TOPICS ON SUPERSYMMETRY AT ELECTROWEAK SCALE

By

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In this thesis, we study different topics relating to supersymmetry at electroweak scale using naturalness and minimality as our guiding principles. We will discuss a new mechanism to generate large $A$-terms in MSSM. This mechanism could be combined with gauge mediation supersymmetry breaking mechanism to explain the MSSM spectrum and the Higgs mass with minimum fine-tuning. Moreover, we will show that these models may have unique spectrum with one of the stops much lighter than other colored particles. We will also discuss how simple vector-like extensions of MSSM can explain the dark matter relic abundance and the Higgs mass in a natural setting. Among other results, we emphasize a tight relation between relic abundance and spin dependent cross-section in our model. Finally we discuss the effect of D-term on the vacuum structure of O’Raifeartaigh models and its possible role in models of direct gauge mediation.
Preface

This thesis is the result of my graduate training at New High Energy Theory Center at Rutgers University, under the direction of professor David Shih. This thesis is organized as follows: chapter one contains a review of weak scale supersymmetry and dark matter, intended to sketch the context in which my work may be relevant. Chapter two, three, and four consist of research on the subject that I conducted with my collaborators under the supervision of my advisor. Finally chapter five is a brief conclusion and outlook. All published papers can be conveniently accessed at http://arxiv.org/.
Acknowledgments

If done correctly, this should be the longest chapter in this thesis. I am grateful to many people who have helped and supported me over the years. Firstly, I am deeply thankful to my advisor, David Shih. Not only because a big part of the physics that I learned in graduate school was directly from David, but also because he was the main source of motivation and support in my graduate studies. In addition to being inspired by his passion for physics, I have learned a lot from his rigorous logic and his attention to details. David always kept his office door open and was always happy to answer my questions or to help me in my research. I am truly thankful to him for his advice, collaboration and generosity in sharing many of his ideas and projects with me.

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Dedication

To my parents,

who have always been there for me
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Chapter 1
Introduction

1.0.1 Background

The Standard Model (SM) of particle physics, has been exceedingly successful in explaining the known particles and their interactions and at present is our best understanding of Nature at the smallest scales. Theoretically, SM is a consistent quantum field theory (QFT) up to energies where we expect gravity to becomes strong at $M_{\text{pl}} = \frac{1}{\sqrt{8\pi G_N}} \approx 10^{18}\text{ GeV}$. Empirically it has endured decades of collider and precision experiments in particle physics. In spite of all this, there has been a general consensus in the community that there should be physics beyond SM. More importantly for this work, there are good reasons to believe new physics exists at $0.1 - 1\text{ TeV}$ (electroweak scale) which is currently being probed by different experiments.

Failures and puzzles of SM might be addressed at very different energy scales and with completely different mechanisms. Let us review the well-known puzzles here and explain why theoretical and experimental evidence show that new physics at electroweak (EW) scale is indeed quite plausible. One drawback of SM is of course its failure to incorporate a full theory of quantum gravity. It is expected that our understanding of QFT must change fundamentally when gravity effects become strong. Neglecting gravity, another intriguing feature of SM is the fact that quarks and leptons of SM fit neatly in irreducible representations of larger gauge groups such as $SU(5)$ or $SO(10)$. It is then very interesting to ask if the gauge group of SM, $SU(3)_C \times SU(2)_W \times U(1)_Y$, is remnant of a larger gauge group and if they unify in the UV. Another puzzle is the smallness of strong CP parameter in SM by nine orders of magnitude. There is also no explanation for the Yukawa structure and the Yukawas are inputs to the model. Aside from neutrinos, all fundamental fermions get their masses from electroweak symmetry breaking (EWSB) in SM. It is then an important
puzzle to ask why fermion masses span six orders of magnitude from electrons to top quarks.

The origin of the neutrino masses is yet another puzzle that is not addressed in SM.

All of the above problems are important to explore but may be solved with known tools and tricks of QFT (as in the flavor structure, strong CP problem, neutrino masses), or may be postponed to much higher energies (as in gauge coupling unification and quantum gravity). There are two puzzles of SM however, that point to EW scale and have been the driving force for much of the work in the field in the past few decades: hierarchy problem and nature of dark matter.

**Hierarchy Problem.** On 4th of July 2012, ATLAS [1] and CMS [2] declared observation of a new boson with a mass of about 125 GeV (see figure 1.1). The discovery was mainly based on $H \rightarrow \gamma\gamma$, $H \rightarrow ZZ \rightarrow 4\ell$ and $H \rightarrow W^+W^- \rightarrow 2\ell + 2\nu$ decay channels (see figures 1.2, and 1.3). Up to the time of this writing, there has been no hints to suggest this new particle is anything other than SM Higgs boson. Independent of our theoretical bias, it has been shown for example that the new particle has the same spin and parity as the SM Higgs and its decay modes and production mechanisms are consistent with those expected from SM [3, 4]. This discovery highlights the urgency of a long standing problem which has been known as “naturalness” or “fine-tuning” problem.

Hierarchy problem of SM, is the fact that the mass of the Higgs (in fact any fundamental scalar) is sensitive to physics at all scales up to the high-energy cutoff ($\Lambda$) of a QFT. In SM
the Higgs field is a complex scalar $H$ with the following potential at tree level:

$$V_H = m_H^2 |H|^2 + \lambda |H|^4$$ (1.1)

The hierarchy problem is usually phrased as the quadratic radiative corrections to $m_H^2$ from
loop diagrams and it appears already at one-loop. The main loop contributions to Higgs mass in SM are:

\[
\delta m_h^2 = \frac{\Lambda^2}{16\pi^2}(-6y_t^2 + 3g_2^2 + g_1^2 + \lambda^2)
\]  

(1.2)

where \(y_t\) is the top quark Yukawa and \(g_1, g_2\) are \(U(1)_Y\) and \(SU(2)_W\) gauge couplings respectively (\(\lambda\) is as defined 1.1). Considering the values of the couplings, the main contribution comes from top quark loops. Here we are parametrizing our ignorance of physics beyond SM in just one high energy cutoff for the theory \(\Lambda\).\(^1\) Barring the possibility of fine-tuning of these contributions to the Higgs mass, we expect each contribution to be of the same size as the Higgs mass itself which gives \(\Lambda \sim \text{TeV}\). That is our main motivation for physics at EW scale beyond SM.

Based on effective field theory perspective, the hierarchy problem is due to the fact that the Higgs mass term in (1.1) is not protected by any symmetry and therefore naturally we expect it to be of the same order as the cutoff. Therefore quite generally, solutions to hierarchy problem in SM either lower the expected cutoff or use a symmetry to stabilize the Higgs mass up to much higher energy scales.

- **Lowering the cutoff.** Hierarchy problem is puzzling only if there is a hierarchy between EW scale and the UV scale of the theory. In particular if we take the scale of underlying UV physics to be near the Planck scale, terms in (1.2) must cancel each other up to one part in \(10^{32}\) which is unnatural. However, if effects of quantum gravity become important not much above EW scale, there would be no hierarchy in the fundamental scales. Models of large extra dimensions solve hierarchy problem in this way. In these models one assumes existence of \(n + 4\) dimensions in nature (where \(n\) dimensions are compact). In such a higher dimensional world, the fundamental gravitational coupling \(G_*\) is related to \(G_N\) in four dimensions via:

\[
G_N = G_* / V_n
\]  

(1.3)

\(^1\)These “quadratic divergences” depend on the regularization that we use for the loop-integrals. In dimensional regularization for example, they do not appear. Nevertheless there will be terms proportional to the quadratic masses of the heaviest particles in the theory that couple to Higgs. Therefore quadratic divergences should be just interpreted as sensitivity of Higgs mass to higher scales.
where $V_n$ stands for the volume of the extra dimensions. Therefore the scale of quantum gravity in these models, $M_\ast = \left(\frac{1}{8\pi G_\ast}\right)^{1/(2+n)}$, might be $O(\text{TeV})$ for large enough $V_n$. As an example, for $n = 2$ extra-dimensions of millimeter-sized we get $M_\ast = 1 \text{ TeV}$. Surprisingly our knowledge of gravity in small scales is so limited that models with millimeter-sized extra dimensions are possible. Of course in such models one must be careful about all the particle physics phenomena that usually invoke high energy scales like gauge coupling unification, proton decay and neutrino masses (for a review of subject and original references see e.g. [9]). One might also worry how much of the techniques that we use in perturbative quantum field theory are valid for such low scales of quantum gravity.

Another scenario with a low cutoff is if Higgs is a bound state of new dynamics that become strong near EW scale. In these models Higgs must be made significantly lighter than other resonances of the strong sector as it has a narrow width and no other resonance has been observed so far. This can be done if Higgs appears as a Nambu-Goldstone boson of this strong sector. Little Higgs models are prime examples of model building in this direction (for a review see e.g. [10]).

- **Stabilizing the Higgs mass via symmetry.** Finally the most motivated and studied idea to stabilize the Higgs mass is supersymmetry. Studying supersymmetry as a solution to hierarchy problem has been the main motivation of this thesis and we will discuss it further in section 1.0.2.

**Dark Matter.** Overwhelming evidence from astrophysics suggests the remarkable fact that most of the matter in the Universe is not composed of baryons or any other of the known particles. The data constrain the energy density of baryonic matter, non-baryonic dark matter (DM), and dark energy to be respectively [11]

$$\Omega_B \simeq 0.0456 \pm 0.0016 \quad (1.4)$$
$$\Omega_{DM} \simeq 0.227 \pm 0.014 \quad (1.5)$$
$$\Omega_{\Lambda} \simeq 0.728 \pm 0.015 \quad (1.6)$$

However, besides their density, we know very little about the properties of DM particles.
We know that candidates of DM must be stable on cosmological time scales, have very small coupling to photons, and have the right relic abundance. These candidates include primordial black holes, axions, sterile neutrinos, and weakly interacting massive particles (WIMP).

WIMPs, as their name suggests, are particles with masses $\sim 10\text{ GeV} - 1\text{ TeV}$ that may interact through $SU(2)_{W}$ gauge group of SM but are neutral under electromagnetism and color. If a WIMP exists and is stable, it can explain the observed relic abundance of DM assuming it is a thermal relic from the early Universe. This tantalizing fact that is sometimes referred to as *WIMP miracle* is one of the important reasons to expect physics beyond SM at EW scale.

If WIMPs are produced thermally in the early Universe, evolution of their number density would be as shown in figure 1.4 (see [12] for a recent review). Initially WIMPs are in thermal equilibrium with other particles. As the Universe cools down and its temperature ($T$) falls below WIMP mass ($m_{X}$), WIMP number density ($n_{\text{eq}}$) follows Maxwell-Boltzman approximation:

$$n_{\text{eq}} \sim (m_{X}T)^{3/2}e^{-m_{X}/T}.$$  \hspace{1cm} (1.7)

If that was the whole story, $n_{\text{eq}}$ would be exponentially suppressed today and WIMPs could not explain DM density. However due to the expansion of the Universe, at some point DM particles do not find each other to annihilate efficiently and their number density in the co-moving coordinates become almost constant. This is called the “freeze-out” process. Following this procedure we get approximately

$$\Omega_{\text{DM}}h^{2} \approx \frac{3 \times 10^{-27}\text{cm}^{3}\text{s}^{-1}}{\langle\sigma v\rangle} = 2.6 \times 10^{-10}\text{GeV}^{-2}$$ \hspace{1cm} (1.8)

What is known as the WIMP miracle is the intriguing order-of-magnitude observation that if we take $\langle\sigma v\rangle \sim \frac{g^{4}}{16\pi^{2}m_{X}}$, we get the right relic abundance for $g \sim 0.5$ (close to $SU(2)_{W}$ gauge coupling) and $m_{X} \sim \text{TeV}$. We emphasize that there is no reason for DM to interact via EW forces or have its mass at EW scale, but considering the fact that we do expect physics at this scale for the hierarchy problem, it is exciting to imagine the same physics explains DM puzzle.
Figure 1.4: The co-moving number density $Y$ (left) and resulting thermal relic density (right) of a 100 GeV, P-wave annihilating dark matter particle as a function of temperature $T$ (bottom) and time $t$ (top). The solid contour is for an annihilation cross section that yields the correct relic density, and the shaded regions are for cross sections that differ by $10$, $10^2$, and $10^3$ from this value. The dashed contour is the number density of a particle that remains in thermal equilibrium. This plot is taken from [13].

1.0.2 Supersymmetry

As mentioned earlier, supersymmetry (SUSY) is probably the most motivated and well-studied solution to the hierarchy problem. Its theoretical discovery however precedes any discussion of its phenomenological applications. Supersymmetric algebras were first discovered in the seventies to be the only non-trivial extension of Poincaré algebra for a consistent relativistic quantum field theory (see [14] for a review and original references). Since then, it has been an important tool in our theoretical understanding of string theory and non-perturbative aspects of QFT. Whether or not SUSY plays any role in the phenomenology of EW scale however, remains to be decided in experiments. At the time of writing this thesis (spring of 2016), there has been no direct evidence for SUSY at EW scale. In particular Run I of LHC has accumulated $\sim 5 \text{ fb}^{-1}$ of integrated luminosity at 7 TeV and $\sim 20 \text{ fb}^{-1}$ of
integrated luminosity at 8 TeV proton proton collisions in ATLAS and CMS detectors and has already been able to explore territories of supersymmetric models at $O$(TeV) without any sign of physics beyond SM. These null results have constrained many of the minimal realizations of SUSY in EW scale. On the other hand Run II of LHC has just started and the most important part of LHC data has yet to be collected in the next couple of years. For a review of the status of SUSY at the end of Run I of LHC we refer the interested reader to [15].

Fortunately there are many excellent pedagogical reviews on the phenomenology of low scale SUSY (the classic review of [16] is one of the best examples) and I do not intend to repeat all the details here. However as all the work in this thesis is related to SUSY model building, in this chapter very briefly I discuss some of the motivations and challenges of low scale SUSY, particularly those related to the work in this thesis, to set the stage for the following chapters. To avoid repeating all the notation conventions here, we use the notation of [16].

The main motivation for SUSY is of course its ability to maintain a natural hierarchy between Planck scale and EW scale. In a supersymmetric world, Higgs and Higgsino (the supersymmetric partner of Higgs) would have the same mass. Since the Higgsino mass is protected by chiral symmetry of fermions, Higgs mass will be protected as well and the hierarchy problem is solved. Of course SUSY can not be an exact symmetry of nature as no supersymmetric partner has been observed (yet) for SM particles. However the idea is to break SUSY “softly”, such that in the UV the theory is approximately supersymmetric and the Higgs mass is shielded from masses at high energy scales. The necessary condition for soft SUSY breaking, is that all SUSY breaking parameters must have positive mass dimensions [17].

In addition to hierarchy problem, there are intriguing consequences of assuming SUSY at EW scale that motivates it further. One of these consequences is a much better unification of gauge couplings in the minimal supersymmetric extension of SM (MSSM). As mentioned before, gauge coupling unification in SM is motivated by the fact that the leptons and quarks fit nicely in representations of larger gauge groups such as $SU(5)$ or $SO(10)$. However evolving the gauge couplings by SM RGEs, we see that they do not cross at any scale. Using
Figure 1.5: Two-loop renormalization group evolution of the inverse gauge couplings $\alpha^{-1}(Q)$ in the Standard Model (dashed lines) and the MSSM (solid lines) taken from [16]. In the MSSM case, the sparticle masses are treated as a common threshold varied between 500 GeV and 1.5 TeV, and $\alpha_3(m_Z)$ is varied between 0.117 and 0.120.

MSSM RGEs however, there is a much better crossing of gauge couplings at $\sim 2 \times 10^{16}$ GeV as shown in figure 1.5. Gauge coupling unification does not make precise prediction of superpartner masses however, as it depends on them only logarithmically. Moreover, one can lift the scale of one whole family without deteriorating gauge coupling unification as each family is in a complete representation. It is still quite striking that assuming SUSY at EW scale for a completely different reason (hierarchy problem) can help gauge coupling unification.

To keep the nice features of SUSY, particularly its solution to the hierarchy problem, SUSY must be broken spontaneously. In fact, studying the possible patterns of SUSY breaking and the theoretical and experimental constraints on them has been the main focus of much of the work done in the field in the past couple of decades. We take the minimal supersymmetric SM (MSSM) as our starting point in all the discussion in this thesis. The superpotential of MSSM written in superspace language (refer to [16] for a pedagogical
review) is
\[
W_{\text{MSSM}} = \bar{u}_i y_{uij} Q_j H_u - \bar{d}_i y_{dij} Q_j H_d - \bar{e}_i y_{eij} L_j H_d + \mu H_u H_d
\] (1.9)

Note that terms like \(L_i L_j \bar{e}_k\), \(L_i Q_j \bar{d}_k\), \(\bar{u}_i \bar{d}_j \bar{d}_k\), and \(L H_u\) that violate baryon or lepton number are also allowed by the gauge symmetries of MSSM, but are not included here. The presence of such terms are not ruled out and have interesting consequences, but they are very constrained. The most obvious constraint comes from proton stability and flavor observables. Interestingly we can exclude all these terms by assuming a single discrete symmetry which is called R-parity (also known as matter parity) and is defined as:
\[
P_R = (-1)^{B-L+2s}. \tag{1.10}
\]

R-parity is also motivated for a different reason, namely to have a stable dark matter candidate. Since all SM particles are even under R-parity and their superpartners are all odd, after postulating R-parity conservation, the lightest supersymmetric particle (LSP) is stable and can be the dark matter! As with gauge coupling unification, we do not need SUSY to explain DM, but it is very economical that the same R-parity that helps to relax constraints from proton decay and flavor observables can help in explaining DM relic density.

From the low energy perspective, the SUSY breaking terms in the Lagrangian must all be dimensionful for SUSY to explain the hierarchy problem. In MSSM, soft SUSY breaking terms consistent with gauge symmetries are
\[
\mathcal{L}_{\text{soft}}^{\text{MSSM}} = -\frac{1}{2} \left( M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B} + c.c. \right)
- \left( \bar{u}_i a_{uij} \tilde{Q}_j H_u - \bar{d}_i a_{dij} \tilde{Q}_j H_d - \bar{e}_i a_{eij} \tilde{L}_j H_d \right)
- \bar{Q}_i m^2_{Qij} \tilde{Q} - \bar{u}_i m^2_{uij} \tilde{u} - \bar{d}_i m^2_{dij} \tilde{d} - \bar{L}_i m^2_{Lij} \tilde{L} - \bar{e}_i m^2_{eij} \tilde{e}
- m^2_{H_u} H_u^* H_u - m^2_{H_d} H_d^* H_d - (bH_u H_d + c.c.). \tag{1.11}
\]

An obvious puzzle is the flavor structure of the SUSY breaking parameters \(a_{ij}\) and \(m^2_{ij}\). In the Yukawa basis, these are generically matrices with non-zero off-diagonal terms. Through loops of SUSY particles, these off-diagonal terms have disastrously large contributions to flavor changing neutral currents, if the scale of SUSY breaking is near \(O(\text{TeV})\) scale. This is called the new physics flavor problem. One solution is to set the soft masses to \(O(100 \text{ TeV})\) where these loop effects are suppressed enough ([18, 19]). However that brings back the
hierarchy problem that we are trying to solve by supersymmetry. Another option is to assume “universality” condition for $a_{ij}, m^2_{ij}$:

$$m^2_{X_{ij}} = m^2_X \delta_{ij} \quad X = Q, \bar{u}, \bar{d}, L, \bar{e}$$

$$a_{X_{ij}} = A_X Y_{X_{ij}} \quad X = u, d, e.$$  (1.12)

In this way all the dangerous flavor changing effects are turned off and the scale of SUSY breaking can be below $O(\text{TeV})$. Interestingly, these universality conditions are automatically satisfied if the effects of SUSY breaking are communicated to MSSM via gauge interactions. This framework is called gauge mediated SUSY breaking (GMSB). The vanilla setup is to assume SUSY to be spontaneously broken in a different sector (usually called the hidden sector) and the effects communicated to MSSM via some fields (called gauge mediation messengers) which are charged under MSSM gauge group (for an excellent review and original references see [20]). Although we do not postulate GMSB in this thesis, most of the following sections are particularly useful when combined with GMSB.

