A Comparison of Trichordal Relations in Milton Babbitt's String Quartet No. 2 and Elliott Carter's A Symphony for Three Orchestras

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Article begins on next page
A Comparison of Trichordal Relations in
Milton Babbitt's String Quartet, No. 2 and
Elliott Carter's *A Symphony for Three
Orchestras*

Nancy Yunhwa Rao

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<table>
<thead>
<tr>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
</tr>
<tr>
<td>Why Trichords?</td>
</tr>
<tr>
<td>Categorization of Trichords</td>
</tr>
<tr>
<td>Communality and Trichordal Generators</td>
</tr>
<tr>
<td>Babbitt’s String Quartet No. 2</td>
</tr>
<tr>
<td>Carter’s A Symphony for Three Orches</td>
</tr>
<tr>
<td>Conclusion</td>
</tr>
<tr>
<td>Abstract</td>
</tr>
</tbody>
</table>
Introduction

The relationship between high-level abstraction, revealed either in music analysis or in composers' sketches, and the actual music has long been enigmatic. Theoretical musical relations often seem so far removed from the actual expressiveness of the music that questions are inevitably raised about their relevance and also about their value in enhancing an understanding of the musical work. For instance, in the introduction to a recent volume on Elliott Carter's compositional sketches, *Settling New Scores: Music Manuscripts from the Paul Scher Foundation*, David Schiff cautioned that "much of that initial process does not find its way down on paper. [...] Indeed the sketches may give the impression of a more theoretical stance than is actually there."[1] Certainly among scholars, performers, and even composers there is little agreement about the role of theoretical abstraction in post-tonal music. Some acerbically dismiss its significance, while others consider it imperative in the shaping and understanding of a musical work.[2]

This discrepancy in viewpoints is due in part to the seldom acknowledged truism that the theoretical abstraction presented in compositional sketches often reveals only fragmentarily the complex ways and multiple levels by which abstract principles relate to the actual music. The explanatory power of musical analyses is by its very nature limited in scope. Consequently, individual compositional strategies, musical relations, theoretical abstraction and analytical models have sometimes become undifferentiated or even mistaken for one another, thereby confusing rather than illuminating the complex issues. This essay is predicated on the assumption that an understanding multi-layered complexity of musical relations be delineated


theoretically and that the connection between abstract musical relations and individual compositional strategies should be analyzed in ways that are sensitive to both the general, non-exclusive, theoretical principles broadly and specifically to their individual contexts. Furthermore, delineating the connection between compositions and various levels of abstract musical relations is germane to conceptualizing the greater web in which such individual compositional worlds are enmeshed as well as to the understanding of individual composers' paths in the musical world.

Rather than being unidirectional, the process in which the relation between the individual compositional strategies and the non-exclusive theoretical principles unfolds is dialectic. Andrew Mead, in his depiction of Milton Babbitt's compositional world, provides an outline that describes such complex interactions among the multi-levels of musical relations:

The dynamic qualities depend on a series of dialectics between the surface moments of a piece and their source in its underlying structures, between a structure's compositional interpretation and its abstract properties, between particular abstract structures and Babbitt's habitual corners of the chromatic universe, and ultimately between Babbitt's preferred perspective of the chromatic universe and the chromatic universe itself.\(^3\)

 Particularly suggestive for our purpose is Mead's concept of "habitual corners of the chromatic universe" which vividly reflects the dialectical and discursive relation between individual compositional strategies and the structures of the chromatic. If we can imagine the chromatic universe as a living space and the compositional strategies as individual approaches to defining and animating that space, we can see clearly how compositional strategies are inevitably engendered by the structure provided by this space. Thus, the world of musical relations supporting each composer's strategies is distinctive yet not exclusive. Two other ideas are particularly relevant to this essay. First, the composers' "preferred perspectives" of the chromatic universe, however

general or trivial they may seem, are themselves the first signs of compositional strategies. Second, so long as compositional strategies are derived from the same chromatic universe a reservoir of "raw" musical possibilities it is inevitable that the composers' resulting worlds of musical relations would overlap; composers' varied interpretations of such overlap provide many clues to their styles. Thus, if it is to get at individual uniqueness, the characterization of any compositional strategies cannot neglect to examine the musical relations provided by the chromatic universe for such strategies. These assumptions significantly ground the analytical approach taken here in examining Babbitt's and Carter's use of trichordal relationships.

Why Trichords?

The trichord has a truly unique place in post-tonal compositions, partly because of its connection with triads in tonal music, and partly because of the way it categorizes the twelve tone universe. For many post tonal composers their worlds of trichordal relations seem intertwined. It is well known that trichordal relations are prominent in the music of Arnold Schoenberg, Anton Webern, Alban Berg, Ruth Crawford, Edgar Varese, Milton Babbitt, Elliott Carter, and others. The roles the trichordal relations play in the formation of musical phrases, harmony, underlying structure, coherence, and narrative reveal much about the characteristics of their individual styles.

The animation of trichordal relations in the chromatic universe is particularly

crucial in the work of Milton Babbitt in the 1960s and in the work of Elliott Carter in the 1970s. The ubiquity of trichordal relations in Babbitt's compositions in the 1960s parallels the frequent references to them in his articles. For example, in a series of seminal essays which elucidate Schoenberg's twelve-tone method, Babbitt uses the concept of the trichord as a starting point for his discussion. Similarly, the omnipresence of trichordal relations in Carter's compositions during the 1970s is matched by recurring allusions to his now well known "Harmony Book" which contains his own catalogue of chords along with analyses of their interval content and relationships. In their writings both composers carefully considered the most fundamental aspects of trichords. Their distinctive interpretations of the trichord space shaped their compositions of the periods.

However, despite the wealth of general literature on trichords in post-tonal compositions, many questions remain unasked. What aspects of the abstract properties of trichords underlie the composers' compositional designs? What are the aesthetic aims of musical connections engendered by their particular use of trichords? What types of motion among trichords and larger chords emerge from their compositional strategies? Why, for example, is 018 important to Carter's work, but of little importance in Babbitt's work? Is such a difference superficial or fundamental in their musical languages? While the two composers share many trichordal relations, there are also many differences between the two composers' various compositional strategies. What are the sources of these differences? This essay attempts to answer these questions through delineating and comparing the trichordal relations that are explicit or implied in the compositional interpretations of the chromatic universe in Milton Babbitt's String Quartet, No. 2 and Elliott Carter's A Symphony for Three Orchestras.

The analyses will be preceded by two sections on trichords. The first focuses

on the issues of categorization, and the second focuses on the means of communality based on trichordal relations, including trichordal generators. In the first section, I examine the ways in which the two composers' categorizations of trichords are engendered by the structure of the chromatic space, and how such categorizations already reveal their individual compositional strategies. In the second section, using the concept of trichordal generators and communality, I theorize the two composers' unique approaches to animating trichordal relations in the chromatic universe. These two sections clarify the theoretical abstraction and the composers' "preferred perspectives" of the chromatic universe, and this will provide the foundation of the musical analysis that follows.

**Categorization of Trichords**

Categorization is, of course, the starting point for an interpretation of musical space or resource. Trichords can be categorized according to a variety of criteria, and every means of categorization inevitably reflects certain compositional strategies. The division of the three note chord into twelve types is based on the concept of inversional/transpositional equivalence, and to some extent on the concept of interval class. Babbitt points out the significance of this categorization when he writes:

> [A]lthough the partitional categorization of the totality of trichords into twelve trichordal types has its origins analytically, synthetically, and historically in the twelve-tone universe, since the criteria for trichordal inclusion in one of the equivalence classes defining a "type" are equivalences of the intervallic content of the associated combinatorial hexachords...these types have come to play decisive roles in music which would and could disavow any taint of twelve-tone serialism, perhaps because of the dependence of the classification solely upon content, not at all upon order.

In other words, while this categorization originates historically from twelve tone music, the compositional strategy implicit in the categorization is not exclusive to...

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6) Babbitt, "Responses," 11-12.
composers of twelve tone music. In his "Harmony Book" compiled between 1963 and 1967, Carter also categorizes trichords into twelve types. His well known chart of trichords is reproduced in Figure 1. It is worth noting that in the ways they categorize trichords into twelve types and therefore establish interval content as a distinctive feature, both composers have similarly assigned a prominent role to the concept of interval class equivalence.\(^7\) For the purpose of comparison let us consider for a moment other possible criteria for the classification of trichords. A categorization of trichords by transpositional equivalence but not inversional equivalence yields nineteen trichordal types for example.\(^8\) Furthermore, an example of categorization based not on the interval class but instead on the contour can be found in the work of Ruth Crawford and Charles Seeger.\(^9\) Their compositional practice, as evidenced by Crawford's composition as well as by Seeger's treatise, places emphasis on the shape (contour) and pitch space rather than pitch class space and interval classes. Accordingly, this perspective results in four different classes of trichords, leaving the sizes of intervals undifferentiated.\(^10\) (Using up and down to describe the contour

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7) This list is based on Schiff, The Music of Elliott Carter. The order and the names of the trichords in this list are Carter's. They are different from Allen Forte's set class numbers. See Forte, Allen. 1973. The Structure of Atonal Music. New Haven: Yale University Press. The first five trichords, arranged by the size of interval spans from large to small, are symmetrical; the remaining trichords are grouped together also by the size of interval spans. In his discussion of trichords, Jonathan Bernard has emphasized the importance of interval spans, and treated them as a significant aspect of equivalence. See his "Spatial Sets."

