what I'm going to share. This is what I'm presenting." And this kind of thing. And so what I want you to do as kind of like a start is, do you all have a sheet of paper? Something you can write on?
5.			Yes
6.		WM	If you don't I'll
7.			She's getting me one, I don't know why, she had to get me one but,
8.	0:50	WM	Haha. Because what I want you to do is just spend a minute and write about basically what you think the most memorable thing about the whole course was. What stands out to you?
9.		RB	Can I have one
10.		SS	You can have
11.		RB	Thank you
12.		SS	You have enough
13.		MS	Please, please
14.		WM	It's fine, you need some paper?
15.		SS	Here. I have paper
16.		RB	So, so you want us to write
17.		WM	What stands out to you most about the whole course. If there's one thing that you're going to remember most, what is it?

18.		RB	Whether it was something we did in our classrooms, or
19.		WM	Something you did in your classroom, something you read, something you watched. Things you thought special while people were working on the projects, anything.
20.		RB	Okay
21.		WM	It's like, "just what did you remember?"
22.		RB	Names on these too, or does it matter?
23.		WM	Yeah, it would be good.
24.		RB	District?
25.		WM	Um, name is fine.
26.		SS	Rich Babst
27.		WM	Proper heading, name, subject, period
28.		SS	We only put one thing?
29.		WM	Yeah, just like a brainstorming, you know, it's supposed to help you out a little bit.
30.	1:56	KF	One thing or just as many, one thing? I was out of the room, so sorry.
31.		WM	One thing is good .
32.		KF	So you want a couple things that we remember from our experience.
33.		WM	One thing would be fine
34.		KF	Okay
35.		WM	Like, if you can't pick one best one, then you could, but one is fine. Just to help get your thoughts flowing, on a Saturday.
36.	2:32	RB	What was the name of that boy? Brandon! Thank you.
37.		MS	That's what I'm doing too.
38.		RB	Oh, you're kidding.
39.	3:18	WM	And it doesn't have to be an essay, so.
40.			Ha ha
41.		WM	About a minute more, or so. Unless you're like really onto something, then just keep going. But don't feel like you have to fill up a whole page or anything. Done!
42.	3:45	SS	Is it alright if we're done?
43.		WM	Yeah, no that's fine, that's fine. I was afraid that some people were like "I gotta keep writing, I gotta keep writing"
44.			Hi

45.		RB	Hello
46.			We um, is it all the schools together?
47.		WM	No, we're doing our separate things.
48.			We're lost.
49.			Nice to see you, though.
50.		SS	We need some pencils.
51.			Murmuring
52.			That's where I'm from
53.		RB	He does. Mitch's the brains of our organization.
54.		WM	Ha ha ha
55.		SS	Oh, okay
56.		RB	You're the lead math teacher.
57.		MS	That's right
58.		RB	Maybe if it was chemistry it would still hold true, right?
59.		SS	So can he be with us any more?
60.		MS	He will.
61.		SS	He will?
62.		MS	He just didn't have enough to take this class. And then he will take this class, probably.
63.			Sidebars
64.			
65.			
66.	5:45	WM	So, one of the things, one of the things we're going to do is, in the long term future, like Carolyn was saying. I might send out an email for an interview. Now I had seen that the thing was Twenty-Five dollars for the interview, so I feel like, I'm kind of obligated to follow that. But the thing is, would anybody prefer to do an interview as opposed to just send out an email and then you just send this thing back?
67.		RB	What do you mean "Prefer an interview?"
68.		WM	Like, you'd rather arrange some time to meet and talk and do that, rather than send you this thing in an email, like fill in a document, and then send it back.
69.		KK	That's fine
70.		WM	That works out?
71.		M	Yeah that's fine

