THREE ESSAYS ON FINANCIAL FRAGILITY AND REGULATION

by

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My academic work explores issues in banking, prudential regulation, and financial fragility. In my first paper, I study how changes in the returns on banks’ assets affect financial fragility using a model in the tradition of Diamond and Dybvig (1983). In my second paper, the analysis is based on this work augmented to include fiscal policy and bailouts as in Keister (2015). I explore how imposing regulations on both sides of banks’ balance sheets can be used to bring about a stable financial system. My third paper further explores the policy implications of my second paper by incorporating the comparative-statics effects studied in my first paper.

What configuration of interest rates will make the banking system most susceptible to a self-fulfilling run? In Chapter 2, I study this question in a version of the model of Diamond and Dybvig (1983) with limited commitment and a non-trivial portfolio choice. I show that the relationship between the returns on
banks’ assets and financial fragility is often non-monotone: a higher interest rate may make banks either more or less susceptible to a run by depositors. The same is true for changes in the liquidation cost and the term premium. I derive precise conditions under which changes in each of these returns increase or decrease financial fragility.

In Chapter 3, I analyze a version of the Diamond-Dybvig (1983) model of financial intermediation in which bailouts create multiple distortions. Banks anticipate that the policy maker will respond to a crisis with transfers that partially cover their losses, which leads them to hold a more illiquid portfolio of assets and to offer higher payments to depositors who withdraw early. These actions, in turn, tend to make the financial system more fragile. In general, fully correcting these distortions requires restricting both banks’ choice of assets (i.e., liquidity regulation) and their short-term liabilities. However, I show that removing the policy maker’s ability to impose liquidity regulation can sometimes promote financial stability.

Chapter 4 studies how the changes in the liquidation cost influence the optimal policy mix. The main result is that implementing a regulation similar to the liquidity coverage ratio is optimal if the liquidation cost is very high. It is optimal to remove the liquidity requirement, in contrast, if the liquidation cost is lower.
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# Table of Contents

Abstract ......................................................... ii
Acknowledgements ................................................ iv
List of Tables ...................................................... viii
List of Figures ...................................................... ix

1. Introduction .................................................... 1

2. Asset Returns and Financial Fragility ......................... 5
   2.1. Introduction .............................................. 5
   2.2. The model ................................................ 10
      2.2.1. The environment .................................... 10
      2.2.2. Financial crises and fragility ...................... 12
   2.3. Equilibrium and financial fragility .................... 13
      2.3.1. The best-response allocation ...................... 14
      2.3.2. Fragility ............................................. 17
   2.4. The case with no liquidation cost ..................... 18
      2.4.1. The impact of $R$ on fragility ................... 19
      2.4.2. Competing effects on fragility ................... 22
   2.5. The full model .......................................... 24
      2.5.1. A general fragility result ....................... 24
      2.5.2. Comparative statics when $1 < \gamma \leq 2$ .......... 26
      2.5.3. Comparative statics when $\gamma > 2$ ............... 29
4.3.1. Full regulation ........................................ 72
4.3.2. Without liquidity regulation .......................... 76
4.4. Comparing policy regimes ............................... 77
  4.4.1. When $\delta = 0$ ..................................... 78
  4.4.2. When $\delta > 0$ ..................................... 80
4.5. Conclusion .................................................. 82

Appendix A. The best-response allocation .................... 83
  A.1. The best-response allocation $A^*$ ....................... 83
  A.2. The best responses under the regime with no regulation .... 85
  A.3. The best responses under the regime with full regulation .... 86
  A.4. The best responses under the regime without liquidity regulation . 88

Appendix B. Proofs of Propositions ............................ 90

Appendix C. A General Definition of Financial Fragility .... 108
  C.1. Authorities never observe the state .................... 108
  C.2. Authorities can observe the state ...................... 110

References ....................................................... 115

vii
List of Tables

2.1. Feasible cases ................................................................. 16
3.1. Four regions ................................................................. 46
B.1. The impact of the return $R$ on $\bar{q}$ with $1 < \gamma \leq 2$ ............... 95
B.2. The impact of asset returns on $\bar{q}$ with $1 < \gamma \leq 2$ .................... 97
List of Figures

2.1. The set of bank’s best-response allocation $A^*$ .......................... 17

2.2. $c_1^*/c_{23}^*$ with $\pi$ varied in the environment with no liquidation cost . 20

2.3. The measure of fragility in the environment with no liquidation cost 21

2.4. Two competing effects in the environment with no liquidation cost 23

2.5. $c_1^*/c_{23}^*$ in the economy ($\gamma = 3, \pi = 0.85, r = 1.1, \rho_1 = 1.25, \rho_2 = 1.2$) with $R$ varied ......................................................... 25

2.6. The impact of $R$ on $\bar{q}$ with $1 < \gamma \leq 2$ ................................. 27

2.7. The impact of $R$ on $\bar{q}$ with $\gamma > 2$ ........................................ 30

2.8. The impact of liquidation cost and term premium on $\bar{q}$ ............... 32

3.1. Timeline of events ................................................................. 43

3.2. The fragile set $\Phi^{NR}$ under the regime with no regulation ........... 52

3.3. The set of the best responses under the regime with full regulation 59

3.4. $\Phi^{FR}$ is strictly contained in $\Phi^{NR}$ ............................... 61

3.5. $\Phi^{NL}$ is strictly contained in $\Phi^{NR}$ .................................. 65

3.6. Adding liquidity regulation may be harmful ............................... 66

4.1. The set of the best responses under the regime with full regulation 74

4.2. Comparing fragility when $\delta = 0$ ........................................ 79

4.3. Comparing fragility when $\delta > 0$ ....................................... 81
Chapter 1
Introduction

As economists, a crucial element of our role is to aid in society’s understanding of how policy changes affect the social, political and economic world around us. My primary area of research is in the fields of financial economics and macroeconomics with particular emphasis on banking, financial fragility and prudential regulation. I have completed three papers to date: (i) studying how changes in interest rates affect the susceptibility of the banking system to a run by depositors, (ii) investigating whether and how liquidity regulation can be used to bring about a stable financial system, and (iii) characterizing the impact of the liquidation cost on the optimal policy mix. My work seeks to shed light on three questions: First, does an increase in the returns on banks’ assets promote financial stability? Second, is it desirable to give policy makers more tools to correct the incentive distortions created by bailouts? Third, is it always optimal to impose restrictions on banks’ choice of asset? My research shows, somewhat surprisingly, that the answer to all questions is negative in some cases.

In Chapter 2, I investigate how changes in the returns on banks’ assets affect financial fragility in a version of the Diamond and Dybvig (1983) model. While there has been a surge of interest in models of bank runs and financial fragility in recent years, surprisingly little is known about the comparative statics of these models. I provide a complete characterization of the comparative statics in a model where, as in Cooper and Ross (1998), there are two assets and banks face a non-trivial portfolio choice with a positive probability of a crisis. I also incorporate the limited commitment approach of Ennis and Keister (2009), which
captures the idea that banks are unable to commit to follow a particular course of action in the event of crisis. My work is the first study to fully characterize the relationship between the returns on banks’ assets and financial fragility in the Diamond-Dybvig framework.

I show the relationship between the returns on banks’ assets and financial fragility is often non-monotone: a higher interest rate may make banks either more or less susceptible to a run. This monotonicity arises because a change in asset returns has two competing effects on banking fragility. Higher long-term interest rates will lead banks to hold more long-term assets and thus become more illiquid. By itself, this change would tend to make the banking system more fragile. At the same time, however, banks will alter their payment scheme to provide a higher payoff for deposits that are held to maturity. By encouraging depositors to not withdraw early, this second change would tend to make the banking system more stable. The net effect of a higher long-term interest rates on banking fragility is thus ambiguous. I provide a complete characterization of the circumstances under which each of these two effects dominates. I then show that similar competing effects arise in other situations, such as when the short-term interest rates or the liquidation value of long-term investment changes, and characterize the net effects of these changes as well.

My interest in policy-relevant research is demonstrated in Chapter 3, which contributes to the literature on financial fragility. Following the recent global financial crisis, promoting financial stability has become an important and explicit goal for governments and central banks around the world. The Basel Committee on Banking Supervision has responded to the crisis by proposing a new liquidity standard for banks, called the liquidity coverage ratio (LCR), as part of the Basel III accords. The primary objective of the LCR is to promote the short-term resilience of banks’ funding liquidity by ensuring that banks hold sufficient liquid assets to survive a significant stress scenario lasting for 30 days. Is this new type of
liquidity regulation desirable? I evaluate this policy instrument using an expanded version of the model in Chapter 2 that includes fiscal policy and bailouts, as in Keister (2015). Anticipating receiving a real transfer from the public sector in the event of a crisis, banks increase their short-term liabilities and hold fewer liquid assets. I show that a regime in which the policy maker places a minimum liquidity requirement on banks’ assets and imposes a tax on banks’ short-term liabilities can fix the resulting distortions on both sides of banks’ balance sheets.

Is having both policy tools desirable to promote financial stability and improve welfare? Such a regime can generate the welfare-maximizing allocation of resources conditional on the financial system being fragile. In other words, if depositors are running on the financial system, the regime with two policy tools always yields higher welfare than other alternative policy interventions. However, in some cases, a regime in which the policy maker only has the ability to levy the liabilities tax and cannot impose liquidity requirements actually provides better incentives for financial stability. The financial system is not fragile when the policy maker uses only one tool in these cases, but becomes fragile when the policy maker uses two tools. There is an obvious cost to taking away the ability to impose an LCR: banks will hold more long-term assets and thus become more illiquid. This action tends to make the financial system more fragile. However, there is a benefit to removing the liquidity regulation as well. Because they hold fewer resources in reserves under the one policy tool regime, banks will alter their deposit contracts to provide a lower payoff to depositors who withdraw early. This latter effect tends to make the financial system more stable by encouraging patient depositors to leave their deposits at their banks. Therefore, while adding liquidity regulation is helpful in some economies, in others it can actually harm rather than stabilize the financial system.

Chapter 4 builds on and extends the work in Chapter 3 for discussing a liquidity regulation similar to the liquidity coverage ratio introduced as part of the
Basel III accords. I develop a general understanding of the relationship between the optimal use of policy tools and the configuration of liquidation costs. When the liquidation cost is sufficiently high, banks tend to hold fairly liquid portfolios even in the absence of restrictions on banks’ asset holdings. In fact, the minimum liquidity requirement chosen by a policy maker without commitment is not a binding constraint on banks’ behavior. In such case, by levying a liabilities tax on banks’ short-term liabilities alone, the policy maker can uniquely implement the fully first-best allocation of resources (i.e. no run-equilibrium exists). What happens if the liquidation cost is smaller? Whenever the liquidation cost is below a precise cut-off, in certain parameterizations, the policy regime with full regulation imposed on banks’ choices of both assets and liabilities can emerge as ones which will always improve outcomes and promote stability relative to a regime without the liquidity regulation. However, the opposite result would naturally arise, if the liquidation cost exceeds the critical value. I derive precise conditions under which the policy regime without liquidity regulation is strictly better than the one with full regulation in terms of financial stability.
Chapter 2
Asset Returns and Financial Fragility

2.1 Introduction

How do changes in interest rates affect the susceptibility of the banking system to a run by depositors? In many situations, the answer is not immediately clear. Suppose, for example, that the yield curve becomes steeper, with the interest rate on long-term investments rising relative to short-term rates. Banks would likely respond to this shift by changing the composition of their asset portfolio and the contracts they offer to depositors, and both of these changes will alter the banking system’s susceptibility to a run. Higher long-term rates may, for example, lead banks to hold more long-term assets and thus become more illiquid. By itself, this change would tend to make the banking system more fragile. At the same time, however, banks may alter their deposit contracts to provide a higher return for deposits that are held to maturity. By encouraging depositors to not withdraw, this second change would tend to make the banking system more stable. The net effect of a steeper yield curve on banking fragility is thus unclear. Similar competing effects arise in other situations, such as when the short-term rate or the liquidation cost of investment changes.

In this chapter, I study how changes in the returns on banks’ assets affect financial fragility using a model in the tradition of Diamond and Dybvig (1983). I show that the effect is often non-monotone: a small increase in a particular asset return may increase the susceptibility of banks to a run, but a larger increase may make the banking system more stable. I derive precise conditions under
which fragility is increasing or decreasing in each asset return. My analysis is
based on a modern version of the Diamond-Dybvig model with the following
features. As in Cooper and Ross (1998), there are two assets and banks face
a non-trivial portfolio choice. Banks make this choice taking into account the
probability of run by depositors, which depends on the realization of a sunspot
variable (as in Peck and Shell, 2003, and many others). I also incorporate the
limited commitment approach of Ennis and Keister (2009, 2010), which removes
the contracting restrictions imposed by Cooper and Ross (1998) while capturing
the idea that banks are unable to commit to follow a particular course of action
in the event of a crisis. My model is the first to combine a non-trivial portfolio
choice with limited commitment and a positive probability of a crisis, which I
show generates particularly rich results.¹

In the environment without commitment, I show that self-fulfilling runs eas-
ily emerge as equilibrium outcomes of the model. I also show that, for given
parameter values, there exists a maximum probability with which a run can oc-
cur in equilibrium. If the probability of a crisis exceeds this cutoff value, the
bank will become sufficiently cautious that running is no longer an equilibrium
behavior for depositors. This cutoff value provides a natural measure of financial
fragility; if a change in parameter values decreases the maximum probability of a
run-equilibrium, I say that it makes the banking system less fragile.

The solution to a typical bank’s maximization problem will lie in one of several
distinct cases, depending on parameter values and the probability of a crisis. If the
probability of a crisis is small, the bank will not hold “excess liquidity”, that is,
liquid assets that the bank holds over two periods if no crisis occurs. However, it

¹Cooper and Ross (1998) study a two-asset model with a positive probability of a crisis,
but with arbitrary restrictions in the banking contract. Ennis and Keister (2009) study a two-
asset model without these restrictions and with limited commitment, but follow Diamond and
Dybvig (1983) in assuming that a crisis is an unexpected event. Peck and Shell (2003), Ennis
and Keister (2010), Bertolai et al. (2014), Sultanum (2014) and many others study models with
varying sources of aggregate uncertainty, but with a single asset and hence no portfolio choice.
would choose to liquidate its investment in order to meet the withdrawal demand if a crisis does occur. As a crisis becomes more likely, the bank begins to hold excess liquidity to mitigate the liquidation costs. Whenever the probability of a crisis is large enough, the bank becomes even more cautious and holds enough resources as a provision against additional withdrawals that liquidating investment to provide liquidity no longer occurs. I derive the precise conditions under which the bank’s best response to depositors’ withdrawal decisions lies in each of these cases. I then use this solution to characterize the conditions under which the banking system is fragile in the sense that a bank run equilibrium can arise.

One interesting result that comes out of this analysis is that, under some conditions, the bank will hold excess liquidity as a precaution to mitigate the effects of a potential run. This idea is very natural, but has been surprisingly difficult to capture in the Diamond-Dybvig framework. Cooper and Ross (1998) establish conditions under which a bank would choose to hold excess liquidity. However, as shown by Ennis and Keister (2006), these conditions only apply when the bank is choosing a run-proof contract. In other words, a bank may choose to hold excess liquidity as a way to make itself immune to runs, but in the Cooper-Ross model a bank will never hold excess liquidity for the purpose of mitigating liquidation costs in the event of a run. In my model, this precautionary motive for holding excess liquidity arises naturally.

To study the impact of changes in asset returns on financial fragility, I begin with a simpler case in which there is no liquidation cost for the long-term asset. In this situation, the portfolio choice is trivial and the model effectively reduces to one with a single asset. This approach makes it easier to determine in which case the solution to a bank’s maximization problem will lie. This approach also highlights, in as simple a setting as possible, two key elements of my study. First, it enables me to fully understand the relationship between the investment return $R$ and the measure of financial fragility I construct. In particular, I show
that the low-return economic system is always stable whenever the fundamental withdrawal demand is sufficiently low. Second, the simpler model is useful for understanding the two competing effects that explain why an increase in \( R \) can either increase or decrease financial fragility. Such an increase in \( R \) does decrease the ex ante incentive for patient depositors to run by providing a higher return for deposits that are held to maturity. However, it also implies that depositors who withdraw in period 1 after it becomes clear that a run has taken place suffer a larger “haircut” relative to depositors who were earlier in the order. This latter effect encourages patient depositors to withdraw early rather than leaving their funds in the banking system, which tends to make the bank more susceptible to a run. I show that either of these two effects can dominate, depending on parameter values.

After establishing these results, I return to the model with liquidation costs and a non-trivial portfolio choice. A unit of good placed into investment in period 0 yields either \( R \) in period 2 or \( r \) in period 1. I also allow the return on storage to differ across periods, with the return between periods 0 and 1 denoted \( \rho_1 \) and that between periods 1 and 2 denoted \( \rho_2 \). This generalization makes it possible to understand broadly the effects of changes in each of the asset returns \( R, r, \rho_1, \) and \( \rho_2 \) on financial fragility. I first study the case where the coefficient of relative risk aversion \( \gamma \) lies in \((1, 2]\), in which case I can solve for the equilibrium allocation in closed form. I provide precise conditions under which there is a non-monotone relationship between asset returns and financial fragility. When depositors are more risk averse (i.e. \( \gamma > 2 \)), closed form solutions are no longer possible, but I provide some limiting results on financial fragility and show that the non-monotone pattern becomes more pronounced with numerical examples. The same principles developed in the special environment with no liquidation cost also apply to this setting. Finally, I study how the changes in the cost of liquidation \( (\rho_1 - r) \) and the term premium \( (R - \rho_1\rho_2) \) influence financial fragility.
I use the comparative static results described above to show how complex, non-monotone patterns can arise in both cases.

While there has been a surge of interest in models of bank runs and financial fragility in recent years, surprisingly little is known about the comparative statics of these models. Sultanum (2014) characterizes the direct mechanism which implements the constrained efficient outcome in a version of Diamond and Dybvig (1983) with aggregate uncertainty. By varying the realization of the impatient fraction of the population, Sultanum numerically characterizes whether the direct mechanism has a run-equilibrium or not. Bertolai et al. (2014) provide a partial answer to the question of how changes in asset return affect banking system’s susceptibility. In an environment with a finite population similar to that in Green and Lin (2003), and the information structure suggested by Peck and Shell (2003), there is a single asset that yields return $R > 1$ if held to maturity. They characterize the conditions under which a bank run equilibrium exists for values of $R$ close enough to one and show that this equilibrium never exists when $R$ is sufficiently large.\(^2\) These results seem to suggest that increases in the return on banks’ investment promote financial stability. However, they leave open the question of what happens for intermediate values of the return $R$, when it is neither close to one nor very large. Moreover, their one-asset model cannot study effects related to changes in banks’ chosen asset portfolios. My work is the first study to fully characterize the relationship between the returns on banks’ assets and financial fragility in the Diamond-Dybvig framework.

The remainder of the chapter is organized as follows. In the next section, I present the environment and describe the definitions of financial fragility and stability. In section 2.3, I analyze equilibrium and characterize the equilibrium measure of financial fragility. I temporarily depart from the assumption that

\(^2\)Andolfatto, Nosal and Sultanum (2014) show, using a mechanism design approach in a similar model, that a fairly high asset return between periods 1 and 2 is a sufficient condition for eliminating the possibility of a bank run equilibrium using an indirect mechanism.
liquidating investment is costly in Section 2.4, a simplification that allows me
to highlight the non-monotonic relationship between the investment return $R$
and the degree of financial fragility. The impact of changes in asset returns on
financial fragility in the environment with portfolio choice introduced in Section
2.5 and the implications for the effects of liquidation cost and term premium are
examined. This chapter concludes with Section 2.6.

2.2 The model

In this section, I construct a version of the Diamond and Dybvig (1983) model
that combines the limited commitment features of Ennis and Keister (2010) with
a non-trivial portfolio choice problem as in Cooper and Ross (1998). I begin by
describing the physical environment and the basic elements of the model and then
define financial fragility and stability in this environment.

2.2.1 The environment

I consider an economy with three periods indexed by $t = 0, 1, 2$. The economy is
populated by a $[0, 1]$ continuum of ex ante identical depositors, indexed by $i$. I
suppose that each depositor has preferences of the form:

$$u(c_1, c_2; \omega_i) = \frac{(c_1 + \omega_i c_2)^{1-\gamma}}{1 - \gamma},$$

where $c_t$ represents consumption in period $t = 1, 2$ and the parameter $\omega_i$ is a binomial
random variable with support $\Omega \equiv \{0, 1\}$. With probability $\pi$ a depositor is
impatient (i.e. $\omega_i = 0$) and only values consumption in period 1; with probability
$1 - \pi$ she is patient and values the sum of period-1 and period-2 consumption. A
depositor’s type $\omega_i$ (impatient or patient) is private information and is revealed
to her at the beginning of period 1. The fraction of depositors in the population
who will be impatient is also $\pi$ due to a law of large numbers. As in Diamond and
Dybvig (1983), the coefficient of relative risk-aversion $\gamma$ is assumed to be greater than one. In period 0, depositors are each endowed with one unit of all-purpose good that can be used for consumption or investment. There are two kinds of assets, a short-term, liquid asset and a long-term, illiquid asset. In what follows, I shall refer to them as the short and long assets, respectively. Each asset is represented by a constant-returns-to-scale investment technology. The short asset is represented by a storage technology that allows one unit of the good placed in period 0 to be converted into $\rho_1$ units of the good in period 1; and one unit of good placed in period 1 yields $\rho_2$ units of good in period 2 as in Wallace (1990). The long asset is represented by an investment technology that allows one unit of the good in period 0 to be converted into $R > \rho_1\rho_2$ units of the good in period 2. If the long asset is liquidated prematurely in period 1, it yields $0 < r < \rho_1$ units of the good for each unit invested.

At the beginning of period 0, depositors pool their resources and set up a bank to insure themselves against individual liquidity risk. The bank then makes a portfolio choice after depositors exit the central location. Depositors cannot directly observe this choice, but they will be able to infer the bank’s portfolio choice in equilibrium. In period 1, upon learning her preference type, each depositor chooses either to withdraw her funds in period 1 or to withdraw until period 2. Those depositors who contact the bank in period 1 arrive one at a time in a randomly assigned order whose value is private information and is only observable when they decide to withdraw. Under this sequential service constraint, as in Wallace (1988, 1990), the bank determines the amount of payment to each withdrawing depositor based on the information received up to that situation when she arrives. As in Ennis and Keister (2009, 2010), the bank cannot pre-commit to future actions, which implies that the bank must always serve depositors optimally depending on the current situation. The objective of the bank is to maximize
welfare measured by the equal-weighted sum of depositors’ expected utilities,

\[ W = \int_0^1 E\{u(c_1(i), c_2(i); \omega_i)\} \, di. \]

As in Peck and Shell (2003) and others, I introduce an extrinsic “sunspot” signal on which depositors can base their withdrawal decisions. The economy will be in one of two states, \( s \in S \equiv \{\alpha, \beta\} \) with probabilities \( \{1 - q, q\} \). Depositors observe the realization of the state of nature at the beginning of period 1. The bank never observes the sunspot state and must infer it based on the observed withdrawal behavior. Notice that there is no restriction on the payments the bank can make to its depositors, which implies that the bank can freely adjust the payment schedule to the remaining depositors when this new information arrives.

### 2.2.2 Financial crises and fragility

After observing her own preference type \( \omega_i \) and the state \( s \), each depositor can choose either to withdraw in period 1, or to wait until period 2,

\[ y_i : \Omega \times S \rightarrow \{0, 1\}, \]

where \( y_i = 0 \) corresponds to withdrawing at \( t = 1 \) and \( y_i = 1 \) corresponds to withdrawing at \( t = 2 \). Let \( y \) denote a profile of withdrawal strategies for all depositors. In this game, an equilibrium is a strategy profile for all depositors, together with strategies for the bank, such that every agent is best responding to the strategies of others.

Ennis and Keister (2010) show that this type of game cannot have a full bank run equilibrium. Without loss of generality, I assume a run only occurs in state \( \beta \). In order to allow a run to occur with non-trivial probability, I assign the value of \( q \) strictly between 0 and 1.\(^3\) All impatient depositors will clearly

\(^3\)If \( q = 1 \), the bank pays each depositor the value of her initial deposit back and there is again no motivation for a run.
choose to withdraw in period 1, since they receive no utility from consuming in period 2. The interesting question is how patient depositors will behave in state $\beta$. Formally, I study the following partial-run strategy profile for depositors:

$$y_i(\omega_i, \alpha) = \omega_i \text{ for all } i,$$

$$y_i(\omega_i, \beta) = \begin{cases} 0 & i \leq \pi \\ \omega_i & i > \pi \end{cases}$$

which corresponds to what Ennis and Keister (2010) call a “one wave” run. Under this specific profile, each patient depositor with $i \leq \pi$ chooses to withdraw early in state $\beta$. Notice that I assume that the remaining patient depositors (those with $i > \pi$) do not withdraw early, even if the state is $\beta$ (a crisis is underway), but instead wait and withdraw in period 2.\footnote{As in Ennis and Keister (2010) a run in this model is necessarily partial.} The following definition provides the notion of financial fragility that I use in the analysis.

**Definition 2.1.** A banking system is said to be fragile if the strategy profile (2.1) is part of an equilibrium; otherwise the banking system is said to be stable.

In principle, there are many possible profiles of withdrawal strategies that involve a partial bank run. I show that the same results are obtained if I use a broader definition of fragility in Appendix C.

### 2.3 Equilibrium and financial fragility

In this section, I first derive the bank’s best response to profile (2.1). I then ask under what conditions the banking system is fragile. Finally, I highlight a key property of the resulting equilibrium allocation: there exists a critical value of $q$ above which the economy is always stable.
2.3.1 The best-response allocation

The bank takes one unit of the good from each depositor in period 0 and invests it in a portfolio consisting of $x$ units of the long asset and $1 - x$ units of the short asset. The bank is initially unable to make any inference about the state of nature and chooses to give the same level of consumption $c_1$ to each withdrawing depositor with $i \leq \pi$. Once $\pi$ withdrawals have taken place, the bank will be able to infer the state of nature and will use this information to calculate the fraction of its remaining depositors who are impatient, which I denote $\hat{\pi}_s$. (Notice that (2.1) generates $\hat{\pi}_\alpha = 0$ and $\hat{\pi}_\beta = \pi$.) Since all uncertainty has been resolved, the bank will choose to give a common amount $c_{1s}$ to each (impatient) depositor who withdraws after the fraction of $\pi$ depositors have been served in period 1. In addition, each of the remaining patient depositors will receive a common amount $c_{2s}$ from the bank’s remaining resources when she withdraws in period 2. Given bank’s portfolio choice $(1 - x, x)$ made in period 0, these common amounts $c_1$, $c_{1\beta}$, $c_{2\alpha}$, and $c_{2\beta}$ will be chosen to solve:

$$\max_{\{x, c_1, c_{1\beta}, c_{2\alpha}, c_{2\beta}\}} \pi u(c_1) + (1 - q)(1 - \pi)u(c_{2\alpha}) + q(1 - \pi) [\hat{\pi}_\beta u(c_{1\beta}) + (1 - \hat{\pi}_\beta)u(c_{2\beta})].$$

(2.2)

I can simplify the constraint set for this problem by first noting that it will never be optimal for the bank to liquidate any of the long assets in state $\alpha$. In such a case, the bank could provide more consumption to all depositors by holding more of the short asset and less of the long asset. Similarly, the assumption $R > \rho_1 \rho_2$ implies that it will never be optimal for the bank to hold units of the short asset until $t = 2$ in state $\beta$. The bank may, however, hold units of the short asset until $t = 2$ in state $\alpha$, and it may choose to meet additional early withdrawal demand by liquidating investment in state $\beta$. Thus, I can write bank’s resource
The constraints as:

\[ \pi c_1 \leq \rho_1 (1 - x), \]
\[ (1 - \pi) c_{2\alpha} = Rx + \rho_2 [\rho_1 (1 - x) - \pi c_1], \]
\[ \rho_1 (1 - x) \leq \pi c_1 + (1 - \pi) \hat{\pi}_\beta c_{1\beta}, \]
\[ (1 - \pi) (1 - \hat{\pi}_\beta) c_{2\beta} = R \left\{ x - \frac{1}{r} [\pi c_1 + (1 - \pi) \hat{\pi}_\beta c_{1\beta} - \rho_1 (1 - x)] \right\}. \]

The first constraint says that the consumption of the first \( \pi \) depositors to withdraw will always come from the resources placed into storage. This constraint may or may not hold with equality at the solution. The second constraint says that in state \( \alpha \), the remaining patient depositors will consume all of the bank’s matured investment plus any resources held in storage for two periods. The third constraint reflects the fact that additional period-1 payments may come only from liquidating investment, since all of the resources in storage have already been depleted. The last constraint is the standard pro rata division of remaining resources that determines the payment in period 2.