8) This would include 012, 013, 023, 025, 024, 035, 027, 057, 014, 034, 015, 045, 037, 047, 016, 056, 026, 046, 036, and 048.

9) Seeger's categorization based on contour can be found in "Tradition and Experiment in the New Music" in Seeger, Charles. 1994, Studies in Musicology II. Edited by A. M. Pescatello. Berkeley: University of California Press. It is also discussed in Straus, The Music of Ruth Crawford Seeger, 20-24. An important distinction should be emphasized. The categorization here is based on contour alone, rather than a combination of contour and interval which Jonathan Bernard described as "contour equivalence class." Bernard wrote, "The intervals are shown in numbers of semitones with 'a' and '2' denoting ascending and descending directions respectively." See Bernard, "Problems of Pitch Structure," 216.

10) See Straus, The Music of Ruth Crawford Seeger. The trichords, which constitute the smallest cells distinguishable in contour, are differentiated into two classes-line and twist. Together with their transformations these trichords can generate from scales, modes, phrase, to the overall structure of a composition. See, for instance, Ruth Crawford, String Quartet 1931, especially the second movement, and Seeger, "Tradition and Experiment in the New Music".
between two notes in a trichord, this method derives the following four groups:
up up, down down, up down, and down up.) In other words, categorization of
trichords by interval class is merely one of many ways to characterize the chromatic
pitch space. That Carter and Babbitt take the same step in this direction is by no
means trivial.

Figure 1. Carter's Trichord Chart in "Harmony Book"

<table>
<thead>
<tr>
<th>Carter number</th>
<th>3-1</th>
<th>3-2</th>
<th>3-3</th>
<th>3-4</th>
<th>3-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prime form</td>
<td>048</td>
<td>036</td>
<td>024</td>
<td>012</td>
<td>027</td>
</tr>
<tr>
<td>Forte no.</td>
<td>3-12</td>
<td>3-10</td>
<td>3-6</td>
<td>3-1</td>
<td>3-9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Carter no.</th>
<th>3-6</th>
<th>3-7</th>
<th>3-8</th>
<th>3-9</th>
<th>3-10</th>
<th>3-11</th>
<th>3-12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prime form</td>
<td>037</td>
<td>016</td>
<td>026</td>
<td>015</td>
<td>025</td>
<td>014</td>
<td>013</td>
</tr>
<tr>
<td>Forte no.</td>
<td>3-11</td>
<td>3-5</td>
<td>3-8</td>
<td>3-4</td>
<td>3-7</td>
<td>3-3</td>
<td>3-2</td>
</tr>
</tbody>
</table>

Inasmuch as trichords are categorized into twelve types by interval class
content, the subsequent ways to classify these twelve trichordal types, (i.e., by their
whole tone content, symmetry, interval span, etc.), inevitably accomplish two
things: they identify other characteristics of trichords, and they further interpret the
interconnection between the twelve trichordial types and the chromatic universe. These
subsequent categorizations of the twelve trichords further define, in their respective
ways, what constitutes equivalence and what characterizes degrees of similitude.
Within these further categorizations, each group of trichordal types renders its
specific range of structural features, as well as musical connections.

In general, for Carter, whose compositions in the 1970s characteristically make
use of all twelve trichords, the individual characters of the twelve trichords are
important.\(^{11}\) They are, however, by no means equally important. Carter's criteria of similitude among trichords is reflected in the order he assigns them in his "Harmony Book:"

I decided to compile a catalogue in which I could look up quickly what interval was held in common by two three-note chords, which three-note chords excluded each other, and which groups of four such chords exhausted the twelve notes. I gave them each numbers for easy reference: the first five, noninvertible, were grouped together; the remaining seven were presented with their mirror inversions, which were numbered the same as the originals, this catalogue was primarily concerned with interval content.\(^ {12}\)

As shown in Figure 1, Carter divides the twelve trichords into the symmetrical (or invertible according to his term) and the non-symmetrical. Just as the consideration given to aggregate completion "which groups of four such chords exhausted the twelve notes" foretells this usage of them in his compositions, the priority Carter gives to symmetrical trichords reflects the unique way that he handles their symmetrical axis in his compositions, a point that will be developed in the next section.

**Communality and Trichordal Generators**

In order to discuss the ways in which Babbitt differentiates the twelve-trichords, we must first visit a closely related concept that of communality. Broadly defined, communality is a connectedness found between two different elements which may seem at first exclusive of each other. In comparing the trichord in

\(^{11}\) For example, Carter's compositional plans for Piano Concerto, String Quartet No. 3, and A Mirror on Which to Dwell detail the distribution of the twelve types into either instrumental groups, tempi or movements.

\(^{12}\) See Bernard, Jonathan. 1990, "An Interview with Elliott Carter," Perspectives of New Music 28 (2): 180-215, p. 701. David Schiff also wrote of the "Harmony Book," "Carter analyzes the chords for their intervallic content in all positions, for their content of small chords, and for their combinational resultants. The goal of these Rameanesque researches is a coherent use of all chordal combinations to produce harmonies at once rich and lucid." See Schiff, David. 1983. The Music of Elliott Carter, London: Eulenburg Books, p. 63.
twelve-tone music to the triad in tonal music, Babbitt characterizes trichords and
trichords as both factors of communality. Yet he notes that their roles in establishing

the trichord in twelve-tone music, very much like the triad in tonal music, is—as
such—much more a factor of communality than a particular creation . . . Tonal
compositions still are explicated . . . by the identification of the changes in function
of intervally the same structures, while comparably, twelve-tone compositions can
be—however incompletely—characterized by intervallically different structures
assuming the same functions, as actual or potential generators of so-derived
hexachords and, thereby, set, for trichords need not be employed explicitly as
generators to effect, by their properties in this respect, the specifics of the relations
between and among set forms, their components, and compounds.\(^\text{13}\)

In tonal music the intervals of a triad, in a particular inversion and spelling, are
signifiers of particularly tonal functions. Because an interval content can be
interpreted in more than one way, a triad thus fulfilling multiple functions, the
interval is the locale of communality.\(^\text{14}\) In post-tonal music, trichords of different
structures can function in similar ways functions determined, of course, by the
individual compositional strategy. For example, all but one of the trichords can
singularly generate a complete aggregate.\(^\text{15}\) When a unique function, such as
generating a complete aggregate, can be fulfilled by eleven different trichords, the
interval content of these trichords is no longer the signifier of this particular function.
The trichords' capability to fulfil the same function becomes the locale for
communality. The intervals become the expression itself.

Integral to this communality in Babbitt's work is the all-combinatorial hexa-

\(^{13}\) Babbitt, "Responses," 11.
\(^{14}\) Of course, the same function can sometimes be fulfilled by different triads—for instance, an
altered chord substitutes for a dominant harmony. The argument remains, however, that this
altered chord comprises certain qualities, which allows it to signify musical functions. The
capability of assuming multiple functions allows the particular triad a kind of ambiguity in
tonal music—its interval content can signify different functions.
\(^{15}\) Only the 036 trichord cannot generate all twelve notes by itself.
chords and their single trichordal generators, stg.\textsuperscript{16} A "family" of trichord-hexachord forms the foundation for the abstract structure in Babbitt's musical world. Part of their interrelationship is summarized in Figure 2.\textsuperscript{17} (This diagram does not include the whole tone members of this family, the trichords, 024 and 041, and the F hexachord, 02468, because their connection to this diagram can be easily conceptualized.) In this diagram hexachords are represented by the vertices of the lattice while the trichordal generators are represented by the edges. Each of the edges (trichords) connected to one vertex represents a single trichordal generator of this hexachord on the vertex. An edge connecting two vertices indicates a stg shared in common between the two different hexachords; it can also be read as a trichord linking two hexachords through its generative property.

\textsuperscript{16} The concept of trichordal generators have been discussed elsewhere. See Babbitt, 1976. "Responses," 3-73; Martino, Donald 1961 "The Source Set and its Aggregate Formations," Journal of Music Theory 5 (2):224-73; Rouse, Steven. 1984-5. "Hexachords and Their Trichordal Generators," In Theory Only 8 (8): 29-42. The following briefly reviews the terminology to be used in this analysis. When a hexachord is partitioned into two trichords, the two trichords can be called the generators of the hexachord. There are two kinds of trichord generators. When a hexachord is derived from a trichord type and its transformation, we call it a single trichordal generator (or stg) of the hexachord. The transformation can be either a transposition, an inversion, or their retrograde. When a hexachord is partitioned into trichords of two different types, we call them a double trichord generator (or dtg).