1.0.3 Higgs mass in MSSM

Measurement of the Higgs mass is an important new constraint on any BSM model and MSSM is no exception. Although in MSSM there are more than $\sim 100$ unknown parameters, only a few of them are relevant for calculating the Higgs mass. The Higgs sector of MSSM is slightly more complicated than the Higgs sector in SM, as in MSSM we need to have two Higgs doublets, namely $H_u = (H_u^0, H_u^0)$ and $H_d = (H_d^0, H_d^-)$. These gauge eigenstates mix with each other to form the mass basis $(h^0, H^0, A^0, H^+)$, as well as Goldstones $(G^\pm, G^0)$. Here $(h^0, H^0, A^0, H^+)$ are the light CP-even, heavy CP-even, CP-odd, and charged Higgs.

The potential of the neutral Higgses in MSSM (above the scale of soft masses) is:

$$V = (|\mu|^2 + m^2_{H_u})|H_u^0|^2 + (|\mu|^2 + m^2_{H_d})|H_d^0|^2 - b(H_u^0 H_d^0 + c.c.)$$

$$+ \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2$$  (1.13)

Note that the quartic coupling of the Higgs comes from gauge couplings $(g^2 + g'^2)$. That is the reason of a famous feature that the lightest CP-even Higgs mass is bounded by $m_Z$ at tree level in MSSM

$$m_{h^0} \lesssim m_Z |\cos 2\beta|$$  (1.14)
where \( \tan \beta := \frac{H_u}{H_d} \). The equality is achieved when \( m_A \to \infty \) which is called the decoupling limit and is also motivated by null results from collider searches. We also have

\[
m^2_Z = \frac{|m^2_{H_d} - m^2_{H_u}|}{\sqrt{1 - \sin^2(2\beta)^2}} - m^2_{H_u} - m^2_{H_d} - 2|\mu|^2
\] (1.15)

There are some comments about (1.13) and (1.15) that deserve being mentioned here as they will be discussed in the following chapters. First, note that the natural scale for the mass terms on the RHS of (1.15) is \( m_Z \). This is puzzling especially because \( \mu \) is a parameter in the superpotential and a priori it is not clear why it should be at the same scale as the SUSY breaking soft masses, both being at EW scale. Second, we can estimate the amount of fine-tuning of the model by the cancellations needed in (1.15). Expanding in large \( \tan \beta \) which is motivated by (1.14), \( m^2_Z \) is simplified to

\[
m^2_Z = -2(m^2_{H_u} + |\mu|^2) + \frac{2}{\tan^2 \beta} (m^2_{H_d} - m^2_{H_u}) + O(1/\tan^4 \beta)
\] (1.16)

Therefore unless \( m^2_{H_u} \) and \( \mu \) are generated by the same underlying dynamics, usually the fine-tuning of the model is dominated by the amount of cancellation between them to get the right \( m_Z \). Finally note that (1.13) is the potential above the scale of SUSY breaking in MSSM and needs to be adjusted for the threshold corrections and running below the scale of superparticle masses. The fact that these corrections can be substantial is fortunate of course because if (1.14) was the whole story, MSSM had already been ruled out!

In MSSM, after including radiative corrections we have

\[
m^2_h \lesssim m^2_Z \cos^2 2\beta + \frac{3m^2_{\tilde{t}}}{4\pi^2 v^2} \left( \log \left( \frac{M^2_S}{m^2_{\tilde{t}}} \right) + \frac{X^2_t}{M^2_S} \left( 1 - \frac{X^2_t}{12M^2_S} \right) \right)
\] (1.17)

where \( M^2_S = m_{\tilde{t}_1} m_{\tilde{t}_2} \) and \( X_t = A_t - \mu \cot \beta \). In large \( \tan \beta \) limit, this means that we can achieve the Higgs mass of 125 GeV either by taking \( 1 \text{TeV} \ll M_S \) or by getting close to “maximal mixing” condition which is when \( A_t \sim \sqrt{6} M_S \) in which case stops \( \sim \text{TeV} \) are enough to get the right Higgs mass.

Having heavy stops however, is disastrous for the fine-tuning of MSSM. In order to see the reason, we need to consider the RGE of \( m^2_{H_u} \)

\[
\frac{d}{dt} m^2_{H_u} = \frac{3}{8\pi^2} \left( |y_t|^2 (m^2_{H_u} + m^2_{Q_3} + m^2_{u_3} + A_t^2) - g_2^2 |M_2|^2 - \frac{1}{5} g_1^2 |M_1|^2 + \frac{1}{10} g_1^2 \text{tr}(Y^2_i m^2_{\phi_i}) \right)
\] (1.18)
where $Y_i$ are the hypercharge and $m^{2}_{\phi_i}$ are the soft mass-squared of all the fields in MSSM.

In complete models, usually $m^2_{H_u}$ is positive in the UV but is driven negative through RG running which makes (1.16) possible. But too large stop masses ($m^2_{\tilde{Q}_3}, m^2_{\tilde{u}_3} \sim (10 \text{ TeV})^2$) makes $|m^2_{H_u}|$ far from EW scale and the fine-tuning necessary in getting the right $m_Z$ in (1.16) would be unnatural. Therefore it is favorable for the tuning purposes to have large $A$-terms to raise the Higgs mass in MSSM or look for sources of raising the Higgs mass outside of MSSM.

The outline of this thesis is as follows: in chapter 2, we build a mechanism to generate large enough $A$-terms in MSSM to achieve maximal mixing and get the Higgs mass with percent level tuning. In combination with GMSB, we will analyze complete models and their phenomenological implications. In chapter 3, we take a different approach and add a minimal extension to MSSM with a vector-like pair of doublets and a singlet. We will show that with this new sector, we can explain the Higgs mass and dark matter relic abundance in a natural way. We check all the constraints from spin dependent and spin independent direct detection experiments as well as LHC and precision measurements. We will discuss the discovery prospects of this model in future direct detection experiments. Finally in chapter 4 we discuss the effect of D-term SUSY breaking in direct gauge mediation. In particular we will show that the usual pseudo-flat direction present in O’Raifeartaigh models is absent when we break SUSY through a combination of F-term and D-term mechanism. As the vacuum structure of these models are different from generalized O’Raifeartaigh models, we expect the light gaugino problem is resolved in this way.
Chapter 2

125 GeV Higgs from Tree-level $A$-terms

With D. Egana-Ugrinovic, S. Knapen and D. Shih


General context of this chapter

As discussed in the Introduction, generating substantial $A$-terms are motivated in MSSM after the observation of Higgs mass at 125 GeV. We present a new mechanism to generate large $A$-terms at tree-level in the MSSM through the use of superpotential operators. The mechanism trivially resolves the $A/m^2$ problem which plagues models with conventional, loop-induced $A$-terms. We study both MFV and non-MFV models; in the former, naturalness motivates us to construct a UV completion using Seiberg duality. Finally, we study the phenomenology of these models when they are coupled to minimal gauge mediation. We find that after imposing the Higgs mass constraint, they are largely out of reach of LHC Run I, but they will be probed at Run II. Their fine tuning is basically the minimum possible in the MSSM.

2.1 Introduction

The discovery of the Higgs boson with a mass near 125 GeV [1, 21] has important consequences for physics beyond the Standard Model, especially supersymmetry. In the MSSM, it implies that the stops must either be very heavy or have a large trilinear coupling (“$A$-term”) with the Higgs [22–30]. The large $A$-term scenario is more interesting from several points of view. It is less fine-tuned and it allows for lighter ($\sim 1$ TeV) stops that are still within reach of the LHC. It also presents an interesting model-building challenge – prior to the discovery of the Higgs, mechanisms for generating the $A$-terms from an underlying model of SUSY-breaking mediation were not well-explored.
In the framework of gauge mediated SUSY-breaking (GMSB) (for a review and original references, see [20]), the problem of how to obtain large $A$-terms becomes especially acute. In GMSB, the $A$-terms are always negligibly small at the messenger scale. If the messenger scale is sufficiently high and the gluino sufficiently heavy, a sizable weak scale $A$-term with relatively light stops may be generated through RG-running [26]. However, this setup is in strong tension with electroweak symmetry breaking (EWSB) [31]. This strongly motivates extending gauge mediation with additional MSSM-messenger couplings that generate $A$-terms through threshold corrections at the messenger scale.

In all models for $A$-terms considered since the observation of a Higgs boson at 125 GeV [32–45], the focus has been on generating $A$-terms at one-loop level through weakly coupled messengers. Integrating out the messengers produces one or more of the following Kähler operators

$$\frac{1}{16\pi^2} \frac{1}{M} X^\dagger H_u H_u, \quad \frac{1}{16\pi^2} \frac{1}{M} X^\dagger Q_3^\dagger Q_3, \quad \frac{1}{16\pi^2} \frac{1}{M} X^\dagger \tilde{\pi}_3^\dagger \tilde{\pi}_3$$

(2.1)

Here $X$ is a field that spontaneously breaks SUSY, and $M$ is the messenger scale. After substituting $\langle X \rangle = \theta^2 F_X$ and integrating out the auxiliary components of the MSSM fields, one obtains the desired $A$-term

$$\mathcal{L} \ni y_t A_t H_u Q_3 \tilde{u}_3, \quad A_t \sim \frac{1}{16\pi^2} \frac{F_X}{M}$$

(2.2)

This setup has the advantage that the $A$-terms come out parametrically the same size as the other soft masses in GMSB (one-loop gaugino masses, two-loop scalar mass-squareds). However, one-loop $A$-terms from (2.1) introduce a host of complications as well. First and foremost is the “$A/m^2$ problem” [33]: in addition to the $A$-terms, one also generates a scalar mass-squared at one-loop, completely analogous with the more well-known $\mu/B_\mu$ problem. A one-loop scalar mass-squared would overwhelm the GMSB contributions and lead to serious problems with fine-tuning and/or EWSB. Previous solutions to the $A/m^2$ problem include taking the messengers to be those of minimal gauge mediation [33], or having the hidden sector be a strongly-coupled SCFT [35, 36].

In this paper, we will explore a new solution to the $A/m^2$ problem: models where the $A$-terms are generated at tree-level in the MSSM-messenger couplings. The advantage with this approach is that there is simply no $A/m^2$ problem to begin with, since at worst any
accompanying sfermion mass-squareds would be tree-level as well. An added benefit of this approach is that it will lead us to consider an interesting new operator for the $A$-terms: one which arises in the effective superpotential, rather than in the Kähler potential. As we will see, this superpotential operator will have qualitatively different effects on the MSSM soft terms as compared to Kähler potential operators.

The basic setup is quite simple. To generate a tree-level $A$-term, either the Higgs or stops must mix with the messengers in the mass-matrix. For example, consider the superpotential

$$ W = X' H_u \tilde{\phi} + \lambda^{ij}_u \phi Q_i \bar{u}_j + M \tilde{\phi} \phi $$

Here $X'$ is another spurion for SUSY-breaking, and $\phi, \tilde{\phi}$ are heavy messenger fields. Upon integrating out the messengers at the scale $M$, one generates the effective superpotential operator

$$ W_{\text{eff}} \supset -\frac{\lambda^{ij}_u}{M} X' H_u Q_i \bar{u}_j $$

Note that because of the SUSY non-renormalization theorem, $W_{\text{eff}}$ can only arise at tree-level, so it is perfectly suited for our purposes. In order to produce an $A$-term of the correct size, one must have

$$ \frac{F_{X'}}{M} \sim O(\text{TeV}) $$

The tree-level $A$-term originating from (2.4) is minimally flavor violating (MFV), provided that the operator in (2.4) generates the full up-type Yukawa coupling of the MSSM. For this to work, $X'$ should acquire a lowest component vev of size $\sim M$.

The interesting complication in these models comes from the fact that when integrating out the messengers, in addition to the superpotential operator (2.4), a Kähler potential operator is also generated at tree-level. For example, in the model (2.3), one generates the term:

$$ K_{\text{eff}} \supset \frac{1}{M^2} X'^* X' H_u^\dagger H_u $$

(For a more general treatment of the Kähler operators, see appendix A.) This leads to a

---

1Note that this is a loop factor smaller than the usual GMSB relation. A smaller $F$-term satisfying this hierarchy can easily be dynamically generated using weakly-coupled messengers, see e.g. [46]. In this paper we will simply assume that $F_{X'}$ of the right size can be obtained somehow and not explore it any further.
soft mass for $H_u$ of roughly the same order as the $A$-term:

$$\delta m_{H_u}^2 = \frac{-y_t^2}{|\lambda_{33}|^2} |A_t|^2$$  \hspace{1cm} (2.7)

For $\lambda_{33} \lesssim 1$, this represents a large, irreducible contribution to $m_{H_u}^2$, and correspondingly to the fine-tuning of the electroweak scale. This is another manifestation of the “little $A/m^2$ problem” encountered in [33], whereby a large $A$-term was accompanied by an equally large sfermion mass-squared. In [33], the situation was even worse, because the contribution was irreducible with a fixed coefficient:

$$\delta m_{H_u}^2 = |A_t|^2$$  \hspace{1cm} (2.8)

There both the $A$-terms and the irreducible contribution to $m_{H_u}^2$ (2.8) originated from integrating out the auxiliary components of the MSSM fields in the first Kähler operator in (2.1). Since we are starting instead with the effective superpotential operator (2.4), the coefficient in (2.7) is free to vary in our present models. Importantly, however, we will see that the sign in (2.7) is always negative, such that (2.7) does not jeopardize electroweak symmetry breaking, in contrast to the relation in (2.8).

In this paper, we will consider various ways to alleviate the fine-tuning problem introduced by the little $A/m^2$ problem (2.7). Clearly, if $\lambda_{33}$ is taken to be large (e.g. $\lambda_{33} \sim 3$), then the little $A/m^2$ problem is ameliorated. This requires a UV completion at a relatively low scale. We will provide such a UV completion in this paper, using a novel application of Seiberg duality [47, 48].

Alternatively, one can consider non-MFV models obtained from (2.3) by exchanging the role played by $H_u$ with $\bar{u}_3$: \footnote{Because these models are not MFV, one should worry about the potential constraints from precision flavor and CP observables. This is beyond the scope of this work (see however [49]). We will assume for simplicity (as in [34]) that the coupling $\kappa$ is real and fully aligned with the third generation. We will also focus on the $\bar{u}_3$ model because then the flavor violation is limited to the up-squark sector and the constraints are much weaker.}

$$W = X'\bar{u}_3\tilde{\phi}_u + \kappa H_uQ_3\phi_u + M\tilde{\phi}_u\phi_u$$  \hspace{1cm} (2.9)

For this model the expression analogous to (2.7) contains $m_{\bar{u}_3}^2$ instead of $m_{H_u}^2$. As in [34], the fine-tuning is greatly reduced with respect to the perturbative MFV case because the
stop contribution to $m_{H_u}^2$ is diluted by a loop factor. Moreover, the situation is even better than in [34], because in that case there were still sizeable two-loop contributions to $m_{H_u}^2$, whereas here the contribution is solely to the squarks.

An important thing to note about the framework for generating tree-level $A$-terms presented in this paper is that it can in principle be tacked on to any mediation mechanism for the rest of the MSSM soft terms; the framework itself does not lead to a particularly compelling choice. This is in contrast to the one-loop models considered previously, whereby the $A$-term messengers also contributed to the MSSM soft spectrum through minimal gauge mediation, and thus GMSB was the most economical choice. Moreover, the tree-level $A$-term module does not affect the overall phenomenology much; the one essential difference occurs in the non-MFV models, where the stops can be split by several TeV due to the non-MFV analogue of (2.8).

For simplicity and concreteness, in this paper we will couple our models to minimal gauge mediation (MGM) [50–52]. We will see that after imposing the Higgs mass constraint, the models are typically out of reach of Run I LHC; however they will be accessible (especially the lightest stop) at 14 TeV LHC. Finally, we will estimate the fine tuning in these models and show that they achieve essentially the best tuning possible in the MSSM (percent level).

The remainder of this paper is organized as follows: Since no strongly coupled UV completion is needed for the non-MFV models, we discuss those first in section 2, as well as their phenomenology when coupled to minimal gauge mediation. In section 3 we analyze the MFV example in a similar way. In section 4, we UV complete the MFV model using Seiberg duality. Finally, in the conclusions we list some potential future directions suggested by our work. A general discussion of the little $A/m^2$ problem and Landau poles in models for tree-level $A$-terms is left for appendix 2.6.

### 2.2 A non-MFV model

As discussed in the introduction, the non-MFV model (2.9) has a less severe version of the little $A/m^2$ problem, and thus does not need an immediate UV completion, unlike the MFV model (2.3). Since the story is simpler here, let us start by analyzing the non-MFV model in detail. Apart from the issues of flavor alignment discussed in the introduction, the
form of the renormalizable superpotential (2.9) is the most general that couples the spurion, messengers and MSSM fields up to terms that are irrelevant for our purposes (powers of the spurion $X'$ and a small soft mass for the messenger pair from $X'\phi_u\tilde{\phi}_u$).

After diagonalizing the mass matrix and integrating out $\phi_u,\tilde{\phi}_u$ at the messenger scale $M$, we obtain the IR effective theory

$$W_{\text{eff}} \supset -\kappa \frac{X'}{M} H_u Q \bar{u}_3$$

$$K_{\text{eff}} \supset \frac{X'^\dagger X'}{M^2} \bar{u}_3^\dagger \bar{u}_3 + \frac{\kappa^2}{M^2} H_u^\dagger H_u Q_3^\dagger Q_3$$

(2.10)

The irrelevant operator induced in the low energy superpotential leads to an $A$-term for the corresponding MSSM fields after substituting $\langle X' \rangle = \theta^2 F_{X'}$. However, an additional contribution to $m_{\bar{u}_3}^2$ from the first term in the Kähler potential is also induced, such that

$$\delta m_{\bar{u}_3}^2 = \frac{y_t^2}{\kappa^2} A_t^2$$

(2.11)

Note that the contribution to $m_{\bar{u}_3}^2$ is negative, so to avoid a tachyonic right handed stop, it must be cancelled off by additional contributions at the messenger scale (e.g. from GMSB) or from MSSM renormalization group running from the messenger scale down to the weak scale. If $\kappa \sim 1$, the fine tuning from (2.11) is comparable to the fine tuning from the $A$-term itself, since both enter the running of $m_{H_u}^2$ in exactly the same fashion. Taking $\kappa > 1$ therefore does not substantially improve the overall fine tuning of the model. One major improvement relative to the non-MFV models considered in [34] is that there are no sizeable contributions generated to $m_{H_u}^2$ from integrating out the messengers.

To study the phenomenology of a model with tree-level $A$-terms and a 125 GeV Higgs, we must add our tree-level $A$-term module (2.9) to an underlying model for the rest of the MSSM soft masses. While in principle any model could be used, GMSB is a particularly well-motivated choice given the SUSY flavor problem. So for simplicity and concreteness, let us now specialize to the case of minimal gauge mediation (MGM) with $5 \oplus \bar{5}$ messengers [50–52].

The parameter space of our model is as follows. The MGM sector of the model is characterized by four parameters: messenger index $N_m$, $\tan \beta$, messenger scale $M$ and SUSY-breaking mass scale $F_X$, where $F_X$ is the highest component vev of the SUSY breaking spurion. We take the masses of the additional messengers in (2.9) to be the same scale $M$. 
Figure 2.1: Contours of the Higgs mass (black), geometric mean of the stop masses (blue) and tuning (dashed), in the \((A_t, m_{\tilde{t}_1})\) (left) and \((A_t, m_{\tilde{\tau}_1})\) (right) planes. The shaded region on the \((A_t, m_{\tilde{\tau}_1})\) plane corresponds to points with tachyonic stops. The black dot on both figures corresponds to the same point in parameter space, with a spectrum presented in figure 2.2. All quantities are evaluated at \(M_{SUSY}\).

for simplicity. We consider \(\mu\) and \(B_\mu\) to be determined by the EWSB conditions and we remain agnostic about their origin. Finally, our model contains additional parameters \(\frac{F_X'}{M}\), which sets the scale for the tree level contribution to \(A_t\), and the coupling \(\kappa\) (see (2.9)).