The best-response allocation to profile (2.1) in the two-asset model is summarized by the vector \( \mathbf{A}^* \equiv \{ x^*, c_{1\beta}^*, c_{2\alpha}^*, c_{1\beta}^*, c_{2\beta}^* \} \) that solves the problem (2.2). The explicit derivation of this allocation is given in Appendix A.1. It is straightforward to show that the solution to this problem will satisfy \( c_{1\beta}^* < c_{2\beta}^* \) as long as

\[ \rho_2 \geq 1. \] (2.3)

If this inequality were reversed, it could be efficient for some patient depositors to consume in period 1 after the bank has rescheduled payments. Since I am interested in bank runs that lead to inefficient outcomes, in what follows, I shall restrict attention to the case (2.3).

The above analysis establishes that this solution to the problem (2.2) will lie in one of the three cases identified in Table 2.1. In the first case, the bank does not hold excess liquidity for the purpose of providing funds to depositors in
the event of a run, and hence it will liquidate investment to provide additional period-1 payments. It is, of course, possible that the bank responds to a run by liquidating investment, even though it holds excess liquidity, which corresponds to the second case. In the third case, the additional early payments come only from the resources in storage without liquidating investment if a crisis occurs. Notice that the bank will never choose to be in the case where there is no excess liquidity and no liquidation. In such a case, the resources in storage have already been paid out to the first $\pi$ depositors who withdrew. Thus, the impatient depositors with $i > \pi$ who have not yet been served would receive no consumption in state $\beta$.

The next result shows when the best-response allocation $A^*$ lies in the different cases in Table 2.1, depending on the probability of a crisis $q$. For notational convenience, I define the following constants, which depend only on parameter values.

$$q_l \equiv \{1 + \frac{\rho_1 - r}{R - \rho_1 \rho_2} [\frac{\pi R}{r} + (1 - \pi)(\frac{R}{r})^\gamma]\}^{-1}, q_u \equiv \{1 + \frac{\rho_1 - r}{R - \rho_1 \rho_2} [\pi \rho_2 + (1 - \pi)(\frac{R}{r})^\gamma]\}^{-1}.$$}

Notice that $0 < q_l < q_u < 1$ holds. I then have the following result.

**Lemma 2.1.** The best-response allocation $A^*$ lies in Case $I$ if $0 < q < q_l$, in Case $II$ if $q_l \leq q \leq q_u$, and in Case $III$ if $q_u < q < 1$.

The intuition for this result is as follows. If a crisis is very unlikely ($q < q_l$), holding excess liquidity is very costly because of $R > \rho_1 \rho_2$. In this situation, additional period-1 payments will come only from liquidating investment since all
of the resources in storage have already been paid out to the first $\pi$ depositors who withdrew. As the probability of a crisis increases, the bank will eventually choose to hold excess liquidity. Having more assets in storage lowers the losses of liquidating investment and thus leaves the bank with more resources in the event of a run. When a crisis is more likely ($q > q_u$), the bank becomes more cautious and leaves the banking system more liquid, and a bank with a very liquid portfolio will avoid liquidation. Combining these discussions and the above result therefore gives me sufficient conditions under which the best response to (2.1) involves holding excess liquidity and/or liquidation, as depicted in Figure 2.1.

### 2.3.2 Fragility

I now verify whether the strategy profile in (2.1) is part of an equilibrium and hence whether the banking system is fragile or stable. Recall that an impatient depositor will always strictly prefer to withdraw early whatever the payment she receives, since she values period-1 consumption only. Therefore, I only need to consider the actions of patient depositors. Assumption (2.3) implies $c^*_{1,\alpha} < c^*_2$, and thus a patient depositor with $i > \pi$ prefers to wait in state $\beta$. For patient depositors with $i \leq \pi$, consider separately each of the two possible sunspot states. In state $\alpha$, a patient depositor receives $c^*_{2,\alpha}$ if she waits until period 2, but receives $c^*_1$ if she withdraws in period 1. It is straightforward to show that $c^*_{2,\alpha} > c^*_1$ always holds, so that a patient depositor will strictly prefer to wait in state $\alpha$ as specified in (2.1). In state $\beta$, a patient depositor with $i \leq \pi$ receives $c^*_1$ if she joins the
run and $c^*_{2\beta}$ if she leaves her funds in the banking system. The discussion above establishes that the profile (2.1) emerges as an equilibrium if and only if the allocation $A^*$ satisfies

$$c^*_1 \geq c^*_2. \tag{2.4}$$

I now define $\bar{q}$ to be maximum probability with which a run can occur in equilibrium, which is the natural measure of financial fragility in this model.

**Definition 2.2.** Given $(R, r, \rho_1, \rho_2, \gamma, \pi)$, let $\bar{q}$ be the maximum value of $q$ such that $c^*_1 \geq c^*_2$ holds. If $c^*_1 \geq c^*_2$ does not hold for any value of $q$, then define $\bar{q} = 0$.

In this way, it is possible for me to establish the relationship between asset returns and financial fragility. Most of this study is concerned with the following question: will an increase in the level of some asset returns would increase or decrease financial fragility? I will first focus on this relationship in a simpler case where the portfolio choice is non-essential, and then I extend the analysis to the full model.

### 2.4 The case with no liquidation cost

To highlight the relationship between the level of investment return $R$ and the degree of financial fragility, I begin by assuming that there is no liquidation cost, that is, $\rho_1 = r$; this assumption is subsequently relaxed. In this special scenario, the portfolio choice becomes trivial because the long asset dominates the short asset in return. By abstracting from the portfolio choice, this environment studied here is analogous to the one-asset Ennis and Keister (2010) model environment with one wave of withdrawals. Considering this limiting case, I show that there exists a non-monotone relationship in the sense that an increase in the return

---

5It is worth noting that $q \leq q_l = q_u \equiv 1$ will necessarily be satisfied if $\rho_1 = r$. In the case, the bank’s best response to (2.1) always lies in Case I. If there is no term premium (i.e. $R = \rho_1 \rho_2$), the bank’s best response to (2.1) only involves Case III, which would lead to exactly the same results in what follows.
on investment $R$ can either increase or decrease the fragility measure $\bar{q}$. Then I illustrate the competing effects that determine how the return affects financial fragility.

### 2.4.1 The impact of $R$ on fragility

Bertolai et al. (2014) show in their model that the economy is fragile if the investment return $R$ is close to 1 under some conditions, and that the economy is always stable whenever the return $R$ is sufficiently large. In my model, I establish the relationship between the return on investment and financial fragility more generally and offer the first complete characterization of this comparative static in the Diamond-Dybvig framework. The answer to the question of whether the fragility of an economy increases or decreases when the return $R$ is raised is not obvious because of the competing effects. I first present the fragility measure $\bar{q}$ in this special case with $\rho_1 = r$, so that the focus is on the remaining asset returns $R$ and $\rho_1$.

When there are relatively few impatient depositors, there will be sufficient assets left for the bank to offer a relatively high payment to patient depositors who wait until period 2, which implies that bank runs never occur in equilibrium. In other words, when $\pi$ is low, the economy is stable for all $q$. In particular, define

$$\pi_F \equiv \frac{(\frac{R}{\rho_1})^{\frac{1}{\gamma}} - 1}{(\frac{R}{\rho_1})^{\frac{1}{\gamma}} - 1 - \pi}$$.  

Note that $0 < \pi_F < 1$ holds whenever $\gamma > 2$.\(^6\) I then have the following result.

**Proposition 2.1.** Suppose $\rho_1 = r$. If $\pi > \pi_F$, then

$$\bar{q} = \frac{\rho_1 \pi - \pi (\frac{R}{\rho_1})^{\frac{1}{\gamma}} + (1-\pi))^{-\gamma}}{1 - \pi (\frac{R}{\rho_1})^{\frac{1}{\gamma}} + (1-\pi))^{-\gamma}}$$; \hspace{1cm} (2.5)

otherwise, $\bar{q} = 0$.

\(^6\)It is straightforward to show that the economy with no liquidation cost is always stable whenever $1 < \gamma \leq 2$.\)
When $\pi$ is large, more depositors have a real need to consume early. As a result, when a run occurs, the bank will realize a crisis is occurring relatively late. After a large number of depositors have been served, the remaining resources are relatively small, which would lead the bank to optimally provide smaller payments to patient depositors who withdraw in period 2. Thus, the banking system tends to be fragile. However, when $\pi$ is small enough, the patient depositors prefer to leave their deposits in the banking system because the bank has sufficient resources to offer after a small amount of early withdrawals have been served.

Proposition 2.1 implies that when both $\pi$ and $R$ are small enough, the economy would be stable (i.e. $\bar{q} = 0$). This result is in sharp contrast to the property in Bertolai et al. (2014), which shows that the economy tends to be fragile as $R$ is close to 1 under some conditions.

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Figure 2.2 plots the ratio $\frac{c_{1,\beta}}{c_{2,\beta}}$ as a function of the probability of state $\beta$. In the left panel, $\pi$ is relative small and, as result, this ratio is always smaller than one; the economy is stable for all values of $q$. In the right panel, in contrast, the economy is fragile when $q$ is small enough. The figure shows how $\bar{q}$ is determined as the point where the ratio $\frac{c_{1,\beta}}{c_{2,\beta}}$ crosses 1.

I now turn to my primary interest: determining how the changes in $R$ influence $\bar{q}$.\footnote{Note that asset returns appear as a form of $\frac{R}{\rho_{1}}$ in $\bar{q}$. Without loss of generality, I focus on} Suppose there is an increase in the investment return $R$. Would financial
fragility tend to increase or decrease? I define
\[ \tilde{\pi} \equiv \frac{2 - \frac{1}{\gamma}}{\gamma - \frac{1}{\gamma}}. \]
I then have the following result.

**Proposition 2.2.** Suppose \( \rho_1 = r \) and \( \pi > \pi_F \). If:

- \( \pi > \tilde{\pi} \), then \( \bar{q} \) is strictly decreasing in \( R \);
- \( \pi < \tilde{\pi} \), then there exists \( \tilde{R} > \rho_1 \) such that \( \bar{q} \) is strictly increasing (decreasing) in \( R \)

as \( R \left( < \right) \tilde{R} \).

This proposition establishes precise conditions under which there is a non-monotone relationship between the return \( R \) and financial fragility \( \bar{q} \), as illustrated in the following figure.

![Figure 2.3: The measure of fragility in the environment with no liquidation cost](image)

\( \gamma = 8, \rho_1 = r = 1 \)

Figure 2.3: The measure of fragility in the environment with no liquidation cost

Figure 2.3 depicts \( \bar{q} \) as a function of \( R \) for three different values of \( \pi \) (note the logarithmic scale for the x-axis). Notice that an increase in \( R \) will make the economy less susceptible to a run if the return on investment is high enough. In some cases, however, (for example when \( \pi \) equals 0.1 or 0.2 and \( R \) is moderate),

the analysis of \( R \). Focusing on \( \rho_1 \) would exactly lead to the reversed result.
the increase in $R$ can either increase or decrease financial fragility, depending on parameter values.

### 2.4.2 Competing effects on fragility

To understand intuitively the non-monotone pattern in Figure 2.3, it helps to write $c_1^*/c_{2\beta}^*$ as

$$\frac{c_1^*}{c_{2\beta}^*} \equiv \frac{c_{1\beta}^*}{c_{2\beta}^*} \times \frac{c_1^*}{c_{1\beta}^*}.$$  

(2.6)

The following proposition explains that the change in $R$ has a basic trade-off between “period” substitution effect and “state” substitution effect in the environment with no liquidation cost.

**Proposition 2.3.** If $\rho_1 = r$, then $\left(\frac{c_{1\beta}^*}{c_{2\beta}^*}\right)$ is strictly decreasing in $R$.

Suppose that I increase $R$, holding other parameter values fixed. Such an increase leads the bank to raise $c_{2\beta}^*$ relative to $c_{1\beta}^*$ because providing consumption in period 2 is now relatively less expensive. This fact decreases the ex ante incentive for depositors to run. At the same time, in contrast, the bank tends to raise $c_1^*$ relative to $c_{1\beta}^*$ because providing consumption in state $\alpha$ is now relatively less expensive compared to the state $\beta$. By itself, this change increases the incentive for a depositor who is early in the order to join the run and collect the payment $c_1^*$, before the bank learns the state. As a result, the latter effect encourages patient depositors to withdraw early rather than leaving their deposits in the banking system, which tends to make the bank more susceptible to a run.

The top panel in Figure 2.4 repeats the solid blue curve from Figure 2.3, but with an expanded scale on the vertical axis. The bottom two panels show that either of these two effects can dominate, as stated in Proposition 2.3. To understand the non-monotonic pattern, I ask the following question. What is the maximum value of $q$ satisfying $\frac{c_1^*}{c_{2\beta}^*} \geq 1$ for a given asset return $R$? As the last two
The impact of $R$ on \( \bar{q} \)

\[ \gamma = 8, \pi = 0.1, \rho_1 = r = 1 \]

Figure 2.4: Two competing effects in the environment with no liquidation cost

Panels illustrate, it is evident that the ratio \( \frac{c_1}{c_{2,2}} \) is strictly decreasing in \( q \). Thus, I can determine whether there exists a cutoff value \( \bar{q} \) such that the ratio \( \frac{c_1}{c_{2,2}} \geq (\leq)1 \) if \( q \leq (>)\bar{q} \), for any given values of \( R \). Panel (b) shows that, for a given value of \( q < q_3 \), the “state” substitution effect is dominant and an increase in \( R \) will increase the ratio \( \frac{c_1}{c_{2,2}} \) when \( R \) is small. This rise, in turn, increases the threshold value of \( q \) from \( q_1 \) to \( q_3 \). Panel (c) illustrates that as the investment return increases further, however, for a given value of \( q > q_1 \), the “period” substitution effect outweighs the “state” substitution effect. In this situation, an increase in \( R \) will decrease the ratio \( \frac{c_1}{c_{2,2}} \), which translates into a decline in the threshold value of \( q \) from \( q_3 \) to \( q_1 \).

In the next section, I turn to the study of how the measure of fragility varies
with comparative statics of $R$, $r$, $\rho_1$, and $\rho_2$ in the presence of an essential portfolio choice.

2.5 The full model

The model of intermediation presented above is, in some ways, rather special: there is no role for excess liquidity in an environment with no portfolio choice. While these features were useful for generating intuition, they are by no means necessary for the non-monotone pattern reported in Proposition 2.2 to obtain. I now demonstrate this fact by removing the assumption of no liquidation cost. The general logic of my analysis here is parallel to that just used. I first identify how changes in asset returns influence financial fragility in this economy assuming that liquidation is costly. Then I illustrate the two competing effects of an increase in asset returns under different cases in the light of Lemma 2.1.

2.5.1 A general fragility result

With an essential portfolio choice in this environment, the structure of equilibrium outcomes is more complex than before. Using Appendix A.1, the measure of financial fragility can be formalized in a general formula. I define

\[
\begin{align*}
    f(\cdot) &= \left[\pi\left(\frac{R}{r}\right)^{-\frac{1}{\gamma}} + (1 - \pi)\right]^\gamma - \frac{R}{\rho_1}; \\
    g(\cdot) &= \left[\pi\left(\frac{R}{r}\right)^{-\frac{1}{\gamma}} + (1 - \pi)\right]^\gamma - \frac{R^2 - (\rho_2 + 1)rR + \rho_1 \rho_2 r}{R(\rho_1 - r)}; \\
    h(q) &= \pi\rho_2 \left(\rho_2 + \frac{1 - \rho_1 \rho_2}{q}\right)^{-\frac{1}{\gamma}} + (1 - \pi) - \left[\frac{(1-q)\rho_1}{(1-q)\rho_1 + 1}\right]^\gamma.
\end{align*}
\]

Proposition 2.4. Given $R$, $r$, $\rho_1$, $\rho_2$ and $\gamma$,

- if $f(\cdot) \leq 0$, then the economy is stable for all $q$ and, therefore, $\bar{q} = 0$;

- if $f(\cdot) > 0$ and $g(\cdot) \leq 0$, then

\[
\bar{q} = \frac{\rho_2 - \left[\pi\left(\frac{R}{r}\right)^{-\frac{1}{\gamma}} + (1 - \pi)\right]^{-\frac{1}{\gamma}}}{1 - \left[\pi\left(\frac{R}{r}\right)^{-\frac{1}{\gamma}} + (1 - \pi)\right]^{-\frac{1}{\gamma}}} \equiv \bar{q}_{\text{Case I}};
\]
• if \( g(x) > 0 \) and \( \{ q \mid h(q) = 0; q \in (q_u, 1) \} = \emptyset \), then

\[
\bar{q} = \frac{(R - \rho_1 \rho_2)r}{(R - \rho_2 r)R} \equiv \bar{q}_{\text{Case II}};
\]

(2.8)

• if \( g(x) > 0 \) and \( \{ q \mid h(q) = 0; q \in (q_u, 1) \} = Q\{q_1, q_2, ..., q_n\} \), then

\[
\bar{q} = q_m \equiv \bar{q}_{\text{Case III}}, \text{ where } q_m \text{ is the biggest element of } Q.
\]

(2.9)

Figure 2.5: \( \frac{c_1}{c_{2,\beta}} \) in the economy \( (\gamma = 3, \pi = 0.85, r = 1.1, \rho_1 = 1.25, \rho_2 = 1.2) \) with \( R \) varied

Lemma 2.1 shows that the bank’s best response lies in Case I, Case II, and Case III respectively as \( q \) increases, and hence there might exist different explicit conditions under which the economy is fragile.

It is straightforward to show that \( \frac{c_1}{c_{2,\beta}} \) is strictly decreasing in \( q \) because of costly liquidation whenever the bank’s best response lies in Case I or Case II (i.e. \( q \leq q_u \)). In such a situation, there exists a threshold value of \( q \) below
which fragility arises. Things are different when the bank’s best response lies in Case III (i.e. \( q > q_u \)). In this case, the bank becomes more cautious and holds sufficient short assets in its portfolio so that it avoids the loss of liquidating investment. Figure 2.5 illustrates that an increase in the probability of a crisis can either increase or decrease the ratio \( \frac{c^*_1}{c^*_2} \) under Case III, depending on parameter values.\(^8\) Moreover, panel (c) of Figure 2.5 shows that the run equilibrium does not exist for some values of \( q \) less than \( \bar{q} \).

As an aside, note that the bank does choose to hold excess liquidity in some cases. Cooper and Ross (1998) analyze the question of when is excess liquidity held. Ennis and Keister (2006) show that – contrary to what Cooper and Ross (1998) suggest – the bank will never hold excess liquidity for the purpose of meeting early withdrawal demand in the event of a run in their model. However, my study here captures the idea of Cooper and Ross (1998) aimed to show: holding excess liquidity to mitigate the loss of liquidating investment is attractive in some situations. This fact can be seen in Figure 2.5 in which \( \bar{q} \) lies in either Case II or Case III.

2.5.2 Comparative statics when \( 1 < \gamma \leq 2 \)

Would the pattern of \( \bar{q} \) as a function of asset returns be non-monotonic as in the environment with no liquidation cost? Do there exist two competing effects on fragility? I first study the case \( 1 < \gamma \leq 2 \) because I can derive the explicit expression for \( \frac{c^*_1}{c^*_2} \) as a function of each asset return.\(^9\) Therefore, starting with this case provides a clear picture of the impact of asset returns on financial fragility,

\(^8\)This fact is similar in spirit to Proposition 2.5 in the Ennis and Keister (2006). In addition, the probability of a crisis can affect the level of investment in a relatively intricate way.

\(^9\)In light of Proposition 2.4, it is straightforward to show that there is a cutoff value of \( q \) below which the economy is fragile and above which it is always stable, given \( R, r, \rho_1, \rho_2, \pi, \) and \( 1 < \gamma \leq 2 \) with \( f(\cdot) > 0 \). Moreover, in this case the equilibrium outcome will never fall in Case III since \( \{ q \mid h(q) = 0; q \in (q_u, 1) \} = \emptyset \) always holds.
but also is useful for providing critical insights into the more complex cases with \( \gamma > 2 \).

In keeping with my earlier analysis, I begin by providing precise conditions under which \( \bar{q} \) is increasing or decreasing in \( R \). It is important to bear in mind that the economy is always stable whenever \( f(\cdot) \leq 0 \) holds, so I restrict attention to the case of \( f(\cdot) > 0 \) through out the remaining study. In addition, the degree of financial fragility \( \bar{q} \) will be given by (2.7) or (2.8) depending on the sign of function \( g(\cdot) \). The following proposition characterizes the outcomes in this case. I define the critical values \( R \) and \( R^* \) by

\[
R = \rho_1\rho_2(1 + \sqrt{1 - r/\rho_1}) \quad \text{and} \quad g(R^*) = 0.
\]

**Proposition 2.5.** Assume \( f(\cdot) > 0 \) holds. For all \( \gamma \in (1, 2] \),

\[
\text{\( \bar{q} \) is strictly}
\begin{cases}
\text{(increasing) in } R \text{ with } R \left( \begin{array}{c}
< \\
>
\end{array} \right) \min \{R^*, R\}, \quad \text{if } \pi < \left(\frac{r}{\rho_1 - r}\right)^{\frac{1}{\gamma}} 1_{\{\gamma = 2\}} \\
\text{(decreasing) in } R \text{ with } R \left( \begin{array}{c}
< \\
>
\end{array} \right) R, \quad \text{if } \pi > \left(\frac{r}{\rho_1 - r}\right)^{\frac{1}{\gamma}} 1_{\{\gamma = 2\}}
\end{cases}
\]

where \( 1_{\{\gamma = 2\}} \) is an indicator function taking the value 1 if \( \gamma = 2 \) and 0 otherwise.

Figure 2.6: The impact of \( R \) on \( \bar{q} \) with \( 1 < \gamma \leq 2 \)
The blue dashed line in Figure 2.6 corresponds to the threshold value of $q$ when the best-response allocation lies in Case I. Similarly, the red dotted line represents the threshold value of $q$ in Case II. It is worth noting that the extreme economy of $R = \rho_1 \rho_2$ is always stable, which is depicted by an empty circle.\footnote{This fact can be easily verified by looking at the condition $\pi \leq \pi_F$.}

Two observations immediately follow. Firstly, the right panel of Figure 2.6 shows that the degree of financial fragility $\bar{q}$ as a function of $R$ can have a non-monotone pattern within the Case II. Secondly, the non-monotonicity in the two-asset model can also stem from the fact that there might be different effects on $\bar{q}$ by varying $R$ in Case I and Case II. For example, the first panel of Figure 2.6 shows that a higher $R$ reduces financial fragility while the economy is in Case I, but it increases instability in Case II. In this case, the change in the bank’s best response regions for $\bar{q}$ generates the non-monotonicity. Hence, I conclude here that the non-monotonicity result reported in Proposition 2.2 is robust to the environment with portfolio choice.

Recall that there are two competing effects – “period” substitution effect and “state” substitution effect demonstrated by equation (2.6) in Section 2.4.2. In Case I, a higher $R$ increases the payment to patient depositors who withdraw in period 2 in state $\beta$, which reduces the incentives for patient depositors to join the run. In the meantime, the bank offers a higher payment to depositors who withdrew before it observes the state of nature. Since the coefficient of relative risk aversion $\gamma \leq 2$ in this subsection, the “period” substitution effect is dominant and thus $c_1^*/c_{2\beta}^*$ is decreasing in $R$ in Case I. Recall that the ratio $c_1^*/c_{2\beta}^*$ decreases as the probability of a crisis $q$ rises in Case I. Therefore, a higher $R$ reduces the fragility of an economy in Case I. When the equilibrium outcome falls in Case II, the “state” substitution is dominant as $R$ is close to $\rho_1 \rho_2$, which implies that $c_1^*/c_{2\beta}^*$ is increasing in $R$. As $R$ increases further, in some economies, however, eventually the “period” substitution effect will dominate the “state” substitution.
effect because providing consumption in period 2 is now less expensive.

Similarly, the following proposition establishes precise conditions under which \( \bar{q} \) is increasing or decreasing in other asset returns. I define the critical values \( \pi^*, r^*, \rho^*_1 \) and \( \rho^*_2 \) by

\[
\pi^* = \left(\frac{(\gamma - 2)R_2 + 2rR}{R_2 - 2rR + \rho_1 r + \rho_2} + 1\right)^\frac{1}{\gamma - 1}, \quad g(r^*) = 0, \quad g(\rho^*_1) = 0, \quad \text{and} \quad g(\rho^*_2) = 0.
\]

I then have the following result.

**Proposition 2.6.** Assume \( f(\cdot) > 0 \) holds. For all \( \gamma \in (1, 2) \),

- \( \bar{q} \) is strictly increasing in \( r \) with \( r < r^* \)
- \( \bar{q} \) is strictly decreasing in \( \rho_1 \) with \( \rho_1 < \rho^*_1 \)
- \( \bar{q} \) is \( \begin{cases} \text{constant} & \text{in } \rho_2 \text{ with } \rho_2 < \rho^*_2, \text{ if } \pi < \pi^* \\ \text{decreasing} & \text{in } \rho_2, \text{ if } \pi > \pi^* \end{cases} \)

This proposition shows that there also can exist a non-monotone effect of the returns \( r, \rho_1, \) and \( \rho_2 \) on financial fragility.

### 2.5.3 Comparative statics when \( \gamma > 2 \)

I now turn to the case where the coefficient of relative risk aversion is larger than 2. The results in this scenario are more complex because the run-equilibrium outcome will sometimes fall in Case III. An increase in the probability of a crisis can either increase or decrease the ratio \( \frac{\bar{c}_2}{\bar{c}_3} \) under Case III as illustrated in Figure 2.5. It is straightforward to show that, when the two-asset model is formalized in the environment with \( \gamma > 2 \), the non-monotonicity with respect to the impact of
changes in asset returns on financial fragility becomes more pronounced at least in the neighborhood of some limiting values.\textsuperscript{11}

Figure 2.7 provides a series of illustrative examples with respect to the impact of $R$ on $\bar{q}$. The red and blue curves represent the same cases as in Figure 2.6, but the green solid line corresponds to $\bar{q}$ in Case III.

![Figure 2.7: The impact of $R$ on $\bar{q}$ with $\gamma > 2$](image)

It is perhaps instructive to point out how the possibility of a bank run that lies in Case III, with holding excess liquidity and no liquidation, plays an important role in determining the scope for non-monotonicity when $R$ is close to $\rho_1 \rho_2$. It is straightforward to demonstrate that $\bar{q}$ is an increasing function of $R$ because the “state” substitution effect dominates the “period” substitution effect in the neighborhood of $\rho_1 \rho_2$ whenever $\pi$ is sufficient small. Panel (a) of Figure 2.7 depicts this situation. It is, however, straightforward to show that the net effect formalizes $\bar{q}$ as a decreasing function of $R$ in the neighborhood of $\rho_1 \rho_2$ if the withdrawal demand is sufficiently large, as depicted in panel (b) of Figure 2.7. Intuitively, holding excess liquidity becomes costly as $R$ rises and, thus, putting more resources in investment will provide a larger return to depositors in period 2. It is clear that an increase in $R$ can promote financial stability in this situation. Also notice that there is a discontinuity between the green and red curve in panel

\textsuperscript{11}I analyze the impact of asset returns on financial fragility for the cases where $R \rightarrow \rho_1 \rho_2$, $R \rightarrow \infty$, $r \rightarrow 0$, $r \rightarrow \rho_1$, and $\rho_2 = 1.$
(b), which relates to the existence of “stable region” in the space of \( q \) as shown in panel (c) of Figure 2.5. In other words, an increase in \( R \) causes the value of \( \bar{q} \) to decrease from \( \bar{q}_{\text{Case III}} \) to \( \bar{q}_{\text{Case II}} \) discontinuously, which translates into a jump directly from the green curve to the red one.

Unlike the analysis in the neighborhood of \( \rho_1 \rho_2 \), there does not exist a closed form expression for \( c_1^*/c_{2/3}^* \) as a general function of \( R \). But I can see it is still likely to depend critically on the two competing effects as discussed above. Note further that the \( \bar{q} \) might display like a roller-coaster in this environment with \( \gamma > 2 \), which is illustrated by the blue curve in the left panel.

### 2.5.4 Implication: liquidation costs and the term premium

Now, I turn my attention to the impact of changes in the liquidation cost and the term premium on financial fragility in the two-asset model. I define the liquidation cost to be the difference \( \rho_1 - r \) and the term premium to be \( R - \rho_1 \rho_2 \). There are several ways one might choose to formalize the relationship between \( \bar{q} \) and these concepts. One might restrict the changes in term premium with respect to the changes in one of \( \rho_1, \rho_2 \) and \( R \) holding the other two fixed. Alternatively, for example, one might assume that the changes in term premium stem from any changes in those asset returns. In what follows, I choose the former approach for two reasons: first, it is simpler to work with; and second, the qualitative aspects of my main results provide insight to the latter method.