A few words about the terminology is necessary here. The composition strategies involving stg and dtg are important to both Babbitt and Carter, although they are used with different senses of ordering and constitute different means of invariance. The generative property is at the same time the partitioning property. While the concept of stg and dtg emphasize the generative property of trichord-hexachordal relations, concepts such as "mosaic" focus on the partitioning property. These two different orientations-generative or partitioning-of theorizing trichord-hexachordal relationships are themselves suggestive in terms of compositional strategy. As will become clear later, our analysis emphasizes the generative property for Babbitt's composition, and gravitates toward the partitioning property for Carter's compositions. The concept of mosaic is discussed in detail in the following: Mead, Andrew. 1988. "Some Implications of the Pitch Class/Order Number Isomorphism Inherent in the Twelve Tone System: Part I." Perspectives of New Music 26 (2): 96-163. The concept of partitions is discussed in Alegant, Brian, and Robert Morris 1988. "The Even Partitions in Twelve-Tone Music," Music Theory Spectrum 10:74-101.

\textsuperscript{17} Robert Morris' Babbitt Network summarizes trichords that partition all-combinatorial hexachords in somewhat different ways. See Morris, Robert 1995 "Compositional Spaces and Other Territories," Perspectives of New Music 33:378-357. Using nodes and arcs to represent trichords and all-combinatorial hexachords respectively, Morris includes all twelve trichords in this network and include both the dtg and stg in this network.
Figure 2. Diagram of stg and five all-combinatorial hexachords
(A: 012345; B: 023457; C: 024579; D: 012678; E: 124589)

The proximity among the trichords and hexachords, as well as their significance, can be further defined according to a number of criteria. For example, degrees of closeness between two hexachords can be decided by the number of stg they share; similarly, degrees of closeness between two trichords can be decided by the number of hexachords they share. In fact, each hexachord shown here is connected to three different stgs; each stg connects at least two hexachords. (Not shown here are 024, stg for hexachords A, B, C, and 048, stg for hexachord E.) Hexachord B is unique in this case because it shares with all the other all combinatorial hexachords at least one stg, including the F hexachord through 024 (not shown on this diagram). This property gives rise to the special place that hexachord B holds in Babbitt's String Quartet No. 2. In contrast, even though hexachord A and hexachord C appear to share no stg except for 024, the two hexachords are M7 related and their stgs map onto one another under M7 operation. Babbitt's trichordal compositions are rooted in the interrelations shown in this diagram, a particular interpretation of the relation.
among the trichords.

To put it differently, for Babbitt the further categorization of the twelve trichord types is linked to the trichords' property as stg of the all-combinatorial hexachords. The relations among trichords are reflected by their relevant positions on this diagram. For example, 012 and 027 trichords share similar relations to the other trichords on this diagram, 012, 013, and 014 are related in ways similar to that of 027, 025, and 037. In this light 015 and 024 are particularly useful trichords in creating communality, whereas 016, 026 and especially 036 have little significance.

Carter noted in 1976 that "[Babbitt's] theoretical work has had an influence on and importance to most American composers working in this domain and even to those like myself who are not centrally involved with it."[18] It is hardly surprising to find that the concept of communality is significant also for Carter's use of trichordal relation in composition. It is through this concept that we can best fathom how symmetrical trichords such as 018 and 027 have special importance for him.

The way that Carter effects communality is related to the concept of the double trichord generator, dtg.[19] Passages of sustained chords using a single hexachord type are often composed of a succession of dtg pairs. In dtgs succession the choice of dtg, from the totality of ten pairs, may at first seem random, yet it is often governed by a unique progression. The most important dtgs in this progression are those that contain one symmetrical trichord and one non-symmetrical trichord. This group of dtg is central to a unique operation-partial inversion. This operation uses the symmetrical axis within the symmetrical trichord of the dtg pair to yield a hexachord of the same type: that is, the symmetrical chord inverts onto itself, and the non-symmetrical trichord inverts onto new pitch classes, either

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19) See fn. 13.
partially or completely. Consequently, the resulting hexachord may have from one to three new pitch classes.

This operation of partial inversion becomes an important means of creating continuity and coherence. Carter often uses this operation with the all trichord 012478 hexachord. Figure 3 lists all ten \( dtg \) pairs derivable from this hexachord, five of which are symmetrical/non-symmetrical pairs.

**Figure 3. All \( dtg \) pairs of the 012478 hexachord**

| \( dtg \) | \( dtg \) pairs with one symmetrical chord | \( dtg \) pairs with no symmetrical chord |
|----------|------------------------------------------|
| 027/037  | 012/014 024/016 036/026 048/016 | 016/014 016/026 016/037 015/025 015/013 |

Figure 4 illustrates the partial inversion operations with music notations. Applying the inversional operation to one \( dtg \) pair yields two new notes for the first three pairs respectively and one new note for the remaining two pairs respectively. The new notes are marked with italics, and the inversional axes are bold face letters, as well as shown above the music examples. As suggested earlier, this type of progression attests to Carter's emphasis on symmetrical trichords. Their function also becomes apparent. With these trichords Carter divides the 12 tone chromatic into a symmetrical space, and conveniently, an inversional one.
Figure 4. Partial inversion shown in five d#g pairs of the 012478 hexachord

<table>
<thead>
<tr>
<th>svmm.</th>
<th>C</th>
<th>D</th>
<th>C</th>
<th>D</th>
<th>C</th>
<th>C#</th>
<th>G</th>
<th>E</th>
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<tbody>
<tr>
<td>trichord</td>
<td>G</td>
<td>D</td>
<td>C</td>
<td>E</td>
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<tr>
<td>non-</td>
<td>C#</td>
<td>C#</td>
<td>E</td>
<td>Bb</td>
<td>C#</td>
<td>Eb</td>
<td>C</td>
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<tr>
<td>svmm.</td>
<td>E</td>
<td>Bb</td>
<td>G</td>
<td>G</td>
<td>A</td>
<td>F#</td>
<td>D</td>
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<tr>
<td>trichord</td>
<td>G#</td>
<td>G#</td>
<td>G#</td>
<td>G#</td>
<td>G</td>
<td>D</td>
<td>C</td>
<td>A</td>
</tr>
</tbody>
</table>

| * symm  | .027 | .012 | .024 | .036 | .048 |
| * non-symm | .037 | .014 | .016 | .026 | .016 |

While the varied ways of categorizing trichords implies compositional strategies, the various means of creating communality further reveal the compositional interpretations. Both Babbitt's and Carter's interpretations of trichordal relations have their origin in the abstract properties of trichord generators. By accentuating different means of creating communality, they generate disparate senses of progressions, adjacency, and ultimately, of community of relations.

**Babbitt's String Quartet No. 2**

Babbitt's *String Quartet No. 2* (1961) and Carter's *A Symphony of Three Orchestras* (1976) are quintessential examples of the two composers' work from their trichord periods. Trichordal connections play an important role in formal design, twelve tone row/chord, aggregate completion, and music narrative. In unique ways their compositional strategies delve deeply into the interaction between trichords and
hexachords. Therefore, a comparison between these two works is very instructive for our exploration of the multi-layered relations between abstract principles and actual compositions.

String Quartet No. 2 exemplifies the most famous common structure of this period of Babbitt's work — trichordal arrays, one of the richest designs built from generative trichords and hexachords. While many writers have discussed trichordal arrays in Babbitt's composition, topics related to their basic formation and their internal relations remain unexplored. This analysis will focus on the distinguishing features of the array construction, including the basic structures of the single trichord generated arrays, patterns of transformational levels and operation group within the arrays, the communality between derived hexachords and the hexachord of the fundamental row, and the choice of trichord-hexachordal relations.

Parts I and III of this four-part composition, of roughly equal length, are comprised of trichordal arrays. The trichords in these arrays are closely derived from the fundamental twelve-tone row. The fundamental twelve-tone row, its full appearance only occurring in Part IV of the quartet, is gradually unfolded through dyads, trichords, tetrachords, and hexachords. Example 1(a) reproduces its full appearance in Part IV, a high point of the quartet.