72.		WM	Okay. Um, I'll try to make it as unintimidating as possible, and pick the really good questions, and if it looks like it's going to take you more than like a half hour, let me know, because it shouldn't take you too long.
73.		RB	What are we doing?
74.	6:44	WM	To fill out the whole email survey questions, you know?
75.		KF	If there's some of us who would rather just talk, is that
76.		RB	Oh!
77.		WM	Yeah, yeah
78.		AC	I was confused when I read the paper. I thought it was respond to the emails one and an interview, so now it's not the case?
79.		WM	Oh, I don't know about that. That's a separate paper. I don't know about that one.
80.		AC	I would rather talk to someone than write something.
81.		WM	Okay, okay, Alright, so then we can arrange that. That's fine.
82.		RB	Okay, yeah, that helps with me too
83.		AC	Yeah, we're all at the same school
84.		RB	Yeah we're all at the same school.
85.		WM	Oh yeah, Okay, so this is the S..
86.			Oldbridge and Sayreville
87.			Yeah Sayreville
88.		KF	Those three are, one, two, three, Oldbridge
89.			Oldbridge, Oldbridge, Oldbridge
90.		RB	Well, Connie, you're healthy today. I could sit next to you.
91.		WM	Okay so yeah, could maybe arrange a time to get to Sayreville. Okay, because one of the things was we were hoping to get some questions done today, but it, knowing that you guys are going to be presenting in, I don't know, about twenty-five minutes, I figured you might want to have an idea of what you have to present. So hopefully this little thing has kind of got you started or given you something, and I want to have you share those, but first I want to see, does anybody anticipate sharing the student work that project that Judy had you do? What was it, the two that were really interesting, two that concerned you, two that… no? Do you remember that thing?
92.		KK	Yes, that's our project, yeah.
93.		WM	Okay, are you going to be sharing that, or do you want to share something else?

94.		RB	I'd rather share something else.
95.		SS	Me too.
96.		RB	Because I have other things to
97.		SS	Mine didn't involve student work
98.		WM	It wasn't
99.		SS	No.
100.		WM	Okay, so if you're not going to share that, I'd like to collect it now before I forget. If that's okay, because that would be good. Oh man!
101.		RB	Yeah, with the books
102.		JL	I'll just hold this last piece.
103.		WM	Okay
104.		KF	Those are two of ours
105.		WM	Okay. Alright, so I'll get yours. Okay, so that's good. So, um who will we start with? Does anybody want to share what they're thinking of sharing over there to kind of get everyone going?
106.	8:50	SS	Well, I was thinking about the thing that stood out to me most was when Brandon was able to make the connection between the pizza problem and the towers problem.
107.		WM	Okay.
108.		SS	Because I would have never in my life made that connection, and I think like the only way was like a connection like that could be made is by having the students, like figuring it out in their own way, with their own representation and I think that's important because often times we just like show them how to do it, like a tree diagram, or, you know, we show them the way. But I think it's so much stronger if you just present them with the problem and have them invent their own way, so I thought that was cool because I thought the pizza problem and the towers problem had nothing to do with each other.
109.		WM	Right
110.		SS	And, you know, then only by the student work, you could see that.
111.		WM	Right, Okay, Cool. Did anybody have anything to share on that one? To add to?
112.		RB	I, picked the same actually video about mine, but I remember something different that I want to see in all my students. At the end of, when his explanation of the task, he's just sitting like this.
113.		WM	Ha ha
114.		RB	And that's the confidence,

115.		WM	Yeah
116.		RB	That's the sense of knowing he did it correctly, and pride. And I want that from my students and that's what I really remember most, and if I can bring that reaction to my students, and get that reaction from even one or two of them, this course was worth every minute, of time. And it's not so much the task. Again, I was really, I remember him going like, at the end of the video and just the sense of confidence that I, that's what I remember and I'm looking for that now with my students.
117.		WM	Actually, that was a pretty good one. Okay. Any others?
118.		MS	I also chose the Brandon video.
119.		WM	Oh, yeah?
120.		MS	But it was more just surprising to me not so much the work that he did, but how she also said that he went on to become a vet, and that when he was in school, he was in like a lower level math class. He was like a lower track or something like that. And just kind of like Rich talked about, I had a girl who when I did this, and the professor came and when you came to watch. The girl who I think did the most impressive work was a special education student. And if you would ask me before this, you know where her mathematical level was, I would have put her at the lower end. But just seeing what she was able to do with this problem, and just the confidence. And, the professor actually asked her at the end "Do you like math?' And she was actually talking about how much she liked math and how much she enjoyed doing it. And I could see that confidence in her also.
121.	11:33	WM	Okay.
122.		JL	Okay, we're supposed to be going around?
123.		WM	Yeah
124.			Ha ha ha
125.		JL	I think, well, the most compelling part to me was that students bring their own individuality to mathematics and I learned that by, like, by some of the readings that we did early on. The young second grade student, like she had to do the task where they had to find like,
126.		AC	Shirts and pants?
127.		JL	How many shirts and pants, you know, could be an outfit. And she, like she was on the right path to solving the problem. But then she stopped short because in her thoughts, she thought that a white colored pants didn't match with a yellow top.
128.			Yellow shirt. Mm hm