A low messenger scale \(M = 250\,\text{TeV}\) and a large messenger number \(N_{m} = 3\) are motivated by the simultaneous requirements of reducing the tuning from the RG while allowing a large enough SUSY scale to be achieved for the Higgs mass. (A different choice of messenger number does not alter the phenomenology heavily, for reasons that will be explained later.) We take \(\tan \beta = 20\) to saturate the tree level bound of the Higgs mass and \(\kappa = 1\) for simplicity and perturbativity. With these choices, the parameter space of our models reduces to \(\left(\frac{F_X'}{M}, \frac{F_X}{M}\right)\). (Recall that we must take \(\frac{F_X'}{M} \sim \frac{1}{16\pi^2} \frac{F_X}{M}\) to achieve \(A\)-terms comparable to the GMSB soft masses.) To make contact with the IR observables, we can trade \(\left(\frac{F_X'}{M}, \frac{F_X}{M}\right)\) by the IR values of \(A_t\) and the mass of the lightest stop \(m_{\tilde{t}_1}\) or the mass of the lightest stau \(m_{\tilde{\tau}_1}\). This parametrization is especially relevant for the LHC phenomenology, since \(\tilde{t}_1\) and \(\tilde{\tau}_1\) are the lightest colored particle and the NLSP respectively, as will be seen shortly.

To generate the IR spectrum we use SOFTSUSY 3.5.1 [53]. Fine tuning \(\Delta_{FT}\) is calculated
according to the measure introduced in [34], given by
\[
\Delta_i \equiv \frac{\partial \log m_{\tilde{t}_1}^2}{\partial \log \Lambda_i} \quad \Lambda_i \in \{g_1^2 \frac{F_X}{M}, g_2^2 \frac{F_X}{M}, g_3^2 \frac{F_X}{M}, \frac{F_{X'}}{M}, \kappa \frac{F_{X'}}{M}, \mu\}
\]
(2.12)
\[
\Delta_{FT} \equiv \max \Delta_i.
\]

The results are presented in figure 2.1 where we show contours of the Higgs mass, tuning and \(M_{SUSY}\), both in the \((A_t, m_{\tilde{t}_1})\) and \((A_t, m_{\tilde{\tau}_1})\) planes. Note that \(M_{SUSY}\) is significantly larger than \(m_{\tilde{t}_1}\). This is because the two stop soft masses are split due to the negative contribution to \(m_{\tilde{u}_3}^2\) in (2.11). In the gray shaded region the GMSB contribution is insufficient to cancel this negative contribution, and the spectrum is invalidated by a stop tachyon. The main source of tuning in this model is the running effect due to the colored spectrum or the \(A\)-term. From the Higgs and tuning contour lines in both figures, we see that the model is able to reproduce the Higgs mass, while keeping fine tuning to the percent level (which is basically the best that can be achieved in the MSSM). Moreover, the Higgs mass can be reproduced in interesting parts of parameter space, where there is both a light colored particle \(m_{\tilde{t}_1}\) and a light slepton \(m_{\tilde{\tau}_1}\).

A typical spectrum for the model is presented in figure 2.2, which corresponds to the black dot indicated in the two different planes presented in figure 2.1.\(^3\) In general, the spectrum across the parameter space of our model is basically that of MGM with \(N_{mess} = 3\) (gaugino unification, colored sparticles heavier than electroweak sparticles, right-handed stau NLSP, etc.). There are, however, two key differences. First, in order to counteract the large negative contribution (2.11) to the right-handed stop, the MGM scale \(\frac{F_X}{M}\) is considerably larger than would otherwise be the case. This results in the other colored sparticles being essentially decoupled. It also results in a higher gravitino mass, which explains [54] why slepton co-NLSPs do not occur in figure 2.1. Second, the right-handed sleptons are a bit lighter than in MGM due to the effects of running induced by the split stops. Amusingly, this effect of running means that the stau is the NLSP even for lower \(N_{mess}\), unlike in MGM, where lowering \(N_{mess}\) leads to bino NLSP.\(^4\)

\(^3\)We choose our benchmark point here and in the next subsection to have \(m_h = 124\) GeV in order to account optimistically for the theory uncertainty on the Higgs mass calculation.

\(^4\)Note that if we exchange the roles of \(\tilde{u}_3\) and \(Q_3\) in (2.9), a negative soft mass for \(Q_3\) would be induced instead, leading to a heavier \(\tilde{\tau}_1\) through running. In this case, it could be possible to have a bino NLSP even for \(N_m > 1\).
Figure 2.2: Spectrum for the point marked with the dot in figure 2.1. The Higgs mass is \( m_h = 124 \text{ GeV} \), with \( A_t = -2.9 \text{ TeV} \). \( \tilde{\tau}_1 \) is 17 GeV lighter than the right handed sleptons. The Higgsino mass is \( \mu = 1.05 \text{ TeV} \). Fine tuning is \( \sim 1/400 \).

Due to the split spectrum, the largest sparticle pair production cross sections at LHC correspond to \( \tilde{t}_1 \) and the right-handed sleptons. Pair production of stops leads to a decay chain with jets, leptons and missing energy. When right handed sleptons are directly pair produced, the decay chain will include relatively soft leptons (due to the moderate splitting of the right handed sleptons and the stau), taus and missing energy. Of course the direct pair production of staus will lead to taus and missing energy.

Of the above signatures, the most spectacular one is given by the decay of pair produced stops, which can contain two jets, 4 leptons (from the decay of the bino to RH sleptons and RH sleptons to stau), and two \( \tau \) jets plus missing energy. A search with a similar topology was carried out in [55], where a limit on the total strong production cross section of \( \sim 1 \text{ fb} \) was obtained. This limit can be used to set an approximate bound on our parameter space, by comparing with our model’s tree level total strong production cross section, which we obtain using MadGraph [56]. This leads to excluding stops roughly below 800 GeV in the parameter space presented in figure 2.1, which corresponds to staus heavier than 150 GeV.

The spectrum presented in figure 2.2 is inaccessible to the LHC run at 8 TeV, but it
Table 2.1: Charge assignments securing (2.13) and (2.14).

<table>
<thead>
<tr>
<th>$Z_3$</th>
<th>$X'$</th>
<th>$Q, \bar{u}, d, L, \bar{e}$</th>
<th>$H_u$</th>
<th>$H_d$</th>
<th>$\phi$</th>
<th>$\bar{\phi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/3</td>
<td>2/3</td>
<td>1/3</td>
<td>2/3</td>
<td>2/3</td>
<td>1/3</td>
<td></td>
</tr>
</tbody>
</table>

will become accessible at 14 TeV. The total SUSY cross section of such point at the 14 TeV LHC is 8 fb, while the total tree level colored production cross section is 2 fb. Relevant searches will be the updated versions of multilepton or GMSB-inspired searches as [57] and [55].

### 2.3 An MFV model

Next we will turn to the MFV model (2.3). Apart from the issues of UV completions to be discussed in the next section, this model is slightly more complicated than the non-MFV model because here we would like to generate the MSSM up-type Yukawas and the $A$-terms from the same operator. To achieve this, it is necessary to turn on a lowest component vev for $X'$, which implies that one must re-diagonalize the messenger mass matrix prior to integrating out the messengers. For later convenience, we will redefine $X'$ so that its lowest component vev is separated out and denoted by $X'_{0}$. Then (2.3) becomes

$$ W = (X'_{0} + X')H_u\tilde{\phi} + \lambda_{u}^{ij}\phi Q_i\bar{u}_j + M\phi\bar{\phi} \quad (2.13) $$

with $\langle X' \rangle = F_{X'}\theta^2$. The form of (2.13) is the most general allowed by a $Z_3$ symmetry, as detailed in table 2.1, which also allows for a $\mu$-term and down type Yukawas,

$$ \delta W = \mu' H_u H_d + \lambda_d^{ij} H_d Q_i d_j + \lambda_e^{ij} H_d L_i \bar{e}_j \quad (2.14) $$

We will not discuss the down sector Yukawas any further.

After diagonalizing the mass matrix and integrating out the heavy messenger states, we are left with the supersymmetric effective action:

$$ W_{eff} \supset y_u^{ij} \left( 1 + \cot \theta_H \cos \theta_H \frac{X'}{M'} \right) H_u Q_i \bar{u}_j + \mu \left( 1 + \sin \theta_H \frac{X'}{M'} \right) H_u H_d $$

$$ K_{eff} \supset \frac{\cos^2 \theta_H X' X'H_u^2 H_u + \cot^2 \theta_H y_u^{ij} y_u^{jk} Q_i^\dagger u_j^\dagger Q_j u_k}{M'^2} \quad (2.15) $$
where

\[ M' = \sqrt{X_0'^2 + M^2}, \quad \sin \theta_H = \frac{X_0'}{M}, \quad y_u^i = -\lambda_u^i \sin \theta_H, \quad \mu = \mu' \cos \theta_H \quad (2.16) \]

and we have everywhere expanded in \( \mu' \ll M, X_0 \), keeping only the lowest nonzero order. In (2.15), the first term in the effective superpotential leads to an \( A \)-term proportional to the up-type Yukawas. The second term in the effective Kähler potential is an MFV interaction suppressed by the messenger scale, so it is safe from flavor constraints [58]. Meanwhile, the first term in \( K_{\text{eff}} \) represents a contribution to the soft mass of \( H_u \):

\[ \delta m_{H_u}^2 = -|A_t|^2 \tan^2 \theta_H \quad (2.17) \]

This is a manifestation of the little \( A/m_H^2 \) problem. Note that this contribution is negative, so it is not dangerous for electroweak symmetry breaking, unlike what was found in the Kähler potential models [33]. However, if \( \tan \theta_H \gtrsim 1 \) it still represents a major contribution to fine-tuning. Taking \( \tan \theta_H \ll 1 \) would alleviate this fine-tuning problem, but at the cost of enlarging the underlying coupling \( \lambda_u^{33} \) according to (2.16). This leads to a Landau pole at low scales and a UV completion becomes necessary. Such a UV completion is the subject of section 2.4, in which we use Seiberg duality [47, 48] to realize the large coupling \( \lambda_u^{33} \).

As in the previous section, to generate the rest of the soft masses we specialize to the case of MGM. The parameter space is essentially the same as before, namely the MGM sector is described by \( N_m, \tan \beta, M \) and \( \frac{F_X'}{M'} \), while our effective theory contains \( \frac{F_X'}{M'} \) which sets the scale for the tree level contribution to \( A_t \), and a coupling \( \lambda_u^{33} \). Again, we consider \( \mu \) and \( B_\mu \) to be determined by the EWSB conditions. We fix most of the parameters to the same values as before - \( N_m = 3, \tan \beta = 20 \) and \( M = 250 \) TeV - for essentially the same reasons. Finally, we consider two values for \( \lambda_u^{33} \): \( \lambda_u^{33} = 1 \) is chosen to illustrate the perturbative case, while \( \lambda_u^{33} = 3 \) is studied since it has a beneficial effect on decreasing tuning. With these choices, the parameter space of our model reduces to \((\frac{F_X'}{M'}, \frac{F_X}{M'})\), which we can trade for the IR values of the \( A \)-term \( A_t \) and the gluino mass \( M_3 \).

\[ ^5\text{The second term in the effective superpotential (2.15) gives rise to } B_\mu = \mu A_t \tan^2 \theta_H \text{ at the messenger scale. While this is parametrically of the right size for EWSB, it has the incorrect sign to lead to the large tan } \beta \text{ EWSB condition } B_\mu \approx 0 \text{ at the weak scale. Thus a more complete model that also aspires to explain the origin of } \mu \text{ and } B_\mu \text{ must include additional contributions to these parameters.} \]
Figure 2.3: $\lambda_{33}^u = 1$ (left) and $\lambda_{33}^u = 3$ (right). Contours of the Higgs mass (black), geometric mean of the stop masses (blue) and tuning (dashed), for two choices of $\lambda_{33}^u$ with $N_m = 3$, $\tan \beta = 20$, $M = 250$ TeV. Different Higgs mass contours are presented to account for the uncertainty in the theoretical Higgs mass calculation. The shaded region corresponds to tachyonic stops/staus. The dot on the figure on the right corresponds to the point in parameter space with the spectrum presented in figure 2.4. The parameter space below the red line on the same figure is excluded by [55]. All quantities are evaluated at $M_{SUSY}$.

In figure 2.3 we show contours of the Higgs mass, tuning and $M_{SUSY}$ in the $(M_{\tilde{g}}, A_t)$ plane for the two choices of $\lambda_{33}^u$. In both figures 2.3 a large Higgs mass can be achieved with moderate values of $M_{SUSY}$ thanks to the large $A$-terms. In figure 2.3 (left) however, the $\mu$-term is very large and induces sizable negative contributions to $m_h$ through the stau and sbottom sectors. This implies that a higher $M_{SUSY}$ is needed to obtain the correct Higgs mass. (see e.g. [59].) The main source of tuning can be either the large induced Higgs soft mass from (2.17) or, for large $M_{SUSY}$, the running effect. We immediately see from figure 2.3 (left) that the first of these sources represents a serious tuning problem for $\lambda_{33}^u = 1$, in which case for a 125 GeV Higgs we obtain a typical tuning of $\sim 10^{-4}$. In figure 2.3(right) we see the beneficial effect of considering a larger value for $\lambda_{33}^u$. This choice suppresses the fine tuning induced by (2.17), in such a way that a 125 GeV Higgs can be achieved while keeping tuning to the one part in $\sim 500$ level.

In figure 2.4 we present a typical spectrum for the model with $\lambda_{33}^u = 3$, which corresponds to the black dot in figure 2.3(right). This model is even more similar to MGM with stau NLSP than the one presented in the previous subsection, since there is no negative contribution to the right-handed stop to counteract. The only difference now with MGM is the large $A$-term, which has a minor effect on the rest of the spectrum primarily through
Figure 2.4: Spectrum for the point shown in figure 2.3. The Higgs mass is $m_h = 124$ GeV, with $A_t = -2.7$ TeV. $\tilde{\tau}_1$ is 32 GeV lighter than the right handed sleptons. The Higgsino mass is $\mu = 1.3$ TeV. Fine tuning is $\sim 1/400$.

the RG. The MGM collider signatures here are potentially spectacular. If colored superpartners are accessible to collider experiments they will lead to a long decay chain including jets, leptons and missing energy. As in our non-MFV model, searches that look for jets, tau final states and large missing energy can be sensitive to this spectrum when the strong production is accessible. In particular ATLAS search [55] analyses a similar spectrum and their results apply directly to our case, setting strong bounds on parts of the parameter space. For $\tan \beta = 20$, gluinos of up to 1.6 TeV are excluded, which corresponds to a total strong production cross section of $\sim 1.5$ fb at tree level [56].

Multilepton searches could also be a leading probe of this model, especially when the colored sparticles are too heavy to be produced. The stau NLSP scenario considered in [57] can be sensitive to our case, but since in our spectrum $\tilde{m}_{e_R} - \tilde{m}_{\tau_1} \sim 20$ GeV and $150$ GeV $< \tilde{m}_{\tau_1}$, the obtained bounds are not currently relevant for us. However, updates of these searches in Run II of the LHC can be very interesting for our models.
2.4 A composite model from Seiberg duality

As discussed in the previous section, the little $A/m_H^2$ problem in the MFV model (2.17) necessitates a large value for $\lambda_{u}^{33}$, and the theory has a Landau pole at a low scale. One way to explain physics above the Landau pole is to build composite models that naturally provide $|\lambda_{u}^{33}| \gg 1$ due to the underlying strong interactions. In general, characterizing such a strongly coupled UV completion is challenging at best, however in the context of supersymmetric gauge theories we can make use of Seiberg duality [47, 48]. We embed the model of section 2.3 in the magnetic side of the duality, where the fields $Q_3, \bar{q}_3$ and $\phi$ will be composite degrees of freedom. Since it is conceptually simpler, we first discuss the electric side of the duality. In a second stage we discuss the mapping to the composite degrees of freedom on the magnetic side, and we complete the model by adding in a number of spectator fields.

2.4.1 Electric theory

The electric theory is defined by SQCD with $N_c = 2$ colors and $N_f = 3$ flavors. Since the fundamental of the electric gauge group $SU(2)_E$ is pseudo-real, this theory is invariant under an $SU(6)$ global symmetry. It is therefore convenient to parametrize its degrees of freedom with a single matter field $q_{a}^i$ in the fundamental of $SU(2)_E$ and $SU(6)$. The standard model gauge group can be embedded in the global symmetry as follows

$$SU(6) \supset SU(5) \supset SU(3)_c \times SU(2)_L \times U(1)_Y$$

(2.18)

With this matter content, the global symmetry is anomalous. In section 2.4.3 we will introduce some spectator fields to cancel the gauge anomalies and give vector-like masses to some exotics. Note that because the global symmetry contains $SU(5)$, grand unification is manifest in this model from the outset. Concretely, the fundamental of $SU(6)$ trivially decomposes as

$$6 = 5 \oplus 1$$

(2.19)

where the 5 further decomposes into standard model representations in the conventional way. The quantum numbers of $q_{a}^i$ are summarized in table 2.2.
Table 2.2: Matter content of the electric theory. \( q = q_c \oplus q_L \oplus q_S \) form a fundamental of the \( SU(6) \) global symmetry.

<table>
<thead>
<tr>
<th>GUT field</th>
<th>( SU(2)_E )</th>
<th>( SU(3)_c )</th>
<th>( SU(2)_L )</th>
<th>( U(1)_Y )</th>
<th>( Z_3 )</th>
<th>( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>( q_c )</td>
<td>( 1 )</td>
<td>( 1 )</td>
<td>(-\frac{1}{3})</td>
<td>( \frac{1}{3} )</td>
<td>(-\frac{1}{3})</td>
</tr>
<tr>
<td></td>
<td>( q_L )</td>
<td>( 1 )</td>
<td>( 1 )</td>
<td>( \frac{1}{3} )</td>
<td>( -\frac{1}{3} )</td>
<td>( -\frac{1}{3} )</td>
</tr>
<tr>
<td>1</td>
<td>( q_S )</td>
<td>( 1 )</td>
<td>( 1 )</td>
<td>( 0 )</td>
<td>( 0 )</td>
<td>( 0 )</td>
</tr>
</tbody>
</table>

Table 2.3: Matter content of the magnetic side of the duality. All fields fill out complete GUT multiplets. Since \( E' \) carries baryon number, it cannot be identified with a right handed lepton.

<table>
<thead>
<tr>
<th>GUT field</th>
<th>( SU(3)_c )</th>
<th>( SU(2)_L )</th>
<th>( U(1)_Y )</th>
<th>composite</th>
<th>( Z_3 )</th>
<th>( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>( Q_3 )</td>
<td>( \bar{u}_3 )</td>
<td>( 1/6 )</td>
<td>( q_c q_L )</td>
<td>( 2/3 )</td>
<td>( 1/3 )</td>
</tr>
<tr>
<td></td>
<td>( E' )</td>
<td>( 1 )</td>
<td>( -2/3 )</td>
<td>( q_c q_c )</td>
<td>( 2/3 )</td>
<td>( -1/3 )</td>
</tr>
<tr>
<td></td>
<td>( \phi )</td>
<td>( 1 )</td>
<td>( 1/2 )</td>
<td>( q_L q_S )</td>
<td>( 2/3 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>5</td>
<td>( d' )</td>
<td>( 1 )</td>
<td>( -1/3 )</td>
<td>( q_c q_S )</td>
<td>( 2/3 )</td>
<td>( -2/3 )</td>
</tr>
</tbody>
</table>

In addition to hypercharge \( U(1)_Y \), the breaking pattern in (2.18) allows for an additional global symmetry which we will denote by \( U(1)_G \). As will be seen in section 2.4.2, it is necessary to consider the MSSM baryon number to be part of the global symmetries for proton stability. It will also be seen that baryon number has a unique embedding in \( U(1)_G \) and \( U(1)_Y \) given by:

\[
B = \frac{4}{5} Y + \frac{1}{10} G \quad \text{with} \quad Y = \text{diag}(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, 0) \\
G = \text{diag}(1, 1, 1, 1, 1, -5)
\]

(2.20)

Note that both the electric and magnetic theories have a \( Z_{N_f} \) discrete symmetry that is leftover from the anomalous global \( U(1) \) symmetry. As we will discuss in the next subsection, we will identify this \( Z_3 \) with the one of table 2.1.