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20) See Zuckerman, Mark, 1976, "On Milton Babbitt's String Quartet No. 2," Perspectives of New Music 14/2-15/1: 85-110. He deals with the first part of this string quartet in great detail. His observations concerning the dynamically projected trichord structure in the dyadic sections and the overall formal design of the dyadic sections are very important. Zuckermann's analysis, however, does not deal with the structures of different trichordal arrays and implied of hexachords, nor does he discuss the operation groups used among the arrays.
Example 1a. Fundamental row of Rabbitt's String Quartet No. 2, mm. 266-268

\[ J = 60 \]

The fundamental row, an all interval row, comprises two chromatic hexachords (in its two halves), each with a distinctive ordering and consequently two interval successions separated by interval 6.\(^{21}\) These two interval successions are unfolded in Parts I and III respectively, both comprising four sections, as shown in Example 1(b). The eight segmental trichords derived from the two halves of the tone row are used, in the forms of trichordal arrays, to unfold the interval successions. Consequently, the trichordal array designs are determined largely by the structure of the segmental trichords.

Example 1b. Segmental trichords and interval successions

\[ (0,1,4) \quad (0,1,3) \quad (0,2,5) \quad (0,1,3) \]
\[ (0,1,5) \quad (0,2,5) \quad (0,1,5) \quad (0,2,4) \]

H1: \(< 3 8 5 9 1 >\) H2: \(< 2 7 4 t e >\)

Figure 5. stk, dtg, dyadic generators and derived hexachords in Parts I and III

PART I

Sec. I II III IV

mm: 1 4 7 19 24 34 47 52 63 76 81

dyad: 03 04 05 03 01

Single trichord (stk): 014 015 025 013

Double trichord (dtg): 014/015 014/015/025

Linear hexachord: E E E E E C C B A A

Vertical hexachord: A B B B B B

Intervals succession: 3 8 5 9 85 59 91 38 / 85 38 / 85 / 59

PART III

Sec. I II III IV

mm: 126 131 153 164 170 181 185 196 217 230 235 244

dyad: 02 07 04 01 0e

stk: 025 015 024 013

dtg/dtg: 025/014 024

Hex.: HB H1/H2 H1/H2

Linear hex: C E E E F A A A

Vert. hex: B B B B

I succession: 2 7 4 t e

27 74 4t te

27194 38591 274t e 38591 274t e
Figure 5 shows the structure of the dyad and trichordal arrays of Part I and Part III. Each of these eight sections distinguishes a new stage of the unfolding by first introducing a new interval (or two intervals if it is a first section) with dyad derived arrays, and then presenting a new segmental trichords with trichord derived arrays. Each section is thus marked by an extended dyadic beginning where pairs of sustained notes appear in unified rhythmic figures expressing similar gestures. Immediately following the slow pace of these dyadic pairs are the trichordal passages where fast three note figures stacked up alternately on different instruments, present at the same time a wealth of tone colors, rhythmic variety and registral disparity. The return of distinctive dyadic gesture signifies the beginning of another section. One by one, all the intervals, as well as all the segmental trichords, are individually presented in the eight sections.\textsuperscript{22} Let us now examine more closely the basic formation of the trichordal arrays and its variation.

1. \textit{Designs of Trichordal Arrays:}

In its most basic definition, trichord array is comprised of an ordered single trichord generator (stg) and trichords which are its transformations—a combination that can also be viewed as trichordal mosaics.\textsuperscript{23} Figure 6 illustrates two common structures of the array as the Latin Square and the Non-Latin Square.

\textsuperscript{22} In several sections this is still followed by double-trichord generated arrays (see sections II and III of Part I, for instance).

\textsuperscript{23} Milton Babbitt defines this group permutation and its multiplication table using dyads as basic units as early as 1960 and 1961. See Babbitt, Milton 1960. "Twelve-Tone Invariants as Compositional Determinants," \textit{Musical Quarterly} 46:245-59, especially pp. 252-253 and 257-258, and Babbitt, "Set Structure as a Compositional Determinant," 77-94. For more recent discussion of the Latin Square from a somewhat different angle, see Alegant and Morris, "The Even Partitions."
**Figure 6** Latin square and Non-Latin square

<table>
<thead>
<tr>
<th>a. Latin Square</th>
<th>b. Non Latin Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>D</td>
<td>C</td>
</tr>
</tbody>
</table>

The Latin Square includes four appearances of one trichordal mosaic, $ABCD$. Each letter represents a distinctive pitch class realization of an ordered trichord. Letter $A$ is a trichordal generator, and the remaining letters, $B, C, D$, in the mosaic represent the classic twelve tone operations of $A$ (i.e., $I, I, R, or RI$ transformations of $A$). The Non Latin Square includes two trichordal mosaics, $ABCD$ and $EFGH$. Similar to the way that $A$ is related to its transformations, $E$ relates to its three transformations, $F, G,$ and $H$. When the ordered trichords $A$ and $E$ are related transformationally, the Non Latin Square becomes a single trichordal generated array, i.e., a combination of $A$, and its seven transformations. We will refer to the hexachord derived from $AB$ as a linear hexachord, and the hexachord derived from $AC$ as a vertical hexachord. Obviously, the choice of a Latin or Non Latin Square is linked to the trichordal generator in question, and the choice of arrays has many implications.

**Figure 7a** Latin square "L" in the quartet

```
L:  Top  11P  R17P  R6P  
    R17P  R6P  Top  11P  
    11P  Top  R6P  R17P  
    R6P  R17P  11P  Top  
```

Array "L", as shown in Figure 7(a), is a basic design of Latin Squares that contains only four different trichordal pc collections (i.e., one trichordal mosaic). This

---

24) The square can be oriented freely; that is, the square can be turned 90 degrees.
design underlies a large portion of the quartet. It is also the best known type of trichordal array. In this design, the same hexachord type is produced by two pairs of trichords; that is, both AB and CD may yield the same type of linear hexachord, while both AC and BD will yield the same type of vertical hexachord. This design, using 1 and 7 as the index numbers of inversion, produces a symmetric axis that is ubiquitous in the Quartet. We will refer to this prevalent operation group, ToP, II P, R17P, and R6P as operation subgroup K, one which Babbitt uses frequently.25)

A Non-Latin Square array, as shown in Figure 6(b), comprises eight different trichordal pitch collections (i.e., two trichordial mosaics). It appears mostly in pairs, as shown in Figure 7(b). Together, two non Latin Square arrays contain sixteen different trichordal pitch class collections (four trichordial mosaics), all of which are transformations of a single trichordal generator P. This specific design thus requires the same pitch class collection of a hexachord type to be generated by three pairs of trichords: ToP with II P; T4P with I5P, or I9P with T8P, as underlined in Figure 7(b).

**Figure 7b. A Pair of Non-Latin square in the quartet**

<table>
<thead>
<tr>
<th>ToP</th>
<th>II P</th>
<th>R4P</th>
<th>R6P</th>
<th>I9P</th>
<th>T8P</th>
<th>R6P</th>
<th>R17P</th>
</tr>
</thead>
<tbody>
<tr>
<td>R17P</td>
<td>6P</td>
<td>T4P</td>
<td>I5P</td>
<td>R2P</td>
<td>RI3P</td>
<td>11P</td>
<td>---ToP</td>
</tr>
<tr>
<td>II P</td>
<td>ToP</td>
<td>RI3</td>
<td>R4P</td>
<td>I9P</td>
<td>T8P</td>
<td>I9P</td>
<td>R17P</td>
</tr>
<tr>
<td>R6P</td>
<td>R17P</td>
<td>I5P</td>
<td>T4P</td>
<td>RI3</td>
<td>R2P</td>
<td>ToP</td>
<td>I1P</td>
</tr>
</tbody>
</table>

Consequently there are at least three ways that the trichord type in question can partition a specific linear hexachord. This special requirement unquestionably restricts the linear hexachord of the non Latin Square to all combinatorial hexachords of the third order or above; that is, to the third or the sixth order.26) As a result in the

25) This operation group is a closed group. It is combined with the transformations of itself to form a larger family of related operations and therefore is considered here as one of the many operation subgroups. For further discussion of operation group see Morris, Robert 1987. *Composition with Pitch-Class*. New Haven: Yale University Press.

Quartet linear hexachords of all the Non Latin Square arrays are either hexachord E or hexachord F. The *s*gs of these third- or sixth-ordered all-combinatorial hexachord include 014, 015, 037, 024, and 048.

A different interpretation of the Non-Latin arrays identifies these mosaics by means of operation subgroups. As shown in Figure 7(b), every Non Latin Square array contains two trichordal mosaics each of which is governed by a twelve tone operation subgroup. One of them is always the operation subgroup K. With a different interpretation of the transformational relations, Figure 7(c) reproduces the pair of arrays in Figure 7(b) as arrays J1 and J2. In this interpretation, the *P*, *Q*, and *S* each stand for a distinct trichordal generator.