129.			And that her thinking, you know, her just normal thinking had just come into mathematics,
130.		WM	Mm hm
131.		JL	In that type of way, and that's something that I tried to bring out in my students. You know, to get their own individual uniqueness how they approach math. I thought that was like, the most, the most relevant to me.
132.	12:51	WM	Okay.
133.		AC	For me, the most, I don't know, surprising experience out of all this was, I kind of revised the way I did cycle one, cycle two and cycle three. During cycle two and three, before the students started their activities, I individually went around and asked them to predict how many towers I thought,
134.		WM	Oh, okay
135.		AC	They thought they were going to make. And it was pretty surprising how many of them were able to correctly predict. But when asked about it, they had no idea. They were like "Well, I know you multiply" like they knew the algorithm
136.		WM	Mm hm
137.		AC	To find it, but couldn't tell me where they've heard it before.
138.		WM	Right.
139.		AC	I said to them "Did you learn this before?" "I don't know, I don't know." So it was just interesting to me that they actually had some memory of this. We've never done it in seventh grade yet. And they knew what it was, but it was hard for them to explain.
140.		WM	Right
141.		AC	So,
142.		WM	Okay.
143.	13:36	KF	Something that stands out for me during this course was experience with one of my students. I initially had to separate him from, I talked about it during our meeting. But I separated him, due to behavior, from his partner. And when he worked on his own, he was actually one of two students out of my total kids that was able to successfully complete Ankur's Challenge.
144.		WM	Oh, wow.
145.		KF	And it was, I don't know. When he was by himself, he was very focused.
146.		WM	Yeah
147.		KF	He actually almost did it like Romina, you know, like where he held

			for constants and then just switched the other two colors. So, I don't know. He was pretty proud of himself. I told him, you know, "I showed your work." Like he was so happy.
148.			Ha ha
149.		KF	And I was extremely surprised. Because he's not at the higher end as far as math, like Brandon.
150.		WM	Right
151.		K	So
152.		WM	Now I'm curious, is he, was he generally like a trouble maker, like in the class?
153.		K	He's not, you know what it is, he's not special ed, but he has a 504 for ad, add.
154.		WM	Okay, yeah
155.		K	So, you know, I think sometimes he just likes to, it's hard for him to focus.
156.		WM	Yeah
157.		K	So, when he actually had somebody else not with him. I'd just see him with the blocks and I went over there and he was actually able to explain orally exactly what he was doing. He said to me, "Well obviously, if it's a tower four high, and you have, you're picking from three colors, one of them has to be repeated."
158.		WM	Huh.
159.		K	So he separated them into those groups, and he was one of the only, one out of two of all my kids, so.
160.		WM	Okay, Great. That leaves you.
161.			Ha ha
162.	14:57	KK	I went with the thing that impressed me the most, or left the, I guess, most lasting impression on me was that, how, specifically Romina. She, we saw, because we saw everything about her. We saw her start to get the idea. And then we saw her have to repeat it on the board to somebody.
163.		WM	Mm hm
164.		KK	And then, in the readings later, she did it again. And it was so organized and so incredible that that made me realize that I really need to give them that time to repeat it.
165.		WM	Yeah
166.		KK	Do it again, and explain it again, Not just, it's a one time thing, "good, that's what you did." We need to give them time to get that

			thought process going so that they can organize it more and. The more exposure they have to it, the better.
167.		WM	Right
168.		KK	Basically is what I got from that.
169.		WM	Okay, Cool. So, um, this I guess is one of those things. The course is called, right, "Lesson Study on Student Reasoning" Right? And so, one of the things we're curious about is just, what examples of student reasoning did you notice? Right? What kind of things, maybe, did you hope to see that you didn't? And what kind of things did you see that you didn't expect, but really amazed you? Basically, what experiences, specifically with the reasoning have you seen? If you could share that.
170.	16:14	K	Like, for me, one of my students on the first task, when they had to build the towers. We hadn't learned tree diagrams or organization, making an organized list. In sixth grade, we do that as a problem solving strategy.
171.		WM	Okay
172.		K	And right away, when I went over to her, she was doing, like what Angela, Tree diagram
173.		WM	Uh huh
174.		K	Initially as a sixth grader, she just wanted to do the tree diagram, wanted to see that out first. And, to her, like she explained to me, that was the most organized way that she'd be able to build the towers. In like, in the least amount of time. That's what she told me.
175.		WM	Huh. Ha ha.
176.		K	So she wanted to be the most efficient math solver.
177.		WM	Right. Did anybody else see something like that? Going for efficiency.
178.	16:47	S	I think, like, well.
179.		RB	No, you go.
180.		S	So I guess I didn't really notice this at first, but when students had certain towers and I couldn't see the progression of like how they got this one and this one. Like what made them think to do this one and then this one. And they talked about how they would take the top block and move it on the bottom.
181.		WM	Mm
182.		S	Because they kind of wanted to use the same block, but cubes that they were using, made color, same number. And I noticed that a lot of students did that. After I noticed the one pair do it. But yet, in their writing and explanation, they never said anything about how