### 2.4.2 Magnetic theory

This theory s-confines in the IR and has a weakly-coupled magnetic dual description in terms of the mesons and baryons of the electric theory as described in table 2.3. These gauge invariants \( q^i q^j \) transform as the antisymmetric tensor \( 15_A \) of the global \( SU(6) \). Under \( SU(5) \) this decomposes as

\[
15_A = 10_A \oplus 5.
\]

(2.21)

The resulting \( SU(5) \) representations allow us to identify \( Q_3, \bar{u}_3 \) and \( \phi \) with composite
degrees of freedom. Note that the baryon numbers of $Q_3$ and $\bar{u}_3$ uniquely determined the coefficients of $U(1)_Y$ and $U(1)_G$ in (2.20). The rest of the composite fields are $E'$ and $d'$, of which $E'$ has the same gauge quantum numbers as right handed leptons, but non-zero baryon number.

The confining electric gauge group dynamically generates a superpotential in the magnetic dual, given by

$$W_{\text{mag}} = \frac{1}{\Lambda^3} \text{Pf}(q^i q^j)$$

$$= \kappa (\phi Q_3 \bar{u}_3 - Q_3 Q_3 d' + d' \bar{u}_3 E')$$

(2.22)

where Pf is the Pfaffian of the antisymmetric matrix $q^i q^j$, and we used the mapping to the magnetic theory in the second line. The coupling $\kappa$ descends from the strong dynamics in the electric theory and can be large (for concreteness we assumed $\kappa \sim 3$ in section 2.3). From the last two operators in (2.22) it should also be clear that rapid, dimension 6 proton decay would be introduced if one were to identify $E'$ with one of the MSSM leptons. The $B$ and $Z_3$ charges for the composite fields are fixed by those of the electric quarks in table 2.2.

2.4.3 Complete model with spectators

Let us now weakly gauge a $SU(3)_c \times SU(2)_L \times U(1)_Y$ subgroup of the global symmetry. To cancel anomalies, fill out complete GUT multiplets, and match the field content of the magnetic theory to the model of section 2.3, we add a number of fundamental fields, which are all spectators as far as the Seiberg duality is concerned. Among these spectators are all three $\bar{d}$, $L$ and $\bar{e}$ generations of the MSSM, as well as the first two generations of the $Q$ and $\bar{u}$ sectors. Finally, the $H_u$ and $H_d$ are spectators as well, but do not come in complete GUT multiplets. This is nothing other than the usual doublet-triplet splitting problem in models with grand unification. The spectators and their quantum numbers are introduced in table 2.4. Aside from the usual baryon number, we also assign the $Z_3$ charges for the spectator fields such that the symmetry in table 2.1 is realized. In addition to the fields we introduced so far, one may choose to add up to three pairs of conventional, 5-$\bar{5}$ gauge mediation messengers without spoiling perturbative gauge coupling unification.\(^6\)

\(^6\)We hereby assume that any uncalculable threshold corrections at the compositeness scale are negligible.
Table 2.4: Spectators of the Seiberg duality required to cancel anomalies and fill out complete GUT multiplets. Primed fields have heavy vector-like masses and are integrated out at the duality scale. The first two generations are also spectators but are not shown here for simplicity.

All the non-MSSM fields have vector-like masses. Some arise from Yukawa interactions in the electric theory, while others are mass terms:

$$W_{\text{elec}} \supset y_{d'} q_L s c d^\prime + y_{E'} q_L q_L E^\prime + M_{Q'} Q^\prime Q^\prime + M_{U'} U^\prime U^\prime$$

$$\rightarrow W_{\text{mag}} \supset y_{d'} \Lambda d^\prime + y_{E'} \Lambda E^\prime E^\prime + M_{Q'} Q^\prime Q^\prime + M_{U'} U^\prime U^\prime$$

(2.23)

Those that are Yukawas in the electric theory are naturally of the same size as the compositeness scale $\Lambda$, and so for unification we must also take $M_{Q'} \sim M_{U'} \sim \Lambda$.

We can see that it is possible to reproduce the model in (2.13) by adding interactions between spectators and the composites and between spectators themselves if we allow the following interactions

$$\delta W = (X'_0 + X') H_u \tilde{\phi} + \tilde{\lambda}^u_{ij} Q_i \bar{u}_j + M \phi \tilde{\phi}$$

(2.24)

where $i, j$ identify quark fields in the gauge eigenbasis. To avoid clutter, we suppressed the mass terms that are introduced in (2.23), as well as the $\mu$-term and the down and lepton Yukawas. This superpotential is generic if we impose the $\mathbb{Z}_3$ symmetry of tables 2.3 and 2.4.

As noted earlier, the first and second generations of the MSSM matter fields are all elementary and spectators as far as the Seiberg duality is concerned. Since $\phi$ is a composite
operator in the electric theory, all up-type Yukawa couplings (other than the top Yukawa) must arise from irrelevant operators in the electric theory. (Recall that the $Z_3$ symmetry of table 2.1 forbids the usual up-type Yukawa couplings $H_u Q \bar{u}$.)

For instance

$$\frac{1}{\Lambda^2_{UV}}(q_L q_S)(q_L q_L) \bar{u}_2 \rightarrow \frac{\Lambda^2}{\Lambda^2_{UV}} \phi Q_3 \bar{u}_2$$

(2.25)

where $\Lambda_{UV}$ is a cut-off scale of the electric theory. In the notation of section 2.3 this yields:

$$\lambda^{ij}_u = \kappa \delta^{i3} \delta^{3j} + \tilde{\lambda}^{ij}_u \sim \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \kappa \end{array} \right) + \left( \begin{array}{ccc} \epsilon & \epsilon & \epsilon^2 \\ \epsilon & \epsilon & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & \epsilon^3 \end{array} \right)$$

(2.26)

with $\epsilon \sim \Lambda/\Lambda_{UV} \ll 1$. The composite sector therefore naturally provides a partial explanation of the texture of the up-type Yukawa matrix. Since $Q_3$ is a composite degree of freedom, it also predicts $\epsilon \sim y_b \sim 0.1$, but the rest of the hierarchies in $y_d$ and $y_\ell$ are not explained.

Upon integrating out the messenger fields, the analysis further reduces to what was presented in section 2.3. There is one exception, in the sense that the model is no longer manifestly MFV since the third generation was given a special treatment. In particular a non-MFV dimension six operator is generated in the Kähler potential from integrating out $d'$ in (2.22)

$$\delta K_{\text{eff}} \sim \frac{1}{\Lambda^2} (Q_3 Q_3)^\dagger (Q_3 Q_3) \sim \frac{1}{\Lambda^2} (u_3 d_3)^\dagger (u_3 d_3).$$

(2.27)

By rotating $Q_3$ to the mass eigenbasis, this operator can in principle couple quarks of different generations. However note that this operator does not introduce any new CP phase into the model and it does not contribute to FCNC processes at tree level. Moreover it is suppressed by the duality scale that is above the messenger scale $\gtrsim 100$ TeV. The effects in the first two generation quarks are further suppressed by powers of $\epsilon$ coming from (2.26).

For instance, the operator contributing to $K$-$\bar{K}$ mixing receives an additional suppression of $\sim \epsilon^8$. Therefore we conclude that it is consistent with the bounds from flavor observables [58].
2.5 Conclusions

In this paper, we presented a new mechanism to generate large $A$-terms through tree-level superpotential operators. We provided explicit examples of both MFV and non-MFV models. In contrast to the conventional setups with one-loop $A$-terms through Kähler potential operators, our tree-level mechanism does not induce any dangerously large soft masses and is therefore manifestly free from the $A/m^2$ problem. Generically, a soft mass of the same order as the $A$-term is nevertheless still generated. For the non-MFV example this contribution greatly increases the splitting between the stop mass eigenstates, but otherwise does not significantly impact the phenomenology or the fine tuning. For the MFV case, the soft mass could potentially lead to disastrous levels of fine tuning, but it can be brought under control by the existence of strong dynamics near the messenger scale. We provide an example of such a composite sector which has a description in terms of Seiberg duality and which explicitly allows for gauge coupling unification.

Some potential future directions suggested by this work include:

- For concreteness, we focused on an MGM setup as a first example, but we emphasize that tree-level $A$-terms are merely a module that can be added to any mechanism for mediating SUSY breaking. In particular, it would be interesting to study whether the mechanism can naturally be embedded in more realistic models of dynamical supersymmetry breaking. In addition one could generalize $X'$ beyond the spurion limit, and study the effects of its dynamics on the phenomenology.

- In the non-MFV case it may be interesting to embed the tree-level $A$-term into a full fledged theory of flavor.

- In the MFV case, we saw that the $A$-term module generated a contribution to $B_\mu$ which unfortunately was of the wrong sign for EWSB. An interesting opportunity here would be to construct a complete model that produces both tree-level $A$-terms and $B_\mu$, perhaps along the lines of the models constructed in [46].

- Finally, the emergence of large $A$-terms from a composite sector in the MFV case may open a new avenue towards constructing a realistic model where large $A$-terms are
generated at the TeV scale, hence further reducing the fine-tuning.

2.6 The little $A/m^2$ problem for arbitrary couplings

In sections 2.2 and 2.3 we concluded that the little $A/m^2$ tuning problem is most serious when a soft mass for the Higgs field is generated. In this appendix we show that this little $A/m^2$ problem is generic for our class of models: it cannot be avoided by increasing the messenger number or considering a more general renormalizable superpotential.

Consider the most general renormalizable superpotential coupling the fields $H_u, Q, \bar{u}$ with $n$ pairs of messengers $\phi_k, \tilde{\phi}_k$ ($k = 1 \ldots n$) with the quantum numbers of $H_u$ and its hermitian conjugate

$$W = M_k \phi_k \tilde{\phi}_k + X_k \tilde{\phi}_k H_u + y_t H_u Q \bar{u} + \lambda_k \phi_k Q \bar{u} + \ldots$$

(2.28)

where we sum over repeated indices. Here, differently from section 2.3, we work in a basis in which the supersymmetric mass matrix has already been diagonalized, so $X_k$ have only F-term vevs. The rest of the interactions included in $\ldots$ do not matter to derive the induced $A$-term and soft mass at lowest order, so we neglect them in what follows. Integrating out the messengers in the small SUSY breaking regime $F/M^2 \ll 1$ we get the low energy superpotential and Kähler potential

$$W = y_t H_u Q \bar{u} - \lambda_k X_k \frac{M_k}{M_k} H_u Q \bar{u}, \quad K = \left( 1 + \left( \frac{X_k}{M_k} \right) \right) H_u H_u^\dagger + \ldots$$

(2.29)

so the $A$-term and induced soft mass are

$$y_t A_t = -\frac{\lambda_k X_k}{M_k}, \quad \delta m^2_{H_u} = -\left( \frac{X_k}{M_k} \right) \left( \frac{X_k}{M_k} \right)$$

(2.30)

To avoid the little $A/m^2$ problem, we need to maximize the ratio of the $A$-term over the soft mass. In particular we are interested in knowing if in doing this, the theory remains perturbative, or if it does not, when does it become strongly coupled. To address this question, note that there is a linear combination of messengers that couples to the light fields with a Yukawa with magnitude given by

$$|\lambda| = \sqrt{\sum_k |\lambda_k|^2}$$

(2.31)
so that the Yukawa beta functions are, at one loop,

\[
\beta_\lambda = \beta_\lambda^0 + \frac{6y_t^2 \lambda}{16\pi^2}, \quad \beta_{yt} = \beta_{yt}^0 + \frac{6y_t\lambda^2}{16\pi^2},
\]

where $\beta_\lambda^0$ is a MSSM-like top Yukawa beta function. We immediately see that the parameter that controls the running of the Yukawas is $|\lambda|$. Fixing this parameter, the ratio of the $A$-term over the soft mass is maximized when $\lambda_k$ and $\frac{X_k}{M_k}$ are parallel vectors in $k$ space. This leads to the bound

\[
\left| \frac{y_tA_t}{\delta m_{H_u}} \right| \leq |\lambda|
\]

where to retain perturbativity $\lambda$ needs to be of order one or smaller. This bound is valid for the most general renormalizable superpotential that couples messengers with the Higgs at tree level. A similar bound relating the squark mass to the $A$-term can be obtained for the non-MFV model of section 2.2. Note that the bound is independent of the messenger number. For messengers at 250 TeV and $\lambda = 1$ as considered in section 2.2 a Landau pole is obtained at $\sim 10^{10}$ GeV. A coupling $\lambda = 3$ as considered in section 2.3 leads to a Landau pole less than a decade above the messenger scale.
Chapter 3

Dark Matter and the Higgs in Natural SUSY

With S. Macaluso and D. Shih

arXiv:1605.08442

General context of this chapter

In the previous chapter we presented a mechanism to obtain large $A$-terms and explain the Higgs mass with minimum fine-tuning in MSSM. In addition to Higgs mass, MSSM is also under pressure from null results of dark matter direct detection experiments. In this chapter, we propose a simple extension of the MSSM that economically solves both problems: a “dark sector” consisting of a singlet and a pair of $SU(2)$ doublets. Loops of the dark sector fields help lift the Higgs mass to 125 GeV consistent with naturalness, while the lightest fermion in the dark sector can be viable thermal relic DM, provided that it is mostly singlet. The DM relic abundance is controlled by s-wave annihilation to tops and Higgsinos, leading to a tight relation between the relic abundance and the spin-dependent direct detection cross section. As a result, the model will be fully probed by the next generation of direct detection experiments. Finally we discuss the discovery potential at LHC Run II.

3.1 Introduction and Summary

The MSSM paradigm is under siege from both the LHC and dark matter (DM) direct detection. The Higgs mass at tree-level in the MSSM is famously bounded by $m_Z$, and relying on radiative corrections from stops and other particles in the MSSM forces the stops to be either at least $\sim 10$ TeV or their $A$-terms to be multi-TeV (for recent reviews and original references, see e.g. [16, 60, 61]). Together with the null direct search results at the LHC, this puts the fine-tuning in the MSSM at the percent level or worse. Meanwhile, to
evade stringent direct and indirect detection bounds, thermal relic neutralino DM in the MSSM must rely on increasingly contrived numerical accidents (well-tempering, blind spots, funnels, co-annihilations) or an increasingly heavy SUSY scale (e.g. $\sim 1$ TeV Higgsinos or $\sim 2-3$ TeV winos) (see e.g. [62, 63] for recent comprehensive studies). The latter constitutes a DM version of the little hierarchy problem, whereby the WIMP miracle’s preference for TeV-scale DM (as opposed to 100 GeV scale DM) is in tension with naturalness.

This strongly motivates looking beyond the MSSM for both the source of the Higgs mass and dark matter. Although it is logically possible that different sectors are independently responsible for the Higgs mass and dark matter, it is interesting to contemplate more elegant and economical models where a single sector generates both. In this paper, we will study such a model. We will show how to achieve a 125 GeV Higgs and thermal relic WIMP DM consistent with all existing constraints, while greatly ameliorating the fine-tuning, by just adding a pair of $SU(2)$ doublets $L, \bar{L}$ and a singlet $S$ to the MSSM. With a $Z_2$ “DM parity” that keeps the lightest state in the dark sector stable, together with matter parity from the MSSM, the most general renormalizable superpotential for this “dark sector” is:

$$W = \frac{1}{2} M_S S^2 + M_L L\bar{L} + k_u H_u LS - k_d H_d \bar{L}S \quad (3.1)$$

Although it would be interesting to also consider phases, we will focus on real couplings in this paper for simplicity. Then without loss of further generality, we can take $M_S$ and $M_L$ to be positive.

The idea of extending the Standard Model (SM) with a “singlet-doublet DM” sector has been studied previously in [64–71], motivated by minimality and by the fact that it is a simple generalization of the well-studied bino/Higgsino system of the MSSM. The idea of lifting the Higgs mass with loops of vector-like matter has also been well-studied [72–89]. But to our knowledge, the two ideas have never been combined before. Combining these two ideas leads to some important differences with previous works.

First, unlike in previous works on lifting the Higgs mass, our dark sector cannot be truly vector-like. The scalar soft mass-squareds of the dark sector must be positive in order to lift the Higgs mass, making our DM the lightest fermion in the dark sector. It cannot be a Dirac fermion, otherwise it would be ruled out by many orders of magnitude by $Z$-mediated
spin-independent (SI) direct detection. Instead, we make the dark sector fermions Majorana (as shown in (3.1)) by having only one singlet and not a vector-like pair of them. This only has a minor effect on the contribution to the Higgs mass in this model, which we fully take into account. We will find that a \( m_h = 125 \text{ GeV} \) Higgs can be achieved with the fine-tuning coming from the DM being only \( \sim 10\% \), provided that \( k_u \sim O(1) \).

Second, we differ from the singlet-doublet DM models in that we are supersymmetrizing everything.\(^1\) A priori, the parameter space of the entire model (MSSM+dark sector) is vast, but most of the soft parameters do not play a significant role in the analysis. As seen in (3.1), our dark sector only couples directly to the Higgs sector and the EW gauge sector of the MSSM. We will keep the Higgsinos light (\( \lesssim 300 \text{ GeV} \)), since they contribute to the fine-tuning of the EW scale at tree level. As a result, DM annihilation to light Higgsinos through superpartners in the dark sector plays a major role in determining the relic abundance of the DM. Meanwhile, it does not change our analysis qualitatively to decouple all other MSSM superpartners (effectively at the \( \sim \text{ TeV} \) scale). This is further motivated by the null results from the LHC.

We will further simplify the analysis of the model by focusing on the regime where the dark matter \( \chi \) is mostly-singlet, i.e. \( M_S < M_L \) and \( v \ll M_L, M_L - M_S \). As we will argue in much more detail in section 3.5, this regime is absolutely necessary in order to evade direct detection bounds while raising the Higgs mass without fine-tuning. A key part of the argument, which distinguishes this from the bino/Higgsino system in the MSSM, is that \( k_u \) must be \( O(1) \) in order to lift the Higgs mass without fine-tuning. This eliminates both the well-tempered regime and the mostly-doublet regime vis a vis DM direct detection. The mostly-doublet regime is further unpromising because (by analogy with pure Higgsino DM in the MSSM) it would require a DM mass in excess of 1 TeV, and this would greatly exacerbate the fine-tuning problem, since the rest of the dark sector would have to be even heavier. This leaves the mostly-singlet regime, where the analysis of the model greatly simplifies, and we are able to understand all the features of the model with simple analytic

\(^1\)Actually, in [65] they also added singlets and doublets to the MSSM. However, they considered soft masses purely from GMSB (whereas we are agnostic) and therefore they never have mostly-singlet fermionic DM. Moreover they fix \( k_u = k_d = 0.3 \) whereas we have them as free parameters. Finally, they do not calculate the contribution to the Higgs mass from the dark sector.
\[ \psi_L, \psi_L, \text{superpartners} \]

\[ \psi_S \]

\[ h, t, \text{Higgsinos} \]

Figure 3.1: A typical, viable spectrum of the model. \( \psi_S, \psi_L, \psi_L \) are the fermionic components of the dark sector fields. Superpartners include scalar components of the dark sector and superparticles in MSSM.

formulas. A cartoon spectrum of the model that describes these hierarchies qualitatively is shown in fig. 3.1.

In this work, we will assume the simplest DM scenario, namely that \( \chi \) is a thermal relic comprising all of the DM. In the mostly-singlet limit with \( k_u \sim 1 \), we will show that the thermal relic abundance is controlled by just two DM annihilation channels: s-wave \( tt \) (through s-channel \( Z \) exchange) and s-wave Higgsinos (through \( t \)-channel superpartner exchange). Assuming \( M_S \ll M_L \) for simplicity, we find:

\[
\sigma v_\chi \approx \frac{3k_u^4 m_t^2}{32\pi M_L^4} + \frac{(k_u^2 + k_d^2)^2 \mu^2}{16\pi (M_L^2 + m^2)^2} \tag{3.2}
\]

where \( m \) is a common soft mass for the dark sector scalars. As noted above, the second term coming from Higgsinos is a major difference from the non-supersymmetric singlet-doublet DM models that have been studied previously. Having more annihilation channels increases \( \sigma v_\chi \), making it possible to have smaller effective couplings between the DM and the SM. This opens up more parameter space that is not ruled out by direct detection experiments and yet still has the correct thermal relic abundance, as compared to the non-SUSY singlet-doublet models.