**Figure 7c. Non-Latin square, J1 and J2 (Reinterpretation of Figure 7b)**

<table>
<thead>
<tr>
<th>J1:</th>
<th>ToP</th>
<th>H1P</th>
<th>ToQ</th>
<th>I9Q</th>
<th>J2:</th>
<th>ToS</th>
<th>I5S</th>
<th>R6P</th>
<th>R17P</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R17P</td>
<td>R6P</td>
<td>R3Q</td>
<td>R6Q</td>
<td></td>
<td>R6S</td>
<td>R6S</td>
<td>H1P</td>
<td>R6P</td>
</tr>
<tr>
<td>H1P</td>
<td>ToP</td>
<td>I9Q</td>
<td>ToQ</td>
<td></td>
<td>I5S</td>
<td>ToS</td>
<td>R17P</td>
<td>R6P</td>
<td></td>
</tr>
<tr>
<td>R6P</td>
<td>R17P</td>
<td>R6Q</td>
<td>R3Q</td>
<td></td>
<td>R6S</td>
<td>R6S</td>
<td>ToP</td>
<td>H1P</td>
<td></td>
</tr>
</tbody>
</table>

index # 1/7 3/9 5/e 1/7

The appearances of operation subgroup K are interrupted by two other operation subgroups using different index numbers, as shown below each columnar pair.

The distribution of *s*gs and array types in Parts I and III is shown in Figure 7(d), where all "L" stands for Latin Square, and all "J" stands for Non Latin Square. Here we will be concerned with the principles underlying Babbitt's choice of array types. As was discussed earlier, the use of all array patterns is restricted to trichordal generators of specific properties, leaving some array patterns opened to more *s*gs than others. Babbitt, because he believes in making music as much as it can be instead of the minimum one can get away with, chooses the more complicated Non Latin square whenever the trichord allows. In other words, whenever the segmental trichord is one of the *s*gs of a third- or sixth-ordered all-combinatorial
hexachord, namely 014, 015, 025, 024, or 018, he will use a Non Latin square. The segmental trichords 025 and 013, which are not stgs for all-combinatorial hexachord of higher order, are associated with Latin Square. The segmental trichords, 014 and 015, both stgs for the all combinatorial hexachord of higher order, are associated with the Non-Latin Square. The segmental trichord 024, although an stg for hexachord F, however, is used in even more complicated designs, J3 and L'.

**Figure 7d. Distribution of array designs in sections of Part I and Part III**

<table>
<thead>
<tr>
<th>Part I</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Section</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>trichord generator</td>
<td>014</td>
<td>015</td>
<td>025</td>
<td>013</td>
</tr>
<tr>
<td>Array</td>
<td>J1</td>
<td>J2</td>
<td>J1</td>
<td>J2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part III</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Section</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>trichord generator</td>
<td>025</td>
<td>015</td>
<td>024</td>
<td>013</td>
</tr>
<tr>
<td>Array</td>
<td>L</td>
<td>L</td>
<td>J1</td>
<td>J2</td>
</tr>
</tbody>
</table>

Arrays J3 and L' are two unique trichordal arrays (shown in Figures 7(e) and 7(f)). Array J3, shown in Figure 7(e), is a non Latin square array which combines two sets of operation subgroup K. These two sets of mosaic collection classes employ trichords P and Q respectively as their single trichordal generators. Also, P and Q are related by inversion at even index number t. The way the two mosaics are conflated with each other makes this array unique.

**Figure 7e. An Unique Non Latin square design, J3**

<table>
<thead>
<tr>
<th>J3:</th>
<th>ToP</th>
<th>ToQ</th>
<th>R1P</th>
<th>R11Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>R17P</td>
<td>R17Q</td>
<td>R6P</td>
<td>R6Q</td>
<td></td>
</tr>
<tr>
<td>ToQ</td>
<td>ToP</td>
<td>R11Q</td>
<td>R1P</td>
<td></td>
</tr>
<tr>
<td>R17</td>
<td>R17P</td>
<td>R6Q</td>
<td>R6P</td>
<td></td>
</tr>
</tbody>
</table>

Array L', shown in Figure 7(f), is a Latin Square, which, unlike other Latin Squares
in the quartet, does not employ the operation subgroup K. It consists of some
twelve tone operations new to the piece. These four twelve tone operation's, 1oP, 12P,
RISP, and R13P, are not a closed operation group; it is a subset of an eight operation
subgroup. The trichordal arrays, L' and J3, present a design unique to the Quartet,
and their uniqueness is a consequence of associating symmetrical trichord, 024.

**Figure 7f.** An Unique Latin square design, L

<table>
<thead>
<tr>
<th>L'</th>
<th>ToP</th>
<th>12 P</th>
<th>R15P</th>
<th>R3P</th>
</tr>
</thead>
<tbody>
<tr>
<td>RISP</td>
<td>R3P</td>
<td>ToP</td>
<td>L2P</td>
<td></td>
</tr>
<tr>
<td>L2P</td>
<td>ToP</td>
<td>R3P</td>
<td>R15P</td>
<td></td>
</tr>
<tr>
<td>R3P</td>
<td>R15P</td>
<td>L2P</td>
<td>ToP</td>
<td></td>
</tr>
</tbody>
</table>

As a consequence, Babbitt's designs of trichordal arrays provide a further
connection in the pitch class structure of this work. The trichordal arrays using the
same types of trichordal generators have identical pitch class collections in their
linear and vertical hexachords, only the inner orderings within each of the trichords
vary. This relation is shown in Figure 7(g) in which letters are used to represent
distinct pitch class designs of the trichordal arrays. The letter with an apostrophe
represents a similar pitch class design in the array when only the internal orderings
of the trichords differ. Sections II, III, and IV of Part I each have a corresponding
section in Part III. The orderings of trichordal generator 015 in section II of Part I
and Part III are further related, namely, the corresponding cell chords of V and V'
can be mapped onto each other by R6. The 024 trichordal array, U, is unique to the
piece, just as the trichord is.27)

---

27) Although the current discussion does not attempt to analyze the foreground connection, with
regard to the pitch choice, it is worth noting that according to Babbitt's own account, the first
vertical hexachord in the trichordal array of Part I shares the same pitch collection with first
hexachord of the row in its first complete presentation at the beginning of the Part IV. See
Figure 7g. Distribution of trichordal arrays with similar pitch class content

<table>
<thead>
<tr>
<th>Part I</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Section</td>
<td>I-1</td>
<td>I-2</td>
<td>I-3</td>
<td>I-4</td>
</tr>
<tr>
<td>trichord generator</td>
<td>014</td>
<td>015</td>
<td>025</td>
<td>013</td>
</tr>
<tr>
<td>Array</td>
<td>U</td>
<td>V</td>
<td>W</td>
<td>X</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part III</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Section</td>
<td>III-1</td>
<td>III-2</td>
<td>III-3</td>
<td>III-4</td>
</tr>
<tr>
<td>trichord generator</td>
<td>025</td>
<td>015</td>
<td>024</td>
<td>013</td>
</tr>
<tr>
<td>Array</td>
<td>W'</td>
<td>V'</td>
<td>Y</td>
<td>X'</td>
</tr>
</tbody>
</table>

An important feature of Babbitt's trichordal compositions is that the trichordal arrays in pitch structure are always juxtaposed with other trichordal arrays characterized by register, dynamics, and articulation groupings. Space considerations do not permit a detailed analysis of the pitch structure defined by the dynamic, registral, and articulation groupings. However, Example 2 illustrates a design based on different types of articulation. Each articulation means is associated with a distinct all combinatorial hexachord. Juxtaposed with the pitch structure derived from trichordal arrays are the hexachords assigned to the four distinct articulation means: tremolo (C), pizzicato (A), sul ponticello (F), and staccato (E). Additionally, juxtaposed over dfg arrays are often stg arrays formed in the other domains. Example 3 illustrates such a juxtaposition of dfgs and stgs.

Example 2. Hexachords associated with different articulation means in mm. 171-176
**Example 3.** Juxtaposition of dtg and stg in mm. 65-71

(a) DTG

(b) STG

2. **All-combinatorial Hexachords and Trichordal Arrays:**

The Quartet's clear sense of unfolding is accomplished by the establishment of the successive pattern of dyads and trichords. With each repetition of the dyad stg dtg sequence, the progression in the unfolding becomes clear. The segmental trichords connect the gradual unfolding of the source hexachord and the local harmony derived from the hexachords on the array, either linear or horizontal. Figure 8 shows the trichords and the derived linear and vertical hexachords.