Panel (a) of Figure 2.8 (note the logarithmic scale for the x-axis) illustrates how changes in the liquidation cost influence financial fragility by reducing \( r \). When the liquidation cost is sufficiently small, the bank will choose not to hold excess liquidity. In this region, an increase in the liquidation cost has no effect on the bank’s portfolio choice and causes a sharp decline in the value of remaining
resources if a crisis occurs. This decline, in turn, increases the incentive for patient depositors to withdraw early. As a result, such an increase in the liquidation cost leads to an increase in $\bar{q}$ depicted by a blue curve. As the liquidation cost increases further, however, eventually the bank will choose to hold excess liquidity. In this case, an increase in the liquidation cost leads the bank to raise $c_{2\beta}$ relative to $c_{1\beta}$ because providing consumption in period 2 is now relatively less expensive. In addition, depositors who withdraw in period 1 after a run has taken place suffer a smaller “haircut” relative to depositors who were earlier in the order since the bank becomes more conservative. In other words, both effects decrease the ex ante incentive for depositors to run, and hence such an increase in the liquidation cost leads to a decline in $\bar{q}$ depicted by a red curve. When the liquidation cost is sufficiently high, no liquidation becomes the best choice for the bank and, as a result, the ratio $c^*_1/c^*_{2\beta}$ becomes independent in the liquidation cost. If the economy were to satisfy the condition $c^*_1 \geq c^*_{2\beta}$ after the solution set switches into Case III, the resulting property would have $\bar{q}$ holding constant as shown by the green curve.

Panel (b) answers the question of what is the effect of a steeper term structure, with a decline in the short-term rate $\rho_1$. Recall that the bank will hold excess liquidity and not liquidate investment if the term premium is close to zero. In this region, an increase in the term premium leads patient depositors to leave their
funds in the banking system if a crisis occurs since the “period” substitution effect is always dominant, which reduces the degree of financial fragility in this region. As the term premium increases further, however, eventually the bank will choose to liquidate investment to meet withdrawal demand. In this case, it is straightforward to show financial fragility is determined only by the “state” substitution effect. An increase in the term premium raises the ex ante incentives associated with \( c_1 \) for patient depositors with \( i \leq \pi \) to run, which increases financial fragility. Decreasing \( \rho_1 \) further and while fixing other parameter values, I can obtain the opposite result as shown by the blue curve in the left panel.

### 2.6 Conclusion

In a two-asset version of the Diamond and Dybvig (1983) model with limited commitment, I have analyzed the question of how changes in asset returns influence financial fragility. The relationship turns out to be surprising complex. I first focused on the relationship between financial fragility and the return \( R \) on the long-term investment in an environment with no liquidation cost. I found that a higher value of this return may either increase or decrease the degree of financial fragility. Indeed, in some cases, the resulting effect of changes in \( R \) on financial fragility is non-monotone. Two competing effects explain this non-monotone pattern. An increase in \( R \) leads the bank to raise \( c_{2,\beta} \) relative to \( c_{1,\beta} \) because providing consumption in period 2 is now relatively less expensive. This fact decreases the ex ante incentive for depositors to run. On the other hand, the bank will choose to hold a more illiquid portfolio since the return on reserves is now relatively lower. This fact implies that depositors who withdraw in period 1 after it becomes clear that a run has taken place suffer a larger “haircut” relative to depositors who were earlier in the order due to the sharp decline in the value of remaining reserves. Thus, the latter effect encourages patient depositors to withdraw early
rather than leaving their deposits in the banking system, which tends to make
the bank more susceptible to a run. I showed that the non-monotonicity be-
comes more pronounced when liquidating investment is costly and the bank faces
a non-trivial portfolio choice. The pattern can again be understood in terms of
two competing effects. Finally, I studied the impact of changes in the liquidation
cost and the term premium on financial fragility. Not surprisingly, the pattern
discovered here can be non-monotonic as well.
Chapter 3
Prudential Liquidity Regulation

3.1 Introduction

The fundamental role of banks is to accept short-term deposits and make longer-term loans. However, this process of maturity transformation exposes them to liquidity risk. During times of financial stress, such as the global financial crisis 2007-2009, banks need to have a sufficient buffer of liquid assets to be able to meet rapid and large withdrawals of funds, motivated by depositors’ own funding needs as well as their concern about banks’ solvency. Insufficient liquidity of banks may accelerate the withdrawal demand of depositors, which can in turn lead to a run on banks. Following the recent global financial crisis, promoting financial stability has become an important and explicit goal for governments and central banks around the world. In order to reduce the likelihood of such runs, the Basel Committee on Banking Supervision (BCBS) has proposed a new liquidity standard for banks, called the liquidity coverage ratio (LCR), as part of the Basel III accords. The primary objective of the LCR is to promote the short-term resilience of banks’ funding liquidity by ensuring that they hold sufficient liquid assets to survive a significant stress scenario lasting for 30 days.

Is this new type of liquidity regulation desirable? There seems to be widespread agreement by policy makers and regulators that liquidity regulation is important and useful. Wellink (2011) praised the new rule as “a landmark achievement” that will help to “significantly reduce the probability and severity of banking crises in the future”. Yellen (2014) called the final rule “an important step” that
will “complement the Federal Reserve’s enhanced supervision and regulation of financial institutions’ liquidity positions” and thus “further bolster financial stability”. Unfortunately, there is less analytical consensus on the need for liquidity regulation. Allen (2014), in his survey of the recent literature on liquidity regulation, concludes his paper by writing “With liquidity regulation, we do not even know what to argue about.”

I study an environment where the role for regulation comes from banks’ anticipation of being bailed out in the event of a crisis. The policy maker will respond to a crisis by diverting some tax revenue away from the production of public good to support the private consumption. These “bailout” payments lead banks to hold a more illiquid portfolio of assets and to offer higher payments to depositors who withdraw early. I ask what role liquidity regulation can play in correcting these distortions. In particular, is liquidity regulation part of an optimal regulatory regime? How does it interact with other regulatory measures such as restrictions imposed on banks’ liabilities?

To address these questions, I study a version of the classic model of Diamond and Dybvig (1983) with the following three features. First, there are two assets and banks face a non-trivial portfolio choice, as in Cooper and Ross (1998). Banks make this choice taking into account the possibility of run by depositors, which depends on the realization of a sunspot variable (as in Peck and Shell, 2003, and many others). A second feature is to introduce fiscal policy in the form of a public good as in Keister (2015). There is a benevolent policy maker who has the ability to tax banks’ deposits and can use the revenue from this tax to produce the public good and, potentially, to make bailout payments to banks. Finally, I study the model in the spirit of Ennis and Keister (2009, 2010) so that the policy maker and banks are unable to pre-commit to follow a particular course of action in the event of a crisis. My model is the first to combine a non-trivial portfolio choice with the features of limited commitment, fiscal policy and a positive probability
of a crisis, which I show generates particularly rich results. These features mean
that the model not only is suitable for capturing distorted incentives on both sides
of banks’ balance sheets, but also can be used to explore potential policy options
(regulations imposed on banks’ choices of both assets and liabilities, prohibition
of bailouts) that are currently being proposed to prevent future crises.

I reach two main conclusions from my analysis. The anticipation of a bailout
distorts banks’ decisions in two dimensions in my model: how much to give to
depositors who withdraw before the sunspot state is revealed plus how to di-
vide their resources between the liquid and illiquid assets. I first show that a
regime in which the policy maker places a minimum liquidity requirement on
banks’ assets and imposes a tax on banks’ early payments can fix the resulting
distortions on both sides of banks’ balance sheets. Such a regime can generate
the welfare-maximizing allocation of resources conditional on the financial system
being “fragile”, that is, self-fulfilling runs emerging as equilibrium outcomes of
the model. In other words, if depositors will run on the financial system with
some probability, the regime with two policy tools always yields higher welfare
than other alternative policy interventions. My second result is that in some cases
the financial system is not fragile if the policy maker only has the ability to levy
a tax on early payments, but becomes fragile if the policy maker is also given
the ability to impose a liquidity requirement. Therefore, while adding liquidity
regulation is helpful in some economies, it can actually harm rather than stabilize
the financial system in others.

My work is closely related to Keister (2015), which studies a version of Diamond-
Dybvig (1983) model with bailouts but with a single asset and hence no portfolio
choice. In that paper, the incentive distortion has only one dimension: when
bailouts are permitted, banks offer larger payments to depositors who withdraw
early than a social planner would choose (i.e. the moral hazard only affects the
liabilities side of banks’ balance sheets). In my framework, the source of the
moral hazard is the same (bailouts), but now banks also make a portfolio choice and, therefore, the distortion affects both sides of banks’ balance sheets. This expanded model allows me to explore some current regulatory tools that Keister (2015) cannot, especially imposing regulations on banks’ asset holdings, and the interaction between regulatory tools.

Two other recent papers argue that a policy mix of bailouts and prudential policy tool(s) is optimal. Stavrakeva (2015) builds a three period model in the spirit of Lorenzoni (2008) where markets are endogenously incomplete. In countries with large fiscal capacity and a concentrated banking sector, bailouts lead bankers to over invest and to pledge too high of a payment in the crisis state relative to what a constrained central planner would optimally pledge. In this framework, regulators in countries with strong moral hazard should impose a second ex-ante regulatory instrument that limits bank liabilities during a crisis when a bail-out is expected, in addition to the minimum bank capital requirement. Bianchi (2016) develops a non-linear DSGE model to assess the interaction between bailouts in credit markets and the build-up of risk ex ante. The optimal policy requires, in general, a mix of ex-post intervention (bailouts) and ex-ante prudential policy (taxes on debt and capital income). Keister (2015), Stavrakeva (2015), and Bianchi (2016) stress that the optimal policy mix involves bailouts combined with macroprudential policy tool(s) correcting distortions associated with bailouts though. In my model, I show that a regime with two policy tools can fix the resulting distortions but sometimes makes the financial system more fragile.

This study also relates to the literature on the optimal combination of different types of regulatory instruments. Perotti and Suarez (2011) suggest that, if the return to the investment activities undertaken by the banks is heterogeneously distributed across banks, a Pigouvian tax on short-term funding will dominate a
liquidity coverage ratio. If some poorly capitalized banks have strong gambling incentives and expand their activity as a way to shift risk to outside stakeholders, an LCR may have better properties. In general terms, an optimal regulatory design may combine price and quantity-based instruments, and the emphasis on each of them will depend on what is the dominant dimension of heterogeneity across banks. Kashyap et al. (2014) expand the standard Diamond-Dybvig (1983) model to explore how capital regulation, liquidity regulation, deposit insurance, loan to value limits, and dividend taxes interact to offset the incentive distortions associated with the possibility of a run. They argue that any attempt to implement the social planner’s allocations using regulation will involve different regulatory tools depending on the weights in the social welfare function.

The reminder of the chapter proceeds as follows. In Section 3.2, I present the model along with the definitions of financial fragility and stability. Section 3.3 characterizes the equilibrium outcomes in the environment in which bailouts affect the ex ante incentives of banks and their depositors in the absence of regulation. Section 3.4 introduces two regulatory tools that can be used to correct distortions: a tax on banks’ early payment and a liquidity requirement. I show that such a regime with full regulation can both promote financial stability and improve welfare relative to the regime with no regulation. In Section 3.5, I study the equilibrium outcomes under a regime in which the policy maker only has the ability to levy the tax on early payments and cannot impose liquidity requirements. I show that adopting such a regime can provide better incentives for financial stability than the regime with full regulation. Section 3.6 provides further interpretations of my results and offers some concluding remarks.
3.2 The Model

I develop a model based on Keister (2015), which is a version of the Diamond and Dybvig (1983) model augmented to include fiscal policy and limited commitment. I incorporate the model of portfolio choice in Cooper and Ross (1998) so that I can study how bailouts distort banks’ asset holdings as well as policy interventions that aim to correct this distortion. I begin by describing the physical environment and the basic elements of the model and then define financial fragility in this environment.

3.2.1 The Environment

As before, there are three periods indexed by $t = 0, 1, 2$, and there is a $[0, 1]$ continuum of ex ante identical depositors, indexed by $i$. Each depositor is endowed with one unit of all-purpose good that can be used for consumption or investment at $t = 0$ and has preferences of the form:

$$U(c_1, c_2, g; \omega_i) = u(c_1, c_2; \omega_i) + v(g) = \frac{(c_1 + \omega_i c_2)^{1-\gamma}}{1-\gamma} + \delta g^{1-\gamma},$$

where $c_t$ represents private consumption at date $t = 1, 2$ and $g$ is the level of public good provided at date 1.

The parameter $\delta \geq 0$ measures the relative importance of the public good. The preference type of depositor $i$, denoted $\omega_i$, is a binomial random variable with support $\Omega \equiv \{0, 1\}$. With probability $\pi$ a depositor is impatient (i.e. $\omega_i = 0$) and only values the date 1 consumption; with probability $1 - \pi$ she is patient and values the sum of date 1 and date 2 consumption. A depositor’s type $\omega_i$ is private information and is revealed to her at the beginning of date 1. The fraction of depositors in the population who will be impatient is also $\pi$ due to a law of large numbers. As in Diamond and Dybvig (1983), the coefficient of relative risk-aversion $\gamma$ is assumed to be greater than one.

There are two types of assets, the liquid asset and the illiquid asset. Each
asset is represented by a constant-returns-to-scale technology. Taking one unit of the liquid asset at date $t$ can convert it into one unit of the good at date $t + 1$, where $t = 0, 1$. Taking one unit of the illiquid asset at date 0 can transform it into $R > 1$ units of the good at date 2; if the illiquid asset is liquidated prematurely at date 1 then it yields $0 < r < 1$ units of the good for each unit invested.

At the beginning of date 0, depositors pool their resources to insure against individual liquidity risk through an intermediation technology. This technology is operated in a central location by a large number of identical and competitive banks with the aim of maximizing their depositors’ expected utility at all times. Banks then make a portfolio choice after depositors exit the central location. Depositors cannot directly observe this choice, but they will be able to infer their bank’s portfolio choice in equilibrium. At date 1, upon learning her preference type, each depositor chooses either to withdraw her funds at date 1 or to withdraw until date 2. Those depositors who contact their bank at date 1 arrive one at a time in a randomly assigned order whose value is private information and is only observable when they decide to withdraw. Under this sequential service constraint, as in Wallace (1988, 1990), banks determine the amount of payment to each withdrawing depositor based on the information received up to that situation when she arrives.

As before, I introduce a “sunspot” signal on which depositors can base their withdrawal decisions. The economy will be in one of two states, $s \in S \equiv \{\alpha, \beta\}$, with probabilities $\{1 - q, q\}$. Depositors observe the realization of the state of nature at the beginning of date 1. After a fraction $\theta \in (0, \pi)$ of depositors have been served, banks and the policy maker observe the state $s$.

Within the environment with limited commitment as in Ennis and Keister (2009, 2010), banks

---

1If $\theta = 0$, there cannot be a bank run in equilibrium; because banks would be able to observe the state before any withdrawals take place and thus give each depositor the after-tax value of her initial deposit back when she withdraws. If $\theta = \pi$, it is equivalent to assuming that banks and policy never observe the sunspot state, but must infer it based on the fundamental withdrawal demand. (See Keister, 2015.)
must always serve depositors optimally depending on the current situation. Moreover, there is no restriction on the payments a bank can make to its depositors, which implies that banks can freely adjust the payment scheme to the remaining depositors when this new information arrives.

There is a technology for transforming units of the private good one-for-one into units of the public good (for simplicity). A benevolent policy maker can tax deposits at date 1 and then transform them into public good by using the above technology. The policy maker is also unable to commit future actions and will choose tax rates as a best response to the situation at hand. The objective of the policy maker is to maximize welfare measured by the equal-weighted sum of depositors’ expected utilities,

\[ W = \int_0^1 E[U(c_1(i), c_2(i), g; \omega_i)] di. \]

### 3.2.2 Bailouts and regulations

At date 1, the policy maker is able to learn the state of nature before collecting taxes. The policy maker will respond to this information by lowering tax rates and the level of the public good, thereby bailing out banks in the event of a crisis. The timing of events is shown in Figure 3.1, which also depicts the timeline of the regulatory decisions. The policy maker is allowed to regulate banks’ short-term liabilities, that is the amount of resources paid out before the state is revealed. Specifically, as the first \( \theta \) withdrawals take place, the policy maker collects a tax on early payments from banks. This confiscated revenue is rebated back to all banks in a lump-sum fashion. The policy maker is also able to require banks to hold a minimum share of liquid assets in their portfolio through a liquidity cover ratio (LCR) chosen before banks make portfolio choices.
3.2.3 Financial crises and fragility

After observing her own preference type $\omega_i$ and the state $s$, each depositor can choose either to withdraw at date 1, or to wait until date 2,

$$y_i : \Omega \times S \rightarrow \{0, 1\},$$

where $y_i = 0$ corresponds to withdrawing at $t = 1$ and $y_i = 1$ corresponds to withdrawing at $t = 2$. Let $y$ denote a profile of withdrawal strategies for all depositors. In this overall game, an equilibrium is a strategy profile for all depositors, together with strategies for the policy maker and banks, such that every agent is best responding to the strategies of others.

Ennis and Keister (2010) show that this type of game cannot have a full bank run equilibrium. Without loss of generality, I assume a run only occurs in state $\beta$. In order to allow a run to occur with non-trivial probability, I assign the value of $q$ strictly between 0 and 1 as before. All impatient depositors will clearly choose to withdraw at date 1, since they receive no utility from consuming at date 2. The interesting question is how patient depositors will behave in state $\beta$. Formally, I study the following partial-run strategy profile for depositors:

$$y_i(\omega_i, \alpha) = \omega_i \quad \text{for all } i,$$

$$y_i(\omega_i, \beta) = \begin{cases} 0 & \text{for } i \leq \theta \\ \omega_i & \text{for } i > \theta \end{cases}, \quad (3.1)$$

which corresponds to what Ennis and Keister (2010) call a “one wave” run. Under this specific profile, each patient depositor with $i \leq \theta$ has an opportunity to
withdraw early in state $\beta$ before banks and the policy maker observe the state. Notice that I assume that the remaining patient depositors (those with $i > \theta$) do not withdraw early, even if the state is $\beta$ (a crisis is underway), but instead wait and withdraw at date 2. The following definition provides the notation of financial fragility that I use in the analysis.

**Definition 3.1.** A financial system is said to be fragile if the strategy profile in (3.1) is part of an equilibrium; otherwise the financial system is said to be stable.

In principle, there are many possible profiles of withdrawal strategies that involve a partial bank run. Keister (2015) shows that, in the one-asset version of the model, there exists an equilibrium in which some depositors run if and only if there exists an equilibrium in which depositors follow the strategy profile in (3.1). Establishing this result is more difficult in my model because of the portfolio choice: when banks have a larger strategy set, the potential for multiplicity of equilibrium is larger. Nevertheless, I show that the same results are obtained if I use a broader definition of fragility in Appendix C.

In the next three sections, I study the equilibrium allocation of resources and fragility under different policy regimes, and ask which regime is most desirable for a given economy.

### 3.3 Equilibrium with no regulation

I begin by studying how the anticipation of bailout payments distorts both sides of banks' balance sheets when the policy maker does not impose any regulation on banks' choices. In the analysis that follows, I derive the best responses of banks and the policy maker to profile (3.1) and to each others' strategies by working backward through the decision points labeled with letters in Figure 3.1. I then ask under what conditions the financial system is fragile in this regime.
3.3.1 Ex post efficient private allocation

First, consider decision point \((d)\) in Figure 3.1: bank \(j\)'s decision after the state has been revealed and any bailout payments have been made. After observing the state of nature, the bank will use this information to calculate the fraction of its remaining depositors who are impatient, which I denote \(\hat{\pi}_s\). (Note that (3.1) generates \(\hat{\pi}_\alpha = \frac{\pi - \theta}{1 - \theta}\) and \(\hat{\pi}_\beta = \pi\).) Since all uncertainty has been resolved, the bank will choose to give a common amount \(c_{1s}\) to each (impatient) depositor who withdraws at date 1. In addition, each of the remaining patient depositors will receive a common amount \(c_{2s}\) from bank \(j\)'s remaining resources when she withdraws at date 2. Bank \(j\) will distribute its remaining available resources to solve

\[
\mathcal{V}_j^s \equiv \max_{\{c_{1s}, c_{2s}\}} \left[ (1 - \theta) \hat{\pi}_s u(c_{1s}) + (1 - \hat{\pi}_s) u(c_{2s}) \right].
\]

(3.2)

I can simplify the constraint set for this problem by first noting that it will never be optimal for banks to liquidate any of the illiquid asset in state \(\alpha\). In such a case, banks could provide more consumption to all depositors by holding more of the liquid asset and less of the illiquid asset. Similarly, the assumption \(R > 1\) implies that it will never be optimal for banks to hold units of the liquid asset until \(t = 2\) in state \(\beta\). Banks may, however, hold units of the liquid asset until \(t = 2\) in state \(\alpha\), and banks may choose to meet additional early withdrawal demand by liquidating some units of the illiquid asset in state \(\beta\). Thus, I can write bank \(j\)'s resource constraints as:

\[
(1 - \theta)\hat{\pi}_\alpha c_{1\alpha} \leq 1 - \tau_\alpha - x^j - \theta c_1^j, \quad (3.3)
\]

\[
(1 - \theta)(1 - \hat{\pi}_\alpha) c_{2\alpha} = 1 - \tau_\alpha - x^j - \theta c_1^j - (1 - \theta)\hat{\pi}_\alpha c_{1\alpha} + Rx^j, \quad (3.4)
\]

\[
1 - \tau_\beta^j - x^j - \theta c_1^j \leq (1 - \theta)\hat{\pi}_\beta c_{1\beta}, \quad (3.5)
\]

\[
(1 - \theta)(1 - \hat{\pi}_\beta) c_{2\beta} = R \left\{ x^j - \frac{1}{\tau} \left[ 1 - \tau_\beta^j - x^j - \theta c_1^j - (1 - \theta)\hat{\pi}_\beta c_{1\beta} \right] \right\}. \quad (3.6)
\]

The first constraint says that the consumption of the remaining impatient depositors in state \(\alpha\) will always come from the resources placed into storage.
This constraint may or may not hold with equality at the solution. The second constraint says that in state $\alpha$, the remaining patient depositors will consume all of bank $j$’s matured investment plus any resources held in storage for two dates. The third constraint reflects the fact that additional date-1 payments may come only from liquidating investment, since all of the resources in storage have already been depleted. The last constraint is the standard pro rata division of remaining resources that determines the payment at date 2.

In solving this problem, it is helpful to divide the constraint set into four regions according to which constraints hold with equality/inequality. In the first case, the bank does not hold excess liquidity for the purpose of providing funds to depositors in the event of a run, and hence it will liquidate investment to provide additional date-1 payments. It is, of course, possible that the bank responds to a run by liquidating investment, even though it holds excess liquidity, which corresponds to the second case. In the third case, the additional early payments come only from the resources in storage without liquidating investment if a crisis occurs. Finally, the bank could choose to be in the fourth case where there is no excess liquidity and no liquidation.

<table>
<thead>
<tr>
<th>State $\alpha$</th>
<th>State $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>no excess liquidity</td>
<td>no liquidation</td>
</tr>
<tr>
<td>excess liquidity</td>
<td>Case-IV</td>
</tr>
<tr>
<td>liquidation</td>
<td>Case I</td>
</tr>
<tr>
<td>Case II</td>
<td>Case III</td>
</tr>
</tbody>
</table>

Table 3.1: Four regions

I provide an explicit derivation of the solution to the problem above, including conditions under which it lies in the different regions, as part of the best-response allocation in Appendix A.2. I show that, since all uncertainty has been resolved at this point, the solution to this problem is the ex post efficient allocation of remaining resources, which will always satisfy $c_{2s}^j \geq c_{1s}^j$. 
3.3.2 State-contingent tax rates

Now consider decision point \((c)\) in Figure 3.1: the best response of the policy maker after the state of nature has been revealed. As in Keister and Narasiman (2015), the policy maker will adjust tax rates and the level of the public good after he learns the state. If the state is \(\alpha\), the policy maker will choose a single tax rate \(\tau_\alpha\) for all banks. If the state is \(\beta\), however, the policy maker then chooses a bank-specific tax rate \(\tau^j_\beta\) according to the financial condition of each bank. In other words, the tax rate must be the same for all banks in state \(\alpha\), but may differ across banks in state \(\beta\). I can interpret the difference \(\tau_\alpha - \tau^j_\beta\) as the bailout payment bank \(j\) receives. In general, the policy maker will respond to a crisis by lowering taxes because he knows that some of the \(\theta\) depositors who have already withdrawn were actually patient, which causes a decline in the amount of the remaining available resources. As a result, the policy maker can mitigate this decline by efficiently allocating the financial system’s remaining resources between public and private consumption, anticipating that banks will optimally distribute their remaining resources as a best response.

Let \(\sigma\) represent the distribution of depositors across banks, so that the total tax revenue in state \(\alpha\) is \(\tau_\alpha\) and in state \(\beta\) is given by \(\tau_\beta \equiv \int \tau^j_\beta d\sigma(j)\). The policy maker will choose the state-contingent tax rates to maximize

\[
\int V^j_s d\sigma(j) + v(\tau_s).
\]  

(3.7)

The policy maker will choose \(\tau_\alpha\) to equate the marginal values of public and private \(t = 1\) consumption for the remaining depositors averaged across banks in the state where no bailouts occur,

\[
v'(\tau_\alpha) = \int u'(c^j_{1\alpha}) d\sigma(j).
\]  

(3.8)

Once the policy maker bails out banks during the crisis, the situation becomes more interesting. In this case, the policy maker will equate the marginal value of
public consumption in state $\beta$ to the marginal value of private $t = 1$ consumption in that state for the remaining depositors in every bank $j$,

$$v'(\tau_\beta) = u'(c^j_{1,\beta}) \text{ for all } j.$$  

(3.9)

In other words, the policy maker will choose to equalize the consumption levels of the remaining depositors who withdraw at date 1 across all banks. For a given size of the total tax revenue $\tau_\beta$ in a crisis, this entails setting

$$\tau^j_\beta - \tau_\beta = \frac{\theta \cdot (\bar{c}_1 - c^j_1)}{\text{lower tax rate}} + \frac{(1 - r)\mathcal{L} \cdot (\bar{x} - x^j)}{\text{excessive payments}} \text{ for all } j,$$

(3.10)

where $\mathcal{L}$ is an indicator such that $\mathcal{L} = 1$ if bank $j$ liquidates some units of the illiquid asset; for otherwise $\mathcal{L} = 0$. Here, $\bar{c}_1 \equiv \int c^j_1 d\sigma(j)$ is the economy-wide average of the payment $c^j_1$ given to the fraction $\theta$ of depositors who withdraw early and $\bar{x} \equiv \int x^j d\sigma(j)$ is defined to be the average amount of the illiquid asset across all banks and all depositors.

As a result, in general, a bank with fewer remaining resources (because it paid more resources to the first $\theta$ depositors and/or chose a more illiquid portfolio) will be charged a lower tax. This fact distorts incentives on the both sides of banks’ balance sheets, as I will show in the next subsection.

### 3.3.3 Distorted incentives

At decision point ($b$) in Figure 3.1, as the first $\theta$ withdrawals take place, bank $j$ will choose to give the same amount $c^j_1$ to each of these depositors based on its portfolio choice $(1 - x^j, x^j)$ which has been made after depositors returned to isolation at decision point ($a$). Since there is no bailout in state $\alpha$, the bank recognizes that giving an extra unit of resources to the first $\theta$ depositors will leave one unit less for the remaining depositors in that state. However, if a crisis occurs, the remaining resources will be determined by (3.10). These bailouts will affect banks’ choices of asset holdings and early payments as I show below.
Scenario I: liquidation

If bank $j$ chooses to liquidate some units of the illiquid asset, its remaining resources will depend only on aggregate conditions,

$$
(1 - \theta)[\hat{\pi}_\beta c_{1,\beta}^j + (1 - \hat{\pi}_\beta) r c_{2,\beta}^j] = 1 - \tau_\beta - (1 - r)\bar{x} - \theta\bar{c}_1,
$$

which the bank takes as given, and not on its own choices of $c_1^j$ and $x^j$. The values of $c_1^j$ and $x^j$ will be chosen to maximize

$$
\theta u(c_1^j) + (1 - q)V_\alpha^j + qV_\beta^j.
$$

The first term corresponds to the fraction $\theta$ of bank $j$’s depositors who withdraw before the realization of state. The sum of the last two terms denotes the expected utility from private consumption for its remaining depositors. The last term is fixed from the individual bank’s point of view and the first-order conditions characterizing the solution to this problem are

$$
u'(c_1^j) = (1 - q)v'(c_{1,\alpha}^j) \quad \text{and} \quad u'(c_{1,\alpha}^j) = Ru'(c_{2,\alpha}^j).
$$

Scenario II: no liquidation

If bank $j$ chooses not to liquidate its illiquid assets, its remaining resource constraints will be

$$
(1 - \theta)[\hat{\pi}_\beta c_{1,\beta}^j + (1 - \hat{\pi}_\beta) r c_{2,\beta}^j] = 1 - \tau_\beta - (1 - r)\bar{x} - \theta\bar{c}_1,
$$

$$
(1 - \theta)(1 - \hat{\pi}_\beta) c_{2,\beta}^j = Rx^j.
$$

The first constraint says that the optimal value of $c_{1,\beta}^j$ is independent of $c_1^j$ and $x^j$ as above. But bank $j$’s payment to depositors who withdraw at date 2 depends on its choice of $x^j$, which is determined by equation (3.15). In this scenario, the values of $c_1^j$ and $x^j$ will be chosen to maximize

$$
\theta u(c_1^j) + (1 - q)V_\alpha^j + qV_\beta^j.
$$
Specifically, I have
\[ u'(c^i_j) = (1-q)u'(c^i_{1\alpha}) \quad \text{and} \quad (1-q)u'(c^i_{1\alpha}) = (1-q)Ru'(c^i_{2\alpha}) + qRu'(c^i_{2\beta}). \quad (3.17) \]

The first-order conditions (3.13) and (3.17) highlight the incentive distortions created by bailouts in two dimensions: (i) bank \( j \) chooses \( c^i_j \) to equate the marginal value of resources before the state is known to the marginal value of resources for later \( t = 1 \) withdrawals in state \( \alpha \) only, and (ii) the optimal choice of \( x^j \) does not balance the expected marginal value of private consumption across periods, instead ignoring the losses of resources in the event of a run.\(^2\) In other words, the bank will tend to give an excessive payment \( c^i_j \) and over invest in illiquid assets \( x^j \) from a social point of view in the anticipation of receiving bailouts.