			they did that. So, I guess, that's almost like recursion when
183.		WM	Mm
184.		S	But I just thought that was interesting when they didn't describe it at all on their paper.
185.		WM	Did anyone, that's actually is a big one that I'm kind of interested in. Did anybody see that? Where they basically could think a lot better with their hands, than they could writing down?
186.			Mm hm
187.			You saw it?
188.			I think all of us did.
189.		RB	I think all of us did. And even with our threaded postings. I know I posted a lot of frustration on that because as I was discussing with my students and asking them questions "Well, how do you do this?" They explained exactly how they did it, and they had their towers set up nicely, but when they wrote their towers from their desk to on the paper, they even flipped some around. And I noticed that when I was going through some of the work, even to select it for, for the project that we just turned in. And it was a little frustrating at times, with that. But I know that verbally, talking to them, they really understood it. And that was something that I was happy with. And I guess that I wish I had video cameras
190.		WM	Ha ha
191.		RB	Set up in my classroom.
192.		WM	Yeah, it would be nice it's true.
193.		RB	Or some tape recorders. And then I could go back and reflect on what they really did. And, um , and looking at their work it did spark, and I'm like "Wait a second! This one and this one were here, when I spoke to them."
194.		WM	Yeah
195.	18:42	RB	And I did have a difficult time. But what I did notice was from the first task to the, to Ankur's Challenge, their writing got much better.
196.		WM	Okay
197.		RB	And their description got better and I see that carrying over into other areas. I gave some open ended word problems in my problem solving class the other day. And their explanations are getting much, much better. Form this class. I only did it in one class. And I would say they're a little more advanced than my other two classes at this point. Probably because they were working on this stuff.
198.	19:10	MS	I saw that a lot of the kids in the first one, they wanted to just list everything. They would just, like, they would use letters. It would

			just be BYYB, BBY. They just wanted to write their answer down, and then be done with it. And a lot of times, I would say, "Well can you explain this?" And they would say it verbally, and I would say "Well, can you write that down?" They'd say "You want me to write ALL of what I just said down?"
199.		WM	Ha ha
200.		MS	And they, they were kind of overwhelmed at first.
201.		WM	Yeah.
202.		MS	But, kind of like Rich was saying, they got a lot better at kind of organizing their thoughts. And if they had something, where they had like, a lot of people saw the staircase pattern. They would talk about, instead of trying to explain it in too much detail, they would say "This is what we did." And then just kind of like "reverse the colors." So instead of, you know. When I first said "Can you explain all that?" After all they just said, to say "Well, put that all down on paper." You know, they were very overwhelmed by that. But as they start to get a little better at it, they can kind of explain, or like what they meant by "Opposites" or they had some other strategy like "we did opposites and then we reversed the opposites." And you know, they kind of
203.		KK	Like what does that mean?
204.		MS	Right. And they kind of realized that they didn't have to explain for every single one, if they just get through
205.			Right. that was neat, actually.
206.		MS	If they could just get you to understand what they mean by "opposites" and "reversing the opposites"
207.		WM	Right.
208.		MS	If you just give an example, or make someone understand that, that's very convincing. And they started to see that after a while.
209.	20:31	KK	I think that was the biggest progression I saw. When they realized like, if they did it for like the red. And this was like going into that last, the last cycle. They realized that if they said, As some of them explained in detail, what they did. I guess "staircase" or whatever you want to say, and they realized that they could say "And we repeated that same pattern for the reds on top," or "with the same pattern for the blues on top." So I thought that was a huge progression. Although, my source of frustration was completely in their writing.
210.		WM	Ha ha
211.		KK	They were so good, they were so good verbally, and it almost nothing transferred into what they, into what they put down on