**Figure 8.** Distribution of stg and hexachords on the trichordal arrays of Parts I and III

<table>
<thead>
<tr>
<th>Part</th>
<th>Section</th>
<th>I-1</th>
<th>I-2</th>
<th>I-3</th>
<th>I-4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>trichord generator</td>
<td>014</td>
<td>015</td>
<td>025</td>
<td>013</td>
</tr>
<tr>
<td></td>
<td>hexachord (linear/vertical)</td>
<td>E/A</td>
<td>E/B</td>
<td>C/B</td>
<td>A/B</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part</th>
<th>Section</th>
<th>III-1</th>
<th>III-2</th>
<th>III-3</th>
<th>III-4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>trichord generator</td>
<td>025</td>
<td>015</td>
<td>024</td>
<td>013</td>
</tr>
<tr>
<td></td>
<td>hexachord (linear/vertical)</td>
<td>C/B</td>
<td>E/B</td>
<td>F/B</td>
<td>A/B</td>
</tr>
</tbody>
</table>
This design relies on the special property of the all-combinatorial hexachords and their stgs. As discussed above, the degree to which the abstract properties of the trichord hexachordal relationship affects his composition is remarkable, and such trichordal paths constitute an important means for Babbitt to navigate through his twelve-tone compositions:

The number of such ... trichordial paths through a set form, of such trichordial characterizers of a set form, beyond suggesting strongly the need for further constraints, carries one from the set as a thing in itself to the set as a member of the closed set society, while combinatorial attributes span the two domains, as greater combinatorial generalization, toward generalized aggregates, depends less and less on the attributes of the set and more and more on the properties of the relations induced by the system's transformation.  

Indeed, the world of musical connections reflected in Babbitt's String Quartet No. 2 is inseparable from the abstract relation of all combinatorial hexachords and stg. The diagram in Figure 2, a way of extracting such relation from the chromatic space, depicts the paths between and interconnections among many of Babbitt's compositions. At first glance, it appears that hexachord A, constituting the fundamental hexachord of the quartet, has the main structural role in the piece. It is, however, the B hexachord, as a derived hexachord, which is prevalent in the arrays. The reason is obvious: every stg of hexachord B is a segmental trichord which is found in the fundamental row of the quartet. (As discussed earlier in the diagram, hexachord B has at least one stg in common with other all-combinatorial hexachords.)

Babbitt's interpretation of the property of this special trichord-hexachordal family gives rise to numerous moldings and remodelings of the interactions among trichord generators, their interval successions, the larger expression of these interval successions in the arrays, the ordered source hexachords, the derived hexachord types,

29) In his book Mead discusses in a similar manner the focal hexachords for various compositions. Hexachord A emerges as the focal hexachord for Composition for Four Instruments, and hexachord F emerges as the focal hexachord for Composition for Viola and Piano, etc. See Mead, An Introduction, chapters two and three in particular.
the operation subgroups, and the Latin/Non Latin Squares. A unique interpretation of the chromatic universe emerges as we study Babbitt's composition strategy, which is not only closely related to the close set family of trichords and hexachord but also to the path of progression connecting trichords and hexachords. In the process, while the fundamental row is used as a compositional determinant, the path among trichords and all combinatorial hexachords remains Babbitt's basic structure. The rich sonority of the multiple layers of trichordal arrays in pitch structures, articulations, and registral space are merged seamlessly to create an expressive whole.

Carter's A Symphony for Three Orchestras

Carter's use of trichords and governing twelve-tone chords brings forth two separate organizing forces in *A Symphony of Three Orchestras*. The overall plan of the symphony is based on their respective forces as well as the interaction between them. The trichordal design gives rise to the individual characters of the three orchestras, whereas the governing chord brings organizing force to the symphony both as a whole and at a local level.

At the outset, the three orchestras are distinguished by their instrumentation, as shown in Figure 9.

**Figure 9. Instrumentation of *A Symphony for Three Orchestras***

<table>
<thead>
<tr>
<th>Orchestra</th>
<th>Designated Instrumentation</th>
<th>Designated Trichords</th>
<th>Designated Intervals</th>
<th>Excluded Interval Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>brass, timpani, strings</td>
<td>024, 012, 048</td>
<td>026</td>
<td>4, 6, 2, 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>symmetrical, non-sym.</td>
<td></td>
<td>ic 5, ic 3</td>
</tr>
<tr>
<td>II</td>
<td>clarinets, piano, nitched percussion, strings (no viola)</td>
<td>027, 036</td>
<td>016, 025</td>
<td>7, 6, 3, 4, ic 1</td>
</tr>
<tr>
<td>III</td>
<td>woodwinds (no cl.), non-nitched percussion, strings</td>
<td>037, 014, 015, 013</td>
<td>e, 4, 5, 9</td>
<td>ic 6, ic2</td>
</tr>
</tbody>
</table>
The symphony has a tripart structure, the main section of which is framed by an introduction with a lengthy trumpet solo and a coda with a series of thunderous twelve tone chords. While the introduction is comprised of long sustained chords on violins which linger in the highest register, the end of the coda consists of sustained chords, played by cellos and contrabass murmuring from the lowest register of the orchestra. Both sections are derived from the governing chord of the symphony.

The governing harmony—an all-interval twelve-tone chord and its three transformations is shown in Example 4(a). Trichords at the top register of the four twelve tone chords form the aggregate of the Introduction, whereas the trichords at the lowest register account for the aggregate for the end of the coda. 30)

**Example 4a. All-interval twelve-tone chord**

![Diagram of twelve-tone chord](image)

A linear articulation of this all-interval twelve-tone chord appears only toward the end of the coda as the melody of a piano solo joined by the clarinets and marimba. This simple yet elegant melody on piano appears in vertical dyads until it is closed off by the strikingly high F#, as shown in Example 4(b).

---

30) One note is an exception to this aggregate.
Example 4b. Lyrical appearance of the all-interval twelve-tone chord in mm. 376-379

While governing harmony constitutes the Introduction and Coda, the main body of the symphony is directed by the twelve trichord types which are distributed among the three orchestras. Figure 10 illustrates the sequence of the trichords' appearance in the three orchestras.

After the first strand of orchestra is established, each subsequent appearance of a new trichord type is introduced by an orchestra against the background of an existing strand. Both the individual trichordal construction and its interaction with the other strands of orchestras contribute importantly to the distinctive characteristics of each trichord. Each of the twelve trichords, in addition to being juxtaposed with another trichord in the duo orchestra section, or with two other trichords in the trio orchestra sections, has a "solo" moment where the articulation of its characteristics is most prominent.

In this design the all-interval twelve-tone chords are the source of unity and contrast in the Introduction and Coda, while the twelve trichord types underlie the three orchestra strands, yielding a constant change of textures in the middle section. How is the referential twelve tone chord related to the distribution of twelve trichord types? How are the trichords manifested in each orchestra and with what musical expression are they associated? What creates the connection among different
expression of a governing harmony? What is the role of aggregate completion? In the following we will focus on (1) the relationships among trichord characteristics, various trichordal constructions, and aggregate completion; (2) the progression within the governing harmony; (3) the relation between the twelve tone chords and the twelve trichordal types.

1. Trichordal Characteristics and Aggregate Completion

The characters associated with each trichord type range from chordal, melodic, and scalar, to dyadic. They appear singularly, in pairs, or interlocking in tetrachord and pentachords. As in Carter's many other compositions, the distinct musical expressions associated with the trichords often reflect the unique interval contents of the trichords, as shown in Figure 9. Exactly two interval classes are excluded from each orchestra. The individual characters of each trichord are indicated in Carter's compositional plan (noted in parenthesis below)31) Two of the three symmetrical trichords, 024(flowing) and 012(angry) in Orchestra 1 are mostly used to generate fast-moving scalar melodies, while the unique sonority of the 048 trichord(sostenuto) constitutes chordal progressions.

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31) The compositional plan for A Symphony of Three Orchestras, is illustrated in a diagram in Schiff, The Music of Elliott Carter, 298.
The 026 trichord (molto espr.), the only non symmetrical trichord in Orchestra I, forms mostly vertical chords with occasional linear motion. Orchestra II has the two remaining symmetrical trichords, 027 (bell like) and 036 (accelerating, piano). The 036 trichord is used almost exclusively in linear melodic motion. The trichords, 027, 025 (grazioso, clarinet) and 016 (cantabile, espr, celi) generate a variety of musical gestures, including dyadic, chordal, and linear motions. An expressive character is associated with the 016 trichord in the strings' cantabile melody. In Orchestra III, the 037 trichord (sostenuto) constitutes most of the substantial chordal motion. The role of sustained 037 chords in Orchestra III is comparable to that of the 018 trichord of Orchestra I. The 014 trichord (giocoso) generates a scherzo like melodic motion characterized by bassoon, while both 015 (fluttered tongued, tremolandi) and 013 trichords (espr., cantabile) are often associated with long stretches of melody accented occasionally by vertical chords of their kind. As the following analysis will show, trichordal generated aggregates and trichordal chains are both important expressive means.