Since all banks face the same decision problem, I omit the index \( j \) to simplify the notation in what follows. The best-response allocation to profile (3.1) under the regime with no regulation is summarized by the vector
\[ \mathcal{A}^{NR} \equiv \left( x^{NR}, c^{NR}_1, \{c^{NR}_{1s}, c^{NR}_{2s}, \tau^{NR}_s\}_{s=\alpha,\beta} \right) \]
that specifies the portfolio choice, the early payments, the private and public consumption levels in each state after all uncertainty has been resolved. The explicit derivation of this allocation is given in Appendix B. It is straightforward to show that the solution to this problem will satisfy \( c^{NR}_1 < c^{NR}_{2\alpha} \) as long as
\[ q < \frac{R - 1}{R}. \quad (3.18) \]

If this inequality were reversed, it could be efficient for some patient depositors to consume at date 1 even if the state is good. Since I am interested in bank runs that lead to inefficient outcomes, in what follows, I shall restrict attention to the case (3.18).

\(^2\)As shown in Allen and Gale (1998), when there are no incentive distortions, banks will choose \( x \) to equate the expected marginal utility at date 1 after they observe the state to the expected marginal utility at date 2 taking into account both states, that is, \( (1-q)u'(c_{1\alpha}) + qu'(c_{1\beta}) = (1-q)Ru'(c_{2\alpha}) + qRu'(c_{2\beta}) \).
3.3.4 Equilibrium and fragility

I now move to asking under what conditions the best-response allocation $A^{NR}$ lies in the different cases in Table 3.1. Presenting the best responses of banks and the policy maker to (3.1) requires two pieces of notation. I define

$$\theta^{NR} = \frac{1}{2} \left[ 1 + \pi + \delta \frac{1}{\gamma} - \frac{(1-\pi)s_\beta}{1-s_\beta} r^{\frac{1}{\gamma}} \right. - \sqrt{(1 + \pi + \delta \frac{1}{\gamma} - \pi r^{\frac{1}{\gamma}})^2 - 4 \left[ \pi + \delta \frac{1}{\gamma} - r^{\frac{1}{\gamma}} \frac{1-\pi}{1-s_\beta} (s_\beta + \delta \frac{1}{\gamma}) \right]},$$

$$r^{NR} = \left[ \frac{(1-\pi)s_\beta}{(1-\pi)s_\beta + \delta \frac{1}{\gamma}} \right]^\gamma.$$

Note that $0 < \theta^{NR} < \pi$ holds whenever $r > r^{NR}$. I then have the following result.

**Lemma 3.1.** The best-response allocation $A^{NR}$ lies in Case

$$\begin{cases} I \\ IV \end{cases} \quad \text{if } \theta \begin{cases} > \\ < \end{cases} \theta^{NR}.$$

The intuition for this result is as follows. Banks have no incentive to hold units of the liquid asset between dates 1 and 2 as a precaution against the liquidation cost, knowing they will receive bailouts from the policy maker in the event of a crisis. As a result, the best-response allocation $A^{NR}$ will be never in Cases II or III. If the speed of banks’ reaction to a crisis is very slow (i.e. $\theta > \theta^{NR}$), all of the resources in storage have been paid out to the first $\theta$ depositors already. In this situation, additional date-1 payments will come only from liquidating investment, which corresponds to Case I. When the speed of bank’s reaction to a crisis is sufficiently fast, there will be sufficient assets left and hence banks with more resources will avoid liquidation in the event of a crisis.

I now verify whether the strategy profile in (3.1) is part of an equilibrium and hence whether the financial system is fragile or stable. Recall that an impatient depositor will always strictly prefer to withdraw early whatever the payment she receives, since she values date 1 consumption only. Therefore, I only need to consider the actions of patient depositors. It is straightforward to show that $c^{NR}_{1s} < c^{NR}_{2s}$ always holds, and thus a patient depositor with $i > \theta$ prefers to wait.
in both states. For patient depositors with \( i \leq \theta \), consider separately each of the two possible sunspot states. In state \( \alpha \), a patient depositor receives \( c_{NR}^{2\alpha} \) if she waits until date 2, but receives \( c_{NR}^{\alpha} \) if she withdraws at date 1. Assumption (3.18) implies \( c_{NR}^{2\alpha} > c_{NR}^{\alpha} \) always holds, so that a patient depositor will strictly prefer to wait in state \( \alpha \) as specified in (3.1). In state \( \beta \), a patient depositor with \( i \leq \theta \) whose opportunity to withdraw arrives before banks observe the state receives \( c_{NR}^{\beta} \) if she joins the run and \( c_{NR}^{2\beta} \) if she leaves her deposit in the financial system.

The discussion above establishes that the profile (3.1) emerges as an equilibrium under the regime with no regulation if and only if the allocation \( A^{NR} \) satisfies

\[
c_{1}^{NR} \geq c_{2\beta}^{NR}.
\]

Let \( \Phi^{NR} \) denote the set of economies in which a run occurs in equilibrium under the regime with no regulation. Figure 3.2 plots the set \( \Phi^{NR} \) as a function of \( q \) and \( \theta \) such that \( c_{1}^{NR} \geq c_{2\beta}^{NR} \) holds given the parameters \((\gamma, \pi, \delta, R, r) = (3, 0.3, 10^{-5}, 1.25, 0.55)\).

![Figure 3.2: The fragile set \( \Phi^{NR} \) under the regime with no regulation](image)

The qualitative features of Figure 3.2 are similar to that in Keister (2015). For a given value of \( q \), the reaction lag \( \theta \) must be sufficiently large for the financial system to be fragile. The threshold value of \( \theta \) decreases with \( q \) as the moral hazard problem becomes more severe.
Recall that when the best-response allocation $A^{NR}$ lies in Case IV, there will be sufficient assets left for banks to offer a relatively high payment to patient depositors who wait until date 2, which implies that the financial system is always stable under the regime with no regulation in such situations. When the best-response allocation lies in Case I, banks realize a crisis is occurring relatively late. After a large number of depositors have been served, the remaining resources are relatively small, which would lead banks to optimally provide smaller payments to patient depositors who withdraw at date 2. Thus, the financial system tends to be fragile in such a situation. Formally, I have the following result.

**Proposition 3.1.** If the financial system is fragile under the regime with no regulation, then $A^{NR}$ must lie in Case I.

The proof of this proposition is straightforward and is omitted. Proposition 3.1 implies that banks under this regime will choose to hold a more illiquid portfolio and hence liquidate some units of the illiquid asset to meet additional withdrawals in a run equilibrium, in the anticipation of being bailed out if a crisis occurs.

In the next two sections, I turn to my primary interest: what types of regulation could potentially improve stability and raise welfare.

### 3.4 Equilibrium with full regulation

In this section, I study a regime in which bailouts are allowed and the policy maker uses two regulations, one imposed on each side of banks’ balance sheets, to correct the distortions created by bailouts. I assume that the setup of the time-line in Figure 3.1 remains the same as the regime with no regulation except that the policy maker is able to both place a minimum liquidity requirement and levy a tax on banks’ early payments, right before decision points (a) and (b) respectively.

The Basel III framework includes the Liquidity Coverage Ratio (LCR), which
requires banks to hold high-quality liquid assets (HQLA) at least equal to their total net cash outflows (NCOF) over a 30-day under a stress scenario:

\[
LCR = \frac{HQLA}{NCOF} \geq 100\% \quad (3.19)
\]

In the context of my model, the high-quality liquid asset is equal to \(1 - x\), that is the liquid asset held in banks’ portfolio. Total expected cash outflows are calculated by multiplying banks’ liabilities by the rates at which they are expected to run off in the stress scenario specified by supervisors. This measure corresponds closely to the term \(\zeta \cdot 1\) in my model – the projected amount of funds that will flow out of banks after endowments deposited. Therefore, the LCR constraint (3.19) can also be written as

\[
1 - x \geq \zeta, \quad (3.20)
\]

which resembles the Basel III Liquidity Coverage Ratio.

In addition, the policy maker collects a tax on banks’ short-term liabilities. When each of the first fraction \(\theta\) of depositors withdraws from bank \(j\) and receives \(c_1^j\), the bank must pay an amount \(\eta c_1^j\) to the policy maker, where \(\eta\) is the chosen tax rate. Then the policy maker gives each bank a lump-sum transfer equal to the average fee collected per depositor, \(\mathcal{N} = \eta \theta \bar{c}_1\).

### 3.4.1 Corrected incentives

To investigate how the policy maker uses these regulatory instruments to correct the distortions associated with bailouts, I characterize equilibrium under this new regime following the same steps as above. For the given partial-run profile (3.1), I first determine the best responses of banks and the policy maker to this profile and to each other’s actions. With these responses in hand, I then ask whether the profile (3.1) is part of an equilibrium under the regime with full regulation.
After a fraction $\theta$ of depositors have withdrawn and taxes on deposits have been collected, each bank’s ex post efficient payment schedule $\{c_{1s}, c_{2s}\}$ is characterized as before. In addition, the state-contingent tax rates on deposits are given by the first-order conditions in (3.8) and (3.9). Since the liabilities tax revenue is given to each bank in a lump-sum fashion (it does not change the aggregate quantity of resources available for private consumption), the remaining resource constraints in state $\beta$ are the same as under the regime with no regulation in expressions (3.11) or (3.14) and (3.15) based on bank’s choice of liquidation or not. I begin, therefore, with studying how the tax on short-term liabilities changes decisions before the state has been revealed. I again solve this problem backward through the decision points in Figure 3.1.

As the first $\theta$ withdrawals take place, bank $j$ will choose the amount it gives to each depositor $c_{1}^{j}$ to maximize

$$\theta u(c_{1}^{j}) + (1 - q)\mathcal{V}_{\alpha}^{j}(\eta) + q\mathcal{V}_{\beta}^{j},$$

(3.21)

The middle term corresponds to the utility from distributing the remaining resources in state $\alpha$, including the tax on banks’ short-term liabilities. The first-order condition characterizing the solution to this problem is

$$u'(c_{1}^{j}) = (1 - q)(1 + \eta)u'(c_{1\alpha}^{j}).$$

(3.22)

Bank $j$ now gives one unit of resources to the first fraction $\theta$ of depositors by reducing $(1 + \eta)$ units of resources available to its remaining depositors in state $\alpha$. As a result, the tax on early payments increases bank $j$’s cost of using the funds to provide early payments.

Like banks, the policy maker is unable to commit to follow a course of action as the tax on banks’ short-term liabilities is collected. The tax rate $\eta$ maximizes

$$\theta u(c_{1}^{j}(\eta)) + (1 - q)[\mathcal{V}_{\alpha}^{j}(\eta) + v(\tau_{\alpha})] + q[\mathcal{V}_{\beta}^{j}(\eta) + v(\tau_{\beta})].$$

(3.23)
The first-order condition that characterizes the policy maker’s optimal choice of \( \eta \) is given by

\[
u'(c^j_1) = (1 - q)u'(c^j_{1\alpha}) + qu'(c^j_{1\beta}),\]

(3.24)

which shows that how the tax on early payments is now used to correct the distortion on the liabilities side of banks’ balance sheets, similar to Keister (2015). Since bank \( j \) undervalues resources paid out before the state is revealed, the tax rate \( \eta \) will be chosen to increase that cost until the marginal utility of early consumption is equated to the expected marginal value of remaining \( t = 1 \) consumption, taking all states into account.

It is worth noting, however, that implementing the liabilities tax cannot fix the distortion on banks’ choice of asset holding since this optimal tax rate \( \eta \) is determined after banks made their portfolio choice, as described in Figure 3.1. The results below show how placing liquidity requirement at the start of point \( (a) \) can address the insufficient incentives for holding liquid portfolio if the crisis occurs.

When faced with the liquidity regulation (3.20), bank \( j \) will now choose \( x^j \) to solve the problem (3.21). Letting \( \mu^j \) denote the multiplier on the constraint (3.20), the solution to this problem is characterized by the first-order conditions

**liquidation in state** \( \beta \):

\[
(1 - q)u'(c^j_{1\alpha}) + \mu^j = (1 - q)Ru'(c^j_{2\alpha}),
\]

(3.25)

**no liquidation in state** \( \beta \):

\[
(1 - q)u'(c^j_{1\alpha}) + \mu^j = (1 - q)Ru'(c^j_{2\alpha}) + qRu'(c^j_{2\beta}).
\]

(3.26)

These conditions implicitly show that the liquidity regulation introduces the incentive for banks to privately provision for a crisis. Before the portfolio choice, the policy maker chooses \( \zeta \) to maximize ex ante welfare,

\[
\theta u(c^j_1) + (1 - q)[V^j_{\alpha}(\zeta) + v(\tau_\alpha)] + q[V^j_{\beta}(\zeta) + v(\tau_\beta)].
\]

(3.27)
This first-order condition for $\zeta$ is
\[
[(1 - q)Ru'(c_{2\alpha}) + qRu'(c_{2\beta}) - (1 - q)u'(c_{1\alpha}) - qu'(c_{1\beta})] \frac{dx}{d\zeta} = \mu^j (1 + \frac{dx}{d\zeta}).
\] (3.28)

It is clear from combining the first-order conditions (3.25) or (3.26) with (3.28) that $\mu^j > 0$ must hold at the solution, and hence by complementary slackness the liquidity requirement must bind, which in turn implies $1 - x^j = \zeta$ and the right-hand side of (3.28) is zero. Using this fact, the runoff rate $\zeta$ will be set to make the left-hand side zero, which requires
\[
(1 - q)u'(c_{1\alpha}) + qu'(c_{1\beta}) = (1 - q)u'(c_{2\alpha}) + qu'(c_{2\beta}).
\] (3.29)

Now the run-off rate is used to modify the distortion on banks’ choice of asset holdings. Combined with (3.24), I have
\[
u'(c_j) = (1 - q)u'(c_{1\alpha}) + qu'(c_{1\beta}) = (1 - q)Ru'(c_{2\alpha}) + qu'(c_{2\beta}),
\] (3.30)
which highlights that bank $j$ now allocates resources to equate the marginal utility of its depositors across periods, taking into account both states.\(^3\)

### 3.4.2 Equilibrium and fragility

As above, all banks face the same decision problem, and I omit the index $j$ subscripts to simplify the notation in what follows. Let $\mathcal{A}^{FR}$ denote the best-response allocation under the regime with full regulation. Using the first-order conditions (3.22), (3.24), (3.28), and (3.29), it is straightforward to show that the optimal values of the liabilities tax rate $\eta$ and the runoff rate $\zeta$ in this allocation are determined by:

\[
\eta^{FR} = \frac{qu'(c_{2\beta}^{FR})}{(1-q)u'(c_{1\alpha}^{FR})}, \quad \text{and} \quad \zeta^{FR} = 1 - x^{FR}.
\] (3.31)

\(^3\)Equation (3.30) characterized precisely what an individual bank chooses to do when there is no bailout and incentives are not distorted.
I now move to asking under what conditions the best-response allocation $A^{FR}$ lies in different cases in Table 3.1. I define

$$\theta^{FR} \equiv \frac{1}{2} \left[ 1 + \pi + \delta^\gamma - \left( \frac{1-\pi}{1-\pi} \right) \hat{\theta} \hat{\pi} \right]$$

$$\theta^{FR} = \sqrt{\left[ 1 + \pi + \delta^\gamma - \pi \left( \frac{r}{R} \right)^\gamma \right]^2 - 4 [\pi + \delta^\gamma - \left( \frac{r}{R} \right)^\gamma] 1 - \left( \frac{1-\pi}{1-\pi} \right) \hat{\theta} \hat{\pi} [\hat{\pi} + \delta^\gamma]};$$

$$q_{l}^{FR} = \left\{ 1 + \frac{1-r}{R-1} \left[ \frac{\left( 1-\theta \right) \hat{\theta} + \left( 1-\theta \right) \left( \hat{\theta} \right)^\gamma + \frac{\delta^\gamma}{\gamma} \right] \gamma \right\} - 1;$$

$$q_{u}^{FR} = \left\{ 1 + \frac{1-r}{R-1} \left[ \frac{\left( 1-\theta \right) \hat{\theta} + \left( 1-\theta \right) \left( \hat{\theta} \right)^\gamma + \frac{\delta^\gamma}{\gamma} \right] \gamma \right\} - 1;$$

$$q_{uu}^{FR} = \left\{ 1 + \frac{1-r}{R-1} \left[ \frac{\left( 1-\theta \right) \hat{\theta} + \left( 1-\theta \right) \left( \hat{\theta} \right)^\gamma + \frac{\delta^\gamma}{\gamma} \right] \gamma \right\} - 1;$$

I then have the following result.

**Lemma 3.2.** The best-response allocation $A^{FR}$ lies in

- Case $I$ if $\left\{ q < q_{l}^{FR} \right\}$ as $\theta > \theta^{FR}$;

- Case $II$ if $\left\{ q_{l}^{FR} \leq q < q_{u}^{FR} \right\}$ as $\theta > \theta^{FR}$;

- Case $III$ if $\left\{ q_{u}^{FR} \leq q \right\}$ as $\theta > \theta^{FR}$;

- Case $IV$ if $\left\{ q_{uu}^{FR} \leq q < q_{uu}^{FR} \right\}$ as $\theta > \theta^{FR}$.

Figure 3.3 shows that this allocation can lie in one of the four cases in the space of $(q, \theta)$. If banks are able to react quickly to an incipient run by adjusting payments to the appropriate level, they will never hold excess liquidity nor will they liquidate investment. Now let us stick to the case in which the delay $\theta$ is large. If a crisis is very unlikely, holding excess liquidity is very costly because of $R > 1$. In this situation, additional date-1 payments can come only from liquidating investment, since all of the resources in storage have already been paid out to the first $\theta$ depositors who withdrew. As the probability of a crisis increases, banks might choose to hold excess liquidity. That is, having more
Figure 3.3: The set of the best responses under the regime with full regulation
assets in storage lowers the cost of providing additional consumption at date 1 and thus leaves banks with more resources in the event of a run. When a crisis is more likely, banks become more cautious, which guarantees that a very liquid portfolio might be able to avoid liquidation. Combining these discussions and the above result therefore gives us sufficient conditions under which the best-response allocation $A^{FR}$ involves holding excess liquidity and/or liquidation, as depicted in this figure.

As in the previous section, I provide the precise condition that determines whether the financial system is fragile by comparing the amount of consumption each patient depositor with $i \leq \theta$ receives in state $\beta$. It can be shown that the financial system is fragile under the regime with full regulation if and only if

$$c_1^{FR} \geq c_2^{FR}.$$  

Notice that if the economy lies in Case IV, the quick reaction of banks ensures that it is optimal for patient depositors with $i \leq \theta$ to wait to withdraw until date 2. If the economy lies in Case III, banks are conservative and hence hold fair excess liquidity as a precaution against the losses because the probability of a
crisis is sufficiently large. If not, the losses created by the run are large and, as a result, it would be attractive for patient depositors to join the run. Then I have the following result.

**Proposition 3.2.** *If the financial system is fragile under the regime with full regulation, then \( A^{FR} \) must lie in either Case I or Case II.*

Formal proofs of all propositions are given in Appendix B unless otherwise noted. This proposition captures the fact that imposing the liquidity regulation ensures that banks will hold a more liquid portfolio as a provision against bad outcomes.

### 3.4.3 The desirability of fully correcting distortions

I now use the allocation \( A^{FR} \) to characterize the set of all economies that are fragile under the regime with full regulation in the space of \((q, \theta)\) given other parameter values, denoted as \( \Phi^{FR} \). If the economy lies in one fragile set but not the other, the optimal policy is to select the non-fragile regime. If the economy is fragile under both regimes, the policy maker chooses the higher-welfare regime by comparing the welfare level \( W \) conditional on the financial system being fragile.

The next proposition shows that the regime with full regulation is unambiguously better than the regime with no regulation: it can both promote financial stability and improve welfare in the run equilibrium.

**Proposition 3.3.** *For all \( q > 0 \), the set \( \Phi^{FR} \) is strictly contained in \( \Phi^{NR} \), and \( W^{FR} > W^{NR} \).*

The first part of this proposition is illustrated in Figure 3.4, which adds the fragile set \( \Phi^{FR} \) to Figure 3.2. When \( q > 0 \), the regime with full regulation generates a considerably higher threshold value for \( \theta \) above which fragility arises than the regime with no regulation, because the policy maker now uses the tax
on banks’ early payment $\eta^{\text{FR}}$ and the run-off rate $\zeta^{\text{FR}}$ to correct the incentive distortions associated with bailouts. For larger values of $q$, the liquidity requirement combined with the tax on banks’ short-term liabilities can eliminate the run equilibrium, as illustrated in the upper-right region of this figure. To understand why, recall that banks under the regime with full regulation choose to hold excess liquidity if $q > q_l^{\text{FR}}$. In this case, when $q$ is large enough, banks become more cautious and hence hold a fairly liquid portfolio and, as a result, make themselves immune to runs.

The proof of the second part of Proposition 3.3 is straightforward. In fact, the competitive equilibrium allocation generated by the regime with full regulation satisfies the welfare-maximizing allocation of resources conditional on the behavior of depositors specified in (3.1), which is proved by Proposition 3.3 in the appendix. Suppose that the policy maker and banks are replaced by a single, benevolent planner whom aims to maximize welfare, facing all of the informational constraints described above. It will allocate resources efficiently conditional on depositors’ behavior and, hence, the planner’s best-response allocation to the strategy profile (3.1) is exactly $A^{\text{FR}}$. 

Figure 3.4: $\Phi^{\text{FR}}$ is strictly contained in $\Phi^{\text{NR}}$
3.4.4 Relation to the literature

Is the above regime with full regulation always best? In the one-asset model, Keister (2015) shows that the policy mix involved with bailouts and a tax on banks’ short-term liabilities can generate higher welfare and a more stable financial system than other policy interventions. This result seems to suggest that using regulatory tools to fully correct distortions created by bailouts is always desirable.

In the two-asset version of the model studied here, Proposition 3.3 does identify situations in which there is no alternative policy mix can do better than the regime with full regulation imposed on both sides of banks’ balance sheets unless it could eliminate the run equilibrium. It is then natural to ask whether this is true: Is it always desirable to give policy makers more tools to correct the incentive distortions created by bailouts? My next analysis shows, somewhat surprisingly, that the answer to this question is negative in some cases.

3.5 Equilibrium without liquidity regulation

I now ask whether, in some cases, financial stability improves if the policy maker is prohibited from placing liquidity requirement. I first compare the policy regime in which the policy maker has the ability to levy a tax on banks’ short-term liabilities but cannot impose liquidity requirements with the regime with no regulation. I show that, in this case, this new regime without liquidity regulation is always desirable. I then study what effects such an intermediate policy regime would have compared to the regime with full regulation. In this case, I show that removing the liquidity regulation is always desirable when it reduces fragility even if it does not fully correct the distortions created by bailouts, providing two analytical results followed by some illustrated examples.
3.5.1 Partially corrected incentive

As in the previous section, the tax on early payments can correct the incentive problem on the liabilities side of banks’ balance sheets, which is characterized by (3.24). Since this regulatory tool is used after the portfolio choice has been made, bank \( j \) chooses the value of \( x^j \) as under the regime with no regulation, meaning that the bank will distribute its resources between liquid and illiquid assets in a way that rationally ignore the losses in state \( \beta \). As a result, the first-order condition of \( x^j \) is given by (3.13) or (3.17), which implies that the distortion on the asset side of the banks’ balance sheets still appears.

As above, all banks face the same decision problem, and I omit the index \( j \) subscripts to simplify the notation in what follows. Let \( \mathcal{A}^{NL} \) denote the best-response allocation to profile (3.1) under the regime without liquidity regulation. Combining the first-order conditions (3.22) and (3.24), it is straightforward to show that the optimal value of the liabilities tax rate \( \eta \) in this regime is determined by

\[
\eta^{NL} = \frac{qu(c^{NL}_1)}{(1-q)w(c^{NL}_1)},
\]  

(3.32)

I now use the allocation \( \mathcal{A}^{NL} \) to identify conditions under which the best-response allocation lies in different cases in Table 3.1. I then have the following result.

**Lemma 3.3.** The best-response allocation \( \mathcal{A}^{NL} \) lies in Case \( \begin{cases} I \\ IV \end{cases} \) if \( \theta \begin{cases} > \\ < \end{cases} \theta^{NL} \equiv \theta^{NR} \).

As in the previous section, there does not exist run equilibrium if the economy lies in Case IV, since banks are able to react sufficiently quick to a crisis. Comparing two elements of the vector \( \mathcal{A}^{NL} \) yields a necessary and sufficient condition for the existence of equilibrium in which depositors follow the strategy profile in (3.1). The financial system is fragile under the regime without liquidity regulation
if and only if
\[ c_{1}^{NL} \geq c_{2}^{NL}. \]

My first result shows that banks under this regime will again not hold excess liquidity and will liquidate some units of the illiquid asset to meet additional withdrawals in a run equilibrium, since an anticipated bailouts still encourages banks to over invest their funds in the illiquid assets.

**Proposition 3.4.** If the financial system is fragile under the regime without liquidity regulation, then \( A^{NL} \) must lie in Case I.

In the one-asset model, Keister (2015) shows that introducing a Pigouvian tax on banks’ short-term liabilities is always desirable. The next result shows that this type of result also obtains in the two-asset model. Let \( \Phi^{NL} \) denote set of economies that are fragile in the regime in which the policy maker is prohibited from imposing liquidity requirement. I then have the following result.

**Proposition 3.5.** For all \( q > 0 \), the set \( \Phi^{NL} \) is strictly contained in \( \Phi^{NR} \), and \( \mathcal{W}^{NL} > \mathcal{W}^{NR} \).

The intuition for this result can be seen in two steps. First, the first-order condition (3.24) illustrates how the introduction of the tax on banks’ short-term liabilities leads banks to become conservative in valuing resources paid out before the state is revealed. For this reason, implementing a liabilities tax alone is sufficient to lower the incentive for a patient depositor to run and thus will decrease financial fragility compared to the regime with no regulation, as illustrated in Figure 3.5. In addition, this type of policy regime without liquidity regulation but with a liabilities tax, can help smooth depositors’ private consumption across states, which raises expected utility. In this way, as in Keister (2015), the regime without liquidity regulation is unambiguously better than the regime with no regulation.
3.5.2 The cost of adding liquidity regulation

My next analysis seeks to shed light on the question of whether it is always desirable to give policy makers more tools to correct the incentive distortions created by bailouts. I show that, in some cases, a regime in which the policy maker only has the ability to levy a tax on banks’ short-term liabilities – and cannot impose liquidity requirements – actually provides better incentives for financial stability. The financial system is not fragile in the regime without liquidity regulation, but becomes fragile in the regime with full regulation.

**Proposition 3.6.** There exist economies in $\Phi^{FR}$ that are not in $\Phi^{NL}$ and vice versa.

First, focus on panel (a) of Figure 3.6. When $q$ is very large, adding liquidity regulation would lead to a large increase in the total resources available to banks in the event of a crisis because of the corresponding decrease in illiquid assets. In this case, the answer is what one would (naturally) expect: imposing restriction on banks’ asset holdings makes the financial system more stable, which is depicted in the upper-right region of panel (a).

However, when $q$ is smaller, adding liquidity regulation can introduce bad...
Figure 3.6: Adding liquidity regulation may be harmful.

Proposition 3.7. If $f(r) > 0$ and $g(r) \leq 0$, then the set $\Phi^{NL}$ is strictly contained in $\Phi^{FR}$. 