Interestingly, the DM mass drops out of the annihilation cross section (3.2) in the mostly-singlet limit. The WIMP miracle becomes one for the mediator scale, not the WIMP mass! With \( k_u \sim k_d \sim 1, m \sim M_L \) and \( \mu \lesssim 300 \text{ GeV} \), mediator scales of \( M_L \sim 1 - 2 \text{ TeV} \) are implied by the thermal relic constraint. Meanwhile the DM can be much lighter than
this, alleviating the DM little hierarchy problem. It is also interesting to contrast this with the mostly bino limit of the bino/Higgsino system in the MSSM. There the annihilation cross section is not large enough, being suppressed by $g_1^4$ instead of $k_u^4$. Our model (and singlet-doublet DM more generally) gets around this in the mostly-singlet regime with $O(1)$ Yukawa couplings that are free parameters, not fixed to be $g_1$ by supersymmetry.

Meanwhile, DM direct detection in these models is completely controlled by the effective couplings of the DM to the Higgs and $Z$ respectively:

$$\delta \mathcal{L} = c_h \bar{\psi}_\chi \psi_\chi + c_Z Z \mu \bar{\psi}_\chi \gamma^\mu \gamma^5 \psi_\chi$$ (3.3)

As is well-known, $c_h$ ($c_Z$) controls the SI (SD) direct detection cross section. For direct detection, as we will review, the current best bounds for our DM mass range of interest ($100 \lesssim m_{DM} \lesssim 1000$ GeV) come from LUX [90, 91] and IceCube [92]. We will convert the official experimental results, which are phrased in terms of the DM-nucleon cross section, into limits on $c_h$ and $c_Z$. Furthermore, in the mostly-singlet limit, we will obtain simple analytic expressions for $c_h$ and $c_Z$. We will see that $c_h$ can be naturally small enough for mostly-singlet DM, due to suppression from the heavier doublet scale, as well as a mild blind-spot cancellation:

$$c_h \approx -m_\chi + \frac{2k_u M_L}{k_\tan \beta} k_d v^2 \sqrt{2v} \frac{k_u v^2}{M_L^2}$$ (3.4)

provided that $k_u \sim -k_d$. We should emphasize here that the Higgs mass depends not just on $c_h$ but also on the effective Yukawa couplings between the Higgs and the other dark sector particles. So even dialing $c_h \rightarrow 0$ does not qualitatively affect the Higgs mass calculation. Meanwhile $c_Z$ is given in the mostly-singlet limit by:

$$c_Z \approx -\frac{g_2}{4c_W} \frac{k_u^2 v^2}{M_L^2}$$ (3.5)

According to our discussion above, after fixing the thermal relic density constraint $\Omega_{DM} h^2 \approx 0.12$, $c_Z$ is essentially fixed to lie within a narrow range which depends primarily on the Higgsino mass $\mu$. Therefore imposing the relic density constraint essentially fixes the SD cross section. Fortunately, this value is not ruled out yet, but the next generation of DM experiments (e.g. Xenon1T [93], LZ [94]) should completely rule out or discover this model.

Although direct detection is controlled by $c_Z$ and $c_h$, the other facets of the model (relic abundance, Higgs mass) depend on more than just these couplings, so our model does not
fit completely into the framework of $Z$- and $h$-portal DM. For instance, we mentioned above that the Higgsino cross section arises entirely from $t$-channel superpartner exchange. Also, we find that DM annihilation to dibosons is suppressed more than would be the case in $Z$ and $h$ portal models, in part due to $t$-channel exchange of doublet fermions. Similar comments apply to the effective operator formalism: our DM is generally not light enough compared to the mediator scale (the doublet mass) for the annihilation to be accurately captured by effective operators. Evidently, the complete model (3.1) is required for an accurate analysis. This illustrates the shortcomings and limitations of both simplified models and effective operator approaches to dark matter.

We have focused primarily on the standard direct detection searches in this work, because other indirect probes of our dark sector are far less sensitive. For example, the Fermi experiment and others have searched for energetic photons produced through DM annihilating at the centers of dwarf galaxies. For DM masses above 100 GeV, Fermi does not constrain any point with the right relic-abundance [95], assuming (as is the case for us) that the relic abundance is determined by $s$-wave annihilation. Meanwhile, searches at colliders and electroweak precision tests (EWPT) could have put constraints on our model. However as we will discuss further in section 3.8.2, LHC bounds [96–99] on $c_h$ and $c_Z$ from monojets+MET and monophoton+MET are orders of magnitude weaker than direct detection for the range of DM masses that we are interested in. We will briefly discuss mono-$(W,Z,h)+\text{MET}$ and show how it could probe the low end of DM masses ($m_{DM} \sim 200$ GeV) in our model, with 300/fb at LHC Run II. Finally, limits from Higgs and $Z$ invisible width do not apply to the mass range of DM that we consider in this work, and we checked that contributions to the $S$ and $T$ parameters are well within the acceptable range, in agreement with previous studies of these variables in closely-related models [68, 76].

In this paper, we will analyze the model using a combination of simple, approximate analytic expressions valid in the mostly-singlet regime, and more precise numerical methods that take into account the full suite of one (and even two) loop threshold corrections. The analytic approach, while being reasonably accurate, is primarily a source of intuition and understanding. The numerical approach is meant to be more accurate and to provide us with the quantitative results. Clearly, having both numerics and analytics is a vital source
of cross-checks, giving us greater confidence in our results.

Our numerical methods are based on publicly available codes. Our starting point was the powerful SARAH 4.5.8 framework [100] for automated analysis of general models. Once we properly defined our model, SARAH automatically generated source code for SPheno 3.3.7 [101, 102] and for micrOMEGAs 4.1.8 [103]. The former calculates the spectrum while the latter calculates the DM relic abundance and direct detection cross sections. In our numerical calculations, all MSSM soft masses as well as gauginos are taken to be at 1 TeV, and the $A$-terms are set to zero. As noted earlier, since $\mu$ appears at tree level in fine-tuning of the electroweak scale we treat it differently. We pick $\mu = 300$ GeV in our numerical calculations which corresponds roughly to 10% fine-tuning. We also consider $\mu = 100$ GeV to see the effect of $\mu$ on our analysis. Finally, to saturate the tree level contribution to the Higgs mass, we take the other Higgses to be heavy and in the decoupling limit, and we take $\tan \beta = 10$.

The outline of our paper is as follows. In section 3.2 we introduce the model. Then in section 3.3, we derive direct detection limits from LUX and IceCube on the effective couplings $c_h$ and $c_Z$. We will emphasize that these results are general and are not limited to the model we consider in this work. In section 3.4 we compute the one-loop corrections to the Higgs mass from the new particles in our model, and we discuss fine-tuning. We argue in section 3.5 that the mostly singlet case is the only viable scenario. In the mostly singlet limit, we provide analytic expressions for dark matter annihilation in the early universe for our model in section 3.6. In section 3.7 we put everything together to show the viable parameter space that satisfies all direct detection constraints while having the right relic abundance and Higgs mass. Here we demonstrate quantitatively that requiring $\chi$ to be all of the DM essentially fixes $c_Z$ (and hence $\sigma^{SD}$) to a unique value which is not yet ruled out by direct detection, but will be fully within reach of the next generation of experiments. We conclude by studying the collider signatures for LHC Run II and the UV behavior of the model, and giving suggestions on future directions on section 3.8. Technical details and validations are reserved for three appendices. In appendix 3.9 we review the derivation

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2We are extremely grateful to Florian Staub for his time and patience in helping us set up the SARAH model and link it to these other codes.
of the direct detection cross sections from effective DM nucleon couplings. We validate our numerical and analytical calculations of the Higgs mass in appendix 3.10. Finally we provide analytical cross sections for DM production at LHC II in appendix 3.11.

### 3.2 The Model

We begin by describing the model in more detail. We add to the MSSM a “dark sector” consisting of a vector-like pair of $SU(2)$ doublets $L, \bar{L}$ and a gauge singlet $S$. The dark sector is equipped with an unbroken $Z_2^{DM}$ parity symmetry under which all new fields are odd and all MSSM fields are even. This makes the lightest new state stable and a DM candidate. Finally, we assume MSSM matter parity, under which all the dark sector fields have the same charge; otherwise there will be additional, potentially dangerous terms. The transformation properties of the dark sector under the gauge and global symmetries is summarized in tab. 3.1.

Table 3.1: Gauge and global symmetries of the dark sector.

<table>
<thead>
<tr>
<th></th>
<th>$SU(3)_c$</th>
<th>$SU(2)_L$</th>
<th>$U(1)_Y$</th>
<th>$Z_2^{DM}$</th>
<th>$Z_2^M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>1</td>
<td>2</td>
<td>$-\frac{1}{2}$</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>$\bar{L}$</td>
<td>1</td>
<td>2</td>
<td>$\frac{1}{2}$</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>$S$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>1</td>
</tr>
</tbody>
</table>

The most generic superpotential consistent with these symmetries is:

$$\delta W = \frac{1}{2} M_S S^2 + M_L L \bar{L} + k_u H_u L S - k_d H_d \bar{L} S$$

(3.6)

The superpotential has four new parameters in addition to the MSSM: $M_L, M_S, k_u, k_d$. There is one physical complex phase, but as discussed in the introduction, we will take these parameters to be real in this paper. In this case, there is still a physical sign. We will take $M_L, M_S$ and $k_u$ to be positive and put the sign into $k_d$.

For the soft SUSY-breaking Lagrangian, for simplicity we take the minimal case with

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$^3$To keep gauge coupling unification as in MSSM, we can assume $L$ and $\bar{L}$ are part of complete $5$ and $\bar{5}$ multiplets of $SU(5)$. We take their colored partners to be heavy and decoupled for simplicity.

$^4$The assumption of matter parity implies another stable particle – either the LSP in the MSSM, or the gravitino. Either way, we assume the parameters are such that this will add a negligible additional component to the thermal relic density. This would be the case, for instance, if the LSP is a light Higgsino.
equal soft mass-squareds and no $A$- or $B$-terms:

$$\delta L_{\text{soft}} = -m^2(|\bar{\ell}|^2 + |\ell|^2 + |s|^2) \quad (3.7)$$

(We denote the scalar components of the dark sector superfields with lowercase letters.)

Allowing different soft masses for the different fields will not change most of the discussion in this paper, only the contributions to Higgs mass.

As we want this new sector to increase the lightest Higgs mass analogous to the MSSM stops, we assume that $m^2 > 0$. This implies that the DM candidate is a fermion. Furthermore it is Majorana, thanks to the fact that we have included only one singlet in the theory. Had we started with a Dirac pair of $S$ and $\bar{S}$ and defined the mass term as $M_S S \bar{S}$, our dark matter would have had a vector-like coupling to the $Z$. In that case it would have been impossible to hide it from SI direct detection experiments while keeping the interesting features of our model.

After EWSB, neutral fields in the dark sector mix through the Yukawa couplings in (3.6). The fermion mass matrix of the neutral states is:

$$\mathcal{M} = \begin{pmatrix} M_S & \hat{k}_u v & \hat{k}_d v \\ \hat{k}_u v & 0 & M_L \\ \hat{k}_d v & M_L & 0 \end{pmatrix}, \quad (3.8)$$

where we have introduced $\hat{k}_u \equiv k_u \sin \beta$ and $\hat{k}_d \equiv k_d \cos \beta$, with $\tan \beta = v_u / v_d$ and $v_u^2 + v_d^2 = v^2 \approx (174 \text{ GeV})^2$ as usual. We take large $\tan \beta = 10$ in this paper to saturate the upper bound on the tree level Higgs mass. The mass matrix is diagonalized by $U \mathcal{M} U^\dagger = \mathcal{M}_{\text{diag}}$. The spectrum of the model consists of three Majorana fermions with masses $m_{\chi_1} < m_{\chi_2} < m_{\chi_3}$ and a Dirac charged fermion with mass $m_{\chi^\pm} = M_L$. The dark matter candidate is then $\chi \equiv \chi_1$.

We note that the fermionic part of our dark sector is analogous to Bino-Higgsino DM in the MSSM (with everything else decoupled), except that in the Bino-Higgsino system, we effectively have $k_u = k_d = g'/\sqrt{2}$, whereas here $k_u$ and $k_d$ are general. In fact, as discussed in the introduction, here we will be primarily interested in $k_u, k_d \sim O(1)$.

After rotating to the mass eigenbasis, DM-$Z$ and DM-Higgs couplings are generated:

$$\delta \mathcal{L} = c_h h \bar{\psi}_\chi \psi_\chi + c_Z Z_\mu \bar{\psi}_\chi \gamma^\mu \gamma^5 \psi_\chi \quad (3.9)$$
where $\psi = (\chi, \chi^\dagger)^T$ is a 4-component Majorana fermion and $c_h$ and $c_Z$ are given by:

$$c_h = \frac{1}{\sqrt{2}} \text{Re}(\hat{k}_u U_{11}^* U_{12} + \hat{k}_d U_{11}^* U_{13})$$

$$= \frac{v}{\sqrt{2}} \left( \frac{m_\chi (\hat{k}_d^2 + \hat{k}_u^2) + 2 \hat{k}_d \hat{k}_u M_L}{M_L^2 + 2 M_S m_\chi - 3 m_\chi^2 + v^2 (k_d^2 + k_u^2)} \right) \quad (3.10)$$

and

$$c_Z = \frac{g_2}{4 c_W} (|U_{12}|^2 - |U_{13}|^2)$$

$$= \frac{g_2}{4 c_W} \left( \frac{(M_L^2 - m_\chi^2) v^2 (\hat{k}_d^2 - \hat{k}_u^2)}{(M_L^2 - m_\chi^2)^2 + v^2 ((\hat{k}_d^2 + \hat{k}_u^2)(M_L^2 + m_\chi^2) + 4 k_d k_u M_L m_\chi)} \right) \quad (3.11)$$

As is well-known [66–70], $c_h$ and $c_Z$ play an important role in the analysis of singlet-doublet DM: they are entirely responsible for SI and SD direct detection, respectively. In the next section, we will review the current direct detection constraints on $c_h$ and $c_Z$.

### 3.3 DM Direct Detection through the $h$ and $Z$ Portals

In the DM mass range of interest ($100 \text{ GeV} \lesssim m_{DM} \lesssim 1 \text{ TeV}$), the LUX experiment currently sets the best bound on SI elastic WIMP-nucleon scattering [91]. Meanwhile, the best limits for SD elastic WIMP-proton (WIMP-neutron) scattering come from IceCube [92] (LUX [91]). The IceCube limits depend on an assumption of DM s-wave annihilation in the sun exclusively to a single SM final state. As we will show in section 3.6, our DM annihilates in the s-wave to both $t\bar{t}$ and Higgsinos. Annihilation to Higgsinos could weaken the limits somewhat if the Higgsinos are stable, but that depends in detail on the other parameters of the model (such as $\mu$, $k_u$ and $k_d$). Here we consider the simplest case where annihilation is only to $t\bar{t}$; this will provide the “worst case scenario” where the SD bound from IceCube is strongest. In section 3.7 we will also take into account annihilation to stable Higgsinos.

In this section, we will recast these constraints in terms of the couplings $c_h$ and $c_Z$. The discussion here can be viewed as an update of the nice treatment in [62] with the latest experimental results (in particular LUX). It is worth emphasizing that these bounds on $c_h$ and $c_Z$ are quite model independent. Any WIMP DM that couples to SM mainly through Higgs and $Z$ (including MSSM neutralinos) should satisfy these bounds.
To convert the results of these experiments into bounds on $c_Z$ and $c_h$, we first translate $c_Z$ and $c_h$ into the couplings appearing in the effective Lagrangian for direct detection:

$$\delta L \supset \sum_q \left( \xi_{q}^{SI} (\bar{\psi} \chi \psi \chi) + \xi_{q}^{SD} (\bar{\psi} \gamma^\mu \gamma_5 \psi \chi) (\bar{q} \gamma^\mu \gamma_5 q) \right).$$  \hspace{1cm} (3.12)

In Higgs and $Z$-portal DM models, the SI (SD) terms arise from Higgs ($Z$) exchange, as shown in fig. 3.2. The coefficients of the effective operators are given in terms of $c_h$ and $c_Z$ as:

$$\xi_{q}^{SI} = y_q \frac{c_h}{m_h^2}, \hspace{1cm} \xi_{q}^{SD} = \frac{g_2 \eta_q}{4 c_W} \frac{c_Z}{m_Z^2}.$$  \hspace{1cm} (3.13)

with $y_q$ being the Yukawa coupling and $\eta_q = 1$ ($-1$) for down-type (up-type) quarks.

Then we use standard formulas that relate the DM-nucleon cross sections to $\xi_{q}^{SI,SD}$ (see appendix 3.9 for our conventions and parameter choices). The result, assuming $m_{DM} \gg m_{p,n}$ is given by:

$$\sigma_{SI} = c_h^2 \times (2.11 \times 10^3 \text{ zb})$$

$$\sigma_{SI}^p = c_h^2 \times (1.17 \times 10^9 \text{ zb})$$

$$\sigma_{SI}^n = c_Z^2 \times (8.97 \times 10^8 \text{ zb})$$  \hspace{1cm} (3.14)

In principle, $\sigma_{SI}^p$ and $\sigma_{SI}^n$ are slightly different but the difference is negligible, so we only take $\sigma_{SI}^p$ to represent both.

The resulting limits on $c_h$ and $c_Z$ are shown in fig. 3.3. \hspace{1cm} 5 Amusingly, we note that although the constraint on the SI cross section is $\sim 10^5$ stronger than the SD cross-section,

---

\[5\text{We agree with the limits from [62] after taking into account a factor of 2 in both } c_Z \text{ and } c_h \text{ from 4-component vs. 2-component notation. We also agree with limits on operators from [104] modulo a factor of} \]
translated constraints on $c_h$ and $c_Z$ are of the same order of magnitude. This is because the Higgs-nucleon effective Yukawa coupling ($y_{hNN}$) is much weaker than the $Z$-nucleon effective coupling ($\sim g_2$). Recall that the Higgs-nucleon coupling is mainly due to Higgs-gluon-gluon loop-induced interaction with heavy quarks running in the loop

$$ y_{hNN} = \frac{\sqrt{2} \alpha_s N_H}{24 \pi v} \langle N | G_{\mu\nu}^a G_{\mu\nu}^a | N \rangle = \frac{\sqrt{2} N_H m_N}{3 b v} \simeq 10^{-3} \quad (3.15) $$

where $N_H = 3$ is the number of heavy quarks and $b = 11 - \frac{2}{3} N_H$ comes from QCD beta function at one loop. The second equality can be calculated using QCD scale anomaly that relates the QCD beta function to nucleon mass (see [106] for the original references).

### 3.4 Higgs mass and Fine-Tuning

In this section we will describe our calculation of the Higgs mass in the model and its implication for the fine-tuning of the EW scale. As described in the introduction, we used SARAH 4.5.8 [100] and SPheno 3.3.7 [101, 102] to include all the loop corrections (contributions up to two loops both from the MSSM and the dark sector [107, 108]). Here

$^4$ between Dirac and Majorana fermions and a factor of a few difference between [104] and the latest LUX bounds. We do not agree with the limits on $g_A$ (related to our $c_Z$ via $g_A = \frac{c_Z}{s_Z}$) reported in fig. 3 of [105]. Their limit on $g_A$, derived from essentially the same LUX results, is over an order of magnitude weaker than ours.
we will describe an analytic treatment of the dominant one-loop contributions from $k_u$ and $k_d$. This will serve as a valuable source of intuition, as well as a validation of the full two-loop numerical calculation (for more details on the validation, see appendix 3.10).

The one-loop Higgs mass was previously computed in the literature using the Coleman-Weinberg potential in closely-related vector-like extensions of the MSSM [74–76, 78]. However, there are some key differences with our case that necessitate a fresh look. First, as noted above, in these past works, the vector-like extension was Dirac, while ours is Majorana (the difference between $W \supset MS\tilde{S}$ and $W \supset \frac{1}{2}MS^2$). This leads to small differences in the formula for the Higgs mass. Second, previous works presented analytic formulas for the one-loop Higgs mass only in the simplified limit with common fermion masses ($M_L = M_S$). Motivated by the DM side of the story, we will need the Higgs mass in a rather different regime, the mostly-singlet regime where $M_S \ll M_L$.