			paper.
212.	21:09	J	I think the, like the whole length of the whole project was challenging for some of my students. Like, they, they weren't used to having one problem take that long.
213.			Ha ha
214.		J	And that sort of thing, so, Like the guys that I teach, they are. Like they are usually, like quick and like, they get frustrated very easily and that sort of thing. So like in twenty minutes, they were. Like the novelty had worn off. And like they pretty much struggled. And it was a challenge to try to get them to actually write more finishing. And finish with care. The same care that they started off with.
215.		WM	Yeah
216.		J	And they, like they often become very frustrated. With that, and that sort of thing. Like the length of it was challenging for them.
217.		WM	And that was, that was consistent throughout the whole, like all three tasks basically, the length was always an issue for that group?
218.		J	Pardon?
219.		WM	Like, did they seem to, Did they seem to get more used to it, or they were still just as frustrated?
220.		J	That's a good question, like, at the beginning, like they had like enough like strength and endurance per se, to carry out the task,
221.		WM	Uh huh
222.		J	But like, by the end they said "Dag, like we got to write this again?"
223.		WM	Yeah, yeah.
224.		J	And that sort of thing, so like, they had that wall that they put up
225.		WM	Yeah, sure
226.		J	Already, like right from the start. Like from the second and the third one. So like the first one, like actually was a little bit better, in some instances. But it didn't last. But, like some of my female students they actually, they did well throughout the whole thing. Like they actually progressed, with their, with their strategies, and thinking and that sort of thing.
227.		WM	Okay.
228.		J	But the fellas, they were all finished pretty quick.
229.		WM	Ha ha, yeah.
230.	23:03	KK	I think maybe they thought "I just explained it to you."
231.		WM	Yeah
232.		KK	"Why do I have to write it?" Like, I think they thought.

233.		WM	"I did all of the work"
234.		KK	It was redundant, you know?
235.		WM	Okay.
236.		KK	"Well, I said that to you" I don't know.
237.		WM	Yeah.
238.		KK	That's the kind of feeling I got.
239.			Yeah
240.		KK	And I had one or two of them say "What is the purpose of this?" And I'm like, and I simply said "To see how you think. I just want to see how you arrange it, how you think, what your thoughts on it are, on how to put this together" But I did have a couple who really were, like you were saying, really thought it was a little bit too… They want an answer. They're so quick to, you know
241.		WM	Yeah
242.		KK	"What's the answer?" And, but they did get better at it, and the more we went on, I think they realized, maybe it wasn't about the answer. It was about how they arranged it, and got their… That they could come up with. Rather than an answer.
243.			You know what I found funny about how you were talking about how, like, their verbal statements didn't match like what they wrote.
244.		KK	Mm hm
245.		A	A lot of times, like, after reading their responses on paper, I feel that they in their heads think that that's clear.
246.		KK	Right.
247.	24:00	A	And it's not until you actually sit down, like with their work and be like "what are they trying to say?" That you realize, like, okay, if you spent like maybe two more sentences on this specific thing, I would have been able to understand it better.
248.		KK	Right
249.		A	Instead of sitting down for twenty minutes and trying to figure out like "What were they saying?"
250.		KK	Mm hm
251.		A	So I think that there's, like a disconnect.
252.		KK	Yeah
253.		A	Between what they think they're conveying to you and what they write.
254.		WM	Yeah.

255.		A	Because I think they think they are clear.
256.			Yeah
257.		A	I really do, I really do.
258.		RB	I think there's some nervousness going on there as well. When I was looking at some of the papers for my project, I actually gave a girl back one of hers. I know it was three weeks after we had done the task, because she put down thirty-four instead of thirty-two. Now, she's a gifted and talented student. She's part of the tag program. And I gave it back to her. I said "Count how many you have here. I just want to make sure that it matches." And she had thirty-four written down in her written response. She clearly had thirty-two towers. You could clearly see what she did, and then I said "What happened?" She said "Oh, I was really nervous, when I was writing it down."
259.		WM	Ha ha
260.		RB	But she had time to go back.
261.		WM	Yeah
262.		RB	And I asked her to change it because I really liked her process and I did use it for one of my, uh, for one of my selections. But um, I think they just get nervous sometimes. And it's more about a grade that they're getting. And I told them that they weren't being graded on it, and
263.		WM	Yeah
264.		RB	And they are pre-programmed form years past, now there's a lot of things I don't assess them on formally,
265.		WM	Yeah
266.		RB	And they have a hard time with that.
267.		WM	Yeah
268.		RB	They really do.
269.		WM	So, one of the things that seems to come up is… In general you guys seem to learn a lot, but there were some frustrations. So I'm curious. If you were to do these again, would you make any modifications or changes? I mean what things would you maybe do differently, or would you do it the same, or would you use the tasks again?
270.	25:43	A	I think, like as Kathleen said, I think it was very difficult for the kids to see the connection between what we're doing, like in class now. Like, for example, with our curriculum. I think it was very difficult that they're like, "Okay, great, we're doing fractions, we're doing proportions. And tomorrow, you want us to build towers?"