Carter uses trichordal-generated aggregates in effecting dramatic gestures and marking sectional boundaries. An especially remarkable dramatic moment is the 018 trichord passage (mm. 246 249) which leads to a climax. As shown in Example 5, these fast moving sixteenth note figures are pointers whose abrupt halts draw attention to the sustained sounding 018 chords. (Perhaps an homage to a passage in the Fourth String Quartet of Schoenberg, mvt 1, m.63.) One after another the 018 trichords, registrally separated, culminate in a climactic twelve tone chord in fortissimo the first of the grand gestures in this piece. Similar stg aggregates can be found elsewhere in this symphony; its construction is not, for example, the constructions of Orchestra I in mm. 237-259.
Example 5. 048 stg aggregate in mm. 246-249 (reduction of Orchestra I)

As in his other compositions of this time, Carter often makes use of complete aggregates to mark sectional boundaries. In the main body of the symphony, the divisions of sections solo orchestra, duo orchestra, or trio orchestra are frequently marked by a complete aggregate either with notes either from one orchestra alone or from all orchestras present at the moment. These complete aggregates are derived from either stg or dtd construction, such as the 015 trichord (Orchestra III) in m. 108 which also marks the beginning of a trio orchestra section, or the 027 trichord (Orchestra II) in m. 145 which begins the piano and pitched percussion solo in the "solo" passage. Because the numerous duo orchestra sections involve two trichordal types at the same time, aggregates formed by dtd are also common as found in mm. 60-69.

Carter also uses the trichordal derived aggregates to smoothen the change of trichords in an interesting way. As discussed earlier, the first twelve tone chord of the composition is derived both instrumentally and registrally from the 018 trichord. The two twelve tone chords derived from the 037 trichords in mm. 206-211, on the other hand, tell a different story. While the 037 trichords appear vividly in the musical surface in Orchestra III, forming complete aggregates, registrally the aggregate is
comprised of the 026 trichords prefiguring the trichords of the entering Orchestra I. Similarly, the aggregate in m. 108 is based on an 015 trichordal construction in Orchestra III; registrally, however, the 013 trichords takes prominence the next trichord in the same orchestra.

The two functions of complete aggregates—as expressive means and as demarcations of sectional boundaries do not exclude one another. As shown in Example 6, the 037 stg aggregate (in square) is simultaneously part of a dts aggregate with 026 trichord (encircled) in mm. 97–99 with the first being an expression of Orchestra III, and the latter marking sectional boundary. The aggregates formed in the vertical and horizontal dimensions can often be distinguished by their different registral distributions, except for the invariance shared among them.

**Example 6.** 037 stg, 037/026 dts aggregate in mm. 88–89

![Example 6](image)

Trichordal chains are also prominent in this work, as well as in other Carter works of this period. In the duo and tri orchestra sections, the trichordal arrays formed by vertical chords are often contrasted with the horizontal appearance ranging from tetrachords, and pentachords to strings of trichordal chains. A characteristic linear motion is seen in the melody of the clarinet in Orchestra II in mm. 107–110, as shown in Example 7, mm.107–110. This expressive clarinet melody comprises series of 025 trichords overlapping mostly by one note, alternating among the three
interval classes 3, 5, 2.

Example 7. 025 trichordal chains in mm. 107-109

2. All-Interval Twelve-Tone Chord:

The function of the referential all-interval twelve-tone chord in A Symphony has intrigued many, as noted in Schiff. Ordered in register, the all interval twelve tone chord and its three transformations distribute pitches in their specific registral locations, and provide a limited pitch space. With all its resemblance to the twelve tone row ordered in time, Carter's twelve tone chord ordered in registral space is unique in the following ways. First, the all interval twelve tone chord allows not only the twelve pitches but also the eleven intervals to have distinct positions in a registral space; as a result, the sonority of pitches and intervals are identifiable with particular registers. Second, the four transformations of the twelve tone chord form a trichordal array of pitches. This trichordal array thus constitutes different aggregates which are distinctive from one another in register concentrations. For instance, the lowest note of the aggregate formed by the trichords at the top register of the array, which constitute the introduction of the Symphony, is [E-flat].

Third, transformations of the twelve-tone chord share invariant pitches. For example, G between the Prime and Retrograde, F between Inversion and Inversion Retrograde, and C between Prime and Inversion. As a result, the twelve tone chord and its three transformations provide forty five different pitches. Not all the complete aggregates or twelve tone chords in the Symphony are related

32) For a comparison of Carter's referential twelve-tone chord and the twelve-tone row see Mead, "The twelve-tone Music and Elliott Carter."

162
to the referential twelve tone chord, but they are mostly derived from this pitch space of forty five note. Most importantly, within the 15 note pitch space, the symphony as a whole moves from its higher register, through its middle range, to its lowest register at the end. Finally, while the degree of closeness to the twelve tone chord or to any of its transformations is more difficult to measure, there is one sharp distinction between those that belong to the pitch space of forty five notes and those that do not. This distinction is used most dramatically at the opening of the Coda. The thunderous twelve tone chords, although not the referential twelve tone chord itself, comprise notes from its limited forty five note pitch space: and the lyrical melodic lines, contains all notes outside of that pitch space.

3. Progression In the Governing Harmony-Partial Inversion:

There are two levels of governing harmony in the Introduction: the all-interval twelve tone chords which are referential on a broader level, as was discussed earlier, and a governing all trichord hexachord which provides unifying sonority at a local level.33) As a governing harmony sustained on the strings of Orchestra I and Orchestra III, this all trichord hexachord, unrelated to the all interval twelve tone chord, persists throughout this passage as a unifying sonority. A sense of progression within the prevalent governing harmony can be felt through the gradual shifting of the chord. Such an progression is rooted in the inter/intra saturation of the subsets of the partitioning. Here in A Symphony, the sense of progression is subsumed by the subtle shifting of sound.

This governing all-trichord hexachord, the 0124/8 hexachord, is expressed by several pairs of its dis.34) To further a misty mood associated with the Introduction,

33) Incidentally, this is the hexachord of the twelve-tone chord of the Piano Concerto. This hexachord is also used extensively in A Mirror on Which to Dwell.
34) At some level it is debatable whether the partitionings that intersect with each other are 5+1, 4+7 or 3+3, because at one time as many as five notes or as few as one note can remain the same. The fact that each orchestra constantly contains a trichord indicates the importance of trichordal partitionings.
the sense of progression is quite subsumed. Two trichords constituting governing harmony are held by the violins of Orchestras I and III, with a gradual subtraction and addition of notes shifting within the designated aggregate. The alternation of a single note or a dyad inevitably results in a different $d tg$ pair distributed in the two orchestras. Since an invariance is held between two pairs of $d tg$ while only one or two notes are altered, the shifting from one $d tg$ pair of the governing chord to the next is created by motion between the non invariant subsets. The carefully controlled sonority here results from the aggregate of fixed register, the unifying sound of the governing hexachord, and the gradual shifting of notes using sustained strings.

The progression of the governing harmony in the Introduction of the Symphony consists of yet another connection engendered by symmetrical chords' the unique partial inversion. Since the two orchestras maintain their respective trichords, on the surface the continuity from one governing hexachord to another is accomplished by the retention of an invariance and the motion between the non invariance. At a deeper level, whether a note is retained and altered depends on its relation to the symmetrical trichord of the $d tg$. Earlier in this essay, I discussed such a progression as partial inversion, (shown in Figure 4). In other words, all the added and subtracted notes are related around an internal symmetry.

Consider for instance, the beginning of A Symphony, shown in Example 8(a).\(^{35}\) The beginning of the governing harmony shown in Example 8(a) in $d tg$ pairs is reproduced here to show its structure surrounding a symmetry. Example 8(b) reproduces H1 through H5 of Example 8(a), illustrating the progression among them resulting from inversiveal symmetry. An invariant symmetrical tetrachord, 0127, is held between H1 and H2 and between H3 and H4. Similarly, 0248 is held between H4 and H5. Each time the symmetrical tetrachord provides an axis around which, the

\(^{35}\) It can include not only the shifting among the ten different pairs of $d tg$s but also the shifting between different pitch class realizations of one $d tg$ pair. One full realization of such a possibility is contained in James Boros' study of the all-trichord hexachord. See Boros, James. 1990. "Some Properties of the All Trichord Hexachord," In Theory Only. 11 (6):19-41. James Boros presents one realization of the $d tg$ pair using a certain trichordal chain.
tetrachord maps onto itself and a non invariant dyad (shown in the boxes) maps onto a new dyad. This means of deriving new dyads (shown by arrows) constitutes the gradual shifting into a new expression of the governing harmony.