The next result identifies situations in which the second effect is clearly dominant and thus the financial system becomes more fragile. For this result, I need to define two expressions:

$$f(r) \equiv (1 - \pi) \pi \left( \frac{\gamma}{R} \right)^{\frac{1}{\gamma}} + (1 - \pi)^2 \frac{r}{R} + \delta \left( \frac{\gamma}{R} \right)^{\frac{1}{\gamma}} - (1 - \pi) \frac{r}{R} R^{\frac{1}{\gamma}} - \delta^{\frac{1}{\gamma}},$$

$$g(r) \equiv \left[ \frac{(1 - \pi) \pi \left( \frac{\gamma}{R} \right)^{\frac{1}{\gamma}} + (1 - \pi)^2 \frac{r}{R} + \delta \left( \frac{\gamma}{R} \right)^{\frac{1}{\gamma}} + \frac{1}{\pi} \frac{r}{R} R^{\frac{1}{\gamma}} + \delta^{\frac{1}{\gamma}}}{(1 - \pi) \frac{r}{R} \pi \left( \frac{\gamma}{R} \right)^{\frac{1}{\gamma}} + (1 - \pi)^2 \frac{r}{R} + \delta \left( \frac{\gamma}{R} \right)^{\frac{1}{\gamma}} + \frac{1}{\pi} \frac{r}{R} R^{\frac{1}{\gamma}} + \delta^{\frac{1}{\gamma}}} \right] ^{\gamma} - \left[ \frac{(1 - \pi) \pi \left( \frac{\gamma}{R} \right)^{\frac{1}{\gamma}} + (1 - \pi)^2 \frac{r}{R} + \delta \left( \frac{\gamma}{R} \right)^{\frac{1}{\gamma}} + \frac{1}{\pi} \frac{r}{R} R^{\frac{1}{\gamma}} + \delta^{\frac{1}{\gamma}}}{(1 - \pi) \frac{r}{R} \pi \left( \frac{\gamma}{R} \right)^{\frac{1}{\gamma}} + (1 - \pi)^2 \frac{r}{R} + \delta \left( \frac{\gamma}{R} \right)^{\frac{1}{\gamma}} + \frac{1}{\pi} \frac{r}{R} R^{\frac{1}{\gamma}} + \delta^{\frac{1}{\gamma}}} \right] ^{\gamma} - \frac{(R-r)(R-1)}{R(1-r)}.$$

To see the intuition behind this result, look at the economy $e^*$ that is in $\Phi^{FR}$ but not in $\Phi^{NL}$. There is an obvious benefit to giving the policy maker an additional ability to impose a minimum liquidity requirement: banks will hold more liquid assets. By itself, this change would tend to make the financial system more stable. At the same time, however, there is a subtle cost to adding the liquidity regulation as well. Because banks hold more resources in reserves as a provision against bad outcomes, they will alter their deposit contracts to provide a higher payoff to depositors who withdraw early. This latter effect tends to make the financial system more fragile by increasing the incentive of patient depositors to join the run. Which of these two effects dominates in terms of financial fragility depends on parameter values, as shown in panel (a) of this figure.
Panel of (b) of Figure 3.6 presents the property that banks find it optimal to not hold excess liquidity if a run occurs because the liquidation cost is relatively small (increasing the value of $r$ from 0.55 to 0.7). In such a case, in fact, the minimum liquidity requirement chosen by a policy maker encourages banks to distribute all of their liquid assets to depositors who withdraw early, which would raise the incentive of patient depositors to join the run.

Propositions 3.6 and 3.7 thus show how adding liquidity regulation is helpful in some economies, but can actually harm rather than stabilize the financial system in others.

### 3.6 Conclusions

This chapter provides a framework for modeling how banks’ incentives are distorted when they anticipate bailouts: (i) banks choose to offer higher early payments and; (ii) banks over invest their resources in the illiquid asset.

My results show that fully correcting distortions created by bailouts requires the policy maker to place a minimum liquidity requirement on banks asset holdings and levy a tax on banks’ short-term liabilities. Such a regime can generate the welfare-maximizing allocation of resources conditional on the financial system being fragile. In other words, if depositors are running on the financial system, the regime with full regulation always yields higher welfare than other alternative policy interventions.

Is it desirable to give policy makers more tools to correct the incentive distortions created by bailouts? My research shows, somewhat surprisingly, that the answer to this questions is often negative. In some cases, a regime in which the policy maker only has the ability to levy a liabilities tax and cannot impose liquidity requirements actually provides better incentives for financial stability. In such situations, the financial system is not fragile under the regime without
liquidity regulation, but becomes fragile under the regime with full regulation.

Note that the limited commitment assumption is crucial here. If the policy maker can commit, then having two policy tools is always at least as good as having only one (since the policy maker could commit to not using the second tool, for example). But without commitment, that logic does not hold. In the environment with limited commitment, the policy is chosen to allocate resources efficiently given that depositors are playing the run strategy profile. One then has to check what effect this response has on the incentive for depositors to run or wait. For example, if I give the policy maker an ability to set liquidity requirements, the policy maker will change the tax rate on banks' short-term liabilities as a best response to the actions of banks and depositors. At the same time, banks would optimally respond to this situation by changing the composition of their asset portfolio and the contracts they offer to depositors, and all of these changes will alter the financial system’s susceptibility to a run.
Chapter 4

When should the liquidity regulation be implemented?

4.1 Introduction

Liquidity regulation has become a key focus for central banks since the financial crisis of 2007-2009. During the crisis, banks experienced large outflows from depositors, which revealed the insufficient asset liquidity buffers of banks relative to their short-term liabilities. This event in part inspired the recent proposals for the regulation of banks, which include the Liquidity Coverage Ratio (LCR).

The purpose of this chapter is to show how the framework developed in Chapter 3 can be used as the basis for assessing the desirability of implementing liquidity regulation. In this chapter, my analysis sheds new light on how the liquidation cost and Basel III type liquidity regulation affect financial stability and the optimal policy mix. I develop a version of the Diamond-Dybvig (1983) model based on the one in the previous chapter. To keep things simple, I eliminate the authorities’ ability of observing the sunspot state as they did in Chapter 2. I believe it is useful for me to focus on the discussion of regulatory policy implications, which is the primary concern in this study.

As is well understood in Chapter 3, in some economies, regulating banks’ asset holdings can make the financial system more susceptible to a run by depositors. Identifying conditions when a liquidity regulation – similar to the LCR introduced as part of the Basel III accords – should be implemented is the focus of the analysis in Chapter 4. Under different policy regimes, there exist distinct explicit
conditions under which the financial system is fragile in the sense that a run can emerge as an equilibrium outcome. In this chapter, I show that there exists a unique maximum probability with which financial fragility arises. If adopting an alternative policy regime decreases this cutoff value, I say that it makes the financial system less fragile. This approach serves as a key channel of evaluating the policy options in term of financial fragility.

In this chapter, I argue that the desirability of imposing a liquidity requirement depends crucially on the configuration of the liquidation cost. If there is no liquidation cost, in fact, the minimum liquidity requirement chosen by a policy maker is not a binding constraint on banks’ behavior. In such case, adding liquidity regulation will has no effect on the financial system. Worse than this, when the liquidation cost is sufficiently low, banks will not have a big incentive to hold fairly liquid portfolios. In this case, paradoxically, requiring banks to put further resources on reserves will increase the early payoff. By encouraging patient depositors to withdraw early, this leaves the financial system more vulnerable to a run. Thus, a restriction imposed on banks’ choice of asset can by itself become the source of deteriorating stability.

One of the contributions of this study is to provide precise conditions under which it is desirable for governments and regulators to impose liquidity requirements. If there is no value associated with the public good, I show that regulating banks’ choice of asset is always desirable whenever the liquidation cost exceeds a precise cutoff point. In contrast, prohibiting liquidity regulation is optimal for promoting stability whenever the liquidation cost is below the critical value. In the general case (i.e. depositors value the public good), I derive precise conditions under which each policy regime is desirable. There exist multiple critical values of the liquidation cost, which depend on the specific features of the economy.

This chapter is structured as follows. Section 4.2 outlines the model. In Section 4.3, I characterize equilibrium outcomes under different policy regimes
along with the measure of fragility. In Section 4.4, I derive precise conditions under which each policy regime is desirable. Section 4.5 concludes.

4.2 The Model

The model used in this chapter combines the limited commitment and fiscal policy features of Keister (2015) with a non-trivial portfolio choice problem as in Cooper and Ross (1998) and is based on that in Chapter 3.

In this framework, I depart from the model in the previous chapter and assume banks and the policy maker never observe the sunspot date and must infer it based on the observed withdrawal behavior. In what follows, I show that once a fraction of $\pi$ depositors have been served, authorities will be able to infer the state of nature. Otherwise, the physical environment and the basic elements of the model are the same as in Chapter 3.

Treating the authorities as learning the realization of the state through inferring the flow of withdrawals simplifies the analysis for two reasons. One is that in the existing literature ignoring the speed of the policy reaction to a crisis seems to be a natural assumption. In addition, this assumption allows me to focus on the object of policies for different types of regulations mix by side-stepping some unnecessary complications that arise from having to solve the large strategy set of agents’ actions.

Within this environment, I study the following partial-run strategy profile for depositors:

$$y_i(\omega_i, \alpha) = \omega_i \quad \text{for all } i, \text{ and}$$

$$y_i(\omega_i, \beta) = \begin{cases} 0 & \text{for } i \leq \pi \\ \omega_i & \text{for } i > \pi \end{cases},$$

(4.1)

which corresponds to that in Chapter 2. Under this specific profile, each patient depositor with $i \leq \pi$ has an opportunity to withdraw early in state $\beta$ before banks
and the policy maker observe the state. As in Ennis and Keister (2010), a run in this model is necessarily partial, that is, the remaining patient depositors (those with \( i > \pi \)) withdraw at date 2. The following definition provides the notation of financial fragility that I use in this chapter.

**Definition 4.1.** A financial system is said to be fragile if the strategy profile in (4.1) is part of an equilibrium; otherwise the financial system is said to be stable.

In the next two sections, I first study the equilibrium allocation of resources and fragility under two different policy regimes, and then ask when liquidity regulation should be implemented.

### 4.3 Equilibrium with regulation

As we saw in the previous chapter, if the policy maker does not have access to the regulatory tool(s) under the policy regime with no regulation, it is obvious that the financial system is unambiguously worse than the regime with full regulation or the one with liabilities tax alone. Thus, I restrict my attention to compare these two policy regimes with regulation(s) in this chapter. I begin by studying equilibrium outcomes under the policy regime with full regulation, in which the policy maker imposes restrictions on both sides of banks’ balances sheets to correct the resulting distortions created by bailouts. I then provide the corresponding analysis under the policy regime without the liquidity regulation, in which the policy maker cannot regulate banks’ choice of asset holdings.

#### 4.3.1 Full regulation

As in Chapter 3, I derive the best responses of banks and the policy maker to profile (4.1) and to each others’ strategies by working backward through the decision points labeled with letters in Figure 3.1. After straightforward algebra,
the best-response allocation is characterized by the same results $A^{FR}$ as that in Chapter 3 with the parameter $\theta$ set equal to $\pi$.

It is also straightforward to show that the solution to this problem will satisfy $c_{1}^{FR} < c_{2\beta}^{FR}$ as long as

$$r \leq R \cdot \left(\frac{(1-\pi)d^\frac{1}{2}}{(1-\pi)\pi + \delta^\frac{1}{2}}\right)^\gamma,$$

which implies that only the no-run equilibrium exists under this regime. Since I am interested in gaining some insights into the complex effects of the prudential regulations, in what follows, I shall assume the above inequality is always reversed in the following analysis.

I now move to asking under what conditions the best-response allocation $A^{FR}$ lies in different cases in Table 3.1. Rewrite those two constant values $q_{l}^{FR}$ and $q_{u}^{FR}$ as:

$$q_{l}^{FR} \equiv \left\{1 + \frac{1}{R-1}\left[\frac{(1-\pi)(1-\pi)^2(\frac{R}{\pi})^{\gamma-1} + \delta^{\gamma}}{(1-\pi)\pi + \delta^{\gamma}}\right]^{-1}\right\}^{-1}, q_{u}^{FR} \equiv \left\{1 + \frac{1}{R-1}\left[\frac{(1-\pi)(1-\pi)^2(\frac{R}{\pi})^{\gamma-1} + \delta^{\gamma}}{(1-\pi)\pi + \delta^{\gamma}}\right]^{-1}\right\}^{-1}.$$

Notice that $0 < q_{l}^{FR} < q_{u}^{FR} < 1$ holds whenever $r > R\left[\frac{(1-\pi)d^\frac{1}{2}}{(1-\pi)\pi + \delta^\frac{1}{2}}\right]^\gamma$. I then have the following result.

**Lemma 4.1.** $A^{FR}$ lies in Case 

$$\begin{cases} 
I & \text{if } q < q_{l}^{FR} \\
II & \text{if } q_{l}^{FR} \leq q < q_{u}^{FR} \\
III & \text{if } q_{u}^{FR} \leq q
\end{cases}.$$ 

The intuition for this result is as before. If a crisis is very unlikely, holding excess liquidity is very costly because of $R > 1$. In this situation, additional date-1 payments can come only from liquidating investment, since all of the resources in storage have already been paid out to the first $\pi$ depositors who withdrew. As the probability of a crisis increases, banks might choose to hold excess liquidity. That is, having more assets in storage lowers the cost of providing additional consumption at date 1 and thus leaves banks with more resources in the event of a run. When a crisis is more likely, banks become more cautious, which guarantees
that a very liquid portfolio might be able to avoid liquidation. Notice that banks will never choose to be in Case IV where there is no excess liquidity and no liquidation. In such a case, the impatient depositors with $i > \pi$ would receive no consumption in state $\beta$ since the resources in storage have already been paid out to the first $\pi$ depositors.

![Diagram](image)

**Figure 4.1:** The set of the best responses under the regime with full regulation

I now verify whether the strategy profile in (4.1) is part of an equilibrium and hence whether the financial system is fragile or stable. Recall that an impatient depositor will always strictly prefer to withdraw early whatever the payment she receives, since she values date 1 consumption only. Therefore, I only need to consider the actions of patient depositors. It is straightforward to show that $c^{FR}_{1s} < c^{FR}_{2s}$ always holds, and thus a patient depositor with $i > \pi$ prefers to wait in both states. For patient depositors with $i \leq \pi$, consider separately each of the two possible sunspot states. In state $\alpha$, a patient depositor receives $c^{FR}_{2\alpha}$ if she waits until date 2, but receives $c^{FR}_{1}$ if she withdraws at date 1. It is straightforward to show that $c^{FR}_{2\alpha} > c^{FR}_{1}$ always holds, so that a patient depositor will strictly prefer to wait in state $\alpha$ as specified in (4.1). In state $\beta$, a patient depositor with $i \leq \pi$ receives $c^{FR}_{1}$ if she joins the run and $c^{FR}_{2\beta}$ if she leaves her deposit in the financial system. The discussion above establishes that the financial system is fragile under the regime with full regulation if and only if the allocation $A^{FR}$ satisfies

$$c^{FR}_{1} \geq c^{FR}_{2\beta}.$$

Using the results in Appendix A.2, I can see that the ratio $c^{FR}_{1}/c^{FR}_{2\beta}$ is strictly
decreasing in $q$, which implies that there exists a threshold value of $q$ below which a run can occur in equilibrium. Formally, I introduce the following definition in this sense.

**Definition 4.2.** Given $(R, r, \gamma, \pi, \delta)$, let $q^{FR}$ be the maximum value of $q$ such that $c_1^{FR} \geq c_2^{FR}$ holds. If $c_1^{FR} \geq c_2^{FR}$ does not hold for any value of $q$, then define $q^{FR} = 0$.

In this way, the measure of financial fragility can be formalized in a general formula. I define

\[
\tilde{f}(r) = (1 - \pi)\pi(\frac{r}{R})^{\frac{1}{\gamma}} + (1 - \pi)^2 \frac{r}{R} + (\frac{r}{R})^{\frac{1}{\gamma}} \delta^{\frac{1}{\gamma}} - (1 - \pi)\frac{r}{R} R^{\frac{1}{\gamma}} - \delta^{\frac{1}{\gamma}},
\]

\[
\tilde{g}(r) = (1 - \pi)\pi(\frac{r}{R})^{\frac{1}{\gamma}} + (1 - \pi)^2 \frac{r}{R} + (\frac{r}{R})^{\frac{1}{\gamma}} \delta^{\frac{1}{\gamma}} - [(1 - \pi)\frac{r}{R} + \delta^{\frac{1}{\gamma}}] \frac{R^2 - 2rR + r}{R^2(1 - r)}^{\frac{1}{\gamma}},
\]

\[
\tilde{h}(q) = (1 - \pi)\pi(\frac{r}{R})^{\frac{1}{\gamma}} + \delta^{\frac{1}{\gamma}}(\frac{1 - q}{1 - q^{FR}})^{\frac{1}{\gamma}} - [(1 - \pi)\pi(\frac{r}{R})^{\frac{1}{\gamma}} + (1 - \pi)^2 \frac{r}{R} + (\frac{r}{R})^{\frac{1}{\gamma}} \delta^{\frac{1}{\gamma}}].
\]

I then have the following result.

**Proposition 4.1.** Given $R, r, \gamma, \pi, \delta$,

- if $\tilde{f}(r) \leq 0$, then the economy is stable for all $q$ and, therefore, $\bar{q}^{FR} = 0$;

- if $\tilde{f}(r) > 0$ and $\tilde{g}(r) \leq 0$, then

\[
\bar{q}^{FR} = \{q | \tilde{h}(q) = 0; q \in (0, q^{FR})\} \equiv \bar{q}^{FR}_{Case \, I};
\]

- if $\tilde{g}(r) > 0$, then

\[
\bar{q}^{FR} = \frac{r(R - 1)}{R(R - r)} \equiv \bar{q}^{FR}_{Case \, II}.
\]

Formal proofs of all propositions are given in Appendix B unless otherwise noted.

This proposition demonstrates that the threshold value of $q$ may lie in either Case I or Case II corresponding to different explicit conditions.
4.3.2 Without liquidity regulation

I now characterize equilibrium outcome under the regime in which the policy maker has the ability to levy a tax on banks’ short-term liabilities but cannot impose liquidity requirements. What effects would such an intermediate policy regime have compared to the regime studied above?

As discussed in Section 3.5, the tax on early payments can correct the incentive problem on the liabilities side of banks’ balance sheets, which is characterized by (3.24). However, the bank will distribute its resources between short and long assets to rationally ignore the losses in state $\beta$. As a result, the first-order condition of portfolio choice is given by (3.13) or (3.17), which implies that the distortion on the asset side of the banks’ balance sheets still appears.

I now adopt the allocation $A^{NL}$ by assuming $\theta = \pi$ to identify conditions under which the best-response allocation lies in different cases in Table 3.1. I then have the following result.

**Lemma 4.2.** $A^{NL}$ lies in Case I.

This result shows that banks under this regime will not choose to hold excess liquidity and will liquidate some units of the long asset to meet additional withdrawals in the event of a crisis, since an anticipated bailouts still encourages banks to over-invest their funds in the illiquid assets.

As usual, comparing two elements of the vector $A^{NL}$ yields a necessary and sufficient condition for the existence of equilibrium in which depositors follow the strategy profile in (4.1). The financial system is fragile under the regime without liquidity regulation if and only if

$$c_{1}^{NL} \geq c_{2}\beta^{NL}. $$

Using the results in Appendix A.3, I can see that the ratio $c_{1}^{NL}/c_{2}\beta^{NL}$ is strictly decreasing in $q$, which implies that there exists a threshold value of $q$ below which
a run can occur in equilibrium. Formally, I introduce the following definition in this sense.

**Definition 4.3.** Given \((R, r, \gamma, \pi, \delta)\), let \(q^{NL}\) be the maximum value of \(q\) such that \(c_1^{NL} \geq c_2^{NL}\) holds. If \(c_1^{NL} \geq c_2^{NL}\) does not hold for any value of \(q\), then define \(q^{NL} = 0\).

In this way, the measure of financial fragility can be formalized in a general formula. I then have the following result.

**Proposition 4.2.** Given \(R, r, \gamma, \pi, \delta\),

- if \(\tilde{f}(r) \leq 0\), then the economy is stable for all \(q\) and, therefore, \(q^{NL} = 0\);
- if \(\tilde{f}(r) > 0\), then

  \[
  q^{NL} = \frac{1}{R} \left[ \frac{(1-\pi)^{\gamma} + (1-\pi)^2 r + (\pi/r)^{\gamma}}{(1-\pi)^{\pi} + R^{-\gamma} \delta^{\gamma}} \right]^{\gamma} - 1,
  \]

As with Proposition 4.1, this result demonstrates that there exists a unique critical value of \(q\) below which bank run emerges as an equilibrium.

### 4.4 Comparing policy regimes

The analysis in the previous section has demonstrated how the policy maker corrects distortions created by bailouts through multiple regulatory instruments. I now turn to the question of what is the optimal policy mix. I first study the limiting case where \(\delta = 0\), that is, depositors do not value the public good. I show that, in such a case, imposing restrictions on both sides of banks’ balance sheets does provide worse incentives for financial stability whenever the liquidation cost is not very high. I then study the case where \(\delta > 0\), meaning that there is value associated with the public good. In this general case, I derive precise conditions
under which removing the liquidity regulation – so that the policy maker has only the ability to levy a tax on banks’ short-term liabilities – can make the financial system less susceptible to a run.

4.4.1 When $\delta = 0$

To be able to streamline better the analysis with more intensive notation, yet rich enough to characterize the theoretical results, I start with assuming there is no value of the public good, i.e. $\delta = 0$. My first result identifies situations where regulating banks’ choice of asset holdings decreases financial stability. I define a critical value $\bar{r}$ by

$$\bar{r} = \{r \mid (1 - r)[\pi(\frac{R}{r})^{1 - \frac{1}{\gamma}} + (1 - \pi)]^\gamma - (R^2 - 2rR + r) = 0; r \in (0, 1)\}.$$ 

When $r > \bar{r}$, removing liquidity requirements is unambiguously better for promoting financial stability.

**Proposition 4.3.** Assume $\delta = 0$. $\bar{q}^{NL} \begin{cases} \geq \bar{q}^{FR} \text{ if } r \leq \bar{r} \end{cases}$. $\bar{q}^{NL} \begin{cases} < \bar{q}^{FR} \text{ if } r > \bar{r} \end{cases}$.

The intuition for this result can be seen by considering the benefits and costs of imposing regulation on banks’ choice of assets. There is an obvious benefit to giving the policy maker an additional ability to place a minimum liquidity requirement on banks’ asset holdings. Banks will hold more short assets and thus become more liquid. By itself, this change would tend to make the financial system more stable. At the same time, however, there is a subtle cost to adding the liquidity regulation as well. Because banks hold more resources in reserves as a provision against bad outcomes, they will alter their deposit contracts to provide a higher payoff to depositors who withdraw early. This latter effect tends to make the financial system more fragile by increasing the incentive of patient depositors to join the run. Which of these two effects of adding liquidity regulation
dominates in terms of financial fragility depends on parameter values, as shown in Figure 4.2.

Figure 4.2: Comparing fragility when $\delta = 0$

Figure 4.2 shows when $r \leq \bar{r}$ holds, adding liquidity regulation would lead to a large increase in the total resources available to banks in the event of a crisis because of the corresponding decrease in illiquid assets. In this case, the first effect always dominates and the answer is what one would (naturally) expect: imposing liquidity restrictions on banks’ asset holdings makes the financial system more stable, which is depicted in the left region of this figure.

However, when $r > \bar{r}$ holds, banks find it is optimal to not hold excess liquidity if a run occurs because the liquidation cost is relatively small. In such case, in fact, the minimum liquidity requirement chosen by a policy maker encourages banks to distribute all of their liquid assets to depositors who withdraw early, which would raise the incentive of patient depositors to join the run. As a result, the cost of liquidity requirement is clearly dominant and adding liquidity regulation makes the financial system more susceptible to a run.

If the economy is fragile under one policy regime but not the other (i.e. all economies with $\bar{q}^{FR} < q < \bar{q}^{NL}$ or $\bar{q}^{FR} > q > \bar{q}^{NL}$), the optimal policy is to
select the non-fragile regime. If the economy is fragile under both regimes, the policy maker chooses the higher-welfare regime by comparing the welfare level $W$ conditional on the financial system being fragile. The next proposition identifies situations where which policy regime is optimal.

**Proposition 4.4.** Assume $\delta = 0$.

- For any economy with $r \leq \bar{r}$, adding liquidity regulation is always desirable;
- For any economy with $r > \bar{r}$, adding liquidity regulation is always desirable, if
  \[
  \left\{ \begin{array}{l}
  q \leq \bar{q}^{NL} \\
  \bar{q}^{NL} < q \leq \bar{q}^{FR}
  \end{array} \right. 
  \]

The proof of this proposition is straightforward. In fact, the competitive equilibrium allocation generated by the regime with full regulation attains the welfare-maximizing allocation of resources conditional on the behavior of depositors specified in (4.1).

### 4.4.2 When $\delta > 0$

My analysis is ultimately based upon a setting with the value associated with the public good. The general logic of my analysis here is parallel to that just used, but the comparison is more complex than before. My next result focuses on the fragility. I define
\[
  k(r) = (1 - \pi) \pi (\frac{r}{R})^{\frac{1}{\gamma}} + (1 - \pi) \frac{2}{R} + (\frac{\pi}{R})^{\frac{1}{\gamma}} \delta^{\frac{1}{\gamma}} - [(1 - \pi) \frac{\pi}{R} R^{\frac{1}{\gamma}} + \delta^{\frac{1}{\gamma}}] R^{\frac{1}{\gamma}(1 - \pi)}
\]

I then have the following result.

**Proposition 4.5.** Assume $\bar{f}(r) > 0$. $\bar{q}^{NL} \left\{ \begin{array}{l}
  \geq \\\n  < \end{array} \right. \bar{q}^{FR}$ if $k(r) \left\{ \begin{array}{l}
  \geq \\\n  < \end{array} \right. 0$.

This result demonstrates that there might exist multiple threshold values of $r$ for comparing the policy regime in terms of fragility.
The example in Figure 4.3 shows that removing liquidity regulation is unambiguously better for financial stability when the value of $r$ is either small or large. When the liquidation cost is sufficiently high (i.e. $r$ is small enough), banks will choose to hold a fairly liquid portfolio. In this region, imposing a liquidity requirement causes an increase in the value of reserves. This increase, in turn, raises the ex ante incentives associated with $c_1$ for patient depositors with $i \leq \pi$ to run, which increases financial fragility. As the liquidation cost decreases further, however, regulating banks’ choice of asset holdings leads banks to raise $c_{2\beta}$ because banks now have sufficient assets to mitigate the losses of liquidating investment. As a result, in this case, adding liquidity regulation decreases financial fragility. When the liquidation cost is sufficiently small, no excess liquidity becomes the best choice for banks. In this region, placing regulation on banks’ asset holdings encourages patient depositors to withdraw early since the costs of adding liquidity regulation always outweigh the benefits, which leads to an increase in financial fragility.

The next result identifies situations where the regime with full regulation is
always desirable in some economies, but it is not in others.

**Proposition 4.6.** Assume $\bar{f}(r) > 0$.

- For any economy with $k(r) \geq 0$, adding liquidity regulation is always desirable;

- For any economy with $k(r) < 0$, adding liquidity regulation is desirable, if

$$ q \leq \bar{q}^{NL}, \quad \bar{q}^{NL} < q \leq \bar{q}^{FR}. $$

**4.5 Conclusion**

In this chapter, I asked when liquidity regulation is a part of an optimal regulatory regime. Even if a bank run could be avoided through placing liquidity requirements on banks’ choice of asset, the costs of such a restriction through altering provision of early withdrawals would be an important concern. Moreover, the changes in the liquidation cost will affect the optimal policy mix through the measure of fragility defined in my model. A key policy conclusion from my analysis is that liquidity regulations like the LCR must be scrutinized carefully to ensure that they promote financial stability. I have shown how, in an environment with no value associated with the public good, removing the regulation imposed on banks’ choice of asset can eliminate bank-run equilibria and hence yield the first-best allocation (i.e. optimal welfare) whenever the liquidity cost is not very high. On the other hand, bank-run equilibria do exist in the same economy under the regime with full regulation. This is in contrast to the widespread agreement by policy makers that liquidity regulation is important and useful. This type of result works for the environment where depositors value the public good. I showed that, in this case, the desirability of prohibiting liquidity regulation can sometimes emerge even if the liquidation cost is very high.
Appendix A

The best-response allocation

A.1 The best-response allocation $A^*$

In this section, I derive the best response of the bank to the strategy profile (2.1). The expressions derived here are used in the proofs of the propositions given in Appendix B as well as in the general definition of financial fragility presented in Appendix C.