Other effects that we will ignore in our discussion here, but that are taken into account in the full numerical SARAH-SPheno calculation, include $g^2_{1,2}$ corrections, two-loop corrections, and the effective $A$-terms due to $\mu$. The effects of $g^2$ are about a 10-20% correction to $\delta m^2_h$, which amounts to a 2 GeV shift in $m_h$. That matters for our calculations quantitively but not qualitatively. The $\mu$ values we consider in this paper motivated by naturalness are small enough that $\mu$ has a negligible effect on the Higgs mass. Finally, we are interested in moderately-large values of $\tan \beta$ (e.g. $\tan \beta = 10$) but for simplicity we will present the $\tan \beta \to \infty$ limit here. The corrections due to $1/\tan \beta$ also do not make a qualitative difference. (In particular, there are no blind-spot cancellations here.)

With all of these simplifying assumptions, the result of our one-loop Coleman-Weinberg calculation is:

$$\delta m^2_h = \frac{1}{4\pi^2}k_u^4v^2 \left( f_1 \log(1 + x_L^2) + f_2 \log(1 + x_S^2) + f_3 \log \frac{x_S^2}{x_L^2} \right)$$

(3.16)

with

$$f_1 = \frac{(2x_L^4 + x_L^6) + 3x_L^4x_S^2 + 3x_L^4x_S^2 - x_S^4) x_S^2}{(x_L^2 - x_S^2)^3}$$

$$f_2 = \frac{(x_L^2 - 5x_S^2) - x_L^2x_S^2 - 3x_S^4) x_L^4}{(x_L^2 - x_S^2)^3}$$

$$f_3 = \frac{x_L^4x_S^2(x_L^2 + 3x_S^2)}{(x_L^2 - x_S^2)^3}$$

(3.17)
where \( x_L = m/M_L \) and \( x_S = m/M_S \). A plot of \( k_u^{-4}\delta m_h^2 \) is shown in fig. 3.4 (left). We see that \( \delta m_h^2 \) asymptotes to a finite value as \( x_L \to \infty \) or \( x_S \to \infty \). In these limits (corresponding to mostly-doublet and mostly-singlet DM respectively), the dependence on the DM mass drops out, and \( \delta m_h^2 \) is controlled by the ratio of the soft mass to the heavier mediator scale (\( M_S \) or \( M_L \) respectively).

To raise the Higgs to 125 GeV in this paper, we rely on a combination of the extra vector-like matter and MSSM stops. For stops at 1 TeV, which satisfy the current experimental bounds and imply about a \( \sim 10\% \) tuning of the EW VEV, the MSSM contribution to the Higgs mass is about 110 GeV (for a recent review see e.g. [61]). Therefore the target for \( \delta m_h^2 \) from the dark sector is:

\[
\delta m_h^2 \approx 3500 \text{ GeV}^2
\]

This selects out a contour in the \( (x_S, x_L) \) plane as shown in fig. 3.4 (left), according to the value of \( k_u \).

This has the following implications for the fine-tuning of the EW scale. Just as the dark sector lifts the physical Higgs mass analogous to stops in the MSSM, it also contributes to the fine-tuning of the EW scale through the renormalization of \( m_{H_u}^2 \). Following [22, 109],
we define the measure of fine-tuning to be:

\[ \Delta = \frac{2\delta m^2_{H_u}}{m_h^2} \tag{3.19} \]

where \( \delta m^2_{H_u} \) is the running of \( m^2_{H_u} \) due to the new fields

\[ \delta m^2_{H_u} = \frac{k_u^2 m^2}{8\pi^2} \log \frac{\Lambda_{UV}}{\Lambda_{IR}} \tag{3.20} \]

Optimistically we take \( \Lambda_{UV} = 10 \Lambda_{IR} \sim 10 \text{ TeV} \). We can combine this with (3.16) and (3.18) as follows. For a given value of \( k_u \) and a given point in the \((M_S, M_L)\) plane, we can solve (3.18) for the soft mass \( m \). Then substituting this into (3.19), we get a value for \( \Delta \). Regions of \( \Delta \leq 20 \) are shown in fig. 3.4 (right) for different representative values of \( k_u \). We see that we need \( k_u \gtrsim 1 \) to have any viable parameter space at all for a natural SUSY 125 GeV Higgs. This is not surprising, since from (3.16), we see that \( k_u \) plays the role that \( y_t \) plays for the MSSM stops. Of course, corrections we have neglected such as the \( D \)-terms and two-loop effects will modify this quantitatively. However, we will see that the same qualitative implications for fine-tuning and \( k_u \) will persist in our final plots.

### 3.5 The need for mostly-singlet DM

In section 3.2, we derived formulas for \( c_h \) and \( c_Z \) in terms of the parameters of the model, while in section 3.3 we showed that direct detection limits on \( c_h \) and \( c_Z \) are at the \( O(10^{-2}) \) level. Finally, in section 3.4, we argued that we need \( k_u \gtrsim 1 \) in order to have any viable parameter space for a natural SUSY Higgs at 125 GeV. Here we will combine these facts and show that the DM must be mostly singlet in order to be consistent with all the constraints.

Basically there are three possibilities: the well-tempered regime where \(|M_L - M_S| \lesssim v\) (recall our convention is that \( M_S \) and \( M_L \) are positive), the mostly-doublet regime where \( M_L < M_S \) and \( v \ll M_S, M_S - M_L \), and the mostly-singlet regime where \( M_S < M_L \) and \( v \ll M_L, M_L - M_S \). Keeping in mind that we need \( k_u \gtrsim 1 \) and large \( \tan \beta \) for a natural Higgs mass, the challenge is to decrease \( c_h \) and \( c_Z \) to the \( 10^{-2} \) level. In fact, \( c_h \) alone is enough to rule out all but the mostly-singlet case. We will comment on the implications for \( c_Z \) in sections 3.6 and 3.7.
Examining the formula for $c_h$ (3.10), we see that for $|M_L - M_S| \sim v$ and $\hat{k}_u \sim 1$, we have $c_h \sim O(1)$. (In particular, there is a cancellation in the denominator, leaving it $O(vM_S)$). This rules out the well-tempered case.

The mostly-doublet case is ruled out separately by two independent considerations. First, from fig. 3.4, we see that in order to be natural and mostly-doublet, we must have the DM mass below $\sim 800$ GeV. However, we know by analogy with pure Higgsinos in the MSSM that the thermal relic density constraint requires $M_L \geq 1$ TeV. (The mostly-doublet DM in this model has additional annihilation modes due to $k_u$ and $k_d$, so $M_L$ will be even larger.) So the mostly-doublet scenario is not promising for naturalness.

Also, from direct detection, we are basically forced into the mostly-singlet regime. In order to lower $c_h$ by two orders of magnitude, we must either (a) raise $M_L$ or $M_S$ to increase the denominator of (3.10), or (b) cancel the two terms in the numerator of (3.10).

(a) Increasing the denominator of (3.10) necessitates either $M_L$ or $M_S \gg v$. In the former, corresponding to mostly-singlet DM, we see that $c_h \propto 1/M_L^2$ and we can achieve the required level of suppression for $M_L \sim 1 - 2$ TeV for $M_S \sim v$ and $k_u \sim 1$. Meanwhile for the latter, corresponding to mostly-doublet DM, we see that $c_h \propto 1/M_S$ and therefore much larger $M_S \sim 2 - 5$ TeV is required for $M_L \sim v$ and $k_u \sim 1$. The latter is greatly disfavored by naturalness (it would likely be as fine-tuned as 10 TeV stops in the MSSM).

(b) Cancelling the two terms in the numerator requires

$$\frac{M_L}{m_X} \sim -\frac{1}{2} \frac{k_u}{k_d} \tan \beta$$

(3.21)

This is the blind spot. Since $k_u \gtrsim 1$ and we are in the large $\tan \beta$ limit, the RHS is generally much greater than one for any reasonable value of $k_d$. Therefore we must be in the mostly-singlet DM regime to realize the blind spot.

We conclude that several different constraints independently point at mostly-singlet DM as the only viable possibility.
Figure 3.5: Values of the coupling $c_h$ while varying $k_d$ for a sample point of the parameter space. The values on the shaded area are excluded by LUX.

For later reference we exhibit $c_h$ and $c_Z$ in the mostly-singlet limit

$$c_h = -\frac{m_\chi + \frac{2k_d M_L}{k_u \tan \beta} \frac{k_u^2 v^2}{M_L^2}}{\sqrt{2} v} + \ldots$$

$$c_Z = -\frac{g_2}{4cW} \frac{k_u^2 v^2}{M_L^2} + \ldots$$

Here we have taken $M_L \rightarrow \infty$ and $\tan \beta \rightarrow \infty$ holding fixed $M_L/\tan \beta$ and all the other mass scales. In fig. 3.5, we exhibit the amount of blind spot cancellation that is required by the SI bounds, for a typical choice of parameters that will lead to a viable relic density. We show this behavior by varying $k_d$ keeping other parameters fixed. We can see that we need only a very mild cancellation to satisfy the constraint on $c_h$. Most of the suppression of $c_h$ is coming from large $M_L$, which as we will see in the next section is fixed by the thermal relic abundance constraint.

In the same mostly-singlet regime, we also exhibit $\delta m_h^2$:

$$\delta m_h^2 = \frac{k_u^4 v^2}{4 \pi^2} \log(1 + x_L^2) - 3 \frac{k_u^4 v^2 x_L^2}{4 \pi^2 x_S^2} \log \frac{1 + x_L^2}{x_L^2} + O(x_S^{-4})$$

As noted in the previous subsection, the Higgs mass in this limit to leading order does not depend at all on the DM mass $M_S$. So the Higgs mass constraint to leading order in the
mostly-singlet regime becomes a constraint on $k_u$ and $m/M_L$. For example, according to (3.23), in order to achieve $\delta m^2_\chi = 3500 \text{ GeV}^2$ for $k_u = 1.6$, we need $x_L \approx 1$.

### 3.6 DM annihilation in the mostly-singlet regime

An attractive feature of WIMP dark matter is its potential to naturally explain the observed relic abundance via the thermal freeze-out mechanism. Following the usual procedure (see e.g. the classic review [110]), we have

$$\Omega_{\text{DM}} h^2 \approx 9.2 \times 10^{-12} \text{ GeV}^{-2} \times \left( \int_{x_f}^{\infty} dx \frac{\langle \sigma v \rangle_{\chi\chi}}{x^2} \right)^{-1}$$

(3.24)

The integral over $x$ takes into account annihilation after freeze-out, and $x_f = m_\chi/T_f \approx 25$ parametrizes the freeze-out temperature. $\langle \sigma v \rangle_{\chi\chi}$ is the thermally-averaged DM annihilation cross section $\chi\chi \to XY$, summed over all final states $X$ and $Y$. This is usually expanded in the small velocity limit:

$$\sigma_{XY} v_\chi = r_{XY} (a_{XY} + b_{XY} v^2_\chi + O(v^4_\chi)),$$

(3.25)

where $r_{XY} = \sqrt{1 - (m_X + m_Y)^2/4m^2_\chi}$ is a kinematic phase space factor. At the time of freeze-out, the DM relative velocity is typically $v^2_\chi \sim 0.1$. Therefore, the annihilation cross section is generally controlled by the $s$-wave contributions $a_{XY}$, unless they are suppressed for some reason.

In our model, the dark matter has many interactions and annihilation channels that should all be considered in full generality. As described in the introduction, for numerical calculations we use micrOMEGAs 4.1.8 [103] source code generated by SARAH 4.5.8 [100] to accurately take these into account. However in the mostly singlet limit that we are interested in, the cross sections simplify and we can have an analytic understanding of the behaviour of our model. We will assume that DM is lighter than all MSSM superpartners except possibly the Higgsinos, which are forced to be light $\mu \sim v$ by naturalness. In this case, the freeze-out process happens only through annihilation to SM particles and the Higgsinos. Including the Higgsinos in the story is a major difference from simplified-model-analyses of singlet-doublet dark matter, which generally just add the singlet and doublets...
to the SM. As we will see, the Higgsinos can be a major part of the DM annihilation in the early universe.

The full cross sections are too complicated to print here. Instead, we will expand in the mostly-singlet limit $M_S < M_L$, $v \ll M_L$, $M_L - M_S$ with the further assumption that $v \ll M_S$. This suffices for our purposes and results in relatively simple expressions. (One exception is the tree-level, $s$-wave $t \bar{t}$ cross section in the next subsection, for which we can write down an extremely simple exact expression in terms of $c_Z$.)

### 3.6.1 DM annihilation to fermions

The fermions have $s$-wave contributions

$$a_{ff} = \frac{3k_u^4}{32\pi} \frac{m_f^2}{M_L^4(1 - \epsilon^2)^2}$$

$$a_{\psi_H \psi_H} = \frac{(k_d^2 + k_u^2)^2}{16\pi} \frac{\mu^2}{M_L^4(1 + \epsilon^2 + x_L^2)^2}$$

(3.26)

where $\epsilon \equiv M_S/M_L$, and $x_L \equiv m/M_L$ was defined in section 3.4. In the second line, we have summed over the various Higgsino final states including both neutralinos and charginos, assuming a pure MSSM Higgsino (i.e. $M_{1,2}$ decoupled). The fermion $b$ coefficients are always subdominant (suppressed by both $v_\chi^2$ and $v^2/M_L^2$), so we have not included them here.

The fermion cross sections are all suppressed by the square of the fermion mass, so $t \bar{t}$ and Higgsinos are the dominant channels. This is the famous $s$-wave helicity suppression of DM annihilation to fermion pairs.

Although $t \bar{t}$ and Higgsinos are parametrically similar, their diagrammatic origin is entirely different. The former (latter) arise from $s$-channel $Z$ ($t$-channel superpartner) exchange. As a result, the Higgsinos are suppressed by the soft mass $m$. For $k_u = 1.6$, we saw in section 3.5 that we need $x_L \approx 1$ for $m_h = 125$ GeV, so the suppression is not large. Also, $\mu$ is constrained to be $\lesssim 300$ GeV by naturalness. So all in all, the Higgsino contribution ends up generally of the same order or smaller than $t \bar{t}$.

The fact that the SM fermions all arise from $s$-channel $Z$ diagrams means that they have a simple exact expression beyond the small $v$ approximation:

$$a_{ff} = \frac{c_z^2}{4\pi m_Z^2} \frac{3y_f^2}{m_f^2}$$

(3.27)
In other words, $c_Z$ controls both the SD direct detection cross section and the annihilation to $t\bar{t}$. Therefore, we expect to see a fairly direct correlation between the SD direct detection limits and the relic density constraint.

### 3.6.2 DM annihilation to bosons

Meanwhile the diboson cross sections are all $p$-wave to leading order:

$$b_{hh} = b_{ZZ} = \frac{k_u^4 \epsilon^2 (3 + 2\epsilon^2 + 3\epsilon^4)}{384\pi M_L^2 (1 + \epsilon^2)^4}$$

$$b_{hZ} = \frac{k_u^4 \epsilon^2}{96\pi M_L^2 (1 + \epsilon^2)^2}$$

$$b_{WW} = 2b_{hh} + b_{hZ}$$

(3.28)

Here we took $\tan \beta \to \infty$ for simplicity; we checked that the $1/\tan \beta$ corrections are irrelevant. The $s$-wave contributions are suppressed by $v^4/M_L^4$ so they are always subdominant to the $p$-wave contributions shown here.

Clearly, the diboson cross sections exhibit some interesting features. They are nonvanishing even in the $v \to 0$ limit, so they can be understood as a consequence of $SU(2)_L \times U(1)_Y$ symmetry. These tree-level cross sections arise entirely due to the longitudinal components of the $W^\pm$ and $Z$ bosons, which by the Goldstone equivalence theorem are also equivalent to the charged and neutral Goldstones $G^\pm$ and $G^0$ respectively. Under a $U(1)_Y$ rotation, $h \to G^0$ and $G^0 \to -h$, while under an $SU(2)_L$ rotation, $W^\pm \to h \pm iG^0$. This explains both relations in (3.28).

Comparing $t\bar{t}$ and Higgsinos to the total diboson cross section, we see that parametrically the latter can be larger than the former, for sufficiently large $M_L$. However the cross over point is generally at very large $M_L$ and $M_S$. For instance, for $\epsilon = 1/2$ and $x_L = 1$, we find the cross over to be in the range $M_L \sim 2.7 - 3.6$ TeV for $\mu \sim 100 - 300$ GeV. This is well beyond the naturalness-motivated part of the parameter space that we are focusing on in this paper. Therefore we conclude that the total $\sigma v_\chi$ is always dominated by $t\bar{t}$ and Higgsinos, and dibosons are always a subdominant part of it.
Figure 3.6: Inverses of the total relic abundance (black) as well as the individual contributions from $t\bar{t}$ (blue) and dibosons (red) as calculated numerically by micrOMEGAs 4.1.8 (solid) and the analytic equations (3.26) and (3.28) (dashed), for $M_L = 1.2$ TeV, $k_u = 1.6$, $k_d = -1.5$ and $\tan \beta = 10$.

3.6.3 Total annihilation cross section

We have shown analytically that the relic density is dominated by $s$-wave annihilation to $t\bar{t}$ and Higgsinos (assuming of course that the DM is above the respective thresholds):

$$\sigma v_\chi \approx a_{t\bar{t}} + a_{\psi_H \psi_H} = \frac{3k_u^4}{32\pi} \frac{m_f^2}{M_L^4(1-\epsilon^2)^2} + \frac{(k_d^2 + k_u^2)^2}{16\pi} \frac{\mu^2}{M_L^4(1+\epsilon^2+x_L^2)^2}$$

A plot comparing our analytics to micrOMEGAs is shown in fig. 3.6 for fixed choices of the parameters; we see there is excellent agreement across the entire range of relevant DM masses. We confirm that the dibosons are never more than $\sim 10\%$ of the relic density across the entire parameter range of interest. Higgsinos and $t\bar{t}$ are comparable for $\mu \sim 300$ GeV, while for $\mu \sim 100$ GeV, $t\bar{t}$ dominates, as expected from the $\mu$ dependence of the Higgsino cross section (3.26).

One very interesting consequence of (3.29) is that in the limit of large $M_L$, the DM mass drops out of the annihilation cross section. Furthermore, we have seen that we need $k_d \sim k_u$ for the blind spot, $x_L \approx 1$ for the Higgs mass, and $\mu \sim m_t$ for naturalness. Thus the WIMP miracle transforms from being a constraint on the WIMP mass to being a constraint on the mediator scale $M_L$! This helps to relieve the “WIMP little hierarchy problem”, whereby the preference of the thermal relic constraint for TeV-scale WIMPs is in tension with naturalness. Comparing with (3.22), we also expect that the relic density
constraint will essentially fix $c_Z$ to a unique value. We will confirm this in the next section with our full numerical scans and discuss its implications for SD direct detection.

### 3.7 Putting it all together

#### 3.7.1 Plots in the $M_L$-$M_S$ plane

Having described the various individual components of the analysis of the model (direct detection, the Higgs mass, and the relic abundance), we will now combine them and describe how the different constraints interact to produce the viable parameter space of the model.

In fig. 3.7 we show contour plots for numerical scans over the $(M_L, M_S)$ plane for fixed values of $k_u$, $k_d$ and $\mu$. We choose four sets of benchmark parameters: large coupling ($k_u = 1.6$, $k_d = -1.5$) and small coupling ($k_u = 1.2$, $k_d = -1$); and large $\mu$ ($\mu = 300$ GeV) and small $\mu$ ($\mu = 100$ GeV).

We see the impact of the direct detection limits on the parameter space of the model. The LUX SI and SD limits are strongest almost everywhere except a tiny sliver in the $k_u = 1.6$, $k_d = -1.5$ case where IceCube has an impact. The SD (SI) limits primarily cover...
the lighter (heavier) DM mass region. The heavier DM region is ruled out because we are holding fixed \( k_d \), so as one increases \( m_\chi \) the blind spot cancellation shown in (3.22) becomes less effective.

For every point in the plane we numerically solved (using SPheno) the \( m_h = 125 \) GeV constraint for the common soft mass \( m \); these contours are shown in fig. 3.7 along with their corresponding tuning. These contours are mostly vertical; as discussed in section 3.4, the soft mass and \( \Delta \) depend primarily on \( M_L \) since the dependence on \( M_S \) drops out to leading order at large \( M_L \).