271.		WM	Ha ha yeah
272.		A	Like, I just felt that, and I realize that life isn't organized in that way,
273.		WM	Mm hm
274.		A	But, at the same time, it was hard for the children to see this concept.
275.		WM	Sure
276.		A	Combinations isn't something in seventh grade that we really focus on, until possibly June, if we even get to it.
277.		WM	Right.
278.		A	So it was very difficult for them to kind of say "Why are we doing this?"
279.		WM	Mm hm
280.		A	So.
281.	26:22	K	For me, actually. Well, I guess I could talk about this later, but.
282.		WM	Ha ha
283.		K	Doing this first, I, we do teach a strategy of making an organized list in sixth grade.
284.		WM	Uh huh
285.		K	And I actually taught it one week after the last cycle. Like, the last cycle's tasks. And they did wonderfully with it.
286.		WM	Alright
287.		K	Better than any of my other years ever.
288.		WM	Mm hm
289.		K	So, I can say that it directly helped improved
290.		WM	Okay.
291.		K	They were like "This is so easy." And now, they were actually able to relate. And I talked about this in my paper. They were able to relate combination problems and to the tower problems and pizza problem.
292.		WM	Oh, cool.
293.		K	And I was like, I said "What does this remind you of?" And they said "The pizza problem." I said "Well how is this answer a little bit different?" "Well, because of this money problem, they're not asking for if you give no money." Whereas with the "no toppings"
294.		WM	Oh, okay.

295.		K	So they were actually able to relate it on their own.
296.		RB	I'd like to do this at the very beginning of the year. Like maybe the first two weeks of school.
297.			Yeah
298.		RB	I, we started this three weeks into, or four weeks, or a month into school. I think it's a good ice breaker for the students.
299.		WM	Yeah
300.		S	Yeah
301.		RB	And it gets them used to working together, possibly.
302.			Yeah
303.		RB	And it gets them comfortable with you being a facilitator in the classroom.
304.		WM	Yeah
305.		RB	Versus the instructor. And it, for our course in problem solving, It would really show them what problem solving's really about. Instead of explaining it, well,
306.		WM	Ha ha
307.		RB	If this is what we do in problem solving, they would actually get to see it hands on. So this might be something I would do at the very beginning of the year, and then maybe one a marking period.
308.			Mm hm
309.		RB	Maybe the four towers, two colors the first marking period, and then go to five high two colors. And then the third marking period, do the pizza problem. And I could work it out that way.
310.		WM	Okay
311.	27:55	RB	But I would like to start it right off the bat.
312.		WM	Right.
313.		S	I think, like a modification I would make, because students in math, I think, are so used to "Either it's right or it's wrong."
314.		WM	Uh huh
315.		S	And, like our students, a lot of them are motivated by what grade they are going to get on the assignment.
316.		WM	Yeah
317.		S	And when you ask them to explain, they're like "is this good enough?"
318.		WM	Ha ha, yeah

319.		S	"Is that the right amount of time?" like "Is this good enough? What I wrote."
320.		WM	Yeah
321.		S	And sometimes kids would write "oh you know, twenty-seven stood to be the right answer because we counted them and couldn't find any more." And it's like "I'm not convinced." And I wrote in the assignment that I would make a rubric for what the explanation should be like. And not particularly whether the number of towers is right or wrong, but you know. More like the justification of it. Because I think, like they are used to seeing rubrics and writing in language arts.
322.		WM	Right
323.		S	But in math, you're used to just getting marked, like right or wrong.
324.		WM	Mm hm
325.		S	So, I said I would create a rubric for it, and, you know, show them ahead of time.
326.		WM	Okay, Justin
327.		J	I think something for me, like that I actually worked on, like with my students was that… That just like regular problems, like on simplifying equations, or expressions, like I would have them say like "Is their thing right?" Like I would have them, like say like a student did something on the board.
328.		WM	Mm hm
329.		J	Like, I would propose the question to the students. Like "Are you guys convinced that she's right?" You know? Or "he's right?" And how are or how aren't. And they would ask me "Is this problem right?" And I'm saying "I don't know. Ask your peers. Ask your peers."
330.		WM	Mm hm Right. Right.
331.		J	Then they would actually have to prove out mathematically, what they did and explaining each step. And then Some students would catch up, like on a step that they did right or did well.
332.		WM	Mm hm
333.		J	And then they could say that "No, that's not correct. And this is why…" And that sort of thing. So just every day math that we did.
334.		WM	Mm hm
335.	29:56	J	Like I had brought in this concept of proving and convincing.
336.			Mm hm
337.		J	And like on student reasoning, so that was helpful in our projects. I