The best response of the bank to profile (2.1) is the solution to the maximization problem (2.2):

$$\max_{\{x,c_1,c_2\beta\}} \pi u(c_1) + (1-q)(1-\pi)u(c_{2\alpha}) + q(1-\pi)[\hat{\pi}_\beta u(c_{1\beta}) + (1-\hat{\pi}_\beta)u(c_{2\beta})]$$

subject to the feasibility constraints

$$\pi c_1 \leq \rho_1(1-x)$$
$$\rho_1(1-x) \leq \pi c_1 + (1-\pi)\hat{\pi}_\beta c_{1\beta}$$
$$\rho_1(1-x) \leq \pi c_1 + (1-\pi)\hat{\pi}_\beta c_{1\beta}$$
$$\rho_1(1-x) \leq \pi c_1 + (1-\pi)\hat{\pi}_\beta c_{1\beta}$$

Letting $\mu_1$, $\mu_{2\alpha}$, $\mu_{1\beta}$, and $\mu_{2\beta}$ denote the multiplier on the constraints, the solution to the problem is characterized by the first-order conditions

$$u'(c_1) = \mu_1 + \rho_2 \mu_{2\alpha} + \frac{R}{\rho} \mu_{2\beta} - \mu_{1\beta} \quad \text{(A.1)}$$
$$u'(c_{2\alpha}) = \mu_{2\alpha} \quad \text{(A.2)}$$
$$u'(c_{1\beta}) = \frac{R}{\rho} \mu_{2\beta} - \mu_{1\beta} \quad \text{(A.3)}$$
$$u'(c_{2\beta}) = \mu_{2\beta} \quad \text{(A.4)}$$
$$\mu_1 + \rho_2 \mu_{2\alpha} + \frac{R}{\rho} \mu_{2\beta} - \mu_{1\beta} = \frac{R}{\rho} (\mu_{2\alpha} + \mu_{2\beta}) \quad \text{(A.5)}$$
The solution to the problem will lie in one of three cases, depending on the value of \( q \). These cases are:

**Case I:** If \( 0 < q < \{1 + \frac{\rho_1 - r}{R - r} [\hat{\pi}_\beta \frac{R}{r} + (1 - \hat{\pi}_\beta) (\frac{R}{r})^{\frac{1}{\gamma}}] \}^{-1} \equiv q_i \), then the solution has \( \mu_1 > 0 \) and \( \mu_{1,\beta} = 0 \), and is given by:

\[
\frac{c_{1,\beta}^*}{c_{2,\beta}} = (\frac{\hat{\pi}}{R})^\frac{1}{7} < 1
\]

\[
\frac{c_i^*}{c_{2a}} = \{(1 - q) \frac{R}{\rho_1} + q R \frac{\hat{\pi}_\beta (\frac{R}{r})^{\frac{1}{7}}} {\rho_1} \}^{-\frac{1}{7}} < 1
\]

\[
\frac{c_{1,\beta}}{c_{1,\beta}} = \{(1 - q) R [\hat{\pi}_\beta (\frac{R}{r})^{\frac{1}{7}}] + (1 - \hat{\pi}_\beta) \}^{-\frac{1}{7}} + q \frac{R}{\rho_1}
\]

\[
\frac{c_{2,\beta}}{c_{2,\beta}} = \{(1 - q) [\hat{\pi}_\beta (\frac{R}{r})^{\frac{1}{7}}] + (1 - \hat{\pi}_\beta) \}^{-\frac{1}{7}} + q \frac{R}{\rho_1}
\]

**Case II:** If \( q_i \leq q \leq \{1 + \frac{\rho_1 - r}{R - r} [\hat{\pi}_\beta \rho_2 + (1 - \hat{\pi}_\beta) (\frac{R}{r})^{\frac{1}{7}}] \}^{-1} \equiv q_u \), then the solution has \( \mu_1 = 0 \) and \( \mu_{1,\beta} = 0 \), and is given by:

\[
\frac{c_{1,\beta}^*}{c_{2,\beta}} = (\frac{\hat{\pi}}{R})^\frac{1}{7} < 1
\]

\[
\frac{c_i^*}{c_{2a}} = \{(1 - q) \frac{R - r \rho_2}{\rho_1 - r} \}^{-\frac{1}{7}} < 1 \text{ as long as } \rho_2 \geq 1
\]

\[
\frac{c_{1,\beta}}{c_{1,\beta}} = (q \frac{R - r \rho_2}{R - r \rho_1})^{-\frac{1}{7}}
\]

\[
\frac{c_{2,\beta}}{c_{2,\beta}} = (q \frac{R (R - r \rho_2)}{R - r (R - r \rho_2)})^{-\frac{1}{7}}
\]

**Case III:** If \( q_u < q < 1 \), then the solution has \( \mu_1 = 0 \) and \( \mu_{1,\beta} > 0 \), and is given by:

\[
\frac{c_{1,\beta}^*}{c_{2,\beta}} : (1 - q) \left( \frac{R}{\rho_1} - \rho_2 \right) [\hat{\pi}_\beta \rho_2 \frac{c_{1,\beta}^*}{c_{2,\beta}} + (1 - \hat{\pi}_\beta) \}^{-\gamma} - q \left( \frac{c_{1,\beta}^*}{c_{2,\beta}} \right)^{-\gamma} + q \frac{R}{\rho_1} = 0
\]

Note that \( c_{1,\beta}^* < c_{2,\beta}^* \) always holds as long as \( \rho_2 \geq 1 \).

\[
\frac{c_i^*}{c_{2a}} = \{(1 - q) \frac{R}{\rho_1} + q R \frac{\hat{\pi}_\beta \rho_2 \frac{c_{1,\beta}^*}{c_{2,\beta}} + (1 - \hat{\pi}_\beta) \}^{-\gamma} - q \left( \frac{c_{1,\beta}^*}{c_{2,\beta}} \right)^{-\gamma} + q \frac{R}{\rho_1} \}^{-\frac{1}{7}} < 1
\]

\[
\frac{c_{1,\beta}}{c_{1,\beta}} = \{(1 - q) \rho_2 \hat{\pi}_\beta \rho_2 + (1 - \hat{\pi}_\beta) \frac{c_{2,\beta}^*}{c_{1,\beta}} \}^{-\frac{1}{7}}
\]

\[
\frac{c_{2,\beta}}{c_{2,\beta}} = \{(1 - q) \frac{R}{\rho_1} \hat{\pi}_\beta \rho_2 \frac{c_{1,\beta}^*}{c_{2,\beta}} + (1 - \hat{\pi}_\beta) \}^{-\frac{1}{7}} + q \frac{R}{\rho_1}
\]
A.2 The best responses under the regime with no regulation

In the following sections, I derive the best responses of banks and the policy maker to the strategy profile (3.1) under each policy regime. The expressions derived here are used in the proofs of the propositions given in Appendix B as well as in the general definition of financial fragility presented in Appendix C. I begin with the regime with no regulation.

The allocation $A^{NR}$ will lie in different cases, depending on the value of $\theta$. These cases are:

**Case-I:** If $\theta^{NR} \leq \theta$, then there is no excess liquidity but liquidation, and the solution is given by:

$$\frac{c_{1\alpha}^{NR}}{c_{2\alpha}^{NR}} = R^{-\frac{1}{\gamma}} < 1 \quad (A.10)$$

$$\frac{c_{1\beta}^{NR}}{c_{2\beta}^{NR}} = (\frac{r}{R})^{\frac{1}{\gamma}} < 1 \quad (A.11)$$

$$\frac{c_{1}^{NR}}{c_{2\alpha}^{NR}} = [(1 - q)R]^{-\frac{1}{\gamma}} < 1 \text{ as long as (3.18) holds} \quad (A.12)$$

$$\frac{c_{2\alpha}^{NR}}{c_{2\beta}^{NR}} = (1 - \theta)\hat{\pi}_{\beta}(\frac{r}{R})^{\frac{1}{\gamma}} + (1 - \theta)(1 - \hat{\pi}_{\beta})\frac{r}{R} + (1 - \theta)(1 - \hat{\pi}_{\alpha})\frac{r}{R} + R^{-\frac{1}{\gamma}}\delta^{\frac{1}{\gamma}} \quad (A.13)$$

$$\frac{c_{1}^{NR}}{c_{2\beta}^{NR}} = [(1 - q)R]^{-\frac{1}{\gamma}} \cdot \frac{c_{2\alpha}^{NR}}{c_{2\beta}^{NR}} \quad (A.14)$$

**Case-IV:** If $\theta > \theta^{NR}$, then there is no excess liquidity and no liquidation, and the solution is given by:

$$\frac{c_{1\alpha}^{NR}}{c_{1\beta}^{NR}} = (1 - \theta)(1 - \hat{\pi}_{\beta}) + \delta^{\frac{1}{\gamma}} \quad (A.15)$$

$$\frac{c_{1}^{NR}}{c_{1\alpha}^{NR}} = (1 - q)^{-\frac{1}{\gamma}} > 1 \quad (A.16)$$

$$\frac{c_{2\alpha}^{NR}}{c_{2\beta}^{NR}} = \frac{1 - \hat{\pi}_{\beta}}{1 - \hat{\pi}_{\alpha}} < 1 \quad (A.17)$$

$$\frac{c_{1}^{NR}}{c_{2\alpha}^{NR}} = \left[(1 - q)R + qR(1 - \hat{\pi}_{\beta})\frac{1}{1 - \hat{\pi}_{\alpha}}\right]^{-\frac{1}{\gamma}} < 1 \text{ as long as (3.18) holds} \quad (A.18)$$
Combing the equations (A.15)-(A.18), I can see that $c_{NR}^{2\beta} > c_{NR}^{2\alpha} > c_{NR}^{1\beta} > c_{NR}^{1\alpha}$ holds if the solution lies in Case IV, which in turn implies that the financial system is always stable if $A^{NR}$ lies in Case IV. Therefore, if the financial system is fragile under the regime with no regulation, the best-response allocation must lie in Case I.

Now, I restrict attention to Case I. It is straightforward to characterize the set $\Phi^{NR}$ by looking at the condition $c_{NR}^{1\alpha} \geq c_{NR}^{1\beta}$, which yields
\[ q \geq 1 - \left( \frac{(1-\theta)\hat{\pi}^{\beta} \left( \frac{r}{R} \right)^{\frac{1}{\gamma}} + (1-\theta)(1-\hat{\pi}^{\beta}) \frac{r}{R} + \frac{1}{\gamma} \delta \frac{1}{\gamma}}{(1-\theta)\hat{\pi}^{\alpha} + (1-\theta)(1-\hat{\pi}^{\alpha}) \frac{R}{R} + \delta} \right)^{\gamma}. \] (A.19)

It is also straightforward to show that the ratio $c_{NR}^{1\alpha}/c_{NR}^{1\beta}$ is strictly increasing in $q$ and $\theta$.

This result is a reflection of the fact that the threshold value of $\theta$ for fragility to raise is decreasing in $q$, as depicted in Figure 3.2.

A.3 The best responses under the regime with full regulation

The best-response allocation $A_{FR}^{NR}$ under the regime with full regulation will lie in one of four cases, depending on the values of $q$ and $\theta$. These cases are:

**Case I:** If $q < \begin{cases} q_1^{FR} \\ q_2^{FR} \\ q_3^{FR} \\ q_4^{FR} \end{cases}$ as $\theta \begin{cases} > \theta_{FR}^{ll} \\ < \theta_{FR}^{lll} \end{cases}$, then there is no excess liquidity but liquidation, and the solution is given by:
\[ \frac{c_{1\alpha}^{FR}}{c_{2\alpha}^{FR}} = \frac{(1-\theta)\hat{\pi}^{\beta} \left( \frac{r}{R} \right)^{\frac{1}{\gamma}} + (1-\theta)(1-\hat{\pi}^{\beta}) \frac{r}{R} + \frac{1}{\gamma} \delta \frac{1}{\gamma}}{(1-\theta)\hat{\pi}^{\alpha} + (1-\theta)(1-\hat{\pi}^{\alpha}) \frac{R}{R} + \delta} \right)^{\gamma} - \frac{1}{\gamma} \] (A.20)

\[ \frac{c_{1\beta}^{FR}}{c_{2\beta}^{FR}} = \left( \frac{\frac{r}{R}}{1-q} \right)^{\frac{1}{\gamma}} < 1 \] (A.22)

\[ \frac{c_{1\alpha}^{FR}}{c_{2\alpha}^{FR}} = \left[ (1-q)R + qR \left( \frac{c_{2\alpha}^{FR}}{c_{2\beta}^{FR}} \right)^{\gamma} \right]^{\frac{1}{\gamma}} < 1 \text{ as long as (3.18) holds} \] (A.23)
\[
\frac{c_{1\alpha}^{FR}}{c_{2\beta}^{FR}} = \left(1 - q \right) R \left( \frac{c_{2\alpha}^{FR}}{c_{2\beta}^{FR}} \right)^{-1/\gamma} + q R \right)^{-1/\gamma}
\]  

(A.24)

Recall that \( q < \left\{ \begin{array}{ll} q_{l}^{FR} & \text{as } \theta \left\{ \begin{array}{l} > \\ \leq \end{array} \right. \theta^{FR} \end{array} \right. \), which in turn implies that

\[
\frac{c_{2\alpha}^{FR}}{c_{2\beta}^{FR}} > \left\{ \begin{array}{l} \frac{(1-\theta) \pi_{\beta} (\gamma) + (1-\theta) \pi_{\beta} (\gamma) + (\gamma) \pi_{\beta} (\gamma)}{1-\pi_{\beta} (\gamma)} \\ (1-\theta) \pi_{\alpha} + (1-\theta) \pi_{\alpha} + \pi_{\alpha} + \pi_{\alpha} \end{array} \right. \end{array} \right. \text{as } \theta \left\{ \begin{array}{l} > \\ \leq \end{array} \right. \theta^{FR}.
\]

Combing this result with the equation (A.20), I can see that \( c_{1\alpha}^{FR} < c_{2\alpha}^{FR} \) always holds if \( \mathcal{A}^{FR} \) lies in Case I.

**Case-II:** If \( q_{l}^{FR} \leq q < q_{u}^{FR} \) and \( \theta > \theta^{FR} \), then there is excess liquidity and liquidation, and the solution is given by:

\[
\frac{c_{1\alpha}^{FR}}{c_{2\beta}^{FR}} = 1 \tag{A.25}
\]

\[
\frac{c_{1\beta}^{FR}}{c_{2\beta}^{FR}} = \left( \frac{r}{R} \right)^{1/\gamma} < 1 \tag{A.26}
\]

\[
\frac{c_{2\alpha}^{FR}}{c_{2\beta}^{FR}} = \left[ \frac{q \left( R - R \right)}{(1-q)(R-1)} \right]^{-1/\gamma} \tag{A.27}
\]

\[
\frac{c_{1\alpha}^{FR}}{c_{2\alpha}^{FR}} = \left[ (1-q) \frac{R - r}{1-r} \right]^{-1/\gamma} < 1 \tag{A.28}
\]

\[
\frac{c_{1\beta}^{FR}}{c_{2\beta}^{FR}} = \left[ \frac{R(R-r)}{r(R-1)} \right]^{-1/\gamma} \tag{A.29}
\]

**Case-III:** If \( q \geq \left\{ \begin{array}{ll} q_{u}^{FR} & \text{as } \theta \left\{ \begin{array}{l} > \\ \leq \end{array} \right. \theta^{FR} \end{array} \right. \), then there is excess liquidity but no liquidation, and the solution is given by:

\[
\frac{c_{1\alpha}^{FR}}{c_{2\beta}^{FR}} = 1 \tag{A.30}
\]

\[
\frac{c_{1\beta}^{FR}}{c_{2\beta}^{FR}} = \left[ R + \frac{1-q}{q} R - 1 \left( \frac{c_{2\beta}^{FR}}{c_{2\alpha}^{FR}} \right)^{\gamma} \right]^{-1/\gamma} < 1 \tag{A.31}
\]
Combining equations (A.32) and (A.33), I can see that $c_{2\alpha}^{FR} > c_{1\alpha}^{FR}$ always holds if the best-response $A^{FR}$ lies in Case III.

**Case-IV:** If $q_{II}^{FR} \leq q < q_{uu}^{FR}$ and $\theta \leq \theta^{FR}$, then there is no excess liquidity and no liquidation, and the solution is given by equations (A.15), (A.17), (A.18), and

$$
\frac{c_{1\alpha}^{FR}}{c_{1\beta}^{FR}} = \left(1 - q\right) + q \left(\frac{c_{1\beta}^{FR}}{c_{1\alpha}^{FR}}\right)^{\gamma} < 1 \text{ since } c_{1\alpha}^{FR} > c_{1\beta}^{FR} \quad (A.34)
$$

$$
\frac{c_{1\beta}^{FR}}{c_{1\alpha}^{FR}} = \left(1 - q\right) - q \left(\frac{c_{1\beta}^{FR}}{c_{1\alpha}^{FR}}\right)^{\gamma} > 1 \text{ since } c_{1\alpha}^{FR} > c_{1\beta}^{FR} \quad (A.35)
$$

$$
\frac{c_{1\alpha}^{FR}}{c_{1\alpha}^{FR}} = \frac{(1 - q)R + qR \left(\frac{1 - \pi_{\beta}}{1 - \alpha}\right)^{\gamma}}{1 - q + \frac{(1 - \theta)(1 - \pi_{\beta})(1 - \pi_{\alpha})}{(1 - \theta)(1 - \pi_{\alpha})} \cdot \frac{1}{\gamma}} \quad (A.36)
$$

It is straightforward to show that $q_{II}^{FR} \leq q < q_{uu}^{FR}$ and $\theta \leq \theta^{FR}$ imply $c_{2\alpha}^{FR} > c_{1\alpha}^{FR}$, which in turn implies that $c_{2\beta}^{FR} > c_{1\beta}^{FR} > c_{1\alpha}^{FR} > c_{1\beta}^{FR} > c_{1\beta}^{FR}$. Thus, the financial system is always stable if $A^{FR}$ lies in Case IV.

Therefore, if the financial system is fragile under the regime with full regulation, the best-response allocation $A^{FR}$ must lie in either Case I or Case II.

**A.4 The best responses under the regime without liquidity regulation**

The allocation $A^{NL}$ will lie in different cases, depending on the value of $\theta$. These cases are:

**Case-I:** If $\theta^{NL} \leq \theta$, then there is no excess liquidity but liquidation, and the
solution is given by (A.10), (A.11), (A.13), and
\[
\frac{c_{NL}^{1}}{c_{NL}^{2}} = \left[ (1 - q)R + q\frac{R}{\pi} \left( \frac{c_{NL}^{2a}}{c_{NL}^{2b}} \right) \right]^{-\frac{1}{\gamma}} < 1 \quad \text{as long as (3.18) holds} \quad (A.37)
\]
\[
\frac{c_{NL}^{1}}{c_{NL}^{2}} = \left[ (1 - q)R - q\frac{R}{\pi} \left( \frac{c_{NL}^{2a}}{c_{NL}^{2b}} \right) \right]^{-\frac{1}{\gamma}} \quad (A.38)
\]

**Case-IV:** If \( \theta > \theta_{NL}^{\alpha} \), then there is no excess liquidity and no liquidation, and the solution is given by (A.15), (A.17), (A.34), (A.35), and
\[
\frac{c_{NL}^{1}}{c_{NL}^{2}} = \left[ R + \frac{q}{1-q} R \left( \frac{c_{NL}^{2a}}{c_{NL}^{2b}} \right) \right]^{-\frac{1}{\gamma}} < 1 \quad (A.39)
\]
It is straightforward to show that \( c_{NL}^{2} > c_{NL}^{2a} > c_{NL}^{1a} > c_{NL}^{1} > c_{NL}^{1} > c_{NL}^{1} > c_{NL}^{1} > c_{NL}^{1} \) holds if the solution lies in Case IV, which in turn implies that the financial system is always stable if \( A_{NL} \) lies in Case IV. Therefore, if the financial system is fragile under the regime without liquidity regulation, the best-response allocation must lie in Case I.

Now, I restrict attention to Case I. It is straightforward to characterize the set \( \Phi_{NL} \) by looking at the condition \( c_{NL}^{1} \geq c_{NL}^{2} \), which yields
\[
q \leq \frac{1}{\frac{R}{\pi}} \left[ \frac{(1-\theta)\hat{\pi}_{\beta} + (1-\theta)(1-\hat{\pi}_{\alpha}) + \frac{1}{\gamma} \frac{1}{\gamma}}{(1-\theta)\hat{\pi}_{\alpha} + (1-\theta)(1-\hat{\pi}_{\beta}) + \frac{1}{\gamma} \frac{1}{\gamma}} \right] - 1 \quad \equiv \bar{q}_{NL}^{\alpha} \quad (A.40)
\]
It is also straightforward to show that the ratio \( c_{1}^{NL} / c_{2}^{NL} \) is strictly decreasing in \( q \) and strictly increasing in \( \theta \).

This result is a reflection of the fact that the threshold value of \( \theta \) for fragility to raise is increasing in \( q \), as depicted in Figure 3.5.
Appendix B

Proofs of Propositions

Proposition 2.1. Suppose \( \rho_1 = r \). If \( \pi > \pi_F \), then

\[
\bar{q} = \frac{\rho_1 R^{1-\frac{1}{\gamma}} \pi + (1-\pi)\pi \left[ \frac{R}{\rho_1} \right]^{1-\frac{1}{\gamma}} + (1-\pi)^{\gamma}}{1-\pi \left[ \frac{R}{\rho_1} \right]^{1-\frac{1}{\gamma}} + (1-\pi)^{\gamma}};
\]

(2.5)

otherwise, \( \bar{q} = 0 \).

Proof. In this economy with no liquidation cost, the solution to the bank’s problem is always in Case I and from Appendix A.1 I have

\[
\frac{c_1^*}{c_{2\beta}^*} = \{(1-q)\frac{R}{\rho_1} \left[ \frac{R}{\rho_1} \right]^{1-\frac{1}{\gamma}} + (1-\pi)^{\gamma} + q \frac{R}{\rho_1} \}^{-\frac{1}{\gamma}}.
\]

Recalling that \( \bar{q} \) is the maximum value of \( q \) such that \( c_1^* \geq c_{2\beta}^* \), I can use this expression to calculate

\[
\bar{q} = \begin{cases} \frac{\rho_1 R^{1-\frac{1}{\gamma}} \pi + (1-\pi)\pi \left[ \frac{R}{\rho_1} \right]^{1-\frac{1}{\gamma}} + (1-\pi)^{\gamma}}{1-\pi \left[ \frac{R}{\rho_1} \right]^{1-\frac{1}{\gamma}} + (1-\pi)^{\gamma}} & \text{if } \pi > \pi_F \\ 0 & \text{otherwise} \end{cases}.
\]

Proposition 2.2. Suppose \( \rho_1 = r \) and \( \pi > \pi_F \). If:

\begin{itemize}
  \item \( \pi > \tilde{\pi} \), then \( q \) is strictly decreasing in \( R \);
  \item \( \pi < \tilde{\pi} \), then there exists \( \tilde{R} > \rho_1 \) such that \( q \) is strictly \( \left( \begin{array}{c} \text{increasing} \\ \text{decreasing} \end{array} \right) \) in \( R \) as \( R \left( \begin{array}{c} < \\ > \end{array} \right) \tilde{R} \).
\end{itemize}
Proof. The proof of this proposition is divided into four steps as follows.

Step (i): By Proposition 2.1, the measure of financial fragility $\bar{q}$ is given by

$$
\bar{q} = \begin{cases} 
\frac{\alpha R - \pi (\frac{R}{\rho_1})^{1-\frac{1}{\gamma}} + (1-\pi)^{-\gamma}}{1-\pi (\frac{R}{\rho_1})^{1-\frac{1}{\gamma}} + (1-\pi)^{-\gamma}} & \text{if } \pi > \pi_F \\
0 & \text{otherwise}
\end{cases}
$$

Note that $1 - [\pi (\frac{R}{\rho_1})^{1-\frac{1}{\gamma}} + (1-\pi)]^{-\gamma} > 0$ always holds, while the condition $\pi > \pi_F$ would be violated if $1 < \gamma \leq 2$. Before deriving the property of $\bar{q}$, I define some useful expressions.

$$
A(R) \equiv \pi (\frac{R}{\rho_1})^{1-\frac{1}{\gamma}} + (1-\pi) - (\frac{R}{\rho_1})^{1-\frac{1}{\gamma}}
$$

$$
B(R) \equiv \frac{\alpha R - \pi (\frac{R}{\rho_1})^{1-\frac{1}{\gamma}} + (1-\pi)^{-\gamma}}{1-\pi (\frac{R}{\rho_1})^{1-\frac{1}{\gamma}} + (1-\pi)^{-\gamma}}
$$

$$
C(R) \equiv (\gamma - 1)\pi (\frac{R}{\rho_1} - 1)(\frac{R}{\rho_1})^{1-\frac{1}{\gamma}}[\pi (\frac{R}{\rho_1})^{1-\frac{1}{\gamma}} + (1-\pi)]^{-\gamma - 1} + [\pi (\frac{R}{\rho_1})^{1-\frac{1}{\gamma}} + (1-\pi)]^{-\gamma - 1}
$$

Differentiating these expressions with respect to $R$, I have

$$
A'(R) = \frac{1}{\gamma} (\frac{R}{\rho_1})^{\frac{1}{\gamma} - 1}[(\gamma - 1)\pi (\frac{R}{\rho_1})^{1-\frac{1}{\gamma}} - 1]
\begin{cases}
\leq 0 & \text{if } R_{\rho_1} \leq [\pi (\gamma - 1)]^{-\frac{\gamma}{\gamma - 2}} \\
> 0 & \text{otherwise}
\end{cases}
$$

$$
B'(R) = \frac{C(R)}{(\frac{R}{\rho_1})^{2}[1-\pi (\frac{R}{\rho_1})^{1-\frac{1}{\gamma}} + (1-\pi)^{-\gamma}]^2}
$$

$$
C'(R) = (\gamma - 1)\pi (\frac{R}{\rho_1} - 1)(\frac{R}{\rho_1})^{1-\frac{1}{\gamma}}[\pi (\frac{R}{\rho_1})^{1-\frac{1}{\gamma}} + (1-\pi)]^{-\gamma - 2} + [\pi (\frac{R}{\rho_1})^{1-\frac{1}{\gamma}} + (1-\pi)]^{-\gamma - 2}
\begin{cases}
\geq 0 & \text{if } R_{\rho_1} \leq [\frac{(1-\pi)(2-\frac{1}{\gamma})}{\pi(\gamma - 2)}]^{\frac{\gamma}{\gamma - 1}} \\
< 0 & \text{otherwise}
\end{cases}
$$

It is straightforward to show that

$$
\lim_{R \to \rho_1} A(R) = 0 \\
\lim_{R \to \rho_1} B(R) = 1 - \frac{1}{\pi(\gamma - 1)} \\
\lim_{R \to \rho_1} C(R) = 0 \\
\lim_{R \to \rho_1} B'(R) = \frac{(2-\frac{1}{\gamma})-(\gamma-\frac{1}{\gamma})\pi}{2\pi(\gamma - 1)}
$$

$$
\lim_{R \to \infty} A(R) = \infty \\
\lim_{R \to \infty} B(R) = 0 \\
\lim_{R \to \infty} C(R) = -1 \\
\lim_{R \to \infty} B'(R) = 0
$$
Step (ii): For $\pi < \frac{1}{\gamma - 1}$, using the results from step (i) above, I have

$$A(R) = \begin{cases} 
\leq 0, & \text{if } \rho_1 < R \leq \tilde{R} \\
> 0, & \text{if } \tilde{R} < R 
\end{cases} , \text{ where } A(\tilde{R}) = 0$$

$$C(R) = \begin{cases} 
\geq 0, & \text{if } \tilde{R} < R \leq \tilde{\tilde{R}} \\
< 0, & \text{if } \tilde{R} < R 
\end{cases} , \text{ where } C(\tilde{R}) = 0$$

and, hence,

$$\bar{q} \text{ is strictly } \begin{cases} 
\text{strictly increasing in } R, & \text{if } \rho_1 < R \leq \tilde{R} \\
\text{strictly decreasing in } R, & \text{if } \tilde{R} < R 
\end{cases}$$

Step (iii): For $\frac{1}{\gamma - 1} \leq \pi < \frac{2 - \frac{1}{\gamma - 1}}{\gamma - 1}$, using the results from step (i) above and the fact of $R > \rho_1$, it is then straightforward to show that $A(R) > 0$, and thus

$$\bar{q} \text{ is strictly } \begin{cases} 
\text{increasing in } R, & \text{if } \rho_1 < R \leq \tilde{R} \\
\text{decreasing in } R, & \text{if } \tilde{R} < R 
\end{cases}$$

Step (iv): For $\frac{2 - \frac{1}{\gamma - 1}}{\gamma - 1} \leq \pi$, using the results from step (i) above and the fact of $R > \rho_1$ again, it then follows that $C(R) < 0$, and I have $\bar{q}$ is strictly decreasing in $R$.

Together, these four steps establish the result.

Proposition 2.3. If $\rho_1 = r$, then $\left(\frac{c_1^*}{c_2^*}\right)$ is strictly decreasing in $R$.