Finally, we used micrOMEGAs to numerically solve the thermal relic density constraint \( \Omega_{DM} = 0.12 \) [111]; this fixes \( M_L \) as a function of \( M_S \) and these contours are shown in green for various choices of the parameters. Note the rapid increase in \( M_L \) across the top and Higgsino thresholds. Here new s-wave annihilation channels open up, and so larger values of \( M_L \) are needed to maintain the overall annihilation rate at the thermal relic value. This effect is more pronounced for larger values of \( k_{u,d} \) and for larger values of \( \mu \). Indeed, in section 3.6 we saw that the annihilation cross sections to \( t\bar{t} \) and Higgsinos are enhanced for greater \( k_{u,d} \), and the Higgsino cross section in particular is proportional to \((k_d^2 + k_u^2)\mu^2\).

Since larger \( M_L \) decreases direct detection cross sections, increasing \( k_{u,d} \) and \( \mu \) also increases the viable parameter space for thermal relic DM. The Higgsino channels in particular allow the model to survive the direct detection limits over a wider range of parameter space than would have been the case for non-supersymmetric singlet-doublet DM. Fig. 3.7 also shows that larger \( k_{u,d} \) is better for fine-tuning, confirming our discussion in section 3.4.

The only potential drawback of the larger coupling choice is (as we will discuss in section 3.8.1) that the former has a considerably lower Landau pole (\( \Lambda \sim 10^2 \) TeV vs \( \Lambda \sim 10^4 \) TeV).

Away from the top and Higgsino thresholds, we see that the relic density contours are mostly vertical, meaning that the relic density constraint becomes a constraint primarily on \( M_L \), once the other parameters (\( k_u, k_d, m, \mu \)) are fixed, i.e. the WIMP DM mass drops out to leading order. This confirms our analytics in the previous section.
Figure 3.8: Fine-tuning for the right relic abundance contours of fig. 3.7 that are allowed by direct detection. We show the case for $\mu = 300$ GeV (solid) and $\mu = 100$ GeV (dashed).

### 3.7.2 Projecting onto the thermal relic contour

Finally let us impose the relic density constraint $\Omega_{DM} = 0.12$ and see how various parameters vary along the green contours in fig. 3.7. In fig. 3.8 we show the fine-tuning for the points with the correct relic abundance. It is remarkable that there are allowed regions of the parameter space with $\Delta \approx 10$, making this model much less tuned than the MSSM.

In fig. 3.9 we show $c_h$ for the points of the parameter space that satisfy $\Omega_{DM} = 0.12$. We see that varying $k_d$ we can move toward the blind spot and satisfy the SI direct detection bounds.

Similarly, in fig. 3.10 we show $c_Z$ for the points of the parameter space with $\Omega_{DM} = 0.12$. We can see that for $m_{DM} > \mu$ contours of constant $\Omega_{DM}$ have an approximately constant $c_Z$. This confirms the discussion based on analytics in section 3.6.3. Indeed, using (3.29) with the parameter choices here, we find that for $100$ GeV $\leq \mu \leq 300$ GeV, $c_Z$ ranges from $0.005 \lesssim c_Z \lesssim 0.008$. These values are clearly illustrated in fig. 3.10.

We conclude that $c_Z$ (and consequently $\sigma^{SD}$) is basically fixed by the relic density constraint. Requiring $\chi$ to be all the dark matter leads to a nearly unique prediction for the SD cross section! Fortunately, as shown in fig. 3.10, these values of $c_Z$ are still allowed by the current direct detection experiments, IceCube in particular.\footnote{It is quite crucial that our DM annihilates almost exclusively to $t\bar{t}$ and Higgsinos. The IceCube bound on $t\bar{t}$ is by a factor of a few weaker than the $W^+W^-$ cross section, and it saves the model from being already}
Figure 3.9: Values of the coupling $c_h$ for the points with $\Omega_{DM} = 0.12$ for different values of $k_d$. We show the values for $k_u = 1.6$ (left) and $k_u = 1.2$ (right). The exclusion region from $\sigma^{SI}$ is in blue.

Figure 3.10: Values of the coupling $c_Z$ for the points with $\Omega_{DM} = 0.12$ for $\mu = 300\,\text{GeV}$ ($\mu = 100\,\text{GeV}$) in solid (dashed) green. The exclusion region from $\sigma^{SD}_n$ is in red and from $\sigma^{SD}_p$ with $\mu = 300\,\text{GeV}$ ($\mu = 100\,\text{GeV}$) with solid (dashed) purple.

improvements in cross section expected from Xenon1T [93] and LZ [94], the next generation of DM direct detection experiments will be sensitive to essentially the entire parameter space of this model (assuming $\chi$ is a thermal relic and is all the DM). A discovery might be right

ruled out (stable Higgsinos would not contribute to DM detection in IceCube).
around the corner!

3.8 Outlook

In this section we briefly discuss the UV behavior of the model (in particular the Landau poles) and the potential sensitivity from LHC Run II. Finally we conclude with some thoughts on future directions.

3.8.1 UV considerations

So far we have been exploring our model at the EW scale and have identified the interesting parts of the parameter space around \( k_u \sim k_d \sim 1.2 - 1.6 \). Here we want to examine the UV consequences of such large Yukawa couplings and comment on possible solutions to the Landau pole problem.

Let’s focus on the most important couplings, \((g_1, g_2, g_3, y_t, k_u, k_d)\) and neglect the effect of the other couplings in finding the scale of Landau poles. Starting from one loop beta functions above the scale of the new fields (including spectator color triplets for unification) we have

\[
\begin{align*}
\beta_{g_i} &= \frac{b_i}{16\pi^2} g_i^3 \\
\beta_{k_u} &= \frac{k_u}{16\pi^2} (2k_d^2 + 4k_u^2 + 3y_t^2 - \frac{3}{5} g_1^2 - 3g_2^2) \\
\beta_{k_d} &= \frac{k_d}{16\pi^2} (4k_d^2 + 2k_u^2 - \frac{3}{5} g_1^2 - 3g_2^2) \\
\beta_{y_t} &= \frac{y_t}{16\pi^2} (6y_t^2 + k_u^2 - \frac{16g_3^2}{3} - \frac{13}{15} g_1^2 - 3g_2^2)
\end{align*}
\]

Solving the RGE’s of our model numerically, we can find the lowest scale at which one of the couplings hits its Landau pole. In fig. 3.11 we show this scale as a function of the Yukawa couplings at 1 TeV.

Note that for \( k_u \lesssim 2 \) (as we have considered in this work), the Landau poles are above 100 TeV. Now we might ask: how can we understand physics above the Landau pole scale, or how can we postpone it to higher energies e.g. the GUT scale? One idea is to use non-Abelian gauge interactions for the new sector to reduce the beta functions of the Yukawa couplings: if we include multiple copies of \( S, L, \bar{L} \) and charge them under a non-Abelian gauge group, the corresponding gauge coupling appears with negative sign in the beta
Figure 3.11: Scale of Landau poles with one-loop RGE’s in terms of $k_u, k_d$ at the IR scale. We are assuming for each point on the plot that $k_u$ and $k_d$ are given at $\Lambda_{IR} = 1$ TeV.

function of $k_u, k_d$ (see e.g. [89] for a recent implementation of this idea). As $S$ is Majorana, we need $S$ to be in a real representation of the new gauge group. A simple example is when the gauge group is $SO(N)$ and $S$ is in the fundamental representation. Another possibility might be to match our model to the magnetic side of a Seiberg duality and interpret physics above the scale of the Landau pole by the electric theory. It will be interesting to explore these ideas further in the future.

3.8.2 LHC Phenomenology

In addition to direct detection experiments, DM models are also probed by the LHC. In principle, monojet+MET [96, 97] and monophoton+MET [98, 99] searches for direct DM production could be sensitive to our model. Since quarks and gluons only talk to $\chi$ through $s$-channel diagrams involving $Z$’s and Higgses, these searches constrain the same $c_Z$ and $c_h$ couplings as direct detection. However, these constraints are weaker by several orders of magnitude than those from direct detection under the assumption that our DM candidate $\chi$ is all of the relic density, for the mass range we consider. See e.g. [112, 113] for a recent discussion in terms of simplified DM models.
Instead, let us briefly consider mono($h$, $W$, $Z$)+MET. This can occur in our model through production of $\chi_1\chi_0^{2,3}$ and $\chi_1\chi^\pm$ and subsequent decay of the (mostly-doublet) $\chi_0^{2,3}$ and $\chi^\pm$. A full treatment including estimation of SM backgrounds, detector acceptances, etc. is beyond the scope of this work. Here we will just present the raw production cross sections in our model.

Diagrams contributing to mono-Higgs/$W$+MET are shown in fig. 3.12 (mono-$Z$+MET is the same as mono-Higgs with the final state Higgs replaced by $Z$). Note that we have included the one-loop gluon fusion diagram.\(^7\) Because of the large, $O(1)$ Yukawas $k_u$, $k_d$ in this model, this contribution can be as much as 60% of the total $\chi_1\chi_0^{2,3}$ cross section. We calculated the gluon fusion contribution analytically, and the tree level contributions both analytically and with MadGraph5 [56] using the model file generated by SARAH and the spectrum files generated by SPheno. More details on the analytics are given in appendix 3.11. In both cases, we used the NNPDF2.3 [114] PDF set. Fig. 3.13 shows the sum of tree level contributions and gluon fusion along the $\Omega_{DM} = 0.12$ contour. We see that LHC13 will ultimately be able to probe the small mass region. Of course, if $\chi$ is all of the dark matter, then direct detection experiments will discover the model first. In that case, the LHC will only be useful as a post-discovery confirmation of the model. However, since the LHC is producing $\chi$ directly, it does not depend on the relic density. Therefore if our dark

\[^7\text{We thank Matt Reece for bringing this to our attention.}\]
sector is only one component of $\Omega_{\text{DM}}$, the direct detection limits could be greatly relaxed while the LHC would remain sensitive.

### 3.8.3 Future directions

The work presented in this paper is a simple realization of a general idea: economically extending the MSSM with a single sector that provides both thermal WIMP dark matter and the 125 GeV Higgs mass. Here we took this sector to be a singlet and a pair of doublets, but one could easily imagine many other possibilities. For instance, very popular ideas for lifting the Higgs mass include the NMSSM (see e.g. [115] for a review and original references) and non-decoupling D-terms [116, 117]. While dark matter in the NMSSM is a well-studied topic, it would be very interesting to try to connect non-decoupling D-terms to dark matter.

Even within the context of our specific singlet-doublet model, there are many interesting open questions. In this work we made some simplifying assumptions in our analysis, and it would be interesting to explore the consequences of relaxing these assumptions. For example, we took all model parameters to be real, but in general there is one physical CP-violating phase. The effect of this phase on direct detection and annihilation cross sections can qualitatively change the model’s behavior. Furthermore, we took all the soft
mass-squareds to be positive to increase the Higgs mass. One might wonder how the phenomenology of the model would change if one of the soft masses is negative and the DM is a scalar instead of a fermion. We also assumed negligible $A$-terms in the dark sector. By analogy to stops, having substantial $A$-terms can help in raising the Higgs mass, see e.g. [76]. This could allow for smaller $k_u, k_d$ and open up more of the parameter space. Additionally, we focused on dark matter above $\sim 100$ GeV. It could be interesting to study the phenomenology of the model for lighter dark matter masses. In particular the annihilations through the Higgs and $Z$ resonances could be large enough while still having suppressed direct detection signals. Finally, one could relax the assumption that $\chi$ is thermal and is all of the DM, and consider non-thermal relics or multi-component DM scenarios. All of these directions will become especially well-motivated if nothing is discovered at the next round of direct detection experiments, as discussed in section 3.7.

There are also many interesting model-building directions in the UV. For example, enlarging the dark sector to accommodate a non-Abelian gauge symmetry could have potentially interesting consequences. As noted in section 3.8.1, this may help postpone the Landau pole of the Yukawa couplings, and the new gauge interactions could play an important role in the dynamics of the dark sector. Additionally we have two supersymmetric masses $M_L$ and $M_S$ at the electroweak scale. Perhaps the same dynamics that generates $\mu$ in the MSSM is responsible for generating these masses as well.

3.9 Connecting model parameters to DD cross sections

In this appendix, we will review how to relate the SI and SD DM-nucleon cross sections to the couplings $\xi^{SI}_q$ and $\xi^{SD}_q$ appearing in the effective Lagrangian (3.12). To check our results we verify that by calculating SI and SD cross sections analytically, we get the same result as the one we get from micrOMEGAs.

Following [110], the SI and SD cross sections are

$$\sigma^{SI}_{p,n} = \frac{x m_r^2}{\pi} f_{p,n}^2, \quad \sigma^{SD}_{p,n} = \frac{3 x m_r^2}{\pi} a_{p,n}^2 \tag{3.31}$$

where $x = 4$ for Majorana ($x = 1$ for Dirac) fermions, $m_r$ is the reduced mass of the
\[
\begin{array}{cccccc}
\Delta_u & \Delta_d & \Delta_s & f_u & f_d & f_s \\
\hline
p & 0.842 & -0.427 & 0.085 & 0.0153 & 0.0191 & 0.0447 \\
n & -0.427 & 0.842 & 0.085 & 0.011 & 0.0273 & 0.0447 \\
\end{array}
\]

Table 3.2: Nucleon quark form factors.

Figure 3.14: Comparing (3.31) with micrOMEGAs 4.1.8 for \( M_S = 200 \text{ GeV}, k_u = 1.6, k_d = -1.5 \).

DM-nucleon system, and \( a_{p,n}^{SD}, f_{p,n}^{SL} \) are the effective DM-nucleon couplings:

\[
f_{p,n} = \sum_{q=u,d,s} \zeta_q^{SI} f_{p,n} \frac{m_{p,n}}{m_q} + \frac{2}{27} \left( 1 - \sum_{q=u,d,s} f_{q,n} \right) \sum_{q=c,b,t} \zeta_q^{SI} \frac{m_{p,n}}{m_q}
\]

\[
a_{p,n} = \sum_{q=u,d,s} \zeta_q^{SD} \Delta_{p,n}^{q,}
\]

(3.32)

Here \( m_q \) is the quark mass, \( f_{q,n}^{p,n} \), and \( \Delta_q \) are hadronic parameters calculated for example by lattice simulations in QCD. We use the values in tab. 3.2 according to [118]. The difference between SI cross sections for proton and neutron is negligible as the main contribution comes from \( f_s \) which is the same in both cases.

In fig. 3.14, we compare our analytic cross sections to micrOMEGAs. We see that the agreement is excellent.
Figure 3.15: Thick dashed: analytical one-loop result (3.16). Dotted: SPheno one-loop. Solid: SPheno two-loop.

3.10 Validating SPheno one loop Higgs mass

In this appendix we validate the contributions to the Higgs mass from the dark sector as calculated by SPheno against the analytic one-loop calculation through the Coleman-Weinberg (CW) potential. We consider the simplified one-loop CW result from (3.16), where we ignored $g_{1,2}$ and $\mu$ and took the $\tan \beta \to \infty$ limit. We will show that both one-loop and two-loop results from SPheno match quite well with our analytical result. As SPheno outputs the total Higgs mass and not just the contributions from the dark sector, we extract these contributions as follows:

$$\delta m_h^2 = m_h^2 - m_h^2|_{MSSM}$$ (3.33)

where $m_h^2|_{MSSM}$ is the contribution to $m_h^2$ from the MSSM with superpartners at 1 TeV.

Since the Higgs mass depends primarily on $k_u, M_L, M_S$, we will demonstrate here that SPheno and our CW calculation agree well as these parameters are varied. From (3.16) we expect $\delta m_h^2 \sim k_u^4$. As we can see in fig. 3.15 (left), SPheno confirms this behavior. After fixing the $k_u$ dependence, we need to check that our analytical equations and SPheno match as we change $M_L$ and $M_S$. That is shown in fig. 3.15 (right) for two values of $M_S$ as we scan over $M_L$. 
3.11 LHC cross section analytics

Here we will provide some analytic details for the calculation of the $pp \to \chi_1\chi_{2,3}$ and $pp \to \chi_1\chi^\pm$ LHC cross sections useful for section 3.8.2. The former receives contributions from both tree-level $Z$'s with $q\bar{q}$ initial state, as well as one-loop gluon fusion. The latter comes from tree-level $W^\pm$'s with $q\bar{q}'$ initial state.

The tree-level, quark-initiated cross sections are given by:

$$
\sigma(q\bar{q} \to \chi_1\chi_i) = \frac{g_4^2 |p_f|}{144 \pi c_W^4 S^{3/2}} (c_{ZqLqL}^2 + c_{ZqRqR}^2) \times
\left[ ((\text{Re} R_i)^2 f_Z(S, m_1, m_i) + (\text{Im} R_i)^2 f_Z(S, m_1, -m_i)) \right]
$$

$$
\sigma(q\bar{q}' \to \chi_1\chi^+) = \frac{g_4^2 |p_f|}{576 \pi S^{3/2}} (f_W(S, m_1, m_+) |R_+|^2 + f_W(S, m_1, -m_+) |R_-|^2)
$$

where

$$
R_i = (U_{1,2}^* U_{i,2} - U_{1,3}^* U_{i,3})
$$

$$
R_{\pm} = U_{1,2} \pm U_{1,3}^*
$$

and

$$
c_{ZuLul} = \frac{1}{2} - \frac{2}{3} s_W^2, \quad c_{ZdLdl} = -\frac{1}{2} + \frac{1}{3} s_W^2, \quad c_{ZuRur} = \frac{2}{3} s_W^2, \quad c_{ZdRdr} = -\frac{1}{3} s_W^2
$$

The parton level gluon fusion cross section (as can be calculated e.g. using [106]) is

$$
\sigma(gg \to \chi_1\chi_i) = \frac{|p_f|m_t^2}{128 \pi S^{5/2}} \left| \frac{\lambda_i \alpha_3 c_{\chi_1\chi_i} h F(S/m_t^2)}{4\pi} \right|^2 g(S, m_1, m_i)
$$

where

$$
g(S, m_1, m_2) = \left( 1 - \frac{(m_1 + m_2)^2}{S} \right) \left( 1 - \frac{m_t^2}{S} \right)^2
$$

$$
F(x) = 2\sqrt{2} \left( 1 + \left( 1 - \frac{4}{x} \right) \left[ \sin^{-1} \sqrt{\frac{x}{4}} \right]^2 \right),
$$

and $c_{\chi_1\chi_i}$ is the coupling between Higgs and $\chi_1\chi_i$ ($i = 2, 3$) defined in the same way as $c_h$ in (3.9):

$$
\mathcal{L} \supset c_{\chi_1\chi_i} h \bar{\psi}_{\chi_1} \psi_{\chi_i}
$$

$$
c_{\chi_1\chi_i} = \frac{1}{\sqrt{2}} \left( \hat{k}_u(U_{1,1}^* U_{1,2}^* + U_{1,2}^* U_{1,1}) + \hat{k}_d(U_{1,1}^* U_{1,3}^* + U_{1,3}^* U_{1,1}) \right),
$$
Chapter 4

F-term and D-term SUSY breaking

General context of this chapter

In previous chapters, we had a more bottom up approach and discussed possible ways to explain the Higgs mass in the MSSM or in close variations of it. Although these mechanisms were particularly useful in conjugation with GMSB, we were mostly agnostic about the dynamics behind SUSY breaking and its mediation.

In this chapter, we discuss the effects of D-term SUSY breaking in O’Raifeartaigh (O’R) models. After discussing the general challenges of breaking SUSY via D-terms alone, we build a renormalizable model that breaks SUSY and $U(1)_R$ symmetry through F-term and D-terms at tree-level. In this way, we show that it is possible to change the vacuum structure of O’R models such that their tree-level flat direction is lifted. Our results are of particular interest for the vanishing gaugino problem in models of direct gauge mediation.

4.1 Introduction and Summary

Studying IR consequences of SUSY breaking could have important results for phenomenology at electroweak scale. It has long been understood that SUSY breaking can be achieved in simple renormalizable models, called O’Raifeartaigh models (O’R models) after the first example [119]. More interestingly it has been shown that these O’R models might be the low energy effective theories of dynamical SUSY breaking [120]. These models are particularly interesting for phenomenology as they can play the role of the hidden sector in the framework of gauge mediation SUSY breaking (GMSB) (for an excellent review and original references see e.g. [20]).