			actually think, so. Like throughout the year I'm going to just keep going back to that type of thing. I'm not saying what's right or what's wrong.
338.		WM	Right
339.		KK	I find myself saying "I don't know, you tell me."
340.		WM	Right
341.		KK	All the time, now. "I don't know, you tell me." And they seem to be doing a little more of that. Or they are more comfortable doing more of that.
342.		RB	As students get more comfortable with that. I'm doing a lot of different things in problem solving this year. I had somebody come in for a PLC group to watch as we tested divisibility rules of seven and eleven.
343.		WM	Oh wow.
344.		RB	And they kept saying "Oh! Look what I found. I found a pattern." I'd go "Okay, well how does this pattern help you with this number?"
345.		WM	Ha ha
346.		RB	And it's really, it's like a nine digit number. "Oh man." But they were persistent, and they know how to convince a little bit better now.
347.			Mm hm
348.		RB	And we've eventually got closure on it. But I see that reaction from them. They want to convince you now. And it's not like "Well, it's wrong." My questioning is better. I'll be like "Well, will it work for this number?"
349.		WM	Right.
350.		RB	Instead of saying "It doesn't make any sense." Then they can make sense out of their own work.
351.		WM	Okay
352.		MS	I think I would also like to, the videos that we saw. I think I'd like to, if we're even allowed to show those to the kids.
353.		WM	Oh, okay.
354.		MS	Because even when we did this problem for the first time. And, you know, they would say "is it, or how do you know?" or "Convince us."
355.		WM	Yeah
356.		MS	At first, you know, my first thought was "I just know it's right."

357.		KK	Right.
358.		WM	Right.
359.		MS	You know, and I'm thinking "How can I…"
360.		KK	Even we were thinking that.
361.		MS	Right. I'm just like "I know it's right, can we move on." And that's a lot of the way that the kids were thinking.
362.		WM	Uh huh.
363.		MS	And just to say "Well how do you know?" or "Can you convince me?"
364.		WM	Uh huh
365.		MS	A lot of times, I just don't think they even know what that means at first.
366.		WM	Right.
367.		KK	Right.
368.		MS	And the more they see it. A convincing argument, or the more they get to practice it, that's when I think they really get better at it.
369.		KK	Mm
370.		MS	And I think I'm just seeing those videos especially, because some of the reasoning in those videos, were just, you know when we saw them, were just really impressive.
371.		WM	Great, yeah. I think that's one of the things
372.		A	That's really great. I'd like it too.
373.		WM	That's a good one, I think to end on. I know that we're just about time. But I think that's something we could probably do.
374.		KK	Yeah. That would be neat.
375.		WM	Thank you.

Appendix E Online Discussion Questions

Prior to September 11, 2010:

Complete pre-assessments using the eCompanion course web site. These assessments must be completed prior to the on-campus class session on Sept. 11th.

September 11, 2010:

Class Activities: Introduction to the course; Engage in 4-tall Towers selecting from 2 colors problem-solving task, with problem extensions and focused discussion about representations. Review syllabus and discuss course requirements.

September 16, 2010:

Discussion of classroom implementation

1. At Rutgers, after building 4-tall towers, selecting from 2 colors, and listening to the arguments shared by your colleagues, what, if anything more did you notice in the video of the children that you watched doing the same task?

2. Before doing the classroom implementation, what do you think your children will predict (without building them) for the 3-tall and 5-tall towers? Do you think they will say that there will be more, fewer, or the same number of towers as there were for towers 4 cubes high? What reasons will they give?

3. Before doing the towers problems with your children, predict how they might arrange their towers and what kind of convincing arguments they might give for their solutions.

Discussion of assigned reading: Combinatorics book chapter 3.

1. Compare and contrast the solutions the students found as second graders to the shirts and pants problem to the solutions they found as third graders.

Discussion of video clips: Stephanie and Dana, grade 3; Stephanie's prediction for 3-tall towers; Meredith removes the top cube

1. You have watched (Video 1) *Stephanie and Dana in grade 3* building 4-tall towers, selecting from 2 colors. Are their arguments convincing? Why?

2. In the other videos (Videos 2 and 3) you have watched, both Stephanie and Meredith make predictions for the number of towers 3-tall. Their predictions are not the same. Does this give you any insight to the way children think or reason?