Proof. In this economy with no liquidation cost, the solution to the bank’s problem is always in Case I and from Appendix A.1 I have

$$\frac{c_1^*}{c_2^*} = \left(\frac{\pi}{R}\right)^{\frac{1}{\gamma}} = \left(\frac{\rho_1}{R}\right)^{\frac{1}{\gamma}}$$

$$\frac{c_1^*}{c_1^*} = \left\{(1 - q)[\pi(\frac{R}{\rho_1})^{1 - \frac{1}{\gamma}} + (1 - \pi)]^{-\gamma} + q\right\}^{-\frac{1}{\gamma}}$$
Differentiating these expressions with respect to \( R \) gives the desired result.

\[\]

**Proposition 2.4.** Given \( R, r, \rho_1, \rho_2, \gamma, \) and \( \pi, \)

- if \( f(\cdot) \leq 0, \) then the economy is stable for all \( q \) and, therefore, \( \bar{q} = 0; \)

- if \( f(\cdot) > 0 \) and \( g(\cdot) \leq 0, \) then
  \[
  \bar{q} = \frac{\rho_2 - \pi(\frac{R}{r})^\gamma + (1 - \pi)}{1 - \pi(\frac{R}{r})^\gamma + (1 - \pi)^\gamma} \equiv \bar{q}_{\text{Case I}};
  \]

- if \( g(\cdot) > 0 \) and \( \{ q | h(q) = 0; q \in (q_u, 1) \} = \emptyset, \) then
  \[
  \bar{q} = \frac{(R - \rho_1 \rho_2) r}{(R - \rho_2 r) r} \equiv \bar{q}_{\text{Case II}};
  \]

- if \( g(\cdot) > 0 \) and \( \{ q | h(q) = 0; q \in (q_u, 1) \} = Q\{q_1, q_2, \ldots, q_n\}, \) then
  \[
  \bar{q} = q_m \equiv \bar{q}_{\text{Case III}}, \text{ where } q_m \text{ is the biggest element of } Q.
  \]

**Proof.** The measure of financial fragility \( \bar{q} \) will lie in Cases I, II, and III depending on parameter values. Define

\[
\begin{align*}
  f(\cdot) &= [\pi(\frac{R}{r})^\gamma + (1 - \pi)]^\gamma - \frac{R}{\rho_1}; \\
  g(\cdot) &= [\pi(\frac{R}{r})^\gamma + (1 - \pi)]^\gamma - \frac{R^2 - (\rho_2 + 1)r R + \rho_1 \rho_2 r}{R(\rho_1 - r)}; \\
  h(q) &= \pi \rho_2 (\rho_2 + \frac{1 - \rho_1 \rho_2}{q})^{-\gamma} + (1 - \pi) - \left[\frac{(1 - q) R}{1 - q \rho_1^\gamma}\right]^\gamma.
\end{align*}
\]

There are four scenarios needed to be considered.

**Scenario (i):** If \( f(\cdot) \leq 0, \) using Appendix A.1, then I see that \( c_1^* < c_{2\beta}^* \) holds in Cases I, II, and III under this condition. Hence, I have \( \bar{q} = 0. \)

**Scenario (ii):** If \( f(\cdot) > 0 \) and \( g(\cdot) \leq 0, \) using Appendix A.1, then I see that \( c_1^* < c_{2\beta}^* \) holds in Cases II, and III under this condition. It is straightforward to show that \( \frac{c_1^*}{c_{2\beta}^*} \) is strictly decreasing in \( q \) when the solution lies in Case I.
In this case, \( c_1^* \geq c_{2,\beta}^* \) if and only if \( q \leq \frac{\alpha_1 \pi(\frac{\rho_2}{\rho_1})^{\frac{1}{\gamma}+(1-\pi)^{-\gamma}}}{1-\pi(\frac{\rho_2}{\rho_1})^{\frac{1}{\gamma}+(1-\pi)^{-\gamma}}} \). Hence, I have 
\[
\bar{q} = \frac{\alpha_1 \pi(\frac{\rho_2}{\rho_1})^{\frac{1}{\gamma}+(1-\pi)^{-\gamma}}}{1-\pi(\frac{\rho_2}{\rho_1})^{\frac{1}{\gamma}+(1-\pi)^{-\gamma}}}.
\]

**Scenario (iii):** If \( g(\cdot) > 0 \) and \( \{q \mid h(q) = 0; q \in (q_u, 1)\} = \emptyset \), using Appendix A.1, then I see that \( c_1^* > c_{2,\beta}^* \) holds in Case I and \( c_1^* < c_{2,\beta}^* \) holds in Case III under this condition. It is straightforward to show that \( \frac{c_1^*}{c_{2,\beta}^*} \) is strictly decreasing in \( q \) when the solution lies in Case II. In this case, \( c_1^* \geq c_{2,\beta}^* \) if and only if \( q \leq \frac{r(R - \rho_1 \rho_2)}{R(R - \rho_2)} \). Hence, I have \( \bar{q} = \frac{r(R - \rho_1 \rho_2)}{R(R - \rho_2)} \).

**Scenario (iv):** If \( g(\cdot) > 0 \) and \( \{q \mid h(q) = 0; q \in (q_u, 1)\} \neq \emptyset \), using Appendix A.1 and Definition 2.2, then I see that \( \bar{q} \) lies in Case III, as shown in panels (c) and (d) of Figure 2.5. It is straightforward to show that there exists a vector \( Q \) such that \( \frac{c_1^*}{c_{2,\beta}^*}(Q) = 1 \). Let \( q_m \) be the largest element of \( Q \). Then, I have \( \bar{q} = q_m \).

\[\Box\]

**Proposition 2.5.** Assume \( f(\cdot) > 0 \) holds. For all \( \gamma \in (1, 2] \),
\[
\bar{q} \text{ is strictly } \begin{cases} \text{increasing} & \text{in } R \text{ with } R \left( < \right) \min \{R^*, R\}, \text{ if } \pi < \left( \frac{r}{\rho_1 - r} \right)^{\frac{1}{\gamma}} \mathbb{1}\{\gamma = 2\} \\ \text{decreasing} & \text{in } \mathbb{1}\{\gamma = 2\} \end{cases}
\]
\[
\text{if } \pi > \left( \frac{r}{\rho_1 - r} \right)^{\frac{1}{\gamma}} \mathbb{1}\{\gamma = 2\}\bigg),
\]
\[1\{\gamma = 2\} \text{ is an indicator function, where } 1\{\gamma = 2\} = 1 \text{ if } \gamma = 2 \text{ and } 0 \text{ otherwise.}
\]

**Proof.** Consider any economy with \( 1 < \gamma \leq 2 \). Notice that \( \frac{c_1^*}{c_{2,\beta}^*} \) is increasing in \( \rho_2 \) in Case III. Suppose that \( \rho_2 = \frac{R}{\rho_1} \), I then have \( \frac{c_1^*}{c_{2,\beta}^*} < 1 \) in Case III since the condition \( \pi > \pi_F \) is violated. In other words, I have the following results:

\[
\text{• if } f(\cdot) \leq 0, \text{ then the economy is stable and, therefore, } \bar{q} = 0;
\]
• if \( f(\cdot) > 0 \) and \( g(\cdot) \leq 0 \), then
\[
\bar{q} = \frac{\frac{1}{\pi} - \frac{1}{\pi} + (1 - \pi)^{-\gamma}}{1 - [\frac{1}{\pi} + (1 - \pi)^{-\gamma}]} \equiv \bar{q}_{\text{Case I}}; \\
\]
• if \( g(\cdot) > 0 \), then
\[
\bar{q} = \frac{(R - \rho_1 \rho_2) r}{(R - \rho_2 \gamma) R} \equiv \bar{q}_{\text{Case II}}; \\
\]
Taking the derivatives of \( \bar{q}_{\text{Case I}} \) and \( \bar{q}_{\text{Case II}} \) with respective to \( R \), yields the following results.

<table>
<thead>
<tr>
<th>( \bar{q}_{\text{Case I}} )</th>
<th>( R )</th>
<th>( \bar{q}_{\text{Case II}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \downarrow )</td>
<td>( \uparrow )</td>
<td>( \downarrow )</td>
</tr>
</tbody>
</table>

where \( \uparrow (\downarrow) \) represents an increasing (decreasing) function;

Table B.1: The impact of the return \( R \) on \( \bar{q} \) with \( 1 < \gamma \leq 2 \)

Next, I can identify \( \bar{q} \) by using the sign of \( f(\cdot) \) and \( g(\cdot) \).

• If \( \pi < \frac{1}{\rho_2 - 1} \left( \frac{1}{\rho_1 \rho_2} \right)^{1 - \gamma} \), I see that \( f(\cdot) \leq 0 \). Thus, I have \( \bar{q} = 0 \).

• If \( \frac{1}{\rho_2 - 1} \left( \frac{1}{\rho_1 \rho_2} \right)^{1 - \gamma} \leq \pi < \left( \frac{r}{\rho_1} \right)^{1 - \frac{1}{\gamma}} \), I see that
\[
\left\{ \begin{array}{l}
\{ f(\cdot) \leq 0 \} \\
\{ f(\cdot) > 0 \ \text{and} \ g(\cdot) \leq 0 \} \text{ if } R \in (R^*, R^{**}) \\
\{ g(\cdot) > 0 \} \text{ if } R \in (\rho_1 \rho_2, R^*)
\end{array} \right.
\]
i.e. \( \bar{q} = \begin{cases} 0 & \text{if } R \in (R^*, R^{**}) \\ \bar{q}_{\text{Case I}} & \text{if } R \in (\rho_1 \rho_2, R^*) \end{cases} \)

where \( f(R^{**}) = 0 \) and \( g(R^*) = 0 \).

• If \( \left( \frac{r}{\rho_1} \right)^{1 - \frac{1}{\gamma}} \leq \pi < \left( \frac{r}{\rho_1 - 1} \right)^{\frac{1}{\gamma}} \), I see that
\[
\left\{ \begin{array}{l}
\{ f(\cdot) > 0 \ \text{and} \ g(\cdot) \leq 0 \} \text{ if } R \in (R^*, R^{**}) \\
\{ g(\cdot) > 0 \} \text{ if } R \in (\rho_1 \rho_2, R^*)
\end{array} \right.
\]
i.e. \( \bar{q} = \begin{cases} \bar{q}_{\text{Case I}} & \text{if } R \in (R^*, R^{**}) \\ \bar{q}_{\text{Case II}} & \text{if } R \in (\rho_1 \rho_2, R^*) \end{cases} \) if \( R \)
• If \((\frac{r}{\rho_1-r})^{\frac{1}{1-\gamma}} \leq \pi\), I see that \(g(\cdot) > 0\). Thus, I have \(\bar{q} = \bar{q}_{\text{Case II}}\).

Combined with the results of Table B.1, I then have this proposition, as desired.

\[\blacksquare\]

**Proposition 2.6.** Assume \(f(\cdot) > 0\) holds. For all \(\gamma \in (1, 2]\),

- \(\bar{q}\) is strictly increasing in \(r\) with \(r \prec r^*\);
- \(\bar{q}\) is strictly increasing in \(\rho_1\) with \(\rho_1 \prec \rho_1^*\);
- \(\bar{q}\) is
  \[
  \begin{cases}
  \text{constant} & \text{in } \rho_2 \text{ with } \rho_2 \prec \rho_2^*, \text{ if } \pi < \pi^* \\
  \text{decreasing} & \text{in } \rho_2, \text{ if } \pi > \pi^*
  \end{cases}
  \]

\[\begin{aligned}
\bar{q} &= \frac{\rho_2}{R} - \frac{\left[\pi\left(\frac{R}{\rho_1}\right)^{\frac{1}{1-\gamma}} + (1-\pi)\right]^{-\gamma}}{1 - \left[\pi\left(\frac{R}{\rho_1}\right)^{\frac{1}{1-\gamma}} + (1-\pi)\right]^{-\gamma}} \equiv \bar{q}_{\text{Case I}}; \\
\bar{q} &= \frac{(R-\rho_1\rho_2)r}{(R-\rho_2)r} \equiv \bar{q}_{\text{Case II}};
\end{aligned}\]

Proof. Consider any economy with \(1 < \gamma \leq 2\). Notice that \(\frac{\dot{c}^2_{2, \beta}}{c_{2, \beta}}\) is increasing in \(\rho_2\) in Case III. Suppose that \(\rho_2 = \frac{R}{\rho_1}\), I then have \(\frac{\dot{c}^2_{2, \beta}}{c_{2, \beta}} < 1\) in Case III since the condition \(\pi > \pi_F\) is violated. In other words, I have the following results:

- if \(f(\cdot) \leq 0\), then the economy is stable and, therefore, \(\bar{q} = 0\);
- if \(f(\cdot) > 0\) and \(g(\cdot) \leq 0\), then
  \[
  \bar{q} = \frac{\rho_2}{R} - \frac{\left[\pi\left(\frac{R}{\rho_1}\right)^{\frac{1}{1-\gamma}} + (1-\pi)\right]^{-\gamma}}{1 - \left[\pi\left(\frac{R}{\rho_1}\right)^{\frac{1}{1-\gamma}} + (1-\pi)\right]^{-\gamma}} \equiv \bar{q}_{\text{Case I}};
  \]
  - if \(g(\cdot) > 0\), then
    \[
    \bar{q} = \frac{(R-\rho_1\rho_2)r}{(R-\rho_2)r} \equiv \bar{q}_{\text{Case II}};
    \]

Taking the derivatives of \(\bar{q}_{\text{Case I}}\) and \(\bar{q}_{\text{Case II}}\) with respective to \(r\), \(\rho_1\), and \(\rho_2\), yields the following results. Next, I can identify \(\bar{q}\) by using the sign of \(f(\cdot)\) and \(g(\cdot)\) with respective to \(r\), \(\rho_1\), and \(\rho_2\).
<table>
<thead>
<tr>
<th>Case</th>
<th>$r$</th>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$\downarrow$</td>
<td>$\uparrow$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>II</td>
<td>$\uparrow$</td>
<td>$\downarrow$</td>
<td>$\downarrow$</td>
</tr>
</tbody>
</table>

where $\uparrow$ ($\downarrow$) represents an increasing (decreasing) function; $\emptyset$ represents no effect;

Table B.2: The impact of asset returns on $\bar{q}$ with $1 < \gamma \leq 2$

- $r$:
  
  \[
  \begin{align*}
  f(\cdot) &\leq 0 \\
  \begin{cases}
  f(\cdot) > 0 \text{ and } g(\cdot) \leq 0 \\
  g(\cdot) > 0
  \end{cases}
  \end{align*}
  \]
  if $r \in \begin{cases}
  (r^{**}, \rho_1) \\
  (r^*, r^{**}) \\
  (0, r^*)
  \end{cases}$

  i.e. $\bar{q} = \begin{cases}
  0 \\
  \bar{q}_{\text{Case I}}
  \end{cases}$ if $r \in \begin{cases}
  (r^{**}, \rho_1) \\
  (r^*, r^{**}) \\
  (0, r^*)
  \end{cases}$

  where $f(r^{**}) = 0$ and $g(r^*) = 0$.

- $\rho_1$:
  
  - If $\pi < \frac{1}{(\frac{R}{r})^{1+\frac{\gamma}{\gamma-1}}}$, I see that $f(\cdot) \leq 0$. Thus, I have $\bar{q} = 0$.

  - If $\frac{1}{(\frac{R}{r})^{1+\frac{\gamma}{\gamma-1}}} \leq \pi$, I see that

    \[
    \begin{align*}
    f(\cdot) &\leq 0 \\
    \begin{cases}
    f(\cdot) > 0 \text{ and } g(\cdot) \leq 0 \\
    g(\cdot) > 0
    \end{cases}
    \end{align*}
    \]
    if $\rho_1 \in \begin{cases}
    (r, \rho_1^{**}) \\
    (\rho_1^{**}, \rho_1^*) \\
    (\rho_1^*, \frac{R}{\rho_2})
    \end{cases}$

    i.e. $\bar{q} = \begin{cases}
    0 \\
    \bar{q}_{\text{Case I}}
    \end{cases}$ if $\rho_1 \in \begin{cases}
    (r, \rho_1^{**}) \\
    (\rho_1^{**}, \rho_1^*) \\
    (\rho_1^*, \frac{R}{\rho_2})
    \end{cases}$

    where $f(\rho_1^{**}) = 0$ and $g(\rho_1^*) = 0$.

- $\rho_2$:
  
  - If $\pi < \frac{1}{(\frac{R}{r})^{1+\frac{\gamma}{\gamma-1}}}$, I see that $f(\cdot) \leq 0$. Thus, I have $\bar{q} = 0$.
If \((R_{\rho 1})_{1}^{-\gamma -1} \leq \pi < (R_{r 1})_{1}^{-\gamma -1}\), I see that
\[
\begin{cases}
  f(\cdot) > 0 \text{ and } g(\cdot) \leq 0 \\
  g(\cdot) > 0
\end{cases}
\]
if \(\rho_2 \begin{cases}
  < \\
  >
\end{cases} \rho_2^*, \text{i.e. } \bar{q} = \begin{cases}
  \bar{q}_{\text{Case I}} \\
  \bar{q}_{\text{Case II}}
\end{cases}\)

where \(g(\rho_2^*) = 0\).

If \((\frac{R_{\rho 2}}{R_{r 1}})^{\frac{1}{\gamma -1}} \leq \pi\), I see that \(g(\cdot) > 0\). Thus, I have \(\bar{q} = \bar{q}_{\text{Case II}}\).

Combined with the results of Table B.2, I then have this proposition, as desired.

Proposition 3.2. If the financial system is fragile under the regime with full regulation, then \(A^{FR}\) must lie in Case I or Case II.

Proof. According to Appendix A.3, I have \(c_1^{FR} < c_2^{FR}\) whenever the economy lies in Case III or Case IV. If a run-equilibrium exists, therefore, the best-response allocation must lie in Case I or Case II.

Proposition 3.3. The set \(\Phi^{FR}\) is strictly contained in \(\Phi^{NR}\), and \(W^{FR} > W^{NR}\).

Proof. The proof of the first part is divided into two steps.

Step 1: Show \(\Phi^{FR} \subset \Phi^{NR}\). For any \(e \in \Phi^{FR}\), I know that \(c_1^{FR} \geq c_2^{FR}\) holds and the best-response allocation \(A^{FR}\) must lie in Case-I or Case-II, which implies that the best-response allocation \(A^{NR}\) must lie in Case-I. Using the equations (A.20), (A.21), and (A.28), it is straightforward to show the ratio \(c_{2\alpha}/c_{2}\beta\) is strictly decreasing in \(q\) if the best-response allocation \(A^{FR}\) lies in Cases I or II. Combing this result with equation (A.13), I can see that \(\lim_{q \to 0^+} \frac{c_{2\alpha}}{c_{2}^{FR}} = \frac{c_{2\alpha}}{c_{2}^{NR}}\), which yields \(\frac{c_{2\alpha}}{c_{2}^{NR}} > \frac{c_{2\alpha}}{c_{2}^{FR}}\) if the economy \(e \in \Phi^{FR}\). Looking at the equations and (3.13) and (3.30), I have \(\frac{c_{2\alpha}}{c_{2\alpha}} > \frac{c_{2\alpha}}{c_{2}^{NR}}\) always holds. Hence, if the economy \(e \in \Phi^{FR}\) then it is also in \(\Phi^{NR}\). Finally, it is easy to show \(e \in \Phi^{NR}\) but \(e \notin \Phi^{FR}\) by looking at Figure 3.4.
**Step 2:** Show the equilibrium allocation of resources under the regime with full regulation attains the highest possible level of welfare conditional on depositors following strategy profile (3.1). It then follows that the welfare levels associated with $A^{NR}$ is strictly lower.

Suppose a benevolent planner could control all endowments and operate both the banking technology and the public sector. However, this planner cannot control depositors’ withdrawal decisions and faces the limited commitment as are banks and the policy maker. At date 0, the policy maker makes a portfolio choice $(1 - x^P, x^P)$. Before the realization of the state, the policy maker chooses to give the common amount of consumption, $c_1^P$, to each depositor who withdraws early. Once she has observed the state $s$, the planner will choose to give common amounts $\{c_{1s}^P, c_{2s}^P\}_{s=\alpha,\beta}$ to each of the remaining impatient and patient depositors, respectively. In addition, she will provide an amount of $g_s^P$ of the public good. Thus, the best response of the planner to the strategy profile (3.1) can be summarized by a vector

$$A^P \equiv (x^P, c_1^P, \{c_{1s}^P, c_{2s}^P, g_s^P\}_{s=\alpha,\beta}).$$

The elements of this vector will be chosen to maximize

$$\theta u(c_1) + (1 - q) \{(1 - \theta) [\hat{\pi}_\alpha u(c_{1\alpha}) + (1 - \hat{\pi}_\alpha) u(c_{2\alpha})] + v(g_\alpha)\}$$

$$+ q \{(1 - \theta) [\hat{\pi}_\beta u(c_{1\beta}) + (1 - \hat{\pi}_\beta) u(c_{2\beta})] + v(g_\beta)\}$$

subject to the resource constraints

$$(1 - \theta)\hat{\pi}_\alpha c_{1\alpha} \leq 1 - g_\alpha - x - \theta c_1,$$

$$(1 - \theta)(1 - \hat{\pi}_\alpha)c_{2\alpha} = 1 - g_\alpha - x - \theta c_1 - (1 - \theta)\hat{\pi}_\alpha c_{1\alpha} + Rx,$$

$$1 - g_\beta - x - \theta c_1 \leq (1 - \theta)\hat{\pi}_\beta c_{1\beta},$$

$$(1 - \theta)(1 - \hat{\pi}_\beta)c_{2\beta} = R \left\{ x - \frac{1}{r} [1 - g_\beta - x - \theta c_1 - (1 - \theta)\hat{\pi}_\beta c_{1\beta}] \right\}.$$

Letting $\mu_{1\alpha}; \mu_{2\alpha}; \mu_{1\beta}$ and $\mu_{2\beta}$ denote the multiplier on the constraints, the solution
to the problem is characterized by the first-order conditions

\[
\mu_{1\alpha} + \mu_{2\alpha} + \frac{R}{\tau} \mu_{2\beta} - \mu_{1\beta} = R(\mu_{2\alpha} + \mu_{2\beta})
\]

\[
u'(c_1) = \mu_{1\alpha} + \mu_{2\alpha} + \frac{R}{\tau} \mu_{2\beta} - \mu_{1\beta}
\]

\[(1 - q) \nu'(c_{1\alpha}) = \mu_{1\alpha} + \mu_{2\alpha}
\]

\[(1 - q) \nu'(c_{2\alpha}) = \mu_{2\alpha}
\]

\[qu'(c_{1\beta}) = \frac{R}{\tau} \mu_{2\beta} - \mu_{1\beta} \]

\[qu'(c_{2\beta}) = \mu_{2\beta} \]

\[(1 - q) \nu'(g_{\alpha}) = \mu_{1\alpha} + \mu_{2\alpha} \]

\[qu'(g_{\beta}) = \frac{R}{\tau} \mu_{2\beta} - \mu_{1\beta} \]

\[
[(1 - \theta) \hat{\pi}_\alpha c_{1\alpha} - (1 - g_{\alpha} - x - \theta c_1)]\mu_{1\alpha} = 0
\]

\[
[1 - g_{\beta} - x - \theta c_1 - (1 - \theta) \hat{\pi}_{\beta} c_{1\beta}]\mu_{1\beta} = 0
\]

\[(1 - \theta)(1 - \hat{\pi}_\alpha)c_{2\alpha} = 1 - g_{\alpha} - x - \theta c_1 - (1 - \theta) \hat{\pi}_\alpha c_{1\alpha} + Rx
\]

\[(1 - \theta)(1 - \hat{\pi}_{\beta})c_{2\beta} = R \left\{ x - \frac{1}{\tau} [1 - g_{\beta} - x - \theta c_1 - (1 - \theta) \hat{\pi}_{\beta} c_{1\beta}] \right\}
\]

The solution to the problem will lie in one of four cases, depending on the value of \((q, \theta)\). It is straightforward to show that the equilibrium allocation vector \(A_{FR}\) under the regime with full regulation solves the problem of the benevolent planner and, since this solution is unique, must create strictly higher welfare than that of the equilibrium allocation \(A_{NR}\).

\[\square\]

**Proposition 3.4.** If the financial system is fragile under the regime without liquidity regulation, then \(A_{NL}\) must lie in Case I.

**Proof.** According to Appendix A.4, I have \(c_{1\alpha}^{NL} < c_{2\beta}^{NL}\) whenever the economy lies in Case IV. The run-equilibrium can only exist, therefore, if the best-response allocation \(A_{NL}\) lies in Case I.

\[\square\]
Proposition 3.5. The set $\Phi_{NL}$ is strictly contained in $\Phi_{NR}$, and $\mathcal{W}^{NL} > \mathcal{W}^{NR}$.

Proof. First, note that the ratio $c_{2\alpha}/c_{2\beta}$ is equal under both regimes, that is $c_{2\alpha}^{NL}/c_{2\beta}^{NL} = c_{2\alpha}^{NR}/c_{2\beta}^{NR}$. Using Lemma 3.1, Lemma 3.3, Proposition 3.1, and Proposition 3.4, it is straightforward to show that if $e \in \Phi_{NL}$ then $e$ lies in Case-I under both policy regimes. In this case, the ratio $c_{1}/c_{2\beta}$ under these two regimes can be written as

$$
\frac{c_{1}^{NR}}{c_{2\beta}^{NR}} = [(1 - q)R(c_{2\beta}^{NR})^{-\gamma}]^{-\frac{1}{\gamma}},
$$

$$
\frac{c_{1}^{NL}}{c_{2\beta}^{NL}} = [(1 - q)R(c_{2\beta}^{NL})^{-\gamma} + qR]^{-\frac{1}{\gamma}} = [(1 - q)R(c_{2\beta}^{NR})^{-\gamma} + qR]^{-\frac{1}{\gamma}},
$$

and, hence, $\frac{c_{1}^{NR}}{c_{2\beta}^{NR}} > \frac{c_{1}^{NL}}{c_{2\beta}^{NL}}$. Therefore, if the economy $e \in \Phi_{NL}$ then it is also in $\Phi_{NR}$.

Finally, it is easy to show that there exist $e \in \Phi_{NR}$ but $e \notin \Phi_{NL}$ by looking at Figure 3.5.

For the second part of the proposition, note that equations (3.32), (A.10), (A.11), and (A.13) imply that the welfare levels associated with the run-equilibrium allocation under the regime without liquidity regulation is given by

$$
\mathcal{W}^{NL}(\eta^{NL}) = \left\{ \theta + (1 - q)\frac{1}{\gamma}(1 + \eta^{NL})\frac{1}{\gamma} - 1 \left[ (\pi - \theta) + (1 - \pi)R\frac{1}{\gamma} - 1 + \delta \frac{1}{\gamma} \right] + q(1 - q)\frac{1}{\gamma} - 1(1 + \eta^{NL})\frac{1}{\gamma} - 1 \left[ R\frac{1}{\gamma} - 1 + \delta \frac{1}{\gamma} \right] \right\} \cdot u(c_{1}^{NL}),
$$

where

$$
c_{1}^{NL} = \frac{1}{\theta} - \frac{1}{\theta}(1 - q)\frac{1}{\gamma}(1 + \eta^{NL})\frac{1}{\gamma} \left[ (\pi - \theta) + (1 - \pi)R\frac{1}{\gamma} - 1 + \delta \frac{1}{\gamma} \right].
$$

It is also worth noting that equations (A.10)-(A.12) imply that the welfare levels associated with the run-equilibrium allocation under the regime with no regulation is given by

$$
\mathcal{W}^{NR} = \left\{ \theta + (1 - q)\frac{1}{\gamma} \left[ (\pi - \theta) + (1 - \pi)R\frac{1}{\gamma} - 1 + \delta \frac{1}{\gamma} \right] + q(1 - q)\frac{1}{\gamma} - 1 \left[ R\frac{1}{\gamma} - 1 + \delta \frac{1}{\gamma} \right] \right\} \cdot u(c_{1}^{NR}),
$$

where

$$
c_{1}^{NR} = \frac{1}{\theta} - \frac{1}{\theta}(1 - q)\frac{1}{\gamma} \left[ (\pi - \theta) + (1 - \pi)R\frac{1}{\gamma} - 1 + \delta \frac{1}{\gamma} \right].
$$
where
\[ c_{1}^{NR} = \frac{1}{\theta} - \frac{1}{\theta}(1 - q)^{\frac{1}{\gamma}} \left[ (\pi - \theta) + (1 - \pi)R^{\frac{1}{\gamma}-1} + \delta^\frac{1}{\gamma} \right]. \]

Together, these two results imply that the welfare levels associated with the run-equilibrium allocations under the two regimes must be equal when \( \eta^{NL} = 0 \), that is \( \mathcal{W}^{NL}(\eta^{NL} = 0) = \mathcal{W}^{NR} \). The final step is to note that the liabilities tax \( \eta^{NL} \) corrects the distortion on the liabilities side of banks' balance sheets to maximize the problem (3.23). Since equation (3.32) establishes that \( \eta^{NL} > 0 \), I must, therefore, have \( \mathcal{W}^{NL} > \mathcal{W}^{NR} \), as desired.

\[ \blacksquare \]

**Proposition 3.7.** If \( f(r) > 0 \) and \( g(r) \leq 0 \), then the set \( \Phi^{NL} \) is strictly contained in \( \Phi^{FR} \).

**Proof.** Recall that if the economy \( e \in \Phi^{NL} \), then the equilibrium outcome lies in Case I. Suppose that the economy \( e \) under the regime with full regulation also lies in Case I. In this case, the ratio \( c_{1}/c_{2\beta} \) for both regimes can be written as
\[ \frac{c_{1}}{c_{2\beta}} = \left[ (1 - q)R^{(\frac{c_{1}}{c_{2\beta}}) - \gamma} + q^{\frac{\beta}{\gamma}} \right] - \frac{1}{\gamma}. \]

It is straightforward to show that \( \frac{c_{1}^{FR}}{c_{2\beta}^{FR}} \) is strictly increasing in \( q \) and the ratio \( \frac{c_{1}^{NL}}{c_{2\beta}^{NL}} \) is identical under both regimes as \( q \to 0 \). In other words, \( \frac{c_{1}^{FR}}{c_{2\beta}^{FR}} > \frac{c_{1}^{FR}}{c_{2\beta}^{FR}} \) \( q \to 0 \) = \( \frac{c_{1}^{NL}}{c_{2\beta}^{NL}} \), and, hence, \( \frac{c_{1}^{FR}}{c_{2\beta}^{FR}} > \frac{c_{1}^{NL}}{c_{2\beta}^{NL}} \).