As first discussed in [121], O’R models are generically equipped with a $U(1)_R$ symmetry. Since gauginos in MSSM have non-zero R-charges, this R-symmetry forbids gaugino
masses and is problematic for phenomenology. Therefore in O’R models that are viable for phenomenology, R-symmetry needs to be spontaneously broken [122–125].

Breaking R-symmetry is not enough to generate large enough gaugino masses in direct gauge mediation, as was first pointed out in models of [126]. It was observed in [126] that gaugino masses vanish at leading order at one-loop even though they are not protected by any symmetry. Making gauginos out of reach of the experiments, the scale of sfermion masses would be too heavy which is problematic if we want to have a natural theory. The origin of this problem was explained in [125] by studying the generalized O’R models and showing that in order to have non-vanishing gaugino masses, the pseudo-moduli space must have a tachyonic direction. This result applies to the minimum of the tree-level potential and therefore one solution is to use quantum effects to have a metastable vacua away from the tachyons as suggested in [127].

The assumptions behind vanishing gaugino masses in direct gauge mediation is to have F-term SUSY breaking in a renormalizable model. Therefore another way to resolve this issue is to relax the renormalizability condition and use non-renormalizable operators in the Kahler potential [128, 129]. Of course in this way, the theory is not UV complete and we still need to explain the dynamics that generate those operators. Another option is to use D-terms (instead of F-terms) to break SUSY [130]. However, breaking SUSY by D-terms alone is challenging [131]. Without having Fayet-Iliopoulos (FI) terms, we can not have D-term SUSY breaking alone [14] and F.I. terms can never be obtained from dynamical SUSY breaking [132].

In this chapter, we try to build models that break SUSY through the interplay of F-terms and D-term in the scalar potential. We will only consider tree-level calculations in O’R models. More accurately, the class of models we consider are renormalizable weakly interacting Wess-Zumino models that are augmented by gauge interactions. Because of its problems, we will not use F.I. terms. The goal is to build models that break SUSY and R-symmetry via F-terms and D-terms together. We will first discuss the challenges of having sizable D-terms in section 4.2. We show that if we have local minimum in the F-term potential, this minimum turns into a runaway after turning on the D-terms. Then starting from a model with no local minimum in its F-term potential, we build a model in
section 4.3 that satisfies our conditions. Although the main benefit of these models are in direct gauge mediation, they could also be used in vanilla gauge mediation as we discuss in section 4.3.

4.2 Need for runaway

As discussed in the introduction, we first prove that on quite general terms, namely for renormalizable weakly interacting Wess-Zumino models, we need to have a runaway in the F-term potentials. The claim is that a local minimum in F-term potential translates into a runaway direction in the scalar potential when we turn on the D-terms. Here we first give the general argument and then in a concrete example we show the runaway direction explicitly.

4.2.1 General argument

In this section we discuss why building a model that (globally) breaks SUSY through F-terms and D-terms is challenging. Our argument is a streamlined proof given in [131], but our discussion highlights how we can circumvent the problems and build a model that has both non-zero F-terms and D-terms in section 4.3.

As noted earlier, the class of models we consider are renormalizable Wess-Zumino models with canonical Kahler potential \((K = \Phi_i \Phi^i)\) augmented with a gauge group (without Fayet-Iliopolous terms). We show chiral superfields with \(\Phi^i = (\phi^i, \psi^i, F^i)\) and complex conjugation is used to lower (or raise in case of the derivatives of the superpotential) of the indices. In general the scalar potential is given by

\[
V = V_F + V_D
\]

\[
V_F = W^i W_i \quad W_i := \frac{\partial W}{\partial \phi^i}
\]

\[
V_D = \frac{g^2}{2} \sum_a (D^a)^2 \quad D^a = \phi_i T^a \phi^i
\]

where \(W^i = W_i^\dagger\). The sum over repeated indices are implicit. Having more than one gauge group does not change our discussion but here we consider having only one semi-simple gauge group for simplicity. We will prove that if \(V_F\) has a local minimum, there is either
a solution to $V_D = 0$ or the total potential has no local minimum at finite field values. All discussion is at tree-level in this chapter.

If $V_F$ has a minimum, it is well known that there is a flat direction for $V_F$ in field space parametrized as follows [133, 125]

$$\phi^i = \phi^{i(0)} + z F^i$$  \hfill (4.2)

where $z \in \mathbb{C}$. Using the equation of motion we know $F_i = -W_i$. In other words, we know that at the minimum of the F-term potential:

$$W_{ij}W^j = 0$$

$$W_{ijk}W^{ij} = 0$$  \hfill (4.3)

To show that we can not have D-terms in the minimum of the total potential, we first need to prove some further identities. First we show that D-terms are constant on the tree-level flat direction of the F-term potential (4.2). In order to do that, we start by the following identities derived from gauge invariance of the superpotential:

$$W_i(D^a)^i = W_i(T^a)^i_j \phi^j \equiv 0$$  \hfill (4.4)

$$W_{ij}(D^a)^i + W_i(D^a)^i_j = 0$$

where the second equation is obtained by taking the derivative of the first one. These identities are valid everywhere in field space as they are merely based on gauge invariance of the superpotential. However along the pseudo-flat direction of the F-term potential, using (4.3) we have

$$W_i T^{ai}W^j = -W_{ij}W^j(D^a)^i_j = 0$$  \hfill (4.5)

Now that we have these identities at hand, we expand the D-term along the flat direction of (4.2)

$$D^a(z) = \phi^{(0)}_i + (z^* W_i T^{ai} \phi^{(0)} + c.c.) + |z|^2 W_i T^{ai}W^j$$  \hfill (4.6)

Here the terms in parenthesis vanish due to (4.4) and the third term vanishes due to (4.5).

We conclude that the total scalar potential is constant along the flat direction of the F-terms. Using these identities, we can show that with an infinitesimal complexified gauge
transformation, if we move slightly away from the flat direction, we have a runaway direction with a limit of no D-terms and same F-terms as the minimum. First note that, we can always rotate the VEV of D-terms into a direction in Cartan generators that we identify with diag{\mu_i}. Since after this rotation, D-term is along the direction of a combination of Cartan generators, the non-abelian nature of the gauge group is of no use and we could just use an abelian gauge symmetry without loss of generality. We now want to make a complexified gauge transformation along this direction. The F-term potential calculated after a complexified gauge transformation (parametrized by \eta) along the pseudo-flat direction (parametrized by z) is

\[ V_F(z, \eta) = W_i(z)e^{(n+\eta^*)\mu_i}W^i(z^*) = V_F(0, 0) + O(\eta^2) \] (4.7)

Note that the dependence on z is implicit in where F-terms are evaluated on the RHS. The fact that the linear term in \eta vanishes can be seen from (4.5). Now we can expand the D-term after the same transformations

\[ D^a(z, \eta) = D^a(0, 0) + \left( |z|^2\eta W^i \mu_i T^{a_j}_i W^j + \eta \mu_i \phi^{(0)} T^{a_j}_i \phi^{(0)}_j + \eta \mu_i z W^{a_i} T^{ai}_j \phi_j + c.c. \right) + O(\eta^2) \] (4.8)

Since \( V_F \sim O(\eta^2) \) and \( D^a \sim O(\eta z) \), there is always an \eta to kill the D-term without changing \( V_F \) for large \( |z| \). We conclude that if F-term potential has a local minimum, either the total potential has a runaway or the D-term vanishes. In the next section we look at an example that shows this claim concretely. More importantly, we will build our main model with non-zero F-term and D-term vevs in section 4.3 based on this example.

### 4.2.2 Example model

Consider the following superpotential augmented with a \( U(1) \) gauge symmetry

\[ W = X(f - \lambda \phi \tilde{\phi}) + m_\phi \phi \tilde{\phi} + m_\varphi \phi \varphi \] (4.9)

We assume \( \phi \) and \( \varphi \) (\( \tilde{\phi} \) and \( \tilde{\varphi} \)) have charge +1 (-1) and \( X \) is neutral. Without loss of generality the parameters \((f, \lambda, m_\phi, m_\varphi)\) can be taken to be positive. We want to show that the D-term either vanish or there is a runaway for the full tree-level potential which is the result we obtained in the previous section.
The F-term potential is given as

\[
V_F = |f - \lambda \phi \bar{\phi}|^2 + |m_\phi \phi - \lambda X \bar{\phi}|^2 + |m_\phi \phi - \lambda X \phi|^2 + |m_\phi \bar{\phi}|^2 + |m_\phi \phi|^2
\]  (4.10)

At tree-level we have two different phases for this potential. If \(\lambda f < m_\phi m_\varphi\) the origin of the field space is the minimum and therefore the claim is verified (D-term vanishes at the origin). If \(m_\phi m_\varphi < \lambda f\), the origin is destabilized. F-term potential is minimized at:

\[
\phi(0) = \frac{m_\varphi (\lambda f - m_\phi m_\varphi)}{\lambda^2 m_\varphi}, \quad \bar{\phi}(0) = \frac{m_\phi (\lambda f - m_\phi m_\varphi)}{\lambda^2 m_\varphi}
\]

\[
\begin{align*}
\varphi(0) &= \bar{\varphi}(0) = X \sqrt{\frac{\lambda f}{m_\phi m_\varphi} \bar{m}_\varphi - 1}.
\end{align*}
\]  (4.11)

When we turn on the D-term, we introduce the runaway direction of section 4.2. Along this runaway we have

\[
\begin{align*}
V_F(z, \eta) &= m_\phi m_\varphi \frac{2 \lambda f - m_\phi m_\varphi}{\lambda^2} + \eta^2 \frac{4 m_\phi m_\varphi (\lambda f - m_\phi m_\varphi)}{\lambda^2} + O(\eta^3) \\
V_D(z, \eta) &= \frac{(m_\varphi^2 - m_\phi^2) (\lambda f - m_\phi m_\varphi)}{\lambda^2 m_\phi m_\varphi} \\
&+ \frac{2 \left( m_\varphi^2 + m_\phi^2 + 2 m_\phi m_\varphi |z|^2 \right) (\lambda f - m_\phi m_\varphi) \eta}{\lambda^2 m_\phi m_\varphi} + O(\eta^2)
\end{align*}
\]  (4.12)

Therefore we see that assuming \(\eta\) is small, when \(|z|^2 = \frac{(m_\phi^2 - m_\varphi^2)}{4 m_\phi m_\varphi}\), D-term vanishes and we get the minimum F-terms. Therefore we conclude that there is a runaway for large \(|z|\).

### 4.3 F-term and D-term Breaking

In this section we want to build a model that has non-zero F-terms and D-terms in the vacuum. Using the discussion of section 4.2, we know that if the F-term potential has a minimum, we will get a runaway after adding the D-terms. Therefore we will build a model with no minimum in the F-term potential. As the scalar potential is continuous in field space, the idea is to start with a potential which has a runaway direction in its F-terms and lift that direction with the help of D-term.

Consider the following superpotential

\[
W = X (f - \lambda \bar{\phi} \phi) + m_\phi \bar{\phi} \phi + \lambda' \bar{\phi} \varphi Y + \frac{1}{2} m_Y Y^2
\]  (4.13)
The $U(1)$-gauge and $U(1)_R$ charges of the model is as in table 4.1. It is easy to see that there is no other symmetry on this superpotential. The F-term scalar potential is:

$$
V_F = |f - \lambda \phi \tilde{\phi}|^2 + |m_\phi \phi|^2 + |\lambda'\phi Y|^2 \\
+ |m_Y Y + \lambda'\tilde{\phi}\varphi| + |m_\phi \tilde{\varphi} - \lambda X \tilde{\phi}|^2 + |\lambda'\varphi Y - \lambda X \phi|^2.
$$

(4.14)

This F-term potential has the following supersymmetric runaway direction for small $\phi^{(0)}$

$$
\tilde{\phi}^{(0)} = \frac{f}{\lambda \phi^{(0)}} \quad \phi^{(0)} \to 0
$$

$$
X^{(0)} = Y^{(0)} = \varphi^{(0)} = \tilde{\varphi}^{(0)} = 0
$$

(4.15)

Now let us turn on the D-term

$$
D = (|\phi|^2 + |\varphi|^2 - |\tilde{\phi}|^2 - |\tilde{\varphi}|^2)
$$

(4.16)

Finding the minimum of this potential analytically is not illuminating, as after minimizing the sum of F-terms and D-term, the VEVs are complicated. Therefore we find the minimum of the potential numerically for representative values of the couplings. The D-term raises the runaway as promised, but we need to make sure that by turning on the D-term, the total potential has a stable minimum. We emphasize that this is just a minimal existence proof that can achieve these criteria, namely breaking SUSY and R-symmetry at tree-level with both non-zero F-term and D-term in a renormalizable Wess-Zumino model. The role of $f$ and $\lambda$ is clear: they destabilize the origin. Without the linear term, the origin of the field space would be a supersymmetric minimum. Without $m_\phi$, we can always choose $\phi^{(0)} = \tilde{\phi}^{(0)} = \sqrt{\frac{f}{\lambda}}$ with all other fields at the origin to have a supersymmetric vacuum even after turning on gauge interactions. The role of $\lambda'$ is to generate an effective mass for $\varphi$ after $\tilde{\phi}$ gets a vev without which the potential has a runaway. If we were interested only in SUSY breaking, we did not need to introduce $m_Y$. In fact the model in (4.13) without the $m_Y$ term is considered in [134]. However, if we want to use this model for direct gauge
mediation, breaking R-symmetry is imperative. We will show shortly that R-symmetry breaking is proportional to $m_Y$.

Let us consider the behavior of the model for different parameters to make sure that SUSY and R-symmetry are broken at tree level and there is a stable vacuum. The model has six parameters: $f, m_\phi, m_Y, \lambda, \lambda', g$ which can all be taken to be positive after field redefinitions. Since we want to minimize the tree-level potential, rescaling fields is not a concern and therefore we can absorb one dimensionful parameter that we choose to be $f$ and one overall scaling parameter that we choose to be $\lambda$. Numerically we find the minimum of the scalar potential for representative values of parameters to show the behavior we explained in section 4.3. First we show the runaway direction of the F-term potential. In figure 4.1, we see that for small $g$, vacuum energy goes to zero which confirms that SUSY is not broken without the D-terms. We also see that $\tilde{\phi}^{(0)}$ goes to infinity and $\phi^{(0)}$ goes to zero which confirms the runaway direction of (4.15) when we have no D-terms.

As discussed, without $m_Y$, we can have SUSY breaking, but the vacuum does not break.
Figure 4.2: Fields with non-zero R-charge. Parameters of the model are taken to be \( \sqrt{\lambda m_\phi} = 2 \), \( \lambda^I = 0.1 \), \( g = 0.2 \).

R-symmetry. We can see this behavior happening in figure 4.2 where we show the vacuum energy and vevs of \( X^0, Y^0, \tilde{\phi}^0 \) that have non-zero R-charges.

As said in the Introduction and Summary, gaugino masses vanish at one-loop (to leading order in SUSY breaking F-term) in models of direct gauge mediation when the hidden sector is an O’R model. Therefore the main advantage of using D-terms, is when we turn \( \phi \) and \( \tilde{\phi} \) as messengers of GM and use SM gauge group instead of the \( U(1) \)-gauge symmetry that we used. We will not follow this direction here, but even for normal GMSB mechanism, this model has the benefit of having no flat direction if used in the hidden sector. In particular if we introduce field \( N (\tilde{N}) \) in fundamental (anti-fundamental) representation of SM gauge group with the following couplings to \( X \) of the model 4.13:

\[
\delta W = \lambda X N \tilde{N} + m_N N \tilde{N}
\]

If \( m_N \) is much higher than the scales involved in the hidden sector, presence of these messengers does not change the discussion of the previous section. Therefore for the gaugino masses we have

\[
M_{\tilde{g}_i} = \frac{g_i^2 f_X}{16\pi^2 m_N}
\]
Chapter 5

Conclusion and outlook

Understanding physics at EW scale was the main focus of this thesis. Although the most naive expectations about naturalness have been rejected by experiments, naturalness is still our organizing principle in looking for physics beyond SM at EW scale. In this regard, SUSY is unique as it is the main framework in which having a large hierarchy, e.g. between Planck scale and EW scale, can be natural. We discussed that in the context of MSSM, Higgs mass is limited by \( m_Z \sim 90 \) GeV at tree level and raising Higgs mass to 125 GeV is only possible if we get large radiative corrections from stops, making stops heavier than \( O(\text{TeV}) \). In fact, if we assume only MSSM, Higgs mass limit on stop masses are equally or more constraining than the limits from direct searches at the time of this writing (for direct limits from LHC Run I see [15, 60]). Therefore it is a theoretical challenge to think about ways to explain the Higgs mass in MSSM with minimal tuning.

As we discussed in chapter 2, in order to explain the Higgs mass with minimum stop masses in MSSM (and hence minimum tuning), stops needs to have large trilinear \( A \)-terms with \( H_u \). These \( A \)-terms also carry flavor indices and their presence highlights the new physics flavor puzzle. GMSB is particularly motivated because it naturally suppresses the new physics contributions to flavor observables. However in GMSB, getting large \( A \)-terms is particularly challenging.

In chapter 2, we discussed a model where large \( A \)-terms are obtained through interaction of MSSM fields with messengers of gauge mediation. We showed that after integrating out the messengers at scale \( M \), we can generate

\[
W \supset y_{ij} \frac{X}{M} H_u Q_i \bar{u}_j
\]

at tree-level where \( X \) is a SUSY breaking spurion. In this way, we can generate \( A \)-terms under the MFV ansatz, i.e. the only source of flavor breaking are MSSM Yukawas, and the
flavor constraints are easily satisfied. We showed that in this way we can have MSSM with
low scale GMSB (to generate the soft masses) with a percent level fine tuning. We further
noticed that integrating out the messengers also generates some contribution to the soft
mass-squared of one of MSSM fields, e.g.

\[ K \supset \frac{X^\dagger X}{\kappa^2 M^2} H_u H_u \rightarrow \delta m^2_{H_u} = -\frac{A_t^2}{\kappa^2} \]  (5.2)

where \(1 \lesssim \kappa\) depends on the details of the model. This is an improved version of the problem
encountered in [33] dubbed little \(A/m_H\) problem where

\[ \delta m^2_{H_u} = A_t^2 \]  (5.3)

In the case of models in chapter 2, this contribution is smaller than (5.3) and its sign is
negative, which is helpful in EWSB. Interestingly we showed that to minimize tuning, we
need to explain the model in the UV with a Seiberg duality. These UV dynamics could
be used to explain why top Yukawa is much larger than the other Yukawas in SM. We
also showed that if we sacrifice the MFV nature of the model, one of the stops (instead
of \(H_u\)) could get this contribution to its soft mass. In this case this contribution has an
interesting effect on the spectrum, namely one stop is much lighter than the rest of the
colored particles.

In addition to Higgs mass, MSSM is under pressure from null results of dark matter
direct detection experiments if we try to explain dark matter relic abundance with MSSM
particles. In chapter 3 we considered these constraints and asked if Higgs-dark matter
interactions are able to raise the Higgs mass in a minimal extension of MSSM. As for a
concrete model, we considered a minimal dark sector at EW scale consisting of a pair of
vector-like doublets and a singlet. Interestingly with an \(O(1)\) coupling to \(H_u\), these new
fields could naturally explain the Higgs mass and dark matter relic abundance. We showed
that although dark matter coupling to Higgs is constrained from spin independent direct
detection experiments to be \(\lesssim 10^{-3}\) for a Majorana WIMP dark matter, the rest of the dark
sector can have sizable couplings to Higgs and raise the Higgs mass. We noticed that in this
chapter too we had the problem of Landau poles for the Yukawas connecting Higgs to the
dark sector. It is conceivable that this Landau pole can be explained by some non-abelian
gauge interactions in the dark sector that we did not proceed any further in this work.
Finally in chapter 4 we discussed possible ways to break SUSY via F-terms and D-terms at the same time. As light gaugino problem that plagues models of direct gauge mediation with F-term SUSY breaking is tightly bound with the vacuum structure of these models, the model in section 4 can be an interesting starting point for a model of direct gauge mediation if we expand the $U(1)$-gauge group of this model to the MSSM gauge group.
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