September 23, 2010:

Discussion of Combinatorics Book, Chapter 4 and video, Stephanie and Dana grade 4

1. How do the children's strategies used to solve the towers problem in third grade look different than the strategies used when they were fourth graders?

2. Which of their arguments did you find convincing?

3. If you asked your students to build 5-tall towers (don't do it yet!), what strategies and convincing arguments do you think they would use?

September 30, 2010

Discussion of Classroom Implementation, Combinatorics Book, Chapter 5, and video, Milan shares his inductive argument video.

1. During your classroom implementation of 4-tall towers, did any solutions surprise, delight, or puzzle you? Talk in detail about the solution of one of your students, so we can understand what the student did and whether you were surprised, delighted, or puzzled about his or her work.

2. In Chapter 5, we see that when children are given the opportunity to share mathematical Ideas, they can contribute to the growth of understanding of their classmates. Talk about what one child in the video did that helped another child grow in their understanding.

3. In the video that you watched, did you find Milan's inductive argument convincing? Did his classmates follow his inductive argument? Give support for your answer.

October 7, 2010:

First regional group meeting activities:

Share classroom experiences and student work from Task I.

Engage in a pizza problem task: pizzas, selecting from 4 toppings. Share how solutions were found and examine representations used in problem solving. Consider how these tasks might be used in classroom instruction.

Discussion Questions:

Discussion of Combinatorics Book, Chapter 6, and video, Brandon's proof and isomorphism

1. In chapter 6, what kind of justifications did the children use to solve the pizza problems? Be sure to talk about the students' work, connecting it with the justification(s) they used when finding their solutions.

2. The researcher in the Brandon video is a very skillful questioner. Talk about some of the questions that Amy asks Brandon to help her understand his mathematical thinking and reasoning. Please be specific.

October 14, 2010:

Discussion of reading, Brandon's Proof and Isomorphism, and classroom implementation

1. In the chapter, *Brandon's Proof and Isomorphism*, we see that skillful teacher questioning can help a student think more deeply about a mathematical idea. What kinds of questions might you ask to learn more about the mathematical thinking of your students? What questions did you ask when you had your children build 5-tall towers?

2. When you implemented towers 5-tall in your classrooms, how did the strategies your children used to solve this problem look the same or different from the ones they used when they solved 4-tall towers? Did any of the solutions look similar to the way you solved the problem with your colleagues?

October 21, 2010:

Discussion of classroom implementation, the pizza problem selecting from 4 toppings

1. When you implemented the pizza task, selecting from 4 toppings, what kinds of strategies did your children use to solve the problem? Did any of their solutions look similar to the way you solved the problem with your colleagues? Talk about one student's solution that you thought was especially neat.

2. When asked if this problem reminds you of any other, how did your children respond?

October 28, 2010:

Discussion of video, Ankur's challenge and Romina's proof

1. In the video you watched, Mike and Ankur come up with 39 as their solution to Ankur's challenge. What method did they use to find their solution?

2. Approaching the problem differently, Romina came up with 36 for her solution. How does she approach solving Ankur's challenge?

3. If you gave Ankur's challenge to your children, do you think any of your middle school students could come up with Romina's proof?

November 4, 2010:

Second regional group meeting activities:

Share classroom implementation experiences and student work from Task II.

Engage in Task III: building 3-tall towers, selecting from 3 colors, and extension problem, Ankur's Challenge. Share how solutions were found and examine representations used in problem solving. Consider how these tasks might be used in classroom instruction.

Discussion Questions: Discussion of Combinatorics book, Chapter 8

1. What are some of the advantages to giving your students more than one opportunity to explain and write about their ideas? Make reference to the chapter and how it was helpful to Romina.

2. Explain why Romina multiplied by two when finding her solution to Ankur's challenge.

November 11, 2010:

Discussion of classroom implementation, 3-tall towers, selecting from 3 colors

1. When you implemented the 3-tall towers task, selecting from 3 colors what kinds of strategies did your children use to solve the problem? Did any of their solutions look similar to the way you solved the problem with your colleagues? Talk about one student's solution that you thought was especially neat.

2. If you got to do the extension problem, Ankur's challenge, what strategies did your students use to solve this problem?

November 18, 2010:

Second regional group meeting activities:

Share classroom experiences and student work from Task III.

Discuss guidelines for preparing poster presentations for December 4th on-campus meeting.

December 04, 2010:

Final on-campus meeting:

Share final projects.