The proposition will be established, therefore, if I can show that the fragile set \( \Phi^{NL} \) is strictly contained in the region of Case I under the regime with full regulation (the black region of Figure 3.3).

It is straightforward to show that \( q_{l}^{FR} \) is strictly decreasing in \( \theta \) and \( q_{ll}^{FR} \) is strictly increasing in \( \theta \). Recall that (A.40) implies that the threshold value of \( \theta \) for fragility to raise under the regime without liquidity regulation is strictly increasing in \( q \).
Together, these results allow me to characterize the proposition that $\Phi^{NL}$ is strictly contained in the Case I region under the regime with full regulation by looking at condition (A.40) as $\theta = \pi$, which yields

$$0 < q^{NL}(\theta = \pi) \leq q^{FR}(\theta = \pi),$$

if $f(r) > 0$ and $g(r) \leq 0$, hence, as desired. Alternatively, it is easy to find examples of economies that belong to $\Phi^{FR}$ but not to $\Phi^{NL}$; see panel (b) of Figure 3.6.

\[\blacksquare\]

**Proposition 4.1.** Assume $r > R \left[ \frac{(1-\pi)\delta^\frac{1}{\gamma}}{(1-\pi)\pi+\delta^\gamma} \right]^\gamma$. Given $R, r, \gamma, \pi, \delta$,

- if $\tilde{f}(r) \leq 0$, then the economy is stable for all $q$ and, therefore, $\bar{q}^{FR} = 0$;

- if $\tilde{f}(r) > 0$ and $\tilde{g}(r) \leq 0$, then

  $$\bar{q}^{FR} = \{q | \bar{h}(q) = 0; q \in (0, q_i^{FR}) \} \equiv \bar{q}^{FR}_{Case \ I};$$

- if $\tilde{g}(r) > 0$, then

  $$\bar{q}^{FR} = \frac{r(R-1)}{R(R-r)} \equiv \bar{q}^{FR}_{Case \ II}.$$ 

**Proof.** The measure of financial fragility $\bar{q}^{FR}$ will lie in Cases I and II depending on parameter values. Define

$$\tilde{f}(r) = (1-\pi)\pi(\frac{r}{R})^{\frac{1}{\gamma}} + (1-\pi)^2 \frac{r}{R} + (\frac{r}{R})^{\frac{1}{\gamma}} \delta^\frac{1}{\gamma} - (1-\pi) \frac{r}{R} R^{\frac{1}{\gamma}} - \delta^\frac{1}{\gamma},$$

$$\tilde{g}(r) = (1-\pi)\pi(\frac{r}{R})^{\frac{1}{\gamma}} + (1-\pi)^2 \frac{r}{R} + (\frac{r}{R})^{\frac{1}{\gamma}} \delta^\frac{1}{\gamma} - [((1-\pi) \frac{r}{R} + \delta^\frac{1}{\gamma}][\frac{R^2-2rR+r}{R(1-r)} \frac{1}{\gamma}],$$

$$\bar{h}(q) = (1-\pi)\frac{r}{R} R^{\frac{1}{\gamma}} (\frac{1-q}{1-qR})^{\frac{1}{\gamma}} + \delta^\frac{1}{\gamma} (\frac{1-q}{1-qR})^{\frac{1}{\gamma}} - [(1-\pi)\pi(\frac{r}{R})^{\frac{1}{\gamma}} + (1-\pi)^2 \frac{r}{R} + (\frac{r}{R})^{\frac{1}{\gamma}} \delta^\frac{1}{\gamma}].$$

There are four scenarios needed to be considered.

**Scenario (i):** If $\tilde{f}(r) \leq 0$, using Appendix A.3, then I see that $c^{FR}_1 < c^{FR}_{2\beta}$ holds in Cases I, II, and III under this condition. Hence, I have $\bar{q} = 0$.

**Scenario (ii):** If $\tilde{f}(r) > 0$ and $\tilde{g}(r) \leq 0$, using Appendix A.3, then I see that $c^{FR}_1 < c^{FR}_{2\beta}$ holds in Cases II, and III under this condition. It is straightforward
to show that the ratio $c_{FR}^1/c_{2\beta}^1$ is strictly decreasing in $q$ when the solution lies in Case I. In this case, there exists a unique value of $q_m$ such that $c_{FR}^1(q_m) = 1$, where $\tilde{h}(q_m) = 0$. Then, I have $c_{FR}^1 \geq c_{2\beta}^1$ if and only if $q \leq q_m$. Hence, I have $\bar{q}_{FR} = \{q | \tilde{h}(q) = 0; q \in (0, q_{FR}^1)\}$.

Scenario (iii): If $\tilde{g}(r) > 0$, using Appendix A.3, then I see that $c_{FR}^1 > c_{2\beta}^1$ holds in Case I and $c_{FR}^1 < c_{2\beta}^1$ holds in Case III under this condition. It is straightforward to show that the ratio $c_{FR}^1/c_{2\beta}^1$ is strictly decreasing in $q$ when the solution lies in Case II. In this case, $c_{FR}^1 \geq c_{2\beta}^1$ if and only if $q \leq \bar{q}_{FR} = \frac{r(R-\rho_1\rho_2)}{R(R-r\rho_2)}$. Hence, I have $\bar{q}_{FR}^1 = \frac{r(R-\rho_1\rho_2)}{R(R-r\rho_2)}$.

\[\text{Proposition 4.2.}\]

Assume $r > R \left[ (1-\pi)^{1/\gamma} \right]$.

- if $\tilde{f}(r) \leq 0$, then the economy is stable for all $q$ and, therefore, $\bar{q}_{NL} = 0$;

- if $\tilde{f}(r) > 0$, then

\[\bar{q}_{NL} = \frac{1}{R} \left[ \frac{(1-\pi)^{\frac{1}{\gamma}} + (1-\pi)^{2} R^{-\gamma} \delta^{\frac{1}{\gamma}}}{(1-\pi)^{\frac{1}{\gamma}} + R^{-\gamma} \delta^{\frac{1}{\gamma}}} \right]^\gamma - 1\]

\[\bar{q}_{NL} = \frac{1}{r} \left[ \frac{(1-\pi)^{\frac{1}{\gamma}} + (1-\pi)^{2} R^{-\gamma} \delta^{\frac{1}{\gamma}}}{(1-\pi)^{\frac{1}{\gamma}} + R^{-\gamma} \delta^{\frac{1}{\gamma}}} \right]^\gamma - 1\]

Proof. The best-response allocation $A_{NL}^1$ is always in Case I. There are two scenarios needed to be considered.

Scenario (i): If $\tilde{f}(r) \leq 0$, using Appendix A.4, then I see that $c_{NL}^1 < c_{2\beta}^1$ holds in Cases I under this condition. Hence, I have $\bar{q} = 0$.

Scenario (ii): If $\tilde{f}(r) > 0$, using Appendix A.4, it is straightforward to show that the ratio $c_{NL}^1/c_{2\beta}^1$ is strictly decreasing in $q$. In this case, $c_{NL}^1 \geq c_{2\beta}^1$ if and only if $q \leq \bar{q}_{NL}$, as desired.

\[\text{Proposition 4.3.}\]

Assume $\delta = 0$. $\bar{q}_{NL} \begin{cases} \geq & \bar{q}_{FR}^1 \text{ if } r \leq \bar{r} \\ < & \bar{r} \end{cases}$.
Proof. This result follows from Proposition 4.5, which is proved below.

Proposition 4.4. Assume \( \delta = 0 \).

- For any economy with \( r \leq \bar{r} \), adding liquidity regulation is always desirable;

- For any economy with \( r > \bar{r} \), adding liquidity regulation is always desirable, if \( \begin{cases} q \leq \bar{q}^{NL} \\ \bar{q}^{NL} < q \leq \bar{q}^{FR} \end{cases} \).

Proof. This result follows from Proposition 4.6, which is proved below.

Proposition 4.5. Assume \( \tilde{f}(r) > 0 \). \( \bar{q}^{NL} \begin{cases} \geq \tilde{q}^{FR} \text{ if } k(r) \geq 0 \\ < \tilde{q}^{FR} \text{ if } k(r) < 0 \end{cases} \).

Proof. Combining Propositions 4.1 and 4.2, there are three cases needed to be considered.

Case (i): If \( \tilde{f}(r) > 0 \) and \( \tilde{g}(r) \leq 0 \) (i.e. \( k(r) < 0 \)), straightforward algebra shows that \( \bar{q}^{NL} < \bar{q}^{FR} \equiv \bar{q}^{FR}_{\text{Case I}} \).

Case (ii): If \( \tilde{f}(r) > 0 \), \( \tilde{g}(r) > 0 \), and \( k(r) < 0 \), straightforward algebra shows that \( \bar{q}^{NL} < \bar{q}^{FR} \equiv \bar{q}^{FR}_{\text{Case II}} \).

Case (iii): If \( \tilde{f}(r) > 0 \), \( \tilde{g}(r) > 0 \), and \( k(r) \geq 0 \), straightforward algebra shows that \( \bar{q}^{NL} \geq \bar{q}^{FR} \equiv \bar{q}^{FR}_{\text{Case II}} \).

Together, these three cases imply that for all economies with \( \tilde{f}(r) > 0 \), \( \bar{q}^{NL} \begin{cases} \geq \tilde{q}^{FR} \text{ if } k(r) \geq 0 \\ < \tilde{q}^{FR} \text{ if } k(r) < 0 \end{cases} \).

The final step is to note that when \( \delta = 0 \), \( k(r) \) can be rewritten as

\[
k(r)_{\delta=0} = \left[ \frac{\nu}{\nu - 1} + (1 - \nu) \right] - \frac{R^2 - 2rR + R}{1 - r}.
\]
It is straightforward to show that the expression $k(r)|_{\delta=0}$ is strictly decreasing in $r$. It is also straightforward to show that

$$\lim_{r \to 0} k(r)|_{\delta=0} = \infty \quad \text{and} \quad \lim_{r \to 0} k(r)|_{\delta=0} = -\infty.$$ 

Together, the results above establish Proposition 4.5.

**Proposition 4.6.** Assume $f(r) > 0$.

- For any economy with $k(r) \geq 0$, adding liquidity regulation is always desirable;
- For any economy with $k(r) < 0$, adding liquidity regulation is desirable, if
  \[
  \begin{cases}
  \{ \text{always} \} & q \leq \bar{q}^{NL} \\
  \{ \text{never} \} & \bar{q}^{NL} < q \leq \bar{q}^{FR}
  \end{cases}
  \]

Proof. First, recall that the equilibrium allocation vector $\mathcal{A}^{FR}$ under the regime with full regulation solves the problem of the benevolent planner, who could control all endowments and operate both the banking technology and the public sector. Since this solution is unique, the equilibrium allocation of resources under the regime with full regulation attains the highest possible level of welfare conditional on depositors following strategy profile (4.1).

Furthermore, if the economy is fragile under one policy regime but not the other, the optimal policy is to select the non-fragile regime. If the economy is fragile under both regimes, the policy maker chooses the higher-welfare regime by comparing the welfare level $W$ conditional on the financial system being fragile.

Consider all economies with $k(r) \geq 0$, in this case, $\bar{q}^{FR} \leq \bar{q}^{NL}$ always holds. In other words, adopting the regime with full regulation can both promote financial stability and improve welfare. Therefore, adding liquidity regulation is always desirable.
Consider all economies with $k(r) < 0$ (i.e. $\tilde{q}^{NL} < \tilde{q}^{FR}$), recalling that the regime with full regulation attains the highest possible level of welfare conditional on depositors following strategy profile, adding liquidity regulation is always desirable as long as it can eliminate run equilibria. It then follows that it is unambiguously better than the regime without liquidity regulation if $q \leq \tilde{q}^{NL}$, but it is never desirable if $\tilde{q}^{NL} < q \leq \tilde{q}^{FR}$. 

■
Appendix C

A General Definition of Financial Fragility

C.1 Authorities never observe the state

In Chapter 2, I used a particular definition of fragility for simplicity; here I show that the same results are obtained if I use a more general definition. Suppose Definition 2.1 is replaced with the following.

**Definition C.1.** A banking system is said to be fragile if there exists an equilibrium strategy profile with \( y_i(1, \beta) = 0 \) for a positive measure of depositors; otherwise the banking system is said to be stable.

The following result shows that focusing on equation (2.4) is also necessary and sufficient for determining whether an economy is fragile under this more general definition.

**Proposition C.1.** The economy is fragile if and only if (2.4) holds.

**Proof.** The proof of this proposition is divided into three steps as follows.

**Step (i):** \( c_1^* \geq c_2^* \) is a sufficient condition for the economy to be fragile based on the discussion in the text. What remains to be proven is that this condition is also necessary for fragility to arise.

**Step (ii):** Let \( \tilde{y} \) be any such strategy profile with \( \tilde{\pi}_\beta < \pi \) and let \( \tilde{A} \) denote the allocation generated by the best response of the bank to this profile. This
allocation is characterized by equations (A.1)-(A.9). If there is an equilibrium in which depositors follow \( \tilde{y} \), it must be the case that \( \tilde{c}_1 = \tilde{c}_{2\beta} \) holds. Suppose that \( \tilde{c}_1 > \tilde{c}_{2\beta} \), which implies that all patient depositors with \( i \leq \pi \) prefer to withdraw. However, the strategy profile \( \tilde{y} \) proposes that the fraction of remaining depositors who are impatient after \( \pi \) withdrawals have been made is less than \( \pi \). Hence, there is an equilibrium in which depositors follow \( \tilde{y} \) as long as \( \tilde{c}_1 = \tilde{c}_{2\beta} \). Therefore, I must show that \( c_1^* \geq c_{2\beta}^* \beta \) holds whenever \( \tilde{c}_1 = \tilde{c}_{2\beta} \) holds for some \( \tilde{y} \) in order to prove that \( c_1^* \geq c_{2\beta}^* \beta \) is a necessary condition for fragility to arise.

- If the best-response allocation lies in Case I: from Appendix A.1, I see that 
  \[ \frac{\tilde{c}_1}{\tilde{c}_{2\beta}} \] is strictly increasing in \( \hat{\pi}_\beta \);

- If the best-response allocation lies in Case II: from Appendix A.1, I see that 
  \[ \frac{\tilde{c}_1}{\tilde{c}_{2\beta}} \] is independent of \( \hat{\pi}_\beta \);

- If the best-response allocation lies in Case III: the impact of \( \hat{\pi}_\beta \) on 
  \[ \frac{\tilde{c}_1}{\tilde{c}_{2\beta}} \] and the nature of 
  \[ \frac{\tilde{c}_1}{\tilde{c}_{2\beta}} \] are determined as follows

  - if \( \rho_2 \leq (\frac{R}{\rho_1})^{\frac{1}{\gamma}} \), then I have that \( \tilde{c}_1 < \tilde{c}_{2\beta} \);
  - if \( (\frac{R}{\rho_1})^{\frac{1}{\gamma}} < \rho_2 < (\frac{R}{\tau})^{\frac{1}{\gamma}} \), then I see that
    \[
    \begin{cases}
      \tilde{c}_1 < \tilde{c}_{2\beta} \\
      \frac{\tilde{c}_1}{\tilde{c}_{2\beta}} \text{ is strictly increasing in } \hat{\pi}_\beta
    \end{cases}
    \text{ as } q \in \left\{ \begin{pmatrix} (q_u, q) \\ (q, 1) \end{pmatrix} \right\}, \text{ where } q = \frac{R - \rho_2}{\rho_2 - \rho_2};
    \]
  - if \( (\frac{R}{\tau})^{\frac{1}{\gamma}} \leq \rho_2 \), then I see that 
    \[ \frac{\tilde{c}_1}{\tilde{c}_{2\beta}} \] is strictly increasing in \( \hat{\pi}_\beta \).

In addition, differentiating \( q_l \) and \( q_u \) with respective to \( \hat{\pi}_\beta \), I have \( q_l \) is strictly decreasing in \( \hat{\pi}_\beta \) and 

\[ q_u \text{ is strictly } \begin{cases} \text{increasing} \\ \text{decreasing} \end{cases} \text{ in } \hat{\pi}_\beta \text{ if } \rho_2 \begin{cases} < \\ > \end{cases} (\frac{R}{\tau})^{\frac{1}{\gamma}} \]

Now, I show how these preliminary results combine to establish the proposition.
• If $\rho_2 \leq \left(\frac{R}{\rho_1}\right)^{\frac{1}{\gamma}}$, then the best-response allocation $\tilde{A}$ with $\tilde{c}_1 = \tilde{c}_{2\beta}$ could be in Case I or Case II.

Recall that $q_l (q_u)$ is strictly decreasing (increasing) in $\hat{\pi}_\beta$, it is straightforward to show the economy cannot be in Case III as $\hat{\pi}_\beta$ increases. Since $\frac{\hat{c}_1}{\hat{c}_{2\beta}}$ is strictly increasing (independent) in $\hat{\pi}_\beta$ in Case I (II), I have $\frac{\hat{c}_1}{\hat{c}_{2\beta}} \geq \frac{\tilde{c}_1}{\tilde{c}_{2\beta}}$.

• If $\left(\frac{R}{\rho_1}\right)^{\frac{1}{\gamma}} < \rho_2 < \left(\frac{R}{\gamma}\right)^{\frac{1}{\gamma}}$, then the best-response allocation $\tilde{A}$ with $\tilde{c}_1 = \tilde{c}_{2\beta}$ could be in Case I, Case II, or $q \in (q, 1)$ of Case III.

Similarly, as $\hat{\pi}_\beta$ increases, the economy would be in Case I, Case II, or $q \in (q, 1)$ of Case III. In this scenario, $\frac{\hat{c}_1}{\hat{c}_{2\beta}}$ is non-decreasing in $\hat{\pi}_\beta$, which implies that $\frac{\hat{c}_1}{\hat{c}_{2\beta}} \geq \frac{\tilde{c}_1}{\tilde{c}_{2\beta}}$.

• If $\left(\frac{R}{\gamma}\right)^{\frac{1}{\gamma}} \leq \rho_2$, then the best-response allocation $\tilde{A}$ with $\tilde{c}_1 = \tilde{c}_{2\beta}$ could be in Cases I, II, or III. Using the results above, I have $\frac{\hat{c}_1}{\hat{c}_{2\beta}}$ is non-decreasing in $\hat{\pi}_\beta$, which implies that $\frac{\hat{c}_1}{\hat{c}_{2\beta}} \geq \frac{\tilde{c}_1}{\tilde{c}_{2\beta}}$.

Therefore, $\frac{\hat{c}_1}{\hat{c}_{2\beta}} = 1$ implies $\frac{\hat{c}_1}{\hat{c}_{2\beta}} \geq 1$, as desired.

**Step (iii):** Now suppose that $c_1^* < c_{2\beta}^*$ holds. Using Appendix A.1 and the results from Step (ii), this inequality implies that $\bar{c}_1 < \bar{c}_{2\beta}$ for the profile $\bar{y}$ in which run never occurs. For the converse, note that if the economy is stable, it follows immediately that $c_1^* < c_{2\beta}^*$ must hold and concludes the proof.

C.2 **Authorities can observe the state**

In Chapter 3, I used a particular definition of fragility in the main text for simplicity; here I show that the same results are obtained if I use a more general definition of fragility. Formally, I introduce the following definition in this sense.
Definition C.2. A financial system is said to be fragile if there exists an equilibrium strategy profile with \( y_i(1, \beta) = 0 \) for a positive measure of depositors; otherwise the financial system is said to be stable.

Proposition C.2. The financial system is fragile under the regime with no regulation if and only if \( c_1^{NR} \geq c_2^{NR} \beta \).

Proof. The proof of this proposition is divided into three steps as follows.

Step (i): \( c_1^{NR} \geq c_2^{NR} \beta \) is a sufficient condition for the economy to be fragile based on the discussion in the text. What remains to be proven is that is this condition is also necessary for fragility to arise.

Step (ii): Let \( \tilde{y} \) be any such strategy profile with \( \hat{\pi} < \pi \) and let \( \tilde{A}^{NR} \) denote the allocation generated by the best responses of banks and the policy maker to this profile. This allocation is characterized by equations in Appendix A.2. If there is an equilibrium in which depositors follow \( \tilde{y} \), it must be the case that \( \tilde{c}_1^{NR} = \tilde{c}_2^{NR} \beta \) holds. Suppose that \( \tilde{c}_1^{NR} > \tilde{c}_2^{NR} \beta \), which implies that all patient depositors \( i \leq \theta \) prefer to withdraw. However, the strategy profile \( \tilde{y} \) proposes that the fraction of remaining depositors who are impatient after \( \theta \) withdrawals have been made is less than \( \pi \). Hence, there is an equilibrium in which depositors follow \( \tilde{y} \) as long as \( \tilde{c}_1^{NR} = \tilde{c}_2^{NR} \beta \). Therefore, I must show that \( c_1^{NR} \geq c_2^{NR} \beta \) holds whenever \( \tilde{c}_1^{NR} = \tilde{c}_2^{NR} \beta \) holds for some \( \tilde{y} \) in order to prove that \( c_1^{NR} \geq c_2^{NR} \beta \) is a necessary condition for fragility.

- If the best-response allocation lies in Case I: from Appendix A.2, I see that \( \frac{\tilde{c}_1^{NR}}{\tilde{c}_2^{NR} \beta} \) is strictly increasing in \( \hat{\pi} \beta \);

- If the best-response allocation lies in Case IV: from Appendix A.2, I see that \( c_1^{NR} < c_2^{NR} \beta \) always holds.
In addition, differentiating the boundary $\theta^{\text{NR}}$ with respective to $\hat{\pi}_\beta$, I have $\theta^{\text{NR}}$ is strictly decreasing in $\hat{\pi}_\beta$.

Now, I show how these preliminary results combine to establish the proposition. First, note that the best-response allocation $\theta^{\text{NR}}$ with $\tilde{c}_1^{\text{NR}} = \tilde{c}_2^{\text{NR}}$ must be in Case I. Recall that $\tilde{\theta}^{\text{NR}}$ is strictly decreasing in $\hat{\pi}_\beta$, it is straightforward to show the economy must lie in Case I as $\hat{\pi}_\beta$ increases. Since $\frac{c_1^{\text{NR}}}{c_2^{\text{NR}}}$ is strictly increasing in $\hat{\pi}_\beta$ in Case I, I have $\frac{c_1^{\text{NR}}}{c_2^{\text{NR}}} > \frac{\tilde{c}_1^{\text{NR}}}{\tilde{c}_2^{\text{NR}}}$.

Therefore, $\frac{\tilde{c}_1^{\text{NR}}}{\tilde{c}_2^{\text{NR}}} = 1$ implies $\frac{c_1^{\text{NR}}}{c_2^{\text{NR}}} \geq 1$, as desired.

**Step (iii):** Now suppose that $c_1^{\text{NR}} < c_2^{\text{NR}}$ holds. Using Appendix A.2 and the result from Step (ii), this inequality implies that $\tilde{c}_1^{\text{NR}} < \tilde{c}_2^{\text{NR}}$ for the profile $\tilde{y}$ in which run never occurs. For the converse, note that if the economy is stable, it follows immediately that $c_1^{\text{NR}} < c_2^{\text{NR}}$ must hold and concludes the proof.

\[\blacksquare\]

**Proposition C.3.** The financial system is fragile under the regime with full regulation if and only if $c_1^{\text{FR}} \geq c_2^{\text{FR}}$.

**Proof.** The proof of this proposition is divided into three steps as follows.

**Step (i):** $c_1^{\text{FR}} \geq c_2^{\text{FR}}$ is a sufficient condition for the economy to be fragile based on the discussion in the text. What remains to be proven is that this condition is also necessary for fragility to arise.

**Step (ii):** Let $\tilde{y}$ be any such strategy profile with $\hat{\pi}_\beta < \pi$ and let $\tilde{A}^{\text{FR}}$ denote the allocation generated by the best responses of banks and the policy maker to this profile. This allocation is characterized by equations in Appendix B.1. If there is an equilibrium in which depositors follow $\tilde{y}$, it must be the case that $c_1^{\text{FR}} = c_2^{\text{FR}}$ holds. Suppose that $c_1^{\text{FR}} > c_2^{\text{FR}}$, which implies that all patient depositors $i \leq \theta$
prefer to withdraw. However, the strategy profile $\tilde{y}$ proposes that the fraction of remaining depositors who are impatient after $\theta$ withdrawals have been made is less than $\pi$. Hence, there is an equilibrium in which depositors follow $\tilde{y}$ as long as $\tilde{c}_1^{FR} = \tilde{c}_2^{FR}$. Therefore, I must show that $c_1^{FR} \geq c_2^{FR}$ holds whenever $\tilde{c}_1^{FR} = \tilde{c}_2^{FR}$ holds for some $\tilde{y}$ in order to prove that $c_1^{FR} \geq c_2^{FR}$ is a necessary condition for fragility.

- If the best-response allocation lies in Case I: from Appendix A.3, I see that $\frac{\tilde{c}_1^{FR}}{\tilde{c}_2^{FR}}$ is strictly increasing in $\hat{\pi}_\beta$;

- If the best-response allocation lies in Case II: from Appendix A.3, I see that $\frac{\tilde{c}_1^{FR}}{\tilde{c}_2^{FR}}$ is independent of $\hat{\pi}_\beta$;

- If the best-response allocation lies in either Case III or Case IV: from Appendix A.3, I see that $c_1^{FR} < c_2^{FR}$ always holds.

In addition, differentiating $\theta^{FR}, q_l^{FR}, q_u^{FR}, q_{lu}^{FR}, q_{uu}^{FR}$ with respective to $\hat{\pi}_\beta$, I have that $\theta^{FR}, q_l^{FR}$ and $q_{uu}^{FR}$ are strictly decreasing in $\hat{\pi}_\beta$, and that $q_u^{FR}$ and $q_{lu}^{FR}$ are strictly increasing in $\hat{\pi}_\beta$.

Now, I show how these preliminary results combine to establish the proposition. First, note that the best-response allocation $\tilde{A}^{FR}$ with $\tilde{c}_1^{FR} = \tilde{c}_2^{FR}$ must be in either Case I or Case II.

- If the best-response allocation $\tilde{A}^{FR}$ with $\tilde{c}_1^{FR} = \tilde{c}_2^{FR}$ lies in Case I: recall that $\theta^{FR}$ and $q_l^{FR}$ are strictly decreasing in $\hat{\pi}_\beta$, and that $q_{lu}^{FR}$ is strictly increasing in $\hat{\pi}_\beta$. It is straightforward to show the economy must lie in either Case I or Case II as $\hat{\pi}_\beta$ increases. Since $\frac{\tilde{c}_1^{FR}}{\tilde{c}_2^{FR}}$ is strictly increasing in $\hat{\pi}_\beta$ in Case I and $\frac{\tilde{c}_1^{FR}}{\tilde{c}_2^{FR}}$ is independent in $\hat{\pi}_\beta$, I have $\frac{\tilde{c}_1^{FR}}{\tilde{c}_2^{FR}} > \frac{\tilde{c}_1^{FR}}{\tilde{c}_2^{FR}}$.

- If the best-response allocation $\tilde{A}^{FR}$ with $\tilde{c}_1^{FR} = \tilde{c}_2^{FR}$ lies in Case II: recall that $\theta^{FR}$ and $q_l^{FR}$ are strictly decreasing in $\hat{\pi}_\beta$, and that $q_u^{FR}$ is strictly increasing
in $\tilde{\pi}_\beta$. It is straightforward to show the economy must lie in Case II as $\tilde{\pi}_\beta$ increases. Since $\frac{c_{1}^{FR}}{c_{2,2}^{FR}}$ is independent in $\tilde{\pi}_\beta$, I have $\frac{c_{1}^{FR}}{c_{2,2}^{FR}} = \frac{c_{2}^{FR}}{c_{2,3}^{FR}}$.

Therefore, $\frac{c_{1}^{FR}}{c_{2,2}^{FR}} = 1$ implies $\frac{c_{2}^{FR}}{c_{2,3}^{FR}} \geq 1$, as desired.

**Step (iii):** Now suppose that $c_{1}^{FR} < c_{2,2}^{FR}$ holds. Using Appendix A.3 and the result from Step (ii), this inequality implies that $\tilde{c}_{1}^{FR} < \tilde{c}_{2,2}^{FR}$ for the profile $\bar{y}$ in which run never occurs. For the converse, note that if the economy is stable, it follows immediately that $c_{1}^{FR} < c_{2,3}^{FR}$ must hold and concludes the proof.

**Proposition C.4.** The financial system is fragile under the regime without liquidity regulation if and only if $c_{1}^{NL} \geq c_{2,3}^{NL}$.

**Proof.** The proof, which follows that of Proposition C.3 closely, is omitted.
References