**Example 8a.** $d_{fg}$ from *A Symphony of Three Orchestras*, mm. 1-9

![Diagram of three orchestras and hexachords](image)

<table>
<thead>
<tr>
<th>Hexachords</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{fg}$</td>
<td>027</td>
<td>027</td>
<td>016</td>
<td>016</td>
<td>016</td>
<td>016</td>
<td>016</td>
<td>037</td>
<td>016</td>
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<td>014</td>
<td>016</td>
<td>048</td>
<td>015</td>
<td></td>
</tr>
</tbody>
</table>

**Example 8b.** Progression within the 012478 hexachord using symmetrical relations

![Diagram of hexachords and progressions](image)

Example 8(c) illustrates the resulting $d_{fg}$ on the surface, distributed in the two orchestra groups; the trichordal pair in the higher staff precedes the trichordal pair in the lower staff. The white note head indicates the two new pitches resulting from each progression. The new dyad may result in a new trichord, or simply yielding two new trichords. Such a connection underlies most of the progression in governing hexachord.36 Undoubtedly, progressions in Example 8 is based on the abstract
musical relation generalized in Figure 4. Although the discussion here has been focused on the all trichord 012478 hexachord, Carter has employed similar techniques with other types of hexachord.\footnote{37}

Example 8c. Resulting trichordal pairs

This brief analysis of Carter's use of trichords in the Introduction, main body, and coda of *A Symphony* reveals by selected examples how significant and complex trichords are in the composer's thinking. While the trichord is used characteristically with a governing hexachord in the Introduction, the distinguishable trichordal sonorities are significant in the main section, as reflected in their distinctive roles in shaping each strand of orchestra. Rather than contributing to a culmination of a larger whole, the unfolding of each of the twelve trichords constitutes an unique sonic event in its own context. At the same time, techniques such as progression based on symmetrical trichordal structure within a hexachord type and simultaneous projection of multiple trichord partition/mosaic in the same aggregate all give us clues to Carter's personal path within the abstract trichordal relation of trichords offered by the chromatic universe.

\footnote{36} The first song from *A Mirror on Which to Dwell*, "Anaphora," adopts a similar technique, using the 012478 hexachord and both 027 and 037 trichords.

\footnote{37} The application of this progression also questions assumptions about equivalence class. I have discussed elsewhere a distinct equivalence class based on this type of progression in the song "Argument" from *A Mirror on Which to Dwell*. See Rao, Nancy Yunhwa. 1984. "Elucidating Stylistic Differences in Post-Tonal Compositions from a Trichordal Perspective: Commonality and Individual Styles in Selected Compositions of Milton Babbitt, Arnold Schoenberg, Ruth Crawford Seeger, and Elliott Carter." Ph.D. dissertation. University of Michigan.
Conclusion

Ranging from the simple categorization of trichords to the use of particular trichords in trichordal arrays and progressions, the theoretical principles at different degrees of abstraction give rise to compositional syntax at multiple levels. Babbitt's and Carter's individual interpretations of the trichord/hexachordal structure inevitably are *distinctive* yet not *exclusive* of one another. The abstract structure of notes and their combinations inevitably become an *internalized* part of the composer's language. Or as Carter puts it:

"Thel compositions are the result of innumerable choices—many unconscious, many conscious, some quickly made, others after long deliberation, all mostly forgotten when they have served their purpose. At some time or other, this sorting and combing of notes finally become a composition. By that time many of its conceptions and techniques have become almost a matter of habit for the composer. . . ." 38

This comparison of the two composers' use of trichordal relations shows the multiple levels these relations are relevant to in their unique compositional syntaxes. During his first compositional period Babbitt positions his musical relations primarily around the interactions among members of a specific family—the generative trichords and the all-combinatorial hexachords; thus, String Quartet No. 2 is unique in the way it emanates from the corner of the B hexachord. Other compositions of the same period, such as Composition for Four Instruments and Composition for Viola and Piano, share similar trichordal space. During the decade after the early 1970's Carter positions his compositions primarily around the meeting point of symmetry and maximum diversity characteristics which are exemplified by the use of the all-trichordal hexachord, all-interval twelve-tone chords, and the twelve distinctive trichords. Compositions such as *A Mirror on Which to Dwell* and Piano Concerto

share similar interpretations of trichordal space.

As shown above, the roles of trichord-hexachordal relationships and the all interval twelve tone chord/row are similarly significant in the musical narratives of Babbitt's String Quartet No. 2 and Carter's *A Symphony of Three Orchestras*. Yet the most obvious gesture that they have in common is the dramatic disclosure of the twelve tone chord/row. In both compositions the lyrical disclosure occurs toward the end, at the climactic moment. However, the meanings of these two climactic presentations must be understood in their respective terms. Babbitt's narrative is culminative in a way that Carter's is not. For Babbitt, the fundamental row is at first hidden in the background; its totality is only suggested by the alternating appearances of the dyadic and trichordal sections. Each appearance of interval successions and trichords, although complicated by its multi-dimensional texture, contributes to an interval unfolding process, and the final appearance of the fundamental row is a dramatic moment of arrival whose lyrical beauty, reinforced by the sudden change into simplified texture, is a true culmination of all the short, intense, fragmented lines that are previously heard in the gradual unfolding of the work.

For Carter, on the other hand, appearances of characteristics of the twelve trichords foster the ever-changing texture and rich sonority. The effect of this multi-layered texture, which is derived from successive appearances of distinctive trichords, is anything but sequential unfolding. Instead, it provides a large, complex, constantly shifting gestures; tone colors, sonorities, characters, and instrumentation. It is to such an overwhelmingly complex main section that the sudden clarity of texture in the Coda poses a sharp contrast—the clear opposition of the thunderous twelve-tone chord and the lyrical melodic lines. Rather than a culmination, the final disclosure of the twelve tone chord is the grand melodic gesture which represents a reflection, a return to the clarity. In both cases, the explicit appearances of twelve tone rows contrast sharply with their generally indirect influence on the musical foreground.

While the two composers' use of *stg* and *dtg* property both originate from
abstract principles of trichordal generators, it is their different means of creating
community that effect their distinctive patterns of progressions their compositional
designs. Babbitt's trichordal arrays, primarily based on the close family of generative
trichords (mostly stg) and all-combinatorial hexachords, convey interrelated
possibilities of traversing the trichordal/hexachordal path. Babbitt's designs thus
animate the relations of not only the particular family of trichords and hexachords,
but more importantly, the path created among them, some weighted more than others.

Comparable to the unique paths in the corners of Babbitt's trichords and hexachords
is Carter's unique use of symmetrical trichords and governing hexachords. The
partial-inversion results from the interactions among the governing hexachord, the
symmetrical subset and its dtg, and the symmetrical subset. In particular the
all-trichord hexachord and its symmetrical subsets and the distinct interval content of
twelve trichords constitute Carter unique interpretation of the trichordal relations.

Furthermore, their different interpretations of trichord-hexachordal relations are
driven further apart by their uses of the all interval twelve tone row. Quite unrelated
to the all-trichord hexachord, the all-interval twelve-tone chord is ordered in register.
It functions for Carter as a means of distributing both the eleven intervals and the
twelve-notes in specific registral locations. With its three transformations, the
twelve-tone chord generates a referential pitch space of forty-five notes—a distribution
of aggregates which Carter uses with great flexibility. The Symphony, at a broader
level, progresses from the higher register of the referential pitch space, through the
middle section, and finally to the lower end of the space. One of the dramatic
elements in Coda is derived from the sharp contrast between notes within it and notes
outside it. Since there is no relation between the governing all-trichord hexachord and
the twelve tone chord, their roles are distinct. The former provide continuity for the
foreground progression, while the latter constitute the background progression.

For Babbitt, on the other hand, the all-interval twelve-tone row ordered in time
results in a succession of intervals that evolves during the course of the composition.
The fundamental row recedes far into the background, but its fragments are always close to the musical surface—the interval succession of its two hexachords functions as a referential norm and is revealed by the dyadic arrays and trichordal arrays. At every level, the use of the all-interval row is closely tied to the use of the hexachord family. Through the ways in which he uses the ordered A hexachord—the fundamental hexachord—and the unordered B hexachord—the ubiquitous derived hexachord—in String Quartet, No. 2, Babbitt seems to point in two different directions: his ordered source hexachord points back to Schoenberg’s twelve-tone method, and his unordered B hexachord as the governing hexachord evokes Carter’s use of governing harmony.
ABSTRACT

A Comparison of Trichordal Relations in Milton Babbitt’s String Quartet, no. 2 and Elliott Carter’s A Symphony for Three Orchestras

Nancy Yunhwa Rao

This paper examines the use of trichords in Milton Babbitt’s and Elliott Carter’s compositions. It analyzes the trichordal structures and their implications in the works. The paper concludes with a discussion on the significance of trichords in modernist music.

171
Babbitt의 String Quartet, no.2의 분석에서는 3화음의 배열과 그것들의 음열과의 관계, 3화음의 생성과 모든 hexachord의 결합, string quartet의 전체적 구조 안에서의 그들의 역할, 그리고 전체적 표현법들에 관해서 설명하였다. 그리고 Elliott Carter의 A Symphony for Three Orchestras의 분석에 있어서는 3화음을 기반으로한 구성적 구조와 부분적 선위에 기초를 둔 확장진행 등에 관해 논하였다. 결과로서는 이 두 작곡가들이 어떻게 3화음적 관계들의 사용을 다양한 방법으로 자신들의 독자적인 작곡적 방주에 연관시켰는지를 보여 주는데 중심을 두었다.